Advanced Multivariable Control Law Design
For Future ACT Rotorcraft

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This thesis considers the use of variable structure and $H_{\infty}$ control system designs for the improvement of the handling qualities of the Lynx helicopter, with full flight envelope operation and enhanced turbulence rejection capabilities.

A model-following variable structure controller is designed, where the 'ideal model' was designed in four different ways using the following methods: eigenstructure assignment, $H_{\infty}$ minimum entropy, $H_{\infty}$ one degree-of-freedom loop shaping, and $H_{\infty}$ two degrees-of-freedom loop shaping. The combination of the ideal model and the loop shaping is seen to allow the time and frequency domain objectives to be specified during the variable structure control system design. The decomposition of the $H_{\infty}$ one degree-of-freedom and two degrees-of-freedom control laws into state-feedback and observer form allows the use of these observers in the implementation of the variable structure controllers. Extensive testing and evaluating of handling qualities, throughout the operating flight envelope, is performed on the non-linear Rationalised Helicopter Model.

Successful Advanced Flight Simulator trials performed at the Defence Research Agency are described, which signify the first real-time 'flight' of a helicopter controlled by a variable structure control law design. A variable structure observer is designed to complete the variable structure observer/controller framework, and shows good handling qualities and time response characteristics throughout the flight envelope.

A novel method of combining established $H_{\infty}$ control system design procedures with modelling of the turbulence entering the linearised system equations, enables generic $H_{\infty}$ control law designs with significantly enhanced turbulence rejection qualities to be successfully demonstrated. The turbulence effects are modelled as perturbing the helicopter's velocity equations, and are included as an extra input to the $H_{\infty}$ Standard Compensation Configuration. Therefore, no statistical knowledge of the turbulence was required. The variable structure control law designs are tested for their turbulence rejection capabilities, and exhibit high levels of handling qualities without having been designed explicitly for turbulence rejection in the design procedure.
Acknowledgements

I would like to express my deepest gratitude to my supervisor, Professor Ian Postlethwaite, for his guidance and encouragement throughout the last seven years of undergraduate and PhD research work. Also, I am indebted to Dr. Sarah Spurgeon for her invaluable guidance and enthusiasm. I wish to thank Dr. Daniel Walker for his help and assistance, particularly during the early stages of the PhD.

The research work was carried out in collaboration with the Defence Research Agency (DRA), Bedford. I am grateful to Dr. Gareth Padfield and Jeremy Howitt for their willingness to communicate their knowledge of the helicopter, and also for their help and interest during the flight simulator trials. I acknowledge, gratefully, the financial support from the U.K. Science and Engineering Research Council (SERC) and the DRA, Bedford.

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Chapter 1

Introduction

The helicopter is unique for flight and control in its ability to ascend and descend vertically, move in any direction horizontally, or hover over a specific spot on the ground and while doing so turn onto a desired heading. A typical single main rotor helicopter, such as the Lynx, has open loop characteristics that are highly nonlinear, unstable and cross-axis coupled. The pilot, therefore, may experience an excessive and fatiguing workload making precise control of the helicopter difficult. Present generation fighter aircraft, such as the Tornado, have Fly-By-Wire (FBW) systems (i.e. electrical links from the pilot’s controls to the on-board computers through to the control surface actuators). However, this is not the case with today’s helicopters. At present, many helicopters make use of stability augmentation systems and complex electro-mechanical interlinks to help the pilot to perform various types of manoeuvre. To carry out the more aggressive manoeuvres the pilot may need to disable these autostabilization systems, due to the inherent limitations of the Single-Input/Single-Output (SISO) classical control techniques used for such designs. Also, technological developments have resulted in further challenges to the control system designer, such as consideration of: replacement of the tail rotor by other designs (for example: NOTAR: NO TAil rotoR, fan in the tail); incorporating all the pilot’s controls into a sidestick; installation of fibre-optic systems (fly-by-light). Therefore, the future generation of helicopters, like the Comanche (Ashley [2]), are looking to full authority Multi-Input/Multi-Output (MIMO) FBW systems to provide the high levels of performance required. This justifies the need to further advance the present state-of-the-art helicopter multivariable control
system design techniques.

The helicopter considered in this work is representative of the Lynx, which is an all-purpose military helicopter, suitable for transportation of equipment and personnel, as well as an attack role. Therefore, the highest levels of performance may be required in the most demanding circumstances, and any FBW control system must be able to meet these. The helicopter has a wide speed range, typically from hover (0 knots) up to approximately 160 knots, which can be roughly described as changing from a hovercraft into an airplane. Therefore, the associated dynamics change significantly over this flight ‘envelope’. There are numerous research literature applications into designing control systems for a small part of this flight range, usually at a single speed. However, for a particular flight control system to be practically applicable to future helicopters, it must meet the stringent performance requirements throughout the operating flight envelope.

The design of a single controller for a nonlinear dynamically changing system which will maintain stability and performance over the full operating envelope is very challenging. The normal procedure used to overcome speed-dependent dynamical nonlinearities with linear controller designs, is to switch or interpolate (Campo et al [9], and Shamma and Athans [88]) between scheduled controllers. This increases the overall system complexity and computing time since the controllers must be either run on-line for bumpless transfer [9], or constantly interpolated. Also, transients will be seen when switching between controllers. This phenomenon appears whether or not bumpless transfer schemes are utilised, due to the inherent difficulty of matching controller outputs and controller output rates. Further, there are unlikely to be any formal stability results relating to the wide-envelope performance of such scheduled linear controllers. This leads to the need to investigate controller design methods which have the theory and potential to (with a single controller) stabilise and maintain performance over a systems full operating envelope (in this particular case, the speed-dependent dynamically varying flight envelope of a helicopter). The stabilization should also be achieved ‘robustly’, that is, it should still be maintained in the face of levels of uncertainty (both unmodelled and unknown) in the system models used for design.

Variable structure control system design methods have been successful in a wide
variety of applications (Zinober [121]), and have robustness and disturbance rejection capabilities which are particularly applicable to aerospace systems. The term ‘variable structure control’ describes the fact that the controller structure around the plant is nonlinear and changes according to certain conditions or disturbances (Decarlo et al [15]). In this work a nonlinear control law is designed in a ‘model-reference’ framework (Spurgeon and Davies [92]). Model-reference control implies that the control system is designed to attempt to force the plant (in this case, the helicopter) to follow a desired ‘ideal’ model. Formal stability results relating to this composite controller are available ([92]). The controller acts on the error vector between the real plant and an ideal model. The model-following approach was selected due to its frequent use in helicopter/airplane controller applications (Durham et al [20]), and also because of the successful flight-simulator testing of an \( H_\infty \) two degrees of freedom designed controller (Walker et al [106]). It was this linear controller ([106]) and an earlier linear controller, using one degree of freedom \( H_\infty \) loop shaping methods (Walker and Postlethwaite [105]), which were used as benchmarks to compare the variable structure controllers. Four likely methods were applied for the design of the ideal model dynamics: eigenstructure assignment, \( H_\infty \) minimum entropy design (which is based on Linear Quadratic Gaussian methods), \( H_\infty \) one degree of freedom design, and \( H_\infty \) two degrees of freedom design. These were chosen since they are all very commonly employed in the literature and research involved with rotorcraft controller design. Initially, state-feedback was assumed when using these four methods, but for the final designs state observers were utilised for the flight simulator trials and for comparison with the benchmark \( H_\infty \) controllers.

\( H_\infty \) optimal control theory has continually developed during the 1980's, with early theoretical contributions from such influential sources as (Zames [117], Zames and Francis [118]), into an extremely powerful design tool, especially when coupled with singular value ideas (Postlethwaite et al [75]), which allow extensive consideration of performance and robustness frequency domain objectives. Therefore, a key element to this thesis is the building on, and use of ideas from, the already established helicopter \( H_\infty \) control system design work (Tombs [96], Yue [114], [105], [106]).

The Method Of Inequalities (MOI) (Whidborne et al [111]) was implemented to
assist with the tuning of the design parameters in the $H_\infty$ minimum entropy design. An $H_\infty$ approach to the ideal model specification has been tested in computer simulation on a flexible manipulator joint (Hashimoto and Konno [46]), but used different $H_\infty$ methods to those implemented here. The military specification of helicopter handling qualities is defined in the document ADS-33C ([4]), and can be used to quantify the performance of control system designs. In fact, the Comanche (Landis et al [57]), which has been designed to be the next generation scout/attack helicopter, was the first helicopter to be procured under these ADS-33C specifications. The nonlinear model used for all the testing of the controller designs in this thesis is the Rationalised Helicopter Model (RHM), provided by the Defence Research Agency (DRA). It was at the DRA, Bedford, that one of the significant aspects of this thesis was carried out: the first ground-based flight simulations of a VS controller, and also the testing of that controller over a wide speed range.

Following the VS controller evaluation on the flight simulator, which had an observer resulting from $H_\infty$ design procedures, an investigation was carried out into the design of a VS observer. This completed the VS controller/observer framework, and provided the opportunity to test whether a VS observer could provide any benefits in the need to estimate the helicopter rigid body states. Again, nonlinear simulations and handling qualities were performed at 20 knot intervals from 1 knot to 120 knots.

Another major factor in a pilot’s workload is the influence of atmospheric turbulence. This is particularly important when flying close to the ground, called Nap-of-Earth (NOE), where wind shear can cause serious problems. The benefits of decreased turbulence effects on the helicopter can be listed as:

- Reduction of pilot workload
- More aggressive manoeuvres with higher precision
- Achievement of improved performance objectives
- Aircraft limits less likely to be exceeded (carefree handling)
- Increased passenger ride comfort and safety
However, it must be noted that there is the usual engineering trade-off: using the active control surfaces (i.e. the two rotor systems) to reduce the buffeting will increase the actuator usage and airframe loadings. Obviously though, the designer must have the capability to increase the handling qualities if so required. One possible approach to decrease the turbulence effects would be to use a filter (so called ‘notch filter’) at the correct frequencies to attenuate the turbulence signals in the control system feedback loop. One disadvantage of this is that gusts are typically in the lower frequency range, where command signal tracking performance is predominantly most important and may therefore be compromised. The design of this filter would also be difficult, since the gust spectrum can vary over a wide range of frequencies. Therefore, it would be a distinct advantage to have a systematic procedure to design control systems, where any knowledge of how the gust effects the helicopter is used explicitly. Such a procedure has been devised in Chapter 7 for a number of $H_{\infty}$ controller design methods. These established $H_{\infty}$ controllers had been proved by earlier authors in piloted flight simulator tests, and include $H_{\infty}$ mixed sensitivity design ([114]) and Loop Shaping Design methods (Walker and Postlethwaite [103]). No information as to the statistical content of the turbulence is required, since it is treated as an extra input to the $H_{\infty}$ standard formulation. The novel element of the design procedure is the modelling of the effect of the turbulence, on the helicopters equations of motion, as perturbations in the velocity states. All the $H_{\infty}$ controllers and VS controllers were simulated to investigate their handling qualities in the face of turbulence.

The main objectives, faced by the designer of advanced full authority control systems for active control technology helicopters, can be summarized as:

- Achieving acceptable levels of stability robustness.
- Meeting the highest performance levels.
- Decoupling the control inputs.
- Accurate tracking of pilot command inputs.
- Pilot workload reduction.
- High bandwidth.
• Design covering the full operating flight envelope.

• Good turbulence rejection handling qualities.

1.1 Major Contributions

These are detailed in the corresponding order to which they appear in the thesis:

• Comprehensive review of the research literature applied to helicopter control law design, concentrating on the full flight envelope and turbulence rejection aspects.

• Application of a model-reference variable structure control law to the helicopter.

• Investigation into various different methods applicable to the design of the ‘ideal’ model required in this VS control system design.

• Use of the Method of Inequalities to tune the design parameters required for $H_\infty$ minimum entropy controller design.

• Utilization of the state-feedback/observer structure of 1 DOF and 2 DOF $H_\infty$ controllers to design the ideal model and provide a state observer.

• Use of the $H_\infty$ Loop Shaping Design Procedures with the model reference VS controller design.

• Extensive simulation and handling qualities analysis on a VS controlled proven nonlinear helicopter model.

• The first and successful testing of a VS controller on a Flight Simulator.

• Comparisons made with established 1 DOF and 2 DOF $H_\infty$ controllers.

• Design of a VS observer (both 1 DOF and 2 DOF) to obtain a complete VS controller/observer framework.

• Comparison of the VS controllers designed using either VS or $H_\infty$ based state observers.
• A working and systematic methodology to design 1 DOF $H_\infty$ controllers for reducing the effect of turbulence.
• Novel combination of turbulence modelling with $H_\infty$ design procedures.
• Enhancement of the turbulence rejection qualities of established 1 DOF $H_\infty$ control system designs.
• Comparison of the turbulence handling qualities of the VS controllers (with either $H_\infty$ or VS designed observers) and the $H_\infty$ controllers applied in the thesis.

1.2 Summary of Chapters

Chapter 2

This first background chapter describes the helicopter system, the issues with regards to control, and the setting for needing to enhance the present state of the art fly-by-wire control system design methods. The pilot inputs and helicopter measurable outputs are discussed. Also, the Military Specification for handling qualities of rotorcraft is introduced, since it will be used extensively in later chapters to quantify the performance of controller designs.

Chapter 3

Building on the helicopter description, this chapter surveys the methods by which the present literature is trying to meet the requirements of future FBW systems.

Chapter 4

This, the final introductory chapter, outlines the major features and design procedures of the control system design methods most commonly applied to helicopters. Details of these methods are required for the controller designs of Chapters 5, 6, & 7.
Chapter 5

An introduction and historical background to Variable Structure (VS) control system design begins this chapter. The VS model reference methodology is then described, with details of how to design the ideal model by a variety of different methods. These include incorporating design details from 1 DOF and 2 DOF $H_\infty$ loop shaping design procedures. Nonlinear simulation results in all four primary output channels are shown at 20 knot trimmed speed intervals covering the flight envelope. An extensive handling qualities evaluation according to Military Specifications, and results from ground based flight simulator trials are presented to analyse the performance of the designed controllers. Also, included in this chapter are comparisons with established $H_\infty$ controllers. The VS controller, with an ideal model designed by two different methods (eigenstructure assignment and $H_\infty$ minimum entropy), and associated simulation results were presented at the European Rotorcraft Forum (Foster et al [34]).

Chapter 6

This chapter contains an investigation into the use of a VS observer coupled with the VS controllers designed in the previous Chapter 5. Nonlinear simulation results and handling qualities analyses throughout the flight envelope are again presented.

Chapter 7

The previous two chapters were concerned with the widely varying flight envelope nature of the helicopter. This chapter demonstrates the enhancement to the controlled helicopters handling qualities which can be made by implicitly incorporating turbulence information into the $H_\infty$ design procedures. An introduction to turbulence fundamentals, the enhanced $H_\infty$ design methods and the nonlinear simulation results are included. The VS controllers of the previous two sections 6 & 7, are also tested for their turbulence handling qualities, and finally all the controllers' results compared. The $H_\infty$ mixed sensitivity based design work was presented at an IEE Control'94
Conference (Postlethwaite et al [76]), and an IEE Colloquium on Flight Control Applications (Foster et al [33]). The 1 DOF $H_{\infty}$ LSDP based designs and VS controller responses were presented at the European Rotorcraft Forum (Foster et al [32]).

Chapter 8

The conclusions and important findings are discussed, with directions for future research indicated.

1.3 Mathematical Notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{R}$</td>
<td>Real Numbers</td>
</tr>
<tr>
<td>$\mathbb{R}^{n \times m}$</td>
<td>Set of real matrices ($n$ rows and $m$ columns)</td>
</tr>
<tr>
<td>$H_{\infty}$</td>
<td>Hardy space of functions analytic and bounded in the open right half plane</td>
</tr>
<tr>
<td>$L_2$</td>
<td>Hilbert space of matrix-valued functions which are square integrable on $j\omega$</td>
</tr>
<tr>
<td>$H_2$</td>
<td>Functions in $L_2$ which are analytic in the open right half plane</td>
</tr>
<tr>
<td>$\forall$</td>
<td>For all</td>
</tr>
<tr>
<td>$\in$</td>
<td>Is an element of</td>
</tr>
<tr>
<td>$\text{sgn}(\cdot)$</td>
<td>Signum function</td>
</tr>
<tr>
<td>$\sigma(\cdot)$</td>
<td>Maximum singular value of ($\cdot$)</td>
</tr>
<tr>
<td>$\sigma_{\min}(\cdot)$</td>
<td>Minimum singular value of ($\cdot$)</td>
</tr>
<tr>
<td>$|G|_{\infty}$</td>
<td>$\sup_{\omega} \sigma[G(j\omega)]$</td>
</tr>
<tr>
<td>$K(s)$</td>
<td>Controller transfer function</td>
</tr>
<tr>
<td>$G(s)$</td>
<td>Nominal plant model</td>
</tr>
<tr>
<td>$</td>
<td>d</td>
</tr>
<tr>
<td>$A^T$</td>
<td>Transpose of the matrix $A$</td>
</tr>
<tr>
<td>$\text{det}(A)$</td>
<td>Determinant of the square matrix $A$</td>
</tr>
<tr>
<td>$A^{-1}$</td>
<td>Inverse of the square matrix $A$</td>
</tr>
<tr>
<td>$\text{rank}(A)$</td>
<td>Rank of the matrix $A$</td>
</tr>
<tr>
<td>$\lambda(A)$</td>
<td>Set of eigenvalues of the square matrix $A$</td>
</tr>
<tr>
<td>$\lambda_{\max}(A)$</td>
<td>Largest eigenvalue of the square matrix $A$</td>
</tr>
</tbody>
</table>
$\lambda_{\text{min}}(A)$  Smallest eigenvalue of the square matrix $A$

$I_n$  Identity matrix (size, $n \times n$)

$\| \cdot \|$  Vector Euclidean norm or matrix Spectral norm

$\equiv$  Equivalence

$\times$  Cartesian product

$\perp$  Orthogonal Complement

$\triangleq$  Equal to by definition
Chapter 2

The Helicopter

2.1 Introduction

The helicopter application is introduced and key factors influencing the control systems engineer, such as the highly nonlinear dynamics, are discussed. This includes attention to the methods by which the pilot is able to fly the helicopter at present through the use of the mechanical linkages between his controls, the hydraulics and the actual control surfaces. Finally, the main elements of the military handling qualities specifications ([1]) are introduced, which will be used in later Chapters 5 & 6 to quantify objectively the controller performance and handling qualities.

2.2 Pilot Controls

Pilots are required to control the flight of a six-degrees-of-freedom helicopter with two hands and two feet. Six pilot control inputs would be unmanageable for the average human being (Kellett [58]), and so four of the inputs are chosen. These are distributed with two axes for the right hand, one for the left hand, and one for both feet in total. To produce changes in pressure distribution and lift forces on the main and tail rotor blades, the pilot has control over the pitch settings of these rotor blades. The different pilot control inputs, corresponding to different blade pitch settings and therefore helicopter movements, are made using the following:
• Collective Pitch Lever - The pitch angle of the main rotor blades is collectively changed to cause vertical movement (Figure (2.1), Prouty [79], [114]).

![Collective Pitch Control System](image)

Figure 2.1: Collective Pitch Control System (Redrawn from [79])

• Foot Pedals - The pitch angle of the tail rotor blades is changed collectively to produce directional control. The main purpose of the tail rotor, apart from heading control, is to counteract the main rotor torque, in the absence of which the helicopter would spin destructively in circles.
Cyclic pitch Lever - The main rotor disc, and therefore the thrust vector, is tilted cyclically by varying the pitch of the main rotor blades individually to cause horizontal movement of the aircraft (Figure (2.2)).

The important difference between cyclic and collective pitch control is that in cyclic pitch control the blades change pitch individually during the cycle of rotation so that at any chosen time the pitch of one blade is increasing, whilst the pitch of the other blade is decreasing.

The collective pitch lever is to the left (and the cyclic stick in front) of the pilot’s seat. The rotor is typically kept at a constant speed, by an automatic engine torque control unit.

2.3 Control System Design Considerations

Typically, there are a number of real-time measurements (Garrard and Low [36]) that are directly available to the flight control system: roll rate, pitch rate, yaw rate. Other
measurements that can also commonly be included are pitch angle, roll angle, heading angle. In total, a significant amount of on-line information as to the state of the helicopter can be provided to the flight control system.

In line with the previous work ([106], [105]), which was also in conjunction with the Defence Research Agency, DRA, Bedford, the following measured outputs were considered to be available for control (Table 2.1):

<table>
<thead>
<tr>
<th>Con. O/p</th>
<th>Descrip.</th>
<th>Pilot I/p</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (y1)</td>
<td>Heave Velocity</td>
<td>Collective - Stick</td>
<td>Ft/s</td>
</tr>
<tr>
<td>$\Theta$ (y2)</td>
<td>Pitch Attitude</td>
<td>Longitudinal Cyclic - Stick</td>
<td>Rad.</td>
</tr>
<tr>
<td>$\Phi$ (y3)</td>
<td>Roll Attitude</td>
<td>Lateral Cyclic - Stick</td>
<td>Rad.</td>
</tr>
<tr>
<td>$\Psi$ (y4)</td>
<td>Heading Rate</td>
<td>Tail Collective - Pedal</td>
<td>Rad/s</td>
</tr>
</tbody>
</table>

Table 2.1: Controlled Outputs

In addition roll and pitch rates were considered to be measurable, and were used as inputs for the controllers designed in the rest of the thesis, but not actively controlled. The above control inputs and controlled outputs result in an ‘Attitude Command Attitude Hold’ (ACAH) response type, which is desirable in hover and bad visibility conditions. An alternative response type called ‘Rate Command Attitude Hold’ (RCAH) is characterized as having a high level of agility, with the capability for extended periods of hands-off control, and being more natural at high forward speed. For control purposes the coordinate system, illustrated below (figure 2.3), is applied, where $(u,v,w)$ are components of the total velocity and $(p,q,r)$ the roll, pitch and yaw angular rates about each of the $(x,y,z)$ axis respectively.

![Figure 2.3: Helicopter Body-Fixed Axes](image)

The phrase 'rigid body states' describes the following list of helicopter states, which were used for state space linearizations at a particular speed (Table 2.2):
Table 2.2: State Vector

<table>
<thead>
<tr>
<th>State</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Θ</td>
<td>Pitch Attitude</td>
</tr>
<tr>
<td>Φ</td>
<td>Roll Attitude</td>
</tr>
<tr>
<td>p</td>
<td>Roll Rate</td>
</tr>
<tr>
<td>q</td>
<td>Pitch Rate</td>
</tr>
<tr>
<td>r</td>
<td>Yaw Rate</td>
</tr>
<tr>
<td>u</td>
<td>Forward Velocity</td>
</tr>
<tr>
<td>v</td>
<td>Lateral Velocity</td>
</tr>
<tr>
<td>w</td>
<td>Vertical Velocity</td>
</tr>
</tbody>
</table>

State space linearizations of the form:

\[
\dot{x} = Ax + Bu \\
y = Cx
\]

used for the controller designs of the later chapters were taken from the nonlinear model, called the Rationalised Helicopter Model (RHM), from the DRA, Bedford. The model is based on a LYNX-like multi-task military helicopter, with a four-bladed semi-rigid main rotor. The model contains eight rigid body states, three engine states, four simple first-order lag actuator states, and six rotor dynamic states amongst other inherent model states. The rotor model is six state, with two flapping modes and one coning mode, each mode represented by a second order system. The rotor dynamics are extremely important, and not at all extensively modelled, which gives good reason for understanding the aerodynamical effects of the rotors. Such effects are the so-called rotor flapping, coning and lead-lag modes ([79]). The coning mode effects can be visualised as due to the shape that the main rotor blade tips sweep out, forming a cone in the air. This coning angle will increase, that is the blade tips will increase in height relative to the helicopter, if a rotor collective input was applied to move the aircraft upwards.

The rotor flapping is due to the asymmetrical velocity distribution of the retreating blades ([79]), compared to the advancing blades.

This is illustrated (Figure 2.4) at a typical forward cruising speed of approximately 120 knots, where the blades over the nose and over the tail have the same velocity distributions, but the retreating tip is revolving 30% slower and the advancing tip 30%
Supposing the helicopter starts off accelerating, with each blade having the same pitch setting (therefore angle of attack), the difference in velocity produces more lift on the advancing side than the other retreating side, thus with a non-rigid rotor the advancing blade lifts up and follows a higher trajectory. This climbing blade, then has a decreased angle of attack, and vice versa on the retreating, which will end up with an equilibrium position when the lift distribution is balanced.

The change in the location of the helicopters system poles can be indicated by plotting the eigenvalues of a rigid body (eight state) linearized system at 0 knots (Figure (2.5)) and 100 knots (Figure (2.6)), which further illustrates the dynamically varying nature of the helicopter, especially with reference to forward speed. Two of the eigenvalues stay in the right half plane, that is they remain in an unstable region. Also, two of the other eigenvalues move into more lightly damped positions as forward speed is increased. Further detailed descriptions of the helicopter aeromechanics can be found in various texts (Prouty [80], Prouty [81], Prouty [82], Gessow and Myers [40]).
2.4 Handling Qualities Specifications

The current US Army Aviation Systems Command specification, in the Aeronautical Design Standard (ADS-33C [1]), details the requirements for the flying and ground handling qualities of military rotorcraft. The requirements are 'intended to assure that no limitations on flight safety or on the capability to perform intended missions will result from deficiencies in flying qualities'. They are structured into three speed regions relative to the hover reference point:

- Hovering Flight - less than 15 knots (7.7 metres/second)
o Low Speed - between 15 and 45 knots (7.7 and 23 metres/second).

o Forward Flight - above 45 knots (23 metres/second)

Each speed region is then sub-divided into small, moderate and large amplitude manoeuvres with respect to the helicopters respective short, medium and long term responses in each control axis.

The acceptability of the flying qualities in each sub-division is quantified in terms of 'levels', which indicates performance attributes equated to pilot ratings on the Cooper-Harper scale (Cooper and Harper [11]). These can be summarized in outline as:

- **Level 1.** Equates to Cooper-Harper ratings 1 - 3.5, and indicates performance satisfactory without improvement.

- **Level 2.** Equates to Cooper-Harper ratings 3.5 - 6.5, and indicates adequate performance with tolerable workload.

- **Level 3.** Equates to Cooper-Harper ratings 6.5 - 8.5, and indicates performance deficiencies requiring improvement.

Further detailed descriptions of the use of the handling quality specifications can be found in previous work ([105]).

### 2.5 Conclusion

The need for a 'robust' controller design, due to the presence of nonlinear dynamics and unmodelled rotor dynamics, has been indicated. Also, the widely varying dynamical nature of the helicopter requires controller designs which can maintain high levels of stability and performance throughout the flight envelope. The military design standard has been introduced as a means of quantifying the handling qualities of particular control system designs. Having justified the need for robust and wide envelope controller designs, the following chapter reviews to what extent the present literature is meeting these requirements.
Chapter 3

Review of Helicopter Control System Design Studies

3.1 Introduction

The review that follows of the past and present literature on helicopter flight control system (FCS) design is required to motivate the need to investigate FCS design methods which may enable stability and performance objectives to be maintained over the full operating flight envelope. Reviews of the helicopter control system design literature were carried out in 1990, ([36], Manness et al [64]), but since then there has been a great deal of significant work done by a variety of authors. Therefore this review will extend, for completeness, up to the present year, 1995. The methods applied to helicopter control system design have been grouped under the following headings:

1. Multi-Input Multi-Output Classical-type Control laws
2. Linear Quadratic Regulator (LQR) / Linear Quadratic Gaussian (LQG) and other $H_2$ Methods
3. $H_\infty$ Design Methods
4. Eigenstructure Assignment Methods
5. Other Methods
Further details of the $H_{\infty}$, $H_{\infty}$ minimum entropy (based on LQR ideas), and variable structure design methods are contained in the chapter 4. Design details for the other helicopter flight control laws can be found in the relevant papers referenced in each section. Tischler [95] presents a comprehensive review of the practical concepts of which control system designers must be aware, such as the importance of high frequency dynamics and the need for full envelope controller designs. This chapter reviews the extent to which previous work has considered the fact that the helicopter has to be able to fly many tasks from hover up to approximately 140 knots. Also, the potential control laws in the literature are investigated for whether they have been shown to work successfully on nonlinear models, ground based simulators or real in-flight simulations. Whether the controller designs include consideration of turbulence effects is discussed in the chapter conclusion.

3.2 Multi-Input Multi-Output Classical-type Control laws

Research effort was bound to investigate whether classically designed Single-Input Single-Output (SISO) control systems could be directly extended to Multi-Input Multi-Output (MIMO) control laws to meet the new handling qualities requirements. The SISO Proportional/Integral (PI) controller is one of the most commonly known and successful structures from classical control. MIMO PI generalisations have been applied to the helicopter control problem (Enns [27]), where most of the control law development was carried out using a eight state model of the Apache YAH-64. The single hover point controller was tested on a ground flight simulator at McDonnell Aircraft Company. The system response to a vertical gust step inputs was tested, and a 10 knot gust disturbance was shown to be attenuated. Real helicopter flight tests of the controller were also demonstrated (Parlier [74]).

A multivariable PID autostabilization controller was developed by Fenton et al [29]) for the EH101 helicopter. This was based on a 146 knot linear model, and showed
the controller maintaining stability of a nonlinear model at hover and at 146 knots. However, the ability to follow pilot command inputs at any speed, other than the design point of 146 knots, was not demonstrated.

3.3 Linear Quadratic Regulator (LQR) / Linear Quadratic Gaussian (LQG) and other $H_2$ Methods

The LQG/LTR techniques have been used extensively in fixed wing aircraft flight control laws (for a review see Wendel and Schmidt [108]). A single fixed linear $H_2$ controller designed at hover was simulated for four low speed hover tasks on the NASA Ames Research Centre flight simulator by Takahashi [94]. The controller performed well, but was found to exhibit an underlying Pilot Induced Oscillation (PIO) in roll for speeds away from hover. Single low speed controllers were designed using Linear Quadratic Gaussian (LQG) methods (Gribble [43]), but only tested on linear helicopter models from 0 to 50 knots. An earlier LQG controller designed by Gribble and Murray-Smith [44] was based on an 80 knot linearization and then tested on a 80 knot higher order linearized model. LQG control system design procedures carried out by Prasad et al [78] were for a single operating point, and they stated that gain scheduling or other methods may be required for full envelope control. Similarly, Duc and Mammar [19] performed a LQG/LTR control law design which was again only designed and tested on a linear model. LQ model-following controllers were designed by McKillip and Perri [67] and evaluated with their optimum trajectory algorithms for an AH-1G helicopter model, but only for near-hover flight conditions.

3.4 $H_\infty$ Design Methods

The use of $H_\infty$ robust optimization methods for helicopter controller design have been particularly notable in the recent literature. The work of Yue and Postlethwaite [115], and Yue [114] showed an $H_\infty$ controlled helicopter system successfully carrying out
hover-based tasks on the Defence Research Agency's Advanced Flight Simulator (AFS). $H_\infty$ design formulations by Young and Lin [112], and Mammar and Duc [63] were applied and tested on single linear state-space models. The control system designs of Walker and Postlethwaite [103], [102], resulted in the recent and very successful AFS simulation of an $H_\infty$ controller which gave the highest level of ratings possible (according to the acknowledged military Cooper-Harper Rating System [1]) for the variety of low speed tasks performed by the real military pilot (Walker et al [106]). The issue of full-envelope design was partially tackled, since part of the flight envelope was covered with five single controllers having been switched between every 20 knots from hover to 80 knots forward speed. The scheduling of $H_\infty$ controllers was discussed by Kellett [58], but not implemented on anything further than linear models.

3.5 Eigenstructure Assignment Methods

Eigenstructure assignment (EA) techniques have been widely applied to the control of fixed wing aircraft (Garrard and Low [36]), therefore it was only natural for them to be applied to rotary wing aircraft. The flight control laws for the Aerospatiale A320 Airbus were developed using these methods (Farineau [28]). One of the earliest helicopter EA contributions (Parry and Murray-Smith [74]) was designed on and tested on a simple eighth order 80 knots linearized model, and it was stated that investigations would be conducted into possible gain scheduling to cover more of the flight envelope. Similarly, early papers (Garrard et al [39], Garrard and Low [36] [37], Garrard and Liebst [35], and Garrard et al [38]) reveal designs for a near-hover linear model and were only tested on higher-order linear models at the same speed. A later and relatively recent paper (Low and Garrard [61]) recorded the testing of their designs on further higher-order linear models, incorporating more explicit rotor and actuator dynamics, but testing at other speeds was confined to a different 10 knot high order linear model. The work of Ekblad [26] used similar EA methods to the Garrard et al research quoted above, but was directed at reduced order modelling rather than at considering the flight envelope. Innocenti and Stanziola [54] were concerned with performance and robustness issues of EA controllers, rather than demonstrating full envelope control law design. Likewise, Smith [91] was concerned with the details of EA design for helicopters and concentrated
the control law testing on a part of the flight envelope (40-100 knots) of a nonlinear LYNX model. Gain scheduling issues were tackled by Manness and Murray-Smith [65], and responses shown compared a fixed hover designed EA controller with constantly interpolated EA controllers designed at various points from -5 to 45 knots. Pitch input responses on the nonlinear model were shown. The authors stated that even though their results showed marginal difference in the responses of these two controllers in this low speed region, the gain scheduled controller would be able to cover a larger portion of the flight envelope before instabilities developed. A combined EA and $H_\infty$ controller was designed for an eighth order linear model by Samblancat et al [86], but was only tested on a 12th order linearized model at the same speed.

### 3.6 Other Methods

A method of designing a full envelope controller using nonlinear transformation theory, to avoid having to use a linearized model in the design procedure, was applied to the helicopter by Heiges et al [47]. However, the authors stated it was tested on a 'rather simple' nonlinear rigid body nonlinear model, and results are shown for only a single speed region (101 feet/second). Approximate Model Inversion was used by Prasad and Lipp [77] for designs on an Apache helicopter model, and tested for a variety of low/medium speed tasks on a nonlinear computer model, but the resulting controller was not tested on a Flight Simulator.

Multivariable Generalised Predictive Controller (GPC) designs (Aslan et al [3]) were only applied to the longitudinal helicopter motion and simulations carried out on perturbed linear models. A strategy of utilising variable structure ('sliding mode') control methods was applied by Fossard [30] to the helicopter control problem, however, the proposed control configuration did not actively control aircraft height and was not tested on a nonlinear model. High gain explicit model-following techniques were compared to eigenstructure assignment methods (Osder and Caldwell [70]), and tested on nonlinear models of the AH-64 helicopter for a variety of low/medium speed tasks.

The multivariable analysis framework (known as Individual Channel Design, ICD) was used (Liceage-Castro et al [59]) to apply control to an 80 knots helicopter model,
but the work was predominantly aimed at analysing the structural and robustness issues of the equations of motion.

Hess and Gorder [48] applied Quantitative Feedback Theory (QFT) to the longitudinal control of an AH-64 helicopter linear model, but no time domain simulation results were shown, and the frequency analysis was carried out on linear models. Decoupling precompensation with a QFT controller (Catapang et al [10]) was tested on UH-60 Black Hawk helicopter linear models from hover up to 15 knots.

3.7 Radio-Controlled Helicopters

There have been a number of different control schemes applied to miniature radio-controlled helicopters. They have similar control problems, but have a smaller flight envelope and are usually heavily mechanically stabilised. Also, the time delay of the radio controller can cause significant problems. The sliding mode design of Sira-Ramirez et al [89] was only applied to altitude stabilization. A two-degree-of-freedom fuzzy-neuro controller structure was applied to a miniature radio-controlled fan helicopter, which was required to 'learn' about fixed computer-generated tasks, i.e. circular trajectories. A variety of control techniques (\(H_2\), eigenstructure assignment, LQG/LTR) have been applied, and used primarily for comparison, by Weilenmann and Geering [107] for the control of an indoor computer-controlled model helicopter. Neural Network control system designs by Pallet and Ahmad [72] were applied to the vertical control and adaption to changes for a miniature helicopter.

3.8 Conclusion

Most of the previous research studies have concentrated on single point designs (i.e. a design at one flight speed), and those that do consider a larger part of the flight envelope have not been proven on nonlinear models throughout the whole speed range. Also, of the advanced controller methods only the \(H_\infty\) controller designs ([115], [106]) have been tested to any great extent in piloted ground based flight simulations to rigorous specifications ([1]).
None of the helicopter controller designs in the literature have included knowledge about the turbulence encountered by a helicopter. Three of the reviewed papers ([65], [64], [30]) considered the gust rejection capabilities of the particular closed loop system, but none of the helicopter controller designs in the literature have directly included in the controller design formulation knowledge about the turbulence encountered by a helicopter.
Chapter 4

Candidate Helicopter Flight Controller Design Methods

4.1 Introduction

This chapter is motivated by the need to describe the key elements of a number of the most commonly used control system design methods. These methods are required as part of the controller designs of Chapters 5 & 6, and used for designing baseline controllers in Chapter 5.1. The previous review Chapter 3 concluded that although many different control system design methods have been applied to the helicopter, little attention has been directed at the dynamically varying full flight envelope nature of the helicopter and also the effects of atmospheric turbulence which can be significant. In the review chapter the control system design methods were grouped under six headings, and it is three of these which are considered further in this chapter. These are as follows:

1. Eigenstructure Assignment Methods

2. Linear Quadratic Regulator (LQR) / Linear Quadratic Gaussian (LQG) and other $H_2$ Methods

3. $H_\infty$ Design Methods
The $H_\infty$ controller design methods have been further split into three categories: one degree of freedom mixed sensitivity design; one degree of freedom loop shaping design; and two degrees of freedom loop shaping design. For an example of the linear quadratic (LQ) design a method called 'H\(\infty\) minimum entropy' is described (the controller is an LQ controller with an $H_\infty$ bound).

Details of the Variable Structure controller design methodology have been included in the following Chapter 5, since they need to be described to a greater extent, and fit naturally alongside the practical issues.

The above methods were chosen since they are the most commonly applied to the helicopter flight control system design problem, and they fit into the design methodologies of the later chapters. Also, to aid the selection of the design parameters for the $H_\infty$ minimum entropy controller, a parameter optimization technique (the method of inequalities) was utilized, and found to speed up design time and further enable easier tuning of the nominal performance characteristics. The controller design methods outlined in this chapter are all well established, and have been applied to a great variety of 'real' aerospace control problems (Lin [60]).

4.2 Eigenstructure Assignment Design

The basic concepts of eigenstructure assignment are well known, and the method relies on experienced and detailed knowledge of the physical system to be controlled (Lin [60]). This then enables the selection of suitable eigenvalues for nominal performance objectives and eigenvectors for appropriate modal decoupling. Knowledge about a system's dominant dynamical modes can be found by inspecting the open-loop system poles. In real systems, especially in aerospace applications, these modes are reasonably well known and characterize special natural dynamical features. For example, airplanes and helicopters exhibit an oscillatory motion called the Dutch roll mode. The practical designer readily translates this modal knowledge and the performance specifications into closed loop pole requirements and associated mode-decoupling eigenvectors.

There are a number of suitable algorithms (for example, refer to [91]) which will
attempt to place the complete eigenstructure as closely as possible to the one desired. To facilitate the matching of the command input to the system output, the use of a Broussard feedforward command tracker matrix (O'Brien and Broussard [69]) can be applied. This method is very straight-forward and requires the formulation of the \( \sigma \) matrix.

\[
\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}
\]  
where \( C \) is the output control matrix of dimension \( m \times n \), and \( K \) is the eigenstructure assignment feedback matrix.

This gives the feedforward gain matrix as:

\[
K_{ff} = (\sigma_{22} - K \sigma_{12})
\]  

and the feedback system can be illustrated by the following diagram (Figure 4.1):

\[ \text{Figure 4.1: Closed Loop System} \]

4.3 \( H_\infty \) Minimum Entropy Design

The Linear Quadratic Gaussian (LQG) Regulator has been used extensively to solve a wide range of design problems ([60]), and most commonly considers the following system description:

\[
\dot{x} = Ax + Bu + \omega
\]  

35
\[ y = Cx + u \]  
(4.4)

where the process noise \( w \) and the measurement noise \( v \) are independent and have constant power spectral density matrices \( W \) and \( V \) respectively. The following stochastic LQG cost function can be formulated as the sum of the steady-state mean-square weighted state \( x \), and the steady-state mean-square weighted actuator signal \( u \):

\[
J_{Iqr} = \lim_{T \to \infty} E(\pi(t)^T Q \pi(t) + u(t)^T R u(t))
\]
(4.5)

where \( Q \) and \( R \) are positive semi-definite weight matrices (shown in Figure (4.2)).

![Figure 4.2: Closed Loop System](image)

A minimum entropy state-feedback controller (Boyd and Barratt [5]) can be derived by taking the above LQG controller, which has the \( H_\infty \) norm inequality specification:

\[
\|H\|_\infty < \gamma
\]
(4.6)

where \( H \) is the closed-loop transfer matrix from \( w \) to \( z \). If this \( \gamma \) is such that a (to be chosen) design specification is feasible, then the following algebraic Riccati equations have unique positive definite solutions \( X_{me} \) and \( Y_{me} \) respectively.

\[
A^T X_{me} + X_{me} A - X_{me}(BR^{-1}B^T - \gamma^{-2}W)X_{me} + Q = 0
\]
(4.7)

\[
AY_{me} + Y_{me} A^T - Y_{me}(C^TV^{-1}C - \gamma^{-2}Q)Y_{me} + W = 0
\]
(4.8)

Also, the symmetric matrix \( X_{me}(I - \gamma^{-2}Y_{me}X_{me})^{-1} \) must be positive definite and the inverse exist for the norm equality specification to be feasible. The state-feedback
controller then has the solution:

$$K_{stb} = R^{-1}B'X_{me}(I - \gamma^{-2}Y_{me}X_{me})^{-1}$$ (4.9)

and the observer gain:

$$L_{est} = Y_{me}C'V^{-1}$$ (4.10)

### 4.4 $H_\infty$ Mixed Sensitivity Design

The $H_\infty$ mixed sensitivity formulation aims to achieve performance and robustness objectives by shaping the singular values of two appropriate closed loop transfer functions. The problem is to design a stabilizing controller $K$ which minimizes the following cost function.

$$\min_{\text{stab} K} \left\| \begin{array}{c} W_1SW_3 \\ W_2KS\tilde{W}_3 \end{array} \right\|_\infty$$ (4.11)

where $S = (I + GK)^{-1}$ is the sensitivity function. The weight $W_1$ is a high-gain low-pass filter, since $S$ must be emphasised at low frequencies for good desensitivity to additive disturbances at the output, and for good tracking. Correspondingly, $W_2$ must be a low-gain high-pass filter to emphasize $KS$ at high frequencies so that actuator activity is reduced. The additional weighting function $W_3$ is primarily to force good tracking from the reference input to the system output. This is illustrated in Figure (4.3):

![Figure 4.3: Mixed Sensitivity Feedback Design Configuration](image)

For completeness and also for comparison with designs in the later Chapter 7, the
$H_\infty$ mixed sensitivity standard regulator framework can be expressed as:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
y \\
e
\end{bmatrix} =
\begin{bmatrix}
A & 0 & 0 & 0 & 0 & B \\
-B_1C & A_1 & 0 & B_1C_3 & B_1D_3 & 0 \\
0 & 0 & A_2 & 0 & 0 & B_2 \\
0 & 0 & 0 & A_3 & B_3 & 0 \\
-D_1C & C_1 & 0 & D_1C_3 & D_1D_3 & 0 \\
0 & 0 & C_2 & 0 & 0 & D_2 \\
C & 0 & 0 & 0 & 0 & 0 \\
-C & 0 & 0 & C_3 & D_3 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
r \\
u
\end{bmatrix}
$$

(4.12)

where $[A, B, C], [A_1, B_1, C_1, D_1], [A_2, B_2, C_2, D_2], [A_3, B_3, C_3, D_3]$, are the state-space descriptions of the plant, weight $W_1$, weight $W_2$, and weight $W_3$ respectively.

### 4.5 One Degree of Freedom $H_\infty$ Loop Shaping Design Procedure

The loop shaping design procedure (LSDP) (McFarlane and Glover [66]) is used to obtain performance/robustness trade-offs, with a robust stabilization technique as a means of guaranteeing closed-loop stability. For the design of the ideal model in the overall sliding mode design it is useful that the controller produced by the LSDP can be separated into a state-feedback and state observer (Kalman filter structure) form (Hyde and Glover [51]).

It is due to the conflicting requirements of performance (tracking and disturbance rejection) requiring high gain, and robustness (sensor noise attenuation) requiring low gain, that there must be a frequency separation of these objectives. An acceptable compromise comes from performance being typically most important at low frequency, and robust stability most pertinent at high frequency. The LSDP consists of three main stages:

1. Loop Shaping. The singular values of the nominal plant are shaped, using filters $W_1$ and $W_2$ to give a desired open-loop frequency shape. The nominal plant, and
the shaping weights are combined (Figure (4.4)) to form the shaped plant, where 
\[ G_s = W_2GW_1. \]

![Figure 4.4: The Shaped Plant Model](image)

2. Robust Stabilization. (a) Calculate \( \epsilon_{\text{max}} = (\gamma_e)^{-1} \). If \( \epsilon_{\text{max}} \ll 1 \), return to (1) and adjust the shaping weights. (b) Choose \( \epsilon \leq \epsilon_{\text{max}} \), and synthesize a feedback controller, \( K_{\infty} \) (Figure (4.5)), which robustly stabilizes the plant with respect to perturbations in the factors of a normalized left coprime factorization of \( G_s \).

![Figure 4.5: \( H_{\infty} \) Robust Stabilization](image)

3. The final feedback controller, \( K \), is then constructed (Figure (4.6)) by opening the feedback loop at (1) and (2) and combining the \( H_{\infty} \) controller, \( K_{\infty} \), with the shaping weights, whereby:

\[
K = W_1K_{\infty}W_2. \tag{4.13}
\]

![Figure 4.6: Final Controller](image)
The $\epsilon_{\text{max}}$ can be thought of as an indicator of the success of the loop shaping, since if $\epsilon_{\text{max}}$ is small, then the desired performance is incompatible with robust stability requirements. The control system structure shown (Figure (4.7)) includes a constant gain matrix pre-filter, $K_0$ which provides the necessary steady state gain on the reference to give zero error when compared with the $K_\infty$ output.

As stated earlier, the state-feedback matrix and Kalman filter structure can be exploited to give the following:

\[ \dot{x} = A_x \ddot{x} + H (C_x \ddot{x} - y) + B_x u \]
\[ u = F \ddot{x} \]

where $[A_x, B_x, C_x]$ is the state-space realization of the weighted plant, $H = -Z C_x^*$, and $F = B_x' (\gamma^{-2} I + \gamma^{-2} X Z - I)^{-1}$. The matrices $X$ and $Z$ are the solutions of the associated control and filtering algebraic Riccati equations ([66]) described below:

For the shaped plant description above, the 'Generalised Control Algebraic Riccati Equation' (GCARE) is given by:

\[ A_x^* X + X A_x - X B_x S^{-1} B_x^* X + C_x^* R^{-1} C_x = 0 \]  

and, the 'Generalised Filtering Algebraic Riccati Equation' (GFARE) is given by:

\[ A_x Z + Z A_x^* - Z C_x^* R^{-1} C_x Z + B_x S^{-1} B_x^* = 0 \]

where $R = I$ and $S = I$.

Again, for comparison with designs in the later Chapter 7, the LSDP standard
regulator framework can be expressed as equation 4.18:

\[
\begin{bmatrix}
\dot{x}_s \\
u \\
y \\
v
\end{bmatrix}
= 
\begin{bmatrix}
A_s & -HR^{1/2}B_s \\
0 & 0 & I \\
C_s & R^{1/2} \\
0 & 0 & R^{1/2}
\end{bmatrix}
\begin{bmatrix}
x_s \\
f \\
u
\end{bmatrix}
\tag{4.18}
\]

4.6 Two Degrees of Freedom $H_\infty$ Loop Shaping Design Procedure

In the 2 DOF design procedure (Hoyle et al [53]) a command prefilter and feedback controller can be designed together by setting the problem in the generalised regulator framework so that the controller uses the same state space for performance and robust stability. The 2 DOF design structure is shown in Figure (4.8).

![Figure 4.8: Two degrees-of-freedom design configuration](image)

This design configuration incorporates coprime factor uncertainty for the robust stabilization of the shaped plant. From the above figure, and the state space equations of the ideal model $[A_0, B_0, C_0, D_0]$ and shaped plant $[A_s, B_s, C_s]$, the generalised
The design procedure consists of the following stages:-

- Select loop shaping weights for the open loop plant to meet performance and robustness design objectives.
- Select a simple ideal, but realistic, model Mo.
- Calculate $\gamma_0$ for the robust stabilization problem from

$$\gamma_0 = (1 + \lambda_{max}(XZ)^{1/2}$$

(4.20)

where $X$ and $Z$ come from the associated GCARE and GFARE (detailed in the previous section 4.5).
- Set $\rho$ in range $1 \leq \rho \leq 3$. The higher the $\rho$, the closer the model-matching obtained (i.e. nominal performance), but the less the degree of stability robustness.
- Select $\gamma$. Ideally in range $1.2\gamma_0 \leq \gamma \leq 3\gamma_0$.
- Calculate controller, post multiply by the weights, and finally rescale the prefilter for perfect model matching in the steady state.

The 'ideal' closed loop model in the above 2 DOF design procedure would conventionally be selected as a diagonal input-output transfer function.

The solution of the algebraic Riccati equation, $X_\infty$, enables the control structure, of a Kalman filter, plus command model and state feedback law, to be completed as
The state vectors $\dot{x}$ and $z_o$ are the observed states and ideal model states respectively.

$$
\begin{bmatrix}
\dot{x} \\
z_o
\end{bmatrix} =
\begin{bmatrix}
A + HC - BB'X_{oo11} & -BB'X_{oo12} \\
0 & A_o
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
z_o
\end{bmatrix}
\begin{bmatrix}
0 & -H
\end{bmatrix}
\begin{bmatrix}
y
\end{bmatrix}
$$

(4.21)

$$
u = -B' \times [X_{oo11} \quad X_{oo12}] \times \begin{bmatrix} \dot{x} \\ z_o \end{bmatrix}
$$

(4.22)

where

$$
X_{oo} = \begin{bmatrix} X_{oo11} & X_{oo12} \\ X_{oo21} & X_{oo22} \end{bmatrix}
$$

(4.23)

and is the non-negative stabilizing solution to the $H_\infty$ state-feedback Riccati equation.

The matrix $H$ is the stabilizing output injection matrix determined by solving the normalization $H_2$ Riccati equation (GFARE in previous section 4.5).

### 4.7 Controller Design with the Method of Inequalities

The method of inequalities (MOI) (Zakian and Al-Naib [116], Whidborne and Liu [109], Maciejowski [62]) is a computer-aided multi-objective design approach, where desired performance is represented by a set of algebraic inequalities, and the aim of the design is to simultaneously satisfy these inequalities. The design problem is expressed as

$$
\phi_i(p) \leq \varepsilon_i \quad \text{for} \quad i = 1 \ldots m
$$

(4.24)

where $\varepsilon_i$ are real numbers, $p \in \mathcal{P}$ is a real vector $(p_1, p_2, \ldots, p_n)$ chosen from a given set $\mathcal{P}$ and $\phi_i$ are real functions of $p$. The functions $\phi_i$ are performance indices, the components of $p$ represent the design parameters and $\varepsilon_i$ are chosen by the designer and represent the largest tolerable values of $\phi_i$. The aim is the satisfaction of the set of inequalities in order that an acceptable design $p$ is reached.

The functions $\phi_i(p)$ may be functionals of the system step response, for example the rise time, overshoot or the integral absolute error, or functionals of the frequency response, such as the bandwidth. They can also represent measures of the system stability. The method has been applied to a wide variety of control design problems (e.g. Katebi and Katebi [56], Rutland [84], Taiwo [93], [109]).
Each inequality \( \phi_i(p) \leq \varepsilon_i \) of the set of inequalities (equation 4.24) defines a set \( S_i \) of points in the \( q \)-dimensional space \( \mathbb{R}^q \) such that

\[
S_i = \{ p : \phi_i(p) \leq \varepsilon_i \}.
\]  

(4.25)

The boundary of this set is defined by \( \phi_i(p) = \varepsilon_i \). A point \( p \in \mathbb{R}^q \) is a solution to the set of inequalities (equation 4.24) if and only if it lies inside every set \( S_i \), \( i = 1, 2, \ldots, m \) and hence inside the set \( S \) which denotes the intersection of all the sets \( S_i \),

\[
S = \bigcap_{i=1}^{m} S_i.
\]  

(4.26)

\( S \) is called the admissible set and any point \( p \) in \( S \) is called an admissible point. The objective is thus to find a point \( p \) such that \( p \in S \). Such a point \( p \) satisfies the set of inequalities (equation 4.24) and is said to be a solution.

Previously, in applications of the MOI, the design parameter \( p \) parameterised a fixed controller with a particular structure. However, in recent applications (Whidborne et al [111], Murad et al [68]), \( p \) parameterised the weighting functions required in \( H_\infty \)-optimisation problems.

The actual solution to the set of inequalities (equation 4.24) may be obtained by means of numerical search algorithms, such as the moving boundaries process (MBP) ([116]). The procedure for obtaining a solution is interactive, in that it requires supervision and intervention from the designer. The designer needs to choose the dimension of the design parameter vector \( p \) and initial values for the design parameters. The progress of the search algorithm should be monitored, and, if a solution is not found, the designer may either change the starting point, amend the size of the design vector, or relax some of the bounds \( \varepsilon \). Alternatively, if a solution is found easily, to improve the quality of the design, the bounds could be tightened or additional design objectives could be included in equation 4.24.

### 4.7.1 Design Procedure

A design procedure to solve the above problem is:

i) Define the plant \( G \), and define the functionals \( \phi_i \).
ii) Define the values of \( \varepsilon_i \).

iii) Define the structure of the controller \( K(p) \). Bounds should be placed on the values of \( p_i \) to ensure that \( K(p) \) is implementable and to prevent undesirable pole/zero cancellations.

iv) Define initial values of \( p_i \).

v) Implement the MBP to find a \( K(p) \) which satisfies a stability condition. If no solution is found, try again with different initial values of \( p \), or change the structure of the controller \( K(p) \).

vi) From a stability point, implement the MBP to find a \( K(p) \) which satisfies inequality equation 4.24, i.e. locate an admissible point. If a solution is found, the design is satisfactory. If no solution is found, relax one or more of the bounds \( \varepsilon_i \), change the initial values of \( p \) by returning to step (iv) or change the structure of the controller by returning to step (iii).

4.8 Conclusion

The fundamental features of the advanced control system design methods, which are most commonly applied to the helicopter stabilization and performance problem, have been described. When relevant, design procedures have also been included.

This chapter, in conjunction with the previous Chapters 2 & 3, has prepared the basis for the following 'Variable Structure Control System Design' Chapter 5.

The \( H_\infty \) controller designs are required for Chapter 7, where they will be built upon to enhance the controlled helicopters turbulence rejection qualities.
Chapter 5

Variable Structure Control System Design

5.1 Introduction

This chapter investigates the potential advancement to the control of the helicopter which could be made by variable structure control (VSC) law design.

Specifically considered is the ability of particular VS controller designs to maintain stability and performance over the helicopters operating envelope. A key feature of the nonlinear controllers employed in this chapter is the use of a 'single' controller, since the normal use of linear controllers being scheduled or switched ([9], [88]) increases system complexity, computing, and there can be undesirable transients when switching between controllers. Furthermore, the VS controllers have associated mathematical theory involving robustness considerations, whereas there is little established controller switching/scheduling theory at present (Slotine and Li [90]). A model following framework is also important to consider, due to the wide-spread and practical use of this scheme in helicopter control system design at an industrial and literature level ([57]). To this end, the following section begins with historical and background details to VSC system design. An ideal model is required to be designed as part of the design procedures, and four methods were applied to this: eigenstructure assignment,
$H_\infty$ minimum entropy design (which is based on Linear Quadratic Gaussian methods), $H_1$ one degree of freedom design, and $H_\infty$ two degrees of freedom design. The design parameters in the $H_\infty$ minimum entropy design were tuned using the Method Of Inequalities (MOI) ([111]). Linear and nonlinear simulation results are presented. The resulting controllers were tested according to military specifications ([1]), and the most promising controllers flight tested on the Defence Research Agency's Advanced Flight Simulator (AFS). These variable structure controllers were also compared to two established candidate future control law designs: a one degree of freedom (Walker and Postlethwaite [105]) and a two degrees of freedom $H_\infty$ (Walker et al [106]) loop shaping design procedure controller. The chapter ends with the results being summarized, conclusions being drawn, and future directions indicated. The next section outlines the history of VSC and its key concepts.

5.2 Introduction to Variable Structure Control System Design

5.2.1 Historical Background

The origins of variable structure control were in relay and bang-bang control system theory (Utkin [99]). The ideas of variable structure systems first appeared in the 1950's with the language of structural circuits, and were mainly restricted to linear second-order systems. Major interests in VSC techniques existed, and exist because of its applicability to linear and nonlinear dynamical systems and also systems with delays and distributed parameters (distributed systems need to be described by partial differential equations, while lumped parameter systems can be detailed by difference equations or by purely algebraic equations; for further details see Brogan [7]). VSC ideas are particularly well suited to the deterministic (i.e. nonrandom) control of uncertain systems. In 1969 early results were established (Draženović [18]) on the invariance of VSC systems to a class of disturbance and parameter variations. The 1970’s VSC research work involved linear scalar cases with the beginnings of multivariable concepts. Through the 1980’s the initial steps towards a general theory for
nonlinear systems were made. At the present day (Zinober [121]) those theoretical results have been transformed into a more intuitive theory with further rigorous formulations. VSC theory does exist for distributed parameter systems (Drakunov and Özgüner [17]), and only recently to a lesser extent for discrete systems (Yu [113]). Throughout the research developments of VSC there have been major studies in the use of VSC and allied techniques in, for example, model-following and model reference adaptive control (further details in Zinober [119]).

The original 1960's ideas are still fundamental to the understanding of VSCS design of the present day. An example outlining the most important details and explaining the concepts of a variable structure control systems is described below.

5.2.2 Sliding Surfaces and Sliding Modes

To provide a degree of complexity and also ease of visualization, a second order linear system was chosen to detail the major concepts of VSC systems. Therefore, consider the state-space model (taken from [15]):

\[
\begin{bmatrix}
    \dot{x}_1(t) \\
    \dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
    0 & 1 \\
    1 & 2
\end{bmatrix}
\begin{bmatrix}
    x_1(t) \\
    x_2(t)
\end{bmatrix} +
\begin{bmatrix}
    0 \\
    1
\end{bmatrix} u(t) \tag{5.1}
\]

with the variable structure control law:

\[
u(t) = k(x_1, x_2) x_2(t) \tag{5.2}
\]

where \(k(x_1, x_2)\) can take the value '2' or '−3'. This nonlinear/variable structure of the control law above can be illustrated in block diagram form as Figure (5.1):

![Figure 5.1: Block Diagram of Variable Structure Control of a Second Order System](image-url)
The system has two possible free responses according to the position of the switch. In the upper position (i.e. \( k(x_1, x_2) = -3 \)), the system has complex eigenvalues and an unstable motion, described by:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-2 & 2
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\tag{5.3}
\]

When the switch is in the lower position the system has real eigenvalues and follows the motion:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
3 & 2
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\tag{5.4}
\]

These separate free motions can be shown diagrammatically on phase-plane plots (Figures (5.2, 5.3)).

![Phase Plane Plot](image)

Figure 5.2: Phase Plane Plot for \( k(x_1, x_2) = -3 \)

The particular location of the switch is defined with respect to a 'switching surface' or 'sliding surface'. This surface 'cuts' the state-space into different regions, and within each region a different control action and therefore motion acts. To illustrate the different motions more formally, a sliding surface can be defined as:

\[
S_v = S_v(x_1, x_2) = s_1 x_1 + x_2 = 0
\tag{5.5}
\]

with \( s_1 < 1 \) and feedback switching law as:
Figure 5.3: Phase Plane Plot for $k_1(x_1, x_2) = +2$

These combined state-space trajectories can be visualized graphically on a phase-plane portrait (Figure (5.4)).

Figure 5.4: Phase Plane Plot of Controlled System when a Sliding Mode exists on the Switching Surface
Since the phase-plane velocity vectors always point to the surface, then for a perturbation off the surface the system is immediately forced back onto the surface again. A particular switching surface has associated with it many possible controllers to attain the respective sliding mode.

The above example illustrates a very important concept in VSCS design: the need to ensure the reachability of the ‘sliding mode’. In Figure (5.4) the system has attained the sliding mode ‘if in the vicinity of the switching surface, the state velocity vector (the derivative of the state vector) is directed towards the surface’ (Further details in [15]). The sliding motion is the central feature of VSC systems (when performance is specified through a reduced order motion, which is insensitive to disturbances entering through the input of the plant), and the assurance of a stable sliding motion, by the selection of suitable switching hyperplanes, forms the first stage of the VSC design process ([119]). This problem of determining the set of hyperplanes is known as the ‘existence problem’ and can be solved separately from the control functions.

The second design stage requires the selection of the control which will ensure that the prescribed sliding motion is attained and maintained. This is called the ‘reachability problem’ and requires the selection of an appropriate control structure and associated gains to ensure the reaching of the sliding mode. This stage is dependent on the knowledge of the existence of the switching hyperplanes, i.e. the first design stage. There are therefore two independent stages in the transient motion of a VSC system: a (preferably rapid) motion onto the sliding surface; and a slower motion while on this sliding surface. Further details of these two design stages for the model-reference framework appear in the following sections 5.3 and 5.4.

The preceding second order example employed a discontinuous control function, that is the control gains were switched directly between ‘2’ and ‘−3’, and therefore discontinuous control chattering (high frequency switching about the switching surface) is present. This would usually be undesirable for practical applications since this may excite unmodelled high-frequency plant dynamics, or be physically impossible to realise or lead to increased wear-and-tear for plant actuators. Therefore a smoothed continuous control function can be substituted to overcome these problems (further details appear in the following section 5.3). A further important notion of ‘equivalent
control' (developed by Utkin [98]) allows the low frequency (average) component of the total control action to be formulated, that is without the high frequency component. This can be useful for analysis.

The key VSCS concepts have been outlined. It should be noted that the previous VS description considered the switching surface as a function of the ‘x’ state space. The following VS controller considers the error ‘e’ state space between the real plant and an ideal model. This model-reference setting of the helicopter applied control scheme, to enable the attainment of the sliding motion with respect to uncertainties and disturbances, is described in the following section 5.3.

5.3 A Model Reference Variable Structure Controller Design Methodology

The following state space description of an uncertain plant will be considered:

\[
\dot{x}(t) = Ax(t) + Bu(t) + F(t, x, u)
\]  

(5.6)

where \( x \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) represent the usual state and input vectors, \( B \) is full rank, \( n > m \) and \( (A, B) \) is a controllable pair. The unknown function \( F(\cdot) \) represents system nonlinearities and model uncertainties in the system (Davies and Spurgeon [14]), and is unknown but assumed to belong to a known class. The decomposition of this uncertainty \( F(\cdot) \) to obtain worst case bounds on its different components is discussed in the next section 5.4.

An associated linear model which has ideal response characteristics is defined by:

\[
\dot{w}(t) = A_m w(t) + B_m r(t)
\]  

(5.7)

where \( w \in \mathbb{R}^n \), \( r \in \mathbb{R}^p \), are the state vector and the reference input of the model respectively. It is assumed that the ideal model is stable so that the poles of the system, equation (5.7), have negative real parts. The associated control system design problem thus involves determining a feedback strategy whereby the output variables of the plant, equation (5.6), faithfully follow those of the model. The following tracking
error state is thus defined
\[ e = z - w \tag{5.8} \]

Differentiating equation (5.8) with respect to time and substituting the plant and model dynamics from equation (5.6) and equation (5.7), the following error dynamics are obtained.
\[ \dot{e} = A_m e + (A - A_m)x + Bu - B_m r + F(t, x, u) \tag{5.9} \]

To satisfy the well-known model matching conditions (Chen [8]) for the nominal error system which will ensure asymptotic decay when \( F(\cdot) \equiv 0 \), the following structure is imposed upon the model.
\[ A_m = A + BL_z \tag{5.10} \]
\[ B_m = BL_r \tag{5.11} \]

The model is thus defined by a constant gain feedback matrix \((L_z)\) for the nominal plant, and an input-output tracking precompensator gain matrix \((L_r)\).

Note that if the control input, \( u \), is defined by
\[ u_1 = L_z x + L_r r \tag{5.12} \]
the nominal error dynamics are asymptotically stable. However, for a very nonlinear, uncertain system the problem of maintaining the tracking performance in the presence of a broad class of uncertainty contributions \( F(\cdot) \) is particularly pertinent. The design of an augmenting control effort to counteract the uncertainty \( F(\cdot) \) is now considered. The methodology employed has its roots in the well known sliding mode approach to controller design, where the error state is constrained to lie on certain surfaces in the error state-space. This method possesses certain inherent robustness properties, and with appropriate switching surface selection enables the designer to prescribe desired error transient behaviour. A set of switching surfaces are defined to be fixed hyperplanes in the error space passing through the origin
\[ s = C_s e \tag{5.13} \]
where \( C_s \in \mathbb{R}^{m \times n} \) is a constant design matrix which determines the ideal rate of decay of the error states.
A sliding mode is achieved when the error states are constrained to the intersection of the hyperplanes (equation (5.13))

\[ s = \{ \mathbf{e} : \mathbf{C}_s \mathbf{e} = 0 \} \quad (5.14) \]

The control required to achieve the desirable sliding mode condition, equation (5.14), was traditionally discontinuous in nature which was clearly undesirable for many applications. However, there are now well-established continuous nonlinear controllers which ensure that the error state, \( \mathbf{e} \), lies within a small region so that \( \mathbf{C}_s \mathbf{e} \approx 0 \) in a robust fashion (Ryan and Corless [85], and [92]). Here the control effort equation (5.12) is augmented by

\[ \mathbf{u}_2 = \mathbf{L}_s \mathbf{e} + \rho(t, x, \mathbf{e}, r) \frac{\mathbf{N}_c}{\|\mathbf{M}_e\| + \delta} \quad (5.15) \]

so that

\[ \mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 \quad (5.16) \]

Here \( \mathbf{L}_s \in \mathbb{R}^{m \times n} \) is an error-feedback to prescribe the rate of decay of the error states onto the switching surfaces. The matrices \( \mathbf{N} \in \mathbb{R}^{m \times n} \) and \( \mathbf{M} \in \mathbb{R}^{m \times n} \), which enable the sliding motion to be attained in finite time and be robust with respect to plant input disturbances, are directly determined from the choice of switching surface \( \mathbf{C}_s \). The parameter \( \delta > 0 \) is a smoothing constant; for \( \delta = 0 \) an undesirable relay type control action would result.

The control strategy employed is relatively straightforward from the point of view of design, with selection of switching surfaces amounting to the solution of a full state-feedback sub-problem. Indeed, a prototype MATLAB toolbox, not used in this work, is now available which includes a number of routines to facilitate the above design and analysis (Zinober [120]). This toolbox allows either eigenstructure assignment or linear quadratic regulator methods as part of the design procedure. The applicability of these two methods, plus two \( \mathcal{H}_\infty \) design procedures, to the helicopter control problem is examined in this chapter. The block diagram (Figure (5.5)) following shows the structure of the complete variable structure controller implemented, for example, on the nonlinear helicopter model. This was built up in the SIMULINK environment, which was found to be a flexible environment to configure the design.
The following section completes the setting up of this VSCS design scheme, by describing in greater detail the switching surface definition, the handling of the uncertainty, and summarizing the major design details for the subsequent helicopter application.

### 5.4 Switching Surface Design

A key consideration is the splitting up of the system into its 'matched' and 'unmatched' parts. The term 'matched uncertainty' relates to the uncertainty which enters the system to be controlled through its input distribution matrix. Correspondingly, 'unmatched uncertainty' is uncertainty which is not explicit in the system's input channels. Since the input distribution matrix $B$ from section 5.3 was assumed to be full rank, there exists an orthogonal transformation matrix $T$ such that:

$$\hat{T}B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$  \hspace{1cm} (5.17)

This transformation matrix can be determined using 'QR reduction' (Dorling and Zinober [16]) and with the MATLAB command 'qr'. The error system can now be
expressed in the transformed coordinates as:
\[
\bar{e} = \bar{T} e
\] (5.18)

Partitioning \(\bar{T}\):
\[
\bar{T} = \begin{bmatrix} T \\ T^1 \end{bmatrix}
\] (5.19)
where \(T \in \mathbb{R}^{(n-m)xn}\) and \(T^1 \in \mathbb{R}^{mxn}\), enables the ideal model dynamics matrix to be compatibly partitioned as:
\[
\bar{T}^T A_m \bar{T} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}
\] (5.20)

A matrix \(M\) is now introduced (which has to be designed) and can be stated as having to specify the required poles of \(A_{11} - A_{12}M\), which will later be shown to yield the sliding mode dynamics. Also, a second linear transformation \(\bar{T}\) can be defined as:
\[
\bar{T} = \begin{bmatrix} I_{n-m} & 0 \\ M & I_m \end{bmatrix}
\] (5.21)

The following equations (5.22) show the resultant error system in terms of the two transformations:
\[
\bar{e} = \bar{T} e = \begin{bmatrix} z \\ \phi \end{bmatrix}
\] (5.22)

Since the error system in the original coordinate system was:
\[
\dot{e} = A_m e + B [u - L_\alpha z - L_\gamma r] + f + g
\] (5.23)
then by applying the two transformations \(\bar{T}\) and \(\bar{T}\) consecutively the error system can be expressed in the \((z, \phi)\) coordinates as:
\[
\begin{bmatrix} \dot{z} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z \\ \phi \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} (u - L_\alpha z - L_\gamma r) + \begin{bmatrix} Tf \\ MTf + T^1 g \end{bmatrix}
\] (5.24)
where:
\[
A_{11} = A_{11} - A_{12}M
\] (5.25)
\[
A_{21} = MA_{11} + A_{21} - A_{22}M
\] (5.26)
\[
A_{12} = A_{12}
\] (5.27)
\[
A_{22} = MA_{12} + A_{22}
\] (5.28)
When $\phi = 0$ (i.e. at point sliding mode condition satisfied) the nominal system is represented by $\dot{z} = A_{11}z$, which means $A_{11}$ gives the sliding mode performance. As stated earlier, the matrix $M$ has to be designed and specifies the required poles of the system $A_{11} - A_{12}M$, which therefore yields the sliding mode dynamics.

The objective is to drive the system error states to a neighbourhood of the switching surface using a continuous control action. First, if $E$ is chosen by the designer to be any stable design matrix, and $P_2 \in \mathbb{R}^{m \times m}$ is the unique symmetric positive definite solution to the Lyapunov equation:

$$P_2 \Phi^* + \Phi^* P_2 + I_m = 0 \quad (5.30)$$

then an appropriate control is:

$$u = L_x x + L_r r - B_1^{-1} [\tilde{A}_{21} | \tilde{A}_{22} - \Phi^*] \begin{bmatrix} z \\ \phi \end{bmatrix} - \frac{B_1^{-1} \Phi^*}{\|P_2 \Phi\|} + \delta \quad (5.31)$$

The $\Phi^*$ matrix is a design parameter and must be chosen to have stable eigenvalues. The matrix $L_s$, taking part of the equation 5.31, required to complete the control law of equation (5.15) in section 5.3 can then be stated as:

$$L_s = -B_1^{-1} [\tilde{A}_{21} | \tilde{A}_{22} - \Phi^*] \tilde{T} \tilde{T} \quad (5.32)$$

The resultant error system in the $(z, \phi)$ coordinates then reduces to:

$$\begin{bmatrix} \dot{z} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ 0 & \Phi^* \end{bmatrix} \begin{bmatrix} z \\ \phi \end{bmatrix} - \left[ \begin{bmatrix} 0 \\ \frac{\phi \delta}{\|P_2 \Phi\| + \delta} \end{bmatrix} \right] + \begin{bmatrix} Tf \\ MTf + T^tg \end{bmatrix} \quad (5.33)$$

This above equation (5.33) illustrates that part of the unmatched uncertainty and the whole of the matched uncertainty can be affected by the unit vector part of the control law.

The specification of $\rho(\cdot)$, which was implemented for the later practical designs described in section 5.7, begins by partitioning the uncertainty function $F(\cdot)$ into the matched and unmatched components. The system uncertainty $F(\cdot)$ can then be expressed as equation (5.34).

$$F(t, x, u) = f(t, x) + g(t, x, u) \quad (5.34)$$
where

\[ f(t, x) = F_1(t, x)x + F_2(t, x) \quad (5.35) \]

\[ g(t, x, u) = G_1(t, x, u)u + G_2(t, x) \quad (5.36) \]

The component \( F_1(\cdot) \) can be interpreted as unmatched parametric uncertainty in the nominal system matrix, and \( F_2(\cdot) \) as including external disturbance affects. Because the exact structure of these uncertain expressions (equations (5.35) and (5.36)) are either varying nonlinearly or just not known, a more practical method for specifying the worst case uncertainty (by using bounds on the size) can be formulated as equations (5.37)-(5.40).

\[ \| F_1(t, x) \| < K_f \quad (5.37) \]
\[ \| F_2(t, x) \| < K_d \quad (5.38) \]
\[ \| G_1(t, x, u) \| < K_g \quad (5.39) \]
\[ \| G_2(t, x) \| < \alpha(t, x) \quad (5.40) \]

These norm bounds enable the engineer/designer to specify \( \rho(\cdot) \): a measure of the uncertainty in the system to be controlled. By suitable manipulations (Davies et al. [13]), the following definition for the practical design can be formulated:

\[ \rho(t, x, e, r) = Q_1\|x\| + Q_2\|u_L(t, e, r)\| + Q_3 \quad (5.41) \]

where \( u_L = L_\sigma x + L_r r + L_e e \), and the \( Q \) parameters are defined as:

\[ Q_1 = \frac{\gamma_1}{\sigma}(K_f\|M\| + K_u) \quad (5.42) \]
\[ Q_2 = \frac{\gamma_1}{\sigma}K_g \quad (5.43) \]
\[ Q_3 = \frac{\gamma_1}{\sigma}(K_d\|M\| + \gamma_2) \quad (5.44) \]

where \( \gamma_1 \geq 1, \gamma_2 > 0, K_u \) norm bounds the size of the matched uncertainty entering the \( A \) matrix of the plant, and

\[ \sigma \overset{\Delta}{=} \lambda_{\min}(I_{nxm} + T^T G_1 B_2^{-1} + (B_2^{-1})^T G_1^T T^T) \quad (5.45) \]

The above theoretical expressions for the \( Q \) parameters may be found to be too conservative when simulations are performed and the values may need to be reduced. The following section will summarize the design procedure details of the described variable structure control system design methodology.
5.5 Summary of Major Design Details

To summarize for the following designs, the major sub-problems involved in the 'existence' and 'reachability' problems must be solved correctly and realistically for a real practical system.

(i) Ideal Model Specification
   Selection of the matrices $L_x$ and $L_r$ from equations (5.10) and (5.11). This is a state-feedback gain matrix design problem.

(ii) Sliding Surface Design
   Selection of matrix $M$ from equation (5.25). Again a state-feedback gain matrix design problem.

(iii) Controller Specification (Robustness)
   Selection of 'Q' parameters from equations (5.42)-(5.44). These values are calculated from the consideration of the system uncertainty, and also the designer selection of values for $\gamma_1$ and $\gamma_2$.

(iv) Controller Specification (Performance onto Sliding Surface)
   Selection of designer chosen $\Phi^*$ matrix of equation (5.30).

5.6 Ideal Model Design Considerations

The following four subsections contain details of the implementational issues of the four design techniques looked at for solving problems (i)-(iv) of the previous section 5.5 in the overall variable structure design framework for the helicopter.

5.6.1 Eigenstructure Assignment Design

Here the ideal model (problem (i) of section 5.5) is specified using eigenstructure assignment, since the method has been widely applied to helicopter (and, in general, aerospace) control system design. The design procedures are well established and
candidate design parameters documented in the research literature. The initial eigenstructure specification (from Manness and Murray-Smith [65]) was tuned to the specific helicopter under consideration here, and can be tabulated as follows (Table 5.1):

<table>
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<th>Eig.</th>
<th>θ</th>
<th>φ</th>
<th>p</th>
<th>q</th>
<th>r</th>
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<td>-0.015</td>
<td>0.995</td>
<td>-0.054</td>
</tr>
</tbody>
</table>

Table 5.1: The specified Eigenvalues and Eigenvectors

A Broussard precompensator matrix was calculated to match the control inputs with the controlled outputs. The eigenstructure above contains two eigenvalues placed at the transmission zeros of the nominal helicopter plant, with the eigenvectors placed at the associated transmission zero directions. This was found to be essential in giving a nominal overall design which had zero steady state error.

The required matrices for the variable structure controller linear control action (equation 5.12 in section 5.3) are shown below.

\[ L_r = K \]  \hspace{1cm} (5.46)

\[ L_r = K_{ff} \]  \hspace{1cm} (5.47)

The ideal model, which has the same input/output characteristics as the eigenstructure assignment controlled linear plant model, is therefore described by:

\[ \dot{x}(t) = (A + BL_x)x(t) + BL_r r(t) \]  \hspace{1cm} (5.48)

The eigenvalues for the state-feedback equation (problem (ii)) were chosen as \(-1, -2, -3, -4\) (these were the first values chosen, and no sensitivity was found to require further tuning). The following design parameters (problem (iii)) were chosen as: \(Q_1 = 97.0, Q_2 = 10.0, Q_3 = 10.0, \delta = 1.2\). The final part (problem (iv)) of the sliding mode control system design procedure requires the specification of the speed at which
the error states reach the hyperplane, which defines how quickly the system attains the required sliding mode (from equation (5.25), which defines the matrix $L_s$ according to equation (5.32) in section 5.4). To this end the selection was $\Phi^* = -10I_{nxn}$.

The addition of actuator dynamics was found to be impractical due to the increased number of eigenvalues/eigenvectors, which had to be chosen by the designer and calculated by the eigenstructure assignment algorithm.

5.6.2 $H_\infty$ Minimum Entropy Design

In this case, the plant was augmented with an integrator state in each input channel to improve the decoupling and steady state performance. Again the inclusion of actuators was not practically possible for the final design. The $H_\infty$ minimum entropy method had been able to provide a good state-feedback matrix, but it was the number of the poles for decay of the switching surface which became too numerous for satisfactory design.

The use of the method of inequalities (as described in section 4.7) was used to tune the ‘$Q$’ and ‘$R$’ matrices to obtain the desired performance and decoupling in the design of the ideal model (problem (i) of section 5.5). The specifications defined for the MOI procedure were:

1. The first requirement being that the resulting controller stabilized the helicopter nominal linearized model.

2. The closed loop step response settling time, with a tolerance of 5%, should be less than 2 seconds for roll attitude, pitch attitude, and yaw rate. For the heave axis the settling time had to be chosen to be slower, i.e. 3 seconds, due to the Lynx main rotor collective actuator bandwidth limits constraints.

3. The closed loop step response overshoot in each channel must be less than 10%.

4. The closed loop step response inter-axis coupling must be less than 10% for each channel.
The closed loop step response actuator demands must be less than 8° in main rotor collective, 4.5° in longitudinal cyclic, 7.5° in lateral cyclic, and 8.5° in tail rotor collective.

The time response specifications were obtained due to the earlier experiences ([106]) with testing candidate control laws on the Defence Research Agency's flight simulator. The resultant 'Q' and 'R' matrices (as defined in section 4.3) were

\[
Q = \text{Diagonal} [0.01, 0.006, 0.35, 0.035] \quad (5.49)
\]

\[
R = \text{Diagonal} [1, 0.025, 0.05, 0.005] \quad (5.50)
\]

The poles for the decay of the switching states were chosen as -1.75, -2.25, -2.75, -3.5, -4.5, -5.55, -5.0, -5.25 (problem (ii)), and the design parameters again had values: \( Q_1 = 97.0, Q_2 = 10.0, Q_3 = 10.0, \delta = 1.2 \) (problem (iii)), and \( \Phi^* = -10I_{m\times m} \) (problem (iv)).

### 5.6.3 \( H_\infty \) One Degree of Freedom Design

In this case, the ideal model (problem (i) of section 5.5) was designed using the \( H_\infty \) one degree of freedom Loop Shaping Design Procedures (LSDP) described in section 4.5.

For the final designs simple first order actuator dynamics were added to the helicopter eight state nominal hover model, to give a twelve state model. An alignment gain was used to shift the frequency response vertically, thus giving an obtainable ‘cut off’ frequency of 5 radians/second. Also, extra premultiplying gain provided more decoupling in certain chosen channels (0.1 in main rotor collective and 0.5 in tail rotor collective). The LSDP, after adding the weights on, resulted in an extra four states.

The weights used were:

\[
W_1 = \text{Diagonal} [\frac{4.5}{2}, \frac{6.5}{2}, \frac{8.5}{2}, \frac{10.5}{2}] \quad (5.51)
\]

\[
W_2 = \text{Diagonal} [1, 1, 1, 0.2, 0.2] \quad (5.52)
\]

The controller was then formulated from the robust stabilization of the normalized left coprime factor shaped plant, and split up into the required state-feedback and observer structure, as described in the earlier section 4.5. For the pole assignment routine
(to specify the speed at which the system will attain sliding mode), the following poles (problem (ii)) were selected as: -0.0054, -0.0014, -13.5, -12.0, -12.5, -11.0, -13.0, -10.0, -19.5, -19.0, -5.0, -7.5. They were chosen to be just faster than the ideal model dynamics, which is desirable for the reasons stated in section 5.6.1. The design parameters implemented had values: $Q_1 = 100.0$, $Q_2 = 100.0$, $Q_3 = 100.0$, and $\delta = 0.3$ (problem (iii)), and $\Phi^* = -13 I_{n \times n}$ (problem (iv)).

5.6.4 $H_\infty$ Two Degrees of Freedom Design

The final method used to design the ideal model (problem (i) of section 5.5) was using $H_\infty$ two degrees of freedom Loop Shaping Design Procedures (LSDP). The appropriate matrices (problems (i),(ii) and (iv)) after following the design procedures of section 4.6 were:

$$L_x = (B'X_{\infty 11} + B'X_{\infty 12})$$  \hspace{1cm} (5.53)

$$L_x = B'X_{\infty 12}$$  \hspace{1cm} (5.54)

Due the success of the piloted simulations of the switched 2 DOF $H_\infty$ controllers, the details of this procedure were incorporated into the variable structure design methodology. Therefore, similar weighting transfer functions were utilised. The chosen weights were:

$$W_1 = \text{Diagonal} [ \frac{\pi \delta}{2}, \frac{\pi \delta}{2}, \frac{\pi \delta}{2}, \frac{\pi \delta}{2} ]$$  \hspace{1cm} (5.55)

$$W_2 = \text{Diagonal} [ 1, 1, 1, 1, 0.2, 0.2 ]$$  \hspace{1cm} (5.56)

The final 2 D.O.F. $\gamma = 4.0$, indicating good robust stabilization properties, and $\rho = 1.5$ so that good model matching had been designed for. For the completion of the controller design: $Q_1 = 1.0$, $Q_2 = 1.0$, $Q_3 = 1.0$, and $\delta = 2.0$ (problem (iii)).
5.7 Helicopter Model Simulation Results

The installation of the full nonlinear helicopter model (Rationalised Helicopter Model (RHM)) in the Simulink environment, coupled with the use of the MATLAB handling qualities toolbox, has enabled the assessment of candidate control laws on the nonlinear simulation model to become an easier and even more integral part of the initial control system design cycle.

The following six sections 5.7.1 - 5.7.6 show step response plots in all four main control axis for the six differently designed controllers, which are:

1. Variable structure controller with an ideal model designed using eigenstructure assignment.
2. Variable structure controller with an ideal model designed using $H_{\infty}$ Minimum Entropy design.
3. One Degree of Freedom $H_{\infty}$ controller.
4. Variable structure controller with an ideal model designed using a one degree of freedom $H_{\infty}$ methods.
5. Two Degree of Freedom $H_{\infty}$ controller.
6. Variable structure controller with an ideal model designed using a two degrees of freedom $H_{\infty}$ methods.

Only linear step response plots are shown for the eigenstructure assignment designed controller since it was unable to maintain stability for the nonlinear model. Also, only plots up to 60 knots are shown for the $H_{\infty}$ Minimum Entropy designed controller due to the further severe degradation at any higher speeds. For the remaining controller designs [3]-[6] the two very different flight conditions of hover and 120 knots for the full nonlinear model were taken as spanning the working flight envelope of the helicopter. The step sizes used were large enough to make significant changes to the trimmed flight conditions and were as follows (Table 5.2):
<table>
<thead>
<tr>
<th>Con. O/p</th>
<th>Descrip.</th>
<th>Commanded Input Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ ($y_1$)</td>
<td>Heave Vel.</td>
<td>10 Ft/s</td>
</tr>
<tr>
<td>$\Theta$ ($y_2$)</td>
<td>Pitch Att.</td>
<td>0.2 Rad.</td>
</tr>
<tr>
<td>$\Phi$ ($y_3$)</td>
<td>Roll Att</td>
<td>0.2 Rad.</td>
</tr>
<tr>
<td>$\Psi$ ($y_4$)</td>
<td>Head. Rate</td>
<td>0.2 Rad/s</td>
</tr>
</tbody>
</table>

Table 5.2: Individual Main Axis Step Size Demand

Where practically possible, for a particular design method, a model for the actuator in each channel was augmented. This consisted of a simple first order lag of the form $\frac{1}{s+\tau}$, where the time constants were as follows (Table 5.3):

<table>
<thead>
<tr>
<th>Control Input Channel</th>
<th>Time Constant (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Rotor Collective</td>
<td>0.0795</td>
</tr>
<tr>
<td>Longitudinal Cyclic</td>
<td>0.0795</td>
</tr>
<tr>
<td>Lateral Cyclic</td>
<td>0.0795</td>
</tr>
<tr>
<td>Tail Rotor Collective</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Table 5.3: Time Constants for Actuator Model in each Channel

For the rest of this section actuator demands in each axis are shown for the single 1 DOF $H_{\infty}$ Controller at hover only. For all the responses in the following sections the actuator demands were within the actuator limits imposed by this particular helicopter.
5.7.1 Eigenstructure Assignment Controller

The following plots (Figures (5.6, 5.7, 5.8, 5.9)) show the good performance and de-coupling characteristics on the nominal linear plant model. Assuming the same initial conditions (which would not always be the case) there would be zero variable structure control action when testing on the linear nominal model, since the error between the ideal model and the controlled nominal model was designed to be zero. But this does provide one method of checking that the design procedure has been carried out correctly and consistently.

Figure 5.6: Heave Axis Step Demand: Variable Structure Controller with Ideal Model designed using Eigenstructure Assignment (0 knots Linear Simulation)

Figure 5.7: Pitch Axis Step Demand: Variable Structure Controller with Ideal Model designed using Eigenstructure Assignment (0 knots Linear Simulation)
However, when this controller was tested on the nonlinear model, these qualities deteriorated so significantly that the system became unstable for any pitch input. Normal eigenstructure procedures for actually implementing the controllers require either augmenting the nominal plant with integrators, or introducing an outer loop containing integrators. This would normally be done to each switched controller every 20 knots. Unfortunately, here the methods did not help, since designs on the augmented plant contained too much coupling even on the nominal plant, and the use of an outer loop contradicts the variable structure philosophy employed here. The difficulty of designing for an augmented plant was not surprising, since the degrees of freedom available in eigenstructure assignment increase with plant dimensions.
5.7.2 $H_\infty$ Minimum Entropy Controller

The nominal linear design results again showed fast response types with minimal cross coupling, and were very similar to those obtained for the eigenstructure assignment approach (the reason they are not shown). However, on the nonlinear model the results were considerably improved as compared to the eigenstructure assignment, and these are shown in the following plots (Figures (5.10, 5.11, 5.12, 5.13)) for the hover flight condition. The enhanced robustness of the minimum entropy design was confirmed at an early stage since the hover-designed controller alone stabilized a 20 knot linearization, while the eigenstructure controller alone failed to do so. Again, it must be noted that the use of integrators in each input channel had enabled these properties, but that the use of an outer integral loop in the eigenstructure assignment design procedure does not fit into the variable structure framework proposed here.

Hover Flight Condition

![Figure 5.10: Heave Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_\infty$ Minimum Entropy (0 knots)](image-url)
Figure 5.11: Pitch Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (0 knots)

Figure 5.12: Roll Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (0 knots)
60 knots Flight Condition

The variable structure controller (with an ideal model designed using $H_{\infty}$ Minimum Entropy methods) could not maintain a reasonable level of performance away from the design condition, which was shown by testing on the nonlinear model at 60 knots (Figures 5.14, 5.15, 5.16, 5.17)).

Figure 5.13: Yaw Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (0 knots)

Figure 5.14: Heave Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (60 knots)
Figure 5.15: Pitch Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (60 knots)

The coupling for a step demand in the pitch axis is shown to be unacceptable by the above plot (Figure (5.15)), and degrades further at higher speeds.

Figure 5.16: Roll Axis Step Demand: Variable Structure Controller with Ideal Model designed using $H_{\infty}$ Minimum Entropy (60 knots)

The time response plots (Figures (5.16, 5.17)) show the degradation of the roll attitude and yaw rate axes performance at 60 knots, and these qualities degraded rapidly for any higher speeds.
5.7.3 Variable Structure Controller Output Components ($H_\infty$ Minimum Entropy designed Ideal Model)

To show that the nonlinear (unit vector) component of the variable structure controller is a key element of the whole framework, the individual parts which are summed to

![Graphs showing variable structure controller output components](image)

Figure 5.18: Individual Components of the Actuator Demand for Pitch Axis Step Demand at Hover: VS controller (unit vector dashed, linear dotted, total control action solid line)
give the total control action have been plotted (Figure (5.18)).

By setting the control output signals from the unit vector part and the $L_a$ part to zero (i.e. set $u_2 = 0$ in equations 5.15 and 5.16 in section 5.3), a pure one degree of freedom $H_\infty$ minimum entropy controller is formed. Therefore, the controller demand of a single $H_\infty$ minimum entropy controller (Figure (5.19)) can be compared to the previous plot (Figure (5.18)). These show how influential the nonlinear part of the controller is on the final control action, illustrating that the VS controller does not just augment an existing linear strategy, and indicating why the potential advantages and disadvantages should be investigated.

![Graphs showing control demands](image)

Figure 5.19: Actuator Demand for Pitch Axis Step Demand: $H_\infty$ Minimum Entropy Controller (0 knots)
5.7.4 1 DOF $H_{\infty}$ Controller

Hover Flight Condition

The first set of time responses (Figures (5.20, 5.22, 5.24, 5.26)) and actuator demands (Figures (5.21, 5.23, 5.25, 5.27)) show the full nonlinear model trimmed at hover and controlled by a single 1 DOF $H_{\infty}$ controller.

Figure 5.20: Heave Axis Demand: Single 1 DOF $H_{\infty}$ Controller (0 knots)

Figure 5.21: Actuator Demands for Heave Axis Step Demand: Single 1 DOF $H_{\infty}$ Controller (0 knots)
Figure 5.22: Pitch Axis Demand: Single 1 DOF $H_\infty$ Controller (0 knots)

Figure 5.23: Actuator Demands for Pitch Axis Step Demand: Single 1 DOF $H_\infty$ Controller (0 knots)
Figure 5.24: Roll Axis Demand: Single 1 DOF $H_\infty$ Controller (0 knots)

Figure 5.25: Actuator Demands for Roll Axis Step Demand: Single 1 DOF $H_\infty$ Controller (0 knots)
The preceding four step response plots in each axis (Figure 5.20, 5.22, 5.24, 5.26) show the good decoupling and performance, within actuator limits (Figures 5.21, 5.23, 5.25, 5.27), attainable for the nonlinear model at the trimmed hover flight condition.
120 knots Forward Flight Condition

To clearly illustrate the fact that the 1 DOF $H_\infty$ controller by itself would not be sufficient, and would have required switching or blending techniques, is shown by the step responses at 120 knots (Figures (5.28, 5.29, 5.30, 5.31)). Also, this shows the level of compensation the variable structure controller would be required to provide.

Figure 5.28: Heave Axis Demand: Single 1 DOF $H_\infty$ controller (120 knots)

Figure 5.29: Pitch Step Demand: Single 1 DOF $H_\infty$ controller (120 knots)
The roll response (Figure (5.30)) was very satisfactory, while the yaw response (Figure (5.31)) gave a 50% steady state error. These plots above of the single 1 DOF $H_{\infty}$ controller at 120 knots were still much improved compared to the previous $H_{\infty}$ minimum entropy controller at the lower speed of 60 knots (Figures (5.14, 5.15, 5.16, 5.17))
5.7.5 Variable Structure Controller (1 DOF $H_\infty$ designed Ideal Model)

The following set of time responses (Figures (5.32, 5.33, 5.34, 5.35)) show the helicopter nonlinear model at hover controlled by the variable structure controller with an ideal model designed using 1 DOF $H_\infty$ procedures.

Hover Flight Condition

Figure 5.32: Heave Axis Demand: Variable Structure Controller (0 knots)

Figure 5.33: Pitch Axis Demand: Variable Structure Controller (0 knots)
These above four main-axis step response plots (Figures (5.32, 5.33, 5.34, 5.35)), show little difference to the corresponding responses of the single $H_{\infty}$ minimum entropy controller (Figures (5.10, 5.11, 5.12, 5.13)), and the $H_{\infty}$ one degree of freedom controller (Figures (5.20, 5.22, 5.24, 5.26)).
120 knots Forward Flight Condition

The next set of responses (Figures (5.36, 5.37, 5.38, 5.39)) are for the same variable structure controller, but at 120 knots.

Figure 5.36: Heave Axis Step Demand: Variable Structure Controller (120 knots)

Figure 5.37: Pitch Axis Demand: Variable Structure Controller (120 knots)

The oscillatory nature of the single 1 DOF $H_{\infty}$ controller (Figures (5.28, 5.29)) was very much reduced and damped by the variable structure controller (Figures (5.36, 5.37)). There is, however, an unacceptable coupling into yaw rate for a pitch axis step demand (Figure (5.37)). This was one of the motivating factors for investigating
the two degrees of freedom $H_\infty$ ideas for the ideal model specification in the VSC law framework.

Figure 5.38: Roll Axis Step Demand: Variable Structure Controller (120 knots)

Figure 5.39: Yaw Axis Step Demand: Variable Structure Controller (120 knots)

The primary roll axis response (Figure (5.38)) was very satisfactory, but the helicopter would be increasingly yawing around, which is not good for pilot workload. The yaw rate axis response (Figure (5.39)) showed a more oscillatory response than the 1 DOF $H_\infty$ controller (Figure (5.31)).
5.7.6 2 DOF $H_\infty$ Controller

The following two subsections show step responses of the full nonlinear model at hover (Figures (5.40, 5.41, 5.42, 5.43)) and 120 knots (Figures (5.44, 5.45, 5.46, 5.47)) controlled by the 2 DOF $H_\infty$ design procedures. These plots enable comparisons to be made with the variable structure controller designed incorporating 2 DOF $H_\infty$ procedures and also with the previously shown variable structure controller designed incorporating 1 DOF $H_\infty$ procedures (section 5.7.5).

Hover Flight Condition

![Graphs showing step responses of the full nonlinear model at hover controlled by 2 DOF $H_\infty$ design procedures.](image)

Figure 5.40: Heave Axis Step Demand: Single 2 DOF $H_\infty$ Controller (0 knots)
Figure 5.41: Pitch Axis Step Demand: Single 2 DOF $H_\infty$ Controller (0 knots)

Figure 5.42: Roll Axis Step Demand: Single 2 DOF $H_\infty$ Controller (0 knots)
Figure 5.43: Yaw Axis Step Demand: Single 2 DOF $H_\infty$ Controller (0 knots)

Again, good responses were obtainable at the hover flight condition, as had been shown with the 1 DOF $H_\infty$ controller (Figures (5.20, 5.22, 5.24, 5.26)).

120 knots Forward Flight Condition

The next set of responses (Figures (5.44, 5.45, 5.46, 5.47)) are for the same 2 DOF $H_\infty$ controller, but at 120 knots.

Figure 5.44: Heave Axis Step Demand: Single 2 DOF $H_\infty$ Controller (120 knots)
The coupling into the heave velocity and yaw rate axis for a step in pitch attitude increased significantly over time (Figure (5.45)), and would not aid the helicopter pilot’s workload.

Figure 5.46: Roll Axis Step Demand: Single 2 DOF $H_{\infty}$ Controller (120 knots)
5.7.7 Variable Structure Controller (2 DOF $H_{\infty}$ designed Ideal Model)

The two following subsections show step responses of the full nonlinear model at hover (Figures 5.48, 5.49, 5.50, 5.51) and 120 knots (Figures 5.52, 5.53, 5.54, 5.55) controlled by the variable structure controller designed incorporating the 2 DOF $H_{\infty}$ design procedures.

Hover Flight Condition

Figure 5.48: Heave Axis Step Demand: Variable Structure Controller (0 knots)
Figure 5.49: Pitch Axis Step Demand: Variable Structure Controller (0 knots)

Figure 5.50: Roll Axis Step Demand: Variable Structure Controller (0 knots)
120 knots Forward Flight Condition

Figure 5.51: Yaw Axis Step Demand: Variable Structure Controller (0 knots)

Figure 5.52: Heave Axis Step Demand: Variable Structure Controller (120 knots)
It was seen during the step response simulations that the VSC controller attempted to force the helicopter model to follow the 'ideal' model very tightly. This meant that the step responses of the VSC controlled nonlinear model exhibited a faster initial risetime, but as a consequence a dip in the step response as the system 'recovered' and slightly more inter-axis coupling. This can be seen when comparing the above plots (Figures (5.52, 5.53)), with the single 2 DOF $H_{\infty}$ controller (Figures (5.44, 5.45)). These also show that the heading rate coupling for a pitch step demand was kept from steadily increasing by the variable structure controller.

Figure 5.53: Pitch Axis Step Demand: Variable Structure Controller (120 knots)

Figure 5.54: Roll Axis Step Demand: Variable Structure Controller (120 knots)
The roll and yaw rate axis responses above (Figures (5.54, 5.55)) were slightly less oscillatory than the corresponding single 2 DOF $H_\infty$ controller (Figures (5.46, 5.47)). The responses of the variable structure controller (with the ideal model designed using 2 DOF $H_\infty$ methods), were improved on the variable structure controller designed incorporating 1 DOF $H_\infty$ methods (section 5.7.5).
5.8 Handling Qualities Evaluation

The Aeronautical Design Standard ([1]) details the latest requirements specification for combat helicopters which is intended to ensure that mission effectiveness will not be compromised by deficient handling qualities. The rating system (outlined previously in Chapter 2) is in terms of three levels, where level-1 is the best and level-3 is the worst. This means that violation of any one specification requirement degrades the handling qualities rating. These levels correspond to the Cooper-Harper ratings, which pilots use to judge the handling qualities when actually flying. The use of a MATLAB Handling Qualities Toolbox (Howitt [52]) enables controller designs to be assessed as part of the design and analysis cycle.

The following results were produced using the off-line simulation model (the complete REM, i.e. including the most comprehensive rotor dynamics model available), which is the correct procedure according to ADS at this stage of the controller evaluation. This also gives some justification for still allowing the use of the ADS when testing the frequency response of the nonlinear control law, since the REM is nonlinear itself anyway.

The flight envelope of the helicopter considered here ranges from hover to approximately 140 knots; these represent the limits of validity of the model. Only the four $H_{\infty}$ LSDP -based controllers are analysed for their handling qualities since the issue was whether these qualities could be maintained throughout the flight envelope, and the other two controllers (eigenstructure assignment and $H_{\infty}$ minimum entropy) did not maintain high enough levels of performance at 120 knots to warrant considering.
5.8.1 Single 1 DOF $H_{\infty}$ Controller

The first selection of handling quality ratings are for the single 1 DOF $H_{\infty}$ controller.

Short Term Response

The bandwidth ($\omega_{bw}$) and phase delay ($\tau_p$) parameters were calculated using frequency sweep inputs on pitch, roll and yaw axes to determine the frequency responses of the closed loop system. These values are tabulated below (Tables (5.4, 5.5, 5.6)) for the hover, 60 knots and 120 knots trimmed speeds.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>5.06</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>7.7</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.4: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>5.00</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>7.62</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.5: Bandwidth and phase delay (60 knots)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>4.81</td>
<td>0.08</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>7.27</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.6: Bandwidth and phase delay (120 knots)

Figure (5.56) shows an example Bode frequency plot obtained from the Fast Fourier Transform analysis of the chirp input applied to the controlled nonlinear model, from which bandwidth and phase parameters were calculated. The bandwidths may seem to be too high to be practically feasible for a real helicopter, but here they show what is possible for the nonlinear model of this particular helicopter, and allow for objective comparisons between controller designs. It must also be noted that the bandwidths and phase delays depend on the magnitude of the chirp input implemented (here being 0.1 radians). Also, in practice, the controller might require lower bandwidths so as not to excite certain unmodelled rotor dynamics which are known to exist around the 10 radians/second frequency. The bandwidths and phase delays are plotted in Figures (5.57, 5.58) following.
Figure 5.56: Bode Frequency Plots (Radians/second)

Figure 5.57: Short Term Pitch Criteria
Mid-Term Response

A damping factor of at least 0.35 is required in the pitch and roll axes to satisfy the level-1 handling qualities criteria. The transient responses to pulse attitude demands were analysed to obtain the following values (Table 5.7), which easily satisfy the Level-1 requirements.

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.72</td>
<td>0.70</td>
<td>0.65</td>
</tr>
<tr>
<td>Roll</td>
<td>0.86</td>
<td>0.92</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Table 5.7: Damping Factor

Moderate amplitude response

The agility parameter \((q_{\text{max}}/\delta \theta \text{ versus } \delta \theta, \text{ and } p_{\text{max}}/\delta \phi \text{ versus } \delta \phi)\) must achieve certain levels for a range of pitch and roll attitude changes from 0 knots to 140 knots. These are plotted at 20 knot intervals in Figures (5.59, 5.60) next. The plotted points get nearer to the Level-1 / Level-2 boundary the higher the speed, with the 140 knot point exactly on the boundary.
Interaxis coupling

The level-1 requirements are for less than 25% pitch-to-roll and roll-to-pitch coupling. The following Tables (5.8, 5.9) contain this information, and show the interaxis couplings to be significantly below the 25% requirement.
<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.63</td>
<td>11.1</td>
<td>11.2</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>0.38</td>
<td>0.49</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Table 5.8: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.7</td>
<td>11.9</td>
<td>11.8</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.39</td>
<td>0.46</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 5.9: Roll-to-pitch Coupling

5.8.2 Variable Structure Controller (1 DOF $H_\infty$ Designed Ideal Model)

The same selection of handling quality ratings as in the previous subsection were applied to the variable structure controller where the ideal model was designed using the 1 DOF $H_\infty$ robust stabilization procedures.

Short Term Response

Similar to the single 1 DOF $H_\infty$ controller (compare to Tables (5.4, 5.5, 5.6)), the bandwidth/phase delay parameters (Tables (5.10, 5.11, 5.12)) were well within the Level-1 boundaries. In fact, when comparing Figure (5.61) to Figure (5.57) the pitch channel had considerably higher bandwidths for all of the tested speeds.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{\text{bw}}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.28</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>8.64</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.10: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{\text{bw}}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.63</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>8.36</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.11: Bandwidth and phase delay (60 knots)
<table>
<thead>
<tr>
<th></th>
<th>$\omega_n$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.62</td>
<td>0.10</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>8.62</td>
<td>0.06</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.12: Bandwidth and phase delay (120 knots)

Figure 5.61: Short Term Pitch Criteria

Figure 5.62: Short Term Roll Criteria
Mid-Term Response

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
</tr>
<tr>
<td>Roll</td>
<td>0.85</td>
<td>0.87</td>
<td>0.86</td>
</tr>
</tbody>
</table>

Table 5.13: Damping Factor

The pitch damping factor was increased slightly, while the roll damping factor stayed the same (Table (5.13)), when compared to the single 1 DOF $H_\infty$ controller (Table (5.7)).

Moderate amplitude response

The plotted points in Figures (5.63, 5.64) get nearer to the Level-1 / Level-2 boundary the higher the speed. The points are further within the Level-1 region for the pitch channel (Figure (5.63)) than the 1 DOF $H_\infty$ controller (Figure (5.59)).

Figure 5.63: Moderate Amplitude Pitch Criteria
Interaxis coupling

The following Tables (5.14, 5.15) show that there was, on average, no change in the interaxis coupling as compared to the single 1 DOF $H_{\infty}$ controller (Tables (5.8, 5.9)).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.5</td>
<td>11.7</td>
<td>12.4</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>0.77</td>
<td>0.63</td>
<td>0.72</td>
</tr>
</tbody>
</table>

Table 5.14: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.9</td>
<td>11.8</td>
<td>11.5</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.48</td>
<td>0.50</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 5.15: Roll-to-pitch Coupling
5.8.3 Single 2 DOF $H_\infty$ Controller

The set of handling qualities tests were carried out on the variable structure controller designed using 2 DOF $H_\infty$ methods.

**Short Term Response**

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>5.07</td>
<td>0.13</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>6.86</td>
<td>0.12</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.16: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>4.87</td>
<td>0.13</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>7.19</td>
<td>0.14</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.17: Bandwidth and phase delay (60 knots)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>5.11</td>
<td>0.12</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>7.18</td>
<td>0.15</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5.18: Bandwidth and phase delay (120 knots)

These values (Tables 5.16, 5.17, 5.18) are plotted in Figures (5.65, 5.66) on the following page. The 1 DOF and 2 DOF $H_\infty$ controllers have very similar bandwidths (compare with Figures (5.57, 5.58)). However, these bandwidths are lower than the variable structure controller bandwidths at the same tested speeds (refer to Figures (5.61, 5.62)).
Mid-Term Response

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.81</td>
<td>0.86</td>
<td>0.93</td>
</tr>
<tr>
<td>Roll</td>
<td>0.96</td>
<td>0.99</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 5.19: Damping Factor

The damping (Table 5.19) was higher for this 2 DOF $H_{\infty}$ controller than both the 1 DOF $H_{\infty}$ controller (Table 5.7) and the variable structure controller (Table 5.13).
Moderate amplitude response

Apart from the 140 knots flight speed in Figures (5.67, 5.68), where the handling qualities have moved into the Level-2 region, the results are similar to the 1 DOF $H_{\infty}$ controller (Figures (5.59, 5.60)), but the variable structure controller plots were more consistently grouped in a higher Level-1 region, especially for the pitch axis (Figures (5.63, 5.64)).

Figure 5.67: Moderate Amplitude Pitch Criteria
Interaxis coupling

The following Tables (5.20, 5.21) show that the interaxis coupling was decreased compared to the 1 DOF $H_\infty$ controller (Tables (5.8, 5.9)), and the variable structure controller (Tables (5.14, 5.15)).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.5</td>
<td>11.6</td>
<td>11.7</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>0.41</td>
<td>0.24</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 5.20: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.6</td>
<td>11.8</td>
<td>11.8</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.33</td>
<td>0.25</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table 5.21: Roll-to-pitch Coupling
5.8.4 Variable Structure Controller (2 DOF $H_\infty$ Designed Ideal Model)

Finally, for this chapter, the set of handling qualities tests were carried out on the variable structure controller designed using 2 DOF $H_\infty$ methods.

Short Term Response

The following values (Tables (5.22, 5.23, 5.24)) are plotted on the subsequent page (Figures (5.69, 5.70)).

<table>
<thead>
<tr>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.02</td>
<td>0.07</td>
</tr>
<tr>
<td>Roll</td>
<td>9.98</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.22: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.07</td>
<td>0.06</td>
</tr>
<tr>
<td>Roll</td>
<td>9.94</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.23: Bandwidth and phase delay (60 knots)

<table>
<thead>
<tr>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.29</td>
<td>0.07</td>
</tr>
<tr>
<td>Roll</td>
<td>9.58</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 5.24: Bandwidth and phase delay (120 knots)

The bandwidths above are higher than the 1 DOF and 2 DOF $H_\infty$ controllers (Tables (5.4, 5.5, 5.6, 5.16, 5.17, 5.18)), while the variable structure controller (with ideal model designed using 1 DOF $H_\infty$ methods) was higher in the pitch channel, but lower in the roll channel (Tables (5.10, 5.11, 5.12)).
Mid-Term Response

The responses (Table (5.25)) gave values lower than the three preceding controllers (Tables (5.13, 5.7, 5.19)), indicating lower damping.

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.54</td>
<td>0.53</td>
<td>0.49</td>
</tr>
<tr>
<td>Roll</td>
<td>0.82</td>
<td>0.82</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Table 5.25: Damping Factor
Moderate amplitude response

The agility parameters are plotted in Figures (5.71, 5.72). The lower damping shown in the previous Mid-Term handling quality test, was a trade-off with these very satisfactory Moderate amplitude tests (and also the previous Short term tests). The Moderate amplitude plots show an orderly grouping of points, which are well within the Level-1 region, better than the other three previous controllers.

Figure 5.71: Moderate Amplitude Pitch Criteria

Figure 5.72: Moderate Amplitude Roll Criteria
Interaxis coupling

The level-1 coupling requirements were satisfied comfortably (Tables (5.26, 5.27)).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.2</td>
<td>11.4</td>
<td>10.7</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>0.91</td>
<td>0.39</td>
<td>0.91</td>
</tr>
</tbody>
</table>

Table 5.26: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.6</td>
<td>11.7</td>
<td>11.7</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.66</td>
<td>0.68</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Table 5.27: Roll-to-pitch Coupling
5.9 Flight-Simulator Trials

The complete simulation model on the Advanced Flight Simulator was written in FORTRAN, and run on an Encore Concept-32 computer. An integration time step of 20 ms was utilised. The visual display was generated by a Link-Miles IMAGE IV CGI system, and gave approximately 48° field of view (FOV) in pitch and 120° FOV in azimuth with full daylight texturing. The variable structure controller designed with a 1 DOF $H_{\infty}$ ideal model was coded in Fortran for the simulator. A switch was included in the code so that the unit vector part and the $L_\infty$ part of the controller could be switched out, leaving just a single 1 DOF $H_{\infty}$ controller.

The sliding mode controller with a 2 DOF $H_{\infty}$ designed ideal model required a simulation sample time of 5 ms, otherwise the performance and coupling deteriorates. The flight simulator will not be able to provide this until 1995. It must be noted that this is not due to the $H_{\infty}$ controller design procedures, but merely due to the fact that the ideal model (in the VS design) required as many states as the weighted nominal plant. Therefore, more calculation time was required than a typical 2 DOF $H_{\infty}$ design which would have a lower order ideal model (eighth order in this case). The VS ideal model calculation should not be a problem in future designs since the error vector would be based on the outputs. However, this controller was simulated, and even with reduced performance as compared to the nonlinear simulation model results, the performance for all the handling qualities tests was very satisfactory, with a good level of performance being attainable over the full speed range covered.

Three outer loops, to improve the handling qualities, were also incorporated ([106], and are also outlined in the ADS ([1]). The different modes were there to be used, when necessary, during the flight simulator tests:

- Turn Coordination: To enable a coordinated turn to be effected as a single axis task at moderate/high speed, the heading rate demand was augmented by a function of bank angle.
- Automatic Trimming: To enable the trimming of the inner loop controller to the correct trim attitude.
o Hover acquisition/hold: To enable the pilot to acquire and hold a hover mode automatically.

The controlled helicopter simulator was ‘flown’ from the desk, by an experienced representative from the DRA with the use of two joysticks. This enabled data to be recorded and comments on the handling qualities to be noted. The CGI databases (developed by DRA for the Euro-ACT programme (Schimke et al [87]) contain three hover/low speed mission task elements (sidestep, quickhop, bob-up) and three moderate/high speed tasks (lateral jinking, hurdles, yaw pointing). Due to simulator time restrictions, the sidestep was selected to test the hover/low speed handling qualities. This was chosen since the more straightforward repeatability of carrying out the manoeuvre enables a more objective comparison of different controllers. For the moderate/high speed region, the yaw pointing exercise was selected, since this puts the full envelope handling qualities under stiff examination. For the same reason, the high/moderate speed lateral jinking task was simulated. The three tasks carried out are detailed below:

o Sidestep: The task is to translate through 150 ft from hover at a height of 30 ft above ground level (AGL) in front of one diamond and square arrangement, to acquire and maintain a stable hover in front of the next sighting system. By maintaining any two of the diamond points within the square the desired ±10 ft lateral position and height tolerances are satisfied. The task aggression depends on the initial bank angle, with 10°, 20° and 30° corresponding to low, moderate and high levels of aggression.

o Lateral jinking: This task is to complete a number of ‘S’ turns through slalom gates followed by a line tracking phase. It must be flown at 60 knots at a height of 2 ft AGL. Again the task aggression depends on the initial bank angle, with 10°, 20° and 30° corresponding to low, moderate and high levels of aggression.

o Yaw pointing: Here the pilot is required to acquire and track one of a number of offset posts, while translating down the runway at 60 knots. The size of the initial offset determines the task aggression.
The first set of Figures (5.73, 5.74) show data collected during a sidestep manoeuvre with a single 1 DOF $H_\infty$ controller. The evident inter-axis couplings were partly due to compensation being attempted by the pilot. This was necessary because the task was carried out very aggressively and was not solely a single axis task.

![Figure 5.73: Sidestep (1 DOF $H_\infty$ Controller)](image1)

For comparison, the next set of Figures (5.75, 5.76) show the same sidestep manoeuvre for the variable structure controller with an ideal model designed using 1 DOF $H_\infty$ control system design procedures. So that the two controllers could be correctly compared the task was carried out to as near the same degree of aggressiveness as was practically possible. There was no pilot compensation required in any channel other
than the required roll stick input, showing the manoeuvre to have been carried out as a single axis task. Comments made by the pilot included: “the nonlinear controller was far more predictable, allowing the pilot to establish a hover very precisely and very quickly”. This comment was related to the fact that both controllers had very similar initial responses, due to the control input being relatively open-loop, but when the loop had to be closed by the pilot to regain hover over a precise point (by cancelling the lateral translation) the variable structure controller performed much more predictably. It is important to note that during the sidestep task the controllers were being asked to operate well away from the nominal design condition (25 knots).

Figure 5.75: Sidestep (Variable Structure Controller)

Figure 5.76: Sidestep (Variable Structure Controller)
The lateral jinking task was shown to be carried out by both the 1 DOF $H_{\infty}$ Controller (Figures 5.77, 5.78) and the variable structure controller (Figures 5.79, 5.80) with a high degree of accuracy and low cross-coupling. The variable structure controller flight simulation was performed with a very slightly higher degree of aggression, and therefore correspondingly shows slightly increased coupling. The major difference in the handling qualities that can be seen between the two different controller responses is the lower amount of activity that occurs in the off-main channel responses of the variable structure controlled system. This would mean a higher degree of predictability and precision would be possible. Again the task was carried out for rotorcraft flight speeds away from the nominal hover design operating point.

Figure 5.77: Lateral Jinking (1 DOF $H_{\infty}$ Controller)
Figure 5.78: Lateral Jinking (1 DOF $H_{\infty}$ Controller)

Figure 5.79: Lateral Jinking (Variable Structure Controller)
The final, but most testing, manoeuvre was the yaw pointing task. Also it is the most practical test of the three selected tasks. The scenario begins with the helicopter traveling at a 60 knots forward speed, turning the nose in a 90° turn, tracking this point in space, and finishing up flying backwards. The difficulty in the repeatability of this task meant that exact comparison of the time results is difficult. The relative aggressiveness of the task carried out using the single $H_{\infty}$ controller was higher. Allowing for this fact, the variable structure controller gave a greater degree of decoupling. Pilot comments again included statements such as: “the variable structure controller did not require as much work to carry out the task”.

Figure 5.80: Lateral Jinking (Variable Structure Controller)
Figure 5.81: Yaw Pointing (1 DOF $H_\infty$ Controller)

Figure 5.82: Yaw Pointing (1 DOF $H_\infty$ Controller)
Figure 5.83: Yaw Pointing (Variable Structure Controller)

Figure 5.84: Yaw Pointing (Variable Structure Controller)
5.10 Performance Evaluation

First, detailed simulation-based conclusions are drawn, and then the major achievements of this work are outlined.

Since the analysis of the control law designs was carried out according to the Military Specification ([1]), comparisons between candidate control laws were made significantly more objective. At hover, all the control laws had very similar step response and actuator demand characteristics in each axis, apart from the eigenstructure assignment controller (which did not contain any integral action).

The following analysis concerns the four $H_{\infty}$ LSDP-based control laws, since they managed to maintain at least some level of performance over 60 knots. At 120 knots the step responses of the four $H_{\infty}$ LSDP-based controllers had a number of primary differences. The 1 DOF $H_{\infty}$ controller responses in heave velocity and pitch were very oscillatory, and there was a significant steady state error in yaw rate response. The variable structure control law (with an ideal model designed using 1 DOF $H_{\infty}$ procedures) did not have these oscillatory affects and achieved the main axis step responses very satisfactorily. The only problem was considerably increased coupling into yaw rate for a pitch input step, which was difficult to reduce. The single 2 DOF $H_{\infty}$ controller and the variable structure control law (with an ideal model designed using 2 DOF $H_{\infty}$ procedures) had very similar and satisfactory step responses at these high speeds. All these controllers were able to satisfy the Level-1 bandwidth and phase delay requirements from hover to 140 knots. The differences arose in the agility and damping specifications. Both the 1 DOF and 2 DOF $H_{\infty}$ controllers had good damping, but were less agile at the higher speeds. In fact both of these controllers became level-2 at 140 knots. The variable structure controllers were less damped, but this was still within specification. Also, they maintained Level-1 handling qualities for agility. This would obviously be very important for a military helicopter. All the controllers pitch-to-roll and roll-to-pitch couplings were easily within the requirements for all the flight envelope. The actuator signals were consistently lower for the $H_{\infty}$ designs, which is a consequence of having the magnitude of the control action penalised in the $H_{\infty}$ design formulation.
The major findings of this research work are described next.

- The work carried out took a variable structure control law formulation through the stages of testing on linear models, testing on a full nonlinear model, examining the handling qualities, and finally implementing on an Advanced Flight Simulator (AFS).

- The 1 DOF $H_\infty$ controller used as a benchmark design showed level-1 handling qualities for the low speed regime, but these qualities degraded at the higher speeds.

- A variable structure control law with an ideal model designed using 1 DOF $H_\infty$ procedures showed no such degradation in the handling qualities at the higher speeds. Also this control law was verified by higher aggression than normal tests on the AFS at the DRA, Bedford.

- A 2 DOF $H_\infty$ control law, which had previously been tested on the AFS [106], was assessed in detail for its handling qualities and step response attributes. This 2 DOF $H_\infty$ controller was found to give high levels of handling qualities and step responses, and which were significantly improved as compared to the 1 DOF $H_\infty$ controller.

- The handling qualities of a variable structure controller (with an ideal model designed incorporating 2 DOF $H_\infty$ procedures) were also examined and found to be at a satisfactorily high level.

- The two variable structure controllers (one with an ideal model designed using 1 DOF $H_\infty$ procedures, and the other with 2 DOF $H_\infty$ procedures) both maintained Level-1 Agility handling qualities, whereas the two single $H_\infty$ controllers (1 DOF and 2 DOF) transgressed into Level-2 at the top speed regions. This was, not surprisingly, at a sacrifice of some damping but the variable structure controllers' damping was still within specification.

- The nonlinear (unit vector) component of the overall variable structure controller was verified as being a very important part of the control action input. This was seen when inspecting the relevant contributions of each component of the whole
control law. This justifies the idea that the variable structure controller is very much a different controller, and not just an amended $H_\infty$ controller. Therefore there may be other possible advantages and disadvantages of this controller that have not been looked at so far.

- The variable structure design procedures provide an alternative method of designing a 2 DOF controller, which has theory for taking into account a varying uncertain system.

- The setting of the controller design into the 2 DOF framework for both the $H_\infty$ and the variable structure controller was seen as being important for the wide envelope nature required of helicopter control system design. This was largely due to the advantages of having time objectives in the ideal model as well as the frequency specifications for the system throughout the flight envelope.

- Since all of the above candidate control laws were designed as part of the work, their associated relative attributes in handling qualities were assessed.

5.11 Conclusions

A variable structure controller has been investigated thoroughly at the off-line pre-flight stage. This chapter has included the necessary theoretical background, testing on linear and nonlinear simulation models, testing against military specified requirements, and finally simulation on an advanced flight simulator. Also, a number of important conclusions were drawn, which further the understanding of the potential of variable structure control laws.

A very important aspect of testing the variable structure controllers which have been designed here must be the affect that the controllers have in rejecting the affects of wind turbulence. The variable structure control law formulation implemented here, has the useful property of being robust to matched uncertainty once the system is in the 'sliding mode', which will be useful when approaching the reduction of gusts as a disturbance.

Also important to this control law development would be the implementation of
the variable structure controller (with an ideal model designed using 2 DOF $H_{\infty}$ procedures) on the AFS at DRA, Bedford. Naturally, the real certification of these advanced controllers would be on the real helicopter, which will surely present a challenging series of further practical issues. The practical issues raise the need for investigation into the combination of the error states making the sliding surface, rather than the error output vector, since it may not be necessary that all the error states tend to zero. This would therefore reduce the size of the ideal model that was required to provide the ideal model state vector.
Chapter 6

A Variable Structure Observer

6.1 Introduction

The complete variable structure (VS) controller framework is completed by this chapter, through the implementation of a VS observer. That is, the outputs of the helicopter system are observed by a VS observer for the VS controller. In the previous chapter the variable structure controllers, which were tested on the Advanced Flight Simulator, incorporated the observers which resulted from the $H_{\infty}$ design procedures. In fact, these observers are known to have the structure of 'Kalman filters'. The Kalman filter (Grewal and Andrews [42]) is an estimator and can be used to estimate the state of a linear dynamic system perturbed by Gaussian white noise, using measurements that are linear functions of the system state, but corrupted by additive Gaussian white noise. The successful applications of Kalman filtering are overwhelming in the literature [42]. Recent developments in the VS observer theory (Edwards and Spurgeon [23]) have shown the potential that a VS observer could have in tackling the problem of estimating a system's state. The advantage of a VS observer being insensitive to matched uncertainty in the so-called 'sliding mode' (when the observer states are following a predefined trajectory) implies that this observer may be preferable in certain applications (i.e. when the uncertainty enters through the input to the system). The previous chapter showed in great detail the successful implementation of a VS controller which has similar properties, and so to keep the VS framework for
a complete helicopter flight control system design, the VS observer design procedure 
([23]) will be applied and investigated. Through the application of the VS theory, it
will also be indicated as to whether any performance improvements can be made with
the use of a different observer. It had been established in [23] that an observer existed
for the nominal helicopter 0 knots linearized state-space model estimation problem,
but the possible observer was not calculated. Also, Edwards and Spurgeon [25] showed
that the robustness properties of a variable structure controller, which requires full
state information, are still maintained in the face of matched uncertainty (uncertainty
entering through the system input distribution matrix) when a variable structure state
observer is implemented.

6.2 A Variable Structure Observer Design Method

The same system description as described in Chapter 5 (i.e. equation 5.6) will be
considered, together with the output equation 6.2:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + F(t, x, u) \\
y(t) &= C_s x(t)
\end{align*}
\]

where \( x \in \mathbb{R}^n \), \( u \in \mathbb{R}^m \) and \( y \in \mathbb{R}^p \), \( B \) and \( C_s \) are full rank, \( n \geq m \) and \( n \geq p \)
and the matrix pair \((A, B)\) are assumed to be known. Again, as in Chapter 5, the
unknown function \( F(t, x, u) \) represents system nonlinearities and model uncertainties
in the system.

6.2.1 Design Constraints and Switching Surface Definition

The following problem constraints and design formulation (from Edwards and Spurgeon
[24]), have been described in relation to the helicopter control problem (as described
in Chapter 2) and with a view to incorporating the observer in the complete variable
structure design formulation. The three constraints which had to be satisfied were:
1. There must be more outputs than inputs (or an equal number), that is \( p \geq m \). The helicopter description considered in Chapter 2 has four inputs and six outputs.

2. The uncertain function must be able to be expressed in the form:

\[
F(t, x, u) = B\xi(t, x, u)
\]

(6.3)

where the function \( \xi : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m \) is unknown but bounded with

\[
\|\xi(t, x, u)\| \leq \rho \quad \forall x \in \mathbb{R}^n, u \in \mathbb{R}^m, t \geq 0
\]

(6.4)

The splitting of the uncertainty function \( F(\cdot) \) into matched and unmatched uncertainty was established in the previous Chapter 5 (section 5.4), and the bounds calculated using equations (5.43) - (5.44) were used for the variable structure controllers applied later in that chapter.

3. A nonsingular matrix must exist which transforms the system (equations 6.1, 6.2) into the form:

\[
\dot{x}_1 = A_{11}x_1 + A_{12}y
\]

\[
\dot{y} = A_{21}x_1 + A_{22}y + B_2(u + \xi)
\]

(6.5)

where \( y \in \mathbb{R}^p \) are the outputs, \( x_1 \in \mathbb{R}^{(n-p)} \), and \( A_{11} \) has stable eigenvalues. The stability of \( A_{11} \) was established in [23] for the helicopter observer existence problem.

Since the three constraints above were satisfied, the observer structure could be considered.

By considering the vectors \((\hat{x}_1, \hat{y}_1)\) to be the state estimates, according to the coordinate system of equations (6.5), the following state estimation errors can be set up:

\[
e_1 = \hat{x}_1 - x_1
\]

(6.6)

\[
e_y = \hat{y} - y
\]

(6.7)
Therefore, the observer has been designed with the intention for \((e_1, e_y) \to 0\) asymptotically with insensitivity to uncertainty. The 'sliding motion' (described in Chapter 5, section 5.1) is induced on the surface described by:

\[
S_o = \{(e_1, e_y) : e_y = 0\} \tag{6.8}
\]

### 6.2.2 Observer Design Procedure

The design procedure ([24]) followed for the helicopter state vector observer design is described next.

**Step 1**

Permute the columns of \(C\) until \(C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}\) where \(C_2 \in \mathbb{R}^{p \times p}\) with \(\det(C_2) \neq 0\).

Then use the nonsingular transformation

\[
T_3 = \begin{bmatrix} I_{n-p} & 0 \\ C_1 & C_2 \end{bmatrix} \tag{6.9}
\]

to obtain a coordinate system in which the outputs appear as the last \(p\) system states.

**Step 2**

In the new coordinate system let the input distribution matrix be written in the form

\[
\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \quad \text{where } B_1 \in \mathbb{R}^{(n-p) \times m} \text{ and } B_2 \in \mathbb{R}^{p \times m} \tag{6.10}
\]

If \(B_2\) is singular, no robust observer exists, and therefore stop.

**Step 3**

Change coordinates with respect to the nonsingular transformation

\[
T_4 = \begin{bmatrix} I_{n-p} & -B_1B_2^{-1} \\ 0 & I_p \end{bmatrix} \tag{6.11}
\]

and write the system matrix as

\[
\begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} \\ \mathcal{A}_{11} & \mathcal{A}_{22} \end{bmatrix} \quad \text{where } \mathcal{A}_{11} \in \mathbb{R}^{(n-p) \times (n-p)} \text{ and } \mathcal{A}_{22} \in \mathbb{R}^{p \times p} \tag{6.12}
\]

If \(\mathcal{A}_{11}\) is unstable then no robust observer exists and therefore stop.
Step 4

The system is now in the canonical form

\[
\begin{align*}
\dot{x}_1 &= A_{11} x_1 + A_{12} y \\
\dot{y} &= A_{21} x_1 + A_{22} y + B_2 u + B_2 \xi
\end{align*}
\]

where \( A_{11} \) is stable

\[
\dot{\gamma} = A_2 x_1 + A_{22} y + B_1 u + B_1 \xi
\]

Define

\[
G_* = \begin{bmatrix} A_{12} \\ A_{22} - A_{22} \end{bmatrix}
\]

(6.14)

where \( A_{22} \) is any stable matrix of appropriate dimension.

Step 5

Compute the gain matrices \( G_i \) and \( G_n \) in the original coordinate system as

\[
G_i = T_3^{-1} T_4^{-1} G_*
\]

(6.15)

\[
G_n = \|B_2^T P\| T_3^{-1} T_4^{-1} \begin{bmatrix} 0 \\ P^{-1} \end{bmatrix}
\]

where \( P \in \mathbb{R}^{n \times p} \) is the solution to the Lyapunov equation below (6.16).

\[
P A_{22}^T + (A_{22}^T)^T P = -Q
\]

(6.16)

where \( A_{22} \in \mathbb{R}^{p \times p} \) is selected to be any stable design matrix.

Step 6

The observer can be written as

\[
\dot{\hat{x}} = A \hat{x} + Bu - G_i(C \hat{x} - y) + G_n \nu
\]

(6.17)

where

\[
\nu = \begin{cases} 
-\rho \frac{\epsilon_v}{\|\epsilon_v\|} & \text{if } \epsilon_v \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

(6.18)

For implementation, the discontinuous unit vector \( \nu \) was replaced by the continuous approximation

\[
\nu_{approx} = -\rho \frac{\epsilon_v}{\|\epsilon_v\| + \delta} \quad \text{where } \delta \text{ is a small positive constant}
\]

(6.19)

Therefore the observer is now no longer completely insensitive to matched uncertainty and only motion within a bounded region of \( S_a \) can be guaranteed [6].
The next section describes the design and implementation details of the VS observer in the complete VS controller/observer framework where the VS controller contains an ideal model designed incorporating:

1. One Degree-of-freedom (DOF) $\mathcal{H}_\infty$ design procedures.
2. Two Degrees-of-freedom (DOF) $\mathcal{H}_\infty$ design procedures.

### 6.3 Observer Design Parameters

The poles of the matrix $A_{22}$ (equation 6.14) were chosen for both of the designs as $-15, -9, -4, -3, -2, -1$, so that the observers had at least partially faster dynamics than the original helicopter system (refer back to list of system poles in Chapter 2). The statement 'partial' relates to the fact that the observer design procedure results in the observer containing poles at the system’s transmission zeros.

The same weighting functions and $\mathcal{H}_\infty$ required parameters (described in section 5.6.3) were used in the design of the variable structure controller parts of both of the whole observer/controller formulations. The only difference was the description of the actuators used (which is something the designer chooses anyway). A first order lag (refer to Table 5.3 section 5.7 could not be used due to the condition that $(C_sB)$ must be full rank. Therefore, a lag compensator was used of the form $\frac{1+T_c s}{1+T_a s}$, where $T_c$ was kept at the values in Table 5.3. The time constant $T_a = 0.02$ was sensibly selected to be significantly into the high frequency region and out of the the range of working frequencies.

Since the observer itself was designed to have the described matched uncertainty rejection qualities, the previously designed controllers would now not be expected to have to reject as much as before. This reduction of controller workload was realised, since the $Q$ parameters of Chapter 5, section 5.5, Equations (5.42)-(5.44) needed to be chosen at considerably lower values (otherwise the overall system became much too sensitive).

The final parameters were selected as follows for the two VS observers/controllers:

1. One DOF $\mathcal{H}_\infty$ designed ideal model: $Q_1 = 0.1$, $Q_2 = 0.1$, $Q_3 = 0.1$, $\rho_s = 1.5$, $\ldots$
δ₀ = 1.5, and δ = 0.3.

2. Two DOF $H_\infty$ design ideal model: $Q_1 = 0.5$, $Q_2 = 0.5$, and $Q_3 = 0.5$, $p_o = 0.5$, $\delta_o = 1.5$, and $\delta = 0.5$.

6.4 Helicopter Model Simulation Results

The step sizes used were large enough to make significant changes to the trimmed flight conditions and were as follows (Table (6.1)):

<table>
<thead>
<tr>
<th>Commanded O/p</th>
<th>Description</th>
<th>Commanded Input Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ (y1)</td>
<td>Heave Vel.</td>
<td>10 Ft/s</td>
</tr>
<tr>
<td>$\Theta$ (y2)</td>
<td>Pitch Att.</td>
<td>0.2 Rad.</td>
</tr>
<tr>
<td>$\Phi$ (y3)</td>
<td>Roll Att.</td>
<td>0.2 Rad.</td>
</tr>
<tr>
<td>$\Psi$ (y4)</td>
<td>Head. Rate</td>
<td>0.2 Rad/s</td>
</tr>
</tbody>
</table>

Table 6.1: Individual Main Axis Step Size Demand

For all the responses in the following sections the actuator demands were within the actuator limits imposed by this particular helicopter, and have been shown for the VS controller with the 1 DOF $H_\infty$ designed ideal model. The following time response plots have been ordered for each VS controller as:

1. One DOF $H_\infty$ designed ideal model

   Hover

   120 knots

2. Two DOF $H_\infty$ design ideal model

   Hover

   120 knots
6.4.1 VS Controller/Observer (1 DOF $H_{\infty}$ designed Ideal Model)

Hover Flight Condition

The first set of time responses (Figures (6.1, 6.3, 6.5, 6.7)) and actuator demands (Figures (6.2, 6.4, 6.6, 6.8)) show the full nonlinear model trimmed at hover and controlled by the variable structure controller/observer pair.

![Graphs showing Heave Velocity, Pitch Attitude, Roll Attitude, and Heading Rate](image1)

**Figure 6.1:** Heave Axis Demand: Variable Structure Controller/Observer (0 knots)

![Graphs showing Main Rotor Collective Actuator, Longitudinal Cyclic Actuator, Lateral Cyclic Actuator, and Tail Rotor Actuator](image2)

**Figure 6.2:** Actuator Demands for Heave Axis Step Demand: Variable Structure Controller/Observer (0 knots)
Figure 6.3: Pitch Axis Demand: Variable Structure Controller/Observer (0 knots)

Figure 6.4: Actuator Demands for Pitch Axis Step Demand: Variable Structure Controller/Observer (0 knots)
Figure 6.5: Roll Axis Demand: Variable Structure Controller/Observer (0 knots)

Figure 6.6: Actuator Demands for Roll Axis Step Demand: Variable Structure Controller/Observer (0 knots)
The preceding four step response plots in each axis (Figure 6.1, 6.3, 6.5, 6.7) show a good level of decoupling and performance, within actuator limits (Figures 6.2, 6.4, 6.6, 6.8), attainable for the nonlinear model at the trimmed hover flight condition. The responses do have a slightly oscillatory tendency, more noticeable in the control action plots, which was also seen in the simulation results of the previously designed variable structure controller (designed with an observer resulting from $H_{\infty}$ design procedures) of Chapter 5, section 5.7.4.
120 knots Forward Flight Condition

The following step response plots illustrate the VS controller/observer operating at 120 knots (Figures (6.9, 6.10, 6.11, 6.12)).

Figure 6.9: Heave Axis Demand: Variable Structure Controller (120 knots)

Figure 6.10: Pitch Step Demand: Variable Structure Controller (120 knots)
The primary channel responses for the 120 knots flight condition (Figures (6.9, 6.10, 6.12, 6.11)) exhibited satisfactory performance, (again with a tendency to be slightly oscillatory), but the coupling was not satisfactory (especially the yaw rate coupling). However, these responses were very much comparable to the previously designed variable structure controller (designed with an observer resulting from $H_\infty$ design procedures) of Chapter 5, section 5.7.4, at 120 knots.
6.4.2 VS Controller/Observer (2 DOF $H_{\infty}$ designed Ideal Model)

Hover Flight Condition

The first set of time responses (Figures 6.13, 6.14, 6.15, 6.16) show the full nonlinear model trimmed at hover and controlled by the variable structure controller/observer pair.

![Graphs showing time responses for hover flight condition](image)

Figure 6.13: Heave Axis Demand: Variable Structure Controller/Observer (0 knots)

![Graphs showing time responses for hover flight condition](image)

Figure 6.14: Pitch Axis Demand: Variable Structure Controller/Observer (0 knots)
The four step response plots in each axis (Figure (6.13, 6.14, 6.15, 6.16)) show a good level of decoupling and performance attainable for the nonlinear model at the trimmed hover flight condition.
120 knots Forward Flight Condition

The following plots show the controlled nonlinear model trimmed and then perturbed from 120 knots (Figures (6.17, 6.18, 6.19, 6.20)).

![Graphs showing Heave Velocity, Pitch Attitude, Roll Attitude, and Heading Rate](image)

Figure 6.17: Heave Axis Demand: Variable Structure Controller (120 knots)

![Graphs showing Heave Velocity, Pitch Attitude, Roll Attitude, and Heading Rate](image)

Figure 6.18: Pitch Step Demand: Variable Structure Controller (120 knots)
The primary channel responses for the 120 knots flight condition (Figures 6.17, 6.18, 6.20, 6.19) exhibited satisfactory performance and decoupling. These responses were very much comparable to the VS controllers (designed with an observer resulting from 2 DOF $H_{\infty}$ design procedures) of Chapter 5, section 5.7.6, at 120 knots (Figures 5.44, 5.45, 5.46, 5.47)).
6.4.3 Further Details of the Variable Structure Controller/Observer
(1 DOF $H_{\infty}$ LSDP designed Ideal Model)

The output error states defined by equation 6.8, section 6.2.1, are plotted below (Figure 6.21).

![Switching States](image)

Figure 6.21: Output Error States for a Pitch Step Demand at 120 knots

The error states could be forced to approach zero faster by increasing the specified observer poles (equation 6.16 in section 6.2.2) or increasing the size of '$p$' (equation 6.19), however the system response becomes more oscillatory. The reason the system takes a reasonably long time to approach zero is due to the observer containing poles situated at the transmission zeros of the original helicopter linear plant. These transmission zeros are at -0.0014 and -0.0054, which causes the slow poles and correspondingly slow decay of the error states. However, the magnitude of the error states is small enough not to cause problems (the largest error state corresponds to heave velocity which has units of feet/second rather than the radians and radians/second of the other channels).
6.5 Handling Qualities Evaluation

As in section 5.8, the handling qualities were assessed for the variable structure controller and observer with the ideal model having been designed using 1 degree-of-freedom $H_\infty$ loop shaping design procedures. Again, the Aeronautical Design Standard (ADS-33C [1]) was used with the rating system in terms of three levels, where level-1 is the best and level-3 is the worst. The following results were produced using the same off-line simulation model (the complete RHM, i.e. including the most comprehensive rotor dynamics model available).

6.5.1 VS Controller/Observer (1 DOF $H_\infty$ designed Ideal Model)

Short Term Response

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>5.01</td>
<td>0.14</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>9.16</td>
<td>0.08</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.2: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>4.96</td>
<td>0.15</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>9.25</td>
<td>0.07</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.3: Bandwidth and phase delay (60 knots)

<table>
<thead>
<tr>
<th></th>
<th>$\omega_{bw}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>4.59</td>
<td>0.60</td>
<td>1</td>
</tr>
<tr>
<td>Roll</td>
<td>9.80</td>
<td>0.07</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6.4: Bandwidth and phase delay (120 knots)

The bandwidths (Tables (6.2, 6.3, 6.4)), and Figures (6.22, 6.23) were lower in pitch and higher in roll than the corresponding variable structure controller, with an ideal model and observer resulting from 1 DOF $H_\infty$ procedures (refer to Tables (5.10, 5.11, 5.12)).

141
Mid-Term Response

The damping factor handling quality requirements were satisfied (Table (6.5)).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.84</td>
<td>0.83</td>
<td>0.80</td>
</tr>
<tr>
<td>Roll</td>
<td>0.83</td>
<td>0.84</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Table 6.5: Damping Factor
Moderate amplitude response

As with the VS controllers of Chapter 5 the plotted points in Figures (6.24, 6.25) get nearer to the Level-1 / Level-2 boundary the higher the speed, with the 140 knot point nearest to the boundary.

Figure 6.24: Moderate Amplitude Pitch Criteria

Figure 6.25: Moderate Amplitude Roll Criteria
Interaxis coupling

The level-1 requirements were easily satisfied (Tables 6.6, 6.7).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.4</td>
<td>11.5</td>
<td>12.0</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>1.90</td>
<td>1.78</td>
<td>1.46</td>
</tr>
</tbody>
</table>

Table 6.6: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.3</td>
<td>11.9</td>
<td>11.8</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.38</td>
<td>0.37</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 6.7: Roll-to-pitch Coupling
6.5.2 VS Controller/Observer (2 DOF $H_\infty$ designed Ideal Model)

Short Term Response

The tables below (Tables (6.8, 6.9, 6.10)) are plotted in Figures (6.26, 6.27).

<table>
<thead>
<tr>
<th>$\omega_{hp}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.20</td>
<td>0.08</td>
</tr>
<tr>
<td>Roll</td>
<td>10.8</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6.8: Bandwidth and phase delay (hover)

<table>
<thead>
<tr>
<th>$\omega_{hp}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.19</td>
<td>0.08</td>
</tr>
<tr>
<td>Roll</td>
<td>10.5</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6.9: Bandwidth and phase delay (60 knots)

<table>
<thead>
<tr>
<th>$\omega_{hp}$</th>
<th>$\tau_p$</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>6.49</td>
<td>0.08</td>
</tr>
<tr>
<td>Roll</td>
<td>10.3</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 6.10: Bandwidth and phase delay (120 knots)

Figure 6.26: Short Term Pitch Criteria
The pitch and roll bandwidths were significantly higher than the calculated bandwidths for all of the controllers in Chapter 5, section 5.8 and for the VS controller/observer with the 1 DOF $H_\infty$ designed ideal model of this Chapter 6, section 6.5.1.

**Mid-Term Response**

Table (6.11) shows level-1 handling qualities.

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>0.78</td>
<td>0.86</td>
<td>0.82</td>
</tr>
<tr>
<td>Roll</td>
<td>0.75</td>
<td>0.80</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Table 6.11: Damping Factor
Moderate amplitude response

The agility parameter are shown below in Figures 6.28, 6.29.

Figure 6.28: Moderate Amplitude Pitch Criteria

Figure 6.29: Moderate Amplitude Roll Criteria
Interaxis coupling

The level-1 requirements were again comfortably satisfied (Tables (6.12, 6.13)).

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch (Degs)</td>
<td>11.5</td>
<td>11.4</td>
<td>10.8</td>
</tr>
<tr>
<td>Max. Roll (Degs)</td>
<td>0.58</td>
<td>0.39</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 6.12: Pitch-to-roll Coupling

<table>
<thead>
<tr>
<th></th>
<th>0 knots</th>
<th>60 knots</th>
<th>120 knots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll (Degs)</td>
<td>11.3</td>
<td>11.3</td>
<td>11.3</td>
</tr>
<tr>
<td>Max. Pitch (Degs)</td>
<td>0.45</td>
<td>0.47</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Table 6.13: Roll-to-pitch Coupling

6.6 Performance Evaluation

Two complete variable structure (VS) controller and observer formulations were designed, and tested in nonlinear simulation and for their relevant handling qualities. The observer/controller pairs were shown to provide good performance and decoupling for the hover flight condition. A slight tendency for the VS controller/observer, with a 1 DOF $H_\infty$ designed ideal model, to exhibit oscillatory behaviour (predominantly seen in the actuator demands), was unfortunately found to be further exaggerated at the higher speeds. However, this degradation of performance at 120 knots for a VS controller, with the ideal model and observer having been designed using 1 DOF $H_\infty$ design procedures, had been already been indicated in the previous Chapter 5. Therefore, it was not considered to be any lack of robustness of the VS observer part, because the states were still being estimated, but more the feedback control part. The handling qualities were all satisfactorily Level-1, and on average very similar to the VS controller (with the ideal model and observer having been designed using 1 DOF $H_\infty$ design procedures).

The VS controller/VS observer, with a 2 DOF $H_\infty$ designed ideal model, maintained high levels of performance, decoupling and handling qualities from hover up to 120 knots. These were very similar to the levels obtained in Chapter 5 for the VS controller.
with an observer resulting from 2 DOF $H_\infty$ procedures. However the handling qualities were on average further improved.

Also, because the VS observer was considering the uncertainty in the same way as the VS controller, both of the VS observers reduced the workload required of the respective VS controllers for rejecting matched uncertainty.

6.7 Conclusions

Variable structure (VS) observer theory has been shown to be successfully applied to the estimation of the state vector from output information in the face of the model uncertainty provided by the full nonlinear Rationalised Helicopter Model (RHM), over the full working envelope (hover to 120 knots). This had enabled the complete VS controller/observer pair to be implemented on the RHM, and for handling qualities tests to be carried out. High levels of performance and decoupling were exhibited by the complete VS system at hover.

In summary, this Chapter has put the complete VS controller and VS observer framework together, and as a consequence investigated whether the implementation of a different observer would affect the performance levels of the helicopter. The VS observers were found to be straight-forwardly implementable, and gave very similar time response results to the observers resulting from the $H_\infty$ design procedures. The handling qualities parameters were improved, and would therefore indicate a reduction in required pilot workload, at the expense of a higher level of control activity.
Chapter 7

Controller Design for Turbulence Rejection

7.1 Introduction

This chapter demonstrates the enhancement to helicopter performance, when affected by atmospheric turbulence, which can be made by modifying two traditional $H_{\infty}$ frequency domain controller design formulations. The controller designs selected to be amended have already been proved in Advanced Flight Simulator (AFS) tests at the Defence Research Agency, Bedford. The reduction of the effects of gusts is very important in reducing the pilots' workload, and enabling them to carry out aggressive manoeuvres whatever the weather conditions. As a consequence of decreased buffeting, the airframe and component lives will be lengthened, and passenger comfort increased. It has also been suggested (Hall and Wereley [45]) that turbulence could be a factor in the random nature associated in the vibration of the rotors.

As was described in Chapter 3, the design of rotorcraft flight control systems, which will maintain system stability and performance, has been receiving attention for many years now, with the $H_{\infty}$ frequency domain controller designs having been particularly successful (Yue [114], Walker [100], Walker et al [106]), and 'flown' in piloted ground-based simulations. These design procedures use frequency information about the disturbances to limit the system sensitivity. In all of these designs there has
not been explicit consideration of the effect that atmospheric turbulence would create. Therefore, by incorporating practical knowledge about the disturbance characteristics, and how they affect the real helicopter, controller designs can be performed to improve the overall performance. For the new designs, the gust was modelled as a perturbation on the velocity states of the helicopter model, and the disturbance included as an extra input signal to the Standard Compensation Configuration (SCC).

The same nonlinear helicopter dynamical model (the Rationalised Helicopter Model (RHM)), as utilised in Chapters 5 and 6, was used for simulation purposes. A turbulence generator module has been included in the most recent version by the Defence Research Agency (DRA), Bedford. This now enables controller designs to be tested for their disturbance rejection qualities at an off-line stage. The performance of particular new designs in minimizing the effect of gusts was judged by comparison to two corresponding traditional $H_\infty$ controller designs: $H_\infty$ mixed sensitivity design, and the one degree of freedom $H_\infty$ Loop Shaping Design Procedure (LSDP). Designs to enhance the disturbance rejection capabilities of a two degrees of freedom $H_\infty$ LSDP controller were made, but no improvement could be found. However, the results are still shown, since this enables comparison to be made to the variable structure controllers designed in the previous Chapter 5. It is important to note that the controller designs were designed using a hover linearization, but the results shown are from testing on the nonlinear model at 20 knots. This tests the robustness of the controller designs with respect to their ability to reduce the effects of turbulence at an off-design point.

It must be noted that the complex model of the gusts and their effect on the helicopter's equations of motion is self contained in the code of the RHM. It is the 'simplified' generic model, of how this turbulence profile was considered to enter the helicopter's equations of motion, which has been developed for use in the controller designs. The following descriptions of the gust modelling will be used to describe the key elements of the gust ideas, but were not used in the design procedures at any stage. The controller designs to take into account gust information are described in section 7.2, but first outlined are the fundamental ideas associated with understanding atmospheric turbulence and its modelling.
7.1.1 Introduction to Turbulence Fundamentals

Turbulence profiles tend to be continuous and irregular (Hoblit [49]), as shown in Figure (7.1).

![Figure 7.1: Typical Turbulence Profile](image)

The gust structure (turbulence profile) is considered, in the terminology of turbulence modelling ([49]), to consist of single gusts or pulses (Figure (7.2)).

![Figure 7.2: Single Gust Profile](image)

Therefore, a continuous turbulence record can be considered as consisting of a series of individual gusts (Figure (7.3)).

![Figure 7.3: Series of Individual Gusts](image)
Not only is atmospheric turbulence continuous, but also "Isotropic". This means, if an aircraft flies through an area of turbulence, the lateral and vertical profiles of that turbulence will have roughly similar characteristics and peak magnitudes.

A major topic in itself is the modelling of turbulence, which can be approached from the mathematical individual blade/blade element techniques (Houston and Hamilton [50], Riaz et al [83]) or from an engineering analysis of real flight velocity data and how a gust is propagated (Turner [97]). Two different applicable approaches are commonly applied to the representation of the turbulence profiles:

2. Power Spectral Density (PSD) Methods (i.e. Continuous).

These will be introduced briefly next, together with some of the many advantages and disadvantages of each method:

**Statistical Discrete Gust (SDG) Methods**

The SDG method uses a simple ramp gust model, the "One-Minus-Cosine", representation (Figure (7.4)). The ramp itself can be of any length or magnitude (i.e. scale or intensity). These are determined by an exponential joint probability distribution, where the parameters are obtained from experimental data.
where:

$$U = \frac{1}{2} U_c (1 - \cos(\frac{2\pi X}{2H}))$$  (7.1)

A further parameter called intermittency, is required in the formulation and relates to the continuous nature of the turbulence record. Pockets of fluctuations embedded in a background of relatively low activity would be considered as highly intermittent as compared to a more continuous record which would have a low intermittency. It is therefore possible to represent relatively unusual large scale events which may have an important effect on aircraft response.

**Power Spectral Density (PSD) Methods**

The turbulence profile of Figure (7.1) can be regarded as a 'stationary Gaussian random process'. It is 'stationary' since it is considered to be of infinite duration and its statistical properties are the same wherever it may be sampled. Being 'Gaussian' implies that if the time history (for example \(y(t)\)) was sampled at random, in practice this is at equally spaced points in time, the resulting probability distribution is Gaussian (or Normal) and defined by the following probability density function:

$$p(y) = \frac{1}{\sqrt{2\pi\sigma_y}} \exp\left(-\frac{y^2}{2\sigma_y^2}\right)$$  (7.2)

where \(\sigma_y\) is a constant.

The Power Spectral Density \(\Phi(\omega)\) can be expressed (for more details refer to [49] or most signal processing texts) as:

$$\Phi(\omega) = \lim_{T \to \infty} \frac{1}{\pi T} \left| \int_{-T}^{T} y(t) \exp^{-j\omega t} \, dt \right|^2$$  (7.3)

Therefore, the PSD approach is to represent the turbulence record as a collection of continuous sine waves at different frequencies. There are many different functions that have been selected to fit the power/frequency distribution of the turbulence (Jones [55]), perhaps the most widely known and used being the 'Dryden' and 'Von Kármán' spectra ([49]).

One disadvantage in PSD techniques is that for each sine wave an 'average' amplitude is calculated at that frequency to represent all the fluctuations. It is therefore possible to obtain identical power/frequency graphs for very different turbulence
records. Also the lack of information about the rate of occurrence of large discrete events is often critical in aircraft or control system design. It is the distribution of changes in turbulence velocities, rather than of absolute velocities, which influence aircraft dynamic quantities such as acceleration, pitch rate and structural loads (Foster and Jones [31]). Both the SDG and PSD methods have been used extensively ([40]) by industry in general.

The SDG method has been recognized ([31]) to be easier for visualising the aircraft response, and has been verified against a large amount of real turbulence data at the Defence Research Agency (DRA), Bedford, who have included an SDG model in their most recent version of the nonlinear Rationalised Helicopter Model (RHM). Again, it must be noted that the ideas above, about modelling the turbulence, have been included for completeness, since the RHM does all the calculation and generation of the turbulence into the helicopter equations of motion automatically. The next section describes a simpler modelling of this complex process, by considering perturbations on the linearized state model. This simple model can easily be incorporated into the controller design process as will be explained.
7.2 $H_\infty$ Controller Design Methodologies

7.2.1 Modelling of How Turbulence affects the Helicopter

For the following controller design procedures the effect of the turbulence disturbance was modelled as gust velocity components perturbing the helicopter's velocity states.

Therefore, the standard state space system description can be modified to include the modelled effects of turbulence:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
d_1 \\
d_2 \\
d_3
\end{bmatrix}
\]

\[
\dot{x} = Ax + A_0 + Bu 
\]

(7.4)

\[
y = Cx 
\]

(7.5)

Now, by defining $d := \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ and $B_d := A(:, 6:8)$ (MATLAB notation for columns 6, 7 and 8 of matrix $A$), the system can be expressed as:

\[
\dot{x} = Ax + Bu + B_d d 
\]

(7.6)

\[
y = Cx 
\]

(7.7)

In transfer function form this can be expressed as:

\[
y = Gu + G_d B_d d 
\]

(7.8)

where $G = C(sI - A)^{-1}B$, and $G_d = C(sI - A)^{-1}$. 

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7.2.2 Disturbance Rejection Design (based on $H_\infty$ Mixed Sensitivity Methodology)

The modelling of the mechanism by which the turbulence enters the helicopter's linear equations of motion enables the design formulation in Figure (7.5) to be set up. This can be compared to the traditional $H_\infty$ mixed sensitivity design formulation of Figure (4.3) in section 4.4, which doesn't include the extra disturbance input.

\[
\text{Figure 7.5: Disturbance Rejection Design Formulation}
\]

The optimization problem is to find a stabilizing controller $K$ that minimizes the cost function:

\[
\min_{\text{cost}} K \begin{bmatrix} W_1 SW_3 & W_1 SG_\omega B_d W_4 \\ W_2 K SW_3 & W_1 K SG_\omega B_d W_4 \end{bmatrix}_{\infty} \quad (7.9)
\]

where $S = (I + GK)^{-1}$ is the sensitivity function.

If $G$ had been square then the formulation could have been solved as a two block problem (Yue [114]). The relation of these transfer functions to the structure involved with the $H_\infty$ mixed sensitivity design (section 4.4) can be seen by setting the disturbance input $d$ (i.e. $W_d$) to zero. For the designs, the same weighting functions $W_1, W_2, W_3$, and scalings (Table 7.1) as used by Yue [114] on the flight simulator at DRA, Bedford, were incorporated. These were:

\[
W_1 = \text{diag} \left\{ \frac{s+12}{(s+0.012)}, \frac{s+2.81}{(s+0.005)}, \frac{s+2.81}{(s+0.005)}, \frac{s+10}{(s+0.01)}, \frac{0.89}{(s+0.005)}, \frac{0.89}{(s+0.005)}, \frac{s+12}{(s+0.012)} \right\} \quad (7.10)
\]
\[ W_2 = \text{diag}\left\{0.5 \frac{(s+0.0001)}{(s+10)}, 0.5 \frac{(s+0.0001)}{(s+10)}, 0.5 \frac{(s+0.0001)}{(s+10)}, 0.5 \frac{(s+0.0001)}{(s+10)}\right\} \]

(7.11)

\[ W_3 = \text{diag}\{1, 1, 1, 0.1, 0.1\} \]

(7.12)

and,

<table>
<thead>
<tr>
<th>Output</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H (y_1) )</td>
<td>10 Feet/second</td>
</tr>
<tr>
<td>( \Theta (y_2) )</td>
<td>0.2 Radians</td>
</tr>
<tr>
<td>( \Phi (y_3) )</td>
<td>0.5 Radians</td>
</tr>
<tr>
<td>( \Psi (y_4) )</td>
<td>0.5 Radians/second</td>
</tr>
<tr>
<td>( p (y_5) )</td>
<td>0.5 Radians/second</td>
</tr>
<tr>
<td>( q (y_6) )</td>
<td>0.2 Radians/second</td>
</tr>
</tbody>
</table>

Table 7.1: Output Scalings

The weight \( W_4 \) was selected to be of the form \( W_4 = \alpha I \), where \( \alpha \) was a simple scaling factor to emphasize disturbance rejection and was chosen for the final designs to be 30. Since \( G_d \) and \( W_4 \) are constant gain matrices then there is no increase in the order of the controller, as compared to the standard \( H_\infty \) mixed sensitivity design.

The final regulator framework (compare with equation 4.12, section 4.4, Chapter 4) used for the design was:

\[
\begin{bmatrix}
\dot{x} \\
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3 \\
y \\
e
\end{bmatrix} =
\begin{bmatrix}
A & 0 & 0 & 0 & 0 & B_3 & B \\
-B_1 C & A_1 & 0 & B_1 C_3 & B_1 D_3 & 0 & 0 \\
0 & 0 & A_2 & 0 & 0 & 0 & B_2 \\
0 & 0 & 0 & A_3 & B_4 & 0 & 0 \\
-D_1 C & C_1 & 0 & D_1 C_3 & D_1 D_3 & 0 & 0 \\
0 & 0 & C_2 & 0 & 0 & D_2 & 0 \\
-C & 0 & 0 & C_3 & D_3 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x \\
x_1 \\
x_2 \\
x_3 \\
y \\
e
\end{bmatrix}
\]

(7.13)

where \([A, B, C]_1, [A_1, B_1, C_1, D_1], [A_2, B_2, C_2, D_2], [A_3, B_3, C_3, D_3]\), are the state-space descriptions of the plant, weight \( W_1 \), weight \( W_2 \), and weight \( W_3 \) respectively, and \( B_p = B_d W_4 \).
7.2.3 Disturbance Rejection Design (based on 1 DOF $H_\infty$ LSDP Design)

The state space system equations 7.6 & 7.7 of section 7.2.1 with an extra turbulence input term can be expressed as two separate state space systems (Green and Limebeer [41]):

\[
\dot{x} = Ax + Bu \tag{7.14}
\]

\[
y = Cx \tag{7.15}
\]

\[
\dot{x}_d = Ax_d + B_d u_d \tag{7.16}
\]

\[
y_d = Cx_d \tag{7.17}
\]

Therefore, equations 7.14 & 7.15 represent the mapping of the control input $u$ to the output $y$, while equations 7.16 & 7.17 represent the mapping of the disturbance $u_d$ to the output $y_d$. It should be noted that the above two systems both have the same $A$ and $C$ matrices.

The normalized left coprime factor (NLCF) description ([66]) of the original (non-disturbed) system, equations 7.14 & 7.15, can be expressed as $M^{-1}N$ where $N$ is described by:

\[
\dot{x}_N = (A + HC)x_N + Bu \tag{7.18}
\]

\[
v = R^{-1/2}Cz_N \tag{7.19}
\]

Matrices $H$ and $R$ were defined in section 4.5. Also, $M$ is described by:

\[
x_M = (A + HC)x_M + Hy \tag{7.20}
\]

\[
u_t = R^{-1/2}Cz_M + R^{-1/2}y \tag{7.21}
\]

Because of the common $A$ and $C$ matrices (stated above), $M^{-1}N_d$ describes a left coprime factorization (not normalized) of the disturbance system (equations 7.16 & 7.17) where $N_d$ is formulated as:

\[
x_d = (A + HC)x_d + B_d u_d \tag{7.22}
\]
This is shown in block diagram form in Figure 7.6.

The above analysis was for a plant description which had not been loop shaped according to the LSDP described in section 4.5. The rest of this section assumes the LSDP has been carried out to form the shaped state space system \([A_s, B_s, C_s]\). The matrix \(B_s\) is defined similarly to equation 7.4, section 7.2.1, with extra selectively stacked zeros according to the number of states in the loop shaping weights. Therefore, in this case \(B_s = A_s(:, 6:8)\).

Operating the NLCF on the shaped plant \([A_s, B_s, C_s]\) combined with \(B_s\) results in the following generalised regulator framework (equation 7.24):

\[
\begin{bmatrix}
\dot{z}_s \\
u \\
y
\end{bmatrix} = \begin{bmatrix}
A_s & -H_sR_{s,4}^{1/2} & \alpha B_s \\
0 & 0 & 0 & 0 & I \\
C_s & R_{s,4}^{1/2} & 0 & 0 & d \\
C_s & R_{s,4}^{1/2} & 0 & 0 & u
\end{bmatrix} \begin{bmatrix}
z_s \\
\phi
\end{bmatrix}
\]

(7.24)

where \(\alpha\) again emphasizes disturbance rejection.

The design plant shaping weights, scalings and other design parameters were selected to be the same as the work of Walker ([100], [104], [106]). These were:

\[
W_1 = \text{diag} \left\{ \frac{s+5}{s}, \frac{s+5}{s}, \frac{s+5}{s}, \frac{s+5}{s} \right\}
\]

(7.25)

\[
W_2 = \text{diag} \{1,1,1,0.5,0.5\}
\]

(7.26)

Also used was an alignment frequency of 5 radians/second and output scalings according to Table (7.2):
### 7.2.4 Disturbance Rejection Design (based on 2 DOF $H_\infty$ LSDP Design)

A similar design procedure to the previous section 7.2.3 was followed. However, this time the model of how the turbulence enters the helicopter equations of motion (equation 7.22 & 7.23) was combined with the two degree of freedom loop shaping design procedure (section 4.6). The design configuration ([41]) is shown in Figure (7.7):

This results in the following generalised regulator framework (equation 7.27):

$$
\begin{bmatrix}
\dot{z}_s \\
\dot{z}_o \\
u \\
y \\
z \\
\beta \\
y
\end{bmatrix} =
\begin{bmatrix}
A_s & 0 & 0 & -H_s R_s^{1/2} & \alpha B_m & B_s \\
0 & A_o & \rho B_o & 0 & 0 & 0 \\
0 & 0 & 0 & R_s^{1/2} & 0 & D_s \\
0 & 0 & -\rho C_o & -\rho^2 D_o & \rho R_s^{1/2} & 0 & \rho D_s \\
0 & 0 & 0 & \rho I & 0 & 0 & 0 \\
0 & 0 & 0 & R_s^{1/2} & 0 & D_s
\end{bmatrix}
\begin{bmatrix}
z_s \\
z_0 \\
r \\
\phi \\
d \\
u
\end{bmatrix}
$$

Table 7.2: Output Scalings

<table>
<thead>
<tr>
<th>Output</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ ($y_1$)</td>
<td>10 Feet/second</td>
</tr>
<tr>
<td>$\Theta$ ($y_2$)</td>
<td>0.2 Radians</td>
</tr>
<tr>
<td>$\Phi$ ($y_3$)</td>
<td>0.2 Radians</td>
</tr>
<tr>
<td>$\Psi$ ($y_4$)</td>
<td>0.2 Radians/second</td>
</tr>
<tr>
<td>$p$ ($y_5$)</td>
<td>0.5 Radians/second</td>
</tr>
<tr>
<td>$q$ ($y_6$)</td>
<td>0.5 Radians/second</td>
</tr>
</tbody>
</table>

Figure 7.7: Disturbance Rejection Design Formulation

This results in the following generalised regulator framework (equation 7.27):
where $\alpha$ emphasizes disturbance rejection and $B_{e4}$ is calculated similarly to section 7.2.3.

The design plant shaping weights, scalings and other design parameters were selected to be the same as the controller tested on the AFS at DRA, Bedford (Walker and Postlethwaite [106]). These were:

$$\begin{align*}
W_1 &= \text{diag}\left\{\frac{s+5}{s}, \frac{s+5}{s}, \frac{s+5}{s}, \frac{s+5}{s}\right\} \\
W_2 &= \text{diag}\{1,1,0.5,0.5\}
\end{align*}$$

with an alignment frequency of 5 radians/second and output scalings according to Table (7.2) from the preceding section 7.2.3.
7.3 Frequency and Time Domain Simulation Results

7.3.1 $H_{\infty}$ Mixed Sensitivity Based Controller Designs

Frequency Domain Results

The following Bode frequency plots demonstrate that the qualities of the $H_{\infty}$ mixed sensitivity design are conserved in the new disturbance rejection design.

The sensitivity plot ($(I + GK)^{-1}$) of the mixed sensitivity design indicates the ability of the system to minimize the effects of an additive disturbance on the plant output.
The insensitivity capabilities are shown to be maintained through the use of the disturbance rejection formulation (Figures (7.8, 7.9)), where the four singular values which are small at low frequencies correspond to the controlled outputs and the other two are a result of inherent interaction between the aircraft attitudes and angular rates. The frequency weighting functions $W_1$ and $W_2$ had been kept identical in both designs.

The Bode plot of $(K(I + GK)^{-1})$ of the mixed sensitivity design (Figure (7.10)) indicated a significantly lower bandwidth compared to the disturbance rejection design (Figure (7.11)).

Figure 7.10: Mixed Sensitivity Design $(K(I + GK)^{-1})$

Figure 7.11: Disturbance Rejection Design $(K(I + GK)^{-1})$
When the robustness to sensor noise plots \((GK(I+GK)^{-1})\) are compared (Figures (7.12, 7.13)), then negligible degradation is seen.

![Figure 7.12: Mixed Sensitivity Design \((GK(I+GK)^{-1})\)](image)

![Figure 7.13: Disturbance Rejection Design \((GK(I+GK)^{-1})\)](image)

**Non-linear Simulation Results**

The nonlinear model was trimmed at the 20 knot forward flight speed, and the effect of the turbulence on the four controlled outputs observed. New designs were tested with turbulence of varying parameters and correspondingly changing characteristics. Also, the trimmed height was selected to be 100 feet, which must be noted, since the RHM includes the information that the scale lengths of the velocity components are shortened as altitude increases. The following plots (Figure (7.14)) show one such gust that was to have its effect on the aircraft minimized. It must be noted that the turbulence profile below was generated and injected into the helicopter model's
nonlinear equations of motion automatically by the RHM, and that this plot of the velocity components in the three ‘x, y, z’ directions was simply a record of these signals.

![Figure 7.14: Velocity Components of Turbulence (15 secs)](image)

Figure 7.14: Velocity Components of Turbulence (15 secs)

The injection of a gust into the helicopter model’s equations of motion was automatically carried out by the RHM, and the output response calculated (Figures 7.15, 7.16). The controller in this case was being required to keep the aircraft flying forward at twenty knots.

![Figure 7.15: Mixed Sensitivity Design Output Responses (15 secs)](image)

Figure 7.15: Mixed Sensitivity Design Output Responses (15 secs)

The disturbance rejection designed controller practically halved the turbulence effect on heave velocity, pitch attitude and roll attitude. There was only a small change in the effect on heading rate. These figures show that in all of the four main axes the
helicopter would be easier to control for the pilot in the presence of turbulence.

![Graphs showing disturbance rejection design output responses](image)

**Figure 7.16: Disturbance Rejection Design Output Responses (15 secs)**

### 7.3.2 1 DOF $H_\infty$ LSDP Based Controller Designs

#### Frequency Domain Results

The same selection of Bode frequency graphs were used for analysis of the one degree of freedom loop shaping design procedure controller and the disturbance rejection controller. The shaped plant weighting functions $W_1$ and $W_2$ and output scalings had consistently been kept identical for both designs.

![LSDP Design graph](image)

**Figure 7.17: LSDP Design $((I+GK)^{-1})$**
The insensitivity capabilities to additive disturbances at the system output are shown to be maintained through the use of the disturbance rejection formulation (Figures (7.17, 7.18)).
The Bode plots of $K(I + GK)^{-1}$ show the LSDP design with lower bandwidth than the disturbance rejection design (Figures (7.19, 7.20)), and no significant change in the robustness to sensor noise plots, i.e. $GK(I + GK)^{-1}$ was demonstrated (Figures (7.21, 7.22)).

![Figure 7.21: LSDP Design ($GK(I + GK)^{-1}$)](image1)

![Figure 7.22: Disturbance Rejection Design ($GK(I + GK)^{-1}$)](image2)

**Non-linear Simulation Results**

As with the testing of the mixed sensitivity designs in the previous section, the non-linear model was trimmed at the 20 knot forward flight speed, and the effect of the turbulence (Figure (7.23)) on the four controlled outputs observed. Again, the controller was being required to keep the aircraft flying forward at twenty knots. The
main reductions in the turbulence effects were seen in the pitch attitude and heave velocity, as illustrated in (Figures 7.24, 7.25)).

Figure 7.23: Velocity Components of Turbulence (15 secs)

Figure 7.24: LSDP Design Output Responses (15 secs)

Figure 7.25: Disturbance Rejection Design Output Responses (15 secs)
7.3.3 2 DOF $H_{\infty}$ LSDP Based Controller Designs

The following plots show the turbulence velocity components (Figure (7.26)) and the associated responses (Figure (7.27)) of the two degree of freedom controller (from [106]).

As with the previous sections 7.3.2 & 7.3.1, the nonlinear model was trimmed at 20 knots, with the controller required to maintain this forward speed in the face of turbulence.
7.4 Variable Structure Controller Turbulence Handling Qualities

7.4.1 Non-linear Simulation Results

The four final variable structure controllers designs of Chapters 5 & 6, which had been extensively tested for their handling qualities and performance, were tested for their turbulence handling qualities. The output responses for this section are shown with regard to turbulence which had the same generating parameters as for the testing of the solely $H_\infty$ designed controllers of the previous sections 7.3.1, 7.3.2 & 7.3.3. The profile of the turbulence was, in each case below, essentially the same to Figure (7.14) in section 7.3.1, which is unsurprising since the calculation of the gust component values comes primarily from the defining parameters rather than from the aircraft motion.

The first two sets of plots are for the variable structure controllers with ideal models and state observers designed using 1 DOF (Figure (7.28)) and 2 DOF $H_\infty$ methods (Figure (7.29)).

![Figure 7.28: Variable Structure Controller (1 DOF $H_\infty$ designed ideal model)](image_url)
The next set of plots are for the coupled variable structure controllers/variable structure observers of section 6 where the ideal models have been designed using 1 DOF (Figure (7.30)) and 2 DOF $H_{\infty}$ methods (Figure (7.31)).
To help give more insight as to the workings of the VS controller, the following plot shows the switching states (Figure 7.32), which gives a measure of how close the error states are to the ideal switching surface. The fact that the switching states are all at small values shows that the controller is operating at near the specified performance. This specified performance depends on the $\rho$, $\delta$, and ideal model characteristics chosen at the design stage.

![Figure 7.32: Variable Structure Controller Switching States](image)

### 7.5 Comparisons of Controller Turbulence Rejection Qualities

Since each of the tested controllers in this chapter were simulated with the same statistically defined turbulence characteristics, they can be directly compared. The one DOF LSDP based turbulence rejection design (Figure (7.25)) performed consistently better in all four channels than the $H_\infty$ mixed sensitivity based disturbance design (Figure (7.16)). However, both these were superseded by the two DOF $H_\infty$ controller (Figure (7.16)), which gave significantly better turbulence rejection qualities, especially in the pitch channel. The variable structure (VS) controller (with a 1 DOF $H_\infty$ designed observer and ideal model) performed better than the 2 DOF $H_\infty$ controller in the pitch and roll channels, but worse in the heave and yaw axes. The VS controller (with a 2 DOF $H_\infty$ designed observer and ideal model, Figure (7.29)) exhibited worse turbulence characteristics than the VS controller (with a 1 DOF $H_\infty$ designed observer and ideal model, Figure (7.28)). Similarly, the VS controller/VS observer (with a 2 DOF
$H_\infty$ designed ideal model, Figure (7.31)) performed worse than the VS controller/VS observer (with a 1 DOF $H_\infty$ designed ideal model, Figure (7.30)).

7.6 Conclusion

Significant improvements to the turbulence rejection capabilities of the $H_\infty$ mixed sensitivity based control system design method have been demonstrated. Therefore, the procedure for modelling the effect of the gust perturbations was successful in representing the complex process involved in the RHM. When applying the same modelling procedure, together with Normalized Left Coprime Factor (NLCF) methods, improvements were also shown for the one DOF $H_\infty$ LSDP based design method. However, no changes were seen in the outputs when carrying out the same procedures with a two DOF $H_\infty$ LSDP based design formulation.

The turbulence handling qualities of the variable structure controllers of the previous two Chapters 5 and 6 were investigated, and the VS controller/VS observer (with a 1 DOF $H_\infty$ designed ideal model) performed, on average, better than all the other controllers (including $H_\infty$) tested in this Chapter. This VS controller (likewise with the other VS controllers) exhibited turbulence rejection qualities without having been designed with an explicit turbulence representation.
Chapter 8

Conclusions

8.1 Concluding Remarks

This thesis has presented the design of a full authority variable structure (VS) flight control system for a Lynx helicopter, with time domain and handling quality testing, and simulation on a ground-based flight simulator. Also, design procedures to enable $H_\infty$ controllers to take into account atmospheric turbulence effects have been formulated and demonstrated by nonlinear simulation. The following are the major findings and accomplishments of this thesis:

• Successful implementation and ‘flight’ on a ground-based Advanced Flight Simulator (AFS at DRA, Bedford) of a VS controller.

• Analysis of a literature-established 1 Degree of Freedom (DOF) $H_\infty$ LSDP controller showed the handling and time response qualities degrade significantly at higher speeds.

• Combining the 1 DOF $H_\infty$ LSDP procedures (to design an ‘ideal’ model) with a VS controller enabled the handling and time response qualities to be maintained at the higher speeds.

• Extensive analysis of a literature-established and previously AFS tested 2 DOF $H_\infty$ LSDP controller showed high levels of handling qualities and step responses, which were a significant improvement upon the 1 DOF $H_\infty$ LSDP controller.
- The VS controller combined with the 2 DOF $H_\infty$ LSDP methods also gave satisfactory high levels of handling qualities.

- The VS controllers maintained the highest level of agility handling qualities, while the $H_\infty$ LSDP controllers transgressed into a lower level at the highest speeds. This VS controller performance was at the expense of increased actuator activity.

- The nonlinear component of the VS controller was verified as being fundamental to the overall operation.

- A disadvantage of the chosen model-reference VS control system design method was that the ideal model had to be of the same order as the shaped plant, which was larger than would normally be used in the standard $H_\infty$ LSDP.

- The model-reference methodology was indicated as being important in giving time response objectives away from the design operating point.

- All the VS and $H_\infty$ LSDP controllers examined satisfied the highest level of inter-axis decoupling handling quality requirements.

- A VS observer was designed to complete the VS controller/VS observer framework, and resulted in high level handling qualities and step responses throughout the speed range 0 knots to 120 knots.

- This VS controller/observer was shown to have good turbulence rejection qualities.

- The combination of turbulence information with $H_\infty$ mixed sensitivity design and the 1 DOF $H_\infty$ LSDP resulted in $H_\infty$ controllers with enhanced turbulence rejection capabilities. This was not the case with the 2 DOF $H_\infty$ LSDP controller.

- The VS controllers showed turbulence handling qualities comparable to the $H_\infty$ responses, even though they had not been explicitly designed with regard to turbulence effects. This demonstrated the robustness aspects of the VS controllers in rejecting disturbances.
As a consequence of the above list of conclusions, the understanding of variable structure control system design in the context of helicopter control has been significantly extended. Also demonstrated was the suitability of this design method to the helicopter stabilization and performance maintenance throughout the operating envelope. The combining of VS control system design with the 'loop shaping' and observer design resulting from 1 DOF and 2 DOF $H_\infty$ procedures has been shown to be a viable and successful technique. The VS observer has also been shown to be practically and successfully applicable to the helicopter rigid body state estimation problem. The enhancement of the turbulence rejection qualities of established $H_\infty$ controllers enables the practical control system designer to have the capability to reduce pilot workload.

8.2 Recommendations for Future Research and Considerations

- The next major step towards verifying such advanced control laws would be the evaluation of the controllers designed in this thesis on a real flying Active Control Technology helicopter. This would provide for a series of exciting and challenging practical issues which have not been considered explicitly in this thesis: actuator anti-windup schemes, vibration problems, further nonlinearities, sensor failures (fault diagnosis), sensor noise, etc.

- Important to this above step is consideration of the effect of time delays (for example, in sensor information) on the VS controller. Also, this thesis used continuous design methodologies, whereas for implementation on the helicopters digital computers a discrete design procedure based on a discrete plant representation will be more applicable ([113]).

- Incorporating the turbulence modelling, as affecting the helicopter's equation of motion through the velocity states, could also be extended into the discrete $H_\infty$ design procedures.

- The design procedure for explicitly incorporating the turbulence information is generic, and can easily be extended to encompass the possibility of considering
the turbulence as affecting the systems rate states as well as, or, instead of, the velocity states.

- This thesis considered the use of an attitude-command/attitude-hold (ACAH) system throughout the whole flight envelope to demonstrate the feasibility of using a VS controller in such a way. In fact, it is most likely that a rate-command/attitude-hold (RCAH) would be required for the higher speeds, and the impact on the VS controlled system of such a switch between command types would have to be investigated.

- The size, in terms of number of states, of control system designs is often an issue when implementing in the real situation (may be less of an issue in the future with increased technology and computing power and speed). Therefore, the question of model-reduction should be addressed. Also, the ideal models designed in this thesis could not be reduced in size due to all the states being required and compared to the helicopter's states to form the error vector. The issue of the size of the ideal model would be addressed by considering the error vector as being the difference between the helicopter output vector and the ideal model output vector. Therefore, the output error vector would form part of the sliding surface, and not all the error states (only the error outputs) would have to become zero, which is more desirable.

- The area of μ-analysis and synthesis (Balas et al [4]) has yet to be applied to the design of a helicopter control system, and has the potential to improve on present $H_{\infty}$ controllers by using knowledge of the uncertainty structures. This could also include the use of the Linear Matrix Inequalities (LMI) routines.
References


[91] Smith, P.R., 1990, 'Application of eigenstructure assignment to the control of powered lift combat aircraft', *D.Phil Thesis*, Southampton University, UK.


