THE MEASUREMENT OF WALL SHEARING STRESS
IN TURBULENT BOUNDARY LAYERS.

by

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Thesis submitted to the University of Leicester
for the Degree of Doctor of Philosophy

AUGUST 1972
For my parents
SUMMARY

The thesis describes the design, calibration and use of a floating element skin friction meter in smooth wall boundary layers under favourable and adverse pressure gradients. The results of an experimental investigation in turbulent, fully developed duct flow are combined with those obtained by BROWN and JOUBERT (1) to give a secondary force contour map for element Reynolds' numbers \( \left( \frac{d_m u_r}{v} \right) \) between 500 and 4000 and element Euler numbers between -16 and +20.

It is shown that these meters can be used in favourable pressure gradient rough wall flows and that the secondary force characteristics are similar to those obtained over smooth walls. Simple physical and mathematical models for the secondary forces are developed which show good qualitative agreement with experiment.

A strongly non-equilibrium boundary layer \( \left( -1.2 < \frac{\delta^*}{\tau_w} \frac{dP}{dx} < 2.6 \right) \) is investigated in detail and tabulated results given. A modified form of COLE'S (4) method for establishing the skin friction coefficient (Cf) from the velocity profile is developed and used to show the sensitivity of log-law methods to the coefficients assumed.

It is also shown that the effects of changes in duct cross-sectional area seriously affect the relationship between wall shear stress and pressure gradient in fully developed flows.
ERRATA

P.3 line 7 - Delete 'of similar magnitude.'

P.4 para 1.3.(4) - Replace semicolon at end of section by full stop.

P.5 para 3 line 3 - For 'along' read 'together'.

P.10 line 13 - Hinze is reference (5).

P.11 para 3 line 2 - Delete comma after 'particularly'.

P.12 line 21 and throughout thesis - For SCHLICTING read SCHLICHTING.

P.14 last line and throughout thesis - For LUDWEIG read LUDWIEG.

P.19 para 1 line 4 - For \( \frac{\Delta U_k}{U} \) read \( \frac{\Delta U_k}{U_k} \).

P.21 para line 5 - Delete 'then'.

P.23 Line 15 - For 2.8.1 read 2.7.1.

P.25 para 2 line 3 - Delete comma after 'knowledge'.

P.25 para 5 line 6 - For 'sparce' read 'sparse'.

P.29 para 1 line 11 - Delete commas around 'irrespective of direction'.

P.37 para 3 line 8 - For 'ease' read 'case'.

P.38 para 3 line 7 - For 'sparce' read 'sparse'.

P.39 para 2 line 7 - For 'is' read 'are'.

P.42 Equation 3.3.3 should read

\[
\mathbf{f}(t) = M \chi + C \dot{\chi} + S \chi
\] 3.3.5

P.44 line 17 - For 3.3.13 read 3.3.12.

P.56 para 1 line 2 - For 'mouths' read 'mouth' and for 'were' read 'was'.

P.63 line 3 - For 'then' read 'than'.

P.64 line 6 - For"(°K)" read '(°K)'.

P.69 Final para line 1 for 'U' read 'U_k'.

P.71 Para 3 line 1 - Insert brackets around "for various .......

........A and B".

P.73 para 2 line 11 - Delete comma after 'and'.

P.75 5.7.2 line 1 - for 'sparcely' read 'sparcely'.

continued overleaf
ACKNOWLEDGEMENTS

My thanks go to all those who helped and advised me during the course of this work: in particular

Professor G.D.S. MacLellan for the use of the Engineering Department facilities at the University of Leicester.

Dr. R. C. Pankhurst for the use of the N.P.L. Boundary Layer Tunnel.

Mr. P. Bradshaw, Dr. J.A.B. Wills and his colleagues for their help and advice on the experimental work at N.P.L.

Mr. K. G. Winter, Adrian Research Fellow at Leicester University, for his advice on the work in general.

The technical staff of Leicester University Engineering Department for their most able technical support: particularly Colin Harris, Eric Hooley and David Granby.

Mrs. Helen Sheppard for typing this thesis so ably.

Most of all I would like to thank my supervisor, Dr. David Cockrell, for his constant encouragement, help and advice during the past four years.
P. 76 Equation 5.8.1. should read

\[ \tau_m = 15.7 I^2 \approx 16 I^2 \]

P. 80 \[ -\epsilon_{ij} \frac{dU_j}{d\lambda} \] is obtained from fig (6.1) (Uδ - x).

P. 83 Para 1 line 4 - Delete sentence 'This condition....etc!'  

P. 86 Line 2 - read \( \frac{\Delta P_s}{\{\Delta P_s\}} \).

P. 87 Line 10 - For 10.6 read 10.5.

P. 90 Para 2 line 14 - For 'independent' read 'independence'.

Fig. (3.7) EULER No. scale for ' - 15' read '-10'

Fig. (6.13) Length scale in cm not m.

Fig. (6.9.f) \( \overline{UVT} \) scale \( m^2/s^2 \).
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NOMENCLATURE

\( a, b \) \{ Constants in various formulations of the Log Law \\
\( A, B \) \\
\( A', B' \) \\
\( A_c \) Flow area of calibration channel \\
\( A_m \) Skin friction meter element area \( \left( \frac{\pi d_m^2}{4} \right) \) \\
\( A_z \) Influence area of single roughness element \\
\( C_f \) Skin friction coefficient \( \left( \frac{\tau_w^{\frac{1}{2}} \rho U_\infty^2}{\frac{1}{2} \rho U_\infty^2} \right) \) \\
\( C_{p_1}, C_{p_2} \) Pitot tube coefficients (Appendix I) \\
\( d \) Pitot tube internal diameter \\
\( d' \) Wall static pressure tapping diameter \\
\( d_d \) Calibration channel depth \\
\( d_m \) Skin friction meter element diameter \\
\( D \) Pitot and Preston tubes - outer diameter \\
\( E \) Euler number \( \left( d_m \frac{dp_W}{dx} / \tau_w \right) \) \\
\( F \) Total force on floating element \\
\( F_{\text{suffix}} \) Forces defined within text \\
\( g \) Skin friction meter gap width \\
\( h, H \) Internal and external heights of rectangular total head probes (Appendix I) \\
\( H \) Hydraulic diameter \\
\( H_{12} \) Two dimensional shape factor \( (\delta^*/\delta) \) \\
\( i, j, k \) Cartesian unit vectors in \( x, y, z \) directions respectively \\
\( k_r \) Representative roughness height \\
\( k_s \) Equivalent sand roughness \\
\( N \) "Universal" constant in PERRY's formulation of wall law \\
\( p \) Local static pressure \\
\( p_m \) Measured total head \\
\( p_s \) Measured wall static pressure \\
\( p_w \) Wall static pressure
REYNOLDS NUMBERS:

- $R'$: Pitot tube gap $\frac{\nu}{\nu}$
- $R_g$: Floating element gap $\frac{gV_1}{\nu}$
- $R_T$: Floating element meter $\frac{d_m u_T}{\nu}$
- $\vec{V}$: Total velocity vector
- $U, V, W$: Mean velocity components in $x, y, z$ directions respectively
- $U_\infty$: Free stream velocity
- $u_T$: Friction velocity $\left(\frac{r_W}{\rho}\right)$
- $\frac{\Delta U_1}{u_T}$: Translation of velocity profile due to pressure gradient
- $\frac{\Delta U_2}{u_T}$: Translation of velocity profile due to roughness
- $U^+$: $\frac{U}{u_T}$
- $V_1$: Meter gap flow velocity
- $x, y, z$: Cartesian streamwise, normal and cross-stream displacements
- $y^+$: $\frac{yu_T}{\nu}$
- $\alpha$: Pressure gradient $\left[\frac{1 dp}{\rho dx}\right]$
- $\beta$: Boundary layer equilibrium parameter $\left[\frac{\delta^*}{r_W} \frac{dp}{dx}\right]$
- $\gamma_{suffix}$: General normalised secondary force $\left[\frac{F_{suffix}}{r_W A_m}\right]$
- $\delta$: Boundary layer thickness ($U = 0.995 U_\infty$)
- $\delta^*$: Boundary layer displacement thickness $\int_0^\infty \left(1 - \frac{U}{U_\infty}\right) dy$
- $\delta'$: Boundary layer-outer limit of flow drawn into meter
- $\delta_t$: Boundary layer-viscous sublayer thickness
- $\delta_t$: Boundary layer-thickness of viscosity influenced layer
- $\delta_{log}$: Boundary layer-outer limit of log law
- $\delta$: Displacement of effective centre of Pitot tube
- $\Delta(x_1)$: Change in variable ($x_1$)
\( \varepsilon \) Normalised Pitot displacement \( \left( \frac{\delta}{D} \right) \)

\( \varepsilon_1 \) Displacement of boundary layer velocity profile origin in rough wall flows

\( \varepsilon_m \) Skin friction meter - element edge width

\( \varepsilon_p \) Pitot - Preston tube displacement effect \( \left( \frac{\delta}{D} \right) \text{ when } y = \frac{D}{2} \)

\( \theta \) Boundary layer momentum thickness \( \int_0^\infty \frac{U}{U_\infty} \left( 1 - \frac{U}{U_\infty} \right) dy \)

\( \kappa \) Von Karman's constant \((0.39 \sim 0.41)\)

\( \mu \) Dynamic viscosity

\( \nu \) Kinematic viscosity \( \left( \frac{\mu}{\rho} \right) \)

\( \Gamma \) Coles wake function

\( \rho \) Air density

\( \rho_m \) Density of manometer fluid (water)

\( \tau \) Shear stress

\( \tau_w \) Wall shear stress

\( \tau_s, \tau_r \) Smooth and rough wall shear stresses - calibration channel

\( \nabla \) Vector differential operator \[ \hat{\mathbf{i}} \frac{\partial}{\partial x} + \hat{\mathbf{j}} \frac{\partial}{\partial y} + \hat{\mathbf{k}} \frac{\partial}{\partial z} \]
CHAPTER 1

INTRODUCTION
1.1. The Importance of the Wall Shearing Stress

The flow of a fluid past a solid surface is of particular interest to engineers because of its ability to transfer momentum, mass and heat between the fluid and the surface. In 1904 Prandtl showed that when the inertia, viscous and pressure forces were of the same order of magnitude interaction between the flow and the surface was confined to a thin 'boundary layer'. Around the same time powered flight became a reality and the major applications of Prandtl's boundary layer concept were aeronautical. The commercial and military potential of aircraft was soon realised and consequently research into boundary layers progressed rapidly. The turbulent boundary layer problem, however, had to wait for the development of the hot-wire anemometer before many of its subtle flow mechanisms could be measured and hence appreciated. In the last few decades the applications of boundary layer theory have steadily multiplied prompting F. H. Clauser to comment, in 1956,

"The concepts of boundary layer phenomena in general and turbulent boundary layers in particular have found application in a wide range of fields including aeronautics, guided missiles, naval architecture, marine engineering, hydraulics, meteorology, oceanography, chemical engineering, sanitary engineering, atomic reactors, astrophysics and the flow of liquids and gases in the human body."

In the last fifteen years this impressive list has become even longer.

There are basic similarities in the transport of momentum, mass and heat which enable one, from a knowledge of the transport of one of these 'properties', to deduce the transport behaviour of the other two. By the application of suitable transformation theories an appreciation of the transport of momentum in incompressible turbulent boundary layers can thus be transformed to an appreciation of mass and heat transfer even in compressible flows.
In aerodynamics the momentum transfer characteristics of the boundary layer are of prime importance since they describe the mechanism by which high grade mechanical energy is degraded, via turbulence and viscous dissipation, to low grade thermal energy (heat). The power consumed in this process must be supplied externally, often at great cost. For example, the power requirements of a vehicle are dictated by its drag and speed. The drag forces may be split into two separate forces of similar magnitude:

(a) Skin friction drag: the integral of the surface shearing stress over the surface of the body.

(b) Form drag: the integral of the surface pressure field over the surface of the body.

The measurement and prediction of the surface pressure is relatively easy in comparison with measurement and prediction of the surface shearing stress. For an aircraft, incorrect estimates of the drag could have disastrous consequences upon the range, payload, capital cost and commercial viability. Thus in this and in other fields mathematical and physical models must give correct estimates of the full scale drag force. Therefore, for economic and technical reasons, the measurement of the wall shearing stress must form a significant part of any boundary layer investigation.

1.2. The Measurement of Wall Shearing Stress ($\tau_w$)

There are two basic categories into which wall shear stress measurement techniques may fall. These are

**DIRECT**  The force upon a small element of the wall is measured, from which the mean value of $\tau_w$ may be deduced.

**INDIRECT**  Theoretical or experimental correlations between the wall shear stress and other boundary layer variables are postulated. Measurement of these variables then yields the wall shearing stress.
The accuracy of the latter depends upon the validity of the correlation used for the particular flow investigated and the accuracy of measurement of the other variables. Very few scientific measurements are direct. This is frequently forgotten because the correlations between desired and measured variables are often simple (e.g. that between tension and extension for an elastic material). Under conditions of high pressure gradient or rough walls the conventional correlations between wall shearing stress and velocity profile are unsatisfactory and the direct method must be used.

1.3. Objectives

The objectives of the work presented are:

1. To describe in detail the various techniques used to measure wall shearing stress and discuss their limitations in pressure gradient and rough wall flows;

2. To describe the design, development and use of floating element skin friction meters, developing a physical model of the dynamic behaviour of the meter;

3. To present original experimental results obtained in boundary layers over rough and smooth walls in pressure gradients whereby direct and indirect methods may be compared.

4. To compare the physical model of the meter characteristics with the experimental results and to consider what improvements might be made in present meter designs;

5. To assess the potential uses of floating element meters in establishing better models of boundary layer flow and make suggestions for future work.
1.4. The Present Investigation

The first objective is accomplished by the review of previous work in Chapter 2. A discussion of the physical structure of the turbulent boundary layer, particularly in the wall region, is a major part of this chapter. The techniques, summarised in Table 2.1, are described and their limitations discussed. Those who wish to measure the wall shearing stress in any particular flow will find within the text sufficient information to enable the most satisfactory technique to be selected. The conclusions of the chapter are that in rough wall flows with pressure gradient the direct method cannot be bettered.

The design criteria for a floating element skin-friction meter and a physical model of the local flow distortion around the meter element are developed in Chapter 3. The physical model predicts qualitatively the effects of pressure gradient and wall roughness upon the measurements obtained. The static and dynamic calibration techniques for these meters are also described.

Experimental work was carried out in an adverse pressure gradient boundary layer and in the fully developed flow between parallel plates. The experimental apparatus used is described in Chapter 4 along with the design details of the floating element meter. The results obtained are given in Chapter 6 together with detailed discussion. The ways in which the results were processed from the raw experimental data and the accuracy of the results are described in Chapter 5.

The conclusions of the thesis and recommendations for future work are presented in Chapter 7.
CHAPTER 2

WALL SHEAR STRESS MEASUREMENT TECHNIQUES.
2.1. Basic Flow Equations - Theoretical Correlations

The motion of a viscous, incompressible fluid is described by the Navier-Stokes equations:

\[
\frac{D\vec{V}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{V} \tag{2.1.1}
\]

where \(\vec{V}\) is the instantaneous velocity vector at a point \((x,y,z)\) given by

\[
\vec{V} = (U + u')\hat{i} + (V + v')\hat{j} + (W + w')\hat{k},
\]

capital letters denoting time mean (steady) components and small (primed) letters denoting the instantaneous value of the fluctuating component.

For continuity of mass flow

\[
\nabla \cdot \vec{V} = 0 \tag{2.1.2}
\]

For steady flow in two dimensions \((W = 0)\) adjacent to a boundary equation 2.1.1 may be simplified using Prandtl's boundary layer approximations:

\[
\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial U}{\partial y} - \rho \frac{U^2 + V^2}{2} \right) - \frac{\partial}{\partial x} \left( \frac{u'^2 + v'^2}{2} \right) \tag{2.1.3}
\]

where \(p\) is the static pressure.

The continuity equation (2.1.2) becomes

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{2.1.4}
\]

Equation 2.1.3 may be integrated with respect to \(y\) to give

\[
\frac{\partial \theta}{\partial x} + \left. \frac{\partial U_\infty}{\partial x} \right|_{y=0} (H_{12} + 2) - \frac{\tau_w}{\rho U_\infty^2} = -\frac{\delta}{U_\infty^2} \int_0^\delta \frac{\partial}{\partial x} \left( \frac{u^2 - v^2}{2} \right) dy \tag{2.1.5}
\]

This relationship was first suggested by von Karman in the form

\[
\frac{\partial \theta}{\partial x} + \left. \frac{\partial U_\infty}{\partial x} \right|_{y=0} (H_{12} + 2) - \frac{\tau_w}{\rho U_\infty^2} = 0 \tag{2.1.6}
\]

which is strictly only accurate for laminar flow but may also be used for turbulent flow when the right hand side is negligible compared with the other terms. In theory equation 2.1.3 could form the basis of a
measurement technique. For practical reasons the integral form (2.1.6) is normally used. It should be remembered that this formulation is for two-dimensional flow and will give incorrect results in three-dimensional situations.

The circumstances in which the method is unreliable in two-dimensional flows may be deduced from the magnitudes of the terms in equations 2.1.5 or 2.1.6. In an adverse pressure gradient the group containing the wall shearing stress \( \frac{\tau_w}{\rho U^2} \) is the small difference between two large numbers leading to large inaccuracies. The effects of turbulence (see equation 2.1.5) have been shown by Bidwell (2) to be significant only when flow separation is approached.

In favourable and zero pressure gradients the technique is intrinsically more accurate. The chief value of von Karman's equation lies in assessing the quality of the experimental flow by inserting other measurements for \( \tau_w \) (in equation 2.1.6). For very slight three-dimensionality the imbalance in the equation is considerable. No experimental flow is ever wholly two-dimensional. Amongst others Clauser (3) has shown that in adverse pressure gradients it is necessary to take extreme precautions to prevent three dimensionality from becoming significant. Coles (4) used the imbalance in equation 2.1.6 as an indicator of the significance of three dimensionality in the flows used as test data for boundary layer prediction methods. Surprisingly, the results of prediction methods dependent upon Prandtl's equation were often satisfactory even when considerable imbalance existed in the test data.

With rough boundary walls the point of origin of the velocity profiles (the wall) becomes uncertain. In suggesting that this origin be viewed as a smooth plane to which roughness elements are attached Hinze (5) indicates that this model will only be valid when these elements are not "too irregular or too large". Since the context in which Hinze makes this comment is concerned with velocity profile similarity wall
roughness will have an uncertain effect upon integrals of velocity profiles. Furthermore this uncertainty is in a region which makes the most contribution to the final integral values. The momentum thickness, $\theta$, is a 'wall dominated' variable since

$$\theta = \int_0^\infty \frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right) dy$$

and the function $\frac{U}{U_\infty} \left(1 - \frac{U}{U_\infty}\right)$ has a maximum of 0.25 at $\frac{U}{U_\infty} = 0.5$ which occurs close to the wall. If the uncertainty in origin ($Ay$) were small compared with the roughness height the resultant error would be small. (Since $[Ay \times 0.25/\theta]$ would be small).

Fully developed flow in two-dimensional or cylindrical ducts may be adequately described by a form of equation 2.1.6 (in which axisymmetric definitions of boundary layer thicknesses are used if appropriate) modified to account for the loss in total head:

$$-\frac{H}{4} \frac{dp_w}{dx} = \tau_w$$

Where $H$ is the hydraulic mean diameter of the duct. TOWNSEND (6) gives the detailed equations and implications with regard to shear stress and turbulence distributions. This equation (2.1.8) forms the basis of a useful calibration technique which will be discussed in detail in Chapter 3.

2.2. Experimental Correlations - Velocity Profiles

Apart from the theoretical correlations based upon Newton's 2nd Law and outlined in Section 2.1 all other correlations rely basically upon the similarity of the boundary layer structure in the region of the wall. These are all ultimately dependent upon the velocity profile structure because of the inter-relation of the transfer of mass, heat and momentum and the effects which the presence of obstacles will have on these transport quantities. BROWN and JOUBERT (1) presented these correlations in the
form of a chart which appears in a modified form in Fig. (2.1). The
general arguments presented in this section for the velocity profile are
capable of extension to the profiles of other dependent variables.

The boundary layer profile at a given section must be defined by
(a) the previous 'life' or experiences of the profile (history)
(b) the external inputs of wall shear stress and free stream conditions and
(c) the local profile 'reaction' to a change in these inputs (represented by
its rapidity of response). Using such a model the local structure may be
deduced by many different 'pseudo-theoretical' techniques which, in their
turn, can be validated by careful experimental measurements. CLAUSER (7),
TOWNSEND (6), HINZE (5) and ROLTA (8) discuss both the empirical and
analytic/empirical approaches in great detail with slightly different
emphasises, HINZE (4) writing a comprehensive review which included the evi-
dence of the other three workers. Their approach is now developed.

The variation of the local velocity ($U$) with the distance from the
wall ($y$) at a given streamwise position ($x$) in a two-dimensional incom-
pressible boundary layer is dependent upon

(i) The fluid properties of density ($\rho$) and kinematic viscosity ($\nu$)
(ii) The free stream conditions represented by the velocity ($U_\infty$) and
the velocity or pressure gradient ($\frac{\partial U_\infty}{\partial x} : \frac{\partial p_w}{\partial x}$)
(iii) The boundary layer thickness ($\delta$)
(iv) The wall shearing stress ($\tau_w$) which may be represented by
$u_f = \left( \frac{\tau_w}{\rho} \right)^{1/2}$, the wall friction velocity.
(v) The wall roughness represented by a characteristic roughness
height ($k_R$)

Thus, generally,

$$U = U \left( \rho, \nu, y, \delta, U_\infty, \tau_w, k_R, \frac{\partial p_w}{\partial x} \right)$$  \hspace{1cm} 2.2.1

Equation 2.2.1 does not include any 'long term' history effects
since all values are localised except for $\frac{\partial p_w}{\partial x}$ which may be viewed as a
'short term history' parameter. Higher derivatives of the pressure field
would include historical implications but for the purposes of shear measurement correlations these are relatively less important.

It has been shown experimentally that the response of the boundary layer to 'step' changes in the inputs (CLAUSER's 'Black Box' analogy ref. (7)) is far more rapid near the wall for changes in wall conditions. This is physically plausible and leads one to the conclusion (experimentally verified) that free stream changes have relatively small effects on the structure of the flow in the wall region). (See for example ANTONIA and LIXTON (9) and BRADSHAW (10)). Based upon arguments like these various models of the boundary layer velocity structure have been developed. Those models applying to the wall region of the flow are generally expressed by the law of the wall

\[
\frac{U}{u_r} = \frac{u}{u_r} \left( \frac{y_r}{\nu}, \frac{k y_r}{\nu} \right)
\]

whereas in the outer region the velocity defect law or law of the wake (COLES (11)) is expressed

\[
\frac{U \infty - U}{u_r} = f \left( \frac{y}{\delta}, \frac{\delta^*}{\nu} \frac{dp}{dx} \right) \text{ (CLAUSER)} \quad 2.2.3
\]

or

\[
\frac{U \infty - U}{u_r} = f \left( \frac{y}{\delta}, \frac{\delta^*}{\nu} \frac{dp}{dx} \right) \text{ (COLES)} \quad 2.2.4
\]

where \( \delta^* \) is Coles' wake function and depends upon \( x \) (history) in general. Strictly both formulations are only universal when applied to near-equilibrium boundary layers \( \left[ \frac{\delta^*}{\nu} \frac{dp}{dx} \Omega \text{ const.} \right] \) since the behaviour of the wake function can only be guessed at in strongly non-equilibrium situations.

Clearly, in order to correlate the wall shear stress with the velocity structure the wall region must be used. The nebulous phrase 'wall region' must therefore be defined more stringently and the functional dependence (2.2.2) mathematically defined.

It is necessary to appreciate the difficulties of measuring fluid velocity accurately, particularly in the sheared, turbulent flow near a
solid boundary. For this reason a brief survey of impact probe technique is given in Appendix I.

2.3. The Law of the Wall

A condition of 'no slip' exists at the wall at the temperatures and pressures encountered in engineering situations. There must therefore be a region in which the flow is viscous since the velocity and the velocity fluctuations normal to the wall must approach zero near the wall. HINZE (5) terms this region the viscous sublayer, having a thickness \( \delta' \). In this very thin sublayer if the shearing stress is assumed constant when the wall is smooth the velocity profile is given by

\[
\frac{U}{u_T} = \frac{y u_T}{\nu} \quad 2.3.1
\]

for \( 0 < \frac{y u_T}{\nu} < 5 - 8 \) approx.

Experimental evidence can be seen in ref. (11) in which the measurements of LAUFER, REICHARDT and KLEBANOFF are given.

Outside this sublayer the effects of viscosity gradually decrease as the local Reynolds number \( \left( \frac{y u_T}{\nu} \right) \) increases. HINZE (5) denotes the distance at which the effects of viscosity become negligibly small by \( \delta^* \).

Beyond this point the familiar log law may be deduced by either local flow structure assumptions or general physical (global) assumptions within a dimensional argument. Naturally the flow structure will depend upon the geometric characteristics of the wall (roughness). SChLICHTING (12) makes the following observations based upon NIKURADSE'S pipe flow experiments.

\[
\frac{U}{u_T} = \frac{1}{k} \ln\left( \frac{y}{k_{S}} \right) + B_T \left[ \frac{k_{S} u_T}{\nu} \right] \quad 2.3.2
\]

where \( k_{S} \) is the equivalent sand roughness. The function \( B_T \) is bounded in the following manner:–
for \( \frac{k_u}{\nu} < 5 \) the flow is hydraulically smooth and

\[
B_r = B + \frac{4}{\kappa} \ln\left(\frac{k_u}{\nu}\right) \tag{2.3.3}
\]

yielding

\[
\frac{U}{u_r} = \frac{1}{\kappa} \ln\left(\frac{y_u}{\nu}\right) + B \tag{2.3.4}
\]

For fully rough flow where \( \frac{k_u}{\nu} > 70 \), \( B_r \) is a constant. The values given by Schlichting are \( \kappa = 0.40 \); \( B = 5.5 \); \( B_r \) (fully rough) = 8.5. More recently Perry (13, 14) used a dimensional approach to deduce the structure of the wall region. This led to better definition of the logarithmic region's outer limit. This definition rests upon the existence of a 'half power' region depending upon the pressure gradient. For smooth walls,

\[
\frac{U}{u_r} = K_1 \left( \frac{\alpha v}{u_r^2} \right)^{1/2} + \frac{\Delta U_1}{u_r} \left( \frac{\alpha v}{u_r^3} \right) \tag{2.3.5}
\]

where \( \alpha = \frac{1}{\rho} \frac{\partial p}{\partial x} \) and \( K_1 \) is a 'universal' constant. The function \( \frac{\Delta U_1}{u_r} \) is described as a 'slip-function' being defined by the intercept of equation 2.3.5 at \( y = 0 \).

For rough walls the log law may be described in the compound form

\[
\frac{U}{u_r} = \frac{1}{\kappa} \ln\left(\frac{y_u}{\nu}\right) + B - \frac{\Delta U_2}{u_r} \left( \frac{k_u}{\nu} \right) \tag{2.3.6}
\]

where \( \frac{\Delta U_2}{u_r} \) is the translation in velocity profile due to roughness. The half power law then becomes

\[
\frac{U}{u_r} = K_1 \left( \frac{\alpha v}{u_r^2} \right)^{1/2} + \frac{\Delta U_1}{u_r} - \frac{\Delta U_2}{u_r} \tag{2.3.7}
\]

The conditions for the existence of a logarithmic region in adverse pressure gradients are given by assuming that the intersection of the log and half power regions occur when \( \frac{\alpha v}{u_r^2} = N \), a universal constant. This constant was found to be 1.41 for the flows investigated by Perry. If the intersection point is at a value of \( y \) outside the region of viscous influence then the log region exists. The inner limit is generally accepted to be given by \( \frac{y_u}{\nu} \geq 50 \). Perry et al (14) give this limit as
and deduce that the outer limit \( \delta_{\log} \) is

\[
\delta_{\log} = 1.4 \frac{u_r^2}{a} \] which is equal to

\[
\delta_t \text{ when } \frac{\partial v}{u_r^2} = 0.05.
\]

The above results are generally valid only when:

(a) \( a \) is the first non-wall variable to affect the flow
(b) the pressure gradient is adverse.

From the above formulations of the law of the wall it is seen that if the velocity profile in the wall region \((U, y)\) is known then several methods may be used to deduce a value for \( u_r \) and hence \( \tau_w \). Measurements in the viscous sublayer of a smooth wall flow are difficult. (When the wall is rough the viscous sublayer as such is non-existent). WILLS (15) gives details of a method which employs a hot-wire anemometer within the viscous sublayer. He points out that the heat loss corrections are based upon the assumption of similar turbulence structures for the sublayers of different flows. He then cites the work of LAUFER who found that the maximum value of \( \sqrt{\frac{u_r^2}{U}} \) ranged from 0.44 to 0.23 depending upon the Reynold's number. When the sublayer is thick the corrections become small thus the method is inherently suitable for flows approaching separation. Unfortunately (for obvious reasons) it cannot be used for rough wall flows.

The values of the constants in the log law are somewhat uncertain since various workers have found differing values. This may either be due to the examination of physically different flow situations or to inaccurate measurements of the wall shearing stress and velocity profiles. The results of several experiments are given in Fig. (2.2). Methods by which the wall shearing stress may be calculated from the velocity profile in the logarithmic region are given in Section 2.4. There is evidence that the log law is generally more reliable in zero and adverse pressure gradients (LUDWEIG and TILLMANN (20)) but is incorrect in high favourable
pressure gradients. PATEL's (16) results show this effect by the poor
fit in the wall region. The results compare with the present investigations
findings near the leading edge of the flat plate boundary layer. COLES (4)
suggests that this apparent breakdown of the wall logarithmic region may, in
many cases, be indicative of the failure of impact probe measurements or to
poorly tripped boundary layers. His view is that the wall similarity laws
are generally more valid than previously suspected.

The similarity laws thus present some problems both to the theoret-
cian and experimentalist. COLES (4) statement "it is probably a waste of
time to attempt detailed velocity measurements near a wall using conventional
impact probe instrumentation" is one which ought to be seriously considered
by any experimentalist working on turbulent boundary layers.

Those who wish to calculate the skin friction from the logarithmic
law need to know:

(i) What constants A,B should be used and/or are there significant
differences in the calculated values for different constants?

(ii) Under what conditions can the log law be assumed valid?

(iii) Is it possible to obtain skin friction values over arbitrarily
rough walls by this method?

These important questions are discussed in Section 2.4.

2.4. Methods Utilising the Log Law

The basic equation used for smooth walls is:-

\[
\frac{U}{U_\infty} = A \log_{10} \left( \frac{\gamma_{1}}{v} \right) + B
\]

CLAUSER's (3,7) method expresses this equation in the form:

\[
\frac{U}{U_\infty} \sqrt{\frac{2}{Cf}} = A \log_{10} \left( \frac{U_\infty v}{\gamma_{1}} \sqrt{\frac{Cf}{2}} \right) + B
\]

where \( Cf \) is the local skin friction coefficient \( \left( \frac{r_{w}}{2pU_{\infty}^{2}} \right) \).
A family of curves of \( \frac{U}{U_\infty} \) vs \( \frac{y U_\infty}{\nu} \) may be plotted for varying values of \( \text{Cf} \) (Clauser chart). Experimental data in the form \( \frac{U}{U_\infty} \) and \( \frac{y U_\infty}{\nu} \) may then be plotted on the Clauser chart and the corresponding value for \( \text{Cf} \) obtained.

In BRADISHOW's (22) method an arbitrary experimental data point \((U, y)\) within the log law region is substituted in equation 2.4.1. The equation may be rearranged and used to give a curve of \( \frac{U}{U_\infty} \) vs \( y \) by inserting different values for \( \text{Cf} \). This curve is then superimposed upon the experimental curve \((\frac{U}{U_\infty} \text{ vs } y)\). The point of intersection satisfies both the experimental data and equation 2.4.1. Hence \( \text{Cf} \) is deduced. Several points should be taken to ensure a reliable result.

Another technique would be to plot \( U \) against \( \log(y) \) and measure the slope of the 'resulting' straight line. It is necessary to check that the region in which the slope is measured is within the accepted limits of \( \frac{y U_\infty}{\nu} \).

RAJARATNAM and FROELICH (21) give another method in which CLAUSER's formulation 2.4.2 is rearranged:

\[
\frac{U}{U_\infty} = A \left\{ \log_1 \left( 10^{B/R_y \sqrt{2}} \right) \right\}^{\sqrt{2}} \tag{2.4.3}
\]

where \( R_y = \frac{U_y}{\nu} \).

This is reduced to the form

\[
\text{Constant} \times Y = X \log X \tag{2.4.4}
\]

where \( Y = \frac{U_y}{\nu} \) and \( X = 10^{B/R_y \sqrt{2}} \).

The values used for \( A \) and \( B \) are 5.6 and 4.9 respectively giving

\[
1.34 Y = X \log X \tag{2.4.5}
\]

whence

\[
X = 7.5 R_y \sqrt{\frac{\text{Cf}}{2}} \tag{2.4.6}
\]
Over the range of interest \((30 < y^+ < 1000)\) the corresponding ranges in \(X\) and \(Y\) are

\[
\begin{align*}
225 < X < 7500 \\
394 < Y < 21700
\end{align*}
\]

Over this range equation 2.4.5 is approximated by

\[
X = 1.19 \, Y^{0.875}
\]

Thus values of \(Y\) can be calculated (within the range specified) from the experimental data and the corresponding value of \(X\) deduced. Hence the value of \(Cf\) is obtained.

COLES (4) expressed equation 2.4.1 in the form:

\[
\text{Residual } R = \frac{U}{u_T} - A \log_{10} \left( \frac{y_{	ext{v}}}{v} \right) - B
\]  

2.4.8

The profile points \((U, y)\) and the unit Reynolds number \(\left( \frac{U}{u_T} \right)\) are fed into a computer together with an arbitrarily chosen value for the skin friction coefficient \((Cf)\). The computer then calculates \(\left( \frac{U}{u_T} \right)\), \(\left( \frac{y_{	ext{v}}}{v} \right)\) and the residual \(R\). Next the profile is plotted in semi-logarithmic co-ordinates and a decision taken to minimise the residuals for specific profile points (presumably in the range \(100 < y^+ < 300\)). A new value for \(Cf\) is then entered and the procedure repeated until, for all the chosen experimental points, the residuals are minimised.

The author has modified this procedure (23) so that no decisions need be taken and so eliminating subjectivity. This is considered a more satisfactory procedure than that above since, in this revised procedure, the computer plots the results and gives 'instant' evaluation of the 'fit' of the data to the law used. The procedure is described in Chapter 5.

From the above calculation procedures and previous discussions on the logarithmic law it should now be possible to answer the three questions posed in the last section. Calculations (5.6.2) show that in the range
(100 < $y^+ < 300$) the differences between values calculated using COLES' constants and the other constants quoted are small. SARNECKI's constants give identical results whilst those of CL/USER and PATEL give results 1% higher and 1% lower respectively. There is little to choose between COLES' values (which are based on uncorrected pitot readings) and SARNECKI's (based upon extensively corrected results). From a practical point of view the differences in the constants may thus be put into perspective.

The range of validity of the logarithmic law naturally depends upon the pressure gradient. Using PERRY's (14) approach, described earlier, the outer limit of the log law ($\delta_{\log}$) is given by

$$\delta_{\log} = 1.41 \frac{u^2}{a}$$

for

$$y^+ = 300, \quad \delta_{\log} = \frac{300v}{u_r}$$

when

$$\frac{300v}{u_r} = 1.41 \frac{u_r^2}{a}$$

or

$$\frac{a \cdot v}{u_r^3} = \frac{1.41}{300} \cdot 0.005$$

thus

$$\frac{a \cdot v}{u_r^3} < 0.005$$

In favourable pressure gradients PATEL (16) observes that the breakdown in the log law (a) seems to propagate from the wall and (b) occurs in weaker pressure gradients than for adverse pressure gradient breakdown. He expresses the criteria for Preston tube 3% error bands:

$$-0.005 < \frac{a \cdot v}{u_r^3} < 0.01$$

Thus, as a working rule it is suggested that the log-region exists in the range $100 < y^+ < 300$ when

$$-0.003 < \frac{a \cdot v}{u_r^3} < 0.005,$$

accepting PATEL's recommendations for favourable pressure gradients.
Unfortunately the last question posed must be answered less positively. When the wall is rough, as PERRY, SCHOFIELD and JOUBERT (25) point out, there are two additional variables introduced which both affect the deduced value of shear stress. These are the roughness 'translation' $\frac{\Delta u}{u}$ and the unknown origin ($\epsilon_1$ below roughness crests). It is considered that, for arbitrary roughness, of unknown characteristics in a pressure gradient flow there is no known satisfactory method based upon the log law. It would be possible to find these characteristics in fully developed flow and then examine the effects of pressure gradient, however no published work is known in this field. PERRY et al (25) approach the problem and show that

$$\frac{\Delta u}{u} = \frac{1}{\kappa} \ln \left( \frac{\epsilon_1 u}{v} \right) + C_r$$

where $C_r$ is a function of the roughness. They further distinguish between roughness types ('grooves' and 'slats') and show that the behaviour of $\epsilon_1$ is not the same in both cases. At present, therefore, there is a dearth of reliable experimental data for incompressible fluid flow over rough walls in adverse pressure gradients.

2.5. Skin-Friction Laws

The primary object of skin friction laws is to correlate the velocity profile parameters with the wall shearing stress. Bearing in mind that the whole structure of the velocity profile must therefore be universal for the law to be valid there must be limitations similar to localised similarity laws. Clearly the integral-type laws should be less satisfactory in marginal cases. Thus the author views the role of skin friction laws as essential within prediction techniques but severely limited for the assessment of raw data. The most commonly used law is probably the LUDWEIG-TILLMANN (2) equation:

$$C_f = 0.246 \left( \frac{\nu}{v} \right)^{-0.268} \times 10^{-0.678H_{12}}$$
Typically the other laws are basically formulated from

\[ \frac{1}{\sqrt{C_f}} = \lambda \log \left( \frac{U_{\infty} \delta^*}{\nu} \right) + B + K \]

where \( K \) is generally a function of the wake structure (see for example NASH (35) or ROTTA (8)). For rough walls the same difficulties as encountered with the log law apply. For prediction methods DVORAK (36) gave a form of 2.5.2 which applies in many cases. BRADSHAW (in a private communication) pointed out that the effects of roughness could be effectively included by adjusting the viscosity term

\[ \frac{U}{U_T} = \frac{1}{\kappa} \left( \ln \frac{U_T}{\nu} + B + f \left( \frac{U_T}{\nu} \right) \right) \]

\[ = \frac{1}{\kappa} \left( \ln \frac{U_T}{\nu_T} + B \right) \] where \( \nu_T = \exp(f) \)

2.6. Heat and Mass Transfer Similarity

The heat or mass transfer rates from elements mounted in the wall can be used to deduce the value of the wall shearing stress under certain conditions.

LUDWEIG (37) describes the first heat transfer device and discusses the theoretical correlations between shearing stress and heat transfer rate. The thermal boundary layer immediately above the element must be confined to the region influenced by wall variables (i.e. wall law region) and the natural convective heat transfer must be low. LUDWEIG expresses this latter condition by stating that the temperature field must not affect the velocity field. The conditions for readings of heat transfer to be a unique function of the wall variables are thus based upon much the same factors governing the existence of a log law region. Since the instrument is calibrated in fully developed turbulent flows the readings obtained under strongly accelerated or retarded flows may not give accurate estimates of the wall shearing stress.
The advantages of such an instrument are

(a) Directional resolution is high thus 3-dimensional flows may be investigated.

(b) If the thermal capacity of the element is low (thin film devices) then fluctuating shear stress measurements can be made (provided the thermal boundary layer is within the viscous sublayer).

Unfortunately these instruments cannot be used in rough-wall flows.

A device used by BRADBURY (38) relies upon heat transfer by pulsing the current to a fine thin film then sensing the resultant downstream temperature pulse by a further thin film. The time between the pulse emission and reception is thus related to the velocity of the medium at the wall and the distance between the elements. The equipment is calibrated in fully developed laminar flow channel. For the calibration to be valid in turbulent boundary layers the temperature pulse must remain within the sublayer. High levels of shearing stress are difficult to measure since the 'time of flight' of a pulse is very small. However the advantages of the technique are (a) that very low shearing stresses may be measured quite accurately (b) that fluctuations in the shearing stress are comparatively easily measured.

MITCHELL and HANRATTY (39) developed an electrochemical shear stress meter in which the mass transfer rate was directly related to the wall shearing stress. The concentration boundary layer must (like the thermal boundary layer) remain in the viscous sublayer and the applied voltage must be sufficiently high for the mass transfer rates to be diffusion controlled. The results of this work show that the turbulence intensity \( \frac{u' \overline{U}^2}{U} \) may not be dependent upon the Reynolds number as indicated by hot-wire investigations. (A value of 0.32 is quoted). The system obviously has limitations in that the fluid investigated must be an electrolyte and that wall roughness cannot be simulated. However COLES (4) points out that the use of water in boundary layer investigations has been neglected, possibly to the detriment of progress.
2.7. Flow Around Obstacles

When an object is fully immersed in a region governed by wall variables the resulting perturbed pressure field may be expected to be correlatable with the wall shearing stress. It should be remembered that the flow is governed by field equations thus a simple 1:1 correlation is not likely.

For such investigations a number of types of obstacles have been used:

1. Preston tube (PRESTON (40))
2. Boundary Layer Fence (HEAD and RECHENBERG (41))
3. Stanton tube (PAGE and FALKNER)
4. Razor Blade (WYATT and EAST (42))
5. Static hole pair (DUFFY and NORBURY)

Ideally any device used should be capable of giving accurate results without prior calibration. This does not mean that the calibration should not be checked merely that the checks are carried out in a limited range. Thus implicitly either the geometry of the obstacle can be produced exactly without too much difficulty or small changes in geometry are unimportant.

Generally the calibration parameters for such devices may be obtained by postulating that the perturbation to the local pressure field \(\Delta p\) is a function of the size of the object \(d\), the fluid properties \((\rho, v)\) and the wall shearing stress \(\tau_w\) provided the pressure gradient does not influence the obstacle's active zone.

Dimensionally:

\[
\frac{\Delta p d^2}{\rho v^2} = \frac{\Delta p d^2}{\rho v^2} \left[ \frac{\tau_w d^2}{\rho v^2} \right]
\]

The form of the above function is such that the pressure reading \(\Delta p\) increases rapidly with the characteristic dimension \(d\). One would
therefore prefer to use the largest possible obstacle with the proviso that its presence should not significantly affect the overall flow structure. The size of the obstacle is governed by the type of flow (i.e. the extent of the wall law region). Stanton (half pitot) tubes, the boundary layer fence and the razor blade device are all primarily intended to operate within the viscous sublayer whereas the Preston tube is used in the logarithmic region. Only the Preston tube and razor blade devices will be discussed here since they can be accurately and easily manufactured to give repeatable calibrations.

**Preston Tubes**

PRESTON (40) suggested the use of wall mounted circular pitot tubes for wall shearing stress measurement and published their calibration curve in 1954 (it is believed that COLES also suggested this method independently around the same time). PRESTON's argument is summarised by equation 2.8.1 and the underlying assumptions. There arose some controversy as to whether PRESTON's calibration (obtained in fully developed pipe flow) could be applied to external flows. This was based upon doubts concerning the similarity near the wall of pipe flows and external flows. In 1958 BRADSHAW and GREGORY (43), the Staff of the N.P.L. (19) and SMITH and WALKER (17) published experimental evidence which appeared to substantiate these doubts. Subsequently (in 1962) HEAD and RECHENBERG (41) showed that there was similarity and validated the Preston tube as a technique. They also noticed appreciable circumferential variations in the wall shearing stress in fully developed pipe flow. (The history of this controversy is summarised in detail in ref. (41)). They (41) found evidence of some error in Preston's original calibration. This led to PATEL's (16) extremely thorough investigation published in 1965.

In addition to giving a definitive calibration for Preston tubes
PATEL was able to show the effects of the breakdown of the wall similarity laws in pressure gradients (see earlier discussions). PATEL's (16) recommendations on Preston tube accuracy are:

(a) Adverse pressure gradients

- Maximum error $3\%$ if $0 < \frac{a \nu}{u_r^3} < 0.01$ ; $\frac{d u_r}{v} < 200$
- Maximum error $6\%$ if $0 < \frac{a \nu}{u_r^3} < 0.015$ ; $\frac{d u_r}{v} < 250$

(b) Favourable pressure gradients

- Maximum error $3\%$ if $0 > \frac{a \nu}{u_r^3} > -0.005$ ; $\frac{d u_r}{v} < 200$ ; $\frac{d}{dx} \left(\frac{a \nu}{u_r^3}\right) < 0$
- Maximum error $6\%$ if $0 > \frac{a \nu}{u_r^3} > -0.007$ ; $\frac{d u_r}{v} < 200$ ; $\frac{d}{dx} \left(\frac{a \nu}{u_r^3}\right) < 0$

Razor blade devices

Although BROWN and JOUBERT (1) express doubts concerning the repeatability of calibration of these devices experiments conducted by the author in a 2-dimensional duct have shown good agreement with previous results obtained by EAST (44). The devices operate within the laminar sublayer and thus are far less susceptible to pressure gradients but then have the disadvantage that the pressure reading ($\Delta p$) is very low. Thus the accuracy is generally limited by the accuracy of the pressure reading. Geometric repeatability in manufacture is achieved by using magnetic clamps to attach the razor blade to the surface thus avoiding the uncertainty of adhesive film thickness.

2.8. Direct Measurement

This technique involves measuring the force upon an isolated section (element) of the wall. All known instruments have achieved this isolation by having a gap around the element. The existence of this gap causes secondary forces particularly when pressure gradients exist. Thus
In low speed flows the practical difficulties are magnified by the very small forces involved (low $\tau_w$ values) and experimental studies in this particular area are not numerous. KEMP (47), SCHULZ-GRUNOW (48) and SMITH and WALKER (17) have presented results for zero pressure gradient flat plate boundary layers. WHITE and FRANKLIN (45) and SMITH et al (46) have given results for flows in concentric annuli. Recently, BROWN and JOUBERT (1) published results of investigations in adverse pressure gradients over smooth walls.

This technique is not in principle restricted by similarity laws and thus is potentially capable of wider application than any other. To the author's knowledge, the method has not been used in rough wall situations or flat plate favourable pressure gradient boundary layers.

The main purpose of this thesis is to describe the development of such an instrument and assess its potential in the practically important case of three dimensional, rough wall flows in pressure gradients. This survey will be developed in more detail in Chapter 3.

2.9. Summary

From the above discussions it is hoped that the reader will have gained the impression that: (a) The measurement of the wall shearing stress is important. (b) The choice of which technique to use depends upon the flow. (c) In certain cases only the direct method is potentially capable of giving conclusive results.

For example the flows over rough walls with pressure gradients have usually been considered in two layer models. These models are outlined in the similarity laws stated earlier. There is very little positive evidence for the validity of this model under extreme flow conditions although in the light of smooth wall studies it seems reasonable. However where the roughness is quite 'sparse' there may be room for doubt. In
strong favourable pressure gradients there is still some uncertainty in the measurements over smooth walls.

An experimental investigation has been conducted which sets out to establish the following points

(1) Can direct measurement be used in rough wall flows?
(2) How do the gap flow characteristics affect readings in favourable pressure gradients?
(3) Can the secondary forces be predicted accurately for different instruments?

The results of these investigations are presented in this thesis.
FIG. (2.1)

WALL SHEAR STRESS MEASUREMENT

after BROWN and JOUBERT (1)

MOMENTUM

Momentum Integral Equation

Special case - fully developed flow

Viscous sublayer

Log. region

Detailed

VELOCITY PROFILES

Overall

Skin-friction Laws

HEAT TRANSFER

Steady heat transfer

Temperature pulse

MASS TRANSFER

Static pressure holes

Preston tube

SIMILARITY OF FLOW AROUND OBSTACLES

Boundary layer fence

Stanton tube

Dye tracer
FIG. 2.2 CONSTANTS A and B

IN THE LOGARITHMIC LAW

\[ \frac{U}{u_*} = A \log_{10} \left( \frac{y}{\delta'} \right) + B \]

(B FOR SMOOTH WALLS)

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CHAPTER 3

SKIN-FRICTION METER DESIGN AND CALIBRATION

"Every instrument requires to be made by experience"

Leonardo da Vinci
3.1. Introduction

A direct skin-friction meter (or floating element meter) is an instrument which measures the total force upon a mechanically isolated section of the wall. Ideally this isolation should not affect the flow investigated and the total force \( F \) would then be the wall shearing stress \( \tau_w \) integrated over the measuring area. In practice the isolation modifies the local flow structure and causes additional forces not present in the undisturbed flow. The total force \( F \) then differs from the integrated shear force \( F_r \).

\[
P = F_r = \int \int \tau_w \, dA
\]

This difference being the sum of several small secondary forces \( \Sigma F_s \) due to the presence of the measuring instrument.

In all previous investigations the mechanical isolation of the measuring section (element) has been achieved by a clearance (gap) around the element. The secondary forces mentioned above are due to the roughening effect of the gap and the penetration of boundary layer fluid into the gap under the influence of the pressure gradient. The complex nature of these secondary flows at present defies analytic solution. Some general trends can be deduced by the approximate methods estimated in Section 3.2.

The force on the element of the meter can be measured mechanically. The mechanical design of the measuring system is developed in Section 3.3 and is compared with previous systems in Section 3.4. The methods by which the force measuring system can be calibrated are discussed in Section 3.5. Finally the meter calibration in boundary layer flows (dynamic calibration) is described in Section 3.6.
3.2. Fluid Dynamic Design Aspects

The method of dimensions may be used in order to establish the main parameters governing the total force \( F \). The floating elements used in previous investigations have either been rectangular (e.g. DHANAN), circular (e.g. SMITH and WALKER) or cylindrical (e.g. WHITE) and have been tailored to the particular flow situation investigated. For flat plate studies the choice is restricted to circular or rectangular shapes. If the instrument is to be used in three dimensional boundary layers it should be directionally sensitive, recording the component of the shear force in the required direction. It would therefore be unwise to use an element shape upon which the secondary forces depend upon flow direction. A circular element offers the same obstruction to the flow, irrespective of direction, and has the added appeal of ease of manufacture. In two-dimensional boundary layer studies the rectangular element possesses several advantages:

(a) The disturbance caused by the gap is considerably less three-dimensional than that caused by the gap around a circular element.

(b) For the same element area, and hence the same total force, the rectangular element gives a more localised indication of the shearing stress.

(c) If the meter is used in rough wall flows the problems concerned with mounting roughness elements in a regular array upon the meter element are significantly reduced.

Although no three-dimensional flows were investigated the meter developed was intended to be suitable for such measurements.
3.2.1. Dimensional Analysis of the Secondary Force Problem

(Fig. 3.1 refers)

The measured total force \( F \) will generally be a function of:

(i) The element geometry -
   - shape - circular
   - size - diameter \( (d_m) \) and gap width \( (g) \)
   - edge width \( (\epsilon_m) \)

(ii) The pressure gradient \( \left( \frac{dp_w}{dx} \right) \)

(iii) The fluid properties - viscosity \( (\nu) \) and density \( (\rho) \)

(iv) The wall shearing stress \( (\tau_w) \)

(v) The position of the element within the gap \( (\chi) \)

Thus \( F = F \left[ d_m, \epsilon_m, g, \nu, \rho, \frac{dp_w}{dx}, \tau_w \right] \)

Dimensional arguments yield:

\[
\frac{u_F}{\pi \tau_w d_m^2} = \frac{F}{\tau_w A_m} = \frac{F}{\tau_w A_m} \left[ \frac{\epsilon_m}{d_m}, \frac{\nu}{d_m}, \frac{\chi}{d_m}, \frac{\frac{dm}{\nu}}{\tau_w}, \frac{\frac{dp_w}{dx}}{\tau_w} \right]
\]

where \( u_F = \sqrt{\frac{\tau_w}{\rho}} \), the friction velocity and \( A_m = \frac{\pi d_m^2}{4} \), the element area.

If the meter is of the 'null-displacement' type then the group \( \left( \frac{\chi}{d_m} \right) \) is constant and equal to zero. The groups \( \left( \frac{d u_F}{\nu} \right) \) and \( \left( \frac{d_m \frac{dp_w}{dx}}{\tau_w} \right) \)
are the Reynolds number \( (R) \) and Euler number \( (E) \) respectively. Equation 3.2.2 thus becomes:

\[
\frac{F}{\tau_w A_m} = \frac{F}{\tau_w A_m} \left[ \frac{\epsilon_m}{d_m}, \frac{\nu}{d_m}, \frac{\chi}{d_m}, R, E \right]
\]

The above expression (3.2.3) does not indicate what form the functional relationship between the non-dimensional groups might take.

The secondary forces, defined in equation 3.1.1, may be minimised by careful choice of the meter geometry (groups \( \frac{\epsilon_m}{d_m} \) and \( \frac{\nu}{d_m} \)) if a reasonable
approximation to the function \( \left( \frac{F}{\tau_w A_m} \right) \) can be found.

### 3.2.2. A Physical Model of the Gap Flow

The simplest flow situation will exist when the meter is used in a zero pressure gradient boundary layer. In Fig. (3.2) intuitive sketches of the flow patterns, shear stress and pressure distribution are given.

It is known that when the gap is small the mean pressure within it is equal to the wall static pressure - the principle used in static pressure measurements. It is likely that the pressure distribution is governed by the wall shearing stress. The wall shearing stress is itself modified by the gap as shown in Fig. (3.2). This effect may be conceptually described as a shear modification force \( F'_T \) defined by

\[
F'_T = \int_{A_m} (\tau_o - \tau_w) dA
\]

where \( \tau_o \) is the modified shear stress distribution due to the presence of the meter.

HÄKKINEN (51) suggested that the total force \( F \) in a zero pressure gradient flow should be regarded as the integral of the wall shearing stress, \( \tau_w \), over the element area plus some proportion of the gap area. This yields:

\[
F'_T \propto \delta d_m \tau_w
\]

If the wall shearing stress is regarded as substantially constant over the element (or a mean value is assigned to \( \tau_w \)) then the total force is \( \tau_w A_m \). Normalising the above relation by \( \tau_w A_m \) gives:

\[
\frac{F'_T}{\tau_w A_m} \text{ defined as } \gamma_T \propto \frac{\delta}{d_m}
\]

The same result may be obtained by postulating that the absence of wall shearing stress over the gap width, \( \delta \), may be viewed as equivalent
to an accelerational force proportional to \( \tau_w \Delta \frac{d_m}{d_m} \). The resultant acceleration of the fluid over the meter element would then lead to higher velocity gradients and enhanced wall shearing stress. Some proportion of the increased momentum would be destroyed by \( \tau_o \) and equation 3.2.5 is obtained once more.

If the pressure forces are assumed to be similarly related to the meter geometry the total secondary force upon the meter element (normalised) may be written

\[
\frac{F_s}{\tau_w \Delta m} \quad \text{defined as } \gamma_o = k_o \left( \frac{\Delta}{d_m} \right) \]

3.2.6

HAKKINEN (51) implicitly gives \( k_o \) of order unity. BROWN's (1) results indicate that \( k_o \) is an order of magnitude greater and may be positive or negative depending upon \( R_T \).

The above model is valid only for zero pressure gradient flows. Where an external pressure gradient exists there is a nett flow through the gap. This is shown in Figures (3.3) and (3.4) for adverse and favourable pressure gradients respectively. As an approximation the physical process may be considered as the superposition of two simpler flows:

(a) Flow through the gap under the action of an externally imposed pressure gradient where the external stream is stationary.

(b) Modifications to the external flow and stress distributions over the element due to the gap flow (estimated by model (a))

The two-dimensional model for case (a) is shown in Fig. (3.5). The following analysis is therefore applicable to a semi-infinite rectangular element of streamwise length \( d_m \). Applying the energy equation between points (1) and (2):-

\[
\psi_{s_1} - \psi_{s_2} = \frac{1}{2} \rho \frac{V_1^2}{V_1} + \text{LOSES} \]

3.2.7

If the pressure gradient is constant then
\[ p_{s_1} - p_{s_2} = - \frac{d}{dx} \left( \frac{dp_g}{dx} \right) \]

Employing the definition of the Euler number \( E = \frac{\frac{dp_g}{dx}}{\tau_w} \) the above equation becomes

\[ p_{s_1} - p_{s_2} = - \tau_w E \]

The Reynolds number of the gap flow \( R_g = \frac{V_1 g}{\nu} \) can be estimated by assuming that the losses are negligible. From equations 3.2.7 and 3.2.8

\[ V_1 = \left( \frac{2 \tau_w E}{\rho} \right)^{\frac{1}{2}} \]

hence

\[ R_g = \left( \frac{2 \tau_w E}{\rho} \right)^{\frac{1}{2}} \cdot \frac{g}{\nu} = \frac{2u_\tau}{\nu} \sqrt{2E} \]

Since \( g = 0[10^{-5}] \); \( \nu = 0[10^{-5}] \) and \( u_\tau = 0[10^0] \)

\[ R_g = 0[\sqrt{2E}] \]

Typically the maximum values of \( E \) encountered in practice are around 25.

Thus \[ R_g = 0[10] \]

The gap flow is therefore laminar and the losses will be proportional to the velocity, \( V \). Equation 3.2.8 may be written

\[ - \tau_w E = \frac{1}{2} \rho V_1^2 + \lambda \rho V_1 \]

where \( \lambda \) is constant of proportionality.

Some of the momentum associated with the gap flow will be destroyed within the meter resulting in a momentum exchange force, \( F_m \). This momentum is proportional to the mass flow rate through the gap multiplied by a representative velocity.

Thus

\[ F_m \propto \rho V_1^2 g d_m \]

From equation 3.2.9

\[ V_1^2 = 2 \lambda^2 + \frac{\tau_w E}{\rho} + 2\lambda \sqrt{\lambda^2 + \frac{\tau_w E}{\rho}} \]
Dividing equation 3.2.10 by the shear force \( \tau_{w,m} \) and substituting \( \psi_f^2 \) for \( V_i^2 \) from equation 3.2.11

\[
\frac{F_m}{\pi \tau_{w,m} d_m} \quad \text{defined as } \gamma_m = -k_c \left( \frac{\psi}{d_m} \right) \left( E + \frac{2\lambda}{u_T} + \frac{2\lambda}{u_T} \sqrt{\frac{\lambda}{u_T}} \frac{\tau_{w,E}}{\rho} \right) 3.2.12
\]

If the losses may be absorbed by the constant, \( k_c \), then

\[
\gamma_m = -k_c \left( \frac{\psi}{d_m} \right) E \quad 3.2.13
\]

(This is not strictly valid since implicitly turbulent gap flow is thereby assumed)

Since the meter is in a pressure field there will be a buoyancy force, \( F_p \), upon the element. This force is given by

\[
F_p \propto (p_{s1} - p_{s2}) d_m \epsilon_m \quad 3.2.14
\]

Normalising by the shear force, \( \tau_{w,m} \), and substituting for the pressure term from equation 3.2.8

\[
\frac{F_p}{\tau_{w,m}} \quad \text{defined as } \gamma_p = -k_p \left( \frac{\epsilon_m}{d_m} \right) E \quad 3.2.15
\]

where \( k_p \) is a constant of proportionality.

When there is an external flow (i.e., the boundary layer) there are additional momentum exchange forces. The fluid drawn into the meter has an associated \( x \)-directed (streamwise) momentum which will be totally dissipated within the meter. In adverse pressure gradients this momentum will be destroyed against the case rather than the measuring system as shown in Fig. (3.6.a). In favourable pressure gradients it is destroyed against the element and measuring system giving the form of pressure distribution shown in Fig. (3.3.(i)). This additional momentum exchange force, \( F_n \), may be approximated in two ways. BROWN (1) suggests that, since the representative velocity of the incoming fluid is \( u_T \)

\[
F_n \propto \text{gap flow rate } x u_T
\]

i.e.

\[
F_n \propto \rho g d_m \left( \frac{dp}{dx} \right)^{\frac{1}{2}} u_T \quad 3.2.16
\]
Normalising
\[ \frac{F_n}{\tau_w \lambda_m} \text{ defined as } y_n \alpha \left( \frac{g}{d_m} \right) \frac{1}{B^2} \quad (3.2.17) \]

This physical process may be considered in more detail by examining the flow in the gap region. There is a mass flow through the gap which will be drawn from the slower moving fluid near the wall. In the two-dimensional model shown in Fig. (3.6(b)) the fluid, at a mass flow rate \( m \) per unit width, is taken from the region

\[ 0 < y < \delta' \]

This region is defined by
\[ \int_{0}^{\delta'} \rho \, U \, dy = m \quad (3.2.18) \]

The extent of the affected region will depend upon the pressure gradient and the velocity profile in the wall region. For a smooth wall flow the velocity profile is given by the wall law:

\[ \frac{U}{u_T} = f \left( \frac{y u_T}{v} \right) \quad (3.2.19) \]

If the outer limit of the affected region is within the viscous sublayer (i.e. \( \delta' < \delta_g \)) the velocity profile is given by equation 2.3.1

i.e.
\[ \frac{U}{u_T} = \frac{y u_T}{v} \]

This is an adequate description of the velocity profile for \( \frac{y u_T}{v} \ll 7 \).

Substituting for \( U \) from equation 2.3.1 in equation 3.2.18 and integrating gives
\[ m = \frac{\delta'}{2} \rho \frac{u_T^2}{v} \quad (3.2.20) \]

Substituting \( m = \rho V_1 \delta \) in the above equation and rearranging
\[ \left( \frac{\delta' u_T}{v} \right)^2 = \frac{2 V_1 \delta}{v} \quad (3.2.21) \]
The limits of validity of equation 3.2.21 are fixed by \( \frac{\delta' u_r}{v} < 7 \). Thus
\[
\frac{2V_1 e}{v} < 49
\]

or
\[
V_1 < 25 \frac{v}{e}
\]

It was shown earlier that, neglecting losses,
\[
V_1 = \left( \frac{2 \tau_w E}{\rho} \right)^{\frac{1}{2}}
\]

thus
\[
\frac{E}{v} \left( \frac{2 \tau_w E}{\rho} \right)^{\frac{1}{2}} < 25
\]

which may be rearranged to give, approximately
\[
\left( \frac{E}{d_m} \right)^2 R_r E < 310
\]

For BROWN's (1) meter \( \frac{E}{d_m} = 0.004 \),

thus
\[
R_r E < 2 \times 10^7
\]

The momentum per unit width, \( M_m \), associated with the gap flow is given by
\[
M_m = \int_0^{\delta'} \rho (U^2 + u'^2) dy
\]

For smooth wall flows MITCHELL (39) gives \( \frac{u_r^2}{U^2} = 0.32 \). Substituting for the velocity, \( U \), from equation 2.3.4 and integrating, equation 3.2.25 becomes
\[
M_m = 1.32 \rho \frac{u_r}{v} \frac{\delta'^3}{3}
\]

Substituting for \( \delta' \) from equation 3.2.21 and normalising by the shear force per unit width, \( \tau_w d_m \)

\[
\frac{M_m}{R_w \tau_w} \text{ defined as } \gamma_n = \frac{0.44}{R_r} \left( \frac{2E}{v} V_1 \right)^{3/2}
\]
Substituting for \( V \) from equation 3.2.23

\[
\gamma_n = 2.1 \left( \frac{E}{d_w} \right)^{3/2} R_T^{-1/2} \ E^{3/4}
\]

3.2.28

The above result is based upon a two-dimensional model. For a circular element much the same flow behaviour is assumed and equation 3.2.28 becomes

\[
\gamma_n = k_n \left( \frac{E}{d_w} \right)^{3/2} R_T^{-1/2} \ E^{3/4}
\]

3.2.29

where \( k_n \) is a constant of proportionality of order unity. The modulus applied to \( E \) corrects for the direction of the force.

The modification in wall shearing stress due to the gap flow is rather complex and no physical model is suggested. The results of DICKINSON (52) may be used to deduce empirically how this secondary force \( F_r \) may vary with the Euler and Reynolds' number. DICKINSON (52) pressurised the meter case whilst the meter was being used in a zero pressure gradient flow. His results are shown in Fig. (3.7) in a modified form. The pressure difference between the case and the external stream has been expressed as \( \frac{d \rho}{d_x} \) and the case over pressure \( (p_c - p_a) \) plotted as \( \frac{2(p_c - p_a)}{\tau_w} \).

Since there can be no buoyancy forces or momentum exchange within the case the secondary forces are purely due to flow impingement and shear stress modification. The nett outward flow for 'adverse' pressure gradient conditions tends to centralise the element thus no conclusions can be drawn from the apparent agreement with BROWN's results. For flow into the case \( (E - V_e) \) the upstream shear modification may be assumed negligibly different from that occurring when the meter is used in a favourable pressure gradient. The momentum destroyed against the leading edge of the element is estimated from equation 3.2.29 by substituting for \( R_T (= 2800) \) and the gap width \( \left( \frac{E}{d_w} = .003 \right) \)

Thus

\[
\gamma_n = 8.7 \times 10^{-3} k_n \left| E^{3/4} \right|
\]
DICKINSON's results yield \( y = 1 - 8 \times 10^{-3} \) \( E \) thus the shear modification forces are small and are similar in character to the momentum force \( \gamma_n \).

BROWN and JOUBERT's (1) results for adverse pressure gradient flows are shown in Figs. (6.16, 6.17) in Chapter 6. At high Reynolds's numbers (\( R \)) the secondary forces become approximately independent of both the Reynolds's number, \( R \), and the Euler number \( E \). The meter then tends to overestimate the wall shearing stress by approximately 10\%. No feature of the physical model proposed can explain the overestimate thus it is assumed that this is a function of the shear modification. This modification must therefore be proportional to the wall shearing stress and independent of both \( E \) and \( R \) at high Reynolds's numbers.

The effect of wall roughness may also be predicted by the physical model described in the preceding text. There are several important practical restrictions since the roughness must be duplicated over the element surface. The element must also encompass a representative sample of the roughness thus the roughness element spacing must be small in comparison with the element diameter. If the roughness is relatively sparse one can use a zone of influence concept. Each roughness element is attached to a flat plane. The zone of influence of the roughness element on that plane is defined by the locus of points bisecting the lines between it and adjacent elements. This zone of influence is inversely proportional to the roughness packing density. If the influence area is denoted \( \Lambda_z \) then the number of roughness elements to be attached to the meter element is \( \Lambda_m/\Lambda_z \). Naturally, the number is rarely an integer so that rounding off is necessary. This gives a useful criterion for whether the roughness sample is representative. If the amount by which the nearest whole number differs from \( \Lambda_m/\Lambda_z \) is greater than 5\% of \( \Lambda_m/\Lambda_z \) the sample size is too small. Thus either a larger element or denser roughness packing must be used.
Assuming that the roughness over the meter element is representative its effects upon the meter performance may be predicted. In a zero pressure gradient it is likely to improve the meter accuracy since the roughening effect of the gap is comparatively lower. Therefore this secondary force is likely to be small. The meter should therefore be capable of giving the wall shearing stress to within ± 1% in rough-wall, zero-pressure gradient flows.

In favourable pressure gradients the momentum exchange force, $F_n$, will be a function of the roughness since both the velocity profile and the turbulence are also functions of the roughness. The force, $F_n$, is likely to be increased and the normalised force, $\gamma_n$, will therefore be correspondingly higher. The shear modification force, $F_{\gamma}$, is likely to exhibit the same increase. Since the mechanisms by which these forces arise is the same in both rough and smooth wall cases the meter characteristics should be similar.

In adverse pressure gradients one could advance the same arguments as above. The meter characteristics are probably similar although it cannot be shown that the estimates of shear stress will be higher or lower than in the smooth wall case.

### 3.2.3. Summary

A physical model of the flow mechanisms around the meter element was presented in the previous section. The findings are summarised below:

$$\frac{F}{\tau_{w,m}} = \frac{F}{\tau_{w,m}} \left[ \frac{\varepsilon}{d_m}, \frac{\varepsilon}{d_m}, E, R, \text{roughness} \right]$$  \hspace{1cm} 3.2.30

or

$$\gamma = \gamma \left[ \frac{\varepsilon}{d_m}, \frac{\varepsilon}{d_m}, E, R, \text{roughness} \right]$$  \hspace{1cm} 3.2.31

and

$$\gamma = 1 + \frac{2F_s}{\tau_{w,m}} \hspace{1cm} 3.2.32$$

$$= 1 + \sum \gamma_s$$  \hspace{1cm} 3.2.33
For zero pressure gradients in smooth wall flow,

\[ \gamma = 1 + \gamma_0 = 1 + k_0 \left( \frac{\varepsilon}{d_m} \right) \]  

For zero pressure gradients in rough wall flow,

\[ \gamma = 1 + k_0 \left( \frac{\varepsilon}{d_m} \right) \]  

where \( k_0 = k_0(\text{roughness}) \)

For favourable pressure gradients in smooth wall flow,

\[ \gamma = 1 + \gamma_m + \gamma_p + \gamma_n \]

where, from DICKINSON's results, \( \gamma_n \) includes the shear modification force i.e.

\[ \gamma = 1 - k_n \left( \frac{\varepsilon}{d_m} \right) E - k_p \left( \frac{\varepsilon}{d_m} \right) E - k_n \left( \frac{\varepsilon}{d_m} \right)^{3/2} R_T^{3/2} E \]  

In rough wall flows equation 3.2.36 is true but the constants \( k_0, k_p, k_n \) become functions of the roughness.

For adverse pressure gradients in smooth wall flow

\[ \gamma = 1 + \gamma_m + \gamma_p + \gamma_r \]

\[ = 1 - k_0 \left( \frac{\varepsilon}{d_m} \right) E - k_p \left( \frac{\varepsilon}{d_m} \right) E + k_r \left( \frac{\varepsilon}{d_m} \right) E + \text{constant} \]  

where \( k_r \) is a function of \( R_T \) such that as \( R_T \) increases \( k_r \) tends to a constant value cancelling the effects of \( \gamma_m \) and \( \gamma_p \). The constant will be a function of \( \left( \frac{\varepsilon}{d_m} \right) \). In rough wall flows all the constants are functions of the roughness.

3.3 System Design

Ideally the force measurement system should be:

(i) Accurate and give repeatable readings;
(ii) Directionally sensitive;
(iii) Robust;
(iv) Easy to manufacture and use;
(v) Free from calibration drift

In addition its dynamic interaction with the fluid dynamics of the element region should be minimal. The following sections describe the static and dynamic analysis, detail design and system accuracy.

3.3.1. Static and Dynamic Response of Measurement System

A suitable measurement system is shown in Fig. (3.8). Its response to an applied fluctuating force $F(t)$ is given by the solution to:—

$$ F(t) = M \dddot{x} + C(\ddot{x} - \dot{x}') + S(x - x') + F_T $$

where $M$ is the suspended mass; $C$ the damping coefficient; $S$ the spring stiffness; $F_T$ the restoring force (which may itself be a cyclic function of time if the force transducer is alternating current powered); $x'$ the element or mass displacement and $x'$ the wall displacement.

For static equilibrium the steady component of the fluctuating force must balance the applied forces:

i.e. if $F(t) = F + f(t)$

then for static equilibrium

$$ F = F_T + Sx $$

and in the measuring (null) position $x = 0$, $F = F_T$.

Since there is a limit to the resolution of any displacement measuring system ($\pm \Delta x$) the limits of resolution of the force ($F$) will be $\pm S\Delta x$. The spring stiffness ($S$) should therefore be small.

For repeatability of the measurement of the static force this first criterion can be expressed
The harmonic response to the fluctuating component \( f(t) \) may be deduced from equation 3.3.1. For simplicity assume that the system is in vibration-free mountings such that \( \dot{\chi}' = 0 \). Then the equation becomes

\[ f(t) = \ddot{\chi} + C \dot{\chi} + S \chi \]  

If \( M, C \) and \( S \) are independent of \( \chi \) and the forcing assumed to be \( f \sin(\omega t) \) where \( f \) is constant then the solution to 3.3.5 is

\[ \chi = \psi_1 e^{-\zeta \omega_n t} \sin(\omega_d t + \phi_1) + \psi_2 (\omega_d - \omega^2) \]

The first part of the solution is the transient response and the second is the steady state response. The terms in the solution are defined as:

- \( \psi_1 \): initial amplitude \( t=0 \) of excitation
- \( \phi_1 \): lag angle determined by initial conditions \( t=0 \)
- \( \zeta \): damping ratio \( C/2M \omega_n \)
- \( \omega_n \): undamped natural frequency \( \sqrt{S/M} \)
- \( \omega_d \): damped natural frequency \( \omega_n \sqrt{1 - \zeta^2} \)
- \( \psi_2 \): steady state amplitude given by:

\[ \psi_2 = \frac{f}{\sqrt{(S - M \omega^2)^2 + (C \omega)^2 \omega_n^2}} \]

\( \phi_2 \): phase angle of resultant oscillation

The maximum amplitude of the steady state oscillation occurs when the forcing frequency is equal to the natural frequency.

when \( \omega = \omega_n = \sqrt{S/M} \)

\[ \psi_{max}^2 = \frac{f}{C \omega_n} \]

A critically damped system is practically advantageous when \( \zeta = 1 \) this gives \( C^2 = 4MS \)
the amplitude expression (3.3.7) becomes

\[ A_2 = \frac{f}{(S + \frac{M}{a^2})} \]

having a maximum

\[ A_{2 \text{ max}} = \frac{f}{2S} \]

The above expression implies that for small amplitude the stiffness, S, should be high whereas the earlier static criterion demanded that S be low. As a compromise the system should be slightly overdamped (high value of C). Equation 3.3.10 indicates that both S and M should be large. Since S must be low, in accordance with 3.3.4, the mass M should be large if the amplitude of oscillation is the critical factor.

### 3.3.2. Suspension Design

The suspension serves two important purposes:

(a) it ensures frictionless motion of the element

(b) it imparts directional sensitivity.

In Fig. (3.9) a typical suspension arrangement is shown. The system's spring stiffness \( S_s \) may be calculated by treating the flexures as encastre beams of length \( l \), second moment of area \( I_{zz} \) and Young's modulus \( E \).

Then

\[ S_s = \frac{24 E I_{zz}}{l^3} \]

The motion of the suspended mass \( M \) is similar to that of a pendulum. Thus there will be an additional restoring force which leads to an effective increase in stiffness of \( \frac{M_o}{l} \). The effective stiffness (at \( \gamma = 0 \) strictly) is

\[ S = \frac{24 E I_{zz}}{l^3} + \frac{M_o}{l} \]
The directional sensitivity can be achieved by arranging that the second moment of area of the flexures is large in the direction perpendicular to the measuring axis.

In Section 3.3.1 the design criterion for adequate force resolution was shown to be

\[
\text{Force resolution } \pm \Delta F = S(\pm \Delta X)
\]

whilst harmonic considerations demanded that the mass, \(M\), should be large. The harmonic criterion is therefore disregarded.

No quantitative calculations on the system have been made since the additional features of structural strength and rigidity cannot easily be quantified. The exercise has shown that the following general guidelines should be followed

1. Both spring and effective stiffnesses should be small. The increased stiffness due to the mass \(M\) should be of the same order as the spring stiffness.

2. For the system shown the spring stiffness can be calculated from equation 3.3.13. The effective stiffness should be about twice this value giving adequate resolution according to 3.3.14.

3. The damping factor should be over-critical.

3.3.3. Force and Displacement Transducers

The displacement resolution of a differential transformer is governed by the stability of the excitation voltage and frequency and the resolution of the voltage output measurement. The Schaevitz 050 DC transducer has an internal solid state oscillator of high stability demanding only a stable 24V dc supply. It is relatively easy to maintain a null position to within \(\pm 2 \times 10^{-5}\) inches \((\pm 0.5 \times 10^{-6}\) m) in practice. The calibration curve for this device is shown in Fig. 3.10.
The force transducer must be capable of exerting force without displacement. The unit chosen is essentially a Kelvin current balance as used by WHITE (45). The force between two concentric coils in series is proportional to the square of the current and the product of the number of turns on each coil.

\[ F \propto N_1 N_2 I^2 \]

The turns product \((N_1N_2)\) can be selected to give the desired force range for a given current range.

### 3.4. Previous Floating Element Skin-Friction Meters

In this section the previous relevant work, briefly discussed in Section 2.8, is summarised. In Table (1) details of previous meters are given in concise form.

Since the present investigation was concerned with incompressible flows the results of particular interest are those of SMITH and WALKER (17), BROWN and JOUBERT (1), DICKINSON and OZER:FOGUL (52) and WHITE and FRANKLIN (45). All the references are relevant from a system design viewpoint.

The basic distinction between the force measurement systems is based upon whether they are passive (allowing deflection) or active (actively applying a force to give a null displacement). The only true null displacement system is that of WHITE (45) who used a Kelvin current balance. BROWN and JOUBERT (1) effectively used a null displacement method although the source of their restoring force was gravitational (i.e. they tilted the meter). All other workers used deflection methods which are likely to be troublesome in flows with pressure variation. DICKINSON (52) used a force transducer which deflected small distances in comparison with the gap. The geometric details of the meters and their static calibration techniques are given in the Table.
The force measuring system is calibrated by the application of small accurately known forces. There are two basic techniques shown in principle in Figs. (3.11) and (3.12). The first employs a pulley (in jewelled bearings) around which a light cord passes. One end of the cord is attached to the measuring system whilst the other end supports the calibration weights. One disadvantage of this system is the frictional torque at the pulley bearings causes the tensions in the cord to differ on either side of the pulley. Also, if the pulley were not balanced, small gravitationally induced torques might be set up. DHAWAN (49) used a carefully manufactured, balanced pulley in which the frictional torque was of the order $10^{-6}$ mg-cm ($10^{-10}$ N-m) and obtained repeatable calibrations with very little scatter. BROWN and JOUBERT (1) found that scatter with the pulley system they used was appreciable and so used a modified version of the technique described below.

The second technique was suggested by HEADLEY (55). The system of three light cords (Fig. (3.12)) is used to transform vertical forces to horizontal forces and has the advantage that frictional effects are absent. Originally the idea was to vary the angles $\alpha$ and $\beta$ at the cord knot to give differing tensions in the cord attached to the element.

Resolving vertically, neglecting cord weight

$$T_2 \sin \alpha - W - T_1 \sin \beta = 0$$  \hspace{1cm} 3.5.1

and horizontally

$$T_1 \cos \beta = T_2 \cos \alpha = 0$$  \hspace{1cm} 3.5.2

From 3.5.1 and 3.5.2 the element load ($L$) is given by

$$L = T_1 \cos \beta = \frac{W}{\tan \alpha - \tan \beta}$$  \hspace{1cm} 3.5.3
The uncertainty in load due to uncertainties in the weight \( (dW) \) and the angles \( (da, dp) \) is given by a differential technique (see Ref(56))

\[
\frac{dL}{L} = \sqrt{\left(\frac{dW}{W}\right)^2 + \left(\frac{\sec^2 da \cdot da}{\tan a - \tan \beta} \right)^2 + \left(\frac{\sec^2 dp \cdot dp}{\tan a - \tan \beta} \right)^2} \cdot \frac{1}{f}
\]

Practically, \( da \approx dp \) and \( \beta \) is nominally \( 0^\circ \) when equation 3.5.4 becomes

\[
\frac{dL}{L} = \sqrt{\left(\frac{dW}{W}\right)^2 + \left(\frac{da}{\tan a} \right)^2 (1 + \sec^2 a)} \cdot \frac{1}{f}
\]

The minimum uncertainty occurs when \( a = 45^\circ \). In addition at large or small values of \( a \) the effects of cord sag increase \( da \) so that better accuracy is achieved if \( a \) is kept between \( 60^\circ \) and \( 40^\circ \).

### 3.6. Dynamic Calibration Techniques

If the meter is used in a flow situation where the wall shearing stress is independently measured a dynamic calibration curve may be obtained. In Chapter (2) the various methods for measuring the wall shearing stress and the situations in which they could be used were discussed. The calibration is thus limited by the alternative measurement of shearing stress.

In favourable pressure gradients the fully developed flow between semi-infinite smooth flat plates enables assessment of the wall shearing stress from the pressure gradient \( \frac{dp}{dx} \). Equation 2.1.9 becomes

\[
\tau_w = \frac{d}{2} \frac{dp}{dx}
\]

since \( H = 2d \), where \( d \) is the distance between the plates.

In Section 3.2 the pressure gradient parameter \( E \) was derived. For this case

\[
E = \frac{d}{m} \frac{dp}{dx} = \frac{2d}{m} \frac{d}{d} \tau_w
\]
Therefore if the plate spacing is variable the influence of $E$ upon the secondary forces can be established.

This method can give misleading results if the spacing of the plates is not maintained constant to a high degree of accuracy. FERRIS (57) showed that the effects of taper might be expected to lead to serious inaccuracies in the deduced values of the wall shearing stress. If the channel cross-sectional area is not constant there will be an imposed contraction or diffusion pressure gradient. The magnitude of this pressure gradient may be approximated by assuming the superimposed contracting flow to be inviscid. Thus for this component of the flow

$$p_c + \frac{1}{2} \rho \bar{U}^2 = \text{constant} \quad 3.6.3$$

Over a length $x$ the change in static pressure $\Delta p_c$ is therefore given by

$$\Delta p_c + \rho \bar{U} \Delta \bar{U} = 0 \quad 3.6.4$$

If the corresponding change in cross sectional area is $A_c$ then from continuity

$$A_c \bar{U} = \text{constant}$$

and

$$A_c \Delta \bar{U} + \bar{U} \Delta A_c = 0 \quad 3.6.5$$

Thus the contraction pressure gradient becomes

$$\Delta p_c = \rho \bar{U}^2 \cdot \frac{\Delta A}{A_c} \quad 3.6.6$$

The pressure gradient is measured by the pressure difference $\Delta p_m$ between two points a certain distance $(\Delta x)$ apart. The assumed value of the wall shearing stress is then $\frac{d}{2} \frac{\Delta p_m}{\Delta x}$. As shown above there is a contraction pressure gradient $\Delta p_c$ superimposed upon the viscous flow. If the taper is small, if the velocity profiles may be assumed 'self similar' and if the value of $\tau_w$ is not changed by the flow acceleration then the correct value of wall shearing stress is given by $\frac{d}{2} \frac{(\Delta p_m - \Delta p_c)}{\Delta x}$. 
Thus there is an error in the assumed wall shearing stress of

\[ \Delta \tau_w = \frac{\Delta p_c}{A_x} d_d \]  \hspace{1cm} 3.6.7

Combining equations 3.6.6 and 3.6.7

\[ \frac{\Delta \tau_W}{\tau_W} = \frac{\Delta A}{A_c} \cdot \frac{d_d}{A_x} \left( \frac{\rho U^2}{\tau_W} \right) \]  \hspace{1cm} 3.6.8

The coefficient \( \left( \frac{\rho U^2}{\tau_W} \right) \) is typically of the order \( [10^2] \) thus there should be large errors for relatively small tapers. (Details of an experiment to verify equation 3.6.8 are given in Chapter 6).

In adverse pressure gradients the shear stress must be deduced from the measurements described in Chapter 2. Preston tubes of varying sizes are generally used (e.g. BROWN and JOUBERT (1)) to ensure that the results are repeatable and that scale effects are absent.

For rough wall turbulent boundary layers there is no known method for measuring the wall shearing stress without making quite drastic assumptions. In the fully developed flow channel the relationship 3.6.1 is still applicable in its original form (2.1.9)

i.e.

\[ \tau_W = \frac{H}{4} \frac{d p_W}{dx} \]  \hspace{1cm} 3.6.9

If one wall is smooth the wall static pressure measurements yield the sum of the rough and smooth wall shear stresses

i.e.

\[ \tau_s + \tau_r = \frac{H}{2} \frac{d p_W}{dx} \]  \hspace{1cm} 3.6.10

Provided the roughness height is small in comparison with the hydraulic diameter \( H \) equation 3.6.10 becomes

\[ \tau_s + \tau_r = \frac{d_d}{2} \frac{d p_W}{dx} \]  \hspace{1cm} 3.6.11
Fig(31) Details of Floating Element
Fig (3.2) Two-dimensional Model: Zero Pressure Gradient
Fig 3.3) Two-dimensional Model:-
Favourable Pressure Gradient
Fig(34) Two-dimensional Model: -
Adverse Pressure Gradient
Fig (35)  **Gap Flow in Absence of Boundary Layer Flow**
Fig.(3.6) Gap Flows Including Boundary Layer Flow

(a) Adverse Pressure Gradient

(b) Favourable Pressure Gradient
Fig. (3.7) DICKINSON's(52) RESULTS FOR THE EFFECTS OF GAP FLOW
Externally applied force $F_o$

Suspended mass $M$

Damper coefficient $C$

Internally applied restoring force $F_r$

Spring stiffness $S$

Fig. (3.8) **Force Measurement System**

Fig. (3.9) **Suspension System**
Excitation voltage 24.00v
Sensitivity 86 mv./0.001" or 340 V/m
Inner core .080" Dia. Mild Steel

Fig(3:10) Displacement transducer calibration
Schaevitz 050 DC
Fig. (3.11) **Static Calibration Technique - PULLEY**

Fig. (3.12) **Static Calibration Technique - HEADLEY**
CHAPTER 4

EXPERIMENTAL APPARATUS AND PROCEDURE
4.1. Introduction

The major objective of the present investigation was to establish the calibration characteristics of floating element skin-friction meters in pressure gradient flows over rough and smooth walls. The first flow investigated was an adverse pressure gradient boundary layer over a smooth wall. This boundary layer was set up in the wind tunnel, described in Section 4.3, and measurements of the velocity profiles, static pressure distribution, turbulent stresses and wall shear stress were made. The experiment was conducted with two basic aims:

1. To examine the performance of the floating element meter in adverse pressure gradient flows in comparison with other wall shear stress measurement techniques.
2. To obtain basic information on the detailed structure of an adverse pressure gradient turbulent boundary layer.

The equipment used in this first experimental programme is described in Sections 4.3-4.6. The meter is described in Section 4.2.

The second experimental programme was designed to examine the floating element meter performance in favourable pressure gradients. A calibration channel, described in Section 4.7, was built and the meter calibrated in the fully developed turbulent flow. The bottom wall of the channel was then roughened, as described in Section 4.10, and the meter calibrated in the rough wall flow. The rough wall shearing stress was deduced from the pressure gradient and the smooth wall shearing stress ($\tau_s$) indicated by the Stanton tube, described in Section 4.8. For details of the method see Chapter 5.
4.2. Floating Element Skin-Friction Meter

4.2.1. Description

Using the design criteria given in Chapter 3 the meter design, shown in Figs. 4.1 and 4.2, was developed. The meter is of the null-displacement type having a flat, circular element made out of perspex. The diameter of the element was chosen as a compromise between localisation of measurement and magnitude of the integrated shear stress. The element is 0.993 inches (2.52cm) in diameter \(d_w\); has an edge width of approximately 0.0015 inches (0.004cm, nominally sharp-edged) and is centrally located in a 1.000 inch (2.54cm) diameter hole, giving a gap width \(g\) of 0.0035 inches (0.009cm). These dimensions were measured with a travelling microscope having a nominal accuracy of ±0.0005 inches (0.001cm).

The edge width was examined with a projection microscope of higher accuracy and was found to be variable (0.001 - 0.002 inches). The edge also had small imperfections invisible to either the touch or naked eye. Several elements were manufactured but no improvement in texture was possible because the perspex deformed plastically when thin sections were machined.

The element is rigidly attached to a perspex carrier \(h\) which supports the inner cores of the force transducer \(J_f\) and the displacement transducer \(E\). The position of the cores relative to the carrier may be varied by adjusting screws \(G\).

The carrier \(h\) is supported by the flexures \(D\) shown separately in Fig. 4.3. The upper ends of the flexures are attached to an adjustable carrier plate \(A\) which may be rotated and translated both horizontally and vertically relative to the fixed wall disc \(C\). The current to the inner core of the force transducer is carried by the flexures. The flexure design (Fig. 4.3) dissipates the resultant internal generation of heat without appreciable temperature rise. The motion of the carrier is
damped by a paddle immersed in a bath of silicone oil (K). The viscosity and level of the oil were set to give an overdamped system.

The meter movement is surrounded by an airtight perspex case. The electrical supplies are carried by B.N.C plugs araldited into the side of the case. The airtightness was checked by sealing the gap (around the element) with sellotape and subjecting the case to a partial vacuum of 400mm H₂O.

4.2.2. Alignment

The element surface must be accurately aligned with the surface of the wall disc. SMITH and WALKER (17) found that the element could be as much as 0.0005 inches below the wall without affecting their meter readings in zero pressure gradient flow. They also found that very slight proudness of the element was immediately noticeable. The element was adjusted to be between flush and 0.0002 inches below the surface using a sensitive dial gauge. The alignment was subsequently checked using a "TALYSURF" and was found to be as indicated by the dial gauge.

4.2.3. Operating Procedure

When a force is applied to the meter element it is displaced. The displacement alters the output voltage of the displacement transducer. (See circuit diagram Fig. 4.4). A current is then passed through the force transducer (see Fig. 4.5) and adjusted until the displacement transducer output returns to its initial value. In this position the external force is balanced by the electromotive force generated by the current (I) through the force transducer. The relationship between force and current is established by the static calibration technique described in Sections (3.5, 4.8).
The National Physical Laboratory 59inch x 9inch Boundary Layer Tunnel was used for the adverse pressure gradient work. The tunnel is described by Bradshaw and Helliens (58). The tunnel was subsequently modified by the insertion of a refrigeration unit in the diffuser blowing box compressed air line.

The tunnel roof was set to give a high, rapidly changing pressure gradient without separation. The boundary layer was tripped 4cm from the leading edge of the floor plate by a transverse rod 1mm in diameter. The resulting boundary layer was strongly non-equilibrium \((-1.2<\frac{\partial \delta}{\partial x}<2.6)\) and is therefore a flow which should tax prediction methods.

The tunnel speed was controlled manually by monitoring the velocity head at the entry to the working section \((\tilde{A})\) using a Betz water manometer. The velocity head was maintained at \(100 \pm 0.2\text{mm H}_2\text{O}\). Tunnel conditions seemed relatively insensitive to the diffuser blower box overpressure. The available adjustment (valve \(V\)) was coarse thus the overpressure was allowed to vary by \(\pm 1\text{cm H}_2\text{O}\) from its mean value of \(20\text{cm H}_2\text{O}\).

All other pressures were measured relative to the static pressure at \((A)\) using either a Betz or an electrical pressure transducer calibrated against the Betz.

4.4. Impact Probes

The local mean velocity in the boundary layer was measured using the total head tubes shown in Fig. (4.7). The probes were manufactured from stainless steel hypodermic tubing mounted in a brass stem. Rectangular mouthed probes were chosen to minimise the displacement errors described in Appendix (I). The angled probe (ii) was used for the measurements close to the wall. This probe design enabled the position of the wall to be sensed by electrical contact. Both probes were traversed
using the traversing mechanism described in Section 4.6.

Surface skin friction measurements were made with the Preston tube shown in Fig. (4.8(i)). The mouths of the tube were carefully lapped to obtain clean, sharp edges.

4.5. Turbulence Measurements

Turbulence measurements were made at several streamwise positions using a DISA 55A01 anemometer with single-sensor hot-wires of the normal (55F 31) and 45° (55F 32) type. For practical reasons it was not possible, in the time available for completion of the tests, to calibrate the hot-wires in a calibration tunnel. The technique used enabled the hot wire calibration to be deduced from the velocity profile measured with the pitot tubes. This technique is described in Section 5.5.

The mean bridge voltages (of the anemometer) were measured by integrating the output over a period of 10 seconds and taking the mean of three consecutive readings. The fluctuating voltages (r.m.s) were measured using either
(a) THERMOSONICS MODEL 1060 RMS Voltmeter coupled to an integrator.
(b) The system shown in Fig. (4.9).

The hot-wire sensors were mounted on a brass stem and moved through the boundary layer using the traversing mechanism shown in Fig. (4.10).

At each measuring position three readings were taken: with the normal wire parallel to the wall; with the 45° wire normal to the wall at ± 45°. These readings give sufficient information, in conjunction with the local velocity, to compute $\overline{u'^2}$, $\overline{v'^2}$ and $\overline{u'v'}$.

4.6. Probe Traversing Mechanism

The pitot and hot-wire probes were traversed normal to the wall
by the mechanism shown in Fig. (4.10). It basically comprises an electrically driven lead screw whose angular position is indicated by a digital encoder (digitiser). The positional accuracy of the mechanism could not be faulted with the measuring equipment available. If the unit is driven in the same direction (eliminating backlash in the gear train) the positional accuracy is believed to be better than ± 0.001 inches (0.0025cm).

4.7. Calibration Channel

4.7.1. Specifications

The second portion of the experimental work was carried out in fully developed duct flow. The basic principles are discussed in Section 3.6. The essential features of a calibration channel are

(1) Fully developed, two dimensional flow must be attained well upstream of the measurement section.

(2) The dimensions (depth $d_3$ and width $w_3$) must be accurately constant along the length of the channel and, for this particular channel,

(3) The channel depth $d_3$ should be capable of adjustment.

The way in which these features may be practically attained are discussed below together with a description of the channel shown in Fig. 4.11. The basic dimensions of the channel were fixed by machining and stock availability. The maximum and minimum sizes of steel strip available were $\frac{1}{2}$ and $\frac{1}{8}$ inch thus the maximum and minimum channel depths were fixed ($\frac{1}{2} < d_3 < \frac{1}{2}$). The steel strip used was remarkably constant in thickness over a single stock length. No variation in thickness could be detected over a 10' length using a 0-1 inch x 0.001 inch micrometer. The maximum length of flat steel plate which could be ground locally was 4ft thus the channel length was fixed. In order that the channel could be easily dismantled and reassembled without special lifting equipment
the weight of the top wall had to be within the lifting capabilities of
the author and yet thick enough to prevent distortion. The top and
bottom walls were thus made from ½ inch steel plate, 8 inches wide and 4
feet long, ground finished on both sides.

The bottom wall was drilled and tapped to enable the ½ inch wide
spacers to be rigidly bolted down so that the width of the channel could
be checked (with slip gauges) prior to fixing the top wall. The top wall
was then clamped to the spacers with G-clamps. Pressure tappings 1mm
in diameter were drilled in the top wall on the centreline every 5cms as
shown (Fig. 4.11). At the entry an inlet contraction and honeycomb
section was fitted. Trip wires were placed immediately downstream of the
contraction to locate transition. The measuring stations were 3½ inch
diameter holes located 3ft 6 inches from the entry. The minimum entry
length and aspect ratio were thus 84 channel depths and 14:1 respectively.

At the Reynolds numbers attainable with the fan used the wall shear stress
reaches its fully developed value within 50 duct heights (see Ref.(61)).
In order to check that the performance of the channel was satisfactory
proving trials were conducted. These are described below.

4.7.2. Proving Trials

The two-dimensionality of the flow was checked by surface flow
visualisation and Preston tube traverses across the measuring section.
Any flow peculiarities are most likely to occur at the maximum aspect
ratio so the Preston tube readings (using the probe shown in Fig. 4.8(ii))
were taken at a channel depth of ½ inch. These results are given in
Chapter 6 (Fig. 6.13). Surface flow patterns were obtained at ½ inch
and ½ inch channel depths by coating the walls with a suspension of lamp
black in paraffin and running the tunnel until the paraffin had evaporated.
The remaining carbon gave a crude indication of the surface flow patterns.
The results obtained initially led to the fitting of the trip wires and improvements in the inlet contraction. After these modifications were made no gross flow peculiarities were observed.

The static pressure gradient along the channel reached its fully developed value within 10 duct heights in all the experiments conducted.

4.8. Stanton Tubes

A Stanton tube of the razor-blade type, shown in Fig. (4.12), was used to measure the shearing stress on the top wall (smooth) for the rough wall investigations. The tube was calibrated in the channel (with both walls smooth) at various channel depths. No detectable blockage effect could be discerned as shown by the calibration curve (Fig. (4.13)). The calibration agreed closely with the results given by EAST (44).

4.9. Meter Static Calibration

A technique based upon HEADLEY's (55) method (described in Chapter 3) was used. Basically it is necessary to apply very small, accurately known forces and find the current necessary to balance them. This was achieved inexpensively by the system shown in Fig. (4.14). One end of a cord (human hair) was attached to the element. To the other end two other pieces of hair were tied, each having a loop at the other end. One loop was attached to a retort stand whilst the other was allowed to hang vertically. To this loop small home-made weights were attached. Using a slide projector an image of the cords at the knot was projected on to a piece of drawing paper secured to a screen. By marking the positions of the cord it was possible to remove the paper, draw in the cords and measure the angles at the knot. Knowing the weight attached the force applied to the element could thus be calculated (see Section 3.5.). The weights
used were manufactured from annealed feeler strip and covered the range 10 - 300 mg. By attaching different weights and changing the angles the calibration curve (Fig. (4.15)) was obtained. In order to check that the meter was not moment sensitive the force was applied at varying distances from the meter surface. No distinguishable moment effect was discovered.

4.10. Wall Roughness

For the rough wall studies the lower wall of the channel was roughened by attaching small glass balls (ballotini) in rectangular grid patterns. The two patterns investigated are shown in Fig. (4.16) (\(\frac{1}{4}\)inch square spacing) and Fig. (4.17) (\(\frac{1}{8}\)inch x \(\frac{1}{4}\)inch spacing). The patterns in the region of the meter element are shown in Figs. (4.18, 4.19).

The diameters of a random sample of 100 ballotini were measured using a micrometer. The heights of a further sample of 100 ballotini attached to the wall were also measured using a travelling microscope. The results are given in Fig. (4.20) and are self-explanatory.
Fig. (4.1) PICTORIAL VIEW OF FLOATING ELEMENT SKIN-FRICTION METER
A. Adjustable carrier plate
B. Element
C. Wall disc.
D. Flexures
E. L.V.D.T.
F. Flexure clamps
G. Core position adjusting screws
H. Moveable core carrier
I. Outer core carrier
J. Inner core
K. Silicone oil

Fig. (4.2) SKIN-FRICTION METER
Fig. (4.3) FLEXURES

Fig. (4.4) DISPLACEMENT TRANSDUCER BLOCK DIAGRAM
Fig. (4.7) TOTAL HEAD PROBES
PRESTON TUBE (PATTERN (i))

PRESTON TUBE (PATTERN (ii))

<table>
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<th>a</th>
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<th>a/b</th>
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<tbody>
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<td>0.27</td>
<td>0.43</td>
<td>0.63</td>
</tr>
<tr>
<td>PRESTON.2</td>
<td>0.13</td>
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Fig.(4.8) PRESTON TUBES
Fig. (4.9) Alternative Apparatus for R.M.S. Voltage Measurement
Fig. (412) Stanton Tube (Razor Blade)
\[ y^* = \log_{10}\left(\frac{xh^2}{\rho v^2}\right) \]

Fig. (4.13) **Stanton Tube (Razor Blade) Calibration Curve**
Fig(4.15) SKIN-FRICTION METER CALIBRATION CURVE
Fig. (4.16) \( \frac{1}{4}'' \times \frac{1}{4}'' \) Roughness Pattern

- General View
Fig.(4.17) \( \frac{1}{4}'' \times \frac{1}{8}'' \) Roughness Pattern
- General View
Fig (4.18) $\frac{1}{4}'' \times \frac{1}{4}''$ Roughness Pattern in Element Region
Fig.(4.19) $\frac{1}{4}$" x $\frac{1}{8}$" Roughness Pattern
-in Element Region
Fig. (4.20) ROUGHNESS DETAILS
CHAPTER 5

DATA REDUCTION AND ACCURACY
5.1 Introduction

The aims of this chapter are:

1. To describe how the experimental measurements were processed to give the results presented in Chapter 6.

2. To estimate the accuracy and uncertainty of these processed results.

All analogue measurements are subject to inaccuracies. These inaccuracies may conveniently be separated into -

(a) Fixed Errors - which lead to a consistently erroneous result

(b) Variable Errors or Uncertainties - caused by uncorrelated random variations in the measurement of variables.

For example an experimental result \(y_1\) is related to a set of measured variables \((x_1, x_2, x_3 \ldots x_n)\) by

\[ y_1 = f(x_1, x_2, x_3 \ldots x_n) \]

If each of the measurements were incorrect by fixed amounts \((dx_1, dx_2, \ldots, dx_n)\) (e.g. due to faulty equipment, consistent observer error or poor calibration of measuring equipment) the value of the experiment result \(y_1\) would also be incorrect by a fixed error \((dy_1)\).

By the chain rule of partial differentiation

\[ dy_1 = \frac{\partial y_1}{\partial x_1} dx_1 + \frac{\partial y_1}{\partial x_2} dx_2 + \ldots + \frac{\partial y_1}{\partial x_n} dx_n \]

or numerically

\[ dy_1 = f(x_1 + dx_1, \ldots, x_n + dx_n) - f(x_1, \ldots, x_n) \]

If the experimental measurements were not consistently in error but were subject to random, uncorrelated errors then the error in the deduced result would also be random and could not be estimated by the above method. KLINE and MCCINTOCK (56) describe a method, based upon statistics, which can be used to estimate uncertainty if certain reasonable
assumptions are made. The main assumption is that the measured value deviates from the true value in a statistically normal manner. The error is thus more likely to be small then large and is as likely to be positive as negative. The probable value of the error, the uncertainty ($w$), depends upon the desired confidence limits. In the following applications of the theory confidence limits of 95% are assumed i.e. there is a 95% probability that the measured value of a variable ($x_n$), whose true value is $x$, will be in the range $x - w < x_n < x + w$ where $w$ is the uncertainty. The uncertainty in this case corresponds to two standard deviations.

The uncertainty in the experimental result ($w_y$) due to equally probable uncertainties in the measured variables ($w_1, ..., w_n$) has the same probability and is given by

$$w_y = \left( \left( \frac{\partial y}{\partial x_1} w_1 \right)^2 + \left( \frac{\partial y}{\partial x_2} w_2 \right)^2 + ... + \left( \frac{\partial y}{\partial x_n} w_n \right)^2 \right)^{\frac{1}{2}}$$

or

$$w_y = \left( \frac{\partial \ln y_1}{\partial \ln x_1} \frac{\partial y_1}{x_1} \right)^2 + \left( \frac{\partial \ln y_2}{\partial \ln x_2} \frac{\partial y_2}{x_2} \right)^2 + ... + \left( \frac{\partial \ln y_n}{\partial \ln x_n} \frac{\partial y_n}{x_n} \right)^2$$

The above methods are used to assess fixed errors and uncertainties for the experimental results presented.

5.2. Mean Velocity

The characteristic equation of the total head probes was assumed to be

$$p_m - p_w = \frac{1}{2} \rho U^2$$

where $p_m$ and $p_w$ are the measured values of the total head and wall static pressure. The true equation is slightly different thus small fixed errors do occur. These are discussed in Appendix I.

The uncertainty in velocity ($w_u$), using equations 5.1.5 and 5.2.1 is given by
5.2.2
\[
\frac{w_u}{U} = \frac{1}{2} \left\{ \left( \frac{w_p}{\rho} \right)^2 + \left( \frac{w_p}{\rho} \right)^2 \right\}^{1/2}
\]

where \( w_u, w_p, w_p, \) and \( w_p \) are the uncertainties in \( U, p, p, \) and \( \rho \) respectively.

The density was calculated from the combined gas law:

\[
\frac{P}{\rho} = RT
\]

where \( P \) and \( T \) are the atmospheric pressure and temperature (°k) respectively. The uncertainty in \( \rho (w_p) \) due to uncertainties in \( P(w_p) \) and \( T (w_T) \) is thus:

\[
\frac{w_p}{\rho} = \left( \frac{w_p}{P} \right)^2 + \left( \frac{w_T}{T} \right)^2 \frac{1}{2}
\]

The estimated values for the above uncertainties are

\[
w_T = \pm 5°k \quad T = 300°k \quad \frac{w_T}{T} = \pm 0.01
\]

\[
w_p = \pm 20mb \quad P = 1000mb \quad \frac{w_p}{P} = \pm 0.02
\]

\[
w_{p_m} = \pm 0.2mm H_2O
\]

\[
w_{p_w} = \pm 0.3mm H_2O
\]

Substituting the above values in equations 5.2.4 and 5.2.2

\[
\frac{w_p}{\rho} = 0.01 \sqrt{5} \Omega 0.02
\]

and, approximately

\[
0.005 < \frac{w_u}{U} < 0.02
\]

5.3. Velocity Profile Integral Parameters

The integral parameters \( \delta^* \), the displacement thickness, \( \theta \), the momentum thickness, and \( H_{12} \), the shape factor were obtained from the mean velocity profiles by the method used for the Stanford data (COLES (4)).
The following integral values were assumed for the region 

\[ \frac{y_{uT}}{v} = 0 \text{ to } \frac{y_{uT}}{v} = 50. \]

\[ \int_0^{50} \frac{U}{u_T} \, d \left( \frac{y_{uT}}{v} \right) = 540.6 \]

\[ \int_0^{50} \left( \frac{U}{u_T} \right)^2 \, d \left( \frac{y_{uT}}{v} \right) = 6546 \]

\[ \int_0^{50} \left( \frac{U}{u_T} \right)^3 \, d \left( \frac{y_{uT}}{v} \right) = 82770 \]

From \( \frac{y_{uT}}{v} = 50 \) to the first experimental data point the velocity profile

\[ \frac{U}{u_T} = \frac{1}{0.41} \ln \left( \frac{y_{uT}}{v} \right) + 5.0 \]

was assumed.

For the remainder of the profile a parabolic integration technique was used. For full details see Ref. (4).

The Ludweig-Tillmann formula was used to calculate the local skin-friction coefficient (\( C_f \)).

\[ C_f = 0.246 \left( \frac{u_\infty \theta}{v} \right)^{-0.268} \quad 10^{-0.6781} \quad 5.31 \]

Since the integral parameters are computed using analytic functions, containing \( u_\tau \), for the wall region the value of \( C_f \) will not strictly be independent of the assumed value of \( u_\tau \).

The effects of fixed errors in velocity on the integral parameters are likely to be small for the rectangular total head probes used. Since the displacement effects vary considerably from probe to probe no estimates are given for this possible error.

The effects of uncertainties are likely to be negligible since the integration of the profile must automatically smooth the data.
5.4 Preston Tube Readings

PATEL's (16) calibration equations for Preston tubes were used to deduce the local wall shearing stress \( \tau_w \) from the Preston tube total head reading \( (p_p) \) and the local static pressure \( (p_w) \). The calibration is generally expressed

\[
y^* = F(x^*)
\]

where

\[
y^* = \log_{10} \left( \frac{\tau_w D^2}{4 \rho \nu^2} \right)
\]

and

\[
x^* = \log_{10} \left( \frac{\Delta p_p D^2}{4 \rho \nu^2} \right)
\]

where \( D \) is the external diameter of the Preston tube and

\[
\Delta p_p = p_p - p_w
\]

PATEL (16) gives the function \( F(x^*) \):

when \( 2.9 < x^* < 5.6 \)

\[
y^* = 0.8287 - 0.1381 x^* + 0.1437 x^*^2 - 0.006 x^*^3
\]

and when \( 5.6 < x^* < 7.6 \)

\[
y^* + 2 \log_{10} (1.95 y^* + 4.10) = x^*
\]

By substituting for \( D \), \( \rho \) and \( \nu \) in the above equations a computer-generated table of \( \Delta p_p \) against \( \tau_w \) was obtained. The density was later corrected for the local static pressure but made no appreciable difference in the values of \( \tau_w \) obtained.

The uncertainty in \( \tau_w \) \( (\Delta \tau_w) \) due to uncertainties in \( \Delta p_p (w_\Delta) \), \( \rho (w_\rho) \) and \( \nu (w_\nu) \) is given by

\[
\left( \frac{\Delta \tau_w}{\tau_w} \right)^2 = \left( \frac{w_\Delta}{\Delta p_p} \right)^2 + (1 + f(x^*)^2) \left( \left( \frac{w_\rho}{\rho} \right)^2 + \left( \frac{w_\nu}{\nu} \right)^2 + \left( \frac{2 w_\nu}{\nu} \right)^2 \right)
\]

5.4.6
where \( f(x^*) \) is the ratio of the uncertainties in \( y^*(w_{y^*}) \) and \( x^*(w_{x^*}) \)

i.e.

\[
\frac{w_{y^*}}{w_{x^*}} = f(x^*)
\]

for \( 2.9 < x^* < 5.6 \), from equation 5.4.5a

\[
f(x^*) = -0.1381 + 0.2874 x^* - 0.018 x^{*2}
\]

for \( 5.6 < x^* < 7.6 \), from equation 5.4.5b

\[
f(x^*) = \left(1 + \frac{2}{(1.95 y^* + 4.1) \ln 10}\right)^{-1}
\]

The following estimates for the uncertainties in \( r_w \) were obtained by substitution in equations 5.4.6 et. seq. for the Preston tube used in this investigation.

At the minimum value of \( x^* \) \( f(x^*) = 0.9 \) hence \( \frac{w_r}{r} \Omega 0.05 \)

At the maximum value of \( x^* \) \( f(x^*) = 0.86 \) hence \( \frac{w_r}{r} = 0.02 \)

5.5. Hot-Wire Readings

The response equation for a hot-wire anemometer may be written

\[
\frac{v^2 - v_o^2}{\Omega(\Omega - \Omega_o)} = b'(pU)^c (\sin^2 x_1 + 2k \cos^2 x_1)^{c/2}
\]

KJELLSTROM and HEDBERG (60) give

\[
c = c_{av} + 0.00078 \{pU - (pU)_{av}\}
\]

and

\[
k^2 = 0.0505 - 0.00015 \rho U
\]

There is, however, considerable scatter in the data from which these curves were obtained so that for accurate results each hot wire should be carefully calibrated to deduce the constants in equations 5.5.2 and 5.5.3. For the flow investigated the variation of the exponent \( c \) for
each velocity profile was assumed negligible. Equation 5.5.1 may be re-
written
\[ v^2 - v_0^2 = k w u^2 \left( \sin^2 x_1 + k^2 \cos^2 x_1 \right)^{\circ/2} \]  
5.5.4

Taking natural logarithms, putting \( x_1 = 45^\circ \)
\[ \ln(v^2 - v_0^2) = \ln k_w + c \ln U + \circ/2 \ln \left( 1 + k^2 \right) \]  
5.5.5

Assuming \( k^2, c \) constant
\[ \frac{d \ln(v^2 - v_0^2)}{d \ln U} = c \]  
5.5.6

(for both \( x_1 = 45^\circ \) and \( x_1 = 0^\circ \))

The calibration curves [\( \log_{10}(v^2 - v_0^2) \) vs \( \log_{10}(U) \)] are shown in
Figs. (5.2). The assumption of constant \( c \) and \( k^2 \) is seen to be
reasonable.

The equations for the normalised Reynold's stresses are obtained
from equation 5.5.4 in the usual manner, differentiating and ignoring
second order terms:

For the normal wire, \( x_1 = 0^\circ \)
\[ \frac{\bar{u}^2}{\bar{v}^2} = \frac{1}{c_1^2} \left( \frac{2 \bar{v}_1}{\bar{v}_1 - v_0} \right) \bar{v}_1^2 = \beta_1 \frac{\bar{v}_1^2}{c_1^2} \]  
5.5.7

For an inclined wire in the x-y plane,
for \( x_1 = + 45^\circ \)
\[ 2 \bar{u'}^2 + \frac{1 - k^2}{1 + k^2} \bar{v}^2 = \frac{1 + k^2}{1 - k^2} \left[ \frac{\bar{u}^2}{c_2^2} \beta_2 \frac{\bar{v}^2}{2 - \bar{u}^2} \right] \]  
5.5.8

for \( x_1 = - 45^\circ \)
\[ 2 \bar{u'}^2 - \frac{1 - k^2}{1 + k^2} \bar{v}^2 = \frac{1 + k^2}{1 - k^2} \left[ \frac{\bar{u}^2}{c_3^2} \beta_3 \frac{\bar{v}^2}{3 - \bar{u}^2} \right] \]  
5.5.9

yielding
\[ \frac{\bar{u'}^2}{\bar{u}^2} = \frac{1}{4} \left( \frac{1 + k^2}{1 - k^2} \right) \left[ \frac{\bar{v}_2^2}{c_2^2} \beta_2 \frac{\bar{v}^2}{2 - \bar{u}^2} + \frac{\bar{v}_3^2}{c_3^2} \beta_3 \frac{\bar{v}^2}{3 - \bar{u}^2} - 2 \bar{u}^2 \right] \]  
5.5.10

and
\[ \frac{\bar{v'}^2}{\bar{v}^2} = \frac{1}{2} \left( \frac{1 + k^2}{1 - k^2} \right) \left[ \frac{\bar{v}_2^2}{c_2^2} \beta_2 \frac{\bar{v}^2}{2 - \bar{u}^2} - \frac{\bar{v}_3^2}{c_3^2} \beta_3 \frac{\bar{v}^2}{3 - \bar{u}^2} \right] \]  
5.5.11
The above equations are used to calculate the turbulence quantities using the values of the exponents \( c_1, c_2, c_3 \) found from the calibration curves (Figs. 5.2) using CHAMPAGNE's (62) value for \( k^2 (0.04) \). Some of the measurements taken showed evidence of equipment malfunctions. Those results showing serious errors in wire alignment have been omitted.


Most procedures using the log-law (see Chapter 2) are somewhat subjective in that they involve 'by-eye' matching of the experimental velocity profile to the log-law:

\[
\frac{U}{u_T} = a \ln \left( \frac{y u_T}{v} \right) + b \quad 5.6.1
\]

The method developed by COLES (4) eliminates much of this subjectivity but relies finally upon a human 'best-fit' decision. The method outlined below is based upon COLES (4) method modified to eliminate any 'best-fit' decision.

5.6.1. Method

The region over which the log-law is valid depends upon the pressure gradient. In this investigation the limits recommended by COLES (4) are used:

\[ 100 < \frac{y u_T}{v} < 300 \quad 5.6.2 \]

For each profile point \((U, y)\) a value of \( u \) is found such that equation 5.6.1 is satisfied. The corresponding value of \( \frac{y u_T}{v} \) is then computed. The \( u_T \) values for points satisfying 5.6.2 are then averaged to give the mean \( u_T \) value for the profile.
The individual values of \( u_r \) are obtained by an iterative solution to equation 5.6.1 which may be rewritten:

\[
\frac{U}{u_r} = a \ln \left( \frac{v u_r}{v} \right) + b \equiv C(u_r) \tag{5.6.2}
\]

The above equation may be written

\[
f(u_r) = u_r - \frac{U}{c} = 0 \tag{5.6.3}
\]

This may be solved by the Newton-Raphson scheme:

\[
u_{r_{i+1}} = u_{r_i} - \frac{f(u_{r_i})}{f'(u_{r_i})} \equiv g(u_{r_i}) \tag{5.6.4}
\]

where

\[
f'(u_r) = \frac{\partial f}{\partial u_r}
\]

An initial value \( u_{r_0} \) is substituted in equation 5.6.4 and successive approximations \( u_{r_1}, u_{r_2}, \ldots \) obtained. The function \( f(u_r) \) satisfies the convergency criteria and the difference between successive approximations decreases rapidly \((u_{r_{i+1}} - u_{r_i} \rightarrow 0)\). The necessary convergency conditions are:

1. A solution must exist i.e. \( f(\phi) = 0 \) where \( \phi \) is a solution

2. \( f(u_r), f'(u_r) \) and \( f''(u_r) \) must be continuous and bounded within an interval, \( I \), containing the solution, \( \phi \), for any initial value, \( u_{r_o} \), in which it leads to the solution, \( \phi \).

3. \( f'(\phi) \neq 0 \). Ideally \( f'(u_r) \neq 0 \) within \( I \) although this does not necessarily preclude convergence.

It can be shown that if \( |g'(u_{r_0})| < 1 \) the scheme 5.6.4 is convergent with \( u_{r_0} \) as an initial value, thus defining the interval \( I \) in terms of \( u_{r_0} \). This may be written

\[
|g'(u_{r_0})| = \frac{|f(u_{r_0}) f''(u_{r_0})|}{[f'(u_{r_0})]^2} < 1 \tag{5.6.5}
\]

Substituting for \( f, f', f'' \)

\[
\left[ \frac{(u_{r_0} - \frac{U}{c})(2\frac{a}{c} + 1)}{[1 + \frac{U^2}{c^2 u_{r_0}}]^2} \right] < 1 \tag{5.6.6}
\]
A value of \( u_{\tau_0} = 1 \) satisfies the above conditions.

When the difference between successive approximations is satisfactorily small equation 5.6.2 has been solved. The corresponding value of \( \frac{y_n}{\nu} \) for the data point is calculated. All points satisfying 5.6.2 are then used to find the arithmetic mean value of \( u_{\tau} \). This value of \( u_{\tau} \) is used to compute the velocity profile in \( \left( \frac{U}{u_{\tau}}, \frac{y_n}{\nu} \right) \) co-ordinates.

The above operations are performed by a digital computer which, from input \((U, y)\) data, computes \( u_{\tau}, \frac{U}{u_{\tau}} \) and \( \frac{y_n}{\nu} \) and plots the graph \( \left( \frac{U}{u_{\tau}}, \frac{y_n}{\nu} \right) \) together with the reference equation 5.6.1. The programme is listed at the end of the chapter and is shown in block diagram form in Fig. (5.1).

5.6.2. Influence of the Log-Law Coefficients

The coefficients \( a \) and \( b \) used in this investigation were those recommended by COLES (4). If different values are used the computed value of \( u_{\tau} \) will be different.

The log-law may be written

\[
\frac{U}{u_{\tau}} = \Lambda \log_{10} \left( \frac{y_n}{\nu} \right) + B \quad 5.6.7
\]

If two other coefficients \( (\Lambda', B') \) are used then, for a given data point \((U, y)\), a different value of \( u_{\tau}(u_{\tau}') \) will be found such that:

\[
\frac{U}{u_{\tau}'} = \Lambda' \log_{10} \left( \frac{y_n}{\nu} \right) + B' \quad 5.6.8
\]

These equations may be rewritten:

\[
\frac{U}{u_{\tau}} = \Lambda' \log_{10} \left( \frac{y_n}{\nu} \right) + (\Lambda - \Lambda') \log_{10} \left( \frac{y_n}{\nu} \right) + B \quad 5.6.9
\]

and

\[
\frac{U}{u_{\tau}} \left( \frac{u_{\tau}}{u_{\tau}'} \right) = \Lambda' \log_{10} \left( \frac{y_n}{\nu} \right) + B' \quad 5.6.10
\]
Subtracting equation 5.6.10 from equation 5.6.9, substituting \( \frac{u_r'}{u_r} = r \) and rearranging:

\[
\frac{1}{1 + \left\{ \frac{A \log_{10} \frac{r}{A} + (A' - A) \log_{10} \left( \frac{y_{ul}}{v} \right) + (B' - B)}{A \log_{10} \left( \frac{y_{ul}}{v} \right) + B} \right\}}
\]

5.6.11

If the experimental points \((U,y)\) are uniformly distributed in the \( \frac{U}{u_r} - \log_{10} \left( \frac{y_{ul}}{v} \right) \) domain then the average value of \( r(r) \) in the matching region \( 100 < \frac{y_{ul}}{v} < 300 \) is given by

\[
\bar{r} = \frac{r_{100} + r_{300}}{2}
\]

5.6.12

where \( r_{100} = r \) at \( \frac{y_{ul}}{v} = 100 \); \( r_{300} = r \) at \( \frac{y_{ul}}{v} = 300 \).

If the distribution is non-uniform the average value \( \bar{r} \), will be weighted toward the region in which most data points are available as is the initial evaluation of \( u_r \).

The ratios \( \bar{r}, r_{100}, r_{300} \) were computed for various quoted values of \( A' \) and \( B' \) using COLES values for \( A \) and \( B \) from equations 5.6.11 and 5.6.12. These results are given below.

<table>
<thead>
<tr>
<th>Originators</th>
<th>Source</th>
<th>( A' )</th>
<th>( B' )</th>
<th>( r_{100} )</th>
<th>( r_{300} )</th>
<th>( \bar{r} )</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLES</td>
<td>(4)</td>
<td>5.62</td>
<td>5.0</td>
<td>REFERENCE VALUES</td>
<td>1</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>CLAUSER</td>
<td>(7)</td>
<td>5.6</td>
<td>4.9</td>
<td>1.008</td>
<td>1.007</td>
<td>1.007</td>
<td>+ 1%</td>
</tr>
<tr>
<td>PATEL</td>
<td>(16)</td>
<td>5.5</td>
<td>5.45</td>
<td>0.989</td>
<td>0.993</td>
<td>0.991</td>
<td>- 1%</td>
</tr>
<tr>
<td>SARNECKI</td>
<td>(18)</td>
<td>5.4</td>
<td>5.5</td>
<td>0.997</td>
<td>1.002</td>
<td>1.000</td>
<td>- 0%</td>
</tr>
<tr>
<td>SCHLICTING</td>
<td>(12)</td>
<td>5.75</td>
<td>5.5</td>
<td>0.961</td>
<td>0.963</td>
<td>0.962</td>
<td>- 4%</td>
</tr>
<tr>
<td>SMITH &amp; WALKER</td>
<td>(17)</td>
<td>5.0</td>
<td>7.15</td>
<td>0.953</td>
<td>0.972</td>
<td>0.962</td>
<td>- 4%</td>
</tr>
<tr>
<td>STAFF OF THE N.P.L.</td>
<td>(19)</td>
<td>4.9</td>
<td>5.9</td>
<td>1.030</td>
<td>1.044</td>
<td>1.037</td>
<td>+ 4%</td>
</tr>
</tbody>
</table>

PATEL (16) points out that the values of \( A' \) and \( B' \) obtained by SMITH and WALKER (17) and the STAFF OF THE N.P.L. (19) did not correct for pitot displacement effects. A re-analysis of SMITH and WALKER's data by
LANDWEBER correcting for displacement gave values \((A', B')\) in accordance with those quoted by PATEL. COLES \((A)\) constants are developed from data without corrections since, he argued, these corrections are not satisfactorily conclusive. The small differences above partially explain the multiplicity of quoted values \((A', B')\) since the measurements of shearing stress must be very accurate to satisfactorily evaluate \(A'\) and \(B'\).

5.6.3. Accuracy

The effects of fixed errors in velocity and position on the computed value of \(u\) may be analytically obtained. Differentiating equation 5.6.1:

\[
\frac{u_T}{u_T} \frac{dU - U}{du_T} = \frac{a}{y^+} \left\{ \frac{ydu_T}{\nu} + \frac{u_T du_T}{\nu} \right\}
\]

where \(y^+ = \frac{y_T}{\nu}\). The above equation may be rearranged

\[
\frac{du_T}{u_T} = \frac{(A \log_{10}(y^+) + B) \frac{dU}{U} - \frac{au_T}{y^+ \nu} \cdot dy}{A \log_{10}(y^+) + a + B}
\]

The fixed errors \(dU, dy, du_T\) are defined:

\[
\begin{align*}
(U)_{true} &= U + dU \\
(y)_{true} &= y + dy \\
(u_T)_{true} &= u_T + du_T
\end{align*}
\]

From Appendix I \(\frac{dU}{U} = 0.008\)

and, provided the pitot tube is not closer to the wall than two tube heights i.e. \(\frac{y}{H_p} < 2\) \(dy \Omega 0.25 H_p = 0.9 \times 10^{-5} m\). Both errors are positive since the true values are higher than those measured. Substituting \(A = 5.62 \left( a = \frac{1}{0.41} \right), B = 5.0\) and \(v = 1.6 \times 10^{-5} m^2/s\) in equation 5.6.14

(a) for \(y^+ = 100\)

\[
\frac{du_T}{u_T} = + 0.007 - 0.0073 u_T
\]
(b) for $y^+ = 300$

$$\frac{du_r}{u_r} = 0.0089 - 0.0026u_r$$  \hspace{1cm} 5.5.16

If the experimental points are uniformly distributed then the above equations may be combined:

$$\frac{\left(\frac{du_r}{u_r}\right)}{\text{average}} = 0.008 - 0.005u_r$$  \hspace{1cm} 5.6.17

Thus for $1 < u_r < 2 \ (m/s)$

$$-0.002 < \frac{du_r}{u_r} < 0.003$$  \hspace{1cm} 5.6.18

At greater values of $y^+$ where the influence of the displacement effect may be disregarded the error $\left(\frac{du_r}{u_r}\right)$ is about 1%. This agrees with the previous findings (Section 5.6.2) since if PATEL’s constants were used together with the above corrections the $u_r$ values obtained would be the same. (With the same data PATEL’s constants give $u_r$ values 1% lower than COLES constants. Correcting the data and using COLES’ constants gives $u_r$ values 1% high)

Uncertainties may be ignored.

5.7. Calibration Channel

5.7.1. Smooth Wall Tests

The wall shearing stress ($\tau_w$) is calculated from equation 3.6.1

$$\tau_w = \frac{d\gamma}{2} \frac{dp_w}{dx}$$  \hspace{1cm} (3.6.1)

The pressure gradient being given by the pressure drop ($\Delta h_m$ H$\_2$O) over a fixed length of the channel ($\Delta x$ m) :-

$$\frac{dp_w}{dx} = \frac{\Delta h_m \rho g}{\Delta x}$$  \hspace{1cm} 5.7.1
From equation 3.6.8 the uncertainty due to taper is:

$$\frac{\Delta \tau_w}{\tau_w} = \frac{\Delta \Delta_c}{\Delta c} \frac{d\beta}{\Delta x} \left( \frac{\rho_v}{\tau_w} \right)^2$$

(3.6.8)

From the experimental investigation of taper effects, the results of which are given in Chapter 6, a taper of \( \frac{\Delta \Delta_c}{\Delta c} = 0.0033 \) gives overestimates in \( \tau_w \) of 4 - 6% for the range of \( \tau_w \) values examined. The accuracy of assembly of the channel in practice leads to an uncertainty of the order \( \pm \frac{1}{2}\% \).

Neglecting taper effects and the uncertainty in \( d_d \) the uncertainty in \( \tau_w \) is

$$\frac{\tau_w}{\tau_w} = \frac{w_h}{2\Delta x D}$$

5.7.2

This may be written, with the aid of equations 3.6.1 and 5.7.1:

$$\frac{\tau_w}{\tau_w} = \frac{w_h}{2\Delta x d} \left( \frac{\rho_m \Omega}{g_0} \right)$$

5.7.3

Substituting \( w_h = \pm 10^{-4} \text{ m H}_20 \); \( \Delta x = 0.2 \text{ m} \); \( \rho_m = 10^3 \text{ kg/m}^3 \); \( g_0 = 9.81 \text{ m}^2/\text{s} \)

$$\frac{\tau_w}{\tau_w} \approx 2.5 \frac{d_d}{\Delta h_s}$$

the uncertainty curves, shown in Fig. (5.3), are obtained.

5.7.2. Rough Wall Tests

When the wall roughness is small \( (\frac{k}{d_d} < 1) \) and sparcely distributed equation 3.6.11 may be used to calculate the shearing stress

i.e.

$$\tau_s + \tau_r = \frac{d_d}{2} \frac{dp_w}{dx}$$

(3.6.11)

where \( \tau_s \) and \( \tau_r \) are the smooth and rough wall shear stresses respectively.

The Stanton tube described in Chapter 4 was used to measure \( \tau_s \), the readings being corrected for local density and viscosity variations.

The shear stress, \( \tau_s \), is related to the kinetic head, \( \Delta h_s \), by
Substituting for \(h(1.1 \times 10^{-4} \text{ m})\), \(\rho(1.2 \text{ kg/m}^3)\), \(\rho_m(10^3 \text{ kg/m}^3)\), \(g(9.81 \text{ m/s}^2)\) and \(\nu(1.6 \times 10^{-5} \text{ m}^2/\text{s})\) equation 5.7.4 becomes

\[
\frac{r_s h^2}{\rho \nu^2} = 0.91 \left( \frac{\Delta h s \rho_m g_o h^2}{\rho \nu^2} \right)^{0.66} \tag{5.7.4}
\]

The uncertainty in \(r_s (w_s)\) due to uncertainty in \(\Delta h s (w_H)\) is thus given by

\[
\frac{w_s}{r_s} = \frac{900 w_H}{r_s} \tag{5.7.5}
\]

The uncertainty in \(r_r (w_r)\) due to uncertainties in \(r_s\) and the channel pressure drop \(w_h\) can be expressed

\[
\frac{w_r}{r_r} = \left[ \left( \frac{w_s}{r_s} \right)^2 + \left( \frac{w_h \rho_m g_o d_d}{r_r 2 \Delta x} \right)^2 \right]^{\frac{1}{2}} \tag{5.7.7}
\]

Substituting for \(\rho_m\), \(g_o\), \(d_d\), \(\Delta x\), \(w_s\) and putting \(\frac{r_s}{r_r} = r_s\)

\[
\frac{w_r}{r_r} = \left[ \left( \frac{900 w_H}{r_s 0.52} \right)^2 + \left( 2.5 \frac{w_h}{r_r} \right)^2 \right]^{\frac{1}{2}} \tag{5.7.8}
\]

Substituting for \(w_H (\pm 4 \times 10^{-5} \text{ m H}_2\text{O})\) and \(w_h (\pm 10^{-4} \text{ m H}_2\text{O})\) the uncertainty curves shown in Fig. (5.4) are obtained.

### 5.8. Floating Element Skin Friction Meter

The shear stress indicated by the meter \((\tau_m)\) is related to the meter current by

\[
\tau_m = 15.7 I^2 \Omega 161 I^2 \tag{5.8.1}
\]

The uncertainty in shear stress \((w_m)\) due to uncertainty in the current \((w_I)\) is thus

\[
\frac{w_m}{\tau_m} = \frac{2 w_I}{I} \Omega \frac{8 w_I}{(r_m)^{\frac{1}{2}}} \tag{5.8.2}
\]

The uncertainty in the ratio \(\tau_m/\tau_w (\gamma)\) is thus
Similarly for the rough wall ratio $\tau_m/\tau_r$ (also $\gamma$)

$$\frac{w_y}{\gamma} = \left[ \left( \frac{w_m}{\tau_m} \right)^2 + \left( \frac{w_r}{\tau_r} \right)^2 \right]^{\frac{1}{2}}$$  \hspace{1cm} 5.8.4

Since $w_1 = \pm 0.002A$, for the range $1 < \tau_m < 6$

$$0.016 > \frac{w_m}{\tau_m} > 0.006$$  \hspace{1cm} 5.8.5

The above equations (5.8.3, 4 and 5) may be used to estimate the significance of the scatter in the meter results presented in Chapter 6.
Fig. (5.1) Block Diagram of Log Law Method for Calculating \( C_f \)
For oil profile point write no values in matching region.

\[ \text{Sum} = 0.0, \quad L = 0 \]

\[ \begin{align*}
\text{Sum} &= \text{Sum} + u_z \\
L &= L + 1
\end{align*} \]

\[ L = 0? \]

\[ u_z = \frac{\text{Sum}}{L} \]

\[ c_f = 2 \left( \frac{u_z}{u_\infty} \right)^2 \]

Write \( u_z \), \( y_z \), and residual for each profile point.

Write \( c_f \).

\[ \frac{U_z}{U_t} \text{ vs } \log \left( \frac{y_z}{U_t} \right) \]

END
Figs. (5.2(a-k))

HOT WIRE CALIBRATION CURVES
Hot Wire Calibration at $X=0.476 \text{ m}$

Fig. (5.2 (a))
Hot Wire Calibrations at $x=0.933 \text{ m}$

![Graph showing calibration results for normal wire and 45° wire.](image)

**Normal Wire**

- Slope: $c = 0.48$
- Figure: (5.2(b))

**45° Wire**

- Slopes: $c = 0.54$, $c = 0.49$
- Figures: (5.2(c))

These graphs illustrate the logarithmic relationship between the variance of the voltage ($V^2 - V_o^2$) and the log of the velocity ($\log_{10} U$) for both normal and 45° wires.
Hot Wire Calibrations at $x=1.086\,\text{m}$

Normal Wire

$\log_{10}(V^2 - V_e^2)$ vs $\log_{10}U$

$c=0.515$

Fig. (5.2(d))

45° Wire

$\log_{10}(V^2 - V_e^2)$ vs $\log_{10}U$

$c=0.52$

$c=0.48$

-45° +45°

Fig. (5.2(e))
Hot Wire Calibrations at $x = 1.391 \text{m}$

**Normal Wire**

$c = 0.48$

Fig. (5.2. (f))

**45° Wire**

$c = 0.48$

Fig. (5.2. (g))
Hot Wire Calibrations at x=1.543m

\[ \log (V^2 - V_o^2) \]

\[ \log U \]

---

Normal Wire

\[ c = 0.50 \]

Fig.(5.2.(h))

---

45° Wire

\[ c = 0.53 \]

Fig.(5.2.(i))
Hot Wire Calibrations at $X = 2.000m$

$\log_{10}(V - V_0)$ vs $\log_{10}U$

$c = 0.50$

**Normal Wire**

Fig. (5.2(j))

$\log_{10}(V - V_0)$ vs $\log_{10}U$

$c = 0.50$

**45° Wire**

Fig. (5.2(k))
Fig. (5.3) Uncertainties - Smooth Wall Channel.

Fig. (5.4) Uncertainties - Rough Wall Channel.
&FORTRAN:

&LIST:

1* C LOG-PLOT METHOD FOR ASSESSING CF..............
2* C FILE IS STATION NUMBER
3* C U12 IS FREE STREAM VELOCITY (M/S)
4* C N IS NO. OF DATA POINTS......
5* C WYE IS DISTANCE FROM WALL (MM)
6* C U2 IS VELOCITY NORMALISED BY U12...............
7* C U IS VELOCITY (M/S)
8* C GUV IS KINEMATIC VISCOSITY (M2/S)
9* C UTAU IS WALL FRICTION VELOCITY......
10* C YUTON IS Y*
11* C RESID IS COLFS RESIDUAL ...CONSTANTS ARE 5.62 AND 5.0.....
12* C UOUT IS U*.............
13* C COMMON YSCALE,YScale,YUTON(40),UOUT(40)
14* C INTEGER A
15* C DIMENSION WYE(40),UN(40),RESID(40),U(40),UTAU(40),WUTON(40),
16* IZ(40)

17* READ(7,1) UIF,UN,HNUM

18* 1 FORMAT(F7.3,213)
19* DO 3 I=1,H
20* READ(7,2)U(I),WYE(I)
21* 2 FORMAT(2F7.3)
22* 3 CONTINUE
23* GNU=0.0001568
24* DO 6 I=1,H
25* UTAU(I)=1.0
26* U(I)=UN(I)*U12
27* 4 CONTINUE
28* YUTON(I)=UTAU(I)*WYE(I)/GNU=0.001
29* B=YUTON(I)
30* UOUT(I)=U(I)/UTAU(I)
31* Z(I)=(U(I)-UTAU(I)*(5.62*ALOG10(B)+5.0))/(UOUT(I)+2.44)
32* UTAU(I)=UTAU(I)*Z(I)
33* D=AHS(Z(I))
34* IF(D.0.T.0.00001) GO TO 11
35* GO TO 4
36* 11 CONTINUE
37* YUTON(I)=UTAU(I)*WYE(I)/GNU=0.001
38* UOUT(I)=U(I)/UTAU(I)
39 RESID(I)=OUT(I)-5.62*ALOG10(B)-5.0
40 WRITE(2,5) YUTON(I),UOUT(I),RESID(I),UTAU(I)
41 FORMAT(4F16.9)
42 CONTINUE
43 L=0
44 SUM=0.0
45 DO 7 I=1,N
46 B=YUTON(I)
47 IF (B.LT.100) GO TO 7
48 IF (B.GT.300) GO TO 7
49 L=L+1
50 SUM=SUM+UTAU(I)
51 CONTINUE
52 WRITE(2,35)
53 FORMAT(9H1UTION)
54 WRITE (2,18) NUM
55 FORMAT(I3)
56 WRITE(?36)
57 FORMAT (9x,2HY*,14x,2HU*,15x,5HRESID//)
58 WRITE(2,117) L
59 FORMAT(I3)
60 IF(L.0.0) GO TO 41
61 DO A I=1,L
62 UTAU(I)=SUM/L
63 UOUT(I)=U(I)/UTAU(I)
64 YUTON(I)=UTAU(I)*MYF(I)/GNI*0.001
65 B=YUTON(I)
66 RESID(I)=OUT(I)-5.62*ALOG10(B)-5.0
67 WRITE(2,9) YUTON(I),UOUT(I),RESID(I)
68 FORMAT(3F16.9)
69 CONTINUE
70 CF=(SUM/L*1.0/ULF)**2*2
71 WRITE (2,19) CF
72 FORMAT(F15.9)
73 CONTINUE
74 : DRAWING AND MARKING AXES......
75 XSCALE=400
76 YSCALE=20
77* NPAGE=0
78* NPLOTE=0
79* CALL ORIGIN(NPLOTE,NPAGE)
80* C AXES
81* CALL MOVE(400,200)
82* CALL DRAW(400,1000)
83* CALL MOVE(400,200)
84* CALL DRAW(1600,200)
85* C MARK Y AXIS...........
86* DO 21 KY=200,1000,200
87* CALL MOVE(400,KY)
88* JIM=KY/20
89* CALL DRAW(390,KY)
90* CALL MOVE(330,KY)
91* CALL WAY(0,4)
92* WRITE(9,20) JIM
93* 20 FORMAT(12)
94* 21 CONTINUE
95* C MARK OFF LOG SCALES..........
96* DO 50 KX=20,90,10
97* A=ALOG10(KX)*XSCALE
98* CALL MOVE(A,200)
99* CALL DRAW(A,190)
100* 50 CONTINUE
101* DO 51 KX=200,900,100
102* A=ALOG10(KX)*XSCALE
103* CALL MOVE(A,200)
104* CALL DRAW(A,190)
105* 51 CONTINUE
106* DO 53 KX=2000,9000,1000
107* A=ALOG10(KX)*XSCALE
108* CALL MOVE(A,200)
109* CALL DRAW(A,190)
110* 53 CONTINUE
111* C MARK X AXIS............
112 C             FIRST MAIN INTERCEPTS........
113 D0 25 KK=400,1600,400
114         JACK =KK/400
115 CALL MOVX(KK,200)
116 CALL UPAX(KK,180)
117 CALL MOVE(KK,140)
118 CALL WAY(0,4)
119 WRITE(9,23)
120 23 FORMAT(2H10)
121         K2=KK*40
122 CALL MOVE(K2,164)
123 CALL WAY(0,3)
124 WRITE(9,24) JACK
125 24 FORMAT(I1)
126 25 CONTINUE
127 C             LABEL AXES:......
128 CALL MOVE(300,700)
129 CALL WAY(0,4)
130 WRITE(9,27)
131 27 FORMAT(2H10)
132 CALL MOVE(1200,100)
133 CALL WAY(0,4)
134 WRITE(9,28)
135 28 FORMAT(2HY+)
136 C             HEADING
137 CALL MOVE(1200,900)
138 CALL WAY(0,4)
139 WRITE(9,31)
140 31 FORMAT(7HSTATION)
141 WRITE(9,32) NUM
142 32 FORMAT(I3)
143 CALL MOVE(1200,800)
CALL WAY(0,4)
WRITE(9,34)
FORMAT(3HCF=)
WRITE(0,33)CF
FORMAT(FB.6)
C
DRAW THEORETICAL LINE
CALL MOVE(400,212)
CALL DRAW(1600,550)
DO 29 I=1,II
CALL GRAPH(I)
29 CONTINUE
CALL MOVE(0,2000)
GO TO 42
WRITE(2,43)
FORMAT(28HNO VALUES IN MATCHING REGION)
42 CONTINUE
STOP
END

SUBROUTINE GRAPH(I)
COMMON XSCLAE,YSCLAE,YUTON(40),UOUT(40)
INTEGER E,F
B=YUTON(I)
E=XSCLAE*ALOG10(B)
F=YSCLAE*UOUT(I)
CALL MOVE(E,F)
CALL CFNCH(1)

RETURN
END
CHAPTER 6

EXPERIMENTAL RESULTS
6.1. Introduction

Ideally any experimental investigation into boundary layer structure should examine a flow which is (a) not already well documented and (b) likely to be a critical test of boundary layer prediction methods. The boundary layer flow described in Section 6.2 satisfies these aims. The numerical results are tabulated in Appendix II.

If significant improvements are to be made to existing boundary layer prediction techniques improved methods of measuring the wall shearing stress are necessary. Although the floating element meter is not new in principle the results presented in Sections 6.2 and 6.3 throw a fresh light in that they show that such a meter may be used in a wider range of flows than indirect measurement techniques.

The physical model, developed in Chapter 3, is compared with experiments in Section 6.4 and gives significant new information on the principle of operation notably showing that these meters are weakly dependent upon wall similarity.

6.2. N.P.L. Boundary Layer Investigation

6.2.1. Free Stream Conditions

The free stream conditions are presented in Figs. (6.1, 6.2, 6.3) in the following forms

(i) \( U_\infty - x \)
(ii) \( \frac{dU_\infty}{dx} - x \)
(iii) \( \frac{dp}{dx} - x \)
The origin for \( x \) is arbitrarily taken as the leading edge of the plate and does not represent the boundary layer origin. The flow is initially accelerated upstream of the first measuring station and is subsequently subject to an increasingly adverse pressure gradient (up to \( x = 0.7 \text{ m} \)). The flow is then permitted to relax in a decreasingly adverse pressure gradient finally accelerating once more. In Fig. (6.3) the pressure gradient was evaluated in two ways. \[ \left( \frac{dp}{dx} \right) \] was obtained from the plot of \( p \) vs \( x \) using a mirror technique to obtain the slope of the normal to the curve at each point plotted. \[ \left( \frac{dp}{dx} \right) \text{ was obtained from the local velocity } (U) \text{ and Fig. (6.2)} \left( \frac{dp}{dx} - x \right) \text{ again using the mirror technique.} \] There is close agreement between the methods.

### 6.2.2. Velocity Profiles

In Figs. (6.4(a-r)) the mean velocity normalised with respect to the free stream velocity \( \left( \frac{U}{U_\infty} \right) \) is plotted against the distance from the wall \( (y) \). The full profile at the beginning of the measuring section rapidly gives way to the centrally flattened profile characteristic of adverse pressure gradient flow. This can be seen more clearly in Figs. (6.5(a-r)) where the mean velocity normalised with respect to the wall shear velocity \( \left( \frac{U}{U_w} \right) \) is plotted against the local Reynolds number \( (\frac{yU}{v}) \). Initially there is apparently a negative wake component shown by the deviation of the experimental points from the solid line \( (u^+ = 5.62 \log_{10} y^+ + 5.0) \). As the boundary layer develops the wake component becomes strongly positive and the experimental points deviate from the log law at progressively lower values of \( y^+ \). The earlier velocity profiles are in agreement with the results of PTEL (16) showing a change in the log-law constants for favourable pressure gradient flows by the steeper slope of a line drawn through the experimental points.

There is evidence of some discrepancy between the two pitot probes.
used (see, for example, Fig. (6.5(m))). This is probably due to minor
discrepancies in the probe positioning and to the effects of turbulence
on narrow mouthed pitots (see BRADSHAW's comments in ref.(4) on flow 2600).
The scatter in the discrepancies would point to probe positioning errors
(for wake flow probe (1)) of the order ± .005 inches.

Figs. (6.5) are the computer output of the log-law method for
calculating the wall shearing stress described in Chapter 5.

6.2.3. Integral Parameters

The streamwise development of the momentum and displacement thick­
ness (θ, δ*) and the shape factor (H_{12}) are shown in Fig. (6.6). There
is evidence that the roof and floor boundary layers merge at x = 1.6 - 1.7m
shown by the fall in both δ* and θ. This is also indicated by the tur­
bulence profiles in Fig. (6.9). BRADLEY (63) first noticed this effect
in developing pipe flow, subsequently confirmed by LEE (64) for developing
flow in a rectangular duct. The subsequent acceleration of the flow
caused by convergence of the roof and floor is of course also partially
responsible for these decreases in integral thicknesses.

The experimental points are somewhat scattered about the mean lines
drawn but this is in no way due to the integration method (see Chapter 5).
Different starting points for the integration (y^+ = 100 and 300) gave
identical integral thicknesses. Thus the scatter is due to slight three
dimensionality and experimental inaccuracy. As pointed out in Chapter 2
the integral thicknesses are wall-dominated variables so that any posi­
tional inaccuracy will be shown in an amplified form on integral plots.
The differences in integral parameters across the flow at the same
streamwise position are also due to a mixture of minor inaccuracies in
measurement and slight three dimensionality of the flow.

The shape factor (H_{12}) is almost constant along the flow. It is
thus a poor indicator of shape since the characteristics of the profiles are quite different near the leading edge.

6.2.4. Skin-Friction

Four methods were used to measure the skin friction: -
(i) 0.065 inch Preston tube
(ii) Skin-friction meter
(iii) Log-law velocity profile matching
(iv) Ludweig-Tillmann skin-friction law

In Fig. (6.7) the results of the first three methods are plotted as the streamwise variation of the wall shearing stress. The skin-friction meter readings have been corrected using BROWN and JOUBERT's (1) correction curves for adverse pressure gradients and the author's corrections for favourable pressure gradients (see 6.3). Bearing in mind the uncertainties discussed in Chapter 5 the agreement between the three methods is good, particularly in the region from $x = 0.6m$ onwards. The agreement up to $x = 0.6m$ is not quite so good with the somewhat surprising result that the Preston tube seems to underestimate. PATEL (16) shows that this is possible with relatively large Preston tubes in favourable pressure gradients. The relevant parameters for estimating Preston tube errors are shown in Fig. (6.11) and (6.12). In Fig. (6.11) the parameters \( \left( \frac{a_1}{u_r^3} \right) \); representing the effects of pressure gradient on the velocity profile near the wall, and the Preston tube Reynolds's number \( \left( \frac{du_r}{v} \right) \) are plotted against $x$ for the tube used in this investigation.

The pressure gradient group does not exceed the 3% error limit set by PATEL

\[
\text{i.e. } \quad \frac{a_1}{u_r^3} > -0.005
\]

although the Reynolds's number is greater than 200. The first three readings correspond to Preston tube Reynolds's numbers in excess of 200.
The gradient \( \frac{d}{dx} \left( \frac{a^\nu}{u^3} \right) \) is positive where PATEL recommends that it should be negative so that the flow should be "far from the commencement of laminar reversion". This is surprising since the developing flow investigated is not tending to reversion. This condition should thus be rephrased

\[ \frac{d}{dx} \left( \frac{a^\nu}{u^3} \right) < 0 \quad \text{and} \quad \frac{d}{dx} (u^-) > 0 \]

In Fig. (6.12) BROWN's (1) recommendations for Preston tube error criteria in adverse pressure gradients are shown. The measurements taken satisfy these criteria. One is forced to conclude that either the measurements are in error or the Preston tube can underestimate the shear stress by approximately 2-4\% when the pressure gradient is rapidly changing in a region of high shearing stress.

The log-law method apparently overestimates the wall shearing stress. This may be due to errors in velocity measurement due to the pitot tube errors described in Appendix I or the failure of the log-law to adequately describe the velocity profile in an initially strongly accelerated flow. The corrected skin-friction meter readings agree very well, within the limits of experimental accuracy, with the other two methods except at the first data point where the pressure gradient is rapidly changing from negative to positive and the effects of the boundary layer trip are likely to persist, (Turbulence effects on skin-friction meters are predicted by the physical model developed in Chapter 3 and additional evidence of pressure fluctuation effects was found in the fully developed channel flow investigation.)

In Fig. (6.8) the skin-friction coefficients \((C_f)\) obtained from the log-law and the Ludweig-Tillmann equation are plotted. The agreement is satisfactory.

\[ \text{6.2.5. Turbulence} \]

The hot-wire anemometer results are given in Figs. (6.9(a-f)) in
the forms
\[
\frac{u'^2}{u^2}, \frac{v'^2}{u^2}, \frac{u'v'}{u^2}, \frac{u'^2}{u'^2}, \frac{v'^2}{u'^2}, \text{and} \ \frac{u'v'}{u'^2}
\]
plotted against the distance from the wall \((y)\) for various streamwise positions \((x)\). The results are similar to those presented by BRADSHAW (10) showing the effect of pressure gradient upon the structure of the turbulence field. The normalised streamwise intensity of turbulence \(\frac{u'^2}{u^2}\) decreases with \(y\) in all cases and generally increases with falling free stream velocity. When the flow accelerates once more the general level begins to fall shown by the profile at \(x = 2m\). A better physical picture of the effects of the pressure field is shown in Fig. (6.9(d)) where the intensity \(\frac{u'^2}{u^2}\) is plotted against \(y\). The influence of the pressure gradient is small near the wall, as expected, but the absolute level begins to increase with \(y\) at \(x = 1m\). The bulge in the turbulence profile being caused by the convection of turbulence from the upstream, high shear, wall region into a region whose mean velocity is decelerated by the pressure gradient.

This convection effect is also shown by the normalised shear stress profiles \(\frac{u'v'}{u'^2}\) and \(\frac{u'v'}{u'^2}\) although, once again, a better physical picture is shown by the absolute \(u'v'\) results. The structure which emerges is again one of convection of upstream turbulence away from the wall. Apparently the shear stress \(-\rho u'v'\) does not decay very rapidly implying that the smaller scale eddies from the wall region strongly retain their identity as they move out from the wall. The local correlation \(\langle u'v' \rangle\) remains high and quite unrelated to the local wall shear as shown by Fig. (6.6(e)). The local turbulence structure is thus strongly dependent upon wall conditions well upstream (i.e. the 'history' of the flow is important). The results are not sufficiently accurate to permit analysis in more depth. This inaccuracy is clearly shown by the curves \(\frac{v'^2}{u'^2}\) vs \(y\) where considerable variation exists at different streamwise positions. The sources of error are wire misalignment and extreme
sensitivity to the values of the exponent (c) in the calibration curve. These curves are presented in Chapter 5 and show strong evidence of misalignment on the 45° wire readings at $x = 0.933m$. Other results obtained nearer the leading edge were abandoned for these reasons. When the boundary layer is thick and the velocity profile known accurately the hot-wire indirect calibration method can be used reasonably successfully. There is also some evidence that the Thermosystems r.m.s. meter was in error due to calibration changes in transit from Leicester to N.P.L.

### 6.2.6. Discussion of Boundary Layer Results

The boundary layer described in the preceding sections is strongly non-equilibrium as shown by the streamwise variation of the equilibrium factor $\left( \frac{\theta^*}{r_w} \frac{dp}{dx} \right)$ in Fig. (6.10). It is thus worthy as a trial flow for boundary layer prediction techniques and should present problems particularly with methods based upon integral forms of the Navier-Stokes equations. The author intends to apply integral prediction techniques to this flow based upon the entrainment method of HEAD.

The results have shown that the modified log-law method is a very convenient, accurate technique particularly when the matching region is realistically restricted ($100 < y^+ < 300$). Naturally it cannot be used under conditions where the log law is invalid and is thus likely to give incorrect estimates of the wall shearing stress in moderately favourable and strongly adverse pressure gradients.

### 6.3. Calibration Channel Investigations

#### 6.3.1. Proving Trials

The variation of wall shearing stress across the measuring section
of the calibration channel is shown in Fig. (6.13) in the form of the local Preston tube kinetic head divided by the centreline value \((\Delta P_t / \Delta P_c)\). There is no evidence of any significant variation of shear stress within the uncertainty of measurement (± 1%) for the two flow rates examined at a channel depth of 0.5 inches. Since this is the minimum aspect ratio one is justified in assuming that no variations will exist for smaller channel depths. Surface flow visualisation at both 0.5 inch and 0.125 inch channel depths did not indicate the presence of longitudinal vortices. The flow was therefore judged to be satisfactorily two-dimensional.

In Chapter 3 a simple mathematical model, first derived by FERRIS (57), indicated that small changes in channel cross-sectional area could lead to large inaccuracies in the deduced wall shear stress. In order to test this analysis a simple experiment was conducted. The side-walls of the channel were moved together to give a small streamwise taper in the channel width over the pressure gradient measuring section. The wall shear stress was measured using a razor blade (previously calibrated with the channel accurately assembled) and compared with shear stress estimated from the pressure gradient. The resultant error \(\left(\Delta \tau_w / \tau_w\right)\) was calculated and plotted against the true wall shear stress \(\tau_w\) shown in Fig. (6.14). The taper in channel width gave an area contraction of \(A_c / A = 0.0033\) in a measurement length of 20 cm at a channel height of 0.5 inches (1.27 cm).

The theoretical curve was calculated from equation 3.6.8 using PATEL's (65) formula \(C_f = 0.0376 Re^{-1/6}\) for the variation of skin friction coefficient with Reynolds's number. The experimental results are similar to those predicted by the theory and indicate that the effects of taper are serious. The discrepancy between theory and experiment is probably due to uncertainty in measurement since the 95% confidence limits are close to the theoretical curve. PATEL (65) gives a lower critical Reynolds's number \(d U / \nu\) for fully developed turbulent flow as 1300 (corresponding to a friction Reynolds number \(d U / \nu\) of 200 approximately).
lowest value of the Reynolds number \( \left( \frac{d \overline{U}}{\nu} \right) \) reached in these tests was 5000 thus the skin friction law \( (C_f - Re) \) is valid and the discrepancies are not due to incorrect estimates of the friction factor.

6.3.2. Smooth Wall Study

In Fig. 6.15 the results of the meter trials in the smooth fully developed flow channel are plotted as the ratio, \( \gamma, \left( \frac{F}{\tau w d_m} \right) \) against the element Reynolds number for various channel depths \((d_d)\). The scatter obtained in preliminary trials was greater than expected (see Chapter 5) and was found to be a function of the air temperature. This is shown in Fig. (6.15) on the curve for \( E = 10.6 \) (channel depth nominally \( 3/16 \) inches). Measurements marked \((\frac{1}{\gamma}, \gamma, \frac{1}{\gamma})\) were taken at room temperatures of 21.5°C, 19.5°C and 20°C. The meter was set up at 20°C. The trends shown occur because the differential expansion of the meter carrier assembly and flexures (perspex and steel respectively) causes the element alignment to change. This may be avoided by replacing the carrier by a specially lightened steel carrier but for these tests all measurements were subsequently taken at 20°C.

The secondary forces on the meter depend only upon the Euler number \((E)\) at Reynolds numbers higher than 2000. The limit of Reynolds's number sensitivity is slightly dependent upon the Euler number, \( E \).

The smooth wall secondary force contours have been combined with those of BROWN and JOUBERT (1) in Figs. (6.16 and 6.17). This is permissible because the ratio of gap width to element diameter is similar \((B A J \frac{g/d_m}{0.004}, \text{ present investigation } \frac{g/d_m}{0.0035})\). The two sets of results seem to match up fairly well with no major discontinuities. Since there is no data in the region \(-4 < E < 0\) for either investigation the characteristics in this zone are conjectured. The striking features of the secondary force contours are the changes in character at high values of \( E \) and \( R_f \). In favourable pressure gradients the secondary forces
are independent of Reynold's number and proportional to the Euler number whilst in adverse pressure gradients they appear insensitive to both $R_e$ and $E$ at Reynold's numbers greater than 1500.

**6.3.3. Rough Wall Study**

The secondary force ratio ($\gamma$) is plotted against the Reynold's number ($R_e$) for both roughness spacings in Figs. (6.18 and 6.19). The increased scatter in these readings is expected since the uncertainties are amplified by the indirect way the rough wall shear stress ($\tau_r$) was measured (see Chapter 5 for details). The secondary forces depend only upon the Euler number ($E$) being independent of Reynold's number ($R_e$).

In Fig. 6.20 it is shown that the secondary forces ($\gamma$) are proportional to the Euler number ($E$). The roughness pattern seems to influence the origin of the secondary force characteristics but not the slope.

For the $\frac{1}{4}'' \times \frac{1}{4}''$ roughness
$$\gamma = 1.0 - 0.017E$$

For the $\frac{1}{4}'' \times \frac{1}{8}''$ roughness
$$\gamma = 0.96 - 0.17E$$

For the smooth wall tests
$$\gamma = 1.0 - 0.009E$$

The roughness thus increases the secondary forces experienced by the meter by about 100%. This is a little surprising if the model developed in Chapter 3 is valid since this implies wall turbulence levels of the order $\left(\frac{U'^2}{U'^2} = 1\right)$. If the method of assessing the rough wall shearing stress is valid the ratio of the smooth to rough wall shear stresses $\left(\frac{\tau_s}{\tau_r}\right)$ should be constant and independent of the relative roughness $\left(\frac{k}{d_d}\right)$ for both roughness patterns. No variation of the ratio $\left(\frac{\tau_s}{\tau_r}\right)$ with Reynold's number (other than that attributable to experimental scatter) was found. The flow was thus fully rough. The $\frac{1}{4}''$ inch $\times \frac{1}{4}''$ inch roughness
Spacing gave \( \tau_s/\tau_r \) constant and independent of the relative roughness but the \( \frac{3}{8} \times \frac{1}{2} \) roughness spacing indicated slight variation at high \( k/d_0 \) values shown by Fig. (6.21). Whilst it is not proven that the rough wall shear stresses \( (\tau_w) \) calculated are correct it has been shown that they are at least consistent and likely to be correct. For the purposes of the immediate objectives of this study they are felt to be sufficiently accurate.

6.4. Evaluation of the Model of the Secondary Force Problem

In Chapter 3 a physical model of the mechanisms by which the secondary forces occur was suggested. This was then used to establish a simple mathematical model. From a comparison of the model with experimental results it is possible to draw important conclusions regarding the design and potential of skin-friction meters.

To recap, the physical model developed suggested that the secondary forces could be attributed to

(a) The roughening effect of the gap giving rise to altered pressure and shear stress distributions on the meter element. These are the sole sources of error in zero pressure gradient flows.

(b) The buoyancy force due to the external pressure field.

(c) The modification of shear stress over the element due to flow through the gap.

(d) The destruction of momentum both within the meter case and, in favourable pressure gradients, directly against the element.

In zero pressure gradients this led to the mathematical model

\[ y = 1 + k_0 \left( \frac{g}{d_m} \right) \]  

where the constant \( k_0 \) may be a function of the Reynold's number \( \left( \frac{gu}{v} \right) \).

Experimental evidence on the pressure distribution within a groove (e.g.
Tani (66), Perry et al (25) shows that the resultant drag may be either positive or negative, depending upon the depth/width ratio and implicitly a Reynolds's number. The shear stress modification is likely to be similar. \( k_o \) must therefore be expected to be a function of the Reynolds's number as shown by Brown's results (Fig. 6.16).

In favourable pressure gradients the model

\[
y = 1 - k_c \left( \frac{g}{\alpha_m} \right) E - k_p \left( \frac{\varepsilon}{\alpha_m} \right) E - k_n \left( \frac{g}{\alpha_m} \right)^{3/2} \frac{1}{R_T} E
\]

was suggested where I represents momentum destroyed within the case, II represents the buoyancy force and III represents the momentum destroyed against the leading edge of the element and also, from Dickinson's results, includes the shear modification effect within the constant \( k_n \). Group III was derived assuming that the fluid drawn into the meter came from within the viscous sublayer. This was conditionally expressed by

\[
\left( \frac{g}{\alpha_m} R_T \right)^2 E < 300 \quad \text{approximately}
\]

For the present investigation this gives

\[
R_T^2 E < 2.5 \times 10^7
\]

Plotting the equality (6.4.2) in Fig. (6.17) it is seen that the line appears to separate the contours into regions of dependence upon and independent from Reynolds's number \( R_T \). This can be shown approximately by repeating the analysis given in Chapter 3 but expressing the velocity profile:

(a) for \( 0 < y^+ < 7.8 \) \( u^+ = y^+ \)

(b) for \( 50 > y^+ > 7.8 \) \( u^+ = 2.5 \ln(y^+ - 5.3) + 5.5 \)

(source Duncan et al (67))

The resulting replacement expression for III in equation 3.2.36 is rather complicated but indicates, for the two dimensional case that
the secondary force ratio becomes progressively less dependent upon $R_T$ and is proportional to $\left(\frac{E^2 e}{d_m}\right)$. The general trends shown by the model are thus in qualitative agreement with the experimental results.

The buoyancy group in equation 3.2.36 may be rewritten $-k_p\left(\frac{E d_m}{d_m}\right)$ without violating the physical model since, as pointed out in Chapter 3, the edge width is ill-defined and only marginally relevant. For high values of $R_T$ and $E$ the secondary forces on a circular element may be written

$$\gamma = 1 - k]\left(\frac{E d_m}{d_m}\right)$$

satisfying the boundary condition that $\gamma = 1$ when there is no gap.

Experimentally, for $\frac{E}{d_m} = 0.0035$, the present investigation gives the constant $k$ as 3 approximately.

The practical implications of the above deductions are

(i) It is not necessarily advantageous to build meters with very small gaps ($\frac{E}{d_m} < .003$) since the Reynolds number sensitivity will persist at higher Reynolds numbers. Whilst the secondary forces are larger they are more easily evaluated.

(ii) Direct skin-friction meters do rely upon wall similarity although to a lesser extent the smaller the gap (smaller $\frac{E}{d_m}$). Only in very strong pressure gradients and at high values of $R_T$ does the reliance upon similarity extend to zones appreciably outside the viscous sublayer.

(iii) The turbulence structure of the wall zone will affect the meter secondary forces. Calibrations obtained in laminar flows are therefore not applicable to turbulent flow.

In rough wall, favourable pressure gradient flows the secondary forces are expected to be somewhat higher since the turbulent intensity in the wall region $\left(\frac{u'}{\tau_x}\right)$ and the velocities $\frac{U}{u_T}$ are considerably higher. Since both these variables depend upon the roughness characteristics the secondary forces will depend upon the roughness. The experi-
mental results suggest that, for both the roughnesses investigated, at
Reynold's numbers \(R_T\) in excess of 2000 the secondary forces may be written

\[ y = 1 - k_1(r) \left( \frac{E}{d_m} \right) + \Delta \]  

where \(\Delta\) represents a shift due to inadequately representative roughness
over the element and \(k_1(r)\) is a factor dependent upon the roughness. For

fully rough flows this is independent of the roughness Reynold's number

\( \left( \frac{k u}{S_T} \right) \). \(k_1(r)\) will be thus a function of the roughness packing density
and the roughness height. For the two roughnesses investigated \(k_1\) is
approximately 2.

The practical implications are

(a) To avoid the shift effect \(\Delta\) rectangular meter elements should be
used in rough wall flow investigations avoiding the difficulty of
placing elements near (or even, theoretically, 'on') the gap.

(b) With prior calibration, the wall shear stress in favourable pressure
gradient boundary layers over rough walls may be accurately measured

\(\pm 2\%\) using floating element skin-friction meters.

In adverse pressure gradients the model fails to indicate why the
secondary force contours become independent of \(R_T\) and \(E\) and the conclusion
drawn (in Chapter 3) is that the secondary forces are dominated by the
shear modification effect. This, unfortunately, means that there is still
much work to be done to establish how roughness might affect the secondary
forces in adverse pressure gradients. Intuitively the secondary forces
are likely to be similar to those in smooth wall flows, perhaps smaller.
Fig. (6.1) FREE STREAM VELOCITY VARIATION

Fig. (6.2) FREE STREAM VELOCITY GRADIENT VARIATION
Fig (6.3) STREAMWISE VARIATION OF STATIC PRESSURE GRADIENT
Figs. (6.4(a-r))

VELOCITY PROFILES

$\frac{U}{U_\infty}$ vs. $y$
Fig. (6.4.(a))

$X = 0.171 \text{m} \ Z = -0.152 \text{m}$

STATION 21

Fig. (6.4.(b))

$X = 0.171 \text{m} \ Z = 0$

STATION 22

Fig. (6.4.(c))

$X = 0.171 \text{m} \ Z = +0.152$

STATION 23
Fig. (6.4. d)

$X = 0.476 \text{m} \ Z = 0$

Station 4

Fig. (6.4. e)

$X = 0.933 \text{m} \ Z = 0$

Station 7

Fig. (6.4. f)

$X = 1.848 \text{m} \ Z = 0$

Station 13
Fig. (6.4.(g))

X = 0.629 m \ Z = -0.152 m
STATION 51

Fig. (6.4.(h))

X = 0.629 m \ Z = 0
STATION 52

Fig. (6.4.(i))

X = 1.391 m \ Z = 0
STATION 10
Fig. (6.4. j))

$X = 1.086 \text{m} \quad Z = -0.152 \text{m}$

STATION 81

Fig. (6.4. (k))

$X = 1.086 \text{m} \quad Z = 0$

STATION 82

Fig. (6.4. (l))

$X = 1.086 \text{m} \quad Z = +0.152 \text{m}$

STATION 83
Fig. (6.4. (p))

\[ X = 2.000m \quad Z = -0.152m \]

STATION 141

---

Fig. (6.4. (q))

\[ X = 2.000m \quad Z = 0 \]

STATION 142

---

Fig. (6.4. (r))

\[ X = 2.000m \quad Z = +0.152m \]

STATION 143
Figs. (6.5(a-r))

VELOCITY PROFILES

$U^+ \text{ vs. } y^+$
Fig. (6.5. (a))

Station 21

\[ X = 0.171 \, \text{m} \quad Z = -0.152 \, \text{m} \]

Fig. (6.5. (b))

Station 22

\[ X = 0.171 \, \text{m} \quad Z = 0 \]

Fig. (6.5. (c))

Station 23

\[ X = 0.171 \, \text{m} \quad Z = +0.152 \]
Fig. (6.5.(d))

STATION 7

\[ X = 0.933m \quad Z = 0 \]

Fig. (6.5.(e))

STATION 10

\[ X = 1.391m \quad Z = 0 \]

Fig. (6.5.(f))

STATION 13

\[ X = 1.848m \quad Z = 0 \]
Fig. (6.5.(g))

**STATION 51**

\[ X = 0.629 \text{m} \quad Z = -0.152 \text{m} \]

---

Fig. (6.5.(h))

**STATION 52**

\[ X = 0.629 \text{m} \quad Z = 0 \]

---

Fig. (6.5.(i))

**STATION 53**

\[ X = 0.629 \text{m} \quad Z = +0.152 \text{m} \]
Fig. (6.5. (j))

Station 81

\[ X = 1.086 \text{m} \quad Z = -0.152 \text{m} \]

Fig. (6.5. (k))

Station 82

\[ X = 1.086 \text{m} \quad Z = 0 \]

Fig. (6.5. (l))

Station 83

\[ X = 1.086 \text{m} \quad Z = +0.152 \text{m} \]
Fig. (6.5.m) 

Station 11

$X = 1.543\text{m} \quad Z = -0.152\text{m}$

Fig. (6.5.n) 

Station 12

$X = 1.543\text{m} \quad Z = 0$

Fig. (6.5.o) 

Station 13

$X = 1.543\text{m} \quad Z = +0.152\text{m}$
Fig. (6.5, (p))

**STATION 41**

\[ X = 2.000 \text{m} \quad Z = -0.152 \text{m} \]

Fig. (6.5, (q))

**STATION 42**

\[ X = 2.000 \text{m} \quad Z = 0 \]

Fig. (6.5, (r))

**STATION 43**

\[ X = 2.000 \text{m} \quad Z = +0.152 \text{m} \]
Fig (6.6) Streamwise Variation of Integral Parameters
Fig. (6.7) Streamwise Variation of Wall Shearing Stress
Fig. (6.8) Streamwise Variation of Skin Friction Coefficient
Figs. (6.9.a-f)

TURBULENCE PROFILES

Legend

+ x = 0.476 m.

0 x = 0.933 m.

Δ x = 1.086 m.

x x = 1.391 m.

▽ x = 1.543 m.

□ x = 2.000 m.
Fig (6.10) Streamwise Variation of Equilibrium Factor

Fig (6.11) Preston Tube Error Parameters
Fig.(6.12) PRESTON TUBE ERROR CRITERIA
(taken from ref.(1))
Fig(6.13) Variation in Wall Shearing Stress across Calibration Channel

Fig(6.14) Taper Effects
Fig. (6.15)  Meter Secondary Force Contours (Smooth Wall)
Fig. (6.16) Secondary Force Contours (Smooth Wall)
Fig. (6.18) Rough Wall Secondary Forces - 1/4" x 1/4" Roughness Spacing

Fig. (6.19) Rough Wall Secondary Forces - 1/4" x 1/8" Roughness Spacing
Fig. (6.20) Secondary Force Contours

Fig. (6.21) Shear Stress Ratio for Rough Wall Channel Flow
CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

FOR FUTURE WORK
7.1. Conclusions - Boundary Layer Investigation

7.1.1. Floating element skin-friction meters give accurate repeatable results in turbulent boundary layer flows provided either

(a) They are previously calibrated for secondary forces

or (b) They are geometrically similar to the present meter and/or that described by BROWN (1) when the readings may confidently be corrected for secondary forces.

7.1.2. The modified form of COLE's method for assessing $u_T$ from the log-law is a rapid, accurate and objective technique when used in flows having the logarithmic velocity profile in the region $100 < y^+ < 300$. For the best results circular pitot tubes, corrected for displacement and turbulence effects, or accurately calibrated hot-wires should be used to measure the velocity profile and PATEL's (16) constants in the log-law should be used ($A = 5.5$ $B = 5.45$).

7.2. Skin-Friction Meter Investigation

7.2.1. (i) The experimental results for favourable pressure gradients indicate that the physical model developed in Chapter 3 qualitatively describes the observed behaviour of the secondary forces.

(ii) For smooth wall flow in favourable pressure gradients the characteristics of a circular floating element meters having a sharp edged element may be described by:

$$\gamma = 1 - 3\left(\frac{g}{d_m}\right)E \ (\pm \ 1\%)$$

provided $\left(\frac{g}{d_m} R_T\right)^2 E > 400$ approximately.
7.2.2. Floating element meters may be used in rough wall favourable pressure gradient boundary layers provided that:-

(a) It is practically possible to duplicate a representative roughness pattern over the meter element.

(b) The meter is calibrated for each roughness pattern.

7.3. Recommendations for Future Work

The present investigation has, in many ways, been of an exploratory nature. Inevitably it has raised a number of interesting problems and also indicated areas in which its results might be relevant. The problems relate to the methods currently adopted for specifying rough-wall structure and assessing its effects on boundary layer development.

The 1968 Boundary Layer Symposium at Stanford University did much to clear up arguments on the merits and faults of various prediction techniques but it dealt only with smooth wall boundary layers. The prediction of boundary layer growth over arbitrarily roughened walls presents difficulties because of the lack of information on precise relationships between the fluid dynamic and geometric characteristics of roughness. Conventionally experiments have been conducted upon various roughness patterns to establish the equivalent sand roughness which is then used to characterise the fluid dynamics of the wall region. DVORAK (36) has collated experimental data for various streamwise spacings of two-dimensional rectangular roughness elements (experimentally studied in depth by FERRY (25)) and gives roughness functions dependent upon roughness height and spacing. In predicting boundary layer growth he uses a two-layer model (wall/wake) of the velocity profile and applies HE/D's entrainment technique (see Ref. (4) for details). However, is the two-layer model valid for rough wall flows and, if so, is there a method by which the roughness functions could be established rapidly and easily to provide
a reliable source of input data for prediction techniques?

The direct method of measuring skin friction may supply the answer to the first question posed although there are formidable experimental difficulties in making measurements in adverse pressure gradients. The method used in this present investigation to calibrate the meter in rough wall flows might well be used to establish the skin friction characteristics of various roughness patterns in fully developed, asymmetric two-dimensional channel flows. At the same time a skin friction meter could be calibrated for roughness effects. The wall shear stress structure of non-equilibrium, favourable pressure gradient boundary layers over that roughness pattern could then be determined with the following potential benefits:

(1) The validity of two-layer models in favourable pressure gradients could be critically examined;

(2) The interesting problem of turbulent boundary layer relaminarisation might be studied under rough wall conditions.

Summarising, there would appear to be four basic areas in which further work could be conducted:

(1) To assess the effects of wall roughness on skin friction meter performance in adverse pressure gradients.

(2) To establish the extent of validity of results from asymmetrically roughened two-dimensional channels with a view to establishing the effects of roughness height and pattern both on the skin friction magnitude and on the calibration of floating element meters.

(3) To use the results of (1) and (2) to examine the validity of two-layer velocity profile models in non-equilibrium rough wall flows.

(4) To use the floating element meter in studies of relaminarisation phenomena occurring over rough walls.
APPENDIX (I)

IMPACT PROBE TECHNIQUE
APPENDIX (I)

IMPACT PROBE TECHNIQUE

The characteristic equation relating the indicated total pressure \(p_m\), the wall static pressure \(p_s\) and the mean velocity vector \(U\) in non-turbulent, inviscid, uniform fluid flow may be written:

\[ p_m - p_s = \frac{1}{2} \rho U^2 \quad I.1.1 \]

where \(p_m\) is the total pressure at the geometric centre of the tube mouth.

In practice, where the flow is viscous, turbulent and sheared, the indicated pressure is not the total pressure nor does it correspond to the geometric centre. The magnitude of the errors involved basically depends upon the design of the pitot-tube mouth provided the stem does not interfere with the flow in the mouth region. If measurements are made near a solid boundary additional errors are incurred.

Thus the selection of a suitable pitot design must be guided by:

(i) The flow in which the measurements are to be made

(ii) The accuracy required.

The most common types of tubes have either circular or rectangular mouths thus the discussion is limited to a comparison of the errors incurred by each type. Refs. (26), (27) survey the previous work in this field.

The sources of error are as follows:

(i) Displacement Effects

A total head tube used in a region of total pressure gradient registers the total pressure at a point displayed from the geometric centre of the tube. The displacement \(\delta\) is in the direction of increasing total pressure. It is found that the magnitude of the total pressure gradient does not affect the displacement. (Hence the reason for ex-
pressing the effect as a displacement error rather than a total pressure error).

When the tube is closer than twice the tube height to a solid boundary the restricting effect of the wall displaces the effective centre towards the wall i.e. in the opposite direction to the shear displacement effect.

For circular, square ended tubes YOUNG and MAAS (28) found that

$$\frac{\delta}{D} = 0.013 + 0.08 \frac{d}{D} = \epsilon$$

Other investigations have reported a weaker dependency upon \( \frac{d}{D} \). These are summarised in the table below. Some have also specified the displacement effect when the tube is touching the wall (e.g. Preston tube). This is denoted \( \epsilon_p \).

<table>
<thead>
<tr>
<th>Investigators</th>
<th>Ref.</th>
<th>Circular Pitot Findings</th>
<th>Rectangular Pitot Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>YOUNG and MAAS</td>
<td>(28)</td>
<td>( \epsilon = 0.131 + 0.082 \frac{d}{D} ) for ( \frac{d}{D} = 0.6; \epsilon = 0.18 )</td>
<td>Suggest ( \epsilon \Omega 0.24 )</td>
</tr>
<tr>
<td>MACMILLAN</td>
<td>(29)</td>
<td>( \epsilon = 0.15 ) for ( \frac{d}{D} = 0.6 ) when ( \frac{y}{D} &lt; 2 \frac{\Delta U}{U} = f(\frac{y}{D}) )</td>
<td>______</td>
</tr>
<tr>
<td>PATEL</td>
<td>(16)</td>
<td>( \epsilon_p = 0.15 )</td>
<td>______</td>
</tr>
<tr>
<td>OZAR and DICKINSON</td>
<td>(32)</td>
<td>( \epsilon \Omega 0.1 ) ( \epsilon_p \Omega 0.6 )</td>
<td>( \epsilon \Omega 0.28 ) for ( \frac{y}{H} &lt; 1.5; \frac{W}{H} = 7.5 ) ( \epsilon \Omega 0.16 ) for ( \frac{y}{H} &gt; 1.5; \frac{W}{H} = 4 ) for ( \frac{y}{H} &lt; 1[-0.1 &lt; \epsilon &lt; 0.3] )</td>
</tr>
<tr>
<td>QUARMBY and DAS</td>
<td>(33)</td>
<td>______</td>
<td>for ( \frac{h}{H} = 0.6 ) ( \epsilon \Omega 0.2 ) wall effect ( \frac{y}{h} &lt; 2 ); at ( \frac{y}{h} = 2; \frac{\Delta U}{U} = 0 ) [± 0.02]</td>
</tr>
</tbody>
</table>

Displacement Errors for Pitot Tubes
(ii) Viscous Effects

MACMILLAN's papers (30,31) cover the effects of viscosity on pitot tubes. Fig. (I.1) is taken from Ref. (30) and shows the variation in the pressure coefficient with Reynolds No. for values below 1000. The curve implies that all rectangular pitot tubes with values of \( \frac{W}{H} \) between 7 and 11 will lie on the lower curve shown.

(iii) Turbulence Effects

The turbulence effects on purely total head tubes are such that the indicated total pressure is too high. The amount by which the indicated reading exceeds the true value is a matter of some uncertainty.

GOLDSTEIN's (34) theoretical analysis suggests that a total head tube reads \( p + \frac{1}{2} \rho \overline{u'^2} + \frac{1}{2} \rho \overline{u'^2} \), where \( p \) is the local static pressure. This can be related to the wall static pressure by considering the \( y \)-directed momentum for incompressible viscous fluid flow including the turbulence terms

\[
p + \rho \overline{v'^2} = p_w
\]

Also the effect of turbulence may cause readings to be low if the yaw sensitivity of the particular pitot tube is high. The velocity vector \( \vec{v} \) comprises the following components (in 2D flow)

\[
\vec{v} = ((U + u')^2 + v'^2)^{\frac{1}{2}} \hat{n}
\]

where \( \hat{n} \) is a unit vector inclined at

\[
\tan^{-1} \left( \frac{v'}{U + u'} \right)
\]
to the \( x \)-wise velocity \( U \).

Bearing in mind that the vector \( \hat{n} \) is neither directionally constant nor is its time average value necessarily equal to \( \hat{i} \) there may exist a steady state yaw angle which can cause a rectangular-mouthed pitot tube to read low in regions where \( \frac{u'v'}{u'^2} \) is high (i.e. wall region). For-
fortunately the turbulence effects oppose each other.

OVER and PANKHURST (26) point out that errors due to pitot tube vibration can be incurred.

The indicated wall static pressure is dependent upon the size of the tapping hole (in fact, instruments utilising this dependence have been used to measure \( \tau_w \)). The pressure tapping holes used had a length to depth ratio greater than 1.5. The error in \( p_w \) can be deduced from Fig. (6.7) in Ref. (26) and is included in the following analysis.

The nett effect of the various corrections may be combined in the form

\[
p_m - p = C_{p_1} C_{p_2} \left( \frac{1}{2} \rho U^2 \right)
\]

where \( C_{p_1} \) is a coefficient accounting for viscous effects and \( C_{p_2} \) accounts for turbulence effects.

In the present investigation the velocity range was \( 10 < u < 50 \text{ m/s} \) and the pitot tube Reynolds Nos. were

\[
50 < R' < 250
\]

From (30) this gives \( C_{p_1} \Omega 0.99 \) over this range for \( \frac{W}{H} = 10 \).

\[
\text{and } C_{p_2} \Omega \left( 1 + \frac{u^2}{v^2} \right)
\]

The errors in \( p \) are due to hole size and pressure gradients within the boundary layer.

\[
i.e. \quad p = p_w + \Delta p_w
\]

where \( \Delta p_w = \tau_w x f(R_s) \)

\[
\text{where } R_s = \frac{d'u_r}{v}
\]

in the present investigation \( 2 > u_r > 1 \text{ m/s} \) and \( d' = 1 \text{mm} \)

thus \( R_s = \frac{10^{-3}}{1.6 \times 10^{-5}} \cdot u_r \Omega 60 u_r \)
hence \( 120 > R_s > 60 \)

over the range \( 150 > R_s > 0 \) 3.1.9 can be approximated \( \frac{\Delta p_w}{r_w} = \frac{1}{3} R_s \cdot 10^{-2} \)

thus \( \Delta p_w = 0.2 u_r \cdot r_w \) \( I.1.11 \)

The error is assuming \( p_w = p \) is given by

\[
p_w = p + \frac{1}{2} \rho \overline{v^2}
\]

hence \( p = p_w - \frac{1}{2} \rho \overline{v^2} \)

thus \( p = p_s - \Delta p_w - \frac{1}{2} \rho \overline{v^2} \) \( I.1.12 \)

thus a measurement \( p_m - p_s \) is effectively

\[
p_m - p - (p_s - p) = C_{p1} C_{p2} \left( \frac{1}{2} \rho \overline{U^2} \right) - \Delta p_w - \frac{1}{2} \rho \overline{v^2}
\]

\[
= \frac{1}{2} \rho \overline{U^2} \left( C_{p1} C_{p2} - \frac{0.2}{\frac{1}{2} \overline{U^2} - \frac{\overline{v^2}}{\overline{U^2}} - \frac{0.2}{\frac{1}{2} \overline{U^2}} \right)
\]

in the free stream \( \overline{v^2} = \overline{v^2} \) and \( U = U_\infty \n\)

hence \( p_m - p_\infty = C_{p1} \frac{1}{2} \rho \overline{U^2} (1 - 0.2 \overline{Cf \cdot u_r}) \)

and \( 0.002 < \overline{Cf \cdot u_r} < 0.008 \)

hence max. error condition \( p_m - p_\infty = C_{p1} \frac{1}{2} \rho \overline{U^2} (1 - 0.00016) \)

\[
= 0.99 \frac{1}{2} \rho \overline{U^2}
\]

near the wall \( U \approx \frac{1}{2} U_\infty \) hence

\[
p_m - p = C_{p1} \frac{1}{2} \rho \overline{U^2} \left[ 1 - 4 \times 0.2 \overline{Cf \cdot u_r} \right]
\]

\[
= 0.99 \frac{1}{2} \rho \overline{U^2} [1 - 0.006]
\]

\[
= 0.984 \frac{1}{2} \rho \overline{U^2}
\]

Thus if the pitot coefficient is assumed to be unity the velocity

is underestimated by \( \frac{1}{2} - \frac{3}{5} \% \).
Fig(I1) Pressure coefficient plotted against Reynolds number based on internal height of the Pitot tube (From MacMillan (30))
APPENDIX II

TABULATED RESULTS OF BOUNDARY LAYER FLOW
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<th>X (m)</th>
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<th>$U_{∞}$ (m s$^{-1}$)</th>
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<th>$\frac{dp}{dX}$ (N m$^{-3}$)</th>
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<th>$\delta^*$ (mm)</th>
<th>$\theta$ (mm)</th>
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TABLE (II.4)

PRESTON TUBE RESULTS

TUBE DIAMETER = 0.065"

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<td>HIDGE, J.M.</td>
<td>&quot;Application of the Von Karman momentum theorem to turbulent boundary layers&quot;. NACA TN 2571 (1951)</td>
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<td>TOWNSEND, A.A.</td>
<td>&quot;The Structure of Turbulent Shear Flow&quot; Cambridge University Press (1956)</td>
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(8) Rotta, J.C.
"Turbulent Boundary Layers in Incompressible Flow"

(9) Antonia, R.A. and Linton, R.E.
"The response of a turbulent boundary layer to a step change in surface roughness".

(10) Bradshaw, P.
"The response of a turbulent boundary layer to the sudden application of an adverse pressure gradient".
A.R.C. R & M 3575 (1969)

(11) Coles, D.
"The law of the wake in the turbulent boundary layer"

(12) Schlichting, H.
"Boundary Layer Theory"

(13) Perry, A.E.
"Turbulent boundary layers in decreasing adverse pressure gradients"

(14) Perry, A.E., Bell, J.B. and Joubert, P.N.
"Velocity and temperature profiles in adverse pressure gradient turbulent boundary layers"

(15) Wills, J.A.B.
"Note on a Method of Measuring Skin Friction"
(16)  PATEL, V.C.
"Calibration of the Preston tube and limitations on its use in pressure gradients"

(17)  SMITH, D.W. and WALKER, J.H.
"Skin Friction Measurements in Incompressible Flow"
N.A.C.A. TN 4231 (1958)

(18)  THOMPSON, B.G.J.
"A New Two-Parameter Family of Mean Velocity Profile for Incompressible Turbulent Boundary Layers on Smooth Walls"

(19)  STAFF OF THE N.P.L.
"On the Measurement of Local Surface Friction on a Flat Plate by Means of Preston Tubes"
A.R.C. R & M 3185.

(20)  LUDWIEG, H. and TILLMANN, W.
"Investigations of the Wall-Shearing Stress in Turbulent Boundary Layers"
N.A.C.A. TM 1285

(21)  R/ARATNAM, N. and PROELICH, C.R.
"Boundary Shear Stress in Turbulent Boundary Layers on Smooth Boundaries"

(22)  BRADSHAW, P.
"Simple Method for Determining Turbulent Skin Friction from Velocity Profiles"
(23) MILLER, B.L.
"Shear Stress Measurements in an Adverse Pressure Gradient Boundary Layer over a Smooth Flat Plate"
Leicester University Engineering Dept. Report 71-16 (May 1971)

(24) CONTE, S.D.
"Elementary Numerical Analysis"
McGraw-Hill Book Company (1965)

(25) PERRY, A.E., SCHOFIELD, W.H. and JOUBERT, P.N.
"Rough wall turbulent boundary layers"

(26) OWER, E. and PANKHURST, R.C.
"The Measurement of Airflow"
Pergamon Press (1966)

(27) BRYER, D.W. and PANKHURST, R.C.
"The Determination of Wind Speed and Flow Direction by Pressure-Sensing Devices"
A.R.C. 30 167 (1968)

(28) YOUNG, A.D. and MAAS, J.N.
"The behaviour of a pitot tube in a transverse total pressure gradient"
A.R.C. R & M 1770 (1936)

(29) MACMILLAN, F.A.
"Experiments on pitot tubes in shear flow"
A.R.C. R & M 3028 (1956)

(30) MACMILLAN, F.A.
"Viscous Effects on Pitot Tubes at Low Speeds"
(31) MACMILLAN, F.A.
"Viscous Effects on Flattened Pitot Tubes at Low Speeds"

(32) ÖZARAPOLA, V. and DICKINSON, J.
"Displacement Errors arising in Boundary Layer Measurements with Pressure Probes"
University of Waterloo, Canada, May 1969.

(33) QUARMBY, A. and DAS, H.K.
"Displacement Effect on Pitot Tubes"
Aeronautical Quarterly May 1969 pp.129-139.

(34) GOLDSTEIN, S.
"A Note on the Measurement of Total Head and Static Pressure in a Turbulent Stream"

(35) NASH, J.F.
"A Note on Skin-Friction Laws for the Incompressible Turbulent Boundary Layer"
A.R.C. CP 862 (1964)

(36) DVORAK, F.A.
"Calculation of Turbulent Boundary Layers on Rough Surfaces in Pressure Gradient"
A.I.A.A. Journal Vol. 7, No.9, p.1752

(37) LUDWIG, H.
"Instrument for measuring the wall shearing stress of turbulent boundary layers"
N.A.C.A. TM 1284.
(38) BRADBURY, L.J.S.
Private Communication

(39) MITCHELL, J.E. and HANRATTY, T.J.
"A study of turbulence at a wall using an electrochemical
wall shear stress meter"

(40) PRESTON, J.H.
"The Determination of Turbulent Skin Friction by Means
of Pitot Tubes"

(41) HEAD, M.R. and RECHENBERG, I.
"The Preston tube as a means of measuring skin friction"
J. Fluid Mech. 14, pp.1-17

(42) WYATT, L.A. and EAST, L.F.
"Low-Speed Measurements of Skin-Friction on a Slender Wing"
A.R.C. R & M 3499

(43) BRADSHAW, P. and GREGORY, N.
"The Determination of Local Turbulent Skin Friction from
Observations in the Viscous Sub-Layer"
A.R.C. R & M 3202 (1961)

(44) EAST, L.F.
"Measurement of Skin Friction at Low Subsonic Speeds by
the Razor-Blade Technique"
A.R.C. R & M 3525

(45) WHITE, J.K. and FRANKLIN, R.E.
"Measurement of Skin-Friction in an Annulus by the
Floating Element Technique"
A.R.C.25 661 (1964)
(46) SMITH, S.L., LAW, C.J. and HAMILIN, M.J.
"The Direct Measurement of Wall Shear Stress in an Annulus"
C.E.G.B. RD/B/N 1232 (1968)

(47) KEMPF, G.
"Neue Ergebnisse der Widerstandsforshung"
Werft Reederei Hafen 10 pp.234-239 (June 1929)

(48) SCHULZ-GRUNOW, VON F.
"Neues Reibungswiderstandsgesetz fur glatte Platten"
Luftfahrtforschung, 17, 239 (1940) - translation NACA TM

(49) DHAWAN, S.
"Direct measurements of skin friction"
NACA TR 1121 (1952)

(50) NALIEID, J.F. and THOMPSON, M.J.
"Pressure-Gradient Effects on the Preston Tube in Supersonic Flow"
J. Aero. Space Sci. 28, 940

(51) HAKKINEN, R.J.
"Measurements of Turbulent Skin Friction on a Flat Plate at Transonic Speeds"
NACA T.N 3486

(52) DICKINSON, J. and OZARAPOLU, V.
"The determination of turbulent skin friction"
Progress report DRB.9550-23 Universite Laval (Canada)

(53) FRANKLIN, R.E.
Contribution to N.P.L. Aero meeting and private communication

(54) FRANKLIN, R.E.
"A force-displacement indicator for a drag balance"
(55) HEADLEY, J.W.

"A Simple Calibration Technique for Skin Friction Balances"
A.I.A.A. Journal Vol. 4 p.1862

(56) KLINE, S.J. and McCLINTOCK, F.A.

"Describing Uncertainties in Single Sample Experiments"
Mechanical Engineering, January 1953.

(57) FERRIS, D.H.

"Preston Tube Measurements in Turbulent Boundary Layers
and Fully Developed Pipe Flow"
A.R.C. C.P. 831.

(58) BRADSHAW, P. and HELLENS, G.E.

"The N.P.L. 59in x 9in Boundary Layer Tunnel"

(59) MILLER, B.L.

"A Floating Element Skin-Friction Meter"

(60) KJELLSTROM, B. and HEDBERG, S.

"Calibration of a DISA Hot-Wire Anemometer etc."

(61) BRADLEY, C.I. and COCKRELL, D.J.

"Boundary Layer Methods applied to Internal Fluid Problems"

(62) CHAMPAGNE, F.H.

"Turbulence Measurements with Inclined Hot-Wires"
(63) BRADLEY, C.I.
"Turbulence Effects in Internal Airflows"

(64) LEE, B.E.
"Some Effects of History on Turbulent Fluid Flow"

(65) PATEL, V.C. and HEAD, M.R.
"Some observations on skin friction and velocity profiles in fully developed pipe and channel flows"

(66) TANI, I., IUICHI, M. and KOMODA, H.
"Experimental Investigation of Flow Separation Associated with a Step or Groove"

(67) DUNCAN, W.J., THOM, A.S., and YOUNG, A.D.
"The Mechanics of Fluids"
Edward Arnold 1962.