PIAGET'S GENETIC EPISTEMOLOGY: A THEORETICAL CRITIQUE OF MAIN
EPISTEMIC CONCEPTS

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Recognition... I'm pleased by it, of course, but it's pretty catastrophic when I see how I'm understood.

JEAN PIAGET

The great man who at any time seems to be launching some new line of thought is simply the point of intersection or synthesis of ideas which have been elaborated by a continuous process of cooperation.

JEAN PIAGET

From the amoeba to Einstein, the growth of knowledge is always the same.

KARL POPPER

How difficult it is! How much more difficult psychology is than physics.

ALBERT EINSTEIN
Foreword

My own view of the work of Jean Piaget is that it embodies an exciting, novel and challenging conception of human knowledge and that his theory possesses a comprehensive breadth of vision that makes it a major intellectual achievement. I mention my personal view at the outset, not in the expectation that it is necessarily shared by others but rather because it is my personal view, one which is not a view that others have to share to want to read further in this study.

As it happens, I started my doctoral research with ambivalent views about the achievements of Piaget's theory. I soon found, however, that my understanding of Piaget's theory was completely inadequate. The present study represents my attempt to repair this deficiency.

I have tried to let Piaget "speak for himself" in those sections which deal with his theory. Passages which tax the ingenuity of the reader abound in his writings. Sometimes, however, Piaget writes with an eloquence and pregnancy of thought and Margaret Boden's advice "read Piaget" well repays the effort expended in establishing the finer points of his theory. It will be apparent that my text is replete with references and there are two reasons why this is so. One reason is because many studies of Piaget are content barely to note the title of a book or paper. The other reason is because if my interpretation of Piaget's views is accepted, or rejected, it is important to know the precise source in the "library" of Piagetian writings. Readers who prefer to have at the outset a synoptic guide to the present study are invited to read the preview in section 1.1 or the review in 10.1.

Since this is a study of Piaget's theory, I have attributed to him alone views which are, in part, the product of work undertaken
with his colleagues and associates. My practice has convenience, rather than accuracy, in its favour and I extend my apologies to Piaget's co-workers if their efforts are passed over in silence.

A good many people, at different universities and conferences, have been kind enough to join argument with me: I sincerely thank them for their interest, suggestions and, most important of all, objections. Naturally, I have wanted to have the last word as to what my own views should be. There are, however, two people that I especially wish to mention for their contribution to my research.

Derek Wright supervised the whole of my research and I have benefitted from the inspiration, support and interest that he has generously given to me. It was his writings that initially aroused, a decade ago, my interest in psychology. Any present understanding that I have of Piaget's theory is now due primarily to the example of sympathetic elucidation combined with critical evaluation set by him. The misunderstanding that remains is, of course, mine. I thank him warmly and commend him to others.

I doubt whether I could have started or completed this research without the encouragement, patience and understanding of my wife. I thank her - what's more she did the typing as well!
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INTRODUCTION

(1.1) Aim of Study

What is the relation between logic and knowledge? The easy way to answer this question is to construe it as a question about the content of knowledge, for there would be general assent to the claim that nobody can have knowledge of that which runs counter to logic. Nobody can simultaneously know both of a pair of contradictory statements, for example, and so conformity to logic is an essential prerequisite of that which knowledge is knowledge of. There is a different way, however, in which the question might be construed, for the question might arise about knowing rather than the content of what is known. Taken in this way, the question is perplexing and difficult for it is not obvious whether and how logic has any application to the process of knowledge-acquisition.

Jean Piaget's theory of genetic epistemology construes the question about the relation of logic and knowledge in this second sense and its primary concern is to show the point of application of logical norms when a subject acquires knowledge. Both the problem and proposed answer have a forbidding degree of complexity as well as powerful fascination. The present study is a contribution to the continuing discussion of Piaget's theory of knowledge. Even though nothing in the sequel will disconfirm the claim that Piaget has made 'the greatest single contribution' (Wright & Taylor 1970, p.497) to the explanation of how intelligent thought arises, it is no easy task to determine the exact nature of that contribution and, in consequence, its value.

The central aim sub-divides into three objectives which can now be stated as a preview to the discussion in the chapters ahead:—

Objective 1.

The first objective is to show how Piaget's genetic epistemology has its source in philosophy and, in particular, in the theory of
Immanuel Kant.

Piaget's theory has its roots in the classical debate between rationalist and empiricist philosophers about the role of reason and experience in knowledge and especially in Kant's mediating theory of knowledge. In chapter 2 a brief review of this debate is followed by a statement of the distinctive aspects of Kant's theory, aspects which are taken over with modification by Piaget. Piaget's central objection to theories such as Kant's is illustrated, namely that it is an empirical question as to which logical norms are used by a subject in the acquisition of knowledge. Genetic epistemology is proposed by Piaget to be a study that provides an empirical resolution to philosophical problems. Grounds for the legitimacy of this proposal, together with a rebuttal of the objection that any such study is incoherent, are provided. Piaget's theory is schematically formulated as a theory which attempts to state empirically necessary conditions of a subject's knowledge of logical necessity which occurs by a process of equilibration that is necessarily constructive.

In short, the first objective is to show how Piaget's theory is one that has philosophical antecedents and psychological consequences. Objective 2.

The second objective is to identify and describe the central constructs employed by Piaget in the statement of his theory, namely action, epistemic subject, functioning, structure, equilibration, abstraction and generalisation.

In chapter 3 it is claimed that Piaget takes a copy theory of knowledge to be one that identifies factors within the object, based upon perception, or factors within the subject, based upon consciousness, to be the ultimate source of knowledge. Piaget rejects copy theories of knowledge which are taken to be flawed because of a failure of any copy exactly to match that which it is a copy of. Piaget refers to the
knowledge that arises from such a source as observable or figurative knowledge and argues that such knowledge is dependent upon the coordinatory or operative knowledge possessed by a subject. This latter type of knowledge is displayed in a subject's action-coordination and so it is action, not perception or consciousness, that is the ultimate source of knowledge. A subject is typically not aware of the coordinatory knowledge present in action and since such knowledge is logical in character, Piaget denies that developing subjects are aware of the logical principles used by them in acting on the world. The epistemic subject of knowledge has coordinatory knowledge, unlike the individual subject who has observable knowledge, arising from action.

In chapter 4 Piaget's stage-theory is discussed. A stage in the development of knowledge is to be characterised by the cognitive structure of a subject's actions. It is argued that whilst there may be logical connections holding between the descriptions of such stages, empirical study is required to establish how, as a matter of fact, development takes place. Piaget's broad conception of action is discussed and it is claimed that the three main stages of development correspond to a subject's gaining practical, conceptualised and propositional knowledge. In general, it is Piaget's claim that knowledge arises from a subject's action upon objects and that the resulting interaction is organised. Organisation, or functioning, of some type there must be, claims Piaget, though not any specific type of organisation. A cognitive structure is an instantiation of organisation and any structure can be described through some set of logical principles. An overview is presented of the salient features of two types of structure, namely action-schemes and operational structures. Piaget's assimilation theory of knowledge is the theory that a subject's knowledge is made possible by the use of an appropriate cognitive structure.

In chapter 5 Piaget's account of a structural change is discussed.
and the process of equilibration, whose presence is sufficient for that of the other necessary conditions of development, is taken to be an essential internal factor in development. A subject's actions always have an assimilatory and accommodatory character and equilibration is their reciprocal change leading to structural enrichment. The sub-processes of equilibration, leading from disequilibrium to disturbance to regulation to compensation and to upper bound equilibration, are taken to occur when a subject makes contradiction-mistakes whose rectification requires the use of a transcending structure, one that permits a subject to gain a better comprehension of the phenomenon generating that mistake. Piaget's account of reflective abstraction and of constructive generalisation are allied processes which occur when development takes place so that a subject's coordinatory knowledge is refined and extended. It is argued that Piaget's account of equilibration, taken as one that presents conditions that are necessary for development, is complementary to accounts which systematise the role of non-equilibratory factors in development.

In short, the second objective is to articulate the distinctive features of the theoretical constructs that Piaget deploys in his psychological investigation of how subject's gain knowledge. The conditions under which knowledge arises are conditions which are detectable by an observer but those conditions are not known by a subject who acquires knowledge as a condition of the subject's acquisition of that knowledge.

**Objective 3.**

The third objective is to evaluate Piaget's theory and test its explanatory scope by establishing its ability to withstand critical objection.

The pursuit of this objective is primarily, but not exclusively, confined to chapters 6 - 9. In chapter 6 the discussion turns to
questions of methodology and Piaget's method of critical exploration is taken to be one that requires the use of linguistically based methods for the attribution of operational knowledge. Two general arguments are stated in defence of this view. Firstly, conceptualised knowledge of a deductive character cannot be attributed to a subject who cannot justify a judgment, who cannot relate that judgment to the body of evidence that is its ground. Secondly, the twin features of universality and necessity, which are characteristic of deductive knowledge, require a subject's use of language for their adequate expression and so a subject must use language for an observer to be able to attribute such knowledge. A subject's explanations, and not merely judgments, are implicated in the attribution of operational knowledge. Two case-studies are discussed in detail and it is argued that Piaget's account of a subject's understanding of transitivity and inclusion is capacious enough to withstand critical attack. Central to this discussion is the distinction between a subject who makes a correct deduction and one who comprehends the necessity of that deduction. These abilities are apparently taken by Piaget and his critics to be concurrent but it is suggested that such abilities may be consecutively acquired.

In chapter 7 the discussion examines Piaget's account of social knowledge. It is argued that a subject's operational knowledge arising from the individual's own actions is concurrent with co-operative knowledge arising from social interaction, since it is Piaget's view that a social life is necessary for a subject's acquisition of knowledge. Two forms of objection are distinguished and rejected. The Social Objection is the objection that Piaget's account is biased in favour of physical to the exclusion of social cognition, whilst the Epistemic Objection is the objection that Piaget's account of cognition, whether physical or social, is biased in favour of individual to the exclusion of inter-individual factors.
In chapter 8 Piaget's account of infancy is taken to be inaccurate. It is argued that Piaget's conception of the infant's Copernican Revolution requires his commitment to some version of phenomenalism and as such development could not take place in the manner suggested by Piaget. It is suggested that Piaget should accept an alternative theory of perception, one which stresses its Intentionality, so that a unitary account of perceptually based knowledge may be stated. Three contradictions in Piaget's account of infancy are identified in his discussion of the infant's observable knowledge, the intentionality of action and representation and reflective abstraction. It is claimed that one way in which such contradictions can be removed is by allowing that an infant has the capacity to conceptualise the world, despite Piaget's own denial that this is so. Piaget's tacit admission that even limbless infants can undergo cognitive development is taken to be evidence for the claim that the infant subject does have the ability to represent the world and not merely to perform physical actions directed upon it.

In chapter 9 Piaget's account of equilibration and necessity is discussed. It is argued that an account which offers necessary conditions alone is one that cannot present a complete account of the development of knowledge. Piaget's account is not trivialised by this claim but its dependence on non-equilibratory accounts is a consequence of it. It is also argued that there is an uneasy tension arising out of Piaget's structuralist and constructivist interests. His account of a subject's understanding of necessity states that such understanding requires the presence of an operational structure; whilst Piaget's constructivist commitment requires him to identify primitive forms of necessity. It is claimed that this tension is not satisfactorily resolved in Piaget's writings. Finally, it is argued that the account of equilibration as a necessarily constructive process cannot be explicated by having recourse to models taken from the main types of logic but it is suggested that
disequilibrating factors are so construed by Piaget that the only way in which their transcendence might occur is by the construction of a new structure. It follows that, if development occurs, equilibration is essentially constructive. It follows, from the first objection presented in this chapter, that Piaget's equilibratory account cannot state the conditions under which development actually does occur.

In short, the third objective is to test the strength and weakness of Piaget's theory. It is argued that the theory is strong enough to withstand objection based upon studies arising from the rival epistemological and psychological theories here discussed but that the theory has internal weakness since it is both contradictory and incomplete.

A final verdict on Piaget's theory is stated in chapter 10. It is claimed that Piaget's theory addresses itself to important questions and that the answers proposed by the theory are distinctive, fruitful and significant. Piaget's own ability to synthesise and put to novel use ideas taken from a wide variety of disciplines is worthy of special note and the inter-disciplinary character of Piaget's theory resists its atomistic dissection. Piaget's own verdict, that his theory presents the skeleton of the answer to questions about the growth of knowledge, is accepted.

(1.2) Methodology of the Study

Two methodological questions arise at this point. The present study is a study of Piaget's theory. Why is it important to study a theory rather than its application to the facts of development? Secondly, given that such a study can be of value, what are the distinctive features of the present study?

To ask why Piaget's theory is worthy of study presupposes that Piaget actually has a theory. Yet Piaget (1978b, p.191) denies that he has an acceptable theory and he has repeatedly (Piaget 1931, p.146;
Stress the foremost critic of the work of Piaget, which consists in a sequence of theoretical trials and revisions. Clearly, an author's verdict on his own work carries weight. Yet the conclusion that Piaget has not stated a theory is premature. Piaget is certainly taken by others to have a theory, as even the cursory study of book-titles (Brainerd 1978a; Brown & Desforges 1979) shows. Moreover, Piaget is proud of the fact that there has been a constant feature of his work that has been dominated by an attempt to understand the nature of real development in terms of a theory of equilibration (Inhelder et al. 1977, pp.6, 22). What, can, therefore, be claimed is that "Piaget's theory" is an attempt to explain certain problems about the development of knowledge by use of a family of constructs whose articulation and deployment has been the subject of revision throughout Piaget's work.

Why, then, are Piaget's theoretical constructs worthy of study? Several reasons readily spring to mind:

1: Philosophers of science, despite their marked differences of opinion on other matters, display unanimity about the role of theory in science. Thus it is claimed that no observation can ever be made without a theory (Popper 1968, p.59; Churchland 1979, pp. 14-5); that any scientific problem is a problem only in the context of some theory (Laudan 1977, p,15); that since no theory can ever agree with all the known facts, a choice can always be made either to abandon a theory or to abandon an observation (Feyerabend 1975, pp.55, 168); and that the abandonment of a theory (or an observation) is possible only when an alternative theory is available (Popper 1968, p.59; Kuhn 1970, p.77; Lakatos 1972, p,130). It is theory, not fact and observation, that has a pre- eminent place in science.

2: Philosophers of psychology are agreed that psychology is still
9. Koch (quoted in Wetherick 1979, p.89) states that psychology always has been, and still is, in a state of crisis; Boden (1977, p.396) declares that psychology is still in a preparadigmatic state; von Wright (1971, pp.5-6) and Gauld & Shotter (1977, pp.4-5) analyse the antithetical claims of mechanistic and hermeneutical approaches in psychology. But if this is so, then psychology should be characterised by the proliferation of theories any of which, at this point in time, might provide a proper basis for the science of psychology. That not all psychologists have realised that this is so is evident, it is claimed (Chomsky 1972, p.xi; Allport 1975, p.152; Bannister 1977, p.9), from the tendency of many psychologists to engage in research that is phenomena-driven rather than theory-based. From the perspective presented here, such a tendency can only be regretted.

3: Psychologists are willing to admit that much psychological research must start from the work of Piaget, for example in developmental research on infancy (Bremner 1980, p.41), childhood (Winer 1980, p.318) or adolescence (Neimark 1979, p.61). Some (Halliday 1981, p.385) even say that Piaget's theory is still the best theory available to developmental psychologists. And even Piaget's critics (Brown & Desforges 1979, p.162; Boden 1979a, pp.154-5) acknowledge the rich and comprehensive scope of Piaget's work.

4: Piaget's empirical findings have been accorded immense scrutiny but the tendency has been to concentrate upon those empirical findings to the exclusion of the theoretical base out of which it is Piaget's claim that they have arisen. Piaget (1963, p.viii; 1977f, pp.88-9) has deplored this tendency and the consequent failure to integrate his theoretical approach into psychological theorising has been noted by others (Furth 1969, p.ix; Halford, 1972, p.190; Rotman 1977, pp. 12-3). Since 'with Piaget the empirical is never separated from the theoretical' (Inhelder 1979, p.ix), such an approach to the work of Piaget can only
lead to its misinterpretation: the serious study of Piaget cannot
divorce the testing of empirical fact from the interpretation of the
theoretical constructs associated with them.

5: Piaget (1963, p.vii) and commentators (Brainerd 1978a, p.3)
agree that Piaget's theoretical constructs resist easy comprehension,
One reason for this difficulty is that Piaget sometimes deploys his
constructs in the absence of explicit definition; another is that
Piaget sometimes clarifies a construct through a series of works with
the result that a later instalment substantially modifies an earlier
one, but without formal notification on the part of Piaget. It is,
therefore, no easy task to determine the intended meaning of those
constructs.

In short, it is admitted that the role of theory is all-important
in science since research remains undirected in its absence. It is
admitted that Piaget does have a theory and that it is taken to be of
current value in psychological research. But the core constructs
utilised by that theory are often ignored by other researchers or are
given radically different interpretations. Thus whilst Piaget's theory
is and must be subjected to the normal methodological requirements
(replication, falsifiability), the theory remains in a state of confusion
to the extent that it is given divergent interpretation. It is,
therefore, not unreasonable to devote a study specifically to the
consideration of the nature and role of the core constructs at the heart
of Piaget's theory.

It must be stressed that the present study is resolutely non-empirical
in character. This study does not seek to provide new factual evidence
to test Piaget's theory nor to impugn the factual evidence provided
by others. Still less does it attempt to provide an alternative
theoretical perspective for the study of development. Rather, the
present study has Piaget's theoretical constructs as its primary focus
and it is the discussion of their inter-connection, consistency and completeness that is the pre-occupation of the chapters ahead. What are the main constructs used by Piaget? What is the role of each within the context of his work? How does one such construct combine with others? Are they given one or a series of uses? Are such uses consistent? And do they offer a complete theoretical perspective? Such are the questions that are raised in the present meta-study of Piaget's theory.

What, then, are the special features of the present study? Two features may usefully be identified at the outset and concern the Piagetian corpus of writing and its English translation. Piagetian Corpus.

Piaget is a prolific author and an attempt has been made to use early (Piaget 1920) as well as recent (Piaget 1980a) writings. There are, however, two limitations on this use. Firstly, no attempt has been made to pursue interesting historical questions which trace the development that has occurred in Piaget's own use of his central concepts. The principle adopted has been to use Piaget's later writings as the vantage-point from which his earlier writings may be surveyed, especially when the latter provide an anticipation, or clarification, of the former. Secondly, no attempt has been made to survey the whole of Piaget's theory. Thus there is no discussion of Piaget's biological, logical, moral, social or educational writings. More importantly, no discussion is provided of the growth of human scientific knowledge, even though Piaget's epistemological position takes such growth to be an extension of human cognitive development. It is claimed, however, that the central constructs taken for discussion are just that, namely constructs central to Piaget's theory of the growth of knowledge. No doubt the constructs chosen for discussion are not of sole importance to the theory. Nonetheless, they raise a formidable range of problems about their nature and value.
Translation.

An attempt has been made to use French in preference to English editions of Piaget's writings. The reason for this preference is that mistranslation, which is by nature insidious, is ubiquitous in English editions. Unfortunately, lack of availability of a complete set of French texts has prevented their systematic use. This defect of the present study, one which it shares with almost all English studies of Piaget, may be offset by the fact that an attempt has been made to identify instances of mistranslation by the adoption of the following procedure:

1: a translation has been offered where no standard translation exists;
2: a standard translation has been used when it has not been found to be defective;
3: a translation of a French edition, coupled with a reference to its standard equivalent (Piaget 1967a, p. 33/1971a, p.14) has been offered where the standard translation has minor defects;
4: a translation of a French edition, coupled with a * reference (Piaget 1975a, p.24/*1978b, p.18) to its standard equivalent, has been offered where the latter has serious defects.

No claim is made that all of the English editions used have been checked for instances of mistranslation. It is claimed that all * references are such instances and their detailed discussion is presented as Appendix A.

Finally, there are numerous, and recent, commentaries dealing with the theory's several aspects. Some examine so as to accept (Ginsburg & Opper 1979) or to reject (Brainerd 1978a; Brown & Desforges 1979) the empirical aspects of Piaget's theory. Some (Boden 1979a) provide an overview of Piaget's work by discussion of its empirical and theoretical aspects. Some (Rotman 1977) submit Piaget's theory to general critical
discussion. The present study duplicates the work of none of these, though is naturally dependent upon them. Rather, it has strong affinities, in aim and approach, to the study carried out by Rita Vuyk (1981a, b). Unfortunately, the present study was substantially complete prior to the publication of her study and this fact accounts for exegetical duplication. Though the two studies are independent, there is substantial agreement about which constructs are Piaget's main constructs as well as the features that are applicable to each. Three differences, however, may be noticed. Firstly, Vuyk's study is more comprehensive than the present study since the former discusses very recent additions to Piaget's theory (procedures, correspondences, possibilities, dialectic). Secondly, Vuyk's study provides a less than ample discussion of many of the constructs, for example structure, functioning, reflective abstraction and consciousness, in contrast to the present study which offers a more detailed discussion. Thirdly, whilst Vuyk is content to report on Piaget's theory and criticisms of it, the present study is more concerned to evaluate the issues which it raises. The two studies, therefore, are complementary.

The present study has been undertaken in the modest assumption that a fertile theory is one that generates attendant commentary and criticism and that proliferation of interest is an indication of its intellectual worth. The work of Jean Piaget contains just such a theory.
PROBLEMS OF EPISTEMOLOGY

(2.1) Introduction

Epistemology is the theory of knowledge and is traditionally concerned with problems of knowledge: problems about the definition of knowledge, problems about the relations between cognitive concepts, problems about whether there can be knowledge at all. Piaget's interest in epistemology is distinct from all of these since he is concerned to establish the empirical conditions under which an individual can attain knowledge. Knowledge is acquired. What, then, are the conditions under which it is possible for the acquisition of knowledge to take place? The central difference between philosophical epistemology and Piaget's genetic epistemology is that the former is taken to be a non-empirical, unlike the latter which is taken to be an empirical, study of the problem of knowledge. Thus the first question for discussion is the question of why Piaget does attach importance to genetic epistemology. The discussion is in two parts. In Section (2.2) the debate between rationalist and empiricist philosophers is reviewed and special attention is given to the mediating position adopted by Kant. In section (2.3) Piaget's central objection to philosophical epistemologies is stated and illustrated, namely that an adequate epistemology is one that is compatible with the facts of development of knowledge. A philosophical defence of genetic epistemology is given to offset the charge that it is incoherent. The final sections contain a discussion of a second question, namely what is the nature of Piaget's theory? The main features of Piaget's thesis are sketched in section (2.4) and his differing use of the notion of necessity is indicated in section (2.5). Finally, in section (2.6), a preview of the subsequent four chapters, in amplification of the outline of Piaget's thesis presented in this chapter, is provided.
The classical debate about the problem of knowledge concerns the relative contributions of reason and experience in knowledge. This controversy is of more than historical interest and in this section a sketch of its main outlines is presented. Consider, then, classical versions of empiricism and rationalism, together with the mediating position of Kant.

The classical problem of knowledge is the problem of the source of knowledge. A typical empiricist answer is given by John Locke:

let us suppose the mind to be, as we say, white paper, void of all characters, without any ideas; how comes it to be furnished?....To this I answer, from experience ....our observation, employed either about external sensible objects, or about the internal operations of our minds....is that which supplies our understanding with all the materials of thinking (Locke, 1881, II,1.2).

On this view, all knowledge arises from experience and no knowledge arises from any other source. Further an individual acquires knowledge from experience by means of observation, which may be of two forms. External observation consists in the sense-perception of physical objects and internal observation is the introspection of the contents of an individual's own mind. Thus Locke would say that to have knowledge is to have an idea and an individual acquires new knowledge (ideas) in cases where perception and introspection take place and in no other cases. The attraction of this account is, no doubt, its simplicity: all knowledge arises by a unitary process of observation and consists in the production of ideas in the individual's mind.

Other empiricists, of course, modify this account. Thus David Hume was led to distinguish two types of mental occurrence in place of Locke's one type:

all the perceptions of the human mind resolve themselves into two distinct kinds, which I shall call IMPRESSIONS and IDEAS. The difference betwixt these consists in the degree of force and liveliness with which they strike upon the mind (Hume 1888, I.I.I.).
Moreover, Hume (1902, sect.18) was anxious to invoke principles other than observation in the production of ideas and insisted that principles of association were important as well. Thus on the empiricist view, there is one source of knowledge (experience) and knowledge (ideas, impressions) arises by the presence of unitary cognitive processes (observation, association). The assumption of empiricism is that all knowledge arises in essentially the same way.

The rationalist position is one that stresses the role of reason rather than experience in knowledge and typically claims that an individual has knowledge only because of the possession of innate ideas, ideas that the mind has (temporally) prior to experience with the world. The main ground on which such a view rests is the claim that experience under-determines the universality of general concepts and the necessity of logical principles. On a rationalist view, to have knowledge is to have an idea and the role of experience consists in the "triggering" of an innate idea that corresponds to it.

Experience is necessary, I admit, in order that the soul... take notice of the ideas which are in us...(Opponents claim that) there is nothing in the soul which does not come from the senses. But you must except the soul itself and its affections. Nihil est in intellectu, quod non fuerit in sensu, excipic: nisi ipse intellectus. Now the soul comprises being, substance, unity, identity, cause, perception, reason, and many other notions which the senses cannot give (Leibniz 1949, p.111).

The dispositional nature of an innate idea, one that is actualised only by way of experience, is emphasised by René Descartes:

in some families generosity is innate, in others certain diseases like gout or gravel, not on this account babes of these families suffer from these diseases in their mother's womb (Descartes 1931a, p.442).

Someone who has an innate idea has a potentiality to form that idea on the occurrence of a relevant stimulus in experience and it is only because of the prior possession of an innate idea that knowledge can arise. On a rationalist view knowledge arises because a person inherits a certain cognitive constitution (innate ideas) and with the onset of
experience forms knowledge of the world. The assumption of rationalism is that it is prior possession of some cognitive constitution that makes possible the possession of knowledge.

It was Immanuel Kant who stated the classical resolution of the problem of the relative contributions of reason (innate ideas) and experience in knowledge (Kant 1933, Axxvii). Since Piaget (1972d, p. 57; 1980, p. 150) has stressed that his theory is Kantian in essence, it will be convenient to review in some detail its main features, namely Kant's view of (1) experience, (2) perception and thought, (3) schematism of the understanding, (4) subject of knowledge and (5) knowledge of inner and outer experience.

(1) Experience

Kant accepts that knowledge is possible and took Euclidean geometry and Newtonian physics to be pre-eminent instances of actual knowledge. Experience does play a role in a subject's attainment of knowledge since

there can be no doubt that all our knowledge begins with experience....it does not follow that it all arises out of experience (Kant 1933, B1).

Kant's claim is important since he distinguishes two senses of priority: temporal and logical. Kant denies that there can be any knowledge that is temporally prior to experience but he affirms that there can be knowledge which is logically prior to experience. Kant rests his case upon the claim that if X is logically prior to Y, what is excluded is the possibility of Y's occurring earlier in time to X but what is not excluded is the possibility of X and Y's occurring at the same point in time. That is, the logical priority of X over Y does not exclude the co-occurrence in time of X and Y.

In consequence of this distinction, Kant distinguishes those concepts that are abstracted from experience from those that are logically prior to experience. The latter are called a priori concepts and
include the concepts — Kant (1933, B60) referred to them as intuitions — of space and time as well as a further twelve concepts, such as unity, negation, causality, possibility, which he extracted from the prevailing Aristotelian logic. The knowledge which a subject has by the application of such concepts to specific, sensory cases is a priori knowledge — knowledge that arises with experience but is also prior to it. Kant identifies two features of a priori knowledge:

necessary and strict universality are... sure criteria of a priori knowledge (Kant 1933, B4).

For example, to have an experience of an object is to experience it as having spatial, temporal and causal properties of some sort: that can be known by a subject a priori since any object has such properties and any object must have them to be an object at all. Of course, the particular spatial, temporal or causal properties of that object cannot be known a priori, since a subject can know what those particular properties are only through experience of that object (Kant, 1933, A207). A priori concepts comprise the form of experience since they are the set of organising principles that make experience what it is and so are conditions for all possible experience (Kant 1933, A20).

(2) Perception and Thought

Kant does not deny that perceptual activity is necessary for the occurrence of knowledge, nor that the deployment of concepts is also necessary. What is distinctive about his view is the claim that knowledge can arise only when perceptual and conceptual elements are inter-linked.

Thoughts without content are empty, intuitions without concepts are blind...the understanding can intuit nothing, the senses can think nothing. Only through their union can knowledge arise (Kant 1933, A51).

Kant refers to sense perception as intuition and so his claim is that perception in the absence of the application of a concept will be blind and so in such a case a subject would know absolutely nothing at all.
Similarly, a subject who deploys a concept in the absence of perceptually mediated elements is a subject whose thought will be empty and so will be in no position to have an objective experience of the world. What is important about Kant's view is that previous philosophers had seemed willing to deny it - the Empiricists in their suggestion that someone who has an impression (Hume) may have knowledge of an object without forming some concept of it, the Rationalists in their suggestion that there are innate ideas that are formed prior to sense-experience. Both suggestions are rejected by Kant for, on his view, a subject may have knowledge of some object only if the subject has a sense-experience of that object and only if the subject applies some concept to that object in experience of it. For Kant, perception and thought are both essential elements in the attainment of any knowledge.

(3) Schematism of the Understanding

Kant takes his view to lead to a problem, that of showing how an a priori concept (which is non-sensory) can be applied to a sense-experience (which is non-conceptual). That is, Kant assumes that some mediating principle must be present which has properties of both perceptual and conceptual elements on the grounds that the absence of such a mediator would be incompatible with their joint inter-play. Such a mediating principle is a schema.

Kant defines the schema of a concept as:

a universal procedure of imagination in providing an image for a concept (Kant 1933, A140).

A schema is not an image. Mindful of Locke's (1881, II.XI.9) difficulty over an abstract idea of triangularity, Kant observes that no image of a triangle could attain the universality of the concept of triangularity, which applies to all triangles and not just (as would be the case with an image) to a right-angled triangle, or an obtuse-angled triangle, or whatever. Rather, a schema is a rule by whose use a subject may
form an appropriate image. Thus someone who possesses the concept, and associated schema, of triangularity may recognise that such and such a figure is an equilateral triangle, or may form an image of a scalene triangle just because the individual can deploy that concept in an indefinite range of cases. Kant is willing to concede that his notion of the schematism of understanding is obscure (A141). What does seem clear, however, is the distinction that he tries to draw by its means, namely the distinction between a subject's possession of a concept and the application of it or what is understood by it.

(4) The Subject of Knowledge

A further claim made by Kant is that knowledge is in all cases the knowledge of a subject. Three points may be noticed in his support of such a claim. Firstly, no act of (sense) experience itself imposes connections upon that which is experience since the application of a set of (a priori) concepts is also required. But secondly, the elements within that experience must be combined (or connected, or organised) in one act of combination, for if they were not so combined they would be experienced in successive and distinct acts. The claim here is not just that elements within the experience must be themselves experienced as unities but also that the experience itself should be a unity, should be one experience (Kant 1933, B130-1). It follows, thirdly, that a condition of this being possible is that there should be a subject who has that experience and who can become conscious of it. Kant states this claim as follows:

only in so far as I can grasp the manifold of representations in one consciousness, do I call them one and all mine (Kant 1933, B134; Kant's emphasis).

By 'the manifold of representations' Kant means the elements within the experience and his claim is the tautology (B138) that all of a subject's experiences are his - and not someone else's - experiences.
And for this to be so, a subject's experience is his own only because he can be aware of it, since it is absolutely necessary that in my knowledge all consciousness should belong to a single consciousness, that of myself (A118).

For Kant, being a subject of consciousness and being a possessor of knowledge are interdependent notions: neither is intelligible in the absence of the other.

Such a claim might seem excessively strong since Kant might be taken to claim that a subject should be conscious of what he knows in every case where he has knowledge — for example in the following statement of his main conclusion:

it must be possible for the 'I think' to accompany all representations; for otherwise something would be represented in me which could not be thought at all, and this is equivalent to saying that the representation would be impossible, or at least would be nothing to me (B131).

What Kant seems to be saying is that someone who makes a judgement, for example, takes his experience to be an experience of a cat on a mat, must be aware of that judgement and so must simultaneously be aware of what his experience is. In fact, this is not Kant's claim. Firstly, Kant claims that it must be possible, and not that it must be actually the case, for the 'I think' to accompany all of my judgements. Secondly, Kant makes it clear that the 'I' is itself completely empty and so must not be taken in the sense of a Cartesian ego, a mind that is conscious of all of the ideas occurring within it. Kant's claim is not the experiential claim that there must be something within consciousness corresponding to the 'I' but rather the formal claim that any such item must be an item within one consciousness:

consciousness in itself is not a representation distinguishing a particular object, but a form of representation in general, that is, of representation as it is to be entitled knowledge (Kant 1933, A346).

It is for this reason that Kant draws a distinction between the empirical
subject, who is conscious of this or that experience, and the pure or transcendental subject, who has all of his own experiences (B132-3). Thus on Kant's view the necessary and sufficient conditions (B137) of there being objectivity and truth is that any experience, putatively objective, must be attributed to a transcendental subject whose impersonal existence as a formal abstraction is required for that experience to be the objective experience that it is.

(5) Knowledge of Inner and Outer Experience

A consequence that must not be drawn from Kant's view is that a subject's inner experience has a priority over his outer experience, a view endorsed by Descartes. Kant claims that there is a strict interdependence between these two types of experience since

consciousness of my existence is at the same time an immediate consciousness of the existence of other things outside me (Kant 1933, B276).

What Kant wishes to deny is that a solitary subject of consciousness, for example a Cartesian ego engaged in a comprehensive doubt about the existence of anything other than itself, could be a subject who possessed knowledge. Again, Kant would reject the view of a Humean sceptic who claimed that his own impressions and ideas enjoyed a primacy over the physical objects that were their causes. On Kant's view, a subject may have inner experience - may form sense-impressions or ideas based upon them - only because and to the extent that he has outer experience. And a subject has outer experience in cases, where he applies his set of (a priori) concepts to the sensory content that he has.

There is no claim that the review has exhaustively stated Kant's position; no claim that Kant's arguments have been analytically discussed; and no claim that Kant's position is acceptable to others. Recent evaluations of Kant's epistemology include sympathetic commentaries (Walsh 1975; Walker 1978) as well as more hostile critiques (Bennett
1966; Strawson 1966). It is clear that Kant's position is still a fruitful, epistemological source. It may be claimed, however, that several strands of Kant's position have been indicated and so his view may be understood, in outline form at least.

Now the reason for wishing to present Kant's position is because of the influence it has exercised over Piaget. It was stated that Piaget himself freely acknowledges that influence and it is also evident that commentators on Piaget have accepted this to be the case (for example Rotman 1977, p.28; Hamlyn 1978, p.47; Boden 1979a, p.134). Perhaps the best way to make this point, however, is to accept the claim:

Piaget would say that what he was really doing... was re-examining the whole question of the Kantian categories. This re-examination formed for him the basis of a new discipline that he called genetic epistemology (Mays 1967, p.306).

What Kant attempted to do was to provide a satisfactory resolution of the problem of knowledge, which he took to be the problem of showing how there could be knowledge at all, and it was evident that Kant saw his task to be that of showing both how there could be an experiential basis to knowledge and yet how knowledge may have the twin features of being universally and necessarily valid. But what must also be stressed is that Piaget's theory represents a modification of that philosophical epistemology that is its starting-point. What will be made explicit, in the subsequent three chapters, is the extent to which Piaget takes the central concepts used by his predecessors - those of experience, abstraction, schematism, subject and knowledge - and uses them for his own theoretical purposes. Why and how does Piaget, then, modify Kant's theory?

(2.3) Piaget's Objection to Philosophical Epistemology

The aim of the present section is twofold, firstly, to state and illustrate Piaget's objection to Kant's theory and, secondly, to rebut the charge that genetic epistemology is radically incoherent.
Piaget is well aware of the classical debate about the problem of knowledge and is also aware that, pace Hamlyn (1978, p.47), that an innate idea is not an a priori concept since he explicitly distinguishes one from the other (Piaget 1971a, pp.53, 269). Nonetheless, he is inclined to classify Kant's theory as being a rationalist theory; or rather, that Kant's theory fails to be an adequate mediating position through its failure to notice that the conditions for the presence of knowledge in a developed subject are different from those that are applicable to the presence of knowledge in a developing subject. In brief, Piaget's objection is that it is one thing to state that there must be some sort of cognitive constitution possessed by a subject for knowledge to arise and quite another to state that one and the same type of cognitive constitution is possessed by all subjects in the acquisition of knowledge.

Consider an example taken from Kant who states that

the universal, though merely negative, condition of all our judgements in general....is that they be not self-contradictory (Kant 1933, A150).

Take any judgement in which knowledge is expressed, then that judgement is not and cannot be self-contradictory. Kant's claim, then, is

(1) necessarily, any judgement expressing knowledge is not self-contradictory.

Clearly, Kant is right; how could someone possess knowledge if the judgement expressing that knowledge was self-contradictory? Thus (1) may be taken as a valid contribution to philosophical epistemology since it is necessarily applicable to any judgement expressing knowledge at all. Moreover, the grounds of (1) are non-empirical. Kant (1933, A53-7) expressly states that his epistemological concern is philosophical and not psychological and this statement must be accepted. It does, however, leave open the question of which of the following is the case:

(2) necessarily, any knowledge that a subject acquires conforms to (1);
(3) necessarily, any knowledge, which conforms to (1), can be acquired by a subject.

(2) is the weaker claim and states that if a subject acquires knowledge, that knowledge cannot be expressed in a self-contradictory judgement. It is clear that (2) makes no claim about the nature and extent of a subject's knowledge at different developmental points. By contrast, (3) is the stronger claim which states that there are no limits on the range and type of knowledge available to a subject, except for those limits stated in (1).

Someone might object that (3) misrepresents the acquisition of knowledge which is not to be described in this way. For (3) is incompatible with

(4) Possibly, there is some knowledge, which conforms to (1), which cannot be acquired by a subject.

It is clear that an empirical approach is required to justify (4), for (4) has a contingent modal status and the extent of a subject's knowledge-acquisition can only be established by recourse to the facts.

That Piaget is committed to (4) can be seen in two ways. Firstly, Piaget (1977d, p.5) asserts that philosophical epistemology is deficient just because of its failure to verify what cognitive instruments are, as a matter of fact, at a subject's disposal in the acquisition of knowledge and he gives several examples of this neglect. For example, it is (in part) a factual question whether a subject's knowledge of necessity arises from language or from some other source (Piaget 1966, p.148; 1969e, p.xii) or whether knowledge arises because of the presence of Platonic forms, Kantian categories or Husserlian essences (Piaget 1971b, p.9). Piaget expressly states that the traditional philosophical problem, "How is knowledge possible?", can be satisfactorily answered only by a scientific epistemology. Secondly, Piaget provides numerous factual studies, covering the whole range of development, to illustrate his claim that subjects are limited in their knowledge-acquisition at different developmental points. Consider just three examples:
(A) The infant's knowledge of object-permanence undergoes development. Thus an infant, at 5 months, does not follow the visible fall of an object with corresponding eye-movements. At 10 months, an infant searches for an object that is hidden at place A but reverts to A if the same object is hidden, in full view, in place B. At 19 months, an infant searches for an object that has been invisibly hidden under another object (Piaget 1954, pp.14, 51, 79). It follows from such reports that an infant's knowledge of the world is radically different from an adult's knowledge of the world since the young infant does not apparently know that objects continue to exist when none of a subject's actions are directed upon them.

(B) Children, aged 5-7 years, are shown a series of bottles each containing a liquid and are asked questions such as "Can a bottle be half-full and half-empty?". Subjects typically deny that this is so: a bottle that is half-full cannot also be half-empty (Piaget 1980b, pp.231-33). The deficiency attributed by Piaget to such subjects is the inability to understand that the properties of being half-full and being half-empty are compatible. The subjects may learn the relevant words. Yet they evidently do not understand the properties to which those words refer since 'half-full' is taken to mean a glass that is almost full, whilst 'half-empty' is taken to mean one that is almost empty. But why should young subjects construe these words in this way? It is Piaget's contention that it is due primarily to an inability to understand that two halves equal, and have to equal, one whole. That is, it is due to an inability to understand that a bottle can simultaneously have the property of being half-full and half-empty and so that such properties are not contradictory.

(C) The converse mistake is also made: subjects take to be non-contradictory that which is contradictory. Subjects, aged 5-10 years, are presented with seven discs, A - G, whose size they are invited to
compare. Only adjacent comparisons are possible and subjects take such pairs to be equal in size. When, however, an A/G comparison is made, subjects find that these discs are of unequal size. The point of the experiment is as follows. The discs are constructed so that a difference in size is detectable only on the A/G comparison. Thus a subject who uses observable information alone claims that adjacent pairs are of equal size, which is falsified when the A/G comparison is made. It follows that the conditions prevent such a subject from making a compatible set of responses in this case and so that coherence can be introduced only by a subject who has the inferential ability to do this. Four typical responses may be noticed:-

(1) The youngest subjects claim that A = (B, C, D, E, F) = G but deny that A = G; which is contradictory. Evidently such subjects do not understand transitivity and so are not embarrassed by their incompatible claims. Such subjects do not know that one response is incompatible with another.

(2) Subjects may claim that A = (B, C, D), that G = (C, D, E, F) and deny that A = G. This response is an improvement on the previous one since there is a lesser degree of contradiction - the mediation of only discs C and D, not B, E and F, generates the contradiction.

(3) Subjects may claim that A = (B, C), that G = (D, E, F), that C = D and deny that A = G. Once again, improvement occurs since no disc is taken to be identical to both A and G. Yet incoherence still arises through the identification of C with D.

(4) Subjects may finally deny that A = B = C = D = E = F = G. In this way subjects make a coherent response to the experimental conditions. By virtue of the understanding of transitivity subjects deduce that the discs are of unequal size and that there must be non-observable differences that produce non-identity (Piaget 1980b, pp.3-20).

Certain features of these reports must be noticed. Firstly,
they are all empirical and report the behaviour of actual subjects in experimental situations. Secondly, even if the claims made in (A), (B) and (C) are vulnerable to further, empirical criticism — for example that Piaget underestimates the competence of the young infant, as Piaget (1976c, p.224) admits — the point to notice here is the empirical nature of the claims, rather than their correctness.

Thirdly, (A) — (C) cover intellectual development in infancy and childhood and the examples are simply examples taken from the massive range of studies carried out by Piaget. Fourthly, a clear example of development is provided by (A). The youngest infants are reported to be uninterested in moving objects and initial search behaviour is confined to the retrieval of visibly disappearing objects. Only at the later stages of infancy is a subject reported to search for an invisibly hidden object. There is a clear dependence of the later instances of these on earlier instances. Thus, fifthly, Piaget's main claim is that, as a matter of fact, a subject's understanding of the world is marked by the serial occurrence of different types of understanding. It is logically possible that a subject should be able to understand object-permanence and should be able to understand contradiction at any developmental-point. It is Piaget's claim that, as a matter of fact, this is not the case.

Thus Piaget's position may be stated as follows. The empiricists are correct to maintain that experience is an indispensable ingredient in the formation of knowledge: a subject who lacks experience is one who is precluded from acquiring knowledge (Piaget 1977d, p.70). And the rationalists are also correct to maintain that the ipse intellectus (Leibniz 1949, p.111; Piaget 1953a, p.2) is not itself a part of experience but is rather that which makes experience what it is. It follows that there is a set of prior conditions of the intellect by the presence of which knowledge is acquired. Kant was correct to maintain
that such conditions arise with experience, and not before it, but he was incorrect in his supposition that the same conditions of the intellect are applicable to all subjects. In particular, even if it is the case that a subject may acquire certain and indubitable knowledge at the terminal stages of development, this is, as a matter of fact, not so at the initial stages (Piaget 1953a, p.3; 1972d, p.57). What is, therefore, needed is a study of the growth of knowledge, one that does not deny the a priori element in intellectual development but one which establishes its different instantiations. Genetic epistemology is taken by Piaget to be just such a study.

Even the bare outline above suffices to show the distinctness of genetic from philosophical epistemology. Piaget makes four main claims relevant to their relation. Firstly, philosophy is the source for many of the problems that the scientist attempts to investigate and he tellingy admits his personal debt with regard to the fact that almost all the questions that we have studied in psychology had a philosophical inspiration (Piaget 1975b, p.18 - my translation).

Secondly, philosophy and science are distinguished neither by aim nor by content, or the problems each tries to solve, but rather by their methods. Philosophy is a search for solutions to problems but a search which fails because the methods appropriate to philosophy do not permit the attainment of an adequate solution. On Piaget's (1972d, pp.11-2) view, philosophy uses neither deductive nor inductive methods. Thirdly, philosophy asks questions such as "What is number?" or "What is life?" unlike the mathematician or biologist who pose questions such as "What sort of numbers are there?" and "How can living organisms be classified?" (Piaget 1977d, p.97). Thus, fourthly, philosophical epistemology poses questions such as "What is knowledge?", unlike genetic epistemology which provides answers to questions such as "How does knowledge arise?". In order to answer such a question, it is not enough to reflect upon
the conceptual problems raised by scientific progress and revolution (Piaget 1972d, p.46) since the use of scientific, i.e. deductive and inductive, methods is necessary as well. Genetic epistemology starts from philosophical epistemology but then proceeds to be a scientific competitor to it.

That Piaget's epistemological problems have a philosophical source is barely noted by some commentators (Flavell 1963, p.38) and roundly condemned by others (Brainerd 1978a, p.16). Notwithstanding the fact that recent commentaries on Piaget's theory have been anxious to stress this aspect of his genetic epistemology (Rotman 1977, p.31; Boden 1979a, p.88) it is sometimes argued that Piaget's theory is incoherent just because it aspires to use scientific methods in an attempt to solve problems which philosophers have traditionally discussed.

It seems to me that Piaget's theory is a blend not only of the empirical and the conceptual (which would be both acceptable and inevitable), but of the empirical and the philosophical. While empirical investigations may throw up suggestions for the philosopher and vice versa, and while these suggestions may well be valuable, I am still inclined to think that a theory that rests directly upon both empirical and philosophical considerations must have a degree of incoherence. This I take to be the case with Piaget's theory (Hamlyn 1971, p.23).

Thus Hamlyn's claim is that Piaget's theory is in principle objectionable and his more recent discussion of Piaget's work (Hamlyn 1978, pp.43, 147) repeats, in a more guarded fashion, the same complaint. It will be worthwhile to show why Hamlyn is incorrect in stating such a complaint so as to make clear how and why Piaget's epistemology is dependent upon, but is also an improvement upon, the epistemology of Kant.

The first point to make clear about Hamlyn's objection is that he provides no criterion for determining what is and what is not a philosophical question as others (Toulmin 1971, pp.26-8, 38; Russell 1978, p.197-8) have pointed out. Cognitive concepts, for example
perceptual parallax (Smith 1981c), evidently raise both psychological and philosophical problems. Two instances which highlight the importance of providing a criterion for distinguishing the philosophical and the psychological may now be noticed. On the one hand, some analytic philosophers would contend that Hamlyn's recent book commits the very mistake that Hamlyn attributes to Piaget, as when a reviewer states:

> there is a lot to be learnt from this book, even though I am not sure how much of it is philosophy and how much is psychology and, therefore, how far readers will feel that it is in danger of falling between the stool of experimentally backed empirical work and that of analytically based conceptual clarification and, as a result, landing fairly and squarely on neither (White 1978, p. 525).

Underlying White's charge that Hamlyn's discussion merges empirical with philosophical issues, is the presumption that it is a mistake to merge the empirical and the philosophical. Thus through lack of a criterion, Hamlyn is hoist with his own petard and the resultant reliance on personal intuition leads to a (presumed) disagreement between Hamlyn and White about the coherence of the former's work. On the other hand, reliance upon the intuitions of the American Psychological Association has led to the bestowal on Piaget of the Distinguished Scientist award, with the citation that

> he has approached questions up to now exclusively philosophical in a resolutely empirical manner, and has made epistemology into a science separate from philosophy, but related to all the human sciences (Piaget 1972c, p. 15).

On this view Piaget's theory is not only not flawed by its combination of the empirical with the philosophical but is actually to be praised for doing precisely that. Such divergent responses to Piaget's work require either that Hamlyn should drop his criticism or that he should substantiate it by supplying a suitable criterion for determining when incoherence arises by the improper relation of the empirical with
the philosophical.

Secondly, several philosophers now reject the view that epistemology and science can be divorced from each other in the way they would have to be divorced if Hamlyn's view is held. The latter holds that a question is either philosophical or empirical but not both. By contrast, it is Karl Popper's (1969, p.216; 1979, p.261) claim that the growth of knowledge, whether in the amoeba or in Einstein, is one that occurs by a process of natural selection and so that epistemology, which deals with the growth of knowledge (Popper 1968, p.22), cannot ignore the biological nature and basis of that growth. Other philosophers (Quine 1969, p.83; Goldman 1978, p.509) contend that epistemology cannot be adequately pursued in the absence of psychology and so they too have no objection to there being a theory that included both philosophical as well as empirical components. As Goldman puts it, 

one cannot give the best advice about intellectual operations without detailed information about mental processes (Goldman 1978, p.509),

though he looks to cognitive psychology, unlike Quine who looks to behaviourism, for such empirical information. Further, any philosophical view that assumes, or is committed, to the view that a subject can always reason correctly about all beliefs held by that subject must be rejected because of its idealised conception of rationality (Cherniak 1981, p.161). On none of these views is there an exclusive opposition between the philosophical and the empirical.

The third reply to Hamlyn's objection is that it can be shown to be false, provided it is accepted that a philosophical statement is taken to be one that is necessarily true (false) and that a scientific statement is taken to be one that is contingently true (false). If this provision is accepted, it can be suggested that Hamlyn's objection is the claim that if a statement is necessary, then the negation of that statement is impossible and so it is pointless to cite empirical
evidence, given by contingent statements in its favour. Moreover, if a statement is necessarily true a search for cases where its negation is true will be futile. Consider, however, the following counter-argument. A philosophical statement is one that may be expressed by

\[(5) \text{ necessarily, } p.\]

Take any statement 'p' then only if 'p' is necessarily true (false) is 'p' a philosophical statement. Since modal concepts are inter-definable (Hughes & Cresswell 1972, p.26), \( (5) \) is equivalent to

\[(6) \text{ not possibly not } p.\]

Similarly, an empirical statement is one with a contingent modal status and may be expressed by

\[(7) \text{ possibly, } p \]

and inter-definability allows

\[(8) \text{ not necessarily, not } p\]

to be taken as its equivalent. It is apparent that

\[(9) \text{ possibly, not } p\]

also expresses an empirical statement.

Let 'p' be the statement 'Today is Monday'. It is clear that 'p' is true on any Monday and false on any other day. It follows that if someone asserts \( (9) \) then the truth of \( (5) \) is excluded. For \( (5) \) is equivalent to \( (6) \) and \( (6) \) is incompatible with \( (9) \). So if someone were to assert \( (5) \), then it would be reasonable to state \( (9) \) as grounds for the falsity of \( (5) \). Presumably, rational people would not assert \( (5) \) - given that 'p' is 'Today is Monday' - but that fact (if it is fact) in no way alters the principle here stated.

Consider a different example and let 'p' be the statement 'Monday is Monday'. Evidently in this example 'p' is a necessarily true statement and its form is given by \( (5) \). From this it follows that a statement such as \( (9) \) could not, in principle, hold true, for \( (5) \) and
(9) are incompatible. Thus prior knowledge of the truth of (5) precludes the possibility of the falsity of 'Monday is Monday'. Presumably rational people who had such prior knowledge would not assert (9) - given that 'p' is 'Monday is Monday' - but that fact (if it is a fact) is not relevant to the point at issue.

It will be noted that, in the last example, the form of 'p' was taken to be given by (5). Yet 'Monday is Monday' is not a philosophical statement. Indeed so; but it was merely claimed that all philosophical statements have a form expressed by (5) and the converse was not claimed.

Thus if someone has independent knowledge of the truth of a statement such as (5), then (5) excludes (9) and so it is futile to look for evidence of the falsity of (5). But there are two features to notice here. Firstly, (5) is taken to be incompatible with (9). Secondly, it is conceded that there is independent knowledge of the truth of (5). What, then, can be said of a case where this second feature does not apply? What can be said of a case where one person asserts a (philosophical) statement, expressed by (5), whilst another person asserts a (scientific) statement, expressed by (9)? In that case, there can be agreement that (5) and (9) are incompatible and disagreement as to which of the two statements - (5) or (9) - should be retained.

It is precisely such cases that Piaget has in mind when he asserts that philosophical epistemology is committed to claims which are incompatible with genetic epistemology. He accepts the validity of the problems posed by philosophers but he maintains that adequate answers to such problems require the use of empirical methods. It is for this reason that his theory combines the philosophical with the empirical. And if the argument here presented is accepted, Hamlyn's contention, that a theory that combines the philosophical and the
empirical is incoherent, can be rejected.

It may be briefly noted that a similar feature holds with respect to philosophical statements of the form

\[(10) \text{ necessarily, not } p,\]

for \((10)\) is incompatible with a statement such as \((7)\). It follows that empirically based affirmations and negations are relevant to the testing of philosophically based affirmations and negations and conversely.

No doubt a philosophical epistemology with truth on its side is one that makes redundant an epistemology that makes an essential appeal to empirical fact. It is Piaget's claim - one for which there is ample inductive support - that no such philosophical epistemology has yet been stated.

It might be argued that whilst the reports, \((A) \rightarrow (C)\), are empirical in nature and whilst they are indicative of the truth of \((4)\), namely

\[(4) \text{ Possibly, there is some knowledge, which conforms to (1), cannot be acquired by a subject,}\]

and whilst \((4)\) is incompatible with \((3)\), namely,

\[(3) \text{ necessarily, any knowledge, which conforms to (1), can be acquired by a subject,}\]

the latter is not a philosophical statement. It follows that the claims made for genetic epistemology are premature. The reply to this objection is that an account such as Kant's does seek to establish the general conditions under which knowledge may arise and thus attempts to make explicit the relations holding between fundamental principles and concepts of experience. The principle of contradiction is one such principle; the concepts of object and number are other examples. But Kant's account is incomplete if it is restricted to claims such as \((1)\), which merely prohibits the occurrence of self-contradiction. For if such principles and concepts are fundamental, the question naturally arises as to the subject's understanding of their instances. Is
their understanding the same throughout development? Or does their understanding change and, in consequence, impose limits on what a subject may know? Kant says nothing which excludes the former's being the case: by contrast, it is Piaget's claim that the latter is in fact the case. Thus (3) is a consequence of a philosophical account and, as such, is open to scientific challenge. The objection, therefore, fails.

In general, it is Piaget's claim that epistemological viewpoints embody factual claims and that philosophers have failed to see that this is so. It must be stressed, however, that Piaget cannot maintain that all epistemological questions are questions for genetic epistemology; or, rather, that Piaget must accept that genetic epistemology must assume a position with respect of certain philosophical questions. Thus Piaget assumes that both subjects and objects exist, assumes that knowledge is possible, assumes that knowledge of the world is spatio-temporal and so on. Doubtless, these are assumptions which are acceptable to common-sense. But it is also the case that philosophers have either denied the truth of such assumptions (e.g. Hume 1888) or have taken seriously the request to justify such assumptions (e.g. Strawson 1959). Moreover, section (3.2) shows that Piaget is too hasty in his dismissal of traditional solutions to the mind-body problem. What can, therefore, be said is that philosophical and genetic epistemology are complementary forms of epistemology: neither can be eliminated in favour of the other.

(2.4) Piaget's Genetic Epistemology

Piaget is not the first genetic epistemologist, as Kaplan (1971) and Russell (1978) notice, but his work is an outstanding contemporary example of genetic epistemology. The aim of this section is to delineate some of the main strands of Piaget's theory.

Piaget's theory is an attempt to state the conditions under which
it is possible for knowledge to grow. Early formulations of this claim show the psycho-analytical influence of Freud's work on Piaget (1920, p.55), as when he states that conscious activity is the outcome of unconscious activity, but later formulations stress a biological interpretation that envisages the acquisition of knowledge as the supreme example of life as such:

cognitive processes seem, then, to be at one and the same time the outcome of organic autoregulation, reflecting its essential mechanisms, and the most highly differentiated organs of this regulation at the core of interactions with the environment (Piaget 1971a, p.26).

Piaget (1967a, p.510/1971a, p.369) takes this to be a banal thesis and yet to be the central feature of his theory. The first claim states that there are cognitive processes which are conditions for the acquisition of knowledge and these processes are the result of self-regulation: the organic and the cognitive represent twin poles of a continuous biological process which may be characterised by a common set of constructs. The second claim is that cognitive processes are superior to, since more complex than, organic ones and are sensitive to a finer range of differences than the latter. Piaget does affirm that one aim has dominated all of his work:

I have pursued a central aim which has always remained the same: to try to understand and explain what living development is in its unceasing construction of novelty and in its progressive adaptation to reality (Piaget 1977e, p.22 - my translation).

Piaget's theory thus purports to offer a unifyingly simple answer to the question of how knowledge can be acquired since the cognitive processes available to a subject are taken to be no different in principle from the organic processes that enable an organism to survive at all.

This thesis is not of course banal, still less simple. An attempt to separate its weak from its strong interpretation is made by Piattelli-Palmarini (1980, pp.4-9) in a discussion of its biological implications.
Piaget's thesis - which is noticed to be similar to theses stated by Waddington in biology and von Foerster in systems-theory and so, it should be noticed, to most of the symposiasts in Köstler (1969) and contributors to Inhelder et al (1977) - might be the (weak) thesis that life is self-regulatory or the (strong) thesis that cognitive self-regulation transcends in a strict sense its organic counter-part. The latter thesis is not banal since it is a denial of the claim, accepted by many, that any self-regulation may operate only upon structures that already exist.

Ignore, for purposes of discussion, the full biological interpretation of Piaget's thesis and concentrate instead on its psychological implications, for example in the following formulation.

We hold that at all levels of mental evolution the mind is in pursuit of some form of coherence....the principle of contradiction is thus present everywhere and so forms a part of the functional invariants of all intelligence. But what appears contradictory to some is not so to others....contradiction or non-contradiction only appear when we interpret experience and in particular when we classify it. Variations in the evaluation of coherence therefore indicate structural variations in the way we interpret or deduce experience and these variations are perfectly compatible with functional invariance (Piaget 1931, pp. 150-1 - my translation).

Piaget takes a subject to be engaged in a pursuit of coherence and so makes a subtle modification of the Cartesian (Descartes 1931a, p.3) view that a subject is engaged in a pursuit of certainty. Piaget will agree that a developed subject may attain certainty but only because the acquisition of certain (indubitable) knowledge presupposes that a subject can acquire knowledge at all. For knowledge to be acquired, a set of prior conditions must be satisfied. If such conditions are universal and so apply throughout the psychological domain - that is, they apply to all cases of knowledge-acquisition - those conditions are functional invariants. They are functional in being conditions of a subject's being a biological organism at all and are invariants in that they apply to all such organisms in all adaptive encounters. The
search for coherence is one such functional invariant, claims Piaget.

Discussion of the biological features of Piaget's theory are beyond the scope of the present study. It may be noticed that whilst some critics (Rotman 1977; Hamlyn 1978) deplore Piaget's adoption of a biologically based model of knowledge-acquisition, some biologists and researchers in artificial intelligence (Boden 1979a, 1980b, 1981) specifically welcome the deployment of cognitive concepts in the attempt to understand organic forms of life.

As a matter of fact, however, the pursuit of coherence is marked by different levels of attainment of coherence, claims Piaget. Reconsider the studies cited in (2.3). (B) shows that subjects may take to be contradictory that which is not so, since subjects refuse to allow that a bottle may be simultaneously half-full and half-empty. And (C) shows that subjects may take to be non-contradictory that which is contradictory, since young subjects assert that each of the discs has the same size and deny that the first and last discs have the same size. Thus even if the principle of contradiction allows consistency and disallows inconsistency, subjects vary in their understanding of its instances and so vary in their assessment of what is the contradictory, and what the non-contradictory, of what. Variation in assessment leads to different modes of understanding and is marked by the subject's use of different cognitive structures.

Functional variance is therefore compatible with structural variation and the growth of knowledge takes place by a subject's construction of new structures on the basis of a structuration of reality through the use of available structures (Piaget 1928b, p.99; 1966, p.159; 1970a, p.703; 1980a, p.23).

In short, Piaget's theory is both structuralist and constructivist. Structures are required for knowledge to arise and new structures are formed out of a subject's use of structures available to that subject.
It is apparent that Piaget's constructivist theory is opposed to an empiricist viewpoint of the sort defended by B.F. Skinner whose environmentalist position is one that

shifts the determination of behaviour from autonomous man to the environment....environmental contingencies now take over functions once attributed to autonomous man....He is controlled by his environment (Skinner 1973, p.210).

Skinner states that his view is capacious enough to cover cognitive acquisition:

perceiving and knowing which arise from verbal contingencies are even more obviously products of the environment (Skinner 1973, p.184).

Thus schedules of reinforcement, as arranged in the environment, are the primary factors responsible for psychological functions. Such a view maintains Piaget, embodies genesis but not structure. Equally, Piaget's theory is opposed to a rationalist theory of the sort proposed by N. Chomsky:

there is a fixed, genetically determined initial state of the mind....The mind passes through a sequence of states under the boundary conditions set by experience, achieving finally a "steady state" at a relatively fixed age (Chomsky 1980, p.187).

Chomsky (1980, pp.3–4) chooses to concentrate on a subject's language—capacities but he too stresses that his account is applicable, in principle, to other cognitive capacities as well. Such a view, maintains Piaget, embodies structure in the absence of genesis. Only a constructivist epistemology may adequately combine both genesis and structure and Piaget (1968a, pp.145–46) takes this to be an important virtue of his theory.

(2.5) Necessity in Piaget's Theory.

Piaget's theory makes important, but ambiguous, use of the concept of necessity and three such uses may now be identified. Piaget's theory formulates necessary conditions for a subject's understanding of necessity by the occurrence of a necessary process of equilibration. Consider
each of these in turn as they arise out of the general claim made by Piaget (1977d, p.1) that knowledge is a process and not fact.

I. Necessary conditions

Piaget's general claim may be taken to state that objective experience and knowledge is always an assimilation to a structure (Piaget 1970d, p.12/*1977d, p.5), that the presence of an assimilating structure is a necessary condition of the growth of experience and knowledge.

The distinction between necessary and sufficient conditions is well-known (Stebbing 1930, p.271; von Wright 1971, pp.43-60). In summary form, if X is a sufficient condition of Y then Y is present in all cases where X is present. But if X is a necessary condition of Y then Y is absent in all cases where X is absent. But if X is sufficient for Y, X may be absent when Y is present (for example when some other factor, say Z, is present and is sufficient for Y); and if X is necessary for Y, Y may be absent when X is present (for example, when other factors that are also necessary for Y are absent).

An example may clarify this point. Consider

(11) Being 18 years old is a necessary condition of being a juror
(12) Being 18 years old is a sufficient condition of being a teenager
and (13) Being 18 years old is a necessary and sufficient condition of being an adult.

Let it be agreed that, as a matter of fact, (11), (12) and (13) are each true statements. It follows from (11) that someone who is less than 18 years old is not a juror: the absence of the former results in the absence of the latter. But even if someone is 18 years old, the satisfaction of that condition alone does not thereby result in that person's being a juror, for that person may never be called to sit on a jury. It follows from (12) that if someone is 18 years old then that person is a teenager: all cases of the former are also cases of the
latter. Clearly, however, individuals who are 17 years old are also teenagers and so the condition stated is one which does not have to be present on all occasions when someone is a teenager. It follows from (13) that a person is an adult in all and only those cases where the condition stated is satisfied: its absence always results in the absence of the person's being an adult and its presence always results in the presence of the person's being an adult.

A related distinction to draw is that between conditions which are logical and those which are empirical (psychological) in character. Thus (11) – (13) state conditions, whether necessary or sufficient, which are empirical conditions: it is conceivable (logically possible) for the relationships to be other than they are, though in fact this is not so. By contrast, a statement such as

(14) Being a bachelor is a necessary and sufficient condition of being an unmarried man

is one which states logical conditions: it is inconceivable (logically impossible) for the relationship to be other than it is.

Piaget's claim is the claim that the presence of a structure is an empirical, not logical, necessary, not sufficient, condition for the acquisition of knowledge. It is possible that cognitive acquisition could have had the presence of an immortal soul, an innate idea or an a priori concept as one of its conditions, though in fact it is none of these at all but is instead a cognitive structure (Piaget 1966, p.145 ff.; 1971b, p.9). Further, such a condition is one that in its absence knowledge is not acquired rather than one in whose presence knowledge does arise. That is, the condition is (empirically) necessary.

Two main reasons can be extracted from Piaget's writings in support of this interpretation. Firstly, a structure consists in what a subject knows how to do (rather than what a subject thinks). This epistemic element of a structure is noted by Inhelder & Piaget (1958, p.260).
A structure is, then, analogous to a capacity consisting in what a subject can do. But a capacity may be attributed to a subject even in cases when it is not displayed: an adult who is not currently walking does not thereby lack the corresponding capacity. By contrast, someone who lacks the capacity to walk, say a legless soldier, could not display walking behaviour. Thus having a capacity is a necessary but not sufficient condition of a corresponding behavioural display. Parity of argument requires that the same conclusion holds with respect to cognitive structures.

Secondly, Piaget's conception of genetic epistemology is such that the study of child psychology is one, but not the only, way to study the growth of knowledge. Thus Piaget praises the editors of his *Festschrift* for stressing this distinction (in Gruber & Vonèche 1977, p.xi) and he states that the genetic epistemologist could, in principle even if not in fact, study the thought and behaviour of primitive man (Piaget 1977f, p.37; 1980c, p.19). The full biological interpretation of Piaget's thesis clearly makes possible such an option. Moreover, Piaget's conception of child psychology is importantly different from the conception accepted by many experimental psychologists. The latter typically investigate the nature and causes of the changes that occur during development from infancy to adulthood (Bryant 1974, p.177) and so concern themselves with the variables that influence behaviour (age, context, task, language, social setting and so on). On such a view, individual differences in cognitive behaviour, in response to the variables identified for study, would be important. By contrast, Piaget roundly declares that on the problem of individual differences, I have nothing to say....That problem has not been my concern (Piaget 1967b, p.279 - my translation).

Piaget here denies an interest in questions about the rate of development in individuals. And this denial is a consequence of his general conception of child psychology (Piaget & Inhelder 1969a, pp.vii–viii) which is
taken to be a study of mental functions in the child. Thus Piaget's study of child psychology cannot, in principle, be a complete study because of its subservience to an interest in the problem of how knowledge grows from lowly biological beginnings to advanced scientific understanding. But if Piaget's study of the fundamental and universal features of the growth of knowledge leaves untouched questions about the particular ways in which specific variables influence behaviour, the account offered by Piaget can, at most, state conditions in whose absence knowledge is not acquired rather than conditions by whose presence knowledge is acquired. That is, his account offers empirically necessary conditions alone.

2. Deductive Necessity

Piaget's claim that knowledge is a process leads to a second claim that a subject may have knowledge of deductive necessity at the terminal, not initial, stages of development (Piaget 1953a, p.3; 1968a, p.157; 1972d, p.57; 1977d, p.6; 1977h, p.xi).

The distinction between knowledge which is deductively necessary, and that which is not, goes back as least as far as Plato (1941, 509D-511E). Quite simply, it is one thing to know, on the basis of observation, that a square has four corners each of which is a rectangle; it is quite another to know, on the basis of deduction from a set of axioms, that a square must have such properties. Thus from a correct claim that an object has such-and-such a property, the further question can always arise as to whether such a claim is contingently or necessarily correct.

Piaget initially thought that an understanding of deductive necessity with the concomitant ability to reason logically could be studied at any age of development.

One can think what one wants of formal logic....But what nobody must dispute is that formal thought can be studied as a psychological fact....I call formal reasoning that reasoning which, from one or several
propositions, draws a conclusion to which the mind assents with certainty without recourse to observation (Piaget 1922, p.272 – my translation and emphasis).

Piaget was led, however, to abandon this claim, no doubt as a consequence of his objection to philosophical epistemology discussed in (2.3): not any type of knowledge may be acquired by a subject at any developmental point. Certainly, however, Piaget retained the essentials of this claim in his mature account of formal thinking (Piaget 1972e, p.159).

Thus in his mature account, Piaget claims that a subject's terminal development is marked by an understanding of necessity, unlike the initial development where such understanding is lacking. As a structuralist, Piaget identifies this difference by appealing to structural differences in what a subject does and, as a constructivist, Piaget characterises the different steps taken in the growth of this understanding. That is, Piaget is committed to there being structures of different types only a sub-set of which arise when a subject understands necessity.

Since Piaget's general thesis claims that a subject's knowledge-acquisitions are bounded by a cognitive structure and since Piaget claims that a structure may be closed only in certain cases, it follows that a knowledge of deductive necessity cannot be acquired by any subject who lacks such a structure: the latter is a psychologically necessary condition of the former.

Piaget's enterprise can be stated as follows. Piaget invokes the distinction between deductive and non-deductive knowledge. In his initial research, say in the 1920s, Piaget, assumed that all subjects could acquire deductive forms of knowledge. In his later research, say from the 1930s onwards, Piaget restricts deductive forms of knowledge to those subjects who use certain types of structures. His central concern, then, is to show how such (operational) structures are generated from previous structures.
3. Necessary Process of Equilibration

A third way in which Piaget interprets the claim that knowledge is a process is by stating that knowledge is always in a state of development and that the use of one structure inevitably leads to the construction of a new structure and so to new knowledge (Piaget 1977d, p.6).

Genetic epistemology is taken to be the study of

the origins of the various kinds of knowledge, starting with their most elementary forms, and (follows) their development to later levels up to and including scientific thought (Piaget 1972c, p.15).

On this view, an assessment - which is doubtless liable to error due to scientific change (Cellerier 1973, p.18) - is made of the highest level of some body of knowledge so that its psycho-genetic antecedents can be traced. Thus the knowledge acquired by some scientist (Einstein) is more general than that acquired by a child, which is in its turn more general than that acquired by an infant. The search for such antecedents may continue indefinitely (Piaget 1971a, p.9) and it is for this reason that Piaget's thesis has a biological interpretation. For the psychologist, however, birth may be taken as a convenient starting-point.

Piaget (1978b, p.3) attempts to explain such cognitive transition by appeal to a central process of equilibration and he takes this to be a necessary process. It is not necessary that equilibration should occur but if it does occur, it is Piaget's (1978b, p.30) claim that it is necessarily constructive. His contention is as follows. For a biological organism to exist, certain processes must be present for that organism to be of the sort it is. Organisation of some sort there must be, though no specific form of organisation is so required. So organisation - or functioning - can have different instantiations, which are the structures of that organism. Thus there is a functional, but
not a structural, a priori (Piaget 1953a, p.3) and Piaget (1971a, p.148) explicitly draws attention to the Kantian conception being invoked here. But he differs from Kant since he claims that the structures corresponding to Kantian a priori concepts occur at the end, not the outset, of development — as the discussion of deductive necessity showed. What is a priori is organisation or functioning, which is an invariant feature of development.

Organisation qua functioning is not transmitted by heredity as are characteristics such as shape, colour, etc.; it continues and succeeds itself qua functioning as a condition necessary to every transmission and not as a transmitted content (Piaget 1967a, p.210/*1971a, p.148).

Piaget is not here claiming that functioning is an empirically necessary condition of transmission, for he denies that functional processes occur as a de facto regularity. Rather, his claim is that there have to be such processes, for de facto regularities to take place in the way that they do and so the occurrence of functional processes is logically prior to the occurrence of de facto regularities.

Since functioning may assume different forms, corresponding to distinct structures, functioning is a process. Since equilibration is taken by Piaget to be the process of structural change, equilibration is also constructive. Further, equilibration is necessarily constructive. The presence of a structure permits a subject's understanding of a certain range of possibilities which are co-extensive with what a subject knows how to do. Piaget is not claiming that a subject may consciously think about these possibilities as possibilities, or even be aware of them at all. It is the observer, not the adolescent subject, who knows the structural possibilities open even to an adolescent subject (Inhelder & Piaget 1958, p.260). But a subject who is confronted by a new structural possibility — one not covered by a currently available structure — may understand that new possibility, and so may gain new knowledge, only by the construction of a new structure.
It is in this sense that equilibration is necessary. The process is not logically necessary (Piaget 1971a, p.322). Nor is it empirically necessary (Piaget 1971a, pp.14, 100). Its necessity is therefore taken by Piaget to be distinct from the previous two senses already discussed. For ease of recognition, this sense may be referred to as constructive necessity.

It is important to keep distinct these three ways in which Piaget uses the notion of necessity. The notion of a necessary condition is invoked in the discussion of structures - in (4.4); that of deductive necessity is central to the discussion of operational structures - in (4.6) and (6.2); equilibration as a necessarily constructive process is discussed in (5.2). A critical discussion of the three notions of necessity appears in chapter 9.

(2.6) Conclusion

The problem of knowledge that is of interest to Piaget is the problem of establishing the conditions which make possible the acquisition of knowledge. There are philosophical theories, notably that of Kant, that have, historically, contributed to the discussion of this question but it is Piaget's view that only genetic epistemology, taken as an empirical study of the growth of knowledge in any of its forms at organic, psychological and scientific levels, may satisfactorily resolve the problem. Piaget's theory is the theory that there are empirically necessary conditions, consisting in the presence of cognitive structures, which lead by a necessary process of equilibration, one that must have occurred for new knowledge to arise and which produces structural enrichment, to a knowledge of deductive necessity at later stages of development. Piaget thus conceives the growth of knowledge to occur as a unitary and continuous biological development.

The outline of Piaget's theory may now be amplified in four ways corresponding to the four subsequent chapters. Firstly, Piaget's
theory emphasises the precedence of action over thought in the growth of knowledge, in striking contrast to Cartesian theories of knowledge. What is Piaget's conception of action and how does the action's of a subject make possible the acquisition of knowledge? Further, what is the role of the subject in the acquisition of knowledge? Such questions are addressed in chapter 3 and the crucial distinction between a subject's own knowledge of what the subject knows and an observer's knowledge is drawn. In chapter 4 the distinction between functioning and structure is taken up. It is shown that on Piaget's view a subject's actions embody functional characteristics and that the growth of knowledge corresponds to the emergence of different structures which can be given a formal or logical description. Thus the chapter tries to show how Piaget takes natural logic to be the precursor of formal logic. In chapter 5, Piaget's account of equilibration is presented and an attempt is made to discuss his account of the inter-play of observable and coordinatory knowledge together with processes of abstraction and generalisation. Finally, in chapter 6, Piaget's methodological position is defended, together with his accounts of the subject's understanding of transitivity and inclusion.
Observed Knowledge

(3.1) Introduction

Piaget will defend an assimilatory theory of knowledge whose discussion will pre-occupy the subsequent two chapters. What requires discussion in the present chapter are Piaget's reasons for rejecting a copy theory of knowledge, one that takes observable knowledge to be the paradigm form of knowledge. In essence, the chapter aims to show why Piaget denies that perception and consciousness can be taken to be the primary source from which knowledge arises. The discussion is in five parts. In (3.2), a sketch of the mind-body problem is presented, firstly to show why Piaget supposes that an adequate answer to that problem requires a commitment to a structuralist theory of knowledge and, secondly, to identify the philosophical view assumed in Piaget's own discussion of theories of knowledge. In section (3.3), Piaget's formulation of the copy theory is presented and his two main objections to it are discussed. It is argued that Piaget's objection that the notion of a perfect copy is contradictory can be rejected but that, even so, his objection to the other claim of the copy theory, namely that all knowledge is observable, is decisive enough to warrant rejection of the copy theory. In (3.4) the distinction between norm and fact is shown to cause a problem for any theory of knowledge which examines the role of logical norms in the acquisition of knowledge. It is argued that an observer may investigate the logical norms used by a subject, even though the subject is unaware of what those norms are. It is also shown why Piaget takes action-coordination to be the source of logical norms in preference to a source in heredity or environment. In section (3.5) the observable nature of perceptual knowledge is discussed and Piaget's account of egocentrism is presented, in outline form, so as to show how Piaget takes observable knowledge to be incomplete and distorting. Egocentric judgements arise when a
subject fails to insert observable knowledge into a sufficiently rich interpretative framework of coordinatory knowledge. In section (3.6), it is shown that Piaget takes consciousness to be a different type of observable knowledge and in consequence that it, too, is incomplete and distorting. It is shown that Piaget takes consciousness to be the conceptualisation of practical knowledge. Finally, it is shown that Piaget is led to distinguish between the individual and epistemic subject, the former being the possessor of observable and the latter the possessor of coordinatory knowledge. It is action and neither perception nor consciousness which is taken by Piaget to be the ultimate source of knowledge.

(3.2) Piaget and the Mind-Body Problem

The mind-body problem - what is the relation between a person's physical, including neural, states and states of mind - is an important problem just because of its centrality to any theory that purports to be an adequate theory of the human subject. The problem is still at the forefront of philosophical (Borst 1970; Wilkes 1978) and interdisciplinary discussion (Popper & Eccles 1977; Pucetti & Dykes 1978). Piaget contributes to the discussion of this problem and it will, therefore, be instructive to review his position.

It is obvious that Piaget does not deny that a subject has a biological - and so in the extended sense physical - inheritance and that his brain and body are organs that ensure a subject's physiological adaptation, just because of his commitment to the continuity between organic and cognitive forms of life. It is for this reason that Piaget says that the central nervous system plays a role which

assures the continuous transition between physiological assimilation in the wide sense, and cognitive assimilation in its sensori-motor form (Piaget 1971a, p.221).

But such states constitute an empirically necessary, not sufficient, condition of knowledge. Piaget adverts to the claim that the nervous
system can be given a description that is isomorphic with the
description of a mathematical group. Since an infant's nervous system
is describable in this way, whilst an adolescent's cognitive structure
is so describable, it may be inferred that other factors are responsible
for this lag. Thus an adequate account of knowledge-acquisition cannot
be given by reference to physical factors (Piaget 1971a, pp.221-23).

Nor does Piaget deny that conscious states exist, since he defines
his legitimate interest to be in the study of the behaviour patterns
and the consciousness of the child (Piaget & Inhelder 1969a, p.vii).
That an infant subject is credited with sensations, needs, desires,
beliefs as well as consciousness as such (Piaget 1953a, p.37). Yet
possession of consciousness is an (empirically) necessary, and not
sufficient, condition for the formation of (conceptualised) knowledge
and Piaget argues that factors other than consciousness are also
required.

Piaget considers several possible ways in which body and mind may
be related, for example identity, causality, correlation and isomorphism.
Identity is rejected since the properties applicable to a subject's
physical states are not those applicable to conscious states and
conversely. For example, neural events are causally related but this
is not true of conscious states. And the judgement that $2 + 2 = 4$
imply, and does not cause, that $4 - 2 = 2$; yet the relation of
implication cannot hold between physical states (Piaget 1968b, p.187;
1971a, p.49). Nor can the relation be causality since it is
incomprehensible how the bearer of physical properties, the brain, can
causally interact with the mind, which lacks such properties. Such
interaction would be conceivable if the mind had physical properties
of some sort – but in that case the interaction would be no longer
between the mental and the physical (Piaget 1968b, p.184). Correlation
fares no better since the claim that the mental and physical are
complementary, since functionally similar, is incomplete: it fails to explain the basis of the very complementarity (Piaget 1968b, p.186). It is for this reason that Piaget opts for isomorphism: even though a neural event and the making of a judgement are quite distinct, each can be functionally similar in the sense that the same set of logical principles is instantiated in each. Thus consideration of the relation of mind and brain reinforces the claim that a structuralist theory is indispensable (Piaget 1968b, p.189).

How adequate is Piaget's position? It might be argued that his rejection of rival positions is too quick since he fails to discuss variant statements of position. Thus physicalism may be stated as the elimination of the mental, rather than as the identity of the mental with the physical, just because of the objections noted by Piaget (Feyerabend 1963). Identity theorists might reject Piaget's use of Leibniz's law, that if X and Y are identical any property possessed by one is possessed by the other, on the grounds that it is admitted to fail in intensional contexts (White 1972). Piaget's discussion of dualism suffers because of its construal of the mental as a form of non-physical thing or state, which view distorts the complexity of mental concepts (Wittgenstein 1958). Moreover, what is incomprehensible is not to be decided by unaided common-sense but rather by a theory and recent formulations of dualism arise out of explicit consideration of empirical theory and research (Popper & Eccles 1977; Puccetti & Dykes 1978). Parallelism is also taken to be a viable position by some psychologists (Gale 1980) just because it allows researchers to postpone their making a commitment to one solution of the mind-body problem rather than another whilst utilising the insights of a wide range of empirical theories. Whilst this view is not accepted by all (Wittgenstein 1967, Sect. 608; Smith 1980b, pp.72-3) it does have the merit of not being question-begging, unlike Piaget's objection to it.
Piaget's defence of isomorphism is also open to reply. Sometimes he states manifest nonsense, as when he is prepared to locate a subject's structures somewhere midway between the nervous system and conscious behaviour (Piaget 1971b, p.138).

Sometimes his general view of philosophical problems is that only science can provide answers to them (Piaget 1972d, p.210) and sometimes that scientific considerations are merely relevant to the possibility of an answer (Piaget 1972c, p.12) and so it is not clear to what extent Piaget takes himself to have "answered" the most notorious of philosophical problems. Piaget, philosophers (Mischel 1979, p.105; Boden 1979a, p.93) contend, does not take seriously enough the philosophical views which he too hastily rejects.

Two conclusions may be safely drawn from this discussion. Firstly, Piaget shows that he is aware of the intellectual tradition out of which his epistemological position has arisen but he is also anxious to retain his own distinctive position. Genetic epistemology has its birth in philosophy but is independent from it. Secondly, Piaget accepts that a subject who acquires knowledge is the possessor of physical and mental states but he rejects any claim that it is the physical, or the mental, which has a primary role in the acquisition of knowledge. It is to the clarification of the consequences of this rejection that the discussion now turns and, specifically, to Piaget's rejection of copy theories of knowledge.

(3.3) The Copy Theory of Knowledge

Sponsors of the copy theory of knowledge can be found in classical times, for example the Greek Atomists, but it is the philosophers in the Empiricist tradition that have given that theory its most striking formulation and it is clear that Piaget has such philosophers in mind when he claims that he will reject the copy theory. That rejection is, however, ambitious enough to include behaviourist psychology (Piaget
The aim of the present section is to discuss the copy theory so as to clarify why Piaget denies that all knowledge can be taken to be observable in nature. The discussion will be in three parts: (A) a statement of the copy theory of knowledge; (B) the rejection of the notion of a perfect copy; and (C) a demarcation of observable knowledge.

(A) The Copy Theory of Knowledge

Piaget claims that the copy theory has two central features, the claim that reality can be reduced to observables and the claim that knowledge is limited to transcribing, and not transcending, those features (Piaget & Inhelder 1969c, p.118). Consider, then, each of these in turn.

The first claim is not transparently clear: it is not clear what Piaget means when he refers to reality, nor what is meant by a reduction to observables. A different formulation of this claim is more helpful, that

all knowledge comes from the senses and results from an abstraction based on sensorial data (Piaget 1977d, p.63)

which is evidently the claim attributed to Locke (1881) in (2.2) above.

On this view, experience is the source of all knowledge which a subject acquires either by observation of sensible objects or by internal observation of one's own mind. Thus physical and mental occurrences take place and knowledge arises by a subject's observation, in either mode, of those occurrences. Moreover, all knowledge arises in this way. It is presumably this last claim which Piaget has primarily in mind when he claims that reality is reducible to the observable.

The second claim states that (observable) knowledge is a transcription of reality and is the claim that

the perceptions and images induced by objects are sufficient to provide knowledge by successive approximations (Piaget & Inhelder 1966a, p.455/1971, p.387).

Once again there are precedents for such a view, for example Hume's (1888)
view cited in (2, 2) above. On this view, a subject's knowledge of reality is always mediated by a set of impressions, faint copies of which occur when an idea is formed. It is for this reason that Piaget claims that subscription to the copy theory requires that the function of knowledge should be to transcribe reality and so not to transcend it: to transcribe reality would be to gain an accurate copy of reality whereas to transcend reality would be to do other than this.

Piaget's dissatisfaction with the copy theory does not stem from a desire to deny that observation is a source of knowledge, nor that some knowledge is observable in nature, nor that a subject forms images – impressions and ideas – in cases where knowledge arises. Physical experience is, and is admitted to be (Piaget 1977d, p. 70) a necessary condition of the acquisition of knowledge. Rather, dissatisfaction arises because copy theorists fail to see that the complexity of the organising factor of the mind is not (with Locke) passive inspection through quasi-optical reflection nor (with Hume) the mere association of ideas (Piaget 1972d, p. 53).

(B) The Notion of a Perfect Copy

Piaget's objection to the second claim of the copy theory – knowledge is acquired when a subject forms an accurate copy of reality – is the claim that a perfect copy is impossible. But if a perfect copy is impossible, then knowledge must also be impossible since acceptance of the copy theory excludes there being any other factor by whose use a subject may correct the imperfections of any copy. Thus if there can be no perfect copies, the copy theory cannot be an adequate theory of knowledge. How strong is this objection?

It is apparent that Piaget is in fact indebted to empiricist philosophers for the manner in which he discusses this question, since the question is posed in the context of a dualist theory of mind. What is especially striking is Piaget's acceptance of their view that images
Hume's ideas - are the mental items whose role is that of copying reality. Representative thought, for Piaget as for the empiricists, consists in the production of images or symbols that have some similarity with that which they are symbols of (Piaget 1951, p.67; 1970a, p.717).

An image, then, is a copy - but a copy of what? One alternative is to say that it is the copy of a geometrical figure, for example a square with sides 5 cm long. But any image-copy of such a figure will be inaccurate since the copy would show a figure whose sides were not exactly 5 cm long. Indeed, the copy would not, strictly, be a square since its apices would not be exactly 90 degrees. It follows that no copy of a geometrical figure could be perfect (Piaget & Inhelder 1971, p.385).

Two comments may be made about this argument. Firstly, the argument discusses one example on the basis of which a general conclusion is drawn, which conclusion would thus be massively underdetermined. Presumably Piaget takes his example to be illustrative of all geometrical figures. But in that case, copy theorists might accept that a perfect copy is impossible, though insist that cognitive acquisition might occur in cases where a subject forms a copy at all, for a less-than-perfect copy of a square is still a copy of a square as opposed to a circle. Secondly, Piaget wishes to show that the notion of a perfect copy is strictly impossible since contradictory:

> an exact copy could not exist without contradiction (Piaget & Inhelder 1971, p.385).

But it is not the copy that is contradictory: the copy does not represent a square with sides both 5 cm and not-5 cm long. Rather the copy is defective since it represents as being more, or less, than 5 cm what is in fact 5 cm. So the claim that the notion of an exact copy is contradictory is false.

The second alternative is the claim that an image is a copy of a
physical object. In discussing this alternative Piaget makes use of de Saussure's distinction between indices, symbols and signs (Piaget 1970a, p.717). An index is causally related to the object that produces it, whereas a symbol is distinct from its object even though similar to it. Suppose, then, that a subject forms an image of blue sky. The image cannot be a perfect copy of blue sky since the image can, at most, resemble the index gained when that subject perceives blue sky. Yet the index, qua effect, may not resemble its cause, qua blue sky, and thus an image copying that index could not be a perfect copy of blue sky. Moreover, blue sky is a complex object since physical theories, for example about light waves or the distances of astronomical objects, are required for its comprehension and these will not normally be available to a (young) subject. Indeed, these theories may even be later replaced by successor-theories and so knowledge of blue sky is incomplete. It follows that any image of blue sky cannot be a perfect copy of blue sky (Piaget & Inhelder 1971, pp.385-6).

One comment to make about this argument is that it is open to defects previously noted: the argument does not show that a copy cannot represent its object at all; and to claim that a perfect copy is an impossibility is not shown by pointing out that a perfect copy does not as a matter of fact occur. Secondly, blue sky is blue sky - whatever the scientists might have to say in explanation of the blueness of sky. For that is an explanandum in competition for the explanation of which different scientific theories are stated. Thus a (young) subject is not required to understand such theories for the perception of the blueness of sky.

(C) The Demarcation of Observable Knowledge

Consider now Piaget's objection to the first claim made by the copy theory, that all knowledge is observable knowledge. Piaget's objection to this claim states that logical and mathematical properties
are not open to observation and thus no subject all of whose knowledge was observable could have knowledge of such properties. Since subjects do have such knowledge, not all knowledge can be observable. Once again, it will be instructive to examine the grounds on which this objection rests.

Piaget (1972f, p.6; 1977d, p.71) sometimes states his objection as the claim that physical objects, say ten pebbles, possess the properties of colour, size or weight but not the property of number (in this case ten). The number of pebbles in a pile, states Piaget, is independent of the means used to count them as too the order of counting and so, strictly, number is a property of the subject's action-coordinations. If Piaget means that a subject can establish the number of pebbles in a pile only by counting them — or only by finding out from someone else who has counted them — then precisely the same conclusion holds with respect to the weight of the pebbles. Yet Piaget affirms that weight is a property of the pebbles. If, however, Piaget means that number is a property of a subject's action-coordination, then the objection is circular, for it evidently presupposes a viewpoint which copy-theorists would reject.

Piaget (1970a, p.715; 1972f, p.4) states the objection in an alternative way, as the claim that what a subject perceives is always dependent upon an interpretative and logico-mathematical framework. Suppose a subject lacks a framework that is robust enough to permit the comprehension of a number. It follows that that subject will not acquire knowledge of the number of objects in a pile. This claim is not analogous to the claim that only a person who has a concept of colour can identify instances of that concept, since such instances are open to observation. Rather, it is the claim that logico-mathematical properties are not open to observation and so their identification requires the presence of a corresponding framework. It is not essential
to this formulation of the objection that Piaget's theory is accepted and so this formulation is distinct from the previous one. What distinguishes the subject who does, from the subject who does not, understand that there are ten pebbles in a pile is the intellectual - and not the perceptual - component of that subject's knowledge.

It is apparent that this objection is strong enough to dispose of the copy theory, whether or not the objection discussed in (B) is accepted. For even a subject who could form perfect mental copies would not thereby be able to understand number. Any such image is merely a copy of perceptually acquired information. And it is precisely that component which is taken to be insufficient as a source of logico-mathematical knowledge. It is for this reason that Piaget commits himself to an alternative theory of knowledge: no theory that construes all knowledge as being a permutation of observable knowledge can account for the presence of non-observable knowledge, and specifically of logico-mathematical knowledge.

Piaget's discussion of observable knowledge, arising from perception and introspection, will be reviewed further in (3.5) and (3.6). A distinction must, however, be stated and clarified between the knowledge accessible to a subject who has knowledge and knowledge accessible to an observer. It is this distinction which is the central topic of (3.4).

(3.4) Normative Facts

A logical norm is a principle which prescribes or proscribes what may be logically combined with what. The principle of contradiction is a paradigm example of a logical norm and the principles of transitivity or class-inclusion would be others. Philosophical problems arise in the grounding of such norms and psychological problems arise in explaining their role in a subject's understanding. Such problems must not be confused, since psychologism occurs when the validity of a logical norm is grounded in facts about how subjects think, whilst
logicism arises when the subject's understanding is viewed as a formal system. Piaget (1966, p.132; 1967b, p.274) is aware of both of these mistakes.

A problem emerges, however, for any theory which attempts to show that the acquisition of knowledge is possible only if a subject uses a logico-mathematical framework for the interpretation of experience. And it is just this claim which is embodied in Piaget's theory. For whilst Piaget rejects the Kantian claim that a subject, at the outset, has available a set of a priori concepts, he does wish to claim that a functional a priori is present throughout experience. For Piaget, the acquisition of any knowledge, at any developmental point, requires a subject to use a logical and interpretative system of some sort. But the problem about such a theory is that it seems to confuse logical norm with psychological fact.

Piaget's reply to such a charge consists in a denial, provided that it is realised that there are normative facts. A norm is not a fact. For example, the moral norm 'Always keep one's promises' is a norm, whatever the facts about the moral behaviour of people and the fact that someone breaks a promise does not justify a claim that such behaviour is permissible. Thus a subject cannot ground a moral norm on the facts of behaviour. By contrast, an observer may correctly record which moral norms, if any, are used by a subject in a moral dilemma, for a normative fact is a fact whose description shows the presence of some norm in a subject's thought and behaviour. Piaget adverts to normative facts as follows:

you experience the norms in yourself; I, the observer, observe them and describe them as the facts they are, if my observation stays faithful and correct, there will never be a contradiction between your norms and my facts, since my facts will be a description, an analysis, a causal explanation of your norms (Piaget 1965, p.49 - my translation).

Piaget's claim, then, is that normative facts are accessible to an
observer and so they may be correctly described by that observer, whether or not they are also accessible to the subject under observation. Genetic epistemology does not ground logical norms in psychological fact but does, instead, seek to describe the role played by such norms in what a subject thinks and does. Genetic epistemology is a third-person study of the role played by logical norms in the (first-person) acquisition of knowledge.

A more intractable problem must now be faced. For a subject to acquire any knowledge, the subject must use an interpretative framework, which is logical in nature. It follows that on this view logical norms always play a part in the acquisition of knowledge. But two questions can be posed. Firstly, does Piaget claim that a subject, at any level, knows which norms are efficacious in that subject's own knowledge-acquisition? Secondly, how do such norms play a role in the acquisition of knowledge?

To the first question, the answer is "No". Even an adolescent subject will, typically, be unaware of the cognitive structure, the operations arising from the use of that structure and so the interpretative framework used. Such a subject uses these

but he does this without enumerating them, or reflecting on them or their relationships, and he is only faintly suspect that they form such a system. He is unaware of this, in the same way that in singing or whistling he is unaware of the laws of harmony (Piaget 1953b, p.40).

This claim is decisive: it is not Piaget's claim that a cognitive structure or a logico-mathematical framework, attributed to a subject by an observer, is consciously known by a subject as a condition of the subject's acquiring the knowledge that he/she does. Of course, a subject may acquire such knowledge at a later date but in that case the presence of some other interpretative framework, distinct from the previous one, may be attributed to that subject by some observer and it is not a condition of the subject's having knowledge of the
earlier framework that the subject should thereby acquire knowledge of this later framework. The conditions for the acquisition of knowledge are not known by a subject as a condition of the acquisition of that knowledge.

But then the second question arises: how do logical norms contribute to the acquisition of knowledge? Piaget's answer is ingenious. To see why, it is instructive to recall the debate about the role of reason and experience in knowledge, Piaget (1971a, pp.
306, 322) denies that perception and learning are an adequate source for the understanding of logical norms and so denies that environmental factors produce such understanding. In general, Piaget accepts the argument stated by rationalist philosophers - see (2.2) - concerning the "poverty of the stimulus" (Chomsky 1980, pp.34-8). Moreover, Piaget rejects the view stated by modern empiricists who accept that the truths of logic and mathematics cannot be grounded in experience. Empiricists claim that such truths are tautologies which provide no knowledge of the world but which instead draw attention to connections which derive from the definitions of a language (Ayer 1946, p.79).

But it is Piaget's (1936, p.132/1953a, p.147; 1967a, p.17/1971a, p.3) claim that intelligence precedes language in that the acquisition of any knowledge requires a subject to use an interpretative and logico-mathematical framework for such acquisition. A pre-linguistic infant would, on the modern empiricist view, be precluded from acquiring knowledge, contends Piaget (1974e, pp.112, 116). Moreover, Piaget (1971a, p.317; 1980a, p.31) denies that logico-mathematical knowledge is hereditary and so he denies that there is complete evolutionary basis for such knowledge. So he rejects any supposition that there is a (Chomskian) Mathematical Acquisition Device (Papert 1980, p.46) in a subject's brain. If such views were accepted, then the universality of logic and mathematics could not be explained and so in consequence
their necessity would be lost. Thus neither heredity nor the environment is an adequate source of such knowledge.

Yet this opposition of heredity and environment is not exhaustive, since there is a third possible source, namely a subject's action-coordination. It is not Piaget's claim that a subject may simply observe his/her own actions and thereby gain logico-mathematical knowledge - for that is to revert to empiricism, since observation of one's own actions is analogous to observation of any other physical phenomenon. Rather, it is the coordination of actions that is the source, for those coordinations constitute an interpretative framework which may be described by an observer - even if not by the subject - in logico-mathematical terms. Thus a subject who acquires knowledge is an agent who acts and the agent coordinates his/her actions in a manner that shows the role of logical norms in the acquisition of knowledge.

In short, Piaget (1966, p.166) uses a logical model to account for a subject's acquisition of knowledge. On the one hand, there are logico-mathematical structures which can be given formal characterisations and Piaget in no way attempts to ground those structures in psychological processes. What he does claim is that a subject's action-coordination can be characterised - by an observer - in structural terms which may also be given a formal characterisation. It is, therefore, a reasonable task to establish whether, as a matter of fact, any characterisation of the former sort is isomorphic with a characterisation of the latter sort. It is also clear, as D.S. Wright (1980, pp.313-4) points out, that Piaget's enterprise is bolder still, since if there is isomorphism between the characterisations of cognitive and formal structures, then the presence of structures of the latter sort both explains and is explained by the presence of structures of the former sort.

The discussion of observable knowledge can now be resumed, given that a distinction is drawn between the knowledge accessible to a
subject and the knowledge accessible to an observer. An acceptance of that distinction allows Piaget to state in detail the limitations of observable knowledge as a source of logico-mathematical knowledge and it is such limitation which is the theme of the two subsequent sections.

(3.5) Observable Knowledge: Perception

Observable knowledge is taken by Piaget to be distorting and incomplete. The present section examines the limitations of perceptual knowledge, the next section will perform the same task with respect to consciousness. It is apparent that Piaget accepts the Lockean view - see (2.2) - that observation is a genus of which perception and internal reflection (consciousness) are its species.

A terminological point is necessary here. Piaget defines an observable as

what may be verified in experience by an immediate reading of the given facts themselves (Piaget 1975a, p. 50/1978b, p. 43).

This definition seems to be applicable to perceptually based forms of knowledge and thus observable knowledge is knowledge of facts that arises from a subject's immediate perception of them. But this definition is clearly incomplete. Firstly, Piaget states that in no case at all can a subject have any, including perceptual, knowledge without the use of a logico-mathematical framework. There can be, then, no immediate perceptual knowledge in the sense of knowledge that is independent of an interpretative, and logical, system for construing it.

"Neat" experience in the sense of a simple registration of external data - in the absence of a contribution from a subject - does not in fact exist... physical experience is always indissociable from a logico-mathematical framework (Piaget 1969c, p. 127 - my translation).

Moreover, the full generality of this view is affirmed:

let us recall that, even from the beginning and already with the youngest subjects, a physical fact is
registered only by means of a logico-mathematical framework, however elementary (Piaget 1977b, p.321—my translation).

Secondly, the definition is incomplete in a different way, as the account of figurative knowledge shows. Figurative knowledge is essentially a copy and is manifest in perception, imitative behaviour and mental imagery. Such knowledge allows a subject to understand some state of an object, and that state alone, and is compared by Piaget to a photograph or to a simple reading of that state: the object has properties which are copied, as faithfully as possible, in that figurative knowledge (Piaget & Inhelder 1969b, p.87). It is clear that the definition of observable knowledge is a special case of figurative knowledge, which includes introspective and not merely perceptual knowledge. In the sequel, figurative and observable knowledge will be regarded as equivalent forms of knowledge.

Piaget's objection to observable knowledge is that it is distorting and incomplete. It is, in short, egocentric. The notion of egocentrism has proved to be vulnerable to misunderstanding claims Piaget (Piaget & Inhelder 1969a, p.61), even though Piaget (1926, pp.9, 38, 48) has used this to characterise the language, thought and behaviour of the child, to account for a subject's moral judgements (1932, p.79) and cognition at all levels of development (1950, pp.160-66). Piaget (Piaget & Inhelder 1969a, p.61) now suggests that the notion of centration is preferable to that of egocentrism, though this notion has earlier uses (Piaget 1950, p.142).

In what respects, then, is observable knowledge egocentric? Discussion of this question permits the understanding of why Piaget takes observable knowledge to be insufficient as a source of all knowledge. The following features are presented, then, as main—though not necessarily all—features of egocentrism:

1) Piaget defines egocentrism as follows:
I have used the term egocentrism to designate the initial inability to decenter, to shift the given cognitive perspective...which stems from a lack of differentiation between one's own point of view and the other possible ones (Piaget 1962, pp.3-4).

A subject has a point of view when the subject gains observable knowledge of an object and so gains knowledge from that point of view. But that point of view is only one point of view and there are an indefinitely large number of alternative points of view. Even though a subject may have knowledge corresponding to two points of view, the subject may not know that these two points of view are related.

To make the same point: an object has different states and a subject may have knowledge of two of those states and yet fail to understand that these two states are related. In particular, a subject may not understand that one point of view (state) enters into certain logico-mathematical relations with the other.

2) It is claimed (Hughes 1975) that Piaget, inconsistently, adopts an alternative account of egocentrism, which arises when a subject fails to differentiate the subject's own point of view from that of other subjects. Certainly, Piaget (1959, p.267) does sometimes so describe egocentrism and so may promote confusion by his failure fully to clarify his position. It is clear, however, that the self-other contrast is but a special case of the one-many contrast, since Piaget (1966, pp.289-90) envisages an individual's growth of knowledge to be co-temporaneous with that individual's knowledge of other subjects. The inconsistency, is therefore, more apparent than real. (For a further discussion of social knowledge, see chapter 7)

3) Egocentrism is an unconscious phenomenon (Piaget 1928b, p.111): a subject who is aware that a point of view is not differentiated from some other point of view is not in that respect egocentric since egocentrism arises precisely because a subject is unable to make that very distinction.

Thus displays of egocentrism are evident to an observer and not to the
subject in question.

4) There are levels of egocentrism and so it is not a subject as such but rather a subject's judgements that are egocentric. Thus egocentrism may be apparent at any of the three main levels of knowledge, namely, sensori-motor action, conceptualisation and formal thought (Piaget 1966, p.157; 1977c, p.349).

5) Egocentrism is manifest when a subject attempts to understand the spatial properties of an object, for example when presented with the Three Mountains task (Piaget & Inhelder 1956). But egocentrism arises through a subject's attachment to one point of view, whether or not that judgement is spatial. For example, egocentrism arises when a young subject attempts to understand transitivity - see (6.3) - where there is no particular influence exerted by spatial factors.

6) An egocentric judgement may, but does not have to, be incorrect. For example, a subject who fails to understand that there are more members of an including class than there are members of its major sub-class makes an egocentric and incorrect judgement; by contrast, the subject who claims that there are more members of that same super-class than there are members of its minor sub-class makes a correct, but presumably egocentric, judgement. (For a discussion of inclusion, see (6.4).)

7) Thus a subject's observable knowledge can still be knowledge - and not some other mode of cognition - and yet be egocentric, for example in a case where a subject correctly represents a given state of affairs. It is not clear, however, whether Piaget takes an incorrect, egocentric judgement to correspond to knowledge or not, since he is silent on this matter. There are two reasons for rejecting this identity. Firstly, Piaget's epistemology is one that assumes that a subject's knowledge is knowledge of reality, that the subject-object relation is primary (Piaget 1970a, p.703; Cellier 1973, p.16; Furth 1980, p.56). But in
that case a subject has no knowledge of reality corresponding to an incorrect judgement, though the subject may have (false) beliefs. Such a view is supported by the contention — discussed in (6.2) — that a subject may have (conceptual) knowledge only of that which is true and so may never know that which is false. Secondly, Piaget's (1971a, p.17) developmental theory envisages the growth of knowledge to be accumulative and integrative in that later stages of development embody all of the knowledge gained at earlier stages. But if there could be false (observable) knowledge, then such knowledge would be integrated into the cognitive system used by the subject at later developmental points. Yet the main point stressed by Piaget is that later stages remedy the omissions and failures of earlier stages. It may, then, be claimed that it is Piaget's view that all observable knowledge is egocentric but not conversely.

8) Observable knowledge is always inserted in some interpretative framework which is a subject's cognitive structure. That framework constitutes a subject's coordinatory knowledge, where a coordination includes necessary inferences and thus transcends the bounds of the observable (Piaget 1975a, p.50/*1978b, p.43).

Coordinatory — or operative — knowledge always incorporates an inferential component and it is for this reason that Piaget distinguishes two types of experience with correspondingly two types of abstraction and generalisation (which is discussed in (5.4) below). That is, observable knowledge arises only if a subject has coordinatory knowledge and conversely.

9) Any knowledge, claims Piaget (1969a, p.356) is tied to a structure and Piaget (Piaget & Inhelder 1971, p.366) distinguishes a scheme and a schema. The presence of an operative scheme allows a subject to generalise an action to a range of cases that are formally similar: a scheme is an instrument of generalisation. A figurative schema is an
instrument of presentation relevant to figuration or evocation. Piaget does not adequately explain this distinction. It may be suggested, however, that a scheme is analogous to the logical skeleton of an advanced, scientific theory and that a schema is analogous to a model of that theory. The distinction between a theory and a model is accepted by philosophers of science (Nagel 1961, p.90). One and the same theory (scheme) may be interpreted by recourse to alternative models (schema). It is presumably for this reason that different subjects represent the series of whole numbers in different ways, for example as stairs or zig-zags or vertical sticks (Piaget & Inhelder 1971, p.380). An image is therefore a representation of some item, whose schema relates that image to a logical form. The Kantian basis – see (2.2) – of a Piagetian schema is clear: a schema is that mediating link that results in the symbolic embodiment of some abstract principle. Action-schemes are further discussed in (4,5).

10) Thus the extent to which a subject is liable to make an egocentric judgement is tied to the logico-mathematical framework – or cognitive structure – used by the subject. The nature of that framework is shown in a subject's coordinatory knowledge, which Piaget sometimes refers to as operative knowledge. This consists in modifying an object or an event so as to grasp the actual transformations and their results and not merely as before the static configurations corresponding to the 'states' linked by those transformations (Piaget & Inhelder 1969b, p.87).

It must be noted that this definition is incomplete since it is Piaget's (Piaget & Voyat 1979, p.18) recent view that a subject's operative knowledge may arise in cases where a subject compares the states of an object and so where no transformations arise. Thus operative (coordinatory) knowledge arises when one point of view, corresponding to the observable knowledge gained on one occasion, is linked by a subject with some other point of view, corresponding to the observable
knowledge gained on some other occasion. That is, the subject connects one state of an object with some other state of that object and so the possibility actualised in one case is located in a cognitive system whereby that possibility is one of several inter-dependent possibilities. It is not claimed - see (3,4) - that a subject will consciously realise that this is so; it is claimed that an observer may attribute such an understanding to a subject on the basis of what a subject does.

Operative knowledge - including its sub-class of operational knowledge - consists in the coordination of actions. Piaget (1931, p.149; 1978b, p.52) sometimes compares such knowledge to a cine film which links discrete snap-shots (observable knowledge) into a continuous sequence that gives order and meaning. Operational knowledge is further discussed in (4,6) and (6,2).

In short, observable knowledge is incomplete because it is not coordinatory knowledge. It fails to incorporate the deductive and inferential properties of the coordinatory knowledge which is present for that observable knowledge to be possible at all. Because observable knowledge is incomplete, it can but does not have to, be distorting since a subject may overlook the logico-mathematical properties of objects. Thus a judgement is egocentric when a subject fails to place the observable knowledge gained on some occasion in some more comprehensive and interpretative framework. That failure is not apparent to the subject in question and is detectable only by an observer.

(3.6) Observable Knowledge: Consciousness

Consciousness is a second type of observable knowledge for Piaget and as such is taken to be an equally incomplete and distorted type of knowledge. The aim of the present section is to show why Piaget adopts this conception of consciousness and how a subject plays a role in the acquisition of knowledge. In essence, the section attempts to clarify the distinction drawn by Piaget between the individual and epistemic subject of knowledge.
Piaget accepts that consciousness as well as perception are forms of observable knowledge in the claim that

there is no real difference between becoming aware of action itself and the acquisition of knowledge of the sequence of events occurring outside the subject (Piaget 1974a, p.271/*1977c, p.342).

If perception is a form of observable knowledge and if there is no difference in essence between perception and consciousness, then the latter too is a form of observable knowledge.

The nature of consciousness is discussed by Piaget in connection with its role in a subject's development of knowledge. At this point a terminological discussion is necessary for Piaget contrasts a subject's prise de conscience with prise de connaissance which may be translated as 'becoming aware' or 'acquiring consciousness' and 'gaining knowledge' or 'acquiring knowledge' respectively. The former French expression is frequently mistranslated (Smith 1981a, 1981b) and is often taken to mean 'cognizance'. But a person may be cognisant of features belonging to the external world - which are, strictly, an object of knowledge for Piaget - whereas a subject is conscious of the features of his/her own action, namely its goal, which are internal to that subject.

In opting to use 'cognizance' Ginsburg & Opper (1979, p.175) and Vuyk (1981a, p.121) seem to overlook this distinction which is central to Piaget's claim that a subject has two available sources of knowledge in acting:

- consciousness of the goal to be attained, in other words consciousness of the intention in its capacity as the overall direction of the act, and the acquisition of knowledge of its outcome, as failure or success (Piaget 1974a, p.263/*1977c, p.334).

Furthermore, Furth (1980, p.59) promotes confusion by claiming that it is Piaget's view that a subject has awareness of an object and cognizance of action, which is evidently the reverse of what Piaget states.

Consciousness, states Piaget (1974a, p.261/*1977c, p.332), is not to be explained by metaphors of illumination. It is not an internal
organ of the senses. This denial is important, because it dissociates Piaget from the Lockean view that internal reflection is an inner sense. Nonetheless, the dissociation is incomplete since Piaget wishes to claim that consciousness is a form of observable knowledge. Rather, consciousness is the conceptualisation of the subject's knowledge at the previous developmental level, namely the action-schemes of the infant.

The acquisition of consciousness consists in making certain elements pass from an unconscious lower level to a conscious upper level,...and hence constitutes a reconstruction on the upper level of what was already organised, albeit in a different way, on the lower level (Piaget 1970b, p.16/*1974e, p.40).

An infant has observable knowledge arising from sensori-motor actions and so must also have coordinatory knowledge, as the discussion in (3.5) made clear. But in becoming aware of those actions a subject conceptualises them and the process of transference is not comprehensive and complete. For conceptualisation is a form of observable knowledge which must have, in its turn, associated coordinatory knowledge. But knowledge gained from sensori-motor action is distinct from its conceptualisation and so there is no reason to require that an automatic transfer of knowledge should occur. It is possible that the transfer might be reliable; but it is also possible that it might not, And it is Piaget's claim that this latter is the case.

An obvious case to cite to illustrate Piaget's view is the study of walking on all-fours. Certainly, this action can be performed by most subjects from infancy onwards and thus the request to older subjects to describe that action, even after a suitable demonstration as a reminder of how it is done, should be an easy request to meet if the former possibility holds, if transfer is reliable and automatic from action to its conceptualisation. The sequence of limb-movements could conform to a Z, N or X pattern but even adult subjects suggest that either of the former pair of this trio is correct, when in fact it is the latter, X pattern, which is correct (Piaget 1977c, pp.1-11). Thus
the presence of incorrect answers is indicative of the fact that the conceptualisation of action is liable to error and so there is no automatic and reliable transfer of knowledge. In general, Piaget (1966, p.202) claims that consciousness is incomplete and preferential. It is incomplete since a subject is not conscious of the genetic mechanism of knowledge acquired at the level of conceptualisation; it is preferential since a subject favours certain beliefs in contrast to others. Piaget also attempts to show why conceptualisation is incomplete and preferential, the latter because of a subject's attachment to the positive, and not the negative, properties of an object—see (5.2) and the former because the growth of knowledge proceeds from periphery to centre—see (5.3).

It may be noticed that some commentators misconstrue Piaget's account of consciousness. Flavell (1963, p.256) restricts the acquisition of awareness to cases where a subject can have knowledge of earlier acquired objects of thought, where a subject has cognitions of the subject's own cognitions. In this claim, he is apparently influenced by Bruner's (1959, p.370) contention that the acquisition of consciousness is the conscious manipulation of what was previously a working assumption. Such construals are, however, too intellectualistic, since they overlook the fact that consciousness is primarily the conceptualisation of practical knowledge displayed in sensori-motor action. Again, Ginsburg & Opper (1979, p.175) take consciousness to be a subject's explicit knowledge of his/her own thought processes but such a claim confuses the reflexivity of knowledge (if $S$ knows $p$, $S$ knows that $S$ knows $p$) with Piaget's claim that practical knowledge is the source of conceptualised knowledge. By contrast Vuyk (1981a, p. 121) clearly contrasts initial conceptualisation and the thematisation carried out by the formal operational subject.

Thus Piaget's account of consciousness is one that accords an
important role to cognitive repression which is taken to be analogous
to affective (Freudian) repression (Piaget 1974e, p.36): a subject
distorts his/her observable knowledge rather than adjusts the cognitive
mechanism - coordinatory knowledge - which generates it. Piaget
(1920, pp.54-8) adverts to the need to integrate psycho-analytical
with developmental concepts in an early paper and his recent study of
consciousness (Piaget 1977c, p.341) proposes the form that integration
should take. A subject does not make an observation or form a belief -
and then reject it. Rather, repression prevents a subject from making
certain observations or forming certain beliefs in the first place.
Piaget in fact uses a variety of terms to refer to the process of
becoming aware of a phenomenon, which is conceptualisation (Piaget 1975a,
p.142/*1978b, p.147), interiorisation (Piaget 1975a, p.86/*1978b, p.84),
construction (Piaget 1978b, p.64) or integration (Piaget 1978b, p.68).
Becoming aware is not a mere occurrence but is the gradual insertion
of incoming material into a cognitive system. Cognitive repression
is the reverse process whereby that material fails to be taken into
a subject's cognitive system.

It may be noticed, however, that Piaget's conception of consciousness
is one that traditional copy theorists, such as Locke and Hume, would
accept, for to be conscious is to form a mental image. Piaget does
not expressly state that this is so. But such a view may reasonably
be attributed to him. He does not deny - indeed, he is pleased to
admit (Piaget & Inhelder 1971) - that a subject forms mental images.
And Piaget (1953a, p.189; Piaget & Inhelder 1969a, p.129; Piaget 1970a,
p.717) takes there to be three types of signification, namely indices,
symbols and signs, which correspond to the three main stages of development,
namely infancy, childhood and adolescence. Since Piaget (1967b, pp.277-
78) denies that children can think in terms of signs, and since children
are taken by him to form images, there is apparently no way other than
by forming images for a child to become conscious. To be conscious, to conceptualise, to construct, to integrate, to interiorise and to form a mental image are, for Piaget, the same cognitive phenomena. Thus Piaget's objection to the copy theory is not that subjects do not form images but rather that any image formed

conveys what one requires it to convey...only because deductive intelligence can understand it, and not because graphic figures, perceived or imagined, are 'copies' of the object (Piaget & Inhelder 1971, p.385).

Piaget is therefore indebted to classical conceptions of consciousness, even if his theory delimits the role of consciousness in the growth of knowledge.

Piaget is therefore led to distinguish between the individual and the epistemic subject of knowledge. That there must be a subject of knowledge is the Kantian - see (2,2) - presumption accepted by Piaget:

an object exists, in so far as it is or can be known, only in its relations with the actions of a subject (Apostel et al. 1957a, p.45 - my translation).

And such a Kantian viewpoint would be accepted by philosophers (e.g. Armstrong 1968, pp.21-2) and psychologists who deplore the adoption of subjectless theories of psychological functioning. It may be noted, however, that the Kantian view is at variance with the proposal made by Karl Popper (1979, p.109), that the subject can be eliminated from epistemology since knowledge may arise which can be attributed to no subject at all. Whatever the merits of Popper's position, it may be doubted that a biological epistemology - which is one that Popper (1979, pp.24-5, 159) wishes to defend - can simply omit the subject in this way, if it is to be applicable to the growth of knowledge in developing, and not just developed, subjects.

Piaget contrasts the individual and epistemic subject thus:

there is the "psychological subject", centred in the conscious ego whose functional role is incontestable but which is not the origin of any structure of general knowledge; but there is also the "epistemic subject"
or that which is common to all subjects at the same level of development, whose cognitive structures derive from the most general mechanisms of the coordinations of actions (Piaget 1966, p.308; see also 1971b, p.139; 1972d, p.49).

The psychological subject is the individual subject and should not be confused with the psychological subject who develops and uses procedures (Inhelder 1978, p. 100; Vuyk 1981a, p.52). The individual subject can be uniquely identified by recourse to that subject's observable knowledge which is possessed by that subject alone. No doubt that knowledge will be indefinitely varied due to factors such as social setting, language, culture, task-content and so on. It will be recalled - from (2.5) - that Piaget expressly denied that his conception of psychology included a study of such individual differences.

By contrast, the epistemic subject can be identified by a subject's operative or coordinatory knowledge and that knowledge will be possessed by all other subjects at that level of development. Thus a subject's observable knowledge is variable due to factors in nature and nurture, unlike a subject's coordinatory knowledge which is invariant to nature and nurture.

Piaget's primary concern is with the epistemic subject (Wright 1980, p.313), even though many have apparently (Inhelder 1980b, p.13) been more concerned to investigate the individual subject. It is the individual subject who has observable knowledge whose acquisition is made possible by the epistemic subject's coordinatory knowledge. No individual subject has conscious knowledge of the knowledge of the corresponding epistemic subject. Only an observer may, therefore, characterise the knowledge of the epistemic subject. In the limiting case, at later stages of development, a subject may act as an observer of his/her own action-coordination.

In summary form, it is Piaget's claim that at any level in the development of knowledge, knowledge may be of two main types. Observable or figurative knowledge is accessible to the consciousness of the
subject and is knowledge of the states of objects. Coordinatory or operative knowledge is not similarly accessible to the consciousness of the subject but is knowledge of the transformations that relate different object-states. The transformations are physical when the physical properties of the object are transformed and logico-mathematical when a subject endows an object with logico-mathematical properties. The individual subject is the possessor of observable, the epistemic subject is the possessor of coordinatory, knowledge. (See Table 3.1)

TABLE 3.1: Schematic outline of types of knowledge at all levels in development.

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>KNOWLEDGE</th>
<th>OBJECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual</td>
<td>Observable - Figurative</td>
<td>States</td>
</tr>
<tr>
<td>Epistemic</td>
<td>Coordinatory - Operative</td>
<td>Transformations - Physical</td>
</tr>
<tr>
<td>Source:</td>
<td>Piaget 1971b, p.139</td>
<td>Piaget 1977d, p.67</td>
</tr>
<tr>
<td>Source:</td>
<td>1978b, pp.43-4</td>
<td>1969a, p.357</td>
</tr>
</tbody>
</table>

Note: For explanation, see text.

In conclusion, it is apparent that several questions have been generated by the account presented in this chapter. Firstly, it is clear that Piaget (1977d, p.67) sees action as the primary source of knowledge and so he firmly rejects a rationalist view which takes thought or an empiricist view which takes perception to be the primary source of knowledge. This difference is obviously important. Yet Piaget does not deny that perception and thought are bound up with a subject's actions. What, therefore, requires clarification is the conception of action used by Piaget. Secondly, the distinction between observable and coordinatory knowledge requires that an account should be given as to how an observer can attribute the latter knowledge to a subject.
and, in particular, to show what Piaget takes to be the psychological correlates of cognitive structures. Thirdly, the growth of knowledge requires the postulation of different types of cognitive structure and so an explanation is owed of nature and cause of structural change and, in particular, of the difference drawn by Piaget between functional and structural properties of psychological processes. Such are the topics for discussion in chapters 4 and 5. What the discussion of observable knowledge shows is Piaget's commitment to an alternative theory of knowledge, for the cognitive system would not be what the environment makes of it: it would be what it makes of what the environment makes of it (Piaget 1977e, p.16 - my translation).

What the environment makes of a cognitive system is shown in the observable knowledge gained by a subject; what the cognitive system makes of that observable knowledge is shown in the coordinatory knowledge of that subject.
COGNITIVE STRUCTURES

(4.1) Introduction

The central aim of the present chapter is to show the connection between Piaget's claim that action is the source of knowledge and his constructivist theory of knowledge. In section (4.2) the discussion concentrates on the five criteria stated by Piaget to be necessary for a phase of development to count as a stage. Two main features of this discussion are, firstly, that one and the same theory can describe a sequence of stages in a logically inter-dependent manner and also incorporate empirical evidence about the facts of development. Secondly, the discussion shows that Piaget is inconsistent in the identification of the number of stages invoked by his theory. Section (4.3) focuses upon the concept of action used by Piaget. An action is, in Piaget's psychological writings, always an intentional action; it is purposeful and not merely purposive behaviour. It is shown that Piaget's conception is a liberal one and covers motor and manipulative behaviour; as well as thinking in terms of concepts and language. In the sequel, the term 'action' should be taken in this broad sense, unless a qualification is introduced to identify a specific type of action. The claim that action is the source of knowledge breaks down, for Piaget, into the claim that practical, representational and propositional knowledge are three distinct types of knowledge which are developmentally linked and that each gives rise to a corresponding form of action-coordination. There are three types of cognitive structure for the three types of knowledge. In section (4.4) Piaget's account of a structure is described and the logical nature of cognitive structures is noticed. Sections (4.5) and (4.6) extend the discussion of (4.4) and concentrate on action-schemes and operational schemes respectively. An action-scheme is an instrument of operative knowledge, corresponding to a subject's practical knowledge.
An action-scheme is a practical concept which, in the absence of an ability to distinguish self and world, groups together a range of actions, rather than objects. The logical properties of action-schemes are shown. An operational scheme is used by a subject in the acquisition of deductive knowledge and as such is the coordination of interiorised actions. Piaget's claim that a subject understands deductive necessity when thinking is reversible is discussed and the distinction between the subject who uses a grouping-structure and one who uses a group-structure is shown to correspond to the difference between representational and propositional knowledge.

(4.2) Stages in the Growth of Knowledge

Knowledge, affirms Piaget (1971a, p.64; 1977d, p.1), is not a state or fact but a process and he proposes:

if all knowledge is always in a process of growth which consist in proceeding from less knowledge to a state that is more complete and efficacious, questions clearly arise of knowing this growth and of analysing it with the greatest possible accuracy (Piaget 1970d, p.13/1977d, p.6).

Knowledge is not a state in the advanced sciences since the history of science is one of scientific revolution and change. And knowledge is not a fact for the psychological subject, since empirical research shows that the cognitive instruments used by subjects at different developmental points are different. In particular, empirical research can establish just which logical norms are available for use by a subject at given developmental points (Piaget 1977d, pp.5,9). So if knowledge is not a state, it is a process and in consequence there must be phases in such a process which are open to description. In general, any state is such that it is the same throughout the duration of that state; by contrast, a process is such that change occurs throughout its duration, such that the process is incomplete during early phases of its occurrence (Armstrong 1968, p.130).

Evidently, Piaget appeals to such a conception when he proposes
that there are stages in the process by the occurrence of which knowledge grows. He is somewhat wayward in stating the criteria for the demarcation of such stages, sometimes offering three (Piaget 1971a, p.17) and sometimes offering five criteria (Piaget 1960, pp.13-4). The latter is preferable and is so taken by Brainerd (1978b), since the former is a shorter - and not inconsistent - version of the latter.

For a phase of development to count as a stage, all of the following criteria must be met:

1: any stage has its own defining structure;

2: the sequence of stages is invariant and remains the same for all inherited and environmental differences;

3: consolidation occurs since earlier stages prepare for later ones;

4: integration occurs since later stages presuppose all of the earlier ones;

5: each stage has its own partial and incomplete level of equilibrium.

Piaget does not say but it is apparent that such criteria are empirically necessary criteria: as a matter of fact, stages of development conform to 1 - 5. Nor does Piaget say whether the criteria are collectively sufficient. Piaget does not rigourously present his own viewpoint and in this case it will be prudent not to attribute the stronger claim, that the criteria are sufficient as well as necessary, to him. Several comments may now be made about these criteria.

Firstly, it is not claimed that a stage is precisely linked with the age of a subject, as Flavell (1963, p.20) points out. Nonetheless, commentators still try to link age with stage, if only because the age of a subject is a convenient indication of the subject's stage of development (Feldman & Toulmin 1976, pp.449-50; Brown & Desforges 1979, pp.107-8). Piaget's position is in fact analogous to that taken by developmental physiologists such as Tanner (1978, p.62) who stress the variability rather than the uniformity of the extent of development and age. Nonetheless, Piaget often specifies the typical age at which
a subject attains a given stage both in early (1928a, p.246) and recent (1980a, p.25) works. And in his own reviews, he sometimes claims that independent research has confirmed such age-findings (Piaget & Inhelder 1969b, pp.159-60). In this respect Piaget is surely too bold since specific age-findings are not essential to his theory. Firstly, Piaget's own research shows the variability of age and stage since some subjects do - at ages 6,0 and 6,6 - whilst others do not - at ages 6,0 and 6,8 - understand the inclusion-relation which is taken to be indicative of a major stage-attainment (Piaget 1952, pp.164-5 and 175-6 respectively). Secondly, and more fundamentally, Piaget admits that factors in nature and nurture are (empirically) necessary conditions of development. Yet Piaget restricts his theory to a discussion of another such condition, namely equilibration. It follows that the growth of knowledge can always be facilitated or impeded by factors other than equilibration and yet such factors, expressed in the individual differences of different subjects, are strictly beyond the scope of Piaget's theoretical concern. In short, the variability of non-equilibratory factors precludes the precise correlation of age with stage, if that correlation is stated on the basis of an equilibratory theory of knowledge. (For a further discussion of this issue, see (5.2) and (9.2).)

Secondly, criterion 2 states that the order of stages is invariant and criterion 4 states that integration occurs, so questions arise as to whether Piaget intends that such order is a logically necessary, and not just an empirical, regularity. No doubt Piaget is remiss in clarifying his position. What does seem clear, however, is that stages can be so defined that one stage is logically linked with another stage and yet the way in which subjects develop may be established only by appeal to the facts of development. Consider a hypothetical example so that questions about the correctness of Piaget's account do not cloud the principle on which it seems to rest. Suppose it is claimed that
the stages of development are A, B and C. Suppose further that each stage has a structure whose formal characterisation is isomorphic with the characterisation of propositional, predicate and modal logic respectively. Now it can be agreed that each is so defined that predicate logic presupposes the whole of propositional logic (Lemmon 1965, p.140) and that modal logic presupposes the whole of predicate logic (Hughes & Cresswell 1972, p.30). It follows that the characterisation of stage B necessarily includes that of stage A and that the characterisation of stage C necessarily includes that of stage B but that neither converse holds. What, then, is excluded by such characterisations of stages A, B and C? Clearly, in no case at all can a subject be at stage B in the absence of the achievements relevant to stage A and in no case can a subject be at stage C in the absence of the achievements of stage B. Nonetheless, there are several possible ways in which development could, as a matter of fact, occur:

1: **Steady State** A subject is at stage C at birth, and so has all the abilities of stages A and B. The subject remains in this state throughout life.

2: **Steady State - Regression** A subject is at stage C at birth, and so has all the abilities of stages A and B. Regression occurs, however, in that the subject passes in childhood to stage B, thus possessing the abilities of stage A but not those of stage C, and finally passes to stage A in adolescence.

3: **Steady State - Fast Regression** As for 2, except that when regression occurs, the subject passes from stage C to stage A directly, i.e. the subject is never at stage B in the absence of stage C abilities but in the presence of stage A abilities.

4: **Progression** A subject is at stage A in infancy at stage B - thus having the abilities of stage A - in childhood and at stage C - thus having the abilities of stage B - in adolescence.

5: **Fast Progression** A subject is at stage A in infancy and passes
to stage C, thus having the abilities of stage B, in childhood.

6: **Slow Progression** A subject is at stage A in infancy and childhood and passes to stage B - thus having the abilities of stage A - in adolescence and passes to stage C in adulthood, thus having the abilities of stage B.

No doubt, this list is incomplete. What is clear is that there are several possibilities here, even though the descriptions of the structures corresponding to each stage are formally linked. Thus a psychological theory can embody claims that such and such constructs are logically tied and can require empirical evidence to establish when, as a matter of fact, such constructs may be applied to describe the behaviour of subjects. It is not, then a "waste of time", *pace* Brown & Desforges (1979, p.90), to submit Piaget's theory to empirical scrutiny. Indeed, not only is such a conclusion supported by others (e.g. Boden 1979a, p.97) but it is even contended that psychology is flawed if it fails to investigate necessary truths since the role of empirical research in psychology is to confirm that a "theorem of common-sense" does have application in such-and-such a case or to show that method so-and-so is in fact a method appropriate for the evaluation of a given theorem (Smedslund 1980, pp.50,72). Moreover, the same conclusion has already been defended - in (2,3) - since it is legitimate to propose a statement with a contingent modal status (factual evidence) as a test of a statement with a necessary modal status.

To claim, then, that a sequence of stages is invariant is to claim that, as a matter of fact, development takes place in this way (for example, possibility 3) rather than that (any other possibility). It is also to claim that development is regular and lawful. As Inhelder (quoted in Furth 1981, p.23) puts it, the question to ask is "Under what laws does knowledge develop and change?" It is apparent
that Piaget's interest in the epistemic, rather than the individual, subject - see (3.6) - and his concern for the general, rather than the specific, features applicable to knowledge-acquisition - see (2.5) - are consonant with this.

Thirdly, the criteria state necessary conditions of the growth of knowledge and it is contended by Hamlyn (1975, pp.28-31) that such conditions are not sufficient: a sequence of events could satisfy the conditions and yet fail to be developmental. In particular, a sequence such as travelling from one place to another could satisfy the conditions - since arrival at the first place is a necessary condition of travel to the second place - and yet the sequence would not be developmental. Piaget's account is deficient, on this view, because it is limited to the statement of such necessary conditions alone. One comment to make about this contention is that Hamlyn apparently relies on Piaget's (1971a) shorter, rather than full (1960), account. A second comment is that Hamlyn seems to ignore criteria 1, 3 and 5 since it is not any sequence but rather only those sequences that can be described in a certain way - via the criteria stated - that count as developmental sequences. A third comment is that Hamlyn's own proposals are satisfied when all of Piaget's criteria are taken into account. Hamlyn proposes that any earlier condition should not merely be necessary for a later condition but should also constitute its rationale. Thus a claim about stage-development is really a teleological claim about what must occur for the sake of the occurrence of some end and Hamlyn cites with approval the analysis of teleological claims provided by Taylor (1964, p.9). Yet Piaget's criterion 3 meets the requirement stated. Consolidation occurs in that a later stage is made possible by an earlier stage and so the presence of an early stage is the reason why a later stage may occur. As von Wright (1971, p.58) puts it, a teleological claim is one that provides an answer to a question "How is X possible?" and any such claim will typically
state necessary conditions. Just as the question "How is blood-circulation possible?" is answered by the teleological claim "Because the heart-beat is for blood-circulation", so too the question "How is it possible for a subject to acquire the abilities corresponding to stage B?" is answered by the teleological claim "Because the abilities corresponding to stage A occur for the sake of those corresponding to stage B". Teleological claims are acceptable in biology and psychology and so Piaget's claims can be taken in such an acceptable sense.

Fourthly, the criticism of Piaget's stage-claims made by C.J. Brainerd (1978b, pp.173-74) is an ignoratio elenchi. Brainerd concedes that the claim that a subject is at a certain stage has a descriptive value but he denies its explanatory value on the grounds that Piaget fails to specify independent variables that are responsible for the changes that are associated with the stages that he specifies, variables that are antecedents of those changes and which are open to manipulation. As Wetherick (1978, p.205) observes, Piaget does not take his stages and their associated changes to be the consequence of independently observable, antecedent variables and so does not try to do what Brainerd criticises him for failing to do. Other commentators concur, for example by pointing out that Brainerd's argument assumes that intelligent behaviour can be atomistically described (Olson 1978, p.198), by his assumption that the "covering-law" model of explanation is the only such model (Berndt 1978; Olson 1978, p.197) or that re-description of behaviour is not a species of explanation (Kendler 1978, p.190). Piaget himself (1978b, p.191) admits that his theory has certain explanatory deficiencies; but it is evident that he (Piaget 1971a, p.328) takes these defects to be due to the lack of an adequate mathematical theory by reference to which his account of functioning can be made more perspicuous. And he would not take his theory to be deficient through its failure to specify variables whether in nature or in nurture.
There are two reasons why this is so. Firstly, Piaget states this when he claims that genetic and environmental factors constitute necessary, but not sufficient, conditions of development (see 5.2 below). Piaget does not deny the importance of such factors but he does deny that his theory should take them to be important to the exclusion of other factors relevant to development, namely equilibration. Secondly, since his claims are taken to be teleological rather than causal, it is a mistake in principle to suppose that the conditions relevant to causal explanation are the same as those relevant to teleological explanation. It is not claimed that Brainerd is wrong to suggest that there are independent variables by whose specification developmental changes may be explained since it is an empirical matter as to whether a phenomenon is to be explained by causal or by teleological laws. It is open to him to seek to specify such variables and so state the causal factors involved. But it is not open to him to claim that Piaget is precluded from explaining the same phenomena by appeal to a different type of explanation.

Fifthly, it is clear that criteria 1 - 5 are criteria based upon Piaget's theory of the development of knowledge. Thus 1 is incompatible with a non-structuralist theory, 3 and 4 are incompatible with a Steady-State or Regression theory and 5 is incompatible with an account that fails to use a concept of equilibration. It is evident, therefore, that further factual evidence could force the withdrawal of criteria 1 - 5. Such a possibility might be dismissed as a mere possibility in the absence of an alternative theory of knowledge-growth. It is, however, relevant to notice that Piaget himself is unclear about precisely how many stages there are. Sometimes he claims, in early and recent writings, that there are three stages (Piaget 1928b, pp.122-3; 1932, p.79; 1953c, p.380; 1970a, p.711; 1971a, p.17; 1977c, p.346; Piaget & Inhelder 1969a, p.152); sometimes he claims, in recent and non-recent writings, that
there are four stages (Piaget 1950, p.123; 1969d, p.117; 1979b, p.17; 1980a, p.25). Commentators similarly disagree, one group favouring the former claim (Rotman 1977, p.39; Inhelder 1956, p.76) and another group favouring the latter claim (Brainerd 1978a, p.95; Ginsburg & Opper 1979, p.26; Brown & Desforges 1979, p.23). Vuyk (1981a, p.192) notes the disagreement and opts for the latter claim as well. There is confusion in Piaget. What is especially confusing is the fact that Piaget's systematic study of the mooted "fourth" stage, the pre-operational period has been in recent times (Piaget et al. - 1968) - yet Piaget still claims that there are only three stages of development after that work.

What is important, however, is not merely the number of stages of development but rather the empirical correlates by reference to which a subject can be deemed to be in one stage rather than another. And that, in turn, raises the question of how an observer is entitled to attribute operative knowledge to a subject. That is, what features of development licence an inference by an observer that a subject has a cognitive structure, appropriate to one stage of development, rather than another? In short, by criterion 1 it follows that any stage has a structure but what are the features on the basis of which any such structure can be inferred? It is to this question that the remaining sections of this chapter are addressed.

(4.3) Action

Action has a double importance for Piaget both since it is action, and neither perception nor thought, that is the source of knowledge and since an observer can attribute operative knowledge to a subject only on the basis of that subject's actions. In this section, a sketch is offered to indicate the central features of the concept of action used by Piaget.

At the outset a distinction must be drawn between bodily movement
and action, a distinction which the concept of behaviour neatly conflates. This distinction has been extensively discussed in recent times (Hamlyn 1953; Melden 1961; Anscombe 1963; Taylor 1964; Armstrong 1968; Davidson 1971; von Wright 1971; McGinn 1979) and it is sufficient here to state that an action can always be intentional, unlike a bodily movement that cannot. When an action is intentional the person knows what the goal of the action is and can know whether the action attains that goal. A bodily movement, by contrast, occurs as the effect of a set of (physical) causes and its occurrence takes place irrespectively of the state of mind of the person. It is evident that behaviourist psychologists (e.g. Blackman 1980) still use a concept of behaviour in this latter, rather than the former, sense.

Piaget uses a concept of behaviour in the former sense but confusingly uses a family of expressions in so doing. Consider four examples:

**Action**: an action is any behaviour pattern...directed upon a goal from a given subject's point of view (Apostel et al. 1957a, p.43 - my translation);

**Conduite**: as far as behaviour patterns are concerned, assimilation is presented in the form of cycles of movements or acts (Piaget 1936, p.129/1953a, p.141);

**Comportement**: we understand by 'behaviour' the set of actions that organisms perform in relation to the external environment so as to change its states or their own situation in relation to it (Piaget 1976a, p.7/1979b, p.ix).

**Réponse**: a stimulus is assimilated to a scheme the manifestation of which constitutes the response (Piaget 1967a, p.25/1971a, p.8).

Piaget evidently uses these terms interchangeably. **Action** is explained in terms of **conduite**; **comportement** is explained in terms of **action**; and **réponse**, which is explained in terms of assimilation, is related to all of these. Interestingly, Piaget defines child psychology as the study of behaviour patterns, or behaviour and consciousness (Piaget & Inhelder 1966a, p.5/1969a, p.vii), where there is a manifest slide from **conduite** to **comportement**.

It is also clear that Piaget's concept of behaviour is one of
goal-directed behaviour. This is explicit in the definition of action and implicit in that of comportement. But there is an important distinction to be drawn (von Wright 1971, p.59) between purposive behaviour - for example a guided missile - where a goal is aimed at but not intended and purposeful behaviour - for example human action - where the goal aimed at is intended. It is claimed (Boden 1979a, p.128) that Piaget uses a concept of purposeful behaviour and this claim must be accepted, in the context of Piaget's psychological writings at least.

In support of this claim, consider that Piaget explicitly states that intentionality is the mark of intelligence and that the behaviour patterns of the infant during stages I and II are not intelligent for this reason:

> the behaviour patterns that we have described in the previous sections form a transition between the organic and the intellectual. They cannot yet be characterised as intelligent since they lack intentionality (Piaget 1936, p.112/*1953a, p.122).

Thus only the infant who engages in intentional action is claimed to be intelligent. Further, Piaget states that intentionality is not to be tightly restricted so that only subjects capable of symbolic thinking may act intentionally. Rather,

> intentionality is defined by an awareness of desire, or the direction of the act, such awareness being itself a function of the number of intermediate acts necessitated by the principal act (Piaget 1936, p.133/*1953a, p.148).

On this view, a subject who engages in means-end behaviour becomes aware of his/her own desires because the action corresponding to a desire can be performed only after the prior performance of instrumental acts that lead to it.

It may be noted that an intentional action is one that has both cognitive and affective elements: a person's intentional actions are related to his/her desires and beliefs. It is well-known that Piaget concentrates on the cognitive rather than the affective aspects.
of action, no doubt for the reason offered by Papert (1980, p.vii),
that affect is certainly not irrelevant but little is known about
it. What can be said is that a person who acts intentionally does
so through the presence of motivating factors internal to that
person rather than through the operation of externally existing
factors. Thus Piaget states that

```
every action involves an energetic or affective aspect
and a structural or cognitive aspect (1950, p.5)
```

and that

```
finality is thus to be conceived not as a special
category, but as the subjective translation of a
process of putting into equilibrium (Piaget 1953a,
p.11).
```

What this means is that a subject who acts intentionally does so for
his/her own reasons: a subject performs free actions directed as
goods set by that subject.

A subject acquires knowledge and does so by the performance of
actions. In what way is action the source of knowledge?

```
Action in and of itself constitutes autonomous and
extensive knowledge, for even if such knowledge
consists in "knowing how to do" and not conscious
knowledge in the sense of conceptualised understanding,
it nonetheless constitutes the source of this latter
since the acquisition of consciousness in almost
every case lags behind, often in a marked way, this
initial knowledge which is thus of remarkable efficacy
despite the fact that it does not embody a conceptualised
346-7).
```

Conceptualised knowledge, claims Piaget, is the outcome of a more
primitive type of knowledge consisting in a subject's knowing how to
do things. Such knowledge is practical knowledge and is illustrated
in an infant's knowing how to make a hanging doll swing (Piaget 1953a,
p.167). It will be recalled, from (3.6), that a subject has two
modes of access to the intentional action he/she performs, namely
prise de conscience and prise de connaissance. It will also be recalled
that these expressions are vulnerable to English mistranslation (Smith
1981a, 1981b). A subject acts and so becomes aware of the goal that action is directed upon and the subject gains (perceptual) knowledge of the success or failure of the action in attaining that goal. Such knowledge is referred to by Piaget — see (3.5) and (3.6) — as observable knowledge. It is knowledge that arises from within the action since the subject can always be aware of what that observable knowledge is. Observable knowledge conforms strictly to the Kantian requirement — see (2.2) — that a subject can actually be conscious of all of his/her knowledge.

But it is Piaget's claim that observable knowledge may arise only if a subject has coordinatory knowledge. Coordinatory knowledge is knowledge that a subject is not, in fact, conscious of though the subject can, in principle, become conscious of it at a later developmental point. Such knowledge arises between actions and consists in their coordination. Action-coordination has, however, a logic and it is the natural logic of action-coordination that gives rise to formal logic itself.

Even logic itself consists in a system of operations (classifications, seriations, placing in correspondence, using a combinatorial system or "transformation-groups", etc.) and the source of these operations is to be found really on the other side of language in the general coordinations of actions (Piaget 1967a, p.23/1971a, p.7).

Thus when Piaget says that intentionality is the mark of intelligence, his claim is that a subject's (intentional) actions display a natural logic in the coordinations made between them. A subject's coordinatory knowledge is to be characterised by an observer who infers from the subject's actions just which coordinations the subject does, and does not, impose upon them. Those coordinations may then be given a formal characterisation that is isomorphic with some formal system, which characterisation is a characterisation of the subject's cognitive structure.
In short, a subject who acquires knowledge is an agent who performs intentional actions. The subject has observable knowledge arising from within the actions performed. When, however, the subject performs a series of actions, action-coordination occurs and the subject has coordinatory knowledge arising from between the actions. Such knowledge can be given a formal characterisation by an observer and is the subject's cognitive structure. Thus Piaget uses a logical model to characterise a subject's structures, even though such structures are the source of logic itself.

It must be stressed that action-coordination is not to be taken as the mere outcome of neural organisation. Piaget, it will be recalled - from (3.2) - admits that possession of a brain is a (physically) necessary condition of knowledge-acquisition. But it is not sufficient, claims Piaget (1971a, pp.221-2) for it is possible to give a description of the brain that is isomorphic with a description of the cognitive structure used by an adolescent subject. Moreover, it is in general a mistake to suppose that neural factors can explain an intentional action as opposed to a bodily movement.

Thus it is Piaget's claim that action is the source of (practical) knowledge and that the latter gives rise to conceptualised knowledge. Moreover, conceptualised knowledge is itself of two main types, Piaget maintains. There is representational knowledge, where a subject employs a symbol that is distinct from what is symbolises but is a (partial) copy of it. Such knowledge may be displayed in overt activity, for example imitation or drawing, or in the formation of mental images as such (Piaget 1951, pp.273-87). A subject may communicate the observable knowledge that he gains through using language but though a subject does acquire a knowledge of language, Piaget denies (1951, p.280) that representative knowledge is the same as propositional knowledge. That is, propositional knowledge is a second and distinct type of conceptualised knowledge, occurring where a subject uses
signs that are both distinct from what is so signified and bearing only a conventional relationship to it. Propositional knowledge is acquired by the adolescent subject (Inhelder & Piaget 1958, pp.251-5).

Piaget's use of the concept of action is a liberal one, covering physical action in infancy, representational activity in childhood and hypothesis-formation in adolescence. A subject is an agent - that is, performs actions in this broad sense - when he/she makes a limb-movement, forms a mental image of an object or makes a statement about the world. The intentionality of action is not confined by Piaget to motor and manipulative interaction with the world but includes as well a subject's construal of the world both conceptually and in language. For Piaget, physical action is exteriorised action, whilst representational and propositionally based activity is interiorised action (Piaget 1978c, p.650; 1980b, p.xv).

**TABLE 4.1: Schematic outline of levels in the development of knowledge.**

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>ACTION</th>
<th>KNOWLEDGE</th>
<th>STAGE</th>
<th>SIGNIFICATION</th>
<th>STRUCTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Exteriorised</td>
<td>Practical</td>
<td>Sensori-</td>
<td>Index</td>
<td>Scheme</td>
</tr>
<tr>
<td>II</td>
<td>Interiorised</td>
<td>Conceptualised:</td>
<td>Pre/Concrete Operations</td>
<td>Symbol</td>
<td>Grouping</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Representational</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Interiorised</td>
<td>Conceptualised:</td>
<td>Formal Operations</td>
<td>Sign</td>
<td>Group</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Propositional</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


NOTE: for explanation, see text.

It is now apparent why Piaget takes there to be three stages in the development of knowledge. Exteriorised action gives rise to practical knowledge whose structure consists in the action-schemes at the disposal of the infant during the sensori-motor period. The infant uses an
index-based form of signification. The next level corresponds to the pre/concrete operational stage when a child may perform interiorised actions and so gain representational knowledge that is symbolic in character. The structure present in the child's actions, at the operational phase of this stage, is that of a grouping. A third level in development is reached at the formal operational stage when an adolescent may gain propositional knowledge that is based on a system of signs. The structure of such actions consists in a group. Table 4.1 summarises Piaget's position here.

It may be noticed that the present work follows Piaget's practice of leaving open the exact type of action under discussion. Thus any claim about action tout court is a claim about action at any of the three levels indicated and so claims about specific levels of knowledge lead to qualified references about the type of action relevant to them (for example, practical knowledge arises from physical or exteriorised action).

(4.4) Cognitive Structures

The aim of the present section is to state an account of the central features of a cognitive structure in the context of Piaget's theory. Sections (4.5) and (4.6) will then distinguish and discuss the two main types of structure identified by Piaget, namely action-schemes and operational schemes. In essence, these sections try to show the contribution of coordinatory knowledge and, in particular, to show the respects in which Piaget takes the growth of knowledge to be dependent upon the structures at a subject's disposal.

Piaget makes the following early claim about a structure:

what we call 'structure' is nothing but a form of organisation of experience, and a form that is continually subjected to revision under the influence of success and failure due to reality...between function which is constant and the content of experience which varies, it is necessary to conceive
of the existence of structures. . . . Whoever says structures, in psychology, does not say, in fact, innate ideas or a priori, but simply forms of organisation (Piaget 1931, pp. 149, 151, 154 respectively - my translation).

Function, which Piaget - see (5.2) - later refers to as functioning, is an invariant feature of knowledge (experience), whilst the content of that knowledge is a variable feature. In fact, a cognitive system is not one that is based on a subject's possession of innate ideas or of a priori concepts and so Piaget clearly distances his position from that assumed by rationalist philosophers and by Kant - see (2.2) and (2.3) above. A structure, then, is a mode or type of functioning, for functioning is instantiated in different ways corresponding to the different levels of knowledge-growth.

Piaget (1971a, pp. 139-40) states five defining features of a structure, in contrast to the incomplete (1966, p. 169) or ambiguous (1971b, pp. 3-16) accounts which he also states. It is apparent that the discussions of Piaget's view by Boden (1979a, pp. 18-9) and Vuyk (1981a, pp. 53-5) use the latter account.

Firstly, a structure is a whole made up of elemental parts. So defined, anything at all can have a structure and it is for this reason that structuralism is stated to be a method rather than a doctrine (1971b, p. 142). A whole can be characterised in terms of laws that are not applicable to its parts: '+', for example, can stand for the addition of elements (in elementary algebra) or their disjunction (in Boolean algebra).

Secondly, the parts of a structure enjoy no existence independently of that structure, and conversely. In this context Piaget's reference to law includes both the law (rule) of contradiction and the (natural) law of gravitation. For example, a pack of cards is a structure with individual cards as parts and neither can exist, qua pack of cards,
without the other. Yet the pack is not reducible to its members since different laws apply to each. Structural laws licence not merely transformations — which is Piaget's (1971b, p.11) early claim — but also correspondences, which is his considered (1978b, p.196; 1977i, p.352; Piaget & Voyat 1979, pp.20-1) claim.

Thirdly, structures differ in degree of generality:

we define "structure" as any logical link that is capable of assuming, consecutively or concurrently, the role of form or of content (Piaget 1972a, p.40 — my translation).

Evidently, Piaget here appeals to the logical distinction between form and content. Thus

(1) all men are mortal

is a content of which its more inclusive form is

(2) (x) (Fx⇒ Gx),

(read: take any 'x', then if it has property 'F', it also has property 'G') which is an expression in predicate logic. Yet (2) may itself be subsumed by

(3) (∀x ∈ M) p(x)

(read: take any 'x' which is a member of set M, then 'p(x)' is a true statement) which is an expression in set theory. (See Lemmon 1965 and Lipschutz 1964). Thus what is a form (2), for a content (1) may itself be a content for some richer form, such as (3). It is, then, claimed that a structure may be a form for some content and yet in turn be a content for some richer structure. Observable knowledge may be subsumed by coordinatory knowledge which may itself be subsumed by some more capacious type of coordinatory knowledge.

Fourthly, two structures may be isomorphic and so give rise to some richer structure whose laws of composition include both (weaker) structures. Fifthly, a sub-structure is a structure that is isomorphic with an integrated structure.
It is clear that Piaget takes a (cognitive) structure to be characterised by recourse to formal, logical systems. Indeed, he offers a clear statement of principle about his theoretical aim:

> it is important to ask what sort of correspondence exists between the structures described by logic and the actual thought structures studied by psychology... the psychologist for his part welcomes the qualitative character of logic, since it facilitates the analysis of the actual structures underlying intellectual operations as contrasted with the quantitative treatment of their behavioural outcome (Piaget 1953b, pp.xvii-xviii).

Piaget intends to use a logical model by the application of which the growth of knowledge can be explained. And he further states that the acquisition of any knowledge, including the practical knowledge of the infant, requires the subject's use of a structure.

All knowledge of objects, of whatever sort it is, is always an assimilation to schemes and such schemes comprise a logical or mathematical organisation, however elementary (Piaget 1967a, p.463/1971a, p.335).

Thus Piaget's claim — see (3.5) — that experience of the world requires a subject to use a logico-mathematical framework, is the claim that knowledge (experience) is always, and at all levels in development, tied to the schemes used by a subject, where a scheme is a (simple) cognitive structure. But if a structure is to be given (in part) a logical characterisation, what characterisation? Which logical systems are to be used, since there are many different such systems?

Two points can be noticed in replying to this question. Firstly, it is Piaget's central point, one that is clear in the early definition of a structure cited at the outset of this section, that there are different structures at different developmental points and so the logical properties relevant to one structure will be different from those relevant to another. Secondly, whilst Piaget hopes that the logical properties that he applies to different cognitive structures are the correct ones, he is prudent enough to notice that a cognitive
structure is distinct from its formalisation (Piaget 1971b, p.5); the theoretician may incorrectly characterise any such structure. Piaget is not, of course, saying that a logician will make logical mistakes in characterising a formal system; he is saying that the actual characterisation given by any logician, and a fortiori by Piaget, may be mistaken. It follows that a commitment to a structuralist theory of knowledge does not exclude a rejection of Piaget's formal description of any given cognitive structure.

A cognitive structure is, then, an instantiation of functioning, since Piaget takes functioning to be a property that is common to all types of organic and psychological life, whilst a structure is a particular type of functioning. The discussion of structure will continue in the subsequent two sections, whilst functioning will be further discussed in (5.2).

(4.5) Action-Schemes

An action-scheme is the simplest type of structure identified by Piaget. An action scheme (Piaget 1967a, p.22/*1971a, p.6) is a scheme used in the sensori-motor behaviour of the infant and so corresponds to practical knowledge (see Table 4.1). Clearly, such knowledge is also possessed by the child and adolescent. Piaget (1978b, p.7) also refers to action schemes as schemes of assimilation.

Salient features of any action-scheme may now be identified:

1) A scheme is a simple, operative structure attributable to infant, as well as to non-infant, subjects, simply because all other structures identified by Piaget are more complex and operative since it is an instrument of generalisation. A scheme is not a thing, object or entity of any sort (Piaget 1953a, p.388), even though some commentators (Brainerd 1978a, p.17) raise misleading ontological suggestions. It is equally mistaken to suppose that a scheme is a rhetorical flourish (Brainerd 1978a, p.19).

2) A scheme is an instrument of generalisation and is
whatever in an action is capable of being transposed, generalised or differentiated from one situation to another, in other words whatever there is in common between the varied repetitions or applications of the same action (Piaget 1967a, p.23/1971a, p.7).

A scheme is a similia in multis, the element in common to a multiplicity of cases. Thus distinct tokens of the same action (grasping) share the same scheme (Piaget 1953a, p.92), whilst more complex schemes are displayed in a range of action-types, for example means-end behaviour (Piaget 1953a, pp. 330-1). Ritualised and regularly occurring cases of social behaviour (playing marbles, having a meal) also share a scheme (Piaget 1932, pp.20, 43; 1951, p.103). To make the same point:

the scheme of an action in relation to a class of actions that are equivalent from a subject's point of view is the common structure that characterises that equivalence (Apostel et al. 1957a, p.46 - my translation).

There need be nothing in common in the tokens of the (possibly) distinct action-types that possess a scheme other than that they are displays of that scheme.

A scheme is a (logical) form which subsumes the content corresponding to different actions. An infant who knows how to swing a doll by shaking his/her legs can do this in an indefinite range of cases and not just in one case, even though the cases are each different. It follows that a scheme confers on a subject a capacity to make a response (Piaget 1970a, p.707): displays of that scheme in behaviour do not exhaust the subject's capacity to display that behaviour and non-displays of the behaviour do not exclude the presence of the capacity.

3) Action-schemes possess logical properties (Piaget 1977c, pp.350-1):

1: Identity The same scheme is present when a subject performs different action-tokens of the same type, for example an infant's grasping a finger on one occasion and then on another (Piaget 1953a, p.91).
2: **Union** Two distinct schemes may be activated in relation to the same object, for example an infant looks at his thumb and sucks his thumb (Piaget 1953a, p.27).

3: **Intersection** Two schemes may be applied together to one and the same object, which Piaget (1953a, p.106) refers to as reciprocal assimilation.

4: **Seriation** When means-end behaviour is present, an infant performs one action first in order to bring about later a certain goal, for example an infant strikes a hanging doll in order to make it swing (Piaget 1953a, p.167)

5: **Associativity** An infant who makes detours is one whose schemes have the property of associativity, for example an infant who moves on a triangular circuit symbolised as AB, BC, CA and then makes the same circuit by moving AC, CB, BA (Piaget 1954, p.197).

6: **Inversion** An infant who can follow an itinerary, symbolised as ABCDEF, and then return to starting point by following the itinerary FEDCBA is one whose schemes are inversely related (Piaget 1954, p.206).

7: **Inclusion** Different objects may belong to, and so may be members of, the same scheme and one scheme may be included in another, for example an infant's looking at a pipe is included in looking at father (Piaget 1953a, p.239; 1977j, p.20).

4) A scheme is not a concept. This is not just because Piaget (1972c, p.25) allows that a concept may be possessed only by a subject who can represent an object in a symbolic way but rather because the infant is incapable, claims Piaget (1954, p.92) of distinguishing an object from an action directed upon it. A scheme is a practical concept and groups together a range of actions in contrast to a concept which groups together a range of objects (Piaget 1977f, pp.44, 69-70). Moreover, a scheme has properties that are analogous to those applicable to a concept, namely intension and extension. The intension of a concept
is its meaning and its extension is the cases to which it is applicable (cf. Stebbing 1930, p.27). It is evident that this distinction requires a subject to know that an object is distinct from an action, which it is hypothesised that the infant is unable to do. Thus Piaget (1980a, p.165) claims that a scheme has intension but no extension, intension because in acting on an object a subject confers (indexical) meaning upon that object, but not extension because the infant fails to differentiate self from the world. Piaget expresses this point as follows:

the extension of a scheme is the union of the extensions of the actions of which it is a scheme. 
The intension of a scheme is the scheme itself...

It is therefore futile to seek to define the meaning of a scheme. As far as its intension is concerned, a scheme "has" a meaning only for an observer; for the subject it "is" a meaning (Apostel et al. 1957a, pp.48-9 - my translation and emphasis).

Since an action-scheme is not a concept, an infant who has practical knowledge does not thereby have conceptualised knowledge and, a fortiori, does not have a conceptualised knowledge of the inclusion of classes (Inhelder & Piaget 1964, pp.9-10).

5) An action-scheme is not an object of thought that an infant subject reflects on or otherwise represents in his/her mind, as Piaget (1972c, pp.25-6) points out. To claim, then, that a scheme is a representation of an event that occurs within a subject (Brown & Desforges 1979, p.43) is to misrepresent Piaget's position. In general, no subject is aware of the coordinatory knowledge available to that subject, yet a subject's coordinatory knowledge is the cognitive structure used by that subject.

An action-scheme is a simple cognitive structure, defining what an infant can do and not what the infant is aware of. It is, therefore, by means of an infant's sensori-motor behaviour that an observer may ascertain which action-schemes are, and which are not, at a subject's disposal. To make the same point: an infant's practical knowledge can be attributed to him/her by an observer in cases where the infant
performs a corresponding physical action. As Inhelder puts it:

structures are inherent in the activity of the subject and are defined as "what he knows how to do" and not "what he thinks of this" (Inhelder 1978, p.100 - my translation; see also Inhelder & Piaget 1958, p.260).

This claim is a general one and is applicable to both exteriorised and interiorised actions - see Table 4.1 - but it can be illustrated here by a discussion of the infant's sensori-motor behaviour (exteriorised action). An infant performs an action. If the infant performs that action, the infant has a corresponding capacity: ab esse ad posse valet consequentia, as traditional philosophers would say.

Practical knowledge is, however, distinct from a (practical) capacity, for the action may have occurred by chance or the outcome of the action may have been a fortuitous one. Consider an example:

as early as 0,2 (26) Laurent, in whose right hand I have put the handle of a rattle, shakes it by chance, hears the noise and laughs at the result. But he does not see the rattle and looks for it in the direction of the hood, at the place from which the sound usually comes. When he finally sees the rattle he does not understand that this is the object which is making the noise nor that he himself made it move (Piaget 1953a, p.166).

In this case, Laurent does shake the rattle and so he can shake the rattle. But Piaget denies that Laurent knows how to shake the rattle since his successful action is an "inadvertent success" and Laurent does not respond appropriately to the rattle when it is later presented to him. An action that is an "inadvertent success" is like a guess: someone who correctly guesses the answer to a problem is not someone who can be relied upon to give the same answer to that problem on some other occasion.

There are two reasons why exteriorised action is tied to practical knowledge. The first reason is that a subject acquires practical knowledge only by performing such an action and so, for Piaget, a subject who never performs some action lacks the corresponding practical knowledge.
The second reason is that an observer may attribute a practical knowledge to a subject only if the subject performs the appropriate action. Thus Piaget (1953a, pp. 32-6) states that practical knowledge arises in cases where a subject repeats an action or where a subject recognises that the same action is appropriate in other situations or where a subject generalises an action to analogous situations. In such cases a subject acquires knowledge and in such cases an observer can attribute knowledge to a subject. The element that is common to these cases, and which distinguishes a practical capacity from practical knowledge, is the subject's response to changes in content.

The sole criterion for the internal character of a structure is the occasion when there can be supplied laws of composition that are capable of functioning in a formal manner such that they remain the same through changes in content (Piaget 1967b, p. 318 - my translation; see also Apostel et al. 1957a, p. 49).

For example, an infant is presented with a centrally pivotted bar, on one arm of which is a toy. If the toy is placed on an arm adjacent to the infant (see Fig. 4.1), no doubt the infant may retrieve the toy. If, however, the toy is placed on the most distant arm, an understanding of rotation is required for successful retrieval, namely that pushing away on one arm produces the same outcome as pulling towards on the other arm (Piaget 1953a, p 284; 1954, p. 196; 1977b, p. 293).

Thus an infant who has practical knowledge of rotation is able to retrieve the toy "through changes in content", that is, whether the toy is placed on either arm and not merely on the nearest arm. The structure of the action, which may be given a perspicuous logical characterisation by an observer, is capacious enough to cover such
variations in content.

Two final comments may now be made about Piaget's conception of a scheme. Firstly, there is an evident Kantian basis to Piaget's conception since a scheme is a form that can be extended to cover a range of cases, unlike a schema which is an instrument for the allocation of a form to a symbolically presented case. Roughly, the Kantian contrast – see (2.2) – of *a priori* concept/schema corresponds to the Piagetian contrast of scheme/schema. Adequate comprehension of Piaget's conception has, however, been flawed in two main ways, namely Piaget's own presentation and English mistranslation. Certainly, Piaget is careless in the presentation of his own theory since his initial, indifferent use of *schème* and *schéma* (e.g. Piaget 1928b) is later replaced by an unsignalled preference for the former of these (e.g. Piaget 1936) to mark a subject's operative structure. Since the German *Schema* is given in French as *schème*, and in English as *schema*, a reader might have been forgiven for supposing that a *schème d'action* is to be construed as a Kantian *schema*. Such a supposition would be incorrect; yet it is only in recent works (Piaget & Inhelder 1966a, p.431) that this supposition is explicitly excluded. Moreover, Piaget's presentation is careless in other ways since he distinguishes between different types of scheme but without always clearly marking the differences. Thus Piaget (1978b, p.156) refers to conceptual schemes, to infra-logical (spatial) schemes (Piaget & Inhelder 1956, p.449) and to operational schemes (Inhelder & Piaget 1958, p.105). No doubt his defence would be that his constructivist theory requires that initial structures are subsumed by later ones. It is at this point that English mistranslation arises (Smith 1981a, 1981f, 1981g). For Piaget's technical use of *schème* has been inconsistent, since interchanged with *schéma*, even though translation has used the English *schema* for both (see Appendix A: Piaget 1936, p.12; Piaget & Szeminska 1941, pp.6, 108; Piaget & Inhelder 1948, p.532; Inhelder & Piaget
Conversely, Piaget's recent and consistent preference for the French *schème* is sometimes given by the English *scheme* (Piaget 1977c) and sometimes by *schema* (Piaget 1978a) and this when such works are companion volumes! Commentators sometimes disregard the distinction (Rotman 1977, pp. 38, 153; Boden 1979a, pp. 17, 26, 34, 93, 110; Brown & Desforges 1979, pp.24-5), even though the distinction is explicit in Furth (1969). Thus English translation has prevented the reader from detecting both inconsistency in Piaget's account and consistency in that account.

Finally, Piaget has recently identified a further type of scheme and the same two defects recur: Piaget himself (1976d, pp.286-7) does not fully chart the relations between the new and old types of schemes and *schème* is mistranslated as *schema* (Piaget & Voyat 1979). Piaget's distinction between presentative, procedural and operational schemes is not discussed here, though salient features are indicated by Vuyk (1981a, pp.175-7). It is clear that Piaget's discussion is extended by the research currently being pursued by Inhelder (1976, 1978), concerning the psychological, rather than the epistemic, subject.

(4.6) **Operational Schemes**

Operational structures are a sub-set of operative structures and Piaget distinguishes two main types, namely cognitive and infra-logical groupings (Piaget 1966, p.174) and group (Piaget 1970a, p.723). Piaget (1967b, p.281) even affirms that there is no property at all that is irreducible to such structures.

An operational structure can be given a formal description. Thus a grouping can be defined through five axioms - combinativity, reversibility, associativity, identity and tautology - in contrast to a group which has only the first four of these (Piaget 1950, pp.40-2; 1970a, p.723). Since a formal system can be stated by different sets of axioms, it is not intrinsically objectionable that Piaget (1972a,
sometimes offers seven axioms. Clear discussions of these structures are provided by Flavell (1963), Brainerd (1978a and Ginsburg & Opper (1979).

Invariance (conservation) is taken by Piaget (1952, p.3; 1980c, p.41) to be the criterion for the presence of a structure. For example, a subject who understands that physical objects exist independently is one who understands the invariance of an object's existence to a subject's actions. Piaget further states the following criterion for the presence of an operational structure:

\[
\text{inferential necessity indicates the closure of an operational structure (Piaget 1975a, p.121/*1978b, p.124; see also 1980c, p.41).}
\]

A subject's structure is operational only in those cases where the subject makes a deductive inference the necessity of which is experienced by that subject. Piaget stresses the fact that an operational structure is manifested in both a subject's consciousness and in behaviour. For example, transitivity appears as "necessary" and such logical "necessity" is not simply identified as an inner feeling — which is impossible to verify — but in the subject's intellectual behaviour (Piaget 1967a, p.337/1971a, p.316).

Piaget does not deny that a subject is aware of the necessity of a deductive link; he does deny that an observer may verify occurrences of that awareness and so asserts instead that an observer may attribute an operational structure to a subject on the basis of that subject's behaviour.

Clearly, the actions corresponding to operational structures cannot be sensori-motor in nature. Sensori-motor action gives rise to practical knowledge and such actions are externally displayed. The infant who pushes one arm of the pivotted bar and then pulls back that arm performs two, discrete actions. Attribution of practical knowledge does not require that an infant should conceptualise the relations of those
actions. By contrast, conceptualised knowledge is such that a subject could understand that the two discrete actions are inter-connected, that pushing and pulling are inversely related. Conceptualised knowledge requires, then, a subject's performance of interiorised actions.

The difference may be illustrated as follows. An infant has practical knowledge of flowers if the infant knows, for example, how to pick or smell or hold a flower and can respond differentially to different flowers. The infant's practical knowledge is made possible by the use of action-schemes. A young child may use a conceptual scheme and use language to describe an array of flowers. The child might correctly classify the flowers as flowers or correctly judge that the flowers fall into two groups, the roses and the daisies, or correctly draw a picture of the flowers on the basis of an ability to represent them. That is, a conceptual scheme is one that is used in a subject's conceptualisation of the world, in the symbolic representations used by the subject, or, in general, in that subject's consciousness (see (3.6) above). But such a child would have an operational structure, only if he/she could correctly understand the deductive relations that hold between such phenomena, for example that the extension of a subclass is necessarily smaller than that of its including class. A subject who uses an operational structure knows not only that there are more flowers than there are daisies in some array but also that there must be more flowers than daisies, if the latter class is included in the other. Thus Piaget (1950, p.168) points out that the content of two interiorised actions might be the same and yet their form, or structure, might be different, for example if one subject uses a conceptual and the other an operational scheme. What is crucially different in these cases is the presence of deductive relations in the thinking of the subject as such, as Piaget stresses in recent (1980a, p.32) as well as non-recent (1950, p.143; 1952, p.162; 1971a, p.14) writings.
It was noticed in (3.4) that subject might use a (logical) norm, even in forming a conceptualisation of the world, and yet not be aware of its presence in thinking. It is the observer who attributes an operational structure to a subject and so even though that subject may experience a feeling of necessity — in making a deduction — the subject will typically not know the nature and extent of that coordinatory knowledge. The presence of an operational structure allows a subject to apply a logical norm, such as transitivity, to cases and so the subject understands instances of transitivity, not the principle of transitivity itself.

Piaget's position is often stated by him as the claim that an operation — or operational scheme (Furth 1981, p.56) — is an interiorised action that is reversible. That is, all operations are interiorised actions but not conversely. A third feature of an operation is that any operation forms part of a system of operations. That there can be no singular operations follows from the previous feature, namely that one operation is reversible by another. Finally, an operation is required for a subject to think in a deductive, and so logical, manner. Thus a subject whose thinking is logical is a subject whose thinking is, in that respect, identical to the thinking of any other subject at all whose thinking is, in that same respect, logical. Thus an operation is a characteristic of the epistemic, rather than individual — see (3.6) — subject (Piaget & Inhelder 1969a, pp.96-7). The central feature here is that of reversibility, since it is the presence of that feature which is taken by Piaget to demarcate deductive from non-deductive thought and Piaget distinguishes between three inter-related phenomena here. An infant may have practical knowledge of reversing (renversement) since instrumental behaviour requires a means to be performed prior to its end and so the order of actions has to be reversed (Piaget 1953a, p.148). Return-behaviour (retour) is another instance of practical knowledge. A child may
have conceptualised knowledge of reversals (renversabilité), since a child might understand that the water level of container A will be the same if all of the contents of A is poured into container B and then back into A (Apostel et al., 1957b, p.44; Piaget 1967c, p.533). A child may have a conceptualised knowledge of reversibility (réversibilité) only if his/her conceptualised knowledge is such that the subject understands that there is an exact correspondence between an object's positive and negative properties (Piaget 1975a, p.116/*1978b, p.119).

Piaget's conception of reversibility can be comprehended in the following way. When an infant performs an exteriorised action, the action is no longer at the infant's disposal after its performance: the infant who pushes the arm of the pivotted bar away can only cancel the effect of that action by performing some other action such as pulling the arm towards him/her. It is Piaget's (1977j, p.116) claim that a pre-operational child performs interiorised actions in the same irreversible way. Given an array of flowers, most being daisies and the rest being roses, such a subject might be asked if there are more flowers or more daisies. An interiorised, irreversible action is one such that the subject classifies the daisies as daisies but then precludes their further classification as flowers by taking the flowers to be what are left when the daisies are removed. Irreversibility consists in the subject's failure both to characterise the daisies through positive, observable features and to characterise them negatively and inferentially as (in this case) the flowers in the array which are not roses. (Class-inclusion is further discussed in (6.4) below.)

Two types of reversibility are identified by Piaget. Inversion (Piaget 1966, p.176) consists in the annulment of one operation by its inverse and allows an initially chosen item to remain unchanged whatever the operations directed upon it. In the flowers-example, a subject who can add the class of daisies A, to the class of roses, A', to form
the class of flowers, B, and who can subtract class A' from class B to form class A performs a reversible operation. The classes remain unchanged through the operations directed upon them. Given an operation of the form \( B = A + A' \), then \( A = B - A' \) is its inverse as too is \( A' = B - A \). Reciprocity is the other type of reversibility and consists in the elimination of a difference (Piaget 1953b, p.29) which may occur in one of three ways: (a) by exchanging terms, (b) by reversing the relation or (c) by both (see Fig 4.2). The effect of these is to yield an equivalence, in (a) and (b), or the original operation, in (c). Thus a subject who sees that container A is taller than container B but that the latter is smaller than the former shows reciprocity (type (c)) in his/her thinking. Further discussion of reversibility in connection with operational intelligence is provided in (6.3) and (6.4) below.

<table>
<thead>
<tr>
<th>Mode of Reciprocity</th>
<th>Relation</th>
<th>Reciprocated</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) terms</td>
<td>( A &lt; B )</td>
<td>( B &lt; A )</td>
<td>( A = B )</td>
</tr>
<tr>
<td>(b) relation</td>
<td>( A &lt; B )</td>
<td>( A &gt; B )</td>
<td>( A = B )</td>
</tr>
<tr>
<td>(c) both</td>
<td>( A &lt; B )</td>
<td>( B &gt; A )</td>
<td>( A &lt; B )</td>
</tr>
</tbody>
</table>

Fig. 4.2: Modes of reciprocity (Source: Piaget 1966, p.177)

An operational scheme is, then, a scheme in whose absence a subject does not understand deductive necessity. It will be recalled, however, that Piaget – see Table 4.1 – distinguishes between two levels of conceptualised knowledge and so between two types of operational structures, namely grouping and group. A grouping is a structure that is limited in its application. For example, a subject who uses conceptual schemes correctly classifies flowers, daisies and roses and the subject who uses an operational scheme correctly understands the deductive relation holding between a sub-class and its including class. Such a subject, however, will not thereby understand that any class has a logical complement such that the complement of an including class is
smaller than the complement of one of the sub-classes of that including class (Inhelder & Piaget 1964, p.291; Piaget 1977a, pp. 82-3). A grouping allows a subject to make limited inferences only which are exclusively of a step by step form (and) express, in effect, a beginning of deductive power, not yet freed from concrete manipulations and only proceeding thus by means of contiguous over-lappings without achieving a combinatorial system (Piaget 1966, p.174).

A second limitation is that a subject who uses a grouping is one who thinks in terms of representations and images rather than in language. Thus children, aged 6 – 8 years, may well give the correct answer to a question that apparently involves an understanding of a rule of inference such as modus ponens ("If John is in school, then Mary is in school. John is in school. What can you say about Mary?") and yet not think in terms of language. It is important to dissociate what comes from language and what language allows as concrete evocation by imagery in general. The question is: about what do the children reason? Do they reason about situations which they succeed in evoking, in imagining, or do they reason by combining verbal terms (Piaget 1967b, pp.77-8 – my translation)?

For a subject to think in verbal terms, it is necessary for the subject to be able to formulate hypotheses and to deduce the consequences that follow, or do not follow, from them. That in turn requires a subject to be able to formulate statements about what is possible and not merely about what is actual. In particular, a subject who uses a group-structure is one who can think combinatorially about any possibility at all, who can formulate statements (propositions) corresponding to such possibilities and who can think in a reversible system about them. The importance of deductive necessity is again stressed: instead of merely introducing initial forms of necessity, as is the case of concrete inferences, formal thought brings about from the outset the synthesis of the possible and the necessary and so deduces with rigour conclusion from their premises (Inhelder & Piaget 1955, p.220/* 1958, p.251).
That is, a subject who uses a group has available a combinatorial system such that the subject may ascertain, in respect of any hypothesised state of affairs, what is and is not compatible with it, what is and is not excluded by it and what is and is not a deductive consequence of it. A combinatorial system of this sort enables a subject to inter-relate systematically instances of the modal concepts of possibility, impossibility and necessity.

Piaget's logical characterisation of such a system is given a detailed and formal articulation, which is taken to be 'an elegant and comprehensive treatment of formal operations' (Neimark 1975, p.542) as well as being the best available account of adolescent thinking (Shayer 1979, p.225; Neimark 1979, p.61). One feature of the account is that each of the sixteen operations (conjunction, disjunction, etc.) which hold between two propositions with binary truth-values can be uniquely specified by its normal disjunctive form (see Table 4.2). For example,

<table>
<thead>
<tr>
<th>OPERATION</th>
<th>NORMAL DISTINCTIVE FORM</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Complete Affirmation</td>
<td>(p * q)</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>2. Complete Negation</td>
<td>(o)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>3. Disjunction</td>
<td>(p v q)</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>4. Conjoint Negation</td>
<td>(-p &amp; -q)</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>5. Incompatibility</td>
<td>(p 1 q)</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>6. Conjunction</td>
<td>(p &amp; q)</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>7. Conditional</td>
<td>(p q)</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>8. Non-Conditional</td>
<td>(-p q)</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>9. Inverse Conditional</td>
<td>(q p)</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>10. Inverse Non-Conditional</td>
<td>-(q p)</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>11. Bi-Conditional</td>
<td>(p q)</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>12. Exclusion</td>
<td>(p w q)</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>15. Affirmation q</td>
<td>q [p]</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

Source: Piaget 1972a, p.254

Truth-values: T = True
F = False
It is apparent that when both 'p' and 'q' are true operation 5 alone does not hold, since that combination also holds in seven other cases (1, 3, 7, 9, 11, 13, 15). On the basis of what is comprehensively observed to be the case (T) and what is not (F), it is possible to ascertain which operation cannot be excluded. Piaget (1966, pp.180-1; Piaget & Inhelder 1969a, p.135) is not saying that a subject himself uses the notation of propositional logic, nor that the subject will formulate the normal disjunctive form of an operation. Rather, an observer can characterise the group-structure used by such a subject in this way. A subject will not typically be aware of the logical norms used by him/her – see (3.4) – even in the case of a subject's propositional knowledge.

Table 4.3: The INRC group of transformations

<table>
<thead>
<tr>
<th>Direct Operation (l)</th>
<th>Negation (N)</th>
<th>Reciprocal (R)</th>
<th>Correlate (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. p * q</td>
<td>(o)</td>
<td>p * q</td>
<td>(o)</td>
</tr>
<tr>
<td>2. (o)</td>
<td>p * q</td>
<td>(o)</td>
<td>(p * q)</td>
</tr>
<tr>
<td>3. p v q</td>
<td>-p &amp; -q</td>
<td>p v q</td>
<td>p v q</td>
</tr>
<tr>
<td>4. -p &amp; -q</td>
<td>p v q</td>
<td>p &amp; q</td>
<td>p v q</td>
</tr>
<tr>
<td>5. p</td>
<td>q</td>
<td>p &amp; q</td>
<td>p v q</td>
</tr>
<tr>
<td>6. p &amp; q</td>
<td>p</td>
<td>q</td>
<td>-p &amp; -q</td>
</tr>
<tr>
<td>7. p → q</td>
<td>p &amp; q</td>
<td>q → p</td>
<td>-p &amp; -q</td>
</tr>
<tr>
<td>8. p &amp; -q</td>
<td>p → q</td>
<td>-p &amp; q</td>
<td>q → p</td>
</tr>
<tr>
<td>9. q → p</td>
<td>-p &amp; q</td>
<td>p → q</td>
<td>p &amp; -q</td>
</tr>
<tr>
<td>10. -p &amp; q</td>
<td>q → p</td>
<td>p &amp; -q</td>
<td>p → q</td>
</tr>
<tr>
<td>11. p ≡ q</td>
<td>p ≡ q</td>
<td>p ≡ q</td>
<td>p ≡ q</td>
</tr>
<tr>
<td>12. p w q</td>
<td>p w q</td>
<td>p w q</td>
<td>p w q</td>
</tr>
</tbody>
</table>

Source: Piaget 1972a, p.258

Piaget claims that the subject who uses a group, and not just a grouping, can think in a way that combines the two types of reversibility
in one use. The INRC group of transformations is the structural basis of this capability (see Table 4.3). Any proposition is identical (I) with itself, has a negation (N), a reciprocal (R) and a correlate (C). Given a proposition (I), N is the transformation leading to its negation, R is the transformation that negates the terms of that proposition (for example, the reciprocal of direct operation 6 is \(-p \& -q\)') and C is the transformation which is the negation of the reciprocal (for example, \(p v q\) is the negation of operation 4 and the correlate of operation 6 since it is the negation of the reciprocal of operation 6). In short, the INRC group enables a subject to interrelate a proposition, its negation, converse and negation of that converse (Piaget 1966, p.182; 1972c, pp.47–8; Piaget & Inhelder 1969a, p.139).

Piaget's account of formal operational thinking has been the subject of complaint. Some (Parsons 1960; Ennis 1975; Hann 1977) complain that Piaget's use of (extensional) logic is contradictory, or at least deviant. It may be briefly noticed, in reply, that logicians such as Papert (1963) and Wermus (Piaget 1972a, addendum) deny this charge and provide a formal description of Piaget's operational structures. Piaget, it seems, employs conventional logic in a possible, even if non-standard, way. A second complaint, initially made by Flavell (1963, p.428) and recently confirmed by Bynum et al. (1972) and Weitz et al. (1973), is that Piaget nowhere provides sound evidence for the claim that an adolescent uses all of the sixteen operations, even though Inhelder & Piaget (1958, p.104) suppose themselves to have provided such evidence. This complaint is evidently a valid one and weakens, though does not destroy, Piaget's account. A third complaint is that Piaget's account makes claims that are too ambitious: the adolescent is supposed to be able to think formally about any hypothesis at all, which is just not the case. Such a complaint is given its conspicuous presentation in the research carried out by
Wason (1966, 1977) Johnson-Laird & Wason (1977) and Wason & Brooke (1979). Piaget could deflect this complaint in several ways: by appeal to the fact that it has been anticipated by his associates (Matalon 1962), by appeal to his own recent investigation of propositional thinking (Piaget 1977a, pp. 81-114) or by appeal to the claim (Piaget 1972e) that a cognitive structure sets limits to what a subject can do rather than to what a subject actually does do.

Piaget, however, is his severest critic. Inhelder & Piaget (1958, p. 254) state that a group-structure enables a subject to think about any possibility (l'ensemble des possibles). Yet the notion of the set of all possibilities is antinomic: such a set is itself but one possibility (Piaget 1975a, p. 174/*1978b, p. 182; 1977h, p. ix; 1980a, p. 25). To make the same point: the development of knowledge is not a process that leads to some fixed terminal state — the stage of formal operations, marked by a group-structure — but is open-ended. The development of knowledge is a process of indefinite change and there is no reason to suppose that scientific thought terminates in some complete and comprehensive description of the world. Thus Piaget's account of formal thinking is deficient in its exclusion of this pervasive feature of knowledge. Piaget's recent research on modal concepts (Piaget 1976d/Piaget & Voyat 1979; 1977g) is an attempt to show how this deficiency can be repaired and a summary account of this on-going research is provided by Vuyk (1981a, pp. 179-82).

Piaget's account of concrete operational thinking has also been the subject of extensive criticism, which will not be pursued here. Two case-studies are, however, presented in chapter 6 where an evaluation of Piaget's and critics' accounts of a subject's understanding of transitivity and inclusion is offered.

Finally, it can be noticed that Piaget's structuralist account is one that sets limits on the knowledge available to a subject: no subject can acquire knowledge of that whose comprehension requires the use
of a structure that is stronger than the structure used by the subject. It is for this reason that Piaget refers to vertical decalage or "intension-lags": with respect to a given level of development — say, sensori-motor action — a subject who has knowledge at that level will not thereby be able to acquire knowledge at the next level — say, conceptualisation. Quite distinct is horizontal decalage or "extension-lags": with respect to a given level — say, conceptualisation — a subject who has knowledge arising in one case may not thereby gain knowledge in a case that is formally similar to it. This contrast is explicit in Piaget's early (1928b, p.126) and recent (1969d, p.126) writings, where Piaget contends that though lags of these sorts do occur, there cannot be a general answer to a question as to why specific lags occur. But it follows from this admission that Piaget's structuralist theory must be coupled with a constructivist account which identifies why it is that a subject is unable to generalise a structure to all cases, at a given level, to which it is applicable. Further, such an account must also explain the genesis of a new structure, one that is richer than a structure used at a lower level of development. It was stated in (2.4) that Piaget's theory is one that requires there to be both structure and genesis and the discussion of functioning — in (4.3) — stated that invariant functioning is compatible with structural differences. In short, if a structure has its own degree of equilibrium, what is needed is an account which shows how any such degree of equilibrium can be transcended by the formation of a new structure that possesses a greater degree of equilibrium. What is needed, therefore, is an account of equilibration, or structural change. Such is the topic for discussion in the next chapter.
119. (5.1)

EQUILIBRATION

(5.1) Introduction

The present chapter will discuss the question "How does the development of a subject's cognitive structures occur?" by outlining the answer proposed by Piaget. Quite simply his answer will be: by a process of equilibration.

Piaget first made use of a concept of equilibrium in his novel *Recherche* (published in 1918) and has used that concept in his psychological work since the 1920s, for example (Piaget 1928a). His first attempt to present a systematic articulation of this concept (Apostel et al. 1957b) has not been translated into English and is taken by Flavell (1963, p.244) to be preliminary and tentative. Whilst his revised attempt (Piaget 1975a) is taken by Inhelder et al. (1977, p.10) to be 'the definitive version of his theses', the English translation of this work (Piaget 1978b) is marred by mistranslation (Smith 1979, 1981b; Furth 1981, p.254). It will be convenient to concentrate on Piaget's recent statement of his position.

At the outset an objection must be considered, that Piaget's concept of equilibrium is so much 'surplus baggage' and may be replaced by the notion of reversibility (Bruner 1959, p.365). Acceptance of this objection would make the present chapter futile and so a comment is needed for its assessment. Firstly, Piaget denies that reversibility and equilibrium are equivalent, since the former is a logical and the latter is a causal concept. The former is used to mark the presence of deductive knowledge at later developmental points; the latter is used to describe how the structure used by a subject at any developmental point is attained. It follows that the latter is a concept with a wider application than the former. Secondly, a distinction should be drawn between a process and a state, between the state of (partial) equilibrium and the process of equilibration leading to that state (Piaget 1971a, p.23). The presence of a
cognitive structure allows a subject to gain knowledge. But the equilibrium of early structures is partial and is incapable of allowing a subject to evade contradiction-mistakes. In consequence such structures lead to their own development and, necessarily, lead to the formation of more stable structures possessing richer forms of equilibrium. Such a process is equilibration. It is evident that since every structure has a genesis, on Piaget's view, an account of genesis is needed. In this way is the account of equilibration important to Piaget.

The discussion will be in three parts. In section (5.2), an exposition will be given of the main sub-processes identified by Piaget in his account of equilibration and so of the roles of disequilibrium, disturbance, regulation, compensation and equilibration. In section (5.3), the equilibratory models specified by Piaget will be discussed and the functional role of observable and coordinatory knowledge will be reviewed. In section (5.4), the roles of abstraction and generalisation will be discussed, since the occurrence of these processes is taken by Piaget to be allied to that of equilibration. Finally, in section (5.5) a preliminary evaluation of Piaget's account of equilibration is presented.

(5.2) Sub-Processes of Equilibration

At the outset it must be said that Piaget's account of equilibration has not been well received and that even Piaget (1972, p.19) admits that he has not been entirely clear about equilibration. The present section is an attempt to provide a clear statement of the main principles of Piaget's recent account.

How does a subject acquire knowledge? Now one main factor in cognitive acquisition is a subject's ability to understand contradiction. There are two ways in which a subject will show an impaired understanding of contradiction and so make contradiction-mistakes, firstly, by
taking to be contradictory that which is not so and, secondly, by falling into self-contradiction when that which is contradictory is taken to be compatible. It follows, then, that the extent to which a subject can acquire knowledge is limited by the ability to understand what is the contradictory of what. Piaget puts this point in an early claim, that we know that if A and B are contradictory we have to choose between them, but we do not know in advance if that is what they are, for the principle of contradiction tell us nothing about that (Piaget 1928c, p.175 – my translation; see also 1980b, p.286).

What the principle of contradiction prescribes is the joint occurrence of A with its contradictory B; what the principle does not describe is just which are, and which are not, its instances. Thus a subject's search for coherence consists in an attempt to extend his/her coordinatory knowledge, for it is coordinatory, and not observable, knowledge that allows a subject to understand logico-mathematical properties. Contradiction-mistakes arise when a subject's coordinatory knowledge precludes the subject's comprehension of the mistake. In this special sense, it is not a contradiction-mistake for a subject to make an error in reasoning such that, once its presence is indicated, the subject can correct it. A contradiction-mistake is such that a subject who makes one is incapable of correcting that mistake, even when another (more advanced) subject indicates its presence.

Piaget's claim is that a subject's structure sets limits to what can, and cannot be known by that subject (Inhelder 1978, p.101). A contradiction-mistake occurs when a subject tries to comprehend that whose comprehension requires the presence of a structure that is more spacious than the structure actually used by the subject. Thus the search for coherence is a search whereby logical norms, such as the principle of contradiction, are applied by a subject to a greater range of cases, when the search succeeds, but whereby a failed search
is one that results in contradiction-mistakes.

Three types of case are identified by Piaget where contradiction-mistakes arise: where an action leads to an outcome that is incompatible with an anticipated outcome, where classes are incompletely opposed and where an incorrect inference is made (Piaget 1980b, p.288). The examples already stated - as (B) and (C) of (2,3) - are alternative ways of making the same point, namely that subjects do not always know what is, and is not, the contradictory of what. Piaget's most general definition of contradiction states that contradiction consists in incomplete compensation between affirmations (attributing quality a to class A) and negations (the attribution of not-a to the complementary class A', given B = A + A' (Piaget 1974d, p.156/1980b, p.288).

A subject can understand the positive properties of an object by virtue of the observable knowledge at his/her disposal. But such knowledge does not allow a subject to understand the negative and inferential properties of an object. To use the flowers example of (4,6): to know that an object is a daisy is not thereby to know that (for a given array) it is also a non-rose-flower. Contradiction-mistakes arise when a subject is unable systematically to inter-relate the positive and negative properties of an object.

At this point, it must be stressed that equilibratory factors, the internal factors of the formation of knowledge (Piaget 1980c, p.18), are not taken to be the sole factors. Genetic, physical and social factors are also taken to be necessary (external) factors as well, as Piaget (1960, p.3; 1970a, pp.719-26) points out. What should also be stated, though in fact it is not, is that such factors are empirically necessary: as a matter of fact, in the absence of genetic factor X, physical factor Y and social factor Z knowledge does not arise. Furth (1981, p.207) takes such factors to be 'absolutely necessary', which claim blurs the distinction between logical and
empirical conditions. Further, equilibratory factors are taken by Piaget to be present, only if factors of all of the other three types are present. That is, the presence of equilibratory factors is an (empirically) sufficient condition of the joint presence of the other factors. Nonetheless, equilibratory factors are themselves only an **empirically necessary** condition of the formation of knowledge, as Furth (1980, p.58) confirms and as Piaget (1970a, p.726) implies. Such factors are the central and necessary factors in the acquisition of knowledge; they coordinate factors of the other three types; but the joint inter-play of all four types of factor does not, in itself, lead to development.

Equilibration is, then, an internal condition of the formation of knowledge and it is by reference to this process that Piaget tries to clarify the opaque claim - see (4.4) - that functioning is an invariant feature of life or that organisation - see (2.5) - is the *ipse intellectus* of Leibniz (1949, p.111; cf. Piaget 1953a, p.2). Functioning is taken by Piaget (1971a, p.141) to be different from function. The latter is used in its biological (Nagel 1961) sense since an item has a function when its activation has a role in tending to ensure preservation. Piaget wants to claim that functioning is adaptive, that it has a function. Physical systems have functions, for example the blood circulatory system, and so cognitive systems can have functions as well. But in that case a problem arises, as commentators (Rotman 1977, p.116; Boden 1979a, p.119) point out. Lower biological kinds, such as insects, are better able to survive than are human subjects. How, then, can it be maintained that cognitive organisation is adaptive? There is an answer to this question. Insects are well adapted to some range of environments. But adaptation requires adaptability and not just adaptedness. Thus however large the number of **actual** environments to which insects are adapted, insects
have no understanding of the infinitely large number of possible environments to which an organism could be adapted. By contrast, man can acquire some conception of the possible, for example at operational levels of development, and so a cognitive organisation that is marked by the presence of operational structures (in man), unlike mere biological structures (in insects) increases the probability of the organism's survival (though it never, of course, guarantees survival). Knowledge that the actual conditions which currently do prevail are a sub-set of the conditions which could possibly prevail makes possible adaptive behaviour on the part of the organism that possess such knowledge, even if such knowledge does not necessitate adaptive behaviour. Thus functioning is adaptive and the use of richer structures does have (some) survival-value.

Certainly, Piaget takes organic and cognitive phenomena to form a continuous biological series - see (2,4) - and his discussion of the functioning invariants is explicit in this respect. Assimilation and accommodation are functional processes, characteristic of any biological organism. Any such organism is an open system whose interactions with the environment can be shown schematically by

\[
(1) (A \times A') \rightarrow (B \times B') \rightarrow (C \times C') \rightarrow \ldots \rightarrow (A \times A')
\]

where A, B and C are elements in the organism and A', B', and C' are elements in the environment. The latter are liberally construed and cover properties such as energy as well as objects qua bearers of properties (Piaget 1971a, p.177). The system shown by (1) is open since no limit is placed on interaction. But an open system can still be closed, though only if its cycle can be shown by

\[
(2) (A \times A') \rightarrow (B \times B') \rightarrow (C \times C')\rightarrow \ldots \rightarrow (A \times A')
\]

An environmental change occurs, since B' replaces B', but in (2) this change does not preclude the system's reversion to one of its previous states, (A x A'), since a corresponding change in the organism,
(C₂ x C), occurs as well. Thus (2) is a system which preserves its own identity but does not exclude change.

Assimilation is integration (Piaget 1972f, p.4) or the feeding of a structure (1971a, p.178) or the incorporation of objects into an action (1953a, p.6) or the subordination of reality to a structure (1980c, p.42). Assimilation is always assimilation to a structure and since a structure is always a structure of action (in Piaget's broad sense), a subject assimilates in acting on the world. But a subject acquires knowledge in acting and so assimilation has an epistemic role.

The notion of assimilation fuses into one whole every use of an object or an element... a scheme of assimilation confers a certain meaning on the objects assimilated and therefore assigns definite goals to the actions which have reference to them (Piaget 1975a, p.24/*1978b, p.18).

Knowledge is a relation linking a subject and object and arises when interaction occurs. A subject's understanding of an object is shown by the action(s) which the subject directs upon that object and so the content of an assimilation is given by the specific character of the action(s). A subject's action may be of different types, corresponding to the three levels of knowledge - see Table 4.1 - and any such action has a corresponding structure. Yet a structure is an instantiation of functioning, which is taken - see (4.4) - to be the invariant feature of experience, of a subject - object interaction. Thus assimilation, which is a functional invariant, is present if action is present; the specific type of assimilation will depend upon the structure used by the subject.

Accommodation is the inverse (Piaget 1953a, p.7) of assimilation since the assimilation of an object requires a change in that which an object is assimilated to, namely a structure. Assimilation and accommodation are twin poles of an action; they do not correspond to different actions (Piaget 1975a, p.45/*1978b, p.39). Since no two
objects can have all their properties in common - two objects cannot
differ solo numero - even successive performances of the same action
require some change in the structure of that action. Thus if there
is action, there is assimilation; and if there is assimilation, there
is accommodation as well.

That is, Piaget's claim is that in acting a subject integrates
particular cases into a general framework which is a cognitive
structure (assimilation) and differentiates that framework to take
into account the specific features of the case in question (accommodation).
A subject is cognisant of neither of these processes as they occur
in that subject. On this conception, a subject has an inherent
capacity to generalise the structure that he/she uses but a capacity
that is restricted by defeating conditions in the cases to which it
is applied.

Functioning is an indefinite process which terminates only with
the death of the subject - when that subject ceases to be an agent
of intentional action (Piaget 1978b, p.5). Functioning is, however,
motivational in the sense that a subject is always active:

the notions of assimilation and accommodation imply
in effect the necessity of continuous functioning,
to assure the alimentation of the first of these
and submission to the second (Piaget 1975a, p.180/
1978b, p.191).

Piaget thus ensures that a subject is always active and so is always
engaged in a search for coherence, though not necessarily in a
successful search (Piaget 1975a, p.13/1978b, p.7). Such a search
would end if assimilation and accommodation were in equilibrium
(Piaget 1953a, p.6). But equilibrium does not occur. And one central
reason for its non-occurrence is a subject's incapacity to avoid
contradiction-mistakes, in the bias within the cognitive system for
the understanding of an object's positive, affirmative rather than
negative, inferential, properties. Contradiction-mistakes are the
expression of disequilibrium in the cognitive system.
What then must be explained is the increasing coherence attained by a subject, the construction of new structures in whose absence new knowledge cannot arise. What is required is a theory of equilibration that shows how disequilibrium leads to equilibrium:

disequilibria at the beginning are a fact and since the search for coherence is another...an explanation is owing of the transition from the first of these to the second, which is the proper concern of a theory of equilibration (Piaget 1975a, pp.20-1/1978b, p.15).

What such an account must do is to specify

the mechanism of transcendence of previous structures by the construction of new ones....(which is) an integration of what is transcended in the transcendence (Piaget 1977e, pp.7, 136 - my translation).

It is clear from the discussion of Piaget's stage-criteria - see (4.2) - that a structuralist theory is complementary to a theory of equilibration.

What, then, are the equilibratory factors proposed by Piaget?
In essence, his claim will be that:
A: a subject makes contradiction-mistakes in cases where that subject fails to inter-relate satisfactorily observable properties with inferential (negative) properties and so the presence of contradiction-mistakes is a necessary condition of a disturbance;
B: the occurrence of a disturbance is a necessary condition of the occurrence of a regulation, where a disturbance occurs because of a subject's failure to attain the goals of his/her own intentional actions;
C: Regulations occur when a subject's later actions are modified by earlier ones and the presence of regulations is a necessary condition for that of compensation;
D: Compensation occurs when a subject is able systematically to inter-relate an object's total set of properties, observable and inferential, and its presence is a necessary condition of the occurrence of transcending forms of equilibration.

It will now be instructive to consider each of these in turn.
A. Disturbance

The presence of a contradiction-mistake is an (empirically) necessary condition of that of a disturbance and so the absence of contradiction excludes there being a disturbance even though its presence does not necessitate one. Thus Piaget is not committed to the obviously false claim that a subject will be disturbed by any and every contradiction-mistake, false because a subject is not so impeccably logical. When a contradiction is disturbing, it will be of one of two types. One type is constituted by the resistance of objects as a result of which an action-goal is not attained. A second type consists in the non-satisfaction of a subject's needs and so is a gap, as when some item whose presence is required for an action-goal to be attained, is absent. Thus disturbance arises either because of the presence of objects in the one case or because of their absence in the other (Piaget 1975a, pp.24-5/1978b, pp.18-9).

The point is evidently that a subject who complacently accepts, in the case of the discs experiment (see (2.3) above), both that $A = G$ and that $A \neq G$ is one who does contradict himself but who is not disturbed by that fact. A subject who is disturbed by that fact is thereby a subject who is motivated to act to (try to) eliminate that contradiction, by performing an action that makes consistent that which is inconsistent. It is an individual matter as to what is a disturbance for a specific subject on a specific occasion. What can be claimed is that a disturbance is need-related and so is a motivating force by its occurrence (Piaget 1978b, p.176). Moreover, it is a force that does not necessitate elimination of the contradiction, whilst also making such elimination more probable by its very occurrence.

B. Regulation

The presence of a disturbance is an (empirically) necessary condition of that of a regulation. Thus all regulations involve
Piaget defines a regulation in the following way: A regulation occurs when a subsequent performance, A', of an action is modified by the results of its initial performance, A, and so by a return effect of the results of A on its new development A'. A regulation can then occur as a correction of A (negative feedback) or as its reinforcement (positive feedback) (Piaget 1975a, pp. 23-4/*1978b, p.18).

Piaget distinguishes between two types of regulation. A regulation (régulation) is an automatic reaction of which the subject has no awareness. By contrast, an adjustment (réglage) is available to a subject's consciousness. The former occurs at sensori-motor levels where conceptualisation is not required for successful action to occur. Adjustments do require the subject to conceptualise and so to comprehend in an orderly manner the organisation inherent in (successful) action (Piaget 1978b, p.21). It would seem that the definition of a regulation just given is to be taken as a definition of adjustment as well.

Schematically, Piaget's position is that a regulation mediates the performances of actions. A useful distinction to use here is that between tokens and types. Thus one action-type will be displayed in an indefinite range of tokens, so a regulation mediates two action-tokens of some action-type. What a subject does on one occasion regulates what he/she does on a later occasion. The point, then, is that if action-token A leads to a disturbance then regulation occurs when action-token A' is modified by the results of A. Some commentators (Vuyk 1981a, p.150) claim that a regulation is simply the modification of A into A' rather than the modification of A into A' because of the affects of A. A regulation consists either in negative feedback in that A' is a correction of A, for example by removing the obstacle taken to prevent the success of A, or in positive feedback - the reinforcement of A - when A' is an attempt
to repair a deficiency taken to prevent the success of A. A regulation embodies a retro-active process, since $A'$ is a token of some action-type of which A is token, as well as a pro-active process, since $A'$ is modification of A.

It is claimed that a regulation, whether positive or negative, consists in a negation. In the case of correction, a subject will remove obstacles or modify a scheme by preferring one movement, or movement with a certain force, to another: this movement, not that one, with this degree of force, not that degree, in this way, and not that. Equally, a reinforcement involves negation since the presence of a gap, defeating the successful performance of A, requires that $A'$ should fill the gap: such and such is not there, but so and so is there. Thus in either case a subject in whom a regulation occurs is one who does grapple with the negative properties of the situation in which he/she acts and does not simply react to observable properties. Yet, in Piaget's view, a subject who does react preferentially to observable properties is one who will fail to inter-relate them to their corresponding negative properties, since the negative, inferential properties of an object, are not open to observation.

C. Compensation

The presence of a regulation is an (empirically) necessary condition of that of a compensation: the absence of the former excludes the presence of the latter. Piaget defines a compensation as being an action in a direction contrary to a given effect and one that therefore tends to annul or neutralise that effect (Piaget 1975a, p.32/1978b, p.26).

It is clear that Piaget here alludes to his notion of reversibility. A compensation is complete only when a subject uses an operational structure (Inhelder & Piaget 1958, p.246). The definition does not, however, exclude there being cases of incomplete compensation, as when a subject uses a non-operational structure. Compensation by
inversion consists in the annulment of a disturbance, that by reciprocity consists in its neutralisation. Thus, faced with a disturbance, a subject who can form an inverse or a reciprocal is one who can act and so compensate that disturbance. But since an inverse or a reciprocal are logical in nature, such a subject is one who can successfully inter-relate an object's observable with its negative, inferential properties. For example, in the discs experiment a subject who understands that disc B is not, contrary to observation, equal in size to disc A and to disc C but is in fact larger than A and also smaller than C is a subject who understands that a certain inferential property applies to disc B: if B is larger than A then A is smaller than B, and if B is smaller than C then C is larger than B, and so A is smaller than C or C is larger than A. In this way does compensation by reciprocity neutralise the disturbance that generated the contradiction.

Evidently, compensations introduce coherence. Piaget makes it clear that coherence within a deductive system of ideas is a final form of coherence of which coherence between a set of actions is its genetic antecedent. The latter is not to be taken as a truncated version of the former but rather as being that whose presence is required for coherence of the final mature type to be imposed at all. Only the subject who can display coherence between actions is one, claims Piaget (1977e, p.48), who can understand how there can be coherence within a deductive system.

D. Equilibration

The presence of compensations is an (empirically) necessary condition of that of equilibration. Piaget here draws a distinction between re-equilibration and equilibration that involves transcendence. The former occurs in any case where the elimination of a contradiction occurs by the use of a structure that in fact was used in the action corresponding to that disturbance in the first place. For example,
a subject in a Training experiment may show an understanding of conservation in the training period but fail to transfer that learning in the test-period. Quite distinct is equilibration which results in the construction and use of a new structure. Following Piaget (1975a, p.9/1978b, p.3) the former is (simple) re-equilibration, whilst the latter is upper bound equilibration (equilibration majorante). Some translators refer to this latter as 'increasing' (Piaget 1978b, p.4) or 'improving' (Vuyk 1981a, p.68) or 'accretive' (Piaget 1980a, p.35) equilibration. But it may be argued (Smith 1981b, 1981f) that the logical insight used by Piaget is lost by these latter translations. The notion of an upper bound (Lipschutz, 1964, p.154) is used by Piaget (1972a, p.90; 1970a, p.723) and can be illustrated by set-inclusion. Consider three sets, where sets A and A' are included in set B. Then B is an upper bound of A and A' since B is that smallest set which includes both A and A'. Piaget can appeal to two properties of this notion. Firstly, the extension of a structure is enlarged, when upper bound equilibration occurs since it may be applicable to a greater range of objects: the objects in set A are now also objects in set B. In general, the use of a new logical framework increases the extension of a structure. Secondly, the intension of the structure changes since the structure itself changes corresponding to the change in its extension: set A is included in set B. In general, the systematisation of the logical links between items improves the intension of a structure. It follows that upper bound equilibration is both conservative and constructive, the former because all objects in A will also be objects in B and the latter because set-inclusion is a richer relation than set-membership.

Piaget specifies six levels at which equilibration occurs. Equilibration at the first of these levels is physical equilibrium, consisting in simple compensation, as when a physical system conserves its equilibrium due to the intervention of forces equal and opposite
to a given disturbance (the Le Chatelier-Brown effect is cited by Piaget). Second level equilibrium also concerns physical phenomena, as in the case of dissipative structures studied by Prigogine, where not all the chemical reactions produced within a system are produced at the same speed so that one part of the system is privileged at the expense of another. Here posteriorly occurring changes conserve those that have previously occurred. Third level equilibration concerns organic phenomena, consisting in the reproduction and multiplication with conservation of an inherited programme and the possibility of occurrence in different environments without loss of that programme. The final three levels are all cognitive rather than biological, level four being that where an organism acts on its environment rather than exchanging energy with it, level five requiring the presence of representation and interiorisation and level six corresponding to scientific thought and the formal articulation of structures (Piaget 1977h, pp.iii-vi; Piaget 1972f, pp.12-13).

Consider the final three levels of equilibration, which Piaget variously refers to as types alpha, beta and gamma (1978b, pp.65-9) or as equilibration that is intra, inter and trans (1977e, p.17). It is evident that these three levels are related to the three levels of knowledge — see Table 4.1 — and correspond to a subject's acquisition of practical, representational and propositional knowledge. By contrast, Vuyk (1981a, pp.66-7, 163-5) apparently takes the first two of these three levels to occur at the sensori-motor level only, whilst Furth (1981, p.272) does not specify levels of knowledge at which such processes occur. No doubt there can be agreement that Piaget fails here, as at many of the key points in this theory, adequately to state his position.

Thus level four equilibration occurs when a subject is faced with a disturbance due to the presence (or absence) of objects. If the disturbance is large, the subject may annul it by neglecting the
disturbance, in which case neither retro-active control nor anticipation are possible. A small disturbance may lead to a response that is contrary to the disturbance. The infant—see Fig. 4.1—who pushes the pivotted bar away so that the other portion of the shaft may move towards him/her displays such a response. Equilibrium occurs at this level as an equilibrium between assimilation and accommodation of action-schemes.

Level five equilibration is the equilibrium of a sub-structure and consists in the integration of a disturbance into a structure which is modified by that incorporation. In the discs-experiment a subject who classifies the discs not as "big" and "small" but rather as "smaller" and "bigger" is one whose structure integrates a disturbance in a way that leads to the acquisition of new knowledge. Representation is thus necessary for this type of equilibration. At this level a subject succeeds in using partial negations: a subject inter-relates only some of the observable and inferential properties in question.

Level six equilibration consists in the anticipation of any possible variation within a system: a deductively inferred variation no longer constitutes a disturbance at all. The stability of this type of equilibration is to be judged by the fact that it is taken by Piaget to be resistant to disturbance since in no case at all can a structure be required to change on grounds of external outcome. Piaget's account of equilibration tries to provide an adequate characterisation of the internal modification that occurs when a subject's structures undergo development. Faced with contradiction-mistakes which constitute a disturbance, a subject acts so as to modify his/her responses—though without being aware of this—and so later actions are regulated by the earlier ones, either as corrections which seek to avoid what is taken to be an (observed) obstacle to the attainment of a goal or as reinforcements which seek
to rectify what are taken to be (observed) deficiencies similarly
preventing goal-attainment. Such actions show the presence of
compensation when a subject can show, in behaviour, an understanding
of an inverse or a reciprocal and when this occurs transcending
equilibration takes place, resulting in an enrichment of structural
states. Thus is the acquisition of knowledge presented as being
not the accumulation of experiences but rather as the integrative use
of past experiences so as to be in a position to have new ones.

(5.3) Equilibratory Models

The distinction drawn between an observable and a coordination
is a fundamental one for Piaget.

An observable is what may be verified in experience
by an immediate reading of the given facts themselves,
whilst a coordination includes necessary inferences
and thus transcends the bounds of the observable....
the criterion of these necessary or pseudo-necessary
inferences is not to be found simply in inductive
generalisation, and thus the extensional move from
some to "all" verifications of observable relations,
but really in the construction of new relations that
transcend the bounds of the observable (Piaget 1975a,
pp.30-1; 1978b, pp.43-4).

An observable is anything that can be observed, where observation is
taken to cover both perceptual and introspective modes of knowledge
(see (3.5) and (3.6) above.) It follows from Piaget's general claim
- stated in (3.5) - that in no case at all can a subject acquire
any knowledge in the absence of an interpretative framework and so
an observable is not that which can be observed without the use of
such a framework. Rather, an observable is any property that is
present in a subject's observation of that property, for example
any of an object's positive, affirmative properties. A coordination,
for example any of the negative or inferential properties of an
object, is the deductive linking of the properties of an object,
whether positive or negative. Because a coordination is not an
observable, a subject cannot have observable knowledge of a coordination.
and so instead must have some other form of knowledge, for example operative or coordinatory knowledge. Simply put, it is one thing to observe that each angle of a square is a rectangle and quite something else to infer that a square must have such a property.

It will be noticed, in the quotation, that a pseudo-necessary inference is not an observable but is rather an inference that can be made by a subject only when an object is presented (in perception) or represented (in thought). The concrete operational subject makes inferences of this type but Piaget denies that the presence of an object, as an intellectual aid, precludes an inference from having the modal status of being necessary (Piaget 1977b, pp.303, 307).

How are observables and coordinations related in the formation of knowledge? To answer this question, Piaget (1978b, pp.47-87) specifies a range of equilibratory models by reference to which such a question may be answered and his account is reviewed by Furth (1980, pp.56-9; 1981, pp.277-80) and Vuyk (1981a, pp.156-63). It is sufficient here to draw attention to two aspects of this account, namely (A) the movement from periphery to centre and (B) the implicatory nature of consciousness.

(A) Periphery and Centre

Piaget (1928a; 1954, p.355) discusses the transition from periphery to centre in early works but his account is more clearly seen in his later writings. Piaget refers to this transition in the following passage.

Fig. 5.1: The transition from periphery to centre
Source: Piaget 1977c, p.335.
Knowledge proceeds neither from the subject nor from the object but from interaction between them, and thus from point P (see Fig. 5.1) which is effectively peripheral in relation to subject S and object 0. From there, the attainment of consciousness proceeds towards the central mechanisms C of the subject's action, whilst the attainment of knowledge of the object proceeds towards its intrinsic properties (and, in this sense, C' which is equally central) (Piaget 1974a, p.263/*1977c, p.335).

A subject gains peripheral knowledge in cases where observable knowledge arises and the subject gains central knowledge in cases where coordinatory knowledge arises. Since neither can occur in the absence of the other, a subject who gains any knowledge gains knowledge of both of these types. Piaget sometimes refers to knowledge from the centre as knowledge of the mechanism or means of an action. The action is made possible by the coordinations present in its display and such coordination represents the mechanism of the action. A subject has to pass through the intermediary of his/her own operative knowledge as a condition of gaining any knowledge at all. The equilibratory models specified by Piaget, and referred to above, attempt to show the interplay of operative (central) and figurative (peripheral) knowledge.

Piaget (1978b, p.58) is thus led to state that there is a set order in the acquisition of knowledge of these types (see Table 5.1). In general, a subject's knowledge may be observable (Obs) or coordinatory (Coord) and in either case it may arise from an object (0) or from the subject (S). Further, there is a definite sequence to such interplay since a subject's observable knowledge precedes the subject's coordinatory knowledge. Finally, there is an asymmetry to this sequence since observable knowledge arises from an object

Table 5.1: The Cyclical Order of Knowledge-Acquisition

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prior to arising from a subject whilst coordinatory knowledge arises from the subject prior to its arising from the object.

It is not clear whether Piaget intends his model to have causal significance since the ordering is logical rather than temporal. Piaget is not claiming that a subject first gains observable knowledge from an object and then later attributes some set of coordinations to it. Rather, Piaget is saying that the latter is impossible in the absence of the former and such a claim is compatible with such knowledge-acquisition's being concurrent. Further, although Piaget's discussion has an example of a subject's gaining causal knowledge of physical objects in mind, he clearly states that the models are applicable to all types of knowledge (1978b, p.50).

(B) The Implicatory Nature of Consciousness

A subject who gains knowledge is one who confers meaning on the world. Piaget claims (see Table 4.1) that there are three vehicles by whose use a subject can confer meaning, namely indices, symbols and signs and a subject may confer meaning (signification) by use of any of these. Increases in knowledge thus result in there being a rich set of inter-connections holding between meanings and Piaget claims that the relation holding between meanings is neither association (correlation) nor causality nor entailment but is instead signifying implication (implication signifiante). Piaget rejects the first member of this trio since he rejects empiricist theories of learning - see (2,4) and (3,4) - whether of the Pavlovian (Piaget 1971a, p.5) or Skinnerian (Piaget 1972b, pp.147-8) varieties. Piaget (1968b, p.182) rejects causality since he rejects interactionist theories of body and mind - see (3,2): it is not by recourse to neurophysiology that a subject's knowledge of addition is to be explained. Finally, it is not the logical relation of entailment (Piaget 1966, pp.154-5) since no vehicle of meaning logically implies any other.
To refer to signifying implication is to name rather than to explain the relation linking different meanings. Certainly, Piaget (1967a, p.78/1971a, p.49) states that consciousness is a system of meanings that are epistemically linked and that the system has two central notions, namely designation (extension) and "implications" (intension). A subject who has conceptualised knowledge is one who has observable knowledge, indexically based, drawn from objects and observable knowledge, symbolically based, drawn from his/her actions. But Piaget fails fully to clarify the nature of the implicatory relation and its nature is apparently sui generis.

Piaget (1978a, p.222) does state what the function of implicatory consciousness might be since he states that the system of signifying implications enables a subject to go beyond practical knowledge and gain conceptualised knowledge since the subject is enabled to give reasons for action-success. A subject who has practical knowledge can achieve certain goals but such achievements remain on a material level and are displayed in overt behaviour. By giving reasons for his/her knowledge a subject transcends that level and can locate knowledge in a system of knowledge which is both internal to the subject and yet shared by any (epistemic) subject at that higher level of development.

The obvious question that arises is how a subject who has conceptualised knowledge can use a system of meanings that are implicatorily related. The answer to this question is given by Piaget in his accounts of abstraction and generalisation and these will be discussed in the subsequent section. It is clear, however, that Piaget does need such accounts for a subject's coordinatory knowledge is not knowledge which is accessible to a subject's conscious awareness. Yet it is clear that mature subjects can gain conscious and conceptualised knowledge of logico-mathematical relations of the
sort that may be used to characterise a structure. The accounts of abstraction and generalisation, then, are attempts to show how such knowledge arises, how a subject conceptualises his/her own coordinatory knowledge.

(5.4) Abstraction and Generalisation

Piaget takes abstraction and generalisation to be important and inter-dependent processes in both early (Piaget 1926, p.158) and recent (Piaget 1980a, p.28) statements of his position. Consider here Piaget's recent accounts of (A) abstraction and (B) generalisation.

(A) Abstraction

Piaget is somewhat wayward in classifying types of abstraction, even in recent accounts. Thus sometimes two (Piaget 1971a, p.320), sometimes three (Piaget 1980a, p.27) and sometimes four varieties (Piaget 1977b, p.303) are offered. It would be prudent to take the latter as representing Piaget's main view.

The four types of abstraction are defined by Piaget as follows.

Empirical abstraction draws its information from objects as such or from a subject's actions in respect of their material character, hence quite generally from observables.

Reflective abstraction is directed upon the coordinations of a subject's actions, which coordinations and the reflective process itself can remain unconscious or give rise to acts of awareness and varied conceptualisations.

Pseudo-Empirical Abstraction (occurs) in relation to an object and its actual, observable properties, as in empirical abstraction, but where the verifications in question, are the products of the coordinations of the subject's actions.

Reflected Abstraction (is) the result of reflective abstraction which has become conscious, at whatever level this is. (Piaget 1977b, p.303 — my translation).

The central distinction that is drawn here is that between observable and coordinatory knowledge. Observable knowledge occurs when a subject acquires, or extends, his/her knowledge of what properties
an object, as a matter of fact, has. Such knowledge is therefore contingent knowledge. By contrast, coordinatory knowledge is non-contingent knowledge, consisting in logical or deductive knowledge of what must, or must not, be the case. In particular, Piaget's position is one that states that a subject may have the former knowledge only if he/she has the latter knowledge and that knowledge of both sorts is open to construction. Thus Piaget's account must differentiate the increase in a subject's observable from an increase in a subject's coordinatory knowledge. And it is this central difference which is invoked in the four definitions of abstraction. Specifically, empirical abstraction is distinct from the other three types of abstraction since empirical abstraction arises from observable knowledge in contrast to the other three types which arise from coordinatory knowledge.

Empirical abstraction is essentially inductive and consists in a subject's abstracting the observable properties of objects. It was implicit, in the definition of observables - see (5.3) - that a subject who makes an inductive inference concludes that all, and not merely some, objects have a given observable property. Empirical abstraction allows a subject to gain knowledge of de facto regularity.

Reflective abstraction is the other main type of abstraction, consisting both in the (physical) projection of what is on one plane to some higher plane as well as in the (mental) occurrence of an act of reconstruction or reorganisation. Consciousness of the occurrence of this process is not an invariable feature of this type of abstraction, though neither is consciousness excluded. It is argued, in (8.6), that Piaget makes contradictory claims about the age/stage at which this type of abstraction occurs.

Reflected abstraction is the result of a reflective abstraction that has become conscious and consists in the systematic articulation of the reorganisation brought about by reflective abstraction. In
early stages of development, it is the latter (reflective) which is the moving force of the former (reflected) abstraction; at later stages, it is reflexion in the mental sense that dominates as operations become objects of thought (Piaget 1977b, pp.303, 306-7; see also 1977c, p.346 and 1974b, p.236/*1978a, p.225). Vuyk (1981a, p.121) seems to regard reflected abstraction as the mere occurrence of awareness when it is the result of reflective abstraction and to regard higher forms of reflected abstraction as metareflexion. There is an evident lack of clarity in Piaget's account since the inter-connections between key concepts are left for the reader to draw.

Pseudo-empirical abstraction occurs when a subject requires the presence of an object, either in perception or in thought, for reflective abstraction to occur and so pseudo-empirical abstraction is an auxiliary to reflective abstraction (Piaget 1977b, p.307).

Since pseudo-empirical and reflected abstraction are variants of reflective abstraction, it will be possible to review their features together. Reflective abstraction is intensional and consists in the introduction of properties, that a subject has constructed, to objects that did not previously possess them and so consists in the application and attribution to objects of their negative and inferential properties. The results of reflective abstraction are not disconfirmable in the light of new experience, since a self-consistent formal system cannot be invalidated by any set of facts, but in view of the termless nature of equilibration those results may lead to the construction of new, richer types of systems (Piaget 1977b, pp.318-324).

The enrichment introduced by reflective abstraction is taken by Piaget to be the establishment of order and correspondence, the transfer of learning, the use of analogy, the generalisation of negation, the construction of operational structures and the capacity to reason and justify. The stratification that occurs in these cases is taken
by Piaget to require the presence of abstraction, with equilibration being the mechanism by which the actual transition from one plane to another may occur. Such stratification is a pervasive feature of thought and in all cases is taken by Piaget to require the presence of this type of abstraction (Piaget 1977b, pp.308-13).

In fact, Piaget identifies the planes which are required for this to occur. The lowest plane occurs in infancy, as shown by an infant's ability to recognise objects or to use indexical meanings. The first plane requires a subject to represent the sequence of his/her actions, for example their serial order of occurrence. A second plane is reached when a subject can reconstitute a sequence of actions and connect their representation in an ordered whole. A third plane requires comparison, as when a total action that is reconstituted at the second plane, is compared to other actions, also reconstituted, so their common properties may be detected or differences established. A fourth plane requires a subject to identify and isolate the structure that is common to actions thus compared. A fifth plane is characterised by the presence of reflected abstraction, when a subject thinks and reasons about the results of previous abstractions. Reflective abstraction, then, requires the serial replacement of form by content: that which is a form to some content for there to be abstraction at one plane becomes in turn the content for an abstraction at a higher plane (Piaget 1977b, pp.304-7).

(B) Generalisation

Abstraction and generalisation are complementary processes. Abstraction is the extraction of that which is common from a multiplicity of cases, whereas generalisation is the application of what is common to a multiplicity of cases.

Piaget distinguishes two types of generalisation, corresponding to the two types of abstraction. Inductive generalisation arises out of empirical abstraction and so from observables. Since it is
extensional, it consists in the enlarging of the range of instances of an observable and so allows the assimilation of new content to a structure already possessed. Constructive generalisation arises out of reflective abstraction and is both extensional and intensional in being the source of both new properties and new instances of them. As such it is the generation of the form that makes new content possible.

Constructive generalisation does not consist in the assimilation of new content to previously constituted forms but rather in the generation of new forms and new content and so in new structural organisation (Piaget 1978e, p.221 - my translation).

Thus do the two types of generalisation differ.

Piaget anticipates a problem due to the inverse relation of extension and intension of a class. For example, there are more quadrilaterals than there are squares, since rectangles are included in this class; yet the intension of the class of quadrilaterals is smaller than that of the class of squares. Thus an increase in extension corresponds to a decrease in intension and conversely (Stebbing 1930, p.29). The problem facing Piaget is that he wishes to claim that constructive generalisation brings an increase in both intension and extension and so that they vary directly (Piaget 1978e, p.223; see also 1978d, p.248). Piaget's reply exploits the distinction between form and content. Any form will have an extension and an intension and their relation will conform to the inverse rule. A similar feature applies in the case of any content. Yet there are other possible combinations, for example between the intension of a form and that of a content - which leads to a paradoxical relation of interest to mathematicians (Piaget 1978e, p.226) - or between the intension of a form and the extension of its content. This latter does vary directly, claims Piaget, and is illustrated in cases of constructive generalisation: the extension of a content at one plane varies directly with the intension of a form at the
next plane. Now since generalisation is allied to reflective abstraction, it is clear why Piaget takes himself to have given a satisfactory reply since it is the role of abstraction precisely to consist in the serial replacement of content at one plane with a form at its successor.

The inter-relation of intension of form and extension of content is taken by Piaget to be an instance of integration, which has three types: totalising integration sub-divides into completive and synthesising integration, both of which are distinct from coordinative integration (Piaget 1978e, p.231). The discussion of this trio hinges, however, on the distinction drawn between intrinsic and extrinsic variations. A variation is extrinsic when differentiation of a property occurs as a result of verification and so of empirical abstraction, for example that mountains may be 1000, 2000 or 3000 metres high. A variation is intrinsic when differentiation occurs on the basis of a necessary deduction, for example that Euclidean triangles will have sides equal or unequal in length, from the meaning of the property. Piaget alludes to the distinction drawn by Frege (1952) between sense and reference: the sense of a word is its meaning in contrast to its reference which is that which the word describes. It follows from the sense of 'bachelor' that a bachelor is unmarried; that Edward Heath is a reference of that term rests upon the verification that he is in fact unmarried.

A coordinative integration is directed upon extrinsic variations, which are thereby classified without the addition of further properties being conferred upon them (Piaget 1978e, p.231). For example, subjects are presented with three vertical bars and some elastic bands and are then asked how many bands would be needed to encircle two adjacent bars. Coordinative integration occurs when a subject understands that 2 bands would be needed for 3 bars, 3 bands for 4 bars and so on. Even though the answer is correct, a subject might not understand the
basis for that answer, as is shown by a subject who correctly identifies the answer but justifies it on the grounds that fewer bands are needed because the bands are bigger than the bars (Piaget 1978e, pp.50-1).

Totalising integration is directed upon intrinsic variations and requires the functioning of a system whose laws are different from those of the sub-systems comprising it. The simpler type of totalising integration is synthesising integration, which occurs when there is isomorphism between distinct sub-systems and which is understood as such by the subject. Completive integration occurs when a total system is thus produced. A related difference between those two sub-types is that the former aims to increase the extension of a system and so increases its intension, whilst the latter aims to increase the intension of a system but does this by increasing its extension (Piaget 1978e, p.234).

(5.5) Conclusion

How adequate is Piaget's account of equilibration? There can be no doubt that his account has been regarded with suspicion and hostility on the part of others. But such a reaction is often misplaced, for even recent accounts (Brainerd 1978a, p.20; Ginsburg & Opper 1979, p.211; Brown & Desforges 1979, pp.44, 141; Mischel 1979, p.93) use Piaget's early (Apostel et al. 1957b) account in preference to his mature one, which Piaget (1978b, p.56) admits to be deficient. It is fair comment to deplore the proliferation of accounts but not fair to ignore a later account when it is stated to be an improvement of an earlier one. It is therefore premature to reject Piaget's account of equilibration in the absence of consideration of its more recent presentation.

Cybernetical psychologists (e.g. Boden 1979a, 1980a) claim that Piaget's recent account of equilibration is seriously incomplete because it does not, and cannot, state which transformations are effected
and in which order they are effected for knowledge to arise. By contrast, research in artificial intelligence can and does make these explicit and so Piaget's account is taken to be verbal (Boden 1979a, p.17) or ambiguous (1979a, pp.79, 138) or dependent upon computational accounts (1979a, pp.85, 131, 135). The reply to this objection is to concede that Piaget's account is incomplete but to deny that computational accounts are superior. A computational account is one that assumes that a cognitive system (man, machine) has a certain logical repertoire: the system has the capacity to make certain logical inferences but has no capacity to make other logical inferences. No doubt systems vary with respect to the extent of their logical powers. Still, there is no system that can perform each and every type of logical inference. But the cybernetical studies of such systems are limited to the study of the system's use of its built-in logical capacities. This is not to say that such studies are easy or trivial. It is to say that it is one thing for a system to acquire new knowledge in the sense of discovering the extent of the deductive capacities inherent in that system; it is quite another for a system to acquire new knowledge by constructing a new set of deductive capacities. Cybernetical research, as currently conceived (Boden 1977), is concerned with the former of these; Piaget's research is concerned with the latter. Piaget's account is one that attempts to show how a less stable and less capacious cognitive structure can develop into a transcending cognitive structure: not only is new knowledge acquired but the structure relevant to new knowledge is enriched as well. But if this is so, Piaget can maintain that his account is complementary to, and is not superseded by, cybernetical accounts. Cellier (1979, pp.89-91) accepts a similar conclusion. He concedes that psychogenetic (Piaget's) theory is not effective in the cybernetical sense but maintains that cognitive science is not explanatory, since its programs show how knowledge can be displayed in action but fail to show how knowledge
has arisen from action in the first place.

The same conclusion can be reinforced since there is an interesting asymmetry between Piaget's and computational accounts. The latter (e.g. Klahr & Wallace 1976, pp.87-96) offer a set of (empirically) sufficient conditions for the presence of knowledge: the presence of such conditions is taken to result in the presence of knowledge. By contrast, Piaget's account is one that offers a set of (empirically) necessary conditions for the acquisition of knowledge: in the absence of such conditions it is held that knowledge does not arise. But if this is so, an account such as Piaget's is more basic than a computational account, for the former is presupposed by the latter but not conversely. Piaget's account is presupposed since that account states conditions in whose absence knowledge-acquisition does not take place. But any specific computational account is not presupposed by Piaget's account since the conditions stated by any such account might be conditions that are severally sufficient for knowledge-acquisition. That is, knowledge arises given the presence of that set of conditions but also given the presence of some other set of conditions that is also sufficient. And the conditions stated by such accounts might not be the same, since such conditions are severally sufficient. It follows that even if Piaget's account is incomplete, in the manner specified by computational psychologists, it cannot be superseded by them (Smith 1982a). It is not here claimed that Piaget's account actually does state conditions that are (empirically) necessary, nor that any computational account does state conditions that are (empirically) sufficient. It is claimed that on the hypothesis that such accounts do state such conditions, the asymmetry noted does arise.

Thirdly, it is argued that Piaget's account of cognitive change is incoherent since the supposition that a subject is a succession
of increasingly more powerful logics – that is, of cognitive structures – is incompatible with development. Consider two logical systems A and B. If A is weaker than B, its weakness can be precisely stated: B possesses, what A lacks, a set of axioms that make possible the derivation of certain theorems. In that case a subject whose structure corresponded to system A could not, in principle, learn anything associated with system B that required the use of those extra axioms and derived theorems just because A is weaker than B in precisely that way (Fodor 1975; 1980, p.148). One reply to this is to claim that if Fodor's argument is accepted, the acquisition of new logical and mathematical systems would be impossible, which is of course historically false (Piaget 1980a, p.150), as too would be the development of biological systems more complex than primitive forms of life (Cellerier 1980, p.87). The decisive objection to such a complaint, however, is that it ignores the special features of Piaget's theory, whereby a distinction is drawn between the knowledge which is accessible to the conscious mind of the subject whose knowledge it is and the knowledge which an observer may attribute to that subject on the basis of the coordinations present in the subject's action. Thus it was stated, (3.4) and in (4.2), that the different stages in development are given richer logical descriptions as development progresses. But such descriptions are provided by the observer and not by the subject in question. No doubt structural change cannot take place if a model of learning is assumed that places reliance on the subject's conscious capabilities and it is for this reason that Fodor's objection — as too the objection stated by Flavell (1977, p.242) — is well-taken. But Piaget's distinction between observable and coordinatory knowledge is one that allows a subject to acquire and extend his/her coordinatory knowledge, even though that knowledge is not consciously accessible. What cannot be denied is
that Piaget does offer an alternative account and as such it is
not invalidated by an objection which assumes some different
conception of development.

The general conclusion to draw from the discussion in this
chapter is that a theory which is both structuralist and constructivist
is committed to an account of structural genesis. The account of
equilibration is an attempt to state such an account and for this
reason alone it is worthy of serious consideration. Some limitations
of Piaget's account are further discussed in chapter 9.
THE ATTRIBUTION OF OPERATIONAL KNOWLEDGE.

(6.1) Introduction

Piaget is firmly of the opinion that his theory should not be discussed in isolation from empirical fact, and conversely. As Inhelder (1979, p.ix) puts it: 'with Piaget the empirical is never separated from the theoretical'. It is then ironical that Piaget (1977f, p.88) should have to complain that the claims of some critics are marred by fragmentation of his work and that this tendency is even present in sympathetic commentaries (Flavell 1963, p.viii). Naturally, Piaget's associates express the same view, for example Mays (1979, p.46). By contrast, some critics (Brown & Desforges 1979, p.100) would accept that Piaget does not hesitate to introduce ideas whose empirical meanings are less than clear. Nor is he averse to introducing more concepts than the minimum needed to describe the phenomena he studies (Brainerd 1978a, p.12).

Others state that there is an inherent bifurcation present in Piaget's work:

Here stands an imposing system of logical propositions and their various transformations; elsewhere stands Piaget's body of empirical data on cognition at different genetic levels; and yet there is not the slightest attempt to effect any real liaison between them (Flavell 1963, p.428).

Evidently, a theory can be elegant and comprehensive but its scientific value must be judged in some, even if not the only, way by its ability to explain empirical facts and by its capacity to withstand the critical assault that arises from competing theories or from experimental studies of the phenomena which it takes to be its base.

The discussion is in two parts. The first part occurs in section (6.2) which discusses the role of language in the attribution of operational (deductive) knowledge. No one seriously doubts that language-based approaches are necessary for the investigation of formal thinking but there is serious doubt expressed as to whether the deductive
knowledge gained at the concrete operational stage of development requires the use of similar methods. The central claim made in this section is that Piaget's position is defensible because it does require the use of such methods. Two arguments are stated in support of this view. Deductive knowledge is knowledge which the subject must be able to justify and such justification may occur only by his/her use of language. Further, the logical properties of deductive knowledge, namely its universality and necessity, can be displayed only in language.

The second part of the discussion occurs in sections (6.3) – (6.5), and is a discussion of two case studies. A subject's understanding of transitivity is discussed in (6.3) where an attempt is made to defend Piaget's position from the arguments marshalled against it from recent experimental studies. A similar discussion occurs in (6.4) with respect to the understanding of class-inclusion.

The concluding section (6.5) briefly states a mediating position whereby Piaget's methods, if not all of his account, should be accepted by any study that is concerned to investigate all types of deductive knowledge at the concrete operational level.

(6.2) Language and Operational Knowledge

Piaget has, on the whole, preferred to use rather than to discuss the clinical methods adopted by him, though associates (Inhelder et al. 1974, pp.19-20; Ginsburg & Opper 1979, pp.91-5, 113-5) present accounts of his methodological position. It is not the aim of the present section to duplicate such accounts but rather to focus attention upon the question of the place of language in the attribution of operational knowledge. In particular, it is argued that the justification-condition relevant to knowledge and the deductive nature of operational knowledge require the uses of language-based approaches.

In order to attribute operational knowledge to a subject, Piaget uses a method - now referred to as the method of critical exploration
(Inhelder et al. 1974, pp.19-20) - which places a subject in some test-condition that typically requires the subject's active manipulation of some set of objects. A question is posed to the subject and a dialogue ensues. The observer (the person in control of the test) poses a set of pre-formulated questions to the subject but conducts the dialogue in a manner which is flexible enough for the observer to investigate the particular (verbal) responses of that subject to those questions. The use of language, by both observer and subject, is thus a central feature of Piaget's method.

At this point, it is important to recall Piaget's general position on the role of language in cognition. Piaget's view is summarised as a two-fold claim:

- some kind of language is essential for the completion of the structures under discussion, i.e. classification and seriation (Inhelder & Piaget 1964, p.293);
- language can constitute a necessary condition for the completion of logico-mathematical operations without being a sufficient condition of their formation (Piaget 1974e, p.113).

This statement of position is clear. Piaget denies that language is a sufficient condition of the formation of operational structures such as classification and seriation; that is, he claims that language is present in the absence of a subject's operational knowledge. But if this is so, some other condition must be present to influence the formation of such knowledge and it is Piaget's claim that this is sensori-motor intelligence. Piaget further affirms that language is a necessary condition of the completion of operational structures; that is, he claims that in the absence of language operation knowledge is absent in its mature forms.

Piaget's claims are the subject of some dispute. Certainly, there are linguists (Brown 1973, p.200) who support his first claim and associates (Inhelder 1980a, p.134) and psychologists (Neimark 1975, p.573) who support the second. There is, however, clear
expression of a dissenting view by those linguists (Chomsky 1980, p.36) who reject the first view and by associates (Furth 1969, p.109; Sinclair-de-zwart 1969, p.320) who evidently reject the second. The latter are supported by those psychologists who deny that language is a necessary condition of operational knowledge.

It was Flavell (1963, p.271) who claimed that, with respect to Piaget's early — and so pre-structuralist— writings, language is a dependent variable in relation to cognition. This claim is readily accepted by other psychologists as a statement of Piaget's more recent — and so structuralist — position (Brainerd 1973, p.176; Brown & Desforges 1979, p.124). In consequence, such psychologists devise experimental procedures for the attribution of operational knowledge which minimises, or even excludes, a subject's use of language, a position which is explicit in the attempt made by Wheldall & Poborca (1980) to assess conservation in the absence of conversation.

The rationale for this extension of Piaget's view is as follows. Firstly, such an approach is compatible with Piaget's stress on action-coordination in development (Wheldall & Poborca 1980, pp.119, 130). Secondly, such an approach is superior since it allows an observer to identify earlier, but equivalent, manifestations of an ability, namely those manifestations which are not occluded by supervening distortions due to language (Miller,1976, pp.425-30). Thirdly, it is more parsimonious to test for a subject's making a correct judgement, which is a display of operational knowledge, rather than also to test for the subject's ability to justify that judgement (Brainerd 1973, 1974). Finally, any method of assessment must avoid the making of false-positive mistakes (taking a structure to be present when it is absent) and the making of false-negative mistakes (taking a structure to be absent when it is present). It is then claimed (Brainerd 1973; Flavell 1977, pp.221,225) that use of Piaget's method tends to result in the latter mistake, and, in
consequence, Piaget is led to offer too pessimistic a view of human ability. The use of a non-verbal (Miller 1976) approach is offered as a corrective to this tendency. Since, however, such an approach is not, strictly, non-verbal - since the observer has to pose a question in language and the subject may express a judgement in language by use of the words "yes" and "no" - such an approach is preferably classified as a judgement-only (Brainerd 1973) approach.

Three initial comments can be made about this viewpoint. Firstly, the psychological critics seem to assume a conception of growth which is linear and atomistic (Larsen 1977, p.1162) since they are concerned to investigate the presence-v-absence of an ability. By contrast, Piaget is concerned to investigate the manner in which an ability is possessed by a subject. The manner in which an ability is possessed is characterised by Piaget's reference to a subject's cognitive structure and any of the logical norms studied by Piaget are taken by him to appear in different ways in a subject's cognitive responses. Secondly, it must be recalled that Piaget's conception of action is a broad one, covering exteriorised and interiorised action. It follows that action-coordination in the latter case is different from that in the former case since it is a subject's conceptualisation of the world, rather than a subject's physical action on the world, that is an observer's focus of attention. But in that case an observer's task is facilitated (Smedslund 1969, p.237) if a subject is asked to explain in language what he/she is doing and thinking. Thirdly, the critics seem to assume that a subject's understanding of a test-question can be divorced from the subject's use of language in response to the question (cf. Wheldall & Poborca 1980, p.130). But in reply to such a view, a remark by Flavell is apposite:

there is probably a point beyond which stripping a concept of its verbal-symbolic accoutrements makes
of it a different, lower-order concept, or even no concept at all (Flavell 1963, p.436).

That is, a subject's failure to use language to express a judgement, in response to a verbally formulated question, can indicate a lower, and not equivalent, level of ability.

Two general arguments can now be stated to support Piaget's view that language is a necessary condition of the completion of operational structures and so the view that operational knowledge can be attributed to a subject only when that subject understands and uses language to express his/her judgement in a manner that justifies that judgement.

The first argument draws its support from an analysis of the concept of knowledge that has a pleasing philosophical tradition in its favour. It is claimed that a subject knows a proposition 'p' only if all of the following conditions are met:

i) 'p' is true,

ii) the subject takes 'p' to be true,

iii) the subject can justify 'p'.

The supporters of this view (Plato 1935, 201D; Ayer 1956, p.35; Armstrong 1968, p.188; Chisholm 1977, p.102; Hamlyn 1978, p.132) are noted for their otherwise philosophical diversity of view and their commitment to the "traditional" analysis of the concept of knowledge is compatible with disagreement as to what conditions other than i) - iii) must also be met for a subject to have knowledge. What matters here is that there is agreement that condition iii) is at least one (logically) necessary condition: a subject who cannot justify a proposition 'p' is a subject who does not know that proposition. The condition does not state that a subject must actually justify 'p' as a condition of the subject's knowing 'p'. The condition does state that an inability to justify 'p' is a reason for denying that the subject knows 'p'. Thus iii) does not exclude a subject's gaining,
or possessing, operational knowledge in the absence of a linguistically mediated display of that knowledge together with its justification. For example, an adult carpenter would presumably have an operational knowledge of transitivity when he cuts two pieces of wood to a measured length in the expectation that each is the same size (subject to cutting-errors which may be ignored here). The carpenter uses language neither to express his knowledge nor to justify his expectation. But the attribution of operational knowledge in such a case is justified on the grounds that on other occasions the carpenter can use language to express and justify his knowledge. In this case, there is presumed to be independent evidence by reference to which the attribution can be justified. By contrast, a psychological study of the development of a subject's knowledge cannot presume that such independent evidence exists or, at least, cannot take it into account in the attribution of operational knowledge. It is for this reason that the subject, during the test itself, should express and justify in language the operational knowledge which is attributed to that subject. Thus language should be used both by the observer and by the subject in the attribution of operational knowledge.

It is not claimed that the justification-condition is relevant to the attribution of all types of operative knowledge, for example the infant's practical knowledge. It is claimed that it is relevant to the attribution of all types of operational knowledge which is a sub-set of operative knowledge. Justification is relevant to the attribution of operational knowledge since such knowledge arises only in the case of reversible actions and this in turn requires a subject to relate different instances of his/her knowledge. For example, the subject who understands the conservation of quantity can justify that knowledge by citing several reasons: "nothing has been taken away or added" (identity); "you can put the water in B back into A" (inversion); or "the water is higher but the glass is
narrower" (reciprocity) (Piaget & Inhelder 1969a, p.98). The subject who cannot give any such reason is one who can not have operational knowledge. Piaget does not endorse the traditional analysis of the concept of knowledge but it is clear that he is aware (Piaget 1926, p.171; 1951, p.237; 1972d, p.11; 1978a, pp. 215, 222) of the relevance of justification to the presence of (operational) knowledge. It is, in particular, clear that Piaget's account of the implicatory nature of consciousness, referred to in (5.3), is compatible with the view that justification is an essential feature of conceptualised knowledge.

The second argument arises from a distinction which should be drawn between a correct judgement and a deductively correct judgement. A subject makes a correct judgement if that judgement is correct. Thus a subject who has (observable) knowledge that some of the glasses on a table are big and that some are small (Piaget 1952, p.6) is a subject who makes a correct judgement. By contrast, a deductively correct judgement is one that is not only correct but which is also taken to be necessarily correct by the subject. That is, it is one that the subject deduces from a given body of data. Since both types of judgements are correct judgements, and since operational knowledge is confined to deductively correct judgements - see (2.5) and (4.6) - the attribution of such knowledge must use a method that allows the observer to ascertain whether or not a subject's correct judgement is the outcome of the understanding of logical necessity.

A subject who gains operational knowledge is one who understands deductive necessity. Consider, for example, a subject's knowledge of transitivity. This property of relations is given a formal definition by logicians (Lipschutz 1964, p.85; Lemmon 1966, p.182) and may be informally presented as follows:

1) \( A = B \)
2) \( B = C \)
3) \( A = C \).
A relation, such as equality, is transitive in all and only those cases where (1), (2) and (3) hold. Two features are to be noticed here. Firstly, in all cases where (1) and (2) hold, then (3) also holds. So one feature of transitivity is its universality. Secondly, if (1) and (2) hold, then (3) must also hold since (3) is a deductive consequence of (1) and (2). Hence necessity is a second feature of transitivity. Correspondingly, a subject who understands the transitivity of equality (in any case) is one who understands that (3) is always true when (1) and (2) are true (universality). Further, the subject understands that if (1) and (2) are true, then (3) is necessarily true as well (necessity). It is not claimed that a subject knows all the cases in which transitivity holds; it is claimed that a subject who knows that (1) and (2) are true knows that (3) is always true as well, if that subject understands transitivity. Similarly, it is not claimed that a subject necessarily knows in which cases transitivity holds; it is claimed that if the subject understands transitivity, the subject knows that (3) is true as a deductive consequence of the truth of (1) and (2). Finally, it is not claimed that if a subject knows (1) and (2), the subject knows and must know (3). It is not Piaget's claim that all subjects are at all times fully logical but only that when a subject is logical (if ever), certain conditions are fulfilled.

It seems that the properties of universality and necessity are such that their display requires a subject's use of language. Suppose a young subject is presented with a square and is asked how many sides it has. Suppose, further, the subject is asked to indicate by pointing which answer is correct from a series of possible answers presented to him/her. And suppose, finally, the subject points to the answer which states "4". Certainly, the subject makes a correct judgement. But that judgement provides under-determining evidence for a claim that the subject deduced that answer. By contrast, a
subject who uses language to claim that all such figures have four sides, that any rectangular figure with equal sides must be a four-sided figure or that there are never any instances of such figures which do not have four sides is one who provides some evidence for a claim that the subject makes the correct judgement on a deductive basis. It is not claimed that the presence of such words is conclusive evidence for such a claim — no doubt other factors are relevant as well. It is claimed that in the absence of such linguistic use, the attribution of deductive knowledge is impossible.

Thus when Piaget (1950, p. 143) claims that a subject who has operational knowledge can make deductions, or claims that such subjects experience a conscious feeling of necessity (1967a, p. 437/1971a, p. 316), his claim is that such knowledge transcends the sphere of observable knowledge. Transcendence (dépassement) is a specific feature of operative, and so operational, knowledge and the account of equilibration — see (5.2) — made important reference to it. It is this feature which Piaget (Inhelder & Piaget 1964, p. 293) refers to in his statement of position concerning the role of language in operational knowledge and it is reasonable to suppose that Piaget has in mind the twin features of deductive knowledge here discussed, namely universality and necessity. His position, then, states that the attribution of completed operational structures is not possible in the absence of a subject's use of language. A good example of the transcending nature of operational knowledge occurs in the discs-experiment — see (2.3) — where older subjects gain knowledge of the correct sizes of the discs on the basis of a transitive inference that utilises the additive value of non-observable differences. Deductively based knowledge of this sort is not available to a subject who has observable knowledge alone of the discs.

What is a completed operational structure? It was argued, in
(4.2), that Piaget is committed to there being three stages in
development and that any stage in development is such that there
is an initial phase whereby a structure is formed and a final
phase whereby it is completed. The pre/concrete operational stage
is a stage of development. The completion of a structure at that
stage occurs when closure occurs – see (4.6). Thus the initial
phase of that stage, when the structure is formed, is the phase
when a child is initially learning to use language and so, in turn,
to conceptualise the world. Language is necessary for conceptualisation
to take place in its completed form, that is, when operational
knowledge arises. Piaget denies that it is necessary for the initial
emergence of conceptualisation.

It follows that a completed operational structure is not to be
construed as the structure used by a formal operational subject.
Certainly, Piaget accords to language an undisputed role at that
stage of development, since such a subject is taken to think in
terms of linguistically stated propositions. The discussion of the
role of language in concrete operational thinking (Inhelder & Piaget
1964) shows that Piaget's view about the necessity of language (but
not its sufficiency) for operational knowledge is applicable at
both operational levels.

When Piaget denies the sufficiency of language for operational
thinking, he has in mind the unreliability of solely verbal criteria
for operational knowledge. Thus he refers to

the emergency of an awareness of necessity: "it has
to be" etc. But the use of a criterion that relies
upon verbal claims and states of consciousness is
dangerous....what is more reliable (solide) is that
this necessity is used in behaviour and that it
corresponds to the "closure" of a structure up to
then incomplete (Piaget 1967b, pp.270-1 – my
translation; cf. 1969d, pp.119-20).

Piaget claims here that the use of verbal criteria is dangerous in
that use of such criteria can lead to the making of false-positive
mistakes - the attribution of operational knowledge to a subject
who lacks such knowledge. But he does not claim that the use of such criteria is prohibited. Rather, use of verbal criteria should be supplemented with use of a criterion which is related to a subject's (interiorised) actions. For example, the subject who is presented with five sticks, each one of which is different in height, displays an understanding of transitivity if that subject systematically examines the sticks to identify the smallest, which is then placed on one side; and then examines the remaining sticks to identify the smallest in that collection. Such a subject understands that the latter stick is both bigger than the first stick removed from the collection and smaller than the other sticks in the collection and evidently understands the reciprocal relationships obtaining in this case. It is no doubt for this reason that Piagetian studies typically require a subject's participation both as the manipulator of test-material and as a dialogist who reports on that manipulation.

There is an obvious objection to consider. Piaget (Piaget & Inhelder 1969a, p.88) and associates (Furth., 1969, p.119), refer with approval to the studies of deaf-mute children which shows that such children, who lack an ability to use a spoken language, nonetheless attain operational knowledge, though such knowledge is usually attained at a later chronological age than is the case with normal children. Does this not show that operational knowledge may occur in the absence of language, contrary to the position here attributed to Piaget?

Three reasons justify a negative response. Firstly, deaf-mute children usually live in a linguistic environment; they are not always congenitally deaf and mute; they are often (in the United States at least) taught to lip-read at the age of 3 years; and they are often taught to use a sign-language. It follows that there might well be no necessity for there to be early language-use in order that cognitive development might take place (Blank 1965, pp.442-3). Secondly, the question of whether a sign-language is a full language
is not a simple one. In response to a question about whether British Sign Language (BSL) has the capacity to allow a user to differentiate between a correct judgement and a deductively correct judgement, the Principal Investigator of the BSL Research Project states:

BSL, the sign language used by the British deaf community, is quite capable of expressing concepts such as necessity and probability, etc. I have no doubt that a deaf person can understand the concepts of a necessary and sufficient condition...all the research we are undertaking at the moment suggests that BSL is a full language (Mary Brennan 1981, personal communication).

Clearly, the statement of a viewpoint by an authority, however eminent, does not settle an issue. What such a statement does show, however, is that it is not unreasonable to claim that while deaf-mute subjects lack a natural (spoken) language they do not lack a language tout court. But if this is so, it can be maintained that such subjects do not attain operational knowledge in the absence of an ability to use a sign-language, such as BSL. Thirdly, the position of a deaf-mute subject is analogous to that of the carpenter whose understanding of transitivity is shown in what he does in cutting two pieces of wood to the same size but without using language in the process. It is not here claimed that a failure to use language precludes a display of operational understanding but only that such a failure precludes the attribution of such understanding by an observer. Similarly, a deaf-mute may display an operational understanding in the absence of a use of language. The attribution of operational knowledge to a deaf-mute subject could proceed in either of two ways. Either, the observer might test for a suitable use of a sign-language in the context of an experimental study for such attribution to take place. Or, given that there is an agreed and standardised procedure for the attribution of operational knowledge to normal subjects, that same procedure might be used in application to deaf-mute subjects, whether or not the latter use a sign-language. Attribution in the latter
case is clearly more dubious than, since parasitic upon, that in the former case.

Piaget's method of critical exploration is, then, an acceptable method for the attribution of operational knowledge. Language is a necessary condition for operational knowledge and any method that permits an observer to attribute such knowledge is one that requires a subject's use of language in reporting on what he/she does. A subject understands a test-question at operational levels only if that subject can use language to express that understanding. The two arguments stated above in support of this contention are arguments that take language to be a logically necessary condition for operational knowledge. It follows from the concept of (conceptualised) knowledge that a subject must be able to justify that knowledge. It is only by a use of language that a subject can show an understanding of the twin features of deductive knowledge, namely its universality and necessity. In consequence, it is denied that a method which requires a subject to make a judgement alone, one that may be non-verbally expressed, is sufficiently robust to permit the attribution of operational knowledge.

It should be noted that the claim here made states that language is a logically necessary condition for the completion of operational structures. It is not clear, however, whether Piaget's bald claim about the necessity of this condition is one that takes the condition to be logically or empirically necessary. Even if it is the latter which is Piaget's actual claim, it is apparently compatible with the stronger position defended here.

It is not a consequence of the arguments here presented that Piaget himself is to be regarded as a paragon of methodological rectitude. It is claimed neither that Piaget is a good practitioner of his method, as Ginsburg & Opper (1979, p.94) confirm, nor that the method is always reliable, as Flavell (1963, pp. 429-32) points out. It is not, of
course, claimed that the method of critical exploration must be used in the attribution of observable knowledge.

(6.3) Case Study: Transitivity

Piaget has carried out numerous investigations into a subject's understanding of transitivity (e.g. Piaget 1952; Piaget et al. 1960). One of the most interesting challenges to his research arises from the experimental studies carried out by Peter Bryant and associates. Reviews of the resulting dispute have been offered by several researchers (Miller 1976; Russell 1978; Brown & Desforges 1979) and these will not be duplicated here. The purpose of the present section is to evaluate those studies in the light of the interpretation of Piaget's theory presented above. Consider, then, two main experimental studies into a subject's making (A) an elicited and (B) a spontaneous transitive inference.

(A) Elicited Inference

A subject who understands conservation of quantity is one who makes a transitive inference but it is Piaget's study of seriation that is the central focus of experimental research. Piaget's study, using sticks of different lengths, has already been mentioned in the previous section and his findings are reviewed by Piaget & Inhelder (1969a, p.101; 1969b, p.171). A general complaint made by Bryant & Trabasso (1971, p.456) about such a study is that it is flawed through inadequate methodological control. In particular, they claim that the study does not ensure that subjects actually remember the information they are asked to combine inferentially and it is due to memory-failure rather than to logical incapacity, that subjects perform the task incorrectly.

What is in doubt is the status of this contention. Sometimes it is claimed that memory is a sufficient condition of the capacity to make a transitive inference (Bryant & Trabasso 1971, p.456). Such a claim is, however, open to rebuttal since Halford & Galloway (1977)
argue that young subjects can remember the premises on which a transitive inference is to be based, and yet fail to make the inference. Their study is itself open to objection, claim Grieve & Nesdale (1979), since Halford & Galloway use the horizontal-vertical illusion as a consequence of which their test-question conflates questions about the apparent and real length of the lines presented to the subjects. Russell (1981a) also objects to the Halford & Galloway study on the grounds that they fail to test for both storage and retrieval of premises. Russell argues that when subjects are presented with a modified version of the towers-task – see (b) below – over a third of the subjects who fail to make an inference actually refer to the correct premise, when asked to do so and about a sixth make such a reference spontaneously. Russell concludes that such subjects are able both to store and to retrieve all the information relevant to the making of a transitive inference and yet fail actually to make one. Memory, Russell concludes, is not a sufficient condition of a subject's capacity to make such an inference.

This conclusion can be accepted since Bryant (1974, p.49) and Trabasso & Nicholas (1978) offer a weaker formulation of their position namely that memory is a necessary condition of the ability to make an inference. There can be no doubt about the validity of this contention: it is analytic, and so logically true, that $A = C$, if $A = B$ and $B = C$ and so a subject who fails to remember that $A = B$ and that $B = C$ is one who is in no position to make a transitive inference. However, as Youniss & Furth (1973) point out, it can be doubted whether Piaget's study is one that overlooks the role of memory, for subjects able simultaneously to view the material they are asked to combine inferentially and so their ability to remember is not in question. This rejoinder evidently begs the question at
issue and it is further weakened on the grounds that the towers-task, to which Youniss & Furth refer, is one that does require the subjects to view information successively rather than simultaneously (Piaget et al. 1948, p.82/1960, p.59).

That memory plays a necessary, but minimal, role in the making of a transitive inference is a conclusion accepted by de Boysson-Bardies & O'Regan (1973). The direct way to make good this claim is to show how a young subject might correctly answer the test-question "Is B taller than D?" but in a non-deductive manner. They argue that subjects can correctly answer the test-question by use of a labelling-strategy. Subjects label rod A as "big" and label rod E as "small" whilst the other rods are neither (see Fig. 6.1). But association, in the training period, of A with B results in the labelling of rod B as "big" and the association of rod D with rod E results in the labelling of rod D as "small". Hence subjects can correctly claim that B is "big" in relation to D which is "small" - but not on the basis of a transitive inference. Certainly, Bryant & Trabasso (1971) introduce important experimental controls. For example, the form of the question is varied so that some subjects are asked "Is B shorter than D?" and the apparatus is sometimes presented so that rod A is on the subject's left-, and sometimes on the subject's right-, side. What remains constant, however, is the internal order of the rods. Thus

A B C D E

Fig. 6.1: Apparatus used to test for subject's understanding of transitivity. Rods of different lengths and colour are presented in a box so that only the same length of each rod is visible to the subject.

Source: Bryant & Trabasso 1971.
despite counter-balancing, subjects could learn that the big rods are at one end of the box and the small ones at the other. That is, because the internal order of the rods is the same, the subjects could learn, and so remember, that the rods are ordered on a continuum "big-small". If A is presented on the subject's left-hand side, then A is "big" and E is "small"; further, by association of presentation, if A is "big" then B is also "big", whilst if E is "small" then D is also "small". Subjects would thus be able correctly to answer the test-question, provided an opportunity was given for correct learning to take place. In fact, such an opportunity was provided since each subject was allowed, during the training period, to learn the sizes of the rods to a criterion of eight out of ten successful choices followed by six correct, successive presentations (Bryant & Trabasso 1971, p.457). Youniss & Furth (1973) claim that such learning is over-learning, whilst de Boysson-Bardies & O'Regan (1973) point out that their replication yielded a successful outcome even though a reduced learning-criterion was used. What can, then, be claimed is that the subjects used by Bryant & Trabasso had the opportunity to learn, and so remember, the sizes of the rods. Such subjects could then give a correct answer to the test-question by use of a non-deductive strategy. The strategy outlined here, for which no experimental evidence is presented, is compatible with the labelling-strategy investigated by de Boysson-Bardies & O'Regan (1973) and so is, to that extent, acceptable.

There is a reply to the claim that subjects perform correctly but in a non-deductive manner. For it is Bryant's (1974, pp.9-15) contention that young subjects use relative, rather than absolute codes. Thus Bryant could claim that subjects who use a labelling-strategy encode the stimuli presented to them by their use of a relative code, not an absolute code. That is, they encode the relation
of A/B as "A is bigger than B" (relative code) rather than encode the actual length of A and the actual length of B. Hence subjects can encode B as "smaller than A" and as "longer than C". And if this is so, such subjects evidently perform a reversible operation since the former is the reciprocal of the latter (see (4,6) above).

What is clear, however, is that Piaget cannot accept this reply. To see why, consider this statement of position:

a perceptual relationship does not as such constitute a relationship. The criterion for the psychological existence of relations is the possibility of their composition, or in other words, the construction of their logical transitivity (or, if they cannot become transitive, the justification of their non-transitivity) (Piaget 1952, p.11).

Logicians (Lipschutz 1964, pp.83-6; Lemmon 1966, pp.179-87) state that any relation must have certain properties. In particular, any relation is either transitive, intransitive or non-transitive. A relation - for example, brotherhood - is transitive if and only if

(4) (x) (y) (z) (Rxy & Ryz¬Rxz)  
(read: for any x, y and z, if x is related to y and if y is related to z, then x is related to z). A relation - for example, fatherhood - is intransitive if and only if

(5) (x) (y) (z) (Rxy & Ryz¬Rxz)  
(read: for any x, y and z, if x is related to y and if y is related to z, then x is not related to z). Given that 'Rxy' and 'Ryz', a relation is transitive if 'Rxz' is always true; intransitive if 'Rxz' is never true; and non-transitive if 'Rxz' is sometimes true. Further, any relation must have at least one of these properties (Lemmon 1966, p.183). It is presumably such logical properties that Piaget has in mind in the quotation just given, for he claims that a subject understands that two objects are related only if the subject understands the transitivity, or non-transitivity, of that relation.

No doubt, Piaget is careless here, since he should differentiate a non-transitive from an intransitive relation. What is clear,
however, is that he claims that a subject who has no comprehension of transitivity, intransitivity or non-transitivity is one who does not understand what a relation is. It is for this reason that Piaget distinguishes between a subject's use of global, intensive, extensive and metric scales. Given the apparatus used in Fig. 6.1 a subject uses: a global scale if the subject registers A as "big" and E as "small"; an intensive scale: if the subject registers that A is "bigger" than E but without being able to quantify the difference; an extensive scale if the subject uses an intensive scale but can quantify the difference in some way, for example "much bigger"; and a metric scale if the subject uses an extensive scale but can assign numerical values to the difference (Piaget 1952, p.244; Piaget et al. 1977, pp.185-6). The main distinction here is that between a global and an intensive scale: the subject who registers that a rod is "big" uses a conceptual scheme: the subject who registers that one rod is "bigger" than another uses an operational scheme.

It follows that Piaget would not accept that young subjects can use a relative code, if by that is meant a code which enables the subject to register that one rod is related to another rod, for example by being bigger than it, if the subject does not understand transitivity, intransitivity and non-transitivity. It is not, of course, claimed that a subject must be able formally to define these logical properties; it is claimed by Piaget - because of his structuralist commitment - that a subject can have operational knowledge of these by their application to specific cases.

The conclusion that emerges is that an investigation into transitivity must indeed control for memory but that even when that is done, a subject may correctly answer a test-question without making a deductive inference. What is also clear is that an investigation must also include a control that excludes a subject's gaining a non-deductive understanding of the test-material and that, in the
In the case of the Bryant & Trabasso (1971) study, the possibility cannot be excluded that subjects answer correctly because of their observable, not operational, knowledge.

(B) Spontaneous Inference

An elicited inference is one that a subject makes on the basis of information presented to the subject; a spontaneous inference is one that requires a subject to gain the information on which the inference is based as well. This distinction is explicitly drawn by Bryant (1973b) and in later research he proposes that young children can spontaneously make inferences: such children can make transitive inferences but they do not always know when to make such inferences (Bryant 1973a, p.418; 1977, p.63). The interest in the study carried out by Bryant & Kopytynska (1976) is that it offers experimental evidence to support this bold proposal.

Piaget's study of spontaneous inference is clearly seen in the towers-task (Piaget et al, 1960). Subjects are presented with a tower, made out of blocks of a given size, and located on a table. The subject is then invited to build a tower, using similar blocks, which has the same height but to do so on a table which is itself smaller in height than the former table. To succeed on the task, subjects must construct a tower whose apparent height is smaller than that of the original tower due to this difference in the height of the tables. Measuring sticks are accessible to the subject though it is left to the subject to decide whether or not to use them. Typically, it is only subjects aged 7 - 8 years who succeed on the task and who do so by their use of a measuring rule.

Bryant & Kopytynska (1976) confirm that subjects do not, typically, succeed on the towers-task until the ages given but further contend that this is due to the subject's failure to realise that the task requires their making a transitive inference. Subjects, aged 5 - 6 years, are invited to perform on the towers-task, on which they fail. They are then presented with an alternative task, consisting in a
wooden box, in the top of which are two holes, and are invited to find out whether the holes are the same depth. A stick is provided and is colour-coded so that its 10 inch length is red for 4 inches on either end and yellow for 2 inches in the middle. In fact, the holes in the block are either 4 or 6 inches in depth and since a direct perceptual comparison is impossible, the subject may correctly answer the test-question only by using the stick. Since a large majority of the subjects do correctly answer the test-question, Bryant & Kopytynska conclude that such subjects do so by spontaneously making a transitive inference: two holes which are equal to a given item are themselves equal to each other.

Four critical comments can be made about this experiment. Firstly, it is noted by Russell (1978, p.150) that the experiment structures the task: the subjects are invited to estimate the depth of the holes; the measuring device is colour-coded so that the estimation is facilitated; and the colour-coding corresponds to the actual depths of the holes. By contrast, the towers-task requires a subject to estimate the height of the towers; sticks made available are either shorter than or longer than the height of the initial towers and they are not colour-coded; and there is no (natural) correspondence between the height of the tower and the sticks. These differences are important. The spontaneity in the Bryant & Kopytynska study arises when a subject decides to use the stick because of the impossibility, rather than unreliability, of a direct, perceptual comparison; whereas the spontaneity in the Piaget et al. study arises when subjects not only decide to use a stick because of the unreliability, rather than impossibility, of direct perceptual comparison but also make active use of a stick which has not been pre-designed to meet the requirements of the study.

Secondly, a subject who has operational knowledge can use the stick provided as a measuring device. For example, a subject might
use the stick which is too long and in consequence the subject must place the stick against the initial tower; mark the place on the stick which corresponds to its height; and then measure the other tower to ascertain whether it has the same height. Thus such subjects spontaneously use the stick but use it as a measuring device since the subjects themselves mark off the correct measurement of the tower. Evidently, the subject performs a reversible operation. Allow that B is the total length of the stick, that A is the portion of the stick corresponding to the height of the tower and that A' is the remaining length of the stick. That is, $B = A + A'$. In order to estimate the height of the tower by use of the stick, a subject must be able to understand inversion, namely $A = B - A'$. That is, the subject must understand that the height of the tower corresponds to length A and that length A is the subtraction of the remaining length, A', from the total length, B. Piaget et al. (1960, p.62) explicitly draw attention to the reversibility of the successful subject's actions. By contrast, in the hole-task the colour-coding of the stick obviates the need for a subject to show a similar understanding: the subject does not have to mark off on the stick, with his/her fingers, the point on the stick corresponding to the depth of the holes.

Thirdly, it is apparent that subjects who perform on the hole-task may succeed but without making a transitive inference. Suppose that one hole $X = 4$ inches, that the other hole $Z = 4$ inches and that the one red portion of the stick $Y = 4$ inches. A subject makes a transitive inference only if the subject gives a correct answer, which answer embodies an understanding that $Y$ is the mediator of $X$ and $Z$. But the correct answer might be given by a subject who uses a global (Piaget) scale. The subject places the stick in hole $X$ and finds that one complete red portion, $Y$, is covered. The subject registers, therefore, that $X$ is "red-size" and remembers this fact. The subject then places the stick in hole $Z$ and finds that one complete red portion, $Y$, is covered. The subject registers, therefore
that Z is "red-size" and remembers this fact. Given that the subject does remember that X is "red-size" and that Z is "red-size", the subject can recall that X and Z have the same size: the subject has attributed the same property, "red-size" to both. Thus it is not clear whether the subject makes this judgement on the basis of a transitive inference: \( X = Y \) and \( Y = Z \) therefore \( X = Z \). Certainly, the subject uses the stick as a mediator (Y) to gain the answer that \( X = Z \). But what is also required, if the answer is to be deductively based, is that a subject should use knowledge of the mediation of the stick (Y) in the measurement of X and Z. A subject compares X with Y and also compares Z with Y. But the subject may also compare, on the basis of memory, X with Z as well.

It is not claimed that subjects actually do perform on the hole-task by their use of a non-deductive strategy: no empirical evidence is provided in support of such a proposal. It is claimed that such a possibility cannot be completely excluded, given the methodology employed by Bryant & Kopytynska. And this leads to the fourth critical comment.

Both the Bryant & Trabasso (1971) and the Bryant & Kopytynska (1976) studies employ a methodology that does not require, for operational knowledge to be attributed, a subject's verbally expressed justification of the judgements he/she makes. But it follows, from the arguments of (6.2), that such studies cannot unambiguously determine whether a subject makes a correct, as opposed to a deductively correct, judgement. Do the subjects who perform correctly on the hole-task understand that it is always the case that \( X = Z \), given that \( X = Y \) and that \( Y = Z \)? Do they understand that it is necessarily the case that \( X = Z \), given these same antecedents? It is not clear. By contrast, a study which employed a methodology similar to that employed by Piaget might be able to resolve this. For example, successful subjects on the towers-task state that:

\[
\text{you have to put one (the stick) up against the other}
\]
Such subjects do use expressions of necessity in contrast to subjects who do not succeed on the task. The latter subjects are content to make visual comparisons of the heights of the towers or to use manual cues to estimate the heights: no doubt such subjects use action-schemes or conceptual schemes but not operational schemes – see (4,5) and (4,6). Thus the use of a different methodology enables an observer to distinguish between those subjects who do, and those who do not, have a deductive understanding.

It will be said: but the instances cited from the Piaget et al. study are unsatisfactory. The subjects use expressions of necessity but in doing so reveal an understanding of instrumental, not logical, necessity. The subjects realise that in order to answer the test-question they must do this or that; but they do not show thereby an understanding of deduction. There are two comments to make about this observation. Firstly, it must be conceded: Piaget et al. are remiss in their failure to provide satisfactory evidence of a subject's understanding of deductive necessity. In general, it can be maintained that Piaget often and uncritically leaves it to the reader to infer the precise sense of a concept used by him. Secondly, in the case of transitivity, however, it is possible to cite alternative evidence, namely from conservation studies. For example, a subject is asked which of two glasses, one being half-full (his own) and the other being one-third full, has the greater quantity of liquid, even when the contents of the latter glass are poured into several other glasses. Suppose that the amount in the former glass = A and that the amount in the second glass = B. Then \( A > B \). Let the total amount in the smaller glasses = C. Then \( B = C \). Subjects who understand transitivity can conclude that \( A > C \). Further, such subjects do not merely make a correct judgement but also express an understanding
of necessity. One subject states that he always (toujours) has more in his glass, A:

No, it's always the same. It's the same all the time (Piaget & Szeminska 1941, p.23/Piaget 1952, p.18; my emphasis).

Another subject asserts:

You've poured it out of the same glass. Like that you can never make the same (Piaget 1952, p.18; my emphasis).

It is clear that such subjects use the modal words 'always', 'all the time' and 'never' and such expressions are used to display an understanding of deductive necessity. Thus Piaget does, sometimes, provide satisfactory evidence to show a subject's operational understanding of transitivity.

The conclusion suggested by this discussion is, therefore, that Piaget's account of a subject's understanding of transitivity is not invalidated by the studies carried out by Bryant and associates; that the latter studies are open to differing interpretation; and that Piaget's theory is robust enough to provide a possible interpretation of their findings.

(6.4) Case Study: Inclusion

Piaget's interest in a subject's understanding of class-inclusion has been long-standing (Piaget 1921) but his mature treatment of the topic is contained in more recent studies (Piaget 1952; Inhelder & Piaget 1964; Piaget 1977a). That account has been influential but has been subjected to critical scrutiny which is reviewed by Winer (1980). A defence of Piaget's position is offered by Smith (1981d, 1982a) whose argument is used in the present section. The aim of this section is to show how Piaget's account can be defended by reference to his overall theoretical position as presented in the chapters above. Consider, then, (A) Piaget's account of and (B) his critics' objection to how children understand class-inclusion.
Piaget's Account

A subject who can attribute a global property - see (6.3) - to an object is not one who understands that:

there are necessarily "more" elements in a whole than in one of the parts (Piaget & Szeminska 1941, p.199/Piaget 1952, p.162; my emphasis).

Such a subject may correctly allocate individuals to classes, draw pictures of the classes so formed, identify members of inter-related classes (an object is a member both of the class of daisies and of the class of flowers) and even, at times, give the correct answer to a question about class-relationships (Piaget & Inhelder 1969b, pp.166-7). It is explicitly stated, however, that any such subject - one who uses conceptual schemes - is one who gives a correct answer to an inclusion-question on the basis of intuitive, and not deductive, modes of thinking (Piaget & Szeminska 1941, pp.202, 215/*Piaget 1952, pp.163, 175). A subject who uses operational schemes understands inclusion and in his most recent account Piaget offers two conditions for such understanding:

inclusion is correctly understood, and so quantifiable in the form $n_A < n_B$ only if two conditions are met...
1) it is necessary that sub-class A (for example daisies) forms a part of a total class B which is resistant and permanent enough to conserve its extension when the subject centres his attention on its sub-divisions....
2) it is further necessary to sub-divide the whole B into sub-classes A and A' which are explicitly characterised by partial negations: $A' = $ the B which are not-A and $A = $ the B which are not-$A'$ (Piaget 1977a, p. 88 - my translation; see also Inhelder & Piaget 1964, p.106).

Several comments may now be made about this claim:

1: It is clear that the conditions stated are (empirically) necessary conditions: the subject understands inclusion only if the conditions are met and not if they are met. That is, a subject does not understand inclusion if they are not met; but a subject might still not understand inclusion, in some given case, even if the conditions are met.

2: The first condition takes up Piaget's distinction between observable and coordinatory knowledge and concerns the extension...
of the classes. A subject who uses conceptual schemes is taken to be one who can distinguish between the intension and extension of a concept, in contrast to the subject who uses action-schemes alone who cannot — see (4.5). Thus a child can attribute the property of being a daisy (intension) to the objects in a given array (extension). Young subjects can, therefore, allocate individuals to classes. For example, in an array of 9 flowers, 7 of which are daisies and 2 of which are roses, young subjects can correctly identify which objects belong to which of these classes. For a subject to understand the inclusion of classes, however, a subject must also be able to form a sub-class and a condition of the subject's doing this is that the extension of that class should be preserved if the subject switches attention to the relevant including class. Specifically, it is claimed that subjects do not understand inclusion if they fail to conserve the extension of an (including) class when attending to a (sub-)class. Such subjects distinguish the properties of being a daisy and being a flower (intension); but fail to conserve the extension of the latter class when they allocate objects to both classes (extension). In short, the condition states that membership of one (sub-) class should not preclude membership of some other (including) class.

3: The second condition concerns the intension of the classes. In the array mentioned, let class \( B = \text{the flowers} \), class \( A = \text{the daisies} \) and class \( A' = \text{the roses} \). The condition states that a subject must have observable knowledge of the objects belonging to these classes and so be able to identify the objects by reference to their positive, affirmative properties. For example, the subject should be able to identify the daisies by reference to the observable properties of daisies. What is also required, however, is that a subject should be able to identify the objects by reference to their negative, inferential properties. The subject observes that the flowers are of two types, daisies and roses: \( B = A + A' \). What the subject should
be able to infer is that, given a universe consisting in class B (flowers), the subtraction of class A' (roses) from class B results in class A (daisies): \( A = B - A' \). Further, the subtraction of class A from class B results in class A': \( A' = B - A \). Thus only when the subject understands that the daisies are flowers-that-are-not-roses and also understands that the roses are flowers-that-are-not-daisies is the subject able to understand inclusion. That is, the subject should understand that the negative and inferential property flowers-that-are-not-roses is a property, and a distinct one, from the property daisies. The two modes of characterising an object are extensionally equivalent but they are clearly intensionally distinct: someone can fail to understand that the same objects can be picked out by both characterisations (Piaget & Szeminska 1941, p.199/*Piaget 1952, p.161).

4: A subject who can identify the objects in both of these ways, by their observable and by their inferential features, is one who performs a reversible operation: given any of the three classes, B, A and A', a subject who understands inclusion can inter-relate the classes such that each individual class can be identified through the other two classes. Such a subject, therefore, understands the necessity inherent in inclusion: if \( B = A + A' \), it is always and necessarily the case that \( A = B - A' \) and that \( A' = B - A \).

5: It is important that the right question should be asked to test a subject's knowledge of inclusion and the question favoured by Piaget has the general form "Are there more members of class B or more members of class A?". What is also important is that the test-material should be such that there are only three classes identified for a subject's consideration. This is not to say that a subject only understands that there are three classes, say flowers, daisies and roses — normal children also understand that there are other classes as well as these. Rather, an understanding of inclusion does
not require a subject to inter-relate any of the three classes in question with any other classes, as the discussion of the step-by-step nature of inferences permitted by a grouping – see (4.6) – showed.

(B) Critics' Accounts

There are two main lines of criticism that may be directed against the statement of Piaget's account here presented. The first one (a) states that class-inclusion may be understood especially when a question not favoured by Piaget is used. The second one (b) states that subjects find some class-relationships that do not involve inclusion to be as difficult to comprehend as those which do. It will be instructive to consider these in turn.

(a) The first criticism states that young children, for example, aged 5 years, can understand some types of inclusion-relation even though Piaget claims that this is a characteristic of older (aged 7 years) children. Several researchers support this claim (Markman & Siebert 1976; Markman 1979; Meadows 1977; Donaldson 1978) but it will suffice to take one study as being illustrative of the others, namely (McGarrigle et al., 1978).

Consider an array consisting of 4 cows such that: class A = the 3 black cows, class A' = the 1 white cow, class B = the 4 cows and class C = the 4 sleeping cows. That is, all the cows are sleeping but only some of them are black since the rest are white. Evidently, class C has the same members as class B. The question asked is "Are there more black cows or more sleeping cows?" and results indicate that subjects can correctly answer this question even though they give an incorrect answer to an equivalent Piagetian question. But it is apparent why this is so. A subject might quantify the membership of class A, quantify that of class C and then subtract the former from the latter to gain the correct answer. But such an answer does not require a subject to satisfy the second of Piaget's conditions: the subject needs to identify neither that A = B - A' nor that A' = B - A. Thus the use of class C, with the same
membership as class B, allows a subject to give a correct answer by appeal to the positive (observable) characteristics of the classes and not by appeal to the negative (inferential) characteristics.

Piaget's test-question is such that it is a condition of a subject's being able correctly to answer it that the subject should be able to perform a reversible operation. Yet the question favoured by McGarrigle et al. does not require, as a condition of a correct answer being given to it, that a subject should be able to perform a reversible operation. Even though class A is included in both class B and in class C, a subject can gain the correct answer without using knowledge of this relationship. A subject who does not understand inclusion can quantify the membership of class A, quantify the membership of class C; and compare the results. Since the membership of class C is the same as that of class B, a subject can correctly answer the test-question "Are there more black (cows) or more sleeping (cows)?", even though the subject is hypothesised not to understand the inclusion of class A by class B (and, a fortiori, the inclusion of class A by class C).

It is not claimed that subjects actually do correctly answer the test-question posed by McGarrigle et al. in the manner stated. The possibility stated here is a mere possibility. Its attractiveness is, however, increased by the experimental studies carried out by Dean et al. (1981) who show that children do use non-deductive strategies to solve questions that apparently require an understanding of inclusion. What is particularly relevant in their studies is the demonstration that children are able to answer correctly certain test-questions by reference to the observable properties of objects rather than to the object's negative properties.

(b) The second criticism is also stated by McGarrigle et al. (1978) but it will be instructive to use the example used by Trabasso et al. (1978). The challenge arises from an array such that there are two systems of classes: class A = the 8 animals, class A1 = the 6 dogs
and class A2 = the 2 cats, whilst class B = the 8 fruits, class B1 = the 4 apples and class B2 = the 4 oranges. Subjects are invited to answer a Piagetian inclusion-question "Are there more dogs or more animals?" and so make an A1 A comparison and also to answer a between-class question "Are there more dogs or more fruits?" and so make an A1 B comparison. Results show that subjects find both questions to be equally difficult and yet only one of them tests for the understanding of inclusion, namely the A1 A comparison.

This challenge can be resisted. Firstly, it is claimed (Trabasso et al., 1978, p. 159) that a subject who counts the membership of A1 will do this by counting the membership of A1 and A2, by adding these counts and then by subtracting the count of A2 from the sum of A1 and A2. It was claimed, in (A), that Piaget's account requires a subject to count A1, count A2 and then, as a condition of being able to add the two sub-classes together to form an including class, that the subject should be able to subtract one of the sub-classes from the including class to yield the other sub-class. Disagreement occurs, therefore, with respect to the interpretation of Piaget's theory. Secondly, even if the interpretation of Trabasso et al. is accepted, it does not follow that Piaget's account predicts that an A1 B comparison should be easier than an A1 A comparison. To count the membership of class B, when making an A1 B comparison, a subject must count the membership of B1, count that of B2 and sum the results. But only a subject who understands inclusion can do this, since B is an including class and B1 and B2 are sub-classes of B. So to quantify B, a subject must understand that B includes B1 and B2. Similarly, on this same interpretation, a subject who wishes to quantify A1 is required to subtract A2 from A and thus understand, as a condition of doing this, that A includes A1 and A2. Thirdly, an inclusion-relation is still involved in an A1 B comparison if the interpretation of Piaget defended here is
accepted. Inclusion would not need to be comprehended to count Al; but it would be for a count of B, which is admitted to be an including class in relation to Bl and B2. Thus a subject who did not understand inclusion would not be able to quantify the classes and so would fail to give a correct answer to the question. Thus both questions require an understanding of inclusion, one explicitly so and the other as a condition of the subject's being able to quantify the relevant classes. It is presumably for this reason that subjects find the questions to be equally difficult.

What conclusion can be drawn from (A) and (B)? Three points may be briefly indicated.

Firstly, Piaget is not always consistent in his selection of an appropriate test-question. Sometimes he uses "easy" questions of the type used by McGarrigle et al. (1978), such as 'Are there more wooden beads or more brown beads?' (Piaget 1952, p.164; see also Inhelder & Piaget 1964, p. 101, Piaget & Inhelder 1969, p.169, Piaget 1977a, p.84), even though he himself takes his "hard" question to be definitive. The accounts presented by the critics are a salutary warning that indifferent use of questions does not lead to an invariable outcome.

Secondly, Piaget does not perhaps signal clearly enough his use, with modification, of concepts taken from the theory of classes. It is not made as clear as it should be that a sub-class A' is to be characterised both positively and negatively (Piaget 1952, p.163 and 172 respectively; Inhelder & Piaget 1964, p.100 and 103 respectively). Piaget's account has to be extracted from his writings and does not transparently stand out from them.

Thirdly, it is evident that Piaget does not give linguistically based evidence for the claim that subjects understand the necessity, and not just the correctness, of their answers. Subjects state that class B has members 'extra' to or 'as well as' those of class A
evidence is cited to show a subject's use of modal notions. On
the interpretation of Piaget's position offered in (6.2), this
omission is to be regretted. It can be claimed, however, that Piaget
does require his subjects to justify the judgements that they make
and that this is compatible with the interpretation offered in (6.2).

(6.5) Conclusion

A final observation can be made about the argument of the
present section. The discussion in (6.3) and (6.4) was committed
to the account of Piaget's theory discussed above and particularly
to the methodological position discussed in (6.2). Suppose, however,
that the experimental studies cited in (6.3) and (6.4) are taken in
the way their authors suggest, namely as experimentally derived
evidence that young children can attain operational knowledge in
apparent conflict with Piaget's claims. Such a supposition can be
supported as follows. The distinction drawn between a correct
(observable knowledge) judgement and a deductively (operational
knowledge) correct judgement must, of course, be accepted. The
latter, however, can be sub-divided into two forms, say incipient
and mature. The incipient form of deductive knowledge occurs when
a subject makes a deduction. The mature form of deductive knowledge
occurs when the subject makes the deduction and also understands the
necessity of the deduction. The latter evidently presupposes the
former but might be consecutive to, and not concurrent with, the
former. Furthermore, Piaget himself does not distinguish between
the two forms of deductive knowledge and his own practice is such
—as the discussion of Piaget's evidence in (6.3) and (6.4) showed
—that the evidence that he sometimes provides is relevant to
incipient rather than mature forms of deductive knowledge. The
experimental studies cited therefore, have a twofold value, on this
view: they show that Piaget's account is one that merges the two forms of deductive knowledge and they also show that the incipient form occurs temporally prior to the mature form.

What must be said about this proposal is that its acceptance requires a commitment to Piaget's method, or an analogue of it, if not to Piaget's account. For the experimental studies cited employ a judgement-only approach and so are clearly suited to the detection of deductive knowledge in its incipient form. But such an approach is not suited, given the first argument discussed in (6.2), to the detection of deductive knowledge in its mature form since the subject's explanations are essentially implicated in such knowledge. Thus even if Piaget's account is vulnerable to the criticisms stated in (6.3) and (6.4) - or, more cautiously, if the version of Piaget's account here presented is so vulnerable - the critics can be in turn criticised for their exclusive use of methods which are appropriate to only one form of deductive knowledge and not to all forms. It should be noticed that Piaget's recent research apparently commits him to the claim that deductive necessity is not the only type of (logical) necessity. The assumption made in the present chapter - as too in (4.6) - is that operational structures enable a subject to gain deductive knowledge. It is also assumed that Piaget's critics, discussed in this chapter, make a similar assumption. A discussion of Piaget's recent research on necessity is contained in (9.3).

In short, the argument of this chapter has been that the attribution of operational knowledge, in its mature form at least, requires the use of the sort of methods used by Piaget. It has further been claimed that, subject to the qualification stated in the present section, Piaget's account of operational knowledge is strong enough to withstand the critical scrutiny that some of the best experimental studies have mounted against it.
THE SOCIAL SUBJECT

(7.1) Introduction

In section (2.2) it was stated that Kant was committed to the view that self-knowledge is impossible in the absence of knowledge of externally existing objects. This claim was modified by J.M. Baldwin who added that there is an important social dimension in knowledge which is apparently not covered by Kant's statement: self-knowledge is impossible in the absence of knowledge of objects and subjects. It is this modified view which is accepted by Piaget:

consciousness of self implies a perceptual comparison of the self with other people (Piaget 1932, p.407);

the self is constituted only by comparison and by contrast with other selves and with the external environment (Piaget 1954, p.233);

object permanency goes hand in hand with that of persons. Persons even constitute the first permanent objects (Piaget 1978b, p.94).

Such claims represent a clear statement of principle on his part. A compact discussion of Baldwin's theory, as too Piaget's, is offered by Russell (1978).

The term 'object' can, however, be used in different senses and confusion can result from their conflation. Consider three ways in which that term is used in the context of Piaget's theory. Firstly, the whole of reality is an object which a cognising subject aspires to know. Piaget uses 'object' in this sense when he refers to the subject-object relation (e.g. 1950, p.5) and this sense is explicit in the French title of Piaget (1937/1954). Associates (Cellerier 1973, p.24; Furth 1980, p.56; Vuyk 1981a, p.50) follow this practice, as too does (3.5) above. Secondly, any physical object is an object and in this sense reality is composed of two main types of member, namely bodies and minds. It is apparent that Piaget (1954, p.3 ff) makes use of this sense in his account of the infant's understanding of the physical universe. Psychologists follow this practice when they refer to object-permanency (e.g. Flavell 1977, p.40). Thirdly, whatever a subject thinks about is an object of thought and the term 'object' is used in this sense when it refers to the object of a thought (e.g. 1950, p.15).
and so an object in this sense is the content, or internal character, of a thought. Piaget (1972d, p.130), following Brentano, uses this sense of 'object' and it is evidently this same sense which is used by Furth (1981, p.60). This notion is further discussed in (8.3) below and is also used in the next section (7.2).

It is for Kantian expositors to determine the sense of 'object' used by Kant in the claim attributed to him above. It is clear that Piaget makes use of all three of these senses and that his main work on permanency (Piaget 1954) can be taken to use both of the first two senses. Such a view is supported by the quotations listed above.

The discussion in the present chapter will be concerned with the social knowledge of a subject at stages posterior to infancy. A brief comment may be made here, however, about the social knowledge of the infant. How, it may be asked, does the infant distinguish the permanence of (physical) objects from that of other subjects? In fact, this question is inappropriate and to see why this is so, consider a parallel case. Suppose a child, who does understand that other subjects exist, is asked to differentiate male from female subjects. If the child does not make that distinction, because he/she has no concept of sex, then the child does not have the problem of determining how the class of females is to be extracted from the class of male subjects. Lacking a concept of sex, the child does not take other subjects to be male subjects. Rather, the child confuses what others keep distinct and so the problem is that of forming two sub-classes out of what was previously one undifferentiated class. Similarly, an infant who does not draw the distinction between objects and subjects does not, through lack of the relevant concepts, take the world to be a world of physical objects. Rather, the world is one of objects-and-subjects and so the task is to form two sub-classes out of what had previously been confused. The infant does not inhabit
a physical, and not social, environment; neither is the infant's initial knowledge a knowledge of (physical) objects, and not subjects. Rather, the infant's knowledge is initially a knowledge which is both physical and social: some of the infant's knowledge is knowledge of physical properties and some is knowledge of social properties. Piaget, in fact, gives several instances of a subject's confusion of what adults take to be distinct classes, for example social rules with laws of nature (1932, p.188) or logical with causal laws (1977j, p.114). And in infancy Piaget specifically claims that the infant initially confuses his/her own existence with that of the universe (1954, p.351) as well as the physical with the social environment (1950, p.158). A limitation on this claim is discussed in (8.2).

The subsequent discussion is in two parts and is pre-occupied with the rebuttal of two types of objection that have been raised against Piaget's account, namely the Social and Epistemic Objections. The Social objection states that Piaget's account is biased in favour of physical forms of cognition and so cannot serve as an account of social cognition, at any level of development. The distinct Epistemic objection states that Piaget's account is a failure because it takes cognition, whether physical or social, to be a phenomenon that is acquired by a subject alone. The first objection restricts the range of cognition open to a subject; the second objection excludes his/her having knowledge at all. The Social objection will be discussed in (7.2) and the Epistemic objection in (7.3), and is based on Smith (1980c, 1982b)

(7.2) The Social Objection

The Social objection states that, whatever merits Piaget's account may have as an account of physical cognition, it is a failure as an account of social cognition because it mistakenly treats the latter as being essentially similar to the former. Thus it is claimed that Piaget fails to take into account the fact that
individuals are immersed in a non-natural world, in an 'environment' of ideas, meanings, intentions, history, symbols, within a matrix of social influence and cooperation (Rotman 1977, p.181).

Again, it is claimed that Piaget's equilibratory models accord a primacy to motor and manipulative behaviour in relation to objects in the world (Grize 1977, p.57); it is therefore unclear whether the same model can be used in cases where a subject interacts with a cultural environment, which is non-manipulative (Grize 1977, p.57). Again, although Piaget's account of formal operational thinking is applicable to social contexts, in that there are similarities between reasoning in physical and inter-personal contexts,

the basic approach to cognitive activity in the social domain has borrowed so heavily from existing cognitive approaches that there is little that is truly 'social' about it (Keating & Clark 1980, p.29).

It is not that Piaget denies social forms of cognition; rather, his account ignores the distinctive feature of social cognition by treating it as a special type of biological phenomena (White 1980, p.469).

There are at least three reasons why this objection can be dismissed:–

1) Piaget explicitly states – see (5.2) – that there has to be a social influence for a subject to acquire knowledge and so the presence of some social environment is one (empirically) necessary condition of the formation of knowledge. If Piaget's theory is taken to be one that embodies a biological conception of knowledge, that conception is such that social factors cannot be left entirely out of account. This is a clear statement of principle on Piaget's part and it is accepted as such by commentators (Mays 1979, p.59; Vuyk 1981a, p.111).

Moreover, it is also clear exactly where Piaget does acknowledge there being a social influence on the subject. It will be recalled – from (3.6) – that Piaget takes consciousness to consist in
conceptualisation and an observer can study the consciousness of
another subject only when each uses language. But Piaget, following
de Saussure, takes language to be a system of signs that are both
collective - since shared by members of a social group - and
conventional - since the meaning of any sign is arbitrary (Piaget
1970a, p.717). Thus inter-personal communication by means of
language requires that two individuals should use the same system
of signs since adequate communication cannot take place if the two
individuals do not assign the same meaning to the same sign. Piaget
states:

the equilibrium of an exchange of thought presupposes
1) a common system of signs and of definitions,
2) conservation of valid propositions which obliges
anyone who recognises them as such and 3) reciprocity
of thought between partners (Piaget 1967d, p.162 -
my translation).

This statement is taken from Piaget's discussion of an equilibratory
theory of communication, whose details may be here ignored. What
should be noticed is that the statement recognises the social element
that is necessary for communication to take place since there must
be a common and so shared system of signs and their definitions.
The system is common in that its use cannot be the exclusive prerogative
of only one subject. No doubt Piaget could here endorse Wittgenstein's
(1958, sects. 244 ff) argument's against the possibility of there
being a private language, since Wittgenstein's arguments are directed
against phenomenalistic theories of perception of the sort referred
to in (8.2). The second condition requires that different subjects
should conserve the same proposition which each is obliged to accept.
The social element here is twofold. A proposition is such that it
is invariant to different subjects and is not the exclusive preserve
of one subject. Further, a subject has an obligation to accept
that such-and-such is a valid proposition (if it is one). The
obligation here is logical, if the proposition is derived as a
logical consequence from other propositions, and moral, if the subject intends to give an honest and truthful characterisation of some state of affairs. It suffices to notice that the discussion of normative facts — in (3.4) — is applicable to moral, and not just to logical, norms; that commentators (Cellerier 1973, p.44; Wright 1982) take Piaget's theory to be a unitary theory applicable to the explanation of legal, moral, and logical as well as physical phenomena; and that Piaget's (1932; p.56) account of conceptualisation is stated by him to be applicable to moral and physical contexts. The third condition explicitly mentions the social element in communication since there must be reciprocity of thought between two subjects.

In short, Piaget takes conceptualisation to occur when a subject uses language and language is taken to be essentially social. Thus any subject who uses language — and this is the majority of subjects at the post-sensori-motor level — is one who is influenced by the social environment. Thus any knowledge gained by such a subject inevitably embodies a social influence upon it.

2) It will be recalled, from (3.6), that Piaget distinguishes between the individual and epistemic subject. The knowledge possessed by the epistemic subject is not the unique possession of any individual subject but is instead knowledge which is common to, and so shared by, any subject at that (operative) level of development. It follows that the epistemic subject is a social subject since the knowledge possessed by that subject transcends any individual subject.

Notice that the claim made is that any epistemic subject is a social subject in that the knowledge possessed by the former is social knowledge: it is knowledge which is open to any individual at a given level. Such a claim is distinct from its converse, that all social knowledge is knowledge possessed by an epistemic subject. Only the former, not the latter claim, is here stated. It is
possible that the failure to draw this distinction results in the sort of complaint attributed above to Keating & Clark (1980).

Piaget's position is stated by him as follows:

a "grouping" is only a system of possible substitutions, whether at the heart of one and the same individual thought (operations of intelligence) or between one individual and another (cooperation). These two sorts of substitutions constitute then a general logic that is both collective and individual (Piaget 1967d, p. 170 - my translation).

A logical form does not have a restricted application to one domain rather than to another and it is Piaget's claim that groupings do have application to inter-subjective transactions.

Two questions arise from this statement: (A) are the operations used by a subject the same as those arising in co-operative relationships with other subjects and (B) what is the temporal ordering of the individual's operations and social co-operation?

(A) The first claim is more fully stated by Piaget as follows:

all logical thought is socialised because it implies the possibility of communication between individuals. But such interpersonal exchange proceeds through correspondences, reunions, intersections and reciprocities, i.e. through operations (Piaget 1962, p.13).

The operations listed in this quotation are all logical in nature and are, of course, referred to in the discussion of action-schemes and operational structures - in (4.5) and (4.6) respectively. An example clarifies this point. Any logical principle, such as transitivity, can have instances in both the physical and social domains. In a conservation experiment, containers A, B, and C have the same quantity of liquid and a subject with operational knowledge can accept that if the amounts in A and B as well as the amounts in B and C are the same, then the amounts in A and C are the same. Thus transitivity can be instantiated in physical contexts.

Suppose, now, that such a subject is presented with not merely these containers, A, B and C, but also with three subjects, S1, S2 and S3. And suppose the subject observes that S1 agrees with S2 that container
A has the same quantity of liquid as container B; observes that S2 agrees with S3 that containers B and C have the same quantity of liquid; and then states that S1 and S3 (should) agree that containers A and C have the same quantity of liquid. A subject who does make this deduction is one who uses a knowledge of transitivity, which in this case is instantiated in a social context.

It is not argued that a subject who understands conservation in the case of the containers will also understand it in the case of the subjects. No doubt a developmental lag, analogous to the sort noticed by Piaget (Piaget & Inhelder 1969a, p.99) with respect to physical cognition, might occur. It is claimed that both of the contexts cited provide instances of the same logical principle, and such a presence is compatible with there being a variety of other differences.

Thus the claim is that the operations referred to by Piaget are applicable to a range of contexts and are not restricted to physical contexts, still less to cases where a subject acts alone upon the world. In particular, they are applicable to inter-personal contexts as well. Thus the operations used by an individual are the same as those arising in cases of social co-operation.

(B) Are an individual's operations the temporal antecedent of social co-operation, does the latter precede the former or do both occur concurrently?

The critics cited at the outset of this section apparently attribute the first of these positions, namely that the individual's operations are constructed and that social co-operation is consecutive to this. But such a position is vulnerable to Piaget's own statement of account:

social life is necessary if the individual is to become conscious of the functioning of his own mind and thus to transform into norms properly so called the simple functional equilibria immanent to all mental life and even all vital activity (Piaget 1932, p.407);
the child first seeks to avoid contradicting himself when he is in the presence of others (Piaget 1950, p.163).

The role of functioning and of contradiction has already been discussed— in (4,4) and (5,2) respectively—and so Piaget's claims here should be taken as claims that arise from his equilibratory theory. Thus his position is that a subject who can conceptualise first avoids self-contradiction when entering social relationships, that social relationships are necessary for the subject's development to take place. For development is taken by Piaget to occur when a structural change occurs in the functioning of a subject. In consequence the subject becomes able to conceptualise logical norms, and so to avoid self-contradiction, through a progressive awareness of the logico-mathematical properties inherent in his/her own action-coordination. Moreover, commentators attribute such a view to Piaget:

subjective forms of necessity will be transformed into normative forms that are objective (that is, they become independent of individual subjectivity) only by their insertion into a network of social exchange (Cellérier 1973, p.44 – my translation).

What is excluded is a case where operations precede co-operation and with this exclusion the first alternative falls.

The second alternative states that social co-operation precedes the formation of an individual's operations. This alternative is compatible with the position just attributed to Piaget, though it is not entailed by it since the third alternative is compatible with that position as well. Recent research provides support for this alternative. Subjects, who fail to perform correctly when presented with an operational task— for example, the three mountains-task (Piaget & Inhelder 1956)— do perform that task correctly when invited to co-operate with a peer. Peers are divided into two groups, those who fully conserve and those who only partially conserve and it is claimed that there is greater cognitive progress in a
test-subject when the latter interacts with a peer who partially conserves rather than when the subject interacts with a total conserver (Mugny & Doise 1978, p.190; Doise & Mugny 1979, p.107).

The conclusion drawn from this research is that:

social coordination of actions facilitates and precedes the individual coordination of actions (Mugby & Doise 1979, p.191).

Socio-cognitive conflict is then a determinant of individual progress.

Three comments can be made about such research. Firstly, the authors do not take themselves to be presenting an interpretation of Piaget's account and in fact state that their view has an explanatory superiority over Genevan-based studies, i.e. those carried out by Piaget's associates. Although these authors do use an equilibratory model to explain cognitive growth and do use Piagetian tasks, together with a clinical methodology, their view can be characterised as neo-Piagetian (if it is Piagetian at all). Even if the authors are correct in stating the conclusion that they do, it would not follow that such a conclusion correctly reflects the view adopted by Piaget.

Secondly, there is research that counter-indicates the conclusion drawn by these authors. A variety of operational tests have been used to investigate the claims made by Mugny & Doise and it is concluded that there is no group-superiority to be found when subjects are presented with the three mountains task (Russell 1981b) or the inclusion-task (Russell, 1981c). Moreover, when social interaction did result in success, it was found that this was often due to mere compliance on the part of an incorrect subject who passively accepted the judgement of a (correct) peer. Social dominance, rather than the coordination of viewpoints, was the prevailing factor. These findings are impressive but not conclusive, for it is not clear whether Russell divided his conserving subjects into partial and total conservers. Mugny & Doise state that social interaction between a
non- and total conserver is small or even non-existent and that
cognitive growth occurs when a non-conserving subject interacts with
a partial conserver, with consequential benefit for both subjects.

Thirdly, even if Piaget's position is taken to be reflected
in the conclusion drawn by these authors, the essential role of
action-coordination cannot be left out of account. Even if socio-
cognitive conflict is superior in the manner suggested, the
individual subject is one who acts, and interacts, in a social context.
But in that case, the individual is one whose social action and
social interaction requires coordination on the part of that individual
subject. Thus the fact (if it is one) that cognitive progress
preferentially occurs when a subject is faced with socio-cognitive
conflict presupposes that the subject in question has the ability
to make coherent the conflicting judgements generated by that group
experience. And that ability is one which that subject must possess.
It is not the mere presence of other subjects nor the consequential
increase in the variety of judgements expressed which facilitates
cognitive change but rather what the individual subject in question
makes of that social presence and of that conflict.

Thus the third alternative remains, namely the view that an
individual's operations are constructed concurrently with social
co-operation. Piaget states this view as follows:

the most generalised forms of thought, those that
can be dissociated from their content, are, by that
very fact, forms of cognitive exchange or of inter-
individual regulation, as well as being produced
by the common functioning which is a necessary part
of every living organisation (Piaget 1971a, pp.360-1;

Piaget's main thesis - it will be recalled from (2.4) - states that
cognitive processes are the outcome of functioning that occurs at
the biological level. Thus Piaget must, of course, accept the
claim that his theory is one that is biological in inspiration. He
is nonetheless able to differentiate the poles of the unitary,
regulatory process which he takes to be basic to life. Operations
cannot be constructed in the absence of a social environment; and the construction of operations is such that they may be applied to any of the contexts in which they are applicable, including social contexts.

3) It will be recalled from (4.3) that Piaget makes use of the notion of intentionality, that the actions of a subject who gains knowledge are purposeful and directed upon the goals that the subject forms. It will also be recalled from (3.5) that a subject who gains knowledge is one who is beset by constricting egocentrism. In particular, a subject who performs an action establishes some end-state which the action is directed upon and gains (observable) knowledge consisting in an awareness of that goal and perceptual knowledge of the outcome of the action. But the observable knowledge gained by a subject is expressed in the judgements that he/she forms and a subject's logical incapacity is revealed through a failure to inter-relate any one judgement with the other possible judgements that are related to it. But the observable knowledge, of either of these two sorts, gained by a subject is shown in the interiorised actions that the subject performs and thus it is the subject's conceptualisation of the world, together with the coordinations that the subject imposes on them, that is crucially important. What matters is how a subject represents the world in acting and not what physical actions the subject performs.

The last point is central. Certainly the concrete operational subject is taken by Piaget to be one who requires the incarnation of an abstract principle in some particular case for its comprehension. To claim this is not, however, to claim that the subject's actions are exclusively motor and manipulative in character since it is the system of representations that underlies such actions which is relevant to the determination of operational levels of thought.

But the capacity to represent is not confined to a subject's interaction with the physical environment since it covers social
interaction as well. A representation is neutral to its object, where the object of a representation is internal to that representation. This notion is more fully discussed in (8.3) below. The object of a representation is the content of the representation and that content can be anything at all. In particular, physical objects are not the only objects that can be represented since a subject can represent any feature of the world at all. It follows that a subject can represent social features of the world.

But once the capacity to represent is granted, Piaget can exploit the conception of action used by him, for the capacity to represent and the ability to engage in intentional action are inter-related. In acting intentionally, a subject conceptualises the world, whether physical or social, and in conceptualising the world the subject encodes the world in some representational system. Thus by virtue of social interaction a subject-agent has the capacity to draw upon the social environment in which that subject lives, upon history, education, ethical, aesthetic, religious and scientific cultures and so on. There is, therefore, no reason in principle why Piaget's account cannot cover a subject's responses to aspects of the social and not just the physical world.

The central conclusion of this discussion can now be stated. Piaget's theory is not biased in favour of physical, to the exclusion of social, forms of cognition. Since this is so, Piaget can rely on the principle of the division of labour and claim that his tendency to study physical forms of knowledge does not exclude the use of his theory in the investigation of social forms of knowledge.

(7.3) The Solitary Knower

The Epistemic objection states that Piaget's account is a failure since it misconstrues the nature of cognition, for a subject who acquires knowledge - whether physical or social knowledge - is taken
to be one who acquires that knowledge through the interplay of factors that are intra-subjective rather than inter-subjective. Knowledge is acquired by an individual whose task is essentially solitary: the subject may acquire that knowledge in the company of other subjects but could acquire it in their absence. It is for this reason that Piaget's subject is a solitary subject of knowledge, one who stands squarely in the Cartesian tradition.

The reference to Descartes may mislead. The Epistemic objection could be stated - as the claim that Piaget takes an infant subject, during stages I and II, to be a solipsist and a phenomenalist (see section (8.2) below). If this is so, that objection is valid. Support for the Epistemic objection is not, however, based on such claims since it is instead stated in relation to subjects at levels posterior to infancy: the non-infant subject is taken to be a solitary knower. It will be expedient to examine Hamlyn's statement of this view, whose position is endorsed by Boden (1979a, pp.95, 151) and Russell (1978, p.238; 1979, p.68).

Hamlyn (1967, p.39) presented an initial statement of his objection to Piaget, which appears in the claim that

the child does not have to systematise phenomena for himself; there are social pressures and influences, generally accepted standards and norms of what is right and correct (Hamlyn 1971, p.10).

Several reasons are offered for making such a claim: that Piaget presents a biologically biased account of cognition which ignores the social context in which such cognition arises; that the account takes knowledge to be an individual rather than a collective phenomenon; that members of a group who possess knowledge have similar interests and objectives and are in broad agreement about the criteria for their successful attainment. Hamlyn claims that his view is an interpretation of Wittgenstein's (1958, sects. 241-2) dictum about forms of life and the need for agreement in judgements as well as in definitions. Hamlyn interprets this view as follows. A subject cannot acquire
knowledge, alone, since any such acquisition is interpersonal. Other subjects are brought in, not because it is a collective decision that decides what is knowledge and what is not, but rather because any criteria that one subject uses are criteria which are available for the use of other subjects. Thus an account of the growth of knowledge must show where and how other subjects enable any one subject to select and reliably use the criteria that he/she does (Hamlyn 1978, pp.58-9). As Wittgenstein puts it:

it is not possible that there should have been only one occasion in which someone obeyed a rule,...to obey a rule (is a) custom (use, institution) (Wittgenstein 1958, sect. 199- author's emphasis).

If knowledge were a solitary affair, a subject could use one criterion on one occasion in some situation but a different one on another occasion in that situation. He could therefore obey a rule on one occasion and a different one on another. And if this is so, the presuppositions of regularity and generality – upon which the attainment of knowledge rests – would be lacking.

In a later paper, Hamlyn expresses his objection in a different way. A subject who has knowledge that 'p' knows that 'p' is true: 'to know something is to know it as true' (Hamlyn 1979, p.3). But this implies that the subject has an understanding of a norm, that is, of some concept of truth. This implies that the subject can distinguish what is true from what is false and so implies that his/her own claims may be corrected by other subjects. Only a subject who takes correction to be correction-by-persons can have knowledge. Thus social relations are required for a subject to have knowledge, which relations are taken into account for the subject to have the knowledge that he/she does. (Hamlyn 1979, p.4). In essence, then, the objection is that although Piaget places his subject in a social setting, his account fails to show how a subject's understanding of this social setting is necessary for knowledge.

There are several comments to make in the rebuttal of this objection:
1) The objection is a general one and is applicable to any, and not merely to Piaget's theory of knowledge. Yet the stipulated conditions are too restrictive and would prevent the attainment of knowledge by computers, animals and infants as well as by young children. For it is unclear how the notion of correction-by-another-subject—which is-construed-as-correction-by-a-subject can be applied in such cases. Yet researchers from a wide range of disciplines such as artificial intelligence (Boden 1977, pp.4, 402), ethology (Koehler 1957, pp. 226-8; Lorenz in Piaget 1971a, pp. 314-5), biology (Boden 1980b), linguistics (Chomsky 1980, pp.48, 187) and developmental psychology (Bremner 1980, p.41; Bryant 1974, p.1) are willing to impute intelligence and understanding to beings other than adult man. Moreover, Hamlyn (1978, p.74) claims that there is a family of epistemic concepts — knowledge, belief, hypothesis — such that a refusal to apply any one of these precludes the application of any of the others and thus the researchers cited cannot respond by attributing some epistemic concept other than knowledge — for example, belief — instead. The choice is clear: either knowledge cannot be attributed to machines, animals and infants since Hamlyn's conditions cannot be satisfied in such cases; or those conditions can themselves be rejected on the grounds that they lead to unacceptable consequences. It is the latter option which is here suggested, one which is endorsed by others (Elliott 1980).

2) It is a misconception to suppose that Piaget's account of the acquisition of knowledge identifies cognitive, and excludes affective, factors:

    there is no behaviour pattern, however intellectual, which does not involve affective factors as motives. (Piaget & Inhelder 1969a, p.158).

This claim is not just the claim that the cognitive and affective life of the subject develop in parallel (Piaget 1951, p.205) but is rather the claim that an intentional action, which constitutes the
basis of a subject's cognitive activity, is always one that includes affective elements. Thus Piaget's account of equilibration (see section (5.2) above) requires that a subject should value coherence, experience needs that arise from his/her own contradiction-mistakes, set goals to overcome a disturbance facing him/her and want to classify or serially order objects in this way rather than that. Clearly, each of these is affective as well as cognitive and it is for this reason that Piaget (1953a, p.9) attempts to present a schematic review of their inter-relation.

Thus when Hamlyn states that a subject of knowledge must have interests and objectives in common with other subjects of knowledge, it is clear that Piaget can accept such a claim just because Piaget's conception of action is one that requires the ascription to an agent of beliefs, desires, values and interests for the subject to be an intentional agent (see (4.3) above). Piaget's biological account of the growth of knowledge must not be taken as being one that uses a mechanical conception of bodily movements of the sort stigmatised by Hamlyn (1953, p.72; 1970, p.147). Yet Hamlyn does not fully discuss Piaget's concept of action and his account of equilibration.

It follows that Piaget's account is not precluded from incorporating affective elements in it and that such factors are specifically included by Piaget. It is therefore open to Piaget to claim that his account is one that has explanatory scope for the more detailed discussion of the role of shared interests and needs. The assertion on the part of Hamlyn that an adequate account must retain such explanatory scope is one that can be readily accepted by Piaget.

3) Hamlyn (1978, p.55) refers to Piaget's notion of egocentrism and states that decentration is a necessary but not sufficient condition for the attainment of an objective judgement, one that expresses knowledge. Hamlyn further implies that an account which is limited
to the statement of necessary conditions is one which is incomplete and that Piaget's account aspires to provide a set of sufficient conditions of objectivity. Two replies can be entered about this view. Firstly, the general claim stated here is a valid one: if an account gives necessary conditions alone and shows neither that the conditions that it offers constitute all of the necessary conditions nor what any sufficient condition might be, that account is incomplete in that respect. The second comment to make however, is that the interpretation of Piaget's account presented here — see (5.2) — suggests that equilibratory factors are necessary and not sufficient conditions of the formation of knowledge. Further, Piaget's distinction between the epistemic and individual subject of knowledge — see (3.6) — is such that it is the epistemic, and not the individual, subject who is the centre of his theoretical concern. But if this is so, Piaget's theory does not give a set of sufficient conditions under which knowledge might arise since that theory is essentially limited to the characterisation of the structural basis of the epistemic subject and the changes therein. (An elaboration of this limitation on Piaget's theory is provided in (9.2).)

4) A distinction is drawn by Cooper (1979, p.28) between an account of how structuring results in knowledge and an account of how a way of structuring — which is in fact knowledge — comes about. He cites an analogy and claims that a physiologist can explain how an athlete can win an Olympic 1000 metres (sic) even though his explanation makes no reference to the rules and conventions surrounding such a race. Similarly, the fact that we can decide to describe a way of structuring as "knowledge" does not entail that the subject in question must see his structuring in such terms as well. Specifically, it is open to Piaget to say:

look, it's not my job to say what knowledge is — but to account for how what in fact is knowledge (by whatever criteria you're laying down) came about (Cooper 1980, p.98 — author's emphasis).
But if this distinction is accepted, it follows that Hamlyn's objection does not provide a reason for rejecting Piaget's account.

In fact, Piaget makes much the same claim himself:

it is not the psychologist's task to determine such and such a level of knowledge, but rather to explain how the transition from one level to another takes place (Piaget 1977f, p.37/1973, p.xlii).

That is, Piaget does not take his task to be that of deciding which knowledge is presupposed by which. Rather, his task is to identify the developmental route whereby a subject can proceed from one level of knowledge to the next, to show how the structure relevant to one level of knowledge generates the structure relevant to the next.

5) Hamlyn (1978, p.91) accepts the traditional view of knowledge, or at least accepts it for ease of discussion of issues in Piaget's theory, that knowledge is true, justified belief. (It will be recalled that this view was discussed in (6.2) above.) On this view a subject can only know what is true and can never know what is false. But there is a difference between a subject's knowing 'p' only if 'p' is true and a subject's knowing 'p' only if the subject knows that 'p' is true. And Hamlyn apparently accepts the latter, and not just the former, of these viewpoints. For example, he states that 'to know something is to know it as true' (Hamlyn 1979, p.3; cf. 1981, p.6). The objection to the latter viewpoint, however, is that it is too strong: a subject can have knowledge and yet not know that this is so. Someone might deny that he knew a certain proposition - say, that St. Helena and not Elba as Napoleon's last exile - or express uncertainty about knowing it or believe that he does not know it and yet know that proposition all the same, as White (1977, p.229) points out. For an observer may attribute such knowledge to a subject despite the lack of conviction, always provided the subject can demonstrate his/her knowledge to that observer in some acceptable way.
It might be claimed that Hamlyn's position has been misinterpreted and that he does not accept the strong claim suggested but is instead content to state the weak claim alone, that knowledge implies truth. Such a claim requires that a subject who has knowledge must have some concept of truth since he/she must be able to distinguish what is correct from what is not. This may be accepted, provided a distinction is drawn between the ascription of a concept to a subject and the attribution of a concept to that subject. A concept may be attributed to a subject in any case where some competent observer can vouch for the fact that the conditions for the application of that concept have been met. That concept is ascribed to a subject in a case where that concept is attributable to the subject and also where the subject knows what (some of) the conditions for the application of that concept are. But if this distinction, between attribution and ascription, is granted, it can be claimed that Piaget's theory allows the attribution of a concept of truth to a subject by an observer, though not thereby the ascription of that concept. This general distinction has been alluded to in several sections above - for example, (3.4) - and it is clear in the following claim that a structural possibility is one which is relative to the operational structures which are available to the subject, which constitute possibility from the observer's point of view (Inhelder & Piaget 1958, p.262).

A subject may not know all of the features of the cognitive system used by him/her as a condition of the acquisition of knowledge.

6) Hamlyn claims that it is a legitimate task for the philosopher to identify conceptual issues relevant to a subject's acquisition of knowledge (Hamlyn 1978, pp.2-4). Evidently, Hamlyn takes such acquisitions to result in the attainment of a state of objectivity and thus seeks to lay down criteria whose satisfaction is required for rational and logical subjects to make objective judgements. Piaget contrasts such an approach with his own dynamic position:
objectivity is a process and not a state...it is impossible to talk about objectivity or object without referring back to the previous condition of cognitive organisation (Piaget 1971a, pp.64-5).

To state that objectivity is a process is to state that questions about the actual level of cognitive organisation — or structure — are inevitably implicated in determining whether a subject can make objective judgements. Consider:

Raoul, have you any brothers? — Gerald. — And has Gerald a brother? — No, only me has a brother (Piaget 1928a, p.84);

Do I have a bottle that is at the same time half-full and half-empty? — No. — Why not? — Because there (\(\frac{1}{2}\)) it's full up, and there (1/2) it's half-full, but there (1/4) it's half-empty (Piaget 1980b, p.233).

Subjects such as these are neither completely rational nor completely logical and the judgements that they make show that they cannot be relied upon to accept the deductive consequences of their own claims. Piaget's concern for the process of objectivity is a concern to establish the respects in which developing subjects fail to satisfy the criteria applicable to their adult counterparts. So although Hamlyn is correct to claim that an empirical inquiry cannot ignore conceptual constraint, it is important to ensure that conceptual prescriptions and proscriptions are stated with the position of the developing, and not merely the developed, subject in mind. A developing subject makes contradiction-mistakes — see (5.2) — and it is in consequence of this that Piaget contrasts genetic with philosophical epistemology — see (2.3). That is, Piaget can claim that even if Hamlyn correctly states the conditions applicable to the formation of knowledge in the case of adult subjects, as a matter of fact these conditions do not apply to developing subjects. And they do not apply not merely for the general reason that Hamlyn does not himself provide empirical evidence in support of his claim but rather because the very claim itself requires a subject's comprehension of factors that Piaget's theory shows to be open to development.
To see this, recall that Hamlyn states that a subject of knowledge must appreciate the force of a norm; must understand that socio-cognitive conflict can lead to correction of a subject's judgements; must understand that corrections have their source in other subjects; and must understand that subjects center into social relations (Hamlyn 1981, p.6). Yet it is precisely the comprehension of such factors as these that Piaget takes to undergo development. And Piaget's view is supported by his study of individual subjects who are undergoing development. Thus if the facts of development are as Piaget says they are, then Hamlyn's conditions are too stringent and do not apply to developing subjects. In the absence of supporting empirical evidence on the part of Hamlyn, Piaget can then conclude that his account can be retained, unless and until an argument that is supported by both philosophical and empirical considerations is forthcoming. No doubt, Hamlyn would not accept such a reply, as the discussion of his criticism of genetic epistemology - in (2,3) - shows. The dispute between him and Piaget is thus a dispute not merely over the conditions under which knowledge arises but also about the sort of epistemology that should be utilised to approach such a question at all.

In sum, the conclusion drawn is that the Epistemic Objection can be set aside since it presents no valid objection to Piaget's account. It is specifically noted that acceptance of this objection would require substantial modification of many scientific theories that investigate the occurrence of knowledge. It is also noted that the conditions for the occurrence of knowledge stated by Piaget's theory arise from his contention that genetic epistemology is a preferential form of epistemology and that the conditions stated by Hamlyn arise from his adherence to philosophical epistemology.
THE INFANT SUBJECT

(8.1) Introduction

Piaget's account of infancy has been subjected to critical review on the part of psychologists and this is well summarised by Bremner (1980, 1981). The present chapter does not seek to contribute directly to this review but is instead centred upon the question of whether the infant subject engages in representation. It seems quite clear what Piaget's answer to this question is, for he states that:

sensori-motor intelligence is so called because it is prior to language...representative intelligence begins, by contrast, only with the semiotic (or symbolic) function (Piaget 1967a, p.17/*1971a, p.3).

That is, Piaget denies that the infant subject has the ability to engage in representation.

Piaget's account of infancy is, however, beset with internal difficulties that arise from this denial. Some of those difficulties are presented in the sections below where a series of contradictions will be identified. Clearly, a contradictory account is unacceptable and so the contradictions will have to be removed, if Piaget's account is not to be abandoned. The conclusion of this chapter, (8.8), offers one possible way in which the difficulties identified can be resolved. The resolution is the claim that the infant is able to engage in representation. Since the conclusion stated runs counter to Piaget's denial that this is so, the grounds on which it rests must be strong enough to warrant drastic change of this sort. The grounds are given in sections (8.2) - (8.7).

The first difficulty arises from Piaget's claim that the infant undergoes a Copernican Revolution. Piaget's account is such - and it is so taken by others - that the infant's initial experience of the world is subjective in a radical sense, namely in being solipsistic and phenomenalistic, but that it is eventually objective in character. It is argued, in (8.2) that a Copernican Revolution, so construed, could not take place. In section (8.3) it is claimed that the use
of the notion of Intentionality is flexible enough to allow Piaget to characterise the subjective aspects of the young infant's perceptual activity but is applicable to the perceptual activity of the older infant as well. Thus the argument of these two sections is that Piaget requires the infant to undergo a Copernican Revolution; and yet such a revolution can take place only if he changes his account of the young infant's perceptual activity.

In section (8.4) it is shown that Piaget is committed to the contradictory statements that a subject can and cannot gain observable knowledge. In section (8.5) it is shown that, in fact, Piaget is committed to the contradictory statements that an infant can and cannot engage in representation. In section (8.6) it is shown that Piaget is committed to the contradictory statements that an infant can and cannot engage in reflective abstraction. Since the contradictions indicated here are explicit, Piaget's account is clearly defective, if it can be shown that he is committed to statements such as these.

In section (8.7) it is shown that Piaget claims that his account of infancy can accommodate cases where a limbless subject undergoes development in much the normal way. It is argued that retention of such a claim requires Piaget's acceptance of the notion that cognitive phenomena are Intentional in character.

In sum, sections (8.2) - (8.7) point to the conclusion that (infant) cognition is Intentional and representational. If this conclusion is accepted, it follows that infancy cannot entirely be explained in the manner suggested by Piaget's main accounts.

(8.2) The Copernican Revolution

Piaget's account of stages I and II of infancy is crucially different from that of stage III and beyond. It is argued that (A) commentators, (B) translators and (C) Piaget himself take the infant subject at stages I and II (hereafter, the young infant) to be a subject who has no perceptual knowledge of objects and subjects
in the world but instead is a solipsistic and phenomenalistic centre of consciousness. But if this is so, the cognitive life of the infant is nothing but 'a blind play of representations, less even than a dream' (Kant 1933, A112). Consider, then, the basis on which this objection rests, which is based on Smith (1981h).

(A) **Commentators**

There can be no doubt that commentators take the young infant to be a solipsist and phenomenalist. The first major (English) work on Piaget expresses this view as follows:

> if the young infant does not conceive of objects as adults do, what are they to him? Piaget believes that they are primarily sensations - images or pictures which, in his egocentricity, he cannot distinguish from the act of assimilating them (Flavell 1963, p.130).

Flavell thus attributes to Piaget the view that the young infant does not have an adult's understanding of objects but instead takes them to be sensations, images or pictures. A similar view is put forward in the following competent review of recent research on infancy:

> according to Piaget, the infant is born in a state of extreme solipsism, experiencing the world as though it were merely an extension of himself, a series of images contingent on his own actions and devoid of independent material existence (Bremner 1980, p.41; cf. Bremner 1981).

Objects do exist independently of the infant and the infant does act upon them. Yet the infant is a solipsist since the young infant's understanding of such objects is one in which they are embodied as a series of images.

(B) **Translators**

It is apparent that such commentaries are dependent upon English editions of Piaget's work on infancy. Unfortunately, the general claim made in (1.2) is applicable in this particular case since some of Piaget's claims are in fact mistranslated. Mistranslation occurs with respect to the French term *tableau* which the same translator sometimes gives as 'picture' and sometimes as 'image'. Consider some
examples (in all cases, the emphasis is mine):

the suckling reflex...does not give rise to perception of objects or even of definite sensorial pictures (1953a, p.35);

when the nursling differentiates between the nipple and the rest of the breast...he does not recognise either an object or a sensorial picture (1953a, p.37);

(moving objects are observed by the infant.) These sensorial images have no meaning...Moreover, such images have neither depth nor prominence (1953a, p.64).

In these four examples two English terms are given for the one French term tableau and so the unity of Piaget's argument is lost. Consider again:

the child distinguishes and quickly recognises certain stable groups which we shall call pictures (1954, pp. 4-5);

there exists at the outset no objective identity of the visual image with the auditory image (which can also be a tactile or gustatory picture, etc.) (1954, p.8).

Once again, the same translator makes the same switch by her use of two English terms for the one French term tableau.

It is, then, probable that the commentators referred to in (A) have made use of an inaccurate version of Piaget's work since the translation permits the view to be formed that the young infant experiences a series of images when in perceptual contact with the world.

Other translators offer the English 'tableau' for this same French term (Piaget & Inhelder 1969a, p.14) or 'entities' (Piaget 1978b, p.86) or 'scenes' (Piaget 1978b, p.87). The middle member of this trio is quite misleading but the first, and preferably the last, members can be offered as a reasonable equivalent.

(C) Piaget

Piaget also commits himself to the view identified in (A) and (B) but the defence of this attribution is more complex. Consider, firstly, the claims made by Piaget about the experience of the young infant; secondly, the type of philosophical theories that are assumed
by Piaget in making these claims; and, finally, the objection to
such claims.

In general, Piaget states that perception is the organisation
of the sensory given (Piaget 1971a, p.2). Care is needed in the
formulation of this (general) claim which is sometimes given in
English as the organisation of sense-data (Piaget & Inhelder 1956,
pp.447, 449, 451). If sense-data is used, then the attribution of
phenomenalistic and solipsistic views follows quite naturally, as
perusal of the literature on the philosophy of perception soon shows
(e.g. Ayer 1956; Austin 1962).

If perception is the organisation of the sensory given, Piaget's
claim is that what is given (presented) in perception must be
organised. Such a reference to organisation is a reference to
functioning - see (4.4) - and so Piaget's claim is that because what
is given in perception is organised, perception is structured. Yet
the structures present during infancy are action-schemes - see (4.5)
- and so perception is structured by action-schemes.

So infancy is sensory and motor in character because it requires
the organisation of perception and action. And what differs, during
infancy, is the type of organisation that links these. In particular,
Piaget claims that the infant undergoes a Copernican Revolution (Piaget
& Inhelder 1969a, p.13). Now this claim is an obvious reference to
the Kantian (1933, Bxvii) claim that a Copernican Revolution is required
for philosophers to construe correctly the relation of knowledge and
reality. Thus Piaget's claim can be taken as the claim that such a
revolution must occur for the infant to have knowledge of reality.

The infant's Copernican Revolution can be presented as follows.
At the early stage of infancy, the infant has experiences consisting
in sensory scenes (tableaux sensoriels). A sensory scene is one that
is fixed by the action of the infant. Thus the scene endures for as
long as the action endures. The existence of the scene begins when
the action begins; and ends when it ends. And the scene does not
exist other than when the corresponding action takes place. Thus a sensory scene lacks substantial permanence. As Piaget puts it:

the universe of the young baby is a world without objects, consisting only of shifting and unsubstantial "tableaux" (Piaget & Inhelder 1969a, p.14).

Piaget in fact states this view clearly in the following claim:

recognition is a very early phenomenon, which I have never denied, (but) it does not tell us whether for the baby the hidden object has momentarily ceased to exist, like an image that can disappear and reappear and then be recognised as having been seen already (Piaget 1968c, p.979).

In view of the mistranslation of tableau already noticed, it is not clear whether the use of 'image' in this (English) extract utilises the equivalent French term or not. On the supposition that it does do so, Piaget's view is that an infant who has experience of a sensory scene is one for whom that scene lasts only as long as the action associated with it is performed. A later performance of that action resulting in the later experience of such a scene may be such that the subject recognises it as having been already experienced. A subject's experience and later experience, of an image is cited as a case which is analogous to, but not identical with, this. Further, the infant will merge into one scene objects which can be discriminated from each other. By contrast, the mature infant is one who can discriminate the different properties of distinct objects and can endow those objects with an existence which endures beyond the duration of the actions directed upon them. The Copernican Revolution is the change from the former to the latter type of experience of the world; the mature infant is one who, unlike the early infant, invests objects with properties that are not as such detectable in the actions which the infant performs upon them.

A qualification must now be introduced. For Piaget sometimes states that the young infant recognises neither sensory scenes nor external objects as such.
Neither could there be a question of purely perceptive recognition or recognition of sensory scenes presented by the external world, even though such recognition considerably precedes the elaboration of objects... to the newborn child, on the contrary, there can only exist awareness of attitudes, of emotions or sensations of hunger and of satisfaction (Piaget 1936, p.38/1953a, pp.36-7).

Here, Piaget distinguishes between the recognition of objects and sensory scenes - and denies that the young infant recognises either. Instead, such an infant experiences attitudes and sensations which are, of course, private and internal to the self. Furthermore, Piaget states that the organisation that takes place, in the form of assimilation - see (5.2) - is an organisation of mental events such as these. For example:

in sucking his tongue or his fingers, the child incorporates the new sensations he experiences into those of former sucking (sucking the breast etc.) - therein is assimilation (Piaget 1953a, p.132).

Thus when assimilation takes place, it is the infant's actions which are taken to be coordinated with the infant's sensations and feelings.

In one of his discussions of assimilation, Piaget denies that a young infant's sensory scene is an external object or a (sensory) scene with independent existence or even an image charged with objective meaning. The young infant's initial perception of movement is such that these sensory scenes have no meaning, being coordinated neither with sucking, grasping or anything which could constitute a need for the subject. Moreover, such scenes have neither depth nor prominence...they therefore only constitute spots which appear, move and disappear without solidity or volume. They are, in short, neither objects, independent scenes, nor even images charged with extrinsic meaning....(later) the rough initial assimilation of the object...becomes recognition and organisation of images (Piaget 1936, pp.63-4/1953a, pp.64-5).

Part of this passage was quoted in (B) and it must be emphasised that the standard translation's reliability fails in a second respect since the French terms tableau and image are both given in English as
'image'. The mature infant is one who forms images that are objective and when such images are formed, assimilation takes place between them, i.e. such images, and the infant's actions.

Piaget's position can then be stated as follows. The young infant has experience of sensory scenes, formed by the feelings and sensations which he/she has, and such experience is subjective in a radical sense. At stage III and beyond, the infant has experience of sensory scenes which are formed by images, though such images have objective reference. Finally, the mature infant is one who has experience of objects as such.

Thus the young infant has experience which is subjective in a radical sense. And it is presumably this radical subjectivity of the infant which Piaget refers to when he claims that the young infant is a solipsist and a phenomenalist.

During the earliest stages the child perceives things like a solipsist who is unaware of himself as subject (Piaget 1954, p.352);

the child proceeds from a sort of initial practical solipsism to the construction of a universe which includes himself as an element (Piaget 1954, p.97);

the mind, then, proceeds from pure phenomenalism whose presentations remain half-way between the body and the external environment, to active experimentation which alone penetrates inside things (Piaget 1953a, p.365).

What, then, is a phenomenalist and a solipsist?

To answer this question, a brief philosophical review is needed to show how phenomenalism is a response to the defects latent in some versions of the causal theory of perception. The causal theory of perception, whose sponsors include Descartes (1931b), Locke (1881) and Russell (1948), states that when a subject perceives an object a causal transaction takes place: the physical properties of the object produce changes in the physiological and neural systems of the subject and these changes in turn produce a change in the mental states of that subject by the formation of a sense-datum (or idea,
or sense-impression, or sensation, or whatever). It is claimed that a subject forms sense-data in each and every perceptual transaction and that it is by their means alone that knowledge of the physical world arises. Now there are at least two objections to such a view. Firstly, if a subject forms a sense-datum of a physical object, and never perceives the physical object itself, then the subject can never establish that any sense-datum accurately corresponds to any physical object. Secondly, a subject can never know whether there actually exists an object corresponding to any sequence of sense-data formed. A subject has no access to such objects except by means of sense-data; yet in no case can a subject slip from behind the "veil of perceptions" (Bennett 1971, p.69) to establish whether such objects do in fact exist.

No doubt such a theory is unattractive and phenomenalism is stated as a theory of perception that lacks such objectionable consequences. This theory states that to perceive a physical object is simply to form a sequence of sense-data. Take any claim about a subject's perception of an object and that claim is really a claim about the sense-data. There can be no claim about a physical object other than one about some set of sense-data and so objections about the goodness of fit between sense-data and objects or about the existence of the latter simply cannot arise. Berkeley (1910) is a leading historical sponsor of such a view, whilst Ayer (1946, 1956) is a more recent proponent.

Phenomenalism has the consequence that a percipient is a solipsistic and solitary centre of consciousness. In perception a subject forms sense-data to which that subject alone has access. Thus when a subject perceives an object, or other subject, that (perceiving) subject always forms sense-data of them. Such a subject has perceptual experience which is radically subjective since any claim about a putative external object, or other subject,
is really a claim about the sense-data which occur in the mind of a perceiving subject (phenomenalism) and in consequence such a subject has no way of knowing that other subjects of experience actually do exist (solipsism).

Phenomenalism is stated as a serious philosophical theory, though modern adherents are quick to point out that

a philosopher who thinks that he directly perceives physical objects does not for that reason expect anything different to happen from what is expected by one who believes that he directly perceives sense-data (Ayer 1956, p.85).

A realist and a phenomenalist do not envisage there being an empirical resolution of their differences. Other philosophers (Austin 1962; Anscombe 1965; Mackie 1969) argue that phenomenalism is inherently flawed. The objection to Piaget's account of early infancy can now be stated as a dilemma. The account claims either that the young infant is a phenomenalist and a solipsist or that the young infant is like a phenomenalist and a solipsist. The objection to the former alternative (identity) is that phenomenalism is a philosophical theory in competition with rival theories. But the infant's Copernican Revolution is such that the young infant is taken to be a phenomenalist at the outset and a non-phenomenalist — since a direct realist — at the end of infancy. Yet sponsors of the philosophical theory of phenomenalism would claim that their theory is applicable to any type of experience and they would not restrict its applicability to a specific type of experience, for example, that of the young infant. A young infant-phenomenalist could develop into a mature infant-phenomenalist but could not develop into a mature infant-non-phenomenalist. That is, the account of infancy, construed as the former alternative, precludes the infant's undergoing a Copernican Revolution of the sort described by Piaget. Consider, then, the second alternative (analogy). This alternative states that a young infant has initial experience which is exclusively subjective, consisting in the sensations,
feelings, impressions and images formed by the infant and nothing else. An observer can take such mental states and events to be (in part) the outcome of causes that occur in the physical world. But the infant in question can have no knowledge at all of such a world, since the infant is taken to have solipsistic experience. Further, the infant has no objective experience of objects in the world, since the infant is taken to have phenomenalistic experience. But such an infant could not undergo development to gain objective experience of the world. For all of the young infant's experience is taken to be of this type and so in no case at all does the infant have experience that enables the infant to distinguish between that which occurs subjectively in the infant and that whose occurrence is independent of that infant. To use a Kantian expression, the experience of the young infant would be 'less even than a dream' (Kant 1933, A112), less than a dream since the adult-dreamer does have objective experience as well as subjective (dream) experience. The adult-dreamer is in a position to contrast that which is, and that which is not, internal to that subject. A subject, all of whose experience is solipsistic and phenomenalistic, could never undergo development to gain objective experience of the world.

Thus both alternatives lead to the same conclusion: the Copernican Revolution could not occur and the infant could never gain objective experience. Since both Piaget's theory and common sense require that subjects do gain objective experience, at least by the end of infancy if not before that, the account which precludes such development must be rejected. Thus Piaget's account, which states that the young infant is (or is like) a solipsist and a phenomenalist, must be rejected.

(8.3) The Intentionality of Perception

An attempt is made in the present section to remedy the deficiency in Piaget's account of early infancy that was identified in (8.2).
In essence, the proposal is that just as Piaget's account is erroneous through its reliance on a (too simple philosophical) theory of perception, such a mistake can be eliminated by acceptance of a (more sophisticated philosophical) theory of perception. It is not claimed that the empirical claims made by Piaget in his account of infancy need to be changed, since it is the construal of those claims which is at issue. The present section is intended, then, to be neutral to any empirical claim about infancy, whether made by Piaget or by other psychologists.

At the outset a clear difference must be drawn between the intentionality of action and the Intentionality of perception. The intentionality of action has already been referred to – in (4,3) – and occurs when a subject engages in purposeful behaviour: the agent of an intentional action has a goal or end which his/her action is performed to bring about. The Intentionality of perception – or of other modes of cognition – arises since perception is always a perception of something and an Intentional object is that which the perception is a perception of. Some authors refer to the intensionality of perception (cognition), though Chisholm (1967) and Anscombe (1965) give reasons for rejecting such a mode of characterisation in their discussion of Intentionality (capitalisation is here used in contrast to 'intentionality of action'). Other reviews of this topic are provided by commentators (e.g. Mackie 1975; Gauld & Shotter 1977; Boden 1977, 1979b).

Since the account of Intentionality presented by Anscombe has been influential, her view is given here. Anscombe cites three criteria any one of which is a (logically) sufficient condition of Intentionality: i) Possible Non-Existence

Many actions are such that the actual existence of some object is required for their successful performance. Thus

(1) John killed a cow
refers to an action that requires the existence of a cow for the cow to be killed by John. By contrast,

(2) John believes that unicorns exist
can truly describe John's belief, even though there are not in fact any existent unicorns. Belief-contexts, unlike killing-contexts, are Intentional because the existence of an object is not required for a statement in that context to be true. From the fact that there might not be an actual object corresponding to what John believes, it does not follow that there never is such an object.

ii) Non-Substitutability of Different Descriptions
A true description of an existing object can be exchanged with any other true description, salva veritate: the substitution of their different descriptions does not change truth-value. Thus

(3) Oedipus killed a traveller
(4) The traveller is Oedipus' father
(5) Oedipus killed his father
are all true statements, if (3) and (4) are true. A true identity-statement, (4), allows the (true) statement (3) to be transformed to yield another true statement, (5). Contrast now:

(6) Oedipus believed that he killed a traveller
(7) Oedipus believed that he killed his father.
Suppose (6) is true; it does not follow, on the basis of (4), that (7) is true. Oedipus did not want to kill his father and if he believed that the traveller was his father he would not have killed him. Thus different true descriptions cannot be substituted in cognitive contexts.

iii) Indeterminacy
When Oedipus killed the traveller, there was a specific cause of death occurring at a specific point in space and time. By contrast, even though (6) is true, it does not follow that Oedipus believed that his action which resulted in the death of the traveller had the
specific features which it actually did have. What Oedipus did may be determinate; what he believes he was doing may have been quite vague and indeterminate.

It is apparent that there must always be an object on which an Intentional phenomenon is directed: there must always be an object of belief, expectation, desire and so on. Such an object is not, however, a physical object nor an entity that intervenes between the subject and the world of physical objects since an Intentional object gives the content of that belief, expectation, desire or whatever. A subject must believe something, however vague, and to claim that there must be an Intentional object of belief is to make this claim.

Thus an Intentional object is truly described only by some description that the subject who has the corresponding form of cognition would assent to. It is sometimes claimed that, in consequence, only language-users can have forms of cognition that are Intentional since it is only they who can describe the objects of that cognition. But this is not so. A bird lands on a twig so as to peck at a seed. In fact, the twig is smeared with bird-lime so this particular landing is a landing on a twig smeared with bird-lime. Clearly, that was not intentional, since the bird did not intend to do that; whereas it did intentionally land on that twig. Since there is only one landing in this case, it follows that what the bird did was intentional under one description but not so under the other. But this fact is not altered by the bird's inability to use language. The bird has the aims, beliefs and wants that it has and these may be truly described only under certain descriptions. No doubt the bird cannot state what these descriptions are but that does not exclude its cognition from having the Intentional character that it does have (Anscombe 1979, p.221).
The claim, then, is that cognitive phenomena, including infantile cognition, are Intentional. Any subject is a subject of experience and experience is always an experience of something, namely the Intentional object of experience. It is not claimed that an Intentional object is an entity, such as a sense-datum or even an image, that is an intermediary between a subject and the world. It is claimed that such an object gives the content of experience in the absence of which it would not be experience at all. It follows that a subject's experience must be described through that content, if its specific character is to be described.

The proposal, then, is the claim that Piaget's account of (infant) experience does not exclude its Intentionality. Indeed, Piaget actually states:

all the data I have tried to analyse in terms of sensori-motor schematism and of assimilating schemes have an intentional character (Piaget 1972d, p.131).

In this claim 'intentional' means 'Intentional' and so the proposed interpretation of Piaget's position is implicitly accepted by him.

That an infant's perception is Intentional is attested by the presence of misperception, the opacity of perception and the indeterminacy of perception (criteria i) - iii) respectively). The first criterion is satisfied if the infant misperceives, for example by hallucinating or by experiencing a perceptual illusion, for in that case what the infant perceives does not actually exist. Presumably the infant Laurent (aged 21 days) has perceptual experience of this sort when, half asleep, his mother's breast is placed near to his cheek. Laurent begins to suck but fails to suck the nipple and it is this failure which awakens him. It is reasonable to claim that in the initial sucking Laurent believed that he was in fact sucking the nipple (cf. Piaget 1953a, p.27). The second criterion is satisfied if the infant fails to inter-relate distinct perceptions. For example, although Laurent (aged 2 months) does smile in recognition of his father, he
does not do so on first seeing him in the morning before his father has been groomed (Piaget 1953a, p.72). Thus, even though

(8) Laurent recognises that a person is present

and

(9) The person is his father

are true statements,

(10) Laurent recognises that his father is present

is a false statement. In general, a subject does not always realise that two distinct perceptions can be perceptions of the same phenomenon. Finally, the third criterion is satisfied if the infant does not determine the precise properties of that which is perceived. For example, Laurent sucks his hand (Piaget 1953a, p.25) but presumably fails to realise that he has four fingers and a thumb, even though he does look at his own hand. Thus what Laurent perceives can be vague and indeterminate.

Adult perception is such that it too satisfies the same criteria. Criterion i) is satisfied in cases of hallucination, for example when Macbeth wondered whether he saw a real dagger or had an hallucination of one. Criterion ii) is satisfied when a person fails to realise that one phenomenon can be alternatively described, as when Priestley, in 1775, took a specimen of gas - which in fact was oxygen - to be a specimen of dephlogisticated air (Kuhn 1970, p.54). And criterion iii) is satisfied in cases of indeterminate perception, for example when someone takes there to be 75 birds in a flock which in fact has 57 birds in it.

It was stated in (7.1) that 'object' can be used in different senses and it is important to contrast the second and third senses distinguished there. For example, a subject is presented with a Muller-Lyer figure (physical object) such that the two lines have the same length. In fact, the subject perceives an illusion and so what he/she perceives (Intentional object) is a figure whose lines are of
different lengths. There is an obvious discrepancy between these objects. In cases of veridical perception this discrepancy is eliminated. This is not to say that an Intentional object is a physical object in this latter case; it is to say that the same descriptions are true of each in this case. And this is so whether or not the subject in question is a language-user.

The attractiveness of the claim that perception is Intentional is twofold. Firstly, an explanation is provided of Piaget's mistake in giving the account of infancy described in (8.2). It is a mistake to confuse the physical object which a person perceives with the Intentional object of perception. An infant subject is taken to have no knowledge of the existence of physical objects and so is supposed to have an experience of a non-physical intermediary, namely a sense-datum or sensory scene that is radically subjective. But the internal content of an infant's perception is not a mental intermediary of this sort. Rather, to claim that an infant's perceptual experience is Intentional in character is to claim that a subject can misperceive the world and can perceive it opaquely or indeterminately.

Secondly, the infant's Copernican Revolution can be construed as follows. The young infant has experience consisting in sensory scenes which are such that the infant fails to separate self from the world, or fails to separate external objects from other subjects, or fails to coordinate distinct and discretely perceived features of the world, or merges into one scene that which the infant will later discriminate as individual entities. The mature infant has experience of individual (physical) objects and can respond differentially to them. Such an infant can coordinate different perceptual experiences, can distinguish physical from social features of the world and can have self-knowledge because of the possession of knowledge of that which is other than self.

In short, there is a difference between what appears to be the
case and what actually is the case. There are better ways of drawing this distinction than by appealing to an account which takes infant-development to be a process that proceeds from radical subjectivity to mature objectivity. To invoke the notion of Intentionality is to use an homogenous concept of perception that is rich enough to draw this distinction whilst being compatible with empirically based claims about the facts of development.

(8.4) Observation and the Infant

Piaget wishes to claim:

(A1) an observable is that which a subject believes that he/she can verify;

(A2) an infant subject engages in observation, during stages I and II, in the absence of beliefs.

It is apparent that (A1) and (A2) are formally incompatible and so it is of interest to see where Piaget commits himself to each.

(A1) Piaget states that an observable is to be defined by what a subject believes that he/she can verify and it is not to be defined as that which is verifiable (Piaget 1974a, p.273/*1977c, p.345; Piaget 1975a, p.50/*1978b, pp.43-4). Presumably, the distinction that Piaget draws here is that one and the same objective stimulus can be interpreted, in different ways, so the nature of a subject's observations is dependent upon the interpretation put on that stimulus. Piaget claims that it is by reference to a subject's beliefs that one observable may be differentiated from another, though which beliefs a subject may form will in turn be dependent upon the recording instruments - that is, cognitive structures - that he/she possesses.

Piaget claims that an observable can be false (Piaget 1978b, p.142).

It follows from the definition of knowledge (see section (6.2) above) that false knowledge is impossible: the proposition stating the content of a subject's knowledge must be a true proposition. Thus someone
who claims that he/she knows a proposition that is false is someone who must retract a claim to have knowledge. A subject may, of course, believe what is false. It is no doubt for this reason that Piaget defines an observable by use of the concept of belief, not of knowledge.

(A2) During stage I or II an infant lacks any knowledge at all. For example, an infant who takes a solid object in his/her mouth to suck it

has no knowledge at all of this mouth nor of his head, except through tactile - kinaesthetic or gustative channels (Piaget 1975a, p.88/1978b, p.86).

The reason for this claim is Kantian (see section (2.2) above): a subject may have knowledge of his/her own internal states only if he/she has knowledge of the states of externally existing objects. Thus a subject who has various sensations cannot know that he/she has them except and until he/she has some knowledge of things other than those states. Piaget shows that he is committed to such a claim when he states:

there is nothing in the states of consciousness of a newborn child which could enable him to contrast an external universe with an internal universe (Piaget 1953a, p.36).

In effect, Piaget is denying the view, held by copy theorists, that knowledge can arise merely by a subject's having a sensory experience. Thus the infant subject whose perceptual life consists in the occurrence of subjective, sensory experience is one who cannot draw a distinction between his/her own experience and that which is not part of that experience and in consequence is a subject who cannot be a possessor of knowledge.

But the concepts of knowledge and belief are inter-dependent in that the non-applicability of either precludes the applicability of the other. An infant who sincerely accepts the (true) belief that he/she has two hands or who sincerely accepts the (false) belief that he/she has two mothers is one who has some understanding
of the relevant concepts. Yet knowledge must be a feature of such understanding. Thus a subject who has no knowledge is one who can form no beliefs. The inter-dependence of these concepts is accepted by Hamlyn (1978, p.74 ff.).

Thus a subject who cannot draw the self-world contrast cannot possess knowledge and so cannot have beliefs. In consequence such a subject cannot cognise any observable property since an observable is defined by Piaget through the beliefs available to a subject. The infant at stages I and II is just such a subject.

It follows that Piaget is committed to both (A1) and (A2); which is contradictory. It may be noticed that the contradiction here stated arises because of Piaget's commitment to the Kantian view that a subject who cannot draw the contrast self-world cannot possess knowledge. It follows that the argument presented here is independent of the argument presented in (8.2), though acceptance of that argument reinforces the argument of the present section.

Two ways of evading this conclusion may be noted. Firstly, Piaget could reply that (A2) is incorrect since a subject's practical knowledge of how to do things (savoir faire) is a type of knowledge (connaissance);

\[ \text{action constitutes autonomous knowledge in the sense of practical knowledge whose conceptualisation occurs only by later acts of conscious awareness (Piaget 1974b, pp.231-2/1978a, p.213).} \]

Thus the infant who can engage in action is one who, on this view, is the possessor of practical, but not conceptualised, knowledge. The comment to make about such a reply is that it makes Piaget's case more difficult to sustain. For if the infant's knowledge is exclusively practical, and is never conceptualised or representational in character, the problem arises in another form. How can a subject who has practical knowledge alone also have beliefs? For a belief can be formed only by a subject who can conceptualise or represent
in some way and, *ex hypothesi*, the infant cannot do this at all.

A second reply takes up the claim made by Piaget that the observables for such an infant are global observables...which are not observables relative to objects, since these are not dissociated from the properties that bind them to his own body (a wuckable object, etc.) nor relative to the subject's actions since he does not know them as such (Piaget 1975a, p.88/1978b, p.86).

A global observable is one that occurs in relation to a subject who cannot draw the distinction between the properties of objects and those arising from his actions on them. But if this is so, it is clear that a global observable is not one consisting in some panoramic survey of the immediate environment - which would be objective - but is instead one that is restricted to subjects who cannot draw the distinction between self and world. Hence a global observable is not an objective observable and so its occurrence does not require the possession of objective beliefs about the world. It follows that this reply too is a failure.

(8.5) *Representation and the Infant*

A second contradiction arises when Piaget commits himself to:

(B1) the infant at stages I - V lacks all power of symbolic representation,

(B2) the infant, during those stages, acquires an understanding whose presence requires some power of symbolic representation.

There is a formal contradiction between (B1) and (B2). Where, then, does Piaget commit himself to these claims?

(B1) Piaget (1951, pp.67-8) distinguishes between representation in its broad and narrow senses. In its narrow sense representation occurs when a subject uses symbols when forming mental images. In its broad sense representation occurs when a subject engages in conceptual thought. It is apparent that the narrow sense is included in the broad sense but not conversely, since conceptual thought may occur when a subject uses signs.
But symbols and signs are only two of the three ways of expressing meaning since indices are a third such way.

The "symbol" and the "sign" are signifiers of abstract meaning such that they imply representation... the "index" is a concrete signifier, tied to direct perception and not to representation (Piaget 1936, pp.168-70/1953a, p.191).

Piaget gives examples of the use of indices on the part of the infant subject.

an index is actually undifferentiated from what it signifies; it constitutes an aspect (whiteness of milk), a part (the visible portion of a half visible object) a temporal antecedent (the door's opening for mamma's arrival) a causal result (a stain), etc. (Piaget & Inhelder 1966b, p.42/1969a, pp.52-3).

Since an index is not differentiated from that which it signifies, it cannot occur in the absence of the latter. Quite different are symbols and signs, which may confer meaning in cases where that which is signified is absent. The semiotic function requires a subject's use of symbols or signs and so requires the ability...to represent an object which is absent or an event that is not perceived (Piaget 1970a, p.717).

Representation in the narrow sense is confined to cases where a subject can use symbols since he/she has the capacity to form mental images. That is, the infant invests meaning on the world by the use of indices. The infant can use neither symbols nor signs. Yet it is Piaget's claim that representation requires the use of symbols or signs. Thus the infant lacks the capacity to represent.

(B2) Piaget does not explicitly state (B2) but there is good evidence for claiming that Piaget is committed to (B2). A key stage in infancy is stage III and that stage is marked by the presence of intentional behaviour in the infant:

intelligence precedes language and every act of sensori-motor intelligence presupposes intention (Piaget 1936, p.132/*1953a, p.147).

Thus Lucienne, Jacqueline and Laurent are each credited with the
striking-scheme at this stage whereby the infant

strikes the hanging dolls in order to make them swing (Piaget 1953a, p.167 - my emphasis).

The infant does one thing with the intention to do something else. Now the intentionality of the infant's behaviour is taken by Piaget to be both important in itself and a criterion by whose use the attribution of understanding - of object-permanence, causality, space and time - is possible.

Certain distinctions-drawn in (4.3) - are crucial here. Behaviour may be purposive in two distinct senses. In an objective sense, purposive behaviour occurs when behaviour is directed upon some state that is actually present, as when a missile surmounts obstacles and corrects deviations in its course so as to hit an existing target. Quite distinct is the subjective sense, where behaviour is directed upon a state that may be non-existent, as in the case of a cat that sits outside a hole awaiting a mouse that is not actually in that hole. It is evidently this latter sense that Piaget needs since a subject's behaviour is directed by his/her beliefs, desires and expectations - and these are Intentional in character since a subject may form a belief, desire or expectation of that which does not exist. Moreover, even in a case where a subject acts on true beliefs, the goal-state does not exist at the time when the action is initiated: for if the goal did exist there would be no point in the subject's acting as he/she does.

But if Piaget is using a subjective concept of purpose, he must credit an infant, at stage III, with an ability to represent a goal that is actually non-existent at the time of action and which may be non-realisable if it is a goal that is formed on the basis of false beliefs, desires and expectations. Moreover, the discussion of egocentrism - in (3.5) - showed that a subject may have observable and false beliefs and so Piaget cannot rule out this possibility. Indeed, it is clearly stated by him that a subject acquires observable
information both in cases where he/she attains a goal and in ones where he/she fails to do that (Piaget 1977c, p. 334). Thus an infant who strikes the dolls in order to make them swing is one who must possess the capacity to represent a goal-state that does not at that time exist, which capacity may be used in cases where a goal is not realisable at all.

The conclusion just stated is accepted by others (Boden 1979a, pp. 42-3, 178) but it may be important to dispel an objection, that an infant cannot be credited with the power to form intentions, prior to or concurrent with, intentional action. Certainly, Piaget does claim that it is the presence of an intention that is the mark of intelligent behaviour, for example in the quotation above stating that general claim. Yet he also, and more cautiously, states this claim in terms of intentionality — for example, in the quotation given in (4.3) above (Piaget 1936, p. 112/*1953a, p. 122). For behaviour can be intentional without there being consciously formed intentions directing that behaviour. A car-driver intentionally brakes his car so as to avoid hitting a child who suddenly runs into the road, even though he consciously formed no intention corresponding to what he did. If an action is intentional, there is an intention with which the subject acts and this states the goal that the action is directed upon. That there is a consciously formed intention is not required for that action's being an intentional action (Anscombe 1963, p. 1; cf. Piaget 1953a, p. 213). It is intentionality, not intention, that is the mark of intelligence.

The discussion has concentrated on the case of intention but it is clear that a similar point applies to inter-related phenomena. For example:

the subject expects his gesture to lead to the desired result. But this expectation is merely based on the belief that the object is at his disposal (Piaget 1954, p. 24; my emphasis).
Expectation, desire and belief are each phenomena that are Intentional and so each may have an object that is non-existent. Such phenomena require the attribution to an infant of a capacity to represent that which is not actually existent.

It follows then that Piaget is committed to (B2), as well as to (B1), and so his account is contradictory.

One way in which this conclusion can be resisted consists in the claim that the infant can only engage in anticipations, expectations or whatever that are brief in duration:

acts of sensori-motor intelligence...can themselves only be reduced to a succession of states linked by brief anticipations (Piaget 1950, p.120).

This claim can be allowed and it is important to distinguish between cases where a subject-object circuit (Piaget 1950, p.5) is short and cases where it is long. But the length of a circuit, whilst important in itself, cannot hide the fact that an anticipation, expectation or whatever is distinct from that existing state corresponding to its object. But it is not the length of the circuit which is important here so much as the nature of its components and in this case the components are representational in character.

(8.6) Reflective Abstraction and the Infant

A fourth contradiction arises when Piaget claims:

(C1) reflective abstraction occurs in stages I - V of infancy even though the infant is not conscious of this;

(C2) the infant does not engage in reflective abstraction through an inability to conceptualise.

Again (C1) and (C2) are incompatible. Where, then, does Piaget commit himself to each?

(C1) Since reflective abstraction is defined (see section (5.4) above) so that it may occur even when a subject is not conscious of its occurrence, it is open to Piaget to claim that this process does occur
in infancy. Indeed, Piaget provides a study of this process, stating that

it is essential in a work on reflective abstraction to give an example of this process at the sensori-motor levels (Piaget 1977b, p.289 - my translation).

The study has already been mentioned (see section (4.5) above) and concerns an infant's ability to inter-relate the movement of a pivotted bar towards himself with that away from himself. Reflective abstraction is taken to have occurred when an infant understands that one of these movements is the inverse of the other (Piaget 1977b, p.295).

(C2) Reflective abstraction requires the presence of reflection in both its mental and physical senses. Thus only a subject who can engage in acts of conscious awareness can engage in this type of abstraction. And Piaget denies that the infant is capable of engaging in such acts:

the schemes of sensori-motor intelligence are not yet concepts, since they cannot be handled in thought... since he lacks the semiotic apparatus for designating them in consciousness (Piaget 1972c, p.25).

Thus it is only the subject who can engage in representation - that is, who can use symbols or signs - who can form concepts, and so can become aware of that which was previously handled in behaviour alone.

The reconstructive, or integrative or reorganisational, element in reflective abstraction is the prerogative of the infant who has the representative power to signify what is absent. Thus Piaget states:

the unconscious comprises everything which cannot be made explicit, through lack of reflective abstraction, of conceptualisation (Piaget 1977f, p.58 - my translation).

This claim is clear support for (C2).

The claim is supported by other passages. Piaget states that reflective abstraction consists in the first place in becoming aware of...actions or operations (Piaget 1967a, p.442/*1971a, p.320).
But the acquisition of consciousness is primarily taken by Piaget - see (3.6) - to occur at stages beyond infancy when representation can occur. It follows that no infant can engage in reflective abstraction.

It is noteworthy that many commentators would defend (C2) rather than (C1) for they take Piaget's account of reflective abstraction to be concerned with the development of mathematical thinking (e.g. Rotman 1977, p.78; Brainerd 1978a, p.213; Boden 1979a, p.90).

It might be claimed that it is only in quite recent research (Piaget 1977b, 1978e) that Piaget states that reflective abstraction can occur at a pre-representational level and so that (C2) is a position that he has now abandoned. The reply to make here is that such a change is possible only if further changes are made as well, for example that the infant can engage in representation, which even in recent accounts Piaget (1980a, p.165) is disinclined to do.

Piaget is therefore committed to both (C1) and (C2), when these are contradictories.

(8.7) Action-Coordination in Infancy

The three contradictions in Piaget's account of infancy can now be related to a test-case, that of an infant subject who lacks the capacity to engage in actions that are motor and manipulative in character. How does a limbless infant or an infant who has no ability to use his/her limbs form knowledge of the world? Lacking an ability to perform physical actions such an infant would apparently be unable to gain practical knowledge corresponding to them and so would in turn be unable to gain conceptualised knowledge at stages beyond infancy. How can Piaget's account accommodate such a case?

At the outset, it must be stated that some care is necessary properly to state this objection, because of Piaget's liberal conception of action - noted in (4.3). It is no valid objection to claim that a subject can display an operational structure in the absence of
manipulative actions upon objects, contrary to Piaget's theory (Anthony 1977, p.23). Anthony bases his case upon an explicit claim made by Piaget in the Foreword of a book on education, though it is apparent that Piaget (1977d, p.28) makes this claim on other occasions as well. Doubts arise about the strength of this case. Firstly, Piaget's use of 'manipulation' sometimes covers cases of mental manipulation (Piaget et al. 1977, p.82). Secondly, Piaget (1970c, p.68; 1976b, p.34) denies that activity is equivalent to physical activity, a claim that is endorsed by Inhelder et al. (1974, p.25). Thirdly, an interiorised action is an action and embodies some conceptualisation of the world. It can in consequence scarcely be denied that an action qua conceptualisation is necessary for a display of an operational structure. The need for caution in interpreting Piaget's use of the concept of action is, rightly, urged by Rotman (1977, p.114).

The difficult case is that of the infant, whose actions are to be taken as exclusively motor and manipulative. Yet cases arise where a subject is unable to perform such actions. Thus Jordan (1972, pp.379-80) cites the case of an adult woman who has the head of an adult but whose body is that of an infant and such that her limbs are functionally useless. Such a subject could not engage in limb-movement as an infant and as an adult. Yet her intelligence and personality are rated as being normal. Further Kopp & Shaperman (1973, p.430) cite the case of subjects who have no limbs at all and yet whose cognitive development proceeds in an apparently normal manner. It follows that subjects do not have to engage in physical activity for cognitive development to take place.

Piaget replied to the challenge issued to him by Jordan; or, rather, invited Sinclair to reply on his behalf. Such an invitation is strange given the importance of the case and the fact that Sinclair apparently interprets Piaget's view of language in a manner which runs counter to what Piaget actually says (see (6.2) above). The reply is the claim that sensori-motor activity is not to be taken too
literally since

it implies that activities are assimilated and
accommodated, and this is the case with any child
that lives, since it has to eat and drink (involving
assimilation and accommodation - one drinks and eats
very different things, and adjusts the movements
according to the substance), and since it has perceptual
activity (movement of the eyes and the head, if that
is possible, also follow the same pattern of

Such a reply seems to invite the death of a theory by a thousand
qualifications and it is certainly regarded with suspicion by others
(Boden 1979a, p.47; Segalowitz 1980, p.139). And that suspicion
is surely well-founded. Consider why this is so.

One way in which the reply works is to rely on the account of
functioning - see (4.4) and (5.2) - which is taken by Piaget to be
an invariant feature of life. Any living organism is taken by Piaget
to be an open system in exchange with its environment. It follows
that a subject must have some action-repertoire, however minimal, and
it is evident that in the cases cited this requirement is satisfied.
But this reply is inadequate, for functioning is an invariant feature
of life and Piaget expressly contrasts functioning with structure,
which is a variable feature of life. Thus whilst the cases cited do,
and indeed must, manifest functioning, no specific structure can be
similarly attributed. It is, therefore, question-begging to claim
that some specific structure can be attributed in these cases simply
because functioning takes place. It follows that a functioning organism
is not thereby one that constructs the structures appropriate to
infancy.

A second feature of the reply is an appeal to the observational
capacities of such an infant. The infant has perceptual abilities and
so can observe what is happening in the world even if that infant is
not itself an agent in the world in the normal sense (initiator of
physical actions) upon the objects in that world). But this reply
too is a failure. One reason is because, as Ginsburg & Upper (1979, p.224) point out, passive observation, unlike active manipulation, does not promote development. A second reason is because actions of this sort have an Intentional component. But the presence of Intentionality is a reason for claiming that a subject performs an interiorised action, which Piaget never admits, even in recent statements of position (1977d, p.40; 1978b, p.84), an infant can perform.

In short, if Piaget's reply to the challenge issued by Jordan is accepted, then the infant subject can gain knowledge even if that subject performs a minimal range of actions. In consequence, the conception of action on which such a reply rests requires a change in Piaget's account of infancy.

(8.8) Conclusion

The main conclusion to be drawn from the discussion in the present chapter can now be stated. Piaget's account of infancy, as presented in the chapters above, is defective since inconsistent and its inadequacy may be primarily attributed to Piaget's denial that an infant subject can engage in representation.

To see this, consider how the contradictions discussed above can be eliminated if it is granted that the infant subject can engage in representation:

i) Piaget's account of the infant's Copernican Revolution, as presented in (8.3), can be accepted if it is agreed that Piaget can use the notion of Intentionality. But the notion of Intentionality can be used only if the phenomena in which it is instantiated are taken to be representational in character (Searle 1979). That is, the infant subject is one who has experience of sensory scenes at the outset of development and experience of physical objects at the end of infancy. But in both cases the infant's experience is Intentional and so representational.
ii) In (8.4) it is clear that (A1), not (A2), should be retained since the infant subject is one who gains observable knowledge. But if the infant does gain such knowledge, the infant must also conceptualise it. Indeed, the equilibratory model specified by Piaget as being applicable to early infancy is one that clearly attributes conceptualisation to the infant (Piaget 1978b, p.93). But if conceptualisation occurs, so too does representation (Piaget 1977c, p.332).

iii) In (8.5) it is clear that (B2), not (B1), should be retained on the supposition currently under consideration since (B2) explicitly attributes representational powers to the infant. Acceptance of (B2) does, however, run counter to Piaget's own claim — noted in (4.3) — that the attribution of intentionality to an infant does not require the co-attribution of a representational capacity. In consequence, the latter claim must be rejected.

iv) In (8.6) it is clear that (C1), not (C2), should be retained since Piaget's constructivist account requires that processes which occur at later points in development should have simple precursors at earlier points. Thus the infant can engage in reflective abstraction which in turn requires some ability to conceptualise and so represent.

v) Finally, in (8.7) it is clear that subjects do undergo development even in the absence of physical activity of a robust sort. In consequence that development must take place by the occurrence of actions which are interiorised and so Intentional. But if this is so, representational powers must be attributed to the infant.

There are two (weak) signs that the conclusion that an infant can engage in representation is not totally unwelcome to Piaget. He admits himself (Piaget 1976c, p.224) that his account of early infancy is inadequate, though he states that this is because of the research of Bower and Mounoud (and not for any of the reasons here discussed). The second sign follows on from this for Piaget (1980c, p.4) is willing to admit that animals and infants have (rudimentary) forms
of consciousness and that he is always ready to study consciousness — when it can be studied. But the study of consciousness requires the mediation of language, claims Piaget (1971a, p.47), and so such a study is apparently excluded. It is for this reason that Piaget (1953a, p. 37) states that he will not specify the states of consciousness that accompany assimilation in the infant. Now this is a defensible (methodological) position to adopt. It does not, however, licence the (epistemological) position that consciousness — and so, for Piaget, conceptualisation and representation — do not occur in infancy. Simply put, Piaget should adopt a position analogous to that taken by many cognitive psychologists (cf. Boden 1979a, p.42) whereby representations and their concomitant transformation are invariable features of knowledge-acquisition.

A rationale for Piaget's adoption of this modified position can now be given. Piaget does not deny that the infant subject is the possessor of a conscious mind and in this respect he accepts a conception of the individual subject which is in conformity with traditional theories about the relation of body and mind (see (3.2) above). But in that case a parallel can be drawn between Piaget's position with respect to consciousness and that with respect to language, for language is present at the outset of the concrete operational period — see Table 4.1 — even though Piaget denies that a subject can think in terms of verbally formulated propositions until the formal operational stage. Just as consciousness is present at a stage prior to its full and effective use, so too language is present at a stage prior to its full and effective use. The infant possesses consciousness but cannot think representationally until the subsequent stage of development; the child possesses language but cannot think in terms of linguistic statements until the subsequent stage of development. Piaget's stage-criteria — stated in (4.2) — can be invoked at this point since early stages prepare for later ones and it is just such a feature which
is instantiated in this modified statement of position.

It is not claimed that Piaget would accept this modified position. It is not claimed that the position so described is a correct one and no empirical evidence is offered in its support. It is clear, however, that acceptance of this modified position requires the re-consideration of Piaget's account of infancy and, in particular, his general claim that the infant subject is unable - through lack of a representational capacity - to contrast completely subject and object. Moreover, Piaget's claim - stated in (4.5) - that an infant uses action-schemes such that no distinction is drawn between intension and extension is also vulnerable to the general claim stated here. It is, however, beyond the scope of the present study systematically to revise and rectify Piaget's account of infancy in the light of the proposals here discussed.
EQUILIBRATION AND NECESSITY

(9.1) Introduction

The present chapter attempts critically to review the three ways in which Piaget's theory employs the notion of necessity (see (2.5) above). Piaget's theory states empirically necessary conditions of a subject's knowledge of necessity which is brought about by a necessary process of equilibration. The central question for discussion in the present section concerns the adequacy of Piaget's theory when it is construed in this way.

In section (9.2) it is argued that Piaget's theory is marked by its failure to specify under just which conditions development takes place since his theory does state empirically necessary conditions alone. It is not claimed that Piaget's work is trivialised by this contention. It is claimed that his theory is, in principle, incomplete since its exclusive reference to the central process of equilibration cannot explain when knowledge is, and when it is not, formed.

In section (9.3) it is argued that there is lack of concordance between Piaget's structuralist and his constructivist interests since the former incline him to the view that operational necessity occurs at the end of development whereas the latter incline him to the view that non-operational forms of necessity occur throughout development. Piaget's recent but discrepant statement of position is noticed and tentative conclusions are drawn from it.

In section (9.4) it is argued that the sense in which equilibration is necessarily constructive is distinct from the previous two senses. It is argued that the transcendence of a contradiction-mistake is possible only by a subject's construction and use of a new structure but that constructive necessity, so understood, requires Piaget's account to be essentially dependent upon accounts which discuss non-equilibratory factors in knowing.
(9.2) **Necessary Conditions of Knowledge-Acquisition**

It was claimed in (2.5) and (5.2) that the equilibratory factors described by Piaget's theory are necessary and not sufficient conditions for the formation of knowledge. Further, such conditions are empirical and not logical conditions. The claim is that as a matter of fact, and not of logic, knowledge does not arise in the absence of such factors. It follows that even if the conditions described by Piaget are present, it is always possible for knowledge not to be formed unless Piaget also claims that the conditions discussed by him represent all of the necessary conditions or represent necessary and sufficient conditions.

Unfortunately, there is no clear statement on Piaget's part by reference to which this matter can be tested. There is, however, indirect evidence which can be cited in defence of the initial claim made, so consider some examples that arise from Piaget's (1978b) mature statement of position:

1. in acting upon objects a subject applies a scheme with the result that, sooner or later, the scheme meets a disturbance (p.82);
2. a subject who faces a disturbance will, sooner or later, make contradiction-mistakes (p.26);
3. sooner or later, reciprocal assimilation between distinct schemes will occur (p.9);
4. a subject's action-coordination will, sooner or later, give rise to reflective abstraction (p.193);
5. this in its turn will, sooner or later, make possible the verification of new observables (p.56);
6. reequilibration will produce upper bound equilibration, sooner or later (p.40).

It is clear that these examples are taken from Piaget's account of equilibration - see (5.2) - which he takes to be the central process in the development of knowledge and the examples cover most of
the main features of that account. Moreover, the examples are related so that the occurrence of earlier members of the series is required for that of later members. The examples, then, are central to Piaget's account of equilibration and so the growth of knowledge.

It can be assumed that the expression 'sooner or later' (tôt ou tard) is a useful expression to use in the statement of a theory which incompletely identifies the conditions for the occurrence of a phenomenon. Presumably, the expression could be used in one of two ways. Either the theory is comprehensive enough to identify all of the conditions which are necessary and sufficient. Or the theory is incomplete and the expression is used to hide deficiency. It is reasonable to claim that it is this latter usage which occurs here. Piaget's account of equilibration has been admitted to be obscure, in its initial formulation, by Piaget himself (Furth 1981, p.219) and his mature account can be taken to represent as clear and as complete account as Piaget is able to provide. Thus the use of the expression 'sooner or later' is an indirect sign that Piaget's account is one that offers (at most) conditions for the formation of knowledge which are (empirically) necessary conditions alone.

The objection to Piaget's theory can be stated as follows. The theory attempts to state conditions under which knowledge arises and to identify processes which are taken to be relevant for the formation of knowledge. But the conditions stated are incomplete: is it the case that the formation of knowledge occurs sooner - and if so, under what conditions - or occurs later - and if so, under what conditions? In consequence, a theory which cannot determine whether knowledge is formed sooner or later is one which cannot determine whether knowledge will be formed at all.

There are a variety of symptoms of this lack of specificity in Piaget's account. One appears in the claim that there cannot be a complete account of why cognitive lags (décalages) occur (Piaget 1969d,
Another appears in the claim that there is an optimal length of time for the construction of a new structure (Piaget 1971a, p.20). Another appears in the claim that there are norms of accommodation of structure such that the structure will break down if the norm is exceeded (Piaget 1975a, p.39/1978b, p.33). Another appears in the claim that language-acquisition sooner or later leads to an understanding of class-relationships (Inhelder & Piaget 1964, p.3). Another appears in the admission that not all subjects reach the stage of formal operations (Piaget 1972e, p.161). Finally, when invited by Alina Szeminska to explain why one scheme, rather than another, is activated on a given occasion, Piaget replied by stating that sooner or later the observables associated with a neglected scheme infiltrate a subject's consciousness (Inhelder et al. 1977, p.120/Smith 1981e, p.282).

A possible objection to the argument here presented arises from Vuyk (1981a, p.149) who implicitly accepts that development might not always take place sooner or later but who nonetheless states that genetic epistemology offers both necessary and sufficient conditions (Vuyk 1981a, p.36). This claim is baldly stated by her, however, and is supported neither textually nor with its rationale. Moreover, even if this claim is accepted, it would represent an ideal which Piaget's account of equilibration aspires to rather than one which it actually attains.

Two final comments can now be made about the central objection stated in this section. Firstly, it is not argued that a theory which provides necessary conditions alone is one which is trivial or uninteresting. The objection seeks to show that Piaget has stated a theory which is limited in the respects specified and it follows from this neither that his theory should be abandoned nor that the conditions stated by the theory are of no value. The point of the objection is that it signals the need for the supplementation of Piaget's theory rather than its replacement. To see why this is so,
it is important to recall the asymmetry which holds with respect to accounts which offer necessary and those which offer sufficient conditions of the formation of knowledge. It was noticed — in (5.5) — that any account that states necessary conditions of the formation of knowledge is presupposed by any account that offers sufficient conditions but not conversely.

Secondly, it is not claimed that the conditions stated by Piaget actually are necessary conditions of the formation of knowledge. To settle that question would require consideration of a host of issues which are beyond the scope of the present discussion.

(9.3) Understanding Necessity

There is an ineradicable tension which is present in Piaget's theory since his structuralist and constructivist interests do not always coincide. This lack of concordance is apparent in Piaget's account of a subject's understanding of necessity and the purpose of this section is to identify some of the ways in which this tension surfaces. Since the understanding of necessity is a central feature of Piaget's theory, acceptance of the present claim constitutes a serious defect in the theory.

Piaget's structuralist interests led him to reject the Kantian view that a subject has a priori knowledge, that is, knowledge which is necessarily and universally valid, at the outset of development. As a matter of fact, such knowledge arises (if at all) only at the terminal stages of development and it is for this reason that Piaget distinguishes between operational and non-operational knowledge — see (4.6). In the chapters above, Piaget's theory has been presented as one which states that deductive knowledge is gained only at operational stages of development.

Piaget's constructivist interests require him to trace the developmental antecedents of mature forms of knowledge. Since Piaget
does accept that deductive knowledge is an important form of knowledge and since any such knowledge will have simple precursors, Piaget claims that even at the outset of development a subject has to use, if knowledge is to arise, a framework that is logico-mathematical in character. Such a claim is explicit in the discussion in (3.4) and (3.5) above.

The question that arises is how a subject can have coordinatory knowledge which is logical in character throughout development and yet have deductive knowledge which has the character of being necessary only at operational levels? How can a subject have logico-mathematical knowledge which is not deductively necessary?

The answer to this question, suggested by the discussion of conceptualised knowledge in (6.2), is that deductive knowledge requires a subject's conceptualised understanding of the reasons for the judgements made by that subject. Further, deductive knowledge arises only in cases where a subject understands the necessity of the relation holding between a judgement and the data upon which it is based. Finally, the discussion attempted to show that the philosophical arguments in question were not incompatible with claims endorsed by Piaget himself.

In his recent work Piaget has this to say about a subject's understanding of necessity.

At the pre-operational level, when possibilities are contiguously generated, little islands of necessity are already constituted. But these are local and are not bound into stable systems...At the heart of such systems, well before operational structures, the first forms of necessity are constituted which we will name "signifying implications"...operational necessities of a higher rank have been drawn, in successive steps, from the "form" characteristic of signifying implications in the initial stages. Their character, which is already necessary, constituted the prior condition of later compositions (Piaget 1977g, pp. 236...240...241 - my translation and emphasis).

Piaget here states that a subject can understand non-operational forms of necessity at stages prior to operational levels; that the necessity in question is localised and not systematic and that such necessity
arises because of the implicatory relation linking the meanings used by a subject. Further, Piaget gives an example to clarify such claims. An infant, aged 10-12 months, may "understand" the meaning of the relation "placed upon" since the infant understands the reason why one meaning (that giving spatial position) implies another meaning (that of its utilisation). Piaget is here committed to the view that an infant can understand "reasons" by virtue of his/her material actions directed upon the world; that signifying implication - see (5.3) - is a relation which holds between perceptual indices and is not confined to cases where a subject consciously forms images or uses verbal signs; and so that even the infant can understand necessity.

It is clear that the position here taken by Piaget is at variance with the position attributed to him in the chapters above and so is at variance with the interpretation of his theory as stated by him prior to his recent research on modal concepts. Since this research is still in progress and since the present account is based on two short reviews written by Piaget (1976d, 1977g), no firm conclusions can be drawn. In particular, publication of Piaget's empirical studies on a subject's understanding of modal concepts is awaited.

The following (tentative) claims can, however, be stated. Firstly, Piaget does distinguish between pseudo-necessity and necessity, between cases where a subject takes something to be necessary (when it is not) and cases where a subject takes something to be necessary (when it is). Pseudo-necessity is illustrated in cases where a subject observes that so-and-so is the case and concludes that so-and-so must be the case or in cases where a subject extends his/her inductive knowledge but does so without transcending the sphere of observable knowledge (Piaget 1977g, p.243). The necessity referred to in the quotation above is necessity, not pseudo-necessity.

Secondly, Piaget (1976d, p.294/Piaget & Voyat 1979, p.80) states that there are varying types of necessity and so denies that there
is a simple dichotomy of necessary and non-necessary. In this respect his position is analogous to the position taken by him in earlier research (Apostel et al., 1972) where he rejects the antithesis of analytic and synthetic truths and defends instead – on theoretical and empirical grounds – a view which is compatible with Quine's (1961) thesis that there is a continuum between the analytic and synthetic. Piaget, in the papers cited, prefers to illustrate his contention rather than to specify the (dis)similarities holding between such senses of necessity.

Thirdly, Piaget's recent claims about necessity are compatible with his (recent) research on abstraction and generalisation. In particular, it was stated in (8.8) that reflective abstraction does occur in infancy and Piaget (1978e, p.222) states that the account of signifying implication enables him to identify precocious manifestations of necessity. Thus the recent position adopted by Piaget is one which is not incompatible with the main conclusion stated in the previous chapter, namely that the infant can engage in representation. By contrast, even in one recent (brief) discussion of signifying implication Piaget (1974b/1978a) does not state that necessity can be comprehended at early levels of development.

Fourthly, it is clear that the discussion of operational knowledge in chapter 6 should be reviewed in the light of Piaget's new position. In that chapter, the interpretation of Piaget's account set forth in (4.6) was assumed and Piaget's critics appeared to accept that account or some analogue of it. In particular, such critics were concerned to investigate deductive forms of knowledge. It is possible, however, that the critics may now be taken by Piaget to have been investigating some non-operational form of necessity. Piaget's failure to specify accurately the nature of such forms precludes adequate consideration of this matter.

In short, it is clear that Piaget's recent statement of position
is distinct from the position attributed to him on the basis of his earlier work. Specifically, his recent position is one that inclines him towards a constructivist thesis whereby operational necessity is the mature outcome of primitive forms of necessity. By contrast, Piaget's earlier writings inclined him towards a structuralist thesis whereby operational necessity occurs only at the end of development. There is, therefore, an in-built source of tension in Piaget's theory in its account of this central feature. Moreover, it is clear that the early account suffers the defect that a subject's coordinatory knowledge is not fully used by him/her, if that subject uses a logico-mathematical framework but does not consciously understand logical necessity. By contrast, the recent account suffers the defect that the distinct types of necessity are incompletely specified by Piaget.

It is assumed that the study of deductive necessity, whether by Piaget or his critics, is distinct from the study of non-deductive necessity. Thus the proposal made in (6.5) that children might have an incipient knowledge of deductive necessity in the experimental studies cited in (6.3) and (6.4) is distinct from Piaget's recent claims that there can be non-deductive necessity which is comprehended even by infants.

(9.4) Necessity for Equilibration

It was claimed in (2.5) that constructive necessity is distinct from the necessity discussed in both of the previous sections. Piaget tries to state (empirically) necessary conditions of a subject's understanding of deductive necessity which occurs on the basis of a necessary process of equilibration. The question posed in the present section concerns the sense in which there can be such a process. Three construals are proposed and rejected and a final construal is delineated and accepted, namely that transgression of a contradiction-mistake necessarily requires the construction of a new structure.
1) Piaget could argue as follows. It follows from the claim made in (3.5) that a subject uses a logico-mathematical framework throughout experience, that is, an interpretative framework which is indeed logical in character. Further, it is admitted, in (9.3), that there are a variety of types of necessity which arise even during infancy and such types are distinct from deductive necessity, discussed in (4.6). It can then be maintained that equilibration is necessarily constructive since a subject who gains knowledge always uses a logical - and so (in some sense) necessary - framework throughout experience. To make the same point: if any structure can be given a formal (logical) characterisation, as was admitted in (4.4), then the construction of a new structure must occur by a process which is itself necessary.

The objection to such a proposal is that it is incompatible with the central claim made in (4.2), namely that two stages (structures) may be logically described and yet their serial ordering may in fact be contingent. In general, it is a mistake to argue that because a stage (structure) may be given a logical characterisation, any realisation of that stage (structure) is similarly logical in character. Consider an analogous case. A gene is a bearer of an hereditary trait and such a characterisation embodies a logical connection. Nonetheless, it is an empirical matter that a gene is a DNA molecule (rather than a molecule of water, say).

2) A second way in which Piaget could defend his claim that equilibration is necessarily constructive is by reference to the account of signifying implication - see (5.3). For Piaget claims that there is a relation of (signifying) implication linking the meanings that a subject confers upon the objects in the world and he denies that such meanings are causally or correlationally linked. Thus Piaget's (1953a, p.405; 1971a, p.5; 1972b, pp.147-8) objection to empiricist theories of learning arises from his commitment to there being a non-contingent -
and so necessary — relation holding between the signifying components of a cognitive system.

The objection to this proposal is that it is opaque since the relation of signifying implication is, apparently, sui generis. To see why this is so, recall that Piaget denies not only that such a relation is not (contingent) causality but also that it is the relation of entailment or any of its logical analogues. This denial is explicit on Piaget's (1966, p.155; 1971a, p.49) part and there is good reason why he states such a denial. Logicians distinguish two types of implication, namely strict and material implication. But it is well-known (von Wright 1957; Hughes & Cresswell 1972) that implication of both types leads to paradoxes. For example, a contradictory proposition strictly implies any other proposition, whilst a false proposition materially implies any other proposition. Logicians accept with equanimity the presence of such inferences, since these types of implication are so defined that they lead to the "paradoxes" stated. Thus if signifying implication is taken to be implication in these senses, then the presence of a contradiction would strictly imply any other proposition whilst that of a false proposition would materially imply any other proposition. Since Piaget claims that self-contradiction — see (5.2) — is an invariable feature of equilibration and since he also claims that egocentric — see (3,5) — and incorrect judgements are a pervasive feature of development, new knowledge could trivially arise in any developing subject. Thus signifying implication cannot be so construed. Indeed, Piaget (1967b, p.273; 1977g, p.242; 1978e, p. 222) stresses that signifying implication does not lead to paradoxical forms of inference. But in that case, the nature of signifying implication is left unexplained since Piaget — as was noted in (5.3) — does not elaborate the specific features that he takes such a form of implication to have.
Piaget could argue that just as entailment is a relation that links propositions, so signifying implication is a relation that links the judgements expressible by a subject. That is, Piaget can deny that these relations are the same and yet maintain that the latter is modelled upon the former. For example, consider:

(1) Things equal to the same are equal to each other
(2) The two sides of this Triangle are equal to the same
(3) The two sides of this Triangle are equal to each other

It is evident that (1) and (2) entail (3). Thus a subject who forms a judgement corresponding to (1) and forms a judgement corresponding to (2) may then form a judgement corresponding to (3). Further, the judgements corresponding to (1) and (2) would signifyingly imply the judgement corresponding to (3).

There is an objection to this proposal, which is clearly stated by Barry Stroud (1979) in a discussion of Lewis Carroll's "Achilles and the Tortoise". Stroud points out that a subject might accept the propositions (1) and (2) and yet might not accept (3) through a failure to realise that (3) is entailed by (1) and (2). Further, even if a subject accepts the propositions (1), (2) and (3), it still does not follow that that subject realises, believes or understands that (3) is entailed by (1) and (2), for that subject might accept such propositions quite independently. Thus even though certain logical relations link various propositions, it does not follow that a subject who forms judgements corresponding to them actually does see what those logical relations are. In general, claims Stroud (1979, p.194), a psychological account must be provided of the process of knowing and that account cannot rely simply upon the logical features of the content of knowledge. In fact, Stroud maintains that cognitive psychologists - he refers to Noam Chomsky - do not meet this requirement. Further, Piaget can correctly claim that his theory does accept, and does attempt to meet, this very requirement. Indeed, this is so; but an attempt is an attempt and in the case under discussion it is claimed
that Piaget's attempt does not succeed since the present proposal is
one that rests upon features of the content of knowledge and not
upon features of the process of knowing as such.

4) Finally, Piaget could argue as follows. It is never the case
that equilibration has to occur: it is not a process which is
necessary in the sense that its absence is impossible. Quite obviously,
there are countless cases where development does not occur, for
example in cases where there are constraints on development which arise
from the non-satisfaction of the other conditions which are necessary
for development's taking place. Suppose, however, a subject does make
a contradiction-mistake. It follows from the admission in question
that a subject might never overcome that mistake since there is never
a necessity for that to occur. Suppose, now, that such a subject
does try to overcome a contradiction-mistake. Since the mistake is
a contradiction-mistake, the only way in which that contradiction-mistake
can be rectified is by the subject's placing his/her knowledge in
an alternative framework. There is never any guarantee - it is never
necessary-that any such attempt will succeed. But if such an attempt
does succeed, its success will have taken place by a subject's use
of a cognitive system that is distinct from the one, used by that
subject, which led to the making of that mistake in the first place. It
is never the case that equilibration has to occur. But given that
it does occur, its occurrence is necessary simply because the only way
to rectify a contradiction-mistake - one that arises by a subject's
use of a structure which is too weak to allow him/her to deal
satisfactorily with some phenomenon - is by the construction and use
of a new structure. Consider, for example, the subjects - referred
to in (2,3) - who deny that a half-full glass is one that is half-empty
(they take to be incompatible that which is compatible); or the subjects
who assert that all the discs can be equal in size to each other but
deny that two of these discs are of the same size (take to be compatible
that which is incompatible). Piaget's claim is not that such subjects
must construct new structures, given that they make such mistakes. Rather, his claim is that the only way in which such mistakes can be rectified is by a subject's construction and use of a new structure.

If constructive necessity is so construed, Piaget's account is one that depends upon accounts of how non-equilibratory factors impinge upon development. For the present construal is one that underscores the main conclusion noted in (9.2), that Piaget's account is one that states (empirically) necessary conditions of development. That is, Piaget is entitled to claim that equilibratory factors are at work in the formation of knowledge and such factors represent conditions in whose absence knowledge does not develop. And Piaget is also entitled to claim that equilibration, when it does take place, is necessarily constructive. But it is clear that the conjunction of these claims leaves open the question of when equilibration does take place and so the conjunction of these claims makes an implicit appeal to an account which offers all of the necessary conditions for the formation of knowledge or an account which offers sufficient conditions. And it is denied that Piaget has stated such an account.

In sum, if equilibration is the central factor in development and if it is construed as a process which is necessarily constructive, it does not follow that a theory which concentrates on the formation of knowledge in the epistemic subject is one that can be independent of accounts of the role played by the individual subject in the formation of knowledge. Piaget's theory is complementary to accounts that show the place of non-equilibratory factors in the development of knowledge. If development occurs, then it is open to Piaget to claim that the necessary process of equilibration occurs. It follows from the conclusion of (9.2), however, that Piaget's equilibratory theory does not state the conditions under which development does occur - but only conditions when it does not - and so his theory is, in principle, dependent upon non-equilibratory theories of knowledge.
Since it was argued, in (5.5), that non-equilibratory theories which state sufficient conditions alone are dependent upon theories that state necessary conditions, then the general conclusion to be stated is that theories of both types are inter-dependent and complementary.
CONCLUSION

(10.1) Main Conclusions

The question of the relation of logic and knowledge, construed as a psychological question about the process of knowing rather than a philosophical question about the nature of knowledge, is obviously important and yet bafflingly complex. The central assumption of the present study has been that Jean Piaget's contribution to the answer to this question is, at least, a major contribution. The present study has concentrated upon the detailed statement of Piaget's theory, as well as upon objections that beset it. It will, therefore, be instructive to review the main conclusions that should be drawn from the discussion in the chapters above and these may be grouped under four headings, corresponding to the objectives and methodology outlined in (1.1) and (1.2), namely (A) Philosophy and Epistemology, (B) Piaget's Theoretical Constructs, (C) Objections to Piaget's Theory and (D) Piaget's Theory.

(A) Philosophy and Epistemology

Philosophical theories have been identified and integrated into the discussion in fulfilment of the first objective, that of showing how Piaget's theory has its basis in philosophy. Two central features of the discussion stand out, namely that 1) not all epistemological problems are philosophical and that 2) not all epistemological problems can be solved by recourse to genetic epistemology. In brief, the discussion indicates that there are two distinct, coherent and interdependent forms of epistemology.

1) That genetic epistemology is a distinct form of epistemology emerges from the discussion, in (2.2), of the classical problem of knowledge. It is Piaget's contention that philosophers have ignored a crucial empirical component of their problems so that even if a given philosophical view, for example that of Immanuel Kant, offers a possible theory about how knowledge arises, such a theory is only a
possible theory and so the actual nature of development can be correctly accounted for only by the use of a theory that has sound empirical backing. Piaget does not deny that logical norms regulate the content of knowledge and these remain unchanged whatever the nature of development. But he does assert that the individual's use of such norms is different in infancy, childhood and adolescence. The examples, which are only examples, referred to in (2.3), (6.3) and (6.4) provide confirming evidence in support of Piaget's position.

Genetic epistemology is, then, taken by Piaget to be a distinct type of epistemology, one which takes seriously the problems posed by philosophers but one which employs scientific methods in an attempt to resolve them. The coherence of this type of epistemology is defended in (2.3) and the argument rests upon the inter-definability of modal notions. A philosophical (necessary) statement, asserting the impossibility of a given negation, is incompatible with an empirical (contingent) statement, asserting the possibility of that same negation. Of course, if the former (philosophical) statement is true, then the latter (empirical) statement is false; but equally, if the latter (empirical) statement is true, then the former (philosophical) statement is false. It is, then, not incoherent to look for empirical evidence to test a philosophically based thesis and conversely.

This general argument is presupposed at two points in the discussion. Firstly, it is applied - in (4.2) - in consideration of Piaget's stage-theory to support the contention that the presence of logical connections linking the characterisations of different stages of development does not preclude empirical testing to establish the actual sequence of development. Secondly, the argument is presupposed - in (7.3) - when D.W. Hamlyn's criticism, that Piaget is committed to the assumption of the solitary knower, is rejected. It is open
to Piaget to argue that if the facts of development are as his theory suggests, then philosophically based counter-claims can themselves be rejected.

2) What is also clear is that genetic epistemology has no monopoly of epistemological problems since Piaget is led to take sides on problems in philosophy. Several examples are apparent. Piaget accepts the Kantian presumption that knowledge is possible and this presumption is incompatible with the position accepted by sceptical philosophers who deny that knowledge can arise at all. This is not to say that the sceptics are right and that Kant is wrong. It is to say that Piaget's epistemological position is one that assumes that scepticism is false. And this assumption should be recognised, if only to prevent a too precipitate claim that genetic epistemology can settle all of the problems of knowledge. This conclusion was stated in (2.3) and is supported by another instance, when Piaget accepts one philosophical theory (realism) and rejects others (phenomenalism and solipsism) in his account of the infant's Copernican Revolution. The argument of (8.2) states that such a construal of infancy is incompatible with development and it is for this reason that an alternative, and philosophical, position based on Intentionality is outlined in (8.3).

Again, Piaget's discussion of the mind-body problem rests upon his conviction that a biological epistemology, one depending upon both philosophical and empirical considerations, is the only acceptable type of epistemology. It is evident that Karl Popper accepts a similar epistemological position. Yet Piaget and Popper reach strikingly different conclusions about the acceptability of a dualist theory of mind. This is not to say that Popper's acceptance of such a theory is justified and Piaget's rejection not so. It is to say that Piaget's position is simply a position.

In general, it can be said that Piaget, due to the reasonable
desire to establish the distinctness of genetic from philosophical epistemology, fails to make comprehensive use of insights formulated by the latter. The distinction between bodily movement and action and between purposive and purposeful action is central to much recent philosophical work - see (4.3) - yet Piaget, unlike researchers such as Margeret Boden, leaves implicit distinctions which can and should be stated in a fully explicit way.

(B) Piaget's Theoretical Constructs

The second objective was to describe the theoretical constructs employed by Piaget in the statement of his theory. There is no serious doubt about the importance of this objective. It follows from the methodological position stated in (1.2) that theory is essential in science and it was also noted that Piaget insists on the need to relate his empirical findings to his explanatory constructs. Three main features of this discussion can be noticed, 1) that an adequate theory of knowledge must show how logico-mathematical knowledge arises, 2) that a structuralist theory accounts for the presence of different types of knowledge and 3) that a constructivist theory shows how stronger structures can be constructed out of weaker structures. In brief, Piaget denies that perception and thought are adequate sources of knowledge and asserts instead that action is the ultimate source of knowledge. A subject uses structures in acting on the world and new knowledge arises with the construction of new structures.

1) Piaget's rejection of the copy (empiricist) theory of knowledge is due to his contention that a subject's observable knowledge, based upon perception or consciousness (conceptualisation) is insufficient as a source of logico-mathematical knowledge - see (3.3). The presence of egocentrism - see (3.5) - or of incorrect and incomplete conceptualisation - see (3.6) - is taken by Piaget to be the manifestation of the asymmetry of the cognitive system which inclines...
a subject towards comprehension of observable reality but without thereby enabling that subject to comprehend the inferential properties applicable to that reality. The studies of transitivity and inclusion - in (6.3) and (6.4) - are taken by Piaget to provide confirming evidence of this asymmetry.

Since Piaget rejects experience, as well as heredity and language, as a source of logico-mathematical knowledge, Piaget has to propose an alternative source. His proposal rests upon the distinction between knowledge which is accessible to a subject's consciousness and knowledge which is accessible to an observer of that subject. In particular, the discussion of (3.4) states that a normative fact is a fact, verifiable by an observer, which records a subject's use of some norm. That norm may be moral, legal or social but Piaget's theory of knowledge primarily attempts to show the role of logical norms in a subject's actions. A subject is an agent and performs intentional actions and so has observable knowledge of their goals, together with associated beliefs and values, as well as observable knowledge of their outcome. A subject's coordinatory knowledge, consisting in the coordinations imposed upon actions, can be described by an observer. Action-coordination is accessible to an observer who can describe those coordinations in logical terms. Such a description is a description of the subject's cognitive structure, a structure which is used by the subject in the acquisition of knowledge. A subject's observable knowledge is, therefore, bounded by a subject's coordinatory knowledge: the latter sets limits to what the subject can and cannot do. The discussion of normative facts, in (3.4), is thus presupposed by the discussion of cognitive structures, in (4.4).

That is, it is Piaget's claim that a subject has two types of knowledge and that coordinatory knowledge of action is the source of logico-mathematical knowledge. Such knowledge arises in a subject's
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acting rather than from the action itself.

2) Piaget's structuralist thesis states that there are different structures used by a subject at different developmental points. The claim that there are different structures is compatible with, but is not equivalent to, the admission that a subject uses structures in acting. Piaget states - see (4.3) - that a subject's actions can be physical or conceptual in character. Physical actions are performed by the infant and action-schemes - see (4.5) - are the corresponding structures. Because such actions are, strictly, taken to be sensory and motor by Piaget, it is denied by him that the infant can engage in conceptualisation: the infant fails to differentiate action from objects (reality) and is taken not to make use of a representational capacity.

Conceptualised actions can be of two types since they can be representationally or propositionally based. Such actions may have some physical embodiment but the underlying system of conceptualisation is their important feature. Operational schemes are the structures of such actions - see (4.6). The criterion of an operational scheme is the presence of deductive necessity in the thought of the subject and corresponding to this is associated behaviour, accessible to an observer. Only the subject whose thinking is propositionally based is taken by Piaget - see (4.6) - to be capable of thinking in a comprehensive and rational manner.

Thus there are three types of action, with three corresponding types of structure. Since such structures are logically characterised and since such characterisations are inter-linked, the stages of the development of knowledge embody logically necessary characteristics. Such an admission is, however, compatible with there being - see (4.2) - alternative ways in which development does as a matter of fact take place and so empirical testing is required for the actual sequence of development to be correctly described. Philosophers and logicians can establish a priori that the descriptions of certain
formal systems are necessarily linked; empirical research is required to establish the application (if any) of such systems to the developing individual.

3) Piaget's constructivist thesis states that there are different structures, that weaker structures lead to the formation of stronger structures and that the process of construction is inherent in the activity of the subject. Such a claim is compatible with, but is not equivalent to, the former claim, namely that there are different structures used at different points in development.

Piaget's constructivist thesis is comprehensively construed — in (2.4) — as the claim that cognitive organisation is the outcome of organic organisation but the full biological implications of this thesis have not been pursued in the present study. The thesis, in its restricted construal, states that the structures of infancy, childhood and adolescence, which correspond — see (4.3) — to the three stages in development, share a common feature, namely functioning (organisation), but also embody differences, namely structural differences. Functioning is thus taken to be — in (4.4) — an invariant feature of life and its presence is taken to be empirically sufficient for the presence of other factors, whether in nature or in nurture. Each such factor is, however, taken — see (5.2) — to be empirically necessary for development. Invariant functioning is compatible with there being different structural instantiations of functioning.

Construction occurs by a process of equilibration, leading from initial disequilibrium to (partial) equilibrium. The discussion of equilibration — see (5.2) — presupposes the discussion of observable knowledge — see (3.5) and (3.6) — and the asymmetry of the cognitive system. Contradiction—mistakes arise when a subject fails to understand what is the contradictory of what and such mistakes can be of two types since the subject can take to be contradictory that
which is compatible and take to be compatible that which is contradictory. A contradiction-mistake arises when a subject tries to understand that whose comprehension requires the use of a structure that is not available to that subject; that is, rectification of such a mistake requires the use of a new structure. Upper bound equilibration occurs when such construction takes place, whereby the identity of a subject's cognitive system is preserved but its internal character is enriched. The enrichment is more fully described by specification of the new structures used by a subject: observable knowledge is supplemented by a more comprehensive type of coordinatory knowledge.

It is Piaget's claim that a subject has an inherent capacity to generalise knowledge, by use of a structure in acting on the world, but that such a capacity is vulnerable to the constraints arising from the specific character of observable knowledge - see (5.2) and (5.3). The presence of cognitive lags is, therefore, an inevitable feature of development.

Processes of abstraction and generalisation are inter-linked with the occurrence of equilibration - see (5.4) - since a subject abstracts the common property from a range of instances and generalises a common property to its new instances. The distinction between observable and coordinatory knowledge is central to Piaget's accounts of these processes.

(C) Objections to Piaget's Theory

Four main types of objection to Piaget's theory are discussed and evaluated and are directed upon 1) Piaget's methodology in the attribution of deductive knowledge, 2) the account of social knowledge, 3) the infant's capacity to engage in representation and 4) Piaget's differing use of the concept of necessity. The first two objections are external and arise from arguments based upon the
work of rival researchers. An attempt is made to defend Piaget's account. The latter two objections are internal arguments derived from Piaget's theory as interpreted in the present study. An attempt is made to show that such objections limit the explanatory scope of the theory.

1) It is argued, in (6.2), that the attribution of deductive knowledge by an observer to a subject requires the use of language by both. The first reason for this requirement is that a subject whose judgement is deductively based is one who should be able to justify that judgement. A subject's complete inability to justify a (deductive) judgement is a ground for denying that that subject has deductive knowledge. Further, deductive knowledge possesses two features, namely universality and necessity, and such features can be expressed only in a subject's use of language. It is not claimed that Piaget always provides good evidence in conformity with the position attributed to him. It is claimed that his own statement of account requires acceptance of this position.

A child's understanding of transitivity and inclusion — see (6.3) and (6.4) — requires the presence of deductive knowledge. Even though Piaget's accounts have been subjected to penetrating experimentally based scrutiny, notably from the work of Peter Bryant, their essential claims can be retained. This is in part due to the possibility of a subject's gaining observable, and so non-deductive, knowledge in the experimental studies. It is also due to over-reliance in such studies on the judgements expressed by the child to the exclusion of the reasons given for that judgement with concomitant expressions of necessity. It is, however, suggested — in (6.5) — that even if the experimental studies are taken in the manner suggested by their authors, namely as indicative of an incipient understanding of deduction, linguistically based studies are still required to test for the child's ability to give reasons for such (incipient) deduction.
2) Piaget's account of knowledge has been criticised for its failure to account adequately for the social dimension in knowledge. Two forms of this objection are considered and rejected. In (7.2) it is argued that Piaget's account is not biased in favour of physical, to the exclusion of social, cognition since the coordinatory (logical) knowledge possessed by a subject can be applied to any, including social, contexts. An individual's operations are concurrent with social cooperation. In (7.3) it is argued that Piaget's account of knowledge, whether physical or social, is not biased in favour of intra-, to the exclusion of inter-, individual factors. It is specifically noticed that such a criticism, derived from the work of D.W. Hamlyn, rests upon a conception of epistemology that is incompatible with Piaget's genetic epistemology and that the nature of development is taken by Piaget to be such that the conditions stated by Hamlyn are, as a matter of fact, incapable of being satisfied.

3) Piaget's account of infancy is beset with contradictions whose removal is made possible, though not necessitated, by a commitment to the claim that the infant has a representational capacity. It is proposed, in (8.8), that Piaget should make such a commitment, though it is noted, in (8.1), that his own account specifically excludes this. The grounds for this conclusion are threefold. It is argued, in (8.2), that Piaget's account of the infant's Copernican Revolution prevents the occurrence of development due to Piaget's uncritical use of the philosophical theories of phenomenalism and scepticism. It is argued in (8.4) - (8.6) that Piaget makes contradictory claims about the infant's capacity to make observations, to engage in representation and to perform intentional actions and to engage in reflective abstraction. And it is noticed, in (8.7), that Piaget does not, and cannot, exclude the occurrence of development in the case of limbless subjects (infants) who have no real
ability to perform actions which are physical in character. It is suggested, in (8.3), that the defects noticed in (8.2) can be rectified by the adoption of an Intentional account of perception. Such an adoption, however, requires the ability to represent. It is also proposed, in (8.8), that the remaining contradictions can be removed if Piaget accepts that the infant can engage in representation.

It is noticed that the proposal in question requires substantial change in Piaget's account of infancy, not least in the claim - already noticed above, and stated in (4.5) - that the infant is unable to differentiate objects from the actions directed upon them.

4) Piaget's theory uses the concept of necessity in three distinct ways and these are identified in (2.5). It is argued, in (9.2), that Piaget's theory offers empirically necessary conditions alone and that in consequence the theory is incomplete, though not rendered trivial or unimportant on that account. It is, argued, in (5.5) - that a theory which offers necessary conditions is one which is more basic than one which offers sufficient conditions.

It is argued, in (9.3), that Piaget's theory states that a subject has knowledge of deductive necessity only by the use of an operational structure, as the discussion in (4.6) showed. Piaget's recent research, based upon his constructivist commitment, results in the claim that at earlier stages in development, for example in infancy, a subject has knowledge of non-deductive necessity. No firm conclusions can be established about such claims, which are stated on the basis of on-going research. It is clear that such claims are, at present, unexplained, and may even constitute a radical revision of Piaget's theory.

Finally, it is argued, in (9.4), that equilibration is a necessarily constructive process but only because the transcendence of a contradiction-mistake requires the construction of a new structure. Piaget's theory, which concentrates on the specification
of equilibratory factors in development, is essentially dependent upon other accounts which specify the non-equilibratory factors in development. That there are other such accounts, and specifically accounts based upon cybernetical research, was noted in (5.5). Piaget's account is, then, complementary to accounts of development that do specify the role of non-equilibratory factors and conversely.

(D) Piaget's Theory

It is generally agreed that Piaget is an author whose work is difficult to understand and the discussion gives ample evidence in corroboration of this aspect of Piaget's work. The difficulty does not arise from the sheer quantity of Piaget's writings, though Piaget is a prolific writer. Difficulty does arise from Piaget's tendency to present his constructs, members from the same family of which are deployed in explanation of the diverse themes which are his theoretical concern, in a manner that promotes mystification rather than clarification. Piaget, it may be said, writes with Piaget rather than other readers in mind. For example, Piaget's distinction between an operative scheme and a figurative schema is not formally codified until recent times; reflective abstraction is initially presented in the context of Piaget's writings on mathematical thinking and is only recently stated to be applicable at all levels in the development of knowledge; and Piaget's conception of action is one that needs to be extracted from his writings and that conception does not perspicuously stand out from them. Thus Piaget's manner of presenting his ideas - in serial form with the minimum of explanatory comment - is one which tends to invite confusion. Another source of difficulty, however, arises because Piaget's work is inspired by a philosophical tradition which is not always shared - or if it is shared, then not always accepted - by others. Thus it was stated in (2.2) that Piaget's interest in the development of knowledge was
due in part to the influence of Kantian philosophy. No doubt that influence was not the only influence on Piaget but it is clear that a Kantian inspired epistemology will be different from one which is inspired by an empiricist epistemology, of the sort which apparently inspires much psychological research. Moreover, the use of logico-mathematical notions in application to the process of knowing is not accepted by other psychologists as willingly as their use in the treatment of experimentally acquired data. Finally, it is apparent from the present study that translation-problems prevent the communication of Piaget's ideas in the English speaking world. Many of Piaget's important, and early, papers are still unavailable in English editions and mistranslation affects much of the writing that has been translated. It may be noticed that the present study has been forced to use French, rather than English, texts in correction of standard editions of the latter. Moreover, in many cases the mistranslation is serious, occurring with respect to crucial, since novel, aspects of Piaget's theory (see Appendix A).

What final conclusion can, then, be stated with respect to Piaget's theory? On the one hand, that theory has been influential; it is one of the major contributions to psychology and epistemology; the theory has provoked an enormous body of research, whether Genevan based or otherwise; and Piagetian, or neo-Piagetian, research - for example, by Bärbel Inhelder, Guy Celliérier and Seymour Papert - is still in an active stage of growth. On the other hand, there is an essential, and perhaps metaphysical, element in the theory since certain sorts of knowledge are taken to be unavailable to subjects, at given developmental points, through lack of the requisite logical knowledge. Piaget's constructivist epistemology requires that such subjects cannot gain such knowledge. Yet there could not be unambiguous and reliable evidence for a negative claim of this sort
which is, strictly, unverifiable. It is no doubt for this reason that many psychologists contend that Piaget's theory contains too pessimistic a conception of human ability.

The overall conclusion that is suggested by this study is that Piaget's theory faces valid and important problems and that genetic epistemology constitutes a suitable discipline for the study of these problems. Such a conclusion is accepted by Rita Vuyk (1981b, p.492) in her own recent discussion of Piaget's theory. As for Piaget's own theory of genetic epistemology, perhaps the best verdict is the one stated by its author and so it is fitting to leave the last words to him:

I have the conviction, which is illusory or well-founded (and only the future will show the contribution of truth or that of sheer, proud obstinacy), of having isolated a general skeleton which is more or less clear but still full of omissions, though these are such that their rectification will lead others to differentiate the articulations in a diversity of ways without thereby contradicting the major strands of the system but rather by integrating them in new interpretations (Piaget 1976c, pp.223-4 - my translation).
APPENDIX A  MISTRANSLATION

The English reader of the works of Piaget faces two problems which the French reader simply ignores. One problem is that many of Piaget's papers, as well as several books, are unavailable in English editions. The second problem is that mistranslation infects many of the English editions of Piaget's works with consequential distortion of his position. Recent translations of Piaget's works seem to be particularly vulnerable in this respect. The second problem is more serious than the first and it is one that confronts both sympathetic and critical readers of Piaget. Interestingly, Piaget shows himself to be aware of the dangers of mistranslation:

I would like to recall that I wrote my first books for readers of the French language without anticipating the possibility of translation. Moreover, I did not dwell on what was well known in French psychology, even though I would have spoken quite differently in English contexts (Piaget 1931, p.146 - my translation).

What is particularly unfortunate is that Piaget's claims are subject to mistranslation at precisely those points in his theory that are central and novel, namely with respect to structuralist and epistemic concepts.

It is the aim of this Appendix to identify and correct instances of serious * mistranslation that have been detected in the chapters above. Works are cited in alphabetical and chronological order by author(s) of the French editions. References are given in the order, for example 220/251/4.6, French edition, standard translation, section-reference respectively. Some of the material in this Appendix is discussed by Smith (1980a, 1981a, 1981b, 1981g).

INHELDER & PIAGET 1955: *De la Logique de l'Enfant à la Logique de l'Adolescent
1958: The Growth of Logical Thinking

1: 220/251/4.6

Piaget's use of the concept of necessity in application to the formal operational subject is misrepresented.
Au lieu d'introduire sans plus un début de nécessité dans le réel, comme c'est le cas des inférences concrètes, elle (la pensée formelle) effectue dès le départ la synthèse du possible et du nécessaire.

Instead of deriving a rudimentary type of theory from the empirical data as is done in concrete inferences, formal thought begins with a theoretical synthesis implying that certain relations are necessary.

'Theory' is incorrectly given for 'nécessité' and Piaget's claim that the formal operational subject produces a synthesis of what is possible with what is necessary is lost.

Far from merely making an initial application of necessity to reality, as is the case with concrete inferences, formal thought brings about from the outset a synthesis of what is possibly with what is necessarily the case.

The French edition uses schéme and schéma with the same meaning, which is inconsistent with Piaget's own practice. The translation offers 'schema' for both and thus prevents detection of Piaget's inconsistency.

PIAGET 1936 La Naissance de l'Intelligence chez l'Enfant
1953a The Origins of Intelligence in the Child

In contrast to earlier works (Piaget 1928b, 100, 110) where scheme and schéma are used interchangeably, Piaget uses the former term only to refer to a sensori-motor structure. The Kantian basis of this new use is shown by the reference to schématisme (1936, pp.12-3). The translation uses 'schema' for all of these terms and thus hides Piaget's innovation.

The translation sometimes offers 'picture' and sometimes 'image' for the same French term, namely tableau. A fuller discussion of this issue is given in (8.2). There are other instances of this confusion, for example, Piaget (1953a, pp. 74, 98, 108, 140-1, 143).
Piaget uses the French image, as well as the French tableau, whilst the translation offers the English 'image' for both. The five instances of 'image' in the translation should, therefore, read 'sensory scene...scene...scene...image...image'. A fuller translation of this passage is given in (8.2).

The generic concept of intentionality has several species of which the concept of intention is but one. Piaget's claim that intentionnalité is the criterion of intelligent behaviour is lost when, in English, 'intention' is given. There are other examples of this mistake, for example Piaget (1953a, pp. 137, 143, 147, 149, 155). This same English term is also used to translate the French intention — for example, the second occurrence of 'intention' (Piaget 1953a, p. 147) or the third occurrence of 'intention' (Piaget 1953a, p. 149) or 'intention' (Piaget 1953a, p. 321).

Omission occurs when a whole line is mislaid:

Or l'intelligence précède la langage et tout acte de l'intelligence sensori-motrice suppose l'intention.

Now intelligence presupposes intention.

The translation should read:

Now intelligence precedes language and every act of sensori-motor intelligence presupposes intention.

The same mistranslation of intentionnalité is apparent — see Note 4 — and a full translation is offered in 4.3.

PIAGET 1937 La Construction du Réel chez l'enfant
1954 The Construction of Reality in the Child

The same mistranslation — see Notes 2 and 3 of Piaget (1936) — of the
French tableau appears which is given both as 'picture' and 'image'. There are other instances of this confusion, for example Piaget (1954, p.9).

PIAGET 1967a Biologie et Connaissance
1971a Biology and Knowledge

A footnote in the French edition is omitted from the English edition and should be attached to the second occurrence of 'intelligence':

sensori-motor intelligence is so-called because it is prior to language. It uses only perceptions and movements and is in consequence distinct ('étrangère') from representation or thought. Representative intelligence begins by contrast only with the semiotic (or symbolic) function.

The translation offers 'schema' for the French schéme, even though Piaget (Piaget & Inhelder 1966) explicitly contrast schéme and schéma.

A crucial negative is omitted from the translation since

as a condition necessary to every transmission and as a transmitted content

should read

as a condition necessary to every transmission and not as a transmitted content (my emphasis).

Prendre conscience is given as 'taking cognizance'. This English expression may cause confusion since it conflates what Piaget (1974a) contrasts, cognition based upon consciousness with cognition based upon perception.

PIAGET 1970b Inconscient affectif et inconscient cognitif
1974e The Child and Reality

The translation offers 'awareness' for prise de conscience. Strictly,
Piaget refers to the process of becoming aware (conscious) rather than to (the state of) awareness. All instances of 'awareness' should read 'becoming aware' or 'the acquisition of consciousness'.

PIAGET 1970d Psychologie et Epistémologie
1977d Psychology and Epistemology

The translation uses 'experiment' for the French expérience when, in context, this means 'experience'.

L'expérience est toujours assimilation à des structures...antérieur à l'expérience.

This extract should read:

Experience is always and assimilation to structures... prior to experience.

The same translator makes the converse mistake: he offers 'experience' (Piaget 1978b, p.95) when, in context, this same French term should mean 'experiment'. The translator is inconsistent with his own practice when he, correctly, uses 'experiment' for this term (Piaget 1978b, p.63).

It may also be noticed that a similar mistake occurs with respect to the French conscience which is given (Piaget 1977d, p.5) as 'conscience' when, in context, it must mean 'consciousness'.

PIAGET 1974a La Prise de Conscience
1977e The Grasp of Consciousness Note: a more accurate translation of this title would be The Acquisition of Awareness.

Piaget claims that psychologists consider that

la prise de conscience ne consiste qu'en une sorte d'éclairage ne modifiant ni ajoutant rien...
Freud va jusqu'à comparer la conscience à un "organe des sens interne";

being conscious of a situation...means that the subject is fully aware of it. The fact that he has become aware of it neither modifies nor adds anything to the situation...Freud even compares consciousness to an "organ of the internal senses".
Piaget uses the metaphor of lightning to characterise this process, unlike the translation which offers a tautology. Grammatical and conceptual grounds require that Freud takes consciousness to be an internal organ of the senses. Thus Piaget's claim is that psychologists consider that

the attainment of consciousness consists only in a sort of illumination which modifies and adds nothing...Freud even compares consciousness to an internal organ of the senses.

2: 263/334/3.6

Piaget claims that there are two peripheral features of an action:

la conscience du but à atteindre, autrement dit de l'intention en tant que direction globale de l'acte, et la prise de connaissance de son aboutissement en tant qu'échec ou réussite;

consciousness of what the goal is — in other words, awareness of the general direction of the action needed to attain it (intention) — and cognizance of its result, either failure or success.

The object of consciousness is a goal or intention. The translation offers 'cognizance' for prise de connaissance, when the former is also used to translate prise de conscience. Thus the distinction drawn by Piaget is lost.

Consciousness of the goal to be attained, in other words consciousness of the intention in its capacity as the overall direction of the act, and the attainment of knowledge of its outcome, as failure or success.

3: 263/335/5.3

In Piaget's discussion of the transition from periphery to centre, the translation conflates the concepts of consciousness and knowledge since 'cognizance' is offered for prise de conscience, in contrast to the practice noticed in Note 2, whilst 'awareness' is offered for prise de connaissance. The difference between prise de conscience and prise de connaissance is discussed in (3.6). Their functional similarity is shown in the following passage, which is also mistranslated (Piaget 1977c, p.336):

le mécanisme des prises de connaissance de l'objet ne se prolonge pas en prise de conscience de l'action;
the mechanism of cognizance of the object must therefore extend into cognizance of the action.

The translation uses 'cognizance' for two quite distinct concepts, which are taken by Piaget to have instances that embody a common mechanism. There is therefore no reason why

the mechanism of the attainment of knowledge of the object does not extend into the attainment of consciousness of the action.

4: 271/342/3.6

From this perspective Piaget claims that

il n'y a pas de différence de nature entre la prise de conscience de l'action propre et la prise de connaissance des séquences extérieures au sujet;

there is therefore no intrinsic difference between cognizance of the action itself and awareness of what is happening outside the subject.

The translation offers 'cognizance' for prise de conscience and 'awareness' for prise de connaissance and thus confuses what Piaget keeps distinct. 'Intrinsic' is used by Piaget (1978e; see (5.4) above) in a technical sense and is a possible source of confusion.

There is no real difference between the attainment of consciousness of the action itself and the attainment of knowledge of the sequence of events occurring outside the subject.

5: 273/345/8.4

Piaget defines an observable as

tout ce qui peut être enregistré par une simple constatation de fait (ou empirique)

'Observation is given for constatation when this latter can be more liberally taken as 'verification'.

6: 275/346-7/4.3

Practical knowledge (savoir faire) is distinct from conscious knowledge or conceptualised understanding and is its source because

la prise de conscience est presque sur tous les points en retard, et souvent de façon très sensible, sur ce savoir initial qui est donc d'une efficacité remarquable, bien que ne se connaissant pas lui-même;
the cognizance lags, and often markedly so, behind this initial knowledge, which is thus of remarkable efficacy despite the lack of understanding.

'Cognizance' is given for prise de conscience and the translation states that practical knowledge lacks understanding, which is ambiguous.

The attainment of consciousness in almost every case lags behind, often in a marked way, this initial knowledge which is thus of remarkable efficacy despite the fact that it does not embody a conceptualised knowledge of itself.

PIAGET 1974b Réussir et Comprendre
1978a Success and Understanding

1: 6/viii/4.5
The translation offers 'schema' for the French schéme, even though Piaget (Piaget & Inhelder 1966) contrasts schéme and schéma.

2: 231-2/213/8.4
Piaget claims that action is an autonomous form of knowledge whose conceptualisation ne s'effectue que par prises de conscience ultérieures;
conceptualised by later, conscious assimilations.
The mistake here is the conflation of the concepts of consciousness and of assimilation. Thus action is a form of knowledge whose conceptualisation occurs only by later acts of conscious awareness.

3: 236/225/5.4
Piaget distinguishes four types of abstraction and differentiates abstraction réfléchissante from abstraction réfléchie. The translation offers 'reflexive abstraction' and reflected abstraction' (p.223) respectively but inconsistently, offers the former English expression for the latter French one as well.

PIAGET 1975a L'Equilibration des Structures Cognitives
1978b The Development of Thought. Note: a more accurate translation of this title would be The Equilibration of Cognitive Structures.

1: 9/3/5.2
The translation offers 'increasing equilibration' for équilibration
majorante. It is suggested, on the basis of Piaget (1970a, p.723; 1972a, p.90) that 'upper bound equilibration' is preferable. Other translators offer 'accretive' (Piaget 1980a, p.33) or 'improving' (Vuyk 1981a, p.68).

2: 13/7/5.2

Piaget states that an appeal to assimilation is limited to assigning

un moteur à la recherche, donc à considérer comme nécessaire une activité du sujet;

a driving force to the process and therefore must assume activity on the part of the subject.

Recherche may be taken to be elliptical for recherche de la cohérence (Piaget 1975a, p.21) which the translation (1978b, p.15), correctly, gives as 'search for coherence'. So the passage reads:

a driving force to the search for coherence and so takes activity on the part of the subject to be necessary.

3: 23-4/18/5.2

A regulation occurs when

la reprise A' d'une action A est modifiée par les résultats de celle-ci, donc lors d'un effet en retour des résultats de A sur son nouveau déroulement A';

when the reaction A', of an action, A, is modified by the original action, i.e., there is a secondary effect of A on the new development.

Reprise is taken to mean 'reaction', when it is clear that a second performance of an action is meant: an action A occurs and then action A', related to but distinct from A, occurs. A regulation thus occurs when

a subsequent performance, A', of an action is modified by the results of its initial performance A, and so by a return effect of the results of A on its new development A'.

4: 24/18/5.2

Omission occurs in the translation (p.18, line 18 - after 'feedback') since a paragraph and a half is left out. Since the translation is,
unfortunately, replete with omission of this sort, it is beyond the scope of this Appendix to rectify this mistake in its entirety. The following may be substituted as a partial correction:

But in this last case there is the possibility of a growth of error (as the material model of a fire illustrates) or of success (formation of habits, etc.). The notion of assimilation fuses into one whole every use of an object or an element...a scheme of assimilation confers a certain meaning on the objects assimilated and therefore assigns definite goals to the actions which have reference to them (such as grasping, balancing, etc., at the practical level or understanding a relation, etc., at the representational level)...all regulations are, from the subject's point of view, reactions to disturbances...though the converse does not hold.

5: 45/39/5.2

Omission occurs and the following should be substituted in the translation (p.39, line 30):

but as assimilation and accommodation constitute two poles, which are always inseparable, and not two distinct behaviour-patterns.

6: 50/43/3.5, 5.3, 8.4

Piaget's distinction between an observable and a coordination is translated in full at the beginning of (5.3). The standard translation omits 'necessary' and thus cannot distinguish between inductive and deductive inferences. It also misconstrues Piaget's notion of transcendence: necessary inferences go beyond the limits of the observable. Finally, 'extensional' is used in its logical sense: a necessary inference is not an extensional move from some to all verified instances but the construction of a new relation which transcends the bounds of the observable.

7: 86/84/3.6

The translation offers 'internalizing' for interioriser and thus conflates the distinction, pointed out by Furth (1969), between the operative and figurative uses of this term.
The translation offers 'entities' for tableaux when this term is correctly given by the same translation (p.87) as 'scenes!.

The translation conflates Piaget's (1962) distinction between the operational concept of reversibility (réversibilité) with the physical concept of reversal (renversabilité).

Without consciously considering reversibility... reversibility without conservation... intuition of reversibility.

It is preferable to translate the last two cases of 'reversibility' as 'reversal'.

'Closing is given for fermeture which is preferably given as 'closure'.

Piaget takes up

la redoutable question de la prise de conscience en ses conceptualisations (puisqu'elle consiste à traduire l'action motrice en termes de représentations);

the fearful question of the conceptualisation of the awareness (since it expresses motivation through representation).

What is fearful is not the conceptualisation of awareness but awareness qua conceptualisation since awareness translates motor behaviour into representations. Piaget thus takes up

the fearful question of the attainment of consciousness in its capacity as conceptualisation (since it translates motor behaviour into representations).

Piaget claims

l'ensemble de tous les possibles demeurant sans doute une notion antinomique puisque le "tout" n'est lui même qu'un possible;

the whole of all possibilities is a paradoxical notion, since the "whole" itself is only possible.
Piaget, in fact, refers to an antinomy that is produced when it is supposed that there is a set of all possibilities: that set is itself one possibility.

The set of all possibilities is undoubtedly an antinomic notion, since the "all" is itself but one possibility.

Note: This translation is seriously defective and the instances of mistranslation noted here are simply instances. It is doubtful whether a reader of the English text can form an intelligent view of Piaget's actual account. It is apparent that Furth (1981, p.254) and Smith (1981a, p.3) independently arrive at the same conclusion as to the value of this translation.

PIAGET & INHELDER 1948 La Représentation de l'Espace chez l'Enfant
1956 The Child's Conception of Space

1: 532/449/4.5

Piaget, inconsistently with his own practice, attributes *schémas opératoires* to the child, which the translation gives as 'operational schemata'. 'Schemata' is also given for *schèmes* (1956, p.452) and thus the translation hides Piaget's inconsistency.

PIAGET & SZEMINSKA 1941 La Genèse du Nombre chez l'Enfant

PIAGET 1952 The Child's Conception of Number

Note: there is no apparent reason for the attribution of sole authorship of this work in the English edition.

1: 6/3/4.5

Piaget, inconsistently with his own practice, attributes *schéma* to the child which the translation gives as 'schema'. 'Schema' is also given for *schème* (1952, p.88) and thus the translation hides Piaget's inconsistency.

2: 199/161/6.4

The translation offers 'comprehension' for *compréhension* when in context, this should be taken to mean 'intension'. A similar mistake occurs in Piaget (1980a, p.165). Piaget (Piaget et al. 1968, p.222-3/1977,
p. 185) is aware of the possible confusion that this French term produces for the English reader (and evidently translator).

3: 202/163/6.4

Omission occurs in the last line which should read:

he makes this discovery intuitively yet without proceeding in a deductive and operational manner.

Similarly, 'non-deductive' is omitted, after 'intuitive', of this translation (p.175).

PIAGET 1976d Le possible, l'impossible et le nécessaire.
PIAGET & VOYAT 1979 The possible, the impossible and the necessary.

Note: there is no apparent reason for the attribution of joint authorship in the English edition

1: 286-7/71/4.5

The translation offers 'schema' for schème when Piaget (Piaget & Inhelder 1966) contrasts schème and schema. The translation prevents the reader from detecting consistency in Piaget's (recent) usage of schème.

Three claims may be stated in conclusion. Firstly, the Appendix has a limited aim, that of inventoring the instances of mistranslation noticed in the chapters above. The Appendix has not set out to identify or correct all instances of mistranslation in the works cited. Secondly, it is evident that there is a lack of sophistication in the translation of Piaget's works since translators show their ignorance of Piaget's own theory, of psychological concepts and of philosophical concepts. It would seem that the serious English student of Piaget should not take too seriously some English translations of his work. Mistranslation is, perhaps, understandable with respect to Piaget's early writings but is, surely, unforgivable with respect to recent works such as Piaget (1974a, 1975a). Thirdly, it is suggested that in view of the wide range of skills - linguistic, psychological, philosophical, mathematical - that a translator of Piaget's work should possess, future translations should be undertaken by a team of translators.
REFERENCES


ABSTRACT

Piaget's theory of genetic epistemology states empirically necessary conditions of a subject's understanding of deductive necessity which is formed by a necessary process of equilibration. The coherence of Piaget's theory, which provides empirical evidence for the resolution of philosophical problems posed by Kant, is defended. Piaget contrasts observable with coordinatory knowledge and denies that logical knowledge arises from the former, based upon perception and consciousness (conceptualisation). Coordinatory knowledge arises in action-coordination. Actions, at physical, representational and propositional levels, utilise different structures. Stages in the development of knowledge are marked by different structures and their logical characterisation does not preclude empirical testing. Deductive necessity is the criterion of the closure of a structure, occurring only at operational levels. Structural change occurs when transcendence of a contradiction-mistake leads to the formation of a new structure. Processes of upper bound equilibration and constructive forms of abstraction and generalisation are necessary for such formation. Functioning is an invariant feature of organic and cognitive life and is manifest in different structural instantiations.

Four lines of objection are considered. Operational knowledge is attributed only when a subject uses language to justify a judgement and expresses the universality and necessity of a deduction. In consequence Piaget's accounts of transitivity and inclusion are defended. The theory does not favour physical over social cognition nor intra- over inter-subjective factors. Piaget's account of infancy, especially the Copernican Revolution, contains contradictions whose removal rests upon the attribution of representation to the infant. Piaget's theory is incomplete because it states necessary conditions alone; it posits and leaves unexplained non-deductive necessity; and necessary equilibration occurs only through the presence of non-equilibratory factors.

Piaget's theory constitutes a skeleton of a theory of knowledge-growth, complementary to non-equilibratory theories.

Mistranslation, especially of structuralist and epistemic concepts, mars English editions of Piaget's work.