Visualization of Streamwise and Crossflow Instabilities on Inclined Circular Cylinders

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Abstract

Experimental observations of streamwise and crossflow streaks on circular cylinders are presented over a range of cylinder inclinations. The spanwise wavelength, $\lambda_s$, of an array of streamwise vortices on a normal circular cylinder was addressed by Kestin and Wood. That theory was for the limiting case of zero sweep and the results are here confirmed experimentally. These observations of streamwise vortices on unswept cylinders confirm the earlier predictions, providing a firm basis for referencing new measurements of vortical behaviour. The work on unswept cylinders provides an excellent benchmark for sweep effects. The principal published collection of results for medium to high sweep angles, $\alpha$, is that of Poll, covering sweep angles between 35$^\circ$ and 71$^\circ$. Surface flow visualization has been undertaken by the authors giving lateral spacing and streak orientation for sweep angles from zero to 61$^\circ$. Results for lateral spacing are consistent with those of Poll, covering the range between the zero sweep results of Kestin and Wood and the medium to high sweep cases addressed by Poll.

Introduction

The circular cylinder in crossflow is an important canonical case for guiding the interpretation of the flow over more complex geometries, such as swept wings and turbine blades. Previous flow visualization investigations have often revealed streamwise vortices and “streaky structures” on flat plates. These were often ignored, considered to be an artefact of the flow visualization medium or of its application. For the cases addressed here, care was taken to eliminate the influence of the visualization medium and its application by systematically varying these.

In flow visualization work at the National Research Council of Canada (NRC) a turbine blade was tested by Mahallati in a planar cascade over a range of Mach numbers [4]. Strong and persistent streamwise vorticity was observed on the suction surface, as shown by the representative close-up visualization in figure 1. This observation instigated the present investigation. Halstead [6] had previously observed fine-scale streaks on the suction surface of a turbine blade in cascade. Fine scale organized and predictable streamwise vorticity has been shown to exist on both the pressure and suction surfaces of turbine blades. Following a blunt leading edge the streamwise vorticity may persist, on a time-average basis, to influence the entire suction surface at the modest Reynolds numbers typical of aircraft cruise conditions. The lateral distance between streaks could be an indicator of the extent of the vortical activity from the surface and the strength of any associated vorticity.

Here the vorticity is considered implicit, whilst still requiring detailed experimental, computational and analytical characterization. Kestin and Wood had demonstrated the existence of streamwise vortices on an unswept cylinder [8]. Poll went on to examine vortex behaviour on a highly swept cylinder [12]. There has existed a gap between the investigations of Kestin and Wood, at zero sweep, and those of Poll, with 55$^\circ$ to 71$^\circ$ of sweep. The aim of the present investigation is to bridge this gap.

The spanwise wavelength of the array of fine-scale streamwise vortices was predicted by Kestin and Wood. The physics of this streamwise vorticity imposes severe requirements on the temporal and spatial resolutions of both experimental and computational methods. Temporal resolution is needed to capture the flow complexity that is fundamental for an understanding of the behaviour of the laminar boundary layer and its separation and transition. However the vortical entities may be quite unstable, the stationary vortices only appearing in an organized fashion after extensive time-averaging. This combination of instantaneous and long duration mapping presents a strong challenge for both instrumentation and computational resources.

For any given flow regime various candidate instability modes exist. In unswept circular cylinders streamwise vorticity may be associated with a high local disturbance level upstream of the leading edge. This is then stretched out into a lengthy vortical structure as it passes along the convex surface [11]. It may well comprise both weakly organized and meandering structures. After a relatively long time the stationary components settle into a clear time-averaged pattern. Halstead has shown that the structures can persist through the turbulent boundary layer to the trailing edge [6]. These fine-scale streaky structures may persist through shock interactions, laminar separation bubbles, the transition region and well into the turbulent layer [4]. Some advanced computations are now showing the same trend.

The introduction of sweep brings consideration of a wide range of instabilities. Prominent is crossflow instability resulting from the inflectional behaviour of the three-dimensional boundary layer. Previous investigations of these structures were conducted on the limiting case of an unswept circular cylinder, by Kestin and Wood and, on a swept circular cylinder, by Poll. The lowest sweep angle considered by Poll was 55$^\circ$. A different approach was taken by Takagi et al. [13] for a sweep angle of 50$^\circ$. There

![Figure 1. Visualization of Suction Surface Flow over Turbine Blade between 80% and 95% Axial Chord [4].](image-url)
had been no previous published attempt to link the experimental data sets of Kestin and Wood and of Poll. Although there are substantial differences between streamwise vortices and crossflow vortices, it has never been clear how, and where, the streamwise vorticity changes to crossflow vorticity or, indeed what other instabilities are active. Nevertheless a gap in sweep of at least 50° between the results has continued to exist. The authors considered it worthwhile and interesting to perform experiments over this unexplored region of parameter space.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$D$</td>
<td>Cylinder diameter</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number, based on $D$</td>
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<tr>
<td>$Tu$</td>
<td>Free-stream turbulence level, %</td>
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<tr>
<td>$A$</td>
<td>Sweep angle, degrees</td>
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<tr>
<td>$\lambda$</td>
<td>Spanwise wavelength of vortex pairs</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Spanwise wavelength under zero sweep</td>
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The Test Facility

The work on turbine blades and circular cylinders has drawn on a wide range of results, from different facilities, and covering a wide range of Reynolds and Mach numbers. In particular work by Kestin and Wood, and by Poll, has provided a framework for analysis. Experiments by Ackerman [1] on a 37.26 mm diameter cylinder in the NRC 1.5 m tri-sonic tunnel are also included.

Complementary new results are included herein from the testing of a 0.152 m diameter cylinder with and without sweep. Tests on the near-surface flow over this circular cylinder were conducted in the Charles Wilson wind tunnel at the University of Leicester. This is a closed-loop low speed tunnel with an aerodynamic working section 0.85 m high, 1.145 m wide, and 4 m long. A downstream fan feeds the return leg of the wind tunnel circuit. The return flow is conditioned through three mesh screens and a honeycomb layer followed by a further screen upstream of the 4:1 area ratio converging test section inlet. This resulted in a free-stream turbulence level of 0.2% at 10 m/s.

The 0.152 m diameter, 2.5 m long, hollow aluminium cylinder was placed in the forward region of the test section. The cylinder was mounted 2.75D above the floor with its axis parallel to the wind tunnel floor; it was pin-mounted on the tunnel far-side wall and simply supported through the near-side wall. This arrangement allows the cylinder to be manually adjusted in fixed increments of sweep over the range 0° to 61°. At sweepback angles less than 61° the cylinder protrudes outside the working section through tight-fit elliptical holes cut in the Perspex near-side wall. In the regime tested a cylindrical model would normally give rise to von Kármán vortex shedding. To suppress this instability the leeward side of the cylinder is fitted with a L-shaped aluminium splitter plate mounted from the rear stagnation line. The effectiveness of this suppression method was verified by hot-wire measurements in the cylinder wake. End plates were used to prevent the wind tunnel side wall boundary layer from interfering with the cylinder flow. The elliptical hole for the cylinder is located 10% mean chord upstream of the ellipse centre to give stability in yaw. At zero sweepback, the cylinder length between the end plates is 1.079 m.

Surface Flow Visualization

A sample of flow visualization on the suction surface of a turbine blade tested in the NRC transonic plant cascade tunnel is given in figure 1. The discharge Mach number was 1.16. The blade was covered with a sheet of self adhesive white vinyl. A mixture of linseed oil and powdered lampblack was applied in a very thin layer. After running for five minutes, the blade was removed and photographed. Flow visualization at subsonic and transonic speeds displayed coherent streamwise vorticity extending from leading to trailing edge. Some results from turbine cascades are presented in reference [5]. When examined in the same way as the surface flow visualization from the NRC cascade, three further results from turbine blading were accessible [2,6,7]. The visualization was performed independently by the different authors using different facilities. The techniques and materials used differed in each case, demonstrating that factors such as surface tension or gravity did not influence the observations.

Ackerman [1] performed surface flow visualization on a 37.26 mm diameter cylinder at a free stream Mach number of 0.5. Streamwise streaks were observed before and after a separation bubble. The application of the visualization medium upstream of separation had been non-uniform. Because the liquid medium collected in a pool in the bubble, a more uniform dispersion of the medium was observed downstream of the bubble. Spanwise periodicity showed up clearly, forming a viable basis for measuring the spanwise wavelength (figure 2).

On the circular cylinder tested at Leicester University the windward surface was prepared for flow visualization by hand polishing the surface to a uniform reflective finish over the arc of -110° to +110° about the upstream stagnation line. The leeward side, left unpolished, is characterized by separated flow. At the test section mid-span, a 0.4 m wide 0.226 mm thick sheet of UV stabilized clear PVC was tightly wrapped around the cylinder. The sheet was held in place by adhesive tape attached to the leeward side of the cylinder. The sheet created a removable surface over which a flow visualization compound was applied. Changing the sheet between tests built a library of flow records for subsequent analysis and comparison.

Poll had considered that surface flow visualisation below the Reynolds numbers he investigated was not feasible. Since the Reynolds numbers used in the present investigation were well below those used by Poll it was not clear that streaks would be observed on the circular cylinder. This represented the biggest challenge for the investigation. Systematic experimentation was applied to this question and eventually a mixture was found that gave streaks consistently over the chosen Reynolds number range. This made flow visualization at low speeds possible. The flow visualization compound comprises a suspension of 1 g of desiccated titanium dioxide powder in a solution of 10 ml of paraffin oil, 0.84 ml linseed oil, and 3 drops of polyoxyethylene 10 oleyl ether. Before each application, the cylinder span was wiped with linseed oil, using a soft cloth, to form a thin film that improves the dispersion of the compound. The latter was applied over the PVC sheet using a soft brush, stroking along the cylinder axis. After applying the mixture, the cylinder was tested by running the tunnel at a constant Reynolds number for about forty minutes. The surface flow visualization pattern on the removable plastic sheet was analysed by removing the sheet from the cylinder, placing it on an overhead projector, and projecting the

Figure 2. Surface Flow Visualization Downstream of Separation Bubble on Circular Cylinder in NRC Tri-sonic Tunnel at a Mach Number of 0.5.
pattern, together with a reference grid, onto a large white screen. The magnification from the optical projection facilitated the measurement of the visualization traces on the screen.

**Circular Cylinders without Sweep - the Benchmark**

The existence of regular streaks on circular cylinders was originally investigated by Kestin and Wood who, in 1970, published a stability analysis. They predicted a theoretical value of spanwise wavelength between vortex pairs, \( \lambda_o \), for a cylinder of diameter, \( D \), given by:

\[
\lambda_o = 1.79\pi D Re^{0.5}.
\]

This result is represented by the \( Tu = 0\% \) line in figure 2. Kestin and Wood also undertook experimental work on cylinders which provided the results for non-zero turbulence levels.

The high speed experiment performed by Ackerman [1] in the NRC tri-sonic tunnel at a Reynolds number of 675,000 provided a point for comparison with the Kestin and Wood prediction (figure 3). More recent testing was undertaken on the 0.152 m diameter aluminium cylinder in the University of Leicester low speed tunnel at three Reynolds numbers. Surface flow visualization has provided a further three points on the Kestin and Wood plot in figure 3. All four results are in reasonable agreement with the Kestin and Wood theory and provide confirmation of this theory for the case of the normal flow over an unswept circular cylinder. The suitability of the Kestin and Wood theory was confirmed as a reliable basis for examining more complex surface geometries, such as turbine blades [5], and variables such as sweep.

**The Effects of Sweep**

Sweep is encountered or employed widely in the natural and physical worlds. It can be used very deliberately on the wings of high-speed aircraft and on turbines and compressors. One familiar use is the alleviation of adverse effects of shock waves. In low pressure turbines the blades might be stacked in a purely radial direction but the through-flow streamlines themselves may have a strong radial component. The expansion of the working fluid dictates the required flow path area increase; this is locally presented as blade sweep. Circular cylinders are being studied to gain an understanding of sweep effects; this has not been fully investigated and further work is needed before a thorough understanding may be claimed. Turbine blades inherently have a more complex geometry than cylinders but would benefit from a more complete understanding of flows about a swept cylinder. Encounters with sweep in the development of wings for high Mach and Reynolds numbers have involved its role in causing early transition in the leading edge region. Much effort has gone into studying this aspect of sweep effects. In turbomachinery, the Reynolds numbers tend to be lower and the situation is less obvious. The frequent occurrence of laminar separation bubbles quite late on the blade shows that leading edge transition is not necessarily a factor [6]. Until further clarification is achieved, it is prudent to assume that a number of modes may be competing.

The results of Kestin and Wood are a good starting point for a discussion of these effects. Although the work is related to unswept cylinders, it also provides an excellent benchmark for sweep effects on cylinders and blades. This was confirmed by the new experimental results of figure 3. These demonstrate that the formula of equation (1), for the lateral spacing of streamwise vortex pairs as a function of Reynolds number, gives a firm foundation for viewing other stability investigations.

The principal published collection of experimental results for high sweep angles is that of Poll [12] on a cylindrical model covering the range 55° < \( \Lambda < 71° \). Data in the useful range of sweep up to 50 degrees are virtually non-existent although the experiments of Takagi et al. [13] and Kohama [9] are of interest. The former gives an experimental point at 50° sweep and the latter gives an intriguing photograph showing two stationary modes interacting. It is of interest to designers to develop an understanding of when the vigorous cross-flow instability mode is expected. Liquid crystal work [10] suggests that in this case cross-flow instability is first observed at around 40° sweep and becomes most strongly amplified at around 57°.

Various first order attempts were made to predict sweep effects. In spite of the restrictive assumptions of these approaches, the same relationship of \( \lambda/\lambda_o = 1/\cos\Lambda \) was reached. This is the traditional Cosine Rule used to predict sweep effects on aerofoils. This approach had been found to be valid only for subcritical flows with a critical Reynolds number that decreases with increasing sweep [3]. It will be seen that \( \lambda/\lambda_o = 1/\cos\Lambda \) is a reasonable descriptor of the measurements over the sweep angle range 0° to 60.1°. The generalization may now be used to plot the lateral vortex spacing, normalized by the unswept case. The predictions of Kestin and Wood are used directly as a benchmark and the results of Poll have been referenced to those.

**Transverse Spacing**

Testing was carried out in the Leicester University wind tunnel over a range of Reynolds numbers from 132,000 to 175,000 and over the range of sweep angles from 0 to 60.1 degrees. These available Reynolds numbers were relatively low. Poll [12] had found that for Reynolds numbers below 339,000 streaks were

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*Figure 3. Measurements of Spanwise Wavelength on Circular Cylinders and Comparison with Kestin and Wood Predictions.*

*Figure 4. Variation of Streaks from Leading Edge to Laminar Separation.*
only visible quite late on the surface. In the current measurements, streaks were faint but visible and consistent much further forward. If care was taken with the surface coating and the optical techniques streaks were visible in all cases.

Figure 4 is a visualization record of both upper and lower quadrants, from the leading edge to laminar separation. It gives an idea of the streak spacings and angles and also acts as a check on the symmetry of the two surfaces. It demonstrates that the flow over the two surfaces behaves in a similar manner and is not noticeably influenced by gravitational effects.

Care was taken to check that the new wind tunnel results and the results of Poll were quoted and normalized in the same way, using equation (1) as a reference. Kestin and Wood’s theoretical result is accessible and represents very closely a regression line fit through our new experimental results for unswept cylinders. Different first order approaches to generalizing the Kestin and Wood prediction of vortex spacing (equation 1) to non-zero sweep angles resulted in the same simple modification:

\[ \lambda = \lambda_0 / \cos \Lambda = 1.79 \eta D / Re^{0.5} \cos \Lambda. \] (2)

Equation (2) is used to plot the lateral vortex spacing, normalized by the unswept case, in figure 5. The results in figure 5 appear to be self consistent and also compatible with Poll’s results, obtained at higher Reynolds numbers. The \( \lambda / \lambda_0 \) theoretical curve is also plotted and demonstrates reasonable agreement with both the Poll data and the new data. At 50° sweep the theoretical and experimental points of Takagi et al. [13] involved ingenious use of theory and hot wire data to discriminate between stability modes. Takagi discovered that at 50° sweep the stationary cross-flow mode dominated; this is the same mode identified by Poll. His results are consistent with the new results of the authors and those of Poll.

**Conclusions**

Experimental work has confirmed the suitability of the zero-sweep Kestin and Wood theory as a basis for predicting streamwise streaks and vortical structures on unswept cylinders. High speed testing was undertaken on a 38 mm diameter cylinder and low-speed testing on a 152 mm diameter cylinder. Although the Kestin and Wood work is related to unswept circular cylinders, it also provides an excellent benchmark for sweep effects. Experimental work, confirming the zero-sweep results, gives a reference for subsequent work over a wide range of sweep angles. No data had been published on streamwise and crossflow vortices in the useful sweep range of up to 50°. Testing has been undertaken over a range of sweep angles from zero to 60° giving results for the lateral spacing and angular orientation of the streaks. At high-sweep angles, the results are consistent with those of Poll. At low Reynolds numbers, first order-theories for circular cylinders predict the effects of sweep quite well.

This work has shown that organized and systemic fine-scale streamwise vorticity may occur more frequently on convex surfaces than hitherto appreciated. Experiments and numerical predictions should be extended to encompass that possibility. The conventional view of two-dimensional laminar boundary layers following blunt leading edges is not realistic. Such boundary layers need to be treated three-dimensionally, particularly when sweep is present. This requires a sufficiently fine spanwise spacing for the streamwise structures to be resolved. Application of computational methods to these problems is likely to be expensive. A combined approach of analysis, computation and experiment is indicated.

Finally it may be noted that the streaks observed by surface flow visualization do have significance in fluid mechanics. They are not mere artefacts of the visualization medium.

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**References**


