Basel Accords and the effect on regulatory capital
The case for Extreme Value Theory during market crises in Emerging and Frontier Stock markets

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Abstract
Since the late 1980s, the Basel Committee has been intending to regulate the financial sector with a view to establish common regulatory standards for the banking industry through the Basel Capital Accords. Successive crises have uncovered several flaws in those directives that were remedied enacting tougher and more sophisticated mandates, particularly regarding the calculation of the Minimum Capital Requirements after the introduction of Value-at-Risk as the official measure to quantify market risks. However, Basel regulations have, in many respects, been incapable to forestall the adverse effects that market turmoil exerts on the banking system. The present thesis aims at analysing the MCR scheme employing the former Basel II and the current Basel III Capital Accords applying the VaR-based Internal Model Approach and the Standardised Approach through a variety of specifications in times of crisis using a sample of Emerging and Frontier stock markets. The findings detected structural glitches in the configuration of the Basel’s MCR formula, given the fact that both the SA and many inaccurate VaR models are allowed to compute MCR. Furthermore, there is clear evidence of the superiority of the Extreme Value Theory to calculate an adequate capital base during abrupt market swings. Basel regulations must act accordingly and reward the accuracy calibrating the extrinsic multiples and additional buffers in line with the behaviour of the models: the thesis underlines that, provided appropriate schemes had been applied, Basel II MCR would have prevented capital shortages in 2007-2008. The thesis also detects the presence of moral hazard and adverse incentives to utilise sharp models like EVT in Basel regulations and proposes a radical overhaul of the SA and a tailor-made evaluation of the parameters of the VaR-based IMA as a methodology to reward and entice the adoption of models that allow the correct estimation of market risk.
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Finally, I state that the PhD thesis expresses only my points of view and, therefore all my errors are my responsibility.
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Chapter 1

Introduction
1.1. Initial considerations

Early in 1825 the then British Prime Minister Lord Liverpool decided to rescue the Pole, Thornton, Free, Down & Scott, a London-based county bank, on the grounds that its collapse may spread the contagion to the whole banking sector. Therefore, banks have a long and respectable history of persuading politicians to rush to their rescue fearing systemic propagation.

The tale offers much resemblance to some contemporary situations witnessed a couple of years ago. In fact, Thornton, Free, Down & Scott, which was the London agent to many other county banks, “…had been inexcusably imprudent in not keeping more cash in the house, instead relying on (the bank’s) credit … which would enable them to borrow whenever they pleased”, as Alkan (2013) didactically expresses. After being reassured that the bank was solvent enough, the Bank of England, at the requirement of Lord Liverpool, replenished the bank’s cash coffers. Parallelisms with the subprime crisis do not require much imagination and creativity to be drawn. In fact, 183 years after, governments all around the world orchestrated controversial colossal bailout manoeuvres that, in a certain fashion, avoided the infection to spread to the otherwise healthier ones. For instance, in the US the federal government saved directly or indirectly1 Bear Sterns, IndyMac, Fannie Mae, Freddie Mac, AIG, Washington Mutual, Wachovia, Citigroup and General Motors;2 in the UK, Northern Rock, Bradford & Bingley, Lloyds and the RBS were fully or partly nationalised under the auspices of the UK bank rescue package. Other Western European countries like Belgium, France, Germany, Ireland, Luxembourg and the Netherlands also observed national governments’ throwing lifelines to collapsed financial institutions.

After such calamitous experiences, politicians around the world have hardened their stance regarding bailouts which, unfortunately, remains to be tested when hard times befall on troubled banks. Nevertheless, nations can take precautionary measures to reduce the probability of bankruptcy and, additionally, to facilitate the winding down of insolvent entities. The only action to reduce the vulnerability of banks is, unquestionably, to increase their safety: this purpose can be attained by determining the right quantity to be borrowed and secondly, increase the amount of capital to be held as a buffer against possible crises. To this end, national regulators have traditionally relied

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1 Indirectly through nonrecourse loans to the eventual purchaser.
2 Emergency measures were not restricted to banks given that AIG is an insurance conglomerate whereas, as is widely known, General Motors manufactures vehicles.
on rules devised by the Basel Committee on Banking Supervision that, regrettably, showed a series of flaws. Alkan (2013) identifies three basic lines failed to be addressed by the incumbent body of regulation contained in Basel II Capital Accord, in force at the time of the subprime disaster in 2008: in the first place, the misconception about the true risk posed by certain kinds of securities that allowed banks to hold less capital against them; secondly, the need to isolate some business lines that are structurally hazardous, like aggressive derivative positions, market making, prime brokerage and high levels of brokerage funding and, finally, the mandate that let banks employ their own models to determine the quantity of capital to constitute. Even though not exempt from obvious glitches, the Basel Committee and the national supervisors have finally acknowledged those snags and are acting accordingly. For the initial concern, the reassessment of the risk-weighted asset methodology could provide some sort of solution; the relief for the second could come in the shape of ringfencing the banks’ transactional operations whereas the enactment of new requisites to compute the Minimum Capital Requirements is aimed at the core of the capital shortage.

The present thesis is entirely devoted to the third track in the context of the relatively neglected Emerging and Frontier markets, particularly in what regards the interrelationship among the now defunct Basel II, the newly born Basel III, the development of own capital models encompassed in the Internal Model Approach and the existence of the simple Standardised Approach, which, alongside representing another avenue to calculate the equity to hold against the entity’s position, acts as the minimum amount of capital to attain. However, and to make matters worse, this lower boundary is also a snapshot of Basel rules ineffectiveness as some banks like Dexia, holding even more than enough capital to satisfy Basel rules, have ended in bankruptcy.

The embracement of the Internal Model Approach implies assessing the market risk involved in the bank’s operation with a view to estimating its Value-at-Risk, the official market risk metric demanded by the Basel Committee on Banking Supervision. As it will be explained in detail in the main body, the above expression is tantamount to bringing some volatility estimation into the risk equation, process that eventually ends in ascertaining the shape of the tails of the distribution of the variations of the prices of the securities forming the trading position of the company. Even though the empirical assessment of a wide range of assets undoubtedly highlighted the need to affix some statistical density like the Generalised Pareto Distribution that fitted more closely the patterns observed, the majority of financial institutions took deliberate steps in order to
simplify the estimation of market risk circumscribing the choice of distributions to the Normal one. Therefore, VaR is much easier to compute, to understand and, most relevantly, brings down the capital base as a result of its reduction, hence pleasing shareholders.

However, the unpleasant events of 2007-2008 painfully reminded the financial community that all those scenarios that had purposefully been excluded from the models had regretfully taken place. In other words, the events that filled the tails of the distributions –i.e., the tail risk- reappeared, thus increasing the market risk at an alarming pace and consequently uncovering capital shortages. Although with the benefit of hindsight, it is clear tail risks must be specifically modelled, switching the focus from common, everyday market conditions to acute turmoil, particularly at the time of constituting an adequate platform upon which capital requirements base.

The thesis aims, then, at tackling the aforementioned issue. Bearing in mind that the model proposed here is not a kind of modern crystal ball to predict the future, it is believed that it can provide greater insight into market risks as well as the regulations dealing with them. The present Chapter accordingly supplies the initial notions that will take shape in the course of the pertaining Chapters; in this regard, the second Section sketches the idea of risk as employed in the main text, the third Section outlines the complex issue of the quantification of risk through risk measures; the fourth one overviews the relationship among three deeply intertwined variables: risk, capital and regulations while the fifth informs the geographical location (i.e., the markets) to be dealt with in the context of the investigation. Furthermore, the sixth Section establishes the research objectives and its translation into research questions, the seventh explains the expected contribution that would emerge from the study, the eighth delivers the general conclusions from the Introduction and, finally, the ninth Section reports the structure of the thesis.

1.2. The concept of risk

According to Holton (2002), any concept about risk must encompass two key components: exposure and uncertainty. The definition, then, appears straightforward as risk is often stated as an exposure to events of uncertain results and consequences, which may be applied to any activity in life.

In general terms, Jorion (1996) states that companies are subject to three broad classes of risks: business risks, strategic risks and financial risks. Synthetically
speaking, the first category refers to those risks assumed in order to gain competitive advantages and add value to the shareholders; the second one derives from changes in the political or economic environment while the third is mainly related to likely losses in financial markets stemming from adverse market movements. More specifically, Penza and Bansal (2001:21) provide an insight into the risks banks actually face, reflected in Graph 1.1 below:

Graph 1.1
A scheme of banking risks

It is relevant to convey an idea about the risks above stated:

- **Credit risk**: risk rooting in the failure of the counterpart to honour its legal obligations belonging to any instrument: unsecured loans, bonds, gilts, notes or bills, and OTC derivatives (typically swaps and forwards);
- **Market risk**: risk of losses incurred in the wake of variations in the value of tradable assets, i.e., changes in the price of market factors. This class is usually denominated price risk;
- **Interest rate risk**: risk of sustaining deficits as a result of adverse movements in interest rates, both for interest rate-sensitive assets and liabilities;
- **Liquidity risk**: risk of failing to make payments or refinance obligations because of shortage of funds;
- **Legal risk**: risk of incurring in deficits arising from nonenforceability of contracts and/or rupture of regulatory mandates;
• **Operational risk:** risk deriving from an ample variety of failures in Information Technology, processing mistakes, fraud, etc.

Financial risk is eventually a category of risks involving the whole spectrum of risks related to the volatility of future cash flows rooting in the value of assets and liabilities: in terms of Chart 1.1, financial risks include market risk, interest rate risk and liquidity risk. In the context of banking management, market risk is increasingly becoming a subject of paramount importance, chiefly due to the following motives highlighted in Penza and Bansal (2001):

- The process of securitisation, which created a secondary market and provided a market price for traditionally illiquid assets. The adoption of mark-to-market pricing techniques ensued, hence allowing the inclusion of these securities in the banks’ trading books and fostering investments in market risk management schemes;
- The exponential growth of derivatives and derivatives-like instruments, both in volume and complexity. It has spurred the necessity to develop an integrated market risk framework;
- The increased volatility that stock prices, interest rates and foreign exchange rates have been displaying during the past 30-35 years, which has prompted the implementation of systems devised to mitigate it;
- The multiplication of trading activities across the globe that heightened the volatility and, consequently, the urgency to execute market risk management strategies.

In light of the increased importance of market risks, the current study is focused on them, with particular emphasis on stock exchanges indices. For the purposes of the present thesis, the risk management process boils down to the precise quantification of the stock market risk and the subsequent construction of adequate risk measures under the framework of the regulations issued by the BCBS. The previous statement is tantamount, then, to the adequate measurement of the volatility displayed by the asset in question—in this case, the stock market index- and its further translation into capital requirements under the framework established by the BCBS. It follows, then, that this
volatility is defined as the dispersion of the relative values (or returns) of the investment.

In this vein, the point may well be appreciated through the following example featuring the behaviour of the FTSE 100 during the period 02/04/1984 – 09/07/2013, encompassing 7392 daily observations which overall and yearly stylised facts are depicted in Chart 1.A.1 in Appendix 1.A. Additionally, Graphs 1.2 and 1.3 display the price and return levels of the FTSE 100 during that period, where the heavy slumps in the values as a result of the 1987 and 2008 crises as well as the 2002 mini-crash are discernible even to the naked eye. Furthermore, Graph 1.3, which depicts the progression of volatility (as measured by the Standard Deviation) exhibits three distinct peaks in those years (1.70% in 1987, 1.73% in 2002 and 2.36% in 2008), which represent the extent of the damage susceptible to be inflicted on the financial position of the bank. It is noteworthy, incidentally, that the nature of the markets looks more ragged in recent times with enhanced jerks and upswings in volatility than in past ones, thus underlining the importance of accurate risk management schemes.

Graph 1.2
FTSE 100. Price levels Apr/84-Jul/13

Graph 1.3
FTSE 100. Return levels Apr/84-Jul/13

Graph 1.3 corroborates the previous concepts by exhibiting the annual progression of the volatility measured by the Standard Deviation. The line seems revealing in some aspects, notably the increase in the volatility\(^3\) (i.e., the distance between peaks and troughs) that occurs in short time spans, which provokes unpleasant consequences.

\(^3\) Even though the Standard Deviation as a measure of volatility has many drawbacks (Sortino (2001)), it is still useful to pinpoint some features of the underlying phenomena. The fact that the annual Standard Deviation peaked an annual 2.36% is indicative of the turmoil recorded in that year.
The concept of risk has been inextricably linked to that of volatility and, commonly, the greater the volatility, the riskier the exposure because its value may spread out over a larger range of values, thus fluctuating dramatically in short periods in either direction. On the contrary, the oscillations in a lower volatility regime are not so extreme, but occur at a steady pace over a longer period of time instead.

1.3. An overview of risk measures
Section 1.2 above has established some clear concepts regarding risk: in the first place, risk means uncertainty and is traditionally understood as volatility, whereas in the second place, volatility needs to be quantified in order to encapsulate that otherwise elusive concept. A risk measure allows the translation of an ethereal notion into numbers, thus conveying an idea of the uncertainty, risk, or danger that the investment may bring about. However, the academic community has been unable to produce an optimal risk measure and is becoming increasingly divided in that respect.

The importance of the risk measures has been escalating over time. The first attempt to put risk into numbers is due to Markowitz, who in 1952 provided an excellent start; in effect, his Modern Portfolio Theory (MPT) explains how assets should be priced in equilibrium in such a way that, on a risk-adjusted basis, all returns are equal. In Markowitz’s framework, risk is understood as the volatility of the returns of the corresponding assets/ portfolios, where that volatility is measured as the Variance or Standard Deviation of those returns. MPT relies heavily on the Mean-Variance Criterium (MVC), which assumes that investors decide solely on the grounds of the first and second moments of the stochastic distribution that explains the behaviour of returns, i.e., the mean and the variance, although the theory is only valid under the framework of the symmetric normal distribution. In this sense, Sortino (2001) states that, even though
the true shape of the underlying distribution remains unknown, it must be positively skewed.

Hadar and Russell (1969) introduced a competing theory to the MVC affirming that the expected utility function is a function of all the moments in the probability distribution. Therefore, given that the schemes that rank the distributions employing only its first two moments are only valid for a limited set of distributions (notably the Gaussian one), they put forward two new rules or stochastic dominance criteria that hold for all the statistical distributions and involve less restrictive assumptions about the expected utility function than the MVC. Sortino (2001) asseverates that, in general terms, Hadar and Russell’s framework (1969) is superior to the MVC for the following reasons:

i. The first stochastic dominance criterium asserts that every investor would choose assets with higher expected returns irrespective of the risk involved while, under the MCV the selection may be blurred by the introduction of the variance, which may be lower in the alternative with the smaller return (Brealey and Myers (2003));

ii. The second stochastic criterium informs that all investors are risk-averse in such a way that, among two alternatives yielding the same expected return, they would pick the one delivering the lower risk. Strangely, MVC is, again, blind to this distinction (Brealey and Myers (2003)).

Even acknowledging the superiority of the stochastic dominance structure over the MVC, the fact that it is constructed upon utility functions plays to its detriment given that real-world examples do not incorporate the Utility Theory. Therefore, the focus must shift to the analysis of more objective and tractable risk metrics.

Hanoch and Levy (1969) warn against the usage of variance as a measure of risk because increased dispersion may seem desirable in those positively skewed distributions. In this sense, Sortino (2001) emphasises the importance of positive asymmetry in the measurement of downside risk\(^4\) while simultaneously fosters the introduction of the Minimal Acceptable Return (MAR) in any risk metric to compute the downside risk. Furthermore, the variance or Standard Deviation could possibly be

\(^4\) It is important to bear in mind the distinction between the two sides of risk: downside risk and upside potential. The entirety of the investors is concerned about the former, given that nobody appears worried about the latter as it represents the ‘potential for success’ (Sortino (2001:15))
utilised to measure the failure to attain that MAR, an example of which is the discrete version of the Fishburn’s $\alpha$-model$^5$:

$$\left[ \sum_{-\infty}^{\infty} (r - MAR)^2 Pr \right]^{1/2} \quad (1.1)$$

An analogous situation is verified with any other risk measure including MAR, or any benchmark instead$^6$. Accordingly, many techniques owe their existence to some sort of benchmark, fact that turns them into metrics informing performance besides risk, no matter the varying degree of sophistication and complexity involved. A couple of typical examples are worth of being noticed.

**a) Beta coefficient:**
The famous $\beta$ coefficient characteristic of the CAPM appears in the SML$^7$:

$$r_i = \alpha_i + \beta_i r_m + \varepsilon_i \quad (1.2)$$

where $r_i$ and $r_m$ denote the expected returns on an individual asset and the market respectively, $\alpha$ the asset’s alpha (intrinsic value, irrespective of the market and non-diversifiable) and $\beta$ represents the measure of volatility or risk of a security in comparison to the market as a whole. Given that beta is a product of regression analysis, it may be understood as the tendency of an asset’s return to respond to movements in the market, or, more specifically, a measure of the correlation between the market and the asset movements: values smaller than one indicate volatility records smaller than the market, $\beta = 1$ replicates market swings whereas $\beta > 1$ means that the security price will be more volatile (i.e., risky) than the market.

**b) The Sharpe Ratio (SR):**
The SR expresses the risk-adjusted performance of a security:

$$SR_i = \frac{r_i - r_f}{\sigma_i} \quad (1.3)$$

where $r_i$ and $r_f$ mark the expected return on an asset and the risk-free rate respectively and $\sigma_i$ denotes the Standard Deviation of the individual security’s returns. As it may be surmised, the greater the SR, the better the risk-adjusted performance whilst, on the

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$^5$ Fishburn (1977) was among the first authors to devise a framework able to capture the failure to attain an MAR as well as the magnitudes involved: $\int_0^t (t - X)^\alpha df(X)\alpha > 0$ where $f(X)$ indicates the cumulative probability distribution of $X$, $t$ the target rate of return and $\alpha$, a proxy for the investor’s risk aversion. However, although it represents a leap forward compared to its predecessors, this downside risk measure still relies on a subjective element, i.e., the risk aversion.

$^6$ In some circumstances, MAR could be treated as a benchmark itself.

$^7$ CAPM and SML stand for Capital Asset Pricing Model and Security Market Line respectively. (Brealey and Myers (2003)).
other hand, negative values indicate that the investment in question fared worse than the riskless asset\(^8\).

c) The Information Ratio (IR):

The IR appears, in broad terms, conceptually similar to the formerly described Sharpe Ratio, notwithstanding which it is built upon the benchmark for the security in question\(^9\). Therefore,

\[
IR_i = \frac{(r_i - r_b)}{\sigma(r_i - r_b)} \quad (1.4)
\]

where \(r_i\) and \(r_b\) denote the expected return on the security \(i\) and its respective benchmark \(b\) and \(\sigma\) the Standard Deviation of the excess return of the asset with respect to the benchmark (also called tracking error). The quotient ascertains whether the asset has beaten the reference through high return, low return of the benchmark or low tracking error (i.e., adding little risk to the exposure).

However, even though instilling some sort of benchmark may seem desirable to inform about performances, subjectivity remains present in the definitions as every business unit within a bank -and the bank itself- respond to different benchmarks or market portfolios, which renders uniform legislation an uphill task. Risk measures, therefore, must be based on objective information susceptible of verification which paves the way for introducing Value-at-Risk (VaR) as a superior risk measure to quantify market risks. VaR is, then, a measure of market risk that tries objectively to blend the sensitivity of a security to market changes and the probability of a given market swing. In spite of its theoretical limitations\(^10\), VaR emerges as the ‘best’ single risk-measurement technique available (Marrison (2002)): this virtue led to its adoption by the BCBS to set the standard amount for the Minimum Capital Requirements (MCR) to be built against market risks in Basel II and Basel III Capital Accords.

1.4. Risk, Capital and Regulations

After abandoning asset regulation, deposit rate ceilings and entry barriers, the 1970s heralded the beginning of an era of banking deregulation which, however, did little to

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\(^8\) The Sortino Ratio (Sortino (2001)) constitutes an interesting variation of the Sharpe ratio exclusively concentrated on the downside risk as it removes the upward price movements when computing the Standard Deviation of the asset.

\(^9\) Sharpe (1994) demonstrates the equivalence between the Sharpe Ratio in its generalised form and the Information Ratio.

\(^10\) Section 2.3.
prevent the increase in the number and severity of banking crises. It is in the context of the search for regulatory instruments compatible with free banking that capital requirements have been introduced in the industry and, since then, achieved popularity among national and supranational supervisors.

Under the notion that capital acts as a buffer against failure, capital requirements were calculated as maximum leverage ratios. Hence, in banking terminology, given the fact that the shareholders’ claims are subordinate to depositors’ claims, banks are solvent if their asset value is not less than depositors’ claims. Besides the riskiness of the banking book, capital ratios, then, determine the probability of failure, and, accordingly, higher capital would lead to higher capital buffers, eventually reducing that probability of default. However, no matter how healthy high capital buffers may appear, banks could be tempted to increase their asset risk in response, thus overshadowing the positive effect of a higher capital cushion.

In an effort to prevent banks from excessive risk-taking, regulators intended to link capital requirements to the risk of the loan portfolio, and in 1988 produced the first attempt to introduce some sort of risk-sensitive capital regulation in the form of the seminal Basel I Capital Accord. This set of rules assigned assets to different risk buckets\textsuperscript{11}, with every risk bracket getting a specific treatment that reflected the inherent risk posed by that security with a total MCR ratio of 8\% of capital to risk-weighted assets (BCBS (1988)). Stolz (2002) credits Basel I with two major feats: firstly, the introduction of an adequate level of capital in the international banking system and, secondly, the unification of several capital regimes into one single scheme, thus becoming the world standard in capital regulation.

However, the 1988 agreement presented several shortfalls which prompted the BCBS to launch a second Capital Accord, popularly known as Basel II. One of the most important snags was represented by the risk bucket framework, as the relationship between risk and capital was insufficiently covered by leaving the door for capital arbitrage ajar. In this sense, Stolz (2002) emphasised that banks could substantially increase the risk without augmenting capital by keeping the allocation between the brackets constant, simultaneously replacing low risk for high risk assets within the bucket. The Second Basel Capital Accord intended, in principle, to remedy that glitch including a system of mutually reinforcing pillars: Pillar 1, Minimum Capital

\textsuperscript{11} The term ‘bucket’ refers to the risk category that the specific asset is included into.
Requirements; Pillar 2, Supervisory Review Process and Pillar 3, Market Discipline. While the main objective of Basel II was still the calculation of MCR, the Accord appeared innovative in its treatment of MCR computation: banks were authorised to choose between Basel I’s calculation (called Standardised Approach –SA-) and a technique denominated Internal Model Approach (IMA), under which the institutions were allowed to develop their own statistical models with a view to quantifying their market risk through the Value-at-Risk metric and translating it into MCR subject to stringent quantitative and qualitative methodological and disclosure standards.\(^\text{12}\)

Undoubtedly, Basel II represented a huge leap forward at the time of forging a relationship between risk and capital courtesy of the VaR-based IMA. The Accord contained, for the first time, interesting elements like a risk metric initially\(^\text{13}\) capable of reflecting the market movements more thoroughly and, even though the calculation of the MCR may appear somewhat opaque in view of its powerful assumptions, the direction towards more risk sensitive capital adequacy measures in banking industry had been clearly established. In this sense, the most important message conveyed by Basel II was the mandate to bank managers, supervisors and every practitioner to get used to deal with and adjust their actions to risk, taking appropriate measures based on the correct and timely assessment of risk of the exposures, to which Saidenberg and Schuerman (2003) add that supervisors must tackle the risks proactively, i.e., before being incurred. BCBS (2004) affirms that the Second Capital Accord intended to generate enough incentives to adopt more risk-sensitive appraisals towards capital sufficiency via more risk-sensitive MCR (the VaR-based IMA) and, moreover, extend its reach via more careful assessment of credit and operational risk ignored in Basel I.

The subprime crisis of 2007-2008 that still lingers pointed out that Basel II was unable to prevent banks from excessive risk taking. The banking sector in many countries had built up excessive on- and off-balance sheet leverage alongside insufficient and gradually declining capital reserves that proved unable to absorb the market shocks. The procyclical nature of the regulations contained in Basel II did little to shield the banking system against the ensuing deleveraging process and the

\(^{12}\) There were also brand new elements in what regards the credit risk. As in the case of market risk, banks were also allowed to choose between the Standardised Approach to credit risk present in Basel I (i.e., the risk bracket structure) and an Internal Rating Based (IRB) approach, where the risk weights belonging to the assets in the loan or credit portfolio were obtained from the ratings provided by an external credit assessment company, Additionally, financial institutions were allowed to employ their own internal estimates of credit worthiness to evaluate the credit risk underlying their portfolio.

\(^{13}\) Sections 2.3 and 2.4 provide a more detailed analysis about VaR and other alternative risk measures.
intertwined ongoing transactions, which eventually led the market to lose confidence in the solvency and liquidity of the banking system as a whole. The weaknesses in the financial industry propagated rapidly, resulting in an astounding contraction of liquidity and credit availability; consequently, many institutions around the world ended in bankruptcy or bailed out by the respective states to prevent a systemic collapse through massive injections of liquidity, capital prop-ups and guarantees to the taxpayers’ detriment.

In response to that plight, the BCBS enacted a new set of regulations collectively known as Basel III, which aimed at improving the capital standards and the liquidity provisions both from a firm-wide and system-wide point of view. In general terms, the new Accord reaffirms Basel II’s inclination towards risk and contains a stricter definition of capital and enhanced the liquidity concerns raising the quality, consistency and transparency of the capital base. While basically maintaining the dual scheme characterised by the SA and IMA to measure market risks, risk is acknowledged as likely to wreak havoc on financial institutions and, accordingly, given more careful treatment. In this vein, VaR-based IMA’s MCR have been hugely increased and liquidity terms vastly modified in order to shield banks against a reiteration of the harmful events sparked by the 2007-2008 market turmoil. The outcome of the latest twist in the regulatory approach will be assessed – unfortunately- during the next crisis.

1.5. Geographical focus
Emerging and Frontier markets have gained increased weight in the world to such an extent that many acronyms have been created to design them or, at least, a bunch of them: for instance, BRIC(S) which refers to Brazil, Russia, India, China and, recently South Korea or MIKT, standing for Mexico, Indonesia, South Korea and Turkey as well as many others that could be added, were coined by Jim O’Neill, global economist at JP Morgan, in an effort to encompass the fastest growing economies with enough potential to become the most dominant ones in a handful of decades.

In order to achieve that growth, it is necessary to develop solid, robust and deep financial markets capable of providing the essential liquidity to foster existing businesses and new ventures. In this vein, Emerging economies typically exhibit financial markets with many characteristics that would enable the expansion of the economy, particularly in terms of the Market Regulatory Environment, Custody and
Settlement and Dealing or Trading\textsuperscript{14}. Frontier markets situate a step behind Emerging ones and lack many of its features, thus hampering the development of the respective economies; their markets are subject to more government interference and abrupt changes in regulations than its counterpart, to which the reduced turnover contributes, in principle, to enhance the volatility levels\textsuperscript{15}.

Even though Emerging markets have been gradually gaining importance, they still lag behind mature ones in terms of the interest they awake in the academic community and, moreover, the intrigue generated by Frontier markets is very far away from them too. A quick look at the FTSE Country Classification (FTSECC) ensures that nations like United States, United Kingdom, Japan, and many other constituents of the list are well studied by practitioners, banks, government agencies and scientists in general. In this light, one additional motivation surges at the time of analysing the FTSECC category to which the components of the BCBS belong (as of September 2012):

- **Developed** (17/27 = 62.96\%): Australia, Belgium, Canada, France, Germany, Hong Kong SAR, Italy, Japan, South Korea, Luxembourg, the Netherlands, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States;
- **Emerging –Advanced or Secondary-** (7/27 = 29.63\%): Brazil, China, India, Indonesia, Mexico, Russia, South Africa and Turkey;
- **Frontier** (1/27 = 3.70\%): Argentina;
- **Unclassified** (1/27 = 3.70\%): Saudi Arabia.

As the above list suggests, the reduced quantity of non-developed participant countries (one third of the total participant nations) drops a hint about the potential curiosity that a study on those nations might entail and simultaneously serves as an inspiring force.

It is customary, too, on the part of the regulator (i.e., the BCBS), to guide their actions by the results of the Quantitative Impact Studies (QIS). QIS constitute, as the name indicates, a data collection exercise which outcome is employed in the determination of the scope of the regulations enacted or soon to come into force: for instance, the effect that the adoption of Basel II or, more recently, Basel III, would exert on the banking institutions operating under BCBS’s framework in what refers to a

\textsuperscript{14} This thesis will follow the Quality of Markets Criteria established in FTSE Global Equity Index Classification. Developed markets must fulfill 21 criteria, Emerging markets between 9 and 15, while Frontier markets require 5 specified criteria (FTSE (2012)). Appendix 1.B.

\textsuperscript{15} Section 4.2.
whole variety of issues, most notably capital structure and levels. In order to gauge the influence of Basel II, the BCBS conducted QIS 2, QIS 2.5, QIS 3 and QIS 5\(^{16}\); while the first two dealt with Operational and Credit Risk respectively, the last two focused mainly on the impact of Basel II on MCR (i.e., Pillar 1).

The quantity of countries participating in QIS 3 totalled 43 and 17 of them belonged to Emerging or Frontier economies: Brazil, Bulgaria, Czech Republic, Chile, China, Hungary, India, Indonesia, Malta, the Philippines, Poland, Russia, Slovakia, South Africa, Tanzania and Turkey. However, even though their presence may be deemed interesting, the BCBS (2003:2) stated: “Outside the G10 and the EU only a small number of countries had any banks completing this (IMA) approach, making it difficult to maintain confidentiality, so these results are not included here...”. The declaration, albeit rather imprecise on which nations fulfilled the report, increases the interest in a study on Emerging and Frontier countries. On the other hand, forty one nations took part in QIS 5, the last assessment under Basel II framework: in this occasion, the exercise aimed at several objectives among which the evaluation of Basel II parameters played a central role. Unfortunately, much in the same fashion as QIS 3, less than a third of the encompassed countries belonged to Emerging markets: Bulgaria, Cyprus, Czech Republic, Hungary, Malta, Poland, Brazil, Chile, India, Indonesia and Peru. The BCBS (2006b) asseverated that not all the banks reported the outcome of the experiment, and, in this vein, a closer look at the information provided shows that Emerging and Frontier countries failed to fully answer the requirements of the study.

After the enactment of Basel III, the BCBS rebranded QIS as Monitoring Exercises (ME) and conducts them every six months. The scope of ME is much broader than their predecessors QIS as they deal with a whole range of capital issues, spanning from sufficiency to liquidity considerations aiming at evaluating the full impact of the introduction of Basel III provisions. ME are carried out and released every semester and many internationally active banks from all over the world take part in them. Unfortunately, only a small number of those institutions belong to Emerging and Frontier economies, as Chart 1.1 exhibits:

\(^{16}\) QIS 4 was not a BCBS’s effort as several member countries decided to carry out a field study on the implementation of Basel II in their respective banks. As far as the results are concerned, only Germany, Japan and the USA reported its outcome.
Therefore, Emerging and Frontier countries account for, at least, 20% of the total sample participating in the Basel III ME, and a more thorough look at the relative weights reveals that only in the June 2012 release a Frontier country performed the evaluation. The academic community in turn reflects this lack of influence and carries out research on Developed nations, leaving the respective Central Banks or national supervisors to rule on and gauge the impact of those regulations. For instance, Jorion (1996), Penza and Bansal (2001), Marrison (2002), Christoffersen (2003), Alexander (2008a, 2008b and 2008c), McNeil, Frey and Embrechts (2005), Embrechts, Kluppelberg and Mikosch (1997), as well as the rest of the authors cited in Chapter 3 carry out their investigations on mature countries.

The lack of involvement of Emerging and Frontier economies in regulation drafting and enactment and the relative neglect on the part of the academics bolsters the development of the present thesis utilising those countries.

1.6. Research questions and objectives
1.6.1. The nature of the research problem
The issue of ringfencing banks against market calamities has always been a controversial one even for national and supranational supervisors like BCBS, which have hitherto been unable to find appropriate procedures to establish adequate capital requirements. Regulations have been patched and corrected in response to market crises
with a clear toughening tendency and, in this vein, the successive Basel Capital Accords undoubtedly display the need to harden the stance in order to prevent further disasters.

The relatively primitive Basel I Capital Accord, which portrayed the seeds of the current Standardised Approach in the manner of the simple Cooke proportion, deployed its inability to fend off the demise of Barings and Metallgesellschaft, among others (Jorion (1996)), thus prompting the emergence of the 1996 Amendment to Basel I that was included in the subsequent Basel II Capital Accord of 2006. The enactment of that Amendment represented a pivotal point in the history of the ‘fight’ against market risks given that, for the first time, these kinds of dangers were tackled using a market-sensible measure like Value-at-Risk. In order to calculate the Minimum Capital Requirements for market risks, Basel II introduced the so-called VaR-based Internal Model Approach, giving freedom to the institutions to use their preferred VaR technique (albeit subject to certain qualitative and quantitative criteria), and simultaneously allowing the comparison with the much-unchanged SA: in the end, the process boiled down to the selection of any VaR-based model or SA according to the respective needs.

Unfortunately, the subprime crisis brought about a reiteration of the film, obliging the BCBS to tighten the rules again by increasing significantly the MCR via the addition of a set of supplements to both schemes (VaR-IMA and SA) so as to avert further market disasters. However, even though the MCR resulted considerably augmented in Basel III, the basic framework remained unaltered as banks are still allowed to choose between any VaR-IMA and SA, once more paying heed to their own particular situations. Regulations, then, look ambiguous and too vulnerable to the discretion of the companies, hence posing high risks to the whole financial system. Besides any theoretical consideration, the outcome of the analysis is of an empirical nature, thus highlighting the need for extensive research.

Basel II and Basel III\(^\text{17}\) constitute the core of the thesis, implying a thorough evaluation of the two avenues to compute the MCR: VaR-based IMA and SA. More specifically, while the former entails delving into VaR models, the latter comprises the simple application of a fixed flat rate.

VaR techniques are inherently difficult to evaluate. Even though Artzner et al. (1999), Acerbi and Tasche (2002), Danielsson and Zigrand (2001) and Danielsson et al.\(^\text{17}\) Investigation on Basel I is embedded in the SA included in Basel II.
(2005) unveiled its limitations particularly regarding the axioms that any risk measure should accomplish, VaR has been enforced by BCBS in Basel II Capital Accord, posterior consultative papers (BCBS (2009b)) and maintained in the Basel III Capital Accord. Therefore, VaR has become the official market risk quantification metric which any financial institution abiding by Basel regulations must comply with, and its importance underlined by the fact that capital charges for every exposure ought to stem from it. Since its inception in the mid-1990s—no doubt fostered by the 1996 Amendment to Basel I to incorporate market risks—academics have carried out substantial investigations to ascertain the existence of a VaR representation capable of overcoming the rest.

However, despite the extensive research on VaR and VaR models, there seems to be no technique susceptible of prevailing (Dowd (2005)), hence rendering the evaluation a subject of empirical concern. Granger (2002) documents the uncertainty stating that the prime position to this imaginary race remains vacant and, furthermore, the outlook looks even gloomier when Emerging and Frontier markets are involved considering the reasons exposed in Sections 1.5 and 4.2. To make matters even worse, regulations enacted by BCBS from the 1996 Amendment up to the time of writing do not recommend a specific approach to compute VaR, only circumscribing to general guidelines adaptable to any assumption and scheme (BCBS (1996, 2004 and 2009)). Furthermore, VaR calculation is bolstered given that the MCR contained in Basel III demand the estimation of a stressed VaR (sVaR) to be added to the base VaR envisaged by Basel II.

The relativity of the conclusions entices continuous research on the peculiarities of the situation and its attractiveness is enhanced by the need to produce accurate VaR forecasts to establish equity levels. In this sense, every system has advantages and disadvantages, supporters and detractors, according to the market, dataset and period measured. For instance, the much battered Historical Simulation seems to work reasonably well when the characteristics of the forecast period do not differ to a large extent from those of the sample period (Finger (2006)), conditional models like GARCH supplemented by the Normal distribution deliver satisfactory results in the

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Following Artzner et al. (1997), Acerbi (2004) and McNeil, Frey and Embrechts (2005), the four properties that a sound risk measure should accomplish are: Monotonicity, Subadditivity, Positive homogeneity and Translation invariance. VaR has demonstrated not to comply with subadditivity, which means that VaR for diversified portfolios may be greater than for undiversified ones. Other measures like Expected Shortfall (ES) are capable of achieving every property, from where it springs the so called 'coherence' term (Artzner et al. (1997)).
event of very high data frequency (Andersen and Bollerslev (1998) and Huisman, Koedijk and Pownall (1998) find that GARCH coupled with a heavy-tailed distribution like Student-t($d$) -hereafter t unless otherwise stated- yields accurate forecasts in some contexts, and McNeil and Frey (2000) make a point of the adequacy of EVT for heteroscedastic financial time series). However, the VaR definition underlines the importance of obtaining an amount enough to cover shortfalls in the event of abnormal market movements or, for example, to protect companies in 99 out of 100 days if applying Basel regulations (Section 2.3). When VaR numbers fail to achieve its mission and losses are posted with higher frequency, the measure becomes useless and enhances the possibility of experiencing disastrous effects (FSA (2009)). Consequently, VaR estimates should shield the company against extreme market swings; intuitively, the higher the VaR, the lesser probability of suffering huge losses.

The dispersion in the recommendations contained in the literature, alongside the vague definitions comprised in Basel II and Basel III and the fact that the majority of research is devoted to Developed markets leave room for potential contributions regarding VaR models applied to those mostly neglected markets. It is important to recall that the research problem is of an intrinsic empirical nature, for the diverse specifications will be tested against real time market movements. Therefore, the approach is inductive rather than deductive: those representations delivering the best results for the purposes intended across the variety of stock exchanges will be deemed more adequate. These conclusions generate far-reaching implications, for capital charges for market risks root in VaR amounts; accordingly, the thesis ventures to critically evaluate the VaR-based MCR formula bound to be endorsed in the future Basel III Accord to extract conclusions about the upcoming framework.

On the other hand, SA does not appear to raise much theoretical enthusiasm apart from its simplicity and implementation devoid of any problem. Although it seems an ideal appraisal for institutions on a limited budget, it is agnostic to market movements, presumably invalidating the response to violent swings. It constitutes, in a nutshell, a very limited measure –yet still available- to compute MCR.

To sum up the discussion, the thesis aims at shedding light on the following issues:

- The adequacy of Basel II and its successor Basel III to establish MCR sufficient enough to fend off market crashes in Emerging and Frontier stock exchange.
The competence of different VaR schemes and the SA alternative under Basel Committee rules for capital constitution purposes in Emerging and Frontier stock markets.

The impact of Basel III modifications on the required capital buffers.

Essentially, there is a strong inkling that VaR values obtained through EVT-based techniques should exhibit better behaviour in Emerging and Frontier markets, which would eventually derive in higher capital cushions enough to shelter banks against violent and abrupt market swings. An additional implication bolsters the application of heavy-tailed distributions as they may result more important than the specification itself. This assertion means that basic models not specially designed to account for abnormal downside jerks but whose random shocks feature leptokurtic distributions might, apparently yield more accurate VaR forecasts than schemes initially intended to tackle anomalous falls with Gaussian innovations.

The events sparked by the bankruptcy of Lehman Brothers in 2007 undoubtedly underlined that banking institutions were grossly undercapitalised in order to face huge losses, from where the necessity to strengthen the capital base springs up even in the very bankers’ words (Financial Times (2010))\(^\text{19}\). On the other hand, the tougher MCR to be implemented by Basel III Capital Accord could be somewhat relaxed were EVT schemes compulsory demanded instead of affixing factors to the original Basel II formula and simultaneously conferring freedom of choice of VaR models. The alterations introduced in Basel III would in principle make sense for inaccurate representations based on empirically incorrect assumptions (e.g., those featuring the Normal distribution). Therefore, the thesis affirms that there seems to be no appropriate incentives for financial institutions to put in practice sharp models, as any structural deficiency on the part of the models could in principle be masked by the multiplication or hysteria factors set at 3 either for the current Basel II or Basel III regulations.

1.6.2. Research questions

The main goal of the current project is to analyse the sufficiency of Basel II and Basel III Minimum Capital Requirements in the context of Emerging and Frontier markets during market crises considering the two avenues available to compute them: the VaR-

\(^{19}\) Additionally, Deutsche Bank, Standard Chartered, BBVA and China Construction Bank quickly moving to raise more capital seem to confirm the degree of capital shortage. (Financial Times (2010)).
based IMA and the Standardised Approach. An immediate objective, not least relevant, is related to assessing the forecasting performance of alternative VaR specifications in the particular environment cited before where few studies are focused on.

The first topic is directed to gauge the effectiveness of Basel II provisions in the two versions –VaR-based IMA and SA\(^{20}\)- to set appropriate MCR to avert a 2008-style market crisis for Emerging and Frontier stock markets. It is deemed that the determination of the adequacy of Basel II MCR formula plays a pivotal part in the remaining of the study given that Basel III Capital Accord largely builds on the results of Basel II’s Backtesting. It is also important to remark that this issue entails the evaluation of the precision of the VaR models embedded in the IMA scheme, which is tantamount to ultimately assessing the sharpness of the VaR techniques themselves.

Therefore, the first question may read as follows:

1.a. *Does Basel II MCR provide adequate capital protection to withstand huge market turmoil? Is the VaR-based IMA superior to the SA?*

From its definition, it is clear that VaR constitutes a metric referenced to the tails of the distribution. Danielsson, Hartmann and de Vries (1998:3) postulate that for a VaR model to be regarded as sound, it must ‘…correctly represent the likelihood of extreme events by providing smooth tail estimates …which extend beyond the historical sample’. These extreme events, which fill the fat tails of the distribution of returns, are possibly ‘structurally’ different from the return generating mechanism in everyday market conditions (Neftci (2000)). Heaney (2008), citing Mandelbrot (1963), adds that these major discontinuities or outliers comprise the substance of price variations. EVT was then developed to deal with atypical episodes of unexpected high volatility, hence:

1.b. *How precise are the different VaR models under Basel framework? Does EVT deliver a more satisfactory outcome for market crises?*

As a result of the abnormal crash that hit the market in 2008 and left many banks operating under Basel II framework with substantial shortages of capital, the BCBS embarked on tightening the capital rules and eventually produced the Basel III Capital Accord that comprises a considerable increase in the MCR for every institution subject

\(^{20}\) While the latter detaches from the market characteristics of the positions, the former ‘has…the advantage of aligning the capital calculation with the risk measurement approach …’ (Jackson, Maude and Perraudin (1997:80)).
to its supervision. Consequently, it is natural for the second question to delve into the
MCR formula and determine whether the alluded modification was well worth being
enforced or, on the contrary, Basel II could have provided an adequate tool to build up
MCR. Again, the issue of the two alternative channels to compute MCR resurfaces in
Basel III, hence the second research question circumscribes to gather information about
the ability of the enhanced schemes involved to help institutions maintain financial
health after violent adverse market shocks. Consequently,

2. Does Basel III MCR contain significant improvements for both VaR-based
IMA and SA in relation to Basel II MCR to forestall market crises? How do
VaR models perform within the new structure?

The consequences of adopting the Basel III regime are yet to be examined for
the whole financial industry. The agreement unquestionably fulfills one of its major
objectives, namely the augmentation of the capital buffer but, given that the full
implementation is planned for January 2019, there seems to be room for consultations
and, perhaps, correction of some of its shortcomings (BCBS (2012a)). Presumably, the
application of Basel III mandates will provoke major changes in the capital levels to be
attained, thus potentially squeezing the credit conduct and restraining the flow of funds
to business projects, redirecting and immobilising them in the banks’ vaults. The third
research proposition explores some ways to flexibilise the otherwise tough requisites
while at the same time maintaining a proper protection to weather a crisis with the likes
of that of 2007-2008. Therefore,

3. Could Basel III MCR be to some extent relaxed—conserving the basic
structure—and simultaneously preserve its main target in the event of market
crisis?

1.7. Expected contribution
Basel Capital Accords have been subject to important controversy since their inception
in 1988. Arguments have continued in the event of Basel II in 2004 and are expected to
constitute a contentious issue under Basel III regime until its full implementation in
January 2019. The debate has been on the rise in parallel with the increasing complexity
of the instruments and transactions involved, to the point that thorough overhauls to the
existing body of regulations have to be brought in after huge crises hit the market. It is
by no means a fact that every Capital Accord surged as a response to major turmoil, like

The BCBS has pledged to regulate as much as possible many aspects of market risk and set up minimum capital requirement directives to be observed by those institutions abiding by the mandates; therefore, market risk is quantified and translated into capital which serves as a defence against adverse market movements. As Section 2.7 indicates, MCR can be calculated through two avenues, SA and IMA, with every specification available in the latter, provided several quantitative and qualitative conditions are met. It is precisely in this direction where the current thesis is directed to and attempts to provide the most relevant contributions.

The 2007-2008 crisis presents an excellent real-time opportunity to evaluate the scope of the present research project. Therefore, it will hopefully shed light on the adequacy of SA and IMA to withstand a plight of that kind, emphasising whether the market-insensitive 8% flat rate characteristic of the SA does a good job at the time of erecting banks’ capital cushions. As aforementioned, given that the IMA is susceptible of being estimated through a wide variety of schemes, the accuracy of the most widespread ones is gauged vis-à-vis a relatively novel one based on Extreme Value Theory (especially devised for these circumstances). On the grounds of the outcomes obtained, the study ventures to determine whether banks are enticed to apply sharp models to compute MCR and, in the event of encountering discouragement, produce a recipe to keep accuracy incentives aligned. The existing body of literature has not so far stressed the question of the incentives implied in Basel II and Basel III in what refers to the alleged ‘bailout’ or financial rescue embedded in the Backtesting procedure (Rossignolo, Fethi and Shaban (2012a, 2012b, 2013).

Finally, the interest conveyed by the thesis appears further highlighted by the geographical focus, i.e., Emerging and Frontier markets, which have not been extensively researched and typically overlooked by the overwhelming majority of the academic community.

1.8. Concluding remarks

The present Chapter has laid the foundations of the thesis. It is firmly believed that the intricate nature of the problem merits an in-depth analysis of the scope of the regulations currently in force and the implications of that design.
Basel mandates constitute responses to massive market turmoil that shook the financial institutions and dented the credibility of the national and supranational supervisors and, in this line, Basel I and Basel II were drafted. On the contrary, even though Basel III could be regarded as a knee-jerk reaction to the subprime crisis of 2007-2008, it may be deemed an important effort to forestall the havoc that market crisis wreaked on banks. Although the last Basel Capital Accord is, by far, the toughest of all, it shares some common traces with its predecessor which might, in some aspects, play to its detriment. The upholding of the two-way route to compute the MCR for market risks constitutes a potential conflict, given that banks can certainly be tempted to opt for this scheme disregarding more accurate ones. The VaR-based IMA, in turn, also presents complications, as the model to calculate the VaR that ultimately derives in the MCR remains ultimately at the discretion of the individual companies. This supposed freedom of choice introduces a bias in the shape of moral hazard as banks are allowed to select the technique delivering the lowest possible MCR—provided it fulfills the regulatory requirement—and, additionally, in almost all occasions, that scheme appears to be an egregious one. Furthermore, although Basel III establishes a healthy approach, its structure fosters the adoption of any technique as its performance—albeit poor—could be salvaged by Basel II’s Backtesting and, additionally, the stressed VaR term.

The current thesis investigates the performance of several VaR models in the context of the subprime crisis recorded in Emerging and Frontier stock markets to analyse their adequacy both in Basel II and Basel III configurations as well as their relationship with the SA; moreover, the study devotes special attention to determine whether Basel Capital Accords sufficiently prompt the banks to stick to accuracy and reward the application of sharp models. The implications of the findings in the light of the former and present mandates and likely suggestions to improve some perceived glitches are put forward. It is judged that some of them could easily be put into practice to improve the reach of the regulations in force.

1.9. Structure of the thesis
As it could be appreciated, Chapter 1 provided the introductory concepts and synthesised the Emerging and Frontier markets as the geographical location of the research project, whereas the remaining of the study aims at developing those ideas thoroughly. Hence, the study unfolds as follows. Chapter 2 describes the path followed by Value-at-Risk to become the official risk measure utilised by the BCBS, thus
detailing VaR’s predecessors and the regulatory route that led to the present Basel III Capital Accord in what regards the scope of the analysis. Chapter 3 prepares the ground for the investigation as it portrays a statistical analysis of the time series involved in order to grasp an idea of the kind of stochastic behaviour to be expected. Additionally, it deals with the VaR models employed along the thesis, from the simple Historical Simulation to the most recent and sophisticated one featuring the Extreme Value Theory. Chapter 4 describes, after characterising the Emerging and Frontier markets object of the study, the methodology to apply the VaR techniques detailed in Chapter 3 to calculate the Minimum Capital Requirements demanded by the former Basel II and the recently introduced Basel III, stressing the prowess of EVT to shield banks against huge turmoil in financial markets. Finally, Chapter 5 closes the investigation reflecting on the coverage that the schemes tried could potentially provide an institution and enquires about the implications that the adoption of the different models could bring about under Basel II and Basel III frameworks. The Chapter also highlights the limits to the research and marks likely ways for further investigations.
Chapter 2

Risk Measures and Regulation
2.1. Introduction
The current Chapter presents the foreword which synthesises two closely related elements dealt with throughout the thesis: risk measures and regulation. Initially, it does not appear easy to find a connection between both, but the common thread may be detected by taking into account the fact that regulators make extensive use of risk measures in order to determine at least the MCR that a financial institution must build based on them.

Throughout the history of financial modelling, practitioners have tried hard to follow the market developments and construct representations which would enable better protection for the bank or institution\[sup]21\[/sup]. In this sense, the Asset & Liability Management (A&LM) as well as its refinements like Duration or Gap Analysis provided a relative shield against interest rate fluctuations and could well be regarded as one of the first attempts to tame market risks. Unfortunately, ongoing market movements uncovered the inappropriateness of A&LM and its related measures because of its limited scope restricted basically to interest rate based instruments such as bonds as well as their accountancy tilted nature.

Another set of measures is represented by the Standard Deviation (SD), typically embodied in the Mean-Variance (MV) framework of the MPT crafted by Harry Markowitz in the 50s. The SD constituted one of the most promising –at that time- schemes to work with every kind of asset in an integrated structure. It introduced the notions of the trade-off between risk and return and risk-aversion subject to a certain utility function describing the preferences of the economic agent. However, the SD as well as Standard Deviation-based measures like Sharpe Ratio (SR) could be considered valid only under restrictive assumptions circumscribed to Normal or, more generally, elliptical distributions of asset returns. Although it represents a significant leap towards a sound risk measure, the aforementioned drawback motivated the search for more appropriate techniques.

Academics eventually produced Value-at-Risk, ultimately endorsed by JP Morgans’s RiskMetrics, system that contributed to enhance its popularity. In a few words, VaR is a quantile of the return distribution of the corresponding asset or portfolios of assets and, accordingly, very easy to explain, understand and implement and has enjoyed an important degree of endorsement and support from practitioners and

\[sup]21\[/sup] The nature of that phrase embodies the fact that no individual market participant is able to move the market (Danielsson (2002)).
regulators as well. However, contemporaneous with its inception and adoption by supervisors, Beder (1995) pointed out that, notwithstanding its undisputed merits, VaR is not a panacea. In fact, it is not able to inform the amount of losses beyond its threshold and lacks one of the most important attributes that a coherent risk measure requires (Artzner et al. (1997, 1999)). Consequently, academics’ attention is gearing towards the so-called coherent risk measures like Expected Shortfall (ES), which, besides retaining VaR’s advantages, reveal themselves as theoretically and technically superior.

It is by no means a fact that regulations are necessary particularly when the free market is unable to ensure an efficient allocation of resources; those mandates, in the banking industry, are often compiled in the form of national and supranational guidelines. In the context of the current thesis, the idea of regulation is confined to the one issued by the Basel Committee of Banking Supervision in what regards market risks and its relationship with Minimum Capital Requirements through the use of models. The road to regulation arising from BCBS’s directives has been evolving from its embryo directives contained in Basel I Capital Accord to the ultimate Basel III. Throughout twenty-five years, the BCBS has varied the approach to market risk modelling though the common denominator is given by its continuous reliance on VaR as the official risk measure. It is a matter of empirical evaluation whether VaR may deliver the correct answers to the market plights and, if so, which estimation technique is the most adequate.

The present Chapter provides a scheme of the relationship between regulations and risk measures, seasoned with the description of the patterns that characterise specific risk measures emphasising Value-at-Risk, i.e., the official risk metric demanded by the Basel Committee. Accordingly, the second section narrates the evolution that market risk measures have experimented, from the early ones and up to the present VaR. Unfortunately, although its construction represents a leap forward in view of its advantages with reference to more primitive expressions, it also entails several disadvantages, both of which are exposed in the third section. The fourth section pictures some alternative risk measures that have been put forward as a remedy to those snags. Section five summarises the analysis featuring some final considerations on the derivation, patterns and glitches of VaR. The next two sections present the general regulatory framework where VaR is fitted; in this vein, Section six discusses the reasons fostering the adoption of rules to regulate the market risk measurement whereas Section
seven explains the structure of the Basel Capital Accords that progressed from the initial Cooke proportion in Basel I to the more sophisticated VaR-related mandates included in Basel III. Section eight reflects on the matter.

2.2. Value-at-Risk predecessors
Before October 1979, the Federal Reserve (FED) generally targeted the price of bank reserves in the financial system. The tightness or ease of monetary policy was evaluated by variations in the federal funds rate, which were set by the Federal Open Market Committee (FOMC) in accordance with the desired values of goal variables. This set of rates, called ‘FED funds target rates’ was accomplished by the Trading Desk at the Federal Reserve Bank of New York through the sale and purchase of securities in open market operations.\(^22\)

However, in October 1979 the FOMC changed its tack and managed the quantity of money, and, more specifically, the nonborrowed reserves (i.e., those reserves not borrowed by depositary institutions). At that time, the FED believed that the change in strategy would deliver a more efficient response to the soaring inflation, though the new monetary policy would result in greater volatility in the FED funds rates. Unfortunately, this was indeed the outcome. Additionally, the late 1970s also marked the beginning of a period of significant interest rate deregulation and financial market innovation that influenced the FED's ability to make monetary policy by targeting the monetary aggregates. A particularly significant development was the growing popularity of money market mutual funds as an alternative savings vehicle and the gradual elimination of interest rate ceilings on all deposit accounts except for demand deposits. As households shifted balances between traditional deposit accounts and the deregulated accounts and money market mutual funds, the impact on the level and growth rates of the various monetary aggregates was profound.

2.2.1. The Asset & Liability Management (A&LM)
As a result of the increased volatility of interest rates recorded in the mid 1970s, the financial industry stimulated the adoption of more adequate techniques for market risk management.

\(^{22}\) Unlike the present times, in the 1970s, the level of the FOMC federal fund rates was not publicly announced but deduced from the amount of open market operations the Federal Reserve carried out.
The first attempts to measure and manage financial risks generated a practice known as Asset & Liability Management, directed to organise risks connected with balance-sheet positions by forecasting current earnings under pre-specified market conditions. While Rachev, Tokat and Schwartz (2003) describe A&LM as the simultaneous consideration of assets and liabilities in strategic investment planning, Penza and Bansal (2001) broaden the scope by defining A&LM as the discipline encompassing the identification, measurement and control of the risk associated with balance-sheet and off-balance-sheet positions, thus bolstering an integrated management of those exposures with a view to maximising the risk-adjusted shareholders’ value.

Marrison (2002) points out that A&LM is mainly concerned with interest rate and liquidity risks, hence dealing with the management of those market risks arising from the bank’s structural position, which is, in turn, a product of the intermediation activity between depositors and borrowers. While the former surges from the fact that profits may change as a result of the variation in interest rates, the latter is represented by the insufficient cash to repay customers. In any case, both risks are due to the difference between the bank’s assets and liabilities, hence giving name to the technique. Additionally, considering that deposits are constituted by checking accounts, saving accounts and fixed deposits whereas the loan department is composed of commercial, personal, home-improvement and car loans, credit card debt and mortgages, A&LM reveals more important for retail banks than for trading or investment institutions.

Marrison (2002) also emphasises a crucial difference between A&LM and the management of trading risks in market operations –where market risk measures like VaR are aimed at- stating that A&LM exposures are relatively illiquid in the sense that, after its birth, the assets and liabilities are commonly held by the bank until they mature. However, the industry is increasingly packing illiquid loanable banking products –mostly mortgages- into securities, thus constituting a kind of backed securities which, in the case of mortgages, receive the name of Mortgage-Backed Securities (MBS). These securities do not belong either to the trading or the A&LM categories, hence the instruments are held in one of the categories and, depending on the bank, measured according to the trading-VaR or A&LM. Marrison (2002) states that the common practice considers trading-VaR instruments as those susceptible of being liquidated in less than one month while A&LM would capture the remaining ones. Trading-VaR, then, focuses on one-day exposures whereas A&LM concentrates on long-term and customer behaviour.
As it may be inferred from its name, A&LM is mostly concerned about the mismatch between short and long horizons in what refers to the equilibrium between deposits and loans, i.e., interest rate risks, as well as the management of the funding liquidity risks. In what follows, both risks are to be succinctly mentioned.

a) Interest Rate risk
Even though A&LM may involve a complex network of tools designed to focus on different types of risks, it is mainly oriented to interest rate management, hence leaving the remaining set of market risks to more market oriented techniques. The objective, then, is to limit, control and possibly reduce the duration gap between long-term loans and quick access to deposits.

However, the evaluation and measurement of (mainly) retail interest-rate exposures reveals as a task of very complicated nature because banks usually receive long-term fixed-interest payments on their loans having to pay short-term floating-rate interest to depositors. That situation is hampered by two facts: i) the ‘indeterminate maturity’, i.e., the uncertainty as to when the customers will perform or demand payments on the respective operations, and ii) the ‘basis risk’, or the risk that surges from holding assets and liabilities that reprice on different basis\(^\text{23}\).

In order to deal with both the indeterminate maturity and the basis risk, A&LM offers three traditional appraisals stated in Penza and Bansal (2001):

a.1) The Current Earnings Approach (CEA)
Once the structure of assets and liabilities is set at a particular point in time, the Current Earnings Approach intends to quantify the risk arising from the margin of interest following a shock to interest rates. The focus appears directed at immunising or profiting from the gap represented by the difference between the rate-sensitive assets (RSA) and rate-sensitive liabilities (RSL):

\[
GAP = RSA - RSL
\] (2.1)

Applying the expectation operator \(E(.)\):

\[
E(GAP) = (RSA - RSL) \Delta i
\] (2.2)

\(^{23}\)Marrison (2002) enumerates a taxative list of main instruments likely to bring about interest rate risk. Hence, on the Assets side: retail personal loans, retail mortgages, credit-card receivables, commercial loans, long-term investments, traded bonds and derivatives whereas on the Liabilities side a bank will probably hold retail checking accounts, retail savings accounts, retail fixed-deposits accounts, deposits from commercial customers and bonds issued by the bank.
where $\Delta i$ denotes the variation in interest rates.

From (2.1) and (2.2) it may be inferred that the main snag about CEA resides in the fact that both assets and liabilities are calculated disregarding the time in which they are produced but, in order to find a remedy, some authors proposed a system of time buckets where the concepts are allocated according to their maturity and priced at the midpoint of each bucket. Hence, the expected total value of the GAP between assets and liabilities becomes:

$$E(GAP) = \sum_{k=1}^{n} \frac{(RSA_k - RSL_k) \Delta i T_k}{365}$$

(2.3)

where $k$ denotes the bucket and $T_k$ represents the number of days from the midpoint of the $k$-th bucket to the end of the time horizon. However, expression (2.3) conveys that sharper approximations can only be obtained by shortening the size of the bucket.

a.2) The Market Value Approach (MVP)

Penza and Bansal (2001) cite a second appraisal to Interest Rate Risk under the A&LM directed to quantify the market value of a firm as the difference between the market value of the financial assets and the corresponding financial liabilities$^{24}$. The Market Value Approach is defined as a method taking into consideration the assets straightforwardly influenced by the interest rate, thus excluding shares and most derivatives or related concepts.

The notion is more clearly understood by evaluating the present methodology in terms of fixed-rate instruments, where the Duration may be employed as an index to the exposure of the firm to changes in interest rates$^{25}$. Hence, stating that the Market Value of a firm is:

$$MV = MVA - MVL$$

(2.4)

where:

- MV: Market Value of the institution
- MVA: Market Value of the assets
- MVL: Market Value of the liabilities

A parallel change in the market interest rate equal to $\Delta i$ brings about a variation in the market value of the assets and liabilities according to the respective Duration$^{26}$:

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$^{24}$ Excluding tangible assets.

$^{25}$ Fabozzi (1989) and Elton and Gruber (1994) analyse Duration and Modified Duration concepts extensively.

$^{26}$ The Duration concept assumes that interest rates movements are always of a parallel nature.
where \( \Delta MV(A/L) \) represents an Asset or Liability and:

\[
\Delta MV = -MD(A/L) \cdot MV(A/L) \cdot \Delta i(A/L) \quad (2.5)
\]

where \( (A/L) \) represents an Asset or Liability and:

\( \Delta \) : change in the respective price

\( MD \) : Modified Duration of the asset/liability

Therefore, the variation in the market value of the institution is calculated as:

\[
\Delta W = \sum_{k=1}^{n} (-MDA_k MV A_k \Delta i A_k) - \sum_{j=1}^{n} (-MDL_j MV L_j \Delta i L_j) \quad (2.6)
\]

supposing that \( k \) and \( j \) denote the \( k \)-th asset and \( j \)-th liability.

**a.3) The Sensitivity Analysis Approach (SAA)**

The last appraisal is based on the concept ‘earnings at risk’, i.e., the amount of potential future losses provoked by interest rate variations. The expression to compute reflects the difference between the interest rate-sensitive exposures, before and after the change in interest rates:

\[
IRE = (PVAs - PVA) - (PVLs - PVL) = (PVA_s - PVL_s) - (PVA - PVL) \quad (2.7)
\]

where:

\( IRE \) : Interest rate exposure

\( PVs \) : Present value of the exposure after the change in interest rates

\( PV \) : Present value of the exposure before the change in interest rates

The current approach basically delivers analogous results to the MVP although the most remarkable difference is due to the possibility to work with nonparallel shifts in interest rates (characteristic of the Duration). However, both appraisals could be equivalent when interest rates move 0.01% considering that:

\[
IRE = (PVAs - PVA) - (PVLs - PVL) = \frac{(-MDA_MVA)}{100} - \frac{(-MDL_MVL)}{100} \quad (2.8)
\]

**b) Funding-Liquidity risk**

Unlike the interest rate risk provoked by the mismatch between assets and liabilities, liquidity risk -usually called funding-liquidity risk or cash-crisis risk- surges because of the difference in the contractual maturities of assets and liabilities. In effect, banks typically obtain funding through instruments with short duration and, after setting aside a small reserve, invest the rest in assets with prolonged duration, thus augmenting the probability of occurrence of the risk.

The mechanics of the funding-liquidity risk are relatively simple, given that it appears once the bank suffers an abnormal wave of withdrawals (event so-called ‘run on
the bank’), therefore being forced to borrow from other banks, sell assets making huge losses or default to the customers and quit the business. Marrison (2002) emphasises that liquidity risk arising from trading operations differs from the funding-liquidity risk; in effect, while the former appears when institutions are locked into positions losing values, the latter arises as a result of the inability to raise cash to pay customers\(^\text{27}\).

Traditional A&LM has long been criticised for its reliance on accrual accounting instead of marking-to-market all the items involved. In this sense, Rachev, Tokat and Schwartz (2003) mark that the contemporaneous version of A&LM is increasingly moving towards a scheme utilising scenario analysis comprising multiple forecasts of variables through, for instance, Monte Carlo simulation or stochastic programming. That methodology would in principle enable to blend both concepts, thus allowing A&LM to manage liquidity, credit, sector and residual risks. However, it would be inappropriate to employ A&LM to deal with market risks, as it is necessary to resort to specially devised metrics.

2.2.2. The Standard Deviation
In some respects, the Standard Deviation of returns can be considered an appropriate measure of risk. It is defined as the squared root of the probability weighted quadratic deviation from the average return:

\[
\sigma = \sqrt{\sum_{t=1}^{n} (p_t r_t - \bar{r})^2} \quad (2.9)
\]

where:
- \(p_t\): probability of occurrence of a return in day \(t\)
- \(r_t\): day \(t\) return
- \(\bar{r}\): average of returns from over the \(n\)-day sample

However, in the typical case of equally probable returns, \(p_t = 1/n\), thus (2.9) becomes\(^\text{28}\):

\[
\sigma = \frac{1}{n-1} \sqrt{\sum_{t=1}^{n} (r_t - \bar{r})^2} \quad (2.10)
\]

Even though SD captures the shape of the underlying Normal (or, more generally, any elliptical) distributions, it does not deliver satisfactory results in case of skewed or

\(^{27}\) However, the BCBS judges both varieties worth of consideration and quickly moved to address the liquidity risk in Basel III introducing a Liquidity Coverage Ratio (LCR) for trading-bound hazards and a Net Stable Funding Ratio (NSFR) for shortages of cash deriving from lack of proper funding sources (BCBS (2013a)).

\(^{28}\) Actually, the present thesis will make use of the unbiased formula which displays the denominator \(n-1\) to discount the degree of freedom corresponding to the average (Da Costa Lewis (2003)).
leptokurtic ones, and, in fact, the outcome may appear deceptive. Graph 2.1 (Panels A and B) conveys an idea of the problems arising from the asymmetry.

Despite the fact that both distributions have been parameterised to portray the same mean and Standard Deviation ($\sigma = 1$), the skewness alters the whole distributions, thus provoking the Largest Extreme Value Distribution to become riskier than the Normal one. This fact remains undetected for SD, which delivers the same outcome for both distributions, thus converting SD in an unreliable risk measure in the presence of non-elliptical distributions. A similar situation is verified in the case of leptokurtic distributions and, in this vein Graph 2.2 exhibits the difficulties encountered by SD to adequately report the true risk conveyed by the distributions. Again, in spite of both distributions delivering the same mean and Standard Deviation ($\sigma = 1$), the $t(4)$ suggests that the leptokurticity increases significantly the risk faced by the portfolio. Hence, the adoption of SD as a measure of risk only works in very restrictive conditions and its application in empirical exercises can deliver misleading estimates of risk.

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29 Panel A exhibits the symmetric Standard Normal Density function whereas Panel B displays a Largest Extreme Value Distribution with location and scale parameters equal to $-0.5772\sqrt{2}/\Pi$ and $6^{1/2}/\Pi$ respectively.

30 Both distributions deliver null mean and unit Standard Deviation.
2.2.3. The Sharpe Ratio

The SR is defined as the quotient between the excess return over a predetermined benchmark and the Standard Deviation of the excess return over the benchmark. Hence,

\[ SR_p = \frac{E(r_p) - E(r_b)}{SD(r_p - r_b)} \]  (2.11)

where:

- \( E(.) \) : expectation operator
- \( r_p \) : portfolio return
- \( r_b \) : benchmark return

Deriving from the CAPM, the SR is consistent with the axioms for rational decision-making and the maximisation of expected utility, consequently fitting into the MV framework (Neumann and Morgenstern (1944)). However, Messina (1995) shows that the SR presents severe difficulties for investors compelled to earn returns over a minimum, therefore constituting a deceptive metric for those who consider risk as a failure to achieve a particular return. Additionally, it holds the same snags attributable to the SD highlighted in 2.2.2.

2.2.4. Early Value-at-Risk measures

VaR has undergone several transformations to become the most widespread market risk measure as is known nowadays. Holton (2002) distinguishes between two different
contexts where early VaR measures were developed: Modern Portfolio Theory and capital regulation.

a) Modern Portfolio Theory
a.1) The Leavens VaR
The first-ever VaR measure could well be attributed to Leavens (1945), who mentioned the ‘spread between probable losses and gains’. Holton (2002) remarks that although Leavens may have had the Standard Deviation in mind, the binomial distribution employed in the example stated in his work gave rise to some sort of VaR.

a.2) The Markowitz and Roy VaRs
Markowitz (1952) and Roy (1952) independently devised two similar VaR-like metrics that incorporated covariances to reflect the benefits of diversification and hedging. However, although conceptually strikingly similar, Holton (2002) stresses that both employed different assumptions: while Markowitz worked with the variance of simple returns and obtained the covariance matrix for risk factors using a primitive version of a Bayesian approach, Roy utilised a kind of shortfall risk representing the upper boundary of the probability of returns failing to achieve a ‘catastrophic level’ and a covariance matrix estimated from past data.

a.3) Further VaR innovations
Holton (2002) cites works by Tobin (1958), Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966) and Schrock (1971) as major contributors to the development of VaR circumscribed to portfolio theory. Notwithstanding the evolution brought about by the aforementioned schemes, Holton remarks that all of them were more suited for equity portfolios, and their application would have raised a host of modelling concerns.

b) Capital regulation
Despite the advances recorded in terms of MPT, the issue of capital regulation provided the quantum leap in VaR development. The origins in VaR-related measures of capital regulation could be traced back to 1980, when the SEC\textsuperscript{31} demanded the constitution of a capital cushion as a safeguard against market losses sufficient to cover, with 95%

\textsuperscript{31} SEC stands for Securities Exchange Commission in the United States.
confidence, the losses that may be incurred during the 30 days that would take to liquidate a troubled financial institution. Holton (2002) points out that, even though the regulation was rudimentarily expressed, in practice it operated like a 30-day 95% VaR upon which the capital holding had to be based. Accordingly, banks started developing more sophisticated ways of calculating VaR, and, in this vein, the current VaR methodology might well be based on two variants:

b.1) The Garbade VaR
In 1986 Garbade elaborated one of the major cornerstones of the modern VaR. Essentially, he calculated the price sensitivity of a portfolio of bonds and, assuming normality on portfolio returns, computed the Standard Deviation of losses and the 99% quantile of losses. One year later, in 1987, the methodology was refined after the introduction of a bucketing scheme which permitted the allocation of large portfolios in a smaller portfolio of representative bonds. Albeit the documents were initially circulated to internal clients of Bankers Trust, Garbade’s calculations were “years ahead of their time” (Holton (2002:10)).

b.2) The Wilson VaR
Later on, in the 1990s, many companies were already working with VaR techniques for a variety of purposes, though the most widespread approach involved the usage of Markowitz’s methodology to build the portfolio of risk factors, the calculation of the corresponding Standard Deviation and the appropriate quantile following the normality assumption.

In 1993, Wilson published a founding article acknowledging the heteroscedastic and leptokurtic nature of financial returns and fostering the application of measures to deal with them, such as the adoption of the heavy-tailed $t$-distribution instead of the typical Normal distribution. Quite straightforwardly, the author recommended a method to adjust the standard risk capital calculations using the $t$-distribution which reflected the fact that market participants possess only limited information about the true composition of the covariance risk matrix and, therefore, the risk should accordingly be enhanced.

b.3) RiskMetrics
At a time of great global concern about derivatives, in 1996 JP Morgan launched a new service called RiskMetrics, which allowed the VaR calculation through a simple methodology. It constituted a system to be employed by all the units within a firm, thus aiming at comprising all the individual VaRs into a single measure of market risk.

In strictly technical terms RiskMetrics did not represent a stellar innovation, given its reliance on a simple non-linear GARCH(1,1)\(^32\) model with normally distributed returns far less sophisticated than its antecedors Garbade and Wilson; its greatest virtue was to promote the VaR measure outside the bank and to endorse the technique to wider audiences to the extent of becoming a kind of synonym to VaR calculation.

### 2.3. Value-at-Risk: definition, advantages and disadvantages

#### 2.3.1. Value-at-Risk definition

Rossignolo, Fethi and Shaban (2012a) mention McNeil, Frey and Embrechts (2005) to define VaR. In this light, the VaR of the portfolio of multiple or single assets at the confidence level \(\alpha\) is the smallest number \(l\) such that the probability that loss \(L\) exceeds \(l\) is no greater than \((1 - \alpha)\):

\[
\text{VaR}(\alpha) = \inf \{l \in R : P(L > l) \leq (1 - \alpha)\} = \inf \{l \in R : F_L(l) \geq \alpha\}
\]  

(2.12)

This risk measure informs the amount of the monetary loss that will only be exceeded \(\alpha\%\) of the time in the next \(k\)-trading days. More specifically, Linsmeier and Pearson (1996) state that, at a given point in time, \(\text{VaR}_{t+1}\) describes the risk in the tails of the conditional distribution of losses over a one-day horizon: it expresses the maximum loss in the value of exposures due to adverse market movements that will not be exceeded within a pre-specified coverage probability \(\alpha\) if portfolios are held static during a certain period of time \(t\), thus making:

\[
Pr (\text{Loss} > \text{VaR}_{t+1}) = 1 - \alpha
\]  

(2.13)

or expressing this definition in relative terms or returns:

\[
Pr (r_{t+1} < \text{VaR}_{t+1}) = 1 - \alpha
\]  

(2.14)

VaR focuses on the tail of the Profits and Losses (P&L) distribution, thus falling under the category that Alexander (2008c) names Quantile Risk Measures. Hence, given

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\(^{32}\)GARCH stands for General Autoregressive Conditional Heteroscedastic model. Section 3.5 treats these kinds of techniques extensively. More specifically, the RiskMetrics model is near an IGARCH(1,1) representation without drift.
that VaR is the loss amount\textsuperscript{33} surpassed with a small probability $1 - \alpha$, holding the portfolio static over the time horizon selected (indicated by $h$ and fixed at 1), it is imperative to calculate the quantile $x_{t+1,\alpha}$ such that:

$$ Pr (r_{t+1} < - x_{t+1}) = 1 - \alpha \quad (2.15) $$

afterwards setting $- x_{t+1} = \text{VaR}_{t+1}$.

VaR representations are usually constructed in returns as models are developed in relative terms, thus being comparable across very long periods of time even when price levels have varied substantially; the calculations, then, are performed using returns and VaR is expressed as a percentage of the current portfolio value\textsuperscript{34}.

As VaR represents simply a quantile of the P&L distribution, the definition depends on two arbitrary parameters: the confidence level and the time horizon. The former denotes the likelihood that the outcome to be obtained will be worse than the threshold set by VaR (must be any value between 0 and 1) whereas the latter represents the period over which the portfolio profit or loss is measured (typically a day, although it may be a week, a month or even longer ones).

Graph 2.3 illustrates some of the aforementioned concepts. It pictures a standard normal P&L over a one-day horizon.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{Graph2.3.png}
\caption{VaR illustration}
\end{figure}

Note: P&L density portrayed using a standard Normal distribution.

\textsuperscript{33} Alexander (2008c) emphasises that VaR is always expressed in present value terms. Therefore, all throughout the current thesis, the P&L distribution and the related concepts are referred to in discounted terms.

\textsuperscript{34} Alexander (2008c) highlights that, even though VaR is computed in present value terms, it is a forward looking risk measure. Therefore, as risk models are estimated to produce an $h$-day ahead VaR, they automatically produce a forecast of the risk over the next $h$ days.
As the figure conveys, the $\alpha$%-VaR is the value on the x-axis (quantile) that intersects the top 95% of the P&L observations from the bottom $(1 - \alpha)\%$ of tail observations. Consequently, it may be read that the 95% quantile amounts to -1.645, thus making $\text{VaR} = 1.645$ whilst the corresponding 99% values are -2.326 and 2.326 respectively.

It is implied in Graph 2.3 that VaR grows in line with the confidence level and Dowd (2005) remarks that it usually augments at an increasing rate (particularly when statistical assumptions are made). The mechanics of the holding period are somewhat more intricate and rely heavily on the mean value of the distribution adopted. In this sense, Dowd (2005) points out that VaR certainly varies with the holding period, although the relationship is not clear cut: if the mean is null, the VaR rises with the square root of the holding period whereas in the case of positive mean, it increases at a decreasing rate and, at some point, would turn down. However, the present thesis will unfold utilising the conventional 99% VaR demanded by the Basel Committee and the assumption of a one-day holding period, thus avoiding the uncertainties posed by the effect of different periods. Moreover, Alexander (2008c) underlines that whenever the risk horizon is relatively short, the impact of the mean on the VaR measure is often negligible—in addition to the fact that, in the case of daily horizons, there appears to be no evidence to consider means different to zero.\footnote{In fact, VaR almost always grows at an increasing rate when a stochastic distribution is employed for the calculus. The only exception is embodied in those techniques that are based upon the empirical distribution of returns, i.e., Historical Simulation and its variant Filtered Historical Simulation, because of their reliance on the observed patterns of the distribution of returns.}

**The VaR basic formula**

It is relevant to derive the classic VaR expression that underpins the well known formulae. Those VaRs are calculated contemplating the former values of the series of returns, obviously disregarding the attribution to different risk factors. The present section follows the rationale in Alexander (2008c) and Christoffersen (2003), thus assuming normally distributed returns and dropping, for the sake of simplicity, any reference to time dependence and extension of time horizon.

In this sense, assuming $r \sim \text{iid } N(\mu, \sigma^2)$ and recalling (2.15): 

$$Pr (r < -x_\alpha) = 1 - \alpha$$

Using the standard normal transformation,

$$P(r < -x_\alpha) = P\left(\frac{r-\mu}{\sigma} < \frac{-x_\alpha-\mu}{\sigma}\right) = P \left(Z < \frac{-x_\alpha-\mu}{\sigma}\right)$$  \hspace{1cm} (2.16)
where $Z \sim N(0,1)$. However, if $Pr(r < -x_\alpha) = 1 - \alpha$, then

$$P\left(Z < \frac{-x_\alpha - \mu}{\sigma}\right) = 1 - \alpha \quad (2.17)$$

The definition of the Normal distribution function states that $P(Z < \Phi^{-1}(\alpha)) = 1 - \alpha$ thus making:

$$\frac{-x_\alpha - \mu}{\sigma} = \Phi^{-1}(\alpha) \quad (2.18)$$

bearing in mind that $\Phi$ represents the standard Normal distribution function. However, given that $-x_\alpha = VaR(\alpha)$, and by means of the symmetry of the standard normal distribution:

$$\Phi^{-1}(\alpha) = -\Phi^{-1}(1 - \alpha) - \mu \quad (2.19)$$

Plugging (2.19) into (2.18) yields the VaR formula for iid standard normal returns:

$$VaR(\alpha) = \Phi^{-1}(1 - \alpha)\sigma - \mu \quad (2.20)$$

It is typical for returns of daily frequencies to exhibit null mean returns, thus (2.20) becomes, after equating $\mu$ to zero:

$$VaR(\alpha) = \Phi^{-1}(1 - \alpha)\sigma \quad (2.21)$$

which is tantamount to writing:

$$VaR(\alpha) = \sigma \Phi^{-1}(1 - \alpha) \quad (2.22)$$

and in a more general fashion:

$$VaR(\alpha) = \sigma F^{-1}(1 - \alpha) \quad (2.23)$$

where $\sigma$ represents the volatility of the distribution of returns and $F^{-1}(1 - \alpha)$ the inverse of the cumulative distribution employed. Furthermore, after reconsidering the conditionality on time $t$, the VaR($\alpha$) for the day $t+1$ is given by:

$$VaR_{t+1}(\alpha) = \sigma_{t+1} F^{-1}(1 - \alpha) \quad (2.24)$$

Expression (2.24) forms the basis of all VaR calculations throughout the current thesis$^{36}$, and additionally highlights the elements involved in its calculation: a) confidence level, b) time horizon, c) volatility forecast, and d) distributional assumption. The BCBS, either in Basel II, Basel 2.5 and in Basel III demands the employment of a 99% confidence level and usage of a daily time span; however, there is no suggestion as to which scheme may be employed to estimate the volatility prediction or distribution of returns. In this sense, the present research aims at shedding light on the differences in the application of several stochastic techniques and its combination with alternative distribution of returns.

$^{36}$ The only exception appears when VaR is estimated through the Historical Simulation. Section 3.3.
2.3.2. Advantages and disadvantages of VaR

2.3.2.1. Advantages of VaR

As every risk measure, VaR also has pros and cons. Beder (1995:12) synthesises in a single sentence, its greatest merit: “VaR enables a firm to determine which businesses offer the greatest expected return at the least expense of risk”, afterwards highlighting its prowess to track, analyse, control and manage market risks. In this line, Linsmeier and Pearson (1996:3) state that VaR constitutes a simple way to describe the magnitude of the likely losses on the portfolio.

The definition mentioned in Section 2.3.1 implies some attractions worth of being enunciated:

i) VaR is a measure susceptible of being applied to any kind of assets and positions, thus enabling the comparison of risks across diverse assets;

ii) VaR allows the aggregation of different subpositions in one single number in a rather simple fashion, taking into consideration the correlations among the assets;

iii) VaR accounts for all the risk factors influencing the portfolio as a whole, as opposed to other traditional appraisals that compute them individually or, furthermore, subsumes all those risk factors into a single measure (e.g., CAPM or Duration/ Modified Duration); furthermore, VaR allows the focus on the firmwide level and not on individual units operating in it;

iv) VaR indicates a probability associated with a certain loss amount, instead of other measures (e.g., CAPM or Duration/ Modified Duration), which do not convey a likelihood assessment;

v) VaR is easily understood and transparently expressed in units of measure.

Stemming from the aforementioned attractions, VaR has many other uses beyond the typical market risk quantification which undoubtedly helped the surge in its popularity. For instance, within the scope of the company:

i) VaR information is often employed to set the overall risk target of a firm (Dowd (1998));

ii) VaR may aid in the optimal allocation of resources and in setting the limits to risk-taking (Dowd (1998))

37 Dowd (2005) gives specific names for the patterns enunciated. Hence, VaR is a common, aggregate, holistic, probabilistic and expressed in units risk measure.
iii) VaR might also be used to determine the remuneration rules for traders, thus rewarding those ones who deliver the highest return per VaR unit (Dowd (1999));
iv) VaR may set the guidelines for macro portfolio or hedging decisions (Dembo (1997) and Jorion (1996));
v) VaR could constitute the founding block of any credit or operational risk technique (RiskMetrics (1996), BCBS (2006a));

whereas from a regulatory point of view:
i) VaR is currently applied to determine the MCR demanded by the BCBS referred to market risk exposures (BCBS (2006a, 2010));
ii) VaR is to be specifically disclosed by institutions operating under the framework of the BCBS (BCBS (2006a, 2010));
iii) VaR is often employed to estimate the probability of distress in Capital Strength ratios.

2.3.2.2. Disadvantages of VaR

Linsmeier and Pearson (1996) stress that, despite representing a quality step evaluated against previous risk quantification measures, VaR cannot be regarded as a panacea.

In this vein, and despite all the points that enhance the magnetism of VaR, it can simultaneously be dangerous. As early as 1995 Beder was the first author to raise concerns about the fact that the multiplicity of schemes to calculate VaR -each one with its respective assumptions- could render significant differences in the results and, perhaps more relevantly, in the MCR level at a time when BCBS was planning to introduce the VaR-based-IMA in the Basel Capital Accord as a result of the Market Risk Amendment of 1996 (Jorion (1996)). Furthermore, Dowd (2005) cites Marshall and Siegel (1997) casting doubts on the implementation risks that VaR models are subject to.

Penza and Bansal (2001) underline two additional limitations which could eventually be shared by many other risk measures. In the first place, the authors remark that VaR is only appropriate for traded assets and liabilities, this way ignoring those positions not subject to market fluctuations. However, the issue may be countered considering that VaR is a measure of market risk, hence disregarding all links to carry

38 Appendix 2.A
39 Appendix 2.A.
assets and liabilities. The second consideration refers to liquidity risk, given the difficulty of VaR (and other risk measures as well) to address it, but, theoretically, it would be feasible to tackle in the case of thin markets\textsuperscript{40}; Alexander (2008c) suggests the transformation of the primary data using, whenever possible, the average between the Highest and Lowest prices recorded in the day instead of applying the Close Price:

\[
p_{t}^{LA} = \frac{p_{t}^{H} + p_{t}^{L}}{2}
\]  

(2.25)

where the superscripts “\(LA\)”, “\(H\)” and “\(L\)” stand for “Liquidity Adjusted”, “Highest” and “Lowest” prices respectively, all recorded in day \(t\). However, though her solution may prove theoretically correct, in practice the difficulty in obtaining that huge amount of information particularly in Emerging and Frontier markets may render the approximation useless. Penza and Bansal (2001) recommended, as a likely response to the perceived lack of depth, the adoption of highly leptokurtic return distributions or the application of penalty factors in a bid to artificially increase VaR. In this line, the BCBS demands that all daily VaR estimates should be calculated considering a ten-day holding period to reflect an average time to abandon any position.

Danielsson and Zigrand (2001) pour concern about tight VaR regulations. They reason that risk modelling affects the distribution of risks, thus rendering risk as an endogenous element instead of an exogenous variable. Therefore, as current directives demand VaR measures, market participants are urged to execute similar trading strategies which change the distributional properties of risk, particularly in crisis times. When shocks hit markets, as all actors deploy the same stratagems they will reduce their exposures by virtue of their VaR-based representations, thus shrinking liquidity and exacerbating the scope of the crisis.

However, VaR as a risk measure portrays much deeper structural deficiencies and, in this vein, Danielsson (2002) points out three major snags. Firstly, it fails to indicate the size of the potential losses beyond the threshold; secondly, it does not belong to the coherent measures of risk and thirdly, its dependence on a single quantile facilitates its manipulation with specially devised strategies, thus nesting a moral hazard problem. The following paragraphs briefly clarify the scope of Danielsson’s asseverations:

\textsuperscript{40} Dowd (2005) classifies markets into four categories according to their liquidity. Therefore (in decreasing order), it would be possible to find: smooth markets (liquid, atomised and with high turnover); choppy markets (less deep, fewer participants and lower turnover); icy markets (even thinner and less liquid, with limited secondary markets) and frozen markets (extremely thin and few –if any- secondary markets). Thin markets would, in principle, encompass the last three.
i) **The size of potential losses:**

In other words, this means VaR’s inability to capture the tail risk because it only states the riskiness of a position considering the likelihood but not the magnitude of losses beyond a certain confidence level or even their expected value (Artzner et al. (1999)). Dowd (2005) stresses that two positions can eventually report the same VaR but deliver very different risk exposures. Other measures like the Tail Conditional Expectation (TCE) or Expected Shortfall (ES) are capable of addressing that challenge.

ii) **Coherent measures of risk:**

Artzner et al. (1997, 1999) proposed a framework to assess every risk measure consisting of four axioms to be complied with: monotonicity, positive homogeneity, translational invariance and subadditivity (also known as superadditivity). The first three are conditions intended to discard embarrassing results while the fourth one reveals as the most relevant and, in this light, VaR fails to show that the diversification, at the very least, does not increase risks.

Additionally, subadditivity conveys more than a theoretical refinement as it may give rise to several unpleasant practical connotations. For instance:

- It implies that widespread risks create a residual risk which existence had not been recorded before;
- Non-additive risk measures might motivate traders in organised exchanges to break up their accounts, therefore reducing their margin requirements;
- Capital requirements set on the basis of non-subadditive risk measurements may tempt banks to break themselves up to reduce the capital demands;
- The perfectly correlated sum of risks could no longer be employed as a conservative estimate of the exposure faced.

Acerbi (2004) stresses that uses of VaR should be restricted to its quantile condition, hence warning against its extended application as a sound risk measure.

iii) **Moral hazard and VaR manipulation:**

---

41 Section 2.3.2.
42 Appendix 2.B.
43 There are, on the other hand, some examples in the literature where VaR displays superadditivity like Danielsson et al. (2005) and Degen, Embrechts and Lambrigger (2007). However, even in BCBS (2012) these instances are regarded as asymptotic and not representative of the general conditions observed in the markets.
Danielsson (2002) and Ahn et al. (1999) note the feasibility to devise permitted trading strategies in order to enhance profits and report –albeit not decrease- a smaller VaR value. The use of options would enable the bank to artificially diminish its VaR and report a lower VaR number to regulators, courtesy of the official focus on a single quantile. Other measures that comprise the evaluation of the whole of the distribution beyond VaR quantile do not allow these tricks\textsuperscript{44}.

### 2.4. Alternative risk measures

Academics and regulators have identified the lack of coherence (i.e., failure to comply with subadditivity) and the absence of information about losses beyond the established quantile as the most important deficiencies of VaR as a risk measure. Therefore, it would be desirable for any replacement risk measure to retain the benefits of VaR while, at the same time, avoid its pitfalls. Furthermore, Dowd (2005) states that those schemes ought to represent functions of the quantiles of the P&L distribution rather than a single and isolated quantile. Accordingly, the financial literature proposes the Expected Shortfall as a natural substitute, which appears endorsed by BCBS (2011b) as the likely substitute for VaR if it eventually gets supplanted. The ensuing lines portray a synthesis of the rationale behind ES.

#### 2.4.1. Expected Shortfall

It constitutes one of the alternatives capable of overcoming VaR’s theoretical deficiencies. According to McNeil, Frey and Embrechts (2005), ES represents a risk measure providing information about the tails of the distribution in depth. Specifically, instead of sticking to a particular confidence level \(\alpha\), the underlying idea boils down to average the VaR for those probability levels \(\zeta\) such that \(\zeta \geq \alpha\), thus making \(ES_\alpha \geq VaR_\alpha\) for identical probability levels. Formally,

\[
ES_\alpha = \frac{1}{1-\alpha} \int_0^1 q_\zeta d\zeta
\]

where \(q_\zeta\) is the quantile function corresponding to the return distribution, thus relating ES and VaR in an analogous fashion:

\textsuperscript{44} The scheme involves writing a call with a strike price right below the correct VaR and purchasing a put with its strike price right above the desired VaR, taking special care in fixing equal absolute differences. Thus, the intended effect is instantaneously achieved and the reported VaR is reduced (Danielsson (2001)).
Additionally, for continuous loss distributions, a more intuitive, easily comprehended interpretation asserts that ES is equal to the expected loss incurred provided VaR is exceeded, i.e., the average of the losses exceeding VaR:

\[
ES_\alpha = \frac{1}{1-\alpha} \int \text{VaR}_\zeta d\zeta \quad (2.27)
\]

for an integrable loss \( L \) (McNeil, Frey and Embrechts (2005)). This expression does not hold for discontinuous loss distributions, in which case the formula below applies (Acerbi and Tasche (2002)):

\[
ES_\alpha = \frac{1}{1-\alpha} \{ E[L; L \geq q_\alpha(L)] + q_\alpha [1-\alpha - P(L \geq q_\alpha)] \} \quad (2.28)
\]

When the distribution is discrete, ES is calculated employing the natural estimator and discrete equivalent of (2.23), i.e., the average of the \( w \) greatest losses:

\[
ES_\alpha = \frac{1}{1-\alpha} \sum_{i=1}^{w} L_i \quad (2.30)
\]

ES becomes an attractive risk measure by means of retaining VaR conceptual simplicity and easiness of implementation and complying with the four axioms of coherence. Likewise, it conveys more precise information about the distribution of losses than VaR, because in addition to the threshold value that represents the quantile-based VaR, it delivers the amount of likely losses in the event of adverse market movements. Graph 2.4 illustrates that principal difference between VaR and ES: while VaR focuses on a single quantile (employing a standard Normal distribution, the 95% VaR equals 1.645), ES gauges the average of all the quantiles exceeding VaR (under analogous assumptions, 2.063). Consequently, ES delivers a figure more than 25% over VaR which would undoubtedly render higher capital requirements were this measure utilised. By construction, any conditional VaR measure like ES can never be less than the corresponding VaR, and the difference between ES and the respective VaR depends upon the heaviness of the tail: the fatter the tail, the greater the difference.
2.5. Concluding remarks

Artzner et al. (1999) defined risk as a random variable that expresses the change in values between two dates due to market variations or, more generally, uncertain events. Therefore, it is a concept linked to the future unknown value of a position which should be quantified in any possible way. At a brushstroke, a risk measure, then, is described as a function that determines whether the future value of the exposure belongs to the subset of attainable risks, either stated by a regulator or a risk manager. More specifically, a risk measure is represented by a relationship that maps the set of possible future results to a single number.

The current section has briefly reviewed the most widespread risk measures spanning from the initial Asset & Liability Management to the latest coherent Expected Shortfall. A&LM and further refinements like Duration or Gap analysis constituted the very first attempts to translate that uncertainty into numbers, although in a much limited fashion given its reliance on accountancy concepts. On the other hand, Standard Deviation represented an improvement in the quantification of market risks; usually presented in the context of the Mean-Variance framework, it is adequate only under certain restrictive assumptions which curb its utility, much like the Sharpe Ratio.

The quest for a suitable risk measure is often tackled through the application of Value-at-Risk, approach endorsed by the supranational financial regulator BCBS. In many respects, VaR constitutes a quantum leap at the time of assessing market risks in an integral way as it can be employed in more general conditions and, given its status as...
a quantile, it can be calculated for any statistical distribution. However, VaR is essentially a theoretically flawed risk measure with serious deficiencies related to the lack of information about the size of losses beyond the established threshold in the first place, and the violation of the diversification principle in the second place.

Coherent risk measures provide a solution to these problems, simultaneously maintaining VaR advantages. In this vein, Expected Shortfall embodies the most apt scheme to overcome VaR glitches and abide by the properties demanded for a sound risk measure, at the expense of an incremental implementation cost by only upgrading a VaR estimation engine. Although VaR remains the official risk measure to gauge market risks and convert exposures into MCR, regulators like BCBS have become increasingly concerned about VaR drawbacks and consequently opened a consultation period aimed at eventually replacing VaR, mainly with ES. Nevertheless, for the time being, VaR is apparently staying for long, at least beyond the year 2019 when Basel III comes definitive and completely into force.

2.6. The need for regulation
In general terms regulation is regarded as necessary when free markets are incapable of allocating resources efficiently, therefore preventing the whole economic and financial system from achieving the Pareto optimum. Jorion (1996) and Santos (1999) reduce the motivations to two situations: externalities and agency problems. A short summary of each alternative ensues.

2.6.1. Externalities
These events appear when institutions fail to fulfill their commitments, affecting other companies and giving birth to systemic risk. The interconnectedness among economic agents allows the propagation of adverse shocks (Kaufman (1996)), provokes bankruptcies, undermines confidence in the banking network and generates instability, therefore posing a threat to the entire financial system and possibly to national or even international economies (Freixas and Saurina (2004)). In addition to the macroeconomic instability, Llewellyn (2002) identifies several elements common to banking crises:

i) inadequate supervision: materialised in non-existent capital requirements, indulgence in loan ratings, solvency problems, etc.;

ii) information inappropriately undisclosed;

iii) corporate governance problems in banks;
iv) reckless banking practices reflected in insufficient capital ratios, excessive growth in
loans, weak risk management and control systems, creative accountancy, etc.;
v) lack of market discipline.

During crises all market participants perceive the underlying danger and
automatically act in a similar way in order to shelter against the increasing risk selling
risky assets and resorting to hedge mechanisms. Danielsson (2002) emphasises that this
kind of herd behaviour increases volatility and restrains liquidity, consequently
exasperating the crisis and establishing a vicious cycle with prices, volatility and
liquidity as distinctive elements. Confronted with this perspective, financial regulations
have progressed in several ways: establishing the Central Banks as lenders of last resort,
creating deposit insurance, imposing restrictions to activity\textsuperscript{45}, issuing solvency
requirements, demanding minimum capital standards or even supervising banks.

One of the most significant schemes mentioned above is the figure of deposit
insurance, given that it is designed to bridge the asymmetry between depositors and
banks as the latter are in possession of more qualified information related to the risk of
their operations (Diamond and Dybvig (1983)). In general, deposit insurance was
supplied by some sort of government guarantee, and its mere existence reassured the
depositors regarding the security of their funds, simultaneously reducing the probability
of massive withdrawals should they infer the market value of the assets of the banks is
falling behind the value of liabilities. Furthermore, even though these programmes were
commonly deemed necessary in order to protect small investors which were unable to
scrutiny the activities their banks engaged in, they gave birth to a host of problems
commonly summarised as moral hazard. Government guarantees minimised the
incentives to monitor the risk of the activities of the banks, in turn motivating the savers
to place their money in those institutions offering higher interest rates at the expense of
taking riskier market positions (Young and Ashby (2001)). Jorion (1996) compares
deposit insurance backed by government guarantees to offering a put to bank owners:
were they to assume risks and obtain gains, profits would be distributed among
investors; on the contrary, the state would bail out them. The government guarantee
behind deposit insurance was typically implemented through premiums related to the
size of the deposits, irrespective of the financial situation, capitalisation levels and risks

\textsuperscript{45} The limitations or complete ban of short selling in UK and Germany in 2009 constitute a
contemporaneous example of those constraints.
positions of the entities, which eventually provoked the reappearance of moral hazard: banks did not internalise the cost of risk and were encouraged to assume more risks, increasing leverage without augmenting the capital base.

2.6.2. Agency problems

Briefly stated, this line of thought is based on the impossibility on the part of depositors to audit the activities of their banks which are subject to moral hazard, as seen above. This monitoring is of a costly nature and demands access to undisclosed information, therefore fostering the idea that investors should delegate their supervision faculties to regulators, already in possession of confidential data and in a more adequate position to perform those duties.

Admitting that the string of bank bankruptcies and their associated costs imply that to a certain extent banks need to be supervised, Beattie et al. (1995) and Dewatripont and Tirole (1993) justify the fact that regulations should be carried out through capital requirements in order to deal with externalities and agency problems respectively. In the first place, capital adequacy rules require institutions to build up minimum levels of own resources, thus reducing insolvency probability and limiting systemic risk. In the second place, minimum solvency requirements are judged as part of an effective regulation as they help to mitigate the agency problems by aligning the incentives of the proprietors, depositors and the rest of the creditors.

Defining capital as the difference between assets and liabilities (mainly deposits), the higher the ratio between assets and liabilities, the more secure savers will feel. Nevertheless, Sharpe (1978) warns against considering ‘adequate capital’ and ‘safe deposits’ as synonyms when he mentions the existence of an optimal capital ratio and characterises capital adequacy as a measure of solvency\textsuperscript{46}. According to Saurina Salas (2002), the road to current regulations evolved from structural norms, strict with regards to prices and quantities, to prudential directives granting some operative leeway, but at the same time demanding the constitution of capital buffers related to the risk assumed by the entity. Last, it is very important to keep in mind that the capital level is always subject to a tug-of-war between shareholders on one side and supervisors and creditors on the other: while the former intend to minimise its size in order to enhance the Return

\textsuperscript{46} Consequently, Matten (2000) affirms that the principal function of bank capital resides in the absorption of financial risk as opposed to fund assets.
Over Equity (ROE), the latter seek to increase it so as to dissuade the bank from reckless behaviour and remain in a stronger position to face losses and prevent systemic risk.

It is beyond the scope of the thesis to analyse the host of regulations that preceded Basel Capital Accords. The following section will feature a synthesis of the provisions of the agreements that bear some relationship with the topics to be treated afterwards, therefore acting as the framework that connects the empirical study with the regulatory instructions presently in force. The stipulations will comprise only the general provisions connected with market risk, without delving into detail with respect to the rest of the risks dealt with by BCBS or instruments encompassed by the definition of capital, for example. For the purposes of the current research, no distinction shall arise between the constituents of capital as stated in Basel Capital Accords (particularly the separation into Tiers): the proportion of capital levels will automatically refer to the concept of “Total Capital”, i.e., the aggregation of all those components of the capital intended to provide coverage against market risks contained in Basel I, Basel II or Basel III\(^47\). For further developments, the interested reader may recur to BCBS (1991, 1996, 2006 and 2009) and Jorion (1996).

2.7. Basel Regulations

Although most of the current regulatory material derives from BCBS, the directives issued by this working group are not legally binding given that it is not entitled to carry out a supranational supervisory activity. Its main function roots in developing guidelines and supervisory standards and providing sets of principles of best practice hoping that they get implemented by national authorities in the form of statutory or other arrangements adequate to their respective local schemes.


In 1988 the members of the BCBS reached an unprecedented agreement (“International Convergence of Capital Measurement and Capital Standards”) tending to supply the commercial banking activity with an equitable framework through minimum capital standards. The Accord, commonly called Basel I, established that capital had to

\[^47\] Hereafter the term ‘capital’ will refer to ‘total capital’.
represent at least an 8% of RWA (Risk Weighted Assets\textsuperscript{48}), figure known as Cooke proportion, but this capital was interpreted as an instrument to protect deposits and consequently represented an attempt to emphasise only credit risk. Needless to say, the most important flaw of Basel I materialised in its very poor handling of market risk, given that assets were recorded at its nominal value instead of their market value: it was possible, then, for an institution to suffer heavy losses at market values and still exhibit an acceptable balance sheet (Jorion (1996)). This exclusion of real-time market values, alongside the unsatisfactory treatment of derivatives (McNeil, Frey and Embrechts (2005)), motivated an Amendment intended to address, among others, those pending issues.

In order to assess the effect that the Basel I Capital Accord exerted on banks belonging to the G-10, the BCBS elaborated a study aimed at evaluating, in the first place, whether the adoption of fixed MCR (i.e., the 8% Cooke proportion) led to capital ratios higher than those potentially held had other schemes been employed and, secondly, at gauging whether the increase in capital ratios was achieved by augmenting capital or reducing lending. The research exercise concluded that the introduction of systematic capital requirements could have led banks to constitute more elevated capital ratios, given that the average ratio of capital to risk-weighted assets had risen from 9.3% in 1988 to 11.2% in 1996, although it was not possible to attribute the increment to the Basel Accord in a direct fashion\textsuperscript{49}.

In addition to the enhancement of the capital base, BCBS (1999) pointed out some other side effects of the Basel I Capital Accord. Two of them are worth being mentioned: initially, it was suggested that the application of uniform capital requirements to a broad class of assets may have prompted banks to engage in riskier activities, this way soaring the overall risk of the portfolio. In the second place, fixed capital ratios might have proven very difficult to meet in some special situations like economic downturns because, when faced with the prospect of being unable to meet the MCR banks may eventually decide to cut back lending, thus aggravating situations like credit crunches.

\textsuperscript{48} Risk Weighted Assets were defined as the sum of the risk assets on- and off-balance sheet. The first had to be classified in one of four risk buckets (0%, 20%, 50% or 100%) where, if an asset belonged to the 100% slot, it would require 8% capital, keeping direct proportionality for the rest of the examples. The second, on the other hand, needed to be converted to an equivalent exposure before being multiplied by their respective risk weights. (BCBS (1988)).

\textsuperscript{49} The investigation strongly suggested that “…banks respond to capital ratio pressures in the manner they believe to be most cost effective” (BCBS (1999:7)).
Besides the reasons exposed, a glimpse at the basic ideas underpinning Basel I Capital Accord is useful to understand the concern of regulators regarding its flaws, therefore highlighting the necessity for further reform:

i) the 1988 Accord was too simplistic to establish a connection between regulatory capital and risk-derived capital;

ii) the agreement did not reward correct risk management practices. Saidenberg and Schuermann (2003) label it as risk-insensitive, as banks could structure their portfolios taking on riskier activities (those delivering greater profit margins) without the necessary capital increase (recall the 8% fixed rate). According to BCBS (1999), there was strong evidence of capital arbitrage;

iii) market discipline and some aspects of internal control routines were thrown into oblivion by the regulatory body;

iv) it was deemed necessary to broaden the scope of the Accord in order to include those new risks like operational risk surged from the evolution of banking activities under the capital umbrella.

In a Consultative Document dated 1999, the BCBS stated that the main idea behind the then new Capital Accord (Basel II) comprised the introduction of the risk concept in directives, as capital levels had to be directly related to the risk profile of the position: the higher the risk, the higher the capital level needed to address it. Simultaneously, the coexistence of the Standardised and Internal Model approaches implied that the next set of mandates would have to achieve the convergence between capital levels deriving from both methodologies: banks working under the former should not see its capital levels increased whereas institutions implementing the latter ought to have recorded a decrease vis-à-vis the SA figures.

2.7.2. The Amendment to Basel I Capital Accord (1996)

The BCBS echoed a series of proposals issued by the G-30 about the treatment of off-balance sheet products and, at the same time, various market practices regarding the market valuation and appropriate risk management of the new financial products. In particular, the famous JPMorgan’s Weatherstone 4.15 report which summarised the bank’s market risk positions in a sole number called Value-at-Risk proved to be of great
influence at the time of defining regulatory policies. The propositions contained in this document eventually constituted a source of inspiration for the revision of the first Basel Capital Accord.

The Amendment (“Amendment to the Capital Accord to incorporate market risks”) basically urged banks to constitute an amount of capital aimed at protecting them against changes in the market value of its exposures, explicitly incorporating market risk to the regulatory process and defining it as “…the risk of losses in and off-balance sheet positions arising from movements in market prices” (BCBS (2005:1)). To tackle the problem, this revision prescribed two different methodologies: standardised and internal, and banks were allowed to choose one to measure market risks. The next two paragraphs provide a very brief synthesis of each scheme, emphasising those aspects connected with linear positions in equity.

2.7.2.1. The Standardised Approach (SA)
It employed a ‘building-block’ approach, understanding the term ‘building-block’ as that source of market risk susceptible of affecting the financial position of the company, namely interest rates, equity, foreign exchange, commodity prices and derivatives. In particular, for equity position risk, the Amendment distinguished between ‘specific’ and ‘general’, where the former encompassed positions in individual stocks and the latter was defined as “…the overall net position in an equity market” (BCBS (2005:19)).

The provision for capital charges varied depending on the kind of risk and its calculation observed a risk-weighted approach, with weights following a categorisation made explicit in the body of the Accord (BCBS (1988)). Specific risk required a minimum of 8% (eight) of the risk-weighted exposure (expressed in market-to-market values), while for general market risk the proportion was fixed at 8%. For the purposes of the current thesis, considering that positions are to be assumed on stock indices, the charge for equity market risk will be set at 8%, corresponding, then, to general market risk.

2.7.2.2. The Internal Model Approach (IMA)

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51 As it could be surmised, riskier positions bore more weight (BCBS (1988) and Jorion (1996)).
52 This charge could well be reduced to 4% provided the portfolio was both liquid and well-diversified, although the extent of the terms ‘liquid’ and ‘well-diversified’ remained at the discretion of the national regulators.
The adoption of the IMA allowed the banks, for the first time, to devise their own risk models in order to determine their capital requirements, subject to a series of qualitative and quantitative requirements, including a regular revision of the methodology at several administrative layers and approval of the respective national regulator.

The most important innovation maintained up to date has been the introduction of VaR as the official measure of market risk. Although the institutions enjoyed some degree of elasticity in developing their specific structures, BCBS demanded several minimum quantitative standards to be met by banks adhering to IMA (BCBS (2006a)):

i) VaR was to be computed on a daily basis;

ii) One-tailed confidence interval anchored at 99% was commanded;

iii) The holding period corresponded to ten (10) trading days. Albeit stipulations also permitted banks to employ shorter horizons, VaR had to be scaled up to ten days applying the square root of time rule\textsuperscript{53};

iv) The minimum length for the sample historical observation period was one-year;

v) Data sets needed to be updated at least every three months, unless national regulators deemed that considerable growth in volatility merited the employment of higher frequency actualisations;

vi) No specific model to calculate VaR was prescribed, thus banks were free to develop their own techniques provided they acknowledged all the significant risks faced;

vii) Capital requirements were to be calculated daily and defined as the higher of the preceding day’s VaR measured according to the above indications and an average of the sixty previous daily VaRs, increased by an add-on factor\textsuperscript{54};

viii) The value of the add-on (or multiplier) factor was set at 3 at minimum and likely to be augmented times a ‘plus’ belonging to the interval [0; 1]. The performance of the model in Backtesting determined the size of the number to be assigned to the ‘plus’;

ix) Banks had to carry out a rigorous and comprehensive Stress Testing programme designed to ascertain the performance of the models in the event of highly strung market conditions. Stress scenarios were identified as those incidents which could provoke extraordinary gains or losses to the bank’s portfolio\textsuperscript{55}.

\textsuperscript{53} Section 3.8 provides further treatment of this rule.

\textsuperscript{54} Sections 4.5 and 4.6.

\textsuperscript{55} The treatment of Stress Testing lies beyond the scope of the present study.
In order to clarify the above concepts, MCR established in items vii) and viii) may well be expressed in terms of the below formula, to be applied in several sections of the current work:

\[
MCR_{t+1}^2 = \max \left( \frac{m_c}{60} \sum_{i=1}^{60} \operatorname{VaR}(99\%)_{t+i-1} ; \operatorname{VaR}(99\%)_t \right) \quad (2.31)
\]

where:

- \(MCR_{t+1}^2\) : Minimum Capital Requirement as of day \(t+1\) in Basel II configuration
- \(m_c\) : multiplication or hysteria factor, where \(m_c = 3(1+k)\) and \(k \in [0; 1]\) according to the result of Backtesting

It is acknowledged that MCR as stated in the IMA contemplates a holding period of 10 (ten) days necessary to liquidate market positions, thus making \(\operatorname{VaR}_t^{10}(99\%)\). However, the thesis will adopt a holding period of 1 (one) day in view of a threefold purpose:

i) the correspondence between the VaR estimation horizon and the constitution of MCR;

ii) the inconsistencies of the square-root-of-time rule detected by Danielsson and Zigrand (2006), among others;

iii) the fact that the square-root-of-time multiplier (\(\sqrt{10}\)) may distort the true results of VaR models and mask its deficiencies (Danielsson et al. (2001)).

Finally, it is important to underline, however, that the Total Capital ratio must reach a minimum of 8% irrespective of the value delivered by formula (2.31): whenever \(MCR_{t+1}\) drills the aforementioned threshold, the 8% floor is applied; proportions superior to that trough are welcomed.

### 2.7.3. The Second Basel Capital Accord (2004)

At the heart of the “International Convergence of Capital Measurement and Capital Standards” (2004), commonly known as Basel II, rested the idea of risk based regulation. Danielsson (2003) stated that this concept meant that the minimum capital required by a bank was to be computed according to a risk scheme internally devised: riskier institutions had to put up higher capital charges, this way probably being inspected more closely by regulators.

The most relevant and innovative contribution of Basel II was certainly constituted by the introduction of the three-pillar concept. It was intended to achieve a
risk management approach focused on the interrelationship among the different sources of risk, however acknowledging the fact that capital standards could not substitute adequate risk management practices, no matter how accurately calculated they might be.

The First Pillar—“Minimum Capital Requirements”—was devoted to the estimation of the MCR, or, simply, Regulatory Capital. While it is appropriate to mention that in BCBS’s eyes the MCR represent the quantity of capital necessary for a bank to be considered as a “viable going concern for creditors and counterparts” (BCBS (2010:1)), the aim of this approach was, apparently, to align the quantification of this minimal capital with the economic loss potential of the banks (McNeil, Frey and Embrechts (2005)). It was established that any class of risk faced by the institutions should have borne a separate capital charge: credit risk, interest rate risk, operational risk and market risk. While market risk directives were fundamentally unchanged relative to the 1996 Amendment of the Basel I Capital Accord (Section 2.7.2. above), the treatment of credit risk was significantly reviewed and operational risk was acknowledged and introduced for the first time in the history of regulations.

The Second Pillar—“Supervisory Review Process”—attempted to incorporate the quantitative approach to risk management (i.e., the calculation of MCR) within the corporate governance structure of the firms. This pillar urged banks to develop and put in practice schemes to monitor and manage risk exposures, establishing constraints on the behaviour of the board of directors, management, employees and internal and external audit processes, establishing obligations for every level and determining, for example, that the board of directors remained the ultimate responsible organ for the oversight of the risk panorama and outlay of the firm’s risk strategy. BCBS expected, then, that senior management used sound judgement in the formulation and assessment of risk policies and set aside enough capital as coverage. Additionally, supervisors were required to guarantee the precise evaluation of the risk that institutions faced and that MCR were enough to match that risk position, furthermore empowering them, as to immediately intervene in the event of insufficiency of funds.

The Third Pillar—“Market Discipline”—embedded the disclosure reporting guidelines necessary to “…fulfill its (BCBS) promise that increased regulation will also diminish systemic risk…” (McNeil, Frey and Embrechts (2005:11)). Gutiérrez López and Fernández Fernández (2006) also highlight that relevant and timely information should help market participants to carry out sensible evaluations about the state of the banks and BCBS (1998) points out that investors, creditors, depositors and other
counterparts may offer better terms and conditions to those companies perceived as safe and well managed. More precisely, information ought to be revealed annually for qualitative aspects of bank management, semi-annually for general data and quarterly for topics related to capital adequacy, thus implying a harmonisation of disclosure standards.

Having sketched a very brief synopsis of the structure of the Basel II Capital Accord, the attention will drift to the treatment of market risks included in the first pillar, specially stressing its characteristics regarding the underlying assumptions employed in the IMA to calculate MCR.

2.7.3.1. The First Pillar and market risk
The chapter narrated the particulars about the evaluation of market risks, mostly maintaining the approach stated in the 1996 Amendment (Section 2.7.2), which represented a “…quantum leap in the prominence of quantitative risk modelling…” (McNeil, Frey and Embrechts (2005:11)). As aforementioned, the leeway to choose between the Standardised and the Internal Model remained, with smaller banks more likely to opt for the former and larger ones selecting the latter, despite the fact that Danielsson and Zigrand (2006) claimed that the uniform application of IMA would render benefits for the whole banking industry.

The BCBS defines market risk as the risk of economic losses arising from adverse market developments. This statement applies to both long and short positions in on and off-balance sheet exposures and investments in instruments linked to interest rates, equities, commodities and foreign exchange are all subject to the present directive. Two broad methodologies are depicted for measuring market risks: Standardised and Internal. In what follows a summary of the most relevant provisions depicted in both approaches are synthetised, specially stressing the equity indices position risk.

a) The Standardised Methodology
The Minimum Capital standard for equity positions is expressed in terms of two separate amounts belonging to specific market risk and general market risk. The former refers to long and/ or short positions in individual equity whereas the latter represents exposures in the market as a whole.
a.1) *Specific market risk*

The calculation is based on gross equity positions -the sum of long and short equity positions expressed on a market-to-market basis- and the capital charge attached amounts 8% \(^ {56} \).

a.2) *General market risk*

Contrary to the previous one, general market risk is estimated in terms of the net equity position in the entire market, for which the capital charge stays at 8%.

Specific risk for equity investments, as defined in Basel II, encompasses positions in an individual security that “…moves by more or less than the general market in day-to-day trading (including periods when the whole market is volatile) and event risk (where the price of an individual debt or security moves precipitously relative to the general market, e.g., on a takeover bid or some shock event; such events would also include the risk of “default”)” (BCBS (2005:163)).

For the purposes of the current thesis, positions in indices fall under the General market risk category; consequently, the capital charge belonging to the Standardised method would amount to 8% of the value of the respective portfolio.

b) *The Internal Model Approach (IMA)*

Institutions applying for IMA should be authorised by the respective national supervisor, subject to general requirements, qualitative and quantitative standards summarised below:

b.1) *General requirements*

The bank ought to, at least, abide by the broad demands below listed:

a) The risk management scheme ought to be theoretically strong and carried out by skilled staff in the implementation, control, audit and back-office areas;

b) The model shall prove reasonably precise (even though it must undergo a real-time test previous to being employed for capital measuring purposes);

c) A regular stress test programme to validate the specification is carried out.

---

\(^ {56} \) Were the portfolio to be sufficiently liquid and well diversified, the capital charge would be dropped to 4%, although the extent of the terms “liquid” and “well diversified” depends on the assessment that national supervisors make of the marketability and concentration of the respective markets.
b.2) Qualitative standards

Qualitative requisites refer to, in general, the internal organisation needed to support the management of the schemes, particularly emphasising the independence between operating, control and audit staff. The following points are specially stressed:

i) The control division in charge of the design and implementation of the risk model should be independent and shall perform detailed real-time and hypothetical scenario validation programmes of the internal model;

ii) Trading and exposure limiting activities must be fully integrated with and closely related to risk management;

iii) The entire risk management process from planning to controlling must pivot around the risk model applied;

iv) Periodic stress tests ought to be carried out in order to supplement the daily results of the internal risk system applied. The output is bound to influence capital adequacy as well as trading limits set by the board of directors; early actions should be adopted were the risk scheme to exhibit weakness to specific circumstances;

v) Frequent independent internal revision of the global risk management process shall be performed to ensure the complete cycle: consistency of data sources, accuracy of position information, adequacy of volatility assumptions, extent of market risks captured by the model applied through Backtesting programme, integration of market risk measures into daily risk management, structure and functions of the risk control unit and appropriateness of the documentation of the entire process, at least.

b.3) Quantitative standards

The framework to develop the risk models in Basel II is coincident with that established in the 1996 Amendment described in Section 2.7.2.b above (items i) to ix)), although banks are granted the application of stricter parameters at their discretion.

b.4) Specific risk

Special provisions regarding the IMA for specific risks are contained in Basel II. It is mentioned that the model ought to deal with the historical price variation of the particular exposure and adverse environments, grasp name-related basis and event risks and address liquidity issues, just to name a few. It remains necessary to carry out
Backtesting and Stress Testing to validate VaR models and achieve a certain soundness standard, though the specific risk may not be subject to a capital surcharge in the event of being adequately captured by the model. However, as the current thesis is concerned with equity indices exposures, the stipulations contained in the last paragraph of Section a.1.3) apply and only General market risk is to be calculated.

b.5) Backtesting
Institutions under the IMA framework ought to punctiliously corroborate the functioning of their respective model on a regular basis through a procedure denominated Backtesting. The term broadly means the “…application of quantitative methods to determine whether the forecasts of a VaR forecasting model are consistent with the assumptions on which the model is based…” (Dowd (2005:321)). Despite the fact that some proofs of a qualitative nature also belong to the category, BCBS is entirely focused on quantitative methods (formal Backtesting), thus circumscribing the treatment to the former sphere.

Backtesting encompasses a wide range of likely applications. Dowd (2005) performs a detailed review of the different alternatives, classifying them in three major groups:

i) Statistical proofs about the quantity of times VaR forecasts are exceeded (Frequency tests);

ii) Statistical proofs stemming from the entire distribution of exceedances, which, in general, suppose a more complete approach to the true evaluation of the models (Distribution equality tests);

iii) Proofs embodying different exposures and values (Alternative positions and data tests).

Categories i) and ii) possess a series of tests of varying degrees of complexity. Several authors have dealt with Frequency tests thoroughly, for instance Kupiec (1995), Christoffersen (1998), Manganelli and Engle (2001) or Brock, Lakonishock and LeBaron (1992). The issue of Distribution equality is typically evaluated employing the Rosenblatt or Berkowitz Transformations alternatively or the more widespread Chi-squared, Jarque-Bera, Kolmogorov-Smirnov, Lilliefors, Kuiper, Anderson-Darling or Shapiro-Wilks Tests, for example. The range of choices to assess the strength of the representations would appear practically unlimited were alternative positions or data
tried. In this vein, practitioners should bring into play their creativity to gauge the behaviour of the risk measures on past portfolios mixtures, different portfolio components, etc., or even changing historical periods or carrying out Monte Carlo simulations or data bootstrapping to profit from the potentially larger data set and the consequent precision of the results.

Amongst all the possibilities to perform these tests, the BCBS is concerned with the comparison of the real trading outcome with VaR values generated by the respective representation —both calculated with daily frequency— with a view to incorporate Backtesting results into the determination of MCR through IMA. In the Committee’s own words, this appraisal has been assumed both on the grounds of the split opinions regarding the theoretical properties of each variant and the imprecision of the verdict delivered by the proofs.

b.6) Backtesting framework in Basel Capital Accords

Given that the priority rested on simplicity and straightforwardness, the approach was designed to hold as few statistical axioms as possible, the most important of which was the independence of the daily results. In concrete, the framework consists of a basic frequency test where the risk measure is compared to the trading outcome on an “exception/ no exception” basis where “exception” is defined as that day when the VaR is surpassed by the corresponding P&L or trading outcome. The total number of exceptions (hereafter also termed excessions or violations indistinctly) is then counted, compared with the desired coverage level and translated into monetary terms for MCR determination.

In order to perform Backtesting under BCBS rules, a series of assumptions must be considered:

i) Backtesting ought to be carried out daily, thus disregarding the ten-day holding period to eschew the possibility of changes in portfolio compositions;

ii) The intended confidence or coverage level stays at 99% to achieve in consonance with the quantitative requirements established for IMA;

iii) Supervisors use the most recent twelve months of observations (tantamount to 250 trading days) to assess the quantity of violations during that period and demand no further actions, revisions or invalidation of the models employed to determine VaR values.
Signals emanated from Backtesting are then interpreted in light of the Three-Zone approach, which eventually defines the level of the multiplier (also hysteria) factor $k$ in (2.31) in Section 2.7.2.b).

The Three-Zone Approach

The results of Backtesting, in terms of the quantity of exceptions in a sample of 250 trading days (the most recent twelve months), is referenced to one of the alternative sectors denoted by colours: Green, Yellow and Red. As it might be appreciated in Chart 2.1, the boundaries of each category are defined using the cumulated binomial probability with a coverage level of 99%. Therefore,

i) **Green Zone**: amounts up to four VaR violations, thus representing a cumulated probability below 95% stated as its upper limit. Models classified in the Green sector are of no concern for supervisors as the probability of Type II error remains low;

ii) **Yellow Zone**: beginning where the cumulative probability equals or surpasses 95% (5 exceptions) and ending below 99.99% (9 exceptions), it encompasses those results which could either belong to accurate or inaccurate models. Surmises of inexactness grow as long as the quantity of violations augments, causing the magnitude of the multiplicative parameter $k$ in (2.31) to increase as well. In general, the imposition of higher capital charges bears the implicit assumption that the scheme selected appears not to be fundamentally unsound and its flaws are susceptible of being corrected through those adjustments, notwithstanding the fact that regulators ultimately assess the likely causes of the failures and decide whether to scrutinise the model more thoroughly or eventually disapprove it for capital setting;

iii) **Red Zone**: were the representation to suffer ten or more exceptions, surcharges would reach its cap ($k = 1$) and subsequent disallowance ensues, as BCBS deems there is an insignificant probability of reaching that number of exceptions (equal or less than 0.01).

For a sample of 250 trading days, the above ideas are deployed in Chart 2.1, extracted from BCBS (2006a):
### Chart 2.1

**Basel II Three-Zone Approach**

<table>
<thead>
<tr>
<th>Zone</th>
<th>Number of exceptions</th>
<th>Increase in scaling factor $k$</th>
<th>Cumulative Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Green Zone</strong></td>
<td>0</td>
<td>0.00</td>
<td>8.11%</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.00</td>
<td>28.58%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.00</td>
<td>54.32%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.00</td>
<td>75.81%</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.00</td>
<td>89.22%</td>
</tr>
<tr>
<td><strong>Yellow Zone</strong></td>
<td>5</td>
<td>0.40</td>
<td>95.88%</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.50</td>
<td>98.63%</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0.65</td>
<td>99.60%</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.75</td>
<td>99.89%</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0.85</td>
<td>99.97%</td>
</tr>
<tr>
<td><strong>Red Zone</strong></td>
<td>10 or more</td>
<td>1.00</td>
<td>99.99%</td>
</tr>
</tbody>
</table>

**Notes:**

1. Cumulative probabilities represent the probability of obtaining up to a quantity of exceptions in a sample of 250 observations when the level of coverage is $p = 0.99$.
2. Green Zone ends where cumulative probability reaches 95%.
3. Yellow Zone starts when cumulative probability equals or exceeds 95% and Red Zone at the number of exceptions where the cumulative probability equals 99.99%.
4. Increases in scaling factor $k$ represent the enlargement necessary to return the model to a 99% coverage, computed using the normality approximation and rounding the results.
5. For other sample sizes the boundaries will follow stipulations on (3).

### b.7) Stress Testing

One of the most important requirements for the respective IMA to be granted approval is the frequent rigorous and comprehensive Stress Testing programme. It constitutes an issue of an inherent contentious nature, as the BCBS demands institutions to perform them without precise guidelines.

With these proofs, BCBS intends to spot circumstances that could compromise the capital position of the bank. By definition, these events occur rarely and with a low probability, but with an intensity capable of rocking the capital base. Even though there is a healthy concern on the part of regulators to ensure the capital soundness through a scheme with clear-cut foundations, the ambiguous indications as to how to develop it make the whole process very diffuse and open to a wide variety of interpretations.

Dowd (2005) affirms the complexity of the procedure highlighting the range of categories of stress tests: type of event, type of risk implicated, risk factor, country or region involved, stress test methodology, model assumptions, kind of exposure, instruments participating, pattern of test, data requirements and portfolio composition. Furthermore, the author separates between two types of stress tests:

i) *Scenario (‘what if’) analysis*: the influence that certain situations and its implications exert on the particular portfolio is assessed;
ii) Mechanical analysis: this proof is directed to identify the most adverse combination of mathematically or statistically defined possible events and their associated losses.

However, BCBS bestows national regulators with enough faculties as to perform their own tests in three different fields:

i) Scenarios not demanding simulations: institutions should provide their respective regulators information about the largest losses occurred during the assessment period as well as its comparison with the capital levels determined under the IMA. Particular relevance is attached to the quantity of days of maximum losses that would have been covered by the calculated VaR levels;

ii) Scenarios requiring simulations: historical real market crashes should be applied on portfolios and their impact on capital levels reported to authorities. This category is focused on the reproduction of real situations and its present implications in terms of solvency;

iii) Specifically developed scenarios: banks ought to spot the most adverse factors bound to affect the particular portfolio and apply them to assess its impact on the positions.

where i) and ii) in BCBS’s list of scenarios correspond to i) and iii) to ii) in Dowd’s (2005) framework.

2.7.4. The Third Basel Capital Accord (2009)

In 2009, the BCBS commissioned a revision to Basel II at the end of which it produced a Consultative Paper where it acknowledged that MCR established in Basel II had been insufficient to cover market losses recorded in the event of the subprime crisis of 2007-2008. In addition, it recognised that MCR displayed a strong procyclicality \(^{57}\) which exacerbated the effects of the predicament: higher VaR values in (2.31) led to increases in MCR, therefore restraining credit and reducing the amount of liquidity necessary to lubricate the financial system and viceversa. This effect may not be surprising as it was already spotted by Danielsson and de Vries (2007) who criticised some aspects of BCBS formulations and suggested alternative measures to estimate MCR.

\(^{57}\) Procyclicality is defined as the tendency to amplify the ups and downs of the real economy (BCBS (2009b)).
In response to the deficiencies in the constitution of MCR, BCBS (2009a) decided to tighten the grip on capital requirements and proposed a new “overlay to tackle system-wide risks” (BCBS (2010b:3)). In the first place, it put forward the introduction of a stressed Value-at-Risk (hereafter stressed VaR or sVaR) with a view to curbing procyclicality and raising significant capital levels in order to fend off similar future crisis. Specifically, the sVaR follows the same guidelines as the current Value-at-Risk (current VaR or cVaR) for MCR, but instead of being calculated over the most recent one year observation period like the cVaR, it must be obtained from a one year observation term inflicting heavy losses to the portfolio of the bank. Unfortunately, BCBS stops short of providing precise details as to the procedure to estimate sVaR for which the following concerns might be raised:

i) there is no indication as to whether the same technique should be employed for the calculation of cVaR and sVaR;

ii) the reference to the stress period appears a bit imprecise, given that it is not clear whether the one year observation period of financial strain is defined in terms of very high daily, or monthly losses or important deficits spanning the overall year.

In the context of the present thesis, the ensuing conventions will be applied:

i) cVaR and sVaR will be estimated using the same techniques. As one of the objects of the work is the comparison between different VaR methodologies, results would undoubtedly be distorted were different models to be employed. Furthermore, schemes delivering high sVaR values could make up for shortages in cVaR, thus conveying a false impression of high capital levels;

ii) The twelve months stress period will be selected as that year with the highest overall loss, i.e., shortfalls are bound to be assessed on an annual basis, irrespective of where the highest daily or monthly deficits took place. It is usually the case that the greatest daily or monthly falls belong to the highest annual loss, but in some opportunities they may correspond to different yearly terms\(^\text{58}\). The adoption of the alternative criterion would surely favour schemes rooting in Extreme Value Theory then compromising the spirit of the present thesis to deal with every model in equal terms.

\(^\text{58}\) When short positions are evaluated, the opposite analysis will be carried out. In this vein, periods with high losses translate into high losses for short positions, i.e., market gains.
As it was aforementioned, the sVaR conserves cVaR guidelines referred to the main calculation parameters, i.e., daily frequency, 99% confidence level and maximum between the 60-day average and last day sVaR. Moreover, the 2009 Revision maintains the essentials of Backtesting and Stress Testing contained in Basel II.

On the grounds of the above specifications, the stricter capital demands reflect in sVaR added to cVaR in the below fashion:

\[
MCR_{t+1} = \max \left( \frac{m_c}{60} \sum_{i=1}^{60} cVaR(99\%)_{t-i+1} ; cVaR(99\%)_t \right) + \max \left( \frac{m_s}{60} \sum_{i=1}^{60} sVaR(99\%)_{t-i+1} ; sVaR(99\%)_t \right)
\]

(2.32)

where:
- \( MCR_{t+1} \) : Minimum Capital Requirement for day \( t+1 \) under Basel III framework
- \( cVaR(99\%)_t \) : 99% cVaR for day \( t \);
- \( m_c \) : multiplier for cVaR
- \( sVaR(99\%)_t \) : 99% sVaR for day \( t \);
- \( m_s \) : multiplier for sVaR

with \( m_s = 3(1+k) \) and \( k \) arises from Backtesting results for cVaR (not for sVaR) indicated in Section 2.7.3 point a.2.6) As \( k \in [0; 1] \), institutions are encouraged to develop precise VaR models in order to keep \( k \approx 0 \) and avoid penalties to establish MCR.

The second major modification implies building an additional countercyclical capital buffer, understanding ‘buffer’ as the amount that would enable the bank to endure a period of noteworthy plummeting values in exposures and still exceed MCR (BCBS (2010a)), thus absorbing shocks to the financial system. This countercyclical capital buffer is expected to lie in the closed interval \([0\%; 2.50\%]\) of the total portfolio; the precise level at every point in time remains undisclosed, although BCBS emphasises its variable nature: it is to be raised in the event of rapid credit growth\(^{60}\) and loosened in the event of a downturn in the credit cycle. However, even though ‘rapid credit growth’ is identified by the augmentation of credit above its long term trend, BCBS neither unveils the exact scale of the expansion in the cushion nor its quantitative relationship with upsides and downsides in the credit cycle.

\(^{59}\) The stipulations stated in Section 2.7.2.b) regarding the ten day holding period and the square-root-of-time rule also result of application here.

\(^{60}\) BCBS hands in the ultimate decision to national supervisors, as the faculty to determine whether the building up in aggregate credit heighten the systemic risk as a whole.
2.7.4.1. A summary of the most salient features of Basel III

Alongside the points enunciated in 2.7.4 regarding the new rules related to market risk capital, this section deals with other prominent points in which the aforementioned ones are embedded. It is therefore useful to contextualise which reforms in the capital structure of the firm have been endorsed. The present section will refer succinctly to the most important topics changed by Basel III in a bid to pave the way for Section 2.7.6\textsuperscript{61}, which will treat in detail the themes dealt with in the current thesis.

In broad terms, Basel III focuses on a stringent definition of the capital base complemented by extra-reliance on Stress Testing aimed at increasing the safety of all the individual banks and, simultaneously, reducing the default risk of the banking system as a whole. The new structure, which substantially reinforces the previous Basel II, is directed to shield banks against acute turmoil from microprudential (firm-specific framework) or macroprudential (systemic risk-based framework) components.

\textit{a) Microprudential components}

The firm-specific variants could be illustrated from the point of view of the capital ratio, defined as:

\[
\text{Capital Ratio} = \frac{\text{Capital}}{\text{Risk Weighted Assets}}
\]  

(2.33)

thus tantamount to expressing:

\[
\text{Capital} = \text{Capital Ratio} \times \text{Risk Weighted Assets}
\]  

(2.34)

Considering the numerator –Capital-, Basel III places strong emphasis on Tier 1 Capital, as it stresses the class of instruments with highest loss-absorbing capacity, i.e., common equity. The preference lies in Tier 1 Capital (common equity), with Tier 2 (subordinated debt) following down in the pecking order and Basel II’s Tier 3 (hybrid debt instruments) subsequently eliminated from the capital base. BCBS also toughened the stance on the assets encompassed in Basel III, given the fact that the trading book, securitisation products and counterpart credit risk on over-the-counter derivatives and repo agreements need to be covered by high quality capital, in stark contrast to its predecessor. Hence, as a side effect, banks will no longer be allowed to take advantage of the loophole that enabled them to switch their positions between banking and trading books, as a big chunk of common equity will be required to match both positions.

\textsuperscript{61} BCBS (2010c) provides extensive treatment of the new elements introduced and the changes with respect to Basel II Accord.
Even though the MCR’s floor remains fixed at 8% like in Basel II, its composition appears much different in terms of the components, as an increase in common equity of 125% in the quantity of Tier 1 capital is demanded under Basel III framework (4.5% of the risk-weighted assets against 2% under its predecessor). Furthermore, banks are required to build an extra buffer of 2.5% -to be held in the form of common equity- called Capital Conservation Buffer (CCB) designed as a bumper against losses that threaten to erode the core capital base of 8%, and a special provision is enacted in order to assure that institutions replenish the CCB through retained earnings or restricted dividends. Consequently, the total capital is driven to 10.5% of risk-weighted assets (8% MCR + 2.5% CCB), thus representing a 31.25% increase with reference to Basel II.

It is useful to consider the framework under which the Capital Conservation Buffer is to be restored in the event of being consumed by eventual losses. The scheme devised by BCBS, called Capital Conservation Standard, implies that the closer the bank moves to MCR (tantamount to a greater deterioration of CCB), the smaller the rate at which profits are handed out or dividends paid until the buffer is fully reloaded, as exhibited in Chart 2.2 below:

```
<table>
<thead>
<tr>
<th>CCB level after losses (% bucket of original CCB)</th>
<th>Percentage of earnings retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>100%</td>
</tr>
<tr>
<td>(0% - 25%)</td>
<td>100%</td>
</tr>
<tr>
<td>[25% - 50%)</td>
<td>80%</td>
</tr>
<tr>
<td>[50% - 75%)</td>
<td>60%</td>
</tr>
<tr>
<td>[75% - 100%)</td>
<td>40%</td>
</tr>
<tr>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>
```

*Source: BCBS (2010a).*

*b) Macroprudential overlay*

Basel III also contains provisions to avoid cataclysms like the subprime crisis of 2008. In this sense, the BCBS introduced the non-risk based leverage ratio as an extra protection to help mitigate the model risk and, additionally, to close any avenue that might appear to dodge the tough risk-based capital requirements. The leverage ratio is defined as the quotient between the Tier 1 capital and the bank’s assets plus off balance sheet positions plus derivatives. This new measure includes total assets as opposed to
the approach in Basel II, where the risk-weighted assets dampened the effectiveness of the ratio.

However, the imposition of the Countercyclical Capital Buffer (CyCB) is arguably one of the most relevant measures included in the body of regulations. This additional cushion aims at mitigating the procyclicality derived from the excessive credit growth placing a buffer ranging from 0% to 2.5% of the risk-weighted assets established at the discretion of the respective national regulators, which should determine the appropriate level considering the stage of the business cycle. The requirement will be calibrated considering the systemic risk, i.e., it shall be increased in the event of risk concerns and released provided the inverse takes place. Furthermore, the BCBS will additionally levy an additional capital requirement for GSIBs (Global Systemically Important Banks), situated in the interval [0% - 3.5%] of the risk-weighted assets and determined applying a bucketing scheme comprising quantitative and qualitative indicators to assess the systemic importance of international financial institutions (BCBS (2013b))

Finally, the macroprudential reform advocates a radical shake-up of the Pillar 2 Supervisory Process, as it underlines the need to strengthen the vigilance of the institutions’ internal model to obtain the respective VaR. Specifically, it recommends the rigourous application of close scrutiny on the part of regulators to guarantee that the models and stress tests employed by banks appropriately capture the systemic and, predominantly, tail risk present in markets. Accordingly, BCBS (2010c) warns explicitly against complacency embodied in the use of the Normal distribution to compute VaR estimates for capital determination purpose, simultaneously suggesting that Stress Testing must play a central role in the evaluation of the adequate capture of the impending risks.

c) Graphical representation of Basel III innovations

The following Chart 2.3 depicts the most prominent variations introduced by Basel III Capital Accord described previously, as well as its comparison with the former Basel II provisions adapted from BCBS (2010b, 2010c). The extent of the changes may be conveyed by the additional Capital Conservation (CCB) and Countercyclical Capital

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62 One of the alternatives that garnered some kind of support proposed to charge those banks with a higher loss-absorbing buffer equivalent to a percentage of the GDP of the country in question once a trigger point was surpassed (Scrivener (2012)). However, it was eventually discarded.
Buffers (CyCB) demanded by the directives, thus representing a significant increase in the capital base. Furthermore, BCBS has also taken every step in order to raise the quality of capital, as the extra cushions are required in the form of common equity, i.e., the class most capable of absorbing losses.

2.8. Concluding remarks
The relationship between market risk modelling and regulations has always been controversial, with supervisors often hardening their stance after market crises of considerable magnitude: proof of that statement can be found in Basel II and Basel III being enacted after the episodes in mid 1990s and the subprime crash of 2007-2008 respectively.

It is acknowledged that some kind of regulation must rein the markets, particularly when market forces are unable to allocate the resources effectively. In this vein, the directives contained in the Basel Capital Accords of 1988, 2004 and 2010 lay the foundations of many aspects in the life of banks, particularly Minimum Capital Requirements, the Supervisory Process and the Market Discipline, of which the first one falls within the scope of the current thesis. BCBS’s directives have developed from the simple Cooke Proportion in 1988 to the dual scheme featuring the Standardised Approach and the Internal Model Approach enhanced by the stressed VaR demanded by Basel III, thus indicating a healthy concern on the part of the authorities to address issues that –either unexpectedly or not- surface when markets are strained.

The new capital standards under Basel III have been designed to considerably increase the quality and quantity of banks’ capital and reduce the probability of systemic risk (oversimplifying tough issues). In broad terms it represents a continuation of the practices observed in Basel II in the sense that MCR depends entirely on the bank cherry-picking between SA and IMA and, in case the latter is selected, the institution needs to adopt their preferred model subject to qualitative and quantitative constraints. However, in fairness to the BCBS, Basel III does exhibit several distinctive features like the sVaR component, the CCB and CyCB as well as the curb to the distribution of earnings and dividend payouts once losses dent the capital buffers, all of which undoubtedly contribute to the soundness of the capital structure and the financial system as a whole.

In 2002 Danielsson stated that risk models often collapse when used for regulatory purposes, citing the herd behaviour displayed by market participants in the
event of panic. Albeit not in response to his comments, the BCBS introduced the CyCB to mitigate the procyclicality exhibited by the MCR when turmoil hits the markets but, apparently, that procyclicality may be attributed to VaR itself (BCBS (2012a)). While the replacement of VaR as a risk measure lies beyond the scope of the current thesis, the next chapters will shed light on the relationship between VaR specifications and the successive formulas to determine the sufficiency of the MCR.
### Chart 2.3
Capital Framework: Basel II and Basel III structures in comparison

<table>
<thead>
<tr>
<th>As a percentage of risk-weighted assets</th>
<th>Capital Requirements</th>
<th>Macroprudential</th>
<th>Overlay</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Common Minimum</td>
<td>Countercyclical Buffer</td>
<td>Additional loss-absorbing capacity for GSIBs</td>
</tr>
<tr>
<td>Basel II</td>
<td>2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Basel III</td>
<td>4.5</td>
<td>2.5</td>
<td>0 – 2.5</td>
</tr>
</tbody>
</table>

**Note:** Values expressed as a percentage of the RWA.

**Source:** BCBS (2006, 2010a, 2010b, 2013b).
Chapter 3

Value-at-Risk models
Methodology and Calculation
3.1. Introduction

The normality paradigm set out in the seminal works of Bachelier (1900) and Osborne (1959) acted as a fulcrum around which the financial theory developed during the first part of the 20th century.

However, the pioneering investigations of Mandelbrot (1963) and Fama (1965) highlighted several aspects related to the stochastic patterns common to financial log return series when the frequency is daily or even higher that contradict the gaussianity assumption. The specialised literature usually refers to these attributes as stylised facts (JP Morgan and Reuters (1996) or Penza and Bansal (2001)) because of the recurrence observed across markets and assets. Given their implications in risk management in the sense analysed in this thesis, the focus will be directed to end of day variations, acknowledging that with longer horizons (for example weekly, monthly or quarterly), the statistical properties may differ from the daily ones, as documented by Richardson and Smith (1993). The main contributions arising from Mandelbrot (1963) and Fama (1965) may well be subsumed in three basic points that keep their validity up to the present times:

a) The distribution of financial return series presents leptokurtic patterns, i.e., it portrays higher or fatter tails and narrower and taller peaks than those predicted by the Normal distribution (effects often referred to as “fat tails” and “thin waists” respectively), which is reflected in empirical distributions reporting kurtosis figures well above the normal level of 3. This is tantamount to saying that extreme swings in financial markets occur more frequently than if movements were driven by the Normal distribution, in turn generating a host of consequences for VaR and, in particular, capital constitution (Krehbiel and Adkins (2006), Danielsson, Hartmann and de Vries (1998), Brooks, Clare and Persand (2000) and Financial Services Authority (2009), hereafter FSA);

b) Returns, in linear form, are only slightly autocorrelated;

c) On the other hand, squared returns have significant autocorrelation, thus hinting at conditional variances, or variances being conditional on time.

Tucker (1992) points out that the absence of regular intervals between trading orders and the infringement of the independence of consecutive returns constitute the major drivers of nongaussianity, simultaneously observing that both features are preconditions of the Bachelier and Osborne model. Duffie and Pan (1997) carry out a
more thorough analysis identifying the former element as unexpected discontinuous changes in price levels—“jumps”—and the latter as persistent variations in volatility over time—“stochastic volatility”—as possible causes of leptokurtic distributions. The authors emphasise that, for typical market behaviour, jumps affect the distribution of returns at a percentile farther out in the tail than the critical value of 1%, while if at least two jumps take place in the same year, the referred quantile will be more seriously affected during few weeks. Alexander (2001, 2008b) and Dowd (2005) note that the stochastic volatility, through persistence (Section 3.8.1), dictates that a relatively high current or recent volatility implies a relatively high volatility forecast in the future, and conversely for low values. Therefore, there is a growing consensus among academics (McNeil, Frey and Embrechts (2005), Manganelli and Engle (2001), Kuester, Mittnik and Paolella (2006), Danielsson and de Vries (2000)) and national regulators as well (FSA (2009) and Japanese Bankers Association (2008)) that VaR as a quantile risk measure is highly sensitive to the impact of distributions with high kurtosis and models accounting for that pattern should in principle deliver more accurate performances.

The discovery of the departure from normality rocked the foundations of financial theory and researchers have since striven to find a plausible explanation for that accounted fact, for which Tucker (1992) singles out two alternative currents of thought. The first school, whose main advocates are Mandelbrot (1963) and Fama (1965), postulates schemes that follow a linear return-generating process with parameters detached from any dependence of past values which, in turn, deliver independent returns as a result.

The second one, supported by Fama and French (1988) and Akgiray (1989) to which Bollerslev (1986), Engle (1982), Jorion (1988) and Andersen and Bollerslev (1998) may well be added, states, on the contrary, that the first and second moments of the distribution of stock returns are conditional on time with persistent effects, possibly generated by nonlinear stochastic processes. The overwhelming empirical proofs collected across assets and markets appear to support the dependence assumptions to the detriment of independence and linearity (Campbell (1987), JP Morgan and Reuters (1996), Penza and Bansal (2001), Christoffersen (2003), McNeil, Frey and Embrechts (2005), among a host of studies). However, even though linear returns may exhibit small autocorrelation and still deliver data portraying leptokurtic features (Jorion (1988), Kon (1984) and Blattberg and Gonedes (1974)), researchers appear not to be concerned with its influence in market risk modelling and, in this vein, Fama and
French (1988) concede that there is not powerful empirical evidence to consider that the
temporal dependence of linear returns is worth of being modelled. Furthermore,
Boudoukh, Richardson and Whitelaw (1994) examine the autocorrelation patterns of
short term stock returns and conclude that they arise from frictions like measurement
errors in data, institutional factors and microstructure effects rather than the
fundamentals of the market, while Tucker (1992) reinforces this view asserting that the
magnitudes of those daily autocorrelations are of such small nature that any profitable
trading rule rooting in them would result very difficult to build. In contrast,
autocorrelations in the second moment and the intertemporal autocorrelation between
the first and second moments arise with crucial relevance at the time of schematising
return processes, and its preeminence in its various forms in modern financial theory
remains virtually unchallenged (Beder (1995), Manganelli and Engle (2001), among
others). Therefore, models that explain the conditional distribution of returns
representing volatility as a time-dependent and persistent process mirror the
leptokurticity and account for the volatility clustering observed in financial markets.

Even though countless numbers of specifications are presently in use in financial
econometrics, the majority of the techniques rely on the basic framework devised by
Engle (1982): the Autoregressive Conditional Heteroscedastic (ARCH) model, which in
essence forecasts the variance of the dependent variable at time $t+1$ using information
available up to time $t$ as a function of past values of the dependent variable with weights
determined as a result of an optimisation process. Further extensions like the
Generalised ARCH (GARCH) introduced by Bollerslev (1986) and Taylor (1986),
which ensures that the weights of the past values of the dependent variable decline
exponentially and never equal zero, and the Exponential GARCH (EGARCH) proposed
by Nelson (1991) that guarantees nonnegative volatility forecasts, are examples of the
great wealth of variants based on the pioneering ARCH. The distinctive characteristic of
the above cited specifications is their ability to track the intertemporal dependence of
variance which justifies their inclusion in this thesis.

The remaining six sections of the current chapter deal with the statistical
characteristics of the representations employed to compute the VaR for every market as
well as the corresponding MCR. Therefore, Sections two, three, four, five and six delve
into Historical Simulation, Filtered Historical Simulation, Conditional Volatility
GARCH and EGARCH, Extreme Value Theory and Linear models respectively
whereas the seventh Section details the techniques employed for VaR computation and the last Section states the final thoughts.

3.2. Historical Simulation (HS)

HS constitutes arguably the simplest route to estimate VaR to such an extent that its mechanics are labelled as ‘deceptively simple’ by Christoffersen (2003:101). HS is, in effect, a non-parametric unconditional and univariate method that supposes that the distribution of returns for the forecast period is sufficiently well approximated by the empirical distribution of a window of past returns of a certain length, hence avoiding the ad hoc assumptions of the (un)conditional return distribution or dynamics of the underlying risk factors. Further inferences about the loss distribution and risk measures are then carried out using the set of historically simulated data.

A good synthesis of HS is provided by Hull and White (1998) who state that HS involves the creation of a database comprising the daily movements of the portfolio in question over a period of time, where the first simulation trial supposes that the relative change in the value of the positions coincides with that of the first day covered by the database, with the rest of the days following suit. In technical terms, McNeil, Frey and Embrechts (2005) explain that assuming that the process that generates the returns is stationary with distribution function (df) $F_r$, the empirical df of the data is a consistent estimator of $F_r$; consequently, the empirical df of the returns $r_{t+s}$, $s = 1, \ldots, n$, is a consistent estimator of the df of the losses under $F_r$. In practical terms, the VaR for the period $t+1$ is obtained by means of the empirical quantile estimation, i.e., the theoretical quantiles are estimated by the sample quantiles of the information set of extension $n$ up to time $t$:

$$\text{VaR}_{t+1}(\alpha) = Q_\alpha(r_t, r_{t-1}, r_{t-2}, \ldots, r_{t+1-n})$$  \hspace{1cm} (3.1)

Therefore, the $\alpha$-VaR of the portfolio is simply the $1 - \alpha$ worst change in the portfolio value recorded in the sample.

As every scheme, it presents a series of advantages and disadvantages, although the latter outweigh the former considerably. The pros of HS justify to a certain extent the reasons why it has become widespread amongst risk managers. First and foremost, the procedure is very easy to understand and of easy implementation nature, given that no statistical estimation of parameters is required. In the second place, the financial literature usually reports that HS dispels the model risk as it essentially avoids any assumption about the dependence structure of the series or, more generally, the process.
that generates the returns: the past \( n \) points fully describe the distribution of the out-of-sample returns eluding any additional modelling restriction. Finally, as a consequence of its inherent structure, it is often claimed that HS is able to capture the fat tailed characteristics of the distribution of returns as it reflects the historical probability distribution of the exposures.

While the first advantage is virtually impossible to contrast, the second one triggers several contentious issues. Finger (2006) asseverates, perhaps rightly, that HS is not a postulate-free technique but, on the contrary, its assumptions are less explicit than in any other specification. Consequently, the sample window is deemed representative of the future returns of the portfolio and, furthermore, the quantity of observations employed in the forecast is sufficient to obtain a significant estimate of the desired quantile of the distribution. In order to achieve that representativeness, the return distribution should necessarily stay constant (or similar enough) to the one verified in the sample period. While this may be the case in specific market contexts, relevant differences are recorded particularly in times of turmoil (Neftci (2001)) when the ability of models to adapt to changing market conditions is questioned with greater detail. It is important to highlight that the assumption of the stability of the return distributions is tantamount to ignoring the volatility fluctuations and the clustering process characteristic of financial time series. These stylised facts—which have been well documented in the academic literature since the early works of Mandelbrot (1963) and Fama (1965)- are at loggerheads with the basics of HS and cannot be solved even if the amount of historical observations in the sample is increased.

The quantity of returns in the sample has always been a controversial issue in HS and, furthermore, one of its prominent intrinsic flaws. In this regard, Christoffersen (2003) points out that it must simultaneously satisfy two contradictory properties for which a reasonable compromise should be reached: large windows imply that the recent past (probably the most important for statistical inference) holds little weight, thus delivering smooth VaR numbers over time; on the other hand, too short samples only comprise the immediate history including or excluding extreme observations, this fact eventually leading to unusual or smooth estimations respectively. No matter how subjective the choice of the length of the window may be, it stands out as a topic of the utmost importance for the magnitude and dynamics of VaR using HS. Christoffersen (2003) mentions that, typically, the number of data points takes some value belonging to the interval \([250; 1000]\) (measured in trading days) which determines striking
differences for VaR estimation purposes. A simulation exercise is therefore proposed to grasp the essentials of that assertion. The simulated stock index daily time series is obtained according to the bilinear model proposed by Penza and Bansal (2001): the first 1000 observations ground on the equation:

\[
I_t = 100 + 0.95 I_{t-1} + 0.01 I_{t-1} \xi_{t-1} + \xi_t \quad (3.2)
\]

while the remaining 9000 points are delivered by

\[
I_t = 100 + 0.99 I_{t-1} + 0.05 I_{t-1} \xi_{t-1} + \xi_t \quad (3.3)
\]

Considering that \( \xi_t \sim \text{iid } N(0;1) \) in both expressions, the latter expression generates a somewhat more jagged series. VaR is computed for day 1001 onwards using HS with rolling windows of the previous 250, 500 and 1000 days. Graph 3.1 illustrates the magnitude of the dilemma faced by the practitioner: the shorter the sample (red lines), the more rapidly HS VaR reacts to abrupt events; conversely, the longer the window (green line), the smoother the string of VaR values over time, hence ignoring the time-varying nature of volatility as well as the clustering patterns. All things considered, the plot reveals that HS produces VaR estimates of a fairly predictable nature owing to the discreteness of the extreme events dropping in or out the sample window. The literature appears to reflect the above mentioned uncertainty, given that, for example, Manganelli and Engle (2001) recommend between six months and two years of daily data, while Kuester, Mittnik and Paolella (2006) propose four years and Christoffersen (2003) suggests some value between one and four years.

Graph 3.1
The effect of window length on HS VaR estimates

*Note:* VaR- and VaR+ mean VaR values for left and right tails of the distribution respectively
As the disparity among VaRs calculated with varying rolling windows suggests, a connection between heavy tails and the volatility configuration appears to exist. In this sense, Finger (2006) raises two points questioning the alleged ability of HS to capture the leptokurtosis of the distribution of returns. In the first place, the author emphasises that the heavy ends could be mistaken for volatility clusters: a series with time-varying volatility and conditional normal distribution might in principle disguise the heteroscedastic volatility with fat tails. If the market is shifting from a term of low volatility to a period of a high one (or vice versa), the VaR rooted in HS will be biased downwards (or upwards) and some time would be required until the data points belonging to the low (high) volatility period leave the sample. Secondly, he goes beyond to stress that historical returns are, in essence, sets of information which should not be used to extrapolate or forecast the future behaviour of the portfolio given the overlooking of the changing variance structure. Moreover, he suggests alternative ways to overcome these shortcomings, like filtering the series with some conditional volatility model, or applying a weighting structure to raw returns.

Dowd (2005) provides a thorough review of HS by linking all its disadvantages to the equal weighting construction of past observations. In effect, the structure of the technique indicates that all the points in the sample bear the same weight, much in the same fashion as those outside the window which do not exert any effect on HS VaR. This approach presents a number of problems that practically invalidate the whole model. Initially, it completely disregards the age of the observation, the particular situation of the market, etc., these aspects revealing important when series are subject to seasonal effects; secondly, it prevents the resulting VaR to answer to abrupt events: market crashes are only accounted for by HS VaR provided the slump repeats in subsequent trading days, i.e., when the relative weight of the decrease in market values is enough to alter the VaR recorded in quiet times (hence ignoring the increase in the underlying risk); in the third place, each observation carrying equal weight is equivalent to assuming that returns are independent and identically distributed, a fact reported absent in financial markets (Section 4.3.1). Fourthly, HS is often criticised for its laxity regarding the volatility structure, as it is assumed constant in time in such a way that any difference in volatility estimates are put down to sampling errors. Thus, even though long window samples will yield less volatile estimates than short windows as conveyed by Graph 3.1, HS only attributes those variations to sample errors as the ‘true’ underlying volatility is supposed to remain unchanged. Finally, it fosters the apparition
of ghost or shadow effects (also highlighted by Penza and Bansal (2001)), given that whenever the sample includes a single (or a small cloud of) heavy loss(es), the resulting VaR would appear disproportionately high (or vice versa) until the unusual observation(s) drops out of the rolling window: at this stage the measured VaR will decrease although the fall will only represent a ‘ghost’ or ‘shadow’ arising from the weighting mechanism and the extension of the sample period.

Even though its adequacy boils down to empirical testing in the appropriate contexts, Manganelli and Engle (2001) consider its snags are sufficiently important to remove HS from the list of reliable methods for VaR calculation purposes. However, despite the aforementioned facts, HS remains hugely popular in the financial industry.

### 3.3. Filtered Historical Simulation (FHS)

Several authors have developed suitable refinements in order to address the flaws presented by HS. Broadly speaking, academics intend to overcome the weaknesses represented by the equal weighting structure of past returns and the lack of any volatility updating scheme, both empirical facts present in financial time series (Section 4.3.1).

For instance, in the Boudoukh, Richardson and Whitelaw (1998) model (BRW), observations are given relative importance in accordance with their age, though depending on a parameter $\lambda$ which plays an analogous role as that of the decay factor $\lambda$ in the EWMA model. The weighting function, expressed as:

$$w_i = \frac{\lambda^{i-1}(1-\lambda)}{1-\lambda^n}$$

(3.4)

where $n$ indicates the length of the sample and $i$ the order of the observation, solves several troubles posed by HS like the overlooking of volatility clusters, the lack of response to large loss observations and the presence of ghost or shadow effects (Dowd (2005)). However, Pritsker (2001) points out that this variant of the Weighted Historical Simulation category still understates VaR and its inability to respond to abrupt adverse changes in market conditions is ‘disturbing’, no matter the value assumed by the parameter $\lambda$. Furthermore, he stresses the absence of any theoretical justification beyond pragmatic considerations to employ the BRW methodology.

Hull and White (1998) incorporate a volatility updating scheme into HS –leading Dowd (2005) to brand their technique Volatility-weighted HS-, finding promising

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63 The response to market changes is measured by the parameter $\lambda$. In this sense, the statistically adequate value delays the manner in which the model tackles the jerks (Pritsker (2001)).
results applying their model to a series of exchange rates and stock indices. They propose to mimic the volatility of the portfolio using a conditional volatility model and scale the values of the observed variable by an estimate of its volatility, hence obtaining an approximately stationary probability distribution of the (modified) historical data that reflects the change in volatility from the past values. In formulas, the returns belonging to the data set, \( r_t \), are replaced with the scaled returns \( r_t^* \) in the following manner:

\[
r_t^* = \sigma_T \left( \frac{r_t}{\sigma_t} \right),
\]

where \( \sigma_t \) denotes the historical volatility forecast for day \( t \) and \( \sigma_T \) the most recent prediction for volatility, both volatility estimates deriving from a GARCH or EWMA model, according to the authors. Actual returns, then, are raised or lowered depending on the relationship between the current volatility forecast and the volatility estimation for the corresponding day after which the traditional HS is applied (Section 3.3), i.e., selecting the appropriate quantile with equal weighting structure of returns in the sample window.

When the time horizon over which VaR is estimated is one period with no scaling factors, Pritsker (2001) states that the volatility-weighted HS is coincident with the FHS to be described below. As this is the case for the present thesis, the focus is analogous and may be subsumed into one, thus justifying the inclusion in the analysis.

Barone-Adesi, Bourgoin and Giannopoulos (1998) and Barone-Adesi and Giannopoulos (2000) proposed the FHS as a route to bridge the flaws that engulf the HS methodology by blending the flexibility of the conditional volatility models with the non-parametric characteristics of HS. The first step towards FHS is the selection of an appropriate representation capable of producing forecasts of volatility acknowledging the stylised facts present in financial time series (e.g., some technique belonging to the GARCH family\(^{64}\)). Only for pragmatic reasons at this stage, it will be assumed that the model describing the dynamics of the return generating process is the conventional GARCH(1,1) which typical equations are:

\[
\begin{align*}
    r_t &= \mu + \sigma_t \varepsilon_t \\
    \sigma_t^2 &= \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2
\end{align*}
\]  

(3.5)  

(3.6)

with \( \varepsilon_t \sim \text{iid } D(0,1) \), \( \alpha, \beta \) and \( \omega > 0 \) and \( \alpha + \beta < 1 \).

---

\(^{64}\) Section 3.5 contains detailed treatment about the conditional volatility models to be employed in the present thesis, either in the conventional conditional volatility modelling, FHS or Extreme Value Theory.

\(^{65}\) Bollerslev (1986) claims that the model is flexible enough to support several distributions, like the Normal, Student-\( t \), Generalised Error Distribution, etc. The most widespread applications of GARCH portray the first two distributions.
Once the volatility predictions ($\sigma_t$) for each of the days in the sample are obtained – under the assumption of null means $\mu$ –, the quotients $\varepsilon_t = r_t / \sigma_t$ are calculated in order to achieve a series of independent and identically distributed standardised returns, thus laying the ground to apply HS as described in Section 3.2.

Pritsker (2001) looks at the method from a different point of view, which is very useful to gain an insight into the technique. The author emphasises that the main asset of the methodology is represented by its ability to capture both the conditional heteroscedasticity and the non-normality characteristic of risk factors. Supposing that the distribution of $\varepsilon_t = r_t / \sigma_t$ (in Barone-Adesi, Giannopoulos and Vosper (1999) the $\varepsilon_t$s are referred to as filtered shocks) has zero mean, unit variance and iid features, FHS relaxes the classical supposition of a particular distribution of $\varepsilon_t$ in the GARCH model (Normal, Student-$t$, etc.) in favour of a much weaker one that expresses that the distribution of $\varepsilon_t$ is such that the parameters of the GARCH can be consistently estimated. Were this assertion to be true, the estimates of $\sigma_t$ at each point in time would be correct and, because the $r_t$s are observable, the equation (3.5) may be employed to obtain the past behaviour of $\varepsilon_t$ in the data. Hence, the fact that these past values of $\varepsilon_t$ are iid allows the usage of the empirical distribution to generate future values of $\sigma_t$.

FHS, then, appears to be able to acknowledge the conditional heteroscedasticity present in financial series data courtesy of the conditional volatility model while, at the same time, takes advantage of the most important element of HS, i.e., the application of the empirical distribution of returns that avoids restrictive assumptions about specific shapes.

Like every method, FHS has its own snags, and, in this sense, Pritsker (2001) highlights a slight degree of understatement of the true risk of the exposures arising from the lack of extreme observations in the dataset employed to calculate the filtered residuals. Testing the model with a simulation exercise where the sample length spans through 500 observations, the author finds that the sample should extend for more than two years of trading days in order to include as many extreme observations as possible. However, a long sample will only exacerbate the model risk or model misspecification for which a separation between errors due to the FHS and errors coming from model risk is recommended. Pritsker’s results differ from Barone-Adesi, Giannopoulos and
Volper (2000) as they found that FHS overstates risk\textsuperscript{66} although the data series upon which the studies are performed belong to assets of different nature: while the former uses Dollar denominated exchange rates, the latter employs interest rates swaps and portfolios composed by derivatives. Consequently, this last fact leads Pritsker (2001) to reaffirm the necessity of further research on the kind of models that deliver the best performance, the length of the sample that should encompass enough quantity of extreme observations as well as some avenue to allocate the source of bias that may appear at the time of volatility forecasting.

3.4. Conditional Volatility models

According to the evidence collected in Section 4.3.1, financial time series appear to be stationary. The stationary condition implies that both unconditional mean and variance are constant and finite\textsuperscript{67}: the former represents the long-term mean value that the series will tend to converge to in the long run and is the best estimate for very prolonged horizons, whereas the latter entails that volatility is tied to that constant mean, i.e., the series are mean reverting. The reason why the conditional variance changes at every point in time stems from its dependence on the history of returns up to that point, and the dynamic properties of the returns are explained by the fact that the distribution of returns at any point in time is conditional on all the information possessed up to that point in time (Alexander (2008b))\textsuperscript{68}.

Although long term means and variances are constant and can be easily estimated using the sample mean and variance respectively, the mechanics for shorter horizons like daily ones to be treated in the context of the present thesis require more precise techniques. Moreover, in the case of the variance, it may be appreciated in Section 4.3.1 that returns in financial time series do not seem to be generated by independent and identically distributed processes, but, on the contrary, exhibit significant amounts of autocorrelation in their squared form\textsuperscript{69}. Additionally,

\textsuperscript{66} It is important to bear in mind that both studies develop on short and relatively ‘long’ horizons, i.e., one day to ten days.

\textsuperscript{67} To be more precise, a process possesses the weak stationary condition if all of its second moments are constant and do not depend on time, hence indicating that the mean and the variance are constants independent of time: $\mu_t = \mu$ and $\sigma^2_t = \sigma^2$. However, albeit a process can present weak stationarity features, its variance may still be conditional on time (Engle, Focardi and Fabozzi (2007)).

\textsuperscript{68} The distribution of a return at time $t+1$ considers that the history of past returns up to and including time $t$ is non-stochastic.

\textsuperscript{69} Traces of autocorrelation may turn out in linear returns occasionally. The bottom line boils down to the fact that lack of independence ought to be taken into account at the time of modelling.
autocorrelation coefficients for squared returns are of positive sign, indicating that volatility appears grouped in clusters, i.e., periods of small returns (low volatility) come alternated with spells of large ones (high volatility). This phenomenon, which receives the technical name of autoregressive conditional heteroscedasticity (Alexander (2001)) was first observed by Mandelbrot (1963) and since then has spurred an enormous quantity of literature on the modelling and forecasting of return volatility.

All throughout the thesis, the conventional notation regarding the time dependence will be followed. In this sense, the information set \( I_{t-1} \) comprising all observable prices and returns up to and including time \( t-1 \) is denoted by \( I_{t-1} \), and returns and variances at time \( t \), conditional on that information set \( I_{t-1} \) are indicated by \( r_t/I_{t-1} \) and \( \sigma^2_t/I_{t-1} \) respectively. For practical motivations the term \( I_{t-1} \) will be dropped, thus \( r_t \) and \( \sigma^2_t \) are to symbolise the aforementioned concepts.

The majority (if not all) the representations acknowledging the conditional nature of volatility derive from the ARCH models introduced by Engle (1982). Using the conventional notation in which \( r_t \) denotes the log return on an asset, \( \sigma^2_t \), the variance of those returns and \( z_t \) the error or innovation term, a brief picture of the ARCH approach is to be sketched in order to build the rest of the models on that seminal framework.

3.4.1. The notion of volatility

According to Andersen et al. (2003), the volatility of an asset return is an unobservable latent concept (latent volatility) susceptible of being approximated through an empirical measure of the daily return variability, termed realised volatility. In order to approximate this somewhat elusive realised volatility, the authors propose a stochastic process \( r_t \) decomposed into a sum of a predictable and integrable mean component \( A_t \) and a local martingale \( B_t \), as follows:

\[
r_t = A_t + B_t \quad (3.7)
\]

Andersen et al. (2003) state that, like all semi-martingales, \( r_t \) carries associated quadratic variations of \( A_t \) and \( B_t \), which act as explanatory variables of its movements;

---

70 The information set is the discrete equivalent to the continuous filtration process (Alexander (2008b)).
71 More precisely, under standard assumptions like lack of arbitrage and finite means, Back (1991) defines \( r_t \) as a semi-martingale, as its two components are embodied in a local martingale \( B_t \) and an adapted finite-variation process \( A_t \).
72 A process \( B_t \) is called a martingale if \( E(B_s/F_t) = B_t \) for all \( 0 \leq t \leq s \), or, equivalently, if the current value \( B_t \) is the best possible prediction of the future value \( B_s \) using the mean square criterion to evaluate the forecasts (McNeil, Frey and Embrechts (2005)).
however, given that the former appears statistically insignificant, only the latter
influences the return volatility process, and specially for short horizons, the variance
of $r_t$ is driven by the ‘…genuine innovations represented by the martingale component’
(2001:5).

The idea of the realised volatility has been the subject of thorough analysis in
Andersen et al. (2003), who name it notional volatility and in Barndorff-Nielsen and
Shephard (2002, 2004) where it receives the denomination of actual volatility. The
former also stress that the quadratic variation in $A_t$ equals zero, hence the notional
volatility is equivalent to the quadratic variation of $B_t$, i.e. the quadratic variation of the
return series $r_t$. Consequently, the notional volatility could result discretionarily
approximated by the accumulation of high-frequency squared returns $r^2_t$ in such a
fashion that it remains independent from the stochastic process determining the
behaviour of returns.

In its more widespread formulation, the process $r_t$ may be written as the sum of
the conditional mean value of $r$ (expected value of $r$ based on the historical information
up to time $t-1$) and the square root of the variance of $r$ (Standard Deviation) multiplied
by the error term $z_t$ corresponding to the present period:

$$r_t = \mu_t + \sqrt{\sigma^2_t} z_t = \mu_t + \sigma_t z_t$$

(3.8)

Even though many efforts have been devoted to develop the mean return and its effect
on forecasting returns (for instance, the GARCH-in-Mean or simply GARCH-M in
Engle, Lilien and Robins (1987)), it is normally the case that the $\mu_t$ factor appears
statistically negligible, thus resembling the integrable mean component $A_t$ in (3.7).
Therefore, the aforementioned rationale leaves only the local martingale $B_t$ in (3.7), or
more specifically, $\sigma_t z_t$ in (3.8) to be specified, and, in this sense, Andersen et al. (2003)
state that plain and straightforward representations have proved successful for this
purpose.

3.4.2. The ARCH framework
One of the most basic approaches to estimate the error variance $\sigma_t$, given its time
dependence (heteroscedasticity) and autocorrelation of the squared values shown in
Section 4.3.1, refers to the employment of short rolling windows, i.e., the Standard

---

73 Andersen et al. (2003) asseverate that $B_t$ is of an order of magnitude superior to that of $A_t$, while Jorion
(1996) goes a step further to claim that the first outnumbers the second 700 times.

74 In terms of the equation (3.5), $\mu_t$, and $\sigma_t z_t$ in expression (3.7) represent $A_t$ and $B_t$, respectively.
Deviation or variance is calculated using a predetermined quantity of the very recent past returns. Even though the underlying idea squares off with the empirical results verified –variance changes slowly over time nevertheless staying approximately constant in a short rolling time window-, the equal weighting structure assumed seems conceptually and technically wrong because of two main reasons: in the first place, it is sensible to assume that more recent events should bear more importance than distant ones, and, in the second place, observations outside the fixed length could also prove relevant for the process.

In order to bridge those deficiencies, Engle (1982) devised the famous ARCH(\(p\)) representation –the letter between parentheses denotes the number of lags-, which portrays the variance \(\sigma^2_t\), expressed as a weighted average of past error terms with the coefficient weights \(\alpha_i\) optimally estimated from the empirical data:

\[
\sigma^2_t = w + \sum_{i=1}^{p} \alpha_i \varepsilon^2_{t-i} = w + \alpha_1 \varepsilon^2_{t-1} + \alpha_2 \varepsilon^2_{t-2} + \ldots + \alpha_p \varepsilon^2_{t-p} \tag{3.9}
\]

where

\[
\varepsilon_t = \sigma_t z_t \tag{3.10}
\]

and the factors \(z_t \sim \text{iid N}(0,1)\).

For the model to yield nonnegative variances it is required that \(w\) should be positive, and all \(\alpha_i\)s nonnegative. Furthermore, if \(\sum_{i=1}^{p} \alpha_i < 1\), the ARCH process enjoys at least the weakly stationary property with the term structure of the dynamic variance \(\sigma^2_t\), converging to a constant unconditional value \(\sigma^2\) given by:

\[
\sigma^2 = \frac{w}{1-\sum_{i=1}^{p} \alpha_i} \tag{3.11}
\]

A more thorough examination of (3.9) indicates that the ARCH representation is, in fact, a moving average of past squared errors and its strongest point, according to Penza and Bansal (2001), resides in the ability to grasp the conditional heteroscedasticity. Hence, if an abnormal market movement (whatever its direction might have been) occurred \(j\) periods ago (\(j \leq p\)), the squared error \(\varepsilon^2_j\) would enlarge, thus increasing the present conditional variance \(\sigma^2_t\) (provided \(\alpha_j > 0\)) and augmenting the chance of a large market jump today. It is relevant to highlight a final point concerning the formula (3.9), which will mark further developments (GARCH and EGA RCH). Given that the majority of the daily financial time series exhibit statistically null unconditional (sample) means (\(\bar{r} = T^{-1} \sum_{t=1}^{T} r_t \approx 0\)), equation (3.9) can be rewritten somewhat differently. In effect, considering that:

\[
\varepsilon_t = r_t - \bar{r} = r_t \tag{3.12}
\]
Then,
\[ \sigma_t^2 = w + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 = w + \alpha_1 r_{t-1}^2 + \alpha_2 r_{t-2}^2 + \ldots + \alpha_p r_{t-p}^2 \] (3.13)
and \( w > 0, \alpha_i \geq 0 \) to make \( \sigma_t^2 > 0 \), from where it may be deduced that the conditional variance amounts to a weighted average of past squared returns, with the weights determined by the model.

In his pioneering paper of 1982, Engle states that ARCH structures are appealing for econometric applications because its power to predict future variations in the random variable changes from time to time according to the context in which it is immersed; furthermore, citing McNees (1979) who documents that large and small errors tend to group together in adjacent periods, the author hints at the adequacy of the ARCH configuration whenever the underlying variance changes over time. However, Alexander (2001) points out that ARCH models have not been widely applied in financial markets because the quantity of lags \( p \) necessary to obtain stable volatility forecasts with appropriate dynamics should increase to such an extent that the likelihood function used in the estimation of the parameters becomes extremely intractable. Additionally, given that increasing lags in ARCH formulations converge to a simple GARCH(1,1)\(^75\) representation which offers only three parameters to compute, the author discourages the application of those techniques for financial volatility forecasting.

### 3.4.3. The GARCH model

Much in the same fashion as the standard time series AR processes extend to the more general ARMA kind, Bollerslev (1986) devised the GARCH (General Autoregressive Conditional Heteroscedastic) model which constitutes a more versatile class of representation than its predecessor, and allows a more adaptable lag configuration as well as a parsimonious structure to estimate.

The GARCH framework is specially designed to address the two most distinctive features of financial time series of daily or higher frequencies, namely the leptokurticity (tails fatter compared to the normal distribution ones) and the heteroscedasticity (volatility clustering). As mentioned in Section 3.5.2, the ARCH differentiates between the unconditional and the conditional variance and explains the variation over time characteristic of the latter employing past errors, or, more precisely

\(^75\) Section 3.4.3.
in financial contexts, using past returns; however, in spite of the advantages, it becomes tedious as a relatively long number of lags is called for to estimate the variance structure. Alexander (2001) stresses that, considering the ARCH structure of equation (3.9), the main contribution of GARCH stems from the second equation representing the dynamics of the conditional variance of the unexpected return process, i.e.:

\[ V_t(\varepsilon_t) = V_t(r_t) = \sigma_t^2 \]  \quad (3.14)

**Conditional Mean Equation**

GARCH models use a return series as input and comprise two equations: the conditional mean equation and the conditional variance equation. It is usually the case that the latter grabs the spotlight, hence the former –the one describing the behaviour of the returns- appears very simple in its structure (Christoffersen (2003)):

\[ r_t = \mu + \varepsilon_t = \mu + \sigma_t z_t \]  \quad (3.15),

where \( \varepsilon_t \sim N(0, \sigma_t) \) and:
- \( \mu \): average of returns during the sample period, and
- \( \varepsilon_t = \sigma_t z_t \): market shock, or, more specifically, mean deviation return.

The term \( \varepsilon_t \) motivates some additional comments. Technically speaking, it denotes the market innovation or unexpected return supposed to follow a zero mean Normal distribution and variance conditional on time\(^77\). Notwithstanding that, as mentioned for the ARCH configuration, \( \varepsilon_t \) is regularly cited as the mean deviation return; hence, after applying (3.12) and its related considerations \( \varepsilon_t = r_t \) implying that:

\[ r_t = \sigma_t z_t \]  \quad (3.16)

again. Alexander (2001) affirms that in some contexts, expressions like (3.8) which feature a time-varying conditional mean may prove helpful, but simultaneously produce some convergence problems were too many parameters included in the estimation phase\(^78\). Therefore, the author recommends the employment of parsimonious models.

Dowd (2005) emphasises that the main advantage of GARCH specifications resides in their ability to reproduce the volatility clustering which, in turn, produces fatter tails than those predicted by the Normal distribution, even though the random

---

\(^76\) Bollerslev (1986) makes clear that the conditional distribution of \( \varepsilon_t \) might be other than the Normal one. The current thesis portrays a variant featuring the \( t \) distribution.

\(^77\) The market shock \( \varepsilon_t \) also embodies the error term from an ordinary linear regression (Alexander (2008b)).

\(^78\) For example, if the return series presents significant autocorrelation characteristics in its linear form, the researcher should resort to an autoregressive conditional mean specification and in the majority of the cases an AR(1) model will serve the purpose intended. Other patterns would of course require different specifications.
shocks are themselves normally distributed. The basic \( \text{GARCH}(p,q) \) setting allows volatility \( \sigma_t^2 \) to depend on \( p \) past errors (or returns) like its ARCH counterpart and \( q \) past volatilities, in a manner called “adaptive learning mechanism” by Bollerslev (1986:309). Therefore, considering the basic equation (3.15) for the mean return, the model postulates that:

\[
\begin{align*}
    r_t &= \mu + \epsilon_t = \mu + \sigma_t z_t \\
    \sigma_t^2 &= w + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_p \epsilon_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2
\end{align*}
\]

(3.17)

where \( w > 0 \) and \( \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q \geq 0 \) in order to ensure that \( \sigma_t^2 > 0 \).

Even though the number of lags \( p \) and \( q \) may extend to fit the data more accurately, the simple \( \text{GARCH}(1,1) \) delivers reasonably good adjustment while simultaneously being easy to apply using few parameters\(^81\). Bollerslev (1986), Penza and Bansal (2001) as well as a plethora of other authors mention that the one-lagged GARCH suffices the majority of purposes since it portrays infinite memory in an extreme parsimonious setting, from where springs the widespread use for financial applications and the motivation for its use in the context of the current thesis. Therefore, (3.17) becomes:

\[
\begin{align*}
    r_t &= \mu + \epsilon_t = \mu + \sigma_t z_t \\
    \sigma_t^2 &= w + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \\
    \text{or} \quad \sigma_t^2 &= w + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2
\end{align*}
\]

(3.18)

and \( \epsilon_t \sim \text{N}(0,\sigma_t^2) \), \( w > 0, \alpha, \beta \geq 0 \) so that \( \sigma_t^2 > 0 \) and \( \alpha + \beta < 1 \) to assure stationarity\(^82\).

**GARCH(1,1) features**

Equation (3.18) constitutes the symmetric Normal version of the GARCH family of models, also termed plain vanilla or simply vanilla by Alexander (2008b) and possesses a number of interesting implications worth of being mentioned.

Notably, GARCH processes are neither identically nor independently distributed as conditional variances (second conditional moments) at diverse moments in time are connected. It also converges to a long run variance value to which it tends to revert, thus

\(^79\) GARCH configurations are generalised ARCH process as they let the variance \( \sigma_t^2 \) to depend on former variances and previous squared values of the model (McNeil, Frey and Embrechts (2005)).

\(^80\) McNeil, Frey and Embrechts (2005) stress that true GARCH processes ought to exhibit the strict stationarity property.

\(^81\) Chou (1988) tests several GARCH\((p,q)\) specifications other than GARCH\((1,1)\) only to find that that Lagrange multiplier tests for lagged terms beyond one are statistically insignificant, thus bolstering the use of GARCH\((1,1)\).

\(^82\) In what follows, the development will apply the terms \( \epsilon_t \) and \( r_t \) interchangeably, by virtue of (3.12) unless otherwise stated.
treated volatility as a mean-reverting phenomenon provided no market shocks take place: whenever volatility increases, it will be inclined to fall over time, whereas the inverse also holds. This unconditional variance is found applying the expectation operator in (3.18):

\[ E(\sigma_t^2) = w + \alpha E(\epsilon_{t-1}^2) + \beta E(\sigma_{t-1}^2) \]

\[ \sigma^2 = w + \alpha \sigma^2 + \beta \sigma^2 \]

since \( E(\epsilon_{t-1}^2) = \sigma_{t-1}^2 \) courtesy of (3.10) and \( E(\sigma_{t-1}^2) = \sigma_{t-1}^2 \).

Using the identity \( \sigma_t^2 = \sigma_{t-1}^2 = \sigma^2 \), the long-term variance is given by\(^{83}\):

\[ \sigma^2 = \frac{w}{1 - \alpha - \beta} \quad (3.19) \]

if and only if \( \alpha + \beta < 1 \), with volatility obtained taking the square root of (3.18). Dowd (2005) posits that the utility of the long-term variance is enhanced at the time of calculating straightforward volatility forecasts. In fact, from (3.19),

\[ w = \sigma^2 (1 - \alpha - \beta) \quad (3.20) \]

and, plugging this expression into (3.18):

\[ \sigma_t^2 = w + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 = \sigma^2 (1 - \alpha - \beta) + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \]

\[ (\sigma_t^2 - \sigma^2) = \alpha (\epsilon_{t-1}^2 - \sigma^2) + \beta (\sigma_{t-1}^2 - \sigma^2) \quad (3.21) \]

Carrying (3.21) \( j \) periods forward:

\[ \sigma_{t+j}^2 - \sigma^2 = \alpha (\epsilon_{t+j-1}^2 - \sigma^2) + \beta (\sigma_{t+j-1}^2 - \sigma^2) \quad (3.22) \]

Making use of the result \( E(\epsilon_{t+j-1}^2) = \sigma_{t+j-1}^2 \) applying (3.10),

\[ E(\sigma_{t+j}^2 - \sigma^2) = (\alpha + \beta) (\epsilon_{t+j-1}^2 - \sigma^2) = \sigma^2 + (\alpha + \beta) (\epsilon_{t+j-1}^2 - \sigma^2) \quad (3.23) \]

Therefore, the prediction for the variance of the future term \( t + j \) is given by:

\[ E(\sigma_{t+j}^2) = \sigma^2 + (\alpha + \beta)^j (\sigma_t^2 - \sigma^2) \quad (3.24) \]

Because of \( \alpha + \beta < 1 \), then \( (\alpha + \beta)^j (\sigma_t^2 - \sigma^2) \) decreases when \( j \) grows -the term structure flattens with \( j \)-, so the forecast of the variance converges to the long-run equilibrium value \( \sigma^2 \) at an exponential rate dictated by \( \alpha + \beta \) and, as expressed above: if \( \sigma_j^2 - \sigma^2 > 0 \), the expected variance for the period \( j \) is greater than \( \sigma^2 \) and viceversa if \( \sigma_j^2 - \sigma^2 < 0 \)\(^{84}\).

However, Christoffersen (2003) points out one important caveat, as even though the one-period ahead conditional distribution is assumed known (e.g., Normal, or \( t \)), the multiperiod conditional distributional shape remains unknown, consequently posing difficulties in forecasting the entire conditional distribution.

\[^{83}\text{McNeil, Frey and Embrechts (2005) treat GARCH properties extensively.}\\
^{84}\text{More details about the importance of the term } \alpha + \beta \text{ can be found in what follows.} \]
It is also noticeable that the apparent simple GARCH(1,1) possesses infinite memory despite its only lag in every term. In fact, after recursive substitution:

\[
\sigma_t^2 = w + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2
\]

\[
\sigma_t^2 = w + \alpha \varepsilon_{t-1}^2 + \beta \{w + \alpha \varepsilon_{t-2}^2 + \beta \{w + \alpha \varepsilon_{t-3}^2 + \beta (\cdots)\}\}
\]

\[
\sigma_t^2 = \frac{w}{(1 - \beta)} + \alpha (\varepsilon_{t-1}^2 + \beta \varepsilon_{t-2}^2 + \beta^2 \varepsilon_{t-3}^2 + \cdots) \tag{3.25}
\]

that is tantamount to an infinite ARCH scheme with exponentially declining weights as mentioned in Section 3.5.2, though easier to compute from a statistical point of view.

GARCH(1,1) representation avows that the most accurate prediction of the variance in the next period is yielded by a weighted average of the long horizon variance, the variance forecast for the current period and the new information available seized by the squared residual/squared return. In this sense, it is of prime importance to analyse the meaning of the parameters \(w\), \(\alpha\) and \(\beta\) in what regards the reaction to market innovations as well as the speed of the convergence to the mean variance value in the aftermath of a market shock, considering that all these elements determine the trajectory of the short-term dynamics of the ensuing volatility series. The below interpretation of the model parameters follows Bollerslev (1986), Alexander (2001, 2008b), Dowd (2005), Christoffersen (2003) and Penza and Bansal (2001):\(^{85}\)

- **GARCH error parameter \(\alpha\):** it represents the reaction of conditional volatility \(\sigma_t\) to unexpected market movements; the higher the number, the more intense the response to market shocks, i.e., the volatility path deploys abrupt jerks or spikes. The question of what ‘high’ means in this context appears of a somewhat subjective nature notwithstanding which Alexander (2008b) states that a large \(\alpha\) exceeds 0.10. In a normal framework, the typical values do not exceed 0.20 (Alexander (2001)) or 0.25 (Dowd (2005));

- **GARCH lag parameter \(\beta\):** it measures how much it takes the conditional volatility to fade away after a market crisis, or, in technical vocabulary, the persistence of the model. When \(\beta\) is large, a relatively long span elapses before conditional volatility returns to normal levels: although the definition looks more precise than the case of the error term \(\alpha\) because ‘large’ may refer to ‘nearer to one’, traces of subjectivity surface given the imprecision in quantifying how close to one it should situate; in this vein, Alexander (2008b) asseverates that a high lag parameter might well be situated above 0.90. The

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common minimum values for $\beta$ start from 0.70 (Dowd (2005)) or 0.80 (Alexander (2001));

- **GARCH constant parameter**: $w$, alongside the rate of convergence below specified influences the unconditional, or long term average volatility towards which the conditional volatility tends in the future;

- **Rate of convergence**: for plain vanilla GARCH models, the sum $(\alpha + \beta)$ determines the speed at which the conditional volatility approximates the long-run equilibrium mean value $\sigma^2$. As a result of the lack of objectivity in determining the qualitative assessment of the levels, the addition reflects this bias, albeit Alexander (2008b) reckons that $(\alpha + \beta)$ greater than 0.99 could be deemed high enough to deliver a GARCH volatility structure relatively flat.

Andersen et al. (2006) provide interesting insights into the mechanics of the GARCH(1,1). In particular, the authors stress the realistic forecasts that drive volatility to a constant long horizon value that arise from the covariance stationary structure (as mentioned in McNeil, Frey and Embrechts (2005)). Effectively, they scrutiny the GARCH(1,1) expression from a different angle: starting from (3.18) and making use of (3.19):

$$
\sigma_t^2 = (1 - \alpha - \beta) \sigma^2 + \alpha \sigma_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (3.26), \\
\sigma_t^2 = \sigma^2 + \alpha (\sigma_{t-1}^2 - \sigma^2) + \beta (\sigma_{t-1}^2 - \sigma^2) \quad (3.27)
$$

which manifests that GARCH(1,1) predicts volatility adjusting the present variance using $\beta$ and also accounting for the impact of the current return on the long-run unconditional variance courtesy of the error parameter $\alpha$. Additionally, after algebraic manipulations of (3.27), the conditional volatility could be elaborated as:

$$
\sigma_t^2 = \sigma^2 + (\alpha + \beta) (\sigma_{t-1}^2 - \sigma^2) + \alpha \sigma_{t-1}^2 (\sigma_{t-1}^2 - 1) \quad (3.28)
$$

thus exhibiting the trajectory of $\sigma_t^2$ driven by the corrections in the average variance controlled by the persistence factor $(\alpha + \beta)$ and the so-called current ‘volatility-of-volatility’ (Andersen et al. (2005:7)) corrected by the level of volatility times $\alpha$. Finally, applying the expectation operator in (3.27), the term $\alpha \sigma_{t-1}^2 (\sigma_{t-1}^2 - 1)$ becomes null, hence obtaining again (3.24) $\sigma_t^2 = \sigma^2 + (\alpha + \beta) (\sigma_{t-1}^2 - \sigma^2)$ where the dynamics of the conditional variance when converging to its long-term horizon value are linked to the adjustments via the persistence factor $(\alpha + \beta)$.  

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Engle and Patton (2001) claim that an adequate volatility model should be able to capture at least the prominent volatility features like mean reversion, persistence and half-life. The GARCH(1,1) scheme appears as a useful tool to deal with these concepts and, in this light, Carroll and Collins (2002) produce attractive alternative views of these facts. For instance, besides measuring persistence by means of the sum \((\alpha + \beta)\) as it is usual practice, they try a somewhat different metric called forward persistence, calculated for \(j\) periods ahead, and given by:

\[
\theta_{t+j} = \alpha (\alpha + \beta)^{j-1} \tag{3.29}
\]

which also delivers a measure of the speed of the mean reversion, given that current information does not exert any impact on the long-term volatility forecast when \(j \to \infty\). Finally, they define the volatility half-life as the time taken by the volatility to return half-way to its average level \(\sigma^2\) following a perturbation from it with its corresponding formula:

\[
\tau = \ln \left( \frac{1}{2} \right) / \ln(\alpha + \beta) \tag{3.30}\]

commonly measured in days for daily time series.

GARCH(1,1) representations have been successfully employed for volatility estimation in a variety of financial time series, from foreign exchange (Andersen and Bollerslev (1998), Andersen et al. (2003)), a sample of market indices and security prices (Akgiray (1989)), futures contracts (Longin (2000), Brooks, Clare and Persand (2000)) and commodity prices (Krehbiel and Adkins (2006)), all of them demonstrating the versatility of the specification. Furthermore, its flexibility is enhanced by switching the distribution of \(\varepsilon_t\) from the usual Gaussian one in (3.18) to another one designed to accommodate more accurately the stylised empirical facts observed in financial markets. For instance, Bollerslev (1987) was the first to propose a GARCH model with symmetric Student-\(t\) innovations, and Guermat and Harris (2002), So and Yu (2006), Ané (2006) and Angelidis and Degiannakis (2005) affix the Generalised Error Distribution (GED) to the GARCH(1,1) scheme. Eventually, following Wilson (1993), who recommends the use of GARCH(1,1)-Student-\(t\) specification for risk management purposes, the current thesis will make use of this variant.

However, as any representation, the GARCH ones also portray some drawbacks. In effect, the literature marks that one of the most significant concerns is given by the difficulty in identifying the proper lag structure (Brorsen and Yang (1993)). Although

\[\text{See Carroll and Collins (2002) for proof of the propositions (3.29) and (3.30).}\]
the optimal approach explained in Bowerman and O’Connell (1993) involves working with the Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) to spot the proper quantity of lags, in practice academics only work with the first order ones as recommended by Bollerslev (1986) because of two main reasons: firstly, higher order models suffer from the so called curse-of-dimensionality (Andersen et al. (2003)) therefore rendering them unsuitable for practical purposes and secondly, GARCH(1,1) appears strong enough to deliver relatively accurate results at least for high frequency samples like daily or even higher ones (Alexander (2008b), Dowd (2005), Christoffersen and Gonçalves (2005), Kuester, Mittnik and Paolella (2006), or McNeil and Saladin (1997)), just to cite a few studies.

A second disadvantage, pointed out by Angelidis, Benos and Degiannakis (2004) refers to the large number of observations necessary to achieve convergence in the estimations where discrepancy seems the common denominator among authors. Alexander (2001) ascertains that even acknowledging its strengths, robustness of results and simplicity of interpretation, GARCH(1,1) is easily influenced by the data used (particularly the constant term \( w \)). Additionally, some authors underline its likely instability for out-of-sample estimations, albeit this fact could be attributed to the presence of structural regime changes in the time series powerful enough to radically alter the values of the parameters (Danielsson (2011)). Alexander (2001) shares these concerns stating that the absence of an optimal number of observations to compute GARCH models produces a kind of trade-off between a long sample necessary to obtain reliable estimates and a prolonged extension that does not reflect the current market sentiment. The author –coinciding with Christoffersen (2003)- hints at the ultimate subjectivity in the selection of the quantity of points to be employed, given that practitioners ought to picture an outlook about the influence that distant past events may exert on GARCH forecasts.

Finally, one of the most relevant hurdles at the estimation stage for GARCH(1,1) schemes resides in the fact that the restrictions in the parameters in (3.18), namely \( w > 0, \alpha, \beta \geq 0 \) and \( (\alpha + \beta) < 1 \), ought to be verified for \( \sigma^2 \) and \( \sigma^2_t \) to be finite and positive. Sometimes the optimisation process yields values that violate the constraints,

\[ \begin{align*}
\text{In fact, Andersen and Bollerslev (1998) stage a strong defense of GARCH(1,1) models defying critics who challenge the lack of accuracy of that representation. They argue that GARCH(1,1) schemes work splendidly with very high frequency data (intraday values like hourly or five minutes ones) because they are capable of detecting the microstructure of the markets.}
\end{align*} \]

\[ \begin{align*}
\text{Actually, this trade-off takes place in every estimation exercise, whether it is Historical Simulation, Filtered Historical Simulation, Conditional Volatility Models or Extreme Value Theory.}
\end{align*} \]
hence leading to suboptimal solutions which indicate that the GARCH(1,1) configuration is inappropriate and a different one should be tried.

However, the fact that the standard GARCH(1,1) representation is still widespread among the academic community and practitioners may well be considered a sign of the lack of alternative models capable of delivering similar or better performances. Accordingly, for the purposes of the present thesis, the referred model carrying two different densities -Normal and Student-\(t\)- will be utilised. The first specification features the classic approach to market risk estimation while the second one attempts to capture the usually non-normal conditional distribution of the market shocks which reflect the fact that market returns exhibit skewed and leptokurtic conditional and unconditional distributions (Alexander (2008b)).

3.4.4. The EGARCH model

One of the ways to check the accuracy of GARCH forecasts, particularly concerning downside risk, is to compare its predictions with a model specifically designed to address the typical negative asymmetry in financial markets. Exponential GARCH (EGARCH) representations were first proposed by Nelson (1991) and treated extensively in Engle, Bollerslev and Nelson (1994), and have since become an interesting alternative to sort out the possible negative variance values of GARCH(1,1): EGARCH does not enforce any restriction on parameters given that the variance equation is expressed in logarithmic terms, thus guaranteeing the positivity of variance.

Following Alexander (2008b), the distinctive feature about the EGARCH model resides in the asymmetric response function \(g(z_t)\), which, in the case of the standard EGARCH specification defined on the grounds of an iid distributed variable \(z_t\), takes the formulation below stated:

\[
g(z_t) = \gamma z_t + \alpha \left[ |z_t| - E(z_t) \right] \quad (3.31)
\]

where, considering that \(z_t = r_t/\sigma_t\) in (3.17) and (3.18) becomes:

\[
g \left( \frac{r_t}{\sigma_t} \right) = \alpha \left[ \frac{|r_t|}{\sigma_t} - E \left( \frac{r_t}{\sigma_t} \right) \right] + \gamma \frac{r_t}{\sigma_t} \quad (3.32)
\]

Alexander (2008b) provides a thorough analysis of the behaviour of the response function \(g(.)\) for different values of \(z_t\), \(\gamma\) and \(\alpha\) which highlights the presence of leverage effects, all of them asymmetric as long as the coefficient \(\gamma \neq 0\).

---

89 Danielsson (2011:45) states “In out-of-sample forecast comparisons, it is often the case that the more parsimonious models perform better, even if a more flexible model is significantly better in sample”, thus bolstering the adoption of the simple GARCH(1,1).
In the general framework proposed by Nelson (1991), the random variable \( z_t \) assumes a Generalised Error Distribution (GED), notwithstanding which in the interests of parsimony and simplicity the present thesis will only feature two particular cases of GED, Normal and Student-\( t \) densities, backed by Alexander (2008b) who asseverates that for practical purposes, the specification in Nelson (1991) proves difficult to estimate. Consequently, the EGARCH setting to be employed hereafter assumes the ensuing formulas:

\[
r_t = \mu + \sigma_t z_t
\]

\[
ln(\sigma_t^2) = w + \beta \ln(\sigma_{t-1}^2) + g(z_{t-1}) =
\]

\[
ln(\sigma_t^2) = w + \beta \ln(\sigma_{t-1}^2) + \alpha \left[ \frac{r_t}{\sigma_t} - E \left( \frac{r_t}{\sigma_t} \right) \right] + \gamma \frac{r_t}{\sigma_t} \tag{3.33}
\]

given that \( z_t = r_t/\sigma_t \) and making use of (3.32).

The EGARCH representation also delivers a long-term variance which bears some resemblance to the GARCH scheme. Hence, equating the conditional and unconditional variances \( \sigma_t^2 = \sigma_{t-1}^2 = \sigma^2 \) and applying the expectation operator, (3.33) becomes:

\[
E[\ln(\sigma_t^2)] = E \left\{ w + \beta \ln(\sigma_{t-1}^2) + \alpha \left[ \frac{r_t}{\sigma_t} - E \left( \frac{r_t}{\sigma_t} \right) \right] + \gamma \frac{r_t}{\sigma_t} \right\} \tag{3.34}
\]

Following Alexander (2008b), given that \( E(r_t/\sigma_t) = 0 \Rightarrow E \left\{ \alpha \left[ \frac{r_t}{\sigma_t} - E \left( \frac{r_t}{\sigma_t} \right) \right] + \gamma \frac{r_t}{\sigma_t} \right\} = 0 \), the long-term variance is given by:

\[
\sigma^2 = \exp \left( \frac{w}{1-\beta} \right) \quad (\beta < 1) \tag{3.35}
\]

where the \( \beta \) parameter accomplishes the same function that the term \( (\alpha + \beta) \) in (3.19).

EGARCH models also portray advantages and disadvantages which manifest more clearly when analysed in comparison with the aforementioned symmetric GARCH. In this vein, the absence of restrictions in the estimation process constitutes one of its most important assets that reveals in the event of pronounced falls in the securities in question. Consequently, the estimated volatility appears not to display a ‘floor’ which typically affects GARCH predictions (Alexander (2008b)). Given the fact that the parameter \( \gamma \) commonly takes negative values, the EGARCH response to increased volatility when prices go up could prove somewhat deficient, and this bias would in principle limit its usage to long positions to the detriment of short ones. Finally, Alexander (2008b) also notes that EGARCH representations usually deliver volatility estimates slightly lower than its symmetric GARCH counterpart hence...
prodding a slower reaction to unexpected market shocks. However, it seems that the only certainties are related to its formulation (namely the lack of constraints in the parameters, the skewed answer to volatility shocks characteristic of the leverage effect\textsuperscript{90} and the difficulty in implementation), whereas the rest are susceptible of empirical verification.

3.5. Extreme Value Theory (EVT)

One of the major strands of the current thesis hinges upon the application of a relatively novel concept, namely the Extreme Value Theory (EVT) –also the Theory of Extremes– to the financial field. In what follows, the basic framework is to be outlined in order to prepare the ground for the development of VaR models based on EVT and, afterwards, a succinct description of that representation is carried out.

3.5.1. Theoretical context

Extreme Value Theory provides a theoretical justification to represent the ultimate quantiles of a distribution (i.e., 1% or less) as an alternative to its whole structure, thus playing the same essential role as the Central Limit Theorem performs when modelling sums of random variables. This section will supply some basic notions indispensable for the rest of the thesis: for a more detailed treatment and/or some theoretical refinements the interested reader may refer to Embrechts, Klüppelberg and Mikosch (1997), McNeil, Frey and Embrechts (2005), or Reiss and Thomas (2007).

Of the two approaches to model extreme values, Block Maxima Models (BMM)\textsuperscript{91} and Peaks over Thresholds (POT), the current study favours the latter on the grounds of the inefficiency of the former arising from the disregard of the intermediate points contained in the block (Coles (2001)), a crucial fact in the relatively short emerging markets data series.

Supposing a sequence of iid returns (random variables) $X_1, X_2, \ldots, X_n$, which have an unknown marginal distribution function $F$, extreme events are defined as those values of $X_t$ exceeding some high value $u$. The rest of this section enunciates theoretical support for the maximum, as long as the results for the minimum can be obtained from those of the maximum by transforming $X_t$ into $-X_t$.

\textsuperscript{90} The leverage effect in financial markets is extensively explained in Chopra, Lakonishok and Ritter (1992) and Cho and Engle (1999).

\textsuperscript{91} Appendix 3.B.
If $X_0$ is the right endpoint of the distribution $F$ (finite or infinite), it follows that:

$$X_0 = \sup \{X \in \mathbb{R}: F(x) < 1 \} \leq \infty$$  \hspace{1cm} (3.36)

Hence, a description of the stochastic behaviour of the excesses over the threshold $u$ is given by the conditional probability:

$$F_u(y) = \Pr \{X - u \leq y \mid X > u\} = \frac{F(y+u) - F(u)}{1 - F(u)}$$  \hspace{1cm} (3.37)

Given that the threshold $u$ is surpassed, $F_u(y)$ represents the probability that a loss exceeds the threshold $u$ by an amount equal or less than $y$. Were the parent distribution $F$ to be known, the distribution of the exceedances $F_u(y)$ would also be known; however, real exercises show the contrary thus obliging to estimate the distribution for high values above the threshold. Consequently, it is necessary to resort to the Balkema and de Haan (1974), Pickands (1975) Theorem, which broadly asserts:

**Theorem 1** [Balkema and de Haan (1974)–Pickands (1975)]: For a large class of underlying continuous distribution functions $F$, it is possible to find a measurable function $\sigma(u)$ as $u$ goes to the right endpoint $x_0$, such that:

$$\lim_{u \to x_0} \sup_{0 \leq x \leq x_0 - u} |F_u(y) - G_{\xi, \sigma(u)}(y)| = 0$$  \hspace{1cm} (3.38)

if and only if $F$ belongs to the maximum domain of attraction of the Extreme Value Distribution $H_\xi$, i.e., $F \in \text{MDA}(H_\xi)$ with $\xi \in \mathbb{R}$. It may be appreciated that the current thesis will employ the von Mises (1936) and Jenkinson (1955) reparameterisation $\xi = 1/\alpha$, which allows the location parameter $u$ to be the left endpoint of the support of the distribution (furthermore, $F_u(y)$ and $G_{\xi, \sigma(u)}(y)$ both possess the same left endpoint $u$). As Coles (2001) states, the principal outcome is embodied in the theorem below:

**Theorem 2:** Supposing:

- $X_1, X_2, \ldots, X_n$ are a sequence of iid random variables with unique distribution function $F$; and $M_n = \max\{X_1, X_2, \ldots, X_n\}$;
- Any (arbitrary) term in the sequence of random variables $X_i$ is denoted by $X$;
- $F$ complies with Fisher and Tippet (1928), Gnedenko (1948) theorem (Fisher and Tippet (1928), Gnedenko (1948)).

Then, for large $n$, $\Pr\{M_n \leq z\} \approx H(z)$ with

---

92 $\mathbb{R}$ denotes Real numbers.
93 Appendix 3.C.
for some $\mu, \sigma > 0$ and $\xi$. $H_\xi(z)$ is the distribution function of the Generalised Extreme Value distribution (standard GEV), where $\xi$ is known as the shape parameter and $z$ must satisfy

$$1 + \xi z > 0 \quad (3.40)$$

For large thresholds $u$, the distribution function of the exceedances $(X - u)$, given that $X > u$, is yielded by the two-parameter limiting distribution function:

$$G_{\xi, \sigma}(y) = \begin{cases} 
1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi} & \text{if } \xi \neq 0 \\
1 - \exp\left(-\frac{y}{\sigma}\right) & \text{if } \xi = 0
\end{cases}$$

as $u$ increases, where $\sigma > 0$, and $y \geq 0$ when $\xi \geq 0$ and $0 \leq y \leq -\sigma / \xi$ when $\xi < 0$.

$G_{\xi, \sigma}(y)$ is the Generalised Pareto Distribution (GPD) family because it includes three other distributions according to the value of the parameter $\xi$:

- $\xi > 0$ : $G_{\xi, \sigma}(y)$ is the classic Pareto distribution;
- $\xi = 0$ : $G_{\xi, \sigma}(y)$ is the exponential distribution;
- $\xi < 0$ : $G_{\xi, \sigma}(y)$ is the short-tailed Pareto type II or Beta distribution

which correspond to the Fréchet, Gumbel and Weibull Extreme Value Distributions respectively. The GPD family can be enhanced if a location parameter $\mu$ is affixed, thus becoming $G_{\xi, \sigma}(y - \mu)$. In general terms, both theorems mean that if block maxima $M_n$ distribution is approximately described by $H$, then the approximate distribution of the threshold excesses is given by $G$, belonging to the Generalised Pareto family. Furthermore, the parameters of the GPD are univocally determined by those parameters of the associated GEV distribution of the block maxima (Coles (2001)).

By deriving $G_{\xi, \sigma}(y)$ with respect to $\xi$, thus making $g_\xi = G'_{\xi}$, it is possible to obtain the corresponding densities. Consequently,

$$g_\xi(y) = (1 + \xi \frac{y}{\sigma})^{-(1+1/\xi)} \quad (3.42)$$

where $0 \leq y/\sigma$ if $\xi > 0$ and $0 \leq y/\sigma < 1/|\xi|$ if $\xi < 0$.

When $\xi = 0$, the exponential (Beta) density becomes:
\[ g_0(y) = e^{-(y/\sigma)} \]  (3.43)

for \( y/\sigma > 0 \) if \( \xi = 0 \) and the convergence \( g_\xi(y) \to g_0(y) \) given that \( \xi \to 0 \) holds. Graph 3.2 depicts the classic Pareto, exponential and Beta densities, corresponding to the Fréchet, Gumbel and Weibull densities belonging to the EVT setting displayed in Appendix 3.C. It may be appreciated that Pareto distribution \( (\xi > 0) \) acquires more relevance for financial applications, given the fact that its density displays fatter tails for higher quantiles.

Graph 3.2

*Generalised Pareto Distribution (GPD) densities*

**Notes:** The continuous line represents the classic Pareto (Pareto) density (Fréchet in GEV configuration), the dashed line the exponential density (Gumbel in GEV setting) whereas the dotted one denotes the Pareto type II or Beta density (Weibull in GEV configuration).

In the present example, \( \xi=1, \mu=0 \) and \( \sigma=1 \) for Pareto; \( \xi=0, \mu=0 \) and \( \sigma=1 \) for Exponential and \( \xi=-0.5, \mu=0 \) and \( \sigma=1 \) for Beta density.

### 3.6. The Linear Approach

Arguably one of the most straightforward appraisals to compute Value-at-Risk is based on the Standard Deviation of the returns, given the simplicity of the Standard Deviation as a measure of the variation of returns. However, despite its limitations, this methodology is still relatively widespread and many institutions employ it as a yardstick against which more complex techniques are evaluated. Its basic characteristics are found in any elementary statistics textbook.

The Standard Deviation (SD) can be interpreted as the expected size of deviations from the mean return, no matter whether these departures are up or down, as
investors worried about SD pay special attention to the average amount of the fluctuations independently of their direction. Furthermore, the Standard Deviation is underpinned by the assumption that asset returns are normally or lognormally distributed, which in most of the cases reveals itself flawed as it does not reflect the leptokurtic features of the return distributions. Finally, the equal-weighting nature of the observations situates this formula (3.7.1) at loggerheads with any Conditional Volatility specification, hence posing further questions about its dynamic assessment of the conditionality on time⁹⁴.

\[
\sigma = \frac{1}{n-1} \sqrt{\sum_{t=1}^{n} (r_t - \bar{r})^2} \quad (3.44)
\]

where, for a sample of \( n \) returns, \( \sigma \) represents the Standard Deviation.

For Value-at-Risk calculation purposes, it is necessary to affix the inverse of a cumulative distribution function to the Standard Deviation of returns to account for the required confidence cutoff level. In this vein, the current thesis will use the two more common alternatives: Normal and Student-\( t \) distributions, thus coinciding with the Filtered Historical Simulation (Section 3.4) or the Conditional Volatility models (Section 3.5) described above. Penza and Bansal (2001) recommend the application of the latter in order to overcome the verified departures of normality⁹⁵.

### 3.7. Value-at-Risk techniques

The objective of the present section resides in dealing with the formulations to be employed in order to obtain the necessary parameters for every model, as well as the ensuing Value-at-Risk expressions and final Minimum Capital Requirements. It contains an enunciation of the indispensable concepts to grasp a general idea of the estimation procedure, as theoretical developments lie beyond the scope of the current thesis and can be found in statistical textbooks that will be indicated whenever necessary.

⁹⁴ Formula (3.44) pictures the unbiased estimator of the Standard Deviation. Da Costa Lewis (2003) describes its advantages over the biased one.

⁹⁵ The authors mention two other alternatives to the use of the Student-\( t \) distribution. Primarily, Pareto stable distributions (in the present work it is to be employed in the context of the Extreme Value Distribution model) and, secondly, the augmentation of the distance from the mean for a certain probability level with reference to the Normal distribution. For example, they suggest that a factor of \( 3\sigma \) instead of the 1.96\( \sigma \) delivered by the Gaussian distribution. However, even though the solution might eventually tackle the matter at hand, it is believed that the proposal appears somewhat tarnished by a more than evident tinge of subjectivity of appreciation and therefore not used for risk quantification purposes in the thesis.
The primary data are formed by univariate price series belonging to a sample of eleven stock market blue-chip indices: seven belong to Emerging markets (Brazil, Hungary, India, Czech Republic, Indonesia, Malaysia and China) and four to Frontier markets (Argentina, Lithuania, Tunisia and Croatia) retrieved from the corresponding stock markets websites and Reuters database. Series were converted into a string of logarithmic returns to achieve stationarity and ergodicity (Bowerman and O’Connell (1993) and Alexander (2008a)). As in Hansen and Lunde (2005) and Mapa (2003), the time series of returns were separated into two periods for the purposes of parameter estimation and evaluation of forecasts respectively.

3.7.1. Measuring financial returns

Risk is often measured in terms of price changes, either raw or related to some initial price, i.e., returns, the latter ones being better indicators of a security’s performance and much easier to model with statistical tools. The simplest definition of price change is expressed uniperiodically:

$$\Delta p_t = p_t - p_{t-1} \quad (3.45)$$

where

$\Delta p_t$: price change (without interests or dividends);

$p_t$: price at $t$;

$p_{t-1}$: price at $t-1$;

As it was already mentioned, throughout this thesis $t$ represents one business day. Therefore, the single period return becomes:

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{\Delta p_t}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1 \quad (3.46)$$

thus making possible the expression

$$p_t = p_{t-1} (1 + r_t) \quad (3.47)$$

For multiperiodal returns, if $p_k$ represents the price of the stock at the end of day $k$:

$$p_1 = p_0 (1 + r_1)$$

$$p_2 = p_1 (1 + r_2)$$

.................

96In order to categorise the markets the study follows the FTSE Global Equity Index Series Country Classification, September 2012 update. (FTSE (2012)).
\[ p_k = p_{k-1} (1+r_k) \quad \Rightarrow \quad p_k = p_0 (1+r_1)(1+r_2)\ldots(1+r_k) \quad (3.48) \]
\[ \Rightarrow r(k) = \frac{p_k}{p_0} - 1 \quad (3.49) \]

Making temporal aggregation:
\[ r(k) = (1+r_1)(1+r_2)\ldots(1+r_k) = \prod_{j=1}^{k} \left(1+r_j\right) - 1 \quad (3.50) \]

Cross-sectional aggregation (across individual returns) at a particular point in time reveals itself more important for VaR to evaluate portfolios. Hence,
\[ r_p = w_1 r_1 + w_2 r_2 + \ldots + w_n r_n = \sum_{i=1}^{n} w_i r_i \quad (3.51) \]

where
\[ r_{i-p} : \text{return of the security } i \text{ or portfolio } p; \]
\[ w_i : \text{weight given to the } i-\text{th. asset in the portfolio } \left( \sum_{i=1}^{n} w_i = 1 \right) \]

Therefore making use of (3.42) for \( t = 1 \),
\[ r_p = \frac{p_1 - p_0}{p_0} \]
\[ \Rightarrow p_1 = w_1 p_0 (1+r_1) + w_2 p_0 (1+r_2) + \cdots + w_n p_0 (1+r_n) = p_0 \sum_{i=1}^{n} w_i (1+r_i) \quad (3.52) \]

However, relative returns appear somewhat unsuitable for statistical modelling. As most of the stochastic developments presuppose the Normal distribution of returns, it is impossible for returns defined as in (3.50) to make use of this assumption, firstly because under the limited liability principle a security yields returns ranging from [\(-1; \infty\)], very different from those predicted by the Gaussian distribution; and secondly, as the product of Normal variables is not Normal in itself, multiperiodal returns do not follow the bell-shaped distribution (despite normality of uniperiodal ones)\(^{97}\).

The unsatisfactory statistical properties of relative returns motivated the use of logarithmic or continuously compounded returns:
\[ r_t = \ln (1+r_t) = \ln \left( \frac{p_t}{p_{t-1}} \right) = \ln (p_t) - \ln (p_{t-1}) \quad (3.53) \]

Aggregating returns in time becomes:

\(^{97}\)Penza and Bansal (2001) treat this topic in more detail.
\[ r_i(k) = \ln \left( \frac{p_t}{p_{t-k}} \right) = \ln (p_t) - \ln (p_{t-k}) \] (3.54)

which is also the sum of \( k \) continuously compounded returns:

\[
\begin{align*}
    r_i(k) &= \ln[1+r_i(k)] = \ln[(1+r_i)(1+r_{i-1}) \ldots (1+r_{i-k+1})] = \\
    &= \ln (1 + r_i) + \ln (1 + r_{i-1}) + \ldots + \ln (1 + r_{i-k+1}) \\
    &= \sum_{i=0}^{k-1} \ln (1 + r_{i-1}) = r_i + r_{i-1} + \ldots + r_{i-k+1} \\
    \Rightarrow r_i(k) &= \sum_{i=0}^{k-1} r_{i-1} \quad (3.55)
\end{align*}
\]

that keeps normality available for modelling.

Unlike relative returns, cross-sectional aggregation is not practical for log-returns:

\[ P_t = w_1 P_0 e^{r_1} + w_2 P_0 e^{r_2} + w_3 P_0 e^{r_3} = P_0 \left( w_1 e^{r_1} + w_2 e^{r_2} + w_3 e^{r_3} \right) \]

If \( r_p = \ln \left( \frac{p_t}{p_0} \right) \Rightarrow r_p = \ln \left( \sum_j w_j e^{r_j} \right) \] (3.56)

However, when returns are measured on a daily basis, the approximation:

\[ r_p \equiv \sum_j w_j r_j \] (3.57)

may well be used instead, as the next example illustrates. Supposing an equally-weighted portfolio composed of the blue-chip stock indices belonging to Estonia (OMXT), Lithuania (OMXV) and Latvia (OMXR) for the period 01/01/2000 to 31/12/2000, the comparison between (3.51) – arithmetic returns – and (3.57) – logarithmic returns – does not exhibit significant differences as the two lines appear barely distinguishable in Graph 3.3:
Furthermore, considering that the current thesis is concerned with univariate time series, the same situation arises when the individual indices are accounted for. In this sense, Graph 3.8 (Panels A, B and C) –which features the three time series for the same time span- shows almost identical lines in what concerns the Logarithmic (L) and the Arithmetic (A) variants:

Graph 3.4
Panel A – OMXT(L) vs OMXT(A)

Graph 3.4
Panel B – OMXV(L) vs OMXV(A)

Graph 3.4
Panel C – OMXR(L) vs OMXR(A)
The visual inspection of the analysis, which coincides with Penza and Bansal (2001) and JP Morgan and Reuters (1996), is susceptible of verification when the statistical profile of every series, depicted in Chart 3.1, is scrutinised:

Chart 3.1
Basic statistics about OMXT(L), OMXT(A), OMXV(L), OMXV(A), OMXR(L), OMXR(A), Portfolio(L) and Portfolio(A)
Period 01/01/2000 – 31/12/2000

<table>
<thead>
<tr>
<th>Statistic</th>
<th>OMXT (L)</th>
<th>OMXT (A)</th>
<th>OMXV (L)</th>
<th>OMXV (A)</th>
<th>OMXR (L)</th>
<th>OMXR (A)</th>
<th>Portfolio (L)</th>
<th>Portfolio (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
<td>257</td>
<td>257</td>
<td>257</td>
<td>257</td>
<td>257</td>
<td>257</td>
<td>257</td>
<td>257</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0001</td>
<td>0.0000</td>
<td>-0.0003</td>
<td>-0.0002</td>
<td>0.0016</td>
<td>0.0017</td>
<td>0.0004</td>
<td>0.0005</td>
</tr>
<tr>
<td>Median</td>
<td>0.0003</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0013</td>
<td>0.0013</td>
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<td>0.0006</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0734</td>
<td>0.0762</td>
<td>0.0458</td>
<td>0.0469</td>
<td>0.0823</td>
<td>0.0858</td>
<td>0.0545</td>
<td>0.0566</td>
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<tr>
<td>Minimum</td>
<td>-0.0587</td>
<td>-0.0570</td>
<td>-0.1022</td>
<td>-0.0971</td>
<td>-0.0528</td>
<td>-0.0514</td>
<td>-0.0397</td>
<td>-0.0380</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0131</td>
<td>0.0131</td>
<td>0.0101</td>
<td>0.0099</td>
<td>0.0154</td>
<td>0.0155</td>
<td>0.0083</td>
<td>0.0083</td>
</tr>
<tr>
<td>Variance</td>
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<td>0.0002</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.1921</td>
<td>0.3516</td>
<td>-3.6144</td>
<td>-3.1678</td>
<td>0.5085</td>
<td>0.6324</td>
<td>0.3892</td>
<td>0.5960</td>
</tr>
<tr>
<td>q(0.0001)</td>
<td>-0.0584</td>
<td>-0.0567</td>
<td>-0.1003</td>
<td>-0.0953</td>
<td>-0.0525</td>
<td>-0.0511</td>
<td>-0.0394</td>
<td>-0.0377</td>
</tr>
<tr>
<td>q(0.01)</td>
<td>-0.0555</td>
<td>-0.0539</td>
<td>-0.0831</td>
<td>-0.0793</td>
<td>-0.0497</td>
<td>-0.0484</td>
<td>-0.0369</td>
<td>-0.0355</td>
</tr>
<tr>
<td>q(0.025)</td>
<td>-0.0415</td>
<td>-0.0407</td>
<td>-0.0200</td>
<td>-0.0198</td>
<td>-0.0378</td>
<td>-0.0371</td>
<td>-0.0234</td>
<td>-0.0229</td>
</tr>
<tr>
<td>q(0.05)</td>
<td>-0.0293</td>
<td>-0.0289</td>
<td>-0.0169</td>
<td>-0.0167</td>
<td>-0.0287</td>
<td>-0.0283</td>
<td>-0.0127</td>
<td>-0.0126</td>
</tr>
<tr>
<td>q(0.10)</td>
<td>-0.0163</td>
<td>-0.0162</td>
<td>-0.0125</td>
<td>-0.0124</td>
<td>-0.0218</td>
<td>-0.0216</td>
<td>-0.0118</td>
<td>-0.0116</td>
</tr>
<tr>
<td>q(0.90)</td>
<td>0.0131</td>
<td>0.0132</td>
<td>0.0082</td>
<td>0.0082</td>
<td>0.0198</td>
<td>0.0200</td>
<td>0.0095</td>
<td>0.0095</td>
</tr>
<tr>
<td>q(0.95)</td>
<td>0.0179</td>
<td>0.0181</td>
<td>0.0122</td>
<td>0.0123</td>
<td>0.0266</td>
<td>0.0270</td>
<td>0.0116</td>
<td>0.0119</td>
</tr>
<tr>
<td>q(0.975)</td>
<td>0.0264</td>
<td>0.0268</td>
<td>0.0155</td>
<td>0.0156</td>
<td>0.0340</td>
<td>0.0346</td>
<td>0.0137</td>
<td>0.0139</td>
</tr>
<tr>
<td>q(0.99)</td>
<td>0.0372</td>
<td>0.0379</td>
<td>0.0242</td>
<td>0.0245</td>
<td>0.0431</td>
<td>0.0440</td>
<td>0.0165</td>
<td>0.0167</td>
</tr>
<tr>
<td>q(0.999)</td>
<td>0.0647</td>
<td>0.0669</td>
<td>0.0412</td>
<td>0.0421</td>
<td>0.0735</td>
<td>0.0764</td>
<td>0.0472</td>
<td>0.0489</td>
</tr>
<tr>
<td>q(0.9999)</td>
<td>0.0725</td>
<td>0.0753</td>
<td>0.0453</td>
<td>0.0464</td>
<td>0.0814</td>
<td>0.0848</td>
<td>0.0537</td>
<td>0.0558</td>
</tr>
</tbody>
</table>

Note: (L) and (A) stand for Logarithmic and Arithmetic returns respectively.

For statistical modelling purposes, the differences appearing in the order of the fourth or fifth decimal place cannot be regarded as sizeable. Therefore, the present thesis will proceed using logarithmic, geometric or continuously compounded returns, as is common practice among academic and practitioners at the time of financial modelling.

3.7.2. Value-at-Risk expressions

As it was aforementioned, many of the aspects to be analysed are closely related to the discussion in Sections 3.3 to 3.7, hence this section will often refer to the previous ones because the characteristics and the dynamics of each representation were dealt with previously. The focus of the present paragraphs hinges on VaR formulas for all the schemes, given the fact that they are easily converted into MCR for the IMA. However, it is useful to provide insights into several assumptions and aspects related to the specifications mentioned in Sections 3.3 to 3.7.
a) Historical Simulation model

Section 3.3 has made clear that calculating Value-at-Risk through Historical Simulation amounts to computing the required quantile of the respective time series of returns. However, the contentious aspect resides in determining the appropriate quantity of returns susceptible of being included in the time window and, in this sense, it is useful to reiterate that the disparity among the theoreticians turns the selection into an issue of subjective nature. Accordingly, Christoffersen (2003) suggests some value in the interval [250; 1000], i.e., between one and four trading years, Manganelli and Engle (2001) advocate between six months and two years of daily data and Kuester, Mittnik and Paolella (2006) promote four years. Even though the choice lies far away from unanimity, the present thesis will be developed considering a rolling sample of 1000 trading days, or four years, following the last study and situating in the upper bound of Christoffersen’s range (2003), thus accounting for Christoffersen’s recommendation which may well be regarded as a rule-of-thumb.

Therefore, given that the information set \( t = 1, 2, \ldots, n \) and \( n = 1000 \):

\[
VaR^{HS}_{t+1}(\alpha) = Q_{\alpha}(r_t, r_{t-1}, r_{t-2}, \ldots, r_{t-n}) \quad (3.58)
\]

where \( VaR^{HS}_{t+1} \) refers to the Value-at-Risk level using Historical Simulation and \( Q_{\alpha} \) denotes the \( \alpha \)-quantile of the \( n \) previous returns states the Value-at-Risk for time \( t+1 \) at the confidence level \( \alpha \).

After developing Filtered Historical Simulation, Conditional Volatility and Extreme Value Theory VaR expressions, it is essential to devote some lines to the estimation routine of the GARCH and EGARCH specifications, both with the Normal or the Student-\( t \) distributions affixed. Therefore,

b) Filtered Historical Simulation models

After fitting some model to compute the required standardised returns, the VaR expression reads:

\[
VaR_{t+1}(\alpha) = \sigma_{t+1} F^{-1}(\alpha) \quad (3.59)
\]

where:

\( \sigma_{t+1} \) : volatility forecast derived from any (GARCH family) volatility model;

\( F^{-1}(\alpha) \) : inverse of the cumulative density function of the empirical distribution of the standardised residuals, i.e., \( \alpha \)-quantile of \( F \)
According to Section 3.4, GARCH and EGARCH volatility models with both Normal and Student-t distributions constitute the combinations under FHS framework. Hence, formula (3.59) may be further specified considering these four possibilities:

\[ b.1) \text{VaR}_{t+1}^{\text{FHS-GN}}(\alpha) = \sigma_{t+1}^{G} F^{-1}(\alpha) \]  
\[ b.2) \text{VaR}_{t+1}^{\text{FHS-Gt}}(\alpha) = \sigma_{t+1}^{G} t^{-1}(\alpha) \]  
\[ b.3) \text{VaR}_{t+1}^{\text{FHS-EN}}(\alpha) = \sigma_{t+1}^{E} F^{-1}(\alpha) \]  
\[ b.4) \text{VaR}_{t+1}^{\text{FHS-Et}}(\alpha) = \sigma_{t+1}^{E} t^{-1}(\alpha) \]  

where \( \text{VaR}_{t+1}^{\text{FHS-XY}} \) denotes the Value-at-Risk number corresponding to the Filtered Historical Simulation scheme using the model “X” (GARCH or EGARCH) with the distribution “Y” appended (Normal or Student-t). The superscript “G” or “E” refers to the GARCH or EGARCH representation employed to obtain the volatility forecast for the day \( t+1 \) respectively whereas the symbols “N” and “t” stand for Normal and Student-t distributions respectively. Finally, the symbol \( F^{-1} \) indicates the inverse of the standardised empirical distribution using the model provided.

c) Conditional Volatility models

Much in the same fashion than FHS, although bearing in mind the differences in the mechanics stated in Sections 3.4 and 3.5, the Value-at-Risk expressions for the Conditional Volatility models are computed as follows:

\[ \text{VaR}_{t+1}(\alpha) = \sigma_{t+1} F^{-1}(\alpha) \]  

where:

\( \sigma_{t+1} \) : volatility forecast derived from any (GARCH family) volatility model;

\( F^{-1}(\alpha) \) : inverse of the cumulative density function of the assumed distribution of the residuals, i.e., \( \alpha \)-quantile of \( F \)

According to Section 3.5, GARCH and EGARCH volatility models with both Normal and Student-t distributions constitute the combinations under the CV framework. Hence, formula (3.64) may be further specified considering those four possibilities:

\[ c.1) \text{VaR}_{t+1}^{\text{CV-GN}}(\alpha) = \sigma_{t+1}^{G} \Phi^{-1}(\alpha) \]  
\[ c.2) \text{VaR}_{t+1}^{\text{CV-Gt}}(\alpha) = \sigma_{t+1}^{G} t^{-1}(\alpha) \]  
\[ c.3) \text{VaR}_{t+1}^{\text{CV-EN}}(\alpha) = \sigma_{t+1}^{E} \Phi^{-1}(\alpha) \]  
\[ c.4) \text{VaR}_{t+1}^{\text{CV-Et}}(\alpha) = \sigma_{t+1}^{E} t^{-1}(\alpha) \]  

where \( \text{VaR}_{t+1}^{\text{CV-XY}} \) denotes the Value-at-Risk number corresponding to the Conditional Volatility scheme using the model “X” (GARCH or EGARCH) with the distribution “Y”
appended (Normal or Student-t). Additionally, the superscript “G” or “E” refers to the GARCH or EGARCH representations employed to obtain the volatility forecast for the day \( t+1 \) respectively whereas the symbols “N” and “t” stand for Normal and Student-t distributions respectively. Finally, the symbols \( \Phi^{-1} \) and \( t^{-1} \) indicate the inverse of the cumulative Normal or Student-t distributions respectively.

d) Extreme Value Theory model

The Value-at-Risk configuration under the framework of the Extreme Value Theory application does not differ from FHS or CV in the sense that, in general terms, it requires the calculation of a volatility forecast and a quantile of the relevant distribution. The distinction arises from the distributional assumption applied given that, in the first case, the standardised empirical distribution is used whereas in the second, a theoretical one is utilised. The Extreme Value Theory variant enrolls in the second category as a theoretical distribution is appended to the volatility forecast, although the characteristics of the distribution mark the difference in performance among the specifications that share analogous volatility forecast, or, in other words, a volatility prediction deriving from the same model.

Consequently, the basic formula is inherently a reiteration of (3.59) and (3.64):

\[
\text{VaR}_{t+1}(\alpha) = \sigma_{t+1}F^{-1}(\alpha) \quad (3.69)
\]

where:

\( \sigma_{t+1} \): volatility forecast derived from any (GARCH family) volatility model;

\( F^{-1}(\alpha) \): inverse of the cumulative density function of the Generalised Pareto Distribution fitted to the standardised residuals, i.e., \( \alpha \)-quantile of \( F \)

Once the choice of the threshold \( u \) is performed\(^{98} \) and the parameters of the GPD are estimated, it is necessary to obtain the expression to calculate the relevant VaR quantiles. From (3.37) and recalling \( x = y + u \), an estimate for \( F(x) \), for \( x > u \) may be:

\[
F(x) = [1 - F(u)]G_{\xi,\sigma(u)}(y) + F(u) \quad (3.70)
\]

Considering \( k \) as the number of observations above the threshold \( u \), \( F(u) \) may easily be non-parametrically approximated by means of the simple empirical estimator:

\[
\hat{F}(u) = \frac{n-k}{n} \quad (3.71)
\]

\(^{98} \) See below.
Plugging (3.70) into (3.69) it is possible to achieve an estimate for $F(x)$:

$$
\hat{F}(x) = 1 - \frac{k}{n} \left[ 1 + \hat{\xi} \left( \frac{x-u}{\hat{\sigma}} \right) \right]^{-\frac{k}{\hat{\xi}}} \tag{3.72}
$$

$\hat{\xi}, \hat{\sigma}$ being estimates for $\xi$ and $\sigma$ respectively. For a level of confidence $\alpha > F(u)$, the expression of the relevant $\alpha$-quantile is computed by inverting $\hat{F}(x)$ and solving for $x$:

$$
\hat{F}^{-1}(\alpha) = u + \frac{\hat{\sigma}}{\hat{\xi}} \left( \frac{1 - \alpha}{k/n} \right)^{-\frac{1}{\hat{\xi}}} - 1 \tag{3.73}
$$

Following McNeil, Frey and Embrechts (2005) and Fernandez (2003) who advocate the use of the GARCH-Normal combination as the model to forecast volatility and standardise the residuals to feed the Generalised Pareto Distribution adjustment, the VaR expression ensues:

$$
VaR_{t+1}^{EV\text{T}}(\alpha) = \sigma_{t+1}^{\text{G}} G_{\tilde{\xi},\hat{\sigma}}^{-1}(y)(\alpha) \tag{3.74}
$$

\textit{e) Linear model}

Once more, the general expression does not differ from the previous ones, in the sense that only a volatility forecast and the quantile of an assumed distribution function are required:

$$
VaR_{t+1}(\alpha) = \sigma_{t+1} F^{-1}(\alpha) \tag{3.75}
$$

where $\sigma_{t+1}$ represents the volatility forecast for day $t+1$ and $F^{-1}(\alpha)$ the inverse of the cumulative distribution function at the appropriate confidence level $\alpha$. Consequently, in view of the enumeration carried out in Section 3.7 above concerning the usage of the Normal and Student-$t$ distributions:

\begin{align*}
\text{e.1) } & VaR_{t+1}^{L-N}(\alpha) = \sigma_{t+1}^{L} \Phi^{-1}(\alpha) \tag{3.76} \\
\text{e.2) } & VaR_{t+1}^{L-t}(\alpha) = \sigma_{t+1}^{L} t^{-1}(\alpha) \tag{3.77}
\end{align*}

where the superscript $L$ stands for Linear (i.e., Standard Deviation) and letters $N$ and $t$ and the symbols $\Phi^{-1}$ and $t^{-1}$ conserve the informed meaning.

\textbf{3.7.3. An insight into the estimation of the model parameters}

The current section is intended to provide additional precisions in what regards the derivation of the preceding Value-at-Risk formulas. In this vein, the focus will be laid upon the general procedures necessary to obtain the volatility forecasts indispensable to
compute the VaR figures. Given the illustrative nature of the following paragraphs, the theoretical gaps that may spring up can be covered resorting to the specific references indicated.

As it is deemed that every consideration concerning Historical Simulation has already been dealt with in Sections 3.3 and 3.8.2.a), provisions regarding the manner in which the parameters that enable the obtention of the $\sigma_{t+1}$ volatility prediction for the Filtered Historical Simulation, Conditional Volatility, Extreme Value and Linear models are to be addressed in this Section. It is however important to bear in mind that the computations referred to the first three techniques only differ in the appended distribution once the volatility has been forecasted, and, given that the issue of the affixed distribution has received appropriate treatment above, formulae to obtain the $\sigma_{t+1}$ term will be presently included on the grounds of its widespread use. On the other hand, a deeper analysis is to be devoted to the Extreme Value Theory technique because of its importance in view of the conclusions drawn in the course of the thesis.

*a) Parameters for Filtered Historical Simulation and Conditional Volatility models*

As it was already mentioned in Sections 3.8.2.b) and 3.8.2.c), the Filtered Historical Simulation and Conditional Volatility estimates will root in the volatility predictions arising from GARCH and EGARCH techniques, both with Normal and Student-$t$ distributions appended. The differences, then, will surface at the time of computing the respective VaR values, given the fact that, for FHS the empirical distribution of the standardised residuals is to be employed whereas for the CV the respective quantile of the Normal or Student-$t$ distributions will determine the fixed multiple to enhance the volatility forecast.

*a.1) The GARCH representation*

The GARCH family acquires relevance when the frequency of the underlying data is intraday, daily (most commonly) and sometimes weekly, but very rarely on monthly or even lower ones. This kind of schemes will otherwise be unable to reflect the volatility clustering effect characteristic of financial time series because those patterns vanish as long as the frequency over which returns are calculated lengthens.

According to Bollerslev (1986), the GARCH regression models are estimated via Maximum Likelihood (ML) which is tantamount to expressing that GARCH parameters are found by maximising the value of the log likelihood function. Reducing
the process to the GARCH with first order lags and assuming initially that financial returns are generated by a normally distributed process with expectation 0 and variance \( \sigma^2_t \), the scheme may well be written as in Bollerslev (1986). Consequently, if \( z \) and \( v \) denote the vector of past information and parameters respectively such that \( z_{t+1} = (1, r_t^2, \sigma_t) \) and \( v = (w, \alpha, \beta) \), \( \theta \in \Theta \), where \( \theta = (b', \nu') \) and “..\( \Theta \) is a compact subspace of a Euclidean space in a manner that \( r_t \) possesses finite second order moments” (Bollerslev (1986:315)). A particular interpretation stated by Bollerslev indicates that the GARCH regression model can be obtained supposing the returns \( r_{t+1} \) are innovations in the linear regression:

\[
r_{t+1} = y_{t+1} - x_{t+1}b
\]

where \( y_{t+1} \), \( x_{t+1} \) and \( b \) are vectors containing the dependent variable, explanatory variables and unknown parameters respectively. Therefore, the model is susceptible of being re-expressed as (3.79):

\[
r_{t+1} = y_{t+1} - x_{t+1}b
\]

\[
r_{t+1} \mid I_t \sim N(0; \sigma_{t+1})
\]

\[
\sigma_{t+1} = z_{t+1}'\nu
\]

where \( I_t \) denotes the information set up to time \( t \).

Using the formulation explained in Alexander (2001), and Alexander (2008b), and setting aside the constant as the optimal asymptotical parameter values remain unaffected (Alexander (2008b) and Bollerslev (1986)), the log likelihood function for a sample of returns of size \( n \) becomes:

\[
L_n(\theta) = n^{-1} \sum_{t=1}^{n} l_t(\theta) \quad (3.80)
\]

\[
\ln L(\theta) = -\frac{1}{2} \sum_{t=1}^{n} \left[ \ln(\sigma_t^2) + \left( \frac{r_t}{\sigma_t} \right)^2 \right] \quad (3.81)
\]

\( \theta \) comprising the parameters of the conditional variance equation \( \alpha \) and \( \beta \). Given that the logarithm is a monotonic increasing function, the same values of \( \theta \) that maximises the likelihood simultaneously maximises the log likelihood which explains the reason why the maximum likelihood estimates are usually computed maximising the log likelihood function (3.81) rather than the likelihood function itself (3.80). Furthermore, maximising (3.81) is tantamount to minimising

\[
-2 \ln L(\theta) = \sum_{t=1}^{n} \left[ \ln(\sigma_t^2) + \left( \frac{r_t}{\sigma_t} \right)^2 \right] \quad (3.82)
\]
where the vector $\theta$ contains the aforementioned variables. For both (3.81) and (3.82) the dependence of the log likelihood function $L$ is ensured by the GARCH equation stated in (3.18).

One of the advantages of the GARCH representation is its flexibility, and, in this sense, it is possible to append different distributions to the basic setting in (3.18) and obtain models with particular characteristics. For the purposes of the current thesis, the Student-$t$ distribution is to be employed alongside the Normal one. Following Alexander (2008b), the log likelihood function of the GARCH-Student-$t$ becomes:

$$\ln L(\theta) = -\sum_{t=1}^{n} \left\{ \ln(\sigma_t) + \left( \frac{d + 1}{2} \right) \ln \left[ 1 + (d - 2)^{-1} \left( \frac{r_t}{\sigma_t} \right)^2 \right] \right\} +$$

$$+ n \ln \left\{ (d - 2)\pi^{-1/2} \Gamma \left( \frac{d}{2} \right)^{-1} \Gamma \left( \frac{d+1}{2} \right) \right\}$$

(3.83)

and $\theta$ includes the parameters of the conditional variance equation stated in (3.18) albeit the standardised residuals follow a Student-$t$ distribution with $d$ degrees of freedom and $\Gamma$ denotes the gamma function used in the derivation of the Student-$t$ density function.

a.2) The EGARCH representation

The same situation arises with the Exponential GARCH model, given that the parameters of both varieties are estimated via the likelihood function. For the EGARCH-Normal, the appropriate likelihood function is similar to (3.81) except for the fact that different parameters are involved considering the equation (3.33) that defines the scheme. Hence, again excluding the constant as the rest of the estimates remain invariant after its withdrawal (Alexander (2008b)),

$$\ln L(w, \beta, \alpha, \gamma) = -\frac{1}{2} \sum_{t=1}^{n} \left\{ \ln(\sigma_t^2) + \left( \frac{r_t}{\sigma_t} \right)^2 \right\}$$

(3.84)

and the dependence of the log likelihood $L$ on the parameters $w, \beta, \alpha$ and $\gamma$ is reflected in (3.33). Analogously, the parameters of the EGARCH model with Student-$t$ distributed standardised errors are obtained through the Maximum Likelihood function $L$, removing the constant term as all other estimates stay unchanged:

$$\ln L(\theta) = -\sum_{t=1}^{n} \left\{ \ln(\sigma_t) + \left( \frac{d + 1}{2} \right) \ln \left[ 1 + (d - 2)^{-1} \left( \frac{r_t}{\sigma_t} \right)^2 \right] \right\} +$$

$$+ n \ln \left\{ (d - 2)\pi^{-1/2} \Gamma \left( \frac{d}{2} \right)^{-1} \Gamma \left( \frac{d+1}{2} \right) \right\}$$

(3.85)
and $\theta$ includes the parameters of the conditional variance equation stated in (3.33) albeit the standardised residuals follow a Student-$t$ distribution with $d$ degrees of freedom and $\Gamma$ denotes the gamma function used in the derivation of the Student-$t$ density function.

\textit{b) The Extreme Value Theory Model}

In order to estimate the parameters of the Generalised Pareto Distribution that define the characteristics of the conditional EVT-GPD representation, the current thesis will adhere to the methodology mentioned in Dowd (2005), McNeil and Frey (2000) and McNeil, Frey and Embrechts (2005), and applied by Rossignolo, Fethi and Shaban (2012a, 2012b, 2013) to different samples. In broad terms, the application of Extreme Value Theory models using a GDP distribution that derive in the corresponding VaR values observe the below enunciation, which resembles the two-stage analysis stated in McNeil, Frey and Embrechts (2005):

1. The first phase comprises the estimation of the dynamic behaviour of the underlying volatility of the series;
2. The second part deals with the modelling of the Generalised Pareto Distribution employing the residuals from the conditional representation fitted in 1. as data.

After performing legs 1. and 2., the calculation of the VaR values considering the conditional volatility structure and the residuals process driven by GPD fitted ensues. In what follows, and in the interests of its influence on the present thesis, some degree of detail will be lavished to the aforementioned points 1. and 2.

\textit{Stage 1: The estimation of the dynamic behaviour of the underlying volatility}

It is often the case that Extreme Value Theory stresses the second phase of the two-stage process above described and, moreover, the specification of a correct likelihood function featuring a Pareto Distribution proves an incredibly complex task. Therefore, the analyst usually resorts to the Quasi-Maximum Likelihood (QML) model fitting procedure\textsuperscript{99}, in a process denominated pre-whitening of the data (McNeil, Frey and Embrechts (2005)).

Briefly stated, the QML process assumes that the dynamic form of the representation has been correctly specified, but the innovations are erroneously supposed to be Gaussian (Normal), as opposed to the Maximum Likelihood (ML)

\textsuperscript{99}The Quasi-Maximum Likelihood (QML) process is also called Pseudo-Maximum Likelihood (PML). However, the former is much more common in the financial literature.
structure which takes both the model and the innovations as correctly specified, thus meaning that the data are indeed engendered by a time series model with both the mechanics and the innovation distribution formerly set.

The QML process exerts a considerable attraction on the practitioners because, provided the precondition is met—i.e., the dynamics of the conditional variance are rightly specified—it allows the parameters of a Normal likelihood to consistently estimate the parameters of the corresponding model, although the rescaled variable (the standardised returns) is not truly Normal or even not independent. More specifically, and in line with the topic dealt with in the current thesis, Bollerslev and Wooldridge (1992) studied the properties of the QML estimators and its related test statistics in dynamic models that jointly estimate the parameters of conditional means, conditional variances and covariances when a normal log-likelihood is maximised under an incorrect assumption of normality. Amid a host of relevant implications, the most important result is conveyed by the fact that, whenever the first two conditional moments are correctly specified, the QML process delivers consistent estimators with a limiting Normal distribution.

However, as Lee and Hansen (1994) point out, the weakest point of the QML roots in the assumption of the process generating the conditional variance; in general, literature employs the GARCH(1,1) and its extensions ((GARCH(p,q))\(^{100}\). For instance, Bollerslev and Wooldridge (1992) use GARCH(p,q), Lee and Hansen (1994) apply GARCH(1,1) and Lumsdaine (1995) utilises GARCH(1,1) and IGARCH(1,1) models to derive the consistency and asymptotic normality of QML estimators in a clear indication that formulas for other class of representations might appear somewhat cumbersome\(^{101}\).

Despite the aforementioned theoretical deficiencies, QML coupled with EVT has surged as a powerful means to address the conditional heteroscedasticity and the fat-tailed nature of data series in finance by a vast body of literature: Embrechts, Resnick and Samorodnitsky (1999), McNeil and Frey (2000), McNeil, Frey and Embrechts (2005), Byström (2004), Chan and Gray (2006) show its appeal by fitting a GPD (via POT) to the standardised residuals after applying GARCH(1,1) (first four articles) or EGARCH(1,1) (last one) filters.

\(^{100}\)GARCH(p,q) refers to a GARCH model with a quantity of lags greater than 1, thus \(p > 1\) and \(q > 1\).

\(^{101}\)In fact, Lee and Hansen (1994) acknowledge that sometimes GARCH models may not mimic the behaviour of the market accurately and pose the development of the properties of QML estimators belonging to sharper representations as a pending challenge.
Therefore, for the purposes of this thesis GARCH(1,1) will constitute the technique assumed to generate the dynamics of the true conditional variance and, on analogous grounds, the Gaussian distribution drives the behaviour of the standardised returns (or, according to Lumsdaine (1995), rescaled residuals). The properties demonstrated in the aforementioned bibliography imply, then, that the maximisation of the normal log-likelihood gives consistent estimators of the parameters of the model and, furthermore, asymptotic normality is obtained for the distribution of the innovations. This result allows the development of the second phase, namely the application of the Extreme Value Theory for the standardised residuals in order to fit a Generalised Pareto Distribution.

**Stage 2: Fitting the Generalised Pareto Distribution**

Despite the theoretical developments comprised by the application of QML estimation, Phase 1 engulfs the determination of the variance forecasts to obtain the standardised residuals. Once the series of standardised residuals is obtained, it is necessary to fit a Generalised Pareto Distribution to that string of standardised residuals via BMM or POT methodologies alternatively. The GDP will be univocally defined by its parameters, thus allowing the calculation of the term $G_{\xi, \sigma}^{-1}(y)(\alpha)$ in (3.69).

1. **The Block Maxima Method**

Block Maxima and Peaks Over Threshold are the only two methods employed to affix a GPD to a series of points. Succinctly stated, to perform the former, the data need to be grouped in homogeneous clusters of equal length in order to select the maximum or minimum values belonging to every group. Coles (2001) points out a series of snags regarding BMM, the first of which is embodied in the selection of the block size given that it amounts to a trade-off between variance and bias: large groups produce few block extremes with the consequent dispersion among their values, whereas small ones engender a poor approximation in terms of the Fisher, Tippett and Gnedenko (1928) theorem exposed in Appendix 3.C, thus reflecting bias in the estimation. Once the choice is carried out, the Block Maxima are assumed to be independent variables extracted from a GEV distribution with parameters to be computed. This independence

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102 In what follows, the extremes will be referred to as ‘maxima’ or simply ‘extremes’. Extreme Value Theory has been developed for maxima, hence the treatment for minima is equivalent to that of the maxima by multiplying all values times -1.
is endorsed by the fact that the original variables distributed as GEV are independent, although it is usually the case that the set of Block Maxima are also independent despite the dependence of the original variables.

The typical techniques employed to estimate the BMM parameters range from graphical methods and moment-based schemes to the most common Maximum Likelihood, which proves its versatility for complex situations. However, regardless of its advantages, Coles (2001) marks that ML does not fulfill the regularity conditions required by the Fisher, Tippett and Gnedenko (1928) theorem, hence conveying the inapplicability of the standard asymptotical likelihood outcomes

The majority of the projects using EVT for VaR estimation rely on the POT technique, consequently disregarding the BMM. Although Longin (2000) and McNeil (1998) offer interesting alternatives using the latter approach, Coles (2001) discourages its application on the grounds of a) the valuable information that may be lost when scores of observations are discarded at the time of setting the blocks; b) the uncertainty about the adequate length of the clusters, and c) the considerable extension of the series needed to construct a statistically significant sample –untenable for markets with short history like Emerging or Frontier ones. Moreover, Marinelli, D’Addona and Rachev (2007) provide evidence that POT outperforms BMM for VaR estimations.

2. The Peaks Over Threshold Approach
In stark contrast with the BMM, the POT appraisal labels as extreme an observation that exceeds a high threshold (hence its name “Peaks Over Threshold”), and the inference consists in fitting the Pareto distribution to the threshold exceedances. The current thesis favours POT on efficiency and pragmatic grounds.

Nevertheless, even accounting for the theoretical refinements, Christoffersen (2003) marks the absence of an appropriate objective method to optimally locate the beginning of the tail –i.e., the selection of the threshold in POT- as the weakest point in its entire construction. Several authors like Beirlant, Vynckier and Teugels (1996), Danielsson and de Vries (2000), Neftci (2001), Christoffersen (2003), Coronel-Brizio and Hernandez-Montoya (2005), Reiss and Thomas (2007) and Cifter (2011) among

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103Smith (1985) linked the results of the estimation process to the value of the tail index $\xi$, finding that whenever $\xi > -0.5$, the ML estimators hold their asymptotic properties.
others, have put forward their proposals to fill the void, although their efficiency and extension for different applications remain a matter of empirical evaluation\textsuperscript{104}.

As in the case of the choice of the block size for BMM, the selection of the appropriate threshold is the weak spot of the POT methodology (Christoffersen (2003)). Though there are several analytic ways to pick the threshold $u$, no theoretical approach with satisfactory performance has hitherto been deemed suitable. Hence, the practitioner must resort to a variety of tools in order to determine the starting point for the GPD to be fitted, always striving to strike a balance between variance and bias because:

- were the threshold to be too low, the asymptotic nature of the model would be violated with values of $\xi$ fluctuating considerably for different number of upper order statistics $k$, leading to bias;
- conversely, too high a threshold $u$ would render few excesses, with high variance.

According to Embrechts, Klüppelberg and Mikosch (1997), notwithstanding it may be theoretically feasible to choose $u$ asymptotically optimal employing a quantification of the bias-variance trade-off, the complicated structure of the data found in finance renders the task uphill. The authors go further to add that any analysis carried out has to be propped up by ‘exploratory data analysis techniques’ (Embrechts, Klüppelberg and Mikosch (1997:351)).

In this vein, the current study prioritises a technique based on the analysis of an array of elements such as the sample Mean Excess Function (MEF), Quantile-Quantile plots (QQ), Sample Kernel Density and Sample Quantile Function on the extremes of the series separately which, albeit not exempt from subjectivity\textsuperscript{105}, seems relatively widespread in the academic world. The method –applied by Rossignolo, Fethi and Shaban (2012a, 2012b) for similar purposes- consists of a mixture of theoretical considerations and eyeball assessment grounded on the linearity of the Mean Excess Function (MEF) which may be synthesised as follows.

In general, supposing a random variable $x$ is distributed according to a Generalised Pareto Distribution $F = G_{\xi,\sigma}$ such that:

\textsuperscript{104}These methods seem to work reasonably well but may eventually result in the selection of a very high number of upper order statistics.

\textsuperscript{105}McNeil and Frey (2000) highlight that EVT confers entity to the parametric estimation of both tails of the distribution separately based on a sound statistical theory, simultaneously allowing the extrapolation beyond the samples. Furthermore, Embrechts, Klüppelberg and Mikosch (1997) underline that EVT makes the best possible use of the scant data about extreme phenomena.
\[ G_{\xi, \beta}(x) = \begin{cases} 1 - \left(1 + \frac{\xi x}{\sigma}\right)^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - \exp\left(-\frac{x}{\sigma}\right), & \xi = 0 \end{cases} \] (3.86)

where \( \beta > 0 \), and \( x \geq 0 \) when \( \xi \geq 0 \) and \( 0 \leq x \leq -\sigma/\xi \) when \( \xi < 0 \). However, considering that the df of the excesses over a threshold \( u \) is given by:

\[ F_u(x) = P(X - u \leq x \mid X > u) = \frac{F(x + u) - F(u)}{1 - F(u)} \] (3.87)

the excess distribution function becomes:

\[ F_u(x) = G_{\xi, \sigma(u)}(x) \quad \text{with} \quad \sigma(u) = \sigma + \xi u \] (3.88)

where \( 0 \leq x \leq \infty \) if \( \xi \geq 0 \) and \( 0 \leq x \leq - (\sigma / \xi) - u \) if \( \xi < 0 \). It is relevant to state that the excess distribution function remains a GPD with the identical shape parameter \( \xi \) although the scaling factor \( \sigma \) grows linearly with the threshold \( u \).

Embrechts, Klüppelberg and Mikosch (1997) and McNeil, Frey and Embrechts (2005) write that, bearing in mind that the mean of the GPD is defined as:

\[ E(X) = \frac{\sigma}{1 - \xi}, \xi < 1 \] (3.89)

the mean excess function of a random variable with finite mean is given by:

\[ e(u) = E(X - u \mid X > u) \] (3.90)

and coupling this result with (3.88), the mean excess function becomes:

\[ e(u) = E(X - u \mid X > u) = \frac{\sigma(u)}{1 - \xi} = \frac{\sigma + \xi u}{1 - \xi} \quad \text{for} \quad \xi < 1 \] (3.91)

with \( 0 \leq u < \infty \) if \( 0 \leq \xi < 1 \) and \( 0 \leq u \leq -\sigma/\xi \) if \( \xi < 0 \), thus highlighting the linearity of the Mean Excess Function in the threshold \( u \), a property that characterises the GPD.

McNeil, Frey and Embrechts (2005) provide an interesting extension for the excess distribution over any higher threshold, besides that of the excess distribution over the threshold \( u \).

In effect, supposing that \( F \) is a loss distribution with right endpoint \( x_F \), and that for some high threshold \( u \), \( F_u(x) = G_{\xi, \sigma}(x) \) for \( 0 \leq x < x_F - u \) for some \( \xi \in \mathbb{R} \) and \( \sigma > 0 \), it follows that \( F_v(x) = G_{\xi, \sigma + \xi(v-u)}(x) \) for all \( v \geq u \), i.e., all thresholds higher than \( u \). Therefore, the excess distribution over higher thresholds still holds a GPD with the same shape parameter \( \xi \) but the scale parameter observes a linear relationship with the threshold \( v \). Under the condition \( \xi < 1 \), the mean excess function is written as:

\[ e(v) = \frac{\sigma + \xi (v-u)}{1 - \xi} = \frac{\xi v}{1 - \xi} + \frac{\sigma - \xi u}{1 - \xi} \] (3.92)
with \( u \leq v < \infty \) if \( 0 \leq \xi < 1 \) and \( u \leq v \leq u - \sigma t \xi \) if \( \xi < 0 \).

The graphical approach for choosing an appropriate threshold \( u \) hinges on the result conveyed by (3.92), i.e., the linearity of the Mean Excess Function in \( v \), and is employed as the tool to fix a GPD model for the distribution beyond the threshold. Embrechts, Klüppelberg and Mikosch (1997) remark that the Mean Excess Function of a given sample \( X_i \), with \( i = 1, 2, \ldots, n \) must be written as:

\[
e_n(u) = \frac{1}{N_u} \sum_{i \in \Delta_n(u)} (X_i - u), \quad u > 0 \tag{3.93}
\]

where \( N_u = \text{cardinal}\{i: i = 1, 2, \ldots, n, X_i > u\} = \text{cardinal} \Delta_n(u) \), which, rooting in the extension given by (3.92) derives in the Sample Mean Excess Function defined as the empirical estimator of the Mean Excess function in (3.90):

\[
e_n(v) = \frac{\sum_{i=1}^{n} (X_i - v) I_{[X_i > v]}}{\sum_{i=1}^{n} I_{[X_i > v]}} \tag{3.94}
\]

where the sample is of size \( n \) and \( I \) denotes the indicator function.

Even though the mechanics of the process appear quite simple, the final determination of the threshold \( u \) involves some subjective elements. The crux of the matter, then, boils down to choosing a threshold \( u \) such that \( e_n(v) \) is approximately linear for \( v \geq u \), for which the mean excess plot \( \{[X_{i,n}, e_n(X_{i,n})] : 2 \leq i \leq n\} \) with \( X_i \) referring to the \( i \)th high order statistic. Were the data adequate for a GPD beyond the threshold \( u \), formula (3.94) ought to exhibit an approximate linear shape, although the interpretation of the term ‘approximate’ seems of a somewhat contentious nature because, in the end, it is reduced to a matter of practice as:

- a linear positive trend indicates a GPD with \( \xi > 0 \);
- a linear negative trend points to a GPD with \( \xi < 0 \) whereas
- a driftless graph is a sign of an exponential distribution with \( \xi = 0 \).

The process of selecting an appropriate threshold can prove tricky at times, and the situations above described are seldom crystal clear, thus fostering the need of considerable empirical work. As McNeil, Frey and Embrechts (2005) remark, the Sample Mean Excess plot is never perfectly linear, particularly towards the right-hand end\(^\text{107} \) as a small number of large excesses are lodged there; in that case, those points are usually excluded from the analysis on the grounds of the distortion they may cause. Consequently, if the researcher finds evidence of the linearity of the mean excess graph

\(^{107}\) It is important to bear in mind that the theoretical development for the theory of extremes is carried out for the maxima, and the equivalent for the minima is found by finding the opposite multiplying every number times -1.
in an entourage of the selected threshold \( u \), the appropriate choice should situate at the beginning of that linear section.

Reiss and Thomas (2007:145) mention that the goodness-of-fit of the fitted GPD is usually assessed in a “…qualitative and subjective manner…” with the aid of several graphical devices like QQ plots, Sample Kernel Density and Sample Quantile Function. Coles (2001) adds that the final ad-hoc test to ensure the appropriate selection of the threshold is the analysis of the stability of the parameters within a range of the chosen statistic \( u \) and, ideally, the estimates should conform the stability requirements and vice versa (Rossignolo, Fethi and Shaban (2012a)). Embrechts, Klüppelberg and Mikosch (1997) emphasise the possibility of finding more than one likely threshold \( u \), for which there are seemingly no antidotes: spotting the right starting point to the GDP ought to be complemented with aiding graphs, judgement and common sense\(^{108}\).

*Parameter estimation*

Reiss and Thomas (2007) list a number of schemes to estimate the shape and scale parameters belonging to the affixed GPD. In broad terms, their enunciation encompasses: Maximum Likelihood (ML), Moment (MM), Drees-Pickands (DP), Slope (S) and L-Moment (LM) appraisals, all of which exhibit corresponding advantages and disadvantages. According to the authors, the first two reveal themselves as more precise and subject to a lesser number of constraints in their application, whereas the last three need special restrictions of the data or in the estimation outcome as some final values of \( \xi \) may provoke the disappearance of the asymptotic properties. For instance, they underline that:

- whenever \( \xi > -1/2 \), ML and MM prove the most accurate techniques and, although DP shows important asymptotic properties, the outcome may not look satisfactory when \( \xi > 0 \);
- LM prove very accurate when the sample size is small and vice versa\(^{109}\);
- the S estimator cannot be employed if \( \xi < 1 \).

Consequently, the necessary preconditions to utilise DP, S and LM convert them into somewhat unreliable techniques or, at least, into estimators in need of further development. Hence, the practitioners resort to ML or MM to ascertain the shape and scale parameters with almost similar results, although the choice appears somewhat

\(^{108}\) Appendix 3.D.

\(^{109}\) The authors stop short of precising the concept of ‘small’.
tilted to the former. McNeil, Frey and Embrechts (2005) asseverate that the Maximum Likelihood scheme offers a more versatile platform for the fitting of a wider range of models encompassing, among others, time dependence (including the affixing of the extremal index (Coles (2001)) or the introduction of explanatory variables. The interested reader may recur to Embrechts, Klüppelberg and Mikosch (1997) and Smith (1987) for a detailed account of its particulars.

However, given that every of the above cited exceptions lie beyond the scope of the current investigation, the adoption of the Method of Moments seems an interesting exercise to test an alternative technique to the most widespread one. In effect, Reiss and Thomas (2007) highlight that the MM has an excellent performance unless it is applied to an extensive set of exceedances (500 or more, Hosking and Wallis (1987)), in which case it gives irregular results. As that, again, is not the context in which the thesis takes place, the employment of this procedure appears further endorsed. Nevertheless, all the estimations in the present work have also been performed using the classical ML procedure with a similar outcome\textsuperscript{110}.

\textit{The Method of Moments}

The Method of Moments (MM) was first engineered by Dekkers, Einmahl and de Haan (1989) when they transformed the widespread Hill estimator for the index of a distribution function with steadily changing tail into an index of an Extreme Value distribution; furthermore, they extended their demonstration to prove the consistency and asymptotic normality of the estimators. Given that the technical developments of the paper lie outside the scope of the current thesis, the methodology may well be summarised following Hosking and Wallis (1987:340-341).

In effect, the two-parameter Generalised Pareto Distribution ($\sigma$ scale, $\xi$ shape) has distribution and density functions given by\textsuperscript{111}:

\begin{equation}
H(y) = \begin{cases} 
1 - (1 - \frac{\xi y}{\sigma})^{1/\xi} & \text{if } \xi \neq 0 \\
1 - \exp(-y/\sigma) & \text{if } \xi = 0
\end{cases}
\end{equation}

and

\begin{equation}
h(y) = \sigma^{-1} (1 - \frac{\xi y}{\sigma})^{(1/\xi - 1)} \quad \text{if } \xi \neq 0
\end{equation}

\textsuperscript{110}For space considerations these results are not included in the study but remain available on request.

\textsuperscript{111}The distribution function $H(y)$ in the case of $\xi \neq 0$ is tantamount to expression (3.41) in Section 3.6.1: $H(y) = 1 - (1 + \frac{\xi y}{\sigma})^{-1/\xi}$ given that Hosking and Wallis (1987) work with $-\xi$ instead of $\xi$. Analogously, the density function $h(y)$ in the case of $\xi \neq 0$ is similar to equation (3.42) in Section 3.6.1: $h(y) = \sigma^{-1} (1 + \frac{\xi y}{\sigma})^{-(1+1/\xi)}$
\[ h(y) = \sigma^{-1} \exp(-y/\sigma) \quad \text{if } \xi = 0 \quad (3.96) \]

where \( 0 \leq y < \infty \) for \( \xi \leq 0 \) and \( 0 \leq y \leq \sigma/\xi \) if \( \xi < 0 \). The authors stress that \( \xi = 0 \) yields the exponential distribution, \( \xi = 1 \) the uniform distribution whereas Pareto distributions appear in the case when \( \xi < 0 \).

The moments of the GDP are obtained setting
\[ E(1 - \xi y/\sigma)^r = 1/(1 + r\xi) \quad (3.97) \]

where \( 1 + r\xi > 0 \), and letting the \( r \)th moment of \( Y \) exist if \( \xi > -1/r \). Therefore, considering the moments exist and are finite, the mean, variance, asymmetry and kurtosis are calculated through the ensuing expressions:

\[
\text{Mean} = \frac{\sigma}{1 + \xi} \quad (3.98)
\]
\[
\text{Variance} = \frac{\sigma^2}{[(1 + \xi)^2(1 + 2\xi)]} \quad (3.99)
\]
\[
\text{Skewness} = 2 \left(1 - \xi\right) \left(1 + 2\xi\right)^{\frac{1}{2}} / (1 + 3\xi) \quad (3.100)
\]
\[
\text{Kurtosis} = \frac{3(1+2\xi)(3-\xi+2\xi^2)(1+3\xi)}{(1+3\xi)(1+4\xi)} - 3 \quad (3.101)
\]

The moment estimators of the scale parameter \( \sigma \) and the shape parameter \( \xi \) become:
\[
\hat{\sigma} = \frac{1}{2} \bar{y} \left( \frac{\bar{y}^2}{s^2} + 1 \right) \quad (3.102)
\]
\[
\hat{\xi} = \frac{1}{2} \left( \frac{\bar{y}^2}{s^2} - 1 \right) \quad (3.103)
\]

with \( \bar{y} \) and \( s^2 \) representing the sample mean and sample variance respectively. Moreover, were \( \xi \geq -0.25 \), \( \hat{\sigma} \) and \( \hat{\xi} \) will be asymptotically normally distributed with variance:
\[
\text{Var}(\hat{\sigma}) = n^{-1} \frac{(1+\xi)^2}{(1+2\xi)(1+3\xi)(1+4\xi)} \cdot 2\sigma^2 (1 + 6\xi + 12\xi^2) \quad (3.104)
\]
\[
\text{Var}(\hat{\xi}) = n^{-1} \frac{(1+\xi)^2}{(1+2\xi)(1+3\xi)(1+4\xi)} \cdot (1 + 2\xi)^2 (1 + \xi + 6\xi^2) \quad (3.105)
\]

while the covariance between \( \hat{\sigma} \) and \( \hat{\xi} \) is defined as:
\[
\text{Cov}(\hat{\sigma}, \hat{\xi}) = n^{-1} \frac{(1+\xi)^2}{(1+2\xi)(1+3\xi)(1+4\xi)} \cdot \sigma(1 + 2\xi)(1 + 4\xi + 12\xi^2) \quad (3.106)
\]

Finally, Hosking and Wallis (1987) carry out an interesting comparison among the Maximum Likelihood, the Method of Moments and a derivation of MM, i.e., the Probability Weighted Method of Moments (PWMM), in order to find the most reliable technique to estimate the parameters of a GPD. However, they underline that the selection very much depends on the problem at hand and, moreover, on the researcher’s insight about the likely range of values that \( \xi \) might assume\textsuperscript{112}. In this sense, they

\textsuperscript{112}It should be recalled that, in Hosking and Wallis’s setting (1987), GPD appears when \( \xi < 0 \).
highlight that ML should be preferred with large samples and $\xi > 0.2$, whilst if $\xi$ is situated in an entourage of 0, or takes ‘small’ negative values, MM delivers the best overall performance. If, by any chance, it would be feasible to encounter large negative $\xi$s, PMM could be singled out. Their results, then, foster the adoption of MM.

3.8. Concluding remarks

The subject of the techniques to model risk has proven a controversial one for many years. In fact, there is no universal recipe to mimic market movements given the utter dependence on the context and the assumptions considered.

Despite the aforementioned vagueness, there is a growing consensus among academics about the empirical behaviour of the time series of financial returns; since the pioneering works of Mandelbrot (1963), Fama (1965) and Fama and French (1988), some certainties have become well-known facts. The departure from normality, which shook the established knowledge, has been followed by the conditionality on time and the persistence effects, thus giving rise to nonlinear generating stochastic processes (Engle (1982) and Bollerslev (1986)). Another salient feature, namely the existence of leptokurtic distributions, also plays an important part in financial modelling (e.g., McNeil, Frey and Embrechts (2005), Embrechts, Klüppelberg and Mikosch (1997)) and is therefore not to be neglected in view of its importance in explaining the success of any risk model.

The basic problem of every model employed to estimate any risk measure is represented by the fact that they deal with a latent, unobservable concept like volatility (Andersen et al. (2003)). Therefore, in order to bridge the difficulty, scientists turned to the notion of Realised volatility (Andersen et al. (2003)) or Actual volatility (Barndorff-Nielsen and Shephard (2002, 2004)), which boils down to the estimation of the variation of the quadratic component of a stochastic process represented by the squared returns.

The concept of VaR is ultimately reduced to the quantification of that elusive concept termed volatility. And although there is agreement as to what to measure, Beder (1995) warned against the high variability in VaR results arising from the diversity in the assumptions required by every model. In this sense, the industry has been very generous at the time of developing techniques for VaR: Historical Simulation, Filtered Historical Simulation, Conditional Volatility Models with several distributions attached, Extreme Value Theory and Standard Deviation, also called linear model, are some of the examples of the ever-growing quantity of schemes.
Historical Simulation constitutes the simplest route to VaR (Christoffersen (2003)), but its over-reliance on the past history of returns and the undefined sample length conspire against its application (Finger (2006)). Filtered Historical Simulation, pioneered by Hull and White (1998) and Boudoukh, Richardson and Whitelaw (1998), represents an evolution to HS and solves many of the shortcomings of its predecessor (e.g., the capture of the heteroscedasticity and volatility clustering effects) through the use of volatility weighting schemes, yet apparently understates the true risk of the positions particularly in crisis times (Pritsker (2001)). Conditional Volatility models root in two dynamics: the volatility, grasped by the model at hand and the distribution that eventually delivers the VaR quantile. Designed to respond to the demands of the empirical time series (conditionality on time, volatility clustering and mean-reversion) and armed with the flexibility to adopt almost any kind of distribution, the two more common specifications, GARCH (Bollerslev (1986)) and EGARCH (Nelson (1991)) have proven immensely popular among practitioners (particularly the former). There are, however, some points that need to be addressed: referring to GARCH, Brorsen and Yang (1989) mention the imprecision in the lag structure and Angelidis, Benos and Degiannakis (2004) pour concern on the convergence rate, while the EGARCH skewed response to volatility shocks prompts Alexander (2008b) to express doubts about its ability to handle abrupt market movements.

Extreme Value Theory is a relatively new concept applied to market risk modelling. Riding on the back of a sound theory (Balkema & de Haan (1974), Pickands (1975), Fisher and Tippet (1928) and Gnedenko (1948)), it is apparently suited to deal with Taleb’s black swans (2007) because it focuses precisely on the farthest values of the distribution of the assets. It combines the advantages of the conditional models (e.g. GARCH) with the leptokurtosis of a Pareto type II density function, thus presenting potentially interesting characteristics for crunch times. Two approaches have hitherto been developed to carry out the application of EVT: BMM and POT and, in view of its optimised use of the data at hand, the latter will be employed throughout the present thesis but, notwithstanding its alleged prowess, the Achilles’ heel of POT is, undoubtedly, the great degree of subjectivity in the selection of the threshold from which the GPD starts. In order to obtain the parameters of the GPD, the guidelines in Hosking and Wallis (1987) shall be followed and, in this sense, the moderately small size of the samples and the values of $\xi$ less than 0.2 support the selection of the Method of Moments.
The final scheme tried in the current thesis is represented by the traditional Standard Deviation. The simplicity of its implementation seems overshadowed by the normality assumption underpinning it and the equal-weighting scheme of the observations, all of which blatantly ignore the adaptive learning environment coined by Bollerslev (1986) and the empirical characteristics of the time series.

Even though the aforementioned considerations convey an idea about the likely behaviour of the models, the ultimate success or failure of all of them to deliver precise VaR estimates able to withstand turmoil is down to the corresponding field tests. Having presented the representations to be employed, the next chapter will shed light on their performance in the event of market turbulence.
Chapter 4

VaR and MCR in Emerging and Frontier Markets
4.1. Introduction

Emerging countries have become an interesting subject to study. Since its inception in the 1980s, the word has been applied to those nations offering attractive investment opportunities alongside market-oriented traces. Their markets also exhibit many of the developed markets characteristics, although some requirements related to liquidity, transparency of regulation and information requirements are somewhat limited. However, throughout the last decade those countries have structured its macroeconomic foundations in such a way to be regarded as strong contenders for developed nations in terms of investment inflows and growth potential.

Given that scenario, it is no surprise that these countries have weathered the effects of the crisis in a much better fashion that its counterparts, thus avoiding many of its unpleasant consequences. Emerging countries may have learned from former crises (the Mexican crash in 1994 or the Russian devaluation in 1998, for instance) to carry out some structural economic and banking reforms, which -blended with their characteristic abundance of commodities-, transformed then into powerful actors in the world economic –and political- concert. Frontier markets, on the other hand, might be regarded as Emerging countries younger sister in the sense that they do not possess some of the patterns embodied in the latter (more opacity in the information disclosure, less deep markets and more government interference with market forces113). Needless to say, they lack many of the attributes that make the Emerging category appealing, but nonetheless they pose an intriguing attraction motivated, precisely, in their somewhat obscure and tightly and discretionary regulated context.

The epicentre of the subprime crisis situated in the developed world, and from a supranational regulatory point of view, almost the entirety of Impact Studies and Stress Exercises carried out by BCBS are referred to European mature markets (BCBS (2012b, 2012c, 2013)), thus leaving behind a fertile field. The current thesis inquires about VaR models in crisis times in Emerging and Frontier stock markets in an effort to gauge the impact that the adoption of different VaR techniques would exert on Basel II and Basel III MCR in those countries.

The framework contained in Basel II Capital Accord has established Value-at-Risk as the official measure of market risk and enforced it to constitute the central point to the determination of capital charges. Moreover, as the BCBS has not hitherto

113 Appendix 1.B.
recommended a particular VaR methodology, the adoption of the most appropriate VaR appraisal becomes a matter of the utmost importance to be decided purely on empirical grounds. However, the magnitude of the plight prompted the BCBS to put forward a proposal to increase the Minimum Capital Requirements for market risks in accordance with the opinion of national regulators. The intended scheme plans the introduction of a stressed VaR which ought to be added to the current base VaR (cVaR) in order to constitute the new MCR in an attempt to curb the procyclicality of the measure in force.

The aforementioned context highlights the significance of developing a precise VaR model to cover market losses and simultaneously build a capital buffer high enough as to allow institutions to distribute dividends in light of a further BCBS directive which restricts the dividend payout unless the capital level exceeds MCR by a quantity called Capital Conservation Buffer equivalent to 2.5% of the amount of the risk weighted assets. Besides the traditional reluctance on the part of the academics to study Emerging and Frontier markets, BCBS’s Consultative Documents have mostly been submitted to developed nations and it is unlikely that these proposals should be evaluated and its impact assessed on the spheres of Emerging and Frontier markets. This study aims at filling that empirical void, as the accuracy of several VaR specifications is analysed and its effect on capital charges gauged from the perspective of non-developed stock markets under Basel II and Basel III regulations, with special emphasis on the relationship between the SA and the various VaR-based IMA, in terms of precision in periods of violent market turmoil, sufficiency of MCR and incentives to their corresponding implementation.

In view of the aforementioned objectives, Section two supplies notions about the patterns that identify Emerging and Frontier markets while the ensuing one assesses and verifies the statistical properties of the series researched, in addition to calculating the parameters corresponding to the models explained in the previous Chapter. Section four depicts the resulting VaR values and the following one tests them under the framework devised in the successive Basel Capital Accords in the event of significant turmoil. Section six determines the Minimum Capital Requirements for every model in the referred stock markets applying Basel II and Basel III configurations, thus paving the way for a more detailed scrutiny of VaR-based Capital Requirements employing

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114 “Capital required against trading activities should be increased significantly (e.g., several times)” (Financial Services Authority (2009:7)).
Extreme Value Theory, often suggested for those circumstances. Finally, Section eight wraps up the entire Chapter.

4.2. Characterisation of Emerging and Frontier Markets

The term ‘emerging markets’ was first coined by A. Van Agtmael in the 1980s to denote those economies in the process of being reconstructed along market-oriented traces. Thus, great business opportunities arise from their vast resources to the extent of becoming the world’s fastest growing markets. Since then they have received a range of definitions, some of which are portrayed by Mody (2004); however, in most occasions, the phrase is employed as a synonym for ‘emerging economies’, ‘emerging countries’ or its replacement by ‘developing’ (markets, economies or countries) indistinctly.

Mody (2004) also emphasises that, despite the wide variety of countries encompassed under the aforementioned umbrella, a series of distinctive elements common to all of them is worth of being pointed out:

a) appreciable growth prospects;

b) high rates of returns on investments;

c) high level of risk, usually represented by important volatility values when compared with their developed counterparts;

d) lack of foreign investment records;

e) transitional character, reflected in the dynamic passage to more open market lines with institutional and political stability.

of which the author singles out c) and e) as the most typical patterns. However, the categorisation is not conclusive when macroeconomic parameters are considered, and in some cases proves confusing. For instance, Taiwan and United Arab Emirates are regarded as Emerging markets even though those countries enjoy a bigger Gross Domestic Product (GDP) per capita than some Developed markets like Spain, Italy or even New Zealand (Chart 4.1). Furthermore, the range of GDP per capita amounts between two different Emerging countries may be capable of reaching astonishing dimensions, as in the case of Czech Republic (24,271) and Côte d’Ivoire (1,672), both measured in 2009 USD.

Setting aside the puzzling nature of the above statistics, it is nevertheless a fact that Emerging markets considered as a whole are erecting themselves as major players in world economic affairs, and its importance should not be neglected. A quick glance at
some macroeconomic variables pictured in the following tables will help to bolster the assertion:

**Chart 4.1**

*Gross Domestic Product per capita*

<table>
<thead>
<tr>
<th>Country</th>
<th>GDP per capita</th>
</tr>
</thead>
<tbody>
<tr>
<td>United Arab Emirates</td>
<td>36,843</td>
</tr>
<tr>
<td>Taiwan</td>
<td>31,776</td>
</tr>
<tr>
<td>Spain</td>
<td>29,625</td>
</tr>
<tr>
<td>Italy</td>
<td>29,068</td>
</tr>
<tr>
<td>New Zealand</td>
<td>26,670</td>
</tr>
</tbody>
</table>


*Notes:* (1): GDP at Purchasing Power Parity (PPP) in USD terms (2009 values).
(2): Emerging economies above the black line; developed markets below.

a) The annual percent increase in combined GDP for Emerging economies largely surpassed those of the world and developed countries in 2000 to 2009 and even in 2010 it was currently posting a rise:

**Chart 4.2**

*Annual increase in combined GDP 2005-2009 - Emerging and Advanced economies*

<table>
<thead>
<tr>
<th>Year</th>
<th>Emerging economies</th>
<th>Advanced economies</th>
<th>World</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>5.97%</td>
<td>4.16%</td>
<td>4.83%</td>
</tr>
<tr>
<td>2001</td>
<td>3.77%</td>
<td>1.39%</td>
<td>2.28%</td>
</tr>
<tr>
<td>2002</td>
<td>4.78%</td>
<td>1.72%</td>
<td>2.89%</td>
</tr>
<tr>
<td>2003</td>
<td>6.24%</td>
<td>1.92%</td>
<td>3.61%</td>
</tr>
<tr>
<td>2004</td>
<td>7.51%</td>
<td>3.18%</td>
<td>4.92%</td>
</tr>
<tr>
<td>2005</td>
<td>7.09%</td>
<td>2.65%</td>
<td>4.48%</td>
</tr>
<tr>
<td>2006</td>
<td>7.92%</td>
<td>2.99%</td>
<td>5.08%</td>
</tr>
<tr>
<td>2007</td>
<td>8.32%</td>
<td>2.76%</td>
<td>5.18%</td>
</tr>
<tr>
<td>2008</td>
<td>6.14%</td>
<td>0.00%</td>
<td>3.02%</td>
</tr>
<tr>
<td>2009</td>
<td>2.39%</td>
<td>-3.16%</td>
<td>-0.60%</td>
</tr>
<tr>
<td>2010 (1)</td>
<td>6.35%</td>
<td>2.33%</td>
<td>4.22%</td>
</tr>
</tbody>
</table>

*Note:* (1) Provisional figures released in April 2010.


b) The contribution of Emerging and Developing markets to the world total GDP\(^\text{115}\) has been augmenting steadily from 37.01% in 2000 to 46.20% in 2009 as opposed to 62.99% and 53.80% from Developed countries. The IMF estimated that in 2013 the participation of the former will overcome the latter and continue afterwards\(^\text{116}\) (Chart 4.3 and Graph 4.1);

\(^{115}\) GDP based on Purchasing Power Parity (PPP).

Chart 4.3

GDP based on PPP - Share of World Total

<table>
<thead>
<tr>
<th>Year</th>
<th>Developed economies</th>
<th>Emerging economies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>62.99%</td>
<td>37.01%</td>
</tr>
<tr>
<td>2001</td>
<td>62.47%</td>
<td>37.53%</td>
</tr>
<tr>
<td>2002</td>
<td>61.79%</td>
<td>38.21%</td>
</tr>
<tr>
<td>2003</td>
<td>60.83%</td>
<td>39.17%</td>
</tr>
<tr>
<td>2004</td>
<td>59.75%</td>
<td>40.25%</td>
</tr>
<tr>
<td>2005</td>
<td>58.83%</td>
<td>41.17%</td>
</tr>
<tr>
<td>2006</td>
<td>57.71%</td>
<td>42.29%</td>
</tr>
<tr>
<td>2007</td>
<td>56.44%</td>
<td>43.56%</td>
</tr>
<tr>
<td>2008</td>
<td>55.10%</td>
<td>44.90%</td>
</tr>
<tr>
<td>2009</td>
<td>53.80%</td>
<td>46.20%</td>
</tr>
<tr>
<td>2010</td>
<td>52.86%</td>
<td>47.14%</td>
</tr>
<tr>
<td>2011</td>
<td>51.90%</td>
<td>48.10%</td>
</tr>
<tr>
<td>2012</td>
<td>50.92%</td>
<td>49.08%</td>
</tr>
<tr>
<td>2013</td>
<td>49.93%</td>
<td>50.07%</td>
</tr>
<tr>
<td>2014</td>
<td>48.90%</td>
<td>51.10%</td>
</tr>
</tbody>
</table>

Note: Projected values below the line.

Graph 4.1

GDP based on Purchasing Power Parity

Note: straight lines denote definitive values; dashed ones projected.

c) the average annual growth in the twenty year period 1990-2009 of the Gross Domestic Product in developing countries doubled that of the mature ones, i.e., 4.8% against 2.4%, with the corresponding per capita gap widening: 3.3% and 1.7%\(^{117}\);

d) the size of the economy, quantified by Gross National Income measured at Purchasing Power Parity in USD billions, has augmented from 8,059 in 1990 to

\(^{117}\) Source: The World Bank (2010).
31,607 in 2009 (292%) for Emerging markets contrasted with 139% for high income countries (17,018 in 1990 and 40,724 in 2009)\textsuperscript{118};
e) the Current Account Balance increased from 7.2 in 2001 to 256.4 in 2008, as measured in USD billions;
f) the Foreign direct investment (in USD millions) in Developing countries raised to 358,605 in 2009 from 21,782 in 1990 (1,546.34%) whereas in mature markets the figures delivered 297.28%, a product of 757,664 and 358,605;
g) the Net Equity Inflows\textsuperscript{119} to Emerging economies have grown an outstanding 288.41% in the 2001-2008 period\textsuperscript{120}.

The above examples may in part support the significant growth prospects as well as convey some ideas about the transitional character mentioned by Mody (2004), but tell little of the inner fabric of these markets. In this sense, the services provided by specialised agencies like MSCI, S&P and FTSE\textsuperscript{121}, among others, greatly contribute to clarify the issue by establishing a series of criteria which ought to be complied with: the quantity of parameters fulfilled eventually determines the category to which every country belongs to. It is important, then, to bear in mind the close connection between countries and markets given that agencies deal with specific market characteristics referred to the way they work; market participants accordingly extrapolate the categorisation to the country itself, thus making both coincide and ultimately become synonyms.

The most complete set of parameters is outlined by FTSE (Appendix 1.B), which assesses the markets according to twenty one criteria grouped in four different clusters: Market and Regulatory Environment, Custody and Settlement, Dealing Landscape and Derivatives. Developed markets require all the 21 criteria to be met; Emerging are separated into Advanced Emerging and Secondary Emerging considering they accomplish 14 or 8 specified criteria respectively, and finally the Frontier category demands 5 items to be satisfied. Even though MSCI and S&P do not produce their own classification with the same degree of detail, the allocation of the countries basically coincides, rendering the choice of the agency a matter of preferences (Appendix 1.B).

\textsuperscript{118} Source: The World Bank (2010).
\textsuperscript{119} Net Equity Inflows includes Foreign Direct and Portfolio Investments.
\textsuperscript{120} Global Development Finance 2010: External Debt of Developing Countries. (The World Bank (2010)).
\textsuperscript{121} MSCI, S&P and FTSE stand for former Morgan Stanley Capital International, Standard & Poor’s and Financial Times Stock Exchange respectively.
Even though supra-national organisations like the IMF only draw two classes of economies (Advanced and Emerging-Developing) and include all non-developed markets in the latter group, investors usually follow the criteria exposed by FTSE, MSCI and S&P at the time of making decisions, not to mention the analogous situation in press reports. For the reasons above mentioned, and even considering that some authors blur the distinction between Emerging and Frontier economies, the current thesis will allocate countries into Developed, Emerging and Frontier classes, specifically working only with the last two to carry out the intended purpose.\(^{122}\)

4.2.1. Special patterns about Emerging and Frontier markets

Having pictured the economic characteristics about Emerging markets in very broad lines, this section focuses on more precise financial matters for which the analysis is centred on performance and risk defined in the Markowitz’s framework\(^{123}\), i.e., the Mean-Variance construction characterised by expected returns and volatility indicated by Standard Deviation. Even though the Standard Deviation has been criticised as a measure of risk\(^{124}\), it is still widely used in practice by researchers, vendors and practitioners, thus providing motivation to employ this element in the present section.

For the upcoming analysis the indices belong to MSCI Barra (hereafter MSCI\(^{125}\)), selected on the grounds of free availability to the general public and widespread usage (Calverley, Hewin and Grice (2000)). Accordingly, the below mentioned indicators comprising the respective countries will be employed to represent the behaviour of Emerging, Frontier, Developed and World stock markets (as of May 2010):

a) **MSCI Emerging Markets Index**: free-float adjusted market capitalisation weighted index designed to measure equity performance of Emerging markets;

b) **MSCI Frontier Markets Index**: free-float adjusted market capitalisation weighted index designed to measure equity performance of Frontier markets;

---

\(^{122}\) The specialised agencies created the Frontier markets category in response to investors’ claims demanding a framework to diversify their positions, given the increasing correlation between Emerging and Developed markets indices. The idea contemplated the creation of a brand new structure capable of exhibiting low correlation levels with indices belonging to mature and developing countries in order to exhibit more balanced portfolios. See [http://www.indexuniverse.com/sections/features/3170-msci-considers-a-new-frontier.html](http://www.indexuniverse.com/sections/features/3170-msci-considers-a-new-frontier.html).

\(^{123}\) Markowitz (1952).

\(^{124}\) Sections 2.2 to 2.5.

\(^{125}\) MSCI is formerly known as Morgan Stanley Capital International. For details about the statistical methodology of the indices, please refer to [http://www.mscibarra.com/products/indices](http://www.mscibarra.com/products/indices)

\(^{126}\) MSCI indices stress industry representation, whereas S&P calculates indices prioritising market capitalisation.
c) **MSCI World Index**: free-float adjusted market capitalisation index designed to measure the equity performance of Developed markets;

d) **MSCI ACWI (All Country World Index)**: free-float adjusted market capitalisation weighted index designed to measure the equity market performance of Developed and Emerging markets. Even though Frontier markets are excluded from this categorisation, it will be employed as a proxy of a global portfolio given that most of the countries are comprised here.

To begin with, markets will be characterised in terms of the individual components: returns and risk, evaluated employing monthly arithmetic returns and its Standard Deviation -in the same fashion as Barry, Peavy III and Rodriguez (1998)- on the relevant indices in the period May/02-Dec/07\(^{127}\), considering May/02 as the Base 100.

### Chart 4.4

**MSCI indices – Markets and components**

<table>
<thead>
<tr>
<th>Index</th>
<th>Components</th>
</tr>
</thead>
</table>
| EM – Emerging Markets  
(May 2010) | Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey. |
| FM – Frontier Markets  
(May 2010) | Argentina, Bahrain, Bulgaria, Croatia, Estonia, Jordan, Kenya, Kuwait, Lebanon, Lithuania, Kazakhstan, Mauritius, Nigeria, Oman, Pakistan, Qatar, Romania, Serbia, Slovenia, Sri Lanka, Tunisia, Trinidad & Tobago, Ukraine, United Arab Emirates, Vietnam. |
| WI – The World Index  
- Developed Markets  
(May 2010) | Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Israel, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States. |
| AC – All Country World Index  
(May 2010) | WI + EM = All developed and emerging countries Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Israel, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, United Kingdom, United States, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey. |

**a) Returns**

The overall picture shows a very marked superiority on the part of Emerging and Frontier markets with regards to Developed ones, then coinciding with Barry, Peavy III

---

\(^{127}\) May/02 was picked as the starting point given the fact that MSCI began compiling the MSCI-FM index on that very date. On the other hand, 31/Dec/2007 corresponds to the last day before the Backtesting period begins in the present study. Section 4.5.
and Rodriguez (1998), Calverley, Hewin and Grice (2000), Aggarwal, Inclan and Leal (1999) and Derrabi and Leseure (2005), just to name a few studies. For comparison and illustrative purposes only, some relevant stock exchanges belonging to each category were selected in order to assess the extent of the performance. Hence, for every group:

**Chart 4.5**

*Selected indices*\(^{128}\)

<table>
<thead>
<tr>
<th>Category</th>
<th>Selected components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emerging markets</td>
<td>Bovespa (Brazil), Shanghai Composite (China), Sensex (India), RTS (Russia)</td>
</tr>
<tr>
<td>Frontier markets</td>
<td>Merval (Argentina), Crobex (Croatia)</td>
</tr>
<tr>
<td>Developed markets</td>
<td>DJIA (USA), S&amp;P500 (USA), Nasdaq (USA), FTSE (UK), Nikkei (Japan)</td>
</tr>
</tbody>
</table>

In this sense, the broad image clearly exhibits that the total return of Emerging and Frontier markets far exceeds those of Developed and Global portfolio, indicated by MSCI-EM, MSCI-FM, MSCI-WI and MSCI-AC respectively. The situation, depicted in Chart 4.6 and Graph 4.2 bolsters the presumption of the extraordinary gains obtained by several stock exchanges, which acted as major drivers for the corresponding group: for instance, India (549%) and Russia (485%) led the way in Emerging markets while Argentina (578%) accomplished analogous task for Frontier ones.

**Chart 4.6**

*The performance of selected and MSCI indices*  
*Total returns May/02-Dec/07 (May/02 = Base 100)*

<table>
<thead>
<tr>
<th>Index</th>
<th>Total return</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJIA</td>
<td>33.65%</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>37.60%</td>
</tr>
<tr>
<td>Nasdaq</td>
<td>72.54%</td>
</tr>
<tr>
<td>FTSE</td>
<td>26.98%</td>
</tr>
<tr>
<td>Nikkei</td>
<td>30.13%</td>
</tr>
<tr>
<td>MSCI –WI</td>
<td>64.16%</td>
</tr>
<tr>
<td>Bovespa</td>
<td>396.74%</td>
</tr>
<tr>
<td>Shanghai Composite</td>
<td>247.13%</td>
</tr>
<tr>
<td>Sensex</td>
<td>549.03%</td>
</tr>
<tr>
<td>RTS</td>
<td>485.42%</td>
</tr>
<tr>
<td>MSCI-EM</td>
<td>259.71%</td>
</tr>
<tr>
<td>Merval</td>
<td>577.60%</td>
</tr>
<tr>
<td>Crobex</td>
<td>327.22%</td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>303.80%</td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>73.20%</td>
</tr>
</tbody>
</table>

**Note:** MSCI indices representing Emerging, Frontier, Developed and World markets in bold letters.  
**Source:** Own elaboration with data obtained from the stock exchanges, Reuters and MSCI websites.

\(^{128}\) Indices belonging to the first category correspond to the BRIC countries (Brazil, Russia, China and India); Argentina and Croatia are two of the most significant Frontier markets, the former being demoted from the Emerging markets category in September 2009 (FTSE) and the latter symbolises one of the emblematic previous Soviet States. Last, DJIA, S&P500, Nasdaq, FTSE and Nikkei embody the most influential stock indices of the world.
A closer analysis of MSCI indices depicted in Charts 4.7 and 4.8 and Graph 4.3 and 4.4 reflects that the trajectory of Emerging and Frontier markets in that term is not exempt from severe swings, thus behaving in a much more unsteady fashion than their counterparts MSCI-WI and MSCI-AC and hinting at their inherent instability (Aggarwal, Inclan and Leal (1999)).

Graph 4.2
Returns May/02-Dec/07
(May/02 = Base 100)

Chart 4.7
Index performance: Annual returns

<table>
<thead>
<tr>
<th>Index</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-EM</td>
<td>-23.47%</td>
<td>51.59%</td>
<td>22.45%</td>
<td>30.31%</td>
<td>29.18%</td>
<td>36.48%</td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>15.96%</td>
<td>42.34%</td>
<td>20.90%</td>
<td>71.63%</td>
<td>-10.81%</td>
<td>38.55%</td>
</tr>
<tr>
<td>MSCI-WI</td>
<td>-27.22%</td>
<td>30.81%</td>
<td>12.84%</td>
<td>7.56%</td>
<td>17.95%</td>
<td>7.09%</td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>-27.07%</td>
<td>31.62%</td>
<td>13.30%</td>
<td>8.83%</td>
<td>18.78%</td>
<td>9.64%</td>
</tr>
</tbody>
</table>

Note: Figures corresponding to 2002 are annualised as the period starts in May/02.

Chart 4.8
Index performance: Cumulated annual returns

<table>
<thead>
<tr>
<th>Index</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-EM</td>
<td>-23.47%</td>
<td>27.87%</td>
<td>56.57%</td>
<td>104.02%</td>
<td>163.56%</td>
<td>259.71%</td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>15.96%</td>
<td>57.48%</td>
<td>90.40%</td>
<td>226.78%</td>
<td>191.45%</td>
<td>303.80%</td>
</tr>
<tr>
<td>MSCI-WI</td>
<td>-27.22%</td>
<td>7.07%</td>
<td>20.82%</td>
<td>29.96%</td>
<td>53.29%</td>
<td>64.16%</td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>-27.07%</td>
<td>7.86%</td>
<td>22.21%</td>
<td>32.99%</td>
<td>57.97%</td>
<td>73.20%</td>
</tr>
</tbody>
</table>

Note: Figures corresponding to 2002 are annualised as the period starts in May/02.
b) Risk

Modern financial theory has been built upon the foundations set by Markowitz (1952) with the now traditional Mean-Variance framework. Despite some detractors like Messina (1995) pointed out several deficiencies, particularly regarding the usage of the Standard Deviation as a measure of risk, it remains widely employed for a variety of issues. It is not in the spirit of the present section to question Markowitz’s hypotheses; on the contrary, they will be considered rather axiomatically because of its utility to portray the intended ideas. More specifically, the first, second and third criteria of
Stochastic dominance are of application in the present context\textsuperscript{129}. Hence, according to the Mean-Variance Criterium (MVC) represented in the risk-return space, high returns are attached to high risk\textsuperscript{130} and vice versa in order to be eligible for optimal portfolios.

Were the risk of the investments to be represented by the Standard Deviation (hereafter also termed volatility indistinctively, unless otherwise stated) in the context of MVC, flying performances reported by Emerging and Frontier markets would undoubtedly be connected to higher Standard Deviation. Accordingly, Charts 4.9 and Graph 4.5 display the previously mentioned fact for the selected indices, from where it surges that 5.36\% volatility of MSCI-EM is fuelled by Shanghai Composite (8.02\%) and RTS (7.74\%) among others, and the 4.73\% of MSCI-FM is driven, for example, by Merval (7.75\%). In contrast, the 3.48\% belonging to MSCI-WI mirrors volatilities around and below 5\% of Developed markets. Finally, the heavier weights of Developed countries in MSCI-AC more than compensate the greater Standard Deviation of Emerging indices to take the overall volatility to 3.55\%.

<table>
<thead>
<tr>
<th>Index</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>DJIA</td>
<td>3.52%</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>3.40%</td>
</tr>
<tr>
<td>Nasdaq</td>
<td>5.62%</td>
</tr>
<tr>
<td>FTSE</td>
<td>3.64%</td>
</tr>
<tr>
<td>Nikkei</td>
<td>4.52%</td>
</tr>
<tr>
<td><strong>MSCI –WI</strong></td>
<td><strong>3.48%</strong></td>
</tr>
<tr>
<td>Bovespa</td>
<td>7.08%</td>
</tr>
<tr>
<td>Shanghai Composite</td>
<td>8.02%</td>
</tr>
<tr>
<td>Sensex</td>
<td>6.43%</td>
</tr>
<tr>
<td>RTS</td>
<td>7.74%</td>
</tr>
<tr>
<td><strong>MSCI-EM</strong></td>
<td><strong>5.36%</strong></td>
</tr>
<tr>
<td>Merval</td>
<td>7.75%</td>
</tr>
<tr>
<td>Crobex</td>
<td>6.02%</td>
</tr>
<tr>
<td><strong>MSCI-FM</strong></td>
<td><strong>4.73%</strong></td>
</tr>
<tr>
<td><strong>MSCI-AC</strong></td>
<td><strong>3.55%</strong></td>
</tr>
</tbody>
</table>

**Note:** MSCI indices representing Emerging, Frontier, Developed, World and All Countries markets in bold letters.

**Source:** Own elaboration with data obtained from the respective stock exchanges and MSCI websites.

\textsuperscript{129} Section 1.3.

\textsuperscript{130} It is important to bear in mind that high risk could also imply low returns; however, those assets are termed inefficient in the sense that it will always be possible to find other securities or combinations of them likely to deliver greater returns for the same risk, or, equivalently, identical returns for a lower level of risk. (Brealey and Myers (2003)).
Like in the case of returns, a decomposition of the cumulative volatility in terms of the individual years yields similar results as before, with MSCI-EM leading clearly and MSCI-FM following (Charts 4.10 and 4.11 and Graphs 4.6 and 4.7).

**Chart 4.10**  
*Index risk: Annual Standard Deviation*

<table>
<thead>
<tr>
<th>Index</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-EM</td>
<td>8.78%</td>
<td>4.38%</td>
<td>4.59%</td>
<td>5.65%</td>
<td>5.41%</td>
<td>5.28%</td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>3.94%</td>
<td>3.25%</td>
<td>2.88%</td>
<td>7.59%</td>
<td>4.92%</td>
<td>2.86%</td>
</tr>
<tr>
<td>MSCI-WI</td>
<td>8.54%</td>
<td>3.54%</td>
<td>2.36%</td>
<td>2.37%</td>
<td>2.15%</td>
<td>2.69%</td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>8.55%</td>
<td>3.55%</td>
<td>2.43%</td>
<td>2.54%</td>
<td>2.36%</td>
<td>2.84%</td>
</tr>
</tbody>
</table>

*Note: Figures corresponding to 2002 are annualised as the period starts in May/02.*

**Chart 4.11**  
*Index risk: Cumulated Standard Deviation*

<table>
<thead>
<tr>
<th>Index</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-EM</td>
<td>8.78%</td>
<td>6.09%</td>
<td>5.48%</td>
<td>5.47%</td>
<td>5.41%</td>
<td>5.36%</td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>3.94%</td>
<td>3.24%</td>
<td>3.08%</td>
<td>4.83%</td>
<td>5.05%</td>
<td>4.73%</td>
</tr>
<tr>
<td>MSCI-WI</td>
<td>8.54%</td>
<td>5.46%</td>
<td>4.48%</td>
<td>3.97%</td>
<td>3.65%</td>
<td>3.48%</td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>8.55%</td>
<td>5.48%</td>
<td>4.50%</td>
<td>4.02%</td>
<td>3.71%</td>
<td>3.55%</td>
</tr>
</tbody>
</table>

*Note: Figures corresponding to 2002 are annualised as the period starts in May/02.*
c) **Emerging and Frontier markets more in depth: efficiency connotations.**

For the purposes of the present paragraphs, the meaning of efficiency will be attached to MVC, thus making a security (or portfolio) efficient if and only if:

i) it delivers the highest return among all of those exhibiting the same level of risk and,

ii) it yields the smaller risk among all of those featuring the same return.

The underlying objective pursued resembles, in many aspects, that performed by Derrabi and Leseure (2005). Even though the authors treat Emerging and Frontier
markets under the same category and employ lower data frequency (weekly), conclusions broadly coincide and appear to foster the notion that both markets are adequate vehicles to achieve more efficient portfolios through diversification.

The first exercise deals with the construction of the different market lines (curves or frontiers\textsuperscript{131}) for Developed, Emerging and Frontier markets through simple regression using monthly arithmetic returns from stock exchanges across the world, selected on the grounds of their relevance to the specific group and world coverage. The indices that form the basis for the statistical adjustment are reflected in Chart 4.12 below:

\begin{center}
\textbf{Chart 4.12}

\textit{Market frontiers – Components}

\begin{tabular}{|l|l|}
\hline
\textbf{Category} & \textbf{Selected components} \\
\hline
Emerging frontier & Bovespa (Brazil), Cetop20 (Hungary), Sensex (India), PX (Czech Republic), JKSE (Indonesia), KLSE (Malaysia), IPCC (Mexico), IGBC (Colombia), Shanghai Composite (China), Taise (Taiwan), Kospi (South Korea), ISE (Turkey), CMA (Egypt), RTS (Russia). \\
\hline
Frontier frontier & Merval (Argentina), OMX-Vilnius (Lithuania), Tunindex (Tunisia), Crobex (Croatia), KSE (Pakistan), KSEPI (Sri Lanka), MSE (Malta). \\
\hline
Developed frontier & DJIA, S&P500, Nasdaq (United States), FTSE100 (United Kingdom), CAC40 (France), DAX (Germany), FTSE MIB (Italy), SMI (Switzerland), OMX-Stockholm (Sweden), Nikkei (Japan), TAS (Israel), IGBM (Spain), AMX (Australia), Strait (Singapore), Hang Seng (Hong Kong). \\
\hline
\end{tabular}
\end{center}

The analysis proposed, which outcome is represented in Graph 4.8 invites some reflections to come to the fore, although in overall they agree with the aforementioned patterns of risk and returns deployed in Sections 2.2.1. and 2.2.2.. The Developed curve (blue line) delivers the most efficient combinations for low returns and low risk, hence conveying their convenience for conservative strategies up to the point (0.0294; 0.4724), from where the Emerging curve (red line) gives the most efficient risk-return mixtures. Its dominium extends until (0.0529; 2.888), starting point for the Frontier line (black line) to extend up into the high risk-high returns space\textsuperscript{132}.

\textsuperscript{131} The term ‘frontier’ without capital ‘f’ refers to a borderline and ought not to be mistaken for the meaning attached to Frontier markets.

\textsuperscript{132} The visual assessment of the graph could be complemented with the steepness of the gradients belonging to the curves involved. Appendix 4.A deals with the outcome of the estimation procedure.
The above observations, then, seem to support the facts pointed out in Section 2.2., as Emerging and Frontier markets appear to take the lead in terms of high yield and high risk, with Developed markets prevailing in the lower part of the space.

To delve into the matter even further, places taken by Emerging, Frontier and Developed markets are again assessed in the risk-return space, although employing portfolios constituted by mixtures of MSCI indices with a view to evaluate the movements of the Efficient Frontiers according to its components. Therefore, four combinations can be outlined\(^\text{133}\):

i) Portfolio EFW (blue line): constituted by MSCI-EM, MSCI-FM and MSCI-WI;
ii) Portfolio EF (purple line): constituted by MSCI-EM and MSCI-FM;
iii) Portfolio EW (yellow line): constituted by MSCI-EM and MSCI-WI and
iv) Portfolio FW (red line): constituted by MSCI-FM and MSCI-WI.

---

\(^{133}\) The following abbreviations apply: EFW, Emerging, Frontier and World; EF, Emerging and Frontier; EW, Emerging and World; FW, Frontier and World; EM, Emerging Markets; FM, Frontier Markets; WI, World Index.
Graph 4.9, which depicts the efficient frontiers\textsuperscript{134}, suggests some interesting implications. In the first place, it makes clear the benefits of diversification: the most efficient frontier belongs to the EFW portfolio, i.e., the one with the three indices. Secondly, it may be appreciated that whenever Developed markets are concerned, the efficiency of the portfolio is increased. Moreover, from the minimum risk portfolio EFW located at (0.0294; 0.0085) onwards, its line represents the most efficient frontier except for a very short segment spanning from (0.0296; 0.0120) to (0.0322; 0.0160) where FW improves EFW performance. Secondly, Emerging and Frontier markets are usually employed to increase yields and risk simultaneously: the equations delivering the corresponding weights show that low risk portfolios are composed mainly of positions in MSCI-WI whereas investors would augment exposures in Emerging and Frontier markets were they to demand higher performance. Accordingly, expressions for weights of Developed markets exhibit negative gradients meaning that, for low risk combinations, the strategy requires going short MSCI-EM and MSCI-FM in varying degrees to leverage positions in MSCI-WI, reversing to long as the risk aversion diminishes along the boundary. Thirdly, setting aside EFW portfolio, FW delivers the best combination from its minimum risk portfolio (0.0308; 0.0124) to (0.0705; 0.0290) where it intersects with EW and the latter becomes the preferred option. This fact should hardly surprise as MSCI-EM delivers slightly better returns with higher Standard

\textsuperscript{134} Appendix 4.A portrays details of the calculations.
Deviation (Charts 4.6 to 4.11). Finally, and easily deducted from the corresponding graphs, the most inefficient frontier is constituted by Emerging and Frontier markets.

In several regards, the aforementioned empirical facts are consistent with the stance adopted by FTSE in the delineation of the criteria to be fulfilled by markets to be classified as Developed, Emerging (Advanced or Secondary) or Frontier. Some of those factors are closely related to the depth, transparency and efficiency of the whole market, particularly those connected with permission of stock lending and short sales, free settlement of operations, efficient trading mechanisms and developed derivatives markets, all of which are missing in Emerging stock exchanges. In this respect, the situation of Frontier markets appears somewhat aggravated as, on top of lacking the former requisites, other equally relevant like free and deep equity and foreign exchange markets, quality of custodian services, brokerage competition, and, most importantly, adequate market liquidity and reasonable transaction costs are absent and therefore increase the probabilities of exploiting those market gaps.

Furthermore, the correlation verified in the period May/02-Dec/07 under analysis may also contribute to explain the investment strategies. Chart 4.A.1 (Panels A-G) in Appendix 4.A depicts the yearly and overall evolution of the correlation coefficients observed in that term between MSCI-WI, MSCI-EM, MSCI-FM and MSCI-AC. A closer look at the figures clarifies the main reason behind the construction of the category Frontier markets, namely the necessity to take positions in a vehicle which movements could detach either from Developed or Emerging markets (as well as MSCI-AC, for it excludes Frontier stock exchanges). Even acknowledging that stock markets appear to exhibit a general positive correlation, and making room for the variations in the composition of MSCI indices through time-which exert influence on the relative weights and the subsequent relationships among its fellow present and former constituents-, the broad picture thus underlines the relatively close liaison between Emerging and Developed markets with Frontier ones offering potential for diversification on a global allocation scale.

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135 Appendix 4.A.
4.2.2. A final reflection on the characterisation of Emerging and Frontier markets

It is a well documented fact in the literature that Developing markets (Emerging and Frontier) commonly yield high returns and high volatility (Divecha, Drach and Stefek (1992), Barry, Peavy III and Rodriguez (1998)). The possibility of taking a share in the growth potential as well as the exploitation of market inefficiencies appear to account for the elevated rates of returns to which volatility itself could contribute to large extent (Mody (2004)). However, the most interesting aspects seem to connect to volatility.

Research is almost certain at the time of supporting the bumpiness of the performance in immature stock exchanges (Aggarwal, Inclan and Leal (1999), Aizenman and Pinto (2004), Barry and Lockwood (1995), among others); however, it may be an interesting exercise to glance at the reasons behind that high volatility. Ranciere, Tornell and Westerman (2003) explain that governments usually adopt some courses of action that may force the natural transition towards an advanced economy, thus bolstering instability. Accordingly, Aguiar and Gopinath (2007) go even beyond to link volatility to powerful fluctuations in business cycles. They reason that extraordinary turnarounds in fiscal, monetary and trade policies typical of underdeveloped nations bring about frequent regime switches which affect the trend growth on a prolonged, almost permanent basis as opposed to mature markets which trends are stable and alterations to the cycle trend are transitory. Their analysis of the dynamics of volatility focuses on a macroeconomic point of view, and volatility comes in the form of variable consumption and countercyclical current account deficits. In their vision, when economic agents residing in immature nations detect a period of high growth, consumption and investment are optimally augmented; however, as shocks to the growth rate push future output even more than current production, consumption swells more than income, thus reducing savings and provoking a current account deficit that instils volatility. On the other hand, when moderate and transitory productivity shocks take place in developed countries, increased incentives to save will overcome greater investment, eventually ending in reduced cyclicality of the current account balance and stable consumption.

The empirical analysis carried out by Aggarwal, Inclan and Leal (1999) highlights that high volatility records in Emerging countries owe to frequent changes in variance driven by economic, political or social events as well. These variations tend to be abrupt and derive from local rather than global events as the October 1987 crash erects as the only international episode fuelling instability to emerging nations. The
authors also coincide with Bekaert and Harvey (1997) as they find that the percentage of the increased variance attributable to world factors is not relevant in Emerging markets but, on the contrary, it remains mostly influenced by local factors.

Barry, Peavy III and Rodriguez (1999) mark that albeit developing countries usually enjoy higher than average returns, they do always endure elevated levels of volatility. Even though it may prove adequate for investors in some occasions—particularly for those risk-taking individuals—, the current study agrees with Derrabi and Leseure (2005) and Barry, Peavy III and Rodriguez (1999) about the diversification benefits of investing in Emerging markets. The underlying idea states, then, that combining their assets with stocks belonging to developed countries, the investment frontier displaces and becomes more efficient in terms of the MVC\textsuperscript{136}.

4.3. Stochastic analysis: time series and model parameters

The present Section features the main characteristics of the time series belonging to the indices and, simultaneously, provides an insight into the values of the parameters of the models to be employed for VaR estimation and subsequent MCR calculation. The analysis will be restricted to the outcome of GARCH and EGARCH estimation and the assessment of the GPD parameters for the EVT approach\textsuperscript{137}. It is important to bear in mind that the knowledge related to the performance of the models in varying market contexts grows alongside the quantity of specifications devised (Beder (1995)), and that the number of models is becoming increasingly unaccountable for (Manganelli and Engle (2001)). However, a useful practice is to analyse the main characteristics of the data series considered in order to grasp the statistical properties of the market series that may prove helpful to understand the behaviour of the models.

The pioneering works of Fama (1963) and Mandelbrot (1965) have highlighted several aspects related to the stochastic patterns common to financial log return series, particularly when the frequency is daily or even higher. The academic literature usually refers to these attributes as stylised facts (JP Morgan and Reuters (1996) or Penza and Bansal (2001)) because of the recurrence observed across markets and assets. Given their implications in risk management in the sense analysed in this thesis, the focus will

\textsuperscript{136} Although the latter cast doubts on the advantages of diversification during crises (however, their conclusion roots in the 1994 Mexican devaluation episode when major Latin American economies, i.e., Argentina, Brazil and Mexico, moved in the same direction thus increasing correlation across Emerging markets; furthermore, they also stress that the upshot would require more evidence to remain a fact).

\textsuperscript{137} For obvious reasons, HS and FHS, in what regards to the empirical estimation of the respective quantiles do not require any special references other than those stated in Sections 3.2 and 3.3.
be directed to end of day variations, acknowledging that with longer horizons (for example weekly, monthly or quarterly), the statistical properties may contrast from the daily ones, as documented by Richardson and Smith (1993). Fama’s (1963) and Mandelbrot’s (1965) main contributions may well be subsumed in three basic points that keep their validity up to the present times:

a) The distribution of financial return series presents leptokurtic patterns, i.e., it portrays higher or fatter tails and narrower and taller peaks than those predicted by the Normal distribution (often referred to as “fat tails” and “thin waists” respectively), which is reflected in distributions reporting kurtosis figures well above the Normal level of 3. This is tantamount to expressing that extreme swings in financial markets occur more frequently than if movements were driven by the Gaussian distribution, in turn generating a host of consequences for VaR and, in particular, capital constitution (Krehbiel and Adkins (2006), Danielsson, Hartmann and de Vries (1998), Brooks, Clare and Persand (2000) and FSA (2009));

b) Returns, in linear form, are not or eventually only slightly autocorrelated;

c) On the other hand, squared returns have significant autocorrelation, thus hinting at the conditional variances, or variances being conditional on time.

Duffie and Pan (1997) identify unexpected discontinuous changes in price levels –“jumps”– and changes in volatility over time with persistence –“stochastic volatility”- as possible causes of leptokurtic distributions. The authors emphasise that, for typical market behaviour, jumps affect the distribution of returns at a percentile farther out in the tail than the critical value of 1%, while if at least two jumps take place in the same year, the referred quantile will be more seriously affected during few weeks. Stochastic volatility, through persistence (Section 4.3.1) dictates that a relatively high current or recent volatility implies a relatively high volatility forecast in the future, and conversely for low values. Therefore, VaR as a quantile risk measure, bears the brunt of the impact of distributions with high kurtosis, and models accounting for that fact should deliver more accurate performances.

Although linear returns exhibit small, and in some cases no autocorrelation, this effect appears not to exert great influence on VaR calculations.

4.3.1. Stochastic analysis of financial time series
Chart 4.B.1 in Appendix 4.B provides the summary statistics for the daily returns series corresponding to the string of prices $P_t$ of the blue-chip indices, computed using the typical log transformation $r_t = \ln \left( \frac{P_t}{P_{t-1}} \right)$ as indicated in Section 3.8.1 belonging to the base periods stated in Chart 4.13. It is important to take into consideration that daily sampling is usually employed to grasp high-frequency fluctuations in return processes for risk management purposes under the BCBS framework while, at the same time, the trading tensions and microstructure twists that plague the intraday return dynamics are avoided in the modelling stages.

Risk modelling procedures require, to a certain extent, the characterisation of the future changes in the value of the asset/s composing the portfolio. In general, this task is accomplished employing past returns to build the predictions even if the approach involves simulations, and, in that regard, it is necessary to model both the distribution of returns at any point in time and the progression of those returns over time, or, alternatively, the temporal dynamics of returns.

With reference to the first point, the summary statistics depicted in Chart 4.B.1 in Appendix 4.B imply the following asseverations:

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Note: Emerging markets above solid line, Frontier markets below.

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<table>
<thead>
<tr>
<th>Stock Exchange</th>
<th>Stock Index</th>
<th>Estimation period</th>
<th>Data points</th>
<th>Forecast period</th>
<th>Data points</th>
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<td>30/12/2008</td>
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<td>02/01/2008</td>
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<td>02/01/2008</td>
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<td>02/01/2008</td>
<td>250</td>
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<td></td>
<td>31/12/2007</td>
<td></td>
<td>31/12/2008</td>
<td></td>
</tr>
</tbody>
</table>

Throughout the current thesis, changes in the value of assets are measured in log terms as indicated.
a) The average return appears not to differ significantly from zero or exhibit slightly positive values (Lithuania, Tunisia and Croatia), as reflected by the \( p \)-values of the corresponding \( t \)-tests. On the grounds of practicality, series could well be assumed driftless with no apparent loss in precision as in McNeil, Frey and Embrechts (2005)\(^ {139} \);

b) The distributions clearly deviate from normality, fact verified through several elements. The value of the skewness coefficient hints at distributions displaced from the central axis: while the majority of the markets exhibit negative asymmetry and only three depict positive shifts (China, Tunisia and Croatia), the figures situate well beyond the null level required by the Gaussian distribution. Secondly, the kurtosis exceeds the appropriate number characteristic of normality, i.e., 3. The joint hypothesis of non-zero skewness and excess kurtosis is confirmed by the Jarque-Bera test, which \( p \)-values situate below \( 10^{-3} \) at most. Following Jondeau and Rockinger (2003), the analysis of the departure from normality is completed evaluating the cut-off points of the empirical quantiles as opposed to those of the standard normal distribution. In effect, after standardising the sample –substracting the mean and dividing every value by the Standard Deviation of the sample–, the leptokurtic characteristics of the empirical distribution become more noteworthy. The departure from the gaussianity assumption in every series is reflected in Graph 4.B.1 in Appendix 4.B, which displays the empirical histograms with the standard normal density superimposed. For every time series the graph reveals the presence of two distinct elements: peaks are higher and narrower and tails are heavier than exhibited by the Normal distribution. While the former could prove more relevant for trading purposes than for risk fencing, the latter suggests that extreme price movements occur much more frequently than under the Gaussian assumption. Although this assertion may at first seem innocuous, it brings about a very relevant implication for risk management: the fact that Taleb’s (2007) ‘black swans’, i.e., sporadic atypical events in the shape of abrupt adverse market movements, are likely to happen often might well lead institutions to

\(^{139}\) In fact, series could be assumed driftless at the modelling stage once the corresponding representations yield statistically insignificant drift terms in the linear return equations. This situation is verified for all the series in the present thesis (Section 4.3).
bankruptcy were these incidents ignored in VaR quantification and its subsequent translation into MCR.

The behaviour of the return generating process over time plays a central role at the time of constructing models, thus aiding in the task of identifying their strengths and weaknesses. Accordingly, it should be determined whether the distribution of returns keeps constant over time and, additionally, if returns are statistically independent over time. It may be recalled that, were both assumptions to be verified, returns could be represented as an IID (identically and independently distributed) stochastic process, this way making life easier for practitioners. In statistical terms, the IID supposition would amount to the following facts:

a) At any point in time \( t \), the mean and variance of returns are unchanged, i.e., homoscedastic;

b) Return values at different points in time \( t \) are uncorrelated, or statistically independent of each other in time.

Visual inspection of every series in Graph 4.B.2 in Appendix 4.B appears to be enough to discard any idea of homoscedasticity, thus simultaneously fostering the notion of volatility changing in time, i.e., heteroscedasticity. A closer scrutiny reveals that volatility comes grouped into clusters: periods of high returns are followed by terms of low returns, which are clustered too. The graph also conveys\(^\text{140}\) that periods of large volatility (indicated by the continuous line) come alternated with spells of low volatility (marked by the dashed pattern) in a fashion that is verified for all series. Given that in the modelling stage volatility is measured by variance, heteroscedasticity is denoted by adding the subscript \( t \) to the variance representation: \( \sigma^2_t \).

The presence of volatility clusters observed in Graph 4.B.2 in Appendix 4.B suggests the idea of time-varying variances, furthermore suggesting that variances could exhibit some degree of autocorrelation, which appears enough to discard any idea of independence of returns over time. In fact, a more thorough and less informal analysis is corroborated by Chart 4.B.1 (Appendix 4.B), from where the following reflections emerge:

\(^{140}\) For practical purposes, only a handful of volatility clusters are marked in every graph. However, this is a pattern that tends to repeat in all the time series analysed.
a) There seems to be only slight partial evidence of autocorrelation in linear returns: the null hypotheses of no autocorrelation in returns cannot be rejected in seven markets with the exception of Brazil, Hungary, China\textsuperscript{141} and Croatia considering the values of the Box-Ljung portmanteau test up to order 10, 15 and 20 assessed at 90% confidence level. However, even though some of the values are statistically significant, the correlograms exhibited in Graph 4.B.3 –Panel A- in Appendix 4.B show that only in sparse occasions (mostly Malaysia and Lithuania) do spikes overcome the Bartlett bands. As the evidence for autocorrelation of daily returns seems to be of weak nature, the omission of any lag effect in the linear equation appears substantiated;

b) Assuming the absence of correlation in linear returns, their independence is thwarted by the fact that variances report significant evidence of autocorrelation, thus confirming the presence of heteroscedasticity. Using squared returns as a proxy for variances\textsuperscript{142}, the figures corresponding to the Box-Ljung statistics for 10, 15 and 20 lags largely overcome the critical values calculated at 90% confidence level (Chart 4.B.2, Appendix 4.B), hence rejecting the null hypotheses of no autocorrelation in variances (correlograms depicted in Graph 4.B.3- Panel B- in Appendix 4.B allow the visual verification of the previous assertions). Moreover, all the correlation coefficients between squared returns and their lags up to 20 exhibit positive sign (not reported here for space considerations), hence fostering the idea of volatility clustering, i.e., that periods of large volatility alternate with periods of low volatility. It is interesting to observe in Graph 4.B.4 in Appendix 4.B, which depicts the series of linear and squared returns superimposed- that terms of large returns are grouped and followed by spells of small returns, in turn reflecting in the behaviour of squared returns, with clusters of tall and short spikes notably discernible. There is, finally, a clear indication that variances are not constant and change with time, or, in a few words, the findings support the heteroscedasticity notion as aforementioned.

\textsuperscript{141} China presents evidence of autocorrelation in lags 15 and 20.

\textsuperscript{142} Assuming driftless series, the expected values of squared returns are variances:
\[ \sigma_t^2 = E[r_t - E(r_t)]^2 = E(r_t^2) - [E(r_t)]^2 \] and given that \( E(r_t) = 0 \) then, \( \sigma_t^2 = E(r_t^2) \).
In view of the proofs collected, the assumption of IID fails to materialise, thus reaffirming the previous results and paving the way for models acknowledging the conditional character of variances.

In order to complete the categorisation of the time series, it is relevant to assess the stationary status of the series, which is a prerequisite for conditional volatility models to be computed (Section 3.5). Eyeball work on Graph 4.B.5 in Appendix 4.B, which displays the level of the data series, discloses that all of them fail to achieve the stationary condition necessary to carry out stochastic modelling: all the series appear to develop a distinct trend upwards, i.e., a typical feature of nonstationary series. On the contrary, the secondary vertical axis of Graph 4.B.5, which plots the log return series, does not give the impression to exhibit any tendency and, furthermore, its values fluctuate around the sample mean, signifying that the nature of the series for the sample periods is of a mean-reverting one: in spite of the amplitude of the fluctuations, the series come back to the mean value. More formally, the stationary pattern of the series is usually gauged by the unit root test and in this sense the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) statistics based on the regression:

$$
\Delta r_t = \mu + \gamma t + \alpha_{t-1} r_{t-1} + \sum_{j=2}^{20} \beta_j \Delta r_{t-j} + \varepsilon_t \quad (4.1)
$$

are computed and their results informed in Chart 4.B.3 (Appendix 4.B) considering the null hypothesis $H_0: \alpha = 1, \gamma = 0$. Both ADF and PP values comfortably exceed the critical values at 1%, 5% and 10% indicating that the unit root null hypothesis is absolutely rejected in favour of stationary returns for every series. The unit root test confirms the back of the envelope assessment of Graph 4.B.5 in Appendix 4.B and highlights the existence of a finite variance, or, alternatively, the presence of a finite fourth moment. Hence, contemplating that the aforementioned conditions are met, usage of standard modelling techniques and conventional diagnostics applies (Alexander (2001)).

4.3.2. Empirical assessment of the model parameters

4.3.2.1. GARCH and EGARCH estimates

Charts 4.C.1 (Panels A and B) and 4.C.2 (Panels A and B) in Appendix 4.C display the outcome of the estimation process for GARCH and EGARCH models respectively with the Normal and Student-t densities fitted. An initial glance indicates that the estimated parameters are significant at 99% according to the $p$-values reported below every figure.
except the $\gamma$ coefficient in Lithuania, Tunisia and Croatia in the EGARCH-Student-$t$ and Tunisia and Croatia in the EGARCH-Normal.

The four parameterisations (GARCH and EGARCH equipped with both probability distributions) accomplish a remarkable job removing the strong temporal dependence in the variance: in effect, Columns [12] and [13] in Panels A and B respectively in Chart 4.C.1 and Columns [10] and [11] in Panels A and B respectively in Chart 4.C.2 in Appendix 4.C give testimony of their prowess, as the reduction in $Q^2_{20}$ for the squared standardised residuals $\hat{e}^2/\hat{\sigma}^2$ compared with the raw squared returns in every market (Column [2] in each Panel of the respective Chart) overcomes 90%. Additionally, Box-Ljung tests on squared original values reported in Chart 4.B.2 in Appendix 4.B hint at the presence of SWN processes and confirm the adequacy of conditional heteroscedastic representations to capture volatility dynamics.

GARCH coefficients –either for Normal or Student-$t$ densities, portrayed in Chart 4.C.1 (Panels A and B) in Appendix 4.C point out the high degree of persistence. For every series –except perhaps Lithuania in GARCH-Normal-, the estimates of the rate of convergence $(\alpha + \beta)$\textsuperscript{143} are high and above 0.80, thus implying that conditional variances are processes with very long memory; therefore, the effects of shocks to volatility smoothly decline over time and the volatility structure of the GARCH series appear relatively flat (although not to the extreme indicated by Alexander (2008b). GARCH parameters also mark that volatility is mostly explained by last period’s variance forecast $\beta$, accounting for the fact that it measures the extent of persistence of the shock: the closer to 1, the more weight is placed on past observations to the detriment of present ones, hence producing very smooth series. The only exception is, however, the case of Lithuania, where, in both configurations, $\beta$ belongs to an entourage of [0.40; 0.50], resulting, thus in a rougher path likely to exhibit sudden jerks. One of the most appropriate ways to appreciate the relationship between the level of the term structure of the volatility and the persistence of volatility after a market shock is conveyed by the volatility half-life\textsuperscript{144}, and, in this regard, the higher the persistence, the longer the time that takes to return half-way to its average level: Malaysia in both specifications reports the greatest quantities (465 and 252 days for the Normal and Student-$t$ densities respectively) and Lithuania –again for both configurations- (2 and 3

\textsuperscript{143} The term $(\alpha + \beta)$ might also shed light on the mechanics of the conditional volatility, particularly on the convergence to the long-run volatility value (Andersen et al. (2003)).

\textsuperscript{144} See Carroll and Collins (2002) and Section 3.4.3.
days for the Gaussian and Student-\(t\) distributions respectively) offer the smallest number of days. The previous figures reflect the highest and lowest \((\alpha + \beta)\) (in the case of Malaysia, 0.998512 and 0.997255 whereas for Lithuania, 0.632842 and 0.817833 for GARCH-Normal and –Student-\(t\) respectively). Malaysia, then, would depict the quietest volatility paths whilst, on the other hand, Lithuania may present the most unstable sequence of volatility forecasts. Finally, it is not possible to infer any regularity from the long-term volatility levels, as it responds to a combination of high constant parameter \(w\) and elevated rate of convergence.

A glance at the value of the estimates would, in principle, show that GARCH-Normal and GARCH-Student-\(t\) do not differ significantly –with the exception of Indonesia and Croatia (Emerging and Frontier markets respectively). It is noteworthy, however, that the log-likelihood values belonging to the ML estimates of the Student-\(t\) density are always greater than their Gaussian counterparts (albeit not too distant), reflecting a better adjustment influenced by the departure from normality exhibited by the raw time series (Chart 4.B.1 in Appendix 4.B). The degrees of freedom of the Student-\(t\) density fitted –Column [6] in Chart 4.C.1 –Panel B- could attest the asseveration as they range from 3.44 (Croatia) to 17.96 (Brazil).

The EGARCH configuration also offers salient features that support the theoretical considerations elicited in Section 3.4.4. As it might be appreciated in Charts 4.C.2, Panels A and B, the estimates deriving from the Normal and the Student-\(t\) settings do not differ to a great extent, exception made of Croatia. A general outlook highlights the fact that the \(\gamma\) coefficient that determines the behaviour of the response function \(g(.)\) in (3.32) is negative for all markets but Tunisia in both models and Croatia in the Student-\(t\) variant (Columns [5] in both panels). The negative value in turn means that the specification accentuates downside movements in the value of the securities in question whereas in Tunisia and Croatia (EGARCH-Normal) the model would react agnostically to the market shocks. On the other hand, it is interesting to trace some parallelisms between the \(\beta\) parameters in the GARCH and EGARCH settings as they denote the weight that the variance of the last period carries in the current variance (or log-variance in EGARCH). In this vein, the EGARCH technique allocates greater weight than its counterpart even in Lithuania where the values are not that high (0.63 vs 0.41 in both Normal models and 0.73 against 0.50 in the Student-\(t\) variant). The ML estimates for the degrees of freedom in the EGARCH-Student-\(t\) gives very similar outcomes to the GARCH ones, except perhaps in Brazil (24.50 against 17.96), albeit
both numbers would tend to the Normal distribution according to the Central Limit Theorem.

The concept of the long-term volatility does not seem as crystal clear as in the GARCH specification, as the figures convey (Column [7] in Panel A and [8] in Panel B in Chart 4.C.2, appendix 4.C). Therefore, in the long-run it appears that the variance tends to lower values than in the GARCH specification with the explanation presumably in the constant \( w \) and the smoothing logarithmic nature of the EGARCH expression.

Finally, the difference between the log-likelihood results corresponding to the Normal and Student-\( t \) technique is not deemed sizeable as to distinctly impact on the ultimate volatility and ulterior VaR estimates (Columns [9] and [10] in Chart 4.C.2, Panels A and B respectively, Appendix 4.C). However, it is important to note that Student-\( t \) values are always greater, albeit with tiny margins, and, furthermore, that the case of Tunisia highlights the adverse effects that may spring up when the EGARCH models struggle to produce significant estimates for the response function \( g(.) \) because the log-likelihood values indicative of the accuracy of the ML fitting process are equalled or even overcome by GARCH figures.

4.3.2.2. GPD estimates via Method of Moments
Chart 4.C.3 in Appendix 4.C reports the results of the estimation of the GPD parameters in accordance with the methodology outlined in Section 3.7.3 b.2). Despite the absence of precise rules regarding the quantity of high order statistics, less than 10% of the observations are devoted to the analysis of the extremes as in Rossignolo, Fethi and Shaban (2012b), well within Christoffersen’s boundary (2003) (Column [8] in Chart 4.C.3, Appendix 4.C).

Recalling that the selection of the threshold \( u \) was carried out abiding by the procedure of Section 3.7.3 illustrated in Appendix 3.D, Chart 4.C.3 portrays the values of \( \xi \) and \( \sigma \), tail index and scale coefficients respectively in Columns [2] and [4]. Positive estimates of \( \xi \) indicate the presence of heavy tails, hence belonging to the classic GPD (equivalent to Fréchet distribution in EVT configuration). Acknowledging the theoretically still imprecise nature of the GPD calibration, following Hosking and Wallis (1987), the Method of Moments provides reliable estimates considering that the samples are not ‘big’ –i.e., less than 151 extremes (the greatest quantity is recorded in Malaysia with 150 points)–. The figures of \( \xi \) only exceed the tentative hurdle set approximately at 0.20 (Czech Republic, 0.2487, Lithuania, 0.2121 and Croatia,
0.26615) and the tail index comfortably lies within 95% positive intervals (Column [3], Chart 4.C.3)\(^{145}\), thus bolstering the case for the presence of the heavy-tailed GPD density. Finally, values of the scaling parameter $\sigma$ (Column [5], Chart 4.C.2 in Appendix 4.C) also reveal significant and belong to the positive domain too in view of the typical 95% intervals reproduced in Column [6] of the same Chart.

### 4.4. Value-at-Risk results

The current section presents the outcome of the Value-at-Risk exercise employing the specifications mentioned in Sections 3.3 to 3.7 and the year 2008 as the Backtesting period. The assessment of the VaR experiment acquires more relevance if the result is evaluated through the outcome of Backtesting in terms of the quantity and proportion of exceptions (Chart 4.D.1, Appendix 4.D) and the increase in the scaling factor $k$ intended as a penalty for inaccuracy, expressed in terms of additional capital (Chart 4.D.2, Appendix 4.D).

Chart 4.D.1 depicts the expected number of exceptions calculated as 1% of the sample in the second column– as demanded by the BCBS- and the empirical number found after performing Backtesting (absolute and relative figures): it should be desirable for the quantities to situate in an entourage of that 1% (Longin (2000))\(^{146}\). Chart 4.D.2 transforms those values into capital surcharges by means of Chart 2.1 in Section 2.7, i.e., the sliding scale defined in Backtesting.

The outcome of the exercise motivates the following reflections, most of them cited in Rossignolo, Fethi and Shaban (2012b):

a) Linear models prove to be inadequate as rejection -irrespective of Normal or Student-$t$ distributions- is obtained in every stock exchange without distinction between Emerging and Frontier markets. The drawbacks of the Standard Deviation as a technique to measure volatility affect its performance given its inability to capture the movements that engulf the markets daily. Perhaps an obvious observation states that the Student-$t$ distribution improves the performance of the Gaussian one on the grounds of the leptokurtic characteristics of the distribution, notwithstanding which the marginal progress reveals insufficient to bring down the

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\(^{145}\) Section 3.7.2 cites Hosking and Wallis (1987), who express that the two first conditions should verify in order to employ ML estimation. Given the rest of the results exhibited, the case for the utilisation of the Method of Moments appears strong. For space considerations, the final VaR calculus have also been carried out under the ML process with very similar results.

\(^{146}\) However, it is important to acknowledge that the BCBS labels a model as ‘accurate’ if the quantity of exceptions is less than four in a typical year of 250 observations (BCBS (1998, 2009)).
number of excessions to levels susceptible of being considered acceptable. For instance, the proportion of VaR violations is floored by 4.07% and 5.69% (Student-\( t \) and Normal distributions in Tunisia), clearly beyond the yardstick 1%;

b) Much in the same fashion as the Linear models, the HS appears unquestionably inaccurate, being discarded in all markets (except Indonesia, which situates on the brink of rejection with 9 excessions representing an 85% capital increase in terms of Backtesting in Column [4], Chart 4.D.2 in Appendix 4.D). Although Emerging markets seemingly situate nearer the required percentage of VaR violations in the sample than Frontier ones, in practice the slight advantage does not represent a significant difference (the former always deliver more than 3.70% of violations whilst the latter cannot drill the 6% floor). The disadvantages mentioned in Section 2.2 represent a clear indication of the limitations of the technique: briefly stated, its inability to incorporate the brusque movements recorded at crisis times makes it unreliable for risk quantification and capital constitution purposes;

c) The application of conditional volatility filters (Columns [5] to [8] in Chart 4.D.2, Appendix 4.D) helps HS to significantly improve its performance, to the extent that only in Croatia does FHS deliver unsatisfactory results (Red or advanced Yellow regions). The rest of the stock exchanges alternate between Green and Yellow colours: the former predominates in Emerging markets whereas Frontier ones get no optimal results for any scheme (outcomes border Red Zone). Models employed to produce standardised residuals yield approximately similar results in terms of excessions: the only likely generalisations indicate that GARCH specifications appear to possess a slender advantage over their exponential counterparts, and, furthermore, the difference between Normal and Student-\( t \) distributions does not affect volatility forecasts considerably;

d) Conditional models represent a significant leap forward, both for GARCH and EGARCH techniques (Columns [9] to [12] in Chart 4.D.2, Appendix 4.D). The Student-\( t \) distribution clearly works better than the Normal one for both models as it avoids the Red Zone in every market, either Emerging or Frontier (except Lithuania in EGARCH setting). The Normal distribution gives mixed results, failing in one Emerging Market (Indonesia) and two Frontier (Lithuania and Croatia) for GARCH and two Emerging (Hungary and Indonesia) and two Frontier (Lithuania and Croatia) employing EGARCH. However, unfortunately, the Normal distribution cannot avoid the Yellow Zone and demands surcharges ranging from 50% (Czech
Republic, Malaysia and Argentina using GARCH and India and Malaysia using EGARCH) to 85% in Brazil (EGARCH). The Student-\( t \) distribution relieves the pressure on shareholders as they escape constituting extra capital in India and Malaysia (GARCH and EGARCH), Argentina (EGARCH) and Croatia (GARCH): GARCH allows it in two Emerging Markets (India and Malaysia) and one Frontier (Croatia), while EGARCH follows suit in two Emerging Markets (India and Malaysia) and one Frontier (Argentina), while falling in the Red Zone in Lithuania;
e) EVT is undoubtedly the best performer (Chart 4.D.1 -Column [14]- and Chart 4.D.2 -Column[13]-, Appendix 4.D). It does not require auxiliary capital in any market, fact reflected in the highest VaR for every Emerging and Frontier market for 01/Jan/09 among those models that avoid the Red Zone and consequent reestimation. Hungary is the country with the highest number of violations (4), although it still falls under the Green Zone. Furthermore, bringing into play the framework devised by Brooks, Clare and Persand (2000), its levels could not be considered excessive or insufficient as the proportion of VaR excessions stays close to the stipulated value (only Hungary-1.59% and India-1.23% exceed the standard 1%, percentages, but not enough to claim capital reinforcement as can be observed in Chart 4.D.3 in Appendix 4.D).

4.5. The effects on regulatory capital

4.5.1. The Internal Model Approach and VaR

As stated in Rossignolo, Fethi and Shaban (2012b), Chart 4.D.2 in Appendix 4.D crowns EVT as the most reliable technique with no capital surcharges while the rest of the specifications deliver mixed results which blur any likely pecking order.

However, in general terms, it should be stressed the failure of the Normal distribution to account for the unexpected market movements, particularly reflected in the behaviour of its CV/GARCH and CV/EGARCH specifications in Hungary, Indonesia, China, Lithuania and Croatia while in the remaining markets the enhancement to the multiplier is fairly high, for \( k \geq 50\% \). The Conditional Volatility models employing the Student-\( t \) distribution moderately improve the Gaussian performance, attaining several null \( k \) factors (India, Malaysia and China CV/GARCH and CV/EGARCH), Croatia (CV/GARCH) and Argentina (CV/EGARCH), although CV/EGARCH would end up invalidated in Lithuania. A closer look at the doubt
(Yellow) zone reveals that CV/GARCH-t implies somewhat lower capital levels than CV/EGARCH-t, for $0.40 \leq k \leq 0.75$ for the former and $0.50 \leq k \leq 0.85$ for the latter\textsuperscript{147}.

The adoption of FHS produces an almost radical change of the picture which, in fact, looks greener and less red\textsuperscript{148}. In effect, joint assessment of Charts 4.D.1 and 4.D.2 in Appendix 4.D conveys a better behaviour on the part of GARCH filter regardless of the distributional assumption\textsuperscript{149}: it is possible to find $k = 0$ using the typical GARCH representation (Hungary, India, Czech Republic, Malaysia and China) and, on the contrary, large multipliers with EGARCH models (Lithuania, Tunisia, Brazil and Indonesia); furthermore, no Green region is attained by FHS/EGARCH alone without FHS/GARCH company (India and China). An overall intra-model\textsuperscript{150} comparison of the performance of the distributional assumption insinuates that capital surcharges are slightly lower using Student-t, although that marginal edge does not deliver a sizeable competitive advantage over the Normal hypothesis.

Finally, high VaR values yielded by EVT (last row, Chart 4.D.3 in Appendix 4.D) translate into absence of extra equity, thus rendering this specification as the most reliable and robust for capital constitution purposes (Green Zones in every market). From the above considerations it is not possible to deduce strong empirical regularities about the behaviour of the schemes: the only certainties seem the supremacy of EVT and the inadequacy of HS when turmoil affects the markets. For the rest of the models, their performance appears inconclusive to classify but, altogether, the results suggest the notion that the structure of the technique prevails over their distributional assumption were the standardised residuals to be employed to determine VaR, i.e., the specification used to find the standardised residuals (in this case, GARCH) gives more accurate VaR values than its counterpart EGARCH irrespective of the distribution utilised to construct the ML function and compute the relevant quantile. On the other hand, the inverse holds if a specific theoretical distribution is attached to estimate the appropriate quantile, i.e., a heavy-tailed leptokurtic distribution like the Student-$t$ proves more effective than the Gaussian hypothesis, no matter which of the two models (GARCH or EGARCH) is applied to calculate the respective VaR. Consequently, the election between FHS and CV schemes remains a topic of more thorough scrutiny in view of the inconsistency of

\textsuperscript{147} Except Argentina, where EGARCH-t locates in the Green Zone and GARCH-t in the Yellow one.

\textsuperscript{148} Except Croatia, which displays a disappointing performance.

\textsuperscript{149} Except Argentina, where EGARCH model yields lower capital charges in either version.

\textsuperscript{150} In the present context, the term ‘intra-model’ comparison refers to the evaluation of the outcome delivered by the same specification but featuring different distributions: for example, a CV/GARCH-Normal against a CV/GARCH-t. Analogous reasoning extends to all possible combinations.
the results: those involved in the decision concerning the selection of the suitable technique must be aware of the dilemma. Therefore, on the grounds of the instability of the outcomes of FHS and CV and the failure of HS and Linear specifications as well, EVT comes out as the most powerful tool for regulatory MCR if the institution aims at a prolonged period of tranquillity.

4.5.2. The Standardised Approach

The existence of SA for capital constitution purposes has been a controversial subject since its inception in the first Basel Capital Accord. Several authors like Penza and Bansal (2001) and Prescott (1997) have raised concerns about its adequacy for MCR, and the empirical evidence fosters their worries.

Prescott (1997) devised a framework which could be applied to gauge the result of Backtesting on capital levels constituted employing the SA\textsuperscript{151}. The author explains that an entity enters bankruptcy when market losses exceed equity provisions; therefore, for an institution to remain healthy it would be essential to keep their excession records at 0. In this sense, the test of the SA levels against market movements in the year 2008 (Chart 4.D.4 – Columns [2] and [3]) yields an outcome far from satisfactory. SA-based capital levels are completely defenceless to shield market losses in crises of important magnitudes: the quantity of breaches occurred reaches 5 (Czech Republic) and 4 (Brazil, Hungary and Argentina)\textsuperscript{152}. The results appear to confirm those of Prescott’s exercise (1997), where SA\textsuperscript{153} delivers the highest failure rate among a range of capital models, particularly when banks adopt risky investment strategies or are subject to very volatile environments. Hence, either from Prescott’s study or the current thesis, the evidence points to the flat rate appraisal being insufficient to build an adequate capital level, mostly because that percentage of the risk-weighted assets is typically low enough to cover abrupt market swings. Despite the outcomes obtained, the BCBS continues offering the SA as a means to compute capital levels; however, it is difficult to figure out precise motivations behind that possibility besides political or cost-driven reasons. Consequently, the issue of moral hazard hovers in the background of SA: directives would induce entities to select SA instead of any tougher VaR-based IMA, this way

\textsuperscript{151} Under IMA, VaR levels are backtested and converted into MCR by means of formula (2.31). On the contrary, as SA delivers a capital level and not a metric to obtain capital levels, the conclusion is drawn on a bankruptcy-survival basis.

\textsuperscript{152} SA would have yielded enough capital to ensure the survival of Tunisia exposures (no exceptions).

\textsuperscript{153} SA or any other method susceptible of constituting capital buffers through the application of a fixed flat rate insensitive to the market movements.
building smaller capital levels (Penza and Bansal (2001)). The adoption of SA could work reasonably well in low volatility environments; however, its underestimation of abnormal movements undoubtedly poses great bankruptcy threats and significantly increases systemic risks.

4.6. Value-at-Risk and Capital Requirements

4.6.1. The situation under Basel II Capital Accord

The procyclical behaviour of conditional VaR prompted the BCBS to demand the calculation of the average 60-day VaR enhanced by the multiplication or hysteria\(^{154}\) factor currently set at 3, supplemented by an add-on coefficient which level is related to Backtesting results (Chart 4.D.2).

Looking at both Chart 4.D.2 and Chart 4.D.5 in Appendix 4.D, it may be appreciated that EVT delivers the highest MCR in four stock exchanges (Malaysia, China, Lithuania and Croatia) even having avoided Backtesting penalties while the remaining models share the honours as a result of the surcharges. This fact signals the potential existence of vestiges of moral hazard that would hamper the adoption of accurate representations like EVT-derived ones.

CV models again display inconsistent behaviour at the time of gauging the correct transmission mechanisms between Backtesting penalties and MCR levels. Briefly stated, it should be desirable that those techniques supporting heavier punishments were reflected in higher VaR values and, consequently, higher capital buffers, hence rewarding sharpness. Brazil (CV/GARCH-N delivers the highest MCR with \(k = 75\%\), whereas CV/EGARCH-N and CV/GARCH-t yield smaller ones, albeit suffering 85% surcharge), Hungary (CV/EGARCH-t, +75%, < CV/GARCH-t, +40%), Czech Republic (CV/EGARCH-N, +75%, < CV/GARCH-t, +40%), Indonesia (CV/EGARCH-t, +75% < CV/GARCH-t, +65%) and Tunisia (CV/EGARCH-t, +50% < CV/GARCH-t, 0%) unfortunately constitute examples of the above asseveration given that higher Backtesting penalties are capable of delivering less MCR\(^{155}\).

FHS again mirrors those distortions referred to the right enticement to apply precise representations given that models with lower penalties end up constituting higher MCR. Some instances are verified in Brazil (FHS/GARCH-t, with 50% yields

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\(^{154}\) Jorion (1996) and Dowd and Hutchinson (2010), for example, employ the name “hysteria factor” to refer to the “multiplication factor”.

\(^{155}\) Space considerations prevent from mentioning more examples in every stock exchange. All instances are reported in Chart 4.D.5, Appendix 4.D.
49.09% while FHS/EGARCH-N and FHS/EGARCH-t, both with 75%, produce 42.26% and 43.07% respectively); Indonesia (FHS/GARCH-N suffers 40% surcharge and delivers 35.08% whereas FHS/EGARCH-N and FHS/EGARCH-t present 65% and 75%, constituting 30.56% and 32.96% respectively) and Lithuania (FHS/GARCH-t, with $k = 40\%$ overcomes FHS/EGARCH-N and FHS-EGARCH-t, which penalties reflected by $k$ are 85% and 75% respectively and render 23.95% and 24.61% respectively against 25.36% in FHS/GARCH-t).

Chart 4.D.5 in Appendix 4.D is judged as pretty descriptive of one of the major concerns regarding the employment of EVT residing in the high amount of capital demanded. However, fears could be averted as MCR do not exceed quantities given by other heavy-tailed specifications like GARCH-t or EGARCH-t (either in CV or FHS versions) to a great extent, with the additional advantage of falling within the Green Zone, this way avoiding periodic revisions demanded by the Yellow Zone (Chart 4.D.2, Appendix 4.D)\textsuperscript{156}. In many stock exchanges, the level may well be deemed elevated (e.g. Brazil, Hungary, Czech Republic, Indonesia and China), whereas in the rest the percentage appears either moderate (India, Malaysia, Lithuania and Croatia) or relatively low (Argentina and Tunisia) notwithstanding the fact that, in all the examples, they would appear enough to cover daily losses of considerable amounts without being admonished.

4.6.2. The situation under Basel III Capital Accord

4.6.2.1. The stressed VaR and the Internal Model Approach

Chart 4.D.6 pictures the outcome of the process regarding the computation of the sVaR for all the models proposed. Recalling Section 2.7.4, the sVaR required the application of the same methodology employed to calculate VaR but, in this particular occasion, on a different dataset constituted by one-year periods when the portfolio would have suffered heavy losses\textsuperscript{157}. Those years, outlayed in Chart 4.14, represent the spans in which the indices features recorded their heaviest cumulated loss before the year 2008, i.e., the Backtesting year. Basel III Capital Accord stipulates that the total amount of capital to be built is obtained as a result of the sum of the two components, the VaR-based Basel II capital base and the sVaR-based Basel III extra capital.

\textsuperscript{156} Except China, where the amount to be constituted by EVT overcomes the second one belonging to the Green Zone (CV/GARCH-t) by 75%.

\textsuperscript{157} The period should be approved by the national regulator.
As in Section 4.6.1. above, disregarding those models falling in the Red Zone in Chart 4.D.2 (highlighted in bold letters in the respective ensuing Charts), EVT gives the highest sVaR amounts in highly strung periods for all Emerging and Frontier markets except Hungary (FHS/EGARCH-t) and Argentina (FHS/GARCH-t). Nevertheless, after applying the Backtesting results as in the current VaR (Chart 4.D.2 in Appendix 4.D) as demanded in Basel III, the balance switches on the grounds of the 60-day average rule and, most importantly, the penalties envisaged for inaccuracy. Therefore, the MCR derived from maximum sVaR charges spread among more specifications (Backtesting surcharges between brackets): FHS/EGARCH-t (Hungary-65%, Indonesia-75% and Tunisia-75%), FHS/GARCH-Normal (Argentina-75%), FHS/GARCH-t (Croatia-85%), CV/GARCH-Normal (India-75%), CV/EGARCH-Normal (Czech Republic-75%), CV/EGARCH-t (Brazil-85%) and EVT retaining three markets (Malaysia, China, and Lithuania). This last turn, depicted in Chart 4.D.7 in Appendix 4.D, eventually gives really high total capital values, of more than 17% in Brazil, 10% in Hungary, Malaysia and Croatia 11% in India and Czech Republic, 20% in Indonesia, 13% in China, 27% in Argentina, 9% in Lithuania and 4% in Tunisia to be added to Basel II levels.
4.6.2.2. Total Capital under Basel III. Basel II and stressed VaR

The new context proposed by BCBS displayed in Chart 4.D.7 -which features the amount of capital deriving from sVaR to be placed first in the VaR-based Basel II-, achieves the intended aim of strengthening the capital base. FHS/GARCH-Normal (Brazil, Argentina and Tunisia), FHS/GARCH-t (Croatia and Indonesia), FHS/EGARCH-t (Hungary), CV/GARCH-Normal (India), CV/GARCH-t (Czech Republic) and EVT (Malaysia, China and Lithuania) top the hypothetical ranking of maximum MCR (Chart 4.D.8 –Panel A- in Appendix 4.D), even though for all models but EVT the result is largely due to the add-in factor verified in Backtesting (Chart 4.D.2).

Rossignolo, Fethi and Shaban (2012b) point out that the question of the incentives to develop accurate VaR models reappears at the time of analysing the final capital charges. In effect, the authors note that the specifications belonging to the Green Zone should be expected to bear smaller Minimum Capital Requirements than those falling under the Yellow one (and, consequently, the smallest capital charges), given that the latter ones are subject to the penalties envisaged because of their poor Backtesting performance. However, the outcome displayed in Chart 4.D.8 –Panel A- does not look so transparent and, moreover, some vestiges of moral hazard appear at the time of assessing the MCR delivered by each specification. In this vein, Chart 4.D.8 –Panel B- contains a classification of the MCR in ascending order, i.e., the model giving the smallest capital level is assigned the number one with the rest following suit. The outcome, then, reflects that shades of moral hazard remain, because models belonging to the Green Zone are usually required to hold more reserve capital than those surcharged for poor Backtesting performance. For example, in Hungary EVT is ranked 3rd, in Indonesia 6th, in Lithuania and Tunisia 6th, in Czech Republic 3rd and in Brazil, India, Malaysia, China, Argentina and Croatia a somewhat more logical order could be observed.

Therefore, although techniques falling under the Green region should be expected to deliver the lowest MCR, FHS/GARCH-t (Hungary, 85%), HS (Indonesia, 85%), FHS/GARCH-t (Czech Republic, 85%), FHS/EGARCH-N (Lithuania, 85%) CV/EGARCH-N and FHS/EGARCH-N (Tunisia, 75%) are sternly penalised for poor

158 The classification excludes schemes falling under the Red Zone.
159 In the rest of the stock exchanges some model belonging to the Green Zone appears first in the pecking order.
Backtesting performance and still exhibit MCR\(^3\) lower than more precise models (Charts 4.D.8 and 4.D.12 – Panel A).

Chart 4.D.9 (Appendix 4.D) portrays a detailed assessment of the increase in the total capital charge for every market as a result of the addition of the sVaR – after the application of Basel III provisions-. As the figures convey, the BCBS unquestionably achieve its declared aim of “…significantly increasing the required level of their (banks’) capital” (BCBS (2010b:2)) notwithstanding which that expansion might be deemed somewhat excessive. In effect, a quick glimpse at the numbers in Chart 4.D.9 reveals that, excluding the models in the Red Zone, the average increase ranges from 32% (minimum in Czech Republic) to 210% (maximum in Argentina), with two countries exceeding 30% (Czech Republic and Hungary), three over 40% (India, Lithuania and Brazil), two surpassing 50% (Tunisia and Croatia), two beyond 60% (China and Indonesia), one over 90% (Malaysia) and Argentina overcoming (200%). Even acknowledging that the stress period selected was the year reporting the heaviest loss in many years and the national regulator could have approved a more lenient one, the swelling of the MCR base seems abrupt and this concern should clearly be addressed by the BCBS.

Chart 4.D.9 also fosters a very important point, again connected with the question of the incentives and the moral hazard that is likely to emerge from its numbers. The last three rows picture the average increase in the minimum capital required by the BCBS for all models lying in the Yellow Zone, those in the Green Zone and the relative difference between the groups, calculated in terms of the quotient between both figures\(^{160}\). The natural rationale would expect the ratio to be always positive, but, unfortunately, in two stock exchanges (Malaysia and Lithuania) it is negative, thus meaning that unpunished techniques would need to add more capital to their already important reserves; furthermore, the outcome is not substantially dissimilar as it situates in the region of 25% (Brazil, Hungary, Czech Republic, Indonesia), 15% (Argentina), 10% or below (India, Tunisia and Croatia) or virtually in the same level (China).

4.6.2.3. Total Capital under Basel III. The Standardised Approach

\(^{160}\) Hence, Difference (%) = \(\frac{\text{Average Increase Yellow Zone}}{\text{Average Increase Green Zone}} - 1\), i.e., the relative difference between the Average Increase in the Capital base for the techniques in the Yellow Zone vis-à-vis those in the Green one.
The Standardised Approach constitutes an alternative for those institutions that do not adopt the Internal Model Approach in Basel III following, in general terms, analogous guidelines to those cited in Basel II. Recalling Sections 2.7.2.a) and 2.7.3.a.1) and 2.7.4, it has some competitive advantages over its counterpart, particularly in the simplicity and easiness of implementation departments but, simultaneously, its serious drawbacks spring up at the time of building a capital level sufficient to withstand powerful adverse market swings.

The SA remains essentially intact in Basel III normative; the main alteration comes as the two extra capital layers are added to the basic 8% flat rate of the risk weighted assets; therefore, the reflections elicited in Section 4.5.2 keep their worth as well. In this sense, under the Basel II Capital Accord, the important infringements to the capital base occurred during the 2008 crisis would surely have placed institutions adopting this approach in bankruptcy position, notwithstanding which the scheme remains upheld by BCBS in Basel III. Chart 4.D.4 –Columns [4] and [5] depicts the outcome of an experiment consisting in the addition of the first of the supplementary buffers, the Capital Conservation Buffer, equivalent to 2.5% of the RWA to the 8% proportion (Section 2.7.4). The results recorded suggest a relative improvement with regards to Basel II (Chart 4.D.4 –Columns [2] and [3]) as the number of excessions diminish in Brazil, Hungary, Czech Republic, Indonesia, Argentina, Croatia, Malaysia, China and Lithuania. Employing Prescott’s construction (i.e., the bankruptcy-survival dichotomy) the last three countries add to Tunisia in avoiding bankruptcy, thus totalling four healthy portfolios (out of eleven, thus amounting 36%). The analysis then shows that the Standardised Approach remains ineffective to provide coverage in the event of major crises and its measure is in need of an urgent overhaul materialised in the form of a considerable enhancement of the 8% (of more than 31.25% given that the flat 10.5% does not exert any positive effect).

Consequently, even though the addition of the CCB envisaged by Basel III renders a marginal edge over Basel II –without neglecting its importance that may determine a safeguard to some institutions- the issue of moral hazard and discouragement to implement accurate VaR models resurface again at this stage. In view of the results, some banks might feel tempted to select the SA over any tougher

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161 India reports the same only one violation to the capital base.
IMA, and furthermore, that temptation could be on the rise once the CyCB is considered.

4.7. Minimum Capital Requirements applying Extreme Value Theory

The present section builds on the outcome of Sections 4.4.2, 4.5.1, 4.6.2.1 and 4.6.2.2 as it intends to cast a more detailed look on the Minimum Capital Requirements demanded by Basel II and Basel III through the Extreme Value Theory prism following a two-pronged strategy, taking each Capital Accord in turn. Therefore, two different stages will be performed under Basel II and Basel III employing EVT as it is considered that this most accurate model is able to provide a clear picture of the quantities to be constituted, the relationship with the extent of the losses they are designed to weather and the link with a key element of the formulas determining the MCR.

The basic idea underpinning the present Section roots in ascertaining the enticement provided by both Basel regulations to operate accurate VaR models for MCR calculation. The mechanics of the first stage boil down to gauging the size of the deficit that the different capital buffers would cover in every country using EVT. On the other hand, the second phase appears of a much more contentious nature as it is directed to evaluate one of the most controversial aspects of formulas (2.31) and (2.32), i.e., the size of both multiplication factors \( m_c \) and \( m_s \) in Basel II and Basel III as well.

4.7.1. Extreme Value Theory and Basel II

4.7.1.1. Maximum Losses Covered under Basel II

As previously explained, Chart 4.D.10 informs the Loss Coverage (Column [2]) and the Maximum daily loss (Column [3]) that the MCR based on Basel II directives (hereafter MCR\(_2\)) and calculated using EVT would deliver. The former concept (hereafter Loss Coverage or LC indistinctly) indicates the times the MCR\(_2\) would cover a loss the size of the maximum deficit recorded in 2008 (Backtesting period) whereas the latter quantifies the Loss Coverage, thus translating that amount into numbers\(^{162}\).

It could be appreciated that EVT provides enough capital to withstand potential daily losses of more than twice the size of the greatest shortfalls suffered by the indices in 2008 (except Argentina, 1.36) (Chart 14-Column [2]). These values mean daily losses

\(^{162}\) For instance, in the Croatian case, if Loss Coverage equals 2.41, the MCR\(_2\) employing EVT would cover the maximum loss posted in 2008 2.41 times. In turn, that statement means that the Maximum daily loss to be matched by EVT under Basel II would amount to 25.97% of the RWA, considering that the maximum daily loss in the Backtesting period was 10.76%. 
averaging 37% -Loss Coverage of 3.3- (minimum of 28.49% or 2.85 Loss Coverage (Malaysia) and maximum of 42.36% or 2.62 Loss Coverage (Czech Republic)) for Emerging markets, with a corresponding average amount of 20% -Loss Coverage of 2.44- (minimum of 10.83% or 2.16 Loss Coverage (Tunisia) and maximum of 26.94% or 3.82 Loss Coverage (Lithuania)) for Frontier stock exchanges (Chart 14-Column [3])163. Brooks, Clare and Persand (2000) asseverate that higher capital cushions constitute the usual response to increased volatility and the subsequent deficits it produces. In this vein, it appears that the Emerging markets experienced an upsurge in volatility much steeper than that of Frontier ones, fact that is translated in average losses augmenting 40% in the former against 12% in the latter (Chart 4.D.11-Columns [5] to [7]).

4.7.1.2. The issue of the multiplication factor m_

It is by no means a fact that the multiplication or hysteria factor m_
emerges as one of the determinants of the formula that establishes the capital buffers under the Internal Model Approach. However, despite its undeniable relevance, it has not been extensively dealt with in the academic literature. One of the few and most significant contributors are Danielsson et al. (1998) who single out the multiplication factor level (m_3=3) as the main snag of Basel II and identify it as a potential source of discouragement at the time of developing accurate VaR models, although falling short of suggesting likely remedies to this compromise.

Rossignolo, Fethi and Shaban (2012b) propose a sensitivity analysis to gauge the impact that alternative levels of m_ would exert on the MCR^2 envisaged in Basel II in light of the technique utilised in 4.7.1.1 above. Hence, the effect of the capital levels corresponding to the respective m_3 is reflected in the Loss Coverage and MCR^2 under that specific m_c, thus bringing about a reiteration of the aforementioned exercise through these alternative values decreasing in a 0.05 step, from m_c=3.00 to m_c=2.25. Chart 4.D.12 in Appendix 4.D compares, then, the result of the sensitivity analysis that pictures the Loss Coverage and Maximum daily losses for alternative levels of m_c in formula (2.31) and gives evidence that, as long as a leptokurtic technique like EVT is employed, the current level of m_c=3 renders excessive capital charges as shown in Columns [2] and [3]. Although m_c<3 brings about a decrease in LC, some values still

163 The index informing the Loss Coverage is related to the maximum daily loss recorded in the Backtesting period (i.e., 2008).
deliver considerable coverage against powerful market swings. For instance, all m","s between 3.00 and 2.50 would potentially demonstrate its worth during crisis events: on average terms, the Loss Coverage does not drill an LC index of 2 (approximately 2.50 in Emerging markets and 2.08 in Frontier stock exchanges), which translates into 31% and 17% MCR respectively. Of course, average values may disguise some strikingly distorting situations: as a token, the case of Argentina might well be handled fairly differently. Local authorities could deem a 15% MCR\(^2\) somewhat low to prevent a powerful adverse market swing and demand the constitution of a 17% buffer (m\(_c\)=3) or even a greater one (thus switching to Basel III regime, as in Section 4.7.2 below).

The task of establishing uniform Minimum Capital Requirements proves contentious given the peculiarities suffered by various countries and, moreover, the way in which everyone weathered the 2008 crisis. However, even acknowledging the limitations of applying averages, if Emerging markets were to obtain a reduction of m\(_c\) to 2.50, they would build a 30% MCR\(^2\) and still be able to cope with maximum daily losses in the region of 2.43 times the greatest 2008 shortfalls, in average values (MCR\(^2\) between 24% (Malaysia) and 35% (China) and LC ranging from 2 (India) and 2.90 (Brazil)). Frontier markets may also get m\(_c\) rather shrunken and still brace adequately for vigorous turmoil, albeit the decrease should perhaps be smaller to make room for Argentina. An m\(_c\)=2.75 might provide an average capital level of 19%, enough to shield average daily losses 2.24 times higher than those previously suffered\(^{164}\) (maximum values in Lithuania, with LC=3.50 with a Maximum daily loss≈25% and minimum amounts of 2 and 10% respectively recorded in Tunisia).

Finally, the existence of the 8% flat rate characteristic of the Standardised Approach surges as a recurrent temptation to adopt and casts serious doubts on the goodwill of banks to develop accurate VaR models for MCR calculation. It appears to be sufficient to cover everyday losses, or deficits that derive from common market movements (those lying in the centre of a hypothetical histogram or, alternatively, those variations with greater frequency). However, the danger remains in what Taleb (2007) denominates the ‘black swans’: sporadic atypical events that would eventually lead institutions to bankruptcy were they not adequately prepared for their occurrence. As emphasised in Section 4.5.2 above, the existence and permanence of the Standardised

\(^{164}\) It is acknowledged that national regulators may consider MCR\(^2\) with m\(_c\)=2.75 exiguous, therefore demanding a somewhat higher multiplication factor. In this sense, m\(_c\) € [2.85;3.00] could be evaluated as satisfactory. See Chart 4.D.12.
Approach in Basel II Capital Accord can only be attributed to some sort of political compromise between the BCBS and the participating national authorities and/or financial institutions: it clearly fails to fulfill the publicly stated aims during highly strung times.

4.7.2. Extreme Value Theory and Basel III

In response to the crisis originated by the subprime mortgage plight, the BCBS enacted new regulations comprised in scattered documents collectively referred to as Basel III Capital Accord that deal with the determination of MCR in a much tougher mode than in its predecessor Basel II (Section 2.7.4).

Setting aside Basel III’s provisions regarding the quality of new capital to be raised\textsuperscript{165}, the central issue about market risk quantification for Basel III directives boils down to the inclusion of the stressed VaR, which would supposedly bring about an increase in MCR of a considerable amount. In effect, the introduction of the sVaR amplified by the multiplier $m_c$ in the MCR\textsuperscript{3} expression (2.32) achieves the purpose proposed under the new capital accord. Furthermore, a closer look at (2.32) conveys the impression that the default level of those factors, the old $m_c$ and the newly-born $m_s$ (both currently set at 3) plays a crucial role in the determination of MCR\textsuperscript{3}. The impact of the addition of the sVaR is depicted in Chart 4.D.13-Column [2] in Appendix 4.D, from where it may be surmised that the Loss Coverage has been driven up by more than four times the maximum loss posted in the forecast year either in Emerging or Frontier markets. These figures represent an increase of more than 52\% and 86\% respectively over MCR\textsuperscript{2} (Chart 4.D.13-Column [3]); consequently, the potential deficits determined by MCR\textsuperscript{3} would look relatively high for both groups compared with shortfalls recorded in the crisis, thus making institutions separate excessively unproductive capital levels\textsuperscript{166}, as suggested by Column [4] in Chart 4.D.13, Appendix 4.D..

The above referred affirmation appears supported by a replication of the EVT-based sensitivity analysis carried out in Section 4.7.1.2 although in this occasion the scenarios encompass alternative values for both multipliers, $m_c$ and $m_s$, much in the same fashion as Rossignolo, Fethi and Shaban (2012b). Chart 4.D.14\textsuperscript{167} in Appendix

\textsuperscript{165} As mentioned in Section 2.6, the allocation of capital into equity and the rest of the components lie beyond the reach of the current thesis.

\textsuperscript{166} Basel III regulations mean an increase in MCR of 190\% in Argentina and 100\% in Malaysia, for example. See Chart 4.D.13.

\textsuperscript{167} MCR equal the maximum daily loss, as capital levels are built to match shortfalls on portfolios.
4.D portrays the MCR obtained as a result of the variation in \( m_c \) and \( m_s \) (levels are stated in the first column) assuming, as a starting point, the situation in Basel II, i.e., \( m_c=3 \), and successively varying the level of \( m_s \), from its exclusion to the current value \( m_s=3 \) envisaged by Basel III, applying a 0.05 ascending step. The fixed multiple of Basel II, \( m_c \), will in turn increase in 0.05 (from \( m_c=3 \) to \( m_c=5 \)) although every \( m_c \) level will be combined with the alternative \( m_s \), thus taking the total number of combinations using \( m_c=3 \) to 8. Chart 4.D.14 in Appendix 4.D, then, informs the MCR that emerge from formula (2.32) given the proposed changes in \( m_c \) and \( m_s \).

The outcome of the experiment brings about several interesting examples, some of which are exposed in Chart 4.D.14 (rows highlighted in bold letters are selected to be dealt with in Chart 4.D.15 in Appendix 4.D). Those individual cases are susceptible to be read as the MCR obtained in Chart 4.D.14 as well as the Loss Coverage indices and potential Maximum daily losses covered by those MCR (like Chart 4.D.13). Charts 4.D.14 and 4.D.15 should be analysed conjointly with Chart 4.D.16 in Appendix 4.D, which exhibits the variation that the instances singled out in Chart 4.D.14 represent over the MCR under Basel II and Basel III.

Initially, evidence points to the fact that \( m_s=3 \) could easily tempt banks to adopt less accurate models because total MCR\(^3\) turn out to be fairly disproportionate and, furthermore, were \( m_s \) to be somewhat reduced, the coverage could still remain substantial to face crises of important magnitudes. For example, under \( m_c=3 \) and considering \( m_s=1.5 \) –Chart 4.D.15-Columns [2] and [3]-, the Loss Coverage matched in terms of the greatest shortfall in the forecast period would edge 4.20 in Emerging markets and 3.43 in Frontier ones (average values that represent Maximum daily deficits beyond 45% for the former and 30% for the latter). Looking at Chart 4.D.16, the cited combination embodies an increase in the region of 27% and 46% respectively over Basel II mandate while simultaneously drives down Basel III demands in 15% and 19% respectively (average values\(^1\))\(^6\). In some occasions, national regulators may ponder the implementation of alternative combinations like \( m_c=3 \) and \( m_s=1 \), or \( m_c=3 \) and \( m_s=0.5 \), with approximately similar (average) results: for Emerging markets, those couples render 4.05 and 3.99 Loss Coverage indices with more than 44% of Maximum daily

\(^{16}\)As in Section 4.7.1.2 above, working with average values may disguise any shortage in MCR. For instance, using the \( m_c=3 \) and \( m_s=1.5 \) combination may be deemed insufficient for Tunisian national regulators: in this case, they could rule a different mixture for MCR such as \( m_c=3 \) and \( m_s=2 \), which automatically takes the MCR\(^3\) to 14.20%, thus setting the Loss Coverage = 2.84. These values represent an increase over MCR\(^2\) of 31.11% and a decrease with respect to Basel III of 10.64%.
loss (Chart 4.D.15-Columns [4] to [7]); furthermore, as Chart 4.D.16-Columns [4] to [7] shows, the first pair yields a 22% increase over Basel II MCR and a reduction of 19% over Basel III whereas the second one takes those values to +20% and -20% respectively. On the other hand, for Frontier stock exchanges the Loss Coverage is 3.27 and 3.25 respectively, thus matching Maximum daily losses in the region of 28% (Chart 4.D.15-Columns [4] to [7]); the increase over Basel II regulations situates around 36%-37% whereas the diminution with regards to the intended Basel III amounts to 23% for \( m_c=3 \) and \( m_s=1 \) and 14% when \( m_c=3 \) and \( m_s=0.5 \) (Chart 4.D.16 Columns-Columns [4] to [7]).

Recapitulating, the above results suggest that the provisions stated in Basel III may appear relatively excessive were accurate heavy-tailed models to be applied. Bearing in mind that there is enough evidence to consider EVT as the sharpest representation, Section 4.7.1.1 already hinted at the possibility that the multiplier \( m_c=3 \) acts as a deterrent for the development of these kind of techniques, consequently provoking a flight-to-inaccuracy given that, in some occasions, schemes falling in the Yellow Zone end up constituting less capital than those in the Green Zone. Unfortunately, the situation only seems to aggravate when Basel III and its stressed VaR comes to the fore: the proofs hitherto gathered have shown that, again, the most reliable model –EVT- remains excessively punished: the facts collected allow to ensure that the fixed multiple \( m_s \) can almost certainly be lowered without affecting the capacity of the institutions to deal with crises of important magnitudes\(^{169}\).

Connected with the previous reflections, and in line with the rationale exposed, a strong inkling regarding the lack of incentives to develop precise models in Basel III emerges. Looking at it from a different angle, the extra conservatism implied in expression (2.32) may also be attained excluding the sVaR term while simultaneously augmenting \( m_c \), provided heavy-tailed models with performances worth of the Green Zone in Backtesting are used.

It is important to emphasise again, that the BCBS discourages precise schemes given that the additional precautions demanded by Basel III may also be accomplished omitting the sVaR term in (2.32) and simultaneously augmenting \( m_c \), provided heavy-tailed models not penalised in Backtesting are employed. Therefore, recalibrating \( m_c \) at

\(^{169}\) Not only do these conclusions appear to apply to Emerging and Frontier markets; Rossignolo, Fethi and Shaban (2013) employ analogous rationale to evaluate long and short positions in PIGS (Portugal, Ireland, Greece and Spain) to find a similar conclusion: \( m_c=3 \) seems too excessive for accurate models already complying with Basel II Capital Accord.
values higher than 3 would drive $MCR^3$ upwards and avoid the unnecessary computations derived from the addition of the sVaR (always taking for granted the application of accurate heavy-tailed representations). Hence, setting $m_c$ to 3.5 or 4 would push $MCR^3$ upwards 17% and 33% respectively with reference to Basel II directives (Chart 4.D.16-Columns [8] and [10]) while simultaneously declining 21% and 10% in Emerging markets and 32% and 22% in Frontier ones (average percentages) evaluated against the sVaR proposition (Chart 4.D.16-Columns [9] and [11]). Although Basel III strengthens the capital base, thus achieving one of its most overarching objectives, an enhanced version of Basel II accomplished setting $m_c=3.5$ or $m_c=4$ might still allow entities to overcome massive losses of around three times the maximum loss recorded in 2008 for both sets of countries. On average, multiples of 3.5 and 4 would shield Emerging markets against a 43% (LC=3.84) and 49% (LC=4.49) daily shortfalls with 24% (LC=2.85) and 27% losses (LC=3.25) in Frontier ones (Chart 4.D.15-Columns [8] to [11]).

The current exercise highlights that it should be desirable to confer national regulators some kind of flexibility to rule over the feasibility to adopt one of the alternative schemes, namely Basel II or Basel III, in accordance with the results delivered by the most accurate model (i.e., the technique attaining the highest capital buffer), or furthermore, to opt for an alternative within one of the frameworks. In this vein, three examples may be cited. In the first place, a glance at Chart 4.D.15-Columns [8] to [11] could suggest that Argentine regulators may deem an enhanced $MCR^2$ below two times the size of the worst loss in the forecast period (LC=1.59 if $m_c=3.5$ and LC=1.84 when $m_c=4$), the sVaR term could be brought in using amounts like $m_s=0.5$ or $m_s=1$ (Chart 4.D.15-Columns [4] to [7]) and drive the Loss Coverage to 2.39 and the $MCR^3$ to a considerable value of 31%. In the second place, Chart 4.D.16-Column [11] depicts that Hungary and Czech Republic record a soar in $MCR^3$ in the region of 3% and 1% respectively were $m_c$ to be set at 4. In this occasion, national regulators might choose to remain under Basel II framework given the fact that the $MCR^2$ amounts to 39% and 42% respectively -judged enough to withstand considerable adverse swings- or, on the contrary, opt to reward institutions applying EVT with a decrease of $m_c$ to some value in the interval $[2.50; 3)$ and still match more than 33% daily losses (for

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170 Given that $k=0$ for EVT models in all markets, $m_c=3(1+k)=3$ or the corresponding figure applied in the sensitivity analysis. Analogously, $m_s=3(1+k)=3$ or the respective constant stated. The former observation applies to Chart 4.D.15.
m_c=2.50 in Chart 4.D.12-Columns [22] and [23]). Thirdly, in Lithuania, Basel III regulations could well be rated as a bit excessive on the grounds of the Loss Coverage index bordering 6 times the maximum loss in 2008, or, equivalently, the Maximum daily loss edging near 42% (Chart 4.D.13-Columns [2] and [4]) applying EVT. In order to reduce this excessive capital buffer, Lithuanian supervisors could scrap the sVaR and stick with an enhanced version of Basel II instead, courtesy of m_c=3.5 (for instance), thus reaching a Loss Coverage of 4.46 tantamount of a MCR of 31% (Chart 4.D.15-Columns [8] and [9]).

4.8. Concluding remarks

The current Chapter compresses several issues worth of analysis, from the characterisation of the markets subject of the study to the performance of the models in light of Basel II and Basel III dispositions.

The first part of the Chapter shows the growing importance of the Emerging markets in the world concert, thus motivating the commitment of the current thesis. The academic community has –understandably- studied the development of the subprime mortgage crisis in the mature markets which originated and spread it around the globe with devastating consequences. Nevertheless, even though research showed that, through the prism of the classical Mean-Variance framework, the Emerging markets bear inherently more risk (and yield higher returns) than mature countries, they endured the subprime plight in a much better shape than their counterparts, thus enhancing its attraction. On the other hand, the Frontier category has rarely drawn attention from the researchers, either because of its lack of depth and relevance in the investment circles or its failure to comply with basic characteristics about regulatory environment, custody, settlement and dealing of trades. Although the preliminary analysis, carried out in terms of the MV structure, exhibited more extreme risk and return profiles than any other group (Developed and Emerging), Frontier markets also avoided a collapse in the style of that of the mature countries in the wake of the 2007-2008 crash. These seemingly conflicting patterns also merit their inclusion in the thesis.

The statistical analysis of the empirical time series belonging to all the stock markets involved confirmed the seminal conclusions of Fama (1963) and Mandelbrot (1965). In this regard, every string of returns exhibited skewed and leptokurtic patterns which, considered together, point to densities diverging from the widespread normality paradigm, which should not be overlooked during modelling stages. Deviation from
gaussianity prop up important consequences that help to explain the results obtained when estimating VaR and the consequent MCR based on them. Unquestionably, then, the leptokurtosis verified fosters the apparition of extreme events more frequently than predicted by the Normal density and, therefore, their concentration on the tails of the distribution. Furthermore, the documented asymmetry indicates that the shape of the distributions ought not to be analysed through the prism of symmetric densities, like the Normal or Student-\( t \), consequently paving the way for specifications like EVT which model both tails separately, paying heed to their specific characteristics\(^{171} \).

Last, but not least, the autocorrelation patterns displayed by the empirical time series bolster the adoption of conditional representations to the detriment of unconditional ones.

Prior to eliciting the final thoughts on the outcome of the IMA exercise, it is essential to mention that, unfortunately, the SA remains utterly powerless to fend off the devastating effects that violent market swings may exert on the financial health of any company relying on it. In effect, even under Basel III framework -which demands supplementing it with the addition of an artificial buffer-, the fixed flat rate that acts as a kind of unconditional technique cannot withstand crises of colossal magnitudes, unfortunately witnessing a ghastly parade of ruinous results. However, as the SA is still eligible under Basel III, banks may feel entitled (and tempted) to switch to the SA regime, erect lower capital base and, eventually, try their luck in the markets. The adverse incentives to pick the Standardised Approach, constitute an area that must be addressed by the BCBS, either by completely overhauling the scheme or, directly abolishing it.

Rossignolo, Fethi and Shaban (2012b) remark that, regarding the behaviour of the models, the evidence garnered appears enough to discard Linear models and HS. Accordingly, although the Standard Deviation for Linear models is allowed to change in time, it is unable to capture the dynamics of the latent volatility. No improvement is virtually recorded employing a heavy-tailed Student-\( t \) distribution instead of the Normal one as the underlying risk measure is inherently flawed. HS reaffirms all the disadvantages mentioned in Section 3.3; for example, the fact that the last VaR overcomes the 60-day VaR average in Hungary reveals the presence of the Ghost or Shadow effect (Penza and Bansal (2001)). The use of the empirical distribution of residuals in the FHS alternative delivers sizeable gains in some Emerging markets (in

\(^{171} \) Rossignolo, Fethi and Shaban (2013) analyse the relative advantage of EVT when dealing with the right and left tails (short and long positions respectively) in Portugal, Ireland, Greece and Spain.
the Student-t distribution setting), albeit in overall it levels the performance of the CV models. One further aspect to underline is, unfortunately, the unreliability of the FHS and CV specifications, as the former perform better in some markets whereas the latter yield more adequate protection in the rest of the stock exchanges.

The significant progress derived from conditional models highlights the need to resort to these schemes. The results imply that the EGARCH technique brings no significant advantage over GARCH, in turn meaning that the leverage effect marked by Chopra, Lakonischok and Ritter (1992) and Cho and Engle (1999) is hardly noticeable, given that the penalties suffered by EGARCH-t are equal or greater than their GARCH-t counterparts (except Argentina). Moreover, the density assumption exerts dominance over the particular specification: GARCH-t and EGARCH-t improve the performance of their Normal similes, therefore making desirable and unavoidable the employment of heavy-tailed distributions. The fact that some configurations reveal more sensitive to the basic model employed whereas other schemes pay more heed to the distributional assumption add to the uncertainty at the time of selecting a technique.

Findings are highly supportive of the EVT approach in comparison with its competitors. Its application would have shielded institutions from huge losses produced in the event of the 2008 crisis and prevented the constitution of extra-capital while simultaneously building up a capital base not excessive in relation to the rest of the specifications, yet escaping unscathed the Backtesting procedure.

In terms of the Basel III Capital Accord, the proofs gathered provide rich quantitative information about the impact of the BCBS’s proposal to increase MCR constituted by banks in Emerging and Frontier markets. The figures obtained suggest several recommendations that, in the future, the BCBS should take into account properly at the time of analysing how well the current normative works and might be included in upcoming Monitoring Exercises.

To begin with, supra-national regulators should demand usage of techniques capable of dealing with large fluctuations in the future -particularly EVT- hence discouraging or banning the employment of traditional methodologies which provide capital buffers only for common market variations (most notably Linear, HS, FHS or Normal specifications). For every market researched, the intended variation characterised by sVaR applying a base multiple of m_s=3 in addition to the current MCR appears somewhat excessive and immobilises funds unnecessarily. Therefore, both factors could be dissociated and calculated independently at the discretion of national
supervisors in view of the particular patterns of the respective markets. In this sense, one viable alternative may involve working out a combination between current MCR and a lighter sVaR value. Another potential version would see sVaR term scrapped and the base multiplication $m_c$ factor lifted to some number between 3 and 4 (for example), at the discretion of the corresponding domestic controller.

This last suggestion echoes the Japanese stance regarding the BCBS’s proposition concerning market risk demands. It is believed that the adoption of the planned stringent measures will surely give birth to disincentives to devise accurate VaR methodologies, as the shortcomings of the models will be compensated by the combination of add-in factors and penalty constants. In contrast, the implementation of sound heavy-tailed techniques like EVT could ensure extensive coverage, avoid the building of superfluous capital buffers and at the same time allow the institutions to match huge future losses without incurring in high development costs to estimate sVaR.
Chapter 5

Conclusion
5.1. Introduction

Financial regulation is an intrinsically cyclical activity constantly oscillating between periods of permissiveness and utterly severe reaction, to such an extent that it becomes increasingly difficult to strike a balance between both extremes. It remains an almost impossible task for politicians, regulators and market participants to reach an agreement about the calibration of the regulatory mandates.

The political side of financial regulation is only tangentially an electoral topic of debate, and relatively long spells of time elapse without proper treatment of those issues. However, that relaxed interval is often marred by market collapses which provoke politically unacceptable costs that barely any government is willing to afford. All things considered, market failures usually foster the political opportunities to carry out a thorough shake-up of the established order usually in the shape of sweeping reforms that, in some circumstances, fail to address the ultimate glitches of the existing regulations. Hence, in some occasions, the radical restructuration frequently aims at pre-existing social, political and economic objectives and, in this fashion, avoids a reiteration of the catastrophes that originated the need for changes.

Modifications in regulations are also subject to a variety of circumstances that alter their course to some extent. In the first place, they often trail political swings and, in this sense, it is not rare to witness a supervisory overshoot after crisis of tremendous magnitudes. Proofs of these are the 1996 Amendment to Basel I, eventually included in Basel II, that was enacted in response to the crashes in late 80s and early 90s and the revision of the Market Risk Framework of 2009 in the wake of the subprime crisis of 2007-2008: in both occasions, substantial increases in capital and liquidity requirements were demanded. In the second place, political times carry a significant delay to come strictly and completely into force after the occurrence of the event: for instance, following the last market disaster, supranational regulators overhauled the existing order in May 2009, and introduced it into the framework of Basel III starting in January 2013, only to operate in full swing in January 2019. Even acknowledging that sometimes the shake-up may be deemed thorough and banks need time to adjust their capital levels, structures and liquidity ratios to converge to the new indicators, market conditions will surely have changed from the time that the necessity was detected to the time the modifications become mandatory. Thirdly, the supervisory extent bears a high negative correlation with the point of the business cycle; in this vein, it is common for politicians to push for light touch regulations when the economy picks up and increase them when
the economy slumps or reaches a trough. In general terms, dispositions are softer when financial industry adds to the growth of the GDP and become tougher after the hitherto allowed unsound practices and the untamed risk burst out of control and begin to dent the perspectives of the economy.

Strangely enough, political reaction could be stated as myopic as it often lacks the strategic vision required because, in some occasions, regulations do not provide what the economy needs in the right moment. In this sense, one of the unpleasant effects brought about by the 2007-2008 market crash has been the so-called credit squeeze, i.e., the reluctance or impossibility on the part of the banks to disburse money to households, SMEs or even large companies. That action, in turn, forced economic agents to postpone expenditures, investment projects or anything that would imply spending. However, regulations contained in Basel III Capital Accord stipulate a massive increase in banks’ capital base, fact that involves immobilising huge amounts of funds in order to abide by the norms enacted instead of applying them to ease the credit crunch. Perhaps justifiably, the BCBS identified the procyclicality of Basel II as one of its worst snags, and tried to amend it by toughening the MCR determination formula. Notwithstanding that, even though the move appears sound with a view to restore the health and soundness of the banks and reduce the risks that unbalanced institutions pose to the whole system, it presents an unwanted side effect by compressing the credit offer even more. Acknowledging the difficulty of the task, the problem boils down to calibrate the phase of the business cycle with the quantity of capital as well as the liquidity requirements, which, unfortunately, has not been considered by the BCBS, influenced as it may appear by political demands. But although regulations are increasingly of international reach, they may also be subject to national political interference, and, in this vein, Basel III Capital Accord may stumble in those countries suffering from weak growth or dark economic perspectives. Nevertheless, it is by no means a fact that supervisors should operate desynchronised with the respective national regimes, thus detaching themselves from any political influence.

As it stands, international regulators do not regard the point in the business cycle to draft new rules, thus rendering their self-adjustment or recalibration in the case of regulatory overshoot an almost impossible task. Furthermore, there does not seem to be

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172 For instance, the creation of the Financial Service Authority (FSA) in 1998 came in the aftermath of the alleged failure of the Bank of England to prevent the demise of BCCI and Barings. However, in the coming years, the Bank of England will regain its former powers and supervise banks again after the FSA was not able to forestall the subprime crisis.
any consensus about what macroeconomic indicators could be considered in order to evaluate whether the mandates hinder the normal development of the economy. Therefore, two potential solutions hover in the background of this apparent cul-de-sac: in the first place, legislation might include sunset clauses or pre-specified dates for revisions, thus generating opportunities to raise the pressure or water down the existing regime; those reviews would, in principle, guarantee an approach more connected with the economic needs. In the second place, assuming that the first alternative lies out of the radar, it would be desirable on the part of the supervisors to separate the wheat from the chaff and introduce an adequate and fair treatment for the great quantity of models available to carry out the compulsory calculations.

Granting that the second alternative will prevail, market risk is bound to be dealt with in the manner envisaged in Basel III Capital Accord, i.e., with the dual SA and VaR-based IMA approaches to constitute MCR selected by each bank according to its criteria; while the former remains as a fixed percentage of the size of the portfolio, the latter can be estimated through diverse representations with results varying to great extents. The current thesis investigates the behaviour of some of those techniques and analyses their performance, alongside that of the SA in the event of the subprime crisis of 2007-2008 in the context of the relatively unexplored Emerging and the even less studied Frontier markets. Crisis times are crucial to gauge the accuracy and reliability of the schemes given that the failure to set up adequate capital buffers may end up in bankruptcy or bailout at the taxpayers’ expense.

The core of the research, then, aims at shedding light on the ability of Basel Capital Accords II and III to establish adequate MCR to withstand market crises in Emerging and Frontier markets. The assessment is carried out for both the SA and the VaR-based IMA, which eventually includes several models employed in its determination. In general terms, the investigation attempts to analyse the precision of some VaR representations and simultaneously ascertain the influence that those VaR schemes exert on Basel’s MCR formulas to eventually determine whether Basel II could have served its purpose had the appropriate scheme been employed. Additionally, the study examines in detail the impact of the introduction of Basel III enhanced mandates and considers a window that would, in principle, allow some sort of relaxation while, at the same time, preserve the intended aim. Finally, the thesis also strives to find whether banks are incentivised to utilise the most adequate VaR models in such a way that Basel
Capital Accords deliver some sort of reward from the application of accurate techniques that could avoid the apparition of moral hazards.

Accordingly, the main and complementary findings may hopefully –even acknowledging the limitations- enlighten likely implications to the problems stated, narrated in Sections 5.2 to 5.4. Section 5.5 states the boundaries to the conclusions put forward, while Section 5.6 sketches likely directions to pursue further investigations.

5.2. Main findings

The facts collected throughout the study support the reflections about the whole process. To begin with, it is interesting to assess the unfolding of the subprime crisis of 2007-2008 from the point of view of the Emerging and Frontier markets. Those countries -mostly commodities exporters- have been aided by the high prices recorded in the last decade, which contributed to enhance the purchasing power of their population and, consequently, augmented the GDP at a time when the GDP levels of the developed world started to shrink. Nevertheless, even though their absolute and relative global importance has been steadily on the rise, not many researchers have focused on them.

Many authors have demonstrated that VaR is not the optimal risk measure; in effect, its lack of coherence places it at loggerheads with the basics of the investment theory, i.e., the diversification. Even though VaR still presents many other (less significant) drawbacks, the BCBS has decided to stick to it, at least until and beyond the year 2019 when the Basel III Capital Accord is to be applied in full force. However, the fact that the BCBS has opened up a consultation period to review some of the dispositions of Basel III and particularly the issue of VaR and its probable replacement for other alternative and more theoretically apt market risk measures like Expected Shortfall or other spectral risk measure conveys some hope that, at the end of the day, regulators have finally grasped VaR’s limitations.

The thesis gathered evidence belonging to Emerging and Frontier stock markets to provide indications to address the main guidelines set in Section 1.6 “Research objectives and questions”. Accordingly, in the context of arguably one of the most powerful market crisis with still lingering effects, it is in principle possible to gauge the aptitude of Basel II and Basel III to erect MCR adequate enough to withstand market turbulence. Furthermore, the elements garnered allow the assessment of several VaR schemes regarding the capital constitution purposes as well as its comparison with the SA still available under BCBS rules. Finally, considering that the Basel III Capital
Framework represents “a decisive breakthrough” (BIS (2010)), the thesis ventured to evaluate the effect that the introduction of Basel III would exert on banks exposed to the Emerging and Frontier stock markets as compared with its predecessor Basel II.

5.2.1. First research question

a) Does Basel II MCR provide adequate capital protection to withstand huge market turmoil? Is the VaR-based IMA superior to the SA?

The surprising news is that Basel II may, under certain conditions, supply with enough coverage to weather huge market turmoil. However, at this stage it is essential to draw out the traditional distinction between the SA and the VaR-based IMA because, in effect, while the SA is unable to withstand market crises of considerable magnitudes, the evidence harvested appears sufficient to affirm that the trouble about Basel II’s MCR does not reside in the formula itself but in the schemes employed to estimate the MCR under the VaR-based IMA instead.

The SA offers an easy route to constituting MCR by means of the application of a flat rate to the RWA of the bank. It might represent a viable alternative as long as markets do not jitter violently, but its performance lies far from satisfactory when trying to forestall sudden jerks, as shown in Rossignolo, Fethi and Shaban (2012b, 2013). The SA has been maintained since Basel I on similar grounds to the Cooke proportion, hence, in view of the market disasters of the late 1990s, it seems almost inconceivable that no major adjustment has been carried out. At a brushstroke, the SA reveals itself clearly inferior to the VaR-based IMA, reality chiefly due to the insensitivity to the fluctuations experienced by the market; in effect, while SA only demands the computation of a fixed percentage of the RWA, IMA requires the calculation of VaR, which is clearly influenced by the ups and downs in market values.

b) How precise are the different VaR models?

Does EVT deliver a more satisfactory outcome for market crises?

The overture of the first research question automatically relates to the performance of the VaR-based IMA. Considering the outcome of the exercise, and acknowledging the shortcomings of the Backtesting procedure designed by the BCBS, the expression (2.31) stated in Section 2.7.2 could deliver adequate protection against massive market crises as long as the right model to estimate VaR is employed. In executing that task, Rossignolo, Fethi and Shaban (2012b) highlight the importance of picking the right
model to raise appropriate MCR to avert further shortages. It would be essential, then, to adopt methodologies capable of mimicking the stochastic behaviour of the time series under analysis: the techniques, then, should pay heed to the highly leptokurtic and asymmetric nature of the empirical distributions verified across a vast quantity of financial assets, particularly measured in daily or even higher frequencies.

The proofs obtained are fairly conclusive in what refers to some of the specifications tried. The Linear models, which feature the widespread Standard Deviation enhanced by the Normal or Student-\( t \) distributions (the latter modification suggested by Penza and Bansal (2001)) employing rolling windows exposes all the disadvantages of the structure, namely the excessive reliance on the Normal density and the equal weighting scheme. HS records a slight improvement over the Linear configurations, even though it is still unable to repel the erosion in the capital base caused by sudden adverse market shakeups in light of all the snags detailed. Perhaps, the lure of the HS resembles a mermaid’s kiss: it may be deceiving because it behaves correctly during common market movements, but end up absolutely incapable of building a sufficient MCR for rainy days.

The adoption of conditional models, coupled with any theoretical distribution like the Normal or the heavy-tailed Student-\( t \) or the empirical one in the FHS variant unquestionably leads to better performances. However, the thesis vindicates Beder (1995) given that, in line with Rossignolo, Fethi and Shaban (2012a, 2012b and 2013) the outcomes are inconclusive at the time of certifying the best model or distribution. In that sense, the imprecision resides in the fact that if the empirical standardised distribution were to be utilised, the specification would reveal more important than the density utilised in the estimation of the volatility forecasts, for the difference between the Normal and Student-\( t \) does not appear significant enough; conversely, in the case of the CV models, the distributional assumption prevails over the particular scheme: GARCH-\( t \) and EGARCH-\( t \) turn out to be more accurate in terms of VaR outcome in Backtesting. Consequently, despite the undeniable improvement in performance, FHS and CV representations showcase inconsistent results.

Findings foster the application of the Extreme Value Theory approach in comparison with its competitors as long as the Generalised Pareto Distribution is correctly fitted. Although the adjustment appears somewhat tricky and subjective, EVT constitutes the most accurate alternative to erect appropriate capital defences to mitigate the adverse effect of high volatility pikes on the equity structure of the firm.
Notwithstanding the relatively novelty of its application to financial purposes, EVT reports an enhanced potential even under unconditionality in Rossignolo, Fethi and Shaban (2012a). The authors venture to show that, detaching the estimation from any dependence on time, the EVT appraisal still delivers adequate MCR under Basel II for a range of Developed and Emerging stock markets considering a Backtesting period that spans two years (2007 and 2008), though the lack of sensitivity with respect to the market momentum –otherwise captured by the GARCH pre-whitening in the current thesis- may end up immobilising excessive funds.

The application of EVT would have shielded institutions from huge losses produced in the event of the 2008 crisis and prevented the constitution of extra-capital while simultaneously building up a capital base not excessive in relation to the rest of the specifications.

5.2.2. Second research question

*Does Basel III MCR contain significant improvements for both VaR-based IMA and SA in relation to Basel II MCR to forestall market crises? How do VaR models perform within the new structure?*

Basel III is, in many respects, an upgrade to Basel II aimed at limiting the havoc that a 2008-style crisis could wreak on the financial health of any institution. To put the improvements in the BCBS terms (2011a), it contains a twofold objective to be carried out through a variety of actions and provisions:

1) The reinforcement of the global capital framework, via:
   a. Increase of the quality, consistency and transparency of the capital base;
   b. Enhancement of the risk coverage;
   c. Constitution of a leverage ratio to supplement the risk-based capital requirement;
   d. Reduction of the procyclicality and promotion of countercyclical capital buffers;

2) The enforcement of a global liquidity standard, via:
   e. Introduction of a Liquidity Coverage Ratio;
   f. Introduction of a Net Stable Funding Ratio;
   g. Introduction of permanent Monitoring Tools.
As aforementioned, the current study focused on strengthening the capital base via the diminution of procyclicality and, tangentially, the presentation of countercyclical capital buffers.

Strictly speaking, the constitution of the extra capital layers CCB and CyCB do constitute an improvement with reference to Basel II as both might eventually come to the rescue of any approach in case of extensive damage to the core capital estimation calculated through SA or VaR-based IMA. However, both are exogenous to the methodology employed and, as such, they cannot be regarded as true ‘improvements’ to the SA or VaR-based IMA, which ultimately means that the appraisals need to be evaluated in isolation as featured in the respective paragraphs of the Basel III Capital Accord.

Setting aside both extrinsic capital layers, Basel III does not alter the situation of the SA at all as the computation remains basically unchanged since the previous mandates with its simplicity and shortfalls. Before the application of CCB and CyCB, SA looks undoubtedly vulnerable to abnormal market swings, fact verified by Rossignolo, Fethi and Shaban (2012a, 2012b) for a sample of different markets.

On the other hand, the important alterations have been operated on the VaR-based IMA before the non-related supplements; most notably, the modifications have been carried out in order to avoid the procyclicality exhibited by the scheme as well as other drawbacks described by Danielsson (2002).

The quantitative information about the impact of the BCBS’s proposal to increase MCR constituted by financial institutions in Emerging and Frontier stock markets did not represent an unquestionable proof of the superior quality of the newly enforced MCR given that, in many occasions where inaccuracy reigns, the increased MCR figures relate to the formula enhanced by the addition of the sVaR term. The performance of the models under Basel III is principally dictated by Basel II’s Backtesting, which allows the disqualification of the nonviable and the scaled penalties for the inaccurate ones. However, the application of Backtesting surcharges to Basel III formula lets inherently weak techniques artificially augment their MCR via add-ons while still leaving relatively powerful market swings uncovered. Consequently, the sVaR term in Basel III bails out several unsound representations disregarding the really accurate ones, which must bear the same complement applied with no concessions for sharpness. To make matters worse, in some opportunities, weak performers end up constituting less capital than those that escape Backtesting penalties, hence propping up
the question about the motivations to use precise techniques (Section 5.3). One of the major inconsistencies of Basel III resides in the likely 'rescue' of some structurally deficient models, hampering the chances of models specially designed to deal with violent market turbulences like those based on EVT or leptokurtic distributions.

The intentions hovering around Basel III could be regarded as healthy, and, simultaneously, it is tempting to asseverate that the performance of both schemes, SA and VaR-based IMA is eventually improved. However, the evidence reaped shows that the two remain essentially intact and further influenced by the extra layers, CCB and CyCB. Setting aside one (and eventually the two) cushions exhibits the resulting capital shortage utilising the SA. On the other hand, the application of the sVaR for the VaR-based IMA does bring a significant swell in the MCR which primarily aids those models with feeble Backtesting outcomes (albeit in the Yellow zone) and unnecessarily hinders the employment of techniques passing Backtesting\textsuperscript{173}.

5.2.3. Third research question

*Could Basel III MCR be to some extent relaxed –conserving the basic structure- and simultaneously preserve its main target in the event of market crisis?*

The former topic paved the way to ponder about likely ways to address the issue of the relationship between MCR and the models that estimate the VaR.

Initially, it is imperative that Basel III should emphasise that there is not any escape route for SA: even though the supplementary buffers may be considered as a step in the right direction, in order to constitute an eligible alternative to provide assurances in case of adverse market swings its levels must be thoroughly raised to some percentage of the RWA similar to those of any fat-tailed scheme or, more precisely, EVT.

Regarding the VaR-based IMA, supra-national regulators must stimulate the usage of techniques capable of dealing with large fluctuations in the future -particularly leptokurtic models or, ideally, EVT- hence discouraging or banning the employment of traditional methodologies which provide capital buffers only for common market variations (most notably Linear, HS, or Normal-bound specifications). Accordingly, the BCBS should calibrate the fixed multiples in relation with the precision of the scheme utilised by the bank as a direct route to its achievement. In this sense, for every market

\textsuperscript{173} It is important to bear in mind that Basel III abides by the results of Basel II with reference to the Red Zone: models that are disqualified under Basel II keep their status under the new agreement.
researched, the intended variation characterised by sVaR applying a base multiple of $m_s=3$ in addition to Basel II’s MCR appears somewhat excessive and would freeze funds unnecessarily as long as institutions were inclined to some sort of heavy-tailed technique, namely EVT. At this stage, two likely alternatives to arrive at some kind of compromise emerge: in the first place, it could seem sensible to work out a combination between the Basel II’s MCR and a decreased sVaR, i.e., a lighter version of Basel III’s MCR. The thesis shows that, applying EVT, the most precise methodology, and leaving intact Basel II’s expression, the fixed factor $m_s$ could well be lowered and still provide substantial coverage for considerable adverse market movements. It is entirely possible that that reduced value should result augmented according to a sliding scale until the upper limit reaches Basel III’s figure of 3 in line with the precision of the scheme involved. The second alternative is directed to the core of the VaR-based IMA in Basel III because, acknowledging the move engineered by the BCBS to increase MCR in a sizeable amount, the proofs obtained justify to waive precise techniques like EVT the application of the provisions established in Basel III. In effect, a similar outcome may be obtained by suppressing the sVaR term in the formula (2.32) and, simultaneously, raising the fixed multiple currently set at 3, which would constitute a more practical and cost-effective solution incidentally endorsed by the Japanese Bankers Association (2008).

5.3. Complementary findings
The aforementioned principal conclusions highlight a couple of interesting side aspects susceptible of being noticed. One potentially relevant topic hidden in Basel Capital Accords refers to the existence of moral hazard; in effect, even though it appears in Basel II, it has not been corrected in Basel III and, therefore, some problems mount.

The question of the moral hazard seems simple to understand. As noted in Rossignolo, Fethi and Shaban (2012a, 2012b), Basel II already contained disincentives to apply precise VaR models and could certainly lead banks to select the SA to the detriment of the VaR-based IMA. Furthermore, it might also have enticed institutions to favour inaccurate schemes like the simple Linear, HS, and even the FHS or the conventional CV models featuring the typical Normal distribution instead of highly leptokurtic ones. In fact, for those models belonging to the Yellow Zone with elevated penalties, it was still possible to obtain MCR below the line established by the one that always situated in the Green Zone, i.e., EVT. Therefore, the MCR configuration in
Basel II acted as a kind of bail out for imprecise representations, though many of the
techniques proved insufficient to fend off massive market slumps.

The advent of Basel III with the enhanced MCR formula and the additional
capital layers could not remedy the predecessor’s glitch and, unfortunately, boosted the
moral hazard and the disincentives predicament. In effect, in the first place, the addition
of CCB and the likely CyCB to the SA still available hampers the chances of
institutions to pick the VaR-based IMA route. Moreover, the mere existence of the
sVaR term in Basel III specifications may, again, tempt banks to resort to models that
comfortably lie in the Yellow Zone, endure penalties for inaccuracy and still deliver less
MCR than EVT which passed a 2008-style crisis with flying colours.

Another potentially relevant complementary finding is given by the somewhat
excessive amount of capital demanded by Basel III, even before the application of CCB
and the contingent CyCB. The figures delivered in the current thesis are descriptive of
the situation, and, in this sense, the outcome of the Monitoring Exercises of June 2011
(BCBS (2012b)), December 2011 (BCBS (2012c)) and March 2012 (BCBS (2013c))
carried out to gauge the impact of the implementation of Basel III capital rules, foster
the conclusions of the present study, although BCBS did not disclose the nature of the
specifications employed by the participating banks in the study.

5.4. Implications

Basel III was enacted as the regulatory response to the shortcomings of the Basel II
Capital Accord, which proved unable to forestall a massive market crash. Nevertheless,
despite the new provisions unquestionably leading to a substantial increase in the capital
base, they also fail to address the disincentives to implement models that correctly
mirror the behaviour of the time series of the assets in the portfolio.

The findings of the current thesis could imply some courses of action that may,
at least partly, remedy the snags in Basel III. They would entail –perhaps- a different
approach towards the constitution of MCR, nevertheless mitigating the otherwise
dangerous incentives problems.

To start with, it should be desirable to phase out or drastically reform the SA.
True as it may be, SA is still necessary because the complexity and the lack of market
values for many exposures do not allow the development of VaR models. SA should
then be preserved for those positions and, simultaneously, the BCBS must bring it to its
demise or complete reformulation in order to avoid banks picking it to soften the MCR.
Secondly, the VaR-based IMA appears in need of being overhauled in order to keep accuracy incentives aligned. In effect, it is high time that supra-national and national regulators began acknowledging the precision of the VaR representations in MCR calculation and rewarded the sharpest ones calibrating the fixed multiples in Basel III configuration, either annulling or softening the sVaR term or granting them permission to operate under Basel II provisions with enhanced \( m_c \) constant.

Finally, given that the objective of the MCR is to protect banks against major market turmoil, the revision may be complemented with the permanent or temporary ban of some specifications that have proven unable to fend off slumps of considerable magnitudes, like the traditional Standard Deviation, the HS and some variants of the FHS and the Conditional Volatility models, particularly those featuring the Normal distribution. The prohibition, then, could come into force in parallel with the employment of the Countercyclical Capital Buffer, which should remain at the discretion of the respective national supervisors who would rule on its application and subsequent size in times of great market strain. Consequently, that hiatus in the usage of representations unfit for turmoil and the obligation to employ more accurate models may spare the banks the CCB and switch to the highly leptokurtic regimes like those based on the Extreme Value Theory suggested in the previous paragraphs.

5.5. Limitations

The thesis highlighted a series of snags present in the fabric of Basel Capital Accords in both versions, Basel II and Basel III. However, it also features own shortcomings which, when addressed, may strengthen the certainty of the conclusions. A brief explanation of some of them is sketched below.

Although the outcome appears valid for a sample of Emerging and Frontier stock markets, it remains to be seen whether their extrapolation still holds in an environment of Developed markets. Despite the study not dealing with mature countries, Rossignolo, Fethi and Shaban (2012a) prove the prowess of EVT-based models in an environment of Developed and Emerging markets showing similar results using the subprime crisis as Backtesting period. These fears, consequently, might result at least partly allayed.

The clear recommendations that emanate from the research experiment may be endorsed through a cross-asset exercise, i.e., commodities, foreign exchange conversion rates, bonds and gilts, interest rates, futures, options and forwards, credit default swaps
(CDS), etc. As the BCBS stipulates different regulations regarding the MCR in either Capital Accord, their adequacy may still be gauged in the light of diverse assets. To the layman, everyday news in the media allow to surmise that some securities are indeed more volatile than stocks and shares (for instance, CDS) thus bolstering the application of EVT for MCR purposes notwithstanding which it must be tried to eventually extending the conclusions.

Room for improvement can be found at the time of dealing with SA. However useless the thesis’s statements render it, SA appears still necessary for some exposures (as noted in Section 5.4 above) as well as for some small banks lacking the appropriate structure to develop sophisticated risk management models. The research has only been conclusive in what regards its demise or complete overhaul, nonetheless falling short of suggesting a compromising figure in those occasions when it must be kept alive as it only indicated the need to increase it thoroughly.

The VaR-based IMA might end up refined by increasing the quantity of models and attached distributions involved. True as it may be, the variety of combinations keeps growing exponentially, thus making the task of encompassing the majority of them an uphill one. Even when the choice of the conventional configurations and the stochastic distributions affixed to estimate the volatility was guided by their popularity among the financial community and practitioners, several representations might provide viable alternatives in terms of the dynamics of variance, like the AGARCH, TGARCH or PGARCH as stated in Alexander (2008b). The application of more sophisticated statistical densities like the Generalised Error Distribution could also prove satisfactory too, but altogether it remains to be evaluated whether the potentially marginal precision is obtained at the expense of very demanding computational burden.

Finally, Andersen et al. (1998) highlight another area worth of being explored. The authors recommend the application of intraday frequencies in order to enable the utilisation of the typical GARCH-Normal model as they show that, by virtue of the Central Limit Theorem, the distribution of the series of returns resembles a Standard Normal, hence easing the modelling stage of MCR estimation. Their finding would unquestionably help the VaR computation particularly when substantial numbers of securities are involved in the context of MLE for conditional volatilities.

5.6. Directions for further research
Studies like the present thesis leave many questions to be answered besides the shortcomings mentioned in Section 5.5. Backed by the evidence collected throughout the whole process, the following paragraphs convey an idea about the future lines of investigation to be addressed in ensuing work.

Even though EVT has been certified as the most efficient scheme to deal with abrupt market crises, it may be blended with a less leptokurtic model that may allow banks to handle the less strained periods constituting less capital (always above the levels demanded by the BCBS) and automatically switch to the heavy-tailed one when the stochastic regime is forecasted to change.

Basel regulations are another department worth of exploring, and, in this vein, the coexistence of SA and the VaR-based IMA with the possibility to employ every single VaR model covers many interesting areas. For instance, it would be essential to determine the scope and limitations to the application of the SA, stating clearly the occasions where it should be used so as not to interfere with the VaR specifications and in this fashion avoid the perverse motivations to erect adequate capital fences. With reference to the IMA, further research should tackle the issue of the special treatment to be dispensed to the different VaR models in accordance with their accuracy in Backtesting and, again, prioritise and reward the incentives to use precise techniques. The idea boils down to evaluate the accuracy of the representations in highly strung times and, after the Backtesting penalties, ascertain the levels of the fixed multiples - currently set at 3- to be used in each case. This strategy might also encompass the analysis of the alternative to spare the application of the sVaR term endorsed by Basel III, augmenting (or not) the fixed Basel II constant used in MCR determination.

An ambitious path has surged after the publication of the Consultative Document of 2012 in which the BCBS first acknowledged the VaR’s pitfalls and recommended the introduction of more complete and theoretically apt risk measures like Expected Shortfall, Tail Conditional Expectation or other spectral ones. Among those innovative metrics, the first one seems the most prone to be involved for capital constitution purposes in relatively similar ways to VaR. In this sense, as BCBS (2012a) underlines, many of the unresolved aspects in VaR hover around and demand close scrutiny; for instance, ES models, ES accuracy tests and their inclusion in MCR could, in principle, reveal as potential dangers to full Basel III implementation scheduled for January 2019.
Appendix 1
# Appendix 1.A

## Basic statistics about FTSE 100

### Chart 1.A.1

Basic statistics about FTSE 100. Period 09/04/1984 – 09/07/2013

<table>
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<tr>
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<td>0.0760</td>
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<td>0.0001</td>
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Appendix 1.B
FTSE Global Equity Index Series
Quality of Markets Criteria – September 2012 Update

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<th>Market and Regulatory Environment</th>
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<th>Advanced Emerging</th>
<th>Secondary Emerging</th>
<th>Frontier</th>
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<tbody>
<tr>
<td>Formal stock market regulatory authorities actively monitor market (e.g., SEC, FSA)</td>
<td>X</td>
<td>X</td>
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<tr>
<td>Fair and non-prejudicial treatment of minority shareholders</td>
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<tr>
<td>Non or selective incidence of foreign ownership restrictions</td>
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<tr>
<td>Free and well-developed foreign exchange market</td>
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<td>Non or simple registration process for foreign investors</td>
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<table>
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<th>Custody and Settlement</th>
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<tr>
<td>Settlement – Rare incidence of failed trades</td>
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<td>Custody – Sufficient competition to ensure high quality custodian services</td>
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<tr>
<td>Clearing &amp; settlement – T+3 or shorter, T+5 or shorter for Frontier</td>
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<tr>
<td>Stock lending is permitted</td>
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<tr>
<td>Settlement – Free delivery available</td>
</tr>
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<td>Custody – Omnibus account facilities available to international investors</td>
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<table>
<thead>
<tr>
<th>Dealing Landscape</th>
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<td>Brokerage – Sufficient competition to ensure high quality broker services</td>
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<tr>
<td>Liquidity – Sufficient broad market liquidity to support sizeable global investment</td>
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<td>Transaction costs – Implicit and explicit costs to be reasonable and competitive</td>
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<td>Short sales permitted</td>
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<td>Off-exchange transactions permitted</td>
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<tr>
<td>Efficient trading mechanism</td>
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<tr>
<td>Transparency – Market depth information / visibility and timely trade reporting process</td>
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<table>
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<th>Derivatives</th>
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<td>Developed derivatives market</td>
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Source: FTSE Group (September 2012 update)
## FTSE Global Equity Index Series

**Country Classification – September 2012 Update**

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<th>Secondary Emerging</th>
<th>Frontier</th>
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<td>Czech Republic</td>
<td>China</td>
<td>Bahrain</td>
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<td>Hungary</td>
<td>Colombia</td>
<td>Bangladesh</td>
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<td>Malaysia</td>
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*Source: FTSE Group (September 2012 update)*
Appendix 2
Appendix 2.A

Alternative uses of VaR: Allocation of resources and Capital Strength

The present Appendix illustrates two widespread side uses of VaR: the risk-adjusted allocation of resources among the different risk-taking units within a bank and the elaboration of a ratio that measures the bank’s Capital Strength, eventually very useful for industry wide comparison purposes.

2.A.1. Allocation of resources

Given that traders often develop a natural tendency to take extra risks, VaR can help to allocate capital and set position limits to control risk-taking and frame the overall institutional risk\(^1\). It simultaneously makes overall limits smaller than the sum of the constituents due to correlation effects, as conveyed by Graph 2.A.1 below:

![Graph 2.A.1](image)

Source: Dowd (1998:206)

Under the VaR-allocation of resources, units will invest in new position \((n)\) if\(^1\):

\[
r_n \geq r_o^p + \frac{\left(\frac{VaR_n}{VaR_o} - 1\right)}{x_n} \quad (2.A.1)
\]

where:

- \(r_n\): return on the new investment
- \(r_o^p\): return on the existing portfolio without the proposed prospect

\(^1\) VaR can also be employed to hedge positions, in which case the approach is exactly the same as in any other speculative investment (Dowd (1998)).

\(^1\) Dowd (1998).
VaR_n : expected VaR including the new alternative
VaR_o : existing VaR excluding the new position
x_n : proportion of funds to be invested in the new exposure

After algebraic manipulations,

\[ r_n \geq r_o^p + \left( \frac{\Delta VaR}{VaR_o} \right) \frac{r_o^p}{x_n} = \left[ 1 + \eta_n (VaR, x_n) \right] r_o^p \]  \hspace{1cm} (2.A.2)

if \( \Delta VaR = VaR_n - VaR_o \) and \( \eta_n(VaR, x_n) \) represents the relative change in VaR provoked by \( n \) divided by the percentage change in the portfolio. This elasticity of VaR with respect to \( x_n \) is approximately a metric of the variation of the portfolio risk adjusted for the size of the change in the portfolio. The implicit condition for investments becomes:

\[ r_n = \left[ 1 + \eta_n (VaR, x_n) \right] r_o^p \]  \hspace{1cm} (2.A.3)

for any level at the organisation. When (2.A.3) does not hold, senior management would shift fund allocation among different units to achieve risk-adjusted returns balanced at the margin. Capital allocation of limits is continually rebalanced, decreasing limits in underperforming units and increasing them in overperforming ones:

\[ \frac{\Delta PL_t}{PL_{t-1}} = \gamma (SR_{t-1}^+ - SR_{t-1}^-) + \frac{(VaR_t - VaR_{t-1})}{VaR_{t-1}} \]  \hspace{1cm} (2.A.4)

where:

\( \Delta PL_t/PL_{t-1} \) : relative change in the position limit
\( \gamma \) : arbitrary feedback parameter\(^{176}\)
\( SR_{t-1}^+ \) : \((t-1)\) Sharpe ratio\(^{177}\) of the institution including the unit in question
\( SR_{t-1}^- \) : \((t-1)\) Sharpe ratio of the institution excluding the unit in question
\( VaR_t \) : institution-wide VaR target for the period \( t \)

From (2.A.4): a) if \( SR_{t-1}^+ > SR_{t-1}^- \) : unit increases position limit (and viceversa);

b) if \( VaR_t > VaR_{t-1} \) : all units augment its limits (and viceversa).

\(^{176}\) Dowd (1998).
\(^{177}\) Sharpe ratio is usually defined as \( SR_t = (r_p - r_f) / \sigma_p \) where \( r_p, \sigma_p \) and \( r_f \) represent the return and Standard Deviation of portfolio \( p \) and the risk-free rate for period \( t \) respectively.
Albeit (2.A.4) depends on discretionary calibration, VaR facilitates setting position limits.

2.A.2. Capital Strength

The core function of an institution’s capital is to cover potential losses while preserving solvency: the greater the capital, the stronger the corporation and the lower the probability of distress \( p(d) \). VaR is also employed to gauge a company’s strength intervening in the calculus of \( p(d) \).

Following Dowd (1998), under restrictive normality assumption for the sake of simplicity\(^{178}\):

\[
Graph 2.A.2
Probability of Distress
\]

```
Given that \( p(d) = p(r) < -K/W \), and assuming \( r \sim N(\mu, \sigma) \) where \( K/W \) represents the Capital/Asset Ratio:

\[
p(d) = \Phi[\alpha(d)] \Rightarrow -\alpha(d) \sigma(p) = K/W \Rightarrow Strength = -\alpha(d) = \frac{(K/W)}{\sigma(p)} \quad (2.A.5)
\]

where:

\( \Phi \) : cumulative distribution function of the standard Normal distribution

\( \alpha \) : VaR confidence level (Section 2.3)

and enterprises will seek low \( \alpha(d) \), considering:

\[
\frac{\partial[-\alpha(d)]}{\partial(K/W)} < 0 \quad and \quad \frac{\partial[-\alpha(d)]}{\partial[\sigma(p)]} > 0
\]

\(^{178}\) Other well-behaved distributions may also be used. It is important to note that, for comparison purposes, the distributions upon which the probability of distress is based must be the same.
Dowd (1998) remarks that $\sigma(p)$ –the Standard Deviation of the portfolio- is usually unavailable, and recalls the definition of VaR:

$$VaR = -\alpha \sigma(p) W \quad (2.\text{A}.6)$$

Operating in (2.\text{A}.6):

$$\sigma(p) = \frac{-VaR}{\alpha W} \quad (2.\text{A}.7)$$

and plugging (2.\text{A}.7) into (2.\text{A}.5):

$$Strength = -\frac{K\alpha}{VaR} \quad (2.\text{A}.8)$$

Therefore, VaR can be applied to assess $p(d)$ over time and compare the relative strength of different institutions\textsuperscript{179}.

\textsuperscript{179} Of course, this measure should be analysed in conjunction with other indicators.
Appendix 3
Appendix 3.A

A detailed view to GARCH and EGARCH parameters

The present Appendix attempts to illustrate graphically the mechanics of both GARCH and EGARCH representations in order to facilitate the understanding of their behaviour as well as the potential differences that may arise at the time of using them. The aim is, then, to wrap the discussion and provide a visual interpretation as to the values of the parameters and the likely implications that may emerge in sudden stress situations, their building up and the ensuing aftermaths.

The current example will deal with the conditional volatility estimates using the FTSE 100 stock index of the London Stock Exchange in the period Oct/86 to Dec/87, which encompasses the October 1987 market crash. The analysis features two different sections: primarily, the GARCH model is evaluated whereas in the second place, a comparison between GARCH and EGARCH configurations is carried out. The examples are computed employing a one-lag variant (GARCH and EGARCH) as mentioned in the main body of the thesis.

Graph 3.A.1 depicts the evolution of the FTSE 100 during the term stated. It is clear from the movements the presence of a large perturbation starting from the middle of October 1987, with consequences lurking until the end of the sample. Two massive slumps heralded the spectacular plummet recorded on October 20th, which sparked a somewhat unstable period until the end of 1987 that reflected in an increase in the volatility levels, thus posing a great challenge to the volatility models that needed to tackle the jerks.

a) GARCH parameters

In order to clarify the consequences arising from the different values of the GARCH parameters, the optimal values estimated through Maximum Likelihood are contrasted with an imaginary hypothetical solution featuring a stark disparity. In this sense, below are depicted both equations in accordance with the configurations stated in Section 3.4.3 in the main body:
The evolution of FTSE 100 October 1986 – December 1987

Graph 3.A.1

Optimal setting ($t$-statistic in brackets):

$$r_{t+1} = \sigma_{t+1} z_{t+1} \quad \text{where} \quad z_t \sim N(0,1)$$

$$\sigma^2_{t+1} = 1.91 \times 10^{-6} + 0.887214 \sigma^2_t + 0.093677 r^2_t \quad (3.A.1)$$

with the long term unconditional GARCH volatility, via formula (3.19) in the main body, amounting to 15.81%.

On the other hand, the equations of the ad-hoc setting are:

$$r_{t+1} = \sigma_{t+1} z_{t+1} \quad \text{where} \quad z_t \sim N(0,1)$$

$$\sigma^2_{t+1} = 0.94 \sigma^2_t + 0.06 r^2_t \quad (3.A.2)$$

where the long term unconditional volatility is not defined given that the sum $\alpha + \beta = 1$, tantamount to a non-stationary IGARCH(1,1) model. This kind of scheme acquires special relevance in view of the fact that it constitutes the platform upon which the arguably most widespread representation is built, i.e., the Exponentially Weighted Moving Average\(^{180}\) (EWMA) RiskMetrics devised by JPMorgan and Reuters in 1996. Its popularity stems from the simplicity of implementation derived from its easy formulation: according to JPMorgan and Reuters (1996), the specific expression is\(^{181}\):

$$r_{t+1} = \sigma_{t+1} z_{t+1} \quad \text{where} \quad z_t \sim N(0,1)$$

$$\sigma^2_{t+1} = \lambda \sigma^2_t + (1 - \lambda) r^2_t \quad (3.A.3)$$

\(^{180}\) More precisely, the EWMA is a special case of the IGARCH(1,1), where the constant $w = 0$.

\(^{181}\) Theoretical developments regarding RiskMetrics may be found in JPMorgan and Reuters (1996).
where $\lambda$ is termed the ‘decay’ factor and only takes the values 0.94 (as depicted in (3.B.2)) or 0.97 in case of daily or monthly frequencies respectively.

Graph 3.A.2 exhibits the behaviour of both volatility forecasts for the aforementioned term whereas Graph 3.A.3 enhances the period 09/10/1987 – 31/12/1987, when the contrast between the respective estimates becomes more striking as a consequence of the turmoil that engulfed the market. Despite the fact that both estimates look pretty similar in the first graph, the process of zooming in exhibited in Graph 3.A.3 acts as a catalyst to verify the statements written in Section 3.5.3 of the text. In this vein, the fact that the error parameter $(1 - \lambda)$ in RiskMetrics setting is smaller than the optimal one $(0.06 < 0.093677)$ induces a less sensitive reaction to market events on the part of the former scheme, given that the blue line does not peak as abruptly as the red one. Complementarily, the persistence lag parameter $\lambda = 0.94$ in RiskMetrics (noticeably higher than $\beta = 0.887214$ in 3.B.1), which measures the persistence in conditional volatility following a market shock determines that the EWMA takes more time to diminish after a crisis (fairly eloquent in the aftermath of the Black Tuesday$^{182}$).

Furthermore, the rate of convergence of the conditional volatility to the long term unconditional average volatility, calculated via the sum $(\alpha + \beta)$ in 3.A.1 (0.980891) or $\lambda$ in 3.A.2 (0.94), would, in principle, point towards uneven term structures of volatility forecasts (as conveyed by Graphs 3.A.2 and 3.A.3). Alexander (2008b) bolsters this assertion asseverating that relatively flat term structures of volatility forecasts typically observe values above 0.99. On the other hand, the RiskMetrics model shows some sort of inconsistency in this department because the expected value of the future variance at horizon $k$ is:

$$E_t(\sigma^2_{t+k}) = \sigma^2_{t+1}, \quad \text{for every horizon } k \quad (3.A.4)$$

because the long term variance is undefined given that $(\alpha + \beta) = 1$ (or $\lambda + (1 - \lambda) = 1$) as opposed to the value found for any typical GARCH setting:

$$E_t(\sigma^2_{t+k}) = (\alpha + \beta)^{k-1} (\sigma^2_{t+1} - \sigma^2), \quad \text{for every horizon } k \quad (3.A.5)$$

$^{182}$In order to state the facts in a more precise fashion, the huge slump in the markets took place on the so-called Black Monday, i.e., 19/10/1987 in New York. However, by the time it occurred the stock exchanges in Europe and Japan had closed, hence the indices plummeted on the following day, 20/10/1987, that constituted, in practice, a Black Tuesday.
The central implication, then, boils down to consider that shocks to variance persist forever in RiskMetrics configuration and an increase in variance will drive the variance forecast upwards by an identical amount for all future prediction terms, which is tantamount to mentioning that RiskMetrics (or any IGARCH specification) dispenses
with the long run variance when carrying out variance predictions. Christoffersen (2003) highlights that the majority of daily financial time series generates values of \((\alpha + \beta)\) close to 1, thus meaning that those specifications might render short term variance predictions similar to RiskMetrics’s ones. However, in the long run, the situation appears very different because GARCH schemes suppose that in the future the volatility will eventually converge to the average value whereas RiskMetrics predicts that all future days will yield the same variance as the present day.

b) GARCH vs EGARCH

Graph 3.A.4 illustrates the comparison between the EGARCH volatility estimates for the FTSE 100 index for the period October 1986 to December 1987 and those of the symmetric GARCH ones, both settings featuring the Normal distribution appended.

The typical behaviour of both specifications resembles the one indicated in Alexander (2008b), and several characteristics are worth of being singled out. Initially, the EGARCH model seems less reactive to volatility peaks than the routine GARCH: this fact may be appreciated perusing the response yielded by both specifications in the interval 20/10/1987 – 30/10/1987 when volatility heightened, depicted in Chart 3.A.1, where the volatility predictions (percentage annual values) computed employing the latter is, on average values, approximately 14.44% higher than those of the former.

<table>
<thead>
<tr>
<th>Date</th>
<th>Volatility forecast GARCH-Normal</th>
<th>Volatility forecast EGARCH-Normal</th>
<th>Difference GARCH-N / EGARCH-N</th>
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<tr>
<td>20/10/1987</td>
<td>73.52%</td>
<td>63.99%</td>
<td>+14.89%</td>
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<tr>
<td>21/10/1987</td>
<td>78.44%</td>
<td>63.83%</td>
<td>+22.89%</td>
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<td>22/10/1987</td>
<td>79.16%</td>
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<td>23/10/1987</td>
<td>75.28%</td>
<td>65.52%</td>
<td>+14.91%</td>
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<tr>
<td><strong>Average</strong></td>
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<td><strong>+14.44%</strong></td>
</tr>
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Note: Volatility forecasts are expressed in annual percentage values. Daily values are converted using the ‘square-root-of-time rule’.

Secondly, and connected with the former point, the EGARCH volatility predictions seem, to some extent, smaller than GARCH ones, as conveyed by the red and blue lines in Graph 3.A.4. Thirdly, given that the EGARCH model does not restrict the variance
computations like the ordinary GARCH does (as explained by the logarithmic structure in Section 3.4.4 in the main body), its volatility estimates do not appear to have an artificial floor at the lower range, and often deliver smaller values at that level.

Graph 3.A.4
GARCH vs EGARCH Volatility Forecasts

The aforementioned statements become clearer looking at Graph 3.A.5, which depicts the same comparison as Graph 3.A.4 but analysing the crisis period 09/10/1987 – 31/12/1987 much in the same fashion that Graph 3.A.3. Therefore, it may be observed that: a) the responses delivered by the EGARCH model are less jerky than those given by the GARCH representation, b) the volatility forecasts deriving from the Exponential GARCH are undoubtedly lower than its counterpart, and c) those predictions are often lower than GARCH ones, particularly at the bottom of the volatility cycle.
Graph 3.A.5

GARCH and EGARCH Volatility Forecasts in detail

Behaviour in crisis or after market shocks
Appendix 3.B

The Extremal Types Theorem and the Block Maxima Approach
According to Embrechts, Klüppelberg and Mikosch (1997), the fundamental result of the classical Extreme Value Theory is the one embodied in the Extremal Types Theorem, commonly known as Fisher-Tippet (1928) theorem, thus named after the authors that derived the limit laws for maxima and Gnedenko (1948) too, who managed to obtain the first rigorous proof of the theorem. This Appendix contains an enunciation of the basic result –the technical demonstration lies beyond the scope of the present research– and, additionally, a succinct explanation of the Block Maxima Method (BMM), useful to picture a overview of the two methodologies to apply EVT (Peaks Over Threshold is described in the main body).

\[ a) \text{Fisher-Tippet (1928) – Gnedenko (1948) Theorem} \]

This theorem establishes a framework that indicates the limit laws for maxima or, in other words, it characterises those distributions which act as the only possible limit laws for the maxima of centred and normalised iid random variables.

Supposing that \( \{X_n\} = X_1, X_2, \ldots, X_n \) is a sequence of iid max-stable random variables with maxima \( M_n \) and common distribution function \( F \), the theory focuses on the statistical behaviour of \( M_n \) which could be determined precisely for all values of \( n \):

\[
\Pr\{M_n \leq z\} = \Pr\{X_1 \leq z, \ldots, X_n \leq z\} = \Pr\{X_1 \leq z\} \times \ldots \times \Pr\{X_1 \leq z\} = \{F(z)\}^n \tag{3.B.1}
\]

Unfortunately, given that the distribution function \( F \) is unknown, academics turned to find approximate families of distributions for \( F^n \), which can exclusively be estimated employing extreme data. But the behaviour of \( F^n \) as \( n \to \infty \) poses further difficulties as, for any \( z < z_+ \), \( z_+ \) being the upper-end point of \( F \), \( F^n(z) \to \infty \), thus degenerating the distribution of \( M_n \) to a point mass on \( z_+ \). Following Embrechts, Klüppelberg and Mikosch (1997) and Coles (2001), the hindrance is bridged assuming the existence of positive norming constants \( c_n > 0 \), \( d_n \in \mathbb{R} \), hence the matter is reduced to determine which distributions satisfy the relationship:

\[
\max(X_1, X_2, \ldots, X_n) = M_n = c_n X + d_n \tag{3.B.2}
\]

for all \( n \geq 2 \), \( c_n > 0 \), \( d_n \in \mathbb{R} \).

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Recalling that \( \{X_n\} \) is a sequence of iid max-stable rvs (a non-degenerate rv \( X \) is called max-stable provided it complies with equation (3.B.2)), (3.B.2) is equivalent to:

\[
(M_n - d_n) / c_n = X \quad (3.B.3)
\]

then allowing to conclude that every max-stable distribution acts as a limit distribution for the maxima of iid rvs and, conversely, max-stable distributions are the only limit laws for normalised and centred maxima. Furthermore, the group of max-stable distributions coincides with the group of all possible non-degenerate limit laws for centred and normalised maxima of iid random variables. The appropriate choice of the constants \( d_n \) and \( c_n \) stabilise the location and scale of \( X \) as \( n \) augments, hence overcoming the difficulties that spring up using the variable \( M_n \). The task subsumes to finding the limit distributions of \( X \) assuming the correct selection of \( c_n \) and \( d_n \). In formulas,

\[
\lim_{n \to \infty} F_n (c_n x + d_n) = H(x) \quad (3.B.4)
\]

for \( x \in \mathbb{R} \) and some non-degenerate distribution function \( H \), continuous on the whole dominium of \( \mathbb{R} \).

The fundamental result elicited in Fisher-Tippet (1928) and Gnedenko (1948), then, states that, considering the string of iid rvs \( \{X_n\} \), the existence of normalising and centring constants \( c_n > 0, d_n \in \mathbb{R} \) and non-degenerate distribution function \( H \) which attain the relationship:

\[
Pr\{(M_n - d_n) c_n^{-1} \leq z\} \to H(z) \quad (3.B.5)
\]

then, that distribution function \( H \) belongs to one of the following types of distribution functions\(^{183}\):

- **Fréchet:** \( \Phi_\alpha(z) = \begin{cases} 0 & \text{if } z \leq d \\ \exp \left\{-\left[\frac{(z - d)}{c}\right]^\alpha\right\} & \text{if } z > d \end{cases} \quad \text{for } \alpha > 0 \quad (3.B.6)\)

- **Weibull:** \( \Psi_\alpha(z) = \begin{cases} 1 & \text{if } z \geq d \\ \exp \left\{-\left[\frac{(z - d)}{c}\right]^\alpha\right\} & \text{if } z < d \end{cases} \quad \text{for } \alpha > 0 \quad (3.B.7)\)

\(^{183}\) Reiss and Thomas (2007) believe the symbols employed for Extreme Value distribution functions in many statistical publications are somewhat confusing in the sense that, for example, the Normal distribution function is also commonly denoted by \( \Phi \), therefore barring its use for the Fréchet distribution function. Therefore, they propose the alternative denominations \( EV_0, EV_1, EV_2 \) and \( G_{1,1}, G_{1,2}, G_{2,2} \) for the Gumbel, Fréchet and Weibull distribution functions respectively.
Gumbel: $A_\alpha(z) = \exp \left\{ -\exp \left[ -\left( \frac{z - d}{c} \right) \right] \right\}$ if $-\infty < z < \infty$ (3.B.8)

In broad terms, the Extremal Types Theorem (Fisher-Tippet-Gnedenko) affirms that the distribution of the rescaled maxima $X = \frac{M_n - d_n}{c_n}$ converges to one of the distributions above cited: Fréchet, Weibull or Gumbel, collectively known as Extreme Value Distributions. A closer look at their respective formulas shows that $d$ and $c$ are the location and scale parameter respectively and, furthermore, the shape parameter appears in the first two distributions.

Coles (2001) asseverates that, provided the maxima of the series $M_n$ ends ups stabilised with constants $d_n$ and $c_n$, the respective centred and normalised variable $X$ has, in the limit, a distribution belonging to the Extreme Value Distribution (Fréchet, Weibull or Gumbel). It is worth of being noted that, no matter what distribution $F$ the population may have, the only limiting distributions of the rescaled maxima $X$ are embodied in the Extreme Value Distribution family.

**The Generalised Extreme Value Distribution**

It is nevertheless useful to recast the aforementioned result stating that all the distributions belonging to the Extreme Value Distribution family (namely Fréchet, Weibull and Gumbel) could be derived from a single model distribution function below expressed$^{184}$:

$$H(z) = \exp \left\{ -\left[ 1 + \xi \left( \frac{z - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}$$

(3.B.9)

where $\{z: 1 + \xi (z - \mu) / \sigma > 0\}$ and $-\infty < \mu < \infty$, $\sigma > 0$ and $-\infty < \xi < \infty$. This distribution is termed Generalised Extreme Value (GEV) family of distributions, which is dependent upon the location parameter $\mu$, the scale parameter $\sigma$ and the shape parameter $\xi$. Furthermore, as long as the shape parameter $\xi$ takes particular values, (3.B.6), (3.B.7) and (3.B.8) are derived. In this regard, $\xi > 0$ leads to Fréchet distribution whereas $\xi < 0$ represents Weibull and $\xi = 0$ is the Gumbel distribution. Coles (2001) underlines the utility of (3.B.9) formulation regarding the simplification of the statistical implementation as the inference on $\xi$ paves the way for the most appropriate determination of the tail behaviour of the distribution.

---

$^{184}$ The reparameterisation $\xi = 1/\alpha$ is due to von Mises (1936) and Jenkinson (1955).
Note: Continuous line represents Fréchet density, dashed line the Gumbel density and dotted one the Weibull density. ($\alpha = 1$ for Fréchet and Weibull functions).

b) The Block Maxima Method

Following Coles (2001), the present section depicts a brief synthesis of the Block Maxima Method, eventually the second appraisal to model the extremes of a distribution. Supposing a string of independent observations $\{X_i\}$, $i = 1, 2, \ldots$ grouped in blocks of observations of length $n$ (with $n$ large), each of which generates series of block maxima $M_{n1}, M_{n2}, \ldots, M_{nj}$, the method deals with fitting the GEV to that series of $M_n$. The estimation of the extreme quantiles of the maxima distribution are obtained inverting expression (3.B.9) after considering $H(z_p) = 1 - p$, $p$ representing the probability level, hence $0 < p < 1$. Thus,

$$1 - p = \exp \left\{ - \left[ 1 + \frac{z - \mu}{\sigma} \right]^{-1/\xi} \right\}$$

$$z_p = \begin{cases} \mu - \frac{\sigma}{\xi} \left( 1 - \left[ -\log(1 - p) \right]^{-\xi} \right) & \text{if } \xi \neq 0 \\ \mu - \sigma \log[-\log(1 - p)] & \text{if } \xi = 0 \end{cases} \quad (3.B.10)$$

It is often the case that the Block Maxima Method poses problems at the time of deciding the appropriate length of the group size, leading to a trade-off between variance and bias of difficult resolution. In this light, Coles (2001) points out that large blocks provide few block maxima, consequently generating large estimation variance whereas small groups imply poor convergence in terms of the Fisher-Tippett-Gnedenko...
theorem, leading to bias in estimation and imprecision in extrapolation. Furthermore, it should be stressed that this methodology may also convey an important misuse of data when some blocks contain more extreme events than others. The author concludes that avoiding the process of blocking can lead to a better employment of the information available, reflection that acquires relevance at the time of considering relatively short time series data like those belonging to Emerging and Frontier stock markets dealt with in the context of the current thesis.
Appendix 3.C

Modelling Excess Losses

The Balkema and de Haan (1974)-Pickands (1975) theorem is also of utility to show that the distribution of excesses over the threshold $u$ also converges to the already fitted Generalised Pareto Distribution even if the threshold is raised.

Recalling that the Theorem states that for a large class of underlying distribution functions $F$, it is possible to find a measurable function $\sigma(u)$, such that:

$$\lim_{u \to y_0} \sup_{0 \leq x \leq y_0 - u} \left| F_u(y) - G_{\xi, \sigma(u)}(y) \right| = 0$$

if and only if $F$ belongs to the maximum domain of attraction of the Extreme Value Distribution $H_\xi$, i.e., $F \in \text{MDA}(H_\xi)$ with $\xi \in \mathbb{R}$, it is possible to employ its enunciation to find the implication mentioned in the first paragraph following McNeil, Frey and Embrechts (2005).

Hence, under the assumptions that $F$ is the loss distribution with right endpoint $y$ and, for some high threshold $u$, $F_u(y) = G_{\xi, \sigma(y)}$ for $0 \leq y < y - u$ given that $\xi \in \mathbb{R}$ and $\sigma > 0$, for any threshold $v$ higher than $u$, $F_v(y) = G_{\xi, \sigma(v-u)}(y)$. Employing (3.37) in the main text,

$$F_u(y) = \Pr \{ X - u \leq y \mid X > u \} = \frac{F(y + u) - F(u)}{1 - F(u)}$$

and considering the distribution function of the GPD given by (3.41) in the body:

$$G_{\xi, \sigma}(y) = \begin{cases} 1 - (1 + \xi y/\sigma)^{-\xi} & \text{if } \xi \neq 0 \\ 1 - \exp(-y/\sigma) & \text{if } \xi = 0 \end{cases}$$

it is possible to deduce:

$$\bar{F}_v(y) = \frac{\bar{F}(v + y)}{\bar{F}(v)} = \frac{\bar{F}[u + (y + v - u)]}{\bar{F}(u)} \frac{\bar{F}(u)}{\bar{F}[u + (v - u)]} \Rightarrow$$

$$\Rightarrow \bar{F}_v(y) = \frac{\bar{F}_u(y + v - u)}{\bar{F}_u(v - u)} = \frac{\bar{G}_{\xi, \sigma}(y + v - u)}{\bar{G}_{\xi, \sigma}(v - u)} \Rightarrow$$

$$\Rightarrow \bar{F}_v(y) = \bar{G}_{\xi, \sigma + \xi(v-u)}(y) \quad (3.C.1)$$

Therefore, the distribution function for thresholds higher than $u$ remains a Generalised Pareto Distribution with analogous shape parameter $\xi$ and a scaling factor that observes an increasing linear relationship with the new threshold $v$. 

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Appendix 3.D

The Peaks Over Threshold Method: An insight into the selection of the threshold

As indicated in Section 2, scientists have hitherto been unable to devise a satisfactory analytic procedure to estimate the optimal threshold which marks the beginning of the fitting of the Generalised Pareto Distribution. Consequently, it is essential to resort to some graphical techniques which aid the practitioner in a kind of trial and error operation which involves a great deal of experience and aptitude on the part of the analyst.

The present appendix aims at providing an illustration of the idea underlying the selection of the appropriate threshold and the level of cross checks necessary to produce a relatively reliable estimate from where the rest of the techniques start. As mentioned in the body of the thesis, the order statistic \( u \) will be approximated using the plot of the sample MEF, defined as an empirical estimator of the true MEF, expressed by (3.94):

\[
e_n(v) = \frac{\sum_{i=1}^{n} (X_i - v)I_{\{X_i>v\}}}{\sum_{i=1}^{n} I_{\{X_i>v\}}}
\]

which represents the average of the exceedances over the upper limit \( u \) for a sample of \( n \) observations. The graphical method for picking a suitable threshold stems from the linearity of the MEF in \( u \), supposing a GPD distribution for the excesses. The upward, downward and unbiased trends exhibited by the slope of the plot (Reiss and Thomas (2007)) correspond to the heavy-tailed, short-tailed and exponential distributions.

As in McNeil and Saladin (1997), the distribution of excesses will follow a GPD with \( \xi > 0 \) in the tail sector over \( u \) if the empirical picture of the MEF exhibits an approximate straight line with positive gradient: the threshold \( u \) can be identified as the value corresponding to the kink from where it is verified an upward gradient. The rationale behind the threshold selection may be illustrated modelling the lower tail of the Shanghai Composite, which sample period spans from 05/01/2000 to 28/12/2007 amounting to 1914 daily observations. The sample MEF displays some evidence of a twist towards the linearly positive section of the plot roughly from \( u = 1.71 \), where a GPD model approximately fitting the largest losses could begin (Graph 3.D.1).
The analysis should be complemented using an array of graphs like the Sample Density Function, the Cumulative Distribution Function and the Quantile Function, all of which are depicted below.

Graphs 3.D.2 and 3.D.3 are highly descriptive of the solutions entailed by the appraisal descriptive. Coles (2001) and Reiss and Thomas (2007), among others, have recommended the above methodology, while Rossignolo, Fethi and Shaban (2012a, 2012b and 2013) have applied to the selection of the threshold for POT estimation in different samples. Furthermore, even though it carries a great deal of subjectivity, it certainly improves Christoffersen’s idea (2003), who assumes that all extremes are
situated in the farthest 10% of the distribution, regardless of the selection of that threshold fulfills the preconditions stated by the theory. The present thesis, then, fosters a distinctive methodology to be applied for every POT process, at loggerheads with pragmatic considerations.

Finally, Coles (2001) argues that the final proof to gauge the accuracy of the threshold selection in the POT method is embodied in the analysis of the stability of the estimated parameters within a range of the chosen order statistic \( u \) and, ideally, the estimate for \( \xi \) should steady in an entourage of the threshold tried\textsuperscript{185}. In this sense, plotting the number of exceedances against MM estimates (Graph 3.D.4) reveals that values of \( \xi \) tend to approximately stabilise near 90 data points, thus bolstering the choice. Analogous procedure is applied for all the series included in the study.

Graph 3.D.4

The stability of the parameter \( \xi \)

\textbf{Note:} Considering estimates obtained through MM, the parameter \( \xi \) roughly stabilises around \( u = 90 \).

\textsuperscript{185} Interpreting the plots is by no means an easy task; on the contrary, McNeil, Frey and Embrechts (2005) warn that experience is required to read mean excess plots.
Appendix 4
Appendix 4.A

Reflections on the estimation process relating the Efficient Frontiers

The present Appendix intends to shed light on the several estimation processes carried out in order to obtain the outcomes related to the Emerging, Frontier and Developed markets displayed in the main body of the thesis.

4.A.1. Analysis of Emerging, Frontier and Developed markets frontiers

The frontiers exhibited in Graph 4.8 derive from a classic Minimum Least Squares (MLS) regression estimation as explained in any standard statistics or econometric textbook (e.g., Gujarati (1995)). In this vein, as mentioned, the three categories are composed by the following indices belonging to the respective stock exchanges:

- **Emerging markets**: Bovespa (Brazil), Cetop20 (Hungary), Sensex (India), PX (Czech Republic), JKSE (Indonesia), KLSE (Malaysia), IPCC (Mexico), IGCB (Colombia), Shanghai Composite (China), Taise (Taiwan), Kospi (South Korea), ISE (Turkey), CMA (Egypt) and RTS (Russia);
- **Frontier markets**: Merval (Argentina), OMX-Vilnius (Lithuania), Tunindex (Tunisia), Crobex (Croatia), KSE (Pakistan), KSEPI (Sri Lanka) and MSE (Malta);
- **Developed markets**: DJIA, S&P500 and Nasdaq (United States), FTSE100 (United Kingdom), CAC40 (France), DAX (Germany), FTSE MIB (Italy), SMI (Switzerland), OMX-Stockholm (Sweden), Nikkei (Japan), TAS (Israel), IGMB (Spain), AMX (Australia), Strait (Singapore) and Hang Seng (Hong Kong).

The regression model employed takes the shape:

\[
r_t = a + b \ln(\sigma_t) + \varepsilon_t
\]

(4.A.1)

where \(\sigma_t\) denotes the Standard Deviation of the log returns of the particular index, considering the end of month values belonging to the sample May/2002 – Dec/2007. The outcome of the whole process is\(^{186}\):

i) **Emerging markets**:

\[
r_t^{EM} = 15.00 + 4.12 \ln(\sigma_t) \quad R^2 = 0.27
\]

(5.52) (1.98)

\(^{186}\) Standard errors are reported between brackets below each estimate.
ii) **Frontier markets:**

\[ r_t^{FM} = 34.87 + 10.87 \ln(\sigma_t) \quad R^2 = 0.71 \quad (4.A.3) \]

\( (8.63) \quad (3.08) \)

iii) **Developed markets:**

\[ r_t^{DM} = 3.41 + 0.83 \ln(\sigma_t) \quad R^2 = 0.62 \quad (4.A.4) \]

\( (1.27) \quad (0.05) \)

From the regression equations (4.A.2) to (4.A.4), two salient features are worth of being mentioned: in the first place, the \( a \) coefficient is higher in Frontier markets, followed by Emerging and Developed markets (3.41 < 15.00 < 34.87) and, secondly, the pecking order is maintained with reference to the three gradients (0.83 < 4.12 < 10.87). The above figures, then, confirm the intuition from Graph 4.8 in the sense that Frontier markets seem the riskiest of the three, followed by Emerging markets, whereas, in general terms, Developed ones are judged as more stable and difficult to reap profits without skilful risk management (as conveyed by the independent term).

### 4.A.2. The Emerging, Frontier and Developed markets Efficient Frontiers

Section 2.2 hinted at the fact that Emerging and Frontier markets can comfortably top the ranks regarding both yield and risk involved in any investment in them. One of the many ways to test that assertion is the analysis of the Efficient Frontier\(^{187}\), i.e., a sensible proof of the movements that those boundaries are subject to as a result of the inclusion of some components.

Section 4.2.1 proposed the construction of the typical two-dimension risk-return space which is obtained via the methodology engineered by Markowitz (1952), which implies obtaining the minimum risk portfolio for every prefixed return level. The mathematical problem, then, boils down to calculating the weights \( x_i \) to be invested in each asset capable of minimising the overall variance of the portfolio \( \sigma_p^2 \), subject to a specific return. The mathematical expression of the former statement is:

Minimise: \[ \sigma_p^2 = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} x_i x_j \sigma_{ij} \quad (4.A.5) \]

Subject to \[ r_p = \sum_{i=1}^{n} x_i r_i \quad (4.A.6) \]

and \[ \sum_{i=1}^{n} x_i = 1 \quad (4.A.7) \]

\(^{187}\) It may be recalled that the term ‘Efficient’ refers to ‘Efficiency’ in the context of the MVC. A portfolio is regarded as efficient when it reports the smallest risk among all those sharing the same return or, conversely, it offers the highest return among all those sharing the same level or risk.
where:

$\sigma^2_p$ : variance of the portfolio

$x_i$ : asset’s weight

$\sigma_{ij}$ : covariance between pairs of assets

$r_p$ : return of the portfolio

$r_i$ : security’s yield

Therefore, after setting the Lagrange function (4.A.8):

$$ F = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} x_i x_j \sigma_{ij} + \lambda_1 \left( \sum_{i=1}^{n} x_i r_i - r_p \right) + \lambda_2 \left( \sum_{i=1}^{n} x_i - 1 \right) $$

and subsequent mathematical manipulations (Alvarez, Mesuti and Graffi (1992)), the system of equations gives the desired weights.

It is also important to derive the minimum risk portfolio because that point allows the separation of the efficient and inefficient subportfolios. The derivation is obtained minimising (4.A.5) subject to (4.A.7) hence dropping (4.A.6). Therefore, the Lagrange function becomes:

$$ F = \sum_{i=1}^{n} x_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} x_i x_j \sigma_{ij} + \lambda \left( \sum_{i=1}^{n} x_i - 1 \right) $$

which, after algebraic operations yields the weights that permit the construction of the Efficient frontier.

4.A.3. Correlation figures

Chart 4.A.1 supports the asseverations stated in the main text of the thesis. It is clear from the numbers exposed throughout the period 2002 – 2007 that the bond between Emerging and Developed markets is stronger than the ones involving the Frontier category. In turn, this finding may account for the diversification benefits that are susceptible of being harvested when allocating some of the available funds in Frontier markets.
### Chart 4.A.1

#### Panel A - Correlation coefficient matrix – Year 2002

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<th>MSCI-WI</th>
<th>MSCI-EM</th>
<th>MSCI-FM</th>
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**Note:** Partial values (May/02-Dec/02).

#### Panel B - Correlation coefficient matrix – Year 2003

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#### Panel D - Correlation coefficient matrix – Year 2005

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<td>MSCI-EM</td>
<td>0.7391</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>0.5992</td>
<td>0.2886</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>0.8310</td>
<td>0.9545</td>
<td>0.4399</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

#### Panel F - Correlation coefficient matrix – Year 2007

<table>
<thead>
<tr>
<th>Index</th>
<th>MSCI-WI</th>
<th>MSCI-EM</th>
<th>MSCI-FM</th>
<th>MSCI-AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-WI</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI-EM</td>
<td>0.2961</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>0.5762</td>
<td>0.5400</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>0.7053</td>
<td>0.8028</td>
<td>0.5279</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

#### Panel G - Correlation coefficient matrix – May/02-Dec/07

<table>
<thead>
<tr>
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<th>MSCI-WI</th>
<th>MSCI-EM</th>
<th>MSCI-FM</th>
<th>MSCI-AC</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI-WI</td>
<td>1.0000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI-EM</td>
<td>0.6633</td>
<td>1.0000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSCI-FM</td>
<td>0.2169</td>
<td>0.2254</td>
<td>1.0000</td>
<td></td>
</tr>
<tr>
<td>MSCI-AC</td>
<td>0.8575</td>
<td>0.8696</td>
<td>0.2314</td>
<td>1.0000</td>
</tr>
</tbody>
</table>
Appendix 4.B

Statistics for the blue-chip indices

The present Appendix informs the values of the basic statistical measures belonging to the sample of Emerging and Frontier markets included in the thesis. Given that these patterns are susceptible of being observed across assets and samples, the literature calls them stylised facts (JP Morgan and Reuters (1996), Penza and Bansal (2001)).

In effect, Charts 4.B.1, 4.B.2 and 4.B.3 below convey an idea of the most relevant notions:

- Non-significant means in every time series. This assertion, in principle, means equations in dynamic volatility environments would not exert sizeable influence on the behaviour of stock market volatility;
- Departure from normality, reflected in the skewness, kurtosis and quantile levels. Distributions are clearly asymmetrical and exhibit narrower and higher centres than predicted by the Normal distribution and, most importantly, a concentration of returns in the tails, particularly in the left one. The aforementioned facts imply that extreme events are more likely to occur than under the Gaussian assumptions and, furthermore, any underestimation of the true characteristics of the empirical distributions could potentially play to the detriment of the sufficiency of the MCR;
- Absence (or only very slight) linear autocorrelation and significant autocorrelation levels for squared returns (or variances) at almost any lag. Furthermore, volatilities are grouped in alternate low-volatility and high-volatility clusters. Therefore, variances conditional on time (heteroscedastic) become an issue of paramount importance at the time of modelling;
- Stationary condition achieved in return series level, consequently facilitating the application of modelling techniques.

The above stated concepts could be graphically confirmed in Graphs 4.B.1, 4.B.2, 4.B.3, 4.B.4 and 4.B.5 which depict the departure from normality, the presence of interspersing volatility clusters, the absence of correlation in linear returns, the existence of significant correlation in squared returns and evidence of stationary patterns in every return series respectively.
### Chart 4.B.1

**Basic statistics about return series**

<table>
<thead>
<tr>
<th>Parameter Index</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Rep.</th>
<th>Indonesia</th>
<th>Malaysia</th>
<th>China</th>
<th>Argentina</th>
<th>Lithuania</th>
<th>Tunisia</th>
<th>Croatia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observ.</td>
<td>1976</td>
<td>1494</td>
<td>2600</td>
<td>3243</td>
<td>1925</td>
<td>2215</td>
<td>1914</td>
<td>2777</td>
<td>2045</td>
<td>2496</td>
<td>2247</td>
</tr>
<tr>
<td>Mean (0.10)</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0004</td>
<td>0.0007</td>
<td>0.0004</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0004</td>
<td>0.0009</td>
<td></td>
</tr>
<tr>
<td>Median (0.01)</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0013</td>
<td>0.0012</td>
<td>0.0006</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0010</td>
<td>0.0008</td>
<td>0.0002</td>
<td>0.0005</td>
</tr>
<tr>
<td>Maximum (0.0734)</td>
<td>0.0859</td>
<td>0.0705</td>
<td>0.0705</td>
<td>0.0671</td>
<td>0.0651</td>
<td>0.0940</td>
<td>0.0112</td>
<td>0.0854</td>
<td>0.0458</td>
<td>0.3941</td>
<td>0.1269</td>
</tr>
<tr>
<td>Minimum (0.0754)</td>
<td>0.0055</td>
<td>-0.0708</td>
<td>-0.1093</td>
<td>-0.0634</td>
<td>-0.0926</td>
<td>-0.1477</td>
<td>-0.1022</td>
<td>-0.0213</td>
<td>-0.0903</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std. Dev. (0.0182)</td>
<td>0.0161</td>
<td>0.0119</td>
<td>0.0139</td>
<td>0.0104</td>
<td>0.0150</td>
<td>0.0222</td>
<td>0.0091</td>
<td>0.0044</td>
<td>0.0144</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skewness (0.2132)</td>
<td>-0.4531</td>
<td>-0.3347</td>
<td>-0.2879</td>
<td>-0.7208</td>
<td>-0.2137</td>
<td>0.0276</td>
<td>-0.1657</td>
<td>-0.7219</td>
<td>0.4002</td>
<td>0.2874</td>
<td></td>
</tr>
<tr>
<td>JarqueBera (55.5327)</td>
<td>209.2272</td>
<td>1291.993</td>
<td>1005.655</td>
<td>1904.667</td>
<td>2901.295</td>
<td>1759.189</td>
<td>3743.358</td>
<td>9698.399</td>
<td>1128.208</td>
<td>9215.895</td>
<td></td>
</tr>
<tr>
<td>q(0.0001) (0.00)</td>
<td>-1.2567</td>
<td>-1.1831</td>
<td>-1.1366</td>
<td>-1.1216</td>
<td>-0.9744</td>
<td>-1.1216</td>
<td>-1.1064</td>
<td>-1.1052</td>
<td>-0.9590</td>
<td>-0.9928</td>
<td></td>
</tr>
<tr>
<td>q(0.01) (0.00)</td>
<td>-2.0972</td>
<td>-2.1934</td>
<td>-2.1823</td>
<td>-1.9295</td>
<td>-2.0029</td>
<td>-2.0473</td>
<td>-1.9949</td>
<td>-1.9291</td>
<td>-1.9576</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q(0.05) (0.00)</td>
<td>-1.7387</td>
<td>-1.6190</td>
<td>-1.5806</td>
<td>-1.6204</td>
<td>-1.4132</td>
<td>-1.5753</td>
<td>-1.5710</td>
<td>-1.5631</td>
<td>-1.4467</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q(0.10) (0.00)</td>
<td>-1.2567</td>
<td>-1.1831</td>
<td>-1.1366</td>
<td>-1.1216</td>
<td>-0.9744</td>
<td>-1.1216</td>
<td>-1.1064</td>
<td>-1.1052</td>
<td>-0.9590</td>
<td></td>
<td></td>
</tr>
<tr>
<td>q(0.90) (0.00)</td>
<td>1.8252</td>
<td>1.9883</td>
<td>1.0948</td>
<td>1.1440</td>
<td>1.0947</td>
<td>1.1035</td>
<td>0.9991</td>
<td>1.0888</td>
<td>1.1499</td>
<td>0.9498</td>
<td></td>
</tr>
<tr>
<td>q(0.95) (0.00)</td>
<td>1.5646</td>
<td>1.5663</td>
<td>1.5302</td>
<td>1.2624</td>
<td>1.5083</td>
<td>1.4864</td>
<td>1.4983</td>
<td>1.5511</td>
<td>1.6334</td>
<td>1.4450</td>
<td></td>
</tr>
<tr>
<td>q(0.975) (0.00)</td>
<td>1.8529</td>
<td>1.8159</td>
<td>1.8976</td>
<td>1.9397</td>
<td>1.9186</td>
<td>2.1111</td>
<td>1.9805</td>
<td>2.0033</td>
<td>2.1418</td>
<td>2.0612</td>
<td></td>
</tr>
<tr>
<td>q(0.99) (0.00)</td>
<td>2.3919</td>
<td>2.4801</td>
<td>2.2987</td>
<td>2.4556</td>
<td>2.5766</td>
<td>2.9018</td>
<td>2.7582</td>
<td>2.8569</td>
<td>2.8418</td>
<td>3.0378</td>
<td></td>
</tr>
<tr>
<td>q(0.9999) (0.00)</td>
<td>3.9221</td>
<td>3.2087</td>
<td>5.2104</td>
<td>5.5463</td>
<td>4.5893</td>
<td>5.5170</td>
<td>6.4017</td>
<td>6.8104</td>
<td>4.9595</td>
<td>8.5666</td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. p-values in brackets.
2. Critical values at 90%: \( X^2_{10} = 15.99 \); \( X^2_{15} = 22.31 \); \( X^2_{20} = 28.41 \)

### Chart 4.B.2

**Box-Ljung statistics for linear and squared returns for referred lags (p)**

<table>
<thead>
<tr>
<th>Box-Ljung BL(p)</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Rep.</th>
<th>Indonesia</th>
<th>Malaysia</th>
<th>China</th>
<th>Argentina</th>
<th>Lithuania</th>
<th>Tunisia</th>
<th>Croatia</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL(10) (0.24)</td>
<td>12.68</td>
<td>15.35</td>
<td>50.92</td>
<td>60.86</td>
<td>26.61</td>
<td>89.94</td>
<td>14.70</td>
<td>37.65</td>
<td>83.02</td>
<td>522.93</td>
<td>8.62</td>
</tr>
<tr>
<td>BL(15) (0.09)</td>
<td>22.80</td>
<td>23.39</td>
<td>60.11</td>
<td>75.49</td>
<td>35.53</td>
<td>105.87</td>
<td>31.20</td>
<td>38.99</td>
<td>110.37</td>
<td>598.01</td>
<td>10.44</td>
</tr>
<tr>
<td>BL(20) (0.06)</td>
<td>30.35</td>
<td>28.39</td>
<td>74.81</td>
<td>81.50</td>
<td>42.24</td>
<td>107.96</td>
<td>37.75</td>
<td>40.72</td>
<td>114.54</td>
<td>601.49</td>
<td>17.18</td>
</tr>
<tr>
<td>BL(10) (0.06)</td>
<td>120.64</td>
<td>172.53</td>
<td>504.67</td>
<td>707.23</td>
<td>143.44</td>
<td>516.76</td>
<td>116.15</td>
<td>1010.30</td>
<td>24.54</td>
<td>1357.50</td>
<td>409.17</td>
</tr>
<tr>
<td>BL(15) (0.00)</td>
<td>146.76</td>
<td>232.96</td>
<td>562.14</td>
<td>937.38</td>
<td>151.05</td>
<td>604.24</td>
<td>164.68</td>
<td>1313.80</td>
<td>28.61</td>
<td>1596.80</td>
<td>419.64</td>
</tr>
<tr>
<td>BL(20) (0.00)</td>
<td>164.20</td>
<td>270.88</td>
<td>616.75</td>
<td>1100.10</td>
<td>154.49</td>
<td>656.12</td>
<td>190.95</td>
<td>1475.90</td>
<td>30.63</td>
<td>1618.70</td>
<td>431.97</td>
</tr>
</tbody>
</table>

Notes:
1. p-values in brackets.
2. Box-Ljung statistics for squared returns are indicated by the superscript 2.
<table>
<thead>
<tr>
<th>Test Statistic Index</th>
<th>Augmented Dickey Fuller</th>
<th>Phillips Perron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>-42.56</td>
<td>-42.53</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Hungary</td>
<td>-31.11</td>
<td>-37.19</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>India</td>
<td>-47.46</td>
<td>-47.40</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>CzechRepublic</td>
<td>-51.08</td>
<td>-51.13</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Indonesia</td>
<td>-39.75</td>
<td>-39.72</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Malaysia</td>
<td>-39.68</td>
<td>-40.11</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>China</td>
<td>-42.70</td>
<td>-42.70</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Argentina</td>
<td>-49.39</td>
<td>-49.68</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Lithuania</td>
<td>-32.54</td>
<td>-41.14</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Tunisia</td>
<td>-32.54</td>
<td>-32.54</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>Croatia</td>
<td>-47.49</td>
<td>-47.49</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: (1): p-values in brackets.
(2): Critical values:
CV(1%) = -3.43
CV(5%) = -2.86
CV(10%) = -2.57
Graph 4.B.1
The departure from normality
Graph 4.B.1 (cont.)

The departure from normality
Graph 4.B.2
The distribution of returns – The volatility clustering effect
The distribution of returns – The volatility clustering effect

Graph 4.B.2 (cont.)

Argentina - Linear returns

Lithuania - Linear returns

Tunisia - Linear returns

Croatia - Linear returns
Graph 4.B.3 – Panel A

Autocorrelation functions (ACF) for linear returns
Graph 4.B.3 – Panel A (cont.)

Autocorrelation functions (ACF) for linear returns
Graph 4.B.3 – Panel B
Autocorrelation functions (ACF) for squared returns
Graph 4.B.3 – Panel B (cont.)

Autocorrelation functions (ACF) for squared returns
Graph 4.B.4
Volatility clustering – The correspondence between linear and squared returns
Graph 4.B.4 (cont.)
**Volatility clustering – Linear and squared returns**

**Argentina - Linear and Squared returns**

**Lithuania - Linear and Squared returns**

**Tunisia - Linear and Squared returns**

**Croatia - Linear and Squared returns**
Graph 4.B.5
Graphical assessment of stationarity at price and return levels
Graph 4.B.5 (cont.)

Graphical assessment of stationarity at price and return levels
Appendix 4.C

A closer look at the model parameters
Charts 4.C.1 and 4.C.2 (Panels A and B) exhibit the details of the outcome of the estimation process in what regards to the Conditional Volatility model parameters – GARCH and EGARCH- in both versions, Normal and Student-t distributions. Additionally, it pictures the details of the Generalised Pareto Distribution parameters affixed to every time series using the POT scheme and after the pre-whitening procedure employing the GARCH-Normal specification.

As it was mentioned in the main text and the first paragraph as well, although the below figures are circumscribed to the CV models, they also correspond to the Filtered Historical Simulation category given that the difference between the two clusters roots in the density appended; in effect, while the former features the Normal and Student-t, the latter relies on the empirical standardised returns, where the Standard Deviation obtained through any of the combinations (GARCH-Normal, GARCH-Student-t, EGARCH-Normal and EGARCH-Student-t) is applied for that purpose.

The current Appendix also includes Chart 4.C.3, which portrays the results of the estimation for the Generalised Pareto Distribution derived from the Method of Moments as explained in Section 3.7.3. It is relevant to recall that the parameters of the GPD are obtained after pre-whitening the original series of log-returns employing the volatility forecasts generated by the GARCH-Normal model whose coefficients are, in turn, displayed below in Chart 4.C.1 –Panel A-. 
### Chart 4.C.1

**Panel A: GARCH-Normal parameters**

<table>
<thead>
<tr>
<th>Market</th>
<th>Raw squared autocorr.</th>
<th>w</th>
<th>α</th>
<th>β</th>
<th>α + β</th>
<th>Half-Life</th>
<th>LT Volatility</th>
<th>Log-likelihood</th>
<th>Filtered squared autocorr.</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>164.20</td>
<td>0.000011 (0.00)</td>
<td>0.053651 (0.00)</td>
<td>0.913557 (0.00)</td>
<td>0.967208</td>
<td>20.79</td>
<td>0.0180</td>
<td>5165.26</td>
<td>14.93</td>
<td>90.91%</td>
</tr>
<tr>
<td>Hungary</td>
<td>270.88</td>
<td>0.000006 (0.00)</td>
<td>0.069415 (0.00)</td>
<td>0.893564 (0.00)</td>
<td>0.962979</td>
<td>18.37</td>
<td>0.0123</td>
<td>4501.93</td>
<td>28.03</td>
<td>89.65%</td>
</tr>
<tr>
<td>India</td>
<td>616.75</td>
<td>0.000009 (0.00)</td>
<td>0.111955 (0.00)</td>
<td>0.857995 (0.00)</td>
<td>0.969950</td>
<td>22.72</td>
<td>0.0169</td>
<td>7277.85</td>
<td>19.05</td>
<td>96.91%</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>1100.10</td>
<td>0.000003 (0.00)</td>
<td>0.125516 (0.00)</td>
<td>0.858603 (0.00)</td>
<td>0.984119</td>
<td>43.30</td>
<td>0.0143</td>
<td>10075.08</td>
<td>17.47</td>
<td>98.41%</td>
</tr>
<tr>
<td>Indonesia</td>
<td>154.49</td>
<td>0.000027 (0.00)</td>
<td>0.172868 (0.00)</td>
<td>0.694188 (0.00)</td>
<td>0.867056</td>
<td>4.86</td>
<td>0.0143</td>
<td>5593.54</td>
<td>18.91</td>
<td>87.76%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>656.12</td>
<td>0.000001 (0.00)</td>
<td>0.099738 (0.00)</td>
<td>0.898774 (0.00)</td>
<td>0.998512</td>
<td>465.48</td>
<td>0.0233</td>
<td>7334.12</td>
<td>27.19</td>
<td>95.86%</td>
</tr>
<tr>
<td>China</td>
<td>190.95</td>
<td>0.000004 (0.00)</td>
<td>0.100768 (0.00)</td>
<td>0.884218 (0.00)</td>
<td>0.984986</td>
<td>48.82</td>
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<td>5483.74</td>
<td>13.72</td>
<td>92.81%</td>
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<td>0.117632 (0.00)</td>
<td>0.854818 (0.00)</td>
<td>0.972450</td>
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<td>15.21</td>
<td>98.97%</td>
</tr>
<tr>
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<td>0.219535 (0.00)</td>
<td>0.413307 (0.00)</td>
<td>0.632842</td>
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<td>0.0092</td>
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<td>94.03%</td>
</tr>
<tr>
<td>Tunisia</td>
<td>1618.70</td>
<td>0.000002 (0.00)</td>
<td>0.251581 (0.00)</td>
<td>0.658635 (0.00)</td>
<td>0.910216</td>
<td>7.37</td>
<td>0.0044</td>
<td>10326.44</td>
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<tr>
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<td>0.057647 (0.00)</td>
<td>0.967344 (0.00)</td>
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<td>0.0142</td>
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| Avg. Reduction | 94.19%                |
| Avg. Reduction EM | 92.32%                      |
| Avg. Reduction FM | 97.01%                      |

**Notes:**
1. p-values between brackets.
3. Emerging markets above solid line, Frontier markets below.
<table>
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<th>Market</th>
<th>Raw squared autocorr.</th>
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<th>α</th>
<th>β</th>
<th>d</th>
<th>α + β</th>
<th>Half-Life</th>
<th>LT Volatility</th>
<th>Log-likelihood</th>
<th>Filtered squared autocorr.</th>
<th>Reduction</th>
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<td>0.000009</td>
<td>0.051909</td>
<td>0.921297</td>
<td>17.964070</td>
<td>0.973206</td>
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<td>0.0180</td>
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<td>91.02%</td>
</tr>
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<td>0.065562</td>
<td>0.897161</td>
<td>9.865319</td>
<td>0.962723</td>
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<td>India</td>
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<td>0.000008</td>
<td>0.112840</td>
<td>0.860118</td>
<td>7.774203</td>
<td>0.972958</td>
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<td>0.0169</td>
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<td>19.11</td>
<td>96.90%</td>
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<tr>
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<td>0.133773</td>
<td>0.861262</td>
<td>7.017832</td>
<td>0.995035</td>
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<td>5.582651</td>
<td>0.904728</td>
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<td>5652.57</td>
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<td>0.0210</td>
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<td>0.0104</td>
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<td>0.256440</td>
<td>0.666247</td>
<td>13.511530</td>
<td>0.922687</td>
<td>8.61</td>
<td>0.0046</td>
<td>10037.22</td>
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<td>0.199345</td>
<td>0.756124</td>
<td>3.443324</td>
<td>0.955469</td>
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<td>0.0187</td>
<td>6837.20</td>
<td>6.53</td>
<td>98.49%</td>
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<td>94.89%</td>
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<tr>
<td>Avg. Reduction EM</td>
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<tr>
<td>Avg. Reduction FM</td>
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<td>97.24%</td>
</tr>
</tbody>
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Notes: (1): p-values between brackets.
(3): Emerging markets above solid line, Frontier markets below.
### Chart 4.C.2

**Panel A: EGARCH-Normal parameters**

<table>
<thead>
<tr>
<th>Market</th>
<th>Raw squared autocorr.</th>
<th>w</th>
<th>α</th>
<th>γ</th>
<th>β</th>
<th>LT Volatility</th>
<th>Log-likelihood</th>
<th>Filtered squared autocorr.</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>164.20</td>
<td>-0.465241</td>
<td>0.081948</td>
<td>-0.091799</td>
<td>0.950157</td>
<td>0.0094</td>
<td>5189.72</td>
<td>28.11</td>
<td>82.88%</td>
</tr>
<tr>
<td>Hungary</td>
<td>270.88</td>
<td>-0.611177</td>
<td>0.140967</td>
<td>-0.058529</td>
<td>0.942870</td>
<td>0.0048</td>
<td>4508.21</td>
<td>30.00</td>
<td>88.93%</td>
</tr>
<tr>
<td>India</td>
<td>616.75</td>
<td>-0.919139</td>
<td>0.255299</td>
<td>-0.130924</td>
<td>0.913726</td>
<td>0.0049</td>
<td>7312.01</td>
<td>23.85</td>
<td>96.13%</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>1100.10</td>
<td>-0.547092</td>
<td>0.229172</td>
<td>-0.039435</td>
<td>0.958742</td>
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<td>10101.19</td>
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<td>98.71%</td>
</tr>
<tr>
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<td>154.49</td>
<td>-1.932786</td>
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<td>-0.139230</td>
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<tr>
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<tr>
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<td>0.196125</td>
<td>-0.026888</td>
<td>0.978879</td>
<td>0.0004</td>
<td>5500.32</td>
<td>13.47</td>
<td>92.95%</td>
</tr>
<tr>
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<td>0.215844</td>
<td>-0.059724</td>
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<td>0.374712</td>
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<td>95.01%</td>
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<tr>
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<tr>
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</table>

**Avg. Reduction** 93.77%

**Avg. Reduction EM** 92.08%

**Avg. Reduction FM** 96.73%

**Notes:**
1. *p*-values between brackets.
3. Emerging markets above solid line, Frontier markets below.
### Panel B: EGARCH-Student-t parameters

<table>
<thead>
<tr>
<th>Market</th>
<th>Raw squared autocorr.</th>
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<th>( \alpha )</th>
<th>( \gamma )</th>
<th>( \beta )</th>
<th>( d )</th>
<th>LT Volatility</th>
<th>Log-likelihood</th>
<th>Filtered squared autocorr.</th>
<th>Reduction</th>
</tr>
</thead>
<tbody>
<tr>
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<td>[2] 164.20</td>
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<td>0.082832</td>
<td>-0.093141</td>
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<td>(0.00)</td>
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**Notes:**
1. \( p \)-values between brackets.
3. Emerging markets above solid line, Frontier markets below.
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<th>Confidence Interval ξ</th>
<th>σ</th>
<th>Confidence Interval σ</th>
<th>N_u</th>
<th>N</th>
<th>Percentage Nu / N</th>
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<tr>
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<td>1925</td>
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<tr>
<td>China</td>
<td>0.179202</td>
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      (2): Emerging markets above solid line, Frontier markets below.
Appendix 4.D

Value-at-Risk and Minimum Capital Requirements results
The present Appendix pictures the outcome of the Value-at-Risk estimations, thus portraying the Backtesting and further Minimum Capital Requirements in accordance with the modelling patterns enunciated in Sections 3.3 to 3.7 in Chapter 3 and in conjunction with the specifications stated in Sections 4.4 to 4.7 in Chapter 4. All throughout the Appendix the following abbreviations apply for the Charts below:

Exp. Num. : Expected Number
HS : Historical Simulation
FHS : Filtered Historical Simulation
CV : Conditional Volatility
EVT : Extreme Value Theory
G : GARCH
E : EGARCH
N : Normal distribution
N : Student-t distribution
Lin. Norm. : Linear model (Standard Deviation) enhanced by Normal density
Lin. t : Linear model (Standard Deviation) enhanced by Student-t density
### Chart 4.D.1

**Backtesting - Quantity and proportion of exceptions in forecast period**

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**Note:** Emerging markets above solid line, Frontier markets below.
Chart 4.D.2  

**Backtesting**

*The Three Zone Approach – Increase in scaling factor k*

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**Note:**  
(1): Increase in scaling factor $k$ pictured in second row of respective stock exchange.  
(2): Emerging markets above solid line, Frontier markets below
## Chart 4.D.3

### Value-at-Risk values

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*Note: Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.*

## Chart 4.D.4

### The Standardised Approach – Number of exceptions

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<th>Market</th>
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<th>Basel II Outcome</th>
<th>Basel III Number of exceptions</th>
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### Chart 4.D.5

**Minimum Capital Requirements VaR MCR(VaR) – MCR² – Basel II directives**

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<tr>
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*Note*: Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.

### Chart 4.D.6

**The stressed VaR (sVaR) proposal – sVaR values**

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<th>Market / Model</th>
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<th>India</th>
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*Note*: Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.
Chart 4.D.7

**Basel III Minimum Capital Requirements - Internal Model Approach (sVaR)**

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<td>12.12%</td>
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<td>11.44%</td>
<td>13.62%</td>
<td>27.18%</td>
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<td>4.77%</td>
<td>9.70%</td>
</tr>
<tr>
<td>EVT</td>
<td>17.10%</td>
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<td>20.39%</td>
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<td>33.50%</td>
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<td>5.06%</td>
<td>13.54%</td>
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</tbody>
</table>

Note: Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.

---

Chart 4.D.8

**Panel A - Basel III Minimum Capital Requirements – Internal Model Approach MCR<sup>3</sup> = MCR<sup>2</sup> (VaR) + MCR (sVaR)**

<table>
<thead>
<tr>
<th>Market / Model</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Republic</th>
<th>Indonesia</th>
<th>Malaysia</th>
<th>China</th>
<th>Argentina</th>
<th>Lithuania</th>
<th>Tunisia</th>
<th>Croatia</th>
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</thead>
<tbody>
<tr>
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<td>47.88%</td>
<td>54.04%</td>
<td>28.69%</td>
<td>14.39%</td>
<td>36.22%</td>
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<td>58.14%</td>
<td>48.96%</td>
<td>51.64%</td>
<td>39.36%</td>
<td>55.37%</td>
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<td>59.19%</td>
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<td>52.70%</td>
<td>44.17%</td>
<td>58.36%</td>
<td>65.94%</td>
<td>32.89%</td>
<td>15.16%</td>
<td>64.51%</td>
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<tr>
<td>FHS-G-N</td>
<td>73.64%</td>
<td>45.11%</td>
<td>41.50%</td>
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<td>56.85%</td>
<td>24.69%</td>
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<td>79.05%</td>
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<td>22.78%</td>
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<td>23.36%</td>
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<td>51.11%</td>
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<td>15.89%</td>
<td>39.50%</td>
</tr>
</tbody>
</table>

Note: Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.
**Chart 4.D.8**  
Panell B - Basel III Minimum Capital Requirements Ranking – Internal Model Approach $MCR^3 = MCR^2 (VaR) + MCR (sVaR)$

<table>
<thead>
<tr>
<th>Market / Model</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Republic</th>
<th>Indonesia</th>
<th>Malaysia</th>
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<th>Lithuania</th>
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<th>Croatia</th>
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<tbody>
<tr>
<td>Linear-N</td>
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<td>Linear-t</td>
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<td>2</td>
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<td>3</td>
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</tbody>
</table>

**Note:**  
(1): Hyphen denotes models belonging to the Red Zone to be eventually excluded by regulators.  
(2): Models are classified in ascending order.

**Chart 4.D.9**  
Increase in $MCR = MCR^3 / MCR^2$

<table>
<thead>
<tr>
<th>Market / Model</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Republic</th>
<th>Indonesia</th>
<th>Malaysia</th>
<th>China</th>
<th>Argentina</th>
<th>Lithuania</th>
<th>Tunisia</th>
<th>Croatia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear-N</td>
<td>95.71%</td>
<td>66.17%</td>
<td>67.45%</td>
<td>84.06%</td>
<td>94.59%</td>
<td>166.72%</td>
<td>69.45%</td>
<td>112.30%</td>
<td>78.77%</td>
<td>85.47%</td>
<td>54.40%</td>
</tr>
<tr>
<td>Linear-t</td>
<td>84.39%</td>
<td>70.88%</td>
<td>100.10%</td>
<td>74.24%</td>
<td>86.50%</td>
<td>159.34%</td>
<td>76.57%</td>
<td>120.91%</td>
<td>78.09%</td>
<td>81.42%</td>
<td>109.61%</td>
</tr>
<tr>
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<td>80.86%</td>
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<td>98.22%</td>
<td>65.09%</td>
<td>86.33%</td>
<td>140.76%</td>
<td>73.73%</td>
<td>122.93%</td>
<td>56.34%</td>
<td>84.49%</td>
<td>92.22%</td>
</tr>
<tr>
<td>FHS/G-N</td>
<td>40.27%</td>
<td>29.30%</td>
<td>37.59%</td>
<td>27.64%</td>
<td>62.06%</td>
<td>96.52%</td>
<td>58.45%</td>
<td>248.88%</td>
<td>39.10%</td>
<td>44.62%</td>
<td>51.64%</td>
</tr>
<tr>
<td>FHS/G-t</td>
<td>38.80%</td>
<td>29.61%</td>
<td>37.12%</td>
<td>26.18%</td>
<td>55.40%</td>
<td>102.23%</td>
<td>57.95%</td>
<td>251.61%</td>
<td>35.29%</td>
<td>44.07%</td>
<td>64.68%</td>
</tr>
<tr>
<td>FHS/E-N</td>
<td>58.75%</td>
<td>41.78%</td>
<td>48.61%</td>
<td>33.82%</td>
<td>83.18%</td>
<td>92.38%</td>
<td>62.68%</td>
<td>226.01%</td>
<td>47.49%</td>
<td>54.78%</td>
<td>46.01%</td>
</tr>
<tr>
<td>FHS/E-t</td>
<td>57.90%</td>
<td>42.00%</td>
<td>44.38%</td>
<td>31.56%</td>
<td>80.60%</td>
<td>92.84%</td>
<td>61.68%</td>
<td>229.14%</td>
<td>45.43%</td>
<td>54.45%</td>
<td>58.13%</td>
</tr>
<tr>
<td>CV/G-N</td>
<td>40.92%</td>
<td>29.50%</td>
<td>41.40%</td>
<td>31.80%</td>
<td>55.16%</td>
<td>99.94%</td>
<td>61.24%</td>
<td>190.37%</td>
<td>54.99%</td>
<td>46.70%</td>
<td>52.12%</td>
</tr>
<tr>
<td>CV/G-t</td>
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<td>28.90%</td>
<td>40.75%</td>
<td>31.19%</td>
<td>54.69%</td>
<td>100.73%</td>
<td>61.97%</td>
<td>190.66%</td>
<td>45.82%</td>
<td>46.08%</td>
<td>57.63%</td>
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<tr>
<td>CV/E-N</td>
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<td>96.15%</td>
<td>61.55%</td>
<td>178.73%</td>
<td>56.94%</td>
<td>46.29%</td>
<td>57.63%</td>
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<tr>
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<td>45.17%</td>
<td>50.83%</td>
<td>39.23%</td>
<td>69.21%</td>
<td>95.94%</td>
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<td>56.00%</td>
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<tr>
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<td>40.92%</td>
<td>29.50%</td>
<td>41.40%</td>
<td>31.80%</td>
<td>55.16%</td>
<td>99.94%</td>
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<td>50.04%</td>
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<td>70.21%</td>
<td>96.31%</td>
<td>61.55%</td>
<td>216.49%</td>
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<td>59.99%</td>
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<td>55.16%</td>
<td>97.96%</td>
<td>60.93%</td>
<td>187.39%</td>
<td>54.99%</td>
<td>46.70%</td>
<td>54.88%</td>
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<td>1.02%</td>
<td>15.53%</td>
<td>-22.49%</td>
<td>8.03%</td>
<td>9.32%</td>
</tr>
</tbody>
</table>

**Notes:**  
(1): Values in bold letters indicate specifications belonging to the Red Zone to be eventually excluded by regulators.  
(2): Abbreviations: AI: Average Increase – Gral: General.  
(3): Yellow and Green encompass all models included in the respective Zone.
**Chart 4.D.10**

*Basel II MCR\(^2\) - Loss coverage and Maximum daily loss*

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<th>Loss coverage</th>
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</thead>
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</tr>
<tr>
<td>Hungary</td>
<td>3.03</td>
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<tr>
<td>India</td>
<td>2.46</td>
<td>28.56%</td>
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<td>Czech Republic</td>
<td>2.62</td>
<td>42.36%</td>
</tr>
<tr>
<td>Indonesia</td>
<td>3.37</td>
<td>36.96%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>2.85</td>
<td>28.49%</td>
</tr>
<tr>
<td>China</td>
<td>5.23</td>
<td>42.06%</td>
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<tr>
<td><strong>Average Emerging</strong></td>
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<td><strong>37.04%</strong></td>
</tr>
<tr>
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</tr>
<tr>
<td>Lithuania</td>
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<tr>
<td>Tunisia</td>
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<td>25.97%</td>
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<tr>
<td><strong>Average Frontier</strong></td>
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Note: Loss coverage = MCR\(^2\) / Maximum loss forecast period

**Chart 4.D.11**

*Standard Deviation and Maximum daily loss – Sample period vs Forecast period*

<table>
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<tr>
<th>Market</th>
<th>Standard deviation Sample period</th>
<th>Standard deviation Forecast period</th>
<th>Standard deviation Variation</th>
<th>Maximum daily loss Sample period</th>
<th>Maximum daily loss Forecast period</th>
<th>Maximum daily loss Variation</th>
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<td>1.82%</td>
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<td>81.28%</td>
<td>7.54%</td>
<td>12.10%</td>
<td>60.45%</td>
</tr>
<tr>
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<td>2.90%</td>
<td>135.72%</td>
<td>5.55%</td>
<td>12.89%</td>
<td>132.11%</td>
</tr>
<tr>
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<td>2.86%</td>
<td>78.53%</td>
<td>11.81%</td>
<td>11.60%</td>
<td>-1.73%</td>
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<td>16.19%</td>
<td>128.70%</td>
</tr>
<tr>
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</tr>
<tr>
<td>Malaysia</td>
<td>1.05%</td>
<td>1.37%</td>
<td>31.06%</td>
<td>6.34%</td>
<td>9.98%</td>
<td>57.34%</td>
</tr>
<tr>
<td>China</td>
<td>1.44%</td>
<td>2.82%</td>
<td>95.78%</td>
<td>9.26%</td>
<td>8.04%</td>
<td>-13.10%</td>
</tr>
<tr>
<td><strong>Avg.Emerging</strong></td>
<td><strong>8.36%</strong></td>
<td><strong>11.68%</strong></td>
<td><strong>39.72%</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argentina</td>
<td>2.22%</td>
<td>2.69%</td>
<td>21.56%</td>
<td>14.76%</td>
<td>12.95%</td>
<td>-12.28%</td>
</tr>
<tr>
<td>Lithuania</td>
<td>1.00%</td>
<td>1.69%</td>
<td>69.14%</td>
<td>5.87%</td>
<td>7.05%</td>
<td>19.95%</td>
</tr>
<tr>
<td>Tunisia</td>
<td>0.44%</td>
<td>0.51%</td>
<td>14.40%</td>
<td>2.12%</td>
<td>5.00%</td>
<td>135.51%</td>
</tr>
<tr>
<td>Croatia</td>
<td>1.43%</td>
<td>2.62%</td>
<td>82.38%</td>
<td>9.03%</td>
<td>10.76%</td>
<td>19.21%</td>
</tr>
<tr>
<td><strong>Avg.Frontier</strong></td>
<td><strong>7.95%</strong></td>
<td><strong>8.94%</strong></td>
<td><strong>12.49%</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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### Sensitivity Analysis – MCR\(^2\) and Loss coverage with varying factor m\(_c\) – EVT-derived values (cont.)

| Country / MCR\(^2\) | m\(_c\) = 3.00 | m\(_c\) = 3.00 | m\(_c\) = 2.95 | m\(_c\) = 2.95 | m\(_c\) = 2.90 | m\(_c\) = 2.90 | m\(_c\) = 2.95 | m\(_c\) = 2.95 | m\(_c\) = 2.90 | m\(_c\) = 2.90 | m\(_c\) = 2.95 | m\(_c\) = 2.95 | m\(_c\) = 2.90 | m\(_c\) = 2.90 | m\(_c\) = 2.95 | m\(_c\) = 2.95 | m\(_c\) = 2.90 | m\(_c\) = 2.90 |
|---------------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Loss Coverage (%)  | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 |
| Brazil              | 36.12 | 2.99 | 35.33 | 2.94 | 34.83 | 2.88 | 34.13 | 2.82 | 33.44 | 2.76 | 32.74 | 2.71 | 32.04 | 2.65 | 31.35 | 2.59 | 30.61 | 2.49 | 30.17 | 2.45 |
| Hungary             | 34.87 | 2.63 | 33.21 | 2.58 | 32.56 | 2.53 | 31.91 | 2.48 | 31.26 | 2.43 | 30.61 | 2.38 | 29.96 | 2.32 | 29.31 | 2.27 | 28.72 | 2.24 | 28.11 | 2.20 |
| India               | 24.76 | 2.13 | 24.28 | 2.09 | 23.80 | 2.05 | 23.33 | 2.01 | 22.85 | 1.97 | 22.38 | 1.93 | 21.90 | 1.89 | 21.42 | 1.85 | 21.00 | 1.82 | 20.73 | 1.81 |
| Czech R             | 36.71 | 2.27 | 36.00 | 2.22 | 35.30 | 2.18 | 34.59 | 2.14 | 33.89 | 2.09 | 33.18 | 2.05 | 32.47 | 2.01 | 31.77 | 1.96 | 31.13 | 1.93 | 30.50 | 1.90 |
| Indonesia           | 32.03 | 2.92 | 31.42 | 2.87 | 30.80 | 2.81 | 30.19 | 2.76 | 29.57 | 2.70 | 28.95 | 2.64 | 28.34 | 2.59 | 27.72 | 2.53 | 27.11 | 2.49 | 26.51 | 2.45 |
| Malaysia            | 24.69 | 2.47 | 24.21 | 2.43 | 23.74 | 2.38 | 23.27 | 2.33 | 22.79 | 2.28 | 22.32 | 2.24 | 21.84 | 2.19 | 21.37 | 2.14 | 20.95 | 2.12 | 20.55 | 2.09 |
| China               | 36.45 | 2.25 | 35.75 | 2.21 | 35.05 | 2.17 | 34.35 | 2.12 | 33.65 | 2.08 | 32.95 | 2.04 | 32.24 | 1.99 | 31.54 | 1.95 | 30.95 | 1.91 | 30.37 | 1.87 |
| Avg. Emg.           | 32.10 | 2.52 | 31.49 | 2.48 | 30.87 | 2.43 | 30.25 | 2.38 | 29.63 | 2.33 | 29.02 | 2.28 | 28.40 | 2.23 | 27.78 | 2.19 | 27.20 | 2.15 | 26.68 | 2.12 |

**Note:** Loss coverage = MCR\(^2\) / Maximum loss forecast period
**Chart 4.D.13**

*Baseline III MCR*² - Loss coverage and Maximum daily loss - Variation over MCR*²*

<table>
<thead>
<tr>
<th>Market / Index</th>
<th>Loss Coverage</th>
<th>Variation over MCR*²</th>
<th>MCR*² = Max. Daily Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>[1]</td>
<td>[2]</td>
<td>[3]</td>
<td>[4]</td>
</tr>
<tr>
<td>Brazil</td>
<td>4.87</td>
<td>40.92%</td>
<td>58.90%</td>
</tr>
<tr>
<td>Hungary</td>
<td>3.93</td>
<td>29.50%</td>
<td>50.60%</td>
</tr>
<tr>
<td>India</td>
<td>3.48</td>
<td>41.40%</td>
<td>40.39%</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>3.45</td>
<td>31.80%</td>
<td>55.83%</td>
</tr>
<tr>
<td>Indonesia</td>
<td>5.24</td>
<td>55.16%</td>
<td>57.35%</td>
</tr>
<tr>
<td>Malaysia</td>
<td>5.71</td>
<td>99.94%</td>
<td>56.96%</td>
</tr>
<tr>
<td>China</td>
<td>8.43</td>
<td>61.24%</td>
<td>67.82%</td>
</tr>
<tr>
<td><strong>Average Emerging</strong></td>
<td><strong>5.01</strong></td>
<td><strong>52.45%</strong></td>
<td><strong>55.41%</strong></td>
</tr>
<tr>
<td>Argentina</td>
<td>3.95</td>
<td>190.37%</td>
<td>51.11%</td>
</tr>
<tr>
<td>Lithuania</td>
<td>5.93</td>
<td>54.99%</td>
<td>41.75%</td>
</tr>
<tr>
<td>Tunisia</td>
<td>3.17</td>
<td>46.70%</td>
<td>15.89%</td>
</tr>
<tr>
<td>Croatia</td>
<td>3.67</td>
<td>52.12%</td>
<td>39.50%</td>
</tr>
<tr>
<td><strong>Average Frontier</strong></td>
<td><strong>4.18</strong></td>
<td><strong>36.05%</strong></td>
<td><strong>37.06%</strong></td>
</tr>
</tbody>
</table>

*Note: Loss coverage = MCR*² / Maximum loss forecast period*

**Chart 4.D.14**

*Sensitivity Analysis - Total MCR with varying factors mₙ and mₑ - EVT-derived values*

<table>
<thead>
<tr>
<th>Case No.</th>
<th>Factors</th>
<th>Brazil</th>
<th>Hungary</th>
<th>India</th>
<th>Czech Republic</th>
<th>Indonesia</th>
<th>Malaysia</th>
<th>Argentina</th>
<th>China</th>
<th>Lithuania</th>
<th>Tunisia</th>
<th>Croatia</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mₙ = 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>mₑ = mₙ</td>
<td>41.80%</td>
<td>39.08%</td>
<td>28.56%</td>
<td>42.36%</td>
<td>36.96%</td>
<td>28.49%</td>
<td>17.60%</td>
<td>42.06%</td>
<td>26.94%</td>
<td>10.83%</td>
<td>25.97%</td>
</tr>
<tr>
<td>2</td>
<td>mₑ = mₙ/0</td>
<td>50.61%</td>
<td>42.30%</td>
<td>35.87%</td>
<td>45.85%</td>
<td>34.32%</td>
<td>35.61%</td>
<td>30.99%</td>
<td>56.53%</td>
<td>37.76%</td>
<td>12.07%</td>
<td>30.35%</td>
</tr>
<tr>
<td>3</td>
<td>mₑ = 3/mₙ</td>
<td>50.61%</td>
<td>42.30%</td>
<td>35.87%</td>
<td>45.85%</td>
<td>43.42%</td>
<td>35.61%</td>
<td>30.99%</td>
<td>56.53%</td>
<td>37.76%</td>
<td>12.07%</td>
<td>30.35%</td>
</tr>
<tr>
<td>4</td>
<td>mₑ = 3/mₙ</td>
<td>50.61%</td>
<td>42.92%</td>
<td>35.87%</td>
<td>46.85%</td>
<td>43.76%</td>
<td>37.98%</td>
<td>30.99%</td>
<td>56.53%</td>
<td>37.76%</td>
<td>12.51%</td>
<td>30.48%</td>
</tr>
<tr>
<td>5</td>
<td>mₑ = 3/mₙ</td>
<td>50.61%</td>
<td>44.84%</td>
<td>35.87%</td>
<td>49.09%</td>
<td>47.16%</td>
<td>42.72%</td>
<td>34.35%</td>
<td>56.53%</td>
<td>37.76%</td>
<td>13.36%</td>
<td>32.74%</td>
</tr>
<tr>
<td>6</td>
<td>mₑ = 3/mₑ = 2</td>
<td>53.20%</td>
<td>46.76%</td>
<td>36.45%</td>
<td>51.34%</td>
<td>47.65%</td>
<td>39.94%</td>
<td>39.23%</td>
<td>59.23%</td>
<td>37.76%</td>
<td>14.20%</td>
<td>34.99%</td>
</tr>
<tr>
<td>7</td>
<td>mₑ = 3/mₑ = 2.5</td>
<td>56.05%</td>
<td>48.68%</td>
<td>38.42%</td>
<td>53.58%</td>
<td>50.95%</td>
<td>45.52%</td>
<td>53.62%</td>
<td>63.52%</td>
<td>39.28%</td>
<td>15.04%</td>
<td>37.25%</td>
</tr>
<tr>
<td>8</td>
<td>m廨 = 3/mₑ = 3</td>
<td>58.90%</td>
<td>50.60%</td>
<td>40.39%</td>
<td>55.83%</td>
<td>57.35%</td>
<td>56.96%</td>
<td>51.11%</td>
<td>67.82%</td>
<td>41.75%</td>
<td>15.89%</td>
<td>39.50%</td>
</tr>
<tr>
<td>9</td>
<td>mₑ = 3</td>
<td>48.76%</td>
<td>45.59%</td>
<td>33.32%</td>
<td>49.42%</td>
<td>43.12%</td>
<td>33.24%</td>
<td>20.53%</td>
<td>49.07%</td>
<td>31.43%</td>
<td>12.70%</td>
<td>30.30%</td>
</tr>
<tr>
<td>10</td>
<td>mₑ = 3/mₑ = 0</td>
<td>57.58%</td>
<td>48.81%</td>
<td>40.63%</td>
<td>52.91%</td>
<td>45.98%</td>
<td>40.35%</td>
<td>39.32%</td>
<td>63.54%</td>
<td>42.25%</td>
<td>13.88%</td>
<td>34.68%</td>
</tr>
<tr>
<td>11</td>
<td>mₑ = 3/mₑ = 0.5</td>
<td>57.58%</td>
<td>48.81%</td>
<td>40.63%</td>
<td>52.91%</td>
<td>45.98%</td>
<td>40.35%</td>
<td>39.32%</td>
<td>63.54%</td>
<td>42.25%</td>
<td>13.88%</td>
<td>34.68%</td>
</tr>
<tr>
<td>12</td>
<td>mₑ = 3/mₑ = 1</td>
<td>57.58%</td>
<td>49.43%</td>
<td>40.63%</td>
<td>53.91%</td>
<td>49.92%</td>
<td>42.73%</td>
<td>33.92%</td>
<td>63.54%</td>
<td>42.25%</td>
<td>14.32%</td>
<td>34.81%</td>
</tr>
<tr>
<td>13</td>
<td>mₑ = 3/mₑ = 1.5</td>
<td>57.58%</td>
<td>51.35%</td>
<td>40.63%</td>
<td>56.15%</td>
<td>53.32%</td>
<td>47.47%</td>
<td>37.93%</td>
<td>63.54%</td>
<td>42.25%</td>
<td>15.16%</td>
<td>37.06%</td>
</tr>
<tr>
<td>14</td>
<td>mₑ = 3/mₑ = 2</td>
<td>60.16%</td>
<td>53.27%</td>
<td>41.21%</td>
<td>58.40%</td>
<td>56.71%</td>
<td>52.22%</td>
<td>42.87%</td>
<td>66.24%</td>
<td>42.25%</td>
<td>16.01%</td>
<td>39.32%</td>
</tr>
<tr>
<td>15</td>
<td>mₑ = 3/mₑ = 2.5</td>
<td>63.01%</td>
<td>55.20%</td>
<td>43.18%</td>
<td>60.64%</td>
<td>60.11%</td>
<td>56.96%</td>
<td>48.45%</td>
<td>70.53%</td>
<td>43.77%</td>
<td>16.85%</td>
<td>41.58%</td>
</tr>
<tr>
<td>16</td>
<td>mₑ = 3/mₑ = 3</td>
<td>65.86%</td>
<td>57.12%</td>
<td>45.15%</td>
<td>62.89%</td>
<td>63.51%</td>
<td>61.71%</td>
<td>54.04%</td>
<td>74.83%</td>
<td>46.24%</td>
<td>17.69%</td>
<td>43.83%</td>
</tr>
<tr>
<td>17</td>
<td>mₑ = 4</td>
<td>55.73%</td>
<td>52.10%</td>
<td>38.09%</td>
<td>56.48%</td>
<td>49.28%</td>
<td>37.98%</td>
<td>23.47%</td>
<td>56.08%</td>
<td>35.91%</td>
<td>14.44%</td>
<td>34.63%</td>
</tr>
</tbody>
</table>

**Notes:**

1. Cases 1, 3, 4, 5, 8, and 17 highlighted in bold letters as referenced in main text.
2. Cases 1 and 8 correspond to Basel II and Basel III directives respectively.
3. Cases 3, 4, 5, 11, and 15: Decreased mₙ values: 1.5, 1.0 and 0.5.
4. Cases 9 and 17: Augmented mₑ values: 3 and 4, no sVAR computed.

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### Chart 4.D.15
**Sensitivity Analysis – Selected examples. Loss coverage and Maximum daily loss**

<table>
<thead>
<tr>
<th>Country / Example</th>
<th>( m = 3 )</th>
<th>( m = 1.5 )</th>
<th>( m = 3 )</th>
<th>( m = 1.5 )</th>
<th>( m = 3 )</th>
<th>( m = 0.5 )</th>
<th>( m = 3 )</th>
<th>( m = 0.5 )</th>
<th>( m = 3 )</th>
<th>( m = 3.5 )</th>
<th>( m = 3.5 )</th>
<th>( m = 4 )</th>
<th>( m = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 1.5 ) Loss coverage</td>
<td>( m = 1 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
<td>( m = 3 ) Loss coverage</td>
<td>( m = 0.5 ) Loss coverage</td>
</tr>
<tr>
<td>Brazil</td>
<td>4.18</td>
<td>50.61%</td>
<td>4.18</td>
<td>50.61%</td>
<td>4.18</td>
<td>50.61%</td>
<td>4.03</td>
<td>48.76%</td>
<td>4.61</td>
<td>55.73%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td>3.48</td>
<td>44.84%</td>
<td>3.33</td>
<td>42.92%</td>
<td>3.28</td>
<td>42.30%</td>
<td>3.54</td>
<td>45.59%</td>
<td>4.04</td>
<td>52.10%</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>India</td>
<td>3.09</td>
<td>35.87%</td>
<td>3.09</td>
<td>35.87%</td>
<td>3.09</td>
<td>35.87%</td>
<td>2.87</td>
<td>33.32%</td>
<td>3.28</td>
<td>38.09%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Czech Rp.</td>
<td>3.03</td>
<td>49.09%</td>
<td>2.89</td>
<td>46.85%</td>
<td>2.83</td>
<td>45.85%</td>
<td>3.05</td>
<td>49.42%</td>
<td>3.49</td>
<td>56.48%</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td>4.30</td>
<td>47.16%</td>
<td>3.99</td>
<td>43.76%</td>
<td>3.96</td>
<td>43.42%</td>
<td>3.94</td>
<td>43.12%</td>
<td>4.50</td>
<td>49.28%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Malaysia</td>
<td>4.28</td>
<td>42.72%</td>
<td>3.81</td>
<td>37.98%</td>
<td>3.57</td>
<td>36.51%</td>
<td>3.33</td>
<td>33.24%</td>
<td>3.81</td>
<td>37.98%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>7.03</td>
<td>56.53%</td>
<td>7.03</td>
<td>56.53%</td>
<td>7.03</td>
<td>56.53%</td>
<td>6.10</td>
<td>49.07%</td>
<td>6.97</td>
<td>56.08%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg.Emg.</td>
<td>4.20</td>
<td>46.69%</td>
<td>4.05</td>
<td>44.93%</td>
<td>3.99</td>
<td>44.31%</td>
<td>3.84</td>
<td>43.22%</td>
<td>4.39</td>
<td>49.39%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg.Ftier</td>
<td>3.43</td>
<td>29.55%</td>
<td>3.27</td>
<td>27.94%</td>
<td>3.25</td>
<td>27.79%</td>
<td>2.85</td>
<td>23.74%</td>
<td>3.25</td>
<td>27.11%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**
1. Loss coverage \([1],[3],[5]\) = MCR\(^3\)/Maximum loss forecast period.
2. Loss coverage \([7],[9]\) = MCR\(^2\)/Maximum loss forecast period.

---

### Chart 4.D.16
**Sensitivity analysis - Selected examples. Variation over MCR\(^2\) and MCR\(^3\)**

<table>
<thead>
<tr>
<th>Country / Example</th>
<th>( m = 3 ) Variation over current directives</th>
<th>( m = 3 ) Variation over proposed directives</th>
<th>( m = 3 ) Variation over current directives</th>
<th>( m = 3 ) Variation over proposed directives</th>
<th>( m = 3 ) Variation over current directives</th>
<th>( m = 3 ) Variation over proposed directives</th>
<th>( m = 3 ) Variation over current directives</th>
<th>( m = 3 ) Variation over proposed directives</th>
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<td>21.10%</td>
<td>-14.07%</td>
<td>21.10%</td>
<td>-14.07%</td>
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<td>-5.38%</td>
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<td>Hungary</td>
<td>17.85%</td>
<td>-11.39%</td>
<td>9.83%</td>
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<td>8.24%</td>
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**Notes:**
1. Loss coverage \([1],[3],[5]\) = MCR\(^3\)/Maximum loss forecast period.
2. Loss coverage \([7],[9]\) = MCR\(^2\)/Maximum loss forecast period.

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