The bridge rail on an overhead crane torsion-box girder is positioned directly above one of the web plates; this web is subjected to wheel loadings from the lifting unit. The study concerns the load carrying capacity of plate box-girder web panels subjected to an in-plane wheel load at the panel midspan.

Investigations are made of the in-plane patch loading produced on crane web panels by the distribution of a wheel load through an overlying rail and flange. Elastic buckling of a short-span model box-girder web panel subjected to an in-plane wheel load is studied. A computer analysis is presented to determine elastic buckling coefficients for flat rectangular plates subjected to a uniform in-plane patch load centrally disposed on one edge and supported by shear stresses on the adjacent edges. Patch loads of various lengths are considered over a range of plate aspect ratios for plates with various combinations of simply supported and clamped edges. Some non-uniform patch loads modelling approximately a distributed wheel load are also considered.

A plastic mechanism analysis originally presented by Roberts and Rockey for predicting collapse loads of patch loaded plate girders is studied and a modified form derived. Results are presented of a series of collapse tests conducted on short-span model box-girders subjected to a wheel load above one of the webs. The effect on the failure load of rail size, web thickness, panel aspect ratio, and longitudinal web stiffening is investigated. Several of the test web panels exhibited snap buckling. From the results, a simple expression is developed for predicting collapse loads of plate girders subjected to narrow patch loads.

A series of recommendations are presented to aid structural designers in taking account of patch loading on slender web panels.
WHEEL LOADINGS ON WEB PANELS
OF OVERHEAD CRANE BOX-GIRDERS

by

Adam Patrick Robertson

A thesis submitted to the University of Leicester
for the degree of Doctor of Philosophy

FEBRUARY
1983
ACKNOWLEDGEMENTS

The author wishes to express his sincere thanks to all those who have contributed towards the work of this thesis in any way, large or small.

The author is particularly indebted to Dr. B. Hayman for his supervision of the work; his comments, suggestions and encouragement have been an invaluable contribution. Special thanks also go to Mr. C. Morrison, Experimental Officer, Leicester University Engineering Department, for his assistance with the experimental work, and to the Technician Staff of the same Department.

The assistance, financial and practical, provided by Herbert Morris Ltd., Loughborough, Leicestershire, is gratefully acknowledged. Without their co-operation, construction of a test frame and test specimens essential to the work might not have been possible. Thanks are extended particularly to Mr. R. Widdowson and Dr. H. Jack for supporting the work programme; to Mr. C. Grimley and Mr. A. Roper for technical advice; and to Mr. T. Pawley for arranging and administering the collaborative work on the part of the firm.

The author is very grateful to Helen Townsend for her excellent work in typing the majority of a difficult manuscript in limited time; to Paul Smith for his careful photography work; and to Doug Pratt, Noreen Berridge and Joyce Meredith for assistance with the figures.

Funding of this CASE Project by The Science and Engineering Research Council is gratefully acknowledged.
SUMMARY

In torsion-box design of twin girder overhead cranes, the bridge rail on which the lifting unit runs is positioned eccentrically on the girder, directly above one of the web plates. This web is subjected to in-plane patch loading produced by the spread of a wheel load through the overlying rail and flange. This study concerns the load carrying capacity of plate box-girder web panels subjected to a wheel load at the midspan of the panel.

Distribution of a wheel load through a rail and flange is investigated from recordings made of in-plane vertical stress distribution profiles along the upper edge of a web panel of a short-span model box-girder. The girder was loaded through various interfaces above the web by a wheel load. A simple method is proposed for relating a distributed wheel load to an equivalent uniform patch load. Methods for estimating distributed wheel loading lengths are investigated. It is shown that crane web panels are generally subjected to patch loads of short length, occupying less than one-quarter of the panel length.

A computer analysis is presented to determine elastic buckling coefficients for flat rectangular plates subjected to a uniform in-plane patch load centrally disposed on one edge and supported by shear stresses on the adjacent edges. Patch loads of various lengths are considered over a range of plate aspect ratios for plates with various combinations of simply supported and clamped edges. Also considered are some non-uniformly distributed patch loads modelling approximately a distributed wheel load. For the large majority of geometries considered, it is the support condition along the loaded edge which has greatest influence.
on the buckling load. Correlation with buckling loads estimated from experimental measurements on a model crane girder web panel indicated that an assumption of simply supported panel edges is over-conservative and that it is probably more representative to consider the edges attached to the flanges as clamped.

Ultimate load carrying capacity is considered. A plastic mechanism analysis originally presented by Roberts and Rockey is studied and a modified form derived which reveals the transition region from collapse initiated by direct web yielding for girders with stocky webs to failure by a mechanism of out-of-plane web deformation for girders with slender webs. Certain approximations in the original analysis are shown to involve the omission of terms which can contribute significantly to the plastic work expression. Inclusion of these terms, however, whilst offering potential refinement, increases considerably the complexity of the analysis.

Results are presented of a series of collapse tests conducted on short-span model box-girders subjected to a wheel load above one of the webs. The effect on the failure load of rail size, web thickness, panel aspect ratio, and longitudinal web stiffening is investigated. Snap buckling was exhibited by several of the test web panels. From the results, a simple expression is developed for predicting collapse loads of plate girders subjected to narrow patch loads.

The main findings of the work are used as a basis for a series of recommendations to aid the structural designer in taking account of patch loading on slender web panels.
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NOTATION

- $a_1, a_2$: openings in web stretching analysis
- $a_i$: coefficient of $i^{th}$ term in assumed displacements
- $b$: length of web panel
- $b_f$: width of flange or effective width of box flange
- $b_r$: width of rail base
- $c$: length of patch load
- $d$: depth of web panel
- $e$: distance from plate midplane to axis of hinge rotation
- $e_0$: value of $e$ in loaded region
- $f$: value of $u$ where $e = \frac{t_w}{2}$
- $f_k$: coefficient of $k^{th}$ term in stress function $\phi$
- $f(Y)$: stress field function defined by Eqn.(3.21)
- $g$: set of constants in assumed displacement functions
- $g(Y)$: function defined by Eqn.(3.22)
- $h, h_1, h_2$: vertical spacings between web hinge lines in loaded region
- $h(Y)$: function defined by Eqn.(3.23)
- $j$: distance between edge of patch load and outer flange hinge
- $k$: distance between point of load application and element
- $l$: term in Eqn.(3.32)
- $m$: number of terms in assumed stress function
- $m', n'$: powers in Eqn.(3.32)
- $n$: number of terms in assumed displacements
- $p, q$: integers in assumed displacement functions
- $p_m$: load per unit length of web
- $r$: cross-section dimension of square rail
- $s$: measurement of depth from rail surface
- $t$: thickness of web
- $t_f$: thickness of flange
- $t_r$: thickness of rail base
$t^*$  reference web thickness of 2.5 mm
$t_w$  thickness of web
$u$  distance from outer flange hinge towards patch load
$v$  small vertical displacement of flange
$w$  transverse (out-of-plane) displacement of web plate
$w_i(x,y)$  $i^{th}$ displacement function
$w_{ix}, w_{iy}$  X-part and Y-part of $w_i(x,y)$
$x, y$  rectangular coordinates in plane of flat web plate
$y_f, y_r$  distance from neutral axis to outermost fibre of flange and rail respectively
$z$  transverse displacement of middle hinge line of web mechanism
$z'$  value of $z$ for steeply inclined web hinge lines

$A$  constant = 3.25 or 4.0 in Eqn.(2.5)
$A_{kl}$  matrix defined by Eqn.(3.19)
$B$  half-depth of web panel
$C$  half-length of patch load
$C_e$  half-length of equivalent uniform patch load
$D$  plate bending stiffness = $E t^3/12(l-v^2)$
$D'$  wheel diameter
$E, E'$  Young's moduli (web, flange)
$F_{ijk}$  vectors defined by Eqn.(3.16)
$G$  web stretching term in collapse equation
$G_{ijkl}$  vectors defined by Eqn.(3.20)
$H$  term in collapse equation representing effect of web hinge line inclination
$H_k$  vector defined by Eqn.(3.17)
$I, I_f$  second moment of area of flange about own neutral axis
$I_{fr}$  second moment of area of integral rail and flange about own neutral axis
\( I_r \) second moment of area of rail about own neutral axis

\( J \) term in Parkes' analysis (Section 2.1)

\( K \) buckling coefficient for patch load : Eqn.(2.6)

\( K' \) buckling coefficient for patch load : Eqn.(2.8)

\( K_b \) buckling coefficient for bending

\( K_e \) buckling coefficient for equivalent uniform patch load

\( K_{nu} \) buckling coefficient for non-uniform patch load

\( L_c \) contact length between wheel and rail

\( M \) bending moment

\( M_f \) plastic moment of resistance of flange or of rail/flange assembly

\( M_w \) plastic moment of resistance per unit length of web

\( N_x, N_y, N_{xy} \) constants in stress field functions

\( O \) centre of hinge rotation

\( P \) applied patch load

\( P_b \) patch load in presence of global stresses

\( P_{cr} \) elastic buckling load (critical value of \( P \))

\( P_{crb} \) patch load which produces \( u_{bcr} \)

\( P_{crc} \) \( P_{cr} \) for panels with clamped horizontal edges

\( P_{crs} \) \( P_{cr} \) for panels with simply supported horizontal edges

\( P_{ex} \) experimental collapse load

\( P_o \) calculated value of \( P \) for which local web yielding occurs

\( P_{sa}, P_{sb} \) measured snap buckling loads for loading and unloading

\( P_u \) predicted collapse load (mechanism solutions)

\( P_{ua} \) predicted collapse load (empirical solution) : Eqn.(5.2)

\( P' \) load per unit thickness of plate

\( Q \) loading parameter in Eqn.(3.12)

\( R \) angle

\( S \) constant for determining \( \cos \psi \)

\( U \) total energy of deformed plate under load
\[ U(B) \] strain energy of bending
\[ U(Q) \] energy term defined by Eqn.(3.12)
\[ U(1), U(2) \] energy terms defined by Eqns.(3.3) and (3.4)
\[ U_{ij} \] second derivative of \( U \) with respect to \( a_i \) and \( a_j \)
\[ V \] dimensionless \( e_o : V = e_o/t_w \)
\[ W \] work per unit length of web hinge
\[ X,Y \] dimensionless \( x \) and \( y : X = x/B, Y = y/L \)
\[ X_r,Y_s \] X-part and Y-part of stress function terms
\[ Z \] dimensionless \( z : Z = z/t_w \)

\[ \alpha \] panel aspect ratio \( = b/d \)
\[ \beta \] patch length parameter \( = c/b \)
\[ \gamma \] angle in web stretching analysis
\[ \varepsilon_{xx}, \varepsilon_{yy} \] in-plane web strains in \( x \)- and \( y \)-directions
\[ \zeta, x, \mu, \rho \] powers in dimensional analysis
\[ \eta \] length of web yielded in direct compression
\[ \theta, \theta_1, \theta_2 \] angles of deformed web from vertical
\[ \theta^a, \theta^b, \theta^c, \theta^d \] angles of rotation of inclined web hinge lines
\[ \lambda \] web slenderness ratio \( = d/t_w \)
\[ \lambda_r, \lambda_s \] series of constants in stress function terms
\[ \nu \] Poisson's ratio (taken as 0.3 for steel)
\[ \sigma_b \] in-plane longitudinal bending stress
\[ \sigma_{bcr} \] critical value of \( \sigma_b \)
\[ \sigma_{cr} \] critical stress under patch load
\[ \sigma_f \] yield stress of flange
\[ \sigma_f^* \] reference stress of 300 N/mm\(^2\)
\[ \sigma_m \] maximum in-plane vertical compressive stress in web
\[ \sigma_r \] yield stress of rail
\[ \sigma_{ra} \] radial stress
\[ \sigma_s \] shear stress
\( \sigma_{scr} \)  critical value of \( \sigma_s \)
\( \sigma_{sm} \)  maximum shear stress
\( \sigma_{su} \)  ultimate shear stress
\( \sigma_u \)  ultimate stress under uniform compression
\( \sigma_w \)  yield stress of web
\( \sigma_y \)  yield stress
\( \sigma_{yy} \)  in-plane vertical compressive stress in web
\( \sigma^*, \sigma_{xy}^*, \sigma_{xy}^* \)  statically admissible stresses
\( \sigma^+, \sigma_{xy}^+, \sigma_{xy}^+ \)  dimensionless statically admissible stresses
\( (\sigma^+)_x, (\sigma^+)_y \)  X-part and Y-part of \( \sigma^+_x \)
\( (\sigma^+)_x, (\sigma^+)_y \)  X-part and Y-part of \( \sigma^+_y \)
\( (\sigma^+)_x, (\sigma^+)_y \)  X-part and Y-part of \( \sigma^+_{xy} \)
\( \phi, \phi_1 \)  angles in web stretching analysis
\( \phi_2, \phi_3 \)  angles of inclination of web hinge lines
\( \psi \)  angle between flange and deformed web
\( \phi(x,y) \)  stress function
\( \phi_k(x,y) \)  kth function is assumed \( \phi_k \)
\( \phi_{kX}, \phi_{kY} \)  X-part and Y-part of \( \phi_k(x,y) \)
Fig. 1.1. ELECTRIC OVERHEAD TRAVELLING CRANE. TWIN TORSION-BOX-GIRDER TYPE.
CHAPTER 1

DISCUSSION OF THE PROBLEM

1.1. Introduction

Electric overhead travelling cranes are now a common feature of factories, warehouses, construction yards, foundries and power generating plants. In early overhead cranes the bridge was generally of riveted truss construction. However, more recently structures of welded steel plate and rolled steel section have become more cost-effective; thin-plate welded box-girders are particularly favoured.

The need for increased knowledge of structural performance, particularly in relation to buckling behaviour, has accompanied these developments. This has been especially so with welded plate structures, for which the strength is frequently governed by buckling considerations.

A popular girder construction is that of the torsion-box. In this the bridge rail is positioned not centrally along the top flange but to one side, directly over a web. This type of structure presents the designer with a buckling condition about which relatively little information is available. It concerns buckling of a web panel under the action of a wheel load which spreads through the rail and flange to produce a localised region of high compressive stress at the top edge of the panel. How this condition might be allowed for in the design of such girders was the question which prompted the investigation reported here.

1.2. Electric Overhead Travelling Cranes

A typical contemporary twin-girder electric overhead travelling crane (EOTC) is shown in Fig. 1.1. The two bridge girders, the length of which defines the span of the crane, are mounted at their extremities onto end-carriages.
The end-carriages contain the wheels on which the complete crane structure runs along rails mounted on the crane gantry girders. This direction of motion is referred to as the 'long-travel'. The crane gantries are structural members designed to carry an EOT crane and as such form a part of the building structure which supports it. The 'crab'\(^*\) is the name given to the trolley unit which accommodates the hoist mechanism. It straddles the gap between the two bridge girders, through which passes the hoist rope, and runs across the span of the crane on rails, one placed on either girder. This motion is known as the 'cross-travel'. The extent of cross-travel and long-travel determines the side and end 'hook-approaches' (the least distances by which the hook approaches the walls of the building housing the crane) which in turn describes the working area of the crane.

Control of the crane shown in Fig. 1.1 is achieved via the push-button pendant which is suspended from an independent track on the crane bridge. This enables the operator to control the crane from any position on the floor beneath the crane span. On many larger cranes, the operator is housed in a cabin suspended below girder level from either the crab or one end of a bridge girder to give the operator an unrestricted view of the load.

Construction of overhead crane bridge girders can take various forms. This is determined primarily by three factors: the required lifting capacity, span of crane, and duty of crane. The qualitative term 'duty' of crane is quantified in crane design codes by a classification of cranes which is dependent upon two measures: the frequency of operation of the appliance as a whole when in service and the extent to which the appliance is used to lift its maximum load or lesser loads expressed as a fraction of the

\(^*\) The German term is the 'kat'.
maximum load (see Ref. [1]). For a crane with a relatively low load carrying capacity, short span, and light duty, a girder constructed from standard rolled steel sections such as universal beams is common. Such a girder may constitute a box-section but need not necessarily do so. As any of the requirements, capacity, span, or duty is increased however, the girder design tends towards a fabricated steel plate box-girder construction. This type of girder structure is consequently employed in a large number of EOT crane applications.

1.3. **Crane Box-Girders**

A crane steel plate box-girder is of rectangular cross-section. Usually a 'narrow box' is used which is typically 2 to 2.75 times as tall as it is wide; in some special cases, however, a 'wide box' is employed which has a cross-section of nearer square proportions. The box comprises a top and bottom horizontal flange plate and two vertical side web plates. Inter-spersed at regular intervals along the length of the box-girders are internal full-depth diaphragms or vertical stiffeners. These members occupy the full cross-sectional area bounded by the four peripheral plates of the box except for a narrow horizontal strip across the bottom which arises from the diaphragms being stopped just short of the tension flange. Unless the girder is a 'walk-through' box with access holes cut in the diaphragms, one of the four diaphragm edges is inaccessible for welding to its adjacent outer plate of the box. Selecting the bottom edge as that to be left unattached avoids introducing possible new sources of fatigue in the tension region. Consequently, the bottom flange is generally the last plate to be welded in place during the box fabrication process.

Diaphragms maintain the integrity and shape of the box, thereby retaining its torsional properties. They also play an important role in designing against shear buckling of the webs by serving to divide the webs up into
discrete panels. For a given web, the resistance to shear buckling of a panel depends on its aspect (length/depth) ratio which is determined by the diaphragm spacing. For cranes, diaphragm spacing is such that web panel aspect ratios lie typically between 1.5 and 1.0, although ratios below 1.0 are sometimes employed.

The crab rails are mounted onto the top flange plates of the two bridge girders at either of two locations: directly above the inner web or midway between the two webs. It is now general practice to mount the rails above the inner webs unless high wheel loads or other special conditions render this impractical. This is discussed further in the following section. When the crane rails are mounted eccentrically, as shown in Figs. 1.1 and 1.2, the crane is known as a twin torsion-box-girder crane.

This rail location is favoured because it usually leads to a more economical crane structure than does its alternative. With centrally located rails, it is often necessary to incorporate additional vertical stiffening between pairs of full-depth diaphragms to prevent undue dishing of the flange as the crab traverses. These stiffeners take the form of short-depth vertical diaphragms which are welded to the underside of the top flange and to the two webs. The entity of the torsion-box, however, eliminates the requirement for these additional stiffeners since each crab rail is supported along its length by the inner web of the girder. With these girders, the full-depth diaphragms provide an additional function in that they transmit shear loadings through to the outer web of each girder.

1.4. Wheel Loads

A consequence of the torsion-box design is that the web panels beneath a rail are subjected to vertical in-plane loadings from the crab wheels, as shown in Fig. 1.3. Before entering the web, a wheel load is distributed
Fig. 1.2. CROSS-SECTIONAL VIEW OF A TWIN TORSION-BOX-GIRDER CRANE

Fig. 1.3. WHEEL LOAD ON WEB PANEL
to some extent along the length of the panel by the rail and upper flange plate. This gives rise to a localised region of high compressive stress along the top edge of a thin panel.

In practice these panels have to withstand not only localised wheel loadings but also global shear stresses and longitudinal in-plane bending stresses. Bending of the bridge girders takes place both in the vertical plane and horizontal plane, owing to inertial loadings. For high wheel loads, these various loadings can combine to over-stress the seam weld between the web and flange beneath the rail. It is for this reason that, currently, the torsion-box concept is generally not applied to higher capacity cranes. The problem can sometimes be overcome by fabricating a structural T-section into the corner of the box where the rail is placed. This section replaces the upper level of the web plate and the section of flange beneath the rail, thereby removing the weld to a less highly stressed region further down the web. The increased fabrication involved in this remedy, however, makes the economics of applying the torsion-box less sound.

High concentrations of stress from wheel loads can arise also in crane gantry girders when end-carriage wheel loads bear onto thin gantry webs. When such loadings become excessive, bogies can be used instead of end-carriages to help alleviate the severity of stressing within the gantry by transmitting the load through more than two wheels. By incorporating bogies, however, a further restriction on the application of torsion-boxes can arise. Torsional loadings on the bridge girders are reacted by the bogies which may cause undesirable inequalities in the bogie wheel loads onto the gantry; in some instances the complications of correcting this outweigh the original benefits of employing the torsion-box.
Whenever the torsion-box construction is adjudged unsuitable for a twin-girder crane application, the alternative centrally mounted rail design is employed. Although the webs are then free from wheel loads, the girder diaphragms are then subjected to wheel loads, again distributed by the rail and flange, centrally disposed on their upper edge. Any additional short-depth flange stiffeners are similarly loaded.

It can be seen that wheel loadings are an important consideration in the design of EOT crane structures; their presence can determine some of the fundamental design features. It is therefore important that design codes provide reliable and accurate means for taking account of these loads. This is especially so in the case of torsion-box web panels when not only is the risk of local yielding of significance but, owing to their geometry, so too is the liability of these panels to buckle.

1.5. Current Design of Crane Bridge Girders

1.5.1. Design Codes

There now exist various British, European, and American Codes of Practice and Standards which deal specifically with cranes (see Refs. [1] - [6]) and others on allied subjects such as bridges and the use of structural steelwork in buildings (see Refs.[7] - [12]). Together they offer guidance to the crane designer on most aspects of crane structure design. As might be expected, certain aspects are covered more comprehensively in some codes than in others. There are, however, some important considerations on which the codes, even when taken collectively, offer only very limited or inelegant recommendations. Two such considerations will now be discussed.
1.5.2. **Bearing Stresses from Wheel Loads**

A wheel load which is spread by a rail and flange plate constitutes an in-plane patch, or partial edge, load on the upper edge of a supporting web panel. Generally it can be assumed that this load is resisted by parabolically distributed shear stresses on the vertical panel edges. The profile of the dispersed wheel load is shown schematically in Fig. 1.4 (a). However, of the codes currently in operation which incorporate wheel load bearing influences, all assume the generation of a uniform patch load as shown in Fig. 1.4 (b).

Code estimations of the length of a distributed wheel load upon entering the panel are based on a rather simplified and somewhat arbitrary 'fan-spread' postulation. This surmise involves only the height of the rail and thickness of the flange. The wheel load is adjudged to spread linearly and symmetrically from its region of contact with the rail, through the rail and flange. The deemed angle of dispersal depends on the code concerned but is generally between $30^\circ$ and $45^\circ$ to the horizontal. By assuming the wheel load to act uniformly over the resulting panel length, the local bearing stress can be evaluated and this used to check against yielding of the web or weld between web and flange.

An additional problem of fatigue cracking exists in this region of welded plate box-girders, a problem which also arises in the upper part of welded crane gantries. For every cycle of load in a bottom, tension, flange there are two or more cycles of local stress in the top flange produced by the passage of each wheel of the crab or end-carriage. Localised loading irregularities along the top flange are caused by misalignment of wheel, rail, and web centrelines, sideways thrusts onto the rail, uneveness of rail, and poor fit between rail and flange. These are the factors attributed by
Fig. 1.4. WEB PANEL SUBJECTED TO PATCH LOAD RESISTED BY SHEAR STRESSES ON VERTICAL EDGES
(a) DISTRIBUTED WHEEL LOAD
(b) UNIFORM PARTIAL EDGE LOAD
Senior and Gurney [13] to the emergence of fatigue cracking in crane girder upper flange regions. Primarily, this cracking originates at the root of fillet welds, particularly at stop-start positions in the weld runs. Consequently, web-to-flange connections which are subjected to localised wheel loading effects are generally made with a good quality, full-penetration, continuous butt weld.

Both local yielding and fatigue cracking in the upper part of crane girders depend primarily on the magnitude of local stress caused by wheel loads. It is therefore important that a good estimate of this be achieved at the design stage but to do so requires a reliable and representative means for taking account of the distribution of wheel loads by the various types of rail and thicknesses of flange that are used; existing code recommendations are considered somewhat basic in their approach to this.

1.5.3. Web Panel Buckling

Current design of crane box-girders focusses on the webs as well as the flanges withstanding the longitudinal bending stresses caused by global bending of the girders. The alternative approach is to consider the flanges alone to carry the bending stresses with the webs being designed to meet shear requirements. The former doctrine tends to be adopted since it enables the second moment of area of the webs to be incorporated into the stiffness value of the girder which is beneficial for reasons of girder deflection.

As a result of this approach it is found that, unless the girder is of a deep section and short-span, bending demands on the web panels around the centre of the span generally exceed those of shear on the panels towards the ends. Consequently it is usually bending considerations, rather than shear, that determine the general proportioning of the web plates.
Crane box-girder webs generally have a depth-to-thickness ratio (d/t) in the region of 150 to 250. This is sufficiently slender for buckling to demand attention and various codes, notably the German DIN 4114 [11], give rules for safeguarding against buckling due to in-plane bending, shear, or a combination of the two. Permissible stresses and dimensional limitations (involving requirements for vertical and longitudinal stiffening) applicable to web plates are largely determined by buckling considerations.

Buckling of plate panels under the action of localised in-plane partial edge loading is now widely acknowledged and some research into the subject has been conducted; various calculations of buckling stresses for plates loaded centrally by uniformly distributed patch loads (see Fig. 1.4 (b)) have been reviewed in a state of the art report by Rockey [14]. Although wheel loads on EOT crane structures produce a very similar form of loading (see Fig. 1.4 (a)) there is, to the author's knowledge, only one major code in the western world that mentions specifically the susceptibility of crane web panels to buckle under wheel loadings. This is the Dutch code NEN 2019 [6]. Its coverage extends no further, however, than to make reference to a German paper by Wilkesmann [15] which contains some buckling coefficient information for panels subjected to either point edge or uniformly distributed full edge in-plane loadings. Clearly there is an inadequacy of information in the codes on this subject and indeed the basic research information available is also rather limited.
1.6. The Present Investigation

Of the various situations where wheel loadings produce in-plane partial edge loadings on crane girder plates, the risk of ensuing buckling is probably greatest in the case of rail-bearing web panels of torsion-box-girders. These panels are generally of a greater d/t than the webs of crane gantries and of a larger aspect ratio than internal full-depth diaphragms of bridge box-girders. Furthermore, they are liable to buckling interactions arising from in-plane compressive bending stresses and shear stresses.

The diaphragms of a crane box-girder in effect divide the girder up into a number of sections, or 'cells'. In the case of a torsion-box, the rail-bearing web panel of each section is subjected to the stresses generated in the girder as a whole and to localised wheel loadings. The global stresses vary in magnitude from one section to another, depending on the position along the girder; the wheel loading condition, however, is common to all the panels. Although the overall performance of these panels will depend upon the complex interaction of all the loadings to which they are subjected, an understanding is first required of the effect of a wheel load acting in isolation.

This simplified situation is conveniently achieved in the present investigation by confining the study to one of short-span sections of crane torsion-box-girders, acted upon by a wheel load. Wheel loads applied at the midspan of the panels are investigated since the load is then furthest from a vertical stiffener when, it is considered, buckling is most likely to develop; this is the situation shown in Figs. 1.3 and 1.4.
Of the existing buckling information on patch loaded plates, the large majority concerns uniformly distributed loads applied symmetrically about the plate midspan. For this information to be applied to the case of a wheel loaded crane web panel requires a reliable means for assessing the distribution of the load into a web for different rail, flange, and web combinations, and a method for equating the resulting non-uniform load to an equivalent uniform patch load. These are therefore the considerations investigated first. The investigation later moves on to evaluate data not previously available on the elastic buckling of patch loaded plates and to consider local plastic collapse, or crippling, of wheel loaded crane box-girder web panels. In conclusion some design recommendations are presented.

In essence, the present investigation concerns the buckling resistance of crane torsion-box-girder web panels, loaded centrally and in the plane of the web by a wheel load which acts through a rail and flange interface.

1.7. Terminology

In an attempt to avoid any confusion over the use of inspecific terminology, the definitions listed below will be adhered to throughout the text unless stated otherwise.

1) Co-ordinates and Directions.

The general co-ordinate system adopted here is shown in Fig. 1.5. Hence, for example, reference is made to longitudinal bending stresses, vertically applied wheel loads, and transverse buckling displacements of web panels. The web panel dimension described by the spacing of the transverse full-depth diaphragms
Fig. 1.5
GENERAL CO-ORDINATE SYSTEM
is referred to as the panel length, the vertical dimension is referred to as the depth. To be consistent, reference is made to the length (as opposed to width) of a patch load on the top edge of a web.

2) Patch load.
Patch, partial edge, and localised load are synonymous.

3) Buckling and Critical Load.
Elastic buckling load and elastic critical load are synonymous.

4) Web Plate Slenderness.
The ratio depth/thickness (d/t) is taken as the measure of slenderness of a web plate. A high slenderness ratio corresponds to a slender plate; a plate with a low slenderness ratio may be described as being stocky.

5) Codes.
No distinction is drawn here between a 'Code', as in a 'Code of Practice', and a 'Standard'; they are use interchangeably.

6) Crippling.
Local plastic collapse of a web resulting from buckling deformations produced by a localised edge load may be referred to as web crippling.

7) Dimensions.
All dimensions given are in millimeters (mm).
CHAPTER 2

PATCH LOADS ON CRANE WEB PANELS

2.1. Introduction

Before the risk of web panel buckling under the action of a wheel load can be investigated, it is necessary to assess the length of the patch load acting onto the top edge of the panel. Existing codes (see Refs. [3] - [6] and [10]) estimate patch lengths by assuming a simple fan-spread of the load through the rail and flange, indicating that lengths depend only on the rail height and flange thickness. It is more likely, however, that distribution of the load into the web will depend on the ratio of stiffnesses of the rail and web plate, as in the elementary theory of a beam resting on an elastic foundation by Timoshenko [16]. It is unlikely though that an actual support to a beam, such as a web panel, can realistically be considered to comprise a number of vertical, independently acting springs, as this theory assumes.

Biot [17] has attempted a more exact solution to the problem of a beam resting on an elastic foundation by developing improved estimates for the stiffness, or modulus, of the foundation. His results for the bending moment distribution in an infinitely long beam subjected to a concentrated load and resting on an infinitely high and long foundation are in close agreement with those produced by Parkes [18] for a similarly loaded flange of a beam with an assumed semi-infinite web. Parkes goes on to show that his analysis is applicable also to beams of practical depths.

Parkes, in what is a classical piece of theory, also derives distributions of vertical stress between the flange and web of a beam. From the distribution for a unit point load, the peak stress directly beneath the
load is given by

\[ \sigma_m = \frac{2}{3\sqrt{3}Jt} \]

where \( J \) is given by

\[ J = \left( \frac{(1+v)(3-v)}{2} \frac{I}{t} \right)^{1/3} \]

where \( I \) is the second moment of area of the flange about its own neutral axis and \( t \) is the web thickness. Assuming the vertical stress to be directly proportional to the applied load \( P \) and substituting \( v=0.3 \) as Poisson's ratio for steel, this becomes

\[ \sigma_m = 0.3191 \frac{P}{t} \left[ \frac{t}{I} \right]^{1/3}, \]

A very similar expression has been derived from another long and detailed theoretical analysis by Girkmann [19] who gives as the maximum load per unit length of web of a beam point loaded on the flange

\[ P_m = \frac{2P}{3\sqrt{3}} \left[ \frac{Et}{2E'\overline{I}} \right]^{1/3}, \]

Where \( E \) is Young's modulus for the web material and \( E' \) that for the flange. Converting this to a direct stress and assuming the web and flange materials to be of the same Young's modulus, the expression becomes

\[ \sigma_m = 0.3055 \frac{P}{t} \left[ \frac{t}{I} \right]^{1/3}. \]
It can be seen that this expression is as that obtained from the theory by Parkes except for a small difference in the constant value.

The analyses by both Parkes and Girkmann show that for a beam loaded on the flange by a point load acting in the plane of the web, the distribution of vertical stress along the web depends on the second moment of area of the flange and the web thickness. This would appear to offer a far more rational basis for estimating patch loading lengths on crane web panels than that currently adopted by the design codes.

In practice, various flange and web thicknesses, together with different types and sizes of rail, are used in crane girders. Three basic rail types are employed: the flat bottom rail, the bridge rail, and the square bar rail. These are shown in Fig. 2.1. Two rails of different section but similar height will therefore have different second moments of area and, it would be expected, produce different distributions of a wheel load onto a given girder web.

Generally speaking, square bar rails tend to be utilized on bridge girders provided the capacity of the crane is not high or the duty severe. This is because square bar is readily and cheaply available as standard steel stock and, being of a non-high-tensile steel, can readily be welded to the flange. Square bar rails therefore tend to be prevalent within standardised crane products. For high wheel loads, local bearing stress considerations often necessitate a sturdier rail and either a flat bottom or bridge rail is then used.

This option may be determined by customer specification although, for torsion-boxes, the restriction on the flange outstand dimension arising
Fig. 2.1 CRANE RAILS
from local buckling considerations may make it possible to fit only the flat bottom type since these have a narrower base than comparable bridge rails.

The dispersal of a wheel load onto a crane web is a rather more complicated situation than that considered in the theories of Biot, Parkes, and Girkmann since, instead of there being a single member which spreads the load, there are in fact two: the rail and the flange. The extent to which the two members act independently or integrally will depend on how the rail is attached to the flange.

Senior [20] and Senior and Gurney [13] have discussed the problems encountered with early crane rail mounting techniques and reported on the two methods currently employed. Although fundamentally different, both are intended to keep track maintenance costs down. The 'fully hard' approach consists of either welding or hard clipping; the 'fully soft' approach involves soft clipping, sometimes with the inclusion of a rail underlay pad. In the fully hard method, the rail is attached in a manner which is sufficiently rigid for it to act with the flange as an integral unit, thereby avoiding fretting wear. The fully soft approach, on the other hand, is designed to accommodate the relative movement between rail and flange produced when the girder deflects, but to do so in such a way that the fastenings do not loosen and fatigue. Square bar rails are generally welded, so too are some of the smaller non-high-tensile steel rails produced commercially. Larger commercial rails are clipped; hard clips are used unless the duty of the crane is arduous when soft spring clips are employed. Whether welded or clipped, the rail is attached by pairs of fastenings positioned at regular intervals along its length.

In this chapter, a study is reported of the effect of different loading
interfaces, primarily of different rail sections, on the transmission of a wheel load through to an underlying web panel of a crane box-girder. The purpose of this is to establish a method for estimating the lengths of simple uniform patch loads that are approximately equivalent to the true patch loads on crane web panels, in order that buckling load calculations may be made.

2.2. Previous Work

Some studies relating to patch loading on crane web panels have been conducted in Germany. Schindler [21] comments on the importance of gaining information on wheel load distribution through the crane rail in order to assess the risk of web panel buckling and to design connections between the flange and web.

He reports a number of rules, calculations, and measurements regarding the distribution of vertical stress produced by a wheel load in a web panel of a 120 tonnes capacity torsion-box-girder foundry crane. Two stress distributions obtained from the rulings of an early German crane code (DIN 120), now superseded by DIN 15018 [5], are presented. In these methods, an initial contact length between wheel and rail of 50 mm is assumed: in one case the load is then assumed to fan-spread through the rail and flange at an angle of 45° to the horizontal, in the other the spread is wider at 30°. Three theoretical curves are also presented: one considers an infinitely long rail supported elastically by the web plate, the other two are based on a Fourier analysis but assume two different initial contact lengths, one 50 mm, the other 230 mm. These distributions are compared with a curve produced from strain gauge measurements.
None of the curves compared favourably with the measured distribution and Schindler dismissed this plot as having been affected by the presence of a "rib" attached to the box. The reasoning behind this dismissal is not apparent. Experimental measurements were repeated on another 70 tonnes crane without such a rib and better agreement was achieved with the Fourier analysis results. However, it is unclear what contact length was assumed between wheel and rail in the analysis but it appears this may have been 230 mm which is surely very unrealistic.

For a number of reasons it is rather difficult to draw any firm conclusions from Schindler's work. His results relate to only two individual cranes and in both cases the type of rail is not specified, making it impossible to assess the influence of different rails. The poor agreement between the experimental and the theoretical results has already been discussed and this throws into question the experimental method. Concern has also already been expressed over the size of the initial contact length assumed in the favoured theory. However, since this theory, and most of the remainder of the source material, is relatively inaccessible German work dating back to the 1920's and 1930's it has not been possible to review Schindler's work further in the present study.

Steinhardt and Schulz [22] have studied the effect on wheel load distribution of incorporating an elastic pad between the rail and flange of a crane girder. Such pads are sometimes used to alleviate the severity of stress which can develop in the girder beneath the rail, thereby reducing susceptibility to the types of damage reported by Senior and Gurney [13]. Experimental measures are presented of the stiffness of three types of rail pad: one made of rubber, one of
poplar wood, the other plywood. Using a modified version of the theory of a beam on an elastic foundation, they produce two curves showing the distribution of load onto the flange of a crane girder resulting from inclusion of the rubber pad; one curve corresponds to a flat bottom rail, the other to a square bar rail. These curves show good agreement with experimentally measured distributions. Two further curves are presented, calculated using the same theory, which illustrate the more concentrated distributions produced by loading of the rails in the absence of the underlay. No corresponding experimental curves are shown for these cases.

As regards direct compressive stresses in the web, Steinhardt and Schulz advocate use of the expression

\[ \sigma_m = 0.318 \frac{P}{t} \left( \frac{t}{I_f + I_r} \right)^{1/3} \]

obtained from the theory by Parkes [18], for determining the maximum stress in the web when no rail underlay pad is present. Steinhardt and Schulz point out that this expression can also be derived from the results of an analysis presented by Rieve[23]. It is of note that in this expression the second moment of area term is given as the algebraic sum of that for the flange, \( I_f \), and that for the rail, \( I_r \). For cases where a rail pad is present, the authors develop a theory which is essentially a combination of Girkmann's theory [19] and the modified theory of a beam on an elastic foundation used by them to determine the distribution of load onto the flange.

They present computed and measured stress distributions at two horizontal levels in the web of an I-section, fabricated plate crane girder. Two rail cases are considered: a flat bottom rail (second moment of area =
888 cm$^4$) and a square bar rail (second moment of area = 32 cm$^4$). The rails were loaded both with and without the rubber underlay present. Generally, reasonable agreement was found between theory and practice, although the results for the flat bottom rail tended to be poorer than those for the square bar rail. It is presumed that, since the rails were interchanged and an underlay incorporated, the rails were fixed by clipping in all cases. Steinhardt and Schulz indicate that only the central 300 mm of the 500 mm wide flange plate was effective in transmitting the wheel load in the case of the small square bar rail, but that the full width was effective in the case of the flat bottom rail. They add, however, that incorporating this into the second moment of area term in the theory leads only to a negligible increase in the predicted maximum stress.

The main conclusion from the work by Steinhardt and Schulz is that by using an elastic underlay pad, maximum local web stresses can be reduced by 20% - 40% compared with when no pad is present. Unfortunately they make no attempt to obtain from their results estimations of equivalent uniform patch loads suitable for buckling calculations on crane web panels.

Mendel [24] has developed the work of Girkmann [19] and Rieve [23] to arrive at expressions for bending stresses produced in the outer fibres of a rail and flange subjected to a wheel load. These expressions closely resemble that derived by Parkes [18] for the bending stress in the outermost fibre of a beam flange subjected to a concentrated load. Parke's equation may be expressed as

$$\sigma_b = 0.4644 \frac{Py_f}{t} \left( \frac{t^2}{I} \right)^{1/4}$$
where \( y_f \) is the distance from the neutral axis of the flange to the outermost fibre. Mendel's final expression for bending stress in the flange of a fabricated I-section plate girder loaded through an overlying rail is

\[
\sigma_b = 0.452 \frac{Py_f}{t} \left[ \left( \frac{t}{1.15 I_f + I_r} \right)^2 \right]^{1/3},
\]

while that for bending stress in the rail is

\[
\sigma_b = 0.485 \frac{Py_r}{t} \left[ \left( \frac{t}{1.15 I_f + I_r} \right)^2 \right]^{1/3},
\]

where \( y_r \) is the distance from the rail neutral axis to its outermost fibre. These two equations apply to the case of a rail that is free to slide (clipped). For a fixed rail (welded), the expressions are as above except that, in each case, the inertia term \((1.15 I_f + I_r)\) is replaced by \((1.15 I_{fr})\) where \(I_{fr}\) is the second moment of area of the integral rail/flange unit. Mendel comments that the requirement for the factor 1.15 lies in the work of Rieve [23] as being an allowance for the fact that the rail and flange are connected to the web. It should be mentioned, however, that no experimental corroboration is presented by Mendel to support his work.

Senior and Gurney [13] have illustrated that the severity of local stresses in the upper part of welded crane girders, including direct stresses in the web, depend on wheel load alignment and the conditions of fit-up between the rail and flange. In practice, crane rails are frequently rolled with a transverse camber on the underside. When
mounted on a flange distorted by welding this leads to the rail
sometimes bearing onto the girder flange along lines towards the
edges of the rail base, when transverse bending of the flange about
the web takes place, and sometimes bearing onto the flange directly
above the web. Transverse flange bending has the effect of distributing
the wheel load over a greater web length than would otherwise result,
thus producing smaller direct stresses in the web. They present
a method, based on idealisations of the stress distributions produced
by Parkes [18] and Biot [17], for calculating local stresses in the
flange and web resulting from extremes of rail and flange fit. Their
calculated results indicate that local stress levels can vary
significantly depending on rail fit and plate thicknesses. The presence
of a rubber pad is shown to equal out these variations in local stress
and lead to overall reductions in stress levels. Mendel [25] and
Sedlmayer [26] have also presented studies of the effects of rail fit
and of incorporating an underlay. Both these pieces of work are based
on the results of Steinhardt and Schulz [22] reviewed earlier.

Unfortunately, a large majority of the previous work reported in the
literature and reviewed here is untranslated German material. Consequently,
this is often difficult to comprehend and much of the reference material,
particularly early or special reports, is relatively inaccessible. The
general conclusions from the present review, however, are that an experimental
investigation is required in order to establish a method for predicting
patch loading lengths on crane web panels; to be reliable, the method
needs to take due account of different crane rails and web thicknesses;
the classical theory by Parkes [18] may provide a sound basis for such
a method.
2.3. **Experimental Investigation of Patch Loads**

2.3.1. **The Test Equipment**

The physical confines of the laboratory and limitations of the available test facilities made it necessary to direct attention towards using a scale model box-girder for the experimental work. Whilst requiring a test box section of manageable size, it was important also to ensure that it represented a true likeness - a small scale model would undoubtedly lead to difficulties over realistic proportioning of individual box elements, weld sizes, and residual stresses. Since this study concerns the localised behaviour of an individual web panel under the action of a wheel load, only a short section of an overall box-girder is required. It was therefore possible to employ a model, based on typical cranes in the 15-20 tonnes range, of one-third scale without exceeding the capacity of available loading equipment.

The model box section is shown in Fig. 2.2. It was fabricated from mild steel plate using small fillet continuous welds. The central (test) web panel, which is bounded on either side by the two inner diaphragms, has an aspect ratio of 1.5. Two outer diaphragms were incorporated to provide stiffened support positions remote from the panel base. A wheel load, simulated by a radiused-head attachment to a hydraulic ram, was applied at the midspan of the section, directly above the test web. The test panel experiences only symmetric shear (no skew shear) and is supported by parabolically distributed shear along its two vertical edges. Owing to the short span of the section, longitudinal bending stresses are small. A box structure was utilised to maintain the same boundary conditions to the panel as exist in practice.
Diaphragm plates 3 thick

Fig. 2.2. SCALE MODEL BOX-GIRDER SECTION
Electrical resistance strain gauges were attached to both the inside and outside faces of the web panel in order to monitor in-plane vertical stress distributions. An access hole had been cut in the rear web panel when the box was fabricated to enable gauges to be mounted on the inner face of the test panel. The locations of the strain gauges are shown in Fig. 2.3. A pair of two-gauge rosettes were mounted at each recording position such that a rosette on one face of the panel had a mirror image on the other. One gauge of each rosette lay in the vertical direction, the other in the longitudinal direction. The upper line of rosettes was positioned close to the top flange but clearing any weld irregularities. The two gauged positions at the bottom of the panel were to facilitate experimental evaluation of bending stresses arising from bending of the section between its supports. In all, 24 rosettes, comprising 48 gauges were used.

Three model steel rails of different cross-sections were selected to test their load distributing effects when mounted on the box. The three rails are shown in Fig. 2.4 and their geometrical properties listed in the accompanying Table 2.1. The two component parts of Rail 2 were welded together to ensure an integral action. Rails 1 and 2 have a similar second moment of area about their respective horizontal neutral axes but different heights, while Rails 2 and 3 have the same height but different second moments of area. These combinations enable the influence of rail height, height to centroid, and rail stiffness to be investigated.

Bolting was chosen as the method for attaching the rails to the box. This models the hard clipping technique used in practice and readily allowed the rails to be interchanged. Pairs of bolts were positioned
Recording position numbers circled

Fig. 2.3. STRAIN GAUGING OF THE TEST WEB PANEL
Table 2.1 MODEL RAIL PROPERTIES

<table>
<thead>
<tr>
<th></th>
<th>Height of Rail mm</th>
<th>Height of Centroid mm</th>
<th>Second Moment of Area $\times 10^4$ mm$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail 1</td>
<td>38</td>
<td>19</td>
<td>4.34</td>
</tr>
<tr>
<td>Rail 2</td>
<td>25.4</td>
<td>9.9</td>
<td>3.96</td>
</tr>
<tr>
<td>Rail 3</td>
<td>25.4</td>
<td>12.7</td>
<td>6.94</td>
</tr>
</tbody>
</table>

Fig. 2.4. MODEL RAIL SECTIONS
symmetrically about the box section midspan at representatively scaled intervals of approximately 200 mm to fix the rails to the flange along the entire length of the box section. This required small feet to be welded onto Rail 1.

2.3.2. The Test Procedure

The in-plane vertical compressive stress at each recording position is given by

\[ \sigma_{yy} = \frac{E}{1-\nu^2} (\varepsilon_{yy} + \nu \varepsilon_{xx}) \]

The vertical and longitudinal strains, \( \varepsilon_{yy} \) and \( \varepsilon_{xx} \), are the average values of the strains from corresponding pairs of gauges on the inner and outer faces of the panel. To avoid recording each individual gauge output and then averaging, the facility to sum (or difference) pairs of strain readings electrically before recording was incorporated. This had two advantages: it halved the amount of data to be recorded and halved the inherent rounding error of the in-plane strains. To minimise the effect of any fluctuations in applied load and ambient temperature during testing, a high-speed data logger was used.

With the box positioned in the test frame beneath the ram as shown in Fig. 2.2, a load was applied and the data logger activated. It recorded the 24 outputs from respective gauge pairs together with the applied load from an electric load cell positioned between the ram and box in 2.5 seconds. Data were thus quickly compiled for a range of loads, both ascending and descending.
Some instability was detected in the readings from strain gauge position 1, a fault which remained undiagnosed. As this occurred at an extreme position in the panel, the results from it were disregarded throughout the testing programme.

Each of the three rails was tested in turn when bolted directly to the box. Loading was restricted to a level of approximately 50 kN in each case to avoid local yielding of the web. The procedure was then repeated for each rail but with a 3 mm thick neoprene rubber underlay incorporated between the rail and flange, the assembly again being bolted together. For comparative purposes, the box when loaded directly onto the flange, in the absence of any rail or underlay, was also recorded. Loading for this case was restricted to 15 kN, again to avoid local yielding of the girder.

2.3.3. Results

A simple computer program was written to calculate the vertical in-plane stress at each of the upper gauged positions from the experimental data. The program provided a graph plotting option which presented the results in the form shown in Fig. 2.5 where the stress at each monitored position is plotted against applied load. The stress for unit applied load is given by the slope of each line.

Plotting these values with respect to position along the web panel gives the in-plane vertical stress distribution at the depth of the gauges for unit applied load. Bending stresses in the panel were estimated from the readings at the two lower recording positions, found to be small, and neglected.
Experimental points shown for recording position 5.

Fig. 2.5. WEB IN-PLANE VERTICAL STRESS RECORDINGS AGAINST APPLIED LOAD
Distributions arising from loading of the rails when bolted directly to the box are shown in Fig. 2.6. The similarity between the plots is apparent: they are symmetric about the line of action of the applied load, there is little variation between the respective peak values which occur beneath the load, and, in each case, the distributions diminish to a negligible stress level within a distance of approximately one-quarter of the panel length to either side of the axis of symmetry.

The stress distributions for the three rails when mounted on the neoprene underlay displayed a similar likeness to one another but were broader and had a less pronounced peak than those of Fig. 2.6. In contrast, direct loading of the box produced a markedly more localised plot with a pronounced peak. Fig. 2.7 compares typical stress distributions in the box web arising from the three loading interfaces investigated: loaded directly onto flange, loaded through rail (Rail 2), and loaded through the same rail but with an underlay pad present.

The distributions presented are of vertical in-plane compressive stress along the length of the web panel at a depth of 15 mm below the underside of the top flange. They correspond to an applied load of 1 kN; the units of stress are N/mm².

2.3.4. Discussion

Generally there was good linearity between the vertical in-plane compressive stress at each recording position and the applied load, as illustrated by the experimental points shown in Fig. 2.5 for recording position ③. In the tests where the rubber underlay was incorporated, however, there was a tendency for deviations from a linear relationship to develop with increasing load. At some of the recording positions stress levels increased more
Fig. 2.6. STRESS DISTRIBUTIONS IN WEB PANEL CORRESPONDING TO THE THREE RAIL LOADING INTERFACES.
Fig. 2.7 COMPARATIVE STRESS DISTRIBUTIONS FOR DIFFERENT LOADING INTERFACES
steeply at higher loads, at others, particularly those adjacent to rail bolting positions, stresses tended to level-off. The underlay was therefore having the effect of introducing a complicated redistribution of stress along the panel as the load increased. Weitsman [27], and Civelek and Erdogan [28], have reported on the tendency for a member such as a rail to lift away from a supporting medium at positions to either side of a local compressive load. The clamping pressure of the rail bolts opposing this would account for the non-linear development of stress detected in the results. Nonetheless, the deviations from linearity were not excessive and developed at higher loads only. Linear measures were thus taken as for the cases when the underlay was not present and the resulting stress distributions for a rail and underlay combination were as shown in the comparison curves of Fig. 2.7.

When the load was applied directly onto the flange, the linearised stress distribution corresponding to an applied load of 1 kN registered a peak stress of 6.4 N/mm², as shown in Fig. 2.7. This is of the order of theoretical expectations obtained by a simple radial distribution method given by Timoshenko and Goodier [29] who consider a concentrated vertical force acting onto a horizontal straight boundary of an infinitely large vertical plate. The distribution of load across the thickness of the plate is assumed uniform. For a load per unit thickness of $P'$, the radial compressive stress at any element distance $k$ from the point of load application is given by

$$\sigma_{ra} = \frac{2P'}{\pi} \frac{\cos R}{k}$$
where $R$ is the angle subtended between the line of action of the load and the line of length $k$ between the point of load application and the element. Now for the model test box, the vertical distance, including the 5 mm flange thickness, between the point of load application and the horizontal centreline of the strain gauge rosettes is 20 mm. For an applied load of 1 kN acting over the 3 mm thick web, the predicted compressive stress directly beneath the load, where $R$ is zero, is $\sigma_{Ra} = 10.6 \text{ N/mm}^2$. A lower experimental value, in this case 6.4 N/mm$^2$, would be expected owing to the load distributing effect of the flange and because the applied wheel load only approximates to a line force.

Fig. 2.7 shows that the presence of a rail (Rail 2) led to a reduction in the recorded peak stress of 43% compared with the value for loading directly onto the flange. The additional presence of the neoprene underlay caused a further reduction of 33% compared with the rail only value. This is in general agreement with the findings of Steinhardt and Schulz [22] whose web stress distribution profiles bear a close resemblance to those presented here.

Integration of the distributions of Fig. 2.6 over the thickness of the web enables the vertical reaction force offered by the web to be determined. For each rail case, this force is found to be 1.0 kN ($\pm$ 4%), illustrating that all the applied load is transmitted into the underlying web panel. Similarly, integration of the stress profiles of Fig. 2.7 for direct loading and loading through a rail and underlay gives forces of 1.0 kN ($\pm$ 8%).
2.4. Estimation of Equivalent Patch Load

The stress distributions of Figs. 2.6 and 2.7 illustrate the nature of the patch loads that bear onto the upper edge of crane web panels. It is more convenient for design purposes if these loads can be represented by an equivalent, or effective, uniform patch load. One method for estimating the length of such a load might be to take the distance between the points where the stress drops to half its maximum value. For the same total force to act, however, this does not necessarily result in the same maximum stress, which is an important design consideration. An improved definition, therefore, of an effective patch length is that length over which the peak stress would have to act in order to balance the total applied force. This can be expressed simply as

\[ c = \frac{P}{\sigma_m t} \]  

(2.1)

Existing codes in Britain and Germany estimate patch lengths by the rather different fan-spread methods briefly discussed earlier. The German DIN 15018 [5] method is shown in Fig. 2.8. It consists of an assumed contact length between wheel and rail of 50 mm followed by a uniform symmetric spread of the load at 45° to the depth of interest - usually the top edge of the web. Thus, at a depth \( s \) below the rail surface, the patch length is given by

\[ c = 2s + 50 \]  

(2.2)

British crane codes BS 466 [3] and BS 3579 [4] specify a similar uniform spread but with zero initial contact length and dispersal at an angle
Fig. 2.8. TYPICAL CODE ESTIMATIONS OF PATCH LOADING LENGTHS RESULTING FROM WHEEL LOAD DISTRIBUTION
which gives a patch length of three times the depth, as shown in Fig. 2.8. Thus, patch length is given by

\[ c = 3s \quad (2.3) \]

Patch lengths determined by Eqs. (2.2) and (2.3) depend, for a given flange, only on the rail height. These expressions are typical of those which appear in codes not only in Britain and Germany but also in America and Australia.

In 1971, a draft was released of the German code DIN 4132 [30] which contains two expressions for the maximum direct stress in the web of a crane gantry girder resulting from a wheel load. The expressions are

\[ \sigma_m = 0.3P \left[ \frac{t}{1.15I_f + I_f} \right]^{1/3} \]

for a rail that is considered free to slide, and

\[ \sigma_m = 0.3P \left[ \frac{t}{1.15I_{fr}} \right]^{1/3} \]

for a fixed rail. \( P \) is the greatest wheel load, including the appropriate dynamic loading factor.

If these expressions for peak stress are substituted into Eq. (2.1), the patch length is obtained as

\[ c = 3.33 \left[ \frac{1.15I_f + I_f}{t} \right]^{1/3} \quad (2.4 \ a) \]
and

\[ c = 3.33 \left[ \frac{1.15 I_f}{t} \right]^{1/3} \]  \hspace{1cm} (2.4b)

respectively. The case where a rail underlay is incorporated appears not to be considered in Ref. [30].

In a recent draft of a British Standard on the structural use of steelwork in buildings [31], which covers crane gantries, an expression for determining the length over which a wheel load is distributed at the web is given as

\[ c = A \left[ \frac{I_f + I_r}{t} \right]^{1/3} \]  \hspace{1cm} (2.5)

where \( A \) is a constant which takes the value of 3.25 when the rail is mounted directly onto the flange (there is, however, no mention of the method by which the rail is assumed to be fixed) and 4.0 when a suitable resilient pad is placed between them. This expression and those in the draft DIN 4132 appear to originate from the works of Parkes [18], Girkmann [19], and Rieve [23].

Eq. (2.1) has been used to evaluate the effective lengths of uniform patch loads on the test web panel equivalent to the experimental stress distributions of Figs. 2.6 and 2.7. These are compared with the estimates obtained using Eqs. (2.2) – (2.5) in Table 2.2.

In applying the DIN 15018 method given in Eq. (2.2) to the model box and rails it has been assumed that the 50 mm contact length between wheel and rail is intended to be representative of full-size crane wheels and rails,
<table>
<thead>
<tr>
<th>Experimental Peak Stress For 1 kN Load N/mm²</th>
<th>Patch Length c mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No Rail</td>
</tr>
<tr>
<td>6.40</td>
<td>52</td>
</tr>
<tr>
<td>3.84</td>
<td>87</td>
</tr>
<tr>
<td>3.66</td>
<td>52</td>
</tr>
<tr>
<td>3.52</td>
<td>136</td>
</tr>
<tr>
<td>2.46</td>
<td>136</td>
</tr>
<tr>
<td>Rail 1</td>
<td>103</td>
</tr>
<tr>
<td>107</td>
<td>107</td>
</tr>
<tr>
<td>107</td>
<td>107</td>
</tr>
<tr>
<td>Rail 2</td>
<td>77</td>
</tr>
<tr>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>77</td>
<td>77</td>
</tr>
<tr>
<td>Rail 3</td>
<td>91</td>
</tr>
<tr>
<td>91</td>
<td>91</td>
</tr>
<tr>
<td>91</td>
<td>91</td>
</tr>
<tr>
<td>91</td>
<td>91</td>
</tr>
<tr>
<td>Rail 2 + Pad</td>
<td>104</td>
</tr>
<tr>
<td>104</td>
<td>126</td>
</tr>
<tr>
<td>126</td>
<td>93</td>
</tr>
<tr>
<td>95</td>
<td>95</td>
</tr>
</tbody>
</table>

Table 2.2: Estimation of Effective Patch Lengths
consequently it has been reduced by a factor of three. Eq. (2.3) can be applied directly to the model box situation. The draft codes from which Eqs. (2.4) and (2.5) are obtained both consider crane gantries and as these are generally of I-section, \( I_f \) and \( I_{fr} \) can readily be determined. For box-girders, however, these values depend on the width of flange considered to be effective in transmitting the wheel load. A section symmetric about the loaded web has been assumed here (given by twice the distance from the web midplane to the nearer flange edge), amounting to a flange width of 80 mm. However, since the flange is always very much thinner than the rail is deep, the effect of the assumed width on the total second moment of area term is only small (as Steinhardt and Schulz [22] have observed).

The experimental results in Table 2.2 show the relatively small variation in effective patch lengths corresponding to each of the three rails. The lengths lie between 87 mm and 95 mm, their average value being 91 mm which is the length recorded for Rail 2. These patch lengths correspond to patch length parameters \( \delta \) (patch length/panel length) of between 0.14 and 0.16 (the average value being 0.15) for the test web panel which has a length of 600 mm. A much wider range exists between the cases for direct loading and loading through a rail-plus-underlay when the effective patch lengths are 52 mm and 136 mm, corresponding to patch length parameters of 0.09 and 0.22 respectively.

The results from the DIN 15018 method of Eq. (2.2) reveal that it rather lacks versatility to cater for anything other than standard rail sections and that it is applicable only over a restricted depth. It overestimated for the tall Rail 1 while underestimating for the other two rails, and overestimated in all cases when extended to the gauge depth. The
assumption on contact length between wheel and rail in this method seems somewhat arbitrary. Formulae for calculating the region of contact between curved and flat surfaces are given by Roark and Young [32]. An expression for the contract length \( L_c \) between a cylinder and a flat surface is given as

\[
L_c = 2.15 \left[ \frac{P D'}{rE} \right]^{1/2}
\]

Thus the contact length is seen to depend on the wheel load \( P \), the wheel diameter \( D' \), the width of the rail surface \( r \), and Young's modulus for the materials \( E \). For a crane with a 50 mm square bar rail, typical values might be \( P = 150 \text{ kN} \), \( D' = 700 \text{ mm} \), \( r = 50 \text{ mm} \), and \( E = 207 \text{ kN/mm}^2 \), giving a contact length of approximately 7 mm. This suggests that as well as being seemingly arbitrary, an assumption of 50 mm for the contact length is rather excessive.

The results from the BS 466 and BS 3579 approach given by Eq. (2.3) follow a similar trend to, but in all cases are greater than, those from the DIN 15018 method. Poorer agreement with the experimental values is thus achieved in all cases except for the predictions at the web weld depth for Rails 2 and 3. Worst agreement occurs for Rail 1 with over-estimations for the patch length of 48% at the web edge and 100% at the depth of the gauges. Schindler [21] has commented on the uniform fan-spread hypotheses as being unsatisfactory for estimating wheel load distributions.

Predictions from Eq. (2.4 a), derived from the draft DIN 4132 expression for maximum web stress for a sliding rail case, and Eq. (2.5), the draft BS specification for a directly mounted rail case \( A=3.25 \), give very
good agreement with the experimental results for the three rails; the predicted values are all within 15%. The draft BS specification for the case where an underlay pad is present (A=4.0) appears to underestimate the patch length but the code does specify "a suitable resilient pad not less than 5 mm thick", which is a typical thickness for crane rail pads (see Ref. [33]); the neoprene rubber underlay used in the tests, being 3 mm thick, probably therefore served to model a pad of excessive load distributing capability. The reformulated draft DIN 4132 expression for a fixed rail case, given by Eq. (2.4 b), overestimates for all the rails. This is not surprising since it is based on the assumption of an integral rail and flange assembly which is likely to be achieved only when the rail and flange are welded together.

Of the methods investigated, equivalent patch loadings for cases of directly mounted rails are best predicted by Eqs. (2.4 a) and (2.5). It may well be, however, that for rails that are welded instead of bolted to the flange then Eq. (2.4 b) would best predict the effective patch lengths. In view of the present findings it is a little surprising to note that the full DIN 4132 [10], published subsequent to the draft [30], reverted to the fan-spread hypothesis for estimating wheel load distributions. There appears to be no meaningful correlation between patch length and rail centroidal height.

As regards the presence of a rail pad, it is unlikely that full account can be taken of the various types and stiffnesses of crane rail pads available (see, for example, Ref. [33]) by simply assigning a single value to a constant as the draft BS specification of Eq. (2.5) suggests. However, on the basis of the present findings and those of Steinhardt and Schulz [22]
it does appear that the draft BS method provides a reasonable and
generally safe allowance for typical pads, as the following discussion
illustrates. By changing $A$ from 3.25 to 4.0 in Eq. (2.5), the predicted
patch length, $c$, is increased by a factor of 1.23, or 23%. Now the
predicted vertical stress, assumed to act uniformly over this length
of web, is evaluated by rearranging Eq. (2.1) thus

$$\sigma_m = \frac{P}{ct}$$

Therefore, a 23% increase in $c$ to take account of a rail pad leads
to a reduction in the predicted bearing stress on the web by a
factor of $1/1.23 = 0.81$, or 19%. This compares with reductions of 33%
found here and 20% - 40% found by Steinhardt and Schulz.

On a more general note, Parkes [18] has shown that although his analysis
is based on plates of infinite depth, it is sufficiently accurate for
most plates of practical depths. His theory considers plates of finite
length but this length is always sufficiently large to accommodate all
the distributed load. It therefore appears that expressions such as
Eqs. (2.4) and (2.5) can be considered to hold generally for crane webs
supporting wheel loads, provided the resultant patch length is shorter
than the panel length - as the present test results indicate. This
implies, however, that patch lengths are independent of the panel
length which, for a given crane box, suggests the patch length is not
affected by the panel aspect ratio.

A consequence of this is that as the aspect ratio of a panel is
changed then the patch length parameter changes also, the patch length
remaining constant. For example, had the aspect ratio of the model box
web panel been 1.0 instead of 1.5, the estimated patch lengths would remain unaltered but the average value of the patch length parameter for the three rail cases would then change from 0.15 to 0.23. Now the British crane code BS 2573/1 [2] limits aspect ratios of web panels to a maximum of 1.5; in practice they generally lie between 1.0 and 1.5. Hence, on the strength of the experimental evidence here, it can be concluded that effective patch lengths on crane web panels with directly mounted rails lie typically between 0.15 and 0.23 times the panel length. Experience of rail and web combinations used in practice, however, suggests that (using Eq. (2.5)) patch length parameters as low as 0.07 may arise for a small rail supported by a thick web panel of aspect ratio 1.5, but that they are unlikely to exceed 0.25. This parameter is important because it plays a direct role in determining the buckling load of the web panel.

2.5. Web Panel Buckling Under Patch Loading

The elastic buckling load of a rectangular plate subjected to an in-plane uniform patch load on one edge which is resisted by parabolic distributions of shear stress on the adjacent edges, as shown in Fig. 2.9, can be given by

\[ P_{cr} = K \frac{\pi^2 D}{d} \]  

(2.6)

where \( P_{cr} \) is the critical value of the applied edge load \( P \), \( D \) is the plate bending stiffness, and \( K \) is a non-dimensional buckling coefficient, the value of which depends on the plate aspect ratio \( \alpha = b/d \), the patch
Fig. 2.9. PATCH LOADED PLATE SUPPORTED BY END SHEARS
length parameter \( \beta = c/b \), and the edge conditions to the plate.

Eq. (2.6) can be expressed in terms of buckling stress by

\[
\sigma_{cr} = \frac{P_{cr}}{ct} = K \frac{\pi^2D}{cd^t}
\]  

(2.7)

A similar expression, leading to a differently defined buckling coefficient, commonly used is

\[
\frac{P_{cr}}{bt} = K' \frac{\pi^2D}{d^t}
\]  

(2.8)

The relationship between the coefficients cited in Eqs. (2.7) and (2.8) is given by

\[ K = \alpha K' \]

The parameter \( K \) defined by Eq. (2.6) is particularly convenient for plotting over wide ranges of panel aspect ratios and patch length parameters and for permitting easy physical interpretation of panel buckling. Khan, Johns, and Hayman [34] have calculated values for this coefficient for various plate and loading geometries. Wilkesmann [15], Rockey and Bagchi [35], and Protte [36] have presented solutions for the coefficient defined by Eq. (2.8). In all cases, a panel with simply supported edges is considered. None of these works, however, covers comprehensively the panel and loading geometries of relevance to EOT crane web panels.
In Chapter 3, the results of an extensive parametric study of patch loaded plates with various edge conditions are presented which provide the required information. In Fig. 3.2, curves are given for the coefficient $K$ in Eq. (2.6) for simply supported plates with aspect ratios from $1/3$ to 4 and patch length parameters from $1/8$ to 1. It is shown there that narrowing the patch load relative to the panel length reduces the coefficient values, and therefore the loads to cause buckling, but that the proportional reduction decreases the narrower the patch load: changing the patch length parameter from 1 to $3/4$ produces significant reductions in $K$ whereas changing from $1/4$ to $1/8$ leads to only slight reductions. An investigation of still narrower loads revealed that the trend continues: coefficients for $c/b = 1/16$ were lower than those for $c/b = 1/8$ by only $1\%$ when $b/d = 1$, $2\%$ when $b/d = 2$, and about $5\%$ when $b/d = 4$. The result for $c/b = 1/33$, and $b/d = 1$ was only $1/4\%$ below that for $c/b = 1/16$.

This indicates that in the range $1.0 \leq \alpha \leq 1.5$ applicable to crane web panels there is very little reduction in the load $P_{cr}$ required to cause elastic buckling if $\beta$ is reduced below about $1/4$. Over this range, the buckling coefficients for $\beta$ can be closely approximated by the formula

$$K = \frac{0.87 + 2.55}{\alpha} \quad 1.0 \leq \alpha \leq 1.5, \beta = 1/4 \quad (2.9)$$

Over the same range, the $\beta = 1/8$ curve is closely fitted by

$$K = \frac{0.95 + 2.30}{\alpha^3} \quad 1.0 \leq \alpha \leq 1.5, \beta = 1/8 \quad (2.10)$$
Results from the preceding Section 2.4 indicated that crane girder webs are subjected to patch loads of lengths generally occupying less than $\frac{1}{4}$ of the panel length. Elastic buckling calculations for crane web panels can therefore be performed using Eq. (2.6) in conjunction with the formulae of Eqs. (2.9) and (2.10); the second formula may be preferred since it gives a more conservative estimate than the first, but the two equations give $K$ values which differ by only 4% to 5%.

For the experimental web panel which measures 600 mm long, 400 mm deep, and 3 mm thick, a typical effective patch length was 90 mm. This gives $a = 1.5$, $b = 0.15$ and thus $K = 2.6$ approximately from Fig.3.2 or Eq. (2.10). Therefore, taking $E = 207$ kN/mm$^2$ and $v = 0.3$, Eq. (2.6) gives for the buckling load, $P_{cr} = 33$ kN.

From Eq. (2.7), the average stress when buckling occurs is $\sigma_{cr} = 122$ N/mm$^2$. A typical yield stress for the web is 250 N/mm$^2$ and so, on this basis, elastic buckling would be able to develop appreciably before any local web yielding occurred.

It is worth noting that had the panel been subjected to a narrower patch load of say $c/b = 0.04$ ($=1/25$) instead of 0.15 then $K$ would change only to about 2.5 and the buckling load drop very slightly to 32 kN but the corresponding average bearing stress on the web leaps to 444 N/mm$^2$ - well above the yield stress. Elastic buckling cannot then take place.

The calculations and discussion in this section relate to a web panel assumed simply supported along all edges and subjected to a uniform patch load equivalent to that arising from a typical distribution of a wheel load by an overlying crane rail and flange. The suitability of the equivalent patch hypothesis as a basis for buckling calculations is
examined at the end of Chapter 3. Other factors relating to the presence of the rail and flange which may have an influence on the critical load of the panel are not taken into account.

Rockey and Bagchi [35], in a paper mainly concerned with finite element calculations for patch loaded plate panels, give a brief account of some further calculations for a case where a uniform patch load is applied to the flange of a plate girder, instead of directly to the web. They comment that in such a case the buckling resistance of the panel would be expected to increase due to the flange causing some further distribution of the patch load and providing some additional in-plane restraint. However, a larger effect is probably produced by the torsional stiffness of the flange restraining to some extent rotation of the top edge of the web, which was previously considered simply supported.

Rockey and Bagchi present results of finite element calculations which incorporate flange flexural and torsional properties for one case only - that of a square web panel with a patch load of $c/b = 0.2$ on the overlying flange. A curve is given of buckling coefficient against the ratio flange thickness/web thickness ($t_f/t_w$) for $t_f/t_w$ values from 2 to 8. The buckling coefficient is that defined by Eq. (2.8) but since the results are for a panel with $a = 1$, the coefficient is identical to $K$ in Eq. (2.6). The presented curve is a smooth, slightly upward-turning curve which shows $K$ to increase from approximately 7.7 at $t_f/t_w = 2$ to approximately 8.8 at $t_f/t_w = 8$. Now the $K$ value for the panel in the absence of a flange (ie $t_f/t_w = 0$) is readily estimated from Fig. 3.2 or Eq. (2.9) to be approximately 3.4. Thus the Rockey and Bagchi curve indicates that a flange of thickness $t_f = 2t_w$
more than doubles \( K \) (and, therefore, \( \frac{P}{Cr} \)), while a flange of thickness \( t_f = 8\ t_w \) has relatively little additional effect. Plotting the point \( \frac{t_f}{t_w} = 0, \ K = 3.4 \) on the graph appears to indicate an anomaly: the form of the presented curve is such that for it to pass through this point also there must be an extremely abrupt change of direction over the range \( \frac{t_f}{t_w} < 2 \).

The calculations by Rockey and Bagchi were performed with conventional plate girders in mind, for which flange stiffness is readily determined. For a box-girder it is more difficult to assign effective values for the in-plane and particularly the torsional stiffness provided by the flange. A further complication is that for a crane box-girder much of the in-plane stiffness is provided by the rail while the torsional stiffness is provided predominantly by the flange, whose thickness is usually not more than twice the web thickness and is often rather less than that. In the absence of further data, particularly for \( \frac{t_f}{t_w} < 2 \), it is difficult to see how Rockey and Bagchi's results can be directly applied to a crane box-girder.

In an attempt to assess the buckling load and mode of buckling of the test box web panel with rail and flange effects included, further experiments were conducted.

2.6. **Experimental Investigation of Panel Buckling**

2.6.1. **Southwell Plots**

Out-of-plane deflections, \( w \), of the web panel were measured for increasing load, \( P \), at a position 160 mm below the underside of the top flange and on the line of action of the load. To locate the deflection transducer,
a cross-member was mounted on the web using electro-magnets placed at the
two inner diaphragm positions. This is shown in Fig. 2.10 which gives
a general view of the experimental equipment used for the investigations
reported in this chapter. Readings were taken with Rail 2 bolted to
the box; an end view of this arrangement is shown in Fig. 2.11.

The development of out-of-plane deflections with increasing load
is shown in Fig. 2.12. When the results are arranged as the Southwell
plot in Fig. 2.13 the buckling load is readily estimated from the
inverse slope to be approximately 85 kN. Repeating the procedure for
direct loading of the box (no rail present) gave the growth of deflections
shown in Fig. 2.14 and the Southwell plot of Fig. 2.15. There is clearly
little possibility of estimating the buckling load from this plot.
Southwell plots were also produced for the cases when Rails 1 and 3
were bolted to the box. Although the linearity of these plots was
not quite as good as that for Rail 2, inverse slope readings indicated
buckling loads of approximately 75 kN for the Rail 1 case and a little
under 70 kN for Rail 3.

The Southwell method offers a very simple procedure for measuring buckling
loads but because it is based on small-deflection approximations (and
thereby falls into the category of linear or neutral buckling theory)
it is applicable strictly only to structures which do not violate
the underlying assumptions. A further assumption inherent in the
Southwell method is that, as the critical load is approached, the fundamental
mode predominates in the deflection. Testing of practical structures
with initial geometric imperfections often means that for the fundamental
mode to predominate, experimental observations have to be taken at values
of deflection well beyond the small deflection range. Roorda [37] has
discussed in general terms the extent to which non-linearities affect
the Southwell plot. He concludes that when the method is applied to a
Fig. 2.10. GENERAL VIEW OF EXPERIMENTAL EQUIPMENT USED FOR ELASTIC INVESTIGATIONS
Fig. 2.11. END VIEW OF TEST BOX WITH RAIL 2 ATTACHED
Fig. 2.12 TRANSVERSE DEFLECTION OF WEB PANEL WHEN LOADED THROUGH RAIL
Fig. 2.13. SOUTHWELL PLOT FOR WEB PANEL WHEN LOADED THROUGH RAIL
Fig. 2.14. TRANSVERSE DEFLECTION OF WEB PANEL WHEN LOADED DIRECTLY
Fig. 2.15. SOUTHWELL PLOT FOR WEB PANEL WHEN LOADED DIRECTLY
structure with a post-critical deflection curve other than the neutral (horizontal line) type, a curved plot results which requires careful interpretation in the knowledge of the post-buckling behaviour of the idealised structure.

Plates have stable-symmetric post-critical deflection paths which are markedly non-neutral. Plates also frequently have large initial geometric imperfections, particularly those which have undergone welding. Spencer and Walker [38], in a paper which looks at various situations where the Southwell technique fails to give a straight line, propose alternative techniques involving a 'pivot-point' concept formulated to help linearise Southwell plot data for plates and provide better estimates of buckling loads. They classify cases where the Southwell plot fails to give a straight line into low-load and higher-load non-linearities. Low-load non-linearities are attributed to errors in registering true zero deflection where only a small error can produce significant inaccuracies in the apparent critical load. An example shows that these initial effects can extend to data points for loads as high as 80% of the critical load and that readings for still higher loads then have to be used to avoid large errors in the Southwell prediction. Non-linearities at higher loads are particularly noticeable for plates owing to their post-buckling behaviour, but are caused also by variations in boundary conditions and plasticity.

Three modifications to the Southwell technique for plates, two graphical and one numerical, have been taken from the work by Spencer and Walker [38] and applied to the data for the model box panel. Each modification progresses in refinement to cater for plates with larger imperfections. Only slight improvement to the plot of Fig. 2.15 for direct
loading of the box was achieved by the graphical methods but this was insufficient to produce any discernable linear slope of the Southwell type. The numerical method predicted buckling loads which varied widely depending on which data points were used as pivot points. Moreover, when applied to the data of Fig. 2.13, the graphical methods actually served to de-linearise the plot.

The failure of these modifications to linearise the Southwell plot of Fig. 2.15, may well be due to the very limited range of load (0-15 kN) which was applied to the box in order to prevent yielding in the loaded region. The Southwell method only claims accuracy as the critical load is approached, which was not the case for the direct loading test, and the modifications were designed more for data extending beyond the critical load. That the alternative techniques produced plots which were less linear than the original Southwell plots for the with-rail cases, however, must throw into question their use in this context.

For both the Southwell and the alternative 'Spencer' plot techniques, imperfections play a crucial role in determining the accuracy of the methods. Other than attempting to introduce a minimum of residual stresses into the model box during fabrication by keeping the welds small and using heat sinks, no special precautions were taken as regards geometric imperfections of the web panel.

Rockey [39] has expressed reservations about the usefulness of Southwell plot results for plates unless an accurate measure is obtained of the initial imperfections.
According to Spencer [40], however, it is not the imperfection in out-of-plane flatness which governs the actual behaviour of a plate but the effective imperfection which takes into account factors such as load eccentricity as well. If this is indeed so then the behaviour of the test web panel is influenced by a loading eccentricity which depends not only on initial wheel/rail/web misalignments but, owing to torsioning of the box as it is loaded, on the magnitude of the applied load also.

Clearly, substantial evidence exists of problems encountered in applying Southwell techniques to plates, much of which has been borne-out here. Nonetheless, good linear Southwell plots were obtained for the test web panel when loaded through the test rails, particularly Rail 2. The estimated buckling load for this case of 85 kN is more than double the theoretical value calculated for the panel in the previous section. Although the evidence is not considered conclusive, the experimental results indicate buckling loads for the box panel when loaded through a rail which are roughly 2 to 2.5 times greater than the value predicted for the panel when assumed simply supported and subjected to a uniform patch load equivalent to a distributed wheel load.

A possible contributory factor to the higher experimental values is the influence of in-plane restraint provided by the flexural stiffness of the rail. However, it could be argued here that it is only once the panel has buckled that any additional in-plane restraint can be provided by the rail because it is not until then that rail flexure develops. It will be recalled that the measured web stress distributions presented earlier illustrated that all the applied load was reacted by the web, indicating the rail simply transmits (rather than carries) the load.
Probably of greater significance is the effect of rotational restraint along the horizontal panel edges provided by the box flanges. As far as the author is aware, however, buckling coefficient information for patch loaded plates with rotationally restrained edges is available only for a few isolated cases, none of which is representative of the crane web panel situation; it was to rectify this situation that the work of the following chapter was conducted.

An indication of the existence or otherwise of edge moment restraint on the test panel can be obtained by monitoring buckling profiles.

2.6.2. Buckling Profiles

Transverse deflections along the vertical centreline of the web panel were measured using ten high precision linear displacement transducers. These were mounted on a special strut attached to the box flanges; the strut was designed to accommodate relative vertical displacement between the top and bottom flanges. Any rigid body movement of the strut relative to the web plane caused by rotation of the top flange was monitored using an eleventh transducer and the outputs from the remaining ten transducers corrected accordingly.

Buckling profiles corresponding to three different loads applied with Rail 2 mounted on the box are shown in Fig. 2.16; similar profiles were obtained with Rails 1 and 3. Each of the profiles has a crest close to the one-third depth position and the plots clearly indicate that the top and bottom edges of the panel experience moment restraint. The profiles are strikingly similar to some theoretical modes for a clamped plate under a partial edge loading calculated by Bagchi and Rockey [41] using a large-deflection finite element solution.
Fig. 2.16. VERTICAL BUCKLING PROFILES AT CENTRE OF WEB PANEL
2.7. **Conclusions**

Rail-supporting web panels of crane girders are subjected to non-uniform patch loads on their upper edge arising from the dispersal of a wheel load by a rail and flange. Experimental recordings of vertical in-plane compressive stress distributions along the top edge of a model box-girder web panel have been presented which reveal the nature of these loads and a simple expression has been used to equate them to equivalent uniform patch loads.

The results indicate that crane web panels are subjected to patch loads of effective lengths that generally occupy between one-quarter and one-fifteenth of the panel length. Although, over this range, the elastic buckling load of a panel is relatively insensitive to the patch length, the average bearing stress on the web is proportional to it.

Experimental estimations of effective patch lengths corresponding to rails of different geometries are well predicted by two similar expressions seemingly based on classical theory by Parkes [18]; one expression featured in a recent draft British Standard, the other in a draft German Standard. Uniform fan-spread estimations used in existing codes do not give particularly satisfactory predictions.

The factors indicated to have a fundamental affect on crane wheel load distribution and, therefore, direct stress in the web are the second moments of area of the rail and flange and the web thickness; however, effective patch lengths are not sensitive to small changes in the values of these terms.

Other factors reported elsewhere as affecting local stress levels
beneath a crane rail are the fit between rail and flange, and rail fixing methods. The presence of a rail pad is seen to lead to increased distribution of a wheel load and reduced local stress levels. For all the loading interfaces considered here, the vertical stress distributions recorded in the web illustrate that all the applied load is transmitted through to the web.

Investigations of panel buckling under patch loading reveal an apparent anomaly. Experimental estimations obtained using the Southwell method for the test panel when loaded through various rails by a wheel load gave buckling loads of 2 to 2.5 times the value predicted by theory for the panel when subjected to an equivalent uniform patch load and assumed simply supported all round. This discrepancy is probably primarily attributable to an inappropriate assumption on boundary conditions in the theory. Simply supported edges are almost certainly over-conservative for a box-girder web panel, particularly along the top and bottom edges where interaction with the flanges occurs. Experimentally recorded buckling profiles suggest moment restraint along these edges.

In the following chapter, the effect on the buckling load of clamping the edges of a patch loaded panel is investigated.
3.1. **Introduction**

Walker [42] has recently drawn attention to certain limitations in existing knowledge on the buckling of patch loaded plates. He has commented that the information available on buckling coefficients used in critical load calculations is generally restricted to plates with simply supported edge conditions. He added that although this may well constitute a safe design basis for plates with elastic moment edge restraints, existing methods for computing coefficients could be extended to provide more relevant results.

This deficiency in information has already been referred to in Chapter 2 where experimental evidence pointed to the simply supported edge assumption being unduly conservative with regard to the buckling load of a crane box-girder web panel subjected to a wheel load.

A simply supported edge condition prevents out-of-plane deflection of the plate edge but provides no rotational restraint. To achieve this physically demands exacting techniques such as that employed by Khan and Walker [43] who, in order to acquire experimental measurements of simply supported plates, mounted their test plates in slotted rollers that rotated freely on needle rollers housed in the plate supporting plattens.

For plates of practical structures this condition is usually somewhat unrepresentative but it tends to be assumed because it is generally conservative and is relatively easy to incorporate into theoretical analyses.

In the case of a fabricated plate crane box-girder, the box section is braced at regular intervals by full-depth diaphragms. Thus there is probably considerable restraint against rotation along the horizontal
edges of a web owing to interaction between the web and flange. The vertical edges of a web panel (provided by the welded connections to the diaphragms) probably experience rather less moment restraint but a simply supported assumption is again likely to be on the conservative side. For both the horizontal and vertical panel edges it is difficult, however, to quantify precisely the degree of elastic moment restraint acting on the panel.

In view of these various factors, a computer analysis has been undertaken by the present author to calculate elastic buckling coefficients for patch loaded plates with rotationally restrained edges. Consideration has been confined to a full elastic, or clamped, edge condition. This provides the upper limit to coefficient values and when compared with coefficients also presented here for simply supported plates gives the range of values applicable to plates with edges confined to remain in-plane (i.e. not free edges). To assess along which edge or edges the provision of rotational restraint has greatest influence on the buckling load, coefficients have been computed for plates as successive plate edges are clamped, the remaining edges being simply supported.

Buckling coefficients are presented for plates of aspect ratios from $\frac{1}{3}$ to 4 subjected to uniform patch loads with patch length parameters from $\frac{1}{8}$ to 1. The presented information is considered sufficiently comprehensive to cover most practical requirements. Some non-uniform patch loads modelling distributed crane wheel loads are also considered. Coefficients for these loads are shown to compare very favourably with those computed for the equivalent uniform loads, calculated by the method of Eqn.(2.1) described in Section 2.4.
3.2. Origin of the Method

The computing method used in the present analysis originates from the work of Alfutov and Balabukh [44]. They suggested an approach to studying buckling of thin flat plates subjected to partial edge loads which overcomes much of the mathematical complexity associated with the need, in such plate problems, to determine the prebuckling stress distribution throughout the plate. This was achieved by the formulation of what is essentially a Rayleigh-Ritz total potential energy method but one in which the stress distribution throughout the plate need satisfy only the conditions of equilibrium and not those of compatibility of the associated strains.

Khan and Walker [43] developed this method and, with the additional simplification of assuming a single buckling mode shape, undertook parametric studies of the buckling of rectangular plates subjected to localized loadings. They verified the approach by comparing their results with those obtained from a computer finite element solution. Previously, the complexity of an analysis involving determination of the actual prebuckled stress distribution had, for the most part, discouraged any parametric studies of this class of buckling problem. The use of finite element methods as an alternative approach for parametric studies is generally prohibitive owing to the attendant demands on computer requirements.

The efficiency and economy of the solution suggested by Alfutov and Balabukh led to this basic method later being further developed by Khan, Johns, and Hayman [34]. They produced a refined version which, although undoubtedly more complex than the earlier format by Khan and Walker, proved sufficiently economical in computer requirements to permit parametric studies of a wider range of patch loading problems involving more extreme plate geometries. Their results compared favourably with those
obtained by other workers. The revised method, which this time assumed the plate displacements to consist of a finite series, was subsequently employed to study the buckling of web plates under combined partial edge, shear, and in-plane bending stresses. The results were presented as interaction curves in a companion paper to Ref. [34] by Khan and Johns [45].

The present author has developed the computer analysis method beyond the stage reached by Khan, Johns and Hayman, to enable it to be used for parametric studies of patch loaded plates with some or all edges clamped. An additional extension, undertaken more with crane wheel loads in mind, enables non-uniform patch loads to be accommodated.

3.3. Method of Analysis

3.3.1. General Description

Since it is appropriate to the method of analysis, notation similar to that adopted by Khan, Johns and Hayman [34] has been used in the present analysis. Fig.3.1 shows the general plate and loading configuration considered. A rectangular plate of length $2L$, depth $2B$, and thickness $t$, is subjected to a uniform patch load $P$, of length $2C$, centrally placed on its top edge. Parabolically distributed shear stresses on the vertical sides of the plate resist the applied load. It should be noted that the $x$-direction is now along the central vertical axis of the plate and the $y$-direction along the central horizontal axis.

From the work of Alfutov and Balabukh [44], the potential energy of a plate subjected to prescribed in-plane stresses on its edges may be written as

$$U = U^{(B)} + U^{(1)} + U^{(2)} \quad (3.1)$$

The term $U^{(B)}$ represents the strain energy of bending of the plate.
Fig. 3.1. PLATE PARTIALLY LOADED ON TOP EDGE
If the plate is polygonal and supported on all edges - as with the rectangular plate with simply supported or clamped edges considered here - then

\[ U^{(B)} = \iint \left( \frac{D}{2} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 \right) \, dx \, dy \]  

(3.2)

The terms \( U^{(1)} \) and \( U^{(2)} \) are integrals which together represent the sum of the membrane strain energy and the potential energy of the external stresses.

\[ U^{(1)} = \iint \frac{t}{2} \left( \sigma_x \left( \frac{\partial w}{\partial x} \right)^2 + \sigma_y \left( \frac{\partial w}{\partial y} \right)^2 + 2\sigma_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right) \, dx \, dy \]  

(3.3)

\[ U^{(2)} = -\iint \frac{t}{E} \left( \sigma_x + \sigma_y \right) \nu^2 \phi \, dx \, dy \]  

(3.4)

Each of the three integrations is taken over the entire area of the plate. The values \( \sigma_x^*, \sigma_y^*, \) and \( \sigma_{xy}^* \) are components of a statically admissible stress field, that is, they must satisfy the equilibrium equations

\[ \frac{\partial \sigma_x^*}{\partial x} + \frac{\partial \sigma_{xy}^*}{\partial y} = 0 \]  

(3.5a)

\[ \frac{\partial \sigma_y^*}{\partial y} + \frac{\partial \sigma_{xy}^*}{\partial x} = 0 \]  

(3.5b)

as well as the conditions for equilibrium with the boundary stresses.

This stress field need not, however, correspond to strains that satisfy compatibility. For this reason, selection of a suitable distribution is not generally difficult, using, for example, simple polynomial functions of the coordinates \( x \) and \( y \). Since \( \sigma_x^*, \sigma_y^*, \) and \( \sigma_{xy}^* \) are in equilibrium with the boundary stresses, they must each be proportional to the intensity of the applied loading. Consequently, integrals \( U^{(1)} \) and \( U^{(2)} \) are also proportional to the load intensity.

Eq.(3.4) contains a separate stress function, \( \phi \), which is related to the out-of-plane displacements, \( w(x,y) \), by the Karman equation.
This stress function must also satisfy the homogeneous boundary conditions

\[ \frac{\partial \phi}{\partial x} = \frac{\partial \phi}{\partial y} = 0 \] (3.7)

The stress function \( \phi \), given by Eq.(3.6), is taken to consist of a further finite series

\[ \phi = \sum_{k=1}^{m} f_k \phi_k(x,y) \] (3.9)

in which each \( \phi_k(x,y) \) is a function satisfying the conditions of Eqn.(3.7).

An approximate solution of Eq.(3.6) may be obtained by the Galerkin method so as to give the coefficients \( f_k \) in terms of the \( a_i \).

The value of \( U \) is then determined entirely in terms of the \( a_i \).

When \( U \) is written as a quadratic function of the coefficients \( a_i \), then for buckling the condition that the matrix

\[ U_{ij} = \frac{\partial^2 U}{\partial a_i \partial a_j} \] (3.10)

be singular is investigated.

From Eqn.(3.1), the matrix \( U_{ij} \) may be expressed as

\[ U_{ij} = \sum_{k} U_{ij}^{(k)} + U_{ij}^{(1)} + U_{ij}^{(2)} \] (3.11)
where the three summed terms are simply the second derivatives of $U^{(B)}$, $U^{(1)}$, and $U^{(2)}$ with respect to the $a_i$. It is in terms of these three matrices that the computer calculations are performed. Since the stress distribution terms $\sigma_x^*, \sigma_y^*$, and $\sigma_{xy}^*$ are proportional to the applied load $P$, then, from Eqn.(3.3) and Eqn.(3.4), so too are matrices $U_{ij}^{(1)}$ and $U_{ij}^{(2)}$. Therefore, if $Q$ represents a loading parameter

$$U_{ij}^{(1)} + U_{ij}^{(2)} = QU_{ij}^{(Q)} \quad (3.12)$$

where $U_{ij}^{(Q)}$ is a matrix not dependent upon the applied loads. The buckling load calculation thus reduces to the linear eigenvalue type of problem

$$\det (U_{ij}) = \det (U_{ij}^{(B)} + QU_{ij}^{(Q)}) = 0$$

which can readily be solved by standard methods. (In the present analysis a simple determinant search method was used.)

When Eqn.(3.8) is introduced into Eqns.(3.2) and (3.3) and differentiated, the matrices $U_{ij}^{(B)}$ and $U_{ij}^{(1)}$ are readily obtained as

$$U_{ij}^{(B)} = \int D \nu^2w_i(x,y)\nu^2w_j(x,y)dxdy \quad (3.13)$$

$$U_{ij}^{(1)} = \int \left\{ \delta_{x} \frac{\partial w_i}{\partial x} \frac{\partial w_j}{\partial x} + \delta_{y} \frac{\partial w_i}{\partial y} \frac{\partial w_j}{\partial y} + \delta_{xy} \frac{\partial w_i}{\partial x} \frac{\partial w_j}{\partial y} + \delta_{xy} \frac{\partial w_i}{\partial y} \frac{\partial w_j}{\partial x} \right\} dxdy \quad (3.14)$$

Matrices $U_{ij}^{(B)}$ and $U_{ij}^{(1)}$ can be evaluated directly once a statically admissible stress field and a displacement distribution which satisfies the boundary conditions have been found.

The third term in Eqn.(3.11), $U_{ij}^{(2)}$, presents more difficulty to evaluate. It can be written in the form
Differentiating Eqn. (3.6) twice with respect to the $a_i$, and writing it in the Galerkin form yields the set of equations

$$ \sum_{k=1}^{m} F_{ijk} A_{k\ell} = G_{ij\ell}, \quad \ell = 1, 2, \ldots m; \quad i, j = 1, 2, \ldots n \quad (3.18) $$

where

$$ A_{k\ell} = \iint \frac{\delta^2 \Phi_k}{\delta a_i \delta a_j} \phi_{\ell} \, dx \, dy \quad (3.19) $$

and

$$ G_{ij\ell} = \iint E \left( \frac{\partial^2 w_i}{\partial x \partial y} \frac{\partial^2 w_j}{\partial x \partial y} - \frac{\partial^2 w_i}{\partial x^2} \frac{\partial^2 w_j}{\partial y^2} - \frac{\partial^2 w_i}{\partial y^2} \frac{\partial^2 w_j}{\partial x^2} \right) \phi_{\ell} \, dx \, dy \quad (3.20) $$

The components of $H_k$ are readily found from Eqn. (3.17); these need be evaluated only once since they are independent of $i$ and $j$. For particular values of $i$ and $j$, the corresponding vector $F_{ijk}$ is obtained by solving the linear equations of Eqn. (3.18). The summation of Eqn. (3.15) can then be performed to find the associated element of the $U^{(2)}_{ij}$ matrix.

Matrix $A_{k\ell}$ and vectors $G_{ij\ell}$ depend only on the geometry of the plate and not on its loading. This feature can be used to advantage when undertaking parametric studies as $A_{k\ell}$ and $G_{ij\ell}$ need be evaluated only once for a plate of a given geometry, irrespective of the range of loadings to be considered.
3.3.2. Functions Employed

The method described above has been used to calculate buckling coefficients for plates loaded as shown in Fig.3.1. The computer evaluations were performed entirely in terms of non-dimensional expressions and so the functions employed are presented here in their non-dimensional form. Coordinates $x$ and $y$ have been non-dimensionalised with respect to the corresponding plate parameters, depth $B$, and length $L$

$$X = \frac{x}{B}$$
$$Y = \frac{y}{L}$$

Stress Function $\phi$

The components of the stress function $\phi$ have been split into two separate parts, an $X$-part and a $Y$-part. These are given by

$$\phi_k(X,Y) = \phi_{kX}(X)\phi_{kY}(Y) \quad \text{(not summed over } k)$$

where

$$\phi_{kX} = X_r \quad \text{and} \quad \phi_{kY} = Y_s$$

Odd-numbered terms of $X_r$ are given by

$$X_r(X) = \frac{\cosh \lambda_r X}{\cosh \lambda_r} - \frac{\cos \lambda_r X}{\cos \lambda_r}$$

where $\lambda_r$ is the $\left(\frac{r+1}{2}\right)$th root of

$$\tanh \lambda_r + \tan \lambda_r = 0$$

and even-numbered terms of $X_r$ are given by

$$X_r(X) = \frac{\sinh \lambda_r X}{\sinh \lambda_r} - \frac{\sin \lambda_r X}{\sin \lambda_r}$$

where $\lambda_r$ is the $\left(\frac{r}{2}\right)$th root of

$$\tanh \lambda_r - \tan \lambda_r = 0$$
These functions provide a mixture of symmetric and antisymmetric components in the x-direction which satisfy the conditions of Eqn.(3.7).

When loading is, as here, symmetrically disposed about the x-axis, it can be shown that those terms in $\phi_k$ which are antisymmetric in the y-direction contribute nothing to the matrix $U_{ij}^{(2)}$ and so these are omitted to improve computing efficiency. The terms in $Y_s$ are therefore given by

$$Y_s(Y) = \frac{\cosh\lambda_s Y}{\cosh\lambda_s} - \frac{\cos\lambda_s Y}{\cos\lambda_s}$$

where $\lambda_s$ is the $s^{th}$ root of

$$\tanh\lambda_s + \tan\lambda_s = 0.$$

These also satisfy the conditions of Eqn.(3.7).

**Stress Field $\sigma^+$**

The non-dimensional form of the stress field, $\sigma^+$, is given by

$$\sigma^+ = \sigma^* \frac{tL}{2P}$$

It is convenient here to introduce three further functions used in the definitions of the stress field. The first of these, $f(Y)$ is defined by

$$-\frac{P}{2ct} f(Y) = \sigma_x$$

where $\sigma_x$ is the stress applied at the top edge of the plate and is taken as positive for a tensile loading. The remaining two functions are successive integrals of the first.

$$g(Y) = \int_0^Y f(Y) dY$$

$$h(Y) = \int_0^Y g(Y) dY$$
The stress field functions used are

\[ \sigma^+_x = N_x (\sigma^+_x)(\sigma^+_y) \]

where \( N_x = -\frac{1}{16} \frac{L}{C} \)

\[ \sigma^+_{xy} = N_{xy} (\sigma^+_{xy})(\sigma^+_{xy}) \]

where \( N_{xy} = \frac{3}{16} \frac{L}{C} \frac{L}{B} \)

\[ \sigma^+_{xy} = 1 - x^2 \]

\[ \sigma^+_{xy} = g(Y) \text{ for } |Y| < C/L \]

\[ \sigma^+_{xy} = g(C/L) = C/L \text{ for } Y > C/L \]

\[ \sigma^+_{xy} = g(-C/L) = -C/L \text{ for } Y < -C/L \]

\[ \sigma^+_y = N_y (\sigma^+_y)(\sigma^+_y) \]

where \( N_y = -\frac{3}{8} \frac{L}{C} \left( \frac{L}{B} \right)^2 \)

\[ \sigma^+_{y} = X \]

\[ \sigma^+_{y} = h(C/L) - h(Y) + \frac{C}{L} \left( 1 - \frac{C}{L} \right) \text{ for } |Y| < C/L \]

\[ \sigma^+_{y} = \frac{C}{L} (1 - Y) \text{ for } |Y| > C/L \]

\[ \sigma^+_{y} = \frac{C}{L} (1 + Y) \text{ for } |Y| < -C/L \]
These stresses correspond to a parabolic distribution of applied shear stresses on the edges \( Y = \pm 1 \) and comply with the conditions of Eqn.(3.5). It is assumed that the stress distribution is symmetrical about \( Y = 0 \) and so \( f(Y) \) is an even function. Note, however, that \( f(Y) \) is not defined for \( |Y| > C/L \). For a uniform patch load, as shown in Fig.3.1, Eqn.(3.21) gives

\[
f(Y) = 1
\]

then from Eqn.(3.22) \( g(Y) = Y \)

and from Eqn.(3.23) \( h(Y) = \frac{1}{2} Y^2 \).

For a non-uniform patch load, the function \( f(Y) \) takes a different form. This feature is utilised later in Section 3.5 when patch loading of the type resulting from a dispersed wheel load is investigated.

**Displacements \( w \)**

The components of the out-of-plane displacements \( w \) have, as with the stress function components, been split into two parts. These are given by

\[
w_{i}(X,Y) = w_{iX}(X)w_{iY}(Y) \quad \text{(not summed over } i)\]

Particular displacement functions have been assumed to suit each of the plate support conditions considered.

(1) All Edges Simply Supported.

\[
w_{iX}(X) = \sin p \frac{\pi}{2} (X + 1) \]
\[
w_{iY}(Y) = \sin q \frac{\pi}{2} (Y + 1)
\]
in which \( p \) and \( q \) are integers.
(2) Edge $X = +1$ Clamped, Edges $X = -1$ and $Y = \pm 1$ Simply Supported.

This corresponds to a clamped top edge with the bottom edge and two vertical sides remaining simply supported.

Replace $w_{ix}$ in (1) by

$$w_{ix} = \frac{\sin g \frac{X+1}{2}}{\sin g} - \frac{X+1}{2}$$

where $g$ is the $p^{th}$ root of

$$\tan g = g$$

(3) Edges $X = \pm 1$ Clamped, Edges $Y = \pm 1$ Simply Supported.

This corresponds to clamped top and bottom edges with the sides remaining simply supported.

Replace $w_{ix}$ in (1) by

$$w_{ix} = 1 - \cos g(X+1) - \frac{(1-\cos 2g)}{2g - \sin 2g} \left[ g(X+1) - \sin g(X+1) \right]$$

where for odd-numbered terms of $w_{ix}$, $g$ is given by

$$g = \frac{(p+1)}{2} \pi$$

and for even-numbered terms of $w_{ix}$, $g$ is the $\left\lfloor \frac{p}{2} \right\rfloor$ root of

$$\tan g = g$$

(4) All Edges Clamped.

The $w_{ix}$ given in (3) are retained and the $w_{iy}$ are similarly assigned

$$w_{iy} = 1 - \cos g(Y+1) - \frac{(1-\cos 2g)}{2g - \sin 2g} \left[ g(Y+1) - \sin g(Y+1) \right]$$

where $g$ is as given in (3) but with $p$ replaced by $q$.
Suitable displacement functions for other combinations of clamped and simply supported edges could also have been incorporated but the four combinations described above were considered to satisfy the large majority of practical situations.

3.3.3. Computational Developments

In the previous work by Khan, Johns and Hayman [34] and by Khan and Johns [45] on simply supported plates, the integrations in Eqns. (3.13) and (3.14) were performed analytically and the resulting expressions then evaluated by computer. This was possible because of the simplicity of the functions which can be assumed for the simply supported edge condition.

The more complex nature of the displacement functions required to describe clamped edge conditions, however, makes it necessary to perform the $U_{ij}^{(B)}$ and $U_{ij}^{(1)}$ integrations numerically by computer. The three remaining integrations of Eqns. (3.17), (3.19) and (3.20) required to find $H_k$, $A_{k\ell}$ and $G_{ij\ell}$ respectively and thus $U_{ij}^{(2)}$ from Eqn. (3.15) were also performed numerically. A simple three-point Simpson integration subroutine was incorporated into the program to compute all the required integrations. More elaborate integration routines available as standard library packages were found to be not particularly well suited to this application since they led to difficulties with integrations that were mathematically zero.

Removing all the numerical integrations to one new Simpson subroutine meant that the procedures for calculating $H_k$, $A_{k\ell}$ and $G_{ij\ell}$ had to be changed despite these having been evaluated numerically in the previous work. The integration method adopted then, however, was less efficient than the new approach and so appreciable savings were made in computer requirements. Changing the integration procedure used to evaluate
U_{ij}^{(B)} and U_{ij}^{(1)} from an analytical to a numerical approach demanded that these routines be completely rewritten also. Thus all the calculation procedures in the work of Ref. [34] were rewritten with only the routines used to invert matrix $A_{k\ell}$ and to evaluate the determinant of the final matrix $U_{ij}$ and find its eigenvalues remaining unaltered.

The new form of the program had to be developed with considerable emphasis on versatility of the end product so that any of the four support situations could be called-up conveniently by means of the external data instructions. This necessitated, in total, the creation of 48 new function subroutines. Fifteen of these described the displacement functions and their first and second derivatives, the stress functions and their second and fourth derivatives, and the stress field expressions. The remaining routines grouped together the various combination of terms into their X-parts and Y-parts that arise from the differentiations in Eqns. (3.13), (3.14), (3.17), (3.19) and (3.20) so that the numerical integration procedure could then be operated.

The program was written in Fortran IV and run on the Leicester University CDC Cyber 73 computer. The compilation time for the final program was approximately 15 seconds. Since the main results were all run from a pre-compiled copy of the program however, a compilation to a higher optimisation was made which took approximately 20 seconds.

3.3.4. Convergence and Accuracy

Verification that the new refined calculation procedure had been successfully implemented was readily achieved by re-running some of the plate and load geometry calculations performed by Khan, Johns and Hayman [34]. For various cases encompassing the whole range of $L/B$ and $C/L$ parameters studied previously, it was found that buckling coefficient values obtained by the two methods differed in only the fifth or sixth significant
figure when 32 strips were used in the new numerical integration program and in only the sixth or seventh significant figure when 64 strips were employed. Hence 32 strips were sufficient in the new integration procedure to reproduce buckling coefficients for simply supported plates to within a very small fraction of one per cent of the values obtained by the previous method.

A three-point Simpson integration routine requires an even number of strips. The integrations performed along the length of the plate which involve stress distribution terms require splitting into three parts: the central loaded region and the two unloaded regions. The new program was therefore developed such that, whatever the nominal total number of strips to be used in the integrations, strips were allocated to the loaded and unloaded regions of the plate pro rata, according to the C/L ratio. This was an improvement on the previous method where the total number of strips used in the numerical integrations depended on the C/L value, more strips being required the smaller the ratio. For many geometries, more strips were thus required for the integrations of $U_{ij}^{(2)}$ in the previous method than were found necessary with the new approach.

Convergence of the method when applied to a clamped edge situation is illustrated in Table 3.1. Buckling coefficients are given which show the effects of varying the numbers of displacement and stress function terms used. A Rayleigh-Ritz analysis provides an upper bound to a critical load calculation. Hence, the use of additional displacement function terms reduces the value of the solution. However, when a non-exact prebuckled stress distribution is used, as in the method described here, the upper bound feature of the Rayleigh-Ritz method is destroyed. Consequently, as additional stress function terms are employed (which have the effect of improving the suitability of the assumed stress distribution) the upper bound feature tends to be restored and generally the solution
Table 3.1 CONVERGENCE OF BUCKLING COEFFICIENT FOR PLATE CLAMPED ALONG THE TOP EDGE WITH OTHER EDGES SIMPLY SUPPORTED:
L/B = 1.25, C/L = 0.25

<table>
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<th>Displacement Terms x. y</th>
<th>Stress Function Terms x. y</th>
<th>Buckling Coefficient K</th>
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</table>
increases. Convergence of the analysis thus depends on the number of displacement terms used in the finite series of Eqn.(3.8) and on the number of stress function terms used in the finite series of Eqn.(3.9).

The critical coefficients were calculated, in each case, to within an accuracy of 2%. The number of terms in $w(x,y)$ and $\phi(x,y)$ required to achieve this accuracy depended on the geometry of the plate and on the edge conditions. Generally it was found that the same number of terms were needed in $\phi$ as were needed in $w$. The relative number of terms needed in the $x$- and $y$-directions depended on the $L/B$ ratio with a greater number of terms being required along the larger plate dimension. Fewest terms were therefore required for the near square plates. For plates simply supported all round it appeared that no more than seven terms were required in either the $x$- or $y$-directions to achieve the desired accuracy.

The effect of clamping the top edge of a plate was to demand additional terms. For a square plate, series consisting of five terms in both the $x$- and $y$-directions were used. This led to matrices of size $25 \times 25$. For a tall, narrow plate with an $L/B$ of $1/3$, nine terms were used in the $x$-direction and five in the $y$-direction, giving matrices of size $45 \times 45$. For $L/B$ values of 3.5 and above, five terms were employed in the $x$-direction and eleven in the $y$-direction, resulting in $55 \times 55$ matrices which were the largest created in the study. Compared with the case of a plate clamped along the top edge only, clamping additional edges demanded increased numbers of terms for $L/B$ values in the range from 1.75 to 3.25.
3.4. Results and Analysis of Results

Buckling coefficients have been evaluated for rectangular flat plates under the loading defined by Fig. 3.1. Coefficients are presented as a function of the plate aspect ratio for L/B values from \( \frac{1}{3} \) to 4. Five different patch loading geometries given by C/L values of 0.125, 0.25, 0.5, 0.75 and 1.0 are considered. The buckling coefficient \( K \) is defined by

\[
P_{cr} = K \frac{\pi^2 D}{2B}
\]

where \( P_{cr} \) is the critical value of the applied edge load \( P \), and \( D \) is the plate bending stiffness. This coefficient is identical to that defined by Eqn. (2.6) and used in the discussions of the previous chapter. Four combinations of simply supported and clamped edge conditions to the plate have been investigated and it was found that for each C/L value the coefficients lay on a smooth curve. These curves are shown in Figs. 3.2 to 3.5 where for clarity the data points are not included. Instead they are given in Tables 3.2 to 3.5 respectively where the numbers of displacement and stress function terms employed are also shown.

Buckling coefficients for plates simply supported on all edges are shown in Fig. 3.2. For each of the five curves, coefficients were computed for each of eleven L/B values. These are listed in Table 3.2. Clamping plate edges resulted in a rather more complex relationship between coefficient values and the plate aspect ratio. Consequently, the number of L/B values for which coefficients were evaluated was increased, from eleven to sixteen. Coefficients for plates clamped along the top edge but simply supported on the other three edges are shown in Fig. 3.3; the computed values are given in Table 3.3. The case when the top and bottom edges are clamped but the sides simply supported is shown in
Fig. 3.2 BUCKLING COEFFICIENTS FOR PLATE SIMPLY SUPPORTED ALL ROUND FOR ASPECT RATIOS FROM \( \frac{1}{3} \) TO 4

\[
P_{cr} = \frac{K \pi^2 D}{2B}
\]

\[
\begin{align*}
\zeta_L &= 1.000 \\
\zeta_L &= 0.750 \\
\zeta_L &= 0.500 \\
\zeta_L &= 0.250 \\
\zeta_L &= 0.125
\end{align*}
\]

BUCKLING COEFFICIENT \( K \)

ASPECT RATIO \( \alpha = \frac{L}{B} \)
Fig. 3.4. BUCKLING COEFFICIENTS FOR PLATE CLAMPED ALONG TOP AND BOTTOM EDGES WITH SIDES SIMPLY SUPPORTED FOR ASPECT RATIOS FROM \( \frac{1}{3} \) TO 4.

\[
P_{cr} = \frac{K\pi^2D}{2B}
\]

![Graph showing buckling coefficients for plate buckling with given aspect ratios.](image-url)
Fig. 3.5  BUCKLING COEFFICIENTS FOR PLATE CLAMPED ALL ROUND FOR ASPECT RATIOS FROM \( \frac{1}{3} \) TO 4

\[ P_{cr} = \frac{K \pi^2 D}{2B} \]

- \( c_L = 1.000 \)
- \( c_L = 0.750 \)
- \( c_L = 0.500 \)
- \( c_L = 0.250 \)
- \( c_L = 0.125 \)

ASPECT RATIO \( \alpha = \frac{L}{B} \)
### Table 3.2 BUCKLING COEFFICIENTS FOR PLATE SIMPLY SUPPORTED ALL ROUND

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### Table 3.3 BUCKLING COEFFICIENTS FOR PLATE CLAMPED ALONG TOP EDGE WITH OTHER EDGES SIMPLY SUPPORTED

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Table 3.4  BUCKLING COEFFICIENTS FOR PLATE CLAMPED ALONG TOP AND BOTTOM EDGES WITH SIDES SIMPLY SUPPORTED

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Table 3.5  BUCKLING COEFFICIENTS FOR PLATE CLAMPED ALL ROUND

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<tr>
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<td>8.63</td>
<td>7.09</td>
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<td></td>
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</tbody>
</table>
Fig. 3.4 and Table 3.4. Coefficients for plates clamped along all four edges are shown in Fig. 3.5 and Table 3.5.

Computer time requirements depended on both the numbers of displacement and stress function terms employed in evaluating a coefficient and on the support conditions to the plate. In the present analysis, this varied from approximately 15 seconds of central processor time for a plate simply supported all round when using four terms in the x-direction and three in the y-direction to 520 seconds for a plate clamped all round requiring five terms by eleven.

Several noteworthy points arise from the result of the present analysis.

It can be seen that for all support conditions, shortening the length over which the patch load acts reduces the K value. Concentrating the patch load therefore lowers the level of loading required to cause buckling of the plate.

Over the range of aspect ratios considered here, the K values for a given C/L attain their maximum value for a tall, narrow, plate with an L/B of $\sqrt{3}$. For larger aspect ratios, the coefficients first fall steeply and then, for all but the narrowest patch length, begin to rise again at an L/B of between 1.0 and 2.0. For cases where at least the top edge of the plate is clamped, the curves then reach a local maximum at an L/B of between 2.5 and 3.5 before finally falling again.

In the region of each local maximum there was a noticeable convergence of coefficient solutions arising from higher mode buckling. The values of the higher mode solutions obtained by the present method were, however, only approximate. To have evaluated them more accurately would have required the use of greater numbers of terms to accommodate the more complex mode shapes; however, as several terms were already being used in this region this was not possible because of the inherent increase in demand on
At L/B values beyond that at which a local maximum occurred, it was observed that two very nearly simultaneous solutions existed: one corresponding to a buckling mode shape in the y-direction which was entirely symmetric, the other to one which was entirely antisymmetric. The lowest eigenvalue was found to leap-frog between a symmetric and antisymmetric mode solution as the number of displacement terms along the length of the plate was increased. For aspect ratios below that at which the local maximum occurred, the solutions arose from entirely symmetric mode buckling along the plate length and adding an antisymmetric displacement term in the y-direction left the solutions unaltered. Because the determinant search method used to find the eigenvalues operated on detecting a sign change in the determinant value, great care was necessary at these large aspect ratios to ensure that the search was run over increments sufficiently small to separate the two near-coincident solutions, otherwise an erroneous third eigenvalue was found as the lowest coefficient. Close observation, coupled with the large numbers of terms used in this region, ensured that the lowest solution was always detected and that this had converged to within the desired accuracy.

The emergence of the local maxima and the accompanying convergence of eigenvalue solutions for plates with at least their top edge clamped can be shown to be attributable to a transition in plate buckling from a mode produced by patch loading to one resulting predominantly from longitudinal in-plane bending stresses. The following example illustrates this.

Consider a long plate of length 2L, depth 2B, and thickness t, loaded uniformly along its upper horizontal edge by a load P. If the plate is assumed to act as a beam simply supported at its bottom two corners, the bending moment is given by
\[ M = \frac{PL}{4} \]

which produces a maximum bending stress in the plate of

\[ \sigma_b = \frac{3PL}{8B^2t} \quad (3.25) \]

Now the elastic critical stress for a plate under pure bending is given by Timoshenko and Gere [46] as

\[ \sigma_{bc} = K_b \frac{\pi^2D}{4B^2t} \quad (3.26) \]

If Eqn. (3.25) is equated to Eqn. (3.26) then the value, \( P_{cr} \), of \( P \) which produces a bending stress sufficient to buckle the plate is obtained

\[ P_{cr} = \frac{2}{3} K_b \frac{\pi^2D}{L} \quad (3.27) \]

which can be expressed as

\[ P_{cr} = \frac{4}{3} K_b \frac{\pi^2D}{\alpha 2B} \quad (3.28) \]

This expression is now of the form of Eqn. (3.24) but with \( K \) replaced by \( 4K_b/3\alpha \). Timoshenko and Gere [46] give values of \( K_b \) for plates with three different edge support conditions: simply supported all round, clamped along the bottom (tension) edge and simply supported along the other three edges, and clamped along top and bottom edges and simply supported along the sides. Minimum \( K_b \) values for the three conditions are 23.9, 24.5, and 39.6 respectively. If the \( K_b \) value of 39.6 for a plate clamped along the top and bottom edges is substituted into Eqn. (3.28) and a plate of aspect ratio 4.0 is considered, then \( P_{cr} \) is given by

\[ P_{cr} = 15.2 \frac{\pi^2D}{2B} \quad . \]

This describes the full edge load on a plate that would cause buckling due to in-plane bending effects. If this is compared with Eqn. (3.24)
then the expression can be seen to correspond to a $K$ value for patch loading of 13.2. Now the $K$ value computed in the present analysis for a plate of aspect ratio 4.0, clamped along top and bottom edges, and subjected to a uniform patch load of $C/L = 1.0$ can be seen from Table 3.4 and Fig.3.4 to be 13.3. In fact the computed coefficient for a patch loaded plate of $L/B = 4.0$ and $C/L = 1.0$ varies only between 13.2 and 13.3 depending on whether the top edge only is clamped or all four edges are clamped. This illustrates that for patch loaded plates where at least the top edge is clamped, the primary cause of buckling as the plate aspect ratio is increased becomes one of longitudinal in-plane bending stresses.

Timoshenko and Gere also show that for a plate subjected to bending, the wavelength of the buckle changes from an $L/B = 0.67$ when all edges are simply supported to an $L/B = 0.47$ when the top and bottom edges are clamped, although very little change occurs when only the bottom (tension) edge is clamped. Thus, for a long plate with a clamped top (compression) edge, buckling due to bending comprises a large number of short wavelength crests: for a plate of aspect ratio 4.0 with its upper edge clamped, the buckling profile in the longitudinal direction consists of 8 or 9 crests. Therefore, very little movement of the plate is required for the buckled shape to change from a symmetric to an antisymmetric mode.

This accounts for the existence in the present analysis of near-coincident solutions arising from symmetric and antisymmetric buckled shapes. It also explains the requirement for large numbers of terms in the longitudinal direction to achieve satisfactory convergence of solutions for long plates with a clamped top edge. The observed convergence of solutions in the region of the local maxima in the coefficient curves is presumably due to a number of transitionary modes producing similar eigenvalue solutions.
Clarification of the precise interaction of symmetric and antisymmetric modes could have been achieved by finding the eigenvectors corresponding to the eigenvalue solutions. However, shortage of time prevented this in the present analysis.

Although not presented here, a brief study of the coefficient curves for simply supported patch loaded plates with aspect ratios greater than 4.0 revealed that they too begin to level off towards local maxima in a similar manner to the curves for plates with a clamped top edge. This investigation of larger aspect ratios was possible because fewer terms are required to achieve convergence for plates with simply supported edges than for plates with clamped edges.

It will be recalled from the previous chapter that crane web panels are subjected to relatively short patch loads; coefficients for typical crane-type patch loads are represented by the lower-most curves in Figs.3.2 - 3.5. It will be noted that these curves each represent an approximately constant $K$ value for aspect ratios between about 1.5 and 4.0. If it is assumed that these remain approximately constant at larger aspect ratios, then the aspect ratio at which bending emerges as the dominant cause of buckling can be estimated by considering again a plate acting as a beam simply supported at its two bottom corners but, this time, subjected to a central concentrated patch load instead of a fully distributed load.

For simplicity consider a central point load $P$. The bending moment is now given by

$$M = \frac{PL}{2}$$

which gives rise to a maximum bending stress of

$$\sigma_b = \frac{3PL}{48^2t}$$  (3.29)
Equating Eqn.(3.29) with Eqn.(3.26) gives the critical value of $P$ for buckling due to longitudinal in-plane bending stresses to take place

$$P_{cr} = K_b \frac{\pi^2D}{3L}$$

which can be expressed as

$$P_{cr} = \frac{2K_b}{3\alpha} \frac{\pi^2D}{2B} \quad (3.30)$$

Equating Eqn.(3.30) with Eqn.(3.24) gives

$$\alpha = \frac{2}{3} \frac{K_b}{K} \quad (3.31)$$

For plates with at least their top edge clamped, $K_b$ has a value very close to 39.6 (see Ref.[46]); the $K$ value, from Figs. 3.3 to 3.5, is approximately 5.7. Substituting these values into Eqn.(3.31) gives $\alpha = 4.6$ as an approximate estimation of the aspect ratio of a plate, subjected to a very narrow patch load, at which buckling is governed primarily by bending stresses. For plates simply supported on all edges, $K_b$ is 23.9 and, from Fig.3.2, $K$ is 2.28 approximately. Thus, for a concentrated patch load on a simply supported plate, bending effects predominate at $\alpha = 7.0$ approximately.

Comparisons of the effect of clamping successive edges of a plate are shown in Fig.3.6 for the case of a very narrow patch load ($C/L = 0.125$) and in Fig.3.7 for a full patch load ($C/L = 1.0$). It can be seen that clamping the top edge of a plate increases the buckling coefficients appreciably compared with the values for a simply supported plate. Additionally clamping the bottom edge has relatively little further effect on the coefficients. Clamping also the sides increases coefficients for plates of small aspect ratio, particularly in the case of long patch loads, but has relatively little effect for plates of larger aspect ratio. For aspect ratios above about 3.0 to 3.5 (depending on the patch loading
Table 3.6  BUCKLING COEFFICIENTS FOR PLATE WITH 
L/B = 1.25 AND C/L = 0.25 SUBJECCT TO 
VARIOUS EDGE CONDITIONS

<table>
<thead>
<tr>
<th>Edge conditions:</th>
<th>K</th>
<th>K for simply supported all round</th>
</tr>
</thead>
<tbody>
<tr>
<td>S = Simply supported</td>
<td>2.93</td>
<td>1.00</td>
</tr>
<tr>
<td>C = Clamped</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S  S</td>
<td>6.52</td>
<td>2.23</td>
</tr>
<tr>
<td>S  C</td>
<td>6.87</td>
<td>2.34</td>
</tr>
<tr>
<td>C  C</td>
<td>7.48</td>
<td>2.55</td>
</tr>
</tbody>
</table>
length), coefficient values for plates clamped all round or clamped on
the top and bottom edges become effectively indistinguishable from those
for plates clamped along the top edge only. These trends are further
illustrated in Table 3.6 where coefficients are shown for a plate of
L/B = 1.25 and C/L = 0.25. For simply supported edges $K$ is 2.93.
When the top edge is clamped, $K$ increases markedly to 6.52. Ad-
ditionally clamping the bottom edge increases $K$ a little more to 6.87.
Clamping also the sides produces a value of 7.48 for $K$. Proport-
tional increases are found by dividing each coefficient by the simply
supported $K$ value; the successive values become 1.0, 2.23, 2.34
and 2.55.

As this investigation represents the first attempt known to the writer to
tackle a parametric study of patch loaded plate buckling problems where
edges of the plate are clamped, it is not possible to corroborate the
results with other work. However, the following points help substan-
tiate the results.

1. The method, when applied to simply supported plates, reproduces
coefficients to within four or five decimal places of the values
obtained with the earlier version; the earlier version has itself
been shown in Ref.[34] to produce coefficients which compare very
favourably with solutions by other methods.

2. The curves for plates with clamped edges are of similar profiles
to the curves for simply supported plates: the coefficients
increase with increases in the C/L value, the C/L curves do not
intersect, the drop-off in $K$ values for clamped plates at large
aspect ratios has been observed to occur also for simply supported
plates when aspect ratios greater than four are investigated.
3. The method effectively takes account of buckling due to in-plane bending stresses: buckling load predictions for patch loaded plates converge towards, but never exceed, those loads at which the associated bending stresses become sufficient to cause buckling of the plate.

4. Khan [47] has calculated coefficients for a plate clamped along the top and bottom edges and simply supported along the sides, but for a square panel only. The coefficients obtained for a range of patch lengths compare to within 3% of the values computed in the present analysis.

5. Bossert and Ostapenko [48] have computed some buckling coefficients for rectangular panels of aspect ratios between 0.8 and 1.5. The panels were subjected to a full edge load along the top edge which was clamped, the other edges were simply supported. They considered combined patch loading and bending for a range of ratios of direct stress to bending stress. For the cases where patch loading only acts onto the panels, the presented coefficients agree to within 3% - 9% of the values in the present analysis. However, in their analysis, Bossert and Ostapenko considered the panels to be supported by uniformly, as opposed to parabolically, distributed shear stresses on the vertical sides which may account for some of the discrepancy.

6. The support condition along the loaded, compression, edge has been shown to be that which has greatest influence on the buckling load. Timoshenko and Gere [46] have shown, similarly, that the support condition along the compression edge of a plate under bending has a far greater influence on the buckling load than that along the opposite edge.
7. The ratios between coefficients for plates with clamped edges and those with simply supported edges appear reasonable. Similar ratios have been found in certain of the cases investigated by White and Cottingham [49] for plates either clamped or simply supported all round. Their analysis considered plates under patch loading but supported by patch loads at the corners of the opposite edge as opposed to parabolically distributed shear stresses on the adjacent edges as considered here.

Numerical Example

Consider again the model box section used in the experimental work described in Chapter 2. The panel measured 600 mm long (2L), 400 mm deep (2B), and 3 mm thick (t). The effective length of the patch load on the web panel resulting from the dispersal of a wheel load through the rail and flange was estimated to be about 90 mm (2C). The geometric parameters are therefore

\[ a = \frac{L}{B} = 1.5 \]

\[ \beta = \frac{C}{L} = 0.15 \]

The critical load is given by Eqn.(3.24) where

\[ D = \frac{Et^3}{12(1-\nu^2)} \]

Taking \( E = 207 \text{ kN/mm}^2 \) and \( \nu = 0.3 \) for the steel of the box web gives

\[ D = 512 \text{ kN mm} \]

Therefore

\[ \frac{\pi^2 D}{2B} = 12.6 \text{ kN} \]

and the buckling load for the web panel is given by

\[ P_{cr} = 12.6 \text{ kN} \]
where $K$ is selected for the particular $L/B$ and $C/L$ from the set of curves in Figs. 3.2 - 3.5 corresponding to the assumed support conditions to the plate. Coefficients for a $C/L$ not given in the presented curves, as in this example, may readily be estimated by interpolating between curves.

Thus, if the panel is assumed simply supported all round then, from Fig.3.2, $K$ is approximately 2.6 and the buckling load $P_{cr}$, 33 kN. If the top edge is assumed to be effectively clamped, $K$ then takes the value 6.2 from Fig.3.3 and $P_{cr}$ becomes 78 kN. Additionally, clamping the bottom edge raises $K$ to 6.6, from Fig.3.4, and the buckling load to 83 kN. Finally, with all edges clamped, from Fig.3.5, $K$ takes the value 6.8 and $P_{cr}$ increases to 86 kN.

Experimental estimations of the panel buckling load were presented in Chapter 2 - using the Southwell Plot method, loads of between 70 kN and 85 kN were obtained. It will be noted that these loads agree very closely with the theoretical predictions for the panel when at least the loaded edge is considered clamped. Now it has been discussed earlier that the boundary conditions to a crane torsion-box web panel are probably better represented by an assumption that the horizontal edges are clamped, rather than simply supported, and the vertical edges simply supported - the close agreement between the theoretical buckling load for this condition of 83 kN and the experimental loads appears to support this hypothesis.

The buckling load may be expressed in terms of a critical stress by

$$\sigma_{cr} = \frac{P_{cr}}{2Ct}$$

For each of the four support conditions, the buckling loads for the web panel equate to critical stresses of 122, 289, 317 and 319 N/mm$^2$ respectively. A typical yield stress for the web would be in the region
of 250 - 300 N/mm². It is clear from this example that the support conditions to a patch loaded plate play an important role in determining whether the elastic critical load is reached before, after, or at about the same time as the plate begins to yield locally.

3.5. Non-Uniform Patch Loading

So far consideration has been given only to uniform patch loads. However, the method of analysis presented here has been developed to enable non-uniform patch loads to be investigated also. The loading considered now is that arising at the top edge of a crane web panel, produced by the dispersal of a wheel load through a rail and flange. This is illustrated in Fig.3.8 where the profile of the patch load represents a typical vertical in-plane web stress distribution (see Fig.2.6).

Adapting the method to cater for loads of this nature requires changes only in the stress field functions. This is conveniently achieved by reassigning the function \( f(Y) \) in Eqn.(3.21) thus

\[
f(Y) = \ell \left[ 1 - \left( \frac{YL}{C} \right)^{2m'} n' \right]
\]

(3.32)

where \( m' \) is a positive integer and \( n' \geq 0 \). The distributions of Fig.2.6 can be represented approximately by assigning \( m' = 1 \) and \( n' = 3 \). The functions \( g(Y) \) and \( h(Y) \) defined in Eqns.(3.22) and (3.23) are obtained by successive integrations of Eqn.(3.32). The value of \( \ell \) is found from the condition that the total applied force be \( P \). This is equivalent to the condition \( g(\pm C/L) = \pm C/L \). For the stated values of \( m' \) and \( n' \) and upon integrating \( f(Y) \) to obtain \( g(Y) \), \( \ell \) is found to have the value 35/16. Thus Eqn.(3.32) becomes

\[
f(Y) = \frac{35}{16} \left[ 1 - \left( \frac{YL}{C} \right)^{2} \right]^3
\]

(3.33)
Fig. 3.8. NON–UNIFORM PATCH LOAD ON PLATE

Fig. 3.9. ESTIMATION OF EQUIVALENT UNIFORM PATCH LOADING LENGTH
This \( f(Y) \) and the associated \( g(Y) \) and \( h(Y) \) were incorporated into the program so that a stress distribution corresponding to either a uniform or non-uniform patch load could readily be generated.

The non-uniform patch load described by Eqn. (3.33) has a maximum at \( Y = 0 \) of \( f(Y) = 35/16 \). From Eqn. (3.21) this corresponds to a maximum compressive stress \( \sigma_m \) of

\[
\sigma_m = \frac{35}{16} \frac{p}{2Ct}
\]

If the curved patch load is equated by the method described in Section 2.4 to an equivalent uniform patch load, as illustrated in Fig. 3.9, the length of the uniform load is given by

\[
C_e = \frac{16}{35} C
\]

or

\[
C_e = 0.4571 C
\]  \hspace{1cm} (3.34)

Buckling coefficients, \( K_{nu} \), have been computed for a range of non-uniform patch loading lengths for plates of different geometries and with different sets of boundary conditions. For each load, the length of the equivalent uniform load was calculated from Eqn. (3.34) and the buckling coefficient, \( K_e \), computed for this case also.

Tables 3.7 and 3.8 show comparisons between \( K_{nu} \) and \( K_e \) for non-uniform patch loading lengths given by \( C/L \) ratios of 1.00, 0.75, 0.50, 0.25 and 0.125. Table 3.7 applies to a square plate, both when all the edges are simply supported and when the top and bottom edges are clamped instead. Table 3.8 concerns a plate of aspect ratio 1.5 both when the top and bottom edges are clamped and the sides simply supported, and when clamped all round. In all cases the coefficients \( K_e \) for the equivalent load are lower than those for the non-uniform load, but only marginally. For a non-uniform patch load occupying the
Table 3.7  BUCKLING COEFFICIENTS $K_{nu}$ AND $K_e$ FOR PLATE WITH $L/B = 1.0$:
(a) WHEN SIMPLY SUPPORTED ALL ROUND
(b) WHEN CLAMPED ON TOP AND BOTTOM EDGES AND SIMPLY SUPPORTED ON SIDES

<table>
<thead>
<tr>
<th>$C/L$</th>
<th>$K_{nu}$</th>
<th>$C_e/L = 0.4571 \times C/L$</th>
<th>$K_e$</th>
<th>$K_e/K_{nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>4.04</td>
<td>0.4571</td>
<td>3.79</td>
<td>0.94</td>
</tr>
<tr>
<td>0.750</td>
<td>3.71</td>
<td>0.3429</td>
<td>3.56</td>
<td>0.96</td>
</tr>
<tr>
<td>0.500</td>
<td>3.46</td>
<td>0.2286</td>
<td>3.38</td>
<td>0.98</td>
</tr>
<tr>
<td>0.250</td>
<td>3.30</td>
<td>0.1143</td>
<td>3.27</td>
<td>0.99</td>
</tr>
<tr>
<td>0.125</td>
<td>3.26</td>
<td>0.0571</td>
<td>3.24</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 3.8  BUCKLING COEFFICIENTS $K_{nu}$ AND $K_e$ FOR PLATE WITH $L/B = 1.5$:
(a) WHEN CLAMPED ON TOP AND BOTTOM EDGES AND SIMPLY SUPPORTED ON SIDES
(b) WHEN CLAMPED ALL ROUND

<table>
<thead>
<tr>
<th>$C/L$</th>
<th>$K_{nu}$</th>
<th>$C_e/L = 0.4571 \times C/L$</th>
<th>$K_e$</th>
<th>$K_e/K_{nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>8.36</td>
<td>0.4571</td>
<td>7.88</td>
<td>0.95</td>
</tr>
<tr>
<td>0.750</td>
<td>7.75</td>
<td>0.3429</td>
<td>7.49</td>
<td>0.97</td>
</tr>
<tr>
<td>0.500</td>
<td>7.33</td>
<td>0.2286</td>
<td>7.19</td>
<td>0.98</td>
</tr>
<tr>
<td>0.250</td>
<td>7.04</td>
<td>0.1143</td>
<td>7.00</td>
<td>0.99</td>
</tr>
<tr>
<td>0.125</td>
<td>6.99</td>
<td>0.0571</td>
<td>6.95</td>
<td>0.99</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$C/L$</th>
<th>$K_{nu}$</th>
<th>$C_e/L = 0.4571 \times C/L$</th>
<th>$K_e$</th>
<th>$K_e/K_{nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>8.81</td>
<td>0.4571</td>
<td>8.14</td>
<td>0.92</td>
</tr>
<tr>
<td>0.750</td>
<td>7.91</td>
<td>0.3429</td>
<td>7.47</td>
<td>0.94</td>
</tr>
<tr>
<td>0.500</td>
<td>7.20</td>
<td>0.2286</td>
<td>6.98</td>
<td>0.97</td>
</tr>
<tr>
<td>0.250</td>
<td>6.70</td>
<td>0.1143</td>
<td>6.63</td>
<td>0.99</td>
</tr>
<tr>
<td>0.125</td>
<td>6.58</td>
<td>0.0571</td>
<td>6.52</td>
<td>0.99</td>
</tr>
</tbody>
</table>
full length of the plate, $K_e$ underestimates $K_{nu}$ by between 5% and 8%; for shorter patch loads the agreement is closer. That the equivalent uniform load produces lower coefficients is to be expected owing to it constituting a slightly more severe (concentrated) load than the non-uniform load - as illustrated in Fig.3.9.

The results indicate that the method for estimating equivalent uniform patch loads leads to acceptable and slightly conservative predictions of buckling loads for plates subjected to non-uniform patch loads of the type produced by wheel loads on crane web panels.

3.6. Conclusions

Theoretical elastic buckling coefficients have been presented for rectangular plates subjected to uniform patch loads symmetrically disposed on one edge and supported by parabolically distributed shear stresses on the two adjacent edges. Coefficients are given for the cases when all edges of the plate are simply supported and when successive edges are clamped.

The results are presented graphically in a form suitable for design calculations of plate buckling loads; wide ranges of plate and loading geometries have been considered such that the majority of practical situations are covered.

When compared with the case when all edges of a plate are simply supported, clamping the loaded edge has a marked effect on the buckling load of the plate - approximately doubling it for the most geometries. Additionally clamping the opposite edge has relatively little further effect, as does clamping the adjacent edges except in cases where these are long compared with the loaded edge.
It is shown that as the aspect ratio of a patch loaded plate is increased, a transition develops between buckling due to the patch load and buckling due to the associated in-plane longitudinal bending stresses.

Plate buckling under the action of non-uniform patch loads of the type produced by wheel loads on crane web panels has also been investigated. The results show that a simple method presented in Chapter 2 for estimating equivalent uniform patch loads leads to very good and safe estimations of plate buckling loads.
CHAPTER 4
COLLAPSE LOAD ANALYSIS

4.1. Introduction

So far consideration has been given only to elastic buckling of patch loaded plates. In Chapter 2, effective lengths of patch loads on crane web panels were assessed. This enabled the critical load of a test box web panel to be calculated. The assumption of a simply supported panel, however, led to a predicted buckling load that was extremely conservative compared with values determined by experiment. Buckling coefficients presented in Chapter 3 enabled the panel buckling load to be calculated assuming various of the panel edges to be clamped instead of simply supported. Vastly improved agreement was achieved between theory and practice when at least the loaded edge was assumed clamped; for panel aspect ratios applicable to crane web panels, clamping the other edges also was shown to have relatively little effect on the predicted buckling load.

Now German designers still insist - see Ref.[50] - that an elastic critical load approach with a tangent modulus correction to allow for possible stressing above the proportional limit (see Bleich [51]) is the only one that can be justified at present for predicting the strength of plate panels with any form of in-plane loading. However, in many other countries including the U.K., emphasis is shifting towards limit state approaches that take account of post-buckled stiffness and strength of a thin plate panel.

For a very slender panel subjected to any in-plane loading other than a highly concentrated patch load, the elastic critical load represents a conservative estimate of strength, sometimes very conservative. Bossert and Ostapenko [48] have shown that for slender plate girders subjected to
full uniform edge loading and in-plane bending, the ultimate load is typically three to four times the elastic buckling load (thus indicating considerable post-buckling strength), even when the buckling load is computed on the basis of clamped edges. For less slender panels, the yield stress or proportional limit may be reached before the elastic critical load is achieved. Numerical examples in Chapters 2 and 3 illustrated that, depending on the length of the patch load and the panel boundary conditions, plasticity may develop in a patch loaded panel before the buckling load is reached.

Fig.4.1 shows a comparison of strength assessments for rectangular plates of thickness $t$, with aspect ratios above 1.0, subjected to uniform compression along two opposite edges of dimension $d$. The German design approach in DIN 4114 [11] consists of a parabolic transition curve between the squash line and the Euler buckling hyperbola. The transition curve arises from the tangent modulus correction and meets the Euler curve at the point corresponding to the limit of proportionality which is taken as four-fifths of the yield stress. A typical design curve based on the ultimate load, such as that offered by the Merrison Rules [12], shows the DIN approach to be generous for all but very slender plates. This arises because of an assumption of flat plates in the DIN method. The comparative underestimation by Merrison exists despite this approach being concerned with ultimate strength and despite its being applicable only if the plate imperfections are within certain tolerances. For very slender plates however, the elastic critical buckling predictions of DIN become more conservative compared with a Merrison type procedure. A similar comparison between the two approaches exists in the cases of in-plane bending and shear loadings on a plate. Over the slender plate region a curve derived by Walker [52] is also shown which represents the ultimate strength of initially flat, perfectly linear elastic plates. Ultimate plate strength is also covered by Timoshenko and Gere [46] for plates
Fig. 4.1. SKETCH OF STRENGTH COMPARISONS FOR PLATES UNDER UNIFORM COMPRESSION
under uniform compression.

Whether elastic buckling provides a representative measure of the load carrying capacity of a patch loaded structure (such as a crane torsion-box) or whether an ultimate strength approach is more suitable can perhaps be assessed most directly by conducting collapse tests: collapse tests on wheel loaded crane torsion-boxes is the subject of the following chapter. For ultimate strength to provide a sound design basis, however, requires a reliable method for predicting collapse loads. The complex geometrical nature of the patch loaded plate structure is difficult to analyse and current predictions consequently rely mainly on empirical formulae. These have been reviewed by Rockey [14]. Although some of the formulae feature in the code recommendations of various countries, none gives entirely satisfactory predictions over the full range of experimental results.

In an attempt to improve this situation Roberts and Rockey [53] have presented a new solution in a mechanism analysis for predicting collapse loads of conventional plate girders under in-plane patch loading. Their analysis is based on the upper bound theorem of plastic collapse and although involving certain approximations and empirical modifications gives results that agree well with experimental data from various sources (see Ref.[53] and also Roberts and Rockey [54]).

It is the purpose of this chapter to look further into the method presented by Roberts and Rockey. Certain of the departures from rigour are discussed and developments of some aspects are proposed. Application of the method to crane torsion-box-girders is also considered which links this chapter closely with the following Chapter 5 where details of a series of collapse tests on box-girders are presented. A brief outline will first be given of the plastic failure mechanism detailed in Ref.[53] by Roberts and Rockey.
4.2. Mechanism Solution by Roberts and Rockey

4.2.1. Slender Webs

Based on observations of conventional plate girders tested to collapse under the action of a patch load, Roberts and Rockey [53] assumed the collapse mechanism shown in Fig.4.2. It consists of four plastic hinges in the girder flange and three yield lines in the web. The locations of the web yield lines and flange hinges are defined by dimensions $h$ and $j$. The angle $\psi$ defines the web deformation just before collapse.

Yield stresses of the flange and web are $\sigma_f$ and $\sigma_w$ respectively; $M_f$ is the plastic moment of the flange and $M_w$ the plastic moment per unit length of the web. For a small downward vertical displacement of the load $\delta v$, the plastic hinges in the flange rotate by $\delta v/j$ and those in the web by $\delta v/2h\cos\psi$ (twice this value along the central yield line).

Equating external and internal work gives

$$ P_u = \frac{4M_f}{j} + \frac{4jM_w}{h\cos\psi} + \frac{2cM_w}{h\cos\psi} - \frac{2\eta M_w}{h\cos\psi} \quad (4.1) $$

where $P_u$ is the collapse load and $\eta$ defines a length of web beneath the load which is assumed to have yielded under direct compression and therefore offers no bending resistance. Minimising $P_u$ with respect to $j$ gives

$$ j^2 = \frac{M_h\cos\psi}{M_w} \quad (4.2) $$

Should spread of the mechanism plastic hinges in the flange be restricted by web stiffener spacing then $j$ is restricted to its maximum value of 0.5 ($b-c$).

Deformation of the flange just prior to collapse can be estimated by using elastic theory. If it is assumed that the flange bending moment varies linearly from $+M_f$ at an outer hinge to $-M_f$ at the adjacent hinge then the deflection is given by $M_fj^2/6EI_f$ where $E$ is Young's Modulus and...
I_f is the second moment of area of the flange about its neutral axis. For compatibility this displacement is equated to that of the web which is given by 2h(1-sin\(\psi\)). Hence

\[
\frac{M_f j^2}{6EI_f} = 2h(1-sin\(\psi\)) \tag{4.3}
\]

Substituting for \(j^2\) from Eqn.(4.2) gives

\[
\frac{cos\psi}{l-sin\psi} = \frac{12EI_f M_w}{M_f^2} \tag{4.4a}
\]

where \(M_w = \sigma_{w} t_w^2/4\). For a standard plate girder, \(I_f = b_t t_f^3/12\) and \(M_f = \sigma_{f} b_t t_f^2/4\) and Eqn.(4.4a) can then be expressed as

\[
\frac{cos\psi}{l-sin\psi} = \frac{4E\sigma_{w} t_w^2}{\sigma_{f} b_t t_f^2} \tag{4.4b}
\]

The last three terms on the right hand side of Eqn.(4.1) represent the web contribution to the collapse load. It is assumed that this contribution is transmitted to a length \(\eta\) of web which then yields in direct compression. If the contribution of the web to the collapse load is equated to the force in the yielded length \(\eta\) of web then \(\eta\) can be determined

\[
(4j + 2c - 2\eta)M_w/hcos\psi = \sigma_{w} t_w \eta
\]

hence

\[
\eta = \frac{(4j + 2c)M_w}{2M_w + \sigma_{w} t_w hcos\psi} \tag{4.5}
\]

The term \(h\) is chosen empirically, enabling the collapse load to be determined. Roberts and Rockey envisaged an upper limit of \(h = d/6\) to be satisfactory for most practical situations but added that further tests were required to substantiate this. Choice of a suitable expression was based on experimental evidence which suggested that the overall depth 2h of the mechanism is not too sensitive to the flange dimensions and
that large changes in $M_F$ are required to significantly alter the collapse load. It was taken to be

$$h = \frac{d}{16} \frac{t_w \sigma_f^*}{t^* \sigma_f} \quad (h \leq d/6) \quad (4.6a)$$

where $t^*$ is a reference web thickness chosen to be 2.5 mm to give good agreement with test results and $\sigma_f^*$ is a reference yield stress taken as 300 N/mm$^2$. The ratio $\sigma_f^*/\sigma_f$ is incorporated so that the solution is not sensitive to small changes in $M_F$ resulting from variations in material yield stress. Hence

$$h = \frac{d t_w \sigma_f^*}{40 \sigma_f} \quad (h \leq d/6) \quad (4.6b)$$

If, however, the value of $j$ calculated from Eqn.(4.2) exceeds the distance allowed by the web stiffener spacing then $j$ is assigned its maximum value of 0.5 $(b-c)$ and $h$ is given by Eqn.(4.6b) but with the yield stress ratio term omitted

$$h = \frac{d t_w}{40} \quad (h \leq d/6) \quad (4.6c)$$

Whenever the calculated value of $j$ exceeds its maximum, $h$ is re-calculated using Eqn.(4.6c) and $\psi$ is evaluated again from Eqn.(4.3) using the new $h$ and maximum $j$ values; Eqn.(4.1) is then used to determine the collapse load.

Roberts and Rockey imposed two restrictions on their solution to prevent unsatisfactory overestimation of experimental collapse loads when the loaded length $c$ becomes large.

(i) If $c > 2j$ as calculated from Eqn.(4.2) then $c$ is limited in the calculations to the value $2j$.

(ii) If $c > 0.2d$ then $c$ is limited in the calculations to the value $0.2d$. 
4.2.2. **Stocky Webs**

Out-of-plane bending resistance of a web is proportional to $t_w^2$; in-plane compressive membrane resistance is proportional to $t_w$. As the thickness of a web increases therefore, direct yielding of the web becomes the more likely cause of stocky girder failure. This situation is analysed by Roberts and Rockey by assuming a simplified mechanism where there are still four hinges in the flange as in Fig.4.2 but the web is considered to yield in direct compression beneath the flange instead of by displacing out-of-plane and forming yield lines.

Equating as before the work done by the applied load as it moves vertically downwards through a small distance $\delta v$ to the internal dissipation of plastic energy gives

$$P_u = \frac{4M_f}{j} + \sigma_w t_w (j+c) \quad (4.7)$$

Minimising $P_u$ with respect to $j$ gives

$$j^2 = \frac{4M_f}{\sigma_w t_w} \quad (4.8)$$

Substituting for $j$ in Eqn.(4.7) gives

$$P_u = 4\sqrt{M_f \sigma_t} + \sigma_w t_w \quad (4.9)$$

The restrictions concerning patch loading lengths in the solution for slender plate girders do not apply to the stocky girder solution. The predicted collapse load is taken to be the smaller of the two values arising from bending failure and direct compression failure of the web.

4.2.3. **Comments**

Part of the appeal of the solution by Roberts and Rockey lies in its potential as a design method, its simplicity readily enabling hand calcula-
tions to be undertaken. Roberts and Rockey have shown that statistically the method gives results that correlate closely with experimental data taken from various sources for 63 different girders. The ratios of predicted to experimental collapse loads have a coefficient of variation (which can be considered as the standard deviation of a set of results with a mean value of unity) of just over 11% which is better than any other method of prediction known to Roberts and Rockey. For the experimental data considered, the extremes of predictions from the solution are an overestimation of nearly 9% and an underestimation of just over 30%. Additional support to the validity of the method is offered in Ref. [53] in the form of predicted and experimental collapse mode comparisons for girder flanges.

The present author has undertaken a study of the Roberts and Rockey solution. Originally this was to assess whether the method could be applied to wheel loaded crane torsion-box-girders as well as to conventional plate girders. However, the following considerations emerged during the initial study and were contributory towards a more detailed investigation being conducted.

1) Deformation of the structure before collapse is estimated by equating elastic deflection of the flange to plastic deformation of the web. In practice there will usually be some complex elasto-plastic behaviour in both the flange and the web which determines the structural deformation at the onset of collapse. Now Horne [55] has illustrated the strong dependence upper bound values of a collapse load have on the assumed deformation just prior to collapse when the upper bound theorem is applied to structural members in compression. For practical structural members, such as web plates, deformation at the onset of collapse will be influenced by the nature and magnitude of initial geometric imperfections and by loading eccentricities. Consequently,
more general application of the mechanism solution (which is based on the upper bound theorem of plastic collapse) may require a more sophisticated estimation of precollapse deformation than that proposed.

2) To an extent, these problematic features have been overcome by empirical selection of an expression for $h$ that suits the available experimental test results. This expression, however, is not dimensionally correct and therefore does not give consistency with scaling of a given structure; the quoted expression - see Eqn.(4.6) - gives $h$ proportional to length squared. It is noted however that in subsequent development of the method by Roberts [56], [57], selection of $h$ was changed to a dimensionally consistent empirical expression given by $h = 25 t_w$.

3) In the assumed mechanism, the distance $h$ between the upper and central web yield lines is equal to that between the central and lower yield lines. Collapse tests conducted at Leicester University on model box-girder crane sections subjected to a wheel load illustrated that for these structures the vertical distance between the upper and middle yield lines is usually somewhat less than that between the middle and lower lines.

4) The assumption that a length $\eta$ of web beneath the applied load yields in direct compression and therefore offers no resistance to web bending while elsewhere the web yield lines offer full plastic moment restraint is an approximation. In practice there will surely be a transition between direct, or near-direct, web yielding beneath the applied load to full moment restraint towards the outer flange hinges.
5) Since the method is applied in two quite distinct parts there is another, more fundamental, transition which is not realised by the solution. The predicted collapse load is taken as the lesser of the two solutions corresponding to the web bending mechanism (slender girders) and the web yielding mechanism (stocky girders). In practice there will be a range of web slenderness ratios over which a transition from stocky yielding to slender bending failure takes place.

6) Roberts and Rockey comment that the solution tends to overestimate collapse loads in cases where the loaded length is relatively large. Satisfactory correlation with the test data is achieved by ensuring the value of $c$ used in the calculations does not exceed either 0.2$d$ or 2$j$. Now for cases where the plate is of short depth or large aspect ratio, or where the predicted mechanism is localised, these restrictions may constitute a significant departure from the true loading and the question arises of whether satisfactory predictions will be achieved in all cases to which the method might be applied.

7) As mentioned by Roberts and Rockey, stretching of the web plate as deformation proceeds is neglected in the solution. The mechanism is not therefore strictly compatible. It is possible, especially in cases where large deformations of the web develop, that the work associated with web stretching may become significant and should then be incorporated into the solution.

8) In evaluating the internal work it is assumed that the web yield lines to either side of the patch load are of length $j$ and that they undergo the same rotation as the yield lines beneath the patch load. This is equivalent to assuming that the dimension $h$
(and 2h) is small compared with \( j \). Although the only true physical restriction on \( h \) is that it cannot exceed \( d/2 \), it is limited in the analysis to \( d/6 \). It is noted again, however, that in a later development by Roberts [56], the limitation on \( h \) was altered to its actual maximum of \( d/2 \). The limitation on \( j \) meanwhile is that it cannot exceed \( 0.5 (b-c) \). If, for a square panel, \( j \) takes its absolute maximum value of \( b/2 \) when \( c=0 \), then the ratio between the maximum of \( h \) (of \( d/6 \)) and \( j \) is 0.33, between 2h and \( j \) it is 0.67. The slant lengths of the middle and lower web yield lines are then 1.05 and 1.20 times the \( j \) dimension respectively. Still larger factors could result from non-zero patch loading lengths and for panels with aspect ratios below 1.0. As these factors change, the true angles of rotation along the inclined yield lines change also. It is therefore possible that the work term for the web in the analysis may be somewhat in error.

With certain of the above points particularly in mind, the remainder of this chapter consists of various developments of, and extensions to, the Roberts and Rockey mechanism solution, together with consideration of its application to crane torsion-boxes.

4.3. **The New Mechanism Solution**

4.3.1. **Basic Hinge Kinematics**

A web yield line is considered as a hinge rotating about an axis through \( 0 \) at a distance \( e \) to one side of the plate midplane as shown in Fig.4.3. If the hinge axis \( 0 \) lies within the plate thickness (i.e. \( e \leq t_w/2 \)) then for a small rotation \( \delta \theta \) as shown in Fig.4.4(a), the plastic work per unit length of hinge is
Fig. 4.3. WEB HINGE ROTATION

Fig. 4.4. AXIS OF ROTATION OF WEB HINGE

(a) $e \leq \frac{tw}{2}$
(b) $e > \frac{tw}{2}$
\[
\delta W = \frac{\sigma_w}{2} \left( \frac{t_w}{2} + e \right)^2 \delta \theta + \frac{\sigma_w}{2} \left( \frac{t_w}{2} - e \right)^2 \delta \theta
\]

which becomes

\[
\delta W = \sigma_w \left( \frac{t_w^2}{4} + e^2 \right) \delta \theta
\]

(4.10)

For \( e > t_w/2 \) as shown in Fig.4.4(b)

\[
\delta W = \sigma_w t_w e \delta \theta
\]

(4.11)

Rotation \( \theta \) about the axis \( O \) is equivalent to a rotation \( \theta \) about the plate midplane plus a squashing displacement \( e \theta \).

4.3.2. The Mechanism

The assumed mechanism, shown in Fig.4.5, is similar to the Roberts and Rockey mechanism except that the distance \( h_1 \) between the upper and middle web yield lines is no longer equal to the distance \( h_2 \) between the middle and lower lines. Transverse displacement \( z \) of the middle hinge line is given by

\[
z = h_1 \theta_1 = h_2 \theta_2
\]

(4.12)

Vertical displacement is given by

\[
v = (h_1 + h_2) - (h_1 \cos \theta_1 + h_2 \cos \theta_2) + 2e_o (\theta_1 + \theta_2)
\]

(4.13)

where \( e_o \) is the value of \( e \) along the central loaded region.

Differentiating Eqn.(4.13) gives

\[
dv = h_1 \sin \theta_1 d\theta_1 + h_2 \sin \theta_2 d\theta_2 + 2e_o (d\theta_1 + d\theta_2)
\]

For small angles and using Eqn.(4.12)

\[
dv = (z + 2e_o)(d\theta_1 + d\theta_2)
\]

(4.14)

Note that on substituting \( z \), the \( h \) and \( \theta \) terms disappear from the displacement expression and hence recognition of unequal web yield line.
spacing is lost.

It is envisaged that $e$ has a maximum value $e_0$ over the central loaded region of length $c$ which decreases linearly to zero at the outer flange hinges. Thus, over the inclined web hinges

$$e = e_0 \frac{u}{j}$$

(4.15)

where $u$ is the distance from an outer flange hinge towards the adjacent inner hinge, as shown in Fig.4.5. For simplicity it is assumed that $e_0$ takes the same value in each of the three web yield lines.

4.3.3. Plastic Work

For the assumed mechanism, internal and external work can be equated for an incremental vertical displacement $dv$.

(a) $e_0 \leq t_w/2$

$$p_u \frac{dv}{j} = 4M_f \frac{dv}{j} + \sigma_w \left( \frac{t_w^2}{4} + e_0^2 \right) (2d\theta_1 + 2d\theta_2) c$$

$$+ \sigma_w \left\{ 2 \left[ \frac{t_w^2}{4} + \frac{e_0 u^2}{j} \right] du \right\} (2d\theta_1 + 2d\theta_2)$$

This becomes

$$\left[ p_u - \frac{4M_f}{j} \right] dv = 2\sigma_w \left[ c \left( \frac{t_w^2}{4} + e_0^2 \right) + 2j \left( \frac{t_w^2}{4} + \frac{e_0^2}{3} \right) \right] (d\theta_1 + d\theta_2)$$

Substituting Eqn.(4.14) gives

$$p_u = \frac{4M_f}{j} + \frac{2\sigma_w}{(z+2e_0)} \left[ c \left( \frac{t_w^2}{4} + e_0^2 \right) + 2j \left( \frac{t_w^2}{4} + \frac{e_0^2}{3} \right) \right]$$

(4.16)

For a given $z$ (including $z=0$) $p_u$ can be minimised with respect to $j$ and $e_0$. Minimising first with respect to $j$ with $e_0$ constant gives
\[ j^2 = \frac{M_e(z+2e_o)}{c\left[\frac{t_w^2}{4} + \frac{e_o^2}{3}\right]} \]  

(4.17)

Minimising with respect to \( e_o \) with \( j \) constant gives

\[ 2j \left[ \frac{ze_o}{3} + \frac{e_o^2}{3} - \frac{t_w^2}{4} \right] = c\left[\frac{t_w^2}{4} - \frac{e_o^2}{2} - ze_o \right] \]

(4.18)

For a given \( z \), Eqns.(4.17) and (4.18) can be solved for \( e_o \) and \( j \). The combined equation is a 6\(^{th}\) order equation in \( e_o \) in the most general case.

(b) \( e_o > t_w/2 \)

In the assumed linear increase in \( e \) with \( u \), let the value of \( u \) where \( e \) becomes \( t_w/2 \) be \( f \). Then

\[ P \ u \ d\ = \ 4M_e \ \frac{dv}{j} + \sigma \ t \ \ e \ c(2d\theta_1 + 2d\theta_2) + \sigma \ t \ w \ \left\{ \frac{j e u}{f} \left[ \frac{f}{j} \right] \right\} (2d\theta_1 + 2d\theta_2) \]

\[ + \ \sigma \ w \ \left\{ \frac{t_w^2}{4} + \left( \frac{e_o^2}{j} \right) \right\} \ (2d\theta_1 + 2d\theta_2) \]

where \( f = \frac{jt_w}{2e_o} \).

The work equation becomes

\[ \left[ P_u - \frac{4M_e}{j} \right] d\ = \ 2\sigma \ t \ w \ e \ c + j \ \left( \frac{jt_w^2}{12e_o^2} \right) (d\theta_1 + d\theta_2) \]

Substituting Eqn.(4.14) gives

\[ P_u = \frac{4M_e}{j} + \frac{2\sigma \ t \ w \ e \ o}{(z+2e_o)} \left[ c + j \left( \frac{t_w^2}{12e_o^2} \right) \right] \]  

(4.19)

Minimising with respect to \( j \) with \( e_o \) constant gives
\[ j^2 = \frac{2M_F e_o (z + 2e_o)}{\sigma_w t} \left( \frac{z^2}{2} + \frac{z e_o}{4} \right) \] (4.20)

Minimising with respect to \( e_o \) with \( j \) constant gives

\[ j \left[ t_e^2 e_o + \frac{zt^2}{4} - 3ze_o^2 \right] = 3cze_o^2 \] (4.21)

Solution of Eqns.(4.20) and (4.21) for \( e_o \) leads this time to a 5th order equation in \( e_o \) in the general case.

If \( z \) and \( e_o \) are non-dimensionalised with respect to the web thickness \( t_w \) thus

\[ Z = \frac{z}{t_w} \]
\[ V = \frac{e_o}{t_w} \]

then for (a) \( V \leq 0.5 \), from Eqns.(4.16),(4.17) and (4.18)

\[ p_u = \frac{4M_F}{j} + \frac{2\sigma_w t_w}{(Z+2V)} \left[ c \left( \frac{1}{4} + V^2 \right) + 2j \left( \frac{1}{4} + \frac{V^2}{3} \right) \right] \] (4.22)

where
\[ j^2 = \frac{M_F (Z+2V)}{\sigma_w t_w \left( \frac{1}{4} + \frac{V^2}{3} \right)} \] (4.23)

and
\[ 2j \left( \frac{ZV}{3} + \frac{V^2}{3} - \frac{1}{4} \right) = c \left( \frac{1}{4} - V^2 - 2V \right) \] (4.24)

and for (b) \( V > 0.5 \), from Eqns.(4.19),(4.20) and (4.21)

\[ p_u = \frac{4M_F}{j} + \frac{2\sigma_w t_w}{(Z+2V)} \left[ c + j \left( 1 + \frac{1}{12V^2} \right) \right] \] (4.25)

where
\[ j^2 = \frac{2M_F V(Z+2V)}{\sigma_w t_w (V^2 + \frac{1}{12})} \] (4.26)

and
\[ j(V + \frac{Z}{4} - 3ZV^2) = 3cZV^2 \] (4.27)
Consider now a limiting case. For an extremely stocky girder, pure squashing will occur. This is equivalent to an infinite $V$ value and a zero $Z$ value. The equations for condition (b) $V > 0.5$ therefore apply. If $V = \infty$ and $Z = 0$ are substituted into Eqn.(4.25) the collapse load is given by

$$P_u = \frac{4M}{j} + \sigma_w t_w (c+j)$$

where, from Eqn.(4.26)

$$j^2 = \frac{4M}{\sigma_w t_w}$$

These are the same expressions as those derived by Roberts and Rockey for the web yielding mechanism (see Eqns.(4.7) and (4.8)).

Consider also the case when $c = 0$. As $V$ is then a function in $Z$ only, it can be evaluated directly for any given $Z$. Eqn.(4.24) becomes

$$V^2 + ZV - 3/4 = 0 \quad V \leq 0.5$$

(4.28)

and Eqn.(4.27) becomes

$$3ZV^2 - V - Z/4 = 0 \quad V > 0.5$$

(4.29)

When $Z = 1.0$, Eqn.(4.28) gives $V = 0.5$, as does Eqn.(4.29). For the special case $c = 0$, the rotational centres of the hinges occur at the surface of the web plate when the middle hinge line of the mechanism has undergone a transverse displacement equal to the thickness of the plate. When the values $Z = 1.0$, $V = 0.5$ are substituted into Eqns.(4.23) and (4.26) the same $j$ value is obtained and the same collapse load is then given by Eqns.(4.22) and (4.25). For $Z$ values greater than 1.0, condition (a) applies and collapse loads are found from Eqns.(4.22), (4.23) and (4.24); for $Z$ values less than 1.0, condition (b) applies and collapse loads are then found from Eqns.(4.25), (4.26) and (4.27).

In the more general case when the patch length $c$ is non-zero, computation
of the collapse loads is more complicated and requires the solution of 5\textsuperscript{th} and 6\textsuperscript{th} order polynomial functions.

4.3.4. Computation of Solution

A Fortran program was written to compute theoretical collapse loads using the new mechanism solution. The 7 polynomial coefficients for the 6\textsuperscript{th} order equation in $V$ for $V \leq 0.5$ obtained by substituting Eqn.(4.23) in Eqn.(4.24) are generated as are the 6 coefficients for the 5\textsuperscript{th} order equation in $V$ for $V > 0.5$ obtained by substituting Eqn.(4.26) in Eqn.(4.27). The roots of both real polynomial functions are found by a standard method for a predetermined range of incrementing $Z$ values. For each $Z$ value there exists only one applicable $V$ solution for a given structure and load. This $V$ value is selected from the 11 solutions that result in the general case. If the value is less than 0.5 then condition (a) applies, $j$ is evaluated from Eqn.(4.23) and the collapse load from Eqn.(4.22); if the applicable solution is greater than 0.5, condition (b) applies and $j$ is then evaluated from Eqn.(4.26) and the collapse load from Eqn.(4.25). The results are presented in the form of a collapse load curve drawn through the loads calculated for each $Z$ value. A summary of the programmed solution is given in the form of a flow chart in Appendix A.

4.4. Collapse Load Curves from New Solution

Using the new mechanism solution, collapse load curves have been produced for the 63 test cases that provided the collapse test data used by Roberts and Rockey [53] to verify their mechanism solution. Details of these test girders are given in Appendix B. Table B.1 refers to a series of collapse tests (the TG and STG girders) conducted mainly in Czechoslovakia by Skaloud and Novak [58]. Table B.2 refers to a number
of tests (the B girders) conducted in Sweden by Bergfelt [59] and by Bergfelt and Hovik [60],[61]. All these girders have web slenderness ratios \( \frac{d}{t_w} \) of between 150 and 400. Table B.3 gives details of collapse tests on stockier plate girders (the TTG girders) with \( \frac{d}{t_w} \) values from 75 to 170 conducted by Drdacky and Novotny [62] in Czechoslovakia. Also given in Table B.3 are the details of three collapse tests (girders BR1-BR3) performed by Bagchi and Rockey [41].

The general form of the test girders listed in Tables B.1 and B.3 is shown in Fig.4.6. They have web panel aspect ratios of between 1.0 and 2.0. Girders B11-B20 in Table B.2 are of a similar construction but have aspect ratios of between 4.8 and 8.0. Tests B1-B10 and B21-B23, however, refer to two very long girders with continuous webs and no intermediate vertical stiffeners. In tests B1-B10 the top flange comprised a number of plates of different cross-sections. Each section was subjected to a patch load with the girder supported on the bottom flange at positions equidistant from the load line. In tests B21-B23 a different girder was supported at its ends and loaded at various positions along a uniform top flange. These tests therefore refer to girders with non-distinct web panel lengths.

Collapse load curves have also been produced for the series of short-span model crane box-girders tested at Leicester University. This collapse programme is described fully in Chapter 5 but details of the girders (referred to as the RH series) are summarised in Table 5.1 and their general form is shown in Fig.5.3. They comprise a number of plate box-girder sections, each with a rail welded to the flange above either web, as shown in Fig.5.3(a); test girders RH1, RH2A and RH2B incorporated a flat steel strip between the square bar rail and flange as shown in Fig.5.3(b). The dimension \( b_f \) (which for an ordinary plate girder is the width of the flange) used in the calculation of \( M_f \) has been taken as twice the
Fig. 4.6: DETAILS OF PLATE GIRDER SPECIMENS
distance from the web midplane to the edge of the nearer flange outstand, as shown in Fig.5.3(a). Girders RH11 and RH12 incorporated a longitudinal web stiffener as shown in Fig.5.5; RH11 had a small flat strip at the mid-depth position while RH12 had a larger steel-angle stiffener at the quarter-depth position. Web slenderness ratios encompassed by the RH series are from 133 to 240 and aspect ratios of the panels are 1.5 except for girders RH9 and RH10 which have square panels.

Selected collapse curves are shown in Figs.4.7 - 4.17. The predicted collapse load $P_u$ is plotted as a factor of the experimental collapse load $P_{ex}$ against $Z$ for $Z$ values from 0 to 10.5. The predicted collapse load was evaluated for $Z$ increments of 0.1 and the plotted points then gave the smooth curves shown in the figures. Where a curve intersects the horizontal broken line drawn through the 1.0 ordinate gives the $Z$ value for which the new mechanism solution predicts the actual experimental collapse load.

Figs.4.7 - 4.9 show the collapse load curves for girders TG5, TG11 and TG15 tested by Skaloud and Novak. The curve for girder TG5, see Fig.4.7, has a $Z$ value at the experimental collapse load of just over 8 (i.e. the maximum out-of-plane displacement of the central web yield line is a little over 8 times the web thickness). Over this region, the slope of the collapse curve is very shallow and the predicted collapse load is relatively insensitive to the precise $Z$ value. In contrast, the curve for girder TG11, shown in Fig.4.8, has a much steeper slope at the intersection point with the broken line which occurs at a $Z$ value of a little over 2. The predicted collapse load is thus sensitive to small changes in $Z$ about this value. Although the curves for girders TG5 and TG11 both give a maximum squash ($Z=0$) load of approximately $2.7 P_{ex}$, their profiles thereafter are quite different with the curve for TG11 falling much more steeply.
Fig. 4.7. COLLAPSE LOAD CURVE FOR S AND N GIRDER TGS.

EXPERIMENTAL COLLAPSE LOAD = 179.00 KN
Fig. 4.8. COLLAPSE LOAD CURVE FOR S AND N GIRDER TG11.

EXPERIMENTAL COLLAPSE LOAD = 93.19 kN
Fig. 4.9. COLLAPSE LOAD CURVE FOR S AND N GIRDER TG15.

EXPERIMENTAL COLLAPSE LOAD = 157.94 KN

PRED/EXPTL COLLAPSE LOAD

Z VALUE
Fig. 4.9 shows the collapse curve for girder TG15. Of all the 63 test cases considered by Roberts and Rockey, this girder has, at just over 4, the greatest ratio between squash load and experimental collapse load. Otherwise, the curve is similar to that for girder TG5 in that it has a shallow slope over the region where it intersects the broken line which is at a relatively large $Z$ value of approximately 9.5. Now apart from small variations in their respective flange and web yield stresses, girders TG15 and TG11 differ only in the size of flange: girder TG15 has a flange which is wider and nearly five times the thickness of that of girder TG11. Girder TG5 has a similar sized flange to girder TG15. A heavy flange therefore appears to lead to a large $Z$ value at collapse and to a situation where the predicted collapse load is relatively insensitive to changes in $Z$ about this value. Conversely, a thin flange tends to lead to a small $Z$ value and to predicted loads that are more sensitive to changes about this value.

Figs. 4.10 and 4.11 show the collapse curves for girders STG7 & 8 and B12; both girders have relatively small flanges. In both cases the $Z$ value at which the collapse curve corresponds to the experimental collapse load is small and in a region where the slope of the curve is steep. Fig. 4.12 shows the collapse curve for girder TTG2, one of a range of stockier girders. In this case, even the squash load corresponding to direct yielding of the web ($Z=0$) gives an underestimation of the experimental collapse load. An example of the curves for the girders tested by Bagchi and Rockey is shown in Fig. 4.13 for BR2.

The collapse curves shown in Figs. 4.14 - 4.17 are taken from the RH series of girders. Figs. 4.14 and 4.15 both apply to RH1. The first shows the collapse curve when the patch length $c$ is taken as zero, the second when assumed to be 10 mm. The curve for the non-zero patch length is slightly higher over small $Z$ values but the difference becomes indistinguishable
Fig. 4.10. COLLAPSE LOAD CURVE FOR S AND N GIRDERS STG7 AND STG8.

EXPERIMENTAL COLLAPSE LOAD = 35.56 kN
Fig. 4.11. Collapse Load Curve for B and H Girder B12.

Experimental Collapse Load = 65.73 kN
**Fig. 4.12.** COLLAPSE LOAD CURVE FOR D AND N GIRDER TTG2.

Experimental collapse load = 147.50 kN
Fig. 4.13. COLLAPSE LOAD CURVE FOR B AND R GIRDER BR2.

EXPERIMENTAL COLLAPSE LOAD = 124.00 KN
Fig. 4.14. COLLAPSE LOAD CURVE FOR R AND H GIRDER RH1.

EXPERIMENTAL COLLAPSE LOAD = 112.00 KN
Fig. 4.15. Collapse load curve for R and H girder RHI (C=10).

Experimental collapse load = 112.00 KN

Pre/d/Expl. collapse load
Fig. 4.16. COLLAPSE LOAD CURVE FOR R AND H GIRDER RH6.

EXPERIMENTAL COLLAPSE LOAD = 80.00 KN
Fig. 4.17. COLLAPSE LOAD CURVE FOR R AND H GIRDER RH9.

EXPERIMENTAL COLLAPSE LOAD = 112.00 KN
over larger $Z$ values. Figs. 4.16 and 4.17 show curves for RH6 and RH9 which are typical of the curves for the RH series. In computing these curves a zero patch length has been assumed.

In Appendix C the $Z$ values for which the collapse curves correspond to the experimental collapse loads are given for all the girders listed in Appendix B and for the RH girders detailed in Table 5.1. Also shown are the $Z$ values at 110% and 90% of $P_{ex}$ which gives an indication of the sensitivity of the predicted collapse load to the $Z$ value. A dash denotes a case where the curve failed to reach the load specified in the column heading as, for example, with girder TTG2 shown in Fig. 4.12. Now a full solution using the new mechanism approach requires some means for selecting $Z$ (or $Z$) - the measure of web deformation at the onset of collapse. Unfortunately this cannot be evaluated in the same way that $e_o$ and $j$ were by minimising the collapse load expressions of Eqns. (4.16) and (4.19). The problem of assessing $Z$ here is directly analogous with that of estimating $h$ and $\psi$ in the method by Roberts and Rockey where $h$ was selected empirically and $\psi$ estimated using elastic theory. The question of determining web deformation at the onset of collapse is returned to later in this chapter. Before that, however, a comparison is now made between the new approach and that by Roberts and Rockey.

4.5. **Comparison of the two Analyses**

In the method of Ref. [53], transverse displacement of the central web yield line is given by $h \cos \psi$ (see Fig. 4.2). This is the dimension $Z$ in the new analysis, as shown in Fig. 4.5. Hence

$$Z = h \cos \psi$$

Substituting in the collapse load expression of Eqn. (4.1) gives
\[ P_u = \frac{4M_f}{j} + \frac{2M_w}{z} (2j + c - \eta) \] (4.31)

Now \( \eta \) is given by Eqn.(4.5); on substituting Eqn.(4.30) this becomes

\[ \eta = \frac{(4j+2c)M_w}{2M_w + \sigma_w t_w z} \]

where

\[ M_w = \frac{\sigma_w t_w^2}{4} \] (4.32)

Hence

\[ \eta = \frac{2j + c}{1 + (2z/t_w)} \] (4.33)

Substituting for \( \eta \) and \( M_w \) in Eqn.(4.31) gives

\[ P_u = \frac{4M_f}{j} + \frac{\sigma_w t_w}{2z/t_w} (2j+c) \left[ 1 - \frac{1}{1 + (2z/t_w)} \right] \]

Using as before \( Z = z/t_w \) this reduces to

\[ P_u = \frac{4M_f}{j} + \frac{\sigma_w t_w}{1 + 2Z} (2j+c) \] (4.34)

where \( j \) is given by Eqn.(4.2), but on substituting Eqns.(4.30) and (4.32) and expressing in terms of \( Z \), \( j \) is given by

\[ j^2 = \frac{4M_f Z}{\sigma_w t_w} \] (4.35)

The solution for direct yielding of girders with stocky webs needs no reformulating for comparison purposes, its original form given by Eqns.(4.7) and (4.8) can be used.

Using Eqns.(4.7) and (4.8), and Eqns.(4.34) and (4.35), collapse load curves can be produced from the analysis of Ref.[53] over a range of incrementing \( Z \) values in the same manner as were produced using the new analysis. Selected comparison collapse load curves are shown in Figs. 4.18 - 4.21 for girders TG11, TG15, TTG2 and RH6 respectively. In each figure two curves are shown: the continuous curve with a smooth transition from a stocky to a slender mode of collapse as obtained with the new analysis and the curve with a squash load cut-off obtained with the method
Fig. 4.18. COLLAPSE LOAD CURVE FOR S AND N GIRDER TG11.

EXPERIMENTAL COLLAPSE LOAD = 93.19 KN
Fig. 4.19. COLLAPSE LOAD CURVE FOR S AND N GIRDER TG15.

EXPERIMENTAL COLLAPSE LOAD = 157.94 KN
Fig. 4.20. COLLAPSE LOAD CURVE FOR D AND N GIRDER TG2.

EXPERIMENTAL COLLAPSE LOAD = 147.50 kN

PRED/EXPTL COLLAPSE LOAD

Z VALUE
Fig. 4.21. COLLAPSE LOAD CURVE FOR R AND H GIRDER RH6.

EXPERIMENTAL COLLAPSE LOAD = 80.00 KN
by Roberts and Rockey. The physical limitation on \( j \) has been applied in the solutions but the restriction on \( c \) normally employed in the original analysis has been omitted here to achieve a more meaningful comparison.

The horizontal squash load line shown in the figures arises from Eqn.\((4.7)\) and extends over the range of \( Z \) values for which the collapse load predicted by Eqn.\((4.34)\) exceeds that predicted by Eqn.\((4.7)\). The smooth curve obtained with the new approach can be seen to give lower estimates of collapse loads over the transition region, although an identical squash load at \( Z=0 \) is obtained. Beyond the transition region, the curve from the new analysis crosses the other and gives slightly greater collapse load predictions but the two curves follow similar paths and converge towards larger \( Z \) values.

Overall, the curves resulting from the two approaches are very similar. The only appreciable difference occurs in the region of transition from a stocky to slender type of collapse. This distinction is of significance only for girders that approach a stocky collapse condition. It is for such girders, however, that the two types of collapse curve are particularly similar, owing to the curves being very flat as shown for example in Fig.\(4.20\) for girder TTG2. For some girders, the restriction in the original theory that \( c \) does not exceed \( 2j \) has the effect of further reducing this difference since the small \( Z \) values in the transition region lead to small \( j \) values when the restriction on \( c \) is more likely to apply; this restriction then lowers slightly the collapse curve from the original solution over the region leading up to the squash line.

Thus it can be seen that the differences between the two analyses are unlikely to be of any great significance.
4.6. Web Deformation at Collapse

Selection of an appropriate $Z$ value represents an estimation of the deformation of a girder at the onset of collapse. In the solution by Roberts and Rockey this is achieved by an empirical evaluation of $h$ and an estimation of $\psi$ (see Fig. 4.2) using elastic theory. If in the new analysis it is assumed that $h_1 = h_2 = h$, then $\theta_1 = \theta_2 = \theta$ (see Fig. 4.5) and the same method can then be employed. From Eqn.(4.12) $z$ is now given by

$$z = h\theta$$  \hspace{1cm} (4.36)

Vertical deflection of the loaded flange, $v$, is given for the new solution in Eqn.(4.13). If this is equated to the elastic deflection of the flange as in the Roberts and Rockey method (see Eqn.(4.3)) and $\theta$ is assumed a small angle so that $\cos \theta$ is given by

$$\cos \theta = 1 - \theta^2/2$$

then Eqn.(4.13) gives

$$v = 2h - h(2-\theta^2) + 4e_o \theta = M_fj^2/6EI_f$$

Hence

$$h\theta^2 + 4e_o \theta = M_fj^2/6EI_f$$  \hspace{1cm} (4.37)

Substituting for $\theta$ from Eqn.(4.36) gives

$$z^2 + 4e_o z = M_fj^2/6EI_f$$  \hspace{1cm} (4.38)

Non-dimensionalising again using $Z = z/t_w$ and $V = e_o/t_w$ gives

$$z^2 + 4VZ = \frac{M_fh}{6EI_f t_w^2} j^2$$  \hspace{1cm} (4.39)

It will be recalled that the collapse load curves presented earlier were produced by evaluating $V$ and $j$ for each of a range of incrementing $Z$ values: for $V \leq 0.5$, $V$ and $j$ are found from Eqns.(4.23) and (4.24); for $V > 0.5$, $V$ and $j$ are found from Eqns.(4.26) and (4.27).
If this procedure is repeated, a curve of $V$ values can be plotted against $Z$. Now for each pair of $V$ and $J$ values, an estimated $Z$ value can then be found from Eqn.(4.39) by substituting for $h$ from Eqn.(4.6). By plotting the estimated $Z$ values so obtained on the same axes as the curve of $V$ values, the $Z$ value at the onset of collapse can be determined from the point where the two curves intersect. An example is shown in Fig.4.22 for RH6 where the curve extending over the entire $Z$ range is that of the $V$ values for incrementing $Z$, and the other curve is that of estimated $Z$ values corresponding to these $V$ values (found using Eqn.(4.39)). Thus, application to the new analysis of the empirical $h$ estimation and the elastic flange deflection condition adopted by Roberts and Rockey leads to an estimated $Z$ value for girder RH6 of approximately 4.3. If this value is used to read-off the collapse load from Fig.4.21 then the predicted load can be seen to overestimate the experimental load by a factor of approximately 1.1.

Generally, however, the predicted collapse loads obtained by this combination of the new and original methods do not agree particularly well with the experimental values and the overall correlation is not as good as that obtained using the original analysis on its own. The increased discrepancy between predicted and experimental loads obtained with the hybrid solution arises because the method used by Roberts and Rockey for estimating $h$ was chosen to suit that particular solution, it is not of general application.

The main distinction between Eqn.(4.37) and its counterpart in the original work given in Eqn.(4.3) is the presence in the new analysis of the squashing term $4e_0^2$. If this term is omitted then the equation for the $Z$ values given in Eqn.(4.39) becomes

$$Z^2 = \frac{M_h}{\bar{f} J} f_w^2$$

(4.40)
Fig. 4.22. $Z$ VALUE SELECTION CURVES FOR R AND H GIRDER RH6.
and this leads to selected $Z$ values that give predicted collapse loads agreeing closely with those obtained from the original solution by Roberts and Rockey.

It should be mentioned that when the patch loading length $c$ is zero then Eqns. (4.24) and (4.27) simplify to the expressions given in Eqns. (4.28) and (4.29) when the $V$ values can be found explicitly in terms of $Z$. When the substitution for $j^2$ is made, either in Eqn. (4.39) if the squashing term is to be incorporated or in Eqn. (4.40) if it is to be omitted, then the estimated $Z$ values are given explicitly in terms of $V$ and the selected $Z$ value can readily be found by an iterative process. For non-zero patch lengths, solution of the equations is more complicated and requires a computer.

4.7. Concluding Remarks on the New Mechanism Solution

The new mechanism solution has been shown to provide improvements to the original solution by Roberts and Rockey in that a smooth transition between stocky and slender collapse is achieved and the need to evaluate the direct yielding term $\eta$ is removed. Apart from over the region of transition, however, the two solutions have been shown to lead to very similar collapse load curves. Accordingly, when girder deformation at the onset of collapse is estimated in the new analysis by the method used in the original work, no improvement in collapse load predictions is achieved with the new method of solution. Until such time as an improved prediction of girder deformation at collapse is found, it therefore appears that as regards a design approach the increased computational complexity associated with the new analysis is not warranted.

Consequently, attention is now refocused on the original analysis for a discussion firstly of the effect of web stretching (which was neglected by Roberts and Rockey in order to simplify the analysis) and secondly of
the inclination of the web yield lines in the assumed mechanism.

4.8. Discussion of Web Stretching

4.8.1. Analysis of Web Stretching

Out-of-plane deformation of the web as collapse proceeds requires, for reasons of compatibility, that the web plate stretches in the longitudinal direction. In the solution by Roberts and Rockey this is neglected.

Fig.4.23(a) shows the out-of-plane deformation of the central, loaded, region of the web. Fig.4.23(b) shows an isometric view of the deformed web to one side of the loaded region. Now if, before loading, the web were cut vertically at the positions aligning with the edges of the patch load and there was no shearing between the web and flange, then the web would open up as deformation proceeded. When projected onto the original flat plane of the web, the total area of the opening is as shown in Fig.4.23(c). This represents the stretching that the web undergoes in practice.

From the sectional view of the central zone of the mechanism shown in Fig.4.23(a) it can be seen that for small rotations $\theta$, compatibility gives

$$v = 2h - 2h \cos \theta = 2h[1 - (1-\theta^2/2)]$$

Hence

$$v = h\theta^2$$

Now from Eqn.(4.36) $z = h\theta$ and so

$$v = \frac{z^2}{h}$$

(4.41)
Fig. 4.23 WEB STRETCHING IN WEB MECHANISM

(a) SECTIONAL VIEW OF CENTRAL WEB ZONE UNDER PATCH
(b) ISOMETRIC VIEW OF WEB TO ONE SIDE OF PATCH
(c) PROJECTED VIEW OF OPENING
(d) AREA OF EXTRA WORK
Fig. 4.23(b) shows that as the web deforms it is required to stretch so as to close the area described by the gap dimensions $a_1$ and $a_2$ at the upper and middle yield lines created by A moving to $A_1$ and B to $B_1$. D is the projection of $B_1$ onto the original plane. The total stretched area when projected onto the original web plane is shown in Fig. 4.23(c) where

$$A^1D = DC = \sqrt{h^2 - z^2} = \sqrt{r^2 - vh}$$

For a small displacement $v$ this reduces to

$$A^1D = DC = h - v/2$$

In Fig. 4.23(b), let angle $AOA^1$ in the plane of the flat web be $\phi$. In the same plane, let angle $AOD$ be $\phi_1$. Let the angle between $OB_1$ and the original plane of the web be $\gamma$. Then

$$a^1 = j(1 - \cos \phi)$$

where $\tan \phi = v/j$ which is a small angle if $v$ is small.

Therefore

$$a^1 = j[1 - (1 - \phi^2/2)] = \frac{v^2}{2j}$$

Using Eqn. (4.41) this then becomes

$$a^1 = \frac{v^2}{2j} = \frac{z^4}{2jh^2} \quad (4.42)$$

Now

$$a_2 = OA - OE$$

where

$$OE = OB_1 \cos \gamma \cos \phi_1$$ and $$OB_1 = \sqrt{j^2 + h^2}.$$ Hence

$$a_2 = j - \sqrt{j^2 + h^2} \cos \gamma \cos \phi_1 \quad (4.43)$$

Now $\gamma$ is a small angle given by

$$\gamma = \frac{z}{\sqrt{j^2 + h^2}} \quad (4.44)$$
\( \phi_1 \) is not necessarily a small angle and is given by

\[
\sin \phi_1 = \frac{ED}{OD} = \frac{h + (v/2)}{\sqrt{j^2 + h^2} \cos \gamma}
\]

where from Eqn.(4.44)

\[
\cos \gamma = 1 - \frac{z^2}{2(j^2+h^2)}
\]

Now \( \cos \phi_1 = \sqrt{1 - \sin^2 \phi_1} \)

Hence \( \cos \phi_1 = \sqrt{1 - \frac{[h + (v/2)]^2}{(j^2+h^2)\cos^2 \gamma}} \)

Therefore

\[
\cos \phi_1 = \frac{\cos^2 \gamma [j^2+h^2] - [h + (v/2)]^2}{\cos \gamma \sqrt{j^2 + h^2}}
\] (4.46)

Substituting Eqn.(4.46) into Eqn.(4.43) gives

\[
a_2 = j - \sqrt{\cos^2 \gamma [j^2+h^2] - [h + (v/2)]^2}
\]

Substituting Eqn.(4.45) gives

\[
a_2 = j - \sqrt{\left[1 - \frac{z^2}{2(j^2+h^2)}\right] [j^2+h^2] - [h + (v/2)]^2}
\]

Neglecting second order small terms this reduces to

\[a_2 = \frac{z^2}{j} = \frac{hv}{j}\] (4.47)

Hence for small displacements \( v \), \( a_2 \) is two orders of magnitude greater than \( a_1 \).

The area of web stretching can therefore be approximated as shown in Fig.4.23(d). Now stretching over this area requires extra work to be done and this can now be assessed.

Extra Work = \( \sigma_{w w} 2h a_2 \)
Substituting Eqn.(4.47) gives

$$\text{Extra Work} = 2a w \frac{h^2}{2} v$$

for an incremental displacement $dv$

$$\text{Extra Work} = 2a w \frac{h^2}{2} dv$$

An additional term $G$ is therefore required in the collapse equation given by

$$G = \frac{2a w h^2}{j}.$$  

(4.48)

This term is similar to the term $4M_F/j$ already present in the collapse equation of Eqn.(4.1). Consequently, the effect of web stretching can be incorporated by modifying the first term on the right hand side of the collapse equation to

$$\frac{4M_F}{j} + 2a w \frac{h^2}{j} = \frac{4}{j} \left( M_F + \frac{a w h^2}{2} \right).$$

This is equivalent to replacing $M_F$ by $[M_F + (a w h^2/2)]$.

If Eqn.(4.2) is substituted into Eqn.(4.1), the first two terms on the right hand side can be seen to in fact be equal

$$P_u = 4 \sqrt{\frac{M_F w}{h \cos \psi}} + 2 \sqrt{\frac{M_F w}{h \cos \psi}} + \frac{2M_w}{h \cos \psi} (c-n).$$

The term involving $c$ and $n$ is generally comparatively small. If its effect is neglected then

$$P_u = \frac{8M_F}{j} = \frac{8jM_F}{h \cos \psi}. \quad (4.49)$$

The significance of the web stretching term in the collapse equation can therefore be assessed directly by comparing the term $a w h^2/2$ with $M_F$ in the knowledge that approximately half the collapse load is attributable
<table>
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<tr>
<th>Test Girder</th>
<th>$M_f \times 10^6$ Nmm</th>
<th>h (Eqn. (4.6)) mm</th>
<th>$\sigma_{w, h^2}^2 / 2 \times 10^6$ Nmm</th>
<th>$\sigma_{w, h^2}^2 / 2M_f$</th>
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<td>5.48</td>
</tr>
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</tr>
<tr>
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</tr>
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<td>4.269</td>
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<tr>
<td>BR2</td>
<td>1.532</td>
<td>61.91</td>
<td>1.557</td>
<td>1.02</td>
</tr>
</tbody>
</table>
to the $M_F$ parameter. Table 4.1 shows the ratio $\frac{\sigma_w t_w h^2}{2M_F}$ for a number of girders.

It is evident that the web stretching contribution can be significant, sometimes very significant, attaining several multiples of $M_F$. The trend in the values of the ratio for girders TG11 - TG15 reveals that web stretching is most significant for girders with light, thin flanges. Girders TG11 - TG15 have progressively heavier, thicker flanges but, apart from small variations in material yield stresses, are otherwise the same. Girder TG11 with a thin flange has a web stretching term which is over 5 times its $M_F$ value while TG15 with a very thick flange has a value of only 0.17 for this ratio. In Table 4.1, $h$ has been evaluated using Eqn.(4.6). The web stretching component is clearly strongly dependent upon this empirical estimation. For a given web and yield stress of flange material, $h$ is constant and so too then is the stretching term. A girder with thin flanges will have a small $M_F$ which leads to a large ratio between the web stretching term and $M_F$, for girders with thicker flanges, $M_F$ is larger and the ratio decreases.

4.8.2. Inclusion of Web Stretching in Solution

If the effect of web stretching is included in the solution then the collapse load equation given in Eqn.(4.1) becomes

$$P_u = \frac{4M_F}{j} + 4jM_w w + \frac{2cM_w}{h \cos \psi} - \frac{2nM_w}{h \cos \psi} + G$$

(4.50)

where $G$ is given by Eqn.(4.48). Substituting $M_w$ into Eqn.(4.48) from Eqn.(4.32) gives for the web stretching term

$$G = \frac{8M h^2}{t_w j}$$

(4.51)

Substituting for $G$ from Eqn.(4.51) into Eqn.(4.50) gives

$$P_u = \frac{4M_F}{j} + \frac{4jM_w}{h \cos \psi} + \frac{2cM_w}{h \cos \psi} - \frac{2nM_w}{h \cos \psi} + \frac{8M h^2}{t_w j}$$

(4.52)
\( P_u \) in Eqn.(4.52) can be minimised with respect to \( j \) as before but now, owing to inclusion of the web stretching term, a minimisation with respect to \( h \) can, in principle, also be made. Minimising first with respect to \( j \) , with \( h \) constant, gives

\[
j^2 = h \cos \psi \left[ \frac{M_f}{M_w} + \frac{2h^2}{t_w} \right] \tag{4.53}\]

Minimising with respect to \( h \) , with \( j \) constant, gives

\[
h^3 = \frac{t_j(2j+c-n)}{8 \cos \psi} \tag{4.54}\]

Eqn.(4.52) can be rearranged thus

\[
P_u = \frac{4M_f}{j} + \frac{2M_w}{h \cos \psi} \left[ 2j + c - n + \frac{4h^3 \cos \psi}{t_j} \right] \tag{4.55}\]

Substituting Eqn.(4.54) for \( h^3 \) gives

\[
P_u = \frac{4M_f}{j} + \frac{2M_w}{h \cos \psi} \left[ 2j + c - n + \frac{(2j+c-n)}{2} \right] \]

Hence

\[
P_u = \frac{4M_f}{j} + \frac{3M_w}{h \cos \psi} (2j+c-n) \tag{4.56}\]

The term \( n \) is again determined by assuming that the web contribution to the collapse load is transmitted to a length \( n \) of web plate in the vicinity of the patch load that yields in direct compression. The web contribution to the collapse load is given by the terms on the right-hand-side of Eqn.(4.55) apart from the first. A new expression for \( n \) is thus obtained by equating the web contribution to the force in the yielded length \( n \) of web

\[
\frac{2M_w}{h \cos \psi} \left[ 2j + c - n + \frac{4h^3 \cos \psi}{t_j} \right] = \sigma \frac{t_w}{w} n
\]

Hence

\[
n = \frac{2M_w[2j + c + (4h^3 \cos \psi/t_j)]}{2M_w \sigma \frac{t_w}{w} \cos \psi} \tag{4.57}\]

Equating elastic deflection of the flange to deformation of the web as
before gives

\[ \frac{M_f j^2}{6E_l} = 2h(1 - \sin \psi) \]

Substituting for \( j^2 \) from Eqn.(4.53) gives.

\[ \frac{\cos \psi}{1 - \sin \psi} = \frac{12M_t w w M_f}{M_f(M_f w w + 2M w h^2)} = S \text{ (say)} \]

(4.58a)

enabling \( \cos \psi \) to be found from

\[ \cos \psi = \frac{2S}{1 + S^2} \]

(4.58b)

Inclusion of the web stretching term \( G \) in the collapse load expression thus enables a minimisation with respect to \( h \) to be conducted and thereby eliminates the necessity for an empirical estimation of this term. Eqn.(4.56) shows that a collapse load expression very similar to the original derived by Roberts and Rockey (see Eqn.(4.1)) is obtained once the substitution for \( h \) is made. However, compared with the original analysis, solution of the equations for \( j, h, \eta \), and \( \cos \psi \) (Eqns.(4.53), (4.54), (4.57) and (4.58) respectively) is very much more complicated and requires a computer. It is therefore difficult to envisage how this complex solution could be utilised as a general design method.

In the following section, attention is turned to a further aspect of the basic method of analysis.

4.9. Influence of Inclination of Web Yield Lines

In the mechanism assumed for girders with slender webs, the web yield lines are considered as plastic hinge lines. The second term on the right-hand side of Eqn.(4.1) represents the internal work attributable to rotation of these hinges along the lines to either side of the patch load (the inclined lines).
In evaluating this work it has been assumed that the lengths of the inclined hinge lines are equal to \( j \) and that these hinges rotate through the same angle as do those beneath the load. If \( h \) is small compared with \( j \), the error associated with these assumptions is small; as \( h \) increases relative to \( j \), however, the error becomes more significant.

The web mechanism to one side of the patch load is shown in Fig.4.24(a) where \( h \) is not small compared with \( j \). The inclined web yield lines are \( OB \) and \( OC \). Let angle \( AOB \) be \( \phi_2 \) and angle \( BOC \) be \( \phi_3 \).

Line \( AKL \) is perpendicular to \( OB \) and line \( BM \) is perpendicular to \( OC \).

Now \[ \cot \phi_2 = \frac{j}{h} \] (4.59)
and \[ OB = \sqrt{h^2 + j^2} \] and \[ OC = \sqrt{4h^2 + j^2} \]

From the sine rule on triangle \( OBC \)

\[ \frac{h}{\sin \phi_3} = \frac{OC}{\sin(90^\circ + \phi_2)} = \frac{OC}{\cos \phi_2} = \sqrt{4h^2 + j^2} \frac{\sqrt{h^2 + j^2}}{j} \] (4.60)

Hence \[ \csc^2 \phi_3 = \frac{4h^4 + 5h^2 j^2 + j^4}{h^2 j^2} \]

Now \[ \cot \phi_3 = \sqrt{\csc^2 \phi_3 - 1} \]

Hence \[ \cot \phi_3 = \frac{\sqrt{4h^4 + 4h^2 j^2 + j^4}}{hj} = \frac{2h^2 + j^2}{hj} \] (4.61)

Let hinge \( OA \) rotate by \( \theta_a \) as shown in Fig.4.24(b)

Then \[ \theta_a = \frac{z}{h} \]

and along this hinge line

\[ \text{Rotation x Length} = j \theta_a = \frac{zj}{h} \]
Fig. 4.24. ROTATION OF INCLINED WEB YIELD LINES WHEN h AND j ARE OF COMPARABLE SIZE
(a) WEB YIELD LINES TO ONE SIDE OF PATCH
(b) SECTIONAL VIEW OF HINGE OA
(c) SECTIONAL VIEW OF HINGE OB
(d) SECTIONAL VIEW OF HINGE OC
Let hinge OB rotate by \((\theta_b + \theta_c)\) as shown in Fig. 4.24(c) where

\[ AK = \frac{z}{j} \sin \phi_2 \]

and

\[ KL = \frac{OK}{OB} \tan \phi_3 = \frac{j}{\tan \phi_2} \cos \phi_2 \tan \phi_3 \]

Then

\[ \frac{\theta_b}{\frac{z'}{j}} = \tan \phi_2 \quad \text{and} \quad \frac{\theta_c}{\frac{z'}{j}} = \tan \phi_3 \]

where

\[ z' = \frac{OK}{OB} z = \frac{j}{\tan \phi_2} \frac{\cos \phi_2}{OB} z \]

Rotation x Length = \(OB(\theta_b + \theta_c)\)

Hence, substituting Eqns. (4.59) and (4.61) gives for hinge OB

\[ \text{Rotation x Length} = z \left[ \frac{j}{h} + \frac{2h^2 + j^2}{hj} \right] = 2z \left( \frac{h^2 + j^2}{hj} \right) \]

Let hinge OC rotate by \(\theta_d\) as shown in Fig. 4.24(d) where

\[ BM = \frac{OB}{\sin \phi_3} \]

and from Eqn. (4.60)

\[ \sin \phi_3 = \frac{\cos \phi_2}{OC} = \frac{hj}{OB \cdot OC} \]

Then

\[ \frac{\theta_d}{\frac{z}{OB \sin \phi_3}} = \frac{\frac{z}{OC}}{hj} \]

Hence

\[ \text{Rotation x Length} = OC \frac{\theta_d}{\frac{z}{hj}} \{ (OC)^2 = \frac{z}{hj} (4h^2 + j^2) \} \]

Summation of the work terms for the inclined web yield lines gives

\[ \sum \text{Rotation x Length} = \frac{z}{hj} [j^2 + 2(h^2 + j^2) + (4h^2 + j^2)] \]

i.e.

\[ \sum \text{Rotation x Length} = \frac{z}{hj} (4j^2 + 6h^2) \]

If \(\phi_2\) and \(\phi_3\) are small, \(h\) is small and

\[ \sum \text{Rotation x Length} = \frac{z}{hj} 4j^2 \]
Hence, the effect of an \( \alpha \) which is not small compared with \( \beta \) is to multiply the work term associated with rotation along the inclined web hinge lines by a factor \( H \) where

\[
H = \left[ 1 + \frac{3}{2} \frac{\alpha^2}{\beta^2} \right] \tag{4.62}
\]

Evaluating \( H \) for selected girders reveals that values well in excess of 1.0 can arise, as shown in Table 4.2. It will be recalled from Section 4.8.1. that the first two terms on the right-hand-side of the collapse equation (Eqn.(4.1)) each represent approximately half the collapse load (see Eqn.(4.49)). Since the factor \( H \) applies to the second of these terms, its effect on the collapse load prediction can be seen to be significant. From Table 4.2 it appears that, as with the web stretching term \( G \), the effect of \( H \) is greatest in the case of girders with light, thin flanges. This is because of the localised mechanisms with small \( \beta \) values that develop for girders with light flanges. The trend in \( H \) values for girders TG11 - TG15 shows \( H \) to be greatest for TG11 with a value of nearly 3, decreasing to just over 1 for TG15. It should be noted that, as with the web stretching term \( G \), \( H \) is a function of \( \alpha^2 \).

If \( H \) is incorporated into the solution together with \( G \), minimisations with respect to \( \alpha \) and \( \beta \) can again be made. The collapse equation becomes

\[
P_u = \frac{4M_f}{3} + \frac{4jM_w}{h \cos \psi} \left[ 1 + \frac{3}{2} \frac{h^2}{j^2} \right] + \frac{2M_w}{h \cos \psi (c-n)} + \frac{8M_h^2}{t_w} \tag{4.63}
\]

Minimising with respect to \( \beta \) gives

\[
\beta^2 = \frac{M_f h \cos \psi}{M_w} + \frac{3h^2}{2} + \frac{2h^3 \cos \psi}{t_w} \tag{4.64}
\]
<table>
<thead>
<tr>
<th>Test Girder</th>
<th>h (mm)</th>
<th>j (mm)</th>
<th>$H = 1 + \frac{3}{2} \left[ \frac{h}{j} \right]^2$</th>
</tr>
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<td>900.0</td>
<td>1.010</td>
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<td>38.92</td>
<td>1.232</td>
</tr>
<tr>
<td>BR2</td>
<td>61.91</td>
<td>127.9</td>
<td>1.351</td>
</tr>
</tbody>
</table>
and minimising with respect to \( h \) gives

\[
\frac{8h^3 \cos \frac{c}{t_w}}{t_w} + \frac{3h^2}{j} - (2j + c - \eta) = 0 \tag{4.65}
\]

\( \eta \) is now given by

\[
\eta = \frac{2M_w[2j+c+(3h^2/j) + (4h^3 \cos \psi/t_wj)]}{2M_w + \sigma_{t_w} h \cos \psi} \tag{4.66}
\]

and equating web and flange deformations gives

\[
\frac{\cos \psi}{1-\sin \psi} = \frac{24M_t E_l f}{M_f[2M_f t_w + 4M_h^2 + (3M_w t_w h/\cos \psi)]} \tag{4.67}
\]

Solution of Eqns. (4.63)-(4.67) is still more complicated than is solution of the equations resulting from inclusion of the \( G \) term alone. Moreover, it has now become difficult to ascertain whether true minimisation of \( P_u \) are being obtained with respect to \( h \) and \( j \). Additional work is required to investigate this before the analysis can be taken any further.

Development of the Roberts and Rockey solution has been taken no further in the present study. It appears attempts at refinement are penalised, often heavily, by increased computational complexity and the attraction of a simple design method is then lost. It is considered that, owing to the large number of parameters involved in plate girder collapse under patch loading, a simple hand calculation method of prediction is desirable for design purposes.

For this reason, the original analysis is returned to in the following section in order to assess its suitability for predicting collapse loads of wheel loaded crane box-girders.
4.10. Roberts and Rockey Solution on the RH Series of Girders

Despite the simplifications and approximations inherent in the solution by Roberts and Rockey, it has been shown in Refs.[53] and [54] to give generally very good collapse load predictions for the test girders detailed in Appendix B. Table 4.3 shows the predictions, $P_u$, obtained for the RH series of short-span model crane box-girders detailed in Chapter 5. Generally, very good agreement with the experimental collapse loads, $P_{ex}$, is again achieved. With one exception, the predictions lie within approximately 10% of the experimental values. The exception is RH1 for which the prediction over-estimates by nearly 50%. As can be seen from Table 5.1, this girder, being of approximately one-third scale, is smaller than the others which are approximately half scale models of typical full-size crane box-girder sections.

Now RH1 is geometrically similar to RH2A and RH2B; linear dimensions of RH1 are between about 0.6 and 0.8 times the corresponding dimensions of RH2A and RH2B, typically the scaling factor is 0.7. Material yield stresses of the components of RH1 are similar to the yield stresses of the corresponding components of RH2A and RH2B. The Roberts and Rockey solution predicted the collapse loads of RH2A and RH2B very closely, but failed to do so for RH1. It is to be expected that for a good prediction to be obtained for RH1, a mechanism (as described by dimensions $h$ and $j$) of approximately 0.7 times the size of that predicted for RH2A and RH2B needs to be predicted for RH1. It can be seen from the $h$ and $j$ values in Table 4.2 that this is not the case. Dimension $h$ for RH1 is approximately 0.4 times that for the RH2 tests, $j$ is approximately 0.54 times the size. Now evaluation of $j$ involves $h$ (see Eqn.(4.2)); therefore, if $h$ is in error, so too will be $j$.

If the solution is re-run for RH1 but with a value of 40 mm for $h$
(approximately 0.7 times the \( h \) value for RH2A and RH2B) instead of 24.7 mm as found from Eqn. (4.6), then \( j \) changes to 203 mm (which is now also approximately 0.7 times the \( j \) value for the RH2 cases). The predicted collapse load then drops from 165.5 kN to 135.5 kN, giving a vastly improved prediction falling now within approximately 20% of the experimental collapse load of 112 kN.

The \( h \) term is estimated empirically using Eqn. (4.6). Now, as has been mentioned earlier, this expression for \( h \) is not dimensionally correct - it gives \( h \) proportional to length squared - which explains the seemingly erroneous \( h \) value (and hence erroneous collapse load) in the case of the small RH1 girder compared with the RH2 girders. The above example illustrates that for two geometrically similar girders of different size, the solution does not give mechanisms of proportions that correspond to girder size, and this then leads to collapse load predictions that vary in accuracy depending on the size of the girder. This suggests that collapse load data on full-size crane girders is required in order to check the validity of the solution for such girders.

In applying the original solution to the RH girders, an experimentally determined \( E \) value (see Chapter 5) of 210 kN/mm\(^2\) has been used. The \( \cos \psi \) term was evaluated from Eqn. (4.4a) using the integral rail and flange term \( I_{fr} \) (given in Table 5.2) for the \( I_f \) term in the equation. It should be mentioned that in calculating the \( h \) values, the values for \( \sigma_f \) substituted into Eqn. (4.6) were those \( \sigma_f \) values given in Table 5.1; this then leads to the good general agreement between predicted and experimental loads shown in Table 4.3. In fact, the major component of the 'flanges' with these girders are the rails which have much higher yield stresses (\( \sigma_r \) in Table 5.1) than do the flange plates. If the \( \sigma_r \) values are used in the evaluation of \( h \), however, much smaller \( h \) values result and large over-estimations of the collapse loads are obtained.
<table>
<thead>
<tr>
<th>Test Girder</th>
<th>$P_u$ kN</th>
<th>$P_{ex}$ kN</th>
<th>$\frac{P_u}{P_{ex}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RH1</td>
<td>165.5</td>
<td>112</td>
<td>1.477</td>
</tr>
<tr>
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<td>240.1</td>
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</tr>
<tr>
<td>RH2B</td>
<td>258.3</td>
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<td>1.072</td>
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<tr>
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</tbody>
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If, for example, $h$ is recalculated for girder RH5 using $\sigma_r$ instead of $\sigma_f$ then the ratio $P_u/P_{ex}$ changes from 1.045 to 1.414. Now since the $M_f$ terms given in Table 5.1 take due account of $\sigma_f$ and $\sigma_r$ and since Eqn.(4.4a) rather than Eqn.(4.4b) was used in the solutions for the RH girders, the over-estimations result entirely from changing the value of the flange material yield stress used to find $h$ as this is then the only place where the $\sigma_f$ term occurs.

One further point relating to the evaluation of $h$ and the strong influence this term has on the load prediction should be raised. It concerns girders RH3, RH11 and RH12, which are all similar except that RH11 has a longitudinal web stiffener at the mid-depth position while RH12 has one at the quarter-depth position; RH3 has no such stiffener. It can be seen from Table 4.3 that there is a slightly increasing trend in collapse load from RH3 to RH11 and from RH11 to RH12. If the full web depth is used to evaluate $h$ in each case then a good prediction of the collapse load is obtained for girders RH11 and RH12, as well as for RH3. If, however, the depth to the stiffeners is used instead, smaller $h$ values are obtained and this again leads to large over-estimations of the collapse loads.

It should be repeated here that in later development of the original solution by Roberts [56],[57], selection of $h$ was changed to $h = 25 t_w$ which is dimensionally consistent. It has not been possible in the present study to re-appraise the solution with this modification incorporated but this could prove a worthwhile exercise.
4.11 Conclusions

A detailed study has been made of the plastic mechanism solution presented by Roberts and Rockey [53] for predicting collapse loads of plate girders subjected to in-plane patch loading.

A new mechanism solution, based on that by Roberts and Rockey, has been developed and this has been used to produce collapse load curves. The new solution offers certain attractions compared with the original version: it illustrates the transition region between failure by a mechanism of direct web yielding for girders with stocky webs and failure by a mechanism of out-of-plane web bending deformation for girders with slender webs; it also removes the requirement (in the solution for girders with slender webs) to evaluate a length of web assumed to yield in direct compression and therefore to offer no resistance to bending deformation.

Comparison between the two methods is achieved by reformulating the original analysis and producing collapse load curves in the same manner as were produced using the new solution. It is shown that the curves differ noticeably only over the relatively small region of transition between stocky and slender failure. It therefore appears that the refinements achieved with the new solution do not justify the increased complexity entailed in operating it. With both methods, prediction of collapse loads requires an estimation of girder deformation at the onset of collapse; in the original work this is achieved by means of an empirical expression (Eqn.(4.6)) for the distance $h$ between web yield lines. Although this leads to generally good collapse load predictions with the original solution, it is not of general application and is not particularly well suited to the new method.

Two extensions to the original method are considered: inclusion of the effects of web stretching and of the inclination of the web yield lines. It is shown that both effects can contribute significantly to the internal
work calculation. In principle, inclusion of one or both of the
developed terms enables an expression for \( h \) to be derived by minimising
the collapse load expression but further work is required to ensure true
minimisations are being obtained. A consequence of incorporating the
terms, however, is a marked increase in the complexity of the solution.

Application of the original solution to a number of model crane box-
girders tested to collapse under the action of a wheel load produced very
good predictions for models of half-scale size. However, the results
aroused further concern over the validity of the empirical expression for
estimating \( h \). It has been illustrated that for girders geometrically
similar in all dimensions, but of different sizes, the expression leads to
predicted mechanisms of sizes that do not correspond to the size of the
girder; this, in turn, leads to collapse load predictions that vary in
accuracy depending on the size of the girder. Predictions for two half-
scale girders were within approximately 7% of the experimental collapse
loads but for a geometrically similar girder of one-third scale, the
solution over-estimated by nearly 50%. This casts some doubt over the
validity of the solution for full-sized crane girders.
5.1. Introduction

In developing a method of analysis for assessing structural strength it is important to have available relevant experimental data so that the validity of the method may be checked. Walker [42] has drawn attention to the growing need for experimental investigations on plates subjected to realistic complex loadings in order to maintain a healthy balance of information with the increasing numbers of theoretical solutions which are arising largely as a result of the greater availability and power of computers.

Although, over recent years, some collapse test programmes have been conducted by various researchers in different countries on conventional plate girders subjected to patch loading, none, to the author's knowledge, has been conducted on the rather different structure of a crane torsion-box-girder.

Appendix B contains the details of 63 collapse tests performed elsewhere on conventional I-section plate girder specimens. The general form of these girders is shown in Fig.4.6. Apart from being box-girders, crane torsion-boxes differ from these girders in a number of ways. Crane girders generally have a flange/web thickness ratio of between 1.5 and 2.5, whereas for conventional girders this ratio generally lies anywhere between 2 and 12; crane girders, however, have additionally a rail attached to the flange directly above a web. Patch loading on the crane girder web arises from a wheel load: at the rail surface the load is of a length close to zero but the load then spreads through the rail and flange to form a non-uniformly distributed patch load at the top edge of the web. Most tests on conventional girders have considered
patch loads uniformly distributed over some length of the flange, the
load then spreading through the flange only before entering the web.

A further distinction which has been discussed earlier is that of the
boundary conditions to the web panel: as a member of a braced box
structure loaded via an eccentrically mounted rail, the crane girder
web panel is likely to experience rather different edge conditions
from its conventional counterpart. In most other respects, however,
the two types of girders are similar. Crane girders generally have
web slenderness ratios in the range 100 - 250 and web panel aspect
ratios between 1.0 and 1.5, both ranges being within those occupied by
conventional plate girders. Longitudinal web stiffening is also often
employed in both types of structure.

In this chapter, the details and results of a series of crane torsion-
box-girder sections tested to collapse under the action of a wheel load
are presented. As the present study is confined to one of the effect
of patch loading only, sections of short-span were employed. For prac-
tical reasons, scale model specimens were used (these generally being
of half scale size). The purpose of the tests was to study the physical
behaviour of the structures and to investigate the influence on the
collapse load of certain parameters, namely the thickness and aspect
ratio of the loaded web panel, the size of the rail, and the presence
of longitudinal web stiffening.

A further reason for the tests was to determine whether the mode of
collapse of these structures resembles that which has been observed in
tests on conventional girders subjected to patch loading, a mode which
forms the basis of the Roberts and Rockey [53] mechanism solution for
predicting collapse loads discussed in Chapter 4. Such tests are also
required in order that the elastic buckling information presented in
Chapter 3 may be assessed as a potential design approach. In essence,
a collapse test programme on crane girder sections is required before a critique of possible design methods can be undertaken.

5.2. The Test Frame

The anticipated loading levels required to collapse half scale crane girders indicated that the loading equipment used in the experimental work of Chapter 2 would not be adequate for this exercise. A suitable hydraulically actuated, servo-controlled ram of 250 kN capacity was available but no suitable test frame existed. It was therefore decided to design and construct a test frame to house the available ram. Facility was to be incorporated for girders of up to half scale size and, should investigations concerning co-existent bending be undertaken at a later date, for long-span as well as short-span girders to be tested.

These requirements indicated a frame too large to be manufactured at Leicester University. The sponsoring firm, Herbert Morris Ltd., however kindly agreed to undertake construction of the frame provided a minimum of skilled man-hours was entailed, particularly regarding demand on welders. The design therefore focused on the use of standard steel sections and, for ease of transportation and final assembly, bolted connections.

The frame was designed to BS 449 [9] for a load capacity of 400 kN (to accommodate a possible future uprating of the capacity of the ram). Design details are given in Appendix D. Figs. 5.1 and 5.2 show the finished product fully assembled in the laboratory and equipped for a collapse test on a short-span, one-third scale box section.
Fig. 5.1. 400 kN LOADING FRAME: GENERAL VIEW
Fig. 5.2. 400 kN LOADING FRAME: END VIEW
5.3. The RH Box-Girder Series

The RH series of test specimens consisted of 7 welded steel plate box-girder sections, each with a rail welded to the top flange above either web. Two collapse tests were obtained from each box by loading either side in turn, giving a total of 14 collapse tests.

Fig.5.3 illustrates the construction of the boxes and Fig.5.4 shows the design details of a typical box. A uniform top flange plate was employed on each box; the flange thickness and yield stress were therefore common to the two tests on any particular box. The web plate and rail on one side of the box could be either similar to, or different from, that on the other. Consequently, one box could be employed to provide two tests involving either similar or quite different parameters. The outer diaphragms were incorporated to provide stiffened support positions remote from the test panel, thereby avoiding the introduction of unrepresentative loadings on the panel.

In the commercial manufacture of crane girders, two small square bars are attached to each flange; these act as backing-bars to the webs and prevent blow-through of the welds between the webs and flanges. As can be seen from Fig.5.4, this feature was retained in the design of the model test boxes in order to provide locations for the webs and, thereby, facilitate the construction of a true box with the webs having straight and parallel longitudinal edges. Details of the test specimens are summarised in Table 5.1.

The first box to be constructed and tested was the smallest of the series. It was an approximately one-third scale model of a typical full-size crane girder and bore a close resemblance to the box section used in the experimental work described in Chapter 2. Initially, only one rail was attached to the flange, as can be seen in Figs.5.1 and 5.2.
Fig. 5.3 DETAILS OF RH BOX-GIRDER SPECIMENS
(a) SQUARE BAR RAIL
(b) SQUARE-BAR-ON-FLAT RAIL
**Fig. 5.4**

**Important Note:**

Web to flange beam welds shall be 3/8" continuous along full length. A groove weld on the extreme edge is to be made to avoid any plate separation during the welding process.
where the box is shown positioned in the loading frame. The rail comprised a square steel bar welded to a flat steel base (modelling a bridge type rail) as shown in Fig.5.3(b). Upon testing the box it was realised that provided no gross plastic deformation was inflicted, the other side of the box could be used for a second test. Another rail, in this case identical to the first, was therefore welded to the box above the unloaded web and this side tested also. Since the two webs were cut from the same steel plate, two tests involving the same parameters were obtained which gave a measure of the variation between collapse loads of similar test specimens. One side of the box collapsed at a load of 114 kN (RH1a), the other at a load of 110 kN (RH1b). Variation about the mean value was therefore less than 2% and these two tests are consequently referred to jointly as RH1 when the mean value of the collapse loads is quoted. Roberts and Rockey [53] have commented that variations between collapse loads of similar test specimens may amount to approximately 7% of the mean value. The outer (support) diaphragms of RH1 were positioned 150 mm beyond the inner diaphragms, giving a girder span of 900 mm (just over twice the overall depth of the box).

The second girder to be constructed and tested measured approximately half as large again as the first and thereby constituted a half scale model girder section. The two rails were of equal size and again of a compound, inverted-tee construction. Although this box, as with the first, was nominally of the same construction on either side, the two webs came from different steel plates and were found to have slightly different thicknesses and yield stresses. These test girders are consequently referred to as RH2A and RH2B. Rather heavy main seam welds were used in the construction of this box and this resulted in particularly pronounced web imperfections. The tests on this girder revealed
that the combination of parameters represented a limiting case for the test facilities since the collapse loads were only slightly below the capacity of the ram. This information assisted in the design of the remaining specimens since it enabled combinations of parameters to be selected with greater confidence that the collapse loads would not exceed the capacity of the testing system.

The remaining box sections were all nominally half scale models and were constructed using welds of a more representative size than those employed with RH2A and RH2B. Each side of each box provided a different combination of parameters (test girders RH3-RH12). A simple square bar rail welded directly to the flange (as shown in Fig.5.3(a) and 5.4) was employed in each case. Panel aspect ratios were 1.5 in all cases except for RH9 and RH10 where the aspect ratio was 1.0. Slenderness ratios of webs were from 133 to 240.

Longitudinal web stiffeners were incorporated in tests RH11 and RH12, as shown in Fig.5.5. RH11 had a small steel flat web stiffener attached at the half-depth position, modelling a 'cosmetic' stiffener as is often used in commercial manufacture to help reduce out-of-plane web imperfections arising from the box fabrication process. RH12 had a heavier angle stiffener attached at the quarter-depth position, modelling a typical structural stiffener. As shown in Fig.5.5, the web stiffeners were mounted conventionally in that their maximum inertia resisted transverse displacement of the web but unconventionally in that they were positioned on the outside, rather than the inside, of the box; this was to facilitate observation of stiffener behaviour during testing. To provide the end restraint to the stiffeners normally afforded by the diaphragms, small plates were welded to the outside of each web at the inner diaphragm locations.
Fig. 5.5  WEB STIFFENERS: TESTS RH11 AND RH12
For all the half scale models, the outer (support) diaphragms were so positioned that the span of each girder was 1380 mm (a little over twice the girder depth).

5.4. Preliminaries

Every steel plate from which a web or flange plate was cut had an additional piece cut from it in order to make a tensile test specimen. Similarly, samples were taken from each of the square bars used for the test rails. The tensile specimens were made and tested according to BS 18 "Methods for Tensile Testing of Metals". To check possible variation in yield stress with the direction of rolling, two samples were tested for certain of the plate thicknesses, one having been cut parallel, the other perpendicular, to the direction of rolling. Yield stresses \( \sigma_w, \sigma_f, \) and \( \sigma_r \) for, respectively, the web, flange, and rail of each test girder are given in Table 5.1.

All plates were grade 43 steel except the thinnest - the 2.45 mm thick plate used for the webs of RH5 and RH6, however, was of grade HR4 since a plate nominally 2.5 mm thick could not be found in a grade 43 steel. The square bar rails were of grade En3B. The actual yield stresses found from the tests attained values of up to nearly 80% in excess of the minimum quoted in the relevant code. From the stress versus strain plots obtained from the tensile tests, an \( E \) value of 210 kN/mm\(^2\) was estimated and this value has been assumed throughout this chapter.

Dimensions of the box components were carefully measured before testing. Several of the important dimensions were found to differ from their nominal values; the measured dimensions are recorded in Table 5.1.

Before testing, the initial out-of-flatness of each web was carefully measured. The initial web imperfections are given in Appendix E.
5.5. **Test Procedure and Instrumentation**

Prior to loading, every care was taken to position the box and moveable box supports within the frame so that the load was applied at the mid-span and in the midplane of the test web panel. The girders were loaded using a Mand closed-loop servo-controlled testing system with a servo-controlled actuator of 250 kN capacity. The feedback signal in the control loop system could be provided by either the displacement or load parameter. In order to monitor the post-ultimate load behaviour of the girders, deflection control of the jack was employed. The facility to apply the load at a constant rate of displacement was also employed; a typical displacement ramp selected for the tests was 0.008 mm/second.

As can be seen from Figs.5.1 and 5.2, load was applied to the boxes via a radiused foot attachment used to simulate a crab wheel. To provide an accurate measurement of load, a load cell was positioned between the foot and the actuator ram. Vertical displacement of the top flange of the girders was measured using a displacement transducer positioned beneath the point of loading. Transverse displacements of the web were measured at selected depths in the central region of the panel, again using displacement transducers.

From the outputs of the load cell and transducers, curves of load against vertical deflection and load against transverse deflection were plotted automatically on a multi-pen chart recorder during the test. This enabled girder behaviour to be monitored continuously and any interesting developments to be detected immediately. The ramp loading could then be reduced or stopped as required, and then either continued or reversed back to zero load.
A number of loading cycles with successively increasing maxima were applied in each test in order to achieve a record of permanent set in the girder. At frequent intervals throughout the tests, the ramp loading was paused and a data logger activated; this recorded on to magnetic tape all outputs from the instrumentation.

Prior to testing RH1a, strain gauges were mounted on to the outside face of the test web panel along the vertical centre line (it was not possible to mount gauges on the inner face owing to the closed construction of the box). The purpose of the strain gauging was to aid the monitoring of yield line formation in the web. However, the outputs from the gauges followed no discernible trends. This indicated that stress redistribution was taking place during loading, probably caused by either changes in buckling mode or the effects of residual stressing. Owing to the rather unfruitful results, no further attempts at strain gauging were undertaken.

Since the principal aim of all the tests was to determine the collapse loads, the instrumentation was kept relatively simple. Three displacement transducers were mounted on a longitudinal bar fixed parallel to the plane of the web by means of electromagnets attached to the web at the two inner diaphragm positions, as shown in Fig.5.6. One transducer measured vertical deflection of the underside of the top flange; the transducer was positioned at the vertical centre line of the web panel and as close to the web as was possible. This procedure for measuring vertical deflection was preferred to simply recording the ram displacement since it eliminated the effects of bedding-in at the box supports. The remaining two transducers measured transverse (out-of-plane) displacements of the web at positions A and B; the transducers were positioned 15 mm to one side of the panel centre-line in order to clear the transducer monitoring vertical deflection of the
Fig. 5.6. MOUNTING OF DISPLACEMENT TRANSDUCERS FOR COLLAPSE TESTS
flange. Positions A and B were altered from one test to another; the depth to A from the underside of the top flange was varied between 35 mm and 120 mm, the depth to B was varied between 125 mm and 280 mm.

After collapse, plastic residual profiles of the rails and of the vertical centre-lines of the webs were recorded using a displacement transducer mounted on a trolley that traversed along a straight beam. The beam housed a long linear potentiometer which monitored the position of the trolley. By levelling the beam over a box rail (or, with the box lying on its side, over the midspan of a web panel) and traversing the trolley, the deformed profile was recorded directly on to an X-Y plotter from the outputs of the transducer and potentiometer.

5.6. Presentation and Discussion of Results

5.6.1. Measured Collapse Loads

The experimental ultimate load carrying capacities, $P_{ex}$, of the test box-girders are given in Table 5.1 where the relevant girder dimensions, material yield stresses, and plastic moments are also given. $M_f$ is the plastic moment of the rail/flange assembly where an effective flange width $b_f$ of twice the distance from the web centre-line to the nearer flange edge has been assumed (see Fig.5.3(a)).

5.6.2. Load Against Displacement Plots

For convenience a computer program was written to plot the experimental load and displacement data recorded on magnetic tape during each collapse test. The program operated by linearly interpolating between successive data points to produce plots of load in kN on the vertical
Table 5.1  DETAILS OF RH BOX-GIRDER TEST SERIES

<table>
<thead>
<tr>
<th>Test Girder</th>
<th>b  mm</th>
<th>d  mm</th>
<th>a</th>
<th>tw  mm</th>
<th>bf × tf  mm × mm</th>
<th>r  mm</th>
<th>br × tr  mm × mm</th>
<th>σw  N/mm²</th>
<th>σf  N/mm²</th>
<th>σt  N/mm²</th>
<th>Mw  N</th>
<th>Mf x 10⁶ Nmm</th>
<th>Pe  kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>RH1</td>
<td>600</td>
<td>400</td>
<td>1.5</td>
<td>3.03</td>
<td>77 x 5</td>
<td>19</td>
<td>50 x 6</td>
<td>317</td>
<td>368</td>
<td>645</td>
<td>728</td>
<td>3.742</td>
<td>112</td>
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<tr>
<td>RH2A</td>
<td>900</td>
<td>600</td>
<td>1.5</td>
<td>4.02</td>
<td>116 x 6</td>
<td>28.5</td>
<td>80 x 8</td>
<td>304</td>
<td>309</td>
<td>595</td>
<td>1228</td>
<td>9.970</td>
<td>230</td>
</tr>
<tr>
<td>RH2B</td>
<td>900</td>
<td>600</td>
<td>1.5</td>
<td>4.08</td>
<td>116 x 6</td>
<td>28.5</td>
<td>80 x 8</td>
<td>322</td>
<td>309</td>
<td>595</td>
<td>1340</td>
<td>9.970</td>
<td>241</td>
</tr>
<tr>
<td>RH3</td>
<td>900</td>
<td>600</td>
<td>1.5</td>
<td>2.95</td>
<td>57 x 6</td>
<td>19</td>
<td>-</td>
<td>324</td>
<td>318</td>
<td>645</td>
<td>705</td>
<td>2.224</td>
<td>107</td>
</tr>
<tr>
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<td>600</td>
<td>1.5</td>
<td>2.95</td>
<td>57 x 6</td>
<td>38</td>
<td>-</td>
<td>324</td>
<td>318</td>
<td>581</td>
<td>705</td>
<td>10.230</td>
<td>164</td>
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<td>1.5</td>
<td>2.45</td>
<td>57.5 x 6</td>
<td>19</td>
<td>-</td>
<td>242</td>
<td>318</td>
<td>645</td>
<td>363</td>
<td>2.232</td>
<td>67</td>
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<tr>
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<td>2.45</td>
<td>57.5 x 6</td>
<td>25</td>
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<td>4.00</td>
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<td>2.96</td>
<td>57 x 6</td>
<td>25</td>
<td>-</td>
<td>325</td>
<td>318</td>
<td>573</td>
<td>712</td>
<td>3.718</td>
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<td>2.95</td>
<td>57 x 6</td>
<td>19</td>
<td>-</td>
<td>324</td>
<td>318</td>
<td>645</td>
<td>705</td>
<td>2.224</td>
<td>112</td>
</tr>
<tr>
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<td>1.0</td>
<td>2.95</td>
<td>57 x 6</td>
<td>25</td>
<td>-</td>
<td>324</td>
<td>318</td>
<td>573</td>
<td>705</td>
<td>3.718</td>
<td>129</td>
</tr>
<tr>
<td>RH11</td>
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<td>600</td>
<td>1.5</td>
<td>2.96</td>
<td>57 x 6</td>
<td>19</td>
<td>-</td>
<td>325</td>
<td>318</td>
<td>645</td>
<td>712</td>
<td>2.224</td>
<td>114</td>
</tr>
<tr>
<td>RH12</td>
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<td>600</td>
<td>1.5</td>
<td>2.96</td>
<td>57 x 6</td>
<td>19</td>
<td>-</td>
<td>325</td>
<td>318</td>
<td>645</td>
<td>712</td>
<td>2.224</td>
<td>118</td>
</tr>
</tbody>
</table>
axis against displacement in \text{mm} on the horizontal axis. Since the tests on girders RH1 and RH2A and RH2B were preliminary tests conducted principally to check collapse load levels to assist in the design of the remainder of the series, no experimental plots are presented for these girders.

Load against vertical displacement for RH3 is shown in Fig. 5.7. Development of transverse (out-of-plane) displacement of the web at positions A and B is shown in Figs. 5.8 and 5.9 respectively; position A was at a depth of 120 mm, position B at a depth of 200 mm (see Fig. 5.6).

Clearly illustrated in these plots is the web snap buckling phenomenon that was evident in several of the RH tests. Sometimes snap buckling occurred dramatically, there being a loud bang accompanying the jump in displacements. As can be seen from Fig. 5.7, snap buckling occurrences were monitored on the vertical displacement plots as well as the transverse displacement plots, owing to the associated rotation of the top flange.

With RH3, snap buckling of the web panel first occurred at a load of 21 kN (approximately 20\% of the collapse load). For each loading cycle, a reverse snap buckle occurred on unloading, this taking place at a slightly lower load than that at which the snap buckle on loading occurred. Loading beyond the elastic limit of the web had the effect of reducing the loads at which snap buckling occurred (see Figs. 5.8 and 5.9).

Fig. 5.7 illustrates that after collapse the load remained very nearly constant for further vertical displacement. The transverse displacement plots reveal that up to collapse, normal displacements at A and B were directed inwards, towards the opposite web (transverse...
Fig. 5.7. Applied load against central vertical displacement for test box RH3.
displacements towards the centre of the box have been taken as positive), but that at collapse there was a reversal in the direction of displacement growth, the new (outward) direction then being followed throughout the post-collapse range. This is particularly evident from Fig.5.8 which shows the displacement at the upper of the two recording positions.

Test girder RH4 incorporated a particularly heavy rail. On first loading, a snap buckle occurred at a load of 61 kN that was so violent the longitudinal bar carrying the displacement transducers was dislodged. The load was therefore removed (no reverse snap buckle occurred), the instrumentation re-positioned and re-zeroed, and the test re-started. This snap buckle was therefore not recorded on the experimental plots. It can be seen from the plots in Figs.5.10 - 5.12 that no further snap buckling took place during the test. The load against vertical displacement plot in Fig.5.10 shows a linear elastic relationship up to a load of approximately 85% of the collapse load. At collapse, a modest peak was exhibited and the load then increased slightly with increasing displacement; the load peak was recorded as the collapse load. The transverse displacement plots shown in Figs.5.11 and 5.12 reveal that there was no change in the direction of recorded buckling displacements during this test (an outward direction being followed throughout). Recording positions A and B were as for RH3.

Fig.5.13 shows the vertical displacement plot for RH5. It can be seen that, as with RH3, the load remained approximately constant after collapse. Transverse displacements for RH5 are shown in Figs. 5.14 and 5.15; recording positions A and B were moved up to depths of 75 mm and 155 mm respectively in this test in an attempt to monitor the web at positions of yield line formation. The plots show that at a load of approximately 35 kN, a snap buckle took place from which there was no recovery on unloading. Apart from differences arising from snap buckling
Fig. 5.10. APPLIED LOAD AGAINST CENTRAL VERTICAL DISPLACEMENT FOR TEST BOX RH4.
Fig. 5.12. APPLIED LOAD AGAINST TRANVERSE DISPLACEMENT AT 'B' FOR TEST BOX RH4.
Fig. 5.14. APPLIED LOAD AGAINST TRANSVERSE DISPLACEMENT AT "A" FOR TEST BOX RHS.
Fig. 5.15. APPLIED LOAD AGAINST TRANSVERSE DISPLACEMENT AT 'B' FOR TEST BOX RH5.
behaviour, the vertical displacement plot for RH5 is broadly similar to that for RH3 - an initial elastic region is followed by a region where some permanent set develops, leading to a fairly well defined collapse point and a near constant load level thereafter. During post-collapse loading of RH5, a web hinge was observed to form just above recording position A (Fig.5.14). It can be seen from Figs.5.14 and 5.15 that transverse displacements initially developed in an inward direction but then changed direction and grew outwards in a manner similar to that monitored in the RH3 test.

Figs.5.16 and 5.17 show the vertical displacement plots for RH6 and RH7 respectively. Both structures exhibited a modest peak in load at collapse; the load then remained nearly constant for further vertical displacement in the case of RH6, but continued to fall-off slightly in the case of RH7. Neither girder exhibited snap buckling behaviour. Fig.5.17 shows the relationship between load and vertical displacement for RH7 to be initially approximately linear elastic but then at a load of about 75 kN the curve becomes non-linear, with the load increasing thereafter at a progressively slower rate up to collapse at 153 kN. The transverse displacement plots for RH7 show a corresponding but more marked change in slope; Fig.5.18 shows the plot recorded at position A (65 mm depth) and Fig. 5.19 that at B (145 mm depth). It can be seen from the plots that permanent set in the web is associated with the changes in slope. It is presumed that the rather different profiles of the RH7 plots arose as a result of the girder having a fairly thick web but small rail (it will be recalled from Chapter 2 that this combination leads to particularly localised web stressing) which, since the yield stress of the web was relatively low, produced significant web yielding before the collapse load was attained (see Section 5.6.5).
Fig. 5.19. Applied load against transverse displacement at 'B' for test box RWP.
RH8 exhibited repeated snap buckling behaviour, as evidenced by the three plots for this test shown in Figs.5.20 - 5.22. On first loading, the web panel snap buckled at a load of approximately 48 kN. On unloading, the panel reverse snap buckled, but not until the load had fallen nearly to zero. For the second cycle of loading, the panel again snap buckled at a load of 48 kN on loading but reverse snap buckled at a load of approximately 10 kN. For further loading cycles, snap buckling occurred at approximately the same loads as in the first loading cycle; however, once some web yielding had taken place, no further snap buckling occurred. Fig.5.20 shows that after collapse, the load again remained approximately constant with increasing vertical displacement. Transverse displacement at A (Fig.5.21) was similar to that at the corresponding position in the RH5 test (Fig.5.14) in that displacement growth changed from an inward to an outward direction. Displacement recorded at the lower position B (Fig.5.22), however, showed no change in direction (unlike the corresponding RH5 plot of Fig.5.15) but maintained an inward direction. Recording positions A and B in the RH8 test were as for RH7 and were thus 10 mm higher than in the RH5 test. The two transverse displacement plots for RH6 (measured at the same positions as in the RH5 test) were broadly similar to Fig.5.22.

Figs.5.23 and 5.24 show the vertical displacement plots for RH9 and RH10, the two girders with square web panels. Both plots display an initial approximately linear elastic relationship, leading to some permanent set and then a well defined turnover at collapse, further displacement then taking place at approximately constant load (particularly with RH10). The transverse displacement plots for these girders (measured at depths of 60 mm and 140 mm for RH9, and 45 mm and 125 mm for RH10) were broadly similar to those of the RH8 test but without evidence of
Fig. 5.23. APPLIED LOAD AGAINST CENTRAL VERTICAL DISPLACEMENT FOR TEST BOX PH9.
Fig. 5.27. APPLIED LOAD AGAINST TRANSVERSE DISPLACEMENT AT 'A' FOR TEST BOX RH12.
snap buckling (although it was noted that displacement at B in the RH10 test grew more rapidly at low loads than at high loads, but not so rapidly as to be regarded a snap-through).

Test girder RH11 was very similar to RH3 except for the addition of a cosmetic stiffener at the mid-depth position of the web - see Fig.5.5. The vertical displacement plot for RH11 in Fig.5.25 shows a rather more gradual turnover prior to collapse than in earlier tests (apart from RH7) and also shows the load to fall-off gradually after collapse. Apart from exhibiting the fall-off in post-collapse load and showing no evidence of snap buckling, the transverse displacement plots for RH11 (monitored at depths of 35 mm and 150 mm) were similar to that recorded at B in the test on RH8 (Fig.5.22).

RH12 exhibited a pronounced load peak at collapse, as can be seen from the vertical displacement plot in Fig.5.26. This girder also resembled RH3 but for the addition in this case of a structural stiffener at the quarter-depth position of the web (150 mm depth) - see Fig.5.5. Transverse displacement was monitored both above and below the stiffener. Position A, Fig.5.27, was at a depth of 50 mm, position B, Fig.5.28, was at a depth of 280 mm. It can be seen from these two plots that displacement was inwards at A but outwards at B, indicating an approximately S-shaped profile of out-of-plane displacement about the longitudinal web stiffener. As with RH11, RH12 exhibited no snap buckling.

5.6.3. Changes in Buckling Displacements

Snap Buckling

The snap buckling phenomenon noted in several of the RH tests has been observed in other similar tests conducted elsewhere. Skaloud and Novak [58], in a detailed and comprehensive study of conventional plate girder
sections under patch loading, have commented on sudden changes in wave pattern of the buckled surface of some of their test girder web panels. Using a stereophotogrammetric method to obtain detailed contour plots of web displacements, they observed that initially there was a tendency for the buckled surface of a web to follow the shape of the imperfection curvature, but at higher loads the characteristics of the contour plots frequently changed "by way of observable jumps".

Snap buckling may be partially attributable to turning moments imposed at the top edge of a web by loads with an effective line of action that is eccentric to the midplane of the web. As loading proceeds, the turning moment increases and may oppose development of the preferred mode of buckling (which depends to some extent on the initial imperfection), causing a sudden change in the buckled shape of the web. Loading eccentricities may arise with crane girders due to misalignments of the wheel, rail, and web centre-lines, or due to a wheel load acting non-uniformly across the width of a rail. With crane torsion-box-girders, loading eccentricity may also arise as a result of overall twisting of the girder under load.

Recently, some discussion has taken place on the introduction of a serviceability limit in steel bridge design to take account of the phenomenon of web snap buckling. Skaloud [63], and Evans [63] have both commented on "snap-through" displacements and accompanying loud bangs occurring in various web buckling tests at loads well below the collapse load; they have emphasised the undesirability of such bangs occurring in working structures, even though such bangs need bear no indication of girder failure. Both Skaloud and Evans link the snap-through phenomenon to the critical load of the web: Skaloud envisaged multiplying the critical load by a correction factor to determine the serviceability limit for webs of steel bridges; Evans suggested the serviceability limit
should ensure that web stresses are not allowed to exceed the critical stress to any great extent.

The loads at which snap buckling occurred with the RH test specimens are given in Table 5.2, together with elastic buckling loads calculated for the panels. \( P_{sa} \) denotes the snap load on loading, \( P_{sb} \) that on unloading (see Section 5.6.2 for an account of snap buckling occurrences); \( P_{crs} \) denotes the elastic buckling load calculated on the basis of all panel edges being simply supported, and \( P_{crc} \) the buckling load calculated on the basis of the horizontal edges being clamped instead. It can be seen that for RH3, \( P_{sa} \) was very close to \( P_{crs} \), whereas for the other girders, \( P_{sa} \) was approximately 2 to 3 times \( P_{crs} \) and much closer to the \( P_{crc} \) value. Whenever snap buckling occurred on unloading it was at loads below \( P_{crs} \).

The test results indicate that a serviceability limit to take account of web snap buckling due to wheel loadings on crane box-girders would be very conservative compared with the load carrying capacities of the girders. Against the introduction of such a limit is the fact that there was no snap buckling in several of the tests. It is worth noting here that snap buckling did not occur in either of the tests where longitudinal web stiffening was incorporated, did not occur in cases where the web slenderness ratio was 150 or below, and did not occur in either of the tests where the panel aspect ratio was 1.0 (as opposed to 1.5). These observations, however, are based on very limited experimental evidence and are presented only by way of giving possible indications of features which, with further investigation, might be employed in design to reduce the likelihood of snap buckling occurrences.

**Changes in the Direction of Displacement Growth**

The above considerations of the causes of snap buckling also provide
<table>
<thead>
<tr>
<th>Test Girder</th>
<th>$P_{ex}$ kN</th>
<th>$I_{fr} \times 10^4$ mm$^4$</th>
<th>$c$ mm</th>
<th>$\beta$</th>
<th>$\lambda$</th>
<th>$P_{sa}$ kN</th>
<th>$P_{sb}$ kN</th>
<th>$P_{crs}$ kN</th>
<th>$P_{crc}$ kN</th>
<th>$P_{o}$ kN</th>
<th>$\frac{P_{ex}}{P_{crs}}$</th>
<th>$\frac{P_{ex}}{P_{crc}}$</th>
<th>$\frac{P_{ex}}{P_{o}}$</th>
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</thead>
<tbody>
<tr>
<td>RH1</td>
<td>112</td>
<td>7.513</td>
<td>102</td>
<td>0.17</td>
<td>133</td>
<td>-</td>
<td>-</td>
<td>34</td>
<td>88</td>
<td>98</td>
<td>3.29</td>
<td>1.27</td>
<td>1.14</td>
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<tr>
<td>RH2A</td>
<td>230</td>
<td>31.901</td>
<td>150</td>
<td>0.17</td>
<td>150</td>
<td>-</td>
<td>-</td>
<td>53</td>
<td>137</td>
<td>183</td>
<td>4.34</td>
<td>1.68</td>
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</tr>
<tr>
<td>RH2B</td>
<td>241</td>
<td>31.901</td>
<td>150</td>
<td>0.17</td>
<td>150</td>
<td>-</td>
<td>-</td>
<td>56</td>
<td>143</td>
<td>197</td>
<td>4.30</td>
<td>1.69</td>
<td>1.22</td>
</tr>
<tr>
<td>RH3</td>
<td>107</td>
<td>3.933</td>
<td>83</td>
<td>0.09</td>
<td>200</td>
<td>21,10</td>
<td>15,6</td>
<td>20</td>
<td>52</td>
<td>79</td>
<td>5.35</td>
<td>2.06</td>
<td>1.35</td>
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<tr>
<td>RH4</td>
<td>164</td>
<td>30.861</td>
<td>165</td>
<td>0.18</td>
<td>200</td>
<td>61</td>
<td>-</td>
<td>22</td>
<td>54</td>
<td>158</td>
<td>7.45</td>
<td>3.04</td>
<td>1.04</td>
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<tr>
<td>RH5</td>
<td>67</td>
<td>3.946</td>
<td>88</td>
<td>0.10</td>
<td>240</td>
<td>35</td>
<td>-</td>
<td>12</td>
<td>30</td>
<td>52</td>
<td>5.58</td>
<td>2.23</td>
<td>1.29</td>
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<td>RH6</td>
<td>80</td>
<td>8.699</td>
<td>115</td>
<td>0.13</td>
<td>240</td>
<td>-</td>
<td>-</td>
<td>12</td>
<td>31</td>
<td>68</td>
<td>6.67</td>
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<td>153</td>
<td>3.906</td>
<td>74</td>
<td>0.08</td>
<td>150</td>
<td>-</td>
<td>-</td>
<td>50</td>
<td>130</td>
<td>74</td>
<td>3.06</td>
<td>1.18</td>
<td>2.07</td>
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<td>RH8</td>
<td>128</td>
<td>8.668</td>
<td>108</td>
<td>0.12</td>
<td>200</td>
<td>48</td>
<td>0.5,10</td>
<td>21</td>
<td>53</td>
<td>104</td>
<td>6.10</td>
<td>2.42</td>
<td>1.23</td>
</tr>
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<td>3.933</td>
<td>83</td>
<td>0.14</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>27</td>
<td>57</td>
<td>79</td>
<td>4.15</td>
<td>1.96</td>
<td>1.42</td>
</tr>
<tr>
<td>RH10</td>
<td>129</td>
<td>8.668</td>
<td>108</td>
<td>0.18</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>27</td>
<td>58</td>
<td>103</td>
<td>4.78</td>
<td>2.22</td>
<td>1.25</td>
</tr>
<tr>
<td>RH11</td>
<td>114</td>
<td>3.933</td>
<td>83</td>
<td>0.09</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>21</td>
<td>53</td>
<td>80</td>
<td>5.43</td>
<td>2.15</td>
<td>1.43</td>
</tr>
<tr>
<td>RH12</td>
<td>118</td>
<td>3.933</td>
<td>83</td>
<td>0.09</td>
<td>200</td>
<td>-</td>
<td>-</td>
<td>21</td>
<td>53</td>
<td>80</td>
<td>5.62</td>
<td>2.23</td>
<td>1.43</td>
</tr>
</tbody>
</table>
an explanation for the reversal in transverse displacement growth observed in several of the RH tests. Eccentric loading of the box-girder produces twisting which tends to move the line of load application towards the edge of the rail nearer the centre of the box; this produces a turning moment on the top edge of the web which acts so as to deflect the web outwards. It will be noted from the transverse displacement plots in Section 5.6.2 that whenever a reversal in the direction of displacement growth took place it was from an inward to an outward direction.

5.6.4. Comparison of Measured Collapse Loads with Calculated Elastic Buckling Loads

The experimental load versus displacement plots presented in Section 5.6.2 show no obvious evidence of elastic web panel buckling. Experimental evaluation of elastic buckling loads by Southwell type techniques was not possible because factors such as the presence of snap buckling and reversals in the direction of displacement growth rendered the transverse displacement data unsuitable for such procedures.

Theoretical elastic buckling loads for the test panels have been calculated from Eqn.(2.6) using the buckling coefficient information presented in Chapter 3. Two sets of loads, \( P_{crs} \) and \( P_{crc} \) (see previous section) are presented in Table 5.2. The procedure for calculating the buckling loads was firstly to evaluate the \( I_f \) terms given in Table 5.2 - the same \( b_f \) dimensions (see Fig.5.3(a) and Table 5.1) were used for this as were used to evaluate the \( M_f \) parameters for the girders. The effective patch lengths \( c \) given in Table 5.2 were then evaluated using Eqn.(2.4b) - the expression for girders with welded rails - and this enabled the patch length parameters \( b = c/b \), also given in Table 5.2, to be calculated. It is of note that the \( b \) values
lie in a range from 0.08 to 0.18 which is within the range of values (0.07-0.25) estimated in Chapter 2 to be applicable to crane torsion-box-girder web panels. Having the $\beta$ values and knowing the panel aspect ratios enables the buckling coefficients to be found from Figs.3.2 and 3.4 (note that in these figures the patch length parameters $\beta$ are denoted by C/L), and thus the buckling loads to be calculated. An $E$ value of 210 kN/mm$^2$ was taken, as discussed in Section 5.4.

The buckling loads presented for RH11 and RH12 ignore the presence of the longitudinal web stiffeners.

$P_{crc}$ exceeds $P_{crs}$ by a factor of between 2.5 and 2.6 for panels with $\alpha = 1.5$, and by a factor of between 2.1 and 2.2 for panels with $\alpha = 1.0$ (see Table 5.1 for aspect ratios of panels). As can be seen from Table 5.2, the experimental collapse loads $P_{ex}$ exceeded the $P_{crs}$ loads by a factor of between approximately 3.1 and 7.5, and exceeded the $P_{crc}$ loads by a factor of between approximately 1.2 and 3.0; the largest factors occurred with RH4, the girder with a particularly heavy rail. Thus, in all cases, the collapse load exceeded both calculated elastic buckling loads.

Owing to the number of variables and limited number of tests in the present work it is difficult to discern any useful relationships between the measured collapse loads and the calculated elastic buckling loads, if indeed any exist. The difference between these two loads may be considered to illustrate the postbuckled reserve of strength. Comparing the ratios $P_{ex}/P_{crs}$ or $P_{ex}/P_{crc}$ for RH7, RH3, and RH5 then indicates that the reserve of strength increases as the web thickness decreases, or - since the girders have a common web depth - that the reserve of strength increases as the web slenderness ratio increases.
5.6.5. **Comparison of Measured Collapse Loads with Loads Calculated for Onset of Local Web Yielding**

Using the \( c \) values listed in Table 5.2 it is possible to estimate the load \( P_0 \) at which the onset of web yielding directly beneath the load takes place. This load, which ignores any effects of web buckling, is given simply by

\[
P_0 = \sigma_w t_w c
\]  

(5.1)

where \( c \) represents the distributed length of the patch load at the web edge (see Section 5.6.4 for procedure for calculating \( c \)). The values of \( P_0 \) and the ratios \( P_{ex}/P_0 \) for the test specimens are given in Table 5.2.

The ratios \( P_{ex}/P_0 \) lie in the range 1.0 - 2.1. The lowest value occurred with RH4, the girder with a heavy rail, and the highest value with RH7, the girder with a relatively thick web of fairly low yield stress. Comparing the ratios for RH3, RH8 and RH4, and for RH9 and RH10, shows the ratio to decrease with increasing \( M_f \) (i.e. increasing rail size - see Table 5.1). Thus \( P_0 \) falls below \( P_{ex} \) for girders with light flanges, but approaches (and may possibly exceed) \( P_{ex} \) for girders with heavy flanges. The loads \( P_0 \), as calculated from Eqn.(5.1), therefore appear to bear no meaningful relationship to the measured collapse loads.

Reference to the experimental load against vertical displacement plots in Section 5.6.2 indicates that permanent set initiated in the test girders at loads generally fairly close to the \( P_0 \) loads; however, initiation of web yielding did not mark the onset of collapse. Nonetheless, the \( P_0 \) load is of importance in design if yielding of the top edge of the web is to be avoided.
5.6.6. **Effect of Flange Plastic Moment**

It can be seen from Table 5.1 that RH3, RH4, and RH8 differed significantly only in the value of their flange plastic moments of resistance $M_f$. RH9 and RH10 also differed only in their $M_f$ values. Fig.5.29 shows the relationship between measured collapse loads $P_{ex}$ and the $M_f$ values for these two sets of girders. It is evident from this plot that increasing $M_f$ increases the collapse load.

5.6.7. **Effect of Web Plastic Moment**

The plastic moment of resistance per unit length of web $M_w$ increases as $\sigma_w$ and as $t_w^2$. RH3, RH5 and RH7 differed in $\sigma_w$ and $t_w$, and therefore $M_w$, but were similar in other respects. Fig.5.30 shows the relationship between $P_{ex}$ and $M_w$ for these girders from which it is clear that increasing $M_w$ increases the collapse load.

5.6.8. **Effect of Web Panel Aspect Ratio**

Comparison of the collapse loads for RH3 and RH9 and for RH8 and RH10 in Fig.5.29 or Table 5.1 shows that a reduction in the web panel aspect ratio from 1.5 to 1.0 increased the collapse load slightly:

RH3 ($\alpha = 1.5$) collapsed at 107 kN and RH9 (similar except $\alpha = 1.0$) at 112 kN; RH8 ($\alpha = 1.5$) collapsed at 128 kN and RH10 (similar except $\alpha = 1.0$) at the marginally higher load of 129 kN. The increases in collapse load are thus less than 5%.
Fig. 5.29 EFFECT OF FLANGE PLASTIC MOMENT $M_f$ ON COLLAPSE LOAD

Fig. 5.30 EFFECT OF WEB PLASTIC MOMENT $M_w$ ON COLLAPSE LOAD
5.6.9. Effect of Longitudinal Web Stiffening

It will be recalled that RH11 and RH12 resembled RH3 except for the addition of longitudinal web stiffening as shown in Fig. 5.5. Reference to Table 5.1 shows that the introduction of a cosmetic stiffener at the mid-depth position of the RH11 web resulted in a collapse load of 114 kN compared with a collapse load of 107 kN for RH3—a decrease of 6.5%. Introduction instead of a structural stiffener at the quarter-depth position of the RH12 web resulted in a collapse load of 118 kN—an increase of 10% on the unstiffened case, and 3.5% on the cosmetic stiffener case. If it is assumed that a stiffener effectively divides a panel up into two distinct sub-panels, such that the upper sub-panel only is acted upon by the patch load, then these results may be interpreted to indicate that the collapse load is relatively independent of the depth of the web.

It is pertinent here to quickly review some of the previous work that has been conducted on the effect of longitudinal web stiffening of patch loaded plate girders. Rockey, Samuelsson and Wennerström [64] have used the finite element method to calculate elastic buckling loads for a square, simply supported plate subjected to an in-plane patch load when the plate is reinforced by a single longitudinal stiffener positioned at different depths. It was demonstrated that the buckling load can be increased significantly by the presence of an effective stiffener in the compression zone of the plate. For the particular case considered of a square panel subjected to a central patch load acting along 2/9 of the panel edge, the buckling load was maximised when the stiffener was close to the one-fifth depth position. For a stiffener placed at this position, a relationship was developed between the buckling resistance of the plate and the flexural rigidity of the stiffener.
Following this work, Rockey, Bergfelt and Larsson [65] carried out an experimental investigation on two pairs of plate girders, one girder of each pair having a longitudinal web stiffener attached at the one-fifth depth position. The presence of the stiffener increased the collapse load by 7.5% in one case and 18% in the other. Rockey et al commented that they considered a greater percentage increase in load carrying capacity would be achieved by placing a longitudinal stiffener closer to the compression flange.

In a state of the art report on the design of stiffened web plates by Rockey, Evans and Porter [66], some more-recent findings are summarised which indicate that there is little benefit in employing a longitudinal stiffener on a girder with a loaded flange that is relatively weak. For girders with stiffer flanges, however, the flange hinges form nearer to the vertical stiffeners, a larger area of web is brought into play, and the presence of a stiffener can, provided it is placed close enough to the compression flange, increase the collapse load considerably. Two examples are given concerning a strong stiffener placed at the one-tenth-depth position: for a girder with a panel aspect ratio of 1.0 the ultimate load was increased by 42.5%, for another with an aspect ratio of 2.0 the ultimate load was increased by 29.7%. It was concluded that to be effective, the stiffener must be located within the buckled region that forms in a restricted zone close to the loaded flange; however, as the stiffener is placed closer to the flange so the flexural rigidity of the stiffener has to be increased.

On a more general note, Rockey, Evans and Porter add that by employing a longitudinal web stiffener, the buckling resistance of the girder web panel is increased and so too then is the serviceability load.

The present findings support this comment in so much as the snap buckling exhibited by RH3 was not exhibited also by RH11 and RH12 (girders
similar to RH3 but for the addition of a longitudinal web stiffener).

5.6.10. Effect of Scale Size of Girder

The collapse load of a structure under simple static loading is a function of the material yield stress and linear dimensions (see Hossdorf [67]). Thus, for two geometrically similar structures of different size, made from materials of the same yield stress and subjected to the same form of loading, the collapse load is proportional to the square of the scaling ratio (neglecting dead weight effects).

Reference to Table 5.1 shows that RH2A and RH2B were approximately half as large again as RH1; of the two, RH2B was slightly nearer a 3/2 scale version of RH1, owing to the web of RH2B being marginally thicker than that of RH2A.

RH1 collapsed at a (mean) load of 112 kN. Thus it would be expected that a geometrically similar girder 3/2 times as large would collapse at a load of $112 \times \frac{9}{4} = 252$ kN. RH2B actually collapsed at a load of 241 kN but it should be noted that both the web and flange thicknesses of RH2B were less than 3/2 times the corresponding RH1 values, and that although RH2B had a slightly higher $\sigma_w$, both $\sigma_f$ and $\sigma_T$ were lower.

Full-size versions of the half scale test specimens would thus be expected to collapse at loads of approximately four times the load recorded for the corresponding model.

5.6.11. Plastic Residual Profiles

Residual profiles recorded along the vertical centre-lines of the RH web panels are shown, for selected cases, in Figs.5.31 - 5.33; displacements
Fig. 5.31. RESIDUAL PROFILES OF WEBS
Fig. 5.32. RESIDUAL PROFILES OF RH4 AND RH5 WEBS
AFTER EXTENSIVE DEFORMATION
to the left signify out-of-plane deformations directed towards the centre of the box (inward displacements). Selected residual profiles of girder rails are presented in Figs. 5.34 and 5.35. Overall plastic deformations are illustrated pictorially in Plates 1 - 10 for several of the RH test specimens where, to highlight the web deformation, a vertical line has been drawn at the midspan of each panel.

Plate 1 shows RHlb after collapse; the residual profile of the vertical centre-line of the web is shown in Fig. 5.31. The remaining residual web profiles in Fig. 5.31 (for RH3, RH4 and RH5), and the web profiles in Fig. 5.33 (for RH6, RH8, RH9 and RH10), were recorded after the girders had undergone the moderate post-collapse deformations indicated in the related load versus displacement plots given in Section 5.6.2. The residual rail profile for RH3 in Fig. 5.34, and the rail profiles in Fig. 5.35 (for RH6, RH8, RH9 and RH10), were also recorded after the girders had undergone the deformations indicated in the related plots in Section 5.6.2. Plates 2 and 3 and Plates 5 - 10 show selected girders, again having undergone the extent of the post-collapse deformation indicated in the related plots in Section 5.6.2.

After the residual web profiles given in Fig. 5.31 for RH4 and RH5 had been recorded, these girders were subjected to further, extensive, plastic deformation. The residual web profiles were then recorded again; these profiles are shown in Fig. 5.32. The residual rail profiles given in Fig. 5.34 for RH4 and RH5 were recorded after the additional deformations had been applied. Plate 4 shows RH5 after having undergone the extensive plastic deformation.

The mechanism of collapse of conventional plate girders subjected to patch loading is illustrated in Fig. 4.2. This shows the mechanism assumed by Roberts and Rockey [53] from observations of several
Fig. 5.34, RESIDUAL PROFILES OF RAILS
collapsed girders and is the mechanism which formed the basis of their solution for predicting collapse loads (see Section 4.2). It can be seen that the mechanism of collapse of the RH girders is broadly similar to this mechanism. Yield (hinge) lines form in a zone in the upper part of the web, and hinges develop in the rail and flange. Not surprisingly, the extremely localised nature of the wheel load on the RH girders led to the formation of a single central flange hinge, as opposed to the two central hinges shown in Fig.4.2 for a girder subjected to a longer patch load of length $c$. Reference to Figs.5.31 - 5.33 and Plates 1-10 shows, however, that the three hinge lines in the RH webs formed such that the spacing between the upper and middle lines ($h_1$) was generally less than that between the middle and lower lines ($h_2$) - as indicated in Fig.4.5 - rather than these two distances being equal, as in Fig.4.2. Unequal web yield line spacing is also evident in several of the tests reported in Refs[56], [58], [62] and [65] on conventional girders. This feature was incorporated into the new mechanism solution presented in Section 4.3 but recognition of unequal (as opposed to equal) spacing became lost as the analysis was developed.

It should also be noted that the web profiles in Figs.5.31 - 5.33 show clear evidence of rotational restraint along the horizontal edges of the webs which tends to maintain the web edge perpendicular to the flange. Earlier evidence of this emerged in the elastic work presented in Chapters 2 and 3 and has already been discussed.

The residual rail profiles given in Figs.5.34 and 5.35 illustrate the influence of web thickness and rail size on the collapse mechanism. Comparing the profile for RH5 with that for RH3, and the profile for RH6 with that for RH8, shows the distance between the two outer hinges to decrease when the web thickness is increased. Comparing the residual profiles for RH3, RH8 and RH4, and for RH9 and RH10, shows the
distance between the two outer hinges to increase as the rail size increases. It can also be seen that for the girders with the larger rails, rail hinge formation is often rather poorly defined with gradual, widespread flexure being exhibited instead of the sharp, localised deformation characteristic of the girders with the smaller rails. In the case of RH4 (Fig.5.34) with its particularly heavy rail, formation of the outer hinges is very poorly defined and smooth rail flexure extending beyond the diaphragm positions is exhibited instead. Plate 2 illustrates that, associated with this, the RH4 web mechanism extended over the full length of the panel and further spread was constrained by the vertical diaphragms. Comparing Plates 7 and 8 (for RH9 and RH10 respectively) shows that the relatively larger rail of RH10 caused the outer flange hinges of RH10 to form nearer to the diaphragms and led the mechanism being distributed over a larger area of web.

Although the mode of failure of all the RH girders was broadly similar, it was observed that significant changes took place in the residual profiles of the webs as displacement of the loaded rail was increased beyond the value at which the maximum load occurred or beyond the value at which vertical displacement began to grow rapidly. Often, it was not until the girders had been subjected to fairly extensive post-collapse deformation that development of all three web hinge lines became discernible.

The point is illustrated by comparing the residual web profiles for RH4 and RH5 given in Fig.5.31 (corresponding to moderate post-collapse deformation) with the profiles given for the same two girders in Fig.5.32 (corresponding to extensive post-collapse deformation). In the case of RH4, it can be seen that evidence of hinge line formation (web rotation) adjacent to the loaded flange emerged only after extensive plastic deformation had been applied to the girder. More noticeable, however,
is the delayed formation of the lower hinge line in the case of RH5. 
Fig. 5.31 shows that having undergone moderate post-collapse deformation, 
RH5 exhibited no lower web hinge line but instead displayed a smooth, 
continuous curve of transverse web displacement below the middle hinge 
position (Fig. 5.31 shows this to be the case for RH3 also). Comparing 
Plates 3 and 4 illustrates pictorially the change in plastic deformation in RH5 due to the additional post-collapse loading. 

These observations indicate that at the maximum load, a part-plastic, 
part-elastic mechanism exists, and that in many cases the mechanism 
becomes fully plastic only after considerable post-collapse deformation 
has been applied. Alternatively, the findings suggest that as regards 
a plastic collapse analysis, a more appropriate mechanism may exist 
than those shown in Figs. 4.2 and 4.5 since these mechanisms appear to be 
representative only of girders which have undergone considerable post-collapse loading. 

5.7. **Empirical Formula for Collapse Loads Using Dimensional Analysis**

The collapse load results presented in Table 5.1 enable empirical solu-
tions of collapse loads to be investigated; in the present investigation 
this is conducted with the aid of dimensional analysis. If it is 
assumed that the collapse load depends on the plastic moment \( M_f \) (of 
the flange for a conventional girder and of the rail and flange 
assembly for a crane girder), the web yield stress \( \sigma_w \), the web thick-
ness \( t_w \), and the web depth \( d \), then 

\[
P_{ua} \propto M_f^\alpha \sigma_w^\chi t_w^\mu d^\sigma
\]

where for a conventional plate girder flange 

\[
M_f \propto \sigma_f b_f t_f^2
\]
When the loaded member is a rail and flange assembly instead of a simple flange, \( b_f \) and \( t_f \) represent the width and thickness of an equivalent rectangular section of yield stress \( \sigma_f \) having stiffness and plastic moment equal to those of the assembly. Hence

\[
P_{ua} \propto \frac{\zeta}{\sigma_f} \frac{\chi}{\sigma_w} b_f t_f \frac{2\zeta}{t_f} t_w d^\rho
\]

This ignores the effect of Young's modulus \( E \), but variations in \( E \) for structural steels is small. Also neglected is the ratio \( b/d \), but \( b/d \) for crane girders varies typically only between 1.0 and 1.5 and in Section 5.6.8 such variations were indicated to lead to changes in \( P_{ex} \) of no more than about 5%. Expressing the relationship dimensionally with \([F]\) denoting force and \([L]\) length gives

\[
[F] \propto [FL^{-2}]^\zeta [FL^{-2}]^\chi [L]^\zeta [L]^2 [L]^\mu [L]^\rho.
\]

Hence for dimensional consistency

\[
\zeta + \chi = 1
\]
\[
\zeta - 2\chi + \mu + \rho = 0
\]

Reference to Table 5.1 suggests a one-third power relationship between the collapse load and \( M_f \), i.e. \( \zeta = 1/3 \).

If \( \zeta = 1/3 \)
then \( \chi = 2/3 \)
and \( \mu + \rho = 1 \)

If the value zero is assigned to \( \rho \), the influence of web depth is neglected. From the discussion of Section 5.6.9, however, it appears the effect of web depth is relatively small. If then \( \rho = 0 \), \( \mu = 1 \) and

\[
P_{ua} \propto t_w \left( \frac{M_f}{\sigma_w} \right)^{2/3}
\]
The choice of powers results in a solution giving \( P_{ua} \propto (\text{length})^2 \) for a given yield of stress of material \( (\sigma_f = \sigma_w = \sigma_y) \), thereby incorporating the effect of scale size (see Section 5.6.10), and gives \( P_{ua} \propto (\text{yield stress}) \) for a given size of girder. An appropriate constant for the relationship is 5.5, giving the equality

\[
P_{ua} = 5.5 t_w (M_f)^{2/3}
\]  

(5.2)

where any consistent set of units may be used.

The predicted collapse loads for the RH test boxes obtained using Eqn.(5.2) are given in Table 5.3. As can be seen, the worst prediction is a 25% underestimation in the case of RH7. The predictions for the two girders with longitudinal web stiffening are underestimations of 15% for RH12 and 12% for RH11. For the remainder of the girders, the predictions are within approximately 10% of the experimental values.

Of the collapse tests from other sources listed in Appendix B, only a few are on girders broadly similar to the RH specimens. Girders B1, B3, B5, B7 and B9 tested by Bergfelt [59] (Table B.2) bear a close resemblance in that they were subjected to a patch load of zero length but, unlike most of the other girders tested by Bergfelt where \( c = 0 \), do not have web panels of excessive aspect ratio. The predictions for these girders using Eqn.(5.2) are also listed in Table 5.3. Generally good (and conservative) estimates are again achieved. The predictions improve as the size of the girder flange increases; particularly good agreement with the experimental collapse load is achieved with girders B7 and B9, the girders which have the heaviest flanges of the five and which have \( M_f \) values similar to those of the RH test specimens. A further comparable test where the patch loading length is very small compared with the panel length is that on girder BR3, tested by Bagchi and Rockey [41], listed in Table B.3. It can be seen from Table 5.3 that Eqn.(5.2) also
Table 5.3. **PREDICTED COLLAPSE LOADS $P_{ua}$ FROM EMPIRICAL FORMULA: EQN.(5.2)**

| Test Girder | $P_{ex}$ kN | $P_{ua}$ kN | $P_{ua}/P_{ex}$ |
|-------------|-------------|-------------|----------------|---|
| RH1         | 112         | 120         | 1.07           |   |
| RH2A        | 230         | 215         | 0.93           |   |
| RH2B        | 241         | 227         | 0.94           |   |
| RH3         | 107         | 100         | 0.93           |   |
| RH4         | 164         | 166         | 1.01           |   |
| RH5         | 67          | 68          | 1.01           |   |
| RH6         | 80          | 81          | 1.01           |   |
| RH7         | 153         | 114         | 0.75           |   |
| RH8         | 128         | 119         | 0.93           |   |
| RH9         | 112         | 100         | 0.89           |   |
| RH10        | 129         | 119         | 0.92           |   |
| RH11        | 114         | 100         | 0.88           |   |
| RH12        | 118         | 100         | 0.85           |   |
| B1          | 95          | 67          | 0.70           |   |
| B3          | 105         | 80          | 0.77           |   |
| B5          | 121         | 98          | 0.81           |   |
| B7          | 126         | 108         | 0.86           |   |
| B9          | 151         | 149         | 0.98           |   |
| BR3         | 89          | 82          | 0.92           |   |
gives a good prediction for this girder.

Eqn.(5.2) therefore appears to offer a very simple method for predicting collapse loads of both crane girders subjected to wheel loads and of conventional plate girders of similar proportions to crane girders subjected to similar, very narrow, patch loads. See Appendix F

5.8. Conclusions

The RH test girders exhibited no obvious evidence of conventional web panel buckling but snap buckling of the panels was frequently in evidence. Sometimes snap buckling occurred as an isolated incident, sometimes reverse snap buckling occurred on unloading and repeated displacement jumps then took place for further loading cycles.

Snap buckling first occurred at loads generally fairly close to the panel buckling load \( P_{\text{crc}} \) calculated on the basis of the horizontal panel edges being clamped; however, an underestimate of the snap buckling load was provided in all cases by the panel buckling load \( P_{\text{crs}} \) calculated on the basis of all panel edges being simply supported. There is some evidence to suggest that longitudinal web stiffening may inhibit snap buckling by increasing the panel buckling resistance, and thereby increases the serviceability limit of the girder.

The collapse loads of the test girders exceeded the \( P_{\text{crs}} \) loads by a factor in the range 3.1 - 7.5, and exceeded the \( P_{\text{crc}} \) loads by a factor in the range 1.2 - 3.0.

There appears to be no useful relationship between the collapse load and the load calculated for the onset of local web yielding - onset of local web yielding does not mark the initiation of girder collapse.
The collapse load increases as the plastic moment of resistance of either the web or flange is increased. The collapse load also increases slightly as the panel aspect ratio is reduced from 1.5 to 1.0. Longitudinal web stiffening increases the collapse load but the effect of the stiffening incorporated in the present study was relatively small: a substantial stiffener mounted at the quarter-depth position produced a 10% increase in the collapse load. The test results also suggest that the collapse load is relatively independent of the depth of the web.

The mechanism of collapse of all the RH test girders was similar, as evidenced by the plastic residual profiles. The mechanism is similar to that which has been observed to develop in conventional girders under patch loading.

Noticeable developments took place in the deformation profiles of the RH web panels as loading was continued beyond the level required to initiate collapse and it was not until the girders had undergone extensive post-collapse deformation that the full plastic mechanism became evident. This indicates that at collapse, both elastic and plastic strains exist within the mechanism zone of the girder and suggests that a more appropriate mechanism may exist for the purposes of a plastic collapse analysis than that indicated by the fully plastic residual profiles. However, since the load against vertical displacement plots developed in the post-collapse region at approximately constant load, a mechanism solution of collapse loads should not prove too sensitive to the precise form of mechanism employed.

A simple expression has been developed using the collapse load data and with the aid of dimensional analysis which gives generally good collapse load predictions for the RH test boxes and for conventional plate girders of similar proportions subjected to similar, very narrow, patch loads.
CHAPTER 6
SUGGESTIONS ON DESIGN PROCEDURE

6.1. Introduction

Based on the findings of the present work, some design recommendations are now presented to help fill the current void in design information on the effect of wheel loadings on web panels of overhead crane box-girders.

It is suggested that three design checks be made:

1) Local yielding
2) Elastic buckling
3) Ultimate load

and the maximum wheel load be restricted to the lowest limit implied by the three cases. Assessment of these three cases will now be considered in turn.

6.2. Local Web Yielding Check

Much of the work of Chapter 2 relates to this consideration. It is suggested that the effective length of the patch load at the top edge of a web be estimated from

\[
c = 3.25 \left[ \frac{I_f + I_T}{t} \right]^{1/3} \tag{6.1}
\]

when the rail is fastened directly to the flange by clips. When a crane rail underlay pad is used, it is suggested the following expression be used

\[
c = 4.0 \left[ \frac{I_f + I_T}{t} \right]^{1/3} \tag{6.2}
\]
When the rail is welded to the flange, it is suggested patch lengths be estimated from

\[ c = \frac{1}{0.3} \left[ \frac{1.15 I_{fr}}{t} \right]^{\frac{1}{3}} \]

which, on combining constants, simplifies to

\[ c = 3.49 \left[ \frac{I_{fr}}{t} \right]^{\frac{1}{3}} \]

Eqns. (6.1) and (6.2) appeared in a recent draft British Standard [31] and Eqn. (6.3) is derived from an expression which appeared in a draft German Standard DIN 4132 [30].

For an I-section girder, the full section of the flange may be considered effective when evaluating \( I_f \) and \( I_{fr} \). For a torsion-box-girder, an assumption is required on the effective flange width. BS 2573/1 [2] stipulates 12 \( t_f \) as the maximum projection for an as-welded flange unstiffened along its outer edge (projections beyond 12 \( t_f \) are considered ineffective because of local buckling). A reasonable assumption for the effective flange width is therefore the full flange projection on the outer side of the rail supporting web (since this should already be no greater than 12 \( t_f \)) plus 12 \( t_f \) on the inner side of the web. This gives an effective flange width of

\[ b_f = (24 t_f + t_w) \]

if the flange outstand were of its maximum value. Alternatively, a width of flange symmetrical about the rail/web centre-line (equal to twice the distance from the centre-line to the nearer flange edge) could be assumed, as shown in Fig. 5.3(a). This gives a slightly more conservative estimate of \( I_f \) and \( I_{fr} \) in cases where the flange outstand is less than 12 \( t_f \). In many cases where the rail is clipped, \( I_f \) can in fact turn out to be very small compared with \( I_r \) (often less than 5%). There is then little penalty in omitting the \( I_f \) term from Eqns. (6.1) and (6.2) which leads to still simpler
expressions for \( c \).

The expressions for the patch length \( c \) enable the patch length parameter \( \delta \) to be evaluated for use in the elastic buckling check (see following section) and also enable the maximum direct stress in the web to be evaluated from

\[
\sigma_m = \frac{P}{ct} \quad (6.4)
\]

Alternatively, \( \sigma_m \) can be evaluated more directly by substituting for \( c \) from Eqns.(6.1) - (6.3) in Eqn.(6.4). Substituting Eqn.(6.1) gives

\[
\sigma_m = \frac{P}{3.25t} \left[ \frac{t}{I_f + I_r} \right]^{1/3} \quad (6.5)
\]

for the direct stress in a web where the rail is clipped and no underlay pad is present. For a clipped rail where an underlay pad is present, substitution of Eqn.(6.2) gives

\[
\sigma_m = \frac{P}{4.0t} \left[ \frac{t}{I_f + I_r} \right]^{1/3} \quad (6.6)
\]

Again, it will be found in many cases that \( I_f \) is small compared with \( I_r \) and omitting \( I_f \) then leads to only slightly more conservative estimations. For a welded rail, substitution of Eqn.(6.3) gives

\[
\sigma_m = \frac{P}{3.49t} \left[ \frac{t}{I_f r} \right]^{1/3} \quad (6.7)
\]

In all cases, the term \( P \) for the wheel load should incorporate the appropriate allowance for dynamic effects.

As regards the shear stress between the flange and web due to the wheel load, draft DIN 4132 [30] gives the maximum shear stress \( \sigma_{sm} \) as

\[
\sigma_{sm} = 0.2 \sigma_m \quad (6.8)
\]
This can be used to check the strength of weld required between the flange and web.

6.3. Elastic Buckling Check

6.3.1. Elastic Buckling: Patch Loading Only

Some of Chapter 2 and Chapter 3 relate to this consideration. It was proposed in these chapters that it is probably more representative when calculating elastic buckling loads of torsion-box web panels to consider the horizontal edges clamped and the sides simply supported, rather than consider all edges simply supported.

The elastic buckling load of a panel under patch loading is given by

\[ P_{cr} = K \frac{\pi^2 D}{d} \]  

(6.9)

where

\[ D = \frac{E t^3}{12(1-v^2)} \]

For a particular set of edge conditions and a given aspect ratio, the buckling coefficient \( K \) depends on the patch length parameter \( \beta = c/b \).

The patch length \( c \) can be calculated using the appropriate expression from Eqns.(6.1) - (6.3) and the panel length \( b \) will be known, hence \( \beta \) can be found. The coefficient \( K \) can then be read-off from the set of curves given in Figs.3.2 - 3.5 corresponding to the assumed edge conditions (note that in Figs.3.2 - 3.5, \( \beta \) is denoted by \( C/L \)).

Alternatively, and more simply, \( K \) can be estimated from the formulae given in Table 6.1. The formulae give \( K \) in terms of the panel aspect ratio \( \alpha \) for panels with four different sets of boundary conditions. The coefficients correspond approximately to a patch length parameter of 0.125. The present work has indicated that for crane web panels,
patch length parameters are typically around 0.125, with a range 0.07 - 0.25, and that over this range the buckling coefficient changes only slightly. The formulae in Table 6.1 thus give K values which are slightly conservative for patch length parameters above 0.125, and very slightly unconservative for patch length parameters below 0.125.

Table 6.1 FORMULAE FOR ESTIMATING BUCKLING COEFFICIENTS K FOR CRANE WEB PANELS SUBJECTED TO A WHEEL LOAD.

<table>
<thead>
<tr>
<th>Plate Support Condition</th>
<th>c = clamped</th>
<th>s = simply supported</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel Aspect Ratio a</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>c</td>
</tr>
<tr>
<td>1.0 ≤ a ≤ 1.5</td>
<td>0.95 + 2.30</td>
<td>0.96 + 5.92</td>
</tr>
<tr>
<td></td>
<td>0.66 + 6.35</td>
<td>1.98 + 6.28</td>
</tr>
<tr>
<td>1.5 &lt; a ≤ 4.0</td>
<td>0.86 + 2.20</td>
<td>1.13 + 5.65</td>
</tr>
<tr>
<td></td>
<td>2.01 + 5.20</td>
<td>2.27 + 5.16</td>
</tr>
</tbody>
</table>

Although the elastic buckling load of a crane web panel is probably better assessed by considering the horizontal panel edges clamped instead of simply supported, it was shown in Chapter 5 that snap buckling may occur at loads below the buckling load calculated on this basis. Until more is known of snap buckling tendencies with full-size crane girders, it is suggested that the buckling load calculated on the basis of all panel edges being simply supported be taken as an estimate of the serviceability limit.
6.3.2. Elastic Buckling: Combined Loading

In practice, web panels subjected to wheel loadings are also subjected to global bending and shear stresses, the bending stresses usually being the more severe global loading on crane girders.

Elastic buckling interaction curves for patch loading with bending and patch loading with shear are given by Khan and Johns [45] and by Rockey, El-gaaly and Bagchi [68] for panels with simply supported edges. These curves show there to be a more detrimental interaction between patch loading and bending than between patch loading and shear; the former interaction is therefore likely to be of greater importance to the crane designer.

1) Patch Loading with Bending
a) Panels with Simply Supported Edges

Both Khan and Johns [45] and Rockey, El-gaaly and Bagchi [68] show the interaction between patch loading and bending to change very little with the patch loading length. Khan and Johns illustrate the effect of aspect ratio on the interaction for aspect ratios between 1/3 and 2.0; greatest interaction occurs for an aspect ratio of 3/4, and least for an aspect ratio of 2.0. A conservative estimate of all the presented curves is provided by the straight line

$$\frac{P_b}{P_{cr}} + \frac{\sigma_b}{\sigma_{bcr}} = 1.0 \quad (6.10)$$

and it is suggested that this expression be used for interaction between patch loading and bending, irrespective of the patch length or panel aspect ratio. For panels subjected to pure bending, $\sigma_{bcr}$ in Eqn.(6.10) is given by

$$\sigma_{bcr} = K_b \frac{\pi^2 d}{d^2 t} \quad (6.11)$$
where for simply supported panels $K_b$ takes the value 23.9.

b) Panels with Clamped Horizontal Edges

The results of a finite element analysis by Bagchi and Rockey [69] on square panels indicate that a weaker interaction between patch loading and bending exists when the longitudinal edges are clamped than when all edges are simply supported. Although then rather more conservative, Eqn. (6.10) may therefore be used for interaction calculations on panels with clamped longitudinal edges. $K_b$ in Eqn. (6.11) then takes the value 39.6, as given by Timoshenko and Gere [46].

2) Patch Loading with Shear

Khan and Johns [45], and Rockey, El-gaaly and Bagchi [68] show that, as with patch/bending interaction, interaction between patch loading and shear is almost independent of the length of the patch load. Khan and Johns show the dependence on aspect ratio for aspect ratios between 1/3 and 2.0 also to be small. A reasonably good, though slightly conservative, fit for all the curves shown is given by the parabola

$$\frac{P_b}{P_{cr}} + \left(\frac{\sigma_s}{\sigma_{scr}}\right)^2 = 1.0$$

(6.12)

where the critical shear stress $\sigma_{scr}$ for $a > 1$ is given by Timoshenko and Goodier [29] as

$$\sigma_{scr} = \left[5.34 + \frac{4.0}{a^2}\right] \frac{\pi^2D}{d^2t}$$

(6.13)

For interaction between patch loading and shear, no results are given by Bagchi and Rockey [69] for the case of a panel clamped along its horizontal edges. However, a curve is shown for a panel supported by a flange along either longitudinal edge which reveals a weaker interaction than for a panel simply supported all round. This indicates that, as with patch/bending interaction, increasing restraint along the
horizontal panel edges reduces interaction between patch loading and shear. Assuming this to be so, Eqn.(6.12) may be used for panels with clamped horizontal edges.

3) **Patch Loading with Bending and Shear**

Three-way interaction curves are given by Khan and Johns [45] for simply supported panels; cases considered are a square panel with a partial edge loading acting over 1/4 of the panel length, and a panel with an aspect ratio of 1/3 with a full edge load.

The curves again show there to be little dependence on the aspect ratio or patch length. It is therefore suggested that in the absence of further information, the curves for the square panel be used if a three-way interaction is required.

6.4. **Ultimate Load Check**

6.4.1. **Ultimate Load: Patch Loading Only**

Chapters 4 and 5 relate to this consideration. In Chapter 4, a mechanism solution presented by Roberts and Rockey [53] for predicting collapse loads of plate girders under patch loading was studied and application of the solution to wheel loaded crane girders was investigated. Although the solution gave good collapse load predictions for a number of half scale crane girder sections, the investigation cast doubt on the confidence with which the solution might be applied to full-size crane girders.

From the results of the collapse tests reported in Chapter 5, and with the aid of dimensional analysis, a simple expression has been developed for predicting the collapse loads of crane girders subjected to a wheel load. The expression is
\[ P_u = 5.5 \ t_w \ (M_f \sigma_w^{2/3}) \]  \hspace{1cm} (6.14)

where any consistent set of units may be used. In calculating \( M_f \) (which represents the plastic moment of the rail and flange combination), the same assumptions on the effective flange section may be made as were proposed in Section 6.2 for calculating \( I_f \) and \( I_{fr} \). See Appendix F

It is suggested Eqn.(6.14) be used for estimating collapse loads, but an additional check could be made using the Roberts and Rockey solution [53] (see Chapter 4) and the more conservative estimate then be adopted as the measure of ultimate load carrying capacity.

6.4.2. Ultimate Load: Combined Loading

1) Patch Loading with Bending

Roberts and Rockey [53] have considered this loading combination and proposed use of the following interaction formula which has been incorporated in a 1976 Czechoslovak code of practice

\[ \left( \frac{P_b}{P_u} \right)^2 + \left( \frac{\sigma_b}{\sigma_w} \right)^2 = 1.0 \] \hspace{1cm} (6.15a)

which gives

\[ P_b = P_u \left[ 1 - \left( \frac{\sigma_b}{\sigma_w} \right)^2 \right] \] \hspace{1cm} (6.15b)

This represents a relatively weak interaction, the described curve being a quadrant of a circle. A still weaker interaction given by Rockey [14] is

\[ \left( \frac{P_b}{P_u} \right)^3 + \left( \frac{\sigma_b}{\sigma_w} \right)^3 = 1.0 \]

but since this expression was derived from experimental results on thin-walled members only it is recommended that interaction be assessed by the safer expression of Eqn.(6.15).
2) Patch Loading with Shear

Rockey [14] quotes for this combination of loadings the interaction formula

\[
\left( \frac{P_b}{P_u} \right)^{1.8} + \left( \frac{\sigma_S}{\sigma_{su}} \right)^{1.8} = 1.0
\]  
(6.16)

which was determined from a series of tests on stiffened, thin, folded plate girders. In the absence of information more applied to crane girders, it is suggested Eqn.(6.16) be used if this interaction is required.

6.5. Examples of the Current Situation

It may be helpful to the crane designer if the present findings are now employed to assess the severity of wheel loadings to which typical existing full-size crane bridge girders are subjected. Ultimate load carrying capacity can be assessed by relating the collapse results for the model box sections reported in Chapter 5 back to the full-size cases from which they were drawn.

Of the model boxes, RH5 and RH8 serve as two appropriate examples since they approximate particularly closely to two full-size crane girders known to the author. These girders are employed in cranes with working loads of 25 and 50 tonnes respectively; for ease of reference, let these full-size cranes be known as FS25 and FS50. The crabs for these two cranes weigh respectively 1.85 and 5.0 tonnes.

Assuming the load to be distributed equally to each of the four crab wheels, the maximum load applied by one crab wheel is given by

\[ P = 0.25 [(\text{Impact Factor} \times \text{Lifting Capacity}) + \text{Crab Weight}] \]

This gives a wheel load of 72 kN for FS25 and 147 kN for FS50 when a
class 1 crane impact factor of 1.1 is used (as given by BS 2573/1 [2]).

**Ultimate Load**

The wheel load that would cause collapse of the underlying region of the full-size girder can be estimated by factoring by 4.0 the collapse load given in Table 5.1 for the corresponding half scale model girder (see Section 5.6.10). This gives a collapse load of 268 kN for FS25 and 512 kN for FS50. Alternatively, similar predictions could be obtained by using Eqn.(6.14) on the full-size girders, provided the yield stresses of the steels in the full-size cranes were comparable with those of the corresponding model. The collapse load estimations for FS25 and FS50 ignore a longitudinal structural web stiffener which is actually present at the quarter-depth position of both crane girders. It has been shown in Chapter 5, however, that such stiffeners have relatively little effect on the collapse load (an increase of approximately 10% is indicated). Hence, ignoring the presence of the stiffener, the ratio between maximum wheel load and estimated collapse load is 0.27 for FS25 and 0.29 for FS50.

Inserting these ratios into the interaction formula of Eqn.(6.15) shows that this level of patch loading has relatively little effect on the allowable bending stress: a reduction of approximately 5% on the level acceptable in the absence of any patch loading is indicated.

**Local Yielding**

Table 5.2 shows the patch length $c$ at the web to be 88 mm for RH5 and 108 mm for RH8. The patch lengths for the full-size girders would be expected to be twice the size, that is 176 mm for FS25 and 216 mm for FS50. Values similar to these could be arrived at by applying Eqn.(6.3) to the full-size girders.
The maximum direct stress can be estimated from Eqn.(6.4); this gives 82 N/mm² for FS25 (5 mm thick web) and 113 N/mm² for FS50 (6 mm thick web). Alternatively, the maximum bearing stress could be estimated directly from Eqn.(6.7). Both bearing stresses are well below the permissible bearing stress to BS 2573/1 of 0.8 $\sigma_y$ for a grade 43 steel. The bearing stress so calculated can then be used to make a combined loading check according to normal practice on the web and web/flange weld.

Elastic Buckling

Unfortunately, the present work does not permit an entirely rigorous check of elastic web panel buckling to be made for the two chosen examples, owing to the presence in either case of the longitudinal web stiffener. The stiffener divides the panel into an upper and a lower sub-panel; the patch load acts on to the upper sub-panel but this panel is no longer supported by a parabolic distribution of shear stress along the vertical edges as was assumed in the calculations of the buckling coefficients presented in Chapter 3. These coefficients are therefore not strictly applicable to a patch loaded sub-panel. However, the following considerations help to illustrate some of the more important points associated with the elastic buckling check.

If a longitudinal web stiffener were not present in the case of FS25 and FS50, the buckling load could readily be estimated from Eqn.(6.9). In the local yielding check, the patch length $c$ was estimated to be 176 mm for FS25 and 216 mm for FS50. FS25 has an overall web depth $d$ of 1150 mm, $d$ for FS50 is 1450 mm. If the web panel aspect ratios for the two girders were 1.0, the patch length parameter $c/b$ would be 0.15 for both girders; if the web panel aspect ratios were 1.5, $c/b$ would be 0.10. In either case, the patch length parameter is close to 0.125 and so the buckling coefficient formulae in Table 6.1 can be used to
evaluate $K$. Taking the values $E = 205 \text{kN/mm}^2$ and $\nu = 0.3$ enables $D$ in Eqn.(6.9) to be evaluated for the two girders; the buckling loads given by Eqn.(6.9) can then be expressed as

$$P_{cr} = 20.1K \text{kN}$$

for FS25 and

$$P_{cr} = 27.6K \text{kN}$$

for FS50.

If clamped horizontal edges are assumed, Table 6.1 gives $K = 7.0$ for $\alpha = 1.0$, and the buckling load is given as $140.7 \text{kN}$ for FS25 and $193.2 \text{kN}$ for FS50; for $\alpha = 1.5$, $K = 6.5$ and $P_{cr}$ becomes $130.7 \text{kN}$ for FS25 and $179.4 \text{kN}$ for FS50.

If the same applied wheel loads were to act on to the unstiffened girders as were estimated earlier ($72 \text{kN}$ for FS25 and $147 \text{kN}$ for FS50) then the interaction formula of Eqn.(6.10) indicates that to avoid panel buckling, bending stresses would have to be reduced significantly below the levels acceptable in the absence of patch loading: the reduction is 51-55% for FS25, and 76-82% for FS50, depending on aspect ratio.

If simply supported panel edges are assumed instead, then buckling loads lower than the applied loads are obtained: for $\alpha = 1.0$, Table 6.1 gives $K = 3.3$, thus $P_{cr} = 66.3 \text{kN}$ for FS25 and $P_{cr} = 91.1 \text{kN}$ for FS50; for $\alpha = 1.5$, $K = 2.3$ and $P_{cr}$ becomes $46.2 \text{kN}$ for FS25 and $63.5 \text{kN}$ for FS50.

With a longitudinal stiffener present at the quarter-depth position of the web, approximate estimations of the upper sub-panel buckling load may be made as follows. The depth $d$ in Eqn.(6.9) becomes $d/4$ and the buckling loads are then given by
\[ P_{cr} = 80.6 \text{ K kN} \]

for FS25 and

\[ P_{cr} = 110.4 \text{ K kN} \]

for FS50.

For an upper sub-panel, it is probably best to assume either all edges are simply supported or the top edge only is clamped and the remaining edges simply supported. If the full panel were of aspect ratio 1.0 then the upper sub-panel will be of aspect ratio 4.0. From Table 6.1, \( K \) is then 2.3 for simply supported edges and the buckling load is given as 185.4 kN for FS25 and 253.9 kN for FS50; for a clamped top edge, \( K = 5.9 \) and \( P_{cr} \) becomes 459.4 kN for FS25 and 629.3 kN for FS50.

Buckling coefficients for panels with aspect ratios greater than 4.0 have not been investigated in the present work. However, Figs.3.2 - 3.5 show that for a narrow patch load, the buckling coefficient is nearly constant for aspect ratios above about 1.5 and an analysis presented in Chapter 3 indicated that the coefficients are likely to remain approximately constant up to an aspect ratio of about 4.7 for a panel with at least the top edge clamped, and up to an aspect ratio of about 7.2 for a panel with all edges simply supported. Thus, in the absence of more relevant data, the coefficient values for \( \alpha = 4.0 \) could be used within the above limits to estimate buckling loads for panels with aspect ratios above 4.0.

When checking buckling interaction with bending for an upper sub-panel, it should be noted that the coefficient \( K_b \) used to evaluate \( \sigma_{bcr} \) in Eqn.(6.11) takes a value corresponding to the linearly-varying distribution of compressive stress acting on the sub-panel. Values of \( K_b \) for this form of longitudinal loading are given in DIN 4114 [11] for
simply supported panels; coefficients for some compressive stress distributions on panels with clamped longitudinal edges are given in the literature (see Refs.[46] and [70]).

6.6. **Concluding Remarks**

The presented suggestions and recommendations should enable generally adequate checks to be made of the effect of wheel loadings on crane girder web panels.

The indications are that wheel loadings on present day cranes are generally not, by themselves, of a level likely to cause local yielding, buckling, or crippling of the girder. However, interaction between patch loading and bending is liable in some cases to reduce significantly the level of bending stress required to cause panel buckling. This is therefore an important point which should be checked in design.
The investigation reported here originates from a practical problem that exists in the design of crane torsion-box-girders. It is a complex problem concerning the load carrying capacity of a plate box-girder when the upper horizontal edge of the web is subjected to an in-plane patch load. With crane torsion-box-girders, the patch load is produced by a wheel load which spreads through a rail mounted on the girder flange directly above one of the webs. This form of loading also arises at the diaphragms of crane box-girders with centrally mounted rails, and is also encountered in other fields of structural engineering, for example, where purlins transmit loads to principal frame members and during the erection of large plate girder structures. In-plane patch loading of plate girder webs is liable to cause buckling and local yielding of a web panel, leading to local failure of the girder; it is therefore an important design consideration.

Owing to its origins, this study has focused attention on the patch loading condition as it arises in crane girder webs; certain of the conclusions are therefore specific to this situation while others apply to patch loading of plates in the more general sense.

The main conclusions, of both a specific and a general nature, will now be summarised. Also offered are some suggestions on further work which should provide useful supplementary information to the present findings.

In-plane vertical stress distribution profiles recorded along the upper edge of a web panel of a short-span model box-girder have been presented. The girder was loaded through various rails and other loading interfaces above the web by a wheel load. The profiles were found to be consistent
in that the total force on the web plate deduced from the measured stresses was, in each case, very close indeed to the applied wheel load. It has been proposed that an equivalent uniform patch load should be defined as an imaginary loading whose intensity is equal to the peak intensity of the true distribution and whose overall magnitude is the same as that of the true loading (the length over which the equivalent uniform patch load acts being taken as the effective length of the true patch load).

Effective patch lengths estimated from the experimental stress distributions by this method were not well predicted by the linear fan-spread methods used in the codes to estimate wheel load distributions. Good predictions were obtained, however, from simple expressions involving the second moment of area of the rail and flange and the web thickness. To the author's knowledge, no expression of this type has, to date, featured in any major design code, although two draft codes have contained expressions of this nature. The expressions can be re-formulated to give expressions for estimating the maximum direct stress in the web beneath a wheel load.

It has been shown that the patch loads to which crane web panels are subjected are generally of short length, occupying less than one-quarter of the panel length.

A computer program has been developed to determine elastic buckling coefficients for flat rectangular plates subjected to a uniform in-plane patch load centrally disposed on the upper horizontal edge and supported by shear stresses on the vertical edges. Patch loads of lengths from 0.125 to 1.0 times the plate length have been investigated for plates with aspect ratios between 1/3 and 4.0. Four sets of plate boundary conditions have been considered involving different combinations of simply
supported and clamped edges. The coefficient results have been presented graphically in a form suitable for design use (Figs.3.2 - 3.5).

For the large majority of geometries considered, it is the support condition along the loaded edge which has greatest influence on the buckling load. For long plates with at least their top edge clamped, there is a fall-off in coefficient values corresponding to the longer patch loads. This can be shown to be attributable to a transition from a local mode of buckling under the patch load to a mode produced essentially by the longitudinal in-plane bending stress resulting from the patch load acting on a long plate.

The computer program has also been used to determine buckling coefficients for a number of patch loads of non-uniform distribution, modelling approximately a distributed wheel load; these showed close agreement with the coefficients computed for the respective equivalent uniform patch loads defined previously.

Coerrelation of the coefficient results with buckling loads estimated from experimental measurements on a model crane girder web panel indicated that an assumption of simply supported panel edges is over-conservative. It was deduced that it is probably more representative to consider the edges attached to the flanges as clamped, with the vertical edges simply supported. Experimental buckling mode plots have indicated moment restraint along the web edges adjacent to the flanges.

Although the short patch loading lengths to which crane web panels are subjected mean the buckling load is not very sensitive to the precise patch length, the buckling load does vary significantly depending on the assumed edge condition along the loaded edge. It would therefore be beneficial to gain an accurate measure of the actual elastic moment restraint along this edge.
To supplement the present buckling information, an investigation is required of patch loading on longitudinally reinforced web panels. Longitudinal web stiffening is frequently incorporated on crane girders to prevent panel buckling under longitudinal in-plane bending stresses but, as Rockey [14] has commented, the influence of patch loading on such panels is an issue which has received little attention.

A study has been presented for predicting the collapse load of patch loaded plate girders. The study was based on an upper bound plastic collapse analysis originally presented by Roberts and Rockey [53] which has previously been shown to give good collapse load predictions for a large number of conventional plate girders. A modified version of the analysis has been developed here which reveals the transition region from collapse initiated by direct web yielding for girders with stocky webs to failure by a mechanism of out-of-plane web deformation for girders with slender webs.

The ultimate load study also showed that certain of the simplifying assumptions in the original analysis lead to the omission of terms which can contribute significantly to the plastic work expression. These omissions are effectively corrected for in the original analysis by the selection of a suitable empirical expression for one of the length parameters of the assumed mechanism. This expression, however, is dimensionally inconsistent and indicates an anomaly in the analysis with regard to scale size of geometrically similar girders. Development of the method to include the omitted terms has shown that the need for an empirical expression for this parameter is removed; in principle, an expression can then be derived by minimising the collapse load expression (as is done for the other length parameter of the mechanism). However, further work is necessary to investigate more thoroughly what is then a complex solution, and to ensure that the derived expressions for the two
length parameters represent true minimisations of the collapse load expression. Once checked, it would be desirable to investigate whether the solution could be reduced to a simpler form more suitable for design purposes.

Collapse tests have been conducted on a series of short-span model box-girders subjected to a wheel load above one of the webs.

In several of the tests, snap buckling of the loaded web panel occurred. This was usually at loads fairly close to the panel buckling load calculated on the basis of clamped horizontal panel edges, but on one occasion was at a load well below this buckling load. Snap buckling occurrences were in all cases at loads greater than the panel buckling load calculated on the basis of all edges simply supported. There is some evidence to suggest longitudinal web stiffening may inhibit snap buckling and so increase the serviceability limit of the girder. Further work on the factors which influence snap buckling of web panels under wheel loadings would be very desirable.

Collapse loads of the test girders increased as the rail size or web thickness was increased, and increased slightly with a reduction in panel aspect ratio from 1.5 to 1.0. A structural longitudinal stiffener placed at the quarter-depth position of the web increased the collapse load by 10%. This implies the collapse load is relatively independent of the depth of the web. Investigations of the effect on the collapse load of stiffeners placed closer to the loaded flange, within the mechanism zone, would be worthwhile.

The measured collapse loads exceeded the calculated elastic buckling loads of the panels. The buckling loads corresponding to clamped horizontal panel edges were exceeded by a factor between 1.2 and 3.0; the buckling loads corresponding to all panel edges simply supported were exceeded by...
a factor between 3.1 and 7.5. There appears to be no meaningful relationship between the measured collapse loads and the loads estimated for local web yielding to occur.

The mechanism of collapse of the test box-girders was similar in all cases and resembled that which has been observed in conventional plate girders subjected to patch loading: plastic hinges formed in the rail/flange and hinge lines in the web. However, in many cases, the full plastic mechanism did not develop until the girders had undergone considerable post-failure deformation. This suggests that a full appreciation of plate girder collapse requires a large-deflection, elastic-plastic analysis of the type developed by Puthli, Supple and Crisfield [71] (using the finite-element method) which takes account of initial web imperfections. A further consideration which may have an influence on the failure of crane torsion-box-girders under wheel loadings is the effect of edge moments created at the box corners by the distortional component of the applied loading.

From the present collapse results, a simple expression has been developed for predicting collapse loads of plate girders subjected to narrow patch loads (Eqn.(5.2)). A feature of the expression is that it takes due account of girder scale size.

Based on the findings of the present work, a series of design recommendations have been presented to provide guidance to crane designers in taking account of wheel loadings on web panels of overhead crane box-girders.
**APPENDIX A**

**FLOW CHART OF NEW MECHANISM SOLUTION FOR PRODUCING COLLAPSE LOAD CURVES**

**ASSIGN CONSTANTS**
- $A_C = \frac{M_f}{\sigma_w t_w}
- B_C = 2A_C
- C_C = \frac{9c^2}{B_C}
- D_C = \frac{3c^2}{4A_C}
- E_C = 2\sigma_w t_w

**Z = 0**

**INCREMENT Z**

**Z = 0 ?**
- **YES**
- **NO**

**GENERATE POLYNOMIAL COEFFICIENTS FOR V > 0.5**
- $F(1) = Z^2(18-C_C)$
- $F(2) = 3Z(3Z^2-4)$
- $F(3) = 2-Z^2(9+(C_c/12))$
- $F(4) = Z(2-1.5Z^2)$
- $F(5) = 5Z^2/8$
- $F(6) = Z^7/16$

**SOLVE POLYNOMIAL EXPRESSION IN V**

**SELECT ANY REAL V > 0.5**

**TEST FOR SPURIOUSNESS**

**WORKING V VALUE FOUND ?**
- **NO**
- **YES**

**SQUASH CASE APPLIES**
- $j = \sqrt{2B_C}$
- IF $j > 0.5 (b-c)$ THEN $j = 0.5 (b-c)$

- $P_u = \frac{4M_f}{j} + \sigma_w t_w (c + j)$

- $j = \sqrt{2M_fV(Z + 2V)}$

- IF $j > 0.5 (b-c)$ THEN $j = 0.5 (b-c)$

- $P_u = \frac{4M_f}{j} + \frac{2\sigma_w t_w V}{Z + 2V} \left[ c + j(1 + \frac{1}{12V^2}) \right]$
GENERATE POLYNOMIAL COEFFICIENTS FOR $V \leq 0.5$

- $F(1) = D_c$
- $F(2) = 2(ZD_c - 1)$
- $F(3) = D_c(Z^2 + 0.25) - 5Z$
- $F(4) = Z(D_c - 4Z) + 3$
- $F(5) = Z(4.5 - 2Z) + 0.25D_c(3Z^2 - 1.25)$
- $F(6) = 0.5(3Z^2 - 0.75ZD_c - 2.25)$
- $F(7) = 0.1875(0.25D_c - 3Z)$

IF $c = 0$

SET $F(1) = F(2)$

ETC.

F(7) = 0

SOLVE POLYNOMIAL EXPRESSION IN $V$

SELECT ANY REAL $V \leq 0.5$

TEST FOR SPURIOUSNESS

WORKING $V$ VALUE FOUND?

YES

$\sqrt{\frac{M_f(Z + 2V)}{\sigma_w t_\text{w}((V^2/3) + (1/12)}}$

IF $j > 0.5 \ (b-c)$ THEN $j = 0.5 \ (b-c)$

$p_u = \frac{4M_f}{j} + \frac{2\sigma_w t_\text{w}}{Z + 2V} \left[ \phi\left( \frac{1}{4} + \frac{V^2}{3} \right) + 2j\left( \frac{1}{4} + \frac{V^2}{3} \right) \right]$

CHECK UNIQUE $p_u$ FOUND FOR EACH $Z$

NO

YES

PLOT $p_u/p_{ex}$ AGAINST $Z$

NO

WARN

YES

$Z = Z_{\text{MAX}}$?

YES

END
## APPENDIX B

### DETAILS OF CONVENTIONAL TEST GIRDERS

Table B.1 TEST DATA OF SKALOUD AND NOVAK [58]

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<th>b (mm)</th>
<th>d (mm)</th>
<th>t_w (mm)</th>
<th>b_f (mm)</th>
<th>t_f (mm)</th>
<th>c (mm)</th>
<th>( \sigma_w ) (N/mm²)</th>
<th>( \sigma_f ) (N/mm²)</th>
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## APPENDIX C

Z VALUES FOR TEST GIRDERS DETAILED IN APPENDIX B AND RH TEST GIRDERS

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APPENDIX D

DESIGN OF 400 kN CAPACITY LOADING FRAME

Standard steel sections and bolted joints were employed wherever possible to facilitate a simple construction requiring a minimum of welding. A general assembly of the loading frame is shown in Fig. D.1, the base unit is shown in detail in Fig. D.2, and details of the remaining components are shown in Fig. D.3.

The test boxes were seated in the frame on a pair of beams which spanned the width of the base unit; onto the top of each support beam was mounted a half-round bar. The beams were independently movable along the base unit, enabling test boxes of different lengths to be accommodated. By interchanging one pair of support beams with another pair of different height, test boxes of different heights could be accommodated. The loading ram was mounted centrally on the underside of the top cross-members and the test boxes were positioned eccentrically in the frame such that the test web panel lay directly beneath the ram.

The main bending stresses in the frame were thus carried by the two longitudinal base members, with the two vertical members being tension members. The frame was designed to BS449 specification [9] for a load capacity of 400 kN; however, a deliberately generous design approach was adopted in order to achieve some reserve capacity should this be required at a later date for the testing of larger boxes.

\[ P = 400 \times 10^3 \text{ N} \]

Allowable bending stress \[ p_b = 165 \text{ N/mm}^2 \]

Allowable tensile stress \[ p_t = 155 \text{ N/mm}^2 \]

Allowable average shear stress in unstiffened web \[ p_s = 100 \text{ N/mm}^2 \]

Steel of Grade 43.
1) **Longitudinal Base Member**

Maximum bending moment \( M = \frac{P \times L}{2} \frac{L}{4} \)

where \( L = \) full length of base member = 3000 mm

Hence \( M = 150 \times 10^6 \) Nmm

Elastic modulus \( Z = \frac{M}{P_b} = 0.9 \times 10^6 \) mm

406 x 178 x 74 kg/m U.B : \( Z = 1.324 \times 10^6 \) mm

Average shear stress in web = \( \frac{P}{4A_w} \)

where \( A_w = \) cross sectional area of web

\[ = 412.8 \times 9.7 \text{ mm}^2 \]

Hence average shear stress in web = 25 N/mm

The two longitudinal members were fixed relative to each other by two cross-members, one at either end of the base unit. To ensure the full section of each longitudinal member was effective in carrying the load transmitted by the vertical members, the central region of each longitudinal member was stiffened and a further bracing cross-member incorporated at the midspan of the base unit.

2) **Vertical Member**

Tensile stress \( p_t = \frac{P}{2A} \)

Where \( A = \) cross sectional area

Hence \( A = \frac{P}{2p_t} = 1290 \text{ mm}^2 \)

305 x 102 x 46 kg/m channel : \( A = 5883 \text{ mm}^2 \).

3) **Top Cross-Member**

Span = 1100 mm, loaded over central 250 mm by ram.

\[ M = \frac{P \times 1}{2} \frac{(425 + 250)}{4} = 48.75 \times 10^6 \text{ Nmm} \]
Z = M/ \( p_b \) = 0.296 x 10\(^6\) mm\(^3\)

305 x 102 x 46 kg/m channel : Z = 0.539 x 10\(^6\) mm\(^3\)

Average shear stress in web = \( \frac{P}{4A_w} \)

where \( A_w = 304.8 x 10.2 \) mm\(^2\)

Hence average shear stress in web = 32 N/mm\(^2\) < \( p_s \)

4) **Box Support Members**

Assume worst possible case of central point load

M = P x \( \frac{L}{Z} \) where L = 920 mm

Hence M = 46.0 x 10\(^6\) Nmm

Z = M/\( p_b \) = 0.279 x 10\(^6\) mm\(^3\)

203 x 152 x 52 kg/m RSJ : Z = 0.471 x 10\(^6\) mm\(^3\)

305 x 127 x 37 kg/m U.B : Z = 0.472 x 10\(^6\) mm\(^3\)

Average shear stress in web of RSJ = \( \frac{P}{4A_w} \)

\[ = \frac{P}{4 x 203.2 x 8.9} \]

\[ = 55.3 \text{ N/mm}^2 < p_s \]

Average shear stress in web of U.B = \( \frac{P}{4A_w} \)

\[ = \frac{P}{4 x 303.8 x 7.2} \]

\[ = 45.7 \text{ N/mm}^2 < p_s \]

5) **Check for Load Bearing Web Stiffener Requirement**

Stiffener required when \( W > p_c t B \) (clause 28)

where \( W = \) concentrated load
\[ P_c = \text{axial stress for struts as given in clause 30, Table 17 of BS 449 for a slenderness ratio of } \sqrt{3d/t} \]

\[ t = \text{web thickness} \]
\[ d = \text{clear depth of web between root fillets} \]
\[ B = \text{dispersed length of bearing load at neutral axis assuming } 45^\circ \text{ spread from edges of bearing load.} \]

i) Top cross-member: 305 x 102 x 46 kg/m channel

\[ W = 200 \text{ kN} \]
\[ \frac{\sqrt{3d}}{t} = \frac{\sqrt{3} \times 239.2}{10.2} = 41 \]
\[ p_c = 138 \text{ N/mm}^2 \]
\[ B = 250 + 305 = 555 \text{ mm} \]
\[ P_{ctb} = 781 \text{ kN} > W: \text{No stiffener required.} \]

ii) Box support member: 203 x 152 x 52 kg/m RSJ

\[ W = 200 \text{ kN} \]
\[ \frac{\sqrt{3d}}{t} = \frac{\sqrt{3} \times 133.2}{8.9} = 26 \]
\[ p_c = 144 \text{ N/mm}^2 \]
\[ B = 304 (\text{Minimum}) \]
\[ P_{ctB} = 267 \text{ kN} > W: \text{No stiffener required.} \]

iii) Box support member: 305 x 127 x 37 kg/m U.B.

\[ W = 200 \text{ kN} \]
\[ \frac{\sqrt{3d}}{t} = \frac{\sqrt{3} \times 264.6}{7.2} = 64 \]
\[ p_c = 122 \text{ N/mm}^2 \]
\[ B = 304 (\text{minimum}) \]
\[ P_{ctB} = 267 \text{ kN} > W: \text{No stiffener required.} \]
6) **Bolted Connection**

M24 close tolerance bolts used; 8 bolts per joint.

Allowable bearing stress = 300 N/mm²

Allowable shear stress = 100 N/mm²

Load/bolt = \( \frac{200}{8} \) kN = 25 kN

Minimum bearing plane thickness = 7.1 mm

Bearing stress = \( \frac{25000}{24 \times 7.1} \) = 147 N/mm²

Shear stress in bolt = \( \frac{2500 \times 4}{\pi \times 24^2} \) = 55 N/mm²

7) **High Strength Friction Grip Bolted Connection**

This method of connection was employed at the base of the vertical members to provide the facility to position and align vertically the two members. The connection was designed in conformity with BS 4604, "The use of high strength friction grip bolts in structural steelwork", Part 1, General grade.

Minimum ply thickness = 10.2 mm

Hence maximum bolt diameter = 20 mm

Allowable load/bolt = \( \frac{\text{Slip factor} \times \text{No. interfaces} \times \text{Proof load}}{\text{Load factor}} \)

\[ = \frac{0.45 \times 1 \times 144 \text{ kN}}{1.4} \]

\[ = 46 \text{ kN for M20 bolt.} \]

With 8 bolts per joint

Load/bolt = 25 kN.
Fig. D2
Out-of-plane imperfections were measured on a grid basis as shown in Fig.E.1. All dimensions are in mm.

Positive out-of-plane imperfections are those towards the opposite web (i.e. into the box).
Table E.1  RH1a : gd = 50

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Table E.2  RH1b : COURSE GRID USED OMITTING ALTERNATE POINTS : gd = 100

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### Table E.3  RH2A: Imperfections Not Recorded Along Diaphragm Grid Lines 0 and 12: gd = 75

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Table E.14  RH12: OWING TO STIFFENER, SPACING BETWEEN GRID LINES A AND B AND B AND C SET AT 55, SPACING BETWEEN C AND D SET AT 115, OTHERWISE gd = 75

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Although the expression

$$P_u = 5.5 t_w (M_f d_w)^{2/3}$$

gives good collapse load predictions for the RH girders and for conventional girders of similar proportions, subjected to similar very narrow patch loads, equally good predictions can be obtained from other formulae which have a wider range of application. Such a formula is given by Roberts [57] as

$$P_u = 0.5 t_w^2 \left[ \frac{E_0}{w} \frac{t_f}{t_w} \right]^{0.5} \left[ 1 + \frac{3c}{d} \left( \frac{t_w}{t_f} \right)^{1.5} \right]$$
REFERENCES


[63] The Design of Steel Bridges, Conference Discussion, Edited by H.R. Evans, University College, Cardiff, 1981, pp. 6.27-6.28 and 6.43-6.44.


