A numerical simulation of a ‘Super-Earth’ core delivery from $\sim 100$ to $\sim 8$ au

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ABSTRACT

We use smoothed particle hydrodynamics simulations with an approximate radiative cooling prescription to model the evolution of a massive ($\sim 100$ au) very young protoplanetary disc. We also model dust growth and gas-grain dynamics with a second fluid approach. It is found that the disc fragments into a large number of $\sim 10M_J$ clumps that cool and contract slowly. Some of the clumps evolve on to eccentric orbits, delivering them into the inner tens of au, where they are disrupted by tidal forces from the star. Dust grows and sediments inside the clumps, displaying a very strong segregation, with the largest particles forming dense cores in the centres. The density of the dust cores in some cases exceeds that of the gas and is limited only by the numerical constraints, indicating that these cores should collapse into rocky planetary cores. One particular giant planet embryo migrates inwards close enough to be disrupted at about 10 au, leaving a self-bound solid core of about 7.5 $M_\oplus$ mass on a low-eccentricity orbit at a radius of $\sim 8$ au. These simulations support the recent suggestions that terrestrial and giant planets may be the remnants of tidally disrupted giant planet embryos.

Key words: accretion, accretion discs – hydrodynamics – methods: numerical – planets and satellites: formation – stars: formation – stars: protostars.

1 INTRODUCTION

1.1 Two competing theories for planet formation

The currently favoured ‘Core Accretion’ paradigm for planet formation (Safronov 1972; Wetherill 1990; Pollack et al. 1996) stipulates that planets grow in a $R \lesssim 10$ au scale disc by accumulation of solids into grains and then into km-sized bodies called planetesimals which then collide and merge into ever larger rocky objects. There is a well-known difficulty with the paradigm, for example, the planetesimal assembly step (e.g. Wetherill 1990). Due to gas–grain friction forces, grains are found to migrate radially inwards with velocities strongly dependent on the grain size (Weidenschilling 1977). Small (mm-sized or less) grains are ‘glued’ to the gas and hence corotate with it; larger bodies (e.g. km-sized rocks) barely note drag forces from the gas and thus orbit the star at the local Keplerian speed, which is slightly higher than the gas orbital velocity. The objects in the middle, for example, about a metre in size, move with respect to gas at velocities approaching a few tens of $\text{m s}^{-1}$. Such objects should migrate inwards rapidly and be lost to the star. Going from cm-sizes to km-sizes is also complicated by the fact that the collisions of $\sim$-m-sized boulders occur at velocities higher than a few $\text{m s}^{-1}$. Experiments show that the fragmentation of solids, rather than their growth, is the most likely outcome for such high-speed collisions (Blum & Wurm 2008). Alternatively, the gravitational instability (GI) of a dense dust layer within a gas-dust disc could form km-sized bodies directly (Safronov 1972; Goldreich & Ward 1973), but turbulence and instabilities are believed to prevent grains from sedimenting to the mid-plane of the disc (Weidenschilling 1980). More recent work suggests that solids may be concentrated into larger structures by the instabilities and turbulence in the disc (e.g. Youdin & Goodman 2005; Johansen et al. 2007; Cuzzi, Hogan & Shariff 2008), allowing them to rapidly grow to km-sized bodies.

Until very recently, the only serious alternative1 to the core accretion scenario was the disc GI model (Kuiper 1951; Cameron 1978) for giant planet formation, most convincingly formulated, based on hydrodynamical simulations, by Boss (1997). In this model, the giant planets form as self-gravitating condensations in a massive protoplanetary disc. Boss (1998) further argued that dust grains inside the young giant planet precursors, which we shall term as the giant (planet) embryo, may sediment to the centre and form moderate-mass solid cores, as observed. Boss, Wetherill & Haghighipour (2002), in addition, pointed out that giant planet embryos may lose their gas-rich envelopes by the photoevaporation from nearby OB stars.

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1 One of us was recently made aware of much earlier work on the subject (starting from McCrea 1960), which is discussed more fully below.
However, the difficulties of this scenario, initially applied to the Solar system giants, were pointed out by a number of authors. Cassen et al. (1981) argued that the gaseous disc must be unrealistically heavy, for example, up to $1M_\odot$, to collapse gravitationally. Cameron, Decampli & Bodenheimer (1982) showed that the thermal bath of the solar radiation at the location of Jupiter is intense enough to result in evaporation and a complete dispersal of a young 1 Jovian (Jupiter) mass self-gravitating gas cloud. Wetherill (1990) stressed the fact that giant planets in the Solar system are significantly more metal rich than the Sun, which could not be the case, in his view, if planets formed from a disc with the same metallicity as the star. This constraint is avoided, however, if planets are allowed to lose volatile elements preferentially, as in the model of Boss et al. (2002). However, perhaps the strongest objection from the Solar system is that the GI theory does not have an answer to forming terrestrial planets, and hence one must appeal to the core accretion model for these.

In addition, improved theoretical understanding of disc fragmentation and hydrodynamical simulations (Gammie 2001; Rafikov 2005; Rice, Lodato & Armitage 2005) showed that protoplanetary discs cannot fragment into clumps on scales less that $\lesssim 50$–100 au from the star. As there are a large number of giant planets observed at au and even sub-au distances from their parent stars (e.g. Baraffe, Chabrier & Barman 2010), these would seem to rule out the GI formation path. Furthermore, Helled & Schubert (2008) and Helled, Podolak & Kovetz (2008) found that dust sedimentation in giant embryos is suppressed by vigorous convective motions and the embryos soon become too hot for dust grains to survive. Therefore, the model of Boss (1997, 1998) did not seem to find support from detailed independent simulations.

Therefore, the core accretion scenario is by far the most widely accepted model for planet formation at the moment, although the existence of exosolar giant planets at $R \sim 100$ au most likely implies that these formed in situ, and hence the GI model cannot be completely excluded (Boley 2009).

1.2 The tidal-downsizing model

Theoretical work on planet migration (e.g. Goldreich & Tremaine 1980; Tanaka, Takeuchi & Ward 2002; Bate, Bonnell & Bromm 2003) and observations of ‘hot jupiters’ too close to the star where they could not have possibly formed (Lin, Bodenheimer & Richardson 1996) puts a serious dent in the critiques of the GI model detailed above and allows for a whole new look at the planet formation process: if we accept that hot Jupiters migrate in their discs, then perhaps planets at $\sim 5$ au also migrated from their birth place at $R \gtrsim 50$–100 au.

Motivated by this, a modified scheme for planet formation has been recently proposed by Boley et al. (2010) and Nayakshin (2010a). In particular, it is suggested that youngest protoplanetary discs are very massive (comparable in mass to their parent star, see Stamatellos & Whitworth 2008; Machida, Inutsuka & Matsumoto 2011) and extended due to a large angular momentum reservoir of typical molecular clouds (e.g. Goodman et al. 1993). Such discs fragment into gas clumps with the mass of a few to a few tens of $M_\odot$ at large distances from the parent star ($\sim 100$ au). It is then proposed that the clumps, also referred to as giant planet embryos, migrate inwards on time-scales of a few thousand years to 10 times that. This migration may be related to the ‘burst-mode accretion’ discussed by Vorobyov & Basu (2005, 2006, 2010).

In a constant clump (giant planet embryo) mass model, Nayakshin (2010b) have shown that dust is very likely to grow and sediment to the centre of the clump (as earlier suggested by Boss 1998). The vigorous convection that resisted dust sedimentation in Helled & Schubert (2008) and Helled et al. (2008) does not occur in these models exactly because the embryos start from afar. At distances of $\sim 100$ au, they are initially fluffy and cool, contracting slowly due to radiative cooling. Nayakshin (2011) continued the calculation into the phase when a massive solid core forms in the centre and found that the energetic feedback from the growing core on to the surrounding gas may significantly impede further growth of the core.

The final step in this ‘tidal-downsizing’ hypothesis is the disruption of the gaseous envelope by tidal or radiative mass-loss when the planet is within a few au from the star (Nayakshin 2010a). If all the gas envelope is removed, then the remnant is a rocky planet; if a part of the massive gas envelope remains, then the outcome is a giant planet (see also Boley et al. 2002).

It is interesting to note that except for the migration of the embryos, the important parts of the tidal-downsizing hypothesis were discussed by a number of authors as early as 50 yr ago (!). For example, McCrea (1960) argued that planet formation begins inside ‘flocules’, and McCrea & Williams (1965) and Williams & Crampin (1971) showed that grains could have grown inside and sedimented to the centre, whereas the outer envelope of volatile elements could have been removed by tidal forces of the Sun. However, Donnison & Williams (1975) realized that the tidal dispersal process is extremely rapid, that is, dynamical, at the present locations of the terrestrial planets, whereas grain sedimentation requires at least $10^3$ yr. Therefore, Donnison & Williams (1975) concluded that ‘terrestrial protoplanets envisaged in this theory are unstable and cannot have existed’. The tidal-downsizing hypothesis is thus late by about 30 yr, given that planet migration was ‘invented’ by Goldreich & Tremaine (1980).

Neither Boley et al., who numerically simulated gas-only discs, nor Nayakshin, who concentrated on isolated embryos, has actually demonstrated that the hypothesis works in a realistic gas-dust simulation of the process. Our goal here is to carry out a two-fluid hydrodynamical simulation of a massive protoplanetary disc to test whether massive embryos do form with appropriate conditions for dust sedimentation and whether they indeed migrate inwards and survive long enough to make it into the inner non-self-gravitating disc. We performed a dozen of such simulations varying the radiative cooling prescription (see below) and initial conditions, indeed finding clump creation and inward migration to be common processes.

While a full study of the parameter space of the simulations remains to be performed, here we report one particular simulation that, in our view, provides a proof of the concept for the tidal-downsizing hypothesis. In this simulation, the inward migrating embryo is dense enough to be disrupted only at around 10 au (and not earlier) and to deposit a $\sim 7.5 M_\oplus$ solid core into a low-eccentricity orbit at about 8 au.

2 THE NUMERICAL METHOD

We employ the 3D smoothed particle hydrodynamics (SPH)/N-body code GADGET-3, an updated version of the code is presented in Springel (2005). We use adaptive SPH smoothing lengths. The dust component is modelled by a two-fluid approach somewhat similar to our radiation-transfer scheme reported in Nayakshin, Cha & Hobbs (2009), although the scheme is much simpler in the present case.

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2.1 Dust-particle implementation

The dust particles in our approach are collisionless particles that experience two forces, one is the gravity due to the gas, the star and themselves, and the other is the aerodynamic drag force between the dust and the gas particles. The standard GADGET-3 machinery is used to calculate the gravitational acceleration, \( \mathbf{a}_{\text{grav}} \), for all the particles.

To calculate the gas density at the location of the dust particle, we use the standard SPH approach, first finding the distance \( h_k \) such that the sphere of radius \( h_k \), centred on the dust-particle position, \( r_d \), contains \( N_{\text{gbh}} = 40 \) neighbours. The distance \( h_k \) is the ‘smoothing length’ at \( r_d \). The gas density, \( \rho_g \), is then calculated by

\[
\rho_g = \sum_j m_j W(|r_j - r_d|, h_d) = \sum_j \rho_j,
\]

where the summation goes over all the SPH neighbours of the dust particles so defined, \( m_j \) and \( r_j \) are the mass and position of particle \( j \), respectively, and \( W \) is the SPH smoothing kernel (Springel 2005).

The quantity \( \rho_j \) is the contribution of the \( j \)th gas neighbour to the gas density around the dust particle, which we shall need below.

Other gas dynamical or thermodynamical properties at the dust-particle position are calculated in a very similar way. For example, the gas velocity is given as

\[
v_g = \rho_g^{-1} \sum_j v_j m_j W(|r_j - r_d|, h_d).
\]

Equations (7)–(9) of Weidenschilling (1977) are used to calculate the aerodynamic drag force (\( \equiv \mathbf{F}_d \)) on the dust particle. We then also define the dust-particle ‘stopping time’, \( t_s \), due to the drag forces as

\[
t_s = \frac{m_d |v_d - v_g|}{|\mathbf{F}_d|},
\]

(equation 10 of Weidenschilling 1977). Having calculated the stopping time, we update the dust-particle velocity in an implicit scheme, according to

\[
v_d^{\text{new}} = \left( \frac{v_d}{\Delta t_s} + \frac{v_g}{t_s} + \mathbf{a}_{\text{grav}} \right) \left( \frac{1}{\Delta t_s} + \frac{1}{t_s} \right)^{-1},
\]

where \( \Delta t_s \) is the block-sized (in powers of 2) time-step for the dust particle. The time-step is determined with the usual accuracy criteria for gravity integration (Springel 2005) and also by the condition that the dust particles do not ‘skip’ ‘interactions with the SPH particles (Nayakshin et al. 2009). Note that this scheme recovers the well-known results for short dust-particle stopping time: if \( t_s \ll \Delta t_s \), then \( v_d^{\text{new}} \to v_g + \mathbf{a}_{\text{grav}} t_s \).

In order to conserve the momentum in the gas–dust interactions, the momentum loss of a dust particle due to the aerodynamic force, \( \Delta \mathbf{P}_d \), is passed back to the respective gas particle neighbours with the negative sign. This involves ‘spreading’ of \( -\Delta \mathbf{P}_d \) over all of the neighbours of the dust particle. Following the method of Nayakshin et al. (2009), this spreading is done proportional to the respective contribution of particle \( j \) to the gas density, \( \rho_j \), at the location of the dust particle. The SPH particle \( j \) is then due to receive the momentum ‘kick’, \( \Delta \mathbf{P}_{d,j} \), given by

\[
\Delta \mathbf{P}_{d,j} = \frac{\rho_j}{\rho_g} \Delta \mathbf{P}_d,
\]

where \( \rho_j \) is given by equation (1). For each SPH particle, all the interactions with its dust neighbours are counted, defining in this way the total change in momentum due to the dust–gas interactions, \( \Delta \mathbf{P}_d \). Note that the SPH particle will in general have its own time-step \( \Delta t_f \). Therefore, the \( \Delta \mathbf{P}_f \) quantity is additive and is updated also during the time the SPH particle itself is ‘inactive’ (its time-step \( \Delta t > \Delta t_f \) of a dust neighbour). For added accuracy, the momentum kick is not passed to the SPH particle directly, but instead is used to define the acceleration acting on the gas particle due to gas–dust drag force, \( \mathbf{a}_d = \Delta \mathbf{P}_d / \Delta t_f \). This also means that the gas-drag acceleration is used as an additional time-stepping criterion for the SPH particles, ensuring accurate time-integration when the gas-drag forces are large.

We emphasize that this scheme is an almost exact copy of the Nayakshin et al. (2009) approach, who ran a number of simulations designed to test the accuracy of the momentum transfer to the gas.

2.2 dust-grain growth

We also implement a simple hit-and-stick dust growth model in the code. Physically, dust particles of different sizes sediment to the centres of the gas clumps or to the disc mid-plane at different velocities, with larger particles settling faster (e.g. Boss 1998; Dullemond & Dominik 2005). Larger grains thus sweep smaller ones, which then stick to the larger ones. As pointed out by Dullemond & Dominik (2005), if such a simple logic were correct, then one would expect that protoplanetary discs would only contain cm-sized particles after just a short time. Instead, small dust particles are clearly observed in the protoplanetary discs, requiring both grain growth and fragmentation.

Such a detailed grain growth model is well beyond the scope of what we can realistically do in this paper. On a numerical level, that would require tracking the smallest microscopic dust particles and then allowing them to accrete on to the large ones. This would necessitate introduction of dust ‘sink particles’ and also an uncomfortably large number of ‘small’ dust particles. One would also have to introduce, likely arbitrary, prescriptions for dust fragmentation.

Therefore, we simply assume that, while we explicitly follow the population of larger dust grains, there is always an associated population of microscopically small grains as well. The density of those is taken to be roughly the same as that of the large grains in the same location. Following Nayakshin (2010b), dust grains grow at the rate given by

\[
\frac{da}{dt} = \left( \frac{4 \rho_g}{\rho_a} (\Delta v + v_{th}) \left( \frac{v_{\text{max}}}{v_{\text{max}} + \Delta v} \right)^2 \right),
\]

where \( a \) is the grain size, \( \rho_a = 2 \text{ g} \text{ cm}^{-3} \) is the material density of grains, \( \rho_a \) is the density of the dust around the grains, \( \Delta v \) is the absolute magnitude of the velocity difference between the gas and dust, \( \Delta v = |v_d - v_g| \), \( v_{th} = 10 \text{ cm s}^{-1} \) is the Brownian velocity of microscopic grains (cf. Dullemond & Dominik 2005) and \( v_{\text{max}} = 3 \text{ m s}^{-1} \) is the critical velocity above which colliding grains are assumed not to stick (Blum & Wurm 2008). The local dust density, \( \rho_a \), is calculated in the same way as the gas density around the dust-particle location (equation 1), but only dust particles are considered as a neighbour.

2.3 Gravitational collapse of dust-particle condensations

Due to the grain sedimentation to the centres of the clumps, the grain population there can become self-gravitating (Boss 1998; Boley et al. 2010; Nayakshin 2010b, 2011). If point-mass gravity law is used to treat their interactions, then such compact grain distributions may collapse to a point. This numerical problem (arising because of the finite and fixed mass of the N-body dust particles) is solved by the introduction of the minimum softening length for the dust particles, as is traditional in most N-body codes (Springel 2005). For
the runs presented in this paper, we use the minimum gravitational softening parameter of $h_{\text{min}} = 0.05$ au.

In contrast, we do not introduce any softening for the dust–gas aerodynamic drag forces. Therefore, we underestimate the physical tendency of the self-gravitating grain condensations to gravitational collapse. This means that our results concerning the formation of bound grain condensations are conservative; a better numerical treatment should lead to even tighter bound solid cores.

The gravitational force from the star is also softened on the same 0.05 au scale. However, as we use a sink particle boundary condition for the star of 1 au, the gravitational softening of the stellar gravitational force is never important in practice (close particles are accreted, that is, added to the star, before the softening becomes significant). This also implies that the tidal field of the star is not softened in practice. Hence, just as the argument above, if a self-gravitating dusty core (a collection of a large number of gravitationally self-bound dust particles) survives near the star for a long time despite the tidal forces, we can be confident that this result will stand even at smaller values of $h_{\text{min}}$.

2.4 Radiative cooling prescription

The radiative cooling scheme we employ follows simple physical considerations. As is very well known from simulations of collapsing molecular clouds (Larson 1969; Masunaga & Inutsuka 2000), low-density gas cools rapidly and therefore it is essentially isothermal, with temperature fixed by an external heating rate. The highest density material, on the other hand, is expected to be found inside the giant planet embryos that are optically thick. For definiteness, we use the Nayakshin (2010b) model for embryos with the opacity $\kappa_0$. With temperature fixed by an external heating (corresponding to the molecular gas held at 10 K for low-density gas, the cooling time is suitably short, so that we can neglect it for the embryo models we consider). The latter density value is the mean embryo density at $t = 0.05$ au. The highest density material is thus at time $t = 0.05$ au.

2.5 Initial conditions and early evolution

We start with a gas disc of mass $M_d = 0.4 M_\odot$ in circular rotation around the star with mass $M_\ast = 0.6 M_\odot$. The disc inner and outer radii are 20 and 160 au, respectively, and the disc surface density profile follows the $\Sigma(R) \propto 1/R^2$ law. At a given radius $R$, the disc initially has a constant density and the vertical scaleheight is $H = R(M_d/M_\ast)$. The initial circular rotation velocity curve is corrected for the disc mass interior to each point to avoid initial strong radial oscillations of the disc, but not for the gas pressure gradient force. Once the simulation starts, we find that gas temperature evolves quickly due to cooling and the compressional heat, and hence any initially imposed pressure profile, erodes rapidly. Boley et al. (2010) used a similar setup and encountered a similar ‘difficulty’ with their initial conditions. We note that a better approach to setting the initial condition is not a better relaxed gaseous disc but rather a more fundamental approach where the disc forms self-consistently, such as in the 2D simulations by Vorobyov & Basu (2005, 2006, 2010). We plan to present results of simulations initialized in a similar fashion in future papers. None the less, our present study is valuable in itself as it shows that the ‘burst mode’ of protostar accretion may start in isolated protostar discs, provided that the disc mass is sufficiently high.

The initial number of SPH particles is $N_{\text{sp}} = 10^6$ and half that for dust particles. The total number of particles is therefore $1.5 \times 10^6$. The initial grain size is $a = 0.1$ cm for all the grain particles, and the total mass of the grains is 0.01 times the disc gas mass. Note that grains of sizes much smaller than this grow rapidly by sticking with small grains due to the Brownian motion of the latter (Dullemond & Dominik 2005); therefore, we neglect that phase of the grain growth.

3 RESULTS: GAS DYNAMICS

Gas is the dominant component in the disc and its evolution affects the dust component greatly. Therefore, we begin by discussing the behaviour of the gaseous component only, with the dust analysed later.

3.1 Fragmentation of the disc into clumps

Fig. 1 shows several snapshots of the gas surface density from the simulation. The earliest one of the snapshots shown (upper left-hand panel) corresponds to time $t = 2320$ yr; all the following snapshots are separated by a time-interval of 120 yr and follow from the left-hand to right-hand side (top to bottom). The rightmost panel at the bottom is thus at time $t = 3280$ yr. The spatial extent of the box in the panel is from $-230$ to $230$ au in both the horizontal and vertical directions. The colour in the panels shows the gas column density, with the black colour corresponding to $0.05$ g cm$^{-2}$ and the yellow showing the maximum set at $\Sigma = 2 \times 10^8$ g cm$^{-2}$. The panels are centred on the star, whose position is not fixed in space and of course varies with time due to disc and disc clump gravitational forces.

Analysing gas dynamics now, we note that, released from the initial condition, the disc contracts vertically and achieves high mid-plane densities. As a result, the disc becomes gravitationally unstable and forms spiral features in the inner disc regions, first, since dynamical time is shortest there. As the disc is very massive, there are only two spiral ‘arms’. Also, there is apparently a radial mode in the instability as well, as one notes a broken nearly-ring-like structure separating the outer and the inner disc regions. This is...
possibly due to the initial disc not being in an exact radial pressure–force equilibrium (see Section 2.5) as it is rapidly evolving.

In the middle panel (top row), we already observe the formation of dense gaseous clumps inside the dense filaments (or arms) in the disc. Most clumps appear to interact strongly with their neighbours which is expected based on analytical arguments of Levin (2007) for gas clumps in an active galactic nucleus disc (his arguments are local and hence scale-free).

Note that the closest distance to the parent star where the clumps are born is about 70 au, which is commensurate with previous analytical and numerical work by a number of authors, indirectly confirming that the radiative cooling prescription used in our simulations is reasonable. As time progresses, clumps are born also at larger radii, out to about 150 au or so.

Clumps interact strongly with each other and also the surrounding gas. Both gas and the clump populations spread radially in either direction. This is apparent from a comparison between the top and the bottom panels of Fig. 1.

3.2 The two phases of the disc

Fig. 1 shows that the originally strongly unstable disc divides into two distinct phases or populations: the clumps and the ‘ambient’ disc. The physical distinction here is that the clumps are
self-gravitating, whereas the disc material is marginally, if at all, self-gravitating. The exact division of the gas into these two phases is definition-dependent and thus somewhat arbitrary, but we suggest two ways to do this that we feel are reasonably meaningful and robust. Both of these are based on the local gas density, \( \rho \).

The simplest one is to say that the gas above some critical density belongs to the clump population and that the gas with a lower density belongs to the disc. A suitable choice for the critical density is \( \rho_{\text{crit}} \) in the radiative cooling prescription (equation 8). To be more quantitative, we define a cumulative distribution function of SPH particles over different gas densities, \( f_\rho(\rho) \), defined as the mass fraction of the gas with density smaller than \( \rho \). By definition, \( f_\rho(0) = 0 \) and \( f_\rho(\infty) = 1 \).

This definition makes most sense when studying the densest end of the gas distribution function, as we find no gas significantly denser than \( \rho_{\text{crit}} \) outside the gas clumps, except in the inner disc, at \( R \lesssim 10 \) au or so. Another useful, and potentially more discriminating, way to ascribe the gas to one of the two populations is to compare the gas density to the local tidal density, \( \rho_{\text{tid}} \),

\[
\rho_{\text{tid}} = \frac{M_*}{2\pi R^3}.
\]

(9)

A marginally self-gravitating disc maintains vertically averaged gas density \( \rho \approx \rho_{\text{tid}} \) (cf. Goodman 2003); therefore, the gas with \( \rho \gtrsim \rho_{\text{tid}} \) may be said to belong to the clumps or perhaps to spiral arms or filaments, whereas gas with \( \rho \lesssim \rho_{\text{tid}} \) belongs to the non-self-gravitating component of the disc. We note that it is entirely possible for some SPH particles to cross from one population into the other and back, as the behaviour of the disc is highly dynamic and the high-density features may appear and later dissipate.

For a more detailed analysis, a function \( F_\rho(x) \) is introduced in analogy to \( f_\rho(\rho) \) as a fraction of the gas whose density is smaller than a given fraction \( x \) of the tidal density, for example, satisfying \( \rho/\rho_{\text{tid}} < x \).

Fig. 2 shows the two cumulative gas density distribution functions for several different times, from \( t = 1800 \) yr (solid curve) to \( t = 5400 \) yr (dot–dashed curve). The left-hand panel of the figure shows that as time goes on, the maximum gas density increases, which physically corresponds to gas clumps contracting with time. The right-hand panel of the figure demonstrates this even more vividly, showing almost a step-function change for \( F_\rho \) at \( \rho/\rho_{\text{tid}} \approx 1 \). This tells us that gas divides very clearly into two distinct phases – the clumps and a non-self-gravitating component. Confirming that the ‘ambient disc’ is indeed non-self-gravitating at later times, there are no significant spiral features (cf. Fig. 8 shown later).

Fig. 3 shows the evolution of the mass in the two disc phases with time. In particular, the red line shows the mass of the gas that has density less than 10 times the local tidal density, \( \rho < 10 \rho_{\text{tid}} \); the black curve shows gas that is at least moderately gravitationally contracted, with \( \rho \gtrsim \rho_{\text{tid}} \); and the green curve shows a strongly bound component with \( \rho > 10 \rho_{\text{tid}} \). By definition, the green and red curves together amount to the total gas mass, that is, 0.4 M\( \odot \) minus the mass of the gas accreted by the star by that time.

Analysing Fig. 3, we recall that early on there is an initial condition relaxation phase. As initially there are no dense and bound gas clumps, there is also no gas exceeding 10 times the local tidal density, and the red curve thus accounts for all the gas mass
initially. Later, spiral density features form in the disc. The black curve increases to encompass as much as 0.3 M$_\odot$ of the material at time slightly after 2000 yr. Most of that gas is weakly bound, as the mass in the green curve component is small at that time.

What is interesting is an ensuing rapid fall in that mass: by about 3000 yr the marginally self-gravitating gas accounts for only $\sim 0.17$ M$_\odot$ of the material. At the same time, the strongly bound component, clearly corresponding to the clumps, increases sharply to about 0.13 M$_\odot$. Taken together, it means that most of the gas is in either one of the phases – either the tightly bound clumps containing about a third of the total mass or the non-self-gravitating part. Indeed, the moderately bound material in the middle, with $\rho_{\rm ad} < \rho < 10 \rho_{\rm ad}$, corresponds to the difference between the black and green curves, and that amounts to only $\sim 0.05$ M$_\odot$ of gas, for example, slightly more than about 10 per cent of the disc mass after $t \sim 3500$ yr.

There are two periods when the green curve decreases with time in Fig. 3, whereas the red curve correspondingly increases. These time-periods correspond to times when the clumps are being tidally disrupted near the star, as we shall see later.

### 3.3 Thermal structure of the disc

Fig. 4 shows the gas column density together with velocity field (left-hand panel) and the gas temperature (averaged over the respective column depth) at $t = 4880$ yr. One notes that maximum gas temperatures are around 1000 K and are reached either in the inner disc (where our cooling prescription, designed to capture the cooling of dense clumps, may underestimate cooling) or inside the gas clumps. There are also shocked gas regions that are heated to a few hundred K typically. The gas density in those regions is relatively low, as a comparison between the left-hand and right-hand panels of the figure shows. These shocked regions are most likely to be found in the space between the clumps or next to isolated clumps, where the ‘ambient’ disc flow runs directly into the clumps (cf. the top left-hand panel of Fig. 5 for a higher resolution velocity field around the embryo).

The fact that the highest density regions are also the hottest in our simulations is crucial for setting the environmental conditions for grain growth. As grain growth is very strongly dependent on the surrounding gas density (see equation 6 and Section 4.1), we see that grains that experienced growth are likely to have done so in a high temperature, $T \sim 1000$ K, environment rather than in a coldish low-density ‘ambient’ disc. Of course, embryos may be temperature-stratified, with cooler material outside the embryos (e.g. Nayakshin 2011), so that icy grains could still form in those regions, but the inner regions of embryos are able to melt and thermally reprocess even the most refractory species.

### 4 GRAIN GROWTH AND DYNAMICS

#### 4.1 Grains grow inside embryos only

We now study grain growth and dynamics. Fig. 5 zooms in on one of the densest embryos, the one closest to the star in Fig. 4. The time of both figures is the same, $t = 4880$ yr. Fig. 5 shows the gas and the dust column densities (top and bottom left-hand panels, respectively), the gas temperature (top right-hand panel) and the dust-particle positions (bottom right-hand panel; only half of the dust particles are plotted in the interest of better clarity of the figure). In the bottom right-hand panel, dust particles of different grain sizes, $a$, are colour-coded as detailed below. The coordinate systems in both the spatial coordinates and in velocity space are centred on the densest part of the giant embryo for convenience.

The shape of the gas embryo is slightly elongated due to the tidal force along the $x \approx y$ direction, which approximately coincides with the instantaneous direction to the star as one can see from Fig. 4.

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**Figure 4.** The column density (left-hand panel) and the column-averaged temperature (right-hand panel) of the gas at time $t = 4880$ yr. Note the clear division between the high-density, high-temperature clumps and the low-density, non-self-gravitating regions, which may be cool or relatively hot due to shocks.
Figure 5. The embryo closest to the star at time \( t = 4880 \) yr (cf. Fig. 4). Gas (upper left-hand panel) and dust (bottom left-hand panel) column densities, with velocity fields, all centred on the densest point of the gas clump. The largest velocity vector in the upper left-hand panel corresponds to velocity of \( 6.2 \) km s\(^{-1}\), whereas the same for the lower left-hand panel is \( 3.9 \) km s\(^{-1}\). Gas temperature is shown in the upper right-hand panel and the lower right-hand panel shows grain particle positions with grains of different sizes marked by different colours as detailed in Section 4.1.

The gas component rotates (spins) nearly as a solid body in the inner few au of the embryo. The direction of the rotation is prograde, that is, the same as that of the disc. This matches the earlier findings by Boley et al. (2010).

Comparing the dust and the gas column densities, there is clearly more than a passing resemblance between the two distributions. Note that even the velocity patterns, normalized on the largest velocity of the given component in the panel (thus slightly different for the gas and the dust), are similar.
The colour-coding in the bottom right-hand panel of Fig. 5 is as follows: $a < 0.5$ cm for the black dots, $0.5 < a < 5$ cm for the red dots, $5 < a < 20$ cm for the green dots and $a > 20$ cm for the blue dots. The approximate mass of these grain components is $\sim 20$, 27, 15 and 7.5 $M_{\oplus}$, respectively. The main point to note from this panel is the very concentrated and segregated dust distribution. The largest grains, $a > 20$ cm, are found exclusively in the very centre of the embryo, as blue dots.

The concentration of the larger grains towards the centre of the gas embryo – the local potential well – is not unexpected. Since grains grow by hit-and-stick collisions in our model (Brownian motion is effective only for very small grains), for example, two-body collisions, grains grow the fastest in a high-density environment. Furthermore, the larger the grain, the faster it settles (Boss 1998).

This result is somewhat different from that of Boley & Durisen (2010) who considered grains of a fixed-size spread evenly in their initial gas disc. Under these conditions, the grains are concentrated to spiral arms and the clumps as well. In contrast, starting with small ($a = 0.1$ cm) grains and making allowance for their growth, we find that grains grow a negligible amount everywhere except the planet embryos, for example, even inside the initial spiral arms. To make this point more explicit, Fig. 6 shows the grain size for the dust particles in the whole simulation domain versus density of the gas, defined on the SPH neighbours of the dust particles. Only a randomly selected fraction of dust particles are plotted for clarity of visualization. There is nearly a one-to-one monotonic relation between the dust size and the density of the surrounding gas, splitting into several nearly vertical ‘tails’ at the upper right-hand part of the figure. This part of the diagram corresponds to several individual clumps.

The monotonic relation between the grain size and the gas density can be broken during periods of clump dispersal. During this time, the smaller grains move with the gas since they experience strong aerodynamic drag forces, so they are ‘frozen in’ with the gas. However, the larger grains experience weak drag forces. Neglecting these weak forces, we can say that the large grains move under the influence of gravity only, whereas the gas experiences both the hydro and the gravity forces. Therefore, the larger grains can be actually separated from the gas due to the different forces that these components experience.

### 4.2 Formation of a terrestrial planet core

Sinking of larger grains to the centre of a gas clump should eventually lead to the grain density in some region, called a ‘grain cluster’ in Nayakshin (2010b), exceeding that of the gas, followed by gravitational collapse of the grains there. This is indeed what happens in the simulation. The panels in Fig. 5 do not have the resolution required to discern this collapse. The blue ‘dots’ in the lower right-hand panel of the figure is actually a spatially compact cluster of grains numbering over 3000 dust particles. Had we not imposed the gravitational softening length of $h_{\text{min}} = 0.05$ au (cf. Section 2), then this cluster would have certainly collapsed, numerically speaking, to a point and, physically speaking, to a terrestrial planet core.

Fig. 7 shows the gas (black dots) and the dust-grain density (colour dots) inside the clump investigated in Fig. 5. The colour scheme used for dust particles in Fig. 7 is as follows: red, $a < 1$ cm; green, $1 < a < 10$ cm; cyan, $10 < a < 100$ cm; and dark blue, $a > 100$ cm. There is a very clear segregation of grain particles by their size, as larger grains sink in more rapidly.

The gas density profile has the constant density shape in the inner part of the clump (cf. the purple curve in Fig. 7), as expected for a polytropic gas cloud, and as found in 1D radiative hydrodynamics simulations by Nayakshin (2010b). The dust density is indeed higher than the gas density in the innermost 0.05 au. We note that further evolution of the simulation shows that the dust particles in this innermost part of Fig. 7 are self-gravitating and self-bound.
When the gas component is disrupted (Section 5), the ‘grain cluster’ survives the disruption and remains a point-like cluster of dust particles. As we argued in Section 2.3, the fact that we smooth out the self-gravity of the dust particles but not the aerodynamic force or the stellar tidal force, both of which oppose grain sedimentation, implies that the cluster would be even stronger self-bound in higher resolution (smaller gravitational softening value) simulations.

One could worry that due to the small physical size of the dust core, we somehow incorrectly estimate the gas density and hence the aerodynamic drag force between the grains and the gas in that region. However, the procedure for finding the SPH neighbours for dust particles is independent of the dust density or the dust concentration, as only the SPH neighbours are searched for. We checked that the number of the gas neighbours for all the dust particles in Fig. 7 varies between the accepted limits, for example, between 39 and 41. Therefore, the gas density drag force does not ‘disappear’ in the centre of the clump and we believe that the grain sedimentation in that region is entirely physical. Of course, due to a finite numerical resolution, we could miss some effects on smaller scales. For example, if the mass of the solids in this small-scale region is very high, say, 20 M⊕, the gas itself could be influenced by the gravity of the dust particles (Nayakshin 2011). The gas could then build up to higher densities around the dust core. In the final disruption of the embryo, some of this inner gas envelope could then survive. At this time, we cannot resolve such small scales. However, it would seem that these effects would make the solid core even more gravitationally bound, due to the increased gas mass in the region.

We also note that we do not find strong convective motions in our models that could resist smaller grain sedimentation. Future higher SPH particle number simulations will allow us to improve resolution at the smallest scales within the clumps. We also note that a better numerical treatment of the problem should include proper radiation transfer, for example, an account of the radiation emitted by the contracting solid core and then its propagation through the surrounding gas. We currently lack numerical capabilities for an explicit radiation transfer, both in terms of software and hardware (as a very high resolution is required near the centre of the embryo), but we plan to improve our methods in the future to address the ‘solid core’ collapse to a numerical singularity better.

5 MAKING THE ‘SUPER-EARTH’

Fig. 1 demonstrates a very dynamic and sometimes violent evolution of gaseous clumps. Clumps merge, some several times, with neighbouring ones. Close interactions may also result in velocity ‘kicks’ to the clumps. The maximum kick velocity possible is a fraction of the sound speed in the clump, for example, \( \sim 1 \) km \( \text{s}^{-1} \) or so. The Keplerian velocity at a distance \( R \) from the star is \( v_{K} \approx 2.3 \frac{R}{300 \text{au}} \) km \( \text{s}^{-1} \). Therefore, such interactions may result in substantial orbital changes for the clumps, as suggested earlier by Boley et al. (2010).

We now concentrate on the evolution of the clump investigated earlier in Section 4 and in Figs 5 and 7. This embryo forms in an initial orbit with the pericentre of \( \sim 70 \) au and the apocentre of \( \sim 100 \) au. The clump makes about three full revolutions around the star, interacting with nearby embryos. Remarkably, its last interaction with another passing embryo sinks it close enough to the star for it to be tidally disrupted. As pointed out in Section 4.2, the densest part of the dust particles inside the embryo (which we call a solid core below) is self-bound and is actually artificially kept from further collapse by the gravitational softening we employ on small scales. It is hence not surprising that the solid core survives the complete dispersal of the gaseous envelope and of the outer dust layers. The end-result of this process is a terrestrial planet core, as envisaged earlier by Boley et al. (2010) and Nayakshin (2010a).

5.1 Dynamics of the Super-Earth embryo

Fig. 8 shows several snapshots of the central 150 au region near the star around the time immediately preceding embryo disruption. The order of the snapshots is from the top to bottom (left-hand to right-hand side). The first one and the last one correspond to times \( t \approx 4680 \) and \( \approx 5000 \) yr, respectively. Concentrating on the inner-most \( \sim 70 \) au of the figure, one notes the close interaction between the closest (marked as ‘S’) and the second closest (marked 1 in the figure) gas clumps to the star. The two happen to be separated by about 20 au for a quarter or so of rotation (see the middle row of panels in the figure) from the innermost clump, whose orbit is only slightly eccentric before the interaction. As a result of the second embryo being positioned behind the first one during this interaction, the first one appears to lose angular momentum to the second. After the interaction, the innermost embryo is thrust inwards on a much more eccentric orbit with a pericentre of about 10 au, whereas the other clump is sent on a wider orbit. The innermost clump is subsequently disrupted and is dissolved inside the innermost disc.

We shall now refer back to Fig. 3 and note that there is a significant depression in the curve of the total masses of both the strongly bound gas component (the green curve) and the mildly bound one (the black curve) at around time \( t \approx 5000 \) yr. We now see that this event corresponds to the destruction of the innermost embryo by the tidal field of the star. The total mass of the embryo is \( \sim 25-50 M_{J} \), depending on where one puts the embryo outer boundary. There is an associated increase in the non-self-gravitating gas component (the red curve), followed by a slower decline. The decline in that curve is mainly due to the accretion of gas on to the star. We note that the clump inward migration and destruction we found here is of course related to the results of Vorobyov & Basu (2005, 2006, 2010).

5.2 Clump disruption: gas dynamics

Expressing the tidal density in terms of numerical values, \( \rho_{\text{tid}} = \frac{M_{\star}}{2 \pi R^{3}} \approx 6 \times 10^{-11} \text{g cm}^{-3} \left( \frac{10 \text{ au}}{R} \right)^{3} \) (10)

Given the outer regions of the gas clump have density of a few times \( 10^{-12} \text{ g cm}^{-3} \), we see that these outer regions can be tidally stripped at a distance of 20–30 au from the star, at which point \( \rho \sim \rho_{\text{tid}} \).

The flow of the gas at earlier disruption phases can be seen in the last three panels of Fig. 8. Fig. 9 shows the inner part of the disc and what is left of the disrupted embryo at time \( t = 5080 \) yr. Overlayed on the gas density plot are the velocity vectors, and also some of the dust particles. The black square-shaped symbol, marked with the label ‘SE’, at \( (x, y) \approx (2, 11) \) shows the only surviving part of the dust core. The core, defined as a very compact \( (R \lesssim 0.01 \text{ au}) \) concentration of about 3000 dust particles with \( a \gtrsim 50 \text{ cm} \), has a mass of about 7.5 \( M_{\oplus} \). As explained earlier, this core should physically be considered a terrestrial-like planetary core of the ‘Super-Earth’ mass range.
5.3 Clump disruption: dynamics of solids

Here we concentrate on the dynamics of the dust particles from and around the `Super-Earth’ clump. The object circles the star on a small-eccentricity ($e \sim 0.1$) orbit with the semimajor-axis of about 8 au until the end of the simulation.

Smaller dust particles are unbound (tidally disrupted) during the clump destruction. The cyan-coloured points in Fig. 9 show some of the ‘small’ dust particles, with size $a$ between 1 and 2 cm. These smaller particles are on the outskirts of the embryo (cf. Fig. 7) and are therefore easily disrupted from the embryo together with the associated gas. There is a leading and a trailing tail in these particles, as expected for a tidally disrupted object.

We shall now analyse the process of dust disruption in more detail. Fig. 10 shows the dust-column-averaged size of the grains at two specific times at the beginning of the embryo disruption, $t = 4968$ and 5056 yr. The figure shows an effect that can be called ‘tidal segregation’, namely smaller grains are strongly bound to the gas (e.g. Weidenschilling 1977) and are also predominantly found...
in the outer reaches of the embryo (cf. Figs 5 and 7). Therefore, as the gas clump is disrupted, the smaller grains are the first to be stripped away; for example, only $a \lesssim 1$ cm grains are unbound from the clump in the left-hand panel of Fig. 10. In the right-hand panel, however, the tidal disruption spreads to more inner regions of the embryo and we have grains of $a \sim$ tens of cm spread into the narrow leading and trailing dust filaments. The Super-Earth core survives, however, as its density is around $10^{-7}$ g cm$^{-3}$ (cf. Fig. 7), that is, orders of magnitude higher than the tidal density at this location. The solid core is visible as a bright yellow dot in both panels. Note that the dimensions of the dot in the figure are given by the size of a pixel, which is much larger than the physical size of the Super-Earth core (see below).

At later times, all of the gas and all the intermediate-size grains are disrupted from the embryo. Fig. 11 shows grain-size maps analogous to Fig. 10 but on smaller spatial scales. The Super-Earth core is now ‘all alone’, with smaller dust particles significantly influenced by the drag forces from the disc. The right-hand panel of Fig. 11 demonstrates that larger grains, $a \gtrsim$ tens of cm, are lost into the star more rapidly than smaller, $a \sim 1$ cm, grains. The Super-Earth core ends up on a slightly eccentric orbit with eccentricity of about 0.1 and a semimajor-axis of $\approx 8$ au.

Fig. 12 concentrates on dust particles within $R \sim 3$ au from the Super-Earth core for different times around the gas clump disruption, as labelled in the figure. The figure shows the radial velocity (defined with respect to the velocity of the Super-Earth core) of dust particles as a function of the distance from the core. The particles of different size are shown with different colours, following the colour convention of Fig. 7. As we underestimate (smooth) gravitational forces on small scales, particles found within 0.01 au from the centre of the dust core, defined as its densest point, are collectively shown with a thick blue dot. The number of dust particles within the dot is shown in the lower left-hand corner of each panel.

Before the tidal disruption of the core, the radial velocities of dust particles are small everywhere except outside the inner couple of au, where the dust may be influenced by external gas velocity

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*Figure 9.* The gas column density with velocity vectors and some of the dust particles. The Super-Earth is the black symbol approximately north of the star.

*Figure 10.* Grain size at two different times at the beginning of the clump disruption. The left-hand panel corresponds to $t = 4968$ yr and the right-hand panel corresponds to $t = 5056$ yr. Note that the largest grains are all in the point-like core that is resistant to tidal shear, whereas the smaller grains are being sheared away. The smaller the grains, the earlier they are sheared away from the embryo.
Figure 11. Same as Fig. 10 but at later times, \( t = 5136 \) and 5360 yr. Note also the smaller spatial scale. The ‘Super-Earth’ core is the bright yellow dot on an orbit with the eccentricity \( e \approx 0.1 \) and semimajor-axis of about 8 au.

Figure 12. The radial velocity of dust particles inside the clump around the disruption time. The colour-coding of the grain sizes is the same as in Fig. 7. The large blue dot at \( R = 0.01 \) au represents the ‘Super-Earth’ core composed of the large grains (>1 m). The numbers in the lower left-hand corners of the panels show the number of dust particles within 0.01 au (represented by the blue dot). Note that all the smaller particles (cyan, green and red) are eventually stripped away from the Super-Earth core.

The disruption of the gas embryo affects the dust particles in two ways. First, the mass enclosed within a given radius decreases, reducing the gravitational attractive force on the dust. Secondly, the gas outflowing from the centre of the clump applies aerodynamical drag forces on the dust. This strips away all but the largest \( a > 100 \) cm particles. In the bottom panel of Fig. 12, the Super-Earth core is a stand-alone feature. We note that the mass of the solid core slightly decreases between the third and fourth panels (from the top to bottom) of the figure, which indicates that some \( a > 100 \) cm particles are also dispersed by the tidal disruption. However, as we soften the self-gravity of the dust within 0.05 au (cf. Section 2), better resolved numerical simulations could have resisted even this slight mass-loss of the core. Finally, we note that the mass of the Super-Earth stabilizes and does not decrease at later times.

6 DISCUSSION

We presented a 3D numerical simulation of a massive gas disc with the usual interstellar mass fraction of dust grains. The grains are treated as a second fluid interacting gravitationally and via gas-drag friction with the gas. The size of the grains is allowed to grow with time by grain–grain collisions. We used a simple density-dependent radiative cooling time prescription allowing the gas to cool rapidly at low densities and increasing the cooling time at higher densities, as found for spherical gas clumps (Nayakshin 2010b). The simulation was selected from a set of \( \sim 10 \) similar simulations with varying initial conditions and/or cooling prescriptions (for details, see below).

While not presenting the results of the other simulations performed, we note that all of them showed the following results, exemplified by the simulation presented here: (i) spiral arms arising in the disc fragment into clumps at large radii due to the cooling time becoming longer than the local dynamical time at high
densities; (ii) the clumps do interact with each other, strongly affecting their orbits. The orbits of clumps may be eccentric due to clump–clump interactions (see also Boley et al. 2010); (iii) bodily interactions of clumps usually result in mergers, although dispersal of the clumps may occur if the relative angular momentum of the clumps is too large; (iv) as found earlier by Vorobyov and Basu (Vorobyov & Basu 2005, 2006, 2010), inward migration of clumps is quite generic, and in the presence of many clumps, may not be easily analytically described; (v) some clumps may remain far out, for example, ~100–200 au for the initial disc size of 200 au, again consistent with Vorobyov & Basu (2010); (vi) the initial inventory of gas divides into two physically distinct components at later times – the high-density strongly self-gravitationally bound clumps, containing about 40 per cent of the material in the simulation presented here, and the ‘ambient’ non-self-gravitating gas disc with density below the local tidal density. No strong spiral arm features remain in the disc at late times, except in the inner 20 au or so; and (g) within the simulation time of ~5000 yr, grain growth occurs only inside the high-density gas clumps. This is not surprising as the hit-and-stick grain growth rate is proportional to density, and the density in the clumps is several orders of magnitude higher than in the ambient disc.

The final fate of the clumps and thus the outcome of the simulation in its entirety does depend on the cooling prescription (as also expected based on analytical models of Nayakshin 2010a,b, 2011), initial conditions, for example, the disc mass, and missing physics (e.g. exact radiative transfer, a better opacity, dust growth and fragmentation models) not yet included into the code. If the radiative cooling of clumps is not suppressed sufficiently strongly at high densities, they may cool and collapse into massive gas giant planets or low-mass brown dwarfs (Stamatellos & Whitworth 2008). Inward migration of such objects would disrupt them only if they migrate very close to the star, for example, sub-au distances that we do not resolve in our simulations. On the other hand, if cooling is suppressed too strongly at high densities, the gas clumps are found to be too fluffy and are disrupted at 30–40 au. There is a further constraint on the planet formation part of our simulations. Grain growth needs to be rapid enough to result in a terrestrial planet core by the time the embryo is tidally disrupted, or else the grains are disrupted as well, and presumably accreted by the star.

The simulation presented in the paper was selected because it did result in one-rocky-core-bearing embryos being delivered to the inner disc and disrupted there. Therefore, as stated in the Introduction, the simulation presented here is only a ‘proof of the concept’: the tidal-downsizing hypothesis may work. Further surveys of the parameter space and improvements in the simulations, both in terms of numerical resolution and especially in terms of the missing physics, are needed to understand how robust the tidal-downsizing route for planet formation might be.

In addition, our simulations spanned 10⁴ yr at the most due to the numerical expense of the calculations. We expect that on longer time-scales the behaviour of the system may change considerably. At lower disc masses, the disc may become only marginally unstable in the sense of bearing spiral arms but not giving birth to high-density clumps we concentrated on here. It is therefore possible that on longer time-scales, for example, millions of years, dust would grow within the spiral arms as well (e.g. Clarke & Lodato 2009; Boley & Durisen 2010). Further migration of planets born in the early massive disc phase might be expected in this case.

One somewhat surprising result was to find that clumps were usually disrupted at greater distances (typically 20–30 au) than expected based on the analytical theory of Nayakshin (2010a). This may indicate that clumps embedded in a massive and rapidly evolved disc are ‘harassed’ by their environment and are thus less compact than similar isolated clumps. The rotation of the embryos, not taken into account in analytical models of Nayakshin (2010a), may be another mechanism providing support against the gravitational contraction of the gas clumps. Alternatively, the simulation results may also mean that clumps are disrupted at densities much lower than the mean embryo density as assumed by Nayakshin (2010a). We plan to investigate these issues in more details in future work.

Another, not unexpected, result is that clumps formed at different parts of the disc evolve differently. The clumps formed the closest (and earliest) migrate inwards too rapidly for their grains to grow large enough for a dense core formation. In the simulation presented, there are two earlier clump disruption episodes, which do not contain a sufficiently dense grain core to survive the disruption. There are also partial disruption events when gas clumps would lose only a part of their mass and then ‘hang around’ at a slightly wider orbit. This indicates that even if the physics of embryo cooling and dust growth and sedimentation are fixed, there is still a wide scope for different outcomes. The result of the planet formation in the tidal-downsizing hypothesis may thus be quite varied if embryo–embryo interactions are important and frequent.

Concerning the missing physics, we believe that the energy released by the collapsing massive dust cores is the most consequential. As found by Nayakshin (2011), and argued much earlier by Handbury & Williams (1975), these cores may release so much binding energy as to remove most of the outer hydrogen-rich envelope even without tidal stripping, perhaps resulting in icy giant planets at relatively large separations, where they could not have possibly been affected by tidal forces of the star. This would require introducing a proper radiative transfer in the simulations. Irradiation from the parent star is another significant effect missing from our work. We plan to include at least some of this physics into our future work and also cover a broader set of parameter space and initial conditions.

7 CONCLUSIONS

We presented one ‘proof of the concept’ simulation (selected from about a dozen) that produced a massive enough giant planet embryo to have migrated to within about 10 au of the parent star. This embryo is dense enough and aged enough to allow grains a sufficient time to grow and sediment to its centre. Future simulations of this kind should include more of the relevant physical processes and sample a broader range of initial conditions and the parameter space to indicate whether such embryo migration and tidal disruption are common enough to explain the abundance and properties of the observed planets.

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REFERENCES

APPENDIX A: CONTRACTION OF THE GRAIN CORE

The size of the grain core in our simulations can be smaller than the SPH smoothing length in the centre of the gas clump (see Section 4.2). There is then a concern about proper numerical treatment of the gravitationally contracting dust-grain core: perhaps the formation of the ‘Super-Earth core’ is somehow artificial and is due to our final SPH resolution.

We believe that an improved SPH resolution would not hinder grain sedimentation to a self-bound core. To back up this statement, and on the referee’s suggestion, we have performed several made-to-purpose simulations. To this end, we have selected the ‘S’ clump that contained the Super-Earth core to initialize these test runs. The selection was done at the time \( t = 4880 \) yr (same as Fig. 7). We then re-sampled the SPH particle distribution to change the SPH particle number, which was varied from one quarter to twice the original SPH particle number. We also varied \( h_{\text{min}} \), the gravitational softening for the dust component, increasing and decreasing it by a factor of 2. The ‘Super-Earth’ clump was then re-simulated for 500 yr.

As we expected (see below), the number of SPH particles did not have a critical influence on the results, whereas changes in \( h_{\text{min}} \) were critical. As we stated in Section 4.2, improving our numerical resolution in treating the self-gravity of the dust (decreasing \( h_{\text{min}} \)) should result in further decrease in the size of the grain core, and in the limit of \( h_{\text{min}} \to 0 \), it should contract to a point. Increasing \( h_{\text{min}} \), degrading our numerical resolution of the gravity force, should result in a more extended grain core.

Fig. A1 shows the innermost 0.003 au for four of these tests at 500 yr. This region contains most of the dust-grain particles of the Super-Earth core. Panel C2 shows the results for the test with the same number of SPH particles and \( h_{\text{min}} = 0.05 \) au as presented elsewhere in this paper. Panels C3 and D3 have \( h_{\text{min}} = 0.02 \) au. Panels D2 and D3 have twice the SPH particle number of the original simulation (C2). Panel D2 has \( h_{\text{min}} = 0.05 \) au, as panel C2.

From the figure, we see that decreasing \( h_{\text{min}} \) leads to a corresponding decrease in the size of the dust core, as it should. At the same time, improving the SPH resolution hardly changes the size of the dust concentration, which has a simple explanation. Within the centre of the clump, our dust-gas neighbour treatment (Nayakshin et al. 2009) leads to a constant density field on scales smaller than the SPH smoothing length. Thus, the grain–gas drag does not ‘disappear’ or gets reduced in the centre of the clump on arbitrarily small scales.

However, due to a finite numerical resolution, we could still miss some effects on small scales. For example, if the mass of the solids in this small-scale region is very high, say, \( 20 M_{\oplus} \), then the gas itself could be influenced by the gravity of the dust particles (Nayakshin 2010a,c). The gas could then build up to higher densities around the dust core. In the final disruption of the embryo, some of this inner gas envelope could then survive. We regret to say that at this point we cannot resolve such small scales. However, it would seem that these effects would make the solid core even more gravitationally bound, due to the increased mass in the region.
Figure A1. The isolated clump simulations performed to check the numerical validity of grain sedimentation.

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