Large-scale current systems in the jovian magnetosphere

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Abstract

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The studies contained within this thesis focus on the large-scale azimuthal and radial current systems of Jupiter's middle magnetosphere, i.e. currents with radial ranges of 20-50 R_J. In the first study using magnetometer data from Pioneer-10 and -11, Voyager-1 and -2, and Ulysses, it is discovered that the azimuthal current in the middle magnetosphere is not axi-symmetric as had been assumed for the last twenty-five years, but that it is stronger on the nightside than on the dayside at a given radial distance. A simple empirical model is formulated, which reasonably describes the data in the domain of interest both in radial distance and local time, and allows direct calculation of the current divergence associated with the asymmetry. In a similar way, in the following chapter the radial currents have been computed for the dawn sector of the jovian magnetosphere along various fly-by trajectories. Combination of these radial current estimations with the azimuthal current model allows the total divergence of the equatorial current to be calculated. These current densities mapped to the ionosphere are surprisingly large at ~1 μA m⁻². In order to carry the current, the magnetospheric electrons must be strongly accelerated along the field lines into the ionosphere by voltages of the order of 100 kV. The resulting energy flux is enough to produce deep, bright (Mega Rayleigh) aurora and thus provides the first natural explanation of the main jovian auroral oval. In the final study, newly-available data from the Galileo orbiter mission are combined with the fly-by data in order to compare them to the model derived in the first study. The model is then re-derived for the entire data set, which significantly improves the associated fractional errors.
Jupiter: The Bringer of Jollity
Declarations

The research undertaken during the course of this doctoral programme has led to the submission and publication of the following scientific papers:


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Chapter 1: Introduction and context

1.1 Introduction

This thesis mainly comprises a discussion of two fundamental current systems within Jupiter's magnetosphere. The first is the azimuthal current system arising from the existence of a continuous plasma source deep within the magnetosphere. The plasma diffuses outwards due to centrifugal instability, stretching the field lines in the process. The second is the system of radial currents directly associated with the coupling between the magnetosphere and the ionosphere, and which cause the magnetic field lines to bend out of their meridian planes. The divergence of each of these current systems, and subsequently the total equatorial current divergence, are in turn and in tandem of great interest because it must close by field-aligned currents (FACs) in the ionosphere. The study of such current systems then leads to an enhanced understanding of such phenomena as the jovian aurora.

This preliminary chapter introduces the Sun, solar wind and interplanetary magnetic field, and the fundamentals of solar-planetary interactions, whilst Chapter 2 will address in detail the current knowledge of the global structure and dynamics of the jovian magnetosphere. This includes plasma sources, current systems, and coupling between the solar wind, magnetosphere, and ionosphere as understood both from theory and via various data from the six missions to encounter Jupiter to date. The information from the recent Jupiter flyby made by the Cassini spacecraft along with concurrent Hubble Space telescope
observations remains to be untangled, and hence Chapter 2 is based on knowledge gained from the six missions only. Chapter 3 looks in detail at the process of modelling the magnetic field of Jupiter due to internal and external sources, and how these models may be exploited in the interpretation of magnetic field measurements. A systematic study of the radial magnetic field vectors from the pre-Galileo flybys is presented in Chapter 4, and a strong local time asymmetry between noon and midnight is discovered. Chapter 5 moves onto the question of the divergence of the total equatorial current in the dawn sector, by way of radial and azimuthal magnetic field averaging. The results are discussed in terms of the magnetosphere-ionosphere coupling current and hence of the implications for the jovian aurora. A similar analysis to that presented in Chapter 4 is repeated using both the flyby data and the first 20 orbits of the Galileo tour of the Jupiter system, in Chapter 6.

1.2 The Sun and the existence of the solar wind

The Sun is the most important object in our solar system, although by spectral standards it is a star of fairly ordinary class, being of type G2V. It is a rotating hot gaseous sphere of mainly hydrogen (~90%) and helium (~10%) and is held together and compressed under its own gravitational attraction (e.g. Priest, 1995). However, it is the proximity of the Sun to the planets and other solar system bodies (e.g. comets) which leads us to consider the way in which the Sun interacts with such obstacles.

Figure 1.1 depicts a sketch of the Sun’s interior and atmosphere, labelled with many of the surface features such as flares, prominences and radio emissions. Also shown are different zones that are understood to exist within the Sun, commencing at the centre with the core. The temperature and pressure at the core are \(14 \times 10^6\) K and \(340 \times 10^{11}\) Pa respectively, and together are of such intense proportions that thermonuclear reactions occur. These nuclear reactions are the Sun’s source of energy. The energy is transported via radiation and convection to the Sun’s visible surface layer (called the photosphere) where photons last interact with atoms before escaping from the Sun. Above the photosphere the solar atmosphere consists of three components: the chromosphere, the transition region and the corona. At the photosphere the temperature is \(5800\) K. Above the photosphere lies the chromosphere, which extends for \(1500\) km. The temperature here first decreases slowly to \(4200\) K and then increases rapidly across its
The properties of the Sun can be understood through the study of its interior and atmosphere. The Sun's surface and atmosphere are particularly dynamic, with phenomena such as solar flares, coronal loops, and solar wind. These features emit various types of radiation, including visible, infrared, and UV light, as well as high-energy particles and neutrinos.

Figure 1.1 Schematic of the Sun's interior and atmosphere [Courtesy NASA/ESA]
Chapter 1: Introduction and context

outer boundary. This boundary is known as the transition region and here the temperature soars to ~10^6 K at the base of the solar corona.

Considering the equations of force balance and mass continuity and assuming the atmosphere to be in static equilibrium, it may be shown that the gas pressure in the corona (~few x 10^3 Pa) falls off to a limiting value of ~10^-5 Pa far from the Sun. If this pressure were less than that of the local interstellar medium (LISM) then the solar corona would become bound by the LISM, and static equilibrium of the solar atmosphere could be achieved. However estimates of the pressure in the LISM indicate that it has a value of approximately 10^-13 Pa. Under such conditions the coronal plasma must flow supersonically outwards in an attempt to "fill" interplanetary space, eventually interacting with the LISM at estimated distances of ~100 Astronomical Units (where 1 AU=150 million km). This supersonic flow of plasma from the corona into interplanetary space is called the solar wind.

The presence of the solar wind was first postulated by Biermann (1951) based upon the existence of comet plasma tails observed to extend far beyond the comet, always pointing away from the Sun. The first mathematical solution for the solar wind outflow was obtained by Parker (1958). The acceleration takes place within ~5 solar radii of the Sun, after which the flow speed remains nearly constant with distance. Thus as the gas expands outwards, the number density and temperature fall with distance. At the orbit of the Jupiter i.e. at 5.2 AU from the Sun, the solar wind has an average temperature T_{sw}, typical number density n_{sw}, and velocity v_{sw} of 10^5 K, 2 x 10^5 m^3 and 450 km s^-1 respectively (see e.g. Huddleston et al., 1998).

1.3 Solar magnetic field structure and the interplanetary magnetic field

For simplicity we may think of the Sun's surface magnetic field as approximately dipolar, and rotating with the Sun with a sidereal rotation period of ~27 days. This magnetic field direction reverses every 11 years, a cycle that is tracked by the number of sunspots there are in the photosphere. At the start of the solar cycle, the sunspot number is low and the field is dipolar in nature. However, the number of sunspots increases to a
maximum after 11 years, and at this time the solar magnetic field structure is disordered, and highly structured. This activity then dies away over the next 11 years, at which point the dipole re-emerges but with the opposite polarity, constituting in total a 22-year cycle. The splitting of spectral lines due to the Zeeman effect suggests that the photospheric field is of the order of 0.1 mT outside of sunspots, and up to 0.3 T inside them. A weak (few nT) remnant of this photospheric field becomes embedded in the solar wind, and is known simply as the interplanetary magnetic field (IMF).

In order to think about the transport of field lines and plasma more quantitatively we first derive the 'equation of motion' of the magnetic field. The simplified Ohm’s law for a plasma of conductivity $\sigma$ is

$$E + V \times B = \frac{j}{\sigma},$$

(1.1)

where $E$ is the electric field, $B$ the magnetic induction, $j$ the electric current density, and $V$ the plasma velocity. Substitution of this into Faraday’s law

$$\frac{\partial B}{\partial t} = -\text{curl } E,$$

(1.2)

and use of Ampère’s law (with the neglect of the displacement current)

$$\text{curl} B = \mu_0 j,$$

(1.3)

where $\mu_0$ is the permeability of free space, then yields

$$\frac{\partial B}{\partial t} = \text{curl}(V \times B) + \frac{1}{\mu_0 \sigma} \nabla^2 B.$$

(1.4)

The first term on the right-hand side may be thought of as the transport term, describing the “frozen-in” transport of magnetic flux with the plasma. The second term is known as the magnetic diffusion term. By taking the ratio of these two terms in dimensional form, we find that the transport term will far exceed the diffusive term if $\mu_0 \sigma L v \gg 1$, where $L$ is the characteristic scale-length of the plasma and $v$ the characteristic velocity. This ratio is known as the magnetic Reynolds number, $R_m$. In the limit of high magnetic Reynolds number, the magnetic field and plasma are frozen together, and as such the field carries along the plasma (or vice versa depending on the relative energy densities) as it evolves in space and time. In the case of the solar wind, the conductivity is high and the characteristic scale length of the order of $\sim 5$ solar radii. Therefore the magnetic field and plasma are frozen together, and expand out together into
interplanetary space. For a more detailed description see, for example, Cowley (1984). If the magnetic Reynolds number is very small, the equation becomes a diffusion equation where there ceases to be coupling between the magnetic field and plasma.

To understand the way in which the plasma and magnetic field propagate into the solar system, it is useful to first neglect the rotation of the Sun. If we first consider the Sun's magnetic field as approximately dipolar and embedded in the Sun's atmosphere, and take the outflow of the solar wind to be constantly radially outwards, after a long time the field near the Sun will point radially inwards or outwards depending on the sense of the field at the Sun's surface. Now consider the rotation of the Sun and the effect that this has on the magnetic field and plasma. Each element of plasma is moving radially outwards carrying the frozen-in IMF, but with the foot of the field line frozen into the Sun's surface. The field lines are therefore wound into an Archimedean spiral, as shown in Fig. 1.2a. This spiral is aligned along the locus of all the plasma elements emitted from a given source point on the Sun. This configuration is sometimes known as the garden sprinkler effect, and was first described mathematically by Parker (1958). Opposite magnetic polarities between northern and southern hemispheres requires the existence of a current sheet at the magnetic equator. As the solar dipole axis is tilted slightly relative to the spin axis, the heliospheric current sheet (HCS) wobbles back and forth across the ecliptic plane. Fig. 1.2b shows the HCS extending far out into interplanetary medium. The inclination of the current sheet defines the width of a cone inside which an observer in space would alternately see different polarities or sectors of the IMF. The tilt angle of this inclination may additionally be thought of as an indication of the level of solar activity. During solar maximum, the sector structure is complex and distorted by a large number of transient disturbances, such as coronal mass ejections (CMEs), shocks, tangential discontinuities and magnetohydrodynamic (MHD) waves, whilst at the minima of the cycle the dipolar nature of the field produces a clear two sector structure.

1.4 Corotating interaction regions

Since the first spacecraft observations it has been known that the solar wind is divided into streams of slow (~400 km s\(^{-1}\)) and fast (>600 km s\(^{-1}\)) wind. At solar minimum, the flow pattern close to the Sun may be thought of as a band of slow wind at
Figure 1.2a A sketch of the Archimedean magnetic field spirals and the spiralling electric field in the current sheet. Magnetic field lines are shown slightly above the equatorial plane, close below they have opposite polarity. From H. Alfvén (1981).

Figure 1.2b Current sheet in the inner heliosphere in the ballerina model. The thick lines indicate the magnetic field lines. From Smith et al. (1978).
low latitudes, centred on the Sun’s dipole equator, with the fast wind at higher latitudes. This flow is disturbed during periods of higher solar activity by flows associated with CMEs for example. Above the surface of the Sun, a region of fixed heliographic latitude and longitude will experience variations in the solar wind speed as a consequence of the solar rotation. The radial propagation of these variations creates compression regions as the fast ‘patches’ catch up with slower ‘patches’, and if slow wind follows fast wind, a rarefied region is created. These density perturbations evolve with increasing distance from the Sun, such that at some distance (between ~2 and 3 AU) shocks develop in both the compressed and rarefied regimes. In the solar wind frame the rarefied, depleted density regions propagate a reverse shock back towards the Sun (of course the overall motion is outward), whilst in the compressed denser regions, a forward shock is sent anti-sunward. This is known as a corotating interaction region (CIR). Depending on the stability of the source locations, the CIR may recur during the following solar rotations: this is called a recurrent CIR. Fig. 1.3 shows an idealised sketch of the evolution of a CIR in the inner heliosphere.

CIRs are the dominant feature in the solar wind between 2-8 AU (Gazis, 2000) and occur on a time-scale of ~2 per solar rotation. In the outer heliosphere (i.e. at Jupiter) the IMF is more azimuthally aligned, and thus CIRs spread, merge and interact and form merged interaction regions (MIRs). In this case the shock structure is converted to a ring-like shell of concentric shock waves travelling outwards like waves from a stone thrown into water. Both CIRs and MIRs play an important role in the modulation of the giant magnetospheres, by introducing large velocity, magnetic field, pressure, and temperature enhancements into the solar wind and IMF.

1.5 Solar wind interaction with a magnetised planet

Of the nine planets in our solar system, six of them are known to have magnetospheres, as described for example by Bagenal (1984). The terrestrial magnetospheres (i.e. those of Mercury and Earth), exhibit very different features to those of the outer planets (i.e. Jupiter, Saturn, Uranus, and Neptune). Fig. 1.4 shows the approximate relative sizes of the planetary magnetospheres in the solar system, starting with Mercury through to Jupiter in ascending order. The ingredients from which a
Figure 1.3 Schematic diagram of two CIRs corotating with the Sun, along with the solar wind and magnetic field signatures associated with it at 1AU. Taken from Kunow (2001).
Figure 1.4 Relative sizes of the planetary magnetospheres. Taken from Russell and Walker (1995).
Chapter 1: Introduction and context

The planetary magnetosphere is formed are as follows. First is the presence of a planetary magnetic field, created by currents flowing azimuthally within the planet's core. These internal magnetic fields are mainly dipolar in nature, but in some cases (i.e. Jupiter) substantial quadrupole and octupole moments are also present. In addition, the planetary magnetic field has its own plasma population, originating e.g. in the planet's ionosphere or from the presence of a volcanically active or surface 'sputtering' moon within the magnetosphere. The latter source supplies a substantial mass of plasma into Jupiter's magnetosphere, which will be discussed in further detail in Chapter 2. Next, we have the solar wind plasma and embedded IMF flowing out into the solar system encountering the planetary magnetic field and plasma population.

As described in Section 1.2 of this chapter, the solar wind and IMF are frozen together, and also the planetary magnetic field and plasma. As such, when the solar wind/IMF encounter the planetary field and plasma they are unable to mix together, since all of the cross-field mixing of plasma elements is suppressed in this limit. As first described by Chapman and Ferraro (1930) and as sketched in Fig. 1.5, a thin current layer forms and separates the two systems, keeping the plasmas apart and forming a magnetospheric cavity. The location of the boundary of the magnetosphere, the magnetopause, is determined by the condition of pressure balance between the shocked solar wind plasma (termed the magnetosheath) on one side, and the magnetospheric plasma and field on the other. Because the speed of the solar wind is supermagnetosonic in the planet's rest frame, a shock wave is formed upstream called the bow shock. Across this shock, the plasma is slowed, compressed and heated. On the inside of the cavity, the magnetospheric magnetic field lines extend down into the ionosphere and upper atmosphere (thermosphere) of the planet, such that the magnetosphere, ionosphere, and thermosphere are strongly coupled together. Collisions between the ions and neutrals in the Pedersen conducting layer of the ionosphere, provide a frictional torque on the magnetospheric flux tubes which drives the plasma up to rigid corotation. This torque is communicated via a large-scale system of FACs which connect and close ionospheric Pedersen currents to radial equatorial magnetospheric currents. The effects associated with this current system will be discussed in detail both in Chapter 2 and Chapter 5.

When the solar wind is fast, the magnetosphere is compressed and when the wind abates, the magnetosphere is able to expand. On the nightside the magnetosphere is shaped
Figure 1.5 Sketch in the noon-midnight meridian plane of the Chapman-Ferraro closed magnetosphere based on the strict application of the frozen-in approximation (Cowley, 1998).
by the solar wind and IMF flowing past, into a long tail consisting of two anti-parallel lobes of magnetic flux stretching out into the anti-solar direction, and connected to the poles of the planet.

It was Dungey in 1961 who first described the effect of a breakdown in the “frozen-in” approximation at the magnetopause. When the spatial scales become small enough, it is possible for the magnetic field lines to diffuse through the plasma. When the fields are oppositely directed on either side of the magnetopause boundary, they are able to “reconnect” across the current layer. This allows the direct entry into the magnetosphere of solar wind plasma, along interplanetary field lines and onto planetary field lines on the planet side of the boundary. This implies that some of the momentum of the solar wind is able to couple to the outer regions of the magnetosphere and sets it into motion away from the Sun. A return flow is set up in the central regions, due to further reconnection on the nightside. This whole process, driven by solar wind convection, is known as the Dungey cycle.

The relative importance of corotation driven dynamics and solar wind-driven convection at Jupiter will be further discussed in Chapter 2, but the precise nature of how these interactions play a role in the jovian magnetosphere is not yet fully understood.
Chapter 2

Review of the Global Structure and Dynamics of the Jovian Magnetosphere

2.1 Introduction

Jupiter's magnetosphere, sketched in Fig. 2.1, is the cavity which contains, and is controlled by Jupiter's magnetic field. Typically, the magnetopause extends to ~60 R\(_j\) on the dayside of the planet (one Jupiter radius, R\(_j\), is taken throughout this thesis to be 71,373 km), and stretches out into a long comet-like tail on the nightside. The magnetotail has a diameter of ~300-400 R\(_j\) and a length of at least 3000 R\(_j\) (Cowley and Bunce, 2001a). As described in the previous Chapter, the magnetic field of Jupiter is confined within this cavity by the solar wind which is flowing past the planet. Jupiter's bow shock stands in the supersonic flow upstream of Jupiter at a subsolar distance of ~75 R\(_j\). The position of the magnetopause boundary is defined by the condition of pressure balance between the magnetosheath plasma and field on one side, and the magnetospheric field and plasma on the other. The magnetosphere, ionosphere and upper atmosphere are coupled together, and as such angular momentum is transferred between the three, via the magnetic field.

The plasma within the magnetosphere has contributions from the ionosphere and solar wind, mainly comprising hydrogen and helium, but by far the most substantial source of plasma within the jovian magnetosphere is provided by the Galilean moon Io. Io orbits at ~5.9 R\(_j\) deep within the magnetosphere, liberating a monumental ~1 tonne s\(^{-1}\) of sulphur
Figure 2.1 Sketch of Jupiter's magnetosphere in the noon-midnight meridian plane, with the Sun to the left and the solar wind blowing from left to right. The arrowed solid lines are the magnetic field lines, while the dashed lines are the magnetopause and bow shock.
dioxide gas, comparable to the outgassing rate of an active comet. As a consequence the magnetospheric plasma contains a substantial fraction of sulphur and oxygen ions. Jupiter has a rotation rate of \( \approx 9 \text{ h } 55 \text{ min} \), and this rotation imparts the main source of momentum and energy into the jovian magnetospheric plasma. As such the most important dynamics arise due to the Io plasma source immersed within the rapidly rotating magnetosphere.

Solar wind coupling at the magnetopause may have significant contributions to the dynamics of the outer magnetospheric regions, but as yet the complexities of the dynamics of this boundary are not yet understood. The solar wind coupling to the outer regions is surely responsible, however, for the formation of the magnetotail.

### 2.2 Discovery and in-situ exploration

In the mid 1950s it was discovered that Jupiter is a major source of strong radio emissions, in the decimetric (\( \approx 1 \text{ GHz} \)) and decametric (\( \approx 10 \text{ MHz} \)) wave bands (Burke and Franklin, 1955). It did not take long to attribute this radio emission to the motion of energetic charged particles in a strong planetary magnetic field, generated by dynamo currents flowing within Jupiter’s interior. This remarkable discovery was made long before Van Allen’s discovery of the Earth’s radiation belts (Van Allen et al., 1958), and the in-situ verification of the solar wind (Neugebauer and Snyder, 1962). These two forms of electromagnetic radiation provided the only means by which information about the jovian magnetosphere could be obtained before the first spacecraft encounters took place.

The continuous decimetric (DIM) component of the emission is generated by synchrotron radiation from gyrating energetic (\( \approx 10 \text{ MeV} \)) radiation belt electrons trapped between equatorial distances of \( \approx 1.3 \) and \( 3 \text{ R}_J \) by the immense jovian magnetic field, close to the equatorial plane. Studies of the DIM emission described the internally produced magnetic field, with which it is associated, as an approximate dipole tilted by \( \approx 10^\circ \) from the rotation axis (Berge, 1965, 1966). The polarisation of the radiation showed that the polarity of the field is opposite to that of the Earth’s, and has field lines running from the northern hemisphere of the planet, via the equatorial plane to the southern hemisphere. Whilst this DIM emission is very steady in time, apart from a modulation at the planetary rotation period (Sloanaker, 1959), the decametric (DAM) radiation (in addition to being
very intense) is observed to be quite sporadic, and exhibits large intensity fluctuations on seconds to tens-of-seconds timescales.

Despite the fact that the DAM bursts were the first of the emissions to be discovered, by Burke and Franklin (1955), the details of the DAM mechanism remains to be understood in detail. However, it is thought to be due to emissions from ~10 keV electrons gyrating at the cyclotron frequency, which have been accelerated along the field lines towards Jupiter's ionosphere. The upper cut-off frequency of 40 MHz then corresponds to the cyclotron frequency in the region of strongest magnetic field encountered by the emitting electrons. This field strength corresponds to $1.4 \times 10^{-3}$ T, a remarkably large field which is about twenty times the strength of the Earth's polar field. In 1964, it was announced by Bigg (1964) that a part of the jovian emission is influenced directly by the innermost of the four Galilean satellites, Io. This discovery was the first evidence that Jupiter and Io have a complicated electrodynamic interaction, between moon and magnetosphere-ionosphere system. Later, R. A. Brown (1974) discovered a toroidal volume near Io's orbit that is made luminous by the multiple optical (and ultraviolet) emissions excited by resonant scattering of sunlight and by electron collisions. This torus of neutral sodium atoms surrounding Io lies between ~5 and 7 $R_J$ from Jupiter near the magnetic equatorial plane.

The first in-situ measurements of the jovian magnetospheric environment began in November 1973 with the fly-by of the NASA Pioneer 10 spacecraft. This was followed by Pioneer 11 soon after in 1974 (Smith et al. 1974, 1975, 1976), Voyager-1 and -2 in 1979 (Ness et al. 1979a, b), followed by ESA's Ulysses in 1992 (Balogh et al., 1992). The NASA Galileo orbiter's insertion into orbit occurred in 1995 (Johnson et al., 1992), and the most recent spacecraft to fly past Jupiter (December 2000) was the joint NASA/ESA Cassini/Huygens orbiter and probe, en-route to Saturn. At the time of writing, the data from the Cassini spacecraft are only recently released, and as such most of the information described here has been derived from the earlier encounters, augmented by radio and optical observations from Earth. The aim of the Pioneer missions was principally to measure the jovian magnetic field and energetic particle environment at energies above a ~1 MeV (Goertz, 1976). The Voyager spacecraft were instrumented to measure the full radio emission spectrum (Carr et al., 1983), the lower energy thermal plasmas (10-100s eV and 10-100s keV) as described for example in Belcher (1983) and Krimigis and Roelof
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(1983), and plasma waves (Gurnett and Scarf, 1983). A similar genre of measurements were collected by the Ulysses spacecraft en route to a successful solar mission, and also by the Galileo orbiter, the first spacecraft to orbit an outer planet. The trajectories of these spacecraft are shown projected onto the equatorial plane in Fig. 2.2, together with the locations of the bow shock and magnetopause as derived by the Voyager spacecraft. The x-axis points positive towards the Sun, whilst the y-axis points from dawn to dusk across the width of the magnetosphere. The z-axis points out of the plane of the diagram, forming a right-handed set in the coordinate system known as Jupiter Solar Orbit (JSO).

It can be seen that all of the fly-by missions explored the pre-noon dayside of the planet on their inbound passes, and that Pioneer-10, and Voyager-1 and -2 passed through the pre-dawn nightside sector during their outbound trajectory. All of these passes were confined to the near-equatorial regions, in contrast to the outbound pass of Pioneer-11 near noon at northern latitudes of ~33°. Similarly, Ulysses exited the magnetosphere approximately along the dusk terminator at southerly latitudes of ~37°. New results from the fly-by data will be presented in Chapters 4 and 5. The Galileo orbiter collected data mainly from the local time sector between midnight and dawn during the main phase of the mission, and it is this data which will be presented in Chapter 6. The Galileo extended mission (GEM) saw the spacecraft progress around the magnetosphere in local time to ~2000 MLT, and it is advancing its local time coverage even further towards dusk during the “millennium mission” (GMM). However, it remains clear from this figure that the post-noon sector of the jovian magnetosphere is as yet relatively unstudied.

2.3 Jupiter's magnetic field morphology

2.3.1 The internal field and the size of the jovian magnetosphere

As indicated above, the main features of the jovian magnetic field produced by internal currents were first determined from the properties of the jovian radio observations. However, it was not until 1973 when Pioneer 10 flew through Jupiter’s magnetosphere, that detailed characterisation of the magnetic environment could begin. Since then in-situ data from the five fly-bys and the Galileo orbiter have provided a far better understanding
Figure 2.2. Trajectories of the first 20 orbits of the Galileo orbiter along with the five fly-by spacecraft relative to Jupiter, shown in Jupiter Solar Orbital coordinates. X points positive sunwards, and Y is orthogonal to X and in the plane of Jupiter's orbit. The solid line indicates the Galileo orbiter and the dashed lines indicate the fly-by spacecraft. The individual fly-by spacecraft are distinguished by the varying symbols shown in the key. A heavy dashed line depicts a model bow shock, while the heavy solid line shows the model magnetopause. Both model positions are derived from the Voyager-2 data. The region of interest for this paper, 20-45 \( R_J \), is highlighted by the grey annulus in the centre of the plot. This figure was kindly provided by Joe Mafi of the Planetary Data System, UCLA.
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of Jupiter's magnetic field, including separating those fields which are due to the internal dynamo currents, and those field perturbations which are due to currents flowing externally to the planet. The most recent of the internal jovian field models, the VIP4 model, described by Connerney et al. (1998), indicates a best-fit centred dipole axis which is inclined at 9.5° to Jupiter's spin axis towards System III longitude 201° (the System III (1965) coordinate system is described in detail in Appendix 1). The corresponding dipole moment is \(4.27 \times 10^{-4} \, \text{TRj}^3\), such that the surface field is \(0.4 \times 10^{-3} \, \text{T}\) at the magnetic equator and twice that at the poles. At Jupiter, higher order terms such as the quadrupole and octupole moments are also important, producing significant asymmetries in the near-planet field and increasing the peak surface field in the northern polar regions to \(\sim 1.5 \times 10^{-3} \, \text{Rj}\). A more detailed description of the magnetic field modelling techniques used for both the planetary field, and also for the magnetic field due to external currents, will be presented in Chapter 3.

In order to estimate the size of the magnetospheric cavity of Jupiter in the solar wind, the above value of the dipole moment may be used (as a good approximation) along with a simple consideration of pressure balance across the magnetopause. If we neglect the magnetospheric plasma pressure, such that the pressure term within the magnetosphere is wholly magnetic, and taking a nominal value of the solar wind dynamic pressure to be \(\sim 0.1 \, \text{nPa}\) (see Huddleston et al., 1998), a simplified calculation will place the position of the sub-solar magnetopause at \(\sim 40 \, \text{Rj}\). However, the flyby observations of the dayside magnetopause show that the boundary, on average, lies nearer to \(60 \, \text{Rj}\). This is due to the fact that at Jupiter, the hot internal plasma within the magnetosphere (originating primarily from Io) cannot be neglected in the pressure balance. The result of this is that the magnetopause has an unusually large range of stand-off distances to the solar wind flow, and is somewhat 'squashier' than the Earth's magnetosphere for example.

2.3.2 The inner magnetosphere

The structure of the magnetic field within the magnetospheric cavity as disclosed by the spacecraft observations is shown schematically in Fig. 2.1, where the magnetic field lines are sketched in the noon-midnight meridian plane. In general, it has become customary to discuss Jupiter's magnetospheric field in terms of four regions (Smith et al.,
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1976). The first is the inner magnetosphere, which is defined by the torus-shaped region of field lines which cross the equatorial plane within ~5 R_J of Jupiter’s centre. The magnetic field in this region is that of a mainly undisturbed planetary field, pointing mainly southwards. The inner magnetosphere similarly contains the trapped radiation zone of energetic electrons which produce the DIM radiation (Fillius, 1976). The inner region is distinguishable from the middle magnetosphere by a lack of any significant electric currents flowing locally. The outer boundary of the inner magnetosphere is thus approximately co-located with the inner edge of the Io plasma torus at ~5 R_J, and is where strong azimuthal currents start to flow. The inner region is, however, affected by the fringing fields of the azimuthal currents, which produce northward fields of strength ~200 nT. This field reduces the southward planetary field in the equatorial plane, although because it is much weaker than the ≥3500 nT equatorial dipole field, the effects are not large.

2.3.3 The middle magnetosphere

One of the most striking discoveries to emerge from the flybys of Jupiter by the Pioneer and Voyager spacecraft in the 1970s was the existence of an equatorial azimuthal current sheet within the jovian magnetosphere, whose magnetic effects were observed at all local times investigated (Smith et al., 1974, 1975, 1976; Ness et al., 1979a,b). Similar effects were also observed during the Ulysses fly-by in 1992 (Balogh et al., 1992) and in the Galileo orbiter data (Russell et al., 1999a, 1999b).

On the dayside, the current sheet was found to extend from jovicentric radial distances of ~5 R_J, just inside Io’s orbit, to within ~15 R_J of the magnetopause, thus defining the extent of the jovian middle magnetosphere region. The radial range of the current sheet on the dayside thus depends on the state of compression of the magnetosphere by the solar wind. During the Voyager inbound passes, for example, it extended to ~45 R_J when the magnetopause was compressed inwards to ~60 R_J, while reaching to ~70 R_J on the Ulysses inbound pass when the last magnetopause crossing was observed at ~90 R_J. The current disc is located near to the magnetic equatorial plane (though less perfectly so with increasing distance, see Khurana (1992) and references therein), and thus executes a quasi-sinusoidal north-south oscillation as the magnetic
dipole, displaced ~10° from the spin axis, rotates with the planet. The equatorial currents are believed to be carried predominantly by plasma originating from Io which is energised by planetary rotation. The current in the equatorial magnetodisc is then carried (a) by the inertia current of near-corotating cold torus plasma which slowly diffuses outwards, and (b) by the pressure-gradient current of low density hot plasma which slowly diffuses inwards (e.g. Hill et al., 1983; Vasyliunas, 1983; Caudal, 1986; and references therein). This low-density hot plasma could have a significant solar wind component too. On the nightside, the current sheet at large distances is found to merge continuously into the equatorial current of the magnetic tail system, and hence becomes associated with solar wind-magnetosphere coupling (Ness et al., 1979a,b). How rotationally-driven dynamics in the inner part of the system interacts with solar wind-driven dynamics in the outer part remains a central issue of jovian magnetospheric physics (e.g. Cowley et al., 1996).

The thickness of the current sheet is estimated to be a few R_J, typically ~2-8 R_J, possibly decreasing from the larger of these figures towards the smaller with increasing distance (Smith et al., 1976; Goertz et al., 1976; Connerney et al., 1981; Acuña et al., 1983; Staines et al., 1996; Dougherty et al., 1996). Since these dimensions are much smaller than the characteristic size of the magnetosphere, many tens of R_J, the current sheet produces a characteristic variation of the radial field component with distance from the equatorial plane. The radial field reverses rapidly in sense across the current sheet from positive values in the north to negative values in the south, and then varies much more slowly outside. The magnitude of the current sheet field is much smaller than the planetary field in the inner part of the current sheet, near Io’s orbit (~6 R_J), such that in this vicinity the total field is still nearly dipolar in form. However, because the planetary field falls off with distance as $r^{-3}$, while the sheet current and associated field falls off much less rapidly, as $\sim r^{-1}$ or $\sim r^{-2}$ (see below), the current sheet fields assume dominance at greater distances, beyond ~15 R_J. In Fig. 2.3a we show a sketch of the equatorial currents in the middle magnetosphere, which are here assumed to close azimuthally around the planet. Also shown are the adjacent equatorial tail currents which flow from dusk to dawn across the system. The tail currents close over the tail lobe magnetopause and form two D-shaped solenoidal current systems, as shown in Fig. 2.3b (to be discussed further below).

Most theoretical models of the current sheet derived to date have assumed that the current is approximately axisymmetric, though often they have been applied only in a
**Figure 2.3a.** Sketch of the current system in Jupiter's magnetic equatorial plane, showing the eastward-flowing current of the middle magnetosphere, which closes around the planet, and the dusk-dawn currents of the tail current sheet, which separates the tail lobes, and closes along the magnetopause.
Figure 2.3b. Sketch of the field and current in a cross-section through the tail, looking away from the planet. The north tail lobe field points away from Jupiter (circled cross), while that of the southern lobe points towards the planet (circled dot).
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piecemeal way to field data from restricted local time sectors. Barish and Smith (1975) used an Euler potential formulation to model the field observed on the pre-noon inbound pass of Pioneer-10 (Fig. 2.2), and found reasonable agreement with a field falling as $\sim r^{-2}$ beyond $\sim 20$ R$_J$. Goertz et al. (1976) and Jones et al. (1981) similarly modelled the outbound Pioneer-10 data near the dawn meridian, and obtained a slightly less steep radial field gradient associated with the current sheet of $\sim r^{-1.7}$. Behannon et al. (1981) considered one-hour averages of the total field strength observed on the nightside outbound passes of Pioneer-10, and Voyagers-1 and -2 over distance ranges of $\sim 20$ to $\sim 150$ R$_J$, and fitted power law variations to the maximum such average in each 10-h planetary rotation interval. They found a continuing trend of reducing radial field gradients with decreasing local time towards midnight, with the field varying as $\sim r^{-1.7}$ at Pioneer-10, $\sim r^{-1.5}$ at Voyager-1, and $r^{-1.4}$ at Voyager-2 (see Fig. 2.2). It must be noted, however, that these fits did not account for the different magnetic latitudes reached on these passes, which will affect the maximum field observed during each planetary rotation cycle.

Khurana (1997) provided a detailed fit to these outbound passes using an Euler potential formulation incorporating a hinged model of the current sheet location. Connerney et al. (1981), on the other hand, modelled the current sheet directly as an azimuthally symmetric distribution of finite thickness (taken as 5 R$_J$), extending from joviancentric distances of 5 R$_J$ to 50 R$_J$, within which the current density falls as $r^{-1}$. The perturbation fields were then obtained by integration, and used to fit both inbound and outbound fields observed by Pioneer-10 and Voyagers-1 and -2 in the inner part of the system, within $\sim 30$ R$_J$. It was found that the magnitude of the current required to fit the Voyager-2 observations is somewhat smaller than that required to fit Pioneer-10 and Voyager-1. In the latter cases, however, the model then over-estimates the radial field observed on the dayside inbound passes, which is weaker at a given radius than the radial field on the nightside outbound passes. Connerney et al. (1981) suggested that this effect might result from the presence of a thicker current sheet on the dayside compared with the nightside, such that the spacecraft did not fully exit the current sheet north-south in the former case. This explanation may be plausible at distances inside $\sim 15$ R$_J$, where the amplitude of the periodic north-south motions of the current sheet are smaller than its thickness, such that a near-equatorial spacecraft can remain immersed within it at all phases of the planetary spin period. However, it cannot explain the asymmetry at larger distances, beyond $\sim 15$ R$_J$, because the amplitude of the current sheet motion is then larger.
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than its thickness, such that spacecraft exit from the current sheet is guaranteed during some part of the rotation cycle. Jones et al. (1981) also concluded from an examination of Pioneer-10 and -11 data that the dayside current sheet field is weaker than that at dawn.

In Chapter 4 we present the first systematic study of the radial magnetic field data from all of the pre-Galileo spacecraft, and compare the field at the different radial ranges and local times of the five passes.

2.3.4 The outer magnetosphere

The outer magnetosphere is a dayside region, defined by the lack of equatorial currents, and was observed during the inbound passes of the flyby spacecraft. The field in this region, although somewhat variable, points on averages to the south and is thus in the direction of the planetary equatorial field. The region is bounded by the magnetopause on one side and by the middle magnetosphere on the other. There is some evidence of a transition region of highly disordered field occurring between the outer and middle magnetosphere, particularly evident on the inbound trajectory of the Ulysses spacecraft (Balogh et al., 1992; Bame et al., 1992). The field in this transition region undergoes sharp changes in direction, indicative of the presence of current layers. These current layers are not however, ordered by latitude or planetary rotation period and as such are easily distinguishable from the current sheet crossings of the middle magnetosphere. The transition region was first observed in the Ulysses data (Haynes et al., 1994), but evidence exists for observations of the so-called magnetic nulls in the Pioneer and Voyager data also (Belcher, 1983; Leamon et al., 1995; Southwood et al., 1995). The possibility of similar phenomena has also been reported as present in the kronian magnetosphere (Goertz, 1983).

2.3.5 Magnetic tail

As discussed by Ness et al. (1979c), the magnetic field data from the outbound Voyager-1 and -2 observations were most naturally interpreted as evidence for a well developed magnetic tail on the nightside of the planet. The tail consists of two lobes of oppositely directed field separated by a ‘neutral sheet’, and is controlled by the external
forces associated with the solar wind interaction. The azimuthal current sheet in the middle magnetosphere is found to merge continuously with the tail’s neutral sheet. Observations of the Voyager-1 outbound magnetopause crossings indicate that the tail has an approximate diameter of 300-400 $R_J$ assuming that it is roughly circular. The two tail lobes are D-shaped in cross-section, as shown in Fig. 2.3b. The strength of the field in the tail lobes falls with increasing radius as the tail expands in radius, and reaches a value of $\sim 2$ nT at a down-tail distance of $\sim 150$ $R_J$. Scarf (1979) pointed out the possibility of the Voyager-2 spacecraft encountering the jovian magnetotail whilst en-route to Saturn and even the further possibility of Saturn being in Jupiter’s tail during the Voyager-2 Saturn encounter. In a study of the distant jovian magnetotail by Lepping et al. (1983), evidence is presented for the jovian magnetotail extending as far as 9000 $R_J$ (which is approximately at the orbit of Saturn).

2.3.6 Magnetosphere-ionosphere coupling

The above-cited studies of the jovian field based on Pioneer, Voyager, and Ulysses fly-by data have demonstrated that the field lines in the middle magnetosphere region dominated by the current sheet are distorted out of meridian planes, associated with an azimuthal field component which reverses about the equator. However, in the near-planet region, within a few tens of $R_J$, the sense of the distortion does not reverse about the noon-midnight meridian as expected for the solar wind-induced effects discussed above. Instead, the distortion is consistently that of a field which “lags” behind planetary rotation, associated with an outward radial equatorial current flow. On the dawn side this “lagging” effect has the same sense as the bending effects induced by the solar wind, such that it is not simple to separate them in this sector. On the dusk side, however, the effects are opposite, and “lagging” fields at smaller distances have been found to give way to “leading” tail-like fields at larger distances, as demonstrated during the outbound pass of the Ulysses spacecraft (Dougherty et al., 1993). The two major effects are illustrated in Fig. 2.4a, which shows a view looking down onto the jovian magnetosphere from above the northern pole, and shows field lines from low latitudes bent out of the meridian planes, i.e. the lagging field described above. Those fields emanating from the higher latitudes are bent away from noon and towards the tail and a result of the interaction with the solar
Figure 2.4a Sketch of field lines emanating from the northern hemisphere of Jupiter projected onto the equatorial plane, showing the bending of the field lines out of meridian planes. High-latitude field lines mapping to the outer parts of the magnetosphere are bent away from noon by the interaction with the solar wind. The current system responsible is the magnetopause-tail system. Lower-latitude field lines mapping to the middle magnetosphere current sheet are bent consistently in the clockwise sense, associated with the transfer of anticlockwise planetary angular momentum from the thermosphere/ionosphere to the magnetosphere. The current system responsible is the magnetosphere-ionosphere coupling circuit shown in Fig. 2.4b.
wind, such that in the north, the perturbation fields are eastwards on the dusk side of the magnetosphere and westward on the dawn side.

The $j \times B$ force associated with the outward radial current of the "lagging" fields in the inner region accelerates the plasma in the sense of planetary rotation, such that the effect is understood to result from the magnetosphere-ionosphere interactions which enforce at least partial corotation of the magnetospheric plasma with the planet (Hill, 1979; Vasyliunas, 1983). The theoretical problem of the inertial limit on corotation was first discussed by Hill (1979), and will be briefly outlined here as it forms the basis of the theoretical discussions in Chapter 5.

Planets which possess both atmospheres and magnetospheres (i.e. the earth and Jupiter for example) are observed, as expected, to exhibit the phenomenon of corotation. Rigid corotation implies that the planetary plasma rotates with essentially the same angular velocity as the planet itself (at Jupiter rigid corotation implies an azimuthal plasma speed of $12.6 \, r_p \, \text{km s}^{-1}$, where $r_p$ is the perpendicular distance from the planetary spin axis in $R_J$). The atmosphere provides a viscous transfer of momentum from the rotating surface of the planet up into the ionosphere, where collisions between ions and neutrals drive the plasma into corotation. The effect of the rotating ionosphere is to produce a corotation electric field

$$E = -(\omega \times r) \times B$$  \hspace{1cm} (2.2)

where $\omega$ and $B$ are the spin angular frequency and the magnetic field vectors, respectively, and $r$ is the radius vector from the spin axis. The electric field is then transmitted outward to enforce corotation of the magnetospheric plasma. For the case of an electrically insulating atmosphere, in order to successfully maintain corotation it must (1) have sufficient viscosity to transport the planetary angular momentum upward into the Pedersen conducting layer of the ionosphere, and (2) the Pedersen conductivity itself must be large enough to impose Eq. (2.2) (equally the ion-neutral collision frequency must be large enough to force the ionosphere to achieve corotation). In early work, the first of the two requirements is assumed to be met for the case of the jovian magnetosphere (Coroniti, 1974; Kennel and Coroniti, 1977; Hill, 1979) and therefore the rate of transport of angular momentum is assumed by Hill (1979) to be limited chiefly by the ionospheric conductivity. If the ionospheric conductivity is high, the currents flow freely and corotation is easily maintained.
Corotation cannot extend to arbitrarily large distances from the planet but must ultimately break down as the result of either external forces or of the inertia of the corotating plasma. For the case of the Earth's magnetosphere, the solar wind imparts large external stresses to the magnetospheric plasma and therefore corotation breaks down beyond ~5 Earth radii, well within the magnetosphere (e.g. Brice, 1967). However, external forces are less important for Jupiter, so that the dynamics of the rotating plasma is of central significance.

If we consider, for example, a given magnetospheric plasma population at rest with respect to the rotating planet, the differential speed between the atmosphere and ionosphere would create an unbalanced torque which will eventually drive the plasma to exact corotation after a sufficient time, and the frictional torque will drop to zero. This end-state will, however, be broken by two effects (i) local plasma production, and (ii) radial transport. If plasma is produced in some region e.g. by ionisation of neutral gas (as in the Io torus) then these newly-ionised particles will take up angular momentum from the plasma, causing it to slow below rigid corotation (Pontius and Hill, 1982). This will result in an atmospheric torque on these flux tubes, and equilibrium is achieved when the latter torque is equal to the take-up of angular momentum by the newly-ionised particles. In such a region, therefore, the plasma will subcorotate. A similar situation exists if plasma is transported outwards from an internal source, as for the Io torus plasma in the middle magnetosphere. Here, the angular velocity of the plasma will tend to drop as it moves radially outwards, as $\rho^{-2}$ if there is no torque on the plasma, such that angular momentum is preserved. Thus outward radial transport will also result in subcorotation of the plasma, leading to the presence of an atmospheric torque which raises the angular velocity of the plasma above the $\rho^{-2}$ profile, towards rigid corotation. Hill (1979) calculates the inertial corotation lag as a function of distance in a magnetosphere which has plasma production and outward transport, for a given ionospheric Pedersen conductivity. The results indicate that the plasma in such a case will depart from corotation at jovicentric distances beyond ~30 R$_J$, for nominal values of the field strength and the ionospheric Pedersen conductivity. The Hill (1979) problem assumes that the neutral atmosphere will corotate with the planet. The ion-neutral collisional drag force exerts not only a prograde force on the ions (balanced by the Pedersen current $j \times B$ force), but also a retrograde force on the neutral gas at ionospheric heights, balanced by the viscous transfer of planetary angular
momentum upward through the underlying atmosphere (Huang and Hill, 1989). In general then, both ions and neutrals are slowed to a speed less than that of corotation. This corotation lag of the neutral atmosphere results in a reduction of the electric field in the neutral rest frame, and hence for a given plasma production or transport rate the corotation lag is enhanced by a factor that depends on the ion-neutral collision frequency and on the coefficient of atmospheric viscosity.

The outward equatorial transport of iogenic plasma thus results in a frictional torque in the ionosphere on the magnetospheric flux tubes which attempts to maintain corotation. This torque is communicated to the equatorial plasma by a large-scale system of field-aligned currents (FACs) which connect and close the ionospheric Pedersen currents to the radial equatorial magnetospheric currents (Hill, 1979; Pontius and Hill, 1982, Parish et al., 1980; Connerney, 1981; Vasyliunas, 1983; Khurana and Kivelson, 1993). The sense of the overall current system is such that the FACs flow away from the planet and into the equatorial current sheet in the inner part of the system, and towards the planet and away from the current sheet at larger distances. This current system is shown in Fig. 2.4b. The $j \times B$ force in the equatorial plane is directed into the diagram and acts to increase the speed of a sub-corotating equatorial plasma, while the $j \times B$ force of the closure currents in the ionosphere acts in the opposite direction as a drag force on the rotation of the thermosphere. The corotation of the thermosphere is maintained to the extent allowed by viscous coupling to the corotating denser atmosphere beneath, as discussed briefly above (Huang and Hill, 1989).

2.4 Plasma populations and their associated dynamics

2.4.1 Sources of plasma mass and momentum

The nature of the plasma dynamics in a planetary magnetosphere depends on the nature of the plasma sources and sinks, and the nature of the transport processes which convey the plasma from the former to the latter. The plasma sources include the solar wind at the outer boundary and the planet's ionosphere at the inner boundary, together with the surfaces and atmospheres of any moons that happen to orbit within the cavity. The sources of momentum include the antisunward flow of the solar wind on the outside, and
Figure 2.4b. Sketch of the current system associated with planetary angular momentum transfer for the case appropriate to plasma sub-corotation. The arrowed solid lines are magnetic field lines, and the dashed lines show the direction of current flow. The circled symbols marked $B_\phi$ indicate the direction of the azimuthal perturbation magnetic field produced by these currents, out of the diagram north of the current sheet, and into the diagram south of the sheet (for the 'lagging' field configuration).
the planet's rotation on the inside. Solar wind interaction at the boundary, e.g. magnetic reconnection, carries magnetospheric flux tubes from the dayside to the nightside in the outer regions of the magnetosphere, where they are stretched out to form the tail lobes. As they are stretched out down-tail, open flux tubes sink in towards the centre plane of the tail, where they reconnect again within the current sheet, returning closed magnetic flux tubes, attached to Jupiter at both ends, back towards the planet (as first described by Dungey, (1961)). However, where the reconnection and return flow takes place at Jupiter is unknown at present. In the absence of such flows, the magnetospheric plasma and field will rotate with the planet, the angular momentum being transferred by ion-neutral collisions at the feet of the field lines in the lower ionosphere, as described in detail in the previous section.

Brice and Ioannidis (1970) were the first to consider the relative importance of these two flow systems at Jupiter. The flows of the magnetospheric plasma and embedded magnetic fields are associated with an electric field, given by \( E = -V \times B \). The overall strength of a flow system can then be measured by the voltage associated with its electric field, since by Faraday's law, 1 volt is equivalent to the transfer of 1 Wb s\(^{-1}\) of magnetic flux embedded in the flow. The electric field associated with the solar wind-driven flow is directed from dusk to dawn across Jupiter's magnetosphere, and by analogy with the Earth, the associated voltage can be estimated to be \(-1\) MV (Kennel and Coroniti, 1977) (i.e. the transfer through this flow system of \(-1\) MWb s\(^{-1}\) from the dayside to the tail in the outer regions, and the return of the same amount, in the steady state, in the central regions). The electric field associated with rotation is directed radially outwards in the equatorial plane, and for rigid corotation with the planet the associated voltage is \(-400\) MV. Rotation with the planet is thus by far the most important flow (and thus source of angular momentum) at Jupiter, although as indicated above, this statement does not preclude the dominance of solar wind-driven effects in the outer regions and magnetotail.

Estimates indicate that both the solar wind and the ionosphere represent sources of a few \(10^8\) ions s\(^{-1}\) for Jupiter's magnetosphere (Hill et al., 1983), consisting principally of hydrogen (i.e. protons, together, of course, with sufficient electrons to keep the gas electrically neutral overall). The corresponding mass sources are a few 10s of kg s\(^{-1}\). The ionospheric source also uniquely provides molecular hydrogen ions, \(H_2^+\) and \(H_3^+\) (Hamilton et al., 1980), as minor constituents, while the solar wind provides \(He^{++}\) and
traces of heavier ions such as carbon. The identification of all these species within the jovian plasma has confirmed the presence of both sources. The major discovery of the Voyager fly-bys (Belcher, 1983; Krimigis and Roelof, 1983), however, was that the jovian system is not dominated by a hydrogen plasma as had previously been anticipated, but by a sulphur and oxygen plasma which originates from the sulphur dioxide atmosphere of the volcanic moon Io, which orbits at a distance of 5.9 R_J. The source rate is estimated to be $\sim 3 \times 10^{28} \text{ ions s}^{-1}$ (Goertz, 1980), similar to the solar wind and ionospheric sources, but because the ions are heavy, with a mean mass of $\sim 21 \text{ amu}$, the corresponding mass source of $\sim 1000 \text{ kg s}^{-1}$ is overwhelmingly dominant. The sodium source at Io, though easily visible in optical emission, is less than this by a factor of around a hundred, and is thus negligible in overall terms. Recent estimates indicate that the moon Europa, which orbits at a radial distance of 9.4 R_J, is also a significant source of oxygen plasma originating from surface water ice, with rates of $\sim 2 \times 10^{27} \text{ ions s}^{-1}$ and corresponding mass rates of $\sim 50 \text{ kg s}^{-1}$ (Ip et al., 1998). The plasma dynamics of Jupiter's magnetosphere is therefore dominated by the consequences of the presence of strong heavy-ion sources lying deep within a rapidly-rotating magnetosphere.

2.4.2. The Io plasma torus

The Io plasma which exists within Jupiter's magnetosphere originates principally from electron-impact ionisation of the clouds of sulphur and oxygen atoms which orbit in the vicinity of Io, originating in the latter's atmosphere. These atoms were first observed both remotely and in-situ by the Voyager-1 EUV instrument and the associated ions by the Voyager-1 PLS instrument (Broadfoot et al. 1979; Bridge et al., 1979). The density of these atoms peaks at a few 10 s cm$^{-3}$ near the orbit of Io (see Schreier et al. 1998), with oxygen being the more numerous species as expected from the sulphur dioxide source, and falls off by an order of magnitude within $\sim 1 \text{ R}_J$ on either side. These neutral particles orbit with Io at speeds of $\sim 17 \text{ km s}^{-1}$, being influenced only by the gravitational force of Jupiter. When these atoms are ionised, however, the resulting ions and electrons suddenly sense the electromagnetic environment as well, that is to say the southward $\sim 2000 \text{ nT}$ magnetic field of the planet, and the $\sim 0.1 \text{ V m}^{-1}$ outward-directed electric field associated with the $\sim 70 \text{ km s}^{-1}$ flow of the near-corotating plasma. The effect of these fields is such as to cause the charged particles immediately to drift with the plasma at the corotation
speed (they are "picked up" by the plasma flow), and also to acquire a gyroratory speed about
the field lines equal to the difference between the corotation and Keplerian speeds, equal to
\(~55 \text{ km s}^{-1}\). This corresponds to a "thermal" energy of \(~250 \text{ eV}\) for oxygen ions and
\(~500 \text{ eV}\) for sulphur, but only 0.01 eV for electrons. Subsequently, the ions are cooled by
Coulomb collisions with the electrons, and the consequently heated electrons are cooled by
collisional excitation of the low-lying energy levels of the ions, thus leading to the
observed optical emission from the torus.

The most detailed information about the low-energy plasma distribution which
results from these processes was obtained during the inbound passage of Voyager-1, and
modelled empirically first by Bagenal et al. (1980), and then improved by Bagenal and
Sullivan (1981), Bagenal et al. (1985), and more recently by Bagenal (1994). The details
of the plasma populations are described in the latter paper, and it is the results from this
paper that are summarised here. Fig. 2.5 shows contours of electron density (equal to the
ion charge density) which have been derived from the Voyager-1 data. The principal
population is the warm plasma torus which, in the equatorial plane, extends outwards from
a jovianentric distance of \(~5.6 R_J\). The ions in this region consist of two populations, a
suprathermal population of recently-ionised few 100 eV particles (increasing in energy
with increasing distance, up to \(~2 \text{ keV}\) at \(~10 R_J\)), comprising \(~10-20\)% of the population,
and a cooled population with a temperature of \(~60 \text{ eV}\) (also increasing with distance up to
\(~300 \text{ eV}\) at \(~10 R_J\)). The electron temperature in this region is \(~10 \text{ eV}\). In the inner part of
the warm torus, within \(~7.5 R_J\), there are approximately equal numbers of sulphur and
oxygen ions, with the oxygen being principally \(O^+\), while the sulphur is roughly equally
divided between \(S^+\) and \(S^{++}\). Outside this distance, the plasma is richer in oxygen,
possibly due to the Europa source, with roughly equal numbers of \(O^+\) and \(O^{++}\), and the
density of \(S^{+++}\) becomes comparable with those of \(S^+\) and \(S^{++}\). Inside \(~5.6 R_J\) the plasma
cools precipitately to form the cold plasma torus near the equatorial plane at distances of
\(~5.0-5.4 R_J\). The ion and electron temperatures in this region are just a few eV, and the
composition is a somewhat sulphur-rich combination of \(S^+\) and \(O^+\).

The distribution of the torus plasma along the field lines is determined by a balance
of forces, namely between the plasma pressure, the magnetic mirror force, the centrifugal
force, the gravitational force due to Jupiter, and a field-parallel electric force which is
required to ensure that the ion and electron charge densities are equal at all points. Apart
Figure 2.5. Contours of the electron density in the Io plasma torus in the meridian plane, determined from Voyager 1 PLS data. The vertical scale is distance from the centrifugal equator, while the horizontal axis is distance from Jupiter's spin axis, both in units of R_J. The numbers on the contours refer to electron density, equal to the ion charge density, in units of electrons cm\(^{-3}\). Taken from Bagenal (1997).
from the latter, the most important physical effect is that the centrifugal force tends to compress the plasma at the centrifugal equator (the point of maximum distance along a field line from the planetary spin axis) while this compression is resisted by the plasma pressure.

The spatial structure of the torus plasma across the field lines reflects both the distribution of the atomic gas sources in the vicinity of the orbits of Io and Europa, together with the nature of the cross-field transport mechanism. Because the outwardly-directed centrifugal force on the plasma is dominant (being much greater, certainly, than the inward force of Jupiter’s gravity), flux tubes containing high-density plasma from the Io source will tend to "fall" outwards to larger distances, restrained by the frictional drag of ion-neutral collisions at the feet of the field lines in the ionosphere. These flux tubes will be replaced by tubes containing lesser densities moving inwards, it being ultimately supposed, of course, that some mechanism (presently unknown) exists for emptying the flux tubes of their plasma content at large distances. Galileo observed both short duration anomalies in the magnetic field near the Io torus (Kivelson et al. 1997), and in addition in the plasma signatures (Thorne et al., 1997), both interpreted as evidence for rapid interchange motions in the vicinity of the Io torus. Observations of apparently ‘empty’ flux tubes have been made more recently by Galileo (Russell et al., 2000). This process has been parameterised in many theoretical models by an empirically-determined spatial diffusion coefficient (Southwood and Kivelson, 1989; Fazakerley and Southwood, 1992; Ferrière et al. 2001).

Observationally, the warm torus plasma is found to pervade the equatorial current sheet out to the outer boundary of the middle magnetosphere at several 10s of R_J. Due principally to the expansion of the flux tubes, the equatorial ion/electron charge density falls from at peak of ~3000 cm^{-3} (during the Voyager 1 flyby) near the inner edge of the torus at ~5.7 R_J, to ~70 cm^{-3} at ~10 R_J (as in Fig. 2.5), and down to ~0.1 cm^{-3} at several 10s of R_J (Scudder et al. 1981). The fraction of suprathermal particles in the population appears to increase with distance, however, such that the average energy also increases, rather than falling as expected for an expanding plasma. Typical values are a few 100 eV. Sporadic enhancements of low-temperature plasma are also observed in the dayside outer magnetosphere, correlated with decreases in the strength of the magnetic field (Southwood
et al., 1993). We may conjecture that these represent plasma fragments which have become detached from the middle magnetosphere current sheet.

2.4.3 The hot plasma population

The outwardly-diffusing Io torus plasma is not, however, the only population which is present in Jupiter's magnetosphere. Observations by the Voyager (Mauk et al. 1996) and Ulysses (Lanzerotti et al. 1993) spacecraft have shown that a low-density but high-energy population is also present, consisting of roughly equal numbers of protons and heavy ions (mainly sulphur and oxygen). The number density and average energy of this population both increase on moving towards the planet, before falling near the inner edge of the warm torus. In terms of the above discussion, we may picture this low-density population to be transported radially inwards as the high-density torus plasma is transported radially outwards. Indeed, evidence has been found in Galileo data for sporadic localised inward injections of hot ions within the current sheet at distances between ~10 and ~30 R_J (Mauk et al. 1999). These injections have aspects in common with substorms at Earth, but at Jupiter they are not confined to the nightside but occur at all local times. Inward transport of this plasma results in compression and heating as observed, the energy required being derived ultimately from the outward "falling" torus plasma. The presence of the hot plasma thus acts partially to suppress this transport.

The hot plasma density is much less than that of the warm torus plasma throughout the system, being \( \sim 10^{-2} \) to \( 10^{-3} \) cm\(^{-3} \) in the outer regions, increasing to perhaps \( \sim 1 \) cm\(^{-3} \) in the inner part of the Io torus. These particles thus make little contribution to the overall mass or charge density compared with the low-energy plasma, except perhaps in the dayside outer magnetosphere. However, their average energy is sufficiently large that they make the dominant contribution to the plasma pressure at all points except for the innermost part of the warm torus where the extreme warm plasma density (Fig. 2.5) and the falling hot ion temperature combine to produce comparable warm and hot plasma pressures. In the outer part of the magnetosphere the hot ion temperature is a few 10s of keV, with the distribution having a non-Maxwellian high-energy tail extending above 1 MeV. The average energy then increases with decreasing distance, reaching a peak of \( \sim 2 \) MeV at ~7 R_J according to Voyager 1 measurements, before falling to ~100 keV inside...
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Io's orbit at 5.9 R\(_J\). Electrons are also present with comparable energies but significantly lower densities, such that they make a smaller contribution to the pressure. It is these energetic particles in the inner part of the warm torus which form the source of the radiation belts within the inner magnetosphere. From the vicinity of Io's orbit these particles are more slowly diffused inwards due to the presence of fluctuating electric fields driven by winds in the thermosphere, gaining further energy as they do so. This input is balanced in the steady state by particle flux and energy losses in the inner magnetosphere which are due to wave-induced particle precipitation into the atmosphere, absorption by ring material and moons, and (for electrons) synchrotron radiation.

A key feature of the hot ion population is that within the outer part of the middle magnetosphere its pressure is comparable with that of the magnetic field. As a consequence it "inflates" the planetary field to form the current sheet structure observed in this region (Fig. 2.1). The distended field lines then provide the inward force (the \( j \times B \) Lorentz force of the azimuthal current) which in the steady state balances the outward pressure gradient of the hot plasma (see Caudal (1986) and references therein). The Io torus plasma also plays a role in current sheet formation, though a lesser one, since the field must also provide the inward force necessary to balance the centrifugal and pressure gradient forces of this population. With regard to the dominant hot plasma population, Voyager observations indicate that the equatorial pressure remains greater than that of the field throughout the outer part of the middle magnetosphere, while falling below that of the field inside ~10 R\(_J\), due to the rapidly increasing strength of the dipole field. Consequently, as noted above, the perturbation fields produced by the plasma at and inside these distances becomes smaller than the planetary field, such that the field geometry then assumes a quasi-dipolar form.

2.4.4 Plasma flow and field bending

Observations of the plasma flow within the middle magnetosphere, extending outwards from the orbit of Io, generally confirm a primary plasma flow in the sense of planetary rotation as discussed above. However, departures from rigid corotation are observed, which as discussed above, are due to two main effects. The first occurs in the main source region of the torus plasma in the vicinity of Io's orbit, where neutral atoms are
ionised and "picked-up" by the plasma. Because angular momentum is continuously provided to the newly-ionised particles, the plasma in this region rotates more slowly than for rigid corotation, by an amount which is just such that the ion-neutral collisions in the lower ionosphere provide the required torque. This coupling between the magnetosphere and the ionosphere was discussed in detail in Section 2.3.6. Ground-based spectroscopic observations of the plasma, together with in situ data from Voyager 1, indicate that the plasma flow is slowed by ~4 km s\(^{-1}\) in a ~2 \(R_J\)-wide region centred near to Io's orbit by this effect (the rigid corotation speed is 74 km s\(^{-1}\)). Outside this region, near-rigid corotation is resumed (Pontius and Hill, 1982; Brown, 1994).

As the torus plasma diffuses radially outwards, however, angular momentum must again be continuously added to maintain plasma rotation at near-rigid speeds. As discussed in the previous section, if no angular momentum is added, conservation of plasma angular momentum requires the azimuthal speed of the plasma to fall inversely as the distance from the spin axis, while for rigid corotation the speed must increase in direct proportion to this distance. In order to maintain near-rigid corotation of an equatorial outwardly-diffusing plasma, the angular momentum flux into the equatorial region must be constant, independent of distance. For radial mass fluxes corresponding to the Io source, it turns out that in the inner part of the torus, the required angular momentum flux can be supplied by ion-neutral collisions in the lower ionosphere for only minimal departures of the flow from rigid corotation. In this region, therefore, the plasma very nearly rigidly corotates with the planet outside the source region. However, with increasing distance from the planet a given area of the equatorial plane becomes connected via the magnetic field to a decreasing area of the ionosphere, which is located nearer to the rotation axis. Consequently, the ionospheric torque ultimately becomes small even for large departures from corotation. Thus beyond a certain radial distance, depending on the Io mass flux and the ion-neutral friction in the ionosphere, the azimuthal velocity is expected to break away from near-rigid corotation, to peak, and then to fall inversely with distance in the regime where the input of angular momentum becomes small. Voyager observations indicate that the flow is near to that expected for rigid corotation to equatorial distances of ~20 \(R_J\), where the azimuthal flow speed is ~200 km s\(^{-1}\), and falls below rigid corotation at larger equatorial distances. Voyager data for a relatively compressed magnetosphere indicate flows in the outer regions which do not fall with distance as then expected, but rather remain at values which are a factor of ~2 lower than rigid corotation speeds (Sands and
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McNutt, 1988; Kane et al., 1995). When the magnetosphere is more extended during intervals of low solar wind dynamic pressure, however, the flow speeds in the outer regions are rather lower than this relative to rigid corotation, as indicated by data from both Pioneer 10 (McDonald and Trainor, 1976) and Ulysses (Cowley et al., 1996). The transfer of angular momentum along magnetic field lines, via a large-scale system of radial currents was discussed in the previous section.

Flows in the outer magnetospheric regions are exceedingly uncertain at the present time. Outbound Voyager 2 measurements established the existence of a layer of plasma adjacent to the dawn tail magnetopause at ~150 R_J which was flowing antisunward, opposite to the direction of planetary rotation. The flow speed was ~500 km s^{-1} in a layer a few tens of R_J wide. The nature of the overall dynamics, however, concerning e.g. the ultimate loss process for Io plasma, and the interaction between planetary and solar wind-driven flows in the tail, are yet to be determined.

2.5 The jovian aurora

The jovian auroral emissions are the observable signatures of the electrodynamic coupling between the ionosphere, the magnetosphere, and possibly the solar wind. Since the discovery of the jovian ultra-violet auroral emissions in 1979 during the flyby of Voyager-1 (Broadfoot et al., 1979), there has been a great deal of research on the nature of the giant planet auroras. We now have now entered an era of highly resolved images of the aurora on Jupiter, obtained at various wavelengths. For example, images in the infra-red wavelengths of the spectrum are obtainable from ground-based telescopes (e.g. Satoh et al., 1996; Connerney et al., 1998), at ultra-violet wavelengths by the Hubble Space telescope (e.g. Prangé et al., 1998; Clarke et al., 1998), and most recently and at the highest spatial resolution in the visible by the Galileo orbiter (Vasavada et al., 1999). The IR aurora (see Fig 2.6) are mainly excited by particle heating, or from Joule heating by large-scale current systems closing in the ionosphere (Prangé et al., 1998), whilst the UV emissions (see Fig 2.7) result from direct excitation of atmospheric species by collisions due to precipitating magnetospheric electron energy flux. There are at present three (potentially four) main types of auroral emission. First is the Io footprint emission (IFT), a ~1 MR emission which maps magnetically to the orbit of Io. The Io oval is quite distinct from the
Figure 2.6. A mosaic of Jupiter (304 deg CML) at 3.4 microns wavelength obtained on 12 January 1992. A meridian plane projection of field lines originating beyond 30 R_J and at the orbital radial distance of Io (5.9 R_J) are superposed on the mosaic. The leftmost field line is traced from the instantaneous orbital position of Io at 79 deg Io phase. Most of the H3+ emission originates from the auroral oval. Two faint emission features due to the Io-Jupiter interaction can seen near the leftmost limb of the planet just equatorward of the aurora in both hemispheres. The featureless background emission distributed across the planetary disc is due to ionospheric H3+ emission. Figure from Connerney et al. (1993)
Figure 2.7. Jupiter's UV aurora, as imaged by the Wide Field Planetary Camera 2 on board the Hubble Space Telescope. Courtesy of John Clarke, University of Michigan.
other jovian auroral emissions, being at its brightest at the position of Io and then remains visible, although fading with distance, downstream of the moon (visible clearly in Fig. 2.7). It is understood that the corotational electric field which is developed by the relative motion between Io and the magnetic field lines swept past the satellite induces a potential difference of approximately 500 kV between the outer and inner faces of the moon. This potential is thought to cause currents to flow from Io toward the ionosphere, both northward and southward, along the outer portion of the magnetic flux tube linking the satellite to the ionosphere. Return currents would then flow along the inner portions of the flux tube towards Io. This circuit would be closed through the ionosphere and presumably, in the ionosphere of the moon (Acuña et al., 1983). This ‘unipolar inductor’ model would explain a spot of aurora associated with the upward directed portion of the current system, but would not however explain the existence of the ‘tail’ of auroral brightness downstream of Io by as much as ∼30°.

The second component of the aurora is the high latitude diffuse emissions, which occur regularly and have a brightness of a few x 100 kR. These polar cap auroras are generally extended across the dusk side of the polar cap. At this time, a production mechanism for these emissions has not been suggested. Third is the main auroral oval (MAO), which has been the subject of a plethora of reports recently. The MAO is the most significant emission in terms of energy output, and arises from circumpolar bands around both the northern and southern poles, consistently observed in all the above wavebands. Although this emission is of variable width (on average ~1000 km) and intensity (up to a few MR), it appears to be essentially continuous in local time, at dipole co-latitudes of ~16° close to the pole than the Io flux tube. It has been known for some time that this auroral region maps magnetically to the middle magnetosphere, it is only recently understood to be associated with the breakdown of corotation of the equatorial plasma in the middle magnetosphere (Cowley and Bunce, 2001b; Hill, 2001; Southwood and Kivelson, 2001), but it is only the former of these models which proposes an empirical model of the aurora, for estimates of the angular velocity of the plasma in the equatorial plane and a suitable magnetic field model. The work presented in this thesis provides the background to the formulation of this model. More recent papers (Pallier and Prangé, 2001; Waite et al., 2001) have observed a fourth feature which is seemingly a regular occurrence in the auroral regions. This feature appears consistently near magnetic local noon, and is reminiscent of the Earth’s polar cusp. Both papers refer to this as a ‘cusp-
like’ feature, which was reported by Waite et al. (2001) to be rapidly evolving, very bright (up to ~40 MR) and localised near noon. This feature lies poleward of the MAO and therefore it is conjectured that it may be controlled by pressure and/or magnetic field changes in the upstream solar wind.
Chapter 3: Modelling the magnetic field in Jupiter's magnetosphere

3.1 Introduction

The subject of this thesis is mainly concerned with the structure of Jupiter's magnetospheric magnetic field, and consists of a series of studies which attempt to understand more about the global morphology and dynamics of the magnetosphere via spacecraft observations of the magnetic field. In order to successfully analyse the measured magnetic field data, we are in need of three specific tools. First, we need to know in detail about the internal magnetic field of Jupiter which is created by currents flowing in the conducting interior of the planet. A good model of the planetary field may be subtracted from those fields measured by spacecraft in order to obtain information about the externally driven currents (mainly the azimuthal equatorial current sheet in the jovian magnetosphere). We therefore provide a brief overview of the spherical harmonic method of modelling the internal field. Furthermore, models of the middle magnetosphere current sheet are also used to gain a better understanding of the external currents. For this purpose we will introduce the current sheet model of Connerney et al. (1981) which is valid out to radial distances 30 R_J. Finally, we need to employ models of the equatorial field (i.e. the north-south B_z component) which are applicable over an extended radial range. Thus we
introduce here the empirical $B_z$ models of Goertz et al. (1976) and Khurana and Kivelson (1993), and how they lead to the calculation of the vector potential of the magnetic field in the equatorial plane. These values can be used to map field lines between the equatorial plane and the planet, as will be described in Section 3.4

3.2. *The internal field*

In the solar system, the magnitude of Jupiter's magnetic field is second only to that of the Sun. As described in Chapter 2, many of the basic parameters of the field were initially derived from ground observations made by radio astronomers observing non-thermal radio emissions at both decimetric and decametric wavelengths. These observations, along with those from the fly-by and orbiter missions to Jupiter (Pioneers 10 & 11, Voyagers 1 & 2, Ulysses and Galileo) have resulted in postulated values being continuously updated and augmented. Early measurements correctly established, however, that the magnetic field has a southward polarity (i.e. opposite to that of the Earth) and the dipole axis is tilted at approximately 9.6° to the rotation axis toward ~202° System III (1965) longitude in the Northern Hemisphere. The magnitude of the magnetic moment of Jupiter is estimated at somewhere between 4.208 and 4.278 GRJ$^3$ (1 G=10^-4 T) and the equatorial surface field intensity is of the order of 420,000nT.

It is generally accepted that the production mechanism for the intrinsic magnetic field is that of a thermal convection-driven dynamo operating in the conducting regions of Jupiter's interior. The hypothesised rocky core of Jupiter is thought to be surrounded by liquid metallic hydrogen, the rotation rate of the magnetic field is then dictated by the rotating core. The internal field is composed of large dipole terms and also significant quadrupole and octupole moments, which may be calculated by way of the standard spherical harmonic analysis method. Any contributions from external sources (i.e. the azimuthal current sheet in the middle magnetosphere) are comparatively negligible in the inner magnetosphere and thus are excluded from the spherical harmonic analysis described in the following section. One method of dealing with the external currents is to assume axi-symmetry and use a vector potential formulation. The current sheet model of Connerney et al. (1981) and the way it is employed in this thesis, will be described in Section 3.3.
Chapter 3: Modelling the magnetic field in Jupiter's magnetosphere

The inner magnetosphere is described as the region which extends in the magnetic equatorial plane out to approximately 6 R_j and where the magnetic field is dominantly planetary. As discussed in the review chapter, this outer limit approximately corresponds to the orbit of the moon Io, whose orbital path lies at ~5.9 R_j in the equatorial plane. A review of the mathematical method used to derive the magnetic field components to third order using spherical harmonic analysis follows. The relative importance of these terms may then be appreciated by considering the standard inner magnetic field models for Jupiter and how these terms affect the total magnetic field within this region.

In general the Ampère-Maxwell equation is

$$\text{curl} \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} ,$$  \hspace{1cm} (3.1)

so that if $\mathbf{J} = 0$ and the displacement current is negligible it follows that

$$\text{curl} \mathbf{B} = 0 ,$$  \hspace{1cm} (3.2)

that is that the curl of the magnetic field in such a region is equal to the null vector.

The general solution to Eq. (3.2) is

$$\mathbf{B} = -\nabla V ,$$  \hspace{1cm} (3.3)

for some scalar field $V$. Furthermore, as a result of Maxwell's equation we have

$$\text{div} \mathbf{B} = 0 ,$$  \hspace{1cm} (3.4)

and so, substituting Eq. (3.3) we have

$$\nabla^2 V = 0 .$$  \hspace{1cm} (3.5)

These equations imply that any conservative magnetic field vector, $\mathbf{B}$, may always be expressed in terms of a scalar potential, $V$, which satisfies Laplace's equation.

When using spherical harmonic analysis to determine the magnetic field components one considers a sum of contributions from internal and external scalar potentials

$$V = V_I + V_E .$$  \hspace{1cm} (3.6)

The magnetic field components shall be derived here for both internal and external contributions (which would include contributions from the fringing fields of the current sheet and the effect of the magnetopause current system). The external terms are neglected.
We now write Laplace’s equation in spherical-polar coordinates

\[ \nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0. \]  

(3.7)

We separate the variables using the standard procedure. Thus we let

\[ V = R(r) \Theta(\theta) \Phi(\phi). \]  

(3.8)

Then Laplace’s equation becomes

\[ \nabla^2 V = \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{dR(r)}{dr} \right) \Theta(\theta) \Phi(\phi) + \frac{R(r) \Phi(\phi)}{r^2 \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta(\theta)}{d\theta} \right) + \frac{R(r) \Theta(\theta)}{r^2 \sin^2 \theta} \frac{d^2 \Phi(\phi)}{d\phi^2} = 0. \]  

(3.9)

Thus multiplying through by

\[ \frac{r^2 \sin^2 \theta}{R(r) \Theta(\theta) \Phi(\phi)}, \]  

(3.10)

we are left with the following equation

\[ \frac{\sin^2 \theta}{R(r)} \frac{d}{dr} \left( r^2 \frac{dR(r)}{dr} \right) + \frac{\sin \theta}{\Theta(\theta)} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta(\theta)}{d\theta} \right) = -\frac{1}{\Phi(\phi)} \frac{d^2 \Phi(\phi)}{d\phi^2}. \]  

(3.11)

From Eq. (3.11), it is clear that the left-hand side is a function of \( R \) and \( \Theta \) only and that the right hand side is a function only of \( \Phi \). This equation may only be satisfied if both sides are equal to each other for any values of the three variables. Thus they cannot depend on \( R, \Theta \) and \( \Phi \) i.e. they must be equal to a constant. The form of this constant is selected according to the nature of the problem in question. When one is dealing with the derivation of the magnetic field of a planet, it is essential that the equations used make physical sense. Therefore it follows that the constant must be positive and real and thus \( m^2 \) is chosen in order to satisfy these conditions. Now the RHS may be solved, and in a similar way we may proceed to solve the rest of the equation.
Chapter 3: Modelling the magnetic field in Jupiter's magnetosphere

\[-\frac{1}{\Phi(\phi)} \frac{d^2\Phi(\phi)}{d\phi^2} = m^2. \tag{3.12}\]

This is clearly a second order differential equation and has the general solution

\[\Phi(\phi) = C \cos m\phi + D \sin m\phi, \tag{3.13}\]

where \(C\) and \(D\) are arbitrary constants.

If \(\Phi(\phi)\) is a periodic function of \(\phi\) then \(m\) must have an integer value. From this equation we can also deduce that the solution for \(m\) positive will be the same as for \(m\) negative and therefore only \(m > 0\) need be considered. Subsequently by substitution of Eq. (3.12) into Eq. (3.11), followed by division by \(\sin^2 \theta\), the following relationship is found

\[\frac{1}{R} \frac{d}{dr} \left( r^2 \frac{dR}{dr} \right) = -\frac{1}{\Theta \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) + \frac{m^2}{\sin^2 \theta}, \tag{3.14}\]

where for ease the functions \(R(r)\) and \(\Theta(\theta)\) are written simply as \(R\) and \(\Theta\). In the same way as before the LHS may be written as some constant value whilst the RHS is solved and vice versa.

Substituting the LHS for some constant we can readily see that the RHS is a standard differential equation, solutions of which are the Associated Legendre Polynomials (ALP’s)

\[\frac{m^2}{\sin^2 \theta} - \frac{1}{\Theta \sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta}{d\theta} \right) = n. \tag{3.15}\]

By equating \(\cos \theta\) to \(x\) in the RHS of Eq. (3.14), and by setting \(\Theta(\theta) = P(x)\) we obtain

\[\left( n - \frac{m^2}{1-x^2} \right) P(x) + \frac{d}{dx} \left[ (1-x^2) P'(x) \right] = 0, \tag{3.16}\]

where the above equation only has physically valid solutions if \(n\) is a product of two successive integers, i.e. \(n=l(l+1)\). Spherical (surface) harmonics are related to the ALP’s by

\[Y_l^m(\theta, \phi) = (-1)^m \left[ \frac{(2l+1)(l-m)!}{4\pi (l+m)!} \right]^{1/2} P_{lm}(x) \left[ C \cos m\phi + D \sin m\phi \right], \tag{3.17}\]

where \(P_{lm}(x)\) are the ALP’s and are given by the recursion formula.
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\[
P_{lm}(x) = \frac{(-1)^m}{2^l l!} \left(1 - x^2\right)^{\frac{m}{2}} \frac{d^{l+m}}{dx^{l+m}} \left(x^2 - 1\right) . \tag{3.18}
\]

In Eq. (3.18) we can see that \(l \geq |m|\) or else \(P_{lm}(x) = 0\), which as a consequence places constraints on the allowed values of \(m\). These values are

\[
m = -l, -l + 1, -l + 2, \ldots, 0, 1, 2, \ldots, l . \tag{3.19}
\]

Now, by substituting \(n = l(l+1)\) into equation 14 we find

\[
\frac{1}{R} \frac{d}{dr} \left(r^2 \frac{dR}{dr}\right) = l(l+1) . \tag{3.20}
\]

By guessing the relationship \(R = r^\alpha\) we find that there are two possible solutions that \(\alpha\) may have: \(\alpha = 1\) and \(\alpha = -(l+1)\). Thus the general solution of this equation is

\[
R(r) = A r^l + B r^{-(l+1)} , \tag{3.21}
\]

where \(A\) and \(B\) are arbitrary constants. The first term, growing with \(r\), describes the external terms, while the second term, decreasing with \(r\), refers to the internal terms.

Finally, using Eqs. (3.17) and (3.21) the general solution of Laplace's equation in spherical polar coordinates is

\[
V(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left[A_{lm} r^l + B_{lm} r^{-(l+1)}\right] Y_l^m(\theta, \phi) . \tag{3.22}
\]

The surface harmonics are orthonormal, with the orthogonalisation condition being

\[
\int_0^{2\pi} \int_0^\pi Y_{l'}^{m'}(\theta, \phi) Y_l^m(\theta, \phi) \sin \theta d\theta d\phi = \delta_{l',l} \delta_{m,m'} , \tag{3.23}
\]

where \(\delta_{l,l'} = 0\) for \(l \neq l'\) and \(\delta_{l,l'} = 1\) for \(l = l'\) (and similarly for \(\delta_{m,m'}\)). \(\delta_{l,l'}\) and \(\delta_{m,m'}\) are known as the Kronecker delta functions.

In magnetism one commonly uses the normalised Schmidt functions, which are related to the ALP's in the following manner

\[
P_l^m = P_{lm} \quad \text{for} \quad m = 0 , \tag{3.24}
\]

and

\[
P_l^m = \left|\frac{2(l-m)}{l(l+m)}\right|^{\frac{1}{2}} P_{lm} \quad \text{for} \quad m > 0 .
\]

Schmidt normalisation is universally used to overcome the very diverse order of magnitude of the ALP's. One replaces the ALP's by numerical multiples of themselves i.e. the Schmidt normalised Associated Legendre Polynomials, which are more nearly normal.
We now have the standard equation for the spherical harmonic expansion of a planetary magnetic field, ignoring the external terms and with the arbitrary constants set to the usual notation, that is the Schmidt coefficients.

\[ V(r, \theta, \phi) = \sum_{l=1}^{\infty} \sum_{m=0}^{l} \frac{1}{r^{l+1}} P_l^m(\cos \theta)(g_l^m \cos m\phi + h_l^m \sin m\phi) . \] (3.24)

Now using Eq. (3.24) the dipole \((l=1)\), quadrupole \((l=2)\), and octupole \((l=3)\) terms are derived. By specifying the value of \(l\) and thus the corresponding values of \(m\), and by way of Schmidt normalisation of the appropriate \(\alpha\)\(L\)\(P\), we arrive at the dipole term

\[ V_{\text{dip}}(r, \theta, \phi) = \frac{1}{r^2} \left\{ \cos \theta g_1^0 + \sin \theta \left[ g_1^1 \cos \phi + h_1^1 \sin \phi \right] \right\} . \] (3.25)

In a similar way the terms for the quadrupole and the octupole are found to be

\[ V_{\text{quad}}(r, \theta, \phi) = \frac{1}{r^3} \left\{ \left[ \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) g_2^0 - \sqrt{5} \sin \theta \cos \theta \left[ g_2^1 \cos \phi + h_2^1 \sin \phi \right] \right] + \right\} , \] (3.26)

and

\[ V_{\text{oct}}(r, \theta, \phi) = \frac{1}{r^3} \left\{ \left( \frac{1}{2} \cos \theta (5 \cos^2 \theta - 3) g_3^0 + \frac{3}{2} \sin \theta (1 - 5 \cos^2 \theta) \left[ g_3^1 \cos \phi + h_3^1 \sin \phi \right] + \right\} + \frac{15}{\sqrt{60}} \cos \theta \sin^3 \theta \left[ g_3^2 \cos 2\phi + h_3^2 \sin 2\phi \right] \right\} - \frac{15}{\sqrt{360}} \sin^3 \theta \left[ g_3^3 \cos 3\phi + h_3^3 \sin 3\phi \right] \} . \] (3.27)

Having determined the potentials, \(V\), the field components themselves are easily obtained via Eq. (3.3), which leads to the following

\[ B_r = -\frac{\partial V}{\partial r} , \] (3.28a)

\[ B_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} , \] (3.28b)

and

\[ B_\phi = -\frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} . \] (3.28c)

To estimate a planetary magnetic field to a reasonable accuracy, in the main it is necessary only to extend this expansion to third order, i.e. that of an octupole, which is described
Table 3.1. Schmidt normalised spherical harmonic coefficients are shown here in Gauss referenced to System III (1965) right-handed coordinates. ‘UR’ highlights unresolved parameters. The VIP4 20eV model values are taken from Connerney et al. (1998), Voyager 1 17eV model coefficients are from Connerney et al. (1982). The Goddard Space Flight Centre(GSFC) O6 model coefficients are reproduced from Connerney et al. (1998) and the Pioneer 11(3,2)A model data are as tabulated by Acuña et al.(1983).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Schmidt Coefficient</th>
<th>VIP4(20eV)</th>
<th>Voyager 1 (17eV)</th>
<th>GSFC O6</th>
<th>Pioneer 11(3,2)A</th>
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<tbody>
<tr>
<td>1</td>
<td>$g_1^0$</td>
<td>4.205</td>
<td>4.208</td>
<td>4.242</td>
<td>4.144</td>
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<tr>
<td>2</td>
<td>$g_1^1$</td>
<td>-0.659</td>
<td>-0.660</td>
<td>-0.659</td>
<td>-0.692</td>
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<tr>
<td>3</td>
<td>$h_1^1$</td>
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<td>0.261</td>
<td>0.241</td>
<td>0.235</td>
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<tr>
<td>4</td>
<td>$g_2^0$</td>
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<td>-0.034</td>
<td>-0.022</td>
<td>0.036</td>
</tr>
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<td>5</td>
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<td>-0.759</td>
<td>-0.711</td>
<td>-0.581</td>
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<tr>
<td>6</td>
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<td>0.483</td>
<td>0.487</td>
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<td>7</td>
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<td>-0.403</td>
<td>-0.427</td>
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<tr>
<td>8</td>
<td>$h_2^2$</td>
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<td>0.107</td>
<td>0.072</td>
<td>0.134</td>
</tr>
<tr>
<td>9</td>
<td>$g_3^0$</td>
<td>-0.016</td>
<td>-0.021(UR)</td>
<td>0.075</td>
<td>-0.047</td>
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<td>10</td>
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<tr>
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<td>0.089(UR)</td>
<td>-0.388</td>
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<tr>
<td>14</td>
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<tr>
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<tr>
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<tr>
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<td></td>
</tr>
<tr>
<td>21</td>
<td>$h_4^1$</td>
<td>0.076(UR)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>$h_4^2$</td>
<td>0.404(.93)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>$h_4^3$</td>
<td>0.166(.89)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>$h_4^4$</td>
<td>0.039(UR)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
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here. As can be seen in Table 3.1 there now exist models in which the Schmidt coefficients have been calculated up to fourth order.

Dipole Terms

\[ B_r = \frac{2}{r^3} \left( g_1^0 \cos \theta + \sin \theta \left( g_1^1 \cos \phi + h_1^1 \sin \phi \right) \right), \quad (3.29a) \]

\[ B_\theta = \frac{1}{r^3} \left( g_1^0 \sin \theta - \cos \theta \left( g_1^1 \cos \phi + h_1^1 \sin \phi \right) \right), \quad (3.29b) \]

\[ B_\phi = \frac{1}{r^3} \left( g_1^1 \sin \phi - h_1^1 \cos \phi \right). \quad (3.29c) \]

Quadrupole Terms

\[ B_r = \frac{3}{r^4} \left[ \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) g_2^0 + \sqrt{3} \sin \theta \cos \theta \left( g_2^1 \cos \phi + h_2^1 \sin \phi \right) \right] \]

\[ + \left( \frac{\sqrt{3}}{2} \sin^2 \theta \right) \left( g_2^1 \cos 2\phi + h_2^1 \sin 2\phi \right) \]

\[ B_\theta = -\frac{1}{r^4} \left( \left[ -3 \sin \theta \cos \theta \right] g_2^0 + \sqrt{3} \left( \cos^2 \theta - \sin^2 \theta \right) \left[ g_2^1 \cos \phi + h_2^1 \sin \phi \right] \right) \]

\[ + \left( \frac{\sqrt{3}}{2} \sin^2 \theta \right) \left[ g_2^1 \sin 2\phi - h_2^1 \cos 2\phi \right] \]

\[ B_\phi = \frac{1}{r^4} \left( \sqrt{3} \cos \theta \left[ g_2^1 \sin \phi - h_2^1 \cos \phi \right] + \left( \frac{\sqrt{3}}{2} \sin \theta \right) \left[ g_2^1 \sin 2\phi - h_2^1 \cos 2\phi \right] \right). \quad (3.30c) \]

Octupole Terms

\[ B_r = \frac{4}{r^5} \left[ \left( \frac{5}{2} \cos^2 \theta - \frac{3}{2} \right) g_3^0 + \frac{\sqrt{5}}{\sqrt{8}} \sin \theta \cos \theta \left[ 5 \cos^2 \theta - 1 \right] \left[ g_3^1 \cos \phi + h_3^1 \sin \phi \right] \right] \]

\[ + \left( \frac{\sqrt{15}}{2} \cos \theta \sin^2 \theta \right) \left( g_3^1 \cos 2\phi + h_3^1 \sin 2\phi \right) + \left( \frac{\sqrt{5}}{\sqrt{8}} \sin^3 \theta \right) \left[ g_3^1 \cos 3\phi + h_3^1 \sin 3\phi \right] \]

\[ (3.31a) \]
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\[ B_\theta = \frac{1}{r^3} \left[ \begin{array}{c}
\left\{ \sin\theta \left( -\frac{15}{2} \cos^2 \theta + \frac{3}{2} \right) \right\} g_0^0 + \left\{ \frac{\sqrt{3} \cos\theta}{\sqrt{8}} \left( 5 \cos^3 \theta - 10 \sin^2 \theta \right) \right\} \left\{ g_1^0 \cos\phi + h_1^0 \sin\phi \right\}
+ \left\{ \frac{\sqrt{15} \sin\theta \left( \cos^2 \theta - \frac{\sin^2 \theta}{2} \right) }{\sqrt{8}} \right\} \left\{ g_3^0 \cos 2\phi + h_3^0 \sin 2\phi \right\}
+ \left\{ \frac{3 \sqrt{5} \sin^2 \theta \cos \theta }{\sqrt{8}} \right\} \left\{ g_3^0 \cos 3\phi + h_3^0 \sin 3\phi \right\}
\end{array} \right] \] (3.31b)

\[ B_\phi = \frac{1}{r^3} \left[ \begin{array}{c}
\left\{ \frac{\sqrt{3} \left( 5 \cos^3 \theta - 1 \right) }{\sqrt{8}} \right\} \left\{ g_1^0 \sin\phi - h_1^0 \cos\phi \right\} + \left\{ \frac{\sqrt{15} \sin\theta \cos\theta }{2} \right\} \left\{ g_3^0 \sin 2\phi - h_3^0 \cos 2\phi \right\}
+ \left\{ \frac{3 \sqrt{5} \sin^2 \theta }{\sqrt{8}} \right\} \left\{ g_3^0 \sin 3\phi - h_3^0 \cos 3\phi \right\}
\end{array} \right] \] (3.31c)

These equations may now be implemented as an internal magnetic field model, with the only unknown parameters being the Schmidt normalised spherical harmonic coefficients. For these we turn to the most recent of the forever improving magnetic field models for the particular planet that we are dealing with. In the case of Jupiter there are four that are in use for comparative purposes, with the VIP 4 (20eV) being the most up to date and thus the most accurate (to highest order) of the models (see Table 3.1).

The spherical polar coordinate system in use for this problem, that is System III (1965) coordinates (see Appendix 1), is a conventional right-handed, spherical system in which the z-axis is oriented along the rotation axis of the planet and the origin is located at the centre of the planet's core. Longitude is thus measured positive eastwards and the angle theta is co-latitude. The Schmidt coefficients for the dipolar part of the spherical harmonic expansion may be understood as follows. The potential term associated with \( g_{10} \) is the same as the potential of a dipole oriented with its magnetic moment along the z-axis. Similary it can be seen that the \( g_{11} \) and \( h_{11} \) terms are equivalent to dipoles with their magnetic moments aligned along the x- and y-axis respectively. It therefore follows that the vector addition of three concentric dipoles, creates another dipole moment, \( M \). Fig. 3.1 shows the orientation of the magnetic axis, \( M \) and thus it becomes clear that the model used dictates the degree of elevation and azimuthal tilt of the magnetic axis. These can be calculated as follows.
Figure 3.1. Plot of the direction of the three dipole components $g_{10}$, $g_{11}$, and $h_{11}$ with respect to System III (1965) coordinates.
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\[ |\mathbf{M}| = \sqrt{\left( g_1^0 \right)^2 + \left( g_1^1 \right)^2 + \left( h_1^1 \right)^2}, \]  \hspace{1cm} (3.32a)

\[ \phi = \tan^{-1}\left( \frac{h_1^1}{g_1^1} \right), \]  \hspace{1cm} (3.32b)

and

\[ \theta = \cos^{-1}\left( \frac{g_1^0}{\sqrt{g_1^{02} + g_1^{12} + h_1^{12}}} \right). \]  \hspace{1cm} (3.32c)

We also show in Fig. 3.2a-c contour plots of the surface field intensity in System III (1965) coordinates, as determined by the VIP4 internal field model (Connerney et al., 1998). In Fig. 3.2a only the dipole terms are shown, and the effect of the tilt and rotation of the magnetic dipole moment is evident in the three field components. In Fig. 3.2b we include the effect of the quadrupole terms, and finally in Fig. 3.2c we add the octupole terms. By adding the terms separately, we can clearly see the effect on the overall surface field intensity according to the VIP4 model due to the different higher-order terms. In Fig. 3.2c, the total field is seen to be significantly stronger in the northern hemisphere than in the south.

3.3 External field modelling

3.3.1 The Connerney et al. (1981) current sheet model

Having modelled the internal magnetic field, the following section aims to briefly describe the vector potential method employed by Connerney et al. (1981) to model the field due to the external azimuthal current sheet.

Although the Connerney, Acuña and Ness current sheet field model (Connerney et al., 1981) was formulated some twenty years ago in response to the measurements collected by the Pioneer and Voyager fly-bys of the early to late 1970s, it is still widely used today in various different studies of the jovian middle magnetosphere (for example Cowley et al., 1996; Dougherty et al., 1996; Satoh et al., 1996; Maurice et al., 1997; Laxton et al., 1997, 1999; Clarke et al., 1998; Connerney et al., 1998; Prangé et al., 1998;
Figure 3.2a. Contour plot of the surface field intensity as determined by the GSFC VIP4 internal field model. In this figure, the only contributing moment to the field is that of the dipole. The top three panels indicate the individual field components in System III (1965) right-handed coordinates, i.e. the radial component $B_r$, the north-south component $B_q$, and the azimuthal component measured positive eastwards. At the foot of the figure is the total magnetic field $|B|$. All components are given in units of Gauss (where 1 Gauss = $10^{-4}$ T).
Figure 3.2b. Contour plot of the surface field intensity as modelled in the GSFC VIP4 internal field model. The figure takes on an identical format to that of Fig.3.2a. Now contributions from the dipole and quadrupole moments of the field are shown.
Figure 3.2c. Contour plot of the surface field intensity as modelled in the GSFC VIP4 internal field model. The figure takes on an identical format to that of Fig.3.2a. Now contributions from the dipole, quadrupole and octupole moments of the field are shown.
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Dougherty et al., 1998; Vasavada et al., 1999). In Chapters 4 and 6 the model is employed to estimate the latitude dependence of the radial field in the middle magnetosphere, and similarly in Chapter 5 to gain knowledge of the variation of the magnetic flux function along the equator (for details see Chapter 5). Thus we briefly review the model here.

In the Connerney et al. (1981) model the equatorial current sheet is taken to be a semi-infinite annular disc of constant half-thickness, in which the azimuthal current varies inversely with the distance from the axis of symmetry. The field of a finite current sheet, extending between two fixed radii, can then be found by adding the field of a second similar semi-infinite sheet in which the current is reversed in sense, which extends to infinity beyond the larger distance. The plane of the current sheet is taken to be either parallel to the magnetic equatorial plane, as in the original formulation, or may be found from fits to spacecraft data, as in the study by Connerney et al. (1982). Although this model current system is highly simplified, it provides an excellent framework within which to understand magnetic effects and magnetic mapping issues in the jovian magnetosphere.

In order to obtain the solution for the vector potential of a current sheet of the above form we begin by considering, in cylindrical co-ordinates \((\rho, \varphi, z)\), the vector potential of an axisymmetric field \(\mathbf{B}(\rho, z)\) which is obtainable from the vector potential \(\mathbf{A}(\rho, z)\), where the field components, given by \(\mathbf{B} = \text{curl}\mathbf{A}\), are

\[
B_\rho = -\frac{\partial A}{\partial z} \quad \text{and} \quad B_\varphi = \frac{1}{\rho} \frac{\partial (\rho A)}{\partial \rho}.
\]  

As indicated above, Connerney et al. (1981) solved Ampère's law for the vector potential of a semi-infinite axisymmetric current sheet, lying between \(\rho = a\) and infinity, of constant half-thickness \(D\) (centred at \(z = 0\)), in which the current density is given by \(j(\rho) = \left(\frac{I_0}{\rho}\right)\hat{\varphi}\), and thus falls inversely with the distance from axis. Integration of this equation through the height of the current sheet \(z\) then gives the value of the current intensity within the current sheet \((I_0\) is given in amps per meter of height in the \(z\)-direction).

Following Connerney et al. (1981), according to Maxwell's equations, the curl of the magnetic field is always zero in a current free region, and thus the following equation arises outside the current sheet.
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\[
-\frac{\partial^2 A}{\partial z^2} + \frac{A}{\rho^2} \frac{1}{\rho} \frac{\partial A}{\partial \rho} - \frac{\partial^2 A}{\partial \rho^2} = 0 ,
\]

(3.34)

which is equivalent to Laplace’s equation in cylindrical coordinates. Now we look for separable solutions, and find those which are physically applicable to be of the form

\[
A^\pm = J_1(\lambda \rho)e^{\pm iz}.
\]

(3.35)

The upper sign is chosen for \(z > 0\) and the lower sign for \(z < 0\), \(J_1(\lambda \rho)\) is the solution of Bessel’s equation resulting from the separation of Eq. (3.34), which remains finite at the origin. The general solution is hence

\[
A^\pm = \int C(\lambda) J_1(\lambda \rho)e^{\pm iz} ,
\]

(3.36)

where the function \(C(\lambda)\) is to be determined such that the boundary conditions are satisfied. To obtain solutions of the appropriate symmetry, we take \(C(\lambda)\) to be the same function both above and below the sheet. Due to the reversal of \(B_\rho\) across the sheet and by application of Ampère’s law, the following boundary condition arises,

\[
\frac{\partial A^+}{\partial z} \bigg|_{z=0} = \frac{\partial A^-}{\partial z} \bigg|_{z=0} - \mu_0 I(\rho) .
\]

(3.37)

where \(I(\rho)\) is the surface current density in the \(z = 0\) plane. By using Eq. 3.36, 3.37 and the generalisation of Neumann’s integral by Hankel (Watson, 1944),

\[
H(r) = \int ud\mu \int H(R) J_\nu(uR)J_\nu(wR)RdR ,
\]

(3.38)

for general order, \(\nu\). Using this relation the function \(C(\lambda)\) is found to be

\[
C(\lambda) = \frac{H_0}{2} \int \lambda J_1(\lambda \rho) I(\rho) \rho d\rho .
\]

(3.39)

We make a specific choice of \(I(\rho)\) such that it is then possible to directly integrate Eq. 3.39. As indicated above, one such simple function is

\[
I(\rho) = \begin{cases} I_0 & \rho > a , \\ 0 & \rho < a , \end{cases}
\]

(3.40)

where \(I_0\) is in Amps. From the basic properties of Bessel functions we have for any Bessel function \(C_\nu(z)\) the indefinite integral,

\[
\int z^{-\nu-1} C_\nu(z)dz = -z^{-\nu+1} C_{\nu+1}(z) .
\]

(3.41)

We obtain an expression for \(C(\lambda)\) and finally the solution for this current distribution,

\[
A^z(\rho, z) = \frac{\mu_0 I_0}{2} \int J_1(\lambda \rho) J_0(\lambda a)e^{\pm iz} \frac{d\lambda}{\lambda} ,
\]

(3.42)
along with the corresponding expressions for $B_{\rho z}$. This corresponds to a current sheet of zero thickness centred at $z = 0$. To obtain expressions for a sheet of finite thickness $\pm D$ about $z = 0$, we consider the current per height, $z$ outside the current sheet. Integrating over $z$ gives an expression for the vector potential with current intensity $I_0/\rho$,

$$A^z = \mu_0 I_0 \int_0^\infty J_1(\lambda \rho) J_0(\lambda a) \sinh(\lambda D) e^{\pm \lambda z} \frac{d\lambda}{\lambda^2}, \quad (3.43)$$

and by implication $B_{\rho}$ and $B_z$. Finally for $|z| < D$, the potential inside the current sheet is,

$$A' = \mu_0 I_0 \int J_1(\lambda \rho) J_0(\lambda a) \left[ 1 - e^{-2D} \cosh(\lambda z) \right] \frac{d\lambda}{\lambda^2}, \quad (3.44)$$

again with $B_{\rho}^i$ and $B_z^i$ implied.

For the purpose of this thesis work, we require values of the vector potential $A$ between 20-50 $R_J$ for two main reasons. First we are able to derive the magnetic field components by using Eq. (3.33a) and (b). We then use the model radial field component $B_{\rho}$ to estimate the latitude dependence of the measured radial field outside of the current sheet in the middle magnetosphere. We also employ the Connerney et al. (1981) model within the range of its validity (out to $\sim 30 R_J$) to map the value of the field from the magnetic equator to the surface of the planet. Beyond the range of validity we simply employ empirical models of $B_z$ from which the magnetic vector potential may be directly inferred (from Eq. 3.33b). In order to do this we first define the flux function, which is given by $F = \rho A$, such that a field line is given by $F = \text{constant}$, and the magnetic flux, $d\Phi$, per radian of azimuth between the field lines $F$ and $F + dF$ is $d\Phi = dF$. It may be simply shown that $F = \text{constant}$ along a field line by considering the following

$$\mathbf{B}.\nabla F = 0, \quad (3.45)$$

This equation is valid of the gradient of $F$ is normal to the plane of constant magnetic flux function $F$, and thus implying that the magnetic field $\mathbf{B}$ must also lie solely in this plane. This equation may be verified simply as shown below

$$\mathbf{B}.\nabla F = B_{\rho} \frac{\partial F}{\partial \rho} + B_z \frac{\partial F}{\partial z}, \quad (3.46)$$

where from eq. (3.33a) and (b) the field components are given by

$$B_{\rho} = \frac{1}{\rho} \frac{\partial F}{\partial \rho} \quad \text{and} \quad B_z = \frac{1}{\rho} \frac{\partial F}{\partial \rho}, \quad (3.47)$$

and therefore
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\[- \frac{1}{\rho} \frac{\partial F}{\partial z} \frac{\partial F}{\partial \rho} + \frac{1}{\rho} \frac{\partial F}{\partial \rho} \frac{\partial F}{\partial z} = 0. \]  \hspace{1cm} (3.48)

In a recent paper, Edwards et al. (2001) have calculated approximate forms for the vector potential of such a current sheet as described above, from which we obtain the following value for the flux function in the equatorial plane \((z = 0)\) at some position \(\rho_0\) lying between \(R_0\) and \(R_1\)

\[ F_{\text{approx}}(\rho_0) = \frac{\mu_0 I_0}{2} \left\{ D \sqrt{\frac{\rho_0^2 + D^2 + \rho_0^2}{2}} \log \left[ \frac{\rho_0^2 + D^2 + D}{\rho_0^2 + D^2 - D} \right] - \frac{DR_0^2}{2\rho_0^2 + D^2} - D^2 \right\} \]

\[ - \frac{\rho_0^2}{2} \log \left[ \sqrt{\frac{R_0^2 + D^2 + D}{R_0^2 + D^2 - D}} \right] - \frac{\rho_0^4}{8} \left[ \frac{D}{(R_0^2 + D^2)^{3/2}} \right]. \]  \hspace{1cm} (3.49)

The values of the spatial model parameters employed by Connerney et al. (1981), and used here, are \(D = 2.5\) \(R_J\), \(R_0 = 5\) \(R_J\), and \(R_1 = 50\) \(R_J\). Similarly, the current parameter is taken to be \((\mu_0 I_0/2) = 225\) nT for the Voyager-1 and Pioneer-10 passes, and \((\mu_0 I_0/2) = 150\) nT for Voyager-2 and Pioneer-11. Edwards et al. (2001) also calculated the error involved in the analytic approximations, relative to the value derived from numerical integration of the exact integrals. For the above parameters and \(\rho_0 = 20\) \(R_J\), the fractional error of the value is found to be less than 0.1%. Thus the analytic form given by Eq. (3.49) represents an excellent approximation to the true value of the flux function for the Connerney et al. (1981) model of the current sheet, which itself was determined from fits to the observed field during the Pioneer-10, and Voyager-1 and -2 passes, within radial distances of \(\sim 30\) \(R_J\). The model should therefore provide a good measure of the flux function at \(\rho_0 = 20\) \(R_J\), for which purpose we will employ it here.

Finally, in Fig. 3.3, we show Connerney et al. (1981) model field lines calculated from the approximate analytic forms as presented by Edwards et al. (2001) given by contours of constant \(F(\rho, z) = \rho A(\rho, z)\) combined with the planetary dipole field, for the axisymmetric case in which the current sheet axis and dipole axis are co-aligned. The parameters of the current sheet are the same as above (i.e. \(D = 2.5\) \(R_J\), \(R_0 = 5\) \(R_J\), and \(R_1 = 50\) \(R_J\)), while the ratio of the current sheet current parameter to the planetary equatorial surface field \(B_J\) has been taken to be \((\mu_0 I_0/2B_J) = (225/420,000)\). This
Figure 3.3. Plots of model field lines, given by contours of constant $F = \rho A$. Distances are normalised in all cases to $R_j$, and we show the domain $0 \leq \rho' \leq 30$ and $-15 \leq z' \leq 15$. Field lines are shown by solid lines, while the dashed lines indicate the boundary of the annular current sheet. The plot shows the field lines of a co-aligned dipole plus current sheet field for parameters $R_0 = 5 \ R_j$, $R_1 = 50 \ R_j$, and $D = 2.5 \ R_j$, corresponding to the "Voyager-I/Pioneer-10" model of Connerney et al. (1981). The ratio of the current sheet current parameter to the planetary equatorial surface field strength $B_j$ has been taken to be $(\mu_0 I_0/2B_j) = (225/420,000)$. Field lines are shown which originate from the planet's polar region every $2^\circ$ of co-latitude, from $0^\circ$ (the $\rho = 0$ axis) to $30^\circ$. 
parameter set corresponds to Connerney et al.’s (1981) “Voyager-1/Pioneer-10” model. Field lines are shown which emerge from the planet’s polar region from 0° to 30° colatitude, at steps of 2°.

3.4 Models of equatorial $B_z$ and the calculation of the equatorial flux function

3.4.1 Models of equatorial $B_z$

For the purposes of mapping the field from the middle magnetosphere to the planet, as will be discussed in Chapter 5, we are required to calculate the flux function in the equatorial plane. In order to obtain this parameter we first discuss simple models of equatorial $B_z$ which are required for the calculation (as seen in Eq. 3.33b). Empirical models of the variation of the total north-south equatorial magnetic field $B_z$ with jovianicentric distance $\rho$ are required in this thesis, and in particular in Chapter 5, for two purposes. The first is to calculate the field-aligned current parameter $(j_i/B)$ from Eq. (5.12), in Chapter 5. The second is to calculate the value of the equatorial flux function $F$ using Eq. (5.35) of Chapter 5, and which is reproduced as Eq. (3.52) in Section 3.4.2 where it will be discussed further. In Chapter 5 we employ empirical model values, valid over the range of interest (20-50 $R_J$), which have been derived for the flyby passes under consideration in previous studies. Specifically, for the outbound Voyager passes we employ the empirical models derived by Khurana and Kivelson (1993). These authors fit a power law expression to total $B_z$ in the equatorial plane of the form

$$ B_z(\rho) = -A\rho^m, \quad (3.50) $$

and found $A = 5.4 \times 10^4$ nT and $m = -2.71$ for the Voyager-1 data, and $A = 4.3 \times 10^4$ nT and $m = -2.56$ for the Voyager-2 data. Similarly, for the outbound Pioneer-10 pass we employ the model derived by Goertz et al. (1976). These authors considered $B_z$ values from which the dipole planetary field had been subtracted, and modelled the remainder as a power law. In effect, the $B_z$ field is taken to be

$$ B_z = B_{z,\text{deg}} + B_z' = -B_j \left( \frac{R_j}{\rho} \right)^3 + A\rho^m, \quad (3.51) $$

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Figure 3.4. Modulus of the various empirical $B_z$ (nT) models employed in this study, plotted versus $\rho$ in a log-log format. The solid line represents a simple dipole, whilst the long-dashed line indicates the Goertz et al. (1976) empirical model fitted to Pioneer-10 data, the short-dashed line the Khurana and Kivelson (1993) Voyager-1 model, and the dot-dash line the Khurana and Kivelson (1993) Voyager-2 model.
where \( A = 6.3 \times 10^4 \) nT and \( m = -2.7 \). The modulus of these fields (since total \( B_z \) is negative) is plotted versus \( \rho \) in log-log format in Fig. 3.4, together with the dipole field (upper solid line) for purposes of comparison. It can be seen that all models give values which are relatively close to each other, thus lending weight, in particular, to the inequality in Eq. (5.4) which leads to Eq. (5.6) as will be discussed further in Chapter 5. No models exist which have been fitted to inbound Pioneer-11 data, but because the values on each pass here are similar, the choice does not seem critical to within factors of \( \sim 2 \). We choose, somewhat arbitrarily, to employ the Pioneer-10 model in this case as well, on the basis that it is the closest to inbound Pioneer-11 in local time.

3.4.2. Calculation of the equatorial flux function

As indicated in Section 5.5 of Chapter 5, the equatorial flux function, \( F_e(\rho) \), for the four spacecraft passes considered here is calculated between 20 and 50 \( R_J \) from Eq. (5.35), reproduced here as Eq. (3.52)

\[
F_e(\rho) = F_e(\rho_0) + \int_{\rho_0}^\rho d\rho \rho B_z.
\]

(3.52)

Here, \( \rho_0 = 20 \) \( R_J \) is the inner boundary of the region of interest. We therefore require to determine the value of the equatorial flux function at this inner boundary, \( F_e(\rho_0) \), and we also require to model the equatorial \( B_z \) field so that the integral in Eq. (3.52) can be performed to find \( F_e(\rho) \) in the region beyond. Here we deal with each of these items in turn.

The value of the equatorial flux function at the inner boundary of the region is determined as the sum of the dipole internal field term given by Eq. (5.12), and a current sheet term obtained from the model of Connerney et al. (1981), i.e.

\[
F_e(\rho_0) = \frac{B_z R_J^3}{\rho_0} + F_{e,\text{CAS}}(\rho_0).
\]

(3.53)

Here we employ Eq. (3.53) in order to compute the second term, as outlined above and as published by Edwards et al. (2001).

To evaluate the integral in Eq. (3.52) we now need to model \( B_z(\rho) \) in the equatorial plane. For the Voyager-1 and -2 passes we employ the empirical fits to the outbound data
obtained by Khurana and Kivelson (1993) as given above in Eq. (3.50). Integrating this expression, we find

$$F_e (\rho) = \frac{B_s R_j^3}{\rho_0} + F_{e_{c10}} (\rho_o) - \frac{A}{m+2} \left[ \rho^{m+2} - \rho_o^{m+2} \right]. \quad (3.54)$$

With the appropriate parameters given above, this is the expression which has been used to derive the $F_e$ curves for the Voyager passes shown in Fig. 5.11. The Pioneer passes are treated a little differently, because the empirical model of Goertz et al. (1976), fit to the Pioneer-10 data, but applied here also to the Pioneer-11 data, considers $B_z$ values from which the dipole planetary field has already been subtracted, as indicated above in Eq. (3.51). Substituting this expression into Eq. (3.52) and integrating, then yields

$$F_e (\rho) = \frac{B_s R_j^3}{\rho} + F_{e_{c10}} (\rho_o) + \frac{A}{m+2} \left[ \rho^{m+2} - \rho_o^{m+2} \right]. \quad (3.55)$$

With the appropriate parameters given above, this is the expression which has been used to derive the $F_e$ curves for the Pioneer passes shown in Fig. 5.11.

### 3.5 Inter-calibration of spacecraft magnetic field measurements

In order to obtain useful data it is essential that the spacecraft noise levels and the noise level of the magnetometers themselves be low enough not to mask the ambient signals. A linear response is also required so that measurements of fluctuating magnetic fields can be made accurately, independent of the strength of the average field. For the Galileo FGM (flux-gate magnetometer) the sensitivity in the dynamic range of interest (i.e. ±512 nT) is 0.03 nT (Kivelson et al., 1992). The Galileo magnetometer was additionally calibrated before flight to within 1 part in $10^4$. Similarly, for the Ulysses magnetic field investigation, the magnetic field values are accurate to within 0.01 nT, and are calibrated prior to launch such that it was thought to be the magnetically cleanest interplanetary spacecraft ever flown (Balogh et al., 1991). The Voyager spacecraft are considered accurate down to the 0.2 nT level for the low field measurements (that is 10s of nT average field strength; Behannon et al., 1977) and similarly the Pioneer spacecraft has an error in the magnetic field of 0.1 nT in a 1 nT field (Smith et al., 1975b). The level of pre-flight calibration for both the Voyager and Pioneer spacecraft is somewhat lower than for Ulysses and Galileo, but the data now available for these missions is of good quality and has undergone significant review processes. It is also felt that the inclusion of any small
constant offset in any of the data sets would not alter the results of this thesis, as we are primarily interested in the differential quantities associated with the divergence of large-scale current systems.
4.1. Introduction

This chapter provides a first systematic comparison of the radial fields associated with the equatorial current sheet in the jovian magnetosphere which were observed during the flybys of the Pioneer–10 and –11, Voyager–1 and –2, and Ulysses spacecraft. These data span a ~210° range of azimuths about the planet, from dusk via noon to the post-midnight sector. The local time coverage of these flybys is indicated in Fig. 4.1, where we have projected the spacecraft trajectories onto Jupiter's orbital plane. The Pioneer and Voyager flybys covered the dawn-sector magnetosphere from near noon (Pioneer–11 outbound) to post-midnight (Voyager–2 outbound), while Ulysses observed the pre-noon sector inbound and the dusk magnetosphere outbound. The jovigraphic latitudes of these passes were near-equatorial in all cases, except for the outbound passes of Pioneer–11 and Ulysses, which exited near noon at ~33°N and near dusk at ~37°S, respectively. Also plotted in the figure are the average position of the bow shock and magnetopause, adapted from Huddleston et al. (1998).

Most theoretical models derived to date have assumed that the current sheet is approximately axisymmetric, though often they have been applied only in a piecemeal way
Figure 4.1. Trajectories of the five fly-by spacecraft relative to Jupiter, projected onto Jupiter's orbital plane. X points towards the Sun and Y from dawn to dusk. P 10 and P 11 refer to Pioneer-10 and -11, V 1 and V 2 to Voyager-1 and -2, and U to Ulysses. Arrows are plotted in the direction of spacecraft motion on the outbound portions of the trajectories. Also plotted are the average positions of the bow shock and magnetopause (adapted from Huddleston et al., 1998).
to field data from restricted local time sectors. Barish and Smith (1975) used an Euler potential formulation to model the field observed on the pre-noon inbound pass of Pioneer–10 (Fig. 4.1), and found reasonable agreement with a field falling as \( \sim r^{-2} \) beyond \( \sim 20 \text{ R}_J \). Goertz et al. (1976) and Jones et al. (1980) similarly modelled the outbound Pioneer–10 data near the dawn meridian, and obtained a slightly less steep radial field gradient associated with the current sheet of \( \sim r^{-1.7} \). Behannon et al. (1981) considered one-hour averages of the total field strength observed on the nightside outbound passes of Pioneer–10, and Voyagers–1 and –2 over distance ranges of \( \sim 20 \) to \( \sim 150 \text{ R}_J \), and fitted power law variations to the maximum such average in each 10-h planetary rotation interval. They found a continuing trend of reducing radial field gradients with decreasing local time towards midnight, with the field varying as \( \sim r^{-1.7} \) at Pioneer–10, \( \sim r^{-1.5} \) at Voyager–1, and \( r^{-1.4} \) at Voyager–2 (see Fig. 4.1). We note, however, that these fits did not account for the different magnetic latitudes reached on these passes, which will effect the maximum field observed during each planetary rotation cycle. Khurana (1997) provided a detailed fit to these outbound passes using an Euler potential formulation incorporating a hinged model of the current sheet location. Connerney et al. (1981), on the other hand, modelled the current sheet directly as an azimuthally symmetric distribution of finite thickness (taken as 5 \( \text{R}_J \)), extending from jovicentric distances of 5 \( \text{R}_J \) to \( \sim 50 \text{ R}_J \), within which the current density falls as \( r^{-1} \). The perturbation fields were then obtained by integration, and used to fit both inbound and outbound fields observed by Pioneer–10 and Voyagers–1 and –2 in the inner part of the system, within \( \sim 30 \text{ R}_J \). It was found that the magnitude of the current required to fit the Voyager–2 observations is somewhat smaller than that required to fit Pioneer–10 and Voyager–1. In the latter cases, however, the model then over-estimates the radial field observed on the dayside inbound passes, which is weaker at a given radius than the radial field on the nightside outbound passes. Connerney et al. (1981) suggested that this effect might result from the presence of a thicker current sheet on the dayside compared with the nightside, such that the spacecraft did not fully exit the current sheet north-south in the former case. This explanation may be plausible at distances inside \( \sim 15 \text{ R}_J \), where the amplitude of the periodic north-south motions of the current sheet are smaller than its thickness, such that a near-equatorial spacecraft can remain immersed within it at all phases of the planetary spin period. However, it cannot explain the asymmetry at larger distances, beyond \( \sim 15 \text{ R}_J \), because the amplitude of the current sheet motion is then larger than its thickness, such that spacecraft exit from the current sheet is guaranteed during some part of the rotation cycle. We note that Jones et al.
(1981) also concluded from an examination of Pioneer–10 and –11 data that the dayside current sheet field is weaker than that at dawn.

In summarising the results of these studies we may conclude that while it is often assumed in modelling that the equatorial current sheet is approximately azimuthally symmetric, some evidence exists that the current sheet field may generally be weaker and fall more rapidly with distance on the dayside than on the nightside. In this chapter we provide a first systematic comparison of the radial field variations observed on the Pioneer and Voyager flybys, and a first cross-comparison with related results from Ulysses. Here we show that a local time asymmetry is indeed present at distances beyond ~20 Rj, with steeper gradients and weaker fields on the dayside than on the nightside. This effect is significantly larger than, and is not masked by, secular changes in the current sheet strength associated e.g. with variations in the Io gas production rate.

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4.2 Data analysis

4.2.1 Current sheet field averages

The starting-point for our study is the magnetic field vectors observed during the five jovian flybys discussed above. These were supplied by the Planetary Data System at UCLA at 10 s resolution for Pioneer–11 and Voyager–2, 48 s for Voyager–1, and 1 min for Pioneer–10 and Ulysses. Our first step was then to form 30 min averages of these fields for intervals when the spacecraft were outside the equatorial current sheet. Examples of these data and our selection procedures are illustrated in Fig. 4.2. In Fig. 4.2a we show one (Earth) day of data from the inbound pass of Pioneer–11 in the pre-noon sector, corresponding to day 335 of 1974, when the spacecraft was located at jovicentric distances of 43.4–27.4 Rj. The magnetic local time (h:min) of the spacecraft and the jovicentric radial distance are indicated at the foot of the figure. The lower panels show the three field components in cylindrical jovimagnetic coordinates, where $B_\rho$ is the radial component.
Figure 4.2a. Example of the high-resolution (10 s) magnetic field data which form the basis of this study. One (Earth) day of data is shown for Pioneer-11 inbound on day 335 of 1974. The top panel shows the distance of the spacecraft, \( z (R_J) \), from the magnetic equatorial plane. Included in this panel are two dotted lines, showing the nominal boundaries of the current sheet ± 2.5 \( R_J \) about the equator. The following three panels show the magnetic field data in cylindrical jovimagnetic coordinates, where \( B_p \) is the radial component perpendicular to the magnetic axis, \( B_\phi \) is the azimuthal component measured positive eastward, and \( B_z \) is along the magnetic axis positive northwards. The measured data are shown as small dots, while larger dots represent the contribution of the internal planetary field derived from the VIP 4 model (Connerney et al., 1998). This model field is shown only for the \( p \) and \( z \) components of the field, since the azimuthal contribution of the internal field is essentially zero in these co-ordinates at these distances. The solid bars shown in the \( B_p \) panel indicate those half-hour intervals in which the spacecraft is judged to have resided outside the current sheet. The field averages obtained during these intervals are those used in subsequent analysis. At the foot of the figure the universal time UT, the magnetic local time of the spacecraft MLT (h:min), and its jovimetric radial distance are shown.
Figure 4.2b. Second example of high-resolution (10 s) magnetic field data which form the basis of this study. Here, one (Earth) day of data is shown for Voyager-2 outbound on day 193 of 1979. The format of the figure is identical to that in Fig. 4.2a.
perpendicular to the magnetic axis, $B_e$ is the azimuthal component positive eastward, and $B_z$ is along the magnetic axis positive northwards. The individual 10 s field values are shown as dots, while the larger dots in the $\rho$ and $z$ component panels show the contribution due to the internal field of the planet, determined from the VIP 4 model (Connerney et al., 1998). The contribution of the internal field to the azimuthal component is essentially zero, and is not shown, because the dipole contribution is identically zero in these coordinates, while at these distances the higher multipole terms are negligibly small. In order to facilitate identification of those intervals when the spacecraft was outside the current sheet, we have also plotted in the top panel the distance $z(R_J)$ from the magnetic equatorial plane. The dotted lines in this panel show the nominal position of the current sheet, taken to lie in the range $\pm 2.5$ $R_J$ about the equator, as in the Connerney et al. (1981) model. It can be seen that Pioneer-11's trajectory lay south of the equator (at 11°S jovigraphic latitude), and approached and entered the nominal current sheet only once per jovian rotation. Correspondingly, the measured field is generally dominated by a negative radial component, corresponding to a location south of the current sheet, which exhibits depressed values and/or enhanced fluctuations indicative of hot plasma currents at ~10 h intervals when the spacecraft approached the magnetic equatorial plane. At other times, when the spacecraft was at larger distances from the equator, the fields are instead stronger and smoothly varying, indicating only weak local currents, and a consequent location outside of the current sheet. Ignoring periods when enhanced magnetic variations are present, therefore, we have averaged the field components over the half-hour intervals indicated by the solid bars in the $B_\rho$ panel, and take these values to represent conditions at the similarly averaged locations outside of the current sheet. Field averages have been taken during these intervals both with and without prior subtraction of the VIP 4 planetary field.

A second example is shown in Fig. 4.2b in the same format as Fig. 4.2a. In this case we show one day of data from the outbound pass of Voyager-2 in the post-midnight sector, corresponding to day 193 of 1979, when the spacecraft was located at jovicentric distances of 34.9–47.9 $R_J$. In this case the spacecraft trajectory was located much closer to the jovigraphic equatorial plane, such that the nominal current sheet passed completely across it twice per 10 h rotation period. Correspondingly, it can be seen that the radial field cycles between intervals of relatively steady positive and negative values, interspersed with periods of field fluctuation and reversal when the spacecraft crossed through the equatorial
current sheet. The 30 min averaging intervals during which the spacecraft was located continuously outside of the current sheet are relatively unambiguous, and are again indicated by the solid bars in the second panel.

4.2.2 Radial variation of the radial field component

In Fig. 4.3 we show representative plots of the 30-min averaged radial field component \( B_\rho \) outside of the current sheet, versus the perpendicular distance from the magnetic axis \( \rho \), in a log-log format. These values correspond to the total field without subtraction of the planetary field. We have chosen to display data derived from the following passes: (a) Pioneer-11 inbound, (b) Pioneer-10 outbound, (c) Ulysses outbound, and (d) Voyager-2 outbound. These passes thus typify observations in the pre-noon, dawn, dusk, and post-midnight sectors, respectively (see Fig. 4.1). Values obtained when the spacecraft was north of the current sheet, such that \( B_\rho \) was positive, are shown as crosses. Those obtained when the spacecraft was south of the current sheet, such that \( B_\rho \) was negative, have been reversed in sense (assuming anti-symmetry in \( B_\rho \) about the centre of the current sheet), and are shown as circles. As in the related study by Behannon et al. (1981), it can be seen that the data can reasonably be fit by a single power law variation, \( B_\rho = A(nT)\rho(R_J)^m \), in which the coefficient \( A \) and the exponent \( m \), determined by least squares, are given in each panel of the figure. These least-squares power law fits are shown by the dashed lines in the figure. It can immediately be seen that the field gradients are largest on the dayside, reduced in value at both dawn and dusk, and are smallest on the nightside, as suggested by the previous studies cited in the introduction. The consequence is that at a given radial distance in the outer part of the middle magnetosphere the radial field values are significantly reduced on the dayside compared with the nightside.

In this chapter we are, however, primarily interested in studying the fields produced by the current sheet itself, and hence the properties of the equatorial currents. As indicated above, we have consequently also derived 30-min averages of the field from which the VIP 4 planetary field has previously been subtracted. Such “current sheet” fields (which will in principle also contain small contributions from other external currents, e.g. those at the magnetopause) will be denoted throughout by a prime, \( B'_\rho \). In Fig. 4.4 we show the radial
Figure 4.3. Log-log plots of the 30-min averaged radial field component $B_p$ outside the current sheet, versus the perpendicular distance from the magnetic axis $\rho$, for (a) the Pioneer-11 inbound pass in the pre-noon sector; (b) the Pioneer-10 outbound pass along the dawn terminator; (c) the Ulysses outbound pass along the dusk terminator; and (d) the Voyager-2 outbound pass post-midnight. The plots show the total field component without subtraction of the internal planetary field. Averages taken north of the current sheet (positive values) are shown by crosses, those taken south of the current sheet (negative values) have been reversed in sense and are shown by circles. The straight dashed lines show least-squares power law fits of the form $B_p = A(nT)\rho(R_J)^{-m}$, where the values of the coefficient $A$ and the exponent $m$ are shown in each panel.
Figure 4.4. Log-log plots of the radial component of the "current sheet" field $B'_r$ (i.e. the total field with the VIP 4 planetary field subtracted), for the same passes as in Fig. 4.3, and in the same format. The dashed lines again show power law fits to these data.
components of these fields versus $\rho$ for the same passes as in Fig. 4.3, and in a similar format. The main effect is that the value of the field is reduced, particularly at smaller values of $\rho$ where the dipole term tends to dominate, such that the slope of the fitted lines is also significantly decreased. It is nevertheless evident that in several cases a single power law does not provide an adequate fit over the full range of $\rho$ values. A second effect observed in Fig. 4.4 is that groups of associated points tend to show local minima towards the centre of the group, rather than local maxima as in Fig. 4.3. As is clear from Fig. 4.2, the major groups of points are associated with individual excursions of the spacecraft above or below the current sheet during given rotations of the planet. The variations within each group are then associated with the latitude of the spacecraft, which reaches maximum values north or south towards the centre of each group. In the case of the total field shown in Fig. 4.3, the value of $B_\rho$ tends to increase with distance from the current sheet, particularly at smaller values of $\rho$, due to the presence of the dipole field. At a given value of $\rho$, the radial component of the dipole field is zero at the magnetic equatorial plane, increases up to a magnetic latitude of $\sim 27^\circ$ (where $|z| = \rho/2$), and then falls again at larger $|z|$. Spacecraft located near the equatorial plane, such as Pioneer-11 inbound and Pioneer-10 outbound, thus tend to show local maxima in total $B_\rho$ as they move to higher latitudes away from the current sheet centre, particularly at smaller values of $\rho$. However, when the planetary field is removed prior to averaging, the remaining “current sheet” field falls with latitude away from the equatorial plane, due simply to the increasing distance from the finite-size current sheet. In the next section we attempt to remove these latitudinal effects in the data by mapping the observed fields to the outer edge of the current sheet using approximate model mapping factors.

4.2.3 Latitude-corrected radial profiles

The benefits to be obtained by correcting the “current sheet” radial fields for latitude effects are two-fold. First, reducing the latitude-related “scatter” in profiles such as those shown in Fig. 4.4, reduces the uncertainty in least-squares fits to empirical field variations. Second, the data from all the passes can be reduced to a common basis independent of the spacecraft latitude, in particular allowing inclusion of the moderately non-equatorial outbound passes of Pioneer-11 and Ulysses (Figs. 4.3c and 4.4c). Our approach to this
task has been to map all the field measurements to the edge of the current sheet using approximate mapping factors determined from the Connerney et al. (1981) model. We note that the values of the exponent $m$ of the power law fit to the current sheet field in Fig. 4.4 are all reasonably close to the value of unity assumed in the latter model, such that the mapping should be valid to a reasonable approximation.

Empirical investigation of the properties of the Connerney et al. model shows that the value of $B_p$ varies only modestly with latitude outside of the current sheet at a fixed jovicentric radial distance. For example, in Fig. 4.5 we show the ratio of the field at a given jovicentric radial distance $r$ at the outer edge of the current sheet $z = D$, $B_p(r,z = D)$, divided by the field at the same radial distance but at latitude $\lambda$, $B_p(r,\lambda)$, plotted versus $\lambda$ at various fixed $r$. The model current sheet employed has an inner edge at $a = 5 R_J$ and a half-thickness of $D = 2.5 R_J$, both standard Connerney et al. values, and an outer edge at $R_i = 70 R_J$. These values thus represent the factors for this model by which the observed values have to be multiplied to map them to the current sheet edge. It can be seen that for near-equatorial spacecraft whose magnetic latitude varies over a range of ±10° during the planetary rotation cycle, the mapping factors differ from unity typically by only a few tens of percent, and are not strong functions of the radius. For Pioneer-11 and Ulysses outbound, at $\lambda = 33^\circ \pm 10^\circ$ and $37^\circ \pm 10^\circ$ respectively, the factor increases to ~2, again not strongly dependent on the radius. Here we have therefore used these factors to map the observed values of $B_p$ to the current sheet edge at fixed jovicentric radial distance. The latitude-corrected field values will be denoted by a zero subscript, as $B_p'$. The single Connerney et al. current sheet parameter which we have adjusted to fit each pass is the distance of the outer current sheet edge $R_i$, which has been varied to agree with the observed outer limit of the current sheet in each case. We have checked individually that the resulting model reproduces the observed field at the spacecraft position with reasonable accuracy, and have found that this is indeed the case. By reasonable accuracy, fits to within an RMS fractional error of 3-6% at small distances and no greater than ~10% at large distances are implied. As expected, the Connerney et al. model produces the best fits to those data which it was designed to replicate. The mapping factors are therefore unlikely to be substantially in error. However, the extent to which this procedure is successful may also be judged pragmatically from the degree to which the latitude-related "scatter" in the data is removed.

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Figure 4.5. Plot of the ratio of the current sheet radial field at a given radial distance $\rho$ at the outer edge of the current sheet $z=D$, i.e. $B'_\rho(r,z=D)$, divided by the field at the same radial distance but at latitude $\lambda$, i.e. $B'_\rho(r,\lambda)$, derived using the Connerney et al. (1981) field model. Values are shown versus $\lambda$ at fixed radial distances of 20, 30, 40 and 50 R$_J$. The parameters of the model are the distance of the inner edge of the current sheet $a=5$ R$_J$, the half-thickness of the current sheet $D=2.5$ R$_J$, and the distance of the outer edge $R1=70$ R$_J$. The figure shows the values by which the observed data must be multiplied in order to correct for latitudinal variations, for this set of Connerney et al. model parameters.
In Fig. 4.6 we thus show the latitude-corrected $B_{p\theta}$ data in a format similar to that of Figs. 4.3 and 4.4. Comparison with Fig. 4.4 shows that the latitude-related field variations are indeed substantially reduced in amplitude, such that the curves are much smoother. This is particularly evident in the Pioneer-11 inbound, Pioneer-10 outbound, and Ulysses outbound passes (panels (a)-(c)), which show the largest latitude-related effects in Fig. 4.4, and which now show almost smooth behaviour in Fig. 4.6, particularly in the radial range $\rho \approx 20-50R_J$. The near-equatorial Voyager-2 outbound pass shows only modest latitudinal effects in Fig. 4.4, and remains almost unchanged in Fig. 4.6. It remains true overall, however, that a single power-law fit is in general inadequate to fit the whole radial range of values observed during each pass. The values tend to decline from the overall fit at values of $\rho$ less than ~20 $R_J$, possibly due to the effects of enhanced current sheet thickness mentioned above (though all the values shown in these figures were obtained at $|z|$ values greater than the nominal 2.5 $R_J$), and on the nightside they also tend to decline at larger values of $\rho$, beyond ~50 $R_J$. In fitting these data we have therefore concentrated on the radial distance range 20–50 $R_J$, both because the data are relatively smoothly behaved in this interval, and because this is the range over which comparisons between dayside and nightside parameters can appropriately be carried out. The power law fits shown in Fig. 4.6 have thus been fitted only to the data lying between 20 and 50 $R_J$, though the remaining latitudinally-corrected data is also shown so that the degree of departure of the data from the fitted curves outside this range can be seen. The power law lines clearly fit the data very well in the 20–50 $R_J$ radial range (and generally, but not invariably, less well outside it), and again show systematic variations with local time. In particular it is notable that the values of $B_{p\theta}$ are all very similar in the inner part of the fitted range, and converge to ~40 nT near ~20 $R_J$. At larger $\rho$ the values then fall at different rates at different local times, such that at a given $\rho$ they are weakest on the dayside and strongest on the nightside. The implication is that the equatorial azimuthal currents are similarly asymmetric, stronger on the nightside than on the dayside at a given radial distance.
Figure 4.6. Latitude-corrected averages of the radial field $B_{\rho 0}$ versus $\rho$, in the same format as Figs. 4.3 and 4.4. The power law fits shown by the straight lines are fitted to data in the range 20-50 R$_J$ only, although the data are shown over the entire range. The exponent $m$ and coefficient $A$ of these lines are shown in each panel as before.
4.2.4 Simple overall model of the radial field dependence on distance and local time

So far we have concentrated on data from the four passes shown in Figs. 4.3, 4.4 and 4.6. In Fig. 4.7 we compare the fitted lines from all eight passes which provide data over a sufficient range that the slope \( m \) and intercept \( A \) (nT) appropriate to distances 20–50 \( R_J \) can be established with confidence, i.e. such that there are sufficient data points to produce a fit to within an \(<5\%\) RMS fractional error (see examples in Fig. 4.6). The passes excluded on this basis are the inbound passes of Pioneer-10 and Voyager-1, whose useable data span too small a radial range for this purpose. It is apparent that these lines tend to converge at \( \rho \sim 20 \, R_J \), as indicated above, and then fall with distance at a rate depending upon the local time, with fastest rates of fall occurring on the dayside. This suggests the possibility of developing a simple empirical model which encompasses the essence of all these results, in which we take the latitude-corrected \( B'_\rho \) field to be independent of local time at a given radial distance \( \rho_0 \) (\( \sim 20 \, R_J \)), then falling as a power law at larger \( \rho \) with the exponent \( m \) being a function of local time. That is we look for a model of the form

\[
B'_\rho(\rho, \phi) = A \left( \frac{\rho_0}{\rho} \right)^m, \tag{4.1}
\]

where \( A \) and \( \rho_0 \) are global constants. To determine these constants we use the eight fitted lines shown in Fig. 4.7, and compute the standard deviation of the eight values at each \( \rho \), normalised to the mean of these values. We then look for the minimum in this quantity, representing the radial distance of least variation in \( B'_\rho \) relative to the mean. We find that the minimum occurs at a radial range \( \rho_0 = 18.8 \pm 1.0 \, R_J \) (where the error has been estimated from the width of the minimum in the normalised standard deviation), and that the average value of the field is \( A = 41.1 \pm 5.1 \) nT (where the error given is the standard deviation of the values). We have therefore used these centre values for \( \rho_0 \) and \( A \) in further modelling.

Using the above values for \( A \) and \( \rho_0 \) as a "hinge" point through which the fitted curves must pass, we have then re-fitted the data to power law curves to determine the form of
Figure 4.7. Plot of the fitted lines as in Fig. 4.6, from the eight spacecraft passes which could be used to determine the dependence on distance in the radial range 20-50 $R_J$. These passes are indicated on the right hand margin. The solid part of each line depicts the radial range over which the fit was determined, while the dashed part (i.e. at radial distances greater than 50 $R_J$ and less than 20 $R_J$) show where the line has been extrapolated outside of the range. An arrow is drawn at the position $\rho_0$ of maximum convergence of the lines, determined from the least value of the standard deviation of the $B_{\rho_0}$ values normalised to the average, while the horizontal bar gives an estimate of the error.
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$m(\phi)$. The results are shown in Fig. 4.8. The points have been plotted at the mean local time of the fitted data (spanning the range $\rho = 20-50 R_J$), an approximation justified by the very modest variations in local time which occur on a given pass (see e.g. Fig. 4.2). It can be seen that a consistent pattern of variation of the $m$ values emerges, with values somewhat less than unity on the nightside increasing to values somewhat less than $\sim 2$ at noon. Given the restricted information available it seems reasonable to fit these values to the periodic function

$$m(\phi) = \alpha \cos \phi + \beta,$$  \hspace{1cm} (4.2)

where $\phi = 0$ at noon, increasing eastward towards dusk, and $\alpha$ and $\beta$ are constants. A least-squares fit to the $m$ values then yields values of $\alpha = 0.48$ and $\beta = 1.26$. The fitted curve is also shown in Fig. 4.8, and clearly represents a reasonable description of the derived values.

Equations (4.1) and (4.2) thus represent our overall empirical model for $B'_{\rho \phi}$, valid in the range $\rho = 20-50 R_J$, with the model constants being given by $A = 41.1 \text{nT}$, $\rho_0 = 18.8 R_J$, $\alpha = 0.48$ and $\beta = 1.26$. It finally remains to check the degree to which this model actually fits the data, given that the “hinged” fit Eq. (4.1) perforce is not the optimum power law fit on each pass, and that Eq. (4.2) represents a further approximation. In Fig. 4.9 we thus show the latitudinally corrected $B'_{\rho \phi}$ data, as in Fig. 4.6, and the model represented by Eqs. (4.1) and (4.2), where we have now employed the actual local time at each value of $\rho$ on the spacecraft trajectory to compute the model value (though the results are almost the same if the averaged local time for the pass is employed). The fits clearly show some relatively minor systematic deviations from the observed values over the expected range of validity, $\sim 20-50 R_J$. The Ulysses outbound data close to 45-50 $R_J$ is an example of this, and the reason for this discrepancy may be as follows. The Connerney et al. model is designed to fit to the Voyager and Pioneer between radial distances of 5 and 30 $R_J$, and hence we would not expect the model to be a good representation of the Ulysses data, particularly at large radial ranges. In addition to this, the high-latitude nature of these data could not be well accounted for by the Connerney et al. model, and as a result have actually been slightly “over-corrected” by the method described previously. However, overall the Connerney et al. current sheet model may be said to provide a reasonable
Figure 4.8. Plot of the exponent m to the "hinged" power law fit to $B'\rho_0$ for each of the eight spacecraft passes employed, versus magnetic local time. The points are plotted at the mean local time of the pass over the radial range 20-50 R$_J$. The solid line depicts the least squares fit to a sinusoidal function assumed symmetric about noon.
Figure 4.9. Comparison between the latitude-corrected $B'_{\rho 0}$ data and the empirical model derived here (dashed line), based on "hinged" power-law fits with $m(\phi)$ modelled by the sinusoidal function shown in Fig. 4.8. The format is the same as Fig. 4.6. The entire range of data is shown, though the model line has been derived only from data in the range 20-50 R$_J$. 
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account of the data. An additional check on validity is shown in Fig. 4.10, where we display the variation of $B_{eq}$ versus local time at fixed radial values of 20, 30, 40, and 50 R$_J$. Here the solid lines show the empirical model given by Eqs. (4.1) and (4.2). The model field indicates only weak variations at 20 R$_J$, but shows increasing local time asymmetry with increasing distance, reaching factors of more than 2 at 50 R$_J$. The solid symbols are derived from the best power law fits to the latitudinally-corrected data over the radial range 20-50 R$_J$, as exemplified by the data and fits shown in Fig. 4.6. In this case we have used the fits to all the spacecraft passes with the exception of Pioneer–10 inbound, but only within the radial range in each case over which we have derived data values. As can be seen in Fig. 4.6, these “best” fits clearly represent an accurate reflection of the observed field at a given distance (within the range) on each pass. The points are plotted at the actual local time of the spacecraft at that radial distance. Clearly the empirical model fits these values very well, and thus is again seen to provide a reasonable overall description.

It can be seen (e.g. near noon), however, that there also exists a significant level of scatter in the data at a given local time, at the level of a few tens of percent. This probably reflects temporal variations in the current sheet strength associated e.g. with variations in the Io source, as found previously in the modelling studies presented by Connerney et al. (1981) and Khurana (1997). While such variations are undoubtedly present, they are clearly not of sufficient amplitude to mask the larger local time asymmetry effects found here. We also note that the basic day-night asymmetry is clearly present in all of the individual spacecraft flybys investigated here (as seen e.g. in Fig. 4.7), which take place over intervals of only several days.

4.2.5 Divergence of the azimuthal current

It seems clear that the results presented above imply a significant divergence of the equatorial azimuthal current in the jovian magnetosphere, with significantly larger currents at midnight than at noon. In general the azimuthal current density (A m$^{-2}$) is given by

$$j_{\phi} = \frac{1}{\mu_0} \left[ \frac{\partial B_{\phi}}{\partial z} - \frac{\partial B_z}{\partial \phi} \right], \quad (4.3)$$

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Figure 4.10. Plot of the variation of $B'_{p0}$ versus local time at fixed radial distances of 20, 30, 40 and 50 R$_J$. The solid lines indicate the empirical model derived here, while the solid symbols are derived from the best power law fits to the latitudinally-corrected data over the radial range 20-50 R$_J$ (as shown in Fig. 4.6). These points are shown only for the radial ranges within which actual data points exist (i.e. the best-fit lines are not extrapolated beyond the range of data on the pass).
where the primed fields again indicate those produced by the current sheet (clearly the curl-free planetary field makes no contribution), and \( \mu_0 \) is the permeability of free space. If we integrate this expression through the current sheet we find the integrated current intensity (A m\(^{-1}\)) is given by

\[
i_{\varphi} = \frac{1}{\mu_0} \int_0^D r J_{\varphi} \, dr = \frac{2}{\mu_0} \left[ B'_{r0} - D \frac{\partial B'_r}{\partial \rho} \right],
\]  
(4.4)

where \( B'_{r0} \) is the radial field just outside the current sheet as above, and \( D \) is the half-thickness. In deriving this expression, we have assumed anti-symmetry in \( B'_r \) on either side of the current sheet, and \( B'_r \approx \text{constant} \). Now in considering the two terms on the RHS of Eq. (4.4), we may reasonably estimate

\[
\frac{\partial B'_r}{\partial \rho} \frac{B'_r}{\rho} \quad \text{and} \quad B'_r \approx B'_{r0},
\]

both expressions being valid e.g. for the Connerney et al. (1981) model. In this case it can be seen that the second term in Eq. (4.4) is less than the first by the ratio \( \sim \frac{D}{\rho} \). In the regime of interest here (\( \rho \) greater than \( \sim 20 \, \text{R}_J \)) this ratio is \( \sim 0.1 \) or less. Consequently, to within less than a \( \sim 10\% \) error we have

\[
i_{\varphi} \approx \frac{2 B'_{r0}}{\mu_0}.
\]  
(4.5)

In this case our model for the radial field outside the current sheet, \( B'_{r0} = B'_{r0}(\rho, \varphi) \), can be approximately but directly converted into a model for the azimuthal current intensity, which thus undergoes the same local time variations as the field (with an amplitude much greater than the \( <10\% \) systematic uncertainties). The divergence of the azimuthal current is then given by

\[
\text{div} \, i_{\varphi} = \frac{1}{\rho} \frac{\partial i_{\varphi}}{\partial \varphi} \approx \frac{2}{\mu_0 \rho} \frac{\partial B'_{r0}}{\partial \varphi},
\]  
(4.6)
and if we introduce the empirical model for $B'_p(\rho, \varphi)$ given by Eqs. (4.1), and (4.2) we find

$$\text{div } i_\varphi \approx -\frac{2\alpha}{\mu_0 \rho} \sin \varphi \ln\left(\frac{\rho_o}{\rho}\right) B'_p(\rho, \varphi). \quad (4.7)$$

In Fig. 4.11a we show a contour map of this function in the equatorial plane, labelled with the divergence values in kA R\textsubscript{j}^{-2}. The divergence is zero at $\rho_o = 18.8$ R\textsubscript{j}, the radius at which the model azimuthal current is axisymmetric, and also at all distances on the noon-midnight meridian, the assumed plane of current symmetry via Eq. (4.2). Divergence values are negative on the dawn side, positive on the dusk side, and peak in magnitude at $\sim 18$ kA R\textsubscript{j}^{-2} at $\sim 30$ R\textsubscript{j} near the dawn-dusk meridian. A negative divergence implies a sink region of azimuthal current, while a positive divergence implies a source region of azimuthal current. Of course the current overall must be continuous, and continuity must be maintained either via the radial current within the current sheet, or via field-aligned currents which flow into or out of the sheet over its upper and lower surfaces and connect with the planetary ionosphere (or both). It is impossible to know from the results presented here which is the case, and this question remains open for future study.

Finally, in order to give an indication of the overall current which must be diverted into one or other directions, we show in Fig. 4.11b the total azimuthal current flowing in given radial ranges versus local time. These have been computed by integration of the overall empirical model for $B'_p(\rho, \varphi)$ (Eqs. (4.1) and (4.2)), combined with Eq. (4.5). Specifically we show the total azimuthal current versus local time in the radial ranges 20-30, 30-40, and 40-50 R\textsubscript{j}, and the sum of these, i.e. the total current flowing in the range 20-50 R\textsubscript{j}. Each of these curves shows a maximum at midnight and a minimum at noon, the difference between the two indicating the amount of azimuthal current which is diverted either into radial or parallel currents in the region between. These differences are 8.2, 12.5, and 13.1 MA for the 20-30, 30-40, and 40-50 R\textsubscript{j} radial ranges, with the total value being 33.7 MA for the range 20-50 R\textsubscript{j}. 
Figure 4.11a. Contours of the divergence of the azimuthal current in the magnetic equatorial plane, in units of kA R$_J^{-2}$, derived from the empirical model of $B'p_0$ derived here. Noon is marked at the top of the plot, with dawn to the right. The dashed rings indicate radial distances of 20, 30, 40 and 50 R$_J$ from the centre outwards, which is the regime of approximate validity of the model. Jupiter is shown in the centre, to scale.
Figure 4.11b. The total current in MA flowing in various radial ranges in the equatorial current sheet is shown versus local time, obtained from the empirical model derived here. The current has been integrated in the ranges 20-30, 30-40, and 40-50 R\(_J\), and over the entire range 20-50 R\(_J\), as indicated on the right hand side of the plot.
4.3. Summary and conclusions

In this chapter we have provided the first systematic study of the properties of the radial field associated with the azimuthal equatorial current sheet in Jupiter's magnetosphere, which has been derived from the flybys of the Pioneer, Voyager, and Ulysses spacecraft. We have found that both individually and collectively these data show a significant local time asymmetry in which the dayside fields and currents are weaker than those at the same distance on the nightside. More specifically, these data suggest that in the radial range 20-50 R_J (encompassing most of the dayside current sheet), the field and current is approximately azimuthally symmetric at ~20 R_J, and then falls more rapidly with distance at noon, as \( \sim \rho^{-1.7} \), than at midnight, as \( \sim \rho^{-0.8} \). The noon-midnight difference in the fields reaches a factor of ~2 at distances of 40-50 R_J. Overall, we find that the data in the radial range 20-50 R_J can be well described by the formulae

\[
B'_{r0}(\rho, \phi) = A\left(\frac{P_0}{\rho}\right)^m(\phi) \quad \text{and} \quad m(\phi) = \alpha \cos \phi + \beta,
\]

where \( B'_{r0} \) is the radial field just outside the current sheet, azimuth \( \phi \) is measured eastward from noon, \( A = 41.1 \) nT, \( P_0 = 18.8 \) R_J, \( \alpha = 0.48 \) and \( \beta = 1.26 \). Secular changes in the current sheet strength may also be present in the data, as found in previous studies, and as indicated here by the scatter in values about the above model curves. However, these are of smaller amplitude than, and do not mask, the consistent local time asymmetry described by the above empirical model.

The above asymmetry in the field implies, via Ampère's law, a related asymmetry in the equatorial azimuthal current, with stronger currents on the nightside than on the dayside at a given equatorial distance. The divergence of the azimuthal current peaks at ~18 kA R_J^{-2} at ~30 R_J near the dawn-dusk meridian, the divergence being negative at dawn and positive at dusk. Over the full range of distances 20-50 R_J considered in this study, the total difference in the azimuthal current flowing at midnight compared with noon computed from our model is ~33.7 MA. Current continuity requires that this current is diverted either into radial currents within the current sheet, or emerges north-south from its surface to flow as field-aligned currents to the planet's ionosphere. We cannot tell from the data...
investigated here which of these possibilities is correct, and this is left as an important matter for future investigation.

The answer to this question does, however, have a bearing on the physical origin of the asymmetry effect. We have to ask why it is that the current-carrying plasma particles in the current sheet do not move on circular drift-paths around the planet to produce an azimuthally symmetric ring-current. Plasma source/loss processes do not seem likely to produce the noon-midnight effect over the wide radial ranges suggested by the data. Rather, two possibilities suggest themselves. The first is that the drift paths of equatorially-confined current-carrying particles are effected by the noon-midnight asymmetry imposed by the solar wind flow around the magnetosphere. As discussed previously e.g. by Goertz (1978), the compressive and confining effect of the solar wind dynamic pressure on the jovian field in the dayside magnetosphere, and its relaxation on the nightside, is such as to cause the current sheet plasma to $\mathbf{E} \times \mathbf{B}$ drift to larger radial distances on the nightside of the planet than on the dayside. A given field line will thus be more stretched out on the nightside than on the dayside, equivalent to an increased equatorial plasma current, and the same will also apply at a given radial distance, as discovered here. In this case, continuity of the azimuthal current will be maintained by radial currents flowing within the current sheet itself, directed outwards at dawn and inwards at dusk. Close to the planet these asymmetric currents will close via noon and midnight wholly within the equatorial current sheet, with the current “streamlines” being located closer to the planet at midnight than at noon. At larger distances, however, the currents at midnight may instead be expected to reach out to the magnetospheric boundary and to close in the magnetopause and boundary layers, such that the effect found here will merge continuously into the formation of the nightside tail. We regard this scenario as the most likely explanation of our results.

A second possibility, however, is that the azimuthal currents close instead in the jovian ionosphere, such that in terrestrial terms, the current system consists of a nightside eastward “partial ring current” in the equatorial plane, closing through the ionosphere via “region-2” field-aligned currents. At Earth, such a current system is generated by time-dependent sunward-directed displacements of the hot plasma distribution in the inner magnetosphere, which result from solar wind-driven convection (e.g. Wolf, 1983). At Jupiter, however, the observed direction of the cross-system electric field in the inner
magnetosphere is opposite to that required to produce such an effect, that is to say, the flow component added to corotation is directed tailward, not sunward. This fact has been deduced from local time asymmetries observed in the photon emission from the Io torus plasma, and holds at least at equatorial radial distances of \( \sim 4-7 \, \text{R}_J \) (Sandel and Broadfoot, 1982; Schneider and Trauger, 1995; Smyth and Marconi, 1998). This electric field is supposed to result from a preferred outflow of the iogenic plasma down the tail, again as a consequence of the confining effect of the solar wind pressure on the dayside (Barbosa and Kivelson, 1983; Ip and Goertz, 1983). Where, and at what distances, such flows may give way to transient solar wind-driven convection effects similar to those at Earth is at present unknown. If the current sheet effect discovered here is indeed due to the latter, however, the implication is that solar wind-driven convection effects are much stronger, and occur much closer to the planet, than previously believed. Overall, we regard the simple flow asymmetry effect described above, and previously by Goertz (1978), as being the most likely possibility.
Chapter 5

Divergence of the Equatorial Current in the Dawn Sector of Jupiter’s Magnetosphere

5.1. Introduction

In the previous chapter, we studied the equatorial radial field measured outside the current sheet in the middle magnetosphere, as a function of local time over the radial range 20-50 R_J. We derived an empirical model of the radial field associated with the azimuthal current system, and were able to quantify the divergence of this current component in the equatorial plane. Here, we perform the complimentary study by taking averaged values of the azimuthal component of the magnetic field observed outside the jovian middle magnetosphere equatorial current sheet which are then used to derive radial profiles of the radial current intensity over the jovicentric distance range 20-50 R_J. Data from four spacecraft flybys have been used, spanning the dawn sector from ~0100 to ~0900 MLT (i.e. inbound Pioneer-11, and outbound Pioneer-10, and Voyagers-1 and -2). These profiles have been combined with the recent empirical model of the azimuthal current intensity, presented in Chapter 4 and published in Bunce and Cowley (2001a), to estimate the total divergence of the current in the current sheet along the trajectory, and hence the density of the field-aligned current that couples the current sheet and the ionosphere.

Studies of the jovian field based on Pioneer, Voyager, and Ulysses fly-by data, cited in Chapter 2, have demonstrated that the field lines in the middle magnetosphere region...
Chapter 5: Divergence of the equatorial current in the dawn sector

dominated by the current sheet are distorted out of meridian planes, associated with an azimuthal field component which reverses about the equator. The distortion is consistently that of a field which "lags" behind planetary rotation, associated with an outward radial equatorial current flow. On the dawn side this "lagging" effect has the same sense as the bending effects induced by the solar wind, such that it is not simple to separate them in this sector. On the dusk side, however, the effects are opposite, and "lagging" fields at smaller distances have been found to give way to "leading" tail-like fields at larger distances, as demonstrated during the outbound pass of the Ulysses spacecraft (Dougherty et al., 1993).

The overall effect is illustrated in Fig. 5.1. Here we show 1/2-hour averages of the field components measured outside of the current sheet during the Pioneer, Voyager, and Ulysses passes, projected onto the magnetic equatorial plane. These data were supplied by the Planetary Data System at UCLA at 10s resolution for Pioneer-11 and Voyager-2, 48s for Voyager-1, and 1 min for Pioneer-10 and Ulysses. The VIP 4 planetary field model (Connerney et al., 1998) has first been subtracted from the data to leave only the fields due to external currents (principally the equatorial current sheet). These vectors have then been rotated through 90° to indicate the approximate direction of the corresponding equatorial current. To take account of the reversal of the equatorial field components across the current sheet, fields measured north of the current sheet have been rotated 90° anticlockwise, while those measured south of the current sheet have been rotated 90° clockwise. Interpreted in terms of a quasi-infinite current sheet with perturbation fields of equal magnitude but opposite direction on either side, a perturbation field of 10 nT corresponds to a sheet current of intensity ~1.1 MA R_j⁻¹. Spacecraft identifiers, P10, P11, V1, V2, and U, are shown as appropriate. Inbound passes are all in the pre-noon sector, and outbound passes are all on the nightside, with the exception of Pioneer-11 outbound which is at noon. All passes are also near-equatorial, with the exception of P11 outbound (33°N) and Ulysses outbound (37°S). The features observed in the figure include (a) the overall eastward sense of the azimuthal current associated with the radial distension of the middle magnetosphere field lines; (b) the larger values of the azimuthal current at a given distance on the nightside than on the dayside, as found by Bunce and Cowley (2001a) and as described in Chapter 4; (c) the outward radial current on the dawn side, associated with the consistently "lagging" nature of the field bending in this sector; and (d) a reversal in sense of the radial current on the dusk side, from outward ("lagging") at smaller distances to inward ("leading") beyond ~40 R_j (for example if one follows the 60 R_j indicator
Figure 5.1. Plot of half-hour averages of the magnetic field components measured outside the current sheet during the fly-bys of Pioneer-10, and -11, Voyager-1, and -2, and Ulysses, from which the VIP 4 planetary field model (Connerney et al., 1998) has been subtracted. The averages have been projected onto the magnetic equatorial plane and rotated through 90° to indicate the approximate direction and strength of the corresponding equatorial current. Fields measured north of the current sheet have been rotated 90° anti-clockwise, while those measured south of the current sheet have been rotated 90° clockwise. The dashed lines show the radial distance from the centre of the planet (Rj), and local time is also indicated. Spacecraft identifiers, P10, P11, V1, V2, and U are also shown. At the bottom right hand side of the plot is the scale for 40 nT.
around in local time, it is evident that the radial component changes from outward to inward in sense).

In the previous chapter, we examined the behaviour of the radial field component observed during the Pioneer, Voyager, and Ulysses fly-bys, and derived a simple empirical model of the azimuthal current distribution in the radial range 20-50 \( R_J \), as briefly reported above. In this chapter we similarly examine the azimuthal field component in the same radial range, and use it to determine the distribution of radial currents on four low-latitude passes from which useful radial profiles can be obtained. These passes are Pioneer–11 inbound, and Pioneer–10 and Voyagers–1 and –2 outbound, spanning the dawn sector from pre-noon to post-midnight. The radial variation of the radial current observed on these passes can then be combined with the azimuthal variation of the azimuthal current from the Bunce and Cowley (2001a) empirical model (herein referred to as the BC model) to determine the overall divergence of the equatorial current in the current sheet. From this divergence we can then estimate the field-aligned current density entering or leaving the current sheet on each of these passes, and also the current densities correspondingly leaving or entering the ionosphere. We then interpret the overall current system as the sum of two physical components. As indicated above, the first of these components closes azimuthally wholly within the current sheet and is associated principally with radial stress balance. The second consists of the field-aligned currents which are taken to be closed by radial currents in the equatorial plane and by Pedersen currents in the ionosphere, and are associated with magnetosphere-ionosphere coupling and angular momentum exchange. The latter radial currents are calculated on the basis of this physical picture, and interpreted theoretically in terms of the angular velocity profile of the equatorial plasma.

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5.2 Radial variation of the azimuthal field

5.2.1 Current sheet field averages

In order to extract useful information concerning the radial current flowing in Jupiter’s equatorial plane from the magnetic field vectors observed during the above jovian fly-bys, averages of the azimuthal field component were computed in order to gain an overall view of their variation with distance from the planet. However, it turned out to be impossible to include data from all of the flybys, for the following reasons. First, because we may expect the azimuthal field to vary systematically with latitude, as well as with radial distance, due e.g. to differential rotation of the flux tubes, we have included data only from the near-equatorial passes, thus excluding the outbound passes of Pioneer-11 and Ulysses. Second, of the equatorial passes, we have also excluded data from the inbound passes of Pioneer-10, Voyager-1 and -2, and Ulysses. The inbound data from Pioneer-10 were found to be very disturbed due to a major compression of the system during the fly-by. In addition, the inbound data from the Voyager-1 and -2 passes were limited because their trajectories were very close to the equatorial plane. It is clear that in order to measure the total current, as required, we need to employ magnetic data from intervals when the spacecraft had fully exited the layer. Insufficient data of this nature exist on the inbound Voyager passes to allow the determination of the radial profile of the current. The inbound Ulysses pass was excluded because those few encounters with the region south of the current sheet indicated that an unusual inter-hemispheric asymmetry existed during the fly-by, such that those azimuthal fields measured above the current sheet are somewhat different in magnitude to those measured below. For this reason we did not feel confident to derive the radial current from northern hemisphere data only during the remainder of the pass (the radial currents are, of course, related to the difference in the azimuthal field across the current sheet). The azimuthal field signatures on this pass merit further separate study. With these omissions, then, the passes included in our study are the inbound section of the Pioneer-11 encounter in the pre-noon quadrant, Pioneer-10 outbound which exited the magnetosphere near the dawn meridian, and the outbound passes of Voyager-1 and -2, both in the post-midnight sector.
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The starting point of our study are field averages formed over complete cycles of time when the spacecraft was outside the current sheet. Such averages provide the general variation of the field as a function of distance, free, for example, from possible local effects of the tilting or bending of the current sheet. The disadvantage of taking such averages is the possible inclusion of latitude dependence of the field during the cycle. Nevertheless examination of the magnetic field values suggests that this is not a dominant effect over the range of latitudes reached by the spacecraft included in this study. Examples of these averages and the way in which they have been selected are shown in Fig. 5.2. In Fig. 5.2a we show one (Earth) day of data from the outbound pass of Pioneer-10, day 340 of 1974 when the spacecraft was located at jovimetric distances between 37.0 and 51.6 R\textsubscript{J}, and at \textasciitilde05:00 MLT. The magnetic local time (h:min) and the jovimetric radial distance are indicated at the foot of the figure. The top two panels show two of the magnetic field components in cylindrical jovimagnetic coordinates, where \(B_\rho\) is the radial component perpendicular to the magnetic dipole axis, and \(B_\phi\) is the azimuthal component positive eastward. We have subtracted the Connerney et al. (1998) VIP 4 internal planetary field from the observed values, as we are interested in the fields due to external currents alone. Such “external” field values are indicated by primes, both here and throughout the remainder of the chapter. The uppermost panel thus shows the \(B'_\rho\) component and the second panel shows \(B'_\phi\), which enables identification of those times when the spacecraft was outside of the current carrying region. To aid identification of the latter intervals we have also plotted in the third panel the distance \(z (R\textsubscript{J})\) from the magnetic equatorial plane. The dashed lines in this panel show the nominal position of the current sheet, taken to lie in the range \(\pm2.5 R\textsubscript{J}\) about the equator, as in the Connerney et al. (1981) model. It can be seen that Pioneer-10’s outbound pass lay, in the main, above the magnetic equator (at \(\sim14^\circ\) N jovigraphic latitude), and approached the current sheet only once per jovian rotation. Correspondingly, the measured field is predominantly positive and radial, interspersed with fluctuating depressions at \(\sim10\) h intervals denoting the presence of hot plasma currents. Thus ignoring times when enhanced magnetic variations were present in the radial field component, averages of the \(B'_\rho\) and \(B'_\phi\) components were taken over complete cycles shown by the solid bar in the top panel.

A second example is shown in Fig. 5.2b in the same format as Fig. 5.2a. In this case we show one day of data from the outbound pass of Voyager-2, day 193 of 1979, when the
Figure 5.2a. First example of the magnetic field data which form the basis of this study. One (Earth) day of data is shown for Pioneer-10 outbound on day 340 of 1974. The top two panels indicate the magnetic field data in cylindrical jovimagnetic coordinates, where $B'_\phi$ (top panel) is the azimuthal component measured positive eastward, and $B'_r$ is the radial component perpendicular to the magnetic axis. The field components are primed to signify that the planetary field has been subtracted, a notation used throughout the paper. Dashed lines show the zero line in both panels, whilst the solid bars in the $B'_\phi$ panel indicate those intervals over which we have chosen to average the data, as used in the subsequent analysis. The bottom panel shows the distance of the spacecraft, $z (R_j)$, from the magnetic equatorial plane. Included in this panel are two dashed lines which show the nominal boundaries of the current sheet $\pm 2.5 \, R_j$ about the equator. At the foot of the figure we give the universal time UT, the magnetic local time of the spacecraft MLT (h:min), and its jovicentric radial distance ($R_j$).
Figure. 5.2h. Second example of the magnetic field data used for this study. Now, one (Earth) day of data is shown from Voyager-2 outbound on day 193 of 1979. The format for this figure is identical to that in Fig. 5.2a.
spacecraft was located at jovian distances between 34.9 and 47.9 R\textsubscript{J}, and at \(-01:30\) MLT. This pass was located much closer to the jovigraphic equatorial plane, such that the nominal current sheet passed over it twice during each 10 h rotation. Here, therefore, the radial field component cycled between steady positive and negative values when the spacecraft was outside the current sheet, interspersed with intervals of weaker and strongly varying field when the spacecraft was located within the sheet. Once more, the solid bars in the top panel indicate those intervals over which averages were taken.

5.2.2 Radial variation of the radial and azimuthal field components

Results for the radial variation of averaged \(B'_\rho\) and \(B'_\phi\) are shown in Fig. 5.3 for the four passes considered here. In the main, we have restricted our attention to the radial range 20-50R\textsubscript{J}. This is the range over which the current sheet can generally be measured on both the dayside and the nightside, and hence also the range of validity of the BC model which is to be used in this study. The upper panels show the averaged radial and azimuthal field components outside the current sheet versus the perpendicular distance from the magnetic axis \(\rho\), in a log-log format. The scale on the left indicates the magnetic field strength in nT, whilst that on the right indicates the equivalent equatorial current intensity (MA R\textsubscript{J}^{-1}), as will be discussed later. Results are displayed for (a) Pioneer-11 inbound, (b) Pioneer-10 outbound, (c) Voyager 1 outbound, and (d) Voyager-2 outbound. These data represent observations in the pre-noon, dawn and post-midnight sectors respectively (see Fig. 5.1). Considering first the azimuthal component of the field, values obtained when the spacecraft was south of the current sheet (such that \(B'_\phi\) was negative and \(B'_\rho\) positive) are shown as circles. Those obtained when the spacecraft was north of the current sheet (such that \(B'_\rho\) was positive and \(B'_\phi\) negative) have been reversed in sense and are shown as crosses. A least squares linear fit to the log data is shown by the straight line. The coefficients of the fit, of the form \(B'_\phi = A(nT)\rho(R_j)^{m}\), are shown at the foot of each panel. It can be seen that the data are generally well fit by this simple form, with relatively little scatter. Typical values of the azimuthal field lie between \(-3\) and \(-8\) nT. The data from Pioneer-10 and -11 both show a decrease of azimuthal field with distance from the planet, which is reflected in the negative sign of \(m\) (-0.53 and -0.61 respectively). Thus during
Figure 5.3. The four panels of the figure show averaged magnetic field data for the four flyby passes analysed here, i.e. for the passes of (a) Pioneer-11 inbound at ~0900 MLT, (b) Pioneer-10 outbound at ~0430 MLT, (c) Voyager-1 outbound at ~0330 MLT, and (d) Voyager-2 outbound at ~0100 MLT. The uppermost plot in each panel shows the averaged $B'_{\rho}$ and $B'_{\phi}$ fields versus the perpendicular distance $\rho$ from the magnetic axis, in a log-log format. The left hand scale indicates the magnetic field strength in nT, whilst that on the right indicates the equivalent current intensity (MA R$^{-1}$) obtained from Eq. (5.6). Those averages taken south of the current sheet are shown as circles, while those taken north of the current sheet have been reversed in sense and are shown as crosses. The opposite procedure has been applied to the $B'_{\rho}$ data, such that all values shown are positive. The $B'_{\rho}$ data has also been corrected by a modest factor for the latitude of the spacecraft, as described in the text. The solid straight lines through the $B'_{\phi}$ data show least squares power law fits of the form $B'_{\phi} = A/nT \rho (R_J)^m$, where the values of the coefficient $A$ and the exponent $m$ are shown in the lower left corners of each panel. The solid lines through the data show values derived from the Bunce and Cowley (2001a) empirical model, as described in Chapter 4. The lower plots in each panel show the ratio $-B'_{\phi}/B'_{\rho}$ of the averaged data in the same log-log format. These values are compared with empirical models derived in previous studies, namely those of Goertz et al. (1976) based on Pioneer-10 data (solid line), and Behannon et al. (1981) (dashed line) and Vasyliunas (1983) (dot-dash line) based primarily on outbound Voyager data. The $B'_{\rho}$ data in these plots have not been corrected for latitude, in order to aid comparison with the previous studies.
these passes, the radial current intensity decreased with increasing radial distance from the planet. Conversely the Voyager-1 and -2 data imply that the azimuthal field and radial current increased with distance in this range, such that $m$ is positive (having values of 0.72 and 0.87 respectively).

The upper panel of Fig. 5.3 also shows the radial component of the field, in a similar format to that for $B'_p$. These data have been averaged according to the same criteria as for $B'_p$, but have in this case been corrected (on a pre-averaging point-by-point basis) by a modest factor for the latitude of the spacecraft. The principal purpose of displaying this component is to exhibit the level of agreement with the BC model, employed here to estimate the divergence of the azimuthal current in the current sheet. This model refers explicitly to the value of $B'_p$ just outside the current sheet, and since $B'_p$ is found to fall slowly with latitude, the observed values have been mapped from the latitude of observation by the spacecraft to the edge of the current sheet, using the Connerney et al. (1981) current sheet model. The procedure is the same as that adopted in the previous section, and the reader is referred to Chapter 4 for full details. Mapping factors are typically ~1.05 for a latitude of ~5°, increasing to ~1.25 for ~15°, such that the corrections are not large. The BC model is shown by the solid line, and, with few exceptions (e.g. Voyager-1 outbound at larger distances), clearly fits the latitude-corrected $B'_p$ data very well.

The lower panels in Fig. 5.3a-d show the ratio of the averaged data as a function of $\rho$, again in a log-log format. Our purpose is to compare the results derived here with previous studies which (for historical reasons) displayed the ratio $-\left(B'_\varphi / \rho B'_\rho\right)$. Consequently, the averaged $B'_\rho$ data employed in this panel are (as in these previous studies) not corrected for latitude effects. Here we simply show the ratio $-\left(B'_\varphi / B'_\rho\right)$, which relates to the twisting of the field lines out of meridian planes, and hence have multiplied the previous empirical models of $-\left(B'_\varphi / \rho B'_\rho\right)$ by $\rho$ in order to undertake a comparison. These models are shown by varying line types in the lower panels of Fig. 5.3a-d. The solid line shows the model derived by Goertz. et al. (1976) from the outbound Pioneer-10 data. This is given by
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\[ \frac{-B_e}{B_\rho} \approx 6.12 \times 10^{-3} \rho(R_j) \exp\left(\frac{\rho(R_j)}{500}\right) . \quad (5.1a) \]

The dashed line shows the fit chosen by Behannon et al. (1981) to represent the Voyager-1 and -2 data, and is given by the equation

\[ \frac{-B_e}{B_\rho} \approx 6.25 \times 10^{-2} \sqrt{\rho(R_j)} . \quad (5.1b) \]

Finally, the dot-dash line shows the Vasyliunas (1983) model derived from the Voyager-1 and -2 data, which is given by

\[ \frac{-B_e}{B_\rho} \approx 9.0 \times 10^{-3} \rho(R_j) \exp\left(-\frac{\rho(R_j)}{260}\right) . \quad (5.1c) \]

We note that these models were all intended to be representative of a rather wider radial range than that studied here, and were thus not necessarily optimised for the nearer distances of ~20-50 R\(_j\). Explicitly, the Goertz et al. model describes the field ratio over the range ~20-80 R\(_j\), whilst Behannon et al. and Vasyliunas provided fits in the range ~20-140 R\(_j\). In addition, we note from their published work that the Behannon at al. model over-estimates the Voyager data at small radial distances. We also note that all data shown here are obtained from field values from which the internal field has been subtracted. This is not true of the previously-published models, however, where averages of the total field were taken. This reduces \(|B_\rho|\) somewhat in the present study, while leaving \(B_\rho\) approximately unchanged.

Fig. 5.3b shows the data from Pioneer-10 outbound compared to the three aforementioned models. As expected, our values are slightly higher than those of the Goertz et al. model, but approach it at larger radial distances. The data fit well to the Vasyliunas model for all distances. The Voyager-1 data, depicted in Fig. 5.3c, reasonably approximate the Goertz et al. model at small distances, whilst increasing in magnitude to fit the Vasyliunas and Behannon et al. models at larger distances. The data for Voyager-2, shown in Fig. 5.3d, fall below all three models close to the planet, but approximates the models of Vasyliunas and Behannon et al. at larger distances, as for Voyager-1. Finally, we also see in Fig. 5.3a that the Pioneer-11 inbound data fit rather well to the Behannon et al. curve, over all distances considered. Overall we may conclude that in general our data are similar to those derived in previous parallel studies. The minor differences are most likely due to differences in the radial range of the fit, and also the fact that the previous authors fit to the total field, whilst
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we have subtracted the VIP 4 internal field model, and concentrated on the "external" field alone.

5.3. Divergence of the equatorial current

To a good approximation, the values of $B'$ derived in the previous section may be converted directly into values of the radial current intensity. In this section we derive these values, and, from the radial profiles of the radial current on each pass, also examine their divergence. By combining these results with the divergence of the azimuthal current derived from the BC model, we also estimate the overall divergence of the equatorial current, and hence the magnitude and sense of the field-aligned currents which couple the current sheet and the ionosphere.

We begin by determining the radial current profiles on the four passes considered here. In general the radial current density ($A m^{-2}$) is given by

$$j_r = \frac{1}{\mu_0} \left[ \frac{1}{\rho} \frac{\partial B'_z}{\partial \phi} - \frac{\partial B'_\phi}{\partial z} \right], \tag{5.2}$$

where, as indicated above, the primed fields indicate those from which the internal field has been subtracted (clearly the curl-free planetary field makes no contribution to the current); $\mu_0$ is the permeability of free space. Integrating this expression through the current sheet we derive an expression for the integrated radial current intensity ($A m^{-1}$)

$$i_r = \frac{1}{\mu_0} \int_D j_r dz = \frac{2}{\mu_0} \left[ \frac{D}{\rho} \frac{\partial B'_z}{\partial \phi} - B'_\phi \right], \tag{5.3}$$

where $B'_\phi$ is the azimuthal field just outside the current sheet in the northern hemisphere as above, and $D$ is the half-thickness. In deriving this expression we have assumed that $B'_\phi(-D) = -B'_\phi(D)$ i.e. that the azimuthal field is anti-symmetric on either side of the current sheet, and that $B'_z$ is approximately constant through the sheet. Let us consider the magnitudes of the two terms on the right hand side of Eq. (5.3). Clearly the first will be negligible compared to the second if the current sheet is approximately axi-symmetrical. More realistically we might assume, for example, that $B'_z$ typically varies by less than half its magnitude in half a turn around the planet (see e.g. Fig. 3.4 in Chapter 3), so that

$$\frac{\partial B'_z}{\partial \phi} \leq \frac{B'_z}{2\pi}. \tag{5.4}$$
In addition, because the current sheet produces fields $B'_r$ which are comparable to $B'_p$, as in Connerney et al.'s (1981) model, we then have

$$\frac{\partial B'_r}{\partial \phi} \leq \frac{B'_r}{2\pi} - \frac{B'_e}{2\pi} \sim B'_e,$$  \hspace{1cm} (5.5)

where the last approximation is justified by reference to the relative magnitudes of $B'_p$ and $B'_e$ which can be seen in Fig. 5.3. Then since $D \sim 2.5 R_J$, we see that the magnitude of the first term in Eq. (5.3) will typically be at least an order of magnitude less than that of the second. Consequently to within better than a $\sim 10\%$ error we have

$$i'_p \approx \frac{2B'_e}{\mu_0}.$$  \hspace{1cm} (5.6)

The values of the equivalent radial current intensity on each pass, given by Eq. (5.6), are indicated on the right-hand scale of the upper panels in Fig. 5.3. Typical values are several tenths of a MA R$_J^{-1}$. These current scales also apply to the azimuthal current intensity derived in an analogous way from the radial field measurements shown in the figure, as justified previously in Chapter 4.

Given the profiles of the radial current intensities we can now calculate their divergence. In general, the divergence of the equatorial current is given by

$$\text{div} i = \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} \left( \rho i'_p \right) + \frac{\partial}{\partial \phi} \left( i'_\phi \right) \right].$$  \hspace{1cm} (5.7)

The first term on the right hand side is the divergence of the radial current, which is determined from our $B'_e$ profiles via Eq. (5.6), assuming that to a sufficient approximation the fly-by passes can be considered to take place at constant azimuth. The second term gives the divergence of the azimuthal current, which is determined from the empirical BC model based on the azimuthal variations of the flyby $B'_p$ profiles. We now consider each in turn.

First we note that the divergence of the radial current involves not $i'_p$ directly, but the product $\rho i'_p$, this being the total current per radian of azimuth (A rad$^{-1}$). For purposes of deriving the divergence we therefore first obtain fits to this quantity. Fig. 5.4 shows $\rho i'_p$ versus $\rho$ in the same log-log format as shown previously in Fig. 5.3. The scale on the left hand side is given in amps per radian (of azimuth), while that on the right is given equivalently in nT-R$_J$, via Eq. (5.6). Results are displayed, as previously, for (a) Pioneer-
Figure 5.4. The equatorial radial current intensity per radian of azimuth, $\rho_i\Phi$, is shown versus $\rho$ for the four flyby passes in the same log-log format as shown previously in Fig. 5.3. The scale on the left is in MA rad$^{-1}$, while that on the right is given equivalently in nT-R$_J$, via Eq. (5.6). A least squares linear fit to the log data is shown by the straight line, the coefficients $A$ and $m$ being shown in the bottom left-hand corner of each panel. The uncertainty of the fit gradient is shown by the dashed lines, which have been drawn to pass through the mean logged $\rho_i\Phi$ and $\rho$ values of the points.
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11 inbound, (b) Pioneer-10 outbound, (c) Voyager-1 outbound, and (d) Voyager-2 outbound. A least squares linear fit to the log data is shown by the straight line. The coefficients of the fit, of the form $\rho i_\rho = A(\text{MA rad}^{-1})\rho(R_J)^m$, are shown at the bottom of each panel. It can be seen that the data are again generally well fit by this simple form, with relatively little scatter. The uncertainty in the gradient is found by the standard least squares method (e.g. J. Topping, 1955), and is shown by the dashed lines in the figure. These have been drawn to pass through the mean of the logged $\rho i_\rho$ and $\rho$ values, through which point the best-fit line itself automatically passes. The data show that the radial current per radian of azimuth increases with distance in each case, such that $m$ is positive, having values of 0.50, 0.40, 1.70, and 1.87 for the four panels respectively. These values are essentially equal to one plus the $m$ values derived from the fits to $i_\rho$ alone in Fig. 5.3, as may be expected. The strongest gradients are the two post-midnight passes of Voyager-1 and -2, while the dawn to noon sector trajectories of Pioneer-10 and -11 show much weaker gradients. Overall, the radial currents on each pass are similar at distances of ~35-40 R_J, with values of ~20 MA rad$^{-1}$. The Pioneer values between dawn and noon are then larger than the nightside Voyager values at smaller radial distances, and vice-versa at larger radial distances.

We now use the fitted lines to calculate the divergence of the radial current, corresponding to the first term in Eq. (5.7). These values are marked as “div $i_\rho$” in the upper panels of Fig. 5.5, which has a similar format to Figs. 5.3 and 5.4. The uncertainty estimates indicated by the dashed lines have been obtained from the uncertainties in the gradients of the fitted lines in Fig. 5.4. It can be seen that the values on the Pioneer passes shown in Figs. 5.5a and b are ~15-20 kA R_J$^{-2}$ at ~20 R_J, falling rapidly with distance to ~5 kA R_J$^{-2}$ at ~50 R_J. The behaviour on the Voyager passes is rather different, having larger values of ~30 kA R_J$^{-2}$ at ~20 R_J, falling more slowly to ~25 kA R_J$^{-2}$ at ~50 R_J. We note, however, that the values are positive throughout i.e. the radial current per radian of azimuth increases monotonically with distance on each pass.

We now consider the divergence of the azimuthal current, corresponding to the second term in Eq. (5.7). Following the arguments in Chapter 4, this current component is similarly obtained from
Figure 5.5. The upper plots in each panel of this figure show the divergence of the radial and azimuthal equatorial currents ($kA R_J^{-2}$) as a function of the perpendicular distance from the magnetic axis of the planet $p$, in the same format as Fig. 5.3. The uncertainty estimates are indicated by the dashed lines. The divergence of the radial current, and its uncertainty limits, have been obtained from the fitted lines in Fig. 5.4. The corresponding quantities for the azimuthal current have been obtained from the Bunce and Cowley (2001a) empirical model, described in Chapter 4. The lower plots show the current density $j_z$ normal to the current sheet at its northern surface required for current continuity, as a function of $p$. An equal but opposite current is assumed to flow out of the southern surface. The uncertainties shown by the dashed lines are the square root of the sum of the squared errors shown in the upper plots of this figure.
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\[
i_\varphi \approx \frac{2 B'_\rho}{\mu_o}, \quad (5.8)
\]

where \( B'_\rho \) is the radial field component (with internal field contribution removed) just outside the current carrying region. The BC model for \( B'_\rho \), obtained from the flyby data is given by

\[
B'_\rho(\rho, \varphi) = A \left( \frac{\rho_0}{\rho} \right)^{m(\varphi)}, \quad (5.9)
\]

where \( A = 41.1\pm5.1 \text{ nT}, \quad \rho_0 = 18.8\pm1.0 \text{ R}_J, \quad \text{and} \quad m(\varphi) = \alpha \cos \varphi + \beta, \quad \text{where} \quad \alpha = 0.48 \quad \text{and} \quad \beta = 1.26. \) Combining Eqs. (5.8) and (5.9) then gives

\[
\text{div} \ i_\varphi = \frac{1}{\rho} \frac{\partial (\rho_i \varphi)}{\partial \varphi} = \frac{2 \alpha A}{\mu_o \rho} \sin \varphi \ln \left( \frac{\rho_0}{\rho} \right) \left( \frac{\rho_0}{\rho} \right)^{m(\varphi)}. \quad (5.10)
\]

Values derived from this expression at the spacecraft position are also shown in the upper panels of Fig. 5.5, marked “\( \text{div} \ i_\varphi \)”. The uncertainty limits, again indicated by the dashed lines, follow through from the uncertainties in the parameters in Eq. (5.9), as indicated above. It can be seen that the values of \( \text{div} \ i_\varphi \) are small near the inner edge of the region investigated, peak near \( \sim 30 \text{ R}_J \), and then fall again with increasing \( \rho \). The small values at small \( \rho \) are due to the increasingly azimuthally symmetric nature of the azimuthal current at smaller distances, the current being exactly symmetric at and within \( \rho \approx 18.8 \text{ R}_J \) in the BC model (see Eq. (5.9) above). The decrease at larger distances following the peak at \( \sim 30 \text{ R}_J \) is then mainly due to the fall in the magnitude of the azimuthal current with increasing distance. The variation of \( \text{div} \ i_\varphi \) with azimuth is such that the largest values occur near dawn (on the outbound Pioneer-10 pass) and fall towards zero at noon and midnight, the assumed axis of symmetry of the azimuthal current. If we now compare these values with those derived from the radial current, it can be seen that \( \text{div} \ i_\varphi \) is generally comparable in magnitude to \( \text{div} \ i_\rho \) but is consistently opposite in sign. The similarity of the magnitudes means first of all that both components of the current must be considered in determining the overall divergence of the equatorial current. The negative sign means that the azimuthal current decreases continuously in moving from midnight via dawn to noon, this decrease partially feeding the increasing radial current with radial distance, as described above. This effect then reduces, or even reverses, the field-aligned current into the sheet that would be deduced from the variation of the radial current alone.
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From current continuity, the north-south current per unit area flowing into or out of the current sheet over its northern surface due to field-aligned currents is given by

\[ j_z = \frac{1}{2} \text{div}_i = \frac{1}{2} \left[ \text{div}(i_\phi) + \text{div}(i_\psi) \right], \quad (5.11) \]

where the factor of a half comes from the assumption that the equatorial current divergence is shared equally between field-aligned currents flowing in the northern and southern hemispheres. The value of \( j_z \) is shown in the lower panels of Fig. 5.5, obtained by combining the divergence of the individual current components shown in the upper panels, as in Eq. (5.11). Note that positive \( j_z \) (at the northern surface of the current sheet) implies current flow out of the sheet towards the ionosphere in both hemispheres, while negative \( j_z \) implies current flow into the current sheet from the ionosphere in both hemispheres.

It can be seen that the current system implied by the Voyager measurements on the nightside is distinctly different from that implied by the Pioneer measurements at dawn and on the dayside. The Voyager measurements both imply that the radial current flowing in the current sheet at the inner edge of the region investigated, \( \sim 20 \) \( R_J \), is relatively small, \( \sim 7 \) MA rad\(^{-1} \) (see Fig. 5.4). We assume that this current had been fed into the current sheet from the ionosphere at smaller distances. With increasing distance, the radial current then grows rapidly due both to an outward diversion of the azimuthal current and continuous field-aligned current input from the ionosphere. For the outbound Voyager-1 pass the two sources were approximately equal at \( \sim 15 \) kA \( R_J^{-2} \) (summing the contribution of the field-aligned currents from the northern and southern hemispheres), such that the radial current grew to \( \sim 35 \) MA rad\(^{-1} \) at \( \sim 50 \) \( R_J \). For Voyager-2 outbound the summed field-aligned current contribution was dominant at \( \sim 20 \) kA \( R_J^{-2} \), compared with \( \sim 5 \) kA \( R_J^{-2} \) for the diverted azimuthal current, such that the radial current grew similarly to \( \sim 30 \) MA rad\(^{-1} \) at \( \sim 50 \) \( R_J \). For the Pioneer passes, however, the radial current was already \( \sim 15 \) MA rad\(^{-1} \) at \( \sim 20 \) \( R_J \), presumably again fed into the sheet from the ionosphere at smaller distances. In the case of Pioneer-11 inbound, the current inflow from the ionosphere dropped to small values, consistent with zero, at and beyond \( \sim 25 \) \( R_J \). The modestly increasing radial current beyond \( \sim 25 \) \( R_J \) was then fed mainly by the diversion of azimuthal current at a rate of \( \sim 10 \) kA \( R_J^{-2} \), reaching \( \sim 25 \) MA rad\(^{-1} \) at \( \sim 50 \) \( R_J \). The situation during the Pioneer-10 outbound pass was similar but more extreme. While the radial current was again \( \sim 15 \) MA rad\(^{-1} \) at \( \sim 20 \) \( R_J \), the inflow of current from the ionosphere had already dropped to small values by \( \sim 20 \) \( R_J \), and was in fact reversed in sense over much of the current sheet.
beyond ~25 R\(_{\text{J}}\). This was due to the large estimated divergence of the azimuthal current on this pass, which peaked at ~20 kA R\(_{\text{J}}\)\(^{-2}\) at ~30 R\(_{\text{J}}\). This divergence was sufficient not only to provide for the modest further increase of radial current beyond ~20 R\(_{\text{J}}\), reaching ~20 MA rad\(^{-1}\) at ~50 R\(_{\text{J}}\), but also required a significant return flow of current to the ionosphere beyond ~25 R\(_{\text{J}}\), at ~5 kA R\(_{\text{J}}\)\(^{-2}\) to each hemisphere.

In the following sections these differing behaviours of the current profile will be related to differing inferred plasma flow conditions prevailing during these passes. In this section, however, we conclude by using the \(j_z\) values shown in Fig. 5.5 to calculate the value of the field aligned current density per unit magnetic field strength, \((j/\mathbf{B})\). This quantity is conserved along the field lines for pure field-aligned current flow between the equatorial current sheet and the ionosphere, and is hence the parameter we need in order to estimate the current density at various points down to the ionosphere. This parameter is given in terms of \(j_z\) by

\[
\frac{j_z}{B} = \frac{J_z}{B_z},
\]

where \(B\) is the strength of the total field outside the current sheet, and \(B_z\) is its \(z\) component. We therefore require knowledge of the \(B_z\) field in the current sheet (taken to be approximately constant within it). For these purposes we have employed corresponding empirical models of \(B_z\) derived by previous authors, specifically the outbound Pioneer-10 model derived by Goertz et al. (1976), and the outbound Voyager-1 and -2 models derived by Khurana and Kivelson (1993). No model has previously been derived to fit the inbound Pioneer-11 data, and so here we have somewhat arbitrarily employed the Goertz et al. model for this case as well. The model expressions are given in Chapter 3 and plotted in Fig. (3.4), where it will be seen that the \(B_z\) values at a given distance typically differ from each other by at most a factor of ~2. Thus the results would not be very different whichever model was used for Pioneer-11. With these \(B_z\) models, then, in Fig. 5.6 we show the variation of \((j/\mathbf{B})\) versus distance \(\rho\) for the four passes, where the sign corresponds to the northern hemisphere, such that positive values indicate outward current along the field from the northern ionosphere into the current sheet. The coloured bands indicate the limits of uncertainty, which follow from the uncertainties in \(j_z\) shown in Fig. 5.5. For the Voyager passes, the values increase from ~2x10\(^{-13}\) A m\(^{-2}\) nT\(^{-1}\) at ~20 R\(_{\text{J}}\), to ~1x10\(^{-12}\) A m\(^{-2}\) nT\(^{-1}\) at ~50 R\(_{\text{J}}\). The values derived from the Pioneer-11 pass are essentially consistent with zero throughout (less than ~10\(^{-13}\) A m\(^{-2}\) nT\(^{-1}\)), while those from
Figure 5.6. Plot of the variation of \( (j/v)B \) versus distance \( \rho \) for the four spacecraft passes. The sign shown corresponds to the northern hemisphere, such that positive values indicate current flowing from the northern ionosphere to the current sheet, and vice versa for negative values. The coloured bands indicate the limits of uncertainty, which follow from the previous figure. The colours also serve as spacecraft identifiers.
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Pioneer-10 indicate a current which is near zero at ~20 R_j, decreasing to ~5×10^{13} \text{ A m}^{-2} \text{nT}^{-1} at ~50 R_j.

We can now use these values to estimate the field-aligned current density at ionospheric heights, where the mean field strength is approximately \(2B_j \approx 860,000 \text{ nT}\). For typical values of \(\langle j \rangle / B \approx 5×10^{13} \text{ A m}^{-2} \text{nT}^{-1}\) in the equatorial plane, the implication is that the field-aligned current density at the ionosphere is ~0.4 \text{ µA m}^{-2}. Such a value is entirely typical of the large-scale ionospheric field-aligned currents which similarly connect the Earth’s ionosphere and magnetosphere (e.g. Iijima and Potemra, 1978). Some implications of this estimate will be discussed in Section 5.6.

5.4. Separation of the equatorial current systems

The aim of this section, and that following, is to provide a physical interpretation of the results derived above on the equatorial and field-aligned current systems observed during the four spacecraft passes in terms of the motion of the magnetospheric plasma. As indicated in the introduction, we base our considerations on a physical model in which the total current system consists of two components. The first is a system of currents which flow on closed (divergence-free) paths around the planet in the equatorial current sheet (and possibly over the magnetopause at larger radial distances). This system is dominated by the azimuthal equatorial current related to radial stress balance, but because these currents are local time dependent, this system also involves radial currents required to maintain current continuity. The second is a system confined (approximately) to magnetic meridian planes, which consists of field-aligned currents flowing between the ionosphere and magnetospheric equatorial plane, which are closed by radial currents in the equatorial current sheet, and latitudinal Pedersen currents in the ionosphere. This system is associated with magnetosphere-ionosphere coupling and the partial enforcement of corotation (e.g. Hill, 1979; Vasyliunas, 1983), and can be interpreted in terms of the angular velocity profile of the magnetospheric plasma via the theory to be derived in the next section. In this section we first accomplish the separation of the total observed current, thus determining the radial current contribution which is associated with the magnetosphere-ionosphere coupling current system.
We begin by pointing out that any equatorial current system can in principle be divided into two systems of the above nature. Suppose, for example, that the equatorial current intensity is \( i(\rho, \phi) \), consisting of radial and azimuthal components given by \( i_r(\rho, \phi) \) and \( i_\phi(\rho, \phi) \), respectively. Then we can determine the radial current \( i_{r,CS}(\rho, \phi) \), which, when combined with \( i_\phi(\rho, \phi) \), produces a total equatorial current \( i_{CS}(\rho, \phi) \) which is entirely divergence-free. From Eq. (5.7), \( i_{r,CS}(\rho, \phi) \) must satisfy

\[
\frac{\partial}{\partial \rho} (\rho i_{r,CS}) = -\frac{\partial \varphi}{\partial \phi},
\]

and, integrating at fixed azimuth, we then find

\[
\rho i_{r,CS} = -\int_0^\rho d\rho' \frac{\partial \varphi}{\partial \phi}.
\]

We can then subtract this radial current from the total radial current to find the radial component which is associated with the magnetosphere-ionosphere coupling circuit. We note that the absolute value of \( \rho i_{r,CS} \) in Eq. (5.14) has been determined by requiring the value to go to zero at \( \rho = 0 \), i.e. that there are no unphysical "sources" of radial current at the origin. In practice, however, the lower limit of the integral need not be taken to be zero, but some finite radial distance where either the azimuthal current becomes axi-symmetric or goes to zero. In the present case, where we employ the BC model of \( B_{pr} \) to determine \( i_\phi \) via Eq. (5.8), the azimuthal current is taken to be axi-symmetric inside a radius of \( \rho_0 \approx 18.8 \text{ R}_\oplus \), and so the lower limit is taken to be \( \rho_0 \). Thus introducing Eqs. (5.8) and (5.9) into Eq. (5.14), and performing the integral from \( \rho_0 \) to \( \rho \), we then find

\[
\rho i_{r,CS} = \frac{2\alpha A \sin \varphi}{\mu_\star} \int_0^{\rho_0} d\rho' \ln \left( \frac{\rho_0}{\rho'} \right) \left( \frac{\rho_0}{\rho} \right)^{m(\rho)} - \frac{2\alpha A \sin \varphi \rho_0}{\mu_\star (m-1)} \left[ \frac{1}{(m-1)} \left( 1 - \left( \frac{\rho_0}{\rho} \right)^{m-1} \right) + \left( \frac{\rho_0}{\rho} \right)^{m-1} \ln \left( \frac{\rho_0}{\rho} \right) \right].
\]

This equation is only valid for \( \rho \geq \rho_0 \); for \( \rho < \rho_0 \), \( \rho i_{r,CS} = 0 \).

In Fig. 5.7 we show the "streamlines" of the divergence-free component of the equatorial current, \( i_{CS} \), thus determined from the BC model. The streamlines are marked with values showing the total amount of current carried in the current sheet between that streamline and radius \( \rho_0 \), the inner radius of validity of the model where the current becomes axi-
Figure 5.7. Streamlines of the divergence-free component of the equatorial current, $i_{CS}$, determined from the Bunce and Cowley (2001a) empirical model, described in Chapter 4. The streamlines are shown by the solid lines and are marked with values showing the total amount of current carried in the current sheet between the streamline concerned and that at radius $\rho_0 = 18.8 \ R_J$ (the innermost solid line marked), where the model current becomes azimuthally symmetric. These lines are shown at equal intervals of 10 MA, so that the distance between them gives an indication of the current intensity. The dashed lines indicate distance from the centre of the planet in steps of 10 $R_J$, from 10 to 50 $R_J$, the outer edge of the region of validity of the model. Local times are as marked.
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symmetric. This has been determined by contouring the current stream-function which corresponds to the BC model, which is readily shown to be

\[
I_{CS}(\rho, \phi) = \frac{2A\rho_0}{\mu_0(m(\phi))^{-1}} \left[ 1 - \left( \frac{\rho \phi}{\rho} \right)^{n(\phi)-1} \right].
\]  
(5.16)

This function (having units of A) is such that

\[
i_{CS} = -\frac{1}{\rho} \frac{\partial I_{CS}}{\partial \phi} \quad \text{and} \quad i_{\phi, S} = \frac{\partial I_{CS}}{\partial \rho},
\]  
(5.17)

so that \( I_{CS}(\rho, \phi) \) is constant on a current streamline (i.e. \( (i_{CS}, \nabla I_{CS}) = 0 \)), while the total current flowing between contours \( I_{CS} \) and \( I_{CS} + dI_{CS} \) is just \( dI_{CS} \). The figure illustrates the spreading of the current contours on the dayside compared with the nightside, corresponding to the observed weaker azimuthal current intensities on the dayside compared with the nightside, and the associated “closure” radial currents directed outwards at dawn and inwards at dusk.

Having thus determined the radial current which combines with \( i_\phi \) to give zero current divergence, we can now subtract this from the observed total radial current. In our physical model this remainder is then the radial current associated with the magnetosphere-ionosphere coupling circuit, \( \rho_{\text{pi}} \), which carries all the equatorial current divergence associated with the magnetosphere-ionosphere field-aligned current system. That is, we define

\[
\rho_{\phi, \text{M}I} = \rho_{\phi} - \rho_{i_{\phi, S}} = \rho_{\phi} - i_{\phi, S}.
\]  
(5.18)

where

\[
j_z = -\frac{1}{2} \text{div}(i_{\phi, M} \rho) = -\frac{1}{2\rho} \frac{\partial}{\partial \rho}(\rho_{\phi, M}).
\]

In Fig. 5.8 we therefore show, in linear-scale format for each pass, the variation with radial distance of the total radial current \( \rho_{\phi} \) (dashed lines, derived directly from the fits shown previously in Fig. 5.4), the “current sheet” contribution \( \rho_{i_{\phi, S}} \) obtained from Eq. (5.15) (dot-dashed lines), and the “magnetosphere-ionosphere” contribution \( \rho_{\phi, M} \) obtained as the difference between these two (solid lines). We note from Eq. (5.18) that the gradient of the latter curves are directly related to the north-south current \( j_z \) shown previously in the lower panels of Fig. 5.5. In each case the outer lines of each set of three indicate the uncertainty estimates for each quantity. For the total current these have been estimated by calculating
Figure 5.8. Plot of the total radial current per radian of azimuth \( \rho_\rho \) (dashed lines) versus radial distance \( \rho \), in a linear scale format for the four spacecraft as shown previously. Also shown are the "current sheet" contribution \( \rho_\rhoCS \) obtained from Eq. (5.15) (dot-dashed lines), and the "magnetosphere-ionosphere" contribution \( \rho_\rhoMI \) obtained as the difference between these two (solid lines). In each case the outer lines of each set of three show the uncertainty estimates for each quantity.
the RMS variance of the residual $\rho_{ip}$ values from the best-fit line in Fig. 5.4. The uncertainty in the "current sheet" contribution has been obtained by combining together the individual errors of the model parameters quoted above in relation to Eq. (5.9). Finally, for the "magnetosphere-ionosphere" contribution we express the uncertainty as the root of the sum of the squared uncertainties contributing from the two sources.

It can be seen in Fig. 5.8 that the radial "current sheet" contribution $\rho_{ip}$ (dot-dashed lines) is negligibly small near the inner edge of the region of interest (i.e. 20 RJ). This arises from the fact that in the BC model the azimuthal current system is taken to be axisymmetric within 18.8 RJ, such that $\rho_{ip}$ = 0 at and within this distance, as previously indicated. However, these currents then grow to significant amplitude at larger distances within the region, consistent with our previous conclusion about the significance of the magnitude of "div $i_p$" in Fig. 5.5, compared with "div $i_p$". For the inbound Pioneer-11 pass in panel (a), the "current sheet" contribution increases gradually with distance at a rate similar to that of the total radial current, reaching a value of ~10 MA rad$^{-1}$ at ~50 RJ. Hence when this is subtracted from the total radial current, we find a "magnetosphere-ionosphere" contribution which is essentially constant with distance, at ~15 MA rad$^{-1}$. The near-constancy of this current is reflected in the negligibly small values of the field-aligned currents derived previously and shown in Figs. 5.5 and 5.6. For outbound Pioneer-10 shown in panel (b), we find the largest contribution to $\rho_{ip}$ of all the spacecraft passes. This is due to the local time of the pass being close to the dawn meridian, which is near the point of maximum divergence of the azimuthal current in the BC model. At ~50 RJ the contribution is ~18 MA rad$^{-1}$, almost twice that on the Pioneer-11 pass. In this case we therefore find that the "magnetosphere-ionosphere" contribution falls strongly with distance from ~16 MA rad$^{-1}$ at 20 RJ, to ~5 MA rad$^{-1}$ at 50 RJ. This fall is related to the significant "return" field-aligned currents deduced previously for this pass, as shown in Figs. 5.5 and 5.6. The results for Voyager-1 outbound shown in panel (c) also indicate a significant contribution from the radial "current sheet" contribution, reaching ~15 MA rad$^{-1}$ at ~50 RJ. Even so, $\rho_{ip}$ is still found to increase sharply with distance from the planet, reaching ~23 MA rad$^{-1}$ at 50 RJ, thus requiring significant field-aligned current input to the equatorial sheet from the ionosphere, as seen in Figs. 5.5 and 5.6. Finally the Voyager-2 outbound data in panel (d) exhibits a much smaller contribution from the "current sheet" current. This is due to the local time of the pass being near the
noon-midnight meridian (see Fig. 5.1), which is the assumed axis of symmetry of the “current sheet” model where the radial current goes to zero (Fig. 5.7). Consequently, $\rho_{\parallel}$ differs from the total radial current by only a small amount on this pass, with the steeply-increasing values with increasing distance thus again requiring significant field-aligned current input from the ionosphere.

In concluding this discussion, we again emphasise the rather differing physical conditions prevailing during the Pioneer passes compared with the Voyager passes, as deduced from the structure of the radial “magnetosphere-ionosphere” currents. In the next section we will interpret this behaviour directly in terms of the prevailing angular velocities of the magnetospheric plasma.

5.5. Plasma angular velocity and ionospheric conductivity

As discussed previously by Hill (1979) and Vasyliunas (1983), the current flowing in the magnetosphere-ionosphere coupling circuit is directly related to the angular velocity of the magnetospheric plasma, and the Pedersen conductivity of the ionosphere to which it is magnetically connected. In this section we first review the derivation of the expression which relates these parameters, specialising the discussion given by Vasyliunas to the conditions appropriate to the jovian environment (i.e. the case of small radial outflow). We will then use the radial profiles of the radial “magnetosphere-ionosphere” current $\rho_{\parallel}$ determined in the previous section to derive angular velocity profiles of the plasma in the equatorial plane for various assumed values of the ionospheric conductivity. Comparison with observed angular velocity values then allows us to set some useful limits on the conductivity.

The magnetic configuration is shown in Fig. 5.9, together with a simple representation of the “magnetosphere-ionosphere” current circuit (following e.g. Hill (1979) and Vasyliunas (1983)). We consider a non-rotating frame and employ cylindrical coordinates referenced to the magnetic axis. The solid lines indicate the magnetic field lines, which we take to be approximately axisymmetric, at least locally within a given sector of local time. The dashed lines show the direction of the current flow, specifically for the case of sub-corotating plasma (the current direction reverses for the case of super-corotation). In the
Figure 5.9. Sketch showing the configuration of the magnetosphere-ionosphere coupling current system after Hill (1979) and Vasyliunas (1983). The solid lines show the magnetic field, while the dashed lines show the current. The equatorial magnetospheric radial current $i_{\rho M}$, the field-aligned current $j_{||}$, and the closure Pedersen current in the ionosphere $i_p$ are indicated. Three separate angular velocities are shown, the angular velocity of the planet, $\Omega_j$, the angular velocity of a given "shell" of magnetic field lines, $\omega$, and the angular velocity of the neutral atmosphere in the Pedersen conducting layer, $\Omega^*_{j}$. 
low-density region between the current sheet and the ionosphere, currents flow as Birkeland field-aligned-currents \( J_b \), which are directed from the ionosphere to the current sheet in the inner part of the system, and away from the current sheet towards the ionosphere in the outer part. These currents are closed by outward radial currents carried by the magnetospheric plasma in the equatorial current sheet at one end, and by equatorward-directed ionospheric Pedersen currents at the other. The relationship to which the continuity condition for this circuit gives rise will be derived below.

Three separate angular velocities are distinguished, as also shown in Fig. 5.9. The first is the angular velocity of rotation of the planet, \( \Omega_p \), which to a sufficient approximation we take to be aligned with the magnetic axis (\( \Omega_p \approx 1.76 \times 10^{-4} \text{ rad s}^{-1} \)). The second is the angular velocity of the plasma on a given “shell” of magnetic field lines, \( \omega \), which we take to be constant along these field lines in the steady state. That is, we assume each shell rotates rigidly without time-dependent distortions taking place, though the shells may rotate differentially with respect to each other. Sub-corotation of the plasma (as anticipated) on a given shell corresponds to the condition \( \omega < \Omega_p \). The third is the angular velocity of the neutral atmosphere in the Pedersen conducting layer of the ionosphere, \( \Omega_j^* \), which can differ from the angular velocity of the planet \( \Omega_p \) due to the torque associated with ion-neutral frictional drag (Huang and Hill, 1989). In this case, we expect that \( \Omega_j^* \) will take a value which is intermediate between \( \omega \) and \( \Omega_p \).

As indicated above, we consider a near-axisymmetric magnetic field whose principal poloidal components can therefore be described by a vector potential \( A \) which has only an azimuthal component \( A = A_\phi \hat{\phi} \). The flux function \( F \) for such a field is given by \( F = \rho A_\phi \), where, as before, \( \rho \) is the perpendicular distance from the dipole axis. This function is such that a particular field shell is given by \( F = \text{constant} \) (and a particular poloidal field line by \( F = \text{constant} \) and \( \phi = \text{constant} \)), while the magnetic flux \( d\Phi \) per radian of azimuth between the field shells \( F \) and \( F + dF \) is given by \( d\Phi = dF \). Knowledge of the flux function allows us to map field lines between the magnetospheric equator and the jovian ionosphere. In general, \( F \) is given by the sum of contributions from the internal planetary field, taken to be approximated by the dipole term only, and the contribution of the external currents, principally that due to the equatorial current sheet (within the jovian middle magnetosphere region). The dipole term is given by
where \( r \) is the joviancentric radial distance, \( B_J \) is the jovian equatorial magnetic field strength (taken to be \( 4.28 \times 10^5 \) nT in conformity with the VIP 4 internal field model), and, as before, \( R_J \) is Jupiter's radius (taken to be 71,373 km). Note that in writing this expression, the absolute value of \( F \) has been fixed by taking it to be zero on the magnetic axis, as we will do throughout. Near the surface of the planet the dipole term is overwhelmingly dominant compared with the external term, such that putting \( r \approx R_J \) to a sufficient approximation, we have the value of the flux function in the ionosphere given by

\[
F_i \approx B_J \rho_1^2, \tag{5.20}
\]

where \( \rho_1 \) is the perpendicular distance from the magnetic axis. For a magnetospheric field line of flux function \( F \), the ionospheric mapping is thus simply given by \( F = F_i \), which determines the value of \( \rho_1 \) on the field line, or equivalently the magnetic co-latitude of the field line at the planet's surface.

We now consider the implication of the current continuity requirement of the "magnetosphere-ionosphere" circuit shown in Fig. 5.9. If we consider, for example, the region between flux shells \( F \) and \( F + dF \), it is evident that the total radial current flowing in the equatorial plane in angular sector \( \Delta \phi \) must be equal to the sum of the northern and southern ionospheric Pedersen currents flowing on the same field lines in the same angular sector. That is

\[
\rho_{p,M} \Delta \phi = (i_{PN} + i_{PS}) \rho_1 \Delta \phi, \tag{5.21}
\]

where \( i_{PN} \) and \( i_{PS} \) are the height-integrated equatorward Pedersen current intensities (A m\(^{-1}\)) in the northern and southern hemispheres, respectively. Mechanically, this condition implies that the magnetic torque on the equatorial plasma on a given flux tube is equal and opposite to the summed magnetic torques on the ionospheric plasma in the two hemispheres. The latter torques just balance the torque on each ionosphere due to ion-neutral frictional drag, such that the sum of these torques is just equal to the torque on the magnetospheric plasma. Here, for simplicity, we assume similar conditions in the two hemispheres, such that the conductivities and currents are equal. Our expressions are easy to generalise to the case of asymmetric ionospheric conditions, but no purpose is served in adding this complexity for present purposes. In this case we therefore have

\[
i_p = i_{PN} = i_{PS} = \Sigma_p E, \tag{5.22}
\]
where $\Sigma_P$ is the height-integrated Pedersen conductivity of the conjugate ionosphere, and $E_i$ is the equatorward electric field in the rest frame of the neutral gas. The electric field is given by the expression $E_i = v_i B_j$, where $v_i$ is the westward ion flow in the neutral gas rest frame. $B_j \approx 2B_j$ is the polar ionospheric magnetic field strength, and we have assumed the field to be directed near vertically in the polar region. In terms of the angular velocities introduced above, $v_i$ is then given by

$$v_i = (\Omega_j^* - \omega)\rho_i , \quad (5.23)$$

so that the electric field is

$$E_i = (\Omega_j^* - \omega)B_j\rho_i \approx 2(\Omega_j^* - \omega)B_j\rho_i . \quad (5.24)$$

The ionospheric current intensity is therefore

$$i_p = 2\Sigma_P(\Omega_j^* - \omega)B_j\rho_i . \quad (5.25)$$

Substitution into Eq. (5.21), and use of Eq. (5.20), then gives for the field line whose flux function is $F$

$$\rho_{\text{p},\text{f}} \approx 4\Sigma_P(\Omega_j^* - \omega)B_j\rho_i \approx 4\Sigma_P(\Omega_j^* - \omega)F . \quad (5.26)$$

The equatorial radial current associated with the magnetosphere-ionosphere coupling circuit on a particular shell of field lines is therefore directly related to the ionospheric Pedersen conductivity at the feet of those field lines, and the difference between the plasma and atmospheric angular velocities on that magnetic shell.

The angular velocity of the atmosphere in the Pedersen layer, as effected by ion-neutral drag, is not at present a well-determined parameter, but in general we may suppose that in the planet’s rest frame it will be some fraction $k$ of the angular velocity of the plasma in that frame. That is, in the planet’s frame the induced wind speed will be some fraction $k$ of the plasma speed. We can therefore write

$$\left(\Omega_j - \Omega_j^*\right) = k(\Omega_j - \omega) , \quad (5.27)$$

for some $0 < k < 1$, or, rearranging

$$\left(\Omega_j^* - \omega\right) = (1-k)(\Omega_j - \omega) . \quad (5.28)$$

Substitution into Eq. (5.26) then gives

$$\rho_{\text{p},\text{f}} \approx 4(1-k)\Sigma_P(\Omega_j - \omega)F . \quad (5.29)$$

Since neither $k$ nor $\Sigma_P$ are well-known quantities, here we combine them to define

$$\Sigma_P^* = (1-k)\Sigma_P . \quad (5.30)$$
where $\Sigma_p^*$ is the "effective" Pedersen conductivity, which is reduced from the true value $\Sigma_p$ by the factor (1 - $k$) due to the "slippage" of the neutral gas in the Pedersen layer from strict corotation (Huang and Hill, 1989). We thus finally have

$$\rho_{\phi,II} \approx 4 \Sigma_p^* (\Omega_j - \omega) F .$$

(5.31)

Preliminary results based on the JIM model of the coupled jovian ionosphere-thermosphere (Achilleos et al., 1998), indicate that $k$ may be as large as ~0.5, or possibly higher (S. Miller, private communication, 2000). In this case the "effective" Pedersen conductivity $\Sigma_p^*$ may be half, or less, of the true value.

Eq. (5.31) provides the expression we require which relates the equatorial radial current associated with the "magnetosphere-ionosphere" current circuit, as determined in the previous section, to the angular velocity of the plasma and the conductivity of the conjugate ionosphere. Before proceeding, however, it is worth noting that the corresponding expression for the field-aligned current density associated with the circuit is given by

$$\frac{\dot{L}}{B} = -2 \frac{d}{dF} \left[ \Sigma_p^* (\Omega_j - \omega) F \right] .$$

(5.32)

This expression has been derived by differentiating Eq. (5.31) according to Eq. (5.18), use of Eq. (5.12), and noting that the equatorial $B_z$ field is related to equatorial flux function $F_e$ by

$$B_z = \frac{1}{\rho} \frac{dF_e}{dp} .$$

(5.33)

Equation (5.33) is obtained directly from $B = \text{curl} A$. In Eqs. (5.31) and (5.32) we thus regard all the variables, i.e. $\omega$, $\Sigma_p^*$, and $\rho_{\phi,II}$, as functions of the field line considered, and hence as functions of $F$, which is constant on a field line. Eq. (5.32) thus shows that the field-aligned current density in the circuit depends upon the variation across the field lines of the quantity $\Sigma_p^* (\Omega_j - \omega) F$.

In Fig. 5.10 we show how simple, physically plausible, variations of the parameters lead to the usual representation of the current system sketched in Fig. 5.9. Fig. 5.10a shows the expected behaviour of $(\Omega_j - \omega)$. In general we expect $(\Omega_j - \omega)$ to be small close to the planet where $\omega \approx \Omega_i$, and to increase towards $\Omega_j$ with increasing distance as corotation breaks down. The equatorial flux function $F_e$ is large close to the planet, and falls with
Figure 5.10. Sketches of the behaviour of the following parameters are shown versus jovicentric distance $\rho$ in the equatorial plane: (a) the angular velocity parameter $(\Omega_J - \omega)$; (b) the equatorial flux function $F_e$; (c) the combined function $\Sigma^*_p(\Omega_J - \omega)F_e$, with the effective Pedersen conductivity assumed approximately constant. In (c), the regions of negative and positive field-aligned current density in the northern hemisphere are indicated (opposite in the southern hemisphere).
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distance, as sketched in Fig. 5.10b, consistent with negative $B_z$ in the equatorial plane. Assuming that the effective Pedersen conductivity does not vary strongly with $F_e$, the combined function $\Sigma_p'(\Omega_j - \omega)F_e$ then increases with increasing $\rho$ for small $\rho$ (i.e. for $F_e$ large and decreasing), then peaks and falls with increasing $\rho$ for large $\rho$, as shown in Fig. 5.10c. As also indicated in Fig. 5.10c, the corresponding field-aligned current given by Eq. (5.32) will then be positive (out of the ionosphere into the current sheet) in the inner part of the system as the angular velocity departs from rigid corotation. It will then go to zero where $\Sigma_p'(\Omega_j - \omega)F$ reaches its maximum value, and then reverses in sense to negative (out of the current sheet and into the ionosphere) at larger distances as the function falls with decreasing $F$. This current pattern is thus consistent with that previously depicted in Fig. 5.9. We emphasise, however, that this represents only the simplest possible case, and more complex patterns of current are clearly possible if either $\omega$ or $\Sigma_p'$ vary in more complex ways. In addition, in an open magnetosphere (i.e. one with an extended magnetic tail) only part of the field-aligned current pattern shown in Figs. 5.9 and Fig. 5.10c may be expressed. In the simplest case of a closed axisymmetric system, conservation of magnetic flux ensures that the flux function $F$ must go to zero in the equatorial plane at the magnetopause. In this case the full current pattern will indeed be expressed, it being easy to show by integration of Eq. (5.32) that (irrespective of all details) the total field-aligned current flowing in any angular sector sums exactly to zero. In an open magnetosphere, however, $F$ will remain finite at the magnetopause, in which case the field-aligned current flowing into the current sheet in the inner part in a given angular sector will not generally be balanced by the return current in the outer part. Indeed, initial investigation using simple physically plausible models to be reported elsewhere, indicates that the current flow can be directed consistently from the ionosphere into the current sheet out to large distances in excess of $\sim 100$ R$_J$. In this case, then, the return current must occur at the outer edge of the current sheet, in the region adjacent to the outer magnetosphere layer and magnetopause.

Here, however, our primary line of enquiry will be directly through the equatorial radial current $\rho_{\parallel}$ and Eq. (5.31), rather than its derivative which gives the field-aligned current density via Eq. (5.32), though our results will, of course, be entirely consistent with the latter. Rearranging Eq. (5.31) then gives an expression for the angular velocity of the plasma in terms of the equatorial radial current.

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\[
\frac{\omega}{\Omega_j} \approx 1 - \frac{\rho_{\text{altf}}}{4 \zeta_j \Omega_j F_i} .
\]  

(5.34)

Using this formula we can now compute radial profiles of the angular velocity of the equatorial plasma, for given \( \Sigma R^* \), from the radial profiles of the radial current \( \rho_{\text{altf}} \) shown in Fig. 5.8. To do this, however, we first need to calculate the value of the equatorial flux function \( F_e \) over the radial range of interest, from 20 to 50 R\( J \). As indicated above, \( F_e \) is obtained from the sum of dipole and current sheet contributions, with the absolute value being fixed by taking its value to be zero on the axis of magnetic symmetry. Here, the value of \( F_e \) at the inner edge of the region of interest, \( \rho_0 = 20 \) R\( J \), has been calculated from the sum of the dipole term \( F_{e \text{ dip}} = B_j R_j / \rho_0 \), given by Eq. (5.19), and a current sheet term obtained from the model due to Connerney et al. (1981). An approximate form for the vector potential of the latter model has recently been derived by Edwards et al. (2001), and has been employed in this calculation. Specifically we have used the Connerney et al. “Voyager-1/Pioneer-10” model parameters for the outbound Voyager-1 and Pioneer-10 passes investigated here, and the “Voyager-2” parameters for both the outbound Voyager-2 and the inbound Pioneer-11 passes. Although no detailed empirical fits to the Pioneer-11 inbound data have been published hitherto, investigation shows that the “Voyager-2” model fits these data tolerably well. Beyond the inner edge of the region of interest we have obtained \( F_e \) by integrating the empirical equatorial \( B_z \) field models for that pass. Integrating Eq. (5.33) we find

\[
F_e(\rho) = F_e(\rho_0) + \int_{\rho_0}^\rho d\rho B_z .
\]  

(5.35)

To evaluate the integral we have used the same \( B_z \) models for each pass as those employed above in relation to the field-aligned current calculation and Eq. (5.12), as given in the Section 3.4 of Chapter 3. Full details of the calculation of \( F_e \) are also given there. Here we summarise in Fig. 5.11, where we show a log-linear plot of the equatorial flux function \( F_e \) (T m\(^2\)), for each of the four spacecraft passes between radial distances of 20 and 50 R\( J \). Specifically, values for the inbound Pioneer-11 pass are shown by the long-dashed line, outbound Pioneer-10 by the solid line, outbound Voyager-1 by the dot-dashed line, and outbound Voyager-2 by the short-dashed line. The dotted line shows the dipole value for purposes of comparison; the elevation of the model \( F_e \) values above the dipole value indicates the degree of inflation of the field lines away from the planet by the equatorial plasma currents. It can be seen that the Pioneer-10 and Voyager-1 values are equal at 20 R\( J \), as are the Pioneer-11 and Voyager-2 values. This results from using the same
Figure 5.11. Log-linear plot of the equatorial flux function $F_e \,(T\,m^2)$, for each of the four spacecraft passes between radial distances of $20\,R_J$ and $50\,R_J$. Pioneer-11 inbound values are shown by the long-dashed line, Pioneer-10 outbound by the solid line, Voyager-1 outbound by the dot-dashed line, and Voyager-2 by the short-dashed line. The dotted line shows the dipole value for purposes of comparison.
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Connerney et al. models for these passes at the inner boundary of the region, as indicated above. As $\rho$ increases, the value of $F$ slowly falls in each case, as previously sketched in Fig. 5.10b. Overall, similar slowly-varying values are obtained for each pass, such that the details of the models employed are not expected to strongly influence the results to be derived from Eq. (5.34).

Combining the results for $F_e$ shown in Fig. 5.11 with those for $\rho_{\text{pl/ll}}$ shown in Fig. 5.8, we are now able to use Eq. (5.34) to derive the radial variation of the equatorial angular velocity of the magnetospheric plasma that would give rise to the observed current distribution, for given values of the effective ionospheric Pedersen conductivity. The angular velocity profiles for the four spacecraft passes are presented in Fig. 5.12, where we plot $\omega' \Omega_i$ versus equatorial radial distance, in the same multi-panel format as employed in previous figures. The solid lines give our best estimates of the values, while the dot-dash lines indicate the uncertainty estimates which have been carried through from the values of $\rho_{\text{pl/ll}}$ shown previously in Fig. 5.8. The values of the “effective” Pedersen conductivity employed are, from the top to the bottom of each plot, 10, 5, 2, 1, 0.5, 0.2, 0.1, and 0.05 mho, respectively. The curve corresponding to 0.5 mho has been marked on the right hand side to ease identification.

We thus emphasise at the outset that the angular velocity profiles shown here correspond to those which would produce the deduced $\rho_{\text{pl/ll}}$ profiles (and field-aligned current distributions) if the effective ionospheric Pedersen conductivity $\Sigma_p^*$ is assumed fixed at the values shown. If, however, the effective Pedersen conductivity varies with latitude, and hence with $F$ (due e.g. to auroral precipitation), then effectively we would move from one curve to another as the distance changes, such that the true angular velocity profile could then differ significantly from those which are shown. Indeed, in principle we could nominate essentially any angular velocity profile, based either on observations or additional theoretical considerations, and then view Fig. 5.12 as indicating the variation of $\Sigma_p^*$ with distance (latitude) which is required for consistency with the inferred currents.

The results shown in Fig. 5.12 should not, therefore, be regarded as being necessarily deterministic of the actual angular velocity variations on these passes, a point to which we return in detail at the end of this section. Nevertheless, a number of useful deductions are
Figure 5.12. Plot of the angular velocity profile of the magnetospheric plasma, $\omega/\Omega_j$ as a function of equatorial distance $\rho$, for various values of the effective Pedersen conductivity, for the four spacecraft passes in the same multi-panel format as in previous figures. The solid lines give our best estimate of the values, while the dot-dash lines indicate the uncertainty estimates which have been carried through from the values of $\rho p_M$ shown in Fig. 5.8. The values of the "effective" Pedersen conductivity employed are, from the top to the bottom of each panel respectively, 10, 5, 2, 1, 0.5, 0.2, 0.1, and 0.05 mho. A label is shown on the right of each panel to indicate the 0.5 mho case.
possible, and we first consider the general dependence of the results on the assumed value of $\Sigma p^*$. In each plot it can be seen that the estimated angular velocity is close to rigid corotation ($\alpha \Omega_\perp \approx 1$) at all distances for the larger assumed values of the conductivity, while decreasing continuously towards lower values as the conductivity decreases. This is a consequence of the fact that as the assumed conductivity is decreased, larger atmosphere-ionosphere relative flows are required to drive the observed currents in the equatorial plane. For the conditions of the jovian magnetosphere, our results show that the observed currents are consistent with very small departures from rigid corotation if the "effective" Pedersen conductivity is as high as $\sim 2-10$ mho (or higher). Although the details vary from pass-to-pass, the inferred angular velocities are indicative of significantly sub-corotating plasma for conductivities between $\sim 0.2$ and $\sim 1$ mho. For conductivities of $\sim 0.1$ mho, or below, the plasma must actually anti-corotate over significant regions of the middle magnetosphere to produce the observed current, i.e. it must flow westward in the inertial frame, opposite to the direction of rotation of the planet.

The more detailed questions which these results prompt thus concern the expected values of both the plasma angular velocity and the ionospheric Pedersen conductivity, and we consider the former issue first. As indicated in the introduction, on the basis of Hill's (1979) theory, and in the absence of external forces, we expect the angular velocity of the plasma will be close to that of rigid corotation at sufficiently small distances, and will then fall towards small values with increasing distance, as $\rho^{-2}$ when the ionospheric torque becomes ineffective. Although observations of the plasma flow in the middle magnetosphere are sparse, and none have been published at all for the four passes investigated here over the radial range of interest, those data which have been published are in rough conformity with this expectation. Specifically, thermal plasma observations during the inbound passes of Voyagers-1 and -2 indicate near-rigidly corotating plasma with $\alpha \Omega_\perp \approx 0.8$ between 10 and 20 $R_J$, falling to $\alpha \Omega_\perp \approx 0.5$ at 40 $R_J$ (Belcher, 1983; Sands and McNutt, 1988). At the larger distances $\sim 30-50$ $R_J$ on the Voyager-2 inbound pass, Kane et al. (1995) have also derived values of $\alpha \Omega_\perp \approx 0.5-0.6$ from the anisotropies of energetic ions. On the Voyager-2 outbound pass they similarly derive values of $\alpha \Omega_\perp \approx 0.5$ at $\sim 70$ $R_J$, beyond the radial range considered here. Overall, therefore, on the basis of these results we may expect flows which are relatively close to rigid corotation at the inner boundary of our region of interest, and which then fall to values of half of rigid corotation, or perhaps a little above, at the outer boundary. Comparing this expectation with the
derived profiles in Fig. 5.12 indicates that effective Pedersen conductivity values above ~1 mho require flows which are too close to rigid corotation throughout, while those below ~0.2 mho require either flows which are too small, or unphysical anti-corotational flows. Generally, the results which most closely resemble those expected on the basis of the (admittedly limited) observational evidence are those derived for ~0.5 mho (say, in the range ~0.3-0.8 mho). If then the atmospheric “slippage” factor is $k \sim 0.5$ or more, as indicated above, the inferred values of the actual Pedersen conductivity given by Eq. (5.30) is ~1 mho or more (i.e. in the range ~0.6-1.6 mho, or larger). Even so, some of the deduced angular velocity profiles are somewhat contrary to expectations, particularly the profiles inferred for Pioneer-10, which show increasing angular velocities with increasing distance from the planet. We will return to a more detailed discussion of this topic below.

Here, we will next enquire about the conductivity values that are expected on the basis of the properties of the jovian ionosphere. On the basis of Pioneer and Voyager radio occultation measurements, combined with a theoretical collision model, Huang and Hill (1989) estimated a value of the “actual” height-integrated Pedersen conductivity of ~0.3 mho, lower by factors of 2-5 than those inferred here. However, as Strobel and Atreya (1983) have pointed out, the electron density in the Pedersen layer, and hence the height-integrated conductivity, will depend significantly on the degree of auroral electron precipitation. For solar illumination only, they estimate a conductivity of only ~0.02 mho. Under conditions of intense electron precipitation, however, the conductivity may be enhanced to ~2 mho, or even higher. Consequently we infer on the basis of our above ~1 mho estimate that the ionosphere to which the spacecraft were connected in this middle magnetosphere region was subject to elevated Pedersen conductivities arising from electron precipitation. The electron precipitation in question is undoubtedly that associated with the main jovian auroral oval and adjacent regions, which is known to be approximately conjugate to the equatorial region considered here (e.g. Connerney et al., 1996; Prangé et al., 1998; Clarke et al., 1998). We note that the values of the equatorial flux function shown in Fig. 5.11 indicate a mapping of the equatorial region between 20 and 50 R_J to an ionospheric region which is only ~1°-2° of latitude (~2000 km) wide, centred near a dipole co-latitude of ~16°. The latter is approximately where the oval is observed. The measured width of the oval, determined as the FWHM of the optical/UV intensity, is typically a few hundred km, somewhat less than that of the region considered here. However, lesser intensities of emission appear to extend over much larger distances.
of one to two thousand km about the primary arc (Prangé et al., 1998; Vasavada et al., 1999). Thus the inference here of elevated and variable ionospheric conductivities over such a region appears entirely plausible.

As indicated above, we finally consider in more detail our results on the angular velocity profiles of the equatorial magnetospheric plasma shown in Fig. 5.12. To aid this discussion, in Fig. 5.13 we reproduce in one plot the angular velocities for the four passes deduced for a single common value of the effective ionospheric Pedersen conductivity, taken here to be $\Sigma_p^* = 0.5$ mho, in line with the above discussion. It can be seen that the Voyager profiles are very similar to each other, and essentially in line with expectation, falling continuously from $\omega/\Omega_t \approx 0.9$ at $-20 R_J$, to $\omega/\Omega_t \approx 0.5$ at $-50 R_J$ (the value of $\Sigma_p^*$ was of course chosen with these rough values in mind). If, however, we now compare these profiles with those for the Pioneer passes for the same value of $\Sigma_p^*$, we see some significant differences. Specifically, the angular velocity of the plasma in the inner part of the region is significantly smaller than those deduced from the Voyager passes, with $\omega/\Omega_t \approx 0.7$. These values then fall less rapidly with distance (Pioneer-11), or even increase with distance (Pioneer-10), to become larger than the Voyager values at larger distances. We could certainly suggest some dynamical processes which might account for such variations. The smaller angular velocities at smaller distances could be due to differing plasma mass loading rates from the Io torus, for example. In addition, the larger angular velocities at larger distances could be due to inward radial motion of the plasma as it sweeps round from the nightside to the dayside and is compressed to smaller distances by the magnetopause. Alternatively, and bearing in mind the cautionary discussion directly after the introduction of Fig. 5.12, the differing angular velocity profiles shown here could instead result from underlying differences in the effective Pedersen conductivity conditions during the flybys. It is not in principle possible to distinguish between these alternatives on the basis of this discussion alone. Specifically, we could roughly transform the Pioneer profiles shown here into the Voyager profiles by taking an effective Pedersen conductivity which fell from $-1$ mho at $-20 R_J$ to $-0.2$ mho at $-50 R_J$ in the former case (see Fig. 5.12). Indeed, such a falling conductivity profile might be consistent with the reversal in the sense of the field-aligned current deduced on the Pioneer passes (see Figs. 5.5 and 5.6), from a current directed out of the ionosphere in the inner part of the
Figure 5.13. Summary of the angular velocity profiles shown in the previous figure, now using a common value of 0.5 mho for the effective Pedersen conductivity for each of the four spacecraft. The solid lines show the best estimates for each pass, while the coloured bands again indicate the level of uncertainty in each case, and also identify the spacecraft.
region (corresponding to auroral electron precipitation), to either small values or a reversed flow into the ionosphere in the outer part.

5.6. Summary and conclusions

In this chapter we have examined the properties of the radial current intensity within the equatorial current sheet in Jupiter's magnetosphere, obtained from averages of the azimuthal field component $B_\phi$ measured outside the sheet. Observations during four near-radial equatorial cuts through the current sheet have been analysed, for which a useful radial current profile could be derived over the range of jovicentric distances between ~20 and 50 R$_J$. These data were obtained during the inbound flyby pass of Pioneer-11, and the outbound passes of Pioneer-10, Voyager-1, and Voyager-2. These data provide coverage of the dawn sector of the jovian magnetosphere, between ~0900 MLT in the pre-noon sector (inbound Pioneer-11), via dawn at ~0430 and ~0330 MLT (outbound Pioneer-10 and Voyager-1 respectively), to ~0100 MLT in the post-midnight sector (outbound Voyager-2). The results show that the radial current intensity per radian of azimuth $\rho I_\rho$ (A rad$^{-1}$), the quantity required to evaluate the divergence, increased with distance $\rho$ on all four passes, thus requiring some source to maintain current continuity. For the Pioneer passes, the radial current is found to be already large at the inner edge of the region investigated, with a value of ~15 MA rad$^{-1}$ at ~20 R$_J$. The current then increased modestly with distance, as $\sim \rho^{0.5}$, reaching values of 20-25 MA rad$^{-1}$ at ~50 R$_J$. For the Voyager passes, the current at ~20 R$_J$ is found to be about half this value, ~7 MA rad$^{-1}$, but then increased much more quickly, as $\sim \rho^{1.8}$, to 30-35 MA rad$^{-1}$ at ~50 R$_J$.

There are two possible sources of current which can account for these increases, namely diversion of the azimuthal current in the current sheet, and field-aligned current input from the ionosphere. The former of these sources was estimated from the empirical model of Bunce and Cowley (2001a), who investigated the azimuthal dependence of the radial field, and hence the azimuthal current, on these and other flyby passes. They found that the azimuthal current is largest on the nightside and decreases continuously via dawn to noon, thus potentially forming a source of outward radial current. Here it has been shown that this current divergence is comparable in magnitude to that of the radial current, and hence must in general be taken into account in evaluating the sense and magnitude of the field-aligned current. Indeed, for the inbound Pioneer-11 pass it was found that the divergence
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of the azimuthal current could account for the whole of the increase in the radial current over much of the range, with no field-aligned currents present (within uncertainty estimates), while for outbound Pioneer-10 the model azimuthal current source is larger than that required, suggesting the presence of "reversed" field-aligned currents on this pass, flowing from the current sheet to the ionosphere. For the Voyager passes, however, this is not the case. For the outbound Voyager-1 pass, the azimuthal current can provide only about one half of the source required, thus also requiring field-aligned current input from the ionosphere. The same is true for the outbound Voyager-2 pass, where the model azimuthal current source provides less than one third of the requirement. Estimates of the field-aligned current density in the latter cases indicates that \((j_B) \approx 2 \times 10^{-13} \text{ A m}^{-2} \text{nT}^{-1}\) at \(-20 \text{ R}_J\), increasing to \(-1 \times 10^{-12} \text{ A m}^{-2} \text{nT}^{-1}\) at \(-50 \text{ R}_J\).

These values also allow us to estimate the magnitude of the field-aligned current flow in the jovian ionosphere which is connected to the middle magnetosphere current sheet, since in the absence of field-perpendicular currents \((j_B)\) is constant along a field line. Taking an ionospheric field strength of \(-2B_J \approx 860,000 \text{ nT}\) yields estimates of the field-aligned current density of \(-0.2-0.8 \mu\text{A m}^{-2}\) at ionospheric heights. Such values are typical also of the ionospheric field-aligned current densities which couple the magnetosphere and ionosphere of the Earth. At the Earth, upward-directed currents of this order can generally be carried by downward-precipitating hot magnetospheric electrons without the need for substantial field-aligned acceleration; the latter is typically required in the terrestrial case if the currents exceed a few \(\mu\text{A m}^{-2}\). At Jupiter, however, the magnetospheric plasma in the region between the current sheet (containing cool relatively dense iogenic plasma) and the ionosphere is much more tenuous than at Earth, such that field-aligned acceleration is required to carry field-aligned currents at lower thresholds. Specifically, the results presented by Scudder et al. (1981) based on Voyager plasma data indicate densities of \(-0.01 \text{ cm}^{-3}\) in this region, with electron temperatures of 2-3 keV. Without acceleration, such an electron gas (assumed isotropic) can carry a field-aligned current of at most \(-0.01 \mu\text{A m}^{-2}\) at the ionosphere, this latter value corresponding to a full downward-going loss cone, and a completely empty upward-going loss-cone. The field-aligned current densities estimated above are one to two orders of magnitude larger than this, and hence certainly require substantial field-aligned voltages to accelerate the electrons downward. In fact, the results presented by Knight (1973) suggest the potential drops required are of order \(-15-60\) times the electron thermal energy, corresponding to voltages of \(-40-150\) kV.
In addition, the top of the acceleration region must reach to a significant altitude, beyond 2-3 \( R_J \) in order that the downward accelerated electron flux be high enough. The energy deposited in the ionosphere by these electrons will then be \( \sim 0.01-0.1 \) W m\(^{-2}\), comparable to the energy inputs into the brightest auroral forms at Earth. On the basis of the UV emission from the bright main auroral oval observed by the Hubble Space Telescope, Prangé et al. (1998) have estimated peak precipitated energy fluxes of \( \sim 0.01-0.2 \) W m\(^{-2}\). We therefore suggest a direct connection between the accelerated electron fluxes inferred here on the basis of our field-aligned current estimates, and the main auroral oval. We note that the region investigated here, between \( \sim 20-50 \) \( R_J \) in the equatorial plane, maps at the planet to a region \( \sim 2000 \) km wide centred near a dipole latitude of \( \sim 16^\circ \). This is indeed where the main oval is located, though the latitudinal width just given is wider than the values usually quoted for the primary arc (a few hundred km). We note, however, that the primary arc is generally set within a wider region of lesser emissions, while the peak currents and hence peak energy fluxes of precipitating electrons occupy less than the whole of the region we have investigated (see Fig. 5.6) (Prangé et al., 1998; Vasavada et al., 1999). On the basis of their location and inferred precipitating electron energy flux, we thus suggest that the field-aligned currents determined here, particularly those observed on the outbound Voyager passes, are those directly responsible for the main auroral oval emissions. Similar currents were presumably present during the Pioneer passes as well, but mainly confined to the region inside of \( \sim 20 \) \( R_J \) in the equatorial plane in these cases.

We have also interpreted the current systems measured here in terms of a physical model consisting of two contributions. The first contains all the azimuthal current associated with radial stress balance, together with just that source-free radial current which is required to make this current contribution overall divergence-free within the current sheet. The second contains the remaining radial currents within the current sheet, together with the system of field-aligned currents that connects the current sheet with the planetary ionosphere, which acts to exchange angular momentum between them. Following Hill (1979) and Vasyliunas (1983), simple theory can then be used to relate the latter radial current to the angular velocity of the magnetospheric plasma and the effective Pedersen conductivity of the conjugate ionosphere. We have therefore constructed radial profiles of the plasma angular velocity from the current values, for various values of the effective Pedersen conductivity. Overall we find reasonable results for the angular velocity profiles, provided that the effective Pedersen conductivity values are several tenths of a mho (in the
range, say, ~0.3-0.8 mho). In particular, the Voyager profiles then show falling angular velocities with increasing distance, from values of ~90% of rigid corotation at ~20 R_J, to values of ~50% of rigid corotation at ~50 R_J. However, because of “slippage” of the neutral atmosphere in the Pedersen conducting layer due to ion-neutral frictional drag, the actual values of the ionospheric Pedersen conductivity will be higher than just indicated, probably by a factor of two or more. The inferred conductivities are thus more likely to be ~0.6-1.6 mho. Such values require the ionospheric density to be significantly augmented by particle precipitation from the magnetosphere, just as inferred above from the field-aligned current estimates.

Using the same effective Pedersen conductivity estimates on the Pioneer passes, however, results in angular velocity profiles which start at significantly smaller values in the inner part of the region, ~70% of rigid corotation, and then remain either approximately constant with distance (inbound Pioneer–11), or even increase with distance (outbound Pioneer–10). The latter could be due to magnetospheric dynamics associated e.g. with the Io mass loading rate and solar wind-induced asymmetry effects. Alternatively, it could also be consistent with a falling angular velocity profile similar to those inferred from the Voyager passes, combined with an effective Pedersen conductivity in the conjugate ionosphere which decreases with equatorial distance. The latter could then be related to the reduced and “reversed” directions of the inferred field-aligned current.
Chapter 6

A simple empirical model of the equatorial radial field in Jupiter's middle magnetosphere, based on spacecraft fly-by and Galileo orbiter data

6.1 Introduction

In this chapter we return to the empirical models of the radial field and azimuthal current in Jupiter's middle magnetosphere region, at distances in the range 20-45 \( R_J \). We first of all compare the model derived previously in Chapter 4 and as published by Bunce and Cowley (Planet. Space Sci. 49, 261, 2001a) using Pioneer, Voyager, and Ulysses fly-by data, with a combined data set that now also incorporates data from the first twenty orbits of the Galileo orbiter.

The local time coverage of the five Jupiter fly-bys mentioned above, and the first 20 orbits of the Galileo mission (between 1996-1999) are shown in Fig. 6.1, where the spacecraft trajectories are shown in Jupiter Solar Orbital (JSO) coordinates, i.e. \( X (R_J) \) is positive sunwards, and \( Y (R_J) \) is orthogonal to \( X \) and in the plane of Jupiter’s orbit. The Pioneer and Voyager fly-bys covered the dawn sector of the magnetosphere from near noon (Pioneer-11 outbound) to post-midnight (Voyager-2 outbound), while Ulysses passed through the pre-noon sector inbound and made unique observations of the dusk meridian magnetosphere outbound. Presently available data from the Galileo mission extend from
Figure 6.1a. Trajectories of the first 20 orbits of the Galileo orbiter along with the five fly-by spacecraft relative to Jupiter, shown in Jupiter Solar Orbital coordinates. X points positive sunwards, and Y is orthogonal to X and in the plane of Jupiter's orbit. The solid line indicates the Galileo orbiter and the dashed lines indicate the fly-by spacecraft. The individual fly-by spacecraft are distinguishable by the varying symbols shown in the key. A heavy dashed line depicts a model bow shock, and a model magnetopause is shown by the heavy solid line. Both model positions are derived from the Voyager-2 data. The region of interest for this paper, 20-45 R_J, is highlighted by the grey annulus in the centre of the plot. This figure was kindly provided by Joe Mafi of the Planetary Data System, UCLA.
dawn through to midnight, and some way into the evening sector. The jovigraphic latitudes of these trajectories were near-equatorial in the main, except for the outbound passes of Pioneer-11 and Ulysses, which exited near noon at ~33°N and near dusk at ~37°S, respectively. Also shown in the figure are the positions of the magnetopause and bow shock as modelled from the Voyager-2 data (Ness et al. 1979b). The shaded region also indicates the domain of interest for this study, that is, the middle magnetosphere region between 20 and 50 Rj. On the dayside, the magnetopause extends on average to ~65 Rj as shown here, but is highly variable depending upon the upstream solar wind conditions. On the nightside the magnetospheric tail extends to ~3000 Rj and has a diameter of ~300 Rj (Ness et al. 1979c).

In recent independent studies, Bunce and Cowley (2001a) (presented in Chapter 4) using magnetometer data from the five fly-by missions mentioned above, and Khurana (2001) also incorporating data from the Galileo orbiter spacecraft, have shown that the azimuthal current in the outer middle magnetosphere depends upon local time. For example, at distances of ~40-50 Rj the current is approximately twice as strong at a given radial distance at midnight than at the same distance at noon. This phenomenon was first noticed by Goertz (1978) in a comparison of the Pioneer-10 inbound and outbound data. The differing gradients of radial field fall-off with distance at the two local times (~1000 MLT inbound and ~0500 MLT outbound for Pioneer-10) were discussed in terms of the asymmetrical compressive and confining effect the solar wind dynamic pressure has on the magnetosphere, compressing the flux tubes on the dayside but allowing them to stretch out on the nightside. This stretching further distends the magnetic field lines, hence increasing the azimuthal current, on the nightside. As discussed in Chapter 4, Bunce and Cowley (2001a) favour this interpretation, which then indicates that azimuthal current closure is enforced via radial currents flowing wholly within the current sheet, flowing away from the planet at dawn and towards the planet at dusk. Khurana (2001) prefers to attribute the divergence of the azimuthal current to an Earth-like partial ring current closing via “region-2 type” field-aligned currents, flowing towards the planet at dawn, closing through the jovian ionosphere and flowing away from Jupiter at dusk.

Whilst previous models of the middle magnetosphere current sheet have been based upon axial symmetry (e.g. Connerney et al., 1981; Khurana, 1992), and are indeed an excellent indicator of the jovian field in the inner region of the middle magnetosphere, it is
now evident that outside this region, roughly beyond ~20 R_j, the current is significantly dependent on MLT as outlined above. In Chapter 4 we derived a simple empirical model of the near-equatorial radial component of the field in the region between 20 and 50 R_j, valid for all magnetic local times, based on the fly-by data alone. This model (herein referred to as the BC model) serves as a useful empirical tool for modelling the middle magnetosphere, and in particular for quantifying the divergence of the azimuthal current. How much current is diverted out of or into the azimuthal current flow, combined with similar information on the radial current derived from the azimuthal component of the magnetic field, provides the necessary information from which the field-aligned currents (FACs) connecting to the ionosphere can be calculated, as described previously in Chapter 5 and by Hill (1979), Vasyliunas (1983), and Khurana and Kivelson, 1993. The nature of the FACs connects in turn with other important magnetospheric phenomena such as the jovian auroras and the decametric radio emission.

In this chapter we compare the BC model of the radial field $B_p$ with newly-available field data from the Galileo orbiter, as a function of both local time and radial distance. We show that while the BC model is generally in good accordance with the Galileo data, some refinements are nevertheless suggested that would bring the model into better accord with the combined fly-by and orbiter data set. We thus derive such a model, using techniques similar to those employed in Chapter 4. As seen in Fig. 6.1a, inclusion of this additional data enhances the overall coverage of the middle magnetosphere region. In particular, Galileo significantly increases the quantity of data in the dawn and pre-midnight sectors of the magnetosphere. However, the evolution of the Galileo orbits has not as yet provided new data from the dayside middle magnetosphere as perijove lies within the inner magnetosphere at this local time.

The work in this study has been submitted for publication in Planetary and Space Science.
Chapter 6: Model of the radial field from Galileo orbiter and spacecraft fly-by data

6.2. Data analysis

6.2.1. Current sheet field averages

We begin our study by presenting magnetic field vectors observed during the five jovian fly-bys and the first 20 orbits of the Galileo mission as discussed above. All data were supplied by the Planetary Data System at UCLA, at 10 s resolution for Pioneer-11 and Voyager-2, 48 s for Voyager-1, and 1 min for Pioneer-10 and Ulysses. Due to telemetry constraints the Galileo data are only available at high time resolution approximately half of the time, and as such there are two distinct time resolutions of magnetic field data. The real time survey (RTS) mode supplies data at 24 s time resolution, whilst the memory read out (MRO) mode provides 32 min averaged data.

As in Chapter 4, the VIP 4 planetary field model (Connerney et al., 1998) has been subtracted from the data to leave only those fields which are due to the external currents (principally the equatorial current sheet). In the case of the spacecraft trajectories lying close to the jovigraphic equatorial plane, the current sheet passes completely across the spacecraft twice per 10 h rotation period. Correspondingly, it can be seen in the magnetic field data that the radial field cycles between intervals of relatively steady positive and negative values, interspersed with periods of field fluctuation and reversal when the spacecraft crossed through the equatorial current sheet. However, in the case of the non-equatorial Pioneer-11 inbound (14°S), Pioneer-11 (33°N) outbound, Pioneer-10 outbound (11°N), and Ulysses (37°S) outbound passes, the measured field is generally dominated either by positive or negative radial components depending upon the latitude of the spacecraft, the former corresponding to a location north of the current sheet and the latter to the south. The radial field then exhibits depressed values and/or enhanced fluctuations indicative of hot plasma currents at ~10 h intervals when the spacecraft approached the magnetic equatorial plane. At other times, when the spacecraft were at larger distances from the equator, the fields are instead stronger and smoothly varying, indicating only weak local currents and a consequent location outside of the current sheet. Ignoring periods when enhanced magnetic variations are present, therefore, we have averaged the field components from both equatorial and non-equatorial passes over 30 min intervals, and take these values to represent conditions at the similarly averaged locations outside of...
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the current sheet. The signature of the changing latitude of the spacecraft outside the current sheet will be discussed further below.

Collectively, the data are shown in Fig. 6.1b. In order to indicate the overall current flow in the equatorial regions, we show the 30 min averages of the total field vectors projected onto the magnetic equatorial plane. The vectors have then been rotated through 90° to indicate the approximate direction of the corresponding equatorial current. To take account of the reversal in the equatorial field components across the current sheet, those fields measured north of the current sheet have been rotated 90° anticlockwise, while those measured in the southern hemisphere have been rotated 90° clockwise. In keeping with the previous study of Chapter 4, every effort has been made to ensure that averages were taken only when the spacecraft were outside of the current carrying region. Since we are interested in estimating the total azimuthal current, inclusion of reduced values obtained when the spacecraft in fact remained in the current-carrying layer would result in under-estimates of the total current. Hence we have chosen to exclude those Galileo data from the MRO mode, whose time resolution was too low to distinguish clearly between such times. If the current layer is then considered to be a quasi-infinite sheet with perturbation fields of equal magnitude but opposite direction on either side, a perturbation field of 10 nT corresponds to an azimuthal sheet current of intensity 1.1 MA R⁻¹.

The individual spacecraft in Fig. 6.1b are identifiable by comparison with Fig. 6.1a. The inbound passes of Pioneer-10 and -11, Voyager-1 and -2, and Ulysses are all in the pre-noon sector, and the outbound passes are all on the nightside, with the exception of Pioneer-11 outbound which is at noon. The Galileo passes (G1-2, C3, E4, E6, G7, C9-10, E11-12, E16-19) included in this study mainly lie between 0900 MLT and 0000 MLT. As described above, all passes are near-equatorial (within ±10° of the jovigraphic equator), with the principal exceptions being Pioneer-11 inbound and outbound, Pioneer-10 outbound and Ulysses outbound as noted above. We see in Fig. 6.1b that the sense of the azimuthal currents are eastward, associated with the radial distension of the magnetic field lines away from the planet in the middle magnetosphere. The larger values of the azimuthal current on the nightside at a particular distance compared with the dayside values are evident. Outward radial currents are also apparent on the dawn side of the magnetosphere, consistent with the magnetic field line “lagging” out of meridian planes. However, on the dusk side of the magnetosphere the outward (“lagging”) currents evolve
Figure 6.1b. Plot of the half-hour averages of the magnetic components measured outside the current sheet during the first 20 orbits of the Galileo orbiter and the five fly-bys of Pioneer-10 and -11, Voyager-1 and -2, and Ulysses, from which the VIP4 planetary field model (Connerney et al., 1998) has been subtracted. The averages have been projected onto the magnetic equatorial plane and rotated through 90° to indicate the approximate direction and strength of the corresponding current. Those fields measured north of the current sheet have been rotated 90° anti-clockwise, while those measured to the south have been rotated in a clockwise sense. Dashed lines indicate the distance from the centre of the planet (R_J), and local time is also shown. The individual spacecraft are identifiable by comparison with Fig. 1a. At the bottom right of the plot is the scale for 50 nT.
into inward ("leading") currents in the outer region at larger distances beyond ~40 R_J, which we take to be associated with solar wind induced effects including that mentioned above.

6.2.2. Latitude-correction of non-equatorial radial field data

Since the equatorial current sheet is of finite spatial extent, the radial field outside the sheet at a given distance will fall slowly with height above the sheet on either side. The field values which give the best indication of the total azimuthal current are those obtained at the outer edge of the sheet, while those obtained at higher latitudes will thus provide an under-estimate. An approximate correction for this effect using a simple theoretical model was implemented in Chapter 4, and in this chapter we follow the same procedure. The benefits of performing such a correction are dualistic. First, we reduce the latitude-related "scatter" in the radial field profiles, thus allowing a more accurate representation by least-squares fitting. Second, we allow inclusion of the non-equatorial data. We are required particularly to correct those data from the Pioneer-11 outbound and Ulysses outbound passes, if they are to be included in this study, but we should also note that much of the data in the fly-by profiles benefit from (albeit modest) corrections. As previously discussed, the Galileo orbiter data were taken close to the jovigraphic equator throughout most of the orbits and therefore do not require substantial correction, although for consistency all data has undergone the same procedure. The approach is to simply map the field measurements to the edge of the current sheet using mapping factors obtained from the approximate forms of the Connerney et al. (1981) model described in a recent paper by Edwards et al. (2001). For precise details of this procedure the reader is directed to Chapter 4, as the method adopted for correction here is identical. Mapping factors depend on radial distance, but are typically ~1.05 for a latitude of ~5°, increasing to ~1.25 for ~15°, such that the corrections are not substantial.

In order to demonstrate the effect of latitude correction, we present in Fig. 6.2 plots of the radial field versus radial distance in a log-log format. Throughout this chapter we employ cylindrical coordinates referenced to the magnetic dipole axis. Thus the ‘radial field’ is the cylindrical component perpendicular to the dipole axis, and the ‘radial distance’ is the perpendicular distance from that axis. In panels (a) and (b) we show
Figure 6.2. Log-log plots of the 30-min averaged radial field component $B'_\rho$ outside the current sheet, versus the perpendicular distance from the magnetic axis $\rho$, with the internal planetary field subtracted. Data are shown before they have been corrected for latitudinal-related effects for (a) 0600-0700 MLT and (b) 0800-0900 MLT and after "correction" for (c) 0600-0700 MLT and (d) 0800-0900 MLT. The Galileo data are shown by stars and the fly-by data are indicated by the diamonds. The solid lines indicate the BC model, whilst the dashed lines indicate the extremes of the model, i.e. noon (upper) and midnight (lower).
30 min "current sheet" radial field averages (i.e. the radial field with the planetary field subtracted), denoted by $B'_p$, before "latitude-correction" for the 1 h MLT intervals 0600-0700 and 0800-0900 MLT, respectively. The same data is shown after correction in panels (c) and (d). Averages derived from Galileo data are indicated by stars, while the fly-by data employed previously by BC are shown by diamonds. For the intervals shown, fly-by data is present only in panels (b) and (d), where it was derived principally from the Pioneer-11 inbound pass. In each panel of the figure the BC empirical model profile corresponding to the limits of the MLT bin are shown by the solid lines, while the extreme profiles of the model are indicated by the dashed lines, for noon (lower) and midnight (upper), respectively. The effect of varying magnetic latitude at the spacecraft is particularly evident in the fly-by data shown in panel (b), where individual groups of points form partial ‘U’-shaped patterns. These groups of points correspond to averages derived from individual spacecraft excursions outside of the current sheet during the planet’s rotation, such that averages obtained near the start and end of each group correspond to values obtained at lower magnetic latitudes relatively close the edge of the current sheet, while those in the middle were obtained at higher magnetic latitudes at larger distances from the current sheet. The effect of falling radial fields with distance from the current sheet is thus very clear in these fly-by data (in the present case reaching $\sim$20° magnetic latitude near the centre of each group), and the need to introduce a latitude correction is correspondingly clear. However, with this introduction, the latitude effect is seen to be present with reduced amplitude in the Galileo data as well, in both panels (a) and (b).

Panels (c) and (d) then show the effect of applying the latitude correction factor derived from the Connerney et al. (1981) model which, as indicated above, maps these data values to the edge of the current sheet. It can be seen that the “scatter” in both data sets is significantly reduced, with two immediate effects. First, the Galileo and fly-by data are brought into much closer agreement with each other. Second, both data sets are brought into better general (if not perfect) agreement with the empirical BC model, which, as indicated above, was derived from and intended to represent the latitude-corrected radial field at the edge of the current sheet. All of the data we will henceforth analyse and display in this chapter will thus correspond to “latitude-corrected” radial field averages mapped to the edge of the current sheet, which will be termed “equatorial” radial field averages.
6.3. **Comparison of the Galileo data with the Bunce and Cowley empirical model**

In this section we will compare 30 min-averaged values of the “equatorial” radial current sheet field, denoted here by \( B'_{\rho_0} \), with the BC model. Data from both Galileo and the fly-bys will be shown in order to facilitate inter-comparison, where the Galileo data correspond to orbits G–1 to C–20 inclusive (between 1996 and 1999). We recall that the BC model is given by the simple function

\[
B'_{\rho_0} = A \left( \frac{\rho_0}{\rho} \right)^{m(\varphi)},
\]

(6.1)

where \( A = 41.1 \) nT, \( \rho_0 = 18.8 \) R\(_J\), and \( m \) is given by

\[
m(\varphi) = \alpha \cos \varphi + \beta
\]

(6.2)

where \( \varphi \) is azimuth measured positive eastward, \( \alpha = 0.48 \) and \( \beta = 1.26 \). The model is thus described by four simple parameters only.

In Fig. 6.3 we thus show model and observed values plotted versus MLT in four radial ranges of width 2.5 R\(_J\), which span the range of validity of the model between 20 and 50 R\(_J\). In panels (a) to (d) these radial ranges are 20.0-22.5, 30.0-32.5, 37.5-40.0, and 47.5-50.0, respectively. Data obtained when the spacecraft were south of the current sheet, such that \( B'_{\rho_0} \) was negative, have been reversed in sense, assuming anti-symmetry in \( B_{\rho_0} \) about the centre of the current sheet. As in Fig. 6.2, the fly-by data previously employed are shown by diamonds, while the Galileo data are shown by stars. The solid lines indicate the BC model for the extremities of each radial range shown. In addition, the RMS residual value of the data points from the equivalent BC model value, normalised to the model magnitude, is given in each panel. This value gives a RMS fractional residual of the data in each panel.

In panel (a) of Fig. 6.3 we see that a majority of the data points lie within or immediately beside the “band” of model values, though a small proportion lie well outside.
Figure 6.3. Representative plots of the "latitude-corrected" radial field $B_{\rho 0}$, as a function of magnetic local time (MLT), are shown for (a) 20-22.5 Rj, (b) 30-32.5 Rj, (c) 37.5-40 Rj, and (d) 47.5-50 Rj. The same symbols are used for the Galileo and fly-by points as indicated in Fig. 6.2. In each case, the two solid lines indicate the BC model for the two extremes of radial range shown. At the foot of each panel the RMS residual of the BC model (expressed as a percentage) is indicated. From this point, all averages shown have been corrected for latitude related variations.
Chapter 6: Model of the radial field from Galileo orbiter and spacecraft fly-by data

As noted previously, this panel corresponds to the radial range of 20-22.5 R_J, and therefore lies at the innermost edge of validity for the BC model. We note, however, that the Galileo and fly-by data correspond well, and that the RMS residual is 15.6%, such that the model represents a reasonable indicator of the radial field strength in this region. Panel (b) shows the data and model for the domain 30-32.5 R_J. Here the BC model fits both Galileo and fly-by data well, with a residual error of 11.2%. The local time asymmetry is now clearly evident in both sets of data, with the radial “current sheet” field being approximately 10 nT stronger at midnight than at noon in this particular radial range. It can also be seen that the Galileo data and fly-by data are closely similar, with rather little scatter about the mean values, despite the fact that the contributing data span ~25 years of time. This indicates that the radial field at a given location is a relatively robust parameter over such intervals.

Moving out further into the middle magnetosphere, panel (c) shows the data and model values between 37.5 and 40 R_J. The day-night asymmetry in the field is still marked, and now the similar 10 nT difference between noon and midnight denotes a factor of almost two in the radial field strength. Once more the two data sets are in close agreement and the model represents a good estimation of the field having a residual error of 13.2%. In panel (d) we finally show data between 47.5 and 50 R_J, the outermost limit of validity of the BC model. We notice that some of the data from the Galileo orbiter do not fit as well to the BC model in this region, and the RMS residual is now 21%. It can be seen that while certain of the Galileo data do follow the model values as they decrease towards magnetic noon, a large percentage of the data population do not. Instead, they remain at an approximately constant value as a function of MLT. Further inspection of the individual radial bins shows that this “flattening” is first observed in the 45-47.5 R_J radial range, suggesting that the local time asymmetry in the “current sheet” radial field does not always exist in this region of the middle magnetosphere. We suppose that these variations of the field strength (presumably from orbit to orbit) may be a signature of the effects of compressions and expansions of the magnetosphere due to changes in the solar wind dynamic pressure, causing the field in the outer regions to change whilst those stronger fields closer to the planet, remain relatively unaffected. For this reason, our revised field model derived below for the combined data set will be restricted to the radial range 20-45 R_J. These residual errors are collected together in the first two columns of Table 6.1, where we show the RMS residuals in 5 R_J radial ranges relative to the BC model values. At the foot of the table the overall RMS residual is shown, which for the BC model is ~13.5% over the radial range 20-45 R_J. Clearly the model provides a reasonably good
Table 6.1. Comparison of the RMS residual of the BC model and the Revised BC (RBC) model for the six radial ranges shown. The radial range 45-50 $R_J$ (shown in italics) has not been included in the calculation in the overall RMS error.

<table>
<thead>
<tr>
<th>Radial range ($R_J$)</th>
<th>RMS Residual (BC model) (%)</th>
<th>RMS Residual (RBC model) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-25</td>
<td>14.2</td>
<td>9.4</td>
</tr>
<tr>
<td>25-30</td>
<td>14.5</td>
<td>14.8</td>
</tr>
<tr>
<td>30-35</td>
<td>11.7</td>
<td>8.6</td>
</tr>
<tr>
<td>35-40</td>
<td>13.4</td>
<td>10.1</td>
</tr>
<tr>
<td>40-45</td>
<td>13.9</td>
<td>10.7</td>
</tr>
<tr>
<td>45-50</td>
<td>19.2</td>
<td>17.0</td>
</tr>
<tr>
<td>Overall RMS residual (20-45 $R_J$)</td>
<td>13.5</td>
<td>9.9</td>
</tr>
</tbody>
</table>
estimate of the radial field in this domain. In Table 6.1 the error for the radial range 45-50 \( R_J \) is also depicted, the italics indicate that the residual error for this range was not included in calculating the overall error for the BC model to the Galileo and fly-by data. The residual error for this 45-50 \( R_J \) displays the need to narrow the range in order to revise the BC model.

In addition to comparing the Galileo and fly-by data and the BC model at fixed radial distance ranges as above, it is also instructive to divide the data into ranges of local time and study them as a function of radial distance, \( \rho \). In Fig. 6.4 representative plots of the equatorial radial field versus radial distance are shown in a log-log format for four ranges of MLT. Panels (a) to (d) correspond to 0900-1000, 0400-0500, 1900-2000, and 0000-0100 MLT respectively, such that they represent observations from the pre-noon sector, and the near dawn, dusk and midnight meridians respectively. The format of the panels of this figure is essentially the same as for Fig. 6.2, with some additional features to be described below. Panel (a) represents data mostly from the inbound pass of Pioneer-11, whilst panel (b) consists of contributions from both Pioneer-10 outbound and from various Galileo orbits. In panel (c) we see the Ulysses outbound data along with a solitary average derived from Galileo orbit C20. Finally, in panel (d) the majority of the data are from various Galileo orbits with a few points from Voyager-2 outbound at smaller distances. In each panel the RMS residual is given for the BC model over the radial range 20-45 \( R_J \), which can thus be seen to be a reasonable measure of the field values. These RMS residuals are collected together in Table 6.2 for 3-h local time intervals, and are in general less than 15\% (with the exception of the 0900-1200 MLT sector). As indicated before, the overall RMS residual is 13.5\%.

In addition to the BC model lines (shown by the dashed and lighter solid lines), we also show in Fig. 6.4 the results of a straightforward least-squares fit to the logged data points, indicated by the heavy solid line. This was fitted to the data only in the radial range 20-45 \( R_J \), for reasons previously discussed. However, we have extrapolated this line to the edge of the plot, as shown by the heavy dashed line, to cover the whole range of the data. It can be seen that the fit actually represents a reasonably good approximation to the data out to at least 60 \( R_J \). The parameters of the fit, namely the coefficient \( A \) and the exponent \( m \), where

\[
B'_{\rho \theta} = A(nT)\rho(R_J)^{-m},
\]

(3)

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Figure 6.4. Log-log plots of the radial field $B_{\rho o}^-$ as a function of radial distance $\rho$, for the four MLT intervals: (a) 0900-1000, (b) 0400-0500, (c) 1900-2000, and (d) 0000-0100. For each panel, the solid lines indicate the BC model for the outer limits of the local time interval shown, whilst the dashed lines show the extremes of the BC model (i.e. noon and midnight). The heavy solid indicates the least-squares power-law fits $B_{\rho o}^- = A(nT)\rho(R_j)^m$ over the radial range 20-45 $R_j$, where the values of the coefficient $A$ and $m$ are shown in each panel. The line is simply extrapolated over the full data coverage range, indicated by the heavy dashed portion of the line. At the bottom left of each plot is the RMS residual of both the least-squares fits and of the BC model values for each point.
Table 6.2. Comparison of the RMS residual of the BC model and the Revised BC (RBC) model for the given local time ranges. The overall RMS residual are indicated in the final row for both BC and RBC models. A dash indicates that insufficient data were available in that local time sector.

<table>
<thead>
<tr>
<th>MLT range (h)</th>
<th>Residual (BC model) (%)</th>
<th>Residual (RBC model) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0000-0300</td>
<td>14.3</td>
<td>10.5</td>
</tr>
<tr>
<td>0300-0600</td>
<td>9.7</td>
<td>8.6</td>
</tr>
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<td>0600-0900</td>
<td>13.2</td>
<td>9.2</td>
</tr>
<tr>
<td>0900-1200</td>
<td>19.8</td>
<td>11.0</td>
</tr>
<tr>
<td>1200-1500</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1500-1800</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1800-2100</td>
<td>12.5</td>
<td>11.0</td>
</tr>
<tr>
<td>2100-0000</td>
<td>12.1</td>
<td>9.5</td>
</tr>
<tr>
<td>Overall RMS residual</td>
<td><strong>13.5</strong></td>
<td><strong>9.9</strong></td>
</tr>
</tbody>
</table>
are given in each plot. We also indicate the RMS residual of the fit, which refers specifically only to the points lying within the fitting range. From panel to panel the error varies by a small amount, and no panel has an error of more than 10%, which is on average significantly less than the overall 13.5% error for the BC model values. Evidently, a simple power law fit to the joint data set provides a good description of the middle magnetosphere "current sheet" field. We can also see from these panels that while the intercepts near ~20 R_J do not vary greatly, each being close to ~40 nT, the field gradients are clearly largest on the dayside, smaller at dawn and dusk, and smallest on the nightside, in accordance with the BC model and other papers cited in the introduction. In Table 6.3 we thus provide for detailed reference the best-fit coefficients $A$ and $m$ derived from the one-hour local time ranges which have sufficient number of radial field averages over the radial range 20-45 R_J. The first column indicates the local time sector, followed in columns 2 and 3 by the exponent $m$ and the coefficient $A$. In addition, the next column shows the RMS residual of each fit. Overall, the RMS residual for these 'best-fit' lines is found to be 7.8% as given at the foot of Table 6.3. This compares with the overall values of 13.5% for the simple four-parameter BC model given by Eqs. (6.1) and (6.2). Although the BC model thus gives a reasonable description of the overall data set, the fact that the residual values are overall ~75% greater than those of the best-fit lines provides motivation to undertake a revision. This will now be attempted in the next section.

6.4.  Revision of the Bunce and Cowley empirical model

6.4.1  Determination of the "hinge" point

Thus far we have shown, in Fig. 6.4, only the best-fit lines to the data in four local time ranges. Now, in Fig. 6.5, we compare the fitted lines from all thirteen of the 1-hour local time intervals which provided data over a sufficient range that the slope $m$ and intercept $A$ relevant to the distance range 20-45 R_J could be determined with confidence. In Chapter 4 we noted that the lines of best fit to the fly-by data seemed to converge at ~20 R_J, and hence used this as a starting point for our model. We assumed that the lines do in fact converge at a certain radial distance $\rho_0$, the distance within which the field may be taken as cylindrically symmetric, and then fall with distance at various rates depending
Table 6.3. This table contains the $m$ and $A$ values, for individual local time ranges, for the full least-squares fit over the radial range 20-45 R$_J$ accompanied by its corresponding normalised RMS residual. Also shown are the $m$ values for the “hinged” fits for the same local time ranges over the range 24.5-45 R$_J$, and the normalized RMS residual error for this fit. Overall residual errors are shown for both categories at the foot of the table.

<table>
<thead>
<tr>
<th>MLT</th>
<th>Full fit over 20-45 R$_J$</th>
<th>“Hinged” fit over 24.5-45 R$_J$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m$</td>
<td>$A$ (nT)</td>
</tr>
<tr>
<td>00-01</td>
<td>0.87</td>
<td>489.5</td>
</tr>
<tr>
<td>01-02</td>
<td>0.74</td>
<td>334.7</td>
</tr>
<tr>
<td>03-04</td>
<td>1.07</td>
<td>1022.2</td>
</tr>
<tr>
<td>04-05</td>
<td>1.22</td>
<td>1689.2</td>
</tr>
<tr>
<td>05-06</td>
<td>1.06</td>
<td>375.8</td>
</tr>
<tr>
<td>06-07</td>
<td>1.47</td>
<td>3702.8</td>
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<td>07-08</td>
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<td>08-09</td>
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<td>3226.2</td>
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<td>19-20</td>
<td>0.91</td>
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<td>21-22</td>
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<td>1205.6</td>
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<td>22-23</td>
<td>1.22</td>
<td>1703.1</td>
</tr>
<tr>
<td>23-00</td>
<td>1.14</td>
<td>1309.3</td>
</tr>
</tbody>
</table>

Overall residual 7.8 8.1
Figure 6.5. Plot of the fitted lines as in Fig. 6.4, from the 13 MLT intervals which could be used to determine the dependence on distance in the radial range 20-45 R₉. These MLT values are shown on the right hand margin. The solid part of each line depicts the radial range over which the fit was determined, while the dashed part (i.e. at radial distances greater than 45 R₉) show where the line has been extrapolated outside of the range. An arrow is drawn at the "hinge" point ρ₀, that is the point of maximum convergence, which was determined from the least value of the standard deviation of the $B'_{ρ₀}$ values normalised to the average, while the horizontal bar gives an estimate of the error in this value.
upon the local time, as described by Eqs (6.1) and (6.2). Here we follow the same procedure, and thus construct a revised empirical model of the same form. To determine the constants \( A \) and \( \rho_0 \) in Eq. (6.1) we use the thirteen fitted lines shown in Fig. 6.5, and calculate the standard deviation of these values at a given radial distance, normalised to the mean of these values. By computing the minimum in this quantity, we then have the radial distance at which there is the least variation in \( B'_{\rho_0} \) relative to the mean. We term this our convergence or “hinge” point, and find that here it occurs at a radial distance \( \rho_0 = 24.5 \pm 1.0 \text{ R}_J \) (where the error has been estimated from the width of the minimum in the normalised standard deviation). The averaged value of the field at this position is \( A = 31.7 \pm 2.2 \text{ nT} \) (where the error given is the standard deviation of the values). These are the values that will be employed in the revised model.

6.4.2 Field model for radial distances greater than the “hinge” point

We now use the above values of \( A \) and \( \rho_0 \) as a “hinge” point through which the fitted lines must pass. In our original study, we found a “hinge” point for the fly-by data alone at a radial distance of 18.8 \( \text{R}_J \), thus lying just outside the region of the fitted data (20-50 \( \text{R}_J \)). All of the data was thus fitted to the form given by Eq. (6.1). Here, however, the best “hinge” point for the full data set lies just within the region of validity of the data at 24.5 \( \text{R}_J \). In this case, therefore, we have modelled the field as axially symmetric between 20.0 and 24.5 \( \text{R}_J \), as discussed further below, and have least-squares fit the data to Eq. (6.1) with \( A=31.7 \text{ nT} \) and \( \rho_0=24.5 \text{ R}_J \) only in the range 24.5 to 45 \( \text{R}_J \). The resulting \( m \) values and RMS residuals of these “hinged” fits for each of the available MLT intervals are given in columns five and six of Table 6.3, for comparison with the “free” fit \( m \) values and RMS residuals. The \( m \) values are seen to be very similar to the “free” fit values, while the residuals are only a little larger, with an overall value of 8.1%. Thus the “hinged” lines fit the data almost equally as well as the “free-fit” lines. The \( m \) values are also shown plotted versus MLT in Fig. 6.6, where the points have been plotted at the central local time of each of the 1-hour ranges employed here. The local time dependence of these gradient values is immediately evident, with values of \(-0.8\) near midnight increasing to \(\geq 1.6\) near noon. This range of values is similar to, but slightly smaller than the range reported in Chapter 4 from the fly-by data alone (\(-0.8\) at midnight increasing to \(\geq 1.7\) at noon). Therefore we choose
Figure 6.6. Plot of the exponent m to the "hinged" power-law fit to $B'_{p0}$ for the 13 MLT ranges employed, versus magnetic local time. The points are plotted at the centre point of each local time interval. The solid line depicts the least-squares fit to a simple periodic function assumed symmetric about the noon-midnight meridian.
to simply follow the procedure described in Chapter 4 and fit to the periodic function given above by Eq. (6.2). Given the range and nature of the results, there seems no reason, for example, to choose any axis of symmetry other than the noon-midnight meridian. The result of least-squares fitting the \( m \) values to the function given by Eq. (6.2), is depicted by the solid line in Fig. 6, and gives values of \( \alpha = 0.50 \) and \( \beta = 1.30 \) (the values determined originally from the fly-by data were \( \alpha = 0.48 \) and \( \beta = 1.26 \)). We also indicate the RMS residual in the \( m \) values to this fitted line, which is 15\%. It can be seen that the least-squares fit depicted by the solid line is a good representation of the points. This then completes our fit to the data in the radial range \( 24.5 < \rho < 45 \, R_j \). They are fitted by Eqs. (6.1) and (6.2) with \( A = 31.7 \, nT, \rho_0 = 24.5 \, R_j, \alpha = 0.50, \) and \( \beta = 1.30 \). The overall RMS residual of the points in this range from this model is found to be 10.8\%.

### 6.4.3 Field model for radial distances less than the “hinge” point

We now return to the issue of including those averages between 20.0 and 24.5 \( R_j \). As indicated above, we assume that the data in this range is independent of MLT, and thus in Fig. 6.7 we show all of the data from all of the local time ranges plotted together. As before, the plot indicates \( B'_{\rho0} \) as a function of \( \rho \), with the Galileo data shown by stars and the fly-by data by diamonds. The “hinge” point, through which all the “outer” fitted lines described by Eqs. (6.1) and (6.2) will pass, is shown by the solid circle at the right hand edge of the plot. We now attempt to represent the “inner” data by a simple power law fit. The first line, shown dashed, is a “free” power law least-squares fit to the data of the form given by Eq. (6.3), with \( A = 11060.7 \, nT \) and \( m = 1.64 \). As indicated at the bottom of the figure the RMS residual is 9.3\%. The second line, shown dot-dashed, is a “hinged” least-squares fit which is forced to pass through the “hinge” point. For this line we find that \( m = 1.13 \), similar to that of the “free-fit”. The RMS residual is 9.5\%, and thus represents a fit which is very nearly as good as the “free” fit. Finally, the third line, shown solid, passes through the “hinge” point with a slope of 1.30, equal to the value of the parameter \( \beta \) in Eq. (6.2). This line thus represents a straight continuation of the “outer” fit lines at dawn and dusk, mid-way between the extreme “outer” behaviours at noon and midnight. The RMS residual for this line is also 9.5\%, thus again being essentially indistinguishable from the “free” and “hinged” fits to the data. In order to keep the model as simple as possible,
Figure. 6.7. Plot of the radial field $B'_{p0}$ as a function of radial distance $p$, for all of the spacecraft data within the "hinge" point. As before, the Galileo data are stars and the fly-by data are indicated by diamonds. A filled circle on the right-hand side of the plot depicts the "hinge" point. Three lines are shown: (a) the dashed line representing a "free" least-squares fit to the data in this range, (b) the dot-dashed line indicating a "hinged" least-squares fit such that the line is forced to pass through the "hinge" point, and (c) the solid line which passes through the "hinge" point with a slope of 1.3, equal to the value of the parameter $\beta$ in Eq. (2). This line thus represents a straight continuation of the fit lines outside of the "hinge" point at dawn and dusk, mid-way between the extreme behaviours at noon and midnight. The associated RMS residuals for each line are indicated at the bottom of the plot.
we thus employ the latter line in the empirical model proposed here. Thus within the “hinge” point, the model field values are again given by Eq. (6.1), with $A = 31.7$ nT and $\rho_0 = 24.5$ R$_J$, but with $m$ taking the fixed value of 1.30.

### 6.4.4 Comparison of model and field data

In this section we finally check how well the overall model defined in Sections 6.4.2 and 6.4.3 fits the data, noting that the “hinged” fits do not represent the optimum fits in a given MLT sector, and that the representation of $m(\varphi)$ by Eq. (6.2) constitutes a further approximation.

In Fig. 6.8a we first show a selection of radial profiles for fixed 1-h MLT ranges, in a format similar to Fig. 6.4. In this case, however, the solid lines show the values implied by our revised model (which we term the RBC model) corresponding to the limits of the MLT range concerned, while the dashed lines show the noon (upper) and midnight (lower) limits of the model. The dot-dashed lines similarly show the values of our original ‘BC’ model. At the foot of each panel we also show the RMS residuals for both models. It can be seen that the models provide a very good overall description of the data, with RMS residuals of typically 11-12%. However, the residuals of the RBC model are generally several percentage points lower than that of the original model. This evidence is borne out in Table 6.2, where the RMS residuals are compared for the two models, over 3-h local time sectors. The RBC model provides an improved description in each MLT sector, and, taking all the data together we find a RMS residual of 9.9% for the RBC model compared with 13.5% for the original BC model. The four-parameter RBC model thus provides a description which is almost as accurate as those of the “free-fit” at intervals of 1-h MLT, for which the RMS residual was 7.8%.

In Fig. 6.8b we provide an alternative presentation, in which are shown the model values versus MLT in fixed ranges of radial distance, similar to Fig. 6.3. Here, the solid lines show the RBC model, and the dashed lines the original model. The RMS residuals for both models are also shown at the bottom of each panel (and over wider radial ranges in Table 6.1). Again the models provide a good overall representation of the field in the middle magnetosphere, with the RBC model giving smaller RMS residuals than the BC model.
Figure 6.8a. In the same format as Fig. 6.4, and for the same MLT intervals, we show the radial field $B'_{\rho 0}$, as a function of radial distance $\rho$ for both Galileo orbiter (stars) and the fly-by spacecraft (diamonds). In each panel the Revised BC model (RBC) is shown by the solid lines for the outer limits of the MLT interval shown. Once again the dashed lines indicate the RBC model limits at noon and midnight. The dot-dashed lines show the original BC model values for the same local times for comparison. At the bottom of each panel, the RMS residuals are given for both the original BC model and for the RBC model, again for comparative purposes.
Figure 6.8b Plots of the radial field $B_{\parallel 0}$, as a function of MLT are shown for (a) 22.5-25 R$_J$, (b) 30-32.5 R$_J$, (c) 35-37.5 R$_J$, and (d) 42.5-45 R$_J$. The same symbols are used for the Galileo and fly-by data as in previous figures. The solid lines indicate the RBC model for the outer limits of the radial range shown, whilst the dashed lines indicate the original BC model values for the same distances. Once more, the RMS residual values are shown for both BC and RBC models.
model in nearly all ranges. In particular the radial range 22.5-25 Rₐ, as can be seen in Fig. 6.8b, actually has a slightly poorer fit after the model revision. This may be due to temporal variations in the “hinge-point” position between the Galileo and fly-by epochs.

6.5. Divergence of the azimuthal current

6.5.1 The azimuthal current and its divergence

As outlined previously in this thesis, a local time asymmetry in the radial field in the middle jovian magnetosphere implies a divergence in the equatorial azimuthal current. Significantly larger currents occur at midnight at a given distance than at noon. In order to quantify the divergence of the azimuthal current we first need to consider the equivalence of the radial magnetic field to the azimuthal current intensity. As described in detail in Chapter 4, we find that, via Ampère’s law, the integrated current intensity (A m⁻¹) in the equatorial current sheet is given by

\[ i_\varphi = \frac{2}{\mu_0} \left[ B'_{\rho 0} - D \frac{\partial B'_z}{\partial \rho} \right] \]

where, as before, the primed fields indicate that the curl-free planetary field has been subtracted, \( D \) is the half-thickness, and \( \mu_0 \) is the permeability of free space. In deriving this expression we have assumed anti-symmetry in the radial field on either side of the current sheet, and that \( B'_z \) remains approximately constant through the thickness of the current sheet. We discussed in Chapter 4 that for jovian current sheet conditions, the second term in Eq. (6.3) is much smaller than the first, such that to within \(~10\%\) the azimuthal current intensity is given by

\[ i_\varphi \approx \frac{2B'_{\rho 0}}{\mu_0} \]

Consequently, our model for the equatorial radial field just outside the current carrying layer given by Eqs. (6.1) and (6.2) above may be approximately but directly converted into an empirical model for the azimuthal current, which will therefore undergo the same local time variations as the radial field. The divergence of the azimuthal current is then simply given by
Introducing the revised empirical model given by Eqs. (6.1) and (6.2), which is clearly only asymmetric outside \( \rho_0 = 24.5 \, R_J \), we find

\[
\text{div} i_\varphi \approx \frac{2\alpha}{\mu_0 \rho} \sin \varphi \ln \left( \frac{\rho_0}{\rho} \right) B'_{\rho_0}. \tag{6.6}
\]

In Fig. 6.9a we show a contour map of this function in the equatorial plane (solid lines), labelled with the divergence values in kA R\(_J\)^2. The divergence is exactly zero at \( \rho_0 = 24.5 \, R_J \), and also at all radial distances on the noon-midnight meridian, as this is the axis of symmetry of the model. The dashed lines in the figure indicate radial distance in the equatorial plane, starting with 20 R\(_J\) and increasing in increments of 10 R\(_J\) to 60 R\(_J\). The range of detailed validity of the RBC model continues only to 45 R\(_J\), and as such the contours outside of this range (which allow comparison with the results of Chapter 4) should be interpreted with caution. We see that the divergence is negative at dawn, implying a sink of azimuthal current in that sector, while reversing to positive at dusk, thus requiring a source of current. The magnitude of the peak divergence in \( i_\varphi \) is 12.2 kA R\(_J^2\), occurring at a radial distance of \( \sim 40 R_J \) near the dawn-dusk meridian. In the original BC model the peak magnitude was \( \sim 18 \, \text{kA} \, R_J^2 \) occurring at \( \sim 30 R_J \) near the dawn-dusk meridian. Necessarily, the current overall is divergence-free, and continuity must therefore be maintained either by radial currents flowing wholly within the equatorial current sheet, or via field-aligned currents which must flow towards the planet at dawn and away from the planet at dusk.

In Fig. 6.9b we give an indication of the overall current which must be diverted into one or other of these directions. The lower three curves in this figure show the total azimuthal current flowing in the model current sheet in the radial ranges 20-30 R\(_J\), 30-40 R\(_J\) and 40-50 R\(_J\) (slightly beyond the outer limit of the model), versus MLT. The upper curve shows the sum of these, that is the total current flowing between 20 and 50 R\(_J\). These curves have been computed by direct integration of Eq. (6.4), combined with Eqs. (6.1) and (6.2). Each of the curves shows, as expected, that the current is maximum at midnight and minimum at noon. Specifically, the amount of current diverted in each case is 1.2, 5.2, and 6.6 MA, the total being 13.0 MA within the range 20-50 R\(_J\). For the original BC model these currents were 8.2, 12.5, and 13.1 MA, respectively, totalling to
Figure 6.9a. Contours of the divergence of the azimuthal current in the magnetic equatorial plane, in units of kA R\textsubscript{j}^{-2}, derived from the empirical model of $B_{\rho 0}$ derived here. Noon is marked at the top of the plot, with dawn to the right. The dashed rings indicate radial distances of 20, 30, 40, 50 and 60 R\textsubscript{j}, a somewhat extended range of validity. Jupiter is shown in the centre to scale.
Figure 6.9b. The total current in MA flowing in various radial ranges in the equatorial current sheet versus magnetic local time, obtained from the Revised BC (RBC) empirical model derived here. The current has been integrated in the ranges 20-30, 30-40, and 40-50 R\textsubscript{J}, and over the entire range 20-50 R\textsubscript{J}, as indicated on the right-hand side of the plot.
33.7 MA. The total diverted current is thus a relatively sensitive function of the model employed.

6.5.2 Current stream-function

As indicated above, the ‘diverted’ azimuthal current must flow either radially in the current sheet itself, or close via field-aligned currents in the ionosphere. In general, both closure paths may be expected to be present. In this case, as described in Chapter 5 and as published by Bunce and Cowley (2001b) we suggest on physical grounds that the total middle magnetosphere current system might best be viewed as the sum of a divergence-free current that flows wholly within the equatorial current sheet itself, $i_{CS}$, which includes all of the azimuthal current, together with additional radial currents that close wholly via field-aligned currents in the ionosphere. The divergence-free equatorial current can then be described by a current stream-function $I_{CS}$ having units of amps, which is such that $I_{CS}(\rho, \phi) = \text{constant}$ defines a current streamline in the current sheet, while the amount of current flowing between $I_{CS}$ and $I_{CS} + dl_{CS}$ is just $dl_{CS}$. This stream-function is related to the current intensity $i_{CS}$ by $i_{CS} = \hat{z} \times \nabla I_{CS}$, where $\hat{z}$ is a unit vector perpendicular the current sheet directed northwards. For the model currents implied by Eqs. (6.1)-(6.2), the stream function for $20 \leq \rho \leq \rho_o = 24.5 \ R_J$ is given by

$$I_{CS}(\rho) = \frac{2A\rho_o}{\mu_0(m' - 1)} \left[ \left( \frac{\rho_o}{20} \right)^{m'-1} - \left( \frac{\rho}{\rho_o} \right)^{m'-1} \right], \quad (6.7a)$$

while for $\rho \geq \rho_o = 24.5 \ R_J$ it is given by

$$I_{CS}(\rho, \phi) = \frac{2A\rho_o}{\mu_0} \left\{ \frac{1}{(m(\phi) - 1)} \left[ 1 - \left( \frac{\rho_o}{\rho} \right)^{m(\phi) - 1} \right] + \frac{1}{(m' - 1)} \left[ \left( \frac{\rho_o}{20} \right)^{m'-1} - 1 \right] \right\}. \quad (6.7b)$$

In these equations, the arbitrary zero of $I_{CS}$ has been set at the inner edge of the region of validity, at $20 \ R_J$. In Fig. 6.10, the solid lines show contours of $I_{CS}$ in the magnetic equatorial plane (i.e. current streamlines), where the noon meridian is at the top and dusk to the left. The dashed lines indicate jovicentric distance, and are shown at intervals of $10 \ R_J$ out to $60 \ R_J$. The stream contours are labelled by the value of $I_{CS}$ in MA, and thus indicate the total amount of current flowing in the current sheet between that
Figure 6.10. Streamlines of the divergence-free component of the equatorial current, $i_{CS}$, determined from the RBC empirical model derived here. The streamlines are indicated by solid lines and are marked with values showing the total amount of current carried in the current sheet between the streamline concerned and that at radius $\rho_0=20 R_J$ (the innermost solid line), the inner edge of the RBC model. The lines are shown at equal intervals of 10 MA, so that the distance between them indicates the current intensity. The distance from the centre of the planet is shown by the dashed lines, in steps of $10 R_J$, from $10 R_J$ to $50 R_J$, the outer edge of the region of interest. Local times are also indicated and Jupiter is shown to scale in the centre of the plot.
Chapter 6: Model of the radial field from Galileo orbiter and spacecraft fly-by data

location and 20 R_J. This diagram shows explicitly how the current streamlines expand outwards on the dayside compared with the nightside, associated with outward radial currents at dawn and inward radial currents at dusk.

6.6 Summary and discussion

In this chapter we have studied the equatorial radial field just outside the jovian middle magnetosphere current sheet, $B'_{p0}$, as a function of radial distance $\rho$ and magnetic local time MLT. We first compared the properties of the empirical model suggested previously in Chapter 4 (the 'BC' model) derived from the fly-by data from the Pioneer-10 and -11, Voyager-1 and -2, and Ulysses spacecraft, with the data obtained from the first twenty orbits of the Galileo orbiter. We find that the Galileo data exhibit the same local time asymmetry described and modelled empirically in Chapter 4 using the fly-by data alone, and as reported independently by Khurana (2001). This indicates that the radial fields, and hence azimuthal currents, are weaker on the dayside than those at the same distance on the nightside. Comparison of the entire Galileo orbiter and spacecraft fly-by data set with the BC model shows that the model is generally a good representation of the radial field in the middle magnetosphere over the range 20-45 R_J, and overall has a RMS residual of 13.5%. Scope for revising the model is, however, evident from the fact that the overall RMS residual from the "best-fit" lines, fitted at 1-h MLT intervals is considerably smaller than this at 7.8%. We thus follow the procedure described in Chapter 4 and revise the model for the larger Galileo and fly-by data set, considering the range of 20-45 R_J.

In the revised model we take the field to be axially symmetric and falling as $\rho^{-1.3}$ within the radial distances of 24.5 R_J, and then beyond to fall off more rapidly at noon, as $\rho^{-1.8}$, than at midnight, as $\rho^{-0.8}$. Overall, we find that the data are well described by the function

$$B'_{p0}(\rho, \varphi) = A \left(\frac{\rho_0}{\rho}\right)^{-m(\varphi)}$$

where $A = 31.7$ nT, $\rho_0 = 24.5$ R_J, and $m = 1.30$ for $\rho \leq \rho_0$, while $m = 0.50 \cos \varphi + 1.30$ for $\rho \geq \rho_0$. The overall RMS residual for this model is 9.9%, now close to the values
obtained from the “free-fits” to data binned by 1-h intervals of MLT. This represents a worthwhile improvement over the fit provided by the original BC model.

The above asymmetry in the field implies a divergence of the azimuthal current, with stronger currents implied on the nightside at a given radial distance than on the dayside. We find that this divergence has a peak value of 12.2 kA R\textsubscript{J}\textsuperscript{2} near the dawn-dusk meridian at distances close to \( ~40 \) R\textsubscript{J}. Over the range 20-50 R\textsubscript{J}, the total difference in the azimuthal current flowing at midnight compared with noon computed from the RBC model is \( ~13.0 \) MA.

An important consideration at this juncture is the effect of the work presented here on the study of the total equatorial current divergence in Chapter 5, which involves employing the BC model in order to estimate the divergence of the azimuthal current. Thus, in Fig. 6.11, we show the re-computed divergence of the azimuthal current for the four spacecraft previously used, i.e. (a) Pioneer-11 inbound, (b) Pioneer-10 outbound, (c) Voyager-1 outbound, and (d) Voyager-2 outbound, marked with “div\textsubscript{i\textsubscript{e}}” in the upper parts of each panel of the figure, together with the divergence of the radial currents calculated from the fitted lines to the \( \rho_i \) data described in detail in the previous chapter. We combine the divergence of both the radial and azimuthal currents for the individual spacecraft trajectories according to Eq. (5.11). The results are depicted in the bottom parts of each panel of Fig. 6.11. The divergence of the azimuthal current in the RBC model is zero within the radial range \( \rho_0 = 24.5 \) R\textsubscript{J} as the model is assumed to be axi-symmetric in this range. Therefore at small radial distances, the contribution to \( j_z \) is produced entirely by the divergence of the radial current, and is \( \sim 10 \) kA R\textsubscript{J}\textsuperscript{2} for both the Pioneer spacecraft and \( \sim 15 \) kA R\textsubscript{J}\textsuperscript{2} for the Voyager spacecraft. We note that, as expected, the divergence of the azimuthal current is less for the RBC model than for the BC model for each of the spacecraft passes. Thus beyond \( \rho_0 \), the effect of the weaker azimuthal current divergence than that predicted by the BC model is evident. For the Pioneer-11 inbound pass \( j_z \) is now seen to be slightly negative (rather than slightly positive compared with Fig. 5.5 of Chapter 5) but approximately zero as previously. Similarly, the Pioneer-10 pass shows that \( j_z \) is less positive (\( \sim 3 \) kA R\textsubscript{J}\textsuperscript{2}) than the value calculated from the BC model in Chapter 5. The two Voyager spacecraft have values which are slightly larger than their previous values, but are not significantly different. We then combine these values of \( j_z \)
Figure 6.11. The upper plots in each panel of this figure show the divergence of the radial and azimuthal equatorial currents (kA Rj⁻²) as a function of the perpendicular distance from the magnetic axis of the planet ρ. The uncertainty estimates are indicated by the dashed lines. The divergence of the radial current, and its uncertainty limits, have been obtained from the fitted lines in Fig. 5.4, of Chapter 5. The corresponding quantities for the azimuthal current have been obtained from the Revised BC empirical model, described in this chapter. The lower plots show the current density \( j_z \) normal to the current sheet at its northern surface required for current continuity, as a function of \( \rho \). An equal but opposite current is assumed to flow out of the southern surface. The uncertainties shown by the dashed lines are the square root of the sum of the squared errors shown in the upper plots of this figure.
with the appropriate $B_z$ models as described in the previous chapter, and hence derive the parameter $\frac{j_{\parallel}}{B}$ versus distance $\rho$ for the four passes. This parameter directly results in an estimate for the field-aligned current density at ionospheric heights. We are therefore interested in quantifying any changes that the RBC model implies for this value. In Fig 6.12 we show this parameter in the same format as it was presented in Chapter 5, with the identical colour coding for the individual spacecraft. We can immediately conclude that the effect of revising the BC model to incorporate the first twenty orbits of the Galileo orbiter data, has no significant effect overall on the value of $\frac{j_{\parallel}}{B}$ for any of the spacecraft passes, although clearly in detail each have slightly different values. Note here that the sign corresponds to the northern hemisphere, such that positive values indicate current flowing from the ionosphere to the current sheet. As such, we conclude here that the estimation of the field-aligned current density of $\sim 0.4 \mu A \ m^2$ at the ionospheric heights remains an entirely valid estimate.
Figure 6.12. Plot of the variation of ($j_{||}/B$) versus distance $\rho$ for the four spacecraft passes discussed in Chapter 5, for the Revised BC model. The sign shown corresponds to the northern hemisphere, such that positive values indicate current flowing from the northern ionosphere to the current sheet, and vice versa for negative values. The coloured bands indicate the limits of uncertainty, which follow from the previous figure. The colours also serve as spacecraft identifiers.
Chapter 7

Summary and Future Work

7.1 Introduction

Planetary magnetospheres provide a natural laboratory for the study of plasma interactions and electrodynamic phenomena. However, not all magnetospheres exhibit the same properties. Those of the outer solar system, the ‘gas giants’, are much larger than that of the Earth. The rapid rotation of all four of the gas giants leads to important centrifugal forces that far exceed those in the Earth’s magnetosphere. The jovian magnetosphere is so large, in fact, that it would appear as large as the Sun if we could see it from Earth. Jupiter itself is ten times larger than Earth, and has a surface magnetic field that is ten times as strong. It has an internal plasma source, the moon Io, which is equivalent to a Halley-like comet persistently out-gassing 1 tonne s\(^{-1}\) of sulphur and oxygen atoms directly into the magnetosphere. Couple this with the planet’s rapid rotation of 9 h 55 min and the fact that the dipole axis is tilted at \(\sim 10^\circ\) to the rotation axis and we are provided with a system to study which is remarkably diverse from the one which we inhabit. In order to make comparisons between magnetospheres we have to visit them. Only relatively rudimentary studies of Jupiter’s magnetosphere have been made to date, consisting of five flybys (Pioneer-10 and -11, Voyager-1 and -2, and Ulysses) and the Galileo orbiter’s continuing presence. The next major project is the Cassini-Huygens mission to Saturn and one which is central to the planetary science programme. Cassini flew past Jupiter in December 2000 leading to orbit insertion at Saturn in 2004, and as such we should regard the investigation of the outer planet magnetospheres as a leitmotiv which
is both relevant and timely. We may also consider the presage of the Jupiter-like planets lying within our solar system and accessible to us for direct measurement, as links with more distant astrophysical objects.

### 7.2 Large-scale structure and dynamics of Jupiter’s magnetosphere

Any interaction between the planet and the magnetospheric plasma, is mediated by the magnetic field. Therefore investigations of the magnetic field configuration provide significant information about the dynamics of the system (for example see Cowley and Bunce, 2001a). One of the most striking discoveries to emerge from the flybys of Jupiter in the 1970’s was the existence of an equatorial azimuthal current sheet, whose magnetic effects (i.e. the distention of the magnetic field lines outwards) were observed at all local times investigated. The inwardly directed $j \times B$ force, related to the stretched magnetic field, opposes and balances the outward force of the plasma inertial and pressure forces. There also exists a radial current system associated with a bending of the magnetic flux tubes out of meridian planes. This is principally caused by the transmission of angular momentum from the rapidly spinning atmosphere to the equatorially confined plasma via a large-scale current system which closes the equatorial current in the ionosphere through substantial field-aligned currents.

The studies presented in this thesis have examined the radial and azimuthal dependence of the equatorial currents by combining all of the spacecraft data pre-Galileo for a first systematic study of this nature. In Chapter 4 we discovered that the azimuthal current in the middle magnetosphere is not axi-symmetric as had been assumed for the last twenty-five years, but that it is stronger on the nightside than on the dayside at a given radial distance. A simple empirical model was formulated, which accurately describes the data for a large portion of the equatorial middle magnetosphere region both in radial distance (20-50 R$_J$) and local time, and allows direct calculation of the current divergence associated with the asymmetry. The total deficit at noon compared with midnight computed from the model is $\sim 34$ MA. Continuity would then be maintained via radial current flowing away from the planet at dawn and inwards at dusk. The physical interpretation of this phenomenon is that the drift paths of equatorially-confined current-carrying particles are affected by the noon-midnight asymmetry imposed by the solar wind.
confinement within the magnetopause. In a similar way, in Chapter 5 the radial currents have been computed for the dawn sector of the jovian magnetosphere along various flyby trajectories. Combination of these radial current estimations with the azimuthal current model allows the total divergence of the equatorial current to be calculated. As previously indicated the overall continuity of the current is maintained by currents flowing along the field lines between the equatorial plane and the ionosphere, directed both into and out of the current sheet over the domain of interest. These current densities mapped to the ionosphere are surprisingly large at $\sim 1\mu$A m$^2$. The field-aligned currents are interpreted as part of a large-scale current system which transmits angular momentum from the planet’s atmosphere to the equatorial plasma. As the ioenic plasma diffuses radially outwards its angular velocity tends to fall off with distance, due to conservation of angular momentum. The differential rotation then causes a frictional torque on the plasma by way of ion-neutral collisions in the ionosphere, which is then transmitted along the magnetic field lines to the equatorial plasma. A simple physical model of this situation has been built, employing empirical models of the current sheet and the angular velocity profiles, which have led to the calculation of the field-aligned current density in the ionosphere. It is found that the current flows upward from the ionosphere to the magnetosphere for much of the middle magnetosphere, and has the same magnitude as indicated above. We have then shown that in order to carry the current, the magnetospheric electrons must be strongly accelerated along the field lines and into the ionosphere by voltages of the order of 100 kV. The resulting energy flux is enough to produce deep, bright (Mega Rayleigh) aurora and thus provides the first natural explanation of the main jovian auroral oval. The electron beams will also form a major source of jovian radio emission via the cyclotron maser instability (Cowley and Bunce, 2001b).

The aforementioned results derived from the flyby data are tested and refined using the newly-available Galileo orbiter data. We first of all compared the model derived previously in Chapter 4 using Pioneer, Voyager, and Ulysses fly-by data, with a combined data set that now also incorporates data from the first twenty orbits of the Galileo orbiter. The overall RMS fractional residual is found to be 13.5%, such that the model does provide a good description of the combined data set. In particular, it is shown that the Galileo data also exhibit the same local time asymmetry as found in the fly-by data, in which the radial field (and azimuthal current) are stronger at a given radial distance on the nightside compared with the dayside. However, it is also shown that if the combined data
are separated into 1 h bins of local time and then fitted to individual power law curves, the overall RMS fractional residual is reduced to 7.8%, thus showing scope for improvement in the empirical model. Based on the combined data set, in our revised model the field is taken as axi-symmetric within 24.5 R_j, and to fall with radial distance within that point with an exponent of -1.3. Outside that distance the exponent is taken to vary sinusoidally with local time, varying between -1.8 at noon and -0.8 at midnight, such that the field becomes increasingly asymmetric with increasing distance. The overall RMS residual for this four-parameter model is found to be 9.9%, only 2.1% higher than those of the free-fits to the 1 h MLT binned data, and representing a worthwhile improvement over the original Bunce and Cowley model. The implied divergence of the azimuthal current for the revised model peaks at \(\sim 15 \text{kA R}_j^{-2}\) near the dawn-dusk meridian at a radial distance of \(\sim 40 \text{ R}_j\). The implied difference in the total azimuthal current flowing in the current sheet between 20 and 50 R_j at midnight compared with noon is 16 MA.

The issue of temporal variations from pass to pass, and orbit to orbit should also be investigated. An obvious logical step forward would be to investigate the radial currents during the Galileo epoch and hence incorporate the improved model of the azimuthal current presented in Chapter 5 and build up a more detailed picture of the field-aligned current densities. Similarly, angular velocity profiles could similarly be obtained via the theory outlined in Chapter 5. Two fundamental issues present themselves at this juncture. The first is the question of the way in which the field-aligned current densities directed into and out of the current sheet, calculated from the field data, vary both temporally and spatially. The results would then allow research into the prediction of the dynamical behaviour of jovian aurora and radio emissions, which will be modulated by the solar wind dynamic pressure. Compressions of the magnetosphere cause the angular velocity of the plasma on a given flux shell to increase. It follows that the friction between the atmosphere and the magnetosphere is then reduced, which in turn reduces the field-aligned current densities and thus the auroral and radio luminosity. The opposite is true for an extended magnetosphere (Cowley and Bunce, 2001b; Southwood and Kivelson, 2001). This prediction should be testable using the Cassini-Jupiter flyby data, which was collected on the 30th December 2000, and provided the first ever multi-point measurements of the jovian magnetosphere. The Hubble Space Telescope (HST) monitored the auroral intensity whilst Cassini provided upstream solar wind conditions to compliment the Galileo data from within the magnetosphere. The identification of the main auroral oval
would suggest that the higher latitude aurora may be directly associated with boundary layer and tail phenomena. These may be modulated by the interplanetary magnetic field, like at Earth, and would also merit further study.

7.3 Structure of the current sheet

As indicated above, the jovian equatorial current is formed by plasma currents associated with both the pressure gradient of the hot plasma and the inertia of the cool plasma. The work proposed thus far has modelled the total currents, but other important questions arise concerning its location and thickness. Particle and field observations from the Pioneer era first discovered that the spacecraft-observed magnetic equator crossings do not coincide with the expected location of the dipole magnetic equator but are delayed in time in proportion to the distance of the spacecraft from Jupiter. This effect is attributed to the finite propagation speed effect that is associated with the communication of information about changes in the magnetic field configuration of the rotating tilted dipole from the foot of a field line in the jovian ionosphere to the outer magnetosphere. An additional effect is that at larger distances the current sheet tends to lie parallel to the solar wind direction because of the force exerted by the solar wind on the jovian magnetotail. The assumption that the current sheet lies in the magnetic equatorial plane is only valid at small distances. Therefore, there is a need for a global model of the current sheet position as a function of radial distance, local time and dipole phase, which would be obtained from the Galileo and flyby data combined.

There exists hinged-magnetodisc models for the nightside current sheet (Khurana, 1992) but there is no present model of the dayside current sheet position. It would thus be useful to develop a new line of study on the location and thickness of the current sheet with the intention of creating a more global prediction of the where the current sheet is expected to be for a given radius, local time and dipole phase. This would clearly facilitate the study of current sheet thickness and distribution of current, which cannot be definitively untangled without knowledge of exactly where the centre of the current sheet lies. There is no information on this subject at present. This work would lead into a new area of study to investigate the dynamical structure and variation within the current sheet on small time-
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scales. Some work has begun in this field (for example see Russell et al., 1999), but by no means have any categorical answers been provided.

7.4 Theoretical modelling

By using a model of the magnetic field in the middle magnetosphere, and of the angular velocity profile of the magnetospheric plasma in the equatorial plane we are able to compute the field-aligned current density at ionospheric heights by assuming a suitable value for the Pedersen conductivity. Models such as these are directly derivable from the magnetic field observations made by spacecraft in the middle magnetosphere, as presented in Chapter 5. Here we found that the field-aligned current density in the ionosphere is of the order of \( -1 \mu \text{A m}^{-2} \), and therefore requires substantial field-aligned voltages of \( \sim 100 \text{kV} \) in order to accelerate the electrons along the field lines and into the ionosphere. The ramifications of such a current density in the ionosphere is bright MR aurora in the regions where the current is directed out of the ionosphere towards the current sheet. A theoretical model of this situation, based upon empirical models of the magnetic field and the angular velocity of the plasma has recently been published by Cowley and Bunce (2001b). This model quantifies for the first time the field-aligned current density in the ionosphere, which arises as a consequence of the break down in corotation of the magnetospheric plasma in the equatorial plane. This model explains the basic properties of the main auroral oval, but has scope for extension in a variety of directions. For example, the model calculations should ideally be made fully self-consistent, by inclusion e.g. of the effect of the field-aligned voltage on the field-perpendicular voltage, the way in which the Pedersen conductivity varies as a function of precipitating electron energy flux. There is also the issue of time-dependency, that is, one might consider the effects of compressions and expansions of the magnetosphere on the angular velocity of the equatorial plasma and indeed upon the magnetic flux function (which essentially describes the amount by which the field lines are extended). This work would substantially improve the existing model and further our understanding of the observations.
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7.5 The kronian magnetosphere

The studies suggested here for possible future work centre on Jupiter due to the superior knowledge and data that currently exist, and furthermore as a result of the Cassini flyby at the end of last year. However, it is appropriate that the attention of the planetary science community will eventually progress from Jupiter to Saturn by way of the Cassini orbiter data, and thus it is opportune to start preparatory studies of this system now. By discovering the physical mechanism of such phenomena as the aurora at Jupiter, we may make similar hypotheses for the aurora which is observed at Saturn. Therefore it would be beneficial to peruse the existing kronian data from the Pioneer-11, Voyager-1 and -2 flybys, and equally to establish the applicability of the model of the jovian main auroral oval to the kronian aurora. Moreover, it is evident that contained within this data are dynamics which are not yet fully explained (for example see Espinosa and Dougherty, 2000) and thus there are exigent issues which should be addressed at the commencement of the Cassini mission.
Appendix

System III (1965) jovian coordinates

Jupiter is a gas giant and therefore the surface features exhibit differential rotation. That is the polar regions rotate around the core at a different rate to the equatorial features. For this reason it is impossible to define a prime meridian that is fixed at zero longitude (equivalent to Greenwich for the Earth), and thus a system must be devised that considers this fact. Such a system is the System III (1965) frame. The rotation axis of Jupiter was easily established from ground observations of its well defined spin equator, but the position of zero longitude was still a problem.

The solution initially was to define two separate longitude grids. System I applies to cloud features to within about 10° of the equator, System II to the higher latitudes. The rotation rates were measured and a Central Meridian Longitude (i.e. sub-Earth longitude) was selected. The prime meridian was measured on 14th July 1897 along with the rotation periods for each region. These systems have origins at the centre of the planet’s core and are orientated such that the z-axis is parallel to the rotation axis.

Half a century later radio signals were measured that are directly indicative of the rotation rate, which drives the magnetic field at the planet’s core. The evidence suggested a magnetic field rotation rate between that of System I and System II. So System III was defined at 00UT 1957, a system that describes the rotation rate of the magnetic field, and it follows that the magnetic field will be at rest in this frame. Soon after this it was discovered that the defined period was incorrect by 1 part in $10^5$ and thus was corrected by the creation of System III (1965), defined at 00UT 1965.
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