AERODYNAMICS OF PARACHUTES AND LIKE BODIES IN UNSTEADY MOTION.

by

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A Thesis submitted in partial fulfilment of the requirements for the degree of DOCTOR OF PHILOSOPHY in Mechanical Engineering.

University of Leicester May 1982.
Dedicated to my Parents and family.
ABSTRACT

As a parachute descends, its axis of symmetry oscillates about a vertical axis. This oscillation implies angular acceleration, with consequential linear acceleration developing. The significance of the acceleration terms in evaluating parachute performance has been long appreciated and commonly allowed for in dynamic stability analysis through introduction of appropriate apparent mass and apparent moment and inertia terms. This research, while dealing specifically with the aerodynamic performance of the parachute, considers the experimental technique to measure the total fluid resistance and apparent mass components of the parachute canopy which are related to the behaviour of any bluff body moving unsteadily through a fluid.

Total fluid resistance and apparent mass components were evaluated by measuring forces and moments with strain gauges during the relative motion of parachute models submerged in water in a ship tank. While being towed by the motion of the carriage, a slider crank mechanism caused the sting-mounted canopies to be harmonically oscillated at a low frequency along any required line which was parallel to the tank bed. Results show that the apparent mass components depend on the shape of the canopy, its angle of attack and the acceleration modulus (the product of the acceleration of the canopy and its diameter divided by velocity squared) and, except at high values of the latter, can be considerably in excess of potential flow evaluations.

A set of differential equations which describe the three-dimensional motion of the parachute canopy-store system during descent were developed. The non-linear equation of motions were solved numerically. The effect on dynamic stability due to the variation of system parameters was studied and appropriate stability criteria were developed. Results show that the resultant dynamic performance is highly sensitive to the chosen values of the apparent mass components.
ACKNOWLEDGEMENTS

First and foremost the author would like to thank his supervisor, Dr. D.J. Cockrell, for the guidance, help and encouragement throughout all stages of this research.

The author wishes to thank all members of the Engineering Department - technical and workshop staff, for their help and tolerance over the past three years, in particular, Mr. P. Williams and Mr. S. Brown for devising and building the experimental apparatus and assisting with the tests, Mr. C. Harris and Mr. R. Gays for making the model canopies and Mr. C.J. Morrison for advice and assistance on strain gauge technology.

The writer would like to thank Mr. D.S. Jorgensen for the many hours he has spent in helping to conduct the tests and for the helpful discussion during the cause of the research, Mr. J.A. Eaton for his valuable suggestions and criticism and Mr. R.D. Dennis and Dr. W.G.S. Lester of the Royal Aircraft Establishment, Farnborough for the interest they have shown in this research and encouragement with the tests.

The author is very deeply grateful to the Ministry of Defence for the financial support, and the Head of the Marine Technology and Naval Architecture Department, Southampton College of Higher Education for permission to use the ship tank and for assistance with the tests.

The author wishes to express his thanks to the Turkish Government for providing facilities to study abroad and the University Research Board for financial support to attend the AIAA Conference in USA.
The writer gratefully acknowledges the help and provision of facilities by Professor G.D.S. MacLellan, the Head of the Engineering Department.

Finally, the author wishes to express his appreciation to Mrs. A. Bilkoo for her excellent typing and Mr. S. Karadeniz and Mr. M. Ozpolat for their help in preparing this thesis.
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LIST OF SYMBOLS

\( A_s \) Canopy surface area, \( A_s = \frac{\pi D^2}{2} \) for hemisphere (m²)
\( = 2\pi r^2 \) for cruciform (m²)

\( A \) Cross-sectional area, \( A = \frac{1}{4} \pi D^2 \) (m²)

\( O \) Origin

\( C \) Centre of volume of canopy

\( R_o \) Centre of reaction of canopy

\( G \) Gravity centre of system

\((x, y, z)\) Body axes with origin

\((x_1, y_1, z_1)\) Earth fixed axes

\((x_a, y_a, z_a)\) Coordinates of centre of reaction, \( R_o \).

\((x_s, y_s, z_s)\) Coordinates of centre of gravity, \( G \).

\( F \) Force

\( \tau \) Torque

\( M_x, M_z \) Bending moments (in Fig. 4.7)

\((X, Y, Z)\) External forces along \((x, y, z)\) axes.

\((L, M, N)\) External moments about \((x, y, z)\) axes.

\((U, W, V)\) Velocity components along \((x, y, z)\) axes.

\((P, Q, R)\) Angular velocity components about \((x, y, z)\) axes.

\((\psi, \theta, \phi)\) Euler angles between Earth and body axes.

\( m \) Total body mass (kg)

\( m' \) Included mass or reference mass of fluid displaced by body (kg).

\( k' \) Apparent moment of inertia coefficient.

\( m_s, M_S \) Store mass (kg)

\( m_c, M_C \) Body mass of parachute canopy (kg).

\( \rho \) Density (kg m⁻³)

\( \nu \) Kinematic viscosity (m² s⁻¹)

\( g \) Acceleration due to gravity (9.81) (m s⁻²)
Pressure

\[ V_c, q \]

Resultant velocity, \( V_c = \sqrt{U^2 + W^2} \) (m s\(^{-1}\))

\[ V_0 \]

Initial or equilibrium velocity (m s\(^{-1}\))

\[ V \]

Linear acceleration (m s\(^{-2}\))

\[ \omega \]

Angular velocity unless stated (s\(^{-1}\))

\[ \dot{\omega} \]

Angular acceleration (s\(^{-2}\))

\[ D \]

Inflated diameter of canopy (m), Drag.

\[ l' \]

Arm length of cruciform canopy (m)

\[ r \]

Arm width of cruciform canopy (m)

\[ \delta \]

Acceleration modulus, \( \delta = \frac{\dot{V}D}{V^2} \)

\[ R \]

Total hydrodynamic force, Equation (4.3)

\[ C_D \]

Drag coefficient, \( C_D = \frac{D}{\frac{1}{2} \rho V_c^2 A_s} \)

\[ C_R \]

Total fluid resistance coefficient, \( C_R = \frac{R}{\frac{1}{2} \rho V_c^2 A_s} \)

\[ C_T \]

Tangential force coefficient, \( C_T = \frac{Z}{\frac{1}{2} \rho V_c^2 A_s} \)

\[ C_N \]

Normal force coefficient, \( C_N = \frac{X}{\frac{1}{2} \rho V_c^2 A_s} \)

\[ C_M \]

Pitching moment coefficient, \( C_M = \frac{\tau}{\frac{1}{2} \rho V_c^2 A_s D} \) (\( D = l' \) for cruciform canopies)

\[ C_{N_a} \]

Normal force coefficient slope, \( C_{N_a} = \frac{\partial C_N}{\partial a} \)

\[ \sigma_o \]

Stress tensor

\[ R_{\alpha ij} \]

Total fluid resistance tensor

\[ t_{\alpha ij} \]

Translation tensor

\[ r_{\alpha ij} \]

Rotation tensor

\[ c_{\alpha ij} \]

Coupling tensor

\[ a_{ij}, a^{A}_{ij} \]

Apparent mass tensor

\[ V_{vol} \]

Reference volume of fluid displaced by body,
\[ V_{vol} = \frac{1}{12} \pi D^3 \] (m\(^3\))

\[ I_h \]

Reference moment of inertia of fluid displaced by body,
\[ I_h = \frac{1}{16} \pi D^2 \rho V_{vol} \] (kgm\(^2\))

\[ I \]

Moment of inertia of body (Subscripts denote
relevant axes) \( (\text{kgm}^2) \)

\[ k_{ij} \] Apparent mass coefficients

\((k_{11}, k_{22}, k_{33})\) Apparent mass coefficient along \((x,y,z)\) axes

\((k_{w}, k_{55}, k_{66})\) Apparent moment of inertia coefficients about \((x,y,z)\) axes.

\( \theta_0 \) Initial pitch angle \((\text{rad., degree})\)

\( \alpha \) Angle of attack measured from symmetry axis of canopy \((\text{rad., deg.})\)

\( \alpha_E \) Equilibrium angle of attack

\( \beta \) Angle between velocity vector and vertical earth axis \((\text{rad.})\)

\( \gamma \) Angle between the direction of acceleration and the \( z \)-axis \((\text{degree})\)

\( t \) Time \((\text{sec})\)

\( t^* \) Non-dimensional time

\( h \) Altitude, Depth \((\text{m})\)

\( \xi \) Suspension line length measured from canopy apex \((\text{m})\)

\( \zeta \) Damping coefficient

\( \omega_n \) Frequency of oscillation

\( \omega_v \) Vortex shedding frequency

\( \omega_t \) Test frequency

\( T \) Period of oscillation \((\text{sec})\), Kinetic energy.

\( \lambda_{1,3} \) Roots of the characteristics equation \((5.35)\)

\( \psi \) Velocity potential function

\( \psi \) Stream function

\( \xi, \eta \) Elliptic coordinates

\( \dot{G} \) Linear momentum

\( H \) Angular momentum

\( f \) Maximum deflection \((\text{mm})\)

\( \theta \) Maximum deflection angle \((\text{deg., rad})\)

\( \text{Re} \) Reynolds number based on the circumferential length, \( zD \), unless stated.
\[ z \quad 3.14 \]

\[ E \quad \text{Young's modulus (N/mm}^2) \]

\[ G \quad \text{Shear modulus (N/mm}^2) \]

\[ \mu \quad \text{Dynamic viscosity (kgm}^{-1}s^{-1}) \]

**Subscripts**

\[ s \quad \text{Store} \]

\[ c \quad \text{Canopy} \]

\[ S \quad \text{Surface} \]

\[ B \quad \text{Body} \]

\[ A \quad \text{Fluid or inertia} \]

**Superscripts**

\[ \times \quad \text{Transpose} \]

\[ \cdot \quad \frac{d}{dt} \]

\[ \dddot{ \quad } \quad \frac{d^2}{dt^2} \]

\[ \frac{dy}{dt} \quad \text{Partial differentiation} \]

\[ \times \quad \text{Vectorel multiplication} \]

\[ \times \quad \text{Non-dimensional unit} \]
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INTRODUCTION

The stability and control characteristics of submerged bodies, such as submarines, balloons, airships and parachutes moving unsteadily through a fluid depend upon a number of parameters. In particular, forces and moments which act on a body in unsteady motion are different from those if the motion were steady. These differences can be related to the apparent mass concept.

The dynamic stability characteristics of parachutes are of great importance in many respects. Advances in Aeronautical Engineering have led to a growing number of aerodynamic decelerator applications, such as for aircraft landing, pilot or capsule escape purposes, atmospheric re-entry, vehicle stabilization, cargo delivery and airborne troop manoeuvres. These all depend on the parachute, possessing good dynamic stability characteristics.

The dynamic characteristics depend upon a number of factors which include both the steady-state aerodynamic characteristics and its apparent masses and inertias, together with a number of geometrical parameters. Extensive aerodynamic data exists for steady-state conditions relating to various parachute canopies. These data expressed as function of the angle of attack and porosity, are well tabulated in the relevant literature.

As the parachute descends, its axis of symmetry oscillates about a vertical axis. This oscillation implies an angular acceleration, with consequential linear accelerations developing. The significance of the acceleration terms in evaluating the performance of parachutes and of other bodies immersed in fluids has been appreciated and commonly allowed
For in dynamic stability analysis through introduction of appropriate apparent mass and apparent moment of inertia terms.

If the oscillatory motion of a body such as a parachute were assumed to occur in an ideal fluid, these apparent mass and moment of inertia terms are calculable. However, in a real fluid, where flow separation occurs from shapes like parachute bodies, these terms can only be determined through suitable experiments. Components which are primarily due to viscous flow are not obtained reliably with existing theory. Though it has been the current practice to assure that a good first approximation to them is obtained by using the ideal fluid calculated values, experimental work by Frazer and Simmons\(^1\), Cowley and Levy\(^2\), Iversen and Balent\(^3\) and by Luneau\(^4\) among others have shown this to be an unreliable procedure.

In this present research, therefore, experimental techniques have been adopted to determine these apparent mass and apparent moment of inertia components. Total fluid resistance in unsteady motion has been evaluated by measuring forces and moments developed on parachute models which were submerged in water in a ship tank and which were undergoing accelerated motion. Results obtained have been compared with theoretical values relevant to ideal flow.

The three-dimensional equations of motion of a parachute rigidly-connected to its load have been derived. These non-linear equations are solved numerically. In this way, the dynamic performance of descending parachutes has been determined.
This thesis is divided into three main parts, which are:

(i) the apparent mass concept, together with an account of both theoretical and experimental developments which have to be made to evaluate apparent masses;

(ii) the definition and the experimental determination of the apparent mass components relevant to parachute behaviour;

(iii) the derivation of the relevant equations of motion and their solution to determine the dynamic performance of a descending fully-deployed parachute.
CHAPTER 1.

THE CONCEPT AND LITERATURE SURVEY CONCERNING APPARENT MASS. THE RESEARCH PROGRAMME OBJECTIVES.
1. THE CONCEPT AND LITERATURE SURVEY CONCERNING APPARENT MASS.

THE RESEARCH PROGRAMME OBJECTIVES.

1.1. Introduction

This chapter aims to develop the apparent mass concept and provide a historical perspective for the present study. The presentation consists of four main parts; the first part considers the idea of apparent mass, the second, the theoretical studies which outline the understanding of apparent mass effects in general fluid mechanics, the third part covers some of the important experimental studies and the fourth summarizes studies which deal specifically with the parachute problem. The chapter concludes with the Research Programme Objectives.

1.2. The Concept of Apparent Mass

If a body is rigidly fixed and fluid moves steadily relative to it, aerodynamic forces and moments are developed on that body. Let us consider a rigid body moving in a straight line and without rotation in an ideal fluid which is unbounded and at rest, except in as far as it is disturbed by the motion of the body. The velocity of the body is \( V \) at the instant considered. Then the velocity of the fluid at any point \( C \) which occupies a fixed position relative to the body is proportional to \( V \) and if \( u_i, v_i \) and \( w_i \) are the components of the velocity of the fluid at \( C \), we can write

\[
\begin{align*}
    u &= u_i V & (1.1a) \\
    v &= v_i V & (1.1b) \\
    w &= w_i V & (1.1c)
\end{align*}
\]

where \( u_i, v_i \), and \( w_i \) are the velocity of \( u, v \) and \( w \) respectively when \( V = 1 \) and are functions only
of the coordinates of \( \mathbf{C} \) relative to the body.

The total kinetic energy of the system comprising both body and fluid is

\[
T = T_1 + T_2
\]

(1.2)
i.e.

\[
T = \frac{1}{2} m V^2 + \frac{1}{2} \rho \int q^2 \, dv
\]

(1.3)

where \( q = (u^2 + v^2 + w^2)^{\frac{1}{2}} \) is the velocity of the fluid at the elemental volume \( \delta v \), \( \rho \) is the density of the fluid and \( m \) is the mass of the body.

Thus

\[
T = \frac{1}{2} m V^2 + \frac{1}{2} \rho \int (u_i^2 + v_i^2 + w_i^2) \, dv
\]

(1.4)

If we define \( k \cdot V_f = \iiint (u_i^2 + v_i^2 + w_i^2) \, dx \, dy \, dz \)

where \( V_f \) is the volume of the fluid displaced by the body and the integral covers the whole region occupied by the fluid, then

\[
T = \frac{1}{2} m V^2 + \frac{1}{2} \rho V^2 \ k \cdot V_f
\]

(1.5)

Hence,

\[
T = \frac{1}{2} V^2 (m + k \cdot m')
\]

(1.6)

where \( m' \) (which equals \( \rho \cdot V_f \)) is the mass of the fluid displaced by the body and \( k \) is termed the apparent mass coefficient.

The quantity \( km' \) is known as the apparent mass of the fluid and \( m + km' \) as the virtual mass of the total system, body plus fluid.

Now suppose the rectilinear velocity of the body were
variable. The kinetic energy of the fluid will vary with time and thus, by the principle of conservation of energy, work must be done upon the fluid. Let \( X \) denote the component in the direction of motion of the resultant force which the body exerts on the fluid, so that \( -X \) is the corresponding component of the force which the fluid exerts on the body. Consider the work done on a body which moves with a velocity \( V(t) \) through a distance \( \delta x \)

\[
\delta T = X \delta x = X.V \delta t = k.m' . V . \frac{dV}{dt} \delta t \quad (1.7)
\]

Hence, the total force equation becomes

\[
F = m \frac{dV}{dt} + k.m' . \frac{dV}{dt} \quad (1.8)
\]

\[
F = \frac{dV}{dt} ( m + km') \quad (1.9)
\]

Consequently, when a body is to accelerate through a fluid, a force must be applied to accelerate not only the mass of the body but also the mass of the fluid set in motion by the body. Thus, the force due to the apparent mass, being the reaction between the body surface and the fluid, constitutes an additional resistance to the body motion in the fluid.

A real fluid is characterised by the presence of its viscosity. It is well-known that the viscosity plays an important role in fluid motion in that the real fluid offers shearing resistance to the motion of the fluid. The shearing resistance of a viscous fluid causes the velocity of flow to be non-uniform developing a boundary layer around the body and a wake behind the body. It is generally assumed that when a viscous fluid
flows past a solid boundary, there is no relative motion between the fluid particles immediately in contact with the solid boundary, i.e. there is no slip of fluid at the solid surface. This condition of no slip between the fluid and the adjacent solid surface is a unique characteristic of viscous fluid flows, that means, the real fluid has a fixed boundary with the solid surface and if a fluid has a fixed boundary with a body, the quantities \( u_1 \), \( v_1 \) and \( w_i \) defined above, will, in general, depend on the position of \( \mathbf{C} \) relative to the boundary.

Thus, the apparent mass coefficient \( (k) \) is not constant, as it is in an ideal fluid, but it varies with the displacement of the body, that is with time and there is therefore an additional term proportional to \( V^2 \) in the force equation developed above. This term would not be present if the fluid were ideal. Hence, the total force on a body accelerating through a viscous fluid becomes

\[
F = m \frac{dV}{dt} + m' \left( \frac{dV}{dt} \cdot k + \frac{1}{2} V^2 \frac{dk}{dx} \right) \tag{1.10}
\]

Similarly for angular motion of the body, effects produced by the presence of the fluid may be represented by an additional moment of inertia \( (I_A) \) associated with the apparent mass to be added to that of the rotating body \( (I_B) \). If the angular velocity of the body is \( \omega \), the total kinetic energy in rotation may be written as

\[
T = \frac{1}{2} (I_B + I_A) \omega^2 \tag{1.11}
\]
\[ I_A \text{ being defined as } I_A = k' \cdot I_f, \text{ where } k' \text{ is the rotational apparent mass coefficient and } I_f \text{ is the moment of inertia of the fluid displaced by the body.} \]

As with the translational apparent mass coefficient, \( k' \) is a constant if the fluid is ideal. However, in a real fluid it varies with displacement.

The moment (\( \tau \)), which must be applied to produce the angular acceleration of the body in an ideal fluid is given by

\[ \tau = I_B \ddot{\omega} + k' I_A \ddot{\omega} \quad (1.12) \]

It is evident that, when a body moves unsteadily through a fluid, it could possess a linear acceleration or it could rotate about one of its axes. In general, unsteady motion will consist of a combination of linear and angular velocity components each of which will vary directly with time or, less directly, through axis displacement which itself is a function of time. Consequently, there are up to 36 apparent mass components each of which is related to the respective body motion and these are discussed later in Chapters 2 and 3.

Forces and moments which arise from unsteady motion can be derived analytically if the fluid is considered ideal but must be determined experimentally if the fluid is real. However, for motion in a given direction through an ideal fluid the relevant apparent mass coefficient will be a constant, whereas if the fluid is real the coefficient will vary with the displacement.

It can be seen that the apparent mass concept is very important when considering the dynamic behaviour of a body which is in unsteady motion through a fluid. For example,
when aircraft or submarines move through water, the inertia of the air or water surrounding the moving body can be significant since work is done to sustain the motion. Similarly, in the dynamic design of tall buildings and bridges appreciable apparent mass effects can arise. If the resultant force required to sustain the motion of the body is $F$, then the rate at which $F$ does work is equal to the rate of increase of the total kinetic energy of the system, for if

$$ T = \frac{1}{2} V^2 (m + km') \quad (1.13) $$

$$ \frac{dT}{dt} = (m + km') V \frac{dV}{dt} = F V. \quad (1.14) $$

Now if a thrust $P$ is required to overcome a drag force $D$, then

$$ F = P - D \quad (1.15) $$

and the rate of work done by the thrust is given by

$$ PV = DV + (m + km') V \frac{dV}{dt}. \quad (1.16) $$

The apparent mass term, $km' \frac{dV}{dt}$ can therefore be considered to be an effective additional drag $D'$ whence

$$ PV = (D + D') V \quad (1.17) $$

1.3. Theoretical Studies

Since the beginning of the 20th century, consideration of apparent mass was embodied in the study of balloon and airship flight and in formulation of the equations of motion for the stability of ships and submarines, because apparent masses play a dominant role in determining the hull natural frequency and its damping.
The problem of calculating the apparent mass for a given body in arbitrary motion in an infinite conservative fluid is essentially that of solving the corresponding potential flow equations. Different approaches to its solution have been developed and most research has been based on ideal flow considerations.

In 1911, Fuhrmann applied and extended Rankine's method which showed how to obtain potential flow around families of bodies of revolution by placing several point sources and sinks of various strengths on the axis. Fuhrmann measured the pressure distributions on airship-like models and found good agreement between these measurements and those obtained by calculation, except for small regions at the downstream end of the bodies.

In 1918, Leathem showed how to determine the motion in two dimensions of an infinite fluid occupying the space outside a body bounded by a closed curve or polygon, due to prescribed motion of the boundary. He used periodic conformal transformations, whereby the doubly-connected space outside the boundary in the physical plane could be represented repeatedly upon successive semi-infinite strips of finite width. The solution was obtained for the case of translational motion.

In 1918, Bickley solved some significant two-dimensional problems connected with the circular arc. Instead of the periodic transformation advocated by Leathem, Bickley used the doubly-connected region of the physical z-plane in the upper half of the ξ-plane, the boundary in the z-plane becoming the real axis in the ξ-plane. Later, he
generalised his method of attack to a wide range of two-dimensional body shapes which have a single closed boundary.

In 1920, Riabouchinsky proposed a method for solving two-dimensional problems for bodies having a plane of symmetry. He applied it to calculate the apparent mass for rectangular shapes. He also obtained the apparent mass for the case where the flow separates from the sides parallel to the direction of motion.

During the period from 1920 to 1930 Lagally, Munk and Taylor separately developed a number of hydrodynamic theorems concerning apparent masses and hence the forces and moments which act on bodies moving through an inviscid fluid. When singularity distributions of sources, sinks and doublets within the body are known, these theorems enable one to determine the apparent masses and the forces and moments.

In 1921, Munk clarified some of the fundamental concepts relating apparent mass, velocity potential, fluid momentum and kinetic energy and derived a number of useful theorems using a general principle relating forces on fictitious sources and sinks immersed in any stream. He showed that, the force developed on a source is equal to the velocity of the fluid at its location multiplied by the source strength and density and is in the direction of the fluid motion for a sink but for a source is in the opposite direction.

In 1922, Lagally investigated the forces and moments acting on a body placed in an arbitrary, steady, potential flow. For the case in which the body can be represented by a system of singularities interior to the surface of the
body, he developed expressions for these singularities and the characteristics of the undisturbed stream in the neighbourhood of the singularities. The statement of the forces and the moments in terms of these singularities has become known as the Lagally Theorem.

In 1927, Von Karman\textsuperscript{12} appears to have been the first to propose a method of solving the problem directly, namely that of the flow over a specified body of revolution. He reduced this problem to that of solving a Fredholm integral equation of the first kind for the axial source-sink distribution. This would generate the given body. He then solved the integral equation approximately by replacing it with a set of simultaneous linear equations. Von Karman pointed out the fact that, only a special class of bodies of revolution can be represented by a distribution of sources and sinks on the axis of symmetry.

In 1928, Taylor\textsuperscript{13} published a relationship expressing the apparent mass coefficients for a body moving in pure translation in terms of a singularity distribution which may be considered to generate the flow field about the body. Taylor's relationship enables the apparent masses to be computed directly from the surface integrals of potential functions. He applied his theory to the determination of the apparent mass of an airship in accelerated linear motion using the source-sink distribution method to generate the airship's form. When the flow can be represented by a system of sources and sinks, Taylor's relationship becomes equivalent to that of Munk.

In 1929, Lewis\textsuperscript{14} published a study of the inertia
characteristics of hull sections. He defined the inertia coefficient as the ratio of the apparent mass for a cylinder which has a particular cross-section to that for a circular cylinder which has the same maximum diameter. Using transformation functions involving two arbitrary constants, he calculated the inertia coefficients for a large number of configurations representing different hull transverse sections, some of curvilinear form and others in the shape of rectangles and rhombuses. Lewis concluded that conditions do not change substantially from a hull-form transverse section to that of a rectangle.

In 1937, Morris\textsuperscript{15,16} developed a general method of formulating directly the solution of two dimensional problems dealing with simple closed cylinders which have curved or rectilinear boundaries. She obtained very compact results by the use of the complex potential function.

In 1940, Milne-Thomson\textsuperscript{17} introduced his famous circle theorem which enabled the disturbance due to placing a cylinder of any form in a uniform stream to be calculated in terms of the conformal transformation which maps the region exterior to the contour of the section on the region external to the cylinder.

In 1947, Polya\textsuperscript{18} discussed the apparent mass for arbitrary two dimensional cylinders in translation and showed that the sum of the translational apparent masses along two orthogonal directions is an invariant. He proved that, for cross sections of equal area, the circular cylinder section has the minimum value of the invariant. Later, with Szego\textsuperscript{19}, he showed that for a three-dimensional case, the sum of the
translational apparent masses along three mutually perpendicular axes is also an invariant. There follows from this result that for the cube, the apparent masses for edge-on and broad side-on motions are identical.

In 1950, Shiffmann and Spencer\textsuperscript{20} studied the flow about a lenticular object formed by two intersecting spheres, moving in the direction of its axis of symmetry. Applying the method of images in the multi-sheeted Riemann space they determined its potential function. They showed that the flow was related to the well-known case of two completely separated spheres which might from a general point of view be considered as two spheres which have an imaginary region of intersection for which the classical procedure for solution is by the method of images.

In 1950, Birkhoff\textsuperscript{21} published in book form, a series of lectures presented at the University of Cincinnati in 1947. In his book, he devoted a chapter to a compact but comprehensive treatment of apparent mass using advanced group-theory interpretations and concepts. Among other concepts Birkhoff contributed an extensive generalisation of Taylor's results using Green's Reciprocal Theorem. He also clarified and generalised Theodorsen's explanation regarding the choice of the appropriate integration space to ensure convergence of Theodorsen's momentum integrals.

In 1953, Darwin\textsuperscript{22} published a paper in which he suggested a physical interpretation of the apparent mass and the apparent moment of inertia involving a marked departure from previously accepted notions. He examined the trajectories of fluid particles resulting from the motion of the body.
and noted a "drift" in the fluid such that the final positions of the particles were different from the initial ones. He then proved quite generally that the drift volume corresponded to the apparent mass. In like manner, for a rotating body, he found that the fluid particles slowly drifted around the body even though the fluid motion is irrotational and without circulation. Darwin concluded that the concept of the hydrodynamic mass "is by no means a mathematical fiction" but is "a genuine physical phenomenon". An amount of fluid corresponding to the hydrodynamic mass is being carried along by the solid body.

In 1955, Landweber and Winzer calculated by means of potential theory the apparent mass coefficients for longitudinal, transverse and rotational motion of a series of 31 streamlined bodies of revolution and compared the calculated values with those of prolate spheroids which had the same length-diameter ratio, the so-called equivalent spheroids. They found that, for the transverse and rotational coefficients values for the equivalent spheroids were satisfactory, but that for the longitudinal coefficient there were large differences between the evaluations.

In 1956, Landweber and Yih published an important study in which they extended the Taylor theorem to include cases with external singularities and boundaries. They presented new results and relationships between apparent masses and singularities. They examined the interconnections that exist between the Taylor theorem (which relates apparent masses to singularities), the Lagally theorem (which relates singularities to forces), and Kirchhoff's equations of motion.
In 1959, Roy developed a mathematical expression for the resistance on a circular cylinder which results when any number of vortices lie in two rows behind the cylinder. His results were obtained by assuming the strength of the vortices to be constant and their locations to be fixed arbitrarily. Later, it was shown by Sarpkaya that one cannot calculate the instantaneous resistance on the basis of Roy's assumptions. Since the strength and the location of the vortices depend on time and since the actual pressure distribution around the cylinder is strongly influenced by its wake, by the vortex formation and by the vortex-feeding layers, without imposing any restrictions on the formation or on the motion or on the growth of these vortices, they can be combined with the experimentally-determined vortex characteristics in order to calculate the instantaneous resistance of the cylinder.

In 1963, Sarpkaya extended the Lagally theorem to moving, growing and arbitrarily situated vortices behind a circular cylinder immersed in a time-dependent flow. Lift, drag and apparent mass coefficient were presented in terms of the characteristics of these vortices.

In 1963, Ibrahim studied the apparent added mass and moments of inertia of cup-shaped bodies in unsteady incompressible flow. These shapes are directly related to parachute canopies. Idealising the parachute canopy into a rigid, non-porous, spherical shell of infinite small thickness and arbitrary concavity and applying the method of images to the case of two intersecting spheres, he determined the apparent added mass for spherical shells of arbitrary
In 1963, Trulie and Walitt\textsuperscript{28} made calculations in which small transverse oscillations of a circular cylinder were simulated. Their methods based on a two-dimensional inviscid flow model in which continuous vortex sheets were represented by arrays of discrete two-dimensional vortices were subsequently successfully applied to several interesting problems. In 1931, this method was first used by Rosenhead\textsuperscript{29}, who investigated the rolling-up behaviour of a vortex sheet. Abernathy and Kronauer\textsuperscript{30} used the same model to study the behaviour of a pair of parallel vortex layers of opposite signs. Their method was also used by Gerrard\textsuperscript{31}, Sarpkaya\textsuperscript{32}, Ujihere\textsuperscript{33} and Laird\textsuperscript{34}, who all describe models of flow behind circular cylinders in which the separation shear layers are approximated by arrays of line vortices.

In 1979, Srokosz\textsuperscript{35} considered submerged spheres as absorbers of surface wave power. In this study, a submerged sphere was considered to be absorbing power from an incident wave through a mooring and power take-off system. He found that the power absorbed depended on the hydrodynamic properties of the sphere, in particular on its apparent mass and damping coefficients. Solving the flow problem for a heaving, surging and swaying submerged sphere, he determined the apparent mass coefficients as a function of the non-dimensional wave number for various depths of the sphere. He found that for a deeply submerged sphere, the effect of the free surface was minimal, and so as submergence is increased the apparent mass coefficients, for both heaving and surging spheres approach the value of 0.5. This is the
non-dimensional apparent mass coefficient for a sphere oscillating in an infinite fluid without a free surface. He also noticed that when a sphere is closer to the free surface, the apparent mass coefficient for a heaving sphere deviates more from the theoretical value than it does for a surging sphere. He concluded that if the ratio of the diameter to depth is less than $\frac{1}{2}$, free surface effects can be considered to be negligible.

It is clear that the theoretical means employed to obtain hydrodynamic coefficients for streamlined bodies are satisfactory. However, coefficients of apparent mass and fluid resistance which depend on viscous separation from bodies are not obtained reliably with existing theories.

1.4. Experimental Studies

The fluid resistance acting on a body in unsteady motion has been studied experimentally over the past century by many workers. In 1917, a pendulum method was first used with the object of estimating the apparent mass effect for a sphere which was suspended by a fine wire, oscillating in both air and water. The apparent mass, defined as the percentage of the water displaced by the sphere was found to be 83 per cent. This is considerably in excess of the theoretical value of 50 per cent for a sphere in an ideal fluid and the result is consistent with the concept that in a viscous fluid a greater mass of the fluid is involved in acceleration effects than in an inviscid fluid.

In 1918, the success of the above experiments led Relf and Jones\textsuperscript{36} to attempt to measure the apparent mass
components for airship models. They measured the lateral and the longitudinal apparent mass components and the apparent moment of inertia of airship models with various fineness ratio in a large tank of still water and in a moving stream of water. Determination of the apparent moment of inertia in yaw was only performed in still water, as the bifilar suspension on which the model was hung could not execute a pure oscillation when the water was moving. Apparent masses and apparent moment of inertia were expressed as percentage ratios of the mass and the moment of inertia respectively of the fluid displaced by the body. It was found that the longitudinal apparent mass component decreases with a fineness ratio as the latter increases up to 4:1 but afterwards increases, whereas the lateral apparent mass component increases with increasing fineness ratio. They also noticed that the longitudinal apparent mass components are approximately independent of forward speed, while the lateral apparent mass components decrease with increasing speed. The apparent moment of inertia was of the same order of size as the lateral apparent apparent mass component. Values obtained are shown in Fig.1.1 together with appropriate theoretical values determined for streamline bodies with appropriate fineness ratios. It is seen that both apparent mass components are considerably higher than their theoretical values the differences arising from viscous effect and flow separation in the case of the real fluid flow.

In 1919, Frazer and Simmons conducted a series of experiments on a sphere and a streamline body, towing them under the water. Fluid resistance and apparent mass
coefficients were obtained and plotted against both velocity and acceleration. It was found that, for the sphere the apparent mass coefficient increases with increasing velocity, reaching a value approximately 7 times higher than theoretical whereas, the apparent mass coefficient for the streamline body was approximately 2.3 times higher than theoretical values and was invariant with velocity. This again indicates that apparent mass components in a real fluid are strongly dependent on the degree of flow separation which occurs around the body.

In 1935, Soule and Miller discussed the application of the pendulum method to the experimental determination of
the moment of inertia of aircraft, with particular reference to the effect of the ambient air on this moment of inertia. They regarded the total moment of inertia of the aircraft as made up of three distinct parts:

a) the structure,

b) the air entrapped within the structure, and

c) the additional mass of external air influenced by the aircraft's motion.

Soule and Miller concluded that the moment of inertia obtained directly through application of the pendulum method are considerably larger than the structural mass moments of inertia on account of the low mass density of the aircraft and relatively large values of the apparent mass effect.

In 1935, Koch\textsuperscript{38} used an electrical analogy as an indirect experimental method for determining the apparent mass for the oscillation of the ships. This determination of the apparent mass is based essentially on the solution of Laplace's equation which governs both the velocity potential and the potential distribution of a plane electric field. Since a real fluid possesses no velocity potential in the boundary layer, Koch's experiment would lead to ideal fluid solution to this problem.

In 1941, Gracey\textsuperscript{39} presented a test procedures and experimental results for determining the apparent mass for the oscillation of the ships. This determination for the apparent masses of flat plates, using data obtained from English, German and Russian sources in addition to experimental results were obtained by the NACA in the United States in 1933 and in 1940. In the NACA tests, the apparent mass of rectangular plates of varying aspect ratio
was determined and the effect of taper of the plan form investigated. In the tests described, the values of the apparent masses were obtained from the difference between the moments of inertia of the plates determined from experiments in air and in vacuum.

In 1941, Yee-Tak Yu\textsuperscript{40} studied the fluid resistance and the apparent masses on discs and cylinders in three different fluids; water, gasoline and carbon-tetrachloride. Using a pendulum method he found that the apparent masses in other fluids were the same as those in water multiplied by the specific gravity of the corresponding fluid. It was shown that the experimental value for the apparent mass coefficient of a disc moving perpendicular to its face is about 1.28 times greater than the theoretical value and about twice the theoretical value for a cylinder.

In 1952, Iversen and Bently\textsuperscript{3} wrote an equation for the force, $F$, acting on a body towed through a fluid in unsteady motion as

$$F = \frac{1}{2} C_R A \rho V^2$$  \hspace{1cm} (1.18)

where $C_R$ is the total fluid resistance coefficient which depends not only upon the shape of the body, its Froude number, its Reynolds number, its Mach number and the roughness of the surface, but also upon the acceleration modulus, $\frac{dV}{dt} \frac{D}{V_T}$ and $A$ is the cross-section area of the body. They then proceeded to compute the apparent mass coefficient, $k$, after assuming that the drag coefficient in unsteady flow was independent of this acceleration modulus. Because its drag coefficient in steady flow is nearly independent of
Reynolds number for their experiments they used a disc. The values obtained for $C_R$ and for $k$ are given in Figs. 1.2. and 1.3. It can be seen that the apparent mass coefficient for a disc approaches to the theoretical value of $\frac{2}{\pi}$ at high values of $\frac{dV}{dt} \frac{D}{V^2}$, that is when the acceleration of the disc is high compared with its velocity. For low values of this acceleration modulus, $k$ was found to be approximately five times its theoretical value, if the steady-flow drag coefficient is assumed independent of this modulus, and again these large discrepancies can be associated with flow separation phenomena.

In addition to Iversen and Balent's study of forces on a disc being accelerated through a fluid, measurements were made by Luneau, for the case of constant acceleration, rather than constant driving force. These data are also presented in Figs. 1.2. and 1.3. There are substantial differences in the results obtained in the two sets of experiments.

In 1952, Stelson, reported on various aspects of the problems of accelerating bodies in fluids, pointing out the conditions when apparent mass effects may be expected to be large. He examined the effects of various experimental parameters and noted that studies using oscillating models were easier to set up and yield more consistent results than studies which use rectilinear accelerating motion. In a later study, Stelson and Mavis presented additional experimental results; they defined a virtual mass which is the body mass together with the apparent mass, as the ratio of the applied force when immersed in the fluid to the acceleration in vacuum.
Fig. 1.2: Apparent mass coefficient, $k$, vs. acceleration modulus, $\delta$, for disc$^{3,4}$.

Fig. 1.3: Total fluid resistance coefficient, $C_R$, vs. acceleration modulus, $\delta$, for disc$^{3,4}$.
Measurements were made on various bodies as they were
immersed in tap water and accelerated in oscillatory motion.
It was found that at the Reynolds number at which they
worked the experimental apparent mass coefficients obtained
for spheres and cubes showed excellent agreement with the
theoretical values.

In detailed comments on the previous paper, Silberman has stressed the importance of distinguishing
between cases involving unidirectional motion and those of
vibratory motion, noting that a body in unidirectional
motion tends to develop both a boundary layer and a wake and
pointing to the difficulty of separating the total resistance
into parts due to viscous and form drag on the one hand and
those due to the apparent mass on the other. He suggested
possible alternative approaches as a means of predicting
fluid resistance in unidirectional acceleration of bodies.

In 1956, Keim tested a series of cylinders accelerating
through water using essentially the equipment of Iversen and
Balent. He determined the total fluid resistance
coefficient as a function of the acceleration modulus in a
way which demonstrated the effect of Reynolds number, as
seen in Fig. 1.4.

In 1958, Keulegan and Carpender studied the forces
exerted on a circular pile and on a flat plate subjected to
standing waves and found that the value of drag coefficient
could not be related to Reynolds number but appeared to be a
function of the parameters, $\frac{V_m \cdot T}{D}$, where $V_m$ is the
maximum fluid particle velocity, $T$ is the period of the wave
motion and $D$ is the diameter of the cylinder. They
developed a technique of obtaining instantaneous values of drag and apparent mass coefficients throughout the complete wave cycle. They found that the apparent mass coefficient decreases first, then increases as the period parameter $\frac{V_{mT}}{D}$ increases but does not exceed its theoretical value.

In 1955, Szebehely studied the effect of frequency, amplitude of oscillation, speed of advance and presence of a free surface on the virtual mass of a prolate spheroid. In his tests, the body performed free oscillations about its shorter axis in the horizontal plane and moved in its plane of oscillation with a uniform forward velocity in the direction defined by the mean position of its longitudinal axis. He concluded that the virtual moment of inertia did not show dependence on the frequency or on the amplitude of oscillation, and therefore, the results of the potential flow calculation might be applied. He also found that if the depth diameter ratio is less than two, the free surface effects become significant and the inertia coefficient rises rapidly.
In 1963, Sarpkaya and Garrison\textsuperscript{47} conducted both theoretical and experimental work on lift, drag and apparent mass coefficients for a circular cylinder immersed in a time-dependent flow. Analysing the strength, growth and motion of vortices behind a circular cylinder immersed in a two-dimensional uniform flow with constant acceleration, they determined equations for lift, drag and inertia force from potential theory in terms of the flow and vortex characteristics. Combining these theoretical equations with the experimental vortex and flow characteristics, they separated drag and apparent mass coefficients and found that both coefficients are function of the relative displacement. They also found a unique relationship between drag and the apparent mass coefficients in which the drag coefficient increases rapidly while the apparent mass coefficient drops from 2 which is the theoretical value of the stationary cylinder, to 1.2 as the relative displacement, s/D, increases, as seen in Fig. 1.5. From dimensional analysis they wrote the total fluid resistance coefficient in terms of the acceleration V as

\[ C'_{R} = \frac{\text{Total Force}}{\rho V \frac{\pi D^2}{4}} \]  

(1.19)

where D is the diameter of the cylinder, and good correlation was found between \( C'_{R} \) and the relative displacement, as seen in Fig. 1.6.

In 1963, Paape and Bruesers\textsuperscript{48} applied both theory and model experiments to establish relations between wave characteristics and the force exerted by the waves on piles.
Fig. 1.5: Correlation of drag and apparent mass coefficients, $C_D$ and $k$, with relative displacement, $s/D$, for cylinder.

Fig. 1.6: Total fluid resistance coefficient, $C_R$, with relative displacement, $s/D$, for cylinder.
of known shapes and dimensions. A number of experiments were carried out in the Delft Hydraulics Laboratory in which they determined the drag and apparent mass coefficients as a function of wave steepness, $\frac{H}{gT}$, where $H$ is the height of wave and $T$ is the period of oscillation. They found that in the range of $H/D \leq 1$ the magnitude of the apparent mass coefficient goes up from 1.5 as high as 3.5.

In 1969, Heiser and Dalton made a series of experiments on cylinders oscillating harmonically in water. They found that there exist several factors influencing the drag and inertia forces in unsteady motion relative to the water particles. These factors are; the roughness of the surface of the cylinder, the vorticity of the fluid, the instantaneous velocity and the instantaneous acceleration, the diameter of the cylinder, the period, the amplitude and the wave height.

In 1971, instead of undertaking a dimensional analysis to regroup all possible factors influencing the drag and inertia into dimensionless parameters, as was done successfully for uniform flow, Hamann and Dalton conducted experiments to investigate the force behaviour for three variables; the diameter of the cylinder, the amplitude of the harmonic oscillations and the frequency of oscillation. To obtain the desired measurements of the forces exerted by stationary water a single circular cylinder was held vertically in the water, oscillating horizontally in harmonic motion. On the basis of these observations they found that with the cylinder in sinusoidal motion forces on the cylinder lie on a base line for low accelerations and all curves relevant to high accelerations merge on to this base line at high
velocities. The base line indicates a higher force than for uniform flow when the velocity is low and the oscillation amplitude small but a force coefficient which is comparable with uniform flow at higher velocities and larger amplitudes. As seen in Fig. 1.7, the difference between the base line and the curves obtained with higher accelerations is a function of velocity, decreasing with increasing velocity. Separating the force into drag and inertia forces, the values of drag coefficient can be correlated with base line values obtained for low accelerations, whereas, the value of the apparent mass coefficient can be correlated with the difference between baseline and acceleration curve values and is a function of velocity. This is the approach which was adopted by Iversen and Balent for their experiments with flat discs.

![Acceleration of cylinder](image)

**Fig. 1.7:** Force behaviour on small cylinder at high amplitude.
In 1974, Sarpkaya\textsuperscript{51} carried out a number of experimentation on circular cylinders and spheres in oscillatory motion. His apparatus consisted of a U-shaped vertical tunnel in which the test bodies were mounted 0.76 m below the free surface of the water level whilst the fluid in the tunnel was oscillated pneumatically by means of a simple slider-crank mechanism which periodically opened and closed the air supply. A Fourier analysis was made of records of forces determined in the direction of oscillation. Drag and apparent mass coefficients were given as functions of the period parameter, $\frac{VT}{D}$. For the sphere, the apparent mass coefficient was at its maximum value of 1.5 for small values of the period parameter, decreasing as the latter increased. However, for the circular cylinder the apparent mass coefficient first decreased then increased again as the period parameter was increased. Neither drag coefficients nor apparent mass coefficients appeared to vary with Reynolds number.

In 1978, Dalton, Hunt and Hassan\textsuperscript{52} made further experiments on the forces on a cylinder oscillated sinusoidally in water. Extending the study of Hamann and Dalton\textsuperscript{50} they found, unlike Sarpkaya and Garrison\textsuperscript{47}, that the total force acting on the cylinder could be expressed as:

$$ F = f\left( \frac{V_m T}{D}, \frac{D}{\sqrt{VT}} \right) \quad (1.20) $$

The total fluid resistance coefficient, $C_R$, was defined in terms of the instantaneous force and maximum velocity, $V_m$. If the instantaneous Reynolds number is also
included, then

\[ C_R = \frac{F}{\frac{1}{2} \rho D V^2} \cdot \frac{1}{m} = f\left( \frac{VD}{\nu}, \frac{VT}{D}, \frac{D}{\sqrt{VT}} \right) \quad (1.21) \]

where \( D \) is the cylinder diameter, \( \nu \) is the kinematic viscosity and \( T \) is the period of oscillation. Dalton, Hunt and Hassan used a scotch-yoke mechanism to oscillate the cylinder below water sinusoidally. The fluid resistance was measured using strain gauges to determine the bending moment incurred by the cylinder. Instead of separating drag and apparent mass coefficient, they determined the variation of the force coefficient with the instantaneous Reynolds number, period parameter and \( \frac{D}{\sqrt{VT}} \) as a parameter. They found that as both Reynolds number and acceleration were increased the value of the total fluid resistance coefficient approached the value obtained under steady flow conditions.

It is apparent that the various investigators have used two main methods to determine the drag and the appropriate apparent mass coefficients in an unsteady motion. The first described by Iversen and Balent\(^3\) takes a steady flow drag coefficient, assuming it to be independent of any unsteady motion. The difference between the total resistance coefficient is then termed the apparent mass coefficient.

The second is that described by Keulegan and Carpenter\(^{45}\) applying the theoretical derived values of velocities and acceleration, the systematic evaluations of the Fourier-averages drag and apparent mass coefficients are determined through measurements of forces on a body in oscillating flow. The curves obtained by Keulagen and Carpenter, as stated by Dalton and Hamann\(^{50}\) seem to represent an upper bound for
the drag and a lower bound for the apparent mass coefficients when related to the period parameter, \( \frac{VT}{D} \) and compared with the values of other investigations.

### 1.5. Studies on Parachute Problems

Advances in parachute design over the last 30 years have led to a concern about the dynamic stability during descent of the parachute-store system. Favourable canopy shapes, having appropriate porosities have been developed empirically. However, attempts to analyse theoretically the dynamic stability of parachutes goes back to Brodetzky\(^53\), some sixty years ago.

In 1943, Jones\(^54\) conducted experimental investigations to consider the unsteady behaviour of model parachutes. In his experiments parachute canopies were mounted as a torsional pendulum and oscillated both in air and in water. He carried out his investigations on three different models, using four different pendulum lengths for each case. Apparent masses were calculated on the assumption that they act at the canopy centre of volume. This assumption leads to values which are approximately the same for all pendulum lengths. He obtained apparent mass coefficients at right angles to the canopy axis of 1.51, 1.42 and 1.36 for rigid canopies, non-rigid canopies with attached loads and non-rigid canopies with loads respectively.

In 1945, Henn\(^55\) using linearised theory and small disturbances assumptions solved the equation of motion of a descending fully-deployed parachute. In his analysis, he took account of apparent mass effects, determining the
parachute's motion consequential on an assumed disturbance. Henn assumed the canopy to be replaced by an air-filled ellipsoid for which analytical expressions applicable to an ideal fluid were available for the apparent mass. Henn also expressed his solutions relevant to a system of co-ordinates the origin of which lay outside the centre of volume of the canopy. Having linearised the equations he solved them. Henn found that variation of the longitudinal and normal apparent mass components, $a_3$ and $a_2$, had considerable effects on both the damping and the frequency of the ensuing lateral oscillations.

Also in 1945, Von Karman stressed the importance of apparent mass effects for parachutes, suggesting a method by which apparent mass components could be measured by instantaneously releasing half of the weight suspended by the parachute and then measuring the resultant deceleration of the remaining parts of the system. His suggestion was adopted by the parachute branch, Wright Air Development Centre (Air Force).

In 1949, O'Hara studied a detailed theoretical analysis of parachute opening characteristics and succeeded in developing a theory from which an appropriate value of the opening shock force could be determined. He used an air mass associated with the parachute which combined the mass inside the canopy, that is, the included mass, together with an additional mass assumed to be same as that of a flat disc which had the same radius as the parachute.

In 1952, Heinrich described the model experiments conducted as Von Karman had proposed, discussing some
significant results which had been obtained. These included from tests which showed the effect of porosity on apparent mass. For different canopy porosities and different parachute types Heinrich calculated the ratio of the apparent mass, to the included mass and found that, for canopies of very little or no porosity, this ratio was larger than unity. His results are given in Fig. 1.8.

In 1962, Lester pointed out certain fundamental inconsistencies in Henn's analysis, mentioning that similar errors had been reproduced in many current studies. He argued that in Henn's derivation of the equations of motion it is assumed that the canopy can be replaced by an air-filled
ellipsoid which posses fore and aft symmetry and thus some apparent mass components are non-zero, contrary to Henn's equation. Unlike Henn, who used rigid dynamical equations of motion, Lester adopted the Kirchhoff equation, deriving the equation of motion in three degrees of freedom. Lester showed that, in general, four apparent mass components should be considered for the parachute canopy. However, as is discussed later in Chapter 3, if the origin is chosen as the canopy centre of reaction, the number of apparent mass components can be reduced to 3. The equations of motion then contain only two apparent mass components, $a_{nn}$ and $a_{nn}$ and one apparent moment of inertia component $a_{nn}$, together with their products.

In 1963, Ludwig and Heins\textsuperscript{60} presented the results of an investigation on the dynamic stability of personnel guide-surface parachutes in which the appropriate system of non-linear equations of motion were integrated numerically, without recourse to linearisation. They followed Henn's method of replacing the canopy by an air-filled ellipsoid of revolution of equal volume. Considering only one single apparent mass component, $a_{nn} = a_{nn}$ in the present notation, they considered the influence on parachute dynamic behaviour of various factors. Among these factors was apparent mass. Their results show the limitations caused by assuming that the oscillation of parachutes can be described by linearised equations of motion. They found that the apparent mass components have a considerable effect on the damping of oscillations but little effect on the oscillation frequency.
In 1964, Ibrahim$^{61}$ conducted experiments to measure the apparent moment of inertia term. Using the pendulum method, he determined the apparent moment of inertia of canopy designs oscillating about different points. He indicated that the effect of the porosity on parachute apparent mass and moment of inertia is very significant and should not be neglected.

In 1964, Wolf$^{62}$ investigated the dynamic stability of parachutes. Assuming, like Ludwig and Heins, that the parachute canopy has a single apparent mass he derived the equation of motion with five degrees of freedom. Non-dimensionalising these equations, he established conditions for the dynamic stability of a parachute-load system. He investigated the effect of a number of parachute parameters on this dynamic stability, finding that increasing the apparent mass has a stabilizing effect.

In 1975, Byushgens and Shilow$^{63}$ derived the equations of motion of parachute in three degrees of freedom which are almost identical with those of Lester. They showed that the apparent mass components, $a_{\|}$ and $a_{\perp}$, exhibit strong effects on the parachute stability.

In 1978, the equations of motion of parachute canopy-load system in six degrees of freedom were given by Tory and Ayres$^{64}$. Considering three apparent mass components, $a_{\|}$, $a_{\|}$ and $a_{\perp}$ in the present notations, they incorrectly derived the equations of motion. Later, Eaton and Cockrell$^{65}$ studying the dynamic sensitivity of the Tory-Ayres equation found that the dynamic performance is sensitive to the apparent mass components in the lateral direction but almost insensitive
to the other two components.

In 1981, Purvis, concerned with parachute inflation effects, analysed the equations of motion for parachute inflation. Equations of motion for a deforming, accelerating control volume were developed and solved from first principles to determine the behaviour of the captured fluid and its interaction with the parachute canopy.

The dynamic stability of the descending parachute-load system depends on many parameters, including assumptions made about the magnitudes of the apparent mass components for the canopy. As can be seen, different investigators have adopted different definitions and have implemented them differently into the equations of motion.

If pendulum methods are used, the Reynolds numbers at which the apparent mass components of a rigid model parachute canopies are obtained, are very much lower than full-scale values. Results obtained are thus unreliable when used to simulate the behaviour of the prototype parachute system.

The need to determine apparent mass components for a parachute system at near full-scale Reynolds number is met for the first time by these current investigations. Components will be defined in Chapter 3 and the method used to determine them will be developed in Chapter 4 of this thesis.
1.6. The Objectives of the Current Research Programme

(i) To define the apparent mass components associated with a fully-deployed parachute canopy.

(ii) To devise and establish an experimental technique to study bluff body behaviour in unsteady motion and to determine appropriate and acceptable values for the parachute.

(iii) Having derived the non-linear equations of motion of a parachute-load system, to determine the dynamic stability criteria and the effect of various parameters, including apparent masses on the dynamic stability of the parachute.

(iv) To solve these non-linear equations of motion using the apparent mass components obtained experimentally and thus to determine the dynamic performance of a descending parachute.
AERODYNAMICS OF PARACHUTES AND LIKE BODIES IN UNSTEADY MOTION

CHAPTER 2.

THEORETICAL EVALUATIONS OF APPARENT MASSES FOR CERTAIN CLASSES OF BODIES.
2. THEORETICAL EVALUATIONS OF APPARENT MASSES FOR CERTAIN CLASSES OF BODIES.

2.1. Introduction

The existing theory for the determination of apparent mass is based on the fact that, when a solid body is moving in an incompressible potential flow, the entire velocity field depends upon the instantaneous velocity of the submerged body and is quite independent of the past history of the motion. Consequently, any change of the motion of the body would be propagated instantaneously to all the particles of the fluid. Thus, the total kinetic energy imparted to the fluid during translation must vary directly with the square of the linear velocity, and during rotational motion, with the square of the angular velocity.

A number of hydrodynamic theorems have been published by Munk\textsuperscript{10}, Lagally\textsuperscript{11} and Taylor\textsuperscript{13} concerning the apparent masses of bodies and the forces and moments which act upon them. These theorems enable the apparent masses and hence the forces and moments to be determined when the singularity distribution of sources, sinks and doublets within the body which may be considered to generate the potential flow about it, are known.

For unsteady flow conditions, an impulse associated with any change in the motion of the body is directly proportional to the change in the value of the velocity potential function, $\phi$, which specifies the flow. Consequently, the determination of the apparent masses of bodies moving through an ideal fluid must depend on the calculation
of the velocity potential or of the corresponding stream function. The apparent masses are determined from this velocity potential function calculating the kinetic energy $T$, by means of the integral,

$$2T = -\rho \int_S \frac{\partial \phi}{\partial n} \, ds \quad (2.1)$$

where $S$ denotes the surface of the body, $\phi$ the velocity potential function, $\rho$ the density of the fluid and $\partial n$ denotes an elemental length drawn in the fluid normal to the surface $\partial s$.

The general motion of a solid body of arbitrary shape has six degrees of freedom represented by three components of linear velocity $(U, V, W)$ and three components of angular velocity $(P, Q, R)$. For each velocity component there is a corresponding fluid velocity potential. Consequently, there are a number of different apparent mass components. If the velocity potential is defined in terms of velocity components $V_i$ of the body, as $\phi = V_i \phi_i$, where $i$ ranges from 1 to 6, the apparent mass components are given from the kinetic energy equation (2.1) as

$$2T = -\rho \int_V \frac{\partial V_i \phi_i}{\partial n} \, ds \quad (2.2)$$

thus,

$$a_{ij} = \rho \int_V \frac{\partial V_i \phi_i}{\partial n} \, ds \quad (2.3)$$

in which $i$ and $j$ range from 1 to 6. $a_{ij}$ represent a
symmetric apparent mass tensor, in other words,

\[ a_{ij} = a_{ji} \quad (2.4) \]

In the quadratic expressions for the energy, there could be six squares and fifteen products of velocity components and therefore 21 distinct apparent mass components as

\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
  a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\
  a_{33} & a_{34} & a_{35} & a_{36} \\
  a_{44} & a_{45} & a_{46} \\
  a_{55} & a_{56} \\
  a_{66}
\end{bmatrix}
\quad (2.5)
\]

However, as shown certain components disappear when the body possesses axes or planes of symmetry.

For an irrotational ideal fluid, the equation (2.3) shows that the apparent mass components depend upon the shape and the orientation of the body as well as on the mode of its motion. However, they do not depend upon its linear or angular velocities and its linear or angular accelerations.

The apparent mass components can also be expressed in terms of a different form of equation (2.3). By applying Green's Theorem, this equation can be written as,

\[
a_{ij} = -\rho \int \int_{S} \frac{\partial \phi_i}{\partial n} ds + \rho \int \left( \phi_i \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_i}{\partial n} \right) ds \quad (2.6)
\]

where the last integral is taken over a surface which approaches infinity. At infinity, \( \phi \) is equivalent to the
strength of the dipole, hence equation (2.6) becomes

\[ a = 4 \pi \rho \sigma_A - \rho \cdot \text{Volume of the object.} \quad (2.7) \]

where \( \sigma_A \) is the dipole strength at infinity of the potential function. This equation is valid for any symmetrical object moving in an infinite fluid. It remains to evaluate the dipole strength, \( \sigma_A \), for a particular flow around a particular body.

If velocity potential singularities are known, i.e., the flow problem about the body has been solved, the apparent mass components can be calculated directly from equation (2.3). Otherwise, it is more convenient to use equation (2.6)\(^6\).

### 2.2. Apparent Masses of an Ellipsoid

Lamb\(^6\) shows how the principle of hydrodynamic theory can be used to determine the values of the apparent mass components for all classes of ellipsoids, moving in an ideal fluid with axial, transverse and rotational motions.

![Dimensions of an ellipsoid.](image)

**Fig. 2.1:** Dimensions of an ellipsoid.
For an ellipsoid, for which

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \tag{2.8}
\]

where if \( a = b > c \), an oblate ellipsoid is obtained which can be used as a model of parachute canopies. However, if \( a = b < c \), a prolate spheroid is obtained, which could be used as a model for an airship hull.

By employing equation (2.3) and considering the ellipsoid motion apparent mass components can be determined directly from the defined velocity potential function on the ellipsoid surface.

For rectilinear motion parallel to the \( x \)-axis with a velocity \( U \), as seen in Fig. 2.1, the velocity potential may be written as

\[
\phi = C x \int_0^\infty \frac{d\lambda}{(a + \lambda)\Delta} \tag{2.9}
\]

where

\[
\Delta = \{ (a^2 + \lambda)(b^2 + \lambda)(c^2 + \lambda) \}^{\frac{1}{2}} \tag{2.9a}
\]

\[
C = \frac{abc}{2-a_0} U \tag{2.9b}
\]

and

\[
a_0 = abc \int_0^{\infty} \frac{d\lambda}{(a^2 + \lambda)\Delta} \tag{2.9c}
\]

The surface condition is

\[
\frac{\partial \phi}{\partial \lambda} = -U \frac{\partial x}{\partial \lambda} \quad \text{for} \quad \lambda = 0.
\]

For the motion in the direction of the other axes,
(i.e. the y and z axes) the corresponding velocity potential and other variables (i.e. C and \( a_0 \)) can be written by substituting the appropriate variables into equations (2.9), (2.9a) and (2.9c). Hence, in the y and the z axes directions, the above equations become

\[
\phi = C y \int_{\lambda}^{\infty} \frac{d\lambda}{\lambda (b^2+\lambda) \Delta} \quad (2.10)
\]

\[
C = \frac{abc V}{2-\beta_0} \quad (2.10a)
\]

\[
\theta_0 = abc \int_{0}^{\infty} \frac{d\lambda}{\lambda (b^2-\lambda) \Delta} \quad (2.10b)
\]

and

\[
\phi = C z \int_{\lambda}^{\infty} \frac{d\lambda}{\lambda (c^2+\lambda) \Delta} \quad (2.11)
\]

\[
C = \frac{abc W}{2-\gamma_0} \quad (2.11a)
\]

\[
\gamma_0 = abc \int_{0}^{\infty} \frac{d\lambda}{\lambda (c^2+\lambda) \Delta} \quad (2.11b)
\]

respectively.

When the ellipsoid rotates about the x axis, with the angular velocity \( \omega \), the velocity potential may be given as,

\[
\phi = C y z \int_{\lambda}^{\infty} \frac{d\lambda}{\lambda (b^2+\lambda)(c^2+\lambda) \Delta} \quad (2.12)
\]

where

\[
C = \frac{c^2 - b^2}{2(c^2-b^2) + (c^2+b^2) (\theta_0 - \gamma_0)} \quad \text{abc} \omega \quad (2.12a)
\]
and the surface condition is
\[
\frac{\partial \phi}{\partial \lambda} = \omega (z \frac{\partial y}{\partial \lambda} - y \frac{\partial z}{\partial \lambda}) .
\]

It may be further seen that, for the motion about any other axis, the corresponding velocity potential can be written by substituting the appropriate variables into equation (2.12).

Thus, the kinetic energy \( T \) of the ellipsoid moving in the \( x \)-axis direction is evaluated from equation (2.1), as

\[
2T = -\rho \int_{S} \left( \frac{\partial \phi}{\partial n} \right) dS = \frac{a_0}{2-a_0} \rho U^2 \int_{S} x \mid dS
\]

where \( \mid \) is the direction cosine in the \( x \) direction. Since the latter integral gives the volume of the ellipsoid, the energy equation becomes

\[
2T = \frac{a_0}{2-a_0} \frac{4}{3} \pi abc \rho U^2
\]

Consequently, the apparent mass in the \( x \)-axis direction may be written from equation (2.14),

\[
a_{xx} = \frac{a_0}{2-a_0} \frac{4}{3} \pi abc \rho
\]

The corresponding apparent mass coefficient which is defined as the ratio of the apparent mass to the mass of the fluid displaced by the ellipsoid is obtained from equation (2.15),

\[
k_{xx} = \frac{a_0}{2-a_0}
\]
By the same approach, the apparent mass coefficients along the other two axes are determined as

\[ k_{yy} = \frac{\beta_0}{2-\beta_0} \]  
\[ k_{zz} = \frac{\gamma_0}{2-\gamma_0} \]  

(2.17)  

(2.18)

Using the velocity potential function for the rotational motion of the ellipsoid, the apparent moment of inertia coefficient which is defined as the ratio of the apparent moment of inertia to the moment of inertia of the fluid displaced by the ellipsoid about the x-axis is found from equation (2.1),

\[ k_f^{\text{xx}} = \frac{e^4(\beta_0-\gamma_0)}{(2-e^2)[(2e^2-(2-e^2)(\beta_0-\gamma_0)]} \]  

(2.19)

where \( e \) denotes the aspect ratio, as

\[ e = 1 - \frac{c^2}{a^2} \] for an oblate ellipsoid

\[ e = 1 - \frac{a^2}{c^2} \] for a prolate ellipsoid.

Then, from equation (2.9b) and (2.10b), the following equations result;

for oblate spheroid

\[ a_o = \beta_0 = \frac{2}{e^2} \left[ 1 - \sqrt{1-e^2} \sin^{-1}e \right] \]  

(2.20a)

\[ \gamma_o = \frac{\sqrt{1-e^2}}{e^3} \sin^{-1}e - \frac{1-e^2}{e^2} \]  

(2.20b)
and, for prolate spheroid

\[ a_0 = \beta_0 = 2\left( \frac{1-e^2}{e^3} \right) \left( \frac{1}{2} \log \frac{1+e}{1-e} - e \right) \] (2.21a)

\[ \gamma_0 = \frac{1}{e^2} - \frac{1-e^2}{2e^3} \log \frac{1+e}{1-e} \] (2.21b)

The result obtained for \( k_{xx} = k_{yy}, k_{zz} \) and \( k'_{xx} \) are given as a function of the aspect ratio in Fig. 2.2.

For \( a = b = c \), the shape of ellipsoid becomes a sphere and \( k_{xx} = k_{yy} = k_{zz} = 0.5, k'_{xx} = 0.0 \). When \( a=b=2c \), it becomes a hemisphere and the apparent mass components are obtained as,

\[ k_{xx} = k_{yy} = 0.3, k_{zz} = 1.1 \text{ and } k'_{xx} = 0.4. \]

Determining the strength of the dipole for a particular flow about spherical shells, Ibrahim\textsuperscript{27}, using equation (2.7) determined the apparent mass coefficient in the direction of the symmetry axis of the spherical cups. He assumed that the included mass which is the mass of the fluid inside the cup is a part of the apparent mass and consequently his results obtained for a sphere and for a hemisphere are higher than those previously calculated. If it is assumed that the included mass is carried by the spherical shell as if it were part of the solid, Ibrahim's results become identical with those developed above, and as seen in Fig. 2.3.
Fig. 2.2: Theoretical apparent mass coefficients for the parachute's aerodynamically related shapes.
2.3. Apparent Masses in Arbitrary Two-dimensional Motion

2.3.1. Elliptic Cylinder in Arbitrary Motion

Introducing the elliptic co-ordinates, $\xi$, $\eta$, the potential function for an elliptic cylinder, moving parallel to its x-axis with a unit velocity, $U$, as shown in Fig. 2.4, is given as,

$$\phi = -U \cdot b \left( \frac{a+b}{a-b} \right)^{\frac{1}{2}} e^{-\xi} \sin \eta$$  \hspace{1cm} (2.22)

where $a$ and $b$ are the major and the minor axis respectively.
If the motion of the elliptic cylinder is parallel to the \( y \)-axis, the equation will be,

\[
\phi = -Va \left( \frac{a+b}{a-b} \right) e^{-\xi} \sin \eta \tag{2.23}
\]

The resultant pressure, i.e. the force exerted by the surrounding fluid on the unit length of the cylinder parallel to the \( x \) and the \( y \) axes respectively are determined by the equations,

\[
F_x = -b \int_0^{2\pi} p \cos \eta \, d\eta \tag{2.24}
\]
\[
F_y = -a \int_0^{2\pi} p \sin \eta \, d\eta \tag{2.25}
\]

where \( p \) is the pressure which is determined by the Bernoulli equation for unsteady flow conditions,

\[
\frac{D\rho}{\rho} = \frac{3\phi}{\rho t} + \frac{1}{2} q^2 \tag{2.26}
\]

where \( q \) is the velocity. Since the fluid is ideal and considering the fact that an ideal fluid exerts zero net force on a body of any shape wholly immersed in it (the
d' Alembert paradox), the second term in the right hand side
of the equation (2.26) does not make any contribution to
force and the first term will be only considered.

Hence, from equation (2.24) and (2.26), the apparent
masses for an elliptic cylinder in the x-axis and the
y-axis directions are found as

\[ a_{xx} = \pi \rho b^2 \]
\[ a_{yy} = \pi \rho a^2 \]

respectively.

Supposing the elliptic cylinder is moving in an
arbitrary direction and U, V are the velocity components in
the x-axis and the y-axis directions respectively. The
potential function can then be expressed as,

\[ \phi = - \left( \frac{a+b}{a-b} \right)^{\frac{1}{2}} e^{-\xi} (Ub \cos \theta + Va \sin \theta) \]  (2.27)

Considering the resultant forces parallel to the x
and the y axes separately, since

\[ \frac{\partial \phi}{\partial t} = - \left( \frac{a+b}{a-b} \right)^{\frac{1}{2}} e^{-\xi} \left[ \frac{\partial U}{\partial t} b \cos \theta + \frac{\partial V}{\partial t} a \sin \theta \right] \]  (2.28)

Substituting this into (2.26) and using equation (2.24)
and (2.25) forces \( F_x \) and \( F_y \) are determined as,

\[ F_x = -\pi \rho b^2 \frac{dU}{dt} \]
\[ F_y = -\pi \rho a^2 \frac{dv}{dt} \]
and finally,

\[ a_{xx} = \pi \rho b^2 \]

\[ a_{yy} = \pi \rho a^2 . \]

Their values remaining the same as if they were determined separately.

### 2.3.2. Ellipsoid in an Arbitrary Motion.

If the ellipsoid moves in any arbitrary direction and \( U, V \) are the velocity components in the \( x \) and \( y \) directions respectively, as seen in Fig. 2.5, the velocity potential function is expressed as

\[
\phi = \frac{a_0}{2-a_0} Ux + \frac{\beta_0}{2-\beta_0} V y
\]

(2.29)

where \( a_0 \) and \( \beta_0 \) are defined in section 2.2.

---

Fig. 2.5: An ellipsoid in an unsteady two-dimensional motion.
The total forces acting on the ellipsoid in the $x$ and $y$ axes directions are evaluated by considering the pressure distribution over the surface.

So,

$$ F_x = -\int p \cos \theta \, dA \quad (2.30) $$

$$ F_y = -\int p \sin \theta \, dA \quad (2.31) $$

where these associated integrals are taken over the surface and $dA$ is small surface element. Since

$$ \frac{-\rho}{\rho} = \frac{3\phi}{\rho} + \frac{1}{2} q^2 $$

then

$$ \frac{3\phi}{\rho} = \frac{a}{2a_0} \frac{dU}{dt} x + \frac{b}{2b_0} \frac{dV}{dt} y \quad (2.32) $$

From equation (2.30),

$$ F_x = \rho \int \theta(\frac{a}{2-a_0} \frac{dU}{dt} x + \frac{b}{2-b_0} \frac{dV}{dt} y) \cos \theta \, dA $$

$$ F_x = \rho \frac{a}{2-a_0} \frac{dU}{dt} \int x \cos \theta \, dA + \rho \frac{b}{2-b_0} \frac{dV}{dt} \int y \cos \theta \, dA \quad (2.33) $$

The first integral gives volume of the ellipsoid. The second is zero, thus

$$ F_x = \frac{a}{2-a_0} \frac{4}{3} \pi abc \rho \frac{dU}{dt} \quad (2.34) $$

and likewise,
\[ F_y = \frac{b_0}{2-b_0} \pi a b c \rho \frac{dv}{dt} \]  

(2.35)

where \( k_{xx} = \frac{a_0}{2-a_0} \)

\[ k_{yy} = \frac{b_0}{2-b_0} \]

which are the same expressions as those obtained when the ellipsoid moves parallel to its main axes separately.

It is concluded that, when the body moves in any arbitrary two-dimensional unsteady pure translational manner, the associated apparent mass components in the direction of the main axes do not change. As further evidence, Ibrahim determined the complex potential function for a circular arc moving in an arbitrary translational direction making an angle \( \gamma \) with one of its main axis. He, thus, evaluated the apparent mass components; namely, the apparent masses in the direction of the two main axes and also the coupling apparent mass component. He found that, the sum of the apparent masses in the directions of the two main axes is independent of the angle, \( \gamma \), showing that the sum of the apparent masses in any two perpendicular directions of translational motion is invariant.
CHAPTER 3.

FLUID RESISTANCE AND DEFINITION OF PARACHUTE APPARENT MASS COMPONENTS
3. FLUID RESISTANCE AND DEFINITION OF PARACHUTE APPARENT MASS COMPONENTS.

3.1. Fluid Resistance on a Body in Unsteady Motion.

3.1.1. Introduction

Attempts to determine the fluid resistance which is caused by the interaction between the fluid and the body moving in it, have been made by many authors. Brenner\(^{70,71}\) sought to characterize the Stokes resistance of a solid body of arbitrary shape which was translating and rotating in an incompressible, unbounded, viscous fluid at rest at infinity. The hydrodynamic resistance of the body could be expressed in terms of two symmetric, second-rank tensors, termed the translation tensor, \(a^t_{ij}\), and the rotation tensor, \(a^r_{ij}\), where \(i,j = 1, 2,3\). The translation tensor depended upon the external configuration of the body. In particular, it was independent of the orientation and speed of the body and also of fluid properties.

As an immediate consequence of the properties of symmetric tensors, every arbitrary body must possess at least three mutually perpendicular axes fixed to it such that if it is moving parallel to one of them it will experience a force only in this direction; that is, there will be no lateral force. Separation of the translational and rotational modes of the motion of the body depends upon the existence of an intrinsic geometrical point, termed the centre of hydrodynamic stress, which plays a fundamental role in hydrodynamic theory, as does the centre of mass in
conventional rigid body dynamics. Such a point exists only for bodies characterized by certain symmetry properties. About the centre of hydrodynamic stress the moment of the hydrodynamic forces is, by definition, zero. The converse is equally true; namely, a body rotating about any axis passing through the centre of hydrodynamic stress in an otherwise quiescent fluid will experience no hydrodynamic force.

If there is no center of hydrodynamic stress, an interaction occurs between translational and rotational motion. This interaction may be described in terms of a third, second-rank tensor, termed the coupling tensor, \( a_{ij}^c \), which is not symmetric in general.

Hence, the resistance of all rigid bodies moving in a fluid may be described in terms of these three fundamental tensors, \( a_{ij}^t \), \( a_{ij}^r \) and \( a_{ij}^c \), where \( i,j \) range from 1 to 3. They can be expressed in a matrix form as,

\[
\begin{bmatrix}
  t & t & t \\
  a_{11}^t & a_{12}^t & a_{13}^t \\
  a_{21}^t & a_{22}^t & a_{23}^t \\
  a_{31}^t & a_{32}^t & a_{33}^t \\
\end{bmatrix}
, \begin{bmatrix}
  r & r & r \\
  a_{11}^r & a_{12}^r & a_{13}^r \\
  a_{21}^r & a_{22}^r & a_{23}^r \\
  a_{31}^r & a_{32}^r & a_{33}^r \\
\end{bmatrix}
, \begin{bmatrix}
  c & c & c \\
  a_{11}^c & a_{12}^c & a_{13}^c \\
  a_{21}^c & a_{22}^c & a_{23}^c \\
  a_{31}^c & a_{32}^c & a_{33}^c \\
\end{bmatrix}
\]

(3.1)

3.1.2. Pure Translational Motion

Defining

\[
a^t = -\int_S ds \, \pi_0
\]
(3.2)

the coupling tensor becomes

\[
a^c = -\int_S \mathbf{r}_p \times (ds \pi_0)
\]
(3.3)
where $S_0$ denotes the surface of the body, $\tau_0$ is the stress tensor, $\delta s$ is the surface element, $r_P$ is the position vector of $P$ fixed to the body.

Thus, the force and moment which act on a body in pure translational motion respectively are,

$$F' = - \mu a^t V$$
$$\tau' = - \mu a^c V$$

where $V$ is its linear velocity and $\mu$ is the dynamic viscosity of the fluid.

Since the stress tensor, $\tau_0$, is independent of the origin, hydrodynamic forces are independent of the choice of the origin. However, the magnitude of the torque obviously depends on the choice of the origin.

The coupling tensor, $a^c$, depends upon the geometry of the body and upon the location of the origin. The rule by which the coupling tensor transforms from point to point is given as,

$$a^c_p = a^c_O - r_{OP} \times a^t$$

where $r_{OP}$ is the position vector of the point $P$ relative to an origin at $O$.

3.1.3. Pure Rotational Motion

The force and torque on a body arising from the pure rotational motion of the body may be expressed in the form

$$F'' = - \mu \Omega_0 \omega$$
$$\tau'' = - \mu a^r \omega$$
where $\omega$ is the angular velocity.

The dyadic $\omega_0$, as proved by Brenner, is the transpose of the coupling tensor. Thus, the hydrodynamic forces experienced by the body rotating about any axis through an origin, $0$, are

$$F'' = -\mu a^c \omega$$  \hspace{1cm} (3.9)

where $^t$ denotes the transpose matrix.

The transformation Law for the rotation tensor is given as

$$\dot{a}_P = \dot{a}_0 - r_{0P} \times a^t \times r_{0P} + a_0 ^c \times r_{0P} - r_{0P} \times a_0 ^c$$  \hspace{1cm} (3.10)

where $P$ is any point fixed to the body and $0$ represents a reference point.

### 3.1.4. Combined Translational and Rotational Motion

In general, where the body may simultaneously translate and rotate, the forces and torque acting on it can be written as

$$F = -\mu a^t V - \mu a^c \omega$$  \hspace{1cm} (3.11)

$$\tau = -\mu a^c \omega - \mu a^c V$$  \hspace{1cm} (3.12)

In contrast to the translation and rotation tensor, which are symmetric at all points, in general, the coupling tensor is not symmetric. Every body, irrespective of shape, possesses a unique, intrinsic geometrical point called the centre of reaction at which this coupling tensor is symmetric. Certain classes of bodies exist whose geometric symmetry is such that the coupling tensor, $a^c$, is zero.
In such cases, translational and rotational motions are un-coupled and the centre of reaction becomes identical to the centre of hydrodynamic stress. The latter then plays a role comparable to that of the centre of mass in rigid body dynamics about which there is no coupling between translation and rotation; at the centre of reaction the hydrodynamic force depends only on its instantaneous translational velocity, while the hydrodynamic torque about this point depends only on the instantaneous angular velocity. Consequently, about any point P which is fixed to the body the rotation and the coupling tensors with respect to the centre of reaction become

\[ a^p_r = a^r_R - r^*_R P \times a^t \times r^*_{PR} \]  
\[ a^c_p = r^*_R P \times a^t \]

respectively.

The coordinates of the centre of reaction, 
\( R_O (X^R_O, Y^R_O, Z^R_O) \) are given in terms of the resistance tensors\(^{71}\), as

\[ X^R_O = \frac{\alpha_{23} - \alpha_{22}}{\alpha_{22} + \alpha_{23}}, \quad Y^R_O = \frac{\alpha_{33} - \alpha_{32}}{\alpha_{33} + \alpha_{32}}, \quad Z^R_O = \frac{\alpha_{12} - \alpha_{13}}{\alpha_{11} + \alpha_{12}} \]  

Hence, the forces, \( F(X, Y, Z) \) and moment \( \tau(L, M, N) \) which act on a body which is translating, \( V(U, V, W, t) \) and rotating, \( \omega(P, Q, R, t) \) in a fluid can be written in matrix form as
\[
\begin{pmatrix}
(F) \\
(\tau)
\end{pmatrix} = -\mu \begin{pmatrix}
(a_{ij}^t)(a_{ij}^c) \\
(a_{ij}^c)(a_{ij}^r)
\end{pmatrix}
\] (3.16)

in which \( \dagger \) denotes the transpose matrix and \( i,j \) range from 1 to 3.

Alternately, this can be written as

\[
(F) = -\mu (a R) (V) \\
(\tau) = -\mu (a R) (\omega)
\] (3.17)

where \( i, j \) range from 1 to 6. It is usual to refer to this 6 x 6 square matrix as the total fluid resistance tensor, \( a_{ij} R \).

This total resistance can be separated into two parts

\[
a_{ij}^R = a_{ij}^D + a_{ij}^A
\] (3.18)

the first denoting the steady resistance and the second, representing the unsteady resistance, being the apparent mass tensor. It is a real, symmetrical matrix, the latter property being a consequence of the symmetry of the sub-matrices, \( a_{ij}^t \) and \( a_{ij}^r \).

In general, there are 36 scalar components of the apparent mass tensor, of which 21 are independent scalar; namely six for the translational tensor, \( a_{ij}^t \), six for the rotational tensor, \( a_{ij}^r \), and nine for the coupling tensor, \( a_{ij}^c \). By making appropriate simplifying assumptions which include symmetry considerations for immersed bodies, these can be reduced in number substantially.
3.1.5. Symmetry Considerations

Case 1. One plane of symmetry

The OXZ plane is the plane of symmetry as shown in Fig. 3.1. The existence of a plane of symmetry implies that the body is identical to itself when reflected in this plane.

Fig. 3.1: A body with one plane of symmetry.

Thus, the relationships among the components are

\[ a_{12} = a_{14} = a_{16} = a_{23} = a_{25} = a_{35} = a_{36} = a_{56} = 0 \]

and the apparent mass tensor becomes

\[
\begin{bmatrix}
  a_{11} & a_{13} & 0 & a_{15} & 0 \\
  a_{13} & a_{22} & 0 & a_{24} & 0 \\
  0 & a_{24} & a_{33} & 0 & a_{35} \\
  a_{15} & 0 & a_{35} & a_{44} & 0 \\
  0 & 0 & a_{35} & a_{46} & a_{55} \\
  0 & 0 & 0 & a_{55} & a_{66}
\end{bmatrix}
\]

(3.19)
Case 2: Two mutually perpendicular symmetry planes

If the body has a second plane of symmetry (the OYZ plane) at right angles to the first, and the origin lies anywhere along the line of intersection of the two planes, then in addition to case 1, the following relationships must also be satisfied.

\[ a_{13} = a_{26} = a_{35} = a_{45} = 0 \]

Thus the apparent mass tensor becomes

\[
\begin{pmatrix}
  a_{11} & 0 & 0 & 0 & a_{15} & 0 \\
  0 & a_{22} & 0 & 0 & 0 & 0 \\
  0 & 0 & a_{33} & 0 & 0 & 0 \\
  0 & 0 & 0 & a_{44} & 0 & 0 \\
  a_{15} & 0 & 0 & 0 & a_{55} & 0 \\
  0 & 0 & 0 & 0 & 0 & a_{66}
\end{pmatrix}
\]

Two principal axes lie normal to the two symmetry planes, while the third principal axes lies parallel to their intersection.

Case 3: Three mutually perpendicular symmetry planes

If the body has a third plane of symmetry (the OXY plane) at right angles to the two former one, then, in addition to case 2, it is also required that

\[ a_{i,j}^C = 0 \]

Then, if \( R_0 \) denotes the point of intersection of the three planes, the apparent mass tensor becomes
so that this point constitutes the centre of reaction of the body, at which the coupling tensor is zero.

Case 4: Spherically isotropic bodies.

If the preceding solid is such that the form of the body is similarly related to each of the three mutually coordinate planes, (e.g. a sphere or a cube), the co-ordinates axes $OX, OY, OZ$ are indistinguishable, and hence

$$a_{11} = a_{22} = a_{33}$$
$$a_{44} = a_{55} = a_{66}$$

therefore,

$$a_{i j}^A = \begin{bmatrix}
a_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & a_{11} & 0 & 0 & 0 & 0 \\
0 & 0 & a_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & a_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & a_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & a_{66}
\end{bmatrix}$$

(3.22)

These bodies are called spherically isotropic. Characterisation of their hydrodynamic resistance requires knowledge of only the two scalar components, $a_{11}$ and $a_{44}$.
Case 5: Bodies of revolution.

If the solid possesses symmetry of rotation about an axis (e.g. the z-axis in Fig. 3.1), the results of case 2 are applicable. Furthermore, the form of the solid would be unaltered if it were rotated through any angle about the other two axes. Thus, in addition to case 2, it is also required that

\[ a_{11} = a_{22}, \quad a_{44} = a_{55}, \quad a_{15} = -a_{24} \]

and hence

\[
A_{ij} = \begin{bmatrix}
  a_{11} & 0 & 0 & a_{15} & 0 \\
  0 & a_{11} & -a_{15} & 0 & 0 \\
  0 & 0 & a_{33} & 0 & 0 \\
  a_{44} & 0 & 0 & a_{44} & 0 \\
  0 & 0 & 0 & 0 & a_{66}
\end{bmatrix}
\]  

(3.23)

Following equations (3.15), the centre of reaction lies along the axis of revolution of this body.

3.2. Definition of Parachute Apparent Mass Components

Considering a parachute canopy to be a hemispherical shell possessing two planes of symmetry, i.e. the -xz and the -yz planes, and also rotational symmetry about the -z axis, as shown in Fig. 3.2., case 5 will be applicable. Consequently, there are eight components of the apparent mass tensor, of which five are independent.

As indicated by Brenner and Dorrepall, all spherical cups possess a centre of reaction \( R_0 \) at which
Fig. 3.2: Apparent mass components of a body of revolution.

\( a_{ij}^c = 0 \). That is, the translational and rotational motions are uncoupled at \( R_o \). Then, the hydrodynamic force on the cup depends only on the instantaneous translational velocity of \( R_o \) while the torque about \( R_o \) depends solely on the instantaneous angular velocity of the cup.

Consequently, if the components of the apparent mass tensor are expressed in terms of \( R_o \) rather than the arbitrary origin, there are only six apparent mass components, i.e. four independent components to be determined; these are,

\[
\begin{align*}
\alpha_{11} &= \alpha_{22} \\
\alpha_{33} &= \alpha_{44} \\
\alpha_{55} &= \alpha_{66}
\end{align*}
\]

3.3. Dimensional Analysis and Correlation Modulus

A characteristic feature of experiments performed to determine the hydrodynamic resistance in unsteady motion, has been the lack of suitable correlation moduli. Usually whole families of curves of the resistance against velocity
and acceleration have been given. The resistance coefficient has been plotted against the acceleration modulus, \( \frac{\dot{V}D}{V^2} \) by Iversen and Balent\(^3\), the period parameter, \( \frac{VT}{D} \) by Kaulegen and Carpender\(^4\), the relative displacement by Sarpkaya and Garrison\(^7\) and different correlation coefficients by other authors. However, for some bodies the scatter of their results is considerable. There follows a proposed correlation modulus which can be applied to the parachute canopy.

3.3.1. Dimensional Analysis

Let us consider the motion of a body through a viscous incompressible fluid. The body is assumed to be axially symmetric and moves through a uniform fluid in the direction of the axis of the symmetry. Suppose that the body is rigid and does not expand and is translating in the x-axis direction with velocity \( U \), as shown in Fig. 3.3.

Fig. 3.3: Dimensional analysis consideration of a rigid body moving with a velocity of \( U \) along the x-axis.
The resistance to motion experienced by the body is assumed to be a function of the fluid density, \( \rho \), kinematic viscosity, \( \nu \), velocity and acceleration and some arbitrary lengths, i.e.

\[ X = f(\rho, \nu, U, U') \] (3.24)

After some manipulations and simplifications, the force exerted on a body is expressed as

\[ X = K \rho U^2 \ell^2 f\left(\frac{UL}{\nu}, \frac{UL}{U^2}\right) \] (3.25)

where \( \frac{UL}{\nu} \) and \( \frac{UL}{U^2} \) are the Reynolds number and acceleration modulus respectively.

The concept of apparent masses arises fundamentally in the premise that \( X \) is independent of powers of velocity higher than the first. In the simplest application where a symmetrical body moves parallel to the x-axis, if the first power of the \( U \) alone is retained the expansion equation in canonical form is

\[ X = \rho U^2 \ell^2 f\left(\frac{UL}{\nu}\right) + \rho \ell^3 U f\left(\frac{UL}{U}\right) \] (3.26)

where the acceleration coefficient \( \rho \ell^3 f\left(\frac{UL}{\nu}\right) \) assumes the character of an effective apparent mass term.

With arbitrary conditions of motion, higher displacement derivatives would appear in the resistance functions, and the dimensional expression of the resultant force acquires the form

\[ X = \rho U^2 \ell^2 f\left[\frac{UL}{\nu}, \frac{UL}{U^2}, \frac{UL}{U^3}, \ldots, \frac{UL^{n-1}U^{n-1}}{U}, \frac{UL^n}{U^n}\right]. \] (3.27)
Some experimental evidence exists for the assumption that, for the motion of a parachute $X$ is independent of the powers of velocity higher than the first in the range of Reynolds number $2 \times 10^5 - 3 \times 10^5$. This is shown in Chapter 4.

3.3.2. Correlation Modulus

Considering, Frazer and Simmon's experimental work conducted on a sphere and a streamlined body with a 4:1 fineness ratio, both of which were towed through water, and for which these results were plotted against velocity and acceleration. From this work the total fluid resistance and the apparent mass coefficients are here determined and plotted against acceleration modulus for various Reynolds numbers.

As seen in Fig. 3.4a, for a sphere, the Reynolds number effect on both the total fluid resistance, $C_R$, and apparent mass, $k$, coefficients, is considerable and it is evident that correlation of the coefficients only with the acceleration modulus is not very convincing. However, as seen in Fig. 3.4b, for a streamlined body which has a 4:1 fineness ratio, scatter is quite small and the effect of Reynolds number on both coefficients is negligible. This is because with sphere the separation points vary considerably with the Reynolds number but little variation occurs with streamlined bodies. Hence, for bodies on which the point of separation is effectively independent of Reynolds number the total fluid resistance and the apparent mass coefficients correlate well with the acceleration modulus. As seen in Fig. 1.4 obtained by Keim, for a circular
Fig. 3.4a: Effect of Reynolds number on apparent mass and total fluid resistance coefficients, $k$ and $C_R$, for sphere.
Fig. 3.4b: Effect of Reynolds number on apparent mass and total fluid resistance coefficients, $k$ and $C_R$, for a streamlined body with a fineness ratio of 4:1.
cylinder the acceleration modulus also correlated well with the total fluid resistance coefficient.

This evidence, whilst by no means conclusive, indicates that for bodies whose steady drag coefficient remains approximately constant over a wide range of Reynolds numbers, such as parachute canopies, the acceleration modulus is a suitable correlation parameter.
CHAPTER 4.

EXPERIMENTAL DETERMINATIONS OF THE APPARENT MASS COMPONENTS AND FLUID RESISTANCE OF PARACHUTE CANOPIES.
4. EXPERIMENTAL DETERMINATIONS OF THE APPARENT MASS COMPONENTS AND FLUID RESISTANCE OF PARACHUTE CANOPIES.

4.1. Introduction

When a viscous fluid accelerated passes a stationary object or vice-versa, the motion which starts from rest is initially irrotational and unseparated. As the velocity increases, a boundary layer develops and a wake forms and grows. The formation of the wake gives rise not only to a form drag, as would be a case if the motion were steady, but also to significant changes in the inertial forces. The effects of unsteadiness depend upon the intensity and duration of the acceleration, the time required for the formation of the wake, and upon the shape of the body. Therefore, the acceleration dependent inertial resistance, i.e., the resistance due to the apparent masses is not the same for unsteady viscous flow as for unsteady inviscid flow and the velocity dependent form drag is not the same as for steady flow of a viscous fluid.

A number of experimental investigations have already been made on the apparent masses of spheres, discs, cylinders and other bodies and results, obtained in the existing literature, have been reviewed in Section 1.

There are few force equations by which the fluid resistance in unsteady motion can be calculated. The so-called Morrison equation

\[ F = \frac{1}{2} C_D \rho A_s |V|V + k \rho V_{\text{Vol}} \frac{dV}{dt} \]  

(4.1)

where \(|V|V\) has been introduced in a place of \(V^2\) in order to
maintain the proper sign for the drag force, $A_s$, in the surface areas and $V_{vol}$ is volume of the body, considers the force to be composed of a drag force which is proportional to the velocity squared and an inertia force which is proportional to the acceleration. The drag is described by the standard semi-empirical steady flow drag coefficient relationships. The inertia force is related to the hydrodynamical apparent mass.

Many authors have applied this equation, determining the coefficients necessary for their own use. In the literature, the force exerted on a body in unsteady motion was initially obtained as a single function of time. Eventually it was divided into two terms relating respectively to velocity and acceleration. A very simple approach would be to divide the force into drag and inertia terms, evaluating the apparent mass coefficient at zero water particle velocity and the drag coefficient at zero water particle acceleration respectively. The value of the drag coefficient (or the apparent mass coefficient) is then assumed constant and applicable even for non-zero acceleration (or non-zero velocity) and changes in force have fully to be worked into the second coefficient. Another method consists of adopting the $C_D$ value from steady flow and assuming that the drag coefficient is not subject to change in the case of unsteady flow.

The second equation indicated by Iversen and Balent, for the force exerted upon a body in accelerated motion through a fluid, can be expressed as the product of one coefficient, and fluid density, $\rho$, the surface area, $A_s$, 
and the square of the velocity, as

\[ F = \frac{1}{2} \rho V^2 C_R A_s \]  \hspace{1cm} (4.2)

where \( C_R \) is the total fluid resistance coefficient which depends upon the flow characteristics.

In the current experimental work, parachute model canopies were towed through water. The total fluid resistance coefficient, \( C_R \), in equation (4.2) was determined. Then, using equation (4.1) and adopting the value of the drag coefficient from steady flow measurement the apparent mass coefficient was evaluated as will be seen in Section 4.4.

As described in Chapter 3, for a parachute canopy which is a hemispherical shell, possessing a symmetry of rotation, there are 8 apparent mass components to be measured. These are (as seen in Fig. 4.1),

\[ a_{11} = a_{22} \]  \hspace{1cm} (motion along either the x-axis or the y-axis)
\[ a_{33} \]  \hspace{1cm} (motion along the z-axis)
\[ a_{44} = a_{55} \]  \hspace{1cm} (motion about either the x-axis or the y-axis)
\[ a_{66} \]  \hspace{1cm} (motion about the z-axis)
\[ a_{24} = -a_{15} \]  \hspace{1cm} (coupling between motion along the x-axis and about the y-axis, or along the y-axis and about the x-axis).

From equation (3.4) the coupling apparent mass components \( a_{15} = -a_{24} \) are zero at the centre of the reaction of the canopy. Consequently, any coupling apparent mass components in the co-ordinate system arbitrarily chosen will either be the products of the apparent masses \( a_{11} \) and \( a_{55} \) and the distance between the centre of reaction and the origin, or else the product of the apparent moment of inertia \( a_{44} = a_{55} \).
Fig. 4.1: Apparent mass components for hemispherical canopies.
and the square of the distance between the centre of reaction of the canopy and the origin. The full equations of motion of the parachute-load system are derived in Appendix C.

Since the z-axis is the axis of revolution of the parachute canopy, $a_{\omega}$ is zero for an ideal fluid. However, because of the existence of the boundary layer it may not be zero for a real fluid. In this present context it has been considered to be negligibly small and therefore, was not determined in the experimental programme.

4.2. Experimental Facilities

Since the fluid resistances due to the apparent mass components are proportional to the density of the medium in which they are measured, experiments were conducted in water using a ship tank at Southampton College of Higher Education.

4.2.1. Water Tank and Towing Carriage

The ship tank, illustrated in Fig. 4.2 and Fig. 4.3, was 61 m long, 3.7 m wide and 1.8 m deep. A towing carriage which accelerates and decelerates rapidly at each end of the tank length can travel over the water surface, maintaining a maximum velocity of about 5 m/s over most of the tank length. The carriage is driven by an electric motor with uniform or time-varying speed as required. Its motion can be adjusted by an automatic speed control system.

4.2.2. Experimental Apparatus

The determination of the apparent mass or the fluid resistance by towing techniques require facilities which
Fig. 4.3: Apparatus mounted on Towing Carriage.
can impart linear and angular velocity and linear and angular acceleration to the model which is being towed.

To impose the appropriate acceleration on the model the apparatus pictured in Fig. 4.4. and Fig. 4.5. and sketched in Fig. 4.6 was developed.

This basically consists of two circular rings, the outer one of which is rigidly attached to the towing carriage while the inner one carries a slider-crank mechanism operated by an electric motor with variable speed drive. A strain-gauged sting which supports the parachute canopy is free to rotate in the slider, but its orientation having been determined, it is then locked to the slider.

By rotating the inner ring relative to the outer the
Fig. 4.5: Hemispherical Parachute Model being accelerated by the Slider-Crank Mechanism.
Fig. 4.6: Principle of the Two Mechanisms.

4.2.3 Model and Mounting

In the measurement of the apparent mass components, nylon fabric models, made of hemispherical 40-104 m in diameter, and cruciform (3-48 m in arm length) canopies with 10 rigging lines were used. The supporting sting carried the models at its one end while the other end was rigidly attached to the...
track, along which the slider moves as the crank rotates, is aligned either parallel or perpendicular to the axis of the tank. If desired, it could also be set at any known intermediate position. Thus, while being towed through water by the motion of the carriage, the slider-crank mechanism can cause the sting-mounted parachute to be harmonically oscillated at a low frequency along any line in a plane parallel to the tank bed. By determining appropriate forces acting on the canopy using strain-gauge balances the apparent mass components, $a_{1}$, $a_{2}$, and $a_{3}$ are evaluated.

To determine the apparent moment of inertia terms, $a_{4}$ and $a_{5}$, the necessary angular motion was obtained using a further mechanism which was added to the existing apparatus, as seen in Fig. 4.6. It consists of a bar, $AB$, which is oscillated by a crank at $B$ and which is pin-jointed at its far end. It is free to move through a swivel block at $D$ mounted with ball bearings to the structure. In this way the original translational slider-crank mechanism was converted to angular motion of the block $D$, to which the strain-gauged sting was attached. In order to obtain the necessary pure rotational motion no translational motion of the towing carriage was required for this part of the tests.

4.2.3. Model and Mounting

In the measurement of the apparent mass components nylon fabric models of made of hemispherical (0.304 m in diameter) and cruciform (0.48 m in arm length) canopies with 16 rigging lines were used. The supporting sting carried the models at its one end while the other end was rigidly attached to the
slider, as seen in Fig. 4.4. and in Fig. 4.7. This sting was constructed from 31.75 mm (1/8") diameter stainless steel tube having a wall thickness of 1.62 mm (0.064"). Its natural frequency when the parachute model was attached was predicted and measured to be approximately 23 Hz. With a 0.304 m diameter model the vortex shedding frequency was estimated to be 1.05 Hz. Thus, the natural frequency of the sting was more than 20 times the vortex shedding frequency and both the vortex shedding frequency and the natural frequency were well above the test frequency, which was about 0.2 Hz. This can be observed in sample graphs Figs. 4.9a, 4.9b and 4.9c obtained when the sting without the model was towed by the carriage.

For the measurement of the apparent moment of inertia terms an aluminium tube, 25.4 mm (1") in diameter and 3.1 mm (0.124") thickness, was selected as the sting and a rigid hemispherical canopy was used, Fig. 4.7, instead of the flexible models employed in the earlier part of the programme.

With the maximum force considered to be acting on the model the maximum deflection and the maximum deflection angle at the end of the sting adjacent to the parachute canopy have been calculated and found to be 6.76 mm and 0.011 radian respectively. That is during the test programme effects caused by sting deflection are considered to be negligible. The dimensions of the sting are given in Table 1 and the relevant calculations are contained in Appendix B.
Fig. 4.7: Determination of forces and moments acting on the model parachute canopies.

Table 1. Dimensions of the Model Supporting Sting.

<table>
<thead>
<tr>
<th>Z (Newton)</th>
<th>d (mm)</th>
<th>η (mm)</th>
<th>L (mm)</th>
<th>f (mm)</th>
<th>θ (radian)</th>
<th>ω_n (Hz)</th>
<th>ω_v (Hz)</th>
<th>ω_z (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120</td>
<td>31.75</td>
<td>1.62</td>
<td>930</td>
<td>6.7</td>
<td>0.011</td>
<td>23</td>
<td>1.05</td>
<td>0.2</td>
</tr>
</tbody>
</table>
4.2.4. Force and Balances

In designing the strain gauge bridge, the aim was to obtain the highest electrical output from the very small forces and moments acting on the sting. Four active gauges formed the arms of a Wheatstone bridge. Two important advantages were to be had from using four active gauges and arranging them as in Fig. 4.8.

(i) The bridge was insensitive, except to forces acting in one direction only. Thus, if forces were measured in the normal direction, any effects arising from forces in the tangential direction would not affect the gauge's output.

(ii) The bridge had complete temperature compensation because the four gauges were symmetrically placed in the same medium, and thus they were subject to the same temperature effects.

To measure bending moments due to both the normal force and the tangential force and the torque due to their couple on the sting supporting the model three independent Wheatstone bridges were arranged as seen in Fig. 4.8.a.b.

The gauges, placed on the sting were to operate about 100 mm above the water level. Because humidity could impair their proper functioning, a tough and flexible rubber, Dow Corning Selastic 738 Adhesive/Sealant, was placed over the gauges. The arrangement proved extremely effective and no trouble was experienced during the measurements.

The static calibration was performed by attaching weights to the sting. It was found that the normal force
influence on the output of the strain-gauge bridge devised to measure the tangential force is negligible, while the tangential force influence on the output of the bridge arranged for the measurements of the normal force and torque is approximately one percent. In the experimental programme no corrections were made for this very small cross-coupling effect.

The electrical outputs from the strain-gauge bridges were suitably amplified and appropriate temperature compensation was applied. The arrangement of a bridge and amplified unit is shown in Fig. 4.8c.

4.3. Experimental Procedure and Data Analysis

In determining the apparent mass components, $a_u$, $a_m$, and $a_n$, the sting-supported model immersed approximately 0.8 m below the water level was harmonically oscillated at a low frequency by the slider-crank mechanism in the appropriate direction in a plane parallel to the tank bed, whilst being towed by the motion of carriage along the tank, as seen in Fig. 4.4. The speed of the carriage along the ship tank was automatically controlled and determined instant by instant through a series of counting pulses initiated by the motion of a permanent magnet attached to one of the carriage wheels. The angular displacement of the electric motor driving the slider-crank mechanism hence, the displacement of the model, was monitored by a potentiometer attached directly to the output shaft of the driving motor.

Hydrodynamic force components in the tangential and normal directions, $Z$ and $X$ respectively, were monitored
Fig. 4.8a: Arrangement of Wheatstone Bridge for the Measurement of Bending Moment.

Fig. 4.8b: Arrangement of Wheatstone Bridge for the Measurement of Torque.

Fig. 4.8c: Bridge, Temperature Compensation Unit and Amplifier System.
by the strain gauges bridges. This was done by measuring the bending moments, \( M_z \) and \( M_x \) acting on the sting (Fig. 4.7) together with their torque, \( \tau \). A differently designed mounting, also shown in Fig. 4.7 was used for the determination of the apparent moment of inertia.

The outputs from the strain-gauge bridge together with signals representing the displacement of the towing carriage and the angular displacement of the electric motor driving the slider-crank mechanism were recorded as analogue signals by a multi-channel chart recorder. From this chart, relevant velocities, accelerating forces and moments could be determined instant by instant during the test programme, as detailed in Appendix A.

4.4. Evaluation of the Apparent Mass Components

As described in the introduction to this chapter the total force acting on a body in unsteady motion is given by the equation (4.1). If \( R \) and \( D \) represent the total fluid resistance in unsteady and in steady flow respectively, equation (4.1) can be rewritten as

\[
R_{ij} - D_{ij} = (m + a_{ij}) \frac{dV}{dt} \tag{4.3}
\]

where \( m \) is the mass of the model and \( a_{ij} \) is the apparent mass component.

Expressing the forces in coefficient form

\[
\frac{1}{2} \rho V^2 A_s C_R = \frac{1}{2} \rho V^2 A_s C_D + (m + a_{ij})V \tag{4.4}
\]

where \( A_s \) is the surface area of the canopy, \( C_R \) is the
total fluid resistance coefficient and $C_D$ is the drag coefficient.

Defining the apparent mass components coefficients $k_{ij}$ (hereafter referred to as the apparent mass coefficient) by the relation

$$k_{ij} = \frac{\alpha_{ij}}{\rho V_{\text{vol}}} \quad (4.5)$$

where $\alpha_{ij}$ is the appropriate apparent mass components, $\rho$ is the density of the fluid in which the parachute canopy immersed and $V_{\text{vol}}$ is the volume of fluid displaced by the body, gives from equation 4.4

$$A = \frac{V^2 A_s}{V_{\text{vol}}} \left( C_{R ij} - C_{D ij} \right) - \frac{m}{\rho V_{\text{vol}}} \quad (4.6)$$

and thus, if appropriate components of $C_R$ and $C_D$ are known $k_{ij}$ is determined.

Similarly, the apparent moment of inertia coefficient, $k_{\omega}$ for a pure rotational motion is given by,

$$\tau = (I_{xx} + a_{\omega}) \dot{\omega} \quad (4.7)$$

Thus,

$$k_{\omega} = \frac{a_{\omega}}{I_{\text{hh}}} = \frac{1}{I_{\text{hh}}} \left( \frac{\tau}{\omega} - I_{xx} \right) \quad (4.8)$$

where $\tau$ is the torque developed on the sting supporting the parachute canopy, and shown in the second diagram of Fig. 4.7, $\dot{\omega}$ is the angular acceleration and $I_{\text{hh}}$ and $I_{xx}$ are the reference moment of inertia of the fluid displaced and the moment of inertia of the body respectively, each about the
x-axis.

Assuming the drag coefficient to be constant at the range of the Reynolds number at which tests were conducted $k_{ij}$ was determined. Since a parachute is approximately hemispherical in shape, in the evaluation, $V_{vol}$ and $I_h$ were taken as the volume of the fluid displaced and the moment of inertia of the fluid displaced respectively by a hemisphere, thus $V_{vol} = \frac{1}{12} \pi D^3$ and $I_h = \frac{1}{16} D^2 \rho V_{vol}$.

In order to estimate the effect of the sting, a few tests were carried out without a model in steady and unsteady flow conditions. Subtracting the forces developed from that measured when the model was attached, the influence of the sting was minimised. Sample graphs obtained from the chart for the steady flow condition and when the sting alone is oscillated in the tangential and normal directions separately are given in Figs.4.9a, 4.9b and 4.9c respectively. Also a sample graph obtained for a cruciform (4.1) canopy oscillating in the tangential direction is given in Figs.4.10a and 4.10b.

4.5. Experimental Results and Discussion

Hemispherical and cruciform shapes of parachute canopy were considered, the latter as a basis for comparison. The arm ratios (ratio of length to width of one of the rectangular members which comprise the cross, as shown in Fig. 4.11) of the cruciform canopies were 2:1, 3:1, 4:1 and 5:1. Steady flow data were obtained not only because of their relevance to the present test programme but because of they were needed to substantiate and extend the Reynolds
Fig. 4.9a: Sample oscillograph record of forces acting on the sting (without model) in steady motion.

Fig. 4.9b: Sample oscillograph record of forces acting on the sting (without model) oscillated in the tangential direction while being towed along the tank.

Fig. 4.9c: Sample oscillograph record of forces acting on the sting (without model) oscillated in the normal direction while being towed along the tank.
Fig. 4.10a: Sample oscillograph record of forces acting on the cruciform canopy with an arm ratio of 4:1 in steady motion.

Fig. 4.10b: Sample oscillograph record of forces acting on the cruciform canopy with an arm ratio of 4:1 oscillated in the tangential direction.
Arm ratio = \frac{l'}{r} 

l'_{\text{model}} = 0.48 \text{ m}

Fig. 4.11: Dimensions of a cruciform canopy.

number range obtained from wind tunnel and full scale data. The results obtained for these models by the steady motion of the carriage without oscillation are given as a function of the angle of attack in Fig. 4.12. They are discussed further by Jorgensen and Cockrell\textsuperscript{74}. It is noted that there is a considerable difference in the values obtained from rigid and flexible models of the same canopy.

Measurements made on the hemispherical canopy 0.304 m in diameter during the acceleration and the deceleration phases of the towing carriage when the slider-crank mechanism
was locked produced apparent mass coefficients under conditions in which the acceleration was approximately constant. Results which are shown as functions of the acceleration modulus in Fig. 4.13, compare well with those obtained when the motion of the carriage was steady but accelerations were imposed by the slider-crank mechanism in Fig. 4.14. In the latter case, the acceleration was not constant.

Values were obtained from the chart recorder at instant of maximum deflections of the slider-crank mechanism. At such instants, zero velocity is imposed on the parachute canopy by this mechanism and thus the angle of attack of the canopy is determined solely by its attitude relative to the longitudinal axis of the ship tank.

By imposing acceleration on the model by the slider-crank mechanism the tangential apparent mass coefficient and total fluid resistance coefficient of cruciform canopies for zero angle of attack were determined. Results obtained are given in Figs. 4.15, 4.16, 4.17 and 4.18 for cruciform 2:1, 3:1, 4:1 and 5:1 canopies respectively. Also values of the normal apparent mass coefficient for hemispherical canopies at zero angle of attack were obtained by oscillating the parachute canopy in the normal direction while is being towed by the steady motion of the carriage. These values are shown in Fig. 4.19.

Providing acceleration using the slider-crank mechanism in either the normal or the tangential direction while it is being towed by the carriage at various angle of attack, enables characteristics of the normal apparent mass
coefficient, $k_{11}$, or the tangential apparent mass coefficient $k_{33}$, v.s. the angle of attack to be determined. Results are given in Fig. 4.20 and in Fig. 4.21.

The parameters which were shown to have the greatest effect on parachute apparent mass and total fluid resistance coefficients were the canopy shapes, as can be seen by comparison of Fig. 4.14 and Fig. 4.17, the angle of attack, shown in Fig. 4.20 and Fig. 4.21 and the acceleration modulus which is the product of the parachute acceleration and its diameter divided by its velocity squared, $\delta = \frac{VD}{v^2}$, as can be seen in Figs. 4.14 .... 4.18. The discontinuity in $C_R$ and $k_{11}$ which appears in Fig. 4.14 and Fig. 4.15 when the acceleration modulus passes through zero is caused by the different shape which is presented to the fluid flow when the parachute accelerates negatively from that when it accelerates positively. A ten-fold variation of the acceleration modulus can produce a three-fold variation in the apparent mass coefficient.

The Reynolds number range based on canopy diameter, at which the results were obtained, varied from $2.10^5$ to $3.10^5$ which were $1/4$ of the full-scale Reynolds number. Results confirmed the conclusions drawn in section 3.6 from the experimental work of both Frazer and Simmons\textsuperscript{1} and Keim\textsuperscript{44} that although the Reynolds number effect on apparent mass and the total fluid resistance coefficient is significant for a sphere, because the point of flow separation varies with Reynolds number, with bodies whose shapes are such that little variation of the point of flow separation occurs with Reynolds number, such as parachute canopies, Reynolds number effects on apparent mass are of little
significance.

With large values of the acceleration modulus the apparent mass coefficients tend to their potential flow values which are 1.3. and 2.1. for normal and tangential apparent mass coefficients respectively. The reason suggested for this result is that a large acceleration modulus can be considered to have been brought about by a high acceleration and a low velocity. When real fluid flow round a body starts from rest it is initially irrotational and the thickness of the laminar boundary layer which develops on the body is proportional to the square root of the viscosity and to time, that is it decreases in thickness as the acceleration increases. With a high acceleration modulus, its thickness tends to be negligible. Under these conditions the flow round the body tends to behave like potential flow. With a decrease in acceleration modulus, however, separation of flow from the body causes a wake to form with a considerable variation in the apparent mass coefficients in consequence.

At the fixed values of the acceleration modulus, large variations in the apparent mass coefficients, especially in the tangential direction, occur with changes in angle of attack. These are seen in Fig. 4.20. The experiments have also shown that when both the angle of attack, and the acceleration modulus are constant the shape of the canopy, as reflected by the reduction in surface area brought about by the increase in arm ratio of the cruciform canopies, significantly affects the values of the apparent mass coefficients, as seen in Fig. 4.22. The steady force coefficient, \( C_T \), obtained from these graphs when the
acceleration modulus is zero compare well with the value obtained by the earlier steady flow measurements.

Tests were conducted on both a rigid hemispherical canopy and a cruciform canopy with a 2:1 arm ratio in which the relative acceleration possessed components in both the x-axis and the z-axis directions. Since the tensorial form of the apparent mass coefficients determined in a real fluid is, at best, only a postulation, the purpose of these tests was to see if in a real fluid the acceleration could be considered as if its x and its z components existed independently, the results being added vectorially together in these tests. This result was proved theoretically for ideal fluid in section 2.3.2. of chapter 2. Good agreement is shown in Fig. 4.23 and Fig. 4.24, between the apparent mass (or total fluid resistance) coefficient $k_{zz}$ (or $C_R$) derived by these tests and those values determined when acceleration was along the z-axis alone but at a corresponding acceleration modulus.

Although no other data are available with which to make direct comparison for the x-directed motion, the value derived in this direction appear to be appropriate too. In classical hydrodynamic theory two-dimensional unsteady flows, $U(t)$ and $W(t)$, produce components of force in both the x-axis and the z-axis direction, as

$$F_x = a_{11} \ddot{U} + a_{13} \ddot{W}$$

$$F_z = a_{33} \ddot{W} + a_{31} \ddot{U}$$

Because of the symmetry of the parachute canopy coupling
apparent mass components, \( a_{13} \) and \( a_{33} \) are zero. Therefore, the total force caused by the apparent masses in either the \( x \)-axis or the \( z \)-axis direction is the product of the apparent mass and the acceleration in the respective direction.

The apparent moment of inertia coefficient, \( k_m = k_{m3} \), was obtained by rotating a rigid model hemispherical canopy which had no rigging lines, using the apparatus in the second sketch of Fig. 4.7. Values were determined at instants in the cycle at which there was zero angular velocity but non-zero angular acceleration. These are shown as functions of the angular acceleration in Fig. 4.25.

At the time of measurement of \( k_m \), the apparent mass coupling coefficient, \( k_{13} \), was also determined. In order to find out the location of the centre of hydrodynamic reaction at which the coupling apparent masses are zero, two different positions of the origin along the axis of symmetry were selected and, within experimental error, the centre of hydrodynamic reaction was found to lie between the centre of volume of the canopy and the canopy hemline.

4.6. Experimental Uncertainties

4.6.1. Free Surface Effect

Since tests were carried out in a water tank with a free surface, it was necessary to study the effect of the free surface on the results. Waves which may be created by the body possess energy which is carried away and dissipated, and therefore, the body experiences a wave resistance. The wave resistance of a circular cylinder moving in a fluid is
given by Lamb\textsuperscript{68} and is dependent upon the Froude number. The maximum wave resistance occurs when this number is

\[
\text{Froude number} = \frac{V}{\sqrt{gh}} = 1 \quad (4.9)
\]

where \( V \) is the maximum velocity, \( g \) is the gravitational acceleration and \( h \) is the depth of immersion measured to the centre of the body. This depth must be large compared with the radius of the cylinder. In the present tests, the range of the velocity varied from 0.8 m/s to 1.3 m/s and \( h \) was 0.85 m, so that the maximum Froude number was approximately 0.3. Referring to Srokosz\textsuperscript{35} and Szebehely\textsuperscript{46}, if the ratio of immersion to the diameter of the body is higher than 2, the free surface effect can be considered to be negligible. Since the ratio of immersion to the diameter of canopy was 2.8, the free surface effect was not considered during the analysis of the test data. No tests were performed at different depths.

4.6.2. Blockage Effect

When a body is immersed in a stream of fluid in a channel of finite breadth, the presence of walls and the wake of the body itself influence the flow round the body. This means that the force differs from that which is experienced in an infinite stream. Keeping the size of the body small in comparison with the size of channel minimises this effect. In the present test programme the ratio of the canopy frontal area to cross-sectional submerged area of the water tank is approximately 1% and the appropriate E.S.D.U. Data Item\textsuperscript{75} shows that under these conditions the blockage
effect can be considered negligible.

In order to avoid errors which may arise in the measurement of $C_R$ and $C_D$, average values were obtained graphically over the range of tests, as seen in Fig. 4.14 .... Fig. 4.18. Having taken the difference between the average values of the total fluid resistance coefficient, $C_R$, and the steady force coefficient, $C_D$, and accounting for possible calibration and zero drift errors of both the strain-gauge bridge amplifier and the chart recorder system, the uncertainty in the values obtained for the apparent mass coefficients are believed to be less than 10%.
Fig. 4.12: Aerodynamic coefficients vs. angle of attack, $\alpha$, for five different canopies tested in water. $Re = 6.1 \times 10^5$. $C_T$, $C_N$, and $C_M$ are based on the surface area.
Fig. 4.13: Variation of \( k_{33} \) and \( C_R \) with acceleration modulus, \( \delta \), for hemispherical canopies accelerated by the towing carriage.
Fig. 4.14: Variation of $k_{33}$ and $C_R$ with acceleration modulus, $\delta$, for hemispherical canopies.
Fig. 4.15: Variation of $k_{33}$ and $C_R$ with acceleration modulus, 6, for cruciform canopy with an arm ratio of 2:1.
Fig. 4.16: Variation of $k_{33}$ and $C_R$ with acceleration modulus, $\delta$, for cruciform canopy with an arm ratio of 3:1.
CRUCIFORM CANOPY

Arm ratio = 4:1

Angle of attack = 0 degree

\[ k_{33} \] versus Acceleration modulus \( \delta(\frac{V_D}{V^2}) \)

\[ x: C_R \]
\[ \Delta: k_{33} = \frac{\alpha_{33}}{\delta \cdot V_{vol}} \]

Fig. 4.17: Variation of \( k_{33} \) and \( C_R \) with acceleration modulus, \( \delta \), for cruciform canopy with an arm ratio of 4:1.
Fig. 4.18: Variation of $k_{33}$ and $C_R$ with acceleration modulus, $\delta$, for cruciform canopy with an arm ratio of 5:1.
Fig. 4.19: Variation of $k_{11}$ (or $k_{22}$) with acceleration modulus, $\sigma$, for hemispherical canopies.
Fig. 4.20: Variation of $k_{33}$ with angle of attack, $\alpha$, for hemispherical and cruciform canopies.
Fig. 4.21: Variation of $k_{11}$ (or $k_{22}$) with angle of attack, $\alpha$, for hemispherical canopies.
Fig. 4.22: Variation of $k_{33}$ with acceleration modulus, $\delta$, for cruciform canopies.
Fig. 4.23: Variation of $k_{11}$ (or $k_{22}$) and $k_{33}$ with the intermediate angle between the direction of acceleration and the z-axis, $\gamma$, rigid hemispherical canopies.
Fig. 4.24: Variation of \( k_{11} \) (or \( k_{22} \)) and \( k_{33} \) with the intermediate angle between the direction of acceleration and the z-axis, \( \gamma \), for cruciform canopy with an arm ratio of 2:1.
Fig. 4.25: Variation of $k_{44}$ (or $k_{55}$) with angular acceleration, $\omega$, for rigid hemispherical canopy.
CHAPTER 5.

THE EQUATIONS OF MOTION AND STABILITY
CRITERIA OF A PARACHUTE-STORE SYSTEM.
5. THE EQUATIONS OF MOTION AND STABILITY CRITERIA OF A PARACHUTE SYSTEM.

5.1. Equations of Motion.

The equations of motion in the absence of any symmetry planes of the canopy store system have been derived in Appendix C, in a form which describes the rigid parachute store-system with six degrees of freedom.

By making the following simplifying assumptions:

(i) the parachute store system has two planes of symmetry, namely, the x-z plane and the y-z plane, as shown in Fig. 5.1. This also implies that the system has a rotational symmetry about the z-axis,

(ii) the apparent mass and apparent moment of inertia components are defined at the centre of reaction of the canopy, \( R_0 \). Neglecting apparent moment of inertia component about the z-axis, 5 apparent mass components remain, namely, \( a_{11} = a_{22} = a_{33} \) and \( a_{44} = a_{55} \),

(iii) the apparent mass and apparent moment of inertia of the store are negligible,

(iv) the aerodynamic forces act at the centre of volume of the canopy which is assumed to coincide with the centre of reaction of the canopy,

(v) a flat earth is assumed,

the equations of motion with six degrees of freedom become
Fig. 5.1: System of axes and definition of terms.
\[ X = (m + a_{11}) \ddot{U} + (m z_s + a_{11} z_a) \dot{Q} + (m + a_{33}) QW + (m + a_{22}) RV - (a_{22} z_a - m z_s) PR \]
\[ Y = (m + a_{11}) \ddot{V} - (m z_s - a_{22} z_a) \dot{P} + (m + a_{11}) RU + (m z_s + a_{11} z_a) PQ - (m + a_{33}) PW \]
\[ Z = (m + a_{33}) \ddot{W} + (m + a_{22}) PV - (m z_s - a_{22} z_a) \dot{P}^2 - (m + a_{11}) QU - (m z_s + m z_a) Q^2 \]
\[ L = (I_{xx} + m z_s^2 + a_{11} z_a^2) \dot{P} + (a_{22} z_a + m z_s) \dot{V} - (a_{11} z_a + m z_s) RU + (a_{22} z_a + m z_s) PW + [(I_{yy} - I_{yy}) - (a_{33} + a_{11} z_a^2) - m z_s^2] QR - (a_{33} - a_{22}) VW \]
\[ M = (I_{yy} + m z_s^2 + a_{33} + a_{11} z_a^2) \dot{Q} + (a_{11} z_a + m z_s) U + (a_{11} z_a + m z_s) QW + [I_{zz} - (I_{xx} + m z_s^2) - (a_w + a_{11} z_a^2)] PR - (a_{22} z_a + m z_s) RV + (a_{11} - a_{33}) UW \]
\[ N = I_{zz} R + (a_{22} - a_{11}) z_a QV + [(I_{yy} - I_{xx}) + (a_{33} + a_{11} z_a^2) - (a_w + a_{22} z_a^2)] PQ + (a_{22} - a_{11}) UV \]

where \( I \) is the moment of inertia of the store about its corresponding axis, \((X, Y, Z)\) and \((L, M, N)\) are the external force and external moments respectively about the \(x, y\) and \(z\) axes; \((U, V\) and \(W)\) are the respective linear velocity components and \((P, Q,\) and \(R)\) are the respective angular velocity components of the origin. The mass of the system is denoted by \(m\) and \(z_s\) and \(z_a\) are the distance along the \(z\)-axis from the origin to the mass centre and to the centre of reaction of the canopy respectively.

Since \((a_{33} - a_{22}) VW\), \((a_{22} - a_{11}) UW\) and \((a_{22} - a_{11}) UV\) terms in the moment equations (5.1) represent the linear steady pitching moment about the corresponding axis of the canopy, by studying the Kirchhoff equation in Appendix C for unaccelerated flow conditions without rotation the components...
of moment become

\[
L = (a_{22} - a_{11}) VW \\
M = (a_{11} - a_{33}) UW \\
N = (a_{22} - a_{11}) VU
\]

and thus, these moments are simply the pitching moments about the corresponding axes and would therefore be included in the experimental pitching moment data. The moments \( L, M \) and \( N \) are defined as external moments which are gravitational and aerodynamic in origin, thus in the following analysis these terms on the right hand side of equation (5.1) should be omitted.

The trajectory calculations are carried out in an earth fixed axes system \((x^1, y^1, z^1)\). The positive \( z^1 \) direction is downwards; the \( x^1 \) and \( y^1 \) axes are in a plane parallel to the surface of the earth, as seen in Fig. 5.1. Both systems, \((x, y, z)\) and \((x^1, y^1, z^1)\), are orthogonal and are right handed. The \((x^1, y^1, z^1)\) axes are aligned with the \((x, y, z)\) axes by a series of three rotations in the following order,

\[
\psi \quad \text{a rotation about } z^1, \\
\theta \quad \text{a rotation about } y^2 \text{ (position of } y^1 \text{ after } \psi \text{)} \\
\phi \quad \text{a rotation about } x^2 \text{ (position of } x^1 \text{ after } \psi \text{ and } \theta) \\
\]

\[
\frac{dx^1}{dt} = U \cos \psi \cos \phi + V (\sin \phi \sin \psi \cos \psi - \cos \phi \sin \psi) + W (\cos \phi \sin \phi \cos \phi) + \sin \phi \sin \psi \\
\frac{dy^1}{dt} = U \sin \phi \sin \psi + V (\sin \phi \sin \psi \sin \psi + \cos \phi \cos \psi) + W (\cos \phi \sin \phi \sin \psi) - \sin \phi \cos \psi \quad (5.2a)
\]
\[
\frac{dz_i}{dt} = -U \sin \theta + V \sin \phi \cos \theta + W \cos \phi \cos \theta
\]

The angular velocities are related to the Euler angles by the relations

\[
P = \dot{\phi} - \dot{\psi} \sin \theta \\
Q = \dot{\psi} \cos \phi - \dot{\phi} \cos \theta \sin \phi \\
R = \dot{\phi} \cos \theta \cos \phi - \dot{\psi} \sin \phi
\]

(5.2b)

5.2. Stability Criteria of the Parachute-Store System

In this section, the static and dynamic stability of the system are considered. The appropriate and exact stability criteria are developed.

5.2.1. Static Stability.

Let 0 be the origin, G the centre of gravity and X and Z normal and tangential force components respectively acting on the canopy, as seen in Fig. 5.2. Resolving force components parallel and perpendicular to the axis of symmetry and taking moment about G gives the condition for equilibrium,

\[
Z = mg \cos \theta \\
X = mg \sin \theta
\]

(5.3) (5.4)

where \( X = \frac{1}{2} \rho V^2 C_N(\alpha)A \), \( Z = \frac{1}{2} \rho V^2 C_T(\alpha)A \).

Thus, \( X(z_a + z_s) + \frac{1}{2} \rho V^2 C_M(\alpha)A.D = 0 \) (5.5)

where \( m \) is the mass of the system, \( \theta \) is the pitch angle, \( D \) is the diameter of the canopy, \( C_N(\alpha) \), \( C_T(\alpha) \) and \( C_M(\alpha) \) are the aerodynamic normal force, the aerodynamic tangential force and the pitching moment coefficients respectively.
Fig. 5.2: Parameters of a descending parachute.
Hence, for static equilibrium the balance between $C_N(\alpha)$ and $C_M(\alpha)$ must be established. From equation (5.5), the steady equilibrium condition is found,

$$\frac{C_N(\alpha)}{C_M(\alpha)} = \frac{D}{(z_a + z_s)}$$

This condition is set at the appropriate angle of attack of the parachute-store system.

$C_N(\alpha) = C_M(\alpha) = 0$ is also equilibrium condition. From equation (5.4), this condition yields $\theta = 0$ and parachute descends in the vertical direction.

But because of the unusual shape of the normal and pitching moment coefficient curves, some parachutes do not descend vertically, even if they are statically stable. Fig. 5.3. shows typical canopy aerodynamic coefficients as function of the angle of attack, $\alpha$. Because the normal force and pitching moment change direction, if $\alpha$ is varied between $-\alpha_E$ and $+\alpha_E$, $C_N(\alpha)$ and $C_M(\alpha)$ characteristics are antisymmetric. Even though $C_N(\alpha)$ and $C_M(\alpha)$ are zero at the point ($\alpha = 0$), a parachute canopy such as one of hemispherical or aeroconical shape is not stable at that point. Any slight disturbances cause a normal induced outward force which will drive the parachute to a new attitude where $\alpha = \pm \alpha_E$. At this point, the weight of the parachute store system is balanced by the tangential aerodynamic force while parachute glides at the $\alpha = \alpha_E$ angle. This angle, $\pm \alpha_E$, is called the equilibrium angle of attack or the glide angle.
If the normal force and the pitching moment coefficients are both zero at the point, \( \alpha = \pm \alpha_E \), i.e. the centre of pressure of the canopy is on the axis of symmetry at that angle, the parachute descends with zero pitch angle \( \theta = 0^\circ \). Otherwise the normal force at the point where \( C_M (\alpha) = 0 \) will cause the parachute to establish a new equilibrium pitch angle where \( \theta \neq 0^\circ \), such as shown in Fig. 5.3 for a hemispherical canopy. The equilibrium angle of attack, \( \alpha_E \) for hemispherical, aeroconical and for a cruciform canopy having a 4:1 arm ratio are approximately 32, 25 and 0 degrees respectively. More details on the static stability are given by Jorgensen \(^7\).  

As will be shown, the glide point at which the parachute is statically stable may not lead to dynamic stability, so that a given parachute may not continue to glide steadily, but may oscillate with increasing amplitude.

5.2.2. Dynamic Stability

This section is divided into two parts. The first part is concerned with the appropriate stability in which by linearizing the equations of motion a small disturbances theory is used and the effect of various parameters are studied. The second part contains the exact stability criteria in which the equations of motion (5.1) are directly solved, without recourse to linearization.

5.2.2.1. Appropriate Stability Criteria

The equations of motion (5.1) in the two planes of symmetry with only three degrees of freedom further reduce to
Fig. 5.3: Aerodynamic forces and moment characteristics based on the cross-sectional area vs. angle of attack, $\alpha$, for aeroconical, hemispherical and 4:1 arm ratio cruciform canopies.
\[ X = (m + a_{11}) U + (m z_s + a_{11} z_a) Q + (m + a_{11}) QW \] (5.7)

\[ Z = (m + a_{11}) W - (m + a_{11}) QU - (m z_s + a_{11} z_a) Q^2 \] (5.8)

\[ M = (I_{yy} + m z_s^2 + a_{11} z_a^2) Q + (a_{11} z_a z_s + m z_s) U + (a_{11} z_a + m z_s) QW \] (5.9)

Expressing the external forces, \( X \) and \( Z \), and moment, \( M \), in terms of their aerodynamic and gravitational components,

\[ X = -\frac{1}{2} \rho V_c^2 C_N(\alpha) A - mg \sin \theta \] (5.10)

\[ Z = -\frac{1}{2} \rho V_c^2 C_T(\alpha) A + mg \cos \theta \] (5.11)

\[ M = -\frac{1}{2} \rho V_c^2 C_M(\alpha) A D - \frac{1}{2} \rho V_c^2 C_N(\alpha) A z_a - mg z_s \sin \theta \] (5.12)

where \( C_N(\alpha) \), \( C_T(\alpha) \) and \( C_M(\alpha) \) are the aerodynamic force and moment coefficients respectively at the centre of volume of the canopy, \( \theta \) is the pitch angle, \( V_c \) is the resultant velocity, \( z_s \) and \( z_a \) are defined in Fig. 5.2 and \( A \) is an appropriate canopy area.

The aerodynamic force components, \( C_N(\alpha) \), \( C_T(\alpha) \) and \( C_M(\alpha) \) are functions of the instantaneous angle of attack, \( \alpha \).

Assuming \( C_M(\alpha) \) is small in comparison with \( C_N(\alpha) \), so that it may be neglected, and \( C_N \) and \( C_T \) can be written as

\[ C_N = \frac{dC_N}{d\alpha_c} \alpha_c \] (5.13)

\[ C_T = C_{T_o} + \frac{dC_T}{d\alpha_c} \alpha_c \] (5.14)

where \( \alpha_c \) is the angle of attack with respect to the centre of volume of the canopy.

For the stability analysis we consider small disturbances
from a reference state corresponding to vertical descent with speed \( V_c = V_0 \). Supposing the disturbance is so small, so that

\[
\tan a_c = a_c = \frac{U_c}{W_c} = \frac{U + z \dot{a}}{W}
\]

where \( U_c \) and \( W_c \) are the velocity components at the centre of volume of the canopy and \( U \) and \( W \) are the velocity components of the origin, \( O \).

Thus,

\[
a_c = a + z \frac{Q}{a} W
\]

(5.15)

Assuming that the variations of \( C_T \), with the angle of attack around the equilibrium is small, the aerodynamic force coefficients can be expressed as

\[
C_N(a) = C_{N_0}(a + z \frac{Q}{a} W)
\]

(5.16)

\[
C_T(a) = C_{T_0} = C_T
\]

(5.17)

where \( C_{N_0} = \frac{dC_N}{da} \).

Thus, the expressions (5.10), (5.11) and (5.12) become

\[
X = -\frac{1}{2} \rho \frac{V_c^2}{c} A C_N(a + z \frac{Q}{a} W) - mg \sin \theta
\]

(5.18)

\[
Z = -\frac{1}{2} \rho \frac{V_c^2}{c} A C_T a + mg \cos \theta
\]

(5.19)

\[
M = -\frac{1}{2} \rho \frac{V_c^2}{c} A z \frac{Q}{a} C_N(a + z \frac{Q}{a} W) - mg z_s \sin \theta
\]

(5.20)

where \( V_c \) is the resultant velocity.

It is supposed that the system is slightly disturbed
from its equilibrium position at which $\theta = \alpha = 0$, and the initial velocity of $V_0$. We assume that $\alpha, \theta, \dot{\theta}, \ddot{\theta}, \dot{\alpha}$ and $\ddot{\alpha}$ are all small and their products and squares may be neglected.

After the disturbance

$$V_c = V_0 + \Delta V$$

$$W = (V_0 + \Delta V) \cos \alpha$$

$$U = (V_0 + \Delta V) \sin \alpha$$

$$\ddot{W} = \Delta \dot{V} \cos \alpha - (V_0 + \Delta V) \sin \alpha \dot{\alpha} = \Delta \dot{V} \quad (5.21)$$

$$\ddot{U} = \Delta \dot{V} \sin \alpha + (V_0 + \Delta V) \cos \alpha \dot{\alpha} = V_0 \dot{\alpha}$$

$$Q = \ddot{\theta}$$

$$\dot{Q} = \ddot{\theta}$$

Substituting into equations (5.7), (5.8) and (5.9) and also considering the time variation of the apparent mass components in the equations,

$$X = (m + a_{31}) V_0 \dot{\alpha} + (m + a_{31}) \dot{\theta}(V_0 + \Delta V) + (mz_s + a_{11} z_a) \ddot{\theta} + a_{31}(V_0 + \Delta V) \theta + z_a \ddot{\theta} \quad (5.22)$$

$$Z = (m + a_{33}) \Delta \dot{V} + \dot{a}_{33}(V_0 + \Delta V) \quad (5.23)$$

$$M = (I_{yy} + mz_s^2 + a_{55} + a_{11} z_a^2) \ddot{\theta} + (mz_s + a_{11} z_a) [V_0 \ddot{\theta} + \dot{\theta}(V_0 + \Delta V)]$$

$$+ a_{31}(z_a \ddot{\theta} + z_a \Delta \dot{V}) \quad (5.24)$$

where $\dot{a}_{31} = \frac{da_{31}}{dt}$ and $\dot{a}_{33} = \frac{da_{33}}{dt}$.

5.2.2.1.1. Vertical Motion

Considering equation (5.23) which represents tangential disturbances, and noting
\[ Z = -\frac{1}{2} \rho V_c^2 C_T A + mg \]

where at equilibrium,

\[ mg = \frac{1}{2} V_o^2 C_T A, \]

hence, equation (5.23) becomes

\[
(m+a_{33}) \Delta V + (\dot{a}_{33} + \rho C_T V_o A)\Delta V = -\dot{a}_{33} V_o. \tag{5.25}
\]

This is a first-order differential equation. The solution is

\[
\Delta V = E e^{-\frac{m+a_{33}}{m+a_{33}}} \tag{5.26}
\]

Since \( \dot{a}_{33} \) is small in comparison with the second term in the equation (5.26), the vertical disturbance, \( \Delta V \) decays exponentially with time.

### 5.2.2.1.2. Lateral Motion

The motion normal to the reference plane (OXZ) is described by the two equations, termed lateral equations, which are (5.22) and (5.24).

Substituting the result previously obtained for \( \Delta V \) into (5.22) and (5.24), and noting,

\[
\frac{1}{2} \rho V_o^2 A = \frac{mg}{C_T}
\]
equations (5.22) and (5.24) yield,

\[
- \frac{mg}{C_T} C_N a \left( \frac{z_a}{V_o} \right) - mg \dot{\theta} = (m+a_{11}) V_o \ddot{a} + (m+a_{11}) V_o \dot{\theta} + (mz_s + a_{11} z_a) \ddot{\theta} + \dot{a}_{11} (V_o a + z_a \dot{\theta}) \tag{5.27}
\]
Non-dimensionalizing all coefficients,

\[ t = \frac{L}{V_o} t^* \quad , \quad \frac{d}{dt} = \frac{V_o}{L} \frac{d}{dt^*} \quad , \quad \frac{d}{dt^2} = \frac{V_o^2}{L^2} \frac{d}{dt^2} \]

and multiplying equations (5.27) and (5.28) by \( \frac{1}{mV_o^2} \) respectively, where \( L \) is a characteristic length, \( mV_o \)

those equations become

\[
\frac{mz_s + \alpha_{11}z_a}{mL} \ddot{\theta} + \left( \frac{mz_s + \alpha_{11}z_a}{m} + \frac{g_z a}{V_o C_T N_a} \right) \theta + \frac{g L}{V_o} \theta + \frac{m + \alpha_{11}}{m} a = 0 \quad (5.29)
\]

\[
\frac{I_{yy} + m z_s^2 + \alpha_{33} + \alpha_{11} z_a^2}{mL^2} \dddot{\theta} + \left( \frac{m z_s + \alpha_{11} z_a}{m} \right) \dot{\theta} + \frac{g z_a}{V_o C_T N_a} \ddot{\theta} + \frac{g z_s}{V_o} \theta = 0 \quad (5.30)
\]

The equations are now in the form of

\[
a_1 \ddot{a} + a_0 a + b_2 \dddot{\theta} + b_1 \dot{\theta} + b_0 \theta = 0 \quad (5.31)
\]

\[
c_1 \dddot{a} + c_0 a + d_2 \dddot{\theta} + d_1 \dot{\theta} + d_0 \theta = 0 \quad (5.32)
\]

where
They are ordinary second order linear differential equations with constant coefficients, apart from the \( \dot{a}_{11} \) term. Solutions of these equations exist only if the determinant of the coefficients is equal to zero,

\[
\begin{vmatrix}
a_1 \lambda + a_0 & b_2 \lambda^2 + b_1 \lambda + b_0 \\
c_1 \lambda + c_0 & d_2 \lambda^2 + d_1 \lambda + d_0
\end{vmatrix} = 0 \quad (5.34)
\]

where \( \lambda \) represents differentiation with respect to time.

From (5.34) the characteristic equation is obtained.

\[
A_{11} \lambda^3 + A_{22} \lambda^2 + A_{11} \lambda + A_{00} = 0 \quad (5.35)
\]

where
\[ A_{33} = a_1 d_2 - c_1 b_2 \]
\[ A_{32} = a_1 d_1 + a_0 d_2 - c_0 b_2 - c_1 b_1 \]
\[ A_{31} = a_1 d_0 + a_0 d_1 - c_1 b_0 - c_0 b_1 \]
\[ A_{00} = a_0 d_0 - c_0 b_0 \]

There are three roots of this characteristic equation; a real root, \( \lambda_1 \), and a complex pair of roots, \( \lambda_2 \) and \( \lambda_3 \). This means that the parachute carries out two motions, namely an aperiodic side-slip motion relative to the earth, and an oscillatory motion.

(i) Aperiodic Side-slip Motion.

If the real root, \( \lambda_1 \), is negative, the aperiodic motion decreases exponentially with time,

\[ a = E_0 e^{\lambda_1 t}, \quad \theta = B_0 e^{\lambda_1 t}. \]  \( (5.36) \)

A short computer program written to solve (5.35) through (5.29), (5.30) is given in Appendix F. The system parameters chosen are: \( m_s = 120 \) kg, \( m_c = 5 \) kg, \( D = 5.1 \) m, \( z_s = 8 \) m, \( z_a = -0.8 \) m, \( V_0 = 8 \) m/s, \( C_T = 0.7 \). It was found that \( \lambda_1 \) is negative for all experimental values of the apparent masses provided that \( \frac{dC_N}{da} \) is positive. Keeping \( \frac{dC_N}{da} \) constant and equal to 0.15, \( \lambda_1 \) reached its maximum value when \( \frac{a_{11}}{a_{11}} = 1 \), then decreased when the latter increased or decreased, as seen in Fig. 5.4. The effect of \( a_{ss} \) was found to be negligible.

The motion of the canopy and the store are defined by the pitch angle, \( \theta \), and the angle, \( \phi \), which is the angle between the resultant velocity and the vertical earth axis respectively, as shown in Fig. 5.2.
where

$$\theta = \alpha + \beta$$

and

$$B_0 e^{\lambda t} = E_0 e^{\lambda t} + C_0 e^{\lambda t}.$$  

Employing the equations (5.31) or (5.32), the ratio of the amplitude, \( \frac{E_0}{C_0} \), may be calculated, as

$$\frac{E_0}{C_0} = \frac{-b_2 \lambda_1^2 - b_1 \lambda + b_0}{a_1 \lambda_1 + a_0 + b_2 \lambda_1^2 + b_1 \lambda + b_0}.$$  

(5.37)

The values of \( \frac{E_0}{C_0} \) obtained are given as a function of \( \frac{a_{ii}}{\sigma_{ii}} \) in Fig. 5.5. It was seen that \( \frac{E_0}{C_0} \) is negative over the range of the apparent mass ratios. This signifies that after a disturbance from which an aperiodic motion relative to air originates, the angle of attack, \( \alpha \), increases as the angle, \( \beta \), but in the opposite direction. As mentioned in Section 5.2., the parachute therefore, drifts sideways while its axis of symmetry remains nearly vertical.

According to the calculations for the given parachute parameters this drift is aperiodically stable.

As mentioned by Henn, the drift originates as follows. Where vertical descent is disturbed in such a way that the parachute drifts sideways, but remains nearly vertical, the resultant air flow acts on the load in a vertical direction, thus compensating for the weight of the load. This means, however, that the parachute carries out a definite sideways drift. This sideways drift increases where the parachute
is statically unstable around the load. In this case the canopy is steadily being pulled sideways and thus there is an effective force perpendicular to the path which steadily increases the drift. When a certain finite drift is obtained, due to this instability, the direction of the relative airflow on the canopy may affect separation phenomena which themselves lead to a new equilibrium state in which the parachute descends with a constant pitch angle. As mentioned in section 5.2, the equilibrium drift angle and pitch angle entirely depend upon the parachute aerodynamic characteristics, i.e., on the design of the parachute canopy.

(ii) Oscillatory Motion

The dynamic stability of the system depends upon the sign of the real part of the complex pair of roots,

$$\lambda_{z,1} = \lambda \pm i \omega$$

where $\lambda_{z,1}$ is the complex root of the characteristic equation (5.35). The solution gives oscillatory motion:

$$\alpha = e^{\lambda t^*} (D \cos \omega t^* + F \sin \omega t^*) \quad (5.38)$$
$$\theta = e^{\lambda t^*} (G \cos \omega t^* + H \sin \omega t^*). \quad (5.39)$$

If $\lambda$ is negative, the system is stable.

According to Etkin, in the cubic equation (5.35) the necessary and sufficient conditions for dynamic stability are:

$$A_{ii}, A_{zz}, A_{ii}, A_{oo} > 0 \quad (5.40)$$

and

$$A_{zz} A_{ii} - A_{ii} A_{oo} > 0 \quad (5.41)$$
The first condition (5.40) requires \( C_{\text{Na}} > 0 \). Again according to Etkin, the dynamic stability boundary is obtained by setting \( A_{22} A_{33} - A_{33} A_{22} = 0 \).

Hence, the second condition (5.41) yields,

\[
K_2 C_{\text{Na}}^2 + K_1 C_{\text{Na}} + K_0 = 0
\]

where

\[
K_2 = (s_1 b_2 - s_1 s_2)(a_1 s_4 + s_1 d_2 - s_1 b_2 - b_2 s_1)
\]

\[
K_1 = (a_1 d_0 - b_2 b_0)(a_1 s_4 + s_1 d_2 - s_1 b_2 - b_2 s_1) + (a_1 b_2 - b_2 s_2)(s_1 b_2 - s_1 s_2)
\]

\[
- (a_1 d_2 - b_2^2)(s_1 d_0 - s_1 b_0)
\]

\[
K_0 = (a_1 b_2 - b_1 s_2)(a_1 d_0 - b_2 b_0)
\]

and

\[
s_1 = \frac{gL}{V_0^2 C_T}, \quad s_2 = \frac{m + \alpha_{\text{au}}}{m}, \quad s_3 = \frac{gz_a}{V_0^2 C_T}, \quad s_4 = \frac{gz_a^2}{V_0^2 C_T L}
\]

Taking the more positive of the two roots, we have the value of \( C_{\text{Na}} \) which defines the dynamic stability boundary for a given set of parameters,

\[
C_{\text{Na}} = -\frac{K_1}{2K_2} \mp \frac{1}{2K_2} \sqrt{K_1^2 - 4K_2 K_0} \quad (5.43)
\]

The degree of dynamic stability depends on the magnitude of \( C_{\text{Na}} \) amongst other parameters. If the parachute were otherwise unstable, a large value of \( C_{\text{Na}} \) at equilibrium would be necessary to achieve dynamic stability.

Neglecting the \( \alpha_{\text{au}} \) term in equations (5.39) and (5.30) a computer program to solve (5.43) is given in Appendix E. Results obtained for \( C_{\text{Na}} \) per radian are given as a function
of $k_u$, $k_3$, $k_5$, $m$, $C_T$ and $z_s$ in Figures 5.6 through 5.11. As seen, the required values of $C_{Na}$ for dynamic stability decrease with increasing $k_u$, $m$ and $z$, whereas $C_{Na}$ increases as $k_3$ and $k_5$ increase.

As seen in Fig. 5.6 for given parachute parameters and when $k_u = 0$ where $k_3 = 2.1$ and $k_5 = 1.4$, the required value of $C_{Na}$ for dynamic stability is 0.32. However, as $k_u$ increases, this value decreases. Eventually, when $k_u = k_3$ the system remains stable even when $C_{Na}$ is zero. For $k_u > k_3$ the system remain stable with all positive values of $C_{Na}$. The effect of $k_3$ and $k_5$ are shown in Figs. 5.7 and 5.8 respectively. It can be seen that $k_3$ has a strong effect on the value of $C_{Na}$ for dynamic stability while $k_5$ has only a slight effect. Large values of $k_3$ require large values of $C_{Na}$ for dynamic stability.

The store mass, $m$, and the rigging line length, $z_s$, have positive effects on dynamic stability. The required value of $C_{Na}$ for dynamic stability decreases as they are increased, as seen in Figs. 5.9 and 5.10. The sensitivity of the required value for $C_{Na}$ with the store mass and the rigging line length is high when the latter two parameters are small. As they increase, then the sensitivity decreases.

The effect of $C_T$ on the dynamic stability is shown in Fig. 5.11. The required value of $C_{Na}$ for dynamic stability increases as $C_T$ increases.

To study the effect of the apparent mass components on the damping coefficient and the natural frequency of the oscillation, the characteristic equation (5.35) is solved
The values obtained of $\lambda$ and $\omega$, where $\lambda_1, = \lambda \pm i\omega$, are given in Fig. 5.12. As seen, both apparent mass components, $k_{11}$ and $k_{33}$ have a considerable effect on the damping and frequency of oscillation. Results obtained with the given parachute parameters are described in the following five cases.

Case (1). For $k_{11} = k_{33} = k_{55} = 0$; the system is unstable at very small values of $C_{\alpha}$ than as $C_{\alpha}$ increases it becomes stable when $C_{\alpha} = 0.007$. The oscillation parameters, $\lambda$ and $\omega$ first increase then as $C_{\alpha}$ continues to increase, they decrease.

Case (2). For $k_{11} = k_{33} = k_{55} = 1.0$; the parameters $\lambda = 0$ and $\omega = 1.005$ when $C_{\alpha} = 0$. The system becomes stable when $C_{\alpha}$ reaches 0.02.

Case (3). For $k_{11} = 1.5, k_{33} = 1.0, k_{55} = 1.0$; the parameters $\lambda$ and $\omega$ are $-0.120$ and $1.009$ respectively when $C_{\alpha} = 0$. The system is stable for all positive values of $C_{\alpha}$.

Case (4). For $k_{11} = 1.0, k_{33} = 1.5, k_{55} = 1.0$; the parameters $\lambda$ and $\omega$ are $0.184$ and $1.005$ respectively when $C_{\alpha} = 0$. The system is unstable for value of $C_{\alpha}$ less than $0.190$, then it becomes stable as $C_{\alpha}$ increases.

Case (5). $k_{11} = 1.0, k_{33} = 1.5, k_{55} = 1.0, k_{11}' = 0.2 \times k_{11}$; a positive time variation $k_{11}'$ has a positive effect. In this case $k_{11}' = 0.2 \times k_{11}$ and other parameters are as given in Case (4), then the system becomes stable when $C_{\alpha} > 0.160$.

For all these cases, the damping coefficient and the natural frequency of the oscillation first increase, then
Fig. 5.4: Variation of the damping coefficient of aperiodic motion, $\lambda_1$, with apparent mass coefficients.

Fig. 5.5: Variation of the ratio of the amplitudes of $\alpha$ and $\beta$ with apparent mass coefficients.
$C_T = 0.7, m_c = 5 \text{ kg}, D = 5.1 \text{ m}$

$z_S = 8.0 \text{ m}, z_a = 0.85 \text{ m}, V_0 = 8 \text{ m/s}$

$k_{33} = 2.1, k_{55} = 1.4, k_{\text{ref.}} = 0.15$

- $m_s = 120 \text{ kg}$
- $m_s = 80 \text{ kg}$

Fig. 5.6: Variation of $C_{N\alpha}$ with $k_{11}$. 
Fig. 5.7: Variation of $C_{N_a}$ with $k_{33}$. 

$C_T = 0.7$
$m_s = 1200 \text{ kg.}$
$m_c = 5.0 \text{ kg.}$
$z_s = 8.0 \text{ m.}$
$z_a = -0.85 \text{ m.}$
$D = 5.1 \text{ m.}$
$V_0 = 8.0 \text{ m/s}$
$k_{11} = 1.5$
$k_{55} = 1.4$
$k_{\text{ref}} = 0.5$
Fig. 5.8: Variation of $C_{N_a}$ with $k_{55}$. 

$C_T = 0.7$
$m_s = 120.0 \text{ kg}$.
$m_c = 5.0 \text{ kg}$.
$z_s = 8.0 \text{ m}$.
$z_a = -0.85 \text{ m}$.
$D = 5.1 \text{ m}$.
$V_o = 8.0 \text{ m/sec}$.
$k_{11} = 1.5$
$k_{33} = 2.1$
$k_{\text{ref}} = 1.0$
Fig. 5.9: Variation of $C_{Na}$ with $m_s$.
\[ C_T = 0.7 \]
\[ m_S = 120.0 \, \text{kg.} \]
\[ m_C = 5.0 \, \text{kg.} \]
\[ z_a = -0.85 \, \text{m} \]
\[ D = 5.1 \, \text{m} \]
\[ V_0 = 8.0 \, \text{m/s} \]
\[ k_{11} = 1.5 \]
\[ k_{33} = 2.1 \]
\[ k_{55} = 1.4 \]
\[ z_{\text{ref.}} = 4.0 \, \text{m} \]

**Fig. 5.10: Variation of \( C_{N\alpha} \) with \( z_s \).**
Fig. 5.11: Variation of $C_{N\alpha}$ with $C_T$. 

stable

unstable
Fig. 5.12: Variation of the approximate dynamic stability criteria with apparent mass components.
they decrease to a minimum value as $C_{Na}$ increases.

No significant effect of $k_s$ on the oscillation parameters was noticed.

The values of $C_{Na}$ at the equilibrium is given for various canopies in Ref. 79. These values were found to vary between 0.0 and +0.4. From experiments conducted jointly by Jorgensen and this author, the values of $C_{Na}$ based on the surface area at equilibrium were found to be approximately +0.41 for a hemispherical canopy and 0.40, 0.16, 0.8, 0.9 cruciform canopies with arm ratios of 2:1, 3:1, 4:1 and 5:1 respectively.

5.2.2.2. Simplified Stability Analysis

The effect of apparent mass components on the dynamic stability may be estimated by the following analysis.

Let $U_o$ and $W_o$ be the velocity components in the $x$ and the $z$ axis directions at the equilibrium condition. If the parachute is displaced so that the small change in its angle of attack from the equilibrium position is expressed

as $\theta$, as seen above, the velocity component in tangential
The lateral motion of the system is expressed by the two equations,

\[ X = (m + a_{11}) U + (m z_s + a_{12} z_a) \dot{Q} + (m + a_{33}) Q W \]  
\[ M = (I_y + m z_s^2 + a_{22} z_a^2) \ddot{Q} + (m z_s + a_{12} z_a) (\dot{U} + Q W) \]

where \( X \) and \( M \) are the external force and external moment about the origin, \( O \). For simplification, the origin is chosen at the centre of volume of the canopy, i.e. \( z_a = 0 \).

\( Q \) and \( \dot{Q} \) are defined in terms of the small disturbance angle, \( \theta \), as \( Q = \dot{\theta} \) and \( \dot{Q} = \ddot{\theta} \). Assuming \( \theta \) and \( \dot{\theta} \) are so small so that their products and squares may be neglected.

Neglecting the pitching moment, \( C_M \), in comparison with the gravitational moment about the origin, the external force and moment can be written as,

\[ X = -\frac{mg}{C_T} \frac{dC_N}{d\theta} \dot{\theta} - mg \sin \theta \]  
\[ = -\frac{mg}{C_T} \frac{dC_N}{d\theta} \ddot{\theta} \quad (5.48) \]

\[ M = -z_s mg \sin \theta = -z_s mg \theta \]  
\[ = -z_s mg \dot{\theta} \quad (5.49) \]

where \( mg = \frac{1}{2} V_o^2 C_T A \) and \( V_o \) is the equilibrium velocity.

From equation (5.46),

\[ U = \frac{1}{m + a_{11}} \left[ X - m z_s Q - (m + a_{33}) Q W \right] \quad (5.50) \]

Substituting these values, i.e. \( X, M \) and \( U \) into (5.47) the final equation becomes as,
\[
[(I + a_m + m \zeta_s - (m \zeta_s^2)/(m + a_{\mu})) \ddot{\theta} + \left[ W_0 (a_{\mu} - a_{\mu}) \frac{m \zeta_s}{m + a_{\mu}} \right] \dot{\theta} \\
+ \frac{m g \zeta_s}{(m + a_{\mu})} \left( -\frac{m}{C_T d \theta} + a_{\mu} \right) \theta = 0 \quad (5.51)
\]

This equation is a second order homogeneous linear differential equation. The auxiliary quadratic of this equation is

\[
K_2 \lambda^2 + K_1 \lambda + K_0 = 0 \quad (5.52)
\]

where

\[
K_2 = I_{yy} + a_m + m \zeta_s - (m \zeta_s^2)/(m + a_{\mu})
K_1 = W_0 (a_{\mu} - a_{\mu}) (m \zeta_s)/(m + a_{\mu})
K_0 = (m g \zeta_s) (-m/d_N + a_{\mu})/[(m + a_{\mu})]
\]

The roots of the equation (5.52) are

\[
\lambda_{1,2} = \frac{1}{2K_2} \left( -K_1 \pm \sqrt{K_1^2 - 4K_1 K_0} \right) \quad (5.53)
\]

If \( \lambda \) and \( \omega \) represent the real and imaginary parts of \( \lambda_{1,2} \), the solution of the equation (5.51) yields

\[
\theta = e^{\lambda t} (A \cos \omega t + B \sin \omega t) \quad (5.54)
\]

For \( \lambda < 0 \), the system is dynamically stable. Since \( K_2 \) is necessarily positive, the system is stable if \( K_1 \) is positive.

Hence, as the parachute oscillates about its equilibrium angle, the oscillation is alternatively,

i) damped when \( a_{\mu} > a_{\mu} \),

ii) undamped when \( a_{\mu} < a_{\mu} \).

This is not the only criterion for dynamic stability. As studied previously, if a parachute has positive and high values of \( \frac{dC_N}{da} \), the oscillation might damp out even when \( a_{\mu} < a_{\mu} \). This occurs with cruciform canopies having arm ratios of 4:1 and 5:1.
Consequently, the most important parameters affecting
dynamic stability of a parachute system are the slope of the
normal aerodynamic force coefficient, $C_{N_a}$, and the positive
difference between the normal and tangential apparent mass
components, $a_n - a_{b}$. 

5.2.2.3 Non-Linear Stability Analysis

Because of the limitations of the previous linearised
analysis, the dynamic stability of the system will be studied
by solving the non-linearised equations of motion (5.1). In
this way, effects of large disturbances are also analysed.
The aerodynamic force and moment coefficients at the canopy
apex obtained from experimental test were fed into the computer
programme which will be discussed in Chapter 6. The origin
is chosen at the centre of volume of the canopy.

a) Apparent mass effect.

To see the effect of the individual apparent mass
components, one apparent mass component was varied while
the others were kept constant (i.e. zero) during the computer
runs. In the computer programme, total fluid mass was
separated into two parts, as included fluid mass and appa-
rent mass terms.

As predicted previously, the normal apparent mass
component, $a_n$, has a stabilizing effect while the other two
components, $a_{\parallel}$ and $a_{b}$, have destabilizing influences. As
seen in Fig. 5.13 for an aeroconical canopy, the damping
coefficient, $\zeta$, which has been determined from the logarithmic
decrement of successive peak amplitudes increases with
increasing $k_{11}$, whereas it decreases with increasing $k_{33}$.
and \( k_{55} \). The effects of the apparent mass coefficients on the natural frequency of the oscillation, \( \omega_n \), is not large. The sensitivity of damping coefficient, \( \zeta \), and natural frequency of the oscillation, \( \omega_n \), to changes in \( k_{11}, k_{33} \) and \( k_{55} \) may be expressed by the relations,

\[
\frac{d\zeta}{d[k/k_{ref}]} \quad \text{and} \quad \frac{d\omega_n}{d[k/k_{ref}]}.
\]

The approximate value in the range of \( 0 < k/k_{ref} < 5 \) are,

\[
\frac{d\zeta}{d[k_{11}/k_{11,ref}]} = +0.0132 \quad \frac{d\omega_n}{d[k_{11}/k_{11,ref}]} = -0.009
\]

\[
\frac{d\zeta}{d[k_{33}/k_{33,ref}]} = -0.060 \quad \frac{d\omega_n}{d[k_{33}/k_{33,ref}]} = -0.0076
\]

\[
\frac{d\zeta}{d[k_{55}/k_{55,ref}]} = -0.002 \quad \frac{d\omega_n}{d[k_{55}/k_{55,ref}]} = -0.0076,
\]

where \( k_{11,ref} = 0.3, k_{33,ref} = 0.2 \) and \( k_{55,ref} = 0.4 \).

As is seen, the damping coefficient, \( \zeta \), is very sensitive to \( k_{33} \). As \( k_{33} \) increases \( \zeta \) decreases and approaches a negative value, consequently, the oscillation of the system becomes unstable, (as seen in Fig. 5.14).

For a cruciform canopy with an arm ratio of 4:1, the influence of the apparent mass components on the damping coefficient and the natural frequency of the oscillation have the same nature, but their sensitivities to changes
in apparent mass coefficients are very much less than they were for the aeroconical canopy, as seen in Fig. 5.15. This is because this cruciform canopy has a high value of $C_Na$ at equilibrium.

Since the parachute descends in an equilibrium state apparent masses have no affect on the descending resultant velocities, as seen in Fig. 5.16.

b) Store Mass and Rigging Line Length Effect

Again, as predicted previously, increasing the store mass, $m_s$, or the rigging line length, $l$, has a stabilizing effect. The damping coefficient, $\zeta$, and the natural frequency of the oscillation, $\omega_n$, increase with increasing $m_s$. Variation of $\omega_n$ with $m_s$ is effectively linear, while the rate of change of $\zeta$ with $m_s$ decreases as the store mass increases, as seen in Fig. 5.17a and Fig. 5.18. Also the resultant descent velocity increases as the store mass increases, as would be expected, as seen in Fig. 5.19.

The effect of the rigging line length variation on the damping coefficient is not high. As seen in Fig. 5.17b and Fig. 5.20, $\zeta$ increases little with increasing $l$. The natural frequency of the oscillation, $\omega_n$, decreases with increasing $l$, varying like a simple pendulum, inversely in proportion to $\sqrt{l}$.

Variation of the pitch angle, theta, angle of attack, alpha, normal and vertical velocity components, U and W and the angular velocity about the y-axis, Q, with the system parameters are given in Figs. 5.15 through 5.27.
c) The Effect of Large Disturbances

To indicate the limitation of the previous linearised analysis, exact computer runs for hemispherical and aeroconical parachutes with various initial disturbances were conducted. Results are shown in Fig. 5.28 and Fig. 5.29. As seen, for the hemispherical canopy (Fig. 5.28) with an initial disturbance of 10°, the parachute is dynamically stable and with an initial disturbance of 20° it is still dynamically stable but with a much reduced damping rate. In contrast, with an initial disturbance of 30°, it is not dynamically stable and the parachute angle of attack increases to a value at which the canopy collapses.

The reason for this last result is that as the parachute oscillates, values of angle of attack are reached at which \( C_{Na} \) becomes negative and dynamic instability results, eventhough the earlier linearized analysis indicates that the system is stable to small disturbances.

5.3. Further Discussion and Conclusions

The experiments conducted in this research programme showed that,

i) the apparent mass (or the total fluid resistance) coefficients are strongly dependent on the shape of the canopy,

ii) the magnitude of the apparent mass coefficients, \( k_{11} \) and \( k_{33} \), tend to their potential value for high values of the acceleration modulus, \( \delta \). For low values of the latter, they can be several times higher than their theoretical values,
iii) the angle of attack, \( \alpha \), has a considerable effect on the apparent mass coefficients. This effect on the tangential coefficient, \( k_{33} \), is especially high.

iv) Variation of the apparent moment of inertia coefficients, \( k_{44} = k_{55} \), with the angular acceleration is not considerable. Therefore, as stated by Szebehely \(^\text{46}\), potential flow evaluations for the apparent moment of inertia terms are acceptable.

Subsequently, it has been shown that during descent because of variation of both the angle of attack, \( \alpha \), and also the acceleration moduli, \( \delta \), the apparent mass coefficients for the fully deployed parachute vary significantly and that values given by a potential flow evaluation, which are relevant to a specific angle of attack, will seriously underestimate the size of some components. A sample of values of these coefficients determined from the computer run are given in Fig. 5.30.

Increased values of apparent mass components such as occur at low acceleration moduli so affect the predicted damping rate of the parachute that contemporary mathematical models of parachute performance which rely on potential flow evaluations of apparent masses can no longer be considered satisfactory.

In the following chapter, the variable experimental apparent mass data obtained by this research programme will be fed into the computer model and subsequent performance of the parachute-store system will be studied.
Fig. 5.13: Effect of the apparent mass coefficients on the damping coefficient, $\zeta$, and the natural frequency of the oscillation, $\omega_n$, for aeroconical parachute.

$M_{\text{Store}} = 115$ kg
$D = 5.1$ m
$l = 10.3$ m
$k_{11\text{ref}} = 0.3$
$k_{33\text{ref}} = 0.2$
$k_{55\text{ref}} = 0.4$
Fig. 5.14: Variation of pitch angle, \( \theta \), and angle of attack, \( \alpha \), with apparent mass coefficients for aeroconical parachute.
CRUCIFORM CANOPY

Arm ratio = 4:1

$m_s = 115$ kg, $m_c = 5$ kg, $l = 10.3$ m, $D = 5.1$ m.

$\theta = 20^\circ$, $k_{33} = 2.1$, $k_{55} = 1.4$

Fig. 5.15: Variation of the damping coefficient, $\zeta$, and the natural frequency of the oscillation, $\omega_n$, with apparent mass coefficients for cruciform parachute with an arm ratio of 4:1.
Fig. 5.16: Variation of the velocity components, U and W, with apparent mass coefficients for aeroconical parachute.
Fig. 5.17a,b: Effect of the variations of the store mass, $m_s$, and the rigging line length, $l$, on the damping coefficient, $\zeta$, and the natural frequency of the oscillation, $\omega_n$, for aeroconical parachute.
Fig. 5.18: Response of the parameters, $\theta$ and $\alpha$, to the variation of the store mass, $m_s$, for aeroconical parachute.
Fig. 5.19: Response of the parameters, U and W, to the variation of the store mass, m_s, for aeroconical parachute.
Fig. 5.20: Response of the parameters, $\theta$ and $\alpha$, to the variation of the rigging line length, $l$, for aeroconical parachute.
Fig. 5.21: Response of the parameters, $U$ and $W$, to the variation of the rigging line length, $l$, for aeroconical parachute.
Fig. 5.22: Response of the parameter, Q, to the variations of \( k \), \( m_s \) and \( l \) for aeroconical parachute.
**Fig. 5.23a**: Response of the parameters, θ and α, to the variation of the store mass, m_s, for hemispherical parachute.
Fig. 5.23b: Response of the parameters, $\theta$ and $\alpha$, to the variation of the store mass, $m_s$, for hemispherical parachute.
Fig. 5.24: Response of the parameters, $U$, $W$ and $V$, to the variation of the store mass, $m_s$, for hemispherical parachute.
Fig. 5.25: Response of the parameters, $\theta$ and $\alpha$, to the variation of the apparent mass coefficients, $k$, for cruciform parachute with an arm ratio of 4:1.
Fig. 5.26: Response of the parameters, $\theta$ and $\alpha$, to the variation of the store mass, $m_s$, for cruciform parachute with an arm ratio of 4:1.
Fig. 5.27: Response of the parameters, $U$ and $W$, to the variation of the store mass, $m_s$, for cruciform parachute with an arm ratio of 4:1.
Fig. 5.28: Response of the parameters, $\theta$ and $\alpha$, to the initial disturbances of $10^\circ$, $20^\circ$ and $30^\circ$ for hemispherical parachute.
Fig. 5.29: Response of the parameters, $\theta$ and $\alpha$, to the initial disturbances of $5^\circ$, $15^\circ$ and $30^\circ$ for aeroconical parachute.
Fig. 5.30: Variation of constant values of $k_{11}$ and $k_{33}$ with time for aeroconical parachute.
AERODYNAMICS OF PARACHUTES AND LIKE
BODIES IN UNSTEADY MOTION

CHAPTER 6.

PERFORMANCE CHARACTERISTICS OF
THE PARACHUTE-STORE SYSTEM
6. PERFORMANCE CHARACTERISTICS OF THE PARACHUTE-STORE SYSTEM

In this chapter, the characteristics of descending parachutes, such as those with aeroconical, hemispherical and cruciform canopies are determined. The effects on flight dynamics of changes in system physical parameters, such as canopy size, the length of the rigging lines, the store mass and the environmental parameters are analysed.

6.1. Mathematical Model

To do this, the experimental data for the apparent mass components determined in this research programme for particular parachute canopies together with their aerodynamic data, \( C_N^a \), \( C_T^a \) and \( C_M^a \) based on the cross-section area were fed into the mathematical computer model and then, the complete equations of motion, described in equation (5.1) were solved. For the computer model, the origin was chosen at the centre of volume of the canopy.

The computer programme detailed in Reference 64 and listed in Appendix D, is divided into a series of discrete subroutines each of which performs a separate physical task. The flow charts are shown in Fig. 6.1.

The system parameters including steady aerodynamic force coefficients are read in the subroutine "INITL" and the operations of the mathematical model are controlled by the subroutine "ADSBH". This is an implementation of the Adams-Bashforth algorithm for solving a set of first-order ordinary differential equations by means of a forward and backward prediction and correction technique.

Using the current values of all variables the subroutine
START

INITL
Reads Input t=0

INTEGRATE DIFFERENTIAL
EQUATIONS FOR
ONE TIME STEP

ERROR ACCEPTABLE ?
YES

ERROR VERY SMALL ?
YES

SHORTEN TIME STEP
AND REPEAT
$\Delta t = \Delta t / 2$

NO

LENGTHEN TIME STEP
FOR NEXT INTEGRATION
$\Delta t = 2 \cdot \Delta t$

$\Delta t$ + $\Delta t$

NO

TIME FOR PRINTOUT ?
NO

END OF RUN ?
YES

STOP

PARA

UNITVCT

WIND

AIR

FORCES

TBL

QER

AITKEN

ADDED

NB: OUTPUT IS OBTAINED
AT 0.2 SEC INTERVALS

Fig. 6.1: Flow diagram of the computer programme.
"PARA" comprises the main part of the model and calls further subroutines; UNTWCT (which computes the trigonometrical functions and unit vectors at a given time), AIR, WIND, FORCES, TRPL, AITKEN, DER and ADDED. It computes each of the 12 variables, namely, $U$, $V$, $W$, $P$, $Q$, $R$, $\theta$, $\phi$, $\psi$, $x_1$, $y_1$ and $z_1$. Providing the predicted and corrected values lie within specified limits, the calculation is repeated to the next time interval. This process continues until a predetermined time has elapsed. The values obtained are printed out using subroutine "OUT" at specified time intervals.

Subroutine "AIR" determines the air velocity and angle of attack relative to the canopy and store. This involves the use of "WIND" which is a user-supplied routine specifying incident air velocities relative to the Earth axes.

Subroutine "FORCES", using the air velocities determined in the subroutine "AIR" in conjunction with user-supplied data for aerodynamic force and moments, computes all external forces and external moments.

Subroutine "ADDED" computes new apparent mass components in conjunction with outputs of the displacements obtained at every time step in subroutine "OUT". These new apparent mass components are called by subroutine "DER", the subroutine to calculate the instantaneous values of derivatives of the 12 differential equations listed in Appendix C. More details are given in Reference 64.

6.2. Assumptions on Apparent Mass Components

As discussed previously in Chapter 5, the dynamic stability of the parachute-store system requires high values
of \( \frac{dC_N}{da} \) or else it requires \( a_n > a_n \) for parachutes with values of \( \frac{dC_N}{da} \) close to equilibrium which are small. Examples of the latter are aeroconical, hemispherical and cruciform canopies with arm ratios of 2:1 and 3:1. If the theoretical apparent mass components determined in Chapter 2 are used in the mathematical model the resulting motion of the parachutes mentioned above is unstable. For cruciform canopies with arm ratios of 4:1 and 5:1 which have high values of \( \frac{dC_N}{da} \) the resulting motion is stable. Taking difference but constant apparent mass values in the range of the experimental values obtained leads to results as if the forces acting on the canopy were varying and system were flying at different altitudes.

Consequently, constant arbitrary values for apparent mass components are inappropriate in the equation of motion. It is necessary instead to allow the apparent mass components in the mathematical model to vary, in the way that they were found to vary in the experimental programme.

Since the apparent mass components are functions of both angle of attack and acceleration modulus to obtain relevant results an iterative technique was adopted. In the dynamic sensitivity analysis made in Chapter 5 the acceleration moduli in both the normal and the tangential directions have been considered. It was found that for conventional fully-deployed load-carrying parachutes the initial values of the acceleration moduli are of the order of 2.0, decreasing to the order of zero some thirty seconds or so later. Considering an aeroconical, hemispherical or cruciform canopy with appropriate arm ratio, as such a parachute fully deploys,
acceleration moduli in both directions are high and a good first approximation to the apparent mass coefficients is to use their potential flow values which are \( k_{11} = 1.3 \) and \( k_{22} = 2.1 \). Then, calculating new acceleration moduli and appropriate values of angle of attack at the end of time step, which was taken to be 0.2 seconds, the values of apparent mass components in accordance Fig. 6.2a and 6.2b are introduced for the next time step. This process continues until the time limit ends. If the ensuing oscillations are only lightly damped the acceleration moduli remain high and little variation in the apparent mass coefficients occurs.

Since these experiments were only performed on hemispherical parachute canopies, the apparent mass components for aeroconical canopies were considered to behave similarly. From experimental evidence, little variation of \( k_{11} \) occurs with angle of attack, \( \alpha \). Therefore, \( k_{11} \) was taken to be only a function of the acceleration modulus, \( \frac{V_D}{V^2} \) while \( k_{22} \) was considered to be a function of both the acceleration modulus and the angle of attack. For simplicity the variations of \( k_{11} \) and \( k_{22} \) with acceleration modulus were considered to be linear and the apparent mass coefficients were fed into the computer programme as idealized in Fig. 6.2a and 6.2b. The apparent moment of inertia terms to be independent of angular acceleration and given by \( k_{44} = k_{55} = 1.8 \).

In this study, only the cruciform canopy with an arm ratio of 4:1 was considered. Since only limited experimental evidence is available, and is shown in Fig. 4.16 and Fig. 4.20 its normal apparent mass, \( k_{11} \) and apparent mass moment of inertia, \( k_{55} \) components were considered to be
Fig. 6.2: Apparent mass coefficients, $k_{11}$ and $k_{33}$, fed into the computer programme vs. angle of attack, $\alpha$, and acceleration modulus, $\delta$, for aeroconical and hemispherical parachute canopies.
constant and equal to 2.1 and 1.8 respectively while its tangential component was taken to be a function of the angle of attack only and not a function of the acceleration modulus.

6.3. Results and Discussion

The dynamic stability criteria of a descending parachute store system were analysed and discussed in Chapter 5. Defining following quantities which are shown in Fig. 6.3,

![Diagram of parachute-store system](image)

Fig. 6.3: Definition of symbols of a parachute-store system.

where

\[ \theta = \alpha_c + \beta_c = \alpha_s + \beta_s \]

and \( \alpha \) is the angle of attack, \( \beta \) is the angle between the resultant velocity, \( V_c \), and the vertical earth axis, subscripts \( s \) and \( c \) represent the store and the canopy.
respectively, the motions of the store and of the canopy were analysed.

The system parameters chosen for the three parachutes considered are: \( m_s = 100 \text{ kg} \), \( m_c = 5 \text{ kg} \), \( D = 5.1 \text{ m} \), \( l = 10 \text{ m} \) (distance between the canopy apex and the store) and the initial disturbance, \( \theta_0 \), is \( 20^\circ \) (0.34 rad.). These are representative values of full-scale conditions.

The time histories of the quantities defined above for these three parachutes, are given in Figs. 6.4 .... 6.12. At the first sight, the pitch angle, \( \theta \), plotted versus time in Fig. 6.4 shows that the hemispherical canopy is much more dynamically stable than the other two models. It reaches its equilibrium stage in approximately 15 seconds, while the aeroconical and the cruciform parachutes achieve equilibrium stage in approximately 30 seconds and 17 seconds respectively. The reason is that, because of the aero-dynamic normal force and the pitching moment characteristics of the hemispherical parachute are not exactly similar, i.e. the centre of pressure of the canopy is not on the axis of symmetry at \( \alpha_E \). Thus, it does not descend with a zero pitch angle, \( \theta \), instead there is non-zero angle at which the parachute is in equilibrium and stable. This angle was found to be approximately \( 5^\circ \). The equilibrium angle of attack at which the hemispherical parachute descends is increased by the pitch angle to about \( 32^\circ \). That is, during descent the hemispherical parachute oscillates about this higher angle of attack which considerably reduces the tangential apparent mass component, \( a_m \). This apparent mass component has a destabilizing effect. By reducing it a high damping rate
results. Also the value of \( \frac{dC_N}{da} \) at equilibrium for the hemispherical canopy is higher than the value of \( \frac{dC_N}{da} \) for the aeroconical parachute.

In 15 seconds, the hemispherical parachute reaches an equilibrium stage and maintains a horizontal velocity of about 4 m/s and a vertical velocity of 6.8 m/s. It sustains an angle of attack of 32° and a pitch angle of 5°. This means that the hemispherical parachute glides at a glide angle (or an incident angle) of 27°. The damping coefficient, \( \xi \), and the period of the oscillation, \( T \), are approximately 0.23 and 4.8 seconds respectively. Time variation of the hemispherical parachute characteristics is given in Fig. 6.5.

The cruciform canopy with a arm ratio of 4:1 reaches an equilibrium stage in less than 20 seconds and maintains a zero horizontal velocity and a vertical velocity of about 8.8 m/sec. Because of its aerodynamic characteristics it descends with zero angle of attack without gliding, as seen in Fig. 6.6. The damping coefficient and the period of the oscillation are approximately 0.19 and 4.4 seconds respectively. As seen, both values are less than they are for the hemispherical canopy.

As shown in Fig. 6.7 the aeroconical parachute without slots becomes stable in about 30 seconds and descends with a horizontal velocity of 3.1 m/sec and a vertical velocity of 6.5 m/sec. After reaching the equilibrium stage it descends with a zero pitch angle and an angle of attack of 25.2° at which the parachute glides. The damping coefficient and the period of oscillation are approximately
0.12 and 5.6 seconds respectively. It can be said that the aeroconical parachute oscillates with a lower damping rate and a higher period than the hemispherical parachute. Also its descent speed is less than that of the hemispherical for the same store mass and the same dimensions. It is emphasized that these results apply to an aeroconical canopy without slots. Its characteristics with slots are discussed by Jorgensen. The horizontal and the vertical displacements for those three parachutes with respect to earth axes are given in Fig. 6.8. This shows that the hemispherical parachute glides more quickly than the aeroconical, whereas the cruciform canopy, after reaching the equilibrium stage, descends vertically without gliding.

The small amplitude of oscillation of the canopy compared to the high amplitude of oscillation of the store is remarkable. This can be seen from the angles \( \beta_s \) and \( \beta_c \) for the corresponding parachutes in Figs. 6.5., 6.6., and 6.7. This means that the motion of the store is approximately similar to that of pendulum with a length equal to the distance between the centre of volume of the canopy and the store. Also, the amplitude of oscillations of \( \alpha_s \) is higher than the amplitude of oscillation of \( \alpha_c \), as seen in Fig. 6.9.

The effect of physical and environmental parameters will be analysed considering the aeroconical parachute only.

The store mass, \( m_s \), as shown previously, has a stabilizing effect. The damping rate increases with increasing store mass. But the rate of change in damping coefficient with the store mass is not as high as it was when the apparent
mass components were considered to be constant in Chapter 5. This can be seen by comparing Fig. 5.23 and Fig. 6.10a. A 100 per cent increase in the store mass now causes 25 per cent increase in the damping factor, compared with 95 per cent with constant apparent mass components. The period of the oscillations is virtually independent of the store mass. The store mass also has a considerable effect on the descending velocity, seen in Fig. 6.10b.

The time histories of the pitch angle, \( \theta \), and the angle of attack, \( \alpha \), in Fig. 6.12 show that the oscillation period strongly depends on the length of the rigging line, increasing with increasing rigging lines length. For three different length (\( L \)), 7 m, 10 m and 13 m the periods of oscillation were found to be 4.6, 5.6, and 6.6 seconds respectively. It can be estimated that the square of the oscillation period is proportional to the distance of the canopy from store, a result in conformity with that obtained for a pendulum. Increasing rigging line length decreases the damping coefficient, values obtained with increasing corresponding lengths being approximately 0.14, 0.12 and 0.11. This trend differs from that obtained in the previous Chapter when the apparent mass components were considered to be constant. Effects of variation of the rigging line length, \( L \), and the store mass, \( m_s \), on the damping coefficient and the period of oscillation are given in Fig. 6.12.

To see the effect of the size of the canopy on the dynamic stability of the system by varying the diameter of the canopy while keeping the ratio of \( \frac{L}{D} \) constant (see Fig. 6.2), several computer runs were conducted. Results,
in Fig. 6.13a, showed that the dynamic stability of the system decreases as the diameter increases. With increasing diameter, the amplitude of the oscillation of the pitch angle, \( \theta \), increases and the damping coefficient decreases. For a 5.1 m diameter canopy, the damping coefficient was 0.12, reducing to 0.08 for a 6.5 m canopy. Eventually, as the diameter increases the oscillation becomes undamped. The same phenomenon would appear if the store mass, \( m_s \), is reduced. Since the aerodynamic forces acting on the canopy and the forces due to the apparent masses all increase with increasing diameter, the effect of increasing canopy diameter is greater than that of reducing the store mass, \( m_s \). The period of oscillation, \( T \), also increases with increasing diameter. The descent speed decreases linearly with the diameter, as shown in Fig. 6.13b.

As pointed out in the previous chapter, the initial disturbance, \( \theta_0 \), also has a considerable effect on the dynamic characteristics. As seen in Fig. 6.14, the damping rate decreases as the initial disturbance is increased. For initial disturbances of 10°, 20° and 30° the damping coefficients are 0.16, 0.12 and 0.10, respectively while the period of oscillation remains approximately unchanged.

In all cases so far considered, the air density at sea level was used in the analysis. Considering three different altitudes, 0 km, 5 km and 10 km, the time variations of the parachute characteristics with altitude, \( h \) are given in Fig. 6.15. It shows that the stability of the system increases as altitude increases, so that at altitude the system has higher values of damping coefficient and lower periods of
oscillation than on the ground. Also the amplitude of oscillation of the store decreases with increasing altitude. It is conceivable that the parachute system initially demonstrates dynamic instability, followed by dynamically stable oscillation as shown in Fig. 6.15. Since the dynamic stability of the system increases with altitude, for the same initial displacement on the ground and at the altitude there is less tendency for the parachute to reach an attitude of collapse when it is at altitude than when it is at ground level. The horizontal and vertical displacements with respect to earth axes in 50 seconds flying time at the three different altitude are given in Fig. 6.16. It can be concluded that the lateral deflection is smaller at altitude and the horizontal displacement is greater than they are on the ground.

Since the magnitude of the apparent mass components for a descending parachute depend on both the acceleration modulus and its angle of attack they must vary with the physical and environmental parameters of the system. The variation of these components, $k_1$ and $k_3$ with time determined from the computer runs for three different rigging line lengths, three different store masses and three different canopy sizes are given in Figs. 6.17a, 6.17b and 6.17c. These indicate that some appropriate constant apparent mass components could well be estimated and used in mathematical model to determine the dynamic characteristics of a parachute having certain initial parameters. But, since the damping rate of the system depends on the system parameters the variation of the apparent mass components does not correspond for different canopies and consequently, it is not possible to predict appropriate constant apparent mass values for given parachutes.
Fig. 6.4: Time histories of $\theta$ and $\alpha$ for aeroconical, hemispherical and cruciform (arm ratio = 4:1) parachutes.
Fig. 6.5: Variation of the hemispherical parachute characteristics with time.
Fig. 6.6: Variation of the cruciform parachute (arm ratio = 4:1) characteristics with time.
Fig. 6.7: Variation of the aeroconical parachute characteristics with time.
Fig. 6.8: Horizontal and vertical displacements with respect to Earth axes for aeroconical, hemispherical and cruciform (arm ratio = 4:1) parachutes.
Fig. 6.9: Comparison of the motions of the store and the canopy for aeroconical parachutes.
$m_C = 5 \text{ kg}, D = 5.1 \text{ m}, l = 10 \text{ m}$

---

: $m_s = 140 \text{ kg}$

: $m_s = 100 \text{ kg}$

: $m_s = 60 \text{ kg}$

---

Fig. 6.10a: Variation of $\theta$ and $\alpha$ with time for aeroconical parachutes, the store mass, $m_s$, being varied.
Fig. 6.10b: Variation of $U$, $W$ and $V_c$ with time for aeroconical parachutes, the store mass, $m_s$, being varied.

$m_c = 5$ kg, $D = 5.1$ m, $l = 10$ m.

$\theta_o = 20^\circ$

- : $m_s = 100$ kg.
- - : $m_s = 70$ kg.
- - - : $m_s = 50$ kg.
Fig. 6.11: Variation of $\theta$ and $\alpha$ with time for aeroconical parachutes, the rigging line length, $l$, being varied.
Fig. 6.12: Damping coefficient, $\zeta$, and period of the oscillation, $T$, vs. store mass and rigging line length for aeroconical parachutes.
Fig. 6.13a: Variation of $\theta$ and $\alpha$ with time for aeroconical parachutes, the size of the canopy, $D$, being varied.
Fig. 6.13b: Variation of $U$, $W$ and $V_c$ with time for aeroconical parachutes, the size of the canopy, $D$, being varied.
Fig. 6.14: Effect of the initial disturbances, $\theta_0$, on the parachute system characteristics.

$m_s = 100 \text{kg}, m_c = 5 \text{kg}, D = 5.1 \text{m}$

$l = 10 \text{m}, U = -6 \text{m/s}, W = 13 \text{m/s}$
Fig. 6.15: Variation of the aeroconical parachute characteristics with time, the altitude, h, being varied.
Fig. 6.16: Horizontal and vertical displacements of aero-conical parachutes with different altitude.
Fig. 6.17a: Variation of $k_{33}$ and $k_{11}$ with time for three different store masses for aeroconical parachutes.
Fig. 6.17b: Variation of $k_{11}$ and $k_{334}$ with time for three
different rigging line lengths for aeroconical parachutes.
Fig. 6.17c: Variation of $k_{33}$ and $k_{11}$ with time for three different canopy sizes for aeroconical parachutes.
AERODYNAMICS OF PARACHUTES AND LIKE BODIES IN UNSTEADY MOTION

CHAPTER 7.

RECOMMENDATION FOR FUTURE WORK
7. RECOMMENDATION FOR FURTHER WORK

7.1. Recommendations for Future Experimentation

Within the limits of the experimental tests conducted, the effects of some parameters, such as shape of the canopy, angle of attack and acceleration modulus, on the apparent mass terms (or $C_R$) were determined. To predict the best stability condition and the optimum values of both $a_n$ and $a_m$ for a descending parachute, the effect of the relevant variables on the apparent mass terms (or $C_R$) are required. To meet all requirements necessary, further experiments suggested are to determine effects of

(i) porosity, vents and drive slots,
(ii) Reynolds number variation,
(iii) higher displacement derivatives (e.g. $V$, $V_1$, etc.).

The experimental facility could also be used to determine the variation of the apparent mass components during the opening and the reefed stages of parachute descent. Its application to problems which are not so directly related to the parachute is also evident. Little or no experimental data exist for the forces developed on bodies of any shape accelerating through fluids, and, with the experiments gained from the present study, it would be possible to contact a more fundamental series of tests on a variety of immersed bodies.

7.2. Recommendation for Future Theoretical-Prediction Studies

(i) The validity of the assumptions made and the resulting validity of the equations of motion can be determined through extensive testing. Ideally, drop-tests need to be conducted under controlled condition for a wide stability range, and the resulting motion can be compared with that simulated by the computer.
model. In this way, the assumptions inherent in the theoretical predictions can be examined and modified. In practice this process is difficult to achieve because the local wind speed relative to the parachute is, in general, both variable and unknown.

(ii) What seems to this author to be a most logical development would be to treat the effect of unsteady motion of bluff bodies such as the descending parachute by introducing into the equations of motion the total fluid resistance coefficient, $C_R$, instead of using both the drag coefficient, $C_D$, and the apparent mass components, $a_i$. At first sight, $C_R$ contains all the characteristics of both the drag coefficient, (as it is a function of the angle of attack) and the apparent mass components (as they are functions of both the angle of attack and the acceleration moduli). If $C_R$ is expressed as a function of both the angle of attack and acceleration modulus in an appropriate direction the final equations would give results of those obtained and analysed by Tory and Ayres. With an appropriate choice of origin the coefficient $C_R$ may be expressed in terms of the angle of attack and the acceleration moduli in two perpendicular directions and the ensuing equations of motion without apparent mass terms can be determined. Theoretical prediction obtained in this way and also by the methods described can be compared and provided they agree the complicated apparent mass concept could be obviated.
CONCLUSIONS

From this research program the following conclusions are drawn.

(1) The apparent mass coefficients, $k_{ij}$, and the total fluid resistance coefficient, $C_{R_{ij}}$, for two-dimensional motion about a bluff body, such as a parachute canopy, are highly dependent on the shape of the body, on its velocity, on the direction of the motion and on the acceleration. For parachute canopies these components can be expressed as functions of shape of the canopy, the direction of motion, the angle of attack, $\alpha$, and the non-dimensional acceleration modulus, $\delta = \frac{V_D}{v^2}$.

(2) At small values of the acceleration modulus, experimentally-determined apparent mass coefficients for parachutes are several times greater than their theoretical values. Magnitudes decrease as the acceleration modulus increases and eventually, at high enough values of the latter, they reach their potential flow values.

(3) For asymmetrical bluff bodies such as parachute canopies the angle of attack has a considerable effect on the magnitude of the apparent mass components, especially on the tangential component.

(4) For hemispherical parachute canopies the potential flow evaluation of the apparent moments of inertia components, $k_{44} = k_{55}$, are sufficiently close to experimental values to be acceptable for prediction purposes.
(5) The dynamic stability and descent characteristics of parachutes depend on physical and environmental parameters as well as on the magnitude of the apparent mass components. The necessary and sufficient parameters for dynamic stability are a high positive value of the slope of the normal force curve v.s. the angle of attack, \( \frac{dC_N}{da} \) at equilibrium and a non-negative difference between the normal apparent mass and tangential apparent mass components, \( a_{11} > a_{33} \). The normal apparent mass components, \( a_{11} \), has a stabilizing effect while the other components, \( a_{33} \) and \( a_{55} \), have destabilizing effects. Apparent masses do not significantly affect either the period of oscillation or the resultant velocity of the system during descent.

(6) The computer model developed allows for a variation in apparent mass components with angle of attack and acceleration modulus. The effect of these variations on performance prediction is shown to be very significant and behaviour cannot be correctly determined if constant apparent mass components are used.

(7) Of the specific canopy shapes considered, an aeroconical canopy without slots is shown to be less stable than is either a hemispherical or a cruciform canopy having an arm ratio of 4 : 1. For both the Aeroconical and Hemispherical parachutes glide with angles to the vertical direction of 25.2° and 27° respectively while the cruciform parachute descends without gliding.
(8) With increased altitude and increased store mass the dynamic stability is improved. Increasing the length of the rigging line has a destabilizing effect. The dynamic stability of the system decreases as the size of the parachute increases for a given store mass.

(9) With the numerous examples which have been considered, it is seen that motion of a descending parachute is nonlinear. Its true behaviour cannot be represented correctly by a linear procedure.
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APPENDIX A

Analysis of the Motion of the Experimental Aparatus
APPENDIX A

A.1. Linear Motion of the Slider Mechanism

Fig. A1.

The displacement of the slider can be expressed in terms of the dimensions given in Fig. A1, as

\[ x = l + r - l \cos \phi - r \cos \theta \]  \hspace{1cm} (A1)

The unknown \( \phi \) can be written in terms of the known variables as

\[ h = l \sin \phi = r \sin \theta \]

\[ \Rightarrow \sin \phi = \frac{r}{l} \sin \theta \]

Hence,

\[ \cos \phi = \sqrt{1 - \left( \frac{r}{l} \right)^2 \sin^2 \theta} \]  \hspace{1cm} (A2)

Substituting (A2) into (A1)

\[ x = l + r - l \sqrt{1 - \left( \frac{r}{l} \right)^2 \sin^2 \theta} - r \cos \theta \]  \hspace{1cm} (A3)

Replacing \( \theta \) by \( \omega t \) and taking first and second derivatives of the equation (A3) with respect to time, velocity and acceleration of the slider, the following results,
\[ x = \omega r \left[ \frac{1}{2} \frac{r \sin 2\theta}{\sqrt{1 - r^2 \sin^2 \theta}} + \sin \theta \right] \] (A4)

\[ \ddot{x} = \omega^2 r \left[ \frac{r \cos 2\theta}{\sqrt{1 - r^2 \sin^2 \theta}} + \frac{1}{4} r^3 \frac{\sin^2 2\theta}{(1 - r^2 \sin^2 \theta)^{1/2}} + \cos \theta \right] \] (A5)

where \( \omega \) is the angular velocity of the driver motor.

The velocity and the acceleration of the slider are plotted as functions of \( \theta \) in Fig. A2. Also a sample of a chart recorder plot for 4:1 cruciform canopy accelerated along the z axis is shown in Fig. A3.

**Fig. A2**

**Fig. A3**

### A.2. Angular Motion of the Model

Let \( OB = s \) and let \( \theta \) be the angle between \( OB \) and the normal axis from \( 0 \) to \( CD \). Let \( e_1 \) and \( e_2 \) denote unit vectors as shown in Fig. A4.
The velocity of AB is

\[
\frac{ds}{dt} = V = s \dot{e}_1 + s \psi \dot{e}_2
\]  

(A6)

Taking the scalar product with \(e_2\) to eliminate the unknown \(s\),

\[
V \cdot \dot{e}_2 = s \dot{e}_1 \cdot \dot{e}_2 + s \psi \dot{e}_2 \cdot \dot{e}_2
\]

\[
\Rightarrow V \cos \psi = s \dot{\psi}
\]

\[
\Rightarrow \dot{\psi} = \frac{V \cos \psi}{s} = \frac{V \cos^2 \psi}{a}
\]  

(A7)

The scalar product of \(V\) with \(e_1\) gives \(s\),

\[
V \cdot \dot{e}_1 = s \dot{e}_1 \cdot \dot{e}_1 + s \psi \dot{e}_1 \cdot \dot{e}_2
\]

\[
\Rightarrow s = V \sin \psi
\]  

(A8)

The linear acceleration of the slider can be written as

\[
\ddot{U}_1 \dot{e}_1 + \ddot{U}_2 \dot{e}_2
\]  

(A9)

where \(\ddot{U}_1\) and \(\ddot{U}_2\) are the components of acceleration in the directions of unit vectors.

Finally, the acceleration of AB is written
\[
\frac{d^2 s}{dt^2} = U_1 e_1 + U_2 e_2 = (\ddot{s} - s \psi^2)e_1 + (s \psi + 2 \dot{s} \psi)e_2
\]  \hspace{1cm} (A10)

As \( e_1 \) and \( e_2 \) are at right angles to one another, the two coefficients of \( e_1 \) and \( e_2 \) can be set equal to zero. Equating the coefficient of \( e_2 \) to zero gives

\[
\ddot{s} = \omega = \frac{V \cos^2 \psi}{a} - 2 \frac{\dot{V}^2}{a^2} \sin \psi \cos \psi \]  \hspace{1cm} (A11)

This relationship was applied at the end of the stroke at which \( V=0 \) when the expression for the angular acceleration is given

\[
\ddot{\psi} = \frac{\dot{V} \cos^2 \psi}{a} . \hspace{1cm} (A12)
\]
APPENDIX B

Stress and Frequency Analysis of the Parachute Model-Sting System
APPENDIX B

STRESS AND FREQUENCY ANALYSIS OF THE PARACHUTE MODEL-STING SYSTEM

B.1. Stress Analysis

\[ E_{St} = 207000 \text{ N/mm}^2 \text{(Young's mod.)} \]
\[ G_{St} = 79900 \text{ N/mm}^2 \text{(Shear mod.)} \]
\[ V_m = 1.5 \text{ m/s} \]
\[ L = 930 \text{ mm} \]
\[ L_1 = 560 \text{ mm} \]
\[ d_1 = 31.7 \text{ mm} \]
\[ d_2 = 28.4 \text{ mm} \]

Fig. B1.

Maximum tangential and normal forces acting on the stainless steel sting are approximately

\[ Z = \frac{1}{2} \rho \frac{V_m^2}{T} \frac{C_i A}{T} = 120 \text{ Newton} \] \hspace{1cm} (B1)
\[ X = \frac{1}{2} \rho \frac{V_m^2}{T} \frac{C_N A}{T} = 35 \text{ Newton} \] \hspace{1cm} (B2)

Maximum bending and torsion strains on the sting are determined by the relations

\[ \varepsilon_z = \frac{M_z}{EI} , \quad \varepsilon_x = \frac{M_x}{EI} , \quad \varepsilon_T = \frac{\tau}{GIp} \] \hspace{1cm} (B3)

Where

\[ M_z = Z \times L \]
\[ M_x = X \times L \]
\[ = X \times L_1 \]
\[ I = \frac{\pi}{12} d_1^3 [1 - \left( \frac{d_2}{d_1} \right)^4] \]
\[ I_p = \frac{\pi}{16} d_1^3 [1 - \left( \frac{d_2}{d_1} \right)^4] \]

and \( d_1 \) and \( d_2 \) are the outer and inner diameter of the tube respectively.
Hence, appropriate bending and torsion strains are obtained as

$$\varepsilon_z = 418 \, \mu \text{strain}$$

$$\varepsilon_x = 111 \, \mu \text{strain}$$

$$\varepsilon_\tau = 100 \, \mu \text{strain}$$

respectively.

**B.2. Vibration Analysis**

i) Natural frequency under beam itself.

![Beam Diagram](image)

Fig. B2. \( P=0.0118 \, \text{New./mm (unit weight)} \)

The natural frequency is given by the expression

$$\omega_1 = 3.53 \sqrt{\frac{gEI}{pL^4}} \quad (B4)$$

ii) Natural frequency under the weight of the model and its attachment.

![Weights Diagram](image)

Fig. B3. \( W = 5.5 \, \text{New. (weight of the model)} \)

The natural frequency is expressed as

$$\omega_2 = \sqrt{\frac{3gEI}{W L^3}} \quad (B5)$$

The natural frequency of the system can be evaluated as

$$\omega = \frac{\omega_1^2 \times \omega_2^2}{\omega_1^2 + \omega_2^2} \quad (B6)$$

From equation (B6), the natural frequency of the system, \( \omega_n \), is determined as

$$\omega_n = \frac{\omega}{2\pi} = 23.0 \, \text{c/s (Hz)}$$
B.3. Vortex Shedding Frequency

\[ D = 0.304 \text{ m (diameter of model)} \]
\[ V = 1.004 \times 10^{-6} \text{ m/s (kinematic viscosity of water)} \]

Reynolds number is found to be:

\[ \text{Re} = \frac{V_mD}{\nu} = 448207. \]

The Strouhal number is taken from graph at the corresponding Reynolds number, as

\[ \text{Str} = \frac{V_mD}{\nu} = 0.21 \quad (B7) \]

where \( \nu \) is the vortex shedding frequency. From (B7), \( \nu \) is found to be

\[ \nu = 1.05 \text{ c/s (Hz)}. \]

It is seen that the natural frequency of the system, \( \omega_n \) is approximately 20 times greater than the vortex shedding frequency, \( \nu \).


Maximum deflection, \( f \) and maximum deflection angle, \( \theta \) are given by the relations

\[ f = \frac{ZL^3}{3EI}, \quad \theta = \frac{ZL^2}{2EI} \quad (B8) \]

From equation (B8), \( f \) and \( \theta \) are obtained

\[ f = 6.76 \text{ mm} \]
\[ \theta = 0.68 \text{ (0.011 rad.)} \] respectively.
APPENDIX C

Derivation of the Equations of Motion of a Parachute-Store system
APPENDIX C

DERIVATION OF THE EQUATIONS OF MOTION OF A PARACHUTE-STORE SYSTEM

For general purposes, the equations of motion of a parachute store system were derived without any symmetry consideration in six degrees of freedom. Then, the equations were reduced to the parachute case in which they were numerically solved.

Kirchhoff showed how to derive the equations of motion of a combined system consisting of the fluid plus the solid body; the external forces and moments acting on a body due to the fluid may be derived from the apparent mass tensor and the velocities of the body in co-ordinated system.

It is a well known theorem of dynamics that for a system without potential energy, the instantaneous external force, \( F(X,Y,Z) \) and moment, \( (L,M,N) \) are related to the instantaneous linear momentum, \( G \), and the instantaneous angular momentum, \( H \), by the relations,

\[
\frac{dG}{dt} = F \quad (C1)
\]

\[
\frac{dH}{dt} = \mathbf{\omega} \times \mathbf{G} \quad (C2)
\]

Furthermore, these momenta are derivable from the kinetic energy of the system, by the relations

\[
G = \frac{\partial T}{\partial \mathbf{V}_i} \quad (C3)
\]

\[
H = \frac{\partial T}{\partial \mathbf{\omega}_i} \quad (C4)
\]

in which \( T \) is the total kinetic energy (body plus fluid), \( \mathbf{V}_i \) and \( \mathbf{\omega}_i \) are the linear and the angular velocities respectively.

Force and moment equations, \( (C1) \) and \( (C2) \), can be written for a system which is translating and rotating simultaneously as

\[
F = \frac{dG}{dt} + \mathbf{\omega} \times \mathbf{G} \quad (C5)
\]
\[ I = \frac{dH}{dt} + \omega \times H + V \times G \]  
(C6)

and in terms of the kinetic energy these equations can be rewritten

\[ \tau = \frac{d}{dt} \left( \frac{3T}{\frac{3}{2}V_i} \right) + \omega \times \left( \frac{3T}{\frac{3}{2}V_i} \right) \]  
(C7)

\[ I = \frac{d}{dt} \left( \frac{3T}{\frac{3}{2}\omega_i} \right) + \omega \times \frac{3T}{\frac{3}{2}\omega_i} + V \times \left( \frac{3T}{\frac{3}{2}V_i} \right) \]  
(C8)

where \( i = 1, 2, 3 \).

Using the kinetic energy of the body and the fluid, the equations of motion of the parachute-store system can be derived, as determined by Cockrell and Doherr. In order to have a better understanding of coupling apparent mass components, which are varying with the position, in the determination of the equations of motion, the linear and angular momenta about the origin arbitrarily chosen will be used instead of the kinetic energy concept.

C.1. Assumptions

To reduce the problem to one which will yield numerical solution, the following simplifying assumptions have been made:

1) The parachute canopy is rigidly attached to the store.

2) The store is a point mass.

3) The canopy and rigging lines masses are neglected in comparison with that of store.

4) The application point of the apparent mass tensor is chosen at the centre of reaction of the canopy which is supposed to coincide with the centre of volume of the canopy, so that coupling apparent mass components, \( a_{15} = a_{16} \), are zero. Consequently, there are 6 apparent mass components, namely, \( a_{11} = a_{22} = a_{33} = a_{44} = a_{55} \) and \( a_{66} \).
C.2. Equations of Motion

The system geometry and co-ordinates frame are shown in Fig.G.1. (X,Y,Z) and (L,M,N) are the external forces and external moments respectively about the x, y and z axes, (U,V,W) are the respective linear velocity components and (P,Q,R) are respective angular velocity components of the origin, O. \( R_0(x_a, y_a, z_a) \) and \( C_G(x_s, y_s, z_s) \) are the centre of reaction of the canopy and the gravity centre of the system which is approximated to be at the gravity centre of the store respectively and \( \alpha_{ij} \) are the corresponding apparent mass components.

The linear momentum, \( G \) due to the system mass and the apparent masses at the origin, \( O \) is given by the relation

\[
G = m \cdot \dot{V}_s + \alpha_{ij} \dot{V}_c
\]  

where \( \dot{V} \) is the linear velocity, \( m \) is the mass of the system, and \( s \) and \( c \) represent store and canopy respectively.

The linear velocity of any point, \( P \) fixed to the system with respect to the origin is expressed as

\[
\dot{V}_p = \dot{V}_o + \omega \times r
\]

where \( \dot{V}_o \) is the linear velocity of the origin and \( r \) is the position vector.

Likewise, the angular momentum, \( H \) due to the system mass and the apparent masses about the origin is given as,

\[
H = r \times (m \times \dot{V}_s + \alpha_{ij} \times \dot{V}_c) + \omega \times (I + \alpha_{ij})
\]

in which \( I \) is the moment of inertia of the body i.e of the store, about its axes and \( \alpha_{ij} \) in the first and second terms in the right hand side of the equation (C11) represents apparent masses, \( (\alpha_{11}, \alpha_{22}, \alpha_{33}) \), and apparent moment of inertia terms, \( (\alpha_{11}, \alpha_{22}, \alpha_{33}) \), respectively.
Fig. C.1: Sketch of System Geometry and Coordinates Frame
Case 2. One Plane of Symmetry

Supposing the parachute canopy-store system has one plane of symmetry, i.e. (OXY) plane in Fig.C.1. Subsequently, \( y_a = y_s = 0 \), and the set of equations, (C12) becomes

\[
\begin{align*}
X &= (a_{11} + m) U + (a_{12}Z_a + mz_s) Q + (a_{15} + m) QW - (a_{22} + m) VR + (a_{33}Z_a + mz_s) PR \\
&\quad - (a_{13}X_a + mx_s) Q^2 - (a_{23}X_a + mx_s) R^2 \\
Y &= (a_{22} + m) V - (a_{22}Z_a + mz_s) P + (a_{23}X_a + mx_s) R - (a_{11} + m) VP + (a_{22}X_a + mx_s) PQ \\
&\quad + (a_{11} + m) UR + (a_{12}Z_a + mz_s) QR \\
Z &= (a_{33} + m) W - (a_{13}X_a + mx_s) Q + (a_{33} + m) VP + (a_{23}X_a + mx_s) PR - (a_{11} + m) QU \\
&\quad - (a_{22}Z_a + mz_s) P^2 (a_{11} + m) + (a_{12}Z_a + mz_s) Q^2 \\
L &= [(a_{22}Z_a^2 + mz_s^2) + a_{12} + I_{XX}] P - (a_{22}X_a + mz_s) Z_s R - (a_{22}Z_a + mz_s) V - (a_{11} + m) UZ_s \\
&\quad + (a_{11} - a_{12}) UU + (a_{22}X_a + mx_s) PV + (a_{13}Z_a + mz_s) QW + [(a_{22}Z_a + mz_s) - (a_{22}X_a \\
&\quad + mx_s^2) + (a_{13} - a_{12})] (I_{ZZ} - I_{XX}) \] PR - (a_{22}Z_a + mz_s) RV + (a_{22}X_a + mz_s) PX + (a_{11} - a_{12}) UV - (a_{22}Z_a + mz_s) PW + (a_{11} - a_{12}) Z_s QU \\
&\quad + (a_{22}X_a + mz_s) Z_s RQ + [(a_{13} - a_{12}) Z_s^2 + (a_{13}X_a + mx_s^2) + (a_{13} - a_{12})] (I_{YY} - I_{XX}) \] PQ \\
M &= [(a_{13}X_a + mx_s^2) + a_{13} + I_{YY}] R - (a_{22}X_a + mz_s) Z_s X_s P + (a_{13}X_a + mx_s) V + (a_{11} + m) U^2 \\
&\quad - a_{11} Z_s PU + (a_{22}X_a + mx_s) RU + (a_{13} - a_{12}) UV - (a_{13}X_a + mx_s) PW + (a_{13} - a_{12}) Z_s QU \\
&\quad + (a_{13}X_a + mz_s) Z_s RQ + [(a_{13} - a_{12}) Z_s^2 + (a_{13}X_a + mx_s^2) + (a_{13} - a_{12})] (I_{YY} - I_{XX}) \] PQ \\
N &= [(a_{13}X_a + mx_s^2) + a_{13} + I_{ZZ}] R - (a_{22}X_a + mz_s) Z_s X_s P + (a_{13}X_a + mx_s) V + (a_{11} + m) U^2 \\
&\quad - a_{11} Z_s PU + (a_{22}X_a + mx_s) RU + (a_{13} - a_{12}) UV - (a_{13}X_a + mx_s) PW + (a_{13} - a_{12}) Z_s QU \\
&\quad + (a_{13}X_a + mz_s) Z_s RQ + [(a_{13} - a_{12}) Z_s^2 + (a_{13}X_a + mx_s^2) + (a_{13} - a_{12})] (I_{YY} - I_{XX}) \] PQ
\end{align*}
\]

The equations derived by Cockrell and Doherr contain one extra term in each of the moment equations, namely, \( \alpha_{13} UV \) in L, \( \alpha_{13} (W-U) \) in M and \( \alpha_{13} VW \) in N. These differences arise because of the assumption made on the application point of the apparent mass tensor.
Hence, from equations (C5) and (C6) through equations (C10), (C10) and (C11) the force and moment components for three cases are obtained, each of which one outlined as follows;

**Case 1.** If the parachute canopy-store system has no symmetry plane, i.e \( x_{a,s} \neq 0, y_{a,s} \neq 0, z_{a,s} \neq 0 \), equations are obtained as,

\[
X = (a_{11} + m) U + (a_{12} z_{a} + m z_{s}) Q - (a_{13} y_{a} + m y_{s}) R + (a_{14} z_{a} + m z_{s}) R - (a_{22} + m) V R + (a_{23} z_{a} + m z_{s}) P R - (a_{24} z_{a} + m z_{s}) R^2 - (a_{12} + m) V Q - (a_{22} + m) V Q - (a_{13} + m) UR + (a_{14} + m) UR - (a_{13} y_{a} + m y_{s}) R^2
\]

\[
Y = (a_{21} + m) V - (a_{22} z_{a} + m z_{s}) P + (a_{22} x_{a} + m x_{s}) R - (a_{31} + m) V P - (a_{33} y_{a} + m y_{s}) R + (a_{33} x_{a} + m x_{s}) P R - (a_{22} + m) V R + (a_{22} + m) V R + (a_{33} + m) UR + (a_{33} + m) UR - (a_{33} y_{a} + m y_{s}) R^2
\]

\[
Z = (a_{31} + m) W - (a_{33} y_{a} + m y_{s}) P - (a_{33} x_{a} + m x_{s}) Q + (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR
\]

\[
L = \left[ (a_{33} + m) Q + (a_{33} x_{a} + m x_{s}) \right] Q - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR
\]

\[
M = \left[ (a_{33} + m) Q + (a_{33} x_{a} + m x_{s}) \right] Q - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR - (a_{33} + m) VP + (a_{33} x_{a} + m x_{s}) PR
\]
Case 3. Two Planes of Symmetry

If system has two planes of symmetry, i.e. (OXY) and (OYZ) planes, in Fig.C.1, the z-axis is the axis of rotation and the origin lies on this axis. In addition to case 1, it requires that $x_a=x_s=0$. Thus, the equations of motion become

$$
X = (a_{11} + m)U + (a_{12}z_a + mz_s)Q + (a_{13} + m)QW - (a_{22} + m)VR + (a_{23}z_a + mz_s)PR \\
Y = (a_{22} + m)V - (a_{22}z_a + mz_s)P - (a_{13} + m)VP + (a_{11} + m)UR + (a_{12}z_a + mz_s)QR \\
Z = (a_{33} + m)W + (a_{12} + m)VP - (a_{11} + m)UQ - (a_{23}z_a + mz_s)P + (a_{33} + m)UQW - (a_{33}z_a + mz_s)PR
$$

in which $a_{11} = a_{22}$, $a_{66} = a_{55}$, and $I_{xx} = I_{yy}$.

C.3. Numerical Solutions of the Equations

Neglecting terms of $(a_{11} - a_{11})WW$, $(a_{33} - a_{33})UW$, and $(a_{22} - a_{22})UV$ in moments components in the equations (C14), as stated in section 5.1, six variables, $U, V, W, P, Q, R$ can be determined. After some rearrangements, these six variables are obtained as,

$$
\frac{dU}{dt} = \frac{[(I_{yy} + a_{55}mz_s + a_{11}z_a)X - (mz_s + a_{11}z_a)Y] + (m + a_{11})-(mz_s + a_{11}z_a)^2]}{[(I_{yy} + a_{55}mz_s + a_{11}z_a)^2]} \\
\frac{dV}{dt} = \frac{[(I_{xx} + a_{66}mz_s + a_{22}z_a)Y + (mz_s + a_{22}z_a)Z] - (m + a_{22})-(mz_s + a_{22}z_a)^2]}{[(I_{xx} + a_{66}mz_s + a_{22}z_a)^2]}
$$
\[
\frac{dW}{dt} = \frac{Z}{(m+a_3)}
\]
\[
\frac{dP}{dt} = \left[ (mz_s^2 + a_{2z} z_a^2) \gamma + (a_{2z} + m) \mu \right] / \left[ (I_{XX} + a_{2z} + m z_s^2 + a_{2z} z_a^2) (m+a_{2z}) - (m z_s + a_{2z} z_a)^2 \right]
\]
\[
\frac{dQ}{dt} = \left[ -(mz_s^2 + a_{1z} z_a^2) X + (a_{1z} + m) M \right] / \left[ (I_{YY} + a_{zz} + m z_s^2 + a_{1z} z_a^2) (m+a_{1z}) - (m z_s + a_{1z} z_a)^2 \right]
\]
\[
\frac{dR}{dt} = \frac{N}{(I_{zz} + a_{zz})}.
\]

From equations (5.2a) and (5.2b),

\[
\frac{dx_1}{dt} = U \cos \theta \cos \psi + V (\sin \theta \sin \theta \cos \psi - \cos \theta \sin \psi) + W (\cos \theta \sin \theta \cos \psi + \sin \theta \sin \psi)
\]
\[
\frac{dy_1}{dt} = U \cos \theta \sin \psi + V (\sin \theta \sin \theta \sin \psi + \cos \theta \cos \psi) + W (\cos \theta \sin \theta \sin \psi - \sin \theta \cos \psi)
\]
\[
\frac{dz_1}{dt} = -U \sin \theta + V \sin \theta \cos \psi + W \cos \theta \cos \psi
\]
\[
\frac{d\phi}{dt} = Q \cos \phi - R \sin \phi
\]
\[
\frac{d\psi}{dt} = P + Q \sin \phi \tan \theta + R \cos \phi \tan \theta
\]
\[
\frac{d\theta}{dt} = (Q \sin \phi + R \cos \phi) \sec \theta
\]

Where

\[
X = X - [(a_{33} + m) QW - (a_{2z} + m) VR + (a_{2z} z_a + m z_s) PR]
\]
\[
Y = Y - [(a_{33} + m) VP + (a_{1z} + m) UR + (a_{1z} z_a + m z_s) QR]
\]
Thus, these twelve simultaneous differential equations were numerically solved for the twelve unknowns; $U, V, W, P, Q, R, \theta, \phi, \psi$ and $x_1, y_1, z_1$, provided that the external forces $(X, Y, Z)$, and external moments, $(L, M, N)$ and the physical properties of the system are known.

The origin of the system was chosen at the centre of volume of the canopy, so that all quantities were defined at that point. The angle of attack, $\alpha$, and the angle, $\beta$, which is the angle between the resultant velocity and vertical earth axis, as seen in Fig. 6.3, with respect to the canopy and the store can be expressed as

$$\alpha_c = \frac{-1}{\sqrt{\frac{U+V}{W}}}$$

$$\alpha_s = \frac{-1}{\sqrt{\frac{U^2 + V^2}{W^2}}}$$

$$\beta_c = \theta - \alpha_c$$

$$\beta_s = \theta - \alpha_s$$

where $U_s = U + z_Q$

$V_s = V + z_P$

$W_s = W_s$
APPENDIX D

Listing of Computer Programs
PROGRAM YAVPARA (INPUT, OUTPUT, TAPE7, TAPE2, TAPE1, TAPE3, TAPE4, 1TAPE5)

PARACHUTES - MAIN PROGRAM

LAST MOD - 17/3/81

THIS PROGRAMME HAS BEEN MODIFIED TO DETERMINE ACCELERATION MODULI (VIA "SUBROUTINE ADDED") ONLY FOR THE CASE WHERE "TIME LIMIT"=50 SEC. AND "TIME INTERVAL"=0.2 SEC.

SOLVES EQUATIONS OF MOTION REFERRED TO LOCAL AXES AND COMPUTES DISPLACEMENTS AND ROTATIONS RELATIVE TO FIXED COORDS

T INDEPENDENT VARIABLE, TIME, STARTING AT TO
Y DEPENDENT VARIABLES, VELOCITIES, DISPLACEMENTS AND ANGLES
-UNITVCT- COMPUTES TRIG FUNCTIONS AND UNIT VECTORS FOR COMMON /T/
-INITL- READS IN DATA AND INITIALISES
-ADSBH- SOLVES SYSTEM OF ORDINARY ODIFF EQUNS
-PARA- SUPPLIES AERODYNAMIC AND GRAVITY FORCES INCLUDING A WIND AND CALCULATES DERIVATIVES FOR -ADSBH-  
-OUT- IS THE PRINTOUT ROUTINE

THE FOLLOWING AERODYNAMIC FUNCTIONS MUST BE SUPPLIED-

CANOPY -
CTF(ANGLE)
CNF(ANGLE)
CMF(ANGLE)

STORE -
CSF(ANGLE)

RUDDER -
CLF(ANGLE)
CDF(ANGLE)

AERODYNAMIC FUNCTIONS - INCL 1/2*RH0*AREA
CTF, CNF, CMF - TANGENT, NORMAL FORCES AND PITCHING MOMENT
ACTING AT TOP OF CANOPY
VELOCITIES MEASURED AT CANOPY C OF G
CSF - DRAG OF STORE
CLF, CDF - ABSOLUTE VALUES OF 1/2*RH0*COEFF FOR LIFT AND DRAG OF A FLAT PLATE ALIGNED WITH AND FIXED TO MOVING X AXIS AT ANGLE OF INCIDENCE IN XY PLANE OF 0 TO 2*PI
THIS PLATE MAKES NO CONTRIBUTION TO THE OVERALL DRAG
COMMON /E/X(16),NIR
COMMON /G/GM1,GM2,XXI,YYI,ZZI,GMC,GMS,OCL,OSL,OC1,VAR,AXR,0,OP
COMMON /P/X(6),WX,WY,WZ,VS,ALPHA,AX,UA,VA,WA,UAS,VAS,HAS,ARM,RA,
VO4,VO5,VAO,MU
COMMON /T/ST,SP,ST,CP,CS,VI(9)
DIMENSION Y(12),G(12)
REAL MU
INTEGER ECV,EOP
EXTERNAL PARA,OUT

CALL INITL(T0,Y,N,ECV,EOP,ERR,ABE,NN,HP,TMAX)
CALL AOSBH(N,Y,PARA,T0,rMAX,HP,ECV,EOP,ERR,ABE,NN,OUT)
STOP
END

C
C
C

C  -----------------
C  SUBPROGRAMME INITL
C
C

SUBROUTINE INITL(T0,Y,N,ECV,EOP,ERR,ABE,NN,HP,TMAX)

READS IN DATA AND INITIALISES

TO START TIME
N NUMBER OF EQUATIONS TO BE SOLVED

NIR-(INTEGER) RUN NUMBER
CANTYP--CANOPY TYPE: AOC0EFS--A/0 COEFFICIENT CURVES ASSUMED
X14=16X8 ALPHAMERIC CHARACTERS, 54 ON FIRST CARD, 64 ON SECOND
ARE PRINTED BELOW NIR ON TOP OF EVERY PAGE OF OUTPUT
GMC,GMS-CANOPY,STORE MASSES;AK3,AK1,AK0=K55-INERTIA COEFFICIENTS
OCL,OSL-DISTANCE OF C OF G OF CANOPY & STORE FROM CANOPY APEX
DIA=CANOPY DIAMETER; MU-PITCH JAMING COEFFICIENT
ARM,RA-DISTANCE OF RUDDER FROM Z AXIS AND ITS AREA
OP-EFFECTIVE CANOPY DIA AND DAMPING COEFF FOR ROTATION
Y(1) TO Y(3)-INITIAL VELOCITIES (U,V,W) IN MOVING AXIS SYSTEM
Y(7) TO Y(9)- INITAL THETA, PHI AND PSI IN DEGREES, CONVERTED
INTERNALLY INTO RADIANS

/INITIAL VALUES OF THE TANGENTIAL APPARENT MASS,AK3=K33, AND THE
NORMAL APPARENT MASS,AK1=K11, COEFFICIENTS FOR HEMISPHERICAL AND
AEROCONICAL PARACHUTE CANOPIES ARE TAKEN 2.1 AND 1.3 RESPECTIVELY.
NEW APPARENT MASS COEFFICIENTS ARE CALCULATED IN SUBROUTINE
"ADDED", APPARENT MOMENT OF INERTIA COEFFICIENT, K55, IS CONSIDERED TO BE CONSTANT, AS 1.31.

HP,TMAX=PRINTOUT TIME INTERVAL AND END TIME
ECV,EOP-(INTEGER) ECV=0 CHECK SUM OF ERRORS IN ALL Y
ECV=j .........................Y(j)
ECV=NE=0 THEN EOP=1 ABS ERROR,2 NORMALISED ERROR,GTE,2 NONE
ERR,ABE=NORMALISED AND ABSOLUTE ERROR PARAMETERS
NN-(INTEGER) =1 TO SUPPRESS PRINTOUT

READ IN AERODYNAMIC DATA AS CO(I,J)=1 NORMAL:I=2 TANGENT:I=3 MOMENT:
J=1,19-INPUT VALUES AT 10 DEGREE INTERVALS (-90 TO 90)-3 SETS OF CARDS

COMMON /COF/ DIA,CO(I,19),RHOA2
COMMON /E/14(16),NIR
COMMON /G/SM1,SM2,X1I,Y1I,Z1I,GMC,GMS,OCL,OSL,OGCL,VAR,AXR,DP
COMMON /P/16,WM,WM2,VS,ALPHA,AX,UA,VA,WA,WA2,VAR,AXR,OP
1 WRITE(7,1000)NIR
READ(7,900)CANTYP(1),CANTYP(2)
READ(7,910)ADCOEFS
READ(7,920)GMC,ICN,ICT,OCL
READ(7,920)GMS,ISN,IST,OSL
READ(7,920)MUA,MU
READ(7,920)ARM,RA
READ(7,920)0,EOP
READ(7,920)Y,1,2,3
READ(7,920)Y,7,8,9
READ(7,920)HP,TMAX
READ(7,920)ECV,EOP
READ(7,920)ERR,ABE
DO 2 I=1,3
2 READ(7,1003)(CO(I,J),J=1,19)
900 FORMAT(2A10)
910 FORMAT(I5)
1000 FORMAT(LF5.0)
1001 FORMAT(8A3/8A8)
1002 FORMAT(11F7,3/8F7.3)
1003 FORMAT(11F7.3/8F7.3)
WRITE(2,700)NIR
WRITE(2,800)CANTYP(1),CANTYP(2)
WRITE(2,810)ADCOEFS
WRITE(2,820)GMC,ICN,ICT,OCL,GMS,ISN,IST,OSL
WRITE(2,830)MUA,MU
WRITE(2,840)ARM,RA
WRITE(2,850)DP
WRITE(2,860)Y,1,2,3,7,8,9
WRITE(2,870)HP,TMAX
WRITE(2,880)ECV,EOP,ERR,ABE
CALL WIND(0,Y)
WRITE(2,890)TI
WRITE(2,900)TF
WRITE(2,910)WXF
WRITE(2,920)WZF
2009 FORMAT(/16H WIND STARTS AT ,F7.2,4H SEC)
DO 10 I=1,9
   Y(I)=Y(I)*3.14159/180.0
10  CONTINUE
RHOA2=0.61*3.1415*OIA*OIA/4.0
NN=0
T0=0
N=12
C
C CONVERT ORIGIN TO CANOPY CENTROID
C
OGCL=OCL
OCL=OCL-OGCL
OSL=OSL-OGCL
C
GMI — MASS OF AIR INCLUDED WITHIN THE CANOPY
C
GMI=(RHO AIR)*VOLUME OF ELLIPSOID A/B=0.5
C
GMI=1.22*3.142*OIA*OIA/12
C
GM1,GM2 — TOTAL EFFECTIVE MASS OF SYSTEM (INCLUDING ADDED APPARENT
C
MASS) IN AXIAL AND NORMAL DIRECTIONS
C
GM1=GM1+2.1*GMI
GM2=GM2+GMC+1.3*GMI
C
XXI,YYI,ZZI — TOTAL MOMENTS OF INERTIA ABOUT X,Y,Z AXES AT THE
C
CANOPY CENTROID
C
XXI=GM1*OSL*OSL+1.8*GMI*OIA*OIA/16.0+ISN
YYI=XXI
ZZI=GM1*OIA*OIA/16.0+IST
C
C initialise TRIG FUNCTIONS AND UNIT VECTORS
C
CALL UNTVCT(Y)
DO 3 J=1,19
   WRITE(2,2009) J,CO(1,J),J,CO(2,J)
2009  FORMAT(6H CO(1,,12 ,2H)=,F7.3,16H CO(2,,12,2H)=,F7.3//)
3  CONTINUE
SUBROUTINE A0SBH(N,Y,DERIV,X,XMAX,HP,ECV,EOP,ERR,ABE,NN,OUT)  

THIS SUBROUTINE SOLVES A SYSTEM OF N FIRST ORDER ORDINARY 
DIFFERENTIAL EQUATIONS. N= NO OF EQNS, Y IS THE ARRAY OF DEPENDENT 
VARIABLES 
MAX VALUE OF N IS LIMITED TO 30 ONLY BY THE DECLARED DIMENSIONS 
OF THE ARRAYS Y(N),F(6,N),Y0(N),Y1(N),G(N) 
DERIV(X,Y,G) IS THE SUBROUTINE WHICH COMPUTES THE DERIVATIVES 
X0 IS THE STARTING VALUE OF THE INDEPENDENT VARIABLE, XMAX IS THE 
FINAL VALUE OF THE INDO. VARIABLE. 
ECV,EOP,ERR,ABE ARE USED FOR ERROR CONTROL 
ERR IS THE RELATIVE ERROR CONTROL, ABE IS THE ABSOLUTE ERR CONTROL 
HP IS THE INTERVAL FOR PRINTING OUTPUT 
NN=1 Suppresses PRINTOUT VIA SUBROUTINE OUT(Y,X) 
NIR IS USED AS RUN NUMBER, NIR(16) HOLDS UPTO 8X16 ALPHAMERIC 
CHARACTERS USED AS A HEADING FOR PRINTOUT 
COMMON /E/X14(16),NIR 
REAL NC 
INTEGER EOP,ECV 
DIMENSION Y(N),F(6,30),Y0(30),Y1(30),G(30) 

DO 10 I = 1,N  
10 Y1(I) = Y(I) 

X = XO 
XP = XO - 0.0004+HP 
H = HP 

ERP = ABS(ERR*H/(XMAX-XO))  
M024 = H/24.0  
N13=50  
NSTEP = 0  
NODES = 0  
NODEXP = 0  
NALG = 1  
NC = 1.0  
NODXC=2000  

PROGRAMME CEASES IF MORE THAN 1000 NODES BETWEEN PRINT POINTS  
100 IF (NODES-NODEXP.GT.NODXC) GO TO 485  
IF(X-XP)120,106,105  
106 IF(NN.EQ.1)GOTO 120
N13=N13+1
NODXP=NODES
IF(N13.LE.42)GOTO 108
N13=0
WRITE(2,1000)NIR,X14
1000 FORMAT(1H1,I7//(16A8/)1)
108 CALL OUT(Y,X,KK,G)
C
XP=XP+HP
C TEST FOR STEP AHEAD
C
120 IF (X- XMAX) 200,700,700
C MARCH AHEAD - 1,2,3,4
C
200 DO 202 I  =  i,N
202 YO(I) =  Y d )
CALL DERIV(X,Y,G,KK)
C
X= IND VARIABLE, Y= DEP VARIABLE (ARRAY) G=ARRAY OF DERIVATIVES
C
DO 210 I  =1,N
210 F(1,I) = G ( I )
GO TO (220,240),NALG
C TWO POINT PREDICTOR FOR STARTING
C
220 DO 222 I  = 1,N
222 Y(I) =  YOd) +  H024*(55.0«F(1,I) -59.0«F(2,I) +37.0«F(3,I)
1  -9.0«F(4,I))
GO TO 250
C
FOUR POINT PREDICTOR FROM AMS P896
C
240 DO 242 I  = 1,N
242 Y(I) =  YO(I) + H024*(55.0*F(1,I) -59.0*F(2,I) +37.0*F(3,I)
1  -9.0*F(4,I))
250 XN =  X  +  H
CALL DERIV(XN,Y,G,KK)
GO TO (260,270),NALG
C TWO POINT CORRECTOR FOR STARTING
C
260 DO 262 I  = 1,N
262 G(I) =  YC(I) +0.5*H*(G(I) + F(1,I))
GO TO 300
C
FOUR POINT CORRECTOR FROM AMS P896
C
270 DO 272 I  = 1,N
272 G(I) =  YO(I) +H024*(9.0*G(I) +19.0*F(1,I) -5.0*F(2,I) +F(3,I))
C
ERROR CONTROL TEST
C
300 CONTINUE
IF(ECV=0)302,302,319
302 S = 0.0
T IS THE DIFFERENCE BETWEEN THE PREDICTED AND CORRECTED VALUES

\[ T = G(I) - Y(I) \]

IF \( |G(I)| \leq 1.0 \times 10^{-4} \) \( N11 = 1 \)
IF \( |ABS(G(I))| \leq 1.0 \times 10^{-5} \) GO TO 308

\[ S = SUM \ of \ ABS(ERRORS), S1 = SUM \ of \ NORMALISED \ ERRORS \]

\[ S = S + ABS(T/G(I)) \]
\[ S1 = S1 + ABS(T) \]
GO TO (318, 310), NALG

308 \( S = S/N \)

CHECKS ON SIZE OF ERROR

IF \( (S-ERP) \leq 338,400,400 \)

319 CONTINUE

\[ T = G(ECV) - Y(ECV) \]
GOTO(320, 330, 600), EOP

320 CONTINUE

\[ S1 = ABS(T) \]
\[ N11 = 1 \]
\[ S = 20G0.0*ERP \]
GOTO 331

330 CONTINUE

IF \( |ABS(G(ECV))| \leq 1.0 \times 10^{-5} \) GO TO 320
\[ N11 = 2 \]
\[ S = ABS(T/G(ECV)) \]
\[ S1 = ABS(T) \]

331 CONTINUE

GOTO(334, 332), NALG

332 CONTINUE

\[ S = 0.25*S \]
334 CONTINUE

\[ S = S/N \]
\[ IF(S-ERP) \leq 338,400,400 \]
\[ IF(S-0.005*ERP) \leq 334,400,400 \]

HALVE THE MESH SIZE- WILL TRAVEL

400 CONTINUE

IF \( (N11 \neq 1) \) GO TO 403
\[ N11 = 4 \]
\[ IF(S1-ABE) \leq 338,400,403 \]
\[ IF(W/\nu \leq 2.0 \times 10^{-6}) \) GO TO 400
\[ GO TO (402, 420), NALG \]

TWO POINT SCHEME CUT OUT AND START OVER
SET \( Y \) EQUAL TO ITS INITIAL VALUES

402 \( 00 \ 404 \ I = 1, N \)

HALVE THE MESH SIZE- WILL TRAVEL

403 \( 00 \ 404 \ I = 1, N \)
C FOUR POINT INTERPOLATION SCHEME AMS P879

C

420 DO 428 I =1,N
F(5,I) = F(3,I)
F(3,I) = F(2,I)
F(2,I) = (15.0*F(1,I) +45.0*F(3,I) -15.0*F(5,I) +3.0*F(4,I))/48.0
F(6,I) = (15.0*F(4,I) +45.0*F(5,I) -15.0*F(3,I) +3.0*F(1,I))/48.0
F(4,I) = (-3.0*(F(4,I) +F(1,I))/27.0*(F(5,I) +F(3,I)))/48.0
428 Y(I) = YO(I)

430 H = 0.5+H
H024 = H024+0.5
NC = 2.0*NC
GO TO (432,434),NALG

432 GO TO 200

C

434 ERP = 0.5*ERP
430 H = 0.5+H
H024 = H024+0.5
NC = 2.0*NC
GO TO (432,434),NALG

C POSSIBLE MESH SIZE INCREASE
C NC INSURES THAT H IS NOT MADE GREATER THAN HP
500 IF (NC - 1.0) 600,600,502
C NSTEP CHECKS TO SEE IF ENOUGH POINTS ARE AVAILABLE TO DOUBLE THE MESH SIZE
502 IF (NSTEP -5) 600,504,504
C MESH SIZE IS DOUBLED ONLY AT PRINTING POINTS (XP)
506 DO 508 I =1,N
Y(I) = G(I)
F(3,I) = F(4,I)
508 F(4,I) = F(6,I)
X = XN
NSTEP = 2
NC = NC/2.0
ERP = 2.0*ERP
H = 2.0*H
H024 = 2.0*H024
GO TO 607

C THE NEW POINT IS SATISFACTORY GET READY TO DO THE NEXT POINT
C

600 DO 606 I = 1,N
Y(I) = G(I)
DO 606 J = 1,5
K = 6-J
605 F(K+1,I) = F(K,I)
X = XN
C

607 NODES = NODES + 1
610 NSTEP = NSTEP + 1
C DO WE HAVE ENOUGH POINTS TO USE THE 4 POINT SCHEME
C
612 NALG = 2
H024 = H/24.0
620 GO TO 100
C
C ACCURACY REQUIREMENT CANNOT BE MET WITHOUT CHANGING PRINTING OUT
C INTERVAL OR STATEMENT NUMBER 100
C
700 IF (NN.EQ.1) GOTO 702
705 CONTINUE
S=S*ERR/ERP
WRITE(2,720) X,ERR,H,S,NODES
C
720 FORMAT(17HTERMINATED AT T=,1PE10.3 ,3X,21HERR OR PARAMETER ERR=,E1AOS 25C
13.3,3X,12HSTEP LENGTH=,E13.3/1H ,10X,17HEST ERROR AT ENO=, E10.3,I1ADS 251
20*X,5STEPS)
C
702 RETURN
C
END
C SUBPROGRAMME UNTVCT
-----------------------------

SUBROUTINE UNTVCT(Y)
C COMPUTES TRIG FUNCTIONS AND UNIT VECTORS FOR COMMON /T/
C ST,SP,SS,CT,CP,CS SIN AND COS OF ANGLES RELATING COORD SYSTEMS
C VI(9) ARE UNIT VECTORS REFERING VARIABLES W.R.T. MOVING AXES
C TO FIXED AXES
C
COMMON /T/ST,SP,SS,CT,CP,CS VI(9)
DIMENSION Y(12)
C
ST=SIN(Y(7))
SP=SIN(Y(8))
SS=SIN(Y(9))
CT=C0S(Y(7))
CP=C0S(Y(8))
CS=C0S(Y(9))
VI(1)=CT*CS
VI(2)=SP*ST*CS-CP*SS
VI(3)=CP*ST*CS-SP*SS
VI(4)=CT*SS
VI(5)=SP*ST*SS+CP*CS
VI(6)=CP*ST*SS-SP*CS
VI(7)=CT
VI(8)=CP*CT
VI(9)=CP*CT
IF(ABS(CT).LT.1.0E-6) CT=SIGN(1.0E-6,CT)
RETURN
END

SUBROUTINE PARA(T,Y,G,K)
EVALUATES TRIG FUNCTIONS AND UNIT VECTORS IN -UNTVCT-
COMPUTES RELATIVE AIR MOVEMENT IN -AIR-
FORCES AND MOMENTS IN -FORCES-
DERIVATIVES FOR ADSBH IN -DER-
T IS TIME, Y DEPENDENT VARIABLES, G DERIVATIVES THEREOF
DIMENSION Y(12),G(12)
CALL UNTVCT(Y)
CALL AIR(T,Y)
CALL FORCES
CALL DER(Y,G,T,K)
RETURN
END

SUBROUTINE OUT(Y,T,K,G)
PRINTOUT SUBROUTINE
COMMON /P/X(6),HX,HWZ,VS,ALPHA,AX,UA,VA,UAU,VAU,VAU,AXA,AYA,RA
COMMON /T/ST,SP,SS,CT,CP,CS,VI(9)
COMMON /B/BALPHA,THETA,PSI
COMMON /F/WF/TI,TF,XXI,YYI,ZZI,GMC,GMS,OCL,OSL,OGCL,VAR,AXR,OR,OP
DIMENSION Y(12),G(12)
BALPHA=ALPHA*57.296
B=COS(Y(7))*COS(Y(7))*COS(Y(8))*COS(Y(3))
A=ATAN(SQRT(A3S(1.0/3-1.0))))*57.296
A=90.0
1 CONTINUE
THETA=Y(7)*57.296
PHI=Y(8)*57.296
PSI=Y(9)*57.296
VRES=SQRT(Y(1)**2+Y(3)**2)
IF(UA.GT.0.)GOTOL
A=ATAN(SQRT(A3S(1.0/3-1.0))))*57.296
GOTO 2
GOTO 1
GOTO100

300 IF(Y(11).GT.0.0)GOTO400
B ALPHA=0
GOTO100

400 BALPHA=90
GOTO100

200 BALPHA=57.296*ATAN(Y(1)/Y(3))
100 CONTINUE
WRITE(2,1000)T,Y(10),Y(11),Y(12),B ALPHA,THETA,PHI,PSI
WRITE(4,1016)T,Y(11),Y(2),Y(3),VRES
WRITE(5,1017)T,Y(10),Y(12),A

1000 FORMAT(1H,4F13.1,2F13.2,3H *,3F13.2)
1115 FORMAT(5F10.6)
1116 FORMAT(5F10.6)
1117 FORMAT(4F10.2)
IF(T.LT.TI)GOTO 1001
IF(T.GT.TF)GOTO 1001
WRITE(2,1002)WX,WZ

1002 FORMAT(5H WX=,F7.2,11H FPS WZ=,F7.2,5H FPS))
1001 CONTINUE
K=INT((T+.00005)*5)+1
DZ=-(GMS/(GMS+GMC+133.0)*OSL)
CALL ADDED(Y,T,K,B ALPHA,THETA,DZ)
RETURN
END

C C C C C C C C C C C
C SUBPROGRAMME ADDED
--------

SUBROUTINE ADDED(Y,T,K,B ALPHA,THETA,DZ)
COMMON /G/GM1,GM2,XXI,YYI,ZZI,GMC,GMS,OCL,OSL,OGCL,VAR,AXR,F,DP
COMMON /A/A<3(251),A<1(251)
COMMON /COF/OIA,C0(3,19),RH0A2
DIMENSION Y(12),X(251),Z(251),XX(251),ZZ(251),XXP(251),ZZP(251)
DIMENSION XXXP(251),ZZZP(251),ACCX(251),ACCZ(251),E(251),EAP(251),PA(251)
DIMENSION TH(251),TTTH(251),TTTH(251)
E(K)=T
X(K)=Y(11)
Z(K)=Y(12)
AP(K)=B ALPHA/57.296
TH(K)=THETA/57.296
IF(K.GT.1)GOTO 100

#include "file"
XX(1) = 0
ZZ(1) = 0
TTH(1) = 0
ACCX(1) = 0.0
ACCZ(1) = 0.0
AK3(1) = 2.1 * COS(AP(1)) ** 5
AK1(K) = 1.3
GOTO 400

100 IF (K, GT. 2) GOTO 200
XX(2) = 0
ZZ(2) = 0
TTH(2) = 0
ACCX(2) = 0.0
ACCZ(2) = 0.0
AK3(K) = 2.1 * COS(AP(K)) ** 6
AK1(K) = 1.3
GOTO 400

200 IF (K, GT. 3) GOTO 300
XX(3) = 0
ZZ(3) = 0
TTH(3) = 0
ACCX(K) = 0.0
ACCZ(K) = 0.0
AK3(K) = 2.1 * COS(AP(K)) ** 6
AK1(K) = 1.3
GOTO 400

300 CONTINUE
K1 = K - 1
K2 = K - 2
K3 = K - 3
C CALCULATE ACCELERATION MODULI
XX(K) = (X(K1) - X(K2)) / 0.2
ZZ(K) = (Z(K1) - Z(K2)) / 0.2
TTH(K) = (T(K1) - T(K2)) / 0.2
XXP(K) = SIN(AP(K)) * SQRT((XX(K) ** 2) + (ZZ(K) ** 2))
ZZP(K) = COS(AP(K)) * SQRT((XX(K) ** 2) + (ZZ(K) ** 2))
IF (K, GT. 5) GOTO 350
XX(K1) = (X(K2) - X(K3)) / 0.2
ZZ(K1) = (Z(K2) - Z(K3)) / 0.2
TTH(K1) = (T(K2) - T(K3)) / 0.2
XXP(K1) = SIN(AP(K1)) * SQRT((XX(K1) ** 2) + (ZZ(K1) ** 2))
ZZP(K1) = COS(AP(K1)) * SQRT((XX(K1) ** 2) + (ZZ(K1) ** 2))

350 XXP(K) = (XXP(K) - XXP(K1)) / 0.2
ZZP(K) = (ZZP(K) - ZZP(K1)) / 0.2
TTH(K) = (TTH(K) - TTH(K1)) / 0.2
ACCX(K) = ((XXP(K) + TTH(K) * ZZP(K) + (TTH(K) + ZZP(K)) * OZ * COS(T(K))) - TTH(K)) ** 2
1 * ABS(OZ) * SIN(T(K)) ** 2
ACCZ(K) = ((ZZP(K) - TTH(K) * XXP(K) - (TTH(K) ** 2) + OZ * COS(T(K))) - TTH(K)) ** 2
1 * ABS(OZ) * SIN(T(K)) ** 2
ACCX(K) = ACCX(K1) + 0.5 * (ACCX(K) - ACCX(K1))
ACCZ(K) = ACCZ(K1) + 0.5 * (ACCZ(K) - ACCZ(K1))
C CALCULATE THE NEW ADDED MASS COEF. IN TANGENTIAL DIRECTION
IF(ABS(ACCZ(K)).LT.0.9)GOTO15
AK3(K)=2.1*(COS(PA(K)))**6
GOTO 25
15 IF(ABS(ACCZ(K)).GE.0.25)GOT016
AK3(K)=(-16.0*ABS(ACCZ(K))+5.0)*COS(PA(K)))**6
GOTO 25
16 AK3(K)=(-2.92*ABS(ACCZ(K))+4.78)*COS(PA(K)))**6
GOTO 25
25 AK3(K)=(AK3(K)+AK3(K1)+AK3(K2))*1.0/3.0
C CALCULATE THE NEW ADDED MASS COEF. IN NORMAL DIRECTION
C
C IF(ABS(ACCX(K)).LT.0.9)GOTO20
AK1(K)=1.3
GOTO 30
20 IF(ABS(ACCX(K)).LT.0.25)GOTO 21
AK1(K)=(-1.53*ABS(ACCX(K))+2.68
GOTO 30
21 AK1(K)=(-14.8*ABS(ACCX(K))+6.00
C
C 30 AK1(K)=(AK1(K)+AK1(K1)+AK1(K2))*1.0/3.0
IF(K.NE.251)GOTO 400
DO 500 KP=4,251
IF(KP.GT.4)GOTO 600
KPP=4
GOTO 500
500 CONTINUE
IF(KPP.LT.44)GOTO700
600 FORMAT(2,2100)
2100 FORMAT(/78H ACCELERATION MODULI (ALL VELOCITIES AND ACCELERATIONS W.R.T. PARACHUTE AXES)://60H TIME (SEC) ACCEL MOD (X) A.
C
C 100 ACCEL MOD (Z) AK3(T) AK1(T),//)
KPP=0
700 CONTINUE
WRITE(2,2200)E(KP),ACCX(KP),ACCZ(KP),AK3(KP),AK1(KP)
WRITE(3,1118)E(KP),AK3(KP),AK1(KP)
2200 FORMAT(F3.1,F15.3,F16.3,F15.3,F15.3)
1118 FORMAT(3F10.3)
KPP=KPP+1
500 CONTINUE
400 CONTINUE
RETURN
END

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SUBPROGRAMME AIR
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SUBROUTINE AIR(T,Y)

COMPUTES RELATIVE AIR MOVEMENT
T IS TIME, Y DEPENDENT VARIABLES

X(6) FORCES AND MOMENTS
WX, WY, WZ WIND VEL COMPONENTS W.R.T. EARTH
VS SQUARE OF RESULTANT AIR VEL, ALPHA IS ANGLE OF ATTACK
AX ANGLE BETWEEN COMPONENT OF RES AIR VEL IN XY PLANE AND X AXIS
UA, VA, WA COMPONENTS OF AIR VEL W.R.T. CANOPY C OF G
UAS, VAS, WAS SAME AS ABOVE BUT OF STORE
VAR VELOCITY NORMAL TO RUDDER, AXR ANGLE OF VEL VECTOR TO RUDDER
VD4, V05 — ANG. VELS. ABOUT Y & X AXES RESPECTIVELY
ARM, RA DISTANCE OF RUDDER FROM Z AXIS AND ITS AREA
VAD AIR VELOCITY AT EFFECTIVE DIAMETER
D, DP EFFECTIVE CANOPY DIA AND DAMPING COEFF FOR ROTATION
GM1, GM2 TOTAL EFFECTIVE MASS OF SYSTEM INCLUDING ADDED APPARENT
MASS) IN AXIAL X NORMAL DIRECTIONS
XXI, YYI, ZZI TOTAL MOMENTS OF INERTIA ABOUT X, Y, Z AXES AT THE
CANOPY CENTROID
ST, SP, SS, CT, CP, CS SIN AND COS OF ANGLES RELATING COORD SYSTEMS
VI(9) ARE UNIT VECTORS REFERING VARIABLES W.R.T. MOVING AXES
TO FIXED AXES

COMMON /G/GM1, GM2, XXI, YYI, ZZI, GMC, GMS, OCL, OSL, OGCL, VAR, AXR, D, DP
COMMON /P/X(6), WX, WY, WZ, VS, ALPHA, AX, UA, VA, WA, UAS, VAS, WAS, ARM, RA,
1 VD4, V05, VAD, MU
COMMON /T/ST, SP, SS, CT, CP, CS, VI(9)
COMMON /VEL/UVEL

DIMENSION Y(12)

UAIR(N) = VI(N)*WX + VI(N+3)*WY + VI(N+6)*WZ - Y(N)

CALL WIND(T, Y)

RESOLVE WIND INTO MOVING SYSTEM

UA = UAIR(1)
UVEL = UA
VA = UAIR(2)
WA = UAIR(3)

AIR VEL ON STORE

UAS = UA - Y(5)*OSL
VAS = VA + Y(4)*OSL
WAS = WA

MOVE TO CANOPY C OF G

UA = UA - Y(5)*OCL
VA = VA + Y(4)*OCL

VS = UA*UA + VA*VA + WA*WA

ALPHA MEASURED FROM Z AXIS

IF(UA*UA+VA*VA+WA*WA)10,20,10
10 ALPHA = ATAN2(SQRT(UA*UA+VA*VA+WA*WA), -WA)
GO TO 40
20 ALPHA = 0.0
40 CONTINUE
VELOCITY NORMAL TO RUDDER

IF(VA)5,6,5
5 AX=ATAN2(-VA,-UA)
   GO TO 7
6 IF(UA)8,8,9
8 AX=0.0
   GO TO 7
9 AX=3.14159265
7 CONTINUE
   VAR=0.0
   AXR=0.0

VD4=-Y(4)
VD5=-Y(5)

C FOR VELOCITIES AT APPEX DUE TO ANG. VELs. Y(4) & Y(5)
C VELOCITY AT EFFECTIVE DIAMETER DUE TO ROTATION ABOUT Z

VA0=-Y(6)*0/2,0
RETURN
END

--------------------
SUBPROGRAMME FORCES
--------------------

SUBROUTINE FORCES

COMPUTES AERODYNAMIC AND GRAVITATIONAL FORCES

X(6) FORCES AND MOMENTS
WX, WY, WZ WIND VEL COMPONENTS W.R.T. EARTH
VS SQUARE OF RESULTANT AIR VEL, ALPHA IS ANGLE OF ATTACK
AX ANGLE BETWEEN COMPONENT OF RES AIR VEL IN XY PLANE AND X AXIS
UA, VA, WA COMPONENTS OF AIR VEL W.R.T. CANOPY C OF G
UAS, VAS, WAS SAME AS ABOVE BUT OF STORE
VAR VELOCITY NORMAL TO RUDDER, AXR ANGLE OF VEL VECTOR TO RUDDER
ARM, RA DISTANCE OF RUDDER FROM Z AXIS AND ITS AREA
VAO AIR VELOCITY AT EFFECTIVE DIAMETER
D, DP EFFECTIVE CANOPY DIA AND DAMPING COEFF FOR ROTATION
MU PITCH DAMPING COEFFICIENT
GM1, GM2 TOTAL EFFECTIVE MASS OF SYSTEM(INCLUDING ADDED APPARENT
MASS) IN AXIAL & NORMAL DIRECTIONS
ST, SP, SS, CT, CP, CS SIN AND COS OF ANGLES RELATING COORD SYSTEMS
VI(9) ARE UNIT VECTORS REFERING VARIABLES W.R.T. MOVING AXES TO FIXED AXES

COMMON /COF/ DIA, CO(3,19), RHOA2
COMMON /G/GM1, GM2, XXI, YI, ZI, GM, GMS, OCL, OSL, OGCL, VAR, AXR, D, DP
COMMON /P/X(5), WX, WY, WZ, VS, ALPHA, AX, UA, VA, WA, UAS, VAS, WAS, ARM, RA,
1 VD4, VD5, VAO, MU
COMMON /T/ ST, SP, SS, CT, CP, CS, VI(9)
REAL MU, MULA
AERODYNAMIC FUNCTIONS INCL 1/2*RHO*AREA
CTF, CNF, CMF - TANGENT, NORMAL FORCES AND PITCHING MOMENT
ACTING AT TOP OF CANOPY
VELOCITIES MEASURED AT CANOPY COF G
CSF - DRAG OF STORE
CLF, COF - ABSOLUTE VALUES OF 1/2*RHO*COEFF FOR
LIFT AND DRAG OF A FLAT PLATE ALIGNED WITH AND FIXED TO MOVING
X AXIS AT ANGLE OF INCIDENCE IN XY PLANE OF 0 TO 2*PI
THIS PLATE MAKES NO CONTRIBUTION TO THE OVERALL DRRAG

GRAVITY
WT = 9.8*(GMS*GMC)
XG1 = VI(7)*WT
XG2 = VI(8)*WT
XG3 = VI(9)*WT

MOMENT DUE TO WT OF CANOPY AND STORE
STARM = 9.8*(GMC*OCL+GMS*OSL)
XG4 = VI(8)*STARM
XG5 = VI(7)*STARM

AERODYNAMIC FORCES ON CANOPY
XC1 AND XC2 ACT AT TOP OF CANOPY
XC1 = -VS*CNF(ALPHA)*COS(AX)
XC2 = -VS*CNF(ALPHA)*SIN(AX)
XC3 = -VS*CTF(ALPHA)
XC4 = -VS*CNF(ALPHA)*SIN(AX) + XC2*OGCL
XC5 = VS*CMF(ALPHA)*COS(AX) - XC1*OGCL

AERODYNAMIC FORCES ON STORE
XT = CSF(ALPHA)*SQRT(UAS*UAS+VAS*VAS+WAS*WAS)
XS1 = XT*UAS
XS2 = XT*VAS
XS3 = XT*WAS
XS4 = XS2*OSL
XS5 = XS1*OSL

MOMENT DUE TO RUDDER
XR6 = 0.0

DAMPING OF ROTATIONS ABOUT X AND Y
MULA = MU*24.0*Dia**4
XD4 = MULA*VD4
XD5 = MULA*VD5

DAMPING OF ROTATION ABOUT Z
XD6 = V4D*VAC*D*SIGN(DP, VAD)/2.0

SUM FORCES
X(1) = XG1 + XC1 + XS1
SUBROUTINE DER(Y,G,T,K)

COMPUTE DERIVATIVES FOR EQUATIONS OF MOTION IN MOVING COORD SYSTEM
AND FOR REFERING TO EARTH AXES

Y DEPENDENT VARIABLES U,V,W,P,Q,R,THETA,PHI,PSI,X,Y,Z
G DERIVATIVES OF ABOVE
X(6) FORCES AND MOMENTS
W,X,Y,Z WIND VEL COMPONENTS W.R.T. EARTH
V SQUARER OF RESULTANT AIR VEL, ALPHA IS ANGLE OF ATTACK
AX ANGLE BETWEEN COMPONENT OF RES AIR VEL IN XY PLANE AND X AXIS
UA,VA,WA COMPONENTS OF AIR VEL W.R.T. CANOPY C OF G
UAS,VAS,Was SAME AS ABOVE BUT OF STORE
VAR VELOCITY NORMAL TO RUDDER, AXR ANGLE OF VEL VECTOR TO RUDDER
ARM,RA-DISTANCE OF RUDDER FROM Z AXIS AND ITS AREA
VAR-AIR VELOCITY AT EFFECTIVE DIAMETER
D,DP-EFFECTIVE CANOPY OIA AND DAMPING COEFF FOR ROTATION
GMC,GMS-CANOPY,STORE MASSES \(1\)AK1,AK2,AKD=K55--INERTIA COEFFICIENTS
OCL,OSL-DISTANCES OF C OF G OF CANOPY \& STORE FROM SYSTEM ORIGIN
GM1,GM2-TOTAL EFFECTIVE MASS OF SYSTEM (INCLUDING ADOEO APPARENT
MASS ) IN AXIAL \& NORMAL DIRECTIONS
XXI,YYI,ZZI-TOTAL MOMENTS OF INERTIA ABOUT XY,Z AXES AT THE
CANOPY CENTROID
ST,SP,SS,CT,CP,CS SIN AND COS OF ANGLES RELATING COORD SYSTEMS
VI(9) ARE UNIT VECTORS REFERING VARIABLES W.R.T. MOVING AXES
TO FIXED AXES

COMMON /G/GM1,GM2,XXI,YYI,ZZI,GMC,GMS,OCL,OSL,OCL,VAR,AXR,DO
COMMON /P/X(6),W,X,Y,Z,V,S,ALPHA,AX,UA,VA,WA,Was,ARM,RA,
1
V04,V05,V0,\mu
COMMON /T/ST,SP,SS,CT,CP,CS,VI(9)
COMMON/COF/DIA,C0(C3,19),RHOA2
COMMON /A/AX(251),AX(251)
COMMON /B/BALPHA,THETA,DZ

DIMENSION Y(12),G(12)
REAL MG1,MG2,XXI,YY1,KA1,KA2,KA3

CALL ADDED(Y,T,K,BALPHA,THETA,DZ)
CALL CALCULATE TOTAL MASS AND TOTAL MOMENT OF INERTIA OF THE SYSTEM
GMI = 1.22 * 3.142 * DIA * DIA / 12.0

MG1 = GMI + (AK3(K) - 2.1) * GMI
MG2 = GMI + (AK1(K) - 1.3) * GMI

KA1 = AK1(K) * GMI * OCL + GMS + OSL
KA2 = AK1(K) * GMI * OCL
KA3 = KA2 + YYI - ZZI

FX = X(1) - MG1 * Y(3) * Y(6) * Y(2) - KA1 * Y(4) * Y(6)
FY = X(2) + MG1 * Y(4) * Y(3) - MG2 * Y(6) * Y(1) - KA1 * Y(5) * Y(6)
FL = X(4) + KA3 * Y(5) * Y(6) + KA1 * Y(5) * Y(1) - Y(5) * Y(3)

F = X(5) - KA3 * Y(4) * Y(6) + KA1 * (Y(4) * Y(2) - Y(5) * Y(3))

CALCULATE THE ORDINARY DIFF. EQ.
G(1) = (KA2 * YYI) * FX - KA1 * FM) / ((KA2 + YYI) * MG2 - KA1 * KA1)
G(2) = (KA2 + XXI) * FY + KA1 * FL) / ((KA2 + YYI) * MG2 - KA1 * KA1)
G(3) = (X(3) - MG2 * Y(4) * Y(2) - Y(5) * Y(1) + KA1 * (Y(4) * Y(4) * Y(5) * Y(5))

1/MG1
G(4) = (KA1 * FY + MG2 * FL) / ((KA2 + YYI) * MG2 - KA1 * KA1)
G(5) = (KA1 * FX + MG2 * FM) / ((KA2 + XXI) * MG2 - KA1 * KA1)
G(6) = (X(6) + (XXI - YYI) * Y(4) * Y(5)) / ZZI
G(7) = Y(5) * CP - Y(6) * SP
G(8) = (Y(4) * (Y(5) * SP + Y(6) * CP) * ST / CT
G(9) = (Y(5) * SP + Y(6) * CP) / CT

J = 1
DO 1 I = 10, 12
G(I) = V(J) * Y(I) + V(J+1) * Y(J) + V(J+2) * Y(J+3)
J = J + 3
CONTINUE

RETURN

END

---

SUBPROGRAMME WIND

---

SUBROUTINE WIND(T, Y)

RELATES WIND GIVEN BY COMPONENTS IN EARTH AXES TO MOVING SYSTEM
T INDEPENDENT VARIABLE TIME
Y DEPENDENT VARIABLES INCL Y(12) VERTICAL DISTANCE FALLEN
X(6) FORCES AND MOMENTS
WX, WY, WZ WIND VEL COMPONENTS W.R.T. EARTH
WS SQUARE OF RESULTANT AIR VEL, ALPHA IS ANGLE OF ATTACK
AX ANGLE BETWEEN COMPONENT OF RES AIR VEL IN XY PLANE AND X AXIS
UA, VA, WA COMPONENTS OF AIR VEL W.R.T. CANOPY C OF G
UAS, VAS, WAS SAME AS ABOVE BUT OF STORE
ARM, PA DISTANCE OF RUDDER FROM Z AXIS AND ITS AREA
VAD AIR VELOCITY AT EFFECTIVE DIAMETER
TI BEGINNING TIME OF GUST
TF FINISHING TIME OF GUST
WXF, WZF MAXIMUM WIND VEL (FPS) IN X AND Z DIRECTION

WND 001
WND 002
WND 003
WND 004
WND 005
WND 006
WND 007
WND 008
WND 009
WND 010
WND 011
WND 012
WND 013
COMMON /P/X(6),W,X,Y,Z,V,W,AS,ALPHA,A1,A1,AUA,WA,UA,US,VS,HAS, ARM,RA,
1
COMMON /WF/T,T,F,WXF,WZF

DIMENSION Y(12)

TI=0.0
TF=0.0
WXF=0.0
WZF=0.0
IF(T.LE.TI)GOTO 10
IF(T.GE.TF)GOTO 10
WCON=((COS(3.1415926+(T-TI)/(TF-TI))*31.415926)+3)*
1(COS(3.1415926+(T-TI)/(TF-TI))*6.28318521+1))/8
WX=WCON*WXF
WZ=WCON*WZF
GOTO 20
10 WX=0.0
20 WY=0.0
WZ=0.0

RETURN
END

SUBPROGRAMME CNF

FUNCTION CNF(A)
COMMON /COF/ OA,CO(3,19),RHOA2
COMMON /VEL/UAVEL
CNF=RHOA2*TRPL(A,1)
IF(UAVEL.GT.0.)CNF=-CNF
RETURN
END

SUBPROGRAMME CTF

FUNCTION CTF(A)
COMMON /COF/ OA,CO(3,19),RHOA2
CTF=RHOA2*TRPL(A,2)
RETURN
END
SUBPROGRAMME CMF

FUNCTION CMF(A)
COMMON /COF/ DIA,C0(3,19),RHOA2
COMMON /VEL/UAVEL
CMF=RHOA2*DIA*TRPL(A,3)
IF(UAVEL.GT.0.)CMF=-CMF
RETURN
END

SUBPROGRAMME CSF

FUNCTION CSF(A)
CD=0.8
CSF=CD*0.22
RETURN
END

SUBPROGRAMME TRPL

FUNCTION TRPL (A,ITYPE)
INTERPOLATES VALUES OF A/DYNAMIC FUNCTIONS FOR ANGLE OF ATTACK A
ITYPE DEFINES TYPE OF A/DYNAMIC FUNCTION
N=NUMBER OF DATA PTS
NO=NUMBER OF PTS FOR AITKEN INTERPOLATION
DIMENSION XP(19),YP(19)
COMMON /COF/ DIA,C0(3,19),RHOA2
COMMON /VEL/UAVEL
N=19
NO=6
NQ1=NO-1
NO2=NO/2
NMAX=N-NO2+1
IF(UAVEL.GT.0.)A=-A
X=18*A/3.14159265+10
A=ABS(A)
IF(A.LT.1.59)GOTO1
WRITE(2,2)
2 FORMAT(/,IX,32HALPHA IS GREATER THAN 90 DEGREES)
STOP
1 CONTINUE
IF(X.LT.19.)X=19.
IF(X.LT.0.)X=0.
SET UP END LIMITS
DO 5 I=1,N
XI=I
IF(XI.LT.X)GO TO 5
IF(I.LE.NO2)GO TO 6
IF(I.GE.NMAX)GO TO 7
II=I-NO2
GO TO 10
5 II=1
GO TO 10
6 II=NMAX-NO2
GO TO 10
7 CONTINUE
10 DO 8 I=1,NO
XP(I)=II
YP(I)=CO(ITYPE,II)
8 II=II+1

C INTERPOLATE
CALL AITKEN(X,Y,XP,YP,NO1)
TRPL=Y
RETURN
END

C C C
C C C
C C
----------
SUBPROGRAMME AITKEN
----------

C C C
C C C
C
SUBROUTINE AITKEN(XC,P,X,Y,N)
DIMENSION X(10),Y(10),VAL(10,10)
DO 5 K=1,N
5 VAL(1,K)=(Y(K)*(XC-X(K+1))-Y(K+1)*(XC-X(K)))/(X(K)-X(K+1))
DO 15 M=2,N
NP1MM=N+1-M
DO 10 K=1,NP1MM
-KPM=K+M
10 VAL(K,K)=((VAL(M-1,K)*(XC-X(K+1))=VAL(M-1,K+1)*(XC-X(K)))/(X(K)-
  X(K))+)
15 CONTINUE
P=VAL(N,1)
RETURN
END
APPENDIX E

Listing of the Computer Program Used to Obtain Stability Limits
APPENDIX E

Listing of the Computer Programme Used to Obtain Stability Limits

Nomenclature Conversion

<table>
<thead>
<tr>
<th>Thesis</th>
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<tbody>
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<td>m</td>
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<td>m_s</td>
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<tr>
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<tr>
<td>K_2</td>
<td>T2</td>
</tr>
<tr>
<td>K_1</td>
<td>T1</td>
</tr>
<tr>
<td>K_0</td>
<td>T0</td>
</tr>
</tbody>
</table>
PROGRAM YAVSEN(INPUT, OUTPUT, TAPE1, TAPE4)
REAL L, IYY
READ(1,1000) GM, G, S, L, D, CT, V, GMC
READ(1,1001) AK3, AK5
1000 FORMAT(4F8.3, F10, 3, 4F9.3)
1001 FORMAT(2F8.3)
WRITE(4,1400) GM, G, S, L
WRITE(4,1410) V, GMC
11X, "L =", 2X, F8.3, //)
C
AK1 = -0.15
00 10 1 =2.50
AK1 = AK1 + 0.15
GMI = 1.22 * 3.14 * 0 * 0 * 0 / 12.0
IYY = GM * L * (1 + AK5) * GMI * D / 16.0 + (1 + AK1) * GMI * S * S + GMC * S * S
M = GM + GMC
C
RAT = AK1 / AK3
A1 = (M + AK1 * GMI) / M
S1 = (G * L) / (V ** 2 * CT)
B2 = (M * L + AK1 * GMI) / (M * L)
S2 = (M + AK3 * GMI) / M
S3 = G * S / (V ** 2 * CT)
B0 = (G * L) / V ** 2
C1 = B2
S4 = G * S / (V ** 2 * CT)
B2 = (IYY) / (M * L)
S5 = G * (S + S) / (V ** 2 * L * CT)
D0 = (G * L) / V ** 2
C
Z3 = S1 * B2 - S4 * S2
Z2 = A1 * S5 + S1 * D2 - S4 * B2 - S3 * B2
Z1 = A1 * 00 - B2 * 90
Z0 = A1 * B2 - S2 * 82
P2 = A1 * D2 - (82) ** 2
P1 = S1 * 00 - S4 * 90
C
T2 = Z3 * T2
T1 = Z1 * T2 + Z0 * Z3 - P2 * P1
T0 = Z0 * Z1
WRITE(4,1002) CT, AK1, AK3, AK5
1"AK5 =", 4X, F8.3, //)
WRITE(4,1100) A1, S1, B2, S2, S3
WRITE(4,1200) C1, S4, D2, S5, 00
WRITE(4,1300) Z3, Z2, Z1, Z0, P2, P1
WRITE(4,1700) T2, T1, T0
1200 FORMAT(1X, "C1 =", 3X, F10.4, 1X, "S4 =", 3X, F10.4, 1X, "D2 =", 3X, F10.4,
11X, "S5 =", 3X, F10.4, 1X, "D0 =", 3X, F10.4, //)
1300 FORMAT(1X, "Z3 =", 3X, F10.4, 1X, "Z2 =", 3X, F10.4, 1X, "Z1 =", 3X, F10.4,
YT = T1 * T1 - 4.0 * T2 * T0
IF(YT<0.0)GOTO 60
CN1=-T1/(2.0*T2)+1.0/(2.0*T2)*SQRT(YT)
CN2=-T1/(2.0*T2)-1.0/(2.0*T2)*SQRT(YT)
GOTO 70
60 CN1=-T1/(2.0*T2)
CN2=1.0/(2.0*T2)*SQRT(ABS(YT))
WRITE(4,1800)
1800 FORMAT(2X,11HIMAG, ROOTS,///)
C
70 WRITE(4,1600)RAT,CN1,CN2
1600 FORMAT(1X,3H"RAT=",4X,F9.5,2X,"CN1="",4X,F8.5,2X,"CN2="",4X,F8.5,///)
10 CONTINUE
STOP
END
APPENDIX F

Listing of the Computer Programme Used to Solve the Characteristic Equation of Motion
APPENDIX F

Listing of the Computer Programme Used to Solve the Characteristic Equation of Motion

Nomenclature Conversion

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<td>$AK5$</td>
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<tr>
<td>$k_{11}$</td>
<td>$AKT1$</td>
</tr>
</tbody>
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PROGRAM YAVSEN (INPUT, OUTPUT, TAPE1, TAPE5)

REAL L, IYY

READ (1,1000) GM, G, S, L, 0
READ (1,1100) CT, V, EST, GMC

1000 FORMAT (4F8.3, F10.3 )

1100 FORMAT (F8.3 )

1001 FORMAT (4F8.3)

WRITE (5,1050)

1050 FORMAT (X, 75HSTORE MASS, GMC - CANOPY MASS, L - STORE DIST. FROM CENT
1ROI0, S - HEMLINE DISTANCE,/) 

WRITE (5,1400) GM, G, S, L, 0, V, GMC

14X, F8.3, / )

WRITE (5,1002) CT, AK1, AK3, AK5, AKT1, EST

1"AKT1 = ", 5X, F8.3, IX, "EST = ", 4X, F8.3, / )

C

GMI = 1.22*3.14*S*S*S/12.0
M = GM

IYY = M*(L**2) + (1+AK1)*GMI*S*S + (1+AK5)*GMI*0*D/16.0 + GMC*S*S
CN = -0.005

D0 10 I = 1, 200

CN = CN + 0.005

A1 = (M+AK1*GMI)/M
A0 = AKTI/M/((G*L)/(V**2*CT)) * CN
B2 = (M+AK1*GMI)/M
B1 = (M+AK3*GMI)/M/((G+S)/(V**2*CT)) * CN +(AKT1*S)/(M*L)
B0 = (G*L)/V**2

C1 = (M+AK1*GMI)/M
C0 = (AKTI*S)/(M*L) + (G*S)/(V**2*CT)) * CN

D2 = (IYY)/(M*L)

D1 = (M+AK1*GMI*S)/(M*L) +(AKT1*S**2)/(M*L**2) +
1(G*S**2)/(V**2*L**2) * CN

D0 = (G*L)/V**2

C

A33 = A1*D2 - C1*B2
A22 = A1*D1 + A0*D2 - C0*B2 - C1*B1
A11 = A1*D0 + A0*D1 - C0*B0 - C1*B1
A00 = A0*D0 - C0*B0

C

WRITE (5, 1100) A1, A0, B2, B1, B0
WRITE (5, 1100) C1, C0, D2, D1, D0
WRITE (5, 1300) A33, A22, A11, A00

1100 FORMAT (1X, "A1 = ", 3X, F10.4, 1X, "A0 = ", 3X, F10.4, 1X, 

1200 FORMAT (1X, "C1 = ", 3X, F10.4, 1X, "C0 = ", 3X, F10.4, 1X, "D2 = ", 3X, F10.4, 1X /


C

TOL = 1.E-8
X = EST
D0 94 I T E R = 1, 600
YT0 = A33*X**3 + A22*X**2 + A11*X + A00
X = X + TOL

C
YT1=A33*X**3+A22*X**2+A11*X+A00
X=X-TOL
EB=(YT1-YT0)/TOL
DELTAX=-YT0/EB
ERROR=ABS(DELTAX)
IF(ERROR.GT.TOL) GOTO 30
GO TO 50
30 X=X+DELTAX
40 CONTINUE
50 X1=X
E=ABS(X1)
C
C
K3=A33
K2=A22-A33*E
K1=A11-E*(A22-A33*E)
C
K0=K2**2-4*K1*K3
RAT=(X1**2*(D2-82)+X1*(D1-81)+D0-90)/(X1*(A1-C1)+A0-C0)
WRITE(5,123)
123 FORMAT(47HRAT IS THE RATIO OF AMPLITUDES OF ANGLE TO PATH, //)
WRITE(5,1600)<3,<2,<1,K0, RAT
1600 FORMAT(1X,"<K3=",3X,F10.4,1X,"<K2=",3X,F10.4,1X,"<K1=",3X,F10.4,
11X,"K0=",3X,F10.4,2X,"RAT=",4X,F10.4,:///)
IF(K0.LT.0.) GOTO 60
X2=-K2/(2.*K3)+(1./(2.*K3))*SORT(K0)
X3=-K2/(2.*K3)-(1./(2.*K3))*SORT(K0)
GO TO 70
60 X2=-K2/(2.*K3)
X3=(1./(2.*K3))*SORT(ABS(K0))
WRITE(5,1005)
1005 FORMAT("NO REAL ROOT=",13X)
70 WRITE(5,1004)<3,<2,<1,X1,X2,X3
1"X2=",3X,F8.3,3X,"X3=",3X,F8.3,////)//
10 CONTINUE
STOP
END
Hence, from equations (C5) and (C6) through equations (C10), (C10) and (C11) the force and moment components for three cases are obtained, each of which one outlined as follows;

Case 1. If the parachute canopy-store system has no symmetry plane, i.e \(x_a, s \neq 0, y_a, s \neq 0, z_a, s \neq 0\), equations are obtained as,

\[
X = (a_{ii} + m)U + (a_{ii}Z + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]

\[
Y = (a_{ii} + m)U + (a_{ii} + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]

\[
Z = (a_{ii} + m)U + (a_{ii} + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]

\[
L = \left(\frac{a_{ii} + m + my}{s} + \frac{a_{ii} + m + mx}{s} + \frac{a_{ii} + m + mz}{s}\right)R + (a_{ii} + m)U + (a_{ii} + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]

\[
M = \left(\frac{a_{ii} + m + mx}{s} + \frac{a_{ii} + m + mx}{s} + \frac{a_{ii} + m + mz}{s}\right)R + (a_{ii} + m)U + (a_{ii} + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]

\[
N = \left(\frac{a_{ii} + m + mx}{s} + \frac{a_{ii} + m + my}{s} + \frac{a_{ii} + m + mz}{s}\right)R + (a_{ii} + m)U + (a_{ii} + m)Q + (a_{ii} + m)W + (a_{ii} + m)PQ - (a_{ii} + m)VR + (a_{ii} + m)WQ - (a_{ii} + m)RU + (a_{ii} + m)PV + (a_{ii} + m)PU + (a_{ii} + m)QW \]
Case 2: One Plane of Symmetry

Supposing the parachute canopy-store system has one plane of symmetry, i.e. (OXY) plane in Fig.C.1. Subsequently, $y_a=y_s=0$, and the set of equations, (C12) becomes

\[
X = (a_{11}+m)U+(a_{11}z_a+mz_s)Q+(a_{11}w_a+mW)QW-(a_{22}+m)VR+(a_{11}z_a+mz_s)PR
- (a_{33}w_a+mW)Q- (a_{22}+m)R^2
\]

\[
Y = (a_{22}+m)V-(a_{22}z_a+mz_s)P+(a_{22}w_a+mW)R-(a_{33}+m)VP+(a_{22}w_a+mW)PQ
+ (a_{11}+m)UR+(a_{11}z_a+mz_s)QR
\]

\[
Z = (a_{33}+m)W-(a_{33}w_a+mW)P+(a_{22}+m)VP+(a_{22}z_a+mz_s)PQ-(a_{11}+m)QU
- (a_{22}z_a+mz_s)Q^2-(a_{11}z_a+mz_s)Q
\]

\[
L = [(a_{22}z_a+mz_s)+a_{01}]P-(a_{22}z_a+mz_s)Q-(a_{11}z_a+mz_s)R-(a_{22}z_a+mz_s)U-(a_{11}z_a+mz_s)V
- (a_{33}w_a+mW)W-(a_{22}+m)PQ+(a_{22}+m)PR-(a_{33}+m)QU
- (a_{22}z_a+mz_s)Q- (a_{22}z_a+mz_s)PQ
\]

\[
M = [(a_{22}z_a+mz_s)+a_{01}]P-(a_{22}z_a+mz_s)Q-(a_{11}z_a+mz_s)U-(a_{11}z_a+mz_s)V
+ (a_{11}-a_{33})Q+ (a_{11}-a_{33})QW+(a_{11}+mW)R+ (a_{22}z_a+mz_s)QU
+ (a_{22}z_a+mz_s)PQ+(a_{22}z_a+mz_s)PQ
\]

\[
N = [(a_{22}z_a+mz_s)+a_{01}]P-(a_{22}z_a+mz_s)Q-(a_{11}z_a+mz_s)U-(a_{11}z_a+mz_s)V
- (a_{22}z_a+mz_s)W+(a_{22}z_a+mz_s)QW+(a_{22}+m)Q
+ (a_{22}z_a+mz_s)R+ (a_{22}z_a+mz_s)QW+(a_{22}z_a+mz_s)QW
\]

The equations derived by Cockrell and Doherr\(^8\) contain one extra term in each of the moment equations, namely, $a_{11}UV$ in $L$, $a_{11}(W-U)$ in $M$ and $a_{11}VW$ in $N$. These differences arise because of the assumption made on the application point of the apparent mass tensor.