Modelling of off-great circle propagation effects for HF radiowaves received over northerly paths

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Abstract

Both the high latitude ionosphere and mid-latitude trough region are essentially inhomogeneous and non-stationary media containing a multitude of large-scale irregularities. Electron density gradients associated with these irregularities form a diverse range of tilted reflection surfaces for high frequency (HF) radio waves. There are also a large number of smaller scale inhomogeneities that scatter the signal.

A series of measurements in the polar cap, auroral and sub-auroral zones of the Earth have been made by researchers at the University of Leicester over a number of years. It has been established that HF radio signals propagating through the high latitude ionosphere often arrive at the receiver over paths well displaced from the great circle direction, sometimes by up to 100°. Another common feature of high latitude HF propagation is the large Doppler and delay spreads imposed on the signal. These together with directional spread of the received signal energy are important parameters to be considered in the design of communication systems and the associated signal processing methods.

The main outcome of the project described in this thesis is the design of a model of the high latitude ionosphere providing numerical investigation of off-great circle propagation of HF signals based on three-dimensional ray tracing. A large number of simulations were carried out for different types of propagation paths, and a comparison of the model results with observations indicates that the model is capable of reproducing the main features of HF signals propagating in the high latitude ionosphere.

A major outcome of the ray-tracing simulations is that paths other than those subject to experimental investigation can be assessed. It is anticipated that the results of this research will be incorporated into prediction tools for forecasting the effects of off-great circle propagation on any path impinging on the northerly ionosphere.
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1. Introduction

Electron density gradients associated with large scale structures within the ionosphere form tilted reflection surfaces for HF (high frequency) radio waves. Consequently HF radio signals propagating through this region often arrive at the receiver over paths well displaced from the great circle direction. Deviations of a few degrees are associated with tilts due, for example, to the solar terminator and to travelling ionospheric disturbances (TIDs) (Jones and Reynolds, 1975). Very large deviations are particularly prevalent in the high latitude regions where signals often arrive at the receiver with bearings displaced from the great circle direction by up to ±100° or more. These large deviations from the great circle path (GCP) are due to the electron density depletion and the associated ionospheric tilts within the mid-latitude trough at sub-auroral latitudes (Rogers et al., 1997), (Stocker et al., 2000), (Warrington et al., 1997), (Siddle et al., 2004 a,b), and in the polar cap, where they are attributed to the presence of convecting patches and arcs of enhanced electron density (Warrington et al., 1997).

Such large deviations of the direction of arrival of the signals from the GCP have serious implications for the operation of communication and radiolocation systems operating within the HF band. For communication systems, directional antennas are often employed and there is a significant possibility that performance will be degraded at times when the supported propagation path is at directions well displaced from the main lobe of one or both of the transmitting and receiving antennas. Radiolocation systems usually operate by measuring the direction of arrival at several receiving sites. The location of the transmitter is then estimated from the intersection of the individual lines of bearing from each receiving site. Deviations from the GCP will therefore adversely affect the estimate of the transmitter location. Although it is not thought possible to correct for these types of bearing error, it is possible to predict the periods during which the large deviations occur.

In addition to the large-scale tilts, which cause gross deviations of the signal from the great circle direction, irregularities in the electron density distribution may be considered as providing a rough reflecting surface for HF radio waves. As a result of this roughness, signals associated with each propagation mode arrive at the
receiver over a range of angles in both azimuth and elevation. The directional spread of the received signal energy is an important parameter to be considered in the design of multi-element receiving arrays and the associated signal processing methods used, for example, in radiolocation or adaptive reception systems.

A series of measurements have been made by researchers at the University of Leicester over a number of years. Results of these experiments are used in this thesis as a basis for the development of a auroral and sub-auroral regions propagation model. The following summarises the principal measurements:

a) One series of measurements was obtained by means of a wide aperture goniometric DF (direction finding) system located at Alert in Canada. Large quasi-periodic bearing variations of up to 100 degrees from the great circle were observed which were attributed to reflection from arcs and patches of enhanced electron density over paths from Iqaluit to Alert (a path length of 2095 km), and Halifax to Alert (a path length of 4182 km).

b) Another series of observations was being carried out to measure the structure of the received signal over a trans-oval path at Kiruna in Sweden. A multi-channel receiver system connected to a spaced aperture antenna array capable of measuring the signal characteristics as function of time of flight, Doppler frequency, Doppler spread, and by application of a superresolution direction finding (DF) algorithm to the multichannel data, direction of arrival. Since the transmitter and receiver systems are synchronised to GPS, absolute time of flight of the signal may be determined. Transmitters were located on Svalbard (a path length of 1152km) and near to Kirkenes in northern Norway (a path length of 430km). The transmitter operated on 3-minute cycle during which time transmissions were made in sequence on 8 frequencies in the range 4 to 20 MHz. A marked relationship between the Doppler frequency and the measured bearing was observed. Signal components arrived at the receiver from directions to the east of the great circle path have had positive Doppler shifts imposed, whereas signal arriving from the west of the great circle direction have had negative Doppler shifts imposed.

c) Observations of the effect of the trough on the azimuth-of-arrival of HF radio signals at several frequencies propagating in the sub-auroral region have been reported by Rogers (Rogers et al, 1997). These authors investigated the effects on two paths, the first between Halifax, Canada and Cheltenham, UK, and the second
between Halifax and Leitrim, Canada. At the lowest frequency (5MHz), very large bearing deviations of up to ±100° were observed.

d) Measurements of the time-of-flight, direction of arrival and Doppler spread have been undertaken for an HF radio signal on a sub-auroral path between Uppsala, Sweden and Leicester, U.K (Siddle et al., 2004 a,b). During the day, the signal usually arrives from the great circle path (GCP) direction. However, at night, especially during the winter and equinoctial months, the signal often arrived at azimuths well displaced from the GCP.

The main objective of the project described in this thesis is the design of a model of the background ionosphere enhanced with a model of the electron density disturbances existing in the high latitude ionosphere and close to the mid-latitude trough.

Studies undertaken using this model indicate that ray tracing is a useful technique in relation to propagation effects associated with this region of the ionosphere.
2. The high latitude ionosphere and solar-terrestrial interactions

2.1 The high latitude ionosphere

The ionosphere is the ionized component of the Earth's upper atmosphere (see, for example Davies, 1990). Two different ionization processes are involved in its creation: photo-ionization, principally by solar extreme ultraviolet and x-ray radiation, and impact ionization by charged particles. During the daytime and at sub-auroral latitudes, photo-ionization is the dominant process, while at high latitudes and at night impact ionization by precipitating auroral electrons plays an important role in the production of ionospheric plasma. The low-altitude ionosphere occupies approximately the same altitude range as the neutral mesosphere and thermosphere and, between 60 and 800 km, is vertically structured in three layers or regions that differ from one another in composition, density, ionization sources, degree of variability, chemistry, and dynamics. There are (see Figure 2.1) the D region (60-90 km), E region (90-150 km) and F region (150-800 km). Within the auroral oval the nighttime E layer plasma densities can be much higher than indicated by the Figure 2.1. The E layer plasma density profiles can also be strongly altered due to the occasional formation of so-called sporadic Es layers. Es layer densities are also very variable because of the spatial and temporal structure in the ionizing particle precipitation. The ionosphere and neutral atmosphere are closely coupled, dynamically as well as chemically. At low and middle latitudes on the Earth's day.
side, thermospheric neutral winds move the conducting plasma of the ionosphere across the geomagnetic field lines, driving an atmospheric dynamo that generates the solar quiet current systems and the equatorial electrojet, a powerful eastward current that flows in the $E$ region along the geomagnetic equator. In the polar regions it is the ions drifting over the polar cap, in response to the imposed magnetospheric convection electric field, that move the neutrals generating neutral winds with speeds up to 1500 km per hour in the high-latitude $F$-region thermosphere. The ionosphere interacts strongly with the magnetosphere (Dungey, 1961) via electric currents flowing along the geomagnetic field lines that connect the ionosphere to the plasma sheet and the magnetospheric boundary layers. These field-aligned currents produce an electric field across the polar cap, which generates the horizontal currents in the polar ionosphere responsible for the convective ion flow. The field-aligned currents are carried by auroral electrons precipitating downward along the field lines.

2.2 Characteristics of solar activity

Solar activity can be quantified in a number of ways, including measurements of the number of sunspots apparent on the solar disk, or 10.7cm (2800 MHz) solar electromagnetic radiation (see for example Davies, 1990). The life of a sunspot is very variable: some spots last for a few days only, while others exist for a few solar rotations (around 27 days each). The most noticeable feature of sunspots is the periodicity in their occurrence, with the time between sunspot maxima varying between 7 and 15 years with an average 11.1 years, and a yearly mean between about 40 and 200. Spectral analysis of sunspot numbers show the presence of periods of about 80 years, 22 years and 11 years, and as a well as a 27 day cycle associated with solar rotation. Sunspots are characterized by a strong magnetic field. The sun emits energy with a slowly varying intensity. This flux originates at heights from sun’s corona to chromosphere. The level of the flux varies gradually from day to day, in response to the number of spot groups on the disk. Solar flux density at a 2800 MHz has been recorded routinely by a radio telescope near Ottawa. Each day levels are determined at local noon (1700 UT) and then corrected to account for factors such as antenna gain, atmospheric absorption and background sky temperature.
2.3 Interplanetary magnetic field

2.3.1 Magnetosphere

The Earth may be considered as a sphere uniformly magnetized in the direction of the dipole axis, at least to a first approximation. However, the interplanetary magnetic field (IMF), part of the Sun’s magnetic field that is carried into interplanetary space by the solar wind, has very non-stationary characteristics.

The concept of continuous solar wind developed in 1950’s (Parker, 1959). The solar wind is the supersonic flow into interplanetary space of plasma from the Sun's corona, the region of the solar atmosphere beginning at about 4000 km above the Sun's visible surface. It extends to several solar radii into space and is composed of approximately equal numbers of ions and electrons. Embedded in the outgoing solar wind plasma is a weak magnetic field (the IMF). The density, velocity, temperature, and magnetic field characteristics of the solar wind vary with the solar cycle, heliographic latitude, distance from the centre of sun and period of the rotation. In the period of the declining and minimum phases of the solar cycle, the solar wind is dominated by high-speed (500-800 km/sec) flows emanating from regions of low coronal density and temperature where the magnetic field is weak and the field lines are open to interplanetary space, known as coronal holes. They occur both at low latitudes and at the poles. The polar holes are largest at solar minimum, extending sun-equatorward and often merging with low-latitude holes of the same magnetic field polarity (Rickett and Coles, 1991). The solar wind also has a dense low-speed (300 km/sec) component associated with the equatorial coronal streamer belt. Near the ecliptic, the high- and low-speed components form streams in the solar wind flow, moving outward into interplanetary space in a spiral, because of the Sun's rotation. As the streams travel away from the Sun, the high-speed streams eventually overtake the slow-speed flows and create regions of enhanced density and magnetic field known as co-rotating interaction regions. These compressed regions play an important role in solar-terrestrial relations. When they encounter the Earth, geomagnetic storms commence (Rostoker et al., 1980). The triggering of a storm recurs usually with a 27-day periodicity, corresponding to the Sun's rotational
period. In the ascending phase of the solar activity cycle and at solar maximum the polar coronal holes shrink (Rickett and Coles, 1991). Eventually the average solar wind speed slows as the high-speed flows weaken. At the same time, the ambient solar wind flow is strongly perturbed by coronal mass ejection following the explosive ejections of coronal plasma from the regions of closed magnetic field lines. The probability of such eruptions reaches the peaks at solar maximum. Fast earthward directed coronal mass ejections are responsible for non-recurrent geomagnetic storms (Rostoker et al., 1987).

The Earth's magnetosphere (see Figure 2.2) is the cavity formed by the magnetic field, dipolar in the first approximation, compressing the field lines on the dayside and stretching them out to form the magnetotail on the night-side. On the dayside, the magnetosphere expands to a distance of about 10 Earth radii whilst the
magneto-tail stretches out a few hundred of Earth radii in the anti-sunward direction (Russell et al., 1980; Crooker et al., 1982).

The IMF is a vector quantity with three directional components, two of them $B_x$ and $B_y$ are oriented parallel to ecliptic. The third component $B_z$ is perpendicular to ecliptic. The Earth's magnetosphere is a highly dynamic structure and responds quite quickly to the changes in the dynamic pressure of the solar wind and the orientation of the IMF. When the IMF and the geomagnetic field lines are oriented in opposite direction, they can reconnect. As a result the transfer of energy, mass, and momentum from the solar wind to the magnetosphere occurs. When the IMF is oriented southward the strongest interaction occurs. Some of the energy extracted from this interaction goes directly into various magnetospheric processes, while some is stored in the magneto-tail and released later in substorms.

### 2.3.2 Geo-magnetic indexes

The intensity of geomagnetic disturbances is quantified by a variety of geomagnetic indices based on the measurements of surface geomagnetic field fluctuations at magnetometer sites around the Earth.

a) The $AE$ index is an auroral electrojet index obtained from a number, usually greater than 10, of stations distributed in the latitude region that is typical of the northern hemisphere auroral zone (Davis and Sugiura, 1966). For each of the stations the north to south component of magnetic perturbation $H$ is recorded as a function of universal time. A superposition of these data from all the stations enables a lower bound or maximum negative temporary variations of the $H$ component to be determined as known as the $AL$ index. Similarly, an upper bound or maximum positive deviation of $H$ is determined, which is known as the $AU$ index. The difference between these two indices, $AU-AL$, is the $AE$ index. Thus the indices $AU$ and $AL$ give some measure of the individual strengths of eastward and westward electrojets, whereas $AE$ provides a measure of the overall horizontal current strength. Signature of deviations of the $AE$ index from a nominal daily level are known as magnetospheric substorms and may have durations of tens of minutes to several hours.
b) The planetary three-hour-range index $Kp$ is the mean standardized K-index (Bartels et al., 1939) from 13 observatories between 44° and 60° northern or southern geomagnetic latitude. It is scaled from 0 to 9, expressed in thirds of a unit. This planetary index was designed to measure solar particle radiation by its magnetic effects. The other indices derived from $Kp$ are the three-hour index $ap$, the daily indices $Ap$, the $Cp$ and $C9$ characteristics that are related to the daily sum of $ap$.

c) The hourly $Dst$ index (Sugiura, 1964) is obtained from magnetometer stations near to the equator but not so close to the region where the E-region equatorial electrojet dominates the magnetic perturbations observed on the ground. At such latitudes the $H$ (northward) component of the magnetic perturbation is dominated by the intensity of the magnetospheric ring current. The $Dst$ index is a direct measure of the hourly average of this perturbation. Large negative perturbations are indicative of an increase in the intensity of the ring current and typically appear on time scales of about an hour. The decrease in intensity may take much longer, on the order of several hours. The entire period defines the duration of magnetic storm. During a storm can be usually observed several isolated or one prolonged substorm signature in the $AE$ index.

d) The Polar Cap index $PC$ (Troshichev et al., 1989) measures geomagnetic disturbances in the polar cap due to ionospheric and field-aligned currents. It is calculated separately for both hemispheres from only one station in each (Thule and Vostok). The $PC$ index was designed to measure the part of the ionospheric current system that is due to magnetospheric field line convection. It is assumed that the $PC$ index correlate with the solar wind input and measures the energy flow from the solar wind into the Earth's magnetosphere. This index gives the same information as the low altitude polar satellites measuring the diameter of the polar cap. It has been shown to be in good agreement with $AE$-index in the wintertime (Vassiliadis et al., 1996).

2.4 Polar cap zone

The electric field from the solar wind IMF system is projected to the high-latitude ionosphere creating horizontal plasma convection. The actual convection pattern depends on the direction of the IMF. In the simplest case, occurring during
southward IMF (Figure 2.3), the F-region plasma flows anti-sunward over the polar cap (open field lines), and returns sunward within the auroral oval (closed field lines). This creates the two-cell pattern relating to the convection electrojets in the highly conducting auroral region (Clauer and Kamide, 1985). The two-cell pattern is most clear during the substorm growth phase.

![Figure 2.3 Two-cell pattern of the convection flow](spidr.ngdc.noaa.gov/spidr/index.html)

The situation is more complex during northward IMF. In this case the ionospheric convection flow patterns are much more structured, confined to much higher latitudes, polar cap is smaller, and the velocities are of smaller magnitude than in the case of southward IMF. Reconnection is possible only between the IMF and the open field lines of the magneto-tail. In this case a four-cell or three-cell ionospheric convection flow patterns are formed (Lockwood, 1993) for strongly or weakly northward IMF, respectively (see Figure 2.4 and Figure 4.1).

From the widespread point of view, the high latitude cells (that are on open field lines all the time) are due to reconnection, and the low latitude cells are formed due to viscous interaction (Axford and Hines, 1961). The measurements of the cross-polar cap potentials and the dependence of the convection flow geometry on $B_y$ component of IMF can prove that the merging process dominates over viscous...
interaction when the IMF has a southward component. However, during northward IMF the relative importance of the viscous interaction is larger. Changing $B_y$ creates asymmetry in the dawn-dusk direction (Reiff and Burch, 1985).

Localised regions of enhanced electron density drifting across the polar cap in F region of the ionosphere known as polar patches (Weber et al, 1989) are the common feature of the high latitude ionosphere. F-layers polar patches are typically about 500km in dawn to dusk direction but they range from 200 to 1000km (McEwen and Harris, 1996) and exhibit electron-density enhancements of up a factor of 10 above background. Normally polar patches are observed in trains with each patch following the last after a period of a few minutes, commonly when $B_z$ component of IMF is northward. Their average occurrence rate is about 0.5/hour (McEwen and Harris, 1996). The polar patches drift from dayside of the Earth across the magnetic pole in accordance with convection flow patterns at speeds of a few hundred metres per second (McDougall et al, 1996). F-layers patches are observed most frequently in winter and at a sunspot maximum (Buchau et al, 1985) although
they have also observed at sunspot minimum (McEwen and Harris, 1996), but they are less pronounced. There is evidence that they can retain their form as they traverse the whole polar cap (McEwen, 1998).

### 2.5 The auroral and sub-auroral zones

The most important feature of the high-latitude ionosphere is the auroral zone, which can be defined as those regions of the Earth where visible auroras have been observed. Visible auroras are the result of excitation of atmospheric atoms and molecules by energetic particles released from magnetosphere.

#### 2.5.1 Auroras

Geographically, most auroras are located within oval shaped regions around both geomagnetic poles of the Earth. The existence of these auroral ovals was derived from the ground-based observations, however, the first image from space was obtained by the Dynamic Explorer 1 satellite (Figure 2.5 from http://sd-www.jhuapl.edu/Aurora/).

*Figure 2.5 Image of the auroral oval*

**Left frame:** image from the Dynamic Explorer-1 satellite.  
**Right frame:** image from Polar UV Imager.  
*Images from http://sd-www.jhuapl.edu/Aurora/*
The ovals are displaced relative to the magnetic poles such that they extend further towards the equator in the midnight sector. The areas contained within the ovals are the polar caps. Within the polar caps the magnetic field lines are open. At lower latitudes they are closed. Because of this, the auroras on northern and southern oval regions can be mirror images of each other, and are known as conjugate auroras. The diameter of the oval depends on the amount of the open flux in the polar cap. During the active periods, especially during major geomagnetic storms, the ovals expand to lower latitudes. In this area, geomagnetic field lines can guide energetic electrons and protons from the magnetosphere down to the Earth's atmosphere. Precipitating particles lose their energy via collisions with the neutral atoms and molecules in the atmosphere and ionize them at approximately the same altitude range as solar UV radiation. Some of the atmospheric components are excited to higher energy levels generating light emissions. The energy spectrum of the particles responsible for these visible auroras has a broad range, from a few hundred eV to a few tens of keV. Brighter arcs and other discrete features are produced by sheets or beams of energetic electrons with relatively narrow spectrum in energy between 1 and 10 keV and accelerated by field-aligned electric potential drops. Most of this activity occurs within the auroral oval. Usually the extension of the auroral zone during active periods is demarked by the high occurrence of the discrete, visual auroral forms. Each auroral oval consists of a more or less continuous band of faint diffuse emissions, within which brighter discrete arcs are embedded on both the dayside and the night side. The diffuse emissions result from a relatively steady sparse flow of electrons and protons that are precipitated out of the central plasma sheet by interactions with plasma waves (Elphinstone et al., 1994). Auroral emissions, both discrete and diffuse, are excited principally by electron precipitation, although some of the emissions that compose the diffuse aurora can be produced by precipitating protons and are known as proton aurora.

Auroral electron precipitation enhances the plasma density and conductivity of the high-latitude ionosphere at times of strong geomagnetic activity (McDougall et al., 1996). These are strongly localized region of high electron density. EISCAT radar measurements exhibit variations in density by factor up to four in the auroral oval region (Jones et al., 1997). As a function of distance along geomagnetic field
In addition to the emissions from the auroral oval, auroral arcs with a noon to midnight orientation are frequently observed inside the polar cap under conditions of northward IMF. Because of their orientation, these emissions are known as sun-aligned arcs (Figure 2.6 from http://sd-www.jhuapl.edu). The first global image of this phenomenon was reported by (Frank et al., 1982). It consists of luminous belt reaching across the polar cap from noon to midnight. The first ground-based all-sky-camera observations originate from the International Geophysical Year of 1957-1958, (Davis, 1962), (Feldstein, 1963). The arcs are about a hundred kilometres or more wide, and have luminosity that may be comparable to the average emissions within the oval.
These features can last several hours, and move slowly across the polar cap in the direction of the IMF $By$ component in the northern hemisphere (Frank et al., 1986). This motion is in the opposite direction in the southern hemisphere for the same sign of $By$ (Craven et al., 1991). The sun-aligned arcs are of much larger scale and weaker than the typical arcs found in the auroral oval.

Electron-density enhancements of up to 3 times the ambient density are frequently observed in the F-region polar cap ionosphere and are attributed to the presence of sun-aligned arcs. These structures are generally aligned in sun-earth direction and have a dawn to dusk width of about 10 to 100 km, and from 1000 km to transpolar length (Ismael et al., 1977; Carlson et al., 1988; Weber et al., 1989). The arcs have been observed drifting across the polar cap with speeds of up to 300 m/sec either in duskward or dawnward direction, according with the sign of $By$ (Weber et al., 1989), whilst plasma flows in the noon-midnight direction along the length of the arcs at up to 1000 m/sec. Auroral arcs appear commonly when $Bz$ is northward (Carlson, 1994).

### 2.5.3 Mid-latitude trough

An ionospheric trough is a region of decreased F layer plasma density. At these altitudes the plasma depletion is due to decreased concentration of O. Ion convection plays a crucial role in the trough development. The mid-latitude trough or the main ionospheric trough is typical for the sub-auroral ionosphere (Moffett and Quegan, 1983), (Roger et al., 1992), (Halcrow, Nisbet, 1977). It has been explained with the stagnation model when the plasma observed has been convecting through night-side in the absence of ionisation sources. The trough is primarily a night-side phenomenon, though it has been observed at all local times. The poleward, field-aligned edge of the trough lies close to the equatorward boundary of the diffuse aurora. As expected, the trough moves to lower latitudes with increasing geomagnetic activity (Collis and Häggström, 1988). The trough is most regularly observed in winter and equinoctial periods whereas in summer it may only be observed occasionally and near local midnight (Moffet and Quegan, 1983).
3. Numeric ray-tracing

3.1 Introduction

Modelling the influence of ionospheric disturbances on HF propagation in the Earth's ionosphere requires a flexible and versatile method of solving of the wave equation and the construction of a model of the electron density. Many ionospheric propagation problems have been investigated using the method of geometrical optics (GO), which is acceptable within the following limitations:

1) The parameters of the propagation medium must only vary slightly on the scale of a wavelength,
2) The scale-size of any irregularities must be large in comparison to the main Fresnel zone.

The following is a brief description of the GO method after Kravtzov and Orlov, (1980). The geometrical optics approximation is transfer from wave equation to eikonal and main transport equations.

The scalar form of the wave equation is:

\[ \nabla^2 E + k^2 n(r,t)^2 E = 0 \]  

(3.1.1)

where \( E \) is electric field, \( k \) is wave number, and \( n(r,t) \) is refractive index.

In a homogeneous medium, \( n \) is constant, and the solutions of equation (3.1.1) are plane waves, i.e.: \( E = \exp(ik \cdot r) \).

If \( n \) varies slowly with position in the medium, it is reasonable to expect that the solution will be similar to a plane wave and hence it is possible to assume a solution with amplitude and phase separated as follows:

\[ E = A(r) \exp(i \cdot k \cdot \varphi(r)) \]  

(3.1.2)

Calculating derivatives and substituting in (3.1.1), it is possible to obtain an equation, which is equivalent to the Helmholtz equation:
\[ k^2 [n^2 - (\nabla \varphi)^2] A + i k [2 \nabla A \nabla \varphi + A \nabla^2 \varphi] + \nabla A = 0 \] (3.1.3)

This is an inseparable equation in the two unknown functions \( A \) and \( \varphi \). In order to solve it, it is possible to expand \( A \) in an asymptotic series in negative power of the wave number:

\[ A(k, r) = \sum_{m} \frac{A_m(r)}{(ik)^m}. \] (3.1.4)

Treating \( k \) as a variable, which formally tends to infinity, we may then equate each power of \( k \) in (3.1.3) to zero. In this way, we replace (3.1.3) by an equation for \( \varphi \) and an infinite sequence of equations for \( A_m \).

\[ (\nabla \varphi)^2 = n(r, t)^2 \] (3.1.5a)
\[ 2 \nabla A_0 \nabla \varphi + A_0 \nabla^2 \varphi = 0 \] (3.1.5b)

\[ \ldots \]

The function \( \varphi \) determining the phase in (3.1.2) is known as eikonal and (3.1.5a) is also called as the eikonal equation. The second equation (3.1.5b) is known as the main transport equation.

Formally the whole series of amplitude terms \( A_m \) can be constructed, but since the series (3.1.4) is asymptotic, and in general it is not convergent. Nevertheless, neglecting all \( A_m \) with \( m > 1 \) it will, however, accurately represent the solution if \( k \) large enough.

From the point of view of the theory of partial differential equations, every first order partial differential equation can be associated with a set of ordinary differential equations describing the trajectories, which are characteristic for the solution of the original equation. These trajectories (the rays) are orthogonal to the surface of constant \( \varphi \). At the same time Equation (3.1.5a) is the Hamilton-Jacobi equation for the motion of a classical particle. In accordance with the method of classical mechanics (Goldstein, 1969), it is possible to write Hamiltonian equations for the same trajectory.

The equation (3.1.5a) can be rewritten in rectangular coordinates \( r = \{x_1, x_2, x_3\} \) as
The conjugate momenta for the coordinates \( r = \{x_1, x_2, x_3\} \) are

\[
p_i = \frac{\partial \varphi}{\partial x_i}, \quad i = 1, 2, 3,
\]

(3.1.7)

With the Hamiltonian

\[
H(r, p) = 0.5[p^2 - n(r)^2]
\]

(3.1.8)

the eikonal equation takes the form of the Hamilton-Jacobi equation of a classical particle at location \( r \) with momentum \( p \) in the potential field:

\[
H(r, p) = 0.5[p^2 - n(r)^2]
\]

(3.1.9)

The trajectories are the solutions of a set of ordinary differential equations:

\[
\frac{dx_i}{\partial H} = -\frac{dp_i}{\partial H} = \frac{d\varphi}{\sum_{j=1}^{3} p_j \frac{\partial H}{\partial p_j}} = d\tau, \quad i = 1, 2, 3
\]

(3.1.10)

where \( \tau \) is the parameter of location along the trajectories. The equations (3.1.10) are the six Hamiltonian equations for the classical particle

\[
\frac{dr}{d\tau} = \frac{\partial H}{\partial r}
\]

(3.1.11a)

\[
\frac{dp}{d\tau} = -\frac{\partial H}{\partial r}
\]

(3.1.11b)

and the equation for the Hamiltonian characteristic function
\[
\frac{d\varphi}{d\tau} = \sum_{i=1}^{3} p_i \frac{\partial H}{\partial p_i} 
\]

(3.1.11c)

with solution

\[
\varphi = \varphi_0 + \int_{\tau_0}^{\tau} \sum_{i=1}^{3} p_i \frac{\partial H}{\partial p_i} d\tau
\]

(3.1.12)

Using the particular Hamiltonian according (3.1.8) the equations (3.1.11a,b) take the form

\[
\frac{dr}{d\tau} = p
\]

(3.1.13a)

\[
\frac{dp}{d\tau} = n\nabla n = \frac{1}{2} \nabla(n^2)
\]

(3.1.13b)

These are six ray equations, which have to be supplemented by six initial conditions. They are given through initial phase distribution \(\varphi_0(\xi, \eta)\) over a boundary surface \(S(\tau_0) = r_0(\xi, \eta)\), described by arbitrary parameters \((\xi, \eta)\), together with initial momenta \(\frac{\partial \varphi_0}{\partial r_0}\) defined from

\[
\frac{\partial \varphi_0}{\partial r_0} \frac{\partial r_0}{\partial \xi} = \frac{\partial \varphi_0}{\partial \xi}, \quad \frac{\partial \varphi_0}{\partial r_0} \frac{\partial r_0}{\partial \eta} = \frac{\partial \varphi_0}{\partial \eta}
\]

(3.1.14)

With (3.1.13a,b) it is possible in principle to construct rays emanating from each point on the boundary. These rays are in general not perpendicular to the boundary, unless this boundary is a surface of a constant phase. The parameters \((\xi, \eta, \tau)\) (Figure 3.1) can be used as new curvilinear coordinates.
(ξ,η), selecting the ray and τ denoting the position along the ray. These are orthogonal to surface of constant phase.

Instead of the parameter τ, it may be convenient to use the distance s along the ray. From equation (3.1.13a) \((ds)^2 = (dr)^2 + (dr)^2 \) and hence \(ds = n \cdot dτ\).

The ray equations take the form:

\[
\frac{dr}{ds} = \frac{p}{n} \quad (3.1.15a)
\]

\[
\frac{dp}{ds} = \nabla n = \frac{1}{2 \cdot n} \nabla (n^2) \quad (3.1.15b)
\]

The main transport equation (3.1.5b) can be rewritten as:

\[
\nabla (A_0^2 \nabla φ) = 0 \quad (3.1.16)
\]

Integrating this expression over an arbitrary volume not containing the source of the waves with Gauss' theorem gives:

\[
\int_v \nabla (A_0^2 \nabla φ) \cdot dV = \oint_{S} A_0^2 \nabla φ \cdot dS = 0 \quad (3.1.17)
\]

This volume is a ray tube as shown in Figure 3.2. The gradient of eikonal \(\nabla φ\) is perpendicular to the surface of the tube except for the end surfaces \(S(τ_0)\) and \(S(τ)\). Furthermore, \(|\nabla φ| = n\), and \(\nabla φ \cdot dS(τ_0) = -\nabla φ \cdot dS(τ)\).

Then (3.1.17) gives:

\[
\int_{S(τ)} A_0^2 \cdot n \cdot dS = \int_{S(τ)} A_0^2 \cdot n \cdot dS \quad (3.1.18)
\]

For infinitesimally thin ray tube, from (3.1.18) the conclusion can be obtain
that $A_0^2 \cdot n \cdot dS = \text{const}$ along the tube. Hence it is possible to express the amplitude for any point along the ray tube if initial value is given:

$$A_0(r) = A_0(r_0) \sqrt{\frac{n(r_0) dS(\tau_0)}{n(r) dS(\tau)}} \quad (3.1.19)$$

Taking into account the expressions for phase (3.1.12) and amplitude (3.1.19) it is possible to construct the field (3.1.2) in the geometrical-optics approximation:

$$E(r) = A_0(r_0) \sqrt{\frac{n(r_0) dS(\tau_0)}{n(r) dS(\tau)}} \cdot \exp\{ik[q_0(r_0) + \int_{r_0}^{r} n(r) ds]\} \quad (3.1.20)$$

### 3.2 Jones 3D ray tracing computer program

A numerical code (Jones and Stephenson, 1975) has been employed in this investigation. This is a versatile FORTRAN program for tracing rays through an anisotropic medium whose refractive index varies continuously in three dimensions. For each path, the program can calculate the group path length, phase path length, absorption, Doppler shift due to a time varying medium, and the geometrical path length.

The program calculates each ray path by numerically integrating the Hamilton-Jacobi equations for a user-specified model of the ionosphere given the transmitter location (longitude, latitude and height above the ground), the frequency of the wave, the direction of the transmission (elevation and azimuth angles), the receiver height, and the maximum number of hops. The program uses a combination of two Hamiltonian's equations three-dimensional Cartesian (Lighill, 1965) with time and spherical (Haselgrove, 1954) to obtain four dimensions (three dimensional spherical with time) equations. For actual calculation, the ray-tracing program uses the group path $P' = c t$, where $c$ is the speed of electromagnetic wave in free space, and $t$ is the travel time of a wave packet. The resulting equations (Jones and Stephenson, 1975) are:
\[
\frac{dr}{dP'} = -\frac{1}{c} \frac{\partial H}{\partial k_r}, \quad (3.2.1)
\]

\[
\frac{d\theta}{dP'} = -\frac{1}{rc} \frac{\partial H}{\partial k_\theta}, \quad (3.2.2)
\]

\[
\frac{d\phi}{dP'} = -\frac{1}{rc \sin(\theta)} \frac{\partial H}{\partial \omega}, \quad (3.2.3)
\]

\[
\frac{dk_r}{dP'} = \frac{1}{c} \frac{\partial H}{\partial \theta} + k_\theta \frac{d\theta}{dP'} + k_\phi \sin(\theta) \frac{d\phi}{dP'}, \quad (3.2.4)
\]

\[
\frac{dk_\theta}{dP'} = \frac{1}{r} \left( \frac{1}{c} \frac{\partial H}{\partial \theta} - k_\theta \frac{dr}{dP'} + k_\phi r \cos(\theta) \frac{d\phi}{dP'} \right), \quad (3.2.5)
\]

\[
\frac{dk_\phi}{dP'} = \frac{1}{r \sin(\theta)} \left( \frac{1}{c} \frac{\partial H}{\partial \phi} - k_\theta \sin(\theta) \frac{dr}{dP'} - k_\phi r \cos(\theta) \frac{d\theta}{dP'} \right), \quad (3.2.6)
\]

\[
\frac{d(\Delta f)}{dP'} = \frac{1}{2\pi} \frac{d\Delta \omega}{dP'} = -\frac{1}{2\pi} \frac{\partial H}{\partial \omega}, \quad (3.2.7)
\]

where \(r, \theta, \phi\) are the spherical coordinates of a point on the ray path, \(k_r, k_\theta, k_\phi\) are the components of the propagation vector, \(f\) and \(\omega=2\pi f\) are the frequency and the angular frequency of the wave respectively, and \(\Delta f\) is the Doppler shift.

The refractive index equations used in the Jones3D ray-tracing program are based on the Appleton – Hartree formula (Budden, 1961), or the Sen - Wyller formula (Sen and Wyller, 1960). The numerical integration subroutine has a built-in mechanism to check errors and adjust the integration step length accordingly.
3.3 Modification of the Jones 3D ray-tracing program

3.3.1 Precision modification

The program was converted from single to double precision in FORTRAN77 and modified to work under UNIX by Stocker (1997). A major change is the addition of an IDL front-end to the program (Stock, 1997) to provide a more flexible and versatile graphical user-interface for plotting the ray-tracing results onto a map of the electronic density.

3.3.2 Time varying ionosphere

An additional loop in time was implemented into the program in order to provide more flexible way of investigating time-dependent processes in the ionosphere. Hence, because many input geophysical parameters are time-dependent, the program recalculates them before each time step.

3.3.3 Calculation of the signal amplitude

It is possible to express the amplitude for any point along the ray tube (Equation 3.1.19) and eventually at the receiver height, but that does not solve the problem of the calculation of the received signal strength. Two different problems arise: constructing the reliable 'homing procedure' and separating the propagation modes.

The high latitude ionosphere is a disturbed region of the Earth’s ionosphere containing irregularities on various scales and amplitudes. This makes it difficult to construct an acceptable convergent algorithm for a homing procedure to distinguish correctly the propagation modes due to the very complicated structure of the rays.

Alternatively, the calculation of the number of rays falling in the area close to the receiver produces reasonable results. The area has the shape of an ellipse, (see Figures 3.3, 3.4) oriented along the bearing angle of arrival, with semi-axis
3.3.4 Variation of the azimuth and elevation step sizes

The program run-time increases rapidly whenever a ray travels through the regions with large gradients of electron density, a situation which commonly arises in the high latitude ionosphere. To improve execution speed, an adaptive procedure to adjust the size steps in both elevation and azimuth angles was incorporated into
the code. In the ray-tracing program the last loop is in elevation angle, and the loop in azimuth angle executes immediately before. In each time step $T_{i+1}$ (excluding the first) the nominal steps in both elevation angle $el_{st_n}(T_{i+1})$ and azimuth, $az_{st_n}(T_{i+1})$ depend on the number of rays reaching the receiver in the previous time step $T_i$ as:

$$az_{st_n}(T_{i+1}) = az_{st_n}(T_i) \frac{N(T_i)/No}{1.72}$$

(3.3.1)

$$el_{st_n}(T_{i+1}) = az_{st_n}(T_i) \frac{el0/az0}{el/az}$$

(3.3.2),

where $N(T_i)$ is the number of rays reaching the receiver in previous time step and $el0$ and $az0$ are user defined steps. $No=10 \ c/D$, where $c$ is the smaller hemi-axis (from Figure (3.4)), and $D$ is the distance between transmitter and receiver.

![Figure 3.5 Schematic to illustrate the adaptive algorithm for angles step](image)

The actual angle steps employed depend on additional conditions. When the loop in elevation angle has been executed, the step changes depend on how close the specific ray goes to the receiver. The shaded area in Figure 3.5 corresponds to the rays with angles of radiation those falling in ellipsis area of Figure 3.4. If the ray hits inside the shaded area then, $el_{st}(T_{i+1}) = el_{st_n}(T_{i+1})/2$, else if it hits inside dashed area, (dashed area is corresponding to a collecting area twice as large as the shaded) $el_{st}(T_{i+1}) = el_{st_n}(T_{i+1})$, otherwise $el_{st}(T_{i+1}) = 2.0*el_{st_n}(T_{i+1})$. The following step in azimuth also changes according to this algorithm. The bold stepped line at the
top of Figure 3.5 separates two ranges in elevation angles of transmitter. If all rays in specific range of azimuth (20 degrees in Figure 3.5) penetrate the ionosphere in previous time step $T_p$, the elevation angle step above stepped line in the same azimuth range at present time step $T_{isj}$ is 3.0*el0.

### 3.3.5 Multiple receivers

Changes to the program were also made to allow ray-tracing simulation for a number of receivers. The locations of the first three are specified in user-input data, and the coordinates a further 60 receivers may be calculated inside the code, based upon input values for them (up to 20 azimuths and 3 ranges).
4. Computational model of electron-density

The ray-tracing program requires the electron density ($Ne$) profile and gradient of the refractive index to be continuous. It is convenient to approximate the electron density with analytical expressions that are amenable to mathematical manipulation. Furthermore, it is important to construct a model of $Ne$, which can be implemented within the ray tracing code since calling an external program at each step of the ray calculation results in unacceptable calculation times.

4.1 The background ionosphere

There currently exist a number of ionospheric models (for example (Bilitza, 1990), (Chasovitin et al., 1987)) that describe the ionosphere under undisturbed conditions. However none of them can reproduce the ionospheric profile in high latitude region with sufficient accuracy (Egorova et al., 1995). Furthermore, they do not reflect the day-to-day variations of the real ionosphere.

For this project, an adjustable model of electron-density profile was constructed based on data from vertical soundings, which are now available via the Internet (e.g. http://spidr.ngdc.noaa.gov/spidr/index.html). The input parameters of the model are the maximum $f_{\text{max}}$ and minimum $f_{\text{min}}$ value of the critical frequencies during the day, heights and height scales (if available) of the F2 and E layers for three reference points and the time of the day when $f_{\text{max}}$ and $f_{\text{min}}$ occur. One reference point was located at the geographical North Pole, while the two others are variable, depending on region of interest and the reliability of the vertical sounding data in database.

The longitudinal dependence of $Ne$ was derived from an approximation of the time dependence of the layer’s critical frequency at the chosen reference points. Three slightly different approximating functions $f_1$, $f_2$, and $f_3$, have been used, reflecting the diurnal variability of the $Ne$ profile.

$$f_1 = \cos(\varphi_i - \varphi + \varphi_{\text{det}}) - 0.15 \cdot \cos(3 \cdot (\varphi_i - \varphi + \varphi_{\text{det}})) / 0.867$$ (4.1.1)
\[
f_2 = \cos(\varphi_i - \varphi + \varphi_{\text{del}}) + 0.2 \cdot \cos(2 \cdot (\varphi_i - \varphi + \varphi_{\text{del}}) + \frac{\pi}{4}) + \\
0.15 \cdot \cos(3 \cdot (\varphi_i - \varphi + \varphi_{\text{del}}) + \frac{2\pi}{3}) - 0.2
\]  \hspace{1cm} (4.1.2)

\[
f_3 = \cos(\varphi_i - \varphi + \varphi_{\text{del}}) + 0.15 \cdot \cos(2 \cdot (\varphi_i - \varphi + \varphi_{\text{del}}) + \frac{\pi}{8}) + \\
0.15 \cdot \cos(3 \cdot (\varphi_i - \varphi + \varphi_{\text{del}}) + \pi)
\]  \hspace{1cm} (4.1.3)

where \(\varphi\) is the current geographical longitude, \(\varphi_{\text{del}}\) is the delay of the moment, when critical frequency reaches it's minimum during the day, relative to local midnight, \(\varphi_i = 15 \cdot t \cdot \pi / 180\), and the \(t\) is universal time in hours. In the actual calculations the function, \(f_1\), \(f_2\), or \(f_3\) that best resembles the experimental profile of \(Ne\) was used. The graph of that function is given in Figure 4.1.

![Figure 4.1 Profile approximating functions](image)

The critical frequency of the layer as a function of longitude and time at the latitude of the \(i\)-th reference point is:

\[
f_{cr}(\varphi, i) = [f_{\text{max}}(i) - f_{\text{min}}(i)] \cdot f_{1,2,3} / 2 + [f_{\text{max}}(i) - f_{\text{min}}(i)] / 2
\]  \hspace{1cm} (4.1.4)

where \(f_{\text{max}}\) and \(f_{\text{min}}\) are maximum and minimum values of critical frequency at the each reference point.
Analysis of vertical sounding data coupled with typical values from various ionospheric models indicates that the latitudinal dependence of the critical frequency of each layer can be approximated as:

\[
fo(\varphi, \theta) = c1 \cdot \theta^2 + c2 \cdot [\cos(k_o \theta + \theta_0) - \cos \theta_0] + f_{pl}
\]  

(4.1.5)

where \( \theta \) and \( \varphi \) are the current geographical latitude and longitude respectively, and \( f_{pl} \) is the critical frequency at North Pole. The parameters \( c1, c2 \) are derived from the values of the critical frequencies at the reference points under conditions:

\[
f_C(r, \varphi) = f_C(r) = 2,3 \quad (4.1.5a)
\]

The parameters \( f_{pl}, k_o, \theta_0 \) are input parameters that allow the user to adjust the model.

A double Chapman profile (see for example Davies, 1990) with the height and half-thickness of the layers dependent on latitude was used as an approximation of the vertical Ne profile.

\[
R_x(h) = \exp(0.5 \cdot (1 - \exp(-z) - z))
\]

(4.1.6)

where \( h \) is current height, \( z = (h - H_m)/H_0 \). \( H_m \) and \( H_0 \) are the height above the Earth and half-thickness of the layer respectively.

The height of the layer, replacing \( H_m \) in formula (4.1.6) is expressed by Epstein function of latitude:

\[
H_m(\theta) = H_{nv} - (H_{nv} - H_{ns})(1 + \exp((\theta_5 - \theta)) / sc_s)^{1} + \Delta_{Ne} \cdot H_{nv} \cdot fNS \cdot fEW^2
\]

(4.1.7)

where \( H_{nv} \) and \( H_{ns} \) are the height of the layers at northern and southern reference point respectively, \( \theta_5 \), and \( sc_s \) are the latitude and latitudinal scale of the southern border of the mid-latitude trough respectively, \( fNS, fEW \) are the functions defining
the structure of the trough for which exact expressions are given in the next section. The half-thickness of the layer $scH$, replacing $H_0$ in expression (4.1.6), depends on $\theta$ and $\varphi$ as:

$$scH(\theta, \varphi) = scH_n + (scH_T - scH_N) \cdot (1 + \exp((\theta_n - \theta) / sc_N))^{-1} -$$

$$(scH_T - scH_s) \cdot (1 + \exp((\theta_s - \theta) / sc_s))^{-1}$$  \hspace{1cm} (4.1.8)

where $scH_T$, $scH_N$, and $scH_s$ are half-thicknesses of the layers inside the trough, and at the northern and the southern reference points respectively, and are user-input parameters (note that $\theta_s$ and $\theta_n$ depend on $\varphi$).

### 4.2 The mid-latitude trough

#### 4.2.1 The Halcrow and Nisbet model

A parameterised model of the location of the walls of the mid-latitude trough based upon top-side sounder measurements was published by Halcrow and Nisbet (1977). The model assumes a constant electron density depletion along the bottom of the trough, and electron density in each of the boundary walls rises linearly from fully depleted at the bottom to undepleted at the top. The invariant latitudes of the top (subscript $t$) and bottom (subscript $b$) of the north (subscript $n$) and south (subscript $s$) are given therein as:

$$\Lambda_n = 54^\circ - 0.5^\circ (Kp - 1/3) + 7^\circ [1 + [(T^2 - 4)/(5.8T + 0.1)]^{22} - 0.5 \{1 + [(T^2 - 8)/(13.5T + 0.1)]^{16}\}^{0.5}]$$

$$\Lambda_s = 64^\circ - 1.0^\circ (Kp - 1/3) + 9^\circ [\exp(-1.9T^{5.42} 10^{-5}) + \exp(-2.15(24 - T)^{4.55} 10^{-3})]$$

$$\Lambda_n = 48^\circ + 5^\circ [1 + [(T^2 - 4)/(5.65T + 0.1)]^{22} - 0.5 \{1 + [(T^2 - 8)/(12.35T + 0.1)]^{14}\}^{0.5}]$$

$$\Lambda_n = 67^\circ - 1.25^\circ (Kp - 1/3) + 9^\circ [\exp(-1.7T^{5.32} 10^{-5}) + \exp(-4.66(24 - T)^{4.10} 10^{-4})],$$

where LT is local time in hours and $T = LT + 12$ if $LT < 12$ and $T = LT - 12$ if $LT \geq 12$.

The closing of the trough at sunrise occurs at local times when the solar zenith angle lies between $95^\circ$ and $87^\circ$.

The opening of the trough begins at a local time 1.5 hours after the solar zenith angle reaches $\chi = 87^\circ - 3^\circ$ (Kp-1/3), and the trough is fully formed at a local time 1.5 hours after solar zenith angle reaches $\chi = 91^\circ - 3^\circ$ (Kp-1/3).
4.2.2 Model used in the simulations

Whilst the Halcrow and Nisbet model has an advantage of being based on observations, it cannot reproduce diurnal variations of the trough due to its statistical character. In order to achieve the required flexibility in simulation, another analytical approximation defining the location of the trough has been used. The input parameters to the model, which define the location of the trough are: geomagnetic coordinates of the central point of the southern ($\Theta_{cs}, \Phi_c$) and northern ($\Theta_{cn}, \Phi_c$) borders, and the eastward $\Delta_E$ and westward $\Delta_W$ extent of the trough, and their scale $sc_E, sc_W$ respectively. The South and North borders of the trough are formed by the composition of the Epstein functions with parameters $\delta\Theta_s, \delta\Theta_n, \delta\Phi_{es}, \delta\Phi_{en}, \delta\Phi_{ws}, sc_{es}, sc_{en}, sc_{wn}, sc_{ws}$. The meaning of these parameters is clear from the equations 4.2.7 - 4.2.10 and Figure (4.2)

The geomagnetic coordinates of the four points defining the position of the trough depend on the level of geomagnetic activity $Kp$ as:

$$\Theta_{EN} = \pi/2 - \Theta_{cn} + (Kp - 1/3) \cdot \pi/180 \quad (4.2.1)$$
$$\Theta_{WN} = \pi/2 - \Theta_{cn} + (Kp - 1/3) \cdot \pi/180 \quad (4.2.2)$$
$$\Theta_{WS} = \pi/2 - \Theta_{cs} + 0.3 \cdot (Kp - 1/3) \cdot \pi/180 \quad (4.2.3)$$
$$\Theta_{ES} = \pi/2 - \Theta_{cs} + 0.3 \cdot (Kp - 1/3) \cdot \pi/180 \quad (4.2.4)$$
$$\Phi_E = \Phi_c + \Delta_E \quad (4.2.5)$$
$$\Phi_W = \Phi_c - \Delta_W \quad (4.2.6)$$

The longitude of the centre of the trough depends on time as $\Phi_c(t) = 15 \cdot t \cdot \pi/180 - \Phi_{GM}$, where $\Phi_{GM}$ is the geographic longitude of the geomagnetic pole, and $t$ is the universal time in hours. The southern border of the trough is given in these terms by:

$$\Theta_{sb} = \Theta_{WN} + (\Theta_{WS} - \Theta_{WN})/4 - \delta\Theta_s/(1 + \exp ES) +$$
$$\delta\Theta_s/(1 + \exp WS) + (\Theta_{WN} + \Theta_{WS} + (\Theta_{WS} - \Theta_{WN})/4) \cdot fEW \quad (4.2.7)$$

where $\exp WS = \exp((\Phi_c + \delta\Phi_{ws} - \varphi)/sc_{ws})$, and $\exp ES = \exp((\Phi_c + \delta\Phi_{es} - \varphi)/sc_{es})$
Figure 4.2 Schematic to illustrate definition of the trough parameters in formulas 4.2.7-4.2.10.

The function \( f_{EW} \) determine the dynamics of opening and closing of the trough in time:

\[
f_{EW} = [1 + \exp(-\varphi + \Delta_E)/sc_E]^{-1} - [1 + \exp(-\varphi + \Delta_W)/sc_W]^{-1} \tag{4.2.8}
\]

where \( \varphi \) is current geomagnetic longitude, \( sc_W \) and \( sc_E \) are the longitudinal scale of the western and eastern ends of the trough respectively.

Similarly for the northern border: \( sc_p \)

\[
\theta_{Nb} = \theta_{WN} + \delta\theta_N[(1 + \exp WN)^{-1} - (1 + \exp EN)^{-1}] + (\theta_{WS} - \theta_{WN})/2 \cdot (1 + \cos(\Phi_e - \varphi)/4) \tag{4.2.9}
\]

where \( \exp EN = \exp((\Phi_e + \delta\Phi_{EN} - \varphi)/sc_{EN}) \) and \( \exp WN = \exp((\Phi_e + \delta\Phi_{WN} - \varphi)/sc_{WN}) \).

The latitudinal dependence of the electron-density depletion inside the trough is derived from that function as:
\[ f_{NS} = \left[1 + \exp(\theta_{nb} - \theta)/sc_N\right]^{-1} - \left[1 + \exp(\theta_{sb} - \theta)/sc_s\right]^{-1} \]  

(4.2.10)

where \(sc_N\), and \(sc_s\) are the latitudinal scale of the wall of the trough, corresponding to \((\Lambda_t - \Lambda_s)\) in Halcrow and Nisbet model.

The electron-density with the presence of the sub-auroral trough according the expressions 4.2.8 and 4.2.9 is given by

\[ N_e = N_{e0} \cdot (1 - \Delta_{Ne} \cdot f_{NS} \cdot f_{EW}) \]  

(4.2.11)

where \(N_{e0}\) is the background electron density and \(\Delta_{Ne}\) is the maximum value of the electron density depletion inside the trough.

The model described above recreates, to a large extent, the Halcrow and Nisbet model, which has a statistical character and can reflect only the mean features of the trough. In the real ionosphere the structure of the trough is significantly more complicated due to vertical and horizontal transport processes, which lead to the development of the irregularities of the electron density at smaller scale. In order to reflect that feature the smooth borders of the trough had superimposed on them a spline approximated random function of longitude \(f_{\text{rand}}\) with range \(\pm 1\) as:

\[ \theta_{Ne} = \theta_{Nb} + \Delta_{Nb}f_{\text{rand}} \]  

(4.2.12)

\[ \theta_{Se} = \theta_{Sb} - \Delta_{Sb}f_{\text{rand}} \]  

(4.2.13)

where \(\Delta_{Sb,Nb}\) are the amplitudes of the latitudinal variations of the southern and northern border of the trough, \(f_{\text{rand}}\) is a random function in the range \(\pm 1\) with the longitudinal scale that is a user-input parameters. The scale in this context means the longitudinal distance between the nodes of random function.

The resulting expressions \(f_{NS t}\) and \(f_{EW t}\) replacing \(f_{NS}\) and \(f_{EW}\) respectively are given by:

\[ f_{NS t} = \left[1 + \exp(\theta_{nb} - \theta)/sc_N\right]^{-1}[1 + \Delta_{N_{wall}} \exp(-\sigma_{\theta_{Ne}}/sc_N)^2)] - \left[1 + \exp(\theta_{sb} - \theta)/sc_s\right]^{-1}[1 + \Delta_{S_{wall}} \exp(-\sigma_{\theta_{Sb}}/sc_s)^2)] \]  

(4.2.14)
\[ f_{EWt} = f_{EW} (1 + \Delta_{EW} f_{EW}) \]  \hspace{1cm} (4.2.15)

Here \( \Delta N_{wall} \) and \( \Delta S_{wall} \) are the random functions of latitude with scale \( scW \) and range \( \pm \Delta N_{wall}, \pm \Delta S_{wall} \) respectively, and \( \Delta_{EW} \) are the random function of longitude with scale \( sc_{EW} \) and range \( \pm \Delta_o_{EW} \). The parameters \( \Delta N_{wall}, \Delta S_{wall}, \Delta_o_{EW} \) and \( \sigma \) are user defined.

The resulting expression for the electron density, \( N_e \), in the ionosphere in the presence of the mid-latitude trough with irregularities is

\[ N_e = N_{e0} \cdot (1 - \Delta_{Ne} \cdot fNSf \cdot f_{EWt} \cdot f_{Rt}) \]  \hspace{1cm} (4.2.16)

The factor \( f_{Rt} \) is included for correction the vertical profile of the depletion \( N_e \). In actual simulation \( f_{Rt} = 1 \), since there is little information about the vertical profile of the electron density inside the trough.

### 4.3 Model of electron density irregularities in the auroral oval

Auroral emissions due to precipitation are observed in oval-shaped bands lying between 65° and 75° of magnetic latitude and centred on the northern magnetic pole. This region is known as the auroral oval, and consists of a more or less continuous band of faint diffuse emissions, within which brighter discrete arcs are embedded on both the dayside and the night side. These diffuse emissions result from a relatively steady flow of electrons and protons that are precipitated out of the central plasma sheet by interactions with plasma waves (see Chapter 2 and references therein). A sharp increase in electron concentration over the height range 100 - 180km, as measured by EISCAT UHF radar (69.6°N 19.2E°), provides a clear signature of auroral precipitation (Jones, et al., 1997).

In the following computational model of the auroral precipitations the southern border of the oval is detached from the northern wall of the mid-latitude trough \( \theta_{Nb} \) at the value of the north wall scale of the trough \( sc_N \).

In latitude-longitude coordinates the expression for electron-density irregularities is:
\[ f_{pr} = \Delta_{pr} \cdot fEW \cdot (fN_p \cdot \delta N_{lat} + fB_p \cdot \delta B_{lat}) \cdot \delta_{lon} \]  \hspace{0.5cm} (4.3.1),

where:

\[ fB_p = [1 + \exp(\theta_{bp} - \theta)/sc_p]^{-1} - [1 + \exp(\theta_{bp} + \delta\theta_{sp} - \theta)/sc_p]^{-1} \]  \hspace{0.5cm} (4.3.2),

\[ fN_p = [1 + \exp(\theta_{np} - \delta\theta_{np} - \theta)/sc_p]^{-1} - [1 + \exp(\theta_{np} - \theta)/sc_p]^{-1} \]  \hspace{0.5cm} (4.3.3)

\( \theta_{np} \) and \( \theta_{bp} \) are the latitudinal positions of the auroral oval and \( \theta_{np} = \theta_{bp} - sc_N \).

In the case of strong events when auroras can be observed at latitudes south the mid-latitude trough \( \theta_{bp} = \theta_{sp} + sc_s \), or \( \theta_{bp} = \theta_{np} - 3 \delta\theta_{np} \) in the situation when additional auroras could be observed moving in northward direction.

The widths of the auroral oval zones are \( \delta\theta_{np} \), and \( \delta\theta_{sp} \), respectively and \( sc_p \) is their latitudinal scale, \( \delta N_{lat} \), and \( \delta B_{lat} \) are the random function with range \( \pm 1 \) and user defined latitudinal scales \( \delta_{lat} \), specifying distribution of the electron-density irregularities in the latitudinal direction, \( \delta_{lat} \) uses for the same purpose for longitude, and \( \Delta_{pr} \) is the intensity of the irregularities.

In the vertical plane a Gaussian distribution along the magnetic field line with a user, specified height scale has been adopted.

### 4.4 Model of the F-layer patches and sun-aligned arcs

At present, there exists no reliable self-consistent model of F-layer patches and sun-aligned arcs of enhanced electron-density in the polar ionosphere. Nevertheless, ray tracing simulation require continuous, three dimensional distribution (with derivatives) of \( Ne \) in the area of calculation. The computational model of the electron-density irregularities responsible for off-great circle HF propagation must adhere to the general understanding of the structure of the high latitude ionosphere, but on the other hand be simple enough to be incorporated into the ray tracing code.

The process of formation and evolution of F-layer patches and sun-aligned arcs has an irregular, stochastic character. The F-layer patches are formed on dayside
of the Earth and travel across the polar cap in accordance with the flow convection patterns. However, the number of patches, their specific trajectory, intensity and spatial scale of the electron-density disturbance are not well defined.

A quasi-statistical approach has been adopted in modelling F-layer patches and arcs, in which their distributions inside the polar cap were determined by one of a number of different scenarios. The size of the region in which the patches and arcs were distributed is a function of $K_p$. The sun-aligned arcs move slowly across the polar cap in the direction of the IMF $B_y$, whilst the trajectories of the patches were adopted as a deformed ellipse, with parameters depending on the IMF.

These trajectories resemble the convection flow patterns in Figure 4.3. The speed of the patches has a Gaussian distribution, where the parameters of the distribution are a user-input parameter.

![Figure 4.3 Model of the cross polar cap convection flow (Lockwood, 1993).](image)

Each patch in the cell is composed of an arbitrary number of three-dimensional Gaussian distributions with approximately equal scale in each of the
horizontal directions. The number of the cells is a user-input parameter, depending on the scenario, which is defined by IMF. The position of each component distribution in the patches is defined as a random function with specific spatial scale around the regularly distributed 'nodes' in the area surrounding the geomagnetic pole. The temporal evolution of the patches depends on the movement of the component distributions forming the patch coupled with the rotation of the Earth beneath the convection flow patterns. A Gaussian distribution for the speed inside each cell was adopted. The maximum value of the speed $V_0$ and the distance from the centre of the cell where this value is attained $\rho_{vm}$ are both a user-input parameters. Two versions of the speed distribution have been incorporated into the code. In the first case, all component distributions move with the linear speed depending on their distance from $\rho_{vm}$. In the second case, the portions belonging to the specific patch travel with equal angular velocity around the centre of the cell. The intensity of the distributions depends exponentially on the time from the moment of their formation with specific user defined lifetime. The numbers of the nodes in Sun-Earth and dawn-dusk directions are user-input parameters.

Each arcs is consist of a number of three-dimensional Gaussian distributions with different scale in dawn to dusk and noon-midnight directions. The position of each distribution in the arcs is defined as a random function with specific spatial scale around the quasi-regularly distributed 'nodes' in the polar cap area.

The size of the convection flow area (polar cap area) $\Delta$ corresponds roughly to the distance between the geomagnetic pole and the northern border of the mid-latitude trough:

$$\Delta = \pi/2 - \theta_{en} + (Kp - 1/3) \cdot \pi/180 + \delta \Theta_N / 2$$  \hfill{(4.4.1)}

and depends on $Kp$.

The positions of the nodes for each cell in quasi-Cartesian coordinates:

$$X_{0_{ij}} = \Delta \cdot (2i - 1 - k) / k$$ \hfill{(4.4.2)}

$$Y_{0_{ij}} = \Delta \cdot (2j - 1 - l) / l$$ \hfill{(4.4.3)}
where $k$ is the number of nodes (patches or arcs) in dawn-dusk direction, and $i$ runs from 1 to $k$, $l$ is the number of nodes in noon to midnight direction, and $j$ runs from 1 to $l$. The positions of the $n^{th}$ component, which form the $ij^{th}$ patch are:

$$x_{ijn} = X_{0ij} + \delta_{ij} \cdot rf(i + j + n)$$

(4.4.4)

$$y_{ijn} = Y_{0ij} + \delta_{ij} \cdot rf(i + j + n + k + l)$$

(4.4.5)

where $rf$ is a random function with range $\pm 1$, and values depending on value its argument, $\delta_{ij}$ is the maximum distance of the portions from each node, and $n$ runs from 1 to $m$, where $m$ is the number of the component forming the patch or arcs. In terms of polar coordinates the expressions for the position of the components are:

$$\alpha_{ijn} = \arctan\left(\frac{x_{ijn}}{y_{ijn}}\right)$$

(4.4.6)

$$\rho_{ijn} = (x_{ijn}^2 + y_{ijn}^2)^{1/2}$$

(4.4.7)

Then expression for angular speed can be given in the first case of the velocity field distribution as,

$$\omega_{ijn} = \exp\left(-\left(\frac{\rho_{ijn} - \rho_{vm}}{\Delta \cdot seV}\right)^2\right) \cdot \rho_{ijn}^{-1}$$

(4.4.8)

and in the second case as,

$$\omega_{ijn} = \exp\left(-\left(\frac{\rho_{ijn} - \rho_{vm}}{\Delta \cdot seV}\right)^2\right) \rho_{ijn}^{-1}$$

(4.4.9)

Current coordinates of the components as function of time $t$ are:

$$\alpha_{ijn}(t) = \omega_{ijn} \cdot Vo \cdot t$$

(4.4.10),

whilst $\rho_{ijn}$ is independent of time.
These formulas give a random distribution of components moving on circular trajectory around the geomagnetic pole. To make the trajectories more realistic, it is necessary to shift the centre of the cell by $\Delta_c$, depending on the $B_y$ component of IMF, and change the shape of the convection flow in accordance with Figure 4.3. In Cartesian coordinates the position of the components will then be:

$$x_{ijn} (t) = \rho_{ijn} \sin(\alpha_{ijn}) \cdot \epsilon - \Delta_c \left(1 - 0.5 \left( \frac{\rho_{ijn}}{\Delta} \cos(\alpha_{ijn}) \right)^2 \right)$$  \hspace{1cm} (4.4.11)

$$y_{ijn} (t) = \rho_{ijn} \cos(\alpha_{ijn})$$  \hspace{1cm} (4.4.12)

where the factor $\epsilon$ squeezes the circle in the dawn-dusk direction to form ellipse, and

$$\Delta_c \left(1 - 0.5 \left( \frac{\rho_{ijn}}{\Delta} \cos(\alpha_{ijn}) \right)^2 \right)$$

shifts the cell centre and adjusts its shape.

In the case of sun-aligned arcs, the numbers of nodes in noon-midnight and dawn to dusk directions are $I$ and $k$, respectively. Each arcs is composed from $m$ components distributed in noon-midnight direction around $k$ nodes. The expressions for the positions of the arcs in Cartesian coordinates as function of time $t$ follow from (4.4.4) and (4.4.5) are:

$$x_{i_n} (t) = X0_n + \delta \cdot rfn(i + n + 1) + V_\infty \cdot t$$  \hspace{1cm} (4.4.13)

$$y_{i_n} (t) = Y0_n + \delta \cdot rfn(i + n + k + 1)$$  \hspace{1cm} (4.4.14),

where $V_\infty$ is the speed of the arcs in dawn to dusk direction.
5. An approach to the determination of the parameters

In order to make credible use of the model it is necessary to find appropriate links between the geophysical parameters that control the HF propagation, and the computational parameters that define the details of the ionospheric model. The process of formation and evolution of high latitude ionosphere involves many different physical mechanisms and is a subject of extensive investigation by many researchers. The progress in auroral global imagery, photometry and spectrometry, in rocket experiments in the aurora, and in the theory give the opportunity to define the main parameters with acceptable reliability. The most of these parameters are defined outside the ray-tracing code to enable flexible use of the model.

5.1 The background ionosphere electron density profile

The input parameters defining the background ionosphere in the model (see Section 4.1) are the maximum \( f_{\text{max}} \) and minimum \( f_{\text{min}} \) value of the critical frequencies during the day of the \( F2 \) and \( E \) layers for three reference points and times of day \( t_{\text{max}} \) and \( t_{\text{min}} \) respectively when \( f_{\text{max}} \) and \( f_{\text{min}} \) occur. Figures 5.1 and 5.2 show the variations of these parameters for two reference points during September 2002. In these figures the vertical soundings data are indicated by dots, whilst the smoothed with two-days size window are shown by lines. The variations of \( f_{\text{max}}, f_{\text{min}}, t_{\text{max}} \) and \( t_{\text{min}} \) during one month (Figures 5.1 and 5.2) are about of 3 MHz even at mid-latitude station (Chilton in this case). Asterisks show the values of the \( f_{\text{max}} \) and \( f_{\text{min}} \) for specific day, which used to define latitudinal distribution of the critical frequency of F2 layer.
Figure 5.1 Variations of \( f_{\text{max}} \) and \( f_{\text{min}} \) (MHz, upper frame), \( t_{\text{max}} \) and \( t_{\text{min}} \) (UT, bottom frame), at Chilton (51.5°N, 358.7°E), UK.

Figure 5.2 Variations of \( f_{\text{max}} \) and \( f_{\text{min}} \) (MHz, upper frame), \( t_{\text{max}} \) and \( t_{\text{min}} \) (UT, bottom frame), at Tromsø (69.7°N, 19.0°E), Norway.
The resulting latitudinal dependence of the critical frequency of the F-layer for one time of day (2400 UT in this case) is shown in Figure 5.3. Asterisks mark the values of the critical frequency at two reference points (Tromsø and Chilton in this case).

![Figure 5.3 Latitudinal dependence of critical frequency of F-layer.](image)

5.2 The mid-latitude trough

The parameters defining regular shape of the mid-latitude trough correspond to the Halcrow and Nisbet model, but they are also subjected to day to day variation in order to approach to the real situation.

The parameters defining statistical properties of the trough depend on geophysical parameters. In a period of high levels of geomagnetic activity the structure of the trough becomes more irregular. The walls of the trough become filled with middle-scale irregularities. Hence, it is reasonable to assume based on observations that the maximum value of the depletion of electron density inside the trough depends on sunspot number. Furthermore, ion convection, which plays a crucial role in the trough development, depends on level of geomagnetic activity. Eventually, the depletion of electron density $\Delta n_e$ increases when $Kp$ increases, and decreases when sunspot number increases, due to processes of transport of the
plasma from the south. These general statements were adopted in the model in the form:

\[
\Delta_Ne = \left( \frac{ssn_0}{ssn + ssn_{\infty}} \cdot \log(Kp + Kp_0) \right)^{1/2} \cdot 0.5 \cdot (1 + \cos(\pi \cdot day_{num})) \tag{6.1}
\]

where \( ssn \) is the current sunspot number, \( \bar{Kp} \) is the diurnal mean value of \( Kp \) index, \( day_{num} \) is the day number, \( ssn_0, ssn_{\infty}, \) and \( Kp_0 \) are adjusting parameters. The last term in (6.1) reflects the seasonal dependence of electron density depletion.

The expression for intensity of the irregularities inside the walls of the trough was adopted in the model as \( \Delta S_{wall} = \Delta N_{wall} = (Kp+4)/20 \), with typical latitudinal and longitudinal scales 0.2° and 2° respectively. These produce a landscape of patches along each wall elongated in the direction of the trough.

The latitude of the both walls of the trough was perturbed by three smooth random functions of longitude with zero mean. The longitudinal scales of fluctuations of the border were set 18°, 6° and 2° with typical latitudinal deviations 1.5°, 0.5° and 0.2° respectively. The map of electron density distribution in the case of presence of mid-latitude trough together with F-layer patches is shown in Figure 5.4 for the height of 300km at 0100 UT.

5.3 The F-layer patches and sun-aligned arcs

Between three and five individual components in the patches are usually used in the model. The typical horizontal scales of the blobs in both directions were about 50km. These produce the patches of diverse shape with size of about 300km. The vertical scale of the patches was set of about 30km. Each arc consists of a number of three-dimensional Gaussian distributions (typically of about 8) with horizontal scale 50km and 400km in dawn to dusk and noon-midnight directions respectively. The vertical scale of the arcs was about 30km. Each fragment of the arc separated in the noon-midnight direction at the distance about 300km, which is close to their scale in this direction. These produce the structure elongated in sun-earth direction. The number of the arcs inside the polar cap is user-input parameters, and the distance between them depend on scenario. The map of the electron density
distribution (plasma frequency in MHz) when sun-aligned arcs are the dominant features in the northerly ionosphere is shown in Figure 5.4(a,b) by colour.

Figure 5.4a The map of the electron density inside northerly ionosphere at a height of 180km. The mid-latitude trough, sun-aligned arc and auroral oval are shown at 0100 UT.
Figure 5.4b The map of electron density inside the northerly ionosphere at a height of 180km. The mid-latitude trough, sun-aligned arc and auroral oval are shown at 0200 UT.
The map of the electron density distribution produced by the model inside the northerly ionosphere at height of 300km is presented in Figure 5.5a. In Figure 5.5b the electron density perturbations are shown.
5.3 Electron density irregularities in auroral oval region

The basic model of the enhancement of electron density inside the auroral oval is trapezoidal in cross section along the constant geomagnetic longitude. As a function of distance in vertical plane along magnetic field lines, the density
enhancements was modelled as starting from 100-120 km from the ground, having several peaks at around 100-150 km, and then decaying slowly towards 200 km. These background enhancements were perturbed by the product of smoothed random functions of longitude and latitude. The typical scales of variation in horizontal projection ($\delta_{\text{lat}}$ and $\delta_{\text{lon}}$) were 0.2° and 2° for latitude and longitude respectively. The peaks value of electron density enhancement depends on sunspot number and $Kp$ as

$$f_{pe} = \log(ssn + ssn_0) + 4 \cdot \log(Kp + Kpo)$$

(6.2)

where $ssn$ is sunspot number, $\overline{Kp}$ is daily mean value of $Kp$, $ssn_0$ and $Kpo$ are the adjusting parameters.

The distribution of the electron density perturbations in vertical plane along constant longitude is shown in Figure 5.6.

Figure 5.6 Vertical section of the ionosphere in the mid-latitude trough region. Plot of electron density perturbations (relative units).

The irregularities in auroral oval region oriented along magnetic field lines that attached to northerly wall of the mid-latitude trough can be seen.
The map of the electron density distribution at height of 150km is shown in Figure 5.7. The mid-latitude trough and auroral oval region together with background ionosphere can be seen.

Figure 5.7 Electron density distributions inside the northern ionosphere at a height of 150km. The mid-latitude trough and auroral oval are shown.
6. Results of simulations and comparison with observations

Several experimental campaigns were undertaken to investigate off-great circle HF propagation effects within high latitude ionosphere (Rogers et al., 1997), (Warrington et al., 1997), (Rogers et al., 2003), (Stocker et al., 2002), (Siddle et al., 2004a,b). Three different types of the paths were used in these experiments: polar cap paths, when the path between transmitter and receiver lies entirely within the polar cap; paths along the mid-latitude trough; and trans-oval paths, in which the transmitter is located inside the polar cap and the receiver predominantly outside the polar oval.

Figure 6.1 Location of the transmitting and receiver sites
6.1 A Polar Cap path – Iqaluit to Alert

6.1.1 Observations

The 2100 km length path from Iqaluit (63.7° N, 291.5° E) to Alert (82.5° N, 297.7° E) is always contained within the polar cap ionosphere. In winter, the path remains in darkness for long periods and the converse is true in summer. There is, consequently, a marked seasonal dependence of the signal behaviour with large (~100°) bearing deviations observed during the winter and equinoctial months and only small (<10°) fluctuations for most of the time during the summer. There is an underlying diurnal trend for propagation to deviate to the west of the GCP (high bearing angles) in the evening sector (local midnight at the GCP mid-point is 0430 UT) with propagation returning from the east of the GCP (low bearing angles) in the morning. This may arise from very large-scale ionospheric gradients in the polar cap. More rapid bearing swings with periods of about 30 minutes are often superimposed upon these trends which are attributed to the presence of convecting patches or arcs of enhanced ionisation.

Figure 6.2 shows an oblique ionogram taken along the Iqaluit-Alert path for daytime on 24 January 1996. As is typical for high latitude ionograms a ‘nose extension’ is evident. This is the patch of diffusive echoes above the junction frequency at 7 MHz, which is also attributed to the presence of convecting patches or arcs of enhanced ionisation (Rogers et al., 2003).

An example period illustrating the rapid bearing swings observed at 9.292 MHz for the period 21-24 February 1994 is presented in Figure 6.3 together
with values of the 3-hourly $a_p$ index and the $B_y$ and $B_z$ components of the IMF. A geomagnetic storm is evident on 21-22 February. The principal bearing swings on the night of the 21-22 February - a period of southward IMF and high $a_p$ values - have a decreasing bearing angle and occur in the six hour period before local midnight (0430 UT) whereas the principal bearing swings on the following night - a period of northward IMF and low $a_p$ - have an increasing bearing and occur principally in the hours following local midnight.

![Figure 6.3 Bearing measurements for the 9.292 MHz transmission from Iqaluit, received at Alert for the period 21-24 February 1994. Three-hourly $a_p$ values and the IMF $B_y$ and $B_z$ values are also shown (middle and bottom panels).](image)

When the IMF is directed southward ($B_z < 0$), patches of ionisation drifting anti-sunwards should lead to a preponderance of decreasing bearing angle swings in the pre-midnight hours and increasing bearing angle swings in the hours after midnight. When the IMF is directed northwards ($B_z > 0$), the principal large scale electron density structures within the polar cap ionosphere are sun-aligned arcs. A series of arcs drifting steadily across the polar cap from dawn to dusk would lead to the expectation that increasing bearing swings would be observed during the time
sector 1800 to 0600 LT, with the largest swings expected in the midnight sector. Decreasing bearing swings would be observed in the local time sector 0600 to 1800 with the largest swings in the noon sector.

6.1.2 Simulation of F-layer patches

The model enables ionograms, directions of arrival and time of flight of the receiving signal to be simulated. The background ionosphere parameters correspond to February 1994 in the following examples of simulations.

6.1.2.1 Ionograms

An example of the simulated oblique ionograms from Iqaluit to Alert for a period dominated by the presence of F-layer patches for four times is presented in Figure 6.4a (note that the simulation was undertaken between 3 and 16 MHz). The depth of shading indicates the power of the signal. Maximum critical frequency in the centre of the patches and their speed are 10MHz and 100m/sec respectively.

![Simulated Ionograms](image)

*Figure 6.4a Simulated ionograms produced by ray tracing through the ionosphere containing F-layer patches of enhanced electron-density.*
The most interesting feature are detached “patches” that are evident on the ionogram between 8 and 13 MHz at 0100, 0700 and 1000 UT, while at 0400 UT the ‘nose extension’ to the 1-hop F-region trace is apparent. Large delay spreads in the detached features are also evident. As noted early these types of features are frequently observed in the experimental ionograms (Rogers, et al., 2003).

Nevertheless it is difficult to make reliable conclusions about the energy distribution from the structure of the ionogram and calculations of the angles of arrival can produce additional information for this purpose. The azimuth direction of arrival of the receiving signal as a function of frequency is shown in Figure 6.4b (each frame corresponds to the ionograms in Figure 6.4a).

It is interesting to note that signal energy associated with the F-layer patches is displaced from GCP by around 40 degrees to the west (negative bearing deviation) at 0700 and 1000 UT, but to the west and to the east at 0100 UT. At 0400 UT, when the ‘nose extension’ observed the structure of bearing deviation also has been changed. Two traces of bearing deviation with different slopes are evident. A large azimuth
spread at the low end of the frequency band is another important feature of these ionograms.

6.1.2.2 Direction of arrival and time of flight

The actual convection flow pattern depends on the direction of the IMF. During southward IMF (Figure 2.5), the F-region plasma flows anti-sunward over the polar caps, and returns sunward within the auroral oval. This creates a two-cell pattern.

It is interesting to know how the behaviour of the structure of the bearing deviations depends on the initial distribution of the patches. Those variations are shown in Figure 6.5 and Figure 6.6, where examples of the time history of the direction of arrival are presented for $By=0$ and $By>0$ respectively. Four different scenarios, corresponding to four different distributions of patches in the polar cap zone, for a simulated 9.3 MHz signal propagating through this time-varying modelled ionosphere are shown. Maximum critical frequency in the centre of the patches and their speed are 10MHz and 100/m/sec respectively. Large bearing deviations are evident in Figure 6.5.

![Figure 6.5](image)

*Figure 6.5 Deviation in azimuth as function of time for a 9.3 MHz signal propagated through a modelled ionosphere containing polar patches. Four different initial distributions. IMF $By=0$.***
The detailed structure of the azimuthal variations depends on the initial distributions of the F-layer patches. As expected, the number of swings, their dynamics and azimuth spread are changed, but the common nature of the azimuth deviations does not change significantly. The traces have predominantly increasing azimuth deviation with time. When \( By \) is equal to zero, the two cell convection flow patterns is symmetric, but movements of the patches relative to the receiver is a composite of the motions of the patches and the rotation of the Earth. Hence the structure of the bearing traces is unlikely to be symmetric relative to local midnight.

![Figure 6.6 Examples of azimuth deviations as function of time of a 9.3 MHz signal propagated through the modelled ionosphere containing polar patches. Four different initial distributions of the patches. \( By > 0 \).](image)

Similar to Figure 6.5, the variation of the structure of bearing deviations depends on the initial distribution of the patches (Figure 6.6). As expected, the azimuth deviations in Figure 6.6 are slightly different from these shown in Figure 6.5. The shape and the size of the convection flow patterns depend on the \( By \) (see Figure 4.1), and eventually influence on the behaviour of bearing traces.

Another examples of time history of the bearing deviation simulated for three orientations of the IMF, \( By < 0 \), \( By = 0 \) and \( By > 0 \) with two cell patterns convection flow are shown in Figure 6.7.
Figure 6.7 Example of the time history of the azimuth of the arrival of a 9.3MHz signal propagating over the Iqaluit-Alert path through ionosphere containing F-layer patches. Bz < InT (two cell convection flow pattern).

Left frame By > 0, middle frame By = 0, right frame By < 0;

When By > 0 (left frame) the principal swings have increasing bearing angle. When By = 0 (middle frame) decreasing swing appears in a few hours before local midnight (local midnight at 0430 UT), and it becomes well pronounced when By < 0 (right frame). The speed of the F-layer patches depends on many geophysical parameters. In the model it is a subject of variation. The speed of the patches in Figure 6.7 is about 300m/sec, while in Figure 6.6 and Figure 6.5 it is approximately 100m/sec.

The situation is more complex during northward IMF: the ionospheric convection is much more structured, confined to much higher latitudes (polar cap is smaller), and the velocities are of smaller magnitude than in the case of southward IMF. This results in a four-cell or three-cell ionospheric convection pattern for strongly or weakly northward IMF, respectively (see Figure 2.6).

It is very interesting to compare the structure of the bearing traces for two, three and four cell convection flow patterns. Left and right frames of Figure 6.8 show the difference of the bearing's structure in the case of the three-cell convection flow pattern. Other parameters of the patches are the same as in Figures 6.4-6.7.
Figure 6.8 Example of the time history of the azimuth of the arrival of a 9.3MHz signal propagating through ionosphere containing F-layer patches.

$B_z > \ln T$ (three and four cells convection flow pattern),

left frame $B_y > 0$ (three cell convection flow pattern),

middle frame $B_y = 0$, (four cell convection flow pattern),

right frame $B_y < 0$ (three cell convection flow pattern).

The sign of $B_y$ changes the direction of the rotation in the central cell of the convection flow (see Figure 2.4 and 4.2). The central cell defines the main features in the structure of the receiving signal on that specific path. It is important to remember that the receiver was located close to geomagnetic pole.

Two cell (Figure 6.7) and four cell (middle frame, in Figure 6.8) convection flow patterns produce similar structure of the received signal (details are different). Similarity in the structure of the bearing deviation in middle frame in Figure 6.7, and 6.8 and reference to the expected convection flow patterns (see Figure 2.3, 2.4, or 4.1) indicates that peripheral cells in four cells pattern can play less important role for this specific path.

The model can also calculate other important characteristics of the receiving signal: elevation angle of arrival and time of flight. These parameters were not measured during the experimental campaign, but have been modelled and are presented in Figure 6.9.
Figure 6.9 Example of the time history elevation angle and time of flight of a 9.3 MHz signal propagating through a model ionosphere containing F-layer patches
Upper row: elevation angle, lower row: time of flight.
Left frame $B_y>0$ (three cell convection flow pattern),
middle frame $B_y=0$, (four cell convection flow pattern),
right frame $B_y<0$ (three cell convection flow pattern).
The bearing deviations are shown in Figure 6.8

The large elevation angle and time of flight spread are evident, especially in the case when four cell convection flow patterns (middle frame in Figure 6.9) are formed.
6.1.3 Simulation of sun-aligned arcs

When geomagnetic activity is low and the IMF is directed northward ($B_z > 0$) the main features within the polar cap ionosphere are sun-aligned arcs with electron density enhancements of a factor 2-3 above the background. An example of the time history of simulated azimuth deviations of a 9.3 MHz signal propagating along the Iqaluit-Alert path is given in Figure 6.10. Maximum critical frequency in the centre of the arcs is 7MHz. The speed of the arcs is about of 100/m/sec. The background ionosphere parameters correspond to March 1994. Large bearing swings (in this case from -80° to +60°) with a time scale of about 3 hours are apparent. A sequence of simulated ionograms corresponding to Figure 6.10 is presented in Figure 6.11.

Figure 6.10 Example of the time history of azimuth deviations for the case of sun aligned arcs.

Figure 6.11 A sequence of ionograms produced by ray tracing simulations through ionosphere containing sun-aligned arcs.
In this case a long horizontal ‘nose extension’ occurs at 0500 UT. Small nose extension with different structure of the traces can be seen also at 0300 UT and 0700 UT. It should be noted that the thin flat traces are a common feature in the ionograms for paths through ionosphere containing arcs. Detached features are less pronounced in these ionograms than is the case for F-layer patches (see Figure 6.3). The azimuth direction of arrival of the receiving signal as a function of frequency, corresponding ionograms in Figure 6.11 is shown in Figure 6.12.

![Figure 6.12 Bearing deviations as function of frequency. The corresponding ionograms are shown in Figure 6.11.](image)

The signal energy associated with long horizontal ‘nose extension’ on an ionogram at 0500 UT is displaced from GCP by around 20° and is independent of frequency. Overall structure of the bearing deviations is less complicated than that for patches (Figure 6.4).
Figure 6.13 shows another example of simulated bearing deviations as a function of time for a path through the polar ionosphere, in which arcs of the enhanced electron density are the dominated feature (maximum critical frequency and speed of the arcs are 7MHz and 25m/sec respectively, background ionosphere parameters correspond to March 1994). A very slow swing with increasing bearing angle is followed by much faster swing with decreasing azimuth deviation.

Figure 6.14 presents the sequence of the simulated ionograms for four different times corresponding to the signal of Figure 6.13.
Similar the ionograms of Figure 6.11 a nose extension is evident at 0300 UT and 0500 UT, however well pronounced detached features at 0700 UT and 0900 UT are also apparent. The bearing deviations as function of frequency for these ionograms are shown in Figure 6.15.

![Figure 6.15 Bearing deviations as function of frequency corresponding to ionograms in Figure 6.14.](image)

The signal energy associated with the long horizontal ‘nose extension’ on the ionogram at 0500 UT is well displaced from GCP and displays internal structure. At 0700 UT a detached feature is evident in this ionogram.
6.2 Mid-latitude trough paths

6.2.1 Period of low solar activity - Halifax to Leitrim path

6.2.1.1 Observations

Observations of the effect of the trough on the azimuth-of-arrival of HF radio signals at several frequencies propagating in the sub-auroral region have been reported by (Rogers et al., 1997). These authors investigated the effects on two paths, the first between Halifax (44.9°N, 296.0°E), Canada and Cheltenham (52.6°N, 324.6°E), UK (4490 km, bearing 286°) and the second between Halifax and Leitrim (45.3°N, 284.4°E), Canada (910 km, bearing 90°). For the longer path this study showed that the occurrence and nature of propagation well displaced from the great circle path is well correlated with the geomagnetic activity index Ap. During the winter and equinoxial months (October to March) the effect of the magnetic disturbances was particularly noticeable on the lowest frequency (5.097 MHz) signal, where during the more disturbed days (i.e. when Ap ≥ 15) a characteristic high-to-low bearing angle swing of about 50° was evident.

The short path from Halifax to Leitrim is at most times sub-auroral and strong effects due to the mid-latitude trough are expected. At the lowest frequency (5.097 MHz), very large bearing deviations of up to ±100° starting at times between midnight UT and up to six hours after midnight UT are a common feature. The deviations can be either positive, negative or include both directions even for geomagnetically quiet days. During disturbed times, the large bearing deviations tend to start a few hours earlier. At higher frequencies the large bearing swings are much less common since the signal is usually lost at 0000 UT due to the low MUF and is not reacquired until about 1200 UT.

Four examples of the observations of the bearing for 5.1MHz signal transmitted from Halifax and measured at Leitrim during four days in 1994 are presented in Figures 6.16a,b.
Despite similar geomagnetic activity on these four days, the structure of the bearing traces is very different. A common feature in these examples is large deviations of up to ±80° from the great circle direction. The bearing deviations are predominantly southward in Figure 6.16a, but they are predominantly northward in Figure 6.16b. Time scale of the bearing traces changes from about to 2 hours (right frame in Figure 6.16a) to up to 6 hours (left frame in Figures 6.16a,b). Time of the commencement of the off-great circle propagation varies from about 2100 UT for 8th March 1994 (southward bearing) to 0600 UT for 27th March 1994 (northward bearing). The overall structure of the bearings is very changeable.
6.2.1.2 Simulation - Halifax to Leitrim path

A ray-tracing study was undertaken in an attempt to reproduce the observed characteristics of the Halifax-Leitrim path. Analysis of the parameters of the received signals show (Rogers et al., 1997; Stocker et al., 2002), that different propagation mechanism could be responsible for northward and southward bearing deviations. Refraction inside the wall of the trough is the more common type of propagation corresponding to the southward trace, while scattering from irregularities within the auroral zone coupled with refraction are the leading mechanism defining the structure of the northward trace of the signal.

Examples of the time history of the azimuth of arrival and time of flight of a 5.1 MHz signal propagating along Halifax-Leitrim path for different $K_p$ are given in Figure 6.17. The background ionosphere parameters correspond to March 1994. The maximum depletion of electron density was equal to 0.6 in this instance.

![Figure 6.17 Example of azimuth deviation (upper row) and time of flight (bottom row) for Halifax-Leitrim path. Left $K_p=1$, middle $K_p=3$, and right $K_p=5$](image)

Large, up to $\pm 90^\circ$, southward (positive deviations of azimuth) and northward bearing deviations are evident in Figure 6.17a. Variation in the time of commencement of deviation from GCP is also seen. As expected, when $K_p$ increases
the northern wall moves nearer to the propagation path and a northward bearing trace appears, whilst the southward azimuth deviation becomes less pronounced. The structure of the azimuth deviations becomes more complicated as \( Kp \) increases. It is important to notice that only one magneto-ionic component was taken into account in this simulation and the structure of the bearing traces was defined by different angular modes of propagation. It is evident from Figure 6.17b that during the day the signal exhibits several modes, corresponding to 1-hop, 2-hop, and 3-hop modes of F-region propagation and bearing remains close to great circle. At night, a wide variety of effects in both azimuth and time of flight have been seen. Southward and northward traces overlap frequently in time of flight.

6.2.1.3 Variation of the path length

The results of modelling seem to be very reminiscent of the characteristics observed in the experimental measurements campaign and enable the nature of off-great circle propagation effects to be estimated for paths which were not subject to experimental investigation.

Some examples for three values of \( Kp \) are shown in Figure 6.18. The transmitter was located in Halifax and the receiver at coordinates 46.1°N, 277E°, resulting in a 1480km propagation path aligned along the Halifax-Leitrim path, whereas the length of Halifax-Leitrim path is 910km and structure of azimuth of arrival over this path is shown in Figure 6.17 (upper row). The length 1480km was chosen because it is close to the length of Uppsala to Leicester path, which is discussed in Section 6.2.2.

![Figure 6.18 Examples of azimuth deviations for 1480km length path aligned along the Halifax-Leitrim path. Left frame \( Kp=1 \), middle frame \( Kp=3 \), right frame \( Kp=5 \).](image-url)
As expected the duration of a skip zone was diminished, but variations in the structure of bearing deviations are not too obvious. The time of commencement off-great circle propagation was changed. Southward bearing deviations are evident for $Kp=1$ and $Kp=3$ at the beginning and the end of the skip zone. Only southward traces of bearing deviation can be seen for $Kp=1$.

### 6.2.1.4 Variation of electron density depletion

It is very important to examine the variations of the structure of the signal from the maximum value of electron density depletion ($dep$) inside the trough. An example of the time history of the azimuth of arrival of 5.1 MHz signal over Halifax–Leitrim path for two values of $dep$ and three values of $Kp$ is shown in Figure 6.19. The background ionosphere parameters have left invariable. As expected, if the depletion of electron density increases, the traces of bearing deviations due to refraction inside the walls of the trough become more pronounced.

![Figure 6.19 Example of bearing deviation simulated for Halifax-Leitrim path, Left column $Kp=1$, middle column $Kp=3$, and right column $Kp=5$, Upper row $dep=0.6$, bottom row $dep=0.4$](image-url)
6.2.2 Period of high solar activity - Uppsala to Leicester path

6.2.2.1 Observations

Experiments were undertaken between October 2000 and January 2002 between Uppsala, Sweden and Leicester, U.K (Stocker et al., 2002), (Warrington and Stocker, 2003), (Siddle et al., 2004 a, b). The transmitter radiated various modulations on frequencies according to a computer based schedule. The signals are received on a 6-channel super-resolution direction finding system located 15 km south of Leicester, U.K, resulting in a 1400km propagation path aligned along the expected position of the mid-latitude trough (see Figure 6.1). As an additional diagnostic, a chirpsounder system was deployed with the transmitter collocated with the main transmitter in Uppsala and the receiver at the University of Leicester. Some typical observations are presented in Figure 6.19.

Figure 6.19. The azimuth (degrees, left frame) measured at the strongest mode and time-of-flight of each mode (msec, right frame) from 1200 UT, 20 November 2001 (day 324) to 1200 UT, 23 November 2001 (day 327). The panels, from the top, represent frequencies of 4.6, 6.9, 10.4, 11.1, 14.4 and 18.4 MHz respectively.
The bearing and time-of-flight measurements made over three days (noon on 20 November (day number 324) to noon on 23 November 2001 (day number 327)) for six operating frequency are shown. The two lowest frequencies are strongly attenuated during the day leading to either no detected signal (e.g. 4.64 MHz at noon on day 325) or a weak signal, which results in a poor measurement of the azimuth. As expected, the daytime propagation begins later and ends earlier with increasing frequency. At night, a wider variety of effects in both azimuth and time-of-flight are observed. On the first night (i.e. day 324–325) the two highest frequencies (14.36 and 18.38 MHz) do not propagate (this is also the case for day 325-326), 10.39 and 11.12 MHz propagate at bearings of about 20° north of the GCP, 6.95 MHz propagate via F-region mode and exhibit northward trace of bearing up to 50° until just after midnight and thereafter on-great circle via an E-region mode, while 4.64 MHz propagates on-great circle throughout the night. On the second night (day 325–326) just after midnight propagation occurs on a mode with a relatively long time-of-flight (8–10 msec) at frequencies 10.39 and 11.12 MHz (and to a lesser extent 6.95 MHz), this mode is associated with bearings predominantly to the north of the GCP. The most unusual behaviour can be observed on the third night (day 326–327) where, on most frequencies, a propagation mode appears at a long time of flight (up to 16 msec) after daytime propagation has finished (about 2030 UT). Over the next six hours the time-of-flight decreases reaching a steady value of about 7 msec by 0230 UT. From about 05 UT until the mode disappears at 0630 UT (for 6.95 MHz, earlier for other frequencies) the time of flight increases by about 0.5 msec. At times around midnight a second mode at longer delays (~11 msec) is present at 10.39 and 11.12 MHz. It is interesting to note that this night is one of only two in over a year of observations in which the signal at 18.38 MHz exhibits this behaviour (albeit rather weakly). The bearing of this mode, which is strongly off-great circle and mostly to the north of the GCP, displays both temporal and frequency dependence. In time, the bearing deviation from great circle decreases as the time-of-flight decreases (e.g. for 11.12 MHz, from about 40° at 2030 UT to 20° at 0230 UT), while the bearing deviation from great-circle observed at 6.95 MHz is smaller than at either 10.39 or 11.12 MHz. The feature observed on day 326–327, where a long time-of-flight is accompanied by bearing deviations and Doppler frequency offsets is a relatively common occurrence. In the winter and spring
months, this phenomenon is observed on over 40% of nights, while in the summer and autumn it is less frequent. It is also reveals that bearing deviations to the north of the GCP are 3-4 times more likely to occur than those to the south and that this is independent of season.

6.2.2.2 Simulation-Uppsala to Leicester path

The main mechanism of the propagation responsible for the effects observed over this path is the scattering from irregularities in the electron density structure either in the poleward wall of the trough or in the auroral oval. Refraction of the rays inside the walls of the trough is another important mechanism by which deflection from GCP could occur at lower end of the frequency band.

Examples of the azimuth deviations simulated in the presence of irregularities with intensity of about 14MHz in auroral region are given in Figure 6.20 for two different values of $Kp$. The background ionosphere parameters correspond to November 2001.

![Figure 6.20 Simulated azimuth deviations of 7.0MHz signal propagating along Uppsala-Leicester path.](image)

*Left frame $Kp=3$, right frame $Kp=4$.*

The northward off-great circle propagation mode is noticeable at all times of the simulation. The ratio of the energy (grey scale in the Figure) of the great circle
mode and that displaced from the GC direction changes sharply at around 2200 UT and 0400 UT. The off-great circle propagation mode become dominant when the skip zone occurs. The mean value of the bearing deviations increases when \( Kp \) decreases (about of 40° for \( Kp=3 \), and ~30° for \( Kp=4 \)), accompanied with changes to the structure of the azimuth of arrival.

Typical measured nighttime traces for time of flight and azimuth deviations for the 7MHz signal is shown in Figure 6.21.

![Figure 6.21 Measured time of flight (left frame) and bearing (right frame) of 7MHz signal signal, propagating along Uppsala-Leicester path.](image)

A propagation mode with a relatively long time of flight between 8 and 10.5 msec (great-circle modes exhibit time of flight 5-6 msec) and bearing deviation to the north of up to 40° is evident at period of time around midnight. The rapid decrease in time of flight (at around 2200 UT) accompanied by small spread of TOF is the most striking behaviour of the signal. It is interesting to note that bearing deviation does not exhibit the specific behaviour in that time. The structured, flat bearing with different azimuth spreads before and after 0000 UT can be seen. As expected, the time of the existence off-great circle propagation is well correlated with times when skip zone is observed. Examples of the simulated time-of-flight of a 7MHz signal for two values of \( Kp \) (corresponding azimuth deviations can be seen in Figure 6.20) are shown in Figure 6.22.
It is evident from Figure 6.22 that during the day the signal exhibits two modes, corresponding in these cases 1-hop and 2-hop modes of F-region propagation. Off-great circle modes of propagation are observed at all time of the simulation. During the daytime these modes are overwhelmed by the great-circle propagation modes, whereas at night the off-great circle modes becomes dominant. Similar effects can be seen in the observations, in which the off-great circle modes exhibit a long time of flight around 6-10 msec near midnight (left frame in Figure 6.22). The rapid decrease in time of flight (at around 2200 UT for $K_p=3$) can also be seen. Overall the structure of the receiving signal is reminiscent of the observations.

As expected and confirmed by observations (see Figure 6.19) the structure of the received signals changes significantly at lower frequencies. The examples of simulated bearing deviations and time of flight for 5MHz signal propagating along Uppsala-Leicester path are shown in Figure 6.23. The great circle propagation mode does not cease in the nighttime and overwhelm the off-great circle traces in bearing deviation and time of flight as well. As a consequence these traces are unlikely to be observed (see Figure 6.19), but can be seen in simulations.
Figure 6.23 Simulated bearing deviations (upper row) and time of flight (bottom row) for 5MHz signal. Left column: Kp=3, right column Kp=4.

6.2.2.3 Variation of the path length

The 920km, 1410km and 1820km links were simulated with transmitter located in Uppsala and three receivers in the points with equal azimuths relatively to the transmitter. It is necessary to remember that the length of the Halifax to Leitrim and Uppsala to Leicester paths are about of 910km and 1400km respectively. The
variations of the structure of the received 8.0MHz signal propagating along the mid-latitude trough are shown in Figure 6.24.

Figure 6.24 Bearing deviations (upper row) and time of flight (bottom row) for 8.0MHz signal propagating along mid-latitude trough.

Left frame: 920km path, middle frame: 1410km path, right frame: 1820km path.

During the daytime, the signals propagate via 1-hop and 2-hop modes of F-region propagation. As expected the length of the skip zone diminishes as the range increases. The off-great circle modes of propagation become more pronounced when the skip zone is observed, but can be seen at all time of simulation for all paths (Figure 6.24a,b). The mean value of the bearing deviations is diminished as the range increased.
6.2.2.4 Variations of the path orientation

It is interesting to investigate the dependence of the structure of the received signal on the orientation of the path relative to the mid-latitude trough. In order to provide information about the structure of the bearing deviations of the signal in the region close to the mid-latitude trough, the simulations of the directional effects over a number of the paths were undertaken. The transmitter is located in Uppsala and ten receivers are disposed in the points shown on the map in Figure 6.25. Nine of them are uniformly distributed at a distance of about 1000km from the transmitter with azimuths between 180° and 340° relative to the transmitter. The tenth receiver is located in Leicester, where experimental data are available.

![Figure 6.25 The location of transmitter and receivers.](image)

The time history of the bearing deviations and time of flight over the nine paths is shown in Figure 6.26a and 6.26b respectively. The intensity of irregularities was about 10MHz with longitudinal and latitudinal scales of about 200km and 25km respectively. The background ionosphere parameters correspond to November 2001.
Figure 6.26a Sequence of the bearing deviations of the 8MHz signal propagating in the region of the mid-latitude trough.

The locations of the receivers are indicated at the top of each frame.

Off-great circle propagation is apparent for all of the paths, however the structure of the bearing deviations depends on the orientation of the path. Over the southerly oriented paths (upper row in Figure 6.26), the weak sparse mode with negative bearing deviations (corresponding to azimuth of arrival from the east) can be seen. In the time sector 0500 – 0800 UT the propagation of the signal occurs via 1-hop, 2-hop and 3-hop F-modes (more pronounced in Figure 6.26b). Along the westward oriented paths (middle row in Figure 6.26) the southward traces of the bearing deviation of up to 80° are well pronounced. The off-great circle mode is dominant at night defining the structure of the received signal. As expected, the size
of the skip zone is enlarged compared with that for the southerly paths. The structure of the received signal over the northerly paths is reminiscent the structure of HF signals observed in the case when propagation occurs via a strong Es mode. The skip zone is diminished and the large azimuth spread is evident.

Figure 6.26b Sequence of time of flight of the 8MHz signal propagating in the region of the mid-latitude trough. The locations of the receivers are indicated at the top of each frame.
6.3 Trans-oval path: Svalbard to Kiruna

6.3.1 Observations

An experimental campaign has been conducted with a receiver system capable of measuring the delay and Doppler spread characteristics and the directional structure of the received signal at Kiruna (67.9.2°N, 20.4°E) in northern Sweden. The transmitter was located on Svalbard (78.2°N, 15.8°E) resulting in a 1152km propagation path usually transverse to the diffuse auroras area. The structure of the signal propagating along trans-oval paths is expected to be the most complicated in the high latitude ionosphere. The rays cross the region in which F-layer patches or arcs are dominated features, the region of diffuse auroras filled in by the middle-scale irregularities, and the mid-latitude trough region. All possible off great-circle propagation mechanisms could occur: refraction inside the polar cap patches, scattering from irregularities due to the precipitations in the auroras area, and refraction in the northern wall of the trough. Owing to orientation of the path the rays cross the region of the auroral oval predominantly transverse to the longer spatial scale of the irregularities, resulting in strong lateral scattering. The prevailing lateral scattering results in an increase in the number of propagation modes and leads to an enlargement of the directional and Doppler spreads of the signal.

In Figure 6.27(a,b) (from http://sd-www.jhuapl.edu/Aurora/ovation) the auroral oval size, position and intensity of precipitation retrieved from satellite and ground based measurements are shown for 23 and 29 September 2002 respectively. The transmitter and receiver positions are indicated in Figure 6.27(a,b) by stars and circles respectively.
Normalized B2i = 69
Flux = 421 MWe
Equivalent Kp = 2.0
Global e - E - Flux = 4.4 MW

NORTH CAP
End Time 23 Sep 2002 - 23:30
DMSP Satellite : F14
No UVI Data for this period.

Figure 6.27b Oval size, position and intensity of precipitation at 29 of September 2002.
Left frame at 2030 UT, right frame at 2330UT.
As it is evident from Figure 6.27(a,b) the layout of the auroral oval region relative to the Svalbard to Kiruna path differs significantly on these two dates. On 29 September 2002, the region of precipitation lies northward of Kiruna and encompass the entire region above the path at 2330 UT. As a result, the most probable mechanism forming the structure of the received signal is scattering from auroral electron density irregularities. This situation is typical for Svalbard to Kiruna path during the experimental campaign.

On 23 September 2002, the auroral oval region was located predominantly southward of the middle point of the path and cannot affect significantly the structure of the received signal.

Examples of some typical measurements of time of flight (TOF), azimuth, and signal to noise ratio (SNR) for three days of observations are presented in Figures 6.28(a,b,c), Figure 6.29, and Figure 6.30.

![Figure 6.28a TOF (upper row), azimuth deviations (middle row), and SNR (bottom row) of the 6 MHz signal propagating along the Svalbard-Kiruna path](image-url)
During the day the signal exhibits two modes, corresponding to the 1-hop and 2-hop modes of F-region propagation. The skip zone is pronounced, especially in the 7MHz signal. Bearing deviations exhibit no specific structure like those, observed on the Iqaluit-Alert or Uppsala-Leicester paths. Quasi-sinusoidal variations in bearing can be observed between 0300 and 0600 UT at 29 September, but with a different time scale for the 6MHz and 7MHz signals.

Figure 6.28b TOF (upper row), azimuth deviations (middle row), and SNR (bottom row) of the 7 MHz signal propagating along the Svalbard-Kiruna path.

Figure 6.28c TOF (upper row), azimuth deviations (middle row), and SNR (bottom row) of the 9 MHz signal propagating along the Svalbard-Kiruna path.
The 9MHz operating frequency signal is propagated via 1-hop and weak 2-hop F-region mode, whereas via more expressed E-region mode during all day 27 and 28 September. The signal exhibits a great circle bearing with a small azimuth spread in the daytime and a much larger angular spread at nighttime.

![Figure 6.29a TOF (upper row), azimuth deviations (middle row), and SNR (bottom row) of the 11 MHz signal propagating along the Svalbard-Kiruna path.](image)

A slightly more distinct structure can be seen in the bearing deviations of 11MHz signal in Figure 6.29a between 0900 UT and about 1830 UT on 30 September 2002. It is interesting to note that the direction of the bearing swing changes from negative (i.e. decreasing azimuth angle with time) at 0900 UT to a positive one at about 1530 UT. Nevertheless the directional structure of the signal is not pronounced. Variations in azimuthal spread rather than bearing traces can be observed.

It is very interesting to examine the directional structure together with the Doppler spread characteristics of the received signal. In Figure 6.29b the time of flight, bearing deviation and SNR are shown for 3 days in September 2002, whilst in Figure 6.29b the Doppler spectra and azimuth of arrival on 30 September 2002 are presented.
The most marked relationship is evident between the Doppler frequency and the measured bearing, as shown in Figure 6.29b. The upper frame of this figure shows the Doppler spectrum of the signal for the period 0900 UT until 2100 UT, whereas the lower frame shows, in the colour scale, the azimuth of arrival for each component in the frequency spectrum.

Figure 6.29b Doppler spectra (upper frame) and azimuth angles of arrival (bottom frame) for the 11MHz signal propagating along the Svalbard-Kiruna path, 30 September 2002.

It is evident from Figure 6.29b that the signal components arriving at the receiver from directions to the east of the great circle path (higher azimuth angle) have positive Doppler shifts imposed, whereas signal arriving from the west of the great circle direction (low bearing angle) have negative Doppler shifts imposed.
The structure of the HF signal shown in Figures 6.27-6.29 is typical of those observed on the Svalbard-Kiruna path during the experimental campaign. Nevertheless, some dramatic situations were observed. An example of such a situation is presented in Figure 6.30, where the power of signal in dBr marked by colour. The auroral oval size, position and intensity of precipitation are shown in Figure 6.27a. It is important to note that such disposition of the auroral oval relative to the propagation path occurs rarely.

*Figure 6.30 Time of flight (upper row) and azimuth deviations (bottom row) of the 9.04 MHz signal propagating along the Svalbard-Kiruna path.*

During the day great circle propagation occurs with a relatively small spread in bearing and time of flight, whereas at night a structured deviation of azimuth up to 50° to the west is evident. The time of flight of this detached bearing trace exceeds 6 msec while the time of flight corresponding to the great circle mode is less than 4.5 msec. The directional structure of the signal indicates that reflection from F-layer patches or arcs of enhanced electron density is the probable propagation mechanism, whilst the influence of the auroral oval is less pronounced.
6.3.2 Simulation

Simulation studies were undertaken in order to provide confirmation that the model would reproduce the observed directional effects over trans-oval paths. Examples of bearing deviations and time of flight are given in Figure 6.31 for two operating frequencies. The parameters of the IMF and background ionosphere correspond to 1 October 2002. The intensity of the irregularities due to precipitation and maximum critical frequency inside the F-layer patches were about of 10MHz.

![Figures showing azimuth deviations and time of flight](image)

*Figure 6.31 Simulated azimuth deviations (left frame), and time of flight (right frame) of the 7.0MHz (upper row) and 9.3MHz (bottom row) signal propagating along Svalbard-Kiruna path.*

The noticeable effect can be seen in the period between 1200 UT and 1700 UT. The intensive azimuth variations with the amplitude of about 10° affecting 1-hop F-region propagation modes for 9.3MHz signal are evident. These features become more pronounced for 7.0MHz signal. Both, 1-hop and 2-hop propagation modes exhibit the bearing and time of flight variations. It is important to note that small bearing deviations correspond to large variations in time of flight. Rather more pronounced azimuth deviations around midnight are apparent, less prominent for
9.3MHz than for 7MHz. It is interesting to note that between 2200 UT and 0200 UT the large azimuth deviations of about 60° for 7.0 MHz (and of about 30° for 9.3 MHz) signal correspond to small variations in time of flight.

In order to find out the main propagation mechanism over this path, the simulations for different initial distributions of the F-layer patches were carried out. In Figure 6.32 bearing deviation and time of flight of a 6.8MHz signal propagated over Svalbard to Kiruna path are presented for three distributions of the patches. The parameters of the ionosphere and the IMF correspond to 22 October 2002 in these simulations. The intensity of irregularities due to precipitation and maximum critical frequency inside the F-layer patches was about 7MHz and 10MHz respectively.

Figure 6.32 Simulated azimuth deviations (left frames) and time of flight (right frames) of the 6.8MHz signal propagating along the Svalbard-Kiruna path.
The off-great circle propagation mode is evident during all day. The structure of bearing deviation depends on the initial distribution of the F-layer patches around midnight, whereas around noon the difference in azimuth is less pronounced. The structure time of flight traces exposed more similarities. Short flat traces in time of flight that start at 0300 UT are due to the scattering from auroral oval irregularities. Consequently, structure of bearing deviation corresponding to these traces does not change, as that can be seen in the left column of Figure 6.32.
7. Concluding remarks

Various measurements of off great-circle propagation effects over a range of high latitude paths and their interpretation have been undertaken over a number of years by researchers at the University of Leicester. These experiments indicate that HF radio signals often arrive at the receiver over paths displaced from the great circle direction due to the presence of gradients in the high latitude electron density distribution. Very large deviations are particularly prevalent in the high latitude regions where signals often arrive at the receiver with bearings displaced from the great circle direction by up to ±100° or more. These are attributed to the presence of large scale electron density structures that are a common feature of the polar cap region ionosphere. Another common feature of high latitude HF propagation is the large Doppler and delay spreads imposed on the signal. These together with directional spread of the received signal energy are important parameters to be considered in the design of systems using multi-element receiving arrays and the associated signal processing methods.

7.1 Polar cap paths

Over the Polar cap path from Iqaluit to Alert in 1994, the main off-great circle propagation mechanism is (reflection) scattering from F-layer patches or from arcs of enhanced electron density. The overall directional structure of the received signal exhibits a high level of repeatability.

The results of simulations are able to reproduce the nature of the experimental data.

Some further developments of the model may be useful. A large amount of information concerning the actual structure of the convection flow is available via the Internet that could be incorporated into the ray tracing code. For example, in Figure 7.1 the convection flow patterns for different orientations of the IMF are shown (from Greenwald et al., 1999).
Figure 7.1 Statistical model of the high latitude electrical potential pattern for various orientations of the interplanetary magnetic field as determined by the APL HF radar at Goose Bay, Labrador. The equipotential contours are in kilovolts and represent the convection paths followed by the ionospheric plasma. The outer circles are at 50° of magnetic latitude. $\Phi_{pc}$ - electrical potential drop across the polar cap. (After Greenwald et al., 1999)
7.2 Paths along mid-latitude trough

The deflections from the GCP over the Halifax to Leitrim path in the period of declining solar activity (1994) at 5.1 MHz occur because the signal propagated via the electron density gradients in the trough walls. Variations in the directional structure of the received signal are large. Nevertheless, the model managed to reflect the main features of the off-great circle propagation.

The structure of the azimuth deviations over the Uppsala to Leicester path in a period of high solar activity (from 2001) between 6 MHz and 18 MHz is defined by scattering in irregularities in the auroral region, whereas refraction inside the walls of the trough is the leading propagation mechanism at low end of the frequency band. Variations in the structure of the bearing traces are smaller compared to the Halifax-Leitrim path.

A comparison of the simulated results with observations indicates that the model is capable of reproducing the main features of the signal, but some further development is desirable, mainly concerned with the actual structure of the auroral zone.

7.3 Trans-oval path

As expected the characteristics of the signal propagating over the Svalbard to Kiruna path are the most complicated. The structure of the signal is defined by a number of propagation mechanisms. Furthermore, combinations of a number of different mechanisms lead to a large diversity in the signal structure.

The model is able to reflect the main features of the signal. In order to reflect variety of the auroral oval zone some modification of the model based on the data available via Internet may be valuable to improve the resemblance between observed and simulated off-great circle propagation effects.

7.4 Ray tracing method

In order to take into account the diffraction effects it would be very crucial to use some advanced method of wave propagation (for example parabolic equation method, see Ishimaru, 1978; Hill, 1985) in regions dominated by irregularities and the GO method outside them.
7.5 Summary of main outcomes

The main aim of the thesis is to report a numerical examination of off-great circle propagation of HF signals based on three-dimensional ray tracing, the results of which may be summarized as follows:

1. A comprehensive computational model of high-latitude ionosphere has been developed which includes models of:
   a) the background ionosphere,
   b) the electron density enhancements associated with the presence F-layer patches and arcs, coupled with a reasonable approximation of the convection flow patterns,
   c) the electron density irregularities in auroral oval region, and
   d) the electron density distribution inside the mid-latitude trough.

2. These models were incorporated into ray tracing code.

3. Development of an adaptive algorithm designed to improve execution speed by varying the steps size in azimuth and elevation based on an analysis of the situation occurring in the previous time step.

4. Modification of the ray tracing code for multiple receivers was made in order to investigate the short-scale and long-scale spatial correlation of the HF signals' characteristics.

5. A large number of simulations were carried out for different types of propagation path in the high latitude ionosphere including:
   a) polar cap paths,
   b) paths along mid-latitude trough,
   c) trans-oval paths.

6. The contribution of a number of mechanisms leading to off-great circle propagation were considered. It was demonstrated that:
a) refraction inside F-layer patches and arcs of enhanced electron density is the main mechanism over the polar cap paths,
b) scattering from electron density irregularities, which are commonly found in the auroral oval, attributed to particle precipitation, is the main mechanism along mid-latitude trough paths at the higher band of operating frequency,
c) refraction in the wall of the trough is the main mechanism at for the lower frequencies over the paths along the trough, and
d) all mechanisms are important over the trans-oval paths.

7. Comparisons with observations were made, the model was found to be capable of reproducing the main features of the directional characteristics of HF signal propagating in high latitude ionosphere.
References


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