TURBULENCE EFFECTS IN INTERNAL AIRFLOWS

by

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for the Degree of Doctor of Philosophy
December, 1970.
For my Mother,

and to the memory

of my Father.
SUMMARY

The effect of inlet conditions on the performance of conical diffusers is considered. The conditions at inlet which were independently varied are the cross-sectional time-meaned velocity distribution and the associated stream turbulence variation. The investigation formed part of a research programme on interaction effects between pipe and duct components.

Sources of data on the effect of such variants include Kline, Winternitz, Livesey, Miller and Cockrell amongst others. Whereas these persons considered the symmetrical profile developments accompanying varying inlet pipe lengths and Reynolds numbers, Tyler and Williamson examined the effects of distorted time-mean velocity profiles at inlet. A very limited amount of experimental work has been published by Kline et al on the effect of varying the free stream turbulence intensity at the diffuser inlet when the associated boundary layer thickness is small.

The thesis considers this previous work, then describes the development of a turbulence generator with which a mean velocity distribution, barely distinguishable from fully-developed flow is obtained in a few pipe diameters. This mean velocity is accompanied by a turbulence intensity which is capable of being increased up to three times that normally associated with fully-developed flow. Conical diffusers were mounted downstream of the turbulence generator and pressure recovery characteristics over a wide range of expansion angle and area ratio were obtained. In general the results show a significant increase in pressure recovery over values associated with normal fully-developed inlet flow.
An integral computational method is developed for ducts, pipes and conical diffusers which leads to successful predictions of boundary layer growth and diffuser pressure recovery for a wide variety of inlet boundary layer thicknesses. Possible extensions of this method to admit inlet turbulence level as a parameter which influences diffuser pressure recovery are considered.

The thesis discusses how variations in mean velocity profiles and turbulence levels at the diffuser inlet are significant parameters in the current industrial context.
ACKNOWLEDGEMENTS

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\( a, c \)  Constants in equation 4.4.1.

\( A \)  Cross-sectional area of duct or pipe

\( b \)  Profile parameter in the quadratic velocity profile expression

\( B \)  Blockage factor, ratio of effective to actual flow area

\( C_p \)  Pressure recovery coefficient in a diffuser \( = \frac{(P_o - P_i)}{\frac{1}{2} \rho U_1^2} \)

\( d \)  Pipe diameter

\( D \)  Hot-wire diameter

\( e \)  Instantaneous fluctuating voltage output of hot-wire anemometer bridge

\( E \)  Hot-wire anemometer bridge D.C. voltage (Chapter 4)

\( E \)  Rate of entrainment into the boundary layer \( = \frac{dI}{dx} \)

\( f, s \)  Coefficients of the ordinary differential equations in Appendix A

\( H \)  Boundary layer mean flow shape factor \( = \frac{\zeta}{\nu} \)

\( I \)  Boundary layer flux, at some prescribed value of \( x \)

\( k \)  A constant depending on the aspect ratio of a hot-wire

\( K \)  A constant in equation 3.2.2.

\( l e \)  Entrance length in a duct or pipe

\( L \)  Length of a hot-wire

\( n \)  Parameter in power law exponent

\( N \)  Axial length of diffuser

\( p \)  Local static pressure at some prescribed point in the flow

\( P \)  Free stream total pressure

\( P_c \)  Centre-line total pressure

\( r \)  Radial ordinate in pipe flow

\( R \)  Pipe radius \( = \frac{\delta}{2} \)

\( Re \)  Duct or pipe Reynolds number, based on the mean velocity \( \bar{U} \) and duct width \( W \) or pipe diameter \( d \) respectively
$R_δ$ Reynolds number based on the maximum velocity $U_c$ and the boundary layer physical thickness $δ$

$s$ Power spectral density

$u_*$ Friction velocity ($= \sqrt{T_W/\rho}$)

$u,v,W$ Longitudinal, lateral and transverse fluctuating components of velocity respectively

$U$ Axial temporal mean velocity

$\bar{U}$ Spatial mean velocity (ratio discharge/cross-sectional area) established in the duct or pipe

$U_c$ Maximum axial temporal mean velocity at some streamwise ordinate $x$

$\hat{U}$ Non-dimensional velocity ($= U/U_c$)

$V$ Vector velocity

$\omega$ Frequency (in Chapter 5)

$W$ Duct width

$x$ Streamwise ordinate, measured from some prescribed datum

$y$ Transverse ordinate, measured from the wall

$a,\beta$ Terms in Coles velocity profile equation

$\delta$ Boundary layer physical thickness

$\delta^*$ Boundary layer displacement thickness

$\theta$ Boundary layer momentum thickness

$\epsilon$ Bödö viscosity

$\nu$ Kinematic viscosity of fluid

$\eta$ Non-dimensional transverse ordinate ($= y/\delta$ or $(R-r)/\delta$)

$\pi$ Wake parameter in the law of the wake velocity profile expression

$\lambda$ Diffuser loss coefficient $= 1 - \frac{C_D}{[1 - (A_t/A_c)^2]}$

$\rho$ Density of fluid

$\zeta$ Vorticity

$\tau$ Local shear stress in the boundary layer

$\tau_w$ Wall shear stress

$\phi$ Total expansion angle of conical diffuser
suffices

i,o Inlet and outlet to conical diffuser respectively
2D,AI Two-dimensional and axially-symmetric values respectively
w Wall conditions
c Centre-line conditions

metric units

Useful conversion factors

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<td>LENGTH</td>
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<td>1 foot</td>
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CHAPTER 1

INTRODUCTION
1.1. INTRODUCTION

If an incompressible fluid is caused to flow through pipework or ductwork the pressure difference between two points $i$ and $o$, downstream of $i$ can be expressed through

$$p_i + \frac{1}{2} \rho U_i^2 = p_o + \frac{1}{2} \rho U_o^2 + \Delta P_{i,o}$$

if the difference in hydrostatic pressure is negligible. The total pressure loss $\Delta P_{i,o}$ is normally calculated by summing tabulated values of losses for individual pipe and ductwork components, such as straight pipe lengths, bends, contractions or diffusers.

Sets of data which tabulate head losses for pipe components usually give pressure-recovery coefficients for diffusers as functions of the angle of expansion and area ratio. More exacting design requirements have shown that the state of the flow at the diffuser inlet is also significant and considerable experimental work has been based on the assumption that boundary layer thickness at the diffuser inlet is the relevant parameter $[1,2]$.

More recently it has become evident that the inlet flow is not fully defined in terms of mean velocity profiles and that the method of generation of the profile is also important $[3,4]$. It seems probable that the method of velocity profile generation affects the structure of turbulence at the diffuser inlet and that this in turn influences the transfer processes which then occur. It therefore seems worthwhile to consider to what extent inlet turbulence affects diffuser behaviour.

The reduction of head losses in pipe systems to a minimum is of vital concern in certain industries, notably atomic energy generation and air-conditioning, because of the need to increase throughputs significantly without altering external dimensions of components. In other industries the requirement is that of making wide-angle diffusers
operate under optimum conditions and the gas turbine designer has this problem in common with the designer of fluidic switching elements. A better understanding of the flow processes in a diffuser could therefore produce dividends over a wide range of applications.

The work contained in this thesis considers, both theoretically and experimentally, the significance of inlet turbulence as a contributory parameter governing the performance of conical diffusers.

1.2. PUBLISHED EXPERIMENTAL DATA

Experimental research on incompressible fluid flow through diffusers owes its origins to Gibson [5] and others at the turn of the century. An uneasy period prevailed in which there were considerable differences between the definitions of diffuser performance employed by experimentalists and a corresponding lack of awareness of the importance of carefully stipulating the way in which dependent parameters were defined.

Nowadays the pressure recovery coefficient $C_p$ is almost universally adopted. Defined by

$$C_p = \frac{(p_0 - p_1)}{\frac{1}{2} \rho \bar{u}_1^2} \quad 1.2.1.$$  

it has the advantages of easy measurement and easy utilisation in pipe and duct flow calculations and the disadvantage that it is only a very approximate measure of the physical process of pressure recovery in the diffuser.

From earliest days the dependence of $C_p$ on the diffuser's inlet conditions was appreciated. Peters [6] showed that the length of inlet duct affected $C_p$. Patterson's [7] data based on work by Gibson, Peters and himself is well known and widely applied. Patterson's curves, which are presented in Figure 1.1 assume that the flow at inlet
to the diffuser has a fully-developed mean velocity profile. It would be inappropriate to apply Patterson's results to a diffuser if the entry boundary layer were very thin (e.g. in a wind tunnel).

Cockrell and Markland [1] extended the range of published conical diffuser characteristics, as shown in Figures 1.2 and 1.3. They considered the diffuser on its own whereas Patterson took into account the length of straight pipe downstream of the diffuser for a fully-developed pipe flow velocity distribution to be re-established. Cockrell and Markland expressed the loss coefficient

$$\lambda = 1 - \frac{C_p}{1 - \left(\frac{A_1}{A_0}\right)^2}$$

as a function of the state of the inlet boundary layer, defined in terms of the two-dimensional momentum thickness, $\theta$. Since the variation of loss coefficient, $\lambda$, with area ratio is only small and since Sprenger [8] shows that the principal contribution of Reynolds number over a very wide range is to the inlet boundary layer thickness, they combined successfully all known conical diffuser characteristics on a single set of curves, Figure 1.3.

The range of momentum thickness ratio in Figure 1.2. is from 0 to 0.05, at which latter value the inlet boundary layer was said to be fully developed. This situation was taken to exist when the inlet flow characteristics, based on temporal mean axial velocity measurements, had become constant (at some 40 pipe-diameters). However Barbin and Jones [9] have shown, for a Reynolds number of $3.88 \times 10^5$, that after 40 pipe-diameters the Reynolds stress distribution does not accord with that for theoretically fully-developed flow. Thus, as Reynolds stress variation appears to be the mechanism promoting the momentum transfer of turbulent fluid flow, a further change in its distribution should produce a corresponding variation of diffuser characteristics.
Work by Winternitz and Ramsay [2] which was later repeated and extended by Cockrell, Diamond and Jones [3] produced conflicting conclusions as to the appropriate parameter for use when the inlet conditions are not due to normal boundary layer growth. The experimenters thickened the inlet boundary layer by two different methods, by increasing the length of inlet pipe and by using screen rings of varying dimensions close to the diffuser inlet. Their results are presented in Figure 1.4. However, for a momentum thickness $\delta/d = 0.037$, the resulting velocity profiles due to the two methods of generation are shown in Figure 1.5. It can be seen that the shape of the profiles is completely different. Thus it is evident that no single integral boundary layer parameter is a unique definition of the diffuser inlet conditions. Furthermore, the turbulence levels (and in particular the Reynolds stress distributions) are different in the above two cases.

Livesey and Turner [4] pointed out that the rate of change of a mean flow quantity is a significant inlet parameter. Since shear stress gradients promote this rate of change they were, in effect, admitting the importance of turbulence information.

Other workers have favoured describing the inlet flow in terms of displacement thickness. This parameter has the merit of being associated (through the definition of spatial mean velocity as the ratio of discharge to cross-sectional area) with the concepts of profile peakness, $U_0/U$ and blockage factor, $(1 - \overline{U}/U_0)$, both of which have been adopted by different schools of experimentalists to define the mean inlet flow. The blockage factor concept was introduced by Caugnower, Kline and Johnston [10] in an only partially successful attempt to reconcile the performance of diffusers having different geometries.

The foregoing discussion implies that there is a fund of experimental data on the way in which the distribution of the time-meaned velocity affects diffuser characteristics. Most of this data is
derived from tests in which carefully-controlled boundary layer growth occurred down long straight ducts and pipes. Tyler and Williamson [11] however allowed a stream of air to flow normal to the axis of a diffuser assembly. By drawing this cross stream into the diffuser the flow separation of the entry lip generated an asymmetrical inlet velocity profile. The peakness necessarily associated with this asymmetrical profile implied a large inlet blockage factor. Assuming that the total head is maintained down the diffuser centre-line, $C_p$ from equation 1.1.1. can be written

$$C_p = \left(\frac{U_0}{\bar{U}}\right)_i^2 - \left(\frac{U_0}{\bar{U}}\right)_o^2 \left(\frac{A_i}{A_o}\right)^2$$

1.2.3.

and the inlet peakness is seen to give rise to a large diffuser pressure recovery coefficient in contradiction to what might be expected on physical grounds. Tyler and Williamson used the inlet blockage factor, described above, as their independent parameter.

Shortly before this, Livesey, Turner and Glasspoole [12] showed that the decay rate, i.e. the distance required for a peaky profile to revert to its equilibrium shape, was strongly dependent upon its means of generation for this affected the turbulence of the flow and hence the magnitude of the Reynolds shear stresses. Since profile decay is accompanied (on energy grounds) by pressure recovery their findings imply that Tyler and Williamson's inlet blockage factor is unsatisfactory as a single independent parameter. Inlet turbulence data, in some way, must accompany the specification of the mean inlet flow. It is therefore desirable to turn to existing experimental evidence on the way inlet turbulence affects diffuser behaviour.

Moore and Kline [13] began the Stanford University's research programme with water table studies of two-dimensional diffuser behaviour. At that time techniques for turbulence measurement in liquids were not well developed and it was not until Waitman, Reneau and Kline's [14]
work that inlet turbulence levels were varied and recorded. In their review of inlet turbulence affects Renou, Johnston and Kline [15] reported that high inlet turbulence increases the peak pressure recovery of diffusers, large scale mixing over the entire throat area producing a significant increase in recovery. However, in the work they described it is unlikely that inlet turbulence levels were varied independently of the mean velocity profile at inlet. It thus appears desirable to try to achieve this independent variation. A method of doing so is described in Chapter 2, and in more detail in Chapter 5.

1.3. INTERNAL FLOW PREDICTION METHODS

This thesis does not attempt to review the various analytical methods that have been developed to predict boundary layer development and pressure recovery in conical diffusers. Early models, such as those due to Gourzhienko [16], Horlock and Lewis [17], Schlichting and Gersten [18] and Ross [19], have been superseded by more generally applicable prediction methods.

These later methods formed the basis of the 1968 Conference at Stanford University on "Computation of turbulent boundary layers". The Proceedings [20] of this Conference fairly represent the present state of the art of boundary layer prediction.

There are two broad classes of prediction methods. The first consists of differential analysis wherein the partial differential equations of motion are used to obtain the resulting flow in detail. The second class comprises integral methods wherein a system of ordinary differential equations derived from integral forms of the equations of motion is employed to carry out the solution for certain main features of the flow. It is the nature of integral methods to allow inclusion of only the overall aspects of the "physics" which govern the behaviour of
the fluid; it is this feature which compels the use of integral tech-
niques when complete understanding of a physical situation is not present. If the rules which govern the overall behaviour of the fluid are ade-
quately formulated, integral techniques can provide quite satisfactory predictions of gross variables such as static pressure rise. An integral technique is presented in Chapter 3 capable of predicting incompressible turbulence boundary layer development in ducts, pipes, and conical diffusers.

1.4. SCOPE OF THE PRESENT WORK

This thesis describes an experimental investigation into the independent assessment of the effect of inlet turbulence on conical diffuser performance for a wide range of expansion angles and area ratios. An integral technique is developed which predicts boundary layer growth in ducts, pipes and conical diffusers for very thin to fully-developed inlet boundary layers. An attempt is made to account for high inlet turbulence levels. The theoretical work reported in Chapter 3 has been published [21], and the experimental work contained in Chapter 5 has been submitted for publication [22].

1.4.1. Outline of the Work

Independent variation of turbulence at the diffuser inlet implies that the mean velocity profile there be kept the same for all the experimental tests. The inlet profile considered was the fully-developed flow case of natural boundary layer growth down a smooth pipe. The need to simulate this inlet profile lead to the design of a velocity profile generating system. This system also produced a high inlet turbulence. Preliminary work comparing the performance of a 21-degree conical diffuser subject to both normal fully-developed flow and the
simulated flow is presented in Chapter 2. As a consequence of this work a more rigorous experimental programme was begun and the results of it are considered in Chapter 5.

The theoretical contribution is presented in Chapters 3 and 6. In Chapter 3 the use of a simple integral technique to predict boundary layer growth in ducts, pipes and conical diffusers is presented and the results compared with existing reliable experimental data. Predictions of conical diffuser performance are made for a wide variety of inlet boundary layer thicknesses. Finally an attempt is made to predict boundary layer growth in conical diffusers with fully-developed flows and high turbulence intensities at their inlet. These predictions, presented in Chapter 6, are compared with the experimental results obtained from Chapter 5.
FIG. 12 VARIATION OF LOSS COEFFICIENT $\Lambda$ WITH INLET MOMENTUM THICKNESS $(\theta/d)$
**Figure 13**

Available Experimental Data on Conical Diffuser Loss Coefficient

<table>
<thead>
<tr>
<th>Key</th>
<th>Investigator</th>
<th>Expansion Angle φ Deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>Peters</td>
<td>10-15 - 20</td>
</tr>
<tr>
<td>□</td>
<td>Gibson</td>
<td>10-15 - 20</td>
</tr>
<tr>
<td>○</td>
<td>Winternitz &amp; Ramsay</td>
<td>4 - 10</td>
</tr>
<tr>
<td>X</td>
<td>Sprenger</td>
<td>8</td>
</tr>
<tr>
<td>X</td>
<td>Squire</td>
<td>5 - 8 - 10</td>
</tr>
<tr>
<td>△</td>
<td>Little &amp; Wilbur</td>
<td>12 - 20</td>
</tr>
<tr>
<td>▽</td>
<td>Nelson &amp; Popp</td>
<td>5 - 12 - 20</td>
</tr>
<tr>
<td>○</td>
<td>Robertson &amp; Ross</td>
<td>5 - 8</td>
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<tr>
<td>○</td>
<td>Fraser</td>
<td>10</td>
</tr>
<tr>
<td>○</td>
<td>Uram</td>
<td>8</td>
</tr>
<tr>
<td>---</td>
<td>Cockrell (mean of fig 1.2)</td>
<td>4-8-10-15-21</td>
</tr>
</tbody>
</table>

**Diagram Notes:**

- **φ = 20-21 Deg**
- **φ = 12-15 Deg**
- **φ = 10 Deg**
- **φ = 8 Deg**
- **φ = 4.5 Deg**
FIG 14 DEPENDENCE OF THE DIFFUSER LOSS COEFFICIENT ON
METHOD OF GENERATION OF THE INLET VELOCITY PROFILE
FIG. 1.5  NON-DIMENSIONAL VELOCITY PROFILES AT $\theta/\pi = 0.037$
CHAPTER 2

PRELIMINARY EXPERIMENTAL WORK:

CONICAL DIFFUSER INLET FLOW PARAMETERS
2.1. INTRODUCTION

The work [23] presented in this chapter was a continuation of the experiments performed by Cockrell [24]. As indicated in Chapter 1, Cockrell had shown that the performance of a conical diffuser was affected by the inlet mean velocity profile. However, as Figure 2.1. shows, Cockrell did not attain truly fully-developed flow at the diffuser inlet. Although at his test Reynolds number bulk boundary layer thickness parameters, such as momentum thickness, reached a maximum at 42 pipe-diameters subsequent values were seen to decrease.

As a consequence of this result the following experiments were conducted using Cockrell's 2-inch diameter pipe and conical diffuser rig which is described in section 4.2.1. and in more detail in reference [24],

(i) boundary layer growth along a smooth pipe 120 pipe-diameters in length,
(ii) the effect of varying the inlet pipe length on the performance of a downstream 21° conical diffuser,
(iii) generation of a highly turbulent velocity profile having the same shape as the fully-developed flow condition of (i),
(iv) the performance of the 21° diffuser with the highly turbulent profile at inlet.

The results of the above experiments will now be presented and discussed.
2.2. VARIATION OF INLET PIPE LENGTH

The experimental arrangement for the variation of inlet pipe length is shown in Figure 4.1(a). The approach length, upstream of a 21° conical diffuser, was varied from 1.5d to 126d. Measurements of the mean velocity profile were made at 6d intervals, the Reynolds number (based on pipe diameter and mean velocity) being approximately \(1.7 \times 10^5\). The wall static pressures \(P_i\) and \(P_o\), at the diffuser inlet and outlet section respectively, were also recorded: thus allowing the diffuser loss coefficient, \(\lambda\), to be calculated. From the mean velocity profile measurements the two-dimensional and axi-symmetric momentum and displacement thicknesses were obtained using a suitable data reduction computer routine.

The variation of the integral boundary layer thicknesses as the approach length of pipe is increased to \(10/\delta = 126\) is shown in Figure 2.2. Here, subscripts (ii) and (iii) refer to the two-dimensional and axi-symmetric definitions respectively. Instead of a steady boundary layer growth until the centre-line is reached, followed by unaccelerated conditions, Figure 2.2. shows that apparently there is a growth to a maximum boundary layer thickness, then with further increase in pipe length a constant but reduced boundary layer thickness is eventually established.

The effect of increasing the approach length on the loss coefficient of a 21° diffuser is shown in Figure 2.3. It is evident that whilst variations in the Reynolds shear stress and mean velocity profile occur there is a corresponding change in the diffuser loss coefficient. It is not until some 90 pipe-diameters or more that constancy of all inlet flow and diffuser performance parameters is attained.
2.3. HIGHLY TURBULENT DIFFUSER INLET FLOW CONDITIONS

The fully-developed flow condition of boundary layer growth down a smooth pipe produces a relatively low turbulence intensity at the diffuser inlet. In many industrial applications the turbulence intensity at the diffuser inlet would be artificially intensified by separation-inducing devices or by rotating machinery. To study the effect of high turbulence intensity a mean velocity profile generator was devised which produced a high turbulence intensity but which gave a mean velocity profile having the same shape as that produced by natural boundary layer growth down a long smooth pipe.

2.3.1. Mean velocity profile generator

The basis for the design of this high turbulence-generating device was work by Armitt and Counihan [25] and later by Counihan [26] at the Central Electricity Research Laboratories. Their problem was the wind-tunnel simulation of the atmospheric boundary layer. Most experimental study in this field at that time had made use of a system of graded rods spanning the wind tunnel upstream of the working section in order to generate the required mean velocity profile. Since the turbulence intensity produced by these devices was considerably less than typical atmospheric values, workers at C.E.R.L. developed a profile generator which consisted of a step mounted on the wind tunnel floor immediately followed by six large triangular-shaped half wings also set on the tunnel floor and extending towards the tunnel ceiling. These vanes were set alternately at positive and negative angles of incidence so as to minimise resultant swirl. The step created a local momentum deficiency and the vortex generators introduced intense low frequency turbulence such as would cause an equilibrium mean velocity profile to be established in a very short distance downstream. Such a device
produced the requisite mean velocity profile with turbulence intensity of the order of that associated with a long upstream duct.

The mean velocity profile generator used in these preliminary experiments was an axisymmetric form of the C.E.R.L. device. Suitable dimensions of the step and the triangular vortex generators were obtained empirically. Since the effect of high intensity Reynolds shear stresses will be to promote an equilibrium mean velocity profile in a short downstream pipe length the effect of adjusting the relative dimensions of the profile generator will be to change mean flow and local turbulence intensities immediately downstream of the generators. This will vary the shape of the equilibrium mean velocity profile and the distance required to achieve such a shape. The resulting mean velocity profile generator dimensions are given in Chapter 4, section 4.2.1., and the experimental set-up in Figure 4.1.(b).

2.3.2. Results

The diffuser inlet mean velocity profile which was generated some 7.5 pipe-diameters downstream of the profile generating system is shown in Figure 2.4. Velocity traverses across two diameters 60° apart are compared with the fully developed flow condition of boundary layer growth along a long smooth pipe. The resulting axial mean intensities of turbulence for the 'artificially-generated' and natural fully developed flow velocity profiles are shown in Figure 2.5.(A). The turbulence structure of the two profiles is very different: the centre-line axial mean intensity of turbulence (based on the spatial mean velocity $U$) being increased from 3.82% to 5.47% with the introduction of the profile generating system.

The loss coefficient of the 21° conical diffuser was determined with the highly turbulent profile at inlet. It can be seen from Figure 2.5.(B) that if the diffuser were placed immediately downstream
of the artificially produced fully developed profile it would have a loss coefficient which is 16% less than the normal. As the distance between the diffuser and profile generator is increased, the loss coefficient gradually reverts to its expected value.

2.4. DISCUSSION OF RESULTS

The 'overshoot' phenomenon of boundary layer growth along a smooth pipe as demonstrated in Figure 2.2. was later studied in more detail at Leicester University by Lee [27]. He investigated boundary layer growth in a 3:1 aspect ratio rectangular duct. Lee's measurements followed the same trend as those of the present author.

From his extensive turbulence measurements Lee gave an explanation of the 'overshoot' phenomenon in terms of a momentum balance in which the acceleration and deceleration of the central region of the flow was shown to be governed by the turbulent shear stress gradient. Since the shear stresses in the central region of the flow have their origins at points on the wall upstream of their present position, the 'overshoot' is shown to be a function of the history of the flow.

Both Lee and the present author have shown that to obtain fully-developed flow conditions at the diffuser inlet some 90 to 100 pipe-diameters is needed upstream. Considering fully-developed flow to be attained when the momentum thickness is at a maximum results in a 5 per cent over-estimation in diffuser loss coefficient for the 21° diffuser from Figure 2.3.

On the basis of the limited results obtained using the velocity profile generating system it appeared that the 16 per cent reduction in diffuser loss coefficient could be directly attributed to the different inlet turbulence structure that it produced: the large-scale eddies generated by the vigorous stirring process influencing the downstream
momentum transfer. However it was considered unlikely that the difference in loss coefficient shown in Figure 1.4. (for the work of Cockrell, Diamond and Jones [3]) could be attributed to the different inlet turbulence structures produced by the two methods of generation. The gauze screens would only produce small-scale turbulence which would decay away rapidly. Anyway the effects shown in Figures 1.4. and 2.5.(B) are of opposite sign. Thus the different loss coefficients in Figure 1.4 are attributed to different inlet mean velocity profiles.

The results of these preliminary experiments indicated that high inlet turbulence intensities at the inlet of conical diffusers should be investigated further. A new experimental rig was designed and constructed for this purpose and the results of a more rigorous investigation are reported in Chapter 5.
TWO-DIMENSIONAL MOMENTUM THICKNESS Vs LENGTH OF PIPE:
COMPARISON OF COCKRELL'S AND BRADLEY'S RESULTS.
EFFECT OF INCREASING THE APPROACH LENGTH ($\ell_e/d$) TO A
21° DIFFUSER ON THE DIFFUSER LOSS COEFFICIENT $\lambda$

Fig 2.3  $Re = 1.7 - 1.77 \times 10^5$

FULLY DEVELOPED FLOW $\lambda$
**Fig 2.4**  GENERATED MEAN VELOCITY PROFILE
Fig 25  AXIAL TURBULENCE IN ARTIFICIALLY-GENERATED PROFILE AND ITS EFFECT ON A DIFFUSER.
CHAPTER 3

THEORETICAL ANALYSIS:

BOUNDARY LAYER METHODS APPLIED TO INTERNAL FLUID PROBLEMS
3.1. INTRODUCTION

This Chapter considers how integral boundary layer techniques may be used to solve problems of incompressible turbulent fluid flow through ducts and pipes. The basic equation used in all the calculations is the Karman Momentum equation. In formulating relationships which are auxiliary to this equation either detail is required about the shear stress development within the boundary layer, or entrainment principles which relate boundary layer rate of growth in various ways to other flow parameters is employed. The worth of the solutions so obtained is considered, and illustrated by results which are compared with the best relevant experimental data known to the author.

In the present context a boundary layer will be assumed to exist after as well as before the velocity defect regions emanating from the walls have merged. The definition will also embrace cases where a shear stress distribution is artificially generated in the stream. However the results for the latter will be presented in Chapter 6.

The prediction method for internal flows is outlined in general terms in Appendix A, and in detail in Appendix B. The method of solution for boundary layer growth in a conical diffuser with fully-developed flow at entry is presented in Appendix C.

3.2. METHODS OF SOLUTION OF THE NAVIER-STOKES EQUATIONS

For incompressible fluid flow the Navier-Stokes equations are written:

\[
\frac{D\mathbf{V}}{Dt} = -\frac{1}{\rho} \mathbf{V} \times \mathbf{p} - \nu \text{ curl curl } \mathbf{V} \]

3.2.1.

Methods of solution, from Thom's [28] work onwards, are summarised in Chu's [29] bibliography. It is attractive to consider
the consequences resulting from the numerical solution of the equation for turbulent flow but this is still scarcely a practicable possibility. The alternative is to write the turbulent flow relationships in their time averaged form. The equation can then be solved directly as though the flow were laminar or else the normal boundary layer simplifications can be made before the equation is put into numerical form. Either method entails the introduction of the eddy viscosity concept in some form to deal with the turbulence-induced inertia stresses. The boundary layer method leads to considerable computational simplification at the physical expense of reducing the problem to two dimensions and constraining the vorticity to a well-defined narrow region. The usual order-of-magnitude analysis is open to some question too, when applied to turbulent flows.

From Prandtl's boundary layer equation either a differential or an integral approach can be adopted and the various techniques commonly employed are well summarised by Reynolds [30]. The differential methods of prediction require an embarrassing amount of information about the physics of Reynolds stress development. Integral methods which have further advantages in simplicity, take for their starting point the Karman momentum equation which can be written as

$$\frac{d\theta}{dx} + \frac{\theta}{U_c} \frac{dU_c}{dx} (2 + H) = \frac{1}{\rho U_c^2} \left( \tau_w + K \frac{dP}{dx} \right) \quad 3.2.2$$

where \( P \) is the free stream total pressure and hence, within the potential core beyond the boundary layers in internal flows, \( \frac{dP}{dx} = 0 \). \( K \) is a constant whose value is dependant upon the boundary geometry, two-dimensional duct flow or axisymmetric flow being considered.

Doubts can be cast on the validity of the Karman momentum equation, particularly for turbulent flows approaching separation, as they do in diffusers. It is noted that for most of the experimental flows (selected for prediction comparison) presented at the 1968
Stanford boundary layer Conference, there is a difference in value between the left and right hand side of the distance-integrated Karman momentum equation. However, in spite of this, the integral boundary layer methods presented at the Conference gave a better prediction of experimental data, even in adverse pressure gradients, than one might have anticipated. Their virtue lies in their lack of sophistication and considerably more effort has to be expanded on a method which will predict boundary layer characteristics to a higher order of accuracy than will the integral techniques.

This Chapter therefore considers what problems in the internal flow domain could lead to successful analysis by such integral boundary layer techniques.

3.3. INTERNAL FLOW PROBLEMS

The solution of pressure drop problems in ducting and piping has been obtained for at least the last fifty years by summing the pressure losses developed across individual components. Appeal is made to the Moody Chart \([31]\) in establishing the loss in commercial piping and to such experimental results as can be found for the corresponding losses in fittings. The information used by practising engineers varies from the College hydraulics texts to quite sophisticated data sources, but the principle adopted is the same in all cases.

This lumped-parameter approach makes it necessary to introduce interference factors to account for the variation in characteristics when components are brought into close proximity. Obtaining sufficient design information to lead to highly accurate prediction calls for experimental data on a vast number of different combinations of fittings. Since evaluation with the precise combination required is rarely available experiment is often necessary. The experiment, conducted at known Reynolds number, would be to determine the pressure drop in a
component with known inlet flow conditions, possibly also simulating full-scale conditions of outlet.

It is rarely possible to compute this pressure drop directly because even in quite simple geometrical configurations such as a pipe bend the turbulent flow is strongly three-dimensional in character. However it is possible to predict behaviour for turbulent fluid flow through straight ducts and pipes and (under certain specified conditions) through diffusers. Though this is a very limited field it is of sufficient concern to warrant serious consideration. To carry out the described experiments for example it would be necessary to know how the developing boundary layer in a duct or pipe is related to its length.

As Barbin and Jones [9] and others [23, 27] have shown there are difficulties in the concept of fully-developed internal flows, but even allowing for these it is not easy to find an estimate for the length (le) required to reach this state from entry. Hinze [32] for example quotes Latzko and gives for turbulent flow in a pipe,

\[ \frac{le}{d} = 0.693 \text{Re}^{1/4} \]

though he states that the expression gives a 21-diameter entry length at \( Re = 9.0 \times 10^5 \), under-estimating Nikuradse's measured 40-diameter entry length.

In smooth wall pipes and ducts the pressure gradient is mildly favourable. Cockrell and Lee [33] using continuity, momentum and energy principles and assuming a one-seventh power law velocity profile obtained the approximate solution of le in a duct

\[ \frac{le}{W} = 2.18 \text{Re}^{1/2} \]

giving a 67-duct width entry length at a Reynolds number of \( 9.0 \times 10^5 \).

The corresponding approximate pipe flow relationship by this method would be

\[ \frac{le}{d} = 1.74 \text{Re}^{1/4} \]
There would seem to be merit therefore in evaluating the length required for turbulent boundary layers to fully develop in straight ducts and pipes. By making the walls diverge corresponding diffuser relationships are obtained. The integral prediction techniques give velocity profiles and corresponding pressure recovery coefficients down the length of the diffuser provided a free stream (or potential core) is still present and, with additional experimental data, even when this last condition is invalidated.

Sovran and Klomp's [34] survey of incompressible flow through diffusers consolidates experimental data in this field. But in generalising their results important assumptions have been made. One of these is that the effectiveness, \[ C_p \times \frac{1}{\sqrt{1 - \left(\frac{A_d}{A_i}\right)^2}} \] of a diffuser is sensibly independent of the inlet boundary layer thickness parameter or blockage factor B. Another is that the flow characteristics of diffusers with different cross-sectional geometries can be correlated using the inlet blockage factor to describe the state of flow there. A third is that when correlating experimental results from conical and two-dimensional diffusers, \( \frac{2N}{d_1} \) and \( \frac{N}{d_1} \) respectively, can be used as the appropriate axial length parameters. There is no doubt that all these generalisations are good engineering approximations for the data available to the author. Further data is not so amenable however, particularly when the boundary layer at the diffuser inlet is either extremely thin or very thick. For these reasons mathematical analysis of diffuser characteristics is desirable, even though this process is difficult for diffusers having other than a circular cross-section.

Since the form of analysis is restricted to two-dimensional or axi-symmetric boundary layer behaviour the main internal flow applications will be to flow development in ducts, pipes and diffusers. However even in this restricted field results which can be validated from reliable experimental data are of some real significance.
3.4. INTEGRAL BOUNDARY LAYER TECHNIQUES

3.4.1. Auxiliary Equations

Consider the solution of equation 3.2.2. when \( \frac{dp}{dx} = 0 \). If integral flow properties are required at known values of \( x \) the unknowns are the free-stream velocity \( U_c \) and the velocity profile parameters \( \delta^*, \theta \) and \( \tau_w \). The velocity profile could be specified by a one-parameter relationship such as a power law

\[
\frac{U}{U_c} = \left( \frac{y}{\delta} \right)^{\frac{1}{n}}
\]

3.4.1.

or quadratic law

\[
\frac{U}{U_c} = 1 - b \left( 1 - \frac{y}{\delta} \right)^2
\]

3.4.2.

(which is a form of Ling's [35] relationship reminiscent of Clauser's [36] constant eddy viscosity model). Thus three additional unknowns are introduced, \( U, \delta \) and a profile parameter \( n \) or \( b \). The usual expressions for \( \delta^* \) and \( \theta \) in terms of \( U, U_c, y \) and \( \delta \) give two more relationships (which now total four) with seven unknowns. Given the appropriate expressions for \( \delta^* \) and \( \theta \) when the flow is axisymmetric the method is as suitable for predicting these flows as it is for two-dimensional conditions.

The free-stream velocity \( U_c \) is expressed by the continuity equation in terms of \( \delta^* \) and the geometry of the problem. If required the pressure is related to the free-stream velocity and the total pressure \( P \) giving

\[
\frac{dp}{dx} + \rho U_c \frac{dU_c}{dx} = \frac{dP}{dx}
\]

3.4.3.

where, as with equation 3.2.2, the right-hand side is at present equated to zero. An empirical relationship such as the Ludwig and Tillmann correlation is required for \( \tau_w \) giving a total of seven relationships to solve for eight unknowns. A further auxiliary equation is therefore necessary to achieve the solution.
A simplification adopted by Bowlus and Brighton \cite{37} is to reduce the number of unknowns by one, thereby making the problem soluble at this stage, by taking the power-law exponent \( n \) as constant and equal to 7.

An alternative velocity profile relationship frequently used for the outer region is the law of the wake (Coles 1956)

\[
\frac{U_c - U}{u_*} = - \frac{1}{0.41} \ln \frac{y}{5} + \frac{\pi}{0.41} \left[ 2 - w \left( \frac{y}{5} \right) \right]
\]

where \( w \) is the wake function, which can be approximated by (Hinze 1959)

\[
w(\eta) = 1 - \cos(\eta)
\]

and the wake parameter \( \eta \) is an unknown function of \( x \). In the inner region it is usually assumed that the velocity profile should be of the "law of the wall form",

\[
\frac{\bar{U}}{u_*} = f \left( \frac{yu_*}{\nu} \right)
\]

In the region away from the wall the inner-region profile normally approaches the "log law"

\[
\frac{\bar{U}}{u_*} = \frac{1}{0.41} \ln \left( \frac{yu_*}{\nu} \right) + 5.0
\]

Furthermore, the condition that the inner and outer profiles asymptotically overlap can be used to obtain the implicit skin friction equation

\[
\frac{U_c}{u_*} = \frac{1}{0.41} \ln \left( \frac{u_*}{\nu} \right) + 5.0 + \frac{2\pi}{0.41}
\]

Thus once again there are seven relationships in toto to solve for eight unknowns, requiring the specification of an auxiliary equation.

Reynolds \cite{30} points out that two ways of generating this auxiliary equation are either to multiply the Prandtl boundary layer momentum equation by a weighting function before integrating it with respect to \( y \) or to integrate over only a segment of the boundary layer. Both are valid techniques, for in these ways some information can be recovered which would otherwise have been lost in the averaging process.
implicit in integration. In this way a large number of alternative relationships could be formed which have little to choose between them. Any one of them renders equation 3.2.2. soluble provided more shear stress information is available.

An alternative approach is to use an entrainment relationship. Entrainment of the non-turbulent by the turbulent flow as defined by Head [38] is the partaking of general motion of the turbulent flow by the neighbouring fluid and is a consequence of turbulent mixing. The entrainment velocity $E$ which is (in two-dimensional flow) the rate of change of the boundary layer volume flux $I = \int_0^5 u dy$, thus $E (dI/dx)$ is related by different methods to boundary layer mean flow parameters. If these are all known then equation 3.2.2. is soluble. Otherwise further supplementary information is necessary.

3.4.2. Choice of velocity profile

In the class of problems under consideration, for two-dimensional flows, the shape factor $H$ could vary from near 2.5 close to transition to about 1.4 in developing turbulent flow, increasing to a value of about 2.4 when the flow separates in a diffuser. If a power-law profile is used and the flow is two-dimensional, corresponding values of the power-law exponent $n$ are $4/3$, 5 and 2. This extreme range of values is in marked contrast to Bowlus and Brighton's [37] simplification where $n$ was maintained equal to 7.0 throughout their calculations.

Both the power law equation 3.4.1 and the Coles velocity profile equation have been used in these calculations. The effect of modelling the laminar sub-layer on boundary layer integral parameters is very small and has therefore been neglected in the present analysis.
3.4.3. Integral Auxiliary Relationship

Following Moses [39] a strip-momentum auxiliary equation was used. This calls for one other additional piece of information, the shear stress in the boundary layer at the top of the strip. Moses found that his method was relatively insensitive to strip width and like him the calculations were based on the region of the boundary layer closest to the wall equal to 0.3 of the total boundary layer thickness. The shear stress \( \tau \) at \( y/\delta = 0.3 \) was therefore required.

Fediaevsky [40] suggested that the non-dimensional shear stress in the boundary layer could be expressed as a function of \( y/\delta \). Writing

\[
\frac{\tau}{\tau_w} = a + b \left( \frac{y}{\delta} \right) + c \left( \frac{y}{\delta} \right)^2 + d \left( \frac{y}{\delta} \right)^3 + e \left( \frac{y}{\delta} \right)^4
\]

\[ 3.4.9. \]
a, b, c, d, and e can be determined from known boundary conditions to the problem. Alternative expressions for \( \tau \) at \( y/\delta = 0.3 \) are given from the assumptions of constant eddy viscosity or of constant shear stress close to the wall (which follows from assuming mixing length is proportional to distance from the wall) within the boundary layer. The simple eddy viscosity model used was that due to Moses

\[
\tau = \rho(\varepsilon + \nu) \frac{\partial U}{\partial y}
\]

and

\[
\frac{\varepsilon + \nu}{U \theta} = 0.0225 + \frac{125}{R_\delta}
\]

the latter equation being an approximation to constant pressure data given by Coles (1962). Fig. 3.1 shows these different shear stress expressions when the boundary layer is developing in a two-dimensional duct. The right hand side of the strip-momentum auxiliary equation is \( (\tau_w - \tau_{0.3}) \), and thus the correct choice of shear stress expression appears important.

In practice results obtained using the constant eddy viscosity
model were in all cases so much closer to experiment than those given by both the Pedeaevsky and constant shear stress models that these latter two were abandoned in favour of the former. The constant eddy viscosity model is discussed in the next section. Presumably the failure of the Pedeaevsky model occurs because of the truncation necessary to give a finite series expression. One would not expect to represent a turbulent mean velocity profile by as simple an expression as a quartic equation even though Pohlhausen was successful in representing a laminar velocity profile in this way.

3.4.4. Entrainment

In Head's [38] original entrainment method, it was postulated that

\[ \frac{E}{U_0} = \phi (H) \]  \hspace{1cm} 3.4.12.

when the function \( \phi \) is specified by experimental data. Various expressions have been proposed for \( \phi \) but since that quoted in reference [38] has been shown to give satisfactory agreement with experiment when the pressure gradient is zero, it was felt to be suitable for the present study where, in a long straight duct or pipe, the pressure gradient is only mildly favourable.

Basing their argument on the turbulent kinetic energy equation and the hypothesis that entrainment rate is proportional to a boundary layer turbulence parameter, Hirst and Reynolds [41] derived as an alternative to equation 3.4.12,

\[ I \frac{dF}{dx} = 0.14 \, u^* \, E^2 - \frac{E^2}{2} \]  \hspace{1cm} 3.4.13.

Since it can be argued that information about the flow history is conveyed through the turbulent kinetic energy equation their equation introduces a degree of sophistication into the prediction technique.
However, as discussed in section 3.4.5, it demands one additional starting condition and for this reason equation (3.4.13) was little used in the present analysis.

The constant eddy viscosity model would seem to be a rough and ready generalisation which can be employed where boundary layer growth is regular and relatively slow. The large eddies which then dominate the shear flow can be considered to extract energy from the mean flow by vortex stretching and in this way the physics of Head's entrainment process is closely related to that of the constant eddy viscosity model. Hama [42] has related the turbulent mean velocity profile shape factor H to the non-dimensionalised wall shear stress, and the latter can be related to local shear stresses and local values of turbulent kinetic energy. Thus the Hirst and Reynolds relationship between entrainment rate and boundary layer turbulence is plausible. The auxiliary relationships are therefore not describing different physical effects but closely-related flow phenomena and it is no accident if they are as good as one another in predicting turbulent layer growth.

3.4.5. Starting Conditions

To start the calculation the velocity profile at the datum stream-wise ordinate must be specified. If equations 3.4.1 or 3.4.2 were used the velocity profile would be specified by stating two independent parameters, say $a$ and $n$ for the power-law profile or, equally well, $b*$ and $g$. But if the profile were specified by equation 3.4.4 a third independent starting parameter would be required, say $b^*$, $g$ and the non-dimensional wall shear stress.

Known values may be taken for these starting parameters from experimental data if comparison is the objective. When evaluating entrance lengths in pipes and ducts however, small consistent values of the
starting parameters must be chosen and the ensuing relationship for $\delta$ in terms of the streamwise ordinate $x$ extrapolated back to the value of $x$ at which $\delta = 0$. The calculations were found to be remarkably insensitive to inconsistent values of the starting parameters.

If equation 3.4.13 is to be used as the entrainment relationship one additional starting condition is needed. This is the value of the entrainment velocity at the datum ordinate. Hirst and Reynolds determined this parameter from known experimental values of the boundary layer volume flux $I$ at several points upstream of datum. The required value of entrainment was then given as the derivative of $I$ at the datum point. They also stated that they found results were fairly insensitive to changes in this value of entrainment.

Since they hypothesise that the entrainment rate is proportional to a boundary layer turbulence parameter it is unlikely that normal conditions in ducts and pipes would give rise to very large changes in initial entrainment rate. But if the turbulence level were to be raised artificially (by vortex generators for example as are sometimes used to simulate an atmospheric boundary layer in a wind tunnel) the change in initial entrainment rate would be considerable.

Using equation 3.4.13 to model boundary layer growth in a long duct Cockrell [43] found that a three-fold increase in initial entrainment rate reduced the non-dimensional boundary layer entrance length $Le/N$ to about three-quarters of its original value.

3.5. NEAR FULLY-DEVELOPED AND FULLY-DEVELOPED TURBULENT FLOW

When laminar boundary layers, developing in ducts or pipes, merge the flow may be considered fully-developed. This situation is not paralleled in turbulent flow. For laminar flows the velocity and the stress fields develop simultaneously for the stress is proportional
to the velocity gradient. But since no such simple relationship exists, for turbulent flows to some extent the two fields develop independently.

Of course the mathematically-convenient concept of a free-stream boundary to the developing viscous region is highly suspect, and conditions based on behaviour at such a boundary often have little physical reality. But assuming such a boundary exists, experimental evidence by Comte-Bellot [44], by Barbin and Jones [9] and by Lee [27] shows that when the boundary layers may first be said to have merged \( \partial \tau / \partial y \) on the duct or pipe centre-line is zero, thus demonstrating that in growing out from the wall the Reynolds stresses can be seen to lag behind the corresponding mean velocity profile. A gradual decrease in value of the shear stress gradient then occurs downstream from the merge point until this value in the duct or pipe becomes constant and equal to \( -2\tau_w / \text{N} \) or \( -2\tau_w / \text{d} \) respectively.

Since \( \partial \tau / \partial y = \partial P / \partial x \) on the centre-line becomes less positive as the centre-line shear stress develops there is a consequent deceleration there, accompanied by an acceleration in the flow closer to the wall. This implies a reduction in the boundary layer thickness parameters. Thus as the turbulent boundary layer develops down the duct or pipe the physical layers will merge and the thickness parameters will then decrease to an equilibrium value as shown by Lee [27] and the present author [23]. It is not until this value has been attained that the flow may be said to be fully-developed.

The simple mathematical models proposed for shear stress behaviour do not concede the possibility of this shear lag and the alternative approach through entrainment principles, would appear to be invalid when there is no free stream to entrain from. Thus the 'overshoot' phenomenon of boundary layer development in internal flows cannot be modelled by these techniques.

Some study of flow development after the boundary layers have
merged is however possible by integral methods. Applying the momentum equation along the duct centre-line gives in two dimensions

$$\frac{\partial}{\partial x} (p + \frac{1}{2} v^2) = \frac{dP_c}{dx} = \frac{\partial r}{\partial y} + \rho v \zeta \quad 3.5.1.$$ 

where $P_c$ is the centre-line total pressure, the transverse velocity $v$ and the vorticity $\zeta$ are all centre line evaluations. Since $v\zeta$ will in general be negligibly small,

$$\frac{dP_c}{dx} = \frac{\partial r}{\partial y} \text{ in two dimensions} \quad 3.5.2.$$ 

$$= 2 \frac{\partial r}{\partial r} \text{ for axi-symmetric flow} \quad 3.5.3.$$ 

$\frac{dP_c}{dx}$ would then be substituted into the Karman momentum equation 3.2.2. in place of $\frac{d}{dx}$ and $K$ equated to $\frac{W}{2}$: for a duct or $\frac{d}{4}$ for a pipe.

Making use of the above analysis it is possible to predict boundary layer variation along conical diffusers with fully-developed flow conditions at inlet. Furthermore, this analysis leads to an estimation of the diffuser pressure recovery.

3.5.1. Diffuser performance with fully-developed flow at inlet

Stevens [45] has indicated that turbulent boundary layer integral methods can be used to study the flow development along conical diffusers with fully-developed flow conditions at inlet. For such cases the axi-symmetric integral equation can be written,

$$\frac{d \theta}{dx} + \theta \left[ R \frac{1}{R} \frac{dx}{dx} + \frac{1}{U_0} \frac{dU_0}{dx} (2 + H) \right] = \frac{1}{\rho U_0^2} \left[ \frac{1}{R} \frac{dP_c}{dx} \right] \quad 3.5.4.$$ 

where $P_0 = \text{total pressure on diffuser centre-line}.$

As the diffuser geometry is known, and the physical thickness can be considered to be equal to the diffuser local radius, then equation 3.5.4 (when coupled with the continuity equation and a suitable skin friction law) is solvable once $\frac{dP_c}{dx}$ is evaluated. The evaluation
of this total pressure drop term will be discussed in the following section.

3.6. RESULTS OF THE INTEGRAL BOUNDARY LAYER PREDICTION

The Karman momentum equation 3.2.2 and the appropriate auxiliary relationship were set in finite difference form and then solved using a fourth-order Adams-Bashforth predictor-corrector technique with a two-point predictor-corrector for starting purposes. The routine employed had a step size control to keep absolute and relative errors within specified bounds. The simple computational method is outlined in general terms in Appendix A. The various flow equations and empirical assumptions used are reduced to a form suitable for computer solution in Appendix B.

For comparison of the theoretical predictions with experiment the data sources of Figure 3.2 have been considered. They are amongst the best relevant experimental information on boundary layer growth in ducts, pipes and conical diffusers.

In Figure 3.3 predictions of two-dimensional boundary layer growth in a duct are compared with Lee's work. The figure shows predicted results using both the strip momentum and entrainment auxiliary relationships. It can be seen that there was good agreement between the predictions. Furthermore, the choice of velocity profile assumption (one parameter power law or two parameter Coles profile) made a negligible difference to the predicted results. However, as expected, the shear stress assumption was very important. The Fediaevsky hypothesis continually proved to be an extremely poor model whereas the constant eddy viscosity model gave surprisingly good predictions and therefore all the results presented here have the latter as a model for the shear stress in the strip-momentum equation.
Lee's experimental results were obtained for a developing turbulent boundary layer in a rectangular duct of aspect ratio 3. As the distance down the duct increases the lack of two-dimensionality in this experimental work becomes evident. Apart from this disparity the agreement between prediction and experiment is good. Figure 3.3 also indicates the boundary layer extrapolation procedure which was necessary to assess the position of the datum boundary layer growth point: the mathematical calculations being started from the axial position of the initial experimental points.

In Figure 3.4 axially-symmetric flow in a pipe is compared with the excellent experimental work of Barbin and Jones. Here experimental and predicted velocity profiles at stated distances from the datum are compared. A log-law velocity profile and strip momentum auxiliary equation were used. All the flow equations and the boundary layer thickness parameters were used in their axially symmetric form. Agreement again is good. It is only for the final velocity profile at \( \frac{X}{D} = 40.5 \) that there is an observable difference between theory and experiment and this is due to neglecting the centre-line total pressure drop term which becomes significant around this axial point.

Fraser's conical diffuser experimental results were presented to the 1968 Stanford Boundary Layer Prediction Conference. They are ideal for the present purpose of prediction validation. The data is well-authenticated whilst the entry boundary layer is very thin so that the potential core is maintained in the diffuser. The flow characteristics for both Fraser, Flow A and Fraser, Flow B were extremely poorly predicted by most of those taking part at Stanford. The experimental results of Fraser are for the flow in a conical diffuser made up of a straight pipe, short transition section, and 10-degree included-angle cone. The turbulence level of the flows is unknown, however, for Flow B it is probably high, as the second honeycomb and the two screens used
For Flow A were not present.

For the Fraser flows, and subsequent diffuser predictions, the auxiliary equation employed was the axially-symmetric strip momentum equation with the shear stress at 0.35 evaluated using the constant eddy viscosity model. The mean velocity profile assumption was that due to Coles.

Figures 3.5 - 3.8 compare mean flow integral boundary layer characteristics and pressure recovery coefficient \( C_p \) for both Fraser flows. In all cases agreement is seen to be extremely good. The method is further vindicated in Figure 3.9 in which predicted velocity profiles along the diffuser are compared with Fraser's (Flow B) experimental measurements.

Robertson and Holl's diffuser experiments were conducted with a boundary layer thickness \( \delta = 0.3R \) at the diffuser inlet. The inlet pipe and 7.5-degree included angle diffuser were coupled by a short transition section. The diffuser was terminated by a tail pipe in which the process of diffusion continued, the change in boundary contour causing an abrupt change in pressure gradient. Predictions are compared with experimental measurements in Figures 3.10 - 3.12. The predictions were started at the diffuser inlet. The letter T marks the position of the diffuser exit and start of the tailpipe for Phase III and Phase IV flows. Though not agreeing so closely as in the case of Fraser, this set of experimental results are well predicted.

Cockrell's experiments were conducted with a variety of inlet boundary layer thicknesses and diffuser included angles. The 4-degree included angle results are chosen for comparative purposes. The variation of diffuser pressure recovery with inlet boundary layer thickness is compared with Cockrell's experimental results in Figure 3.13. It can be seen that agreement is less good than in the other two examples quoted. This may be due in part to the poorer quality of the experimental data. However, as the predicted results diverge increasingly
from the experimental points as the inlet boundary layer is thickened this is probably an indication of potential core decay. The fully-developed inlet flow experimental points of the present author are also plotted in Fig. 3.13 and can be seen to give consistently higher pressure recovery's than the corresponding points of Cockrell.

To predict diffuser characteristics with fully-developed flow at the inlet, Stevens [45] used a power law boundary layer velocity profile with exponent parameter $n = 7.0$ at entry and assumed that $\frac{\partial \tau}{\partial y} = -2\tau/\delta$. Published results by Robertson and Calehuff [50] show that this assumption is a considerable underestimate of the actual algebraic value. Alternative assumptions have been examined by the author. They were that $\frac{dP}{dx}$ was given by

(i) Stevens experimental results
(ii) Constant eddy viscosity model

and finally
(iii) that it could be neglected.

The estimation of $\frac{dP}{dx}$ and the equations used to predict diffuser performance with fully developed flow at inlet are outlined in Appendix C. The above $\frac{dP}{dx}$ estimates are compared with Stevens' experimental results in Figure 3.14. Not surprisingly the experimentally-derived evaluation lead to the best fit for the development of shape factor ($H$) down the 10-degree total angle diffuser. What was surprising was that when it was assumed that the total pressure drop was zero the results obtained for the shape factor development were comparable with the theoretical results by Stevens, the maximum error between experiment and theory being some $6\%$.

Stevens diffuser efficiency ($\eta$) characteristics were also examined. Since these are a function of pressure recovery and the pressure recovery is related to the mean velocity characteristics through the centre-line total pressure loss it was anticipated that these too would vary with the assumptions made about this loss. However insensitivity to the assumptions was found, in no case was the error from the
experimental assessment greater than 5%. Fig. 3.14 shows the comparison of predicted and experimental diffuser efficiency ($\eta^*$) for the case where $\frac{dP_o}{dx}$ was given by an equation which fitted Stevens measured centre-line total pressure drops. Once again the diffuser performance is well predicted.

3.7. DISCUSSION OF RESULTS

From consideration of developing flow in smooth two-dimensional ducts and smooth pipes, covering a Reynolds number range from $10^3$ to $10^6$, the present prediction method (utilising the strip momentum equation and eddy viscosity shear stress model) gives for the length, $l_e$, taken for the boundary layers to merge, in two-dimensional flow

$$\frac{l_e}{\lambda} = 14.34 (Re)^{0.09}$$  \hspace{2cm} 3.7.1.

instead of equation 3.3.2 and for axi-symmetric flow

$$\frac{l_e}{d} = 9.71 (Re)^{0.11}$$  \hspace{2cm} 3.7.2.

instead of equations 3.3.1, and 3.3.3. Equation 3.7.1 gives a 50-duct width entry length at a Reynolds number of $9.0 \times 10^5$ compared to 67-duct widths from equation 3.3.2. Equation 3.7.2 gives a 47-diameter entry length at the same Reynolds number compared to Nikuradse's 40-diameter entry length.

With the limitations previously stated the integral technique appears very suitable for predicting conical diffuser characteristics. It should be noted that although it differs in only minor details from Moses' method[39] the predicted results for Fraser's diffuser characteristics are considerably better than those Moses obtained. Possibly Moses followed the approximate procedure suggested by the Stanford Conference organisers who proposed that an axi-symmetric flow problem
should be regarded as pseudo-two-dimensional rather than using axi-symmetric definitions for quantities in the appropriate flow equations.

It is surprising that such imprecise expression for shear stress distribution as is given by the constant eddy viscosity model satisfactorily predicts flow characteristics in a strongly adverse pressure gradient. This may be due in part to the fact that the strip momentum equation, only demanding knowledge of the shear stress at one point in the boundary layer, minimises the errors inherent in estimating this quantity.
FIG. 3.1. SHEAR STRESS DISTRIBUTION IN THE BOUNDARY LAYER
<table>
<thead>
<tr>
<th>Type of Flow</th>
<th>Originator</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-dimensional duct flow</td>
<td>LEE</td>
<td>[27]</td>
</tr>
<tr>
<td>Axi-symmetric pipe flow</td>
<td>BARBIN &amp; JONES</td>
<td>[39]</td>
</tr>
<tr>
<td>Conical diffuser characteristics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thin inlet boundary layer</td>
<td>FRASER</td>
<td>[46]</td>
</tr>
<tr>
<td>Variety of inlet boundary layer thicknesses</td>
<td>ROBERT &amp; HOLL</td>
<td>[47]</td>
</tr>
<tr>
<td>Variety of inlet boundary layer thicknesses</td>
<td>URAM</td>
<td>[48]</td>
</tr>
<tr>
<td>Variety of inlet boundary layer thicknesses</td>
<td>COCKRELL</td>
<td>[24]</td>
</tr>
<tr>
<td>Variety of inlet boundary layer thicknesses</td>
<td>MILLER</td>
<td>[49]</td>
</tr>
<tr>
<td>Fully developed inlet boundary layer thickness</td>
<td>STEVENS</td>
<td>[45]</td>
</tr>
</tbody>
</table>

**FIG. 3-2** SOURCES OF EXPERIMENTAL DATA FOR PREDICTION COMPARISON.
FIG. 3-3  TWO-DIMENSIONAL BOUNDARY LAYER GROWTH IN A DUCT
FIG. 3-4. AXI-SYMMETRIC BOUNDARY LAYER GROWTH IN A PIPE
FIG. 3-5 FRASER: FLOW A. BOUNDARY LAYER GROWTH IN A 10-DEG. TOTAL ANGLE CONICAL DIFFUSER
FIG. 3-6 FRASER: FLOW A. PRESSURE RECOVERY AND WALL SHEAR STRESS VARIATION IN 10-DEG. CONICAL DIFFUSER
FLOW B

FIG. 3-7 PREDICTION OF FRASER'S MEAN FLOW PARAMETERS IN A 10-DEG. DIFFUSER
FIG. 3-8 PREDICTION OF FRASER S CP IN A 10-DEG. DIFFUSER
FIG. 3-9 PREDICTION OF FRASER'S VELOCITY PROFILES IN A 10-DEG. DIFFUSER
FIG. 3-10  PREDICTION OF ROBERTSON AND HOLL'S (Phase II) MEAN FLOW PARAMETERS IN A 7-5-DEG. DIFFUSER.
FIG. 3-11 PREDICTION OF ROBERTSON AND HOLLIS (Phase III)
MEAN FLOW PARAMETERS IN A 7.5-DEG. DIFFUSER
FIG. 3-12. PREDICTION OF ROBERTSON AND HOLL'S (Phase IV)
MEAN FLOW PARAMETERS IN A 7-5-DEG. DIFFUSER
FIG. 3.13 PREDICTION OF COCKRELL'S $C_p$ IN A 4-DEGREE DIFFUSER
FIG. 3.14. PREDICTION OF FLOW IN A 10-DEGREE CONICAL DIFFUSER WITH FULLY-DEVELOPED FLOW AT INLET

\[ \frac{dP_c}{dx} \text{ ASSUMPTION} \]

- a - Using stevens exp.tl.values
- b - Pipe flow (2Tw/D)
- c - Constant eddy viscosity
- d - Equal to zero

Experiment (STEVENS)
CHAPTER 4

EXPERIMENTAL LAYOUT AND TECHNIQUE
4.1. SCOPE OF EXPERIMENTAL WORK

At a known and approximately constant Reynolds number, Re, it is required to determine the performance of a series of conical diffusers of varying expansion angle, $\phi$, and area ratio, $A_o/A_i$. This performance is to be determined with known mean and turbulence parameters at inlet to the diffusers. In the course of experimental investigation it is required to measure mean velocity profiles, the accompanying Reynolds stresses, $-\overline{\rho u'v'}$, and the longitudinal and transverse intensities of turbulence, $\overline{u'^2}$ and $\overline{v'^2}$.

The diffuser inlet flow conditions will be due to

1. the fully-developed flow of natural boundary layer growth along an upstream smooth pipe, or
2. a velocity profile having the same shape as the natural fully-developed profile but with a much higher level of turbulence, produced by a generator as illustrated in Fig. 4.5.(a) and (b), or
3. the fully-developed flow of natural boundary layer growth along an upstream roughened pipe.

4.2. DESIGN OF EXPERIMENTAL APPARATUS

4.2.1. Experimental Apparatus used for preliminary measurements

Preliminary experimental tests were carried out at a Reynolds number of $1.8 \times 10^5$ in a 2-inch diameter perspex pipe airflow rig previously used and described in detail by Cockrell[24]. Fig. 4.1. shows the experimental set-up for (a) approach length variation, and (b) the fully-developed mean velocity profile generating system. The profile generating system consisted of an 8-inch high annular step followed by eight $\frac{3}{4}$-inch high $45^\circ$ triangular vortex generators. The vortex generators were equally spaced around the pipe circumference and
set in pairs at approximately $\pm 5^\circ$ angle of incidence to the pipe axis. The generating system produced considerable streamwise vorticity in local regions but the net streamwise vorticity was very small.

4.2.2. Apparatus used for main experimental measurements

A new airflow rig was designed and constructed which produced good axially symmetric flow and provided suitable access for measurements close to the wall.

Sufficient length of piping was required to enable the mean and turbulence parts of the resulting airflow to be in equilibrium, i.e. further lengths of pipe made no detectable changes to the mean and turbulence flow characteristics. From the preliminary tests 100 pipe-diameters was considered adequate. A pipe bore of 8-inches was chosen as this was sufficiently large for accurate manufacturing and setting of the vortex generators. Furthermore, an 8-inch inlet contraction was readily available.

The airflow rig comprised an inlet contraction; piping; conical diffuser; settling length of pipe; and fan and hydraulic motor. Its maximum overall length was 110-feet approximately. As the rig was too long for the existing low speed Aerodynamics Laboratory it was constructed on an open gallery directly below the laboratory. The fan and motor were coupled to the hydraulic system which powered the large wind tunnel in the Aerodynamics Laboratory.

As many of the rig components as possible were constructed in fibreglass. This material was very suitable for the present rig as it is relatively inexpensive, light in weight, and can be manufactured accurately.

The general layout of the airflow rig is shown in Figs. 4.2 and 4.3. The 8-inch diameter pipe lengths, the diffuser and settling length of pipe between the diffuser and fan are supported by Handyangle
frame\textsuperscript{5} with foam rubber at the points of contact between the pipes and the frame, and cork pads at the base of each frame. These precautions minimised any pipe vibrations.

The 8-inch thick, 8-inch dia. fibreglass piping was constructed in 17,4-feet lengths (6 pipe-diameters) and flanged at each end. Thus a possible 102 pipe-diameters in length could be utilised. Care was taken to ensure that all duct components fitted well. All joints had a rubber gasket and a dowelling peg fitted to ensure that they were air-tight and accurately aligned. Each pipe section had a measuring port midway along. On certain pipes two ports perpendicular to each other were constructed at the mid-section. The pipe length shown in Fig. 4.4 had an extra measuring port for velocity traverses at the diffuser inlet. When the measuring port was not in use an aluminium bung (shown in the foreground of the photograph) replaced the automatic traverse gear.

Four static pressure taps at 90\degree to each other were positioned at the cross-section of the measuring element. The dimensions of each tap gave a ratio of hole diameter to length of approximately 3, as recommended by McDiou & Rankin\textsuperscript{3}. Static pressure taps were also positioned along each diffuser and settling length pipe.

The fibreglass conical diffusers which had total angles of 4\degree, 8\degree, 10\degree, 15\degree and 21\degree were constructed in three sections to give possible area ratios, \(A_0/A_1\), of 2.25, 5.05 and 7.55. The three pipe settling lengths corresponding to the outlet diameter of each diffuser section were constructed in steel. These outlet diameters were 12, 18 and 22-inches, and each pipe was 6 pipe-diameters in length.

The diffuser outlet settling length was connected by flexible hose to the eye of a radial flow fan. With a mean velocity of 150ft/sec and a discharge through the apparatus of about 3000 ft\textsuperscript{3}/min the losses from the system were approximately 8-inches of water. A suitable fan was selected having a circular flanged inlet of 22-inches. The fan was
driven by a hydraulic motor, which was coupled to the Aerodynamics laboratory hydraulic system.

Air entered the 8-inch dia. pipe through a large 8:1 shaped fibreglass contraction via a 52-feet square wood settling chamber. This chamber had Dufaylite honeycomb fitted at its entrance and exit, a cloth sheet at its entrance and a stretched linen sheet midway along it to filter dirt particles present in the air.

The experimental set-up for the profile generating system to produce a "fully-developed" shaped mean velocity profile with a much higher turbulence level at the diffuser inlet is shown in Fig. 4.4. The brass protractors, which allow for accurate setting of the vortex generator angle of incidence, are located around the circumference of the 8-inch length of 8-inch dia. aluminium piping. Fig. 4.5. (a) shows the profile generating system in detail. It consists of eight vortex generators spaced 7-inches apart; an annular step with a thin inner circular rod attached; and 7 pipe-diameters of aluminium oxide paper around the inside of the pipe leading to the diffuser inlet. The paper had an approximate roughness height of 0.0075. The final dimensions of the profile generating system are given in Chapter 5.

For the fully-developed pipe flow case of boundary layer growth along a roughened pipe, the inside of the pipe was lined with the same aluminium oxide paper as described above.

The radial traverse gear shown in Fig. 4.4. and in more detail in Fig. 4.6. is basically a travelling nut on a rotating lead screw of 20 turns per inch driven by a 24 volt motor. The distance traversed across the pipe is automatically controlled by a stepping circuit which stops the traverse after any pre-set number of turns. The circuit also has a variable delay switch thus allowing an adjustable delay time to elapse when making turbulence measurements but giving a shorter delay when mean velocity measurements are being made.
4.3. MEASUREMENT OF MEAN FLOW PARAMETERS

Mean velocity profiles at various cross-sections along the pipe were measured with a pitot tube. Wall pressure tappings were used for static measurements in the smooth straight pipe. However, with roughened pipes and behind the velocity profile-turbulence generating system, static tubes were used. Pitot and static tubes were manufactured from stainless steel hypodermic tubing having outside diameters of 0.065-inches and 0.1-inches respectively. Static holes were sited in accordance with the recommendations of Pankhurst and Holder [52]. For the wall shear stress measurements the pitot tube had an outside diameter of 0.065-inches, the inside to outside diameter ratio being approximately 0.6 as recommended by Preston [53]. To measure swirl after the vortex generators along the pipe leading to the diffuser inlet, a claw-type yaw probe with maximum sensitivity to flow direction was produced to the design specifications given by Schulze, Ashby and Erwin [54].

All pressure measurements were recorded on a methylated spirit multitube manometer inclined at 30° to the horizontal. However the pressure measurements, used to evaluate the pressure recovery data for the conical diffusers, was recorded on the more accurate Betz distilled water manometer.

The barometer temperature and pressure, and the air temperature at the inlet settling chamber were recorded before and after each traverse. Using the above data the raw experimental measurements of pressure and velocity were corrected for (i) density variations along the rig and (ii) static pressure variation due to changes in atmospheric pressure. A data reduction computer programme corrected the raw experimental velocity measurements and then evaluated the resulting boundary layer integral thicknesses.
4.4. MEASUREMENT OF TURBULENCE PARAMETERS

Hot-wire anemometry was used for the measurements of turbulence in the experimental programme. The anemometer system consisted of a DISA 55A01 constant temperature anemometer in conjunction with a DISA 55D10 Lineariser. An oscilloscope and a Thermo-Systems Inc. model 1060 R.M.S. voltmeter were used as turbulence output devices. The true R.M.S. voltmeter utilised a wide variety of integrating times ranging from 0.1 to 100 seconds. The DISA 55A25 straight-wire and the DISA 55A29 slant-wire miniature probes were used as they were the smallest hot-wires which could be repaired. The repairs being accomplished using a DISA 55A11 Micromanipulator in conjunction with a suitable welding current generator. The static calibration of the hot-wires was performed in the DISA 55Q41/42 calibration tunnel.

The hot-wire probes were made from 5-micron diameter platinum-plated tungsten wire. They had an aspect ratio of approximately 200 and were operated at a resistance 1.8 times the cold resistance of the wire. It was found that an integrating time of 30 seconds was sufficient for consistent readings of the hot-wire output as a longer time left the reading unaltered. Both the airflow rig and the electronic equipment were allowed to warm up before readings were recorded.

To minimise contamination of the hot-wires by dust particles filter cloths were fitted to the inlet settling chamber of the rig. These proved quite successful and as a further precaution each wire was ultrasonically cleaned in trichloroethylene before and after each run.

In turbulence measurement the accuracy of the final result depends considerably on the calibration of the hot-wire. Hot-wires need to be individually calibrated in a velocity field having a moderately low level of turbulence. In this investigation the hot-wires were calibrated at the centre of the pipe. However, as the hot-wire output
was linearised using the DISA Lineariser each wire was initially tested in the DISA calibration tunnel to choose the lineariser exponent setting giving the best linearity. Typical exponent settings were in the range 2.1 to 2.3. The non-linear response of hot-wires corresponding to an exponent setting of 2.3 was,

\[ E^2 = a + b u^{0.435} \]  \hspace{1cm} 4.4.1.

Fig. 4.7. shows a typical linearised calibration curve for a normal wire and a check on its linearisation made during a traverse.

There have been many publications dealing with the causes of the errors that arise in the measurement of turbulence. As they have been summarized by Lee [27] and dealt with in more detail by Hinze [32] they will only be listed here.

Major causes are (i) high turbulence intensity (ii) directional sensitivity of inclined wires.

Minor causes are (a) ambient temperature fluctuations (b) wall proximity (c) drift caused by dust (d) finite wire length (e) end effects

Whilst the error in the results due to (a) to (e) has not been corrected for, it has been kept to a minimum by the choice of the hot-wire probes and the precautions taken against dust deposit. Furthermore, no correction has been made for the effect of high turbulence intensity which would occur directly after the vortex generators and probably in the wall region of a pipe. Turner [55] has shown that the magnitude of the correction to the anemometer signal is significant whenever the turbulence intensity exceeds 20%. However the error due to the directional sensitivity of the inclined wire will now be discussed further and accounted for in the analysis.
Assuming that the hot-wire output has been linearised the response equation for the hot-wire can be written as,

\[ E = AU + B \]

where \( E \) is the hot-wire anemometer bridge D.C. voltage, and \( A \) and \( B \) are constants (\( B \) is usually biased to zero). If the wire is yawed through an angle \( \alpha \) then the effective cooling velocity is a function of both the normal and parallel components of mean velocity. Thus for a slanting-wire response equation 4.4.2 may be written as,

\[ E = A(\alpha)U + B \]

In many applications normal component cooling or cosine law cooling has been assumed, i.e.

\[ A(\alpha) = A_1 \cos \alpha \]

However it has been shown that for \( \alpha > 60^\circ \) this assumption can introduce a considerable error. The two most applicable alternative expressions for \( A(\alpha) \) are

\[ A(\alpha) = A_1 (\sin^2 \alpha + k^2 \cos^2 \alpha)^{\frac{1}{2}} \]

and

\[ A(\alpha) = A_1 \cos^{m} \alpha \]

where in general the yaw parameters

\[ (k, m) = \text{function } (L/D, \alpha, U, T_w/T) \]

where \( L/D \) is the length to diameter ratio of the hot-wire, and \( T_w/T \) is the ratio of average temperature of the hot-wire to the temperature of the air.

According to Hinze [32], \( k \) lies between 0.1 and 0.3, increasing as Reynolds number decreases. Webster [56] gives \( k \) equal to 0.2. From the measurements of Champagne [57] \( k = 0.2 \pm 0.04 \) for platinum wires having a length to diameter ratio (as in the current investigation)
of 200. In general Champagne found that $k$ decreased as $L/D$ increased
tending to zero as $L/D$ becomes large.

Equations 4.4.3 and 4.4.5 combine to give,

$$E = A_1 (\sin^2 \alpha + k^2 \cos^2 \alpha)^{1/2} U + B$$  \hspace{1cm}  4.4.8.

and for small fluctuations in velocity about the mean equation 4.4.8 becomes,

$$\delta E = A_1 \left[ \frac{(\sin^2 \alpha + k^2 \cos^2 \alpha) \delta U + \frac{1}{2}(1-k^2) \sin 2\alpha U \delta \alpha}{(\sin^2 \alpha + k^2 \cos^2 \alpha)^{3/2}} \right]$$  \hspace{1cm}  4.4.9.

If the fluctuating components $u$, $v$ and $w$ are small compared with the
mean velocity $U$ then,

$$\delta E = e, \quad \delta U = u \quad \text{and} \quad U \delta \alpha = v$$  \hspace{1cm}  4.4.10.

Substituting equations 4.4.10 into equation 4.4.9,

$$e = A_1 \left[ \frac{u (\sin^2 \alpha + k^2 \cos^2 \alpha) + \frac{1}{2} v (1-k^2) \sin 2\alpha}{(\sin^2 \alpha + k^2 \cos^2 \alpha)^{3/2}} \right]$$  \hspace{1cm}  4.4.11.

Squaring equation 4.4.11 and taking the time average yields,

$$\overline{e^2} = A_1^2 (\sin^2 \alpha + k^2 \cos^2 \alpha) \left[ \overline{u^2} + (1-k^2) \sin \cos \alpha \cdot 2 \overline{uv} + \frac{(1-k^2) \sin^2 \alpha \cos^2 \alpha}{(\sin^2 \alpha + k^2 \cos^2 \alpha)^2} \cdot \overline{v^2} \right]$$  \hspace{1cm}  4.4.12.

Using the two readings of $\overline{e^2}$ taken by reorientating the single
slanting wire in the plane defined by the wire and the mean velocity $U$
from an angle of $+\alpha$ to $-\alpha$ equation 4.4.12 becomes for the two cases
respectively,

$$\overline{e_1^2} = A_1^2 (\sin^2 \alpha + k^2 \cos^2 \alpha) \left[ \overline{u^2} + (1-k^2) \sin \cos \alpha \cdot 2 \overline{uv} + \frac{(1-k^2) \sin^2 \alpha \cos^2 \alpha}{(\sin^2 \alpha + k^2 \cos^2 \alpha)^2} \cdot \overline{v^2} \right]$$  \hspace{1cm}  4.4.13.
and,

\[
e_2^2 = A_1^2 (\sin^2 \alpha + k^2 \cos^2 \alpha) \left[ u^2 - \frac{(1-k^2) \sin \alpha \cos \alpha}{(\sin^2 \alpha + k^2 \cos^2 \alpha)} \cdot 2uv + \frac{(1-k^2 \sin^2 \alpha \cos^2 \alpha)}{(\sin^2 \alpha + k^2 \cos^2 \alpha)^2} \cdot v^2 \right]
\]

4.4.14.

By subtraction and addition of equations 4.4.13 and 4.4.14 and substitution of equation 4.4.5 both the Reynolds shear stress and the transverse intensity of turbulence are evaluated, giving

\[
\overline{uv} = \left[ \frac{\overline{e_1^2} - \overline{e_2^2}}{4A^2 \cot \alpha} \right] \left[ \frac{1 + k^2 \cot^2 \alpha}{1 - k^2} \right] 4.4.15.
\]

and

\[
\overline{v^2} = \left[ \frac{\overline{e_1^2} + \overline{e_2^2}}{2A^2 \cot \alpha} - \frac{u^2}{\cot^2 \alpha} \right] \left[ \frac{1 + k^2 \cot^2 \alpha}{1 - k^2} \right]^2 4.4.16.
\]

and furthermore, for \( \alpha = 90^\circ \), a normal wire, equation 4.4.13 gives,

\[
\overline{u^2} = \frac{\overline{e_1^2}}{A^2} 4.4.17.
\]

The second bracket in equations 4.4.15 and 4.4.16 represents the correction due to the effect of the parallel component of velocity.

Choosing a representative value for \( k \) of 0.2 and using \( \pm 45^\circ \) slant-wires the deviation from the simple normal component cooling assumption is 8.4% in \( \overline{uv} \) and 17.8% in \( \overline{v^2} \). Bruun[58] using the alternative assumption for \( A(\alpha) \) given by equation 4.4.5, obtained the corresponding deviations in \( \overline{uv} \) and \( \overline{v^2} \) of 11% and 21%. Thus it can be seen that the two assumptions for \( A(\alpha) \) taking into account both normal and parallel component cooling are in close agreement. However the present investigation follows Champagne[57] and uses equation 4.4.15 to 4.4.17, to determine the Reynolds normal and shear stresses.
(a) $(\frac{L_e}{d_i})$ varied from 1.5 to 126

APPROACH LENGTH $L_e$

6d₀ SETTLING LENGTH

Pitot traverse

2" DIA. PIPE

21° DIFFUSER

AREA RATIO $A_0 = \frac{7.55}{A_0}$

AIR

(b) 7½ pipe diameters

2" DIA

DISC, 1 3/4"

Internal dia.

Eight 3/4" x 3/4" Triangular plates, set at approx.

+5 deg. & -5 deg. alternately

**FIG 4.1** EXPERIMENTAL SET-UP

(a) FOR APPROACH LENGTH VARIATION.

(b) FOR MEAN VELOCITY PROFILE GENERATION
A. Hydraulic Motor
B. Radial flow fan
C. Flexible coupling
D. 6 diameters of settling pipe
E. Conical diffuser

F. Variable length of 8 inch dia. piping
   102d used for producing natural "Fully-dev.Flow"
   7d used for artificially produced "Fully-dev. Flow"

G. 8:1 Circular inlet contraction.

H. Square inlet settling chamber with dust filter cloth
   of entry and midway along

I. Dufaylite honeycomb at entry and exit

FIG 4-2 8 inch DIA. PIPE - DIFFUSER RIG
FIG. 4.3. LAYOUT OF PIPE-DIFFUSER RIG
FIG. 4.4. EXPERIMENTAL SET-UP FOR PRODUCTION OF HIGHLY TURBULENT FULLY-DEVELOPED INLET VELOCITY PROFILE
FIG. 4.5(a) MEAN VELOCITY PROFILE GENERATOR
FIG. 4.5.(b) MEAN VELOCITY PROFILE GENERATOR
FIG. 4.6. TRAVERSE GEAR
Fig. 4.7  Typical linearized calibration for a normal hot-wire.
CHAPTER 5

EXPERIMENTAL WORK:

THE RESPONSE OF DIFFUSERS TO FLOW CONDITIONS

AT THEIR INLET
5.1. INTRODUCTION

The available experimental data on the response of conical diffusers to the time-meaned velocity distribution and the turbulence intensity at inlet has been considered in Chapter 1.

The experimental tests reported in this chapter have as their object the independent variation of the diffuser's inlet turbulence and its subsequent effect on diffuser performance. To this end two types of experiment were conducted. In the first tests high inlet turbulence was achieved by the use of a long inlet pipe that had been artificially roughened. Aluminium oxide paper was used for this purpose which gave a roughness ratio (d/roughness height) = 250 which was comparable with commercial piping. However the fully-developed flow mean velocity profile associated with rough-wall pipes is different from that attained when the walls are smooth. In the case considered this difference was significant. Though results obtained with rough wall upstream pipe are of value for their own sake they did not lead to the required independent appreciation of the effect of inlet turbulence. A second series of tests was therefore conducted.

For these later tests a mean velocity profile generator was devised. This generator replaced the long smooth pipe and was designed to produce a mean velocity profile which was barely distinguishable from fully-developed flow whilst also producing a greatly enhanced turbulence intensity.

The chapter discusses the production of these different inlet conditions and their effect on diffuser pressure recovery for a family of conical diffusers covering a wide range of expansion angle and area ratio.

Incompressible fluid flow is assumed throughout.
5.2. FULLY-DEVELOPED FLOW ALONG A SMOOTH PIPE

5.2.1. "Inlet flow region" and Fully-developed flow

The preliminary experiments described in Chapter 2 had demonstrated that fully-developed flow conditions at the diffuser inlet were not obtained unless the approach length was more than approximately 80 pipe-diameters. When the experimental programme, using the new test rig, was decided upon it was felt that the preliminary results should be re-examined and extended.

Measurements of the static pressure variation around the circumference and along the 100 pipe-diameters long rig were made. A typical static pressure variation is shown in Figure 5.1. At corresponding 6 pipe-diameter intervals mean velocity traverses were taken. The variation of two-dimensional momentum thickness, which was evaluated from each of the velocity profiles, is shown in Figure 5.2. Once again the "overshoot" phenomenon of internal boundary layer growth is demonstrated, thus confirming the earlier measurements. From the mean velocity measurements selected non-dimensional velocities are presented in Figure 5.3 as a function of their non-dimensional position from the pipe centre line as the inlet pipe length is varied. Again it is not until some 80 pipe diameters that the position of these mean velocities is independent of further pipe addition: whereas the "inlet flow region", which is that length from inlet to the region where the boundary layers first merge, occurs after some 40 pipe diameters. This latter length, at which incidentally the boundary layer thickness parameter have their maximum value, has been successfully predicted by the theoretical method for pipe flows described in Chapter 3.
5.2.2. Wall shear stress

From the velocity measurements at the walls an estimate of the wall shear stress was obtained by Preston's method [53]. This makes use of a simple pitot tube resting on the surface, and depends upon the assumption of a universal inner law (or law of the wall) common to boundary layers and fully developed pipe flow. Preston shows that the non-dimensional relationship between Preston-tube reading and wall shear stress, \( \tau_w \), can be presented in the practically convenient form

\[
\frac{\tau_w d^2}{4 \rho v^2} = f \left( \frac{\Delta p}{4 \rho v^2} \right)
\]

where \( \Delta p \) is the Preston-tube reading (i.e. the difference between pitot and static pressures), \( d \) is the diameter of the Preston tube, and \( f \) is determined from measurements in fully developed pipe flows. Using the calibration curve for the Preston tube obtained by Patel [59] the wall shear stress variation with inlet pipe length was evaluated and is shown in Figure 5.4. The wall shear stress attains its fully developed flow value after some 20 pipe diameters. This result compares well with that of Barbin and Jones [9], and with the theoretical prediction of Chapter 3.

5.2.3. Reynolds shear stress and longitudinal intensity distributions

Using an inlet length of 99 pipe-diameters Reynolds shear stress and longitudinal turbulent intensity distributions were obtained for fully-developed flow along a smooth pipe at a Reynolds number of \( 4.5 \times 10^5 \) and are shown in Figures 5.5 and 5.6 respectively. The longitudinal turbulent intensity (non-dimensionalised by the friction velocity, \( u_w \) obtained from the wall shear stress measurements above) compares well with the other experimenters' results. The Reynolds shear stress has attained its fully-developed linear profile.
5.3. SIMULATED MEAN VELOCITY PROFILE

5.3.1. Mean velocity profile simulation devices

As has been shown in the previous section, if a flow is allowed to develop unimpeded down a long pipe then eventually a stable situation is reached, in which the flow parameters remain constant with increasing distance. Since it is intended to simulate this fully-developed mean velocity profile and at the same time produce a corresponding high turbulence intensity, it is worth noting the various available simulation methods.

The literature contains many accounts of these methods and therefore only the more significant ones will be listed here. They are the use of (i) wall roughness

(ii) grids of rods normal to the flow

(iii) curved, angled to the flow, and overlapping mesh screens

(iv) shaped honeycombs

and (v) forced mixing devices

of these (i) and (v) are the most suitable for use in an axisymmetric flow situation.

Schubaner and Spanenburg [60] considered the use of a number of forced mixing devices to alter velocity profiles and prevent flow separation in a region of adverse pressure gradient.

One of the devices discussed by the above authors, the vortex generator, was devised by H.D. Taylor [61] and his associates at the Research Department of the United Aircraft Corporation in 1946. Taylor applied the vortex generator mixing qualities to the prevention of separation inside a wind tunnel diffuser by selective positioning of the generators along the diffuser. The purpose of the generated vortices which had axes essentially parallel to the direction of flow was to intermix the higher free stream energy with that of the retarded boundary
layer. The re-energised layer so obtained, as shown in Figure 5.7, had the ability to flow further against a given adverse pressure gradient. If the vortex generators of Figure 5.7 are positioned in a counter-rotating configuration consisting of left- and right-handed airfoils alternately placed the adjacent airfoils generate vortices which are opposed in direction. Fig. 5.8, in which the assumed flow direction is into the paper, describes the mechanics of the mixing process. Taylor suggests that this is essentially a displacing action whereby the energy of the fluid next to the wall is increased at the expense of that in the free stream.

Armitt and Counihan [25] used the vortex generator mixing principle to produce an accelerated version of a model of the flow processes in an atmospheric boundary layer as proposed by Townsend [62]. The concept implied the existence in the boundary layer of large, slow-moving, contra-rotating vortex pairs with their axes parallel to the main flow. The vortex generators used by Armitt and Counihan were intended to produce Townsend’s proposed vortex system, the generated vortices spreading some of the ground level turbulence upwards in order to overcome the initial lack of turbulence in the outer section of the simulated boundary layer.

An axisymmetric form of Armitt and Counihan’s velocity profile generator has been described in Chapter 2. The following two sections describe how this device was redesigned and improved to produce the required simulated fully-developed mean velocity profile shape at the diffuser inlet in some 6\(\frac{1}{2}\) pipe diameters.
5.3.2. Variable parameters of the profile generator

The axi-symmetric form of Armitt and Counihan's velocity profile generator is to consist of an annular step downstream of which are triangular vortex generators followed by \( \frac{6}{5} \) pipe-diameters of roughened pipe. The generators are to be set alternately positive and negative angles of incidence so as to minimise resultant swirl. Using such a system a separation bubble, and thus a local momentum deficiency, will be caused behind the step. This flow is immediately confronted by the generators which introduce intense low frequency turbulence and vigorous rapid mixing occurs. The resulting turbulence produced is then supplemented by that produced by the surface roughness, the boundary layer grown on the roughness blending with the flow produced by the step and vortex generators to form the required simulated mean velocity profile at the diffuser inlet.

The required profile can be generated by varying a number of parameters. These include

(i) the internal diameter of the step and its width,
(ii) the distance between step and vortex generator,
(iii) the size, number, spacing, shape and angle of incidence of the vortex generators,
(iv) the form of roughness e.g. discrete elements or continuous roughness. Roughness height and spacing are therefore variables.
5.3.2. Simulating a fully-developed flow velocity profile

Since the effect of high intensity Reynolds shear stresses will be to promote the equilibrium mean velocity profile in a short downstream pipe length the effect of adjusting the relevant dimensions of the profile generator is to change mean flow and local turbulence intensities immediately downstream of the generator.

It is proposed to simulate the fully-developed profile in some 6π pipe diameters. Various parameters were kept constant for the tests: the remainder being systematically altered. Unaltered parameters were,

(i) horizontal distance between step and vortex generator centre of gravity, being 0.1d

(ii) 8 vortex generators spaced π-inches apart around the pipe circumference.

Two types of right angled triangular vortex generator were used. Generator A made an angle of 45° with the vertical side, which had length 0.75R, whereas generator B made an angle of 30° with the 0.875R long vertical side. The internal radius of the step (which was 1/8-inch thick) was varied from 1.0R to 0.843R. For the downstream roughness aluminium oxide paper was used which had a roughness ratio (pipe dia./roughness height) of 250. Mean velocity traverses were made at positions 3-, 6- and sometimes 9- and 15-pipe diameters downstream of the vortex generators with and without the roughness length (6πd) in position. The resultant mean velocity profiles are compared with the case of natural fully-developed pipe flow in a smooth pipe in Figures 5.9 to 5.15 for various combinations of the above parameters. The flow Reynolds number was approximately 2.2 x 10^5.

The vortex generators produce a boundary layer thickness slightly in excess of their height. The effects of varying the
generator angle of incidence and the internal radius of the step on the resulting velocity profile shapes are shown in the above figures for both of the generators with and without downstream roughness.

Although the effect on the flow of the different components of the profile generator is a complex one, because of the interactions which occur, systematic variation of the relevant parameters led to a successfully simulated mean velocity profile as shown in Figure 5.15. This simulation was achieved with the addition of a small annular rod set at a diameter of 0.825 d at the same cross-section as the step.

The final geometric data of the profile generator (a photograph of which has been shown in Figure 4.5.(a)) is tabulated below:

<table>
<thead>
<tr>
<th>NON-DIMENSIONALISED MEAN VELOCITY PROFILE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENERATOR DATA:</td>
</tr>
</tbody>
</table>

**Annular Step**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal diameter</td>
<td>0.875 d</td>
</tr>
<tr>
<td>Distance upstream of vortex generators</td>
<td>0.125 d</td>
</tr>
<tr>
<td>7.8 x 10^{-3} d dia. rod, set at a diameter of</td>
<td>0.825 d</td>
</tr>
</tbody>
</table>

**Vortex Generators**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>8</td>
</tr>
<tr>
<td>Shape Triangular, leading edge swept back by 30-deg., trailing edge radial, 0.438 d high</td>
<td></td>
</tr>
<tr>
<td>Angle of incidence alternately + and - 5 deg.</td>
<td></td>
</tr>
<tr>
<td>Distance Upstream of Diffuser from trailing edge of vortex generators</td>
<td>6.5 d</td>
</tr>
<tr>
<td>Covered by aluminium oxide paper with (d/roughness height) = 250</td>
<td></td>
</tr>
</tbody>
</table>
Using the rather crude spectral distribution facility afforded by filtering the hot-wire sensor's output to the DISA 55401 anemometer, the turbulence spectral densities produced by the long smooth inlet pipe and the mean velocity profile generator were compared.

The power spectral density, $s(w)$, can be defined in terms of the energy contained within a small frequency band, i.e.

$$\text{energy associated within range } w, w + dw = \frac{1}{\pi} s(w)dw$$

Integrating this expression gives the total energy associated with the flow,

$$\frac{1}{2} u^2 = \frac{1}{2\pi} \int_0^\infty s(w)dw$$

Figure 5.16 shows the result of filtering the hot-wire output. Here the resulting mean square values have been non-dimensionalised by the maximum mean square reading which corresponds to the OUT position of the HIGH and LOW pass DISC filters. The simulated flow has a higher energy level associated with low frequencies than the naturally-developed profile.

With the limitations of the instrumentation that was available at the time of this research it is apparent that the turbulence generated by the mean velocity profile generator is generally similar in its spectral distribution to that developed naturally, but low frequency energy levels are higher. The procedure now being adopted at Leicester University to determine spectral density levels is to record the turbulence in analogue form on tape and then input this data to a Fenlow spectral density analyser set to average the signal over as long a time interval as possible.

Flow direction measurements at 3- and 6- pipe diameters behind the profile generator were made using the yaw probe described in Chapter 4. At neither of these measuring stations were swirl velocity components detected, and thus the simulated and natural diffuser
inlet profiles differed only in their corresponding turbulence intensities.

5.3.4. Flow conditions at the diffuser inlet

The various mean velocity and turbulence diffuser inlet parameters resulting from the three different methods of generation will now be presented and discussed.

The simulated fully-developed mean velocity profile is shown in Figure 5.17 for three traversing diameters at each of the two cross-sections, 3- and 6- pipe diameters downstream of the profile generator. It can be seen that at 6d downstream the profile due to a long length of smooth pipe and that due to the generator system have the same shape. Corresponding axial turbulence intensities and Reynolds shear stress distributions are given in Figures 5.18 and 5.19 respectively. The Reynolds shear stress distribution so produced is seen to be nearly linear.

As mentioned earlier initial experiments investigated the fully-developed flow condition of boundary layer growth down a roughened pipe. To this end 96 pipe-diameters of roughened pipe with a pipe diameter/roughness height ratio of 250 resulted in the velocity profile shown in Figure 5.20. The roughened pipe produced a significantly more peaked distribution than that produced by the smooth pipe.

Finally the inlet axial turbulence intensities (non dimensionalised with respect to the local mean velocity) due to those three methods of generation are compared in Figure 5.21. The distributions due to the rough and smooth pipes are very similar, whereas that due to the profile generator has a significantly high magnitude across the central portion of the pipe. Comparison of centre-line axial turbulence intensities show that the roughened pipe and the profile generator
values are, respectively, some 40 per-cent and 140 per-cent in excess of the corresponding smooth inlet pipe value.

5.4. RESPONSE OF CONICAL DIFFUSERS TO THE INLET FLOW

This section outlines the criterion adopted for the response of conical diffusers to the inlet flow. Following this, some of the extensive data obtained by the British Hydromechanics Research Association is presented and discussed. Finally, the author's results for the performance of conical diffusers subjected to highly turbulent inlet conditions are presented.

5.4.1. Significant response of diffusers to the inlet flow

There is of course a range of degrees of complexity with which internal fluid flow problems can be solved. Though for some very large installations small improvements in pressure recovery could result in significant financial returns, in the huge majority of design studies this desired effect is outweighed by calculation uncertainties in the system. Thus although it is unlikely, for example, that the inlet blockage factor outlined in Chapter 1 is entirely adequate for the task of defining highly asymmetric mean velocity profiles for most practical purposes it is a satisfactory parameter. Neither are engineers often concerned with very thin layers at the diffuser inlet (to which its performance is extremely sensitive) because even in the unlikely event of a diffuser being sited directly downstream of a contraction very great care indeed has to be taken to achieve inlet boundary layer thicknesses such as Little and Wilbur [63] obtained in their carefully controlled laboratory experiments.

Much uncertainty in past data has occurred because of inadequately
designed experiments and confusion about experimental procedure. But where reliable data now exists for certain inlet flow conditions it will be considered that a significant response to a change in those inlet conditions would be one for which a change in pressure recovery coefficient of the order of 10 per-cent is detectable. It is with such a criterion that the following experiments results are analysed.

5.4.2. B.H.R.A. experimental data

As part of an extensive experimental investigation into diffusor behaviour Miller [49] examined the response of conical diffusers to variations in mean velocity distribution. His results have been replotted in Figure 5.22 where the pressure recovery coefficient is applicable to a conical diffuser with an open downstream end.

In general in pipe and duct work systems the entry boundary layer will be fully developed. The diagram shows that where it is thin however (for example downstream of the working section in a wind tunnel) there can be a significant improvement in the pressure recovery. The regime in which this is most pronounced is discussed with reference to Figure 5.23.

Figure 5.23, reproduced from Figure 5.22, and therefore also based on Miller's work, records values of maximum $C_p$ (denoted $C_p^*$ by Tyler and Williamson [11]) for given values of non-dimensional diffuser length. If this area ratio is not attained the design length of the diffuser will be excessive and the surface friction unnecessarily high, whereas if it is exceeded premature separation will result.

Comparing Figure 5.22 and 5.23 it is evident that the optimum value of $C_p$ would be significantly increased if the entry boundary layer were thinned. The thickness of the entry boundary layer is also significant if the proposed specification lies in the "excess flow
separation" regime. In the "excess surface friction" regime however this effect is insignificant. This is borne out by analytical studies of diffuser behaviour, which are normally only applicable to the "excess surface friction" regime. As boundary layer thicknesses vary calculated values of $C_p$ in general lie within ± 5 per-cent of the mean, except when the inlet boundary layer is exceptionally thin.

In many applications a diffuser is not open-ended. Where an outlet length of pipe exceeding some five or six pipe diameters in length is present Figure 5.22 is inapplicable because the process of diffusion continues down that pipe as the mean velocity profile reverts to its fully-developed flow shape. This is discussed in more detail by Reneau, Johnston and Kline [15]. A conservative rule for designers would be to use the pressure recovery coefficient values from Figure 5.22 if the characteristics lie in the "excess surface friction" regime (Figure 5.23) but increase $C_p$ by 40 to 50 per-cent if the characteristics lie in the "excess flow separation" regime.

5.4.3. Conical diffuser experimental results

The results presented here demonstrate the effect of the three different inlet conditions on the pressure recovery of downstream conical diffusers. In all cases the change in geometry from pipe to diffuser is sharp. The pressure recovery $C_p$ is defined as

$$C_p = \frac{P_o - P_i}{\frac{1}{2} \rho \overline{U}_1^2}$$

$P_o$ being taken as the recorded static pressure at some axial position along the diffuser, $P_i$ is the extrapolated value, at the diffuser inlet, of the upstream static pressure gradient, and $\overline{U}_1$ is obtained from the inlet mean velocity profile. A Betz water manometer was used to determine both static and total pressures.
All the tests relate to fully developed flow at the diffuser entry, either natural or simulated. The pressure recovery variation with area ratio for smooth pipe inlet conditions is shown in Figure 5.24 and extends Miller's work to larger area ratio diffusers. Figure 5.25 shows the pressure recovery data for the five diffusers when subjected to smooth walled, rough-walled and artificially simulated inlet flows. These results are then presented in a different form in Figure 5.26 and 5.27.

Comparison with Figures 5.26 and 5.27 shows that flow separation within these diffusers has occurred earlier than in Miller's programme but that there is a fair measure of agreement when the fully developed inlet flow was produced by means of a long smooth pipe.

A limited range of tests was conducted with the fully developed flow generated by roughened pipe; the roughness ending at the diffuser inlet. In these cases measured values of $C_p$ were less than with the smooth inlet pipe, this decrease being up to 10 per-cent in magnitude.

A substantial range of tests was conducted with the mean velocity profile generator at the diffuser inlet. In these cases the measured values of $C_p$ were greater than with a smooth pipe at inlet. This increase varied, being greater (10 - 12 per-cent) for the larger-angled diffusers.

In internal flow calculations the pressure drop developed down the long smooth inlet pipe would normally be independently assessed in the calculation. Thus even though the pressure drop associated with the system comprising mean velocity profile generator and diffuser is only about 20 per-cent of that associated with the long smooth inlet pipe and diffuser, there is no possibility, by first introducing the mean velocity profile generator, of increasing the pressure recovery of a diffuser mounted downstream of a normal fully-developed pipe flow profile. The losses associated with the generation of high intensity turbulence more than outweigh the gains resulting from increased pressure recovery.
5.5. DISCUSSION OF EXPERIMENTAL RESULTS

The two sets of experiments conducted on the effects of inlet turbulence have produced apparently anomalous results. The diffuser pressure recovery coefficient fell when the smooth entry pipe was replaced with a roughened one, but rose when the smooth entry pipe was replaced by the mean velocity profile generator. In both cases the inlet turbulence intensity was raised above the smooth entry pipe level but whereas the roughened inlet pipe produced a markedly different mean inlet profile, that produced from the mean velocity profile generator was substantially the same as it was with a smooth entry pipe.

The explanation suggested is that with the roughened inlet pipe the effect of the change in mean inlet profile dominates any effect from increased turbulence intensity. Figure 5.20 shows that the profile generated by the rough wall is peakier and therefore more prone to separate under the influence of an adverse pressure gradient than that generated by much smoother walls: earlier separation resulting in reduced diffuser pressure recovery.

The blockage factors for the smooth and rough wall profiles respectively are 0.15 and 0.20. With such an inlet blockage factor variation Tyler and Williamson's results [11] indicate no detectable trend in $C_p$ for diffusers with non-optimum geometry.

With the mean velocity profile generator however all of the change in $C_p$ can be ascribed to inlet turbulence. Presumably the mechanism for the increased pressure recovery is the re-energising action of the turbulent stream delaying flow separation and thereby raising the effectiveness of the diffuser. This would conform with Reneau, Johnston and Kline's findings [15].

Referring again to Figures 5.25 - 5.27 it is observed that although there is only a small amount of data for the first test the
effect of wall roughening in the entry pipe can be seen to reduce $C_p$ and this effect is particularly pronounced with wide-angled diffusers, a result which is indicative of earlier separation which would then result. It should be noted that roughness extended to the inlet pipe only and not into the diffuser. Polzin [64] has shown that if the diffuser walls also are roughened there is a pronounced movement of the separation point upstream, with a consequent marked reduction in pressure recovery coefficient.

The effect of the mean velocity profile generator is beneficial, particularly so for wide-angled diffusers of high area ratio, but significant with all diffuser geometries.

5.6. INDUSTRIAL APPLICATION

Assuming the significance criterion that has been suggested in this chapter, the data presented may be used for design purposes as follows:

Where it is proposed to use a conical diffuser as part of a pipework system suitable dimensions can first be determined from Figure 5.23. If compromises are necessary Figure 5.22 will give the pressure recovery coefficient, suitably corrected for downstream pipework.

In general the entry boundary layer will be fully developed having been formed along smooth and straight entry piping. Except for diffusers with unusually large expansion angles roughened inlet pipes have little effect. If there is considerable asymmetry at the diffuser inlet (as there might be immediately downstream of a bend for example) the designer needs to remember that there is evidence that $C_p$ is significantly increased.

In rarer cases where the inlet boundary layer is thin, such as in the design of wind and water tunnels and other applications where
a wide, uniform potential core is desirable, Figures 5.22 and 5.23
give detail which can be supplemented from Miller's [49] report.

In applications where there is considerable inlet turbulence,
such as downstream from rotating machinery, Figures 5.22 and 5.23 are
inappropriate. Where the mean velocity profile is not significantly
changed the pressure recovery coefficient of the diffuser could be
increased by some 10-12 per-cent. But if a diffuser were placed
immediately downstream of a very peaky profile which was also highly
turbulent a considerable increase in $C_p$ from the values given in Figure
5.22 could be anticipated.
Fig. 5.1 TYPICAL STATIC PRESSURE VARIATION ALONG RIG AND AROUND CIRCUMFERENCE

\[
\left( \frac{P_x - \text{pref}}{\text{pref}} \right)
\]

- 4 Static-pressure tappings at each cross section equally spaced around circumference.
FIG. 5.2  TWO-DIMENSIONAL MOMENTUM THICKNESS VARIATION ALONG A SMOOTH PIPE

$Re \approx 4.4 \times 10^5$
Fig. 5.3 TURBULENT BOUNDARY LAYER DEVELOPMENT IN A SMOOTH PIPE

Re = 4.4 x 10^5
FIG. 5.4 WALL SHEAR STRESS VARIATION ALONG INLET REGION OF SMOOTH PIPE
COMPARISON OF LONGITUDINAL TURBULENT INTENSITY DISTRIBUTIONS

- LAUFER
  - Red = 0.5 \times 10^5
  - Red = 5.0 \times 10^5

- SANDBORN
  - Red = 1.0 \times 10^5
  - Red = 2.0 \times 10^5
  - Red = 3.0 \times 10^5

- PATEL
  - Red = 2.74 \times 10^5
  - Red = 1.93 \times 10^5
  - Red = 3.16 \times 10^5

- BRADLEY
  - Red = 4.47 \times 10^5

Fig 5-5 FULLY-DEVELOPED SMOOTH-WALLED PIPE FLOW LONGITUDINAL TURBULENT INTENSITY
Fig. 5.7
Purpose of Vortex Generators (After Taylor)
COUNTER ROTATING VORTEX GENERATOR CONFIGURATION

Energy Levels Before Mixing

Energy Distribution during mixing

Energy distribution after mixing

FIG. 5-8 MECHANICS OF MIXING (After TAYLOR)
FIG. 5-9  
DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITHOUT ROUGHNESS  
(Variation of generator angle of incidence)
FIG. 5-10  DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITHOUT ROUGHNESS

(Variation of the step internal radius)
For vortex generator A and no step.

FIG. 5-11  DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITH ROUGHNESS
(Variation of generator angle of incidence)
For vortex generator A
Angle of incidence ± 10°

Internal rad. step:
- No step
- 0.937R
- 0.875R
- Fully-dev. flow.

FIG. 5.12  DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITH ROUGHNESS
(variation of the step internal radius)
For vortex generator B
Angle of incidence ±10°

**FIG. 5.13** DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITH ROUGHNESS
(variation of the step internal radius)
For vortex generator B

- $\pm 7^\circ$
- $\pm 10^\circ$
- Fully-dev. flow

Internal rad. step 0.875R
Wall roughness ($d$/roughness height = 250)

FIG. 5.14. DOWNSTREAM MEAN VELOCITY DISTRIBUTIONS WITH ROUGHNESS
FIG. 5.15  PRODUCTION OF SIMULATED FULLY-DEVELOPED PROFILE
FIG. 5.16  EFFECT OF VARYING THE DISA HIGH PASS FILTERS ON THE HOT-WIRE MEAN SQUARE LONGITUDINAL INTENSITY OUTPUT
Long smooth-walled pipe
Mean velocity profile
Generator
Traverses made at 3 and 6 pipe diameters downstream
DIFFUSER INLET AT 6.5d

FIG. 5-17 MEAN VELOCITY PROFILES PRODUCED BY GENERATOR
FIG. 5:18 AXIAL TURBULENCE INTENSITY BEHIND VORTEX GENERATORS
FIG 5.20 FULLY DEVELOPED MEAN VELOCITY PROFILES FOR SMOOTH & ROUGH PIPE FLOW
FIG. 5-21 AXIAL TURBULENCE INTENSITY AT DIFFUSER INLET

X Smooth inlet pipe
+ Rough inlet
O Artificial profile generator
FIG. 5-22  EFFECT OF INLET BOUNDARY LAYER THICKNESS ON DIFFUSER PRESSURE RECOVERY COEFFICIENT
FIG 5:23  OPTIMUM AREA RATIO FOR A GIVEN LENGTH OF DIFFUSER
FIG 5.24 PRESSURE RECOVERY $C_p$ VARIATION WITH AREA RATIO FOR SMOOTH PIPE FULLY-DEVELOPED FLOW INLET CONDITIONS
FIG. 5.25 VARIATION ON $C_p$ WITH DIFFUSER INLET FLOW CONDITIONS

(Dept shifted origins)
FIG. 5:26 EFFECT OF INLET TURBULENCE ON DIFFUSER PRESSURE RECOVERY COEFFICIENT

(a) artificially simulated inlet
FIG. 5·27  EFFECT OF INLET TURBULENCE ON DIFFUSER PRESSURE RECOVERY COEFFICIENT
(b) Rough-walled inlet pipe
CHAPTER 6

DIFFUSER PRESSURE RECOVERY PREDICTION

WITH HIGH INLET TURBULENCE
6.1. DIFFUSER PRESSURE RECOVERY WITH HIGH INLET TURBULENCE

Successful analytical prediction of experimental results for a diffuser's pressure recovery coefficient would imply a proper appreciation of the flow processes involved. With care it might also lead to meaningful extrapolation of existing diffuser test data.

Methods of procedure with fully-developed inlet boundary layers have been discussed by McMillan and Johnston [65] and by the present author in Chapter 3 and Appendix C. It has been shown that if integral methods are used the Karman momentum equation is written as

$$\frac{\partial \phi}{\partial x} + \frac{6}{U_o} \frac{\partial U}{\partial x} (2 + H) = \frac{1}{\rho U_o^2} \left[ r_w - R \left( \frac{\partial r}{\partial r} \right)_o \right]$$

where subscript (o) represents centre-line conditions.

Whereas as mean inlet parameters vary such a procedure can and does predict diffuser characteristics satisfactorily when the turbulence intensity is normal, the effects of a high intensity of inlet turbulence can only enter the system indirectly, largely via the centre-line total pressure loss and thus these will be much more difficult to predict. With highly turbulent inlet conditions the centre-line shear stress gradient, \( \frac{\partial r}{\partial r} \), will normally be significantly different from the smooth inlet pipe value.

The method used to predict the pressure recovery with high inlet turbulence is outlined in Appendix C, Section (b). It differs from the previous predictions for normal fully-developed flow only in the values ascribed to the coefficients \( k_1 \) and \( k_2 \) in equations C10 and C11; these being estimated from the experimental results. Furthermore, assuming that the inlet velocity profiles are represented by a simple power law of varying exponent, \( n \), their initial conditions are similarly estimated giving:
(i) smooth-walled profile, \( n = 9, \ k_1 = 1, \ k_2 = 1 \)

(ii) rough-walled profile, \( n = 6, \ k_1 = 2, \ k_2 = 3.5 \)

(iii) simulated profile, \( n = 9, \ k_1 = 2, \ k_2 = 3.5 \).

The pressure rise in a diffuser can be related to the change in total pressure along the streamline of maximum velocity by

\[
\rho_o - \rho_i = (\frac{1}{2} \rho U_c^2 - \frac{1}{2} \rho U_0^2) - (\rho_i - \rho_o) \quad \text{(6.1.2)}
\]

for incompressible flow free from curvature effects (such that static pressure is uniform at any cross-section).

Hence,

\[
C_p = \frac{\rho_o - \rho_i}{\frac{1}{2} \rho U_i^2} = \left( \frac{U_c^2 - U_0^2}{U_i^2} \right) - \left( \frac{\rho_i - \rho_o}{\frac{1}{2} \rho U_i^2} \right) \quad \text{(6.1.3)}
\]

The first term in equation 6.1.3. is determined by changes in velocity profile, while the second term is determined by shear losses.

The resulting predicted pressure recovery coefficients for the 4- and 10-degree conical diffusers are shown in Figure 6.1. Both the rough pipe and generator inlet conditions have resulted in lower \( C_p \) values than those due to the smooth pipe, experimental \( C_p \) values being predicted to within 10 per-cent. The method is, of course, a gross over-simplification of the physical situation, however it is surprising that the normal and highly turbulent inlet conditions should produce such similar values of \( C_p \). This is due to the relatively insignificant role that the \((\partial T/\partial r)\) term is playing in equation 6.1.1, for changing its magnitude by a factor of 3.5 has a negligible effect on the pressure recovery.

*Here \( \rho_i - \rho_o \) is the total pressures such as could be determined using two pitot tubes. The more manipulatively convenient and commoner used expression, equation (1.1.1) contains a term \( \Delta P \) which is dependent on velocity profile shape.
Another significant factor in the discrepancy between prediction and experiment is the assumption implicit in equation 6.1.1. that normal Reynolds stress terms are negligible. Integration of this equation with respect to $x$ and construction from reputable experimental data of the numerical values of the resulting left and right hand sides independently by Coles and Hirst [66] has shown that the Karman momentum equation in the form here expressed may fail to satisfy established experimental conditions, particularly when the pressure gradient is strongly adverse.

Successful prediction of diffuser characteristics when the inlet flow is very highly turbulent is thus dependent on more experimental data concerned with the nature of the flow within the diffuser. Since integral prediction techniques are only dependent on bulk flow parameters and boundary conditions they are much less demanding than differential ones in this respect. To use differential techniques successfully much more experimental data is required about shear stress and turbulence intensity variation within the diffuser. Though turbulence data obtained within diffusers having thin inlet boundary layers has been published the considerable asymmetry and unsteadiness with fully-developed entry flow results in a dearth of experimental data for the required condition.
FIG. 6.1 PREDICTION OF $C_p$ VARIATION WITH DIFFERENT INLET TURBULENCE CONDITIONS

$\phi = 4^\circ$

$\phi = 10^\circ$

- $C_p$ vs $N/Re$
- Smooth pipe experiment
- Rough pipe and generator predictions
- Smooth pipe predictions

$Re = 2.15 \times 10^5$
CHAPTER 7

TURBULENCE EFFECTS IN INTERNAL FLOWS :

SUGGESTIONS FOR FURTHER WORK
Any new research programme on internal fluid flow should aim at extending our physical understanding of the role of turbulence on the resulting flow development. This can best be done by a series of basic experiments, the results of which, whilst being sufficient in their own right, should also produce more general information. This would result in:

(a) a challenge to the prediction capabilities of the current successful calculation methods;

(b) the incorporation of more relevant "physics" (i.e. better turbulence models) into new calculation methods, thus contributing to their development;

(c) a better understanding and a quicker solution of specific industrial problems.

It has been demonstrated in this thesis that further analysis of internal flows is hampered by the inadequacies in existing experimental data, in particular in reliable turbulence information. Unfortunately Laufer's [67, 68] classic papers on Internal Flows only dealt with conditions when boundary layers were fully-developed. What is chiefly required to exploit the integral technique to its full potential is information on shear stress and its variation, first to reappraise the non-dimensional profiles given in Figure 3.1, then to make a more vigorous attack on flow behaviour when the boundary layers are about to merge. This experimental data is needed for two-dimensional duct flow and for axisymmetric flows in pipes and in diffusers.

The experimental work reported in this thesis has demonstrated the degree to which the inlet turbulence will affect diffuser performance. Any extension of this work, be it by further use of the vortex generator device described or by some other means of turbulence generation, should certainly seek to provide detailed data on Reynolds normal and shear stress variation and the corresponding boundary layer development in the diffuser.
An extended research programme would make extensive turbulence and mean flow measurements both along smooth and roughened pipes or ducts upstream of conical or two-dimensional diffusers, and within the diffusers themselves. Variation of the roughness height and length of inlet pipe or duct would produce significantly different inlet conditions for which a meaningful correlation with diffuser performance could be attempted. Such an experimental programme would also produce information on the relationship between the mean flow and the turbulence structure of turbulent boundary layers on rough surfaces, and would be contributory to the development of boundary layer prediction techniques for rough surfaces. This data might also be used to further adequate prediction of flow behaviour when boundary layers are about to merge.

The better present-day computational methods, such as those of Bradshaw [69] and Hirst and Reynolds [41] have considerable potential because they are capable of extension to axisymmetric flows and to conditions of extreme inlet turbulence. To apply the operationally-simpler Hirst and Reynolds technique a necessary starting condition is the datum entrainment rate into the boundary layer, which could be obtained from their hypothesised relationship between the entrainment rate and the boundary layer turbulence parameters. From the suggested experimental programme this Hirst and Reynolds hypothesis could be fully investigated. If it proved acceptable the resulting relationship could be incorporated into what would be an unsophisticated but extremely reliable prediction method, capable of being used in high turbulence level situations.
CONCLUSIONS
1. The 'overshoot' phenomenon of boundary layer growth along a smooth pipe has been demonstrated. As a consequence of this phenomenon, although the boundary layers growing on the walls of the pipe merge in the region $35 < le/d < 40$ (at a pipe Reynolds number of $4.4 \times 10^5$) the flow is not stable (i.e. unchanging with increasing distance) until a value of $le/d = 90$ is reached.

2. Integral boundary layer prediction techniques may be very successfully applied to internal fluid problems provided that

(a) the flow is sensibly two-dimensional or axisymmetric

and (b) flows in the separation region are treated with some reserve.

Other limitations arise from the paucity of valid experimental data. For example, flow prediction of diffuser characteristics with near fully-developed inlet conditions is dependent on experimental data on the centre-line shear stress distribution.

3. In the light of the results shown, developing boundary layers in ducts and pipes and the characteristics of conical diffusers under conditions when a potential core is present can be predicted with confidence. Prediction can be extended to cases where the potential core is absent if the necessary experimental shear stress data is available. Although some attempts have been published, integral techniques do not lend themselves to the prediction of the behaviour of diffusers whose cross-sectional areas are non-circular.

4. Problems of component interference can be considered by seeking to isolate the effect of

(a) the mean velocity profile,

(b) the turbulence distribution

and (c) the three-dimensionality.
Integral prediction techniques certainly enable cause (a) to be considered as an independent parameter. There are prospects that they may also contribute to understanding cause (b). They can never be used to investigate cause (c).

5. A successful mean velocity profile generator has been described which affords a means of investigating inlet turbulence as an independent parameter affecting the performance of downstream conical diffusers. For inlet conditions due to the above generator and long smooth and roughened pipes, subsequent pressure recovery data has been obtained for conical diffusers covering a wide range of expansion angle and area ratio.

6. Using the above results and available sources of reliable experimental data on the response of diffusers to conditions at their inlet it is concluded that

(a) the thickness of the entry boundary layer may significantly affect the values of pressure recovery coefficient,
(b) except when the diffuser expansion angle is unusually high roughness of inlet pipe has little effect, but
(c) that asymmetry of the inlet profile and high intensities of turbulence at the diffuser inlet cause the pressure recovery coefficient to increase.

7. There is still insufficient reliable information about the flow processes in diffusers, particularly when the potential core is absent, for computational analysis to proceed reliably. Once again shear stress characteristics would be valuable, as would Reynolds normal stresses. This information would then replace the over-simplified assumptions of the previous section, and thus afford a more accurate prediction of the effect of high inlet turbulence on diffuser performance. However, owing to the
effect of asymmetry and flow separation which are prevalent this information is extremely difficult to obtain.

8. Of the auxiliary equations discussed the one offering greatest potential is that by Hirst and Reynolds, equation 3.4.13, since this affords the possibility of varying the boundary layer turbulence level at the inlet of a duct or pipe component and predicting the resulting effect. Before this can be done with any confidence however the hypothesis relating the entrainment velocity to boundary layer turbulence level must be carefully tested by a full experimental programme.
APPENDIX A

OUTLINE OF PREDICTION METHOD FOR INTERNAL FLOWS
(a) **Flow Equations**

The prediction technique presented in this appendix is as applicable to axially-symmetric flows as it is to the two dimensional case. The following is a listing of the flow equations used to provide a closed set leading to the solution of various boundary layer problems encountered in ducts, pipes and conical diffusers.

**EQUATION A1 :**

The Momentum Integral equation for the whole of the boundary layer, used here as a basis for the prediction technique.

**EQUATION A2 :**

The integral continuity equation which is used to express the variation of free-stream velocity $U_c$ with $x$.

**EQUATION A3 :**

The mean velocity profile equation. This assumption is used to simplify the main flow equations to a set of common variables with $x$. The profile assumptions tested were (i) the simple one-parameter power law, and (ii) the Coles two-parameter profile. The basic difference between (i) and (ii) is that a separate skin friction equation (Ludwig-Tillmann) must be used with the power law, whereas the Coles profile provides an implicit skin friction relationship. This relationship is presented as the ordinary differential equation A3 in the present analysis.

**EQUATION A4 :**

An Auxiliary equation. The two equations tested were the momentum integral equation integrated over a strip of the boundary layer, and Head's original entrainment equation. The strip equation demands extra knowledge of the turbulent shear stress at the edge of the strip. This shear stress information was obtained from, 1) an eddy viscosity model, and 2) $T/\tau_w=\text{function}(\sqrt{\delta})$ after Fediasvsky.
(b) Computer Equations

The above equations must now be put into a form suitable for analysis by a digital computer. Thus consider, for example, the Coles velocity profile relationship,

\[ \frac{U}{U_0} = 1 + \alpha \log \left( \frac{y}{\delta} \right) + \beta \left( 1 + \cos \frac{\pi y}{\delta} \right) \]

where \( \alpha = \frac{1}{0.41} \sqrt{\frac{C_f}{2}} \), and \( \beta = -\pi(x) \alpha \)

\( \pi(x) \) being Coles wake parameter.

Now \( U_0, \delta, \alpha \) and \( \beta \) are all functions of \( x \).

Substitution of the Coles profile into equations \( A1, A2 \) and \( A4 \) and after integration with respect to \( y \), and use of the relationship,

\[ \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial \delta} \frac{\partial \delta}{\partial x} + \frac{\partial \phi}{\partial U_0} \frac{\partial U_0}{\partial x} + \frac{\partial \phi}{\partial \alpha} \frac{\partial \alpha}{\partial x} + \frac{\partial \phi}{\partial \beta} \frac{\partial \beta}{\partial x} \]

where \( \phi \) is any function of \( x \) yields the flow equations in the following general form,

\[ S_1 + S_2 \frac{d\delta}{dx} + S_3 \frac{dU_0}{dx} + S_4 \frac{d\alpha}{dx} + S_5 \frac{d\beta}{dx} = S_6 \]

\[ \frac{dU_0}{dx} = S_7 + S_8 \frac{d\delta}{dx} + S_9 \frac{d\alpha}{dx} + S_{10} \frac{d\beta}{dx} \]

\[ \frac{d\beta}{dx} = S_{11} \frac{d\delta}{dx} + S_{12} \frac{dU_0}{dx} + S_{13} \frac{d\alpha}{dx} \]

\[ S_{14} + S_{15} \frac{d\delta}{dx} + S_{16} \frac{dU_0}{dx} + S_{17} \frac{d\alpha}{dx} + S_{18} \frac{d\beta}{dx} = S_{19} \]

where,

equation \( A3 \) is obtained by differentiating the implicit skin friction relation with respect to \( x \), and rearranging into the above form.

The coefficients \( S_1 \) to \( S_{19} \) are functions of \( \alpha, \beta, \delta, U_0 \). Furthermore the coefficients \( S_1, S_7 \) and \( S_{19} \) are only non-zero for internal flows in which the area of cross-section, varies with \( x \), as for example, in a
diffuser.

At this point the above four relationships could be set in finite difference form and then solved using an available fourth-order Adams-Bashforth predictor-corrector technique with a two-point predictor-corrector for starting purposes. This routine employs a step size control to keep absolute and relative errors within specified bounds. However by substituting for \( \frac{d\delta}{dx} \) from eqn. A3 into eqns. A1, A2 and A4 and solving those three equations simultaneously for \( \delta \), \( a \) and \( U_c \) we get three simple ordinary differential equations of the form,

\[
\begin{align*}
\frac{d\delta}{dx} &= \Phi_1(S_1, S_2, \ldots, S_{19}) \\
\frac{da}{dx} &= \Phi_2(S_1, S_2, \ldots, S_{19}) \\
\frac{dU_c}{dx} &= \Phi_3(S_1, S_2, \ldots, S_{19})
\end{align*}
\]

and the errors due to the finite-difference technique are minimised. Thus it was at this point that the solution to the problem was computed using an Elliott 4130 digital computer. The constituent sub-routines of the integral boundary layer prediction routine are outlined in Figure A1.

The Power Law profile \( \frac{U}{U_c} = \left( \frac{\delta}{\delta} \right)^{1/n} \) is treated in the same manner. This time \( \delta \), \( U_c \) and \( n \) vary with \( x \) and the resulting Momentum Integral equation, Continuity equation and Auxiliary equation can be written respectively,

\[
\begin{align*}
f_1 \frac{dU_c}{dx} + f_2 \frac{d\delta}{dx} + f_3 \frac{dn}{dx} &= f_4 \\
\frac{dU_c}{dx} &= f_5 \frac{d\delta}{dx} + f_6 \frac{dn}{dx} \\
f_7 \frac{dU_c}{dx} + f_8 \frac{d\delta}{dx} + f_9 \frac{dn}{dx} &= f_{10}
\end{align*}
\]

The Ludwig-Tillman skin friction equation replacing equation A3.
FIG. A.1. CONSTITUENT SUB-ROUTINES OF THE INTEGRAL BOUNDARY LAYER PREDICTION ROUTINE (AFTER HIRST)
APPENDIX B

DETERMINATION OF THE COEFFICIENTS FOR THE FLOW EQUATIONS
Appendix A has outlined in general terms the various flow equations and empirical assumptions used in the present theoretical investigation, and shown how these equations are finally reduced to a set of ordinary differential equations in one variable. In this Appendix the flow equations are presented in more detail, and the coefficients $S_1$ to $S_{19}$ and $f_1$ to $f_{10}$ are evaluated. The alternative sets of closed equations are summarised in block diagram form in Figure B1. Various internal flows were modelled using the selection of equations presented in Figure B1 so that the importance of the following assumptions could be investigated:

(i) velocity profile  
(ii) auxiliary equation  
(iii) shear stress  

(a) Two-dimensional Momentum Integral Equation

The turbulent boundary layer equations for two-dimensional flow can be written,

\[
\begin{align*}
\text{x-momentum} & : \quad U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial y^2} - \frac{\partial \overline{u^2}}{\partial x} - \frac{\partial \overline{uv}}{\partial y} \quad \text{B1} \\
\text{y-momentum} & : \quad 0 = - \frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\partial \overline{v^2}}{\partial y} \quad \text{B2} \\
\text{Continuity} & : \quad \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad \text{B3}
\end{align*}
\]

A set of ordinary differential equations can be obtained from those boundary layer equations by integrating in $y$ and neglecting the Reynolds normal stress:
where $\eta_1$ can have values $0 < \eta_1 < 1.0$

Evaluation of equation B4 for $\eta_1 = 1.0$ results in the two-dimensional Momentum Integral equation. Substitution of the Coles velocity profile results in equation A1,

$$S_1 + S_2 \frac{d\delta}{dx} + S_3 \frac{dU_c}{dx} + S_4 \frac{d\alpha}{dx} + S_5 \frac{d\beta}{dx} = S_6$$

where,

$S_1 = 0$ (in constant area ducts)

$S_2 = \alpha - \beta - 1.5\beta^2 - 2\alpha^2 + 3.17888\alpha\beta$

$S_3 = (3\alpha - 3\beta - 3\beta^2 - 4\alpha^2 + 6.35776\alpha\beta)/U_c$

$S_4 = (1 - 4\alpha + 3.17888\beta)\delta$

$S_5 = (-1 - 3\beta + 3.17888\alpha)\delta$

$S_6 = 0.1681\alpha^2$

Similarly, substitution of the power law velocity profile results in equation A6,

$$f_1 \frac{dU_c}{dx} + f_2 \frac{d\delta}{dx} + f_3 \frac{dn}{dx} = f_4$$

where,

$$f_1 = \frac{(2 + 3n)}{(1 + n)(2 + n)} \cdot \frac{\delta}{U_c}$$

$$f_2 = \frac{n}{(1 + n)(2 + n)}$$

$$f_3 = \frac{(2 - n^2)}{(1 + n)^2(2 + n)^2} \cdot \delta$$

$$f_4 = \frac{\tau_w U_c^2}{\mu}$$

$f_4$ being obtained from the Ludwig-Tillmann skin friction relationship.

$$\frac{\tau_w U_c^2}{\mu} = 0.123 R_0^{-0.268} \times 10^{-0.678H}$$
(b) Two-dimensional Continuity Equation

Continuity considerations state that for two-dimensional flow
\[ \frac{d}{dx} \int_0^h U \, dy = 0 \]

where \( h \) is the duct half-width.

Substituting the Coles profile in equation B5 we obtain equation A2,
\[ \frac{dU_c}{dx} = S_7 + S_8 \frac{d\delta}{dx} + S_9 \frac{d\alpha}{dx} + S_{10} \frac{d\beta}{dx} \]

where,
\[ S_7 = 0 \quad \text{(in constant area duct)} \]
\[ S_8 = \frac{(a - \beta)U_c}{(h + \delta(\beta - a))} \]
\[ S_9 = \frac{\delta U_c}{(h + \delta(\beta - a))} \]
\[ S_{10} = -S_9 \]

Similarly if the power law profile is substituted into equation B5 then equation A7 is obtained,
\[ \frac{dU_c}{dx} = f_5 \frac{d\delta}{dx} + f_6 \frac{dn}{dx} \]

where,
\[ f_5 = \frac{U_c}{(h(1 + n) - \delta)} \]
\[ f_6 = -\frac{\delta U_c}{((1 + n)(h(1 + n) - \delta))} \]

(c) Two-dimensional Auxiliary Equation

(i) Entrainment Equation

Head assumed, by analogy with jets and wakes, that the entrainment in the boundary layer was determined by the velocity defect in the outer part of the layer and independent of viscosity. Furthermore, the rate of entrainment of fluid into the boundary layer was
given by

\[
\frac{d\delta}{dx} = \frac{d}{dx} \int_0^\delta Udy = \frac{d}{dx} \left[ U_c \left( \delta - \delta^* \right) \right] = \frac{dF}{dx} \text{ say B6}
\]

The function \( F \) was determined from experiment and well represented by the equation,

\[
\frac{1}{U_c} \frac{d}{dx} \left[ U_c \left( \delta - \delta^* \right) \right] = 0.0306 \left( (H' - 3)^{-0.653} \right)
\]

where

\[
H' = 1.535 \left( H - 0.7 \right)^{-2.715} + 3.3 \quad \text{B8}
\]

Using the Coles profile to evaluate the boundary layer thickness parameter \( \delta^* \) and \( H \) in equation B7 equation A4 is obtained,

\[
S_{14} + S_{15} \frac{d\delta}{dx} + S_{16} \frac{dU_c}{dx} + S_{17} \frac{d\alpha}{dx} + S_{18} \frac{d\beta}{dx} = S_{19} \quad \text{B9}
\]

where,

\[
S_{14} = 0 \quad \text{(in constant area duct)}
\]

\[
S_{15} = 1 - \alpha + \beta
\]

\[
S_{16} = S_{15} \frac{\delta}{U_c}
\]

\[
S_{17} = -\delta
\]

\[
S_{18} = \delta
\]

\[
S_{19} = 0.0306 \left( (H' - 3)^{-0.653} \right)
\]

Similar use of the power law results in equation A8,

\[
f_7 \frac{dU_c}{dx} + f_8 \frac{d\delta}{dx} + f_9 \frac{dn}{dx} = f_{10} \quad \text{B10}
\]

where,

\[
f_7 = \frac{\delta n}{U_c}(1 + n)
\]

\[
f_8 = \frac{n}{1 + n}
\]

\[
f_9 = \frac{\delta}{(1 + n)^2}
\]

\[
f_{10} = 0.0306 \left( (H' - 3)^{-0.653} \right)
\]
(ii) Two-dimensional Strip Momentum Equation

The strip momentum equation is formed from equation B4 by evaluating the integrals over a strip of the boundary layer from the wall to some fraction of the boundary layer thickness, \( \delta \). For the Coles profile assumption the strip size was \( \eta_1 = \sqrt{y/\delta} = 0.3 \), and the resulting equation which was once again of the form of equation B9 had coefficients

\[
S_{14} = 0 \quad \text{(in constant area duct)}
\]
\[
S_{15} = 0.3\alpha - 0.08106\beta + 0.79251\beta - 0.9613\alpha^2 - 0.15517\beta^2
\]
\[
S_{16} = (1.6224\alpha - 1.19566\beta + 3.057\alpha^2 - 1.7186\alpha^2 - 1.19564\beta^2) \frac{\delta}{U_c}
\]
\[
S_{17} = (0.5612 + 1.4634\delta - 2.7186\alpha) \delta
\]
\[
S_{18} = (-0.5574 + 1.323\alpha - 1.1956\beta) \delta
\]
\[
S_{19} = 0.1681\alpha^2 - \tau (0.3)^\frac{1}{\rho U_c^2}
\]

Various strip sizes (\( \eta_1 = B \) say) were used with the power law profile, and the resulting equation B10 had coefficients:

\[
f_7 = \left( \frac{1}{B^2/n} - \frac{n^2}{(1+n)(2+n)} \right) \frac{\delta}{U_c}
\]
\[
f_8 = \frac{n}{(1 + n)(2 + n)}
\]
\[
f_9 = \left( \frac{2 - n^2}{(1 + n)^2(2 + n)^2} + \frac{\log B}{(1 + n)(2 + n)} \right) \delta
\]
\[
f_{10} = B^{-1/2/n} \times \left( \frac{\tau_w - \tau_B}{\rho U_c^2} \right)
\]

(d) Shear Stress Relationship

The coefficients \( S_{19} \) and \( f_{10} \) of the strip momentum equations demand both knowledge of the wall shear stress and the shear stress at the edge of the strip.
(i) Wall Shear Stress

If the wall shear stress cannot be obtained from the velocity profile, as is the case for the power law profile, then some empirical equation is necessary. The one used here is the equation due to Ludwig and Tillmann which was derived from an examination of a number of experimental flows and relates the skin friction to properties of the mean flow,

\[ \frac{C_f}{2} = \frac{\tau_w}{\rho U_c^2} = 0.123 R_{\theta}^{-0.268} x 10^{-0.678H} \quad \text{B11} \]

The Coles Law profile, however, gives the implicit skin friction relationship,

\[ \frac{U_c}{U_*} = \sqrt{\frac{2}{C_f}} = \frac{1}{0.44} \log \left( \frac{\delta U}{\nu} \right) + 5.0 + \frac{2\pi(x)}{0.44} \quad \text{B12} \]

as demonstrated in section 3.4.1. Furthermore, substituting

\[ \alpha = \frac{U_*}{0.44 U_c} \quad \text{and} \quad \beta = -\alpha \pi(x) \]

we obtain,

\[ \beta = 0.5\alpha \left[ \log \left( \frac{\alpha \delta U_c}{\nu} \right) + 1.1584 \right] - 0.5 \quad \text{B13} \]

By differentiating equation B13 with respect to \( x \) we can obtain an equation of the form,

\[ \frac{d\beta}{dx} = S_{11} \frac{d\delta}{dx} = S_{12} \frac{dU_c}{dx} = S_{13} \frac{d\alpha}{dx} \quad \text{B14} \]

where

\[ S_{11} = \frac{\alpha}{28} \]
\[ S_{12} = \frac{\alpha}{2U_c} \]
\[ S_{13} = 0.5 \left[ 2.1584 + \log \left( \frac{\alpha \delta U_c}{\nu} \right) \right] \]

which is used as the fourth differential equation \( A3 \) of the closed set and leads to evaluation of the wall shear stress. This potential dual role of the profile would seem to provide an important basis for distinction between boundary layer prediction methods.
(ii) Shear Stress at edge of strip

Fediasovsky [40] expressed the shear stress as a power series

\[ \frac{r}{r_w} = a + b(y/\delta) + c(y/\delta)^2 + d(y/\delta)^3 + e(y/\delta)^4 \]  

subject to the following boundary conditions

(y = 0) (1) \( r = r_w \)

(2) \( \frac{\partial r}{\partial y} = \frac{dp}{dx} \) from the momentum equation written for the wall streamline.

(y = \delta) (3) \( r = 0 \) by definition of \( \delta \)

(4) \( \frac{\partial r}{\partial y} = 0 \) from the assumption that the derivative with respect to \( x \) of the total head is a continuous function.

The derivative of the momentum equation for the wall streamline yields a fifth condition,

(5) \( \frac{\partial^2 r}{\partial y^2} = 0 \) at \( y = 0 \)

A shear stress consistent with these conditions is

\[ \frac{r}{\rho U_c^2} = (1-4(y/\delta)^3+3(y/\delta)^4) \frac{r_w}{\rho U_c^2} \left( \frac{(y/\delta)^3 - 3(y/\delta)^3 + 2(y/\delta)^4}{U_c} \right) \frac{\delta}{U_c} \frac{dU}{dx} \]

A second shear stress assumption that was used was the simple eddy viscosity model as outlined in section 4.4.3. This relation for the shear stress

\[ \frac{r}{\rho U_c^2} = \frac{\theta}{U_c} \left[ 0.0225 + \frac{125}{R_b} \right] \frac{dU}{dy} \]

gave consistently better predictions than the Fediasovsky expression.

(a) Boundary Layer Thickness Parameters

Defining the wall conditions as \( y = 0, r = R, U = 0 \) and the conditions at the edge of the boundary layer as

\( y = \delta, r = R - \delta, U = U_c \)

then the two-dimensional thickness parameters can be written as,
\[ \delta^* = \int_0^\delta (1 - \hat{U}) \, dy, \quad \theta = \int_0^\delta \hat{U} (1 - \hat{U}) \, dy, \text{ where } \hat{U} = U/U_c \]

After substitution of the velocity profile and evaluation of the integrals we have

(i) power law profile;

\[ \delta_{2D}^* = \delta/1 + n, \quad \theta_{2D} = \delta n / (1 + n)(2 + n), \quad H_{2D} = \delta^*/\theta = 2 + n/n \]

(ii) Coles Law profile;

\[ \delta^* = (\alpha - \beta)\delta, \quad \theta = (\alpha - \beta + 3.17888\alpha^2 - 2\alpha^2 - 1.5\beta^2)\delta, \quad H = \delta^*/\theta \]

However for the predictions of axially symmetric flows the true axi-symmetric flow equations must be used and consequently the correct axi-symmetric definitions for the thickness parameters i.e.

\[ \delta_{AXI} = \int_R^R (1 - \hat{U}) \, \frac{\delta r}{R} \text{ and } \theta_{AXI} = \int_R^R \hat{U} (1 - \hat{U}) \, \frac{\delta r}{R} \]

Thus for the Coles Law profile,

\[ \delta_{AXI}^* = \delta_{2D}^* + (.29748 - .25\alpha)R^2 \text{ and } \theta_{AXI} = \theta_{2D} + (.29748 - .25\alpha + .25\alpha^2 + .34485 - .57112\alpha^2)R^2 \]

and \( H_{AXI} = \delta_{AXI}^*/\theta_{AXI} \)

(f) Axially-symmetric Momentum Integral Equation

Writing the Navier-Stokes equation in cylindrical polar coordinates and making the usual Prandtl boundary layer assumptions the \( x \)-direction momentum can be written

\[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{1}{\rho r} \frac{\partial}{\partial r} (\rho r r) \]

and the Continuity equation in cylindrical polar coordinates can likewise be stated as,

\[ \frac{\partial U}{\partial x} = - \frac{1}{r} \frac{\partial}{\partial r} (r U) \]
By multiplying equation B18 by \( r \) and substituting for \((V_r)\) from equation B19 and then integrating with respect to \( \eta = \sqrt{\frac{R - r}{b}} \) we obtain the Momentum Integral Equation,

\[
\frac{U(\eta_1)}{R} \frac{d}{dx} \left[ \delta R \int_0^{\eta_1} U d\eta - \delta^2 \int_0^{\eta_1} \eta U d\eta \right] - \frac{1}{R} \frac{d}{dx} \left[ \delta R \int_0^{\eta_1} U^2 d\eta - \delta^2 \int_0^{\eta_1} \eta U^2 d\eta \right] + \frac{U_0}{R} \left[ \delta R \int_0^{\eta_1} \eta d\eta - \delta^2 \int_0^{\eta_1} \eta^2 d\eta \right] \frac{dU_0}{dx} = \frac{r \eta_1 \delta}{\rho} - \left( \frac{R - \eta_1 \delta}{R} \right) \frac{r(\eta_1)}{\rho} \tag{B20}
\]

Substitution of the Coles velocity profile in equation B20 leads to the equation A1,

\[
S_1 + S_2 \frac{d\delta}{dx} + S_3 \frac{dU_0}{dx} + S_4 \frac{d\alpha}{dx} + S_5 \frac{d\beta}{dx} = S_6
\]

Here \( S_1, S_2 \) etc. refer to the axi-symmetric coefficients, where

\[
S_1 = (\alpha - \beta - 1.5 \beta^2 + 2.5 \beta^5 + 3.17888 \alpha \beta) \frac{\delta \delta R}{\rho} \text{ for diffuser flows}
\]

\[
S_2 = S_2(2D) + (1.5948 \beta - 6.96 \beta^2 - 5 \alpha + 5 \alpha^2 - 1.5 \delta - 32 \alpha \beta) \delta R
\]

\[
S_3 = S_3(2D) + (1.822 \beta + 6.96 \beta^2 - 5 \alpha + 5 \alpha^2 - 1.5 \delta - 32 \alpha \beta) \frac{\delta^2}{\rho R U_0}
\]

\[
S_4 = S_4(2D) + (25 \alpha - 25 - 5711 \beta) \frac{\delta^2}{\rho R}
\]

\[
S_5 = S_5(2D) + (2974 \alpha - 6.96 \beta - 5711 \alpha) \frac{\delta^2}{\rho R}
\]

\[
S_6 = S_6(2D)
\]

(g) Axially Symmetric Integral Continuity Equation

Considering flow through a conical diffuser a mass balance from a reference point i say to any other point yields the equation

\[
\frac{U_1 \pi R_1^2}{U_0 \pi R_0^2} = R^2 \tag{B21}
\]

and it follows from Continuity considerations that

\[
\frac{U}{U_0} = 1 - \frac{2 \delta^*}{\rho R} \tag{B22}
\]
Eliminating $U$ between equations B21 and B22 and substituting for $\delta_{MK}$ for the Coles profile and finally differentiating with respect to $x$ gives

$$\frac{dU_c}{dx} = \frac{d}{dx} \left[ \frac{U_i R_i^2}{R^2 - 2(a - \beta)R - (0.5948\beta - 0.5a)^2} \right]$$

which can once again be written in the form of equation $A^2$,

$$\frac{dU_c}{dx} = S_7 + S_8 \frac{d\delta}{dx} + S_9 \frac{d\alpha}{dx} + S_{10} \frac{d\beta}{dx}$$

where, if we define $S = \frac{U_i R_i^2}{[R^2 - 2(a - \beta)R - (0.5948\beta - 0.5a)^2]^2} \left[ R^2 - 2(a - \beta)R - (0.5948\beta - 0.5a)^2 \right]^2$

then,

$$S_7 = S \left[(2a - 2\beta)R - 2R \right] \frac{dR}{dx} \text{ for diffuser flows}$$

$$S_8 = S \left[2R(\alpha - \beta) + (1.1895\beta - a)\delta \right]$$

$$S_9 = S\delta \left[2R - 0.5\delta \right]$$

$$S_{10} = S\delta \left[0.5948\delta - 2R \right]$$

(h) Axially-symmetric Strip Momentum Integral Equation

Evaluation of equation B20 to the edge of the strip at $\eta_1 = 0.3$ after substitution of the Coles profile yields the equation $A_4$,

$$S_{14} + S_{15} \frac{d\delta}{dx} + S_{16} \frac{dU_c}{dx} + S_{17} \frac{d\alpha}{dx} + S_{18} \frac{d\beta}{dx} = S_{19}$$

where the axisymmetric coefficients are given by,

$$S_{14} = (3a - 0.08106\alpha - 0.7925\alpha^2 - 0.9613\alpha^2 - 0.15517\beta^2) \frac{dR}{R} \text{ for diffuser flows}$$

$$S_{15} = S_{15}(2D) \left(+0.08106\beta + 0.22094\beta^2 - 0.3837\alpha - 0.045\alpha^2 + 0.0916\alpha^2 \right) \frac{d\beta}{\beta}$$

$$S_{16} = S_{16}(2D) \left(+0.17004\alpha + 0.34874\beta^2 - 0.60248\alpha - 0.17586\alpha + 0.19148\beta^2 \right) \frac{d^2}{RU_c}$$

$$S_{17} = S_{17}(2D) \left(+0.1948\alpha + 0.2888\beta - 0.07668 \right) \frac{d^2}{R}$$

$$S_{18} = S_{18}(2D) \left(+0.0805 + 0.34874\beta - 0.31368\alpha \right) \frac{d^2}{R}$$

$$S_{19} = S_{19}(2D) \left[0.3\delta R + (0.3)\right] \frac{dU_c}{dx}^2$$
TWO-DIMENSIONAL DUCT FLOW

2-DIM. MOMENTUM INTEGRAL EQUATION

2-DIM. CONTINUITY EQUATION

POWER LAW VELOCITY PROFILE

COLES VELOCITY PROFILE

LUDWIG-TILLMANN SKIN FRICTION EQUATION

COLES IMPLICIT SKIN FRICTION EQUATION

2-DIM MOMENTUM STRIP AUXILIARY EQUATION

HEAD'S ENTRAINMENT EQUATION

2-DIM MOMENTUM STRIP AUXILIARY EQUATION

HEAD'S ENTRAINMENT EQUATION

SHEAR STRESS ASSUMPTION

CONSTANT EDDY VISCOSITY

\( \tau = f(y) \)

\( \tau_w = f(y/w) \)

AXIALLY SYMMETRIC PIPE AND CONICAL DIFFUSER FLOWS

AXI-SYM. MOMENTUM INTEGRAL EQUATION

AXI-SYM. CONTINUITY EQUATION

COLES VELOCITY PROFILE

COLES IMPLICIT SKIN-FRICTION EQUATION

AXI-SYM. MOMENTUM STRIP AUXILIARY EQUATION

SHEAR STRESS

CONSTANT EDDY VISCOSITY

\( \tau/\tau_w = f(y/w) \)

FIG. B1. EQUATIONS FOR INTERNAL FLOW PREDICTIONS
APPENDIX C

THEORETICAL ANALYSIS OF BOUNDARY LAYER DEVELOPMENT
IN CONICAL DIFFUSERS WITH FULLY-DEVELOPED FLOW
AT INLET
The prediction of boundary layer development in conical diffusers with fully-developed flow at inlet is relatively simple as the physical thickness, \( \delta \), of the boundary layer is equal to the local diffuser radius. Therefore, no auxiliary equation is needed. However, the momentum integral equation for axisymmetric flow must now contain a centre-line total pressure drop term \( \left( \frac{dP_c}{dx} \right) \), and can be written

\[
\frac{d\theta}{dx} + \theta \left[ \frac{1}{R} \frac{dR}{dx} + \frac{1}{U_c} \frac{dU_c}{dx} (2+H) \right] = \frac{1}{\rho U_c^2} \left[ \tau_W + \frac{R}{2} \frac{dP_c}{dx} \right] + \text{Reynolds normal stress terms}
\]

In this analysis the Reynolds normal stress term has been neglected because of the sparse amount of experimental information from which to model it. However, it is known to be of significance as separation is neared.

Using the Continuity considerations as described in Appendix B, section (g) and the well tested Ludwieg-Tillmann skin friction equation together with the power law velocity profile assumption,

\[
\frac{U}{U_c} = \left( \frac{R - r}{R} \right)^H
\]

then equation C1 can be solved by the method described in Appendix A, once the centre-line total pressure drop term has been estimated.

(a) Estimation of the term \( \frac{R}{2\rho U_c^2} \left( \frac{dP_c}{dx} \right) \)

There is no reliable method available for the estimation of total pressure loss along the streamline of maximum velocity, particularly at high values of shape factor, \( H \). In view of the lack of data \( \left( \frac{dP_c}{dx} \right) \) was accounted for using the following alternatives:

(i) Skin friction \( \tau_w \) at the wall:

\[
\frac{dP_c}{dx} = \frac{2\tau_w}{R_1} \quad \text{as used by Stevens}
\]
(ii) Constant eddy viscosity model for outer part of the boundary layer:

\[ \frac{\varepsilon}{U_c \delta^*} = G \ (G = 0.018) \]

writing \( \tau = \rho\varepsilon \frac{dU}{dy} \)

and using \( \frac{dP_c}{dx} = -2 \frac{\partial r}{\partial r} \)

whence after substitution of the power law velocity profile,

\[ \frac{R}{2\rho U_c^2} \left( \frac{dP_c}{dx} \right) = 0.009 \frac{(1 - n)(1 + 3n)}{n^2(1 + n)(1 + 2n)} = \text{function (n)} \]

(iii) Stevens experimental measurements for \( \left( \frac{dP_c}{dx} \right) \):

Using the Lagrangian formula an equation was obtained which gave an exact fit to the experimental values. This equation could be written as a function of \( n \) as follows,

\[ \frac{R}{2\rho U_c^2} \left( \frac{dP_c}{dx} \right) = 0.0007 + \frac{0.0026}{n} + \frac{0.0244}{n^2} - \frac{0.0284}{n^3} \]

(iv) \( \frac{dP_c}{dx} = 0 \)

(b) Highly Turbulent Flows

An attempt has been made at predicting the conical diffuser pressure recovery characteristics presented in Chapter 5 for fully developed inlet conditions due to,

(i) long smooth pipe

(ii) long rough pipe

(iii) velocity profile generating system.

assuming that the constant eddy viscosity model of (a)(ii) represents the \( \frac{dP_c}{dx} \) term. However, the inlet wall shear stress andcentre-line shear stress gradient are different for each method of inlet mean velocity profile generation, and this must be taken into account. Modifying the Ludwieg-Tillmann equation and equation C7 we can write,
\[ \frac{\tau_w}{\rho U_g^2} = 0.123 K_1 \times R_g^{-.268} \times 10^{-678H} \quad \text{C10} \]

and

\[ \frac{R}{2\rho U_g^2} \left( \frac{dP_o}{dx} \right) = 0.009 K_2 \frac{(1 - n)(1 + 3n)}{n^2(1 + n)(1 + 2n)} \quad \text{C11} \]

where \( K_1 \) and \( K_2 \) are constants which fit equations C10 and C11 to their inlet values.

By this very approximate model the high inlet turbulence of (ii) and (iii) is partially accounted for.
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