SECTOR BOUNDARY PASSAGES AND
JUPITER'S DECAMETRIC RADIATION

A Thesis Submitted for
The Degree of Doctor of Philosophy

by

E. R. PEKUNLU

Department of Astronomy
University of Leicester

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CHAPTER I
INTRODUCTION

Jovian radio observations of both DAM and the weaker
decimetric emission have provided the only information about the
plasma environment of Jupiter until the recent Pioneer flybys. At the
same time, our understanding of terrestrial magnetospheric processes
has now become very extensive. Since the magnetospheric features and
the plasma environment of the Earth and Jupiter are quite similar, it
therefore seems reasonable to use methodology based on the former to
explore qualitatively and quantitatively the characteristics of the
latter.

Jovian DAM bursts show high intensities, limited bandwidths,
short time-scales, and a sporadic nature. These characteristics of
the emission indicate that the origin is stimulated emission from
micro-instabilities. The dynamic spectra repeat in a remarkable
manner under similar geometrical conditions. This implies that the
structure of the ambient medium is stable in the long-term. According
to Smith (1976), "the repeatability of DAM spectra suggests that the
instability occurs for waves of frequency at, or related to, one of
the characteristic frequencies of the plasma, as is commonly the case.
In the high-frequency regime, these are the electron plasma frequency
$f_{pe}$, the electron gyrofrequency $f_{ce}$, and the upper hybrid frequency
$f_{UH} = (f_{pe}^2 + f_{ce}^2)^{1/2}$. At Jupiter, the DAM frequency range may be
identified with these characteristic frequencies only near the planet:
roughly speaking, at altitudes of $\lesssim 1 R_J$.

Finally, the wave propagation effects are expected to exert
an observable influence on the fine structure of the Jovian dynamic
spectra.
The present thesis explores three phenomena related to Jovian DAM emission. In Chapter II, the effect of the IMF sector boundary crossing on the occurrence probability of Jovian DAM emission will be examined, and compared with the comparable phenomena for the Earth. After considering the effect of sector boundary crossings on populating the radiation belts, we naturally move on to the problem of radial diffusion, to be examined in Chapter III. The results from this can be used in the final relevant subject - the wave propagation characteristics. In Chapter IV, using the "cold" and "warm" plasma models, we therefore give some explanation of the observed fine structure of Jovian DAM bursts.
CHAPTER II
THE INTERPLANETARY MAGNETIC FIELD (IMF) AND ITS RELATIONSHIP TO THE JOVIAN DECAMETRIC (DAM) EMISSION

1. SOLAR ORIGIN OF IMF

The magnetic field of the Sun shows two distinct large-scale features: the polar fields and the "solar sector structure" (see Ch. II.2). Both features exhibit spatial and temporal recurrence on time scales up to several years. The magnetic field intensity is weak in both cases, and, hence, the structures are characterized by "open" field lines carried out by the solar wind (see Ch. III.1). Neutral sheets, or current sheets, occur between regions containing oppositely directed open field lines, and separate the fields. Neutral sheets are identified as sector boundaries and they have a considerable extent in latitude; perhaps up to 40° on both sides of the equator. The existence of these boundaries have been confirmed by Wilcox (1968) from spacecraft observations at 1 A.U. Parker (1958) showed in his theoretical study of the hydrodynamic expansion of the corona that the kinetic energy of the solar wind plasma as it leaves the Sun decreases as $r^{-2}$, whereas the magnetic energy density must decrease as $r^{-4}$. It follows, therefore, that the weak magnetic field of the Sun will not significantly influence the motion of the outflowing gas once the gas leaves the solar corona. The large electrical conductivity of the solar wind plasma causes the solar magnetic field lines to be "frozen into" this streaming plasma (i.e., $E + V/c \times B$), and so stretched out radially away from the Sun. Parker (1958) showed that the combination of a solar wind and the solar rotation actually distorts the interplanetary magnetic field lines into an Archimedean spiral.
2. LARGE-SCALE SPATIAL STRUCTURE OF IMF

IMP-1 observations (Wilcox and Ness, 1965) suggest that the direction of the IMF, as observed near the Earth, is either predominantly away from the Sun, or towards the Sun (along Parker's theoretical spiral angle) for several days (on the average 6 - 7 days). This pattern repeats itself every 27 days, and is referred to as the "sector structure". The polarity of the field in the sector structure is labelled as positive when the average vector field is directed outward along the spiral, and as negative when the field direction is inwards towards the Sun. Interplanetary magnetic probes - Mariner 2 (Neugebauer, M. and Snyder, C.W., 1966); IMP-1 (Wilcox, J.M. and Ness, N.F., 1965) and many others - have reported that the polarity can fluctuate backwards and forwards in a few hours, but can also remain the same for 6 - 7 days, even for two weeks. IMP-1 (Ness et al., 1964) measurements have confirmed that the field lines have the same configuration as the Archimedean spiral geometry, and the IMF sector structure corotates with the Sun. Later, IMP-3 and Pioneer-VI (Ness, 1966) observations clearly established that much of the structure of the interplanetary field is indeed corotating with the Sun.

The boundaries separating the "toward" and "away" sectors are reported to be remarkably thin: Wilcox and Ness (1965) observed that the 180° field reversal at the boundary crossing takes place between successive 5-minute readings. Hence, it is important to consider the possible effect of magnetic field merging between the two neighbouring and oppositely directed sectors of the IMF. Hill (1975), using typical observed parameters at the Earth's orbit, concluded that the IMF sector structure "would survive to at least the orbit of Jupiter even if merging proceeded at the maximum possible rate". Hence, in this study we will assume that the IMF sector
structure is corotating with the Sun, and that it preserves its pattern as far as Jupiter's orbit.

3. IMF SECTOR STRUCTURE EFFECTS ON PLANETARY MAGNETIC FIELDS.

a) GEOMAGNETIC ACTIVITY

It has long been clear that changes in geomagnetic activity and the radiation belts are closely related to the passage of the IMF sector boundary. It appears from the IMP-1 (Wilcox and Ness, 1965) observations that geomagnetic activity reaches a maximum at about the second day of the sector, and then declines monotonically. There is a remarkable similarity between geomagnetic activity variations in "toward" sectors and "away" sectors. Mariner-IV (Rothwell and Greene, 1966) measurements have clearly shown that the boundary of the stable trapping zone moves outwards towards higher L values (where L is the distance measured from the centre of the Earth in Earth radii) around the time of each sector boundary crossing, and reaches its maximum distance from the Earth about a day and a half after the boundary. Williams (1966) found that the major electron intensity increases are observed at the arrival of sector boundaries in which the field changes from towards the Sun to away from the Sun. Near a sector boundary where the IMF reversal occurs, disturbances in magnetic fields, solar wind velocity, particle density distribution are more likely to occur. The energy input to the magnetosphere is greatly increased. Lin and Anderson (1966) and Krimigis et al. (1967) noted that low energy electrons ( \( > 40 \text{ KeV} \)) and protons (approximately \( 0.5 \text{ MeV} \)) have full and essentially immediate access from interplanetary sector boundary to some geomagnetic field lines. Hence, geomagnetic activity is expected to be organized in a similar manner within a sector. Figure (II.1) shows that this is indeed the case. The
Reproduced from Wilcox & Colburn, 1972. Superposed epoch analysis of the magnitude of the planetary magnetic 3-hour range indices Kp as a function of position with respect to a sector boundary. The abscissa represents position with respect to a sector boundary, measured in days, as the sector pattern sweeps past the Earth.
geomagnetic activity has a minimum just before the boundary and increases to a maximum approximately one day after the boundary. The source of recurrent geomagnetic activity can therefore be identified with the IMF sector structure.

What is more interesting is the close correlation between the "terrestrial kilometric radiation" (TKR) and the geomagnetic activity. The IMP-6 (launched on March 13, 1971) and IMP-8 (launched on October 26, 1973) satellites have shown that the Earth emits intense radio waves in the frequency range of about 50 - 500 kHz. The total power emitted in this frequency range is $10^9$ W, and comparable with the decametric radio emission from Jupiter, $2 \times 10^7$ W (Gurnett, 1974; Gurnett, 1975; Palmadesso, 1976; Benson, 1975). Kurth et al. (1975) have observed temporal variations in the source position of TKR: these variations are thought to be associated with the spatial variation of sector boundary, more precisely with the position of sector boundary with respect to Earth. Since the TKR has many features in common with the Jovian DAM emission, we shall look, in the next subsection, into the question of whether any correlation exists between the IMF sector boundary passage across Jupiter's orbit and Jovian DAM emission. (They are both believed to be generated at low planetary altitudes where the characteristic frequencies of the plasma are comparable to the radiation frequency; they are both sporadic, beamed, elliptically polarised, and the total power emitted by TKR in the frequency range 50 - 500 kHz is, as has already been pointed out above, comparable.)

b) JOVIAN ACTIVITY

Our primary goal is to determine whether there is an appreciable influence exerted by the IMF sector boundary on the
occurrence probability of Jovian DAM emission.

The first step in the procedure is to find out the times at which sector boundary passage occurs at the orbit of Jupiter. For this, we assume that the IMF sector structure is corotating with the Sun (an assumption that we have justified in Section II.2). We need to determine for each 27-day period at the Earth the corresponding period during which Jupiter would be most likely to encounter solar wind plasma from the same sources, and in the same sequence, as the Earth. Gosling et al. (1976) have shown that changes in the yearly average speed of solar wind in the 13-year period 1962-1974 were not large. Thus, we can assume that the solar wind has a constant radial streaming velocity, \( \dot{V}_s = V_{sr} \). Then \( \Delta t \), the time interval from the arrival at the Earth to the arrival at Jupiter of a long-lived plasma stream from a particular solar meridian, is given by

\[
\Delta t = \left[ \frac{(r_J - r_E)}{V_{sr}} \right] + \left[ \frac{(\theta_J - \theta_E)}{\Omega} \right]
\]

where the subscripts \( J \) and \( E \) refer to Jupiter and the Earth, respectively; \( \Omega \) is the angular velocity of the Sun, assumed here to be constant and independent of the heliocentric latitude, \( \theta \); \( r_J (r_E) \) and \( \theta_J (\theta_E) \) are the heliocentric distance of Jupiter (Earth) and the heliocentric longitude of Jupiter (Earth). It is readily seen from equation (II.3.1) that the delay time, \( \Delta t \), is strongly dependent on the heliocentric longitudes of the two planets. Heliocentric longitudes of Jupiter and the Earth are tabulated in the Astronomical Ephemeris. Jupiter's heliocentric longitude is given by 10-day intervals; and knowing its daily motion we have first found the daily heliocentric longitude of Jupiter throughout the period 1967-1974. The difference, \( r_J - r_E \), between the Sun-Jupiter distance and the Sun-Earth distance is also found in the Ephemeris, on a daily basis. We have thus found the
daily delay time, $\Delta t$, which we have used in finding the time at which the sector boundary passage occurs at Jupiter.

Svalgaard (1976) has tabulated the sector boundary crossings at Earth in "An Atlas of Interplanetary Sector Structure 1947-1975". Svalgaard compiled his data from the interplanetary sector polarity observations made by spacecraft over various periods [ Wilcox and Colburn (1972) for the interval 1962-1969; Fairfield and Ness (1974) for the interval 1970-1972; Hedgecock (1975) for the interval 1969-1974 ]. Svalgaard used additional spacecraft data obtained from the National Space Science Data Centre to fill in the gaps and to check the accuracy of the compilations. A small part of the data contained in the Atlas was inferred from geomagnetic observations made at Alert, Godhavn, Mould Bay, Pionerskaya, Resolute Bay, Thule and Vostok. The time resolution of the compilations varies; it is three hours for spacecraft data and one day for inferred polarity.

Svalgaard has chosen one day as the time resolution for the Atlas in order that data from various sources may be combined to describe the large-scale sector structure. It should also be mentioned that, for a field reversal to be classified as a sector boundary, Svalgaard generally required that the polarity be the same for at least 4 days before the reversal and that it remain reversed for at least 4 days following the reversal.

By using Svalgaard's data and the delay time, $\Delta t$, we have found the corresponding sector boundary crossings at Jupiter.

Systematic observations of Jovian DAM emission have been made at a number of stations operated by the Goddard Space Flight Centre [ viz. Goddard Space Flight Centre (Greenbelt, Maryland), Clark Lake Radio Observatory (Sorrego Springs, Calif.) and at Manned Space Flight Network stations in Kauai, Hawaii; Carnarvon, Australia;
II.7

and Grand Canary Is., Spain]. Observations are made at 16.7 and 22.2 MHz. In addition to the above data, we employ here information obtained by the University of Colorado's radio spectrograph. This station is operated independently of the Goddard Space Flight Centre, but the frequency range covers the DAM emission of Jupiter.

Knowing the time at which IMF sector boundary crossings occur at the orbit of Jupiter, and the time at which Jupiter emits DAM emission, we can plot histograms showing the relationship between the two: the sector boundary and the DAM emission. Table (II.1) shows the times at which sector boundary crossings take place at the Earth and at Jupiter. Also shown in Table (II.1) is how the IMF polarity changes at times of boundary crossings.

Table II.1 This table is reproduced from Svalgaard (1976), except the last column which gives "sector boundary crossing times at Jupiter". The computer program, prepared in order to determine the values in the last column, is given in Appendix I.

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4. STATISTICAL ANALYSIS OF RESULTS

Observational results seldom agree exactly with theoretical predictions. Let us suppose that we observe Jovian DAM emission to occur with frequencies $O_1, O_2, O_3, ... O_k$, called the observed frequencies, and that they are expected to occur with frequencies $E_1, E_2, E_3, ... E_k$, called expected, or theoretical, frequencies. In this particular case, we have no definite theoretical (or expected) result to fit our observations. We assume here that the occurrence probability of Jovian DAM emission has no favoured position in the IMF sector structure. Although in the Earth's case, the passage of a sector boundary across the Earth's orbit increases the occurrence probability of geomagnetic activity, and causes it to have a peak within one day after the passage, we have no a priori reason to believe that the same pattern will be observed for Jupiter. Therefore our initial assumption is that Jovian DAM emission may be observed at any time, irrespective of the position of sector boundary. It means that our expected frequency is constant for each particular case (i.e., for each year). In terms of the average of observations made on that particular year, i.e.

$$ E = \frac{\sum O_k}{n} \quad (II.4.1) $$
where \( O_i \) is the number of observations contained in each cell (in our case, a one-day interval) and \( n \) is the number of cells (in this example, \( n = 10 \)).

We wish to know how our observations differ from the expected frequencies, and what is the most likely distribution of occurrence probability. For the goodness of fit test, we employ a chi-squared test, which is given by

\[
\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}
\]

where \( E_i \) is the expected frequency, given by equation (II.4.1).

In passing we should mention that, in order to have statistically significant results, each cell on each year should have at least four data points. It is clearly seen in the figures II.2 through II.17 that some histograms are lacking sufficient data points. What we have done is to amalgamate the neighbouring cells to meet the above-mentioned requirement, that is, to have at least four data points in a single cell. In Fig. (II.2), +4th and +5th cells; in Fig. (II.4), -5th and -4th; +3rd, +4th and +5th cells; in Fig. (II.5), -5th and -4th; +4th and +5th cells; in Fig. (II.12), -5th, -4th and -3rd; -2nd and -1st; +3rd, +4th and +5th cells; in Fig. (II.13), -5th, -4th and -3rd; -2nd, -1st and +1st; +2nd, +3rd, +4th and +5th cells; in Fig. (II.14), -5th and -4th cells are amalgamated into one single cell.

Next we have found the expected frequency, \( E_i \), of each year. For those cells which are amalgamated into one, expected frequency becomes twice, three times (depending on the number of cells amalgamated) as big as the original expected frequency of that particular year. Result of amalgamation is to reduce the number of degrees of freedom. For instance, for 16.7 MHz there would have been 72 degrees of freedom if there had not
FIG. (II.2)
FIG. (II.3)

NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

1968

16.7 MHZ.
FIG. (II.4)

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

NUMBER OF JOVIAN DECAMETRIC ACTIVITY

1969
16.7 MHZ.
FIG. (II.5)
1971
16.7 MHz.

NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

FIG. (II.6)
1972
16.7 MHZ.

FIG. (II.7)
1973
16.7 MHZ.

Fig. (II.8)
FIG. (II.9)

The histogram represents the number of jovian decametric activity with respect to sector boundary days. The y-axis indicates the number of occurrences, and the x-axis shows the position relative to the sector boundary in days. The data is for the year 1974 with a frequency of 16.7 MHz.
FIG. (II.10)

NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

1967
22.2 MHZ.
FIG. (II.11)

NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

1968
22.2 MHZ.
FIG. (II.12)
FIG. (II.13)
Figure (II.15)

Number of Jovian Decametric Activity

Position with respect to sector boundary, days

1972

22.2 MHz.
NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

FIG. (II.16)
NUMBER OF JOVIAN DECAMETRIC ACTIVITY

POSITION WITH RESPECT TO SECTOR BOUNDARY, DAYS

FIG. (II.17)
been any amalgamation. But, now, n has been reduced to 66 for 16.7 MHz and to 59 for 22.2 MHz.

After finding the $\chi^2$ and n, the number of degrees of freedom, of each year for particular frequency (16.7 MHz or 22.2 MHz) we add all the values of $\chi^2$ and n to find a single value of $\chi^2$ and n. $\chi^2 = 94.48$; n = 66 for 16.7 MHz and $\chi^2 = 71.09$; n = 59 for 22.2 MHz. Then, entering the tables (see Ostle, 1963) we find the probability of getting a flat distribution. P = 0.025 for 16.7 MHz and P = 0.1 for 22.2 MHz are the values we have found.

Results clearly show that we have every reason to believe that Jovian DAM emission increases at sector boundary passages.
CHAPTER III

THE "IMF" AND THE PHYSICS OF THE JOVIAN MAGNETOSPHERE

1. MAGNETIC FIELD LINE MERGING

In the previous chapter we have seen that the solar wind and the sector boundary crossing play an important role in populating and driving various activities at the Earth's and Jupiter's magnetospheres. The magnetosphere contains electrons and protons created internally through the decay of albedo neutrons ejected from the upper atmosphere by energetic solar protons and galactic cosmic-ray particles colliding with atmospheric atoms.

The estimate made by Kennel and Coroniti (1974) on the comparability of Jupiter's particle and energy sources with the solar wind particle and energy fluxes across Jupiter's magnetopause suggested that the solar wind could be a significant source of particles and energy. (Their estimate was consistent with the Pioneer 10 measurements that reported $10^{28}$ particles per sec. and at least $10^{21}$ erg s$^{-1}$.) This conclusion can be supported by estimating the energy input due to "merging" at Jupiter's magnetopause. In order to better visualize the process of "merging", let us consider magnetic field lines in the vicinity of Jupiter. In a simple model for the magnetic field configuration in Jupiter's magnetosphere, there are three classes of field lines: (1) "closed" field lines that connect to Jupiter in both directions; (2) "open" field lines that connect to Jupiter at one end and to the distant interplanetary magnetic field at the other, and (3) interplanetary field lines that do not connect to Jupiter at all. Magnetic field line merging or "reconnection" is the process whereby plasma flows across a surface that separates regions containing topologically different magnetic field lines. The reconnection model of Alfven (1968) and Hill (1973) emphasize the importance of the interplanetary magnetic...
field, and especially the NS (north-south) component which is perpendicularly to the ecliptic plane, because the merging efficiency is strongly dependent on the amount of southward flux (see Fig. III.1). That is, the potential drop across the dayside magnetosphere (a measure of the merging efficiency) attains its maximum value when the angle between $B_1$ and $B_0$ (the fields just inside and outside the magnetopause) is $180^\circ$ (T.W. Hill, 1973).

In the interplanetary medium, the configuration of the interplanetary magnetic field lines are twisted into an Archimedean spiral. Thus, $B$ the magnetic field vector, has a radial component $B_r$ which is either inward (toward the Sun) or outward (away from the Sun), and an azimuthal component $B$ the EW (east-west) component, which is directed westward in the "toward sector" and eastward in the "away sector" (see Chap. II.2). Kawasaki et al. (1973) pointed out that since the magnitude of the IMF is relatively steady (the average magnitude rises to a peak of $6\gamma$ early in the sector, and declines to about $4\gamma$ in the trailing portion of the sector) changes of the EW component are often associated with corresponding changes of the NS component. Thus, when the EW component decreases the NS component increases. (The IMF vector of constant magnitude rotates clockwise, or counter-clockwise, in the plane perpendicular to the Archimedean spiral.) The NS component is the most crucial parameter for the interplanetary magnetic field.

If we examine the theoretical consequences of a significant field component perpendicular to the ecliptic plane, there seems to be a serious discrepancy between the theory and the experiments. IMP-1 findings by Ness and Wilcox (1966) indicated that the measured field had a southward component during two-thirds ($\frac{2}{3}$) of the 3-hour intervals of the IMP-1 observations, whilst the average NS component attained a
Reproduced from Hill, 1973. Potential drop across the dayside magnetopause as a function of the angle between the fields outside and inside the magnetopause.
magnitude of 0.5 - 1.0 $\gamma$. Davis (1966) showed that the existence of such a field component should have serious consequences for the observed solar magnetic field. In his analysis, using the concept of "frozen-in" flux (wherein the field lines are constrained to move with the plasma flow and obey the equation, $E + V/c \times B = 0$), Davis concluded that, if the magnitude of the NS component is even 0.1 $\gamma$, then the outwardly convected flux in this component alone would be $10^{23}$ maxwell/year ( ~ $10^{15}$ weber/year). This would be very difficult to explain: it is equivalent to the flux of 100 typical sunspots.

Nevertheless, we may compare results from Pioneer 5 (Coleman et al. 1960), which was launched on March 11, 1960. The orbit of the vehicle is an ellipse of aphelion $1.5 \times 10^8$ km and perihelion $1.2 \times 10^8$ km. The measurements suggested the existence of a significant NS component of the interplanetary magnetic field. A series of satellites have confirmed the Pioneer 5 findings, e.g. the measurements obtained from IMP-1 (Ness and Wilcox, 1964), which was launched on 17 November, 1963, into a highly eccentric orbit with a geocentric apogee of $1.97 \times 10^5$ km; Mariner 2 (Davis, L. Jr. et al. 1966; Coleman, P.J., 1966) launched on August 27, 1962, and reached $9.0 \times 10^7$ km geocentric range at maximum; Mariner 4 and 5 (Coleman, P.J. et al. 1969; Coleman, P.J. and Rosenberg, R.L., 1971) - Mariner 4 was launched on November 28, 1964, and covered the range of heliocentric distances from 1.0 to 1.5 AU, whilst Mariner 5 was put into operation on June 14, 1967, and made measurements in the heliocentric range 0.65 - 1.0 AU; Explorer 33 and 36 (Rosenberg, R.L. et al. 1971) - Explorer 33 was launched on July 1, 1966, and Explorer 35 on July 19, 1967.

Coleman and Rosenberg (1971) have found from the magnetometer data of Mariner 2, 4 and 5 that, for a field of outward (positive) polarity, the mean value of NS component is less than zero north of the
solar equator, zero at, or near, the equator, and greater than zero south of the equator. For inward (negative) polarity, the sign of the mean value of NS component is reversed.

The IMF sector structure of solar origin with its organised magnetic field and embedded high speed plasma streams is identified as the source of some recurrent geomagnetic disturbances. J.H. Wolfe (1971) reported that solar wind streaming controls the IMF sector structure. These high velocity streams are apparently the interplanetary manifestations of coronal temperature inhomogeneities. According to the classical Parker model (1958), higher temperature regions in the corona would lead to higher interplanetary solar wind velocities. Due to the rotation of the Sun, these higher velocity streams would be expected to interact with the lower velocity gas associated with the quiescent corona. Based on the observations, this interaction region is postulated to form along the Archimedean spiral of the IMF. The conductivity of the interplanetary medium is essentially infinite. The streams cannot penetrate one another and the plasma density increases, in a cumulative manner, forward of a hypothetical boundary. This boundary is, perhaps, associated with the initial velocity increase at the leading edge of a new stream.

Since the interplanetary boundary separates different plasma regimes, the change in the dominant IMF polarity might be expected most likely to occur at this boundary. Indeed, in agreement with our expectation Bahnsen, A. and D'Angelo, N. (1976) have noted that geomagnetic activity often occurs within a few hours of a sector boundary passage. The field polarity may change from being away from, to being towards the Sun within 3 hours. Geomagnetic activity tends to decline monotonically in the trailing portion of a sector and to increase rapidly near and after the time that the sector boundary passes the
Earth. This finding suggests that a stable and recurrent "driving function" for geomagnetic activity and, by extension, for a wide range of other geomagnetic, ionospheric and magnetospheric phenomenon is provided by the sector boundary and adjoining regions. The solar wind electric field appears to be an important part of the "driving function (Wilcox and Colburn, 1972).

Heos-2 magnetometer results have shown electric field increases at each sector boundary from a pre-boundary value of \( \sim 1 \text{mV/m} \) to peak values between 3mV/m and 6mV/m.

Any plasma flow across magnetic field lines is associated with an electric field, according to the usual equation:

\[
\mathbf{E} + \frac{1}{c} \mathbf{V} \times \mathbf{B} = 0
\]

In particular, this is true of a plasma flow across the "separatrix". (The regions of space traversed by the different classes of field lines are bounded by a surface made up of field lines, which is called the separatrix.) It is associated with an electric field lying in the separatrix surface at right angles to \( \mathbf{B} \); the existence of such an electric field is equivalent to saying that magnetic merging is occurring. The modulation of the electric field in a sector is associated with a modulation of the magnetic field, as well as with a modulation of the velocity. Due to the "frozen-in" magnetic field configuration of interplanetary space and the solar wind density "pile-up" in front of high velocity streams, peaks of \( \mathbf{B} \) and \( \mathbf{E} \) precede peaks of \( \mathbf{V} \) by 1-2 days.

The magnitude of the interplanetary field as a function of position within an average sector rises to a peak magnitude of greater than \( 6 \gamma \) (\( 1 \gamma = 10^{-5} \) gauss) early in the sector and declines to about \( 4 \gamma \) in the trailing portion of the sector. The average velocity also shows a similar variation. It reaches a maximum in the early portion of the sector and declines monotonically in the trailing portion. The
density distribution within a typical sector varies in a quite different manner. It reaches a maximum in the early portion of the sector, shows a steep decline to less than half of the maximum value in the central portion, and then increases in the trailing portions of the sector. Rothwell and Greene (1966), using Mariner IV data on the interplanetary sector structure, have studied the daily changes in intensity and spatial extent of the radiation zone electrons with energies of approximately 1.5 MeV. The spatial extent and intensity of energetic electrons in the radiation belts increased after every sector boundary. They suggested that the replenishment of particles in the magnetosphere probably occurs near the sector boundaries. The regions of increased density then drift, or diffuse, inwards.

The inward diffusion of solar wind particles is an important transport process, since it is associated with the creation of the radiation belts. This is especially true of radial diffusion, in which \( \mu \) (the first adiabatic invariant; see Ch. III.2) and \( J \) (the second adiabatic invariant) are conserved. Particles then gain energy in the process of diffusing toward the surface from an external source. Radial diffusion at constant \( \mu \) and \( J \) thus plays the dual role of injecting particles into the magnetospheric interior and accelerating the particles to the energies observed. It is known that the internal hydromagnetic convection from the tail is the fastest radial transport process in the Earth's outer zone. For Jupiter, however, Pioneer 10 measurements have shown that the plasmapause is beyond, or, at most, coincides with, the expected location of the magnetopause at 60 R\( _J \) (C.F. Kennel, 1973). Hence, convection could not transport electrons inside 60 R\( _J \). Inward radial diffusion would thus be the only transport mechanism for charged particles. Indeed, inner zone data obtained by Pioneer 10 yield the clearest evidence that radial diffusion is the
dominant transport process at Jupiter.

Unsteady flow of solar wind plasma causes time variations of the magnetospheric field configuration, and these have a considerable effect on the particles trapped in the field. Slow variations induce so-called "adiabatic effects" (see Ch. III.2), which, in principle, do not alter the dynamical balance. Sudden variations - in particular those associated with magnetic fluctuations - have a profound and irreversible effect on the trapped particle population. In the study of radiation belt dynamics, adiabatic theory and diffusion theory are the two important aspects. The adiabatic theory deals in an approximate manner with charged particle motion in magnetic fields. The theory can be used satisfactorily for radiation belt particles. Diffusion theory is applied to the study of the interactions of trapped particles with the medium (atmosphere, ionosphere, etc.), with random field fluctuations, etc. These processes control the dynamics of the radiation belts (i.e. the balance between injection and loss, their spatial distribution and their energy spectra).

2. ADIABATIC THEORY OF CHARGED PARTICLE MOTION

The equation of motion of a charged particle in a magnetic field, \( \mathbf{B} \), under the action of an electric field, \( \mathbf{E} \), (external and induced) and an external non-electromagnetic force, \( \mathbf{F} \) (\( \mathbf{N} \)), is given by

\[
\frac{d^2 \mathbf{r}}{dt^2} = \frac{e}{mc} \mathbf{r} \times \mathbf{B}(\mathbf{r}, t) + \frac{e}{m} \mathbf{E}(\mathbf{r}, t) \quad \text{(III.2.1)}
\]

where \( \mathbf{r} \) is the particle's position, \( e \), \( m \) and \( c \) are the charge, mass, and velocity of light, respectively, and dots imply a time derivative. In a uniform and constant magnetic field, the solution to (III.2.1) is a helix. In a static "mirror-type" magnetic field, there are three distinct components of the motion of a trapped particle - cyclotron motion, bounce motion, and drift motion.
a) CYCLOTRON MOTION

In cyclotron motion, the particles trace a circular path perpendicular to the magnetic field with gyroradius \( \rho_c = \frac{p}{qB} \), period \( T_c = \frac{2\pi m}{qB} \) and cyclotron angular frequency, \( \omega_c = \frac{2\pi}{T_c} = \frac{qB}{m} \).

The stronger the magnetic field, the higher will be a particle's cyclotron frequency, and the smaller its gyroradius. An "adiabatic invariant" is associated with each component of the motion. This quantity is conserved at all times – even for time-dependent fields – so long as any spatial variations are very small over a domain of size \( \rho_c \), and time variations are very small during intervals of time of order \( T_c \). For cyclotron motion, the quantity conserved is the "magnetic moment", or first "adiabatic invariant". The magnetic moment is expressed by

\[
\mu = \frac{p}{2m} B = \frac{p}{2} \sin^2 \alpha / 2m B = \text{const.} \quad (\text{III.2.2})
\]

where \( p \) is the particle's momentum, \( p_\perp \) is the component of momentum perpendicular to the magnetic field, and \( \alpha \) is the "pitch angle" (the angle between the particle's velocity vector and the magnetic field).

A charged particle in cyclotron motion is equivalent to a current loop of intensity \( I = \frac{|q|}{\tau_c} = \frac{q^2 B}{2\pi m} \), and area \( A = \pi \rho_c^2 \). This current loop has a magnetic moment

\[
\mu_n = I \cdot A = q^2 \pi \rho_c^2 B / 2\pi m = q^2 \phi_c / 2\pi m \gamma = p^2 / 2mB, 
\]

where \( \phi_c \) is the external magnetic flux through the cyclotron orbit, and \( \gamma = (1 - v^2/c^2)^{\frac{1}{2}} \).

The magnetic moment can be written in vector form:

\[
\vec{\mu}_n = -p_\perp / 2mB \hat{e},
\]

where \( \hat{e} \) is the unit vector in the direction of \( \vec{B} \). The direction of the magnetic moment is antiparallel to that of the applied \( \vec{B} \) field. The magnetic fields created by the magnetic moments of charged particles act together to alter the applied magnetic field. Thus, plasma in a
magnetic field behaves like a diamagnetic gas. Magnetic moment can be written in terms of the external magnetic flux as \( \mu = q^2 \frac{\mathbf{u}}{c} / 2 \pi m_0 \), which clearly shows that the conservation of the relativistic moment is equivalent to conservation of the magnetic flux through the particle's cyclotron orbit: \( \frac{\mathbf{u}}{c} = \text{const} \).

b) **BOUNCE MOTION**

If we assume that there are no parallel components of external forces (i.e. that field lines are equipotentials), we deduce from the conservation of the magnetic moment that

\[
\sin^2 \alpha(s)/B(s) = \sin^2 \alpha_1/B_1 = \text{constant} \quad (\text{III.2.3})
\]

where \( s \) is the field line arc length, and \( \alpha_1, B_1 \) are values at an arbitrary initial point. An injected particle with a pitch angle, \( \alpha_1 \), at a point where the field intensity is \( B_1 \), spirals along a field line with a parallel velocity given by

\[
v_{11}(s) = v \cos \alpha(s) = v \left[ 1 - \frac{B(s)}{B_1} \sin^2 \alpha_1 \right]^{\frac{1}{2}} \quad (\text{III.2.4})
\]

If the particle is moving in a "dipole-like" magnetic field, where \( B \neq 0 \), it bounces between magnetic mirror points situated where the field has a value \( B_m = B_1/\sin^2 \alpha_1 \). This bounce motion, with period

\[
\tau_b = 2 \int_{M_0}^{M_N} ds/\sqrt{v_{11}(s)} = \frac{2}{v} \int_{M_0}^{M_N} ds/[1 - B(s)/B_m]^{\frac{1}{2}} \quad (\text{III.2.5})
\]

constitutes the second periodicity of the particle's motion.

Whilst a particle bounces along a field line, it also drifts perpendicularly to it. Adiabatic theory requires that during one bounce a particle drifts only of the order of one Larmor radius away from the initial field line. The drift motion of a particle has three components arising from: (a) an external force, given by \( \mathbf{E} + \mathbf{v}_F \times \mathbf{B}/c = 0 \);
(b) the gradient of field strength normal to $\vec{B}$, given by

$$\vec{V}_G = \left( \frac{\mu}{qB^2} \right) \vec{B} \wedge \nabla \vec{B};$$

(c) curvature of the field lines, given by

$$\vec{V}_C = \left( 2w / qB \right) \hat{e}_1 \wedge \left[ (\hat{e}_1 \cdot \nabla) \hat{e}_1 \right].$$

Here, $w$ is the parallel kinetic energy, and $\hat{e}_1$ is the unit vector tangential to a magnetic field line.

In a dipole magnetic field, both $\vec{V}_G$ and $\vec{V}_C$ are westward for positive particles and eastward for negative particles. The quantity,

$$J = \oint \frac{p}{\parallel} ds = \oint p \cos \alpha ds$$  \hspace{1cm} (III.2.6)

is the second adiabatic invariant.

It is conserved during the drift of a trapped particle only if the field changes little during a time interval of the order of the bounce period.

c) THE DRIFT MOTION

In this motion, particles move along the surface generated by the field lines successively occupied by the "guiding centre" (the centre of the circular orbit of a particle is called the guiding centre). Under static conditions (i.e. when time variations in the fields acting on the particles are small compared with the drift period), this surface is closed. The drift motion on a closed shell is the third periodicity of particle's motion in a dipole-like magnetic field.

The quantity $\tau_d = \oint ds / < V_o >$ is the drift period. It is computed along the closed drift shell; $s$ being the arc length of the intersection of the drift shell with a reference surface. When the adiabatic time variations satisfy the condition $\tau_d B/B << 1$, the magnetic flux, $\Phi$, encompassed by the guiding drift shell of a particle remains constant (i.e. $\Phi = \oint A_o \cdot dx = constant$). $A_o$ is the magnetic vector potential and the integration is carried out along a curve which lies in the guiding drift shell of the particle.
There are four processes acting simultaneously in the radiation belts and keeping them in existence. These are: (a) injection of charged particles into the trapping region of the magnetosphere from the solar wind through the dayside neutral points; (b) acceleration; (c) diffusion; (d) loss. Observations by Simpson et al. (1974) with Pioneer 10 have given the clearest evidence that radial diffusion is the dominant source of electrons and transport process in the Jovian radiation belts. The type of radial diffusion that conserves both $\mu$, the first adiabatic invariant, and $J$, the second adiabatic invariant, can be caused by "magnetic sudden impulses" (Schulz, M. and Lanzerotti, L.J., 1974) and by other magnetospheric disturbances operating on the same time scale as the drift period of a particle $\tau_d \gg \Delta t \gg \tau_b \gg \tau_c$ (the typical time scale, $\Delta t$, of the perturbation is of the order of minutes).

If mechanisms of radial diffusion violate $\mu$ and/or $J$, particles cannot be energised efficiently in the process, and these processes are less important than sudden impulses in radiation belt dynamics. Magnetic impulses in $B$ correspond to sudden changes in the jovian centrifugal stand-off distance from the subsolar point on the magnetopause. The stand-off distance is governed by the momentum flux of the solar wind. A decrease in the stand-off distance, the time scale of which is less than, or of the same order as, the drift period, causes a sudden contraction of the magnetosphere. This contraction consists of both an azimuthally symmetric compression, which induces reversible changes or adiabatic effects, and an azimuthally asymmetric distortion, which violates the third invariant, thereby producing radial diffusion. The repeated action of magnetospheric compressions (probably at the sector boundaries), even if small, will lead to acceleration and radial
diffusion. Radial diffusion requires that at least the third invariant should change. Since the longitudinal drift period is the longest of the three periods, the third invariant is the most likely candidate to be affected by electric and magnetic fluctuations. In the computations described below, radial diffusion is applied to equatorially trapped particles only. The reason is that the adiabatic approximation predicts that conservation of the first and second invariants and violation of the third produces both an acceleration of trapped particles and a motion of their mirror points toward the equator, as particles move to lower field lines. The assumptions we make here about the equatorially trapped particles are supported by the observations obtained by Pioneer 10 (Van Allen et al., 1974). These indicate that the electrons are confined to a narrow region around the equatorial plane, as is required in the radial diffusion model proposed here.

In this type of computation, it seems most convenient to work with $\mu$, $J$, and $L$, as independent variables (where $\mu$ and $J$ are the first and second adiabatic invariants, respectively, and $L$ is McIlwain's magnetic shell parameter, $R = L/R_j$). If $f(\mu, J, L, t)$ represents the particle distribution function, such that $\int f d\mu dJ dL$ is the number of particles in the differential volume $d\mu dJ dL$ at the point $\mu, J, L$ and time, $t$, then the diffusion equation becomes

(Walt, M., 1971)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial L} \left[ \frac{D_{LL}}{L^2} \frac{\partial}{\partial L} (L^2 f) \right] - Q \quad (III.3.1)$$

where $D_{LL}$ is the diffusion coefficient, and $Q$ is the loss rate of particles in $d\mu dJ dL$.

The source term is not included in this diffusion equation. Since the solar wind is assumed to be the only source, the source term can be expressed as a boundary condition for the integration of the equation. In the present instance, since both $\mu$ and $J$ are conserved,
there is a one-to-one correspondence between radial position and particle energy. The diffusion is therefore one-dimensional, and can be represented by one spatial variable, $L$.

We will assume that synchrotron radiation provides the primary electron energy loss mechanism. Farley and Walt (1971) have shown that the loss rate, $Q$, is given by:

$$Q = \frac{\partial}{\partial \mu} \langle \Delta \mu \rangle + \frac{\partial}{\partial J} \langle \Delta J \rangle$$

(III.3.2)

where $\langle \Delta \mu \rangle$ and $\langle \Delta J \rangle$ are the average change of $\mu$ and $J$ per unit time caused by synchrotron radiation. Since we have assumed that the particles are equatorially trapped, the second term in the loss rate, $R$, becomes negligibly small compared with the first term. Hence, in solving the diffusion equation, we will neglect changes in $J$, and the electron distribution function will be considered in $(\mu, L)$ phase space.

The average change of $\mu$ is:

$$\langle \Delta \mu \rangle = d\mu / dt = d\mu / dE \cdot dE / dt$$

(III.3.3)

Inserting the relativistic expression for magnetic moment,

$$\mu = (pc)^2 / 2m_o c^2 B = [E^2 - (m_o c^2)^2] / 2m_o c^2 B$$

(III.3.4)

where $p$ is the electron's momentum; $m_o$ - its rest mass; $c$ - the velocity of light; and $B$ - the magnetic field, we find that

$$d\mu / dE = E / m_o c^2 B = (1/B) \left[ (2 \mu B / m_o c^2) + 1 \right]^{1/2}$$

(III.3.5)

The total power of the synchrotron radiation over all frequencies can be calculated from (cf. Ginzburg and Syrovatskii, 1965):

$$P(E) = \frac{2e^4 B^2 v^2}{3a^2 c^5 (1-v^2/c^2)} = \frac{2e^4 B^2}{3m_o^2 c^3} \left[ \left( \frac{E}{m_o c^2} \right)^2 - 1 \right]$$

(III.3.6)

In the case of continuous radiation losses - as by synchrotron radiation - the energy loss rate is given by
\[
\frac{dE}{dt} = -P(E) = -\beta B^2 [(E/m_0 c^2)^2 - 1] \tag{III.3.7}
\]

where \( \beta = 2e^4 / 3m_0 c^3 \). The total loss rate \( R \) can now be written as:

\[
Q = -2\beta B^2 \left[ \frac{(3\mu B/m_0 c^2) + 1}{(2\mu B/m_0 c^2) + 1} \right] f + \mu \left( \frac{2\mu B}{m_0 c^2} + 1 \right) \frac{\partial f}{\partial \mu} \tag{III.3.8}
\]

Because of its rapid drop-off with distance \( (B \sim R^{-3}) \), this loss term is significant only over the synchrotron emitting region close to Jupiter. Smith et al. (1974) have reported that Jupiter's magnetic field cannot be represented by a simple centred dipole. Three models—one off-set dipole; two off-set dipoles; and a centred dipole plus quadrupole—have been developed recently. Smith et al. (1974) point out, however, that it should be borne in mind that there is no unique way to model the observed field.

Examination does not indicate great differences between magnetic field models for diffusion. Hence, taking the simplest model, we have assumed that a centred dipole magnetic field can represent the field configuration in the equatorial plane \( B = B_0 / L^3 \) (where \( B_0 = 11.5 \) G is the equatorial surface field, as has been found from Pioneer 10 and Pioneer 11 data). The radial diffusion coefficient, we assume, is in the form \( D_{\text{LL}} = D_0 L^n \). Here, \( D_0 \) is associated with non-radial dependence, and is calculated from the Pioneer data as having a value \( D_0 \approx 2.5 \times 10^{-6} \) \( R_J \) \( s^{-1} \); with \( n \) indicating the power dependence on radial position. We will use a value of \( n = 2 \), since it provides a very good fit to the Pioneer 10 data (Baker and Goertz, 1976). Finally, the steady state assumption \( \partial f / \partial t = 0 \) will be used. This assumption may seem to be a crude one, since there is evidence that short-term variations of the synchrotron emission are correlated with solar activity (Gulkis et al. 1973; Roberts and Huguenin, 1963). E. Gerard (1970) carried out observations on the synchrotron radiation of Jupiter.
and found that there is a positive correlation between the flux density from Jupiter around 21 cm wavelength and solar activity, as measured by 10.7 cm solar flux. The 11.13 cm total flux density of Jupiter underwent changes up to 30% from December, 1967, to August, 1968. It has already been said that the solar wind is the source of electrons for Jupiter's radiation belts, and that solar activity can influence the equilibrium conditions within the belts. The physical mechanism by which solar activity may cause the Jovian flux density to change is unknown, but could involve changes in electron density, or magnetic field strength, in the solar wind, in general, and at the sector boundaries, in particular. The short time variation in Jupiter's synchrotron output can be accounted for by making the source and loss terms a slowly varying function of time. We do not include such a time dependence in this study, however.

If \( L \) derivatives are evaluated and the loss rate added, the diffusion equation becomes:

\[
\frac{\partial}{\partial L} \left[ (\frac{D_L}{L^2}) \frac{\partial}{\partial L} \right] \frac{\partial f}{\partial L} = R = 0 \quad \text{(III.3.9)}
\]

and,

\[
\frac{\partial^2 f}{\partial L^2} + \left( \frac{n+2}{L^2} \right) \frac{\partial f}{\partial L} + \left[ \frac{2(n-1)}{L^2} + \frac{3 \epsilon}{D_L L^{6+n}} \cdot \frac{2B_0 (\mu/L^3)^{1+1}}{[\Omega (\mu/L^3)^{1+1}]^2} \right] f
\]

\+
\frac{2 \epsilon \mu}{D_L L^{6+n}} \left( \frac{\Omega \mu + 1}{L^3} \right)^{1/2} \frac{\partial f}{\partial \mu} = 0 \quad \text{(III.3.10)}
\]

where, \( \epsilon = \beta B_0^2/m_o c^2 \), and \( \Omega = 2B_0/m_o c^2 \). We now consider the numerical integration of the diffusion equation over the interval \( L = 1 \) to \( L = 5 \). Reasons for this choice are given below. In this numerical calculation, it is necessary to assume boundary values for the distribution function at the ends of the \( L \)-interval, and to make estimates of the source and loss mechanisms that occur over the same interval. Since the diffusion
equation is a second-order differential equation in the variable \( L \),
and first order in the variable \( \mu \), two boundary conditions are required
for \( L \) and one for \( \mu \). The first boundary condition is \( f(\mu, L = 1) = 0 \).
This means that, if particles with different energies hit the surface
of the planet (\( L = 1 \)), they will be absorbed. Electrons have a magnetic
moment, \( \mu_1 \), at the injection distance, \( L_1 \). In the process of diffusion,
electrons suffer some \( \mu \) degradation. This \( \mu \) degradation is, however,
quite small except in the synchrotron radiation region (\( L = 1 - 5 \));
and electrons can travel from \( L_1 \) towards \( L = 5 \) with \( \mu \) still approximately
\( \mu_1 \). Hence \( F(\mu) \), the solar wind spectrum, is taken to be the spectrum
of diffusing particles at \( L = 5 \), i.e. \( f(\mu, L = 5) = F(\mu) \).

Particles may undergo acceleration before reaching the
radiation belt. Thus \( F(\mu) \) may not be the solar wind spectrum. It is
well known that synchrotron radiation causes a loss of energy in the
motion perpendicular to the local magnetic field. On the other hand,
electrons gain energy in moving to the region of stronger magnetic
field (\( \mu = p^2/2m_0^2 = \text{constant} \)). In the process of diffusion into
\( L = 1.5 - 2 \), the electron energy degradation by synchrotron radiation
is faster than the energy gain by radial diffusion into the stronger
magnetic field. Thus, values of \( \mu = 100 - 2000 \text{ MeV/G} \) are required at
the outer zone in order to explain the high frequency synchrotron
radiation. The boundary condition for \( \mu \), which is taken to be
\( f(\mu = 150 \text{ MeV/G}, L) = 0 \), reflects the fact that all particles are
injected at \( \mu = 150 \text{ MeV/G} \), and that radiation causes them to lose
energy.

4. DERIVATION OF THE DIFFERENCE EQUATION AND ITS NUMERICAL SOLUTION

The diffusion equation derived in the previous sub-section is
a linear, parabolic, partial differential equation which holds within
the prescribed region of the \((\mu, L)\) space. The region consists of a rectangle \([1 \leq L \leq 5] \times [0.4 \leq \mu \leq 150 \text{MeV/G}]\). To find a finite difference replacement of the diffusion equation, the above region is covered by a rectilinear grid with sides parallel to the \(L\) and \(\mu\)-axes, with \(\Delta L = 0.02\) and \(\Delta \mu = 0.4 \text{MeV/G}\) as the grid spacings in the \(L\) and \(\mu\)-directions, respectively. The next step is to find the partial derivatives \(\partial^2 f/\partial L^2\), \(\partial f/\partial L\) and \(\partial f/\partial \mu\) of the diffusion equation at the grid point \((i,j)\), where \(L_j = 1 + j\Delta L\) and \(\mu_j = i\Delta \mu\).

The diffusion equation can be put in a more general form, i.e.

\[
\frac{\partial f}{\partial \mu} = a(\mu, L) \frac{\partial^2 f}{\partial L^2} + b(\mu, L) \frac{\partial f}{\partial L} + c(\mu, L)f
\]

(III.4.1)

where

\[
a(\mu, L) = -\frac{D_o L^{6+n}}{2 \epsilon \mu (\mu^3/L^3+1)^{1/3}}
\]

and

\[
b(\mu, L) = -\left[\frac{2(n-1)}{L^2} + \frac{3 \epsilon}{D_o L^{6+n}} \frac{2 \Omega (\mu^3/L^3+1)^{1/3}}{[\Omega (\mu^3/L^3+1)]^{1/3}}\right] \cdot a
\]

We take the Taylor expansion of the distribution function, \(f\), with respect to \(\mu\),

\[
f_j^{i+1} = f_j^i + \Delta \mu \left(\frac{\partial f}{\partial \mu}\right)_j^i + o(\Delta \mu^2)
\]

(III.4.2)

and rearrange it to give

\[
\left(\frac{\partial f}{\partial \mu}\right)_j^i = \frac{(f_j^{i+1} - f_j^i)}{\Delta \mu}
\]

(III.4.3)

If (III.4.3) is substituted into the diffusion equation, we find that:

\[
\left(\frac{\partial f}{\partial \mu}\right)_j^i = \left[a(\mu, L)\frac{\partial^2 f}{\partial L^2} + b(\mu, L)\frac{\partial f}{\partial L} + c(\mu, L)f\right]_j^i
\]

(III.4.4)

The next step is to carry out a Taylor expansion of \(\partial f/\partial L\) and \(\partial^2 f/\partial L^2\) at the point \((i,j)\).
\[ f_{j+1}^1 = f_j^1 + \Delta L (\partial z / \partial L)_j^1 + O(\Delta L^2) \]  
\[ (\partial z / \partial L)_j^1 \] can be found by subtracting (III.4.6) from (III.4.6), i.e.
\[ (\partial z / \partial L)_j^1 = \frac{1}{2\Delta L} \left( f_{j+1}^1 - f_{j-1}^1 \right) \]  
Following the same procedure, we then find \( (\partial^2 z / \partial L^2)_j^1 \)
\[ f_{j+1}^1 = f_j^1 + \Delta L (\partial z / \partial L)_j^1 + \frac{\Delta L^2}{2!} (\partial^2 z / \partial L^2)_j^1 + O(\Delta L^3) \]  
\[ f_{j-1}^1 = f_j^1 - \Delta L (\partial z / \partial L)_j^1 + \frac{\Delta L^2}{2!} (\partial^2 z / \partial L^2)_j^1 - O(\Delta L^3) \]  
Finally,
\[ (\partial^2 z / \partial L^2)_j^1 = \frac{1}{\Delta L^2} \left( f_{j+1}^1 - 2f_j^1 + f_{j-1}^1 \right) \]  
By substituting (III.4.7) and (III.4.10) into (III.4.4), the following finite difference equation is found:
\[ \frac{1}{\Delta \mu} \left( f_{j+1}^4 - f_j^4 \right) = a_j^1 \left[ \frac{1}{\Delta L^2} \left( f_{j+1}^4 - 2f_j^4 + f_{j-1}^4 \right) \right] + \]
\[ b_j^1 \left[ \frac{1}{2\Delta L} \left( f_{j+1}^4 - f_{j-1}^4 \right) \right] + c_j^1 f_j^4 \]  
On rearranging (III.4.11), the implicit difference formulae is given its final form, i.e.
\[ - \left[ \frac{b_j^1}{2\Delta L} - \frac{a_j^1}{\Delta L^2} \right] f_{j-1}^4 + \left[ c_j^1 - \frac{2a_j^1}{\Delta L^2} + \frac{1}{\Delta \mu} \right] f_j^4 - \]
\[ \left[ - \frac{b_j^1}{2\Delta L} - \frac{a_j^1}{\Delta L^2} \right] f_{j+1}^4 = \frac{1}{\Delta \mu} f_j^{4+1} \]  
(III.4.12)
A system of linear algebraic equations arises from the implicit difference formulae which must be solved at each time step. Since the region we are considering is \([1 \leq L \leq 5] \times [0.4 \leq \mu \leq 150 \text{ MeV/G}]\), the indices have the ranges \(1 \leq i \leq 37\) and \(0 \leq j \leq 200\). The difference equation can now be written as:

\[-\alpha_j f_{j-1} + \beta_j f_j - \gamma_j f_{j+1} = \delta_j\]  

(III.4.13)

where

\[
\alpha_j = \left[ \frac{b^i}{2\Delta L} - \frac{a^i}{\Delta L^2} \right], \quad \beta_j = \left[ \frac{c^i}{\Delta L^2} - \frac{2a^i}{\Delta \mu} \right], \quad \gamma_j = \left[ -\frac{b^i}{2\Delta L} - \frac{a^i}{\Delta L^2} \right],
\]

and \(\delta_j = \frac{1}{\Delta \mu} f_{j+1}^i\), for \(1 \leq j \leq 199\),

where \(f^i_0\) and \(f^i_{200}\) are known from the boundary conditions.

Since \(\alpha_j > 0, \beta_j > 0, \gamma_j > 0\), and \(\beta_j \geq (\alpha_j + \gamma_j)\), for \(1 \leq j \leq 199\), a highly efficient method, Gaussian elimination, is available for solving the tridiagonal system. This system acquires its name from the fact that, if written in matrix form, namely,

\[
\begin{array}{cccc|c}
\beta_1 & -\gamma_1 & 0 & f_1 & \delta_1 + \alpha_1 f_0 \\
-\alpha_2 & \beta_2 & -\gamma_2 & f_2 & \delta_2 \\
0 & -\alpha_{j-1} & \beta_{j-1} & f_{j-2} & \delta_{j-2} \\
\end{array}
\]

the matrix is tridiagonal.
Gaussian elimination can be carried out as follows. The relationship, \( f_j = \omega_j f_{j+1} + g_j \), is considered for \( 0 \leq j \leq 199 \).

It follows that \( f_{j-1} = \omega_{j-1} f_j + g_{j-1} \), and that this can be used to eliminate \( f_{j-1} \) from the original difference formula. The result obtained is

\[
 f_j = \frac{\gamma_j}{\beta_j - \alpha_j \omega_j^{-1}} f_{j+1} + \frac{\delta_j + \alpha_j g_j^{-1}}{\beta_j - \alpha_j \omega_j^{-1}} (\text{III.4.15})
\]

where

\[
 \omega_j = \frac{\gamma_j}{\beta_j - \alpha_j \omega_j^{-1}}, \quad g_j = \frac{\delta_j + \alpha_j g_j^{-1}}{\beta_j - \alpha_j \omega_j^{-1}}
\]

Since \( f_0 = 0 \) (boundary condition), then \( \omega_0 = g_0 = 0 \), in order that the difference relation \( f_0 = \omega_0 f_1 + g_0 \) holds for any \( f_1 \). The remaining \( \omega_j, g_j \) \( (j = 1, 2, \ldots, 199) \) can now be calculated from

\[
 \omega_1 = \frac{\gamma_1}{\beta_1} \quad g_1 = \frac{\delta_1}{\beta_1}
\]

\[
 \omega_2 = \frac{\gamma_2}{\beta_2 - \alpha_2 \omega_1^{-1}} \quad g_2 = \frac{\delta_2 + \alpha_2 g_1^{-1}}{\beta_2 - \alpha_2 \omega_1^{-1}}
\]

\[
 \vdots
\]

\[
 \omega_{j-1} = \frac{\gamma_{j-1}}{\beta_{j-1} - \alpha_{j-1} \omega_{j-2}^{-1}} \quad g_{j-1} = \frac{\delta_{j-1} + \alpha_{j-1} g_{j-2}^{-1}}{\beta_{j-1} - \alpha_{j-1} \omega_{j-2}^{-1}}
\]

\( f_{200} \) is given as a boundary condition, that is, \( f_{200} = F(\mu) \), then \( f_1, f_2, \ldots, f_{199} \) are calculated from

\[
 f_{j-1} = \omega_{j-1} f_{j-1} + g_{j-1}
\]

\[
 f_{j-2} = \omega_{j-2} f_{j-2} + g_{j-2}
\]

\[
 \vdots
\]

\[
 f_1 = \omega_1 f_1 + g_1
\]
5. RESULTS

The main objective of this study is to see whether there is a quantitative agreement between the "phase space density" found from the above diffusion process and the phase space density observed by Pioneer 10. The phase space density is defined so that \( f d^3x d^3p \) is the number of particles contained in the spatial volume element \( d^3x \) and in the momentum volume element \( d^3p \), and is found for constant first and second adiabatic invariants. Radial phase space density profiles for equatorially mirroring particles are shown in Fig. (III.2) through Fig. (III.4). These figures show that \( f \) (the phase space density) varies considerably with L. On the other hand, Liouville's theorem states that \( f \) is constant along a particle's orbit for a lossless process, during the L drifting motion. In other words, in Hamiltonian mechanics, the temporal evolution of phase space density is specified by Liouville's theorem, which asserts that

\[
\left( \frac{df}{dt} \right) = \sum_i \left( \frac{\partial f}{\partial n_i} \right) + \sum_i \left( \frac{\partial f}{\partial q_i} \right) = 0
\]

along any dynamical trajectory in phase space. Considering Liouville's theorem, we can evaluate sources and losses in the radiation belts by calculating the phase space density. An interesting feature of Figs. [(III.2) - (III.4)] is that \( f \) increases monotonically with L; this indicates that the particle source is at large L and loss processes reduce \( f \) further in. The larger slopes, \( \left( \frac{\partial f}{\partial L} \right) \mu J \), of Figs. (III.2; III.3; III.4) at the larger L value imply that loss processes are probably relatively important there; the much smaller slopes at the lower L values imply that loss processes are relatively less important in this region. This study strengthens the suggestion by many authors (Coroniti, F.V., 1974; Birmingham, T. et al., 1974; Stansberry, K.G. and White, R.S., 1974) that the source of the outer-
FIG. (III.2)

PHASE SPACE DENSITY, $f$ (RELATIVE UNITS)

RADIAL DISTANCE, $L = R/R_J$

1: 150 MeV/N
2: 140 MeV/N
3: 130 MeV/N
4: 120 MeV/N
5: 110 MeV/N
RADIAL DISTANCE, $L (=R/RJ)$

PHASE SPACE DENSITY, $f$ (RELATIVE UNITS)

6: 100 MeV/G
7: 80 MeV/G
8: 70 MeV/G
9: 60 MeV/G
10: 50 MeV/G

FIG. (III.3)
PHASE SPACE DENSITY, $f$ (RELATIVE UNITS)

RADIAL DISTANCE, $L (= R/R_J)$

FIG. (III.4)

$11: 50 \text{ MeV/G}$
$12: 40 \text{ MeV/G}$
$13: 30 \text{ MeV/G}$
$14: 20 \text{ MeV/G}$
$15: 10 \text{ MeV/G}$
$16: 0.4 \text{ MeV/G}$
belt electrons is near the magnetopause, rather than near Jupiter, and that some process that violates the third adiabatic invariant is important in moving electrons into the region of the outer radiation belt. Although uniqueness cannot be claimed, the similarity of the calculated phase space density to measurements [Pioneer 10 results by McIlwain, C.E. and Fillius, R.W. (1975); Baker, D.N. and Goertz, C.K. (1976)] supports the idea that the diffusion process described here is important (see Fig. III.5). Full confirmation that diffusion is caused mainly by magnetic disturbances due to IMF sector boundary crossings must await further experimental results.

In dealing with the radial diffusion process, we have had in mind the high intensities, limited bandwidths, short-time scales, and sporadic nature (see Chapter IV) of individual decametric bursts which indicate that they originate in stimulated emission from one or more collective micro-instabilities.

The free energy resides in some non-thermal feature of the particle distribution function micro-instabilities. Such a non-thermal feature manifests itself in either the configuration space or velocity space dependence of $f$. For example, density gradients (probably caused by the radial diffusion of particles from the solar wind) in a magnetized plasma create drift currents, which may drive a variety of instabilities.
Radial dependence of phase space densities $f(\mu)$ of near equatorial electrons having a constant value of the first adiabatic invariant $\mu$.

Reproduced from Van Allen, 1976 (in "Jupiter").
CHAPTER IV

THE PLASMA HYPOTHESIS AND OBSERVATIONS OF JOVIAN DAM EMISSION

1. OBSERVATIONS

The great accumulation of information concerning terrestrial magnetospheric processes has made it possible for magnetospheric physicists to interpret a large number of the detailed observations of Jovian DAM emission. However, despite all these theoretical efforts, no comprehensive convincing interpretation of DAM has as yet been advanced. Most of the theoretical investigations up to the present deal with one or two different aspects of the problem only.

Warwick and Dulk (1964) have presented the evidence for the presence of Faraday rotation in some DAM emission from Jupiter, and have discussed its origin. Their spectrograph at the High Altitude Observatory operated in the range 7.6 to 41 MHz. Horizontal fringes in the 22 MHz to 35 MHz range can be clearly seen in their spectrograph recordings of a major Jupiter emission observed on 13 September 1963. These fringes are clearly different from interferometer fringes. The authors have concluded that the structure of the bands is caused by the reception of elliptically polarized radiation, in which the direction of the major axis of the ellipse is rotated as a function of frequency, and that the cause of the band structure is Faraday rotation. Faraday rotation may occur in Jupiter's ionosphere, interplanetary space, the Earth's ionosphere or in all three. From the Faraday fringes on the dynamic spectrograph records, Warwick and Dulk determined the orientation of the polarization ellipse at the receiver over the frequency range of 20 to 35 MHz. They attributed the rotation to effects in the Earth's ionosphere. They also computed the electron content from equation (IV.1.1), and compared the result with estimates of electron content from Boulder ionosonde data.
\[ \Omega = \frac{K}{\tau^2} \left( H \cos \Theta \sec \chi \right) \int N dh \] (IV.1.1)

where \( \Omega \) is the total amount of Faraday rotation in radians; \( K = 2.36 \times 10^4 \) (C.G.S. Gaussian units); \( \tau \) - the wave frequency in Hz; \( H \) - the magnetic field intensity in gauss; \( \Theta \) - the angle between the ray direction and magnetic field direction; \( N \) - the electron density in \( \text{el/cm}^3 \); \( \chi \) is the zenith angle of the ray. The integral is taken along a vertical path through the Earth's ionosphere.

From the measured position of the major axis of the polarization ellipse at the receiver and a sufficiently accurate determination of the total Faraday rotation, one can extrapolate backwards to find the position of the major axis of the polarization ellipse at the source.

The event of 12 August 1963 showed that the major axis of the polarization ellipse at Jupiter was parallel to the equator of the planet. The authors found the presence of observable Faraday effect in Jupiter's radio emission from 15 to 40 MHz surprising in view of the fact that Faraday rotation on these low frequencies can be produced by quite moderate plasma densities and magnetic fields. The fact that they could explain all of the rotation in terms of the Earth's Faraday effect led them to suggest several possible interpretations:

a) Jupiter has no ionosphere; hence, it produces no Faraday effect;

b) the emission is generated above Jupiter's ionosphere;

c) both modes of polarization are generated, but only one mode escapes through the planet's ionosphere;

d) only one mode is generated, and this mode escapes.

It should be borne in mind that for Faraday rotation to occur, both magnetoionic modes must be present (see Ratcliffe, 1972). The polarization characteristic of a single mode is fixed by the geometrical
relation between the magnetic field and the direction of propagation. Radiation in one "characteristic mode" will remain in that mode, so long as the ray path traverses a "slowly varying medium" (see Budden, 1961). The "characteristic modes" are such that, for the "ordinary mode", the major axis of the polarization ellipse lies in the same direction as the magnetic field. For the "extraordinary mode", it lies perpendicular to the magnetic field. The observed emission had its major axis perpendicular to the rotation axis. (Jupiter's magnetic field is inclined about $10^\circ$ with respect to the rotational axis.) It was therefore concluded that only one mode - the extraordinary mode - is generated and escapes. Later, in this chapter, we will show that in order to explain the absence of Jovian Faraday rotation to assume only one characteristic mode is loss of generality. Wave conversion mechanism predicts a phase shift between the extraordinary mode and the plasma mode. Due to the electrostatic character of the plasma wave the Faraday rotation is obscured by the "transition region". From their series of observations, Warwick and Dulk found that the apparent axial ratio (assuming that the waves are 100% polarized) varies from burst to burst, but averages about 0.6 in the frequency range from 20 to 35 MHz.

From observations of polarization diversity in millisecond bursts, Gordon and Warwick (1967) found an electron density of about $10^4$ - $10^5$ per cm$^3$, and a magnetic field intensity of about 13 Gauss, to be characteristic of the lower Jovian ionosphere through which the radiation passes. Polarisation diversity describes a phenomenon in which the sense of polarisation flips back and forth periodically as a function of time and frequency. If emission were produced in more than one mode, or if mode coupling occurred, and if quasi-longitudinal Faraday rotation occurred in Jupiter's ionosphere, Gordon and Warwick argue that such rotation would far exceed its equivalent in the Earth's
ionosphere. Faraday rotation in interplanetary space is always negligible, because of the low electron densities and magnetic field strength.

In their later study, Parker and Warwick (1969) confirmed the earlier results that nearly all of the observed Faraday rotation occurs in the Earth's ionosphere, and that, if the radiation is emitted in the northern hemisphere and near the magnetic flux tube to Io, the major axis of the polarization ellipse is nearly perpendicular to the field, corresponding to the extraordinary mode. They suggested two possible origins of the right-hand elliptical polarization:

1) The radiation originates and escapes in one magnetoionic mode which is right-hand elliptical because of the characteristics of the source region and the direction of propagation;

2) The radiation is emitted in the extraordinary mode nearly perpendicular to the local field lines, and is almost linearly polarized at right angles to the field. Provided the electron density is not too small, or too rapidly varying, the radiation tends to remain in the extraordinary mode as it propagates out through Jupiter's ionosphere. As the magnetic field orientation changes and the local gyrofrequency decreases, this mode tends to become right-hand elliptical. No Faraday effect occurs, since only one mode is present. After the radiation leaves the ionosphere, the electron density is too low to affect the polarization and it remains right-hand elliptical.

Ever since the discovery of the Jovian DAM emission in 1955, it has been observed that reception of the emission is strongly correlated with the rotation of the planet. This periodic relationship has been expressed by one dimensional histograms in which occurrence probability
of emission is plotted as a function of CML. Certain localised peaks appear in such diagrams, and they are called the sources of the radiation.

Wilson et al. (1968) have examined the simultaneous dependence of the Jovian emission probability and intensity on $\lambda_{111}$ (System 111 longitude, epoch of 1957.0) and $\gamma_{Io}$ (departure of Io from superior geocentric conjunction). They analyzed the data obtained from the University of Colorado radio spectrograph, which had an effective frequency range from 11 to 41 MHz. From this analysis one can easily visualise the spectral ranges over which the various Jovian sources are identified (see Fig. IV.1). Io-B which is known as the "early source" is shown to be well defined at all frequencies between 11 and 39 MHz. Io-A, the main source, extends from about 14 to 36 MHz. Io-C, the late source, and Io-D, the fourth source, cover the ranges from 11 MHz to 26 and 18 MHz, respectively. Between 11 and 28 MHz, a component of Source A emission appears which does not favour any particular Io phase. This is named non-Io-A, or fifth source. Desch et al. (1979) have uncovered an additional non-Io-related source. They made their observations at a single frequency (26.3 MHz) but with a highly directional antenna array. The new source, non-Io-B, is clearly shown in Fig. IV.1, where it accounts for 30 to 40% of the total activity time. Desch et al. argue that every source may have an Io-controlled component and a (usually weaker) Io-independent component. Warwick (1963, 1967) has shown that the regular recurrence of narrow-band features in dynamic spectra when the same longitude of Jupiter faces the Earth implies that the radiation originates near the ionosphere or surface. For a non-Io source the presence of the appropriate central meridian longitude is a necessary, but not sufficient, condition for the occurrence of emission (Carr and Desch, 1976). The obvious question is - what additional
Reproduced from Carr & Desch, 1976. Identification of emission sources in the $\lambda_{III} - \gamma_{lo}$ plane.

parameters are involved in the triggering of the emission? This question led to the present studies reported in Chapter II. We have shown there that the IMF sector boundary passage is an important parameter in the occurrence probability of DAM emission.

At the lower frequencies, however, the morphology of Jupiter's sources is quite different from that at higher frequencies (Lebo, 1964; McCulloch and Ellis, 1966; Dulk and Clark, 1966; Zabriskiu, 1970). Sources A and B, which are most prominent at high frequencies, are not evident below about 10 MHz. These low frequency sources do not overlap in longitude with higher frequency sources (see Fig. IV.2). Dulk and Clark (1966) have argued that Jupiter may be continuously active at all central meridian longitudes, if observed with a sufficiently sensitive radio telescope. Observations of Duncan (1966), McCulloch and Ellis (1966), Wilson et al. (1968) have revealed that the Io modulation has been more prominent at the higher frequency, up to the 39.5 MHz cut-off. Desch et al. (1975) concluded that over most of its spectrum the emission consists of an intense Io-controlled component and a much weaker, but relatively more observed, non-Io-controlled component.

It was observed that each of the radio features drifts backward relative to System II longitudes at the rate of 0.2885 per day (Douglas and Smith, 1963; Carr and Gulkis, 1969). This corresponded to a rotational period for the radio centres which is 11.8 sec. shorter than that of System II. A new longitude system, based on this rotational period, has been designated as "System III". The new system was defined as follows: a) System III coincided with System II at 0^h U.T. on January 1st, 1957; b) the rotational period for System III longitude is 9h 55m 28.8s. Subsequently, however, Gulkis and Carr (1966) have shown that this value of the rotation period is \( \sim 0.3 \) sec. too short. The CML (Central Meridian Longitude) of the centre of source A, the main
source, clearly exhibited a sinusoidal drift, with an amplitude of about 17°, and a period which is believed to be 11.9 years. Two possibilities have been put forward: a) The source A is related in some way to the solar cycle, which has a period of about 10 to 11.5 years; b) There is a close correlation between the longitude of the centre of Source A and $D_E$, the Jovicentric declination of the Earth. This is the angle of the Jupiter-Earth line with respect to the Jupiter's equatorial plane. In one orbital period it ranges between a maximum of $+3^\circ.3$ and a minimum of $-3^\circ.3$. Possibility (b) appeared more likely than (a). This conclusion has been supported by the subsequent work of Carr et al. (1970), Carr (1972) and Mitchell (1974).

Jupiter's dynamic spectra show a high degree of complexity. The most commonly encountered type of burst is composed of random noise with a smoothly rising and falling envelope and a duration of 1 to 10 sec. These are the L bursts. Most noise storms consist of a randomly occurring sequence of L bursts. S bursts are characterised by considerably shorter durations, typically between 1 and 50 milliseconds each. Their narrow bandwidth is typically 50 kHz, but sometimes as small as 3 kHz. High resolution spectral recordings (e.g. 0.1 sec. in time and 50 kHz in frequency) of Jupiter's DAM emission during the years 1963-1968 exhibit several different spectral types. Riihimaa (1970) classified the spectra into three components: a) the radiation envelope, which has a duration imposed by the interplanetary medium; b) the substructure within the envelope, which is inherent to the source, and c) the superimposed, interference fringe-like modulation lanes, which have an unknown cause, but seem to be related to conditions at Jupiter (see Fig. IV.3). A prominent feature of bursts in the spectral records is the envelope. The envelope of a burst may appear as a vertical, or slightly tilted, bar in the spectrum. The envelope may contain basically
Fig. (IV, 3a)

Fig. (IV, 3b)
Reproduced from Carr & Desch, 1976. High resolution dynamic spectra of S bursts.
two types of emission. One of them lasts longer than 0.5 seconds and looks like continuum emission: the other is composed of short-duration (millisecond) pulses in a rapid succession, each lasting less than the time resolution used.

The third feature consists of repeated lanes of emission, which modulate the spectra with alternating maxima and minima of intensity. The lanes are slightly curved and tilted in the time-frequency plane, in the sense that the drift rate is less at 21 MHz than at 23 MHz. The separation between the maxima is of the order of 300 kHz, while the average drift rate is 100 kHz sec\(^{-1}\). The lanes are not always equally spaced from one another, and there may be slight variations in the drift rate from lane to lane and group to group. These tilted lanes are clearly distinguished from modulation due to Faraday rotation in the Earth's ionosphere, which appears as nearly horizontal lanes on the records. So far as the modulation of emission is concerned there are four common types of records: 1) no modulation present; 2) modulation lanes drift positively (increase of frequency with increase of time); 3) both positively and negatively drifting lanes are simultaneously present and 4) lanes drift negatively.

The spaced-spectrograph experiment has indicated that smooth narrow band bursts (about 50 kHz for L and S bursts, and for S bursts sometimes as small as 3 kHz) and trains of millisecond pulses received at spaced locations are identical (Riihimaa, 1968d). This strongly suggests that the narrow band of the radiation is not of ionospheric origin. The stability of the narrow band emissions, lasting for tens of seconds, also rules out interplanetary effects which have shorter durations. Therefore, the narrow bandwidth of the radiation is caused either by wave escape conditions at the planet, or radiation characteristics inherent to the source or its excitation by Io. Most of
the negatively drifting lanes occur at the central meridian longitudes of Source A, with drift rates between about -70 and -140 kHz sec\(^{-1}\).

A considerably smaller number of negative drifts, ranging between about -30 and -100 kHz sec\(^{-1}\), are found at the Source B longitude. Most of the positive drifts are found in the Source B region, with values mainly between 100 and 150 kHz sec\(^{-1}\). As we have seen, Jovian pulses are narrow band and drift rapidly in frequency; dispersion in the interplanetary medium is inadequate to explain this phenomenon.

Riihimaa (1970b) therefore concluded that the substructure, whether smooth or pulsed, is a basic property of the radiation, and is related to the circumstances of the emitting source at Jupiter.

It is well known that the radiation from Sources A and B is mostly right elliptically, or right circularly, polarized. However, Io-C radiation displays left elliptical polarization at 22.2 and 16 MHz, and left elliptical, or circular polarization, at 10 MHz (Kennedy, 1969). Io-D source is also found to be strongly left-handed polarized.

Gordon and Warwick (1967) observed one instance of S burst activity during which the dynamic spectrum displayed an apparent alternation of polarization sense as a function of frequency, which was referred to as "polarization diversity". They interpreted this as a form of Faraday effect caused by elliptically polarized magnetoionic modes which have different phase velocities. If the interpretation of the polarization diversity effect by Gordon and Warwick is correct, there must be a coupling of energy between the extraordinary mode and the ordinary mode.

Riihimaa (1970) has shown that the repeated lane structure can be produced by any of the three effects: a) Faraday rotation effect; b) polarization diversity effect; c) intensity variation of the emission.

In a spectral experiment by Riihimaa (1968a), in which a linearly
polarized antenna was used, the Faraday fringes and modulation lanes clearly appear as two distinct and independent patterns. In a later experiment, he used a circular antenna and observed no Faraday fringes. He therefore concluded that the modulation lanes cannot be Faraday fringes. They are not produced by polarization diversity effect either. The modulation lanes must therefore be produced by variations in the apparent intensity of the emission. There are three possible regions which could be responsible for the intensity variation: 1) the terrestrial ionosphere; 2) interplanetary space; 3) the Jovian ionosphere. If the lanes are produced by terrestrial ionospheric scintillation the patterns recorded at spaced sites should therefore be independent. Because the lane patterns observed are virtually identical, they cannot be due to the scintillation effect in the terrestrial ionosphere. The interplanetary scintillation gives rise to the radiation envelopes: it is not involved in the production of the modulation lanes. The lanes must therefore be produced in the vicinity of the planet. Modulation lanes are sharply bounded regions of the dynamic spectra within which radiation is conspicuously, but temporarily, absent. Riihimaa refers to them as "shadow events".

Our present knowledge of the fine structure of Jovian bursts is insufficient to draw any definite conclusion as to how they are produced. Although some investigators are beginning to introduce a certain degree of order from the study of the simpler S bursts and of the L burst modulation lanes, they are still far from giving a satisfactory explanation of the factors involved. However, there is no doubt that the high-resolution dynamic spectra of the Jovian events are providing significant details of the emission and propagation process.
2. MODELS OF JOVIAN DAM RADIATION

The high intensities, limited bandwidths, short time-scales, and sporadic nature of individual DAM bursts have prompted many theoretical attempts at explanation. However, most of the theoretical works until now have dealt with one or two different aspects of the problem.

a) CERENKOV EMISSION

Warwick (1963) analysed the dynamic spectra of Jupiter's DAM emission, observed at the High Altitude Observatory, and proposed an auroral-zone source for the emission. According to Warwick, emission is produced near the electron gyro-frequency by a Cerenkov process. The permanent dynamic spectrum, he concludes, indicates strongly that the emission frequency is not determined by the electron density of Jupiter's ionospheric plasma. Warwick based his hypothesis on the magnetic field control of the DAM emission. He assumed that the radiation is generated, and escapes, in a fixed geometric relation to lines of force in Jupiter's ionosphere. Emission occurs if a line of force of the proper strength and orientation contains fast particles. If localised disturbances of the magnetic field lines take place at distances of 2-3 \( R_J \) (Jupiter radii) from the planet, the subsequent precipitation of fast electrons moving along lines of force excites Cerenkov radiation. This radiation, directed at generation toward the surface of the planet, reflects off the ionosphere, or surface, of Jupiter at an oblique angle toward the Earth. Warwick argues that we therefore observe the emission only when the magnetic field and surface of the planet lie in the correct orientation relative to the Earth.

In addition to this condition, particles should at this moment precipitate along the same line of force. In order to give a satisfactory
interpretation of the observed small asymmetry of the dynamic spectra about CML, $\lambda_{III} = 200^\circ$, Warwick determined the magnetic moment of Jupiter from synthetic dynamic spectra. By a suitable choice of position and strength (but not orientation, which is defined by the decimetric observations of Morris and Berge, 1961) for the magnetic moment, synthetic dynamic spectra could be made to correspond closely to his observations. The derived strength of the moment ($|M| = 4.2 \times 10^{30}$ gauss cm$^3$) agreed well with the result of Davis and Zhang (1961). The location of the dipole moment $M$ was found at $M(X = +0.15178; y = 0.11384; z = -0.73007)$.

However, as a consequence of the Pioneer 10 and 11 missions, Smith et al. (1974) have shown that two parameters of the magnetic field - the magnetic dipole moment, $M_y$, and the displacement of the dipole from the centre of Jupiter - are not well established from ground-based measurements. They derived a model and showed that Jupiter's magnetic field cannot be represented by a simple centred dipole. Three models have been proposed from the results of Pioneer observations: a) an off-set dipole; b) two off-set dipoles; c) a centred dipole plus a quadrupole.

Warwick (1970) has developed his early hypothesis, and has concluded that even the obvious candidate - lines of force involved in emission - fails to give a good interpretation of the emission mechanism. There is nothing special about either the Io-L-shell, or the L-shell whose line of force at Jupiter's surface points toward the Earth. Furthermore, the Io-independent emission suggests that even Io's longitude is not an essential component.

Warwick then goes on to discuss the Jovian Faraday effect. If the waves originate in just one of the two characteristic modes for
the medium, then, knowing that the reflection occurs in a magnetoactive medium, it can be expected that, after reflection, both characteristic modes will be excited. The observations show that the Jupiter polarization ellipse rotates, but only due to the terrestrial effect. This suggests that the observed elliptical radiation is a base mode of the emission (Warwick, 1970). Warwick's conclusions from DAM emission are that the emission is produced by the Cerenkov process, that the field is essentially dipolar and tilted with respect to the rotational axis, and that the sense of the dipole is parallel (rather than anti-parallel) to Jupiter's rotation axis.

b) CYCLOTRON EMISSION

In cyclotron emission theory, radiation is assumed to be produced by bunches of electrons trapped in the Jovian magnetic field (Ellis, 1962; Ellis, 1963; Ellis and McCulloch, 1963). The theory of radiation by an electron travelling along a helical path in a magnetic field predicts a relationship between the wave frequency and the direction of emission, $\Theta$, in the form

$$\omega_s = \frac{s \omega_{ce}}{1 - \beta^2} \sqrt{1 - \frac{v^2}{c^2}}$$

Here $\beta = \frac{v}{c}$, with $v$ - the perpendicular velocity component of the electron with respect to the magnetic field; $n$ is the refractive index of the medium, $\omega_{ce}$ is the electron cyclotron frequency; $\Theta$ is the angle between the axis of emission cone and the magnetic field; $s = 0, 1, 2, ...$

For cyclotron radiation ($s = 1$), the above equation shows that the frequency of the emitted wave will be Doppler-shifted to a frequency greater than the local cyclotron frequency in the forward direction ($0 < \Theta < \pi/2$), and to a lower frequency in the backward direction.
Cyclotron radiation in the forward direction is possible only where the plasma frequency is relatively small compared with the cyclotron frequency. As the electron travels along the field line towards the mirror point, it will radiate at increasing frequencies. The forward radiation will also travel towards the mirror point until reflected at the appropriate extraordinary mode reflection level. After the electron is reflected at the mirror point, it will emit descending frequency radiation in an outward direction. The bounce motion of the electron through the magnetic field changes the emitted frequencies with time.

Since Ellis assumed the emission and propagation of cyclotron radiation, the wave will be elliptically polarized. The propagation properties of the medium will ultimately determine the observed axial ratio of the polarization ellipse. Above all, if successive noise bursts originated from electron bunches in different magnetic longitudes, the axial ratio of the polarization ellipse will vary from burst to burst.

Ellis and McCulloch (1963) refined Ellis's equation for the Doppler-shifted frequency by including the pitch angle of radiating electrons. They thus formulated the Doppler-shifted observed frequency as:

\[ \omega = \omega_{ce} \left(1 - \frac{v^2}{c^2}\right)^{\frac{i}{2}} \left[1 - (nv/c)\cos \phi \cos \Theta\right] \]  

where \( \phi \) is the pitch angle of the electrons, and the remaining parameters are the same as before. It is well known that in this theory, the radiation is limited to a cone. The axis of the cone lies on the magnetic field vector. The radius of the cone depends on the ratio \( \omega_{ce}/\omega_{pe} \) (\( \omega_p \) is the local plasma frequency) and on the pitch angle, \( \phi \); but only to a small extent on the electron energy, providing the latter
Wu et al. (1973a) proposed a generation mechanism for Jupiter's DAM emission based on a cyclotron instability of magnetospheric origin. Furthermore, they pointed out that the existence of several types of DAM emission with different time scales, radiation intensities and frequency drift rates, makes it highly likely that more than one type of instability is involved in the DAM source region. They assumed that magnetospheric plasma of Jupiter contains two kinds of electrons - thermal and energetic. The characteristic energy range for the trapped energetic electrons is of the order 1 - 100 keV, and their density is small in comparison with that of the thermal electrons. These trapped electrons in the Jovian magnetic field must possess a loss-cone distribution in momentum space (Wu et al., 1973a). It is well known that the loss-cone distribution is given by

\[ F = F(p_{\perp}, p_z) \quad \text{for} \quad p_{\perp}^2 > p_z^2 \tan^2 \theta_c \]

\[ = 0 \quad \text{for} \quad p_{\perp}^2 < p_z^2 \tan^2 \theta_c \]

where \( p_{\perp} \) and \( p_z \) are the perpendicular and the parallel components to the magnetic field of the momentum, \( p \); and \( \theta_c \) is the loss-cone angle, which is given by the ratio

\[ \theta_c = \sin^{-1} \left[ \left( \frac{B_o}{B_m} \right)^{1/2} \right] \]

\( B_o \) and \( B_m \) are the local magnetic field and the field at the mirror point. Trapped electrons have a velocity vector outside the loss cone. They found that these energetic particles can be unstable, so that part of their energy can be transferred to the extraordinary mode. Cyclotron resonance of weakly relativistic electrons thus causes an instability.

Wu et al. (1973) attributed the spectral differences between
the main and the early-source to the differences in ray propagation from
the two sources. They argue that the spectral difference indicates the
asymmetry of the magnetosphere with respect to the planet. Frequency
drifts were assumed to be governed by the manner in which the waves
propagate close to the source region.

c) **Io-MODULATED EMISSION**

Ever since the discovery of the modulating effect of Io on
Jupiter's DAM emission (Bigg, 1964), many authors have devoted attention
to the Io-modulated emission. The problem has been dealt with under
three headings: the mechanism coupling Io's orbital motion to the
inner exosphere, the consequent instability mechanism by which electro­
magnetic waves are amplified, and the subsequent propagation of the
waves in the source region and the Jovian plasmasphere.

In his wave model, Ellis (1965) speculated that the coupling
might be due either to MHD (magnetohydrodynamic), or to whistler waves
generated by Io. There was, however, no investigation in his model as
to how such waves might be excited. He hypothesised that the Io­
generated waves would interact with particle streams to stimulate the
electromagnetic radiation observed as DAM, in analogy to the phenomenon
of stimulated very low frequency (VLF) emissions in the terrestrial
magnetosphere.

McCulloch (1971) developed Ellis's ideas by assuming whistlers
to be generated by Io, and carried out ray tracing in a model magneto­
sphere. However, the parameters which McCulloch found necessary to
describe his model magnetosphere, in order to obtain good agreement
between the coupling mechanism and observational morphology, are not in
accord with the Pioneer observations. For example, he used an
ionospheric electron temperature, $T_e$, of 2000 K, whereas Pioneer results
give $T_e = 750^\circ K$. Furthermore, he used a density model for the thermal plasma derived by Melrose (1967), but stated that "the most satisfactory models, however, were those for which the electrons were concentrated in a disk about the magnetic equator, and the plasma frequency and gyrofrequency at Io were nearly equal". The latter condition, 

\[ f_e \approx f_{pe}, \]

is actually in accord with observations. However, the feature of a plasma disk in the equatorial plane is not contained in Melrose's density model. Therefore, McCulloch's model may possibly be internally inconsistent. McCulloch classified the theories proposed for Io-modulated emission according to the location of emission. In the first theory, the radiation is produced near Io, with the emission probabilities and frequencies depending upon the position of Io within Jupiter's magnetosphere (Gledhill, 1967). The second class of theories, which requires some coupling mechanism, produces the radiation close to the surface of Jupiter (Dulk, 1965; Warwick, 1967). In McCulloch's model, whistler mode waves are generated near Io by low energy electrons which are disturbed by Io's motion relative to Jupiter's magnetosphere. The crucial point in this theory is the gyroresonant wave-particle interaction. The gyroresonant interaction requires that incoming whistler mode waves must interact with outgoing electron streams, which are thereby excited to emit Doppler-shifted cyclotron radiation. It is this secondary emission that is observed as the DAM at the Earth.

By considering Io to be perfectly conducting, Warwick (1967) made the first quantitative estimate of MHD-wave coupling. He estimated that for resonant waves of phase velocity $V_{ph} \approx V_{Io}$ (where $V_{Io} = 56 \text{ km sec}^{-1}$ is the speed of Io relative to the co-rotating plasma) and wavelength, $2R_{Io}$, waves of amplitude $B_o = (V_A/\pi V_{Io}) B_o$ might be produced, where $V_A$ is the Alfvén velocity near Io. Since $V_A$ is currently
estimated to be 700 km sec\(^{-1}\), this gives \(B_\omega \approx 4 B_o\). Smith (1976) argues that waves of this magnitude are nonlinear and would ultimately grow into shocks as they propagate towards Jupiter. Warwick estimated that, if the MHD radiation from Io were isotropic, the radiation would be coming from an area of \(1.6 \times 10^5 \text{ km}^2\) (the presumed size of the DAM source region) at the surface of Jupiter.

Goertz and Deift (1973) assumed current leakage from Io into the ambient plasma. The resulting distortions of the ambient magnetic field are considered as MHD wave fields. They asserted that an X-type neutral point behind Io occurs when the field lines leading Io are stretched too far. Magnetic field line merging then occurs behind the satellite, leading to the propagation of Alfven waves up to IFT (Io flux tube). They then determined a transmission coefficient for the waves into the Jovian ionosphere. From the Alfven wave frequency they found the depth - and, therefore, the gyrofrequency at this depth - to which the Alfven waves penetrate. By equating the gyrofrequency at the local altitude of penetration, to the maximum DAM frequency at that longitude, Goertz (1973) has shown good agreement with observation.

Piddington and Drake (1968), in their acceleration model, suggested that an induced electric field, \(\mathbf{E}_{\text{ind}} = \mathbf{V}_I \times \mathbf{B}_I/c\), appears in Io's frame, where \(\mathbf{B}_I\) is the ambient magnetic field at Io. This induced electric field results from the motion of Io with velocity \(|\mathbf{V}_I| = 56 \text{ km sec}^{-1}\) relative to the co-rotating plasma. They assumed that Io is highly conducting, and so its interior would be screened from the induced field by surface polarization charges. Consequently, they assert, an electromagnetic field of 400 KV could result across the IFT, which are assumed to be equipotentials (see Fig. IV.\(\dagger\)). This potential is propagated along the field line at the Alfven speed,

\[
V_A = \frac{B_o}{(4\pi \rho_m)^{\frac{1}{2}}},
\]

where \(B_o\) is the magnetic field strength and \(\rho_m\) the
Collected thermal electron flux

Accelerated photoelectron flux

Jovian magnetic field lines

Jovian ionosphere conductivity

Plasma potential

400 kv

Positive sheath

Negative sheath

Atmosphere

Ionosphere

FIG. (IV, 4)

mass density of the plasma. If the conductivity, $\sigma$, is large enough to stop the field from diffusing through Io in a time shorter than the propagation time, $\tau_A$, of Alfvén waves through the Jovian ionosphere, a DC circuit, which closes in the Jovian ionosphere, will be formed. This DC circuit model has been developed by Goldreich and Lynden-Bell (1969), Gurnett (1972) and Hubbard et al. (1974). Shawhan et al. (1974) extended the model to account for the existence of an ionosphere at Io. They argued that the conductivity required to close the current system is provided by the ionosphere of Io, rather than by the solid satellite, and that the sheaths should be assumed to form at the top of the ionosphere.

Wu (1973), in contrast to the MHD-wave and acceleration models, assumed that the conductivity of Io is low enough to let the Jovian magnetic field diffuse through it. Trapped particles on the field lines threading Io would therefore impinge on the satellite and be swept from the magnetosphere. The time during which energetic particles in a dipole magnetic field experience a complete bounce, from one mirror point to its conjugate and back again, is given (Mead and Hess, 1973) by

$$\tau_b \equiv (1.3 - 0.56 \sin \alpha_o) \frac{R_J}{\nu}$$

where $\alpha_o$ is the equatorial pitch angle; $L$ is the magnetic shell parameter; and $\nu$ is the speed of the particle. $\tau_s$ ($= 60$ seconds) is the time during which Io moves a distance equal to its own diameter. Therefore, particles completing half-a-bounce in a time shorter than $\tau_s$ will be swept away by Io. This sweeping of energetic particles by Io results in a cavity in the density distribution of such particles. The resulting density gradient of energetic protons across the walls of the IFT causes drift currents to appear. These currents drive instabilities
at the upper and lower hybrid resonances.

3. PLASMA HYPOTHESIS OF DAM AND ITS IMPORTANCE

In this subsection we are doing the critical review of the up-to-date Jovian DAM hypothesis and showing why the plasma hypothesis is more powerful to explain the fine structure of the Jovian dynamic spectra. This will be followed, in the next chapter, by a quantitative study giving an explanation to modulation lanes and the Faraday rotation.

In the plasma hypothesis of the Jovian DAM emission, it is believed that the most likely source of the bursts of emission are plasma waves in Jupiter's ionosphere, excited by some mechanism and then partially converted into electromagnetic waves. The main problem is then the origin of the radio emission, i.e. the problem of the actual generation mechanism and the explanation of its action. On the other hand, a significant part is also played by the problem of the propagation of electromagnetic waves in the ionosphere of Jupiter. Since early observations on the Jovian radio emission and Pioneer 10 and 11 flybys have revealed that Jupiter's ionosphere is strongly ionized (i.e. is in a plasma state) and the plasma is embedded in a strong dipole-like magnetic field, it is quite reasonable that current ideas on the origin and propagation of the radio emission of Jupiter are based on plasma physics. One of the most important problems of extensive range in this sphere, is the "wave conversion" mechanism, that is, the conversion of one wave mode into another wave mode.

It must be borne in mind that, under cosmic conditions, problems such as wave conversion and related phenomena have their own specific features determined by the way the problems are stated and the magnitude of the characteristic parameters. Plasma in general, and
cosmic plasma in particular, are very easily excited and enter a
turbulent state. The energy density of turbulent plasma pulsations
can be quite large, and its conversion into electromagnetic waves can
serve as a powerful source of radiation.

In the Jovian ionosphere, which may be assumed to be in-
homogeneous and magnetoactive, interaction between the "normal waves"
sets in, accompanied by exchange of energy, both between waves of the
same type, but with different wave numbers, and between waves of
different types. This means that, if sufficiently intense waves of at
least one type are excited in the plasma for some reason, the mutual
exchange of energy can ultimately lead to radiation of electromagnetic
waves. These leave the plasma and are observed in the form of very
powerful radio emission. Thus a detailed analysis of the plasma
radiation mechanisms should include both an investigation of the
excitation of various types of waves, and the conversion of wave modes
into one another. The smooth inhomogeneity of the Jovian ionosphere
can alter the conditions for the propagation of the waves so that they
cease to be strictly independent and consequently can interact with
one another. In the ionospheric plasma where the wave frequency becomes
nearly equal to the local plasma frequency $\omega \approx \omega\text{pe}$, even small changes
of the plasma parameters lead to an appreciable change in the value of
the dielectric properties of the medium, and consequently also to a
change in the structure of the field of the radio wave.

The problem of the conversion of plasma waves into electro-
magnetic waves is particularly important for the plasma hypothesis of
the origin of the Jovian DAM emission. According to this hypothesis,
the source of the observed radio emission is plasma waves excited in
the Jovian ionosphere at local plasma frequencies, $\omega = \omega\text{pe}$
(Zhelezniakov, 1958; Oya, 1974). Whether wave conversion can be
realized with sufficient efficiency under the actual conditions of the Jovian ionosphere is an arguable requirement of this plasma hypothesis. Generally, when looking for ways in which plasma waves change into electromagnetic radiation, the main point to be remembered is that plasma waves are purely longitudinal and therefore have no magnetic field ($\nabla \times \mathbf{E} = 0$, i.e. $\mathbf{H} = 0$). In electromagnetic waves, on the contrary, $\mathbf{H} \neq 0$. It thus becomes clear that, for excitation in a certain region of a system of plasma waves, no electromagnetic emission can escape from the region. In the vicinity of the ionospheric region where the wave conversion occurs, the frequency of the plasma waves (and the radio emission frequency), $\omega$, is nearly equal to the plasma frequency, $\omega_{pe} = \left(\frac{4\pi e^2 N}{m}\right)^{\frac{1}{2}}$. Therefore, the bursts of radio emission from Jupiter can only occur at frequencies lying below the critical frequency, $\omega_{cr}$, of the Jovian ionosphere:

$$\omega = \omega_{pe} < \omega_{cr} = \left(\frac{4\pi e^2 N_{\text{max}}}{m}\right)^{\frac{1}{2}}.$$ Here, $N_{\text{max}}$ is the maximum electron concentration in the Jovian ionosphere. This conclusion is confirmed by the fact that the intense radio emission from Jupiter is observed in the frequency range $f \leq 27$ MHz, but not on frequencies $f \geq 41$ MHz.

From the above formulation, it can easily be seen that the frequency range of the emission is closely related to the electron density of the medium.

When studying the propagation of radio waves in the ionosphere, we must allow for the fact that the dependence of the electron concentration on the altitude, $N(z)$, is not monotonic. This quantity becomes maximum at a certain altitude, $Z_{\text{max}}$, and falls as $z$ moves away from $Z_{\text{max}}$. This plasma parameter is very important, not only for the determination of the frequency range of the emission, but also for the determination of the transformation bands and the directivity of the emission. It is well known that the character of the directivity of
radio emission will differ according to its position with respect to the layer maximum. If the source is separated from the observation point by a layer $z = z_{\text{max}}$, the directivity of the emission does not depend on the actual position of the source, but is determined by the value of the plasma frequency at the layer maximum. Otherwise, it depends on the value of the plasma frequency at the point where the source is located.

If plasma waves are responsible for the DAM emission of Jupiter, we must consider the conditions under which conversion is possible. These conditions state that, at least one point $z = z_0$ at which the refractive index of a wave propagating in the ionospheric plasma becomes infinite, must be contained inside the layer. If this condition is fulfilled, three frequency bands appear in which wave conversion is possible. The limits of the transformation frequency bands are determined by the distribution of the plasma concentration (Golant and Piliya, 1972). Another important plasma parameter playing a decisive role in wave conversion is the angle between the concentration gradient and the external magnetic field, $\mathbf{H}$. When this angle is not zero, $\alpha \neq 0$, there is a certain frequency interval, separating the upper and middle bands, in which wave transformation cannot take place.

It must be emphasized that, in the case of an isotropic plasma, wave conversion is possible only if the wave vector, $\mathbf{k}$, of the plasma wave does not have the same direction as $\nabla N$ (the gradient of the electron density). However, Jupiter's ionospheric plasma is anisotropic, and the conversion of plasma waves into electromagnetic radiation can also occur when the propagation is along the gradient. Such conversion becomes possible when the thermal motion of electrons in a magnetoactive plasma is taken into account. The effect of including the thermal motion of electrons is to bring a new wave mode
into existence - namely, the plasma wave. Gershman (1957) showed that, for a particular set of parameters, the plasma wave and the extra-ordinary wave are represented in the parameter space not by separate curves, but by what appears to be a continuation of the same curve (see Fig. IV.5).

Owing to the strong magnetic field of Jupiter, the previous theories were deeply concerned with the magnetic field control of the decametric wave emission. Oya (1974) has demonstrated that the external magnetic field inhomogeneity in the vicinity of the region where \( \omega = \omega_{ce} \) makes practically no change in the conversion rate. He concluded that it is not the magnetic field inhomogeneity that gives a high conversion rate, but the region where the wave frequency coincides with the plasma frequency, \( \omega \sim \omega_{pe} \).

Experiments on plasma resonances which occur at the point where \( \omega \sim \omega_{pe} \), offer evidence of excitation of slow longitudinal waves in the plasma under the influence of electromagnetic waves incident from the outside (Stern and Tzoar, 1965; Aksornkitti et al., 1969; Gregory and Purbhakar, 1969). Thus, the data available on the excitation of plasma resonances in a magnetic field confirm the existence of efficient conversion of electromagnetic waves into longitudinal waves. It can be noted that resonances are observed only when there exists in the volume of a plasma a region of conversion connected with the field singularity. A situation is possible wherein the plasma wave produced upon transformation of an electromagnetic wave is not absorbed in the plasma, but reaches a second singular point and is then reconverted into an electromagnetic wave.

Elliptical polarization has been recorded when studying certain components of the Jovian radio emission. It is possible that coherent waves may occur in the emission source itself, or in the "interaction
FIG. (IV, 5)

Reproduced from Gershman, 1957. Solid curves - $n_1^2$ (extraordinary wave); Dotted curves - $n_3^2$ (plasma wave). $\omega_{ce}^2/\omega^2 = 0.5; \beta_e^2 = 10^{-4}$. 
region", where there is a partial change of waves of one type into another. Conversion of waves of one type into another is accompanied by a significant alteration in the polarization of the waves.

There is no Faraday rotation observed in the DAM emission. This is not surprising at all. Since the mutual conversion of plasma and extraordinary waves is assumed to be responsible for the Jovian bursts, and since the plasma waves cannot propagate into the vacuum, it is only the extraordinary mode that escapes beyond the ionosphere and is received on Earth. Jovian Faraday rotation, if any, should occur within the limiting region, which is of limited spatial extent. The absence of Faraday rotation outside the interaction region may mean that the effect of the magnetic field need not be taken into consideration.

If we summarize what has been said so far in general terms: the models of the Jovian magnetic field and plasma and cyclotron frequencies in the magnetoplasma and thereby provide a basis for estimating the local electromagnetic emission in the ionosphere of Jupiter. Regions of absorption, mode conversion and wave amplification are readily identified. In order to study systematically the propagation properties of a magnetoplasma, the well-known CMA (Clemmow-Mullaly-Allis) diagram of plasma physics may be utilized. This diagram divides the complex modes of propagation into various regions in which a characteristic type of propagation is readily identified (see Boyd and Sanderson, 1969). Similar propagation regions, for specified propagation frequencies and selected magnetoplasma models, may be identified in the configuration space around Jupiter. Loci of propagation cut-offs and resonances help in determining the distribution of radio noise. These properties of propagation provide a useful basis for speculation about the distribution of local radio noise and its source mechanism. Within each of the volumes in parameter space, bounded by the cut-offs and
resonances, the type of a wave remains the same. It is only when these bounding surfaces in the parameter space are crossed that the type of the wave can change. It is, therefore, clear that the possibility of explaining the DAM emission in terms of a plasma mechanism is closely associated with the solution of the conversion problem.

4. WAVE PROPAGATION CHARACTERISTICS IN JUPITER'S IONOSPHERE

a. WAVE MODES

In this section we shall briefly review the wave modes that are important for understanding the nature of the emission and the changes it undergoes. Later on we shall look for the "cold" and "warm" solutions of the wave propagation equation. By "matching" these two solutions we expect to gain an insight into the fine structure of Jovian DAM emission.

In the previous subsection we have emphasized that the propagation characteristics of waves in plasmas are equally important as the generation mechanism. Abrupt changes occur in the topological genera and in the modes as the bounding surfaces in parameter space are crossed. Before examining the problem of wave propagation and wave conversion, it is necessary to define the wave modes in plasmas.

Plasma physics borrows the concept of a "wave-normal surface", or, briefly, a "normal surface" from the field of optics. This surface is the locus of the tip of a vector which has the direction of the propagation vector and the amplitude of the phase velocity. Let us assume a space in which the scale lengths in the different directions are proportional to the plasma parameters (electron density, magnetic field strength, etc.). In this parameter space, it is only when certain surfaces are crossed that the modes of the wave normal surfaces can change. Otherwise the topological genus of a wave normal surface remain
the same within these volumes in parameter space which are bounded by
the bounding surfaces (see Fig. IV.6). Any wave can, therefore, be
immediately identified and labelled. Furthermore, we know precisely
the regions of parameter space for which this identification is valid.

It is necessary to distinguish between the wave-normal
surfaces and the bounding surfaces. Bounding surfaces are the ones in
parameter space. On the other hand, wave normal surfaces are in the
space embedding the phase-velocity vector. Waves may be classified
into four groups in a cold plasma\(^1\) depending on the following factors.
(a) The topological genus of their wave-normal surface; (b) The
magnitude of the phase velocity at angles between 0 and \(\pi/2\) [Fast (F)
or Slow (S)]; (c) The circular polarization for parallel propagation
to the static magnetic field [Right (R) or Left (L)]; (d) The
dispersion relation for propagation at right angles to the static
magnetic field [Ordinary (O) or Extraordinary (X)]; (e) The region
of parameter space in which the waves occur (Stix, 1965).

(i) EXTRAORDINARY AND ORDINARY MODES

The equation of motion of an electron in a magnetic field is

\[
\ddot{\mathbf{r}} + e \frac{\partial \mathbf{r}}{\partial t} \wedge \mathbf{B} = m \frac{\partial^2 \mathbf{r}}{\partial t^2} + m \nu \frac{\partial \mathbf{r}}{\partial t}
\]  

\text{(IV.4.1)}

where \(\mathbf{E}\) and \(\mathbf{B}\) are the electric field and the magnetic induction,
respectively; \(m\) is the mass of an electron; \(\nu\), the collision frequency.
Now all quantities vary with time through the factor \(e^{i\omega t}\), so that
the operator \(\partial / \partial t\) can be replaced by \(i\omega\). If the equation IV.4.1 is
multiplied by \(N_e/m \omega^2\), it then becomes

\[\text{(IV.4.1)}\]

\(^1\) For a cold plasma model, \(\psi\), the kinetic pressure dyad, can be
taken as zero. More precisely, it is a model in which the
thermal motions of the particles are neglected.
Reproduced from Boyd and Sanderson, 1969. CMA diagram; wave-normal surfaces for waves in a cold plasma. The figures are not plotted to scale but the dotted circle represents the velocity of light in each region.
\[
\frac{\varepsilon_0}{\varepsilon} \frac{\partial^2 E}{\partial t^2} + \frac{i\omega}{\varepsilon} \mathbf{P} \wedge \mathbf{H} = -\mathbf{P} (1 - i\omega) \tag{IV.4.2}
\]

where \( \mathbf{P} = Ne\mathbf{r} \) is the electric polarisation; \( Z = \nu/\omega \).

Let \( \mathbf{Y} = \frac{eB}{m\omega} \mathbf{B} \) \tag{IV.4.3}

We rearrange (IV.4.2) to give

\[-\varepsilon \mathbf{X} \mathbf{E} = \mathbf{P} (1 - i\omega) + i\mathbf{P} \wedge \mathbf{Y} \tag{IV.4.4}\]

where \( \mathbf{X} = \frac{\varepsilon_0}{\varepsilon} \frac{2}{\varepsilon} \mathbf{m} \mathbf{n} \).

Let \( \mathbf{Y} = |eB/m\omega| \), and let \( \xi, \mu, \nu \) be the direction cosines of the vector \( \mathbf{Y} \). Then (IV.4.4) may be written in Cartesian coordinates (Budden, 1961);

\[-\varepsilon \mathbf{X} \mathbf{E} = \mathbf{U} \mathbf{P} + i\mathbf{N} \mathbf{Y} = i\mathbf{N} \mathbf{P} \]

\[-\varepsilon \mathbf{X} \mathbf{E} = \mathbf{U} \mathbf{P} + i\mathbf{N} \mathbf{Y} \]

where \( \mathbf{U} = 1 - i\omega \).

We now choose the coordinate system in such a way that the z-axis coincides with the direction of the wave normal, and the static magnetic field lies in the X-z-plane ("magnetic meridian plane"). This makes \( \mathbf{m} = 0 \) (\( -\xi, -\mu, -\nu \) are the direction cosines of the static magnetic field). With this choice of axes, (IV.4.5) becomes in matrix form

\[
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
= \begin{pmatrix}
U & i\xi \nu \\
-i\xi \nu & U & i\xi \mu \\
0 & -i\xi \mu & U
\end{pmatrix}
\begin{pmatrix}
\mathbf{P}_x \\
\mathbf{P}_y \\
\mathbf{P}_z
\end{pmatrix} \tag{IV.4.6}
\]
Let us seek a solution of Maxwell's equations which represent a "progressive plane wave". Then \( \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \) becomes in Cartesian coordinates

\[
\begin{align*}
\text{i}\kappa \mathbf{H}_y &= \frac{\text{i}\kappa}{\varepsilon_0} \frac{\partial}{\partial x} D_x \\
\text{-i}\kappa \mathbf{H}_x &= \frac{\text{i}\kappa}{\varepsilon_0} \frac{\partial}{\partial y} D_y \\
D_z &= 0
\end{align*}
\]

where \( \mathbf{D} \) is the electric displacement, and is defined as (Budden, 1961):

\[
\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}
\]

On the other hand, the wave polarisation is defined by

\[
\rho = \frac{\mathbf{E}}{\mathbf{E}} = \frac{\mathbf{P}}{\mathbf{E}} = \frac{\mathbf{D}}{\mathbf{E}} = -\frac{\mathbf{H}}{\mathbf{H}}
\]

The last equation of (IV.4.7), together with (IV.4.8), gives

\[
\varepsilon_0 \mathbf{E} + \mathbf{P} = 0
\]

Substitution of (IV.4.10) in the third equation of (IV.4.6) then leads to

\[
(U - X)P_z = \text{i}\xi Y P_y
\]

A combination of (IV.4.11) with the first two equations of (IV.4.6), gives:

---

A plane wave is defined as a wave in which there is no variation of any field component in any plane perpendicular to the wave normal. If the wave normal is in the \( z \)-direction, the derivatives \( \partial / \partial x \), \( \partial / \partial y \) are then zero for all field components. A plane wave in which the dependence of all field quantities upon \( z \) is the same (e.g. through the term \( e^{-\text{i}kxz} \)) is called a "progressive plane wave".

---

\( ^2 \)
If we divide the second equation of (IV.4.12) by the first, and use (IV.4.9), the result is

\[ \rho = \frac{-\ln Y + \rho \left[ U - \frac{\epsilon^2_Y}{U-X} \right]}{U + \ln Y \rho} \] (IV.4.13)

and

\[ \rho^2 - \frac{i \epsilon^2_Y}{n(U - X)} \rho + 1 = 0 \] (IV.4.14)

Designating the longitudinal and transverse components of \( Y \) by:

\[ Y_L = nY, \quad Y_T = \epsilon Y, \quad \text{respectively,} \] (IV.4.14) may be written as

\[ \rho^2 - \frac{i Y_T^2}{Y_L(1 - X - iZ)} \rho + 1 = 0 \] (IV.4.15)

The solutions of the quadratic equation (IV.4.15) are

\[ \rho_{1,2} = \frac{iy_T^2}{2Y_L(1-X-iZ)} \pm i \left[ \frac{Y_T^4}{4Y_L^2(1-X-iZ)^2} + 1 \right]^{\frac{1}{2}} \] (IV.4.16)

In a homogeneous anisotropic medium, a progressive plane wave must have a polarisation given by one of the two values of (IV.4.16).

We now turn to Maxwell's third and fourth equations:

\[ \nabla \times E = -\mu_0 \frac{\partial H}{\partial t} \quad \text{and} \quad \nabla \times H = \frac{\partial D}{\partial t} \], respectively. Written in Cartesian coordinates, these become

\[ nE_y = -H_x \] (IV.4.17)

\[ nE_x = H_y \] (IV.4.18)

\[ H_z = 0 \] (IV.4.19)
and

\[
\begin{align*}
    n_H &= \frac{D_x}{\varepsilon_0} \quad \text{(IV.4.20)} \\
    -n_H &= \frac{D_y}{\varepsilon_0} \quad \text{(IV.4.21)} \\
    D_z &= 0 \quad \text{(IV.4.22)}
\end{align*}
\]

Elimination of \( H_y \) from (IV.4.18) and (IV.4.20) gives

\[
\varepsilon_0 n^2 E_x = D_x \quad \text{(IV.4.23)}
\]

Similarly, the elimination of \( H_x \) from (IV.4.17) and (IV.4.21) gives

\[
\varepsilon_0 n^2 E_y = D_y \quad \text{(IV.4.24)}
\]

Combining (IV.4.8), (IV.4.9) and the first equation of (IV.4.12) with (IV.4.23) leads to

\[
n^2 = 1 - \frac{X}{U + in\rho} \quad \text{(IV.4.25)}
\]

Since \( \rho \) has two possible values (IV.4.16), \( n^2 \) also has two values.

If the value of \( \rho \) is substituted from (IV.4.16), the result is

\[
\begin{align*}
    n^2 &= 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{1 - X - iZ}} \\
    &= 1 - iZ - \frac{\frac{1}{2} Y_T^4}{(1 - X - iZ)^2} \pm \left[ \frac{\frac{1}{2} Y_T^4}{(1 - X - iZ)^2} + Y_L^2 \right]^{\frac{1}{2}}
\end{align*}
\]

From this equation it is easily seen that, for propagation perpendicular to the external magnetic field, either \( n^2 = 1 - \frac{X}{1 - iZ} \) or

\[
n^2 = 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{1 - X - iZ}}
\]

The mode satisfying the first of these two dispersion relations, which does not involve \( B_0 \), is labelled 'Ordinary' (O); that satisfying the second dispersion relation, namely
a $B_0$-dependent dispersion relation, is labelled 'Extraordinary' (X).

(ii) PLASMA OSCILLATIONS AND PLASMA WAVES

As is well known, the basic property of a plasma is its overall electrical neutrality. If any significant imbalance of charge is introduced, large electrostatic (Coulomb) forces come into play and these forces are responsible for the electron plasma oscillations. The equations of plasma dynamics are useful in deducing the properties of the electron plasma oscillations. We will examine those properties of a cold plasma model where the thermal motions of electrons are not taken into consideration. It is also assumed that there is no magnetic field due to external sources, and the ions have been assumed to be immobile. With these approximations, the linearized forms of the equation of continuity and the momentum transport equations become

\[
\frac{\partial}{\partial t}N(r,t) + N_o \left[ \nabla \cdot \vec{v}(r,t) \right] = 0 \quad (IV.4.27)
\]

\[
\frac{\partial}{\partial t}\vec{v}(r,t) = - \frac{e}{m} \vec{E}(r,t) \quad (IV.4.28)
\]

where $N_o$ and $N$ are the equilibrium and perturbation electron number densities; $\vec{E}$ is the electric field, $m$ - the electron mass and $\vec{v}$ - the average velocity. The time-varying charge and current densities give rise to electromagnetic fields which satisfy the following Maxwell's equations (Stix, 1965):

\[
\nabla \times \vec{E} = -\mu_o \frac{\partial \vec{H}}{\partial t} \quad (IV.4.29)
\]

\[
\nabla \times \vec{H} = \epsilon_o \frac{\partial \vec{E}}{\partial t} + \vec{J} \quad (IV.4.30)
\]

\[
\nabla \cdot \vec{E} = \frac{\rho_o}{\epsilon_o} \quad (IV.4.31)
\]

\[
\nabla \cdot \vec{H} = 0 \quad (IV.4.32)
\]
where $\mu_o$ and $\epsilon_o$ are the vacuum permeability and permittivity, respectively. Charge density, $\rho$, is defined as $\rho(\hat{\mathbf{r}}, t) = -eN(\hat{\mathbf{r}}, t)$.

Thus, (IV.4.31) can be rewritten

$$\nabla \cdot \vec{E}(\hat{\mathbf{r}}, t) = -\left(\frac{e}{\epsilon_o}\right) N(\hat{\mathbf{r}}, t)$$

(IV.4.33)

If we differentiate (IV.4.27) with respect to time and substitute (IV.4.28), we obtain

$$\left(\frac{\partial^2}{\partial t^2}\right) N(\hat{\mathbf{r}}, t) - \left(N_e/m_e\right) \nabla \cdot \vec{E}(\hat{\mathbf{r}}, t) = 0$$

(IV.4.34)

By using equation (IV.4.33), equation (IV.4.34) can be reduced to

$$\left(\frac{\partial^2}{\partial t^2}\right) N(\hat{\mathbf{r}}, t) + \omega_{pe}^2 N(\hat{\mathbf{r}}, t) = 0$$

(IV.4.35)

where $\omega_{pe} = \left(N_e^e/m_e \epsilon_o\right)^{\frac{1}{2}}$.

(IV.4.36)

We see from (IV.4.35) that the perturbation number density of the electrons has a simple harmonic variation with time. The solution of (IV.4.35) is given by

$$N(\hat{\mathbf{r}}, t) = N(\hat{\mathbf{r}}) \exp(-i \omega_{pe} t)$$

(IV.4.37)

where $\omega_{pe}$ is called the electron plasma angular frequency. It can easily be verified that all the first-order perturbations (electron number density, velocity, electric field) vary with time at the electron plasma angular frequency, $\omega_{pe}$. Therefore we shall proceed with the assumption that all the field quantities have a harmonic time variation as expressed by

$$f(\hat{\mathbf{r}}, t) = f(\hat{\mathbf{r}}) \exp(-i \omega t)$$

(IV.4.38)

where $f(\hat{\mathbf{r}}, t)$ is any field quantity. Then, equations (IV.4.27), (IV.4.28) and (IV.4.33) can be reduced to give
IV.34

\[ N(r) = (-\frac{i}{\omega})N_0 \nabla \cdot \mathbf{v}(r) \]  \hspace{1cm} (IV.4.39)

\[ \mathbf{v}(r) = (-\frac{i e}{\omega m_e}) \mathbf{E}(r) \]  \hspace{1cm} (IV.4.40)

and

\[ \nabla \cdot \mathbf{E}(r) = -\left(\frac{e}{\varepsilon_0}\right)N(r) \]  \hspace{1cm} (IV.4.41)

With the help of equations (IV.4.39) and (IV.4.40), (IV.4.41) can be expressed in the form

\[ (1 - \frac{\omega^2}{\omega^2}) [\nabla \cdot \mathbf{E}(r)] = 0 \]  \hspace{1cm} (IV.4.42)

The non-trivial solution of (IV.4.42) gives \( \omega = \omega_{pe} \), which justifies the assumption that all the perturbations have a harmonic time variation at the electron plasma angular frequency, \( \omega_{pe} \). We see from (IV.4.40) that the electron velocity, and, hence, electron motion are parallel to the electric field. The electron plasma oscillations are therefore "longitudinal" in character.

Current density may be defined as

\[ \mathbf{J}(r) = -N_0 e \mathbf{v}(r) = (i N_0 e^2/\omega m_e) \mathbf{E}(r) \]  \hspace{1cm} (IV.4.43)

The right-hand side of (IV.4.30) is \( -i \omega \varepsilon \mathbf{E}(r) + \mathbf{J}(r) \) which together with (IV.4.36) and (IV.4.43) can be rewritten as

\[ -i \omega \varepsilon \mathbf{E}(r) + \mathbf{J}(r) = -i \omega \varepsilon \left[ 1 - \frac{\omega^2}{\omega^2} \right] \mathbf{E}(r) \]  \hspace{1cm} (IV.4.44)

For harmonic time variation, as expressed in (IV.4.38), equations (IV.4.29) and (IV.4.30) can be simplified with the help of (IV.4.44) to yield

---

3 In plasma physics, the term "longitudinal" reflects the relation between the velocity vector, \( \mathbf{v} \), or the propagation vector, \( \mathbf{k} \), and the electric field of the perturbation (plasma oscillation, plasma wave or electromagnetic wave). When \( \mathbf{v} \parallel \mathbf{E} \), or \( \mathbf{k} \parallel \mathbf{E} \), is satisfied, the perturbation is called "longitudinal".
\[ \nabla \times \vec{E}(r) = i \omega \mu_0 \vec{H}(r) \quad \text{(IV.4.45)} \]

\[ \nabla \times \vec{H}(r) = -i \omega \varepsilon_0 \varepsilon \vec{E}(r) \quad \text{(IV.4.46)} \]

where

\[ \varepsilon_r = 1 - \frac{\omega^2}{\omega_p^2} \quad \text{(IV.4.47)} \]

For the electron plasma oscillations, \( \omega = \omega_p \), \( n = 0 \), and, therefore, equation (IV.4.46) reduces to

\[ \nabla \times \vec{H}(r) = 0 \quad \text{(IV.4.48)} \]

Since \( \nabla \times (\nabla a) = 0 \), for any scalar function, \( a \), it follows that the magnetic field can be sought in the form

\[ \vec{H}(r) = -\nabla \psi(r) \quad \text{(IV.4.49)} \]

where \( \psi(r) \) is a scalar potential. Since \( \nabla \cdot (\nabla \times \vec{A}) = 0 \) for any vector function, \( \vec{A} \), it can be deduced with the help of equations (IV.4.45) and (IV.4.49) that

\[ \nabla \cdot [\nabla \psi(r)] = \nabla^2 \psi(r) = 0 \quad \text{(IV.4.50)} \]

The only solution of Laplace's equation which is not singular at infinity is given by \( \psi(r) = \text{constant} \). Then equation (IV.4.49) indicates that

\[ \vec{H}(r) = 0 \quad \text{(IV.4.51)} \]

Equation (IV.4.51) implies that the magnetic field associated with the electron plasma oscillations is zero, and, therefore, these oscillations are electrostatic in character. Moreover, it is seen from equation (IV.4.38) that all the field variables associated with the perturbation number density change with time in phase, and there is no relative phase variation from point to point in space. This implies the absence
of any wave propagation. In other words, electron plasma oscillations are also stationary oscillations. Hence, the plasma oscillations are longitudinal, electrostatic, and stationary.

It is not difficult to establish that the stationary electron plasma oscillations that can exist in a cold plasma become propagating disturbances on account of the thermal motions of the electrons. It may be assumed that the thermal motions of the electrons are adequately described by a scalar pressure (Seshadri, 1973). Thus, the linearized form of the momentum transport equation (IV.4.28) needs to be modified to include the pressure term. The result is

\[
\frac{\partial}{\partial t} \vec{V}(\vec{r}, t) = -\left(\frac{e}{m} \right) \vec{E}(\vec{r}, t) - \left(\frac{1}{N_0 m} \right) \nabla P(\vec{r}, t) \tag{IV.4.52}
\]

All the field quantities are assumed to have a harmonic time dependence as expressed in equation (IV.4.38). Then, (IV.4.52) reduces to

\[
\vec{V}(\vec{r}) = \left(\frac{-ie}{\omega m} \right) \vec{E}(\vec{r}) - \left(\frac{i}{\omega N_0 m} \right) \nabla P(\vec{r}) \tag{IV.4.53}
\]

Combining equations (IV.4.36), (IV.4.47) and (IV.4.53) yields

\[
-\frac{ie}{\omega} \vec{E}(\vec{r}) - N_0 e \vec{V}(\vec{r}) = -i\omega \epsilon \vec{E}(\vec{r}) + \frac{i\epsilon}{\omega m} \nabla P(\vec{r}) \tag{IV.4.54}
\]

The electric field may be separated into two parts as follows

\[
\vec{E}(\vec{r}) = \vec{E}_{em}(\vec{r}) + \vec{E}_{p}(\vec{r}) \quad \vec{E}_{p}(\vec{r}) = \frac{e}{\omega^2 m \epsilon_0 \epsilon_r} \nabla P(\vec{r}) \tag{IV.4.55a,b}
\]

where the subscripts 'em' and 'p' stand for electromagnetic and plasma, respectively. The substitution of (IV.4.55) into (IV.4.54) gives

\[
-\frac{ie}{\omega} \vec{E}(\vec{r}) - N_0 e \vec{V}(\vec{r}) = -i\omega \epsilon \vec{E}(\vec{r}) \tag{IV.4.56}
\]

The right-hand sides of equation (IV.4.30) and equation (IV.4.56) suggest the separation of the magnetic field also into two parts as follows:
IV.37

\[ \hat{H}(r) = \hat{H}_e(r) + \hat{H}_p(r) \quad \hat{H}_p(r) = 0 \quad (IV.4.57a,b) \]

Since \( \nabla \cdot \hat{V} P(r) = 0 \), the substitution of equations (IV.4.55) - (IV.4.57) into equations (IV.4.29) and (IV.4.30) may be shown to give

\[ \nabla \cdot \hat{E}_e(r) = i \omega \mu \hat{H}_e(r) \quad (IV.4.58) \]

\[ \nabla \cdot \hat{H}_e(r) = - i \omega \varepsilon \hat{E}_e(r) \quad (IV.4.59) \]

If we take the divergence of both sides of equation (IV.4.59) the result is

\[ \nabla \cdot \hat{E}_e(r) = 0 \quad (IV.4.60) \]

since \( \nabla \cdot [\nabla \cdot \hat{H}_e(r)] = 0 \). Together with equations (IV.4.57), equation (IV.4.32) becomes

\[ \nabla \cdot \hat{H}_e(r) = 0 \quad (IV.4.61) \]

If equations (IV.4.55) are substituted into equation (IV.4.53) and the resulting equation simplified with the help of equations (IV.4.36) and (IV.4.47), it is found that

\[ \hat{V}(r) = \hat{V}_e(r) + \hat{V}_p(r) \quad (IV.4.62) \]

\[ \hat{V}_e(r) = - \frac{i e}{\omega m_e} \hat{E}_e(r) \quad \hat{V}_p(r) = - \frac{i}{\omega m_e} \nabla P(r) \quad (IV.4.63a,b) \]

From equations (IV.4.60) and (IV.4.63a), we obtain

\[ \nabla \cdot \hat{V}_e(r) = 0 \quad (IV.4.64) \]

Using the adiabatic gas law, \( P N^{-\gamma} = \text{constant} \), and \( \gamma = 1 + 2/\delta \), where \( \delta \) is the number of degrees of freedom, the equation of continuity may be put in a new form

\[ - i \omega P(r) + m_e e^2 N_0 \left[ \nabla \cdot \hat{V}(r) \right] = 0 \quad (IV.4.65) \]
where \( a_e^2 = \frac{P(r,t)}{m_e N(r,t)} = \gamma P_e/m_e N_e \) is the sound velocity in the electron gas. An examination of equations (IV.4.65) and (IV.4.64) suggests the separation of \( P(r) \) into the following two parts.

\[
P(r) = P_{em}(r) + P_p(r) \quad P_{em}(r) = 0 \quad \text{(IV.4.66a,b)}
\]

The substitution of equations (IV.4.62)-(IV.4.64) and (IV.4.66) into equation (IV.4.65) may be shown to yield the following result:

\[
\nabla^2 P_p(r) + \frac{\omega^2}{a_e^2} \left( 1 - \frac{\omega^2}{\nu_e^2} \right) P_p(r) = 0 \quad \text{(IV.4.67)}
\]

with the help of equations (IV.4.66b), (IV.4.66), and (IV.4.67), it can be deduced that

\[
\nabla \cdot \overrightarrow{E}_p(r) = -\frac{e}{m \varepsilon_0 a_e^2} P_p(r) \quad \text{(IV.4.68)}
\]

In the light of the preceding discussion, based on Seshadri (1973), it is clear that all the field variables may be separated into two parts which are associated with the electromagnetic and the plasma modes.

The electromagnetic mode consists of the components \( \overrightarrow{H}_{em}(r), \overrightarrow{E}_{em}(r), \overrightarrow{V}_{em}(r) \), and with \( P_{em}(r) = 0 \) which are governed by equations (IV.4.58)-(IV.4.61), (IV.4.63a), and (IV.4.64). The plasma mode consists of the components \( \overrightarrow{E}_p(r), \overrightarrow{V}_p(r), P_p(r) \), and with \( \overrightarrow{H}_p(r) = 0 \) which are governed by equations (IV.4.67), (IV.4.55b) and (IV.4.63b). It is clearly seen that the plasma mode contains all the charge accumulation and no magnetic field. The electron pressure has variation only in that direction towards which the plasma wave propagates. We can easily infer from equations (IV.4.55b) and (IV.4.63b) that \( \overrightarrow{E}_p(r) \) and \( \overrightarrow{V}_p(r) \) have components only in the direction of propagation. In other words, the plasma mode represents longitudinal waves. Moreover, since there is no magnetic field associated with it, \( \overrightarrow{H}_p(r) = 0 \), the plasma mode is electrostatic.
Summing up, the longitudinal, electrostatic, and stationary electron plasma oscillations that exist in a cold plasma retain their longitudinal and electrostatic character, but become propagating disturbances when the thermal motion of electrons is taken into account. The electrostatic character of plasma waves may enable us to explain the modulation lanes, observed by Riihimaa (1970), in Jovian dynamic spectra.

b. ANALYSIS OF WAVE PROPAGATION CHARACTERISTICS BY THE "W.K.B. APPROXIMATION"

In the light of the foregoing analysis, a new approach can be made to the problem of wave propagation via the WKB approximation. Our primary concern, now, is not the generation mechanism of DAM, but rather the effect of "wave conversion" on the phenomenology and the morphology of DAM. Time-varying charge and current densities, probably caused by the radial diffusion of particles from the IMF sector boundaries, give rise to electromagnetic fields which satisfy the Maxwell's equations. Before going into the formulation, it is constructive to consider the geometry of the problem. It is assumed that the particle distribution gradient is in the direction of the rotational axis. This assumption is, in fact, justified by the observations of many authors (i.e. Ellis, 1965; Melrose, 1967; Gledhill, 1967; Piddington, 1967; Brice and Ioannidis, 1970). It was observed that centrifugal forces would dominate the plasma configuration beyond a few Jupiter radii. We are considering a Cartesian coordinate system with the z-axis directed along the particle distribution gradient, VN, so that the electron number density and collision frequency are functions only of z. We assume that there is a plane wave which is incident on the ionosphere from the less denser side, with its normal
in the x-z plane. Let $S = \sin \Theta_1$, $C = \cos \Theta_1$; where $\Theta_1$ is the angle of incidence. We shall also assume that the ionosphere is replaced by a number of thin, discrete strata in each of which the medium may be taken as homogeneous. Then, in accordance with the definition of a plane wave, in each stratum $\partial / \partial x = -ikS$; $\partial / \partial y = 0$. We shall adopt for the magnetic field, $\mathbf{H}$, a different measure which simplifies the equations and will be used throughout this chapter, i.e., $\mathbf{H} = (\mu_o / \epsilon_o)^{1/2} \mathbf{H}$. Using the derivatives and the above notation in the Maxwell equations, we get

$$\nabla \times \mathbf{E} = -ik\mathbf{H}, \quad \nabla \cdot \mathbf{H} = \frac{ik}{\epsilon_o} \mathbf{D}$$

when written in full, the Maxwell equations become:

$$- \frac{d\mathcal{E}_y}{dz} = -ik\mathcal{H}_x \quad (IV.4.69)$$

$$\frac{d\mathcal{E}_x}{dz} + ik\mathcal{E}_z = -ik\mathcal{H}_y \quad (IV.4.70)$$

$$- ik\mathcal{E}_y = -ik\mathcal{H}_z \quad (IV.4.71)$$

$$- \frac{d\mathcal{H}_y}{dz} = \frac{ik}{\epsilon_o} \mathcal{D}_x \quad (IV.4.72)$$

$$\frac{d\mathcal{H}_x}{dz} + ik\mathcal{H}_z = \frac{ik}{\epsilon_o} \mathcal{D}_y \quad (IV.4.73)$$

$$- ik\mathcal{H}_y = \frac{ik}{\epsilon_o} \mathcal{D}_z \quad (IV.4.74)$$

For vertical incidence, $S = 0$ and $C = 1$ and the above equations reduce to a simpler form, (where $k = \omega / c = 2\pi / \lambda$ is the normalized propagation constant with respect to free space).
\[
\begin{align*}
\frac{dE_y}{dz} &= ik\mathcal{H}_x, \quad \frac{dE_x}{dz} = -ik\mathcal{H}_y, \quad \mathcal{H}_z = 0 \quad (IV.4.75a,b,c) \\
\frac{d\mathcal{H}_y}{dz} &= -\frac{ik}{\varepsilon_0} D_x, \quad \frac{d\mathcal{H}_x}{dz} = \frac{ik}{\varepsilon_0} D_y, \quad D_z = 0 \quad (IV.4.76a,b,c)
\end{align*}
\]

If \( \mathcal{H}_x \) and \( \mathcal{H}_y \) are eliminated, we find:

\[
\begin{align*}
\frac{d^2 E_x}{dz^2} + \frac{k^2}{\varepsilon_0} D_x &= 0 \quad (IV.4.77) \\
\frac{d^2 E_y}{dz^2} + \frac{k^2}{\varepsilon_0} D_y &= 0 \quad (IV.4.78)
\end{align*}
\]

In these second-order differential equations, the two variables, \( E_x \) and \( E_y \), refer to the total fields. Each of these, now, must be expressed as the sum of the fields for the ordinary and extraordinary waves, that is:

\[
\begin{align*}
E_x &= E_x^{(o)} + E_x^{(x)} \quad (IV.4.79) \\
E_y &= E_y^{(o)} + E_y^{(x)} \quad (IV.4.80)
\end{align*}
\]

From (IV.4.9) we infer

\[
\begin{align*}
\frac{E_y^{(o)}}{E_x^{(o)}} &= \rho_o, \quad \frac{E_y^{(x)}}{E_x^{(x)}} &= \rho_x \quad (IV.4.81a,b)
\end{align*}
\]

Since the wave normal does not coincide with the z-axis, (IV.4.6) becomes:

\[
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
= \begin{pmatrix}
U & \text{in}Y & -\text{in}Y \\
-\text{in}Y & U & i\varepsilon Y \\
\text{in}Y & -i\varepsilon Y & U
\end{pmatrix}
\begin{pmatrix}
P_x \\
P_y \\
P_z
\end{pmatrix}
\]

(IV.4.82)
or,

\[-\varepsilon o E_x = UP_x + \text{in}YP_y - \text{im}YP_z\]
\[-\varepsilon o E_y = -\text{in}YP_x + UP_y + i\varepsilon YP_z\]
\[-\varepsilon o E_z = \text{im}YP_x - i\varepsilon YP_y + UP_z\]  \hspace{1cm} (IV.4.83)

From (IV.4.76c) we have, \(D_z = 0\), \(\vec{D} = \varepsilon o \vec{E} + \vec{P}\) becomes \(D_z = \varepsilon o E_z + P_z = 0\),
or, \(E_z = \frac{-1}{\varepsilon o} P_z\). If we substitute this into the last equation of (IV.4.83), we find

\[XP_z = \text{im}YP_x - i\varepsilon YP_y + UP_z\]
or,

\[(X-U)P_z = \text{im}YP_x - i\varepsilon YP_y \Rightarrow P_z = \frac{\text{im}YP_x - i\varepsilon YP_y}{X - U}\]

Let us substitute \(P_z\) into the first two equations of (IV.4.83), that is:

\[-\varepsilon o E_x = UP_x + \text{in}YP_y - \text{im}YP_z \frac{\text{im}YP_x - i\varepsilon YP_y}{X - U}\]  \hspace{1cm} (IV.4.84)
\[-\varepsilon o E_y = -\text{in}YP_x + UP_y + i\varepsilon YP_z \frac{\text{im}YP_x - i\varepsilon YP_y}{X - U}\]  \hspace{1cm} (IV.4.85)

Let us rearrange (IV.4.84) and (IV.4.85) from which we can find a relation between \(\rho_o\) and \(\rho_x\)

\[-\varepsilon o E_x = \left[ U + \frac{m^2 Y^2}{X - U} \right] P_x + \left[ \text{in}Y - \frac{\varepsilon Y^2}{X - U} \right] P_y\]  \hspace{1cm} (IV.4.86)
\[-\varepsilon o E_y = \left[ -\text{in}Y - \frac{\varepsilon m^2 Y^2}{X - U} \right] P_x + \left[ U + \frac{\varepsilon Y^2}{X - U} \right] P_y\]  \hspace{1cm} (IV.4.87)

\[
\frac{E_y}{E_x} = \frac{P_y}{P_x} = \rho = \frac{\left[ -\text{in}Y - \frac{\varepsilon m^2 Y^2}{X - U} \right] + \left[ U + \frac{\varepsilon Y^2}{X - U} \right] \rho}{\left[ U + \frac{m^2 Y^2}{X - U} \right] + \left[ \text{in}Y - \frac{\varepsilon m^2 Y^2}{X - U} \right] \rho}\]  \hspace{1cm} (IV.4.88)
Finally, we find a quadratic equation for $\rho$:

$$\rho^2 \left[ \frac{\varepsilon m^2 Y^2}{X-U} + \left( \frac{m Y^2}{X-U} - \frac{\varepsilon m Y^2}{X-U} \right) \rho + \frac{\varepsilon m Y^2}{X-U} \right] = 0$$

(IV.4.89)

If $\rho_1$ and $\rho_2$ are the roots of the above equation, then

$$\rho_1 \rho_2 = \frac{\text{in}(X-U) + \varepsilon m Y}{\text{in}(X-U) - \varepsilon m Y}$$

(IV.4.90)

Using (IV.4.90), equation (IV.4.81b) can be rewritten thus

$$\rho \frac{E_x(x)}{E_y(x)} = \rho \frac{E_1}{E_2} = A \implies E_x(x) = \frac{\rho \rho_1}{A} E_y(x)$$

Equation (IV.4.79) becomes

$$E_x = E_x^{(o)} + \frac{\rho \rho_1}{A} E_y^{(x)}$$

(IV.4.91)

and,

$$E_y = \rho \frac{E_x^{(o)}}{E_y^{(x)}} + E_y^{(x)}$$

(IV.4.92)

Elimination of $\mathcal{H}_y$ from (IV.4.18) and (IV.4.20) gives, for the ordinary and extraordinary wave:

$$D_x^{(o)} = \epsilon \frac{n^2 E_x^{(o)}}{E_x^{(x)}}$$

$$D_y^{(o)} = \epsilon \frac{n^2 E_y^{(o)}}{E_y^{(x)}}$$

(IV.4.93a,b)

$$D_x^{(x)} = \epsilon \frac{n^2 E_x^{(x)}}{E_x^{(x)}}$$

$$D_y^{(x)} = \epsilon \frac{n^2 E_y^{(x)}}{E_y^{(x)}}$$

(IV.4.94a,b)

$D_x$ and $D_y$ must also be expressed as the sum of the fields for ordinary and extraordinary waves:

$$D_x = D_x^{(o)} + D_x^{(x)} = \epsilon \frac{n^2 E_x^{(o)}}{E_x^{(x)}} + \epsilon \frac{n^2 \rho \rho_1 E_y^{(x)}}{A}$$

(IV.4.95)

$$D_y = D_y^{(o)} + D_y^{(x)} = \epsilon \frac{n^2 \rho \rho_1 E_x^{(o)}}{E_x^{(x)}} + \epsilon \frac{n^2 E_y^{(x)}}{A}$$

(IV.4.96)
Equations (IV.4.91), (IV.4.92), (IV.4.95) and (IV.4.96) can now be substituted in (IV.4.77) and (IV.4.78). We will put \( s = k z \) (height measured in units of \( \lambda / 2 \pi \)). The derivatives \( \frac{dE_x}{dz} \) and \( \frac{d^2E_x}{dz^2} \) become:

\[
\frac{dE_x}{dz} = \frac{dx}{ds} \cdot \frac{ds}{dz} = \left[ E(x)' + \frac{\rho_o 'A - \rho_o A'}{A^2} E(x) + \frac{\rho_o E(x)'}{y} \right] k \tag{IV.4.97}
\]

\[
\frac{d^2E_x}{dz^2} = \left[ \frac{E(x)''}{x} + \frac{(\rho_o 'A - \rho_o A')A^2}{A^4} E(x) - \frac{2A^2(\rho_o 'A - \rho_o A')}{A^4} \right] \frac{E(x)}{y} + \frac{2}{\rho_o A^2} E(x) + \frac{\rho_o E(x)''}{Ay} \right] k^2 \tag{IV.4.98}
\]

Equation (IV.4.77) can be rewritten as

\[
\frac{\rho_o E(x)''}{Ay} + \frac{2(\rho_o 'A - \rho_o A')}{A^2} E(x)' + \left[ \frac{(\rho_o 'A - \rho_o A')A^2(\rho_o 'A - \rho_o A')}{A^4} \right] E(x) = - \frac{E(x)' + \rho_o E(x)''}{y} \tag{IV.4.99}
\]

Similarly,

\[
\frac{dE}{dz} = \frac{dx}{ds} \cdot \frac{ds}{dz} = \left[ \rho_o E(x)' + \rho_o E(x)'' + E(x)'' \right] k \tag{IV.4.100}
\]

\[
\frac{d^2E}{dz^2} = \left[ \rho_o E(x)'' + \rho_o E(x)'' + \rho_o E(x)'' + \rho_o E(x)'' + E(x)'' \right] k^2 \tag{IV.4.101}
\]

Using (IV.4.100) and (IV.4.101) in (IV.4.78), we find:

\[
\rho_o E(x)'' + 2\rho_o E(x)'' + (\rho_o'' + \frac{2}{\rho_o})E(x) = \frac{E(x)'}{y} - \frac{2}{x} \frac{E(x)}{y} \tag{IV.4.102}
\]
In the case of normal incidence for the components $E_x$ and $E_y$, we obtain from Maxwell's equation two linear differential equations of second order, (IV.4.99) and (IV.4.102). The coefficients of these equations can be expressed in terms of the values of the refractive indices and the polarisation of the ordinary and extraordinary waves. In order to proceed so far, we have assumed that the spatial dispersion is negligible if the path, $2\pi v/\omega$, traversed by a particle in one period is small compared with the characteristic dimensions of the inhomogeneity of the field (see Ginzburg, Propagation of EM waves in plasmas). This assumption calls for the concept of a "slowly varying medium". All studies of wave propagation to date have been in the context of geometrical optics, in which the medium is slowly varying, i.e. $\frac{c}{\omega n^2} |\nabla n| \ll 1$. (Where $n$ is the local refractive index of the medium; $\omega$ - the frequency of the wave; $c$ - the velocity of light.)

In a smoothly changing, plane-stratified medium, geometrical optics are applicable everywhere for waves propagating along the gradient of the particle distribution. This approximation breaks down in the vicinity of the point where $\frac{2}{\omega^2} = 1$ (i.e. $n = 0$), since, as we approach the $n = 0$ level, the wavelength, $\lambda = \frac{2\pi c}{\omega \sqrt{\varepsilon}}$, steadily increases. Hence, the relative change in the properties of the medium at a distance of the order of $\lambda/2\pi$ grows (see Budden, 1961; Zhelezniakov, 1970; Ginzburg, 1964).

We may suppose that the number density of electrons, $N$, varies slowly with height above Jupiter's surface (that is, the direction of the particle density gradient is away from the surface). Both $\rho^1_0$ and $\rho^2_0$ are sufficiently small to be neglected. Equations (IV.4.99) and (IV.4.102) reduce to:
We now multiply (IV.4.103) by \( \rho_o \), and substitute (IV.4.104) in (IV.4.103)

\[
\frac{\rho_o}{A} E(x)^n - \frac{2 \rho_o A'}{A^2} E(x) + \left[ \frac{-\rho_o A'' A + 2 \rho_o A'^2}{A^3} + \frac{\rho_o}{A} n \frac{2}{A} \right] E(x)
\]

\[
= - E(x)^n + 2 \frac{\rho_o E(x)}{A} - E(x)^n - \frac{n}{A} E(x)
\]

(IV.4.104)

Rearranging (IV.4.105), we find:

\[
\left( \frac{\rho_o}{A} - 1 \right) \frac{d^2 E(x)}{ds^2} - \frac{2 \rho_o A'}{A^2} \frac{dE(x)}{ds} + \left[ \frac{-\rho_o A'' A + 2 \rho_o A'^2}{A^3} + \frac{\rho_o}{A} n \frac{2}{A} \right] E(x) = 0
\]

(IV.4.106)

In order to solve (IV.4.106), we have to make some assumption about the profile of the particle density distribution. S-band radio occultation measurements from Pioneer 10 and 11 at various magnetic latitudes of Jupiter have provided the basis for comparison between theoretical models and observations. Fig. 7 of Ashihara and Shimizu (1977) and Figs. 2,3 and 5 of Fjeldbo et al. (1975) clearly show that the less dense part of the ionosphere (which is closer to the surface) may be approximated by a parabolic layer. A parabolic law may be expressed in the form:

\[
\rho(x) = \rho_0 \left( 1 - \frac{x}{a} \right)^2
\]
\[ N = N_m \left[ 1 - \left(\frac{z-z_m}{a}\right)^2 \right] \quad \text{for} \quad \left| \frac{z-z_m}{a} \right| \leq a \]

\[ N = 0 \quad \text{for} \quad \left| \frac{z-z_m}{a} \right| \geq a \]

where \( N_m \) is the maximum value of \( N \) at \( z = z_m \). The constant, \( a \), is the "half thickness" of the ionosphere. Since \( X \) is proportional to \( N(X = N_e^2 / \varepsilon m M \omega^2) \), we may write

\[ X = X_m \left[ 1 - \left(\frac{z-z_m}{a}\right)^2 \right] \quad \text{for} \quad \left| \frac{z-z_m}{a} \right| \leq a \]

We can now determine the coefficients of the differential equation (IV.4.106) in terms of the height, \( z \).

\[ A = \frac{\ln(X-U) + \xi mY}{\ln(X-U) - \xi mY} \]

Let us substitute \( X \) into \( A \),

\[ A = \frac{\ln \left[ X_m - \frac{X}{a} \left(\frac{z-z_m}{a}\right)^2 - U \right] + \xi mY}{\ln \left[ X_m - \frac{X}{a} \left(\frac{z-z_m}{a}\right)^2 - U \right] - \xi mY} \]

The first and the second derivative of \( A \) become

\[ A' = \frac{4\ln X_m (z-z_m) \xi mY}{\left[ \frac{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY}{\frac{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY}{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY} \right]^{\frac{1}{2}}} \cdot \frac{dz}{ds} \]

and

\[ A'' = \frac{4\ln \xi mY X_m}{2} \frac{\ln \left[ \frac{2\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY}{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY} \right]^{\frac{1}{2}}} \cdot \frac{dz}{ds} \]

\[ \left[ \frac{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY}{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY} \right]^{\frac{3}{2}} \cdot \left( \frac{dz}{ds} \right)^2 \]

\[ \left[ \frac{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY}{\ln X_m (z-z_m)^2 - \ln X_m + \ln U + \xi mY} \right]^{\frac{3}{2}} \cdot \left( \frac{dz}{ds} \right)^2 \]
respectively.

In the ionospheric plasma, $|X_m| \sim |U|$ (see Boyd and Sanderson, 1969). If we carry out the analysis away from the reflection point ($z = z_m$), which should be valid for the W.K.B. approximation, we may assume that the terms containing $(z-z_m)$ dominate. Accordingly, the approximate values of $A$, $A'$ and $A''$ become:

$$A \sim 1$$

$$A' \sim \frac{4\ell m a^2 Y}{\ln X_m^3} \cdot \frac{1}{k}$$

$$A'' \sim \frac{-12\ell m a^2 Y}{\ln X_m^4} \cdot \frac{1}{k^2}$$

By substituting (IV.4.107) into (IV.4.106) we find,

$$\left(\rho_o^2 - 1\right) \frac{d^2 E}{ds^2} - \frac{8\ell m a^2 \rho_o^2 Y}{\ln X_m^3} \cdot \frac{1}{k} \frac{dE}{ds} + \frac{n^2 (\rho_o - 1) n^2 m X_k Z}{12 i \ell m a^2 \rho_o^2 X Y Z + 32 \ell m a \rho_o^2 Y^2}$$

$$E = 0$$

(IV.4.108)

Let us put $z = \frac{a}{k}$ and rearrange (IV.4.108)

$$\left(1 - \rho_o^2\right) n^2 m Y_s^6 \frac{d^2 E}{ds^2} - 8 i \ell m a^2 \rho_o^2 Y_s^3 \frac{dE}{ds} + \frac{\left[n^2 (1 - \rho_o^2) n X_s^6 + 12 i \ell m a^2 \rho_o^2 X Y_s^2 + 32 \ell m a \rho_o^2 Y^4\right]}{E = 0}$$

(IV.4.109)

We now put (IV.4.109) into a more general form, i.e.

$$A_1 s^6 \frac{d^2 E}{ds^2} + iB_1 s^3 \frac{dE}{ds} + [C_1 s^6 + iD_1 s^2 + F_1] E = 0$$

(IV.4.110)
where

\[ A_1 = (1 - \rho_0^2) n x_m^2 \]

\[ B_1 = -8 \varepsilon m n x_m a^2 \rho_0^2 k y \]

\[ C_1 = n^2 (1 - \rho_0^2) n x_m^2 \]

\[ D_1 = 12 \varepsilon m n a^2 \rho_0^2 x y k^2 \]

\[ F_1 = 32 \varepsilon m a^2 \rho_0^2 y k^2 \]

Since we let the plane wave propagate along the particle density gradient (which is inclined at about 10° with respect to the z-axis), equation (IV.4.110) may be simplified by using the "quasi-longitudinal" approximation. The condition necessary for this approximation to hold is given by (Helliwell, 1969):

\[
\frac{y^2 \sin^4 \theta}{4 \cos^2 \theta} \ll |(1 - x - iZ)| \quad (IV.4.111)
\]

This is derived from (IV.4.16). When (IV.4.111) is applied, (IV.4.16) gives \[ |\rho| = 1, \] which indicates that the wave mode is circularly polarised. Under the quasi-longitudinal approximation (where \( \rho_0 \approx 1 \)), \( A_1 \) and \( C_1 \) become negligibly small. Let us put

\[ n^2 (1 - \rho_0^2) x_m^2 = \eta \]

(IV.4.110) becomes

\[
\eta s E'' + iB_1 s^3 E' + [\eta n^2 x_s^6 + iD_1 s^2 + F_1] E = 0 \quad (IV.4.112)
\]

In the first approximation, neglecting the terms with \( \eta \), we have
\[ iB_1 s^2 E^{(o)} + [iD_1 s^2 + F_1] E^{(o)} = 0 \]  \hspace{1cm} (IV.4.113)

or

\[ \frac{dE^{(o)}}{E^{(o)}} + \left[ \frac{D_1}{B_1} \frac{1}{s} + \frac{F_1}{iB_1 s^3} \right] ds = 0 \]  \hspace{1cm} (IV.4.114)

If we take the integral of this differential equation, we find

\[ \log E^{(o)} + \frac{D_1}{B_1} \log s - \frac{F_1}{2iB_1 s^2} = \text{const.} \]  \hspace{1cm} (IV.4.115)

Finally,

\[ E^{(o)} = \alpha s^2 \exp \left[ \frac{F_1}{2iB_1 s^3} \right] \]  \hspace{1cm} (IV.4.116)

Equation (IV.4.116) is the solution for the \( y \)-component of the electric field of the extraordinary wave; where \( \alpha \) is the constant of integration, superscript \( (o) \) indicates the first approximation.\(^4\)

We have assumed a slowly varying plane-stratified medium, where geometrical optics are applicable everywhere for waves propagating along the \( z \)-axis. This assumption can be justified everywhere except in the vicinity of the point, \( z \), where \( \omega^2 / \omega_p^2 = 1 \) (i.e., \( \epsilon = 0 \)).

The breakdown of the W.K.B. approximation in some region implies that a full-wave solution (that is, the wave equation obtained by the inclusion of thermal motions of electrons) must be carried out in that region and matched to the W.K.B. solution in regions where geometrical optics are valid (Piliya and Federov, 1970; Kuehl, 1967; Kuehl and O'Brien, 1970; Golant and Piliya, 1972). It is clear from the above argument that near the point where \( \epsilon = 0 \) it is necessary to take account of the "spatial dispersion", i.e., it may not be possible

\(^4\) The approximate solution for (IV.4.112) should be of the form:

\[ E(s; \eta) = E^{(o)}(s) + \eta E^{(1)}(s) + \ldots \] We are concentrating here on the zero-order approximation in equation (IV.4.112).
to use only the local properties, $\varepsilon$, of the medium. In other words, we need to make the thermal correction to the wave equation.

In the vicinity of the point where $\omega_{pe}^2 = \omega^2$, we can rewrite the Maxwell's equations including the electric permittivity of the medium

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad (IV.4.117)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (IV.4.118)$$

A wave equation for the electric field vector can be formed by operating with $\nabla \times$ on the first equation, and then substituting from the second. We obtain in this case (Golant and Piliya, 1972):

$$(\nabla \times \nabla \times \vec{E})_i - \frac{\omega^2}{c^2} \varepsilon_{ij}(z) E_j(z) = 0 \quad i = 1,2 \quad (IV.4.119)$$

(We have introduced here the electric permittivity tensor, $\varepsilon_{ij}(z)$, which relates $D$ and $E$, as $D_i = \varepsilon_{ij}E_j$; see Shafranov, 1967). $i = 1,2$.

This is so because a plane wave propagating along the magnetostatic field in a magnetoionic medium is seen as a transverse electromagnetic wave, since there is no component of either electric or magnetic fields in the direction of propagation (Seshadri, 1973). This electric permittivity tensor is useful for estimating the importance of the thermal motion of the charged particles. By allowing thermal motion of the electrons, Ginzburg (1964) has indicated that the extraordinary wave travelling towards the upper hybrid resonance (this resonance occurs when $\omega_{pe}^2(z) = \omega^2 - \omega_{ce}^2$) is simply converted into a longitudinal electron plasma wave. It is the upper hybrid resonance for which we are seeking a solution to equation (IV.4.119).
\[ \varepsilon_{ij} = \delta_{ij} - \sum \frac{\omega^2}{\omega^2 - \omega^2 - k^2 v_T^2} \]

where \( \delta_{ij} \) is the Kronecker delta; \( v_T \) is the thermal velocity of the particles and the summation is taken over all species of charged particles. We may rewrite the electric permittivity as:

\[ \varepsilon_{ij} = \delta_{ij} - \frac{x}{(1-Y^2) - k^2 v_T^2} \]

Substituting (IV.4.121) into the wave equation (IV.4.119), we obtain two coupled equations for \( E_x \) and \( E_y \):

\[
\begin{pmatrix}
\begin{pmatrix}
1 & iY & 0 \\
-1Y & 1 & 0 \\
0 & 0 & 1-Y^2
\end{pmatrix}

+k^2 v_T^2 \begin{pmatrix}
\begin{pmatrix}
1 & k^2 v_T^2 \\
-k^2 v_T^2 & 1
\end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
0
\end{pmatrix}
\end{pmatrix} \]

\[
\begin{pmatrix}
\begin{pmatrix}
1 & iY & 0 \\
-1Y & 1 & 0 \\
0 & 0 & 1-Y^2
\end{pmatrix}

+k^2 v_T^2 \begin{pmatrix}
\begin{pmatrix}
1 & k^2 v_T^2 \\
-k^2 v_T^2 & 1
\end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
0
\end{pmatrix}
\end{pmatrix} \]

(IV.4.121)
The first coupled equation becomes:

\[
\left(\frac{v_T}{c} - 1\right) \frac{\partial^2 E_x}{\partial z^2} + \frac{\omega^2}{c^2} \left(\frac{X}{(1-Y^2)-k_{zT}^2} - 1\right) E_x = -\frac{i\omega}{c} \frac{\partial^2 E_y}{\partial z^2} + \frac{\omega^2}{c^2} \frac{iXY}{(1-Y^2)-k_{zT}^2} E_y
\]  

(IV.4.123)

The second coupled equation becomes:

\[
\frac{\partial^2 E_y}{\partial z^2} \left(\frac{v_T}{c} - 1\right) + \frac{\omega^2}{c^2} \frac{X}{(1-Y^2)-k_{zT}^2} E_y = -\frac{\omega}{c} \frac{v_T}{c^2} \frac{\partial^2 E_x}{\partial z^2} + \frac{\omega^2}{c^2} \frac{iXY}{(1-Y^2)-k_{zT}^2} E_x
\]  

(IV.4.124)

By differentiating equation (IV.4.124) twice, and using equation (IV.4.123) to eliminate \(E_x\), a fourth-order differential equation can be obtained:
The prime on $X$ denotes the derivative with respect to $z$. Several conditions must be satisfied for equation (IV.4.125) to be valid; namely, (i) $v_T^2 = KT/m \ll 1$, (ii) near the upper hybrid resonance ($X - 1 + Y^2 \approx 0$), the inclusion of the term $\frac{\omega^2}{c^2} \left( \frac{X}{(1-Y^2) - k^2 v_T} - 1 \right) E_x$ does not change the solution of equation (IV.4.125) more than by the order $O(v_T^2)$ (see equation (IV.4.123)), (iii) in the region where the spatial dispersion is taken into account, $X(z)$ is assumed to be a linear function of $z$, (iv) $k = k_0 z$, and we have replaced $k_0^2$ by the differential operator $d^2/dz^2$, thus the electric permittivity tensor has become an operator. It is necessary to go beyond the second-order equation since the effect of the inclusion of thermal motion of electrons is to bring a plasma wave into existence, at $\omega_{pe} = \omega$ (see Ginzburg, 1964). These resonances are particularly interesting phenomena to study. We have already seen that the W.K.B. approximation breaks down and reflection and/or absorption of plasma waves can occur. The question that then arises is whether the wave will reach the high-density region of the resonance, or whether it will be reflected at some region of intermediate density. If no such intermediate reflection occurs, the resonance is said to be "accessible" (Stix, 1965). Stix has also shown that as a wave approaches a region of resonance, the group velocity (which is also the signal velocity) in the direction of the density gradient will become slower, and the transverse component of energy flow vector gradually increases. Tsytovich and Shvartsburg (1967) have shown that the maximum generation of the plasma waves is at right angles to the transverse electromagnetic waves. Therefore, in the region where the spatial dispersion becomes important, we assume
that the density gradient vector is at right angles with respect to
the external magnetic field; and we attempt to solve equation (IV.4.125)
at the upper-hybrid resonance (i.e. \( X - 1 + Y^2 \approx 0 \)). If we rearrange
equation (IV.4.125), we find:

\[
\lambda^2 \frac{\partial^4 E_y}{\partial z^4} - \frac{z(1+2Y^2)}{c} \frac{\partial^2 E_y}{\partial z^2} - 2 \frac{\partial E_y}{\partial z} + \frac{z^2 Y^2}{\lambda^2} E_y = 0 \quad (IV.4.126)
\]

where \( \lambda^2 = \frac{c^2}{\omega^2} \left[ (1-Y^2) - k_T^2 \right] \), and \( X = z \). We should bear in mind
that the independent variable, \( z \), differs in direction from the previous
coordinate system. In the new coordinate system, positive \( z \) is
directed towards the higher density side of the plasma.

By using the Laplace transformation, we attempt to find a
solution to equation (IV.4.126) (see, Ince, 1956). Equation (IV.4.126)
is satisfied by the definite integral

\[
E_y = \int_a^\beta e^{zt} v(t) \, dt \quad \text{where } z \text{ enters as a parameter. Then,}
\]

\[
\mathcal{L}(E_y) = \int \left[ \lambda^2 t^4 - \frac{z(1+2Y^2)}{c} t^2 - 2t + \frac{z^2 Y^2}{\lambda^2} \right] e^{zt} v(t) \, dt
\]

(IV.4.127)

By repeated integration by parts, we have:

\[
- \int \frac{z(1+2Y^2)}{c} t^2 e^{zt} v(t) \, dt =
\]

\[
- \frac{1+2Y^2}{c} e^{zt} t^2 v(t) + \frac{1+2Y^2}{c} \int e^{zt} \frac{d}{dt} (t^2 v) \, dt
\]

(IV.4.128)

\[
\int \frac{z^2 Y^2}{\lambda^2} e^{zt} v(t) \, dt = z \frac{Y^2}{\lambda^2} e^{zt} v(t) - \frac{Y^2}{\lambda^2} \frac{e^{zt} dv}{dt} +
\]

\[
\frac{Y^2}{\lambda^2} \int e^{zt} \frac{d^2 v}{dt^2} \, dt
\]

(IV.4.129)

Substituting equations (IV.4.128) and (IV.4.129) into equation (IV.4.127)
we find:

\[ L(E) = \left[ -\frac{1 + v^2 r^2}{c} e^{zt} t^2 v(t) + z \frac{v^2 r^2}{\lambda^2} e^{zt} v(t) - \frac{v^2 e^{zt}}{\lambda^2} \frac{dv}{dt} \right]_x + \int_\alpha^\beta \lambda^4 t^4 v(t) + \frac{1 + v^2 r^2}{c} \frac{d}{dt} (t^2 v) - 2tv(t) + \frac{v^2 e^{zt}}{\lambda^2} \frac{dv}{dt^2} e^{zt} dt \]

\[ (IV.4.130) \]

If the definite integral, \( E = \int_\alpha^\beta e^{zt} v(t) dt \), is to be the solution of equation (IV.4.126), then (i) the non-integral part of equation (IV.4.130) should be zero, and (ii) the associated equation should be of order two.

We now turn to the solution of

\[ \frac{v^2 d^2 v}{\lambda^2 dt^2} + t^2 \frac{1 + v^2 r^2}{c} \frac{dv}{dt} + t(\lambda^2 t^3 - 2) v = 0 \quad (IV.4.131) \]

By the method of solution in series (that is, the solution of \( v(t) \) assumes the form \( v = A_o + A_1 t + A_2 t^2 + A_3 t^3 + \ldots \)), we find:

\[ v(t) = A_o + A_1 t + A_2 \frac{\lambda^2}{3r^2} t^3 + \ldots \quad (IV.4.132) \]

where \( A_o \) and \( A_1 \) are the constants of integration. Then the solution of equation (IV.4.126) becomes

\[ E_y(z) = \int \left( A_o + A_1 t + A_2 \frac{\lambda^2}{3r^2} t^3 \right) e^{zt} dt \quad (IV.4.133) \]

\[ E_y(z) = \frac{A_o \lambda^2}{r^2 z} \left[ \frac{t^3}{3} - \frac{t^2}{z} + \frac{2t}{z^2} - \frac{2}{z^3} \right] e^{zt} + \frac{A_1}{z} \left[ t - \frac{1}{z} \right] e^{zt} \quad (IV.4.134) \]

We have found two solutions, namely equation (IV.4.116) and equation (IV.4.134) for two different regions separated by the point \( \epsilon = 0 \). If Jupiter's decametric radio emission is produced upon
transformation of plasma waves into extraordinary electromagnetic waves, the above-mentioned two solutions should "match". Crudely speaking, the idea of matching is that the behaviour of the "cold solution" [equation (IV.4.116)] as z \to 0 (i.e., \varepsilon = 0) and the "warm solution" [equation (IV.4.134)] as z \to \infty is in agreement. More formally, there is a domain in which both solutions are valid - an overlap domain - and in which these solutions agree (for as many terms as are considered, see Nayfeh, 1973). Matching to the first order consists in obtaining agreement of the two solutions to terms of order one; higher order matching is defined correspondingly. Thus, to first order, we have

\[ \text{Im} \eta \to 0 \left[ \frac{F_1}{2iB_1} + \frac{2A_0 \lambda^2 \omega^4}{\nu^2 \omega^4} + \frac{A_1 \omega^2}{\omega^2} \right] = 0 \]

If we assume that the amplitude of the original wave is unity (i.e. \( \alpha = 1 \)), matching provides the value of \( A_1 \) in terms of \( A_0 \), that is:

\[ A_1 = -\frac{F_1 \omega^2}{\omega^2 2iB_1} - \frac{2A_0 \lambda^2 \omega^6}{\nu^2 \omega^6} \]

The coefficient of transformation to plasma waves is defined as the ratio, \( T \), of the energy flux in the warm solution

\[ S_{\text{pl.}} = \nu \frac{E_y^2}{\text{gr.} 8\pi} \sim \frac{E_y^2}{T \Delta z \omega 8\pi} \quad \text{(IV.4.135)} \]

to the energy flux in the incident wave, \( S_o = cH_o^2/8\pi \). Here, \( \Delta z \) is the dimension of the transition layer, \( \nu \), is the group velocity of the plasma wave, and \( H_o \) is the magnetic field of the incident wave.

\[ T = \frac{S_{\text{pl.}}}{S_o} \sim \frac{E_y^2(\nu)}{c^2 \Delta z E_y^2(\nu)} \quad \text{(IV.4.136)} \]

where \( E_y(\nu) \) and \( E_y(\nu) \) are the "warm" and "cold" solutions of \( E_y \).
respectively. When the absorption for the incident wave is sufficiently small, and the thickness of the layer is not too small (near \( \varepsilon \sim 0 \)), the amplitude coefficient of reflection, \( R \), for the incident wave, is given by (Ginzburg, 1964):

\[
| R | = 2(1 - T)
\]

or,

\[
| R | = 2 \left( 1 - \frac{v_T^2 E_y(w)}{c^2 \Delta z E_y(c)} \right)
\]

Finally, the energy absorbed near the point \( \varepsilon \rightarrow 0 \) is given by (Ginzburg, 1964):

\[
|A|^2 = 1 - |R|^2 - |T|^2
\]

or,

\[
|A|^2 = 1 - 4 \left( 1 - \frac{v_T^2 E_y(w)}{c^2 \Delta z E_y(c)} \right)^2 - \left( \frac{v_T^2 E_y(w)}{c^2 \Delta z E_y(c)} \right)^2
\]

We have previously seen that, when spatial dispersion is taken into account, the plasma waves and the extraordinary waves become the branches of one single curve (see Fig. IV.6). On the other hand, reflection and absorption play very important roles in the efficiency of conversion from one mode into another. In order to get high conversion efficiency, the conditions \(| R | \ll 1 \) and \(| A | \ll 1 \) must be fulfilled. An examination of \(| R | \) and \(| A | \) shows that this condition becomes:

\[
\frac{v_T^2 E_y(w)}{c^2 \Delta z E_y(c)} \approx 1
\]

The thickness, \( \Delta z \), of the transition region can easily be determined. According to Ginzburg (1964), for quasi-longitudinal approximation (i.e. \( \alpha \leq 20^\circ \)), \( \Delta z \approx 0.03/\left( \frac{1}{N} \frac{dN}{dz} \right)_{x=1} \).
For a parabolic layer \( N = N_{\text{max}}(1 - z^2/z_m^2) \), we have at the point \( X = 4\pi e^2 N/m \omega^2 = 1 \)

\[
\left( \frac{1}{N} \frac{dN}{dz} \right)_{X=1} = \frac{4\pi e^2}{m \omega^2} \left( \frac{dN}{dz} \right)_{X=1} = \frac{4\pi e^2 N_{\text{max}}}{m \omega^2 z_m^2} \cdot 2 \left| z(X=1) \right|
\]

\[
= \frac{2 \omega^2}{z_m \omega} \sqrt{\frac{1 - \omega^2}{\omega}} = \frac{2 f_{\text{cr}}^2}{z_m f_{\text{cr}}^2} \sqrt{1 - \frac{f^2}{f_{\text{cr}}^2}} \quad (\text{IV.4.142})
\]

where \( \omega_{\text{cr}} = 2\pi f_{\text{cr}} = (4\pi e^2 N_{\text{max}}/m)^{1/2} \) is the critical frequency for the extraordinary wave, and we have used the fact that reflection of the extraordinary wave with frequency, \( \omega \), at the point \( X = 1 \) occurs when

\[
|z(X=1)| = z_m \sqrt{\left(1 - \frac{f^2}{f_{\text{cr}}^2}\right)} \quad (\text{IV.4.143})
\]

Using equations (IV.4.142) and (IV.4.143) in equation (IV.4.141) we find

\[
\Delta z \approx \frac{0.03 z_m f^2}{2 f_{\text{cr}}^2 \sqrt{1 - f^2/f_{\text{cr}}^2}} \quad (\text{IV.4.144})
\]

From the Pioneer 10 observations it was found that the maximum particle concentration occurs at about \( z_m \sim 1000 \text{ km} \) from the surface of Jupiter. For \( f/f_{\text{cr}} = 1/3 \), the thickness of the transition layer is found as

\[
\Delta z \sim 17 \text{ km} \quad (\text{IV.4.145})
\]

Hence, it is clear that the variation of \( (dN/dz)_{X=1} \) over a distance of 17 km leads to a considerable change in the transformation coefficient, \( T \). Substituting this value of \( \Delta z \) in equation (IV.4.140) we can find the unknown coefficient, \( A_0 \), and, subsequently, the exact solution of \( E_y(w) \). When \( |E_y(c)| = 1 \) is a boundary condition, equation (IV.4.140) becomes:
By extracting $A_o$ from equation (IV.4.146) and substituting into equation (IV.4.134), we find the exact solution of $E_y(w)$.

c. DISCUSSION

We are now in a position to discuss the fine structure of the dynamic spectra of Jovian L bursts which display repeated, tilted lanes. The explanation of the modulation lanes, which are a very noticeable feature of the Jovian radio phenomena, can be given in terms of the wave conversion mechanism. Riihimaa (1970) has suggested that, "the modulation lanes, void of emission, must be intrinsic to the source".

In order to prove Riihimaa's speculation we need to know the quantity, $q = \frac{\omega_{pe}}{\omega_{ce}}$. By using the Pioneer 10 and Pioneer 11 data, we find

$$q = \left(\frac{4\pi N e^2}{m}\right)^{\frac{1}{2}} \cdot \frac{mc}{eB} = 0.72 \quad (IV.4.147)$$

where $N = 5 \times 10^5$ cm$^{-3}$; $B = 10$ Gauss; $m$ (electron rest mass) = $9 \times 10^{-28}$ g; $c$ (speed of light) = $3 \times 10^{10}$ cms$^{-1}$.

When $q < 0.8$, the minimum value for the group velocity of
the wave at the upper hybrid frequency is less than, or is equal to
0.04 $v_T$, where $v_T = (KT/m)^{\frac{1}{2}}$ is the electron thermal velocity (Oya, 1971). In Jupiter's ionosphere, $T = 1000$ K hence the group velocity
of the wave $v_{gr} = 12.8$ km/s. This calculation shows that the time
required for the plasma wave to pass through the transition region
is $\Delta t = \frac{\Delta z}{v_{gr}} \sim 1.4$ sec. This may be compared with Riihimaa's
(1970) observations on L-bursts. What he found was that the modulation
lanes intersect a given fixed frequency at intervals varying from 1 to
5 s, the average being 3 s. Therefore our hypothesis - that the wave
conversion mechanism is an inseparable part of the wave generation
mechanism - seems to be consistent with the observations.

When the extraordinary mode wave is converted into plasma
wave, it travels through the transition region in a time which is equal
to the time interval of a modulation lane. Since plasma waves cannot
propagate in space ($\nabla \times \vec{E} = 0, \vec{H} = 0$), there will be an interruption
in the continuity of a single L-burst. We conclude, therefore, that
wave conversion shows itself as a modulation lane in the dynamic spectra
of Jovian L-bursts. The conversion between the long-wavelength
electromagnetic modes and the extremely short-wavelength plasma modes
may be considered from a more general point of view. In order to
examine more precisely how one mode can be converted into another, it
is necessary to obtain an overall connection formula which connects
the propagating plasma mode on the higher-density side of the plasma to
the propagating electromagnetic mode on the lower-density side. An
overall connection formula requires finding the asymptotic solutions
of the cold and the warm models. The connection formula enables us to
find the phase shift experienced by the extraordinary mode in traversing
the plasma from $\omega = \omega_{pe}$ to the upper hybrid resonance. Remembering
that the orientation of the resultant polarization ellipse is determined
not only by the form and orientation of the wave modes, but also by
the phase shift between them. Any change in the latter along the
direction of propagation will appear as Faraday rotation. The absence
of Faraday rotation, therefore, must be explained in a more careful
way. Reception of right-handed elliptically polarised emission
(extraordinary mode) at all times does not necessarily mean that the
radiation is emitted only in one mode, and that therefore there is no
Faraday rotation. Jovian Faraday rotation, if any, should occur
internally in the transition region which has a limited spatial extent.

The treatment of this problem is not a dead end. The future
work we envisage is the asymptotic solution treatment of the same
problem.
CHAPTER V

CONCLUSION

The conclusion we have drawn from the present study may be outlined as follows. In order to give a sound explanation to the DAM emission from Jupiter, two major aspects of the problem - the generation mechanism and the propagation effects - should be dealt with in a single context. In Chapter II we have shown that the IMF sector structure has a definite modulating role on the source and generation mechanism. We have observed a peak within 1 or 2 days after the IMF sector boundary crossings, provided that the IMF sector structure is corotating with the Sun. In the case of the Earth, sector boundary crossings populate the radiation belt with a high flux of low-energy particles and increase the geomagnetic activities. The same mechanism is expected to be in operation at Jupiter, since both planets have magnetospheric features in common. The statistical study presented in Chapter II gives support to our expectation, that is, the histograms show an increase in the amount of DAM emission around the time of the sector boundary passage. Figs. (II.2), (II.3), (II.4), (II.6), (II.7), (II.8), (II.10), (II.11), (II.12), (II.14) and (II.16) show this particularly clearly. Since many similarities exist between TKR and Jovian DAM radiation, it has been widely suggested that their source mechanism could be the same. It would therefore be more appropriate to examine the effect of IMF sector boundaries on terrestrial kilometric radiation, and compare this with the Jupiter sample. But, to the best knowledge of the author of the present study, appropriate data on TKR are not currently available. Hence, this problem awaits further study.

It has also been shown that a close correlation exists between the occurrence of TKR and that of discrete auroral arcs (Gurnett, 1974).
Discrete auroral arcs are known to originate from electron precipitation through radial and/or pitch-angle diffusion. In Chapter III, we have attempted to solve the radial diffusion problem in the inner radiation belt (i.e. $1 \leq L \leq 5$). We have assumed that solar wind particles find an access to the radiation belt of Jupiter through IMF sector boundaries. These excess particles, diffusing radially inward, upset the electrical neutrality. The re-establishment of the electrical neutrality is accompanied by the appearance of plasma oscillations. These radiate the electromagnetic waves that are detected on Earth as sporadic radio bursts from Jupiter. The solution of the radial diffusion problem has given us the particle density distribution profile at the inner radiation belt of Jupiter. We have plotted, in the text, the phase-space density ($f$) vs. the radial distance ($L$). According to Hess (1968), the particle distribution function is proportional to the phase-space density. This fact has allowed us to use the ($f$, $L$) profile later on in Chapter IV as one of the plasma parameters for the Jovian ionosphere. Our theoretical result for radial diffusion has been confirmed by Pioneer 10 observations, that is the source of particles is at the magnetopause (probably at the sector boundaries), and, further in, processes act to reduce $f$. We have supposed that sector boundary crossings and radial diffusion are important phenomena in explaining Jupiter's DAM emission because the free energy resides in some non-thermal feature of the particle distribution function. Such a non-thermal feature manifests itself as a peak in the phase space density, and this, in turn, in the configuration space dependence of $f(L)$ affects the process of radial diffusion.

However, knowing the source and generation mechanism is not sufficient, by itself, to explain the nature of Jovian DAM emission. We have therefore looked into the problem of wave propagation. First,
we assumed that the change in the physical parameters of a medium over distances of the order of a wavelength is negligible. We then solved the problem of wave propagation in the approximation of geometrical optics. It must be noted, however, that the solution by the perturbation method we presented in Chapter IV is lacking in completeness, due to the complex nature of the differential equation (IV.4.112). It suggests a solution in the form:

$$E(s, \eta) = E^{(0)}(s) + \eta E^{(1)}(s) + \eta^2 E^{(2)}(s) + \ldots$$

but our solution consists only of $E^{(0)}(s)$. Nevertheless, even this approximation is sufficient to obtain the necessary information from the "matching condition". The geometrical optics approximation breaks down, however, at those points where the wave vector vanishes, or the wave vectors corresponding to different oscillations coincide ("points of intersection of solutions"). If the wave propagates through a region of "interaction", a new wave is induced whose dispersion parameters are different from those of the incident wave. Therefore, in our second approximation (rather correction), we have taken the thermal motions of electrons into account, and have solved the full wave equation. A system of two weakly coupled, second-order differential equations in the neighbourhood of the "upper hybrid resonance"

$$\omega^2 = \omega_{pe}^2 + \omega_{ce}^2$$

is replaced by a fourth-order differential equation. A powerful method to solve this fourth-order differential equation is provided by the Laplace transform method. It gives us asymptotic solutions that are expressible in terms of tabulated functions. The above argument, however, applies to differential equations with linear coefficients. Such differential equations give a first-order "associated equation" which makes it possible to find a solution by the method of "steepest descent", or by "the saddle point integration" (see
Piliya and Fedorov, 1970; Budden, 1961). Having an associated equation of order two, we had to resort to the method of series expansion. The essential difference between the asymptotic solution and the series expansion method is that the former can have different forms on different sides of the interaction region (in other words, the solution can be obtained in the complex plane), whereas the latter gives us the solution only on the real axis.

The wave conversion problem reduces to a "matching" of the solutions. This matching has provided us with the values of the unknown coefficients, transformation, reflection and absorption coefficients and the dimensions of the transition region. Since plasma waves, which cannot be radiated directly, are excited in Jupiter's ionospheric plasma, the observed radiation is connected with wave transformation. It should be remarked that the transformation can lead to a delay of the induced radiation. The propagation of plasma waves from the excitation region to the radiation region can last a relatively long time, since the group velocity of these waves is low. The transition time of the plasma wave has been found to equal the time interval of a modulation lane. In order to explain the fine structure of Jovian DAM emission, we should bear in mind the following points.

(1) At the point \( \omega = \omega_{pe} \), the extraordinary electromagnetic wave becomes an electrostatic plasma wave; 
(2) A plasma wave passes through the transition region in \( \sim 1.4 \text{ sec.} \); 
(3) The plasma wave must be reconverted into the extraordinary mode at the upper-hybrid resonance; 
(4) In order to observe the modulation lanes, plasma waves should go through total reflection at the point \( \omega^2 = \omega_{pe}^2 + \omega_{ce}^2 \).

Most of the assumptions we made in the wave conversion problem could be firmly established using the asymptotic method. This would be possible only if we were able to find the asymptotic solutions of the
wave equations. This indicates the course that future study should take.
APPENDIX I (continued)

SUBROUTINE FMINE 73/71 OPT=1 TRACE MANTRAP FTN 4.5*41C

1 SUBROUTINE FMINE (ML0J,DMJ0J,DM00J,UT,DM05J)

5 DIMENSION ML0J(36),DMJ0J(36),DM00J(36),UT(36)

10 DO 5 J=1,36

15 N=INT(UT(J))

20 RETURN

SUBROUTINE SMINE 73/72 OPT=1 TRACE MANTRAP FTN 4.5*41C

1 SUBROUTINE SMINE (L,M,EN,GLS,HLOE,FHLOE,IN,FM,FMN,INFMN,S,FS)

5 DIMENSION L(365),EN(365),IN(365),FMN(365),FS(365)

10 DO 996 I=1,365

15 IF((GLS(I) GT 360.0) .AND. (L .LT. 81)) HLOE(I)=GLS(I)-360.0

20 CONTINUE

996 CONTINUE

RETURN

END
PROGRAM MINE 73/72 OPT=1 \( ROUNDS=**\) MANTRAP FTN 4.6*452

1

PROGRAM MINE (INPUT, OUTPUT, TAPE2=INPUT, TAPE7=OUTPUT)

DISTION : ((1201, 1206) 1200, AAA(200), BAA(201), 21C(201), CC(200), F(200),
1 FF(20), ALPHA(200), BETA(200), GAMMA(200), DELTA(200), N(200),
2 G(20), J(199)

M=13.1416
DELMS=0.02
RMS=1.2
R=1.6
CL=1.6
DEL=1.9
F(0)=1.6
D=1.8

BETA1=(1.0**4)/((1.0*EM**2)*CL**3)
EPS=((1.0**2)/(1.0*EM**2))
ETA=2.0
F(0)=EXP(-(RMS-100.0)/50.0)**2
DEL=1.6
DELSP=1.0
A(J)=-(M*ZETA*(RMS**3)+1)**1
B(J)=-(M*ZETA*(RMS**3)+1)**1
C(J)=-((M*ZETA*(RMS**3)+1)**1)*AA(J)
F(J)=0.0
CONTINUE

CALL PCSEN(X,Y,24HRADIAL DISTANCE,L(=R/RJ) ,24)X=130.0 Y=-0.125
CALL PCSEND(X,Y,11HFIG. (III.3),11)
X=-30.0 Y=0.6
CALL CTRORI(1.0)
CALL CTRPHASE SPACE DENSITY,F (RELATIVE UNITS),37)
CALL PSPACE(C.0,1.0,0.0,0.7)
CALL AXES

END
Carr and Desch, 1976, Jupiter, p.693.
Davis, L. Jr. et al., 1966, "The Solar Wind", Ed. Mackin, R.J. and Neugebauer, M.
B.2

Helliwell, R.A., 1965, "Whistlers and related ionospheric phenomena".


Hill, T.W., 1975, University of Rice (preprint).


Kennedy, D.J., 1969, Polarization of the DAM radiation from Jupiter,

Ph.D. Dissertation, Univ. of Florida, Gainsville, Fl.


Kennel, C.F. and Coroniti, F.V., 1974, "The Magnetosphere of the Earth and Jupiter".


Mitchell, J.L., 1974, The rotation period of the Jovian magnetosphere,


Peters, C.C., and Von Voith, R.W., 1940, "Statistical procedures and their mathematic bases."
Piddington, J.H., 1967, Jupiter's magnetosphere, Univ. of Iowa, Tech. rept., p.63.
Ratcliffe, J.A., 1972, "An introduction to ionosphere and magnetosphere".
Smith, R.A., 1976, "Jupiter".
Zhelezniakov, V.V., 1970, Radio emission of the Sun and Planets.
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This thesis presents descriptions of phenomena, namely, interplanetary magnetic field (IMF) sector boundary crossings, radial diffusion and the wave conversion mechanism, that have important places in the explanation of the phenomenology and the fine structure of Jovian decametric (DAM) emission.

The IMF sector structure is examined first. Assuming that this sector structure is co-rotating with the Sun, we have found the triggering effect of the IMF sector boundary crossings on Jovian DAM emission. Histograms (number of occurrences versus the time with respect to sector boundary passage) show peaks within one or two days following the sector boundary.

Next, the question of radial diffusion is reviewed. Using the Pioneer 10 and Pioneer 11 data on the plasma parameters and magnetic field of Jupiter, we have found the phase-space density profile of the inner-radiation belt, \( 1 \leq L \leq 5 \), and have compared it with the observational results. Our theoretical result is consistent with the observations, and has been used in the wave conversion problem.

Finally, "cold" and "warm" plasma models have been used to solve the wave equation. By "matching" the two solutions, we have obtained information about the "transition region". The wave conversion mechanism has given us an insight into the fine structure of Jovian dynamic spectra, such as the "modulation lanes". Finally, the role of asymptotic solutions for obtaining more accurate results is considered.