Computational Models Of Financial Markets

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A thesis submitted for the degree of

Doctor of Philosophy

September 2013
To Clare, Jessica, and Lucas
Acknowledgements

Special thanks and gratitude to my advisors—Subir Bose and Dan Ladley. They have been a constant source of encouragement and support, and I am indebted to both for their patience during my difficult times, and for my subsequent success in the doctoral job market.

I would also like to thank Emmanuel Haven and Javier Rivas, and participants at the Conference on Computing in Economics and Finance (2012) and Eastern Economic Association Meetings (2013) for their helpful comments.
Abstract

The three chapters of this thesis share the common theme of computational approaches to modeling financial markets. Chapter 1, “Market Ecologies: The Interaction and Survival of Technical Trading Strategies”, finds its place in the boundedly-rational heterogeneous agent literature. Market prices result from the interaction of fundamental and technical trading strategies. We show that the way in which traders process information is critical in determining the long-run profitability of individual strategies. More realistic auction settings—in which price information is incorporated into trading methods in “real time”—demand computationally demanding techniques. The main conclusion of the chapter is that contrarian technical traders inadvertently mimic the role of arbitrageurs in more realistic auction settings.

Chapter 2, “Capital Allocation in a Delegated Trading Model”, develops a model of capital allocation that removes the need for full mean-variance optimization of the firm-level portfolio. The strategies explored within the artificial setting of Chapter 1 are used to test the model against empirical foreign exchange data. We observe that the proposed capital allocation scheme yields economically and statistically significant returns, even when traders choose rules without the
benefit of hindsight.

Chapter 3, “Portfolio Choice: The Costs and Benefits of Asymmetric Information”, continues the theme of artificial markets, with the auction process departing from the fictitious auctioneer of Chapter 1, toward a market making model in which risk-neutral dealers quote bid-ask spreads to compensate them for the losses incurred by trading with informed agents. We obtain the intriguing result that, in multiple markets, there is an “optimal” level of inside information. In individual markets, portfolio managers incur higher transaction costs as asymmetric information increases, but benefit from an externality at the portfolio level, as inside information aids price discovery.
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Chapter 1

Market Ecologies: The Interaction and Survival of Technical Trading Strategies

Technical trading strategies make profits by identifying and exploiting patterns in market prices—patterns generated by the interaction of market participants. This paper examines model markets composed of traders using a range of trading rules, and identifies the ecologies under which different strategies are profitable and persist. We show that the presence of technical traders may be beneficial, in some cases reducing volatility and increasing price efficiency. In particular, contrarian traders who base their decisions on high frequency data have the largest positive effect. It is also found that if technical traders condition their actions using ‘real time’ information, they partially emulate arbitrageurs and make positive profits. If this is not the case, trend following traders may make higher returns.
1.1 Introduction

There is a long-standing debate surrounding the ability of rational arbitrageurs to drive noise traders out of financial markets. Whilst prominent work has forcibly argued the case for noise traders being driven out of markets e.g., Friedman [1953] and Fama [1965], others have identified conditions under which they may persist, e.g. De Long et al. [1990]. In this paper, we analyze market ecologies, combinations of traders using different strategies, in order to understand the situations in which technical traders may make long term positive returns. We conclude that whilst in many circumstances fundamentalist traders make the highest returns, both trend following and contrarian traders may make positive profits, dependent on the market mechanism.

We consider a market populated with homogeneous fundamentalists, each hypothesizing the same rate of adjustment of price to some known fundamental value (such as purchasing power parity, in the case where the underlying asset is a currency). The market also consists of heterogeneous technical traders, each of whom follows a particular style of technical trading, such as the commonly used moving average and breakout rules studied in the empirical trading rule literature. Fundamentalists trade on departures of the realized price from its underlying value, whilst chartists\(^1\) trade on patterns in historical prices. Trader demands—channeled through a Walrasian auctioneer—determine the realized market price, which in turn determines the flow of wealth between different strategy types. In implementing the technical trading rules, we use the notion of an acceptable risk-adjusted level of profitability. This enables buy/sell signals to be mapped to price

\(^1\)We use the terms ‘technical trader’ and ‘chartist’ interchangeably. It is also common to refer to fundamentalists as arbitrageurs.
expectations, thus enabling the construction of an ecology of diverse, realistic trading rules. By allowing the inclusion of a large variety of trading strategies, we are able to assess the impact of a complex range of strategy interactions and draw conclusions about the likelihood of their survival.

Fundamentalists observe the same freely available fundamental value. Technical traders pay no attention to this value, instead relying on a variety of trading rules commonly investigated in the empirical literature. These include the ‘trend following’ momentum, moving average and channel rules. Natural counterparts to these trend following rules are contrarian rules, which we model by reversing trend following signals from ‘buy’ to ‘sell’ and from ‘sell’ to ‘buy’. Each rule has a single parameter, which is drawn from a set of values that correspond to common data window sizes, such as weekly, monthly, quarterly and semi-annual windows. Together with an ever-present fundamentalist strategy, we run a simulation for each permutation of parameter values for the trend following and contrarian strategy types.

Markets clear through a Walrasian auctioneer, who is tasked with finding an equilibrium price in each period. Although the Walrasian auction is an abstraction from reality, it allows a relatively simple comparison to be drawn between information that is processed with a lag and information that is processed in ‘real time’. Alternative constructs, such as a market maker or an electronic limit order market, move the analysis away from part-analytic/part computational to fully computational. A fully computational model is used as the auction process in Chapter 3 of this thesis.

Within the Walrasian auction, it has been common practice to construct traders’ demand schedules using prices up to—but not including—the current price. This
has the convenience of producing a smooth aggregate demand curve and a unique market clearing price. However, what looks at first glance like a fairly innocuous assumption, creates a market conducive to the profitability of positive-feedback strategies. We contrast this setting with one in which traders condition their behavior on all prices up to the current period, and henceforth we refer to ‘lagged prices’ and ‘real time prices’ Walrasian auction protocols.

We first present results showing the volatility and efficiency of the markets. We find that under the lagged information setup, volatility is reduced in the presence of trend followers, whilst the difference between the fundamental price and the market price is lower when there are more technical traders basing their decisions on short price windows. If markets are dominated by contrarian traders using up to date information\(^1\), there is a risk of disconnection between the market price and the fundamental. The market may experience low volatility, but a higher tracking error. In contrast, if trend following traders are present in these markets, volatility is increased, but the disconnection is decreased. We go on to analyze the effect of the market ecology on the profitability of trading rules. We find that some initial trading rule endowments are more conducive to the profitability of individual strategies than others, which we illustrate through the use of sensitivity tables which capture average interaction effects. There is a clear message: under the assumption of conditioning with respect to lagged prices, both fundamentalists and trend followers accrue profits— at the expense of contrarians. In contrast, when all technical trading strategies use the most up-to-date price information, not only do contrarian strategies outperform trend followers, but

\(^{1}\)Here we are referring to prices; contrarians share the same information set as trend-followers.
they also accumulate long-run profits. Crucially, the overall level of profits and losses is much reduced in the real time protocol: the market seems better suited to its primary purpose of providing prices that properly reflect fundamental value.

1.2 Related Literature

The Efficient Markets Hypothesis [Fama, 1965] and the related Rational Expectations Hypothesis [Lucas, 1972; Muth, 1961] formalized the Friedman [1953] notion that irrational speculators will eventually be driven out by rational arbitrageurs. Critics of this approach, however, argue that there is too high a computational burden imposed on agents in order for them to calculate the rational expectations equilibrium. Real world examples demonstrate this. For instance, investors seemingly failed to learn the lessons of the NASDAQ crash: barely a decade later a new bubble developed, this time in the United States real estate market, with subsequent dire consequences for the global financial system.

The idea that agents operate within ‘bounds of rationality’, or that they use rules of thumb instead of complex calculations, can be traced back to Simon [1957] and Tversky and Kahneman [1974]. Sargent [1993] and Evans and Honkapohja [2001] provide more recent examples of models in which heterogeneous agents learn using boundedly rational rules. Within a simple cobweb model, Brock and Hommes [1997] describe an ‘adaptively rational equilibrium’ in which heterogeneous agents choose between a costly rational expectations forecast or a costless naive forecast. The composition of the population evolves according to feedback on the success of each strategy. The responsiveness of agents to this feedback—the ‘intensity of choice’ parameter—determines whether there is a stable rational
expectations equilibrium. The Brock and Hommes [1998] adaptation of this evolutionary learning mechanism to a simple asset pricing model is one contribution to a large literature on simulation-based approaches to heterogeneous agent modeling. There are a number of recent surveys of this literature, including Hommes [2006], LeBaron [2006], Chiarella et al. [2009] and Hommes and Wagener [2009].

A common feature of many models is the role played by technical analysis. A number of influential survey articles have pointed to the emphasis placed by foreign exchange professionals on this approach. Allen and Taylor [1990] and Taylor and Allen [1992] present survey evidence from the London foreign exchange market. They find that 90 percent of correspondents use technical analysis in forming exchange rate expectations, with a particular emphasis on its use at short forecasting horizons. They suggest that chartists may have an intuitive handle on nonlinearities in short-term exchange rate dynamics, but that linear fundamental models prevail in the long run. For a recent discussion of heterogeneity in exchange rate expectations, see Menkhoff et al. [2009]. Early empirical studies focusing on foreign exchange include Sweeney [1986] and Levich and Thomas [1993], whilst Menkhoff and Taylor [2007] surveys the technical strategies used in these markets.² Qi and Wu [2006] examine the common stock market rules of Brock et al. [1992] and Sullivan et al. [1999] and apply them to foreign exchange markets.

In early exchange rate models, prices were formed by weighting the expectations of heterogeneous agents [Frankel and Froot, 1990]. Frankel and Froot [1986] proposed a model in which portfolio managers weight the forecasts of chartists and fundamentalists, using Bayesian updating to refine those weights. The model

² For a broader survey, encompassing other asset classes, see Park and Irwin [2007].
is used to construct a hypothesis for the excessive strength of the US dollar in
the 1980s. De Grauwe et al. [1993] suggest that the influence of fundamental-
ists’ expectations becomes more important the farther the exchange rate is from
its equilibrium. In their model, heterogeneous expectations are used in the con-
struction of a chaotic model of the exchange rate. These types of models have
been used to examine the consequences of policy intervention [Bauer et al., 2009;
Wieland and Westerhoff, 2005] or in the case of De Grauwe and Grimaldi [2005],
to examine the effect of frictions on exchange rate dynamics.

Whilst the interaction of technical traders has been considered in foreign ex-
change markets (and markets for assets more broadly) the scope of those strategies
analyzed has been limited. The majority of models have focused on limited trader
types and so have simplified the complex interactions which may occur in these
markets. In this paper, we analyze in detail the effect of the ecology of trading
strategies, examining how the presence of different trader types may increase or
decrease the profits of an investor following a different approach. This analysis
allows us to comment both on the profitability of different strategies, but also on
the effect of technical traders on market behavior. There have been many recent
calls for restrictions on speculation, e.g., Krugman [2009], but the supporting ev-
dence for these constraints is relatively weak. This paper will contribute to this
debate by showing the impact of different types of traders within the market.

1.3 The Model

We model the exchange of a risky and a risk-free asset. We do not specify the
type of asset—this could equally well be an equity or foreign currency. Trade
occurs daily, with all positions closed before the beginning of the following day and settled at today’s price. Trading takes place between a large number of fundamentalist and technical traders through a single Walrasian auctioneer. At the beginning of each trading day, the underlying fundamental value of the asset is revealed to the fundamentalists. This information is private and free.

Both fundamentalists and technical traders are mean-variance optimizers, with constant absolute risk aversion (CARA) preferences. The auctioneer quotes a sequence of prices and collects and aggregates the demand schedules of all traders. With no outside supply of the risky asset, it is the auctioneer’s role to set the price that induces aggregate excess demand to be as close to zero as possible.

The model setup is similar to most models in the chartist-fundamentalist literature. Here, the approach has been to derive closed-form solutions for equilibrium prices, with further parsimony achieved by CARA utility preferences that yield asset demands independent of agents’ endowments. See, for example, Brock and Hommes [1998] and the references contained in Hommes and Wagener [2009].

1.3.1 Demand

All agents are modeled as having the exponential utility function

$$U(W) = -\exp(-\lambda W),$$

(1.1)

where the utility function $U(.)$ is defined over wealth $W$ and $\lambda$ is the coefficient of absolute risk aversion. If it is assumed that conditional wealth at time $t + 1$ is distributed normally with mean $\mu$ and variance $\sigma^2$, then it can be shown that to
maximize expected utility is equivalent to maximizing the value of

\[ \mu - \frac{\lambda \sigma^2}{2}. \]  

(1.2)

The one-period-ahead wealth equation is specified as

\[ W_{t+1} = (1 + r^*) S_{t+1} Q_t + (1 + r)(W_t - S_t Q_t), \]  

(1.3)

where \( r^* \) and \( r \) are the rates of return on the risky and risk-free assets, respectively. \( Q_t \) is the quantity of the risky asset purchased by the agent at time \( t \) and \( S_{t+1} \) is the spot price realized at time \( t + 1 \). Solving the agent’s maximization problem, subject to this wealth constraint, gives the optimal quantity of risky asset demanded as

\[ Q_t^* = \frac{(1 + r^*) \mathbb{E}_t[S_{t+1}] - (1 + r)S_t}{\lambda(1 + r^*)^2 \mathbb{V}_t[S_{t+1}]}, \]  

(1.4)

where \( \mathbb{E}_t[S_{t+1}] \) and \( \mathbb{V}_t[S_{t+1}] \) are conditional expectations and variances of the future spot price.

1.3.2 Expectations

Equation 1.4 gives the demand for the optimal quantity of the risky asset. The numerator is interpreted as the expected excess return beyond the risk-free rate. This is composed of dividends (or foreign interest earned) and capital gain from price movements. Traders have heterogeneous expectations of the price, with fundamentalists expecting a constant rate of convergence to a fundamental value and technical traders expecting an acceptable risk-adjusted level of profitability.
Technical traders differ in their strategies and through the use of a diverse range of sample sizes of historical prices.

1.3.2.1 Fundamentalists

Fundamentalists’ expectations are driven by privileged access to the underlying fundamental value of the asset. For simplicity, we assume that they have free access to this value. We model the underlying fundamental value as a geometric Brownian motion. After applying Ito’s Lemma, this gives the following process for the log of the value in discrete time:

\[ \ln Y_t - \ln Y_{t-1} = \mu - \frac{1}{2} \sigma^2 + z_t, \]  

where \( Y(t) \) is fundamental value, \( z_t \) is an i.i.d. draw from a standard normal distribution and \( \mu \) and \( \sigma \) are constants. Under the assumption that there is no drift term, \( \mu = 0 \), the equation for the evolution of the fundamental value is

\[ \ln(\frac{Y_t}{Y_{t-1}}) = z_t - \frac{1}{2} \sigma^2. \]  

Fundamentalists have privileged access to the realization \( z_t \), and hence have access to current fundamental value. The volatility of the underlying process, \( \sigma \), is assumed to be constant, and represents the (exogenous) volatility of a well-diversified portfolio. The volatility of the well-diversified portfolio is common knowledge, and is incorporated both by fundamentalists and technical traders in their asset demand equations.

Fundamentalists buy the risky asset when it is under-priced relative to its

\[ \text{The impact of a cost to acquiring this information would be an interesting extension.} \]
fundamental value, and sell it when it is overpriced. They do so with increased confidence the farther the asset is trading from its true level. They form expectations according to the rule

$$
E_t[S_{t+1}] = S_t + \gamma(Y_t - S_t), \quad \gamma \in [0, 1]
$$

(1.7)

where $Y_t$ is the realization of fundamental value at time $t$ and $\gamma$ is the rate at which fundamentalists expect the asset price to converge to its true level.

1.3.2.2 Technical Traders

A difficulty with the types of technical rules commonly examined in the empirical literature is that signals of the type ‘buy’, ‘sell’ or ‘hold’ are generated with no reference to the quantity to be traded. A key contribution of this paper is to propose that simple trading rules can be mapped into price expectations by using the notion of an acceptable risk-adjusted level of profitability. Risk-adjusted performance metrics (RAPMs) in the financial industry are dominated by the use of the Sharpe ratio [Sharpe, 1964], with variations such as the Sortino ratio [Sortino and van der Meer, 1991].

Under the assumption that returns are i.i.d., the mean and standard deviation of the daily returns of a strategy or portfolio are translated into an annualized Sharpe ratio using the formula

$$
SR_{\text{annual}} = \frac{252 \cdot (\mu_{\text{daily}} - r_{\text{daily}})}{\sqrt{252 \cdot \sigma_{\text{daily}}}},
$$

(1.8)

The Sortino ratio belongs to a general class of statistics which only consider the volatility of returns below some target rate of return. Another example is the semi-variance measure, which only includes those returns below the sample mean.
where $\mu_{\text{daily}}$ is the mean daily return, $r_{\text{daily}}$ is the risk-free rate and $\sigma_{\text{daily}}$ is the daily volatility of returns. Lyons [2001] points to the use of the Sharpe ratio as a performance metric in banks and hedge funds. Grinold and Kahn [2000] and Menkhoff and Taylor [2007] suggest that the benchmark used for committing capital is a Sharpe ratio of 0.5. The daily expected return implied by a prior Sharpe ratio of 0.5 and Equation 1.8 corresponds to an expected appreciation or depreciation of the asset price. The direction depends on the current trading signal, leading to an expression for technical trader expectations of

$$
\mathbb{E}_t[S_{t+1}] = \left(1 + r_{\text{daily}} + \frac{I_t \cdot \text{SR}_{\text{annual}} \cdot \sigma_{\text{daily}}}{\sqrt{252}}\right) S_t,
$$

(1.9)

where $I_t$ is the value of the trading signal $\in \{-1,0,1\}$.

### 1.3.3 Trading Rules

We select three classes of technical trading rules that tend to dominate the empirical literature. As discussed above, the three rules do not offer obvious ways of using extrapolation to derive point forecasts. Instead, they have been designed with a view to signaling possible changes of trend, rather than being used as forecasting methods. We consider trend following versions of these rules—as set out below—and contrarian versions, which use the same rule, but simply reverse the sign of the output.

#### 1.3.3.1 Momentum

In this study, momentum is defined as a simple threshold rule in which a buy signal is given if the rate of change of prices is positive over a defined period, a sell
signal is given if the rate of change is negative, and in the rare case where there is no rate of change, a neutral signal. Rate of change is measured with respect to the endpoints of the sample only; no attempt is made to model the rate of change by incorporating all of the sample observations. Much of the literature on momentum is equity-based [Chan et al., 2000; Conrad and Kaul, 1998], but the general principle is the same across all asset types—strong price movements are expected to follow strong price movements. The rule measures whether the rate of change is positive or negative over a time period $n$, which includes today’s price $S_t$

$$I_t(n) = \begin{cases} 
+1 & \text{if } S_t > S_{t-n} \\
0 & \text{if } S_t = S_{t-n} \\
-1 & \text{if } S_t < S_{t-n} 
\end{cases}$$

(1.10)

1.3.3.2 Moving Average

Moving average rules vary with respect to the weighting scheme used (e.g. exponential and simple) as well as with respect to the method of implementation, which can involve several averages in combination. Menkhoff and Taylor [2007] discuss the method in which a long position is taken upon a short period moving average crossing a long period moving average from below, and a short position when the short period moving average crosses the long period from above. Moving average combinations are also considered in Brock et al. [1992], Sullivan et al. [1999] and Qi and Wu [2006], amongst others. We use the simple moving average

$$\text{SMA}_t = \frac{1}{n+1} \sum_{t=t-n}^{t} S_t$$

(1.11)
The indicator function for the simple moving average rule is

\[
I_t(n) = \begin{cases} 
+1 & \text{if } S_t > \text{SMA}_t \\
0 & \text{if } S_t = \text{SMA}_t \\
-1 & \text{if } S_t < \text{SMA}_t 
\end{cases}
\] (1.12)

The simple moving average is similar to the momentum rule except that the test condition relates to the average of all the prices in the sample, rather than just the start and end points.

1.3.3.3 Trading Range Breakout

Also known as a support and resistance or channel method, this trading style is event-driven, in that the breakout is intended to signal a change in market conditions. Early investigations of channel rules include Irwin and Uhrig [1984], Lukac et al. [1988] and Taylor [1994]. Several variants appear in the modern bootstrap-based studies. Brock et al. [1992] consider channels of 50, 150 and 200 days, where a channel is defined as those prices contained by a local maximum and minimum. It is argued that at these extremes many investors are willing to buy or sell, with the minimum acting as ‘support’ and the maximum acting as ‘resistance’; the eventual ‘breakout’ is deemed to be a significant event. We use an \(n\)-day breakout indicator, where the indicator takes a positive value if the most recent close is greater than the highest high of the previous \(n\) prices.

\[
I_t(n) = \begin{cases} 
+1 & \text{if } S_t > \max(S_{t-1}, S_{t-2}, ..., S_{t-n}) \\
-1 & \text{if } S_t < \min(S_{t-1}, S_{t-2}, ..., S_{t-n}) \\
I_{t-1} & \text{otherwise}
\end{cases}
\] (1.13)
At the start of the simulation, the indicator will be zero until the first breakout. The indicator then maintains this value until a breakout occurs in the opposite direction.

### 1.3.4 Walrasian Auction Protocols

Trade is conducted through a Walrasian auctioneer. In the literature, the standard implementation assumes traders use prices up to, but not including, the current price being quoted by the auctioneer. This is a matter of convenience, as it provides smooth individual demand functions and a unique market-clearing price. However, it is a simplified view of how technical traders behave. In this paper, we also consider an auction mechanism in which expectations are adjusted contemporaneously with the sequence of prices quoted by the auctioneer. We refer to the two auction protocols as ‘lagged’ and ‘real time’.

In both the lagged and the real time versions of the model, the current fundamental value is revealed before trade. In both versions, fundamentalists adjust their expectations contemporaneously with the auctioneer’s quote. The difference between the two mechanisms lies in the way technical traders update their rules. For example, a trader using a lagged version of the breakout rule has already decided whether to be long or short, irrespective of the auctioneer’s quote. His quantity depends on the difference between his expectation of the price and the auctioneer’s quote. In contrast, a trader using the real time version of the same breakout rule notes that quotes lying above (below) the current maximum (minimum) channel threshold will result in a buy (sell) signal. The trading signal for the current round of trade updates at the same time as the auctioneer’s quote.
These threshold values create discontinuities in the real time breakout trader’s demand schedule. Discontinuities in the aggregate demand schedule follow.

Note we are still striving for a market clearing solution in each time step, so there is a material difference between this paper’s real-time Walrasian auction and a market making model in which the current price is known to traders. The market making model perpetually trades out of equilibrium, with the market maker’s inventory taking up the slack of excess demand or supply. Under the assumption of a risk-neutral market maker, there are no bounds to the size of this inventory.

1.3.4.1 Lagged Price Information

With expectations formed from a combination of a trading signal \( I_{t,i} \) and a prior Sharpe ratio \( \text{SR}_{\text{annual}} \), it can be shown that technical trader aggregate demand, \( D^T(S_t) \), a function of the current price quote \( S_t \), is given by

\[
D^T(S_t) = \sum_i I_{t,i} w_i \cdot \frac{\text{SR}_{\text{annual}}}{\lambda \sigma S_t \sqrt{252}},
\]

where \( i \) indexes the technical rules and \( w_i \) is the proportion of the population using a particular rule. Similarly, fundamentalist aggregate demand, \( D^F(S_t) \), is

\[
D^F(S_t) = \frac{w_f \gamma(Y_t - S_t)}{\lambda \sigma^2 S_t^2},
\]

where \( w_f \) is the proportion of the population employing the fundamentalist strategy. The trading population consists entirely of technical traders and fundamentalists, with technical proportions and fundamentalist proportions summing to 1:
Figure 1.1: Market Clearing Scenarios. When technical traders condition their rules on lagged prices, aggregate demand is smooth, yielding a unique market clearing price (panel A). But when trading rule signals are synchronized with the auctioneer’s quote, discontinuities in individual demand curves lead to discontinuities in aggregate demand. A unique clearing price is still possible (panel B), but there is also the possibility of disequilibrium trading (panel C) and multiple equilibria (panel D).
\[
\sum_i w_i + w_f = 1. \tag{1.16}
\]

With no outside supply of foreign currency, adding \( D^T \) and \( D^F \) and setting to zero yields an expression for the market clearing rate, \( S_t^* \), of

\[
S_t^* = \frac{w_f \gamma Y_t}{w_f \gamma - \frac{1}{\sqrt{2\pi}} \cdot \lambda \sigma \sum_i I_{t,i} w_i \cdot SR_{annual}}. \tag{1.17}
\]

In the lagged prices auction protocol, technical traders calculate their trading signals without reference to the auctioneer’s current quote. For example, if the market clearing price produces a ‘buy’ signal for a technical trader using a breakout rule, then that trader is committed to using a ‘buy’ signal in the next auction round—regardless of the auctioneer’s quote. Hence, \( I_{t,i} \) is fixed in each time step, which has the convenience of producing a unique market clearing price, \( S_t^* \). An examination of Equation 1.17 reveals that if all technical traders carry forward ‘buy’ signals into the current auction round, then the denominator of Equation 1.17 is smaller, leading to a higher market clearing rate. The market clearing rate is the rate that induces the fundamentalist to take the opposite position of the combined technical trader position. Large order imbalances are easier to offset if (a) the size of the fundamentalist population is large and (b) if the price is far away from fundamental value. The reason that the lagged prices protocol is prevalent in the literature is that it has the advantage of a low computational burden for the calculation of the market clearing price in each time step. Panel A of Figure 1.1 illustrates this case.
1.3.4.2 Real-time Price Information

The setting of prices is more complex if traders condition their demand on the current trade price. In the real time setting, $I_{t,i}$ now becomes an endogenous variable of Equation 1.18:

$$S_t^* = \frac{w_f \gamma Y_t}{w_f \gamma - \frac{1}{\sqrt{252}} \cdot \lambda \sigma \sum_i I_{t,i}(S_t^*) \cdot \overline{SR}_{annual}}.$$  (1.18)

Equation 1.18 makes explicit the joint determination of the market clearing price and trading signals. The equation is identical to Equation 1.17, except for the dependence of trading signals on the current quoted price, $I_{t,i}(S_t^*)$. Technical traders now use their rules more intelligently, allowing the trading signals to vary with up-to-date information.

Real time information processing may produce discontinuities in the demand curve which are a result of discontinuities in the trading rules, i.e., points at which traders change the nature of their demand. The result of this is that there are three qualitatively different scenarios which do not fit with the concept of a single market clearing price. The first, in Panel B of Figure 1.1, shows an example in which the discontinuities do not preclude the existence of an equilibrium or market clearing exchange rate. The curve crosses the $D(S_t) = 0$ line at a unique point. The next possible outcome (Panel C) shows the non-existence of an equilibrium crossing point. This raises economic as well as computational questions. If the market cannot clear at equilibrium, where does the market trade? And if the market trades out of equilibrium, how is excess demand dealt with? In this example, just as the demand curve is about to intersect the zero line, certain strategies adjust their signals so as to create gaps both before and after the likely
crossing point. Finally, by updating their information sets to include the current quote, traders can generate multiple equilibria (Panel D) leading to the question of which price should be chosen.

Additional assumptions need to be made about how the auctioneer goes about calculating the traded price for each potential scenario. Whilst in some cases it may be possible to use a technique such as the bisection method, in general the potential multiple or zero crossing points mean this is not possible. It is therefore necessary to use a computationally demanding search. We define the set of prices considered as: \( \{S_L, S_L + \delta, \ldots, S_U - \delta, S_U\} \), where \( S_L \) marks the lower limit of the search range, \( \delta \) is the price increment, and \( S_U \) is the upper limit. In all cases, \( \delta = 0.001 \) or 10 basis points, which we judge to be a fair trade-off between precision and speed\(^5\). Our simulations were performed using multiple clusters on ALICE, the University of Leicester’s High Performance Computing (HPC) facility; our choice of precision reflects the economic cost of processing time, and as such is the level that keeps processing time within one business day. We note little qualitative difference in our results, however, from those obtained using coarser precision levels such as 50 basis points and 100 basis points.

The boundary conditions are obtained by computing the market clearing price that would occur if all technical traders were buyers or if they were all sellers. This is achieved by setting all trading signals, \( I_{t,i} \), to either +1 or −1 in Equation 1.18. In each case, we select the price with minimum excess demand. Since the set of search prices is discrete, the excess demand will never be zero. The auctioneer deals with this by rationing members of the larger group in proportion

\(^5\)Our simulations are intended to represent trading at the daily frequency, so it may be more appropriate to use a finer precision for simulations which are intended to capture activity at higher frequencies.
to individual demands. In essence, the auctioneer operates as an order matcher. An alternative would be for them to store excess quantity in a similar manner to a market maker. However, this would potentially add or remove wealth from the trading environment, and complicate the dynamics.

1.4 Results

We consider markets containing combinations of the six classes of trading strategies described above (momentum, momentum contrarian, moving average, moving average contrarian, breakout and breakout contrarian) using four different historical sample sizes: \( n \in \{5, 21, 63, 126\} \). These values were chosen to represent psychologically important periods of a week, a month, a quarter and half a year. In total, this gives \( 6 \times 4 = 24 \) different technical trading strategies. Assuming the fundamentalist strategy is present within all simulations, this gives \( 2^{24} = 16,777,216 \) possible strategy combinations. Each strategy within a simulation is allocated an equal share of the population i.e., \( 1/n \). For each combination of trading strategies, we perform 1000 repetitions of the simulation, with different underlying time series of fundamental value. The same underlying price series are used for all trader combinations, ensuring that variations are due to agent interactions. Each simulation runs for 3024 days (12 years), with each day consisting of a single market-clearing time step. Data is only recorded for the final 2520 days (10 years) in order to allow the rules with longer windows to base their behavior on market data. The volatility of log returns in the underlying geometric Brownian motion is 1% per trading day, which equates to approximately 16% per annum using a square-root of time scaling rule. We assume that all traders
use this value as their estimate of future volatility. Hence, heterogeneous expectations of future volatility are not driving realized volatility. All risk aversion parameters are set to 1, and all prior Sharpe ratios are set to 0.5. Note, as long as they are identical across individuals, neither of these parameters qualitatively affect results as they merely scale the size of investment that traders make. The fundamentalists use a convergence parameter of 0.01: they expect 1% of the difference between the actual price and the underlying value to be eradicated each day. Note that this is a significant departure from a belief that prices should adjust instantaneously to their fair values. A slow adjustment parameter can be justified by the empirical observation of Lyons [2001], for example, that foreign currencies decay with a half-life of approximately 2 years to Purchasing Power Parity (PPP). Our convergence parameter implies that fundamentalists expect a half-life in the order of 4 months—faster than the observed half-life of foreign currencies, which reflects the certainty with which fundamentalists receive information in our model. The dimensionality of the simulation problem is reduced by holding the fundamentalists’ parameter fixed at this value. These parameters are summarized in Table 1.1.

1.4.1 Market Statistics

We first present results showing the effect of the composition of the population of traders on market behavior. We focus on two aspects: efficiency—as measured by the mean squared error between the market prices and the underlying fundamen-

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6Endogenous estimation of volatility is an interesting avenue for future research. In the same manner in which there are many ways to estimate prices, there are also many ways to estimate volatility, and potentially many complex ways in which they could interact. We therefore leave this aspect to future work, in order to focus on the dynamics of price rules.
Table 1.1: Table documenting those parameters common to all simulations. Traders share the same utility function, with coefficient of absolute risk aversion $\lambda$. The volatility of underlying fundamental value and traders’ expected conditional volatility of returns is given by $\sigma$. Chartists’ prior notion of an acceptable strategy is $\text{SR}_{\text{annual}}$ and fundamentalists expect the exchange rate to converge to fundamental value at the rate $\gamma$. Populations are allocated equal fractions of the arbitrage strategy and $[1,25]$ technical strategies. Each simulation runs for 12 years.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
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<tr>
<td>$\lambda$</td>
<td>Risk aversion</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>Daily volatility</td>
<td>1%</td>
</tr>
<tr>
<td>$\text{SR}_{\text{annual}}$</td>
<td>Target risk-adjusted return</td>
<td>0.50</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fundamentalist convergence</td>
<td>1%</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of strategies</td>
<td>$[1, 25]$</td>
</tr>
<tr>
<td>$w_i$</td>
<td>Population shares</td>
<td>$1/n$</td>
</tr>
</tbody>
</table>

Table 1.2: Table presenting the average price volatility of markets in which a particular trader participates. The first column contains the average volatility of all simulations for the lagged and real time cases. The remaining columns detail the volatility relative to the base case when the specified trader type is present in the market.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Trend Following</th>
<th>Contrarian</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>63</td>
<td>126</td>
<td>5</td>
<td>20</td>
<td>63</td>
<td>126</td>
</tr>
<tr>
<td>Lagged Breakout</td>
<td>0.1201</td>
<td>0.930</td>
<td>0.965</td>
<td>0.981</td>
<td>0.986</td>
<td>1.070</td>
<td>1.035</td>
<td>1.019</td>
</tr>
<tr>
<td>Lagged Momentum</td>
<td>-</td>
<td>0.855</td>
<td>0.869</td>
<td>0.895</td>
<td>0.940</td>
<td>1.145</td>
<td>1.131</td>
<td>1.105</td>
</tr>
<tr>
<td>Lagged Moving Average</td>
<td>-</td>
<td>0.833</td>
<td>0.845</td>
<td>0.872</td>
<td>0.898</td>
<td>1.167</td>
<td>1.155</td>
<td>1.128</td>
</tr>
<tr>
<td>Real Time Breakout</td>
<td>0.0288</td>
<td>1.075</td>
<td>1.053</td>
<td>1.041</td>
<td>1.027</td>
<td>0.925</td>
<td>0.947</td>
<td>0.959</td>
</tr>
<tr>
<td>Real Time Momentum</td>
<td>-</td>
<td>1.093</td>
<td>1.078</td>
<td>1.061</td>
<td>1.050</td>
<td>0.907</td>
<td>0.922</td>
<td>0.939</td>
</tr>
<tr>
<td>Real Time Moving Average</td>
<td>-</td>
<td>1.113</td>
<td>1.095</td>
<td>1.073</td>
<td>1.060</td>
<td>0.887</td>
<td>0.905</td>
<td>0.927</td>
</tr>
</tbody>
</table>

Table 1.2 presents the results for price volatility. In line with previous findings [Chiarella et al., 2009; Hommes and Wagener, 2009], the technical traders have
a significant effect on the dynamics of the market price. There is, however, a surprising difference between the real time and lagged specifications. Volatility is approximately four times greater under the lagged specification than the real time equivalent. This indicates that if traders use real time information, the destabilizing effect of technical traders may be lower than previous estimates suggest. Notably, however, the real time price volatility is itself approximately three times higher than the underlying fundamental volatility. The use of real time information allows traders to avoid some trading mistakes which destabilize prices. For instance, a trader may avoid buying into a trend which was previously present, but which does not persist in the current period.

The effect of different types of traders on volatility is not uniform. In the real time setting, those markets which contain trend followers have higher volatility than those containing contrarians. The contrarian traders act to dampen the market, resisting trends in price changes. In contrast, trend followers actively move the price away from its current value. In the lagged setting, the pattern is reversed. The contrarians, using out of date information, destabilize the price. In many cases this is because their efforts ignore those of the fundamentalist trader. For instance, if a trend appeared in period $t-1$ away from the fundamental, the fundamentalist would take a position which would act to correct this. Contrarians would take a similar position (in reverse of the existing trend) making the reversion larger. In contrast, trend followers would oppose the fundamentalist, reducing the size of the reversion and consequently the overall volatility.

The mean squared error between the market price and the fundamental price gives a measure of the efficiency of the financial market: its ability to price an asset fairly. Table 1.3 shows that this value is higher for the lagged case than...
the real time setting. This is unsurprising, as the real time protocol uses more up-to-date information in the calculation of prices, and therefore it should be expected that the fundamental price and the market price will be closer.

Under the lagged protocol, short-period momentum and moving average traders decrease tracking error, whilst those with longer periods increase it (and vice versa for contrarians). In this setting, it is those traders who are most volatile in their opinions who do most to make the market price accurate. This is because changes in the fundamental value often take time to be incorporated into the price. The fundamentalist only expects mis-pricing to be corrected by 1% per day, meaning that, other things being equal, there is some persistence in deviations. Short period trend followers may inadvertently capture the trend associated with these deviations, speeding up the mean reversion. Longer period traders, in contrast, may capture deviations which have already been corrected, and so move the price away from the fundamental.

In the real time case, when contrarian traders are present, the tracking error is increased. However, as Table 1.2 shows, this is accompanied by lower volatility.
Whilst the market may have been relatively calm, there is a disconnection between the market price and fundamental value. In contrast, the higher volatility evidenced in this setting when trend followers are present is associated with lower tracking error. The trend followers, whilst causing volatility, must be moving the price back towards the fundamental price.

1.4.2 Profits

In this section, we consider the average profits of each trading strategy across all possible market ecologies. We collect results for fundamentalists, trend followers and contrarians, aggregating across time windows. We calculate average annual profits for each permutation of the $2^{24}$ technical strategies. Given that the number of strategies varies in each ecology, we present our results as profits per trader.\(^7\)

Our experimental design is such that the distributions of profit illustrated in Figure 1.2 are robust to a wide variety of fundamentalist population proportions. Given that the fundamentalist is present in all ecologies, the variability in population composition is a result of the make-up of the technical trading population in each of the $2^{24}$ permutations. Specifically, we use the following method for determining population weights. First, the make-up of the technical trader permutation implies the total number of trading strategies in the market ecology. If, for example, there are 5 technical trading strategies, then there are 6 trading strategies in total, once the fundamentalist has been included. Second, we allocate an equal population share according to the number of strategies in each permutation. Continuing with the example of 5 technical trading strategies, the fundamentalist makes up 1/6 of the population, as do each of the 5 technical

\(^7\)Calculated as gross profit divided by population share.
trading strategies.

In this way, the fundamentalist population share varies from a maximum of 50% (when there is only 1 technical trading strategy in a specific permutation) to 4% (when there are 24 technical trading strategies present). Hence, it is clear that the population share of the fundamentalist does not explain the positive mean and skew of the fundamentalist profit distributions shown in Figure 1.2. Also, population proportions are constant across simulations, with CARA utility ensuring that trader demands are independent of accumulated profit. An interesting avenue for future research is to examine whether the fundamentalist loses this advantage in the more realistic setting of a limit order market.

The left column of Figure 1.2 presents the results for lagged information. The losses of contrarian strategies largely feed the profits of the fundamentalists, who make positive returns in all scenarios. There is also evidence that trend-followers gain from some of these losses, as they benefit from positive-feedback effects. In this setting, there is little difference between the trend-following momentum and moving average rules: both distributions are centered around zero and have long right tails. In contrast, the breakout rule exhibits a tighter, symmetrical distribution. This is because the breakout rule, only being concerned with the high or low of a range of prices, is insensitive to erratic price movements within those bounds. The endpoints of the momentum rule and the value of the moving average change with every new price—resulting in higher profit volatility, which in some circumstances can result in very high gains.

In line with the price deviation results of the previous section, the right column of Figure 1.2 shows that the absolute levels of profits and losses are much reduced when technical traders update their orders contemporaneously with prices. This
Figure 1.2: Histograms of average annual profits for traders using lagged information (Left) and real time information (Right). Top: Breakout rules, Middle: Momentum rules and Bottom: Moving average rules. In each case the dashed line is for fundamentalists, the solid line is for trend followers and the dotted line is for contrarians.
change means that in each time step each trader is behaving in a way they believe is the optimal manner—trading at a price consistent with their beliefs. In the lagged price scenario, a trader’s actions are in essence out of date. The direction of their trade may be the opposite of what would be desired if the current market price were included. This seemingly innocuous change in the way traders process price information limits the number of profit opportunities.

An examination of the composition of winners and losers reveals the driving force behind this increased market efficiency. All of the trend-following strategies now accumulate losses, with the fortunes of contrarians also reversing. In particular, the distribution of the moving average contrarian strategies almost exactly matches that of the fundamentalists, suggesting that technical strategies can mimic arbitrage strategies if information is processed in a timely manner.

It is clear that the ability of trend-following traders to push prices for their own benefit is largely removed when the population as a whole updates prices in real time. In the absence of positive-feedback effects, temporary deviations from fundamental value are quickly eradicated by fundamentalists, who now face fierce competition from contrarians for these limited wealth transfers. The limits to arbitrage have been overcome by those members of the noise trading population who push prices back to equilibrium.

1.4.3 Strategy Interaction

In this section, we consider the interaction of trading strategies, identifying those ecologies that are most conducive to profit or loss-making for each type. Strategies are classified both by rule type and by the size of the data window they use.
The first column of each table presents the unconditional mean of annual profits by strategy type. White cells represent profitable strategies, whilst grey cell represent loss making approaches. Subsequent columns measure the incremental impact of the inclusion of strategy types in the population. Cells are shaded grey if the inclusion of that strategy decreases profitability, and white if their inclusion is beneficial. For example, in Table 1.4 fundamentalist profits have an unconditional mean of 122.28. The inclusion of trend following rules diminishes these profits, whereas the inclusion of contrarian rules—in particular moving average rules—increases profitability. We have highlighted those strategies that have the largest impact on profitability in bold-type.

Table 1.5 confirms the reduction in the scale of wealth transfers when traders use up-to-date price information. Otherwise, the shading of the table matches that of Table 1.4, but with one major difference: fundamentalists accrue greater profits when the rest of the population is dominated by trend-following rules. The histograms demonstrate a similarity between the distributions of fundamentalists and contrarians—Table 1.5 adds further evidence that contrarians trade in the same direction as fundamentalists. Even though the contrarians can be regarded as noise traders, they behave much the same as fundamentalists, alleviating the limits to arbitrage evident in the lagged prices protocol.

Fundamentalists and trend followers benefit from the inclusion of contrarian rules, but suffer from the inclusion of trend followers. Sensitivities are the opposite for contrarians, who benefit from the presence of trend followers, but suffer in the presence of other contrarians. The inclusion of traders of an opposite class provides strategies with individuals willing to take opposite positions—leading to potentially greater volumes. However, traders of the same type serve as compe-
tition. Within each class, it is evident that in the majority of cases strategies are most sensitive to the inclusion of moving average rules, closely followed by momentum rules and, to a much lesser extent, breakout rules\(^2\). This reflects the frequency with which indicator functions change sign. As highlighted above, the breakout indicator is measured with reference to a range, which can remain unaltered for long periods. The indicators of the other rules, however, are measured against values that change with each subsequent price.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Breakout</th>
<th>Fundamental</th>
<th>Trend Follower</th>
<th>Momentum</th>
<th>Moving Average</th>
<th>Breakout</th>
<th>Momentum</th>
<th>Moving Average</th>
<th>Contrarian</th>
<th>Trend Follower</th>
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<th>Moving Average</th>
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<tr>
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Table 1.4: Table presenting the average profits of traders. The first column contains the average yearly profits of each trader type over all simulations in which they are present. The remaining columns detail the effect on the average profits of the row trader of the presence of traders of the column type. Traders are grouped by their rule type. Results are calculated for markets using lagged price information.

\(^2\)The current model outputs a single measure of profitability for each simulation run of each ecology. A more micro-based approach would track the time series behavior of market participants, potentially yielding further insights into the dynamics of strategy interaction.
### Table 1.5

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Table 1.5: Table presenting the average profits of traders. The first column contains the average annual profits of each trader type over all simulations in which they are present. The remaining columns detail the effect on the average profit of the row trader of the presence of traders of the column type. Traders are grouped by their rule type. Results are calculated for markets using up to date price information.

### 1.5 Conclusions

This paper has considered an environment in which ecologies of traders, consisting of multiple varieties of heterogeneous technical strategies, interact with fundamentalist traders. We have utilized two different market protocols, one of which assumes that market participants include the currently-being-determined price in their decision making. Fundamentalists submit demands based on an assumed rate of convergence to a known fundamental level of a risky asset. Technical traders make decisions using historical windows of market prices and by
invoking the notion of the annual risk/reward profile of an ‘acceptable’ trading strategy.

We find that in many settings the presence of a technical trader reduces market volatility or increases the accuracy with which the market price tracks the underlying fundamental value. This is particularly true for trend following strategies based on short term price patterns. In terms of the long-run profitability of strategies, when technical traders use a lagged information set, trend following traders induce positive-feedback effects—effects that benefit themselves, but cause contrarians to incur losses. The levels of profits and losses in the market are substantial, indicating that market prices are doing a poor job of instantaneously adjusting to the arrival of new information. The consequences of the ‘real time’ setting, in which technical traders update their rules using the most up-to-date information, are profound. Not only are the absolute levels of profits and losses significantly reduced—as arbitrage opportunities are quickly eradicated—but contrarian traders now join fundamentalists in profiting from this return to fundamental value. We also show that the inclusion of competing strategies within an ecology can help or hinder the profitability of individual strategies.

Our approach invites further development. The set of trading strategies considered in this paper was constrained to a number which could be analyzed effectively. In reality, there are many different classes of strategy which we do not consider, but whose presence could radically affect the behavior of the market. This paper also focuses primarily on profits, but an important next step is to analyze market dynamics over time in the presence of changing population shares. If individuals adapt their strategy based on recent profits, then the profile of profits in a particular ecology gives the instantaneous derivative of population
change. A detailed analysis of this type of dynamic would potentially allow the identification of groups of strategies which lead to fixed populations, and those which lead to chaotic mixes.
Chapter 2

Capital Allocation In A Delegated Trading Model

We present a model of capital allocation in a foreign exchange proprietary trading firm. The owner allocates capital to individual traders, who operate within strict risk limits. Traders specialize in individual currencies, but are given discretion over their choice of trading rule. The owner provides the simple formula that determines position sizes—a formula that does not require estimation of the firm-level covariance matrix. We provide supporting empirical evidence of excess risk-adjusted returns to the firm-level portfolio, and we discuss a modification of the model in which the owner dictates the choice of trading rule.
2.1 Introduction

We propose a model of a foreign exchange trading firm owner sharing capital amongst a group of traders. His objective is to earn excess risk-adjusted returns to the firm-level portfolio, but under the constraint that his employees trade as individuals. They specialize in individual currencies, concentrating solely on the exchange rate between their designated currency and the U.S. dollar.

We contribute to the literature on risk-adjusted performance measurement (RAPM) in two distinct ways. First, we derive a simple plug-in formula for capital allocation that may be adapted to different risk measures, and second, we highlight the impact organizational constraints may have on the allocation problem.

A full mean-variance optimization exercise (based on the firm-level covariance matrix) would likely generate a volatile capital allocation scheme. This is due in part to the high sensitivity of the optimal weights to changes in the problem setup brought about by time-varying sample estimates of variances and covariances, but also due to estimation error. A literature has developed that deals with the problem—the shrinkage literature. We propose that the owner adopts a simple risk budgeting scheme that is based on the conditional volatilities of individual currencies, but not on the correlations between them.

We discuss two versions of the model: the ‘discretionary’ and the ‘automated’ model. In the former, each trader is tasked with trading a single currency, but is given discretion over his choice of trading rule. In the latter, owners dictate

\[1\] For a survey of the problem approached from a Value-at-Risk perspective see Aziz and Rosen [2010].

\[2\] See, for example, Ledoit and Wolf [2003] and Ledoit and Wolf [2004].
both the risk limits and the trading rule choices. In both versions, the owner provides the formula that determines position sizes. The key distinction between the models lies in the way information is processed. The discretionary model is an ‘as-if’ model, in which optimal rules are chosen with the benefit of hindsight—accordingly, a higher statistical threshold is put in place if the results are to be deemed significant. In the automated model, the owner’s choice of rule is adapted to the information actually available in real time. His choice of rule is based on an ever-expanding information set, and can be seen as an exercise in statistical learning.

The remainder of the paper is set out as follows. In Section 2.2, we develop the position sizing formula for the case of a single trader. In the case of multiple traders, we introduce a simple method of risk budgeting that is based on an equitable distribution of capital. We assume that risk budgets are binding, and that variations in position sizes are driven primarily by time variation in conditional volatilities. Section 2.3 introduces the owner’s forward-looking risk-adjusted performance target—the target Sharpe ratio acts as the transmission mechanism from conditional volatilities to position sizes. Section 2.4 provides supporting empirical evidence of the ability of both versions of our model to deliver statistically and economically significant excess returns to the firm-level portfolio. Section 3.6 concludes, and discusses concurrent research in which we allow traders to use their full risk allocations selectively.
2.2 Position Sizing Formula

We begin by solving the owner’s capital allocation problem for the case of one trader. In this case, the solution provides an optimal level of leverage. The model follows Campbell and Viceira [2002], modified to the choice between a foreign currency and a risk-free domestic asset.

The owner exhibits constant relative risk aversion (CRRA), with power utility defined over next-period wealth:

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma} - 1}{1 - \gamma},$$  \hspace{1cm} (2.1)

where $W$ is wealth, and $\gamma$ is the coefficient of relative risk aversion.

We assume portfolio returns are log-normally distributed, which implies that next period wealth is also log-normally distributed. It is a standard result of the log-normal density function that, if $\ln (W_{t+1})$ is normally distributed, then

$$\mathbb{E} [W_{t+1}] = \exp \left( \mathbb{E} [\ln (W_{t+1})] + \frac{1}{2} \mathbb{V} [\ln (W_{t+1})] \right)$$  \hspace{1cm} (2.2)

where $\mathbb{E}$ and $\mathbb{V}$ are the expectation and variance operators, respectively. Taking natural logs of both sides of Equation 2.2 yields

$$\log (\mathbb{E} [W_{t+1}]) = \mathbb{E} [\ln (W_{t+1})] + \frac{1}{2} \mathbb{V} [\ln (W_{t+1})]$$  \hspace{1cm} (2.3)

The owner’s objective is to maximize the expected utility of next-period wealth:

$$\max \frac{\mathbb{E} (W_{t+1}^{1-\gamma})}{1 - \gamma}$$  \hspace{1cm} (2.4)
subject to the budget constraint

$$W_{t+1} = (1 + R_{p,t+1}) W_t, \quad (2.5)$$

where $R_{p,t+1}$ is the simple portfolio return.

To maximize $E \left[ W_{t+1}^{1-\gamma} / (1 - \gamma) \right]$, one can instead maximize $E \left[ \log (W_{t+1}^{1-\gamma}) \right]$, since $1 - \gamma$ is a constant. Furthermore, the same maximum is obtained by maximizing $\log \{ E \left[ \log (W_{t+1}^{1-\gamma}) \right] \}$. Therefore, the owner's objective can be restated equivalently as

$$\max \log \{ E \left[ \log (W_{t+1}^{1-\gamma}) \right] \} = E \left[ \log (W_{t+1}^{1-\gamma}) \right] + 1 \left[ (1 - \gamma) \right] V \left[ \log (W_{t+1}^{1-\gamma}) \right]. \quad (2.6)$$

Now, by dividing throughout by $(1 - \gamma)$, and by applying natural logs to the budget equation, Equation 2.5, the owner’s equivalent maximization problem is

$$\max E \left[ \log (W_{t+1}) \right] + \frac{1}{2} (1 - \gamma) V \left[ \log (W_{t+1}) \right] \quad (2.7)$$

subject to the constraint

$$\log (W_{t+1}) = \log (R_{p,t+1}) + \log (W_t). \quad (2.8)$$

Since $\log E [1 + R_{p,t+1}] = E [\log (1 + R_{p,t+1})] + \frac{1}{2} V [\log (1 + R_{p,t+1})]$, then substituting the budget constraint in the objective function yields the mean-variance
objective function

$$\max \mathbb{E} \left[ \log (1 + R_{p,t+1}) \right] + \frac{1}{2} (1 - \gamma) \mathbb{V} \left[ \log (1 + R_{p,t+1}) \right].$$  \hfill (2.9)

It is now convenient to introduce the lower case notation $r_{p,t+1} \equiv \log (1 + R_{p,t+1})$, and the simpler-looking objective function

$$\max \mathbb{E} [r_{p,t+1}] + \frac{1}{2} (1 - \gamma) \mathbb{V} [r_{p,t+1}].$$  \hfill (2.10)

The maximization problem is now stated in terms of log portfolio returns, which are a non-linear combination of the individual assets in the portfolio. We follow the Campbell and Viceira [2002] linear approximation of these returns, again adapted to the special case of foreign exchange. The equation of the simple return $R_{t+1}$ to the foreign currency is

$$1 + R_{t+1} = (1 + R_{0,t+1}^* \frac{S_{t+1}}{S_t}),$$  \hfill (2.11)

where $R_{0,t+1}^*$ is the foreign risk-free interest rate, and $S_t$ is the exchange rate expressed as domestic currency units per unit of foreign currency. Taking natural logarithms throughout Equation 2.11 gives

$$\log (1 + R_{t+1}) = \log (1 + R_{0,t+1}^*) + \log (S_{t+1}) - \log (S_t).$$  \hfill (2.12)

The trader allocates $\alpha_t$ of his wealth to the foreign currency, and $1 - \alpha_t$ to the domestic risk-free asset, giving an equation in terms of the portfolio simple return
where $R_{0,t+1}$ denotes the rate of interest on the domestic risk-free asset. This equation can be rearranged to give

$$
\frac{1 + R_{p,t+1}}{1 + R_{0,t+1}} = 1 + \alpha_t \left( \frac{1 + R_{t+1}}{1 - R_{0,t+1}} - 1 \right).
$$

(2.14)

Substituting for $1 + R_{t+1}$ from Equation 2.11 in Equation 2.14 and taking natural logarithms throughout, yields

$$
r_{p,t+1} - r_{0,t+1} = \log \left\{ 1 + \alpha_t \left[ \exp \left( r^*_{0,t+1} - r_{0,t+1} + s_{t+1} - s_t \right) - 1 \right] \right\}.
$$

(2.15)

Here we have introduced the lower-case notation $r_{p,t+1} \equiv \log (1 + R_{p,t+1})$, $r_{0,t+1} \equiv \log (1 + R_{0,t+1})$, $r^*_{0,t+1} \equiv \log (1 + R^*_{0,t+1})$, and $s_t \equiv \log (S_t)$. Equation 2.15 shows that portfolio excess returns are a function of the log interest rate differential, $r^*_{0,t+1} - r_{0,t+1}$ and the log return on the exchange rate, $s_{t+1} - s_t$. Let

$$
f \left( r^*_{0,t+1} - r_{0,t+1}, s_{t+1} - s_t \right) = \log \left\{ 1 + \alpha_t \left[ \exp \left( r^*_{0,t+1} - r_{0,t+1} + s_{t+1} - s_t \right) - 1 \right] \right\}.
$$

(2.16)

The second-order Taylor expansion of the function $f$ around the point $r^*_{0,t+1} -$
\[
r_{0,t+1} + s_{t+1} - s_t = 0 \Rightarrow \]
\[
f \left( r_{0,t+1}^* - r_{0,t+1} + s_{t+1} - s_t \right) \approx f(0) + f'(0) \left( r_{0,t+1}^* - r_{0,t+1} + s_{t+1} - s_t \right) \]
\[
+ \frac{1}{2} f''(0) \left( r_{0,t+1}^* - r_{0,t+1} + s_{t+1} - s_t \right)^2, \tag{2.17}
\]

with \( f'(0) = \alpha_t \) and \( f''(0) = \alpha_t (1 - \alpha_t) \). After replacing \( \left( r_{0,t+1}^* - r_{0,t+1} + s_{t+1} - s_t \right)^2 \) with its conditional expectation \( \sigma_t^2 \), the linear approximation for excess log returns can be written as

\[
r_{p,t+1} - r_{0,t+1} = \alpha_t \left( r_{0,t+1}^* + -r_{0,t+1} + s_{t+1} - s_t \right) + \frac{1}{2} \alpha_t (1 - \alpha_t) \sigma_t^2. \tag{2.18}
\]

The final stage is to substitute Equation 2.18, and the corresponding variance of portfolio log returns, \( \alpha_t^2 \sigma_t^2 \), into the objective function Equation 2.10:

\[
\max \alpha_t E_t \left( \left( r_{0,t+1}^* - r_{0,t+1} + s_{t+1} - s_t \right) + \frac{1}{2} \alpha_t (1 - \alpha_t) \sigma_t^2 + \frac{1}{2} \gamma \right) \alpha_t^2 \sigma_t^2. \tag{2.19}
\]

The first-order condition yields an optimal allocation of wealth to the foreign currency of

\[
\alpha_t = \frac{E_t \left( s_{t+1} - s_t \right) + r_{0,t+1}^* - r_{0,t+1} + \sigma_t^2 / 2}{\gamma \sigma_t^2}. \tag{2.20}
\]

The proportion of wealth allocated to the foreign currency is increasing in the expected appreciation of the foreign currency, and increasing in the differential between the foreign interest rate and the domestic risk-free rate. The proportion is decreasing in conditional volatility, and decreasing in the owner’s coefficient of relative risk aversion.
Portfolio Construction

Of course, the owner’s allocation problem is complicated by there being multiple currencies, and by each currency being assigned to an individual trader. Although we do not discuss the principal-agent problem in this paper, employees in proprietary firms commonly have individual contracts that detail percentage profit splits with the owner, who in return offers substantially reduced commissions that reflect the large trading volumes the firm places with third-parties. That traders act as individuals—rather than as part of a team—restricts the ability of the owner to allocate capital in accordance with a full mean-variance optimization scheme. The allocations would likely be too volatile, with certain traders being allocated large shares of capital purely on the basis of pairwise correlations between currencies. We suggest that the owner is more likely to adopt an equitable scheme, with departures reflecting the discipline of traders in staying within their risk limits, or by their ability to choose ‘good’ trading rules. As an abstraction, we adopt the purely equitable approach, and leave for future research the interesting questions of how to measure and reward individual performance.

We suggest that the owner achieves an equitable allocation simply by multiplying his coefficient of relative risk aversion by the number of traders in the firm. Diversification benefits to the firm-level portfolio ensure that portfolio risk lies within the owner’s original risk tolerance. An equitable allocation can be thought of as a risk-adjusted naive diversification\(^3\) strategy without short-sale constraints. A justification for this approach could be to avoid the problem of distinguishing current profitability due to chance from that of trading skill. Logistically, the

\(^3\)Naive diversification is a strategy which allocates an equal share of capital to the constituents of a portfolio constructed with a no-sales constraint. See, for example, DeMiguel et al. [2009].
owner then presents a simple position-sizing formula to each trader, with variations in position size reflecting changes in conditional volatility and the level of the firm’s capital:

\[
\alpha_t = \frac{1}{N} \frac{\mathbb{E}_t (s_{t+1} - s_t) + r^*_{0,t+1} - r_{0,t+1} + \sigma_t^2 / 2}{\gamma \sigma_t^2},
\]

(2.21)

where \(N\) is the number of traders. It now remains to discuss how the owner calculates conditional volatility, and how traders use trading rules in forming expectations.

### 2.3 Expectations and Conditional Volatility

[Jackson and Ladley, 2013, Chapter 1 of this thesis], in their study of an artificial asset market, introduce the notion of a ‘target Sharpe Ratio’. The idea offers a practical solution to the problem of mapping binary signals into expectations, but also offers a practical insight into the way traders use simple rules-of-thumb in their decision-making. The method is grounded in several references in the literature to firms’ use of threshold levels of risk-adjusted profitability. Lyons [2001] provides anecdotal evidence that foreign exchange trading firms only allocate capital to those strategies expected to yield annualized Sharpe ratios in the range 0.5 to 1.0; Grinold and Kahn [2000] and Menkhoff and Taylor [2007] suggest that 0.5 is a common benchmark used for identifying ‘good’ trading rules.

We propose, uncontroversially, that the owner expects to earn a risk premium as compensation for being exposed to exchange rate risk. Owners adopt a target Sharpe ratio, which when rearranged and augmented by a trading rule signal
gives an expression for the expected appreciation of each foreign currency:

\[ E_t(s_{t+1} - s_t) = I_t \frac{\text{SR}_{\text{target}} \times \sigma_t}{\sqrt{250}}. \] (2.22)

Here \( \text{SR}_{\text{target}} \) is an annualized measure of the Sharpe ratio, \( I_t \in \{-1, 1\} \) is a binary signal, and we assume that there are 250 trading days in a year. Owners use an exponentially-weighted moving average (EWMA) estimate of volatility. The advantage of the method—which is well established in the risk management industry—is that it can be used to produce estimates extremely quickly. The EWMA estimator is defined by

\[ \hat{\sigma}_t^2 = (1 - \lambda) \mu_{t-1}^2 + \lambda \hat{\sigma}_{t-1}^2, \] (2.23)

where \( \lambda \) is a smoothing parameter and \( \mu_{t-1} \) is last period’s return. A smoothing parameter of 0.94 is generally regarded as appropriate for daily observations [Alexander and Sheedy, 2010].

Substituting Equation 2.22 and Equation 2.23 into the capital allocation equation (Equation 2.21) yields the bottom-line position sizing formula:

\[ \alpha_t = \frac{1}{N \gamma} \left( I_t \frac{\text{SR}_{\text{target}} \times \sigma_t}{\sqrt{250}} + \frac{r^*_{0,t+1} - r_{0,t+1}}{\hat{\sigma}_t^2} + \frac{1}{2} \right). \] (2.24)

Let us examine the sensitivity of the position size to each variable in Equation 2.24. First, we use a simple risk-budgeting scheme (rather than full mean-variance optimization). To allocate \( \frac{1}{N} \) of optimal position sizes to each currency is to be conservative; likely there would be benefits to diversification at the portfolio level that would allow larger position sizes in each currency. Second, position
sizes are inversely related to the risk aversion, \( \gamma \), of the owner. Now, examining the terms in the parentheses, a higher target Sharpe ratio implies higher expected returns, and hence larger position sizes. The target Sharpe ratio—or the market price of risk—is the mechanism that maps volatility into expected returns. On balance, however, higher estimates of conditional volatility are accompanied by smaller position sizes. Even though the target Sharpe ratio heuristic maps higher volatilities into greater absolute expected exchange rate movements, smaller position sizes result, as the risk term dominates. Larger position sizes are taken when trading rule signals act in the same direction as the interest rate differential—the ‘carry trade’ effect.

### 2.3.1 Trading Rules

In the ‘discretionary’ version of our model, traders are free to choose the trading rule that generates signals in their particular currency. In the ‘automated’ version, the owner dictates the choice of trading rule. We now describe the choice set of trading rules.

We include four types of rule—designed to broadly follow those of Qi and Wu [2006], who in turn apply the stock index rules of Sullivan et al. [1999]. With the exception of the ‘filter’ rule, the variable of interest is the number of days of sample data. There are four types of trading rule, each having 250 possible parameter values, giving a total trading rule universe of 1000 rules. The ‘momentum’, ‘moving average’, and ‘trading range break’ rules are trend-following rules, whereas the ‘filter’ is a contrarian rule. Each rule is described below.
Momentum
The momentum indicator signals whether the rate of change of the exchange rate has been positive or negative over a historical time period \( n \in \{1, 2, 3, ..., 250\} \):

\[
I_t(n) = \begin{cases} 
+1 & \text{if } S_t > S_{t-n} \\
0 & \text{if } S_t = S_{t-n} \\
-1 & \text{if } S_t < S_{t-n} 
\end{cases}
\]  
(2.25)

Moving Average
The moving average indicator offers a slightly more complicated version of the momentum rule; it includes all sample points in its calculation:

\[
SMA_t = \frac{\sum_{t=n}^{t} S_t}{n+1}.
\]  
(2.26)

The indicator function for the simple moving average rule is

\[
I_t(n) = \begin{cases} 
+1 & \text{if } S_t > SMA_t \\
0 & \text{if } S_t = SMA_t \\
-1 & \text{if } S_t < SMA_t 
\end{cases}
\]  
(2.27)

Trading Range Break
The \( n \)-day breakout indicator generates a positive signal if today’s close is greater
than the highest high of the previous \( n \) prices:

\[
I_t(n) = \begin{cases} 
+1 & \text{if } S_t > \max(S_{t-1}, S_{t-2}, \ldots, S_{t-n}) \\ 
-1 & \text{if } S_t < \min(S_{t-1}, S_{t-2}, \ldots, S_{t-n}) \\ 
I_{t-1} & \text{otherwise}
\end{cases}
\] (2.28)

The indicator is zero until the first breakout, and maintains this value until a breakout occurs in the opposite direction.

**Filter**

The filter rule focuses on recent highs and lows. Consider a falling market. The low is reset at subsequent lower lows until an \( n \)-percent rise generates a buy signal. We consider \( n \in \{1.000\%, 1.006\%, 1.012\%, \ldots, 2.494\%\} \), designed to capture 250 rules in a range consistent with the previous literature. Sell signals in a rising market are generated similarly.

The trading rule signals provide the final piece of information required by the position sizing formula, Equation 2.24. We provide empirical evidence that the owner is able to allocate capital equitably, whilst still generating firm-level excess risk-adjusted returns. Our exchange rates are drawn from the Federal Reserve Board’s H.10 series, and interest rates are British Bankers Association 3-month LIBOR rates. They cover the period from 4th January 1999 to 20th January 2012, and comprise six major foreign currencies: the Australian dollar, British pound, Canadian dollar, Euro, Japanese yen and Swiss franc. We now compare the returns to the ‘discretionary’ and ‘automated’ versions of our model.
2.4 Results

In the discretionary model, the owner dictates the position sizing formula, but allows traders discretion in their choice of trading rule. This version offers the owner diversification across methods, as well as diversification across currencies. The discretionary model raises the question of how traders choose their trading rule. Although this is an interesting question in itself, we sidestep the modeling problem by allowing traders to choose the optimal in-sample rule for their particular currency. Clearly this is not achievable in reality, so any statistical inference drawn from the exercise must take into account the so-called ‘data snooping’ problem [Lo and MacKinlay, 1990].

The Reality Check [White, 2000] tests whether the best rule beats the null hypothesis of zero excess profitability. The idea is that the researcher can search aggressively across a wide variety of rules, safe in the knowledge that the distribution of the test statistic under the null hypothesis adjusts to compensate for the increased chance of achieving ‘lucky’ results across many searches. The time series bootstrap [Politis and Romano, 1994] generates pseudo-time series of returns by sampling blocks of observations from the empirical series, where the size of each block is drawn from a geometric distribution with mean size $q$. The size of the block is an increasing function of the dependency evident in the empirical data—we use a conservative block size of $q = 10$, as in Sullivan et al. [1999].

The following iterative procedure [White, 2000] obtains the $p$-value for the best model. Starting with the first model, and $B = 1000$ bootstrap replicate
series, the test statistic $\bar{V}_1$ is defined as

\[ \bar{V}_1 = n^{1/2} \bar{R}_1, \quad (2.29) \]

where $\bar{R}_1$ denotes the mean excess return of the first model, and $n$ is the number of returns. For each of the $B = 1000$ bootstrap replicate series, one calculates the statistic

\[ \bar{V}_{1,i}^* = n^{1/2}(\bar{R}_{1,i}^* - \bar{R}_1), \quad i = 1, ..., 1000 \quad (2.30) \]

where the superscript \(^*\) identifies simulated series. The $p$-value of the first rule is obtained by comparing $\bar{V}_1$ to the percentiles of $\bar{V}_{1,i}^*$. One then proceeds to examine the second trading rule. Compute

\[ \bar{V}_2 = \max\{n^{1/2} \bar{R}_2, \bar{V}_1\} \quad (2.31) \]

and

\[ \bar{V}_{2,i}^* = \max\{n^{1/2}(\bar{R}_{2,i}^* - \bar{R}_2), \bar{V}_{1,i}^*\}, \quad i = 1, ..., 1000 \quad (2.32) \]

noting that Equation 2.32 uses the same replicate series as in Equation 2.30.

One proceeds recursively through the remaining $k$ models, obtaining

\[ \bar{V}_k = \max\{n^{1/2} \bar{R}_k, \bar{V}_{k-1}\} \quad (2.33) \]

and

\[ \bar{V}_{k,i}^* = \max\{n^{1/2}(\bar{R}_{k,i}^* - \bar{R}_k), \bar{V}_{k-1,i}^*\}, \quad i = 1, ..., 1000. \quad (2.34) \]

The $p$-value of the optimal rule is obtained by comparing $\bar{V}_k$ with the percentiles
Table 2.1: Table presenting the excess returns, Reality Check $p$-values, and Sharpe ratios of individual traders in the ‘discretionary’ model. All simulations use a target Sharpe ratio of 0.5 and deduct 0.05% proportional round-trip transaction costs, as in Qi and Wu [2006]. Levels of significance are * = 5 percent and ** = 1 percent.

Table 2.1 presents the results for discretionary traders. Panel A presents, for comparison, the results for the time series of returns conditioned on the trading rule signals of the optimal rule. To enable meaningful comparisons between absolute levels of excess return, we calibrate the coefficient of relative risk aversion, $\gamma$, to a restricted version of Equation 2.24, in which the conditional volatility estimate is fixed and the trader ignores the interest rate differential:

$$\alpha_t = \frac{1}{\gamma} \left( \frac{I_t \text{SR}_{\text{target}}}{\sqrt{250 \tilde{\sigma}_t}} \right) \equiv k,$$

where $k$ is a constant. Now $\gamma$ merely acts as a leverage parameter—the same percentage of capital is invested or borrowed for all signals. Reality Check $p$-values and Sharpe ratios are unaffected by the particular value of $\gamma$ chosen, but
the calibrated value of $\gamma = 3.7$ ensures that, on average, position sizes are equal with and without the owner’s position sizing formula. An equivalent exercise—and the one followed by most studies of technical trading rules—is to condition the time series of returns by a sequence of ones and minus-ones (corresponding to long and short positions), and to then analyze the conditioned time series of returns. Our restricted version of Equation 2.24 merely changes the sequence of conditioning variables to $+/-k$, where $k$ is chosen to generate position sizes that are, on average, equal to those generated by the owner’s formula.

Panel B presents the results for discretionary traders using the optimal rule in combination with the owner’s position sizing formula. This is an ‘as-if’ analysis, where we study the situation in which each trader chooses the single in-sample rule that is optimal for their currency. The improvement in Reality Check $p$-values—as traders actively manage their position sizes—is evident across all currencies. The economic significance of the results—the Sharpe ratio—increases markedly once the traders actively manage their position sizes. However, the results in Panel B present an interesting dilemma to the owner. After adjusting for data snooping bias, half of the traders appear to generate excess returns. In practice, the owner may have to balance the competing claims of the star traders (those delegated with the responsibility of trading the Australian dollar, Euro, and British pound) with those of the under-performers—traders who still offer value to the owner in terms of method and currency diversification. This principal/agent problem is one we are actively researching, but from which we abstract in the current paper.

The corresponding performance of the firm-level portfolio is described in Table 2.2. The value to the owner in imposing the position sizing formula is striking. Reality Check $p$-values for constant position sizing are 0.193 with transaction
Table 2.2: Table presenting excess returns, Reality Check \( p \)-values and Sharpe ratios for the firm-level portfolio in the ‘automated’ model. All simulations use a target Sharpe ratio of 0.5 and deduct 0.05% proportional round-trip transaction costs, as in Qi and Wu [2006]. Levels of significance are * = 5 percent and ** = 1 percent.

<table>
<thead>
<tr>
<th>Panel A: Without Position Sizing</th>
<th>Best trading rule</th>
<th>Excess return (annualised)</th>
<th>Reality check ( p )-value</th>
<th>Sharpe (annualised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00% round-trip costs</td>
<td>108-day momentum</td>
<td>4.4</td>
<td>0.193</td>
<td>0.60</td>
</tr>
<tr>
<td>0.05% round-trip costs</td>
<td>108-day momentum</td>
<td>4.0</td>
<td>0.287</td>
<td>0.54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: With Position Sizing</th>
<th>Best trading rule</th>
<th>Excess return (annualised)</th>
<th>Reality check ( p )-value</th>
<th>Sharpe (annualised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00% round-trip costs</td>
<td>108-day momentum</td>
<td>11.1</td>
<td>0.013(^*)</td>
<td>0.99</td>
</tr>
<tr>
<td>0.05% round-trip costs</td>
<td>108-day momentum</td>
<td>10.0</td>
<td>0.024(^*)</td>
<td>0.89</td>
</tr>
</tbody>
</table>

costs, and 0.287 without transaction costs. The Sharpe ratio of the best rule—the 108-day momentum rule—is 0.60, reducing to 0.54 when transaction costs are properly taken into account. In stark contrast, the returns to the firm-level portfolio when traders are charged with using the owner’s position sizing formula are both statistically and economically significant. The 108-day momentum rule is still the optimal in-sample rule, but the Reality Check \( p \)-value is now 0.024, with a corresponding Sharpe ratio of 0.89. That the Sharpe ratio has also increased provides evidence that the results have economic, as well as statistical, merit.

Table 2.3 examines the impact on individual currency traders of being forced to move away from the currency-specific optimal trading rule to the portfolio optimal trading rule, the 108-day momentum strategy. It can be seen that, with the exception of the Japanese Yen, which has been forced into losses, individual currency trading remains profitable. In Panel B, one observes consistent improvement in returns once the position-sizing formula is applied. Interestingly, position sizing turns the Japanese Yen returns into profit, suggesting that economic benefit is being derived from estimating time-varying volatility in accordance with
Table 2.3: Table presenting the excess returns and Sharpe ratios of individual traders when all traders use the 108-day momentum rule. All simulations use a target Sharpe ratio of 0.5 and deduct 0.05% proportional round-trip transaction costs, as in Qi and Wu [2006]. Levels of significance are * = 5 percent and ** = 1 percent.

In the ‘automated’ model, the owner assumes control of the trading rule choice. We assume the owner instructs every trader to use the same trading rule—the best-performing rule on the available historical data. At the beginning of the sample—where there is little historical information—the trading rule choice is volatile. But eventually the owner’s choice converges to the 108-day momentum rule that was found to be the best in-sample rule in Table 2.2.

Table 2.4 presents the results for the firm-level portfolio in the ‘automated’ model. Naturally, mean returns are lower than for the in-sample ‘discretionary’ model: the best net returns in Table 2.2 were 10.0 percent, whereas they have decreased to 7.5 percent in Table 2.4. Nevertheless, the $p$-value has decreased further to 0.021. The reduction in mean returns has been more than offset by the elimination of data-snooping bias—the owner only uses historical informa-

<table>
<thead>
<tr>
<th>Best trading rule</th>
<th>Excess return (annualised)</th>
<th>Sharpe (annualised)</th>
<th>108-day momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Without Position Sizing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australian dollar</td>
<td>103-day momentum</td>
<td>9.0</td>
<td>0.63</td>
</tr>
<tr>
<td>Canadian dollar</td>
<td>113-day momentum</td>
<td>5.9</td>
<td>0.60</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>22-day breakout</td>
<td>5.1</td>
<td>0.45</td>
</tr>
<tr>
<td>Euro</td>
<td>26-day momentum</td>
<td>9.0</td>
<td>0.86</td>
</tr>
<tr>
<td>British pound</td>
<td>101-day breakout</td>
<td>6.7</td>
<td>0.67</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>176-day breakout</td>
<td>5.5</td>
<td>0.54</td>
</tr>
<tr>
<td><strong>Panel B: With Position Sizing</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australian dollar</td>
<td>1.7% filter</td>
<td>19.6</td>
<td>1.00</td>
</tr>
<tr>
<td>Canadian dollar</td>
<td>113-day momentum</td>
<td>11.0</td>
<td>0.67</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>108-day momentum</td>
<td>7.9</td>
<td>0.46</td>
</tr>
<tr>
<td>Euro</td>
<td>26-day momentum</td>
<td>15.6</td>
<td>0.95</td>
</tr>
<tr>
<td>British pound</td>
<td>101-day breakout</td>
<td>15.3</td>
<td>0.84</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>176-day breakout</td>
<td>14.7</td>
<td>0.70</td>
</tr>
</tbody>
</table>
tion in the ‘automated’ model. The qualitative conclusion of the previous section remains—the position sizing formula is crucial in generating excess returns to the firm-level portfolio.

Table 2.4: Table presenting excess returns and Sharpe ratios for the firm-level portfolio in the ‘automated’ model. Levels of significance are * = 5 percent and ** = 1 percent.

<table>
<thead>
<tr>
<th>Panel A: Without Position Sizing</th>
<th>Excess return (annualised)</th>
<th>Time-series bootstrap (p-value)</th>
<th>Sharpe (annualised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00% round-trip costs</td>
<td>1.5</td>
<td>0.240</td>
<td>0.21</td>
</tr>
<tr>
<td>0.05% round-trip costs</td>
<td>0.9</td>
<td>0.344</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: With Position Sizing</th>
<th>Excess return (annualised)</th>
<th>Time-series bootstrap (p-value)</th>
<th>Sharpe (annualised)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00% round-trip costs</td>
<td>8.0</td>
<td>0.020*</td>
<td>0.70</td>
</tr>
<tr>
<td>0.05% round-trip costs</td>
<td>7.5</td>
<td>0.021*</td>
<td>0.66</td>
</tr>
</tbody>
</table>

Sensitivity Analysis

The owner’s coefficient of relative risk aversion, and his choice of target Sharpe ratio, are leverage parameters that do not affect the results, other than by scaling the mean returns. Variations in capital allocation are driven by time-varying estimates of volatility. The degree to which the EWMA estimator (Equation 2.23) discounts past information is determined by the smoothing parameter \( \lambda \). Setting \( \lambda = 1.00 \) is equivalent to using a constant volatility estimate, which in combination with a target Sharpe ratio, implies expectations of a constant risk premium. It is evident from the first row of Table 2.5 that the worst results follow from an assumption of constant volatility. If, however, the trader uses a value of \( \lambda \) in the region of 0.94—as suggested by J.P.Morgan/Reuters [1996]—then the results are robust to variations around this level. The results are more forgiving of a trader
who errs on the side of placing greater weight on recent returns, than of one who
errs on the side of treating volatility as constant. The implication is that the
owner should be confident using the J.P.Morgan/Reuters [1996] parameter.

<table>
<thead>
<tr>
<th>EWMA parameter (λ)</th>
<th>Excess return (annualised)</th>
<th>p-value (annualised)</th>
<th>Sharpe (annualised)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: ‘Discretionary’ Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Volatility</td>
<td>1.00</td>
<td>4.8</td>
<td>0.147</td>
</tr>
<tr>
<td></td>
<td>0.98</td>
<td>3.7</td>
<td>0.037*</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>9.7</td>
<td>0.022*</td>
</tr>
<tr>
<td>RiskMetrics™</td>
<td>0.94</td>
<td>10.0</td>
<td>0.024*</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>10.1</td>
<td>0.026*</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>10.1</td>
<td>0.028*</td>
</tr>
<tr>
<td></td>
<td>0.88</td>
<td>10.0</td>
<td>0.030*</td>
</tr>
<tr>
<td></td>
<td>0.86</td>
<td>9.9</td>
<td>0.035*</td>
</tr>
<tr>
<td><strong>Panel B: ‘Automated’ Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant Volatility</td>
<td>1.00</td>
<td>3.2</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td>0.98</td>
<td>6.4</td>
<td>0.030*</td>
</tr>
<tr>
<td></td>
<td>0.96</td>
<td>7.2</td>
<td>0.021*</td>
</tr>
<tr>
<td>RiskMetrics™</td>
<td>0.94</td>
<td>7.5</td>
<td>0.021*</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>7.6</td>
<td>0.021*</td>
</tr>
<tr>
<td></td>
<td>0.90</td>
<td>7.7</td>
<td>0.021*</td>
</tr>
<tr>
<td></td>
<td>0.88</td>
<td>7.7</td>
<td>0.022*</td>
</tr>
<tr>
<td></td>
<td>0.86</td>
<td>7.7</td>
<td>0.023*</td>
</tr>
</tbody>
</table>

Table 2.5: Table demonstrating the importance of the EWMA estimator. The smoothing parameter \( \lambda = 1.00 \) is equivalent to a constant volatility assumption; \( \lambda = 0.94 \) is the level recommended by J.P.Morgan/Reuters [1996]. All simulations use a target Sharpe ratio of 0.5 and deduct 0.05% proportional round-trip transaction costs, as in Qi and Wu [2006]. Levels of significance are * = 5 percent and ** = 1 percent.
2.5 Conclusion

We consider an organizational structure in which a trading firm owner shares capital amongst a group of traders, each of whom trades in isolation from his colleagues. We propose an equitable risk-budgeting scheme, a key strength of which is its computational simplicity. The allocation scheme offers some stability, with variations in position sizes resulting from changes in conditional volatility, rather than from traders’ past performance. We show that this abstraction of the owner’s capital allocation problem generates statistically and economically significant excess returns to the firm-level portfolio in both the ‘discretionary’ and ‘automated’ versions of the model.

We are actively researching extensions to the model that reflect the finer organizational details of actual proprietary trading firms. One departure from our abstract model is in the way owners compensate traders—traders tend to be self-employed, with formal profit-sharing contracts. Successful traders see their capital accounts grow, whereas unsuccessful traders see their accounts shrink, creating interesting intra-firm capital dynamics.

A second extension lies in the way traders use their risk limits. In this paper, we assume risk limits are binding, as the owner dictates the position sizing formula. A relaxation of this assumption creates an incentive for traders exhibiting the traits associated with Prospect Theory [Kahneman and Tversky, 1979] to trade within their limits when in the domain of profits, and only at their limits when in the domain of losses. Should the owner wish to adopt our equitable structure, his contracts will need to be structured accordingly.
Chapter 3

Portfolio Choice: The Costs And Benefits Of Asymmetric Information

In dealership markets, asymmetric information feeds through to higher transaction costs as dealers adjust their bid-ask spreads to compensate for anticipated losses. In this paper, however, we show that the presence of asymmetric information can be beneficial to those who operate in multiple markets: portfolio managers. Specifically, the actions of insiders can lower the estimation errors of portfolio selection methods, thus improving asset allocation. We develop a model of multiple markets, in which portfolio managers trade alongside informed and uniformed speculators. We contrast the performance of ‘volatility timing’ portfolio selection—a method that relies on efficient price discovery—with that of naive diversification. Volatility timing is shown to consistently outperform naive diversification on a risk-adjusted basis.
3.1 Introduction

In market microstructure models, transaction costs arise endogenously—either through the inventory management process of the monopolist [Ho and Stoll, 1981], or through the asymmetric information advantage of insiders [Glosten and Milgrom, 1985]. Repeated iteration of the Glosten and Milgrom [1985] model generates intra-day price dynamics via the price setting behavior of a market maker responding to the flow of orders arriving from a large pool of informed and uninformed traders. The degree to which intra-day prices ‘discover’ true fundamental value depends on how sensitive the dealer’s priors are to the flow of new orders—the dealer adjusts prices most rapidly when the proportion of informed trade, and the volume of orders, is high.

In this paper, we study the effects of asymmetric information in the wider context of multiple asset markets. In an individual market, a higher probability of informed trade unambiguously leads to higher transaction costs; in this respect, our findings are entirely in line with those of Glosten and Milgrom [1985]. We suggest, however, that there are subtle benefits of asymmetric information that accrue to those who operate across many markets: portfolio managers. The reason is that portfolio selection methods rely to various degrees on efficient price discovery—the ability of the market mechanism to accurately reflect underlying fundamentals. We argue that private information counteracts the impediment to price discovery inherent in low trading volume, and that there appears to be an optimal level of private information, given the other characteristics of a particular market.

Our approach is to simulate multiple assets with correlated fundamentals. In
our dealership markets, insiders act as the conduit between fundamentals and
prices. To assess the costs and benefits of asymmetric information to portfolio
managers, we contrast the performance of a strategy that relies on efficient price
discovery—the volatility timing strategy—with naive diversification. This choice
partly reflects recent developments in the portfolio choice literature, but also
reflects our preference for methods that offer the practical advantage of rapid
computation.

Attention has recently focused on portfolio management strategies that avoid
the problems associated with full mean-variance optimization: singularity in the
covariance matrix of returns, and excessive volatility in asset allocations. Restric-
tions are placed on elements of the covariance matrix, or ‘shrinkage’ estimators
are formed as weighted sums of the sample covariance matrix and a simple ‘tar-
get’ matrix; see, for example, Jagannathan and Ma [2003] and Tu and Zhou
[2011]. The naive diversification strategy [DeMiguel et al., 2009] entirely removes
the need for an estimated covariance matrix, instead allocating an equal share
of capital to all portfolio constituents. The volatility timing strategy [Kirby and
Ostdiek, 2012] is a little more involved, basing its allocation on relative volatil-
ities calculated using moving windows of asset prices. Both strategies share the
characteristics of full capital allocation and no-short-sales.

The approach of this paper is to take advantage of the simple Bayesian up-
dating mechanism offered by the binomial branching structure of the sequential
trade model [Glosten and Milgrom, 1985], while retaining the original statistical
properties of the full multivariate simulation of underlying fundamental values.
This is achieved by mapping multivariate normal returns into their Bernoulli
equivalents, a process that requires boosting the elements of the original covari-
Covariance matrices are randomly generated using a wide range of parameter values within a single-index factor model. We generate multivariate asset returns using the Cholesky factorization of these matrices, which requires matrix inversion, but we address the potential singularity problem by reconstructing those matrices with negative eigenvalues [Rebonato and Jackel, 1999].

A further innovation of this paper is to borrow the recombining tree structure of the Cox et al. [1979] binomial options pricing model. We replace the risk-neutral probabilities of Cox et al. [1979] with the probabilities implied by a single-index model with drift. Multiple markets are linked together by the correlations between their fundamental values. The recombining tree structure lays the foundation for future research on the stochastic arrival of information, as it keeps the dealer’s Bayesian updating task manageable. Information arrives at the beginning of each trading period, with true values revealed at the end of each period.

We draw an important distinction between the trading population that generates prices (uninformed and informed speculators), and portfolio managers who act upon multiple asset prices. A feature of the Glosten and Milgrom [1985] model is that as the dealer processes orders, the uncertainty of the true underlying value diminishes, in turn leading to narrower bid-ask spreads. If we were to posit portfolio managers as arriving randomly during the session—like the rest of the population—we would also randomly vary the impact of transaction costs. We prefer instead to place all portfolio manager trades at the opening bid-ask spreads of each period, which enables transaction costs to be a pure function of
the probability of informed trade. This abstraction also enables us to sidestep the tricky issue of strategic behavior when market participants trade more than a single unit. Portfolio managers in our model are able to accurately signal to the dealer that they are uninformed. In concurrent research, we consider the liquidity cost that must be borne by portfolio managers who are unable to naturally differentiate themselves from the rest of the population. In this version, portfolio managers operate in multiple ‘Kyle’ auction markets [Kyle, 1985].

The final bid-ask spread of each session is used to calculate the session ‘close’. Portfolio managers mark-to-market their holdings using closing prices. The day-to-day changes in account value imply a series of strategy returns, with mean returns and risk-adjusted returns (Sharpe ratio) following. In addition, volatility timing managers use closing prices in the volatility calculations that determine their asset allocations. This is why trading volume and the probability of informed trade have a joint influence on the performance of the volatility timing strategy. A large flow of orders makes it easier for the dealer’s posterior probabilities to converge to the true probabilities, but unless there is a sufficient level of informed trade, even high volume may be insufficient for efficient price discovery. In the extreme, with an entirely uninformed population, a competitive, risk-neutral dealer quotes a single bid/ask price, and sees no reason to adjust the price in response to trading volume. Instead, the price jumps each time the changes in fundamental value become common knowledge.

The model of fundamentals presented in Section 3.2 generates multivariate normal returns using a single index factor model. Individual assets are characterized by the sensitivity of their returns to movements in the market index, and through the portfolio’s correlation matrix. These data determine the sizes and
probabilities of ‘up’ and ‘down’ movements in our Cox et al. [1979] discretization scheme. Intra-day trade takes place in individual competitive markets that are indirectly connected by the insiders who make decisions based on private access to fundamental information. The latest change in fundamental value is made common knowledge at the end of each day, with dealers adjusting their opening spreads accordingly. Although beyond the scope of the current paper, the recombining structure of the Cox et al. [1979] scheme allows the revelation of information to occur stochastically, whilst keeping the dealer’s updating task manageable. A natural way to do this is to use a geometric distribution to randomly select the release of ‘news announcements’. Specifically, if one thinks of a simple Cox et al. [1979] binomial lattice with two branches, one can imagine tossing a coin to determine if the lattice extends to four branches (and three nodes). The lattice continues to expand with each affirmative coin toss—a ‘head’, say—until the lattice terminates with a ‘tail’. The ‘tail’ represents the arrival of news, the revelation of value, and the beginning of a new two-branch lattice. If the probability of a news event is independent and identically distributed, then the number of nodes in the tree is a random draw from a geometric distribution.

Section 3.2 also describes the Einrich and Piedmonte [1991] procedure for transforming multivariate normal random variates into their Bernoulli equivalents. We describe the Rebonato and Jackel [1999] method for dealing with singular correlation matrices, and list the parameter assumptions used in constructing our various portfolios.

Section 3.3 describes the model we use to create intra-day price dynamics and closing prices. We derive probability updating equations in terms of the probabilities of informed trade and the probability of value rising. The sizes of
price movements, and their probabilities of occurrence, feed from Section 3.2.

Once the time series of opening and closing prices has been generated, we test the performance of the naive diversification and volatility timing strategies. In Section 3.4, upon observing the vector of opening bid-ask quotes, each manager revalues his current positions, and calculates his desired holdings. The naive diversification manager allocates capital equally between assets, whereas the volatility timing manager allocates capital using rolling estimates of volatility.

In Section 3.5, we present the results, and we use nonparametric methods to identify the key drivers of portfolio performance. The key driver of mean returns is the probability of informed trade, while the key driver of the Sharpe ratio statistic is the strategy type.

The determinants of the highest mean return are intuitively straightforward: substantial volume in illiquid states, combined with low probabilities of informed trade. The determinants of a strategy’s Sharpe ratio offer a more interesting story. The Sharpe ratios of the volatility timing strategy dominate those of the naive diversification strategy across all market conditions. Since mean returns are not driven by strategy type, it must be that the volatility timing strategy offers improved risk-adjusted returns via lower risks. There are substantial improvements in the volatility timing strategy’s risk-adjusted performance as the number of assets in the portfolio is increased, but the most intriguing driver is the probability of informed trade—the Sharpe ratios corresponding to a 1% probability of informed trade are lower than those corresponding to higher probabilities. Evidently, the volatility timing strategy benefits from the improved price discovery offered by ‘reasonable’ levels of asymmetric information, but these gains are eventually overwhelmed by higher transaction costs.
The paper concludes with suggestions for future research. In particular, our recombining tree structure allows for staggered news arrivals, without the need for great complexity in the dealer’s Bayesian updating problem. The use of a geometric distribution for the timing of news arrivals would seem a sensible start, with insiders maintaining their informational advantage at all times.

3.2 Fundamentals

The log-returns of the portfolio constituents’ fundamental values are multivariate normally distributed. The returns generating process is assumed to be a single-index model, where the return on the risk-free asset is normalized to zero. An individual asset’s expected returns are a simple function of its beta coefficient and the expected return to the market index:

\[ \bar{r}_i = \beta_i \bar{r}_m, \]  

where \( \bar{r}_i \) denotes the expected return to asset \( i \), and \( \bar{r}_m \) denotes the expected return to the market index. The beta coefficient \( \beta_i \) is defined by

\[ \beta_i = \frac{\sigma_{i,m}}{\sigma_m^2}, \]  

and measures the ratio of the covariance of the returns to an asset and those of the market index to the variance of the returns to the market index.

The expected return to the market index is assumed to be constant, \( \bar{r}_m = 10\% \) p.a., with a constant annual volatility of \( \sigma_m = 20\% \) p.a. Individual volatilities \( \sigma_i \), betas \( \beta_i \), and pairwise correlations \( \rho_{i,j} \) are drawn independently from various
uniform distributions. Table 3.1 lists the various specifications. Each asset’s annual volatility is assumed to lie in the range 5% to 40%, and its beta coefficient in the range 0.50 to 1.50. The pairwise correlation coefficient between assets lies in the range 0.00 to 1.00. These parameter distributions are chosen to allow for a wide range of volatilities, as well as a variety of relationships with the market index.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Number of portfolios</td>
<td>1000</td>
</tr>
<tr>
<td>$N$</td>
<td>Number of portfolio constituents</td>
<td>{2, 5, 10}</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Market Index volatility (p.a.)</td>
<td>20%</td>
</tr>
<tr>
<td>$\bar{r}_m$</td>
<td>Market Index expected return (p.a)</td>
<td>10%</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Asset $i$ volatility (p.a.)</td>
<td>Uniform(5%, 40%)</td>
</tr>
<tr>
<td>$\beta_i$</td>
<td>Asset $i$ beta</td>
<td>Uniform(0.50, 1.50)</td>
</tr>
<tr>
<td>$\rho_{i,j}$</td>
<td>Correlation($i, j$)</td>
<td>Uniform(0.00, 1.00)</td>
</tr>
</tbody>
</table>

Table 3.1: Table presenting the parameters used in the fundamental value simulations. Portfolios consist of 2, 5, and 10 stocks. Correlation matrices are randomly generated, with each element $\rho_{i,j}$ ($i \neq j$) drawn independently from a continuous uniform distribution with range $[0, 1]$.

Each portfolio consists of 2, 5, or 10 stocks. For each of these different portfolio sizes, we simulate 1000 portfolios using randomly-generated correlation matrices. We assume that each $\rho_{i,j}$ ($i \neq j$) is drawn independently from a continuous uniform distribution with range $[0, 1]$. The elements along the main diagonal are set to 1, and those below the main diagonal are set (by symmetry) to $\rho_{j,i} = \rho_{i,j}$. The resulting correlation matrix $C$ is used to generate multivariate normal random variates. In order to be compatible with the simple intra-day sequential trade model, these multivariate random variates are then transformed to Bernoulli
random variates.

The sizes of fundamental value movements are described by the following equations:

\[ u = \exp(\sigma \sqrt{\Delta t}) \]
\[ d = 1/u, \]
\[ \text{(3.3)} \]

where \(\sigma\) denotes annual volatility, and \(\Delta t \equiv 1/250\) denotes a single day in which prices can move up \(u\) or down \(d\), where the size of the down move is simply the reciprocal of the up move.

The probabilities of the moves are calculated using a modified version of the Cox et al. [1979] discretization scheme, in which the risk-neutral drift rate is replaced by the stock’s expected return, \(\bar{r}_i\):

\[ \text{Prob}(u_i) = \frac{\exp(\bar{r}_i/250) - d_i}{u_i - d_i}. \]
\[ \text{(3.4)} \]

This enables the design of a procedure that starts by generating correlated multivariate random variates, and then maps those variates into a simpler Bernoulli distribution. The binomial process for fundamental value fits comfortably with the sequential trade model of Section 3.3, which—when iterated over many time periods—recaptures the statistical properties of the original distribution.

**Multivariate Bernoulli Transformation**

The square matrix \(C\) can be expressed in terms of its diagonal eigenvalue matrix...
Λ, and the corresponding unit-length eigenvector matrix S:

\[ CS = SA. \] \hfill (3.5)

Provided the matrix C has only non-negative eigenvalues, Equation 3.5 can be post-multiplied throughout by the inverse matrix \( S^{-1} \) to yield

\[ C = SAS^{-1}. \] \hfill (3.6)

Furthermore, since the eigenvector matrix has been defined in terms of unit-length vectors, Equation 3.6 may be written as

\[ C = SAS^T, \] \hfill (3.7)

with \( S^T \) replacing \( S^{-1} \). Now define \( B = S\sqrt{\Lambda} \). Then Equation 3.7 may be rewritten as

\[ C = S\sqrt{\Lambda}\sqrt{\Lambda}S^T = BB^T, \] \hfill (3.8)

the spectral decomposition of the correlation matrix. A matrix of correlated standard normal random variates \( X \) is constructed using the transformation

\[ X = BZ, \] \hfill (3.9)

where \( Z \) is a matrix of independent standard normal random variates.

Our objective is to use a simple mapping from the matrix \( X \) of correlated normal random variates into a matrix \( P \) of correlated Bernoulli random variates, which in turn are used in the binomial branching structure of the Glosten and
Milgrom [1985] sequential trade model. We denote the multivariate Bernoulli distribution’s marginal probabilities by \( \pi_1, \pi_2, \cdots, \pi_n \). These probabilities correspond to each asset’s probability of an up move, as defined by Equation 3.4. If a stock’s characteristics are such that it has a high expected rate of return, then its probability of an up move will be higher—the magnitude of the move is given by Equation 3.3.

The resulting correlation matrix of returns has pairwise correlation coefficients that are significantly lower than the original correlation matrix \( C \). The problem is overcome by first increasing the off-diagonal elements of \( C \) using the procedure proposed in Einrich and Piedmonte [1991]. First, the quantiles of the standard normal distribution, \( z(\cdot) \), are evaluated at the Bernoulli marginal probabilities, \( \pi_i \):

\[
z_{\pi_i} = z(\pi_i).
\]  

Then, the pairwise correlation coefficients of \( C \) above the main diagonal are replaced by numerically solving for \( r_{i,j} \) in the following equation:

\[
\Phi_2(z_{\pi_i}, z_{\pi_j}; r_{i,j}) = \rho_{i,j} \sqrt{\pi_i(1 - \pi_i)\pi_j(1 - \pi_j) + \pi_i\pi_j},
\]

where \( \Phi_2(\cdot) \) is the c.d.f. of the bivariate standard normal distribution. The correlation coefficients below the main diagonal are set as \( \rho_{j,i} = \rho_{i,j} \), ensuring that the new ‘boosted’ correlation matrix \( C' \) is square-symmetric.

Using Equations 3.5 through 3.8, spectral decomposition is performed on \( C' \). However, it is well known (especially for larger portfolios) that the correlation matrix is likely to have at least one negative eigenvalue, making it impossible to invert the correlation matrix in the first step of the decomposition. One method.
of addressing this problem is to follow Rebonato and Jackel [1999] in setting any negative eigenvalues to zero, and then reconstructing a new correlation matrix as an approximation to the original. The eigenvector matrix $S$ is post multiplied by the square-root of the corrected eigenvalue matrix $\Lambda'$ to yield the adjusted factor matrix, $B$:

$$\mathbf{B} = \mathbf{T} \mathbf{S} \sqrt{\mathbf{\Lambda}'}.$$  (3.12)

where $\mathbf{T}$ is a diagonal scaling matrix with elements $t_i = \sum_{j=1}^{n} s_{ij} \lambda_j$, i.e., the row-wise eigenvectors multiplied by the adjusted eigenvalues. The adjusted correlation matrix $\mathbf{C}''$ is defined by

$$\mathbf{C}'' = \mathbf{B} \mathbf{B}^T.$$  (3.13)

Finally, the boosted matrix of correlated standard normal random variates $\mathbf{X}' = \mathbf{B}' \mathbf{Z}$ is mapped into a matrix of correlated Bernoulli random variates $\mathbf{P}$ using the rule

$$p_{ii} = 1 \text{ if } x_{ii} \leq z_{\pi},$$

$$= 0 \text{ otherwise.}$$  (3.14)

To summarize: we randomly create a target correlation matrix $\mathbf{C}$ that describes the original multivariate distribution of fundamental returns. The pairwise correlation coefficients of $\mathbf{C}$ are boosted in order to construct a new matrix $\mathbf{C}'$ to be used in the generation of multivariate Bernoulli random variates. If the eigenvalues of $\mathbf{C}'$ are all non-negative, then spectral decomposition is performed on $\mathbf{C}'$; otherwise, a new correlation matrix $\mathbf{C}''$ is constructed from the ‘corrected’ diagonal matrix of eigenvalues. The adjusted matrix of correlated standard nor-
mal random variates is then mapped into a matrix of correlated Bernoulli random variates, which when used in conjunction with Equations 3.3 and 3.4 recovers the properties of the original correlation matrix $C$.

### 3.3 Intra-day Trading

The intra-day model is based on Glosten and Milgrom [1985]$^1$, and is used to generate time series of opening and closing prices, with a view to testing various portfolio strategies. Opening prices are used to revalue current positions, and to determine the prices at which fresh purchases and sales are transacted; the opening bid-offer spread determines transaction costs. Closing prices are prices at which it is not possible to trade, but are commonly the ones used to calculate the returns to a strategy. They also play a central role in the volatility timing strategy, as the strategy uses volatility estimates calculated from rolling windows of closing prices. Closing prices are determined by the set of dealer quotes after the final trade of the day. The price discovery mechanism is expected to function better in high-volume conditions, with aggregate order imbalances reflecting asymmetric information.

There are four market participants: informed traders, uninformed traders, portfolio managers, and risk-neutral dealers. Risk-neutral dealers do not exhibit inventory aversion; they are entirely motivated by expected returns. Price competition between dealers ensures that each dealer exactly offsets the expected losses from trading with informed traders with the expected gains from trading with uninformed traders. Provided the details of individual trades are made available

$^1$Other references include Easley and O’Hara [1992], who extend the model to include the possibility of infrequent information asymmetry, and Back and Baruch [2004].
to all dealers, the problem reduces analytically to that of one dealer.

Trading volume $\lambda$ determines the number of trades that take place each day. The sequential trade model deals with daily trading volume as a sequence of single-unit transactions between individual traders and the dealer. Traders are randomly selected, one at a time, from a large pool of informed and uninformed traders, with $q$ denoting the probability of drawing an informed trader. The dealer quotes an ask price at which traders may buy a single unit of the asset, and a bid price at which they may sell. When presented with these quotes, traders have the option to buy, sell, or pass on the trading opportunity. The dealer knows that informed traders will choose to buy only if $\text{Ask} < V_t$ (the ask is below fundamental value), and will choose to sell only if $\text{Bid} > V_t$ (the bid is above fundamental value). Uninformed traders choose to trade for reasons unrelated to private information. They are, for example, motivated by hedging requirements, or by the need to meet liabilities. We assume that, for all quotes, uninformed traders randomly buy or sell with probability $1/2$.

In the basic version of the model, the dealer learns the true value of the asset at the end of each trading period. In the meantime, his ability to keep track of value depends on liquidity (the number of trades each period), and the proportion $q$ of informed traders. Figure 3.1 illustrates the unconditional probabilities of various events, each organized by the trading decisions of informed and uninformed traders—and two possible changes in value. We assume that the dealer is fully conversant with the structure of the model, and that his specialist knowledge ensures that he uses correct values for volatility and expected returns. As a consequence, he correctly calculates the unconditional probabilities $p$ and $1 - p$ of up and down moves. Informed traders never pass, because the presence of
uniformed traders \(((1 - q) > 0)\) ensures that if the next trader buys, expected value must lie below the ‘up’ value \(V^1\). This is because the buy trade could come from an uninformed trader in the ‘down’ value state of the world. Similarly, if the next trader sells, expected value must lie above the ‘down’ value \(V^2\).

The asset price is initially set to fundamental value \(V^0\), and the returns generating process determines whether value moves up to \(V^1\) or down to \(V^2\). A trader is chosen at random from the pool of informed and uninformed traders, with \(q\) denoting the probability of selecting an informed trader, and \(1 - q\) the probability of selecting an uninformed trader. Informed traders immediately receive a signal of the new value. The dealer’s risk-neutrality, and the zero-profit condition, lead the dealer to set his quotes according to

\[
\text{Ask} = E[V|\text{next trader buys}] \tag{3.15}
\]

and

\[
\text{Bid} = E[V|\text{next trader sells}] \tag{3.16}
\]

The ask is set such that the dealer expects to make zero profit if the next trade is a buy. Because of the presence of uninformed traders, buy trades can occur for both values of \(V\). Equation 3.15 therefore expands to

\[
\text{Ask} = V^1 \text{Prob}\{V^1|\text{buy}\} + V^2 \text{Prob}\{V^2|\text{buy}\} \tag{3.17}
\]

where \(\text{Prob}\{V^1|\text{buy}\}\) and \(\text{Prob}\{V^2|\text{buy}\}\) serve as the dealer’s updating equations.
Figure 3.1: Figure presenting the probability of different trade types in the sequential trade model. Each terminal node of the tree corresponds to an informed or uninformed trader making a buy, sell, or pass decision.
in a dynamic setting. Using Bayes’ Rule, we obtain

\[ \text{Prob}\{V^1|\text{buy}\} = \frac{\text{Prob}\{\text{buy}|V^1\} \text{Prob}\{V^1\}}{\text{Prob}\{\text{buy}\}} \]  
(3.18)

and

\[ \text{Prob}\{V^2|\text{buy}\} = \frac{\text{Prob}\{\text{buy}|V^2\} \text{Prob}\{V^2\}}{\text{Prob}\{\text{buy}\}}. \]  
(3.19)

Using the probabilities in the rightmost column of Figure 3.1,

\[ \text{Prob}\{V^1|\text{buy}\} = \frac{\pi_1(1 + q)}{\pi_1(1 + q) + \pi_2(1 - q)} \]  
(3.20)

and

\[ \text{Prob}\{V^2|\text{buy}\} = \frac{\pi_2(1 - q)}{\pi_1(1 + q) + \pi_2(1 - q)}. \]  
(3.21)

The bid price is derived in a similar manner. Using the probabilities in the rightmost column of Figure 1,

\[ \text{Prob}\{V^1|\text{sell}\} = \frac{\pi_1(1 - q)}{\pi_1(1 - q) + \pi_2(1 + q)} \]  
(3.22)

and

\[ \text{Prob}\{V^2|\text{sell}\} = \frac{\pi_2(1 + q)}{\pi_1(1 - q) + \pi_2(1 + q)}. \]  
(3.23)

Trading volume determines the quantity of random draws from the trading population in each session. The dealer updates his bid-ask spread using the updating Equations 3.15 and 3.16. After the final trade of the day, the mid-price of the bid and ask prices is used as the closing price of the day. With a small probability \( \zeta = 0.1 \), trading volume may change from a low-volume regime to a high-volume
regime, and *vice versa*. The motivation behind this assumption comes from the empirical application of the time series bootstrap [Politis and Romano, 1994] to weakly dependent returns data. In constructing pseudo-time series, blocks of observations are drawn with replacement, in an attempt to retain the statistical properties of the empirical data series. In applying the time series bootstrap to Dow Jones daily returns, Sullivan et al. [1999] suggest using blocks of ten observations. We use this observation as a tentative motivation for a one-in-ten probability of a regime shift in our model.

Finally, the fundamental value becomes common knowledge in the time between the close of the current session and the open of the next. The dealer’s opening spread reflects this update in public information.

### 3.4 Portfolio Strategies

We consider two portfolio strategies: naive diversification and volatility timing. Both strategies are fully-invested, and exclude the possibility of short sales. The naive diversification strategy allocates a weight $w_j$ to each portfolio constituent,

$$w_j = 1/n, \quad j = 1, \ldots, n$$

(3.24)

whereas the volatility timing strategy allocates capital on the basis of a modified version of the minimum variance portfolio.

The problem set-up for the volatility timing strategy is

$$\min_{w \geq 0} w'Vw$$

(3.25)
subject to

$$\sum_{j=1}^{n} w_j = 1,$$  \hspace{1cm} (3.26)

where \( w \) is the vector of portfolio weights, and \( V \) is the variance-covariance matrix of returns. It can be shown that the solution to the problem is

$$w_j = \frac{\psi_j}{\sum_{i=1}^{n} \psi_i},$$  \hspace{1cm} (3.27)

where \( \psi_j \) is the sum of the elements of the \( j \)th column of \( V^{-1} \), the inverse of \( V \). However, the volatility timing strategy removes the need to compute the inverse by setting the off-diagonal elements of \( V \) to zero. The elements of the inverse matrix are now simply the reciprocal of the elements of the original matrix, and the solution to the problem is

$$w_j = \frac{1/\sigma_j^2}{\sum_{i=1}^{n} 1/\sigma_i^2},$$  \hspace{1cm} (3.28)

where \( \sigma_j^2 \) is the variance of the returns to asset \( j \). As with the naive diversification portfolio, the weights of the volatility timing portfolio are non-negative. Both strategies are fully-invested, and both strategies attempt to reduce the high turnover and estimation errors associated with full mean-variance optimization.

We adopt the simplifying assumption that the trades made by portfolio managers do not influence the intra-day dynamics of price\(^2\). Instead, we assume that orders are good for any size at the opening bid-ask spread, and that portfolio managers place all their orders at the open. The justification for this assumption

\(^2\)An interesting enhancement would be to include portfolio managers as part of the trading population, with their orders contributing to daily trading volume—and hence price dynamics. However, the order-splitting strategy of managers needs to be carefully addressed in such a setting.
is that portfolio managers, in our model, are uninformed. If the dealer knows
with certainty that a trader is uninformed, then he will not adjust his bid-ask
price in response to the trader’s order. Moreover, risk neutral dealers are un-
afraid of holding inventory, so do not require compensation for taking the other
side of large uninformed trader orders. The assumption that portfolio managers
place uninformed orders at the open has the additional analytical advantage of
separating the influence of private information on transaction costs from its influ-
ence (in combination with trading volume) on ‘price discovery’. A richer model
specification would analyze the trading impact of portfolio manager trades. One
way of doing so, without having to explicitly model market maker risk aversion,
is to move the auction process to that of an electronic limit order market. In
this setting, liquidity and transaction costs arise endogenously from the order
setting behavior of market participants. This is a development we are actively
researching.

Portfolio managers use the closing price to value positions at the end of each
day, which in turn allows calculation of the daily returns to each strategy. Clos-
ing prices also provide the information that the volatility timing strategy uses
for calculating rolling estimates of daily volatilities—the estimates that in turn
determine the desired weights in each asset for the next session. The following
algorithms describe the daily activities of the naive diversification and volatility
timing strategies.
Algorithm 1: Naive Diversification

*Step 1: Revalue account using opening mid-prices*

Positions $q_j$ are revalued at the dealer’s opening mid price $p_j$:

$$\text{account} = \sum_{j=1}^{n} q_j p_j + c,$$  \hspace{1cm} (3.29)

where $c$ is the value of short-term cash balances. We allow small temporary negative cash positions, but do not allow strategies to manage leverage strategically.

*Step 2: Calculate Desired Positions*

The account size is multiplied by $1/n$, and divided by the dealer’s opening mid-price to yield a new desired holding $\hat{q}_j$:

$$\hat{q}_j = \left(\text{account} \times \frac{1}{n}\right)/p_j \quad j = 1, \ldots, n.$$  \hspace{1cm} (3.30)

*Step 3: Calculate Orders*

New orders are calculated as the difference between desired positions $\hat{q}_j$ and current positions $q_j$:

$$o_j = \hat{q}_j - q_j, \quad j = 1, \ldots, n.$$  \hspace{1cm} (3.31)

where $o_j$ denotes today’s order in asset $j$.

*Step 4: Calculate Expenditure and Income*

For buy orders, expenditure is calculated using the dealer’s ask price, and for sell orders, income is calculated using the bid price:
\[ c_j = \begin{cases} 
  a_j \cdot \text{ask}_j & \text{if } a_j > 0 \\
  a_j \cdot \text{bid}_j & \text{if } a_j < 0 
\end{cases} \quad (3.32) \]

The change in the cash position is the sum of expenditure and income over all assets:

\[ \Delta c = \Sigma_{j=1}^n c_j. \quad (3.33) \]

**Step 5: Revalue Account at Closing Mid-Prices**

At the conclusion of intra-day trading, the final dealer quotes are used to calculate closing prices—the final mid-prices for each asset. The account is revalued, and the daily return to the naive diversification strategy is calculated using

\[ r_t = (\text{account}_t/\text{account}_{t-1}) - 1. \quad (3.34) \]

**Algorithm 2: Volatility Timing**

**Step 1: Revalue account using opening mid-prices**

Positions \( q_j \) are revalued at the dealer’s opening mid price \( p_j \):

\[ \text{account} = \Sigma_{j=1}^n q_j p_j + c, \quad (3.35) \]

where \( c \) is the value of any cash holdings.

**Step 2: Calculate Desired Positions**

The account size is multiplied by the weights calculated in Equation 3.28, and
divided by the dealer’s opening mid-price to yield a new desired holding $\hat{q}_i$:

$$\hat{q}_j = \left( \text{account} \times \frac{1/\sigma_j^2}{\sum_{i=1}^{n} 1/\sigma_i^2} \right) / p_j \quad j = 1, \ldots, n. \quad (3.36)$$

**Step 3: Calculate Orders**

New orders are calculated as the difference between desired positions $\hat{q}_j$ and current positions $q_j$ in each asset:

$$o_j = \hat{q}_j - q_j, \quad i = j, \ldots, n. \quad (3.37)$$

where $o_j$ denotes today’s order in asset $j$.

**Step 4: Calculate Expenditure and Income**

For buy orders, expenditure is calculated using the dealer’s ask price, and for sell orders, income is calculated using the bid price:

$$c_j = \begin{cases} 
  o_j \cdot \text{ask}_j & \text{if } o_j > 0 \\
  o_j \cdot \text{bid}_j & \text{if } o_j < 0 
\end{cases} \quad (3.38)$$

The change in the cash position is the sum of expenditure and income over all stocks:

$$\Delta c = \sum_{j=1}^{n} c_j. \quad (3.39)$$

**Step 5: Revalue Account at Closing Mid-Prices**

At the conclusion of intra-day trading, the final dealer quotes are used to calculate closing prices—the final mid-prices for each stock. The account is revalued, and
the daily return to the volatility timing strategy, $r_t$, is calculated using

$$r_t = (\text{account}_t/\text{account}_{t-1}) - 1.$$  \hspace{1cm} (3.40)

It now remains to examine large-sample returns and risk-adjusted returns to the two strategies for various market conditions. Each simulation generates 10 years of intra-day trade and closing prices. Each market condition is tested for 1000 simulations of fundamental values.

### 3.5 Results

Each cell of Table 3.2 contains the mean annual return and Sharpe ratio for 1000 multivariate simulations of fundamental values using the single-index model of Section 3.2. The upper panel contains the results for 2-stock portfolios. The middle panel contains the results for 5-stock portfolios, and the lower panel the results for 10-stock portfolios. Within each panel, the results are split horizontally into those results for the naive diversification strategy, and those for the volatility timing strategy. Vertically, the results are arranged by increasing levels of asymmetric information or probabilities of informed trade. Within each of these sections, an individual cell corresponds (vertically) to the level of trading volume in an illiquid state (10, 50, or 250), and (horizontally) to a level of trading volume in a liquid state (50, 250, 1000). For example, the upper-left cell of the top panel reports a mean return of 10.21\%, and a Sharpe ratio of 0.59 for the naive diversification strategy. This corresponds to 1000 underlying simulations of markets in which the probability of informed trade is 0.01, trading volume in the
illiquid state is 10, and trading volume in the liquid state is 50. An alternative to
the two-state model is a single-state model with constant volume, but we prefer
to allow for the possibility of the price discovery mechanism being disrupted at
the points where regime shifts occur. The probability of switching is $\zeta = 0.1$,
which is intended to reflect our intuition that markets ‘remember’ the current
regime.

For the 2-stock portfolio, an interesting pattern develops as the level of asym-
metric information increases from 0.01 through 0.20. For the (10, 50) volume
combination, the mean return for the naive diversification strategy is at its high-
est when $q = 0.01$. This is to be expected, as the dealer quotes narrow spreads
when the probability of adverse selection is low. As the probability of informed
trade rises from 0.01 to 0.05, and from 0.05 to 0.10, the naive diversification
strategy’s mean return falls to 9.85%, and then to 9.31%. The interesting change,
however, occurs when the probability of informed trade rises from 0.10 to 0.20:
the mean return rises to 9.53%, despite the dealer’s wider spreads. A similar
pattern occurs for the corresponding Sharpe ratio: 0.59, 0.58, 0.56, and then an
increase to 0.60. As this pattern disappears for portfolios with more assets, we
suggest that the pattern is linked to the naive diversification strategy’s in-built
tendency to over-allocate capital to high beta stocks.

The pattern for mean returns is broadly similar for the volatility timing strat-
agy, with apparently little difference between the volatility timing and naive di-
versification strategies’ mean returns under similar market conditions. The most
striking difference, however, is in the levels of the Sharpe ratio—the volatility tim-
ing strategy consistently produces results approximately 0.20 in excess of those
of the naive diversification strategy. In the illiquid volume combination (10, 50),
the highest Sharpe ratio of the volatility timing strategy (0.81) occurs when the level of asymmetric information is at its highest. In the most liquid combination (250, 1000), the highest Sharpe ratio occurs when $q = 0.05$, and falls thereafter. The volatility timing strategy outperforms the naive diversification strategy, not because of its similar mean returns, but because of its lower risk. Efficient price discovery is essential to its success, with the presence of asymmetric information offsetting the impediment to price discovery inherent in low trading volume. That the volatility timing strategy consistently outperforms the naive diversification strategy confirms our view that those strategies that rely on accurate prices, and in turn returns, benefit most from a reasonable level of asymmetric information.

In the next section we more formally identify the drivers of portfolio performance.

### 3.5.1 Data Visualization and Interpretation

Table 3.2 presents our results in finely-classified samples. While it is clear from the table that the Sharpe ratios of the volatility timing strategy dominate those of the naive diversification strategy, it is not easy to determine whether strategy type, or some other characteristic of market conditions, is the key driver of portfolio performance. For instance, the probability of informed trade may be important, as may be the simple diversification effect from increasing the number of portfolio constituents. Classification trees—a technique from the nonparametric statistics literature—offer an excellent way of ranking the determinants of portfolio performance, as well as providing a neat visual representation of the data. They are ideally suited to a ranking task, with the data being repeatedly partitioned according to those elements of the sample space that most reduce
Table 3.2: Table reporting portfolio performance statistics (mean returns and Sharpe Ratios) for portfolios employing the Naive Diversification and Volatility Timing strategies. The upper panel reports results for portfolios containing 2 stocks, the middle panel reports results for portfolios containing 5 stocks, and the lower panel reports results for portfolios containing 10 stocks. Column headings refer to volume in the liquid state; row headings refer to the proportion of informed traders, and volume in the illiquid state.
prediction error. The key reference is Breiman et al. [1984] 3.

The response variable $Y$ is predicted using a multivariate set of predictors $X$. In this paper, we consider two response variables—the mean return and the Sharpe ratio of a strategy. The set of predictors includes the strategy type, the number of stocks in the portfolio, the probability of informed trade, and the volume of trade in the illiquid state.

Consider first the two trees for the mean prediction task. Figures 3.2 and 3.3 are in fact drawn from one tree, but have been separated to improve legibility. Each sample of observations is represented by an ellipsis or rectangle. The ellipses represent samples that will be divided further into smaller groups; the rectangles, known as the ‘leaves’ or ‘terminal nodes’ of the tree, represent the finest partitions of the data. Theoretically, the samples can be partitioned into ever-decreasing samples, until each terminal node contains only one observation, but in practice a tree ceases to be grown (or is ‘pruned’) according to a statistical or normative criterion. With a view to clarity and parsimony, we terminate the trees using a maximum depth criterion: the number of levels below the initial sample is set to 5, meaning that each of the sub-figures, Figure 3.2 and Figure 3.3, has 4 levels. When compared with alternative statistical pruning procedures, we find that this level of detail errs on the side of parsimony—further nodes, by definition, improve the in-sample predictive accuracy of the tree, but do so with increased risk of over-fitting.

There are 144,000 observations in the full sample, which correspond to the 144 cells of Table 3.2. The pooled mean return is 9.55% for these observations. The procedure examines all possible partitions across the predictor variables,

3Software-based tutorials include Martinez and Martinez [2008] and Torgo [2011].
and chooses the best binary split—defined as the partition that most reduces the total mean-squared-error of the tree. Formally, the mean of the full sample, $\bar{y}$, is defined by

$$
\bar{y} = \frac{1}{n} y_i, \tag{3.41}
$$

and the mean-squared-error, $R$, by

$$
R = \frac{1}{n} \sum (y_i - \bar{y})^2. \tag{3.42}
$$

After the first split, there are two nodes, each with its own $\bar{y}$ and $R$. Denote these within-node squared errors by $R_1$ and $R_2$. After the first split, the mean-squared-error of the tree is the sum of $R_1$ and $R_2$.

The first partition of the mean returns classification tree splits the full sample into two groups. The first contains observations in which the probability of informed trade $q \in \{0.01, 0.05\}$, and the second contains observations in which $q \in \{0.10, 0.20\}$. These are the nodes at the top of the trees of Figures 3.2 and 3.3. Interpreting these two new samples as ‘low’ and ‘high’ asymmetric information samples, the low asymmetric information tree of Figure 3.2 has a sample mean of 9.76%, with 72,000 observations. The high asymmetric information tree also contains 72,000 observation, but with a mean return of 9.35%. The 0.41% fall in mean returns represents a flow of wealth from uninformed to informed traders.

We next consider the low asymmetric information and high asymmetric information trees. For each tree, we describe a particular path down the tree. The highest mean return of Figure 3.2 is 9.97%, represented by the second terminal node from the left. The first partition of the tree divides observations into probabilities of informed trade of $q = 0.01$ and $q = 0.05$. In markets with the lowest
probability of informed trade \((q = 0.01)\), mean returns are 9.89%, 0.27% higher than the mean returns generated by markets with \(q = 0.05\). Continuing down the \(q = 0.01\) branch of the tree, the next most important driver of performance is the number of portfolio constituents, with those portfolios containing 2 or 5 stocks generating higher returns than those with 10 stocks. An explanation for this phenomenon could be that the naive diversification strategy allocates equal levels of capital to high-volatility and low-volatility assets, thus boosting mean returns when the number of portfolio constituents is small. More clearly, in the next partition, mean returns are higher when trading volume is high in the illiquid state. Note that this path down the tree does not distinguish the returns to the naive diversification strategy from those to the volatility timing strategy.

The story is different, however, in the high asymmetric information tree. Figure 3.3 demonstrates that, conditional on the probability of informed trade being 10% or 20%, the next partition that most reduces the tree’s mean-squared-error is strategy type. The returns to the naive diversification strategy are 9.18%, whereas they are 9.52% for the volatility timing strategy. This would suggest that the naive diversification strategy generates a higher turnover of trade than does the volatility timing strategy—a feature that most impacts on performance when bid-ask spreads are wide. Continuing down the volatility timing strategy path, the next partition is according to the volume in the illiquid state, with volumes of 50 or 250 generating higher returns than a volume of 10. The final partition, branching to the farthest-right terminal node (9.66% across 6,000 observations), further distinguishes illiquid state trading volume of 50 from trading volume of 250. In sum, in markets characterized by high levels of asymmetric information, a volatility timing strategy applied under conditions of high overall liquidity gen-
erates on average mean returns of 9.66% p.a.

We conclude this section by examining the classification tree in which the response variable is the Sharpe ratio of the volatility timing strategy (Figure 3.5). We describe the path that leads to the highest Sharpe ratios. The root node of Figure 3.5 contains 72,000 observations, with a mean Sharpe ratio of 1.07. The next partition is with respect to the number of portfolio constituents, with the Sharpe ratios of 5 or 10 stock portfolios, substantially exceeding those of 2-stock portfolios (1.21 versus 0.79). Of those portfolios with 5 or 10 stocks, the next partition distinguishes the 5-stock portfolios from the 10-stock portfolios. On average, 10-stock volatility timing portfolios generate Sharpe ratios of 1.33, 0.24 higher than 5-stock portfolios. It is interesting to note, however, that the terminal nodes across the tree are partitioned according to the probability of informed trade. For 2-stock portfolios there appears to be little difference between the average Sharpe ratios in each sample, with terminal nodes containing average Sharpe ratios of 0.78, 0.79, 0.81, and 0.78. However, for better-diversified portfolios, the optimal partition splits the observations by probabilities of informed trade of 1%, and probabilities of informed trade of greater than 1%. Even though higher levels of private information lead to wider spreads and higher transaction costs, they lead to higher Sharpe ratios for both the 5-stock and 10-stock portfolios. This ‘price discovery’ effect is most pronounced in the 10-stock portfolios, with an improvement in the Sharpe ratio from 1.21 to 1.37 when the probability of informed trade exceeds 1%.

That asymmetric information aids price discovery is not an unusual finding, and is indeed a key finding of the Glosten and Milgrom [1985] model, upon which our auction process is based. The intriguing contribution of our analysis, how-
Figure 3.2: Figure presenting the mean returns tree. The tree contains the results for levels of informed trade of \( q \in \{0.01, 0.05\} \). Results are classified according to various paths down the tree. The boxes represent the terminal nodes or ‘leaves’ of the tree.
Figure 3.3: Figure presenting the mean returns tree. The tree contains the results for levels of informed trade of $q \in \{0.10, 0.20\}$. Results are classified according to various paths down the tree. The boxes represent the terminal nodes or 'leaves' of the tree.
Figure 3.4: Figure presenting the Sharpe Ratios tree. The tree contains the results for the Naive Diversification strategy. Results are classified according to various paths down the tree. The boxes represent the terminal nodes or ‘leaves’ of the tree.
Figure 3.5: Figure presenting the Sharpe Ratios tree. The tree contains the results for the Volatility Timing strategy. Results are classified according to various paths down the tree. The boxes represent the terminal nodes or ‘leaves’ of the tree.
ever, is that there appears to be an ‘optimal’ level, at which portfolio managers earn an externality. It would appear that the reason an optimum exists is because asymmetric information works to the disadvantage of portfolio managers through higher transaction costs, but works to their advantage by making their estimates of conditional variance more accurate. An interesting extension to the paper would be to examine these opposing channels in more depth, perhaps by examining individual portfolio managers, their volatility estimates, and the trading costs they incur.

3.6 Conclusion

We develop a framework in which multi-asset fundamentals are mapped into binomial processes compatible with the Glosten and Milgrom [1985] sequential trade model. Intra-day price dynamics are generated by dealers’ Bayesian updating equations, with closing prices determined by the average of the final bid and ask prices of each session. The degree to which closing prices track fundamental value is determined by the joint interaction between private information and trading volume. Higher levels of private information reduce mean returns, as dealers widen spreads to compensate for the losses incurred from informed trade. But private information also helps to improve the price discovery process, thus improving the risk-adjusted returns of strategies that rely on accurate volatility estimates.

We use nonparametric classification trees to identify and rank the determinants of portfolio performance. Mean returns are primarily driven by the probability of informed trade, whereas the strategy type—naive diversification or
volatility timing—is the key driver of risk-adjusted returns. This suggests that the higher Sharpe ratios of the volatility timing strategy arise because of its objective of minimizing risk; this does not appear to sacrifice mean returns. The diversification effect from increasing the number of portfolio constituents is the next most important driver of risk-adjusted returns, with the highest Sharpe ratios of both strategies occurring in the 10-stock portfolios. We note the interesting dominance of the volatility timing Sharpe ratios in markets when the probability of informed trade is greater than 1%. Indeed, looking down the columns of Table 3.2, it is evident that the lower mean returns associated with wider spreads are often accompanied by higher Sharpe ratios, there being an apparent ‘optimal’ level of asymmetric information, beyond which Sharpe ratios decline. These declines occur as higher levels of transaction costs begin to dominate improvements in the price discovery mechanism.

With regard to extensions and future research, we have deliberately designed the framework with flexibility in mind. We have used a single-index model, but envisage more elaborate factor models in the data generation stage. The recombining tree structure of our sequential trade model allows for stochastic news arrivals, whilst keeping the dealer’s updating task manageable. We would maintain the informational advantage of the insiders during the ‘no news’ days, thus making a distinct contribution to the literature.
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