X-ray constraints on the number of stellar mass
black holes in the inner parsec

Patrick Deegan and Sergei Nayakshin
University of Leicester, University Road, Leicester, LE1 7RH, UK

Abstract. Due to dynamical friction stellar mass black holes should form a cusp in the inner parsec. Calculations [5, 6] show that approximately 20 thousand black holes would be present in a sphere with radius of about a parsec around Sgr A*. The presence of these objects opens up the possibility that they might be accreting “cool” gas (i.e. the Minispiral) as discussed by Morris [6]. Here we calculate the X-ray emission expected from these black holes as a method to constrain their population. We find that the data limits the total number of such black holes to around $10^{20}$ thousand. Even a much smaller number of such black holes, i.e. 5 thousand, is sufficient to produce several sources with X-ray luminosity above $L_X \sim 10^{33}$ erg s$^{-1}$ at any one time. We suggest that some of the discrete X-ray sources observed by Muno [7] with Chandra in the inner parsec may be such “fake X-ray binaries”.

1. Introduction
Morris [6] noted that dynamical friction of massive stellar remnants on a population of less massive background stars will initiate transfer of these remnants closer to the nucleus of the Galaxy. In particular, he estimated that for stellar mass black holes near our Galactic Centre, the radius from which these will be collected into a central cluster is about 4 parsecs, and the total mass of the black holes is as high as $10^6 M_\odot$. More recently, Miralda-Escud & Gould [5] updated and quantified Morris’s [6] estimates and showed that black holes will force most of low mass stars out of the central region with size $R_{bh} \approx 0.7$ parsec around Sgr A*. In their model, black holes form a central cluster of this size and number about 24 000. The total mass of these black holes could be as high as $5 - 10\%$ of Sgr A*’s mass. In addition to this, more recent observations of young massive stars near Sgr A* suggest another potentially important channel that can contribute to this population of stellar remnants in the central parsec of Sgr A*: in-situ star formation that seems to be heavily biased towards forming many more massive stars per unit stellar mass than in normal galactic environment [8, 9, 11]. Formation of a cluster of black holes surrounding the SMBH thus appears not just plausible but inevitable.

Morris [6] also discussed the possibility that compact remnants will be accreting gas from molecular clouds that may be unusually dense near our Galactic Centre. He noted that if gas density within the central parsec of Sgr A* were much higher than that estimated from the current observations, then these black holes may be collectively rather bright. In this paper we set out to explore this idea in more detail and also speculate on the implications for observations of AGN in general. In particular, we improve on the Bondi-Hoyle accretion rate estimates of Morris [6] in two ways. First of all, we include in the model the formation of small scale discs around accreting black holes due to non-zero residual angular momentum of the accreting gas.
In addition to this, we use a more sophisticated form for the radiative efficiency motivated by the observations of radiatively inefficient accretion flows.

2. Numerical approach

2.1. Bondi-Hoyle gas capture rate estimates

Consider a stellar mass black hole ploughing through a gas cloud or a disc. The black hole will be accreting, or rather capturing, gas at the Bondi-Hoyle accretion rate [3]:

$$\dot{M}_{\text{capt}} \sim 4\pi \rho \frac{(GM_{\text{bh}})^2}{(\Delta v^2 + c_s^2)^{3/2}},$$

where $\rho$, $c_s$ and $\Delta v$ are the ambient gas density, sound speed and the relative velocity between the black hole and the gas, respectively. The above picture is complicated by the presence of the SMBH. The area of influence of the stellar mass black holes is limited by the Hill radius,

$$r_H = R \left(\frac{M_{\text{BH}}}{3 M_{\text{SMBH}}} \right)^{1/3},$$

where $R$ is the distance between the stellar mass black hole and the SMBH. This imposes a limit onto the capture rate, given by the Hill accretion rate,

$$\dot{M}_H = 4\pi r_H^2 \rho c_s.$$

2.2. Time-dependent disc accretion

The gas captured by a stellar mass black hole will in general have a net angular momentum, resulting in the formation of a small scale accretion disk. We will assume that a disc of size $r_c$ is formed, where $r_c = \zeta r_{\text{capt}}$. The capture radius is

$$r_{\text{capt}} = \min \left[ r_H, \frac{GM_{\text{BH}}}{\Delta v^2 + c_s^2} \right],$$

and $\zeta$ is a parameter $\leq 1$. In fact, below we only consider values of $\zeta \leq 0.1$ because larger values of $\zeta$ are possible but not very realistic as it would imply that the captured flow has a maximum attainable asymmetry. For the same reason very small values of $\zeta \ll 10^{-3}$ would also seem quite unlikely.

The viscous time scale in the small scale disk about the stellar mass black hole is $t_{\text{visc}} = \alpha^{-1} \Omega_d^{-1} (r/h)^2$, where $\alpha$ is the viscosity parameter, $\Omega_d$ is the angular velocity at $r = r_c$ and $h$ is the height of the disk. This can be expressed as,

$$t_{\text{visc}} = 1.5 \times 10^3 \text{ year} \alpha_{0.01}^{-1} \mu_d r_c^{1/2} T_{d,3}^{-1},$$

where $T_{d,3}$ is the disk temperature in units of $10^3$ K, the viscosity parameter is $\alpha_d = 0.01 \alpha_{0.01}$, $\mu_d$ is the mean molecular mass in units of Hydrogen mass, and $r_c,12$ is the circularisation radius in $10^{12}$ cm. This is to be compared with period (the orbital time) about the SMBH. At the distance of 0.1 pc, $P \sim 3000$ year $M_6^{-1/2}$. Thus, the gas captured in the small scale disk about a stellar mass black hole accretes on the black hole after a delay of perhaps a fraction to a few orbital times.

We model the small scale disc evolution in the following way. We estimate the time-dependent accretion rate onto the black hole as $\dot{M}_{\text{acc}} = (\dot{M}_d/t_{\text{visc}}) \exp^{-t_{\text{visc}}/12t}$. The mass of the disc then evolves according to the addition and accretion of mass as $\dot{M}_d = \dot{M}_{\text{capt}} - \dot{M}_{\text{acc}}$. The luminosity of the accretion flow is assumed to be given by $L_X = \epsilon \dot{M}_{\text{acc}} c^2$, where $\epsilon$ is an analytical “fit” to the radiative efficiency in both non-radiative and standard radiative regimes. $\epsilon = 0.01 M / (M_0 + M)$, where $M_0 = 0.01$ is the critical accretion rate where the switch from one regime to the other occurs. We also assumed that X-ray emission visible in the Chandra band constitutes 10% of the bolometric efficiency, which would be a lower limit for typical spectra of X-ray binaries in their hard state.
2.3. Orbital evolution of accretors

We model the velocity and space distribution of stellar mass black holes as a cusp that follows the Bahcall-Wolf [2] distribution for heavier species in a mass-segregated cusp. This distribution results in the black hole number density obeying a power law of the form $R^{-7/4}$. Both space and velocity distributions are isotropic. The most recent Monte-Carlo simulations [4] broadly support these classical results.

Generating a series of orbits consistent with this distribution, we randomly prescribe the initial phases of the black holes, and then we follow their spatial motion. When one of these orbits intersects the disc of the Minispiral, the black hole in question starts capturing gas and builds up a disc around it as described in §2.2.

2.4. The model for the Mini-Spiral

In our simple model, the Minispiral is modelled as a disk in circular rotation around Sgr A* with the total gas mass of 50 Solar masses, in accord with estimates in [10]. It extends from a radius of 0.1 pc from the SMBH to a radius of 0.5 pc. [10] suggests that the Minispiral is a dynamical feature almost in free fall onto Sgr A*. The dynamical age of the feature is a few thousand years. Therefore, we ran our calculations for 3 000 thousand years with these assumptions, and then we “remove” the Minispiral instantaneously. This is done as a very rough model of time evolution of the system. In a future work a more complicated, but unavoidably model-dependent, dynamic of the Minispiral might be included.

3. Few tests

The upper panels in Figure 1 display the total X-ray luminosity of a cluster of 5 000 black holes as a function of time for two values of the circularisation radius parameter, $\zeta$, as labelled in the Figure. The lower panels show the number of X-ray sources with luminosity higher than $10^{33}$ erg s$^{-1}$. Such sources could be observed by Chandra. Several conclusions can be made. With a larger value of $\zeta = 0.1$, the accretion discs around black holes are larger, and thus viscous times are long. As a result, the X-ray emission varies smoothly with time, first increasing as the discs are built up, and then decreasing on $\sim$ a thousand years time scale. Thus the sources are rather steady in time, and are rather dim.

For the smaller value of $\zeta = 10^{-3}$, viscous times in small scale discs are much shorter. Therefore, the X-ray emission from the sources vary on much shorter time scales, i.e., of few years to tens of years. The sources are also brighter as the peak accretion rates are higher – each individual source shines much brighter for a shorter time, of course, as compared with the larger $\zeta$ case. Figure 2 shows the same experiments as Figure 1 but for 20 000 black holes. Comparison between the two different values of $\zeta$ shows similar trends as before.

It is interesting to compare the two Figures. While the results depend significantly on the apriori unknown value of $\zeta$, both low and high $\zeta$ tests show same tendency of a significant luminosity increase with increase in the number of black holes. In fact, the luminosity increased by a larger factor than the black hole number did. The number of sources above the chosen luminosity threshold also increased. This suggest that by performing tests across all reasonable parameter space for $\zeta$ we should be able to find the maximum allowed number of stellar mass black holes in the cusp.

4. Results

Following this idea, we ran a number of models for a range of values of $\zeta$ and for the total black hole numbers of $N = 5, 10, 20$ and 40 thousand. During the time period modelled, the results vary considerably in each test. For example, it is possible for just one single source to dominate the X-ray luminosity output of the cluster. Thus we have chosen to average several
Figure 1. X-ray light curves (upper panels) and number of individual sources (bottom panels) where $L_X < 10^{33}$ erg s$^{-1}$, for 5000 black holes in the inner parsec. The left and right panels correspond to $\zeta = 0.1$ and $\zeta = 0.001$ respectively.

Figure 2. X-ray light curves and number of individual sources where $L_X < 10^{33}$ erg s$^{-1}$, when the total number of black holes in the inner parsec is 20000. The left and right panels correspond to $\zeta = 0.1$ and $\zeta = 0.001$ respectively.

Table 1. Characteristics of black hole cusp averaged between 2000 and 3000 years.

<table>
<thead>
<tr>
<th>Number$^a$ [10$^3$]</th>
<th>$\zeta^b$</th>
<th>$&lt; L_X &gt;^c$ [10$^{35}$ erg s$^{-1}$]</th>
<th>P($L_X &gt; 10^{36}$)$^d$</th>
<th>P($L_X &gt; 10^{35}$)$^e$</th>
<th>$&lt; N &gt;^f$</th>
<th>P(N&gt;20)$^g$</th>
<th>P(N&gt;10)$^h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.001</td>
<td>5.39 ± 3.06</td>
<td>0.16 ± 0.11</td>
<td>0.58 ± 0.19</td>
<td>4.28 ± 0.97</td>
<td>0.0 ± 0.0</td>
<td>0.05 ± 0.03</td>
</tr>
<tr>
<td>5</td>
<td>0.010</td>
<td>1.33 ± 0.64</td>
<td>0.02 ± 0.02</td>
<td>0.27 ± 0.15</td>
<td>3.04 ± 0.75</td>
<td>0.0 ± 0.0</td>
<td>0.0 ± 0.0</td>
</tr>
<tr>
<td>5</td>
<td>0.100</td>
<td>1.52 ± 0.39</td>
<td>0.00 ± 0.00</td>
<td>0.62 ± 0.22</td>
<td>6.13 ± 1.36</td>
<td>0.0 ± 0.0</td>
<td>0.15 ± 0.11</td>
</tr>
<tr>
<td>10</td>
<td>0.001</td>
<td>7.06 ± 4.31</td>
<td>0.59 ± 0.17</td>
<td>1.0 ± 0.01</td>
<td>9.79 ± 1.59</td>
<td>0.001 ± 0.001</td>
<td>0.35 ± 0.21</td>
</tr>
<tr>
<td>10</td>
<td>0.010</td>
<td>7.53 ± 2.11</td>
<td>0.27 ± 0.15</td>
<td>1.0 ± 0.0</td>
<td>10.69 ± 1.30</td>
<td>0.001 ± 0.001</td>
<td>0.50 ± 0.17</td>
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<tr>
<td>10</td>
<td>0.100</td>
<td>1.74 ± 0.69</td>
<td>0.01 ± 0.004</td>
<td>0.50 ± 0.21</td>
<td>11.82 ± 0.65</td>
<td>0.002 ± 0.001</td>
<td>0.72 ± 0.12</td>
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<tr>
<td>20</td>
<td>0.001</td>
<td>20.8 ± 5.3</td>
<td>0.55 ± 0.12</td>
<td>1.0 ± 0.0</td>
<td>12.96 ± 0.24</td>
<td>0.0 ± 0.0</td>
<td>0.92 ± 0.02</td>
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<tr>
<td>20</td>
<td>0.010</td>
<td>20.2 ± 2.65</td>
<td>0.96 ± 0.02</td>
<td>1.0 ± 0.0</td>
<td>18.55 ± 0.48</td>
<td>0.23 ± 0.06</td>
<td>1.0 ± 0.0</td>
</tr>
<tr>
<td>20</td>
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<td>8.89 ± 2.20</td>
<td>0.32 ± 0.23</td>
<td>1.0 ± 0.0</td>
<td>19.79 ± 1.15</td>
<td>0.42 ± 0.11</td>
<td>1.0 ± 0.0</td>
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<tr>
<td>40</td>
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<td>84.1 ± 23.9</td>
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<td>1.0 ± 0.0</td>
<td>29.32 ± 0.37</td>
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<td>1.0 ± 0.0</td>
</tr>
<tr>
<td>40</td>
<td>0.010</td>
<td>101 ± 16.3</td>
<td>1.0 ± 0.0</td>
<td>1.0 ± 0.0</td>
<td>34.28 ± 2.90</td>
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<tr>
<td>40</td>
<td>0.100</td>
<td>12.0 ± 3.30</td>
<td>0.42 ± 0.211</td>
<td>1.0 ± 0.0</td>
<td>44.32 ± 0.74</td>
<td>1.0 ± 0.0</td>
<td>1.0 ± 0.0</td>
</tr>
</tbody>
</table>

$^a$ Total number of black holes in the cusp

$^b$ Circularisation parameter

$^c$ Time-averaged luminosity of the cusp

$^d$ Probability that the total luminosity of the cusp is greater than 10$^{36}$ erg s$^{-1}$

$^e$ Probability that the total luminosity of the cusp is greater than 10$^{35}$ erg s$^{-1}$

$^f$ Average number of sources with X-ray luminosity greater than 10$^{33}$ erg s$^{-1}$ ($n$)

$^g$ Probability that $N$ is greater than 20

$^h$ Probability that $N$ is greater than 10.
observables and then present their mean values. In particular, we calculated the average total X-ray luminosity and the number of black holes brighter than $10^{33}$ erg s$^{-1}$, as such sources would have been resolved by Chandra into separate point sources. The averaging is done between $2000 < t < 3000$ years as to look at a state that may be similar to the present state of the mini-spiral.

A summary of the results is presented in Table 1, where a number of time-averaged quantities are presented. In order to reduce and estimate statistical noise of the results, for each of the values of $\zeta$ and the total black hole number considered in Table 1, the tests were repeated 3 times, each time generating a different random black hole orbit distribution. The mean values of the quantities and their deviation are listed in Table 1.

Observations of the inner parsec by Chandra have placed upper limits on the total luminosity of sources of approximately $10^{35}$ erg s$^{-1}$ [1] and the number of individual X-ray sources with a luminosity greater than $10^{33}$ erg s$^{-1}$ of a dozen or so (F. Baganoff, private communication at the meeting). With these constraints in mind, we can immediately rule out the possibility that the cusp contains 40,000 black holes. For any reasonable value of $\zeta$, the total luminosity and the number of individual sources with $L_X > 10^{33}$ erg s$^{-1}$ are too large compared to observations. A cusp containing 5,000 or 10,000 black holes is well within the limits imposed by observations. A cusp containing 20,000 black holes is less likely but cannot be ruled out completely at this time.

5. Conclusions
The presence of stellar mass black holes in a cusp around a SMBH is a robust theoretical prediction [5, 6]. We describe an approach that can begin to constrain the numbers of these objects. Thus far the results presented suggest an upper limit of 20,000 for the number of stellar mass black holes in the inner parsec, consistent with theoretical predictions. Further refinements of the model of the X-ray emission from the black hole cusp, combined with updated observational data, may significantly improve the limits we put on these objects.

We suggest that at least some of the X-ray sources visible in the central parsec may be isolated black holes and neutron stars accreting gas from the Minispiral. Such sources should be preferentially located close to the Minispiral if viscous time is short ($\zeta$ is small). In addition, binary systems containing a black hole and a normal low mass star can also accrete gas in roughly the same way as we calculated here. In the case of low values of circularisation parameter, $\zeta$, the size of the disc around the primary (the black hole) can be smaller than the size of the binary itself. Thus, these systems may appear as “fake X-ray binaries”, where the gas supply comes from outside rather than from the low mass secondary. Observational signatures of such systems might be warped and out of binary plane accretion discs, “too short” or “too weak” accretion outbursts for the size of the binary.

References