INVERTER-FED INDUCTION MACHINE DYNAMICS

Thesis submitted to the University of Leicester
for the Degree of Doctor of Philosophy

by

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The accompanying thesis, "Inverter fed Induction Machine Dynamics" is submitted in support of an application for the degree of Doctor of Philosophy from the University of Leicester.

This work has not been submitted, in part or in whole, for another degree in this University, or for an award of a Degree or Diploma of any other Institution. The help received from others has been fully acknowledged in Section 1 of this thesis.

The work includes the following important points:

(1) Two novel methods of analogue simulation derived by the author of inverter fed induction machine are described and are shown capable of predicting the steady state and transient behaviour of a real system.

(2) An established method of predicting the steady-state behaviour is shown to be incomplete and is improved.

(3) An established method of predicting the stability of the system is shown to be inaccurate under certain circumstances.

A paper describing the analogue simulation of the system has been accepted for publication (1.1) as has a paper describing the traction system used for comparison purposes (3.1).

I hereby declare that the statements in this Memorandum are true in all particulars.

M. Lockwood
Senior Scientific Officer
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Technical Centre
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JUNE 1978
The study includes the analysis and investigation of inverter-fed squirrel cage induction machine drives. The particular drive used was a 120° square wave inverter feeding a Tubular Axle Induction Motor developed for rail traction by British Railways.

The system and its operating modes, including self-excited braking, are described. The conditions under which self-excited braking can be achieved are investigated both theoretically and experimentally and upper and lower limits to the range of permissible rotor speeds are found by several analytical methods.

An original analogue model of the inverter is developed which is suitable for the investigation of inverter firing algorithms. A simpler and more efficient model is developed for the investigation of commonly used inverters.

A two-axis model of the induction machine is described and used to produce an analogue simulation.

State-variable analysis is used to predict the steady-state waveforms and the transfer functions of the system.

A simple method of predicting the frequency response of a linear or linearised system is described. Steady-state sinusoidal analysis is used to predict the limits to self-excitation.

Results from the various methods of analysis are compared with each other and with results from the real system in both the transient and steady-state modes of operation. The results from the analogue model are found to give best agreement with those from the real system in both modes.

Inverter losses are found to affect the boundaries of self-excitation.

The possibility of using the analogue model in the development of micro-processor control for the traction system is discussed.
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<td>thyristor x</td>
</tr>
<tr>
<td>Dx</td>
<td>diode x</td>
</tr>
<tr>
<td>T x F</td>
<td>logical trigger signal for THx</td>
</tr>
<tr>
<td>T x O</td>
<td>logical forced commutation signal for THx</td>
</tr>
<tr>
<td>T x S</td>
<td>logical SET signal for a thyristor equivalent bistable</td>
</tr>
<tr>
<td>T x R</td>
<td>logical RESET signal for a thyristor equivalent bistable</td>
</tr>
<tr>
<td>I</td>
<td>logical direction indicator for i</td>
</tr>
<tr>
<td>H</td>
<td>logical absolute magnitude indicator for i</td>
</tr>
<tr>
<td>V_p</td>
<td>logical indicator for v = v_d</td>
</tr>
<tr>
<td>V_ocl</td>
<td>logical indicator for v = v_{oc} limit</td>
</tr>
<tr>
<td>V_N</td>
<td>logical indicator for v = 0</td>
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<tr>
<td>FAULT</td>
<td>logical indicator for dc-link short circuit</td>
</tr>
<tr>
<td>Q</td>
<td>logical indicator for wave form type</td>
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<td>R_s</td>
<td>Resistance of one stator phase</td>
</tr>
<tr>
<td>R_r</td>
<td>Resistance of one rotor phase</td>
</tr>
<tr>
<td>R_L</td>
<td>dc-link braking resistance</td>
</tr>
<tr>
<td>R_B</td>
<td>Equivalent ac braking resistance</td>
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<tr>
<td>X_{ss}</td>
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</tr>
<tr>
<td>X_{rr}</td>
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<td>X_{sm}</td>
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</tr>
<tr>
<td>X_{rm}</td>
<td>Mutual Reactance between two rotor phases at base frequency</td>
</tr>
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</table>
\( X_{mm} \)  
Peak value of the mutual reactance between stator and rotor phases measured at base frequency

\( \alpha \)  
Angle between arbitrary reference frame q axis and stator a phase

\( \beta \)  
Angle between arbitrary reference frame q axis and rotor a phase

\( \phi \)  
Angle between rotor and stator a phases

\[ X_s = X_{ss} - X_{sm} \]  
Apparent three phase stator self reactance at base frequency

\[ X_r = X_{rr} - X_{rm} \]  
Apparent three phase rotor self reactance at base frequency

\[ X_m = 1.5 X_{mm} \]  
Apparent three phase mutual reactance at base frequency

\[ X_{so} = X_{ss} + 2 X_{sm} \]  
Apparent three phase stator zero sequence reactance at base frequency

\[ X_{ro} = X_{rr} + 2 X_{rm} \]  
Apparent three phase rotor zero sequence reactance at base frequency

\( L_s \)  
Apparent three-phase stator inductance

\( L_r \)  
Apparent three-phase rotor inductance

\( M \)  
Apparent three-phase mutual inductance

\( C \)  
dc-link capacitor

\( L \)  
dc-link inductor

\( A \)  
\( B \)  
Coefficients of the inverse inductance matrix
\( T_e \)  
*instantaneous electromagnetic torque*

\( PP \)  
*pole pairs*

\( \omega_b \)  
*frequency at which the reactances are measured.*  
*(Base frequency)*

\( \omega_e \)  
*angular frequency of the machine supply*

\( \omega_r \)  
*rotational frequency of the machine rotor*

\( \omega_s = \omega_e - \omega_r \)  
*Rotor angular slip frequency*

\( \bar{H} \)  
*A bar above a logical variable indicates the logical inverse: NOT(H)*

\( \bar{A} \)  
*A bar above a non-logical term indicates the use of matrix notation*

\( i \)  
*Instantaneous current*

\( i_c \)  
*Instantaneous filter capacitor current*

\( i_d \)  
*Instantaneous dc-link current*

\( i_s \)  
*Instantaneous dc-source current*

\( i_{\text{hold}} \)  
*Thyristor/diode threshold current. If the device current is below \( i_{\text{hold}} \) the device is assumed to be open circuit*

\( v \)  
*Instantaneous voltage*

\( v_d \)  
*Instantaneous dc-link voltage*

\( v_s \)  
*Instantaneous dc-source voltage*

\( v_{oc} \)  
*Open-circuit e.m.f. due to inverter load*

\( v_{oc}^{\text{limit}} \)  
*Open-circuit e.m.f. due to load clamped by the inverter input voltage \( v_d \)*

\( v_{AB} \)  
*Instantaneous potential difference between points A and B terminals.*

\( i_s' \)  
*Instantaneous dc-source current referred to the motor terminals.*
$\psi$ Instantaneous phase flux-linkages

$\phi$ Angle in cycle at which an inverter phase current becomes zero

$\Theta$ Angle in a cycle at which an inverter phase open circuit emf exceeds the dc-link voltage

$s, r$ Suffices indicating stator and rotor quantities respectively

$q, d$ Suffices indicating q and d axes respectively

$R, S, T$ Suffices indicating inverter output (and motor input) terminals

$oc$ Suffix indicating open circuit emf

$P$ Suffix indicating inverter dc-link positive rail

$N$ Suffix indicating inverter dc-link negative rail

$O$ Suffix indicating star point of inverter load
Symbols used in state variable analysis

\( \mathbf{A} \)  System matrix

\( \mathbf{B} \)  Input matrix

\( \mathbf{C} \)  Output matrix

\( \mathbf{D} \)  Direct coupling matrix

\( \mathbf{x} \)  Vector of state variables

\( \dot{\mathbf{x}} \)  Differential of \( \mathbf{x} \) with respect to time

\( \mathbf{u} \)  Input vector

\( \mathbf{y} \)  Output vector

\( \mathbf{b} \)  Input column vector

\( \mathbf{c} \)  Output column vector

\( d \)  Direct coupling scalar

\( u \)  Input scalar

\( y \)  Output scalar

\( \mathbf{P} \)  Non-singular transformation matrix

\( \mathbf{I} \)  Unit diagonal matrix

\( \exp{\mathbf{A} t} \)  Matrix exponentiation function

\[
\exp{\mathbf{A} t} = \mathbf{I} + \mathbf{A} t + \frac{\mathbf{A}^2 t^2}{2!} + \frac{\mathbf{A}^3 t^3}{3!} + \ldots
\]

\( \lambda(t) \)  Matrix convolution integral

\[
\lambda(t) = \mathbf{I} t + \mathbf{A} \frac{t^2}{2!} + \frac{\mathbf{A}^2 t^3}{3!} + \ldots
\]
1.4 ACKNOWLEDGEMENTS

The author wishes to thank Mr. J. McKay, Head of the School of Electrical and Electronic Engineering, Leicester Polytechnic for support in the early part of this work.

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CHAPTER 2

INTRODUCTION
2. INTRODUCTION

The development of high power thyristor and transistor inverters has opened many possibilities in the control of power electrical machinery. One of the most rewarding fields for the use of such inverters is in the production of variable speed squirrel cage induction motor drives.

The squirrel cage induction machine does not have a commutator or any sliding electrical contacts. This means that the machine requires little maintenance. The rotor construction can be made simple and robust and a high torque to mass ratio can be achieved. These characteristics are particularly useful in the rail traction environment where reliability and, in some cases, power to mass ratio are of prime importance. However to utilise these advantages over a wide speed range it is necessary to supply the machine from a variable frequency power source such as a three-phase inverter.

Inverter-fed squirrel cage induction machine systems can display instabilities under some circumstances. It is convenient to study these instabilities and other aspects of the system dynamic behaviour using computers.

It is the aim of this study to develop techniques for investigating the steady-state and dynamic behaviour of inverter-fed induction machines. Comparison between these techniques is made utilising a particular mode of instability. Five techniques are studied:

(1) Analogue simulation of inverter/induction machine systems including load commutation effects.
(2) Analogue simulation of inverter/induction machine systems in
which the thyristors have well defined periods of permissible conduction.

(3) Steady-state inverter waveform analysis implemented with the aid of a digital computer program.

(4) Transfer function analysis of the system linearised about a steady-state operating point.

(5) Steady-state limitations to self-excitation.

These techniques are developed for a particular inverter/induction machine traction system but they are valid for a much wider range of systems.
CHAPTER 3

SYSTEM STUDIED

3.1 THE MOTOR TYPE

3.1.1 Motoring Performance

3.1.2 Braking Performance

3.2 THE INVERTER TYPE

3.3 BRAKING RESISTOR

3.4 SYSTEM RATING

3.5 STATOR MECHANICAL DYNAMICS
3.0 **SYSTEM STUDIED**

The system studied was one under development, for rail traction purposes, by the Research Department of the British Railways Board.

The Advanced Electric Traction Equipment (3.1) project was created by B.R. Research Department in early 1974. The programme of the project involves the design and development of a tubular axle induction motor, Figs (3.1) and (3.2), and its interface with commercial power conditioning equipment.

3.1 **THE MOTOR TYPE**

Conventional electric traction uses d.c. series or separately-excited motors mounted on the bogie. The drive to the axle is via reduction gears. Considerable saving in bogie design and size can be made if the motor is incorporated into the wheel/axle assembly. However, d.c. motors can be ruled out for use within the axle because the commutator and brushes would be unlikely to survive the severe vibration and shock in the axle environment and brushes would be difficult to maintain. Similar considerations mitigate against the use of a gear box. Therefore, the only practical way of building an axle motor is to use some form of direct driving brushless motor.

Operating conditions limit unsprung mass, while gauge clearances constrain the shape and size of an axle motor. These limits make an "inside-out motor" (i.e. stator on the inside) most suitable to give the largest possible airgap diameter within the given physical envelope. Physical space limitations mitigated against the use of a brushlessly excited synchronous machine and so an inverter fed asynchronous squirrel cage induction machine was chosen as most suitable for this application.
The rotor (Fig 3.3), attached to the inside of the axle tube, rotates around the stator (Fig 3.4) and drives the wheelset directly. Fig 3.5 shows the overall mechanical arrangement of the TAIM schematically. Power to the stator is supplied via cables which pass through one main axle box bearing. The stator is torsionally restrained by a reaction torque tube which runs through the other axle box bearing. Cooling air is fed to the stator via this tube. After entering the stator at its mid-point, the air splits into two main paths. Approximately half the air is channelled along the back-iron of the stator and then over the stator end-windings. The rest of the air is channelled down the air-gap. The air is exhausted via holes in the axle tube (Fig 3.1). A flow rate of 0.19 m$^3$/s of air at 3 K Pascals is used. Gear boxes and transmission components are eliminated and, as the motor is supported directly by the rail, powered and unpowered bogies can be identical. The reduction in capital cost resulting from these mechanical simplifications should be sufficient to offset most or all of the inverter cost. The unsprung mass of the TAIM compares favourably with that of conventional powered axles and is only marginally heavier than conventional trailing axles.

The principal dimensions of the motor are given in Appendix A. Variants of the motor which have solid rotors or copper-sleeved rotors are currently under investigation.

3.1.1 Motoring Performance

The torque-speed characteristic of a squirrel cage induction motor operated on a constant-voltage fixed-frequency supply is shown in Fig (3.6). If the supply voltage is changed the torque (for a given speed) is scaled in proportion to the square of the voltage. A change of supply frequency modifies the torque speed characteristic.
FIG 3:4 TAIM STATOR ASSEMBLY
FIG 3.6 SQUIRREL CAGE INDUCTION MOTOR TORQUE SPEED CHARACTERISTIC
so that the zero torque condition occurs at a new synchronous speed. If the air gap flux in the machine can be maintained constant (to the first order by keeping voltage proportional to frequency) during this frequency change the general shape and peak torques of the torque speed curve will be unaffected. Thus for operation at constant torque with varying speed the induction motor's voltage to frequency ratio is kept approximately constant, while the supply frequency is varied. Figure 3.7 shows the torque/speed characteristics of the induction machine at several discrete supply frequencies and indicates how the induction machine can be made to produce a constant torque over a wide range of speeds.

The performance of an electric traction motor is usually constrained by a torque limit and a power limit. The torque limit is determined by track adhesion characteristics and by axle loadings, while power limit is determined by the onboard power conditioning equipment and by the power supply system.

These constraints produce a torque-speed envelope as shown in Fig. 3.8.

When an inverter fed induction motor is operated in the constant torque region, it is possible to maintain the flux at an optimal value. In this form of control, the terminal voltage of the motor rises approximately proportional to speed up to a limit imposed by either the supply voltage or the inverter device ratings. This constant flux-constant torque mode of operation demands a supply current maintained at the system limit to provide maximum torque. The slip frequency is also constant. Beyond the constant-torque speed limit, voltage constraints make it necessary to reduce the flux in the motor. Torque then falls inversely with the square of speed and the corresponding power proportionally to the inverse of speed. In this type of control,
FIG 3.7 CONSTANT TORQUE LOADING IN A VARIABLE FREQUENCY DRIVE
FIG 3.8 TORQUE / SPEED ENVELOPE FOR RAIL TRACTION
the current in the motor will fall as speed increases.

It is possible to maintain the current and power constant by increasing the slip frequency as speed increases. This has the effect of keeping the power constant and therefore the torque is proportional to the inverse of speed. The peak torque value of the torque-speed characteristic is the limiting point for this operation. At higher speeds, the current can no longer be maintained constant and the torque then begins to be proportional to the inverse of the square of speed.

In this type of operation the motor can be considered to be badly matched to the inverter in that its impedance is too high for the inverter rated voltage to drive the inverter rated current into the motor. A partial solution to this problem is to reconfigure the motor to give lower impedance. The method chosen for the system under discussion is to design the motor with phase windings in two equal parts. These are initially connected in series and are then re-connected in parallel to lower the motor impedance. This re-connection gives another region of constant torque (with current at its limit) and then another region of constant power where slip is varied (both current and voltage at their limits). Also as the parallel windings have a lower effective resistance, the power available for traction is increased. Thus there are five modes of motoring:

\[
\begin{align*}
(i) & \text{ Constant torque, constant current, constant slip frequency, voltage increasing - proportional to speed.} \\
& \text{Power increasing - proportional to speed.} \\
\end{align*}
\]

\[
\begin{align*}
(ii) & \text{ Constant voltage, constant current, constant power.} \\
& \text{Slip frequency increasing, torque decreasing - proportional to the inverse of speed.}
\end{align*}
\]
Coils in parallel

(iii) As (i)

(iv) As (ii)

(v) Constant voltage, constant slip frequency, current decreasing - proportional to the inverse of speed.

Power decreasing - proportional to the inverse of speed. Torque decreasing - proportional to the inverse of the square of speed.

The torque-speed envelope produced by these modes is shown in Fig 3.9. The associated variations in voltage and slip are shown in Fig 3.10.

3.1.2 Braking Performance

With suitable control, the inverter fed induction machine can operate as a generator and produce a retarding torque. However, system limits of torque, voltage and current imply that any regeneration through the inverter will be constrained into a torque-speed envelope similar to that for the motoring case (Fig 3.8). This will give low braking torques at high speed and high braking torques at low speed. The requirement for braking is high constant torque throughout the braking mode. This requirement can be met if the inverter is designed with a sufficiently high rating so that it can handle the total system power. However, this is an impractically expensive solution and for the system under discussion a cheaper alternative was chosen. The inverter is used to provide the magnetisation current demanded by the motor and the power generated is dissipated in resistance on the a.c. side of the inverter.

The phase winding sections are connected in parallel to allow adequate fluxing of the machine at high speeds and the inverter is operated at its current limit throughout braking. To achieve constant
FIG 3-10 VARIATION IN SLIP FREQUENCY AND MOTOR VOLTAGE WITH MOTOR SPEED
full braking torque the motor must run at approximately constant
current and constant flux, which implies that the terminal voltage of
the motor must be approximately proportional to speed. This further
implies that the braking resistance must vary approximately with speed.
In practice a stepped resistance was chosen as it is easier to implement
at these power levels than a continuously variable resistance.

The inverter power losses are provided by the machine and so the
braking integrity is independent of the electrical source. Fig 3.11
is a schematic diagram of the braking mode. The control of the
inverter in braking and the limits to braking are discussed further in
Chapter 8. Table I summarises the various operating modes of the
system.

3.2 THE INVERTER TYPE

The choice of a squirrel cage induction machine as the traction
motor precluded the use of a line-commutated inverter. Consequently
a force-commutated inverter was required. A commercially available
inverter of the ratings required was chosen. Paper studies and capital
cost considerations resulted in the choice of the 120° square wave
voltage source inverter manufactured by ASEA Ltd, Milton Keynes (formerly
Harland Simon Ltd). Figure 3.12 shows output waveforms of the inverter
when it is feeding the TAIM at 5% slip. An analysis of alternative
inverters was carried out.

The source for the inverter was a three-phase controlled
rectifier feeding an inductive-capacitive filter. A d.c. chopper is
under development for feeding the d.c. link of the inverter from a
fixed voltage d.c. source (Southern region third rail system) or
from a rectified single phase a.c. source (25 kV overhead system).
<table>
<thead>
<tr>
<th>Mode</th>
<th>Stator coil Connection</th>
<th>Voltage</th>
<th>Current</th>
<th>Slip frequency</th>
<th>Torque</th>
<th>Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Series</td>
<td>Increasing as speed increases</td>
<td>Constant</td>
<td>Constant</td>
<td>Constant</td>
<td>Proportional to speed</td>
</tr>
<tr>
<td>(ii)</td>
<td>Series</td>
<td>Constant</td>
<td>Constant</td>
<td>Increasing as speed increases</td>
<td>Inversely proportional to speed</td>
<td>Constant</td>
</tr>
<tr>
<td>(iii)</td>
<td>Parallel</td>
<td>Increasing as speed increases</td>
<td>Constant</td>
<td>Constant</td>
<td>Constant</td>
<td>Proportional to speed</td>
</tr>
<tr>
<td>(iv)</td>
<td>Parallel</td>
<td>Constant</td>
<td>Constant</td>
<td>Increasing as speed increases</td>
<td>Inversely proportional to speed</td>
<td>Constant</td>
</tr>
<tr>
<td>(v)</td>
<td>Parallel</td>
<td>Constant</td>
<td>Decreasing as speed increases</td>
<td>Constant</td>
<td>Inversely proportional to speed</td>
<td>Inversely proportional to speed</td>
</tr>
<tr>
<td>Braking</td>
<td>Parallel</td>
<td>Decreasing as speed decreases</td>
<td>Constant</td>
<td>Constant</td>
<td>Constant</td>
<td>Proportional to speed</td>
</tr>
</tbody>
</table>
FIG 3.11 SCHEMATIC DIAGRAM OF BRAKING MODE.
FIG 3-12 INVERTER OUTPUT WAVEFORMS AT 5° SLIP
3.3 BRAKING RESISTOR

The braking resistance is required to be approximately proportional to speed. Discrete resistance steps were chosen as the most economical solution to that requirement. Costing factors also determined that the resistance should be a rectifier-fed single resistor chain having six elements. Five of these elements can be shorted out by parallel-connected thyristors.

3.4 SYSTEM RATING

The basic system was designed to give a maximum torque of 4.125 kNm with a wheel diameter of 0.75 m. An inverter rating of 200 kVA gives a speed of 43 kph at the transition point between constant torque and constant power (knee point). The system has a designed top speed of 200 kph. This choice of top speed constrained the mass of the axle motor to be less than 2.2 tonnes. The final mass of the motor is approximately 2 tonnes. The braking requirement is a torque of 4.125 kNm throughout the speed range giving a peak power of 610 kW.

3.5 STATOR MECHANICAL DYNAMICS

The stator assembly of glued laminations, windings and support member is restrained from rotating by a torque reaction member called the torque tube. This, as its name implies, is a thin walled tube. The stator and torque tube thus form a torsional mass/spring system which has very little inherent damping. This type of system exhibits resonant behaviour with large gains in the region of the resonant
frequency. Design calculations predicted that the resonant frequency would be in the order of 45 Hz with a damping coefficient estimated to be less than 1%. The non-sinusoidal nature of the inverter output waveforms cause a torque ripple or pulsation whose fundamental component has a frequency which is a sixth harmonic of the inverter frequency. For the system considered the inverter frequency sweeps from 3 Hz to 75 Hz as the train accelerates from rest to 200 kph. This implies that the torque pulsation frequency sweeps from 18 Hz to 450 Hz and so passes through the resonant frequency of the stator/torque tube system. If no remedial action were taken the amplification effect of exciting at the resonant frequency would produce destructively high torques in the torque tube. The remedial action chosen in the system studied was to put a resilient coupling between the torque tube and mechanical earth to lower the resonant frequency to 12 Hz and so avoid the range of frequencies excited by the torque pulsations. The resilient coupling chosen utilised rubber bushes and so included appreciable damping giving a low amplification at resonance.

Unfortunately the remedial action described had the side effect of making it difficult to measure the dynamic electrical torque of the machine. This was because the inertia of the stator absorbed the torque pulsations and did not allow them to be measured by the torque transducer which was a strain gauge assembly mounted on the torque tube.
CHAPTER 4

ANALYSIS OF INVERTERS

4.1 ANALYSIS OF INVERTER WITH LOAD-INDUCED COMMUTATION

4.1.1 The bistable model of the thyristor

4.1.2 Inverter model

4.1.3 Determination of dc-link current

4.1.4 Fault conditions

4.1.5 Firing information

4.1.6 Model Implementation - Inverter with load-induced commutation

4.2 ANALYSIS OF THE INVERTER WITH DEFINED FIRING INFORMATION

4.2.1 Inverter model

4.2.2 Determination of dc-link current

4.2.3 Fault conditions

4.2.4 Firing information

4.2.5 Model Implementation - Inverter with defined firing information

4.3 INVERTER CONTROL LOGIC

4.4 DC-LINK SIMULATION INCLUDING BRAKING RESISTOR

4.5 COMPARISON OF WAVEFORMS
4.0 ANALYSIS OF INVERTERS

There are various types of voltage source inverter (V.S.I.) presently realisable and they can be classified and described in the following manner.

(i) **Line-commutated inverters**

In line-commutated inverters the load, usually a synchronous machine, provides all the commutation capability of the inverter. The inverter in this form is basically a phase-controlled rectifier. This type of inverter draws reactive power from the ac terminals.

(ii) **Fixed dc-link inverters or Pulse Width Modulated Inverters (P.W.M.)**

In P.W.M. inverters the output voltage amplitude is controlled by varying the width of the output pulses. The output waveform can be synthesised in the form of a square-wave or a sine wave.

(iii) **Variable dc-link inverters**

In variable dc-link inverters the output voltage amplitude is controlled by the amplitude of the dc input voltage to the inverter. This is varied by the input control element which can be a dc chopper or phase-controlled rectifier.

Variable dc-link inverters can be further sub-divided into classes as follows:

(a) **Those with auxiliary commutation thyristors.** These are defined as "vertically commutated" or 180° - square wave inverters.

(b) **Those without auxiliary commutation thyristors.** These are defined as "horizontally commutated" or 120° - square wave inverters.
The basic circuit analysed in this section is the voltage-source inverter shown in Fig 4.1. This circuit is capable of operating in modes (ii) and (iii) above, but it does not represent the line-commutated inverter.

One phase of the 3-phase voltage-source inverter can be represented as shown in Fig 4.2. In this case, the output waveform depends upon the firing and commutation information supplied to it and upon the capability of the associated commutation circuitry. Two output waveforms, the 180° square wave and the 120° square wave, were studied but the models produced are valid for others including P.W.M.

In the case of the 180° square-wave inverter the permissible period of conduction of each thyristor is half a cycle or 180° and so the inverter output voltage is determined only by the dc-link voltage and the firing information. The voltage regulation effects produced by the load and the reflected effects of the dc-link source impedance are second order effects in the inverter circuitry. In the case of the 120° square-wave inverter the permissible period of conduction of each thyristor is one third of a cycle or 120° and so the inverter output has two periods of 60° in each output cycle when both thyristors in a limb are off. Under these conditions the output voltage is very dependant upon the nature of the load and in cases where the current falls to zero during such periods the output voltage of the inverter becomes the open-circuit emf of the load.

The assumptions made with respect to the inverters are:

(i) that thyristors have negligible leakage current in the non-conducting states and negligible forward voltage drop in conduction,

(ii) that diodes have negligible reverse leakage current and negligible forward voltage drop, and
FIG 4.2 INVERTER CIRCUIT FOR 1 PHASE OF OUTPUT
(iii) that the effects on the load of commutation transients and inter-phase reactors are negligible. The actual implementation of the commutation circuitry is therefore not included in the models.

Under some circumstances the load is capable of commutating the inverter. This could be the case where the load is a synchronous machine. The simulation of an inverter with such a load involves making the inverter model capable of accepting such load commutation. Such a model is described in Section 4.1. Under other circumstances the effects of load induced commutation are not noticeable or not present as the thyristors have clearly defined periods of permissible conduction and have their gates excited during such periods. With such a system a much simpler and more efficient model can be developed and this is described in Section 4.2.

Previous studies of systems where load induced commutation is possible have modelled the thyristors individually. This approach is inherently inefficient because it fails to take advantage of the operational constraints set by the circuit geometry. Beattie et al (Refs. 4.1-4.4) developed a model of the thyristor using two bistables and four logic gates. The modelling of the six thyristor and six diode inverter in this study would therefore be very inefficient in terms of the number of computational elements used.

Several authors (Refs 4.5-4.8) have derived models of the 180° square wave inverter. Mayer and Lipo (Ref 4.9) developed a model of a P.W.M. inverter but ignored the possibility of discontinuous output currents.

De Carli and Raimondi (Ref 4.10) developed a hybrid model which permits the output current to become zero for short periods but they
erroneously assumed that the inverter output voltage would also be zero. Their model used 37 logic gates and required a further 15 gates and 3 monostable circuits to overcome a problem in zero current detection. The model developed in this study (Section 4.2) uses 16 logic gates and avoids that problem.

Buchner (Ref 4.11) describes an analogue computer model of a 180° square-wave inverter which uses 11 switching elements whereas the model described in Section 4.2 needs only 9 and is capable of representing both types of square wave inverter. Buchner also states that the model can be modified for the case of the 120° inverter but does not describe how this is achieved. The results from this model are not very accurate. Inspection of the results given shows that the model inverter output current rises to 30% of peak value during the period for which it should remain zero.

4.1 ANALYSIS OF THE INVERTER WITH LOAD-INDUCED COMMUTATION

4.1.1 The bistable model of the thyristor

The behaviour of the thyristor is a function of the history of its inputs; in other words the thyristor is a memory device. The thyristor begins to conduct when it is forward biased and a gate signal is maintained until the current through the device exceeds \( i_{\text{hold}} \). The thyristor then continues to conduct until the current through it falls below \( i_{\text{hold}} \) either naturally or due to externally forced commutation. Thus the thyristor can be modelled by a SET/RESET bistable of the type whose truth table is given in Table II.
TABLE II  Truth Table for SET/RESET bistable

<table>
<thead>
<tr>
<th>SET</th>
<th>RESET</th>
<th>T_t</th>
<th>T(t + Δt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>X</td>
</tr>
</tbody>
</table>

The bistable is SET when the conditions for the initiation of conduction are satisfied and is RESET when the current falls below \( i_{\text{hold}} \) naturally or when a turn-off signal \( T_{x0} \) is present.

Referring to Fig 4.2 and using Boolean variables we find:

\[ T1S = T1F \cdot I_R \cdot H_R \]  \hspace{1cm} (4.1)

The term \( I_R \) indicates that for \( T1H \) to conduct, the inverter limb output current, \( i_R \), must be positive.

\[ T4S = T4F \cdot \overline{I}_R \cdot H_R \]  \hspace{1cm} (4.2)

The term \( \overline{I}_R \) indicates that for \( T4H \) to conduct \( i_R \) must be negative.

\[ T1R = T1F + \overline{H}_R + \overline{I}_R \]  \hspace{1cm} (4.3)

\( \overline{I}_R \) is included in the RESET term for \( T1H \) to ensure that the bistable is RESET if \( i_R \) goes rapidly negative. Inclusion of this term avoids the possible timing difficulties that might occur with short duration RESET signals.
Similarly
\[ T_{4R} = T_{4o} + \frac{H_R}{I_R} + I_R \]  \hspace{1cm} (4.4)

4.1.2 Inverter Model

From Fig 4.2 it can be seen that there are three possible states for the inverter limb output voltage \( v_{RN} \). They are:

(i) \( v_{RN} = v_d \)

(ii) \( v_{RN} = v_{RNoc} \)

(iii) \( v_{RN} = 0 \)

(i) \( v_{RN} = v_d \) when either TH1 or D1 is low impedance. TH1 is low impedance when \( i_R \) is positive and TH1 is conducting or is turning on. D1 is low impedance when \( i_R \) is negative and TH4 is not conducting nor being gated on.

Thus \( v_{RNp} = T_{1F} + T_1 + \frac{I_{R}}{T_{4F}} \cdot T_4 \) \hspace{1cm} (4.5)

(D1 is also low impedance when \( i_R \) is zero and the open-circuit emf, \( v_{RNoc} \), due to the load exceeds the dc-link voltage \( v_d \).

The method of including this condition is shown in (ii).)

(ii) \( v_{RN} = v_{RNoc} \) when no devices are conducting. The open-circuit emf of the load is impressed on the load model in the simulation so constraining the current in the load to remain zero. However, this is only true for \( v_{RNoc} \) between zero and \( v_d \). For excursions of \( v_{RNoc} \) outside this range the diodes D1 and D4 will clamp the load terminal voltage \( v_{RN} \) to \( v_d \) and zero respectively. This effect is included in the inverter model by introducing the variable \( \left[ v_{RNoc} \right] \) where:

\[ \text{limit} \]
\[
\begin{align*}
\text{\text{limit}} \begin{bmatrix} v_{\text{RNoc}} \end{bmatrix} &= 0 \quad \text{for } v_{\text{RNoc}} < 0, \text{ or} \\
&= v_{\text{RNoc}} \quad \text{for } 0 < v_{\text{RNoc}} < v_d, \text{ or} \\
&= v_d \quad \text{for } v_d < v_{\text{RNoc}}
\end{align*}
\]

The condition for \( v_{\text{RN}} = \begin{bmatrix} v_{\text{RNoc}} \end{bmatrix} \) is given by the following equation,

\[
v_{\text{RNocl}} = \overline{H_\text{R}} \cdot \overline{T1F} \cdot \overline{T4F}
\]

(iii) \( v_{\text{RN}} = 0 \) when \( i_{\text{RN}} \) is zero and \( v_{\text{RNoc}} \) is negative as shown above or when either \( \text{TH4} \) or \( \text{D4} \) is low impedance. \( \text{TH4} \) is low impedance when \( i_{\text{R}} \) is negative and \( \text{TH4} \) is conducting or being fired. \( \text{D4} \) is low impedance when \( i_{\text{R}} \) is positive and \( \text{TH1} \) is not conducting nor being fired,

i.e. \( V_{\text{RNN}} = T4F + T4 + \overline{I} \cdot \overline{T1F} \cdot \overline{T1} \)

Alternatively

\[
v_{\text{RNN}} = \overline{V_{\text{RNP}}} \cdot \overline{V_{\text{RNocl}}}
\]

4.1.3 Determination of dc-link current

Referring to Fig 4.2 it can be seen that when either \( \text{TH1} \) or \( \text{D1} \) conducts \( i_{\text{dR}} = i_{\text{R}} \). Under all other circumstances \( i_{\text{dR}} = 0 \). When \( \text{TH1} \) or \( \text{D1} \) conducts \( v_{\text{RN}} = v_d \) and \( V_{\text{RNP}} \) is true. Thus \( i_{\text{dR}} \) can be produced in the model using a switch controlled by \( V_{\text{RNP}} \). If required \( i_{\text{dR}} \) can then be used for the simulation of the dc-link filter elements.

4.1.4 Fault Conditions

If thyristors \( \text{TH1} \) and \( \text{TH4} \) begin to conduct simultaneously a "shoot-through" fault is said to have occurred. It was assumed in this study that the inverter would cease to operate in such an event, although
if necessary it is possible to include any protection circuitry in the model. The "shoot-through" condition can be expressed as:

\[ \text{FAULT}_R = (T1 + T1F)(T4 + T4F) \]  \hspace{1cm} (4.9)

The simulation was arranged so that \( \text{FAULT}_R \), being true, stopped the computation.

4.1.5 Firing information

The firing and turn-off information for the thyristors can be produced either by a computer sub-model or by linking the computer to actual inverter control logic. The production of firing information for this study is shown in Section 4.3.

The analysis shown in Section 4.1 is particularly useful for the study of the operation of inverter firing control algorithms.

4.1.6 Model Implementation - Inverter with load-induced commutation

The flow diagrams for the simulation of the analysis shown in Section 4.1 are given in Figs 4.3 to 4.6. Triplication of these gives the flow diagrams for the three phase inverter shown in Fig 4.1, assuming that the suffix \( R \) is replaced by \( S \) and then \( T \).

The total dc-link current, \( i_d \), is given by:

\[ i_d = i_{dR} + i_{dS} + i_{dT} \]  \hspace{1cm} (4.10)

and the general fault signal by:

\[ \text{FAULT} = \text{FAULT}_R + \text{FAULT}_S + \text{FAULT}_T \]  \hspace{1cm} (4.11)

When implementing equations 4.5, 4.6 and 4.9 it was found to be more efficient, in terms of gate utilisation, to use De Morgan's Theorem (Ref. 4.1) and to re-write the equations in the form:

\[ V_{\text{RNP}} = T1F + T1 + \frac{1}{T1 \cdot T2} + T4F + T4 \]  \hspace{1cm} (4.5a)
FIG 4-3 PRODUCTION OF IR AND HR TERMS
FIG 4.4. SIMULATION OF TH1 AND TH4
FIG 4-5. PRODUCTION OF $[\nu_{RNOC}]$ LIMIT TERM.
FIG 4.6. LOGIC AND SWITCHING IMPLEMENTATION
4.2 ANALYSIS OF THE INVERTER WITH DEFINED FIRING INFORMATION

4.2.1 Inverter Model

The usual mode of operation of force-commutated thyristor inverters involves clearly defined intervals of permissible conduction for the thyristors. The actual conduction period of a thyristor depends on the load. Highly inductive loads cause the shortest periods of thyristor conduction, the current being carried in the fly-back diodes for the rest of the time.

It is normal practice to excite the thyristor gate with firing pulses during the whole of the permissible conduction period of the device. At the end of this permissible conduction period, the force-commutation circuit is used to turn the thyristor off. Thus the output voltage of the inverter becomes defined during the periods of gate excitation. Referring to Fig 4.2 it can be seen that the inverter output voltage $V_{RN0c1}$ equals $v_d$ if $TH_1$ is being fired as, if $i_R$ is positive, $TH_1$ will conduct and connect $R$ to $P$ or, if $i_R$ is negative, $D_1$ will conduct also connecting $R$ to $P$. Similarly if $TH_4$ is being fired $R$ will be connected to $N$ via $TH_4$ or $D_4$.

If both $TH_1$ and $TH_4$ are being fired a "shoot-through" fault condition will occur and remedial action will be necessary.

If neither $TH_1$ nor $TH_4$ is being fired and the phase current $i_R$ is greater than the leakage current of the diodes then $R$ will be connected to $P$ via $D_1$ if $i_R$ is negative, or $R$ will be connected $N$ if $i_R$ is positive. If neither $TH_1$ nor $TH_4$ is being fired and $i_R$ is less
than the leakage current of the diodes then R will be effectively open-
circuit with respect to P and N and \( v_{RN} \) will equal the load open-
circuit emf \( v_{RNoc} \). However, if \( v_{RNoc} \) in this condition exceeds \( v_d \)
diode D1 will become forward biased, conduct and therefore limit \( v_{RN} \)
to \( v_d \). Similarly if \( v_{RNoc} \) becomes negative D4 will conduct clamping
\( v_{RN} \) to zero.

Thus in Boolean terms:

\[
V_{RNP} = T1F + \overline{T1F} \cdot T4F \cdot \overline{I_R} \cdot H_R
\]

Therefore

\[
V_{RNP} = T1F + T4F \cdot \overline{I_R} \cdot H_R
\]

and using De Morgan's Theorem

\[
V_{RNP} = T1F + T4F + I_R + H_R \tag{4.12}
\]

Also \( V_{RNocl} = \overline{T1F} \cdot T4F \cdot H_R \)

i.e. \( V_{RNocl} = \overline{T1F} + T4F + H_R \) \tag{4.13}

Similarly

\[
V_{RNN} = T4F + T1F + \overline{I_R} + \overline{H_R}
\]

or \( V_{RNN} = V_{RNP} \cdot V_{RNocl} \) \tag{4.14}

giving \( v_{RN} = v_d \) when \( V_{RNP} \) is true,

or \( v_{RN} = \left[ \begin{array}{c}
\nu_{RNoc} \\
\text{limit}
\end{array} \right] \) when \( V_{RNocl} \) is true

where

\[
\left[ \begin{array}{c}
\nu_{RNoc} \\
\text{limit}
\end{array} \right] = v_d \text{ when } v_d < \nu_{RNoc}, \text{ or }
\]

\[
= \nu_{RNoc} \text{ when } 0 < \nu_{RNoc} < v_d, \text{ or }
\]

\[
= 0 \text{ when } \nu_{RNoc} < 0
\]

also \( v_{RN} = 0 \) when \( V_{RNP} \) and \( V_{RNocl} \) are both false.
4.2.2 Determination of dc-link current

Referring to Fig 4.2 it can be seen that when \( R \) is connected to \( P \), \( i_{dR} = i_R \) and under all other circumstances \( i_{dR} = 0 \). When \( R \) is connected to \( P \), \( v_{RNP} \) is true. Thus \( i_{dR} \) can be produced in the model using a switch controlled by \( v_{RNP} \).

4.2.3 Fault Conditions

In this mode of operation "shoot-through" faults occur only if commutation fails or if the firing information becomes erroneous. The former case is not included in this study as it is assumed that the commutation circuitry is capable of commutating any current encountered in the operation of the inverter. The model is capable of giving information about peak current levels, commutation currents etc. and can therefore be used in the design process for the commutation circuitry. The case of erroneous firing information is recognised by:

\[
\text{FAULT}_R = T1F \cdot T4F \text{ being true} \quad (4.15)
\]

4.2.4 Firing information

The firing and turn-off information in this mode is included in the signals \( TxF \). \( TxF \) must be true for the whole of the permissible conduction period of the thyristor. This firing information may be generated by a computer sub-model or by linking the computer to actual inverter control logic. Section 4.3 contains the methods used in the production of the firing information used in this study.

The analysis covered in Section 4.2 is useful for inclusion in simulations of dc-supply-inverter-load simulations. It is less complex in terms of equipment requirements than that shown in Section 4.1 and consequently it is the analysis used in the system studies in the remainder of this thesis. However, it is less useful than that shown
in Section 4.1 for studying prototype firing algorithms.

4.2.5 Model Implementation - Inverter with defined firing information

Fig 4.7 shows the flow diagram for the implementation of the analysis shown above. The substitution of S and R in the suffixes gives the flow diagrams for the other two phases of the three-phase inverter shown in Fig 4.1.

Equations 4.10 and 4.11 give the dc-link current, \(i_d\), and the overall fault signal, FAULT, respectively.

Fig. 4.8 summarises the information flow for this model of the inverter.

4.3 INVERTER CONTROL LOGIC

The firing information for the inverters in this study was produced by a computer sub-model. It was desired to produce firing information suitable for simulating both 120° square-wave and 180° square-wave inverters and a combined sub-model was developed. 120° square-wave firing information was produced when the control signal Q was true and 180° square-wave firing information was produced when Q was not true. Following actual practice it was assumed that the frequency demand would be an analogue signal and a voltage controlled oscillator was produced to give the inverter clock frequency \(f_{\text{clock}}\) from the inverter frequency demand voltage \(v_f\). The flow diagram showing how this was achieved is given in Fig 4.9. The square wave thus produced was used as the shift signal to the six-stage circulating shift register shown in Fig 4.10. The initial condition for the shift register is \(\overline{A}.A'.B.\overline{B}'.\overline{C}.\overline{C'}\). Shift by one place occurs on each input edge. From this it is apparent that the outputs are high for 120° of each cycle and that adjacent elements such as A and A' overlap by 60°. From
FIG 4.8 INFORMATION FLOW FOR INVERTER MODEL
this $A + A'$ is high for $180^\circ$ of each cycle.

This gives: for the $120^\circ$ square-wave inverter

$$\begin{align*}
T1F &= A \\
T3F &= B \\
T5F &= C \\
T2F &= A' \\
T4F &= B' \\
T6F &= C'
\end{align*}$$

\hspace{1cm} (4.16)

and for the $180^\circ$ square-wave inverter

$$\begin{align*}
T1F &= A + A' \\
T3F &= B + B' \\
T5F &= C + C' \\
T2F &= A' + B \\
T4F &= B' + C \\
T6F &= C' + A
\end{align*}$$

\hspace{1cm} (4.17)

The combination of these equations gives:

$$\begin{align*}
T1F &= A + A' \cdot \overline{Q} \\
T3F &= B + B' \cdot \overline{Q} \\
T5F &= C + C' \cdot \overline{Q} \\
T2F &= A' + B \cdot \overline{Q} \\
T4F &= B' + C \cdot \overline{Q} \\
T6F &= C' + A \cdot \overline{Q}
\end{align*}$$

\hspace{1cm} (4.18)

Fig 4.11 is the logic diagram for these equations.

The information flow for the inverter control logic is shown in Fig 4.12.
FIG 4:11 PRODUCTION OF INVERTER FIRING INFORMATION
120° SQUARE-WAVE AND 180° SQUARE-WAVE.
FIG 4.12 INFORMATION FLOW FOR INVERTER FIRING LOGIC MODEL
4.4 DC-LINK SIMULATION INCLUDING BRAKING RESISTOR

It is assumed that the dc-link source can be controlled to operate either as a perfect voltage source or as a perfect current source. The effects of source impedance and source produced harmonics are neglected. A more detailed simulation of the source chopper is possible but it was found to be unnecessary in the present study.

It can be shown easily that the dc voltage across the braking resistance $R_L$ is equal to the dc-link voltage, $v_d$. Therefore the dynamics of the system are unaffected by whether the braking resistor is connected across the inverter dc-link or connected via a rectifier to the terminals of the induction machine. It is convenient, for the purposes of analysis and simulation, to assume that the braking resistor is connected across the inverter dc-link. This has no effect on the validity of the model and markedly reduces the number of computing elements required to simulate the braking resistor.

If the inverter power device currents need to be studied in detail it is necessary to simulate the braking rectifier separately.

Applying Kirchoff's laws to the dc-link shown in Fig 4.1 gives

$$v_d = \frac{1}{C} \int_0^t i_c \, dt \quad (4.19)$$

$$i_c = i_s - i_d - \frac{v_d}{R_L} \quad (4.20)$$

$$i_s = \frac{1}{L} \int_0^t (v_s - v_d) \, dt \quad (4.21)$$

Fig 4.13 shows the flow diagram for the implementation of equations 4.19, 4.20 and 4.21 on the analogue computer. Fig 4.14 shows the information flow for the dc-link model.
FIG 4.13 FLOW DIAGRAM FOR THE SIMULATION OF THE dc LINK
FIG 4.14 INFORMATION FLOW FOR D.C. LINK MODEL
4.5 COMPARISON OF WAVEFORMS

Fig 4.15 shows the output waveforms of the 180° square wave inverter. Fig 4.16 shows the corresponding outputs for the 120° square-wave inverter. For the purposes of this comparison the dc-link was assumed to be an infinite dc-bus. The operating conditions were:

\[ V_d = 1.57 \text{ pu} \]
\[ W_e = 1 \text{ pu} \]
\[ W_r = 0.95 \text{ pu} \]

The discontinuous nature of the 120° square-wave inverter output current can be seen clearly. When the phase current is zero, the machine open-circuit emf appears across the output terminals. This reduces the fundamental component of output voltages, thus leading to a degraded value of average torque. Fig. 4.17 shows the output waveforms of the real inverter feeding the TAIM. The operating conditions were:

\[ V_d = 0.83 \text{ pu} \]
\[ W_e = 1 \text{ pu} \]
\[ W_r = 0.95 \text{ pu} \]

The TAIM windings were connected in the series mode and the average torque was found to be 0.18 pu.

Fig 4.18 shows the results of the simulation under the corresponding conditions. Although the general form of the waveforms are in good agreement, the results of the real system contain some artifacts caused by the instrumentation. The type of instrument transducer used contains an internal oscillator and transformer to maintain both linearity and electrical isolation at low frequency and dc. The high frequency spikes seen on the waveforms are due to these oscillators. Some source produced ripple is also in evidence. The torque predicted
FIG 4.15 OUTPUT WAVEFORMS FOR THE 180° SQUARE WAVE INVERTER
FIG 4.16 OUTPUT WAVEFORMS FOR THE 120° SQUARE WAVE INVERTER (SIMULATION)
FIG 4.17 OUTPUT WAVEFORMS FOR THE 120° SQUARE WAVE INVERTER (REAL MACHINE)
FIG 4.18 OUTPUT WAVEFORMS FOR THE 120° SQUARE WAVE INVERTER FED FROM A FINITE d.c.-LINK SOURCE

[Simulation]
by the simulation was 25\% higher than the actual torque. This discrepancy is due to simplifying assumptions used in the analysis of the inverter and in the analysis of the induction machine (Chapter 5).
CHAPTER 5

INDUCTION MOTOR ANALYSIS

5.1 ASSUMPTIONS USED

5.2 GENERAL EQUATIONS

5.3 TRANSFORMATION TO AN ARBITRARY REFERENCE FRAME

5.4 TRANSFORMATION TO PARTICULAR REFERENCE FRAMES

5.4.1 Stationary reference frame

5.4.2 Synchronously rotating reference frame

5.4.3 Induction Motor Model
5.0 **INDUCTION MOTOR ANALYSIS**

The study of the transient performance of 3-phase induction machines and their behaviour on non-sinusoidal supplies is facilitated by the transformation of the machine variables to a reference frame in which there is a single pair of orthogonal axes (Ref 5.1). The choice of these axes is determined by the particular aspect of behaviour to be studied. In this analysis the equations describing the transformation to a general reference frame will be developed. Particular cases of this general reference frame will then be described.

5.1 **ASSUMPTIONS USED**

It is useful to make certain simplifying assumptions when analysing the induction machine in this manner. These assumptions are:

(i) The air-gap is assumed to be uniform, i.e. teeth effects are neglected as are any effects of eccentricity.

(ii) The magnetic circuit is assumed to be linear i.e. Saturation and hysteresis effects are neglected.

(iii) The stator phase windings are assumed to be identical and distributed in a manner to produce a single rotating sinusoidal MMF space wave when the phases are supplied with balanced 3-phase currents.

(iv) The rotor phases or bars are assumed to be arranged so that, at any instant, the rotor MMF space wave is a sinusoid having the same number of poles as the stator MMF wave.

(v) The values of resistances in the machine are assumed to be independent of temperature and frequency, i.e. the effects of eddy currents are neglected.
The present study is concerned only with singly-fed machines and so all rotor phases are assumed to be short-circuited. The analysis can easily be extended to the doubly-fed case if desired. All rotor parameters and variables are referred to the stator.

5.2 GENERAL EQUATIONS

Using the above assumptions the machine windings can be represented by single coils with appropriate physical displacements (Fig 5.1). (Fig 5.2) shows the induction machine equivalent circuit fed from a sinusoidal supply at base frequency. Such an idealised machine can be described by the following equation:

\[
\bar{V}_{3\text{ph}} = \bar{R}^\prime_{3\text{ph}} \bar{i}_{3\text{ph}} + \frac{p}{\omega_b} \bar{X}^\prime \bar{i}_{3\text{ph}}
\]  

(5.1)

where

\[
\begin{bmatrix}
V_a \\
V_b \\
V_c \\
0 \\
0 \\
0
\end{bmatrix}^T
\]

\[
\begin{bmatrix}
ia, ib, ic, ie, if, ig
\end{bmatrix}^T
\]

\[
\begin{array}{ccccccc}
R_s & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_s & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & R_s & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & R_r & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_r & 0 \\
\end{array}
\]

\[
\begin{array}{ccccccc}
R_s & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & R_s & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & R_s & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & R_r & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & R_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & R_r & 0 \\
\end{array}
\]
FIG 5.1: DIAGRAMATIC REPRESENTATION OF THE 3PHASE INDUCTION MACHINE
FIG 5.2 INDUCTION MACHINE EQUIVALENT CIRCUIT FOR SINUSOIDAL SUPPLIES

\[ X_s = X_m + X_1 \]
\[ X_r = X_m + X_2 \]
\[ X_s = w_b \ L_s \]
\[ X_r = w_b \ L_r \]
\[ X = w_b \ M \]
The cosinusoidal terms involving the instantaneous angular displacement of the rotor, $\psi = \omega t$, used in the inductance matrix $X$ make the analysis of this equation extremely laborious. The following transformation eliminates the time-dependent terms.

5.3 TRANSFORMATION TO AN ARBITRARY REFERENCE FRAME

The arbitrary reference frame contains an orthogonal pair of axes, the $d-q$ axes. These axes are rotating at an arbitrary angular velocity, $\omega$, so that when $\omega$ is positive the $q$ axis leads the $d$ axis. The choice of time-zero is also arbitrary but it is most convenient to choose time-zero at an instant when the $q$, $a$ and $e$ axes are all coincident. The relationship between the $d-q$ axes and the machine phase axes is shown in Fig. 5.3. Trigonometrical resolution gives the following relationships.

$$
\begin{align*}
\bar{V}_{dq0} &= \overline{T} \bar{V}_{3ph} \quad (5.2) \\
\bar{I}_{dq0} &= \overline{T} \bar{I}_{3ph} \quad (5.3)
\end{align*}
$$
FIG 5.3 MACHINE PHASE AXES AND THE ARBITRARY d-q AXES
where

\[
\overrightarrow{V}_{dqo} = \begin{bmatrix} V_{qs}, V_{ds}, V_{os}, 0, 0 \end{bmatrix}^T
\]

\[
i_{dqo} = \begin{bmatrix} i_{qs}, i_{ds}, i_{os}, i_{qr}, i_{dr}, i_{or} \end{bmatrix}^T
\]

\[
\begin{bmatrix}
\frac{2}{3} \cos \alpha & \frac{2}{3} \cos(\alpha \frac{2\pi}{3}) & \frac{2}{3} \cos(\alpha \frac{4\pi}{3}) & 0 & 0 & 0 \\
\frac{2}{3} \sin \alpha & \frac{2}{3} \sin(\alpha \frac{2\pi}{3}) & \frac{2}{3} \sin(\alpha \frac{4\pi}{3}) & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0
\end{bmatrix}
\]

and

\[
\frac{2}{3} \cos \beta & \frac{2}{3} \cos(\beta \frac{2\pi}{3}) & \frac{2}{3} \cos(\beta \frac{4\pi}{3}) & 0 & 0 & 0 \\
\frac{2}{3} \sin \beta & \frac{2}{3} \sin(\beta \frac{2\pi}{3}) & \frac{2}{3} \sin(\beta \frac{4\pi}{3}) & 0 & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0
\]

Assuming that the displacement of the arbitrary reference frame is continuous, the relationships between the machine variables in the d-q axes are now

\[
\overrightarrow{V}_{dqo} = R_{dqo} i_{dqo} + \frac{p}{w_b} \overrightarrow{x}_{dqo} i_{dqo}
\]

(5.4a)

\[
\begin{bmatrix}
R_s & \frac{w}{w_b} X_s & 0 & 0 & \frac{w}{w_b} X_m & 0 \\
-\frac{w}{w_b} X_s & R_s & 0 & -\frac{w}{w_b} X_m & 0 & 0 \\
0 & 0 & R_s & 0 & 0 & 0 \\
0 & \frac{w_p}{w_b} X_m & 0 & R_r & \frac{w_p}{w_b} X_r & 0 \\
-\frac{w_p}{w_b} X_m & 0 & 0 & -\frac{w_p}{w_b} X_r & R_r & 0 \\
0 & 0 & 0 & 0 & 0 & R_r
\end{bmatrix}
\]

where

\[
\frac{w}{w_b} = \begin{bmatrix} w_s, w_d, w_o \end{bmatrix}
\]

and

\[
\frac{w_p}{w_b} = \begin{bmatrix} w_{p_s}, w_{p_d}, w_{p_o} \end{bmatrix}
\]
In the case where there are no neutral connections the zero sequence terms are zero and equation (5.4a) simplifies to equation (5.4b):

\[
\bar{V}_{dq} = \bar{R} \bar{i}_{dq} + \frac{p}{w_b} \bar{X} \bar{i}_{dq}
\]

where \( \bar{V}_{dq} = \begin{bmatrix} V_{qs}, V_{ds}, 0, 0 \end{bmatrix}^T \)

\( \bar{i}_{dq} = \begin{bmatrix} i_{qs}, i_{ds}, i_{qr}, i_{dr} \end{bmatrix}^T \)

\[
\bar{R} = \begin{bmatrix}
R_s & \frac{w}{w_b} X_s & 0 & \frac{w}{w_b} X_m \\
-\frac{w}{w_b} X_s & R_s & -\frac{w}{w_b} X_m & 0 \\
0 & \frac{wB}{w_b} X_m & R_r & \frac{wB}{w_b} X_r \\
-\frac{wB}{w_b} X_m & 0 & -\frac{wB}{w_b} X_r & R_r
\end{bmatrix}
\]

and \( w_B = p (\beta) \)

i.e. \( w_B = w - w_r \)
The application of the principle of virtual displacements (Ref 5.2) gives the electromagnetic torque:

\[ T_e = \frac{3}{2} \frac{3}{PP} \frac{X_m}{w_b} (i_{qs} i_{dr} - i_{ds} i_{qr}) \]  

(5.5)

in per unit terms this becomes

\[ T_e = X_m (i_{qs} i_{dr} - i_{ds} i_{qr}) \]  

(5.6)

5.4 TRANSFORMATION TO PARTICULAR REFERENCE FRAMES

5.4.1 Stationary Reference Frame

When the details of stator waveforms are important it is useful to use a stationary reference frame. In this way the transformation between d q variables and real stator variables is time invariant.

For the stationary reference frame, \( w = 0 \),

and \( w_p = w - w_r \)

therefore \( w_p = -w_r \)

i.e.

\[ \vec{V} = \left( \bar{R} + \frac{P}{w_b} \bar{X} \right) \vec{I} \]  

(5.7)

where \( \vec{V} = \begin{bmatrix} V_{qs}, V_{ds}, 0, 0 \end{bmatrix}^T \)

\( \vec{I} = \begin{bmatrix} i_{qs}, i_{ds}, i_{qr}, i_{dr} \end{bmatrix}^T \)
\[ \bar{R} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & -\frac{w_r}{w_b} X_m & R_r & -\frac{w_r}{w_b} X_r \\ \frac{w_r}{w_b} X_m & 0 & \frac{w_r}{w_b} X_r & R_r \end{bmatrix} \]

\[ \bar{X} = \begin{bmatrix} X_s & 0 & X_m & 0 \\ 0 & X_s & 0 & X_m \\ X_m & 0 & X_r & 0 \\ 0 & X_m & 0 & X_r \end{bmatrix} \]

From equations (5.2) and (5.3),

\[ V_a = v_{qs} \quad \quad \quad i_a = i_{qs} \]
\[ V_b = \frac{1}{2} v_{qs} - \frac{\sqrt{3}}{2} v_{ds} \quad i_b = \frac{1}{2} i_{qs} - \frac{\sqrt{3}}{2} i_{ds} \quad (5.8a) \]
\[ V_c = \frac{1}{2} v_{qs} + \frac{\sqrt{3}}{2} v_{ds} \quad i_c = \frac{1}{2} i_{qs} + \frac{\sqrt{3}}{2} i_{ds} \]

and

\[ V_{RT} = \frac{3}{2} v_{qs} - \frac{\sqrt{3}}{2} v_{ds} \quad (5.8b) \]
\[ V_{SR} = \frac{3}{2} v_{qs} - \frac{\sqrt{3}}{2} v_{ds} \]
\[ V_{TS} = \sqrt{3} v_{ds} \]

or conversely

\[ i_{qs} = \frac{2}{3} (i_a - \frac{1}{2} (i_b + i_c)) = i_a \]
\[ i_{ds} = \frac{i_c - i_b}{\sqrt{3}} \]
\[ V_{qs} = \frac{V_{RT} - V_{SR}}{\sqrt{3}} = \frac{2}{3} (V_a - \frac{1}{2} (V_b + V_c)) = V_a \quad (5.9) \]
\[ V_{ds} = \frac{V_{TS}}{\sqrt{3}} \]
If the phase currents become zero in the discontinuous current mode, the open circuit emf appears across the corresponding terminals of the machine. For zero $i_a$, there is an induced emf in phase a due to rotor currents. This may be expressed as follows:

When $i_a = 0$, $\frac{p}{w_b} i_a = 0$, $i_{qs} = 0$ and $\frac{p}{w_b} i_{qs} = 0$

Thus $V_{qs} = (R_s + \frac{p}{w_b} X_s) i_{qs} + \frac{p}{w_b} X_m i_{qr}$

becomes simply

$$V_{qs} = \frac{p}{w_b} X_m i_{qr}$$

also when

$i_a = 0$ and $\frac{p}{w_b} i_a = 0$,

$$V_a = \frac{p}{w_b} X_m i_{qr}$$

Similarly for

$i_b = 0$ and $\frac{p}{w_b} i_b = 0$,

$$V_b = \frac{p}{w_b} X_m (\frac{i_{qr}}{2} - \frac{\sqrt{3}}{2} i_{dr}) \quad (5.10)$$

and for

$i_c = 0$ and $\frac{p}{w_b} i_c = 0$,

$$V_c = \frac{p}{w_b} X_m (\frac{i_{qr}}{2} + \frac{\sqrt{3}}{2} i_{dr})$$

Thus in the case of the inverter fed induction machine (Fig 5.4):

when $i_a = 0$, $V_{RN} = \frac{3}{2} V_a + \frac{1}{2} (V_{SN} + V_{TN}) \quad (5.11)$
FIG 5.4 INVERTER-FED INDUCTION MACHINE SYSTEM.
Similarly when
\[ i_b = 0, \quad v_{SN} = \frac{3}{2} v_b + \frac{1}{2} (v_{RN} + v_{TN}) \]  
(5.12)
and when
\[ i_c = 0, \quad v_{TN} = \frac{3}{2} v_c + \frac{1}{2} (v_{RN} + v_{SN}) \]  
(5.13)

For the six-step inverters considered in this study, inspection shows that when any phase is open-circuit, one of the other inverter outputs is at the dc-link negative potential and the third output is at the dc-link positive potential, i.e. equations (5.11), (5.12) and (5.13) then become.

\[ v_{RNOC} = \frac{3}{2} \left( \frac{p}{w_b} \right) x_m i_{qr} + \frac{1}{2} v_D \]  
(5.14)
\[ v_{SNOC} = \frac{3}{2} \left( \frac{p}{w_b} \right) x_m (-\frac{1}{2} i_{qr} - \frac{\sqrt{3}}{2} i_{dr}) + \frac{1}{2} v_D \]  
(5.15)
and
\[ v_{TNOC} = \frac{3}{2} \left( \frac{p}{w_b} \right) x_m (-\frac{1}{2} i_{qr} + \frac{\sqrt{3}}{2} i_{dr}) + \frac{1}{2} v_D \]  
(5.16)
respectively.

5.4.2 Synchronously Rotating Frame

The electro-mechanical dynamics of the induction machine on non-sinusoidal waveforms can be approximated by the response of the machine to the fundamental of the applied waveform. If the assumption that the effects of supply harmonics are negligible is made, the synchronously rotating reference frame can achieve considerable simplification of the resultant analysis. If only the fundamental of the applied waveform is considered, the use of a synchronously rotating reference frame eliminates the periodic terms from the d-q variables and so can simplify the solution of the machine equations. In some cases a suitable choice of time-zero can also reduce the order of the equations by eliminating the stator d-axis variables. The
following equations result from using a synchronously rotating reference frame and neglecting supply harmonics. Time zero is chosen so that \( V_{ds} = 0 \).

\[
\bar{V} = \bar{R} \bar{I} + \frac{d}{w_b} \bar{X} \bar{I}
\]  

(5.17)

where \( \bar{V} = \begin{bmatrix} V_{qs} & 0 & 0 \end{bmatrix}^T \)

\[\begin{align*}
\bar{I} &= \begin{bmatrix} i_{qs} & i_{ds} & i_{qr} & i_{dr} \end{bmatrix}^T \\
\bar{R} &= \begin{bmatrix}
R_s & \frac{w_e X_s}{w_b} & 0 & \frac{w_e X_m}{w_b} \\
\frac{-w_e X_s}{w_b} & R_s & \frac{-w_e X_m}{w_b} & 0 \\
0 & \frac{w_s X_m}{w_b} & R_r & \frac{w_s X_r}{w_b} \\
\frac{-w_s X_m}{w_b} & 0 & \frac{-w_s X_r}{w_b} & 0
\end{bmatrix}
\end{align*}\]

\[
w_e = \text{angular frequency of the supply} \\
w_s = \text{angular slip frequency} \\
\omega_r = w_e - w_r
\]

and

\[
\bar{X} = \begin{bmatrix}
X_s & 0 & X_m & 0 \\
0 & X_s & 0 & X_m \\
X_m & 0 & X_r & 0 \\
0 & X_m & 0 & X_r
\end{bmatrix}
\]

5.4.3 Induction Motor Model

The induction machine analysis with respect to a stationary reference frame (Section 5.4.1) was used as the basis for the model of the machine. This model was implemented on the analogue computer and linked to the inverter model (Chapter 4). For convenience, flux
linkages are used as variables in this model. The flux linkages are defined as:

\[ \bar{\psi} = \frac{1}{\omega_{b}} \bar{X} \bar{I} \]  

(5.18)

where \( \bar{\psi} = \begin{bmatrix} \psi_{qs}, \psi_{ds}, \psi_{qr}, \psi_{dr} \end{bmatrix}^T \)

and \( \bar{X} \) and \( \bar{I} \) are as defined in Equation (5.7).

The inverse \( L^{-1} \) of the inductance matrix has the form:

\[
L^{-1} = \begin{bmatrix}
A & 0 & C & 0 \\
0 & A & 0 & C \\
C & 0 & B & 0 \\
0 & C & 0 & B
\end{bmatrix}
\]

(5.19)

where \( A = \frac{L_{r}}{L_{s} L_{r} - M^2} \)

\( B = \frac{L_{s}}{L_{s} L_{r} - M^2} \)

\( C = -\frac{M}{L_{s} L_{r} - M^2} \)

Fig. 5.5 shows the block diagram for the simulation of equations 5.6, 5.7 and 5.18 and Fig 5.6 is a similar diagram for the simulation of equations 5.14, 5.15, and 5.16.

The implementation of the axes transformations is shown in Fig. 5.7.
FIG 5.5 INDUCTION MOTOR SIMULATION
FIG 5.6 PRODUCTION OF OPEN-CIRCUIT VOLTAGES
FIG 5.7 AXES TRANSFORMATIONS
CHAPTER 6

STATE VARIABLE ANALYSIS OF THE SYSTEM

6.1 INVERTER WAVEFORMS

6.1.1 Inverter mode (i)
6.1.2 Inverter mode (ii)
6.1.3 Inverter mode (iii)
6.1.4 Steady-state Solution
6. **STEADY STATE ANALYSIS OF THE SYSTEM**

Inverter-fed induction machines can be studied using state-variable analysis techniques to give steady state waveforms. The assumptions used in this analysis included those detailed in Sections (4.0) and (5.1). Also it was assumed that both the inverter frequency and the motor speed were constant.

6.1 **INVERTER WAVE FORMS**

Lipo and Turnbull (Ref 6.1) have used state variable analysis to predict the behaviour of an inverter-fed induction machine under steady-state conditions and assumed constant speed. However, their analysis is invalid when a flyback diode begins to conduct before its corresponding thyristor is fired. This condition occurs at low positive slips and during regeneration. The following analysis of the present author is valid for all regions of operation:

In the steady-state operation of the inverter-fed induction machine shown in (Fig 6.1) each cycle can be split into six similar periods of \( \frac{\pi}{3\omega_e} \) duration. \((\omega_e \text{ is the inverter angular frequency})\). In each of these periods there are three possible modes of conduction:

(i) One motor phase is open circuited since the thyristors and diodes associated with that phase are all blocking (Fig 6.2a).

(ii) The open-circuit voltage of the non-conducting phase exceeds the inverter link voltage and causes the associated diode to conduct (Fig 6.2b).

(iii) The next thyristor in the sequence is fired and commutation of the current from one phase to the next takes place (Fig 6.2c).
FIG. 6.1 INVERTER-FED INDUCTION MACHINE SYSTEM.
FIG 62 INVERTER CIRCUIT MODES
If the open-circuit voltage of the non-conducting phase does not exceed the inverter link voltage before the next thyristor is fired, the inverter does not enter mode (ii). This is the case analysed by Lipo and Turnbull.

If the current does not become discontinuous (due to the open circuit emf of the motor exceeding the inverter link voltage at the instant the phase current becomes zero) the inverter does not enter mode (i).

6.1.1 Inverter Mode (i)

For the period \( \varphi < \omega t < \varphi \)

\[
i_R = 0
\]

Therefore \( i_{qs} = 0 \)

giving

\[
V_{qs} = \frac{p}{\omega_b} X_m i_{qr}
\]

Now

\[
V_d = V_{to} - V_{s0}
\]

thus

\[
V_{ds} = \frac{V_d}{\sqrt{3}}
\]

Also \( i_d = i_T \)

giving

\[
i_d = -\frac{1}{2} i_{qs} + \frac{\sqrt{3}}{2} i_{ds}
\]

Substituting into the equations of the motor with a stationary reference frame (Section 5.4.1) and including equations for the d.c. link components gives:

\[
\bar{V} = \bar{R}_1 \bar{I} + \frac{p}{\omega_b} \bar{X}_1 \bar{I}
\]  

(6.1)

where

\[
\bar{V} = \begin{bmatrix} 0, 0, 0, 0, 0, V_s \end{bmatrix}^T
\]

\[
\bar{I} = \begin{bmatrix} i_{qs}, i_{ds}, i_{qr}, i_{dr}, V_d, i_s \end{bmatrix}^T
\]
Re-arranging gives the state equations:

\[ \frac{p}{\omega_b} \bar{x} = \bar{A}_1 \bar{x} + \bar{B}_1 \bar{u} \]  \hspace{1cm} (6.2) \\

where \( \bar{x} = \bar{r} \) \\
\( \bar{u} = \bar{v} \) \\
\( \bar{A}_1 = -X^{-1} R_1 \) \\
and \( \bar{B}_1 = X^{-1} \)
6.1.2 Inverter mode (ii)

For the period $\varphi < \omega t < \frac{T}{3}$

\[ V_d = V_{T0} - V_{S0} \]
\[ V_{RO} = V_{T0} \]
\[ i_d = -i_s \]

giving:

\[ V_{qs} = \frac{V_d}{\sqrt{3}} \]
\[ V_{ds} = \frac{V_d}{\sqrt{3}} \]

and

\[ i_d = \frac{1}{2} i_{qs} + \frac{\sqrt{3}}{2} i_{ds} \]

This leads to:

\[ \bar{V} = \bar{R}_2 \bar{I} + \frac{p}{\omega_b} \bar{X}_2 \bar{I} \] \hspace{1cm} (6.3)

where $\bar{V}$ and $\bar{I}$ are as defined in equation (6.1),

\[ \bar{R}_2 = \begin{bmatrix}
R_s & 0 & 0 & 0 & -\frac{1}{3} & 0 \\
0 & R_s & 0 & 0 & -\frac{1}{\sqrt{3}} & 0 \\
0 & -\frac{w_r X_m}{\omega_b} & R_r & -\frac{w_r X_r}{\omega_b} & 0 & 0 \\
w_r X_m & 0 & w_r X_r & R_r & 0 & 0 \\
\frac{w_r X_m}{\omega_b} & \frac{1}{2} & \sqrt{3} & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix} \]
and \( \bar{x}_2 \) =

\[
\begin{bmatrix}
X_s & 0 & X_m & 0 & 0 & 0 \\
0 & X_s & 0 & X_m & 0 & 0 \\
X_m & 0 & X_r & 0 & 0 & 0 \\
0 & X_m & 0 & X_r & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\bar{X}_c} \\
0 & 0 & 0 & 0 & 0 & X_1
\end{bmatrix}
\]

re-arranging gives:

\[
\frac{p}{w_b} \bar{x} = \bar{\Lambda}_2 \bar{x} + \bar{\beta}_2 \bar{u}
\]

(6.4)

where \( \bar{x} = \bar{I} \),\( \bar{u} = \bar{v} \)\( \bar{\Lambda}_2 = -x_2^{-1} \bar{R}_2 \)
\( \bar{\beta}_2 = x_2^{-1} \)

6.1.3 Inverter mode (iii)

For the period \( \frac{\pi}{3} < w_c t < \phi + \frac{\pi}{3} \)

\[
V_d = V_{R0} - V_{S0}
\]
\[
V_{S0} = V_{T0}
\]
\[
i_d = i_R
\]

These relationships lead to:

\[
V_{qs} = \frac{2}{3} V_d
\]
\[
V_{ds} = 0
\]
\[
i_d = i_{qs}
\]
which in turn lead to:

\[ V = R_3 x + \frac{p}{w_b} \bar{x}_3 \]

(6.5)

where

\[ R_3 = \begin{bmatrix}
R_s & 0 & 0 & 0 & -\frac{2}{3} & 0 \\
0 & R_s & 0 & 0 & 0 & 0 \\
0 & -\frac{w_r}{w_b} X_m & R_r & -\frac{w_r}{w_b} X_r & 0 & 0 \\
\frac{w_v}{w_b} X_m & 0 & \frac{w_r}{w_b} X_r & R_r & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix} \]

and \( \bar{x}_3 = \bar{x}_2 \)

This gives the state equations:

\[ \frac{p}{w_b} \bar{x} = \bar{A}_3 \bar{x} + \bar{B}_3 \bar{u} \]

(6.6)

where \( \bar{x} = \bar{\bar{x}} \)

\( \bar{u} = \bar{\bar{v}} \)

\[ \bar{A}_3 = -\bar{x}_3^{-1} R_3 \]

\[ \bar{B}_3 = \bar{x}_3^{-1} \]

6.1.4 Steady-State Solution

The general solution (Ref 6.1) to the state-equation is:

\[ x(w t) = \exp A(w t - w t_0) x(w t_0) + \frac{\lambda(w t - w t_0)}{f_R} B \bar{u} \]

(6.7)
where \( f_{w} = \frac{w}{w_{b}} \) and \( \lambda_{w}(w_{e} - w_{t}) = \frac{1}{f_{R}} \int_{w_{t}}^{w_{e}} \exp \lambda_{w}(t - r) \, d(w_{r}) \)

Thus, for the period \( \varphi < w_{e} < \Theta \) we get

\[
\bar{x}(w_{e}) = \exp \lambda_{1}(w_{e} - \varphi) \, \bar{x}(\varphi) + \frac{\lambda_{1}(w_{e} - \varphi)}{f_{R}} \, B_{1} \, \bar{u} \quad (6.8)
\]

and when \( w_{e} = \Theta \),

\[
\bar{x}(\Theta) = \exp \lambda_{1}(\Theta - \varphi) \, \bar{x}(\varphi) + \frac{\lambda_{1}(\Theta - \varphi)}{f_{R}} \, B_{1} \, \bar{u} \quad (6.9)
\]

Similarly for the period \( \Theta < w_{e} < \frac{\pi}{3} \),

\[
\bar{x}(w_{e}) = \exp \lambda_{2}(w_{e} - \Theta) \, \bar{x}(\Theta) + \frac{\lambda_{2}(w_{e} - \Theta)}{f_{R}} \, B_{2} \, \bar{u} \quad (6.10)
\]

and when \( w_{e} = \frac{\pi}{3} \),

\[
\bar{x}(\frac{\pi}{3}) = \exp \lambda_{2}(\frac{\pi}{3} - \Theta) \, \bar{x}(\Theta) + \frac{\lambda_{2}(\frac{\pi}{3} - \Theta)}{f_{R}} \, B_{2} \, \bar{u} \quad (6.11)
\]

and for the period \( \frac{\pi}{3} < w_{e} < \varphi + \frac{\pi}{3} \),

\[
\bar{x}(w_{e}) = \exp \lambda_{3}(w_{e} - \frac{\pi}{3}) \, \bar{x}(\frac{\pi}{3}) + \frac{\lambda_{3}(w_{e} - \frac{\pi}{3})}{f_{R}} \, B_{3} \, \bar{u} \quad (6.12)
\]

and when \( w_{e} = \varphi + \frac{\pi}{3} \),

\[
\bar{x}(\varphi + \frac{\pi}{3}) = \exp \lambda_{3}(\varphi) \, \bar{x}(\frac{\pi}{3}) + \frac{\lambda_{3}(\varphi)}{f_{R}} \, B_{3} \, \bar{u} \quad (6.13)
\]

Substitution from equations (6.9) and (6.11) into (6.13) gives:

\[
\bar{x}(\varphi + \frac{\pi}{3}) = \exp \lambda_{3}(\varphi) \, \bar{x}(\frac{\pi}{3}) + \frac{\lambda_{3}(\varphi)}{f_{R}} \, B_{3} \, \bar{u} \quad (6.13)
\]
However, in the steady state, symmetry considerations dictate that the conditions at any instant are related in a time-invariant manner to the corresponding conditions one sixth of a cycle earlier. For example the d.c. link variables must have a basic sixth harmonic variation giving:

\[ V_d (w_e t + \frac{\pi}{3}) = V_d (w_e t) \]

and \[ i_d (w_e t + \frac{\pi}{3}) = i_d (w_e t) \]

Resolution of the dq variables at time \((w_e t + \frac{\pi}{3})\) gives the relationships:

\[ i_{qs} (w_e t + \frac{\pi}{3}) = \frac{1}{2} i_{qs} (w_e t) + \frac{\sqrt{3}}{2} i_{ds} (w_e t) \]
\[ i_{ds} (w_e t + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2} i_{qs} (w_e t) + \frac{1}{2} i_{ds} (w_e t) \]
\[ i_{qs} (w_e t + \frac{\pi}{3}) = \frac{1}{2} i_{qr} (w_e t) + \frac{\sqrt{3}}{2} i_{dr} (w_e t) \]
\[ i_{ds} (w_e t + \frac{\pi}{3}) = -\frac{\sqrt{3}}{2} i_{qr} (w_e t) + \frac{1}{2} i_{dr} (w_e t) \]

Combining these relationships gives:

\[ x(w_e t + \frac{\pi}{3}) = S \ x(w_e t) \]

and thus

\[ \bar{x}(\phi + \frac{\pi}{3}) = S \ x(\phi) \]  

(6.15)
where

\[
S = \begin{bmatrix}
\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 \\
-\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\
0 & 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Substitution of equation (6.15) into equation (6.14) gives:

\[
x(\phi) = \left[ \begin{array}{c}
S - \frac{\exp A_3(\phi)}{\tau_R} \cdot \frac{\exp A_2(\frac{\phi}{3} - \phi)}{\tau_R} \cdot \frac{\exp A_1(\phi - \phi)}{\tau_R}
\end{array} \right]^{-1}
\]

\[
\begin{array}{c}
\exp A_3(\phi) \cdot \exp A_2(\frac{\phi}{3} - \phi) \\
\exp A_3(\phi) \cdot \lambda_2(\phi - \phi) B_1 u \\
\exp A_3(\phi) \cdot \lambda_2(\phi - \phi) B_2 u + \lambda_3(\phi) B_3 u
\end{array}
\]  

Equation (6.16) can be used to find the values of the variables at the instant \( \phi \). These initial conditions can be used in equations (6.8, 6.10, 6.12) to compute the variable values through the rest of the cycle, if the values of \( \phi \) and \( \phi \) are known.

Compatible values of \( \phi \) and \( \phi \) can be computed using a two-variable iteration.

Two conditions must be satisfied:

(a) At \( \omega t = \phi \), \( i_{qs} \) is zero, and

(b) for the period \( \phi < \omega t < \phi \) the inverter phase open circuit voltage \( V_{RNOC} \) must be less than, or equal to, the inverter d.c. link voltage \( V_d \). The value of \( \phi \) is constrained to be between \( \phi \) and \( \frac{\phi}{3} \). If \( V_{RNOC} \) does not exceed \( V_d \) before \( \omega t \) becomes greater
than $\frac{\pi}{3}$ the value of $\Theta$ is $\frac{\pi}{3}$. This indicates that inverter mode (ii) is not entered. If $V_{RNOC}$ exceeds $V_d$ at $\Theta$ the value of $\Theta$ is $\emptyset$ indicating that the inverter does not enter mode (i).

In this type of operation modes (ii) and (iii) can be considered to be similar and the equations for mode (iii) used to produce the solution (Ref 6.1). (Fig. 6.3) shows a flow-diagram of a computer program which predicts the steady state behaviour of the system utilising the algorithm described in this section. (Fig 6.4) shows in graphical form, one set of results from this programme. The operating conditions of 1 pu frequency, 1.57 pu d.c. link voltage and 5% slip are similar to those used for the production of Figs 4.15 and 4.16.

This analysis has the advantage of being independent of dynamic instabilities to which the physical system and any dynamic models may be prone. Losses and loading effects due to the non-sinusoidal inverter waveforms in regions of instability can now be estimated.
INPUT DATA
FORMULATE STATE EQUATIONS
ASSUME \( \theta = \frac{\pi}{3} \)
ASSUME \( \phi = 0 \)
CALCULATE \( \infty (0) \)
ITERATE VALUE OF \( \phi \)

ITERATION FAILS?

YES

ASSUME MODE (iii) ONLY
SET \( \phi = \theta - \frac{\pi}{3} \)

NO

ITERATE VALUE OF \( \theta \)

YES

VRNOC < \( V_d \)

NO

OUTPUT

VALUES OF MODE (iii) ONLY

FIG 6.3 FLOW CHART FOR STEADY-STATE ANALYSIS PROGRAMME
FIG 6-4 WAVEFORMS FOR STATE VARIABLE INVERTER
ANALYSIS AT 5% SLIP
CHAPTER 7

TRANSFER FUNCTION ANALYSIS

7.1 CONTROL TECHNIQUES

7.2 ASSUMPTIONS

7.3 DERIVATION OF STATE EQUATIONS
7.0 TRANSFER FUNCTION ANALYSIS

Several authors (7.1 - 7.6) have developed transfer functions for inverter fed induction machines operating in various control modes. The method used for analysing the particular system under consideration was based on the techniques developed by Lipo and Plunkett (7.1) and by Lipo and Krause (7.2).

7.1 CONTROL TECHNIQUES

The system analysed had the following control loops:

(i) a variable voltage d.c. power source control loop which provided a constant current, $i_s$, into the d.c. link, and

(ii) an inverter frequency proportional to the d.c. link voltage $V_d$. This form of operation is described in a little more detail in Chapter 8.

Supervisory control modes were included in the system to prevent destructively large excursions in current and voltage. These modes had no effect under normal operating conditions.

7.2 ASSUMPTIONS

In addition to the assumptions used in the analysis of the induction machine (Section 5.1) the following assumptions have been made:

(i) The dynamic behaviour of the system is determined primarily by the nominal fundamental component of the inverter output and so all harmonic effects are neglected as are the effects of changes in inverter output voltage waveform due to changes in load conditions.

(ii) The d.c. link appears to be fed by a perfect current source.
i.e. the control loop on the source current has negligible error. This assumption is particularly valid when the source current is zero i.e. during self-excitation.

7.3 **DERIVATION OF STATE EQUATIONS**

The differential equations describing the behaviour of an induction motor with finite rotor inertia are non-linear and are therefore unsuitable for the direct derivation of transfer functions. However, the equations can be linearised about an operating point. The resultant equations can be used to derive transfer functions for the system and these transfer functions are valid for small perturbations about the operating condition. In this analysis per-unit values, and the notation developed by Lipo & Plunkett (Ref 7.1) are used.

Lipo & Krause (Ref 7.2) show that the relationship between the fundamental component amplitude of the line voltage and the dc-link voltage $V_d$ is:

$$V_{\text{line fundamental}} = \frac{2\sqrt{3}}{\sqrt{3}} V_d$$

(7.1)

Choosing a synchronously rotating reference frame, neglecting all harmonics, and choosing time zero to coincide with peak $q$-axis stator voltage leads to:

$$V_{qs} = \frac{2}{\sqrt{3}} V_d$$

(7.2)

$$V_{ds} = 0$$

(7.3)

There must be a power balance across the inverter (neglecting losses) and this gives:

$$V_d i_d = \frac{3}{2} V_{qs} i_{qs}$$

(7.4)
Substitution of equation (7.2) into (7.4) gives:

\[ i_{qs} = \frac{\tau T}{3} i_d \]  

(7.5)

The proportionality between dc-link voltage and frequency leads to:

\[ v_{qs} = d \frac{w_e}{w_b} \]  

(7.6)

Referring to figure 4.1 and applying Kirchoff's Current Law

\[ i_s = i_d + V_d + i_c \]  

\[ i_s = \frac{3}{\tau T} i_{qs} + V_d + \frac{1}{X_c} \frac{p}{w_b} V_d \]  

\[ i_s = \frac{3}{\tau T} i_{qs} + \frac{\tau T}{2} \frac{v_{qs}}{w_b} + \frac{\tau T}{2} \frac{1}{X_c} \frac{p}{w_b} v_{qs} \]  

\[ i_s = \frac{3}{\tau T} i_{qs} + \frac{\tau T}{2} \frac{w_e}{w_b} + \frac{\tau T}{2} \frac{1}{X_c} \frac{p}{w_b} w_e \]  

(7.7)

Combining these equations with those of the motor and including torque terms gives:

\[ \bar{u} = \frac{\bar{v}}{R_L} \bar{x} + \frac{\bar{p}}{w_b} \bar{v} \bar{x} \]  

\[ \bar{u} = \begin{bmatrix} 0, 0, 0, 0, i_s, T_L \end{bmatrix}^T \]  

(7.8)

where \( \bar{u} \) = mechanical load torque

\[ \bar{x} = \begin{bmatrix} T, w_e, \frac{w_e}{w_b} \end{bmatrix}^T \]

\[ \bar{I} = \begin{bmatrix} i_{qs}, i_ds, i_{qs}, i_dr \end{bmatrix}^T \]
\[
\begin{align*}
\bar{R}_{\text{n}} &= \begin{bmatrix}
(R + \omega e) \bar{F} \\
\frac{3}{T} & \bar{G} & \bar{I} \\
\frac{T}{\omega_b} & 0 & -\omega_b D
\end{bmatrix} \\
\bar{x}'' &= \begin{bmatrix}
\bar{x} \\
0 \\
-\frac{T d}{2x_c}
\end{bmatrix}
\end{align*}
\]

\[
\bar{I} = \begin{bmatrix} 1, 0, 0, 0 \end{bmatrix}^T
\]

\[
\bar{\sigma} = \begin{bmatrix} 0, 0, 0, 0 \end{bmatrix}^T
\]

\[
\bar{R} = \begin{bmatrix}
R_s & 0 & 0 & 0 \\
0 & R_s & 0 & 0 \\
0 & 0 & R_r & 0 \\
0 & 0 & 0 & R_r
\end{bmatrix}
\]

\[
\bar{F} = \begin{bmatrix}
0 & x_s & 0 & x_m \\
-x_s & 0 & -x_m & 0 \\
x_m & 0 & x_r & 0 \\
x_m & 0 & -x_r & 0
\end{bmatrix}
\]
For the steady-state operating condition equation (7.8) becomes:

\[
\bar{u}_o = \bar{R}^0 \bar{x}_o
\]  
(7.9)

where the suffix \( o \) indicates steady-state values and:

\[
\bar{u}_o = \begin{bmatrix} 0 \end{bmatrix}^{T}, 0, 0, 0, i_{s0}, T_{L0}
\]

\[
\bar{x}_o = \begin{bmatrix} \bar{i}_o^{T}, w_{eo}, w_{ro} \end{bmatrix}^{T}
\]

\[
\bar{i}_o = \begin{bmatrix} i_{qso}, i_{dso}, i_{qro}, i_{dro} \end{bmatrix}^{T}
\]

and

\[
\bar{R}_o = \begin{bmatrix} \bar{R} + w_{co} \bar{F} & -d \bar{T} & \bar{G} \bar{T}_o \\
\frac{3}{2} \bar{T} & \frac{T_{1d}}{2R_L} & 0 \\
\frac{1}{w_{b}} \bar{T} & 0 & -w_{b} D
\end{bmatrix}
\]
Perturbing all variables about the steady-state and subtracting the steady-state terms leads to:

\[
\vec{u} = \overline{R^T} \vec{x} + \frac{p}{w_b} \overline{X^T} \vec{x}
\]

(7.10)

where

\[
\vec{u} = \begin{bmatrix} 0,0,0,0,\triangle i_s, \triangle T_L \end{bmatrix}^T
\]

\[
\overline{x} = \begin{bmatrix} \overline{i}^T, \triangle \overline{w}_e, \triangle \overline{w}_r \end{bmatrix}^T
\]

\[
\triangle \overline{i} = \begin{bmatrix} \triangle i_{qs}, \triangle i_{ds}, \triangle i_{qr}, \triangle i_{dr} \end{bmatrix}^T
\]

\[
\overline{X^T} = \overline{x}^T
\]

and

\[
\overline{R^T} = \begin{bmatrix}
3 \overline{I}^T \\
\overline{I^T} \\
\overline{I_o^T} \\
\frac{\pi d}{2R_L} \\
\end{bmatrix}
\]

Equation (7.10) can be manipulated to give the state-equations.

\[
\frac{p}{w_b} \vec{x} = \overline{A} \vec{x} + \overline{B} \vec{u}
\]

(7.11)

where

\[
\overline{A} = -\overline{X^T}^{-1} \overline{R^T}
\]

\[
\overline{B} = \overline{X^T}^{-1}
\]

For the purpose of computing transfer functions it is usually acceptable to consider single input/output pairs. Super-position can be applied in linear multivariable systems. Thus in equation
(7.11) the input vector becomes \( \bar{v} \) \( u \) where:

- (i) \( \bar{v} = \begin{bmatrix} 0,0,0,0,1,0 \end{bmatrix}^T \) for \( u = \triangle i_s \)

or

- (ii) \( \bar{v} = \begin{bmatrix} 0,0,0,0,0,1 \end{bmatrix}^T \) for \( u = \triangle T_L \)

In general the output variable is given by:

\[
y = \bar{c} \bar{x} + z u \tag{7.12}
\]

where \( z \) is a direct coupling scalar. None of the outputs considered has any direct coupling with the inputs so equation (7.12) becomes

\[
y = \bar{c} \bar{x} \tag{7.13}
\]

Table III contains the \( \bar{c} \) vector for the outputs considered.

Having derived the state-equations in this linearised form the corresponding transfer functions can be computed using the program written by Melsa (See Appendix C).

Table IV gives the location of the poles and zeros of the transfer function between source current and electrical torque for the system assuming infinite inertia.

Fig 7.1 shows the root loci for the system considered. The stator resistance used for these results was 0.075 pu and the d.c. link braking resistance was 0.3125 pu. The case for infinite inertia was chosen for simplicity. Fig 7.2 shows the root locus of the dominant pole drawn to an enlarged scale.
<table>
<thead>
<tr>
<th>Output $y$</th>
<th>$c$ vector</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator voltage amplitude</td>
<td>$[0,0,0,0,d,0]$</td>
<td></td>
</tr>
<tr>
<td>Stator current amplitude</td>
<td>$\begin{bmatrix} i_{qso} \ i_{dso} \ 0,0,0,0 \end{bmatrix}$</td>
<td>$i_{ss0} = i_{qso}^2 + i_{dso}^2$</td>
</tr>
<tr>
<td>Real component of stator</td>
<td>$[1,0,0,0,0,0]$</td>
<td></td>
</tr>
<tr>
<td>current</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stator Power</td>
<td>$\left[ \frac{3}{2} d \ w_e, 0,0,0,0, \frac{3}{2} i_{qso}, 0 \right]$</td>
<td></td>
</tr>
<tr>
<td>Air gap flux linkages</td>
<td>$\begin{bmatrix} (i_{qso} + i_{qro}), (i_{dso} + i_{dro}), (i_{qso} + i_{qro}), (i_{dso} + i_{dro}) \end{bmatrix}$</td>
<td>$i_{mo} = (i_{qso} + i_{qro})^2 + (i_{dso} + i_{dro})^2$</td>
</tr>
<tr>
<td>Electrical Torque</td>
<td>$\begin{bmatrix} i_{dro}, -i_{qro}, -i_{dso}, i_{qso}, 0,0 \end{bmatrix} \frac{X_m}{w_b}$</td>
<td></td>
</tr>
<tr>
<td>Mechanical Speed</td>
<td>$[0,0,0,0,1]$</td>
<td></td>
</tr>
</tbody>
</table>
TABLE IV

Transfer function for inverter-fed induction motor with dc-link voltage to frequency feedback.

Operating conditions

$i_{so} = 0 \quad w_{ro} = 1 \text{ pu} \quad w_{eo} = 0.99972 \text{ pu} \quad T_{eo} = -0.0095 \text{ pu}$

Transfer function between a change in source current and a change in electrical torque

<table>
<thead>
<tr>
<th>Gain</th>
<th>Poles pu</th>
<th>Zeros pu</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5743</td>
<td>$-0.121 + 0.914 \text{ j}$</td>
<td>$-0.163 + 0.915 \text{ j}$</td>
</tr>
<tr>
<td></td>
<td>$-0.121 - 0.914 \text{ j}$</td>
<td>$-0.163 - 0.915 \text{ j}$</td>
</tr>
<tr>
<td></td>
<td>$-0.43 + 2.68 \text{ j}$</td>
<td>$-0.454$</td>
</tr>
<tr>
<td></td>
<td>$-0.43 - 2.68 \text{ j}$</td>
<td>$-0.458$</td>
</tr>
</tbody>
</table>
FIG 7.1 Root loci for $R_s = 0.075\, \text{pu}$ and $R_g = 0.3125\, \text{pu}$
CHAPTER 8

LIMITS TO BRAKING

8.1 ANALYSIS OF BRAKING MODE

8.2 PARTICULAR CASES

8.2.1 Infinite Braking Resistance Case

8.2.2 Finite Braking Resistance Case - Neglecting Leakage Reactance

8.2.3 Finite Braking Resistance Case - Including Leakage Reactance Effects

8.2.4 Limiting Value of Braking Resistance

8.3 PRACTICAL BRAKING LIMITS
8.0 **LIMITS TO BRAKING**

In general system description given in Chapter 3 various modes of motoring and braking were described. In the rail traction environment safety considerations are paramount and so great emphasis is placed on braking performance. This emphasis is reflected in this work; the remainder of which concentrates on operation in the braking region. The general approach and techniques are, however, applicable to the other operating modes.

Braking integrity must be maintained even in the event of supply failure and this necessitates that the inverter – induction machine system is capable of maintaining self-excitation. To achieve this the frequency is derived from the dc-link voltage \( V_d \) using a voltage controlled oscillator (v.c.o.). If the source current fails the dc-link capacitor tends to discharge into the motor causing the dc link voltage, \( V_d \), (Fig 5.4) to fall. As \( V_d \) falls the output frequency of the v.c.o. falls. Inertia maintains the rotor speed approximately constant and so the motor slip reduces and then goes through zero to a negative value. As the slip goes negative the motor begins to regenerate and power flows into the dc-link capacitor tending to restore \( V_d \) to its original level. In this way, in the event of supply failure, the system uses its kinetic energy to supply the electrical losses in the system. The addition of the braking resistor means that the power being dissipated is higher and therefore the retardation torque is higher. Braking effort can be controlled by changing the value of the braking resistor. However, control is more easily achieved by varying the voltage to frequency ratio \( V/f \) of the v.c.o.

The rotor speed defines the approximate value of frequency and so changes in the \( V/f \) ratio of the system produce changes in dc-link
voltage giving rise to changes in torque.

8.1 ANALYSIS OF BRAKING MODE

The requirement of braking in the absence of a power source implies a speed limit below which such braking cannot be achieved. To produce a braking torque the induction machine must be magnetised. In the braking mode the inverter acts as a source of reactive power for the induction machine. The magnetisation current flowing in the inverter and in the induction machine causes power losses. The absence of an electrical power source for these losses implies that they must be generated by the induction machine. Therefore there is a retardation torque associated with the electrical losses. If the air-gap flux in the induction machine is maintained approximately constant by using a constant V/f ratio there is an associated constant peak torque capability. If the magnetisation current is constant the associated power losses will also be approximately constant. Therefore the retardation torque caused by these losses will increase with decrease in speed to maintain the power constant. Thus there must be a speed below which the peak torque capability of the machine, under constant fluxing conditions, is insufficient to provide the losses incurred in magnetising the machine and self-excitation cannot be maintained.

The criterion for self-excitation being maintained is that there is net zero power flow in the dc-link.

Using the approach detailed in Section 7.3, referring to Fig 5.4 and assuming that steady-state conditions prevail it can be seen that:

\[ i_s = \frac{v_d}{R_L} + \frac{3}{\frac{2}{11}} i_{qs} \]
\[ i_s = \frac{\pi}{2} \frac{V_{qs}}{R_L} + \frac{3}{\pi^2} i_{qs} \]

\[ \frac{6R_L}{\pi^2} i_s' = v_{qs} + \frac{6R_L}{\pi^2} i_{qs} \]  

This leads to:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
6R_L \frac{i_s'}{\pi^2}
\end{bmatrix} = \begin{bmatrix}
R_s & wL_s & 0 & wM & -1 \\
-wL_s & R_s & -wM & 0 & 0 \\
0 & wM & R_T & wL_T & 0 \\
-\omega M & 0 & -\omega L_T & R_T & 0
\end{bmatrix} \begin{bmatrix}
i_{qs} \\
i_{ds} \\
i_{qr} \\
i_{dr}
\end{bmatrix}
\]

(8.1)

(8.2)

Where \( w_s \) is the slip frequency, this can be written as:

\[ \bar{u} = \bar{R} \bar{I} \]

For the zero power flow condition \( i_s = 0 \)

this implies either \( v_{qs} = 0 \) or \( \bar{R} \) is singular. For the non-trivial case, \( \text{Det}(\bar{R}) = 0 \).

Therefore:

\[ w^2 \left( w_s^2 \left( M^2 - L_s L_T \right)^2 + L_s^2 R_T^2 \right) + w w_s R_T M^2 (2 R_s + R_B) \]

\[ + \left( R_T^2 + w_s^2 L_T^2 \right) R_s \left( R_s + R_B \right) = 0 \]  

(8.3)

where \( R_B \) is the value of the dc-link braking resistor referred to the ac terminals of the motor.

The relationship is:

\[ R_B = \frac{6}{\pi^2} R_L \]  

(8.4)
Solving the quadratic equation (8.3) gives:

\[ w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]  

(8.5)

\[ a = w_s^2 (M^2 - L_s L_r)^2 + L_s^2 R_r^2 \]

where \( b = w_s R_r M^2 (2R_s + R_B) \)

\[ c = (R_r^2 + w_s^2 L_r^2) R_s (R_s + R_B) \]

8.2 PARTICULAR CASES

8.2.1 Infinite braking resistance case

If \( R_B \) is open-circuit the zero power flow criterion implies that:

\[ w = -R_s w_s \frac{L_r^2}{R_r} \frac{w_s}{2} - \frac{R_s R_r}{w_s M^2} \]  

(8.6)

Differentiating equation (8.6) with respect to \( w_s \) and equating to zero leads to the value of \( w_s \) for minimum supply frequency.

i.e. \( w_s(\text{crit}) = -\frac{R_r}{L_r} \)  

(8.7)

This occurs at the supply frequency:

\[ w(\text{crit}) = 2R_s \frac{L_r}{M^2} \]  

(8.8)

and therefore the rotor speed \( w_r(\text{crit}) \) is given by:

\[ w_r(\text{crit}) = 2 R_s \frac{L_r}{M^2} \frac{L_r}{L_r} \]  

(8.9)

Thus self-excitation cannot be maintained at a supply frequency less than \( w \) (crit).
This analysis is independent of the level of flux in the machine assuming that saturation is not reached. Saturation effectively reduces the size of the magnetisation inductance $M$ and so tends to increase the critical value of rotor speed.

### 8.2.2 Finite braking resistance case - neglecting leakage reactance

If $R_B$ is finite and if the leakage reactances are considered to be negligible a procedure similar to that in Section 8.2.1 leads to equations (8.10), (8.11) and (8.12)

\[
\omega_s(\text{crit}) = -\frac{R_r}{M} \left(1 + 2R_s \right) \quad (8.10)
\]

and

\[
\omega(\text{crit})_{\text{min}} = \frac{2R_s}{M} \left(1 + R_s \right) \quad (8.11)
\]

or

\[
\omega(\text{crit})_{\text{max}} = \frac{R_B}{M} \left(1 + 2R_s + 2R_s^2 \right) \quad (8.12)
\]

The critical rotor speeds $\omega_r(\text{crit})_{\text{max}}$ are given by equations (8.13) and (8.14)

\[
\omega_r(\text{crit})_{\text{min}} = \frac{2R_s}{M} \left(1 + \frac{R_r}{2R_s} + \frac{R_s + R_r}{R_B} \right) \quad (8.13)
\]

\[
\omega_r(\text{crit})_{\text{max}} = \frac{R_B}{M} \left(1 + 2R_s + 2R_s^2 \right) + \frac{R_r}{R_B} \left(1 + 2R_s \right) \quad (8.14)
\]

Thus for a finite value of braking resistance there is a range of frequencies outside which self-excitation cannot be maintained.

### 8.2.3 Finite braking resistance case - including leakage reactance effects

The values for $\omega_r(\text{crit})_{\text{min}}$ obtained using equation (8.13) were found to be in good agreement with values determined using equation 8.5. However, values for $\omega_r(\text{crit})_{\text{max}}$ obtained using equation (8.14)
were found to be very different from the values predicted using equation 8.5. This indicates that the assumption of negligible leakage reactance is not valid in this case.

Solving equation 8.3 as a quadratic in $w_s$ leads to the requirement that for a solution to exist the inequality 8.15 must be satisfied.

\[
(2R_s + R_B)^2 \frac{M^4}{4} \geq ((L_s L_T - M^2)^2 + L_r^2 R_s (R_s + R_B)). \frac{w^2}{w^2} (w^2 L_s^2 + R_s (R_s + R_B))
\]  

(8.15)

Manipulation converts this into inequality (8.16)

\[
w R_B \frac{M^2}{2} \geq (w^2 (L_s L_T - M^2) L_s + R_s (R_s + R_B) L_T) \quad (8.16)
\]

Setting the two sides equal leads to the limiting value of $w$ that satisfies the inequality (8.16).

\[
w(\text{crit}) = R_B \frac{M^2}{2} \left( 1 + \sqrt{1 - \frac{16(L_s L_T - M^2) L_s L_T R_s (R_s + R_B)}{R_B^2 M^4}} \right) \quad (8.17)
\]

If $R_B$ is two or three times greater than $R_s$ this can be approximated by:

\[
w(\text{crit})_{\text{max}} = R_B \frac{M^2}{2} \left( \frac{R_B}{2L_s K} \right) \quad (8.18)
\]

where $K = \frac{L_s L_T}{M^2} - 1$

Substitution for $w$ from equation 8.18 into equation 8.3 and assuming $\frac{L_s}{M} \ll \frac{L_T}{M} \ll 1$ leads to:

\[
w_s(\text{crit}) = -\frac{R_T}{\frac{1}{MK} (2e + 1)}
\]  

(8.19)
where $e = \frac{R_s}{R_B}$

Combining equations 8.18 and 8.19 gives:

$$w_{r(crit)}_{\text{max}} \approx \frac{R_B}{K^M} \left( \frac{1}{2} + \frac{R_r}{(2R_s + R_B)} \right)$$  \hspace{1cm} (8.20)

### 8.2.4 Limiting value of braking resistance

For small values of braking resistance the minimum frequency for self-excitation, $w_{r(crit)}_{\text{min}}$, given by equation 8.11 becomes approximately equal to the maximum frequency for self-excitation $w_{r(crit)}_{\text{max}}$ (Equation 8.18) indicating that self-excitation is not possible at all.

From equation 8.5 it can be seen that the condition for self-excitation to be possible is given by the inequality 8.21.

$$\left( w_s^2 R_r M^2 (2R_s + R_B) \right)^2 \geq 4(w_s^2 (M^2 - L_s L_r)^2 + L_s^2 R_r^2). \hspace{1cm} (8.21)$$

or:

$$\frac{R_r^2 M^4 (2R_s + R_B)^2}{4 R_s (R_s + R_B)} \geq \left( \frac{L_s L_r - M^2}{L_r} \right)^2 + \frac{L_s^2 R_r^2}{w_s^2 R_r^2}(R_r^2 + w_s^2 L_r^2) + \frac{L_s^2 R_r^2}{w_s^2 R_r^2}(R_r^2 + w_s^2 L_r^2)$$

Differentiation with respect to $w_s$ of the right hand side of this inequality and equating to zero indicates that the right hand side is a minimum when $w_s^2 = \frac{L_s R_r^2}{L_r (L_s L_r - M^2)}$ \hspace{1cm} (8.22)

Substitution into the inequality 8.21 gives:

$$\frac{(2R_s + R_B)^2}{R_s (R_s + R_B)} \geq 4(2L_s L_r - 1)$$ \hspace{1cm} (8.23)
This gives the limiting value of braking resistance

\[ R_{\text{b}}^{\text{min}} = 8 R_s m \left( 1 + \sqrt{1 + \frac{1}{4m}} \right) \]  

(8.24)

where \[ m = \frac{L_s I_r}{M^2} \left( \frac{L_s I_r}{M^2} - 1 \right) \]

Figure 8.1 shows the results from equations (8.9), (8.13), (8.20) and (8.24) plotted against the dc-link braking conductance \[ \frac{1}{R_L} \].

Additionally equation 8.5 was used to plot the boundary of self-excitation. It can be seen that equations (8.13), (8.20) and (8.24) are useful in describing this boundary using relatively simple expressions.

At rotor speeds between \( \omega_r^{(\text{crit})_{\text{min}}} \) and \( \omega_r^{(\text{crit})_{\text{max}}} \) there will be two inverter frequencies at which the zero power flow criterion may be satisfied. Fig 8.2 shows plots of \( \omega_e \) against \( \omega_r \) for various values of dc-link braking resistance. For any given value of braking resistance self-excitation can be maintained only with operating values of frequency and speed which lie within the area bounded by the appropriate curve.

8.3 PRACTICAL BRAKING LIMITS

In practice self-excitation fails at a speed significantly higher than \( \omega_r^{(\text{crit})} \). In their study of self-excitation in inverter fed induction machines Novotny et al (8.1) suggest that this is accounted for by the increased losses due to the non-sinusoidal waveform and by the losses in the inverter. This accounts for only a proportion of the discrepancy. In practice an instability appears as the rotor speed reduces and de-excitation occurs due to this instability. Fig 8.3
FIG 8.1 THEORETICAL AND APPROXIMATE BOUNDARIES TO SELF-EXCITATION
FIG 8.2 SELF-EXCITATION BOUNDARIES IN THE FREQUENCY/SPEED PLANE
FIG 8:3 DYNAMIC DE-EXCITATION OF REAL SYSTEM WITH CONSTANT ROTOR SPEED
shows recordings of the inverter waveforms during such instability and the resultant de-excitation. Table IV shows some result taken from the practical system and compares them with the speed predicted by steady state theory (Section 8.1).

### TABLE IV

<table>
<thead>
<tr>
<th>Test Conditions</th>
<th>Speed at de-excitation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux in the induction machine</td>
<td></td>
</tr>
<tr>
<td>Low flux</td>
<td>Not connected</td>
</tr>
<tr>
<td>Rated flux</td>
<td>Not connected</td>
</tr>
<tr>
<td>Saturation flux</td>
<td>Not connected</td>
</tr>
<tr>
<td>Rated flux (0.87) pu</td>
<td></td>
</tr>
<tr>
<td>Rated flux (0.8) pu</td>
<td></td>
</tr>
</tbody>
</table>

The semi-conductor devices in the inverter have an approximately constant voltage drop when they are conducting. As the fluxing level in the machine is reduced the current levels in the machine reduce proportionately and the associated losses change in proportion to the square of the flux. However, the losses in the inverter are approximately proportional to current and therefore flux and so the losses in the inverter are more important at low levels of fluxing. This effect causes the de-excitation speed of the low fluxed machine to be higher than the machine with rated flux. If the machine is in saturation the apparent mutual inductance decreases and this has the effect of raising the de-excitation speed.
CHAPTER 9

VALIDATION

9.1 STEADY-STATE VALIDATION OF THE ANALOGUE MODEL

9.2 DYNAMIC VALIDATION OF THE ANALOGUE MODEL

9.3 VALIDATION OF THE TRANSFER FUNCTION ANALYSIS

9.4 VALIDATION OF THE STATE-VARIABLE WAVEFORM ANALYSIS
9.0 VALIDATION

Before using models of physical systems, it is important to compare the behaviour of the model with that of the real system in order to ensure that the model is valid. In practice the validation can be against either the real system or another previously validated model. Both methods are used in this study.

The following comparisons between the system and the models have been made:

(i) The analogue model was validated against the real system in the steady-state.

(ii) The analogue model was validated against the real system in the dynamic condition of de-excitation (Section 8.3).

(iii) The transfer function analysis (Chapter 7) of the system was compared with the analogue model for the 180° inverter.

(iv) The state-variable waveform analysis (Chapter 6) was validated against the real system and against the analogue simulation.

9.1 STEADY STATE VALIDATION OF THE ANALOGUE MODEL

The experimental induction machine was supplied by the inverter at a frequency of 1 pu and with a 0.05 pu slip. The dc-link voltage was 0.83 pu and the measured average torque was 0.18 pu. This measured torque was 20% less than the model prediction, but this magnitude of error can be expected when the assumptions used in deriving the model are considered. Losses in the inverter plus hysteresis, eddy currents and stray losses account for this discrepancy. In this respect the simulation is similar to the conventional equivalent circuit approach. The waveforms taken from the real machine are reproduced in Fig 9.1 and those for the simulation under equivalent
FIG 9.1 WAVEFORMS FROM THE REAL SYSTEM
AT 5% SLIP
operating conditions are shown in Fig 9.2. It can be seen that there is excellent correlation between the two sets of results. The presence of commutation transients and phase-controlled converter supply ripple are apparent in Fig 9.1.

9.2 DYNAMIC VALIDATION OF THE ANALOGUE MODEL

The performance of the analogue model was compared with that of the real system for the condition of dynamic de-excitation. The mechanical behaviour of the torque tube and stator assembly (Section 3.5) was included in the analogue model. In order to demonstrate the effects of the mechanical resonance, the resilient coupling described in Section 3.5 was removed and the torque tube was connected rigidly to mechanical earth. In order to prevent destructively large torques building up under resonance conditions, the motor was operated with a low voltage to frequency ratio. This had the effect of making the torque ripple extremely small and so the resultant amplified torque at resonance was kept within the machine mechanical design limits.

The mechanical resonant frequency of the stator was found experimentally to be 34 Hz and so the design values of stator inertia and torque tube stiffness were modified to make the equivalent resonant frequency of the analogue model 34 Hz. The inverter voltage to frequency ratio was set to give 1 pu frequency at .57 pu dc-link voltage. This was chosen to ensure that the torque ripple amplitude was less than 2% of the rated torque of the machine.

The motor was run unloaded at 1 pu frequency and steady-state conditions were achieved. The dc-source was then removed causing the system to go into the self-excited mode of operation. Friction and the retardation torque due to the electrical losses caused the machine to reduce in speed. Fig 9.3 shows traces taken of dc-link voltage,
FIG 9.2 WAVEFORMS FROM SIMULATION AT 5% SLIP
FIG 9.3 DYNAMIC DE-EXCITATION OF THE SYSTEM
induction machine line voltage, dc-link current, induction machine line current and torque in the torque tube. Unfortunately the latter recording is corrupted by an intermittent fault in the telemetry system. However, most of the trace remains intelligible.

The reduction in speed caused a reduction in both inverter frequency and inverter dc link voltage. As the voltage to frequency ratio was fixed, the flux in the machine was nominally constant and so the line current drawn by the machine was also constant. As the frequency of the torque pulsation is six times the inverter frequency the point was reached where the torque pulsation frequency approached the resonant frequency of the stator assembly. This caused the stator assembly to be mechanically excited and the resultant torque measured in the torque tube reached .7 pu which was greater than the design figure for machine output torque (.66 pu) and approaching the mechanical design torque (1.0 pu). During this vibration the movement of the stator affected the electrical excitation of the machine and some of the effects can be seen in the recordings of dc-link current and voltage. The machine continued to slow and eventually the stator mechanical excitation frequency was passed. The system then dynamically de-excited.

The analogue model was then used with equivalent operating conditions and the corresponding results are reproduced in Fig 9.4. It can be seen that whilst there are some superficial differences between the model response and that of the real system there is good correspondence on the major features.

9.3 VALIDATION OF THE TRANSFER FUNCTION ANALYSIS

The transfer function analysis was validated using the analogue computer model of the 180° square wave inverter. With this
FIG 9.4 SIMULATED DYNAMIC DE-EXCITATION OF THE SYSTEM
type of inverter the output voltage is fully defined by the dc-link voltage and the inverter firing information. This implies that the harmonic content of the output voltage waveform is, to a first order approximation, independent of load conditions and so the assumptions used in the transfer function analysis are reasonably valid.

The system was modelled on the analogue computer using effectively infinite inertia. The rotor speeds at de-excitation for various values of braking resistance were measured. These speeds are plotted in Fig 9.5 against the dc-link braking conductance.

The system transfer function was calculated for a range of braking resistance values and rotor speeds. The values of rotor speed for which the real part of the dominant pole became positive were found. The real part of the dominant pole becoming positive indicates the onset of instability. The values of rotor speed for this onset of instability were plotted in Fig 9.5. It can be seen that the predictions from the transfer function analysis are optimistic compared with those from the analogue simulation but there is agreement on the general form of the results. The fact that the transfer function analysis does not include losses due to the harmonic content of the inverter output waveform accounts for some of the difference between the two sets of results. However, the assumption that the inverter output waveform is effectively a sine wave implies that there will be no delay between a change in inverter frequency demand and an actual change in inverter output frequency. In practice in the square wave inverter there will be such a delay. For example, if the inverter frequency is $w_e$ the period between changes of inverter output state will be $\frac{1}{6w_e}$. If half way through such a period the frequency demand is changed by a small amount $dw_e$, there will be a delay of approximately $\frac{1}{12w_e}$ before the next output event can indicate a change in frequency to the motor.
FIG 9.5 VALIDATION OF TRANSFER FUNCTION ANALYSIS

KEY
RESULTS FROM ANALOGUE SIMULATION OF A 180° INVERTER.
RESULTS FROM TRANSFER FUNCTION ANALYSIS.
There is, therefore, a delay between changes in frequency demand and change in actual frequency. The magnitude of this delay lies between zero and $\frac{1}{\omega_e}$ and will depend upon the time and amplitude of the change in frequency demand. As this delay is, on average, inversely proportional to inverter frequency, the effects of the delay become greater at low frequencies. The delay in the feedback between dc-link voltage and frequency has an adverse effect on stability and therefore it is to be expected that the transfer function analysis, which does not include the delay, will give optimistic results in terms of stability at low frequencies.

9.4 VALIDATION OF THE STATE VARIABLE WAVEFORM ANALYSIS

A computer programme using the analysis detailed in Chapter 6 was developed and used to produce the waveforms shown in Fig 9.6. The conditions used were those detailed in Section 9.1 except that the dc-link voltage used was 1.57 pu. As magnetic linearity is assumed, detailed comparison can be made by scaling all electrical quantities by $\left(\frac{0.83}{1.57}\right)$ and by scaling torque by $\left(\frac{0.83}{1.57}\right)^2$. It can be seen that the correlation between this analysis and the analogue simulation is very good. This is because the same assumptions were used for both models. It can also be seen that the results correlate well with those of the real system. The discrepancies between the waveforms in Fig 9.1 and those in Fig 9.6 are again due to hysteresis, eddy current and stray losses.
FIG 9.6. WAVEFORMS FOR STATE VARIABLE INVERTER ANALYSIS AT 5% SLIP
CHAPTER 10

INVESTIGATION OF THE ZERO-POWER FLOW CONDITION

10.1 STEADY-STATE LIMITS TO SELF-EXCITATION

10.2 STABILITY LIMITS TO SELF-EXCITATION
10. INVESTIGATION OF THE ZERO POWER FLOW CONDITION

The various techniques described in previous chapters were used to investigate the system operation in the self-excited mode.

10.1 STATIONARY LIMITS TO SELF-EXCITATION

The real system was operated in an open-loop mode. Frequency was independent of dc-link voltage and the limiting rotor speeds for zero power flow were measured. Great care had to be taken to prevent excessive saturation of the induction machine and the associated destructively high currents.

The inverter used has an auxiliary electrical supply for the commutation circuitry. This supply in association with the commutation process injects pulses of energy into the terminals of the induction machine. Under normal operation these pulses have negligible effect. However, if the machine is operated at a suitable slip these pulses of energy are sufficient to initiate the process of self-excitation. Self-excitation in the induction machine would then occur at the appropriate externally applied speed and inverter frequency, thus indicating the minimum rotor speed for self excitation.

The results from this investigation are shown in Fig. 10.1. Limiting rotor speeds for self-excitation are plotted against the reciprocal of the dc-link braking resistance. Also plotted in Fig. 10.1 are the boundaries predicted by equation 8.17 and the boundaries predicted by the analogue simulation. It can be seen that there is appreciable discrepancy between the measured and predicted values. However, in the analysis of the inverter the inverter was assumed to be without losses. In practice the real inverter has losses associated with semi-conductor junction potentials and these losses
FIG 10.1 STEADY STATE LIMITS TO SELF EXCITATION

\( R_s = 0.0414 \text{ pu.} \)
add to the losses in the stator windings. If the value of the stator resistance used in the analogue model is increased to partly compensate for the losses in the inverter much better correlation between the real system performance and the performance predicted by the simulation can be achieved. Fig. 10.2 is similar to Fig. 10.1 except that the machine stator resistance is assumed to be 0.075 pu. The order of this increase in resistance can be justified by the following calculation:

\[
\text{Inverter rated dc link voltage} = 650 \text{ V} \\
\text{Inverter rated power} = 250 \text{ Kw} \\
\text{Therefore Inverter rated dc-link current} = 384 \text{ A}
\]

Assuming an efficiency of 96% for the inverter:

\[
\text{Inverter power loss} = 10 \text{ kW}
\]

Therefore the equivalent dc series resistance in the dc link,

\[
R_{dc} = \frac{\text{loss}}{(\text{current})^2} = 0.068 \text{ pu. This gives an equivalent ac phase resistance } R_{ac} = R_{dc} \frac{x_0}{n^2} = 0.036 \text{ pu, which leads to a total stator resistance in the order of 0.075 pu.}
\]

10.2 STABILITY LIMITS TO SELF-EXCITATION

The real system was operated with dc-link voltage to frequency feedback. Measurements were made of the rotor speed at which dynamic de-excitation took place for several values of braking resistance. The results are plotted as points in Fig. 10.3.

The analogue model was used to simulate the system under similar conditions and Fig. 10.3 shows the rotor speeds at dynamic de-excitation plotted against dc-link braking resistor conductance. The discrete nature of the braking resistance limited the number of points for the real system which could be taken. However, there is excellent correlation between the results of the simulation and those of the real
FIG 10.2 STEADY STATE LIMITS TO SELF EXCITATION

Rs = 0.075pu
KEY

--- SIMULATION OF 180° SQUARE WAVE INVERTER

X MEASURED

--- SIMULATION OF 120° SQUARE WAVE INVERTER

FIG 10.3 DYNAMIC LIMITS TO SELF-EXCITATION

$$Rs = 0.075_{pu}$$
system. The high de-excitation speeds of low levels of braking (low braking conductance) are due to the effects of the notch in the 120° waveform. The size of this notch is dependant upon the induction machine operating condition. In particular the size of the notch is dependant upon the effective phase angle of the induction machine. If the phase angle is large, the notch disappears entirely and the inverter output waveform becomes similar to that of the 180° inverter. Further increases in phase angle have no effect on the inverter output waveform. Thus the notching has the effect of a non-linear output impedance of the inverter. This non-linearity has a destabilising effect on the system and increases the minimum rotor speed for self-excitation. Increasing the braking effort, i.e. decreasing the braking resistance, has the effect of reducing the size of the notch and therefore its effect. As shown in Fig. 10.3 the rotor speeds for de-excitation reduces with increasing braking resistor conductance until the notch disappears entirely. Further increase in braking conductance causes the rotor speed for de-excitation to increase. For comparison purposes comparable results for a 180° inverter are also shown in Fig. 10.3. The value of stator resistance of 0.075 was used for the simulation results. This value included an allowance to take into account some of the effects of inverter losses (see Section 10.1).
CHAPTER 11

CONCLUSIONS
11. CONCLUSIONS

11.1 SUMMARY OF ANALYSES

The dynamic behaviour of inverter-fed induction machine systems has been studied. Five methods of analysis were used:

(a) An analogue computer model of the inverter-fed induction machine system suitable for investigating prototype inverter firing algorithms was developed.

(b) An alternative computer model of the inverter was developed for the case of inverters in which the active devices have permitted periods of conduction which are well defined.

(c) State-variable analysis was used to develop a digital computer program for the prediction of steady-state current and voltage waveforms of the system. Constant rotor speed was assumed.

(d) State-variable analysis was used to develop a digital computer program which produced transfer functions of the system.

(e) The steady-state limits to self-excitation were investigated analytically in terms of frequency and braking resistance.

The model (a) is capable of realistically simulating load-induced commutation. The model (b) uses appreciably fewer logic and switching components than model (a) and is therefore implemented more easily. The lower number of computing elements implies increased reliability.

The use of these analyses was demonstrated by investigating the behaviour of the system in the self-excited mode at low rotor speeds.
11.2 CONCLUSIONS

(1) The digital computer program (c) required a two dimensional iteration and therefore used appreciable computer central processor time (in the order of one minute using an IBM 370/168). The results from this program are easily organised into a form suitable for harmonic analysis and rms value determination.

(2) The digital computer program (c) produced similar results to those produced by the model (b) when that model was operated with assumed infinite inertia.

(3) It was found that the assumption that frequency can change instantaneously was implicit in the transfer function analysis (d) of the system and was not valid for the type of inverter under investigation. The errors caused by the invalidity of this assumption become large for low inverter frequencies.

(4) The assumption used in (d) that the dynamic behaviour of the system is independent of inverter waveform changes due to loading effects is valid for a 180° square wave inverter but is invalid for a 120° square wave inverter for some regions of operation.

(5) In general transfer function analysis is useful when devising stabilising control schemes but does not give a great deal of insight into the transient behaviour of the system.

(6) The steady-state limits to self-excitation are markedly different from the limits imposed by the system dynamics at low rotor speeds and low values of braking resistance.

(7) For high values of braking resistance and the upper limiting values of rotor speed the dynamic and steady-state limits are similar.
(8) The analogue models give results which are as good as or, in most cases, better than the alternative analyses when operated with similar constraints.

(9) The analogue models give results for the system dynamics which correlate well with those from the real system. Improved correlation can be achieved by making allowance for inverter losses. This can be implemented simply by assuming an appropriate increase in effective stator resistance.

(10) The steady-state waveforms given by state variable analysis are restricted to the constant speed case whereas the analogue models can also cope with the case where torque ripple can vary the rotor speed.

(11) Modifications to the analogue model can be readily effected; for example the dynamic mechanical behaviour of the stator and torque tube was included for some parts of the investigation.

(12) The analogue model provides a good interface between the investigator and the system under study. The model can be partitioned easily for checking purposes. Interaction between the investigator and the model is readily achieved as the high solution speed of the analogue computer can give rapid results for a change in a system parameter. This is not possible with the current generation of digital computers, when a study of a dynamic system of this size and type is undertaken.

(13) Potentially dangerous operating conditions, such as stator mechanical resonance, can be investigated safely with an analogue computer model of the system.
11.3 SUGGESTIONS FOR FURTHER WORK

Further work in this field could include the design and implementation of alternative control strategies. Of particular interest in this respect is the application of microprocessors to the control of inverter-fed induction machines which should lead to better utilisation of electronic power components and to better utilisation of the motor for little extra expense. The design and commissioning of such a micro processor controller would be facilitated by using a system analogue model. The micro processor could be linked into the analogue computer enabling the electronics and programming to be "de-bugged" without putting expensive hardware at risk.

Problems in program timing could be isolated effectively from problems in program operation by running the analogue computer with a timescale slower than real time.

The techniques described in this work could be applied to the study of other types of inverter such as the Pulse Width Modulated voltage fed inverters or current source inverters.
APPENDIX A

MOTOR DESIGN DATA
DESIGN DETAILS FOR EXPERIMENTAL TAM

Rating
Max line voltage, V 430
Max line current, A 293
No of phases 3
Max frequency, Hz 75
Max speed, rev/min 1480
No of poles 6
Insulation Class H

Loading
Specific magnetic loading, T 0.46
Specific electric loading, A/m 71 000
\(D^2L\) product, m³ 0.107

Main Dimensions
Diameter at the air gap, mm 378
Gross core length, mm 750
Pole pitch, mm 198
No of ducts, radial 1
Width of duct, mm 25
Iron length, mm 689
Air gap, mm 1

/Continued......
### Stator Details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inside diameter, mm</td>
<td>248</td>
</tr>
<tr>
<td>Type of winding</td>
<td>Double layer</td>
</tr>
<tr>
<td>Connection</td>
<td></td>
</tr>
<tr>
<td>No of parallel paths, a) series mode</td>
<td>3</td>
</tr>
<tr>
<td>b) parallel mode</td>
<td>6</td>
</tr>
<tr>
<td>No of turns per phase in series</td>
<td></td>
</tr>
<tr>
<td>a) series mode</td>
<td>48</td>
</tr>
<tr>
<td>b) parallel mode</td>
<td>24</td>
</tr>
<tr>
<td>No of slots</td>
<td>54</td>
</tr>
<tr>
<td>No of slots/pole/phase</td>
<td>3</td>
</tr>
<tr>
<td>Winding factor</td>
<td>0.94</td>
</tr>
<tr>
<td>Slot pitch, mm</td>
<td>21.99</td>
</tr>
<tr>
<td>Slot size, mm</td>
<td>11.5 x 30</td>
</tr>
<tr>
<td>Conductors per slot</td>
<td>16</td>
</tr>
<tr>
<td>Conductor size, mm</td>
<td>4.9 x 2.55</td>
</tr>
<tr>
<td>Current density, A/mm$^2$</td>
<td>7.8</td>
</tr>
<tr>
<td>Mean length of turn, m</td>
<td>2.05</td>
</tr>
</tbody>
</table>

### Rotor Details

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter, mm</td>
<td>450</td>
</tr>
<tr>
<td>Type of winding</td>
<td>Cage</td>
</tr>
<tr>
<td>No of Slots</td>
<td>62</td>
</tr>
<tr>
<td>Slot pitch, mm</td>
<td>19.25</td>
</tr>
<tr>
<td>Conductor cross section, mm$^2$</td>
<td>100</td>
</tr>
</tbody>
</table>
APPENDIX B

PER-UNIT NOTATION.

The per-unit system was used to simplify the scaling problems for the analogue computer and to render the results, as far as possible, independant of actual system size and rating.

All rotor quantities are referred to the stator.

The base values used were chosen at the point of transition between maximum torque operation and constant power operation (the so-called "knee-point").

Base current, \( I_B \) = peak value of rated phase current = \( \sqrt{3} \times 220 = 311.127 \) A

Base voltage, \( V_B \) = peak value of rated phase voltage = \( \sqrt{3} \times 250 = 353.553 \) V

Base frequency \( f_B = 12.5 \) Hz

Base electrical angular frequency \( \omega = 2 \pi f_B = 78.5398 \) rad/sec

Base time \( t_B = \frac{1}{\omega} = \frac{1}{\omega_B} = 0.0127323 \) sec

Base mechanical angular frequency \( \omega_{mb} = \frac{\omega}{p} = 26.18 \) rad/sec.

Pole Pairs.

Base Power, \( P_B = \) Rated apparent power = \( 3 \times \frac{V_B I_B}{\sqrt{2}} = \frac{3}{2} \times 2 \times 353.553 \times 311.127 = 165 \) kW

Base Torque = \( \frac{P_B}{\omega_{mb}} = \frac{6302.5}{26.18} = 6.302.5 \) Nm
Base Impedance = \frac{\text{Base Voltage}}{\text{Base current}} = \frac{V_B}{I_B} = 1.13636 \text{ ohms.}

Base Inertia = \frac{\text{Base torque x Base time}}{\text{Base mechanical speed}} = \frac{T_B \times t_B}{w_{MB}} = \frac{P_B}{w_B(w_{MB})^2} = 3.0652 \text{ kg m}^2

The per-unit values of the system parameters were calculated to be:

\[ R_S = 0.04136 \]
\[ R_R = 0.04136 \]
\[ L_S = 2.55726 \]
\[ L_R = 2.52629 \]
\[ M = 2.48814 \]
\[ J_{Stator} = 3.1424 \]
\[ K_{Stator} = 43.353 \]
\[ W_{os} = 3.7143 \]
\[ J_{rotor} = 14.56 \]
APPENDIX C

STATE - VARIABLE ANALYSIS

C.1  BASIC CONCEPTS

C.1.1  The System State

C.1.2  A State Variable

C.1.3  A Controllable System

C.1.4  An Observable System

C.1.5  The State-Equations

C.2  TRANSIENT RESPONSE

C.2.1  Unforced response

C.2.2  Forced response
C. **STATE VARIABLE ANALYSIS**

State variable analysis (C.1) is a convenient tool for the study of the transient and steady-state response of \( n \)th order systems. It can also be utilised for the production of the transfer functions of such systems and this facilitates the study of system stability and control.

The state-variable representation of an \( n \)th order system results in \( n \) first order differential equations which describe the behaviour of the system.

C.1 **BASIC CONCEPTS**

C.1.1 **The System State**

The system state is a physical concept. At any time \( t_0 \) the system state represents the minimum amount of system information which is required to determine the system response to inputs at any time after \( t_0 \) without any additional knowledge of the form of the inputs before \( t_0 \).

C.1.2 **A State Variable**

A state variable is one of the set of \( n \) variables the knowledge of which constitutes the system state. At \( t_0 \) the values of all the state variables of a system are usually called the system initial conditions. To a certain extent the choice of the state variables is arbitrary and the possibility exists of an inadequate set being chosen. However, there are standard checks which can be applied to ensure that the choice of state variables makes the system controllable and observable.
C.1.3 Controllable System

A system is said to be controllable if any initial state can be transferred to any final state in a finite time by some control. Controllability implies that each state variable is affected by the system inputs.

C.1.4 Observable System

A system is said to be observable if every state variable at any time $t_0$ can be determined from knowledge of the output over a finite interval of time $t_0 < t < t_f$.

In other words, a system is observable when each state variable can influence the output of the system.

C.1.5 State Equations

It is convenient to use matrix notation to write the n first order differential equations which result from the state-variable analysis of an $n^{th}$ order system:

$$\dot{x}(t) = \bar{A} \ x(t) + \bar{B} \ u(t) \quad (C.1.1)$$
$$y(t) = \bar{C} \ x(t) + \bar{D} \ u(t) \quad (C.1.2)$$

where

- $\bar{x}(t)$ is an n dimensional vector of state variables
- $\bar{u}(t)$ is an r dimensional input vector
- $\bar{y}(t)$ is an m dimensional output vector
- $\bar{A}$ is an n by n system matrix
- $\bar{B}$ is an n by r input matrix
- $\bar{C}$ is an m by n output matrix
- $\bar{D}$ is an m by r direct coupling matrix (which is often a null matrix)
In the above and subsequent work it is assumed that the system considered is either linear or linearised about an operating point. This assumption makes $\bar{A}$, $\bar{B}$, $\bar{C}$ and $\bar{D}$ time-invariant.

In the special case of the consideration of a single input and single output system the equations become:

$$\dot{x} = \bar{A} \bar{x} + \bar{B} u$$  \hspace{1cm} (C.1.3)

$$y = \bar{C} \bar{x} + d u$$  \hspace{1cm} (C.1.4)

where

$\bar{x}$ is an n dimensional time variant vector of state variables
$u$ is a time-variant input scalar
$y$ is a time-variant output scalar
$\bar{A}$ is an n by n time-invariant system matrix
$\bar{B}$ is an n dimensional time-invariant input column vector
$\bar{C}$ is an n dimensional time-invariant output row vector
$d$ is a direct-coupling scalar

C.2 TRANSIENT RESPONSE

As the state equations are n first order differential equations it is reasonable to expect the solution to be the sum of the unforced response and the forced response of the system.

C.2.1 Unforced Response

The unforced response of the system is the response of the equation:

$$\dot{x} = \bar{A} \bar{x}$$ \hspace{1cm} (C.2.1)

defining $\exp(\bar{A}t) = \sum_{i=0}^{\infty} \frac{(\bar{A}t)^i}{i!} = I + \bar{A}t + \bar{A}^2 \frac{t^2}{2!} + \ldots$ \hspace{1cm} (C.2.2)
Where \( \bar{I} \) is the unit diagonal matrix then

\[
\frac{d(\exp(At))}{dt} = \bar{A} + \bar{A}^2 t + \frac{\bar{A}^3 t^2}{2!} + \ldots
\]

\[
= \bar{A} \exp(At)
\]

Thus if the initial conditions are \( \bar{x}(0) \) then it can be seen that

\[
\bar{x} = \exp(At) \cdot \bar{x}(0)
\]

satisfies equation C.2.1

i.e.

\[
\dot{\bar{x}} = \bar{A} \exp(At) \cdot \bar{x}(0)
\]

for the general interval \( t_0 \) to \( t \) we get

\[
\bar{x}(t) = \exp(A(t-t_0)) \cdot \bar{x}(t_0)
\]

C.2.2 Forced Response

Using the standard result for a single order system as a guide let:

\[
\bar{x} = \int_{0}^{t} \exp(A(t-r)) \bar{B} u(r) \, dr
\]

This is a matrix convolution integral.

Differentiating gives:

\[
\dot{\bar{x}} = \exp(A(t)) \bar{B} u(t) + \int_{0}^{t} \frac{d}{dt} \exp(A(t-r)) \bar{B} u(r) \, dr
\]

therefore

\[
\dot{\bar{x}} = \bar{A} \bar{x} + \bar{B} u(t)
\]

\[
\dot{\bar{x}} = \bar{A} \bar{x} + \bar{B} u(t)
\]

i.e. \( \bar{x} = \int_{0}^{t} \exp(A(t-r)) \bar{B} u(r) \, dr \)

satisfies the state equations.
Combining the forced and unforced responses to give the
transient response of the system over the general interval \( t_0 < t < t_p \):

\[
\bar{x}(t) = \exp(A(t-t_0)) x(t_0) + \int_{t_0}^{t} \exp(A(t-r)) . B. \bar{u}(r) \, dr \quad (C.2.6)
\]

C.3 STEADY STATE RESPONSE

If we consider the response of the system to single frequency
sinusoidal inputs the state equations take the form:

\[
\dot{x} = A \bar{x} + \bar{B} \bar{U} \sin \omega t
\]

where \( \bar{U} \) is a time-invariant input vector. From the previous section
we know the solution will be of the form:

\[
\bar{x} = \exp(At) x(o) + \int_{t_0}^{t} \exp(A(t-r)) \cdot \bar{B} \cdot \bar{U} \sin \omega r \, dr
\]

Integrating by parts gives:

\[
\bar{x} = \exp(At) x(o) + \frac{1}{\omega} \left[ \exp(At) - I \cos \omega t - \frac{A}{\omega} \sin \omega t \right] . \left[ I + \frac{A^2}{\omega^2} \right]^{-1} \cdot \bar{B} \cdot \bar{U}
\]

But in the steady state:

\[
\bar{x}(2\pi) = \bar{x}(o)
\]

\[
x(o) = \exp(A2 \pi) \bar{x}(o) = \frac{1}{\omega} \left[ \exp(A2 \pi) - I \right] \left[ I + \frac{A^2}{\omega^2} \right]^{-1} \cdot \bar{B} \bar{U}
\]

\[
\bar{x}(o) = - \frac{1}{\omega} \left( w^2 + A^2 \right)^{-1} \cdot \bar{B} \bar{U} \quad (C.3.1)
\]

and it follows that

\[
\bar{x}(t) = (I \cos \omega t + \frac{A}{\omega} \sin \omega t) \bar{x}(o) \quad (C.3.2)
\]
The steady-state response to any periodic input can be found from the summation of the responses of the system to the individual harmonic components of the input. Confirmatory results have been published by Rogers and Ioannides (C.2). However, if the periodic input can itself be reduced to periods of constant input a more efficient approach can be used. An example of this approach is given in Section 6.1.

C.4 TRANSFER FUNCTION ANALYSIS

The determination of the transfer function poles for a system is relatively easy and these poles are not dependent upon the input and outputs variables considered. However, the determination of the transfer function zeros is more difficult and they are different for each choice of input and output combination. This section describes a technique for producing the system transfer functions from the state equations.

C.4.1 Transformation of Variables

It is often convenient to transform the system equation to a new set of variables such that

\[ \bar{x} = \bar{P} \bar{Z} \]  

(C.4.1)

where \( \bar{P} \) is a non-singular transformation matrix.

Therefore

\[ \dot{\bar{x}} = \bar{P} \dot{\bar{Z}} \]

and

\[ \bar{P} \dot{\bar{Z}} = \bar{AP}\bar{Z} + \bar{B} \bar{u} \]

It follows that

\[ \dot{\bar{Z}} = \bar{P}^{-1} \bar{A} \bar{P} \bar{Z} + \bar{P}^{-1} \bar{B} \bar{u} \]  

(C.4.2)

i.e. \[ \dot{\bar{Z}} = \bar{A}\bar{Z} + \bar{B}\bar{u} \]
where \( \overline{A^*} = \overline{P^{-1} A P} \)

and \( \overline{B^*} = \overline{P^{-1} B} \)

Also \( y = \overline{C^*Z} \) \hspace{1cm} (C.4.3)

where \( \overline{C} = \overline{C P} \)

It can be proved that \( \det(sI - \overline{A^*}) = \det(sI - \overline{A}) \); that is the eigenvalues are unchanged by the transformation. Also it can be proved that the transfer function is unchanged by the transformation.

**C.4.2 (Phase Variable) Canonical Form**

We require a transfer function in the form

\[
\frac{y(s)}{u(s)} = \frac{P_n(s)}{P_m(s)} + d \tag{C.4.4}
\]

where \( P_n(s) \) is an \( n \)-th order polynomial in \( s \) of the form

\[
n_1 s + n_2 s^2 + \ldots + n_{n-1} s^{n-1}
\]

\( P_m(s) \) is an \( m \)-th order polynomial in \( s \) \((m>n)\) of the form

\[
m_1 s + m_2 s^2 + \ldots + m_{m-1} s^{m-1} + s^m
\]

and \( d \) is the direct coupling scalar

Therefore

\[
y(s) = \frac{P_n(s)}{P_m(s)} \frac{u(s)}{P_m(s)} + du(s) \tag{C.4.5}
\]

Let

\[
\frac{z_1}{u(s)} = \frac{1}{P_m(s)} \tag{C.4.6}
\]

i.e. \( z_1 = \frac{u(s)}{P_m(s)} \)

Giving \( y(s) = \frac{P_n(s)}{P_m(s)} z_1 + du(s) \)
Also let \( z_2 = sz_1 \)

\[ z_3 = sz_2 \]

\[ \vdots \]

\[ z_m = sz_{m-1} \]

From equation (C.3.5)

\[ u(s) = PM(s)z_1 \]

\[ = m_1z_1 + m_2sz_1 + m_3s(sz_1) + \ldots + s^mz_1 \]

\[ = m_1z_1 + m_2sz_2 + m_3sz_3 + \ldots + m_mz_m + sz_m \]

giving \( sz_m = -m_1z_1 - m_2z_2 - \ldots - m_mz_m + u(s) \) \hspace{1cm} (C.4.7)

In matrix form this is:

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_n
\end{bmatrix}
= s
\begin{bmatrix}
  0 & 1 & 0 & \cdots & 0 \\
  0 & 0 & 1 & \cdots & 0 \\
  \vdots \\
  0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
  z_1 \\
  z_2 \\
  \vdots \\
  z_n
\end{bmatrix}
+ \begin{bmatrix}
  0 \\
  0 \\
  \vdots \\
  0
\end{bmatrix}u
\]

i.e. \( sz = \alpha^* z + \beta^* u(s) \) \hspace{1cm} (C.4.8)

Also from equation C.4.5

\[ y(s) = n_1z_1 + n_2sz_1 + n_3s^2z_1 + \ldots + n_n(s^{n-1})z_1 + du(s) \]

i.e. \( y(s) = n_1z_1 + n_2z_2 + n_3z_3 + \ldots + n_nz_n + du(s) \)

giving

\[ y(s) = \gamma^* z + du(s) \] \hspace{1cm} (C.4.9)
where \( \mathbf{C}^* = (c_1, c_2, c_3, \ldots, c_n, 0, 0, \ldots) \).

As \( z_v = s z_{v-1} = z_{v-1} \) this form of state variables is called the
(phase variable) Canonical form (Ref. C3, C4).

Thus if the transformation matrix \( \mathbf{P} \) can be determined the
coefficients of the numerator can be found and the positions of the
zeros evaluated using a polynomial root algorithm.

Melsa (Ref. C.5) used the recursion relationship given by Tuel
(Ref. C.3) and Rane (Ref. C.4) as a transfer function algorithm. This
algorithm is:

\[
\mathbf{P} = (\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \ldots, \mathbf{P}_m) \tag{C.4.10}
\]

where \( \mathbf{P}_i \) is an \( m \) dimensional column vector

\[
\mathbf{P}_m = \mathbf{b}
\]

and

\[
\mathbf{P}_{m-i} = \mathbf{A} \mathbf{P}_{m-i+1} + \mathbf{M}_{m-1+i} \mathbf{b} \quad ; \quad i = 1, 2, \ldots, m-1
\]

The programme written by Melsa (Ref. C.5) includes checks on
controllability and observability. It also includes a check of
accuracy to prevent misleading results arising from ill-conditioned
problems.
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