THE PROPAGATION OF VERY LOW FREQUENCY RADIO WAVES WITH SPECIAL REFERENCE TO THE OMEGA NAVIGATION SYSTEM

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## CONTENTS

### CHAPTER 1: Introduction

1.1 THE STRUCTURE OF THE EARTH'S ATMOSPHERE ........................................ 1
1.2 THE DISCOVERY OF THE IONOSPHERE ..................................................... 2
1.3 STRUCTURE OF THE IONOSPHERE, AND VLF RADIO WAVES .......................... 6
1.4 AIMS OF THE PRESENT STUDY ...................................................................... 9

### CHAPTER 2: The D-Region of the Ionosphere

2.1 INTRODUCTION ............................................................................................ 12
2.2 THE FORMATION AND CHEMISTRY OF THE D-REGION ............................... 13
   2.2.1 Sources of ionization - daytime ......................................................... 13
   2.2.2 Sources of ionization - night-time ..................................................... 15
   2.2.3 Positive and negative ion chemistry .................................................. 16
   2.2.4 Electron attachment and detachment ................................................. 16
2.3 HEIGHT DISTRIBUTION OF ELECTRON DENSITIES ................................ 18
   (undisturbed conditions)
   2.3.1 Experimental methods ......................................................................... 18
   2.3.2 Night-time distributions ...................................................................... 19
   2.3.3 Variation prior to ground sunrise ....................................................... 21
   2.3.4 Daytime variation .............................................................................. 21
   2.3.5 Seasonal variations ............................................................................ 26
   2.3.6 Latitude variations .............................................................................. 26
   2.3.7 Solar cycle variation ........................................................................... 30
2.4 D-REGION COLLISION FREQUENCIES ..................................................... 31
2.5 THE DISTURBED D-REGION ...................................................................... 36
   2.5.1 Sudden ionospheric disturbances (SIDS) ........................................... 36
   2.5.2 Polar cap absorption events (PCA) ..................................................... 38
   2.5.3 Storm effects in the D-region ............................................................... 40
2.6 SUMMARY .................................................................................................. 43
CHAPTER 3 : The Theory of VLF Propagation

3.1 INTRODUCTION

3.2 IONOSPHERIC PROPAGATION
   3.2.1 The constitutive relations of the ionosphere
   3.2.2 The wave equations
   3.2.3 Full wave integration

3.3 THE TERRESTRIAL PROPAGATION OF VLF RADIO WAVES
   3.3.1 Theory of propagation - isotropic model
   3.3.2 Inhomogeneous, anisotropic ionosphere
   3.3.3 Waveguide mode theory
   3.3.4 Spherical wave-geometric series solution
   3.3.5 Wave Hop solution

3.4 SUMMARY

CHAPTER 4 : The Propagation Characteristics of VLF Radio Waves

4.1 INTRODUCTION

4.2 PROPAGATION DURING UNDISTURBED CONDITIONS
   4.2.1 Experimental studies
   4.2.2 Theoretical considerations
   4.2.3 Effect of ground conductivity
   4.2.4 Influence of the geomagnetic field
   4.2.5 Mode conversion

4.3 PROPAGATION DURING IONOSPHERIC DISTURBANCES
   4.3.1 Sudden ionospheric disturbances (SIDs)
   4.3.2 Polar cap absorption (PCA)
   4.3.3 Storm effects
4.4 NUMERICAL COMPARISON OF THEORETICAL TECHNIQUES

4.4.1 Comparison of Spherical wave, Waveguide mode and Wave Hop techniques at 16 kHz and 20 kHz

4.4.2 Waveguide mode and Spherical wave comparison (19.8 kHz)

4.4.3 Frequency dependence study with the Spherical wave and Wave Hop methods

4.4.4 Comparative study as a function of ground conductivity

4.4.5 Influence of the geomagnetic field (Spherical wave method)

4.5 CONCLUSIONS

CHAPTER 5: The Omega Navigation System and the ONSOD Prediction Program

5.1 INTRODUCTION

5.2 SYSTEM CONFIGURATION

5.3 THE SEMI-EMPIRICAL MODEL FOR PHASE VARIATION

5.4 PATH SEGMENTATION AND GEOPHYSICAL MODELS

5.5 THE DIURNAL FUNCTIONS

5.6 SUMMARY

CHAPTER 6: Experimental Measurement of VLF Phase and Omega Lines of Position

6.1 INTRODUCTION

6.2 THE LEICESTER VLF/OMEGA MONITORING STATION

6.2.1 The receivers

6.2.2 Microprocessor controlled interface and memory

6.2.3 Wang mini computer system

6.3 THE FARNBOROUGH AND BUTT OF LEWIS MONITORING STATIONS

6.4 EXPERIMENTAL ERRORS AND SUMMARY
CHAPTER 7 : Presentation of Experimental Data

7.1 INTRODUCTION 150

7.2 PHASE AND AMPLITUDE RECORDS COLLECTED AT LEICESTER 150
  7.2.1 Norway-Leicester path 155
  7.2.2 Liberia-Leicester path 161
  7.2.3 North Dakota-Leicester path 164
  7.2.4 Other propagation paths monitored 168
  7.2.5 Influence of disturbed conditions 168

7.3 MEASURED LINES OF POSITION (LOPs) AT LEICESTER 175

7.4 MEASURED LINES OF POSITION AT FARNBOROUGH 179
  7.4.1 10.2 kHz data 179
  7.4.2 13.6 kHz data 184

7.5 MEASURED LINES OF POSITION AT THE BUTT OF LEWIS 189

7.6 CONCLUSIONS 189

CHAPTER 8 : Comparative Studies of the Accuracy of ONSOD Predictions with Experimental Measurements and the Effectiveness of Omega/Satellite Systems

8.1 INTRODUCTION 199

8.2 COMPARISON OF MEASURED AND PREDICTED LOP ERRORS 199
  8.2.1 Farnborough monitor 200
  8.2.2 Leicester monitor 202
  8.2.3 Butt of Lewis monitor 202

8.3 ERRORS IN GROUND POSITION FIXES 204
  8.3.1 The position of the Farnborough receiver 205
  8.3.2 The position of the Leicester receiver 206
  8.3.3 The position of the Butt of Lewis receiver 206
  8.3.4 Diurnal variation in mean positions 207
  8.3.5 Concluding note 215
<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.4 COMPARISON OF LOP ERRORS MEASURED AT THE RECEIVING STATIONS (10.2 kHz)</td>
<td></td>
</tr>
<tr>
<td>8.4.1 Leicester-Farnborough LOP error differences</td>
<td>217</td>
</tr>
<tr>
<td>8.4.2 Farnborough-Butt of Lewis LOP error differences</td>
<td>217</td>
</tr>
<tr>
<td>8.4.3 Implications for Differential Omega</td>
<td>217</td>
</tr>
<tr>
<td>8.5 REAL TIME UPDATING OF OMEGA POSITION FIXES</td>
<td></td>
</tr>
<tr>
<td>8.5.1 Omega/Satellite receiver simulation</td>
<td>224</td>
</tr>
<tr>
<td>8.5.2 Radial error distributions (hourly data, updates six-hourly)</td>
<td>226</td>
</tr>
<tr>
<td>8.5.3 Comparisons with nominal Omega accuracy</td>
<td>232</td>
</tr>
<tr>
<td>8.6 CONCLUSIONS</td>
<td></td>
</tr>
</tbody>
</table>

CHAPTER 9: Propagation Path Modelling and Concluding Notes

9.1 INTRODUCTION 236

9.2 PROPAGATION PATH MODELLING WITH THE SPHERICAL WAVE TECHNIQUE 237

9.2.1 Norway-Leicester path 238

9.2.2 Liberia-Leicester path 242

9.2.3 Liberia-Butt of Lewis path 244

9.3 SOURCES OF ERROR WITHIN THE ONSOD PREDICTION PROGRAM 249

9.4 CONCLUDING NOTES 250

9.5 SUGGESTED FURTHER STUDIES 253

APPENDIX A: Comparisons of Measured and Predicted LOP Errors

APPENDIX B: Position Fix Distributions

REFERENCES
CHAPTER 1

Introduction

1.1 THE STRUCTURE OF THE EARTH'S ATMOSPHERE

Surrounding the surface of the earth is the gaseous atmospheric envelope. The chemical composition of the atmosphere is well known, and is in agreement with expectations of the ability of the Earth's gravitational field to retain gases of a given molecular weight. The principal constituents are nitrogen (75.5% by weight), oxygen (23.1%) and argon (1.3%). Smaller amounts of neon, helium and heavier inert gases (krypton and xenon) are also present. Variable amounts of water vapour (0.1 - 0.7%) and carbon dioxide (0.03%) make up the remainder. Apart from these latter two constituents the composition of the atmosphere is remarkably uniform to a height of at least 100 km. This testifies to the efficiency with which the gases are mixed by turbulence.

The earth's atmosphere has a number of reasonably distinct regions, with different atmospheric properties at various altitudes. The primary classification is based on temperature, the height variation of this parameter and the associated regions being illustrated in figure 1.1. The lowest atmospheric region, the troposphere, extends to an altitude of eleven to fourteen kilometers and consists of a relatively turbulent zone. Within this layer the temperature falls off at a rate of about $10^{-6}$ K km$^{-1}$. The troposphere is bounded by the tropopause, above which is the stratosphere. This less turbulent region was originally thought to be isothermal but is now known to be a layer of increasing temperature. A local temperature maximum due to ozone absorption appears at about 50 km altitude and this marks
the stratopause. The temperature decreases above in the mesosphere to a minimum at 80-85 km (the mesopause). With a temperature of around $180^\circ K$ this represents the coldest part of the atmosphere. Above the mesopause temperatures increase throughout the thermosphere but then become constant at about $1500^\circ K$ at altitudes over 400 km.

The turbulent lower region of the atmosphere is termed the homosphere, above which lies the heterosphere where the mixing of gases is relatively weak. Within the heterosphere layers form of dominant gases (e.g. helium) while at the highest levels, above 600 km, individual molecules (e.g. hydrogen) can escape the earth's gravitational attraction.

The upper atmosphere (>50 km altitude) is ionized by energetic radiation from the Sun. Once produced, the free electrons and ions tend to recombine and a balance is established between the electron-ion production and loss rates. The net concentration of free electrons is greatest at heights of several hundred kilometers (see figure 1.1). Although the electron concentration at these heights amounts to only 1% of the concentration of neutral gases, it is the ionized component that gives rise to important characteristics in the upper atmosphere that are negligible or absent in the troposphere. Such characteristics include the existence of electric currents and fields, the reflection of radio waves and various plasma processes. This ionized region of the upper atmosphere is called the ionosphere and its discovery dates back to the late 19th and early 20th centuries.

1.2 THE DISCOVERY OF THE IONOSPHERE

As early as 1839 Gauss had speculated that the cause of variations in the earth's magnetic field was due to atmospheric electric
Fig. 1.1 Schematic representation of atmospheric regions (after Whitten & Poppoff, 1971)
currents. Balfour Stewart (1878) carried this theory further, calculating (incorrectly) a height of 5-10 miles for the layer containing the flowing electric currents. It was not however until the development of radio that conclusive evidence for the existence of the ionized region was obtained.

Early work by Marconi reported the transmission of radio electromagnetic waves over short distances. At the time, Marconi's major contribution was in the use of a ground antenna system as a radiating element coupled to a resonant oscillating source. However, in 1901 Marconi conducted one of the best known experiments in radio physics. On the 12 December of that year he succeeded in receiving in Newfoundland, Canada, a wireless signal transmitted from Cornwall, England.

Marconi's experiment conclusively proved that radio waves could travel around the curvature of the earth and prompted a considerable revival of interest in the electrical structure of the upper atmosphere. Working independently, Kennelly (1902) and Heaviside (1902) suggested that the radio signals must have been deviated by a conducting layer of ions at approximately 80 km altitude. This layer was known as the Kennelly-Heaviside layer for many years.

Theoretical work carried out by Nicholson (1910), Poincare (1904) and March (1912) failed to explain consistently the experimental results. Watson (1918) reformulated the integrals devised by March by means of a technique now called the Watson transformation. He again concluded that the waves decayed exponentially around the earth's curvature, the experimental evidence not withstanding.

In an important paper Watson (1919) further postulated a reflector of concentric plasma of high conductivity and reworked the
theory. With this model the electromagnetic waves did indeed decay at a rate comparable to that observed by the experimentalists. Watson's transformation technique has been the basis of most theoretical work at radio frequencies below about 300 kHz in subsequent years (e.g. Bremmer, 1949; Wait, 1957, 1960; Budden, 1961).

Experiments with a wave interference technique by Appleton and Barnett (1925) and a pulse sounding method by Breit and Tuve (1925, 1926) proved that a reflecting layer or layers existed and their approximate height was established. The latter technique (in which the time delay is measured between transmission of a pulse and the receipt of an echo from the reflecting layer) is used today for the exploration and monitoring of the ionosphere from ground stations. Appleton is credited with naming the reflecting layer, the E-layer.

In 1925 Appleton, and Nicholas and Schelleng drew attention to the importance of the earth's magnetic field in influencing the return of waves from the ionosphere. It was Appleton who proposed the name magneto-ionic theory to describe the propagation of electromagnetic waves through an ionized medium.

Between 1927 when Appleton published the outline of his theory and 1932 when he gave it in full, several other papers were produced on the subject (e.g. Hartree, 1929; Chapman, 1931). Although it was widely held that the ionosphere consisted of gas ionized by solar radiation, it was Chapman (1931) who first formulated a quantitative theory of the formation of ionospheric layers. His work still forms the basis of many contemporary ionospheric calculations.
1.3 STRUCTURE OF THE IONOSPHERE, AND VLF RADIO WAVES

The vertical structure of ionospheric electron density has been observed and studied for many years and is understood fairly well in terms of the physical and chemical processes at work there (e.g. Whitten & Poppoff, 1971). Typical vertical structures are illustrated in figure 1.2. The various layers were originally identified from ionograms originating from pulse sounding experiments which tend to emphasize the inflection of the electron density profiles. In fact, the layers are not necessarily separated by minima and are now referred to as regions;

- D-region, 50-95 km \( \text{Ne} \sim 10^0 - 10^4 \text{ cm}^{-3} \) by day
- E-region, 105-160 km \( \text{Ne} \sim 10^5 - 10^6 \text{ cm}^{-3} \) by day
- F-region, > 180 km \( \text{Ne} \sim 5 \times 10^6 \text{ cm}^{-3} \) at peak of day

For a detailed explanation of the structure of the ionosphere it is necessary to take account of the atmospheric composition, of the solar spectrum and of the detailed photochemistry. Much of this is beyond the limits of this present study. It is particularly difficult to determine the resultant electron density variation with height in the lower ionosphere in view of the complexity of the photochemical processes present. Studies have however indicated that the important ionizing solar radiations are those in the x-ray band \( (1 - 170 \text{ A}) \) and the extreme ultra-violet or EUV \( (170 - 1750 \text{ A}) \) as illustrated by figure 1.3. These emissions originate in the solar chromosphere and corona and some are greatly enhanced during solar flares. It is important therefore to understand that many changes and disturbances in the ionosphere are intimately linked to changes within the solar atmosphere.
Fig. 1.2 Typical vertical profiles of the mid-latitude ionosphere (from Hargreaves, 1979)

Fig. 1.3 The role of various solar emissions in the upper atmosphere (from Hargreaves, 1979)
A variety of techniques are available to sense the upper atmosphere and especially the ionosphere, a radio pulse method (ionosonde) having already been mentioned. Direct sensing usually consists of probes and other instruments carried on rockets as this part of the atmosphere is too high to be reached by balloon and generally too low to permit stable satellite orbits. The most important technique for observing the ionosphere is still by radio waves, the frequency of the radio signal utilized being dependent upon the altitude of the region to be studied. Of particular interest in this thesis are radio waves in the Very Low Frequency (VLF) band which lies between three and thirty kilohertz (100 - 10 km wavelength).

The propagation of VLF radio waves has been of considerable importance since the earliest days of radio. Radio waves at these frequencies are reflected from the lowest part of the ionosphere (the D-region) which is normally penetrated by High Frequency (HF) transmissions. Little attenuation of VLF signals occurs on reflection in the ionosphere and consequently they propagate to considerable distances around the earth. The major disadvantage of using these low frequencies is the very limited bandwidth available and the high cost and low efficiency of the transmitters. However, because of their inherent stability they are utilized for navigation, timing and frequency comparison systems. As a tool for sensing the ionosphere, VLF waves transmitted at steep incidence are known to be very sensitive to changes throughout the D-region.

This thesis is concerned with VLF waves propagating in the frequency range 10 - 14 kHz. Operating within this range is a VLF radio navigation system developed by the U.S. Navy, known as Omega.
Implicit in any study of such a VLF system is an understanding of the nature and behaviour of the D-region and the propagation characteristics of VLF waves. In particular, the diurnal and seasonal changes observed in the D-region electron density influence the phase and amplitude of VLF waves. The present investigation is concerned with the ionospheric propagation of VLF waves and the major objectives are outlined below.

1.4 AIMS OF THE PRESENT STUDY

In the Omega navigation system, position is determined by comparing the phase of the signals received from two pairs of VLF transmitters. The locus of geographic positions which give rise to the same phase difference with respect to two transmitters defines a closed curve on the earth's surface called a line of position (LOP). The intersection of two such LOPs fixes a navigator's position. However, changes in the D-region will influence the measured position and the extent of the resulting error will be a function of the deviation of the phase velocity of the VLF signals from its nominal assumed value (see Chapter 5). The Omega Navigation Systems Operations Detail (ONSOD) issue predictions of the expected LOP errors on a world wide basis based upon both physical and empirical modelling of the D-region. These predictions are available in simple tabular form which allows the navigator to correct his measured position and reduce the error involved.

This thesis describes the work undertaken on behalf of the Admiralty Compass Observatory (ACO) as part of an experimental and theoretical investigation of the performance of the Omega system in the United Kingdom area. Early trials by ACO revealed some limitations on the accuracy of the system which clearly indicated the need for
a better understanding of the propagation characteristics of the VLF waves.

To achieve this aim, monitoring at two fixed sites in the UK of various Omega transmitters was conducted by ACO. In addition to these facilities (located in Farnborough, England and on the Butt of Lewis, Scotland) a further, more sophisticated monitor was developed at Leicester. The arrangements of the various receiver sites, their equipment and a summary of data collected is presented in Chapter 6.

Data collection and reduction has formed a major part of the monitoring program. This analysis may be classified as follows:

(a) An examination of the phase and amplitude records collected at Leicester. This is the principal subject of Chapter 7 in which important characteristics of each of the observed propagation paths are discussed.

(b) Presentation of LOPs as a means of determining the effective day-to-day variability of the ionosphere and as a method of comparing the experimental data with the ONSOD predicted LOPs.

(c) LOP data has been combined in a manner later explained in Chapter 8 to obtain ground position fixes. These are compared with the known location of the receiver and the temporal and spatial differences and errors quantified. The same analysis is applied to data that has been corrected by the ONSOD method and the residual errors described.

(d) A comparison of the LOP variations recorded at each of the three UK receiver locations has been undertaken, the object being to quantify the similarity (or difference) in LOP at the various sites.
(e) A method of improving the performance of the Omega System by combining it with a satellite navigation aid is discussed. Recorded LOP data is analysed in a study to investigate the possible improvements from such a hybrid system.

A discussion of the results of studies (b) - (e) above is presented in Chapter 8, much of the data being illustrated in Appendix A and B. The conclusions drawn from this analysis require further explanation in terms of the differences in propagation conditions along the various paths involved and a modification of the ONSOD prediction technique. An attempt to model some of the features observed in the experimental data with existing analytical techniques described in Chapters 3 and 4 is presented in the final chapter of this study.
CHAPTER 2

The D-Region of the Ionosphere

2.1 INTRODUCTION

The D-region is that part of the earth's ionosphere lying below about 95 km altitude. This region is characterized by small ionization densities and high collision frequencies of electrons and ions with neutral molecules. During normal daytime conditions the concentration of free electrons increases nearly exponentially with height from about $1 \text{ cm}^{-3}$ at 50 km to $10^4 \text{ cm}^{-3}$ at 90 km, whereas the electron neutral collision frequency decreases from $10^8 \text{ s}^{-1}$ to about $10^5 \text{ s}^{-1}$ in the same height range. The electron densities in the D-region are large enough to reflect extremely low (ELF), very low (VLF) and low frequency (LF) radio waves. Because of the large collision frequencies medium and high frequency radio waves which penetrate the region are strongly absorbed. This chapter is concerned principally with a presentation of experimentally determined electron density profiles obtained for different times of day, season, latitude and period of solar cycle. It is important that the characteristics of the electron density, and collision frequency profiles are established since any theoretical modelling of VLF radio wave propagation through the D-region requires a knowledge of these features. A brief review of the nature of the processes that control the formation of the D-region is presented in Section 2.2. The concluding section of the chapter describes situations during which the normal 'quiet' structure of the region becomes disturbed. At such times abnormal radio propagation conditions may exist.
2.2 THE FORMATION AND CHEMISTRY OF THE D-REGION

2.2.1 Sources of ionization - daytime

Ion pair production rates for the quiet daytime D-region (solar zenith angle, \( \chi = 60^\circ \)) during minimum solar sunspot conditions are indicated in figure 2.1. It is evident that below 90 km the most important source of ionization is the photo-ionization of nitric oxide by Lyman-\( \alpha \) radiation at 1216 \( \AA \) (Nicolet and Aikin, 1960). This radiation is one of the strongest emission lines presented in solar UV emissions of wavelength 1000 - 1300 \( \AA \), originating principally from the solar photosphere. The production rate indicated in figure 2.1 for this source of ionization has been calculated for a radiation flux of \( 4.3 \text{ erg cm}^{-2} \text{ s}^{-1} \).

Donahue (1966) recognised the need for an additional source of \( \text{O}_2^+ \) ions from comparisons of its production and loss rates and observed concentrations. Hunter and McElroy (1968) proposed that this additional source could be provided by the photo-ionization of the metastable molecules \( \text{O}_2(1^\text{Ag}) \) by solar radiation (UV) in the range 1027-1118 \( \AA \). Results by Huffman et al (1971) are presented in figure 2.1. Also indicated in figure 2.1 is the total production rate curve derived from the results of Bourdeau et al (1966) for the C VI line at 33.7 \( \AA \), X-rays between 2-8 \( \AA \) and 40-75 \( \AA \), the C III line at 977 \( \AA \) and the Lyman-\( \beta \) radiation at 1026 \( \AA \). These sources become increasingly important in the upper D-region and lower E-region.

At heights below about 70 km cosmic rays provide the principal source of ionization. This is caused by secondary particles produced by the primary radiation, and the ionization rate increases with atmospheric density (Webber, 1962). The production rates, presented in figure 2.1, due to galactic cosmic rays are from Webber, 1962.
Fig. 2.1 Ionization rates in the quiet daytime D-region for solar minimum conditions and solar zenith angles near 60° (from Thomas, 1974)

Fig. 2.2 Ionization rates in the quiet nighttime D-region (from Thomas, 1974)
It is of interest to consider the possible changes in production rates during maximum solar sunspot conditions. This is of particular relevance here as the experimental data presented in subsequent chapters has been collected during a period approaching a sunspot maximum. The greatest change is expected to be in the contribution of $2.8 \times 10^9$ X-rays. Poppoff et al (1964) have presented results of rocket and satellite observations of X-ray spectra over a large part of the solar cycle and the corresponding photo-ionization rates as functions of height for an overhead sun. Their results indicate that the rates can vary by more than three orders of magnitude over the solar cycle. On this basis the production rates due to hard X-rays may approach those due to Lyman-α during solar maximum.

The variation of electron production by photo-ionization of NO or $O_2(\text{^1}A\text{g})$ during the solar cycle is difficult to assess. Weeks (1967) has suggested an increase in mean values from a radiation flux of $4.3 \text{ erg cm}^{-2} \text{ s}^{-1}$ used in figure 2.1 to about $6.1 \text{ erg cm}^{-2} \text{ s}^{-1}$. It would appear likely that the change in NO concentration could be more significant.

Webber (1962) has also given a value to the solar cycle variation of production rates due to cosmic rays; the values for solar maximum being reduced by a factor of two from those of figure 2.1.

2.2.2 Sources of ionization – night-time

Figure 2.2 presents the ionization rates in the quiet night-time D-region. The photo-ionization of NO by Lyman-α radiation in the night-glow has been demonstrated to be a likely source of ionization in the night-time D-region. The ionizing effects of precipitating high energy particles have also been considered to be important (Potemra and Zmuda, 1970). These results imply that the ionization by
corpuscular radiation might be even more important than that due to Lyman-α radiation. The possible contributions of galactic X-ray sources to the night-time D-region have also been examined following reports of changes in propagation characteristics associated with the transit of sources such as Scorpius XR-1 (Poppel and Whitten, 1969). As expected, the contribution due to cosmic ray ionization is assumed to be the same during night-time conditions as during the day.

2.2.3 Positive and negative ion chemistry

On the basis of the model proposed by Nicolet and Aikin (1960) it was expected that NO⁺ and O₂⁺ would be the major ions in the D-region. O₂⁺ is derived from the mesospheric molecular oxygen while NO⁺ is derived from photo-ionization of NO and in addition by ionization of molecular nitrogen to N⁺ which then undergoes rapid charge exchange with NO to form NO⁺ (Reid, 1973). Mass spectrometer observations (Narcisi and Bailey, 1965) have confirmed the predominance of these ions above about 82 km. This work has also demonstrated that below about 80 km the predominate positive ions are water cluster ions.

Mass spectrometer measurements of negative ion composition in the D-region have produced conflicting results and as a result our present knowledge of this part of D-region chemistry has been largely derived from laboratory measurements. Fehsenfeld et al (1967) have used such measurements to devise a model for the negative-ion chemistry of the D-region involving ions O⁻, O₂⁻, O₃⁻, CO₂⁻, NO₂⁻ and NO₃⁻. The importance of negative ions will be discussed further in Section 2.2.4, however a detailed discussion of this branch of D-region chemistry is beyond the scope of this study.

2.2.4 Electron attachment and detachment

The precise physical representation of the D-region requires many photo-chemical reactions and some uncertainties exist regarding
the reaction rates. For our present purpose it is more convenient to represent these processes by a 'bulk parameter' approach in which the global production and loss rates are considered rather than those of the individual constituents.

The continuity equation for electrons can be written, (Thomas, 1971):

\[
\frac{dN_e}{dt} = \sum \frac{q_i}{(1 + \lambda)} - \left( a_d + \lambda a_i \right) N_e^2 - \frac{N_e}{(1 + \lambda)} \frac{d\lambda}{dt} \quad (2.1)
\]

where \( q_i \) represents the ionization rate,

\( \lambda \) = ratio of negative ion to electron concentrations,

\( a_d \) = dissociative recombination coefficient for molecular ions and electrons,

\( a_i \) = recombination coefficient for positive and negative ions.

It is often assumed that equilibrium exists at certain times and if this is the case, equation (2.1) reduces to:

\[
\sum \frac{q_i}{N_e^2} = (1 + \lambda) (a_d + \lambda a_i) = \psi \quad (2.2)
\]

where \( \psi \) = effective loss coefficient for electrons.

In the D-region the electron-ion recombination coefficient \(( a_d )\) has a magnitude of about \( 10^{-13} \, m^3 \, s^{-1} \), appropriate to dissociative recombination and the ion-ion recombination coefficient \(( a_i )\) is of the same order. In parts of the ionosphere where \( \lambda \) is of the order of unity or greater, the ion-ion 'recombination' is thus significant.

Observations of electron concentrations during the sunrise period (Smith et al, 1966) indicate that prior to ground sunrise the greatest changes in electron density occur at heights below 80 km (geometric sunrise at 80 km altitude occurs when the solar zenith angle \( \chi = 99^0 \)). It is thought that during the night, electrons...
become attached to form negative ions and the changes during dawn are
due to photo-detachment from these negative ions. Doherty (1968) has
proposed that electron attachment and detachment processes involving
\( \text{O}_3 \) and \( \text{O} \) respectively can play a major role in explaining the diurnal,
seasonal and latitude variations observed in low frequency radio
propagation measurements. Although the dissociative attachment of
\( \text{O}_3 \) is now thought to be less important (Fehsenfeld and Ferguson, 1968)
the conclusions drawn are probably not altered.

Further details of electron attachment and detachment processes
will not be presented here. However, the effect of these processes
in determining electron density distributions and hence the propagation
characteristics of VLF radio waves will be made apparent in subsequent
parts of this study.

2.3 HEIGHT DISTRIBUTION OF ELECTRON DENSITIES (undisturbed conditions)

2.3.1 Experimental methods

Various methods exist for measuring the electron densities in
the D-region, however, because of the low electron concentration and
short mean free path, such measurements are difficult, scarce and
often unreliable. The ground based techniques capable of providing
directly the height distributions of electron concentrations make
use of the partial reflection of radio waves from D-region irregularities
(Belrose and Burke, 1964) or the cross modulation between two signals
propagating simultaneously (Barrington et al, 1963). In both of these
methods measurements are made over several minutes to minimise the
effects of noise. Other indirect ground based observations are made
with VLF and LF transmissions (Bracewell et al, 1951) or by vertical
incidence phase absorption (Appleton and Piggot, 1954). Partial
reflection and cross modulation techniques provide reliable results
in the upper regions of the D-region, while the indirect methods are sensitive to changes throughout the D-region. However, with the indirect methods determination of actual D-region electron density profiles involves an ambiguous iterative procedure (Deeks, 1966; Piggot and Thrane, 1966).

The most important and increasingly reliable source for D-region electron density height distributions are the rocket-borne techniques using either probes or radio propagation. However, even here the interpretation of data can be difficult as the mean free path is comparable with, or less than, the Debye length. Mechtly et al (1967) have devised a combined rocket experiment in which a d.c. Langmuir probe is operated simultaneously with Faraday rotation and differential absorption radio propagation experiments. The measurements of electron current collected at the probe indicate the detailed height distribution of electrons, while the propagation measurements provide a method of calibrating the current in terms of electron concentration.

2.3.2 Night-time distributions

Few reliable measurements of the height distribution of electron concentration at night are available. Three profiles derived by Deeks (1966) from an interpretation of VLF reflection data are presented in figure 2.3(a). Since VLF reflection is essentially from the first Fresnel zone the actual spatial resolution of this method is not good. The results however indicate a very rapid decrease in electron concentration just below 90 km during the summer under sunspot maximum conditions. During winter conditions a more developed D-layer at about 80 km is present with a prominent ledge at about 84 km during sunspot minimum.

Profiles by Mechtly and Smith (1968a) for a solar zenith angle (\(\chi\)) equal to 108° and Thomas and Harrison (1970) for \(\chi = 105°\) are
Fig. 2.3(a) Nighttime D-region electron density profiles (from Deeks, 1966)

Fig. 2.3(b) Nighttime D-region electron density profiles (from Mechtly & Smith, 1968a and Thomas & Harrison, 1970)
reproduced in figure 2.3(b). Both of these results show the presence of a distinct ledge of ionization between about 82 and 90 km.

2.3.3 Variations prior to ground sunrise

Early observations of VLF and LF transmissions indicate that pronounced changes occurred in the middle-latitude D-region prior to ground sunrise, beginning when the solar zenith angle was near 98° (Bracewell et al, 1951). Such changes have been confirmed by direct measurements from rocket borne experiments (Mechtly and Smith, 1968a) and by cross modulation measurements (Smith et al, 1967). Mechtly and Smith (1968a) reported five rocket flights carried out during the early morning hours. Their results (figure 2.4) indicate that major changes in electron density occur between $X = 96°$ and $X = 94°$. Furthermore, a marked C-layer, below the D-layer with a peak ionization at about 65 km altitude was established by the time $X = 85°$.

Distributions similar to that of figure 2.4 have been deduced by Thomas and Harrison (1970) from an interpretation of VLF and LF propagation data, and are illustrated in figure 2.5. Again a prominent C-layer at 65 km altitude develops with changing solar zenith angle.

2.3.4 Daytime variation

The variations in daytime electron concentration prior to noon determined by Deeks (1966) are reproduced in figure 2.6. They indicate a stable C-layer at 65 km altitude and a gradual increase in D-layer electron densities (75 to 85 km) from dawn to mid-day. Measurements by Mechtly and Smith (1970) for $X = 60°$ and $X = 18°$ are presented in figure 2.7 which also includes more of their measurements for larger solar zenith angles. These results are of interest as they indicate a C-layer almost totally obscured by the D-layer for $X = 60°$. This is to be contrasted with Deeks'
Fig. 2.4 Dawn electron density profiles (from Mechtly & Smith, 1968a)
Fig. 2.5 The height distributions of electron density deduced for night-time (N) and the period up to ground sunrise for Summer, high solar activity conditions at middle latitudes (from Thomas & Harrison, 1970)
Fig. 2.6 Local time variation of electron density at March equinox, sunspot minimum, and middle latitudes. (from Deeks, 1966)
Fig. 2.7 Diurnal electron density variations in the D-region (from Mechtly & Smith, 1970)
(1966) results of figure 2.6 where the lower layer is not fully hidden. However, in this situation seasonal effects may be important since Deeks' measurements are for March while those of Mechtly and Smith were recorded during July.

Measurements reported by Thrane (1969) from partial reflection experiments are presented in figure 2.8. The solar angle control is evident at all altitudes.

### 2.3.5 Seasonal variations

Mechtly and Smith (1968b) have investigated seasonal changes in D-region electron densities. Figure 2.9 illustrates results for five months during quiet solar activity. The summer day electron concentrations are larger than those for winter by a factor of about four, between altitudes of 65 and 85 km. A similar seasonal effect has been described by a comparison carried out by Gregory and Manson (1967) of electron densities measured with the partial reflection technique.

Results from Deeks (1966) presented in figure 2.10 also indicate increases in electron densities from winter to summer, however, there is little variation above 80 km.

### 2.3.6 Latitude variations

As yet there have been no complete and consistent studies or measurements of the latitudinal electron density behaviour. The few profiles which have been obtained at different latitudes for comparison at a nearly constant zenith angle show no general pattern of behaviour. Thrane et al (1968) have utilized the partial reflection and cross-modulation techniques in Europe and Africa. They have indicated that the mean distributions with \( \chi = 50^\circ \) were similar in shape and magnitude over the latitude range 69°N to 19°S. However, measurements
Fig. 2.8 Diurnal variation of D-region electron concentrations (N) at middle latitude from (a) measurements of partial reflections (b) measurements of cross modulation (from Thrane, 1969)
Fig. 2.9 A seasonal comparison of five International Quiet Sun Years (IQSY) electron density profiles, (from Mechtly & Smith, 1968b)
Fig. 2.10 Seasonal variation of electron concentration at local noon, middle latitudes and sunspot minimum conditions (from Deeks, 1966)
made in Canada (Belrose et al, 1966) have demonstrated that at high latitudes very marked differences are found in height distributions at similar zenith angles but different latitudes.

At geomagnetic latitudes between about 60\textdegree{}N and 80\textdegree{}N intense interaction between solar particles and the ionosphere takes place resulting in photo-luminescence, generally referred to as the aurora. In this zone major disturbances in the D-region electron density profiles frequently occur (see also Section 2.5.3(a)). During the day (ionospherically quiet conditions) the electron density tends to increase with height at roughly the same rate as at middle latitudes, although the details of this increase usually differ. In particular, the electron density profile for the auroral zone does not undergo as large a change with zenith angle as that for lower latitudes, especially at heights below 80 km. At night, the differences between the auroral latitudes and others are even more pronounced. While the profiles for the auroral region and mid-latitudes have roughly the same shape, the former are shifted downwards by about 10 km, so that the electron density is much larger in the auroral zone at night. In addition, the collision frequency (see Section 2.4) for the D-region appears to be slightly higher in the auroral region at all heights.

2.3.7 Solar cycle variation

A cyclic variation in the strength of UV and X-ray radiation from the sun has been well established. This variation, initially discovered from counts of sunspots, has a period of about eleven years and is called the solar cycle. Through the cycle the energy flux in the ultra-violet part of the spectrum increases by a factor of about two. Changes in X-ray flux are greater. Between wavelengths of 50 and 100\AA{} solar power output increases by a factor of about three.
from solar minimum to solar maximum and for wavelengths less than 10 Å the increase is of the order of 30. By contrast the power output of Lyman-α radiation changes by only 50% and that in the visible part of the spectrum hardly at all. These variations in solar radiation are known to change the undisturbed structure of the D-region. In addition, during solar maximum the number of solar disturbances, such as flares, are expected to increase.

A comparison of electron density profiles during solar minimum and solar maximum conditions has been carried out by Mechtly et al (1972). Data from five rocket experiments made during the minimum of 1964/65 have been averaged and compared with a similar set of data obtained during the maximum of 1968/69. The results are presented in figure 2.11. During solar maximum there appears to be an increase in electron concentrations between about 65 and 80 km compared with the solar minimum situation. Conversely, below about 60 km the solar minimum profile indicates a greater electron concentration than the solar maximum conditions. Above 80 km there is an overlap in the original data sets due to daily and seasonal variations, however, the mean profiles indicate an increased electron concentration during solar maximum conditions.

Changes in D-region structure during the solar cycle will only be of importance in this study in that all the experimental data to be presented subsequently has been obtained during a period of solar maximum and as a consequence the level of solar activity is relatively high.

2.4 D-REGION COLLISION FREQUENCIES

Important to any discussion of radio wave propagation through the ionosphere is the effect of collisions between free electrons and
Fig. 2.11 Median profiles representing the mean behavior of five solar cycle minimum and five solar cycle maximum electron density profiles ($\chi \sim 60^\circ$) (from Mechtly et al., 1972)
other constituents of the medium. The atmosphere at D-region heights is only weakly ionized, the ratio of the number density of free electrons to the number density of neutral particles being in general less than $10^{-7}$. Collisions of free electrons with neutral molecules will therefore be much more numerous than collisions with ions. It is the average frequency, $\bar{v}$, of such electron-neutral collisions which will determine the absorption a radio wave suffers when it passes through the D-region.

The free electrons of the D-region are produced by ionizing radiations or by detachment from negative ions as previously discussed. Their initial energies are considerably larger than the mean thermal energy of the neutral gas particles. Several authors (e.g. Belrose and Hewitt, 1964) have therefore suggested that the D-region electrons may not be in thermal equilibrium with the neutral gas. Dalgarno and Henry (1965) have considered the theoretical aspects of this problem and concluded that the fast electrons must cool very rapidly through inelastic collisions with heavy molecules. We can therefore assume that the D-region electrons have a Maxwellian velocity distribution characterized by the neutral gas temperature. Thus the mean collision frequency can be defined as follows:

$$\bar{v} = \int n \sigma(v) v f(v) dv$$  \hspace{1cm} (2.3)

where $n$ is the number density of neutral particles, $\sigma(v)$ is the collision cross section of electrons with velocity $v$ and $f(v)$ is the normalized Maxwell-Boltzmann distribution function. Integration is over all velocity space. It has become customary to work in terms, not of the mean collision frequency, $\bar{v}$, but of the collision frequency, $v_m$, of monoenergetic electrons with energy $kT$. 

The collision frequency of electrons in the D-region has been deduced from experiments made by rockets and from ground observations of reflected radio waves. In addition, laboratory measurements of collision cross sections of free electrons in air combined with measurements of temperature, pressure and composition of the air in the mesosphere have been made.

Laboratory measurements by Phelps and Pack (1961) have led to the following result:

\[ \nu_m = 6.34 \times 10^5 p \ \text{s}^{-1} \]  \hspace{1cm} (2.4)

where \( p \) is the total atmospheric pressure (N\text{m}^{-2}). Since the composition of the atmosphere in the D-region is essentially the same as for ground levels, estimates of the height variations of \( \nu_m \) may be obtained from the above equation in combination with profiles of atmospheric pressure against altitude.

Experimental values of collision frequency deduced from rocket and ground based techniques derived by Thrane and Piggott (1966) are presented in figure 2.12. The two experimentally determined curves of figure 2.12 are also compared with the collision frequency profiles deduced from the CIRA Standard Atmosphere 1965 and cross sections for collision by Phelps and Pack (1961). There is very close agreement between the profile derived by Thrane and Piggott (1966) for winter and spring and that from the CIRA Standard Atmosphere.

Thrane (1968) presents data illustrating the seasonal change of collision frequency at a latitude of 69°N. The best fit between collision frequency and pressure curves is obtained when:

\[ \nu_m = 8 \times 10^5 p \ \text{s}^{-1} \]  \hspace{1cm} (2.5)
Fig. 2.12 D-region collision frequency profiles

a - monoenergetic, from CIRA (1965) mean model atmosphere

b - Thrane & Piggott (1966), summer, monoenergetic

c - Thrane & Piggott (1966), winter, monoenergetic
Thrane concludes that the differences between this value and that obtained by laboratory measurements (equation 2.4) may be due to an underestimation of the collision cross section of electrons in oxygen. Also the fact that the observed collision frequencies are proportional to pressure over a wide height range, suggests that the adopted form of the velocity dependence of the electron cross section in air is correct. In addition, Thrane points out that the experimental values of $v_m$ display little scatter while Belrose et al (1967) have indicated that collision frequencies deduced from partial reflection measurements during winter in Ottawa show a day to day variability which can be greater than the seasonal change.

Recent rocket measurements by Dickinson et al (1976) of collision frequencies at 80 km altitude above South Uist, Scotland imply that, at D-region heights, the electron collision frequency is more nearly proportional to atmospheric density than to pressure, at least for seasonal variations. This is in contrast to the behaviour predicted from laboratory observations.

2.5 THE DISTURBED D-REGION

The lower ionosphere is sensitive to a number of geophysical disturbances of solar origin. There are many ways of classifying ionospheric disturbances, and in this section the classification by Rishbeth and Garriott (1969) will be adopted. Table 2.1 lists the disturbances included in this scheme.

2.5.1 Sudden ionospheric disturbances (SIDs)

The most familiar type of disturbance in the D-region is that associated with solar flares. This Sudden Ionospheric Disturbance or SID derives its name from its rapid effect on radio propagation conditions
<table>
<thead>
<tr>
<th>Disturbance</th>
<th>Propagation effects</th>
<th>Time and duration</th>
<th>Possible cause</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sudden Ionospheric Disturbance (SID)</td>
<td>In sunlit hemisphere, strong absorption, anomalous VLF-reflection, F-region effects</td>
<td>All effects start approx. simultaneously. Duration ∼ 1/2 hour</td>
<td>Enhanced solar x-ray and EUV flux from solar flare</td>
</tr>
<tr>
<td>Polar Cap Absorption (PCA)</td>
<td>Intense radiowave absorption in magnetic polar regions. Anomalous VLF reflection</td>
<td>Starts a few hours after flare. Duration one to several days</td>
<td>Solar protons 1-100 MeV</td>
</tr>
<tr>
<td>Magnetic storm</td>
<td>F-region effects; increase of foF2 during first day, then depressed foF2, with corresponding changes in MUF. E-region effects; storm Es D-region effects; enhanced absorption, VLF-anomalies</td>
<td>May last for days with strong daily variations</td>
<td>Interaction of solar low energy plasma with earth’s magnetic, causing energetic electron precipitation</td>
</tr>
<tr>
<td>Auroral Absorption (AA)</td>
<td>Enhanced absorption along auroral oval in areas of 100-1000km in extent. Sporadic E may give enhanced MUF.</td>
<td>Complicated phenomena lasting from hours to days.</td>
<td>Precipitating electrons with energies of a few tens of keV</td>
</tr>
<tr>
<td>Relativistic Electron Precipitation (REP)</td>
<td>Enhanced absorption, VLF anomalies at sub-auroral latitudes</td>
<td>Duration 1-2 hours</td>
<td>Precipitating electrons with energies of a few hundred keV</td>
</tr>
</tbody>
</table>

Other disturbances not detailed above - Travelling Ionospheric Disturbances (TID's), Winter anomaly, Stratospheric warming.

Table 2.1 Ionospheric disturbances (after Rishbeth & Garriott, 1969)
(Budden et al, 1939). It has since been shown that SIDs are caused by increased electron concentrations (Belrose and Centiner, 1962) produced by enhanced X-ray and UV emissions from solar flares. Satellite observations (Lindsay, 1963) have indicated no significant increases of solar Lyman-\(\alpha\) intensity during flares, but marked increases in hard X-ray flux have been reported. The changes in D-region ionization during such disturbances are produced by radiation in the 0.5 to 1027 Å range and extend over the sunlit hemisphere. Detailed studies of SID effects (Rowe et al, 1970; Montbriand and Belrose, 1972; Deshpande and Mitra, 1972) have established that the X-rays not only cause enhanced ion production, but that they also influence the ion chemistry and produce changes in the ionization loss rate. In figure 2.13 are presented electron density profiles measured during a flare of moderate strength (Montbriand and Belrose, 1972). The reaction time of the lower ionosphere is evident from figure 2.14. This diagram indicates the ionization rates at 80, 70 and 65 km and the electron densities for the same altitudes computed from the continuity equations and reaction rates listed by Potemra et al (1969). The peak electron density at 70 km lags the peak ionization rate by about 2 minutes and at 65 km by about 3 minutes. Recovery times to pre-flare electron densities vary with altitude, but are generally slower at lower altitudes.

2.5.2 Polar cap absorption events (PCA)

During certain flares the sun emits energetic particles, mainly protons, but also electrons, alpha particles and other positive ions of high atomic mass number. In the vicinity of the earth the charged particles are geomagnetically deflected to the high latitude regions.
Electron density versus height profiles for a large solar X-ray event on July 8, 1969 (from Montbriand & Belrose, 1972)

Fig. 2.14 Ionization rates and electron densities at 65, 70, and 80 km altitudes during the solar flares of August 1, 1967, (from Potemra, 1974)
where they produce ionization enhancements in the D-region below about 90 km. The observed absorption of HF (3-30 MHz) transmissions led to designating these events as Polar-Cap Absorptions or PCAs. The frequency of occurrence of PCA events varies from a few events per year during sunspot minimum conditions to one or more per month in sunspot maximum years. Events start a few hours after the associated flare and last from one to several days. During one of the strongest ever recorded PCA of August 1972, Regan (1977) has indicated that below 80 km the ion production rate is enhanced by a factor of $10^3$ to $10^4$ relative to quiet daytime conditions. From a knowledge of the proton energy spectrum the ion production profile may be computed but such events are also believed to have a profound influence on the chemistry of the D-region. Figure 2.15 illustrates typical electron density profiles for different D-region conditions ranging from undisturbed night to a PCA disturbed day (Larsen & Thrane, 1976).

2.5.3 Storm effects in the D-region

In the previous sub-section the effects of PCA events caused by solar protons were discussed. In addition to these events are a number of disturbances produced by electron precipitation. These depend strongly upon geomagnetic latitude and longitude and are characterized by a modification of the geomagnetic field called a geomagnetic storm. They may be divided into three groups as follows:

(a) **Auroral absorption**

The aurora itself is produced by electrons of energies 1-10 keV interacting with the atmospheric gases. During an auroral absorption event or substorm, enhanced absorption of HF waves occurs, caused by electrons with energies in excess of 10 keV penetrating into the D and lower E-regions. Rees (1969) has reviewed the characteristics of
Fig. 2.15 Typical electron density profiles for different D-region conditions ranging from undisturbed night to PCA (from Larsen & Thrane, 1976)

Fig. 2.16 Latitudinal flux profiles of the locally-trapped (HEES) and locally-precipitating (REES) electrons >130 keV before (upper panel) and after (lower panel) the magnetic storm of December 17, 1971 (from Larsen et al. 1977)
auroral electrons and computed ionization rates while Berkey et al (1974) have summarized the auroral absorption effects during sixty substorms.

(b) **Mid-latitude storm after effects**

At sub-auroral latitudes less dramatic ionospheric disturbances are often observed which have been correlated with geomagnetic activity and attributed to precipitating energetic electrons (Lauter and Knuth, 1967; Belrose and Thomas, 1968). The energetic electrons which are presumed to cause these mid-latitude events differ from the auroral electrons by virtue of the different magnetospheric processes which accelerate and precipitate them. From a summary of a number of satellite and rocket measurements Potemra and Zmuda (1970) have indicated the continual existence of precipitated electrons of energies in excess of 40 keV over the geomagnetic latitude range 40° to 60°. A continual build-up of electron flux is observed at magnetic B and L values appropriate to the South Atlantic anomaly (Imhof, 1968) and Zmuda (1966) has shown energetic particles to be a potentially important ionization source in this region. Simultaneous observations of particle spectra by satellites and ground based observations of electron density have been made in order to study the response of the lower ionosphere to the storm particles (Larsen et al, 1977).

Illustrated in figure 2.16 is the latitudinal variation of electron flux (\(> 130 \text{ keV}\)) before and after the storm, indicating the enhanced precipitation down to lower latitudes after the storm. Strong variability in electron loss rates over Ottawa in the height range 75-90 km have been observed during moderately disturbed conditions (Larsen et al, 1977). It must be concluded that even when a detailed particle spectrum is available and hence the ion production is known, the electron density profile cannot be predicted with reasonable accuracy.
(c) **Relativistic electron precipitation events (REP)**

Bailey and Pomerantz (1965) first noted a type of ionospheric disturbance affecting the very lowest part of the D-region causing various abnormal radio propagation conditions. The disturbances are closely correlated with substorms and occur at sub-auroral latitudes and are most frequent between 06:00 and 18:00 hours local time. Most events last from one to six hours and the monthly number may be as large as 25. It is clear that the effects are caused by precipitation of relativistic electrons (Matthews and Simons, 1973) with energies in excess of 500 keV. Thorne and Larsen (1976) conclude that substorm activity is a necessary condition for REP events but that not all substorms lead to the intense precipitation causing REPs. Daytime events are often delayed by several hours relative to the onset of a substorm, whereas night-time events are directly correlated with substorm activity.

### 2.6 SUMMARY

This chapter has been concerned with a description of the processes involved in the formation of the ionospheric D-region. A detailed presentation of experimentally determined electron density profiles, including their diurnal, seasonal and latitude variation has been made. Changes in D-region electron density are known to strongly influence the stability of propagating VLF radio waves, and therefore must form a fundamental part of any successful VLF phase (or amplitude) prediction model. Profiles illustrated above will be utilized in a comparison of available analytical VLF modelling techniques (Chapter 4) and in comparisons between theoretical and experimental results (Chapter 9).
In addition to the regular diurnal variations in electron density, disturbances, such as those described above (section 2.5), may radically alter the structure of the D-region. Such phenomena are known to have dramatic effects upon radio waves propagating through, or reflected from this layer. Examples of the influence of some of these disturbances at VLF are illustrated in subsequent parts of this study.
CHAPTER 5
The Theory of VLF Propagation

5.1 INTRODUCTION
In order to evaluate the performance of any radio system it is necessary to calculate the propagation characteristics of the signal. The analytical techniques described below enable the phase and amplitude of a propagating VLF wave to be determined as a function of distance for a given D-region model. In the theory of the long distance terrestrial propagation of VLF (and LF) signals it is customary to divide the paths of the waves into regions inside and outside the ionosphere and to treat these separately. Integration techniques enable the calculation of a reflection coefficient matrix for a realistic model ionosphere, i.e. anisotropic and vertically inhomogeneous. The model can be analytic or based on experimentally measured electron density and collision frequency profiles such as those presented in the previous chapter.

Propagation below the ionosphere is considered in terms of a spherical waveguide formed by the earth as one wall and the ionosphere as the other. Three analytical procedures are available for this region, viz: Waveguide Mode Theory, Spherical Wave Theory and Wave Hop Theory. All three methods are briefly described below following the derivation of the fundamental zonal harmonics series.

3.2 IONOSPHERIC PROPAGATION
In the following analysis the ionosphere is assumed to be a horizontally stratified cold magnetoionic plasma. At high and medium radio frequencies propagation through such a medium may be considered by solving Maxwell's electromagnetic equations approximately with analogy to geometric optics. At LF and VLF however the ionosphere
can vary considerably within the space of one wavelength (~30 km for a 10 kHz signal) as indicated in the previous chapter. Conventional ray optics break down in these circumstances, 'full wave' solutions (Budden, 1961a) become necessary, in which the wave fields are calculated at many points in the course of one wavelength.

The constitutive relations of the ionosphere, relating the electric polarization $P$ to the electric field $E$, are completely generalized to account for the collision frequency dependence on electron energy in the D-region (Sen & Wyller, 1960). This generalization is arrived at by a consideration of Boltzmann's transport equation. The susceptibility tensor, $M$, is found from the constitutive relations, and it, along with Maxwell's equations defines the differential equation governing radio wave propagation through the medium.

3.2.1 The constitutive relations of the ionosphere

Appleton (1932) and Hartree (1931) are credited with derivation of the expression for the refractive index of radio waves through a magnetoionic medium such as the ionosphere. Their technique, however, does not account for the dependence of the collision frequency on electron energy. Laboratory measurements (Huxley, 1959; Phelps & Pack, 1960) indicate that the collision frequency of monoenergetic electrons with nitrogen gas is directly proportional to the electron energy to a high degree of accuracy. This result is a consequence of the ease with which fast electrons can excite rotational states in nitrogen, so that significantly more collisions occur than had been previously expected. Consequently there is no single collision frequency that can rightfully be substituted into the Langevin equation of the so-called Appleton-Hartree theory. This equation is reproduced below, from a consideration of the motion of a free
electron lying in the path of a propagating radio wave through the ionosphere model described above

\[
\dot{m}v + v_e \frac{dv}{dt} = eE + e(v \times B_o) \tag{3.1}
\]

where \( m \) is the electron mass, \( e \) its charge, \( v \) the electron's velocity, \( v_e \) is the effective collision frequency, \( B_o \) the geomagnetic field and \( E \) the applied electric field. For the reasons outlined above this equation is not generally valid and the Boltzmann transport equation which statistically describes the state of a gas by the distribution function \( f(t, \mathbf{r}, v) \) must be employed. This function is defined so that the mean number density of particles, \( dN \), in the volume \( d\mathbf{r} \, dv \) is \( dN = f \, d\mathbf{r} \, dv \) where \( v \) is the particle velocity and \( \mathbf{r} \) is the position vector. The Boltzmann equation from which \( f(t, \mathbf{r}, v) \) is determined is

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f + \frac{e}{m} (E + v \times B_o) V_v f - S = 0 \tag{3.2}
\]

where

\[
V_v f = i \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z} \tag{3.3}
\]

\[
\nabla f = i \frac{\partial f}{\partial x} + j \frac{\partial f}{\partial y} + k \frac{\partial f}{\partial z} \tag{3.4}
\]

and

\[
S = \left( \frac{\partial f}{\partial t} \right)_{\text{collisions}} \tag{3.5}
\]

The term \( S \) represents the change in \( f \) due to electron-molecule encounters; because the gas is weakly ionized, electron-ion collisions are considered negligible. The following assumptions are also required: the equilibrium distribution of electrons is Maxwellian; the medium is homogeneous; spatial dispersion and non-linear effects due to strong electric fields are negligible. Noting that the electric polarization \( \mathbf{P} \) is related to the current density, \( \mathbf{J} \) by,

\[
\mathbf{J} = \frac{\partial \mathbf{P}}{\partial t} = i \omega \mathbf{P} \tag{3.6}
\]

it can be shown that the constitutive relations for the ionosphere may
be obtained in matrix form,

\[ P = \epsilon_0 M \equiv \mathbf{E} \quad (3.7) \]

where \( M \) is the susceptibility tensor, and \( \epsilon_0 \) is the permittivity of free space. In the co-ordinate system used by Pitteway (1965) and illustrated in figure 3.1, the elements of \( M \) are, (Deeks, 1966a)

\[
\begin{align*}
M_{11} &= \frac{1}{2} (M_{1} + M_{2}) \\
M_{12} &= -i/2 \cos \theta (M_{1} - M_{2}) \\
M_{13} &= -M_{12} \tan \theta \\
M_{21} &= -M_{12} \\
M_{22} &= \frac{1}{2} \cos^2 \theta (M_{1} + M_{2}) + \sin^2 \theta M_{3} \\
M_{23} &= -\sin \theta \cos \theta \left[ \frac{1}{2} (M_{1} + M_{2}) - M_{3} \right] \\
M_{31} &= -M_{13} \\
M_{32} &= M_{23} \\
M_{33} &= \frac{1}{2} \sin^2 \theta (M_{1} + M_{2}) + \cos^2 \theta M_{3}
\end{align*}
\]

where \( M_{n} = -\frac{X}{Z_m} (b_q + i a_q) \) for \( q = 1,2,3 \)

\[
\begin{align*}
X &= N \frac{e^2}{(\epsilon_0 \mu_0 \omega)^2} \\
Z_m &= \frac{\nu_m}{\omega} \\
\nu_m &= \nu (kT), \text{ collision frequency of monoenergetic electrons of energy } kT.
\end{align*}
\]

\( \theta \) = angle between magnetic field direction and direction that is vertically upwards (= 90° + dip angle)

\[
\begin{align*}
b_q &= \frac{5}{2} \left( \frac{c}{v_q} \right)(V_q) \\
a_q &= V_q \left( \frac{c}{2} \right)(V_q) \\
c_p (x) &= \frac{1}{p!} \int_0^x \frac{y^p \exp(-y) dy}{y^2 + x^2} \\
V_q &= \frac{\omega q}{\nu_m} \quad \text{for } q = 1,2,3 \\
\omega_1 &= \omega + \omega_H \quad (\omega_H = \text{angular gyro frequency } |eB| m)
\end{align*}
\]
Fig. 3.1 Ionospheric coordinate system (after Pitteway, 1965)

The incident wave normal has direction cosines, \( l = \sin I \sin \alpha \)
\( m = \sin I \cos \alpha \)
\( n = \cos I \)
\[
\frac{\omega_2}{\omega_3} = \frac{\omega - \omega_H}{\omega_H}
\]

3.2.2. The wave equations

Assuming an incident plane wave of form \(\exp(i\omega t - i \mathbf{k} \cdot \mathbf{r})\), combining equation (3.7) with Maxwell's equations,

\[
\begin{align*}
\nabla \times \mathbf{E} &= -ikz_0 \mathbf{H} \\
\nabla \times \mathbf{H} &= \left(\frac{\mathbf{k}}{\varepsilon_0} \right) = \frac{ik}{\varepsilon_0} (\varepsilon_0 \mathbf{E} + \mathbf{P})
\end{align*}
\]

where \(z_0\) is the characteristic impedance of the medium, yields the matrix equation,

\[
\frac{d}{dz} \mathbf{e} = -ik T \mathbf{e}
\]

where \(\mathbf{e}\) is the wave field vector,

\[
\mathbf{e} = \begin{bmatrix} E_x \\ -E_y \\ z_0 H_x \\ z_0 H_y \end{bmatrix}
\]

and \(T\) is given by (Deeks, 1966a),

\[
T = \begin{bmatrix}
-iB_1 & 1B_1 & 1mB_3 & 1-l^2B_3 \\
imB_1 & -mB_2 & 1-m^2B_3 & 1mB_3 \\
-lm + iB_4 & 1-l^2-B_6 & -mB_2 & lB_2 \\
1 - m^2B_5 & -lm-iB_4 & -imB_1 & ilB_4
\end{bmatrix}
\]

where

\[
\begin{align*}
B_1 &= -iM_{31}B_3 \\
B_2 &= M_{32}B_3 \\
B_3 &= 1 / (1 + M_{33}) \\
B_4 &= i \left( M_{21} - M_{23}M_{31}B_3 \right) \\
B_5 &= M_{13}M_{31}B_3 - M_{11} \\
B_6 &= M_{23}M_{32}B_3 - M_{22}
\end{align*}
\]
The terms $M_{ij}$ refer to elements of the susceptibility matrix, $M$, while the incident wave normal has direction cosines $(l,m,n)$. The time and horizontal space variation factor, $\exp(\omega t - ik(lx + my + nz))$ has been omitted.

To describe correctly the VLF wave fields equation (3.10) must be integrated numerically for a given electron density and collision frequency model of the ionosphere. Such a technique is described below.

### 3.2.3 Full wave integration

The numerical integration technique adopted for this study is that originally due to Pitteway (1965) but modified in order to take into account the energy dependence of collision frequencies as described above. The set of four linear differential equations (equation 3.10) are in a form suitable for direct numerical integration by computer. This process takes the values of the wave-field parameters $(E_x', E_y', H_x', H_y')$ at some specified height $z$, produces new values for a slightly different height and then repeats the procedure. The integration is started at a height where the electron density is large. The boundary condition to be satisfied here is that all the incident wave energy comes from below, so that there is no downgoing wave. Once the correct starting values have been obtained the main integration procedure is carried out down through the ionosphere to a level where the electron density is negligible. Here it is possible to separate the solution into an upgoing and downgoing part and to define a reflection coefficient matrix which consists of the ratios of the incident to reflected wave;
Here \( E_{\parallel}^{(I)} \) and \( E_{\perp}^{(I)} \) are the components of the incident electric field parallel and perpendicular to the plane of propagation respectively, and \( E_{\parallel}^{(R)} \) and \( E_{\perp}^{(R)} \) are defined similarly for the reflected wave. However, since the wave equations are linear and homogeneous the reflection coefficients for any particular incident polarization can be found by combining two different solutions. Thus it is necessary to calculate another solution of the wave equations, starting with a different polarization for the upgoing wave high in the ionosphere. Care must be taken during the numerical integration to ensure that the two solutions remain independent (see Pitteway (1965)).

### 3.3 THE TERRESTRIAL PROPAGATION OF VLF RADIO WAVES

The previous section was concerned with a theoretical treatment of the propagation of a radio wave through the ionosphere. Propagation below the ionosphere is usually considered in terms of a terrestrial waveguide formed by the ground as one wall and the ionosphere as the other. The reflection properties of the upper, ionospheric boundary have already been defined in terms of a reflection coefficient matrix (equation 3.13).

#### 3.3.1 Theory of propagation - isotropic model

The rigorous mathematical treatment of the propagation of a radio wave, from a Hertzian dipole source, around a finitely conducting spherical earth, surrounded by a concentric isotropic electron-ion plasma, can be expressed as a series of zonal harmonics. Solutions of this type, but with no concentric plasma, were obtained many years ago.
(e.g., Love, 1915). The introduction by Watson (1918, 1919) of the ionospheric boundary resulted in a new series whose summation, even at long wavelengths, was considered impractical. This series will be derived here, for the model presented in figure 3.2. Here the propagation media are characterized by their electrical constants which can be expressed in concise form as the wave number $k$.

For the region $a \leq r \leq c$, $k = k_1 = \frac{\omega}{c} \eta_1$ \hfill (3.14)

where $\eta_1 \approx 1.0001$ to 1.0003

The angular wave number of the ground (of conductivity $\sigma$, mhos/m) is,

$$k = k_2 = \frac{\omega}{c} \sqrt{\frac{\varepsilon_2 - i \sigma \mu c^2}{\omega}}$$ \hfill (3.15)

where $\varepsilon_2$ is the relative dielectric constant (permittivity, $\varepsilon = \varepsilon_2 \varepsilon_0$).

Also the angular wave number of the plasma is defined as,

$$k_3 = \frac{\omega}{c} \sqrt{1 - i \frac{\omega^2 N}{\omega (\nu + i \omega)}}$$ \hfill (3.16)

where the plasma frequency squared, $\omega_N^2 = \frac{Ne^2}{\varepsilon_0 m}$

and $\nu$ is the electron-neutral collision frequency.

The electrodynamic fields $E$ and $H$ are described by Maxwell's equations,

$$\nabla \times E + \mu_0 \frac{\partial H}{\partial t} = 0$$ \hfill (3.17)

$$\nabla \times H - \varepsilon_0 \frac{\partial E}{\partial t} = J$$ \hfill (3.18)

The fields are treated as continuous time harmonic waves, for example,

$$E = |E| \exp(i\omega t - ikD)$$ \hfill (3.19)

where $D$ is the distance from the source of frequency $f = \omega / 2\pi$.

A spherical co-ordinate system (figure 3.3), with unit vectors $\hat{r}$, $\hat{\theta}$ and $\hat{\phi}$, is introduced for convenience since the earth and
Fig. 3.2 Earth-ionosphere model for a terrestrial sphere of radius \( r = a \), surrounded by a concentric plasma from \( r = c \) to \( r = \infty \).

Fig. 3.3 General spherical coordinate system
ionosphere present approximately spherical boundaries.

There are six components of the electrodynamic fields in such a co-ordinate system (namely $E_r$, $E_\theta$, $E_\phi$, $H_r$, $H_\theta$, $H_\phi$) which can be calculated by a differentiation process from a single vector $\Pi$, called the Hertz vector, provided the scalar $\Pi$ satisfies the wave equation,

$$ (\nabla^2 + k^2) \Pi = 0 $$

where $-k^2 = \mu_0 i \omega (\epsilon i \omega + \sigma)$

The field components are (Hertz, 1889; Debye, 1909),

$E_r = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \sin \theta \partial \Pi^e \over \partial \theta \\ \frac{1}{rb} \partial^2 \Pi^e \over \partial \theta \partial \phi \\ \end{bmatrix}$ \quad (3.21)

$E_\theta = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \frac{1}{rb} \partial^2 \Pi^e \over \partial \phi \partial \theta \\ \end{bmatrix}$ \quad (3.22)

$H_\phi = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \frac{1}{rb} \frac{k^2}{\mu_0 i \omega} \partial \Pi^e \over \partial \theta \\ \end{bmatrix}$ \quad (3.23)

$E_\phi = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \frac{\mu_0 i \omega}{b} \partial \Pi^m \over \partial \theta \\ \end{bmatrix}$ \quad (3.24)

$H_r = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \frac{-1}{rb \sin \theta} \partial \Pi^m \over \partial \theta \\ \sin \theta \frac{\partial \Pi^m \over \partial \phi} \end{bmatrix}$ \quad (3.25)

$H_\theta = \begin{bmatrix} \exp(i\omega t) \end{bmatrix} \begin{bmatrix} \frac{1}{rb} \partial^2 \Pi^m \over \partial \phi \partial \theta \\ \end{bmatrix}$ \quad (3.26)

where $\Pi^m$ refers to vertical magnetic source dipole and $\Pi^e$ refers to vertical electric source dipole, provided the problem can be reduced to a two-dimensional form, i.e.

$$ \frac{\partial \Pi}{\partial \phi} = 0 $$

(3.27)

The model for the source or transmitter is specified by the primary Hertz vector (Johler & Berry, 1962),

$$ \Pi_o^e = I_o \frac{\mu_0 c}{4\pi} \frac{\exp\left[i\omega t - ik_1D\right]}{-ik_1D} $$

(3.28)

where the source dipole current moment, $I_o = \frac{4\pi}{\mu_0 c}$.
The solution of the wave equation (3.20), given the condition (3.27) depends upon the separation of the variables $\theta$ and $r$,

$$\Pi = f(r) F(\theta)$$  \hspace{1cm} (3.29)

It can be shown that, (Stratton, 1941),

$$F(\theta) = P_n(\cos \theta)$$  \hspace{1cm} (3.30)

where $P_n(\cos \theta)$ is the Legendre function.

Also since,

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[ r^2 \frac{\partial F}{\partial r} \right] = \frac{1}{r} \frac{\partial}{\partial r^2} \left[ r F \right]$$  \hspace{1cm} (3.31)

then,

$$\frac{d^2 \zeta}{d \xi^2} + \left[ \frac{1}{\xi} \frac{d}{d \xi} \right] \frac{d \zeta}{d \xi} + \left[ 1 - \frac{(n + \frac{1}{2})^2}{\xi^2} \right] \zeta = 0$$  \hspace{1cm} (3.32)

which is Bessels equation ($z = \sqrt{\xi} f(r)$, $\xi = kr$)

Thus the solution of equation (3.20) may be written as a series of zonal harmonics consisting of terms of the form,

$$\left[ \beta_n \Psi_n(kr) + \gamma_n \zeta_n^{(2)}(kr) \right] P_n(\cos \theta)$$  \hspace{1cm} (3.33)

where $\beta_n$ and $\gamma_n$ are constants and,

$$\Psi_n(z) = \sqrt{\frac{\pi z}{2}} J_{n+\frac{1}{2}}(z)$$  \hspace{1cm} (3.34)

$$\zeta_n^{(2)}(z) = \sqrt{\frac{\pi z}{2}} H_{n+\frac{1}{2}}^{(2)}(z)$$  \hspace{1cm} (3.35)

$J_{n+\frac{1}{2}}(z)$ and $H_{n+\frac{1}{2}}^{(2)}(z)$ are Bessel and Hankel (2nd type) functions of order $(n + \frac{1}{2})$ and argument $z$ (Watson, 1958; Abramowitz, 1972).

The $\zeta^{(2)}$ waves are outgoing and vanish at $r = \infty$ while the $\Psi$-waves are ingoing and vanish at $r = 0$.

In particular we may write, (Johler & Berry, 1962),

$$\Pi_o = \frac{1}{k_1^2 r b} \sum_{n=0}^{\infty} (2n+1) \zeta_n^{(2)}(k_1 b) \psi_n(k_1 r) P_n(\cos \theta)$$  \hspace{1cm} (r < b)  \hspace{1cm} (3.36)

$$\Pi_o = \frac{1}{k_1^2 r b} \sum_{n=0}^{\infty} (2n+1) \zeta_n^{(2)}(k_1 b) \psi_n(k_1 r) P_n(\cos \theta)$$  \hspace{1cm} (r > b)  \hspace{1cm} (3.37)
The secondary field is a result of the presence of the ground and ionosphere and hence a solution for the Hertz vector within the waveguide is,

$$\Pi_{\text{total}} = \Pi_{o} + \Pi_{1}$$  \hspace{1cm} (3.39)

The terms $b_{n}^{e}$ and $c_{n}^{e}$ in equation (3.38) are determined from the boundary conditions by equating the tangential E and H fields at $r = a$ and $r = c$. If the effects of the coupling in the region $a \leq r \leq c$ are neglected (Johler & Walters, 1960; Johler, 1961; Johler, Walters & Harper, 1960) these conditions may be expressed in matrix form,

$$\begin{pmatrix}
\frac{\psi_{n}(k_{1}a)}{1} & \zeta_{n}^{(2)}(k_{1}a) & \frac{\psi_{n}(k_{1}a)}{1} & 0 \\
-\frac{1}{k_{2}} \psi_{n}(k_{2}a) & 1 & \frac{\psi_{n}(k_{2}a)}{k_{2}} & 0 \\
0 & \frac{\zeta_{n}^{(2)}(k_{1}c)}{k_{1}} & \frac{\psi_{n}(k_{1}c)}{1} & \frac{\zeta_{n}^{(2)}(k_{2}c)}{1} \\
0 & \frac{1}{k_{1}} \zeta_{n}^{(2)}(k_{1}c) & \frac{1}{k_{1}} \psi_{n}(k_{1}c) & -\frac{1}{k_{2}} \zeta_{n}^{(2)}(k_{2}c)
\end{pmatrix}
\begin{pmatrix}
a_{n} \\
b_{n} \\
c_{n} \\
d_{n}
\end{pmatrix}
= \begin{pmatrix}
-\zeta_{n}^{(2)}(k_{1}b) \psi_{n}(k_{1}a) \\
-\frac{\zeta_{n}^{(2)}(k_{1}b)}{k_{1}} \psi_{n}(k_{1}a) \\
-\frac{\zeta_{n}^{(2)}(k_{1}c)}{k_{1}} \psi_{n}(k_{1}b) \\
-\frac{1}{k_{1}} \zeta_{n}^{(2)}(k_{1}c) \psi_{n}(k_{1}b)
\end{pmatrix}
$$

where $a_{n}$ and $d_{n}$ are constants (determined by boundary conditions) referring to the Hertz vector field within the earth and ionosphere respectively; also,

$$b_{n} = b_{n}^{e}, \quad c_{n} = c_{n}^{e} \quad \text{and} \quad \zeta_{n}'(z) = \frac{d}{dz} \left[ \zeta_{n}^{(2)}(z) \right]$$

Casting the solution in the form given by equation (3.39) and utilizing (5.21) leads to, (Johler & Berry, 1964),

$$E_{r} = \sum_{n=0}^{\infty} \frac{A_{n}}{k_{1}^{2}} \left[ \psi_{n}(k_{1}b) \psi_{n}(k_{1}r) + B_{n} r^{(2)}_{n}(k_{1}r) + C_{n} \psi_{n}(k_{1}r) \right]$$

$$\left\{ \begin{array}{ll}
\psi_{n}(k_{1}b) \zeta_{n}^{(2)}(k_{1}r) & \text{for} \quad r < b \\
\psi_{n}(k_{1}b) \zeta_{n}^{(2)}(k_{1}r) & \text{for} \quad r > b
\end{array} \right. \hspace{1cm} (3.41)$$
where \( A_n = \left[ \frac{n(n+1)(2n+1)P_n(\cos \theta)}{r^2 b^2} \right] \left[ \frac{I_0 \mu_0 \epsilon}{4\pi} \right] \)

Again \( B_n \) and \( C_n \) depend upon the boundary conditions set up in the matrix (3.40).

For the special case of \( b = a = r \) we obtain, (Johler, 1966 and 1970)

\[
E_r = \frac{I_0 \mu_0 c}{k_1^2 a^4} \sum_{n=0}^{\infty} n(n+1)(2n+1)P_n(\cos \theta) \zeta_{1a}^{(2)} \psi_{1a} \left[ 1 + \left( R_n, - \frac{\zeta_{1a}^{(2)}}{\psi_{1a}} \right) \right] \\
\times \left[ 1 + \left( T_n - \frac{\psi_{1a}}{\zeta_{1a}^{(2)}} \right) \right] \left[ 1 - \left( R_n, T_n \right) \right]^{-1} 
\]

(3.42)

where \( \zeta_{1a}^{(2)} = \zeta_n^{(2)}(k_1 a) \)
\( \psi_{1a} = \psi_n(k_1 a) \)

and the reflection coefficients \( R_n \) and \( T_n \) contain focusing or convergence-divergence factors,

\[
R_n = \begin{bmatrix}
- \zeta_{1a}^{(1)} \\
- \zeta_{1a}^{(2)} \\
\end{bmatrix} \begin{bmatrix}
R_n^S \\
\end{bmatrix} 
(3.43)
\]

\[
T_n = \begin{bmatrix}
- \zeta_{1c}^{(2)} \\
\zeta_{1c}^{(1)} \\
\end{bmatrix} \begin{bmatrix}
T_n^S \\
\end{bmatrix} 
(3.44)
\]

Here, the spherical reflection coefficients are defined,

\[
S_{R_n} = \begin{bmatrix}
\zeta_{1a}^{(1)} & -k_1 \\
\zeta_{1a}^{(2)} & k_2 \\
\end{bmatrix} \begin{bmatrix}
\zeta_{2a}^{(1)} \\
\zeta_{2a}^{(2)} \\
\end{bmatrix} 
\]

(3.45)

\[
S_{T_n} = \begin{bmatrix}
\zeta_{1c}^{(2)} & -k_1 \\
-\zeta_{1c}^{(1)} & k_2 \\
\end{bmatrix} \begin{bmatrix}
\zeta_{3c}^{(2)} \\
\zeta_{3c}^{(1)} \\
\end{bmatrix} 
\]

(3.46)
where \( \psi_n(z) = \frac{1}{2} \left[ \zeta_n^{(1)}(z) + \zeta_n^{(2)}(z) \right] \) (3.47)

Equation (3.42) represents the solution for the vertical electric field strength, \( E_r \), for the situation presented in figure 3.2.

3.3.2 Inhomogeneous, anisotropic ionosphere

The solution described in the previous subsection relates to the propagation of a radio wave below a simple model ionosphere. Attention is now turned to the generalization of this solution to the case of an inhomogeneous anisotropic ionosphere.

The anisotropic reflection coefficients for a variety of D-region electron density profiles can be derived by means of integration techniques such as that outlined in section 3.2. A consequence of the anisotropic ionosphere is that a pure transverse-magnetic (TM) source will generate waves propagating in both transverse magnetic (TM) and transverse electric (TE) modes in the waveguide. In this situation (Johler & Harper, 1962; Johler, 1962) the reflection coefficient \( R \) of equation (3.45) is replaced by \( R_e \) and \( R_m \), the vertical electric and vertical magnetic reflection coefficients. The coefficient \( T \) (equation 3.46) is also replaced by \( T_{ee} \), \( T_{em} \), \( T_{me} \), \( T_{mm} \) where \( T_{ee} \) is a vertical electric incident wave with a corresponding vertical electric reflected wave. \( T_{em} \) is the abnormal or vertical magnetic component resulting from the vertical electric field at the anisotropic boundary.

Similarly, \( T_{mm} \) refers to the vertical magnetic incident and reflected and \( T_{me} \) corresponds to an abnormal component. The coefficients \( T_{ee} \), \( T_{em} \), \( T_{me} \) and \( T_{mm} \) are denoted \( R_{||} \), \( R_\perp \), \( R_{||} \) and \( R_\perp \) respectively by Budden (1961) and others, and in equation (3.13).

Equation (3.42) becomes, (Johler & Berry, 1964),

\[
E_r = \frac{\mathbb{I} \mathbb{L}}{\mathbb{K}^2 \mathbb{K}^4} \frac{\mu_0 c}{8\pi} \sum_{n=0}^{\infty} F_n (2a)(a) (1+R_{e,n}) \frac{|I - \rho_n T_n|}{|I + \rho_n T_n|} \frac{|I + \rho_n T_n|}{|I - \rho_n T_n|} (3.48)
\]
where
\[ F_n = n(n + 1)(2n + 1) P_n(\cos \theta) \]

Unit matrix,
\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\] (3.49)

\[ R_e = \begin{bmatrix} R_e & 0 \\ 0 & -1 \end{bmatrix} \] (3.50)

\[
\mathbf{p} = \mathbf{P} \begin{bmatrix} 1 & 0 \\ 0 & -R_m \end{bmatrix}
\] (3.51)

\[
\mathbf{P} = \begin{bmatrix}
\zeta_{1a}^{(1)} \\
\zeta_{1a}^{(2)}
\end{bmatrix}
\begin{bmatrix}
\zeta_{1g}^{(1)} \\
\zeta_{1g}^{(2)}
\end{bmatrix}
\] (3.52)

\[
\mathbf{T} = \begin{bmatrix}
T_{ee} & T_{em} \\
T_{me} & T_{mm}
\end{bmatrix}
\] (3.53)

The series (3.48) is slowly convergent and may be used directly to calculate vertical electric field strengths. However, even at VLF it is necessary to sum the series over several thousand terms to reach a sufficiently accurate solution. Before the advent of digital computers such a summation was considered impossible and therefore further approximations were introduced to obtain a solution. Principally these involved a transformation developed by Watson (1919), and lead to two techniques called the Waveguide Mode and the Wave Hop theories respectively. A third technique, the Spherical Wave theory, essentially avoids the approximations involved in the Watson transformation, and relies on the summation of more slowly convergent series.

These three alternative solutions to the problem of the propagation of terrestrial VLF waves are briefly discussed below.

### 3.3.3 Waveguide Mode Theory

Watson (1919) proposed a representation of the zonal harmonics series as a contour integral essentially to aid computation. Thus,
\[ E_r = \left[ \frac{i}{\kappa_1 a} \right] \frac{F(v - \frac{1}{2})}{\cos \nu \pi} \sum_{s=0}^{\infty} \frac{\zeta_{v - \frac{1}{2}}(k_1 a)}{\cos \nu \pi} \left[ I^+ \left( \rho_{v - \frac{1}{2}} T e, v - \frac{1}{2} \right) \right] \left[ I^- \left( \rho_{v - \frac{1}{2}} T e, v - \frac{1}{2} \right) \right] \]

where \[ F(v - \frac{1}{2}) = 2v(v^2 - \frac{1}{4})P_{v - \frac{1}{2}}(-\cos \theta) \]

Stretching the contour \( C \), in a semicircle on the infinity of the right half of the complex \( \nu \)-plane, it can be concluded that the field can be represented as a series of residues or waveguide modes, \( s = 0, 1, 2, 3 \ldots \),

\[ E_r = \left[ \frac{\mu_0 c}{4\pi} \right] \frac{1}{i \nu_{s \frac{1}{2}}} \sum_{s=0}^{\infty} \frac{F(v - \frac{1}{2})}{\cos \nu \pi} \left[ \zeta_{v - \frac{1}{2}}(k_1 a) \right] \left[ \zeta_{v - \frac{1}{2}}(k_1 a) \right] \]

\[ \times \left[ I^+ \left( \rho_{v - \frac{1}{2}} T e, v - \frac{1}{2} \right) \right] \left[ I^- \left( \rho_{v - \frac{1}{2}} T e, v - \frac{1}{2} \right) \right] \]

It is necessary to locate the poles, \( \nu = \nu_s \), in the complex \( \nu \)-plane with the now well known modal equation,

\[ \left| I - \rho_{v - \frac{1}{2}} T e, v - \frac{1}{2} \right| = 0 \]

These equations (3.56) and (3.57) form the basic waveguide mode solution which has been found especially applicable at VLF and ELF. Further details can be found in Wait's \( (1962) \) treatise. The solution may be expressed in the following form,

\[ E_r \sim k \sum_{s=1}^{\infty} \left[ \Lambda_s \exp(-iv \theta) \right] \]
\( \Lambda_s \) is known as the 'excitation factor' of the s-mode and is essentially a measure of its magnitude upon launch from the transmitter. The real and imaginary parts of \( \nu_s \) represent the phase velocity and attenuation rate respectively. At frequencies of about 10 kHz only two modes of propagation are generally important. As the frequency increases it becomes necessary to include an increasing number of modes and solution of the modal equation becomes more and more tedious. Some of the problems encountered with the waveguide mode technique are discussed in Chapter 4.

### 3.3.4 Spherical wave-geometric series solution

As an alternative to the Waveguide Mode solution Johler (1964, 1966) and Berry (1964) formulate the expansion of the determinant ratio in equation (3.56) in the form of a geometric series,

\[
\frac{|I + \rho n T_n|}{|I - \rho n R_{e,n} T_n|} = |I + (I + R_{e,n}) \sum_{j=1}^{\infty} (\rho n R_{e,n} T_n)^{j-1} \rho n T_n| \quad (3.59)
\]

This leads to the spherical wave-geometric series solution, subsequently referred to simply as the Spherical Wave solution,

\[
E_r = E_{r,0} + \sum_{j=1}^{\infty} E_{r,j}
\]

(3.60)

where \( j = 0 \) is the ground wave and the \( j \)-terms, \( j = 1, 2, 3 \ldots \) are the ionospheric waves.

**ground wave**

\[
E_{r,0} = \left[ \frac{\mu c}{8 \pi} \frac{I l}{k_1^2 a^4} \right] \sum_{n=0}^{\infty} F(n) \left[ \begin{array}{c} (2) \\ 1a \\ (1) \\ 1a \\ (1 + R_n) \end{array} \right]
\]

(3.61)

\[
E_{r,j} = \left[ \frac{\mu c}{8 \pi} \frac{I l}{k_1^2 a^4} \right] \sum_{n=0}^{\infty} F(n) \left[ \begin{array}{c} (2) \\ 1a \\ (1) \\ 1a \\ (1 + R_n)^2 P C_j \end{array} \right]
\]

(3.62)
where \((p_n T_n R_{e,n})^{j-1} p_n T_n = \begin{bmatrix} c_j & x_j \\ y_j & z_j \end{bmatrix}\)

Then it can be readily shown that, (Johler, 1961),

\[
\begin{align*}
C_1 &= T_{ee} \\
C_2 &= R_{ee}^2 + R_{em}^m T_{ee} \\
C_3 &= 2R_{em}^m T_{ee}^2 + R_{ee}^2 + R_{em}^m T_{ee}^3 + R_{em}^m T_{ee}^2 \\
C_4 &= \text{......}
\end{align*}
\]

The geometric series representation permits the introduction of local reflection coefficients as depicted in figure 3.4. It should be noted here that this series representation is a wave solution and not a geometric-optical ray. The geometric-optical ray approach is approximate and only valid at short distances. The difference between the two methods is particularly important near the transmitter's radio horizon. In this region, the so-called 'caustic' radio waves arrive tangentially to the earth's surface and the geometric-optical approach predicts an infinite field strength (Bremmer, 1949). Beyond this range the field strength apparently falls to zero. This situation does not occur as diffraction effects become important at grazing angles of incidence. A study of the angles of incidence of the j-series waves of equation (3.62) leads to the interesting conclusion that local reflection regions at the ionosphere can be considered. Thus, as indicated in figure 3.4, the wave \(j = 1\) has a reflection region centered about the path midpoint and denoted \((1,1)\). A reflection coefficient, matrix \(T(1,1)\) is then ascribed to this region and in this manner an inhomogeneous earth-ionosphere duct may be modelled. The technique has become practical since the advent of fast digital computers. For example,
Fig. 3.4 Schematic representation of ionospheric waves, $j=1,2,3...$ using optical rays, illustrating local reflecting regions $(j,k)$, where $j=$ order of term of the geometric series and $k=$ order of the reflection region.

Fig. 3.5 Physical interpretation of wave hop paths in the region of the caustic. Downcoming waves ($a$ and $b$) excite ground waves ($a'$ and $b'$) which propagate into the shadow region.
at a radio frequency of 20 kHz a summation over ten j-terms may be

needed, each of which requires the n-series (equation 3.62) to
be summed over approximately 28,000 terms.

3.3.5 Wave Hop solution

Berry (1964) and Berry and Chrisman (1965) have developed
another computation technique which initially follows that of the
Spherical Wave approach. Equations (3.61) and (3.62) can be written
as contour integrals as in Watson's (1919) analysis,

\[ E_{r,0} = \frac{i}{k_1^2 a^4} \cdot \frac{\mu_0 c}{8\pi} \int_C \frac{F(u-\frac{1}{2})}{\cos \psi} \left[ \frac{(2)(k_1 a)}{u-\frac{1}{2}} \right] \left[ \frac{(1)(k_1 a)}{u-\frac{1}{2}} \right] (1+R_e)^2 d\psi \]

\[ E_{r,j} = \frac{i}{k_1^2 a^4} \cdot \frac{\mu_0 c}{8\pi} \int_C \frac{F(u-\frac{1}{2})}{\cos \psi} \left[ \frac{(2)(k_1 a)}{u-\frac{1}{2}} \right] \left[ \frac{(1)(k_1 a)}{u-\frac{1}{2}} \right] (1+R_e)^2 c_j d\psi \]

where \( R_e = R_e (\psi - \frac{1}{2}) \)
\( c_j = C_j (\psi - \frac{1}{2}) \)

These equations form the Wave-Hop method which has been found
especially applicable at LF and high VLF bands. In order to solve
the equations numerically it is convenient to divide the propagation
path into several regions and apply various approximate methods as
follows, (see also figure 3.5),

(i) Close to the transmitter - geometric-optics approximation
(ii) Beyond region (i) to a point near the caustic - saddle point
approximation.
(iii) At the caustic - numerical integration
(iv) Beyond the caustic - residue series.

At about 60 kHz this technique requires a summation over
about ten j-terms, however there is no n-series summation to be
carried out as in the case of the Spherical Wave approach. Further
discussion of the application of this method, in addition to those
outlined in the preceding subsections, is presented in the following
chapter.

3.4 SUMMARY

The theory of the propagation of VLF waves through and below
the ionosphere has been reviewed. A generalized magneto-ionic theory
coupled with full wave integration techniques leads to an expression
for the ionospheric reflection coefficient matrix (R). Propagation
within the spherical duct below the ionosphere and above the earth's
surface has been examined in terms of a series of zonal harmonics.
Application of the Watson(1919) transformation leads to the Waveguide
Mode solution. Expansion in terms of a geometric series yields the
Spherical Wave approach while a further application of the Watson
transformation gives the Wave Hop method.

Waveguide Mode theory has received much attention (e.g. Wait
& Spies, 1964) but is generally restricted to the VLF band. At
higher frequencies the number of propagating modes required to obtain
solutions of sufficient accuracy increase to the extent of rendering
the technique prohibitively cumbersome. Application of the theory
to situations involving transmitter and/or receiver elevated above
ground level requires the introduction of height gain functions
(Pappert et al., 1967). Further developments have been completed
(Pappert & Morfitt, 1975) to include the influence of discontinuous
waveguides such as those traversing a dawn or dusk line. Some of
the results of work in this area are presented in the next chapter.
The Wave Hop method has been applied to frequencies in the LF band where only a small number (typically ten) 'hops' are required to obtain sufficiently accurate solutions. It is not however in a form that easily enables the introduction of horizontally inhomogeneous waveguides. However, the Spherical Wave method is applicable in such situations (Johler, 1970) at both VLF and LF. In the final part of this thesis the Spherical Wave technique will be combined with experimentally measured D-region models in a study of the propagation characteristics of VLF waves through a number of inhomogeneous waveguides (Chapter 9).

Under conditions of high attenuation both the Wave Hop and Spherical Wave approach may lead to erroneous results unless a sufficient number of terms of the geometric series are accounted for. This is further discussed in the following chapter, where comparisons of the outputs from the three methods are presented for given ionospheric models and wave parameters.
CHAPTER 4

The Propagation Characteristics of VLF Radio Waves

4.1 INTRODUCTION

The development of highly stable atomic frequency standards has enabled the phase of VLF signals to be measured at great distances from the transmitter. Observations at fixed locations have provided information on the diurnal and seasonal variations of the D-region electron density, in particular, complex changes in phase and amplitude have been recorded at dawn and dusk. These variations have, to a certain extent, been explained by models of the D-region such as the Waveguide Mode approach, described in the previous chapter. Measurements over long paths (> 1000 km) have also proved to be a sensitive monitor of the stability of the lowest ionospheric layer.

Recordings of signal phase and total electric field strength, as a function of distance conducted along a radial from a VLF transmitter provide valuable spatial information on the influence of ground conductivity, geomagnetic field, near field effects and modal interference. Some of these geophysical effects are described in this chapter.

Most numerical treatments of VLF propagation have been concerned with the Waveguide Mode model. The early work assumed the earth to be perfectly conducting (e.g. Budden, 1961). While this is reasonable for propagation over sea water, it is certainly not valid for land paths and especially for propagation over ice. Wait and Spies (1964) have carried out many modal calculations for a variety of ionospheric models and ground conductivities and some of their results are presented below.
While a large number of Waveguide Mode calculations have been undertaken, attention has recently turned to the application of the Spherical Wave method. Work with the Wave Hop formulation has generally been restricted to LF. A comparative study, indicating the numerical equivalence of the three analytical approaches (Waveguide Mode, Spherical Wave and Wave Hop) is presented in the last part of this chapter.

4.2 PROPAGATION DURING UNDISTURBED CONDITIONS

The characteristics of propagating VLF waves under ionospherically quiet and undisturbed periods are now reviewed.

4.2.1 Experimental Studies

Early experimental measurements comparing the phase of the downcoming VLF sky wave with that of the direct ground wave by an interferometer method, have enabled accurate observations of the changes in the apparent height of the reflecting layer (Best et al., 1936). An example of these results, from Bracewell et al., 1931, is presented in figure 4.1 which indicates the diurnal phase change at 16 kHz at a distance of 90 km from the transmitter. During the day, the pattern is very regular and appears to have a weak solar zenith angle (\(\chi\)) dependence given by:

\[ h = h_0 + A(t) \log_\varepsilon (\sec \chi) \]  

(4.1)

where \(h\) and \(h_0\) are the apparent reflection heights at \(\chi^0\) and \(\chi = 0^0\) respectively. \(A(t)\) is a constant for a given day. Differences from winter to summer were observed in the magnitude of the diurnal change in apparent reflection height. Generally, at 16 kHz, the day to night variation was found to be largest during the summer months.

The variation of signal strength as a function of distance from a VLF transmitter was studied by Budden et al., 1939. The
Fig. 4.1 Change of phase of skywave (16kHz) with time of day (Bracewell et al., 1951)

Fig. 4.2 Variation of signal strength as a function of distance from an 85kHz transmitter (Bracewell et al., 1951)
technique adopted was similar to that of Hollingsworth (1926) and a series of maxima and minima were observed as indicated in figure 4.2 (Bracewell et al., 1951). The measurements suggested that beyond 500 km it is necessary to invoke a two layer model of the D-region.

Early investigations of VLF propagation to great distances were confined to measurements of amplitude only (e.g. Austin, 1929). Since the development of atomic frequency standards, many studies have been conducted of VLF phase variations over long paths (e.g. Burgess, 1964; Volland, 1964; Belrose, 1968). Very marked changes in the diurnal phase pattern with location have been recorded and summarised by Belrose (1968). Figure 4.2a illustrates some of these results, from which it is apparent that the greatest changes occur at sunrise and sunset. Variations at dawn and dusk were investigated by Reiker (1963), Crombie (1964), Walker (1965) and Ries (1967). A theoretical explanation of their observations is presented in section 4.2.5.

Further experimental work has been concerned principally with propagation in the polar regions. Blackband (1964) demonstrated some unusual high latitude features which are the result of long periods of dusk.

More recently, experimental studies of VLF propagation have been related to specific systems (e.g. navigation aids, frequency and time signals). For example, an investigation into the influence of the ionosphere on VLF propagation with particular reference to systems applications was undertaken by Burgess and Jones (1975). Anomalies in phase and amplitude of night-time signals propagating at 10.2 kHz and 13.6 kHz (Omega navigation frequencies) have been recorded by Steele and Diede (1977).

4.2.2 Theoretical Considerations

Analytical modelling of terrestrial VLF radio waves may conveniently be considered in terms of propagation within a concentric
Fig. 4.2(a) Diurnal variation in the phase of VLF waves observed at various places for paths of differing length (after Belrose, 1968)
waveguide formed between the earth and ionosphere. The mathematical development of this technique was presented in the previous chapter where the concept of propagating modes was introduced. If only one propagation mode is supported by the waveguide, then there will be a simple relationship between phase (or amplitude) and distance at a distance ($\geq 1000$ km) from a transmitter. In general however all resonant modes will propagate although with different characteristic phase velocities and attenuations.

The spherical waveguide model leads to several valid conclusions regarding the nature of the propagating VLF wave. For example, energy will propagate around the globe in all directions and may reinforce at the antipodal point from the transmitter. This phenomenon has been observed and illustrates the extreme range obtainable at VLF. Also, since the D-region controls the reflection of VLF waves, severe attenuation is expected when the wavelength is comparable with the waveguide height of 70 to 90 km. This situation is observed at a frequency of about 4 kHz. Another characteristic which has an important bearing on VLF systems is that, assuming a single dominant mode, then phase and amplitude should vary regularly as a function of distance without fluctuations due to interference between different modes. In practice, this condition is more likely to occur at frequencies near 10 kHz for distances greater than about 1000 km from the transmitter during daytime. This phenomenon is illustrated later in section 4.4 (figure 4.24) which also indicates that in the near field zone close to the transmitter many modes are present and large and rapid amplitude (and phase) variations are expected.

Many studies using the Waveguide Mode theory have been conducted by Wait, Budden, Galejs, Pappert and Synder (e.g. Wait, 1960). Over long paths with only one dominant mode, waveguide theory, assuming a
sharply bounded isotropic ionosphere, yields,

\[ E = \frac{3 \times 10^5}{h} \left( \frac{P \lambda}{a \sin(d/a')} \right)^{\frac{1}{2}} e^{-a' d} A \]  

(4.2)

where \( E \) = received field strength (\( \mu \) V m\(^{-1} \))
\( h \) = height of the ionosphere (km)
\( P \) = radiated power (kW)
\( \lambda \) = radio wavelength (km)
\( a \) = radius of the earth (km)
\( d \) = path length, i.e. great circle distance (km)
\( a' \) = attenuation coefficient (Napiers Mm)
\( A \) = excitation factor

Note that the field strength decreases exponentially with distance, modified by a 'focussing' term, \( \sin(d/a) \), to take into account the convergence of the spherical field in the far half of the earth.

Wait and Spies (1964) have made extensive computations to determine the expected characteristics of VLF propagation under a variety of conditions. Important parameters to be considered are propagating mode, characteristic attenuation, velocity and excitation. Theoretical assumptions include mode (n), effective ionospheric height (h), effective ionospheric gradient (\( \beta \)), ground conductivity (\( \sigma_g \)) and, in the case of anisotropic models, path orientation with respect to the earth's magnetic field. Other parameters include an ionospheric conductivity parameter related to the effective electron collision frequency.

Exponential electron density profiles developed by Wait and Spies (1964) assume,

\[ \omega_r = \frac{\omega_p}{\nu} = (2.5 \times 10^5) \exp \left[ \beta (z - h) \right] \sec^{-1} \]  

(4.3)
where \( \beta = 0.3 \text{ km}^{-1} \) for daytime profiles
= 0.5 km\(^{-1}\) for night-time profiles
z = height above the ground
\( \omega_p \) = plasma frequency
\( \nu \) = electron collision frequency
\( \omega_r \) = conductivity parameter
h = reference height

It is further assumed that:

\[
\nu = 1.816 \times 10^{11} \exp(-0.15z) \text{ sec}^{-1} \tag{4.4}
\]

Typical computed mode one attenuation rates (Wait and Spies, 1964) that would apply over seawater paths are indicated in figure 4.3. The \( h = 70 \text{ km} \) curve is approximately applicable during the day while that with \( h = 90 \text{ km} \) represents night conditions. Note that the minimum attenuation occurs around 16 kHz, increasing slowly for higher frequencies, but increasing more slowly below 16 kHz. Mode two results are presented in figure 4.4. At night the 10 kHz second mode attenuation rate is 9 dB/Mm compared with a first mode attenuation rate of 1 to 0.5 dB/Mm. However, at 20 kHz these rates are 3dB/Mm and 1.7dB/Mm respectively.

Mode one and mode two excitation factors are illustrated in figures 4.5 and 4.6 respectively. In both examples it is easier to excite the second mode, however the difference at 10 kHz is small (only 2.3 dB) compared with the 12.5 dB predicted for 20 kHz. The 10 kHz first mode will equal the second mode after 0.3 Mm but at 20 kHz this does not occur for nearly 10 Mm. Consequently, the interference pattern formed between the first and second modes is much less apparent at the lower VLF frequencies.
Fig. 4.3 Theoretical attenuation rates for the first mode (from Wait & Spies, 1964)

Fig. 4.4 Theoretical attenuation rates for the second mode (from Wait & Spies, 1964)

Fig. 4.5 Theoretical excitation factors for the first mode (from Wait & Spies, 1964)

Fig. 4.6 Theoretical excitation factors for the second mode (from Wait & Spies, 1964)
4.2.3 Effect of Ground Conductivity

An important parameter affecting propagation at VLF is ground conductivity. However, because the surface of the earth consists of many different substances, the conductivity ($\sigma$) and permittivity ($\varepsilon_r$) of the lower boundary of the waveguide is not uniform. The highest values of these parameters are for seawater as indicated in Table 4.1 (Watt, 1967). The ratio $\sigma/\omega\varepsilon$ indicates whether the physical mechanism underlying the propagation is due to conduction currents (ratio $\gg 1$) or displacement currents (ratio $\ll 1$). The data of Table 4.1 demonstrates that most of the waveguide's lower boundary will behave as a conducting, lossy medium at VLF. Polar regions however will behave as regions of poor conductivity.

Wait and Spies (1964) have calculated Waveguide Mode characteristics for a wide range of ground conductivities. Their results indicate that attenuation rates of the dominant modes, as a function of $\sigma$ have a maximum value which depends on frequency. Frequencies of the order of 10 kHz may be more adversely affected by very low conductivity than higher frequencies of, for example, 30 kHz.

When propagation occurs over land, higher attenuation rates are expected than for a seawater path. However, it is only over a large icecap that attenuation rates become excessive. This effect is illustrated in figure 4.7 which indicates changes in both signal strength and time delay at 13.6 kHz over the Greenland icecap (Burgess and Jones, 1975).

4.2.4 Influence of the Geomagnetic Field

In the presence of the earth's magnetic field, the ionosphere behaves as a bi-refrangent medium, and propagation is anisotropic, the phase velocity being a function of direction. The theoretical aspects of anisotropic propagation have been discussed in the previous
Table 4.1 Earth's surface electrical characteristics at VLF (Watt, 1967)

<table>
<thead>
<tr>
<th>$\sigma$ (mhos/m)</th>
<th>$\varepsilon/\varepsilon_0$</th>
<th>Type of surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 to 5</td>
<td>80</td>
<td>Sea-water</td>
</tr>
<tr>
<td>$10^{-2}$ to $3\times10^{-2}$</td>
<td>15 - 30</td>
<td>Rich damp soils, not heavily leached</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>10 - 20</td>
<td>Dry sandy soils or heavily leached areas</td>
</tr>
<tr>
<td>$10^{-4}$ to $10^{-3}$</td>
<td>5 - 10</td>
<td>Very thin soil, over rock</td>
</tr>
<tr>
<td>$10^{-3}$ to $3\times10^{-3}$</td>
<td>80</td>
<td>Fresh water, average lake</td>
</tr>
<tr>
<td>$\approx4\times10^{-5}$</td>
<td>10 - 20</td>
<td>Glacier ice, 0°C, 20 kHz</td>
</tr>
<tr>
<td>$\approx2\times10^{-5}$</td>
<td>4 - 8</td>
<td>Greenland ice, 20 kHz</td>
</tr>
</tbody>
</table>

(a) Time delay of 13.6 kHz relative to 10.2 kHz as a function of distance from the Omega Norway transmitter showing the effect of Greenland ice-cap.

(b) Signal strength at 13.6 kHz as a function of distance from the Omega Norway transmitter. Effect of Greenland ice-cap is shown.

Fig. 4.7 Influence of the Greenland ice-cap at 13.6 kHz (Burgess & Jones, 1975)
chapter. At VLF the influence of the geomagnetic field is to introduce a strong non-reciprocity between east-west and west-east propagation (Barber and Crombie, 1959; Hanselman et al., 1964).

Synder and Pappert (1969) have utilised a computer program based upon Budden's (1961a) formalism for calculating reflection coefficients (refer to Pappert, Gossard and Rothmuller, 1967). This method provides for a quite general ionospheric electron density profile, exponential variation of gyro frequency with height, magnetic dip angle, magnetic field strength, magnetic path orientation and ground conductivity. Specific calculations (Synder and Pappert, 1969) for the San Diego - Hawaii path (isotropic and anisotropic) were obtained. Attenuation rates and excitation factors for seven modes are indicated in figures 4.8 to 4.11. It can be seen that attenuation rates vary depending upon:

1. whether or not the model includes the geomagnetic field, and
2. whether the signals travel from east to west or vice-versa.

Specifically attenuation rates are greater on the east-west path. Theory also indicates that this effect is more prominent near the equator where the field direction is approximately perpendicular to the direction of propagation. At the magnetic equator it also appears that there is little dependence of the phase velocity on propagation direction. At slightly higher latitudes there is some evidence (Crombie, 1966) supported by theory (Galejs, 1967) that the difference in phase velocity of the first two modes is greater over west-east paths than in the opposite direction at night.

Contributions to the mode sum (Synder and Pappert, 1969) from the first few modes for three separate situations are presented in table 4.2. As previously mentioned, the isotropic night results from Wait and Spies (1964) indicate equality between first and second
Fig. 4.8 Attenuation versus frequency
Hawaii to San Diego path
Solid curves - anisotropic
Dashed curves - isotropic
(Snyder & Pappert, 1969)

Fig. 4.9 Attenuation versus frequency
San Diego to Hawaii path
Solid curves - anisotropic
Dashed curves - isotropic
(Snyder & Pappert, 1969)

Fig. 4.10 Magnitude of the excitation factor (db) versus frequency
Hawaii to San Diego path
Solid curves - anisotropic
Dashed curves - isotropic
(Snyder & Pappert, 1969)

Fig. 4.11 Magnitude of the excitation factor (db) versus frequency
San Diego to Hawaii path
Solid curves - anisotropic
Dashed curves - isotropic
(Snyder & Pappert, 1969)
<table>
<thead>
<tr>
<th>MODEL</th>
<th>FREQUENCY (kHz)</th>
<th>PROPAGATION CONSTANTS</th>
<th>RELATIVE FIELD STRENGTH (db) AT:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( n )</td>
<td>( \alpha ) (db/Mm)</td>
</tr>
<tr>
<td>Wait and Spies h = 90 km</td>
<td>10</td>
<td>1</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>8.8</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3.0</td>
</tr>
<tr>
<td>Snyder and Pappert San Diego - Hawaii Isotropic, ( h' = 84 ) km</td>
<td>10</td>
<td>1.2</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1.2</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3.7</td>
</tr>
<tr>
<td>Snyder and Pappert San Diego - Hawaii Anisotropic, ( h' = 84 ) km</td>
<td>10</td>
<td>2</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>11.5</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Table 4.2 Comparison of relative amplitudes of various modes (Swanson, 1976)
mode contributions at distances of 0.3 Mm for 10 kHz and 10 Mm for 20 kHz. Modal constants from Synder and Pappert (1969) for this particular model are in general agreement except that the second mode is more attenuated. With an anisotropic model, however, mode two has the lowest attenuation rate but is very weakly excited. At 10 kHz mode three is nearly equal to mode one at a distance of 1.5 Mm, but is rapidly dominated by the first mode at greater distances. Mode one, 20 kHz, is suppressed at all distances, but there is a change in dominance between modes two and three. As before, the propagation characteristics at the higher frequency appear more complicated, and hence predicting attenuation rates and phase velocities could be considerably more difficult than at a frequency nearer 10 kHz.

4.2.3 Mode Conversion

Idealised diurnal phase variations of VLF signals over long paths can generally be considered in terms of a trapezium (Pierce, 1955). In this situation day and night conditions correspond to two steady state phase levels separated at dawn and dusk by a smoothly changing signal phase. Deviations from the idealised transition pattern in the form of steps in phase accompanied by deep signal fading have been recorded by many observers, e.g. Pierce (1957); Crombie (1964); Walker (1965).

Crombie (1964) accounted for these experimental results (illustrated from Walker, 1965 in figure 4.12) by recognising that at sunrise the transmitter (which is in darkness) excites a first and second order mode. Because of the difference in the phase velocity of these modes an interference pattern is produced. At the sunrise line it was postulated that the two modes are converted into one first order mode whose amplitude is proportional to the sum of the two modes on the dark side of the terminator. At sunset the
Fig. 4.12 NBA 18.0kHz phase ($\phi$) and amplitude (A) records, 6th December 1963 (Walker, 1965)

Fig. 4.13 Ionospheric model for sunrise and sunset; west-to-east path (after Crombie, 1964 and Walker, 1965)

Fig. 4.14 Model for numerical simulation of terminator (after Pappert & Morfitt, 1975)
first order mode, in the daytime part of the waveguide, is converted into a first and second order mode, again producing an interference pattern. This phenomenon is schematically illustrated in figure 4.13. As the sunrise or sunset line moves along the propagation path the interference pattern causes the periodic amplitude and phase variations observed. Measurements made by Walker (1965) using a mobile shipboard monitor station fully supported Crombie's (1964) theoretical model of mode conversion.

Pappert and Synder (1972) have developed a mode-conversion program which allows for the vertical inhomogeneity of the ionosphere as well as its anisotropy. Essentially, their technique of modelling inhomogeneity in the direction of propagation involves dividing the waveguide by many vertical sections into shorter homogeneous guides as indicated in figure 4.14. It is necessary to calculate orthogonality conditions (Pappert and Smith, 1972) satisfied by the height gain function (see Chapter 3) governing the VLF fields within each short waveguide. From these conditions, mode conversion coefficients can be evaluated. Pappert and Synder (1972) assumed a 1000 km thick terminator with a linearly varying ionospheric scale height, $\beta$, and reflection height, $h$, and obtained good agreement with Walker's (1965) experimental data.

Discontinuities in the lower boundary are formed by changes in ground conductivity and/or variations in height above sea level (e.g. mountain ranges). Dobrott (1966) has derived and analysed the fundamental solutions for VLF waves propagating across a shoreline, but the method is very complicated and is applicable only to a limited number of situations. Chang (1969) utilising the same technique has considered propagation over a land-sea boundary, including the influences of homogeneous ionosphere and the earth's curvature.
Extending their earlier work, Wait and Spies (1970) have derived mode conversion coefficients for propagation over an elevated coastline and have indicated that the land elevation above sea level will modify the conversion coefficients, particularly when propagation is towards the sea.

Mobbs (1978), utilizing a variety of anisotropic ionospheric models, conducted a theoretical comparison with the experimental results illustrated in figure 4.7 for propagation over a sea/ice boundary. Subject to the suitable selection of D-region electron density profiles the analytical technique is capable of achieving close agreement with the experimental data.

4.3 PROPAGATION DURING IONOSPHERIC DISTURBANCES

The ionospheric D-region can undergo rapid and dramatic changes during geophysical disturbances such as those described in Chapter 2 (section 2.5). Generally these result in an enhancement of electron density within the region, which in turn produces changes in radio wave propagation conditions. Of particular relevance to this study are events such as Sudden Ionospheric Disturbances (SIDs), geomagnetic storms and polar cap absorption disturbances which are now described.

4.3.1 Sudden Ionospheric Disturbances (SIDs)

These disturbances are associated with x-ray bursts which occur during solar flares (see Chapter 2). Several changes are observed in radio propagation conditions during such events.

At VLF sudden phase anomalies (SPAs) and sudden field anomalies (SFAs) occur (see below). In the LF band sudden enhancements of atmospherics (SEAs) are recorded while at HF sudden frequency deviation (SFD), short wave fade out (SWF) and sudden cosmic noise absorption
(SCNA) may be observed. In addition, an intense SID may induce a simultaneous small change in the geomagnetic field, sometimes called a crochet. A summary of these phenomena is presented in figure 4.15.

Many experimental measurements of the influence of SIDs on VLF signal phase have been recorded (Burgess, 1964a; Chilton et al., 1964; Albee and Bates, 1965; Burgess and Jones, 1967; Potemra and Rosenberg, 1973). Reder and Westerlund (1967) report substantial phase changes but only small signal amplitude variation during SIDs. Solar flare effects have been interpreted by Chilton et al. (1963) in terms of a change in the reflection height for a sharply bounded homogeneous ionospheric model.

(a) Sudden phase anomalies (SPAs)

The enhancement of solar x-ray flux during a flare event produces an increase in electron density within the D-region (as indicated in figure 2.13) resulting in a lowering of the effective reflection height. Phase advances at VLF are observed as a consequence of this which gives rise to a sudden increase in phase velocity (Bates and Albee, 1965). Burgess and Jones (1967) have recorded changes in phase and amplitude of VLF waves propagated over a variety of distances during flare conditions. In general the phase of the received signal advances rapidly at onset (a few minutes) while recovery times are longer depending upon the magnitude of the x-ray event, solar zenith angle and latitude. Figure 4.16 illustrates the effect of a Class 2 (optical) flare on a 16 kHz VLF wave propagating from GBR to Nairobi (Burgess and Jones, 1967). A rapid phase advance of about 60° is recorded, followed by a slow recovery (1½ hours) to pre-flare conditions.

(b) Sudden field anomalies (SFAs)

Associated with the SPA, the total field intensity of VLF
**FLARE**

**ELECTROMAGNETIC RADIATION**
Delay time 0-3 min.
- UV and X ray
- Visible light
- Radio waves $\lambda \sim 3$ cm - 10 m

**COSMIC RAY PARTICLES**
Delay time 1/4 up to several hours
- Mostly protons
- Noise bursts recorded by radio telescope

**MAGNETIC STORM PARTICLES**
Delay time 20-40 hours
- Protons and electrons
- Polar cap absorption (PCA)
  - Cosmic ray increases (balloon heights - sea level)
  - Magnetic storms
  - Ionospheric storms

**SIMULTANEOUS EFFECTS**
- Occasional F layer increase.
- Frequent E layer increase.
- D layer increase (SWF, SID)
- Magnetic crochets
- SCNA
- SEA
- SPA

**DELAYED EFFECTS**

---

Fig. 4.15 The terrestrial effects of a solar flare (after Davies, 1965)
Fig. 4.16 Solar flare effect of October 26, 1963 at 10:50 UT as recorded for the GBR-Nairobi and GBR-ROME paths (16kHz; 7000 and 1500km respectively). Burgess & Jones, 1967

Fig. 4.17 Phase and amplitude variations at VLF during PCA for paths across the polar cap. The shaded areas indicated deviations from the normal diurnal variation (Larsen, 1977)
waves undergoes changes during an SID. Although many observations (Burgess and Jones, 1967; Crombie, 1965) indicate that often the amplitude of the received signal increases by a few decibels during a flare event, a certain percentage of SFAs result in decreases in amplitude. It has been suggested (Belrose, 1968) that these variations are the result of two separate changes which produce opposite results; firstly, an increase in electron density decreases the reflection height and increases the attenuation of the signal, secondly, an increase in electron density/height gradient improves the reflection of the wave.

4.3.2 Polar Cap Absorption

The energetic particle bombardment of the polar ionosphere during PCAs causes long lasting and large changes of plasma density particularly in the lower ionosphere (see Chapter 2).

In the lower frequency bands (VLF and LF) both phase and amplitude changes are observed over trans-polar paths (Westerlund et al, 1969). Amplitude changes, particularly for paths crossing the Greenland ice cap, seem to be erratic and difficult to predict, however the major effect on all paths is a phase advance due to the lowering of the effective reflection height.

VLF phase changes may be several cycles in magnitude with attenuation rates of 30 dB or more, as indicated in figure 4.17 (Larsen, 1976). Westerlund et al (1969) have investigated the phase shift $\Delta \phi$ of trans-polar VLF transmissions as a function of proton flux, $F$. Theory predicts a linear relationship between phase shift $\Delta \phi$ and $\ln(\ln F)$, which is confirmed by the experimental evidence.

4.3.3 Storm Effects

(a) Auroral Absorption

VLF radio propagation at auroral latitudes (60° to 75°) is
frequently disturbed by precipitated electrons. Egeland and Naustvik (1967) have correlated auroral absorption events with VLF disturbances. A reduction of signal strength and of diurnal phase changes are noted, particularly for frequencies below 12 kHz. Aurora caused phase fluctuations of 5-10 μsec duration and unstable signal amplitudes in observations carried out by Reder and Westerlund (1967). Egeland et al (1969) have discussed the main findings from VLF measurements during these events. The reflection layer is markedly depressed and at night large and rapid fluctuations in reflection height may occur simultaneously over distances from 500 to 1000 km.

(b) Mid-latitude storm after effects

During magnetic storms, anomalies are produced in both the phase and amplitude of VLF (and LF) waves propagated over both short and long paths. These events are of long duration, lasting up to ten days or more. Lauter and Knuth (1967) have noted that the sudden commencement of the geomagnetic storm does not generally affect the received signal strength. During the main period of the disturbance deep fading, known as the primary storm effect, occurs at night. For large storms high absorption levels are recorded three or four days after commencement; this is the so-called storm after effect. This situation continues for several days. Similar behaviour is noted at steep incidence (Belrose, 1968). Figure 4.18 illustrates the influence upon the diurnal pattern of phase for a 16 kHz signal steeply reflected from the ionosphere at middle latitudes (Belrose, 1968). The primary storm effect is evident during the night of 16-17 May 1956 while changes over the night of 19-20 May correspond to the storm after effect.

(c) Relativistic electron precipitation (REP)

Potemra and Rosenberg (1973) have demonstrated that long distance
Fig. 4.18 Diurnal phase changes during and after a geomagnetic storm (Belrose, 1968)

Fig. 4.19 Collection of data for periods of geomagnetic activity, December 16–23, 1971
(a) Integral fluxes of electrons (>130keV) precipitating at L=3.65 near the longitude of APL during daytime (open circles) and nighttime (solid circles). The passage of the 1971-089A satellite at L=3.65 on orbit 905 is marked.
(b) Diurnal phase variations at 16.0kHz for the GBR-APL path.
(c) Diurnal phase variations at 18.6kHz for the NLK-APL path.
(d) Kp values with times of sudden commencements (SC) indicated.

Larsen et al., 1977
Location of VLF transmitters - NKL = Seattle, Washington
GBR - Rugby, England
Receiver location - APL = Washington, D.C.
night time VLF transmissions can serve as a very sensitive detector of low intensity electron precipitation at middle latitudes. Helliwell et al (1973) have observed amplitude fluctuations in VLF waves propagated between Annopolis and Sao Paulo. These are thought to be due to particle precipitation into the South Atlantic geomagnetic anomaly during solar proton events. Westerlund and Reder (1973) have utilized VLF radio wave propagation to study the characteristics of electron precipitation in the auroral zone and have indicated a good correlation of VLF disturbances with optical aurora, geomagnetic disturbance and riometer absorption, especially at night. Larsen et al (1977) have described how VLF phase disturbances at mid-latitudes and energetic electron precipitation events occur following magnetic storms. Some of their results are presented in figure 4.19 which illustrates the disturbances on VLF phase over the GBR-APL (Washington D.C.) path at geomagnetic latitude, L = 3 to 4, and over the NKL (Seattle)-APL path at L = 3. Prolonged daytime and night-time phase advances are observed, which appears to indicate widespread precipitation at low L-shell values of nearly relativistic electrons which would be required to penetrate to altitudes below about 70 km.

4.4 NUMERICAL COMPARISON OF THEORETICAL TECHNIQUES

Three alternative methods are available for calculating the variation of phase and amplitude of VLF waves as a function of distance from the transmitter. A systematic comparison of these analyses has hitherto not been undertaken and it is the object of this section to present such a comparison.

In all but one computation (figure 4.22) the following parameters are assumed:
(a) A vertical electric source current moment of $I_0 = \frac{4\pi}{\mu_0}c$
    situated on the earth's surface.
(b) A vertically inhomogeneous, anisotropic concentric plasma.
(c) D-region electron density profile from Deeks (1966) for noon,
    summer, sunspot minimum conditions.
(d) Monoenergetic collision frequency profile based on the CIRA
    (1965) model atmosphere.
(e) Full wave calculations carried out with the Pitteway (1965)
    method.

4.4.1 Comparisons of Spherical Wave, Waveguide Mode and Wave Hop
    techniques at 16 kHz and 20 kHz

In figures 4.20 and 4.21 the results of calculations with
all three theoretical methods at 16 kHz and 20 kHz respectively are
presented. The model adopted is as given above, the geomagnetic
parameters being appropriate to northward propagation at U.K.
latitudes, and the ground parameters being those of sea water.

At 16 kHz (figure 4.20) all three techniques indicate a
large minima in vertical electric field strength at a range of
approximately 400 km from the transmitter. This minimum is the
result of strong destructive interference between the ground wave
and first hop (skywave, $j = 1$) propagation modes of the Spherical
Wave theory. Beyond this distance undulations in field strength as
a function of distance result from further interference between
higher order terms of the geometric series. Differences in the field
strengths determined by the three methods are less than 3 dB, except
in the region of the first deep minimum. Here the Wave Hop approach
predicts a deeper fade, the signal level being approximately $4\frac{1}{2}$ dB
below that calculated by the other two techniques.
Fig. 4.20 Comparison of Spherical wave, Waveguide mode, and Wave-hop techniques at 16 kHz
Frequency = 20kHz
Electron density profile - Deeks (1966), noon, summer, sunspot minimum
Collision frequency profile - mono-energetic, CIRA (1965)
Dip angle = 68°
Azimuth = 0°
Gyro frequency = 1.28 MHz
$\sigma = 4.64 \text{ mhos/m}$
$E_r = 81$

- Spherical wave
- Waveguide mode
- Wave-hop

Fig. 4.21 Comparison of Spherical wave, Waveguide mode, and Wave-hop techniques at 20kHz
The situation at 20 kHz (figure 4.21) differs from the 16 kHz results in that the first deep minimum is less pronounced and an almost equally deep fade is apparent at a distance of about 1600 km. Amplitude differences between the three analytical methods at 20 kHz are as large as 3 dB at some ranges from the transmitter.

4.4.2 Waveguide Mode and Spherical Wave Comparison at 19.8 kHz

A further comparative study between the Spherical Wave and Waveguide Mode formulations is presented in figure 4.22. Synder and Pappert (1969) using Wait and Spies (1964) exponential type electron density and collision frequency profiles have conducted a parametric study of VLF modes. A waveguide mode result by Synder and Pappert (1969) for southwards propagation, across sea, at a geomagnetic dip angle of 60° is reproduced in figure 4.22 (broken line). The radio frequency is 19.8 kHz and the exponential D-region electron density profile is parameterised by \( \beta = 0.5 \text{ km}^{-1} \), and \( h = 90 \text{ km} \). Note that the amplitude of the electric field is enhanced beyond 19,000 km, near the transmitter antipode due to the constructive interference of the two waves travelling around the earth in opposite directions.

A Spherical Wave calculation (conducted over \( j = 0 \) to \( j = 10 \) terms of equation 3.60) for the same ionospheric model and path parameters is indicated by the solid line of figure 4.22. Beyond about 6000 km the Spherical Wave method predicts slightly higher amplitude levels than the Waveguide mode approach, however in general the agreement is good over distances as great as 18,000 km (better than 4 dB, except in the region of the first minima). At the extreme range (> 18,000 km) the Spherical Wave skywave \( j \)-series summation (equation 3.62) requires more than ten terms for accuracy and is therefore not carried out here.
Frequency = 19.8 kHz
Exponential profiles, $\beta = 0.5 \text{ km}^{-1}$
$h' = 90 \text{ km}$
Dip angle = 60°
Azimuth = 180°
Gyro frequency = 1.25 MHz
$\sigma = 4.64 \text{ mhos/m}$, $\varepsilon_r = 81$

- Spherical wave
- Waveguide mode (Snyder & Pappert, 1969)

Fig. 4.22
Comparison of Spherical wave and Waveguide mode (Snyder & Pappert, 1969) techniques
4.4.3 Frequency Dependence Study with the Spherical Wave and Wave Hop methods

A comparison of Spherical Wave and Wave Hop formulations at several frequencies between 10 and 60 kHz is presented in figures 4.23 and 4.24. The model adopted is the same as that in section 4.4.1, but here propagation is towards the east. Figures 4.23(a) and (b) illustrate the nature of the changes in vertical electric field strength with distance for 10, 20, 30, 40, 50 and 60 kHz. For ranges greater than 1000 km the decrease in field strength at 10 kHz is almost exponential, however at 30 and 60 kHz deep fades are present. As the radio frequency is increased differences between the two theoretical methods become apparent. In general, amplitude minima occur slightly further away from the transmitter with the Wave Hop approach compared with those of the Spherical Wave theory. The depth of the minima are also different, particularly at the higher frequencies (e.g. 60 kHz). For frequencies in the 10 kHz range however the degree of agreement between the two techniques is excellent (better than $2^{1/2}$ dB anywhere over a 6000 km range). The reason for the discrepancies at higher VLF and LF is not at present understood.

Illustrated in figures 4.24(a) and (b) are the variations in phase (relative to a wave of phase velocity = c) as a function of distance and frequency. As expected, rapid phase variations are well correlated with the amplitude fades indicated above. Phase differences (in radians) between the two techniques are again more prominent at higher frequencies, however actual time ($\mu$ sec) differences are not; for example at the extreme range of 6000 km the time difference between the two theoretical approaches is 7 $\mu$ sec at 10 kHz but 4 $\mu$ sec at 60 kHz.
frequency = 10 kHz
electron density profile - Deeks (1966),
oon, summer, sunspot minimum.
collision frequency profile - monoenergetic,
from CIRA (1965) mean model atmosphere,
dip angle = 68°
azimuth = 90°
gyro frequency = 1.28 MHz
σ = 4 mhos/m, ξ = 80
- Spherical wave
- Wave-Hop

Fig. 4.23(a) Comparisons of the Spherical wave and Wave-Hop techniques at 10, 20 and 30 kHz
Fig. 4.23(b) Comparisons of the Spherical wave and Wave-Hop techniques at 40, 50 and 60 kHz
Fig. 4.24(a) Comparisons of the Spherical wave and Wave-Hop techniques at 10, 20 and 30 kHz
Fig. 4.24(b) Comparisons of the Spherical wave and Wave-Hop techniques at 40, 50 and 60 kHz
4.4.4 Comparative Study as a Function of Ground Conductivity

The changes in vertical electric field strength against distance predicted by the three analytical methods for three different ground conductivities are reproduced in figures 4.25a, b and c. The radio frequency is 13.6 kHz and propagation is towards the east. The results presented in figure 4.25a (sea, $\sigma = 5$ mhos/m) and figure 4.25b (land, $\sigma = 0.01$ mhos/m) indicate good agreement between the three methods, the largest discrepancies occurring in the region of the first deep minimum at a range of about 400 km.

Propagation over ice ($\sigma = 3 \times 10^{-5}$ mhos/m) is represented in figure 4.25c. The Spherical Wave method (solid line) and Waveguide Mode approaches are in reasonable agreement (better than 7 dB) until a range of about 2300 km is reached. At this distance, due to the large attenuation over ice, the Spherical Wave technique predicts the total vertical electric field strength to be of comparable magnitude to the amplitude of the $j = 10$ term of the summation (equation 3.62). As $j = 10$ was prescribed as the upper limit for this particular calculation, the Spherical Wave result breaks down beyond 2300 km. A similar effect is observed with the Wave Hop technique which was summed over nine hops. Here the break down point is at a range of 1800 km. This diagram illustrates the importance of including a large number of $j$-series terms ($> 10$) or hops ($> 9$) in situations of high attenuation.

4.4.5 Influence of the geomagnetic field (Spherical Wave method)

Finally, the effects of path geomagnetic azimuth and dip angle on a 10.2 kHz wave are presented in figures 4.26 and 4.27. Four azimuths (figure 4.26) are reproduced for a U.K. latitude, indicating propagation to the north, south, east and west. The non-reciprocity
frequency: 13.6 kHz
electron density profile - Deeks (1966),
noon, summer, sunspot minimum
collision frequency profile - mono-
energetic, CIRA (1965)
dip angle = 68°
azimuth = 90°
gyro frequency = 1.28 MHz
\( \sigma = 5 \) mhos/m
\( \xi_r = 80 \)
- - - - - - - - Spherical wave
- - - - - - - - Waveguide mode
- - - - - - - - Wave-Hop

Fig. 4.25(a) Comparison of Spherical wave, Waveguide mode, and Wave-
Hop techniques - Propagation over sea

frequency: 13.6 kHz
profiles & magnetic field as above
\( \sigma = .01 \) mhos/m
\( \xi_r = 15 \)
- - - - - - - - Spherical wave
- - - - - - - - Waveguide mode
- - - - - - - - Wave-Hop

Fig. 4.25(b) Comparison of Spherical wave, Waveguide mode, and Wave-
Hop techniques - Propagation over land
Fig. 4.25(c) Comparison of Spherical wave, Waveguide mode, and Wave-Hop techniques - Propagation over ice
Spherical wave technique

Frequency = 10.2 kHz
Electron density profile - Deeks (1966), noon, summer, sunspot minimum.
Collision frequency profile - monoenergetic, CIRA (1965)
Dip angle = 68°
Azimuth = 0°, 90°, 180°, 270°
Gyro frequency = 1.28 MHz
σ = 4 mhos/m
ε_e = 80

Fig. 4.26(a) VLF field strength against distance for paths orientated at 0°, 90°, 180° & 270° geomagnetic azimuth

Spherical wave technique

Model as above

Fig. 4.26(b) VLF phase against distance for paths orientated at 0°, 90°, 180° & 270° geomagnetic azimuth
**Spherical wave technique**

Frequency = 102 kHz

Electron density profile - Deeks (1966), noon, summer, sunspot minimum.

Collision frequency profile monoenergetic, CIRA (1965)

Dip angle = 10°, 30°, 60°, 80°

Azimuth = 90°

Gyro frequency = 0.94 to 1.5 MHz

σ = 4 mhos/m

ε_r = 80

---

**Fig. 4.27(a)** VLF field strength against distance for paths at 10°, 30°, 60° & 80° geomagnetic dip angle.

---

**Spherical wave technique**

Frequency = 10.2 kHz

Model as above

---

**Fig. 4.27(b)** VLF phase against distance for paths at 10°, 30°, 60° & 80° geomagnetic dip angle.
in east-west and west-east propagation is very pronounced in the amplitude data, the total vertical electric field strength changing by 5 dB at a range of 6000 km. A change in signal phase is also apparent (\(\sim 3\mu\) sec at 6000 km range). Similar changes in field strength and phase are predicted by the Waveguide Mode and Wave Hop formulations.

The influence of geomagnetic dip angle on a 10.2 kHz, eastwards propagating signal as calculated by the Spherical Wave technique is illustrated in figure 4.27. At the higher dip angles the total vertical electric field is attenuated at slightly greater rates; for example attenuation rates at a dip angle of 80° are about 1 dB per 1000 km greater than those at 10°. A small difference in phase is also predicted by the changes in dip angle.

### 4.5 CONCLUSIONS

This chapter has been concerned with a brief description of the propagation characteristics of VLF waves within the earth-ionosphere waveguide. An insight into many of the geophysical parameters influencing propagation in the VLF band may be gained through the Waveguide Mode technique. It is demonstrated that this method is equivalent, certainly at frequencies between 10 and 20 kHz with both the Spherical Wave and Wave Hop approaches.

During ionospheric disturbances a variety of changes are observed in the normal undisturbed diurnal patterns of VLF phase and amplitude. Prominent changes are produced at mid-latitudes as the result of solar flares and geomagnetic storms. At high latitudes polar cap absorption events can dramatically influence VLF propagation conditions.
Both experimental and theoretical aspects of D-region physical processes, morphology and terrestrial VLF propagation have been presented in Chapters 2 and 4. This review has been necessary in order to put into context the variations observed in experimental data to be presented in following chapters, and to provide a background for the modelling work to be undertaken.
5.1 INTRODUCTION

In this chapter a description of the configuration and operation of the Omega navigation system is presented. Details are given of system geometry, signal format, position fix determination, the construction of navigation charts and the elimination of lane ambiguities by multi-frequency operation.

Both temporal and spatial changes of the earth-ionosphere waveguide produce variations in received signal phase and hence position errors. A necessary requirement of the Omega system is a method of predicting the magnitude of these errors, and presenting the information in a form that a navigator can readily utilize. The prediction technique usually employed is discussed below in detail. In a later chapter the accuracy of this prediction method will be determined by comparing its output with experimentally measured Omega data.

5.2 SYSTEM CONFIGURATION

Omega is a radio navigation system employing Very Low Frequency (VLF) signals which provide positional information to users on a worldwide basis. Position is determined by a phase technique and the stability of VLF propagation to great distances via the ionosphere is therefore an essential requirement. Omega is generally considered to have its origin in a system proposed by J. A. Pierce in 1947 which utilized VLF phase differences rather than time differences. When fully implemented (by mid-1982) eight transmitting stations will be
deployed to provide coverage over the whole globe (refer to figure 5.1 and table 5.1).

The Omega primary navigational signal is transmitted at a frequency of 10.2 kHz at a power of 10 kW. Additional frequencies are also provided at 13.6 and 11.7 kHz. The signals from each transmitter are synchronized (by means of Caesium standards), unmodulated, continuous VLF waves, which in order to prevent signal interference are time-sequenced as in figure 5.2. Four unique transmission interval widths of 0.9, 1.0, 1.1 and 1.2 seconds are defined within a ten second time frame. Each Omega station thus transmits a unique sequence of signals within the time frame and it is this feature that enables conventional Omega receivers to identify the individual signal sources. Figure 5.2 indicates additional transmissions to the three navigational frequencies. These are primarily station identification frequencies and are not essential to the operation of Omega.

Outside the near field zone of each Omega station a stable signal pattern exists which is repeated in a radial direction every wavelength (approximately 30 km at 10.2 kHz). The cumulative phase is defined as the total intervening phase between a transmitter and an observation point, and is therefore approximately proportional to the corresponding great-circle distance. In practice the reference oscillator of an Omega receiver does not ordinarily provide the time epoch of the transmission bursts and hence only the fractional portion of the great circle distance in wavelength units is known. The remaining integral number of wavelengths must be determined by external means.

The system is customarily used in a hyperbolic mode, that is the phase difference between the signals received from two transmitters
<table>
<thead>
<tr>
<th>Station Letter</th>
<th>Location</th>
<th>Latitude / Longitude</th>
<th>Antenna type</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Aldra, Norway</td>
<td>66°25'N / 13°09'E</td>
<td>Valley span</td>
</tr>
<tr>
<td>B</td>
<td>Monrovia, Liberia</td>
<td>6°18'N / 10°40'W</td>
<td>Tower</td>
</tr>
<tr>
<td>C</td>
<td>Haiku, Hawaii</td>
<td>21°24'N / 157°49'W</td>
<td>Valley span</td>
</tr>
<tr>
<td>D</td>
<td>La Moure, North Dakota</td>
<td>46°21'N / 98°20'W</td>
<td>Tower</td>
</tr>
<tr>
<td>E</td>
<td>La Reunion</td>
<td>20°58'S / 55°17'E</td>
<td>Tower</td>
</tr>
<tr>
<td>F</td>
<td>Golfo Nuevo, Argentina</td>
<td>43°03'S / 65°11'W</td>
<td>Tower</td>
</tr>
<tr>
<td>G</td>
<td>Trinidad</td>
<td>10°42'N / 61°38'W</td>
<td>Tower</td>
</tr>
<tr>
<td>H</td>
<td>Tsushima, Japan</td>
<td>34°36N / 129°27'E</td>
<td>Tower</td>
</tr>
</tbody>
</table>

Table 5.1 Omega transmitting station locations

Proposed full format is indicated—
+ is unique tx frequency
11-05 is fourth navigation frequency
• Trinidad until 31/12/80

Fig. 5.2 Omega transmitted signal format (all frequencies in kHz)
is measured. The locus of geographic positions which give rise to the
same phase difference with respect to the two transmitters is a
closed curve (actually a 'spherical hyperbola') on the earth's
surface. The curves are analogous to the interference fringes set
up by two coherent light sources (figure 5.3) and are referred to as
Lines of Position (LOP's). A navigator can thus fix his position at
the intersection of two LOP's obtained from a minimum of three
transmitters. The distance between LOP's differing by $2\pi$ radians
in phase is termed a 'lane' (about 15 km for 10.2 kHz signals). In
general lanes become wide for points off the baseline joining two
transmitters, with lane boundaries near the transmitters diverging
faster than those equidistant from the transmitting stations.

Finding the intersection of two LOP's on the surface of a
non-spherical earth is generally intractable to all navigators except
those equipped with computers. As a result, charts have been
constructed for various areas of the globe which depict families of
LOP's associated with several transmitter pairs. Omega charts are
computed on the basis of an ellipsoidal earth (Fischer Spheroid of
1960) and as the LOP's are parameterized by phase difference rather
than distance difference the introduction of a phase velocity is
necessary. It is well known (Chapter 4) that the phase velocity of
a VLF wave is a function of frequency, ionospheric height, ground
conductivity, season, etc., and hence the assumption of any single
value will nearly always yield erroneous results. To minimize this
error, a nominal value is selected for constructing Omega charts
which is a rough average of (dominant mode) phase velocity over all
paths and time. The numerical value is given by,

$$v_0 = \frac{c}{0.9974}$$  \hspace{1cm} (5.1)
Fig. 5.3 Lines of Position (LOP's) formed by two radio transmitters A and B (the baseline is indicated by the broken line)
The charts usually depict lane boundaries, however the lane count or total number of cycles of phase difference with respect to a transmitter cannot be measured by a conventional receiver. For a user performing enroute navigation this difficulty can be overcome by initializing the receiver read-out at a known starting location, a ship's harbor for example. While enroute, the receiver will then automatically increment the lane read-out as each cycle of $2\pi$ radians is passed. Circumstances may however arise in which this procedure is not possible, for example if a one-off position fix is required or in the event of an equipment breakdown when the lane count is lost. For a navigator utilizing Omega in the hyperbolic mode and located near the intersection of two baselines, absence of lane information implies that the user must resolve his position by some other means within a 15 km x 15 km 'square'(assuming he is receiving the 10.2 kHz transmissions only). If such resolution is not achievable the additional navigation frequencies of 13.6 kHz and 11\(\frac{3}{4}\) kHz may be used as follows. Assume a highly idealized propagation model in which signal phase is isotropic for all geographic signal source locations. For this model it follows that,

$$\phi = \frac{s}{\lambda} = \frac{sf}{v_p}$$  \hspace{1cm} (5.2)

where $\phi$ is the cumulative phase, $s$ is the shortest great-circle distance between the signal source and receiver, $f$ and $\lambda$ are the frequency and wavelength respectively of the radiation and $v_p$ is the phase velocity (assumed to be frequency independent). The cumulative phase difference at 10.2 kHz for a receiver at great-circle distances $d_1$ and $d_2$ from two transmitters is,

$$\phi_{10.2}^{12} = \frac{10.2 \times 10^3}{v_0} (d_1 - d_2)$$  \hspace{1cm} (5.3)
Similarly, the phase difference at 13.6 kHz is,

\[ \phi^{13.6}_{12} = \frac{13.6 \times 10^3}{v_0} (d_1 - d_2) \]  \hspace{1cm} (5.4)

Hence, the difference in phase readings at these two frequencies is,

\[ \phi^{13.6}_{12} - \phi^{10.2}_{12} = \frac{3.4 \times 10^3}{v_0} (d_1 - d_2) = \phi^{3.4}_{12} \]  \hspace{1cm} (5.5)

Thus by differencing phase difference values at 10.2 kHz and 13.6 kHz, one obtains a number which is equivalent to the difference of two 3.4 kHz signals. The wavelength of a 3.4 kHz signal is approximately 90 km, and therefore the baseline lane width is about 45 km, i.e. three times the 10.2 kHz lane width. Since the phases of the two signals at the transmitter are the same, the lane boundaries for the 3.4 kHz signal coincide with every third 10.2 kHz lane boundary. Hence, if the uncertainty in the 10.2 kHz LOP is one or two lanes, this procedure may be applied to resolve the lane ambiguity. If we substitute the 11.5 kHz signal for the 13.6 kHz transmission (equation 5.4) a phase difference reading corresponding to a frequency of 112/15 kHz is obtained. The wavelength associated with this frequency (assuming a phase velocity near that of free space) is approximately 270 km and hence the baseline lane width is about 135 km, which essentially overcomes lane ambiguity problems.

5.3 THE SEMI-EMPIRICAL MODEL FOR PHASE VARIATION

The construction of Omega charts requires the assumption of a constant phase velocity (Section 5.2), however this parameter is sensitive to a wide range of geophysical effects (see Chapter 4). A navigator's position as determined by received phase information and charts alone will therefore not necessarily coincide with his true position. The magnitude of this error in position is related to the difference between the actual phase velocity and that given by equation
A method for predicting these errors has been developed by the Omega Navigation System Operations Detail (ONSOD) and this technique is discussed in detail below.

The ONSOD prediction program (Morris & Cha, 1974; Swanson & Brown, 1972) calculates the phase of the VLF signals from an Omega transmitter at a point anywhere on the earth's surface for anytime of day or season. The approach utilized differs from previous analytical methods, such as the Waveguide Mode Theory (refer to Chapter 3), in that it does not seek to relate analytically, observed radio wave characteristics (e.g. phase or amplitude) to a given ionospheric electron density/collision frequency model. Instead values of attenuation, phase velocity and mode excitation are assumed to be specified (via relevant geophysical functions) in terms of readily defined characteristics of the path (e.g. orientation, latitude, ground conductivity, diurnal and seasonal period). The required specification of relevant geophysical functions has been derived by both observational results and information generated by computer programs based upon full wave analysis and waveguide mode theory (for example, Pappert, Gossard & Rothmuller, 1967; Snyder & Pappert, 1969). It is the combination of experimental data with theoretical calculations that distinguish this so-called semi-empirical approach from purely analytical techniques. The relative contributions of the various geophysical effects have been determined by regression analysis on thousands of hours of data. The physical model has undergone continued refinement at the Naval Electronics Laboratory Center (NELC), San Diego, and since 1970 for limited areas has also received the benefit of a 'force fit' method (Calvo & Bortz, 1974).
The basic equation in the semi-empirical model has the following general form,

\[ \Delta \phi(p,x,t) = \sum_{i=1}^{n} \Delta \phi_i(p,x,t) \] (5.6)

where \( p \) is a given path, \( x \) is a point along the path, \( t \) is time and \( \Delta \phi \) is the total variation of cumulative phase from the nominal value along the path between the transmitter and the given receiver at a specified time. The term \( \Delta \phi_i \) represents the contribution to the phase variation (from the nominal) due to the \( i \)-th geophysical function. Thus a total of \( n \) geophysical functions are considered significant.

Since the geophysical functions and the signal phase do not vary excessively over a short spatial increment, each propagation path is divided into many short segments and equation (5.6) is applied to each segment. The total phase variation from the nominal value at a receiver position is then the sum over the \( \Delta \phi_i \)s for all segments between the transmitter and the given receiver.

At any given time the magnitude of the phase variation arising from diurnal effects varies from path segment to path segment depending upon the amount of illumination at each segment. Thus in determining the time component of phase variation to be included in the \( \Delta \phi_i \) terms for a particular segment, account must be taken of the variation as a function of solar zenith angle. In the ONSOD model, this has been accomplished by determining the phase variation assuming the sun is directly overhead and then applying an appropriate weighting function, \( W_i \), to correctly describe the zenith angle dependence. Thus the total phase variation from the nominal value for a given segment at a given time may be written as:

\[ \Delta \phi(p,s,t) = \sum_{i=1}^{n} W_i(A_i, \chi_i) \psi_i(p,s) \] (5.7)
where \( s \) denotes the particular segment and \( \chi \) the solar zenith angle measured at the segment. \( \psi_i(p,s) \) is the phase variation due to the \( i \)-th geophysical function assuming \( \chi = 0^\circ \). Each weighting function has a general form \( W_i(a_i, \chi) \). Regression analysis has been used to determine the set of \( a_i \)'s which give the best fit to the available experimental data. \( a_i \) is constant for all paths, geophysical locations and time.

5.4 PATH SEGMENTATION AND GEOPHYSICAL MODELS

Each propagation path is divided into a series of short segments. In the ONSOD program the angular extent of each segment is 0.01 radian \( (\approx \frac{1^\circ}{2}) \), and these are classified into groups or zones according to their proximity to the transmitter or receiver. Presented in figure 5.4 is a schematic representation of this path segmentation. Three zones are defined as follows:

(a) Excitation zone
(b) Mid-path zone
(c) De-excitation zone

The terms of the equations in the semi-empirical model are parameterized by three space-dependent quantities; ground conductivity, geomagnetic latitude (or equivalently, dip angle) and magnetic path bearing.

The equations for the phase variations in each of three path zones are given below:

Excitation zone

\[
\Delta \phi_e = \sum_{i=1}^{i_e} \sum_{j=1}^{j_e} \left[ K_{jk}^D + f_i \left( K_{jk}^N - K_{jk}^D \right) \right] h_{ji} \quad (5.8)
\]

Mid-path zone

\[
\Delta \phi_m = R \sum_{i=i_e}^{i_m} \sum_{j=i_e}^{j_m} \left[ K_{jk}^D + f_i \left( K_{jk}^N - K_{jk}^D \right) \right] h_{ji} \quad (5.9)
\]

De-excitation zone

\[
\Delta \phi_d = \sum_{i=i_d}^{i_N} \sum_{j=i_d}^{j_d} \left[ K_{jk}^D + f_i \left( K_{jk}^N - K_{jk}^D \right) \right] h_{ji} \quad (5.10)
\]
where, \( f_i \) = diurnal interpolation function evaluated at path segment \( i \)

\[ (f_i = 0 \text{ for normally illuminated day, } f_i = 1 \text{ for night}) \]

\( h_{ji} \) = geophysical function \( j \) evaluated at path segment \( i \),

\( i \) = general path segment index,

\( j \) = general geophysical function index,

\( i_d \) = index for the initial path segment of the de-excitation zone

(see figure 5.4),

\( i_e \) = index for the final path segment of the excitation zone

(see figure 5.4),

\( \{j_e\}, \{j_m\}, \{j_d\} = \text{set of indices indicating all geophysical functions to be evaluated in the excitation, mid-path and de-excitation zones respectively,} \]

\( K^D_{jk}, K^N_{jk} \) = day (D) and night (N) phase coefficients for geophysical function \( j \) and frequency \( k \) (determined by regression analysis),

\( N \) = total number of segments in the propagation path,

\( R \) = number of wavelengths at frequency \( k \) per radian distance along the propagation path,

\( \Delta \phi_e, \Delta \phi_m, \Delta \phi_d \) = total phase variation in the excitation, mid-path and de-excitation zones respectively.

The final output from the ONSOD prediction program is a value termed the predicted propagation correction (PFC). These are determined from the equation,

\[ CF = -100(\Delta \phi_e + \Delta \phi_m + \Delta \phi_d) \] (5.11)

where, \( CF \) is the value of the PFC in centicycles (i.e. 0.01 of a phase cycle, abbreviated "cec"). A negative sign is required since the correction is taken in the opposite sense to the predicted variation (i.e. to reduce the error between the
Fig. 5.4 Schematic of the propagation path indicating segmentation
measured values and the charted nominal values. A sample PFC table is presented in figure 5.5. Each table contains a set of 24-hourly corrections, each set being valid for a 15-day (half-monthly) period.

Further details of the geophysical models utilized in the various path zones are given below:

(a) Excitation zone and (b) De-excitation zone

These two regions are similar to the Fresnel zones of antenna theory and it is assumed that propagation in these regions is weakly affected by the presence of the ionosphere. The characteristics of the signals in these regions are however dependent upon the amount of energy transferred to (from) the dominant mode at the transmitting (receiving) antenna. This effect is described by the excitation factor mentioned in Chapter 3 and figure 5.6 indicates how the phase of this factor varies with ground conductivity. The shape of the curves differs, depending on whether the region is fully illuminated (day) or not illuminated (night). In either case however, the phase changes rapidly at low conductivities (0.1 mmho/m or less) but less rapidly at high conductivities (2 mmho/m or more).

The variation of the phase of the excitation factor with geomagnetic latitude and path bearing at night is indicated in figure 5.7. The east-west non-reciprocity mentioned in Chapter 4 is clearly evident in these curves. This effect is particularly large at low geomagnetic latitudes, but decreases as the latitude increases. Since the phase of the excitation factor only varies between 7° and 15° for any combination of geomagnetic latitude and path bearing, the corresponding curves for daytime conditions will not be presented.
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Fig. 5.5 A typical Omega propagation correction table
Fig. 5.6 Theoretical behavior of the Excitation factor phase as a function of ground conductivity and parameterized by frequency (after Gallenberger and Swanson, 1971)
Fig. 5.7 Theoretical behavior of the Excitation Factor Phase (10-2 kHz) as a function of bearing and parameterized by geomagnetic latitude (dip angle) for a non-illuminated path (Swanson & Brown, 1972)

Fig. 5.8 Theoretical behavior of the Relative Phase Velocity variation as a function of ground conductivity and parameterized by frequency for both normally illuminated and non-illuminated paths (Gallenberger, 1968)
(c) **Mid-path zone**

The mid-path region constitutes the area where propagation is strongly affected by the presence of the ionosphere. Signals propagating in this zone are assumed to consist essentially of a single dominant mode. It is important to distinguish the area between 60°N and 60°S geomagnetic latitude from those at higher latitudes, where auroral activity greatly affects VLF propagation. Presented in figure 5.8 is the variation in relative phase velocity as a function of ground conductivity for non-auroral propagation paths. Again, the largest and most rapid variations occur at low ground conductivities (regardless of frequency and time). The agreement between theoretical values of relative phase velocity and observational values determined from phase measurements is generally good for conductivities above 0.1 mmho/m as can be seen from figure 5.9 (the variations in this figure have been taken relative to the phase velocity for sea-water). For conductivities less than 0.1 mmho/m there is a sharp divergence of the theoretical curve from the experimental results.

The variations in relative phase velocity produced by the geomagnetic field are presented in figures 5.10 and 5.11 which are for day and night conditions respectively. Clearly phase velocity in the mid-path region (below the auroral zones) will be critically dependent on time, geomagnetic latitude and magnetic path bearing. The curves again indicate that propagation along east-west paths will differ considerably depending upon the direction of propagation.

As mentioned in Chapter 2 the D-region ionospheric electron density is greatly enhanced at latitudes where auroras occur. The shaded ring in figure 5.12 represents this region for the northern hemisphere. For paths extending into this region the relative
Fig. 5.9 Observed and theoretical behavior of the relative phase velocity variation as a function of ground conductivity for a 10.2 kHz signal (Swanson & Brown, 1972)
Fig. 5.10 & 5.11 Relative phase velocity variations vs. azimuth and parameterized by latitude for a 10.2 kHz signal. $h' = 86$ km, $\beta = 0.3$ km$^{-1}$, ground conductivity = 4.64 mho/m (Gallengerger & Swanson, 1971)
phase velocity varies with geomagnetic latitude as indicated in figure 5.13.

5.5 THE DIURNAL FUNCTIONS

Mathematically the diurnal functions are normalized interpolation functions which use the predicted day and night phase variations to give the required variation at any given intermediate time. They have been derived from a simple model relating electron density at a particular ionospheric height to time (Swanson and Bradford, 1971). The model indicates that rapid changes occur at sunrise and sunset while slow variations occur during the day. Electron density changes are associated with an ionospheric time constant related to the effective attachment coefficient (Davis, 1965). The form of the equations of continuity (Swanson and Bradford, 1971) are in general simpler than those presented in Chapter 2 (equations 2.1 and 2.2). Variations in electron density with time are assumed to be related to changes in ionospheric height resulting in a change in the phase of the propagated signal, thus:

\[
\frac{d\phi}{dt} = \frac{S_{\text{eff}} - \phi}{\tau}
\]

(5.12)

where \( \phi \) = phase (a function of time),

\( S_{\text{eff}} \) = effective source function, related to solar flux,

\( \tau \) = ionospheric time constant, reciprocally related to the effective attachment coefficient.

A normalized diurnal phase function \( g(t) \) is defined as:

\[
g(t) = \frac{\phi(t) - \phi_o}{\phi_N - \phi_o}
\]

(5.13)

where \( \phi(t) \) = phase observed at time \( t \),

\( \phi_o \) = phase which would be observed if the entire path were normally illuminated (i.e. \( \chi = 0^\circ \)).
Indicated latitudes are relative to magnetic dipole axis.

\[ a = \text{geographic north pole} \]
\[ b = \text{magnetic dipole axis} \]
\[ c = \text{magnetic north pole} \]

Fig. 5.12 Effective northern hemisphere auroral zone

Fig. 5.13 Relative phase velocity variation in the auroral zone (Rothmuller, 1968)
\( \phi_N \) = phase which would be observed if no portion of the path were illuminated.

In order to develop a differential expression for \( g(t) \), let \( f(t) \) be the diurnal function which would be observed if the ionosphere reacted instantaneously to changes in solar flux. If \( f(t) \) is identified as being proportional to \( S_{eff} \), equation (5.12) may be written using equation (5.13) as:

\[
\frac{dg(t)}{dt} = \frac{f(t)}{\tau} - \frac{g(t)}{\tau} + \frac{D(t)}{\tau} \tag{5.14}
\]

where the forcing function \( D(t) \) has been added to account for the fact that phase behaviour has been observed at the onset of sunrise or sunset which cannot be explained by the function obtained from the instantaneous solar response function \( f \) and the time constant \( \tau \). This anomalous behaviour is assumed to occur as weakly attached electrons are suddenly 'dumped' into the ionosphere (see also Section 2.2.4).

Both the function \( f \) and the anomalous 'dump schedules' have been experimentally determined and are parameterized as functions of \( \cos(\chi) \) where \( \chi \) is the solar zenith angle. The resulting linearized diurnal function, \( f \) is presented in figure 5.14. The sunrise and sunset dump schedules (figure 5.15) are given in terms of incremental changes to the diurnal function \( g \). In the ONSOD prediction program \( g(t) \) is calculated from equation (5.14) with the following assumptions:

(a) \( \tau \) varies with time but is invariant over a small interval \( \Delta t \).

(b) \( f(t) \) changes linearly over the integration interval.

(c) the integration interval \( \Delta t \) is of the order of one hour.

5.6 SUMMARY

A general description of the Omega navigation system has been presented and the need for a phase prediction program emphasized. The
Normally illuminated day

Fig. 5.14  Linearized diurnal function, $f$
(Morris & Cha, 1974)

Fig. 5.15  Dump schedules (Swanson & Bradford, 1971)
model currently in use by users of Omega has been described in some
detail with particular reference to path segmentation, geophysical
functions and diurnal functions. The semi-empirical approach to VLF
phase prediction adopted for the Omega system has considerable economic
advantages over traditional, purely analytical techniques such as those
discussed in Chapter 3. Essentially the data input required consists
of the specification of a propagation path (spatial and temporal) and
a set of regression coefficients. A major portion of the computer
program consists of book-keeping tasks and the summing of the various
functions applied to each path segment. A necessary part of the
development of such a model however, is a data base of experimentally
measured phase information and a theoretical analysis to provide guidance
on the separation of variables and to suggest appropriate geophysical
functional forms.

In the following chapters measurements of phase and lines of
position derived from several Omega transmitters, at three locations
in the U.K. will be discussed. These data will then be compared with
the ONSOD predicted phase and LOPs for the particular propagation paths
concerned and an assessment of the accuracy of the predictions made.
6.1 INTRODUCTION

In the previous chapter a description of the existing VLF Omega navigation system was presented. A theoretical treatment of phase prediction was outlined and this forms at present the only well established means of providing corrections for the changes in VLF/Omega phase velocity over a particular propagation path. A very necessary part in the assessment of the performance of the system is therefore a comparison of theoretically predicted diurnal and seasonal, phase and amplitude changes with the corresponding values of these parameters determined experimentally. The description of such a comparison carried out in the U.K. in conjunction with the Admiralty Compass Observatory (A.C.O.) forms the basis of the following chapters. In this chapter the equipment and data collection techniques developed for this project are described.

In order to assess the performance of the Omega navigation system in the U.K. area, three fixed monitoring sites were established at the following locations (see also figure 6.1):

(i) Leicester 52° 37' 19" N, 1° 7' 22" W
(ii) Royal Aircraft Establishment 51° 17' 17" N, 0° 45' 15" W (R.A.E.), Farnborough
(iii) The Butt of Lewis 58° 30' 55" N, 6° 15' 37" W

At all three locations measurements of various Omega lines of position were made at more than one frequency. Additionally, at Leicester, the phase and amplitude of the received signals were also recorded. A detailed summary of the data collected at the three U.K. monitoring sites is presented in a later section.
Fig. 6.1 The location of three U.K. monitoring sites
6.2 THE LEICESTER VLF/OMEGA MONITORING STATION

For several years a number of VLF transmitters have been monitored at Leicester and the data collected on chart recorder paper. Major problems were encountered in analysing these analogue records due to the time required to scale charts and convert them to a usable format. In order to overcome these problems an automated digital recording system was developed for the Leicester monitoring site. This system consisted principally of three sections:

(i) the VLF and Omega receivers, aerials, atomic frequency standard and Omega gate;
(ii) a microprocessor controlled interface and memory;
(iii) a Wang 2250 and 2200 mini computer system and floppy disc unit.

In addition to the above, three six colour chart recorders were maintained as part of the system to provide a 'quick-look' facility for monitoring the performance of the receivers and as a backup system in the event of a breakdown in the automated logging equipment. A schematic diagram of the arrangement is presented in figure 6.2. Details of the equipment hardware and program software are given below.

6.2.1 The receivers

Three types of receiver have been employed at the Leicester monitoring site:

(i) a set of five Tracor 599-J VLF tracking receivers;
(ii) a single Redifon Omega receiver, type NV1;
(iii) two Tracor model 700 Omega receivers.

The all solid-state Tracor model 599-J tracking receiver has been
Fig. 6.2 The Leicester VLF/OMEGA monitoring system
expressly developed to utilize the highly stabilized carrier frequency signals of the VLF transmitters for time and frequency calibration, and phase and amplitude measurements. The unit itself incorporates a VLF receiver/frequency synthesizer, phase comparator, electronic phase shifting servo and power supply. The only external requirements are an antenna and frequency standard. Several loop antenna were employed, orientated to obtain maximum signal strengths from the particular VLF transmitters being monitored. An Efratom model FRK rubidium atomic frequency standard provided an external highly stable oscillator for the Tracor 599-J receivers. This device has a long term stability of less than one part in $10^{10}$ per month for a 10 MHz output.

In order to monitor a specific Omega transmitter with this type of receiver a gating system is necessary in order to select the appropriate time interval in the transmitted Omega format (refer to Chapter 5, figure 5.2). The gate ensures that each receiver tracks the signal from only one particular transmitter. This device, driven by the rubidium frequency standard was synchronized by monitoring the nearest Omega station to Leicester (Norway) on one of the Tracor 599-J's, and a manually operated delay adjusted until a maximum signal strength was registered by that receiver. Other Omega stations were then checked in a similar manner to ensure full synchronization over the whole 10 second cycle of the transmissions. Each Tracor 599-J receiver was also modified to enable it to operate efficiently on a non-continuous input signal.

Two analogue signal outputs were available from each Tracor 599-J, one being proportional to the received signal strength, the other proportional to the phase of the incoming signal (relative to the atomic standard). In the latter case a nine volt maximum output
level represented a phase change of 100 µs. Each analogue output (ten in total) provided an input to one channel of two Foster chart recorders, in addition to an input channel of the microprocessor controlled interface.

The Admiralty Compass Observatory (ACO) provided Leicester with a Redifon Omega Navigator (type NVI). This instrument is a commercial Omega receiver operating at 10.2 kHz. It tracks automatically, and continuously all eight transmitters of the Omega system, synchronization being made with the nearest Omega station (specified by the operator). The output display presents the user with three Omega lines of position, accurate to one centilane. These three LOP's are derived from any combination of operator selected transmitters. In addition to this visual display, the NVI receiver provides three analogue outputs representing each LOP being monitored. The device was operated from a 2.4 m whip antenna, via a pre-amp. It was also modified in order to provide further analogue outputs representing the relative signal strengths of the three operator selected transmitters. Thus a total of six analogue outputs were formed, these being connected to the microprocessor interface and a further Foster chart recorder.

Two Tracor model 700 Omega receivers (on loan from ACO) were later incorporated into the Leicester monitoring system. One instrument was operated on 10.2 kHz, the other on 13.6 kHz. Each device provided a digital display and analogue output for two Omega LOP's, accurate to one centilane. Automatic synchronization was provided and each receiver was operated from the whip antenna. Output levels were also monitored on two small chart recorders.
6.2.2 Microprocessor controlled interface and memory

The analogue outputs from the receivers discussed above formed the input data to a microprocessor controlled interface which had the following three main functions:

(a) the interface could continually scan specified input data channels, converting data to digital form and storing it in appropriate memory locations;

(b) every fiftieth of a second, a pulse was derived from the 50 Hz mains cycle to interrupt the channel data scan and cause the processor to increment a software counter. In this way, the memory contained the current time in the following form: day of the year, hour, minute, second and fractions of second (GMT).

(c) the channel scan could also be interrupted by an appropriate command from the Wang 2250 mini computer (see below) which caused the microprocessor to retrieve the current value in any specified memory location and send it back to the Wang computer. This value could be either a specific channel datum or a time datum (e.g. day of the year). In addition provision was made so that on interruption by another Wang command the software clock could be reinitialized.

A block diagram of the interface is presented in figure 6.3. Further details of this device will not be given here.

6.2.3 Wang mini computer system

A Wang mini computer was integrated into the Leicester monitoring system to provide a means of linking the microprocessor interface to an existing magnetic floppy disc storage device. The Wang 2250 unit (16K memory and BASIC compiler) was programmed in the following manner:
Fig. 6.3 Block diagram of the VLF/Wang microprocessor controlled interface.
(a) Initialization procedure

An initialization program requested the user to enter in on a keyboard the day number (Julian calendar), hour, minute and second at which the microprocessor software clock is to be started. At this time the operator depressed a key and the clock commenced at the pre-defined setting. The user was then instructed to input the following information:

(i) the number of data channels to be monitored (24 maximum);

(ii) a series of channel descriptions. This enabled the operator to subsequently display on the Wang 2250 a list of Omega stations and frequencies being monitored, and the relevant data channel numbers;

(iii) the time interval (minutes) between each scan of all data channels. This scan rate had a default value of 15 seconds;

(iv) the time interval (minutes) between each logging of the data on to floppy disc. This logging rate was generally set equal to five minutes;

(v) a further request, asked the user if the logging rate was to be held constant. This feature enabled the user to define a rate of change of Omega signal phase (microseconds/minute) above which the logging rate would be increased so that it equalled the scan rate. Thus at times of very rapid phase change being monitored by one of the receivers the logging program would automatically increase the rate at which data was recorded.

Once the initialization procedure was completed the Wang system then loaded into memory the auto-logging program and monitoring commenced. A flow chart of this program is presented in figure 6.4 and a description of its main features is given below.
START

Obtain current time (days, hours, minutes, seconds)

User interrupt?

If interrupt = 'V' set k=0
If interrupt = 'D' set k=1
If interrupt = '1' STOP

no

Scan data?

yes

Obtain channel data
Average with previous scan

Is phase change (usec/min) > user defined limit?

yes

Set logging rate equal to scan rate

no

If k=0 print out channel data
If k=1 print out channel descriptions
print out scan time

Log data?

yes

Obtain next disc sector number

is disc full?

yes

Log data & increment sector number

Print scan interval
Print log interval
Print next disc sector to be used

no

Fig. 6.4 Flow chart of the Leicester automated logging program
(b) **Automatic logging program**

Essentially this program performed two tasks. Firstly, it interrogated the microprocessor system (section 6.2.2) to obtain the current time and, when necessary, the current data value held on each data channel. Then at specified times this information was stored onto the floppy disc unit. Secondly, the program presented the user with a visual display of information related to the monitoring in progress. The mode of operation of the first task is evident from figure 6.4 and further details will not be presented here. It is however necessary to mention briefly the format of the data.

Data read from the microprocessor data channels was in the form of an integer number ranging from zero to 200, representing a voltage change from zero to nine volts. As the analogue to digital converter (ADC) of the microprocessor is not perfectly linear (especially at small voltages) a calibration was made before any monitoring took place. This calibration curve was included in the Wang software. The resultant resolution of phase of the system was therefore 0.5 µsec. In order to smooth out very short period fluctuations in the data, averaging over 15 second periods was carried out. No other averaging or manipulation of the data took place at this stage.

The format and content of the visual information that was presented on the screen of the Wang 2250 is indicated in figure 6.5.

(c) **Data transfer to main frame computer**

A program specifically written for the Wang 2200 unit enabled data transfer from a filled floppy disc to the University main frame computer (a CDC Cyber 73). Each floppy disc was capable of storing 3½ days of Omega data, and after transfer was overwritten and re-used. A temporary file on the main frame enabled ten days of data to be stored...
CURRENT TIME IS 12 HOURS, 36 MINS, 42 SECS GMT, DAY 134

<table>
<thead>
<tr>
<th>CHANNEL DATA (VOLTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 3.24</td>
</tr>
<tr>
<td>2. 4.71</td>
</tr>
<tr>
<td>3. 8.56</td>
</tr>
<tr>
<td>4. 5.19</td>
</tr>
</tbody>
</table>

24 CHANNELS SCANNED AT 12. 36. 30. GMT, DAY 134.
24 CHANNELS LOGGED AT 12. 35. 00. GMT, DAY 134.

SCAN INTERVAL = 0.25 MINUTES  LOG INTERVAL = 5.00 MINUTES
NEXT LOG ON DISC SECTOR 95

KEYIN D=DESCRIPTION, V=VALUES, I=INTERRUPT, S=STATUS

Fig. 6.5 Typical example of monitor information displayed by the Wang 2250
prior to copying onto 9-track magnetic tape for archiving. The results of subsequent analysis, carried out on the University machine are the subject of following chapters.

6.3 THE FARNBOROUGH AND BUTT OF LEWIS MONITORING STATIONS

The two U.K. Omega monitor sites at Farnborough and the Butt of Lewis had been set up and controlled by A.C.O. Data from these two sites (recorded on to paper tape) have been forwarded to Leicester for analysis.

Although various types of receiver have been in operation at the two sites the data used in this thesis has been recorded on Magnovox MX1104 type Omega receivers. This commercial device provides automatic phase synchronization and tracking of the three primary Omega navigation frequencies and all eight transmitting stations. Data at both sites were measured at hourly intervals and upon receipt at Leicester, transferred on to magnetic tape.

6.4 EXPERIMENTAL ERRORS AND SUMMARY

The greatest contribution to system errors in a navigation aid such as Omega is due to ionospheric variability along a particular propagation path and this problem is considered in subsequent chapters. The error contribution due to instrumental accuracy is generally small in comparison, but is estimated below.

Digital phase measurements made against the Rubidium frequency standard at Leicester were accurate to ± 0.5 μs which at 10.2 kHz is approximately equivalent to ± 0.5 centicycles (cecs). Measurements of signal amplitude made on the Tracer 599-J receivers may be calibrated in decibels to an accuracy of ± 1 dB. The limiting factor in both digital measurements of phase and amplitude, at Leicester, is governed by the characteristics of the A-D converter. Line of position
<table>
<thead>
<tr>
<th>Recording period</th>
<th>Receiving equipment</th>
<th>Type of data (recorded every 5 minutes)</th>
</tr>
</thead>
</table>
| **1st March 1978 - 30th June 1980** | Five Tracor 599-J's | a) Omega station A (Norway), 10.2 kHz, amplitude (db)  
b) Omega station B (Liberia), 10.2 kHz, phase (μs)  
c) Omega station C (North Dakota), 10.2 kHz, amplitude (db)  
d) Omega station D (North Dakota), 10.2 kHz, phase (μs)  
e) Omega station E (Norway), 13.6 kHz, amplitude (db)  
f) Omega station F (Liberia), 13.6 kHz, phase (μs)  
g) Omega station G (North Dakota), 13.6 kHz, amplitude (db)  
h) Omega station H (Liberia), 13.6 kHz, phase (μs)  
i) Omega station I (North Dakota), 13.6 kHz, amplitude (db)  
j) Omega station J (Liberia), 13.6 kHz, phase (μs) |
| **26th November 1978 - 30th June 1980** | Redifon - NV1 | a) Omega LOP AB, 10.2 kHz (cels)  
b) Omega LOP AD, 10.2 kHz (cels)  
c) Omega LOP BD, 10.2 kHz (cels)  
d) Omega station A, signal level  
e) Omega station B, signal level  
f) Omega station D, signal level |
| **9th January 1979 - 30th June 1980** | Tracor 700 | a) Omega LOP AB, 10.2 kHz (cels)  
b) Omega LOP AD, 10.2 kHz (cels)  
c) Omega LOP BD, 10.2 kHz (cels) |
| **9th July 1979 - 30th June 1980** | Tracor 700 | a) Omega LOP AB, 13.6 kHz (cels)  
b) Omega LOP AD, 13.6 kHz (cels) |

* At certain periods (e.g., during transmitter down times) the following Omega stations were monitored - La Reunion (E), Argentina (F), Trinidad (temporary G).

** At certain periods - LOP's AF, AG & BG: signal level of Trinidad station; phase of stations B & D.

Table 6.1 Summary of Omega data collected at Leicester during the period March 1978 to June 1980
<table>
<thead>
<tr>
<th>Recording period</th>
<th>Receiving equipment</th>
<th>Type of data (recorded every hour)</th>
</tr>
</thead>
</table>
| 1st September 1977 - 2nd July 1979 | Magnavox MX1104 | a) Omega LOP AB, 10.2kHz (cels)  
 b) Omega LOP AD, 10.2kHz (cels)  
 c) Omega LOP BD, 10.2kHz (cels)  
 d) Omega LOP AF, 10.2kHz (cels)  
 e) Omega station A, signal level, 10.2kHz  
 f) Omega station B, signal level, 10.2kHz  
 g) Omega station D, signal level, 10.2kHz  
 h) Omega station F, signal level, 10.2kHz  
 i-p) As (a) to (h) above but at 13.6kHz |

**Table 6.2 Summary of Omega data collected at Farnborough during the period September 1977 to July 1979**

<table>
<thead>
<tr>
<th>Recording period</th>
<th>Receiving equipment</th>
<th>Type of data (recorded every hour)</th>
</tr>
</thead>
</table>
| 24th March 1979 - 13th August 1979 | Magnavox MX1104 | a-h) Omega stations A to H, phase, 10.2kHz (measured against a crystal controlled local oscillator), cels.  
 i-p) Omega stations A to H, signal level, 10.2kHz  
 a-h) As (a) to (h) above but at 13.6kHz  
 i-p) As (i) to (p) above but at 11.5kHz |

**Table 6.3 Summary of Omega data collected at the Butt of Lewis during the period March to August 1979**
measurements derived from the Redifon NV1 and Tracor 700 receivers were estimated to be accurate to ± 0.5 centilanes (cels). Signal levels measured on these receivers were not calibrated in terms of decibels and therefore just provided an indication of signal to noise levels and transmitter power levels.

LOP data recorded at Farnborough and the Butt of Lewis were limited by the receivers to ± 0.5 cels. Again signal levels were not directly convertible into a decibel scale and provided only an approximate indication of signal level.

Summaries of all the Omega data collected at the three U.K. monitoring sites during the period of this investigation are denoted in Tables 6.1 to 6.3. Examples of typical diurnal variations and the illustration of seasonal changes derived from these data are presented in the following chapter.
CHAPTER 7

Presentation of Experimental Data

7.1 INTRODUCTION

The unprocessed data collected at the three monitoring stations described in the previous chapter are now presented and discussed. A major part of these data (refer to Table 6.1) consists of phase and amplitude measurements recorded at Leicester, and as such provide a very important monitor of propagation path conditions and variations from several Omega transmitters. LOP measurements on the other hand are the result of the combination of signal phase recorded over two, probably very dissimilar paths, and are therefore more difficult to interpret.

Diurnal and seasonal variations in VLF phase and amplitude are examined from the Leicester data set. Evidence for modal interference (Chapter 4) under certain conditions is identified. Examples of some of the ionospheric disturbances discussed in Chapter 2 are also illustrated in these data. It is expected that all of these variations and disturbances will influence the accuracy of a navigation system utilizing these signals.

It is apparent from the discussions of Chapter 2 that ionospheric conditions at D-region heights vary spatially as well as temporally. Differences in the propagation paths monitored are summarized by figures 7.1 to 7.4 which illustrate great circle routes and quantify path length, bearing angles and magnetic field parameters.

7.2 PHASE AND AMPLITUDE RECORDS COLLECTED AT LEICESTER

As indicated in Table 6.1 Omega phase and amplitude data collected by the Leicester monitoring equipment spans a time period of over two years. The principal features of these data for each propagation path are now discussed.
Great circle routes & path mid-points from the Omega Norway transmitter to the U.K. monitoring sites.

<table>
<thead>
<tr>
<th>Path</th>
<th>$L$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$I$</th>
<th>$\beta$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Norway-Leicester</td>
<td>1727 km</td>
<td>214°</td>
<td>22°</td>
<td>72°</td>
<td>211°</td>
<td>-4.9 gauss</td>
</tr>
<tr>
<td>Norway-Farnborough</td>
<td>1857 km</td>
<td>212°</td>
<td>20°</td>
<td>72°</td>
<td>208°</td>
<td>-4.9 gauss</td>
</tr>
<tr>
<td>Norway-Lewis</td>
<td>1324 km</td>
<td>238°</td>
<td>40°</td>
<td>73°</td>
<td>235°</td>
<td>-5.0 gauss</td>
</tr>
</tbody>
</table>

$L =$ path length  $\theta_1 =$ bearing of Rx from Tx  $\theta_2 =$ bearing of Tx from Rx  
$I =$ magnetic dip at path mid-point  $\beta =$ path magnetic azimuth at path mid-point  
$F =$ total magnetic field intensity at the path mid-point

Fig. 7.1 Omega Norway - U.K. propagation paths
Great circle routes & path mid-points from the Omega Liberia transmitter to the U.K. monitoring sites.

<table>
<thead>
<tr>
<th>Path</th>
<th>L</th>
<th>$\Theta_1$</th>
<th>$\Theta_2$</th>
<th>I</th>
<th>$\beta$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberia-Leicester</td>
<td>5209 km</td>
<td>8°</td>
<td>193°</td>
<td>42°</td>
<td>17°</td>
<td>-39 gauss</td>
</tr>
<tr>
<td>Liberia-Farnborough</td>
<td>5070 km</td>
<td>9°</td>
<td>194°</td>
<td>41°</td>
<td>18°</td>
<td>-39 gauss</td>
</tr>
<tr>
<td>Liberia-Lewis</td>
<td>5804 km</td>
<td>3°</td>
<td>186°</td>
<td>47°</td>
<td>13°</td>
<td>-40 gauss</td>
</tr>
</tbody>
</table>

$L =$ path length  $\Theta_1 =$ bearing of Rx from Tx  $\Theta_2 =$ bearing of Tx from Rx  
$I =$ magnetic dip at path mid-point  $\beta =$ path magnetic azimuth at path mid-point  
$F =$ total magnetic field intensity at the path mid-point

**Fig. 7.2 Omega Liberia – U.K. propagation paths**
Great circle routes & path mid-points from the Omega North Dakota transmitter to the U.K. monitoring sites. Geomagnetic L-shells 3 and 8 are indicated.

<table>
<thead>
<tr>
<th>Path</th>
<th>L</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>I</th>
<th>$\beta$</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>N. Dakota-Leicester</td>
<td>6524 km</td>
<td>45°</td>
<td>233°</td>
<td>78°</td>
<td>120°</td>
<td>56 gauss</td>
</tr>
<tr>
<td>N. Dakota-Farnborough</td>
<td>6634 km</td>
<td>46°</td>
<td>233°</td>
<td>78°</td>
<td>121°</td>
<td>56 gauss</td>
</tr>
<tr>
<td>N. Dakota-Lewis</td>
<td>5893 km</td>
<td>41°</td>
<td>240°</td>
<td>80°</td>
<td>113°</td>
<td>56 gauss</td>
</tr>
</tbody>
</table>

$L =$ path length $\theta_1 =$ bearing of Rx from Tx $\theta_2 =$ bearing of Tx from Rx $I =$ magnetic dip at path mid-point $\beta =$ path magnetic azimuth at path mid-point $F =$ total magnetic field intensity at the path mid-point

Fig. 7.3 Omega North Dakota - U.K. propagation paths
Great circle routes & path mid-points from the Omega Trinidad and Argentina transmitters to the U.K. monitoring sites.

<table>
<thead>
<tr>
<th>Path</th>
<th>L</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$I$</th>
<th>$\beta$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trinidad-Leicester</td>
<td>7097 km</td>
<td>36°</td>
<td>252°</td>
<td>58°</td>
<td>65°</td>
<td>0.46 gauss</td>
</tr>
<tr>
<td>Trinidad-Farnborough</td>
<td>7078 km</td>
<td>38°</td>
<td>253°</td>
<td>57°</td>
<td>67°</td>
<td>0.46 gauss</td>
</tr>
<tr>
<td>Trinidad-Lewis</td>
<td>7032 km</td>
<td>29°</td>
<td>245°</td>
<td>61°</td>
<td>58°</td>
<td>0.47 gauss</td>
</tr>
<tr>
<td>Argentina-Leicester</td>
<td>12248 km</td>
<td>36°</td>
<td>225°</td>
<td>14°</td>
<td>44°</td>
<td>0.30 gauss</td>
</tr>
<tr>
<td>Argentina-Farnborough</td>
<td>12160 km</td>
<td>37°</td>
<td>225°</td>
<td>12°</td>
<td>45°</td>
<td>0.30 gauss</td>
</tr>
<tr>
<td>Argentina-Lewis</td>
<td>12498 km</td>
<td>29°</td>
<td>223°</td>
<td>24°</td>
<td>39°</td>
<td>0.31 gauss</td>
</tr>
</tbody>
</table>

$L =$ path length $\theta_1 =$ bearing of Rx from Tx $\theta_2 =$ bearing of Tx from Rx
$I =$ magnetic dip at path mid-point $\beta =$ path magnetic azimuth at path mid-point
$F =$ total magnetic field intensity at the path mid-point

Fig. 7.4 Omega Trinidad and Argentina - U.K. propagation paths
7.2.1 Norway-Leicester path

The location of the Omega Norway transmitter relative to Leicester (and the other two monitoring sites) is indicated in figure 7.1, together with the great circle path from the transmitter to the receiver. The Norwegian Omega station is the nearest to the U.K. (1727 km from Leicester) and consequently high signal levels are obtained. All commercial Omega receivers (i.e. those not utilizing atomic frequency standards) use this transmitter for synchronization purposes in the U.K. area.

(a) 10.2 kHz data

Figures 7.5(a) and (b) illustrate the diurnal variations in phase and amplitude respectively observed at 10.2 kHz. A typically quiet day has been selected from each month of the year (either 1978 or 1979) in order to illustrate the seasonal variations. Included in these figures are the times of ground level sunrise and sunset at both transmitter and receiver. The time period that a particular path is sunlit or in darkness will be shown to be a controlling factor influencing the diurnal patterns. Figures 7.5(a) and (b) demonstrate the generally stable nature of VLF phase and amplitude signals over a day and over several months. The stability of the phase of a VLF signal is clearly related to the stability of the effective reflection height of the D-region (see Chapter 4). Consequently, during dawn and dusk when ionospheric conditions are changing most rapidly (see also, section 2.3.3, Chapter 2) the records show greatest variability. The phase data for 15 March 1978 illustrates this phenomenon. The phase is relatively stable except at the dawn and dusk transitions where changes of approximately 15° sec occur. The amplitude variations are also greatest during the dawn and dusk periods (see figure 7.5(b)).
Fig. 7.5(a) Diurnal phase variations of the Omega Norway 10.2kHz signal measured at Leicester.
<table>
<thead>
<tr>
<th>Date</th>
<th>Diurnal Amplitude Variations</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>15th Jan 1979</td>
<td>No data 5db</td>
<td>Leicestershire</td>
</tr>
<tr>
<td>13th Feb 1979</td>
<td>tr</td>
<td>Transmitter</td>
</tr>
<tr>
<td>15th Mar 1978</td>
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<td>Transmitter</td>
</tr>
<tr>
<td>13th Apr 1978</td>
<td>tr</td>
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</tr>
<tr>
<td>16th May 1978</td>
<td>ts</td>
<td>Transmitter</td>
</tr>
<tr>
<td>8th Jun 1979</td>
<td>tr</td>
<td>Receiver</td>
</tr>
<tr>
<td>14th Jul 1978</td>
<td>ts</td>
<td>Transmitter</td>
</tr>
<tr>
<td>15th Aug 1979</td>
<td>tr</td>
<td>Receiver</td>
</tr>
<tr>
<td>17th Sep 1979</td>
<td>ts</td>
<td>Transmitter</td>
</tr>
<tr>
<td>19th Oct 1978</td>
<td>tr</td>
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</tr>
<tr>
<td>22nd Nov 1978</td>
<td>ts</td>
<td>Transmitter</td>
</tr>
<tr>
<td>22nd Dec 1978</td>
<td>tr</td>
<td>Receiver</td>
</tr>
</tbody>
</table>

Fig. 7.5(b) Diurnal amplitude variations of the Omega Norway 10.2kHz signal measured at Leicester.
During the summer months (e.g. 8 June 1979) the diurnal pattern is less pronounced. The difference in phase from day to night for the 8 June 1979 is about 10 μsec. Over the summer period the path is in darkness for about two and a half hours of the day compared with some ten and a half to eleven hours at the equinoxes. Thus the period of full ionospheric night conditions is greatly reduced. This seasonal change is also apparent in the amplitude variations of figure 7.5(b).

The diurnal and seasonal changes of short period (< 1 hour) variations in phase and amplitude are also evident in figure 7.5. In general, such variations (excluding those due to flare or geomagnetic storm activity) are more prominent during the night, particularly during the winter months.

(b) 13.6 kHz data

The diurnal variations in phase and amplitude of the 13.6 kHz signal from the Omega Norway transmitter received at Leicester are reproduced in figures 7.6(a) and (b). Comparisons with the 10.2 kHz results for the same path reveal some interesting differences in propagation characteristics. The variation on March 15 1978, for example, is remarkably different in both phase and amplitude at the two frequencies. Several peaks in the phase record, representing phase retardations, occur which are accompanied by amplitude fades (e.g. 02:00, 05:30, 19:00, 21:00 GMT). At certain times of the year very deep fades (10-15 dB) are observed in the 13.6 kHz phase records (e.g. 14 May, 1978 at 22:00 GMT, 15 August, 1979 at 02:30 and 21:30 GMT). These features are the result of strong modal interference (refer to Chapter 4). The transmitter-receiver distance is such that the amplitudes of the first two TM modes of the spherical waveguide are of approximately equal amplitude at 13.6 kHz, and destructive interference occurs giving rise to an anomalous diurnal pattern at the monitor site.
Fig. 7.6 (a) Diurnal phase variations of the Omega Norway 13.6kHz signal measured at Leicester.
Fig. 7.6(b) Diurnal amplitude variations of the Omega Norway 13.6 kHz signal measured at Leicester.
At 10.2 kHz the second mode is more strongly attenuated than the first and has little effect, resulting in a normal diurnal variation.

An interesting comparison may be made between the 13.6 kHz and 10.2 kHz records for June 1979 and July 1978. During these months the phase variations at the two frequencies exhibit similar diurnal patterns. However, at 13.6 kHz the daytime amplitude level exceeds that observed at night in contrast to the behaviour at 10.2 kHz. This feature is produced at 13.6 kHz by the combination of two amplitude interference fades at this time of the year. During August, the two fades have separated in time producing, for example, the type of record obtained during the 15 August 1979 (figure 7.6(b)).

7.2.2 Liberia-Leicester path

The propagation path from Omega station B, (Liberia) crosses only about 10° of longitude and is therefore similar in this respect to the Norway-Leicester path. For both of these paths the dawn or dusk transition is relatively rapid (see figure 7.2).

Illustrated in figures 7.7(a) and (b) are the 10.2 kHz diurnal phase and amplitude variations recorded by the Leicester monitor for the signals received from transmitter B. The dawn transition in diurnal pattern is particularly sudden and a phase advance of about 25 μs occurs over a time interval of 40 to 50 minutes. A small reversal in the direction of change in phase lasting between 30 and 60 minutes sometimes follows. This anomalous behaviour can be accounted for in terms of the weakly attached electrons that are suddenly 'dumped' into the ionosphere at sunrise (refer also to sections 2.2.4 and 5.5).

After the dawn transition the daytime diurnal phase pattern exhibits a strong zenith angle control. This daytime pattern is very stable from day to day, especially during the summer months. In
Fig. 7.7(a) Diurnal phase variations of the Omega Liberia 10.2 kHz signal measured at Leicester.
Fig. 7.7 (b) Diurnal amplitude variations of the Omega Liberia 10.2kHz signal measured at Leicester.
contrast to the Norway data little seasonal variation is evident. This is a result of the lower latitude of the Liberia-Leicester path and hence smaller seasonal changes in sunrise/sunset times. Figure 7.7(a) does however indicate the manner in which the slope of the dawn transition in phase is strongly controlled by sunrise times. During June when sunrise at the receiver precedes that at the transmitter by some three hours, the rate of change of phase is at its yearly minimum (≈12 µs/hour). In mid-October however, sunrise at transmitter and receiver occurs almost simultaneously and consequently the rate of change of phase is much greater (about 65 µs/hour). The sunrise 'dump' effect is also seasonally dependent, being more prominent around the equinoxes when the dawn transition is at its most rapid. The phenomenon is absent during the months of May and June.

The amplitude variations of figure 7.7(b) indicate rapid changes at dawn, usually a decrease from a high night-time value followed by a recovery to a daytime level that generally never exceeds that observed at night.

The 13.6 kHz phase and amplitude records for the Liberia-Leicester path are not reproduced here. Generally the variations exhibit similar features to those observed at 10.2 kHz. The dawn transition is again rapid and the anomalous behaviour in phase at sunrise is also evident.

7.2.3 North Dakota-Leicester path

The great circle path from Omega North Dakota to Leicester (figure 7.3) traverses the auroral zone (Chapter 2) and the Greenland ice cap which is an area of low ground conductivity (refer to Chapter 4 for the effect of ground conductivity upon the propagation of VLF waves). As a result, the propagation characteristics of this path are somewhat different to those already described. The effect of path length and low ground conductivity are to produce appreciable attenuation of
received signal (∼ 20 dB) and hence signal to noise ratios over this path are less than those observed on the previously described paths. In addition to the transmitter-receiver great circle routes, lines of constant values of the geomagnetic L parameter are included in figure 7.3. The ionospheric regions located at high L-values are more disturbed by solar energetic particles than those at low and mid-latitudes (see sections 2.5.2 and 2.5.3). The diagram indicates that the path from station D reaches a L-value of eight though the transmitter and receiver are at geomagnetic mid-latitudes (L ~ 2). The path will therefore be sensitive to auroral zone disturbances causing disruptions in both phase and amplitude of reflected VLF waves.

Figures 7.8(a) and (b) illustrate diurnal phase and amplitude variations in the North Dakota 10.2 kHz signal recorded at Leicester. No rapid dawn or dusk variations are observed as this is an east-west path and hence the terminator line takes several hours to traverse the path. In general, after receiver sunrise, a steady phase advance accompanied by a decrease in signal strength occurs until sunrise at the transmitter. During the summer months the path remains fully sunlit for some nine hours and the phase record is relatively stable. The amplitude records for this period however exhibit fairly rapid fluctuations on a time scale of several minutes. A slow return to night-time conditions commences after receiver sunset. During December full daytime conditions only exist for some two hours, while during mid-summer full night-time conditions only exist for approximately one and a half hours. Consequently for much of the time, the path is in transition between sunlit and dark conditions. Larger fluctuations in phase and amplitude associated with solar flare and geomagnetic storm activity are noted and these are described in section 7.2.5.
Fig. 7.8(a) Diurnal phase variations of the Omega North Dakota 10.2 kHz signal measured at Leicester.
Fig. 7.8(b) Diurnal amplitude variations of the Omega North Dakota 10-2kHz signal measured at Leicester.
7.2.4 Other propagation paths monitored

During periods when Omega stations A, B or D have been undergoing maintenance certain other Omega transmitters were monitored. Principally these were station F (Argentina) and temporary station G (Trinidad). Figure 7.4 denotes the propagation paths from these two transmitters to the U.K. monitor sites. The Argentina-U.K. path is the longest that has been monitored in these experiments, being over 12000 km. Figure 7.9 illustrates the diurnal phase and amplitude variations recorded at Leicester on 10.2 kHz from Argentina on the 17 April 1979, and Trinidad on 25 September 1978. The night to day phase change over the Argentina path is approximately 130 µs. During periods when this particular path is fully sunlit the phase record indicates a similar solar zenith angle control to that previously described for the Liberia-Leicester path. The daytime amplitude levels are in general lower than the night-time values for this path.

The phase variation for the Trinidad-Leicester path is in some respects similar to that observed on the Liberia-Leicester path. The daytime phase variation is almost identical (compare figure 7.9 with the phase variation shown for 20 September 1978 in figure 7.7(a)). The dawn and dusk transitions are less rapid over the Trinidad-Leicester path and extend over a longer time interval compared with the Liberia-Leicester path. In common with the Liberia data the Trinidad signal level exhibits a minimum near transmitter sunrise.

7.2.5 Influence of disturbed conditions

Chapter 2, section 5 was concerned with changes in the D-region due to a number of geophysical disturbances of solar origin. The influence of two of these particular phenomena on propagating VLF waves are well illustrated in the Leicester phase and amplitude data and are discussed below.
Fig. 7.9 Diurnal phase and amplitude variations of the Omega Argentina and Trinidad signals measured at Leicester.
(a) **Sudden Ionospheric Disturbances (SIDs)**

These disturbances are associated with solar flare activity and can have dramatic effects on VLF propagation. Figure 7.10 illustrates the effect of a large solar flare on several phase and amplitude records collected at Leicester. The data is for 28 April 1978 and the solar disturbance commenced at 13:00 UT. Solar observations made on that day (Solar-Geophysical Data, Prompt Reports, NOAA, Boulder, Colorado) recorded a X-5 class X-ray burst that was associated with a large flare of importance 3B. Rapid phase advances are produced in the measured VLF signals followed by a gradual return to pre-flare values. A phase advance of some 15 μs was observed on the Norway 10.2 kHz phase record but at 13.6 kHz the advance is of the order of 8 μs. In general SID induced phase advances over this path were greater in magnitude at 10.2 kHz than 13.6 kHz. Recovery times at 10.2 kHz are also generally longer than those at 13.6 kHz (4$\frac{1}{4}$ hours compared with 45 minutes for the flare of figure 7.10).

The largest phase advance was recorded on the Liberia 10.2 kHz trace. Over a period of approximately 30 minutes a phase change of 57 μs was observed (comparable to the magnitude of the normal undisturbed diurnal phase change). Recovery time is about six hours, by which time full night-time conditions are achieved. The low latitude Liberia-Leicester path is particularly sensitive to the influence of solar flares.

A relatively large phase advance (≈ 40 μs) is also observed on the North Dakota, 10.2 kHz record. Recovery time is shorter than for the Liberia-Leicester path, being of the order of two hours. The influence of flare activity on the VLF waves propagating over this high latitude path is variable. The flare of the 28 April 1978, because
Fig. 7.10 Comparison of VLF phase and amplitude data measured at Leicester on 28th April 1978.
<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>Transmitter</th>
<th>Recorded phase advance</th>
<th>Amplitude change</th>
<th>Recovery time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2</td>
<td>Norway</td>
<td>15 µs</td>
<td>-1 dB</td>
<td>4.5 hours</td>
</tr>
<tr>
<td>13.6</td>
<td>Norway</td>
<td>8 µs</td>
<td>+4 dB</td>
<td>0.75 hours</td>
</tr>
<tr>
<td>10.2</td>
<td>Liberia</td>
<td>57 µs</td>
<td>-25 dB</td>
<td>6 hours</td>
</tr>
<tr>
<td>10.2</td>
<td>North Dakota</td>
<td>40 µs</td>
<td>-14 dB</td>
<td>2 hours</td>
</tr>
</tbody>
</table>

**Table 7.1** Summary of the disturbances at VLF caused by the large solar flare recorded at Leicester on 28th April 1978
of its magnitude, has a dramatic effect, but flares of smaller size present in the Norway and Liberia data may be absent in the North Dakota data. The short time period variability already noted in the North Dakota data is expected to mask the influence of small flares.

Generally the rapid phase advance associated with a solar flare outburst is accompanied by a decrease in signal amplitude as indicated in three of the four amplitude records of figure 7.10. The exception occurs at 13.6 kHz over the Norway-Leicester path. Here an increase in signal strength of about 4 dB is observed. It is interesting to note that the change in signal strength at 10.2 kHz over this path is hardly detectable. Recovery times for signal levels are usually shorter than those for signal phase.

A summary of the phase and amplitude disturbances produced by the solar flare of 28 April 1978 is presented in table 7.1.

(b) Geomagnetic Storm Activity

The influence of particle emissions on the lower ionosphere following certain solar disturbances has been outlined in Chapter 2. Associated changes in the D-region occur most frequently in the auroral zones and are therefore expected to produce changes in the propagation characteristics of VLF waves passing through this zone. Figure 7.11 illustrates the changes in the phase of the North Dakota signal (10.2 kHz) recorded by the Leicester monitor during a geomagnetic disturbance. After commencement of the storm on 10 November 1978 the normal diurnal pattern is shifted downwards (a phase advance) by approximately 40 μs. After this time the pattern slowly shifts back to its original position, the recovery period being of the order of five days. The very strong shift in phase was almost certainly caused by ionization enhancement by precipitating electrons in the auroral zone. Several similar events to that displayed in figure 7.11
Fig. 7.11 Phase variations of the Omega North Dakota 10.2 kHz signal measured at Leicester during the geomagnetical active period of November 1978.
are present in the North Dakota phase data. During extremely active
gemagnetic periods the Norway-Leicester path may also be disturbed
resulting in a distortion of the normal diurnal pattern.

7.3 MEASURED LINES OF POSITION (LOPs) AT LEICESTER

Examples of the diurnal variations in LOPs measured at Leicester,
on 10.2 kHz, are presented in figures 7.12 a, b and c. Diurnal patterns
for LOP AB (Norway phase minus Liberia phase), figure 7.12(a) are
clearly dominated by the phase change in the Liberia signals. The
patterns are inverted with respect to the normal diurnal variations
of figure 7.7(a) as this phase is subtracted from the Norway phase.

LOP measurements are presented in terms of centilanes, the size of
which may be related to actual distances along the ground (see figure
B.1, Appendix B). Referring to figure 7.12(a), the magnitude of
diurnal change in LOP is of the order of 40 to 50 centilanes. This
converts to a ground distance of 5.9 to 7.4 km and represents the
measured variation in the position fix of the Leicester monitor site
as determined by Omega. It is important to note that this variation
is not necessarily centered on the monitor site. In certain situations,
indicated below, there is no time when the experimentally measured LOP
coincides with the true position of the receiver. This observation
clearly demonstrates the requirement for a means of providing additional
corrections if the errors in position fixes are to be reduced in
magnitude from several kilometers. The relationship between the
measured lines of position and the presently available ONSOD prediction
program (Chapter 5) is dealt with in the following chapter.

Data for LOP AD (Norway minus North Dakota) are presented in
figure 7.12(b). For this particular LOP the North Dakota diurnal
component is dominant and therefore the controlling factor in the
variations. As a consequence of this and the known lower signal levels
Fig. 7.12(a) Diurnal variations of the Omega AB LOP (10-2 kHz) measured at Leicester.
Fig. 7.12 (b) Diurnal variations of the Omega AD LOP (10.2 kHz) measured at Leicester.
Fig. 7.12 (c) Diurnal variations of the Omega BD LOP (10.2 kHz) measured at Leicester.
over this path, the LOP is expected to exhibit a greater variability over a short time scale than LOP AB.

Diurnal variations in LOP BD (Liberia minus North Dakota) are illustrated in figure 7.12(c). This LOP demonstrates the result of subtracting one large diurnal phase pattern from another of comparable magnitude but different shape. As this LOP involves transmitter D (North Dakota) it is expected to exhibit similar short period variations as LOP AD.

7.4 MEASURED LINES OF POSITION AT FARNBOROUGH

7.4.1 10.2 kHz data

In the present study the majority of LOP measurements will be presented in terms of the differences between the position of the LOP determined by an Omega receiver and the known location of that receiver; i.e. the 'LOP error'. Figure 7.13(a) illustrates the LOP errors for station pair AB measured on 10.2 kHz at Farnborough during September and October 1978. This data consists of hourly values and consequently less detail is present compared with the Leicester data. The regular variation in LOP AB is clearly evident and is of the order of 40 cels (very similar to the Leicester LOP AB). The data also indicate that around midday the LOP error is at a minimum (< 5 cels), and at midnight a maximum (~ 35 to 40 cels). Evidence of SID disturbances is also present in the data in the form of sudden upward going spikes, for example at 11:00 a.m. on 23 September 1978.

Diurnal patterns are also presented for the LOP error variations in LOPs AD, BD and AF (figures 7.13b, c and d respectively). These patterns, in common with that for LOP AB are very similar in nature to the corresponding variations measured at Leicester. The period illustrated in figure 7.13 is noteworthy in that a large geomagnetic storm occurred on the 23 September 1978. Only a minor perturbation
(after the flare event of that day) is present on LOPs AB and AF, but major effects are evident on both LOP AD and BD. This is caused by disturbances over the North Dakota-Farnborough path of a similar nature to those discussed in section 7.2.5(b). Recovery time for the two LOPs is approximately eight days, during which very significant position errors occur (60 to 90 cels maximum).

7.4.2 15-6 kHz data

The 10.2 kHz and 15-6 kHz signals are reflected from almost the same region of the ionosphere and therefore the shapes of the diurnal patterns in LOP are nearly the same on both frequencies. Any modal interference differences in the two Norway signals tends to be lost in the Farnborough hourly data or masked by the larger diurnal variation in the Liberia (LOP AB) or North Dakota phases (LOP AD). In addition to the diurnal patterns, disturbances in the records (e.g. SIDs) tend to be of similar magnitude at 10.2 and 13.6 kHz. By taking the difference in LOPs measured at the two frequencies (see also Chapter 5, section 5.2) the influence of such disturbances may be greatly reduced.

Examples of LOP errors measured at Farnborough on 13.6 kHz are presented in figures 7.14(a) to (d). An important systematic difference between for example LOP AB at 10.2 kHz and 13.6 kHz exists. The mean LOP error at the former frequency is about -20 cels while on the latter frequency its value is about -70 cels. It is clear from the figures that a navigator using the 13.6 kHz LOP AB at Farnborough without additional corrections would never record a fix that actually coincided with the true position of the receiver. Such systematic differences have to be modelled in addition to the shape of the diurnal patterns in any Omega phase prediction technique.
RAE FARNBOROUGH LOP ERROR DATA, OMEGA LOP AB, 13.6KHZ
(HOURLY DATA POINTS, 1978 DATA SET)  
Fig. 7.14(a)
RAE FARNBOROUGH LOP ERROR DATA, OMEGA LOP BD, 13.6KHZ
(HOURLY DATA POINTS, 1978 DATA SET)
RAE FARNBOROUGH LOP ERROR DATA, OMEGA LOP AF, 13.6KHZ
(HOURLY DATA POINTS, 1978 DATA SET)

Fig. 7.14(d)
7.5 **MEASURED LINES OF POSITION AT THE BUTT OF LEWIS**

Examples of LOP data collected at the Butt of Lewis (10.2 kHz and 13.6 kHz) are presented in figures 7.15 and 7.16. Detailed comparisons between the LOP measurements at the three U.K. monitor sites are made in a later chapter. It therefore suffices at this point to note that, in general, the diurnal variations at the three sites are comparable to within about five to ten centilanes. It will be demonstrated later that important detailed differences below this level exist. In common with Leicester and Farnborough systematic differences in the mean value of LOP error are apparent, both between different frequencies and different monitor stations.

7.6 **CONCLUSIONS**

Typical examples of the various types of data collected at each of the three U.K. monitoring sites have been presented. In particular the importance of solar zenith angles and sunrise/sunset times in controlling the type of diurnal pattern has been emphasized. Seasonal variations were discussed in a similar manner. Anomalous behaviour observed at dawn under certain conditions is thought to be due to the 'dumping'of weakly attached electrons.

Comparisons between data collected at different frequencies or over different paths have been made. Path length, latitude and orientation are found to be of considerable importance in determining the nature of the phase and amplitude variations at the receiver. Examples of further anomalous behaviour, such as modal interference and SID induced changes are present in the data. These anomalies introduce errors into the Omega system that are particularly difficult to overcome. One of the paths monitored reaches high latitude and frequently exhibits phase disturbances, of geomagnetic storm origin, in the received signal.
BUTT OF LEVIS LOP ERROR DATA, OMEGA LOP AB, 10.2KHZ
(HOURLY DATA POINTS, 1979 DATA SET)

Fig. 7.15 (a)
Fig. 7.15 (b)

BUTT OF LEVIS LOP ERROR DATA, OMEGA LOP AD, 10.2KHZ
(HOURLY DATA POINTS, 1979 DATA SET)
BUTT OF LEWIS LOP ERROR DATA, OMEGA LOP AF, 10.2KHZ
(HOURLY DATA POINTS, 1979 DATA SET)

Fig. 7.15 (d)
The diurnal and seasonal variations in Omega lines of position have been measured and how these may be related to actual errors in position on the ground discussed. It was observed that reliance on Omega LOP measurements alone can lead to very large position fix errors. These errors may be of short duration (e.g. those associated with SIDs) or may be systematic in nature and always present. These errors are discussed further in the next chapter which contains a comparison between the experimentally recorded LOPs and those predicted by the ONSOD prediction program. In addition, LOP measurements are converted into actual ground position fixes and the observed system errors will be described.
CHAPTER 8

Comparative Studies of the Accuracy of ONSOD Predictions
with Experimental Measurements and the Effectiveness
of Omega/Satellite Systems

8.1 INTRODUCTION

Examples of line of position data recorded at three sites in the
U.K. have been presented in the previous chapter. It was noted that
the observed diurnal changes in LOP would produce significant errors
in navigator's position fix unless some form of correction was applied.
Such a correction method has been described in Chapter 5. This present
chapter is concerned in part with a comparison of experimentally
measured LOPs and those predicted by the ONSOD technique. The influence
of the correction method on position fixes as opposed to LOPs is also
examined in detail.

It was noted in the previous chapter that small differences
may occur between the LOPs recorded at each monitoring station. These
differences are examined in more detail here, together with their
relationship to Differential Omega (Section 8.4.3). High accuracy
navigation can be provided by satellite systems such as NAVSAT, and
commercial receivers combining these systems with Omega are currently
available. Using the data recorded in the present study a composite
navigation system employing a receiver of this type was simulated
and the improvements in radial errors quantified.

8.2 COMPARISON OF MEASURED AND PREDICTED LOP ERRORS

The predicted diurnal LOP error variations at each of the U.K.
monitor sites have been determined by means of the ONSOD prediction
program. The predicted propagation correction (PPC) tables (e.g.
figure 5.5) provide hourly diurnal corrections for each half monthly
period, thus the computer generated PPCs were available at this interval throughout 1978 and 1979. Direct comparison between the predicted corrections (broken lines) and the experimentally measured LOP errors (solid lines) are presented for convenience in Appendix A (figures A.1 to A.18). The experimental data reproduced in the figures correspond to the same half monthly periods as in the PPC tables and the diurnal patterns within each period are overplotted. This presentation illustrates the following two important parameters:

(a) the magnitude of the day to day variability of the measured LOP data within each half monthly period;
(b) the degree of agreement between the measured and predicted LOP error within each half monthly period.

8.2.1 Farnborough monitor

Comparisons between the 1978/79 LOP data collected at Farnborough and the ONSOD predicted corrections for the same period are presented in figures A.1 to A.8. Both 10.2 kHz data (figures A.1 to A.4) and 13.6 kHz data (figures A.5 to A.8) are included for LOPs AB, AD, BD and AF. Quantitative details of the comparisons are not presented here, but some general comments regarding these data are given below.

(a) Although the Farnborough LOP data are recorded hourly and hence are less detailed than those obtained at Leicester, seasonal changes in the diurnal variations are apparent for all the LOPs presented.

(b) The day to day variability is seasonally dependent and is generally greater during the winter months than the summer. This is particularly apparent for LOP AB, but less so for LOP AD or AF.

(c) LOPs involving station D (i.e. AD or BD) exhibit large day to
day variabilities for the reasons given in Chapter 7.

(d) The influence of SIDs is evident in the LOP data involving stations B or F (i.e. AB, BD and AF). For example, LOP AB, 10.2 kHz, 1-15 July, 1978 (figure A.1(b)).

(e) Very large disturbances to the normal quiet LOP diurnal variations can occur when station D is involved due to geomagnetic storm activity. For example, LOP AD, 10.2 kHz, 16-30 September 1978 (figure A.2(c)).

(f) The comparison of the ONSOD predictions and experimental data suggests that the former is overestimating both the rate of change and the magnitude of the dawn, and to a lesser extent the dusk, transitions over the Liberia and Argentina paths.

(g) The predicted change in phase from day to night over the Liberia and Argentina paths is too large. This effect is probably related to the discrepancies produced at dawn and dusk mentioned above.

(h) The relative positions of the experimental data and the prediction curves differ at 10.2 kHz compared with 13.6 kHz for LOPs AB and AF. This implies that the accuracy of the ONSOD program in estimating the systematic or mean LOP errors varies from one frequency to another. The existence of systematic differences in LOP data has been discussed in Chapter 7, sections 7.4.2 and 7.5.

(i) A combination of effects (f), (g) and (h) given above lead to very large (＞20 cels) systematic errors for certain AB and AF LOPs (e.g. LOP AB, 10.2 kHz at night throughout 1978)

(j) Systematic differences between the experimental and predicted data of LOP AD (10.2 kHz and 13.6 kHz) are relatively small
except during very disturbed days. The maximum systematic
differences occur when the North Dakota path is fully sunlit
and are therefore more prominent during the summer than in
winter.

(k) LOP BD presents a complicated diurnal pattern for reasons given
in the previous chapter. Differences between the experimental
data and the ONSOD predicted curves are caused by a combination
of many of the above effects.

8.2.2 Leicester monitor

The Leicester monitor as pointed out in Chapter 6 differs from
those at Farnborough and the Butt of Lewis in that LOP information is
recorded at five minute intervals instead of hourly. Consequently the
Leicester data is particularly suitable for studies of short duration
transient changes in LOP.

The LOPs recorded at Leicester show very similar behaviour to
those measured at Farnborough and the differences between the measured
and predicted LOP errors closely correspond to those already discussed
above (section 8.2.1). In order to illustrate the short period changes
in LOP error a limited number of examples are reproduced here, however a
detailed account is not presented. Figures A.9 to A.11 contrast
winter and summer conditions for LOPs AB, AD and BD measured at 10.2 kHz
during 1979. As expected short period variations are much more evident
in the AD and BD LOPs as these include transmitter D (North Dakota).

8.2.3 Butt of Lewis monitor

Reliable data from the Butt of Lewis site have only been obtained
for the summer months of 1979, and therefore comparisons with the
predicted diurnal variations are restricted to this period. Examples
of these results are presented in figures A.12 to A.15 (10.2 kHz) and
and A.16 to A.19 (13.6 kHz). Some important features of these data
and comparisons with the results obtained at the other two monitor sites are given below:

(a) Seasonal changes in the Butt of Lewis experimental data are less apparent than those at the other receiver stations due to the shorter time period over which LOPs were recorded.

(b) In common with the Farnborough and Leicester data LOPs AD and BD exhibit larger day to day variability compared with other LOPs, and are particularly sensitive to geomagnetic disturbances (e.g. 1-15 June 1979).

(c) Also in common with the previous data the effects of SIDs are clearly evident in the data for LOP AB and AF.

(d) The magnitudes of the day to day variabilities in LOP error for a particular LOP and frequency are in close agreement at all three receiver sites.

(e) Differences between LOP AB data and the predicted values are less pronounced at the Butt of Lewis than at the other monitor stations. It appears that the magnitude of the phase change from day to night over the Liberia path and the changes at dawn and dusk are more accurately predicted by the ONSOD program at this northerly receiver. This very important feature will be discussed in further detail in the following chapter.

(f) In common with data from the other monitor sites the relative positions of experimental and predicted data for LOP AB change slightly with radio frequency. However, large systematic errors do not result because of the accuracy of the predictions discussed above.

(g) Systematic differences are observed between the experimentally obtained LOP AD data and the predicted values. The 10.2 kHz differences are smaller than those at 13.6 kHz while the latter
are comparable in magnitude to those observed at Farnborough and Leicester.

(h) Differences between predicted and measured AF LOP errors are similar at the Butt of Lewis compared with the other receiver sites.

(i) As for the Farnborough and Leicester data, the variations in LOP BD and the correlation with ONSOD predictions is caused by a complicated amalgam of many of the above effects.

8.3 ERRORS IN GROUND POSITION FIXES

The data presented up to this point in this chapter have been concerned with errors in particular LOPs, since the LOP is the basic parameter measured by an Omega receiver and predicted by the ONSOD program. A navigator is however primarily interested in his position fix as measured by a system such as Omega. Since the location of each monitor site (static) is accurately known, the errors in position fixes determined from Omega can be readily ascertained. To evaluate the effectiveness of the ONSOD corrections, ground position fixes and their associated errors have been calculated from both the raw data set and from data to which the appropriate corrections have been applied. The majority of these results are presented graphically in Appendix B and are discussed below.

As previously noted the Omega lane widths at each of the three receiving locations varies from site to site. Three LOPs (AB, AD and BD) are combined in the following analysis and these are depicted in figure B.1 for both 10.2 kHz and 13.6 kHz. The transmitter positions are such that LOP AB lies in a north-west to south-east direction while LOP AD is orientated almost north-south in the U.K. Lanes at 13.6 kHz are closer together than the corresponding ones at 10.2 kHz.
Omega lane AD is the widest of the three, while lane AB is the narrowest. The remaining figures of Appendix B are based upon this geometry, each position fix being marked by a cross. Position fixes are generally determined from all three LOPs producing an error triangle of uncertainty in position. However, as no error triangle of side greater than $\frac{1}{2}$ km has been observed in these data a cross is placed at the centroid. Positions determined from only two LOPs will not produce an error triangle.

8.3.1 The position of the Farnborough receiver

(i) 10.2 kHz (figure B.2)

The positions of the Farnborough receiver determined from the 10.2 kHz signals at two hourly intervals are illustrated in figure B.2. All the data collected in the period January 1978 to June 1979 for each time (GMT) stated are included. No attempt has been made to distinguish between data collected at various seasons or months. Data to the left of each diagram is that uncorrected by ONSOD while that to the right has had PPCs applied. Several important features are apparent from these data, and are briefly discussed below:

(a) The distribution of position fixes is spread along an approximately east-west line, along the great circle path from the North Dakota transmitter to the receiver. This is a result of the large variability in signal phase from this transmitter.

(b) Variability in the north-south direction is related to the stability of the AB LOP. Only at dawn (\(\sim 06:00\) GMT) and to a lesser extent, dusk (\(\sim 18:00\) GMT) is the scatter in this direction large, due to the rapid phase changes in LOP AB at these times.

(c) Uncorrected position fixes drift diurnally towards the receiver's true location and then away again. This phenomena is a natural consequence of the strong diurnal phase change patterns present.
in all Omega LOPs (especially AB). Further reference to this effect is made in section 8.3.4.

(d) ONSOD corrected data in general shows improvements in position fix accuracy over the uncorrected data, but the scatter in the east-west direction is not reduced. At times the corrected data overshoots the receiver's true location (e.g. 00:00 GMT) and this is a consequence of the large systematic differences recorded in section 8.2.1 between LOP AB and the prediction curves at night.

(ii) 13.6 kHz (figure B.3)

Position fixes determined from the 13.6 kHz Farnborough data are presented in figure B.3. In general, similar comments to those above apply at this frequency. However, the mean position of fixes is different at the higher frequency (refer also to section 8.3.4). In addition, the overall scatter of position fixes appears to be less at 13.6 kHz (section 8.3.4).

8.3.2 The position of the Leicester receiver

As a consequence of the similarity in LOP data between Farnborough and Leicester the ground position fix distributions agree closely. Improvements obtained by application of ONSOD corrections are also similar in magnitude.

For comparison of these distributions and those recorded at the Butt of Lewis, the Leicester 10.2 kHz data reproduced is restricted to the period 24/3/79 to 13/8/79 (figure B.4) which coincides with the availability of the North of Scotland observations.

8.3.3 The position of the Butt of Lewis receiver

Butt of Lewis position fixes are presented in figures B.5 (10.2 kHz) and B.6 (13.6 kHz), 10.2 kHz results being compared with corresponding Leicester measurements in figure B.4.
(i) 10.2 kHz (figure B.5)

(a) Butt of Lewis night-time fixes are in general distributed at a greater distance from the receiver's true location than those recorded in Leicester.

(b) The scatter of position fixes in an east-west direction appears slightly greater at certain times at Leicester (e.g. 14:00 GMT). This may be a consequence of remaining differences between the coverage of data in the period indicated (i.e. there is no Butt of Lewis LOP data from 16 June to 14 July).

(c) Following point (b), the north-south scatter appears slightly greater at the Butt of Lewis (e.g. 16:00 GMT). This may also be related to the differences in the diurnal variations of LOP AB at the two locations.

(d) ONSOD corrected fixes are generally more accurate at the Butt of Lewis than at Leicester, as expected from the discussion in section 8.2.3. Differences between the sites (corrected data) appear more prominent at night (20:00-04:00 GMT) than during the day (06:00-18:00 GMT).

(ii) 13.6 kHz (figure B.6)

Position fixes determined from 13.6 kHz Butt of Lewis LOPs are illustrated in figure B.6. The comparative statements made above between Leicester and Butt of Lewis 10.2 kHz data are generally valid at 13.6 kHz. Differences between the position fixes recorded at the two frequencies in Scotland are also apparent as was the case for Farnborough (compare figures B.5 and B.6).

8.3.4 Diurnal variation in mean positions

The greatest ionospheric influence which affects VLF radio wave propagation is the diurnal variation in electron density. It is this change which must be modelled if accurate predictions of the propagation errors are to be achieved. The existing ONSOD predictions partly meet
this objective, but as indicated above, the degree of success is rather variable. To quantify these differences the diurnal variations of errors in position fix have been investigated in detail.

Illustrated in figures 8.1 to 8.5 are mean hourly position fix plots, recorded at each monitoring location. Both uncorrected and ONSOD corrected data are included, the former always exhibiting a clockwise rotation. These diagrams essentially quantify the mean diurnal pattern in position fixes for the data presented in Appendix B. The form of this pattern is particularly important since it could well provide the basis of a simple prediction program for the U.K. area.

When ONSOD corrections are applied the diurnal variations in mean position fixes are destroyed, the remaining distribution being a function of the accuracy of the PPCs.

Histograms, illustrating radial standard deviations (RSDs) about the mean positions are also included in the figures. Generally only small reductions in RSD are apparent after application of ONSOD corrections.

The behaviour of these data are summarized below.

(i) Farnborough monitor, 10.2 kHz (figure 8.1)

Farnborough, 10.2 kHz mean position fix distributions are indicated in figure 8.1. A minimum radial error (< 0.5 km) occurs around 10:00 GMT prior to correction by ONSOD PPCs. The maximum radial error (~ 9 km) present in the uncorrected data set occurs at 20:00 GMT. ONSOD corrections reduce radial errors to 5 km or less, however, RSDs at dawn and dusk are larger than at other times. This is a consequence of the difficulties in predicting LOP variations at these times.
Onsod corrected Farnborough hourly mean position fixes, Jan 1978 - Jun 1979, 10.2kHz

Fig. 8.1
UNCORRECTED FARNBOROUGH HOURLY MEAN POSITION FIXES, JAN 1978 - JUN 1979, 13.6KHZ

ONSOD CORRECTED FARNBOROUGH HOURLY MEAN POSITION FIXES, JAN 1978 - JUN 1979, 13.6KHZ

UNCORRECTED DATA
CORRECTED DATA
RADIAL STANDARD DEVIATION ABOUT THE MEAN POSITIONS

Fig. 8.2
(ii) **Farnborough monitor, 13.6 kHz (figure 8.2)**

The magnitude of the 13.6 kHz diurnal pattern (figure 8.2) is reduced and displaced some 10 km from the receiver's true location. RSDs of both corrected and uncorrected data are approximately 1 km less than those at 10.2 kHz. There is however a prominent increase in RSD around 08:00 after PPCs have been added.

(iii) **Leicester monitor, 10.2 kHz (figure 8.3)**

Similar diurnal patterns in uncorrected mean position fixes are observed at Leicester. A more direct comparison of the Butt of Lewis and Leicester 10.2 kHz results (24/3/79 - 13/8/79) is presented in figure 8.3. Variations between this diurnal pattern and that of figure 8.1 (Farnborough) are due principally to the differences in the time periods over which the data originate rather than due to change in receiver location.

Radial standard deviations increase slightly during the dawn period (06:00 - 07:00 GMT) after addition of ONSOD corrections.

(iv) **Butt of Lewis monitor, 10.2 kHz (figure 8.4)**

Figure 8.3 (Leicester, 10.2 kHz) may be compared with a similar data set recorded in Scotland (figure 8.4), which has the following features. Firstly the shape and location of the diurnal variation is different at the two locations. Secondly, application of ONSOD PPCs generally results in more accurate position fixes at the Butt of Lewis, and their distributions differ from one site to the other. Thirdly, RSDs are similar at the two locations, the maximum differences being of the order of 1 km (at dawn and dusk).

(v) **Butt of Lewis monitor, 13.6 kHz (figure 8.5)**

In common with the Farnborough measurements at this frequency (figure 8.2) uncorrected fixes form a diurnal pattern located some distance (~10 km) from the receiver's true location. ONSOD corrections
UNCORRECTED LEICESTER HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 10.2 KHZ

ONSOD CORRECTED LEICESTER HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 10.2 KHZ

RADIAL STANDARD DEVIATION ABOUT THE MEAN POSITIONS

Fig. 8.3
UNCORRECTED BUTT OF LEWIS HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 10.2KHZ

ONSOD CORRECTED BUTT OF LEWIS HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 10.2KHZ

UNCORRECTED DATA
CORRECTED DATA
RADIAL STANDARD DEVIATION ABOUT THE MEAN POSITIONS

Fig. 8.4
UNCORRECTED BUTT OF LEWIS HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 13.6KHZ

ONSOD CORRECTED BUTT OF LEWIS HOURLY MEAN POSITION FIXES, 24/3/79 TO 13/8/79, 13.6KHZ

Fig. 8.5
destroy the variation and RSDs are slightly less (≤ 1 km) at 13.6 kHz than those observed on 10.2 kHz in Scotland.

8.3.5 Concluding note

The analysis presented in this section (8.3) has illustrated both the systematic and random variations in Omega position fixes derived from three LOPs (AB, AD and BD). Such variations have been quantified in terms of radial distances from the true position and radial standard deviations about hourly mean position fixes. The distributions observed are a result of the combination of some or many of the effects described in the previous section (8.2), and the above analysis has to a certain extent indicated their relative magnitude. For example, variations in LOP AD produce a very prominent trend in position fix distributions, and contribute substantially to the observed level of RSD, at Leicester and Farnborough.

Furthermore, the results presented above provide a clear indication of the nature of the improvements possible with the application of ONSOD PPCs and highlight the differences in system performance in the north and south of the U.K.

8.4 COMPARISON OF LOP ERRORS MEASURED AT THE RECEIVING STATIONS (10.2 kHz)

The discussions of LOP error patterns and ground position fix distributions suggest that the performance of the Omega system differs even within the confines of the U.K. An attempt was therefore made to compare the LOP errors measured at the various monitoring sites. The object of this study was to quantify the similarity (or difference) in LOP errors. Caution must be exercised in interpreting some of these results, especially the biases, as ambiguities in LOP error difference may occur (refer to Table 8.1). Six different possible situations are illustrated in Table 8.1, giving rise to the same absolute value
<table>
<thead>
<tr>
<th>LOP Error Location X</th>
<th>LOP Error Location Y</th>
<th>Difference in errors (X-Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+5</td>
<td>+1</td>
<td>+4</td>
</tr>
<tr>
<td>X</td>
<td>Y</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>-5</td>
<td>-4</td>
</tr>
<tr>
<td>+1</td>
<td>-1</td>
<td>-4</td>
</tr>
<tr>
<td>-5</td>
<td>-1</td>
<td>+4</td>
</tr>
<tr>
<td>+3</td>
<td>-1</td>
<td>+4</td>
</tr>
<tr>
<td>+1</td>
<td>+3</td>
<td>-4</td>
</tr>
</tbody>
</table>

Table 8.1 The possible variations of LOP error and the resulting differences
for the difference in errors. The change in sign does reduce this example to two groups of indistinguishable results (i.e. +4 cels and -4 cels, Table 8.1).

8.4.1 Leicester-Farnborough LOP error differences

(i) LOP AB, 10.2 kHz (figure 8.6)

The differences in AB LOP error (10.2 kHz) measured at Farnborough and Leicester are indicated in figure 8.6. There is a small systematic bias of about -5 cels, and the variations appear fairly repeatable over each half monthly period indicated.

(ii) LOP AD and BD 10.2 kHz (figures 8.7 and 8.8)

The variations for LOP AD (figure 8.7) and LOP BD (8.8) appear to be quite random. A small bias of ~5 cels is apparent in the BD data which coupled with that noted for LOP AB appears to indicate an association with the Liberia path.

8.4.2 Farnborough - Butt of Lewis LOP error differences

In contrast to the variations presented in figures 8.6 to 8.8 the LOP error differences recorded between Farnborough and the Butt of Lewis exhibit very marked diurnal patterns, and some large systematic biases (refer to figures 8.9 to 8.11). For example, a diurnal variation of 15 - 20 cels is evident in the AB data (figure 8.9) and at night the bias is approximately 25 cels.

8.4.3 Implications for Differential Omega

These comparisons serve to illustrate the very real differences in Omega performance within the U.K. and are of great importance in assessing the operational characteristics of any Differential Omega system proposed for the U.K. Differential Omega consists of a series of fixed ground monitor stations, not unlike that operated on the Butt of Lewis. The monitoring equipment determines the errors in local Omega LOPs automatically from a knowledge of the known receiver
Fig. 8.6  HALF MONTHLY LOP ERROR DIFFERENCES, LEICESTER - FARNBOROUGH DATA FOR LOP AB, 10.2KHZ, 1979
Fig. 8.7  HALF MONTHLY LOP ERROR DIFFERENCES, LEICESTER - FARNBOROUGH
DATA FOR LOP AD, 10.2KHZ, 1979
Fig. 8.8  HALF MONTHLY LOP ERROR DIFFERENCES, LEICESTER - FARNBOROUGH 
DATA FOR LOP BD, 10.2KHZ, 1979
Fig. 8.9  HALF MONTHLY LOP ERROR DIFFERENCES, FARNBOROUGH - BUTT OF LEWIS DATA FOR LOP AB, 10.2KHZ, 1979
Fig. 8.10 HALF MONTHLY LOP ERROR DIFFERENCES: FARNBOROUGH - BUTT OF LEWIS DATA FOR LOP AD, 10.2KHZ, 1979
Fig. 8.11  HALF MONTHLY LOP ERROR DIFFERENCES, FARNBOROUGH - BUTT OF LEWIS DATA FOR LOP BD, 10.2KHZ, 1979
location in a similar manner to that presented in section 8.3 above. The magnitudes of these errors are then broadcast at VHF in the form of additional corrections to be applied to position fixes made within a limited range of the ground stations. For economy one requires the minimum number of ground stations and situated such that their acceptable 'correction range' is as large as possible.

These design parameters require an examination, like that presented above, of the LOP error differences and their biases between the proposed ground sites. It is clear that within the U.K. at least one Scottish Differential Omega station would have to be established to supplement any in the south of England.

8.5 REAL TIME UPDATING OF OMEGA POSITIONFixes

The magnitude and variability of the diurnal LOP errors have been investigated and the effectiveness of ONSOD predictions discussed. An alternative method of correcting the navigation errors can be postulated if the position of the receiver is accurately determined by some other method at certain times. For example, real time updates of position could be obtained from a satellite navigation system or from the re-transmission in a Differential Omega system. Errors due to the differences in receiver positions in the latter system have been examined in the previous two sections. The alternative technique of combining Omega with satellite navigation has already been developed by some commercial companies, and therefore a simulation of this type of updating method is of interest.

8.5.1 Omega/satellite receiver simulation

A navigation satellite can provide a very accurate position fix at certain times of the day, i.e. when the satellite is above the operator's horizon. From these fixes the errors in Omega LOPs at
the particular time and location of the observation can be determined. This information then provides an additional correction to the Omega position fixes and this can form the basis of a correction to be applied during the period when the satellite is not available.

The possible improvements achieved with such an Omega/satellite receiver have been examined by combining Omega LOP data previously presented with a simulated satellite aid. The satellite derived positions (or updates) are assumed to have no associated error and to be available at regular six hourly intervals (00:00, 06:00, 12:00 and 18:00 GMT). One further simulation for which the satellite updates are available hourly is also included.

The results of this analysis are presented in figures 8.12 to 8.16 in the form of circular error probability (cep) versus radial error in kilometers. Navigation radial errors are usually quantified in terms of a particular cep level; a radial error of 7 km (95% cep) being generally quoted for Omega (Maenpa, 1978). That is 95% of all Omega derived fixes (including PPCs) should be less than 7 km from the receiver's true location. The Admiralty have adopted a 98% cep level for their statements of system performance and therefore both 95% and 98% levels are indicated in the figures. A summary of these radial errors is presented in Table 8.2.

The data have been assessed by the four methods indicated below and illustrated in figures 8.12 to 8.16:

(a) Position fixes determined from totally uncorrected LOP data.
(b) Position fixes determined from LOP data combined with ONSOD PPCs.
(c) Position fixes determined from LOP data with satellite updates only.
(d) Position fixes determined from LOP data combined with ONSOD corrections and satellite updates.
LOPs AB, AD and BD have been utilized in this analysis.

8.5.2 Radial error distributions (hourly data, updates six-hourly)

(i) Farnborough monitor, 10.2 kHz (figure 8.12)

Radial error distributions recorded at Farnborough during the period January 1978 to June 1978 on 10.2 kHz are indicated in figure 8.12. The improvements possible through the application of methods (b) – (d) above are immediately apparent from the diagram, method (d) providing the most accurate position fixes. Note that with techniques (c) and (d), i.e. those involving satellite updates, about 17% of the position fixes have no associated radial error, and correspond to the times when the satellite update correction is first applied in each six-hourly interval.

(ii) Farnborough monitor, 13.6 kHz (figure 8.13)

Distributions at 13.6 kHz differ from those at 10.2 kHz, large radial errors being recorded in the 13.6 kHz uncorrected data. ONSOD corrections are more accurate at this higher frequency, producing small radial errors. Satellite updating combined with uncorrected data results in a radial error distribution not unlike the most accurate distribution indicated in figure 8.12 (satellite updates plus ONSOD corrected data, 10.2 kHz).

(iii) Leicester monitor, 10.2 kHz (figure 8.14)

Leicester 10.2 kHz radial error distributions over the period 24/3/79 to 13/8/79 are presented in figure 8.14. These data are compared below with those from the Butt of Lewis.

(iv) Butt of Lewis monitor, 10.2 kHz (figure 8.15)

Comparing figures 8.14 (Leicester, 10.2 kHz) and 8.15 (Butt of Lewis, 10.2 kHz) an interesting difference in the distributions for uncorrected data is apparent. The shallow ledge present in figure 8.15 is probably due to the more dominant phase change present over the
RADIAL ERROR DISTRIBUTIONS AT FARNBOROUGH, JAN 1978 - JUN 1979, 13.6KHZ
SATELLITE UPDATES EVERY 6 HOURS.

Fig. 8.13
RADIAL ERROR DISTRIBUTIONS AT LEICESTER, 24/3/79 - 13/8/79, 10.2KHz
SATELLITE UPDATES EVERY 6 HOURS, HOURLY DATA

Fig. 8.14
RADIAL ERROR DISTRIBUTIONS AT THE BUTT OF LEVIS, 24/3/79 - 13/8/79, 10.2KHZ
HOURLY LOP DATA. SATELLITE UPDATES EVERY 6 HOURS.

Fig. 8.15
Liberia path which tends to segregate the position fixes more effectively into two regions; one close to, and the other further away from, the receiver's true location.

These data further quantify the improved performance of the ONSOD corrections at the Butt of Lewis. Their accuracy here is such as to render unnecessary the inclusion of satellite updates at six hourly intervals.

(v) Butt of Lewis monitor, 13.6 kHz (figure 8.16)

A large bias is evident in the 13.6 kHz uncorrected Scottish data as indicated by figure 8.16. ONSOD corrections appear more effective in reducing radial errors at this site compared with those obtained on 10.2 kHz. The combination of both correction techniques (method (d) above) results in greater radial errors at large ceps. This is a result of the introduction of overcorrection by the satellite updates.

8.5.3 Comparisons with nominal Omega accuracy

The nominal value of radial error in Omega is 7 km at a cep of 95%. It is apparent from Table 8.2 that this is never achieved in the U.K. except when some form of correction is applied. Navigation at Farnborough with 10.2 kHz transmissions only just meets this nominal figure when both ONSOD PPCs and satellite updates are combined. The situation at 13.6 kHz is better due to greater accuracy of the ONSOD PPCs. Three situations are considered from the Leicester 1979 data, and discussed below:

(a) 10.2 kHz Leicester, hourly data for the period January to December 1979 is examined. Nominal Omega performance is achieved with either ONSOD PPCs or with satellite updates plus PPCs.

(b) 10.2 kHz Leicester, hourly data for the period 24/3/79 to 13/8/79. The trends here are essentially the same as those
<table>
<thead>
<tr>
<th>LOCATION</th>
<th>FREQ. (kHz)</th>
<th>DATA PERIOD</th>
<th>DATA TYPE</th>
<th>SATELLITE UPDATE INTERVAL</th>
<th>UNCORRECTED DATA</th>
<th>ONSOD CORRECTED</th>
<th>SATELLITE CORRECTED</th>
<th>ONSOD PLUS SATELLITE CORRECTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farnborough</td>
<td>10.2</td>
<td>Jan. 1978 - Jun. 1979</td>
<td>Hourly</td>
<td>6 hourly</td>
<td>11.5</td>
<td>8.6</td>
<td>8.9</td>
<td>7.0</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2</td>
<td>Jan. - Dec. 1979</td>
<td>Hourly</td>
<td>6 hourly</td>
<td>10.6</td>
<td>8.5</td>
<td>7.7</td>
<td>6.0</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2</td>
<td>24/3/79 - 13/8/79</td>
<td>Hourly</td>
<td>6 hourly</td>
<td>10.8</td>
<td>6.0</td>
<td>8.1</td>
<td>5.1</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2</td>
<td>Jan. - Dec. 1979</td>
<td>5 minute</td>
<td>Hourly</td>
<td>10.7</td>
<td>8.7</td>
<td>7.5</td>
<td>3.4</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>10.2</td>
<td>24/3/79 - 13/8/79</td>
<td>Hourly</td>
<td>6 hourly</td>
<td>11.2</td>
<td>5.7</td>
<td>7.6</td>
<td>4.2</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>13.6</td>
<td>24/3/79 - 13/8/79</td>
<td>Hourly</td>
<td>6 hourly</td>
<td>14.3</td>
<td>5.4</td>
<td>5.5</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 8.2 Measured radial errors at the three UK monitoring sites at 95% and 98% circular error probability levels (all radial errors in km)
above (January - December 1979), differences being accounted for by the reduction in the size of the data set.

(c) 10.2 kHz Leicester, five minute data recorded during 1979 but unlike situation (a) and (b), with hourly satellite updates. Under these conditions very marked improvements are possible with a Omega/satellite system, radial errors at best being reduced to 2.4 km (95% cep).

Finally, Butt of Lewis results (hourly data, six hourly updates) are indicated in Table 8.2, where at both 10.2 kHz and 13.6 kHz radial errors are about 4.2 km (95% cep) with just the addition of ONSOD corrections.

8.6 CONCLUSIONS

A detailed analysis of the experimentally determined performance of the Omega system at three locations within the U.K. has been undertaken. The validity of the ONSOD LOP corrections for these locations has been assessed by comparison with the observations and their diurnal and seasonal variations quantified. From this study a number of systematic differences and biases are apparent between the ONSOD predicted LOPs and the experimental data. Major predicted errors are evident at dawn (and to a lesser extent at dusk) for some of the propagation paths. Under certain situations the prediction program overestimates the day/night phase changes leading to further large systematic errors. The limitation of any long-term prediction program due to the day-to-day variability of the ionospheric D-region is also noted.

The changes in the receivers' apparent positions were determined from three LOPs. The variation in Omega performance accuracy was investigated by calculating the differences in the measured error of
the same LOP at three U.K. locations. From these analyses it is apparent that the performance of Omega at Farnborough and Leicester is very similar, but there are marked differences between the Butt of Lewis errors and those for the two southerly receivers.

The differences in radial errors in position have been further quantified in the context of an Omega/Satellite combined system. This analysis highlights the possible advantages and limitations of such a system, and has further emphasised the differences in performance within the U.K.

Further work is required to determine the origins of the errors noted in the prediction technique and to explain the differences observed in accuracy within the U.K. area. An attempt will be made in the next chapter to account for some of these features.
CHAPTER 9

Propagation Path Modelling and Concluding Notes

9.1 INTRODUCTION

The experimental data presented in the preceding two chapters have indicated that many factors affect the propagation of VLF radio waves of the Omega navigation system. These include signal frequency, path orientation (azimuth), path latitude, ground conductivity, solar zenith angle, solar and geomagnetic conditions. The relative importance of many of these parameters varies from path to path and with time, however it is clear from the propagation data that the dominant regular variation at VLF is the shape and size of the diurnal phase pattern.

The diurnal changes of the D-region of the ionosphere have been the subject of much research as indicated in Chapter 2. An understanding of the processes controlling the diurnal variations of the D-region is of prime importance in the development of any VLF prediction method such as that produced by ONSOD.

It is apparent from the study described in this thesis that a number of systematic errors are present in the ONSOD predictions for the U.K. In particular, predictions calculated for dawn periods may involve large discrepancies between Omega position fixes and a navigator's true position.

The ONSOD technique as described in Chapter 5, utilizes both VLF theoretical methods and experimental data collected at many different locations across the earth. VLF mode theory and full wave calculations (see Chapter 3) have both provided a means of determining the relative importance of the various geophysical functions within the ONSOD program. However, the implementation of these techniques during dawn and dusk transitions is complex and cumbersome. The Spherical Wave method (Chapter 3)
however, provides a means of incorporating discontinuities within the waveguide in a relatively simple manner. This more fundamental series is applied below in an attempt to model propagation conditions at dawn over both the Norway-U.K. and Liberia-U.K. paths described in Chapter 7. Comparisons with the experimentally obtained phase and amplitude variations over these paths reveals possible areas of D-region physics inadequately described by the ONSOD prediction program.

9.2 PROPAGATION PATH MODELLING WITH THE SPHERICAL WAVE TECHNIQUE

The Spherical Wave approach, described in Chapter 3, has been employed here in an attempt to model the propagation conditions over two particular great circle paths. Results from a series of different models are compared directly with experimental measurements made at Leicester as part of the Omega monitoring program and presented in Chapter 7.

The theoretical modelling follows the analysis of Johler (1970) in which an anisotropic ionosphere with vertically and horizontally varying electron density and a varying ground conductivity profile are considered. A number of D-region electron density profiles were selected from those presented in Chapter 2 and a collision frequency profile described earlier (Chapter 4) and based on the CIRA model atmosphere was adopted throughout. The Pitteway (1965) method was employed in determining the ionospheric reflection coefficients by full wave integration.

A ground conductivity map forming part of the ONSOD prediction program and described by Morris and Cha (1974) was incorporated to enable ground parameter profiles to be constructed. All Spherical Wave calculations were summed over ten terms \( j = 1 \) to \( 10 \) of the zonal harmonics series of equation 3.62. The approach of assigning local
reflection regions follows that described in Chapter 3 and discussed by Johler (1970) and Johler and Mellecker (1970).

9.2.1 Norway-Leicester path

From the waveguide and ionospheric models discussed above the temporal phase and amplitude variations in the Omega Norway 10.2 kHz signal at Leicester have been calculated for the 27 July 1978. The results of this analysis are illustrated in Figure 9.1 which also includes the corresponding experimental data recorded at Leicester for that day.

Five theoretical models are considered based upon the following assumptions:

Model 1

(i) Horizontally uniform ionospheric electron density structure. A single profile is assigned to the whole path according to the solar zenith angle at the path mid-point. The D-region profiles incorporated in this model are denoted in table 9.1.

(ii) Geomagnetic field parameters at the path mid-point are assumed constant over the whole path.

(iii) An appropriate ground conductivity (sea water for the Norway-Leicester path) is assumed constant over the whole path.

Model 2

(i) As assumption (i) of Model 1.

(ii) As assumption (ii) of Model 1.

(iii) A variable ground conductivity along the path, based upon the ONSOD conductivity map is incorporated. Spatial resolution of the map is $\frac{1}{2}^{\circ}$ in latitude and longitude (refer to Morris and Cha, 1974).

Model 3

(i) As assumption (i) of Model 1.

(ii) A horizontally variable geomagnetic field is added. The path
Fig. 9.1 Theoretical modelling - Norway-Leicester path, 10.2 kHz, 27th July 1978

Norway-Leicester, 10.2 kHz
27th July 1978

Phase

- Experimental data
- Models 1 to 4
- Model 5

Amplitude

- Experimental data
- Models 1 to 4
- Model 5

Sunrise (at 80 km altitude)
Sunrise (at 80 km altitude)
path mid-point

10 μs

5 db
is divided into ten regions, each of which is assigned the correct dip angle, geomagnetic azimuth and gyro frequency. Values of these parameters have been obtained from standard geophysical data maps (e.g. Valley, 1965).

(iii) As assumption (iii) of Model 1.

Model 4

(i) As assumption (i) of Model 1.
(ii) As assumption (ii) of Model 3.
(iii) As assumption (iii) of Model 2.

Model 5

(i) A variable D-region electron density profile along the propagation path is catered for. Ten regions are constructed, each being assigned an electron density profile according to the solar zenith angle within the region, following the scheme of table 9.1.
(ii) As assumption (ii) of Model 3.
(iii) As assumption (iii) of Model 2.

The theoretically calculated variations in phase and amplitude derived with models one to four above demonstrate very little change from model to model. These essentially do not include a horizontally inhomogeneous electron density and hence indicate that at 10.2 kHz over this path the influence of variable geomagnetic field and ground conductivity is small. Thus mid-path values of both of these parameters may be applied over the whole path.

Calculations derived with the fifth model yield a different result, particularly for signal phase. The transition in phase from night to day is slightly less rapid when a horizontally varying electron density profile is introduced. An important difference occurs just after D-region (80 kms) sunrise over the receiver (note that at this altitude, latitude and time
<table>
<thead>
<tr>
<th>Range in solar zenith angles (ground level values in degrees)</th>
<th>D-region electron density profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>-105°</td>
<td>Thomas &amp; Harrison (1970), ( \chi = 105° ) (see fig. 2.5, chapter 2)</td>
</tr>
<tr>
<td>98.5° - 105°</td>
<td>( \chi = 99° )</td>
</tr>
<tr>
<td>97.5° - 98.5°</td>
<td>( \chi = 98° )</td>
</tr>
<tr>
<td>96° - 97.5°</td>
<td>( \chi = 97° )</td>
</tr>
<tr>
<td>94° - 96°</td>
<td>( \chi = 95° )</td>
</tr>
<tr>
<td>92.25° - 94°</td>
<td>( \chi = 93° )</td>
</tr>
<tr>
<td>90.75° - 92.25°</td>
<td>( \chi = 91.5° )</td>
</tr>
<tr>
<td>88° - 90.75°</td>
<td>( \chi = 90° )</td>
</tr>
<tr>
<td>72° - 88°</td>
<td>Mechtly &amp; Smith (1970), ( \chi = 84° ) (see fig. 27)</td>
</tr>
<tr>
<td>40° - 72°</td>
<td>( \chi = 60° )</td>
</tr>
</tbody>
</table>

Table 9.1 Electron density profiles employed in models 1 to 5

<table>
<thead>
<tr>
<th>Range in solar zenith angles</th>
<th>D-region electron density profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>-90.75°</td>
<td>As in table 9.1 above (first seven profiles)</td>
</tr>
<tr>
<td>88.5° - 90.75°</td>
<td>Mechtly &amp; Smith (1970), ( \chi = 90° ) (see fig. 27)</td>
</tr>
<tr>
<td>85.5° - 88.5°</td>
<td>Profile obtained by averaging above (( \chi = 84° )) and below (( \chi = 60° )) profiles</td>
</tr>
<tr>
<td>78° - 85.5°</td>
<td>Mechtly &amp; Smith (1970), ( \chi = 84° )</td>
</tr>
<tr>
<td>66° - 78°</td>
<td>Profile obtained by averaging above (( \chi = 84° )) and below (( \chi = 60° )) profiles</td>
</tr>
<tr>
<td>40° - 66°</td>
<td>Mechtly &amp; Smith (1970), ( \chi = 60° )</td>
</tr>
</tbody>
</table>

Table 9.2 Modified ionospheric profile arrangement
of year the D-region above the transmitter is permanently sunlit).

After about 03.00 GMT models one to four indicate a phase advance for a period, followed by a phase retardation to stable daytime conditions (after about 04.30 GMT). This feature is not observed in the results obtained from the more sophisticated fifth model. It is also clear that the final model is in closer agreement with the experimental data recorded at Leicester on that day and included in the figure.

The importance of these results is clear; despite the small azimuth of the path (about 22° west of south as illustrated by figure 7.1), horizontal variations in electron density must be accounted for. The magnitude of the path azimuth is sufficient to produce a tilt in reflection height along the propagation path which affects the phase and amplitude as demonstrated above.

9.2.2 Liberia-Leicester path

A similar set of calculations to those of section 9.2.1 have been applied to the Omega Liberia to Leicester path. The models incorporated the same D-region profile structure as denoted by table 9.1. 10.2 kHz data recorded on the 27th July 1978 at Leicester are presented together with the theoretical results from the five models in figure 9.2. As before, very little difference is observed in the predicted variations from models one to four.

Model five results indicate a more gradual dawn transition in phase compared with the theoretical results from the earlier models. Agreement between the experimental data and the final model appear excellent. There is as before no large sunrise phase reversal predicted by the more sophisticated model.

The results of these calculations for both the Norway-Leicester and Liberia-Leicester paths demonstrate the possible improvements in prediction accuracy when local changes in solar zenith angle, and
Fig. 9.2 Theoretical modelling - Liberia-Leicester path, 10.2 kHz, 27th July 1978
associated D-region electron density, along near north-south or south-north paths are accounted for. It is important to note however that the phase reversal features observed in the simpler models are not thought to be connected with the 'sunrise dump' phenomena described in previous chapters (2, 5 and 7). The more sophisticated model five does not include this feature and is in fact in closer agreement with the experimental data, which also does not include a sunrise 'dump'. In the simpler models the choice of the 'correct' D-region profile to describe the whole path is in practice impossible and leads to the erroneous results indicated in figure 9.2.

These calculations confirm the secondary importance of variations in the geomagnetic field and ground conductivity for these paths.

9.2.3 Liberia-Butt of Lewis path

It is apparent from discussions in the previous chapter that within the U.K. differences are observed in the magnitude of the diurnal change in LOP AB. A line of position requires phase measurements on two propagation paths, in this case, Norway-U.K. and Liberia-U.K. As two LOPs have been recorded and compared (e.g. Leicester LOP AB and Butt of Lewis LOP AB) a total number of four propagation paths are involved.

The phase measurements at Leicester (Chapter 7) are relative to a rubidium frequency standard, and records exist for each individual propagation path. At the Butt of Lewis, however, the frequency standard was a crystal oscillator, therefore rigorous long period (> a few days) phase records could not be obtained due to oscillator drift. A single day has thus been selected from the data to serve as an example for the following modelling calculations.

The Norway and Liberia phases and LOPs AB recorded at Leicester and the Butt of Lewis for the 12th June 1979 on 10.2 kHz are reproduced in figure 9.3. This figure demonstrates that much of the contribution
Fig. 9.3 Diurnal phase and LOP variations recorded at Leicester and the Butt of Lewis on 12th June 1979 at 10:2 kHz
of the difference in magnitude of the diurnal shape of LOP AB at the
two sites is a consequence of differences over the two Liberia-U.K.
paths. Thus at Leicester a diurnal phase change of about 52 cecs is
recorded while at the Butt of Lewis it is approximately 68 cecs. The
difference is 16 cecs or about 16 µs. The Liberia-Leicester and Liberia-
Butt of Lewis paths are separated by only about 5° in azimuth, (figure
7.2), however the latter path is longer by 595 km. The contribution to
the 16 cecs diurnal pattern difference due to the change in path length
is examined below.

Burgess and Jones (1975) have indicated the manner in which
the magnitude of the diurnal phase variation changes with distance at
10.2 kHz and 16 kHz. These results, reproduced in figure 9.4, demonstrate
that under single moded propagation conditions for their waveguide model
there is an increase in diurnal phase delay variation of about 8 µs
over the distance ranges between Leicester and the Butt of Lewis. This
accounts for only about half of the 16 cecs change observed in the data.

The waveguide model utilized by Burgess and Jones (1975)
incorporated exponential Wait profiles as described in Chapter 4.
While these may give a reasonable representation of the propagation
characteristics it is of interest to determine what effects more realistic
D-region profiles may produce. An attempt has therefore been made to
calculate the phase changes over the two Liberia-U.K. paths for the day
indicated in figure 9.3.

The Spherical Wave modelling approach described above has again
been employed with an extended D-region profile arrangement illustrated
in table 9.2. The calculations are made for model five only, which
includes both vertically and horizontally inhomogeneous electron
densities, and varying ground conductivity and geomagnetic field
parameters. The results are presented in figure 9.5 which also includes
Fig. 9.4 Magnitude of diurnal phase delay variation versus distance for 10.2 kHz and 16 kHz (after Burgess & Jones, 1975)
Fig. 9.5 Theoretical modelling -
Comparisons of predicted dawn phase variations (broken line) and experimental recorded data (full line) collected at Leicester and the Butt of Lewis

Data - Liberia, phase,
10.2 kHz, 12th June 1979
the experimental data recorded on the 12th June 1979. The theoretical and experimental results agree quite closely for the Liberia-Leicester path. Over the Liberia-Butt of Lewis path there is still a difference of some 6 cecs.

9.3 SOURCES OF ERROR WITHIN THE ONSOD PREDICTION PROGRAM

Phase errors due to propagation effects are of two kinds. First, there are regular and generally slow changes due to diurnal and seasonal variations in D-region electron density profiles. Secondly, there are transient changes which result from ionospheric disturbances associated with events such as solar flares and geomagnetic storms. These events are generally short-lived ranging in duration from a few minutes to a few days. The ONSOD program is constructed to predict errors of the former type and is unable to account for transient phenomena. However, data presented in the previous chapter have indicated that there must be other areas where the technique inadequately predicts VLF phase. In particular, the dawn transition over the Liberia-U.K. path is thought to be a major source of systematic errors at certain times of the year.

The modelling carried out above indicates that through a more suitable choice of D-region profiles and a greater understanding of dawn processes it may be possible to predict more accurately the phase changes at these times.

In order to quantify the sources of error within the prediction program, reference is made to equation 5.7 of Chapter 5. This equation demonstrates how spatially and temporally dependent terms are partitioned and how the variations of these parameters are determined through a weighting function. The temporally dependent terms are therefore calculated by summing a series of relevant geophysical functions by
the diurnal function illustrated in figure 5.14.

To illustrate the relative magnitudes of the terms of this summation the ONSOD program was modified to produce predictions every five minutes. One day (1 March 1978) was chosen to indicate the contributions of the time dependent terms for the Liberia-Leicester path on 10.2 kHz. These are presented in figure 9.6 (a). The following figure, 9.6 (b), is reproduced from the experimentally recorded Liberia data for that day. It is clear from the figures that a major source of error within the prediction program is the geophysical function included to account for ionospheric time constants and anomalous 'dumps'.

9.4 CONCLUDING NOTES

In earlier chapters of this study evidence was presented for a wide variety of factors affecting the propagation characteristics of VLF radio waves. These discussions are of relevance in determining the performance of the VLF Omega navigation aid and such previous work has to some extent determined the present configuration and resultant operation of the current system. This is particularly true of the ONSOD prediction program.

The experimental data collected within the U.K. and described in subsequent chapters of this thesis have demonstrated that an increased knowledge of the quiescent ionosphere including its spatial and temporal variations is needed if Omega is to be improved to an acceptable level through the application of predicted corrections. Greater understanding of and improved predictions for sunrise and sunset variations, high latitude propagation (auroral zone and polar cap) and modal interference conditions are required. It is apparent from the results presented in this final chapter that the incorporation of more realistic dawn ionospheric models would immediately produce improvements in many
Time dependent terms

Term 1 - variation over all seawater path with mid-path geophysical functions
Term 2 - ground conductivity dependence
Term 3 - excitation/de-excitation zone dependence
Term 4 - ionospheric time constants and dump schedule dependence

Fig. 9.6(a) ONSOD predicted propagation correction — Liberia-Leicester path, 10.2 kHz, 1st March 1978
Fig. 9.6(b) Diurnal phase variation of the Omega Liberia 10.2kHz signal measured at Leicester on 1st March, 1978
position fix accuracies.

Ultimately however, even with these improvements the day-to-day variability of the D-region has been demonstrated in the previous chapter to set a limiting factor on any long term prediction method based on mean models. Significant (to a navigator) variability may exist even on ionospherically 'quiet' days while, during solar and geomagnetic disturbances, very large errors in position may be introduced.

Real time updating provides one means of reducing the influence of day-to-day variability and transient phenomena. The Differential Omega system described in the previous chapter is one such updating technique, however results obtained in the U.K. and presented in this study indicate that the operational range of this method may be uneconomical.

The investigation into the performance of a hybrid Omega/Satellite navigation aid has revealed that accurate ONSOD predictions (e.g. those at the Butt of Lewis) may result in position fixes of equal quality to a combined system. The controlling factor for such a combination is the updating interval; hourly updates providing substantially improved fixes compared with Omega alone. The results of such investigations are of great importance as hybrid Omega/Satellite receivers are envisaged for the future.

9.5 **SUGGESTED FURTHER STUDIES**

In view of the comments made above, it is apparent that further work is required to clarify the causes of the errors within the ONSOD prediction technique when applied in the U.K. This present study has served to indicate the general areas in which further investigations should be directed. It is clear that an extension of the North of Scotland data set (e.g. to cover the winter months) and, in particular,
the recording of phase rather than IOP would be considered a priority. The establishment of additional monitoring sites, especially to supplement the Butt of Lewis, would provide valuable data.

With an extended data set a statistical analysis of the occurrence of transient phenomena such as solar flares may be correlated with the known characteristics of the underlying cause (e.g. X-ray flux). From such an investigation an estimate of the duration of such events following commencement and detection may be possible.

Further theoretical work is required, particularly as the diurnal functions and sunrise/sunset dump schedules within the ONSOD algorithm appear to require modification. Investigations with the Spherical Wave technique may aid this work, but more particularly, this analytical method is considered to be of value in predicting situations of modal interference.

It is expected that further simulations of hybrid systems would be possible with more data, including further frequencies and propagation paths. Such calculations may be of great value to the users of the Omega system.
## APPENDIX A

### Comparisons of Measured and Predicted LOP Errors

#### List of Contents

<table>
<thead>
<tr>
<th>Location</th>
<th>Frequency</th>
<th>Time Period</th>
<th>Figure No(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farnborough</td>
<td>10.2 kHz</td>
<td>LOP AB, January 1978-August 1979</td>
<td>A.1(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>10.2 kHz</td>
<td>LOP AD, January 1978-August 1979</td>
<td>A.2(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>10.2 kHz</td>
<td>LOP BD, January 1978-August 1979</td>
<td>A.3(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>10.2 kHz</td>
<td>LOP AF, January-December 1978</td>
<td>A.4(a)-(c)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>13.6 kHz</td>
<td>LOP AB, January 1978-August 1979</td>
<td>A.5(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>13.6 kHz</td>
<td>LOP AD, January 1978-August 1979</td>
<td>A.6(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>13.6 kHz</td>
<td>LOP BD, January 1978-August 1979</td>
<td>A.7(a)-(e)</td>
</tr>
<tr>
<td>Farnborough</td>
<td>13.6 kHz</td>
<td>LOP AF, January-December 1978</td>
<td>A.8(a)-(c)</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2 kHz</td>
<td>LOP AB, 16-30 June &amp; 1-15 December 1979</td>
<td>A.9(a),(b)</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2 kHz</td>
<td>LOP AD, 16-30 June &amp; 1-15 December 1979</td>
<td>A.10(a),(b)</td>
</tr>
<tr>
<td>Leicester</td>
<td>10.2 kHz</td>
<td>LOP BD, 16-30 June &amp; 1-15 December 1979</td>
<td>A.11(a),(b)</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>10.2 kHz</td>
<td>LOP AB, May-August, 1979</td>
<td>A.12</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>10.2 kHz</td>
<td>LOP AD, May-August, 1979</td>
<td>A.13</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>10.2 kHz</td>
<td>LOP BD, May-August, 1979</td>
<td>A.14</td>
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<td>Butt of Lewis</td>
<td>10.2 kHz</td>
<td>LOP AF, May-August, 1979</td>
<td>A.15</td>
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<tr>
<td>Butt of Lewis</td>
<td>13.6 kHz</td>
<td>LOP AB, May-August, 1979</td>
<td>A.16</td>
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<tr>
<td>Butt of Lewis</td>
<td>13.6 kHz</td>
<td>LOP AD, May-August, 1979</td>
<td>A.17</td>
</tr>
<tr>
<td>Butt of Lewis</td>
<td>13.6 kHz</td>
<td>LOP BD, May-August, 1979</td>
<td>A.18</td>
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<td>Butt of Lewis</td>
<td>13.6 kHz</td>
<td>LOP AF, May-August, 1979</td>
<td>A.19</td>
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Fig. A.1(a) Half Monthly LOP Errors Measured at Farnborough. Data for LOP AB, 10.2kHz, 1978.
Fig A.1(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 10.2KHZ, 1978

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
Fig.A.1(c) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 10.2KHZ, 1978
SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION

Fig.A1(d) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 10.2KHZ, 1979
Fig. A.1(e) Half Monthly LOP Errors Measured at Farnborough.

Data for LOP AB, 10.2 kHz, 1979
Fig. A.2(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 10.2KHz, 1978.
Fig A.2(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 10.2KHZ, 1978

SOLID LINE - LOP DATA, BROKEN LINE - D.N.S.O.D. PREDICTION
SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION

Fig. A.2(c) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 10.2KHz, 1978
Fig. A.2(d) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 10.2KHz, 1979
Fig. A.2(e) Half monthly LOP errors measured at Farnborough. Data for LOP AD, 10.2kHz, 1979

- Solid line - LOP data
- Broken line - O.N.S.O.D. prediction
Fig. A.3(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 10.2KHZ, 1978
SOLID LINE - LOP DATA, BROKEN LINE - D.N.S.O.D. PREDICTION

Fig. A3(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 10.2KHZ, 1978
Fig. A.3(c) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.  
DATA FOR LOP BD, 10.2kHz, 1978
Fig. A.3(d) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 10.2KHz, 1979
Fig. A.3(e) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.  
DATA FOR LOP BD, 10.2KHZ, 1979
Fig. A.4(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AF, 10.2KHZ, 1978
Fig A.4(b) Half Monthly LOP Errors Measured at Farnborough.

Data for LOP AF, 10.2 KHz, 1978
Fig. A.4(c) Half monthly LOP errors measured at Farnborough.
Data for LOP AF, 10.2kHz, 1978
Fig.A.5(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 13.6KHZ, 1978
Fig. A5(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 13.6 KHZ, 1978
SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION

Fig. A.5(c) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 13.6KHz, 1978
Fig. A5(d) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 13.6KHZ, 1979

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION

1-15 JAN.

1-14 FEB.

1-15 MAR.

1-15 APR.

16-31 JAN.

15-26 FEB.

16-31 MAR.

16-30 APR.
Fig. A.5(e) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AB, 13.6KHZ, 1979
Fig. A.6(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 13,600KHZ, 1978
Fig. A.6(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 13.6kHz, 1978
Fig. A.6(c) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 13.6KHz, 1978
Fig. A.6(d) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 13.6KHZ, 1979
Fig. A.6(e) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AD, 13.6KHz, 1979
Fig. A.7(a) Half Monthly LOP Errors Measured at Farnborough. Data for LOP BD, 13.6KHz, 1978
Fig. A.7(b) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH. DATA FOR LOP BD, 13.6KHZ, 1978

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION

Fig. A.7(c)  HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 13.6KHZ, 1978
Fig. A.7(d)  HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 13.6KHZ, 1979

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
Fig. A.7(e) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP BD, 13.6KHz, 1979
Fig. A.8(a) HALF MONTHLY LOP ERRORS MEASURED AT FARNBOROUGH.
DATA FOR LOP AF, 13.6KHZ, 1978
Fig. A.8(b) Half Monthly LOP Errors Measured at Farnborough. Data for LOP AF, 13.6kHz, 1978.
Fig. A.8(c) Half Monthly LOP Errors Measured at Farnborough. Data for LOP AF, 13.6KHz, 1978

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
- Fig. A.9(a) -

LEICESTER NV1 LOP ERROR AB, 10.2 KHz, 16-30 JUN. 1979

- Fig. A.9(b) -

LEICESTER NV1 LOP ERROR AB, 10.2 KHz, 1-15 DEC. 1979
LEICESTER NV1 LOP ERROR BD, 10.2 KHz, 16-30 JUN. 1979
Fig. A.11 (a)

LEICESTER NV1 LOP ERROR BD, 10.2 KHz, 1-15 DEC. 1979
Fig. A.11 (b)
Fig. A.12  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.  
DATA FOR LOP AB, 10.2KHZ, 1979
Fig. A.13  Half Monthly LOP Errors Measured at the Butt of Lewis.
Data for LOP AD, 10.2 kHz, 1979
Fig. A.14  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.  
DATA FOR LOP BD, 10.2KHz, 1979.
Fig. A.15  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.
DATA FOR LOP AF, 10.2KHZ, 1979
Fig. A.16  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.
DATA FOR LOP AB, 13.6KHZ, 1979

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
Fig A.17  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.
DATA FOR LOP AD, 13.6KHz, 1979
Fig. A.18  HALF MONTHLY LOP ERRORS MEASURED AT THE BUTT OF LEWIS.  
DATA FOR LOP BD, 13.6 KHZ, 1979

SOLID LINE - LOP DATA, BROKEN LINE - O.N.S.O.D. PREDICTION
Fig. A.19. Half monthly LOP errors measured at the butt of Lewis. Data for LOP AF, 13.6 KHz, 1979.
## APPENDIX B

**Position Fix Distributions**

**List of Contents**

<table>
<thead>
<tr>
<th>Omega lane boundary geometry and lane widths at the U.K. monitoring sites</th>
<th>Figure No(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Farnborough, 10.2 kHz, January 1978-June 1979</td>
<td>B.2(a)-(c)</td>
</tr>
<tr>
<td>Farnborough, 13.6 kHz, January 1978-June 1979</td>
<td>B.3(a)-(c)</td>
</tr>
<tr>
<td>Leicester, 10.2 kHz, 24 March-13 August 1979</td>
<td>B.4(a)-(c)</td>
</tr>
<tr>
<td>Butt of Lewis, 10.2 kHz, 24 March-13 August 1979</td>
<td>B.5(a)-(c)</td>
</tr>
<tr>
<td>Butt of Lewis, 13.6 kHz, 24 March-13 August 1979</td>
<td>B.6(a)-(c)</td>
</tr>
</tbody>
</table>
Fig. B.1 Omega lane boundary geometry and lane widths at the U.K. monitoring sites.
Fig. B.2(a)

Position Fix Distributions - Farnborough, Jan 1978 - Jun 1979, 10.2KHz

Uncorrected Data  Corrected Data
Fig. B.2(b)  

POSITION FIX DISTRIBUTIONS - FARNBOROUGH, JAN 1978 - JUN 1979, 10.2KHZ
POSITION FIX DISTRIBUTIONS - FARNBOROUGH, JAN 1978 - JUN 1979, 10.2KHZ
POSITION FIX DISTRIBUTIONS - FARNBOROUGH, JAN 1978 - JUN 1979, 13.6KHZ
Fig. B.3(b)

Position Fix Distributions - Farnborough, Jan 1978 - Jun 1979, 13.6kHz
Fig. B.3(c)

POSITION FIX DISTRIBUTIONS - FARNBOROUGH, JAN 1978 - JUN 1979, 13.6KHZ

UNCORRECTED DATA

ONSOD CORRECTED DATA
-B.4(a)-

Fig. B.4(a)

UNCORRECTED DATA

DMSOD CORRECTED DATA

POSITION FIX DISTRIBUTIONS - LEICESTER, 24/3/79 - 13/8/79, 10.2KHZ
UKICORRECTED DATA  Fig B.4 (c) ONSDD CORRECTED DATA

POSITION FIX DISTRIBUTIONS - LEICESTER, 24/3/79 - 13/8/79, 10.2KHZ
Fig. B.5(a)

Position Fix Distributions - Butt of Lewis, 24/3/79-13/8/79, 10.2KHz

- Uncorrected Data
- Corrected Data
UNCORRECTED DATA

Fig B.5 (b)

ONSOD CORRECTED DATA

POSITION FIX DISTRIBUTIONS - BUTT OF LEWIS, 24/3/79-13/6/79, 10.2KHZ
POSITION FIX DISTRIBUTIONS - BUTT OF LEWIS, 24/3/79-13/8/79, 10.2KHZ
Fig. B.6(a) UNCORRECTED DATA
POSITION FIX DISTRIBUTIONS - BUTT OF LEWIS, 24/3/79-13/8/79, 13.6KHZ

UNCORRECTED DATA

ONSOG CORRECTED DATA

0.00 GMT
2.00 GMT
4.00 GMT
6.00 GMT
Fig. B.6(b)

POSITION FIX DISTRIBUTIONS - BUTT OF LEWIS, 24/3/79-13/8/79, 13.6KHZ
Fig. B.6(c)

POSITION FIX DISTRIBUTIONS - BUTT OF LEWIS, 24/3/79-13/8/79, 13.6KHZ

UNCORRECTED DATA

ONSDD CORRECTED DATA
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Radio waves in the Very Low Frequency (VLF) band (3-30kHz) are reflected from the lowest part of the ionosphere, the D-region. Due to their relative stability and low attenuation they are utilized for navigation, timing and frequency comparisons.

Any changes in D-region electron density structure are known to influence the phase velocity of VLF waves. Both regular diurnal and irregular behaviour have been extensively reported.

A comparison of theoretical techniques is presented, demonstrating the influence of changes in radio frequency, ground conductivity, geomagnetic path azimuth and latitude.

Described in this thesis is an analysis of the experimentally determined performance, within the U.K., of the VLF navigation system 'Omega'. The system relies on the stability of the phase velocity of the transmitted signals to provide a navigation fix. Both diurnal and seasonal variations in signal phase are investigated while the effects of modal interference and ionospheric disturbances are noted.

In order to reduce errors resulting from regular variations in phase velocity, a prediction technique has been developed by the Omega Navigation System Operations Detail (ONSOD). An assessment of the validity of ONSOD predictions within the U.K. is presented and it is apparent that a number of differences exist between them and the experimental data.

Major errors are evident at dawn over certain near north-south paths and marked differences may occur between data collected at particular sites. The relevance of these differences to Differential Omega is noted and an investigation into the performance of an Omega/Satellite combined system is conducted.

The variations within the U.K. are not completely accounted for by changes in propagation path length although theoretical investigations indicate the importance of changes in electron density structure along the whole path. It is evident that the sunrise schedules of the ONSOD prediction program require modification.