Magnetospheric period magnetic field oscillations at Saturn: Equatorial phase “jitter” produced by superposition of southern and northern period oscillations

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[1] We investigate magnetic field oscillations near the planetary rotation period in Saturn’s magnetosphere observed during the initial near-equatorial phase of the Cassini mission. Phase determinations on 28 periapsis passes during this ~2 year interval display pronounced nonrandom “jitter” relative to the ~10.8 h modulations in the dominant southern Saturn kilometric radiation (SKR) emissions. Phase deviations in the radial and azimuthal components are strongly positively correlated, while being anticorrelated with the phase deviations in the colatitudinal component. This suggests the presence in the equatorial magnetosphere of superposed weaker field oscillations at the ~10.6 h period of the northern SKR modulations, the phase deviations being shown to be periodic near the corresponding ~23 day “beat” period. Modeling the effect of the northern period oscillations shows that their amplitude is ~30%–40% of the southern period oscillations, producing phase deviations of ~±25°. The relative phasing of the northern period radial and azimuthal fields is such as to form a rotating quasi-uniform field, as for the southern period oscillations, while the phasing of the colatitudinal component indicates perturbation field lines arched with apices pointing to the south, opposite to the southern period field lines that are arched with apices pointing to the north. The northern period field draws toward northern SKR maxima, consistent with previous observations of the northern polar oscillations and opposite to the southern period field that points tailward at southern SKR maxima. The results support the view that the field oscillations are due to two auroral current systems that rotate with differing periods in the two hemispheres.


1. Introduction

[2] Oscillations associated with Saturn’s magnetosphere that occur close to the planetary rotation period were first discovered in Voyager observations of Saturn kilometric radiation (SKR) emissions, which were found to undergo large “strobe-like” modulations in intensity, independent of the position of the observer, with a period of ~10.7 h [Kaiser et al., 1980; Desch and Kaiser, 1981; Warwick et al., 1981]. Subsequent investigations using Ulysses and Cassini have shown that the modulation period can vary over yearly intervals by fractions of a percent, far too large a change to be associated directly with the planet [Galopeau and Lecacheux, 2000; Gurnett et al., 2005; Kurth et al., 2007, 2008]. In addition, Zarka et al. [2007] have shown the presence of comparable variations in SKR modulation period on time scales of 20–30 days, which they relate to variations in the solar wind speed. SKR emissions are believed to be generated by auroral electrons on high-latitude field lines in each hemisphere, with primary sources in the dawn-to-noon sector [Lecacheux and Genova, 1983; Galopeau et al., 1995; Lamy et al., 2008a, 2008b, 2009, 2010; Cecconi et al., 2009]. Most recently it has been shown that emissions from the two hemispheres are modulated at differing periods, at ~10.6 h in the north and ~10.8 h in the south during the southern summer conditions that prevailed during the initial Cassini era considered here (2004–2006) [Gurnett et al., 2009a; Lamy, 2011], with the overall power generally being dominated by emissions from the south. Corresponding distinct modulation periods have subsequently been reported in magnetospheric energetic electron fluxes [Carbary et al., 2009], auroral hiss emissions [Gurnett et al., 2009b], and...
northern and southern ultraviolet (UV) auroral emissions [Nichols et al., 2010a, 2010b].

[5] In situ observations within Saturn’s magnetosphere, initially during the Pioneer 11 and Voyager flybys, and subsequently in greater detail during the Cassini era, have demonstrated that related oscillations are also ubiquitously present in charged particle and magnetic field data [e.g., Carbary and Krimigis, 1982; Espinosa et al., 2003; Cowley et al., 2006; Southwood and Kivelson, 2007; Carbary et al., 2007a, 2007b; Andrews et al., 2008, 2010a; Provan et al., 2009; Burch et al., 2009; Clarke et al., 2010]. Unlike the strobe-like SKR modulations, however, these perturbations rotate around the planet with a synodic period close to the SKR period. Specifically, Andrews et al. [2010b] have employed Cassini data to examine the magnetic oscillations occurring in the northern and southern polar regions of Saturn’s magnetosphere over the pre-equinox interval to mid-2009, together with those occurring within the “core” magnetosphere, meaning the region of quasi-dipolar field lines extending to ∼10 Rₚ in the equatorial plane. They showed that the period of the oscillations on polar field lines in each hemisphere agree with those of the corresponding SKR modulations to within ∼0.01%, while the oscillations in the near-equatorial “core” have the longer period of the dominant southern SKR emissions. The latter finding is in agreement with the prior “core” region results of Andrews et al. [2008, 2010a] and Provan et al. [2009].

[4] This paper is concerned with the magnetic field oscillations observed within the near-equatorial “core” magnetosphere. Despite the level of agreement found by Andrews et al. [2008, 2010b] and Provan et al. [2009] between the period of these oscillations and the southern SKR modulations, two phenomena remain unaccounted for. One is the presence of slow long-term drifts in relative phase between these oscillations, implying slight differences in the period of order ∼10 s (thus being two orders of magnitude smaller than the difference in periods between the two hemispheres). While taking account of the existence of these drifts, their physical origin is not examined here. The second is a significant short-term “jitter” or deviation in the phase of the “core” field oscillations about the slowly drifting mean, first noted by Southwood and Kivelson [2007]. Andrews et al. [2008] showed that this effect is not simply due to measurement error, however, since the phase deviations determined independently from different components of the field vector are found to be highly correlated. Here we examine this phase “jitter” phenomenon, and suggest that it is due principally to the simultaneous presence of weaker northern period field oscillations superposed on dominant southern period oscillations within the “core.” On this basis we employ the measured phase deviations to determine the polarization characteristics of the northern period oscillations, their amplitude relative to the southern period oscillations, and their phasing relative to the northern SKR modulations.

2. Physical Picture

2.1. Southern and Northern Period Field Oscillations

[5] We begin by outlining the nature of the oscillatory perturbation fields in Saturn’s magnetosphere revealed by the recent studies of Andrews et al. [2008, 2010a, 2010b] and Provan et al. [2009], which forms the basis of subsequent analysis here. This is illustrated schematically in Figures 1a and 1b for the southern and northern period oscillations, respectively, taken from Andrews et al. [2010b]. The colored lines in Figures 1a and 1b show the perturbation field lines in the principal meridian of the perturbation at some instant of time, where the vertical axis Z represents the spin (and magnetic) axis of the planet. The black dashed lines indicate the quasi-static “background” magnetospheric field, with closed field lines at lower latitudes (gray region) and open field lines at high latitudes (clear region) mapping into the northern and southern polar regions of the planet. To a first approximation the perturbation field lines out of the meridian plane shown can be obtained simply by displacing the colored loops directly into or out of the plane. The overall perturbation field pattern then rotates approximately rigidly about the axis at the southern SKR period in Figure 1a, and at the northern period in Figure 1b, each giving rise to magnetic field oscillations at the appropriate period at a fixed point.

[6] We first examine the magnetic perturbations associated with the southern system shown by the red lines in Figure 1a, focusing initially on the near-equatorial region shaded gray. It can be seen that the perturbation field in this region consists of a quasi-uniform field pointing to the left at the instant shown in Figure 1a, combined with a north-south field directed south on the left and north on the right which is such that the perturbation field lines in this region form arches with apices pointing to the north, as first deduced by Andrews et al. [2008, see Figure 10]. Figure 1c shows these fields in the equatorial plane viewed from the north, where the solid red lines show the quasi-uniform field, the circled dots and crosses the directions of the north-south field, and the arrowed black dashed circle indicates the direction of rotation of the perturbation field pattern. Throughout this paper we consider spherical polar magnetic field components in a planet-centered system referenced to Saturn’s northern spin and magnetic axis. At the instant depicted in Figure 1c, a maximum in the radial (r) component occurs at point P in the equatorial plane, together with a maximum in the colatitude component (θ), with corresponding minima on the opposite side of the planet (this defining the principal meridian shown in Figure 1a). As the perturbation field pattern then rotates about the axis, the r and θ components oscillate in phase with each other at any point, as found by Andrews et al. [2008, 2010a] and Provan et al. [2009]. The maximum in the r component at P is also accompanied by a zero in the azimuthal component (φ), which instead reaches a maximum at P one quarter cycle later. The φ component thus oscillates in lagging quadrature with r as found by Espinosa et al. [2003], Southwood and Kivelson [2007], and Andrews et al. [2008, 2010a], this being the characteristic signature of a rotating quasi-uniform field.

[7] These perturbation (red) field lines in the closed field (gray) region then close over the southern open (clear) region as shown in Figure 1a, where they now have a form similar to that of a planet-centered transverse dipole as found by Provan et al. [2009]. Although it is difficult to represent the true perturbation field geometry in a simple 2-D sketch, we emphasize that the intention in Figure 1 is to indicate that the quasi-uniform equatorial field corresponds to the inner region of closed field lines, while the quasi-dipolar field...
which closes these loops over the southern pole corresponds to the outer magnetosphere including the polar region of open field lines. It can then be seen from Figure 1a that in the polar region the $r$ and $\theta$ components continue to oscillate in phase with each other, and with the corresponding components in the equatorial region as found by Provan et al. [2009] and Andrews et al. [2010b]. However, the sense of the azimuthal perturbation field is reversed compared with the equatorial region, such that it now oscillates in leading quadrature with both the $r$ and $\theta$ components, this being the characteristic of a rotating transverse dipole field. The direction of the effective dipole corresponding to the southern polar field perturbations is shown by the central red arrow pointing to the left at the instant depicted, having the same direction as the quasi-uniform field in the equatorial region. We emphasize, however, that this “effective dipole” picture is used here principally as an expositive device that allows us to refer in simple terms to the structure and orientation of the southern polar perturbation field at any instant. It does not, of course, represent a true planet-centered transverse dipole that would produce equivalent perturbations in both hemispheres, though we note that the southern auroral current system proposed by Andrews et al. [2010b] to account for these perturbations does indeed have a magnetic dipole moment in the direction shown. SKR maxima in the southern hemisphere are then found to correspond to times when this effective dipole and the quasi-uniform field point down-tail and somewhat toward dawn, toward a local time (LT) of $\sim$1.5 h [Andrews et al., 2008, 2010a, 2010b; Provan et al., 2009].

Turning now to the northern perturbation field system illustrated by the green lines in Figure 1b, it can be seen that quasi-dipolar field perturbations are also present in the northern open field region as found by Provan et al. [2009], which rotate at the northern SKR modulation period as shown by Andrews et al. [2010b]. The resulting field oscillations are again such that the $\varphi$ component is in leading quadrature with $r$, characteristic of a quasi-dipolar field, but now the $\theta$ component is reversed in sense and oscillates in antiphase with $r$. At the instant depicted, the planet-centered effective transverse dipole corresponding to these perturbations is shown by the central green arrow pointing to the right. SKR maxima in the northern hemisphere are found to correspond to times when this effective dipole points nearly sunward at $\sim$11.7 h LT [Andrews et al., 2010b]. Thus the field configurations shown in Figures 1a and 1b correspond approximately to the situation for SKR maxima in both hemispheres if the direction to the Sun is to the right in both cases, such that the system is being viewed from dawn.

Although not previously detected directly, the perturbation field lines depicted in Figure 1b also suggest the likely presence of perturbation fields oscillating at the northern period within the equatorial “core” region, additional to the dominant southern period field shown in Figure 1a. According to the sketch, these field oscillations should take the form of a rotating quasi-uniform equatorial field having the same direction as the effective dipole of the northern polar oscillations (green arrow), together with a north-south field directed south on the left of the diagram and north on the right, which is such that the field lines within the “core” form arches with apices pointing to the south. These anticipated northern period “core” region perturbation fields are shown.
in the equatorial plane in Figure 1d in the same format as Figure 1c, from which it can be seen that oscillations in the \( s \) component are again in lagging quadrature with \( r \), as for the southern period equatorial field, while the \( \theta \) component is reversed in sense as in the polar region and oscillates in antiphase with \( r \). The effects due to the superposition of the northern and southern period field oscillations in the equatorial region is the subject of this paper.

### 2.2. Equatorial Phase “Jitter” Due to Superposition of Northern and Southern Period Fields

[10] Assuming that the perturbation fields of the northern (Saturn winter) system are somewhat weaker than those of the southern (Saturn summer) system, then in the equatorial region at the instant depicted in Figure 1 the quasi-uniform field of the northern system would partially cancel that of the southern system, corresponding to destructive interference of the oscillations in the \( r \) and \( \varphi \) components, while the north-south fields would add, corresponding to constructive interference of the oscillations in the \( \theta \) component. However, because the two field systems rotate with modestly differing periods, these phase relations will change over time from in phase via quadrature to antiphase for each component and back again, thus giving rise to modulations in both the amplitude and phase of the combined oscillations relative to those of the southern period oscillations alone. On the above picture the phase deviation (or “jitter”) at any instant will clearly have the same sense as each other for the \( r \) and \( \varphi \) components, since these components have the same phase relationship with each other in the two oscillations. The phase deviation for the \( \theta \) component will have the opposite sense, however, due to the opposite phase relationships of \( \theta \) with both \( r \) and \( \varphi \) in the two oscillations. Thus the characteristic feature of the equatorial phase “jitter” expected on the basis of Figure 1 are deviations from the long-term behavior that are in the same sense as each other for the \( r \) and \( \varphi \) components, while simultaneously having the opposite sense for the \( \theta \) component. The phase values for the three field components will thus become characteristically “split” by opposite phase deviations for \( \theta \) compared with \( r \) and \( \varphi \) on either side of the long-term trend.

[11] To quantify this discussion, the amplitude and phase modulations caused by the proposed superposition of northern and southern period oscillations in the equatorial region are illustrated specifically for the equatorial quasi-uniform field in Figure 2, where we view the equatorial plane from the north. Vectors \( B_s \) and \( B_n \) correspond to the quasi-uniform fields of the southern and northern period oscillations, respectively. The northern period field then rotates relative to the southern period field at an angular velocity equal to the difference in the angular velocities of the two oscillatory systems, antiphase as indicated by the arrowed dashed circle if the northern period is shorter than the southern period, as in the present case. As it does so, the quasi-uniform field formed by the combined southern and northern vectors, \( B^* \), rocks about the direction of the southern period field and varies in magnitude from a maximum of \((B_s + B_n)\) to a minimum of \((B_s - B_n)\). The period of these cyclic modulations is the “beat” period of the two oscillations, given by

\[
\tau_B = \frac{\tau_s \tau_n}{\tau_s - \tau_n},
\]

where \( \tau_s \) and \( \tau_n \) are the rotation periods of the southern and northern perturbation fields, respectively. For \( \tau_s \sim 10.8 \) h and \( \tau_n \sim 10.6 \) h, we thus find a “beat” period of \( \tau_B \sim 23 \) days, a result that will be refined for the SKR modulation periods in section 3.

[12] The two combined vectors shown in Figure 2, \( B^* \) and \( B^{**} \), correspond to the two extreme displacements of the field direction on either side of \( B_s \), which occur when the combined field and \( B_s \) are orthogonal as shown. The maximum angular deflection of the combined quasi-uniform field, \( \Delta \psi_{\text{max}} \), as shown in Figure 2, equal to the maximum...
phase deviations of the corresponding combined $r$ and $\varphi$ component oscillations, is thus given by

$$\Delta \psi_{\text{max}} = \pm \sin^{-1}(R),$$  

(2)

where $R$ is the ratio of the two field strengths, $R = (B_r/B_\varphi)$. This maximum angular displacement occurs when the angle between the two fields, $\Delta \Phi_{\text{max}}$ as also shown in Figure 2, satisfies

$$\Delta \Phi_{\text{max}} = \cos^{-1}(-R),$$  

(3)

e.g., just past quadrature on the two sides if $R$ is small.

[11] These considerations show that the presence of field oscillations at both northern and southern periods in the equatorial magnetosphere should result in the appearance of distinctive deviations at the “beat” period in both the magnitude and phase of the combined oscillations. Here we focus specifically on the phase deviations, since the amplitude of the oscillations is known also to depend significantly on both radial distance and local time in the equatorial plane [Andrews et al., 2010a], while the phase deviations depend only the relative magnitudes of the two fields, which may not vary as greatly if the spatial form of the two fields is similar. In section 3 we briefly describe the methods employed to determine the phase of the equatorial field oscillations from Cassini magnetic field data, following the previous related analyses of Andrews et al. [2008, 2010b] and Provan et al. [2009], and examine them for the presence of such effects.

3. Derivation of Core Region Oscillation Phases From Cassini Magnetic Field Data

3.1. Northern and Southern SKR Phases

[14] As in previous studies, we determine the phase of the core region magnetic field oscillations (to modulo 360°) relative to the phase of the dominant southern period SKR modulations, employed as an exact “guide phase.” In section 5 we also use the phase of the northern period SKR modulations to model the observed phase “jitter.” Here we therefore begin by outlining the nature of the SKR modulation phase data employed in this study, determined by Lamy [2011] using data from the Cassini RPWS instrument [Gurnett et al., 2004]. Briefly, the radio data were first separated into northern and southern emissions on the basis of their polarization and the latitude of the spacecraft, integrated in the SKR frequency band 40–500 kHz, and normalized with respect to radial distance from the planet. The dominant period in each hemisphere, $\tau_{\text{SKR, n,s}}(t)$, where “n” indicates north and “s” south, was then computed using a Lomb-Scargle analysis with a 200 day sliding window, such that variations on shorter intervals such as those described by Zarka et al. [2007] are not included. The corresponding SKR phases $\Phi_{\text{SKR, n,s}}(t)$ required for the magnetic field analysis are then given by

$$\tau_{\text{SKR, n,s}}(t) = \frac{360}{(\Phi_{\text{SKR, n,s}}(t))},$$  

(4a)

where the phases are measured in degrees. (Note that a list of principal mathematical symbols employed in the paper is provided for easy reference in the notation section.) The time series for the phase is thus determined by numerically integrating equation (4a) to obtain

$$\Phi_{\text{SKR, n,s}}(t) = 360 \int_0^t \frac{dt}{\tau_{\text{SKR, n,s}}} + \Phi_0_{n,s},$$  

(4b)

where $\Phi_0_{n,s}$ is a constant determined (to modulo 360°) by requiring the corresponding SKR power maxima to occur at times given by

$$\Phi_{\text{SKR, n,s}}(t) = 360N \text{ deg},$$  

(4c)

where $N$ is an integer.

[15] Results of this analysis are shown in Figure 3, where Figure 3a shows the period (hours) of the northern (blue) and southern (red) SKR modulations versus time $t$ (days) over the study interval. Specifically, as for all the time series plots shown in this paper, Figure 3 covers the interval $t = 250–1022$ days, where time $t = 0$ corresponds to the start of 1 January 2004. Year boundaries are shown at the top of Figure 3. As will be discussed in section 3.2, the core region magnetic field data employed span $t = 300–1016$ days (27 October 2004 to 12 October 2006 inclusive). Over this interval the mean values of the northern and southern SKR periods are found to be 10.59 and 10.80 h, respectively. The difference between these periods is shown in Figure 3b, varying between ~0.20 and ~0.23 h, averaging 0.210 h (or ~760 s) over the same interval. The “beat” period of the two modulations is shown in Figure 3c, given by equation (1), this being the interval between successive times when the SKR modulations in the two hemispheres have specific phases, modulo 360°, relative to each other. This period varies between ~21 and ~24 days over the interval, with a mean value of 22.7 days.

[16] With regard to the integrated phase values, we note that while the related study by Andrews et al. [2010b] used exactly the same SKR modulation phase data as those employed here, the earlier studies by Andrews et al. [2008] and Provan et al. [2009] used the phase of what is now understood to be the dominant southern SKR modulation derived by Kurth et al. [2007, 2008], expressed as polynomial functions of time $t$. However, the difference between these phases and the southern period phases employed here are small, typically within ±20° over the interval of their validity, and relatively slowly varying over hundreds of days [see Andrews et al., 2010b, Figure 2]. These differences produce no significant effect in the pass-to-pass magnetic oscillation phase deviation results obtained here.

3.2. Derivation of Core Region Phase Data From Magnetic Field Observations

[17] Following the previous analyses of Andrews et al. [2008, 2010b] and Provan et al. [2009], we now derive the phase of the magnetic field oscillations in the core region, defined here as dipole $L \leq 12$, relative to the southern period SKR modulations. Assuming that the field variations rotate around the planet in the sense of planetary rotation as a disturbance varying with azimuth $\varphi$ as $e^{-jm\varphi}$ with $m = 1$, they
can be expressed for spherical polar field component \(i\) by the form

\[
B_i(\varphi, t) = B_{i0} \cos(\Psi_{Mi,i}(\varphi, t)),
\]

where \(i\) can be \(r\), \(\theta\), or \(\varphi\). In this expression the magnetic phase \(\Psi_{Mi,i}(\varphi, t)\) is given by

\[
\Psi_{Mi,i}(\varphi, t) = \Phi_{SKR,i}(t) - \varphi - \Psi_{Mi,i},
\]

where \(\varphi\) is azimuth measured from noon positive toward dusk (equivalent to LT), \(\Phi_{SKR,i}(t)\) is the phase of the southern hemisphere SKR modulations obtained as outlined in section 3.1, and \(\Psi_{Mi,i}\) defines the relative phase of the oscillations for field component \(i\). For a given Cassini periapsis pass through the core region, the value of \(\psi_{Mi,i}\) is determined independently for each field component by cross correlating the form given by equation (5) with suitably processed magnetic field data. Specifically, the “Cassini SOI” model of the internal planetary field [Dougherty et al., 2005] is first subtracted from the data, and the resulting residual field band-pass filtered between 5 and 20 h using a standard Lanczos filter to extract the periodic signal. A constant pass-to-pass value of \(\psi_{Mi,i}\) then indicates a rotating perturbation with the same synodic period as the southern SKR modulation, whose value is determined by the specific phase relation between the magnetic and SKR modulation cycles. In particular, we note that at the times of southern SKR maxima given by equation (4c), the individual field components have perturbation maxima at azimuths given (to modulo 360°) by

\[
\varphi_{\text{max } i} = -\psi_{Mi,i}.
\]

In order to maintain a reasonably homogeneous magnetic field data set with comparable phase measurement uncertainties from pass to pass, here we consider only the magnetic field data from the initial ~2 year interval of near-equatorial orbits of the Cassini mission, spanning spacecraft orbits from Rev A in October 2004 to Rev 30 in October 2006. More exactly, as noted above the magnetic field data employed in our analysis span \(t = 300–1016\) days, an interval of 716 days centered on \(t = 658\) days. This slightly extends the interval to Rev 26 analyzed by Andrews et al. [2008], but excludes the later interval with highly inclined orbits also considered by Provan et al. [2009]. During the interval to Rev 30 considered here, the periapsis passes through the core lasted ~1.5 days, separated from each other by ~20–40 days. Each pass thus encompassed ~3 full oscillations of the field, from which the phase \(\psi_{Mi,i}\) can be determined to within a few degrees. Phase data for a few Revs are missing due to magnetic field data gaps, while as discussed previously by Andrews et al. [2008, 2010b] and Provan et al. [2009], \(\theta\) and/or \(r\) (but not \(\varphi\)) component data are omitted in other cases due to the presence of field variations at the spacecraft associated with the traversal of the ring current which introduce power into the filter band that is unrelated to the field oscillations. With regard to the possibility of phase deviations varying with the ~23 day SKR modulation beat period, we note that the ~1.5 day periapsis pass intervals are short compared with the beat period, so that phase deviations should be essentially constant during a given pass. However, the ~20–40 day separations between passes are comparable with and larger than the beat period, so that the phase deviations are expected generally to vary strongly from pass to pass, thus having more the appearance of “scatter” in the pass-to-pass phase data values (though correlated between the field components as discussed above).

[8] Magnetic phase values so determined are shown in Figure 4a, where we plot \(\psi_{Mi,i}\) versus time \(t\) for each field component \(i\) for each contributing Cassini pass. “Error bars” on the phase data are not shown in Figure 4 or Figures 5–14, but an analysis of the phase deviations is provided in section 4.2 which indicates that the RMS value of the random “measurement errors” does not exceed ~±3° (i.e., ~±1% of a full rotation). This is essentially the same vertical size as the data points plotted. Rev numbers, defined from apoapsis to

Figure 3. SKR modulation periods versus time for \(t = 250–1022\) days spanning our analysis interval, where \(t = 0\) corresponds to the start of 1 January 2004, together with related parameters. Year boundaries are shown at the top of the plot. (a) The modulation periods (hours) of the integrated SKR powers from the northern (blue) and southern (red) hemispheres, (b) the difference in these periods (hours), and (c) the “beat” period of these modulations given by equation (1).
apoapsis, are shown at the times of periapsis at the top of Figure 4. Phase data for the \( r \), \( /C_{18} \), and \( \varphi \) field components are shown by the red, green, and blue data points, respectively. These data immediately illustrate the main findings of Andrews et al. [2008, 2010b] and Provan et al. [2009] outlined in sections 1 and 2. First, the \( r \) and \( \theta \) oscillations are approximately in phase with each other, while \( \varphi \) is approximately in lagging quadrature (\( \psi_{Mi,s} \) values larger by \( \sim 90^\circ \)). Second, a slow phase drift to larger values with increasing time is present, implying a small (few seconds) difference in the periods of the magnetic and SKR oscillations, quantified in section 3.3. However, the overall change in phase is restricted to several tens of degrees so that a relatively constant phase relationship is maintained between the field oscillations and the southern SKR modulations over this 2 year interval. Third, a significant pass-to-pass "jitter" in the phase data is also present, which we will show below contains a major nonrandom component.

3.3. Long-Term Phase Drift

[19] To quantify these short-term phase deviations ("jitter"), we first define the long-term behavior by fitting smooth lines to the phase data in Figure 4, representing the overall trends. Andrews et al. [2008] fitted a single straight
line to their combined data set, assuming the \( r \) and \( \theta \) components are exactly in phase, while the \( \varphi \) component is exactly in lagging quadrature, such that \( 90^\circ \) was first subtracted from those data. Provan et al. [2009] similarly fitted a single cubic polynomial to their more extended data set using the same procedure. Here we again employ linear fits over the more restricted interval (it being found that use of quadratic or cubic polynomials did not significantly change the results), but while assuming that the relative phases of the components remain fixed over the interval such that the slope of the fit to each component is the same (thus implying a common period), we did not assume exact in-phase or quadrature behavior. We thus assume long-term trend lines relative to the southern period SKR modulation phase given for field component \( i \) by

\[
\psi_{Mi,s}(t) = \psi_{Mi,0,s} + \psi_{Mi,1,s} t,
\]  

where \( \psi_{Mi,1,s} \) is the common slope, while the three \( \psi_{Mi,0,s} \) values are the individual intercepts. A least squares fit then yields a common slope of \( \psi_{Mi,1,s} = 0.133 \pm 0.015 \) deg d\(^{-1}\), similar to those obtained previously over the same interval by Andrews et al. [2008, 2010b] and Provan et al. [2009], with intercepts for the \( r \), \( \theta \), and \( \varphi \) components of 75°, 74°, and 161°. The uncertainty in the mean phase values deduced from the fitted lines, determined as the standard error of the mean based on the RMS deviations of the points about the lines (see section 4.1), is about \( \pm 5^\circ \). These trend lines are those shown by the appropriately colored dashed lines in Figure 4a, though the lines corresponding to \( r \) and \( \theta \) are so close that they nearly overlap. Overall the \( r \) and \( \theta \) component oscillations are thus closely in phase, while the \( \varphi \) component oscillations are in lagging quadrature, the phase difference being \( +86^\circ \) (with an uncertainty of \( \sim 6^\circ \)).

[20] The implication of the long-term positive phase gradient is that during this interval the measured period of the southern SKR modulation is slightly shorter than the synodic period of the magnetic field oscillations. From equation (5) the long-term temporal phase of the magnetic field oscillations over the interval is

\[
\Phi_{Mi,s}(t) = \Phi_{SKR,s}(t) - \psi_{Mi,1,s} t,
\]

such that the period of the oscillations is

\[
\tau_{Mi,s} = \frac{360}{\frac{d}{dt}(\Phi_{Mi,s}(t))} = \frac{360}{\frac{d}{dt}(\Phi_{SKR,s}(t)) - \psi_{Mi,1,s}},
\]

\[
\approx \tau_{SKR,s} \left(1 + \frac{\psi_{Mi,1,s}}{360} \right),
\]

where in the last expression we have used equation (4a) and the fact that the second term in the bracket is small compared with unity. With \( \tau_{SKR,s} = 10.80 \) h during the interval as shown in Figure 3, and a magnetic phase gradient of \( \sim 0.133 \) deg d\(^{-1}\), the difference in the periods given by equation (8) is only \( \sim 6.5 \) s (i.e., about 0.02% of the period). Over the 716 days of the study, the total drift in phase is \( \pm 48^\circ \), corresponding to \( \pm 13\% \) of one cycle. At the center time of the study, corresponding to \( t = 658 \) days, the specific phase values of the trend lines for the \( r \), \( \theta \), and \( \varphi \) components are 163°, 162°, and 248°, respectively. From equation (5c) we then note that at the center time, at southern SKR maxima the \( r \) and \( \varphi \) field components have maxima at azimuths of \( -163^\circ \) and \( -248^\circ \) (or \( +197^\circ \) and \( +112^\circ \), respectively, corresponding to LTs of 1.1 and 19.5 h. Thus as indicated in section 1, at southern SKR maxima the quasi-uniform field formed by the \( r \) and \( \varphi \) components points down tail and somewhat toward dawn, radially outward at about 1.3 h LT (taking the mean indicated by the above phases). The variations in these values of \( \pm 48^\circ \) over the study interval then imply changes in these LTs by \( \pm 3.2 \) h, to later LTs prior to the center time, and to earlier LTs after the center time.

4. Phase Deviations Relative to the Long-Term Drift

4.1. Correlations of the Phase Deviations Between Field Components

[21] Using the linear trend lines \( \psi_{Mi,0,s}(t) \) to define the long-term behavior, the deviations in the phase data are taken to be given by

\[
\Delta \psi_{Mi,s} = \psi_{Mi,s} - \psi_{Mi,0,s}(t),
\]

and are shown in Figure 4b using the same color code as in Figure 4a. The overall RMS values of these quantities are comparable for the \( r \) and \( \varphi \) components at 19.6° and 21.9°, respectively, while for the \( \theta \) component it is somewhat larger at 28.1°. However, inspection of these data shows that the values are by no means random, it being seen, for example, that the deviations in the \( r \) and \( \varphi \) component phases for a given Rev generally have the same sign and similar values to each other, while the sign of the deviation of the \( \theta \) component phase is generally opposite, this being the effect anticipated in section 2 for the superposition of weaker northern period field oscillations on dominant southern period oscillations according to Figure 1.

[22] The correlations in the phase deviations are demonstrated more clearly in Figure 5, where we show scatterplots of the deviations for one field component plotted against another for all three simultaneous pairs of values. Figure 5a shows the deviation of the \( r \) component phase, \( \Delta \psi_{Mr,s} \), plotted versus the deviation of the \( \varphi \) component phase, \( \Delta \psi_{M\varphi,s} \). The two independently determined phase deviations are positively correlated as found previously by Andrews et al. [2008], with a cross-correlation coefficient of +0.94. This shows that most of the phase scatter in these components is not due to measurement error, but instead results from physically caused deviations that are common to the two field components. The dotted lines in Figure 5a show least squares fits to the data constrained to pass through the origin, where the lines of smaller and larger slope are the regression lines of \( \Delta \psi_{Mr,s} \) on \( \Delta \psi_{M\varphi,s} \) and vice versa, respectively, which have slopes of 0.80 and 0.92. The dashed “best fit” line is then taken to be their angular bisector, which has a slope of 0.86, meaning that the phase deviation of the \( r \) component is comparable with but typically modestly smaller than that of the \( \varphi \) component. Figure 5b similarly shows \( \Delta \psi_{Mr,s} \) plotted versus \( \Delta \psi_{M\varphi,s} \). In this case a negative correlation is evident with a cross-correlation coefficient of −0.55, meaning that the phase deviations are
The pairs of dotted regression lines in this case have slopes of \(-0.67\) and \(-2.22\), with the slope of the “best fit” line being \(-1.18\), meaning that the magnitude of the phase deviation of the \(\theta\) component is comparable with but typically modestly larger than that of the \(\varphi\) component. For completeness, Figure 5c shows \(\Delta \psi_{\text{MRH}}\) plotted versus \(\Delta \psi_{\text{MRD}}\), again showing the presence of a negative correlation, with a cross-correlation coefficient of \(-0.59\). The pairs of dotted regression lines have slopes of \(-0.39\) and \(-1.21\), with the slope of the “best fit” line being \(-0.72\), in agreement with expectations based on the slopes in Figures 5a and 5b.

### 4.2. Analysis of Phase Deviations

[23] In this section we analyze these phase deviation data to provide estimates of the magnitude of the correlated phase deviations compared with those due to random (e.g., measurement) errors. We begin by assuming that the phase deviation in magnetic field component \(i\) at any instant (where \(i\) can again be \(r, \theta,\) or \(\varphi\)) is given by the sum of three components

\[
\Delta \psi_i = \Delta \psi_{iS} + \Delta \psi_{iC} + \Delta \psi_{iR},
\]

where for simplicity we have dropped inessential subscripts. In this equation \(\Delta \psi_{iS}\) is the “splitting” phase deviation noted above (section 2.2) in which the \(r\) and \(\varphi\) component phases characteristically deviate in one direction about the long-term trend lines while the \(\theta\) component phase deviates in the other, \(\Delta \psi_{iC}\) is a possible “common” phase deviation, and \(\Delta \psi_{iR}\) is the “random” phase deviation.

[24] In the “splitting” deviation the phases for each component are taken to deviate by fixed ratios relative to each other. Arbitrarily taking the deviation of the azimuthal component \(\Delta \psi_{\varphi S}\) as the “base” value, the deviation of the \(r\) component can then be written as

\[
\Delta \psi_{rS} = \alpha_r \Delta \psi_{\varphi S},
\]

and that of the \(\theta\) component as

\[
\Delta \psi_{\theta S} = \alpha_\theta \Delta \psi_{\varphi S},
\]

where \(\alpha_r\) and \(\alpha_\theta\) are constants to be determined. The regression analysis above suggests \(\alpha_r \sim 0.9\) and \(\alpha_\theta \sim -1.2\) if this is the only nonrandom effect present in the data, to which the \(\alpha_i\) values determined from the present analysis will be compared below. For general field component \(i\) we thus write

\[
\Delta \psi_{iS} = \alpha_i \Delta \psi_{\varphi S},
\]

where \(\alpha_i \equiv 1\).

[25] The “common” phase deviation \(\Delta \psi_{iC}\) in equation (10a) is then taken to have the same value for each field component at a given time, acknowledging the possibility, for example, that the magnetic oscillation phases may deviate gradually from the linear behavior assumed in equation (6) over the \(~700\) day interval of the study. Alternatively, the phases may also undergo short-term fluctuations, due, e.g., to some external forcing (such as that found in SKR phase data by Zarka et al. [2007] related to the solar wind velocity). We note in Figure 4b that the phase deviation triads for some

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**Figure 5.** Scatterplots of the phase deviations from the long-term linear trend lines in Figure 4, \(\Delta \psi_{\text{MR}}\), showing the deviation of one component plotted versus another for all available corresponding pairs of data. (a) \(r\) versus \(\varphi\), (b) \(\theta\) versus \(\varphi\), and (c) \(r\) versus \(\theta\). The pairs of dotted lines show the two linear least squares regression lines of ordinate on abscissa and vice versa, while the dashed “best fit” line is their angular bisector. The slope of the “best fit” line is given in each plot (“Grad”), together with the value of the cross correlation coefficient of the data (“XCC”).
Revs appear overall to deviate to positive or negative values (see, e.g., the relative values of the triads for Revs A and B). We assume that these deviations are physically distinct from and uncorrelated with those of the “splitting” deviation. The “random” error \( \Delta \psi_{i.r} \) in each field component in equation (10a) then includes all other uncorrelated sources of phase deviation, including measurement error.

[26] We thus take each of the five phase deviation parameters in the model, namely, the “splitting” deviation for the \( \varphi \) component \( \Delta \psi_{i.\varphi} \), the “common” deviation \( \Delta \psi_{i.c} \), and the three “random” errors \( \Delta \psi_{i.r} \), to be uncorrelated with each other and to have zero long-term means (about the trend lines in Figure 4). We can then form the averages of the three cross products of the phase deviation values for field components \( j \) and \( k \) (values related to the cross-correlation coefficients quoted in section 4.1), giving (for \( j \neq k \))

\[
\delta_{jk} = \langle \Delta \psi_{j.r} \Delta \psi_{k.r} \rangle = \alpha_{j.k} \langle \Delta \psi_{j.s}^2 \rangle + \langle \Delta \psi_{k.s}^2 \rangle,
\]

(11a)

where we note again that \( \alpha_{r.r} = 1 \). Similarly, we can form the averages of the three autoproducts (the squares of the RMS values quoted in section 4.1), giving

\[
D_i = \langle \Delta \psi_{i.s}^2 \rangle = \alpha_{i.\varphi} \langle \Delta \psi_{i.s}^2 \rangle + \langle \Delta \psi_{i.r}^2 \rangle.
\]

(11b)

We thus have six quantities whose values can be estimated from the phase deviation data shown in Figure 4, i.e., the three \( \delta_{jk} \) values and the three \( D_i \) values, and seven unknowns, i.e., the mean squared values of the five phase deviation parameters noted above, together with the two “splitting” ratios \( \alpha_{\varphi} \) and \( \alpha_{\theta} \). We therefore require one further constraint in order to solve these equations. Given that the random measurement errors in the \( \varphi \) and \( \theta \) component phases are prima facie likely to be similar to each other, noting the similar overall RMS deviations in these data, we write

\[
\langle \Delta \psi_{j.r}^2 \rangle = \langle \Delta \psi_{k.r}^2 \rangle + K.
\]

(11c)

where constant \( K \) is the expectedly small difference in the mean square random errors in the \( \varphi \) and \( \theta \) component data. We then determine the solutions of the seven equations given by equations (11a)–(11c) as a function of \( K \). Although this analysis does not, therefore, give a unique solution, the range of \( K \) and of the other parameters is found to be strongly constrained by the requirement for a physically valid solution that the mean squared values of each of the five phase deviation parameters must simultaneously be positive.

[27] Elimination of the unknowns between equations (11a)–(11c) is then found to yield a quadratic for \( \alpha_{\varphi} \), whose required root is

\[
\alpha_{\varphi} = \frac{\left( \delta_{r.r} - \delta_{\theta.r} \right) + D^*}{\sqrt{\left( \delta_{r.r} - \delta_{\theta.r} \right) + D^*}^2 + 4\left( \delta_{r.r} - \delta_{\theta.r} \right) \left( \delta_{r.r} - 2\delta_{\theta.r} \right)}}{2\left( \delta_{r.r} - \delta_{\theta.r} \right)},
\]

(12a)

where

\[
D^* = (D_r - D_\theta) + K.
\]

(12b)

The solution for \( \alpha_{\theta} \) is then

\[
\alpha_{\theta} = -\frac{(\delta_{r.r} - \delta_{\theta.r})}{D^*},
\]

(12c)

the mean squared values of the “splitting” and “common” phase deviation parameters are

\[
\left\langle \Delta \psi_{i.s}^2 \right\rangle = D^*/(\alpha_i^2 - 1) \quad \text{and} \quad \left\langle \Delta \psi_{i.r}^2 \right\rangle = \delta_{i.r} - \alpha_i \left\langle \Delta \psi_{i.s}^2 \right\rangle.
\]

(12d)

while the mean squared values of the three “random” phase deviations for each field component \( i \) are given by

\[
\left\langle \Delta \psi_{i.r}^2 \right\rangle = D_i - \left\langle \Delta \psi_{i.s}^2 \right\rangle - \alpha_i^2 \left\langle \Delta \psi_{i.s}^2 \right\rangle.
\]

(12e)

The values of the cross products \( \delta_{jk} \) corresponding to the data shown in Figure 5 are

\[
\delta_{r.r} = 419 \text{ deg}^2 \quad \delta_{r.c} = -375 \text{ deg}^2 \quad \delta_{r.\theta} = -314 \text{ deg}^2.
\]

(13a)

while the values of the autoproducts including all of the data shown in Figure 4 are

\[
D_r = 384 \text{ deg}^2 \quad D_\theta = 789 \text{ deg}^2 \quad D_\varphi = 478 \text{ deg}^2.
\]

(13b)

Results for these values are shown in Figure 6 plotted versus \( K \) (equation (12b)), where the curves are plotted only over the range of \( K \) for which the solution values for all five squared phase deviation parameters are simultaneously positive. This corresponds to the modest range of \( K \) between \(-15.8 \text{ and } +11.9 \text{ deg}^2\), constrained at the lower end by negative values of the “random” error parameter for the \( \varphi \) component \( \left\langle \Delta \psi_{i.\varphi}^2 \right\rangle \), and at the upper end by negative values of the “random” error parameter for the \( \theta \) component \( \left\langle \Delta \psi_{i.\theta}^2 \right\rangle \).

[28] Figure 6a shows the values of the “splitting” ratios \( \alpha_{r} \) (red) and \( \alpha_{\theta} \) (green). It can be seen that these values vary only modestly over the allowed range of \( K \), between 0.86 and 0.88 for \( \alpha_{r} \), and \(-1.03 \text{ and } -1.39 \) for \( \alpha_{\theta} \) (from the lower to the upper limit on \( K \)), with values at \( K = 0 \) being 0.87 and \(-1.21 \). We note that these values are in excellent accord with the “best fit” gradients of 0.86 and \(-1.18 \) found from the corresponding scatterplots in Figures 5a and 5b.

[29] Figure 6b then shows the square roots of the mean squared phase deviation parameters, i.e., the RMS values of these parameters. The red, green, and blue solid lines show the RMS values of the “splitting” phase deviations for the \( r, \) \( \theta, \) and \( \varphi \) components, respectively, the values for the \( r \) and \( \theta \) components being related to that for the \( \varphi \) component, \( \Delta \psi_{i.\varphi} \), by equations (10b) and (10c). The RMS value of the “common” phase deviation \( \Delta \psi_{i.c} \) is shown by the black solid line. The red, green, and blue dashed lines then show
The RMS values of the three “random” phase deviations \( \Delta \psi_R \) for the \( r \), \( \theta \), and \( \varphi \) components, respectively.

[30] The results show that the “splitting” phase deviations are the largest nonrandom effect over the whole range of \( K \) values, with RMS values of \( \sim 20^\circ \) for all three field components, compared with values of \( \sim 10^\circ \) for the “common” phase deviations. Specifically, at \( K = 0 \) the RMS values of the “splitting” phase deviations are 17.0\(^\circ\), 19.5\(^\circ\), and 23.6\(^\circ\) for the \( r \), \( \theta \), and \( \varphi \) components, respectively, while that for the “common” phase deviation is 9.3\(^\circ\). The “random” phase deviations in the \( r \) and \( \varphi \) components are both much smaller, having RMS values at \( K = 0 \) (where by definition of \( K \) these are equal) of 3.0\(^\circ\). We note that in addition to other possible effects, these “random” phase deviations include the measurement errors of the phase values from the filtered residual field data, which we thus conclude do not exceed RMS values of a few degrees as noted above in section 3.2. For the \( \theta \) component, however, the estimated RMS value of the “random” phase deviation is generally much larger, equal to 12.0\(^\circ\) at \( K = 0 \), though still rather smaller than the RMS value of the “splitting” phase deviation in this component. Since these phase values were determined in exactly the same way as those for the \( r \) and \( \varphi \) components, for which the “random” errors are much smaller as we have just seen, the reason for the larger “random” phase deviations in the \( \theta \) component remains unclear, possibly associated with small residual effects of field variations associated with the ring current remaining in the filtered data. Overall, however, the principal conclusion from this analysis is that the dominant phase deviation effect present in our data set is a “splitting” phenomenon in which the \( r \) and \( \varphi \) component phases deviate in one sense and the \( \theta \) component phase simultaneously in the other, all with comparable RMS values of \( \sim 20^\circ \). In what follows we therefore concentrate on this dominant effect.

4.3. Phase Deviation Spectra

[31] As indicated above, a likely physical origin of the phase “splitting” phenomenon is the perturbing presence in the “core” magnetosphere of northern period field oscillations superimposed on dominant southern period oscillations. If so, the phase deviations should be quasi-sinusoidal in nature, with a period close to the \( \sim 23 \) day beat period of the northern and southern SKR oscillations shown in Figure 3. In this case, our phase data set consisting of values determined at typical orbital intervals of \( \sim 20–40 \) days represents significant undersampling of the oscillatory values.

[32] As a simple initial approach to determining whether such periodicity exists in our magnetic oscillation phase data, we have formed spectra of the phase deviations for each field component for fixed periods spanning the expected “beat” period. In effect we plot the phase deviation data for each field component versus the linearly increasing phase corresponding to fixed period \( \tau \), i.e., \( \Phi(t) = 360(\tau/\tau_0) \) deg, modulo \( 360^\circ \), and least squares fitted a sinusoid \( \Delta \psi_{Mi} = A \cos(\Phi - \Phi_0) \) to the resulting distribution to determine amplitude \( A \) and phase \( \Phi_0 \). The amplitude spectra so determined are shown in Figures 7a–7c for the \( r \), \( \theta \), and \( \varphi \) field components, respectively, for values of \( \tau \) between 10 and 35 days at 0.25 day resolution. Investigation of spectra formed from randomly generated phase deviation data with the same time sequence and RMS variability as the present data sets shows that with the modest number of data points at our disposal, “accidental” peaks comparable in amplitude to the RMS values occur relatively frequently. Correspondingly, the spectra in Figure 7 show a number of such peaks at a variety of periods \( \tau \), which are generally different in the differing components. However, we are interested in peaks that occur simultaneously at the same period in all three field components, conspicuous among which are the peaks near a period of 23 days marked by the vertical dashed lines. This is the most prominent feature in the spectrum for the \( \varphi \) component, which is the most sampled data set, and is also relatively clear in the \( \theta \) and \( r \) component spectra, though the

**Figure 6.** Plots showing the results of the phase deviation data analysis presented in section 4.2. Parameters are shown plotted versus \( K \), the difference (in deg\(^2\)) between the mean squared values of the “random” phase deviations in the \( r \) and \( \varphi \) field components (equation (11c)). Results are shown only over the physically relevant range of solutions where all five squared phase deviation parameters are positive. (a) The ratios of the phase deviations for the \( r \) component (red) and \( \theta \) component (green) relative to the \( \varphi \) component for the phase “splitting” term (i.e., \( \alpha_r \) and \( \alpha_\theta \) in equations (10b) and (10c)). (b) The RMS values of the five phase deviation parameters, namely, the “random” phase deviations \( \langle \Delta \psi^2_{R \phi} \rangle^{1/2} \) for the \( r \), \( \theta \), and \( \varphi \) field components (red, green, and blue dashed lines), the “common” phase deviation \( \langle \Delta \psi^2_{C \phi} \rangle^{1/2} \) (black solid line), and the “splitting” deviation for the \( \varphi \) field component \( \langle \Delta \psi^2_{s \phi} \rangle^{1/2} \) (blue solid line). We also show the corresponding RMS values of the “splitting” deviations for the \( r \) and \( \theta \) field components given by \( \alpha_r \langle \Delta \psi^2_{s \theta} \rangle^{1/2} \) and \( \alpha_\theta \langle \Delta \psi^2_{s \theta} \rangle^{1/2} \), respectively (red and green solid lines).
latter has its most prominent peak at 25 days where the component amplitude is small.

[33] In Figures 7d–7f we thus show the phase deviation data for the three field components plotted versus phase $\Phi$ for a fixed period of 23 days, together with the fitted sinusoids. Oscillations with an amplitude of $\sim 20^\circ$ are present in each component, which have phases $\Phi_0$ of 206°, 322°, and 167° for the $r$, $\theta$, and $\phi$ components, i.e., approximately in phase (within $\sim 40^\circ$) for the $\phi$ and $r$ components, and antiphase (within $\sim 25^\circ$) for the $\phi$ and $\theta$ components. This simple analysis thus provides supporting evidence that the phase deviation values are oscillatory near the north-south “beat”
Figure 8. Argand plane diagram showing the “vector” combination of two sinusoidal oscillations in magnetic field component $i$ having different amplitudes and periods. The red vector with amplitude $B_{0n}^i$ and phase $\Psi_{Mi}^i$ corresponds to the southern period oscillation as given by the first term in equation (15a), while the blue vector with amplitude $B_{0n}^i$ and phase $\Psi_{Mi}^i$ corresponds to the northern period oscillation as given by the second term in equation (15a). The instantaneous phase difference of these components is $\Delta \Phi_{Mi} = \Psi_{Mi}^i - \Psi_{Mi}^i$ as given by equation (16d). The black vector then corresponds to their instantaneous vector sum, given by equation (15d) with instantaneous amplitude $B_{0n}^i$ and phase $\Delta \Psi_{Mi}^i$ relative to the southern period vector phase $\Psi_{Mi}^i$.

Period of ~23 days. However, it is clear that a more sophisticated treatment is desirable, incorporating the phase variations expected for superposed oscillatory signals, together with the somewhat variable “beat” period that is expected to be present on the basis of the results shown in Figure 3. Such an analysis is now pursued.

5. Phase Deviations Due to Superposed Northern and Southern Period Field Oscillations

5.1. Phase Deviation Model

[34] In the above discussion the observed oscillations for field component $i$ have in effect been expressed by the form

$$B_i(\varphi, t) = B_{0i}\cos(\Psi_{Mi}(\varphi, t) - \Delta \Psi_{Mi}(t)), \quad (14a)$$

where $\Delta \Psi_{Mi}(t)$ is the pass-to-pass phase deviation, while $\Psi_{Mi}(\varphi, t)$ is the smoothly varying magnetic phase related to the southern period SKR modulations

$$\Psi_{Mi}(\varphi, t) = \Phi_{Nn}(t) - \varphi = \Psi_{Mi}(\varphi, t) - \varphi - \Psi_{Mi}(t). \quad (14b)$$

slightly modified in period by inclusion of the slowly varying linear “phase drift” $\Psi_{Mi}(t)$ given by equation (6). If the phase deviations are in fact due to the presence of a weaker smoothly varying perturbation rotating near the northern SKR period, superposed on a dominant smoothly varying perturbation rotating near the southern SKR period, then we can also write the observed oscillation in equation (14a) as

$$B_i(\varphi, t) = B_{0i}\cos(\Psi_{Mi}(\varphi, t) + \Delta \Psi_{Mi}(t)), \quad (15a)$$

The first term on the right side of this expression is the dominant oscillation near the southern SKR modulation period (which we continue to call the “southern period” field oscillation), whose phase, $\Psi_{Mi}(\varphi, t)$, is taken to be given by equation (14b). The second term on the right is then the perturbing oscillation near the northern SKR modulation period (which we call the “northern period” field oscillation), whose phase is similarly taken to be given by

$$\Psi_{Mi}(\varphi, t) = \Phi_{Mi}(t) - \varphi = \Psi_{Mi}(\varphi, t) - \varphi - \Psi_{Mi}(t), \quad (15b)$$

where the relative phase $\Psi_{Mi}(t)$ in general includes some slowly varying “phase drift,” to be determined, of the northern period field oscillations relative to the northern SKR modulations. By analogy with the results for the southern period oscillations we simply assume a linear dependency over the interval considered

$$\Psi_{Mi}(t) = \Psi_{Mi}(t) + \Delta \Psi_{Mi}(t), \quad (15c)$$

where again the gradient $\Psi_{Mi}(t)$ is assumed to be common between the three field components so that their relative values remain fixed in time, implying a common oscillation period.

[35] The two oscillations in equation (15a) are represented as vectors in the Argand plane in Figure 8, where the southern period oscillation with amplitude $B_{0n}^i$ and phase angle $\Psi_{Mi}(t)$ is shown by the red vector, while the northern period oscillation with amplitude $B_{0n}^i$ and phase angle $\Psi_{Mi}(t)$ is shown by the blue vector. The combined oscillation is then given by the sum of these two vectors, shown by the black vector, whose instantaneous amplitude is $\Delta \Psi_{Mi}(t)$ and whose phase relative to the dominant southern period oscillation is $\Delta \Psi_{Mi}(t)$ as shown. That is, we can write equation (15a) as

$$B_i(\varphi, t) = B_{0i}(t)\cos(\Psi_{Mi}(\varphi, t) - \Delta \Psi_{Mi}(t)), \quad (15d)$$

where $\Delta \Psi_{Mi}(t)$ is thus the model phase deviation of the combined oscillation relative to the southern period magnetic phase due to the presence of the northern period oscillation, as can be seen directly by comparison of equations (15d) and (14a).

[36] It is then evident that as the northern period vector rotates relative to the southern period vector, both the amplitude and phase of the combined oscillation will be modulated about the values for the dominant southern period oscillation at the “beat” period of the two oscillations given by equation (1), similar to the discussion of Figure 2 in section 2. We note in passing that the vector geometry shown in Figure 8 is essentially the same as that shown in Figure 2, but applied here to a single oscillatory field component rotating in the Argand plane, instead of to the vector addition of two rotating uniform fields in the equatorial plane in Figure 2. Figure 8 transforms into Figure 2 in the case of
exact quadrature and equal amplitudes of the $r$ and $\varphi$ components for both the southern and the northern period equatorial oscillations, in which case $\text{Re}(B_i)$ and $\text{Im}(B_i)$ can be taken to correspond to two orthogonal field components in the equatorial plane, with, e.g., $\text{Re}(B_i)$ corresponding to the $r$ component and $\text{Im}(B_i)$ to the $\varphi$ component.

[37] The value of the model phase deviation $\Delta \Psi_{M_i}^{*}$ is readily determined using Figure 8 by resolving the northern period vector into components parallel and perpendicular to the southern period vector, and combining with the latter to obtain

$$\tan \Delta \Psi_{M_i}^{*} = \frac{B_{0,\varphi}^{i} \sin (\Psi_{M_i}^{*} - \Psi_{M_i})}{(B_{0,r}^{i} + B_{0,\varphi}^{i} \cos (\Psi_{M_i}^{*} - \Psi_{M_i}))}. \quad (16a)$$

Substituting for the magnetic phases from equations (14b) and (15b) and rearranging we then have

$$\Delta \Psi_{M_i}^{*} = -\tan^{-1} \left[ \frac{R_i \sin (\Delta \Phi_{M_i}(t))}{1 + R_i \cos (\Delta \Phi_{M_i}(t))} \right]. \quad (16b)$$

where $R_i$ is the ratio of the amplitudes of the northern and southern period oscillations for field component $i$

$$R_i = \left( \frac{B_{0,\varphi}^{i}}{B_{0,r}^{i}} \right). \quad (16c)$$

and $\Delta \Phi_{M_i}(t)$ is the instantaneous relative phase of the two magnetic field oscillations for field component $i$, independent of azimuth $\varphi$, i.e., the magnetic “beat” phase given by

$$\Delta \Phi_{M_i}(t) = \Psi_{M_i}^{*} - \Psi_{M_i} = \Psi_{M_i} - \Phi_{M_i} = \left( \Phi_{SKR_n}(t) - \Phi_{SKR_s}(t) \right) - \left( \Psi_{M_i} - \Psi_{M_i}(t) \right). \quad (16d)$$

It is readily shown from equation (16b) that the maximum model phase deviation is given by $\Delta \Psi_{M_i}^{* \text{ max}} = \sin^{-1} R_i$, which occurs when the relative phase satisfies the condition $\Delta \Phi_{M_i}(t) = \cos^{-1} (-R_i)$, consistent with equations (2) and (3) in section 2.

[38] The “beat” period of the phase and amplitude modulations of the combined magnetic field oscillations, $\tau_{BM_i}$, is given by

$$\tau_{BM_i}(t) = \frac{360}{\frac{d}{dt} \Delta \Phi_{M_i}} = \frac{360}{\frac{d}{dt} \left( \Phi_{SKR_n} - \Phi_{SKR_s} \right) - \left( \Psi_{M_i} - \Psi_{M_i}(t) \right)} \approx \frac{\tau_{SKR}}{1 + \left( \frac{\Psi_{M_i} - \Psi_{M_i}(t)}{360} \right) \tau_{SKR}}. \quad (17a)$$

where the phases are again expressed in degrees, $\Psi_{M_i}$ and $\Psi_{M_i}(t)$, are the gradients of the phase drift of the northern and southern period field oscillations in equations (6) and (15c), and $\tau_{SKR}$ is the corresponding “beat” period of the SKR modulations. The latter (shown in Figure 3) is given by

$$\tau_{SKR}(t) = \frac{360}{\frac{d}{dt} \left( \Phi_{SKR_n} - \Phi_{SKR_s} \right)} = \frac{\tau_{SKR} \tau_{SKR}}{\tau_{SKR} + \tau_{SKR}}. \quad (17b)$$

in conformity with equation (1). We note from the approximate form in equation (17a) that if the magnetic phase drifts are of order $-0.1 \deg d^{-1}$, as they are for the southern period oscillations, then with $\tau_{BM_i} \approx 23$ days as shown in Figure 3, the second term in the bracket is less than or of order $-0.01$. Thus with phase drifts of this magnitude, the beat period of the magnetic field oscillations will remain essentially that of the SKR modulations, to better than $1\%$.

5.2. Determination of Northern Period Oscillation Parameters From Phase Deviation Data

[39] If we examine the parameters that enter the expressions in equation (16) for the model phase deviation $\Delta \Psi_{M_i}^{*}(t)$, we see that the known quantities are the SKR phases for the northern and southern period modulations $\Phi_{SKR_n}(t)$ (section 3.1), together with the long-term phases $\Psi_{M_i}(t)$ of the southern period magnetic field oscillations for each component $i$ relative to the southern period SKR phase (section 3.3). The unknowns are the corresponding phases $\Psi_{M_i}(t)$ of the northern period magnetic oscillations for each field component $i$ relative to the northern period SKR phase, taken to be linear functions of time as in equation (15c), together with the amplitude ratios $R_i$ for each component, assumed for simplicity to be constant over the study interval. There are therefore three unknowns for each field component, i.e., $\Psi_{M_i}$, $\Psi_{M_i}$, and $R_i$, though as indicated above, ultimately the phase gradient $\Psi_{M_i}$ is taken to be common for each component so that their relative phases remain fixed with time.

[40] Estimates of the unknown parameters have been determined by computing the RMS deviation between the $\Delta \Psi_{M_i}$ phase deviation data for each field component shown in Figure 4b and the model values given by equation (16), seeking the set of model parameters that give the minimum value. Specifically we have considered $R_i$ values between 0 and 1 with a resolution of 0.01, $\Psi_{M_i}$ values between $-180^\circ$ and $+180^\circ$ with a resolution of 1° (any 360° range is equivalent), and (after initial exploration to locate the minima) $\Psi_{M_i}$ values between $-0.2$ and $+0.4 \deg d^{-1}$ with a resolution of 0.01.

[41] The results show that while the RMS values are sensitive to the values of $R_i$ and $\Psi_{M_i}$, as demonstrated below, they are relatively insensitive to the phase gradient $\Psi_{M_i}$ within this range. Figure 9 shows the minimum values of the RMS deviation over the whole of the above ranges of $R_i$ and $\Psi_{M_i}$ plotted versus $\Psi_{M_i}$, where the red, green, and blue lines show results for the $r$, $\theta$, and $\varphi$ components. In each case we find a relatively shallow minimum marked by the arrows, which occur at $\Psi_{M_i}$ values of $-0.05, 0.20,$ and $0.08 \deg d^{-1}$ for the $r$, $\theta$, and $\varphi$ components, respectively. The black arrow in the top part of the plot indicates for comparison the phase drift of the southern period oscillations, $\Psi_{M_i} = 0.133 \deg d^{-1}$, which is thus of comparable order. As indicated above, however, we expect physically that the relative phases of the oscillations in the three field components should remain fixed with time implying a common period, such that we impose one common value of $\Psi_{M_i}$ that is in line with these results. We have chosen the round value $\Psi_{M_i} = 0.1 \deg day^{-1}$, with a likely uncertainty of about $\pm 0.1 \deg d^{-1}$ (thus essentially also consistent with zero), which we can see from Figure 9 does not strongly increase the minimum value of the RMS deviation for
any field component. From the northern equivalent of equation (8), we note that the implication of a value of $\psi_{M1,n}$ of $+0.1$ deg d$^{-1}$ is that the synodic period of the northern period magnetic field oscillations is slightly longer than the period of the corresponding SKR modulation, by $\sim 5$ s (i.e., by $-0.01\%$ of the SKR modulation period). We also note from equation (17a) that with these values, the beat period of the northern and southern period magnetic field oscillations is slightly shorter than the $\sim 23$ day beat period of the northern and southern period SKR modulations, by $\sim 1$ h (i.e., by $-0.2\%$ of the SKR beat period).

In Figure 10 we thus show color-coded plots of the RMS deviation between the model and observed phase deviations in the $R_t$-$\psi_{M1,n}$ plane for each field component, with $\psi_{M1,n}$ held fixed at 0.1 deg d$^{-1}$. In each case it can be seen that a well-defined minimum is present, shown by the orange and red colors, from which the best fit values of $R_t$ and $\psi_{M1,n}$ can be determined in each case. The position of the minimum value of the RMS deviation is marked by the black cross in each plot. The minimum values themselves are $14.8^\circ$, $20.8^\circ$, and $13.9^\circ$ for the $r$, $\theta$, and $\varphi$ components, respectively, compared with initial values for the RMS deviations from the long-term trend lines in Figure 4 of $19.6^\circ$, $28.1^\circ$, and $21.9^\circ$. Thus with the inclusion of the phase deviations due to the northern period oscillations the best fit models show marked reductions in RMS deviation. For comparison, we have investigated how the use of such a model improves the fit to similar sets of randomly scattered data with the same initial RMS deviations. The results show that the RMS deviations are typically decreased by $\sim 1^\circ$ or less, i.e., by a few percent, while the significant reductions found here are by 24% for the $r$ component, 26% for $\theta$, and 37% for $\varphi$.

We now consider the phase intercept values $\psi_{M1,n}$ at the minima, which determine the polarization characteristics of the northern period oscillations, and (together with the common phase gradient $\psi_{M1,n}$) the oscillation phases relative to the northern SKR modulations (discussed in section 5.3). To modulo $360^\circ$ these values are $-57^\circ$ for the $r$ component, $+86^\circ$ for the $\theta$ component, and $+23^\circ$ for the $\varphi$ component. The uncertainties in these values have been estimated by considering the change in parameters that would result in a $0.5^\circ$ increase in the RMS deviations, shown by the black dashed contours in Figure 10. The uncertainties indicated by this limit are $\pm 23^\circ$, $\pm 18^\circ$, and $\pm 14^\circ$ for the $r$, $\theta$, and $\varphi$ components, respectively. The difference in phase between the $r$ and $\varphi$ components of the northern period oscillations is thus determined to be $80^\circ \pm 27^\circ$, such that $\varphi$ is near to lagging quadrature with $r$, certainly within the estimated uncertainties. This result is essentially the same as the corresponding phase difference of $86^\circ \pm 6^\circ$ between the $\varphi$ and the $r$ components of the southern period oscillations determined in section 3.3 (Figure 4), and again indicates the presence of a quasi-uniform field in the equatorial plane now rotating near the northern SKR period. However, the phase difference between the $\theta$ and $r$ component oscillations is determined as $143^\circ \pm 29^\circ$, such that these oscillations are now closer to antiphase within the errors, rather than in phase as for the southern period oscillations (Figure 4). These results are thus in conformity with expectations based on the discussion in section 2.

Turning now to the values of the relative amplitudes $R_t$ at the minima, we find these are 0.28 for the $r$ component, 0.44 for the $\theta$ component, and 0.39 for the $\varphi$ component, with uncertainties estimated as above of about $\pm 0.08$ for the $r$ and $\varphi$ components, and about $\pm 0.11$ for the $\theta$ component. These results thus indicate that the amplitude of the northern period oscillations is $\sim 30\%-40\%$ of the southern period oscillations, thus implying a similar ratio in the strengths of the northern and southern rotating current systems. We recall that the interval examined here corresponds to northern winter and southern summer conditions. The corresponding amplitudes of the phase deviations, given by $\sin^{-1}(R_t)$ as in equation (2), are $16^\circ$, $26^\circ$, and $23^\circ$ for the $r$, $\theta$, and $\varphi$, respectively.

### 5.3. Phase Relations Between Northern Period Field Oscillations and SKR Modulations

We now discuss the implications of these results for the relationship between the northern period field oscillations and the corresponding northern period SKR modulations. For clarity of discussion we plot in Figure 11 the linear phases $\psi_{M1,n}(t)$ given by equation (15c) for the three field components versus time $t$ over the interval of our observations, in the same format as Figure 4a. The red, green, and blue lines thus correspond to the $r$, $\theta$, and $\varphi$ components, respectively, each plotted over the interval $t = 300$–1016 days spanned by the magnetic field data employed. At the center time of the interval, $t = 658$ days, the oscillation phases for the $r$, $\theta$, and $\varphi$ components given by equation (15c) are $+9^\circ$, $+152^\circ$, and $+89^\circ$, as marked by the dots on the lines in Figure 11. With a common phase
gradient $\psi_{M1n} = 0.1 \text{ deg d}^{-1}$ the variation of the phases over the interval is $\pm 33^\circ$ for each component (thus being comparable with, but greater than the uncertainties in these phases estimated in section 5.2). We note that Andrews et al. [2010b] found no evidence for long-term phase drifts in the high-latitude northern period oscillations during the interval between $t \approx 1000–2000$ days subsequent to that investigated here ($t = 300–1016$ days). However, their results are also not inconsistent with phase drifts of comparable magnitude to that inferred here occurring over shorter intervals of a few hundred days. From the northern period equivalent of equation (5c) (i.e., $\varphi_{\text{max}_i} = -\psi_{M1n}$) we thus find that at the times of northern SKR maxima, the maxima in the $r$, $\theta$, and $\varphi$ components occur at azimuths $-9^\circ$, $-152^\circ$, and $-89^\circ$, corresponding to local times of 11.4, 1.9, and 6.1 h. The variations in these values of $\pm 33^\circ$ over the interval then correspond to variations in LT of the positions of the maxima of $\pm 2.2$ h, to later LTs prior to the center time, and to earlier LTs after the center time.

We thus find that the northern period rotating quasi-uniform field in the equatorial plane points approximately sunward at the times of northern SKR maxima, with a maximum in the $r$ component near noon and a maximum in the $\varphi$ component near dawn. This is essentially opposite to the case for the southern period field oscillations, for which (as noted in section 3.3) the quasi-uniform field points approximately tailward and a little toward dawn on average at southern period SKR maxima, with maxima in the $r$ and $\varphi$ components typically in the postmidnight and postdusk sectors at LTs of 1.1 and 19.5 h ($\pm 3.2$ h), respectively. In both cases, however, maxima in SKR power correspond to maxima in the related $\theta$ component oscillation in the postmidnight sector, at $\sim 1.9 \pm 2.2$ h LT for the northern oscillation and $\sim 1.2 \pm 3.2$ h LT for the southern oscillation. We note that these results are entirely compatible with those derived previously by Andrews et al. [2010b] for magnetic field oscillations at northern high latitudes, as will be discussed further in section 6.

5.4. Residual Phase Deviations

We now consider the degree to which our model fits the data in Figure 4. This is illustrated in Figure 12 in three related formats. Figures 12a–12c show the original phase data $\psi_{Mi}$ versus time as in Figure 4a, but now separately
for the \( r \), \( \theta \), and \( \varphi \) components in Figures 12a–12c, respectively. The dashed lines similarly show the linear fits of the long-term phase drift for each component \( \psi_{Mi}^s(t) \) given by equation (6), while the solid oscillatory lines show the effect of the inclusion of the northern period oscillations, i.e., the combined phase model

\[
\psi_{Mi}^s(t) = \psi_{Mi}^b(t) + \Delta \psi_{Mi}^s(t),
\]

where \( \Delta \psi_{Mi}^s(t) \) is given by equation (16b) using the “best fit” parameters \( \psi_{Mi}^{b0} \) and \( \Delta \theta_{Mi}^s \), together with \( \psi_{Mi}^s = 0.1 \) deg day\(^{-1} \). Careful inspection shows that in each case the variations in the data about the dashed line follow the solid line reasonably well.

[48] While these plots provide an illuminating view of the phase variations in the combined model and their overall relation to the phase data, a clearer view of the fit between the data and the model is provided in Figures 12d–12f, where we plot the phase deviations \( \psi_{Mi}^s(t) \) versus the “beat” phase \( \psi_{Mi}^b(t) \) for each component, modulo 360°, given by equation (16d) using the “best fit” values of \( \psi_{Mi}^{b0} \) and \( \psi_{Mi}^s = 0.1 \) deg day\(^{-1} \). The solid lines then show the model phase deviations \( \psi_{Mi}^s(t) \) given by equation (16b) using the “best fit” values of \( \psi_{Mi}^{b0} \) and \( \psi_{Mi}^s \). It can be seen that the correspondence between the data and the model is reasonably good, though with considerable remaining scatter. In Figures 12g–12i we also show scatterplots of the observed phase deviations \( \Delta \psi_{Mi}^s(t) \) versus the model phase deviations \( \psi_{Mi}^s(t) \) for each field component. The inclined dashed line in each plot is a line of unit slope. It is seen that the values are positively correlated in each case, with cross-correlation coefficients for the \( r \), \( \theta \), and \( \varphi \) components of 0.67, 0.78, and 0.68, respectively, thus demonstrating good overall correspondence.

[40] We finally consider the nature of the residual phase deviations, shown in Figure 13. In Figure 13a we show the initial phase deviation data \( \Delta \psi_{Mi}^s(t) \) versus time as in Figure 4b, with RMS deviations of 19.6°, 28.1°, and 21.9° for the \( r \), \( \theta \), and \( \varphi \) components, respectively. Figure 13b then shows the best fit model values \( \Delta \psi_{Mi}^s(t) \) for each field component in the same format, showing a clear “splitting” effect in the phase values, though with smaller RMS deviations of 11.9°, 18.1°, and 16.3°. As outlined in section 2.2, the phase “splitting” effect results from the \( \varphi \) component being close to lagging quadrature with \( r \) for both the southern period oscillations and the model northern period oscillations, such that their superposition produces similar phase deviations in the two components, while the \( \theta \) component is in phase with \( r \) for the southern period oscillation and nearly antiphase with \( r \) for the northern period oscillation, producing simultaneous phase deviations in the opposite sense. Figure 13c then displays the residual phase deviations for each field component \( \Delta \psi_{Mi}^s = \psi_{Mi}^s - \psi_{Mi}^s(t) \), showing clearly reduced scatter compared with the initial phase deviations, particularly for the \( \varphi \) component, with the RMS values now being reduced to 14.8°, 20.8°, and 13.9° for the \( r \), \( \theta \), and \( \varphi \) components, respectively. It can be seen that the deviations in the\( r \) and \( \varphi \) components are still correlated, however, though with a less evident relation to the simultaneous deviations in \( \theta \). This is shown explicitly in Figure 14, where we show scatterplots of the residuals of one field component relative to the other in the same format as Figure 5. The \( r \) and \( \varphi \) component deviations are still strongly

Figure 11. Plot of the linear variations versus time \( t \) of the phases \( \psi_{Mi}^s(t) \) of the core region northern period oscillations relative to the phase of the northern SKR modulation for the three field components \( i \), given by equation (15c). The phase gradient is fixed at \( \psi_{Mi}^s = 0.1 \) deg d\(^{-1} \) for each component, while the intercepts are determined from the results shown in Figure 10. As in Figure 4, results for the \( r \), \( \theta \), and \( \varphi \) field components are shown by the red, green, and blue lines, respectively. The plot covers the same time interval as Figures 3 and 4 (\( t = 250–1022 \) days), while the lines are shown over the interval spanning the magnetic field data employed in the analysis, corresponding to \( t = 300–1016 \) days. The colored dots highlight the phases at the center time of this interval, \( t = 658 \) days.
Figure 12. Plots illustrating the fit of the phase deviation data to the model deviations based on superposition of southern and northern period oscillations. The phase data $\psi_{M_{i,t}}$ versus time $t$ in the same format as Figure 4a but now separately for the (a) $r$, (b) $\theta$, and (c) $\varphi$ components. The black dashed lines show the long-term linear fits $\psi'_{M_{i,t}}(t)$ for each component, also as in Figure 4, while the black solid oscillatory lines show the effect of the inclusion of the northern period oscillations, $\psi'_{M_{i,t}}(t) + \Delta\psi'_{M_{i,t}}(t)$, using the “best fit” parameters $\psi'_{M_{160,1}}$ and $R_{0}$, together with $\psi'_{M_{1,1}} = 0.1 \text{ deg d}^{-1}$, in equation (16b). (d–f) The phase deviations from the long-term linear trend lines $\Delta\psi'_{M_{i,t}}$ versus the “beat” phase $\Delta\varphi_{M_{i,t}}$ for each component, modulo $360^\circ$, given by equation (16d) using the “best fit” values of $\psi'_{M_{160,1}}$ and $\psi'_{M_{1,1}} = 0.1 \text{ deg d}^{-1}$. The solid lines in these plots then show the model phase deviations $\Delta\psi_{M_{i,t}}$ given by equation (16b) using the “best fit” values of $R_{0}$. (g–i) Scatterplots of the observed phase deviations $\Delta\varphi_{M_{i,t}}$ versus the model phase deviations $\Delta\psi_{M_{i,t}}$ for each field component. The cross-correlation coefficients (“XCC”) are given in each plot, while the inclined dashed lines are lines of unit slope.

positively correlated with a cross-correlation coefficient of 0.90, the “best fit” line having a gradient of 0.99. However, no significant correlation remains between $\theta$ and $\varphi$, where the cross-correlation coefficient is now $-0.05$.

[30] To take this discussion further we briefly consider the relationship between the model results found here and the analysis of section 4.2, whose results for the “splitting,” “common,” and “random” phase deviations are shown in Figure 6. In these terms it is evident that the model results shown in Figure 13 relate principally to the dominant “splitting” phase deviations, as expected. In particular we note that the ratios of the model phase deviation amplitudes between the three components are similar to those of the “splitting” effect derived previously. The amplitudes of the model phase deviations shown in Figures 12a–12c are $16^\circ$, $26^\circ$, and $23^\circ$ for $r$, $\theta$, and $\varphi$, respectively, such that the ratio of the amplitudes of the $r$ and $\theta$ components to the amplitude of the $\varphi$ component are $\sim 0.72$ and $\sim 1.14$, respectively. The corresponding values for the “splitting” effect are $\sim 0.87$ and $\sim 1.21$ (parameters $a_{r}$ and $a_{\theta} \varphi$ in equations (10b) and (10c) shown in Figure 6a). However, we also note that the magnitudes of the model phase deviations are insufficient to account for all of the “splitting” deviation deduced in section 4.2, which would have resulted in the RMS deviations being reduced in the residual data to minimum values of $\sim 10^\circ$ for the $r$ and $\varphi$ components and $\sim 15^\circ$ for the $\theta$ component, defined by the combined “common” and “random” phase deviations (Figure 6b). These values compare with residual RMS values of $14.8^\circ$, $20.8^\circ$, and $13.9^\circ$ for the $r$, $\theta$, and $\varphi$ components, respectively, as indicated above. These considerations suggest that the model accounts for roughly half of the “splitting” deviation deduced in section 4.2, presumably due to short-term variations in the relative amplitude and phase of the northern and southern period oscillations not included in the model. The remaining “splitting” deviation is then comparable in magnitude with the “common” deviation, while the “random” deviation is also of comparable magnitude for the $\theta$ component, but remains much smaller for $r$ and $\varphi$. The remaining “splitting” and “common” deviations then both produce positive correlations of compa-
rable magnitude between $r$ and $\varphi$, such that overall they remain strongly positively correlated as seen in Figure 14a, while producing opposite correlations of comparable magnitude between $\theta$ and $\varphi$, leading to the lack of overall correlation seen in Figure 14b. Further analysis of the residual effects, including the origin of the “common” phase deviations awaits additional study.

6. Summary

[51] We have studied the phase of the “magnetospheric period” magnetic field oscillations observed on 28 periapsis passes through the quasi-dipolar “core” region of Saturn’s magnetosphere during the initial near-equatorial season of Cassini orbits spanning October 2004 to October 2006. These phases have been related to those of the SKR power modulations from the northern and southern hemispheres with the following principal findings.

[52] 1. Although the phase of the near-equatorial field oscillations is well organized by that of the dominant southern summer SKR emissions as found previously by Andrews et al. [2008, 2010a, 2010b] and Provan et al. [2009], a pronounced pass-to-pass phase “jitter” is also present about the slow long-term drift, with RMS amplitudes $\sim 20^\circ$ for the $r$ and $\varphi$ components, and $\sim 30^\circ$ for the $\theta$ component. The deviations in the $r$ and $\varphi$ component phases are found to be strongly positively correlated (cross-correlation coefficient $\sim +0.9$), while the deviations in the $\theta$ component phases are somewhat less strongly negatively correlated with both $r$ and $\varphi$ (cross-correlation coefficients $\sim -0.6$). The main effect

Figure 13. Time series plots of phase deviation data in a format similar to Figure 4. (a) Initial phase deviation data $\Delta \psi_{\theta \varphi}$ as in Figure 4b, (b) best fit model values $\Delta \psi_{\theta \varphi}^*$ from equation (16), and (c) residual phase deviations $\Delta \psi_{\theta \varphi}$. The RMS values of the data in each plot are indicated in red, green, and blue for the $r$, $/C_{18}$, and $\varphi$ field components, respectively.
observed is thus a “splitting” of the phases of the three field components about the long-term trends, which an overall analysis indicates has RMS amplitudes of ~17° for the r component, ~24° for θ, and ~20° for φ. The same analysis also suggests the presence of phase deviations common between the three components, uncorrelated with the splitting, which have an RMS amplitude of ~9°, while the RMS amplitude of the random (e.g., measurement error) deviations are ~3° for r and φ, and ~12° for θ. The larger random deviation for θ could be due to simultaneous traversals of the ring current perturbation fields.

Figure 14. Scatterplots of the residual phase deviations Δψ_M, ∈ in the same format as Figure 5.

We finally note that while the “best fit” simple model produces significant reductions in the RMS deviations between data and model of ~30% compared with the initial data set, our overall analysis indicates that it does not account for all of the “splitting” phase deviation present. This is presumably due to short-term variations in the rel-
ative phase and/or amplitude of the northern and southern period oscillations that are not included in the model, which thus remain to be elucidated. Incorporation of observed short-term fluctuations in corresponding SKR phases [e.g., Lamy, 2011] represents a potentially fruitful prospect for future progress. In addition, the nature of the secondary uncorrelated “common” phase variations suggested by the overall analysis requires further investigation. If this effect is validated, it remains to be determined whether it is again associated with some significantly undersampled short-period variation, due, e.g., to external solar wind forcing near the solar rotation period. Alternatively it may relate to some intermediate time scale longer than the typical orbital period but shorter than the overall data interval of 700 days investigated here.

**Notation**

\( t \) time in days since start of 1 January 2004.

\( \tau_{SKR} (t) \) modulation periods of the northern (\( n \)) and southern (\( s \)) SKR emissions.

\( \Phi_{SKR} (t) \) modulation phases of the northern (\( n \)) and southern (\( s \)) SKR emissions (equation (4)).

\( \varphi \) azimuth about Saturn’s rotation axis measured from noon (\( \varphi = 0 \)) positive toward dusk (\( \varphi = 90^\circ \)).

\( \Psi_{Mi} (\varphi, t) \) model phase of magnetic field oscillations based on the southern SKR phase (equation (5b)), employed to determine the oscillation phases \( \psi_{Mi} \) on each periapsis pass; \( i \) indicates the field component which can be \( r, \theta, \) or \( \varphi \).

\( \psi_{Mi} \) oscillation phase (modulo 360°) of field component \( i \) for a given periapsis pass based on the southern SKR guide phase (equation (5b)), determined by cross correlation with magnetic field data.

\( \psi_{Mi} (t) \) long-term trend model of oscillation phase \( \psi_{Mi} \) versus time \( t \) for field component \( i \), taken to be a linear function of \( t \) (equation (6)).

\( \psi_{Mi} (t) \) gradient ("1") and intercept ("0") of \( \psi_{Mi} (t) \) for field component \( i \) (equation (6)), where the gradient is taken to be common for all \( i \).

\( \Phi_{Mi} (t) \) long-term temporal phase of the oscillations of field component \( i \) obtained by combining \( \Phi_{SKR} (t) \) with the long-term linear trend model \( \psi_{Mi} (t) \) (equation (7)).

\( \tau_{Mi} (t) \) period of the oscillations for field component \( i \) corresponding to \( \Phi_{Mi} (t) \) (equation (8)).

\( \Delta \psi_{Mi} \) deviation of measured phase \( \psi_{Mi} \) from the long-term trend line \( \psi_{Mi} (t) \) (equation (9)).

\( \Delta \psi_{iR} \) model “random” ("\( R \") phase deviation for field component \( i \) in section 4.2 (equation (10a)); \( \langle \Delta \psi_{iR} \rangle \) is the mean squared value.

\( \Delta \psi_{C} \) model “common” ("\( C \") phase deviation in section 4.2 (equation (10a)); \( \langle \Delta \psi_{C} \rangle \) is the mean squared value.

\( \Delta \psi_{iS} \) model “splitting” ("\( S \") phase deviation for field component \( i \) in section 4.2 (equation (10a)); \( \langle \Delta \psi_{iS} \rangle \) is the mean squared value.

\( \alpha_{r, \alpha_{0}} \) model “splitting” ratios for the \( r \) and \( \theta \) field components relative to \( \varphi \) in section 4.2 (equations (10b) and (10c)).

\( \delta_{jk} \) mean value of the products of the phase deviations \( \Delta \psi_{Mi} \) for field components \( j \) and \( k \) (with \( j \neq k \)) (equation (11a)).

\( D_{i} \) mean squared value of the phase deviations \( \Delta \psi_{Mi} \) for field component \( i \) (equation (11b)).

\( K \) assumed difference between the mean squared values of the “random” phase deviations for the \( \varphi \) and \( r \) magnetic field components (equation (11c)).

\( B'_{00} \) model amplitude of the “southern period” field oscillations for field component \( i \) in section 5.1 (equation (15a)).

\( \Psi_{Mi} (\varphi, t) \) model phase of the “southern period” field oscillations for field component \( i \) in section 5.1 (equation (14b)).

\( B'_{00} \) model amplitude of the “northern period” field oscillations for field component \( i \) in section 5.1 (equation (15a)).

\( \Psi_{Mi} (\varphi, t) \) model phase of the “northern period” field oscillations for field component \( i \) in section 5.1 given by \( \Phi_{SKR} (t) - \varphi - \psi_{Mi} (t) - \varphi \) (equation (15b)).

\( \Psi_{Mi} (\varphi, t) \) assumed long-term linear phase variation of the “northern period” field oscillations relative to the northern SKR modulation phase.

\( \Psi_{Mi} (\varphi, t) \) gradient ("1") and intercept ("0") of \( \psi_{Mi} (t) \) for field component \( i \) (equation (15c)), equivalent to \( \psi_{Mi} (\varphi, t) \) for the “northern period” oscillation.

\( B'_{00} (t) \) time-dependent amplitude of the combined northern and southern period field oscillations (equation (15d)).

\( \Delta \psi_{Mi} (t) \) model of the phase deviation \( \Delta \psi_{Mi} \) due to combined northern and southern period field oscillations (equations (15d) and (16b)).

\( R_{i} \) ratio of the northern and southern period oscillation amplitudes (\( B'_{00} / B'_{00} \)) (equation (16c)).

\( \Delta \Phi_{Mi} (t) \) difference between the temporal phases of the northern and southern period field oscillations for field component \( i \), \( \Phi_{Mi} (t) - \Phi_{Mi} (t) \), the magnetic “beat” phase (equation (16d)).

\( \tau_{B} (M) \) beat period of the magnetic oscillations corresponding to phase difference \( \Delta \Phi_{Mi} (t) \) (equation (17a)).

\( \tau_{B} (SKR) \) beat period of SKR modulations (equation (17b)).
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