THE FAILURE OF PERSPEX
IN LUBRICATED CONTACTS

by

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The failure of Perspex in lubricated contacts

ABSTRACT

The catastrophic failure of lubricated Perspex disks and cylinders rotating in heavily loaded point and line contacts in conditions of both rolling and sliding has been investigated. Observation of specimens in various stages of failure suggested that both thermal effects and mechanical stresses were involved in the failure mechanism. It has been shown that temperatures generated by conventional "flash temperature" mechanisms are not likely to be a significant factor in the mode of failure. Moreover, since this form of failure occurs under conditions of pure rolling the generation of heat by hysteresis loss appears to be a significant factor.

The quasi-elastic theory of rolling friction developed by Greenwood, Minshall and Tabor has been used to derive the intensity of heat generation, and hence the temperature distribution, in the sub-surface regions; this theory has also been developed to cover a wider range of materials and conditions. The theoretical sub-surface temperatures have been compared with those measured, over a range of loads and speeds, in experiments using embedded thermocouples. The heat transfer coefficient at the surface of a rotating disk (which is a factor influencing sub-surface temperatures) has been estimated, using the Chilton-Colburn relationship, from experimental measurements of the mass
transfer coefficient. Experiments to measure the coefficient of rolling friction are described, and the relationship between hysteresis loss factor, frequency and temperature is derived from the results. The effect of the variation of hysteresis loss factor with temperature upon the subsurface temperature has been discussed.

The relevance of these theories and experiments to the original observations of failure of Perspex, and to the successful operation of rolling systems using polymers, has been discussed.
PREFACE

This project has been carried out in the Research Laboratories of the Engineering Department, University of Leicester, under the supervision of Dr. J. F. Archard to whom I give my sincere thanks for encouragement, advice and helpful suggestions. I also wish to thank Mr. D. Whittaker for his help in taking the experimental measurements described in Section 6.5 as part of an undergraduate project.

The work embodied in this thesis is the product of the author's own research except where otherwise stated.

I am also indebted to the technical and workshop staff of the Department, in particular Mr. D. Gilbert, for their help in the construction and modification of apparatus. Thanks are also due to Mrs. J. J. C. Westerman and Miss J. M. Boulsover for their help in the preparation of this thesis.
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NOMENCLATURE

The meaning of terms used in the text, unless otherwise stated, is as follows

A  Constant
B  Constant
C  Constant
D  Diameter
E  Youngs modulus
F  Frictional force
G  Modulus of rigidity
H  Dimensionless parameter = $\frac{\tau_1}{\tau_3}$
J  Mechanical equivalent of heat
K  Thermal conductivity
L  Dimensionless parameter = $\frac{\tau_1}{\tau_2}$
L_0 = $\frac{1}{p_0}$ Dimensionless parameter related to energy loss in one stress cycle
P  Force
Q  Heat flux (per unit area)
R  Radius of curvature
V  Velocity
W  Normal load
Y = $\frac{Y}{b}$ Dimensionless coordinate (depth below surface)
a  Radius of area of contact
b  Half width of line contact
c  Specific heat
g  Acceleration due to gravity
h  Oil film thickness (unless otherwise specified)
\[ k = \frac{(1 - v^2)}{\pi E} \] Elastic constant

\( l_0 \) Perimeter of the closed loop on a stress diagram

\( p \) Pressure

\( p_o \) Maximum Hertzian contact stress

\( q \) Heat generated per unit volume

\( r \) Radius (radial coordinate)

\[ s = \frac{1}{2}(\sigma_y - \sigma_x) \] Shear stress

\( t = \tau_{xy} \) Shear stress

\( w \) Displacement

\( xyz \) Cartesian coordinates

\( x'y'z' \) Cartesian coordinates (at 45° to \( x,y,z \) coordinates)

\( x,y,z \) Distance in the \( x, y \) or \( z \) direction, where specified

\( a \) Hysteresis loss factor

\[ \beta = \frac{q}{\sigma} \] Function of the hysteresis loss factor

\( \epsilon \) Strain

\( \eta \) Oil viscosity

\( \theta \) Temperature

\[ k = \frac{\kappa}{\rho c} \] Thermal diffusivity

\[ \lambda = \frac{\eta}{G} \] Relaxation time

\( \mu \) Coefficient of friction

\( \nu \) Poisson's ratio

\( \rho \) Density

\( \sigma \) Direct stress

\( \tau \) Shear stress

\( \phi \) Work done in moving unit distance

\( \Phi \) Energy loss

\( \tau_1 \) Time taken for generated heat to diffuse to a depth \( a \)
\( \tau_d \) Time taken for the contact to move a distance a
\( \tau_s \) Time between successive passages of the contact
\( \omega \) Revs per second
\( d \varepsilon \) Incremental energy loss

**SUBSCRIPTS**

- m Mean or average
- f Oil film
- v Volume
- max Maximum
- xyz'y'z' Directional
- A Ambient

**Note**

Appendix 2 lists the notation and relationships specific to the problems of heat and mass transfer discussed in Chapter 7.
CHAPTER 1
INTRODUCTION

Knowledge of the details of the mechanisms of wear of non-metallic materials is quite incomplete. The great majority of wear studies have been concerned with the behaviour of metallic materials. Similarly knowledge of the failure of rolling and sliding contacts under concentrated loads is almost entirely confined to studies of metals and alloys. Under the heavily loaded conditions which occur in the lubricated contact of non-conforming metallic surfaces, two major forms of failure occur. In scuffing, metallic contact occurs through the lubricant film and severe welding and tearing of the surfaces results. In pitting failure, discrete lumps of material are detached from the surfaces by a process which, according to the generally accepted view, involves a fatigue process associated with the stress distribution below the heavily loaded region.

The work described in this thesis is related to the failure of polymeric materials under these same conditions of concentrated load. It has been concerned entirely with Perspex. This material was used because the starting point of the work was a new form of failure observed when using crossed cylinders of this material. A large lump of material was removed from one of the surfaces without any apparent damage to the other. Examination of the specimens suggested that both mechanical stresses and thermal effects played their part in the mechanism of failure.
From these original observations of failure the work developed into a more detailed study of some thermal effects which occur in the rolling and sliding of polymeric materials. Although the experiments were confined to Perspex, it seems clear that these thermal effects have a wider significance since they are likely to occur with all polymeric materials.

In the experimental work described in this thesis only one type of Perspex was used, namely the polymethyl - methacrylate product sold under that name by I.C.I. Some of the physical properties of the material, applicable to this work, are described in Appendix 1.
Fig 2.1  PERSPEX CYLINDER AFTER CATASTROPHIC FAILURE IN THE CROSSED CYLINDERS MACHINE. A LARGE LUMP OF MATERIAL HAS BEEN REMOVED
CHAPTER 2
EXPLORATORY INVESTIGATIONS

2.1 Introduction

The observation that the failure of Perspex at lubricated contacts differs from that of steel, or other metallic materials, was first made by Archard (1963). In a brief unpublished note, based upon experiments performed with a crossed cylinders machine, he reached the following tentative conclusions.

(1) The load-carrying capacity of heavily loaded contacts between Perspex specimens is high compared with that of steel specimens run under similar conditions.

(2) The high load-carrying capacity of Perspex specimens is due to the low elastic modulus and the consequent reduction in the pressure and the coefficient of friction.

2.2 Failure Experiments using the crossed cylinders machine

Archard (1963) observed that, whereas steel specimens in lubricated contact failed by scuffing, the failure of Perspex specimens was sudden and catastrophic, a large lump of material being removed from one surface with little apparent damage to the other specimen, other than that caused by debris. A photograph of a failed specimen is shown in Figure 2.1. A more extensive series of tests was undertaken by the present author in an attempt to elucidate the mechanisms of failure of Perspex.
Fig 2.2 ARRANGEMENT OF CYLINDERS IN THE CROSSED CYLINDERS MACHINE

DIRECTION OF TRAVERSE AT 45° TO THE TWO CYLINDER AXES
The first series of experiments used the crossed cylinders machine. In this apparatus (Figure 2.2) two rotating cylinders are loaded together with their axes mutually at right angles. The lower cylinder is traversed slowly to and fro in a direction at 45° to the two cylinder axes and this ensures that the specimens retain their original cylindrical shape despite wear. When the lower specimen is traversed the area of contact traces a helical path around the specimens. If the lower cylinder is not traversed then the area of contact traces a circumferential path around the specimens. A fuller description of the machine is given in a paper by Archard and Kirk (1961). A most important consideration in the present work is the speed of traverse of the lower specimen (Figure 2.2) compared with the surface speed of the rotating cylinders. At higher speeds of traverse the contact region traces out a widely spaced helical path on each of the specimens, so that, in successive rotations, the tracks on both the specimens are completely separated. As the speed of traverse is reduced the distance between these tracks gets smaller until, at intermediate speeds of traverse, successive rotations of the cylinders cause the helical path of the contact region to overlap the path of the previous rotation. At the extreme, with no traverse, the contact region remains at the same axial position on the cylinder, and in successive rotations the load is then applied to the same point on the specimens. At intermediate speeds of traverse, when, in successive rotations of the cylinder, the contact region is passing over the same
point on the cylinder more than once, the number of successive stress cycles to which each point on the surface is subjected, can be calculated from the speed of traverse, the surface velocity of the rotating cylinders and the width of the helical track. This width can be calculated from the size of the loaded region and is given by the well known Hertzian equations. (Timoshenko and Goodier 1951).

In the crossed cylinders machine experiments, two rotating Perspex cylinders, of 2.54 cm diameter, were loaded together and run over a range of loads (0 to 270kg) in attempts to achieve failure. The surface speed in these experiments was varied between 20 and 120 cm/sec and the cylinders were lubricated with a jet of a plain mineral oil to Admiralty specification O.M.100. The physical properties of this lubricant have been defined by Crook (1958) Archard (1958/9) and Archard and Kirk (1961). In these experiments the surfaces of the Perspex specimens were polished to a surface finish, as measured on a Talysurf, of better than \(5 \times 10^{-9}\) cm C.L.A.

In the first series of experiments the specimens were run without traverse so that repeated load applications occurred at each point of the track. In these experiments the application of light loads at moderate speeds did not cause failure, and no obviously detectable deformation of the specimens occurred. At heavier loads, and higher speeds, failure occurred but no clear pattern of relationships between speed and failure load could be found. There was
Fig. 2.3 TALYSURF TRACE OF SURFACE FLOW ON A 3.8 cm DIA. PERSPEX CYLINDER AFTER PASSAGE OF THE CONTACT REGION.
some indication that rate of lubricant supply affected the results, and the total time of running under any given set of conditions appeared to be a significant factor influencing the onset of failure.

In a second series of experiments the speed of traverse was set so as to ensure no overlapping of successive portions of the helical track of the loaded region. Within the limitations of the conditions possible (maximum load 270 kg, maximum speed 120 cm/sec), no failure of the specimens was achieved. It is of interest to note that in similar experiments, and somewhat less severe conditions, Archard (1963) was able to produce a scuffed helical track upon the surface of steel cylinders with one passage of the helical track of the contact region. Therefore, to increase the severity of the contact conditions, 3.8 cm 5 cm and 7.6 cm diameter Perspex cylinders were used, internally heated by oil flowing through them to raise their temperature. Due to its poor thermal conductivity the temperature of the Perspex close to the surface, measured by an embedded thermocouple, could only be raised to 45°C. Surface flow after one passage of the contact under the severest conditions was found, but no catastrophic failures occurred. The surface flow was in the form of an indentation conforming to the shape of the other cylinder. A Talysurf trace of such an indentation is shown in Figure 2.3.

In other experiments the speed of traverse was slowed so that the helical path of the contact region overlapped in successive rotations. Catastrophic failure then occurred.
Fig. 2.4  SPECIMENS FROM THE CROSSED CYLINDERS MACHINE SHOWING:

A. Tracks of surface flow traced out by the contact region.

B. Cracks along a track of surface flow.
Fig. 2-5  A LUMP OF MATERIAL IS BEING FORCED OUT OF THE SPECIMEN AT C

Fig. 2-6  CATASTROPHIC FAILURE OF A PERSPEX SPECIMEN. MATERIAL HAS BEEN TORN FROM THE SPECIMEN IN A TRACK AROUND THE CYLINDER. THE WIDTH OF THE TRACK IS COMPARABLE WITH THE DIAMETER OF THE CONTACT AREA.
Fig 2.7 THE SECTIONS CUT FROM A CYLINDER FOR EXAMINATION ON A PHOTO-ELASTIC BENCH.
**Fig. 2.8a**

The longitudinal section of Figure 2.7 photographed on a photoelastic bench. The two light areas show regions of residual strain beneath areas of surface flow.

**Fig. 2.8b**

The cross section of Figure 2.7 photographed on a photoelastic bench. The section has been cut at D in Figure 2.8. The light areas show, once again, regions of residual strain beneath areas of surface flow.
Fig. 2-9  
TWO FURTHER SPECIMEN CROSS SECTIONS PHOTOGRAPHED ON A PHOTOELASTIC BENCH.

(a) CROSS SECTION CUT IN A SPECIMEN THROUGH A CIRCUMFERENTIAL TRACK OF SURFACE FLOW. THE WHITE AREAS SHOW REGIONS OF RESIDUAL STRAIN.

(b) CROSS SECTION CUT IN A SPECIMEN AT A POINT OF SEVERE SURFACE DAMAGE. REMOVAL OF MATERIAL FROM THE SURFACE HAS RELIEVED THE STRAINED REGION BENEATH THE SURFACE.
The width of the lump of material removed was generally comparable with the diameter of the Hertzian contact, and the depth was usually about half that size, i.e. comparable with the position of the maximum shear stress (Timoshenko and Goodier 1951). Figures 2.4 2.5 and 2.6 show photographs of Perspex specimens in various stages of failure. A 3.8 cm diameter specimen which had failed was sectioned (Figure 2.7) and the sections examined on a photo-elastic bench. Figure 2.8a shows the longitudinal section through the specimen. The white spots show regions of residual stress beneath tracks of visible surface flow. Figure 2.8b shows the cross-section at D in Figure 2.8a. Once again the region of residual stress beneath the surface can be clearly seen. Figure 2.9 shows two more cross-sections through specimens. It can be seen that the removal of a lump of material from the surface has relieved the residual stresses beneath the surface. At this point in the investigation, from an analysis of failed specimens, it seemed apparent that surface flow and deformation occurred when the surface temperature reached the softening point of Perspex, and that after further traversals of the contact over the same point the excessive stresses caused cracks to form and catastrophic failure to follow.

The observations of specimens at various stages of failure suggested that both thermal effects and mechanical stresses were involved in the failure mechanism. The following stages of a mechanism of failure were tentatively advanced on the basis of these observations.

(1) The load is applied and the region of contact is approximately Hertzian.
(2) Under sliding the Perspex specimens and the oil film become heated.

(3) Flow occurs on the surface and ridges are thrown up on either side of the track traced out by the contact areas. The width of the surface damage is comparable in size to the diameter of the contact area.

(4) Cracking occurs along the track.

(5) A lump is torn out of the track.

Because thermal effects play a large part in this proposed mechanism of failure it is necessary to examine, in some detail, the magnitude of the temperatures generated by sliding. In the next three sections the theory of temperatures generated by sliding will be examined and the theoretical predictions for the conditions of these crossed cylinders experiments will be critically examined.

2.3 Generation of heat by sliding; the flash temperature

The average temperature $\theta_m$ at the contact area may be predicted using the flash temperature concept. This concept assumes that the two bodies come into rubbing contact; the heat is generated at the area of true contact and conducted away into the bulk of the rubbing members. Blok (1937), Jaeger (1943) and later Archard (1958/9) have presented a theory to predict $\theta_m$, the mean rise in temperature over the contact area, for a range of surface speeds. Jaeger showed that the type of equation used to calculate the temperature depends upon a non-dimensional parameter.
where, \( V \) = velocity of heat source
\( a \) = radius of circular area of contact
\( \kappa \) = thermal diffusivity

Archard (1958/9) emphasised the fact that the type of equation appropriate for the prediction of the flash temperature depended upon the nature of the heat conduction in the two rubbing members. Thus, when applying the theory, one has to decide whether the velocity of the contact (regarded as a heat source) over the surface is slow enough for the temperature distribution of a stationary contact to be established in the body, and, therefore, for steady state conditions to exist, or whether the contact is moving so fast that the sideways flow of heat can be neglected and the problem can be treated as one of linear heat flow. Thus it was shown that the Jaeger non-dimensional parameter \( L \) could be expressed as

\[
L = \frac{V a}{2\kappa} \tag{2.3.1}
\]

Where \( \tau_1 \) is the time taken for the maximum effect of the surface generated heat to penetrate to a depth \( a \), and \( \tau_2 \) is the time taken for the contact to move a distance \( a \). Thus for large values of \( L \) the time taken for the heat to penetrate to a depth \( a \) is large compared with the time for which the heat source is applied.

In the experiments under consideration the values of \( L \) are very large indeed. For example, when two 2.5 cm diameter Perspex cylinders were loaded together in the crossed cylinders...
<table>
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<th>$\mu$</th>
<th>W Kg</th>
<th>$\frac{1}{W^2}$</th>
<th>$\frac{1}{gm^2}$</th>
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machine under a load of 118 kg, the value of $a$, derived from the Hertzian equations, was 0.15 cm; assuming a surface velocity of $106.8 \text{ cm/sec}$ and taking the value of $\nu$ as $10.7 \times 10^{-4} \text{ cm}^2/\text{sec}$ (Appendix 1) the calculated value of $L$ is 7,500.

Therefore the conditions of the high speed equations apply to all of these experiments. It is appropriate to use the simplified form deduced by Archard

$$\theta = \frac{\mu g}{J} \frac{1}{3.8} \left( \frac{E}{K \rho c R} \right)^{\frac{1}{2}} W^{\frac{1}{2}} V^{\frac{1}{2}} \quad (2.3.3)$$

This assumes that all the heat generated is conducted to one surface. In our experiments the heat may be assumed to be divided equally between the two surfaces so that the values of temperature are reduced by a factor of 2. Thus

$$\theta = \frac{\mu g}{J} \frac{1}{7.6} \left( \frac{E}{K \rho c R} \right)^{\frac{1}{2}} W^{\frac{1}{2}} V^{\frac{1}{2}} \quad (2.3.4)$$

This equation gives the average temperature in a circular contact. Archard shows that the maximum temperature $\theta_{\text{max}}$ is 1.65 times the average temperature. Values of $\theta_{\text{max}}$ predicted from equation $(2.3.4)$ for a series of experimental conditions at which failure occurred are shown in Table 2.1. It will be observed that these theoretical values of the maximum temperatures are hardly sufficient to cause thermal softening of the Pergpex. Moreover the mean temperatures in the contact region are significantly less than those shown in Table 2.1, which occur only at the trailing edge of the contact region.

2.4 Generation of heat by sliding: sub-surface temperatures.

The theoretical analysis of flash temperatures given by Jaeger (1943) also provides values of temperatures below the
surfaces of the rubbing bodies. However, it is convenient to develop here a simplified treatment, similar in concept to that of Archard (1958/9), assuming that at high speeds the flow of heat is linear.

The appropriate analysis is concerned with the temperature distribution in a semi-infinite solid when a plane heat source of strength \( q \) is applied at the surface, starting at time \( t = 0 \). The temperature \( o_y \) at a depth \( y \), at time \( t \) is given (Ingersoll, Zobel and Ingersoll, 1948) by the equation.

\[
0_y = \frac{q x}{K \sqrt{\pi t}} \int_{\eta x}^{\infty} \frac{e^{-\beta^2}}{\beta^2} \, d\beta \quad (2.4.1)
\]

where

\[
\eta = \frac{1}{2\sqrt{\pi t}}, \beta = \frac{x}{2\sqrt{\kappa (t - \tau)}}, \kappa = \frac{K}{\rho c}
\]

and \( \tau \) is a time variable with limits 0 and \( t \).

Now the integral in equation (2.4.1) can be rearranged as follows:

\[
\int_{\eta x}^{\infty} \frac{e^{-\beta^2}}{\beta^2} \, d\beta = \left[ \frac{e^{-\beta^2}}{\beta} \right]_{\eta x}^{\infty} - 2 \int_{\eta x}^{\infty} e^{-\beta^2} \, d\beta
\]

\[
= \left[ \frac{-e^{-\beta^2}}{\beta} \right]_{\eta x}^{\infty} - \int_{0}^{\eta x} e^{-\beta^2} \, d\beta + 2 \int_{0}^{\eta x} e^{-\beta^2} \, d\beta
\]

\[
= \left[ \frac{-e^{-\beta^2}}{\beta} \right]_{\eta x}^{\infty} - \sqrt{\pi} \left[ \beta - \frac{\beta^3}{3} + \frac{\beta^5}{10} - \cdots \right]_{\eta x}^{\infty}
\]

\[
\int_{\eta x}^{\infty} \frac{e^{-\beta^2}}{\beta^2} \, d\beta = -\frac{1}{\sqrt{\pi}} + \frac{1}{\eta x} + \sum_{n=1}^{\infty} \frac{(\eta x)^{2n-1}(-1)^{(n-1)}}{(2n-1) \, n!} \quad (2.4.2)
\]
Thus, substituting this expression for the integral in equation (2.4.1) we can write for the first three terms of the expression for $0y$:

$$0y = \frac{2a\theta^2}{(K\pi^2 c)^2} - \frac{ax}{K} + \frac{ax^2}{2K(\kappa^2 t)^2}$$ \hspace{1cm} (2.4.3)

It is interesting to compare this solution, for the passage of a rectangular heat source over a semi-infinite solid, with that of Jaeger (1942) for a band source. The difference between the two expressions for temperature are negligible for values of $L$ greater than 5.

A more detailed examination of this solution including an interpretation of the physical meaning and a justification for dropping all but the first three terms of equation 2.4.3, will be given elsewhere. (Baglin and Archard - to be published). It is sufficient to say at present that if, as is usual, one talks of the distance beneath the surface at which the temperature drops to $1/e$ of the surface temperature, then the simplification of equation (2.4.3) to the first three terms is justified. This depth for, say, loaded Perspex cylinders rotating at 100 cm/sec will be of the order of $3 \times 10^{-3}$ cm (when the radius of the contact area $a=0.1$ cm). Clearly, this depth is small compared with the depth of the maximum shear stress.

2.5 Temperatures generated in the lubricant film.

Although flash temperature theory assumes that the effect of the oil film can be neglected, the temperatures
existing within the elasto-hydrodynamic film can be predicted from Archard's analysis. He shows that the temperature difference required within the oil film to ensure that all the heat generated is conducted to the surfaces is

$$\theta_f = \frac{q_v h}{8K}$$

(2.5.1)

where $K_s$ is the thermal conductivity of the oil and $q_v$ is the heat generated per unit volume in the oil film. Inserting experimental results into this equation shows that this temperature difference is so small that it will have negligible effect upon the surface temperatures.

An approximate value of $h$ derived from Archard and Kirk (1963) is $5.5 \times 10^{-5}$ cm. The following are a typical set of experimental conditions. Load $W = 118.5$ Kg, sliding velocity $V = 106.8$ cm/s, coefficient of friction $\mu = 0.004$. Thus $q_v = 3 \times 10^6$ cal.sec.$^{-1}$cm.$^{-3}$. Taking $K$, as $4 \times 10^{-5}$ cal.sec.$^{-1} \text{°C}^{-1}$. Equation 2.5.1 gives a value of $0.3 ^\circ \text{C}$ for the temperature difference $\theta_f$ within the oil film. Thus it would appear that the temperatures generated within the film make a negligible contribution to overall temperatures.

The above calculation is based upon the assumption that the heat generated within the film is conducted to the surfaces. This is obviously true when metal specimens are used but it is less clear that the argument is valid when (as in the present experiments) the specimens are of a material having thermal properties almost identical with those of the lubricant. A more complete analysis (Baglin

* see Section 2.6
and Archard, to be published) shows that this equation (2.5.1) is indeed valid when Perspex specimens are used. In this case the depth of heat penetration into the Perspex is more than an order of magnitude larger than the thickness of the lubricant film. The thickness of the film is about $5 \times 10^{-3}$ cm and, as shown in Section 2.4, the depth of penetration of the heat into the Perspex is of the order of $10^{-6}$ cm. Therefore most of the heat generated in the film by viscous dissipation is conducted to the surfaces, despite their poor thermal conductivity.

2.6 Discussion of experiments using the crossed cylinders machine.

The examination of specimens in various stages of failure suggests that both thermal softening and mechanical stresses are involved in the mechanism of failure. The depth of material removed is comparable with the depth of the maximum Hertzian shear stress. On the other hand it has been shown that thermal effects associated with the flash temperature are confined to depths below the surface which are small when compared with the dimensions of the Hertzian stress distribution. We may conclude that the thermal softening of the Perspex, from the surface down to a depth comparable with the position of maximum shear, is not due to surface generated flash temperatures.

It remains to consider the possible influence of asperity contacts through the lubricant film. When elastohydrodynamic conditions occur in heavily loaded contacts it has been
shown, Dowson and Higginson (1966), that the thickness of the oil film may be greater than that predicted by classical hydrodynamic theory. Archard and Kirk (1963) show that, for a material such as Perspex with a low Young's Modulus, as the load is increased, deformation of the surfaces becomes significant before the pressures have become large enough to cause any appreciable increase in viscosity, and that the film thickness is given by

$$h = 0.92\left(\frac{b V R}{E}\right)^{\frac{1}{2}}$$  \hspace{1cm} (2.6.1)

Where $V$ is the vector sum of the surface speeds of the specimens. If the experimental parameters of failure tests on 2.5 cm diameter cylinders are inserted into this equation, then the predicted film thickness will be typically $5.5 \times 10^{-6}$ cm when $V = 106.8$ cm/sec. Values of film thickness of this size compare favourably with those measured experimentally by Archard and Kirk (1963). Using an optical technique they measured film thickness between $3 \times 10^{-6}$ cm and $5 \times 10^{-5}$ cm over a range of loads at $V = 21$ cm/sec for Perspex cylinders in the crossed cylinders machine.

When this value of film thickness is compared with the surface finish of the specimens, namely $5 \times 10^{-6}$ cm, the likelihood of sufficient asperity contacts through the oil film to produce some form of failure seems most unlikely. The measured value of surface roughness may, in fact, be reduced when passing through the contact region. Blok (1959) has shown that there are indications
that there might be elastohydrodynamic cases in which full hydrodynamic lubrication (complete contact inhibition) is achieved with minimum film thickness that are smaller than the height of the irregularities on the rubbing surfaces bounding the film. This is due to a smoothing of the irregularities, when they pass through the contact region, by local rises of hydrodynamic pressures (superimposed on the overall distribution of hydrodynamic pressures) in the film.

Although in the case of lubricated Perspex contacts the pressures generated in the contact region are smaller than those between steel rubbing bodies, there is still a possibility of a small amount of smoothing of the surface irregularities as they pass through the contact region. Finally, while considering the possibility of contact through the oil film between the surfaces of cylinders in the crossed cylinders machine, it should be noted that the values of friction measured during the experiments were very low. Typical values are shown in Table 2.1. If significant contact between the surfaces was occurring, far higher values of $\mu$ would be expected.

To conclude, it has been shown that substitution of experimental parameters into equation 2.3.3 indicates that it is unlikely that sufficiently high surface temperatures could be produced by a flash temperature mechanism to cause softening of the surface layers of the specimens. Such temperatures might, however, be produced as a result of two or more successive passages of the contact over the same
Fig. 2-10 GEOMETRIC ARRANGEMENT OF THE DISKS IN THE TWO DISK MACHINE

Fig. 2-11 DISC USED TO INVESTIGATE BODILY ROTATION OF OUTER SURFACE

0.034 CM DIA. COLOURED PERSPEX PLUGS
point on the surface. But such temperatures, if they occur, would be confined to layers very close to the surface; such layers of softened material would be very thin compared with the depth of the maximum Hertzian shear stress. Moreover, it has been shown that it is unlikely that failure is initiated by the existence of appreciable solid contact; it has been shown that the thickness of the lubricant film and the surface finish of the specimens suggests that this is most unlikely to occur.

2.7 Failure Experiments using a two disk machine.

The remainder of the experiments to be described have been performed with Perspex disks in line contact using the Two Disk Pitting Wear Machine that was used by Crook (1957) and Hamilton (1963) in the well known experiments concerned with sub-surface deformation and the forward flow of material loaded beyond the elastic limit. The geometric configuration of the disks in the machine is shown in Fig. 2.10. The two disks, which were 1.59 cm wide and 7.6 cm diameter, were loaded together by a lever system arranged so as to be self-aligning about the contact area. The lower shaft was driven by a \( \frac{1}{2} \) HP a.c. motor, through a Kopp variator and two pulleys. By changing the size of the pulleys a speed variation from 150 r.p.m. to 2,500 r.p.m. could be achieved. The upper specimen was driven through a pair of gears mounted on the two shafts. With two gears
of the same size the disks could be run in pure rolling, and by changing the ratio of the gears the disks could be run over a range of rolling with sliding conditions. The whole system, disks, gears and bearings, was lubricated with oil to Admiralty specification QM100, identical with that used in the crossed cylinders experiments (section 2.2).

The advantages of testing disks in this machine rather than in the crossed cylinders machine were twofold. Firstly a greater range of kinematic conditions, ranging from pure rolling, to rolling with appreciable sliding, could be achieved. Secondly, it was hoped to reduce the buttressing effect of material which is not under stress beside the contact region i.e. to conduct experiments under conditions of plane stress. A series of tests on 7.6 cm diameter Perspex disks, in conditions of pure rolling and rolling with sliding, confirmed the form of the failures in the cross cylinders machine. Once again surface flow occurred, followed by cracking and removal of a lump of material.

Herrebrugh (1968) has presented a numerical solution for the hydrodynamic and elasticity equations in elastohydrodynamic lubrication for a large range of loading conditions for constant viscosity. Archard and Kirk (1963) have shown that the increase in viscosity with pressure in highly loaded, low elastic modulus materials, such as Perspex, is small and so as a first approximation to the film thickness between Perspex disks in line contact Herrebrugh's solution, assuming constant viscosity of the
lubricating oil, should be adequate. For a range of loads between 14 Kg/cm and 200 Kg/cm on Perspex disks of 7.6 diameter, rolling together at a speed of 590 cm/sec Herrebrugh's solution predicts that the film thicknesses will range between $5 \times 10^{-4}$ cm and $2 \times 10^{-4}$ cm. These values of film thickness are, once again, much larger than the surface roughness.

A series of tests were devised to see whether the phenomenon of forward flow of material, previously investigated by Crook (1957) and Hamilton (1963) using soft metallic disks, could be reproduced when using Perspex disks. In the earlier experiments mild steel and copper disks were run at loads exceeding the yield point. It was found that flow of material occurred below the surface at a depth comparable with the position of the maximum Hertzian shear stress. Under conditions of pure rolling the outer rim of material, above the deformed region, was rotated forward with respect to the body of the disk. The shear necessary for this movement occurred in the plastically deformed subsurface region.

Fig. 2.11 shows the type of Perspex specimen used in these tests. Six holes 0.034 cm in diameter were drilled into the disk to a depth of 0.75 cm, and small coloured Perspex plugs were glued into the holes. Five such disks were tested in the 2 disk machine until they failed catastrophically. When the disks were removed and later sectioned there was no evidence of the outer surface layer rotating
bodily forward, and the only regions in which surface movement was detected were beneath areas of visible damage caused by catastrophic failure.

2.8 Discussion of experiments using the disk machine.

These experiments have shown that the mechanism of failure can be reproduced under conditions of plane stress. Once again it has been shown that failure occurs under conditions where the lubricant film thickness is much larger than the surface finish. It has also been shown that the phenomenon of forward flow, previously investigated by Crook and Hamilton, does not have a major influence upon the failures.

The most important aspect of the experiments with disks is the finding that the mechanism of failure can be reproduced under conditions of pure rolling, and this fact has determined the whole direction of the further work to be described in this thesis. Under conditions of rolling the only mechanism of energy dissipation arises from rolling rather than sliding friction. When using polymeric materials a major mechanism of rolling friction is hysteresis loss and this occurs in regions below the surface of the bodies. The earlier calculations, which are concerned with flash temperatures from energy dissipated at the surface, are not relevant to this situation. We therefore consider, now, some general principles which will influence the thermal effects of hysteresis loss.

In section 2.3 it was explained that the equations
which are appropriate for the calculation of flash temperatures are determined by a non-dimensional parameter, \( L \).

Equation 2.3.2 shows that this parameter is the ratio of two times, \( \tau \), the time for the generated heat to diffuse through a distance \( a \) (the half width of the Hertzian contact) and \( \tau_2 \), the time for the contact to move a distance \( a \).

It is now appropriate to define a non-dimensional parameter \( H \).

\[
H = \frac{\tau_1}{\tau_2} = \frac{V_a^2}{4\pi R} \tag{2.8.1}
\]

Where \( \tau_1 \) is the time required for heat to diffuse through a distance \( a \), and \( \tau_2 = \frac{2\pi R}{V} \) is the time between successive passages of the contact over a given region of the surface.

Thus, \( L \) (equation 2.3.2) indicates the extent to which, during the passage of a contact over a given region, the heat has diffused through distances comparable with the size of the contact region itself. The parameter \( H \) (equation 2.8.1) indicates the extent to which, in the period between successive contacts, the heat has diffused through distances comparable with the size of the contact region. Now elastic hysteresis losses must cause the dissipation of energy in regions below the surface, at depths comparable with the size of the contact region, to be manifested as temperature rises in these regions. Therefore, if the non-dimensional parameter \( H \) is large, the rise in temperature, associated with one passage of the contact remains, largely unchanged by thermal diffusion, at the instant of the next passage of
the contact. This means that, when $H$ is large, the temperature rise in sub-surface regions, although brought about by a series of pulses of energy dissipation, is determined by the conduction of heat over periods of time which are long compared with the time between successive passages of the contact. Thus the major factor to be considered is energy dissipation as a continuous process rather than as a transient phenomena.

Consider a typical failure experiment. Two disks of 7.5 cm diameter were loaded together and run in pure rolling with surface velocities of 250 cm/sec. Under a load of 110 Kg/cm the half width of the contact region, derived from Hertzian equations, is 0.39 cm. Since we can write from equation (2.8.1)

$$H = \frac{Vb^3}{4\kappa \pi R}$$  \hspace{1cm} (2.8.2)

then the parameter $H$ has a value of 1,500.

The discussion of the last two paragraphs sets the mechanism of failure in a new light. It is seen that the hysteresis losses associated with rolling friction may cause a rise in temperature of the sub-surface regions, and it is now possible to consider a mechanism of failure in which thermal effects are significant in those regions where the mechanical stresses are high. We shall therefore be concerned, in later chapters of this thesis, with a closer examination of these thermal effects.

As far as can be discovered there is little discussion in the literature of the thermal effects of hysteresis losses
in rolling friction. The only relevant information is contained in descriptions of experiments by Buckingham (1943) and Atack and May (1961).

Buckingham (1943), investigating the surface fatigue of plastic materials, reported the results of an experiment in which a hardened steel disk was loaded against a phenolic laminated disk (bakelite) and run in rolling conditions. At the end of 107,000 cycles a blister appeared on the bakelite surface with no cracks or other signs of failure on the surface. When the disk was sectioned to examine the blister a crack, about one quarter of an inch long about 0.15 in below the surface was found to have developed under the blister. In addition, a circular band of the typical dark brown colour of overheated phenolic resins, about 0.030 in wide and 0.15 in below the surface, was present, running entirely around and concentric with the surface. A similar dark brown band of about one half this width was also observed, extending from the surface to slightly below it. Buckingham concluded that the material itself made an automatic record of the heat concentration built up by the internal friction created by the stresses imposed; that the testing of such materials is complicated by these thermal conditions, by the material having high internal friction, low heat conductivity, and appreciable loss of strength at higher temperatures.

Similarly Atack and May (1961) conducted experiments which attempted to simulate the pulping process, in which pulp is produced by rubbing wood against a grindstone. It
had been noted that when the process was operating successfully the grits of the stone were rounded. In their experiments the scale of the process was increased in size, the rounded grits being represented by cylindrical steel sliders of radius 0.75 in, which rubbed against the surface of wood saturated with water. A significant finding in these experiments was a layer of charred wood below the surface, at depths comparable with the position of the maximum shear stress.
CHAPTER 3

MEASUREMENTS OF TEMPERATURES OF ROLLING DISKS

3.1 Introduction

In Chapter 2 the importance of thermal effects in the failure of Perspex was established. It was also suggested that the high temperatures which were significant in the mechanism of failure, arose from the generation of heat by hysteresis losses. Moreover, an elementary calculation showed that these temperatures were achieved as the cumulative effect of a large number of passages of the contact over the surface. Because hysteresis losses cause the generation of heat below the surface, a consideration of the sub-surface temperatures becomes of critical importance in any further, more detailed, examination of this subject.

This chapter is concerned with the measurement of surface and sub-surface temperatures in rolling Perspex disks. The various techniques used in these measurements will be described. Chapters 4 and 5 will then provide a parallel theoretical analysis of sub-surface temperatures generated by hysteresis losses. At this stage it is only necessary to explain that hysteresis losses are associated with the Hertzian stress distribution; therefore regions where the energy dissipation is greatest occur at depths comparable with $b$, the half width of the band of contact. The maximum losses occur near the position of maximum shear stress, at a depth of $0.78b$.

In a typical disk machine experiment to be described, loads between 40 kg/cm and 100 kg/cm were applied to 7.5 cm
diameter disks; and thus the values of \( b \), derived from the Hertzian equations, were in the range \( 8 \times 10^{-2} \) cm to \( 12 \times 10^{-2} \) cm. Therefore it was decided to measure the sub-surface temperatures at depths up to a maximum value of about 0.4 cm.

3.2 Measurements of surface temperatures using trailing lead thermocouples

In some early experiments attempts were made to measure the temperature of the disks using a trailing lead thermocouple. Two 7.5 cm diameter Perspex disks were loaded together and driven in pure rolling at a range of speeds between 500 r.p.m. and 1000 r.p.m. A trailing lead thermocouple was lightly loaded against the lower disk and another thermocouple was placed in the jet of lubricating oil being pumped to the disks. After running for 30 minutes at a constant load, the machine was stopped and a mercury in glass thermometer was quickly pressed against the surface of the disk. In a typical experiment the oil jet temperature remained constant throughout the experiment at 21°C, while the temperature recorded by the trailing lead thermocouple rose from 21°C to 32°C. The mercury in glass thermometer, pressed against the lower disk as soon as the machine was stopped, reached a maximum temperature of 48°C. Let us assume that the thermometer, when pressed against the surface, was measuring the true temperature. Since a finite time had elapsed between stopping the machine and pressing the
thermometer against the disk, the surface temperature would already have started to fall. The only inference to be drawn from this experiment is that the surface temperature, immediately prior to the machine being stopped, was in excess of 48°C, and that a trailing lead thermocouple (which in this experiment read 32°C) is not suitable for measurement of surface temperature using Perspex disks.

Some explanation for this can be offered by a brief examination of the use of trailing lead thermocouples. Several examples of their application to the measurement of surface temperatures of steel disks in hydrodynamically lubricated experiments can be found in the literature (for instance O'Donoghue, Manton and Cameron (1967) ). When using metal disks it is usually assumed that thermal equilibrium between thin oil films and the disks is rapidly established, so that the temperatures of the disk and the oil film are the same. When using disks of materials of low thermal conductivity, heat generated at or near the surface cannot be conducted away rapidly into the disk. Consequently the time taken to establish conditions of thermal equilibrium in the surface regions is much longer. In many of the reported experiments using metal disks the rate of lubricant supply to the disks has been sufficiently small for the temperature of the oil film on the surface to be the same as the disk surface temperature. In the author's experiments, however, large quantities of fresh oil were being pumped over the disks and the surface oil
temperature was closer to the bulk lubricant temperature than to the disk surface temperature. Heat generated near the surface of the Perspex disks tended to flow to the surface, rather than through the specimen to the steel shaft, as may well be the case when metal disks are used. Finally it should be noted that the effectiveness of trailing lead thermocouples for surface temperature measurement on steel disks has been questioned by Dyson, Naylor and Wilson (1965), who have reported that the temperatures recorded from a trailing lead thermocouple on steel specimens was as much as 10°C lower than that of an embedded thermocouple 0.32 cm below the surface.

Clearly, from the author's own experiments there is justifiable doubt as to what temperature a trailing lead thermocouple is recording, in the experimental conditions prevailing (i.e. low thermal conductivity, disks, flooded with fresh cool oil) and, taken in conjunction with the evidence of Dyson, Naylor and Wilson, no further measurements were made with trailing lead thermocouples.

3.3 A consideration of possible techniques of temperature measurement

Some time was taken in a careful consideration of techniques which might be used to measure the sub-surface temperatures of rolling Perspex disks. A convenient review of temperature measuring techniques has been made by Watson (1964) and we now consider briefly the advantages and disadvantages of the methods discussed in this review under
three broad categories. The primary problem is to ensure that the actual temperature measured is the one required, and that the presence of the sensor does not affect other conditions in the vicinity of the measuring position.

(a) **Sensors Mounted on the Surface**

The techniques of temperature measurement grouped under this heading are, in general, only suitable for surface temperature detection and, because of difficulty in mounting them, cannot be used for sub-surface measurements. It is still of interest, though, to consider whether they can be used to measure surface temperatures.

1) **Melting.** A substance of known melting temperature is placed on the surface. The substance can be in the form of a paper, paint or pellet or it can be applied as a crayon. A range of such temperature sensitive materials is available commercially. This group, although easily applied are clearly not suitable for the present application.

2) **Colour.** There are several types of paint or lacquer commercially available which can be applied to a surface and change colour at a known temperature. Since the work on Perspex is restricted by its physical properties to the temperature range 20°C to 100°C, only the lower part of the range in which paints and lacquers can be used (40°C to 1400°C) is applicable. A typical paint will only change colour
at one temperature, and that temperature can usually be defined only within 2 or 3°C. Clearly these materials are once again not suitable.

(3) **Contact thermography.** When certain phosphors are excited by ultra-violet radiation, the degree of luminescence varies significantly with temperature. Hence, if a thin film of phosphor is applied to a surface and excited uniformly and steadily, the amount of re-radiated energy i.e. the brightness of the phosphor, can be measured as an indication of the temperature. A phosphor film could be applied directly to the surface by spraying or painting, or, alternatively, on coated paper with an adhesive backing, and it is claimed that the technique enables changes of temperature of 1°C to be detected up to 100°C. Although it is necessary to work in partial darkness, the technique could possibly be used to detect surface temperatures, but its application to the measurement of sub-surface temperatures would obviously create difficulties in resolution in depth.

(b) **Radiation Techniques.**

There are several forms of pyrometer available which, in general, will measure the change, with temperature, of heat radiated from a surface. Although the temperature range in which pyrometers find most of their application is higher (up to 3000°C) than that of the present application, modern developments with
radiation techniques have brought the operating range of radiation thermometers down below 50°C. The advantage of such a technique is that the sensing device could be mounted away from the surface of the disk, thus causing no interference with local heat conduction. The chief disadvantage of a radiation thermometer would be in trying to resolve in depth. It is doubtful whether the technique could, for instance, discriminate between the surface temperature and the oil film temperature, or 0.1 cm beneath the surface.

(c) Electrical Techniques. The obvious advantage of these techniques, involving either the measurement of electrical resistance, or thermoelectric voltage, is that they can be used for both surface and sub-surface detection, and their percentage accuracy (usually better than 1%) makes them suitable for the present application. They do, however, suffer from the disadvantage of requiring leads connecting the sensing body to the sensing equipment, mounted remotely from the point of measurement. The presence of leads can tend to weaken the body and interfere with local heat conduction.

The deposition of a thin evaporated temperature sensitive film upon the surface (Kannel et al. 1966) is a possible technique for the measurement of surface temperature. However, exploratory experiments using evaporated films quickly revealed the well known difficulties associated with
the evaporation of thin adherent films upon the surfaces of polymers. The production of surface films, which would remain intact for the duration of the experiments reported here, is clearly a difficult and specialist problem. The possibility of using thermistors (which are commercially produced and readily available) was considered. However those available (usually larger than 0.2 cm diameter) are too bulky to be embedded below the surface and still measure accurately the temperature at a given depth.

Two techniques have been used by the author and employ thin wires used as resistance thermometers and thermocouples. Three stages in the development of these techniques are described below.

3.4 Embedded thermocouples; radial orientation.

The first exploratory measurements of sub-surface temperature were made using embedded thermocouples. Fig. 3.1 shows how the thermocouples were placed in the disks. A 0.155 cm diameter hole was drilled diametrically into the disk, and a Perspex rod was turned on a lathe to fit the hole. At a specified distance \( t \) from the end of the rod a 0.035 cm diameter hole was drilled through the rod and the junction of a Copper/Constantan thermocouple glued into the hole with Perspex Cement No. 6. The thermocouple junction was formed by butting together the ends of 0.034 cm diameter Copper and Constantan wires and effecting the joint by silver soldering. The wires were laid in grooves scored either side of the rod, which was then glued into the hole in the
ARRANGEMENT OF LOWER DISK AND SHAFT FOR TEMPERATURE MEASUREMENT

Fig 3.2

GEAR LOCATED AGAINST TUFNOL COLLAR

LOCKING NUT

STEP B

COLLAR A

TUFNOL COLLARS WITH ACCESS TO HOLE THROUGH CENTRE OF SHAFT

DISK E

LOCKING NUT C

MERCURY CONTACTS

COLLAR D

MERCURY CONTACTS
KENT RECORDER

COPPER COLD JUNCTION

CONSTANTAN

WIRES PASS THROUGH MERCURY SLIP RINGS

THERMOCOUPLE JUNCTIONS EMBEDDED IN DISK

Fig. 3.3 CONNECTION OF DISK THERMOCOUPLES TO RECORDER
Perspex disk. The outside of the disk was then turned between centres on a lathe to an outside diameter of 7.5 cm, and the surface was polished to better than $5 \times 10^{-6}$ cm, measured on a Talysurf.

The arrangement of the disk on its shaft is shown in Fig. 3.2. A collar A is located against the step B. The disk E was then secured next to this collar by the locking nuts C and collar D. The gear, driving against a gear on the other shaft, was located against collar A. The thermocouple wires then passed from inside the disk, along a groove in the side of the disk, and were connected to terminals on either collar A, or collar D. From these terminals wires were fed through holes in the collars to the hollow shaft, and along this to a mercury slip ring at the end of the shaft. All the mercury slip rings used in the author's experiments are of a type described by Kenyon (1954).

Two thermocouples were embedded in this manner in the lower disk of the 2 Disk Machine. Because a 3-pole mercury slip ring was being used, and only 3 wires could be taken from the disk, the Constantan wires of the two thermocouples were joined at collar A. Connections were made from the terminals on the mercury slip rings to a cold junction and a Kent, 2-channel, strip chart recorder. A diagram for the connections of the thermocouple wires is shown in Fig. 3.3. The Kent recorder samples, and prints, the input from only one channel at a time.

The depth of the thermocouple junction beneath the disk surface was measured optically. Modifications were
made to the mounting of a microscope to allow disks to be examined. A two-dimensional movement, incorporating an attachment for holding disks, was built on to the conventional vertical movement of a microscope. Once a disk was under the microscope it could be manipulated with the three-dimensional movement to bring the surface into focus, without touching the disk. Provision was also made so that once the surface had been brought into focus, the disk could be rotated and, without further adjustment to the vertical or horizontal movement of the microscope, any other point on the disk surface examined. To measure the depth of a thermocouple the microscope was focused on the disk surface above the junction, using x 100 power magnification. The disk was then moved vertically upwards to bring the junction into focus. The distance moved vertically was recorded from a dial gauge (graduated in 0.0001 inch divisions) located against the disk holder. The depth of the thermocouple below the surface could now be determined. Since the refractive index of Perspex is 1.5,

\[
\text{Thermocouple depth} = 1.5 \times (\text{movement of dial gauge})
\]

Experiments with disks fitted with these types of thermocouples were performed over a range of loads and speeds. The temperatures recorded at depths comparable with the position of maximum shear stress were found to be greater than those at points closer to the surface. However, the more detailed value of these results was in doubt; attempts to reproduce temperature readings in repeated runs under identical experimental conditions, were not particularly successful.
These first experiments with embedded thermocouples prompted a consideration of the possible influence of the embedded wires upon the distribution of temperature within the Perspex. There is a wide disparity between the thermal conductivity of Perspex and that of the metals used in the thermocouples (Perspex $4.5 \times 10^{-2}$-Constantan $5.4 \times 10^{-2}$-Copper $0.92$ Cal/$^\circ$C cm). Thus, in a simple linear heat flow situation a 0.034 cm diameter rod of Constantan will have the same thermal acceptance as a 0.37 cm diameter rod of Perspex. Similarly a 0.034 cm diameter rod of Copper will have the thermal acceptance comparable with a 1.54 cm diameter rod of Perspex. Clearly wires of 0.034 cm diameter can have a considerable disturbing effect upon the temperature distribution.

The orientation of the wires with respect to the direction of heat flow is also of some importance. The disturbing effects are likely to be greatest if the wires are laid in the direction of heat flow, at right angles to the isotherms. Similarly one would expect the disturbing effect to be a minimum when the wires are laid along the isotherms, at right angles to the direction of heat flow. Consider, therefore, the probable temperature distribution in rolling disks in which heat is generated by hysteresis losses. In the experiments described here the disks were 1.59 cm wide and the heat was thought to be generated, with appreciable intensities, down to depths of about 0.4 cm, with a maximum intensity, typically, at a depth of 0.05 cm.
Fig 3.4
TEMP. MEASUREMENT - RESISTANCE WIRE TYPE (AXIAL ORIENTATION)
This suggests that in a disk not subjected to any disturbing thermal influences the flow of heat is radial and the isothersms are axial. Indeed, this simplifying assumption is made in deriving the theoretical temperature distribution in Chapter 5.

With these considerations in mind in the further development of the temperature measuring techniques, thinner wires were used and they were arranged in an axial orientation.

3.5 Embedded resistance thermometry; axial orientation.

The second technique which was adopted used Tungsten wire \(0.5 \times 10^{-8}\) cm diameter (5 micron) as a resistance thermometer; the wire was arranged in the axial orientation. It was hoped in this way to overcome the major drawbacks of the thermocouple experiments described in Section 3.4. An axial slot, 0.32 cm wide, was cut in the disk as shown in Fig. 3.4. The wire was then laid across the bottom of the slot and a Perspex peg was glued into the slot using Perspex Cement No. 6. Since only the portion of the wire embedded in the disk was to be used as a resistance thermometer, the ends of the wire protruding from either side of the disk where copper plated to 0.0025 cm diameter before the wire was fixed on the disk. To do this the 5 micron wire was threaded into a copper plating bath, and the central 1.59 cm of the wire was raised out of the Copper Sulphate solution in the bath. When set into the disk the protruding Copper plated portions of the wire were attached to the sides of the disk using conducting silver paint. The disks were
turned between centres on a lathe and polished to a surface finish better than \(5 \times 10^{-8}\) cm, measured on a Talysurf. The disks were assembled on the shafts in the same manner as before, as shown in Fig. 3.2. Using fine wires, connections were made from the conducting silver paint on the sides of the disks to the terminals on the collars A and D, from these through the hollow shaft to the mercury slip rings. The electrical resistance of the unplated 1.59 cm length of Tungsten wire was approximately 45 ohms when measured 'cold' — with no current passing through it (the resistivity of Tungsten is \(5.51 \times 10^{-6}\) ohms cm). The resistance of a 1 cm length of the Copper plated wire was approximately 0.25 ohms, and in the measurements made, the variation of this with temperature, compared with the change in resistance with temperature of the unplated portion, was assumed to be negligible.

The resistance of the wire embedded in the specimen could now be measured on a Wheatstone Bridge. Fig. 3.5 shows the arrangement for recording readings from one embedded resistance wire. Into the unknown arm of the bridge was connected, by a three way switch, either \(R_T\) (the wire in the disk) or resistances \(R_0\) or \(R_{FG}\). To compensate for ambient temperature changes the resistance \(R_3\) was a wire in a dummy specimen at room temperature. \(R_3\) also compensated for the heating effect of the current in the embedded wire. (The current was always less than 0.1 milliamp). The resistance \(R_0\) was a fixed resistance equal to the resistance of the embedded wire at ambient
Fig 3.6 BRIDGE CIRCUIT TO RECORD 2 WIRES
temperature (taken to be $20^\circ$C). When this resistance was connected in the unknown resistance branch of the bridge the zero adjustment potentiometer was adjusted until the recorder read zero. The fixed resistance $R_{FS}$ was equal to the resistance of the embedded wire at $100^\circ$C and when it was connected into the unknown branch of the bridge the full scale calibrating resistor was adjusted until the recorder read full scale. With the recorder calibrated in this fashion the temperature of the embedded wire could now be recorded, with $R_T$ switched into the unknown arm of the bridge. By modifying the bridge circuit it was possible to record the temperature of several embedded wires simultaneously. Fig. 3.6 shows the bridge circuit for recording 2 wires. It can be seen that the left hand side of the bridge is common to both wires. The advantage of this arrangement, when recording several wires at once, is that one side of the wires can be connected to a common terminal on collar $A$, beside the disk, and that only one connection to this terminal through the mercury slip rings is required, however many embedded wires there are.

For the experiments using embedded resistance wires a 6-pole mercury slip ring was fitted to the end of the shaft and the temperatures from four embedded wires were recorded. Several disks were run in the 2-disk machine and measurements of sub-surface temperatures made using this technique. The results again suggested that the highest sub-surface temperature occurred at depths comparable with that of the position
of maximum shear stress. However, few consistent results of quantitative significance were obtained, because of a fresh series of problems which were revealed.

Two problems contributed to the limitations of the experimental results.

(a) The wires used in this work were extremely fragile when subjected to the very large deformations encountered in the experimental conditions. The elastic modulus of Perspex is about two orders of magnitude less than that of typical metals, so that, even if the stresses are small, the strains are very large. Moreover, the Tungsten wire, being a sintered composition of a hard refractory metal, is extremely brittle. It was found that the wires tended to break at the end of the Copper plated section after a relatively small number of stress cycles. In the course of these experiments the work of Lines, Lawrie and O'Donoghue (1966) was reported, in which similar difficulties were encountered with Tungsten wires having diameters five times greater than the wires used in the present work. (In the work of Lines et al the wires were embedded in rotary shaft garter spring seals, flush with the surface, and were subjected to rubbing contact stresses under lubricated conditions).

(b) The Perspex plugs glued over the slots cut in the disks, showed a tendency to work loose after only a small number of stress cycles. The glue used in this work, Perspex Cement No. 6, consists basically
of Perspex dissolved in a suitable solvent, and the establishment of a satisfactory joint would seem to require the evaporation of the solvent from a thin film, to the surface. Experience of many attempts to make satisfactory joints and, at a later stage, the advice of I.C.I. Plastics Division, suggested that it was not possible to produce a satisfactory result using this type of glue.

Two conclusions can be drawn from the experience with this technique of temperature measurement. Firstly, it is necessary to effect a compromise in the choice of the diameter of the wire; if the wire is too thick it will unduly distort the temperature distribution, if the wire is too thin it cannot withstand the cycles of strain to which the disks are subjected. Secondly, a solution must be found to the problem of embedding the wire within the disk in such a way that the resultant structure gives a good approximation to the mechanical behaviour of a solid disk of Perspex.

3.6 Embedded Thermocouples: axial orientation.

In the final technique used to measure sub-surface temperatures the axial orientation of the wires was again adopted. To overcome the problem of the fragility of the wires Copper/Constantan thermocouples were used, since these materials are more ductile than Tungsten. The wire diameter was increased to $7.5 \times 10^{-3}$ cm. The use of
thermocouples rather than resistance thermometry has two additional advantages. Firstly, the thermocouple allows a simple direct reading arrangement. Secondly, the thermocouple junction could be placed at the centre of the disk, where the disturbance to the temperature distribution is likely to be least.

The thermocouples were produced by discharge welding 0.0075 cm diameter Copper and Constantan wires together. One of the wires was placed in a Perspex holder attached to the three-dimensional movement on the microscope, which has already been described (Section 3.4). A further three-dimensional manipulator was attached to the microscope and the other wire held in a Perspex holder attached to it. The ends of the wires were now adjusted, using the three-dimensional movements, to bring them both into focus when viewed through the microscope. With the two wires butted together in this manner a 1000 μF capacitor, charged by applying a potential of 25 V across it, was discharged through the wires. The ends of the wires melted and fused together.

The problem of embedding the wires into slots cut in the disks was overcome by polymerising Perspex 'in-situ' in the slots. Perspex Cement No. 7 consists of a solution of polymer dissolved in monomer. The monomer dissolves the Perspex surface in the same manner as do other solvents, but it is then polymerised in the joint instead of evaporating. The physical properties of Perspex polymerised in this manner
are similar to those shown for Perspex in Appendix 1. When the slots in a disk had been filled using Cement No. 7, the disk was stress relieved by heating to 80°C in an oven for four hours, and then slowly cooled.

Disks with 8 thermocouples embedded in this manner were run in the 2-disk machine. On one end of the lower shaft was a 6-pole mercury slip ring and on the other end a 3-pole mercury slip ring. The Constantan wire from each thermocouple was connected to a common terminal on collar A. By doing so, only 9 connections through mercury slip rings needed to be made to the disk for 8 thermocouples. A recording system, similar in nature to that shown in Fig. 3.3, was used. A multi-channel Kent strip-chart-recorder sampled and printed the voltage input from each of the eight thermocouples in turn.

A number of disks, each with 8 thermocouples embedded at different depths, were run over a range of load and speed conditions in the 2-disk machine. When removed from the machine the depths of the thermocouples were measured by the optical technique already described.

3.7 Results of temperature measurement.

In all tests two 7.6 cm diameter Perspex disks of 1.59 cm width were loaded together and both driven at the same speed to give pure rolling conditions. The experimental technique was as follows. The oil supply to the disks, bearings and gears was switched on and the oil allowed to run over the disks until the disk temperature reached that
Fig 3.7
RECORDED SUB-SURFACE TEMPERATURES VERSUS TIME THERMO-COUPLE DEPTHS X-0.05 cm 0-0.03 cm
Fig 3.8 EQUILIBRIUM TEMPERATURE VERSUS DEPTH FOR PERSPEX DISKS ROLLING AT 13.1 REVS/SEC

- X 41.5 kg/cm
- O 56.5 kg/cm
- △ 72 kg/cm
- □ 87 kg/cm
- ● 94 kg/cm
of the oil supply. (The oil supply temperature was monitored by a thermocouple across the outlet of the oil supply pipe to the disk).

The motor driving the disks at the appropriate test speed was switched on and a load of 41.5 kg/cm applied to the disks. Each of the recorded sub-surface temperatures increased exponentially to an equilibrium value. Fig. 3.7 shows a typical set of results, the sub-surface temperature at two different depths being plotted as a function of time. As a guide the top of the graph shows the number of rotations, and therefore the number of stress cycles, to which each point in the disk has been subjected. It will be observed that this graph fully justifies the earlier calculation which forecast that the sub-surface temperature distribution was attained as the result of a large number of successive cycles. The equilibrium temperature reached by each thermocouple was recorded. The load was now increased in steps to a maximum load of 94 kg/cm. At each load the disks were run until all the thermocouples had reached equilibrium and these temperatures were recorded. Fig. 3.8 shows a typical set of results, the equilibrium temperatures for Perspex disks rolling at a speed of 13.1 revs/sec. (surface velocity 312 cm/s).

Further discussion of the results of these experiments will be delayed until Chapter 8, when the experimental values of the sub-surface temperatures will be compared with those derived from the theoretical analysis developed in Chapter 5.
CHAPTER 4

THEORETICAL ANALYSIS OF HYSTERESIS LOSS

4.1 Introduction

If a hard body moves over a softer body energy will be expended in forming a depression. If the deformation is ideally elastic an equal amount of energy will be recovered from the rear part of the contact region between the bodies. Such an elastic deformation could be repeated continuously without any net energy loss. Most materials, however, do not conform to such ideal behaviour, and some of the energy expended in forming the deformation is not recovered from the rear of the contact area. The mechanism involved in the dissipation of this non-recoverable energy is attributed to internal friction or elastic hysteresis.

In this chapter we first outline two differing approaches which have been adopted in the analysis of hysteresis losses for cylinders rolling in nominal line contact. The first of these uses a visco-elastic model for the material of the cylinders. The second, due primarily to Greenwood, Minshall and Tabor, takes as its starting point the stress distribution which would exist if the materials of the cylinders were perfectly elastic; this treatment will therefore be described as a quasi-elastic theory. After an outline of the theory of the stress distributions which exist in the contact of perfectly elastic cylinders, two developments of the quasi-elastic theory will be described. The first is concerned with the fact that the Greenwood, Minshall and Tabor theory
Fig 4.1 TWO MODELS WHICH SIMULATE THE MECHANICAL RESPONSES OF VISCOELASTIC MATERIALS
is derived for the special case of a material with Poisson's ratio, \( \nu = 0.5 \), which is effectively incompressible under hydrostatic stresses; since the value of \( \nu \) for Perspex is 0.35 an extension of the theory for other values of \( \nu \) is necessary. The second extension of the theory shows how the existence of tangential surface stresses affects the sub-surface hysteresis losses.

4.2 Hysteresis losses for visco-elastic materials

If a hard roller passes over a visco-elastic material the recovery from the deformed state will be time dependent. Thus, the amount of energy regained from the deformed material will depend upon the relaxation characteristics of the material. These characteristics are, in turn, dependent upon temperature and the rate at which the material is strained. Analysis of problems involving this type of behaviour—visco-elastic behaviour—are usually made by postulating a 'model' of the material using springs and dashpots.

The term 'relaxation characteristics' of a material may be described by examining one such simple model—the Maxwell model—consisting of a coupled spring, with spring constant \( G \), and dashpot of viscosity \( \eta \) (Figure 4.1(a)). If a constant force \( P \) is applied to the system and the displacements of the two components are \( s_1 \) and \( s_2 \), then

\[
\begin{align*}
\dot{s}_1 G &= P \\
\dot{s}_2 \eta &= P \\
\dot{s}_1 G &= \dot{P}
\end{align*}
\]

and since the total displacement \( s = s_1 + s_2 \)
Then
\[ \dot{s} = \frac{\dot{P}}{G} + \frac{P}{\eta} \]  \hspace{1cm} (4.2.2)

Hence
\[ \eta \dot{s} = \frac{\eta \dot{P}}{G} + P \]  \hspace{1cm} (4.2.3)

or
\[ \eta \dot{s} = \lambda \dot{P} + P \]  \hspace{1cm} (4.2.4)

where \( \lambda = \frac{\eta}{G} \) is the relaxation time of the material. If the material was represented by a series of different Maxwell models it would have a series of discrete relaxation times which would form a "relaxation spectrum".

Some recent theoretical work on the rolling of hard (completely rigid) disks over visco-elastic materials, has been presented. May, Morris and Atack (1959) have analysed the rolling of a hard cylinder over a visco-elastic plate. They have presented an approximate, and a more rigorous, solution for a material with a single relaxation time. They represent the material as an infinite number of independent linear Maxwell elements which undergo compression and recovery during rolling. By representing the material in this manner, as a series of vertical columns, only compressive stresses in the deformation process are considered, and shear stresses are ignored. It will be shown, later in the chapter, that shear stresses do indeed play an important part in contributing to the energy losses in rolling. Their approach has, however, taken into account the frequency dependence of the bulk properties of the material, and they have demonstrated the
dependence of rolling friction upon velocity. They have further shown that the rolling friction reaches a maximum value at a velocity determined by the peak of the relaxation spectrum of the material. They also suggest that, since a plot of rolling friction against velocity follows the same shape as the relaxation spectrum, the latter could be determined experimentally by the measurement of rolling friction over a range of velocities.

Flom and Bueche, (1959), when considering a hard sphere rolling on a viscoelastic material have represented the properties of the visco-elastic materials by a Voigt model (Figure 4.1(b)) and once again the material is considered to have a single retardation time. They showed that the friction-speed relationship resembles closely the mechanical loss factor against frequency relationship.

More recently a formal analytical treatment of the problem has been developed, by Hunter (1961), Morland (1962) and Morland (1967). Hunter (1961) has shown that the coefficient of friction, resulting from the rolling of a rigid cylinder over a visco-elastic half space, is a function of the rolling velocity and tends to zero for small and large values of velocity, and attains a single maximum at an intermediate value. This treatment is, however, limited to materials characterised by a single relaxation time. Morland (1967) has extended his earlier analysis to include a treatment of the problem of rolling contact between linear
visco-elastic cylinders with different radii and different quantitative mechanical responses. His analysis is restricted by considering only conditions of slow rolling and, due to the mathematical complexity of the analysis, numerical techniques are used to obtain a solution. Application of the analysis to a visco-elastic material characterised by a wide spectrum of retardation times, would therefore be difficult.

While much work has been devoted to the interpretation of visco-elastic behaviour using models, and explanations in terms of molecular movements have been forthcoming, little data is available to enable a prediction of the hysteresis losses in such materials for a complex deformation process. It can be seen that the major problem of predicting the energy losses caused by the rolling of a hard cylinder over a softer material (or the rolling together of two cylinders) is in obtaining a suitable model, whose behaviour in rolling motions closely resembles that of real materials. The behaviour of most elastomers and polymers (including Perspex) can be described only by a spectrum of relaxation times. Consequently solutions in which it is assumed that the behaviour of the material may be characterised by a mean relaxation time can only be regarded as necessarily an approximation to the behaviour of real materials. Because these treatments are also rather complicated there is a real need for a simpler treatment.
Fig 4.2 (a) REPRESENTATION OF A THIN WALLED RUBBER TUBE SUBJECTED TO A STRESS CYCLE SUCH THAT AT ANY STAGE BETWEEN II and IV $\sigma^2 + 3\tau^2 = \sigma_0^2$

(b) STRESS DIAGRAM FOR THE CONSTANT ENERGY CYCLE IN (a)
4.3 Quasi-elastic theory of Greenwood, Minshall and Tabor

Tabor and co-workers (Tabor (1952), (1955), Greenwood and Tabor (1958), Atack and Tabor (1958)) have developed a theory of rolling friction in which it is assumed that the stress distribution under the rolling body can be regarded as identical with that which would occur if the materials were perfectly elastic. It was argued that the frictional losses can be represented by a simple proportion of the energy expended in deforming the material at the front of the rolling body. The constant of proportionality, which is the fraction of input energy lost, is called the hysteresis loss factor, \( \alpha \). It was suggested that this loss factor, \( \alpha \), was identical with that which could be determined for the same material in a simple uniaxial stress cycle.

Greenwood, Minshall and Tabor (1961) argue that the hysteresis energy loss in complex stress cycles cannot be described in terms of the total elastic energy per unit volume, as assumed in the earlier work. The misconception of this earlier assumption was demonstrated. Consider a thin walled rubber tube subjected to a stress cycle. Throughout the cycle the elastic strain energy may be maintained at a constant value. Such an arrangement is shown in Figure 4.2(a). The tube in its original State I is subjected to a pure tension \( \sigma = \sigma_0 \) (State II). Then, as \( \sigma \) is reduced, a torque \( \tau \) is applied such that at any stage

\[
\sigma^2 + 3\tau^2 = \sigma_0^2 \quad \text{(State III)} \quad (4.3.1)
\]
The elastic stored energy per unit volume is (Timoshenko and Goodier 1951)

\[ \frac{1}{2G} \cdot r^2 + \frac{1}{2E} \sigma^2 \]  \hspace{1cm} (4.3.2)

and since, for a material for which Poisson's ratio \( v = 0.5 \), we may write \( E = 2G (v + 1) = 3G \), the elastic energy may be written

\[ \frac{1}{2G} \left[ r^2 + \frac{1}{3} \sigma^2 \right] \]  \hspace{1cm} (4.3.4)

Thus, between State II and State III, the elastic stored energy may be maintained constant. Finally, when the tensile stress is reduced to zero the torque has its maximum value \( \tau_o \) where

\[ \tau_o = \sigma_o \sqrt{3} \quad \text{(State IV)} \]  \hspace{1cm} (4.3.5)

Cycles may now be made between States IV and II; the path on a \( r, \sigma/\sqrt{3} \) diagram is shown in Figure 4.2(b). Thus, although there is no cyclic input and withdrawal of elastic energy there is clearly a hysteresis loss. This example, in itself, shows that when considering complex stress cycles the cyclic input energy provides no direct measure of the energy losses involved. By considering this, and other stress cycles, Greenwood, Minshall and Tabor suggest that stresses causing the deformation of an element beneath the surface of a rubber block, as a hard cylinder is rolled
Fig 4.3 (a) DEFORMATION OF AN ELEMENT OF MATERIAL \((\nu = 0.5)\) AS A HARD CYLINDER ROLLS OVER IT.
(b) STRESS DISTRIBUTION DERIVED BY PORITSKY AT A DEPTH \(b\).
(c) THE INDEPENDENT STRESSES ACTING ON THE ELEMENT
Fig 4.4 STRESS DIAGRAM TO DESCRIBE THE STRESSES ON AN ELEMENT BENEATH THE SURFACE

(a) ν = 0.5
(b) ν < 0.5
over it (Figure 4.3(a)) should also be represented on a stress diagram, similar in nature to Figure 4.2(b).

As a first approximation to the stress distribution created by a hard cylinder rolling over a rubber block, they use the stress distribution occurring under static loading. Since this is a case of plane strain there are three independent stresses on any element in the plane \( \sigma_x, \sigma_y, \tau_{xy}, \) or \( \tau_{yx} \), as shown in Figure 4.3(c). This is equivalent to a shear stress \( t = \tau_{xy} \), another shear stress \( s = \frac{1}{2} (\sigma_y - \sigma_x) \) at 45° to \( t \), and a two dimensional hydrostatic component \( \frac{1}{2} (\sigma_x + \sigma_y) \). In addition the stresses \( \sigma_x \) and \( \sigma_y \) produce a stress normal to the plane, of magnitude \( \nu (\sigma_x + \sigma_y) \). For a material such as rubber for which Poisson's ratio \( \nu = 0.5 \) the latter is \( \frac{1}{2} (\sigma_x + \sigma_y) \), so that we are left with a three dimensional hydrostatic pressure of amount \( \frac{1}{2} (\sigma_x + \sigma_y) \) together with \( s \) and \( t \). Since a material having \( \nu = 0.5 \) is incompressible under a hydrostatic stress, this does no work and we may assume that it does not affect the energy loss. We thus need only consider the two shear stresses \( s \) and \( t \), at 45° to each other.

The stress distribution under a loaded cylinder has been given by Poritsky (1950), and is shown in Figure 4.3(b) for a depth \( b \) below the surface. The stress cycle may also be represented on a stress diagram in which the two axes are \( s \) and \( t \). Figure 4.4 shows the complete set of shear stresses for an element at a depth \( b \) as the cylinder rolls over it. It should be emphasised that, for an isotropic
material, the axes are completely irrelevant to the stress cycle. There is no difference in kind between s and t; both are shear stresses and the distinction is simply that t is a shear parallel to the x, y axes and s is a shear stress at 45° to these axes. It is indeed clear that if the curve of Figure 4.4 is drawn without axes or origin this will contain all the information relevant to the calculation of the energy loss.

Greenwood, Minshall and Tabor suggested that, according to the hypothesis that they present, the energy loss in the element considered can now be found from the squares of the perimeter l of the closed loop in the stress diagram, and may be written $\beta t^2$ (where $\beta$ is a constant of proportionality, and a function of hysteresis loss factor $\alpha$).

To determine $\beta$ for plane strain they note that in a pure shear cycle the elastic energy input per unit volume is $r^2/2G$ where $r$ is the maximum shear stress. Since the energy loss for such a cycle is the elastic energy multiplied by a hysteresis loss factor $\alpha$, and the length of the 'loop' for such a cycle is simply $l = 2r$ then

$$\alpha \left( \frac{r^2}{2G} \right) = \beta t^2 = 4\beta r^2 \quad (4.3.6)$$

and hence $\beta = \frac{\alpha}{8G}$

Now the incremental length of the stress path of an element at the point x, y due to a movement dx of the roller is

$$dl = \left[ \left( \frac{\partial \sigma}{\partial x} \right)^2 + \left( \frac{\partial \tau}{\partial x} \right)^2 \right]^{\frac{1}{2}} \cdot dx \quad (4.3.7)$$
Integrating between \( x = \pm \infty \) gives the total length of the stress path of an element at depth \( y \) as a roller moves completely across it, from an infinite distance away on one side to an infinite distance on the other. Then for an element at depth \( y \) the energy loss per unit volume is given by

\[
q = \beta t_o^2 \quad (4.3.8a)
\]

where

\[
l_o = \int_{-\infty}^{+\infty} \left( \left( \frac{\partial t}{\partial x} \right)^2 + \left( \frac{\partial s}{\partial x} \right)^2 \right)^{\frac{1}{2}} \, dx \quad (4.3.8b)
\]

Thus the energy loss in a strip of rubber of unit width is

\[
\Phi = \beta \int_{0}^{\infty} l_o^2 \, dy \quad (4.3.9)
\]

Clearly this will be the same for all strips and will be equivalent to the total energy lost in the rubber for steady-state rolling when the cylinder rolls forward unit distance. We may note here that the physical argument which is implicit in equations 4.3.8 and 4.3.9 is that the incremental loss of energy associated with any change in the total stress system is derived as the sum of the incremental losses associated with each of the stress modes (each stress mode being considered as an independent system) i.e., incremental loss of energy,

\[
d\varepsilon = \beta \left( \frac{ds}{dx} \right)^2 + \beta \left( \frac{dt}{dx} \right)^2 = \beta \left( d\ell_o \right)^2 \quad (4.3.10)
\]

Greenwood, Minshall and Tabor point out an obvious defect of the treatment, namely that a 'double cycle' would
involve a fourfold increase in the loss instead of a two-fold increase. In an Appendix to their paper they present an alternative treatment which has the merit of giving only a twofold increase in the loss for a 'double cycle'. The results from it are very similar to those discussed above. However, when the analysis is applied to closed cycles, such as those occurring in rolling friction, where only a single cycle is involved, it predicts satisfactorily the losses that occur.

The analysis presented by Greenwood, Minshall and Tabor is, strictly, applicable only to materials which have a Poisson's ratio of 0.5, such as rubber. Nevertheless the theory explains a major discrepancy between the earlier theoretical analysis (Tabor, 1955) and experimental values of rolling friction. Greenwood, Minshall and Tabor have reduced equation (4.3.9) to an elliptic integral using the complex stress distribution of Poritsky (1950). (It should be noted that in their paper there is a printing error and their equation (22) should be written \( s = \frac{1}{2} (Yy - Xx) \). Unless presented in this form, the elegant technique they use for reducing equation (4.3.9) cannot be applied). For a hard cylinder rolling on rubber they show that

\[
\text{energy lost/unit distance} = 0.74a \frac{Wb}{R} \quad (4.3.11)
\]

(This equation will be derived by the author in section 4.5 using a numerical technique).

They also show that their earlier analysis, assuming that the energy loss is a fraction of the elastic input
energy, will lead to the result:

$$\text{energy loss/unit distance} = 0.21a^{(WFR)} (4.3.12)$$

Comparison with equation (4.3.11) shows that the true energy loss in rolling is 3.5 times larger than that based on the 'total energy' theory. Experimental results reported in their paper confirm this difference and show a very good agreement with the theoretical result of equation (4.3.11).

The main merit of the quasi-elastic approach of Greenwood, Minshall and Tabor is the relative simplicity of the mathematics. It uses as its starting point the well-known Hertzian equations for a perfectly elastic material. Therefore one would expect this approach to have validity when applied to materials whose behaviour is only weakly visco-elastic. Perspex, for most of the temperatures involved in our experiments, meets this requirement. The hysteresis loss factor is very small ($\approx 0.04$). However, the fundamental behaviour of the material is viscoelastic and therefore $\alpha$ is not a simple material constant but a function of the rate of deformation. In terms of our experiments, $\alpha$ is, in principle, both a function of temperature and rolling speed.

4.4 Sub-surface stresses for elastic contact between cylinders

In this section existing theory, giving the sub-surface stresses in the elastic contacts between cylindrical bodies, will be outlined. In particular we shall include the general case (not considered by Greenwood, Minshall and
The influence of superposed tangential stresses, which may occur in rolling with sliding, will also be considered. Analytical solutions for the stress distribution within cylindrical bodies in contact have been presented by several authors, Poritsky (1950); Smith and Liu (1953); Hamilton and Goodman (1966). In the present work the equations used are in approximately the form obtained by Smith and Liu (1953) using the method of real variables. This form is the most convenient for computational purposes.

The pressure distribution between two cylinders pressed together in line contact is given (Hertz (1881)) by

\[ p = \frac{2W}{\pi b} \left( 1 - \frac{x^2}{b^2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (4.4.1)

where \( p \) is the pressure at any point \( x \) in the surface and \( W \) is the compressive force between the cylinders. The semi-width of the contact \( b \) is given by

\[ b = \left( \frac{4W(k_1 + k_2)}{R_1 + R_2} \right)^{\frac{1}{2}} \]  \hspace{1cm} (4.4.2)

where \( R_1 \) and \( R_2 \) are the radii of the two cylinders and \( k_1 \) and \( k_2 \), their elastic constants, are defined by

\[ k_1 = \frac{(1 - \nu_1^2)}{\pi E_1}, \quad k_2 = \frac{(1 - \nu_2^2)}{\pi E_2} \]  \hspace{1cm} (4.4.3)

Clearly from equation (4.4.1) the maximum pressure \( p_0 \) will be

\[ p_0 = \frac{2W}{\pi b} \]  \hspace{1cm} (4.4.4)
The stress distribution, in approximately the form obtained by Smith and Liu (1953), using the directional axes of Figure 4.3 may now be written

\[
\begin{align*}
\sigma_x &= -p_0 y \left[ H(1 + 2x^2 + 2y^2) - 3Gx^{2} \right] \\
\sigma_y &= -p_0 y \left[ H - Gx \right] \\
\tau_{xy} &= -p_0 \, G y^2 
\end{align*}
\]  

(4.4.5)

where

\[
\begin{align*}
G &= \frac{1 - M}{KM} \left( \frac{2MK + K + L - 4}{K} \right)^{-\frac{1}{2}} \\
H &= G \left( \frac{1 + M}{1 - M} \right) \\
K &= (1 + x)^2 + y^2 \\
L &= (1 - x)^2 + y^2 \\
M &= \left( \frac{L}{K} \right)^{\frac{1}{2}}
\end{align*}
\]  

(4.4.6)

The application of this technique may also be extended to include the effects of tangential surface forces. The stress distribution due to normal contact with tangential forces is first known to have been derived by Karas (1941) and Weber (1949) but their solutions, published in Germany during the war, did not get much publicity. Poritsky (1950) solved the problem in terms of Airy Functions, and Smith and Liu (1953) by the method of real variables. The latter is again the most convenient for computational purposes.
On the surface it is assumed at every point that the tangential stress \( q \) is given by

\[
q = \mu p
\]  

(4.4.7)

where \( \mu \) is the coefficient of friction or coefficient of traction. Then the stresses due to the tangential force are

\[
\begin{align*}
\sigma_x &= -p_o \mu \left[ (2x^2 - 3y^2 - 2)G + 2x + 2Hx(1-x^2 - y^2) \right] \\
\sigma_y &= -p_o \mu y^2 \\
r_{xy} &= -p_o \mu \left[ H(1 + 2x^2 + 2y^2) - 2 - 3x^2 \right]
\end{align*}
\]

where \( G \) and \( H \) have the same significance as in equations (4.4.6). The final stress distribution in the body is obtained by adding the stresses corresponding to the normal and tangential surface forces.

4.5 Development of 'quasi-elastic' theory

Using the stress distribution outlined in Section 4.4 we now provide two developments of the quasi-elastic theory of Greenwood, Minshall and Tabor, which was outlined in Section 4.3.

4.5.1. Variations of Poisson's Ratio

If we consider the stresses acting on an element of material beneath the point of contact, (Figure 4.3) there will be three independent stresses

\[
\sigma_x, \sigma_y \text{ and } r_{xy}
\]
The fourth stress $\sigma_z$ may be determined from $\sigma_x$ and $\sigma_y$ by the value of $\nu$, since the stress condition is one of plane strain.

$$\sigma_z = \nu (\sigma_x + \sigma_y) \quad (4.5.1)$$

Resolving stresses $\sigma_x$ and $\sigma_y$ using co-ordinates $x'$ and $y'$ at $45^\circ$ to the $x$ and $y$ axes we may write:

$$\sigma_{x'} = \sigma_{y'} = \frac{(\sigma_x + \sigma_y)}{2} \quad (4.5.2)$$

$$\tau_{x'y'} = \frac{(\sigma_y - \sigma_x)}{2}$$

Adopting a notation similar to that of Greenwood, Minshall and Tabor we may now write

$$t = \tau_{xy}$$

$$s = \tau_{x'y'} = \frac{1}{2} (\sigma_y - \sigma_x) \quad (4.5.3)$$

$$\sigma' = \sigma_{x'} = \sigma_{y'} = \frac{1}{2}(\sigma_x + \sigma_y)$$

Considering the energy, $\epsilon$, associated with each of these stresses we may write

$$\epsilon_1 = \frac{1}{2}G (t)^2 \quad (4.5.4)$$

$$\epsilon_2 = \frac{1}{2}G (s)^2$$

and $\epsilon_3$, associated with the stress $\sigma'$, may be derived in the following manner.
Consider an element of material subjected to the three stresses \( \sigma_x', \sigma_y', \) and \( \sigma_z \).

We may write for the strains in the \( x' \) and \( y' \) directions
\[
\epsilon_{x'} = \epsilon_{y'} = \frac{(1 - \nu)}{E} \sigma' - \frac{\nu}{E} \sigma_z \quad (4.5.6)
\]
and for plane strain
\[
\epsilon_z = \frac{1}{E} \sigma_z - \frac{2\nu}{E} \sigma' = 0 \quad (4.5.7)
\]
thus \( \sigma_z = 2\nu \sigma' \)

and
\[
\epsilon_{x'} = \epsilon_{y'} = \frac{(1 - \nu - 2\nu^2)}{E} \cdot \sigma' = \frac{(1 - 2\nu)}{E} \cdot \frac{(1 + \nu)}{E} \sigma' \quad (4.5.8)
\]
and hence
\[
\epsilon_s = 2\frac{(1 - 2\nu)}{2E} \frac{(1 + \nu)}{E} \cdot (\sigma')^2 = \frac{1}{2} \frac{2(1 - 2\nu)}{E} \frac{(1 + \nu)}{E} \cdot (\sigma')^2 \quad (4.5.9)
\]
(The factor 2 is necessary since one considers energy associated with both \( \sigma_x' \) and \( \sigma_y' \),
using the relationship \( E = 2G (1 + \nu) \)
then
\[
\epsilon_s = \frac{1 - 2\nu}{2G} (\sigma')^2 \quad (4.5.10)
\]
The hysteresis energy loss can be calculated from a representation of the stress cycle on a three dimensional plot (Figure 4.4(b)), similar in nature to the two dimensional plot (Figure 4.4(a)) used in the treatment of Greenwood, Minshall and Tabor. Once again the energy lost will be proportional to \( \ell^2 \), where \( \ell \) is the length of the closed loop representing the stress path at any given
depth. The incremental length of the stress path of an element at the point \( x, y \), due to a movement \( dx \) of the roller is

\[
dl = dx \left[ \left( \frac{\partial \sigma_x}{\partial x} \right)^2 + \left( \frac{\partial \sigma_y}{\partial x} \right)^2 + (1 - 2\nu) \left( \frac{\partial \sigma'}{\partial x} \right)^2 \right]^{\frac{1}{2}}
\]

(Equation 4.5.11)

Once again, integrating between \( x = \pm \infty \) gives the total length \( l_o \) of the stress path, of an element at depth \( y \), as a roller moves completely across it, from an infinite distance away on one side to an infinite distance on the other

\[
l_o = \int_{-\infty}^{+\infty} \left[ \left( \frac{\partial \sigma_x}{\partial x} \right)^2 + \left( \frac{\partial \sigma_y}{\partial x} \right)^2 + (1 - 2\nu) \left( \frac{\partial \sigma'}{\partial x} \right)^2 \right]^{\frac{1}{2}} \, dx \quad (4.5.12)
\]

and the energy loss in a strip of unit width is, as before

\[
\dot{\psi} = \beta \int_0^\infty l_o^2 \, dy \quad (4.5.13)
\]

We can derive the coefficient \( 1 - 2\nu \) of the \( \frac{\partial \sigma'}{\partial x} \) term in an alternative manner. Consider once again an element of material subjected to the three stresses \( \sigma_x', \sigma_y', \) and \( \sigma_z' \) (\( \sigma_x' = \sigma_y' = \sigma' \))

For the case of plane strain where \( \sigma_z' = \nu (\sigma_x' + \sigma_y') \) we have (Equation 4.5.3)

\[
\sigma_x' = \sigma_y' = \sigma'
\]
and thus $\sigma_z = 2\nu \sigma^*$

Then the stored energy in the element (Timoshenko and Goodier p. 148)

$$= \frac{1}{2E} \left( \sigma^* + \sigma'^* + 4\nu^2 \sigma'^* \right) - \frac{\nu}{E} \left( \sigma'^* + 2\nu \sigma'^* + 2\nu \sigma'^* \right)$$

$$= \frac{\sigma^*}{2E} \left[ 2 + 4\nu^2 - 2\nu - 8\nu^2 \right]$$

$$= \frac{\sigma^*}{2E} \left[ 2 - 2\nu - 4\nu^2 \right]$$

$$= \frac{2\sigma^*}{2E} (1 + \nu) (1 - 2\nu)$$

$$= \frac{1 - 2\nu}{2G} (\sigma'^*)^2 \quad (4.5.14)$$

which may be compared with Equation (4.5.10).

We should also note that when $\nu = 0.5$, as in the case of rubber, then Equation (4.5.12).

$$\mathcal{E}_o = \int_{-\infty}^{+\infty} \left[ \left( \frac{\partial \mathcal{E}}{\partial x} \right)^2 + \left( \frac{\partial \mathcal{E}}{\partial t} \right)^2 + (1 - 2\nu) \left( \frac{\partial \sigma'^*}{\partial x} \right)^2 \right] \frac{1}{2} \mathrm{d}x$$

becomes

$$\mathcal{E}_o = \int_{-\infty}^{+\infty} \left[ \left( \frac{\partial \mathcal{E}}{\partial x} \right)^2 + \left( \frac{\partial \mathcal{E}}{\partial t} \right)^2 \right] \frac{1}{2} \mathrm{d}x \quad (4.5.8)$$

which is the result obtained by Greenwood, Minshall and Tabor (Equation 4.3.8).

The manner in which the stresses $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ acting on an element are reduced to the independent stresses
Figure 4.5: Representation of the way in which the stresses on an element can be reduced to 'independent' stresses.
s, t, and \( \sigma' \) is repeated diagramatically in Figure 4.5.

The remaining problem is now to express the energy losses, written in Equation (4.5.13) as

\[
\bar{\tau} = \beta \int_{0}^{\infty} \ell_{0}^{2} \, dy
\]

where

\[
\ell_{0} = \int_{-\infty}^{\infty} \left[ \left( \frac{\partial \delta}{\partial x} \right)^{2} + \left( \frac{\partial \tau}{\partial x} \right)^{2} + (1 - 2\nu) \left( \frac{\partial \sigma'}{\partial x} \right)^{2} \right]^{{\frac{1}{2}}} \, dx
\]

in terms of physical parameters which may be related to experimental conditions.

To summarize; it has been argued that if the stress system in an element in a semi-infinite solid, beneath a contact, can be reduced to a number of independent stresses, then these stresses can be represented on an s - t type diagram, such as that used by Greenwood, Minshall and Tabor. Further, as before, if \( \ell_{0} \) is the length of the closed loop representing the stress path of an element at a given depth, the energy lost is \( \beta \ell_{0}^{2} \). The term "independent stress" is used in this context to describe a stress whose action upon the stress system does not change the energy loss due to another stress. Thus, in this theory s and t correspond to the two independent shear stresses of the Greenwood, Minshall and Tabor analysis. The third stress is described by them as a "two dimensional hydrostatic stress" i.e. equal direct stresses \( \sigma'_{x} = \sigma'_{y} = \sigma' \) and a stress \( \sigma'_{z} = 2\nu \sigma' \) such as to maintain the plane strain conditions.
The physical meaning of the modification to the earlier theory should now be clear. In the earlier theory, energy changes in any element are associated with the two shear stresses $s$ and $t$. No energy changes are associated with the compressive stress $\sigma'$ since a material with $\nu = 0.5$ is effectively incompressible. For $\nu \neq 0.5$ energy changes are associated with the compressive stress $\sigma'$. Moreover, in the theory developed above, it is assumed that the proportion of energy lost, $\alpha$, is the same for a compressive system of stresses as for a shear system. Whether this assumption is justified may be questioned but there is little available experimental evidence upon which to justify any decision here.

In the event, the result of computations using this theory shows that the influence of Poisson's ratio (and therefore the influence of the additional energy associated with compressive forces) is fairly small. The computation has been made for non-dimensional depths $Y = \frac{y}{b}$ and stress paths $L_0 = \frac{\sigma_0}{P_0}$. Values of $L_0^2$ (proportional to the energy loss at a depth $y$) may now be determined by substituting Equations (4.4.5) and (4.4.6) into Equation (4.5.12). The integration of Equation (4.5.12) has been performed numerically on an Elliot 4130 computer for values of $y$ between $y = 0$ and $y = 20b$. The limits of the integral in Equation (4.5.12) are between $x = +\infty$ and $x = -\infty$. Obviously for a numerical solution of this integral, finite limits must be chosen and in the programme are from $x = \pm 20b$. At this distance from the contact region the stresses $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ are
Fig 4.6 \( L_0^2 \) VERSUS DEPTH \( Y: \mu = 0 \)
Fig 4.7 \[ L_0^2 \text{ VERSUS DEPTH } \gamma : \mu = 0.1 \]
Fig 4.8 \[ L_0^2 \text{ VERSUS DEPTH } Y: \mu = 0.2 \]
At $Y=0$ $L_0^2 = 6.01$ for $\nu = 0.25$

Fig 4.9 $L_0^2$ VERSUS DEPTH $Y$: $\mu = 0.3$
Fig 4.10 \[ \frac{L_0^2}{Y} \text{ VERSUS DEPTH } Y : \mu = 0.4 \]
At $Y=0$, $L_0^2 = 9.29$ for $\nu = 0.35$
and $L_0^2 = 11.37$ for $\nu = 0.25$
so small that their contribution to $L_0$ can only be detected as a change in the third decimal place.

The computation of $L^2_0$ was made for values of Poisson's ratio $\nu$ between 0.25 and 0.50 in steps of 0.05. Figure 4.6 shows the results of the computation when only normal Hertzian stresses are applied (without superposed tangential stresses). Results for $\nu = 0.25$, 0.35 and 0.5 only have been shown; curves for the other values lie between the plotted graphs and are omitted to facilitate clearer presentation. It will be observed that the deduced influence of Poisson's ratio is small but not negligible. The total energy loss (from equation 4.5.13) is proportional to the area under these curves, and the effect of changing $\nu$ from 0.5 to 0.25 is to change the loss by only 25%.

4.5.2 Variations of tangential surface stresses

Using the Equations (4.4.6 and 4.4.8) derived from the work of Smith and Liu (1953) it is now possible to superimpose the effect of tangential surface stresses upon the sub-surface stress distribution. By adding the stresses corresponding to the normal and tangential surface forces (Equations 4.4.5 and 4.4.8) and substituting these into the expression for $\ell_0$ (Equation 4.5.12) it is possible to see the effect of both $\nu$ and $\mu$ upon $\ell_0^2$. The computation was made for the values of $\nu$ used in the last section and values of $\mu$ between 0 and 0.5 in steps of 0.1. Figures 4.7 to 4.11 show the results of these computations, the results for $\mu = 0$ having been shown in Figure 4.6. Once
again, the presentation is in the non-dimensional form where \( L_0 = \frac{F_0}{P_0} \) and \( Y = \frac{y}{b} \). It should be noted that in Figures 4.10 (\( \mu = 0.4 \)) and 4.11 (\( \mu = 0.5 \)), the scale of the vertical axis has been reduced to make it possible to present data for regions near to the surface. It will be observed that as the tangential forces are increased the position of the maximum intensity of energy dissipation moves towards the surface. At intermediate values of \( \mu \) (e.g. \( \mu = 0.2 \), Figure 4.8) there are two maxima, one at the surface and another corresponding to the modified position of the maximum for normal loading. This behaviour corresponds, in broad terms, to that of the position of the maximum shear stress under conditions of loading, as shown by Poritsky (1950) Smith and Liu (1953) and Hamilton and Goodman (1966). Of course, the intensity of energy dissipation at any level is determined by the total stress cycle which an element undergoes; however, the results of Figure 4.6, discussed in Section 4.5.1 show that the shear stresses make the greatest contribution to the energy dissipation. The energy lost per unit distance may now be found from Equation 4.5.13, which we can write in non-dimensional form as:

\[
\phi = \frac{p_0^2 b^2}{\beta} \int_0^{\infty} L_0^2 dY \quad (4.5.15)
\]

The integral in this equation can be evaluated by finding the area under the curve of a plot of \( L_0^2 \) against \( Y \). Values of this area between \( Y = 0 \) and \( Y = 20 \) have been
Table 4.1  Effects of coefficient of traction ($\mu$) and Poisson's ratio $\nu$ upon hysteresis energy loss. The area under the curves in Figures 4.6 to 4.11 is shown for different values of $\mu$ and $\nu$.

<table>
<thead>
<tr>
<th>$\mu \ \frac{}{\nu}$</th>
<th>0.25</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
<th>0.45</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.10</td>
<td>5.80</td>
<td>5.50</td>
<td>5.20</td>
<td>4.90</td>
<td>4.60</td>
</tr>
<tr>
<td>0.1</td>
<td>6.15</td>
<td>5.85</td>
<td>5.55</td>
<td>5.25</td>
<td>5.00</td>
<td>4.70</td>
</tr>
<tr>
<td>0.2</td>
<td>6.35</td>
<td>6.05</td>
<td>5.70</td>
<td>5.40</td>
<td>5.10</td>
<td>4.80</td>
</tr>
<tr>
<td>0.3</td>
<td>6.65</td>
<td>6.30</td>
<td>6.05</td>
<td>5.70</td>
<td>5.35</td>
<td>5.05</td>
</tr>
<tr>
<td>0.4</td>
<td>7.05</td>
<td>6.75</td>
<td>6.45</td>
<td>6.10</td>
<td>5.80</td>
<td>5.60</td>
</tr>
<tr>
<td>0.5</td>
<td>7.75</td>
<td>7.35</td>
<td>7.00</td>
<td>6.60</td>
<td>6.30</td>
<td>5.90</td>
</tr>
</tbody>
</table>
determined for different coefficients of traction and Poisson's ratios and are shown in Table 4.1. Once again, the use of a numerical technique to evaluate an integral with limits between \( Y = 0 \) and \( Y = \infty \) has necessitated the choice of finite limits.

When \( Y = 10 \) the value of \( L_1^3 \) is less than 1\% of its maximum value and the contribution of losses at depths greater than \( Y = 20 \) does not make a significant difference to the total energy loss. Hence it is now possible to write energy lost/unit distance = \( \beta \times \) appropriate number from Table 4.1 \( \times p_0 \times b \)

For example when \( \nu = 0.5 \) and \( \mu = 0 \)

Energy lost/unit distance =

\[
\beta \times 4.6 \times \frac{4W^2}{\pi b} = \frac{\alpha b^2}{8WR} \times 4.6 \times \frac{bW^2}{\pi b}
\]

since \( \beta = \frac{\alpha}{G}, \) and \( G = \frac{WR}{\pi b} \), when \( \nu = 0.5 \), then the energy lost/unit distance = 0.74 \( \alpha \frac{Wb}{R} \); the result found by Greenwood, Minshall and Tabor (Equation 4.3.11).

Finally it should be emphasised that the work described in this section is concerned with the sub-surface dissipation of energy by hysteresis losses and the effect which changes in the tangential surface stresses have upon this dissipation. Tangential surface forces may also produce sliding, and the generation of heat at the surface, by sliding friction; these latter effects have not been considered.

4.6 Discussion and Conclusions

This chapter has been concerned with the way in which
energy dissipated is distributed in depth. The quasi-elastic approach of Greenwood, Minshall and Tabor has been adopted and their theory has been extended to cover situations where Poisson's ratio has values other than 0.5 and where tangential surface forces are superposed upon the normal Hertzian pressure distribution. Whatever the limitations of this approach, the broad conclusions which have been reached must be valid; this is particularly true since in the present work we are concerned with a material whose behaviour is close to that of a perfectly elastic material.

It has been shown that for materials for which Poisson's ratio has values other than 0.5, a component of the energy stored, and presumably of the energy dissipated, is associated with the compressive components of stress. There is some uncertainty as to whether this component should be treated in the analysis in the same way as the shear component. For instance, when \( \mu = 0 \) a change from \( \nu = 0.5 \) to \( \nu = 0.35 \) produces a change of about 15% in the total energy dissipation, with a small change in the shape of the energy dissipation curve; thus it can be seen that the value of Poisson's ratio has only a small effect upon the overall energy dissipation, and thus upon the coefficient of rolling friction.

The effect of superposed tangential surface stresses upon the energy dissipation is to cause small increases in the total magnitude, up to an increase of approximately 27% for surface stresses corresponding to a coefficient of
traction of 0.5. Increases in tangential stresses also cause a marked reduction in the depths at which most of the energy is dissipated. For tangential forces greater than values corresponding to a coefficient of traction of 0.3 a significant proportion of the energy dissipation occurs very close to the surface.

Under conditions of pure rolling, with which this investigation is mainly concerned, the tangential tractions are very small. The maximum dissipation of energy then occurs at a depth close to the position of maximum shear stress. If this energy manifests itself as heat, then the sub-surface temperatures are likely to be greater than those on the surface. Chapter 5 will be concerned with the derivation of the temperature distribution corresponding to the energy dissipation pattern derived in this chapter.
CHAPTER 5
THEORETICAL ANALYSIS OF SUB-SURFACE TEMPERATURES

5.1 Introduction

In this chapter the sub-surface temperature distribution is determined from the predicted energy loss of Chapter 4. Since the author is primarily concerned with the failure of Perspex the temperature distributions shown later in this chapter are for a material of Poisson's ratio 0.35 in conditions of pure rolling. The heat conduction equation for one-dimensional heat flow, and the analysis which follows, could equally well be applied to the energy loss for any of the other conditions of Poisson's ratio and coefficient of traction, derived in Chapter 4.

We shall consider the case of long cylinders loaded together in line contact. The width of the Hertzian contact, $2b$, is much smaller than the length of the cylinders. Likewise the depth at which the heat is generated will be much smaller than the length of the cylinders. It therefore seems obvious that the heat, generated in a relatively thin layer, will be conducted to the surface. The heat conduction problem will therefore be treated as one of steady state linear heat conduction.

5.2 Steady state linear heat conduction theory

The general differential equation of conduction in a rectangular co-ordinate system may be written (Simonson 1967)

$$\frac{\partial \theta}{\partial t} = \frac{K}{\rho c} \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right] + \frac{q}{\rho c} \quad (5.2.1)$$
HEAT FLUX AT DEPTH

\[ -\bar{Q} - q = -\int_{\bar{y}} y q(y) dy \]

(Shaded Area)

\( \bar{y} \)

\( \Theta - \Theta_0 \)

\( \Theta_{\text{max}} - \Theta_0 \)

\( y \)

**Fig 5.1** THE HEAT FLOW PROBLEM.
where \( \rho \) is the density, \( K \) thermal conductivity, \( c \) specific heat and \( q \) the heat generated per unit volume.

For steady state conduction \( \frac{\partial \theta}{\partial t} = 0 \) and the equation for one dimensional heat flow, using the co-ordinate system of Figure 5.1, may be written

\[
q = -K \frac{\partial^2 \theta}{\partial y^2} \quad (5.2.2)
\]

where the origin of the \( y \) co-ordinate is taken at the surface and \( y \) is the depth, of any point below the surface.

By integrating this equation the temperature gradient may be written

\[
\frac{\partial \theta}{\partial y} = \frac{1}{K} \int_0^y q \, dy + A \quad (5.2.3)
\]

where \( A \) is a constant.

Further integration yields the temperature

\[
-\theta = \frac{1}{K} \int_0^y \int_0^y q \, dy \, dy + Ay + B \quad (5.2.4)
\]

where \( B \) is another constant. We note that to arrive at a complete solution of the physical problem two boundary conditions are required to evaluate the constants \( A \) and \( B \).

At a point of maximum temperature, the temperature gradient will be zero. Denoting the depth of such a point by \( \bar{y} \) then for equation (5.2.3) we may write

\[
0 = \frac{1}{K} \int_0^{\bar{y}} q \, dy + A \quad (5.2.5)
\]
and if the heat generated per second between \( y = 0 \) and \( y = \bar{y} \) (per unit cross sectional area) is denoted by \( \bar{Q} \)

\[
\bar{Q} = \int_{0}^{\bar{y}} q \, dy \quad (5.2.6)
\]

then from equation (5.2.5) we may write

\[
A = -\frac{1}{K} \bar{Q} \quad (5.2.7)
\]

Noting, also, that \( \bar{Q} \) is the heat flux at the surface, and denoting the temperature at the surface \( (y = 0) \) by \( \theta_0 \), then when \( y = 0 \) in equation (5.2.4)

\[
B = -\theta_0 \quad (5.2.8)
\]

Further let the heat generated (per unit cross sectional area) between \( y = 0 \) and \( y = y \) be denoted by \( Q \)

\[
Q = \int_{0}^{y} q \, dy \quad (5.2.9)
\]

Then equation (5.2.3) can be rewritten as

\[
\frac{dQ}{dy} = -\frac{1}{K} (Q - \bar{Q}) \quad (5.2.10)
\]

Noting that \( (Q - \bar{Q}) \) is the heat flux at a depth \( y \) (see Figure 5.1). Similarly equation (5.2.4) becomes

\[
\theta - \theta_0 = \frac{1}{K} \bar{Q}y - \frac{1}{K} \int_{0}^{\bar{y}} Q \, dy \quad (5.2.11)
\]

Equation (5.2.11) allows a calculation of the difference
in temperature between any sub-surface point and the surface. The two boundary conditions used here have been the surface temperature \( \theta_0 \) and the position of the maximum temperature \( \bar{y} \). We shall see later the extent to which the value of \( \bar{y} \) is a critical parameter.

In section 3.2 the difficulties associated with the experimental determination of \( \theta_0 \), the surface temperature, were discussed. An alternative approach is to use as our reference temperature, \( \theta_A \), the temperature of the medium surrounding the disc. Then (since the heat flux at the surface is \( \bar{Q} \)) we have

\[
\bar{Q} = h ( \theta_0 - \theta_A ) \tag{5.2.12}
\]

where \( h \) is the heat transfer coefficient between the surface and the surrounding medium. Then equation (5.2.11) becomes

\[
\theta - \theta_A = \frac{\bar{Q}}{h} + \frac{1}{K} \bar{Q} \bar{y} - \frac{1}{K} \int_0^\bar{y} Q \, dy \tag{5.2.13}
\]

Chapter 7 contains a discussion of the heat transfer coefficient for rotating disks, together with an experimental determination of \( h \). In the present chapter we shall therefore confine the discussion to the determination of \( \theta_A \) from equation (5.2.11).

5.3 Application of the theory to rolling Perspex disks

The linear heat conduction theory of the last section will now be applied to the rolling disk experiments of
Chapter 3. The hysteresis loss calculations of Chapter 4 corresponding to $\mu = 0$ and $\nu = 0.35$ will be used. As explained above it will also be assumed that the cylinders are long compared with the dimensions of the contact region.

To apply the heat conduction equations to the energy loss distribution for a material in conditions of pure rolling, some assumptions about the form of the energy dissipation must be made. Firstly, it must be assumed that all the energy dissipated, manifests itself as heat. We note that when two disks roll together, the dissipated energy could appear as either heat or stored energy. Although, under conditions of near-failure, the evidence of the photo-elastic experiments suggest that some of the energy is stored as elastic energy, under less severe conditions no such effects occur. The fact that no 'forward flow' of the Crook-Hamilton type was detected by embedded pegs (section 2.6) also suggests that the stored elastic energy is small.

The second assumption is concerned with the use of steady state heat conduction equations. If we consider an element of material beneath the disk surface, significant energy loss will occur in it only when the area of contact passes over the surface above it. To apply the derived equations for heat conduction we must assume that heat flow through such an element has reached a steady state. As a first approximation to the situation occurring in practice this may not be far wrong. In Chapter 2 a non-dimensional parameter $H$ was derived. (Equation 2.8.1)
\[ H = \frac{V b^2}{4 \kappa \pi R} = \frac{t}{\tau_s} \frac{(\text{Time for heat to diffuse through a distance } b)}{(\text{Time between successive passages of the contact})} \]

Over the range of speeds and loads in the experiments of Chapter 3 this parameter is always large - typically greater than 1,500. We may thus conclude that although the process of energy dissipation is not continuous, much of the heat generated by the passage of the contact region is in the same place when the contact region returns to the same point on the disk surface after one revolution. This being so, we shall assume that the process of heat conduction is continuous. In this case we must consider, not the heat generated for each passage of the contact, but the heat generated per unit time.

From the theory of hysteresis losses described in Chapter 4 we may write that the energy dissipation per unit volume at a depth \( y \) associated with one complete passage of the contact is

\[ \beta \ell_o^2 \]

where \( \ell_o^2 \) has the value corresponding to the depth \( y \). Since, from the arguments above, we are assuming that the energy dissipated manifests itself solely as heat, and that the process of heat conduction is continuous, we may now write that the heat generated per unit volume per second at a depth \( y \) is given by

\[ q = \beta \ell_o^2 \quad (5.3.1) \]
where \( \Lambda \) is the number of passages of the contact region per second.

Rewriting the equations of Section 5.2 in a similar notation, the heat generated between \( y = 0 \) and \( y = y \) (per unit area) is given by

\[
Q = \Lambda b p_o^2 \int_0^Y L_o^2 \, dy \quad (5.3.2)
\]

In this equation both \( L_o \) and \( Y \) have been expressed in the non-dimensional forms:

\[
Y = \frac{y}{b}, \quad \bar{Y} = \frac{\bar{y}}{b}, \quad L_o = \frac{L_o}{p_o} \quad (5.3.3)
\]

The heat generated between \( y = 0 \) and the position of the temperature maximum \( y = \bar{y} \) is given by

\[
\bar{Q} = \Lambda b p_o^2 \int_0^{\bar{Y}} L_o^2 \, dY \quad (5.3.4)
\]

The heat flux at a depth \( y \) is given by

\[
Q_y = Q - \bar{Q} \quad (5.3.5)
\]

and the heat flux at the surface \( (y = 0) \) is

\[
Q_0 = -\bar{Q} \quad (5.3.6)
\]

Then using equation (5.2.11) the temperature at a depth \( y \) is given by

\[
\theta - \theta_o = \frac{b}{K} \left[ \bar{Q} Y - \int_0^Y Q \, dy \right] \quad (5.3.7)
\]
Fig 5.2 $(\theta - \theta_0)^\circ$ VERSUS DEPTH $Y$ FOR PERSPEX DISKS.
SPEED 13.1 REV/S SEC AND $W = 87$ Kg/cm
and substituting the expressions for $\tilde{Q}$ and $Q$ from equations (5.3.2) and (5.3.4) into equation (5.3.7)

$$\frac{(\theta - \theta_0)K}{\Lambda \beta b^2 p_0^2} = Y \int_0^Y L_0^2 \, dY - \int_0^Y \int_0^Y L_0^2 \, dY \, dY \quad (5.3.8)$$

As discussed earlier, to calculate the temperature distribution it is necessary to assume the depth $\bar{y}$ of the maximum temperature. In Figure 5.2 values of $(\theta - \theta_0)$ found from equation (5.3.8) have been plotted as a function of the depth $Y = \bar{Y}/b$ for assumed values of $\bar{Y} = \bar{Y}/b$ of 6 and 10. It will be observed that the influence of the assumed value of $\bar{y}$ has little influence upon the temperature distribution over the range $Y = 0$ to $Y = 4$.

In the case of two long cylinders of a material of poor thermal conductivity, loaded and rolling together, most of the heat generated will be conducted to the surfaces rather than to the centre of the cylinders, because fresh cool oil applied to the surface will tend to act as a heat sink for the heat. In these circumstances a choice of $\bar{Y} = 10 \, \bar{Y}/b$ would be applicable. (Approximately 98% of the energy dissipation occurs between $Y = 0$ and $Y = 10$). However, in our experiments (Chapter 3) the disks are not infinitely long and there will be some flow of heat in the axial directions. The influence of this axial flow of heat might be expected to increase with increasing depth. Therefore in the following calculations we shall make the arbitrary assumption that the maximum temperature occurs at a depth of $6a$ ($\bar{Y} = 6.0$).
Fig. 5.3 $(\theta - \theta_0)\degree$ VERSUS DEPTH Y: SPEED 9.7 REVS/SEC
Fig 5.4 $(\theta - \theta_0)^\circ C$ VERSUS DEPTH $Y$: SPEED 11.7 REVS/SEC
$\left(\theta - \theta_0\right)_C \text{ versus depth } \gamma$: speed 13.1 revs/sec
Fig 5.6  \((\theta - \theta_0)^0\text{C}\) VERSUS DEPTH Y: SPEED 15.5 REVS/SEC
Fig 5.7 \((\theta - \theta_0)^\circ\) VERSUS DEPTH \(Y\): SPEED 18.6 REVS / SEC
To provide some indication of the magnitudes of the sub-surface temperatures forecast by this theory Figures (5.3) to (5.7) show values of \( (\theta - \theta_o) \) deduced from equation (5.3.8) assuming \( Y = 6 \). Assumed material constants are those taken from Appendix 1. \( K = 4.5 \times 10^{-4} \text{cal/cm}^0 \text{C sec} \), \( \beta = c/aG \) where \( G = 1.11 \times 10^{10} \text{dynes/cm}^2 \). The assumed value of the hysteresis loss factor is 0.04 and is based upon experimental measurements of the coefficient of friction to be described in Chapter 6.

3.4 Discussion

This chapter has been concerned with an approach to the prediction of the theoretical sub-surface temperature distribution, derived from the energy loss theory presented in Chapter 4. From the theoretical temperature distributions of Figures (5.3) to (5.7) it can be seen that an increase in speed, increases the magnitude of the temperature difference \( (\theta - \theta_o) \) and that an increase in load causes both an increase in \( (\theta - \theta_o) \) and an increase in the depth of the thermal effects. This latter point is less clearly seen from the figures, where, for ease of computation, a non-dimensional depth axis has been chosen. This tends to mask the point that the real depth \( y \) will be greater at, say, \( Y = 5 \) for a load of 100 Kg/cm than for 50 Kg/cm because the half width of the contact \( b \) \( (Y = b) \) increases with increasing load.

We should also note, briefly, that equation (5.3.8) may be rewritten:-

\[
(\theta - \theta_o) \propto b^2 p_o^2 \beta C \quad (5.4.1)
\]
where \( C \) is a constant. The temperature difference \( (\theta - \theta_o) \) is thus clearly proportional to \( W^2 \). In Chapter 8 we shall refer back to this relationship when presenting a hypothesis for failure.

The author's presentation of the sub-surface temperatures in the form of Figures (5.3) to (5.7) is only one approach to the problem. An alternative approach to the boundary conditions would be possible if \( \theta_A, \theta_o \) and the heat transfer coefficient at the surface (discussed in Chapter 7) were known. Then, equating the heat flux at the surface found from equation (5.2.12) with that from equation (5.2.15), it would be possible to find an absolute value of \( \theta \) and relate the sub-surface temperatures to a known surface temperature \( \theta_o \). The difficulties involved in measuring the surface temperature of Perspex disks has been discussed at some length in Section 3.2 and have proved to be beyond the scope of this present work. The problem of measuring \( \theta_A \), the temperature of the medium surrounding the disk, stems largely from defining just what the surrounding medium is in the experimental conditions. In an unlubricated situation \( \theta_A \) would obviously be the temperature of the surrounding air, and for disks or gears running in an oil bath, \( \theta_A \) would be the temperature of the oil. In the present experiments, and in many other practical situations, the disks were running in an 'atmosphere' of oil and air, to which cool oil was continuously being applied. Since the merit of experimental measurements in such a situation appear very doubtful, an assumed value of \( \theta_A \) would have to be applied to the theoretical analysis.
CHAPTER 6

MEASUREMENT OF THE HYSTERESIS LOSS FACTOR FOR PERSPEX

6.1 Introduction

In Chapter 4 the energy dissipated in a sub-surface element of material was related to the stress cycle to which the material is subjected, by the hysteresis loss factor $\alpha$ (section 4.5). In Chapter 5 the sub-surface temperature distribution in Perspex disks was predicted theoretically from the energy dissipated, using an assumed value of $\alpha$ of 0.04. In this chapter two experimental techniques to determine $\alpha$ for a rolling stress cycle are described. Since the hysteresis loss factor is dependent on temperature and frequency, Perspex, like many other polymers, exhibits higher mechanical losses at certain temperatures and frequencies, than at others. When the percentage energy loss for stress cycles in Perspex is plotted against temperature, a peak in the loss spectrum occurs between $140^\circ C$ and $180^\circ C$ depending upon the frequency of deformation. A great amount of work has been done on relating the frequency and temperature spectrums for visco-elastic materials, notably by Ferry (1961).

An estimate of the equivalent frequency of deformation in rolling can be made by considering a cylinder rolling over a semi-infinite block of material. For a band of contact, say of width $2b$, the time taken for a rolling cylinder to pass over it would be $2b/V$ where $V$ is the speed of the
cylinder. The frequency of deformation of the material can thus be roughly estimated as \( \frac{V}{2b} \) cycles per second.

Measurements of dynamic mechanical losses in Perspex have been reported by several authors. Maxwell (1955) has determined the loss factor for a range of frequencies and temperatures using a rotating Perspex beam. Gordon and Grieveson (1958) record measurements of percentage energy absorption over a range of temperatures using a falling ball technique, however, the frequency range over which rebound experiments can be made is limited. Typical experiments, where the contact time for the falling ball on the test piece is of the order of \( 10^{-5} \) sec, give frequencies of approximately \( 10^{4} \) cycles/sec. Schmieder and Wolf (1952) have made measurements of dynamic losses with a torsion pendulum at frequencies between 1 and 60 cycles per second.

An insight into the mechanical loss behaviour of a material can be gained by examination of its electrical properties, particularly those relating dielectric losses to frequency and temperature. Deutsch, Hoff and Reddish (1954) have shown that the electric and mechanical losses for a material exhibit similar maxima in their loss spectrums, although the frequencies and temperatures may be different. Bueche and Flom (1958/59) have re-plotted the dielectric loss data of Telfair (1954) and shown that the tan \( \delta \) versus frequency curves are very similar to the friction versus speed curves that they obtain for the sliding of steel riders on Perspex,
Fig. 6.1

A HARD BALL ROLLING ON A SOFTER MATERIAL
- CALCULATION OF WORK DONE
lubricated with Sodium stearate. However, the present investigation is concerned with energy losses in deformation caused by rolling, and since the frictional energy loss, or hysteresis loss factor, will be dependent upon the stress cycle that an element of material is subjected to, values of \( \alpha \) applicable to the present work can only be found from experiments involving rolling on Perspex. Useful comparisons can, however, be made with the results of other authors who have used rebound or pendulum techniques.

Flom (1962) has recorded values of coefficient of rolling friction for balls rolling over a range of temperatures and speeds on a number of different polymers, including Plexi-glass, which, like Perspex, is a glassy poly-methyl-methacrylate polymer. He has measured coefficients of rolling friction of about 0.002 for a steel ball rolling between 2 Plexi-glass plates at a rolling frequency of 280 cps. In his experiments \( \alpha \) remained constant between 20\(^\circ\)C and 80\(^\circ\)C and then rapidly increased to a peak at about 150\(^\circ\)C.

For the case of a hard ball rolling over a semi-infinite body, Greenwood, Minshall and Tabor have related the frictional force to the work done in moving unit distance, as follows: Consider a sphere of radius \( R \) under a total normal load \( W \) (Figure 6.1). The diameter of the circle of contact is \( 2a \)

\[
a = \left[ \frac{2WR}{E} \left( 1 - \frac{\nu^2}{\sqrt{E}} \right) \right]^{\frac{1}{3}} \quad (6.1.1)
\]
The pressure at a point distance \( r \) from the centre of the contact circle is

\[
p = \frac{3W}{2\pi a^2} \left(1 - \frac{r^2}{a^2}\right)^{\frac{1}{2}}
\]  
(6.1.2)

and the displacement \( w \) of the surface of the rubber is, for \( |r| < a \),

\[
w = C + \frac{r^2}{2R} = C + \frac{(x^2 + y^2)}{2R}
\]  
(6.1.3)

The change in displacement at \((x, y)\) in moving forward a distance \( \Delta \) is \( x\Delta/R \), so the total work done by the front half of the sphere is

\[
\frac{\Delta}{R} \int_{-a}^{a} \int_{-a}^{a} \rho \, dx \, dy
\]  
(6.1.4)

taken between the limits \( x = 0 \) to \((a - y^2)\); \( y = -a \) to \( +a \). This equals

\[
\frac{\Delta}{R} \frac{3W}{2a^2} \frac{a^3}{8} = \frac{\Delta}{R} \frac{3Wa}{16}
\]  
(6.1.5)

so the work done in moving unit distance is

\[
\phi = \frac{3}{16} \left(\frac{3}{4}\right)^{\frac{1}{2}} \frac{W^\frac{1}{3}}{R^\frac{1}{3}} \left(\frac{1 - \nu^2}{E}\right)^{\frac{1}{2}}
\]  
(6.1.6)

and thus the frictional force will be

\[
F = a \frac{3}{16} \frac{Wa}{R}
\]  
(6.1.7)

Some experiments by Greenwood, Minshall and Tabor with a hard sphere rolling on a block of rubber, have shown that this simple treatment agrees well with the observed results,
except that the effective loss is 2.5α. The reason for this is clear when account is taken of the arguments of Chapter 4, namely, that the rolling friction can only be related to the cyclic energy changes and not the total input energy for rolling.

A similar simple theory for the rolling of a cylinder shows that the frictional force will be,

\[ F = \frac{\alpha^2}{2\pi} \frac{Wb}{R} \quad (6.1.8) \]

almost 3.5 times smaller than that predicted by the cyclic hypothesis for rubber (Equation 4.3.11).

Work is at present in hand to extend the hypothesis of Chapter 4 to the case of a sphere rolling on a material for which Poisson's ratio need not be 0.5. Until such time as this is completed a direct calculation of the energy loss in each element is impossible and a prediction of the rolling friction in these cases can be nothing more than a close guess. It seems relevant to note that a proportion of deformed material will pass round the sides of a rolling sphere, and the stress paths for these elements will be straighter and shorter than for elements which pass immediately underneath the sphere. As a result the loss factor might be expected to be less than that for a long cylinder. In the absence of more direct evidence we will write for the rolling of a sphere:

\[ F = 2.5\alpha \frac{3}{16} \frac{Wb}{R} \quad (6.1.9) \]

This should be a good approximation to a more rigorous
Upper slab restrained by strain rod

Lower slab mounted on traversing sledge

Fig. 6.2  SIMPLIFIED ARRANGEMENT OF BALL ROLLING BETWEEN TWO PERSPEX SLABS, USED IN SLOW SPEED EXPERIMENTS.
(cyclic type) approach. Applying this equation to the results of Flom (1962) for Plexiglass gives a loss factor, \( a \), of approximately 0.1 at a temperature of 50°C and a frequency of 280 cps.

Two simple experiments were devised to measure the frictional force for a hard ball rolling over Perspex, one at low rolling frequencies (1 to 50 cps) and the other at higher frequencies (600 to 6000 cps) over a temperature range of 20°C to 100°C. Both investigations were limited, unfortunately, to the rolling of spheres. From the descriptions of the apparatus that follow, it can be seen that only in the low frequency experiments could measurements of rolling friction for a cylinder be attempted. Here it was found that the accurate alignment of a cylinder between two Perspex blocks (similar to the technique used for a ball in section 6.3) proved too difficult to yield acceptable results. The reason for restricting measurements to the rolling of balls and not cylinders can be seen by examining the simplified geometric arrangement shown in Figure 6.2. Suppose the ball shown here was, in fact, a cylinder with its axis running into the paper. As long as this axis is at 90° to the direction of rolling, results similar to those of a rolling ball would be obtained. If, however, the axis of the cylinder deviated by, say, one degree from an orientation normal to the direction of motion, then slip between the cylinder and the flats must occur and a component of sliding motion would be introduced into the
system. We would therefore measure both the rolling friction force and the sliding friction force, and be unable to distinguish between them.

Examination of equations (6.1.9) and (6.1.1) shows that before the loss factor can be found values of $E$ and $v$ at the appropriate temperatures must be known. Young's Modulus for Perspex varies over the temperature range $0$ to $100^\circ C$ from $3.5 \times 10^6$ kg/cm$^2$ to $1.5 \times 10^6$ kg/cm$^2$. Appendix 1. The value of Poisson's ratio $(0.35 -$ Appendix 1) is only tabulated between $-25^\circ C$ and $+50^\circ C$. To establish whether large variations of Poisson's ratio are likely to occur in the temperature range $50^\circ C$ to $100^\circ C$ a simple experiment was devised.

6.2 Measurement of the area of contact between a hard ball and a Perspex plane for a range of temperatures

The radius of the area of contact between a ball and a flat is given by Timoshenko and Goodier, as:

$$a = \left[\frac{3\pi PR (k_1 + k_2)}{4}\right]^\frac{1}{3}$$  \hspace{1cm} (6.2.1)

where

$$k_1 = \frac{1 - \nu_1^2}{\frac{\mu E_1}{\nu E_1}}$$ and $$k_2 = \frac{1 - \nu_2^2}{\frac{\mu E_2}{\nu E_2}}$$

relate to the elastic constants of the ball and the flat, $R$ is the radius of the ball and $P$ is the compressive force between the ball and the flat.

A 2.54 cm diameter hardened steel ball was pressed
Fig. 6.3 MEASUREMENT OF THE AREA OF CONTACT BETWEEN A STEEL BALL AND A PERSPEX FLAT
Fig. 6.4  RADIUS OF CONTACT AREA VERSUS LOAD FOR A STEEL BALL
ON A PERSPEX FLAT  SOLID LINE — Theory
Experimental values • 22.5 °C  ○ 50 °C  ◊ 80 °C  □ 100 °C

Radius of Contact Area $\times 10^{-2} \text{cm}$

Load kg
(Figure 6.3) against a block of Perspex by a 100lb proof ring. The area of contact was viewed by internal reflection, and the diameter measured using a travelling microscope. Measurement of the temperature of the Perspex block was made using 2 embedded thermocouples, one near the side of the block 4 cm from the contact region, and the other 2 cm beneath the contact region.

The Perspex block was heated by a battery of infra-red lights around it. Measurements of contact diameter were made only when the temperature indicated by both thermocouples was within $\pm 0.5^\circ C$ of the required measuring temperature. The diameter of the area of contact was measured for a range of temperatures between $22.5^\circ C$ and $100^\circ C$ for loads between 1 Kg and 9 Kg. In Figure 6.4 the radius of the contact area is plotted against the applied load at four different temperatures. The lines for the theoretical radius of the contact area, determined from equation (6.2.1) using a value of Poisson's ratio of 0.35 for all temperatures and the appropriate value of Young's modulus (Appendix A), are also shown. It can be seen that there is fairly close agreement between theoretical and experimental values, although the experimental results are between 2 and 3% lower than the theory for the three highest temperatures. In Table 6.1 the equation for the theoretical value of radius is compared with the equation for a line drawn through the experimental points.
Table 6.1

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Theoretical line</th>
<th>Experimental line</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.5</td>
<td>a = 3.00 W^\frac{1}{3}</td>
<td>a = 3.10 W^\frac{1}{3}</td>
</tr>
<tr>
<td>50</td>
<td>a = 3.23 W^\frac{1}{3}</td>
<td>a = 3.20 W^\frac{1}{3}</td>
</tr>
<tr>
<td>70</td>
<td>a = 3.43 W^\frac{1}{3}</td>
<td>a = 3.37 W^\frac{1}{3}</td>
</tr>
<tr>
<td>80</td>
<td>a = 3.58 W^\frac{1}{3}</td>
<td>a = 3.45 W^\frac{1}{3}</td>
</tr>
<tr>
<td>90</td>
<td>a = 3.72 W^\frac{1}{3}</td>
<td>a = 3.65 W^\frac{1}{3}</td>
</tr>
<tr>
<td>100</td>
<td>a = 3.91 W^\frac{1}{3}</td>
<td>a = 3.80 W^\frac{1}{3}</td>
</tr>
</tbody>
</table>

Thus the measurements of the area of contact as a function of temperature lie close to those calculated using the values of elastic modulus given by the manufacturers of Perspex and assuming a value of 0.35 for Poisson's ratio. The measurements suggest a small increase in Poisson's ratio at higher temperatures. However, the agreement between theory and experiment lies within the accuracy of the optical measurements. Therefore a value of 0.35 for Poisson's ratio seems valid and this value has been assumed in the analysis of experiments throughout this thesis.

6.3 Measurement of rolling friction: the frequency range 1 to 50 c.p.s.

The simplest arrangement to measure rolling friction is simply for a ball to roll on a flat plane. Great care must be taken when devising experiments to measure rolling friction coefficients in the order of 0.001 to ensure that
SLOW SPEED MEASUREMENTS

(a) The ball between the two slabs
(b) Apparatus with oil bath removed
no external constraint is applied to the ball. To avoid any such constraint the author adopted the geometric configuration of Figure 6.2 onto the basic framework of the crossed cylinders friction machine described by Archard (1958). A 2.54 cm diameter steel ball rolled between two Perspex slabs, both 2.5 cm thick by 5 cm wide by 20 cm long. The lower slab was mounted on the traversing carriage of the machine, driven by a variable speed motor. The upper slab was mounted from the loading arm of the machine. Loads were applied to the loading arm, thus loading the upper slab, ball and lower slab together. The rolling frictional force was determined by measuring the deflection of a strain rod restraining the loading arm from moving. A Talysurf head was mounted in contact with the strain rod and deflections of the rod were displayed using conventional Rank Taylor Hobson equipment on a strip chart recorder. The chart was calibrated by applying known forces to the strain rod. This technique for measuring small frictional forces is extremely sensitive; displacements of the Talysurf head are transposed into electrical signals, which are amplified and displayed on the chart recorder. The amplifier used in these experiments had six ranges, giving a span of sensitivity of 0.2 grams/division or 4 grams full scale deflection on the most sensitive range, and 6 grams/division or 120 grams full scale deflection on the coarse scale. The apparatus is shown in Figure 6.5.

The temperature of the two slabs and the steel ball
was controlled by total immersion in an oil bath, which was built onto the lower carriage in such a way as to enable the upper slab to be immersed without interfering with the loading arm. Two rheostat controlled strip heaters were mounted on either side of the bath to maintain the oil at the required temperature and a peristaltic pump was used to circulate the oil in the bath and to minimise thermal gradients. The temperature in the centre of both of the slabs was recorded on a chart recorder from thermocouples buried in the centre of the slabs. Rolling friction measurements were only made when the temperature of both slabs was within $\pm 1^\circ C$ of the appropriate test temperature. Measurements of rolling friction were made at three speeds of traverse of the lower slab, 0.615 cm/sec, 0.308 cm/sec and 0.125 cm/sec. The range of loads applied to the ball ranged between 1.5 Kg and 12.0 Kg at 22.5°C and 1.5 Kg and 3.6 Kg at 100°C.

The experimental procedure was as follows. The appropriate range of the R.T.H. amplifier was selected and the chart recorder was calibrated by applying known forces to the strain rod. When both the Perspex slabs had reached the appropriate temperature a load was applied to the loading arm and the motor started, to traverse the carriage. At the full extent of the traverse the motor was reversed and a record of the friction force in the other direction was made. A typical trace is shown in the photograph in
Figure 6.6

Traversing speed: 0.615 cm/sec. Temp 22.5°C. Load 2.4 kg

Talysurf Trace Slow Speed Experiments

Scale 0.234 division
Figure 6.6. To overcome problems of zeroing the trace before and after the traverse, the friction force was found by halving the difference between the readings in the two directions.

To examine the possibility of surface effects influencing the friction force, measurements of rolling friction were made, firstly with oil in the bath, and then without oil under identical conditions of load, speed and temperature. The rolling friction forces in both cases were the same and it was concluded that surface effects were negligible.

6.4 The loss factor for low speed rolling (1 to 50 cps)

In Section 6.1 the relationship between rolling friction force $F$ and the hysteresis loss factor $\alpha$ was discussed and as a first approximation equation (6.1.9) gives:

$$F = 2.5 \alpha \frac{3}{16} \frac{Wa}{R}$$  \hspace{1cm} (6.4.1)

In the experiments described in the previous section the rolling friction force $F$ was measured for a known load $W$ applied to a steel ball of radius $R$. Knowing the experimental temperature, the radius of the area of contact $a$, between the ball and the Perspex slab can be found from equation (6.2.1). Rearranging equation (6.4.1) we can write

$$\alpha = 2.1 \frac{F/W}{(a/R)}$$  \hspace{1cm} (6.4.2)
Fig 6.7 COEFFICIENT OF FRICTION PLOTTED AGAINST $\frac{a}{R}$
Rolling speed 0.61 cm sec; Temperature 20°C
Slope = 0.013 Thus $\alpha = 0.027$
By plotting $F/W$ against $a/R$ for a given temperature and rolling speed, the hysteresis loss factor can be found from the slope of the curve. Typical results are shown in Figure 6.7 for a rolling speed of 0.61 cm/sec and a temperature of 20°C. The slope of the graph gives a loss factor of 0.027. In Table 6.2 the loss factor is tabulated for the three experimental speeds at four different temperatures.

Table 6.2

<table>
<thead>
<tr>
<th>Temperature</th>
<th>0.61 cm/sec</th>
<th>0.30 cm/sec</th>
<th>0.12 cm/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>22.5°C</td>
<td>0.027</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>68.5°C</td>
<td>0.030</td>
<td>0.030</td>
<td>0.031</td>
</tr>
<tr>
<td>80°C</td>
<td>0.030</td>
<td>0.030</td>
<td>0.030</td>
</tr>
<tr>
<td>100°C</td>
<td>0.030</td>
<td>0.034</td>
<td>0.037</td>
</tr>
</tbody>
</table>

From this table it can be seen that there is a general trend of increasing loss factor with increasing temperature, as would be expected. The frequency range of these experiments, estimated in the manner described in Section 6.1, was between 1 cycle/sec and 50 cycles/sec. Although this is approximately two orders of magnitude less than the frequencies of the two-disk machine experiments, the relevance of these experiments has been in finding the order of magnitude of the hysteresis loss factor for Perspex applicable
Fig. 6-8  DIAGRAM OF APPARATUS TO MEASURE ROLLING FRICTION: HIGH SPEED RANGE
to the earlier work, and in demonstrating a technique to measure very small coefficients of rolling friction caused by hysteresis losses of polymers. In Section 6.5 a further series of experiments is described to measure rolling friction at higher frequencies of rolling.

6.5 Measurement of rolling friction: the frequency range 600 to 6000 cycles/sec

The ideal experimental arrangement to measure rolling friction is for the ball to roll in a straight line on a flat surface. If, however, high rolling speeds over a range of temperatures are required such an arrangement presents some obvious experimental difficulties, and it is more satisfactory to resort to a circular motion. The apparatus used for these measurements (Figure 6.8) was similar to that used by Flom (1960) and (1962) in his experiments using a wide range of polymers. Three hardened steel balls rolled between two concentric Perspex disks. The upper disk had a flat surface and was maintained in position by a hollow brass shaft glued into the centre of the disk and passing through an air bearing. The lower disk had a small circumferential groove, having a radius of curvature of 7.5 cm, at a radius of 7 cm from the axis of rotation; this groove was to constrain the balls between the disks. The lower disk was rotated by a chain drive using a rheostat controlled, $\frac{1}{2}$ h.p. 1500 rpm D.C. motor. Torques exerted on the upper disk, by the rolling frictional
forces, were measured by a strain gauge attached to a 0.025 cm thick copper strip, one end of which passed through a slit in the hollow brass shaft and the other end was rigidly attached to a stop. Changes of resistance of the gauge were measured on a strain gauge bridge, reading directly in percentage strain. The strain gauges were calibrated by applying known torques to the upper disk. For measurements at elevated temperatures a furnace, heated by two strip heaters, was built around the disk assembly, giving a temperature range of 20°C to 110°C. The mean temperature of the upper disk was measured by a thermocouple embedded in the disk. The leads of the thermocouple were passed from the disk through the hollow tube.

Before making any measurement of rolling friction, the effects of spin on the balls rolling in a circular path was examined. Forcing the balls in a circular path must impart a certain amount of spin, however minute. If the lower disk did not have a circumferential groove in it the balls would tend to 'creep' outwards as the disk was rotated. Plom has shown that the spin effects, representing deviations from pure rolling, can be minimised by using small balls and large track radii. In the author's experiments the track radius of 7 cm was approximately twice that used by Flom over the same speed range. The groove radius on the lower disk (again larger than that widely used by Flom) was large enough to ensure essentially sphere-on-plane contact between the balls and disk. Thus additional friction due
Fig 6.9 (a) COEFFICIENT OF FRICTION VERSUS ROLLING SPEED $\theta = 25^\circ \text{C}$ $R = 0.62 \text{ cm}$

(b) LOSS FACTOR VERSUS FREQUENCY

- $x$ $W = 1.82 \text{ Kg}$
- $\circ$ $W = 1.15 \text{ Kg}$
- $-$ $W = 0.33 \text{ Kg}$ $R = 0.16 \text{ cm}$ (Results from FLOM)
Fig. 6.10 (a) COEFFICIENT OF ROLLING FRICTION VERSUS TEMPERATURE
(b) LOSS FACTOR VERSUS TEMPERATURE

X  R = 0.32 cm  V = 184 cm/sec  w = 0.65 kg
Δ  R = 0.32 cm  V = 73 cm/sec  w = 0.65 kg
O  R = 0.62 cm  V = 110 cm/sec  w = 0.65 kg
to sliding was not introduced unless the speed became high enough for centrifugal forces to become appreciable. The rolling friction was measured for the balls running at constant speed in the apparatus both dry, and lubricated by OM100 oil. No difference in rolling friction measurement was recorded, over the speed range at which the tests were conducted. It was thus concluded that surface effects were negligible.

6.6 The loss factor for high speed rolling 600 to 6000 r.p.m.

The frequency range of the investigation was roughly the same as that of the 2-disk machine experiments, namely 600 to 6000 cps. At higher rotational speeds of the disk, inertia effects became appreciable, the balls tending to slip and catch up with each other. Measurements of rolling friction, at constant load and temperature, were made over the speed range 30 cm/sec and 300 cm/sec and are plotted in Figure 6.9a. The results of Flom for steel balls rolling on Plexiglass at a load of 330 grams per ball at 25°C are shown as a dotted line. It can be seen that his results are in reasonable agreement with the author's, and that over the speed range covered there is little variation in rolling friction with speed.

In Figure 6.10a the coefficient of rolling friction for rolling at constant speed and load, is plotted against temperature. It can be seen that between about 20°C and 70°C there is little change in the coefficient of friction.
Above 70°C the coefficient of rolling friction increased sharply with increasing temperature. Similar results were obtained by Plom although they are not shown in Figure 6.10a because the scale upon which he presents his results in his paper is too small to allow accurate interpolation.

Plom's results for Plexiglass show that the rolling friction continues to increase with temperature to a peak of $8 \times 10^{-2}$ at about 160°C and then fall again. Unfortunately the furnace used by the author was not sufficiently powerful to raise the disk temperature above 110°C, and consequently an interesting comparison between the work of Plom and that of the author could not be made. One feature of particular interest in the work reported here is the inflexions in the curves of Figure 6.10a. Gordon and Grieveson (1958) report an inflexion in the energy absorption spectrum (measured by a falling ball technique) for Perspex at about 100°C and attribute this to the onset of rotation of one of the polymer's side groups. It is possible that the inflexions in Figure 6.10a at 100°C could also be caused by this mechanism.

The loss factor $\alpha$ can be derived from these experimental results using equation 6.4.1. In Figure 6.9b the variation of loss factor with frequency at constant load and temperature is replotted from the curves drawn through the results of Figure 6.9a. It can be seen that there is very little difference between the loss factors for the two loads of the author's experiments. A more detailed series of experiments over a wider range of loads are clearly required to
investigate the dependence of loss factor upon load. It is interesting to compare the author's results with those derived from Flom's work, shown as a dotted line in Figure 6.9b. In calculating the loss factor from Flom's results for rolling friction it has been assumed that the elastic modulus for Plexiglass is the same as that of Perspex. It can be seen that there is reasonable agreement, the loss factor from Flom's work being about 1.3 times greater than that found by the author.

In Figure 6.10b the variation of loss factor with temperature at constant load and frequency has been replotted from the curves drawn through the results of Figure 6.10a. The difference between the curves for the 0.635 cm diameter ball rolling at 184 cm/sec and 73 cm/sec is small, and only the curve for rolling at 184 cm/sec has been shown. In this case the frequency, which is dependent upon the diameter of the area of contact (which in turn varies with temperature) changes from about 5,600 cycles/sec at 20°C to about 4,400 cycles/sec at 95°C. Also shown in Figure 6.10b is the curve for a 1.25 cm diameter ball rolling at frequencies of 2,600 cycles/sec at 20°C to 2000 cycles/sec at 95°C, under a load of 0.65 Kg. The difference between the two curves shown in this figure is more distinct at the lower temperatures. It is interesting to note when comparing Figures 6.10a and 6.10b that the inflexions in the curves between 90°C and 100°C are for very similar values of loss factor (Figure 6.10b) whereas the rolling friction coefficient (Figure 6.10a) shows almost a factor of 2 difference for the two sizes of
Changes in temperature effect both the elastic modulus and the diameter of the contact area; and hence the ratio $\frac{a}{R}$.

6.7 Discussion

The loss factor for Perspex, like that of other viscoelastic materials is a function of both frequency and temperature. The experiments described in this Chapter have demonstrated techniques for determining the relationship between frequency, temperature and the coefficient of rolling friction, the results of which are plotted in Figures 6.9 and 6.10. As a first approximation the loss factor has been found using Equation 6.4.1, and its relationship with frequency and temperature has been plotted, from both the author's results and those of Flom.

Clearly in a rigorous treatment of the work described in Chapters 4 and 5, computation should include the variation of loss factor with both frequency and temperature. However, as a first approximation to predict the sub-surface temperature distribution, the author has used a single value of loss factor for the computation in Chapter 5. The value taken was 0.04. In Figure 6.10b it can be seen that variations of loss factor with temperature are small between 20°C and 60°C. In Figure 6.9b the variations over the range of frequencies of the experiments are also small. Clearly, more detailed experimental work is required to explore, in more detail, the pattern of the variation of loss factor.
for rolling over a wide range of conditions. However, for the work described in this thesis, the measurements described here provide an adequate indication of the magnitude of the loss factor and its variation with temperature.
CHAPTER 7

HEAT TRANSFER FROM ROTATING SURFACES USING
A MASS TRANSFER ANALOGY

7.1 Introduction

One of the difficulties in the estimation of local heat transfer coefficients at the surface of solids rotating in fluids, is the determination of temperatures near the interface. In section 5.2 the relationship between the heat flux at the surface of a disk, $\bar{Q}$, the disk surface temperature $\theta_0$ and the temperature of the surrounding medium $\theta_A$ was given in equation (5.2.12) as

$$\bar{Q} = h (\theta_0 - \theta_A) \quad (7.1.1)$$

where $h$ is the heat transfer coefficient between the surface and the surrounding medium. The difficulties involved in experimentally measuring $\theta_0$ and $\theta_A$ have been discussed in sections 3.2 and 5.4. Consequently the straightforward approach in determining heat transfer coefficients, namely that of measuring $\theta_0$ and $\theta_A$ and, knowing the heat flux at the surface, finding $h$ from equation (7.1.1) is very limited. This chapter, taken largely from a report by Maxwell and Wannop (1969), presents the results of an experimental technique; mass transfer rates from the surfaces of rotating disks have been measured and heat transfer coefficients estimated by invoking the Chilton-Colburn relationships. The experiments have involved the measurement of mercury evaporated from amalgamated areas on the surface of disks,
rotating in a controlled atmosphere, using a very sensitive Mercury Detection Meter. Because of the somewhat different nature of the theoretical derivations in this chapter, compared with the terms used in the other chapters of this thesis, the author has included in Appendix 2 a glossary of the relationships and terms applicable to heat and mass transfer used in this chapter.

Maxwell and Storrow (1957) demonstrated the technique of mercury evaporation from amalgamated surfaces to determine overall and local mass transfer coefficients from the surfaces of stationary spheres, and cylinders, in air and nitrogen streams. Mercury surfaces have the advantage, compared with other evaporation systems, of being readily formable to various geometries (the geometry can be formed first in copper or brass before the mercury is amalgamated on to the surface), and of having a low vapour pressure—giving evaporation rates and requiring low energy transfer rates.

Recent investigators, Iguchi and Maki (1967) and Kreith et al (1959), have determined the mass transfer coefficients from the side faces of rotating naphthalene disks. These experiments using naphthalene involve difficult measurements in weighing or microscopic examination.

The present work has used the mercury evaporation technique to determine the mass transfer coefficients from disks rotating in air. The experiments involve the flow, at very low velocity, of air past a partially amalgamated rotating disk and the measurement of the mercury vapour in
Fig. 7.1 DIAGRAM OF APPARATUS FOR MASS TRANSFER EXPERIMENTS
the air before and after passing over the disk.

7.2 Experimental Measurements of Mass Transfer

The experiments were carried out in a Perspex duct shown in Figure 7.1. Air from a compressor passed through a valve, rotameter $R_1$ (See Table 7.1), and then to the working section. In all of the tests described the velocity of the air over the disk was less than 2% of the circumferential velocity of the rotating disk. It was assumed that the flow pattern near the surface of the disk was not significantly affected by the overall low velocity in the duct. A continuous sample of the air was drawn from the duct and passed through rotameter $R_2$ into the Mercury Detection Meter E.3472 (manufactured by Hendrey Relays, Slough). All the air containing mercury vapour was exhausted to atmosphere outside the laboratory. In order to maintain the mercury concentration at a level suitable for measurement by the meter, the sample could be diluted by mixing with mercury free air from rotameter $R_3$ before passing to the detection meter.

**TABLE 7.1 : FLOW METERS**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R.1</td>
<td>Rotameter 24 $P+$ duralumin float</td>
</tr>
<tr>
<td>R.2</td>
<td>&quot; $7X+$ &quot; &quot;</td>
</tr>
<tr>
<td>R.3</td>
<td>&quot; $7X+$ &quot; &quot;</td>
</tr>
</tbody>
</table>

Figs 7.2 and 7.3 show the working section. Air entered the square working section of side 20 cms through a glass pipe (diameter 7.3 cm) and a diffuser. A honeycomb was
placed at the entrance of the working section to straighten and smooth the air flow.

The disk C was mounted on a shaft E running in bearings A (Figure 7.3). The bearing housings B fitted flush to the internal walls of the working section and were located and secured by the embedded bolts and nuts, D. The shaft was driven through a rubber coupling F by a motor G. For rotational speeds of 30 to 1000 rpm the shaft was driven by a D.C. motor controlled by a series rheostat, and for speeds between 1000 and 4000 rpm an A.C. motor controlled by a series rheostat was used.

The chosen areas on the disks were amalgamated by immersion in an acidic (nitric) solution of mercuric chloride. The disks were previously lacquered to prevent unwanted deposition of the mercury on other areas of the disks.

During some preliminary experiments on disks of three different base materials (as listed in Table 7.2) it was observed that the transfer characteristics, under similar conditions, varied for the three materials.

**TABLE 7.2 : BASE MATERIALS**

1. Cu. BSS 1433  
   Cu. 99.9% Pb 0.04%.
2. Brass Rod BSS 249 (Freecutting)  
   Cu. 56-60% Pb 2-3.5% Zn remainder.
3. Brass Tube BSS 885  
   Cu. 70-73% Zn remainder.
Fig 7.4  DECAY OF TRANSFER RATES FROM DIFFERENT MATERIALS
Measurements were made of the downstream mercury concentration over a period of time, for constant disk speed, air flow and temperature. The results are shown in Figure 7.4 where the meter reading of mercury concentration is plotted against time, for tests on disks of the different materials.

It can be seen from Figure 7.4 that the concentration of mercury using a copper base falls slightly over a period of one hour, while the concentration using the two brass bases falls off more rapidly in the same period of time. The concentration using the brass tubing base falls slowly over the first ten minutes but more rapidly over longer periods of time. The mercury transfer rate from the brass rod base, after a short initial period of slow linear fall, rapidly decays exponentially. Similar results were obtained by Maxwell and Storrow. This decrease of mercury transfer rate from the brass specimens can probably be attributed to the diffusion of zinc from the brass base, which affects the oxidation rate and interfacial transfer processes. It is hoped that in future work that this phenomena can be clarified by measuring the vapour pressure of mercury over various amalgamated substrata, using a mass spectrometer.

Copper and brass tubing were used as a base material in subsequent experiments and mercury concentration measurements were extrapolated to zero time. The mercury surface was immersed in the amalgamating solution and the coating kept "mirror clean" for each test.
Fig 7.5  DIAGRAMMATICAL REPRESENTATION OF WORKING SECTION

\[ x \] is the mole fraction of mercury vapour in the air

\[ \{ x_0 \text{ on the surface} \]
\[ x_1 \text{ downstream} \]
\[ x_2 \text{ upstream} \]
Measurements were made of the air flow through rotameter \( R_1 \) and the rotational speed of the disk (using a tachometer for speeds between 100-4000 r.p.m. and timing with a stopwatch for speeds between 30-100 r.p.m.). The mercury concentration in the sampled air stream was read from the meter on the mercury sampling device. The background concentration of mercury vapour in the air entering the working section through rotameter \( R_1 \) was measured without the shaft and disk in the section and the instrument was zeroed. When measurements were made with the disk in the working section the resulting reading gave the concentration of mercury in the sample due to the rotating disk and not the background mercury concentration in the atmosphere. The surface temperature of the disk and the temperature of the air flow were the same and therefore only the air flow temperature was recorded for each test.

7.3 Results of Mass Transfer Experiments

The local mass transfer coefficient \( (k_{x_A} \text{ loc}) \) for a disk (shown diagramatically in the working section in Figure 7.5) may be defined (Bird, Stewart and Lightfoot, 1962) by the equation:

\[
W_A - x_A (W_A + W_B) = (k_{x_A} \text{ loc}) \delta A \Delta x_{A0} \quad (7.3.1)
\]

where \( W_A \) is the molar flow rate of mercury leaving the surface and \( W_B \) is the molar flow rate of air moving towards the surface from downstream in the duct. \( \delta A \) is the area of
the amalgamated surface, \( A x_{A0} \) is the saturation concentration for mercury in air on the disk surface and \( x \) is the mole fraction of a species.

Since \( x_A \) is very small, equation (7.3.1) can be written

\[
W_A = (k^* x_A \Delta c) \Delta A_\Delta x_{A0}
\]  
(7.3.2)

The mercury (species \( A \)) mass balance for the duct, assuming \( x_1 \) (the upstream mole fraction of mercury in the air approaching the disk) is small, can be written

\[
W_A = c_T S V x_1
\]

where \( c_T \) is the total molar concentration, \( x_1 \) is the downstream mole fraction of mercury in the air leaving the disk, and \( V \) the duct air velocity. \( S \) is the cross-sectional area of the duct.

Hence from equations (7.3.2) and (7.3.3) we can write

\[
(k^* x_A \Delta c) = \frac{c_T S V x_1}{\delta A_\Delta x_{A0}}
\]  
(7.3.4)

and since the Sherwood Number \( Sh \) is defined by

\[
Sh = D(k^* x_A \Delta c)
\]  
(7.3.5)

we can now write

\[
Sh = \frac{S V D x_1}{\delta A_\Delta x_{A0} D^*}
\]  
(7.3.6)
Fig 7.6

DIFFUSION COEFFICIENT VERSUS TEMPERATURE
MERCURY SATURATION CONCENTRATION VERSUS TEMPERATURE

Fig 7.7
Where $D$ is the duct diameter, $S$ the cross-sectional area of the duct and $D^*$ the diffusivity of mercury in air. In Figure 7.6 values of $D^*$ predicted from Hirschfelder, Curtiss and Bird (presented by Bird, Lightfoot and Stewart) are shown plotted against the appropriate temperature range. In Figure 7.7 the saturation concentration of mercury on the amalgamated surface $\Delta x_{A^*}$ is shown plotted for the range of temperatures encountered in the experiments. (Data from the International Critical Tables). In equation (7.3.6) $S$, $D$ and $\Delta A$ are constants of the experimental conditions. The experimental measurement of the duct velocity $V$, by reading rotameter $R$, and the air temperature in the duct, enable $D^*$ and $\Delta x_A$ to be found from Figures (7.6) and (7.7). Thus, with the reading of $x_i$ the downstream mole fraction of mercury in air from the Mercury Detection Meter, it is possible to evaluate the Sherwood Number from equation (7.3.6).

In order to correlate the mass transfer results with the available data of other workers (Iguchi and Maki, 1967) and Kreith, Taylor and Chong (1959), and to enable extrapolation to be made to heat transfer problems it is necessary and viable to invoke the Chilton-Colburn analogy that

$$j_H = j_D = a \text{ function of } Re, \text{ geometry and boundary conditions.} \quad (7.3.7)$$

This has proved invaluable for a number of flow situations such as flow over spheres and cylinders (Maxwell and Storrow), with the low transfer rate used in the present
\[ J_D = 0.87 \frac{\text{Re}^{0.52}}{\text{Re}_D} \]

Fig 7.8  PLOT OF $J_D$ VERSUS $\text{Re}_D$
Fig 7.9  PLOT OF $J_D$ VERUS $Re_D$
experiments, $j_H$ and $j_D$ are defined by the relationship

$$j_H = \frac{\text{Nu}}{\text{RePr}^{\frac{1}{3}}} = j_D = \frac{\text{Sh}}{\text{ReSc}^{\frac{1}{3}}} \quad (7.3.8)$$

$j_H$ and $j_D$ are further defined in Appendix 2, such that equation (7.3.8) may be expressed in the form

$$\frac{h}{\rho C_p V} \left( \frac{C_p \mu}{k} \right)^{\frac{2}{3}} = \frac{k^*}{C_p V} \left( \frac{\mu}{\rho D^*} \right)^{\frac{2}{3}} \quad (7.3.9)$$

In Figure 7.8 $j_D$ is plotted against $\text{Re}_D$ (Reynolds number based on diameter) for rotating cylindrical disks, the curved surfaces of which have been amalgamated with mercury. Experiments were carried out with three disks of different diameters, and over the range of diameters covered the results can be expressed by the relationship

$$j_D = \frac{0.87}{\text{Re}_D^{0.52}} \quad (7.3.10)$$

for Reynolds numbers between 600 and 30,000.

In Figure 7.9 $j_D$ is plotted against $\text{Re}_D$ for tests on disks, where only a small portion of the curved surface of the disk was coated, and for a gear. The gear was copper plated and one side of every seventh tooth was amalgamated. The gear was run with the amalgamated area facing both backward and forward. These tests on the disks were an attempt to assess the influence of position of a coated band upon the disk surface. From Figure 7.9 it would seem that the transfer rate varies across the face of the disk, the
Fig 7.10 Diagram of disks and gears
Fig. 7.11  PLOT OF $\dot{J}_D$ VERSUS $Re_R$

$X$ 5.00 cm DIA. DISK-COATING ON 1 SIDE FACE

$\bullet$ 7.26 cm DIA. DISK-COATING ON 1 SIDE FACE

Iguchi and Maki

Kreith et al.
divergence being greater at low Reynolds numbers. Figure 7.10 shows the form in which the coating was applied to the surfaces of the disks and gears.

The spread in the results of Figure 7.9, when only a small area of the disk was coated is, as would be expected, greater than those shown in Figure 7.8 when the complete curved surface of the disk was coated with mercury. The results of Figure 7.9 for gears and partially coated disks do, however, indicate that interesting surface transfer effects in the lubrication field could be investigated, using this technique employing mercury films.

7.4 Mass transfer from the side faces of disks

The experimental work of both Iguchi and Maki (1967) and Kreith, Taylor and Chong (1959) who have measured mass transfer from rotating disks has been restricted to transfer from the side face of disks. A further series of experiments by the author was undertaken to investigate the correlation between the results of these authors, who have used napthalene and measured either by weighing or microscopic examination the mass transfer, and the mercury evaporation technique employed by the author. In Figure 7.11 values of $j_P$ determined by measuring mercury evaporation from the coated side faces of rotating disks, are plotted against $Re_R$ (Reynolds number based on disk radius).

It can be seen that there is good agreement between the present results and those of Iguchi and Maki. The
The disk was driven using only the DC motor and hence the speed range was limited to 30-1400 rpm. As a result the range of $Re_R$ in Figure 7.11 did not include the results of Kreith et al.

Iguchi and Maki give their results for mass transfer experiments on napthalene disks in the form $Sh=1.58\ Re^{0.4}$ for Reynolds numbers between 200 and 30,000.

The present results are expressed in terms of $j$ factors which give correlations with a wide range of similar geometries. In order to effect a comparison between the results of Iguchi and Maki and those of the author, the results of Iguchi and Maki have been expressed in the form

$$j_D = \frac{Sh}{Re\ \frac{Sc^3}{Re}} = \frac{1.58\ Re^{0.4}}{Re\ \frac{Sc^3}{Re}} = 1.15\ Re^{-0.6} \quad (7.4.1)$$

where $Sc^\frac{1}{2}$ for napthalene = 0.73

Similarly the results of Kreith et al, who present their results in the form $Sh=0.67\ Re^{0.5}$, may be written

$$j_D = \frac{Sh}{Re\ \frac{Sc^3}{Re}} = \frac{0.67\ Re^{0.5}}{Re\ \frac{Sc^3}{Re}} = 0.49\ Re^{-0.5} \quad (7.4.2)$$

From Figure 7.11 it can be seen that there is good agreement between the work of the author and the work reported by Iguchi and Maki, and that the relationship $j_D = 1.15\ Re^{-0.6}$ is valid for Reynolds numbers between 200 and 20,000.
7.5 Discussion and Conclusions

It is convenient at this point to briefly review the work presented in this chapter and to summarise its application to heat transfer from the surface of rotating disks. A technique for measuring the mass transfer of mercury, coated on copper and brass surfaces, has been described. The Sherwood number for disks rotating in an open environment has been determined for a range of disk diameters and over a range of Reynolds numbers. These results are presented in Figure 7.8 where $j_D$ for the disk is plotted against $Re_D$. Invoking the Chilton-Colburn analogy it is possible to find $j_H$ and thus $Nu$, the Nusselt Number for a disk. Since (Appendix 2)

$$Nu = \frac{hD}{K} \quad (7.5.1)$$

the heat transfer coefficient for a disk at a given Reynolds number in known surroundings can now be found.

As an example of the use of this technique it is of interest to examine a problem encountered by Ibrahim and Cameron (1963). The work that they presented was concerned largely with the measurements of oil film thickness between heavily loaded running gears. They did, however, attempt to make an estimate of the equilibrium surface temperature of the gears, to show that the temperature of the inlet region, and hence the oil film thickness, is below a certain critical value determined by the surface roughness. They found that the main problem posed when equating the
heat balance at the surface of the gears, was in giving a suitable value to $h$, the heat transfer coefficient at the surface. They quoted a value empirically derived by Esvert (1950) of $1.36 \times 10^{-4} \text{Btu in}^3 \text{sec}^0 \text{F}$. The gears in their experiments were of 5" diameter, run at a constant speed of 1,500 rpm. Taking the details of oil D from their experiments (viscosity 32cS at 100°F and specific gravity 0.871) it is possible to obtain a value for Reynolds Number

$$Re_D = \frac{VD}{\nu} = 3.9 \times 10^4$$

From Figure 7.8, reading from the solid line, $j_D = 0.0037$. Invoking the Chilton-Colburn analogy $j_H = j_D$, equations (7.3.8) and (7.3.9) can be rearranged in the form

$$h = \frac{j_H C_p V \rho}{\left( \frac{C_p \mu}{K} \right)^{\frac{3}{2}}} \quad (7.5.2)$$

Assuming values of $C_p = 0.5 \text{ Btu/lb}^0 \text{F}$ and $K = 9.7 \times 10^{-2} \text{ Btu/ft.hr.}^0 \text{F}$ for the oil, then

$$h = 4.8 \times 10^{-4} \text{ Btu/in}^3 \text{ sec}^0 \text{F}$$

If, however, we use the details of oil A from their paper (viscosity 237 cS at 100°F and specific gravity 0.896) then the value for Reynolds Number

$$Re_D = \frac{VD}{\nu} = 5.25 \times 10^8$$

From Figure 7.8 we obtain a value of $j_D = 0.01$, and upon substituting values into equation (7.5.2), we then get a
value for \( h \) of

\[
h = 3.4 \times 10^{-6} \text{ Btu/in}^2 \text{ sec}^0 \text{F}
\]

The results for \( h \) obtained using the analysis of this chapter, for the oils A and D in the experiments of Ibrahim and Cameron compare favourably with the value that they quote. (When allowance is made for assuming values of \( C_p \) and \( K \) for the oil, and using the relationship between \( j_D \) and \( Re \) for disks rather than gears).

One important point should, however, be noted from the above comparison. The values of heat transfer coefficient for the two sets of experiments (which differ only with the use of 2 different oils) will not be the same. This point is evident from an examination of equations (7.3.8) (7.3.9) and (7.5.2) where it can be seen that the heat transfer coefficient is a function of Reynolds Number, Prandtl Number and thermal conductivity. As a consequence it should be borne in mind that when values of \( h \) are derived, they are only applicable to one set of experimental conditions (in terms of speed, disk diameter, oil viscosity, density etc.).

In conclusion it should be said that the results of the mass transfer experiments described in this chapter are for disks rotating in an open environment for Reynolds Numbers of laminar flow, and transfer to only one medium. To find, for instance, the heat transfer coefficient for a gear in a gear-box we would have to assume the gear-box was either completely full of oil or that the gears were
unlubricated and rotating in air.

Finally, in discussing the relevance of the work described in this chapter to lubrication experiments it is worth recalling the importance of heat transfer in the work described in this thesis. In the experiments of Ibrahim and Cameron metal gears were used and heat transfer at their surface to the surroundings could well have been only a small part of the total heat transfer equation; conduction to the shafts could well have played a significant role. However when, as in the present work, disks or gears of poor thermal conductivity are used, the heat transfer problem discussed here becomes of the greatest significance.

The work described in this chapter has further demonstrated the technique of mercury evaporation from amalgamated surfaces to determine overall and local mass transfer coefficients from the surfaces of disks and gears. Good agreement has been shown between the results of this technique and those of other workers (Iguchi and Maki and Kreith et al) who have measured transfer from the side faces of napthalene disks (Figure 7.11). The author has also shown that mass transfer from the curved faces of disks between 3.18 cm and 7.6 cm diameter may be described by the relationship $J_D = 0.67 \text{Re}^{-0.52}$ between Reynolds Numbers of 600 and 30,000.

Further work is indicated for the determination of mass transfer coefficients from rotating gears, and an investigation into relative transfer rates across the
faces of a disk could be successfully attempted using this technique. It is felt, however, that data over a wider range of conditions, but similar in nature to that of Figures 7.8, 7.9 and 7.11, could prove a useful aid to the designer of rotating machinery.
CHAPTER 8
DISCUSSION AND CONCLUSIONS

8.1 Introduction

The original observations of the failure of Perspex cylinders in lubricated point contacts were made by Archard, during an experimental investigation of film thickness in elastohydrodynamic lubrication. The starting point of the present work was a series of experiments in the crossed cylinders machine, in which rotating Perspex cylinders were loaded together in nominal lubricated point contact until catastrophic failure occurred. Observations of specimens in various stages of failure suggested that both thermal effects and mechanical stresses were involved in the failure mechanism. Because of the apparent importance of thermal effects upon the failure mechanism, the magnitudes of temperatures generated by sliding were examined in some detail.

Theoretical predictions of surface temperatures, subsurface temperatures and lubricant film temperatures were made from experimental observations, by developing the "flash temperature" concept. This concept assumes that the two cylinders come into rubbing contact, and that the heat is generated at the area of true contact and conducted away into the bulk of the rubbing members. It was shown that the generation of heat by a "flash temperature" mechanism was unlikely to create sufficiently high surface temperatures to cause softening of the surface layers of the specimens,
and that such temperatures, if they occurred would be confined to layers very close to the surface, such layers being very thin compared with the depth of the maximum Hertzian shear stress.

The next series of experiments proved instrumental in determining the direction of the further work described in this thesis. Perspex disks were loaded together in a two-disk machine and the failure mechanism, found in the point contact experiments, was reproduced under conditions of pure rolling in nominal line contact. Clearly, under conditions of rolling the mechanism of energy dissipation arises from rolling rather than sliding friction. For polymeric materials a major mechanism of rolling friction is hysteresis loss, occurring in regions below the surface. The remaining work described in this thesis was concerned with the measurement of sub-surface temperatures generated in rolling Perspex disks, and the prediction of such temperatures.

In Chapter 3, experimental techniques for measuring sub-surface temperatures are described and the results obtained from different methods are discussed. The results from experiments using embedded thermocouples with an axial orientation in the disks are recorded for a range of conditions of load and speed.

Two approaches to the analysis of hysteresis losses for cylinders rolling in nominal line contact are outlined in Chapter 4. The first of these uses a viscoelastic
model for the material of the cylinders, and the second, due primarily to Greenwood and Tabor, takes as its starting point the stress distribution which would exist if the material of the cylinders was perfectly elastic. The theory of stress distribution is outlined for the contact of perfectly elastic cylinders. Following this, two developments of the 'quasi elastic' theory of Greenwood, Minshall and Tabor are described. Their theory is for the special case of a material with Poisson's ratio $\nu = 0.5$, which is effectively incompressible under hydrostatic stresses. In Chapter 4 the theory is developed to include materials where Poisson's ratio is less than 0.5, and further, to take account of the influence that tangential surface stresses have upon the sub-surface hysteresis loss.

The theoretical sub-surface temperature distribution has been derived from the energy loss distributions presented in Chapter 4. As a first approximation to the problem, the author has assumed steady state linear heat conduction beneath the surface. An indication of the magnitudes of the sub-surface temperatures forecast by the theoretical treatment have been given in Chapter 5. When computing these temperatures it was assumed that the hysteresis loss factor had a constant value of 0.04 over the range of temperatures and speeds considered.

Although direct measurement of the hysteresis loss factor for Perspex, subjected to the rolling of a hard ball, cannot be made directly, the author derived values
of a from measurements of the rolling friction force. In Chapter 6 experimental measurements of the rolling friction force for a hard ball rolling on Perspex are described, over a range of load and speed conditions. The hysteresis loss factor for Perspex has been derived from these measurements, and its relationship with frequency and temperature is shown for a limited range. When deriving the loss factor, the values taken for Young's Modulus and Poisson's ratio for Perspex were those given in Appendix 1. A simple experiment was devised which established that no large variations of Poisson's ratio occurred in the temperature range of 50°C to 100°C. To simplify the computation of sub-surface temperatures in Chapter 5 the hysteresis loss factor was assumed to have a constant value of 0.04. The values of a derived in Chapter 6 show that this is clearly not the case and that at elevated temperatures a significant increase in the loss factor can be expected. The effect of this variation upon the failure mechanism of Perspex will be discussed in Section 8.3.

In Chapter 5 an alternative approach to the boundary conditions for heat transfer from a rotating disk was presented in terms of $\theta_o$, the surface temperature, $\theta_A$, the temperature of the surrounding 'atmosphere' and $h$, the heat transfer coefficient at the surface. This latter parameter is very difficult to determine with any accuracy from heat transfer data, for the case of a rotating disk. In Chapter 7 the results of an experimental technique for measuring
mass transfer coefficients are given. Mass transfer rates from the surfaces of rotating disks have been measured, and from them heat transfer coefficients can be estimated by invoking the Chilton-Colburn relationships. Clearly, knowledge of the heat transfer coefficient is a considerable aid when faced with problems of a heat balance at the surface of a rotating disk or cylinder. For instance, the heat flux $Q$ at the surface of a disk may be written as (Equation 5.2.12):

$$Q = h(\theta_o - \theta_A)$$

(8.1.1)

In the experiments described in this work, where Perspex disks have been loaded together, measurement of $\theta_o$ and $\theta_A$ has proved very difficult. In the case of steel disks or cylinders, however, where trailing lead thermocouples can give a meaningful reading, the value of the heat transfer coefficient should prove a useful tool in the analysis of a thermal balance between the oil film and disk surfaces.

8.2 Sub-surface temperatures. Comparison of theory and experiment.

The sub-surface temperatures in rotating Perspex disks loaded together in the two-disk machine have been measured using embedded thermocouples with an axial orientation. The experiments were described in Chapter 3 and typical results, shown in Figure 3.8, have been plotted in terms
TEMPERATURES DERIVED FROM EXPERIMENTAL MEASUREMENT

- $41.5 \, \text{Kg/cm}$
- $56.5 \, \text{Kg/cm}$
- $72.0 \, \text{Kg/cm}$
- $87.0 \, \text{Kg/cm}$
- $94.0 \, \text{Kg/cm}$

**Fig. 8.1** $\Delta \theta^\circ \, \text{C}$ VERSUS $\frac{y}{b}$ : SPEED 13.1 rev/sec

THEORY — Solid Lines
of $\theta^\circ C$, the sub-surface temperature, against depth $y$(cm).

From the theoretical analysis of hysteresis loss given in Chapter 4, predictions of the sub-surface temperatures for Perspex disks have been made in Chapter 5. In Figures 5.3 to 5.7 the difference between the sub-surface temperature $\theta$ and the surface temperature $\theta_0$ have been plotted against non-dimensional depth $y/b$. Since insufficient data is available to enable the heat balance of Equation (5.2.12) to be solved, an absolute value for the sub-surface temperature cannot be predicted; only the temperature difference $(\theta - \theta_0)$ can be found. Consequently, to make a comparison between the theoretical predictions and the experimental results, the latter will have to be replotted in terms of $(\theta - \theta_0)$. Examination of Figure 3.8 shows that by drawing the best line between the recorded temperatures, and extrapolating the curve to the line $y = 0$, then values of $(\theta - \theta_0)$ can be found.

Figure 8.1 shows such a comparison between theory and experiment. The points show values of $(\theta - \theta_0)$, derived from the experimental measurements of Figure 3.8, as explained above, for loads of 41.5, 56.5, 72, 87 and 94 Kg/cm. The lines show theoretical values of $(\theta - \theta_0)$ derived for these same loads. As in the theory of Chapter 5, the temperature distributions are plotted as a function of the non-dimensional depth $y/b$. It is clear that there is good general agreement between theory and experiment; the form
of the temperature distributions are similar and the measured sub-surface temperatures, at a given speed, increase, very much as predicted by the theory, with increasing load.

In the theory of Chapter 5 and in the theoretical distributions plotted in Figure 8.1 a constant value of the hysteresis loss factor $\alpha$ of 0.04 has been used. It is clear that rather better agreement between theory and experiment could be obtained in Figure 8.1 by using a slightly higher value of $\alpha$ and, indeed, a higher value may well be justified if one considers the variation of $\alpha$ with temperature shown in Figure 6.10b. Moreover, it should be emphasised that, against our present background of knowledge, there arise a number of uncertainties in the derived values of $\alpha$. These uncertainties include the validity of assuming that hysteresis losses are associated with the hydrostatic components of stress and the absence of any complete theory for hysteresis losses for a rolling ball. It will be recalled that in Section 6.1 the differences between the "total stored energy approach" and the "complex stress cycle approach" were compared. For a rolling cylinder a numerical factor of 3.5 is introduced in this way and this seems well justified by the work of Greenwood, Minshall and Tabor. In the present work values of $\alpha$ have been derived from experiments with rolling balls and for this situation a numerical factor of 2.5 is introduced into
the theory of rolling friction; this is much less well established. Clearly, to resolve these issues one requires a more rigorous theory for a rolling ball. Alternatively, measurements of friction for a rolling cylinder, free from the problems discussed in Section 6.1, are needed to establish more exact values of the hysteresis loss factor.

Despite these difficulties, and in addition those associated with the accurate measurement of the sub-surface temperatures, the general agreement between theory and experiment must be regarded as satisfactory. It is clear that, in the rolling of materials of low thermal conductivity, relatively high temperatures can be generated below the surface even when the hysteresis loss factor, and its associated coefficient of rolling friction, is quite low. The theory of the sub-surface temperature distribution, which has been derived in Chapter 5 from the work of Greenwood, Minshall and Tabor, seems to provide an adequate method of estimating temperatures which are generated in this way.

8.3 The Mechanism of Failure

The generation of heat beneath the surface and the increase in sub-surface temperatures has been considered in some detail. It is now appropriate to examine the part played by thermal effects in the failure mechanism. The increasing sub-surface temperatures will have an effect
upon the mechanical properties of Perspex. For instance
the variation of Young's Modulus with temperature is shown
in Figure A.1. In Chapter 6 the variation of loss factor
with temperature was examined for a range of loads and
frequencies and the results plotted in Figures 6.9 and
6.10. The theoretical sub-surface temperatures were
predicted in Chapter 5 assuming that the loss factor had
a constant value of 0.04 and showed no variation with
frequency and temperature. The experiments described in
Chapter 6 showed that such an assumption was only valid
for temperatures between 20°C and 60°C. At higher
temperatures the loss factor increases rapidly for small
increases of temperature. Increases of loss factor will
in turn cause more heat to be generated, which will raise
the sub-surface temperature and cause the loss factor to
increase further.

This influence of temperature dependent hysteresis
loss could well be a significant factor in the mechanism
of failure. Consider a typical failure experiment in
which the load was increased in small steps, some time
being allowed to elapse between each incremental increase
in load. Each increment in load would cause an increase
in the sub-surface temperatures, some time being required,
at each stage, to establish the equilibrium temperature
distribution. At a certain load the sub-surface temper-
atures will be such that the values of $a$ at certain depths
will increase rapidly with temperature. A small increment
of load could then cause cumulative and catastrophic increases in temperature which would persist until failure occurred. This could well explain the sudden and catastrophic nature of the failure as originally observed and would also explain why one region of a specimen showed this form of failure, whilst other parts of the same specimen and the opposing specimens showed no indications of damage.

A rigorous treatment of this cumulative mechanism of failure is fairly complex since, at any stage, the temperature and $\alpha$ will be inter-related and both will be dependent on the depth below the surface. However, the most important features involved can be illustrated by a simplified theory which will be given below. The relationship between the sub-surface temperature $\theta$ and the load $W$ was given in Chapter 5, Equation (5.4.1) as

$$(\theta - \theta_0) \text{ proportional to } -b^2 p_o \beta C$$

which can be written as

$$\Delta \theta = C \bar{\alpha} W^\gamma$$

(8.3.1)

where $\Delta \theta = (\theta - \theta_0)$ is the difference between the sub-surface temperature and that at the surface and $C$ is a function of the depth and the experimental conditions. In this equation the loss factor has been denoted as $\bar{\alpha}$ to indicate that a constant value, independent of temperature as in the theory of Chapter 5, is assumed. Since $\bar{\alpha}$ is independent of temperature we may now write

$$\frac{\partial (\Delta \theta)}{\partial W} = 2C \bar{\alpha} W$$

(8.3.2)
Fig. 8.2 THE EFFECT OF LOSS FACTOR UPON TEMPERATURE FOR BOTH A MEAN VALUE OF LOSS FACTOR (broken line) AND LOSS FACTOR AS A FUNCTION OF TEMPERATURE (solid line.)
The relationship between $\bar{a}$ and $(\Delta \theta)$ is shown by the broken line in Figure 8.2a, and that between $\partial (\Delta \theta)$ and $W$ is shown by the broken line in Figure 8.2b. The broken line in Figure 8.2c shows the relationship between $(\Delta \theta)$ and $W$, assuming a constant value of loss factor, $\bar{a}$.

We can now show the effect of the variation of $\alpha$ with temperature. The solid line in Figure 8.2a shows the relationship between $\alpha$ and temperature indicated by the experiments described in Chapter 6. The value of $\alpha$ varies with temperature and, consequently, at different depths beneath the surface there will be different loss factors.

To simplify the problem, we shall consider the sub-surface temperature $\theta^*$ at a depth $y^*$ beneath the surface and a corresponding value of $\Delta \theta^*$. We now write,

$$ (\Delta \theta^*) = C W^2 $$ \hspace{1cm} (8.3.3)

where $C$ is once again a constant. However, since $\alpha$ is a function of temperature,

$$ \frac{\partial (\Delta \theta^*)}{\partial W} = C \frac{\partial \alpha}{\partial W} W^2 + 2C \alpha W $$

$$ = C \frac{\partial \alpha}{\partial (\Delta \theta^*)} \frac{\partial (\Delta \theta^*)}{\partial W} W^2 + 2C \alpha W $$

and using equation (8.3.3)

$$ \frac{\partial (\Delta \theta^*)}{\partial W} = \frac{\Delta \theta^*}{\alpha} \frac{\partial \alpha}{\partial (\Delta \theta^*)} \frac{\partial (\Delta \theta^*)}{\partial W} + 2 \frac{\Delta \theta^*}{W} $$

Thus

$$ \frac{\partial (\Delta \theta^*)}{\partial W} \left( 1 - \frac{\Delta \theta^*}{\alpha} \frac{\partial \alpha}{\partial (\Delta \theta^*)} \right) = 2 \frac{(\Delta \theta^*)}{W} \hspace{1cm} (8.3.4) $$

we see that $\frac{\partial (\Delta \theta^*)}{\partial W} \to \infty$ (and $\Delta \theta^* \to \infty$) as

$$ \frac{\partial \alpha}{\partial (\Delta \theta^*)} \to \frac{\alpha}{\Delta \theta^*} $$
The effect of this will be that as the gradient of the solid curve of Figure 8.2a tends towards \( \frac{\alpha}{(\Delta \theta)} \), the value of \( \frac{\partial(\Delta \theta)}{\partial W} \) will increase, as shown by the solid curve in Figure 8.2b. The effect on the relationship between \((\Delta \theta)\) and \(W\), shown in Figure 8.2c, (solid line), will be similar to that outlined above. As the sub-surface temperature approaches a given critical value small increments in the load will cause larger and larger increases in the sub-surface temperature. The simple analysis given above suggests that the critical value of sub-surface temperature, beyond which catastrophic and cumulative rises in temperature occur, is related to the value of \( \frac{\partial \alpha}{\partial \theta} \).

We can now summarise the role which these factors play in the failure mechanisms for Perspex. With increases of load on the disks the sub-surface temperatures will at first increase slowly, causing small increases in the loss factor. These small increases in loss factor and progressive increases of load will build up the sub-surface temperatures. Eventually the point will be reached where small increases of temperature cause larger increases of loss factor. These large increases of \( \alpha \) cause the temperature to rise more rapidly, until a point is reached where increases in load are no longer needed to cause increases of sub-surface temperatures. Eventually these cumulative increases cause the temperature to reach the softening point of Perspex and mechanical failure will occur.
This approach to the failure mechanism explains the difficulties experienced in the original experiments. In the preliminary two-disk machine experiments it was found that the disks would run for extended periods under certain loads with no surface damage occurring. However, a small increase in the load led to surface failure occurring after a few hundred more passages of the contact. This becomes clearer in the light of the preceding discussion. Steady state conditions had apparently been reached within the disk, the heat generated within the disk being balanced by the heat loss from the disk. Only when the additional load had been applied did the cumulative heat generation effect start to increase the sub-surface temperatures rapidly. A similar explanation can be applied to the crossed cylinders experiments where it was found that repeated traversals of the contact region were required to initiate failure.

The earlier observations about the effect of lubricant supply to the disks having an apparent effect on the failure can also be explained in this context. Clearly, by increasing the supply of lubricant to the surfaces the heat balance within the disk is affected. The increased supply of lubricant to the surfaces, has the effect of a heat sink on the surface, thus affecting the heat balance within the disk and reducing the sub-surface temperatures. Consequently higher loads were required to initiate failure.

Relating the preceding discussion to the experiments
described in Chapters 2 and 3, an outline of the physical characteristics of the failure mechanism can now be seen, as follows:—

1. The load is applied and the region of contact is approximately Hertzian.
2. The sub-surface temperatures of the specimens increase. (This is the case for both point and line contact experiments, the latter under conditions of pure rolling). The sub-surface temperature is higher than the surface temperature; both theoretical predictions and experimental measurements indicate this.
3. If the load has been increased in gradual steps, with a break between each application to allow thermal equilibrium to be reached, a critical load is reached where a further small increase causes the sub-surface temperatures to increase rapidly and, instead of reaching a higher equilibrium temperature, to continue to increase until the softening point of Perspex is reached.
4. The material now becomes weak in those regions which normally bear the greatest elastic stresses.
5. Cracking now occurs along the surface of the disk or cylinder.
6. A lump is torn out of the surface to a depth approximately equivalent to that of the maximum shear stress.
8.4 The failure of high hysteresis loss materials

The implications of the present work are not only confined to Perspex. Thermal effects due to hysteresis losses are present in a range of applications of high loss materials where the predominant motion is rolling rather than sliding. For instance, in the design and choice of material for automobile tyres the build-up of heat due to hysteresis loss has to be considered, and the present work has a possible application in this field. For example, when operating at high speeds tyres can show a form of failure called 'shelling', in which a layer of material is removed from their surface. It seems likely that the work described in this thesis is relevant to this form of failure. (Grosch, British Natural Rubber Producers Research Association—private communication).

Hysteresis losses may well play a significant part in the application of plastics to roller bearings, but in other situations, such as journal bearings, where the movement between load bearing surfaces is largely sliding, rather than rolling, the effects of surface heating by sliding contact between surfaces will predominate.

In gears, where rolling and sliding occur, the applicability of the work described in this thesis is less clear. The work of Buckingham (1943), outlined in Section 2.8, was motivated by an interest in surface fatigue in gears and showed features similar to those described in our own failure experiments. Housz (1967)
has examined the failure of Nylon gears and found that a form of scuffing failure occurs, caused by softening of the surfaces of the gears under high loads.

The author has also found (Archard and Wannop, 1967), in a series of unlubricated pin and ring experiments using Nylon, that although low coefficients of friction occur at low loads, at higher loads the coefficient of friction increases rapidly, caused by thermal breakdown of the surface. The failure conditions of such materials, when used in bearings and gears, will be affected by the rate of lubricant supply. There is an important contrast here between thermal effects in the lubrication of metals and polymers. In some earlier work on elastohydrodynamic lubrication of metals, the relatively small influence of the temperature and rate of lubricant supply was emphasised because of the poor efficiency of the lubricant as a method of removing heat from the surfaces. Although polymers will often show a low coefficient of friction (particularly under ehl conditions) it is more difficult to remove the generated heat because of their poor thermal conductivity. Therefore with this class of material the rate of lubricant supply and its role as a heat transfer medium becomes of much greater significance.

8.5 Suggestions for further work and Conclusions

The greatest difficulties experienced in the present work were those described in devising satisfactory methods
for the measurement of sub-surface temperatures. The final methods adopted were fairly satisfactory and Figure 8.1 supports this view. Nevertheless, in future work, it might be desirable to improve the accuracy, and to reduce the disturbance to the flow of heat, by using thermocouple wires of smaller diameter. Similarly a better approximation to the assumed linear heat conduction would be obtained by the use of cylinders, rather than disks, in line contact.

There also is a need for development of the theory discussed in Chapter 4. The quasi-elastic theory of Greenwood, Minshall and Tabor for line contact needs to be compared, in detail, with the results of visco-elastic theories. At the same time the quasi-elastic approach could be extended to the case of a ball rolling on a flat surface.

The application in Chapter 5 of hysteresis loss theory to the generation of sub-surface temperatures was limited to the case of a constant value of the hysteresis loss factor. The discussion of Section 8.3 suggests a need for a more rigorous analysis including a hysteresis loss factor dependent upon temperature.

In Chapter 6 the need for further experimental measurements of the rolling friction was discussed. In the author's laboratory some work has been performed with a four-disk machine using a central disk of Perspex, (Wolveridge 1970). Work is also in hand to extend the
range of conditions for measurements of mass transfer from rotating disks and cylinders.

The work described in this thesis took as its starting point a particular form of failure of lubricated Perspex cylinders under heavily loaded conditions. Further investigations of this failure revealed that nothing seemed to be known about the thermal consequences of hysteresis losses in rolling contact. Therefore two major objects of the work have been the development of an appropriate theory, which would forecast the sub-surface temperatures, and the devising of suitable experiments to measure the temperatures. The relevance of these theories and experiments to the original observations of failure has been demonstrated. Improvements both to the theories and to the experiments will, no doubt, be made in the future. However, it is thought that the work described in this thesis is the first serious attempt to understand the magnitudes of the temperatures generated in rolling contacts by hysteresis losses, and to assess their significance for the successful operation of rolling systems using polymers.
Fig. A1. COMPARISON OF COMPRESSIVE AND TENSILE MODULI OF PERSPEX OVER A RANGE OF TEMPERATURE
Appendix 1

The Physical Properties of Perspex
(derived from publications of ICI Ltd.,
Plastics Division Welwyn Garden City)

Thermal Properties

Heat Distortion Temperature 100°C.
Specific Heat 0.35 cal/g °C.
Coefficient of Thermal Conductivity at 20°C. 4.5 x 10^{-4} cal cm/cm^2 °C sec.
Coefficient of Thermal Expansion at 20°C. 2.2 x 10^{-4} cu cm/cu cm °C.
Density 1.2 g/cm^3

Mechanical Properties

Property Mean Value
Tensile strength 840 Kg/cm^2
(straining rate: 1% per sec)
Young's Modulus in Flexure 3.0 x 10^4 Kg/cm^2
Shear Modulus 1.13 x 10^4 Kg/cm^2

(Note moduli in flexure tension and compression are substantially the same) Fig. (A1) shows values of compressive and tensile moduli over a range of temperature.

POISSON'S RATIO FROM - 25°C. to + 50°C. 0.35

The value remains essentially constant for this range of temperature. At temperatures above 100°C. where large deformations become possible, Poisson's Ratio approaches 0.5.
Appendix 2

Glossary of notation and relationships concerned with heat and mass transfer used in Chapter 7

Nomenclature

$x$ is the mole fraction of Mercury vapour in air:

- $x_a$: upstream
- $x_0$: at surface
- $x_1$: downstream

S: Cross sectional area of working section

$\delta A$: Area of amalgamated surface

$C_p$: Specific heat

D: Diameter

R: Radius

$D_s$: Diffusion coefficient

$h$: Heat transfer coefficient

$k_x$: Mass transfer coefficient

$k_{x_A}^{\text{loc}}$: Local mass transfer coefficient for species A

$\theta$: Temperature

V: Velocity

W: Molar flow rate

$c_T$: Total molar concentration

$\nu$: Kinematic viscosity

$\rho$: Fluid density

$\mu$: Viscosity

K: Thermal conductivity

A,B: Subscript for species

2,1,0: Subscripts for upstream, downstream and surface
Dimensionless Groups

Heat Transfer Quantities

Reynolds \( N^o \) \( \text{Re} = \frac{VD\rho}{\mu} \)

Nusselt \( N^o \) \( \text{Nu} = \frac{hD}{k} \)

Prandtl \( N^o \) \( \frac{C_p\mu}{k} \)

Stanton \( N^o \) \( \text{St} = \frac{\text{Nu}}{\text{PrRe}} = \frac{h}{\rho C_p V} \)

Chilton Colburn factor

\( j_H = \frac{\text{Nu}}{\text{Pr}^{\frac{1}{3}} \text{Re}} \)

Binary Mass Transfer Quantities

Reynolds \( \text{Re} = \frac{VD\rho}{\mu} \)

Sh \( = \frac{k_D}{c_T D^*} \)

Sc \( = \frac{\mu}{\rho D^*} \)

Stanton AB \( = \frac{Sh}{\text{ReSc}} = \frac{k_x}{3 c_T V} \)

\( j_D = \frac{Sh}{\text{ReSc}^{\frac{1}{3}}} \)
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