TURBULENT FLUID FLOW THROUGH LOW LOSS BENDS

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SUMMARY

In the Introduction to this thesis the steady flow of Newtonian fluids in duct systems is discussed, with particular consideration to rectangular section bends. The considerable economic savings, both capital and annual that can be obtained by a small reduction in the head loss of an individual system element is indicated.

Reliable experimental data on the value of head loss coefficients for incompressible fluid flow through rectangular section bends is very limited and correlation of different experiments is almost impossible because of the different definitions of loss coefficient and the various experimental procedures used. At present theoretical methods of analysing the three-dimensional flow problem occurring in a bend are non-existent and only a limited contribution is made by two-dimensional approaches. Reasons for this situation are discussed together with possible areas for future development.

A computer based data logging instrumentation system is used to record the necessary data required to calculate the value of the bend loss coefficient for various combinations of inner and outer radius ratio. The aspect ratio and Reynolds number of 2 and $2 \times 10^5$ respectively are representative of a number of practical situations.

An indication of the accuracy of the values of bend loss coefficient quoted is given by the corresponding uncertainty interval which is based on that selected, for given confidence limits, in the collected data.
For the combinations of inner and outer radii the variable area bends are modelled by a combination of a diffuser, constant area bend and a contraction in the above or reversed order. The calculated value of the model loss coefficient agrees surprisingly well with the experimental values. Based on the extension of the model calculations the optimum geometrical shapes for circular cross-section 90° bends are suggested.
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CONTENTS

Summary i
Acknowledgements iii
List of Contents iv
List of Figures vii
Notation xi

Chapter 1. Introduction - Internal Fluid Flow 1

Chapter 2. Available Information 6

2.1. Introduction
2.2. Experimental Data
2.3. Theoretical Methods of Attack
2.3.1. Nature of the Flow in Bends
2.3.2. Various Approaches for Analysing the Flow Through Bends
2.4. Scope of Present Work

Chapter 3. Experimental Facilities 35

3.1. Design Philosophy
3.2. The Test Rig
3.2.1. Duct Sections
3.2.2. Bend Section
3.2.3. Duct Traverse Section
3.2.4. Inlet Nozzle
3.2.5. Control Box and Fan
3.3. Instrumentation

Chapter 4. Calculations and Uncertainty Analysis 42

4.1. Introduction
4.2. Theory of the Uncertainty Analysis
4.3. Definition of Bend Loss Coefficient

4.4. Application of Uncertainty Analysis
   4.4.1. Uncertainty Intervals and Odds for the Collected Data
   4.4.2. Other Parameters involved in the Calculation

Chapter 5. Experimental Work

5.1. Preliminary Tests
   5.1.1. Alignment of Duct Sections
   5.1.2. Static Head Tappings
   5.1.3. Nature of the Flow
   5.1.4. Mean Velocity Measurement
   5.1.5. Reynolds Number Range

5.2. Duct Friction Factor Reynolds Number Relationship

5.3. Bend Loss Coefficient

Chapter 6. Analysis of Results

6.1. Introduction

6.2. Duct Friction Factor Reynolds Number Relationship
   6.2.1. Experimental Results
   6.2.2. Uncertainty Interval

6.3. Values of Bend Loss Coefficient
   6.3.1. Experimental Results
   6.3.2. Uncertainty Intervals
   6.3.3. Calculation Procedure
   6.3.4. Selected Cross-Sections

6.4. Optimisation of the Bend Geometry
### Chapter 7. Bend Loss Coefficients from Models of Variable Area Bends

7.1. Model Calculations for Experimental Bend Geometries  

7.2. Model Extensions  
   7.2.1. Rectangular Cross-Section Bends  
   7.2.2. Circular Cross-Section Bends  

7.3. Discussion

### Chapter 8. Suggestions for Further Research

### Chapter 9. Conclusions

### Appendix 1. General Equations of Motion and Continuity

- A.1.1. Equation of Continuity  
- A.1.2. The General Equations of Motion

### Appendix 2. Computer Programs

- A.2.1. Program H15E - Friction Factor - Reynolds Number  
- A.2.2. Program H15F - Bend Loss Coefficient

### Appendix 3. Computer Graphical Representations

- A.3.1. Duct Friction Factor - Reynolds Number Relationship  
- A.3.2. Bend Loss Coefficient

### References
LIST OF FIGURES

2.1 Schematic Representation of Head Loss Attributable to a Bend.
3.1 Typical Duct Section.
3.2 Schematic Representation of the Construction of the Duct Section.
3.3 Example of the Three Types of Bend Configuration.
3.4 Duct Traverse Section.
3.5 Cylindrical Yawmeter Used to Measure Local Velocity.
3.6 General Photograph of Instrumentation.
3.7 Instrumentation - Collection and Assembly of the Collected Data on Punched Paper Tape.
4.1 Definition of the Bend Loss Coefficient $k$.
4.2 Histogram and Normal Distribution Formed by the Static Head Readings at a Given Tapping.
4.3 Flow Chart Outlining Calculation Procedure.
5.1 Calibration of Each Static Head Tapping.
5.2 Co-ordinate System $x, y, z$.
5.3 Velocity Profiles at Cross-Section along a Straight Duct.
5.4 Velocity Distribution in a Straight Duct at a Distance of 48 Hydraulic Diameters Downstream of the Inlet Nozzle.
5.5 Velocity Profiles from Fig.5.4 - Variations of local velocity $v$ with $\alpha$ for values of $\beta$.
5.6 Total Quantity of Air Through the Duct - Area $A_{\beta}$ Under the Profiles of Fig.5.5 against $\beta$.
5.7 Schematic Representation of Duct Friction Factor - Reynolds Number Test Rig.
5.8 Experimental Arrangement for Duct Friction Factor - Reynolds Number Tests.
5.9 Schematic Representation of Bend Loss Coefficient Test Rig.

5.10 Experimental Arrangement for Bend Loss Coefficient Tests

6.1 Experimental Duct Friction Factor – Reynolds Number Relationship.

6.2 Moody Chart of Colebrook-White Transition Formula.

6.3 Computer Line Printer Output for a Typical Duct Friction Factor-Reynolds Number Test.

6.4 Graphical Representation of the Variation of $H(j)$ with $L/Dh(j)$ for a Typical Duct Friction Factor-Reynolds Number Test.

6.5 Computer Line Printer Output of Results for a Typical Bend Loss Coefficient Test.

6.6 Graphical Representation of the Variation of $F(j)a$ with $L/Dh(j)$ for a Typical Bend Loss Coefficient Test.

6.7 Variation of Bend Loss Coefficient with $Ro/D$ for Values of $Ri/D$.

6.8 Comparison of Experimental Values of Bend Loss Coefficient.

6.9 Variation of Aspect Ratio Correction Factors with the Bend Radius Ratio.

6.10 Correlation of up-to-date Values of Bend Loss Coefficient for Constant Area Bends of Aspect Ratio 2 at a Reynolds Number of $1.8 \times 10^5$.

6.11 Percentage Uncertainty in Bend Loss Coefficient.

6.12 Value of Bend Loss Coefficient Calculated without the Duct Friction Factor – Reynolds Number Relationship.

6.13 Variation of Bend Loss Coefficient with Value Calculated as shown in Fig.6.12.
6.14 Effect of Calculating Bend Loss Coefficient from Friction Factor Check.

6.15 Value of Bend Loss Coefficient Calculated for $H(J)$ Value at a Single Cross-Section Downstream of the Bend.

6.16 Variation of Bend Loss Coefficient from Fig. 6.15 with Distance Downstream of the Bend.

6.17 Variation of Bend Loss Coefficient with Mid-Plane Cross-Sectional Area of the Bend.

6.18 Optimum Geometry for Rectangular Cross-Section $90^\circ$ Bends of Aspect Ratio 2.

7.1 Diffuser - Constant Area Bend - Contraction Representations for Variable Area Rectangular Cross-Section $90^\circ$ Bends.

7.2 Rectangular Diffuser Static Pressure Recovery Coefficient $C_p$.

7.3 Comparison of Model and Experimental Values of Bend Loss Coefficient.

7.4 Loss Curves for the Model Bend Representations shown in Fig. 7.2.

7.5 Variation of Model Bend Loss Coefficient from Extension of Variable Area Rectangular Cross-Section $90^\circ$ Bends.

7.6 Static Head Recovery Coefficient $C_p$ for Conical Diffusers.

7.7 Circular Cross-Section Constant Area Bend Performance Chart ($R_n = 10^5$).

7.8 Variation of Model Bend Loss Coefficient for Variable Area Circular Cross-Section $90^\circ$ Bends.

8.1 Circular Cross-Section $90^\circ$ Bend formed by Combination of Two $45^\circ$ Constant Radius Bends of Different Radius.
A.1.1 Cylindrical Co-ordinates, \( r, \theta, z \).

A.1.2 Flow through an Infinitesimal Volume Formed by Three Pairs of Adjacent Co-ordinate Surfaces.

A.1.3 Projections of the Velocity at a Point on the Axes of the Cylindrical Co-ordinates.

A.2.1 Flow Chart for Computer Program H15E - Friction Factor-Reynolds Number Relationship.

A.2.2 Flow Chart for Computer Program H15F - Bend Loss Coefficient.
NOTATION

D  duct width in plane of bending.
H  duct height normal to plane of bending.
AR aspect ratio = H/D.
Dh duct hydraulic diameter = 4 Area/Perimeter.
L  duct length.
Ld duct length downstream of the bend.
Ri bend inner wall radius.
Ro bend outer wall radius.
s static head.
V̅ bulk mean velocity.
TH total head at a given duct cross-section.
K bend loss coefficient defined in equation 4.6.
Rn Reynolds number.
f duct friction factor.

s(J)a static head at a given tapping J on the duct face a.
ś(J)a average of five values of s(J)a
F(J)a mean static head ś(J)a minus a reference
static head ś(R)a divided by the mean inlet velocity
head, V̅^2/2g.

H(J) average of four values of F(J)a at a given cross-section.
m arithmetic mean of observed values of a given
quantity
N number of observed values.
w uncertainty interval.
b odds.
P barometric pressure.
Nu nozzle upstream static head.
- xii -

**Nt** nozzle throat static head.

**AD** air density.

**NH** nozzle differential head.

**PR** nozzle pressure ratio.

**AEF** nozzle adiabatic expansion factor.

**c** ratio of specific heats of air = \( c_p/c_v \)

**A** duct cross-sectional area.

**DR** density ratio (water/air).

**CU** upstream constant.

**CD** downstream constant.

**Q** total quantity of air through the duct = \( A \cdot \bar{v} \)

**v** local velocity

**k** duct hydraulic roughness.

**K_{AR}** bend loss coefficient for an aspect ratio AR.

**C_{AR}** aspect ratio correction factor.

**K_{FFC}** bend loss coefficient from friction factor check-defined in Fig.6.12.

**K'** bend loss coefficient from single cross-section downstream of the bend - defined in Fig.6.15.

**K_{CA}** bend loss coefficient for constant area bend.

**\bar{q}** mean velocity head = \( \bar{V}/2g \).

**Cp** diffuser static pressure recovery coefficient.

**HL_{CA}** head loss caused by a constant area bend.

**Kc** model bend loss coefficient defined in equation 7.10.

**Am** bend mid-plane cross-sectional area.

**\theta** air temperature

**\mu_0** absolute viscosity of air.

**\sigma** standard deviation.

**\beta** ratio of nozzle throat to pipe diameter.
x, y, z. co-ordinate system defined in Fig. 5.2.

α, β, γ co-ordinate system defined in Fig. 5.6

ρ density

v_r, v_θ, v_z local velocities in radial, azimuthal and axial directions, see Fig. A.1.2.
CHAPTER 1

INTRODUCTION - INTERNAL FLUID FLOW
Internal duct flow systems are used extensively in all branches of engineering. Distribution systems in the fields of heating, ventilating and air conditioning, civil, mechanical, automobile and aeronautical engineering are obvious examples. Internal fluid flow is quite an old subject for study. A highly interesting historical book by Rouse & Ince (1) (1957) contains some surprising details. Around 1500, Leonardo de Vinci made many fluid mechanical experiments and presented perceptive flow sketches.

In 1965 a symposium (2) on the fluid mechanics of internal flow was held in U.S.A. The papers and discussions presented represent a general appreciation of internal fluid flow behaviour at that stage.

A proper appreciation of any such internal duct flow system entails consideration of three factors of varying degrees of precision. First there is a general understanding of the physical processes occurring in the individual components an understanding of which gives a warning of possible difficulties and means whereby they may be overcome. Typical of these processes are the prediction of boundary layer growth, the location of separation and reattachment, the occurrence of large scale unsteady phenomena such as oscillating stall in a diffuser, the size, frequencies and intensities of unsteady forces on flow boundaries, and noise, both due to structural vibration induced by unsteady forces at the boundaries and that caused directly from open jets and shear layers. Second there is the available experimental data, such as the efficiency of diffusers or the values of the loss coefficient for a bend, by which the performance of
the overall system may be predicted. Third there is the knowledge and methods by which the designs may be optimised. In most designs it is only the first and second stages which can be achieved to any significant extent, optimisation techniques for internal flow systems being in their formative stages. This was observed by Hawthorne\(^{(2)}\) (1965) who stated:

"From the utilitarian point of view theories are of greatest assistance when they can be used to optimise the design of engineering equipment. However, the degree of refinement with which such optimisation can be performed, even with the assistance of well correlated experimental data, is often disappointing to engineers designing equipment in which fluid mechanics play a significant role. In many instances the theory is incomplete and the theoretical procedures appear to be too complicated and their results too uncertain to be worth the effort of a genuine process of optimisation. On the other hand, the use of theory to interpret and correlate experimental data from which overall performance can be estimated, although a less ambitious objective, has been more useful, even though the theoretical basis of such correlations is often no more complicated than dimensional analysis".

Significant interest exists in internal flows and many of the unresolved problems in this field are important, challenging and interesting. Any duct element such as a bend acts as an obstruction to the flow, resulting in additional head loss. With the increasing size and complexity
of modern internal duct flow networks marginal improvements in the design of a component can lead directly to significant economic savings. For example in 1967 Lewis & Holmes\textsuperscript{(3)} quote a figure of £150,000 as the approximate capitalised value of each lbf/in\textsuperscript{2} of pressure drop saved on the first generation of gas-cooled nuclear reactors. Since a typical stated fall in pressure through the circulation system was of the order of 12 lbf/in\textsuperscript{2}, uncertainties of 1 or 2 lbf/in\textsuperscript{2} are well within the bounds of possibility given the general status of knowledge on pressure losses. Another instance where the loss of head caused by a duct element has very important economic significance is in the tunnel of a hydro-electric scheme. In such systems the water usually has a high velocity and, therefore, high kinetic head in order to keep the required duct size to a minimum. Water flowing through a tunnel of 20ft. diameter at 12ft/sec. would be equivalent to almost 1,000 horsepower. A 90\degree bend is unlikely, though not unknown, in such a pipeline, but even a 30\degree deflexion would result in a loss of some hundreds of horsepower. Each 100 horsepower lost might be worth £1,000 per annum assuming a one-third load factor. In such cases a reduction of the loss caused by a component or a more accurate estimate of the loss coefficient and hence a reduction in the load factor is clearly of importance.

In the early (pre 1960) designs of internal duct flow systems, fluid mechanical considerations were not overriding and the requirements of a number of factors such as space, weight and production costs usually resulted in the acceptance of simple geometrical shapes. These designs were incompatible with the aim of optimising fluid mechanical designs and
resulted in excessive head losses through the system. Over the last decade the situation has been changing slowly and the reductions in head losses that can be obtained by specially designed components in conjunction with the accurate determination of the loss coefficients have become significant.

Determining the values of loss coefficients under steady flow conditions is thus a most important practical problem in the field of internal duct flows. The purpose of the present experimental investigation is intended to reflect this situation. Throughout the thesis attention is confined to the single-phase, steady flow of an incompressible fluid in ducts running full, these conditions being representative of many practical situations.
CHAPTER 2

AVAILABLE INFORMATION
FIG. 2.1 - Schematic Representation of Head Loss Attributable to a Bend
2.1 INTRODUCTION

The nature of the flow of fluids around bends has stimulated much interest in the past. This has resulted in extensive literature, there being well over 200 major papers on the subject. In the present context effort has been concentrated on previous reviews and papers concerned with rectangular section bends. For this reason it is not necessary to list all the available references since they are adequately covered by the major survey papers*.

Before discussing the available experimental data it is convenient to define the head loss which is attributable to the bend as used in the definition of the bend loss coefficient presented in this thesis. A schematic representation of a typical hydraulic gradient is shown in Fig. 2.1. When a fluid is forced through a bend a considerable portion of energy is lost in the downstream tangent as shown by the actual hydraulic gradient A-B-C-D. The hydraulic gradient in a corresponding straight duct is represented by A-B'-C'-D' and if the bend were removed, the gradient would be represented by A-B'-C''-D''. Therefore, the total head loss chargeable to the bend is given by DD''. The calculation process is simplified by constructing the actual hydraulic gradient with the bend assumed to have zero system length. This means that points D' and D'' become the same.

2.2 EXPERIMENTAL DATA

The first recorded experiment on the losses in a bend is that of Weisbach\(^4\) in 1850. Since then a considerable number of researchers have carried out experiments to obtain

* References 5,11,13,15,18,21,28 and 30.
data or derive an expression for the calculation of losses in bends.

The first authoritative review of this early work was made by Nippert\(^5\) in 1929 as a pre-requisite to his own experimental work. He tried to correlate the results from his 33 quoted references but found considerable scatter in the values of bend loss coefficient. From his own experimental work, including variable area bends (discussed in detail later in this thesis), Nippert came to the following conclusions:

"In spite of the vast amount of test material, a complete explanation of the motion and of the flow losses in bends could not be achieved. The problem proved to be far more complicated than initially expected".

It should be noted that the results obtained by Nippert underestimate the bend loss for most practical applications. The bends were connected to an accelerating nozzle at inlet giving a practically uniform velocity profile. Such conditions would be expected to favour the bend performance because of the thin boundary layer present at inlet. Also the bends were exhausted to atmosphere with little or no downstream tangent length, thereby neglecting the head loss in this component which is attributable to the bend (see Fig.2.1). Because of his complicated experimental procedure (water was supplied from a variable head tank) Nippert had to make a number of simplifying assumptions in order to determine the value of the loss coefficient.
In 1937 Patterson\cite{6} produced a review of the data on corner losses. He did not include the results of Nippert and based his conclusions primarily on the work of Wirt\cite{7}(1927) and Hofmann\cite{8}(1929). Wirt used a similar experimental arrangement as Nippert's described above. His results indicated a loss in a squared corner bend of about 10% less than that in a constant area rectangular bend. This conclusion was reversed by McLellan \& Bartlett\cite{9} in 1941, apparently because the invalid assumption was made by Wirt that the losses of the two bends would maintain their relative values when the aspect ratio and other proportions were changed from those tested. McLellan \& Bartlett's conclusions have been confirmed by the recent experiments of Sprenger\cite{10}(1969), who shows that the loss coefficient for a square corner bend is greater than that for a constant area bend. The increased loss is more marked for an aspect ratio of 2 than 0.5, but in each case it is dependent on the bend inner wall radius to duct width ratio and the Reynolds number of the flow. Hofmann's experiments using steel pipes and water were designed to investigate the variation of the 'extra loss coefficient' with Reynolds number for both smooth and rough pipes. Considerable scatter appeared in the results caused by the rusting of the pipes and Hofmann found that a reading error of only 0.2 m.m. on the manometer scales could produce a difference of 12% in the loss coefficient for the worst cases. The effect of Reynolds number on the value of the bend loss coefficient has been fully investigated by the recent experiments of Keinhofer \& Smith\cite{11}(1966) in U.S.A., Sprenger\cite{10}(1969) in Germany and Miller\cite{12}(1970) in Great Britain.
The next notable review was by Henry\textsuperscript{(13)} which appeared in 1944. This review, based on 19 references, included Patterson's earlier review, but excluded the work of Nippert. In conclusion Henry states:

"The pressure loss through a duct component is affected by the nature of the entering flow and, when unsymmetrical velocity distributions occur, the pressure loss coefficients are higher than those given herein for conditions of uniform flow. This consideration raises the question of the accuracy with which the overall losses for a duct system can be predicted by summation of component losses obtained from the material in this report"

Although his statement on the effects of the inlet velocity distribution are an over-simplification they showed his awareness of general interaction problems. During experiments on standard 6 inch diameter 90° pipe bends Yarnell & Nagler\textsuperscript{(14)}(1935) (not included in Henry's review) found that, for the same quantity of flow, with the velocity in the approach duct high towards the inner side and low towards the outer side of the bend, the loss of head may be two to four times as much as would be measured for the same bend with a uniform velocity distribution existing in the approach duct. On the other hand with the approach velocity high toward the outer and low toward the inner side of the bend, the loss of head may be less than that caused by the same bend when a uniform velocity distribution prevails in the approach tangent.

In 1945 an extensive survey and correlation of the available data was produced by Gray\textsuperscript{(15)}. This included the
earlier reviews apart from Henry’s which appeared as a restricted wartime N.A.C.A. report. Gray’s correlation of loss coefficients from his 35 quoted references shows that experimental work after Nippert’s increased rather than decreased the amount of loss coefficient scatter. Gray offers many good reasons for this variation and from the following extract from his summary it can be seen that he is fully aware of the parameters affecting the flow in bends.

"Information on plain bends of circular arc curvature is surveyed in order to correlate the existing data as far as possible. The nature of the flow in bent channels, the variables affecting the losses and the influence of experimental methods on results are also discussed. The flow in bend channels is complex and the disturbances introduced in the fluid stream may persist well downstream, producing interference effects on other components. Performance is affected by geometrical construction, the fluid flow characteristics and the situation of the bend. Existing information is inadequate. Tests have been largely of an ‘ad hoc’ nature and correlation is complicated by discrepancies introduced by different experimental methods. In general compactness is incompatible with good performance, which is very susceptible to surface roughness and upstream disturbances, such as may be caused by badly fitting joints”.

As stated by Gray the scatter in the values of the bend loss coefficient was too great to be attributed to pure experimental error. Details of the experimental arrangements,
methods of measurement, definition of bend loss coefficient and flow conditions were incomplete in many cases.

Typical of the 'ad hoc' test report by Gray were those performed by Stack\(^{(16)}\)(1931). His results for circular cross-section bends indicated that an increase or decrease in the excess loss coefficient compared with that for a constant area bend was obtained by increasing the bend mid-plane cross-sectional area by varying amounts. No attempt was made to optimise the bend geometry and no experimental details are presented.

The correlation of the available data presented in the review by Anderson & Straub\(^{(17)}\) in 1948 covers more or less the same ground as Nippert and Gray. They quote 27 references but do not include the work of Nippert, Patterson, Henry or Gray. Their conclusion below is somewhat similar to that presented in the previous review:

"This review of literature pertaining to the flow through pipe bends has indicated that a great deal of additional research is required to solve experimentally the general problem. The literature, however, designates the principal variables that enter the problem and suggests their influence on the bend coefficient".

Locklin\(^{(18)}\) produced a survey and analysis of available information in 1950 as part of the program of the Technical Advisory Committee on Air Distribution. The survey covers the flow energy losses in single 90° duct elbows assembled in duct systems and having substantially uniform cross-section throughout the bend. He gives 62 references including Nippert, Patterson and Henry but not Gray or Anderson & Straub.
Although Locklin probably oversimplified two aspects of the problem, firstly the replacement of the relative roughness and Reynolds number by the friction factor and secondly the use of equivalent pipe lengths instead of a loss coefficient this does not reduce the good quality of his work, as is shown by his own concluding comments:

"An exact correlation of existing data is practically impossible owing to differences in test methods, flow conditions, and elbow construction. No single investigation has covered the entire field of flow conditions and elbow geometry and construction. Each has instead been confined to a relatively limited range of application. The effects of Reynolds number and upstream velocity distribution on the energy loss in elbows have not been accurately established in this survey because of the inadequacy and inconsistency of existing data.

It is believed that the curves presented in this paper for energy losses in 90-degree duct elbows are as accurate as may be deduced from the data presently available. If data of greater accuracy are required a comprehensive experimental programme would seem necessary. The survey should be extended to cover losses in vaned elbows, compound elbows and elbows involving change of area."

As a prelude to his extensive experimental work Itō(19)(1956) reviews 27 references concerned with the turbulent flow in smooth pipe bends of circular cross-section. This survey was primarily concerned with work on bends of radius to diameter ratio greater than 5. It only
contained the review of Nippert's from those discussed above, although a number of the individual papers such as Richter, Hofmann and Wasielewski are included. The results of his survey and experimental work are given in his own synopsis.

"It was found both theoretically and experimentally that the loss coefficients for turbulent flow in smooth pipe bends of circular cross-section are a function of two dimensionless numbers, $Rn(a/R)^2$ and $\theta \sqrt{R/a}$ *, provided that the radius of curvature of the pipe axis is comparatively large so that a separation does not take place. Pressure losses were determined for ten drawn-copper pipe bends, and a formula which expresses the pressure losses for turbulent flow in smooth pipe bends of radius ratio greater than 5 is given. The formula agrees within $\pm 5\%$ error with the experimental results on smooth pipe bends hitherto obtained by many investigators under similar test conditions".

During further experimental work on smooth pipe bends of circular cross section Itō (1960) extended the range of radius of curvature to pipe diameter to as low as 1.0. The good quality of his experimental work can be seen from his own abstract:

"The results of extensive experimental studies to determine the pressure losses for turbulent flow in smooth pipe bends of circular cross-section are presented in this paper. To make the data usable in practical design problems, the results are discussed in relation to those found by previous

*In Itō's work $R=\text{mean radius of the bend}, a=\text{duct radius}$.\)
investigators and empirical formulas for the bend loss coefficient are given. The general correlation of the test data appears to be as good as our present test information will permit”.

Ito's work combined with the results obtained by Miller\(^{(12)}\) (1970) provide sufficient accurate design data for smooth, constant area, pipe bends of circular cross-section.

A comprehensive survey, based mainly on Russian references, was presented by Idel'chik\(^{(21)}\) in 1960 (Translation published in 1966). This included only the work of Nippert from the above reviews. In the design data presented for circular and rectangular section bends the influence of Reynolds number and the roughness is based on the data from Abramovich\(^{(22)}\) (1935), Idel'chik\(^{(23)}\) (1953) and Evdomikov\(^{(24)}\) (1940), and in Idel'chik’s own words - “To be used until refined by experimental data”. Also the influence of aspect ratio based on the work of Hofmann\(^{(8)}\) (1929), Wasielewski\(^{(25)}\) (1932) and Richter\(^{(26)}\) (1953) is incorrectly presented as independent of the radius ratio of the bend and the Reynolds number of the flow. This dependence of the aspect ratio correction factor on the bend radius and Reynolds number is shown by the experimental work presented in this thesis and also by the work of Sprenger\(^{(10)}\) (1969) and Miller\(^{(12)}\) (1970).

In order to help the Royal Aeronautical Society with the presentation of their Data Sheets\(^{(27)}\), Ward-Smith\(^{(28)}\) produced a review in 1963. This contained 23 references including Patterson, Henry, Anderson & Straub and Gray but not Nippert, Locklin or Idel'chik (not published as a translation until 1966). The results of his survey are
The flow in bends is complex. In addition to the bend geometry, duct cross-sectional shape and Reynolds number, both the length of the tangents and the position of the pressure measurements relative to the bend are important when determining or specifying bend losses. An attempt has been made in this paper to account for these factors in a rational manner and particular attention has been paid to the downstream tangent. A basis for considering the upstream tangent is shown but has not been pursued in detail through lack of suitable experimental data. The effect is probably smaller than that associated with the downstream tangent.

A correlation of experimental data is made in Section 6. The effects of different factors on the bend loss will be found there under the appropriate heading. The inadequacy of existing experimental data is an obstacle to the presentation of comprehensive results. It should be reiterated that the bends which were considered in the correlation of experimental data were carefully made to have an accurate profile. In practice it is not usually possible to maintain such high standard during production and a reduction of cross-sectional area in the bend is a common fault. By increasing the adverse pressure gradients this effect increases the losses of the bend. Efforts to minimise the distortion in the bend are often worthwhile therefore.
Ward-Smith's comments on the upstream tangent are not justified by Yarnell & Nagler's work discussed earlier. The effect of the downstream tangent on the flow and pressure losses in a bend were further investigated by Ward-Smith (29) in 1968. The following extract is taken from his concluding remarks:

"A large increase in the loss coefficient for sharp bends having short downstream tangents has been found, and this has been shown to be closely associated with the effect of the boundary conditions on the pressure gradients developed within the bend. For bends of rectangular cross-section, the variation of the loss coefficient with aspect ratio has been shown to be principally determined by geometrical rather than fluid-mechanical factors.

An appraisal is made of the conditions to be satisfied by theoretical calculation methods. It is concluded that the lack of knowledge of the structure of the turbulence and complexity of the governing equations prevents the development of realistic methods appropriate to the experimental study".

As stated above, based on the experimental work presented in this thesis and from Miller's and Sprenger's work, it appears that the aspect ratio correction factor is a function of the Reynolds number of the flow and the radius ratio of the bend. Ward-Smith gives the correction factor as independent of both variables.

The most recent reviews appear to be those of Kelnhofer & Smith (11) (1966) in U.S.A. and Zanker & Brock (30)
(1967) in Great Britain. The first of these, prepared for the Naval Ship Engineering Center, contains 175 references and was made to determine what design data were available and the areas where more design data were needed. The reviews of Nippert, Patterson, Henry, Anderson & Straub and Itō are included, the notable exceptions being those of Gray and Locklin. From the extensive review and their experimental work Kelhofer & Smith concluded that:

"The static pressure loss coefficient for square cross-section elbows, although dependent on a number of variables including a subsonic Mach number up to 0.4, can be predicted for elbows in a system made up of other components. Two factors, however, are based on a limited amount of data found in the literature, and these factors should be investigated more thoroughly. They are: the Reynolds number effect over a large range \(1 \times 10^5 \leq Re \leq 5 \times 10^6\), and elbow position effect in systems with more than one elbow where relative radii are less than one.

The static pressure loss coefficient for elbows with aspect ratio greater than one appears to be more complicated than reported in the literature. Based on the limited testing reported herein the effect of aspect ratio is not independent of the length of duct following the elbow. A more comprehensive investigation of this effect is warranted.

A simple procedure for determining system pressure loss including incompressible and subsonic compressible
flow with Mach number up to 0.4 can be formulated. This includes the effects of the many parameters of importance in a system. However, the literature upon which much of the data for this procedure is based is continually reflecting improvements in the prediction of component pressure loss coefficients, and for this reason the procedure reported herein must be periodically revised.

The value of aspect ratio correction factor presented by Kelnhofer & Smith differs from that given by Ward-Smith for a given value of aspect ratio and is also presented as independent of the bend radius ratio and Reynolds number.

In order to conform to the definition of the bend loss coefficient used in this thesis (see Fig. 2.1) and by Itô, Ward-Smith and Miller the head loss in a length of straight duct equivalent to the centreline length of the bend must be added to the values of loss coefficient presented by Kelnhofer & Smith.

The second of the recent reviews is that prepared by Zanker & Brock as a preliminary to extensive experimental work at the British Hydromechanics Research Association in conjunction with the Central Electricity Generating Board. This review contains a bibliography of 226 references including all the above reviews (except Kelnhofer & Smith, which was probably published after the literature search had been completed). The overall situation regarding correlation of experimental data is not improved as can be seen from Zanker's concluding remarks:

"Despite the vast amount of published material the general problem of turbulent flow in bends of
closed conduits remains unsolved. The main reason for this is the large number of variables that enter into the problem. The use of different test set-ups, different measuring stations and different definitions of loss coefficient by previous experimenters makes correlation of their results most unsatisfactory and does little to increase understanding of the basic problem".

The experimental work performed at B.H.R.A. on constant area circular arc bends of circular and rectangular cross-section is presented by Miller (12)(1970). The tests were performed over a range of Reynolds number of $0.5 \times 10^6$.

In 1960 Eastwood & Sarginson (31) performed tests to determine the effect of a transition curve on the loss of head at a $90^\circ$ bend in a pipeline. The 2 inch square cross-section bends were constructed from perspex. A reduction in the value of the loss coefficient (expressed in terms of equivalent duct length) was obtained by the use of a transition curve in place of a quarter circle but no definition of loss coefficient or measuring procedure is presented.

The most recent experimental work on variable area $90^\circ$ rectangular section bends was performed by Sprenger (8) (1969) in Germany. A maximum inner bend wall radius to duct width ratio of 1.0 was used and no attempt was made to optimise the geometrical shape of the bend. The main aim of the experimental work was to determine the variation of the bend loss coefficient for bends of aspect ratio of 2 and 0.5 over a range of Reynolds number of $0.3 - 6.0 \times 10^5$. 
The definition of bend loss coefficient used by Sprenger neglects the friction loss in a length of duct equal to the centreline length of the bend. Applying an appropriate correction Sprenger's results may be compared with Miller's at a Reynolds number of $5 \times 10^5$. Miller's results are about 3% of the mean inlet velocity head below Sprenger's, the difference being probably due to the roughness effects in Sprenger's tests (At a Reynolds number of $5 \times 10^5$ Sprenger's friction coefficient was about 1.15 times the smooth duct coefficient).

2.3 THEORETICAL METHODS OF ATTACK

2.3.1 Nature of the Flow in Bends

Consider the flow of fluid in a duct upstream of a bend. Because of boundary layer growth the flow is non-homogeneous as far as the velocity is concerned but since the flow is purely axial, the pressure will be uniform across the duct. As the bend is approached the momentum of the individual fluid elements will be responsible for the continuence of straight line motion, hence instead of following the bend walls fluid will move towards the outside wall. The pressure is thereby increased at the outer wall and reduced by the deficiency of fluid at the inner wall. The pressure gradient now existing across the fluid will promote a centripetal force enabling the fluid to move in the arc of a circle of radius $r$. Since $\frac{dp}{dx} = \frac{\nu^2}{r}$ for a given pressure gradient, the faster moving fluid near the centre of the duct will be impelled to move around the circumference of a larger circle than the slower moving fluid and so faster moving fluid will be displaced toward the outer wall and the slower moving fluid displaced, via the duct walls,
to the inner wall. This is the so-called centrifugal effect on the fast moving fluid stream which, under suitable conditions, results in a twin spiral motion called the 'secondary flow' superimposed on the main flow.

This secondary flow phenomena has been recognised for a number of years. In 1877 Thomson\(^{(32)}\) demonstrated secondary flows in a curved water channel eight inches wide and one or two inches deep by means of threads and small particles moving with the fluid.

The term secondary flow has been used to describe several different phenomena. It was used by Prandtl\(^{(33)}\)(1952), Goldstein\(^{(34)}\)(1938) and others to describe the flow in the corners of straight ducts of non-circular cross-section. The same authors also used the term to describe the spiralling flow in bent ducts and pipes.

Two essentially similar explanations of the origins of secondary flow have been proposed in which the effect of viscosity is neglected except in so far as it is responsible for the establishment of the initial velocity gradient. The first of these, proposed by Squire & Winter\(^{(35)}\) in 1951 directs attention to the behaviour of a rotating element of fluid approaching a bend.

"A cylinder of fluid with its axis normal to the flow and in the plane of the bend will have a rotation about its axis due to the non-uniform velocity in the approaching flow. As it passes round the bend the axis of the rotating cylinder will be turned about its axis perpendicular to the plane of the bend. By analogy with the gyroscope a rotation will be set up about a third axis perpendicular to the other two".
The second explanation by Kronauer (1952) concentrates attention on a similar vortex filament in the approaching flow.

"At inlet to a bend the vorticity is normal to the flow and is the result of viscosity in the flow upstream of the bend. As the filament passes round the bend, the end of the filament at the inner radius move ahead of that at the outer radius so that downstream the filament is no longer normal to the flow. A component of vorticity in the direction of flow is thereby obtained and this secondary vorticity induces the secondary flow".

The inviscid flow theory suggested by Squire & Winter was extended by Hawthorne (1961) in 1961. Assuming that as the fluid enters the bend the inertial effects initially predominate over the viscous effects over most of the cross-section, hence viscosity can be neglected in the first portion of the bend and regarding the departure from uniformity of the velocity approaching the bend as small, Hawthorne's approximate analysis of the inviscid flow in bent pipes shows that the important effect is the generation of a component of vorticity in the direction of flow and the consequent setting up of a secondary velocity field which displaces the streamlines about the axis of the pipe.

The secondary vorticity produced in a flow around a bend depends on the vorticity in the approaching flow. In normal pipe flow this vorticity reaches its maximum at the wall of the inlet duct. The closer this flow is to the duct walls the larger the fluid shear and the more likely, first, that the small shear assumption becomes invalid,
but secondly, and more important, that viscous effects become significant. Squire & Winter recognised the inadequacy of the inviscid theory by neglecting a portion of the approaching velocity profile upon which the secondary vorticity and velocity depend. They assumed that the approaching velocity profile was "cut off" at $45\%$ of the mean velocity.

The same problem was met by Detra\(^{(38)}\)\((1953)\) in a theoretical and experimental study of the flow in a bent pipe in which it was found necessary to "cut off" the upstream velocity profile at the wall of $65\%$ of the mean velocity.

In 1948 Weske\(^{(39)}\) postulated a description of the flow in curved ducts at large Reynolds numbers ($5 \times 10^5$) as follows:

"Three distinctive regions of flow in curved ducts may be defined as follows: (a) The central body or "core" of the fluid in which velocities parallel to the axis of the duct are large compared with traverse velocity components. (b) The layer near the wall, in which velocity components normal to the wall of necessity are small, the transverse components parallel to the wall, i.e. in peripheral direction, however, being of the same order as the components parallel to the axis of the duct. This peripheral motion serves to shed the fluid of lower velocity surrounding the core, hence it will be referred to as shedding motion, and the layer as the shedding layer. (c) The region near the inner portion of the
duct area occupied by fluid of relatively low
total energy moving in swirling or random turbulent
motion".

The division of the flow into three distinct regions is
an over-simplification of the complex three-dimensional
nature of the flow. This was shown by Prandtl(33) in 1952
who states in his own words:

"The phenomenon may be regarded as a combination
of the main flow with a"secondary flow" at right angles
to it, which for reasons of continuity is not usually
confined to the boundary layer, but affects the "core"
of the flow and may considerably influence it".

Whether the above secondary flow pattern is completely
developed or not depends on the radius of the bend, the
angle of deflexion and the velocity distribution of the
approaching fluid. Since on continuity grounds the
mean axial velocity of the fluid must remain unchanged the
mean vector velocity will now be greater because of
the superposition of the spiral motion. Thus there will be
an increase in the kinetic energy of the fluid and a
Corresponding drop in the static head. It is unlikely
that much of this kinetic energy is converted back into
potential energy (static head) after the fluid leaves the
bend. It will probably be dissipated in extra boundary
friction and turbulence losses in the length of duct
immediately downstream of the bend outlet.

The special case of turbulent fluid pipe flow through
a mitred, right-angle bend has received relatively little
attention. A potential flow method has been applied by
Lichtarowicz & Markland(40)(1963) to the flow in a right-
angled elbow with separation from the inner corner as considered by Haase\(^{(41)}\) in 1954. The nature of the flow in mitred right-angled pipe bends has recently been investigated by Tinstall & Harvey\(^{(42)}\)(1968). The results of their work are indicated by their own summary:

"It has been found experimentally that the turbulent pipe flow through a mitred, right-angle bend produces a downstream secondary circulation which does not conform to the twin-circulatory flow usually to be found in pipe bends. The secondary flow is dominated by a single circulation about the axis in either a clockwise or an anticlockwise sense, between which it switches abruptly at a low, random frequency. The phenomenon is explained in terms of the asymmetry of the inner wall separation and the turbulent axial circulation generated in the upstream flow".

Experimental investigations into the phenomenon of secondary flow in curved pipes by Joy\(^{(43)}\)(1950), Eichenberger\(^{(44)}\)(1953) and Squire\(^{(45)}\)(1954) show that in fact for bends of prolonged curvature the secondary flow develops in an oscillatory fashion whilst still maintaining a twin spiral motion.

The nature of incompressible turbulent flow in bends has recently been described in terms of static pressure distribution by Gillard\(^{(46)}\)(1967) who states in his own words:

"When the flow passes round a bend the fluid particles experience centrifugal forces. These
forces cause an increased static pressure at the outer wall and a decreased static pressure at the inner wall of the bend, thereby setting up a static pressure gradient across the bend. This phenomenon starts at a distance of about one duct diameter upstream of the bend inlet, where the static pressure on the outer wall begins to increase and that on the outer wall begins to decrease. At about one diameter downstream of the bend exit the static pressure on the outer and inner walls becomes equal again. The pressure which acts on the outer wall and the suction which acts on the inner wall together constitute the force which is required to cause a change in the momentum of the fluid in being turned through an angle*.

Adverse pressure gradients (increasing static pressure in the direction of flow) normally occur in a bend on the inner wall at the outlet and on the outer wall at the inlet, the former gradient being the more severe one. If a fluid flows into a region of adverse pressure gradient part of its kinetic energy is converted into potential (pressure) energy in overcoming the pressure gradient. In the boundary layer the velocity and hence the kinetic energy of the fluid is reduced by friction. If the adverse pressure gradient is too severe all the kinetic energy of the fluid layers closest to the wall will be converted into potential energy. Unless these layers can be supplied with energy from other fluid layers by a process of turbulent mixing the adverse pressure gradient can even cause this almost stationary fluid near the wall to flow in a direction opposing
the main flow. This phenomena, known as boundary layer separation or stall, is responsible for considerable head loss.

The small radii of curvature and short axial length of 'short' bends produce high adverse pressure gradients, and therefore separation is usually more severe in such cases.

2.3.2. Various Approaches for Analysis of Turbulent Flow Through Bends.

In external aerodynamics the use of the concept of the boundary-layer as a thin region of viscous flow close to the wall of a body, which is immersed in an otherwise inviscid fluid, has been so successful that it is only natural that every effort should be made to make the same approach in internal aerodynamics. This possibility is not promising since in flows bounded by walls, as for instance in long straight ducts, the region of flow affected by viscosity covers the entire width of the passage. There is no region in which viscous or turbulent shear stresses can be neglected.

From a detailed review of 68 references Huang (1966) concluded that:

"Theoretical investigations have been few in number and comparisons between theoretical and experimental results have either been inconclusive or unsuccessful. It is necessary to employ different theoretical approaches in order to bring theoretical results more in line with observed performances"

The main objectives of Huang's work was to develop methods of predicting potential flow patterns of fluid flow in two-dimensional bends. These are then used to delineate
theoretical flows in various bends under various conditions with specific regard to pressure distributions along the inner and outer walls, the locations of regions of separation and distributions of velocity.

By using a conformal mapping technique Kamiyama (1963) developed a two-dimensional potential theory on the correlation between bend profile and its surface pressure distribution. He also proposed analytical methods for both surface pressure distribution and of bend profile for a given arbitrary surface pressure distribution.

A mathematical solution to the problem of designing the profile of a two-dimensional bend by potential flow methods was presented by Szczeniowski (1944). It postulated that the velocities never exceed the uniform velocity assumed at entry and exit. Exact equations were derived for any angle of turn. The application of these two-dimensional results to rectangular and circular cross-section bend is given somewhat incompletely.

These analyses of the inviscid rotational flows as a perturbation of a potential flow are limited due to the usual difficulty that, although the secondary motion may remain small compared with the basic potential flow, when the flow has continued for long enough, the inviscid assumption no longer gives a satisfactory representation of the motion. Such inviscid analyses give very poor agreement with measured static head losses. This is because these theories assume that the static head losses are entirely due to the irreversible increase of rotational kinetic energy at the expense of the static head (whilst the total head remains constant), whereas the main cause of the losses is the
dissipation of mechanical energy due to more direct effects of viscosity. Further, such analyses cannot be calculated independent of, and subsequently reconciled with, an analysis of the viscous region in the way that boundary layer theory is used to calculate attached external flows. This is due to the fact that there is a strong interaction between the viscous and inviscid regions (assuming that two such regions can be identified) both as regards the axial development of the flow and because the viscous flow conditions in the neighbourhood of the wall change appreciably around the circumference of the bend.

The division of the flow into a thin boundary-layer region near the wall and inviscid potential flow outside this boundary layer has not aided analysis as much in internal flow problems as it has in external aerodynamics. In many internal flows the boundary layer effectively covers the entire width of the passage as for instance in the 'fully developed' turbulent flow in a straight pipe. When considering the flow in bends, both inviscid flow theory and boundary layer theory are of little value, the former because there is no region of genuine inviscid flow and the latter because the boundary layer theories are two-dimensional in character whereas the flow round a bend is strongly three-dimensional.

Where flows are markedly three-dimensional, limited progress has been made in solving the Navier-Stokes equations directly, though not in as complicated a flow situation as a bend. For example, in an analysis of laminar flow Chorin (1967) worked from the three rectangular cartesian co-ordinate Navier-Stokes equations. In three
dimensions it is not possible to eliminate the pressure terms (as it is in two dimensions by differentiating the x-directed equation with respect to y and the y-directed equation with respect to x and subtracting). The pressure terms must be retained, forming a $\nabla^2 p$ term, and velocity and pressure are then used as primary variables.

Since the Navier-Stokes equations are elliptic, in order to achieve a solution flow conditions must be fully specified on all the field boundaries. But with a problem in which the flow pattern is highly distorted (as it would be for a 90° bend) it is virtually impossible to specify the time-mean flow conditions achieved downstream, corresponding to some given upstream flow. An alternative approach would be to employ a time-dependent method of solution. In this case the entire flow field would be assumed to be initially subjected to some constant impulsive speed. Since the stream changes direction round the bend the stream velocity would vary with the streamwise co-ordinate, but could nevertheless be determined. A suitable choice of co-ordinate system, orthogonal cartesian, is presented in Appendix 1. A doubly iterative solution technique would then be employed in which the (approximate) finite difference equation formed from the Navier-Stokes equations would first be iterated with respect to spatial co-ordinates at one instant of time, then this solution, obtained over the whole field, would be updated over a small finite time interval over the field, the process then being continuously repeated. Eventually the required Reynolds number would be obtained.

A recent guide to the solution of turbulent flow problems by the numerical integration of the equations of
motion by an explicit finite-difference method has been proposed by Nash (51) 1969. His present method for the calculation of three-dimensional turbulent boundary layers is restricted to incompressible flows over flat surfaces (or developable surfaces of large radius). In his own words:

"The differential equation of motion (the equation expressing the momentum balance in the two orthogonal directions parallel to the wall), and the continuity equation, are integrated numerically using an explicit finite-difference method. The shear stress is determined from the empirically modified turbulent energy equation, following the work of Townsend (1955), Bradshaw et al. (1957) and McDonald (1968). As in two dimensions the assumption is made that the magnitude of the shear stress is directly proportional to the turbulent intensity, and the additional assumption is made that the shear stress acts in the direction of the maximum rate of strain of the mean motion. These assumptions are regarded as being of a provisional nature and, when it appears to be necessary, the overall method can be updated to embrace a more sophisticated flow model with little difficulty.

In principle the extension to more general geometries requires only the inclusion of the curvature terms in the equations of motion, so long as the body radii remain large compared with the boundary-layer thickness".
A method of numerically integrating the Navier-Stokes equations for certain three-dimensional incompressible flow is described by Williams (1969):

"The technique is presented through application to the particular problem of describing thermal convection in a rotating annulus. The equations, in cylindrical polar co-ordinate form, are integrated with respect to time by a marching process together with the solving of a Poisson equation for the pressure. A suitable form of the finite difference equations gives a computationally-stable long-term integration with reasonably faithful representation of the spatial and temporal characteristics of the flow".

There is as yet little or no guide to the solution of the complex three-dimensional turbulent flow problem that occurs in the flow of fluid round bends. An alternative, writing the Navier-Stokes equations in their time-averaged form (Appendix 1) and using an assumed eddy viscosity, does not permit a time-dependent solution to be adopted unless the parameters are time-averaged over relatively short periods of time, the resulting expression being updated for finite time increments like Chorin's laminar solution.

Thus, although the problem under review is theoretically capable of solution it is more complex by several orders than the types of problems at present being successfully solved. Even if a viable computer program were written to solve the problem at a representative Reynolds number, the computer store it would require would be enormous.
It is concluded, therefore, that since it is impossible at the present time, to make accurate mathematical analyses or predictions for the observed phenomena, it is to be expected that carefully selected and controlled experiments will lead the way.

2.4. SCOPE OF PRESENT WORK

For a 90° rectangular cross-section bend this thesis provides:

a) values of bend loss coefficient for constant area bends at a constant representative Reynolds number,

b) the optimum geometrical shape for variable area 90° bends,

c) a measure of the accuracy associated with the experimental values of the bend loss coefficient.
CHAPTER 3

EXPERIMENTAL FACILITIES
3.1. DESIGN PHILOSOPHY

In order to optimise the shape of a bend it is necessary to be able to detect small variations in the value of the bend loss coefficient. The required accuracy both in the construction and alignment of the bends and associated ducting and also in the instrumentation was indicated by the author's introductory work at Leicester University.

Repeated tests on a rectangular section bend formed by a given combination of inner and outer radii produced a variation in the value of the bend loss coefficient as high as 15%. This was primarily caused by the variations in alignment of the bend walls with the upstream and downstream ducts when the sections were disconnected and reconnected. Another contributory factor was the lack of sophistication in the experimental procedure. The values of wall static head were measured on a multi-tube manometer and the hydraulic gradients upstream and downstream of the bend were constructed by eye.

As a result of these findings a new test rig was designed and constructed at the British Hydromechanics Research Association. The details of the rig are described in the following sections together with the computer-based data logging instrumentation system.

3.2. THE TEST RIGS

The test rigs were composed of long straight ducts bolted to the upstream and downstream flanges of a bend section. Air was circulated from the laboratory, via an inlet nozzle, through the ducts by means of a fan. The flow rate was controlled by a door arrangement on a control box
FIG. 3.1 - Typical Duct Section
FIG. 3.2 - Schematic Representation of the Construction of the Duct Sections
immediately upstream of the fan. A duct traverse section could be positioned at various cross-sections along the straight ducts.

The overall design was governed by the available fan capacity and economic considerations.

3.2.1. Duct Sections

The capacity of the available fan was approximately 430 cubic feet of air per minute. To represent a variety of practical situations a Reynolds number of approximately $2 \times 10^5$ and an aspect ratio of 2 were selected. These were satisfied, from the available fan capacity, by the duct dimensions of 4 inches normal to the plane of bending and 2 inches in the plane of bending.

A typical section of ducting is shown pictorially in Fig. 3.1 and schematically in Fig. 3.2. The faces of the ducts were made from $\frac{1}{4}$ inch plywood and provided with hardwood flanges at each end. The internal surface of the plywood was sanded and varnished to produce a smooth consistent finish. A total of sixteen sections were constructed, twelve having a length equivalent to twelve hydraulic diameters, two six diameters and two three diameters.

Each section of ducting was fitted with static head tappings along the centreline of each face an integral number of hydraulic diameters apart. These were made from stainless steel hypodermic tubing 0.020 inch internal diameter ground flush with the plywood surface.

3.2.2. Bend Section

The variable area $90^\circ$ bends were formed by combination of inner and outer radius blocks. These were made from hardwood sanded and varnished to give the same internal surface as the
FIG. 3.3 - Example of the Three Types of Bend Configurations
FIG. 3.5 - Cylindrical Yawmeter Used to Measure Local Velocity
duct sections. An example of the three types of bend configuration, (a) converging - diverging, (b) constant area and (c) diverging - converging are shown in Fig.3.3. For the cases (a) and (c) above straight lengths of duct wall were added to either the inner or outer radius block to complete the bend section. The various blocks forming the bend sections were provided with accurately drilled hardwood flanges thereby eliminating any gaps or shoulders when assembled.

3.2.3. Duct Traverse Section

The duct traverse section shown in Fig.3.4 was constructed in a similar manner as the duct sections described above. A circular-cylinder yawmeter shown in Fig.3.5 was used to measure the local axial velocity at a given point in the cross-section being traversed. This yawmeter consisted of a cylindrical tube with three head holes positioned as shown in Fig.3.5, each connected to a separate internal tube. The instrument was supported with its axis perpendicular to the plane of the flow and rotated about its axis until the head readings at the two outer holes were equal. The line of the central hole was then along the direction of the flow. The sensitivity of the instrument depends on the angle between the two outer holes. From reference 53 an angle of approximately 90° is quoted for maximum sensitivity at low Mach number over a wide range of Reynolds number. The value of the local velocity was obtained from the difference in head between one of the balanced outer holes and the central hole.

Before any velocity distributions were measured the instrument was calibrated against a standard N.P.L. pitot-static tube. The head readings from the two outer holes
were balanced as shown by a zero reading given by a Betz manometer and the velocity head reading measured by a second Betz manometer.

3.2.4. Inlet Nozzle

The mean velocity of the flow through the duct for each test was calculated from the static head difference between the upstream and throat sections of the nozzle. This nozzle was designed to the specifications given in reference 54 with a throat cross-sectional area equal to that of the duct sections. Hardwood blocks were used to make the nozzle, the internal surface finished in the same manner as the bend and duct sections. A ring of four static head tappings were located at both the upstream and throat sections. Any maldistribution of the inlet flow was straightened by a honeycomb of hexagonal cell size \( \frac{1}{8} \) by \( \frac{1}{4} \), positioned in the straight duct upstream of the nozzle. The flow was transferred from the nozzle to the duct sections by means of a constant area transition section made from \( \frac{1}{4} \) plywood.

3.2.5. Control Box and Fan

The fan used was a constant speed centrifugal fan made by K. Blackman. A sliding door arrangement on the control box enabled the rate of flow through the duct to be varied as required. Steady flow conditions in the duct were established by the addition of a fine mesh screen at the junction of the duct sections and control box. Any vibrational effects in the duct caused by the fan were reduced to an insignificant level by the introduction of a flexible coupling between the fan and control box.
FIG. 3.6 - General Photograph of Instrumentation
FIG. 3.7 - Instrumentation - Collection and Assembly of the Collected Data on Punched Paper Tape
3.3. INSTRUMENTATION

The Computer based data logging system shown pictorially in Fig.3.6 and schematically in Fig.3.7 was used to record the values of the collected data on punched tape. This data consisted of the test number, air temperature, barometric pressure and the values of static head across the nozzle and along the duct measuring sections. Each of the above static head tappings were connected to the face of the data logging box by plastic tubing. The individual tappings were selected by the selector unit actuated by the scan drive, and the value of the static head converted to a voltage reading by the scanner valve. This voltage was first amplified and then displayed on a digital voltmeter. Controls on the amplifier enable the voltage reading to be adjusted to the corresponding static head, in inches water gauge, as measured by an accurate Betz manometer. The output from the amplifier was linear over the required range. Therefore, only the zero, intermediate and maximum static head readings were required to calibrate the amplifier for a given test.

Each static head tapping location corresponds to a channel number on the scan drive unit and data logging box. This value and the corresponding calibrated static head reading were recorded by a line printer and punched on data tape ready for analysis by a computer. The full range of static heads for a given test were selected on the scan drive unit. On command each calibrated value and channel number were recorded as the range was scanned in a step by step manner. This scanning procedure was repeated any number of times, the average static head value at each location calculated by the procedures incorporated in the computer programs.
At the start of each test the values of test number, air temperature and barometric pressure are fed into the system by means of a variable potentiometer. These values are recorded on the above outputs by a single command instruction.
CHAPTER 4

CALCULATIONS AND UNCERTAINTY ANALYSIS
4.1. INTRODUCTION

The fundamental aim of the experimental work described in this thesis is to optimise the geometrical shape of a 90° rectangular section bend. This requires not only very accurate determination of the values of the bend loss coefficient but just as important some measure of the accuracies achieved. An uncertainty analysis, based on Kline and McClintock's (55) method has been adapted to suit this investigation. The analysis postulates a rational means for estimating and describing the uncertainties both in the collected data and in the variables involved in the problem. A method for calculating the propagation of these uncertainties into the final result is also presented.

In this Chapter the theory associated with the above uncertainty analysis is discussed. The definition of the bend loss coefficient is presented. Finally, a step by step description of the application of the above uncertainty analysis to the calculations involved in the determination of the duct friction factor - Reynolds number relationship and the values of the bend loss coefficient is presented.

4.2 THEORY OF THE UNCERTAINTY ANALYSIS

From the point of view of reliable estimates, experiments fall into two overlapping categories, single-sample and multiple-sample experiments. In multiple-sample experiments the measurements are repeated a sufficient number of times, using enough observers and diverse instruments, so that the reliability of the results can be assured by the use of statistics.

In most engineering experiments, including those described in this thesis, it is impractical to obtain the uncertainties by repetition because of the cost and
time involved. These type of experiments are called single-sample experiments and it is inevitable that the statement of reliability will be based in part on estimates. This must be true since by definition statistics cannot be applied to all the errors. Even so, these estimates may be entirely satisfactory, particularly if the uncertainty is of the order of a few per cent or less of the original data.

Kline and McClintock define 'uncertainty' as a possible value the error might have and 'propagation of uncertainty' as the way in which uncertainties in the variables affect the uncertainty in the result. Also they postulate that a good concise means of describing the uncertainty in each variable is to specify the mean of the readings and an uncertainty interval based on specified odds. Representing the mean by $m$ (arithmetic mean of observed values), the uncertainty interval by $w$, and the odds by $b$, this becomes:

$$ v = m \pm w \ (b \ to \ 1) \quad (4.1) $$

For example, the wall static head at a point along a duct might be given as follows:

Wall static head = 10.32 ± 0.05 inches w.g. (20 to 1)

This states that the value for the wall static head is believed to be 10.32 inches w.g. and the odds are 20 to 1 that the true value is within ± 0.05 inches w.g. of the above value. The uncertainty interval, $w$, is not a variable but a fixed value selected so that the experimenter would be willing to wager $b$ to 1 that the error is less than $w$. The selection of the odds $b$ to 1 is discussed later in this Chapter.
Equation 4.1 postulates a method by which the uncertainties in each of the basic variables can be described in an accurate and simple manner for routine use. It is then necessary to determine the propagation of these uncertainties into the final result. Kline and McClintock suggest that the second-power equation below, equation 4.2, might be used directly as an approximation for calculating the uncertainty interval in the result.

$$\omega_R = \left[ \left( \frac{\partial R}{\partial \omega_1} \right)^2 + \left( \frac{\partial R}{\partial \omega_2} \right)^2 + \cdots + \left( \frac{\partial R}{\partial \omega_n} \right)^2 \right]^{1/2} \tag{4.2}$$

Where \( R \) is a linear function of \( n \) independent variables, each of which is normally distributed, \( \omega_i (i = 1, \ldots, n) \) are the uncertainty intervals for the variables and \( \omega_R \) the interval for the result having the same odds as for the variables.

This method summarized above provides a means for describing and analysing the uncertainties in single-sample experiments. In this method the actual estimation of the uncertainty intervals must still depend largely on the judgement of the experimenter. Such estimates are, of course, not pure guesses. Factors such as instrumentation backlash, sensitivity and fluctuations, as well as the accuracy of the basic theory of operation of the instrumentation, can be given specific uncertainty intervals. Calibration of the instrumentation against some type of standard may be possible. Alternatively, experience based on prior auxiliary experiments could be used.

4.3. DEFINITION OF THE BEND LOSS COEFFICIENT

The total head, \( TH \), of fluid at any cross-section along a duct is considered equal to the sum of the static head \( s \)
FIG. 4.1 - Definition of the Bend Loss Coefficient, $K$. 
and the mean velocity head \( \frac{v^2}{2g} \).

\[
\theta H = s + \frac{v^2}{2g} \tag{4.3}
\]

For the flow round a bend the total head loss between two cross-sections upstream and downstream of the bend is thus:

\[
\Delta \theta H = s_u - s_d - \frac{v_u^2}{2g} - \frac{v_d^2}{2g} \tag{4.4}
\]

If the bend is connected between upstream and downstream ducts of equal cross-sectional area the mean velocity head in each duct is the same, therefore:

\[
\Delta \theta H = s_u - s_d \tag{4.5}
\]

The bend loss coefficient, \( K \), is defined through:

\[
\Delta \theta H = K \cdot \frac{v^2}{2g} \tag{4.6}
\]

Thus:

\[
K = \frac{(s_u - s_d)}{\frac{v^2}{2g}} \tag{4.7}
\]

In order to calculate the total head loss in the whole system attributable to the bend, allowance must be made for the head losses in the straight ducts between the measuring cross-sections upstream and downstream of the bend. This procedure is avoided by constructing the static head gradients upstream and downstream of the bend as shown in Fig.4.1.

The static head drop, \( \Delta s \), along a straight duct between two cross-sections, distant \( L \) apart, in the region of fully developed flow is given by D'arcy's equation

\[
\Delta s = f \cdot \frac{L \cdot v^2}{2g} \tag{4.8}
\]
where $f$ is the duct friction factor.

Since:

$$\frac{\Delta s}{\sqrt{\frac{V}{2g}}} = f \cdot \frac{L}{\Delta h} = H(J), \text{ see section Bl, page 58}$$

the upstream and downstream static head gradients where the flow is 'fully developed' can be obtained using the value of the duct friction factor. Extrapolating these static head gradients as shown in Fig.4.1 gives the value of the bend loss coefficient $K$. This definition of $K$ implies that the bend is considered to have zero system length. Also the upstream and downstream tangents of the bend are of sufficient length for the flow to be 'fully developed' immediately upstream and some way downstream of the bend.

**4.4. APPLICATION OF THE UNCERTAINTY ANALYSIS**

**4.4.1. Uncertainty Intervals and Odds for the Collected Data**

Similar measurements are made to specify either the duct friction factor - Reynolds number relationship* or the values of the bend loss coefficient. These consist of the values of the wall static head (inches w.g.) along the four faces of the duct and upstream and at the throat of the inlet nozzle, the air temperature at the inlet to the nozzle and the barometric pressure in the laboratory.

The value and odds selected for the uncertainty interval in the value of the wall static head at a given tapping location are determined from the following auxiliary test. Using the instrumentation described in Chapter 3 a sufficient number of recordings of the static head $s(J)a$ at a given station were taken to enable the construction of the

* The necessity for the experimental determination of this relationship is discussed in Chapters 5 and 6
The Normal Frequency Distribution

\[ y = \frac{N}{\sigma \sqrt{2\pi}} \exp\left(\frac{- (x - \mu)^2}{2\sigma^2}\right) \]

**Fig. 4.2** - Histogram and Normal Distribution formed by the Static Head Readings at a Given Tapping
The normal distribution curve, equation 4.9, is superimposed on the histogram.

\[ y = \frac{N}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right) \]  \hspace{1cm} (4.9)

The quantity \( y \), which is the height of the curve at any point along the scale of \( x \), is known as the relative frequency of that particular value of the variable quantity. The symbol \( \sigma \) stands for the standard deviation defined by equation 4.10 and \( \bar{x} \) is the arithmetic mean for the distribution.

\[ \sigma = \sqrt{\frac{(x-\bar{x})^2}{N}} \hspace{1cm} \sigma = \sqrt{\frac{(x-\bar{x})^2}{N-1}} \]  \hspace{1cm} (4.10)

In Fig.4.2 the value of \( \bar{x} \) and the scale for the standard deviation are shown by the lower line. As shown in any textbook in statistics, approximately 95% of the area under the normal distribution curve is contained between the range \( \pm 2\sigma \). This is equivalent to saying that if the range of \( x \) values forming the histogram is within \( \pm 2\sigma \) then a 95% confidence limit can be attributed to the recorded value of \( x \). From Fig.4.2 it is evident that the above condition is satisfied by the recorded values of static head forming the histogram. To form a statement compatible with equation 4.1 the 95% confidence limit can be represented by specifying the uncertainty odds of 20 to 1.

The uncertainty interval in the value of the wall static head is given as \( \pm 0.03 \) inches w.g. since this is the range contained within \( \pm 2\sigma \) of the mean value of the histogram shown in Fig.4.2. In order to allow for
any anomalies throughout the test program a value of ± 0.05 inches w.g. is used throughout the calculation described in this Chapter. Therefore, the statement of the uncertainty in the value of the wall static head is given by:

$$\omega _{h} = \pm 0.05 \text{ inches w.g. (20°F)} \quad (4.11)$$

Assuming that a number of recorded values of air temperature and barometric pressure for a given test are normally distributed as above, their estimated uncertainties $\omega _{T}$ and $\omega _{p}$ respectively, are given by equation 4.12 and 4.13.

$$\omega _{T} = \pm 0.25^\circ C \quad (20°F) \quad (4.12)$$

$$\omega _{p} = \pm 0.01 \text{ inches Hg (20°F)} \quad (4.13)$$

4.4.2. Other Parameters Involved in the Calculations

The calculation is divided into three sections as follows:

Section A: Parameters common to both the determination of the duct friction factor - Reynolds number relationship and the bend loss coefficient.

Section B: Parameters associated with the first of these alone.

Section C: Parameters associated with the second of these alone.

The necessary calculations for the uncertainty interval in the various parameters and their propagation into the final results are incorporated in the two data-reduction computer programs. A flow chart and listing of each program are included in Appendix 2.
In order to calculate the uncertainty interval in any parameter from equation 4.2 the parameter must first be expressed as a function of \( n \) independent variables. To assist in this process and illustrate the overall calculation procedure the flow chart shown in Fig.4.3 was constructed. The value of the bend loss coefficient is traced through the various parameters back to the collected data.

The equations for the value of the various parameters and their uncertainty interval are developed in the following sections. For each final equation the corresponding computer program statement numbers are given in squared brackets immediately after the equation.

SECTION A

A 1. Mean Wall Static Head, \( s(j)a \) at a given location, \( J, \) on the Duct Face \( a. \)

For a given test the mean values of wall static head at a given tapping location, \( J, \) on the duct face \( a \) is calculated from five readings of the static head. Therefore:

\[
s(j)a = \frac{1}{5} \sum_{i=1}^{5} s(i)a_i \tag{4.14}
\]

Since \( s(j)a \) is a function of five independent variables the uncertainty interval \( w_s(j)a \) is given from equation 4.2 by:

\[
w_s(j)a = \sqrt{\left( \frac{\partial s(j)a}{\partial s(j)a_1} w_s(j)a_1 \right)^2 + \left( \frac{\partial s(j)a}{\partial s(j)a_2} w_s(j)a_2 \right)^2 + \ldots}
\]

\[
+ \left( \frac{\partial s(j)a}{\partial s(j)a_5} w_s(j)a_5 \right)^2 \right)^{1/2} \tag{4.15}
\]

* The values of \( s(j)a \) are determined in statements 3 to 32 and 8 to 37 in computer programs H15E and H15F respectively.
The partial derivatives of $\delta(j)a$, from equation 4.14, with respect to $i$ are equal to $1/5$. Substituting these values in equation 4.15 together with the values of $w_{s}(j)a$ from equation 4.11, the value of $w_{\delta}(j)a$ is obtained as:

$$\omega_{\delta}(j)a = \pm 0.0234 \ \text{inches} \ \text{w.g.} \ (20 \ \text{to} \ 1) \ (4.16)$$

A2. Nozzle Upstream and Throat Mean Static Heads,

Nu and Nt respectively.

The values of Nu and Nt are calculated as the average of four meaned static heads obtained from equally spaced pressure tappings at each cross-section.

Therefore:

$$Nu = \frac{1}{4} \sum_{a=1}^{4} \delta(u)a$$

$$[H_{IS} = 104 \ \text{and} \ H_{SF} = 156]$$

and

$$Nt = \frac{1}{4} \sum_{a=1}^{4} \delta(t)a$$

$$[H_{IS} = 105 \ \text{and} \ H_{SF} = 157]$$

Since Nu and Nt are functions of four independent variables their respective uncertainty intervals $w_{Nu}$ and $w_{Nt}$ are given by:

$$w_{Nu} = \left[\left(\frac{\partial Nu}{\partial \delta(u)_{1}} \omega_{\delta}(u)_{1}\right)^{2} + \left(\frac{\partial Nu}{\partial \delta(u)_{2}} \omega_{\delta}(u)_{2}\right)^{2} + \ldots + \left(\frac{\partial Nu}{\partial \delta(u)_{4}} \omega_{\delta}(u)_{4}\right)^{2}\right]^{1/2} \ (4.19)$$

and

$$w_{Nt} = \left[\left(\frac{\partial Nt}{\partial \delta(t)_{1}} \omega_{\delta}(t)_{1}\right)^{2} + \left(\frac{\partial Nt}{\partial \delta(t)_{2}} \omega_{\delta}(t)_{2}\right)^{2} + \ldots + \left(\frac{\partial Nt}{\partial \delta(t)_{4}} \omega_{\delta}(t)_{4}\right)^{2}\right]^{1/2} \ (4.20)$$

The partial derivatives of Nu and Nt with respect to $a$ are equal to $1/4$. Substituting these values in equations 4.19 and 4.20 together with the values of $w_{\delta}(u)a$ and $w_{\delta}(t)a$ from equation 4.16 the values of $w_{Nu}$ and $w_{Nt}$ are obtained as

$$w_{Nu} = w_{Nt} = \pm 0.0112 \ \text{inches} \ \text{w.g.} \ (20 \ \text{to} \ 1) \ (4.21)$$
A3. Air Density, AD.

For a given test the value of AD is calculated from the measured values of air temperature ($\theta$) and barometric pressure ($P$)

$$AD = \frac{0.0764 \cdot P \cdot 288.7}{30 \cdot (273+\theta)}$$

(4.22)

[HISE - 106 and H15F - 158]

Since the value of AD is a function of two independent variables, see Fig.4.3, the value of the uncertainty interval $\Delta AD$ is given by:

$$\Delta AD = \left[ \left( \frac{\partial AD}{\partial P} \Delta P \right)^2 + \left( \frac{\partial AD}{\partial \theta} \Delta \theta \right)^2 \right]^{1/2}$$

(4.23)

[HISE - 107 and H15F - 159]

A4. Nozzle Differential Head, NH

The values of NH is calculated from equation 4.24 given in reference 54.

$$NH = \frac{(NT - NW)(62.4 - AD)}{62.317}$$

(4.24)

[HISE - 108 and H15F - 160]

Equation 4.24 is a function of three independent variables, see Fig.4.3, therefore, the uncertainty interval $\Delta NH$ is given by:

$$\Delta NH = \left[ \left( \frac{\partial NH}{\partial NT} \Delta NT \right)^2 + \left( \frac{\partial NH}{\partial NW} \Delta NW \right)^2 + \left( \frac{\partial NH}{\partial AD} \Delta AD \right)^2 \right]^{1/2}$$

(4.25)

[HISE - 109, 110 and H15F - 161, 162]

A5. Nozzle Pressure Ratio PR.

The value of PR is calculated from equation 4.26

$$PR = \frac{P - NH}{13.6}$$

(4.26)

[HISE - 111 and H15F - 163]

where $P$ is the barometric pressure.
In order to obtain \( PR \) as a function of \( n \) independent variables it is necessary to combine equations 4.24 and 4.22 with equation 4.26. The value of \( PR \) is thereby obtained as a function of four independent variables as shown in Fig. 4.3.

\[
PR = \phi(NT, Nu, P, \theta)
\]  
\( (4.27) \)

Therefore, the uncertainty interval \( W_{PR} \) is calculated from:

\[
W_{PR} = \left( \frac{\partial PR}{\partial NT} \omega NT + \frac{\partial PR}{\partial Nu} \omega Nu + \frac{\partial PR}{\partial P} \omega P + \frac{\partial PR}{\partial \theta} \omega \theta \right)^{1/2}
\]  
\( (4.28) \)

\[ H15E = -112, 113 \quad \text{and} \quad H15F = -164, 165 \]

A6. Nozzle Adiabatic Expansion Factor, \( AEF \)

From reference 54 the value of \( AEF \) is calculated from equation 4.29:

\[
AEF = \left( \frac{2c}{c-1} \right) \left( \frac{1 - \gamma^2}{1 - \beta^2} \right)^{1/2}
\]  
\( (4.29) \)

\[ H15E = -114 \quad \text{and} \quad H15F = -166 \]

where \( c \) is the ratio of specific heats of air - \( Cp/Cv \) and \( \beta \) is the ratio of the nozzle throat to pipe diameter. Since the values of \( c \) and \( \beta \) are constant equation 4.29 is a function of a single variable and therefore the value of the uncertainty interval is given by:

\[
W_{AEF} = \left( \frac{\partial AEF}{\partial PR} \omega PR \right)^{1/2}
\]  
\( (4.30) \)

\[ H15E = -115 \leq 119 \quad \text{and} \quad H15F = -167 \leq 171 \]

A7. Mean Velocity, \( \bar{V} \).

The inlet nozzle is designed to the specifications given in reference 54 and the rate of flow of air through the nozzle is given by:

\[
W_h = \frac{359 \cdot c_d \cdot F_d \cdot \bar{V} \cdot A_E F \cdot (N H \cdot A_b)}{3600}
\]  
\( (4.31) \)
where \( W_h \) = rate of flow of air through the nozzle.

\[
\begin{align*}
C_d &= \text{nozzle discharge coefficient.} \\
F &= \text{velocity of approach factor} = \frac{1}{\sqrt{(1 - \beta^4)}} \\
d &= \text{diameter at a nozzle throat.} \\
F_a &= \text{thermal expansion factor.} \\
AEF &= \text{adiabatic expansion factor.} \\
NH &= \text{nozzle differential head.} \\
AD &= \text{air density.}
\end{align*}
\]

By definition the rate of flow of air through the nozzle is given by:

\[
W_h = \bar{V} \cdot A \cdot AD \quad (4.32)
\]

where

\[
\begin{align*}
\bar{V} &= \text{mean velocity of the flow} \\
A &= \text{cross-sectional area of the nozzle throat.}
\end{align*}
\]

Combining equation 4.31 and 4.32 the following equation for the value of the mean velocity is obtained:

\[
\bar{V} = \frac{359 \cdot C_d \cdot F \cdot d^2 \cdot F_a \cdot AEF \cdot (NH \cdot AD)}{3600 \cdot A \cdot AD}^{1/2} \quad (4.33)
\]

For a given nozzle the values of \( C_d, F, d, F_a \) and \( A \) in equation 4.31 are constant. Therefore, the value of \( \bar{V} \) is a function of three variables:

\[
\bar{V} = \phi(AEF, NH, AD) \quad (4.34)
\]

In order to obtain the value of \( \bar{V} \) as a function of \( n \) independent variables it is necessary to combine equations 4.33, 4.29, 4.26, 4.24 and 4.22. This results in the value of \( \bar{V} \) as a function of four independent variables as shown in Fig.4.3:

\[
\bar{V} = \phi(NT, NU, \theta, \rho) \quad (4.35)
\]
The uncertainty interval $\omega V$ is therefore given by:

$$
\omega V = \left[ \left( \frac{\partial V}{\partial N} \omega N \right)^2 + \left( \frac{\partial V}{\partial N_u} \omega N_u \right)^2 + \left( \frac{\partial V}{\partial \omega} \omega \right)^2 + \left( \frac{\partial V}{\partial \rho} \omega \right)^2 \right]^{1/2}
$$

(4.36)

[A8. Absolute Viscosity of Air $\mu_0$.]

From reference 56 Sutherland's expression for the variation of the absolute viscosity with the value of the air temperature is given by:

$$
\mu_0 = \mu_{273} \left[ \frac{273 + c'}{\theta' + c'} \right] \left[ \frac{\theta'}{273} \right]^{3/2} \text{ c.g.s. units (4.37)}
$$

where $c'$, Sutherland's constant, equals 117. Also the mean value of the absolute viscosity of air at $23^\circ C$ is given by:

$$
\mu_{23} = (1830 \pm 2.5) \times 10^{-7} \text{ c.g.s. units (4.38)}
$$

An expression for $\mu_{23}$ can be obtained from equation 4.37 by substituting $273 + 23^\circ C$ for the value of

Therefore:

$$
\mu_{23} = \mu_{273} \left[ \frac{273 + 117}{296 + 117} \right] \left[ \frac{296}{273} \right]^{3/2} \text{ c.g.s. units (4.39)}
$$

and by substituting for $\mu_{273}$ from equation 4.37 we obtain:

$$
\mu_0 = \mu_{23} \left[ \frac{4.13}{4.00} \right] \left[ \frac{273}{296} \right] \left[ \frac{4.00}{9.1 + 117} \right] \left[ \frac{\theta'}{273} \right]^{3/2} \text{ c.g.s. units (4.40)}
$$

Finally, substituting for $\mu_{23}$ from equation 4.38 the value of the absolute viscosity of air is obtained as:

$$
\mu_0 = (1830 \pm 2.5) \times 10^{-7} \left[ \frac{4.13}{9.1 + 117} \right] \left[ \frac{\theta'}{296} \right]^{3/2} \text{ c.g.s. units (4.41)}
$$

After multiplying equation 4.41 by the appropriate conversion factors the final form of the equation is obtained:
\[
\mu_g = \left( \frac{1830 \pm 2.5}{453.59 \cdot 32.17} \right) \frac{4.13}{(273+6)+117} \frac{273+6}{296} \text{ slug/kg} \cdot \text{sec}^{3/2} \quad (4.42)
\]

\[
\text{[HISE - 136 and HISE - 188]}
\]

Since the value of \( \mu_g \) is a function of a single variable the uncertainty interval \( \mu_g \) is given by:

\[
\omega \mu_g = \left( \frac{\partial \mu_g}{\partial \omega} \right)^2 \frac{1}{2} \quad (4.43)
\]

\[
\text{[HISE - 137, 138 and HISE - 189, 190]}
\]

\( \text{A9. Reynolds Number, } R_n. \)

The value of \( R_n \) determined during each test is based on the mean velocity and the duct hydraulic diameter and is given by:

\[
R_n = \frac{\bar{v} \cdot D_h}{\mu_g / \pi D}
\quad (4.44)
\]

Multiplied by the appropriate conversion factors equation 4.44 becomes:

\[
R_n = \frac{\bar{v} \cdot \pi D \cdot D_h}{32.17 \mu_g}
\quad (4.45)
\]

\[
\text{[HISE - 139 and HISE - 191]}
\]

For the duct sections used in the experiments it is assumed that the value of \( D_h \) is constant. This is discussed in detail in Chapter 5. The value of \( R_n \) is therefore a function of three variables:

\[
R_n = \phi(\bar{v}, \pi D, \mu_g)
\quad (4.46)
\]

In order to obtain \( R_n \) as a function of \( n \) independent variables it is necessary to combine equations 4.33, 4.29, 4.26, 4.24 and 4.22 with equation 4.45. \( R_n \) is thereby obtained as a function of four independent variables as shown in Fig.4.3.

\[
R_n = \phi(\rho, \phi, N_t, N_u)
\quad (4.47)
\]
The uncertainty interval is therefore given by:

\[ \omega h = \left[ \left( \frac{\partial h}{\partial \rho} \omega h \right)^2 + \left( \frac{\partial h}{\partial y} \omega y \right)^2 + \left( \frac{\partial h}{\partial N} \omega N \right)^2 \right]^{1/2} \tag{4.48} \]

\[ \left[ H_{15E} - 140 \text{ to } 154 \text{ and } H_{15F} - 192 \text{ to } 206 \right] \]

**A10. Density Ratio, DR.**

The ratio of the density of water to the density of air, DR is calculated from equation 4.49:

\[ DR = 84.9 \left( \frac{273 + y}{\rho} \right) \tag{4.49} \]

\[ \left[ H_{15E} - 155 \text{ and } H_{15F} - 225 \right] \]

Since DR is a function of two independent variables the uncertainty interval, \( \omega DR \) is given by:

\[ \omega DR = \left[ \left( \frac{\partial DR}{\partial y} \omega y \right)^2 + \left( \frac{\partial DR}{\partial \rho} \omega \rho \right)^2 \right]^{1/2} \tag{4.50} \]

\[ \left[ H_{15E} - 156 \text{ and } H_{15F} - 226 \right] \]

**All Values of \( \Delta \bar{s}(j) a / \sqrt{g} \) at Static Pressure Tapping Locations along the Duct.**

Along the four faces of the duct, the value of the mean static head, \( \bar{s}(R) a \) at a reference tapping location is subtracted from the value of the mean static head, \( \bar{s}(J) a \) at a given tapping location. Therefore:

\[ \Delta \bar{s}(J) a = \bar{s}(J) a - \bar{s}(R) a \tag{4.51} \]

In order to form the values of \( F(J) a \) at each tapping location the value of \( \Delta \bar{s}(J) a \) is multiplied by a constant \( VH \) given by:

\[ VH = \frac{y \cdot 2g}{12 \sqrt{\bar{V}^2}} \tag{4.52} \]

\[ \left[ H_{15E} - 157 \text{ and } H_{15F} - 378 \right] \]

To obtain a value of \( VH \) as a function of \( n \) independent variables it is necessary to combine equations 4.33, 4.29, 4.26, 4.24, and 4.22 with equation 4.52. As shown in
Fig. 4.3 the value of VH is thereby obtained a function of four independent variables:
\[ VH = \phi \left( N_l, N_u, \rho, \theta \right) \]  
\[ (4.53) \]
and the uncertainty interval \( \Delta V_H \) is given by:
\[ \Delta V_H = \left[ \left( \frac{\partial VH}{\partial N_l} \right)^2 + \left( \frac{\partial VH}{\partial N_u} \right)^2 + \left( \frac{\partial VH}{\partial \rho} \right)^2 + \left( \frac{\partial VH}{\partial \theta} \right)^2 \right]^{1/2} \]
\[ (4.54) \]

\[ [H_{15E} - 158, 164 \text{ and } H_{15F} - 279, 285] \]

Multiplying equation 4.51 by equation 4.52 we obtain the required values of \( \Delta \tilde{V}(s) \) which will be denoted by \( F(j)a \).
\[ F(j)a = \Delta \tilde{V}(s) \frac{\sqrt{2}}{2j} = VH \left( \tilde{V}(s)a - \tilde{V}(r)a \right) \]
\[ (4.55) \]
\[ [H_{15E} - 166, 169, 171, 173 \text{ and } H_{15F} - 287, 290, 292, 294] \]

Since the value of \( F(j)a \) is a function of three independent variables the uncertainty interval is given by:
\[ \Delta F(j) = \left[ \left( \frac{\partial F(j)}{\partial \tilde{V}(s)a} \right)^2 + \left( \frac{\partial F(j)}{\partial \tilde{V}(r)a} \right)^2 + \left( \frac{\partial F(j)}{\partial VH} \right)^2 \right]^{1/2} \]
\[ (4.56) \]
\[ [H_{15E} - 168, 170, 172, 174 \text{ and } H_{15F} - 289, 291, 293, 295] \]

**SECTION B: Parameters Associated with the Determination of the Duct Friction Factor - Reynolds Number Relationship.**

**B1. Mean Value of \( F(j)a \) at each Cross-Section along the Straight Duct Test Section, \( H(j) \).**

The mean value \( H(j) \) at a given cross-section, of the four values of \( F(j)a \) is given by
\[ H(j) = \frac{1}{4} \sum_{a=1}^{4} F(j)a \]
\[ (4.57) \]
\[ [H_{15E} - 176] \]

Since the value of \( H(j) \) is a function of four independent variables the uncertainty interval is given by:
\[ \omega H(j) = \left[ \left( \frac{\partial H(j)}{\partial F(1)} \omega F(1) \right)^2 + \left( \frac{\partial H(j)}{\partial F(2)} \omega F(2) \right)^2 + \ldots + \left( \frac{\partial H(j)}{\partial F(n)} \omega F(n) \right)^2 \right]^{1/2} \quad (4.58) \]

\[ \left[ H_{5E} - 177 \right] \]

B2. Duct Friction Factor, \( f \).

The value of \( f \) is obtained, as shown in Section 4.3, from a least mean squares procedure which calculates the value of \( f \) as the slope of the 'best' straight line through the \( n \) points formed by the values of \( H(j) \) and their corresponding values of \( L/Dh(j) \). Equation 4.59 gives the equation for calculating the value of \( f \):

\[ f = \frac{\sum_{j=1}^{n} H(j) \cdot \sum_{j=1}^{n} L/Dh(j) - \sum_{j=1}^{n} [L/Dh(j) \cdot H(j)]}{\left[ \sum_{j=1}^{n} L/Dh(j) \right]^{2} - \sum_{j=1}^{n} H(j)^2} \quad (4.59) \]

\[ \left[ H_{5E} - 32 \text{ to } 48 \text{ and } 178 \right] \]

Since the values of \( L/Dh(j) \) are constant the value of \( f \) is a function of the \( n \) independent variables \( H(j) \), \((j = 1 \text{ to } n)\). Therefore, the uncertainty interval \( w_f \) is given by:

\[ \omega_f = \left[ \left( \frac{\partial f}{\partial H(1)} \omega H(1) \right)^2 + \left( \frac{\partial f}{\partial H(2)} \omega H(2) \right)^2 + \ldots + \left( \frac{\partial f}{\partial H(n)} \omega H(n) \right)^2 \right]^{1/2} \quad (4.60) \]

\[ \left[ H_{5E} - 186 \text{ to } 198 \right] \]

SECTION C: Parameters Associated with the Method of Calculating the Value of the Bend Loss Coefficient.

Cl. The Value of the Duct Friction Factor \( f \) Corresponding to the Value of the Reynolds Number \( R_n \).

From the calculated value of \( R_n \) the corresponding value of \( f \) is obtained by the interpolation procedures QINDATA and QINTER (H15F = 53 to 90). The experimental relationship between \( f \) and \( R_n \) forms the data for the procedure QINDATA.
In order to calculate the uncertainty interval $\omega f$ in the interpolated value of the duct friction factor the experimental relationship is represented by the quadratic equation:

$$f = a_0 + a_1 R + a_2 R^2$$  \hspace{1cm} (4.61)$$

Therefore the uncertainty $\omega f$ is given by:

$$\omega f = \left[ \left( \frac{\partial f}{\partial R} \omega R \right)^2 \right]^{1/2}$$  \hspace{1cm} (4.62)$$

$C2. \text{Upstream and Downstream Constants } CU \text{ and } CD$

The value of $CU$ is calculated from the mean values of $F(j)a$ at eight cross-sections upstream of the bend. As shown in Bl and in Fig. 4.3 the mean values $H(j)$ are given by:

$$H(j) = \frac{1}{4} \sum_{a=1}^{4} F(j)a$$  \hspace{1cm} (4.63)$$

$$\left[ H(1)F - 297 \right]$$

and the uncertainty interval $\omega H(j)$ obtained from the equation:

$$\omega H(j) = \left[ \left( \frac{\partial H(j)}{\partial F(j)} \omega F(j) \right)^2 + \left( \frac{\partial H(j)}{\partial F(j)} \omega F(j) \right)^2 + \cdots + \left( \frac{\partial H(j)}{\partial F(j)} \omega F(j) \right)^2 \right]^{1/2}$$  \hspace{1cm} (4.64)$$

$$\left[ H(1)F - 298 \right]$$

From the last mean squares procedure the value of the upstream constant is given by:

$$CU = \frac{1}{8} \left[ \sum_{j=1}^{8} H(j)^2 - \frac{2}{3} \sum_{j=1}^{8} L \frac{\partial H(j)}{\partial F(j)} F(j) \right]$$  \hspace{1cm} (4.65)$$

$$\left[ H(1)F - 301, 306 \right]$$

Since $CU$ is a function of nine independent variables the uncertainty interval $\omega CU$ is given by:
\[ \omega_{CU} = \left[ \left( \frac{\partial \mu}{\partial f} \right)^2 + \left( \frac{\partial \mu}{\partial H(1)} \right)^2 + \left( \frac{\partial \mu}{\partial H(2)} \right)^2 + \ldots + \left( \frac{\partial \mu}{\partial H(8)} \right)^2 \right]^{1/2} \quad (4.66) \]

The value and uncertainty interval in the downstream constant CD are calculated in a similar manner from the values of \( H(1) \) at seven cross-sections downstream of the bend:

\[ \omega_{CD} = \frac{1}{\pi} \left[ \sum_{j=38}^{34} H(j) - \sum_{j=38}^{34} \lambda \mu(H(j)) \cdot f \right] \quad (4.67) \]

\[ \left[ H_{15, 3.0, 3.3} \right] \]

and

\[ \omega_{CD} = \left[ \left( \frac{\partial \mu}{\partial f} \right)^2 + \left( \frac{\partial \mu}{\partial H(28)} \right)^2 + \left( \frac{\partial \mu}{\partial H(29)} \right)^2 + \ldots + \left( \frac{\partial \mu}{\partial H(34)} \right)^2 \right]^{1/2} \quad (4.68) \]

\[ \left[ H_{15, 3.1, 3.4} \right] \]

### C3. Bend Loss Coefficient, \( K \)

As shown in Section 4.3 the value of \( K \) is obtained from the values of the upstream and downstream constants:

\[ K = \mu - \mu_{CD} \quad (4.69) \]

\[ \left[ H_{15, 3.4, 3.6} \right] \]

Combining equations 4.65 and 4.67 the value of \( K \) is a function of sixteen independent variables and the uncertainty interval \( \omega K \) is given by:

\[ \omega K = \left[ \left( \frac{\partial K}{\partial f} \right)^2 + \left( \frac{\partial K}{\partial H(1)} \right)^2 + \left( \frac{\partial K}{\partial H(2)} \right)^2 + \ldots + \left( \frac{\partial K}{\partial H(8)} \right)^2 + \left( \frac{\partial K}{\partial H(28)} \right)^2 + \left( \frac{\partial K}{\partial H(29)} \right)^2 + \ldots + \left( \frac{\partial K}{\partial H(34)} \right)^2 \right]^{1/2} \quad (4.70) \]

\[ \left[ H_{15, 3.2, 3.2} \right] \]

This value of \( \omega K \) is the required uncertainty interval, for the odds of 20 to 1 (confidence limit of 95%) in the value of the bend loss coefficient.
CHAPTER 5

EXPERIMENTAL WORK
FIG. 5.1 - Calibration of Each Static Head Tapping
5.1. PRELIMINARY TESTS.

5.1.1. Alignment of the Duct Sections.

A detailed description of the construction of the duct sections was presented in Chapter 3. The clearance in the bolt holes connecting the duct sides to the top and bottom faces allowed sufficient movement to enable the precise alignment of the duct side walls. At each end of the duct sections the height at the flange between the top and bottom faces was measured with an internal micrometer. When the duct sections were bolted together to form the straight measuring lengths, duct flanges of similar internal height were placed together. This resulted in the measuring sections aligned along the top and bottom faces to within 0.5% of the duct height.

5.1.2. Static Head Tappings.

Static head tappings were located, as shown in Chapter 3, along the centrelines of each face of the duct sections. They were positioned such that at a given distance along the duct axis the four tappings were in one cross-sectional plane. This enabled each individual tapping reading to be compared with three readings which were theoretically the same.

Each section of ducting was connected, in turn, to the downstream flange of a long length of straight ducting as shown in Fig. 5.1. At each tapping cross-section along the test duct the four average values of the static head were compared. The deviations were minimised by successive modifications to the tappings such as re-drilling and rubbing down until flush with the duct surface. Duct sections for which the above deviations were reduced to a minimum value
(approximately ± 0.01 inches w.g.) were selected to form the measuring sections for the tests described later in this Chapter.

5.1.3. Nature of the Flow.

To conform to the definition of bend loss coefficient presented in Chapter 4 the value of static head in the region of 'fully developed flow' upstream and downstream of the bend are required. The statement 'fully developed' flow is widely used to mean different things. It is sometimes taken to mean a duct flow whose parameters do not change as observations are made further downstream; whereas in some cases it is taken to mean a duct flow where the boundary layers may, by some criteria, be said to have just met. Lee states:

"although the boundary layers growing on the opposite wall of a rectangular duct merge in the region \(21 < \frac{L}{D_h} < 25\) the flow is not stable (i.e. unchanging with increasing distance) until a value of \(\frac{L}{D_h} = 60\) is reached".

In the above conclusion the parameters based on the mean velocity and turbulence are all taken into account.

For the present investigation it is sufficiently accurate and more representative of practical situations to consider only the mean velocity parameters. Therefore, a sufficient duct length is required upstream and downstream of the bend such that the velocity profile based on the mean velocity does not change with distance downstream. To determine this required duct length velocity profiles were obtained at various cross-sections along a straight
FIG. 5.2 - Co-ordinate System $x, y, z$. 

DIRECTION OF FLOW
FIG. 5.1 - Velocity Profiles at Cross-Sections along a Straight Duct
FIG. 5.4 - Velocity Distribution in a Straight Duct at a
Distance of 48 Hydraulic Diameters Downstream of the
duct formed by the duct sections. Both the velocity profile in the xz and yz plane, defined by Fig. 5.2, was measured using the duct traversing section described in Chapter 3. These profiles are shown in Fig. 5.3. It is evident that, in terms of mean velocity, the profiles do not change appreciably beyond a duct length of 24 hydraulic diameters.

5.1.4 Mean Velocity

In the calculations described in Chapter 4 the value of the mean velocity, $\bar{V}$, is given as a function of the static head difference across the inlet nozzle. Since the value of the bend loss coefficient is based on the mean inlet velocity head it is essential to determine the value of $\bar{V}$ as accurately as possible. Therefore, as a check on the accuracy of the inlet nozzle the value of $\bar{V}$, for a given test, was calculated from the 'fully developed' velocity distribution shown in Fig. 5.4. This was measured at a distance of 48 hydraulic diameters downstream of the inlet nozzle using the duct traverse section described in Chapter 3.

The value of $\bar{V}$ was calculated from the velocity distribution as follows. Considering the co-ordinate system shown in Fig. 5.4, the value of the local velocity $v$ is a function of $\alpha$ and $\beta$.

$$v = \phi(\alpha, \beta) \quad (5.1)$$

If the velocity distribution is assumed to be made up of rectangular columns of length $v$ and sides of infinitesimal length $\delta\alpha$ and $\delta\beta$, the total quantity $Q$, of air is given by:

$$Q = \sum_{\beta=0}^{b} \sum_{\alpha=0}^{a} v \delta\alpha \delta\beta \quad (5.2)$$
FIG. 5.5 - Velocity Profiles from FIG. 5.4 - Variations of Local

Velocity v with $\phi$ for Values of $A$
FIG. 5.6 - Total Quantity of Air Through the Duct - Area Under the Profiles of FIG. 5.5 against $\beta$ ft
This expression for \( Q \) may be evaluated by the following graphical procedure:

For each value of \( \beta \) the value of \( \sum_{\alpha=0}^{b} v \, d \alpha \) is equal to the area under the curve formed by the variation of \( v \) with \( \alpha \).

Therefore, denoting the area by \( A_\beta \).

\[
\sum_{\alpha=0}^{b} v \, d \alpha = A_\beta \quad (5.3)
\]

Substituting in equation 5.2 the flow rate \( Q \) becomes:

\[
Q = \sum_{\beta=0}^{a} A_\beta \, \delta \beta \quad (5.4)
\]

This expression is the area under the curve formed by plotting the above values of \( A_\beta \) against \( \beta \).

Fig. 5.5 shows the variation of \( v \) with \( \alpha \) for the various values of \( \beta \) corresponding to the measured velocity distribution given in Fig. 5.4. The value of \( Q \) is obtained as the area under the curve shown in Fig. 5.6, formed by the variations of \( A_\beta \) (from Fig. 5.5) with \( \beta \).

Therefore, the value of \( \bar{V} \) is given by:

\[
\bar{V} = \frac{Q}{A} = \frac{7.0}{2/12 \times 4/12} = 12 \omega \, \text{ft/\( \omega \)} \quad (5.5)
\]

The values of the nozzle static head difference, air temperature and barometric pressure were recorded at the same time as the above velocity distribution was measured. These values (3.73 inches w.g., 5.5°C and 28.88 inches Hg respectively) were substituted in equation 4.33 resulting in a value of 126.1 ft/\( \omega \) for \( \bar{V} \). Thus very good agreement for the value of \( \bar{V} \) has been obtained, reflecting the accuracy in the construction of both the inlet nozzle and the static head tappings.
FIG. 5.7 – Schematic Representation of Duct Friction Factor –

Reynolds Number Test Rig
FIG. 5.8 - Experimental Arrangement for Duct
Friction Factor - Reynolds Number Tests
5.1.5. Reynolds Number Range

For each bend formed by the combinations of the inner and outer bend radius to duct width ratios, Ri/D and Ro/D, the resistance to the flow will vary. This will cause a slight variation in the value of the Reynolds number Rn and a consequent variation in the value of the duct friction factor f. In order to determine the f - Rn relationship the approximate range of Reynolds number was determined from equation 4.45 for the two Ri/D and Ro/D combinations assumed to give the upper and lower bounds. These tests resulted in a Reynolds number range of 1.65 - 1.8 x 10^5. Therefore, in order to cover the possible range occurring in the full range of Ri/D and Ro/D combinations the f - Rn relationship was determined for a Rn range of 1.5 - 1.9 x 10^5.

5.2. Duct Friction Factor - Reynolds Number Relationship

From the results of the preliminary tests the duct sections were bolted together to form a long straight duct, as shown schematically in Fig.5.7 and pictorially in Fig.5.8. An approach duct length of 25.5 hydraulic diameters was placed upstream of the duct test section. This is in excess of the recommended length of 24 hydraulic diameters obtained in Section 5.1.3.

The required range of Rn values were obtained by different settings of the sliding door arrangement on the control box. For each setting the values of test number, air temperature, barometric pressure and the values of the static head, s(J)a, were recorded using the instrumentation described in Chapter 3. The values of s(J)a at each station along the faces of the duct and across the inlet nozzle were recorded five times by the automatic scanning procedure.
FIG. 5.9 - Schematic Representation of Bend Loss Coefficient Test Rig
FIG. 5.10 - Experimental Arrangement for Bend Loss Coefficient Tests
All the above readings were recorded on punched paper tape. The required values of $f$ and $R_n$ were obtained from the calculation procedure described in Chapter 4 which is incorporated in the data-reduction computer program $H15E$ listed in Appendix 2.

5.3. BEND LOSS COEFFICIENT

The experimental arrangement shown schematically in Fig. 5.9 and pictorially in Fig. 5.10 was constructed from the duct sections. Duct lengths of 31.5 and 49.5 hydraulic diameters respectively were provided between the nozzle and the upstream measuring section and between the bend and the downstream measuring section. Variations in the bend geometry were formed by combinations of $R_i/D$ and $R_o/D$ within the bend section as described in Section 3.1.2.

For each bend combination similar readings as for the $f - R_n$ tests were recorded on punched tape. The calculation procedure described in Chapter 4 which is incorporated in the data-reduction computer program $H15F$ (listed in Appendix 2) was used to evaluate the value of the bend loss coefficient defined in Chapter 4.

Using the existing experimental facilities a few values of the bend loss coefficient for constant area bends of aspect ratio 0.5 were determined in the same manner as above. These were obtained by rotating the duct sections through $90^\circ$ and making small modifications to the bend sections.

The experimental results from both the duct friction factor - Reynolds number and bend loss coefficient tests are discussed in detail in Chapter 6.
CHAPTER 6

ANALYSIS OF RESULTS AND EXPERIMENTAL PROCEDURE
FIG. 6.1 - Experimental Duct Friction Factor - Reynolds Number Relationship

$\sqrt{f} = 2 \log_{10} \left[ R_n f \right] - 0.8$

Von-Karman Prandtl Smooth Pipe Relationship
6.1. INTRODUCTION

The optimum geometrical shape for the 90° rectangular cross-section bends are developed from the experimental values of the bend loss coefficient, $K$. Individual values of $K$ are compared, where possible, with the up-to-date available data discussed in detail in Chapter 2. This correlation is based on the definition of $K$ presented in Chapter 4.

When considering any published experimental results, the reader inevitably requires to know the degree of accuracy or uncertainty associated with the values quoted. Because of the small order of magnitude, a statement of the value of $K$ is meaningless without an indication of the corresponding uncertainty interval. In this Chapter the values of the uncertainty intervals in the various parameters and in the final values of $K$, obtained from the analysis presented in Chapter 4, are discussed.

The accuracy of the overall experimental procedure is discussed with particular reference to the separate determination of the duct friction factor - Reynolds number relationship.

For ease of analysis the results are divided into two sections in line with the experimental work.

6.2. DUCT FRICTION FACTOR - REYNOLDS NUMBER RELATIONSHIP

6.2.1. Experimental Results

Fig.6.1 shows the duct friction factor - Reynolds number relationship ($f - R_n$) over the required range of Reynolds number.

It is theoretically possible to obtain the $f - R_n$ relationship from the Moody chart of the Colebrook-White (58)
\[ f^{\frac{1}{2}} = \frac{2}{\log_{10} \left[ \frac{2.51}{R_{nf}^{0.4}} + \frac{k}{3.7 dh} \right]} \]

**Fig. 6.2 - Moody Chart of Colebrook - White Transition Formula**
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<tr>
<th><strong>IN TEMPERATURE</strong></th>
<th>12.0</th>
<th>OR = 0.25 DEG. C (20 TO 1)</th>
<th>2.083 PER CENT UNCERTAINTY</th>
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<td><strong>AROMETRIC PRESSURE</strong></td>
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<td>OR = 0.01 INCH. HG. (20 TO 1)</td>
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<th><strong>EYNOLES NO.</strong></th>
<th>1.859230%</th>
<th>S</th>
<th>OR = 4.236726384%</th>
<th>(20 TO 1)</th>
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<td><strong>RIFCTION FACTOR</strong></td>
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<th>OR = -2.021844734%</th>
<th>-1</th>
<th>FT./SEC. (20 TO 1)</th>
<th>0.158 PER CENT UNCERTAINTY</th>
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<td>OR = 7.2794818%</td>
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<td>LB./CU.FT. (20 TO 1)</td>
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**FIG.6.3** - Computer Line Printer Output for a Typical Duct

**Friction Factor - Reynolds Number Test**
### VELOCITY HEAD RATIO

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**FIG. 6.3 - Continued**
Transition Formula, equation 6.1. This would require a knowledge of the hydraulic roughness $k$, of the duct material.

$$f^{-1/2} = -2 \log_{10} \left[ \frac{2.51}{R_n f^{1/2}} + \frac{k}{3.7 h} \right] \quad (6.1)$$

Substituting the experimental values of $f$ and $R_n$ in equation 6.1 a mean value of 0.0000188 is obtained for the value of $k$. Assuming this value could be measured or evaluated by some independent means the required $f$-$R_n$ relationship would then have to be interpolated from the Moody chart shown in Fig.6.2. This is obviously impractical considering the small range of Reynolds number covered by the experiments and the accuracy required in the value of the duct friction factor.

The accuracy of the experimental $f$-$R_n$ relationship is shown in Fig.6.1 by its comparison with the Von-Karman Prandtl smooth duct relationship, equation 6.2. This is obtained by setting the value of $k$ in equation 6.1 to zero.

$$f^{-1/2} = 2 \log_{10} \left[ R_n f^{1/2} \right] - 0.8 \quad (6.2)$$

The computer line printed output of the results for a given $f$-$R_n$ test is shown in Fig.6.3. Values of the various parameters and the required duct friction factor and Reynolds number are printed with their corresponding uncertainty intervals. The values of the mean wall static head, $\bar{s}(J)a$ at each tapping location, $J$, along the four faces, $a$, of the duct are tabulated under the heading "TESTDATA" followed by the values of $H(J)$.
FIG. 6.4 - Graphical Representation of the Variation of $H(J)$ with $L/Dh(J)$ for a Typical Duct Friction Factor - Reynolds Number Test
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### TABLE 6.1 - Percentage Uncertainties in Parameters and Results for Duct Friction Factor - Reynolds Number Tests
(the mean of the four values of $\bar{s}(j)a$ at each tapping cross-section minus a reference static head $\bar{s}(R)a$ divided by the mean inlet velocity head) along the duct measuring section tabulated under the heading "VELOCITY HEAD RATIO".

A typical computer graphical representation of the variation of $H(j)$ with the value of $L/Dh(j)$ along the test section is shown in Fig.6.4. The value of the duct friction factor is determined, as shown in Chapter 4, from the least mean squares straight line through the above points. This is represented by the dotted line in Fig.6.4. A graphical representation of $H(j)$ against $L/Dh(j)$ for each $f-Rn$ test is included in Appendix 3. From these plots it is evident that the deviation between the data and line values of $H(j)$ are very small; typical values are tabulated under the heading "DEV" in Fig.6.3. This small scatter is indicative of the accuracy in the construction and alignment of the duct sections and also the reliability of the individual wall static head tappings.

6.2.2. Uncertainty Intervals

The percentage uncertainty, for the odds of 20 to 1 (95% confidence limit) in the values of the various parameters, duct friction factor and Reynolds number for each test are tabulated in Table 6.1. For each test the maximum uncertainty interval* occurred in the value of the nozzle differential static head, $NH$. This shows the necessity for the accurate construction of both the inlet nozzle and its wall static head tappings as discussed in Chapters 3 and 5.

* The absolute value of the air temperature is used throughout the calculations and its corresponding uncertainty interval is given in Table 6.1.
H15E
BEND LOSS COEFFICIENT

ASPECT RATIO = 2.0
TEST NO. 42

AIR TEMPERATURE  9.0 + OR - 0.25 DEG. C (20 TO 1)  2.7778 PER CENT UNCERTAINTY
BAROMETRIC PRESSURE  29.88 + OR - 0.01 INCH. HG. (20 TO 1)  0.0335 PER CENT UNCERTAINTY
MEAN VELOCITY  1.2309868  2 + OR - 2.148705788 -1 FT./SEC. (20 TO 1)  0.1746 PER CENT UNCERTAINTY
REYNOLDS NO.  1.7998028  5 + OR - 4.387829108 -2 (20 TO 1)  0.2438 PER CENT UNCERTAINTY
FRICTION FACTOR  -1.5996368 -2 + OR - 8.302006228 -6 (20 TO 1)  -0.0519 PER CENT UNCERTAINTY
DENSITY  7.7875338 -2 + OR - 7.379408448 -5 LB./CU.FT. (20 TO 1)  0.0948 PER CENT UNCERTAINTY
NOZZLE DIFFERENCE  -3.6257988  0 + OR - 1.584049508 -2 INCH W.G. (20 TO 1)  0.4369 PER CENT UNCERTAINTY
PRESSURE RATIO  9.9107768 -1 + OR - 2.448709258 -3 (20 TO 1)  0.2471 PER CENT UNCERTAINTY
ADIABATIC EXPANSION FAC  9.9618298 -1 + OR - 1.323054698 -3 (20 TO 1)  0.1328 PER CENT UNCERTAINTY
DENSITY RATIO  8.0126518  2 + OR - 7.592727948 -1 (20 TO 1)  0.0948 PER CENT UNCERTAINTY
VISCOITY  3.6792968 -7 + OR - 6.382732338 -13 (20 TO 1)  0.0002 PER CENT UNCERTAINTY

CASE 1: UPSTREAM FF = DOWNSTREAM FF = -1.5996368 -2

CU= -6.522574 -2 + OR - 1.598586368 -3 (20 TO 1)  -2.4509 PER CENT UNCERTAINTY
CD= -7.151099 -1 + OR - 2.361263708 -3 (20 TO 1)  -0.3302 PER CENT UNCERTAINTY

BEND LOSS COEFFICIENT = 6.498842 -1 + OR - 2.834479908 -3 (20 TO 1)  0.4362 PER CENT UNCERTAINTY

CASE 2: FRICTION FACTOR CHECK

2.1 UPSTREAM
FRICTION FACTOR  -1.6755268 -2  DEV=  7.58898 -4
CONSTANT  -5.7257358 -2  DEV=  7.96848 -3

2.2 DOWNSTREAM
FRICTION FACTOR  -1.7302598 -2  DEV=  1.30628 -3
CONSTANT  -5.9754948 -1  DEV=  1.17568 -1

BEND LOSS COEFFICIENT = 5.4029218 -1  DEV= -1.09598 -1

FIG. 6.5 - Computer Line Printer Output for a Typical Bend Loss Coefficient Test
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FIG. 6.5 - Continued
FIG. 6.6 - Graphical Representation of the Variation of $F(J)a$ with $L/Dh(J)$ for a Typical Bend Loss Coefficient Test
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<tr>
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<td>0.663</td>
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<td>86</td>
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<td>0.311</td>
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<td>4</td>
<td>0.223</td>
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<td>88.7</td>
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<td>4.35</td>
<td>0.199</td>
</tr>
<tr>
<td>810</td>
<td>4</td>
<td>5</td>
<td>0.222</td>
</tr>
</tbody>
</table>

**TABLE 6.2 - Combinations of Inner and Outer Bend Radius to Duct Width Ratios Ri/D and Ro/D and corresponding Value of Bend Loss Coefficient**
In Fig.6.1 the uncertainty intervals in the values of \( f \) and \( R_n \) are represented by the vertical and horizontal bands respectively centred about the calculated values.

The small range of values covered by the above bands and in the percentage uncertainties in the various parameters is indicative of the reliability of the experimental rig and calculations.

6.3. VALUES OF THE BEND LOSS COEFFICIENT, \( K \).

6.3.1. Experimental Results

A typical computer line printer output of the results for a given test is shown in Fig.6.5. The value and uncertainty interval of the various parameters, \( \bar{s}(J)a \) and \( H(J) \) are given in the same way as for Fig.6.3. The calculated value of the bend loss coefficient, \( K \), is printed under the heading "CASE 1: UPSTREAM FF = DOWNSTREAM FF = A VALUE". Values of friction factor, constant and \( K \) printed under the heading "CASE 2: FRICTION FACTOR CHECK" are discussed later in this Chapter. The values of \( F(J)a \), (mean static head at a given tapping location \( \bar{s}(J)a \) minus a reference mean static head \( \bar{s}(R)a \) divided by the mean inlet velocity head), along the four faces of the duct are printed under the heading "VELOCITY HEAD RATIO (STATION-REF.STATION)". A typical graphical representation of the variation of \( F(J)a \) with \( L/Dh(J) \) along the duct is shown in Fig.6.6. Similar plots for each combination of inner and outer bend radius to duct width ratios, \( R_i/D \) and \( R_o/D \), are included in Appendix 3 and these are discussed later in this Chapter.

For the range of combinations of \( R_i/D \) and \( R_o/D \) the experimental values of \( K \) are given in Table 6.2. These values are plotted in Fig.6.7 as a family of curves of given
Fig. 6.7: Variation of Bend Loss Coefficient with Ro/D for Values of Ri/D
FIG. 6.8 - Comparison of Experimental

$\frac{R}{D} = 1$

$\frac{R}{D} = 2$
values of $\frac{R_i}{D}$, against $\frac{R_o}{D}$. The curve for $\frac{R_i}{D}=1$ is shown dotted because it is composed of only three experimental values. It is reasonable to assume that the curve is of the same general shape as for higher values of $\frac{R_i}{D}$. The variation in the value of $K$ shown in Fig. 6.7 is discussed later in this Chapter.

In Fig. 6.8 the experimental values of $K$ are compared, where possible, with the available up-to-date data and also the results obtained by Nippert$^{(5)}$. This data is discussed in detail in Chapter 2 and only reasons for the discrepancies need to be detailed here. Nippert's results for bends of aspect ratio 2.4 are included because they show the same trend as the present experimental results. The scatter in the individual values is due to the absence of a downstream duct and the complex calculation procedure used in Nippert's work. In Sprenger's$^{(10)}$ work a piece of duct was removed from the upstream section of a calibrated straight line of ducting, the length of the duct being the same as the centreline length of the bend which replaced it. Therefore, an approximate correction has been made to Sprenger's results to allow for the head loss in this missing piece of straight ducting. It is difficult to make an overall comparison between Sprenger's results and the present experimental results because only two of Sprenger's bends fall within the range given in Table 6.2.

The values of $K$ obtained from Kelnhofer & Smith$^{(11)}$ and the Engineering Sciences Data Sheet$^{(27)}$, for constant area bends only, are calculated from a loss coefficient, (a function of the radius ratio) multiplied by a series of correction factors for deflection angle, aspect ratio,
FIG. 6.9 - Variation of Aspect Ratio Correction Factors with the Bend Radius Ratio
Reynolds number and surface roughness. There appears to be some discrepancy in the value of the aspect ratio correction factor. The value given in the Engineering Sciences Data Sheet (0.83 for $AR = 2$) differs from that given by Kelnhofer & Smith (0.95 for $AR = 2$) and both references give the value as independent of the radius of the bend.

However, from the experimental work of Miller (12) and that presented in this thesis it is evident that the aspect ratio correction factor is a function of the bend radius ratio. Assume the value of the bend loss coefficient for a given aspect ratio, $K_{AR}$, is calculated by multiplying a basic loss coefficient $K_{1.0}$ (aspect ratio = 1.0) by a correction factor, $C_{AR}$, thus:

$$K_{AR} = C_{AR} K_{1.0} \quad (6.3)$$

The ratio of any two aspect ratio correction factors is therefore given by the corresponding ratio of the values of $K_{AR}$. For the aspect ratios considered in this thesis this becomes:

$$\frac{K_{0.5}}{K_{2.0}} = \frac{C_{AR = 0.5}}{C_{AR = 2.0}} \quad (6.4)$$

The variation of the ratio $C_{AR = 0.5}/C_{AR = 2.0}$ obtained from the above experimental work is shown in Fig.6.9. From these curves it is evident that the aspect ratio correction factor is dependent on the radius ratio of the bend and also appears to be a function of the Reynolds number. This accounts, in part, for the large deviation between the results from the Engineering Sciences Data Sheets and the present experimental values.
FIG. 6.10 - Correlation of up-to-date values of bend loss coefficient for constant area bends of aspect ratio 2 at a Reynolds number of $1.8 \times 10^5$. 
A correlation of the up-to-date experimental data for the values of the bend loss coefficient for constant area 90° bends, aspect ratio 2, at a Reynolds number of $1.8 \times 10^5$ is shown in Fig. 6.10. The values of $K$ from the Engineering Sciences Data Sheet and Kelnhofer & Smith can be approximately corrected for the aspect ratio correction factor using the results presented in the table below based on the work of Miller for a Reynolds number of $10^6$.

<table>
<thead>
<tr>
<th>BEND</th>
<th>BEND LOSS COEFFICIENT</th>
<th>$C_{AR=2.0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{R_i}{D}$</td>
<td>$\frac{R_o}{D}$</td>
</tr>
<tr>
<td>0.5</td>
<td>1.5</td>
<td>0.241</td>
</tr>
<tr>
<td>1.5</td>
<td>2.5</td>
<td>0.147</td>
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<tr>
<td>2.5</td>
<td>3.5</td>
<td>0.142</td>
</tr>
</tbody>
</table>

The approximate values of $C_{AR=2.0}(Rn=10^6)$ are obtained by multiplying the values of $C_{AR=2.0}(Rn=1.8 \times 10^5)$ by the ratio of the Reynolds number correction factors

$C_{Rn=10^6} = 0.70$, $C_{Rn=1.8 \times 10^5} = 0.92$

given by Kelnhofer & Smith and the Engineering Sciences Data Sheet. A proportion of this correction is used for a bend inner radius ratio, $Ri/D$, of 2.5 because the Reynolds number effect shown in Fig. 6.9 is very small.

Therefore, the values of $K$ quoted in the above references for a Reynolds number of $1.8 \times 10^5$ are approximately corrected for aspect ratio by multiplying by the value of $C_{AR=2.0}(Rn=1.8 \times 10^5)$ from the above table and dividing by the aspect ratio correction factor quoted in the corresponding reference. These corrected values of $K$ are included in Fig. 6.10. Although it is only
<table>
<thead>
<tr>
<th>Test No.</th>
<th>Bend Loss Coefficient</th>
<th>Air Temp. °C.</th>
<th>Air Temp. Absolute</th>
<th>Barometric Pressure</th>
<th>Mean Velocity</th>
<th>Reynolds Number</th>
<th>Friction Factor</th>
<th>Air Density</th>
<th>Nozzle Difference</th>
<th>Pressure Ratio</th>
<th>Nozzle Adiabatic Exp.Fac.</th>
<th>Density Ratio</th>
<th>Absolute Viscosity</th>
</tr>
</thead>
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<td>22</td>
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<td>2.174</td>
<td>0.088</td>
<td>0.034</td>
<td>0.174</td>
<td>0.243</td>
<td>0.051</td>
<td>0.064</td>
<td>0.437</td>
<td>0.248</td>
<td>0.133</td>
<td>0.094</td>
<td>0.0002</td>
</tr>
<tr>
<td>24</td>
<td>0.995</td>
<td>3.125</td>
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<td>0.035</td>
<td>0.164</td>
<td>0.236</td>
<td>0.051</td>
<td>0.095</td>
<td>0.418</td>
<td>0.255</td>
<td>0.137</td>
<td>0.095</td>
<td>0.0002</td>
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<td>0.052</td>
<td>0.095</td>
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<td>0.425</td>
<td>0.256</td>
<td>0.138</td>
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<td>0.095</td>
<td>0.417</td>
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<td>0.439</td>
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<td>0.137</td>
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</tr>
</tbody>
</table>

**TABLE 6.3 - Percentage Uncertainties in Parameters and Results**

*for Bend Loss Coefficient Tests*
FIG. 6.11 - Percentage Uncertainty in Bend Loss Coefficient
possible to correct the value of K for two combinations of Ri/D and Ro/D it is evident from Fig. 6.10 that the amount of scatter is reduced. A more accurate correlation could be achieved as the relationship between the aspect ratio correction factor and the bend radius ratio is further investigated at various values of Reynolds number.

6.3.2. Uncertainty Intervals

The percentage uncertainties in the various parameters and the values of K are shown in Table 6.3. It is evident that the conclusions given above for the values given in Table 6.1 are applicable to those of Table 6.3. The small uncertainties in the values of the duct friction factor is indicative of the accuracy of the values used in the determination of the values of K.

Fig. 6.11 shows the variation of the percentage uncertainty in the values of K, a maximum uncertainty of 1.27% being obtained. These uncertainty intervals are represented by the vertical bands in Fig. 6.7 centred about the experimental value of K. The small range of these intervals shows the accuracy of the quoted results.

6.3.3. Calculation Procedure

The calculations involved in the determination of the value of K are discussed in detail in Chapter 4. The overall procedure may be summarised as follows:

1) From the calculated value of Rn the corresponding value of f is obtained from the relationship shown in Fig. 6.1.
FIG. 6.12 - Value of Bend Loss Coefficient Calculated Without the Duct Friction Factor - Reynolds Number Relationship
FIG. 6.13 - Variation of Bend Loss Coefficient with Value Calculated as shown in Fig. 6.12
2) Using this value of \( f \) the 'best' straight lines are constructed through the values of \( H(J) \) (measured static head at each cross-section minus a reference static head divided by the mean inlet velocity head), plotted against \( L/Dh(J) \), at the selected cross-sections upstream and downstream of the bend.

3) From the constants of the two straight lines the value of the bend loss coefficient \( K \) is calculated as defined in Chapter 4.

Assume the experimental \( f-Rn \) relationship is unknown and the value of \( f \) upstream and downstream of the bend is to be calculated from the values of \( H(J) \) at the selected cross-sections. This would be necessary in order to determine the value of \( K \) as defined in Chapter 4. In most cases the calculated value of \( f \) (using a least mean squares procedure) upstream and downstream of the bend will not be equal to each other and will differ slightly from the value interpolated from the experimental \( f-Rn \) relationship. This is shown diagramatically in Fig. 6.12 where the difference in slopes is magnified for the sake of clarity. The value of \( K \) obtained from the calculated values of \( f \) is denoted by \( K_{FFC} \) in Fig. 6.12. For a typical test these calculated values of \( f \) and constants upstream and downstream of the bend and the resultant value of \( K_{FFC} \) are tabulated in Fig. 6.5 under the heading "CASE 2: FRICTION FACTOR CHECK". The difference between the calculated value of \( K \) and \( K_{FFC} \) is shown in Fig. 6.13 which shows that a random error can be introduced into the value of \( K \) when calculated from the FRICTION FACTOR CHECK. The discrepancies between the values of \( K \) is caused by the deviations of the calculated values of \( f \) from the interpolated
FIG. 6.14 - Effect of Calculating Bend Loss Coefficient from Friction Factor Check
value. This is shown in Fig.6.14 in which the dotted lines join the values of the ratios of the respective friction factors for each test. The value of the ratio $K_{FFC}$ to $K$ is given by the cross on each dotted line. As shown by Figs.6.12 and 6.14, the major contribution towards the deviation between the values of $K$ and $K_{FFC}$ is the discrepancy in the value of the downstream friction factor.

6.3.4. Selected Cross-Sections.

To conform to the definition of $K$ presented in Chapter 4, the selected cross-sections upstream and downstream of the bend must be in the regions of 'fully developed' flow. For the purpose of this investigation this is satisfied when the gradient of the line through the values of $H(J)$, plotted against $L/Dh(J)$, has attained the constant value of the interpolated duct friction factor and is unaffected by the presence of the bend.

For each bend combination of $Ri/D$ and $Ro/D$ the variations of $F(J)a$ (mean static head at a given tapping location minus a reference static head divided by the mean inlet velocity head) against $L/Dh(J)$ along each face of the duct are shown by the computer graphical representations included in Appendix 3. A typical plot is shown in Fig.6.6 for ease of reference. On each plot the dotted lines represent the 'best' straight lines of slope equal to the interpolated duct friction factor constructed from the values of $H(J)$ (average of four values of $F(J)a$) at the upstream and downstream selected cross-sections. On each plot the number near the bottom right-hand corner corresponds to the test number in Table 6.2.
FIG. 6.15 - Value of Bend Loss Coefficient Calculated from $H(J)$

Value at a Single Cross-Section Downstream of the Bend
The selected cross-sections upstream and downstream of the bend cover the \( L/D_h(j) \) ranges of 0 to 21 and 81 to 99 respectively. This downstream range is equivalent to a downstream duct length to mean hydraulic diameter ratio \( L_d/D_h(j) \), range of 49.5 to 67.5. From the graphical representations shown in Appendix 3 it is possible to make the following conclusions:

1. At the selected cross-sections there is a very small amount of scatter in the values of \( F(j)a \) from the four faces of the duct. In a few tests the bend appears to have a marginal effect on the last selected upstream cross-section, \( L/D_h(j) = 21 \).

2. The bend has a marked effect on the values of \( F(j)a \) for a duct length of approximately ten hydraulic diameters upstream of the bend. This is greater than the value quoted by previous experimenters and possibly accounts for some of the scatter in the published results.

3. The values of \( H(j) \) at the selected cross-sections upstream and downstream of the bend form a constant gradient represented by the interpolated value of the duct friction factor \( f \).

4. The necessity for the long downstream duct length is shown by considering the effect of calculating the value of \( K \) from the value of \( H(j) \) at the individual cross-sections downstream of the bend. Fig.6.15 shows the value of the bend loss coefficient, \( K' \), obtained from a typical cross-section downstream of the bend using the interpolated value of \( f \). For each test the variation in the ratio of \( K' \) to \( K \) with the value
FIG. 6.16 - Variation of Bend Loss Coefficient
FIG. 6.16 - Continued
FIG.6.17 - Variation of Bend Loss Coefficient with Mid-Plane Cross-Sectional Area of the Bend
FIG. 6.18 - Optimum Geometry for Rectangular Cross-Section $90^\circ$ Bends of Aspect Ratio 2
of $L_d/D_h(J)$ is shown in Fig. 6.16. Considerable scatter exists until the flow has travelled a downstream duct length, $L_d$, of approximately 45 hydraulic diameters. The value of the ratio $K'$ to $K$ then oscillates with small amplitude about the value of unity. This oscillation shows the advantage of fitting the interpolated value of $f$ to the seven selected cross-sections downstream of the bend.

6.4. OPTIMISATION OF THE BEND GEOMETRY

The values of $K$ shown in Table 6.2 are plotted in Fig. 6.17 as a function of the cross-sectional area at the mid-plane of the bend. It is evident that a reduction in the value of $K$ compared with that obtained for a constant area bend, $K_{CA}$, was obtained by increasing the bend mid-plane cross-section area, the optimum value being dependent on the inner radius to duct width ratio, $R_i/D$, of the bend. For each value of $R_i/D$ the optimum value of the outer radius to duct width ratio $R_o/D$, is given by the curves in Fig. 6.7. These optimum values of $R_o/D$ are plotted in Fig. 6.18 and compared with the constant area bend value as shown by the dotted line. Assuming a straight line relationship the optimum $R_o/D$ ratio for a given value of $R_i/D$ is given by equation 6.5

$$R_o/D(\text{optimum}) = 0.87 \frac{R_i}{D} + 0.9$$  \(6.5\)

As shown in Fig. 6.17 a maximum reduction of 10% in the value of $K$ was obtained for the combinations of $R_i/D$ and $R_o/D$ given in Table 6.2. This reduction in the value of $K$ could be of significant economic importance with the increasing size and complexity of modern internal duct flow systems. Also an important fact that is evident
from Fig. 6.17 is that any increase in the bend mid-plane cross-section area above the optimum value or a reduction below that corresponding to a constant area bend, results in a considerable increase in the value of $K$. 
CHAPTER 7

BEND LOSS COEFFICIENTS FROM MODELS
OF VARIABLE AREA BENDS
<table>
<thead>
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<th>DIFFUSER</th>
<th>CONSTANT AREA BEND</th>
<th>COMBINED LOSS COEFFICIENT</th>
</tr>
</thead>
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<td>$\phi$</td>
<td>$N/D_1$</td>
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<td>22 [0.566]</td>
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<td>1.333</td>
<td>11°</td>
<td>0.625</td>
</tr>
<tr>
<td>4.2 [0.655]</td>
<td>0.588</td>
<td>22°</td>
<td>0.875</td>
</tr>
<tr>
<td>4.4 [0.347]</td>
<td>0.715</td>
<td>15°</td>
<td>0.75</td>
</tr>
<tr>
<td>48 [0.678]</td>
<td>1.33</td>
<td>7°</td>
<td>1.125</td>
</tr>
<tr>
<td>6.2 [0.655]</td>
<td>0.50</td>
<td>22°</td>
<td>1.25</td>
</tr>
</tbody>
</table>

FIG. 7.1 - Diffuser-Constant Area Bend-Contraction Representative for Variable Area Rectangular Cross-Section 90° Bends
<table>
<thead>
<tr>
<th>BEND</th>
<th>DIFFUSER</th>
<th>CONSTANT AREA BEND</th>
<th>COMB LD Coeff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{A_1}{A_2}$</td>
<td>$\phi$</td>
<td>$\frac{N}{B_1}$</td>
</tr>
<tr>
<td>64 [0.443]</td>
<td>0.654</td>
<td>10$^\circ$</td>
<td>1.00</td>
</tr>
<tr>
<td>66 [0.239]</td>
<td>0.80</td>
<td>11$^\circ$</td>
<td>0.625</td>
</tr>
<tr>
<td>67 [0.214]</td>
<td>0.87</td>
<td>16$^\circ$</td>
<td>0.25</td>
</tr>
<tr>
<td>610 [0.744]</td>
<td>1.360</td>
<td>16$^\circ$</td>
<td>0.5</td>
</tr>
<tr>
<td>82 [0.663]</td>
<td>0.40</td>
<td>22$^\circ$</td>
<td>1.75</td>
</tr>
</tbody>
</table>

FIG. 7.1 - Continued
<table>
<thead>
<tr>
<th>Bend</th>
<th>Diffuser</th>
<th>Constant Area Bend</th>
<th>Combined Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_1/A_2$</td>
<td>$\phi$</td>
<td>$N/D_1$</td>
</tr>
<tr>
<td>84 [0.629]</td>
<td>0.50</td>
<td>15°</td>
<td>1.75</td>
</tr>
<tr>
<td>86 [0.311]</td>
<td>0.571</td>
<td>15°</td>
<td>1.50</td>
</tr>
<tr>
<td>88 [0.223]</td>
<td>0.80</td>
<td>8°</td>
<td>0.875</td>
</tr>
<tr>
<td>88.7 [0.199]</td>
<td>0.85</td>
<td>8°</td>
<td>0.875</td>
</tr>
</tbody>
</table>

FIG. 7.1 – Continued
FIG. 7.2 - Rectangular Diffuser Static Pressure

Recovery Coefficient $C_p$
7.1. MODEL CALCULATIONS FOR EXPERIMENTAL BEND GEOMETRIES

The analysis of the experimental results presented in this thesis shows that, for a given bend inner radius to duct width ratio $R_i/D$, there is an optimum value for the bend outer radius to duct width ratio $R_o/D$. This optimum value of $R_o/D$ is shown in Fig. 6.18 and given by equation 6.5. The obvious question that follows from the above discussion is "why is there an optimum value for $R_o/D"?"

In an attempt to answer the above question the bend configurations formed by the combination of $R_i/D$ and $R_o/D$ are represented by the arrangements shown dotted in Fig. 7.1. These arrangements are composed of a diffuser, a constant area bend and a contraction in that order for the diverging-converging bends and in the reverse order for the converging-diverging bends. The very approximate representation of the bend geometries enables the values of the overall loss coefficient $K_c$ to be calculated from the diffuser static head recovery coefficient $C_p$ and the loss coefficient for the constant area bend, $K_{ca}$. In each case the contraction is assumed to have a negligible loss coefficient. The values of $C_p$ are interpolated from Fig. 7.2 which is taken from reference 59 and the values of $K_{ca}$ are obtained from Fig. 6.10. The overall loss coefficient, $K_c$ is defined as the static head difference, $s_1-s_4$, between stations 1 and 4 divided by the mean velocity head $\bar{\nu}$ at station 1. Therefore:

$$K_c = \frac{s_1 - s_4}{\bar{\nu}}$$  \hspace{1cm} (7.1)

For each bend combination the values of the various diffuser and constant area bend parameters are tabulated in Fig. 7.1. The static head difference $s_1-s_4$ is obtained by calculating the static head at each section 1, 2, 3 and 4 round the bend representation.
a) **Section 1 to Section 2 (Diffuser)**

The static pressure recovery coefficient $C_p$ is defined by:

$$C_p = \frac{s_2 - s_1}{\bar{q}_1}$$

Thus:

$$s_2 = C_p \bar{q}_1 + s_1 \quad (7.2)$$

The mean velocity head at Station 1 is obtained from the mean velocity calculated from equation 4.33 for each bend.

b) **Section 2 to Section 3 (Constant Area Bend)**

The Bernoulli equation between Sections 2 and 3 may be written as:

$$s_2 + \bar{q}_2 = s_3 + \bar{q}_3 + HL_B \quad (7.3)$$

where

- $\bar{q}_2 = \text{mean velocity head at Section 2}$.
- $\bar{q}_3 = \text{mean velocity head at Section 3}$.
- $s_3 = \text{static head at Section 3}$.
- $HL_B = \text{head loss caused by the bend based on } \bar{q}_2$.

Since $A_1 \bar{v}_1 = A_2 \bar{v}_2 \quad (7.4)$

the value of $\bar{q}_2$ is given by:

$$\bar{q}_2 = \left(\frac{A_1}{A_2}\right)^2 \bar{q}_1 \quad (7.5)$$

The value of $HL_B$ is given by the interpolated loss coefficient, $K_{ca}$ for the constant area bend from Fig. 6.10 multiplied by $\bar{q}_2$

$$HL_B = K_{ca} \bar{q}_2 \quad (7.6)$$

By substituting equation 7.6 in equation 7.3 and noting that $\bar{q}_2 = \bar{q}_3$ the value of $s_3$ is obtained:

$$s_3 = s_2 - K \bar{q}_2 \quad (7.7)$$
c) Section 3 to Section 4 (Contraction).

A similar relationship as equation 7.3 can be written assuming there is negligible head loss across the contraction:

\[ s_3 + \tilde{q}_1 = s_4 + \tilde{q}_4 \]  

(7.8)

Since \( \tilde{q}_4 = \tilde{q}_1 \) the value of \( s_4 \) is given by:

\[ s_4 = s_3 + \tilde{q}_2 - \tilde{q}_1 \]  

(7.9)

Combining equation 7.1, 7.2, 7.5, 7.7 and 7.9 the required value of \( K_c \) is obtained:

\[ K_c = (1 - C_p) + \left( \frac{A_1}{A_2} \right)^2 (k_{w 1} - 1) \]  

(7.10)

The calculated values of \( K_c \) are tabulated in Fig.7.1 and compared with the experimental values of the bend loss coefficient (given in brackets under the test number in Fig.7.1) in Fig.7.3. Considering the approximations in the model representations the comparisons between the experimental and calculated loss coefficients are surprisingly good.

It is evident from the values of the parameters presented in Fig.7.1 that the value of \( K_c \) is strongly dependent on the area ratio \( A_1/A_2 \). This is demonstrated by the static head 'loss curves' for values of \( Ri/D \) of 3 and 4 shown in Fig.7.4. The lines join the points giving the change in static head occurring between successive sections. The line for test number 6.10 is shown dotted because the model representation is reversed (contraction - constant area bend - diffuser). Fig.7.4 shows that the major contribution to the overall loss of static head is that occurring across the contraction although this occurs with negligible total head loss. This
is because the velocity head at the inlet to the contraction is proportional to the square of the area ratio \( A_1/A_2 \) as shown by equation 7.5. Therefore, as the area ratio decreases or increases from the optimum value the static head loss across the contraction progressively increases as shown in Fig.7.4.

7.2. MODEL EXTENSION

7.2.1. Rectangular Cross-Section Bends

The model calculations presented above may be extended to higher values of the inner radius ratio, Ri/D. These calculations are shown in Table 7.1 for Ri/D values of 5 and 6. The approximate optimum outer radius ratio are calculated from equation 6.5 and indicated by the asterisks in Table 7.1. Values of the bend loss coefficient for the constant area bends and the diffuser static head recovery coefficients are interpolated, as before, from Fig.6.10 and Fig.7.2 respectively. The model loss coefficient \( K_c \) is calculated from equation 7.10.

<table>
<thead>
<tr>
<th>Ri/D</th>
<th>Ro/D</th>
<th>( A_1/A_2 )</th>
<th>( \phi )</th>
<th>N/D</th>
<th>( C_p )</th>
<th>( Rm/D_2 )</th>
<th>( K_c )</th>
<th>( K_{CA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 4</td>
<td>0.572</td>
<td>17°</td>
<td>1.250</td>
<td>0.31</td>
<td>2.43</td>
<td>0.240</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>5 5</td>
<td>0.780</td>
<td>15°</td>
<td>0.625</td>
<td>0.23</td>
<td>3.61</td>
<td>0.225</td>
<td>0.30</td>
<td></td>
</tr>
<tr>
<td>5 5.25</td>
<td>0.840</td>
<td>11°</td>
<td>0.500</td>
<td>0.20</td>
<td>5.00</td>
<td>0.240</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>5 6</td>
<td>1.430</td>
<td>12°</td>
<td>0.750</td>
<td>0.21</td>
<td>5.00</td>
<td>0.240</td>
<td>0.265</td>
<td></td>
</tr>
<tr>
<td>6 5</td>
<td>0.625</td>
<td>15°</td>
<td>1.150</td>
<td>0.32</td>
<td>5.35</td>
<td>0.255</td>
<td>0.39</td>
<td></td>
</tr>
<tr>
<td>6 6.1*</td>
<td>0.800</td>
<td>13°</td>
<td>0.650</td>
<td>0.22</td>
<td>5.85</td>
<td>0.275</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>6 7</td>
<td>1.43</td>
<td>13°</td>
<td>0.65</td>
<td>0.20</td>
<td>7.0</td>
<td>0.325</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 7.1 — Model Calculations for Rectangular Cross-Section Bends for Ri/D Values of 5 and 6**
FIG. 7.5 - Variation of Model Bend Loss Coefficient from Extension of Variable Area Rectangular Cross-Section 90° Bends
FIG. 7.7 - Circular Cross-Section Constant Area Bend Performance Chart (Rn = 10^5)
<table>
<thead>
<tr>
<th>$R_1/D$</th>
<th>$R_0/D$</th>
<th>$D_1/D_2$</th>
<th>$\phi^0$</th>
<th>$N/r_1$</th>
<th>$C_p$</th>
<th>$R_m/D_2$</th>
<th>$K_{ea}$</th>
<th>$K_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.740</td>
<td>26</td>
<td>0.75</td>
<td>0.55</td>
<td>0.815</td>
<td>0.35</td>
<td>0.28</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1.333</td>
<td>11</td>
<td>1.25</td>
<td>0.55</td>
<td>2.500</td>
<td>0.21</td>
<td>2.03</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.588</td>
<td>22</td>
<td>1.75</td>
<td>0.50</td>
<td>0.955</td>
<td>0.30</td>
<td>0.42</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.715</td>
<td>15</td>
<td>1.50</td>
<td>0.57</td>
<td>1.250</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
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<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1.333</td>
<td>7</td>
<td>2.25</td>
<td>0.62</td>
<td>3.390</td>
<td>0.22</td>
<td>2.06</td>
</tr>
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<td>3</td>
<td>1</td>
<td>0.500</td>
<td>22</td>
<td>2.50</td>
<td>0.43</td>
<td>1.125</td>
<td>0.27</td>
<td>0.53</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.654</td>
<td>10</td>
<td>2.00</td>
<td>0.60</td>
<td>1.780</td>
<td>0.22</td>
<td>0.26</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.800</td>
<td>11</td>
<td>1.25</td>
<td>0.49</td>
<td>2.300</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>3.5</td>
<td>0.870</td>
<td>18</td>
<td>0.50</td>
<td>0.36</td>
<td>2.850</td>
<td>0.21</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>1.360</td>
<td>16</td>
<td>1.00</td>
<td>0.55</td>
<td>5.330</td>
<td>0.24</td>
<td>2.10</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.400</td>
<td>22</td>
<td>3.50</td>
<td>0.35</td>
<td>1.10</td>
<td>0.28</td>
<td>0.63</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>0.500</td>
<td>16</td>
<td>3.50</td>
<td>0.47</td>
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<td>0.24</td>
<td>0.48</td>
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<td>3</td>
<td>0.570</td>
<td>15</td>
<td>3.00</td>
<td>0.60</td>
<td>1.715</td>
<td>0.22</td>
<td>0.32</td>
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<td>4</td>
<td>4</td>
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<td>8</td>
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<td>0.53</td>
<td>2.900</td>
<td>0.21</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>4.35</td>
<td>0.850</td>
<td>8</td>
<td>1.75</td>
<td>0.45</td>
<td>2.900</td>
<td>0.21</td>
<td>0.14</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1.380</td>
<td>14</td>
<td>1.10</td>
<td>0.55</td>
<td>5.800</td>
<td>0.26</td>
<td>2.23</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>0.570</td>
<td>17</td>
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<td>0.55</td>
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</tr>
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<td>5.000</td>
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<td>0.19</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>7</td>
<td>1.430</td>
<td>12</td>
<td>1.25</td>
<td>0.56</td>
<td>5.600</td>
<td>0.24</td>
<td>2.74</td>
</tr>
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<td>6</td>
<td>5</td>
<td>0.625</td>
<td>15</td>
<td>2.30</td>
<td>0.62</td>
<td>5.350</td>
<td>0.25</td>
<td>0.27</td>
</tr>
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<td>6</td>
<td>0.800</td>
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<td></td>
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<tr>
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<td>1.30</td>
<td>0.57</td>
<td>7.000</td>
<td>0.29</td>
<td>2.53</td>
</tr>
</tbody>
</table>

Table 7.2 - Model Calculations for Variable Area 90° Bends of Circular Cross-Section.
FIG. 7.8 - Variation of Model Bend Loss Coefficient for Variable Area Circular Cross Section 90° Bends
These calculated model loss coefficients, $K_c$, are plotted in Fig. 7.5 and a dotted curve for each value of $R_i/D$ is drawn to represent the same shape as the experimental curves presented in Fig. 7.3. It appears that the minimum value of the loss coefficient is increasing with increasing values of the bend inner radius ratio $R_i/D$. This is, of course, due to the extra friction loss caused by the increase in the effective centreline length of the bend.

7.2.2. Circular Cross-Section Bends

The values of the model bend loss coefficients for variable area 90° bends of circular cross-section are calculated from equation 7.10 assuming the same diffuser, constant area bend and contraction configurations, only two-dimensional, as those shown in Fig. 7.1 and Table 7.1. Values of the diffuser static head recovery coefficient $C_p$ are interpolated from Fig. 7.6 based on the work of Miller\(^{59}\) and the values of the bend loss coefficient, $K_{ca}$ for constant area 90° bends of circular cross-section interpolated from Fig. 7.7, also based on the work of Miller\(^{12}\).

The values of the various diffuser and bend parameters are tabulated in Table 7.2 together with the calculated values of the model combination loss coefficient, $K_c$. These values of $K_c$ are plotted in Fig. 7.8 as a family of curves of given $R_i/D$ values against $R_o/D$. The curves show the same trends as the experimental curves shown in Fig. 7.3 for the rectangular cross-section bends. Compared with the rectangular cross-section bend results it appears that for the circular cross-sectional bend a larger reduction in the value of the bend loss coefficient (compared with that for a constant area bend) is achieved by increasing the bend
mid-plane cross-sectional area, two-dimensionally, by an amount dependent on the inner radius ratio \( R_i/D \) of the bend. The optimum outer radius ratio, \( R_o/D \), of the bend is given by equation 6.5 for a given value of \( R_i/D \).

Also it is evident from Fig. 7.8 that any reduction in the bend mid-plane cross-sectional area produces a considerable increase in the value of \( K_c \). This is likely to occur when a circular cross-section bend is fabricated from a straight length of pipe and possibly accounts for some of the scatter in the correlations of the early experimental results as presented by Gray.

7.3. DISCUSSION

Bearing in mind the approximate nature of the above model approach, the calculated values of loss coefficient for circular cross-section bend indicate that a further reduction (below that presented in Fig. 6.17) in the value of the bend loss coefficient for the rectangular cross-section bends may be achieved by increasing the mid-plane cross-sectional area two-dimensionally as in the model circular cross-section bends. For a given value of the bend inner radius ratio, \( R_i/D \), a possible criterion might be to keep the aspect ratio constant throughout the bend whilst maintaining the optimum outer radius ratio, \( R_o/D \), given by equation 6.5.
CHAPTER 8

SUGGESTIONS FOR FURTHER RESEARCH
The experimental results of Sprenger's show that the effect of Reynolds number on the value of the bend loss coefficient for rectangular cross-section bends is sometimes quite large, especially in cases where the bend inner radius is not sharp but has a small radius. He shows that at a certain value of Reynolds number (dependent on the bend geometry) there is a peculiar decrease in the loss coefficient which is followed by a sudden increase. In order to determine the reasons for this seemingly erratic behaviour of some bend geometries, further work is required in which the flow patterns upstream, in the bend and downstream are investigated for the range of Reynolds number. Improved methods of flow visualisation will help reduce the amount of velocity traverse, both mean and turbulent, that would be required.

The model bend calculations presented in this thesis suggest that a "low loss" bend is obtained by varying the bend cross-sectional area two-dimensionally. This would be easier to investigate experimentally for a rectangular cross-section bend since economic and production considerations would have to be borne in mind when constructing a variable area circular cross-section bend. The optimum outer radius for a rectangular cross-section (aspect ratio $\frac{h}{w} = 2$) bend of given inner radius is formulated in this thesis therefore a possible criterion for increasing the cross-sectional area two-dimensionally might be to keep the aspect ratio constant throughout the bend whilst maintaining the above optimum radius geometry.

Bearing in mind the difficulties in the manufacture of a variable area bend of circular cross-section a reduction in the loss coefficient may be achieved by constructing a $90^\circ$ constant circular cross-sectional area bend from two $45^\circ$
FIG.8.1 - Circular Cross-Section 90° Bend
formed by Combination of Two 45° Constant
Radius Bends of Different Radius
bends of different by constant radius ratios as shown in Fig.8.1. By placing the 'tight' \(45^\circ\) bend at the inlet the secondary flow in the outlet \(45^\circ\) bend and hence the downstream tangent may be reduced. Care will be necessary to define the 'low loss' since in the type of combination shown in Fig.8.1 an extra length of straight duct would be required to connect the combination bend to the upstream tangent assuming a constant area - constant radius bend is the basis for comparison.

Due to the possible economic savings the systematic experimental determination of the head losses caused by duct elements when interference effects caused by other components upstream and/or downstream are present or not, will generate useful data. In the long term, however, we can never determine the head losses for all combinations of duct elements and associated parameters. Our goal must be to predict quantitatively the flow in an unfamiliar configuration from existing data in other situations. In order to achieve this, extensive work is required to relate the head loss in a given duct element to parameters which define the inlet and outlet conditions. These conditions require defining in terms of all the variables, in particular parameters which define the three-dimensional nature of the turbulent flow. At the present time this is difficult to describe in a wholly satisfactory manner, especially if separation occurs. Therefore, extensive research on improved methods of flow visualisation at a representative Reynolds number would provide very useful information on the nature of the flow.
Although the mathematical analysis of the three-dimensional flow in a bend is theoretically capable of solution, it is more complex by several orders than the types of problems at present being successfully solved. Improved methods of analysis, combined with an increase in the size of computers, are required before a solution is obtained at a representative Reynolds number.
CHAPTER 9

CONCLUSIONS
1. A rational means of estimating the uncertainty in the values of the bend loss coefficient is presented. It is shown that the overriding factor is the accurate determination of the value of the mean velocity based on the differential static head across the inlet nozzle. A maximum value of $1.284\%$ was obtained for the uncertainty in the experimental values of the bend loss coefficient.

2. The three-dimensional flow in a bend is so complex that a complete rational theory is unattainable. There is, therefore, no present means of simplifying the problem which does not lose either the three-dimensional nature of the flow or the viscous terms from the Navier-Stokes equations.

3. The aspect ratio correction factor applied as one correction to a basic loss coefficient is more complex than the constant value previously assumed. It is a function of both the radius to duct width ratio of the bend and, less significantly, the Reynolds number of the flow.

4. Using the above correction for the aspect ratio effect a good correlation is obtained between the up-to-date published data and the present experimental results for constant area $90^\circ$ bends of aspect ratio 2 at a Reynolds number of $1.8 \times 10^5$.

5. Measurements of wall static head upstream and downstream of the bend indicate that the presence of the bend has a marked effect for duct lengths of 10 and 45 hydraulic diameters upstream and downstream respectively. This upstream length is approximately three times that previously suggested.
6. The importance of using an interpolated value for the duct friction factor, from the calculated value of the Reynolds number, in the determination of the bend loss coefficient is presented. A maximum random error of 10% was introduced into the value of the bend loss coefficient when calculated without the above value of this duct friction factor.

7. A reduction as high as 10% in the value of the bend loss coefficient, compared with that obtained for a constant area bend, is obtained by increasing the mid-plane cross-sectional area of the bend by an amount dependent on the inner radius to duct width ratio of the bend. For a rectangular cross-section bend of aspect ratio 2 the optimum outer radius, $R_o/D$, for a given inner radius ratio $R_i/D$ is given by:

$$R_o/D_{(optimum)} = 0.87 R_i/D + 0.9$$

8. Any increase in the bend mid-plane cross-sectional area above the optimum value or a reduction below that of a constant area bend results in a considerable increase in the value of the bend loss coefficient over the range of bend inner radius to duct width ratios examined.

9. A simple model of the rectangular cross-section variable area bends, formed by a combination of a diffuser, constant area bend and a contraction, represents the flow behaviour sufficiently accurately. Based on the extension of the model calculations the optimum geometrical shapes and reductions in head loss for circular cross-section 90° bends are developed.
APPENDIX 1

THE GENERAL EQUATIONS OF MOTION AND CONTINUITY
FIG.A.1.1 - Cylindrical Co-ordinates $r, \theta, z$
FIG.A.1.2 - Flow through an Infinitesimal Volume Formed by Three Pairs of Adjacent Co-ordinate Surfaces.
A.1.1. EQUATION OF CONTINUITY

For the flow of a fluid round a rectangular section bend the equation of continuity can be obtained by considering an infinitesimal volume formed by three pairs of adjacent co-ordinate surfaces. The cylindrical co-ordinates shown in Fig.A.1.1 form such a volume. The flow of fluid through the sides of the volume ABCDEFGH is shown in Fig A.1.2 and are as follows

Flow through the boundary AEHD = \( -\rho v_r \, \delta \theta \, \delta z \)

Flow through the boundary BCGF = \( \left[ \rho v_r + \frac{\partial (\rho v_r)}{\partial r} \right] \, \delta \theta \, \delta z \)

Flow through the boundary ABFE = \( -\rho v_\theta \, \delta r \, \delta z \)

Flow through the boundary DCGH = \( \left[ \rho v_\theta + \frac{\partial (\rho v_\theta)}{\partial \theta} \right] \, \delta r \, \delta z \)

Flow through the boundary ABCD = \( -\rho v_z \, \delta \theta \, \delta r \)

Flow through the boundary EFGH = \( \left[ \rho v_z + \frac{\partial (\rho v_z)}{\partial z} \right] \, \delta \theta \, \delta r \)

Therefore the total flow through the entire surface is given by

\[
\left[ \frac{\partial (\rho v_r)}{\partial r} + \frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} \right] \, \delta r \, \delta \theta \, \delta z \quad (A.1.1)
\]

The decrease of the mass of fluid within the volume is given by

\[-\frac{\partial \rho}{\partial t} \, \delta r \, \delta \theta \, \delta z \quad (A.1.2)\]

Equating A.1.1 and A.1.2 we obtain the equation of continuity

\[
\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (\rho v_r)}{\partial r} + \frac{1}{\theta} \frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0 \quad (A.1.3)
\]
FIG.A.1.3 - Projections of the Velocity at a Point on the Axes of the Cylindrical Co-ordinates
For incompressible steady flow equation A.1.3 reduces to
\[ \frac{1}{\nu} \frac{\partial (\nu \cdot \nu_x^2)}{\partial t} + \frac{1}{\nu} \frac{\partial \nu_\theta}{\partial \theta} + \frac{\partial \nu_z}{\partial z} = 0 \tag{A.1.4} \]

A.1.2. THE GENERAL EQUATIONS OF MOTION

From kinematics, the projections of the velocity of the point M, Fig.A.1.3 on the axes of the cylindrical co-ordinates \( r, \theta, z \) are
\[ \nu_r = \dot{r}, \quad \nu_\theta = r \dot{\theta}, \quad \nu_z = \dot{z} \tag{A.1.5} \]
Introducing along the axes of the cylindrical co-ordinates the three unit vectors \( \hat{i}_r, \hat{i}_\theta \) and \( \hat{i}_z \) we can represent the velocity vector of the point M in the form
\[ \nu = \dot{r} \hat{i}_r + r \dot{\theta} \hat{i}_\theta + \dot{z} \hat{i}_z \tag{A.1.6} \]
and differentiating with respect to time, we obtain for the acceleration
\[ \omega = \dot{\dot{r}} \hat{i}_r + \dot{r} \dot{\theta} \hat{i}_\theta + \ddot{z} \hat{i}_z \tag{A.1.7} \]
however,
\[ \frac{d \hat{i}_r}{dt} = \dot{\theta} \hat{i}_\theta, \quad \frac{d \hat{i}_\theta}{dt} = -\dot{\theta} \hat{i}_r \text{ and } \frac{d \hat{i}_z}{dt} = 0 \tag{A.1.8} \]
therefore
\[ \omega = \ddot{r} \hat{i}_r + \dot{r} \dot{\theta} \hat{i}_\theta + \ddot{z} \hat{i}_z \tag{A.1.9} \]
Substitution equation A.1.5 in equation A.1.9 we obtain
\[ \omega = \left[ \nu_r - \frac{\nu_\theta^2}{\nu} \right] \dot{r} + \frac{1}{\nu} \frac{d (\nu \nu_\theta)}{dt} \hat{i}_2 + \nu_z \hat{i}_3 \tag{A.1.10} \]
whence we can conclude that
\[ \omega_r = \nu_r - \frac{\nu_\theta^2}{\nu}, \quad \omega_\theta = \frac{1}{\nu} \frac{d (\nu \nu_\theta)}{dt}, \quad \omega_z = \dot{\nu}_z \tag{A.1.11} \]
Further, since by a basic property of the gradient
\[ (\nabla \cdot \nu)_s = \frac{\partial \nu}{\partial s} \]
where $S$ is an arbitrary direction, and since the elements of the co-ordinate lines for cylindrical co-ordinates will be $\partial r$, $\partial \theta$ and $\partial z$ respectively, we have

\[
\begin{align*}
(q\cdot \nabla \ p)_r &= \frac{\partial p}{\partial r} \\
(q\cdot \nabla \ p)_{\theta} &= \frac{1}{\ell} \frac{\partial p}{\partial \theta} \quad (A.1.12) \\
(q\cdot \nabla \ p)_z &= \frac{\partial p}{\partial z}
\end{align*}
\]

If we now project on the axes of the cylindrical co-ordinates the equation of motion

\[
\omega = F - \frac{1}{\ell} q\cdot \nabla \ p \quad (A.1.13)
\]

where $F$ is the body force we obtain for the three scalar equations of motion

\[
\begin{align*}
\dot{v}_r - \frac{v_\theta^2}{r} &= F_r - \frac{1}{\ell} \frac{\partial p}{\partial r} \\
\frac{d(\gamma v_\theta)}{dt} &= \gamma F_\theta - \frac{1}{\ell} \frac{\partial p}{\partial \theta} \quad (A.1.14) \\
\dot{v}_z &= F_z - \frac{1}{\ell} \frac{\partial p}{\partial z}
\end{align*}
\]

The total derivations can be represented by the following equations A.1.15, A.1.16 and A.1.17.

\[
\begin{align*}
\dot{v}_r &= \frac{\partial v_r}{\partial r} \dot{r} + \frac{\partial v_r}{\partial \theta} \dot{\theta} + \frac{\partial v_r}{\partial z} \dot{z} + \frac{\partial v_r}{\partial t} \\
\dot{v}_r &= \frac{\partial v_r}{\partial r} v_r + \frac{\partial v_r}{\partial \theta} v_\theta + \frac{\partial v_r}{\partial z} v_z + \frac{\partial v_r}{\partial t} \quad (A.1.15) \\
\frac{d(\gamma v_\theta)}{dt} &= \frac{\partial (\gamma v_\theta)}{\partial r} \dot{r} + \frac{\partial (\gamma v_\theta)}{\partial \theta} \dot{\theta} + \frac{\partial (\gamma v_\theta)}{\partial z} \dot{z} + \frac{\partial (\gamma v_\theta)}{\partial t}
\end{align*}
\]
Substituting equations A.1.15, A.1.16 and A.1.17 into equation A.1.14 we obtain the following differential equations of motion.

\[
\frac{d}{dt}(r\dot{v}_\theta) = v_r\dot{r} + \frac{\partial v_r}{\partial r} + \frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial v_r}{\partial z} \quad \text{(A.1.16)}
\]

\[
\frac{d}{dt}(z\dot{v}_r) = v_r\dot{z} + v_\theta \frac{\partial v_r}{\partial \theta} + v_z \frac{\partial v_r}{\partial z} + \frac{\partial v_r}{\partial t}
\]

\[
\dot{v}_z = \frac{\partial v_z}{\partial t} + v_\theta \frac{\partial v_z}{\partial \theta} + v_r \frac{\partial v_z}{\partial r} + \frac{\partial v_z}{\partial z}
\]

\[
\dot{v}_z = v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t}
\]

These are the Euler equations of motion for a non-viscous fluid. Each term had the dimension of force per unit mass, or acceleration, and the total acceleration in a given direction is seen to be equal to the sum of the gravitational component and the component due to the existence of a pressure gradient in that direction.

If the viscous terms are included in equations A.1.18 we obtain the following Navier-Stokes equations in the cylindrical co-ordinates \(r\) (radial), \(\theta\) (azimuthal) and \(z\) (axial).

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} + v_z \frac{\partial v_r}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial r} +
\]

\[
\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + v_z \frac{\partial v_\theta}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial \theta} +
\]

\[
\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} = \frac{1}{\rho} \frac{\partial p}{\partial z} +
\]
\[
\mu \left[ \frac{\partial^2 \nu_0}{\partial t^2} + \frac{1}{r} \frac{\partial \nu_0}{\partial r} - \frac{\nu_0}{r^2} + \frac{1}{r^2} \frac{\partial^2 \nu_0}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial \nu_r}{\partial \theta} + \frac{\partial^2 \nu_\theta}{\partial z^2} \right]
\]

\[
\frac{\partial \nu_z}{\partial t} + \nu_r \frac{\partial \nu_z}{\partial r} + \nu_\theta \frac{\partial \nu_z}{\partial \theta} + \nu_z \frac{\partial \nu_z}{\partial z} = F_z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\partial^2 \nu_z}{\partial z^2}
\]

(A.1.19)

and also the continuity equation

\[
\frac{\partial \nu_r}{\partial t} + \nu_r \frac{\partial \nu_r}{\partial r} + \frac{1}{r} \frac{\partial \nu_\theta}{\partial \theta} + \frac{\partial \nu_z}{\partial z} = 0
\]

Let us now consider the velocity in a turbulent flow as consisting of a time averaged portion and a fluctuating part. For example, the radial component of velocity is given by

\[
\nu_r = \overline{\nu_r} + \nu_r'
\]

(A.1.20)

where the averaged value \(\overline{\nu_r}\) is defined by

\[
\overline{\nu_r} = \frac{1}{T} \int_{t}^{t+T} \nu_r \, dt
\]

(A.1.21)

It also follows, by definition that

\[
\overline{\nu_r'} = \frac{1}{T} \int_{t}^{t+T} \nu_r' \, dt = 0
\]

(A.1.32)

The time \(T\) represents the interval over which the averaging is to be carried out. The exact value of \(T\) is difficult to prescribe, but will have to be long compared with any of the frequencies of the fluctuations. For sufficiently large values of \(T\) the variation of \(\overline{\nu_r}\) with \(T\) is negligible, consequently \(\overline{\nu_r}\) is essentially independent of \(T\) provided \(T\) has been selected as specified.
Consider the Navier-Stokes equation for incompressible fluid flow in the absence of body forces. Only one component is discussed here as the steps are analogous for the remaining components.

\[
\frac{\partial \bar{v}_r}{\partial t} + \bar{v}_r \frac{\partial \bar{v}_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial \bar{v}_r}{\partial \theta} - \frac{v_\phi}{r} \frac{\partial \bar{v}_r}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial r} + \left(\frac{\mu}{\rho} \left[ \frac{\partial^2 \bar{v}_r}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_r}{\partial r} - \frac{v_r}{r^2} \frac{\partial^2 \bar{v}_r}{\partial \theta^2} - \frac{2}{\gamma^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 v_r}{\partial z^2} \right] \right) \tag{A.1.23}
\]

Substituting in equation A.1.23 for the velocities and pressure in terms of the time averaged and the fluctuating components the following equation is obtained

\[
\frac{\partial \bar{v}_r}{\partial t} + \left[ \bar{v}_r + v'_r \right] \frac{\partial (\bar{v}_r + v'_r)}{\partial r} + \frac{(\bar{v}_\theta + v'_\theta)}{r} \frac{\partial (\bar{v}_r + v'_r)}{\partial \theta} = - \frac{1}{\rho} \frac{\partial (\bar{p} + p')}{\partial r} + \left( \frac{\mu}{\rho} \left[ \frac{\partial^2 (\bar{v}_r + v'_r)}{\partial r^2} + \frac{1}{r} \frac{\partial (\bar{v}_r + v'_r)}{\partial r} - \frac{(\bar{v}_r + v'_r)}{r^2} \right] \right) \tag{A.1.24}
\]

In a further operation each term in equation A.1.24 is averaged over a time interval \(T\). It is important to note that for the type of averaging process under consideration the average of the derivative of a function is equal to the derivation of the average.

\[
\frac{\partial \bar{v}_r}{\partial t} + \bar{v}_r \frac{\partial \bar{v}_r}{\partial r} + \bar{v}'_r \frac{\partial \bar{v}_r}{\partial r} + \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_r}{\partial \theta} + \frac{\bar{v}_\phi}{r} \frac{\partial \bar{v}_r}{\partial z} - \frac{\bar{v}_r^2}{r} + \frac{v_\theta^2}{r^2} \frac{\partial v_\theta}{\partial \theta} = - \frac{1}{\rho} \frac{\partial \bar{p}}{\partial r} + \left( \frac{\mu}{\rho} \left[ \frac{\partial^2 \bar{v}_r}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{v}_r}{\partial r} \right] \right) + \left( \frac{\mu}{\rho} \left[ - \frac{\partial^2 \bar{v}_r}{\partial \theta^2} - \frac{2}{\gamma^2} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial^2 \bar{v}_r}{\partial z^2} \right] \right) \tag{A.1.25}
\]
Rearranging equation A.1.25 gives

\[
\frac{\partial \bar{v}_x}{\partial t} + \bar{v}_x \frac{\partial \bar{v}_x}{\partial x} + \bar{v}_y \frac{\partial \bar{v}_x}{\partial y} + \bar{v}_z \frac{\partial \bar{v}_x}{\partial z} - \frac{\bar{v}_x^2}{\bar{v}} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \\
\frac{1}{\rho} \frac{\partial}{\partial z} \left[ \mu \frac{\partial \bar{v}_x}{\partial z} - \rho \bar{v}_z \bar{v}_y \right] + \frac{\partial}{\partial z} \left[ \mu \frac{\partial \bar{v}_x}{\partial z} - \rho \bar{v}_y^{2} \right] + \\
\frac{\partial}{\partial \theta} \left[ \mu \frac{\partial \bar{v}_x}{\partial \theta} - \rho \bar{v}_y \bar{v}_\theta \right] - \left[ \mu \bar{v}_x^{2} - \bar{v}_\theta^{2} \right] - \frac{\mu^2}{\rho \gamma^2} \frac{\partial \bar{v}_\theta}{\partial \theta}
\]  
(A.1.26)

Comparing equation A.1.26 to the Navier-Stokes in its original form, equation A.1.19, great similarities are observed. In fact, except for the terms \( \rho \bar{v}_z \bar{v}_y \), \( \rho \bar{v}_y^{2} \), \( \rho \bar{v}_y \bar{v}_\theta \) and \( \bar{v}_\theta^{2} \) in equation A.1.26, it is seen that the average velocity components and the average pressure satisfy an equation identical to the Navier-Stokes equation. In the case of laminar flow the fluctuating components are all equal to zero and the Navier-Stokes equation is exactly satisfied by the average velocities and pressure. This fact is, of course, not surprising because in laminar flow there is no difference between the average and the instantaneous components. In turbulent flow, however, the equation for the average quantities contain extra terms such as

\( \rho \bar{v}_z \bar{v}_y \), \( \rho \bar{v}_y^{2} \), \( \rho \bar{v}_y \bar{v}_\theta \), \( \bar{v}_\theta^{2} \) and in the way in which the equation A.1.26 has been arranged these terms may be regarded as additional stresses. The additional terms are called turbulent or Reynolds stresses.
APPENDIX 2

COMPUTER PROGRAMS
Procedure Data

Calculates the values of the mean wall static head \( \bar{\sigma}(J)a \) from any number of readings of the individual static head \( \sigma(J)a \) at a given location \( J \) along the duct face \( a \).

Procedure Line

Calculates the slope and constant of the least mean squares straight line through the \( N \) points, the co-ordinates of which are contained in the elements of the arrays \( X \) and \( Y \).

Procedure Qindata and Qinter

Procedure Qindata reads the co-ordinates of the data forming (in this application) the experimental duct friction factor - Reynolds number relationship. Procedure Qinter is an interpolation procedure which calculates, from the above data, the value of the duct friction factor for a given value of Reynolds number.

Procedures Newplot, CCplot, Pen To, Incr, Number, Curve and Endplot

These procedures are used to form the graphical representations of the variations of \( F(J)a \) (mean static head at a given tapping location minus a reference static head divided by the mean inlet velocity head) and \( H(J) \) (average of four values of \( F(J)a \)) against \( L/Dh(J) \) for the duct friction factor - Reynolds number and bend loss coefficients tests respectively.

The calculations for these procedures are compiled and stored on magnetic tape and only the procedure declarations are required in the main heading of each program.
A.2.1, Computer Program H15F

Duct Friction Factor — Reynolds Number Relationship
FIG.A.2.1 - Flow Chart for Computer Program H15E - Friction Factor - Reynolds Number Relationship
'PROGRAM'(H15E)
'INPUT'0=TR0
'OUTPUT'0=LPO
'OUTPUT'1=TP0
'CONTINUE'

'BEGIN'
'INTEGER'J,K,Z;
'REAL'DENS,NOZD,PR,AEF,V,RR,SLOPE,CONST,DR,VR,DS,WS,WN,
WPR,WR,WVIS,WD,WH,WSLOPE,XX1,XX2,YY1,YY2,WRN;
'ARRAY'R,WAEF,V,FH,DRN,KN,DR,VV,DSV[1:10],INSIDE,BOTTOM,OUTSIDE,
TOP,BO,0,T,H,DL,Y,L,MR,WO,WT,WH,DSLOPE[1:50];
'PROCEDURE'DATA(M,A,B,K);
'INTEGER'A,B,K;
'ARRAY'M;
'BEGIN'
'INTEGER'H,N,J;
'ARRAY'Q[1:50];
'BOOLEAN'SCAN,TESTDATA;
'REAL'D;
'FOR'J:=1'STEP'1'UNTIL'50'DO'M[J]:=0;
'SCAN'='TRUE';
'H:=0;
'FOR'H:=H+1'WHILE'SCAN'DO'
'BEGIN'
D:=READ;
'IF'ENTIER(ABS(D)*10^(-4))=A'THEN'SCAN'='FALSE''ELSE'
'BEGIN'
TESTDATA='TRUE';
N:=H;J:=0;
'FOR'J:=J+1'WHILE'TESTDATA'DO'
'BEGIN'
'IF'ENTIER(ABS(D)*10^(-4))=B'THEN'TESTDATA='FALSE''ELSE'
'BEGIN'
M[J]:=M[J]+D;
D:=READ;K:=J
'END'
'END'
'END';
'FOR J := 1 STEP 1 UNTIL K DO'
'BEGIN'
M[J] := M[J]/N;
'IF M[J] < 0 THEN Q[J] := ENTIER(M[J]*10^(-4)) + 1 ELSE'
Q[J] := ENTIER(M[J]*10^(-4));
M[J] := 100*(M[J]*10^(-4) - Q[J])
'END'
'END' DATA;
'PROCEDURE LINE(N,X,Y,SLOPE,CONST);
'VALUE' N; 'INTEGER' N; 'REAL'SLOPE,CONST; 'ARRAY' X,Y;
'BEGIN'
'INTEGER' J; 'REAL' X1,X2,Y1,Y2;
X1 := X2 := Y1 := Y2 := 0;
'FOR J := 1 STEP 1 UNTIL N DO'
'BEGIN'
X1 := X1 + X[J]; X2 := X2 + X[J]*2;
Y1 := Y1 + Y[J]; Y2 := Y2 + Y[J]*Y[J]
'END';
SLOPE := (Y1*X1*N*Y2)/(X1*2-N*X2); CONST := (Y1-SLOPE*X1)/N
'END' LINE;
'PROCEDURE POM;
WRITE TEXT('("%OR% =")');
'PROCEDURE ODDS;
WRITE TEXT('("%20%TO%1")');
'PROCEDURE PCU;
WRITE TEXT('("%PERCENT%UNCERTAINTY")');
'PROCEDURE NEWPLOT(A,B,C,D,E,F,G,H,I);
'VALUE' A,B,C,D,E,F,G,H,I; 'INTEGER' A,B,I; 'REAL'C,D,E,F,G,H;
'ALGOL';
'PROCEDURE CCPLT(N,X,Y,TYPE,MES);
110 VALUE N, TYPE, MODE; INTEGER N, TYPE, MODE; ARRAY X, Y;
111 ALGOL;
112 PROCEDURE PEN TO(X, Y, MODE);
113 VALUE X, Y, MODE; REAL X, Y; INTEGER MODE;
114 ALGOL;
115 PROCEDURE INCR(X, Y, MODE);
116 VALUE X, Y, MODE;
117 INTEGER X, Y, MODE;
118 ALGOL;
119 PROCEDURE NUMBER(X, M, N, STYLE, SIZE);
120 VALUE X, M, N, STYLE, SIZE; INTEGER M, N, STYLE; REAL X, SIZE;
121 ALGOL;
122 PROCEDURE ENDPLOT;
123 ALGOL;
124 Z := 0;
125 L1: WRITE TEXT("%"%"PC"%"C")BHRA/APB-AB6""%"3S")%"06/3478
126 ""%"%"C")%"FRICTION FACTOR=REYNOLDS NUMBER""%"2C")%"%")%"");
127 FOR J := 1 STEP 1 UNTIL 3 DO R[J] := READ;
128 R[1] := ENTER(R[1], 1); R[2] := ENTER(R[2], 1)*1;
130 DATA(INSIDE, 29, 28, K);
131 DATA(BOTTOM, 29, 28, K);
132 DATA(OUTSIDE, 29, 28, K);
133 DATA(TOP, 29, 28, K);
134 WRITE TEXT("%"%"TEST NO."%")% PRINT(R[1], 7, 0);
135 WRITE TEXT("%"%"C")%"AIR TEMPERATURE")% PRINT(R[2], 10, 1);
136 POM: WRITE TEXT("%"%"S")%"0.25 DEG. %C"%"2S")%"%")%""); ODDS;
137 PRINT(25/R[2], 3, 3); POM;
138 WRITE TEXT("%"%"C")%"BAROMETRIC PRESSURE")%;
139 PRINT(R[3], 6, 2);
140 POM: WRITE TEXT("%"%"S")%"0.01 INCH. HG."%"2S")%"%")%""); ODDS;
141 PRINT(1/R[3], 3, 3); POM;

DENS:= 0.0766*[R[3]*288.7/(30*273+R[2])];

WD:= DENS*SORT((0.01/R[3])+(0.01/R[3])+(0.25/(273+R[2]))

(0.25/(273+R[2]));

NOzd:= (R[5]-R[4])*(6.4-DENS)/62.317;

WN:= (0.012/ABS(R[5]-R[4])){2;

WN:= NOzd*SORT((2*WN)+(WD/62.4-DENS))+2;

PR:= (R[3]-ABS(NOzd)/13.6)/R[3];

WPR:= (6.4-0.0766*288.7*R[3]/(30*(273+R[2]))/(13.6*R[3]);

WPR:= SORT(ABS(2*(WPR*.012)+2+(.25*.0766*288.7*(R[5]-R[4])/13.6

*(273+R[2])+30)))+2+(.01*(13.6*R[3])*(13.6+(R[5]-R[4]))*

13.6*R[3]-(R[5]-R[4])*13.6+R[3])]}{2);*

AEF:= SORT(PR*.43*3.5*(1-PR*.286)/(1-PR)*(1-.009835*PR*.43));

WAEF[1]:= 1.009835*PR*.43-PR+.009835*PR*.43;

WAEF[2]:= 1.43*3.5*PR*.43-1.716*3.5*PR*.716;

WAEF[3]:= 3.5*PR*.43-3.5*PR*.716;

WAEF[4]:= 1.43*.009835*PR*.43-1.2.43*.009835*PR*.43;

WAEF[5]:= WPR*(WAEF[1]*WAEF[2]*WAEF[3]*WAEF[4])/

(WAEF[1])1/WAEF[1])2*AEF;

V:= 359.99*1.009492*576*(AEF*SORT(ABS(NOzd*DENS)))/

(360*3,1416*DENS);

KONST[1]:= 1.005*359.99*1.009492*576/(3600*3,1416*

SORT(.7353*SORT(62.317)));

R[6]:= 273+R[2];

V[5]:= SORT(ABS(R[5]-R[4]));

V[1]:= .7353*R[3]/R[6];

V[2]:= 62.4-VV[1];

V[3]:= VV[5]*VV[1]/(2*R[6]*SORT(VV[2]));

V[6]:= VV[5]+3*SORT(VV[2]);


/(13.6*R[3]))/(2*SORT(R[6])))/SORT(R[3]);

V[4]:= -.7353*VV[5]/(2*R[6]*SORT(VV[2]));
KONST[5] := 32.17*84.9/(6*KONST[1]#2);
DVH[8] := 2*KONST[5]/VV[7]*#3;
WVH := SORT((DVH[1]*.25)*#2+(DVH[2]*.01)*#2+(DVH[3]*.0122)*#2);
*FOR J := 1*STEP 1*UNTIL*K-5'DO*
'BEGIN'
I[J] := (INSIDE[J+5]-INSIDE[J]+5)*#1;
WI[J] := SQRT((VH*+.0224)*#2+(VH*-.0224)*#2+(VH*+INSIDE[J+5])
-INSIDE[J]+5)*#2);
B[J] := (BOTTOM[J+5]-BOTTOM[J]+5)*#1;
WB[J] := SQRT((VH*+.0224)*#2+(VH*-.0224)*#2+(VH*+BOTTOM[J+5])
-BOTTOM[J]+5)*#2);
O[J] := (OUTSIDE[J+5]-OUTSIDE[J]+5)*#1;
WO[J] := SQRT((VH*+.0224)*#2+(VH*-.0224)*#2+(VH*+OUTSIDE[J+5])
-OUTSIDE[J]+5)*#2);
T[J] := (TOP[J+5]-TOP[J]+5)*#1;
WT[J] := SQRT((VH*+.0224)*#2+(VH*-.0224)*#2+(VH*+TOP[J+5])
-TOP[J]+5)*#2);
L[J] := 3#J-3;
WH[J] := SQRT((WI[J]/4)*#2+(WB[J]/4)*#2+(WO[J]/4)*#2+(WT[J]/4)*#2)
*END*;
LINE(K-5,L,H,SLOPE,CONST);
YY1 := YY2 := XX1 := XX2 := WSLOPE := 0;
*FOR J := 1*STEP 1*UNTIL*K-5'DO*
'BEGIN'
XX1 := XX1+L[J];
XX2 := XX2+H[J]*H[J];
YY1 := YY1+H[J];
YY2 := YY2+L[J]*H[J]
*END*;
*FOR J := 1*STEP 1*UNTIL*K-5'DO*
187 'BEGIN'
188 DSLOPExJ:=(XX1*XX1-(K-5)*XX2)*(XX1-(K-5)*L(J))+(YY1*YY1
189 -(K-5)*YY2)*2*(K-5)*H(J))/((XX1*XX1-(K-5)*XX2)*2);
190 WSLOPE:=WSLOPE+(DSLOPExJ)*WH(J)
191 'END';
192 Y[J]:=L(J)*SLOPE+CONST;
193 DH[J]:=H[J]-Y[J]
194 'END';
195 WRITE TEXT('"3C"' 'REYNOLDS' 'NO.' '12S"') PRINT(2N,0,6);
196 POM: PRINT(WRN,0,8); ODDS;
197 PRINT(WRN*100/RN,3,3); PCU;
198 WRITE TEXT('"3C"' 'FRICITIONFACTOR' '9S"') PRINT(2N,0,6);
199 PRINT(SLOPE,0,6); POM: PRINT(WSLOPE,0,8); ODDS;
200 PRINT(WSLOPE*100/SLOPE,3,3); PCU;
201 WRITE TEXT('"3C"' 'MEANVELOCITY' '11S") PRINT(V,0,6);
202 POM: PRINT(WV,0,8); WRITE TEXT('"FT./SEC."') ODDS;
203 PRINT(WV*100/V,3,3); PCU;
204 WRITE TEXT('"3C"' 'DENSITY' '17S") PRINT(DENS,0,6);
205 POM: PRINT(WD,0,8); WRITE TEXT('"LB./CU.FT."') ODDS;
206 PRINT(WD*100/DENS,3,3); PCU;
207 WRITE TEXT('"3C"' 'NOZZLEDIFFERENCE' '7S") PRINT(WN,0,6);
208 POM: PRINT(ABS(WN),0,8);
209 WRITE TEXT('"3C"' 'INCHW.G."') ODDS;
210 PRINT(WN*100/WN,3,4); PCU;
211 WRITE TEXT('"3C"' 'PRESSURERATIO' '10S") PRINT(PR,0,6);
212 POM: PRINT(WPR,0,8); ODDS;
213 PRINT(WPR*100/PR,3,4); PCU;
214 WRITE TEXT('"3C"' 'ADIABATICEXPANSIONFAC' 'S") PRINT(AEF,0,6);
215 PRINT(AEF,0,8); POM: PRINT(AEF,0,8); ODDS;
216 PRINT(AEF[5]*100/AEF,3,4); PCU;
217 WRITE TEXT('"3C"' 'DENSITYRATIO' '11S") PRINT(DR,0,6);
218 POM: PRINT(WDR,0,8); ODDS;
PRINT(WDR*100/DR,3,4); PCU;
WRITE TEXT('("C")"VISCOISITY"("15S")")'); PRINT(VIS,0,6);
PO:s; PRINT(VIS,0,6); ODDS;
PRINT(WVIS*100/VS,3,4); PCU;
WRITE TEXT("("3C")"TESTDATA%OR%-%0,0224(20%TO%1)"("CS")"
INSIDE"("3S")"BOTTOM"("3S")"OUTSIDE"("3S")"TOP"("2C")")
FORJ=1STEP1UNTILK'DO'
BEGIN
PRINT(INSIDE[J],2,3); PRINT(BOTTOM[J],2,3);
PRINT(OUTSIDE[J],2,3); PRINT(TOP[J],2,3);
PRINT(J=1,2,0); NEWLINE(1)
END';
WRITE TEXT("("3C")"VELOCITY%HEAD%RATIO"("C")"STATION
"("11S")"DATA"("17S")"LINE"("11S")"DEV"("C")"NO"("3S")"
L/D"("5S")"MEAN"("4S")"OR-%(20%TO%1)"("2C")")
FORJ=1STEP1UNTILK=5'DO'
BEGIN
PRINT(J,2,0); PRINT(EL[J],2,0); PRINT(H[J],0,6);
PRINT(ABS(WH[J]),0,4); PRINT(Y[J],0,6);
PRINT(DH[J],0,4); NEWLINE(1)
END';
FORJ=1STEP1UNTILK=5'DO' H[J]=ABS(H[J]);
NEWPLOT(1,1,10,0,12,5,0,0,0,0,1,8,69,0,1);
END';
END';
END';
END';
END';
END';
END';
END';
END';
END';
END';
PEN TO(0,0,0,0); INCR(-10,0,1);
INCR(-40,-5,0); NUMBER(D,3,0,0,0,15)
END';
END';
PEN TO(V,0,0,0); INCR(0,-10,1);
INCR(-20,-20,0); NUMBER(V,1,1,0,0,15)
END';
PEN TO(1,0,9,0,0); NUMBER(R,1,2,0,0,0,2);
CCPLOT(K-5,H,L,7,0);
PEN TO(ABS(CONST),0,0,0);
PEN TO(ABS(LLK-5)*SLOPE+ CONST),LLK-5,2);
SELECT OUTPUT(0);
D:=READ;
'IF'ENTIER(ABS(D)*10^(-4))=92'THEN'GOTO'L1; PAPERTHROW;
SELECT OUTPUT(1); ENDPLT;
'END';
SEGMENT H15E LENGTH 4593
21/51/04 20/02/70 COMPIL ED BY XAL M MK. 14

0 'LIBRARY'
0 'READ FROM'(MT,001 OP RES 1,GRAPHPLOTTER);
0 'FINISH';

CORE 19392
COMPIL ED #H15E EC
A.2.2. Computer Program H15F

Bend Loss Coefficient
FIG. A.2.2 - Flow Chart for Computer Program H15F

Bend Loss Coefficient
21/53/54

20/02/70

COMPARED BY XALM MK. 14

STATEMENT

0 'PROGRAM (H15F)
0 'INPUT 0 = TR 0
0 'INPUT 1 = TR 1
0 'OUTPUT 0 = LP 0
0 'OUTPUT 1 = TP 0
0 'CONTINUE

21/54/16

20/02/70

COMPARED BY XALM MK. 14

STATEMENT

0 'BEGIN
1 'INTEGER U, J, K, Z:
1 'REAL DENS, NOZD, PR, AEF, VN, SLOPE, DR, VH, X1, X2, SU, SD, CU,
2 CD, D, Y1, VIS, WD, W, WPR, W, WVIS, WDR, W, WVIS, WSLOPE, WR,
2 WCONST1, WCONST2, WBL:
2 ARRAY (RE[1:30], FF[1:30, 1:3], INSD, BOTTOM, OUTSIDE, YOP,
3 TOP, XT, XY, DEV, XV, XV, R, H1, B, 0, T, L, W, MB, W, W0, WT,
3 WH[1:50], WAEF, DRN, VR, KONST, V, DVH, D, DVV, CONST[1:10]:
3 'PROCEDURE COPY;
4 'BEGIN
4 'INTEGER M;
4 START: M = READ CH;
6 'IF M = CODE ('('')-> THEN 'GOTO FIN' ELSE 'PRINT CH(M);
7 'GOTO START;
8 FIN: 'END COPY;
8 'PROCEDURE DATA (M, A, B, K);
10 'INTEGER A, B, K;
12 'BEGIN
12 'INTEGER H, N, J; 'ARRAY Q[1:50]:
13 'BOOLEAN SCAN, TESTDATA; 'REAL D;
15 'FOR J = 1 'STEP 1 'UNTIL '50 'DO M[I] = 0;
18 'SCAN := 'TRUE'; H := 0;
20 'FOR H := H + 1 'WHILE 'SCAN 'DO*
21 'BEGIN

118
D:=READ;
END.

IF ENTIER(ABS(D)*10↑(-4))=A THEN SCAN:="FALSE" ELSE

BEGIN
TESTDATA:="TRUE"; N:=M; J:=0;
FOR J:=J+1 WHILE TESTDATA DO
BEGIN
IF ENTIER(ABS(D)*10↑(-4))=B THEN
TESTDATA:="FALSE" ELSE
BEGIN
M[J]:=M[J]+D; D:=READ; K:=J
END
END
END;
END;
FOR J:=1 STEP 1 UNTIL K DO
BEGIN
M[J]:=M[J]/N;
IF M[J]<0 THEN Q[J]:=ENTIER(M[J]*10↑(-4))+1 ELSE
Q[J]:=ENTIER(M[J]*10↑(-4));
M[J]:=100*(M[J]*10↑(-4)-Q[J])
END;
ENDDATA;

PROCEDURE LINE(N,X,Y,SLOPE,CONST);
VALUE N; INTEGER N; REAL SLOPE,CONST; ARRAY X,Y;
BEGIN
INTEGER J; REAL X1,X2,Y1,Y2;
X1:=X2:=Y1:=Y2:=0;
FOR J:=1 STEP 1 UNTIL N DO
BEGIN
X1:=X1+X[J]; X2:=X2+X[J]*Y[J];
Y1:=Y1+Y[J]; Y2:=Y2+X[J]*Y[J]
END;
SLOPE:=(Y1*X1-N*Y2)/(X1*X2-N*X2); CONST:=(Y1-SLOPE*X1)/N
END;
END;

PROCEDURE QINDATA(N,X,Y);
120 INTEGER N; ARRAY X, Y;

BEGIN

INTEGER J;
N := READ;
FOR J := 1 STEP 1 UNTIL N DO
BEGIN
    X[J] := READ; Y[J, 1] := READ;
    IF J > 1 THEN Y[J-1, 2] := (Y[J-1, 1] - Y[J, 1]) / (X[J-1] - X[J]);
    IF J = N THEN
        BEGIN
            Y[N, 2] := Y[N-1, 2];
        END;
    IF J > 2 AND J < N THEN
        Y[J-1, 3] := (Y[J-1, 2] - (Y[J, 1] - Y[J-2, 1]) / (X[J] - X[J-2])
        / (X[J-1] - X[J-2])
    END;
END QINDATA;
PROCEDURE QINTER(N, X, Y, XX, YY, L);
VALUE N, X; INTEGER N; REAL X, Y;
ARRAY XX, YY; LABEL L;
BEGIN
    INTEGER J, P; P := 0;
    IF X = XX[1] THEN P := 1 ELSE
        FOR J := 1 STEP 1 UNTIL N-1 DO
            BEGIN
                IF X > XX[J] AND X LE XX[J+1] THEN
                    BEGIN
                        P := J; J := N-1
                    END;
            END;
        IF P = 1 THEN Y := YY[1, 1] + (X-XX[1]) * (YY[1, 3] + (X-XX[2]) * YY[1, 3]);
    IF P = N-1 THEN
        BEGIN
            YY[1, 3] := YY[1, 3] + (X-XX[1]) * YY[1, 3];
        END;
END.

120
BEGIN
P:=N;
Y:={Y[N,1] + (X-XX[N]) * (Y[N,2] + (X-XX[N-1]) * Y[N,3])
END;
IF P>1 AND P<N-1 THEN
Y:={Y[P,1] + (X-XX[P]) * (Y[P,2] + (X-XX[P+1]) * Y[P,3])
IF P=0 THEN
BEGIN
WRITE TEXT("("C")"INTERPOLATION("C")"ARGUMENT
OUTSIDE TABLED DOMAIN("C")"ARGUMENT="");
PRINT(X,0,4);
GOTO 1
END
END QINTER;
PROCEDURE PQM;
WRITE TEXT("("%OR-%")");
PROCEDURE ODDS;
WRITE TEXT("("20% TO 1")");
PROCEDURE PCU;
WRITE TEXT("("PERCENT UNCERTAINTY")");
PROCEDURE NEXPLOT(A,B,C,D,E,F,G,H,I); 
VALUE A,B,C,D,E,F,G,H; 
INTEGER A,B; 
REAL C,D,E,F,G,H;

ALGOL;
PROCEDURE CCPLOT(N,X,Y,TYPE,MODE); 
VALUE N,TYPE,MODE; 
INTEGER N,TYPE,MODE; 
ARRAY X,Y; 
ALGOL;
PROCEDURE CURVE(N,X,Y,MODE); 
VALUE N,MODE; 
INTEGER N,MODE; 
ARRAY X,Y; 
ALGOL;
PROCEDURE PEN TO(X,Y,MODE); 
VALUE X,Y,MODE; 
REAL X,Y; 
INTEGER MODE; 
ALGOL;
PROCEDURE INCR(X,Y,MODE);
VALUE X,Y,MODE;
INTEGER X,Y,MODE;
ALGOL;
PROCEDURE NUMBER(X,N,STYLE,SIZE);
VALUE X,N,STYLE,SIZE; INTEGER N,STYLE; REAL X,SIZE;
ALGOL;
PROCEDURE END_PLOT;
ALGOL;
QINDATA(U,RE,FF); XT[1]:=READ; XT[2]:=READ;
SELECT INPUT(1); Z:=0;
L1:WRITE TEXT("**PC"H15E"C")BEND%LOSS%
COEFFICIENT("2C")**1);
COPY;
FOR J:=1 STEP 1 UNTIL 3 DO R[J]:=READ;
R[1]:=(R[1]*.1); R[2]:=ENTER(R[2]*.1)*.01;
R[3]:=2*R[3]*.01;
DATA(INSIDE,40,39,K);
DATA(BOTTOM,40,39,K);
DATA(OUTSIDE,40,39,K);
DATA(TOP,40,39,K);
WRITE TEXT("TEST NO."); PRINT(R[1],7,1);
WRITE TEXT("**2C")AIR%TEMPERATURE")
PRINT(R[2],10,1);
PRINT(R[3],6,2);
WRITE TEXT("**S")0.25%DEG.-%C%"); ODDS;
PRINT(25/R[2],3,4); PCU;
WRITE TEXT("**C")BAROMETRIC%PRESSURE")
PRINT(1/R[3],3,4); PCU;
DENS:=.0764*R[3]*288.6/(30*(273+R[2]));
WD:=DENS*SQRT((0.01/R[3])*(.01/R[3]+.25/(273+R[2]))*
\[ \sqrt{VW(2)} - VW[6]/(13.6*V[3])/2*R[3]^1.5; \]

\[ VW[10]; = \sqrt{VW(2)}/(2*VW(5))/VW[2]^3*V[5]/(27.2*R[3]); \]

\[ DV[3]; = \text{CONST}[1]*\sqrt{R(6)} + V[10]/\sqrt{R(3)}; \]

\[ VW[1]; = \sqrt{(DV[1]^2 + 2*VW(2)*VW[1])*(DV[3]^2 + 2*VW(3)^2)*VW(2)^2}; \]

\[ VWS[1]; = 1830*30.48*10^(-7)/(413/(390*R[2]) + (273 + R[2])); \]

\[ /296^11.5/(453,59*32,17); \]

\[ KONST[4]; = 1830*30.48*10^(-7)/(453,59*32,17*296^11.5); \]

\[ VW[2]; = 0.25*\text{KONST}[4]*(117+R[2])*1.5*\sqrt{R(2)} = R[2]^1.5; \]

\[ ((117+R[2])*1.5/(36*32,17*VWS[1]); \]

\[ RN[1]; = 8*V*DEN^S/(36*32,17*VWS[1]); \]

\[ KONST[3]; = 8*7,353*453,59*32,17*296^11.5; \]

\[ VR[1]; = (R[6]^11.7)/R[6]^1.5; \]

\[ VR[3]; = \sqrt{VW(2)}; \]


\[ DR[1]; = 7,353*VW[5]^2/(13.6*62,137*R[6]^1.5); \]

\[ DR[2]; = 7,353*R[1]/(2*\sqrt{VW(2)}*R[6]^1.5); \]

\[ DR[3]; = (R[6]^2*2*(R[6]^1.5))/R[6]_3; \]


\[ (13.6*62,137*R[3]^1.5); \]

\[ DR[6]; = 7,353/(2*VW[3]^2*R[6]); \]


\[ DRN[8]; = VW[2]/(13.6*62,137*R[3]); \]

\[ DRN[9]; = 1/(2*VW[5]); \]


\[ VW[5]^2*DRN[8]; \]

\[ WRN; = \sqrt{(DRN[4]*25)+2*(DRN[7]^2+01)^2+2*(DRN[10]^2+0112)^2}; \]

\[ WRITE TEXT('""C"" MEAN VELOCITY""(""11S"" )); \]

\[ PRINT(V,0,6); \]

\[ POM; = PRINT(V,0,8); \]

\[ WRITE TEXT('""FT./SEC."" )); \]

\[ ODDS; \]

\[ PRINT(VW*100/V,3,4); \]

\[ PCU; \]
215 WRITE TEXT("**(0)" REYNOLDS NO.**(12S)**)1; 
216 PRINT(RN,0,6); 
217 POM; PRINT(WRN,0,8); ODDS; 
220 PRINT(WREN*100/RN,3,4); PCU; 
222 QINTER(U,WRN,SLOPE,REF,F,L2); 
223 SLOPE:=SLOPE; 
224 WSLOPE:=WREN*0.01*(2*XT[1]*RN*10+(-16)-XT[2]*10+(-5)); 
225 DR:=84.9*(273+R[2])/R[3]; 
226 WDR:=SORT((84.9*25/R[3])**(2)+(84.9*(273+R[2])*0.01/ 
227 (R[3]*R[3]))**(2)); 
228 WRITE TEXT("**(0)" FRICTION FACTOR**(9S)**)1; 
229 PRINT(SLOPE,0,6); 
230 POM; PRINT(WSLOPE,0,8); ODDS; 
232 PRINT(WSLOPE*100/SLOPE,3,4); PCU; 
234 WRITE TEXT("**(0)" DENSITY**(17S)**)1; 
235 PRINT(DENS,0,6); 
236 POM; PRINT(WD,0,8); WRITE TEXT("**(0)" LB./CU.FT.**)1; 
239 ODDS; PRINT(WD*100/DENS,3,4); PCU; 
242 WRITE TEXT("**(0)" NOZZLE DIFFERENCE**(7S)**)1; 
243 PRINT(NOZD,0,6); POM; PRINT(ABS(WN),0,8); 
246 WRITE TEXT("**(0)" INCH%G.6.**)1; ODDS; 
248 PRINT(ABS(WN*100/NOZD),3,4); PCU; 
250 WRITE TEXT("**(0)" PRESSURE RATIO**(10S)**)1; 
251 PRINT(PR,0,6); 
252 POM; PRINT(WPR,0,8); ODDS; 
255 PRINT(WPR*100/PR,3,4); PCU; 
257 WRITE TEXT("**(0)" ADIABATIC% EXPANSION FAC**)1; 
258 PRINT(AEF,0,6); POM; PRINT(WAEF[5],0,8); ODDS; 
262 PRINT(WAEF[5]*100/AEF,3,4); PCU; 
264 WRITE TEXT("**(0)" DENSITY RATIO**(11S)**)1; PRINT(DR,0,6); 
266 POM; PRINT(WDR,0,8); ODDS; 
269 PRINT(WDR*100/DR,3,4); PCU; 
271 WRITE TEXT("**(0)" VISCOSITY**(15S)**)1; PRINT(VIS,0,6);
273 POM; PRINT(WVIS,0,8); ODDS;
276 PRINT(WVIS*100/VIS,3,4); PCU;
278 VH:=32.17*DR/(6*V*V);
279 KONST[5]:=32.1784.9/(6*KONST[1]*2);
280 VV[7]:=VV[5]*SORT(VV[2])=VV[6]/(13.6*R[3]);
281 DVH[8]:=2*KONST[5]/VV[7]*3;
282 DVH[1]:=DVH[8]*VV[8];
283 DVH[2]:=DVH[8]*VV[9];
284 DVH[3]:=DVH[8]*VV[10];
286 'FOR J=1*STEP*1*UNTIL*K=5*DO';
287 'BEGIN'
288 I[J]:=(INSIDE[J+5]-INSIDE[J])*VH;
289 WI[J]:=SQRT((VH+.0224)*2+(VH+.0224)*2+(WWH*INSIDE[J+5]*2);
290 B[J]:=(BOTTOM[J+5]-BOTTOM[J])*VH;
291 WB[J]:=SQRT((VH+.0224)*2+(VH+.0224)*2+(WWH*BOTTOM[J+5]*2);
292 O[J]:=(OUTSIDE[J+5]-OUTSIDE[J])*VH;
293 WO[J]:=SQRT((VH+.0224)*2+(VH+.0224)*2+(WWH*OUTSIDE[J+5]*2);
294 T[J]:=(TOPIE[J+5]-TOPIE[J])*VH;
295 WT[J]:=SQRT((VH+.0224)*2+(VH+.0224)*2+(WWH*TOPIE[J+5]*2);
296 L[J]:=3*J+3;
298 WH[J]:=SQRT((WI[J]/4)*2+(WB[J]/4)*2+(WO[J]/4)*2+(WT[J]/4)*2);
299 'END'; X1:=Y1:=WCONSTU:=WCONSTD:=0;
300 'FOR J=1*STEP*1*UNTIL*8*DO';
301 'BEGIN'
302 X1:=X1+L[J]; Y1:=Y1+H[J];
303 WCONSTU:=WCONSTU+WH[J]*WH[J]/64
304 'END';
305 YOP[9]:=X1;
306 LINE(8,L,H,SU,CA); CONSt[1]:=(Y1-X1*SLOPE)/8;
'FOR J = 1 *STEP* 1 *UNTIL* 8 *DO'

'BEGIN'
  XY[J] := L[J] * SLOPE + CONST[1];
  'END';

'FOR J = 28 *STEP* 1 *UNTIL* 34 *DO'

'BEGIN'
  X[J] := L[J];
  Y[J] := H[J];

'END';

LINE(7, X, Y, SDI, CD);

WBL := SORT(WCONSTD + WCONSTD + (651 / 7 - 84 / 8) * WSLOPE * 12);

WCONSTU := SQRT(WCONSTD + YOP[9] * YOP[9] * WSLOPE * WSLOPE / 64);

WCONSTD := SQRT(WCONSTD + X[1] * X[1] * WSLOPE * WSLOPE / 49);

'FOR J = 28 *STEP* 1 *UNTIL* 34 *DO'

'BEGIN'
  XY[J] := L[J] * SLOPE + CONST[2];

'END';

WRITE TEXT('***'('CASE 1: 'UPSTREAM%FF% = 'DOWNSTREAM%FF%')***'): PRINT(SLOPE, 0, 6);

WRITE TEXT('***'('2CS')***'CU='): PRINT(CU, 0, 6);

PRINT(CONST[1], 0, 6); PRINT(WCONSTD, 0, 8); ODDS;

PRINT(WCONSTD * 100 / CONST[1], 3, 4); PCU;

WRITE TEXT('***'('2CS')***'CD='): PRINT(CONST[2], 0, 6);

PRINT(CONST[2], 0, 8); ODDS;

PRINT(WCONSTD * 100 / CONST[2], 3, 4); PCU;

WRITE TEXT('***'('2CS')***'BEND%LOSS%COEFFICIENT='): PRINT(BEND, 0, 6);

PRINT(ABS(CONST[1] - CONST[2]), 0, 6); PRINT(WBL, 0, 8);

ODDS;

PRINT(WBL * 100 / ABS(CONST[1] - CONST[2]), 3, 4); PCU;

WRITE TEXT('***'('2CS')***'CASE 2: %FRICTION%FACTOR%CHECK='): PRINT(CS2, 0, 6);

WRITE TEXT('***'('2CS')***'FRICITION%FACTOR%**')');
353 PRINT(SU,0,6); WRITE TEXT('""("2S")'/'DEVI"');
355 PRINT(ABS(SU-SLOPE),0,6);
356 WRITE TEXT('""("9S")'/'CONSTANT"'); PRINT(CU,0,6);
358 WRITE TEXT('""("2S")'/'DEVI"'); PRINT(ABS(CU-CONST1),0,4);
360 WRITE TEXT('""("9S")'/'2.2%DOWNSTREAM"'); PRINT(CU,0,6);
362 PRINT(ABS(SD-SLOPE),0,4);
364 WRITE TEXT('""("9S")'/'CONSTANT"'); PRINT(CD,0,6);
366 WRITE TEXT('""("2S")'/'DEVI"'); PRINT(ABS(CD-CONST2),0,4);
368 WRITE TEXT('""("9S")'/'BEND%LOSS%COEFFICIENT%"');
369 PRINT(ABS(CU-CD),0,6); WRITE TEXT('""("%DEV")"');
371 PRINT(ABS(CU-C):-ABS(CONST1-CONST2),0,4);
372 WRITE TEXT('""("C")'/'TESTDATA%OR%0.0224%(20%TO%)"');
372 INSIDE('""("3S")'/'BOTTOM"'); WRITE TEXT('""("3S")'/'OUTSIDE"');
373 'FOR'J:=1'Step'1'UNTIL'K'DO'
374 'BEGIN'
374 PRINT(INSIDE(J),2/3); PRINT(BOTTOM(J),2/3);
377 PRINT(OUTSIDE(J),2/3); PRINT(TOP(J),2/3);
379 PRINT(J-1,2/0); NEWLINE(1)
380 'END';
381 WRITE TEXT('""("C")'/'VELOCITY%HEAD%RATIO"');
381 'LINE'('""("1S")'/'DATA"');
381 'LINE'('""("1S")'/'NO"');
381 'L/D'('""("6S")'/'MEAN"');
381 'OR%'('""("20%TO%)"');
382 'FOR'J:=1'Step'1'UNTIL'K'=5'DO'
383 'BEGIN'
383 PRINT(ABS(J),2,0); PRINT(L(J),2,0); PRINT(H(J),0,6);
387 PRINT(ABS(WH(J),0,4);'
388 'IF'J'GE'1'AND'J'LE'8'OR'
388 'J'GE'20'AND'J'LE'34'THEN'
388 'BEGIN'
388 PRINT(XY(J),0,6); PRINT(DEV(J),0,6)
390 'END';
391 NEWLINE(1)
'END';
WRITE TEXT((''PC'')'VELOCITY%HEAD%RATIO%(STATION-REF.%STATION)
  *'('2C')'STATION'('10S')'VELOCITY'('6S')'HEAD'('7S')'RATIO
  *'('CS')'NO.(1S)=L/D('4S')'INSIDE'('9S')'BOTTOM'('9S')'
  OUTSIDE('8S')'TOP('2C')'');
'FOR'J:=1'STEP'1'UNTIL'K=5'DO'
'BEGIN'
  PRINT(J,2,0); PRINT(L(J),3,0); PRINT(I(J),0,6);
  PRINT(B(J),0,6); PRINT(O(J),0,6);
  PRINT(T(J),0,6); NEWLINE(1)
'END';
'FOR'J:=1'STEP'1'UNTIL'34'DO'
'BEGIN'
  I(J):=ABS(I(J)); B(J):=ABS(B(J));
  O(J):=ABS(O(J)); T(J):=ABS(T(J))
  I(35):=1.4; B(35):=1.9; O(35):=2.4; T(35):=2.9; L(35):=9.0;
  NEWPLOT(1,1,10.0,18.5,0,0,0,3,6,105,0,1);
  'FOR'D:=0.0'STEP'3.0'UNTIL'102.1'DO'
  'BEGIN'
    PEN TO(0,0,0,0); INCR(-10,0,1);
    INCR(+0,-5,0); NUMBER(0,3,0,0,0,15)
  'END';
  'FOR'V:=0.0'STEP'0.20001'UNTIL'3.5'DO'
  'BEGIN'
    PEN TO(V,0,0,0); INCR(0,-10,1);
    INCR(-16,-20,0); NUMBER(V,1,1,0,0,15)
  'END';
  CCCPLOT(35,I,L,7,0);
  CURVE(34,I,L,1);
  CCCPLOT(35,B,L,12,0);
  CURVE(34,B,L,1);
  CCCPLOT(35,O,L,6,0);
  CURVE(34,O,L,1);
  CCCPLOT(35,T,L,8,0);
433    CURVE(34,T,L,1);
434    PEN TO(ABS(CONST[1]),0,0,0);
435    PEN TO(ABS(L[8]*SLOPE+CONST[1]),L[8],2);
436    PEN TO(0,0,31.5,0); PEN TO(2,0,31.5,2);
438    PEN TO(2,0,3,0,0); NUMBER(R[1],3,1,0,0.25);
440    PEN TO(ABS(CONST[2]),0,0,0);
441    PEN TO(ABS(L[34]*SLOPE+CONST[2]),L[34],2);
442    L3:SELECT OUTPUT(0);
443    L2:'IF ENTIER(ABS(READ)*10^(4))=92 THEN"GOTO'L1;
444    PAPERTHROW;
445    SELECT OUTPUT(1); ENDPLOT;
447    'END';
SEGMENT H15F   LENGTH 6425

21/57/04  20/02/70     COMPILLED BY XALM MK. 14
STATEMENT
0    'LIBRARY'
0    'READ FROM'(MT,001 OP RES 1,GRAPHPLOTTER);
0    'FINISH';
CORE 21504
COMPILLED #H15F   EC
APPENDIX 3

COMPUTER GRAPHICAL REPRESENTATIONS
### Duct Friction Factor - Reynolds Number Relationship

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Friction Factor $x10^{-2}$</th>
<th>Reynolds Number $x10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.6567</td>
<td>1.5102</td>
</tr>
<tr>
<td>16</td>
<td>1.6490</td>
<td>1.5513</td>
</tr>
<tr>
<td>11</td>
<td>1.6329</td>
<td>1.6270</td>
</tr>
<tr>
<td>2</td>
<td>1.6094</td>
<td>1.7519</td>
</tr>
<tr>
<td>3</td>
<td>1.6020</td>
<td>1.7978</td>
</tr>
<tr>
<td>4</td>
<td>1.5959</td>
<td>1.8211</td>
</tr>
<tr>
<td>6</td>
<td>1.5902</td>
<td>1.8592</td>
</tr>
</tbody>
</table>

Each graph represents the variation of $H(J)$ (mean static head at each cross-section minus a reference static head divided by the mean inlet velocity head) with the duct length to hydraulic diameter ratio $L/Dh(J)$ along the duct.
Wall Static Pressure / Mean Velocity Head - $H(\varphi)$

Duct Length / Hydraulic Diameter - $L/D_h(\varphi)$
DUCT LENGTH / HYDRAULIC DIAMETER ~ L/Dh(T)

WALL STATIC PRESSURE / MEAN VELOCITY HEAD ~ H(T)

16
WALL STATIC PRESSURE / MEAN VELOCITY HEAD - H(ζ)
Duct Length / Hydraulic Diameter ~ L/Dh(τ)

Wall Static Pressure / Mean Velocity Head ~ H(τ)
A.3.2.

Bend Loss Coefficient

Each graph represents the variation of $F(J)a$ (mean static head at a given tapping location minus a reference static head divided by the mean inlet velocity head) with the duct length to hydraulic diameter ratio $L/Dh(J)$ along the duct.
Wall Static Pressure / Mean Velocity Head ~ F(s)α
WALL STATIC PRESSURE / MEAN VELOCITY HEAD = F(3x)
Wall Static Pressure / Mean Velocity Head = F(\theta)\alpha.
- 159 -

Duct Length / Hydraulic Diameter = L/d_out

X INSIDE  O BOTTOM  + OUTSIDE  □ TOP

WALL STATIC PRESSURE / MEAN VELOCITY HEAD = F(3)x
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26. Richter, G. *Hydraulics of Pipelines*, 1953, ONTI.


49. Szczeniowski, B. Design of Elbows in Potential Motion, *Journal Aeronautical Sciences*, 1944, 11, 73-75


