Effects of small isolated roughness elements on turbulent boundary layers

by

H H. M. Nigim

The Thesis submitted to the University of Leicester for the Degree of Doctor of Philosophy

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Dedicated to my parents and family
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS ON TURBULENT BOUNDARY LAYERS

By H.H.M. NIGIM

ABSTRACT

A series of six equilibrium turbulent boundary layer flows has been established with values of $H$ from 1.3 to 2.3, and measurements made in them of the effect on the boundary layer development of a single two-dimensional roughness element of mainly square cross-section mounted near the start of the equilibrium region. It is shown that the local increment of the momentum thickness caused by the element is well-predicted by the flat-plate correlation of Gaudet and Johnson, a correlation which is here shown to be universally valid and that, for all flows except for the most adverse pressure gradient, a satisfactory prediction of the subsequent boundary layer development can be made with the aid of relationships proposed by Professor Bradshaw, for the change in $H$ at the roughness element. For the flow with the largest value of $H$ the prediction method for the development fails even in the absence of the element which, in fact, has little influence on the flow. The discrepancy between calculation and experiment is much larger than can be accounted for by normal stress terms and the reasons for this discrepancy are not entirely evident. However, the essential outcome of the experiment is clear that the incremental drag of a roughness element depends on wall variables. In consequence, the effect of an element which is of small height compared with the boundary layer thickness is negligible in flows with strongly adverse pressure gradients. It is also demonstrated that the length of the separation region behind small roughness elements decreases as the pressure gradient increases adversely.
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CONTENTS

ACKNOWLEDGEMENTS ii

CONTENTS iii

LIST OF SYMBOLS V

LIST OF FIGURES Vi

1. HISTORICAL SURVEY AND INTRODUCTION TO CURRENT RESEARCH PROGRAMME

1.1. Small Roughness Elements Within Turbulent Boundary Layers 1

1.2. Classification of Roughness Element 2

1.3. Measurement of Roughness Element Drag 5

1.4. Objectives of the Current Research Programme 7

2. EXPERIMENTAL FACILITIES REQUIRED TO DEVELOP THE APPROPRIATE TURBULENT BOUNDARY LAYERS AND THE MEASUREMENT OF ROUGHNESS ELEMENT DRAG

2.1. Experimental Facilities 15

2.1.1. Introduction 15

2.1.2. The Working Section 15

2.1.3. Instrumentation 16

2.2. Turbulent Mean Velocity Distribution 17

2.2.1. Introduction 17

2.2.2. The Inner Part of the Boundary Layer 17

2.2.3. The Outer Part of the Boundary Layer 18

2.3. Experimental Measurements 19

2.3.1. Mean Velocity Determination 19

2.3.2. Local Skin Friction 21

2.3.3. The Flow Two-Dimensionality 21

2.4. Turbulent Equilibrium Boundary Layers 23

2.4.1. Introduction 23

2.4.2. Equilibrium Parameters 24

2.4.3. Equilibrium Locus 26

2.4.4. Equilibrium Conditions 27
4.4. Comparison of the Experimental Results with Prediction

4.4.1. In the Absence of Roughness Elements

4.4.2. In the Presence of Roughness Elements

4.4.3. Results of Comparisons with Experiment

5. DISCUSSION OF RESULTS

5.1. Flow in the Absence of Roughness Elements

5.1.1. The Equilibrium Locus

5.1.2. Expressions Required in Auxiliary Equations for Boundary Layer Prediction Purposes

5.1.3. The Mixing Length Validity

5.2. Flow over Small Roughness Elements

5.2.1. The Mechanism Controlling the Reattachment Length

5.2.2. Mean Velocity Profiles

5.3. \( \frac{C_p}{C_f} \) Similarity with the Law of the Wall

5.4. Prediction Methods

5.4.1. The Prediction of Flows which have High Shape Parameters

5.4.2. The Power Law Velocity Profile Method

5.4.3. Two-Parameter Velocity Profile Method

6. RECOMMENDATION FOR FUTURE WORK

6.1. Improvements in Measurement Facilities

6.2. Recommendations for Future Experimentation

6.3. Recommendations for Future Computer-Prediction Studies

CONCLUSIONS

REFERENCES

APPENDIX

A  Tabulation of Experimental Data in the Absence of Roughness Elements

B  Tabulation of Experimental Data Behind Isolated Roughness Elements

C  Listing of Computer Programs

D  Summary Chart for the Experimental Roughness Elements Flows
### List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_D$</td>
<td>Roughness element drag coefficient, $C_D = \frac{D_x}{\frac{1}{2} \rho U_e^2 h}$</td>
</tr>
<tr>
<td>$C_E$</td>
<td>Entrainment coefficient, Equation (4.13)</td>
</tr>
<tr>
<td>$c_f$</td>
<td>Local skin friction coefficient, $c_f = \frac{\tau_o}{\frac{1}{2} \rho U_e^2}$</td>
</tr>
<tr>
<td>$c_{f_o}$</td>
<td>Local skin friction coefficient in equilibrium flow at zero pressure gradient</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Maximum shear stress coefficient, $\tau_{\text{max}} = \frac{1}{2} \rho U_e^2$</td>
</tr>
<tr>
<td>$D_x$</td>
<td>Drag force per unit width</td>
</tr>
<tr>
<td>$E_f$</td>
<td>Equilibrium friction parameter, Equation (2.22)</td>
</tr>
<tr>
<td>$E_P$</td>
<td>Equilibrium pressure gradient parameter, Equation (2.23)</td>
</tr>
<tr>
<td>$E_Q$</td>
<td>Equilibrium condition</td>
</tr>
<tr>
<td>$E_{Q_o}$</td>
<td>Equilibrium condition in the absence of secondary influences on turbulent structure</td>
</tr>
<tr>
<td>$F$</td>
<td>Function of $C_E$ and $c_{f_o}$, Equation (4.27)</td>
</tr>
<tr>
<td>$F_{C}$</td>
<td>Scaling functions in skin friction law, Equation (4.22)</td>
</tr>
<tr>
<td>$F_R$</td>
<td></td>
</tr>
<tr>
<td>$G$</td>
<td>Clauser defect shape parameter, Equation (2.23)</td>
</tr>
<tr>
<td>$H$</td>
<td>Shape parameter, $H = \delta^*/\theta$</td>
</tr>
<tr>
<td>$H_1$</td>
<td>Entrainment shape parameter, $H_1 = (\delta - \delta^*)/\theta$</td>
</tr>
<tr>
<td>$H_{32}$</td>
<td>Energy shape parameter, $H_{32} = \delta E/\theta$</td>
</tr>
<tr>
<td>$h$</td>
<td>Height of roughness element</td>
</tr>
<tr>
<td>$h^+$</td>
<td>Roughness Reynolds number, $h^+ = h U_T / \nu$</td>
</tr>
<tr>
<td>$J$</td>
<td>Equilibrium shape parameter, $J = (H - 1)/H$</td>
</tr>
<tr>
<td>$K$</td>
<td>Recirculation length</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number at the edge of the boundary layer</td>
</tr>
<tr>
<td>$m$</td>
<td>Flow parameter, $m = (x/\bar{U}_e) \frac{\partial \bar{U}_e}{\partial x}$</td>
</tr>
<tr>
<td>$p$</td>
<td>Dynamic pressure</td>
</tr>
</tbody>
</table>
\( P_a \)  
Atmospheric pressure

\( P_L \)  
Left-hand side of the momentum balance Equation (2.12)

\( P_R \)  
Right-hand side of the momentum balance, Equation (2.12)

\( P_R \)  
Right-hand side of the momentum balance with normal stress terms present, Equation (2.14)

\( Q \)  
Volume flow rate per unit span

\( R_\theta \)  
Reynolds number based on momentum thickness

\( T_a \)  
Atmospheric temperature

\( U \)  
Local x-component of mean velocity

\( U_e \)  
Freestream velocity

\( U^+ \)  
Ratio \( U/U_T \)

\( U, V \)  
Velocity components of turbulence

\( V_E \)  
Entrainment velocity, Equation (4.4)

\( W \)  
Uncertainty value

\( w \)  
Coles' wake function, Equation (2.7)

\( X \)  
Streamwise co-ordinate measured from the start of the working section

\( X_0 \)  
Virtual origin of equilibrium flow \( X = X_0 \) at \( x = 0.0 \)

\( X_R \)  
Reattachment length

\( X_{R_0} \)  
Reattachment length at zero pressure gradient

\( x \)  
Streamwise co-ordinate of equilibrium flow

\( x' \)  
Reseparation length

\( y \)  
Co-ordinate normal to the surface

\( y^+ \)  
Reynolds number, \( y^+ = y U_T / \nu \)

\( \beta \)  
Clauser's pressure gradient parameter for equilibrium flow, Equation (2.15)

\( \Delta \)  
Defect thickness parameter, Equation (2.16)

\( \Delta_D \)  
Increase of drag due to a discrete element of roughness
\[ \Delta \theta \]
Increase of momentum thickness due to a discrete element of roughness

\[ \delta \]
Boundary layer thickness, \( y = \delta \) at \( U/U_e = 0.995 \)

\[ \delta_E \]
Energy thickness, \( \delta_E = \frac{1}{U_e^2} \int_0^\delta U(U_e^2 - U^2) \, dy \)

\[ \delta^* \]
Displacement thickness, \( \delta^* = \frac{1}{U_e} \int_0^\delta (U_e - U) \, dy \)

\[ \theta \]
Momentum thickness, \( \theta = \frac{1}{U_e^2} \int_0^\delta U(U_e - U) \, dy \)

\[ \epsilon \]
Mixing length scale, \( \epsilon = U_e / \frac{\delta U}{\partial y} \), Equation (5.6)

\[ \kappa \]
von Karman's constant

\[ \mu \]
Dynamic viscosity

\[ \nu \]
Kinematic viscosity \( \nu = \mu / \rho \)

\[ \Pi \]
Coles' wake parameter, Equation (2.8)

\[ \rho \]
Fluid density

\[ \tau \]
Shear stress

\[ \tau_{\text{max}} \]
Maximum shear stress

\[ \tau \]
Local shear stress at the wall
<table>
<thead>
<tr>
<th>LIST OF FIGURES</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1. Diagrammatic representation of grooves and steps on air foils</td>
<td>9</td>
</tr>
<tr>
<td>1.2. Bradshaw and Wong mean velocity profiles behind backward-facing step [8]</td>
<td>10</td>
</tr>
<tr>
<td>1.3. Drag coefficient for rectangular roughness elements, as measured by Wieghardt [10]</td>
<td>11</td>
</tr>
<tr>
<td>1.4. Drag coefficient of elementary surface imperfections as a function of their height ratio from Hoerner [4]</td>
<td>12</td>
</tr>
<tr>
<td>1.5. Drag of small isolated roughness elements at zero pressure gradient from Gaudet and Winter [15]</td>
<td>13</td>
</tr>
<tr>
<td>1.6. Drag coefficient from adverse pressure gradient test - Good and Joubert [17]</td>
<td>14</td>
</tr>
<tr>
<td>2.1. Photograph of wind tunnel and working section</td>
<td>36</td>
</tr>
<tr>
<td>2.2. Boundary Layer wind tunnel</td>
<td>37</td>
</tr>
<tr>
<td>2.3. The traversing unit</td>
<td>38</td>
</tr>
<tr>
<td>2.4. Flow Chart of data evaluation cycle</td>
<td>39</td>
</tr>
<tr>
<td>2.5. Regions of a turbulent boundary layer velocity profile</td>
<td>40</td>
</tr>
<tr>
<td>2.6. Photograph of working section setting for flow 6 containing the two pins</td>
<td>41</td>
</tr>
<tr>
<td>2.7. Normal stress integrals in low speed boundary layers</td>
<td>42</td>
</tr>
<tr>
<td>2.8. Equilibrium defect velocity profiles for $\beta &gt; 0$ from Clauser [6]</td>
<td>43</td>
</tr>
<tr>
<td>2.9. The shape parameter for equilibrium turbulent boundary layers</td>
<td>44</td>
</tr>
<tr>
<td>2.10. Conditions before and after a small roughness element and general behaviour of a reattaching flow</td>
<td>45</td>
</tr>
<tr>
<td>3.1. Tunnel roof setting for all flows</td>
<td>56</td>
</tr>
<tr>
<td>3.2. Freestream velocity distribution for all flows</td>
<td>57</td>
</tr>
<tr>
<td>3.3. Growth of the momentum thickness for all flows</td>
<td>58</td>
</tr>
<tr>
<td>3.4a-f Pins test results for all flows</td>
<td>59-64</td>
</tr>
<tr>
<td>3.5a-f Momentum integral equation balance</td>
<td>65-67</td>
</tr>
<tr>
<td>3.6a-b Spanwise variation of pressure distribution across the flows 5 and 6</td>
<td>68-69</td>
</tr>
<tr>
<td>3.7. $y/X$ vs $U/U_e$ equilibrium velocity profiles</td>
<td>70</td>
</tr>
<tr>
<td>3.8. Logarithmic plots of mean velocity profiles</td>
<td>71</td>
</tr>
<tr>
<td>Section</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
<td>-------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>3.9</td>
<td>Replot of the velocity profiles using inner part variables</td>
</tr>
<tr>
<td>3.10a-b</td>
<td>Universal plot of turbulent velocity defect profiles</td>
</tr>
<tr>
<td>3.11</td>
<td>Skin friction distribution for all flows</td>
</tr>
<tr>
<td>3.12</td>
<td>Shape parameter distribution for all flows</td>
</tr>
<tr>
<td>3.13</td>
<td>Defect shape parameter distribution for all flows</td>
</tr>
<tr>
<td>3.14</td>
<td>Equilibrium locus in general co-ordinates</td>
</tr>
<tr>
<td>3.15a-d</td>
<td>Pins test for flows 2, 3, 4, 5 with square roughness elements</td>
</tr>
<tr>
<td>3.16a-f</td>
<td>Mean velocity profiles behind square roughness elements</td>
</tr>
<tr>
<td>3.17a-f</td>
<td>Replot of the velocity profiles behind square roughness element using inner variables</td>
</tr>
<tr>
<td>3.18</td>
<td>Skin friction coefficient versus distance from single square roughness elements</td>
</tr>
<tr>
<td>3.19</td>
<td>Shape parameter H versus distance from single square roughness elements</td>
</tr>
<tr>
<td>3.20a-b</td>
<td>Growth of momentum thickness behind single square roughness elements for flows 1 - 4 and 3 - 5</td>
</tr>
<tr>
<td>3.21a-d</td>
<td>Shape defect parameter G versus distance from single square roughness elements</td>
</tr>
<tr>
<td>3.22</td>
<td>Dependence of $C_D/c_f$ on $h^+$ for square roughness element</td>
</tr>
<tr>
<td>3.23</td>
<td>Dependence of $C_D/c_f$ on $h^+$ for square roughness element using Nash and Bradshaw's equation (2.30)</td>
</tr>
<tr>
<td>3.24a-b</td>
<td>Results for different shape roughness elements in flows 5 and 6</td>
</tr>
<tr>
<td>3.25</td>
<td>Flow visualization photographs in flow 6 for single roughness elements</td>
</tr>
<tr>
<td>4.1a-c</td>
<td>Comparison of prediction and experimental data from Stanford Conference [1968]</td>
</tr>
<tr>
<td>4.2a-f</td>
<td>Comparison of prediction and experiment for all flows</td>
</tr>
<tr>
<td>4.3</td>
<td>Comparison of prediction and experiment for Tillmann ledge flow</td>
</tr>
<tr>
<td>4.4a-b</td>
<td>Comparison of prediction and experiment for all flows with square isolated roughness elements</td>
</tr>
<tr>
<td>4.10</td>
<td>Comparison between prediction and experiment for flow 6 with roughness element (prediction started at $x = 1781$ mm)</td>
</tr>
<tr>
<td>5.1 - 5.4</td>
<td>Comparison between the experimental results with the turbulent equilibrium locus analysis of Green, Weeks and Brooman</td>
</tr>
<tr>
<td>5.5</td>
<td>Plot showing that $H_1 = 2/J$ is a close fit to the data for equilibrium boundary layers</td>
</tr>
</tbody>
</table>
5.6. Relationship between energy shape parameter and J  
5.7. Variation of reattachment length with H for small square isolated roughness elements  
5.8. Dependence of $C_D/c_f$ on $h^+$ for square isolated roughness elements  
5.9. Comparison of prediction with Ludwieg and Tillmann strong adverse pressure gradient flow  
5.10. Comparison of prediction with Chu and Young flow 2  
5.11. Comparison between prediction with the suggested entrainment shape parameter equation (5.1), and experiment flow 6
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 1

HISTORICAL SURVEY AND
INTRODUCTION TO CURRENT RESEARCH
1. HISTORICAL SURVEY AND INTRODUCTION TO CURRENT RESEARCH PROGRAMME

1.1. Small Roughness Elements within Turbulent Boundary Layers

There are a number of contemporary engineering situations in which small roughness elements are seen to lie within turbulent boundary layers. In the atmospheric boundary layer, for example, buildings can be considered to act as small roughness elements on an otherwise much smoother earth's surface. In rivers and estuaries, the sand river bed forms into patterns of ridges, which comprise small roughness elements. In underground passageways formed in mines the roof supports form small roughness elements whose drag must be evaluated if efficient ventilation is to be provided and in many heat transfer applications, such as in the design of nuclear fuel elements, roughness elements are deliberately introduced to increase the local heat transfer rate to the surrounding fluid.

Although the research described in this thesis may be applicable to any of these applications, the situation which led to its commencement was the need to establish the drag resulting from the presence of discrete surface discontinuities on aircraft wings. The work, which began in July 1978, was monitored by Mr K.G. Winter, of the Royal Aircraft Establishment, Bedford. Following on from earlier research in this field at Leicester University conducted by Lacey [1] and Abd Rabbo [2], particular emphasis has been placed in this current work on the effects which single small two-dimensional square or triangular cross-section roughness elements may have on equilibrium adverse pressure gradient turbulent boundary layers. In such boundary layers the shape factor, $H$, in the absence of roughness elements varies from about 1.3 to about 2.3. This situation is characteristic of that arising on a modern high-lift aircraft wing where the complex extending surfaces which form leading-edge slats and trailing-edge flaps can cause small two-dimensional surface discontinuities when they retract into the main aerofoil. Most of these surface discontinuities occur in regions of high adverse pressure gradient,
as shown in Fig. (1.1).

In this particular situation the need is to be able to predict with confidence the gross characteristics of the turbulent boundary layer through and downstream of the small roughness element. It is this need which the current research programme is designed to meet.

1.2 Classification of Roughness Elements

Roughness elements, as Table 1 indicates, can be classified according to their geometry as well as according to their effect. The current investigation is concerned with two-dimensional roughness elements, that is elements which perturb the flow field in two dimensions only, variations in velocity in the direction parallel to the ridge being considered negligible. Characteristics of arrays of three-dimensional ridges are considered elsewhere, notably by Schlichting [3] and by Hoerner [4].

Isolated roughness elements develop drag which is dependent on the size and shape of the elements and on the characteristics of the boundary layers in which they are immersed. The drag resulting from arrays of elements also depends on the spacing between the elements. Perry, Schofield and Joubert [5] showed that these could be classified into K-type roughness arrays and d-type roughness arrays, the first giving a Clauser-type [6,7] roughness function which depends on a Reynolds number based on the shear velocity and on a length associated with the length, k, of the elements. The second, arising when the roughness elements are much closer together, is described as a d-type roughness array because early experiments were conducted in pipes having diameter, d. These give roughness functions which correlate with pipe diameter. Abd Rabbo's [2] work on ridge arrays is related to both K-type roughness, in which the spacing-to-height ratio of the elements exceeded 3.6 and vortices are shed into the flow from the spaces between the elements, as distinct from d-type roughness, in which element spacing-to-height is a smaller ratio, when the small spaces between the roughness elements
can be regarded as cavities in which non-time-varying vortices, which are not shed into the flow, will form.

The classification of isolated roughness elements shown in Table 1 is based on the work of Bradshaw and Wong [8]. Here, the significant parameter is the ratio of the roughness element height to the undisturbed boundary layer thickness. What characterises the effect of the small roughness elements, which are the subject of the present investigation, is that although the velocity-scale and the length-scale of the flow may be altered by the presence of the roughness element, no significant changes occur to the dimensionless properties of the boundary layer turbulence structure downstream of the element. This implies that the effects of such roughness elements can be determined by conventional calculation techniques, and this implication is at the root of the research programme. Tani [9] has reviewed the effects of such small roughness elements, observing that downstream of the roughness elements recovery to equilibrium is almost instantaneous in the flow near to the wall but rather slower in the outer part of the boundary layer.

Larger roughness elements, in relation to the boundary layer thickness, by contrast cause significant changes to the turbulence structure of the flow. Bradshaw and Wong [8] studied the characteristics of redeveloping boundary layers behind backward-facing steps having height to boundary layer thickness ratios of eight. They found that the redeveloping boundary layer did not follow the law of the wall, but behaved instead as shown in Fig. (1.2). The wake component of the mean velocity profile was abnormal and there was a rapid decrease in the turbulent shear stresses just downstream of flow reattachment. Since normal turbulent boundary layer characteristics had not been fully recovered at a distance of 52 step heights downstream of the step, the unusual turbulence structure of the downstream flow was apparent.

If the roughness elements become even larger, Bradshaw and Wong show that the boundary layer mutates into either a wake or a mixing layer.
ROUGHNESS ELEMENTS

TWO-DIMENSIONAL

Elements which span the flow field, so that flow variations occur in planes containing the free stream and perpendicular to the element only e.g. steps, ridges, grooves.

THREE-DIMENSIONAL

Discrete elements. Flow occurs in three mutually perpendicular planes. e.g. rivets, holes, buildings

(Characteristics not considered further)

Arrays of Elements
(Distributed roughness)

Discrete Elements
(Isolated elements)

k-type roughness
d-type roughness

(Characteristics not considered further)

Small roughness elements

Medium-sized roughness elements

Large roughness elements

h/6 < 1 causing weak flow perturbation

h/6 = O(1) significantly alters turbulence structure downstream

h/6 > 1 boundary layer mutates into a wake or mixing layer

Subject of current investigation

(Characteristics not considered further)

Table 1 CLASSIFICATION OF ROUGHNESS ELEMENTS
1.3 Measurement of Roughness Element Drag

In considering the effects of small roughness elements, observers have had to contend with the two related problems of the methods available to them by which the drag of such elements can be measured and what is the most rational way of non-dimensionalising this drag once it has been measured. Wieghardt's [10] work in 1942 forms a suitable starting-point for a review, as it was both comprehensive and thorough. Wieghardt measured the drag using a steelyard balance attached to a test plate, on which the roughness elements were mounted. The drag of these elements was determined by subtracting the drag of the test plate without the element from the value obtained when the element was present. This value was non-dimensionalised by dividing it by the roughness element height, h, and a mean kinetic pressure, \( \bar{q} \), of the undisturbed boundary layer, formed by assuming the boundary layer to be represented by a one-seventh power law, deducing from it the mean velocity, \( U_m \), and writing \( \bar{q} = \frac{1}{2} \rho U_m^2 \). His results clearly indicate the significance of the parameter h/6. In Fig. (1.3), some of his results for two-dimensional roughness elements are shown.

Tillmann [1] extended Wieghardt's measurements, as did Hoemer [4]. The latter non-dimensionalised the drag by dividing it by the roughness element height, h, and the mean kinetic pressure developed in the absence of the element over a distance equal to the height of the element. Using this independent drag coefficient, as he described it, Hoemer showed that his results were approximately independent of element height. Some experimental results as plotted by Hoemer are shown in Fig. (1.4).

Plate [12] calculated the drag of a smooth flat plate whose turbulent boundary layer is disturbed by a two-dimensional, sharp-edged fence. The calculations were based on knowledge of the drag coefficient for the fence, and the friction coefficients along the plate in the disturbed boundary layer downstream of the fence. In the calculation, experimental data were used to provide a relation between the drag coefficient of the fence and the
characteristics of both the fence and the boundary layer. Plate found the drag coefficient of the fence to depend on the $h/b$ ratio, paralleling the results of Wieghardt [10].

The most extensive series of relevant experimental investigations are those carried out by Winter and Gaudet [13], Johnson and Gaudet [14], and Gaudet and Winter [15]. In their measurements of roughness element drag, the elements were mounted on balances fitted in the side walls of the wind tunnel working section. The investigations cover subsonic as well as supersonic data for different types of two-dimensional and three-dimensional roughness elements. Considering that a power law velocity profile was an over simplification of the problem, they presented a drag coefficient for their different configurations by plotting $C_D/c_f$ versus $h^+ = hU_\infty/v$, where $c_f$ is the skin friction coefficient at a given location in the absence of the roughness element. For two-dimensional sharp edges, steps, and ridges their drag measurements were calculated by

$$\frac{C_D}{c_f} = A \log h^+ + D$$  \hspace{1cm} (1.1)

where $A$ and $D$ depend on Mach number and roughness element configuration. Reproduction of the results reported in [14] for forward steps, backward steps, and square ridges at 0.2 Mach number (i.e. effectively incompressible flow) is shown in Fig (1.5), together with the corresponding relationships between $C_D/c_f$ and $h^+$ in the form of equation (1.1).

Pugh and Hutton [16] pointed out that any roughness element will affect the skin friction of the boundary layer formed downstream of it. If this downstream effect is considered to be separate from the roughness element, then the drag balance must be constructed so as to include a minimum length of downstream surface.

Good and Joubert [17] made measurements of pressure distribution over the forward and rear faces of a fence and then integrated them to determine
the fence drag. The results obtained were correlated with the characteristics of the smooth wall boundary layer in which the fence was immersed. For zero pressure gradient flows correlations obtained for the variation of form drag with plate height were analogous in form to the law of the wall and the velocity defect law. Figure (1.6), which is based on their work, shows some inconsistency in the way in which $C_D/c_f$ varies with pressure gradient.

Lacey [1], and later Abd Rabbo [2] used a momentum defect technique to determine the drag for isolated and uniformly distributed small square-section roughness elements. While Lacey made his measurements in a negligible pressure gradient, Abd Rabbo established two mild equilibrium adverse pressure gradients. The results for isolated roughness elements reported by Lacey are in good agreement with other published data, Abd Rabbo's data, like Good and Jourberd's [17] data, contain inconsistencies. Both Lacey [1] and Abd Rabbo [2] used Nash and Bradshaw's [18] relationship to calculate the change in momentum thickness across the roughness element. The application of this momentum defect technique as well as Nash and Bradshaw's momentum relationship is discussed in more detail in Section (2.5.1).

1.4. Objectives of the Current Research Programme

The overall aim is to investigate the effects of small isolated two-dimensional, mainly square cross section, roughness elements on turbulent boundary layers in strong adverse pressure gradients. Specific objectives are:

1 - To set up a series of two-dimensional equilibrium turbulent boundary layers, and to compare their characteristics with other investigators' results.

2 - To determine the change in the boundary layer integral parameters (specifically $\theta$, $H$, $c_f$ and $G$) downstream of isolated small two-
dimensional roughness elements, and to consider the additional drag
developed at the element position.

3 - To establish suitable empirical formulae to relate these integral
parameters with the corresponding adverse pressure gradient.

4 - To model the effects of these flow developments in a conventional
turbulent boundary layer prediction programme, and to compare the
resulting predictions with experimental measurements.
Fig. (1-1) Diagrammatic representation of grooves and steps on airfoils
Fig.(1.2) Mean velocity profiles. Semi-logarithmic plot, (a) $x/h = 16-34$

Scales of $U/u_r$ refer to $x/h = 16$, subsequent curves displaced by one unit each time. (b) $x/h = 40-52$. Bradshaw and Wong[8]
Fig(1-3) Drag coefficient for rectangular roughness elements, as measured by Wieghardt [10]
$C_{D_{ind}} = \frac{C_D}{3} \sqrt[3]{h/x_1}$

Figure (1-4) Drag coefficient of elementary surface imperfections as a function of their height ratio. Hoerner [4]
Fig. (1-5) Drag of small isolated roughness elements at zero pressure gradient

Gaudet and Winter [15]

$\frac{C_D}{C_f} = 150 \log h - 190$
$\frac{C_D}{C_f} = 60 \log h - 80$
$\frac{C_D}{C_f} = 16 \log h - 6$

Wiegardt

$\frac{C_D}{C_f}$

$10^3$
FIG. (I-6) DRAG COEFFICIENT FROM ADVERSE PRESSURE GRADIENT (PG) tests
Good and Joubert [17]
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 2

EXPERIMENTAL FACILITIES REQUIRED TO DEVELOP THE APPROPRIATE TURBULENT
BOUNDARY LAYERS AND THE MEASUREMENT OF ROUGHNESS ELEMENT DRAG
2. EXPERIMENTAL FACILITIES REQUIRED TO DEVELOP THE APPROPRIATE TURBULENT BOUNDARY LAYERS AND THE MEASUREMENT OF ROUGHNESS ELEMENT DRAG

2.1. Experimental Facilities

2.1.1. Introduction

In order to carry out the present investigation, mainly the same facilities were used which have been described by both Lacey [1] and Abd Rabbo [2].

Because setting up strong pressure gradients is one of the subjects of the investigation, the working section of the open-return boundary layer wind tunnel which they used was modified. The side walls were extended as was the range over which the flexible roof could be adjusted. This was done by making the upstream part, over which the pressure distribution had previously been constant, variable by using four additional adjustable screw jacks. These jacks made it possible to impose a wide range of pressure distributions over the flat lower surface, where the boundary layer measurements are made.

The working section was carefully fitted and realigned with the upstream 10:1 screened two dimensional contraction (described in detail by Lacey [1]) whose outlet to the working section has an aspect ratio 2:1, the width there being 456 mm and the height 228 mm. The air was provided by a contra-rotating fan downstream of the working section, driven by single 10 horse power motor. The fan is connected to the working section by a suitable diffuser. Fig. (2.1) shows a photograph of the wind tunnel and the modified working section. The dimensions of the working section are shown in Fig.(2.2).

2.1.2. The Working Section

A rectangular cross sectional working section of 3.6 metres length,
measured from the upstream transition trip wire, was employed. The diameter of the trip wire was chosen to satisfy the limiting condition reported by Schlichting [3].

The floor surface selected satisfied Schlichting's criteria, so that a hydraulically smooth test surface was obtained. The floor of the working section is constructed of three sheets of Tufnol. Along the axial centre line of the floor ten removable blank plugs of 88 mm diameter were fitted at spacing shown in Fig. (2.2). These plugs can be replaced by others on which traverse gears and other forms of measuring instrumentation are mounted. To determine the static pressure distribution static tappings were drilled both longitudinally and laterally in the working section floor. The upper surface of the working section was constructed from flexible sheets of plywood faced with laminate, its position being maintained by fifteen manually-operated screw jacks, so that the local height of the tunnel could be varied between 150 mm and 850 mm, thus obtaining a wide range of pressure gradients. The gap size between the flexible roof and the side walls is no more than 1 mm. If desired, this gap can be sealed.

2.1.3. Instrumentation

Moveable and stationary stainless steel Pitot tubes of 1.27 mm external diameter and internal to external diameter ratio of 0.6 were employed to determine the dynamic pressures in the boundary layer and in the free stream respectively. The moveable Pitot tube, mounted on a standard wall disc, is driven across the boundary layer by an automatic traversing unit powered by a 6-volt stepping motor. The whole unit is shown in Fig. (2.3). Stepping increments of 0.127 mm, 127 mm and 12.7 mm can be obtained from the unit, which is activated by signals received from the monitor.

Pitot tube pressures are measured relative to the wall static pressure by two micromanometer pressure transducer units, which convert the dynamic
pressures into millivolt signals. These units show sensible linearity when calibrated against a Betz water manometer. The resulting millivolt signals are fed to a data logger which derives mean values by integration. Time for the fluctuating voltage used in this investigation is six seconds. The foregoing cycle is repeated for each measuring position across the boundary layer from the wall surface up to the free stream. A flow chart of the cycle is shown in Fig. (2.4).

2.2. TURBULENT MEAN VELOCITY DISTRIBUTION

2.2.1. Introduction

The mean velocity profiles of turbulent flows over surfaces are normally divided into two main parts, as shown in Fig. (2.5). The inner part is the region very close to the surface in which the flow is determined by viscous shear, while the outer part is the flow in the remaining part of the profile which is turbulent shear dominated.

2.2.2. The Inner Part of the Boundary Layer

The thickness of the inner part is about 10 - 20% of the complete boundary layer thickness. In this region the mean velocity depends upon the wall shear stress, the physical properties of the flow, and the distance y from the wall. Dimensional analysis of the inner part gives

\[ U^+ = f(y^+) \quad (2.1) \]

where \( U^+ = \frac{U}{U_T} \), and \( y^+ = \frac{yU_T}{\nu} \).

In a sub-region very close to the wall called the viscous sublayer, where the shear stress dominates, the velocity profile is linear, thus

\[ U^+ = y^+ \quad (2.2) \]

Outside this viscous sublayer, but still within the inner part of the boundary layer, turbulent motion approaches a state of equilibrium Rotta [19], and as \( y^+ \) continues to increase the turbulent shear stress becomes
at least as effective as the viscous stress. In this region the rate of strain \( \frac{\delta U}{\delta y} \) becomes independent of molecular viscosity (Cebeci and Bradshaw [20]). The velocity distribution in this fully developed turbulent region can be derived from equation (2.1) by differentiation with respect to \( y \);

\[
\frac{\delta U^+}{\delta y} = \frac{U_T}{v} \frac{\delta f}{\delta y^+}
\]  

(2.3)

In this relationship, since the right-hand side must be independent of viscosity it follows that

\[
\frac{\delta f}{\delta y^+} = \frac{v}{\kappa y U_T} \quad \text{and} \quad \frac{\delta U^+}{\delta y} = \frac{1}{\kappa y}
\]

(2.4)

where \( \kappa \) is a universal constant. Integration of equation (2.4) gives when \( y^+ > 50 \)

\[
U^+ = \frac{1}{\kappa} \ln y^+ + B
\]

(2.5)

where \( B \) is a constant of integration. This semilogarithmic law is widely known as the universal law of the wall. Initially Ludwieg and Tillmann[21] demonstrated experimentally the existence of this universal law of the wall in boundary layers with non-zero pressure gradient. Because of scatter in the data the value of constants \( \kappa \) and \( B \) are controversial. In this current investigation the Coles [22] values of \( \kappa = 0.41 \) and \( B = 5.0 \) have been used. The most significant property of the law of the wall is that it provides a plausible method for estimating skin friction.

2.2.3. The Outer Part of the Boundary Layer

The outer part of the turbulent boundary layer contains 80 - 90% of the
layer's thickness. In this part the velocity profile is free from direct effects of viscosity, but depends upon the wall shear stress, $\tau_w$, the boundary layer thickness $\delta$, and the free stream pressure gradient, $\frac{dp}{dx}$. Thus the outer part can be expressed by the velocity defect law (Clauser [6,7])

$$\frac{U_e - U}{U_T} = g\left(\frac{\nu}{\delta}, \frac{\delta}{\tau_w}, \frac{dp}{dx}\right)$$

(2.6).

A refined approximation to the defect profile has been given by Coles [22], in which the departure of the flow from the semilogarithmic law equation (2.5), is described by a universal function $w(y/\delta)$, called the wake function. Coles' approximation is

$$w(y/\delta) = 2\sin^2\left(\frac{\pi}{2\delta}y\right)$$

(2.7).

This leads to the following velocity profile relationship throughout the boundary layers

$$U^+ = \frac{1}{k} \ln y^+ + B + \frac{H}{k} \left[2\sin^2\left(\frac{\pi}{2\delta}y\right)\right]$$

(2.8),

where $H$ is known as the Coles' wake parameter.

2.3. Experimental Measurements

2.3.1. Mean Velocity Determination

At a known distance from the floor the dynamic pressures at both the traversing and the free stream Pitot tubes were measured and recorded simultaneously relative to the static pressure, so that if variations in free stream velocity occurred, local values of dynamic pressure could be
corrected readily to a value corresponding to a datum free stream velocity.

MacMillan's [23] correction for the disturbing effect of the floor on the Pitot tube reading was employed in the present work. That is, for a rounded cross-section Pitot tube a correcting displacement of 0.15 times the external diameter of the tube was used. To obtain the required integral thickness parameters, since measurements in the region of viscous sublayer are generally not possible, Coles [24] standard sublayer profile integrations were used up to \( y^* = 50 \), where \( y^* = \frac{y u_l}{v} \). These are:

\[
\begin{align*}
\int_0^{50} \left( \frac{u_l}{u_T} \right) dy^* &= 540.6 \\
\int_0^{50} \left( \frac{u_l}{u_T} \right)^2 dy^* &= 6546 \\
\int_0^{50} \left( \frac{u_l}{u_T} \right)^3 dy^* &= 82770
\end{align*}
\]  

(2.9)

Any discontinuity between \( y^* = 50 \) and the first experimental point was bridged by adopting the law of the wall relationship.

The experimental data was integrated, using a modified Simpson's rule method, as first reported in Coles and Hirst [25]. Calculations by this method were carried out using the raw experimental data, without smoothing. The accuracy of this integration method depends strongly on the input data and is hard to estimate, Coles and Hirst reporting that their modified method has clear superiority over both the trapezoidal rule and the conventional Simpson's rule. The program computing integral thickness and shape factors is listed in Appendix (C). Its output includes machine generated graphs of velocity profiles.
2.3.2. Local Skin Friction

As in Coles and Hirst [25], the basic assumption that the law of the wall relationship is universally valid was used for estimating the local skin friction. In the computations only the parts of experimental velocity profile in the range $100 < y^+ < 300$ were normally used. The value of the friction velocity was adjusted by taking the arithmetic mean value of the values of friction velocity computed for each experimental point in that defined region. The fitting was carried out with the aid of computer subroutine SKIN, listed in Appendix (C), together with subroutine PT1, which represented graphically the experimental values of $\frac{U}{U_r}$ against $\frac{y}{\nu}$ in semi-logarithmic coordinates, together with the values calculated from the law of the wall.

Additionally, Ludwieg and Tillmann's [21] empirical relationship was employed to evaluate the local skin friction from the boundary layer integral parameters $\theta$ and $H$

$$c_f = 0.246 (R_{\theta})^{-0.268} (10)^{-0.678H} \quad (2.10)$$

2.3.3. The Flow Two-Dimensionality

During the course of this investigation, three methods were used to check the overall two-dimensionality of the flow. The first, called the two pins technique, uses two ordinary needles, typically 100 mm length and 1 mm diameter, fixed an equal distance from and perpendicular on the wind tunnel centre line in the working section floor at an upstream position (station 1). Fig. (2.6) shows a photograph of the working section containing the two pins. The wake generated by the pins was then traced in the downstream flow. Any lack of flow two-dimensionality could be observed by an angular deviation of the wake centre line formed by each pin. The sensitivity of this technique is high, a deviation of 0.1 degree being easily determined.
The second method used was to test the overall constancy of the experimental results in satisfying the two dimensional momentum integral equation

\[ \frac{d(\nu \bar{u}_e^2)}{dx} + \left( \frac{1}{2} \right) \frac{d(\delta \nabla_x^2)}{dx} = \frac{\tau_\omega}{\rho} \]  

(2.11).

This test was carried out by normalizing and integrating the equation in the downstream direction over the region of interest from \( x = x_0 \) to \( x = x \), where subscript 0 denotes the initial condition. The result of this integration is

\[ \left( \frac{\bar{u}_e^2}{\bar{u}_{e0}^2} \right) - 1 \left( \frac{2}{\theta_0} \right) \int_{x_0}^{x} \left( \frac{\delta \nabla_x^2}{\bar{u}_e^2} \right) dx = \int_{x_0}^{x} \left( \frac{\bar{u}_T^2}{\bar{u}_{e0}^2} \right) dx \]  

(2.12)

\[ P_L = P_R \]

Newman [26] cautioned that the integral momentum equation is not a very precise test for boundary layer development in strong adverse pressure gradients because of the absence of normal stress terms. As an extension to this method, an empirical relation which was first suggested by East, Sawyer and Nash [27], and which had been shown to agree with the published data both of Schubauer and Klebanoff [28] and of Bradshaw [29], was used to estimate these missing normal stress terms. Data together with the suggested relationship is shown in Fig. (2.7). The relationship takes the form

\[ \left( \frac{1}{\theta} \right) \int_{0}^{\delta} \left( \frac{\bar{u}_T - \bar{v}_T}{\bar{u}_e^2} \right) dx = 0.072 J \]  

(2.13)

where \( J = (H-1)/H \), \( H \) being the boundary layer shape parameter.

When this relationship is introduced to equation (2.12), the right hand side of the equation becomes
The computer program TEST 2D, listed in Appendix (C), calculates individually each term on both the left and the right hand sides of equation (2.12), both with and without the normal stress terms by applying the modified Simpson's rule to the unsmoothed data. The left hand side is abbreviated to PL, the right hand side to PR and PR denotes the presence of normal stress terms. Subroutine PLOT, compares the two sides of the equation graphically.

Flow two-dimensionality can also be investigated by observing the degree of spanwise uniformity of the pressure distributions across the wind tunnel working section.

2.4. Turbulent Equilibrium Boundary Layers

2.4.1. Introduction

Turbulent boundary layers for which the velocity defect profiles at various positions in the streamwise direction $x$ are similar in shape and differ only by a scale factor in the velocity and in the distance from the wall $y$ (analogous to a family of laminar boundary layers with similar profiles) are known as equilibrium turbulent boundary layers. These kinds of boundary layers have been widely discussed by many investigators from the theoretical point of view because of their comparative mathematical simplicity, and this simplicity can be utilized as a basis for efficient prediction calculations for general flows, because a streamwise direction term resulting from non-equilibrium in the flow does not arise. Experimentally, the

\[
\frac{P_R}{x_0} = \int \left( \frac{U_r^2}{U_{e0}^2} \right) d \left( \frac{x}{\theta_{e0}} \right) + (0.072) \left( \frac{U_r^2 \theta}{U_{e0}^2 \theta_{e0}} - 1 \right)
\]  

(2.14)
advantage of doing measurements in equilibrium boundary layers is that because of the velocity defect profile similarity measurement need only be made at one station.

Townsend [30] defined equilibrium regions in turbulent flow past rigid boundaries as those regions close to the wall in which the local rates of energy production and dissipation are so large that aspects of the turbulent motion concerned with these processes are determined almost solely by the distribution of shear stress within the boundary layer and are independent of conditions outside of it. He called these regions equilibrium layers because of the equilibrium existing between local rates of energy production and dissipation.

Equilibrium flows are here defined as flows in which the velocity defect profiles in the boundary layers do not vary with streamwise direction x.

2.4.2. Equilibrium Parameters

In 1954 Clauser [6] succeeded in verifying experimentally the existence of boundary layers in adverse pressure gradients, which had velocity defects of similar shape. He associated these flows with a free-stream pressure distribution characterised by the parameter $\beta$, the Clauser equilibrium parameter, defined as

$$\beta = \left( \delta^*/\tau_\omega \right) \left( \frac{dP}{dx} \right) = \left( \frac{2\delta^*}{c_f} \right) \frac{dU}{dx} = \text{constant} \quad (2.15)$$

Although boundary layer thickness, $\delta^*$, is a convenient length scale for comparison of data, there is an uncertainty in determining its value. Instead, Clauser introduced the boundary layer defect thickness $\Delta$

$$\Delta = \int_0^\infty \left( \frac{U_e - U}{U} \right) \frac{dy}{\left( \frac{c_f}{2} \frac{\sqrt{c_f}}{2} \right)} = \delta^*$$

(2.16)
By analogy with the shape parameter, $H$, he introduced the defect shape parameter, $G$. For the universality of these boundary layer profiles $G$ must be constant throughout the region of interest

$$ G = \left[ \int_0^\infty \left( \frac{U_e - U}{U} \right)^2 \, dy \right] / \Delta . $$

(2.17)

This defect shape parameter can be related to the conventional shape parameter $H$, according to the following relationship:

$$ H = \frac{\delta^*}{\delta} = \left[ 1 - \left( \frac{G}{2} \right) \right]^{-1} . $$

(2.18)

It should be noted that, while $G$ is a constant for a given equilibrium layer, $H$ depends strongly on $c_f$.

Various useful empirical correlations of $\beta$ as a function of $G$ or of the wake parameter, $\Pi$ have been proposed. The equilibrium locus relation reported by Green, Weeks and Brooman [31], which relates $G$ to $\beta$

$$ G = 6.432(1 + 0.8 \beta)^{0.5} $$

(2.19)
together with White's [32] empirical correlation of Coles wake parameter $\Pi$ and Clauser's equilibrium parameter, $\beta$, for which he considered the thirteen near equilibrium flows prepared for the 1968 Stanford boundary layer conference are used where necessary.

$$ \Pi = 0.8(\beta + 0.5)^{0.75} . $$

(2.20)

Fig. (2.8) shows the velocity defect profiles measured by Clauser [6] for two different experimental pressure distributions corresponding to two equilibrium boundary layers together with the velocity defect profile for zero pressure gradient.

Townsend [33] and Mellor and Gibson [34] have shown that approximate equilibrium is obtained if

$$ U_e \propto x^n $$

(2.21)
where \( m \) is a flow parameter equal to \( \left( \frac{x}{U_e} \right) \frac{dU_e}{dx} \).

In Fig. (2.9) East, Smith and Merryman [35] show the variation of the shape parameter, \( H \), with the flow parameter \( m \), for equilibrium flows in adverse pressure gradients with values of momentum thickness Reynolds numbers \( R_\theta \), from \( 10^3 \) to \( 10^6 \). Flows in favourable pressure gradients are not shown since \( H \) changes very little over the range \( 0 < m < \infty \).

The main feature of this graph is that for values of \( m \) near \(-0.25\) there is a wide range of values of \( H \) for which the flow is near equilibrium. For any particular value of Reynolds number \( R_\theta \) there is a minimum value of \( m \) of approximately \(-0.25\). For higher values of \( H \), \( m \) increases and tends to infinity, \( m \) tends to zero.

In the present work no distinction has been made between equilibrium boundary layers in which \( U_e \propto x^m \), and equilibrium boundary layers in which \( \beta = \left( \delta^*/\nu \right) (dp/dx) \) = constant, because Bradshaw [29] has shown that the difference between these two definitions of equilibrium is small and likely to be within experimental error.

### 2.4.3. Equilibrium Locus

East, Smith and Merryman [35] have pointed out that Clauser's equilibrium parameters \( G \) and \( \beta \) are unsuitable for separated flows where \( c_f \) may tend to zero or even be negative, and that equivalent, but more general parameters are an equilibrium friction parameter, \( E_f \), defined by

\[
E_f = G^{-2} = c_f/2J^2
\]

(2.22)

where \( J = \) equilibrium shape parameter = \((H-1)/H\), together with an equilibrium pressure gradient, \( E_p \), defined by

\[
E_p = \beta /G^2 = \left( \frac{-\delta^*/J^2U_e}{U_e} \right) \left( \frac{dU_e}{dx} \right)
\]

(2.23)
In these new coordinates East, Sawyer and Nash [27] point out that an empirical equilibrium locus proposed by Green, Week, and Brooman [31] takes the following simple linear relationship:

\[ E_f = 0.024 - 0.8 \frac{E}{P} \]  \hspace{1cm} (2.24)

2.4.4. Equilibrium Conditions

Rotta [19] in his theoretical analysis has showed that turbulent boundary layers are in equilibrium only if the skin friction coefficient \( c_f \) is constant, and if the boundary defect thickness \( \Delta \) is linearly proportional to \( x \). These conditions are in addition to the constancy of Clauser's pressure parameter, \( \beta \). In consequence, the shape parameter \( H \), which is a function of \( c_f \) (equation 2.10) must be constant, and from the momentum integral equation, which may be expressed as

\[ \frac{d\theta}{dx} = \frac{c_f}{2} - \frac{\beta \cdot U_e}{U_e} \frac{dU_e}{dx} \] \hspace{1cm} (2.25)

or

\[ \frac{d\theta}{dx} = \frac{c_f}{2} \left[ 1 + \beta(1 + \frac{2}{H}) \right] = \text{constant} \] \hspace{1cm} (2.26)

the momentum thickness must grow linearly in the streamwise direction \( x \).

In spite of Rotta's [19] observation that such flows are strictly possible only on surfaces with an appropriate streamwise distribution of roughness, in practice flows which are acceptably close to equilibrium can be obtained for two-dimensional turbulent layers in incompressible flow if they satisfy the following conditions:

(i) The shape parameter \( H \) must be either constant or must decrease very slowly as the Reynolds number, based on momentum thickness \( R_\theta \), increases.

(ii) The momentum thickness must grow linearly in the streamwise direction \( x \).

(iii) The freestream velocity distribution must take the form \( U_e \propto x^m \).
It is necessary for the virtual origin of $x$ in conditions (ii) and (iii) to be at the same point, i.e. $x = x - x_o$, where the coordinate $x$ is measured from the start of the working section and $x_o$ is the effective start of the equilibrium region and the virtual origin of $x$. The value of $x_o$ can be determined from the point at which the momentum thickness can be assumed to grow linearly.

2.4.5. The Downstream Stability of Equilibrium Boundary Layers

One of the important results of Clauser's [6,7] investigation is that, with more adverse pressure gradients the development of an equilibrium layer becomes more difficult to establish experimentally, because the flow tends to instability, i.e. small changes in initial conditions may lead to large changes in the subsequent history of the layer. This type of instability was observed theoretically. Townsend [30] in his study of equilibrium boundary layers concluded that for values of $m$ less than $-0.25$, two courses of development appear to be possible with the same external conditions. East, Smith and Merryman [35] show in Fig. (2.9) that for all negative values of $m$ two boundary layer flows are possible, one which is always attached, and the other attached or separated, depending on the value of $m$.

In the current investigation the values of $H$ and $G$ must be determined in order that flow instability, that is the flow with the undesired value of $H$, is prevented.

2.5. Drag Measurement on Single Roughness Elements

In boundary layer flows, the difference measured between the drag of a test plate with and without a roughness element present gives the increase in the drag $D$ caused by the roughness element. This increase consists of two parts. The first part is the form drag of the roughness element itself and the second part arises because the roughness element causes changes to the velocity profile in its neighbourhood and hence affects the wall shear stress.
As outlined in Section (1.3), the need is to define a dimensionless drag coefficient for the roughness element where its drag is divided by a representative area and a suitable dynamic pressure. For two-dimensional elements, frontal area is the obvious choice for the former, but the choice for the latter is less clear. Whilst Wieghardt[10] adopted a mean dynamic pressure, \( q \) averaged over the element,

\[
q = \frac{1}{h} \int_0^h \frac{1}{2} \rho U^2 \, dy \tag{2.27}
\]

Morris [36] and Townsend [37] used the dynamic pressure, \( q_h \), at the height of the element. Both of these evaluations are made in the absence of the element. The merit of these two determinations for the drag coefficient is that they take into account that the shape of the boundary layer velocity profile must affect the drag coefficient. Their disadvantage is that this shape will not necessarily be known in a situation in which it is necessary to predict the roughness element area drag coefficient. For this latter reason Plate [12], Winter and Gaudet [13] and Good and Joubert [17] defined the drag coefficient in terms of the dynamic pressure of the local freestream, \( q_e = \frac{1}{2} \rho U_e^2 \). For roughness elements which are small compared with the local boundary layer thickness the disadvantages of this definition are evident. In this present work, the drag is that caused by the roughness element itself and the drag coefficient \( C_D \) is defined by,

\[ C_D = \frac{D}{\frac{1}{2} \rho U_e^2} \]

2.5.1. The Relation Between \( \Delta D \) and \( \Delta \theta \)

In two-dimensional flow across a control volume surrounding the roughness element the change in momentum thickness, \( \Delta \theta \), measured with and without its presence is partly caused directly by the drag of the roughness element and partly by the change in the skin friction which it causes.

Schlichting [3] showed that the drag per unit width on a flat plate, \( D_x \), can be expressed as
\[ D_x = \rho \int_0^\infty U(U_e - U) \, dy \]
\[ = \rho U_e^2 \theta \]  
(2.28)

Since the roughness element has been assumed to be small, \( x_1 = x_2 \) in Fig. (2.10) and consequently \( U_{e2} = U_{e1} \).

Applying equation (2.28) upstream and downstream of the roughness element gives

\[ \Delta D = \rho U_e^2 \Delta \theta \]  
(2.29)

This expression relates the increase in the drag which is associated with an isolated two-dimensional roughness element on a smooth surface to the resulting momentum thickness increase.

It is independent of the element shape and the only restrictions are that the value of \( \Delta \theta \) must be measured immediately downstream of the element position, and that it must be small enough not to affect the freestream pressure distribution either in its vicinity or downstream of it. It demonstrates the necessity to identify the change in momentum thickness very close to the roughness element but, because of the complex nature of the flow in this region, direct measurement by traversing the velocity profile is, at the best, difficult and, at the worst, meaningless. The alternative is to measure the change in the boundary layer momentum thickness far downstream of the roughness element and then to extrapolate these values upstream to the element position, and that is what has been done here.

A relationship which can be used instead of this upstream extrapolation process was developed by Nash and Bradshaw [18]

\[ \frac{\Delta \theta}{\Delta \theta_0} = \left( \frac{\theta_e}{\theta} \right)^{0.2} \left( \frac{U_{e0}}{U_e} \right)^{4.2} \]  
(2.30)
where \( \Delta \theta, \theta, \) and \( U_e \) are the momentum deficit far downstream of the roughness element, the clean surface momentum thickness and the freestream velocity at the same position respectively. Subscript \( o \) denotes conditions at the roughness element position. The Nash and Bradshaw formula must be used with caution since elementary boundary layer techniques were used in its development.

2.6. Experimental Uncertainty

2.6.1. Introduction

The purpose of this section is to provide some measure of the reliability of the reported results. Here, as in most engineering experiments, because of time and cost requirements repetition of the same experiment with different equipment and observers is not a practical possibility. From the point of view of reliability estimates this kind of experiment is known as a single-sample experiment. Because of the lack of knowledge about the true value of measured quantities only the uncertainty, i.e. the possible value of error, and not the error of experimental results can be estimated.

There are three kinds of errors which might cause experimental uncertainties. First, fixed errors are those which are constant for a given procedure, typically arising from the inherent construction of the observing instrument. Second, random errors are those which vary from one reading to another. Third, human errors are those attributable to mistakes made by the observer rather than by the sampling technique employed.

2.6.2. Uncertainty Estimation Methods

Kline and McClintock's [38] estimation method can be summarized in three steps as follows:

(1) Estimate and record the uncertainty in each of the variables. 'Variable' implies any quantity which is measured directly in the laboratory.
(2) Calculate the uncertainty in the result due to the uncertainty in each of the variables. The expression for $n$ independent variables is,

$$W_R = \sqrt{\left( \frac{\delta R}{\delta V_1} \cdot W_1 \right)^2 + \left( \frac{\delta R}{\delta V_2} \cdot W_2 \right)^2 + \cdots + \left( \frac{\delta R}{\delta V_n} \cdot W_n \right)^2} \quad (2.31)$$

where $W_R$ is the uncertainty in the result, while $V_1, V_2, \ldots, V_n$ and $W_1, W_2, \ldots, W_n$ are $n$ variables and the uncertainties in the $n$ variables respectively.

(3) Combine the uncertainties found in step 2 to give the total uncertainty in the result.

If there are intermediate results, then steps 2 and 3 must be repeated to obtain the final result.

When a Pitot tube is traversed across a region, the uncertainty in the obtained data is due to the uncertainty of the Pitot tube reading relative to the static tube.

The uncertainty in the integral

$$R_s = \int_0^y P \, dy \quad (2.32)$$

was expressed by Moffat[39] as

$$W_{R_s} = \left\{ \left[ \frac{y_1 - y_0}{2} \right] w_{P_1} \right\}^2 + \left[ \left( \frac{P_1 + P_0}{2} \right) w_{y_1} \right]^2 + \sum_{n=2}^{N-2} \left[ \left( \frac{y_{n+1} - y_{n-1}}{2} \right) w_{P_n} \right]^2 + \sum_{n=2}^{N-2} \left[ \left( \frac{P_{n+1} - P_{n-1}}{2} \right) w_{P_n} \right]^2 + \left[ \left( \frac{y_N - y_1}{2} \right) w_{P_N} \right]^2 + \left[ \left( \frac{P_N + P_{N-1}}{2} \right) w_{y_N} \right]^2 \right\}^{1/2} \quad (2.33)$$

where $n$ is the number of the traverse points, and the subscript $0$ and $N$ implies the first and the last traverse point respectively. In order to get the above result Moffat had approximated the integral relation by a series of trapezoidal elements which he summed.
The methods thus summarized provide means for describing and analysing the uncertainties in single sample experiments, the only important restriction is that the uncertainties in each of the variables must be independent.

### 2.6.3. Calculations and Uncertainty Considerations

The following list contains the measured quantities, the uncertainty symbol and the estimated uncertainty intervals for the current investigations.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Uncertainty Symbol</th>
<th>Estimated Uncertainty Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a$</td>
<td>$W_T_a$</td>
<td>$\pm 1, \text{K}^\circ$</td>
</tr>
<tr>
<td>$p_a$</td>
<td>$W_{p_a}$</td>
<td>$\pm 1, \text{mm of mercury}$</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>$W_{\Delta p}$</td>
<td>$\pm 2%$</td>
</tr>
<tr>
<td>$h$</td>
<td>$W_h$</td>
<td>$\pm 0.1, \text{mm}$</td>
</tr>
<tr>
<td>$y$</td>
<td>$W_y$</td>
<td>$\pm 0.5%$</td>
</tr>
</tbody>
</table>

where $T_a$ is the atmospheric temperature measured by a calibrated mercury in glass thermometer, $p_a$ is the atmospheric pressure measured by a Fortin barometer, $h$ is the height of the roughness element, $\Delta p$ is the pressure difference between the Pitot tube pressure and the wall pressure and $y$ is the distance of the pitot tube from the surface. The values of uncertainty in the pressure difference are the summation of uncertainties from the calibration of the pressure transducer against the water manometer and from the calibration of the pressure transducer mean voltage measured by means of the integrating data logger.

Kline and McClintock's method, together with the listed measured values were used to calculate the uncertainty intervals associated with the quantities.
of primary interest. Calculated quantities and typical approximate uncertainty intervals are listed below.

(a) Uncertainty in the measured velocity profile

At each measured point in the velocity profile both Bernoulli's equation and the perfect gas law were employed to calculate the velocity at that position as,

\[
U = \sqrt{\frac{2\Delta p R T}{P a}}
\]  

where \( R \) is the perfect gas constant for dry air. The corresponding uncertainty in the velocity profile ratio, \( \frac{U}{U_e} \) is \( W_u = \pm 1.4\% \).

(b) Uncertainty in the momentum thickness

Since the integral equation for momentum thickness is

\[
\frac{\delta}{\delta} = \int (\frac{U}{U_e}) dy - \int (\frac{U}{U_e})^2 dy
\]

(2.35)

to evaluate the uncertainty in \( \theta \) equation (2.33) was applied to each term of the integrand individually. Applying the uncertainty in \( \frac{U}{U_e} \) and in \( (\frac{U}{U_e})^2 \), together with the uncertainty in \( y \) it was found that \( \frac{\Delta \theta}{\theta} = \pm 3\% \) for all flows except flow 6, where the value has gone up as high as \( \pm 6\% \).

The consequent uncertainty in the difference of the momentum thickness \( \Delta \theta \), \( \frac{\Delta \theta}{\theta} = \pm 4\% \), except for flow 6 when it was \( \pm 8\% \).

(c) Uncertainty in the Drag Coefficient of the Roughness Element

Since \( C_D \) is defined in section (2.5) as

\[
C_D = \frac{2\Delta \theta}{h}
\]

(2.36)

* Flow 1 to 6 will be described in Chapter 3.
by combining the uncertainty of the intermediate result $\frac{\Delta \theta}{\theta}$ with the uncertainty in the measuring of the roughness element height the uncertainty in the drag coefficient is seen to be between $\pm 4\%$ to $\pm 5\%$ for flows 1 to 5 and between $\pm 8\%$ to $\pm 9\%$ for flow 6 depending on the height of the roughness element. These values will be used as an intermediate result to calculate the $C_D/c_f$ ratio.

Coles and Hirst [25] pointed out that the difference between friction coefficients obtained by fitting the mean velocity profile to the law of the wall (which is the case here), and that obtained by other means is unlikely to be more than one to two per cent. Hence the uncertainty in $C_D/c_f$ ratio becomes $\pm 4\%$ to $\pm 5\%$ except for flow 6 when it was $\pm 8\%$ to $\pm 9\%$, dependent on the height of the roughness element.

The above results are summarised in the following Table.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Per cent Uncertainty</th>
<th>Per cent Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>flows 1 to 5</td>
<td>flow 6</td>
</tr>
<tr>
<td>$U/U_e$</td>
<td>$\pm 1.4$</td>
<td>$\pm 1.4$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\pm 3$</td>
<td>$\pm 6$</td>
</tr>
<tr>
<td>$C_D$</td>
<td>$\pm 4 - 5$</td>
<td>$\pm 8 - 9$</td>
</tr>
<tr>
<td>$C_D/c_f$</td>
<td>$\pm 4 - 5$</td>
<td>$\pm 8 - 9$</td>
</tr>
</tbody>
</table>
FIG(2.1) Photograph of wind tunnel and working section.
Fig.(2:2) Boundary layer wind tunnel

ALL DIMENSIONS IN mm.
FIG (2.3) The traversing unit
Fig. (2.4) FLOW CHART OF DATA EVALUATION CYCLE
Fig. (2.5) Regions of a turbulent boundary layer. Outer part profile shown is for $U_e =$ constant.
FIG. (2-6) Photograph of working section setting for flow 6 containing the two pins.
Fig. (2.7) Normal stress integrals in low speed boundary layers

$$J = \frac{H-1}{H}$$
Fig. (2.8) Equilibrium defect velocity profiles for $\beta \geq 0$
from Clouser [6]
Fig. 29 The shape parameter for equilibrium turbulent boundary layers

Eost, Smith and Merryman [35]
Fig. (2-10) Conditions before and after a small roughness element and general behaviour of a reattaching flow.

\[ U_{e2} = U_{e1} \]
\[ X_2 = X_1 \]
\[ \theta_2 = \theta + \Delta \theta \]
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 3

EXPERIMENTAL RESULTS
3. EXPERIMENTAL RESULTS

3.1. Flow Through the Working Section in the Absence of Roughness Elements

3.1.1. Setting up the Equilibrium Flows

In the current investigation the roof of the tunnel has been adjusted to give six different equilibrium flows as shown in Fig. (3.1). In the first of these the pressure gradient was zero and in the remaining flows adverse. It is a characteristic of equilibrium boundary layers that for the same value of flow parameter \( m \) in the expression \( U_e \propto x^m \), corresponding to the adversity of the pressure gradient, two flows which have different values of shape parameters and rates of growth are possible [see Section (2.4.2) and Fig. (2.9)]. Thus in order to distinguish between flows, a designation number rather than the flow parameter will be given.

Flows 1 to 5 were initially developed from a constant pressure region upstream. With flow 6, the constraints of height and usable length of the working section made it necessary to contract the upstream section. The gaps between the flexible roof and the side walls were completely sealed for flows 1 to 5 but with flow 6 for stability reasons, (see section 2.4.5), the gaps were not sealed.

The actual shape of the flexible roof was calculated from a one-dimensional analysis such that in the absence of boundary layers

\[
U_e \propto x^{m_0}
\]

by selecting different values of \( m_0 \) \((0.0, -0.16, -0.25, -0.38, -0.42, -0.5)\). This method was also used by East, Sawyer and Nash [27]. Measured values of the flow parameter \( m \) differed considerably from the prescribed \( m_0 \) but, as will be shown later, this method of setting up gave at least 1.5 metres of the required equilibrium flow in downstream region of the 3.5 metres long working section.

In Fig. (3.2) the measured freestream velocity distributions are
plotted. Also shown in Fig. (3.2) are the fitted curves of the form

\[ U_e \propto (X - X_o)^m \propto x^m \]  

where the co-ordinate \( X \) is measured from the start of the working section, and \( X_o \) is the effective start of the equilibrium flow. The values of \( m \) were obtained by fitting a straight line to plots \( \log(U_e/U_{Ref}) \), \( U_{Ref} \) being the reference freestream velocity, against \( \log(X - X_o) \) using the value of \( X_o \) determined from the growth of the momentum thickness, \( \theta \), shown in Fig. (3.3). For all flows the actual values of the flow parameter \( m \) which were obtained were 0.0, -0.16, -0.19, -0.25, -0.25 and -0.23 respectively. Fig. (3.3) demonstrates that in all cases when \( X \) exceeded 1.5 metres, the growth of the momentum thickness with \( X \) was sensibly linear.

3.1.2. Two-Dimensionality of the Flows

The two-dimensionality of these flows was initially investigated using the two-pins technique, as described in Section (2.3.3). When enough degree of two-dimensionality was evident[Fig. (3.4)], then checks of overall consistency of the results in satisfying the two-dimensional momentum integral equation with and without suggested normal stress terms were made. Results, which were encouraging, are shown in Fig. (3.5). These compare well with East, Sawyer and Nash [40]. Since the two-dimensional momentum integral balance method is not a very precise test when pressure gradients are high, within the equilibrium region the two-dimensionality of flows 5 and 6 were investigated separately, using the two-pins in that region. The object of this exercise was to obtain a spanwise uniformity of the pressure distribution. However, because of the high turbulence intensity, it proved to be extremely difficult to detect the wake forming behind the pins other than in the outer part of the boundary layer, as shown in Fig. (3.6). These figures also show that the boundary layers from the side walls extended over about one quarter of the wind tunnel cross-section, and this result was unaffected by the
presence of the pins.

3.1.3. Velocity Profiles

Numerical results for the mean velocity distributions are given in Appendix (A). For each flow the profile at \( X = 2829 \text{ mm} \) has been selected as typical of the equilibrium flow as a whole. These profiles, which were deduced from the Htot tube measurements relative to wall static vents, are plotted in Fig. (3.7) and shown in log-linear co-ordinates in Fig. (3.8). The normal co-ordinate, \( y \), is plotted in the non-dimensional form of \( y/x \) in Fig. (3.7), as this illustrates the very great thickness of the layer under the increasingly adverse pressure gradient. The log-linear relationship in Fig. (3.8) illustrates that all profiles exhibit a substantial linear region, with flow 6 following the law of the wall up to \( y/\delta = 0.1 \) and the remainder closely following the law of the wall up to at least \( y/\delta = 0.2 \). Selected velocity profiles are represented on semi-logarithmic axes in Fig. (3.9): here mean velocity has been non-dimensionalised with the local friction velocity, \( U_\tau \), and \( U/U_\tau \) has been expressed as a function of the local friction Reynolds number, \( \frac{y U_\tau}{\nu} \). Values of \( U_\tau \) were calculated by fitting velocity profile points to the law of the wall (see section (2.3.2)). For all flows the outer layer velocity defect profiles are shown in Fig. (3.10) as \( (U - U_e)/U_\tau \) versus \( y/\delta \) at each traversing station within the equilibrium region. These figures demonstrate that for each family of equilibrium flows the velocity defect profiles collapse up to a single curve, and are thus self similar.

3.1.4. Boundary Layer Integral Parameters

The streamwise variation of the momentum thicknesses derived from integrations of the boundary layer velocity profiles is given in Fig. (3.3). This graph demonstrates that the growth of momentum thickness over the region in which equilibrium has been established is approximately linear. The
corresponding values of local skin friction coefficients, $c_f$, shape parameter, $H$, and the defect shape parameter, $G$, are shown in Fig. (3.11), Fig. (3.12) and Fig. (3.13) respectively. The skin friction coefficients, $c_f$, have been estimated by fitting velocity profiles to the law of the wall. In the parts where the freestream velocity distributions follow the equilibrium relationship (3.2), the values of local skin friction coefficient are sensibly constant, while those of the shape parameter and the defect shape parameter remain either constant or decrease slowly as the Reynolds number increases. These graphs show that a considerable length of flow is required to achieve equilibrium conditions, (see section (2.4.4.)), particularly if the value of shape parameter, $H$, is high. It will also be noted from the value of $c_f$ that flow 6 is close to separation. In Appendix (A) the boundary layers characteristics are listed. These show that the agreements between the local skin friction coefficients estimated from the law of the wall and those calculated from the Ludwig and Tillman empirical relationship (2.10) are within $\pm 3\%$.

3.1.5. The Equilibrium Characteristics of the Flows

It is evident that the equilibrium conditions which were discussed in Section (2.4.4.) have been satisfied in the present investigation, see Fig. (3.3), Fig. (3.1), Fig. (3.12) and Fig. (3.13).

From Fig. (3.13) and Appendix (A), it can be seen that for each flow Clauser's defect shape parameter $G$ remains constant to within $\pm 3\%$, while Clauser's pressure equilibrium parameter $\beta$ is constant to within less than $\pm 10\%$ for all flows except flow 6, for which the variation in $\beta$ is as much as $\pm 20\%$. The scatter of the pressure parameter values of $\beta$ and of the wake parameter values $\Pi$ was first recognised by Good and Joubert [17], when they reported that an error of $\pm 1\%$ in $\sqrt{\frac{c_f}{2}}$ produces an error of nearly $\pm 17\%$ in $\Pi$. East, Smith and Merryman, in their analysis of flow
separation, pointed out that Clauser's parameters were unsuitable for separating flow, because under these conditions values of $c_f$ are almost zero or negative. Instead they used the equilibrium locus parameters which were defined in Section (2.4.3). In Fig. (3.14) the present data have been presented in the form of the equilibrium friction parameter, $E_f$ against the equilibrium pressure gradient parameter, $E_p$. A discussion of this figure and the equilibrium locus follows in Section (5.1.1).

3.2. Measurements Behind Single Roughness Elements

3.2.1. Introduction

Experiments were performed mainly with a series of different sizes of two-dimensional square roughness elements, whose heights were very small compared with the boundary layer thickness in which they were placed, thus $h/\delta \ll 1$. The element height ranged from 0.125 inch (3.18 mm) up to 0.5 inch (12.7 mm) in increments of 0.0625 inch (1.59 mm). These roughness elements were placed across the working section in or near the equilibrium region of each flow, hence the size and number of roughness elements which were used with each flow varies with the flow conditions.

3.2.2. Two-Dimensionality of the Flows

The two-pins technique was again used to check the two-dimensionality of the flows in the relaxation region downstream of the roughness elements. As in the case of smooth surface tests the two pins were mounted at traverse station number one, Fig. (2.6), and the wakes were traversed at different stations downstream of the roughness element. In Figs. (3.15) results of the two pins test for flows 2 to 5 presented.

3.2.3. Velocity Profiles Behind the Roughness Elements

The local dynamic pressure distributions in the boundary layer and in
the free stream were determined in exactly the manner as described in
Section (2.3.1.). For all flows, the mean velocity values at each measuring
station downstream of the roughness element are given in Appendix (B) and
eamples from these profiles are plotted in the form of \( \frac{U}{U_e} \) vs. \( y/b \) in
Figs. (3.16), together with the corresponding smooth surface velocity
profile. From these graphs it is seen that the inner part of each velocity
profile quickly relaxes to the usual turbulent boundary layer profile, while
the outer part lags behind the sudden change of the boundary layer condition
caused by flow reattachment. The same velocity profiles are shown in terms
of the non-dimensionalized variable \( \frac{U}{U_f} \) against \( \frac{y}{\delta_f}/\nu \) in Fig. (3.17).

Local friction velocity values were determined in the manner described in
Section (2.8.2.). These graphs demonstrate that the profiles depart uniformly
from the law of the wall as \( y \) increases and recover as \( X \) increases.
This is less evident in strong adverse pressure gradients and also when the
roughness element height is very small compared with the boundary layer
thickness, \( \delta \), when recovery is extremely rapid.

3.2.4. Boundary Layer Integral Parameters

From the measured mean velocity profiles the integral parameters were
calculated using the data reduction program listed in Appendix (B). Results
are shown in Figs. (3.18), (3.19), (3.20): in these figures the streamwise
co-ordinate, \( X \), is plotted in the non-dimensionalised form of \( X/\delta \), where
\( \delta \) is the height of roughness elements. Tabulation of the integral parameters
is given in Appendix (B). In these tables, close agreement between values
of the skin friction coefficient derived from the law of the wall and
calculated from the Ludwieg and Tillman formula (equation 2.10) is evident.
This demonstrates the strong relationship which exists between the surface
shear stress and the integral parameters when the law of the wall relationship
is valid.
3.2.5. Departure from Equilibrium

As in Bradshaw and Wong's [8] early work, Clauser's defect shape parameter, G, has been used to measure the departure of the boundary layers from equilibrium caused by the presence of the roughness element. As shown in Fig. (3.21), in all the cases measured G does not recover to its equilibrium value immediately downstream of the roughness element, but after a considerable distance along the working section. This recovery distance appears to decrease with increasingly adverse pressure gradient. In addition to flow 1 data, Bradshaw and Wong's [8] data and Lacey's data [1] are all plotted in Fig. (3.21a).

3.3. Drag of Square Cross-Section Roughness Elements

The two methods reported in Section (2.5) were employed to determine the drag of the roughness elements. With the Nash and Bradshaw method station 10 Fig. (1.2) was employed as the boundary layer traverse point except for flows 4 and 6. In these latter cases station 9 was used instead, because for flow 4 the free stream velocity distribution far downstream in the working section was not in equilibrium and also because of the rapid increase in boundary layer thickness for flow 6, it is conjectural whether or not the flow is two dimensional. The corresponding boundary layer momentum thickness at the position of the roughness element was determined from that at the traversing station by applying Nash and Bradshaw's equation (2.30) and by extrapolating forwards values of the momentum thickness made at traverse stations.

Data obtained have been analysed in terms of drag coefficient divided by the smooth surface skin friction coefficient, \( \frac{C_D}{C_f} \), and the roughness Reynolds number \( h^+ = \frac{hU_L}{\nu} \). Fig. (3.22) gives the drag coefficients in linear-logarithmic co-ordinates, values of \( C_D \) having been determined by extrapolation from the experimental data. Fig. (3.22) also shows a proposed relationship which fits the data and takes the form
\[ \frac{C_D}{c_f} = A \log_{10} h^+ + D \]  
\text{(3.3)}

where \( A \) and \( D \) are constants. Typical values of \( A \) and \( D \) are 150 and -190 respectively.

Scatter in the data around the suggested relationship is not surprising since the estimated uncertainty in \( \frac{C_D}{c_f} \) may well be as high as \( \pm 9\% \). Within the experimental range of \( h^+ \), equation (3.3) demonstrates the validity of wall similarity and the independence of the drag coefficient of flow history, provided that the boundary layer has developed normally and that the changes in pressure caused by the presence of the roughness element occurred suddenly. The similarity with the law of the wall is discussed in detail in Section (5.3.).

Values of \( \frac{C_D}{c_f} \) calculated from the experimental data and using the Nash and Bradshaw relationship (2.30) are shown in Fig. (3.23). In addition to the experimental uncertainty the high scatter present may well relate to the fact that Nash and Bradshaw [18] based their analysis on a technique which assumed mild adverse pressure gradient flows when calculating momentum thickness. Greater uncertainty should be anticipated from these results which are based on only one traverse station measurement rather than extrapolated values from all traverse stations.

Although both Good and Joubert [17], and Abd Rabbo [2] reported that the value of \( \frac{C_D}{c_f} \) may be pressure gradient dependent, Fig. (3.22) does not substantiate their view. Their belief stems from the high uncertainty of their experimental data since their values of drag are some \( \pm 5\% \) to \( -15\% \) greater than those deduced by other workers.

3.4. Drag of Two-Dimensional Roughness Elements having Different Cross-Sections

Roughness elements of different sectional shape were placed in the equilibrium region in two strong adverse pressure gradient flows (flows 5
and 6). The same techniques were employed to measure the integral parameters and the resulting measurements are tabulated in Appendix (B). In these tables the shape and the size of the elements are clearly defined. Figs. (3.24) show comparisons between experimental data obtained for roughness elements having the same height but different cross-section. For strong adverse pressure gradients, it is evident from these graphs that, provided that the roughness element size is small compared with the thickness of the layer in which it is immersed, it is the height rather than the shape of the roughness element which has the main influence on the boundary layer.

3.5. Flow Visualization

An oil flow technique similar to that reported by Abd Rabbo [2] was used in flow 6 to determine the reattachment length after flow separation around the roughness element. Details of the mixture used are given in the following table.

<table>
<thead>
<tr>
<th>Item</th>
<th>Volumetric per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine grain titanium dioxide</td>
<td>40</td>
</tr>
<tr>
<td>Paraffin</td>
<td>40</td>
</tr>
<tr>
<td>Oleic acid</td>
<td>10</td>
</tr>
<tr>
<td>Silicon oil</td>
<td>10</td>
</tr>
</tbody>
</table>

As in any qualitative observation, the distinction between flow regions is somewhat arbitrary, and a classification similar to that of Abd Rabbo[2] was used to interpret this flow visualization. The reattachment length measured by examining a series of photographs, was found to be 5.5 roughness element heights with an uncertainty of one element height. Kim, Kline and
and Johnston [41] suggest that large, unsteady fluctuations near reattachment occur. This contrasts with Bradshaw and Wong's [8] suggestion that large eddies are split into two in the reattachment zone. The general feature of flow photographs shown in Fig. (3.25) are in good agreement with the results of Abd Rabbo and indicate that when the roughness element is small compared with the boundary layer thickness, reattachment length decreases as the free-stream pressure gradient increases adversely. The reattachment length for flow 6 is 35 per cent less than that for the zero pressure gradient flow reported by Abd Rabbo.

The photographs in Fig. (3.25) show the existence of a considerable region of the three-dimensional spanwise flow spreading from the side walls into the centre of the working section. These three-dimensional regions increase in size as the roughness element height increases.
Curves $U_e \propto (X - X_0)^m$

Fig. (3-2) Free stream velocity distributions for all flows.
Fig.(3.3) Growth of the momentum thickness for all flows
Fig. (3.4a) Pins test for equilibrium flow, stream wise variations of pitot pressure at $y = 60$ mm.
Fig. (3.4b) Pins test for equilibrium flow 2, stream wise variations of pitot pressure at $y = 60$ mm
Fig. (3.4c) Pins test for equilibrium flow 3, stream wise variations of pitot pressure at $y = 60\text{ mm}$
Fig. (3.4.4) Pins test for equilibrium flow 4, stream wise variations of pitot pressure at \( y = 60 \text{mm} \)
Fig. (3.4 e) Pins test for equilibrium flow 5, stream wise variations of pitot pressure at $y = 60$ mm
Fig. (3.4f) Pins test for equilibrium flow at stream wise variations of pitot pressure at $y = 60$ mm
Momentum integral equation balance
Fig. (35d) Flow 3

Momentum integral equation balance
Figure (3.5b) Flow 5

Momentum integral equation balance
Fig. (3.6a) Spanwise variation of pitot pressure across flow 5 at $X = 2620\text{ mm}$, downstream of two pins.
Fig. (3.6 b) Spanwise variation of pitot pressure across flow 5 at $X = 2620\, mm$, downstream of two pins
Fig. (3.7) Equilibrium velocity profiles at $X = 2829$ mm
Fig.(3.8) Logarithmic plots of equilibrium velocity profiles at X=2829 mm
Fig. (3.9) Equilibrium velocity profiles in terms of inner-region variables at $X = 2829\,\text{mm}$. 

For symbols 
See Fig. (3.7)
Fig. (3.10a) Universal plot of turbulent velocity defect profiles for flows 1, 3 and 5.
Fig. (3.10b) Universal plot of turbulent velocity defect profiles for flows 2, 4 and 6.
For symbols
See Fig. (3-7)

Fig. (3-11) Stream wise variation of skin-friction coefficient for flows 1 to 6
Fig. (3.12) Stream wise variation of the shape parameter, $H$, for flows 1 to 6

For symbols
See Fig (3-7)
Fig. (3-13) Stream wise variation of the defect shape parameter, G, for flows 1 to 6

For symbols, see Fig. (3-7)
Fig. (3.14) Equilibrium locus in general coordinates, data for flows 1 to 6 and from other sources.
Figure 3.15a) Pins test for flow 2 with single roughness element at X = 1420 mm spanwise variation of pitot pressure at y = 60 mm
Fig. (3.15 b) Pins test for flow 3 with single roughness element at $X = 1620$ mm. Spanwise variation of pressure at $y = 60$ mm.
Fig (3-15c) Pins test for flow4 with single roughness element at $X=1620\text{mm}$ spanwise variation of pitot pressure at $y=60\text{mm}$
Fig. (315d) Pins test for flow 5 with single roughness element at $X=2020 \text{mm}$ spanwise variation of pressure at $y=60 \text{mm}$
Fig.(3.16a) Mean velocity profiles behind square roughness element at $x = 1781$ mm in Flow 1
Fig. (3.6b) Mean velocity profiles behind square roughness element at $x = 1781$ mm in Flow I
Fig. (3) (d) Mean velocity profiles behind square roughness element at x=1620 mm in Flow 3.
Fig (3.16b) Mean velocity profiles behind square roughness element at $x^*=2020\text{mm}$ in Flow 5
Fig. (3.16f) Mean velocity profiles behind square roughness element at x = 2020 mm in Flow 5
Fig. (317a) Mean velocity profiles behind square roughness element at $X = 1781 \text{ mm}$ in flow 1, in terms of inner-part variables. (--- law of the wall)
Fig. (317b) Mean velocity profiles behind square roughness element at $X = 1781 \text{mm}$ in flow 1, in terms of inner-port variables. (---law of the wall)
Fig. (3.17c) Mean velocity profiles behind square roughness element at $X = 1620$ mm in flow 3, in terms of inner-part variables ($-$ law of the wall)
Fig. (3.17d) Mean velocity profiles behind square roughness element at \( X = 1620 \text{ mm} \) in flow 3, in terms of inner-part variables, (--- law of the wall)
Fig. (3.17e) Mean velocity profiles behind square roughness element at $X = 2020\,\text{mm}$ in flow 5, in terms of inner-part variables (--- law of the wall)
Fig. (3·17 f) Mean velocity profiles behind square roughness element at $X = 2020 \text{mm}$ in flow 5, in term of inner-part variables, $(\cdots$ law of the wall $\cdots)$
Fig. (3.18) Skin friction coefficient versus distance from single square roughness elements
Fig. (3.19) Shape parameter $H$ versus distance from single square roughness elements.
Fig.(3.20a) Growth of momentum thickness behind single square roughness elements for flows 1 and 4
Fig (3.20b) Growth of momentum thicknesses behind single square roughness elements, for flows 3 and 5
Fig. (3.21a,b) Shape defect parameter $G$ versus distance from single roughness element for flows 1 and 3
Fig. (3.21 c,d) Shape parameter G versus distance from single roughness elements for flows 5 and 6
Fig. (3.22) Dependence of $\frac{C_D}{C_f}$ on $h^+$ for square isolated roughness elements. $C_D$ estimated using forward extrapolation.
For symbols [See Fig. (2.23)]

Fig. (3.23) Dependence of $\frac{C_D}{C_f}$ on $h^+$ for square isolated roughness elements using Nash and Bradshaw's equation (2.30)
Fig(3.24a) Boundary layer properties downstream of differently shaped roughness elements in flow 5 with \( h=6.35 \text{mm} \).
Fig(3.24b) Boundary layer properties downstream of differently shaped roughness elements in flow 6 with $h=9.53\mu m$. 
Fig. (3.25) Flow visualization photographs in flow 6 for single roughness elements.

<table>
<thead>
<tr>
<th>PHOTOGRAPH</th>
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<td>5</td>
<td>6</td>
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<tr>
<td>b</td>
<td>1</td>
<td>1</td>
<td>45</td>
<td>5</td>
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</table>
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 4

BOUNDARY LAYER PREDICTION METHODS
AND THE CONSEQUENCES OF PREDICTION
4. BOUNDARY LAYER PREDICTION METHODS AND THE CONSEQUENCES OF PREDICTION

4.1. Methods of Turbulent Boundary Layer Prediction

Because of the complex nature of the differential equations which describe the behaviour of the turbulent boundary layer, exact analysis can only be made for certain limiting cases. Such methods of analysis can be divided into two categories. The first category is of integral methods, averaged across the boundary layer, and the second of differential methods which seek to solve the full partial differential equations of the boundary layer. At the Stanford Conference [42] held in 1968 a total of twenty-nine different turbulent boundary layer prediction methods were tested. Twenty of these methods were integral.

Although integral methods were developed to provide quick but reasonably accurate predictions of boundary layer growth, the Conference results failed to show that there was any one method which was uniquely successful. In particular, they did not show that differential methods were significantly more, or less, accurate than integral methods.

Most integral methods employed the von Karman integral relationship,

$$\frac{d \theta}{dx} + (H + 2) \frac{\theta}{U_e} \frac{dU_e}{dx} = \frac{c_f}{2}$$

(4.1)

but auxiliary equations must be used with this relationship because it contains three significant variables \( \theta \), \( H \), and \( c_f \).

Thus at least two additional independent relations are needed to provide closure. These two new relations will usually contain a fourth variable, and hence another auxiliary equation is required. If this were to uncover still another new parameter the process must continue. Possible auxiliary equations are:
1) The law of the wall
2) An empirical skin friction correlation formula
3) The turbulent energy integral equation
4) Polynomial or exponential velocity profile approximation
5) An entrainment integral equation

The last relationship is discussed in greater detail in the following sections.

4.2. The Entrainment Integral Relationship

A relationship is derived from the assumption that turbulent boundary layers grow by a process of entrainment from the freestream into the boundary layer of non-turbulent fluid at the outer edge of the layer. This notion was first proposed by Head in 1958 [43] but at that time it was not evident that the entrainment concept had any particular merit other than simplicity and a certain physical appeal.

4.2.1. Head's Original Entrainment Method

In his analysis Head assumed that entrainment into the boundary layer should depend on the velocity defect in the outer part of the layer and be independent of viscosity. To specify this Head defined the entrainment shape parameter as

$$ H_1 = \frac{\delta - \delta^*}{\delta} \quad (4.2) $$

In the boundary layer, the volume flow rate per unit span \( Q \), between \( y = 0 \) and \( y = \delta \) is given by

$$ Q = \int_0^\delta Udy = U_e \left[ \int_0^\delta dy - \int_0^\delta \left( 1 - \frac{U}{U_e} \right) dy \right] $$

$$ = U_e (\delta - \delta^*) \quad (4.3) $$
The entrainment velocity \( V_E \), defined as the component of velocity normal to the edge of the boundary layer, is the rate at which the volume rate per unit length changes with \( x \), so that, using the foregoing equation

\[
\frac{V_E}{U_e} = \frac{d}{dx} \left[ U_e (\delta - \delta^*) \right]
\]  
(4.4)

The non-dimensional form of entrainment velocity \( \frac{V_E}{U_e} \) is known as entrainment coefficient parameter and may be written

\[
C_E = \frac{1}{U_e} \cdot \frac{d}{dx} \left[ U_e (\delta - \delta^*) \right] = f(H_1)
\]  
(4.5)

In addition, to relate \( H \) and \( H_1 \), Head suggested that

\[
H_1 = g(H)
\]  
(4.6)

The functions \( f \) and \( g \) were obtained from an analysis of the experimental data obtained by both Newman [26] and Schubauer and Klebanoff [28]. The corresponding fitting relationships were

\[
f(H_1) = 0.0306 (H_1 - 3)^{-0.653} 
\]  
(4.7)

and

\[
g(H) = H_1 = 1.535 (H - 0.7)^{-2.715} + 3.3 
\]  
(4.8)

These relationships combined with the von Karman integral equation (4.1) and the Ludweg and Tillmann skin friction formula (2.10) give what is known as Head's integral analysis of turbulent boundary layers. Although Head's analysis has failed to give good predictions for equilibrium boundary layers, it proposed a significant new approach to the turbulent boundary layer prediction problems.

### 4.3. Lag-Entrainment Prediction Method

Since 1958 many attempts have been carried out to improve Head's
entrainment method, making it more flexible and giving better agreement with experiment. Green, Weeks and Brooman [31] developed an integral method for predicting the behaviour of turbulent boundary layers in two-dimensional and axisymmetric, incompressible and compressible flows. Their method was a significant improvement upon Head's original entrainment method. Their technique, widely described as the lag-entrainment integral method, uses three differential equations; the von Karman integral equation, the entrainment equation and a rate equation for the entrainment coefficient, $C_E$.

The method is derived primarily from a model of the boundary layer turbulence structure proposed by Bradshaw et al [44] and from the analysis of equilibrium boundary layers by Mellor and Gibson [54] so that the method is rapid but also accurate. Explicit analytic approximations have been obtained to represent the flat plate skin friction relation. Allowances for the effects of flow divergence and for longitudinal curvature on the turbulence structure as well as for the change of turbulence structure in a wake have been introduced in a simple and logical form. The effect of lack of two-dimensionality on the development of momentum and displacement thicknesses has also been taken into account to provide a firmer base for comparison between prediction and experiment. The extension of the method to compressible flow is based on the same arguments as in Bradshaw and Ferriss [45], who stated that turbulence structure is essentially unaffected by compressibility. An empirical factor to account for the variation of the integral parameters with Mach number in equilibrium flow is also employed.

In the present investigation the lag-entrainment method was compared with the experimental results described in Section (4.4), as well as with the proposed prediction of the boundary layer growth in the presence of small roughness elements.

4.3.1. Summary of the Lag-Entrainment Method

The following summary of the method is taken from Green, Weeks & Brooman[31].
Definition of the main parameters

In compressible flow, the parameters which occur in the present method are defined as: boundary layer thickness \( \delta = y \) at \( \frac{U}{U_e} \sim 0.995 \), but with the precise value depending on Mach number and Reynolds number.

Displacement thickness
\[
\delta^* = \int_0^\infty \left( 1 - \frac{\rho \frac{U}{U_e}}{\rho_e e} \right) dy
\]  
\[\text{(4.8)}\]

Momentum thickness
\[
\delta = \int_0^\infty \frac{\rho \frac{U}{U_e}}{\rho_e e} \left( 1 - \frac{U}{U_e} \right) dy
\]  
\[\text{(4.9)}\]

Shape parameters
\[
H = \frac{\delta^*}{\delta}
\]
\[
\bar{H} = \frac{1}{\delta} \int_0^\infty \frac{\rho}{\rho_e e} \left( 1 - \frac{U}{U_e} \right) dy
\]  
\[\text{(4.10)}\]

\[
H_1 = \frac{1}{\delta} \int_0^\infty \frac{\rho}{\rho_e e} dy
\]
\[
\text{= } \frac{\delta - \delta^*}{\delta}
\]  
\[\text{(4.11)}\]

Skin-friction coefficient
\[
c_f = \frac{\tau}{\frac{1}{2} \rho e U_e^2}
\]  
\[\text{(4.12)}\]

Entrainment coefficient
\[
C_E = \frac{1}{\rho e U_e} \frac{d}{dx} \left( r \int_0^\delta \rho U dy \right)
\]
\[
\text{= } \frac{1}{\rho e U_e} \frac{d}{dx} \left( r \rho e U H_1 \right)
\]  
\[\text{(4.13)}\]

Basic Equations

The boundary layer is defined by three independent parameters; momentum thickness \( \delta \), shape parameter \( H \), and entrainment coefficient \( C_E \).

The development of these in a given pressure distribution, with their initial values known, is predicted by the forward integration of three simultaneous ordinary differential equations.
1. The momentum integral equation

\[ \frac{d(r\theta)}{dx} = \frac{rc_f}{2} - \left( H + 2 - M^2 \right) \frac{r\theta}{U_e} \frac{dU_e}{dx} \]  \hspace{1cm} (4.14)

2. The entrainment equation

\[ \theta \frac{dH}{dx} = \theta \frac{dH}{dH_1} \left[ C_E - H\left( \frac{c_f}{2} - (H+1) \frac{\theta}{U_e} \frac{dU_e}{dx} \right) \right] \]  \hspace{1cm} (4.15)

3. The rate equation for entrainment

\[ \theta \frac{dC_E}{dx} = F \left[ \frac{2.8}{H + H_1} \left( C_f \frac{4}{E_{Q_e}} - \sigma C_t \frac{4}{E_{Q_o}} \right) \right] + \left( \frac{\theta}{U_e} \frac{dU_e}{dx} \right) E_Q \]

\[ - \frac{\theta}{U_e} \frac{dU_e}{dx} \left( 1 + 0.075 M^2 \right) \left( \frac{1+0.2 M^2}{(1+0.1 M^2)} \right) \]  \hspace{1cm} (4.16)

where \( r \) is the body radius in axisymmetric flow set to unity for two-dimensional flow, \( \sigma \) is a secondary influence correction term; if extraneous effects are to be ignored \( \sigma \) may be set equal to 1.0. The \( x \) co-ordinate along the surface, the subscript \( E_Q \) denotes equilibrium conditions, \( E_{Q_o} \) denoting equilibrium conditions in the absence of secondary influences on turbulent structure. The various dependent variables and functions in the equations are evaluated from the following relationships:

(a) For skin friction coefficient \( c_f \): from the known surface pressure distribution the local freestream properties are evaluated, then

\[ R = \frac{\rho U_e}{\mu_e} \theta \]  \hspace{1cm} (4.17)

\[ F_C = \left( 1 + 0.2 M^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (4.18)

\[ F_R = 1 + 0.056 M^2 \]  \hspace{1cm} (4.19)
\[ F_c \frac{c_f}{c_{f0}} = \frac{0.01013}{\log_{10}(F_c R_\theta^{0.3}) - 1.02} - 0.00075 \quad (4.20) \]

\[ 1 - \frac{1}{H_o} = 6.55 \left( \frac{c_f}{2} \left( 1 + 0.04 M^2 \right) \right)^{\frac{1}{2}} \quad (4.21) \]

\[ c_f = c_{f0} \left[ 0.9 \left( \frac{H}{H_o} - 0.4 \right)^{-1} - 0.5 \right] \quad (4.22) \]

In these relationships the subscript \( o \) implies zero pressure gradient flow at a corresponding Reynolds number \( R_\theta \).

(b) For shape parameter \( H \):

\[ H = (\bar{H} + 1)(1 + \frac{M^2}{5}) - 1 \quad (4.23) \]

(c) For \( H_1 \) and \( \frac{d\bar{H}}{dH_1} \):

\[ H_1 = 3.15 + \frac{1.72}{\bar{H} - 1} - 0.01(\bar{H} - 1)^2 \quad (4.24) \]

\[ \frac{d\bar{H}}{dH_1} = \frac{(\bar{H} - 1)^2}{1.72 + 0.02(\bar{H} - 1)^3} \quad (4.25) \]

(d) For \( C_T \) and \( F \):

\[ C_T = (0.024 C_E + 1.2 C_E^2 + 0.32 c_{f0}) (1 + 0.1 M^2) \quad (4.26) \]

\[ F = \frac{(0.02 C_E + C_E^2 + 0.8 c_{f0}/3)}{(0.01 + C_E)} \quad (4.27) \]
(e) For equilibrium quantities:

\[
\left( \frac{\theta}{U_e} \right) \frac{dU_e}{dx} \bigg|_{E_Q} = \frac{1.25}{H} \left\{ \frac{c_f}{2} - \left( \frac{H - 1}{6.432H} \right)^2 (1 + 0.04 M^2) \right\}
\]

(4.28)

\[
(c_e')_{E_Q} = H_1 \left\{ \frac{c_f}{2} - (H + 1) \left( \frac{\theta}{U_e} \right) \frac{dU_e}{dx} \bigg|_{E_Q} \right\}
\]

(4.29)

and

\[
(c_T')_{E_Q} = \left( 0.024 (c_e')_{E_Q} + 1.2 (c_e')^2_{E_Q} + 0.32 c_f \right) (1 + 0.1 M^2)
\]

(4.30)

Defining

\[
c = (c_T')_{E_Q} (1 + 0.1 M^2)^{-1} c_f - 0.32 c_f
\]

(4.31)

\[
(c_e')_{E_Q} = \left( \frac{c}{1.2 + 0.0001} \right)^2 - 0.01
\]

whence

\[
\left( \frac{\theta}{U_e} \right) \frac{dU_e}{dx} \bigg|_{E_Q} = \left( \frac{c_f}{2} - (c_e')_{E_Q} / H_1 \right) / (H + 1)
\]

(4.32)

These equations, arranged in a subroutine in the computer program in the order presented here, provide the dependent variables needed to evaluate the basic equations (4.14), (4.15) and (4.16) at each stage of the numerical integration.

The Computer Program

On the University of Leicester computer some preliminary technical adjustments were necessary before using the original lag-entrainment Fortran program. The program consists of a driver and ten subroutines.

To specify the problem, freestream stagnation properties $P_a$ and $T_a$ and streamwise distributions of $U_e(x)$ and $r(x)$, or some equivalent information,
must be given. Sometimes longitudinal curvature of the surface \( R(X) \) will also be required. Initial values of \( \theta \) and \( H \) are also needed. In the program the initial value of entrainment is usually assumed to be an equilibrium value or its value obtained by specifying an initial value of \( \frac{dH}{dx} \). A facility is available for the insertion of the measured value of \( R_\theta(X) \). An additional subroutine makes it possible to compare graphically the experimental results with the corresponding prediction.

4.4. Comparison of the Experimental Results with Prediction

4.4.1. In the Absence of Roughness Elements

Solid lines in Figs. (4.1a to 4.1c) show predictions by the lag-entrainment method of a sample of test cases from the Stanford Conference [42]. In Fig. (4.1a), the equilibrium retarded flow studied by Bradshaw is shown. The method is seen to be in good agreement with experiments. In Fig. (4.1b) the severely retarded non-equilibrium flow of Schubauer and Spangenberg is given. This flow was chosen for comparison purposes: the severity of the pressure gradient increases with \( x \), and this typically occurs on the upper surface of a lifting aerofoil. The final test case from the Stanford Conference shown in Fig. (4.1c), is the nominally axisymmetric boundary layer studied by Moses. This boundary layer is brought close to separation by a severe adverse pressure gradient which then abruptly falls to zero, allowing the boundary layer to relax back to its flat plate condition. These comparisons confirm earlier claims by Green, Weeks and Brooman [31] that predictions by lag-entrainment are fully comparable in accuracy with the best of the methods assessed at the Stanford Conference in 1968.

In Figs. (4.2a to 4.2f) the experimental data for the present equilibrium flows were also compared with predictions by the lag-entrainment method. The agreement between prediction and experiments is very satisfactory.
in the case of flow 1 to 4, as shown in Figs. (4.2a to 4.2d). For flow 5 Fig. (4.2e), the prediction method is seen to be more satisfactory for $c_f$ and $H$ than it is for $R_e$. Finally, comparison for flow 6 is shown in Fig. (4.2f); the discrepancy here between prediction and experiment is significant. This may well be accounted for by the departure from two-dimensionality revealed by the substantial momentum imbalance in data shown in Fig. (3.5f).

4.4.2. In the Presence of Roughness Element

The main object of this section is to demonstrate modifications which if introduced into the lag-entrainment prediction method enable it to deal satisfactorily with the effects of the presence of a small roughness element.

In the course of this approach it is necessary to specify both the shape parameter developed downstream of the element, $H_2$, and the increase, due to its presence, in the momentum thickness $\Delta \theta$ at the roughness element position. Since direct measurements in the recirculation region behind the roughness element are far from easy, the necessary boundary layer parameters were developed semi-empirically.

The Increase in Momentum Thickness, $\Delta \theta$:

This parameter can be identified either directly by extrapolating measured values of momentum thickness from downstream up to the roughness element position, or, in the case of small square roughness elements, by employing the linear-logarithmic correlation relationship (equation 3.3) discussed in Section (5.3). In this case for the clear surface, $c_f$, $\nu$, $U_T$ are known but the height of the roughness element must be specified.

The Shape Parameter Developed Downstream of Element, $H_2$:

To estimate the growth of the shape parameter consequent to the element, Bradshaw [46] suggested two possible methods, both of which were based on the assumption that the mass flow thickness, $\delta - \delta^* = \int_0^\infty \left( \frac{U}{U_e} \right) dy$, is the same at the roughness element position, without and with its presence,
i.e. \( \delta - \delta^* = \text{constant} \).

The flow parameters without and with the presence of the roughness element are denoted by subscripts 1 and 2 respectively.

In the first method in addition to the mass flow thickness assumption a power-law velocity profile is used;

\[
\frac{U}{U_e} = \left( \frac{y}{\delta} \right)^n
\]

(4.33)

where \( n = \frac{H-1}{2} \)

which produces

\[
\frac{\delta^*}{\delta} = \frac{H-1}{H+1}
\]

(4.34)

\[
\frac{\theta}{\delta} = \frac{H-1}{H(H+1)}
\]

(4.35)

If \( \delta - \delta^* = \delta_2 - \delta_1 \)

then

\[
\frac{H_1 \theta_1}{H_1-1} = \frac{H_2 \theta_2}{H_2-1}
\]

or

\[
H_2 = 1 + \frac{(H_1 - 1)}{H_1 \left[ \left( \frac{\theta_1}{\theta_2} \right) - 1 \right] + 1}
\]

(4.37)

where \( \theta_2 = \theta_1 + \Delta \theta \)

This simple relationship can be used directly in the lag-entrainment method, since \( \theta_1 \) and \( H_1 \), are known and \( \Delta \theta \) can be obtained by one of the methods previously mentioned.

In the second method Coles' two-parameter velocity profile, equation (2.8), together with Bradshaw's [46] mass flow assumption and additional...
relations are found to be needed to provide closure.

Integration of Coles' two-parameter velocity profile gives

\[
\frac{5^*}{\kappa} = S \lambda \quad \quad (4.38)
\]

and

\[
\frac{\theta}{\kappa} = S \lambda - \left[ \frac{2 + 3.179 \Pi + 1.5 \Pi^2}{\kappa^2} \right] \lambda^2 \quad \quad (4.39)
\]

where \( \lambda = \frac{\sqrt{2f}}{2} \), and \( S = \frac{1 + \Pi}{\kappa} \).

In addition, definition of the shape parameter,

\[
H = [1 - \lambda G]^{-1} \quad \quad (4.40)
\]

was employed. Substitution of these equations into equation (4.36) gives

\[
\theta \left[ \frac{1 - \lambda_1 S_1}{\lambda_1 S_1(1 - \lambda_1 G_1)} \right] = \theta \left[ \frac{1 - \lambda_2 S_2}{\lambda_2 S_2(1 - \lambda_2 G_2)} \right] \quad \quad (4.41)
\]

To solve this last equation, two additional relationships are needed. The first is given by eliminating \( \beta \) from equations (2.19) and (2.20). To obtain the second additional relation Bradshaw suggested that the upstream and downstream profiles be assumed to have the same velocity at some small value of \( y/\delta \) such as \( y/\delta = 0.2 \). To give another equation the defect velocity profile

\[
\frac{U e - U}{U} = -\frac{1}{\kappa} \, \xi_n(y/\delta) + \Pi \left[ 1 + \cos \left( \frac{\Pi y}{\delta} \right) \right] \quad \quad (4.42)
\]

was applied before and after the roughness element, these give
\[ \lambda_1 \left[ -\frac{1}{\kappa} \ln \left( \frac{y}{b} \right) + \frac{1}{\kappa} \left( \frac{U_Y}{b} \right) \right] + 1 + \cos \left( \frac{\pi Y}{b} \right) \]  
\[ \lambda_2 \left[ -\frac{1}{\kappa} \ln \left( \frac{y}{b} \right) + \frac{2}{\kappa} \left( 1 + \cos \left( \frac{\pi Y}{b} \right) \right) \right] \]

(4.43)

where \( y/b \) is constant and equal to 0.2 (or some other convenient value).

The evaluation of the corresponding value of the shape parameter, \( H_2 \), from the foregoing analysis is not straightforward. A numerical iteration method was used, with a control on the residual value to ensure numerical accuracy.

A subroutine has been added to the lag-entrainment prediction program to enable it to predict the effect of the presence of single square roughness elements. While the value of \( \Delta \theta \) is calculated from the linear-logarithmic equation (3.3), the corresponding shape factor \( H_2 \) may be calculated by either of the two methods described above. Only the position and the height of the roughness element need to be specified in this method in order to predict its effect on the boundary layer behaviour.

4.4.3. Results of Comparisons with Experiment

This approach has been tested for two-dimensional incompressible flow with both constant pressure and adverse pressure gradients. Fig. (4.3) shows predictions upstream and downstream of a ledge for the boundary layer measured by Tillmann. The method is seen to be in good agreement with experiments, and has superiority over the methods used at the Stanford Conference to predict this flow, since at the Stanford Conference it was necessary to specify experimental values of \( \Delta \theta \) and \( H_2 \) at the ledge position. Comparisons between the prediction and the present data for a zero pressure gradient are shown in Figs. (4.4) in which different sizes of square ridges are defined by \( h^+ \) (where \( h^+ = \frac{hU}{v} \)). Similar comparisons are demonstrated in Figs. (4.5) to Figs. (4.9) for all flows. In case of flow 6, Fig. (4.9),
the lag-entrainment prediction failed to satisfactorily predict the higher value of \(H\), since it predicts separation at \(H/H_0 = 2.2\). So Ludwieg and Tillmann's\[21\] skin friction formula (equation 2.10) relating \(H\) to \(c_f\) and \(R\) was used instead. As remarked previously, the lag-entrainment method does not predict flow well.

In order to test the validity of the method in the equilibrium region with such highly adverse pressure gradients, the prediction of flow was started from traversing station 5\((X = 1781\) mm\). Results are shown in Fig. (4.10).

In all figures described, the solid lines and the broken lines show the results of prediction using the power law and by the two-parameter velocity profile methods to predict \(H_2\) immediately downstream of the element respectively. Where the pressure gradient is not strongly adverse and the effect of a roughness element on flow characteristics downstream of it is of real significance, the two-parameter method gives as good prediction as does the power law method. When the pressure gradient is strongly adverse the two-parameter method is superior. The methods predict the effects of the presence of small elements acceptably well, the quality of the prediction depending on the roughness element size but not on the free-stream pressure distribution.
Fig (4.1a) Comparison of lag entrainment prediction with Bradshaw's data for equilibrium flow $A = -0.255$ reported in Stamford Conference [28] under code (ident 2600)
Fig. (4.1b) Comparison of log-entrainment prediction with Schubauer and Spangenberg for non-equilibrium flow E, reported in Stamford Conference [28] under code (ident 4800)
Fig. (4.1c) Comparison of lag-entrainment prediction with Moses axisymmetric flow 5, reported in Stamford Conference [28] under code (ident 4800)
Fig(4.2a) Comparison of Lag-Entrainment prediction with experiment for equilibrium flow 1.
Fig. (4-2b) Comparison of prediction and experiment for equilibrium flow.
FIG. (4.2c) Comparison of prediction and experiment for equilibrium flow 3.
Fig. (4.2d) Comparison of prediction and experiment for equilibrium Flow 4
FIG. (4.2e) Comparison of prediction and experiment for equilibrium flow (5)
Fig. (4-2f) Comparison of prediction and experiment for equilibrium flow 6
Fig.(4.3) Comparison between prediction and Tillmann ledge flow, reported in Stamford Conference under code (ident 1500)
Fig. (4-4a) Comparison of prediction and experiment for flow 1 with single square ridge $h' = 226$
Fig. (4-4b) Comparison of prediction and experiment for flow 1 with single square ridge $h^* = 678$
Fig. (4.5a) Comparison of prediction and experiment for flow 2 with single roughness element (ht = 205)
Fig. (4.5b) Comparison of prediction and experiment for flow 2 with single ridge (h = 410)
Fig. (4.6a) Comparison of prediction and experiment for flow 3 with single roughness element (ht = 175)
Fig. (4.6b) Comparison of prediction and experiment for flow 3 with single roughness element ($h^+ = 350$)
Fig. (4.7a) Comparison of prediction and experiment for flow 4 with single roughness element (ht = 175)
Fig. (4·7b) Comparison of prediction and experiment for flow 4 with single roughness element ($h^t = 350$)
Fig. (4-8o) Comparison of prediction and experiment for flow 5 with single roughness element (ht = 112)
Fig. (4.8b) Comparison of prediction and experiment for flow 5 with single roughness element (h = 2.25)
Fig. (4.9a) Comparison of prediction and experiment flow 6 with single square ridge $h^t=138$
Equation (3.3)

Roughness element position

Fig.(4.9b) Comparison prediction and experiment for flow 6 with single roughness element
Equation (3-3)

Experiment

Log entrainment with power law equation (4-37)

Log entrainment method with two parameter equation (4-41)

Fig. (4-10) Comparison between prediction and experiment for flow 6 with roughness element (ht = 136) (prediction start downstream at x = 1781)
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 5

DISCUSSION OF RESULTS
5. DISCUSSION OF RESULTS

5.1. Flow in the Absence of Roughness Elements

5.1.1. The Equilibrium Locus

Although the equilibrium flows are defined unambiguously by the equilibrium locus shown in Fig. (3.14) this form of representation does not bring out the more important physical features of the flow.

In East, Smith and Merryman [35], the normal boundary layer parameters were evaluated by obtaining an approximate analytic solution of the momentum integral equation. In this solution they made use of the equilibrium locus of Green, Weeks and Brooman [31], together with the skin friction law (equation 4.22), and the assumption of a constant Reynolds number flow. The results of that analysis have been extracted from their report without modification and are given in Figs. (5.1) to (5.4). Also shown in the figures are both Bradshaw's [29] and East, Sawyer and Nash's [27] experimental data together with the present results. The figures show that the present data fit the prediction closely. The general level of agreement may be taken as indirect evidence that in the present investigation adequate two-dimensional flows have been achieved.

The general agreement between the present experimental data and the equilibrium locus employed by East, Smith and Merryman [35] displays the following dominant features:

(i) For all negative values of flow parameter, \( m \), there are two possible boundary layer flows, one of which is always attached and the other may be either attached or separated, depending on the value of \( m \). As shown in Fig. (5.1) flows 4 and 5 are attached and have the same value of flow parameter, \( m = 0.25 \), but with different values of shape parameter, \( H \), of 1.63 and 1.72 respectively.

(ii) The thickness of the boundary layer increases rapidly with the shape parameter, \( H \). For flow 6, \( H = 2.28 \), in the equilibrium region the momentum thickness, \( \theta \), is eight times that for flat plate flow.
(flow 1, $H = 1.33$). The corresponding growth of the displacement thickness, $\delta^*$, is even more rapid. For the same flow it has reached fourteen times the flat plate flow value.

(iii) The present experimental data accords well with the results of earlier experiments and supports the simple linear equilibrium locus of equation (2.24) though the constants of this expression could be modified to give a better fit to the data. Instead of 0.24 and 0.8, the values 0.225 and 0.75 are proposed respectively. Thus equation (2.24) is replaced by

$$E_\ell = 0.225 - 0.75 \, E_p$$

5.1.2. Expressions Required in Auxiliary Equations for Boundary Layer Prediction Purposes.

In integral calculation methods, as discussed in Section (4.1), it is necessary to use auxiliary equations in addition to the integral equation (4.1). Two possible principles on which to base relationships are entrainment and the kinetic energy in the boundary layer. Using either of these, expressions are required relating the shape factors which arise to the conventional shape factor, $H = \delta^*/\theta$.

As in East, Sawyer and Nash [27], the entrainment shape factor $H_1$, defined as $H_1 = \frac{\delta - \delta^*}{\delta}$, has been evaluated from the present data and is represented together with both Bradshaw's [29] and East, Sawyer and Nash's [27] evaluations in Fig. (5.5). These are all shown to fit closely to the relationship

$$H_1 = \frac{2H}{H-1}$$

(5.1)

which can be derived from the power law velocity profile relationship
\[ \frac{U}{U_e} = \left( \frac{y}{\delta} \right)^n \]  

(5.2)

where \( n = \frac{H-1}{2} \).

The energy shape parameter, \( H_{32} \), defined as the ratio of energy thickness, \( \delta_E \), to the momentum thickness, \( \theta \), \( H_{32} = \frac{\delta_E}{\theta} \), is shown in Fig. (5.6) plotted against \( J \), where \( J = \frac{H-1}{H} \). The line drawn through the experimental points has the equation

\[ \frac{4}{H_{32}} - 2 = J \]  

(5.3)

which again can be obtained from the power law velocity profile relationship (5.2).

5.1.3. The Mixing Length Validity

Calculation of the turbulent boundary layer development relies upon one or more physical assumptions to make the governing equations determinate. Such physical assumptions have tended to take the form of either relatively simple models, describing the distribution of eddy viscosity or mixing length through the layer. The latter model is expressed as

\[ \tau = \rho \xi^2 \left( \frac{\delta U}{\delta y} \right)^2 \]  

(5.4)

where \( \tau \) is boundary layer shear stress and \( \xi \) is the mixing length scale. This scale is widely assumed to be equal to \( k \gamma \) in the wall region, where \( k \) is the von Karman's constant.

The plotting of the experimental results on linear-logarithmic co-ordinates [Fig. (3.8) and Fig. (3.9)] demonstrate that the law of the
wall holds for all flows examined, as discussed in Section (2.2). This
is possible if \[ \frac{U}{\tau} = k y \frac{\delta U}{\delta y}, \]
which implies that,
\[ \tau_\omega = \rho (k y)^2 \left( \frac{\delta U}{\delta y} \right)^2 \]  \hspace{1cm} (5.5)
where \( \tau_\omega \) is the wall shear stress. This result is inconsistent with the
mixing length model unless \( \tau = \tau_\omega \). It is not difficult to show analytically
that, if a universal law of the wall is valid, then the corresponding
mixing length distribution should be given by
\[ f = \kappa y \sqrt{ \frac{\tau}{\tau_\omega} } \]  \hspace{1cm} (5.6)
This relationship was also reported in Galbraith and Head [47]. After
extensively studying mixing length validity Head and Galbraith demonstrated
that the law of the wall is the more universally valid expression.

5.2. Flow over Small Roughness Elements

5.2.1. The Mechanism Controlling the Rattachment Length

The reattachment point downstream of the roughness element was defined
by Eaton and Johnston [48] as the point where the dividing stream returns to
the wall and where the wall skin friction is zero, see Fig. (2.10). To
determine the location of the reattachment point Eaton and Johnston [48]
proposed the following model:
(i) The boundary layer growth rate is controlling the reattachment length.
(ii) The strength of the streamwise adverse pressure gradient in the
reattachment zone must also be considered.

They expected that an increase in boundary layer growth rate corresponds
to a decrease in reattachment length, as first suggested by de Brederode
and Bradshaw [49], while an increase in the adverse pressure gradient causes
an increase in the reattachment length. The model satisfies mass conservation,
as follows. When the boundary layer growth rate is increased, the entrainment rate from the recirculation zone also increases, thus the boundary layer requires less length to entrain the fluid back from the reattachment zone. While, if the adverse pressure gradient is increased in the reattachment zone, more fluid flows back into the recirculating zone, and the boundary layer therefore requires a greater length to entrain the backflow.

The reattachment length $X_R$, obtained for flow 6 from the flow visualization technique together with reattachment lengths determined by Abd Rabbo [2] are shown in Fig. (5.7) in non-dimensional form $X_R/X_{R_0}$, against the corresponding clean surface shape parameter, $H$. $X_{R_0}$ is the reattachment length obtained for zero pressure gradient. This figure demonstrates that, in the case of small roughness elements, the dominating feature in the Eaton and Johnston model must be the boundary layer growth rate.

### 5.2.2. Mean Velocity Profiles

The mean velocity profiles shown in Figs. (3.16) exhibit typical flow after reattachment features. Downstream of the reattachment, the velocity profile returns towards that for the undisturbed turbulent boundary layer. The inner part of the profile adjusts quickly while the outer part requires a larger distance to return. This indicates that the outer part of the boundary layer is more strongly influenced by the larger turbulent eddies, which have a longer life time than small eddies which are present nearer to the surface. The decrease in the velocity gradient in the centre of the layer shown in Figs. (3.16) is due to the rapid increase of the velocity after reattachment and its slower response further away from the surface. The profile deviates from the law of the wall, indicating that the flow is not in local equilibrium.

To explain this decreased velocity Bradshaw and Wong [8], considering
flat plate flow, used the local equilibrium form of the shear stress mixing length formula (Townsend [30]),

$$\frac{\partial U}{\partial y} = \frac{\sqrt{\tau / \rho}}{\epsilon}$$  \hspace{1cm} (5.7)

where $\epsilon = k \gamma$ in the wall region.

In the region $y/6 < 0.2$ this gives a higher velocity gradient than the value consistent with the law of the wall, $\sqrt{\tau / \rho / k \gamma}$. The failure to satisfy this local equilibrium made Bradshaw and Wong suspect that the turbulent mixing length in the region downstream of excrescences may not be proportional to $y$, but rather increase much more rapidly. Under similar conditions to those of Bradshaw and Wong, Kim, Kline and Johnston [41] found experimentally that the mixing length is higher than $k \gamma$ at $y/6 < 0.2$. They believed this to be because part of the separated shear layer, which has a larger mixing length than the usual value near the surface, came very close to the surface, through the reattachment process. Therefore, the turbulent mixing length can be larger very close to the surface but decrease rapidly to zero at the surface, causing a lower velocity gradient than that predicted by equation (5.7).

The figures (5.17) show that the dip in the data below the universal law of the wall is less significant either in flows with small roughness elements or in flows with adverse pressure gradients. This is due to the fact shown by East, Sawyer and Nash [27], that the region in the smooth surface boundary layer in which the flow has a higher value shear stress or mixing length, see equation (5.4), is an increasing distance from the surface as the pressure gradient increases adversely. It is also because the length of the separation region decreases as the pressure gradient increases (see 5.2.1. and Fig. 5.7). This apparently allows insufficient time for the effects of the turbulent structure which has a larger mixing length to diffuse towards the surface and affect flow there.
5.3 $C'_D/c_f$ Similarity with the Law of the Wall

If wall-variable similarity is to hold when small roughness elements are present, then both the upstream pressure gradients caused by the roughness element and the separation behaviour of boundary layers under the action of these pressure gradients must be independent of the history of the flow. In their analysis of the possibility of wall variable similarity Good and Joubert [17] pointed out that this could occur provided that the boundary layer developed naturally, the adverse pressure gradient caused by the flow over the roughness element were approximately constant and the pressure rise induced by the roughness element were sudden. In the present work these conditions have been met, and thus wall-variable similarity should occur.

In Fig. (3.22) the experimental values of $C'_D/c_f$, at various values of $h^+ = h U_r / \nu$, are plotted in linear-logarithmic co-ordinates. Within an acceptable range of scatter in the data of ±6%, wall-variable similarity is seen to occur. The appropriate relationship is

$$C'_D/c_f = 150 \log_{10} h^+ - 190 \quad (5.8)$$

This relation was first reported by Gaudet and Johnson [14] for zero pressure gradient flows. It is now shown to be universally valid.

If the roughness element height is comparable with the viscous sublayer thickness, Good and Joubert [17] pointed out that the height of roughness elements produces changes in pressure distribution over the roughness element which depend on the sublayer thickness. The length scale for the sublayer is $\nu/U_r$. But as the roughness element height increases and becomes large compared with the sublayer thickness, say $h^+ > 50$, then the change of the pressure distribution over the roughness element with roughness element height is no longer influenced by the sublayer thickness.

The relationship between $C'_D$ and $c_f$ can be expressed:
Differentiation of this relationship with respect to \( h \) gives

\[
\frac{C_D}{c_f} = \frac{\Delta D/h}{\frac{1}{2} \rho U_r^2} = f \left[ \frac{hU_r}{v} \right] = f[h^+] \tag{5.9}
\]

\[
\left( \frac{1}{\frac{1}{2} \rho U_r^2} \right) \frac{\partial (\Delta D)}{\partial h} = \left( \frac{U_r}{v} \right) \frac{\partial f}{\partial h^+} \tag{5.10}
\]

Since the right-hand side of the differentiation must be viscosity independent, this is only possible if

\[
\frac{\partial f}{\partial h^+} = A \left( \frac{v}{hU_r} \right) \tag{5.11}
\]

where \( A \) is a universal constant. Integration of equation (5.11) gives

\[
\frac{C_D}{c_f} = A \log_{10} h^+ + D \tag{5.12}
\]

where \( D \) is also a universal constant. The argument to establish the logarithmic variation of the drag coefficient is analogous to that given in Section (2.2) to explain the shape of the turbulent boundary layer velocity profile.

Fig. (5.8) shows in addition to the present data, the data of Wieghardt [10], Gaudet and Johnson [14] and Lacey [1]. All these appear to follow the suggested universal empirical equation (5.8).

Good and Joubert [17] have expressed the drag coefficient of a plate under a zero pressure gradient as
\[ \frac{C_D}{C_f} = 277 \log_{10} h^+ - 268 \]  
(5.13)

This gives values of drag between 5% - 15% higher than those deduced by other workers. Neither Good and Joubert [17] (see Fig. 1.6), nor Abd Rabbo [2] were able to correlate results for the drag coefficient obtained in flows in which there is a pressure gradient. This was due, in part, to the limited pressure gradient range in which they performed their experiments. The maximum value of the pressure parameter \( \beta \) achieved by Good and Joubert was 1.75, and by Abd Rabbo was 2.4. In the present study this parameter reached as high a value as 53. They also did not take into account the uncertainty in their estimates of \( C_D/C_f \). For example, for the present work the uncertainty in \( C_D/C_f \) for small roughness elements could be as high as ± 9%.

It should be noted that \( C_D/C_f \) data from flow 6 have not been shown in Fig. (5.22) and Fig. (5.8). This is because the measured changes in the momentum thicknesses obtained are negligible compared with the values for smooth surface momentum thickness. This is consistent with the universal validity of equation (5.12). As shown in Figs. (3.8) and (3.9) the part of the velocity profile for flow 6 which follows the law of the wall does not exceed \( \gamma^+ = 600 \), therefore the corresponding height of roughness element should not exceed \( h = 20 \text{ mm} \). Such a size of roughness element does not cause an increase in momentum thickness of more than 2% of the clean surface momentum thickness. Since the estimated value of uncertainty of \( \frac{\Delta \theta}{\theta} \) for flow 6 is ± 8 - 9%, this estimation of 2% is well within the uncertainty band.

It is thus that a boundary layer which is subjected to a strong adverse pressure gradient is not at all sensitive to the presence of small roughness elements. Provided that roughness elements remain small the strong adverse pressure gradient boundary layer is much more influenced by the height of
the element than it is by its shape.

5.4. Prediction Methods

5.4.1 The Prediction of Flows which have high shape parameters on smooth walls.

The failure of the lag-entrainment method to predict flow 6 satisfactorily, as was shown in Fig. (4.2f), can partly be related to a lack of flow two-dimensionality Fig. (3.4f) and Fig. (3.5), and partly to the failure of the prediction method itself. More experimental results obtained from flows with very strong adverse pressure gradients have been employed to check the prediction technique. Comparisons between the corresponding prediction and, the experimental data from Ludwieg and Tillmann's very strong adverse pressure gradient flow [25], and from Chu and Young's flow 2 [50] are shown in Fig. (5.9) and Fig. (5.10) respectively. These figures demonstrate that the prediction technique is unsatisfactory in the regions where the shape parameter is high. Introducing the empirical relationship for the normal stress terms, equation (2.13), into the momentum equation causes a limited improvement to the prediction, however. Agreement between prediction and the corresponding experimental data is still poor.

The suggested empirical relationship equation (5.1) which relates the shape parameter, \( H \), to the entrainment shape parameter \( H_1 \), gives a better fit to the experimental data than the equation (4.24) which is used in the lag-entrainment method, see Fig. (5.5). This was introduced, together with its first derivative and the Ludwieg and Tillmann skin friction formula (2.10), in the original lag-entrainment method. As shown in Fig. (5.14) these modifications result in no significant improvement. This failure indicates the need for a review of the method, if better prediction in strong adverse pressure gradients is to be achieved.

The following factors support this approach.

The dissipation scale length used in the lag equation (4.16), which was
first employed by Bradshaw [44] was shown experimentally by East, Sawyer and Nash [27] to be over-simplified. For this parameter, Bradshaw used the mixing length value in a zero pressure gradient and assumed its value to be universally valid. Experimentally, East, Sawyer and Nash [27] have shown that the mixing length must be dependent upon the pressure gradient (also see Section 5.1.3.), and that Bradshaw's dissipation length scale becomes independent of pressure gradient only if the diffusion in the region of shear stress towards the wall is taken into account. Also in the lag equation (4.16) the diffusion term has been over-simplified because it was believed that this term was very small and that local equilibrium in the wall region results from a balance of turbulence dissipation and production. East, Sawyer and Nash [27] show that when the flow has a high value of shape parameter, the diffusion term exhibits a high shear stress gradient near the wall and this implies that in such a case local equilibrium results from a balance of turbulence dissipation and diffusion.

5.4.2. The Power Law Velocity Profile Method

Effects of the presence of a roughness element can be predicted by calculating the change of momentum thickness using the universal drag coefficient relationship (equation 3.3) and the corresponding shape parameter, based on a power law velocity profile, equation (4.37).

As shown in Figs. (4.4) to (4.9) the accuracy of prediction depends on the roughness element height, compared with the boundary layer thickness. When the roughness element is very small, say \( \frac{h}{D} \approx 0.1 \), this method predicts well but as the height of the element increases the prediction becomes less good. It is suggested that the reason for this lack of agreement is the increasing length of the separation region downstream of the element, as the element height increases. With this increased length, flow with a high rate of shear could diffuse into the region close to the wall, causing significant changes in boundary layer structure there.
5.4.3. Two-Parameter Velocity Profile Method

This method, together with the necessary auxiliary equations, was reported in Section (4.4.2). An explanation for the superiority of this method over the power law method in the case of strongly-adverse pressure gradients follows. The power-law method depends only on the increase of the momentum thickness caused by the presence of the element, while the two-parameter method depends on the skin friction coefficient, $c_f$, in addition to the increase in the momentum thickness. As discussed in (5.3) the increase in the value of momentum thickness is negligible, hence in the two-parameter method the change in skin friction becomes the dominant factor.

For flow 6, no precise assessment can be made as to flow prediction in the presence of roughness elements, since at such an adverse pressure gradient the lag-entrainment method in the absence of the roughness elements does not predict flow adequately.
Fig(5.1) The shape parameter for equilibrium turbulent boundary layers
Turbulent equilibrium locus

For symbols
See Fig.(5.1)

Fig.(5.2) Equilibrium locus skin friction coefficient, $c_f$ for different values of $R_\theta$, compared with data for flows 1 to 6 and other sources.
Fig. (5.3) The growth rate of equilibrium locus momentum thickness for different values at $R\theta$, compared with data for flow 1 to 6 and other sources.
Fig. (5.4) Equilibrium locus pressure gradient parameter, \( \frac{\theta}{U_e} \frac{dU_e}{dx} \) for different values of \( R\theta \) compared with data for flow 1 to 6 and other sources.
Fig. (5·5) Plot showing that $H_1 = \frac{2}{J}$ is a close fit to the data for equilibrium turbulent boundary layers.
Fig (5.6) Relationship between energy shape, $H_{32}$, and $J$, data for flow 1 to 6 and other sources.
Fig. (5.7) Variation of reattachment length with $H$ for small square roughness elements

- ▲ Abd Rabbo [2], zero and adverse pressure gradients
- ■ Flow 6
Fig. (5-8) Dependence of $\frac{C_D}{C_f}$ on $h^+$ for square isolated roughness elements
Fig(5.9) Comparison of Lag-Entrainment prediction with Ludwieg and Tillmann
strong adverse pressure gradient flow, reported in Stanford Conference
[28] under code (Ident 1200)
Fig (5.10) Comparison between prediction and Chu and Young’s separating flow 2
Cf

0.0035

0.0030

0.0025

0.0020

0.0015

0.0010

0.0005

0.0000

0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5

Fig. (5.11) Comparison between prediction and experiment for equilibrium flow 6
EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS
ON TURBULENT BOUNDARY LAYERS

CHAPTER 6

RECOMMENDATION FOR FUTURE WORK
6. RECOMMENDATION FOR FUTURE WORK

6.1. Improvements in Measurement Facilities

During the course of the present investigation two major structural alterations were made to the wind tunnel working section. Even so, flow 6 constitutes the most adverse pressure gradient that can be obtained in the present wind tunnel facility, for the following reasons:

(i) Because of the low aspect ratio of the working section (about 2:1 at the inlet to the working section) a high non-uniformity in pressure distribution across the working section has to be tolerated when the pressure gradient is strongly adverse. In the case of flow 6, for example, the boundary layer formed on the side walls extends over all but the mid quarter of the tunnel cross-section, as is shown in Fig. (3.6).

(ii) The region occupied by the freestream in the downstream region of the working section is uncertain, because of the rapid increase in thickness of the roof boundary layer.

If pressure gradients which are more adverse were required, a facility having a considerably larger area ratio working section would be needed. Suction, applied to both the roof and to the side walls of the working section, would also be desirable as a means of limiting the boundary layer growth. If a new facility were constructed, the traverse mechanism should be re-designed so that more flexibility could be introduced in the length of the traverse steps. Magnetic tape data storage would offer considerable advantages over the present paper tape.

6.2. Recommendations for Future Experimentation

The following recommendations are made for gaining an increased understanding of flow behaviour behind small roughness elements.

(i) As discussed in Section (5.3) and shown in Fig. (5.8), the results
of the present investigations reveal that the additional drag generated by the presence of small roughness elements can be considered as a drag analogue to the law of the wall for turbulent boundary layer velocity profiles. This analogy can be extended for higher values of $h^+$ to establish a drag defect function, in a manner proposed by Good and Joubert [17].

(ii) A larger range of pressure gradients needs to be investigated in order to test the universality of the $C_p/c_f$ vs $h^+$ relationship, quoted in equation (5.8). Relaxing, reattached, favourable flows, both equilibrium and non-equilibrium should all be considered.

(iii) The present study should be extended to include more work on roughness elements with different shapes, such as backward-facing and forward-facing steps and grooves cut into the surface. The investigation might also consider three- dimensional forms of roughness elements as well as effects when the flow is three-dimensional.

6.3. Recommendations for Future Computer-Prediction Studies

With regard to the prediction of the effects of roughness elements by the lag-entrainment technique, the following suggestions are made for improvement of the method adopted.

(i) In the present prediction model, the imbalance in the momentum integral equation was assumed to be due to the lack of normal stress terms. However, the discrepancy between experimental and prediction values for flow 6 is shown to be larger than can be accounted for by these normal stress terms. Other effects, such as flow three-dimensionality, may have had significant effects and it could be desirable to directly evaluate the normal stress terms experimentally. However, this is a difficult experimental task demanding sophisticated techniques and instruments. Since turbulence intensity is high, hot-wire anemometry is an unsuitable technique. Coles and Hirst [25] estimate the
uncertainty of the extensive hot-wire measurements made by Schubauer and Klebanoff to be as high as ± 40%. In such flows pulsed-wire or laser-Doppler anemometers are more suitable instruments than hot-wire anemometers. Eaton and Johnston [48] estimate the uncertainty of turbulence intensity measurements made by pulsed-wire anemometry to be less than ± 5% when the turbulence intensity is over 20%.

(ii) It would be desirable to examine the modelling of the roughness elements by different prediction methods such as the Coleman-Cross and Bradshaw's field method; as well as by the lag-entrainment method. In such cases results of predictions would be compared with the experimental data.

(iii) For larger roughness elements better prediction might well be obtained by seeking to model the flow behaviour in both the recirculation and the reattachment regions. Although these influence downstream flow characteristics they are the most difficult regions in which to make detailed measurements. One possible model could be based on Tanner's [51] theoretical prediction of base pressure, which depends on the application of momentum principles to the recirculation region.
CONCLUSIONS

(1) The six turbulent boundary layers studied are shown to be good approximations to two-dimensional equilibrium flows. The mean flow parameters obtained are consistent with existing published data, supporting the equilibrium locus developed by Green, Weeks and Brooman [31].

(2) There is strong evidence that the law of the wall holds for all flows and that in consequence, the scale length in the mixing length model of shear stress must vary appreciably. The values of the skin friction coefficient which have been deduced by fitting velocity profiles to the law of the wall are in good agreement with those calculated by the Ludwieg and Tillmann formula.

(3) The drag of square cross-section two-dimensional roughness elements, which are small compared with the boundary layer thickness, has been analysed in terms of a drag parameter and roughness Reynolds number. For all the flows the results obtained are self-consistent. For the same roughness Reynolds number the drag coefficient, $C_D$, decreases as the pressure gradient increases adversely. Within an acceptable approximation of ± 6%, the data can be represented by a single linear-logarithmic relationship.

(4) The effects of a ridge which is of small height compared with the boundary layer thickness is negligible in flows with strong adverse pressure gradients, i.e. shape parameter, $H$, in excess of 1.7.

(5) Although the lag-entrainment prediction method is fully comparable in accuracy with the best of the methods assessed at the Stanford Conference of 1968, its accuracy is impaired when pressure gradients are highly adverse.

(6) In the case of small roughness elements the size of the separation region behind the element decreases as the freestream pressure gradient increases adversely, from some 8.5 element heights at zero pressure gradient to some 5.5 element heights when the shape parameter, $H$, is 2.3.
(7) Modifications have been made to the lag-entrainment prediction method to enable it to predict effects caused by the presence of isolated square small two-dimensional roughness elements. The method predicts the presence of roughness elements well, and is not strongly dependent on the freestream velocity distribution. However, in highly-adverse pressure gradients the method fails because in the absence of these elements the lag-entrainment technique cannot predict such flows adequately.
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Chu, J., 50.
Clauser, F.H., 6, 7.
Cockrell, D.J., 42.
East, L.F., 27, 35.
Eaton, J.K., 48.
Ferriss, D.H., 44, 45.
Galbraith, R.M.McD., 47.
Gaudet, L., 13, 14, 15.
Gibson, D.M., 34.
Good, M.C., 17.
Granville, P.S., 58.
Green, J.E., 31, 57.
Head, M.R., 43, 47.
Hirst, E.A., 25.
Hoerner, S.F., 4.
Hutton, P.G., 16.
Johnson, P., 14.
Joubert, P.N., 5, 17.
Kim, J., 41.
Klebanoff, P.S., 28.
Kline, S.J., 38, 41, 42.
Lacey, J., 1.
Ludwieg, H., 21.
Lumley, J.L., 54.
MacMillan, F., 23.
McClintock, F.A., 38.
Mellor, G.L., 34.
Merryman, P.J., 35.
Moffat, R.J., 39.
Morkovin, M.V., 42.
Morris, H.M., 36.
Nash, C.R., 27.
Nash, J.F., 18.
Pankhurst, R.C., 53.
Perry, A.E., 5.
Plate, E.J., 12.
Pugh, P.G., 16.
Rotta, J.G., 19.
Sawyer, W.G., 27.
Schlichting, H., 3.
Schubauer, G.B., 28.
Sovran, G., 42.
Spence, D.A., 56.
Tani, I., 9.
Tanner, M., 51.
Tennekes, H., 54.
Tillmann, W., 11, 21.
Townsend, A.A., 30, 33, 37, 55.
Weeks, D.J., 31.
White, F.M., 32.
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EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS ON TURBULENT BOUNDARY LAYERS

APPENDIX A

Tabulation of Experimental Data in the Absence of Roughness Elements
| X (mm) | Ue (m/sec) | Uz (m/sec) | G | H | Rθ | ε | ε* | εE | εf | εf_TBL |
|-------|------------|------------|---|---|----|---|----|----|----|-----|-------|
| 332   | 31.59      | 1.31       | 6.77 | 0.05 | 1.390 | 1.782 | 1.20 | 2528.1 | 15.9 | 1.66 | 2.13  | 0.00344 |
| 694   | 32.17      | 1.27       | 6.99 | 0.51 | 1.383 | 1.774 | 1.74 | 3730.7 | 21.0 | 2.40 | 3.08  | 0.00313 |
| 1054  | 32.45      | 1.23       | 7.28 | 0.54 | 1.380 | 1.771 | 2.21 | 4004.3 | 28.6 | 3.06 | 3.92  | 0.00287 |
| 1417  | 28.88      | 0.94       | 10.32 | 2.45 | 1.502 | 1.709 | 3.83 | 7402.3 | 32.4 | 5.76 | 6.55  | 0.00210 |
| 1781  | 25.56      | 0.74       | 12.46 | 4.20 | 1.569 | 1.681 | 6.33 | 10815.4 | 54.0 | 9.93 | 10.64 | 0.00169 |

| 2149  | 23.69      | 0.63       | 14.27 | 4.43 | 1.617 | 1.663 | 9.07 | 14366.5 | 67.9 | 14.67 | 15.09 | 0.00143 |
| 2509  | 22.23      | 0.59       | 14.30 | 4.95 | 1.608 | 1.662 | 11.72 | 17410.7 | 80.6 | 18.85 | 19.48 | 0.00140 |
| 2829  | 21.12      | 0.54       | 15.07 | 5.12 | 1.632 | 1.655 | 14.78 | 20858.6 | 95.9 | 24.12 | 24.46 | 0.00136 |
| 3149  | 20.39      | 0.53       | 14.57 | 5.02 | 1.611 | 1.660 | 17.39 | 23894.7 | 117.5 | 28.02 | 28.86 | 0.00134 |
| 3459  | 20.20      | 0.57       | 12.26 | 1.05 | 1.530 | 1.686 | 18.56 | 25070.3 | 126.4 | 20.40 | 31.29 | 0.00160 |

| 332   | 31.60      | 1.28       | 7.25 | 0.15 | 1.417 | 1.768 | 1.18 | 2508.2 | 12.1 | 1.67 | 2.08  | 0.00329 |
| 694   | 31.36      | 1.20       | 7.58 | 0.37 | 1.411 | 1.757 | 1.82 | 3782.5 | 17.1 | 2.56 | 3.19  | 0.00296 |
| 1054  | 27.19      | 0.89       | 10.40 | 2.00 | 1.520 | 1.703 | 3.33 | 5963.9 | 28.6 | 5.06 | 5.66  | 0.00216 |
| 1417  | 23.80      | 0.68       | 13.59 | 4.75 | 1.630 | 1.659 | 5.95 | 9406.3 | 41.3 | 9.70 | 9.88  | 0.00152 |
| 1781  | 21.68      | 0.58       | 14.75 | 5.41 | 1.654 | 1.649 | 8.78 | 12903.0 | 57.8 | 14.52 | 14.48 | 0.00147 |
| 2149  | 20.11      | 0.50       | 16.26 | 6.36 | 1.681 | 1.641 | 12.34 | 17178.2 | 80.6 | 26.74 | 20.25 | 0.00124 |
| 2509  | 19.19      | 0.45       | 17.75 | 7.76 | 1.703 | 1.636 | 16.49 | 21296.6 | 106.0 | 26.99 | 26.98 | 0.00108 |
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| 332   | 44.10      | 2.04       | 5.59 | -0.33 | 1.348 | 1.859 | 0.60 | 1767.0 | 8.3  | 0.81 | 1.11  | 0.00427 |
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### Table: Smooth Surface Flow 

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**SMOOTH SURFACE FLOW:**

**E-6**

**ALL DIMENSIONS (mm):**

- X: 525 mm
- U_{in} (y): 30.5 mm
- All other dimensions are measured in millimeters.

**NOTES:**

- Dimensions are exact as per the table.
- All measurements are taken from the image.
<table>
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### Smooth Surface Flow 6

**A 8**

**SMOOTH SURFACE FLOW 6**

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### ALL DIMENSIONS IN (mm)

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EFFECTS OF SMALL ISOLATED ROUGHNESS ELEMENTS ON TURBULENT BOUNDARY LAYERS

APPENDIX B

Tabulation of Experimental Data Behind Isolated Roughness Elements
### INTEGRAL PARAMETERS

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## INTEGRAL PARAMETERS

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<td>(\theta) mm</td>
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Flow Direction: Upward

Flow: \(X_t\) to \(h\)
### FLOW 3

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<th>G</th>
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<th>cf T &amp; L</th>
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### INTEGRAL PARAMETERS

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#### h = 6.36 mm

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#### h = 9.54 mm

### Flow Direction

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<th>( H_32 )</th>
<th>( G )</th>
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\( h = 9.54 \text{ mm} \)

\( h = 6.36 \text{ mm} \)

\( h = 4.77 \text{ mm} \)
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<th>H32</th>
<th>G</th>
<th>cf</th>
<th>cf T&amp;B</th>
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Flow Direction: [Diagram]
### INTEGRAL PARAMETERS

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<th>(H_{32})</th>
<th>(G)</th>
<th>(c_f)</th>
<th>(c_f_{T&amp;L})</th>
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\(h = 6.36\) mm

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\(h = 9.54\) mm

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<th>(\theta)</th>
<th>(H)</th>
<th>(H_{32})</th>
<th>(G)</th>
<th>(c_f)</th>
<th>(c_f_{T&amp;L})</th>
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\(h = 12.7\) mm

Flow Direction

\(2310 \text{mm} \quad \xi_1\)
### INTEGRAL PARAMETERS

#### FLOW 5

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<th>$G$</th>
<th>$c_f$</th>
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#### FLOW 6

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#### FLOW 8

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</table>

Note: The table above represents the flow and density values for different temperatures and pressures. The values are in m/s. The table includes data for temperatures ranging from 100G to 1000G and densities corresponding to each flow value.
### Flow 3

<table>
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<tr>
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<th>X/h = 256</th>
<th>X/h = 512</th>
<th>X/h = 1024</th>
<th>X/h = 2048</th>
<th>X/h = 4096</th>
<th>X/h = 8192</th>
<th>X/h = 16384</th>
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<td>0.128</td>
<td>0.256</td>
<td>0.512</td>
<td>1.024</td>
</tr>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.08</td>
<td>0.16</td>
<td>0.32</td>
<td>0.64</td>
<td>1.28</td>
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<td>5.12</td>
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<td>0.015</td>
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<td>0.12</td>
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<td>2.56</td>
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</table>

### Flow 4

- **Direction:**
  - X: 0.002, 0.01, 0.015, 0.02, 0.025
  - Y: 0.004, 0.01, 0.015, 0.02, 0.025

### Flow 5

- **Direction:**
  - X: 0.002, 0.01, 0.015, 0.02, 0.025
  - Y: 0.004, 0.01, 0.015, 0.02, 0.025

### Flow 6

- **Direction:**
  - X: 0.002, 0.01, 0.015, 0.02, 0.025
  - Y: 0.004, 0.01, 0.015, 0.02, 0.025
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<th>X/L = 40</th>
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Flow Direction

<table>
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<td>X/h = 16</td>
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<td>---------</td>
</tr>
<tr>
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<td>0.9100</td>
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<tr>
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<td>0.9100</td>
</tr>
<tr>
<td>0.941</td>
<td>0.9100</td>
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Notes:
- Flow direction is indicated by arrows.
- Dimension h = 9.34 mm.
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<th>X/h = 480</th>
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<td>0.674</td>
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**Flow Direction**

1.000

**Flow**

- **Direction:**
  - X

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<td>1.000</td>
<td>2.705</td>
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<td>X/h</td>
<td>X/h</td>
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<td>-----</td>
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Flow Direction

h = 4.77 mm
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**Flow 4**

**b = 0.36 mm**

- Flow Direction
- X1

*Flow 4 Diagram*
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**Flow Direction:**

Flow is directed from the left to the right along the X-axis.

**Unit:** h = 9.84 mm

**Legend:**

- Flow Direction: Arrows indicate the direction of flow.
- Scale: 1820 mm.
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**Flow Function**

- 2000mm
- 4000mm
- 6000mm
- 8000mm
- 10000mm

- X: position
- X+: next position
- X++: next next position
- X+++: next next next position

- Y: vertical position
- Y+: next vertical position
- Y++: next next vertical position
- Y+++: next next next vertical position

- Z: depth
- Z+: next depth
- Z++: next next depth
- Z+++: next next next depth

**Flow Function Diagram**

- Graphical representation of flow function over different positions and vertical levels.
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Flow Direction

Direction

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**FLOW**

- **X/h**
- **X/h + 63
- **X/h + 6.36**

- **B 31**
APPENDIX C

Listing of Computer Programs
DATA REDUCTION PROGRAM

THIS PROGRAM CONSISTS OF A DRIVER (DRAG) AND 5 SUBROUTINES (DATAIN, MENT, UNKIN, MEAN, AND PT1).

NP IS NUMBER OF DATA POINTS IN A PROFILE
NIN IS THE NUMBER OF INPUT CHANNEL
NOUT IS THE NUMBER OF CHANNEL
UZ IS SHEAR VELOCITY FOR THE PROFILE
UFRE(I) IS ARRAY OF NORMALIZED VELOCITY VALUES OF THE PROFILE
UINF IS THE MEAN VALUE OF UFRE(I)
Y(I) IS ARRAY OF VALUES FOR THE DISTANCE FROM THE WALL (MM)
DENSITY IS DENSITY OF AIR (KG/MM**3)
X IS STATION POSTITION (MM)
YUNC(I) IS ARRAY OF UNCORRECTED VALUE OF Y (MM)
UPLUS IS U/UZ
YPLUS IS UZ*Y/VKINEM
UPL IS UPLUS FOR THE FIRST DATA POINT JOINED WITH LOG LOW FROM THE WALL

PROGRAM DRAG(OUTPUT, TAPE5, TAPE6)
DIMENSION U(250), Y(250), UFRE(250), UINF, VKINEM, VEL, IVEL, lUFRE, YUNC, BG, AA, NP
COMMON /ASS/UZ

CALL PAPER(1)
READ (5,*) NP
CALL DATAIN(Y, U, UFRE, UINF, VKINEM, VEL, IVEL, iUFRE, YUNC, BG, AA, NP)
CALL SKIN(U, UFRE, Y, VKINEM, YP1, UP1, NP, UPLUS, YPLUS, RESID, UINF, ULOG)
CALL MENT(Y, U, DP, UP, YPLUS, UPLUS, ULOG, NP)
CALL PT1(Y, U, DP, UP, YPLUS, UPLUS, ULOG, NP)
CALL GREN
STOP
END

SUBROUTINE DATAIN

R IS AIR CONSTANT (KJ/(DEGREE K* KG))
T IS AIR TEMPERATURE (DEGREE K)
AP IS ATMOSPHERIC PRESSURE (NEWTON/MM**2)
HG IS ATMOSPHERIC PRESSURE (MM OF HG)
D IS EXTERNAL DIAMETER OF PITOT TUBE
YGAP IS THE GAP BETWEEN THE WALL AND THE PITOT TUBE AT LOWEST FDS
IVEL(I) IS ARRAY OF PITOT TUBE PRESSURE DIFFERENCE (MILLIVOLTE)
IUFRE(I) IS ARRAY OF PITOT TUBE PRESSURE DIFFERENCE (MILLIVOLTE)
CFR1 IS THE CALIBRATION FACTOR OF MICROMANOMETER NUMBER 1 (WATER)
CFR2 IS THE CALIBRATION FACTOR OF MICROMANOMETER NUMBER 2 (WATER)

DIMENSION IVEL(250), lUFRE(250), VEL(250), BG, AA

READ (5,30) T, HG, YGAP
YDEL=1.27
D=1.27
R=0.2871
AP=13.6*HG
DENSITY=(AP*9.81)/(T*R*1000.)
WRITE (6,60)
WRITE (6,65) T, HG, DENSITY
READ (5,35) IVEL(I), IUFRE(I)
WRITE (6,60)
WRITE (6,65) IVEL(I), IUFRE(I)
CONTINUE

END

SUBROUTINE MENT

R IS AIR CONSTANT (KJ/(DEGREE K* KG))
T IS AIR TEMPERATURE (DEGREE K)
AP IS ATMOSPHERIC PRESSURE (NEWTON/MM**2)
HG IS ATMOSPHERIC PRESSURE (MM OF HG)
D IS EXTERNAL DIAMETER OF PITOT TUBE
YGAP IS THE GAP BETWEEN THE WALL AND THE PITOT TUBE AT LOWEST FDS
IVEL(I) IS ARRAY OF PITOT TUBE PRESSURE DIFFERENCE (MILLIVOLTE)
IUFRE(I) IS ARRAY OF PITOT TUBE PRESSURE DIFFERENCE (MILLIVOLTE)
CFR1 IS THE CALIBRATION FACTOR OF MICROMANOMETER NUMBER 1 (WATER)
CFR2 IS THE CALIBRATION FACTOR OF MICROMANOMETER NUMBER 2 (WATER)

DIMENSION IVEL(250), lUFRE(250), VEL(250), BG, AA

READ (5,30) T, HG, YGAP
YDEL=1.27
D=1.27
R=0.2871
AP=13.6*HG
DENSITY=(AP*9.81)/(T*R*1000.)
WRITE (6,60)
WRITE (6,65) T, HG, DENSITY
READ (5,35) IVEL(I), IUFRE(I)
WRITE (6,60)
WRITE (6,65) IVEL(I), IUFRE(I)
CONTINUE

END
SUM=0.0
DO 20 I=1,NP
PRESS1=CRF1*IVEL(I)
PRESS2=CRF2*UFRE(I)
VEL(I)=SORT((2.0*PRESS1)/DENSITY)
UFRE(I)=SQRT((2.0*PRESS2)/DENSITY)
VEL(I)=SORT((2.0+PRESS1)/DENSITY)
UFRE(I)=SQRT((2.0*PRESS2)/DENSITY)
U(I)=VEL(I)/UFRE(I)
L=L+1
SUM=SUM+UFRE(I)
10 CONTINUE
MACMILLAN DISPLACEMENT CORRECTION
IF (I.GT.1) GOTO 10
YUNC(I)=YGAP+D/2.0
Y(I)=YUNC(I)+0.15»D
GOTO 15
10 CONTINUE
YUNC(I)=YUNC(I-1)+YDEL
Y(I)=YUNC(I)+0.15*D
15 CONTINUE
FF=YUNC(I)/D
IF (FF.GT.1.7) GOTO 20
MACMILLAN DISPLACEMENT CORRECTION NEAR THE WALL
A(1)=0.1384
A(2)=0.546
A(3)=0.896
A(4)=-0.234
A(5)=0.294
A(6)=-0.046
Z=A(1)+A(2)*FF+A(3)*(FF**2)+A(4)*(FF**3)+A(5)*FF**4+A(6)*FF**5
U(I)=U(I)+Z
20 CONTINUE
UINF=SUM/L
WRITE (6,50)
DO 25 J=1,NP
WRITE (6,55) J,IVEL(J),1UFRE(J),VEL(J),UFRE(J),U(J),YUNC(J),Y(J)
25 CONTINUE
RETURN
30 FORMAT (4(F8.4,4X))
35 FORMAT (20A14)
40 FORMAT (1H0,11H POINT NO.,2X,16H PITOT PRESSURE,3X,15H FREE PRESSURE,3X,13H VELOCITY)
1,H1,9H IN MVOLTE )//)
45 FORMAT (6X,13I2,9X,I8)//
50 FORMAT (2X,3HNG,3X,4HIVEL,2X,5HIUFRE,4X,3HVEL,7X,4HUFRE,9X,1H1,9X
1,4HYUNC,9X,1HY,8X,4HU-02///)
55 FORMAT (1H0,13I2(2X,15)6(2X,F8.4))
60 FORMAT (4X,1H1,8X,2HM,8X,6HVKJNEM,6X,7HDENSITY)//
65 FORMAT (4E12.6,///)
END

-----------------------------
SUBROUTINE SKIN
-----------------------------
LOG-PLOT METHOD FOR ASSESSING SURFACE FRICTION COEFFICIENT
SUBROUTINE SKIN(U,UFRE,Y,VKJNEM,YP1,UP1,NP,PLUS,FLUS,RESID,UINF,
LUX,BY,ULUG)
DIMENSION PLUS(250),FLUS(250),RESID(250),UX(250),UFRE(250),Y(250)
1,UN(250),BY(250),ULUG(250)
DIMENSION /AG/US
L=0.0
SKH=0.0
1I=0
DO 10 I=1,NP
IF (((1I),EQ.Y(I)))) GOTO 5
IF (((1I),EQ.0.0)) DEY=Y(I)
5 CONTINUE
10 CONTINUE
DO 25 I=1,NP
UZ=0.01
U(I)=U(I)*UFRE(I)
25 CONTINUE
UPLUS(I)=U(I)/UZ
PLUS(I)=UZ*(Y(I))/VKJNEM*0.001
END
CONTINUE
RESID(I)=UPLUS(I)-(5.62*ALOG10(B)+5.0)
IF (B .LT. .100.) GO TO 25
IF (B .GT. .300.) GO TO 25
L = L + 1
SUM = SUM + UZ
IF (L .EQ. .1) WRITE (6,35)
WRITE (6,40) I, U(I), Y(I), UZ, UPLUS(I), YPLUS(I), RESID(I)
25 CONTINUE
UZ=SUM/L
WRITE (NOUT,45) UZ
WRITE (6,55)
DO 30 1=1,NP
UPLUS(I)=U(I)/UZ
YPLUS(I)=UZ*Y(I)/VKINEM*0.001
B=YPLUS(I)
ULOG(I)=5.62*ALOG10(B)+5.0,
RESID(I)=UPLUS(I)-ULOG(I)
DY(I)=Y(I)/DEL
UD(I)=(U(I)-UFRE(I))/UZ
U(I)=U(I)/UFRE(I)
UP1=UPLUS(1)
YP1=YPLUS(1)
IF (I .EQ. .1) WRITE (6,35)
WRITE (6,40) I, U(I), Y(I), UZ, UPLUS(I), YPLUS(I), RESID(I), UD(I), DY(I)
30 CONTINUE
CF=(UZ/UINF)**2*2
WRITE (6,50) CF, UINF
WRITE (6,55)
WRITE (6,60) YP1, UP1
RETURN
35 FORMAT (5H N0,9X,5H U EL ,9X,3H Y ,9X,3H UZ,9X,4H U + ,9X,4H Y+ ,1
11X,6H RESID,4X,7HUDEFECT,5HY/DEL,//)
40 FORMAT (IH , 2X,1 3 ,9 ( 3X, F I1 .6))
45 FORMAT (IH ,18H SHEAR VELOCITY = ,F8.4)
50 FORMAT (IH ,32H SURFACE FRICTION COEFFICIENT =
12F0.6,6H ÜINF=,F8.4,1//)
55 FORMAT (IH ,100(H-))
60 FORMAT (1OH,4HYF1=,FB.4,4HUP1=,FB.4,1//)
END

SUBROUTINE MOMENT

IN THIS SUBROUTINE INTEGRAL PARAMETERS HAVE BEEN CALCULATED.

SUBROUTINE MOMENT(U,Y,UPLUS,YPLUS,UINF,X,VKINEM,NP,G,GG,G1,GG1,G2,GG2
DIMENSION G(250),GG(250),G1(250),GG1(250),G2(250),GG2(250),X(20),U
1(250),Y(250),DY(250),UD(250)
COMMON /ASS/UZ
SUM1=0.0
SUM2=0.0
SUM3=0.0
NP2=NP-2
DO 5 1=50,NP2
CALL INTEGR(U,Y,SUM1,SUM2,SUM3,G,GG,G1,GG1,G2,GG2,NP,I,NP2)
CONTINUE
CALL NEAR(YP1,VKINEM,UINF,S1,S2,S3)
WRITE (6,20) S1,S2,S3
SUM1=SUM1+B
SUM2=SUM2+B
SUM3=SUM3+B
DG=UINF-UPLU
RTHETA=(UINF*THETA) / (VKINEM*1000.)
RDSTAR=(UINF*DSTAR)/(VKINEM*1000.)
PW1=-0.67B**H12
PW2=-0.268
CF=0.246*10.**PW1*RTHETA**PW2
UPLUS=UINF/UZ
CF2=2.*(1./UPLUS)**2
CD=2*UPLUS/UPLUS
DO 10 1=1,NP
DY(I)=Y(I)/CD
UD(I)=UPLUS*(1.-U(I))**(-1.)
WRITE (6,35) I,Y(I),UD(I),DY(I)
10 CONTINUE
END

SKI0036
SKI0037
SKI0038
SKI0039
SKI0040
SKI0041
SKI0042
SKI0043
SKI0044
SKI0045
SKI0046
SKI0047
SKI0048
SKI0049
SKI0050
SKI0051
SKI0052
SKI0053
SKI0054
SKI0055
SKI0056
SKI0057
SKI0058
SKI0059
SKI0060
SKI0061
SKI0062
SKI0063
SKI0064
SKI0065
SKI0066
SKI0067
SKI0068
SKI0069
SKI0070
SKI0071
SKI0072
SKI0073
SKI0074
SKI0075
SKI0076
MDM0001
MDM0002
MDM0003
MDM0004
MDM0005
MDM0006
MDM0007
MDM0008
MDM0009
MDM0010
MDM0011
MDM0012
MDM0013
MDM0014
MDM0015
MDM0016
MDM0017
MDM0018
MDM0019
MDM0020
MDM0021
MDM0022
MDM0023
MDM0024
MDM0025
MDM0026
MDM0027
MDM0028
MDM0029
MDM0030
MDM0031
MDM0032
MDM0033
MDM0034
MDM0035
MDM0036
MDM0037
MDM0038
MDM0039
MDM0040
MDM0041
MDM0042
MDM0043
MDM0044
MDM0045
MDM0046
MDM0047
SUBROUTINE INTEGR(U, Y, SUM1, SUM2, SUM3, G, GG, G1, G2, GG2, NP, I, NP2)

IN THIS SUBROUTINE NUMERICAL INTEGRATION HAS BEEN USED BY FITTING A POLYNOMIAL TO THREE SUCCESSIVE DATA POINTS USING A SECOND ORDER EQUATION, AND INTEGRATING THIS EQUATION USING THE INTEGRATION OVER TWO STRIPS METHOD.

DIMENSION G(250), GG(250), G1(250), G2(250), GG2(250), U(250), Y(250)

U1 = U(I)
U2 = U(I + 1)
U3 = U(I + 2)
Y1 = Y(I)
Y2 = Y(I + 1)
Y3 = Y(I + 2)
Y21 = Y2 - Y1
Y32 = Y3 - Y2
Y212 = (Y2**2 - Y1**2) / 2
Y312 = (Y3**2 - Y1**2) / 2.0
Y213 = (Y2**3 - Y1**3) / 3.0
Y214 = (Y2**4 - Y1**4) / 4.0
Y215 = (Y2**5 - Y1**5) / 5.0
Y216 = (Y2**6 - Y1**6) / 6.0
Y217 = (Y2**7 - Y1**7) / 7.0
Y322 = (Y3**2 - Y2**2) / 2.0
Y323 = (Y3**3 - Y2**3) / 3.0
Y324 = (Y3**4 - Y2**4) / 4.0
Y325 = (Y3**5 - Y2**5) / 5.0
Y326 = (Y3**6 - Y2**6) / 6.0
Y327 = (Y3**7 - Y2**7) / 7.0

Y31 = Y3 - Y1
U31 = U3 - U1
U32 = U3 - U2

AA = U1 * Y2 * Y3 * Y32 - U2 * Y1 * Y3 * Y31 + U3 * Y1 * Y2 * Y21

DELTA = Y31 * Y32 * Y21

CC = U31 * Y21 - U21 * Y32

A = AA / DELTA
B = BB / DELTA
C = CC / DELTA

G(I) = A * Y21 + B * Y212 + C * Y213

G1(I) = A * Y32 + B * Y322 + C * Y323

G2(I) = A * Y21 + B * Y212 + C * Y213

IF (I.EQ.1) GOTO 5
IF (I.EQ.NP2) GOTO 10

SUM1 = SUM1 + 0.5 * (GG(I) + G(I-1))
SUM2 = SUM2 + 0.5 * (GG1(I) + G1(I-1))
SUM3 = SUM3 + 0.5 * (GG2(I) + G2(I-1))
GOTO 15

CONTINUE
END
SUBROUTINE NEAR
***************
IN THIS SUBROUTINE INTEGRAL VALUES FROM THE WALL UPTO THE FIRST DATA
POINT HAVE BEEN CALCULATED FROM THE LAW OF THE WALL EQUATION.
SUBROUTINE NEAR(YP1, VKINEM, UV, S1, S2, S3)

COMMON /ASS/UZ
COLE1=540.6
COLE2=6546.
COLE3=82770.
IA=VKINEM/UV
F=UZ/UV
A=1./0.41
B=5.0
DIFF=YP1-50.
EE=ALOG(YP1)
AE=ALOG(50.)
EG1=EE*YP1-AE*50.0-DIFF
EG2=EE*EE*YP1-AE*AE*50.0-2.0*EG1
EG3=(EE**3)*YP1-(AE**3)*50.-3.*EG2
E1=A*EG1+B*DIFF
E2=A*A*EG2+2.*A*B*EG1+B*B*DIFF
E3=(A**3)*EG3+2.*A*A*B*EG2+2.*A*B*B*EG1+(B**3)*DIFF
S1=(COLE1+E1)*DA*1000.
S2=(COLE2+E2)*DA*F*1000.
S3=(COLE3+E3)*DA*F*F*1000.
RETURN
END

SUBROUTINE PTK AA, BB, CC, DD, EE, FF, ULOG, NP)
DIMENSION AA(250), BB(250), CC(250), DD(250), EE(250), FF(250), ULOG(250)

AP=AA(NP)*1.15
BP=BB(NP)
CP=CC(NP)*1.15
DP=DD(1)*1.15
EP=EE(NP)*1.15
FP=FF(NP)*1.15
CALL PSFACE(0.35*0.65*0.1*0.375)
CALL MAP(0.0, AP, 0.0, BP)
CALL CURPOS(AA, BB, NP)
CALL PSFACE(0.35*0.65*0.425*0.7)
CALL MAP(0.0, CP, DP, 0.0)
CALL CURPOS(CC, DD, NP)
CALL PSFACE(0.3*0.7*0.75*0.95)
CALL MAPX(10., EF, 10., FP)
CALL GRAXL
CALL CURVED(EE, ULOG, 1*NP)
CALL CURPOS(EE, FF, NP)
RETURN
END

SUBROUTINE CURPOS(FA, EA, NP)
DIMENSION FA(250), EA(250)

CALL SCALES
CALL THICK(2)
CALL BORD
CALL THICK(1)
CALL CTRSET(4)
CALL CTRMAG(12)
DD 5 KK=1*NP
PX=FA(KK)
PY=EA(KK)
CALL POSITN(PX, PY)
CALL TYPENC(53)
CONTINUE
RETURN
END
THIS PROGRAM TO TEST THE TWO DIMENSIONAL GROWTH OF A TURBULENT BOUNDARY LAYER FOR INCOMPRESSIBLE FLOW. THE PROGRAM EVALUATE EACH SIDE OF THE MOMENTUM INTEGRAL EQUATION INDEPENDTLY. IF A BALANCE OF THE TWO SIDE OF THE EQUATION IS OBTAINED THE FLOW ASSUMED TO BE TWO DIMENSIONAL.

PROGRAM TEST2D
-----------
DIMENSION X(20),U(20),UZ(20),DSTAR(20),THETA(20),SUM1(20),SUM2(20),AJ(20),S

READ (5,30) (X(I),I=1,NPP)
READ (5,*) (THETA(I),I=1,NPP)
READ (5,*) (UZ(I),I=1,NPP)
READ (5,*) (U(I),I=1,NPP)
READ (5,*) (DSTAR(I),I=1,NPP)
READ (5,*) (CF1(I),I=1,NPP)
CALL FAPI(1)
M=5
N=10
DO 5 I=1,NPP
WRITE (6,25) I,X(I),U(I),THETA(I),DSTAR(I),UZ(I),CF1(I)
SUM1(I)=0.0
SUM2(I)=0.0
AG(I)=0.0
AG1(I)=0.0
5 CONTINUE
DO 10 I=1,NPP
UU(I)=U(I)**2
UU(I)=UU(I)/U(I)**2
UZU(I)=UZ(I)**2
CONST=(1.0/(THETA(I)*U(I)**2))
F(I)=UU(I)*THETA(I)
F(I)=F(I)*CONST
X(I)=X(I)/THETA(I)
AJ(I)=DSTAR(I)/THETA(I)
DSTAR(I)=DSTAR(I)/THETA(I)
NORMAL STRESS TERMS
AJ(I)=0.072*((AJ(I)-1.0)/AJ(I))
AJ(I)=F(I)-1.0+0.5*SUM1(I)
10 CONTINUE
CALL ANTG3P(X,UZU,SUM2,NPP,K,AG,AG1)
CALL ANTG3P(UU,DSTAR,SUM1,NPP,K,AG,AG1)
DO 15 I=1,NPP
11=I+10
X(II)=X(I)/(1000.0)
X(II)=X(II)/THETA(II)
WRITE (6,*) I,X(II),F(I),SUM1(I)
PLIFT=F(I)-1.0+0.5*SUM1(I)
CONTINUE
R=PLIFT-PLIFT
IF (I.EQ.1) WRITE (6,35)
WRITE (6,40) I,R,PLIFT,PRIGHT,R*X(II),AJ(I)
15 CONTINUE
WRITE (6,30) (X(J),J=1,NPP)
CALL EQ2ACF(X,F,9,A,B,REF)
A1=A(1)
A2=A(2)
A3=A(3)
A4=A(4)
A5=A(5)
A6=A(6)
A7=A(7)
A8=A(8)
SUBROUTINE ANTG3P(VV, DS, SUM, NPP, K, AG, AGI)

DIMENSION SUM(K), VV(K), DS(K), AG(K), AGI(K)

NPP2=NPP-2

DO 10 I=1,NPP2

DS1=DS(I)
DS2=DS(I+1)
DS3=DS(I+2)

VV1=VV(I)
VV2=VV(I+1)
VV3=VV(I+2)

DS21=DS2-DS1
DS31=DS3-DS1
DS32=DS3-DS2

VV31=VV3-VV1
VV32=VV3-VV2
VV21=VV2-VV1

VV322=(VV3**2-VV2**2)/2.0
VV312=(VV3**2-VV1**2)/2.0
VV212=(VV2**2-VV1**2)/2.0

DELTA1=VV31*VV32*VV21

AA=DS1*VV2*VV3*VV32-DS2*VV1*VV3*VV31+DS3*VV1*VV2*VV21

BD=(DS21*VV312-D31*VV212)*2.0

CC=DS31*VV21-DS21*VV31

A=AA/DELTA1

B=BB/DELTA1

C=CC/DELTA1

AG(I)=A*VV21+B*VV212+C*VV213

AGI(I)=A*VV32+B*VV322+C*VV323

IF (I.GT.1) GOTO 5

SUM(I+1)=AG(I)

GOTO 15

5 CONTINUE

IF (I.EQ.NPP2) GOTO 10

SUM(I+1)=SUM(I)+0.5*(AG(I)+AGI(I-1))

GOTO 15

10 CONTINUE

SUM(I+1)=SUM(I)+0.5*(AG(I)+AG1(I-1))

SUM(I+2)=SUM(I+1)+AGI(I)

15 CONTINUE

20 CONTINUE

RETURN

END

C

SUBROUTINE PLOT

-----------------------------

IN THIS SUBROUTINE NUMERICAL INTEGRATION HAS BEEN USE
BY FITTING A PARABOLA TO THREE SUCCESSIVE DATA POINTS USING A
SECOND ORDER EQUATION (A+B*X+C*X**2) AND INTEGRATE THIS EQUATION
USING THE INTEGRATION OVER TWO STRIPS METHOD.

SUBROUTINE ANTG3P(VV, DS, SUM, NPP, K, AG, AGI)

DIMENSION SUM(K), VV(K), DS(K), AG(K), AGI(K)

NPP2=NPP-2

DO 10 I=1,NPP2

DS1=DS(I)
DS2=DS(I+1)
DS3=DS(I+2)

VV1=VV(I)
VV2=VV(I+1)
VV3=VV(I+2)

DS21=DS2-DS1
DS31=DS3-DS1
DS32=DS3-DS2

VV31=VV3-VV1
VV32=VV3-VV2
VV21=VV2-VV1

VV322=(VV3**2-VV2**2)/2.0
VV312=(VV3**2-VV1**2)/2.0
VV212=(VV2**2-VV1**2)/2.0

DELTA1=VV31*VV32*VV21

AA=DS1*VV2*VV3*VV32-DS2*VV1*VV3*VV31+DS3*VV1*VV2*VV21

BD=(DS21*VV312-D31*VV212)*2.0

CC=DS31*VV21-DS21*VV31

A=AA/DELTA1

B=BB/DELTA1

C=CC/DELTA1

AG(I)=A*VV21+B*VV212+C*VV213

AGI(I)=A*VV32+B*VV322+C*VV323

IF (I.GT.1) GOTO 5

SUM(I+1)=AG(I)

GOTO 15

5 CONTINUE

IF (I.EQ.NPP2) GOTO 10

SUM(I+1)=SUM(I)+0.5*(AG(I)+AG1(I-1))

GOTO 15

10 CONTINUE

SUM(I+1)=SUM(I)+0.5*(AG(I)+AG1(I-1))

SUM(I+2)=SUM(I+1)+AG1(I)

15 CONTINUE

20 CONTINUE

RETURN

END

C

SUBROUTINE PLOT

-----------------------------

SUBROUTINE PLOT HAS BEEN ESTABLISHED TO REPRESENT
AND TO COMPARE EACH SIDE OF THE MOMENTUM INTEGRAL EQUATION
GRAPHICALLY.

SUBROUTINE PLOT(AG, AGI, X)

DIMENSION AG(20), AGI(20), X(20)

CALL FRAME

CALL MAP(0.2,0.8,0.2,0.6)

CALL MAP(0.0,4.0,0.0,6.5)
CALL AXES
CALL THICK(2)
CALL BORDER
CALL CTRSIZ(1)
CALL CTRSIZ(0.19)
CALL THICK(1)
CALL FLOTCS(-0.9,4.2,5HFL FR,5)
CALL FLOTCS(0.4,5.2,34HMOENTUM INTEGRAL EQUATION BALANCE,34)
CALL FLOTCS(1.2,-1.0,24HAXIAL DISTANCE X (METER),24)
CALL BLKREN
CALL CTRENET(4)
CALL CTRENET(0.25)
CALL FOSITN(0.0,0.0)
DO 5 I=1,9
X1=X(I)
Y1=Y(I)
CALL FOSITN(X1,Y1)
CALL TYPENC(59)
CALL FOSITN(X1,Y1)
CALL TYPENC(59)
CALL FOSITN(X1,Y1)
CALL TYPENC(53)
CALL FOSITN(X1,Y1)
CALL TYPENC(53)
5 CONTINUE
RETURN
END
APPENDIX
`D`