Fluctuation Phenomena in the Proportional Counter

by

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Department of Physics
University of Leicester
To my Mother and Father
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ABSTRACT

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J P Sephton

The energy resolution of a proportional counter depends fundamentally on (i) fluctuations in the number of electron-ion pairs produced in the primary ionization and (ii) fluctuations in the size of the subsequent electron avalanche. The avalanche fluctuation is typically 3-4 times larger than the ionization fluctuation. Previous theoretical and experimental work has indicated that fluctuations in both processes can be reduced by improving the efficiency of the ionization mechanisms involved. The ionization efficiency can be increased by means of the Penning effect. Energy spent exciting metastable states is used to increase the ionization yield. This is achieved by introducing a second gas whose first ionization potential is slightly lower than the first metastable level of the main gas. Cross ionization of the admixture gas by metastable states in the main gas can thus occur. With appropriate conditions, the Penning effect becomes the dominant ionization process.

The operation of a proportional counter filled with a range of Penning mixtures is investigated experimentally. Penning mixtures are shown to give improved resolution over the more conventional filling of argon-methane. The Penning mixture, argon-xenon, seems to be a practical alternative filling.

In order to make rather more fundamental studies of avalanche fluctuations in Penning mixtures, a uniform field chamber is constructed. With this chamber it is possible to directly compare experimental measurements of $f$ (the relative variance of the avalanche size) with values predicted by various avalanche models. (With a cylindrical counter the ionization efficiency varies from cathode to anode and such a comparison is less clear.) The models assume different variations of the local value of Townsend's first ionization coefficient $a_1$ with the distance travelled by an electron from the previous ionization. The results obtained are in reasonable agreement with two of the models but these are unsatisfactory from a theoretical point of view. The results are, however, reasonably close to predictions based on experimental measurements of ionization probability.
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1.6 The Nature of this Investigation
1 Introduction - The Proportional Counter

1.1 Historical

Following Rontgen's discovery of X-rays in 1895 it was soon found that charged bodies rapidly became discharged when an X-ray tube was operated nearby. C.T.R. Wilson showed that the charge loss was via the surrounding air rather than the supporting insulation. The radiation was causing ion pairs to be produced in the gas which thus became conducting. This effect was utilized in the first gas based radiation detector - the ionization chamber. This was of the form of a gold leaf electroscope. Modern forms of the ionization chamber consist of a cylindrical or parallel plate electrode system, across which a high potential is applied. A charge integrator measures the ionization current. An ionization chamber responds smoothly to the integrated effect of a large number of ionizing events and such "current chambers" are used extensively for flux determination or source intensity measurement for all types of radiation.

The use of the phenomenon of ionization by collision to amplify the small primary charge deposited in the gas was first developed by Rutherford and Geiger (1908). Their apparatus consisted of a charged metal cylinder with an insulated axial wire electrode connected to one plate of an electrometer. The gas pressure in the chamber and the applied potential difference were adjusted so that the original charge was multiplied sufficiently to allow the use of a relatively insensitive but short period electrometer. This first proportional counter could measure about four α-particles a minute though an improved version (Geiger and Rutherford (1912)) could count a thousand a minute.
Whilst having the advantage of proportionality these early counters were only sufficiently sensitive to respond to \(\alpha\)-particles. The only way to get a measurable response to \(\beta\)-particles was to operate just below the breakdown potential. This was most easily physically realized by replacing the central wire by a short pointed needle (figure 1.1)

![Early form of point counter](image)

**Figure 1.1** (From Curran and Craggs (1949))

The formation of just a few ion pairs in the sensitive volume of the gas near the point initiates a discharge, the magnitude of which is independent of the number of ion pairs triggering it off, i.e. proportionality has been completely lost. The sensitivity of this instrument, the Geiger counter, depends on the sharpness of the point and on the nature and pressure of the gas. With a sufficiently fine point the instrument is capable of detecting a single \(\beta\)-particle.

Much work was done on Geiger counters from 1908 onwards but little progress in the understanding of the mechanisms of operation was made for about thirty years. Reviews of early work have been given by Rutherford, Chadwick and Ellis (1930) and Bothe (1942).
Until the work of Trost (1935 to 1937) it was customary to use individual gases (helium, hydrogen or nitrogen) or even air in these early detectors. With such gases it was found that the resistance $R$ in figure 1.2 had to be a very high value ($\geq 10^8 \Omega$) to prevent the counter breaking down into a steady glow discharge before counting.

![Basic counter circuit](image)

Figure 1.2 (From Curran and Craggs (1949))

This, however, resulted in long time constants and prevented fast counting. For example, if the capacitance of the detector and associated circuitry is 10 pf and if $R = 10^9 \Omega$ then the time constant is $10^{-2}$ seconds. To overcome this problem, circuits were developed which, following a pulse, rapidly reduced the counter voltage to a value too low to permit a glow discharge. The voltage was reduced for about $10^{-4}$ seconds — the time needed for the counter to become clear of ions after the entry of a particle.

Trost, however, found that the addition of small amounts of complex polyatomic vapours, e.g. ethyl alcohol to the more usual counter gases, such as argon, provided quenching in the gas itself. This allowed the use of much lower values of $R$ ($< 10^6 \Omega$) and hence much shorter time constants. These became known as the so-
called "fast" counters.

One of the more recent applications of the proportional counter has been as an instrument for X-ray Astronomy. For reasons relating to detection area, absorption efficiency, background rejection and operating temperature the proportional counter has been widely used for such studies.

Having provided a brief historical introduction to the field, the actual operation of the proportional counter will be considered in more detail.
1.2 The operation of the proportional counter

A schematic diagram of a conventional cylindrical proportional counter is shown in figure 1.3. The anode is normally held at a potential of one to two thousand volts. The cathode can serve as the gas container and main structural member, as well as supporting a thin entrance window for the radiation.

![Conventional proportional counter](image)

Figure 1.3 (From Rice-Evans (1974))

The electric field at a distance \( r \) from the anode is given by

\[
E(r) = \frac{V_{ac}}{r \gamma n(r_c/r_a)} \tag{1.1}
\]

where \( V_{ac} \) is the anode to cathode voltage.

\( r_c \) and \( r_a \) are the cathode and anode radii respectively.

Typical field strengths are 200 V/cm at the cathode and 200 W/cm at the anode.

An X-ray photon which is absorbed in the counter gas ionizes an atom of the gas by ejecting a photo-electron and the release of a fluorescence photon. An Auger electron may also be ejected. The
photo-electron may interact with the gas and result in additional electron-ion pairs. Its energy will eventually be dissipated and a cloud of electron ion pairs will remain along its path. Auger electrons will also interact with the gas and result in electron-ion pairs. If no fluorescence photons escape the counter the total number of electron-ion pairs is, on average, proportional to the incident photon energy.

1.2.1 Voltage characteristic

Under the influence of the electric field the electrons diffuse towards the central wire and the positive ions towards the outer cathode. Let us now consider the effect of gradually increasing the anode voltage from zero. At very low voltages the ion pairs will tend to recombine. Increasing the voltage moves the ions across more quickly and less recombination occurs. This is indicated in figure 1.4 by the initial rising section of the curve. Eventually all the ions and electrons move across without recombining and we enter the so-called "plateau region" where the device operates as an ionization chamber. The charge produces a change in potential across an anode resistor and its distributed capacity, which may be amplified externally or integrated to measure the total dosage of ionizing radiation over some time interval.

If the anode voltage is further increased, the field gradient becomes large and the electrons, after diffusing toward the central wire, may gain more than the ionization potential of the gas in one collision mean free path and therefore produce additional ion pairs. The new electrons may themselves produce more ion pairs producing a total "charge multiplication" which may be of the order of $10^3$ to $10^5$. 
Pulse size obtained from a counting chamber as a function of voltage applied. Curves are drawn for a heavy initial ionization, e.g., from an α-particle and for a light initial ionization, e.g., from a fast electron. For voltages less than $V_p$, only the primary ion pairs are collected; for $V > V_p$, ionization by collision takes place and the counter ceases to be a conventional ionization chamber. Very roughly the electric field in argon at atmospheric pressure corresponding to $V_p$ is 10,000 V cm\(^{-1}\).

**Typical Voltage Characteristic of a Proportional Chamber**

*Figure 1.4 (From Burcham (1963))*
The development of such an "avalanche" is shown schematically in figure 1.5.

![Diagram of avalanche development](image)

Figure 1.5 (From Charpak (1977))

The work of Mathieson (1979) suggests that the avalanche develops in a more radial manner. The avalanche is usually detected by means of a charge sensitive amplifier connected to the anode. A voltage sensitive amplifier may be used but the capacitance of the detector has to be taken into account in calibration. In uniform field and cylindrical geometry proportional counters the signal measured is largely due to the motion of the positive ions away from the anode. Only a minor contribution is made by the electrons produced in the multiplication process. In both types of counter the growth of the avalanche is such that the majority of electrons are produced close to the anode - the effect is more pronounced in the cylindrical counter due to the rapidly rising electric field. Thus the electrons only travel a small fraction of the potential difference maintained between the electrodes of the chamber. The positive ions, on the other hand, travel through almost all the chamber potential.
In this respect the signal obtained from a proportional counter differs basically from an ionization chamber signal which is produced by the motion of electrons and positive ions.

The rise time of the observed pulse is typically a few microseconds and this can be regarded as the dead time of the instrument. The preamplifier output is normally further amplified and shaped and fed to a pulse height analyser where the spectrum can be displayed.

If the anode voltage is increased further the gas gain will rise. It is desirable to keep the gas gain of a proportional counter reasonably low in the interest of stability of operation. The counter loses its proportionality for high initial ionizations or high multiplications owing to space charge effects in the avalanche. In the limit of high multiplications the output pulse height becomes independent of the primary ionization (due to secondary avalanches) and the instrument starts to operate in the Geiger region. If the voltage is increased still further then a continuous discharge occurs, the magnitude of which is largely governed by the external circuit.

1.2.2 Nature of Gas Filling

A noble gas at about one atmosphere pressure is usually chosen for the primary counter gas constituent because of the higher gas gain and initial ionization which can be obtained. This is because non-ionizing energy loss mechanisms, such as excitation of low level rotation or vibration states in a polyatomic molecule are absent or less probable in the noble gas. A counter filled with a single noble gas may be very unstable, though, since a trace of impurity may
dominate the behaviour of the gas. In addition the noble gas is transparent to the ultra-violet light emitted by the excited gas atoms. This u.v. radiation may be of sufficient energy to eject electrons from the cathode and start an additional "secondary avalanche". The gas atoms also may be excited to metastable states in the electron multiplication process. These states have long lifetimes and the atom often de-excites by ionizing collisions with the counter walls. This may again lead to electron emission and a secondary avalanche. Secondary avalanches tend to result in longer pulses and increased avalanche fluctuation.

A polyatomic quench gas, such as methane, carbon dioxide or alcohol, is usually added to the counter to absorb the u.v. light from the noble gas and allow the noble gas atoms in metastable states to de-excite by collisions with the quench gas molecules. The mean free path for these events is small and electron ejection from the quench gas takes place near the anode during the original pulse and therefore does not start a secondary avalanche at a later time. A common gas composition is 90% argon and 10% methane, known as P-10.

1.2.3 Initial Ionization

The mean number of ion pairs $\bar{N}$ created when an X-ray photon of energy $E$ interacts with the gas is given by

$$\bar{N} = \frac{E}{W}$$

(1.2)

where $W$ is the mean energy required to produce an electron-ion pair.

1.2.4 Gas Gain

The mean value of the amplification factor $\bar{K}$ is given to a first approximation by
\[ \bar{K} = \exp (\alpha d) \quad \text{for a uniform field} \quad (1.3a) \]

\[ \bar{K} = \exp \int_{r_a}^{r_c} \alpha \, dr \quad \text{for cylindrical geometry} \quad (1.3b) \]

where \( \alpha \) is the number of ion pairs formed by one electron per cm of path (the first Townsend coefficient)
\( d \) is the distance between cathode and anode planes for a uniform geometry device.
1.3 Proportional Counter Resolution

The energy resolution of a proportional counter (defined as the full width at half the maximum of the pulse height spectrum produced, divided by the pulse height corresponding to the peak) depends upon many factors. These are conveniently divided into two groups:

(i) **Fundamental factors**

These are the statistical fluctuations occurring (a) in the number of primary ion pairs produced in the initial ionization and (b) in the number of secondary electrons produced in the avalanche initiated by each primary electron. The resolution of an ideal detector would be governed solely by these effects.

(ii) **Non-fundamental factors**

These are effects which degrade the resolution from its fundamental theoretical value and include anode wire non-uniformity, electron attachment to impurities, amplifier noise etc. These effects can, at least in principle, be reduced to the point where they do not alter the resolution significantly.

This study is concerned with the fundamental effects involved with a view to their reduction. Considering first the fluctuation in the initial ionization, it was first noted by Fano (1947) that the standard deviation in the number of initial electrons was smaller than the $\sqrt{N}$ predicted by Poisson statistics. Fano has shown that in the case of hydrogen the standard deviation is between one third and one half that expected if the process was governed by a Poisson distribution. This is as a result of the fact that particles of a certain energy always lose exactly that much energy when they are brought to rest in the
gas of a detector. The requirement that the particle must have lost a definite energy in coming to rest restricts the number of impacts. Thus a particle that experiences an unusually large number of collisions in travelling a certain distance will also have lost an excessive amount of energy. This particle has less than the usual amount of energy left over in the remaining portion of the path, and the fluctuation in the total number of ionizing collisions is less than would be expected from a simple Poisson distribution.

The variance in the number of ion pairs can be expressed:

\[
\frac{\delta N^2}{N^2} = \frac{F}{N}
\]

where \( F \) is the so-called Fano factor (the factor by which the variance is reduced from that of a Poisson distribution).

\( \delta N \) is the standard deviation of \( N \).

It can be seen that the relative standard deviation is proportional to \( \frac{1}{N} \).

In an ionization chamber this is the sole fundamental effect governing the resolution. At high energies the fluctuation is very small. For example, for the mean number of ion pairs produced by 5.298 MeV polonium alpha particles in hydrogen, namely \( \bar{N} = 140,000 \) the standard deviation of the actual number of ion pairs would be approximately \((0.4 \bar{N})^2 = 240\) (using the factor 0.4 suggested by Fano) which is 0.17% of the mean number of ion pairs liberated. Instrumental spreads (introduced for example by amplifier noise) are typically an order of magnitude larger than this value.
However, in the case of a proportional counter operating in
the few KeV X-ray range, the number of initial ion pairs may be so
small that the fluctuation in this number makes a significant contribution
to the spread of the detector signal. For example, a 1 KeV X-ray
liberates, on average, only 38 ion pairs in argon and the standard
deviation is 10% of this value.

A simple expression for the theoretical resolution of a
proportional counter can be derived in the following way – (see for
example, Sipila (1976))

The resolution is defined as

\[ R = \frac{\text{FWHM}}{\bar{P}} \times 100\% \quad (1.5a) \]

where FWHM = FWHM of spectrum (in electrons)
\[ \bar{P} = \bar{N} \cdot \bar{K} = \text{average final charge collected at anode} \]
(in electrons).

For a Gaussian distribution FWHM = 2.36δ so we can rewrite (1.5)

\[ R = 2.36 \times \frac{\delta P}{\bar{P}} \times 100\% \quad (1.5b) \]

where \( \delta P \) is the standard deviation of P.

The fluctuation in P depends on the fluctuations in N and
K in the following way:

\[ \frac{\delta P^2}{\bar{P}^2} = \frac{\delta N^2}{\bar{N}^2} + \frac{1}{\bar{N}} \frac{\delta K^2}{\bar{K}^2} \quad (1.6) \]

If we define f as being \( \frac{\delta K^2}{\bar{K}^2} \) and also using (1.2) we have

\[ \frac{\delta P^2}{\bar{P}^2} = \frac{(P + f)W}{E} \quad (1.7) \]

Hence we can rewrite (1.5) as
\[ R_{th} = 236 \sqrt{\frac{(F + f)W}{E}} \]  

(1.8)

where \( R_{th} \) is the theoretical resolution expected from a proportional counter at an energy \( E \). Secondary effects which can degrade the resolution are not included.

With conventional gas mixtures such as argon-methane a resolution of about 14% at 6 KeV can be expected with a very uniform anode wire (Charles and Cooke (1968)). To improve the resolution it is apparent that the quantity \((F + f)W\) must be reduced.
1.4 Ionization Fluctuation

In the initial ionization we need to increase the number of ion pairs produced per KeV and to reduce the Fano factor, i.e. we must reduce $W$ and $F$. The overall effect of an interacting X-ray is to produce a number of ions, electrons and excited atoms. In general these excited states are short lived $\sim 10^{-8}$ seconds and will de-excite, with the emission of a quantum of radiation, before undergoing a collision. The rare gases, however, have metastable states which are much longer lived $\sim 10^{-3}$ seconds or more. Metastable atoms, therefore, are much more likely to lose energy in a collision. For a pure gas this energy will not be high enough for ionization. If, however, a second gas is present then one of its atoms may be ionized if the metastable level of the host gas is above the ionization level of the added gas. This cross ionization process is known as the Penning effect. The ionization process is made more efficient as energy previously only going to excitation is now spent in ionization. The net result is that more ion pairs are produced and, as will be examined in more detail in Chapter 2, the fluctuation in the number of ion pairs is reduced.

A classic example of the Penning effect occurs in a mixture of neon and a small amount of argon. Neon has a metastable level at 16.53 eV, which is greater than the ionization potential of argon (15.76 eV). Hence in a collision the following reaction can take place

$$\text{Ne}^* + \text{Ar} \rightarrow \text{Ne} + \text{Ar}^+ + e^- \quad (1.9)$$

The probability of the effect taking place is large here - almost unity per collision. The energy difference is small and the metastable state has a long lifetime. In the more detailed analysis presented in Chapter 2 it will be shown that, with Penning mixtures, values of $F$ as low as 0.05 are theoretically possible.
1.5 Avalanche Fluctuation

Investigations into avalanche fluctuations have been carried out since the 1930's - a full analysis will be provided in Chapter 3.

The main mechanism of avalanche growth is impact ionization by electrons, i.e. electrons moving towards the anode gain sufficient energy between collisions to ionize the gas:

\[ e^- + \text{Atom} \rightarrow e^- + \text{Ion}^+ + e^- \]  

(1.10)

The new electrons themselves produce more ion pairs and so the avalanche develops. The amount of charge present in the avalanche increases exponentially with distance \( x \) from the cathode. For a uniform field

\[ K(x) = \exp(\alpha x) \]  

(1.11)

Note that \( \frac{1}{\alpha} \) is the mean distance between ionizing collisions.

In a proportional counter an avalanche develops from each initial electron. It is simpler, however to consider the avalanches in isolation.

The earliest stages of the avalanche are the most critical as far as fluctuations are concerned. Any fluctuation early on will be amplified by the later stages of the avalanche. For example, if it happens that the primary electron travels several values of \( \frac{1}{\alpha} \) before ionizing, then the final number of electrons will be considerably reduced. On the other hand many early collisions will result in a large final charge. Uncertainty in the charge multiplication of later stages of the avalanche will still affect the total spread but to a lesser extent as it does not subsequently affect the rest of the avalanche to the same degree.
Various attempts have been made to correlate the relative variance of the avalanche size with known parameters of the gas. The general conclusion seems to be that $f$ depends essentially on the quantity

$$\chi = \alpha U_1 / E$$ (where $U_1$ is the first ionization potential of the gas, $E$ is the electric field). We can regard $\chi$ as a measure of the efficiency of ionization in the avalanche growth. It has been suggested that low values of $f$ can be obtained by arranging the conditions of the avalanche (with regard to gas filling and $E/p$ where $p$ is the gas pressure) so that the value of $\chi$ is large. In order to reduce the uncertainty in the charge multiplication it is necessary to maximize the probability of ionization taking place at each collision - hence the requirement for large values of $\chi$.

It has been shown experimentally that Penning mixtures can be used to give considerably higher values of $\chi$ than can normally be obtained with conventional proportional counter fillings. As electrons move towards the anode they can excite atoms without ionizing them. In a Penning mixture cross ionization of the added gas by the host gas can occur (as in the primary ionization process). Ionization efficiency is increased markedly (this is examined more fully in chapter 4).

By using Penning mixtures in a proportional counter one can expect to observe reduced fluctuation in the avalanche size and hence improved resolution. In a cylindrical counter the correlation of $f$ to $\chi$ is made rather unclear due to the changing nature of the field. Close to the anode the value of $\chi$ may vary over a single ionization mean free path. A uniform electric field is indicated for making such correlation studies.

$\dagger$ The quantity $\chi$ is discussed more fully in chapter 3.
1.6 The nature of this investigation

This investigation is concerned with examining experimentally the principles just outlined. A cylindrical proportional counter is operated with a range of Penning and conventional gas mixtures and the resolutions measured and compared.

To examine the behaviour of Penning mixtures in a proportional counter in a more fundamental way a uniform field research chamber was constructed. This allows the study of avalanche development for a constant electric field. This field value can be set to give a particular value on $\chi$ over the whole avalanche path. A schematic diagram of the apparatus is shown in figure 1.6.

![Schematic diagram of research chamber](image)

Figure 1.6 Schematic diagram of research chamber

The chamber has separate absorption and avalanche regions as shown. A low voltage is provided on the grid to transport the electrons to the high field avalanche region.

Using published values of $W$, the efficiency of charge multiplication $\alpha/E$ can be measured and compared with previous data. The contribution to the gas gain from secondary avalanches can also be investigated. More importantly, by measuring the resolution and using published values of $F$ and $W$, the quantity $f$ can be obtained. These are the first known measurements of resolution and
avalanche fluctuation in Penning mixtures in a uniform electric field. Comparison can be made with conventional fillings. The correlation of $f$ to $\chi$ can be examined and compared with the theoretical correlations suggested by various researchers (to be discussed fully in Chapter 3).

The variation of resolution with gas gain can also be measured. To a first approximation the absolute value of avalanche standard deviation is proportional to the avalanche magnitude. In a proportional counter it is the relative variance which governs the resolution, so, in the simplest analysis the gas gain should not matter. For both fundamental and practical reasons, however, the gas gain may be very important. It has been suggested by Alkhazov (1970) that $f$ should decrease with $K$, especially at gas gains $< 10$. At low gas gains, however, the final charge of the fully developed avalanche may be comparable with the noise charge of the preamplifier and the actual spectrum obtained on the pulse height analyser will be broadened. At high values of gas gain space charge and secondary avalanches become important and also degrade the resolution. In a Penning mixture the lack of a quenching agent increases the predominance of secondary avalanches.

In an attempt to quantify the above phenomena all measurements are made over the widest possible range of avalanche gain, i.e. from the point where the final charge obtained is just above the preamplifier noise right up to where breakdown takes place. In this way the optimum gas gain can be determined.

By way of summary, the scope of this work is to investigate the improvement of proportional counter resolution via the Penning effect. Fundamental information concerning the avalanche growth is obtained from a uniform field device.
In the following chapters ionization and avalanche fluctuations are analysed in detail. The Penning effect, involved in their reduction, is also considered. The main experimental techniques involved in the study are described. Results from the cylindrical counter are presented and discussed. The design and construction of the research chamber is described along with the subsequent development of the device. As with the cylindrical counter the results are presented and discussed.
CHAPTER 2: IONIZATION FLUCTUATIONS

2.1 Introduction

2.2 Ionization Fluctuations in Individual Gases

2.3 Ionization Fluctuations in Penning Mixtures
2 Ionization Fluctuations

2.1 Introduction

In chapter one the main processes that are involved when X-radiation interacts with a gaseous medium were briefly outlined. The fundamental dependence of proportional counter energy resolution on variations in the primary ionization and subsequent avalanche was shown. An experimental programme to measure avalanche size variation in Penning mixtures was indicated.

A single-electron study of avalanche fluctuations (using a U.V. source with a photo-electric cathode) is not possible with Penning mixtures. This is because the absence of a quenching agent means that the gas gain has to be kept low (around 100) to reduce secondary avalanches. The noise level of a good modern preamplifier is of the order of 200 electrons (with no capacitance loading on the input) and so the measurements cannot be made with a satisfactory signal to noise ratio.

By using an X-ray source to ionize the gas, however, a large number of ion pairs can be formed and so the final charge developed in the gas is proportionally higher. By suitable choice of X-ray energy it is thus possible to operate with an adequate signal to noise ratio.

The pulse height spectrum obtained with such an arrangement will of course be broadened by fluctuations in the number of primary ion pairs. The overall resolution was expressed in the following way in chapter one:

\[
R_{th} = 236\sqrt{\frac{(F+f)w}{E}}
\]

where \( R_{th} \) is the theoretical resolution.
By measuring the experimental resolution, the relative variance in the avalanche size, $f$, can be obtained using equation 1.8 (assuming that any disturbing effects are not significant). This procedure, however, requires a knowledge of the Fano factor $F$ and the mean ionization energy $W$ of the Penning mixture.

Some measurements of $W$ for Penning mixtures have been reported but $F$ has only been measured for the Penning mixture argon-acetylene. Alkhazov (1967) however has made theoretical calculations of $F$ and $W$ for such mixtures. These calculations were made by applying the analysis of Fano (1947) to Penning ionization.

In this chapter a brief account is provided of the theoretical analysis of ionization fluctuations and the most significant results are presented.

2.2 Ionization fluctuations in individual gases

When an X-ray of energy $E$ is absorbed in a gas with a mean ionization energy $W$ then the average number of ion pairs produced, $\bar{N}$, is given by:

$$\bar{N} = \frac{E}{W} \quad (2.1)$$

We are interested in the variation of and in particular the variance of $N$, denoted by $\Delta^2 N$.

Ionization fluctuations were first considered by Fano (1947) who regarded the problem as being analogous to the straggling in range of $\alpha$-particles. This had been examined previously by Bohr (1915). Assuming that the ionization process does not vary with the energy of the primary ionizing photon and that the results of successive impacts are independent of one another, Fano has used Bohr's analysis to express $F$ in the following way:

$$F = \frac{1}{n} \left( \frac{1}{\bar{n}} \{ n - (c/W) \}^2 \right) \quad (2.2)$$
Where $c$ is the energy loss per impact,
n is the number of ion pairs produced by the
absorption of the energy $c$.

If the number of ion pairs produced was governed by a Poisson
distribution then $F$ would equal unity. In practice, values of the
order of $\frac{1}{3}$ are observed. The reason for this reduced dispersion was
given in section 1.3.

Fano used cross section data available at the time to
evaluate $F$ for hydrogen and obtained a value of about 0.4. In
making the calculation impacts leading to excitation, ionization
and secondary ionization were considered.

Alkhazov et al (1967) made use of more recent data to
calculate $F$ for the noble gases. A summary of their results is
given in table 2.1. The quantity $FW$ is also shown as this is
important in determining the resolution of a proportional counter
(equation 1.8).
<table>
<thead>
<tr>
<th>Gas</th>
<th>$U_1^a$ (eV)</th>
<th>$U_1^b$ (eV)</th>
<th>$\bar{W}$ (eV)</th>
<th>$F_{\text{calc}}$</th>
<th>$F_{\text{exp}}$</th>
<th>$F_{\text{calc}} W$</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>19.83</td>
<td>24.58</td>
<td>42.8$^c$</td>
<td>0.17</td>
<td>7.3</td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>16.54</td>
<td>21.56</td>
<td>36.2</td>
<td>0.17</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>11.49</td>
<td>15.76</td>
<td>26.2</td>
<td>0.17</td>
<td>0.19$^d$</td>
<td>4.5</td>
</tr>
<tr>
<td>Kr</td>
<td>9.9</td>
<td>13.94</td>
<td>24.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xe</td>
<td>8.28</td>
<td>12.08</td>
<td>21.9</td>
<td>$\leq 0.17^e$</td>
<td>3.7$^f$</td>
<td></td>
</tr>
</tbody>
</table>

a Frish (1963).  
b Whyte (1963).  
c Impurities may have reduced this value - Bortner and Hurst (1954) obtained $\bar{W} = 46$ eV.  
d Measured by Alkhazov (1967).  
f Value given is $F_{\text{exp}} W$.

**Notation**  
$U_1$ is the first excitation energy.  
$U_1$ is the first ionization energy.  
$\bar{W}$ is the mean ionization energy.  
$F$ is the Fano factor.

---

**Table 2.1**

(Table is from Alkhazov et al (1967) except reference e.)
2.3 Ionization fluctuations in Penning mixtures

Alkhazov et al have also applied Fano's analysis to the ionization processes observed in Penning mixtures. A summary of their results is provided in table 2.2. By comparison with table 2.1 it can be seen that the FW product is considerably reduced in Penning mixtures. Table 2.2 also shows values of FW for the standard (non-Penning) counter mixture of argon-methane. The Penning mixture argon-acetylene has an FW product approximately three times less than for argon-methane.

Agreement between Alkhazov's calculations and the available experimental data seems to be reasonable.

Using Alkhazov's data one can determine f from resolution measurements using equation 1.8.

Alkhazov et al have summarized the underlying reasons for reduced ionization fluctuations in Penning mixtures: In noble gases, ionization fluctuations are determined by alterations in the number of excited and ionized atoms. If \( N_i \) and \( N_{ex} \) are, respectively, the number of ionized and excited atoms in the main gas, then the sum \( N_i + N_{ex} \) should fluctuate less than \( N_i \) and \( N_{ex} \) separately. In Penning mixtures excited states in the main gas give rise to ionization of the added gas and so the total number of ion pairs \( N \) will be similar in number to \( N_i + N_{ex} \). Fluctuations in \( N \) can thus be expected to be less than in individual gases.

The increased ionization efficiency also leads to a reduction in the value of the mean ionization energy \( W \).
<table>
<thead>
<tr>
<th>Main(^{\text{a}}) Gas</th>
<th>Added(^{\text{a}}) Gas</th>
<th>(E_1^{\text{b}}) (main(^{\text{a}}) gas) eV</th>
<th>(E_1^{\text{c}}) (added(^{\text{a}}) gas) eV</th>
<th>(W) (mixture) eV</th>
<th>(\sigma)</th>
<th>(P^{\text{f}}) (\text{calc}) (mixture)</th>
<th>(P^{\text{f}}) (\text{exp}) (mixture)</th>
<th>(P^{\text{g}}) (\text{calc}) (W) (mixture)</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>Ar</td>
<td>19.83</td>
<td>15.76</td>
<td>30.1(^{\text{c}})</td>
<td>1</td>
<td>0.055</td>
<td>1.66</td>
<td></td>
</tr>
<tr>
<td>He</td>
<td>Xe</td>
<td>19.83</td>
<td>12.08</td>
<td>28.7(^{\text{h}})</td>
<td>1</td>
<td>0.06</td>
<td>1.72</td>
<td></td>
</tr>
<tr>
<td>He</td>
<td>CH(_4)</td>
<td>19.83</td>
<td>13.04(^{\text{c}})</td>
<td>30.3(^{\text{c}})</td>
<td>0.83(^{\text{e}})</td>
<td>0.075</td>
<td>2.27</td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>Ar</td>
<td>16.53</td>
<td>15.76</td>
<td>25.3</td>
<td>1</td>
<td>0.05</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>Ar</td>
<td>CH(_4)</td>
<td>11.49</td>
<td>13.04</td>
<td>26.0(^{\text{f}})</td>
<td>0.04(^{\text{f}})</td>
<td>0.17</td>
<td>0.19</td>
<td>4.42</td>
</tr>
<tr>
<td>Ar</td>
<td>C(_2)H(_2)</td>
<td>11.49</td>
<td>11.4(^{\text{d}})</td>
<td>20.3(^{\text{i}})</td>
<td>0.76(^{\text{e}})</td>
<td>0.075</td>
<td>0.09</td>
<td>1.52</td>
</tr>
</tbody>
</table>

- **a** Pressure of main gas is 760 torr, impurity concentration is that required for maximum ionization increase.
- **b** Frish (1963).
- **c** Kraus et al (1955).
- **d** Lindholm et al (1964).
- **e** Platzman (1960).
- **f** Alkhazov (1967).
- **g** Bortner and Hurst (1954).
- **h** Jesse and Sadauskis (1955) and Bortner and Hurst (1954).
- **i** Moe et al (1957).

**Notation**

- \(U_1\) is first excitation energy of main gas.
- \(U_1^{\text{c}}\) is first ionization energy of added gas.
- \(\sigma\) is probability that de-excitation of a main gas atom results in ionization of an added gas atom.

**Table 2.2**

(Table is from Alkhazov et al (1967))
CHAPTER 3 : AVALANCHE FLUCTUATIONS

3.1 Introduction

3.2 The Electron Avalanche
   3.2.1 Gas discharge tube voltage-current characteristic
   3.2.2 The Townsend theory of avalanche growth
   3.2.3 Ionization efficiency

3.3 The Secondary Avalanche

3.4 Avalanche Fluctuations
   3.4.1 Snyder's model
   3.4.2 Legler's model
   3.4.3 Byrne, and Lansiart and Morucci's model

3.5 Alkhazov's Analysis
   3.5.1 Theoretical estimation of $f_0$
   3.5.2 The relationship between $f$ and $\bar{K}$

3.6 Avalanche Fluctuations in Penning Mixtures

3.7 Overall Discussion
3 Avalanche Fluctuations

3.1 Introduction

In this chapter a review is provided of previous investigations into avalanche fluctuations. Such work has tended to concentrate on the development of theoretical models of avalanche fluctuation phenomena which are in agreement with experimental results. Using these models it is possible to calculate the relative variance of the avalanche size, $f$, in terms of the quantity $\chi = \alpha U_i / E$. The general trend of the models, and experimental results, suggests that low values of $f$ should be obtained with high values of $\chi$. The theoretical values of $f$, though, vary from model to model and so the theoretical resolution of a proportional counter varies accordingly. High values of $\chi$ can be achieved using Penning mixtures. The use of such mixtures to obtain low values of $f$ and hence improved energy resolution is discussed.
3.2 The Electron Avalanche

The electron avalanche is only one of the gas discharge effects that can take place when an electric field is applied across a volume of gas. In this section the various possible mechanisms are briefly described - thus showing how the electron avalanche fits into the general discharge scheme.

The Townsend theory of avalanche growth is then outlined. Ionization efficiency is also discussed.

3.2.1 Gas discharge tube voltage-current characteristic

The overall gas discharge picture is conveniently described by reference to a typical gas discharge tube voltage-current characteristic - see figure 3.1. The curve may alter with the various parameters, but is general enough to apply to most cases. The characteristic is obtained using fixed voltages and depends also on the outer electric circuit of which the discharge is only a part.

The early stages of the curve were introduced in chapter one. When the voltage across the electrodes is raised up slowly from zero the first measurable current will be random pulses of less than $10^{-16}$A. However if enough free electrons are present in the gap, as a consequence of external ionization, a steady current may be observed without the onset of the pulses. The irradiation of the cathode with a suitable ultra-violet source also leads to the production of electrons and the passage of a photoelectric current. Under a constant radiation level, the current will increase with voltage until it reaches a plateau known as the saturation current. Here all electrons emitted from the cathode and/or produced in the gas are collected.

Increasing the voltage across the gap does not, at first, affect the current. The current starts to rise though when the energy gained by electrons between collisions with atoms is high enough for ionization to occur. This increase is exponential and the area is known as the
Townsend discharge region. This chapter is devoted to an analysis of fluctuations of charge multiplication in the Townsend region.

As is discussed in section 3.3 further increase of the voltage leads to an over-exponential increase in the current followed by the collapse of the voltage across the gap. This abrupt transition is known as breakdown and is characterized by an increase in current of several orders of magnitude with almost no increase of voltage. The current is now independent of the external ionizing source and is thus self sustained. This is in contrast to regions A, B, and C where the current ceases if the ionizing source is removed.

If the current is allowed to increase further, by reducing the series resistance, the voltage across the discharge will begin to drop to a low value. Regions F and G are known as the subnormal and normal glow regions respectively. An exponential increase in current follows, giving rise to the abnormal glow.

Finally, when the current is allowed to increase further, another transition occurs and a new form of discharge, known as the arc discharge (because of its extreme brightness), develops.

The Townsend discharge region of the curve was first investigated by J.S. Townsend at the beginning of the century. He found that in this area the current starts to increase exponentially with voltage. Despite the lack of advanced experimental techniques and precise knowledge of atomic structure, Townsend was able to clarify many aspects of gas discharge phenomena.

Townsend and colleagues measured the first ionization coefficient \( \alpha \) (the number of ion pairs formed by one electron per cm of path) active in region C of the curve in air, CO\(_2\), H\(_2\)O and HCl (Townsend (1914) and Loeb (1955)). Problems with gas purity tended to make the results somewhat unreliable but corrections were made later by Townsend's colleagues.
In the thirties, extensive work to measure the ionization coefficients was conducted by Loeb in California and Penning in Holland. Improvements in gas purity led to more consistent results. Impurities tend to increase the number of ionized particles, via the Penning effect and by direct ionization of the impurities. A pure gas may therefore have a lower value of ionization coefficient than an impure one.

![Typical voltage-current relation of a gaseous gap. This curve is for neon at 1 torr with disk electrodes of 2-cm diameter and 50-cm separation.](image)

**Figure 3.1**

*(From Nasser (1971))*
3.2.2 The Townsend theory of avalanche growth

The theory of avalanche growth has been reviewed by a number of authors including Penning (1965), Kuffel and Abdullah (1970) and Rice-Evans (1974). In the E/P region of interest, an electron is likely to gain sufficient energy to cause ionization when it collides with a gas atom in its path. An additional electron is thus liberated which, after acceleration, will be able to ionize too. This, and further multiplication, result in rapid development of the avalanche. The electrons move quickly towards the anode at about $10^7$ cm/sec whereas the much heavier positive ions move towards the cathode at about $10^5$ cm/sec. In plate 3.1 a cloud chamber photograph of a single electron avalanche is shown. Gas breakdown occurs at a sufficiently high value of voltage. Breakdown may occur due to over-exponential charge multiplication (section 3.3) or, if the avalanche is sufficiently large ($>10^8$ electrons), via the streamer mechanism (figure 3.2).

The avalanche process may be described with Townsend's first ionization coefficient (defined in 3.2.1). The number of electrons produced by n electrons in a distance dx will be

$$dn = na \, dx$$  \hspace{1cm} (3.1)

and hence the number produced by one electron in a length x will be

$$n = \exp (ax)$$  \hspace{1cm} (3.2)

A simple expression for the coefficient may be obtained from kinetic theory. Two assumptions are involved:

(1) The ionizing electron starts, between two collisions, with a velocity component in the field direction equal to zero. This means that it loses all its kinetic energy, gained from the electric field, to the atom or molecule at every collision.
The transition of a Townsend (generation) type of breakdown to a streamer breakdown. (a) An avalanche emits photons to produce photoelectrons at the cathode. (b) The photoelectrons produce avalanches of varying size with electrons drifting at the same velocity. (c) Positive space charge accumulated at the anode, enhancing the field and augmenting the avalanche size to reach the critical size. (d) A streamer grows from the avalanche toward the cathode by the oncoming avalanches.

Figure 3.2
(From Nasser (1971))
(2) The probability of ionization by collision is unity as soon as the kinetic energy of the electron, at impact, is equal to or exceeds, the ionization energy of the atom.

While these assumptions may not be strictly justifiable, the resulting expressions for $\alpha$ are extremely close to the much more complicated expressions obtained from the ionization cross sections.

For an avalanche to develop successfully, some electrons must have sufficiently long free paths ($x_i$) to acquire the energy necessary to ionize a gas atom on collision, i.e.

$$eE x_i \geq U_{1e}$$

where $U_1$ is the ionization potential.

$e$ is the electronic charge.

Assuming that if an electron gains this energy it will ionize, the probability of ionization is determined by the probability of the path being greater than $x_i$.

From Boltzmann's law, the number $n$ of electrons that have free paths $> x_i$ is given by:

$$n = n_o \exp(-x_i/\lambda)$$

$$= n_o \exp(-U_{1e}/kE)$$

(3.4)

where $n_o$ is the number of free electrons (both $n$ and $n_o$ are per unit length of particle path).

$\lambda$ is the mean free path.

The coefficient $\alpha$ is the number of free paths multiplied by the chance of a free path being $> x_i$:

$$\alpha = \frac{1}{\lambda} \exp\left(-\frac{U_{1e}}{kE}\right)$$

(3.5)

we know that the mean free path is inversely proportional to the gas pressure $p$:

$$\lambda = \frac{1}{\lambda p}$$

(3.6)

where $\lambda$ is a constant for a particular gas.
Hence  
\[ \alpha = A \rho \exp(-A\rho U_1/E) \]

or  
\[ \alpha/\rho = A \exp(-B\rho/E) \]  (3.7)  

where A and B are constants.

This indicates that the number of ionizations caused by electron collision depends upon the product \( E\rho \) which is the average energy gained between two successive collisions. Townsend had already shown by experiments at low pressure that \( \alpha/\rho \) depends, for any particular gas, uniquely upon the ratio \( E/\rho \). Experiments have shown that equation (3.7) holds for a number of gases for a wide range of \( E/\rho \) (see table 3.1).

The general shape of \( \alpha/\rho \) versus \( E/\rho \) curves is indicated by typical examples of \( H_2 \) and \( N_2 \) shown in figure 3.3. The dots represent actual measurements while the curve was obtained using (3.7) with \( A = 5.1 \text{ cm}^{-1}\text{torr}^{-1} \) and \( B = 138.8 \text{ Vcm-torr} \). The curves usually have a typical \( s \)-shape when drawn on a linear scale as shown in figure 3.3.

In the above curve for \( H_2 \), the slight deviation of the theoretical curve from the measured points at high values of \( E/\rho \) is a consequence of the simplified assumptions made in the derivation of (3.7). For limited ranges of \( E/\rho \) equation (3.7) may normally be used quite successfully. Empirical equations can also be used to fit the experimental curves.

The general shape of the curves can be explained as follows. The curve starts at an \( E/\rho \) value dependent on the ionization energy of the gas. As the field is increased, the electrons gain more energy and are able to ionize further so \( \alpha/\rho \) rises. However the probability of ionization does not increase indefinitely with electron energy but declines after reaching a maximum (see figures 3.4 and 3.5) so the increase in \( \alpha/\rho \) slows down. This decrease in ionization probability arises because at high energies (and hence high velocities) the electron is only close to the atom for a short time.
Table 3.1
(Badereu and Popescu (1965))

<table>
<thead>
<tr>
<th>GAS</th>
<th>A (cm(^{-1}) Torr(^{-1}))</th>
<th>B (V cm(^{-1}) Torr(^{-1}))</th>
<th>E/p validity range (V cm(^{-1}) Torr(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>3</td>
<td>34</td>
<td>20 - 150</td>
</tr>
<tr>
<td>Ne</td>
<td>4</td>
<td>100</td>
<td>100 - 400</td>
</tr>
<tr>
<td>Ar</td>
<td>14</td>
<td>180</td>
<td>100 - 600</td>
</tr>
<tr>
<td>Kr</td>
<td>17</td>
<td>240</td>
<td>100 - 1000</td>
</tr>
<tr>
<td>Xe</td>
<td>26</td>
<td>350</td>
<td>200 - 800</td>
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<tr>
<td>H(_2)</td>
<td>5</td>
<td>130</td>
<td>150 - 600</td>
</tr>
<tr>
<td>N(_2)</td>
<td>12</td>
<td>342</td>
<td>100 - 600</td>
</tr>
<tr>
<td>CO(_2)</td>
<td>20</td>
<td>466</td>
<td>500 - 1000</td>
</tr>
<tr>
<td>Air</td>
<td>15</td>
<td>365</td>
<td>100 - 800</td>
</tr>
<tr>
<td>H(_2)O</td>
<td>13</td>
<td>290</td>
<td>150 - 1000</td>
</tr>
</tbody>
</table>

The variation of a/p\(_0\) with E/p\(_0\) in hydrogen and nitrogen, according to recent measurements [2, 6, 7]: p\(_0\) is the normalized pressure, that is, the pressure reduced to 0°C, and is calculated from p\(_0\) = p(T\(_0\)/T), where p and T are the actual pressure and temperature (K), respectively, and T\(_0\) = 273 K.

Figure 3.3
(From Nasser (1971))
Electron energy, eV

Figure 3.4
(From Nasser (1971))

Ionization cross section, $\sigma_{\text{ion}}$, for $O_2$, $N_2$, and $H_2$ due to electron impact given in units of $\pi a_0^2$.

Electron energy, eV

Figure 3.5
(From Nasser (1971))

Ionization cross section $\sigma_{\text{ion}}$ for the rare gases due to electron impact given in units of $\pi a_0^2$. 
3.2.3 Ionization Efficiency

The ratio $\alpha/E$ is often useful as it represents physically the number of ion pairs produced by electron collision per unit potential drop. This ratio is referred to as the "ionization efficiency" $\eta$.

As will be discussed in section 3.4 the fluctuation of the avalanche size is dependent on the quantity $\chi = \alpha U_i/E$. Physically $\chi$ is the number of ion pairs produced by an electron travelling through an energy equal to the ionization potential of the gas. If $\chi$ were to equal unity then a new ion pair would always be generated by such an electron. The variation in the avalanche size would therefore be zero.

In practice $\chi$ is less than unity and the uncertainty in the charge multiplication gives rise to a distribution of avalanche size. To minimize the avalanche fluctuation $\chi$ should be as large as possible.

An example of an $\alpha/E$ versus $E/p$ curve, for hydrogen, is shown in figure 3.6. From equation (3.7) $\eta$ can be expressed in terms of the constants $A$ and $B$:

$$\eta = \frac{\alpha}{E} = \frac{\alpha/\epsilon}{E/p}$$

$$\Rightarrow \eta = \frac{A}{E} p \exp \left( -\frac{Bp}{E} \right) \quad (3.8)$$

The point of maximum ionization efficiency ($\eta_{\text{max}}$) is of interest. Nasser (1971) has deduced by differentiation that $\eta$ is a maximum when $E/p = B$ and that $\eta_{\text{max}} = \frac{A}{Be}$ where $\epsilon$ = base of natural logarithms = 2.718.

$\eta (= \alpha/E)$ as a function of $E/p$ for hydrogen. The figure was obtained using the values of figure 3.3.

Figure 3.6

(From Nasser (1971))
3.3 The Secondary Avalanche

If, in the parallel plate system under discussion in section 3.2.1 we have a fixed rate of electron production at the cathode \( i_0 \) then from equation (3.2) we can write down the gap current \( i \) (assuming operation in the Townsend discharge region).

\[
i = i_0 \exp (\alpha x)
\]  

where \( i_0 \) is the current being released from the cathode. According to this equation a graph of \( \log i \) against the gap length should give a straight line of slope \( \alpha \) if \( E \) is kept constant. In his early measurements of current in parallel plate gaps Townsend (1948) observed that at higher voltages the current increased at a more rapid rate than given by equation (3.10). Fig.3.7 shows the kind of curves obtained by plotting \( \log i \) against electrode separation at a constant pressure.

\[\text{Slope = } \alpha \]

\[\text{Variation of gap current with electrode spacing in uniform field gaps.}\]

**Figure 3.7**

(From Kuffel and Abdullah (1978))

To explain this departure from linearity at higher voltages Townsend postulated that a secondary mechanism must be affecting the current. The two main secondary processes are as follows:

(a) The positive ions on striking the cathode are likely to eject secondary electrons.

(b) Photons arising from the discharge may also release electrons through the photoelectric effect.
Previously, attempts have been made to define coefficients for these and other secondary mechanisms. Nearly all significant processes can be ultimately described as an equivalent number of emitted electrons at the cathode for every positive ion produced in the gas - this number $\gamma$ (the second ionization coefficient) can therefore be taken to represent all such secondary effects. It is readily shown (see for example Howatson (1965)) that the current may be expressed in the following way:

$$i = i_0 \frac{\exp qx}{1 - \gamma (\exp qx - 1)}$$

(3.11)

Secondary avalanches are of importance in the study of avalanche fluctuations due to their disruptive effect. In a proportional counter the gas gain at which optimum resolution occurs may at least partly be determined by the effect of secondary avalanches. This is considered experimentally in chapters six and nine.

3.4 Avalanche Fluctuations

Equation (3.2) predicts the average value of the multiplication factor of the avalanche. In practice a distribution of final avalanche charge is obtained. This variation in charge multiplication arises because the probability of ionization by electron impact at a distance of $U_i/E$ is less than unity. This uncertainty in the local multiplication of charge results in a dispersion in the final avalanche size.

In this section avalanche fluctuations are considered in detail. It is assumed that processes such as recombination, electron attachment to impurities, secondary avalanches and space charge do not significantly affect the process of avalanche growth.
3.4.1 Snyder's Model

Fluctuations in the electron multiplication process were first considered by Snyder (1947) and then by Wijsman (1949), Legler (1955), Kharitonov (1956) and Frisch (1959). They assumed that electrons are produced only in ionizing collisions (implying that the photoelectric effect is negligible) and that the instantaneous probability of ionization is independent of the distance travelled by the electron since its previous ionizing collision (i.e. the probability of ionization during avalanche growth is constant and the electrons are in equilibrium with the electric field).

Considering the case of a uniform electric field the energy distribution of the electrons will depend in general upon \( x \) (the distance from the cathode - see figure 3.8) so the probability \( \alpha(x)dx \) that an electron ionizes while drifting through a region between \( x \) and \( x+dx \) also depends on \( x \).

![Diagram](image)

**Figure 3.8**

As discussed previously, in section 1.5, it is the early ionizations that have the greatest influence on the final avalanche size.
We are interested in the probability $P(K,x)$ that a single electron starting at the cathode grows to an avalanche of $K$ electrons at a distance $x$ from the cathode. It is assumed that in every ionization only one additional electron is created.

If there is one electron at $x=0$ then the probability of having one electron at $x$ is denoted by $P(1,x)$ and the probability of still having one electron at $x+dx$ is:

$$P(1,x+dx) = P(1,x)(1-\alpha dx)$$

(i.e. the probability of having one electron at $x$ multiplied by the probability of not generating another electron along $dx$). The probability of having two electrons at $x$ $dx$ is given in a similar way by:

$$P(2,x+dx) = P(1,x)\alpha dx + P(2,x)(1-2\alpha dx).$$

The rate of change of $P(1,x)$ and $P(2,x)$ with $x$ can be expressed in the following way:

$$\frac{d}{dx} P(1,x) = -\alpha P(1,x)$$

$$\frac{d}{dx} P(2,x) = -2\alpha P(2,x) + \alpha P(1,x).$$

In general:

$$\frac{d}{dx} P(K,x) = -\alpha K P(K,x) + \alpha (K-1) P(K-1,x)$$

with the boundary conditions:

$$P(1,0) = 1; P(K,0) = 0 \text{ for } K>1.$$

Furry (1937), considering a rather different physical process (the energy loss of high energy electrons passing through lead), solved the equations by successive integration and application of the boundary conditions to give:

$$P(1,x) = \exp(-\alpha x)$$

$$P(2,x) = \exp(-\alpha x)(1-\exp(-\alpha x))$$

In general

$$P(K,x) = \exp(-\alpha x)(1-\exp(-\alpha x))^{K-1} \quad (3.12a)$$

The mean number of electrons is $\bar{K} = \exp(\alpha x)$ so

$$P(K,x) = (1/\bar{K})(1-1/\bar{K})^{K-1} \quad (3.12b)$$
In particular, by setting \( x = d \), we obtain the probability that the avalanche develops to \( K \) electrons at the anode. Equation (3.12) is referred to as the Furry distribution. For \( K \gg 1 \), which is the usual case, equation (3.12) can be approximated by

\[
P(K, x) = \left( \frac{1}{\sqrt{K}} \right) \exp \left( -\frac{K}{\sqrt{K}} \right)
\]

(3.13)

It can be shown that, \( f \), the relative standard deviation of \( K \), governed by such a distribution, takes the value unity. The formula shows that the probability of an avalanche producing \( K \) electrons declines exponentially with \( K \) and also that the fluctuations of \( K \) about \( \mu \) are large. An example of the distribution function is shown in figure 3.9, for the case \( K = 5 \). The corresponding Poisson distribution is also shown for comparison. Figure 3.10 shows the total probability for \( K \) or more electrons - again for \( K = 5 \) and for Furry and Poisson distributions. It can be seen from figure 3.9 that the mean value is not marked by any particular feature of the curve, while the Poisson curve, of course, has a well-defined maximum.

Electron avalanche distributions have been measured by many experimenters. (Genz (1973) has provided a useful review of such work.) A typical experimental configuration is shown in figure 3.11. The apparatus is normally arranged so that a single electron is released from the cathode. A similar arrangement may of course be used with a uniform electric field.

Some experimental data obtained by Schlumbohm (1958) and Cookson and Lewis (1966) is shown in figures 3.12 to 3.14. This data was obtained under uniform electric field conditions. For each curve the quantity \( \chi = aU_i/E \) is shown.

† The Furry distribution is also known as the exponential distribution.
A Furry distribution compared with a Poisson distribution for $\bar{K} = 5$. The functions have significance only for integral values of $K$, but for convenience continuous curves are plotted.

**Figure 3.9**
(From Furry (1937))

The total probability for $\bar{K}$ or more electrons for Furry and Poisson distributions with $\bar{K} = 5$. As in figure 3.9 smooth curves are plotted although only discrete points are defined.

**Figure 3.10**
(From Furry (1937))
A typical arrangement for measuring electron avalanche size distributions. Ultra-violet radiation is used to release a single photo-electron from the cathode. The resulting electron avalanche is measured with a multi-channel analyser. A large number of such measurements are made. In this way a complete size distribution is obtained.

Figure 3.11
(From Carver and Mitchell (1967))
Distributions of avalanche size in dimethoxymethane CH$_2$(OCH$_3$)$_2$.

(a) (b) (c) (d) (e)

$E/p$ (v/cm/torr)  70  76.5  106  186.5  426

$p$ (torr)  106  76.6  86.7  17  11

$X$  0.039  0.044  0.095  0.19  0.24

Figure 3.12

(From Schlumbohm (1958))
Distributions of avalanche size in acetane \((\text{CH}_3)_2\text{CO}\) and ethanol \(\text{CH}_3\text{CH}_2\text{OH}\).

<table>
<thead>
<tr>
<th>Gas</th>
<th>acetone acetone</th>
<th>ethanol</th>
<th>ethanol</th>
<th>ethanol</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{E}{p} (\text{v/cm/torr}))</td>
<td>95</td>
<td>104</td>
<td>76</td>
<td>92</td>
</tr>
<tr>
<td>(p \text{ (torr)})</td>
<td>71</td>
<td>54</td>
<td>36</td>
<td>30</td>
</tr>
<tr>
<td>(x)</td>
<td>0.036</td>
<td>0.044</td>
<td>0.062</td>
<td>0.095</td>
</tr>
</tbody>
</table>

*Figure 3.13*

(From Schlumbohm (1958))
Distribution of avalanche sizes in methane

(a) (b) (c) (d) (e) (f)

<table>
<thead>
<tr>
<th>E/p (v/cm/torr)</th>
<th>48.2</th>
<th>51.3</th>
<th>78.9</th>
<th>120</th>
<th>156</th>
<th>218</th>
</tr>
</thead>
<tbody>
<tr>
<td>p (torr)</td>
<td>692</td>
<td>373</td>
<td>187</td>
<td>9.1</td>
<td>11.5</td>
<td>11.5</td>
</tr>
<tr>
<td>X</td>
<td>0.026</td>
<td>0.034</td>
<td>0.088</td>
<td>0.13</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>b (equation 3.17)</td>
<td>1</td>
<td>1.1</td>
<td>1.3</td>
<td>1.4</td>
<td>2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Figure 3.14
(From Cookson and Lewis (1966))

Pulse size distributions obtained from a cylindrical proportional counter at two values of anode voltage.

Figure 3.15
(From Curran et al (1949))
It can be seen that in weak uniform electric fields, where $X$ is low, the exponential distribution described above is observed. However, with higher electric fields the distributions depart from this form and exhibit a pronounced maximum and reduced dispersion. The same form of distribution is observed in a cylindrical device - Curran et al (1949), for example, found that the pulse height spectrum (figure 3.15) could be described approximately by the empirical relation

$$F(K) = \left(\frac{K}{K_0}\right)^{1/2} \exp\left(-\frac{3}{2} \frac{K}{K_0}\right)$$

(3.14)

The maximum was at $K = \frac{1}{3} K_0$ and the relative variance of the avalanche size, $f$, was $2/3$.

To explain the discrepancy between the theory of Snyder, Wijisman etc. and the experimental results at high values of $E/p$, one can consider the validity of the assumption that "the probability that an electron will ionize is independent of its distance from the cathode". After each ionizing collision the electron must travel a distance $U_i/E$ before ionization is again possible. If $U_i/E$ is much smaller than the mean distance between ionizing collisions ($1/\alpha$) then the idea of a constant ionization probability seems reasonable - the electron velocities are able to achieve equilibrium in non-ionizing collisions. This condition holds for low values of $E/p$ because the ionization efficiency $\alpha/E$ is small. Fluctuations are therefore large as observed.

However at high values of $E/p$ a much greater fraction of the energy gained by the electrons from the electric field is now spent on ionization as opposed to excitation. With high values of ionization efficiency, i.e. $\alpha U_i/E = 1$ (which implies $\frac{U_i}{E} = 1/\alpha$) the assumption is no longer valid. The ionization mean free path ($1/\alpha$) is now so reduced that it has become comparable with $U_i/E$. The instantaneous probability of ionization will now depend on the previous history of the electron, in particular on the distance $\xi$, which this electron has covered towards the anode after the last ionization.
3.4.2 *Legler's Model*

Legler (1961) was the first to develop an avalanche model which did not assume that the ionization coefficient remained constant throughout the development of the avalanche.

Legler suggested that for high values of \( \chi \), a local ionization coefficient \( a_i \) can be defined which depends on the distance \( \xi \) an electron has travelled since the previous ionization, in the following way:

\[
a_i(\xi) = \begin{cases} 
  a & \text{for } \xi < U_0/E \\
  a_0 & \text{for } \xi > U_0/E
\end{cases}
\]

where \( a_0 = \text{constant} = \frac{\alpha}{2\exp(-\alpha U_0/E) - 1} \)

\( U_0 \) is the so-called "model parameter". For the model to fit as Legler supposed, \( U_0 \) should take the value \( U_i \), the first ionization energy of the gas. Figure 3.16c shows the above step function variation of \( a_i \) with \( \xi \). Figure 3.16a is a suggestion of how \( a_i \) might be expected to vary in a "real" gas - the function is less abrupt.

Figure 3.16b is simply the "variation" of \( a_i \) with \( \xi \) for low values of \( \chi \) (i.e. \( \chi \ll 1 \)) where \( a_i \) is constant (Snyder's model).

The probability distribution of the avalanche size was calculated for the model gas with the following result

\[
P(K, d) = \left( \frac{K}{K(d)} \right) \phi \left( \frac{K}{K(d)} \right)
\]

(3.15)

where \( \phi \) is a function which depends on \( \alpha U_0/E \) but can only be evaluated numerically. At high values of \( E/p \) Legler's model predicts a reduced dispersion of avalanche size - as measured experimentally.
Legler compared, calculated and measured avalanche distributions and found that $U_0$ agreed within 20% with the expected value $U_1$.

Legler's model is further discussed later on.

![Graph showing the variation of the local Townsend coefficient $\alpha$ with distance $\xi$ travelled by an electron from the previous ionization. Three cases are shown:]

(a) A 'real' gas
(b) Snyder's model
(c) Legler's model

Figure 3.16

(From Legler (1961))
3.4.3 Byrne, and Lansiart and Morucci's Model

To explain the results obtained at high values of \( E/p \) by Curran and others, an alternative model was proposed by Byrne (1962) and Lansiart and Morucci (1962). In this model it is assumed that the probability that an electron will ionize depends only on the total number of electrons already produced in the avalanche (and not on the distance from the previous ionization). The local ionization coefficient \( a_i \) is assumed to decrease with \( K \) in the following manner:

\[
 a_i = \alpha (1 + \mu/k) \quad (3.16)
\]

where \( \mu \) is an empirically defined parameter which depends on conditions in the gas.

Byrne bases this variation of \( a_i \) with \( K \) on the fact that "the average energy of the two electrons coming from an ionizing collision must be less than the energy of the colliding electron". Their reduced energy implies a reduced ionization coefficient.

Whilst this idea may be valid locally along individual ionization mean free paths, it is difficult to see why it should apply to the whole avalanche as electrons will regain lost energy whilst travelling between collisions. Their ability to ionize is thus restored. The physical basis of Byrne's model is further discussed later in this section.

Using a method similar to that involved in obtaining the Furry distribution, it was found that the above assumption concerning \( a_i \) leads to a Polya distribution of the form:

\[
 P(K,\alpha) = \frac{b}{K} \frac{1}{(b-1)!} \left(\frac{bK}{K}\right)^{b-1} \exp\left[-\frac{bK}{K}\right] \quad (3.17)
\]

where \( b = 1 + \mu \).
Strictly speaking, the avalanche size $K$ should really be replaced by $(K-1)$ in equation (3.17) but in practical applications the error is negligible. The limiting cases of the distribution are obtained for $b = 1$ (i.e. $\mu = 0$) and $b = \infty$ leading to the Furry and Poisson distributions respectively. The Polya distribution has a maximum at

$$K_m = \frac{1}{b} \quad (3.18)$$

which thus shifts to larger $K$ as $b$ (or $\mu$) increases. Byrne noted that when $b = 3/2$ the distribution coincides with the experimental distribution obtained by Curran et al (equation (3.14)). Byrne interpreted $1/b$ as the fraction of the electron swarm with energies greater than $eU_1$ and concluded that $b$ must be restricted to values between 1.64 and 1. Cookson and Lewis (1966) criticised this interpretation by pointing out that it implies that at low values of $E/p$, where Furry distributions are recorded ($b = 1$), all the electrons must have ionizing energies. In the same way, at high values of $E/p$ where modified distributions with $b > 1$ are obtained, the above interpretation implies that fewer electrons have ionizing energies and that therefore the ionization coefficient should decrease.

Clearly these effects are not what one would tend to expect.

Cookson and Lewis, however, found good agreement between their experimental results and appropriate Polya curves. A typical comparison is shown in figure 3.17. All the different distributions of figure 3.14 could be fitted to Polya distributions by appropriate choice of $b$.

Cookson and Lewis also found that the Polya distributions closely resembled the theoretical functions obtained by Legler (1961) as can be seen from the typical comparison shown in figure 3.18. By making many such comparisons between the Polya distributions, their own results and those of Legler, Cookson and Lewis suggested that one could assume a simple reciprocal relationship between $\chi$ and $\mu$, see figure 3.19. The constant of proportionality is slightly different
Comparison of a measured distribution of avalanche size with an appropriate Polya curve.

- \( P(K, d) \) for \( E/p = 156 \, \text{v/cm/torr} \), \( \chi = 0.19 \)
- Polya distribution for \( b = 2 \)

Figure 3.17
(From Cookson and Lewis (1966))

Comparison of Legler and Polya functions.

- The Legler function for \( \chi = 0.05 \)
  - The Polya function for \( b = 1.2 \)

Figure 3.18
(From Cookson and Lewis (1966))
Relationship between the parameters \( \mu \) and \( X \left( = \frac{\alpha U_i}{E} \right) \) for methane from the work of Legler and Schlumbohm (\( \bullet \)) and Cookson and Lewis (\( \bullet \)).

**Figure 3.19**
(From Cookson and Lewis (1966))

The variation of \( \alpha /p \) with distance from the cathode for neon and hydrogen. \( E/p \) is in \( \text{v/cm/torr} \), \( p \) is in \( \text{torr} \).

- Chanin and Rork (1963)
- mean variation according to equation (3.19)

**Figure 3.20**
(From Cookson and Lewis (1966))
for the two cases shown. Thus, once $X$ has been determined for the
given gas condition, the form of the expected pulse height
distribution can be found. The expected variation of $a_1$ can be
found using equation (3.16). Writing $\mu = cX$, where $c$ is the
constant of proportionality in figure 3.19, we obtain

$$a_1 = \alpha \left(1 + \frac{cU_1}{EK}\right)$$

(3.19)

It is interesting to consider why the ionization coefficient should
vary in the manner suggested by equations (3.16) and (3.19).

Lansiart and Morucci (1962) proposed that the effect could
be explained by changes in the ionization cross section: With the
high $E/p$ values that we are concerned with an electron does not lose
all of its energy in an ionizing collision. This means that over a
number of collisions the average electron energy increases. At these
high energies the ionization cross section starts to decrease and so
$a_1/p$ will also decrease in value.

The cross section data, for methane, for example, does not
however support the above hypothesis. The peaks of the total
collision and ionization cross sections occur at energies of 6.5
and 67 eV respectively (Brüche 1930, Tozer 1958) with the ionization
cross section changing relatively slowly for energies considerably
higher than 100 eV. For methane, Polya distributions are obtained
when $E/p$ is about 50 v/cm/torr. The electron energy is about 4 eV
under these conditions (Haylen (1962), Schlumbohm (1965)) and so is
too low to fit in with Lansiart and Morucci's explanation.

Equations (3.16) and (3.19) allow $a_1$ to tend to a limiting value
but at high energies the cross section will decrease and so this
aspect of the argument does not seem valid either.
Some support for Byrne's model comes however from the work of Chanin and Rork (1963) who have made experimental measurements of $a_i/p$ as a function of $px$ (the pressure $p$ was kept constant for each set of measurements and so $px$ here is proportional to $x$, the distance into the gas from the cathode). Measurements were made for neon and hydrogen for a variety of $E/p$ values – figure 3.20. For the higher values of $E/p$ a damped oscillatory variation in $a_i/p$ can be seen. At large values of $px$ $a_i/p$ tends to the limit $a/p$. Chanin and Rork (1963) and Kruithof (1940) suggested that this type of behaviour was due to a fluctuating electron energy distribution. The potential difference between successive peaks is approximately equal to the ionization potential of the gas. When the electrons gain an energy equal to the ionization energy the probability of ionization reaches a maximum value. Note that it is only the mean variation of the curves that seems to follow equation (3.19). The cyclic variation is not allowed for in Byrne's model.

The results obtained by Chanin and Rork were later criticized by Golden et al (1965) because only three values of voltage were used to obtain each value of $a_i/p$ (using a method outlined in section 4.2.1). This may have led to large experimental errors.

Figure 3.21 shows an attempt by Cookson and Lewis to depict the variation in the energy distribution of electrons as they move away from the cathode. It is assumed that $\chi \approx 1$ so that the mean distance between ionizing collisions is similar to the distance required to obtain the ionization energy from the field, i.e. $1/\alpha = U_i/E$. A large fraction of collisions will therefore result in ionization. It is also assumed that the energy range of the electrons released from the cathode is fairly small – not above about 2eV.
Changes in the probability distribution of electron energy as an electron moves away from the cathode for $x \gtrsim 1$. Electrons are emitted photo-electrically from the cathode with a distribution (a) and ultimately attain the stationary distribution (f). The excitation and ionization energies of the gas are $U_1$ and $U_i$ respectively.

**Figure 5.21**
(From Cookson and Lewis (1966))
The probability of ionization is indicated in figure 3.21 by the fraction of the energy distribution above $eU_i$ (shown shaded).

At increasing distances from the cathode the electron energy distribution will move up the energy axis and will tend to broaden due to collisions in the gas. If the probability of ionization is high, however, $(\chi \approx 1)$ the energy distribution will not change significantly until a distance of $U_i/E$ into the gas has been reached. The probability of ionization will now increase considerably and the two low energy electrons which result from the ionization process will now appear low on the energy distribution scale. Just beyond $U_i/E$ the probability of ionization will become small but with increasing distance it will increase again as in the previous cycle. As the probability of ionization fluctuates so of course will $a_i$.

As a result of collisions which do not lead to ionization and the randomness of the processes, the peaks in the distribution will become broadened and so the oscillation of $a_i$ will decrease. Cookson and Lewis have suggested that eventually the stationary distribution shown in figure 3.21f will be obtained.

Whilst it may be reasonable to suppose that a stationary distribution will ultimately be obtained at large distances into the gas, it is not clear why the average probability of ionization should decrease. This is indicated in figure 3.21f by the very small shaded fraction. Space charge effects will cause local reductions in the electric field strength which will thus reduce the local value of the ionization coefficient. Such effects are most prominent though at large avalanche sizes and will not therefore lead to large reductions in $a_i/p$ near the cathode.
With lower values of \( X \) (i.e. when non-ionizing collisions are more prominent) the changes in the probability distribution shown in figure 3.21 become more gradual. The initial energy distribution will alter to the stationary form in a shorter distance and so \( a_1 \) will decrease more quickly. The results obtained by Chanin and Rork support this idea.

The sequence shown in figure 3.21 corresponds with the variation of \( a_1 \) with \( x \) measured by Chanin and Rork and with equation (3.19) which defines \( a_1 \) at integral values of \( K \). According to equation (3.19) the value of \( a_1 \) varies abruptly by an amount 
\[ cU_4a_2/\epsilon K(K+1) \]
as the number of electrons in the avalanche increases by one. As \( K \) gets larger the changes in \( a_1 \) become progressively smaller.

Cookson and Lewis have compared equation (3.19) with the data obtained by Chanin and Rork shown in figure 3.20. Using the values of \( X \) given by Chanin and Rork, the corresponding values of \( \mu \) can be obtained from figure (3.19) (assuming that the relationship applies to neon and hydrogen as well as methane). Values of the mean ionization coefficient \( \alpha \) were taken from measurements made with large values of avalanche size (so that the early fluctuation effects were not present). It was possible to plot \( a_1 \) as a function of \( px \) (as opposed to \( K \)) by using the fact that the areas under the \( (a_1/p, px) \) curves in figure 3.20 are equal to \( \int a_1 dx \) (\( K \) is given by \( K = \exp \int_0^x a_1 dx \)).

From figure (3.20) one can see that there is reasonable agreement between the experimental results of Chanin and Rork and equation (3.19). Cookson and Lewis suggest that the differences between equation (3.19) and the experimental results obtained at the highest values of \( E/p \) are due to the varying electron energy distribution mentioned earlier.
Byrne (1969) later developed his model so that $\mu$ was quantified in terms of a new independent variable $s$, defined by the relation

$$s = \int_0^\chi a_i(x) \, dx \tag{3.20}$$

Physically $s$ is the number of ionization mean free paths traversed by the avalanche and is a measure of the average number of electron generations. A Polya distribution is predicted as before.

Before proceeding further it may be useful to summarize the main points of this section:

Snyder (1947) and others proposed an avalanche model in which it is assumed that the probability that an electron will ionize does not vary with the distance from the previous ionization. This model leads to a Furry distribution and is in agreement with measurements made at low values of $\chi$.

At high values of $\chi$, however, experimental distributions become more peaked and Snyder's model is no longer valid.

Legler (1961) suggested that under such conditions the probability of ionization is zero until the electron has gained an energy equal to the ionization energy. The probability then undergoes a step-function like increase. Legler found reasonable agreement between his model and experimental results.

Byrne (1962) and others suggested that $a_i$ should initially decrease with $K$ and then tend to a constant value. Such a model leads to a Polya distribution. As with Legler's model reasonable agreement exists between Byrne's model and the experimental data. The justification for assuming such a variation of $a_i$ however remains unclear.

The analysis presented in the next section allows a more quantitative consideration of these and other avalanche models.
3.5 Alkhazov's Analysis

3.5.1 Theoretical Estimation of $f$

In section 3.4 we have examined various model representations of the development of an electron avalanche. These models assume different local variations of the Townsend coefficient $\alpha$ with distance from the cathode. In the form given above, the models do not directly give explicit expressions for $f$ as a function of $X$ and so non-empirical calculations of proportional counter resolution are not possible. Alkhazov (1970), however, has developed a technique whereby explicit expressions for $f$ can be obtained for a given model relationship between $a_i$ and distance $x$. In this section we will outline Alkhazov's analysis and its application to the avalanche models.

The method employed by Alkhazov essentially involves the derivation of a general expression for the moments of the avalanche size. This may be used to obtain the relative variance $f$. The assumptions made at the start of section 3.4 are considered to apply here.

Alkhazov begins by dividing all the avalanche electrons into two groups and designating them as "s-electrons" and "f-electrons". He defines s-electrons as being "those which have been created in an ionizing collision with a small amount of kinetic energy and have not yet attained sufficient energy from the electric field to produce further ionization". Because the energy of these electrons is less than the ionization energy the term "slow" may be accurately used. However f-electrons are not necessarily "fast". In the models of Legler (1961) and Byrne (1969), for example, f-electrons are those which have come into equilibrium with the electric field but only a fraction of these have an energy above the ionization energy.
Probability Notation

Let \( p_s(l) \) be the probability distribution of the distance \( l \) towards the anode from the point of an s-electron origin to the point at which this electron is transformed into an f-electron (see figure 3.22a). In a similar way let \( p_f(l) \) be the probability distribution of the distance \( l \) towards the anode from the point of an f-electron origin, to the point at which with probability \( 1 - p_i(l) \) this electron is transformed into one s-electron, and with probability \( p_i(l) \) it is transformed into two electrons (see figure 3.22b).

Two separate forms of ionizing collision are considered. In the first case an f-electron is transformed in an ionizing collision into two s-electrons. In the second case one f-electron is transformed in an ionizing collision into one s- and one f-electron. In the models to be considered only one of these cases is allowed and we will refer to these as first and second case models respectively.

\[
\begin{align*}
\text{Non-Ionizing} & \quad \text{Ionizing} \\
\bullet \rightarrow \bullet + \text{Non-Ionizing} & \quad \bullet + \bullet \rightarrow \text{Ionizing} \\
\end{align*}
\]

Figure 3.22  Probability notation

- \( \bullet \) represents an s-electron
- \( \circ \) represents an f-electron

Let \( P_s(K,x) \) and \( P_f(K,x) \) be the probabilities that the avalanches, having developed to the point \( x \) from one s-electron and one f-electron respectively, will contain \( K \) electrons. It is assumed that all the avalanches develop independently under the influence of a steady uniform electric field.
With reference to figure 3.23 the following expression can be written down for $P_b(K, x)$:

$$P_b(K, x) = \int_{-\infty}^{\infty} \rho_b(\xi) P_f(K, x-\xi) \, d\xi \quad (3.21)$$

**Figure 3.23** The individual probabilities involved in obtaining $K$ electrons at a distance $x$ from one $s$-electron are shown. The total probability is denoted by $P_b(K, x)$.

The expression for $P_f(K, x)$ is more complicated due to the number of possible routes involved and to the allowance that has to be made for first and second case models. It is obtained by summing the individual probabilities of obtaining $K$ electrons at a distance $x$ from an $f$-electron. In general it is possible to obtain $K$ electrons from an $f$-electron in three ways:

(i) From one $s$-electron after a non-ionizing collision.

(ii) From two $s$-electrons after an ionizing collision (first case models only).

(iii) From an $s$- and an $f$-electron after an ionizing collision (second case models only).

Figure 3.24 shows the complete avalanche development scheme from one $f$-electron. From the diagram the following expression for $P_f(K, x)$ can be deduced:

$$P_f(K, x) = \rho_f(\xi) \left( \prod_{k=1}^{K-1} \frac{1-P_i(\xi)}{P_i(\xi)} \right) \cdot P_b(K, x-\xi) + \sum_{K'=1}^{K-2} P_i(\xi) P_b(K', x-\xi) \cdot P_{b,f}(K-K', x-\xi) \, d\xi \quad (3.22)$$

where $P_{b,f} \equiv P_b$ in the first case.

$P_{b,f} \equiv P_f$ in the second case.
The individual probabilities involved in obtaining K electrons at a distance x from one f-electron are shown. First case and second case models are indicated. The total probability is denoted by $P_f(K, x)$. 
The moments \( k^m_S(x) \) and \( k^m_F(x) \) are given by the following expressions:

\[
k^m_S(x) = \sum_{k=1}^{\infty} k^n P_S(k,x)
\]

\[
= \int_{-\infty}^{+\infty} \rho_S(\xi) k^n F_S(x-\xi) \, d\xi \tag{3.23a}
\]

\[
k^m_F(x) = \sum_{k=1}^{\infty} k^n P_F(k,x)
\]

\[
= \int_{-\infty}^{+\infty} \rho_F(\xi) \{[1-p_i(\xi)]\} k^n S_S(x-\xi)
+ p_i(\xi) \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \frac{k^m_S(x-\xi)}{k^m_S(x)} \, k^m_F(x-\xi) \, d\xi \tag{3.23b}
\]

where \( k^m_S, k^m_F \) = \( k^m_S \) in the first case.

\( k^m_S, k^m_F \) = \( k^m_F \) in the second case.

† We wish to evaluate the moments of an avalanche initiated by one \( s \)-electron. The mean value of \( k_S(x) \) is given simply by

\[
k_S(x) = \exp[\alpha (x-\delta)] \tag{3.24a}
\]

where \( \delta \) is the distance the electron must travel before it can be regarded as having come into equilibrium under the electric field.

Alkhazov re-expresses \( k^m_S(x) \) in terms of a new variable \( z \) where

\[
z = \frac{k}{\bar{k}}:
\]

\[
k^m_S(x) = \sum k^n P_S(k,x)
= \frac{1}{\bar{k}} k^n \sum \left( \frac{k}{\bar{k}} \right)^n P_S(k,x)
= \frac{1}{\bar{k}} k^n \sum z^n P(z/\bar{k}) \tag{3.24b}
\]

where \( P(z) = P_{S}(k,x) \).

For \( \bar{k} > 1 \) we can re-write our expression for \( k_S^m(x) \) in the form of an integral:

\[
k^m_S(x) = \frac{1}{\bar{k}} k^n \int_{0}^{\infty} z^n \cdot P(z) \, dz \tag{3.25}
\]

† Moments are briefly outlined in Appendix 1.
Alkhazov denotes the integral term by $A_n$:

$$A_n = \int_0^\infty Z^n p(z) \, dz$$

$A_n$ is the $n^{th}$ moment of $Z$ ($A_0 = A_1 = 1$).

Alkhazov quotes the following expression for $A_n$ which has been derived from equation (3.23):

$$A_n = \sum_{m=0}^{n} \frac{n!}{m!(n-m)!} \frac{A_{m} A_{n-m} J_0(n) J_2(n) J_1(n) \rho_{m}(n) \rho_{n-m}(n)}{1 - J_0(n) J_1(n)}$$

(3.26)

where

$$J_0(n) = \int_{-\infty}^{\infty} \rho_{0}(\ell) \exp(-\alpha n \ell) \, d\ell$$

$$J_1(n) = \int_{-\infty}^{\infty} [1 - P_1(\ell)] \rho_{f}(\ell) \exp(-\alpha n \ell) \, d\ell$$

$$J_2(n) = \int_{-\infty}^{\infty} P_1(\ell) \rho_{f}(\ell) \exp(-\alpha n \ell) \, d\ell$$

$B(m) = 1$ for first case models.

$B(m) = [J_0(m)]^{-1}$ for second case models.

For first case models (i.e. where ionization by an $f$-electron results in two $s$-electrons) the following simpler expression for $A_n$ results:

$$A_n = \sum_{m=1}^{n-1} \frac{n!}{m!(n-m)!} \frac{A_{m} A_{n-m} J_1(n)}{1 - 2J_1(n)}$$

(3.27)

where

$$J_1(n) = \int_{-\infty}^{+\infty} \rho_{1}(\ell) \exp(-\alpha n \ell) \, d\ell$$

$$J_1(n) = \frac{1}{2} - \int_{-\infty}^{\infty} \rho_{1}(\ell) \exp(-\alpha n \ell) \, d\ell$$

$\rho_{i}(\ell)$ is the probability distribution of the path $\ell$ at which one electron is duplicated. The relationship between $\rho_{i}(\ell)$ and $a_{i}(\ell)$ is considered in Appendix 2.

By using appropriate expressions for the probability distributions involved in equations (3.26) and (3.27) Alkhazov has evaluated $A_n$ for six different avalanche models. In particular by evaluating the second moment he has obtained explicit expressions for $f_0$ as a function of $\chi$ ($f_0$ is the value that $f$ tends to as $F$ tends to infinity - this is discussed in section 3.5.2).
A listing of the expressions for $f_0$ is provided below.

The local variation of the ionization coefficient for models 1 to 4 is shown in figure 3.25. Models 1 to 4 are first case models; models 5 and 6 are second case models.

In the following listing $U_0$ is the "model parameter". For a given model to fit as proposed by the originator, $U_0$ should take the value $U_1$.

**Model 1** This is the step function model proposed by Legler (1961) and outlined in section 3.4.2. For this model $f_0$ varies with $\chi$ in the following way:

$$f_0 = \frac{(2b-1)^2}{4b^2-1}$$

where $b = \exp(-\chi)$

**Model 2** This model was proposed by Alkhazov (1970). It is assumed that the local probability of ionization takes the form of a $\delta$-function at integral multiples of $U_0/E$. For this model a simple relationship exists between $f_0$ and $\chi$:

$$f_0 = 2\exp(-\chi) - 1$$

**Model 3** This model is of the form later proposed by Legler (1967). Some account is now taken of changes in the electron energy distribution at integral multiples of $U_0/E$. The ionization coefficient thus decreases after a distance $U_0/E$ and then increases again at $2U_0/E$. At a distance of $\approx 3U_0/E$
the coefficient stabilizes at the value proposed by
Legler's original model. For model 3 $f_o$ varies with $X$
in the following way:

$$f_o = \frac{(3 + \frac{3}{2}X - 2b) b - X - 1}{X + 1 - (1 + \frac{3}{2}X)b} \quad (3.30)$$

**Model 4**

With model 4 it is assumed that the ionization coefficient
increases smoothly with distance from the origin and
tends asymptotically to a constant value. This is the converse
to Byrne's model. The relationship between $f_o$ and $X$ is as follows:

$$f_o = \frac{1 - X + 2X^2}{1 + \frac{3}{2}X - 2X^2} \quad (3.31)$$

**Model 5**

Model 5 is a modified version of Legler's model in which
an $f$-electron is transformed in an ionizing collision
into one $s$- and one $f$-electron (it is thus a second case
version of model 1).

With model 5 $f_o$ is related to $X$ by the following simple
expression:

$$f_o = \frac{\exp(-X)}{2 - \exp(-X)} \quad (3.32)$$

**Model 6**

This is the model proposed by Byrne (1962 and 1969) and
Lansiaert and Korucci (1962) and outlined in section 3.4.3.
The local variation of the ionization coefficient was
illustrated in figure 3.20. For Byrne's model $f_o$ varies
with $X$ in the following way:

$$f_o = \frac{1 + X + 2X^2}{1 + \frac{3}{2}X} \quad (3.33)$$
The ionization probability $a_i$ as a function of distance travelled by an electron from its point of origin. Examples of four models are shown. For Model 1 $\chi = 0.105$, $f_0 = 0.645$; for Model 2 $\chi = 0.195$, $f_0 = 0.645$; for Model 3 $\chi = 0.145$, $f_0 = 0.645$; for Model 4 $\chi = 0.144$, $f_0 = 0.645$.

Note that $x_o = U_o/E$.

$\xi/x_o = (\xi s_k/U_o)$ is the number of multiples of $U_o$ that the electron has travelled through.

$x_o a_i(\xi) = a_i(\xi) U_o/E$ is the number of electrons generated by an electron travelling through $U_o$ (at a particular value of $\xi$).

*Figure 3.25*

(From Alkhazov (1970))
The variation of $f_0$ with $X$ for models 1, 3, 5. The curves were obtained using equations (3,28), (3,30) and (3,32) respectively.

Data obtained by Schlumohm (1958).


Figure 3.26a
The variation of $f_0$ with $\chi$ for models 2, 4 and 6. The curves were obtained using equations (3.29), (3.31) and (3.33) respectively.

- Data obtained by Schlumbohm (1958).
- Data obtained by Stumpel et al. (1973).

Figure 3.26b
In figures 3.26a and 3.26b $f_0$ is plotted as a function of $\chi$ for the six models. Note that with models 1, 2 and 3 $f_0$ becomes zero at certain finite values of $\chi$. With model 5 $f_0$ tends to zero as $\chi$ tends to infinity. Models 4 and 6 do not allow $f_0$ to take the value zero. With model 4 $f_0$ reaches a minimum value of 0.49 at $\chi=0.414$; the minimum value of $f_0$ with model 6 is 0.78 at $\chi=0.33$.

3.5.2 The Relationship between $f$ and $\overline{K}$

Equations (3.28) to (3.33) give $f_0$ as a function of $\chi$ for the six avalanche models. Knowing the relationship between $\chi$ and $E/p$ for a particular gas, one can therefore obtain theoretical values of $f_0$. In practice proportional counters are, of course, operated with finite values of $\overline{K}$ so we must examine the variation of $f$ with $\overline{K}$.

Alkhazov has examined the situation by considering the avalanche at two separate distances from the cathode. At $y_1$ the avalanche contains $K_1$ electrons and at $y_2$ ($y_2>y_1$) the avalanche contains $K=\sum_{j=1}^{\infty} (K_2)_j$ electrons where $(K_2)_j$ is the number of electrons formed from the $j$th electron along the path $y_2-y_1$ (figure 3.27). $\overline{K}_2$ is the average multiplication along $y_2-y_1$.

If space charge and secondary avalanche effects are not significant then $(K_2)_j$ will not depend on $K_1$. Assuming also that the various branches $(K_2)_j$ develop independently of each other then we may write

$$\overline{K} = \overline{K}_1 \cdot \overline{K}_2$$

(3.34)

$$\frac{\delta K}{K} = \frac{\delta K_1}{K_1} \cdot \frac{\delta K_2}{K_2}$$

(3.35)

setting $\overline{K}_1 = \overline{K}(y_1)$; $\overline{K}_2 = \overline{K}(y_2-y_1)$; and expressing the variances in the following way:
Schematic representation of avalanche development. At $y_1$, the avalanche contains $K_1$ electrons. At $y_2$, it contains $K_2$ electrons formed from the $j$th electron along the path $y_1 - y_2$. In the example shown, $K_1 = 3$, $K_2 = 9$, and $(K_2)_1 = (K_2)_2 = (K_2)_3 = 3$. The electron $K_1$ contains $(k_1)$ electrons, where $(k_1)$ is the number of electrons formed from the $j$th electron along the path $y_1 - y_2$.
\[
(\delta k)^2 = f(\bar{k}) \bar{k}^2 \quad (f(\bar{k}) \text{ is the relative variance of } \bar{k})
\]
\[
(\delta k_1)^2 = f(\bar{k}_1) \bar{k}_1^2
\]
\[
(\delta k_2)^2 = f(\bar{k}_2) \bar{k}_2^2
\]

then from (3.34) and (3.35) one can deduce:
\[
\begin{align*}
\bar{k}(y_2) &= \bar{k}(y_1) \cdot \bar{k}(y_2-y_1) \\
f(\bar{k}) &= f(\bar{k}_1 \cdot \bar{k}_2) \\
&= \frac{(\delta k)^2}{(k_1 k_2)^2} \\
&= \bar{k}_1 (\delta k_2)^2 + (\delta k_1)^2 \bar{k}_2^2 \\
&= \bar{k}_1^{-1} f(\bar{k}_2) + f(\bar{k}_1)
\end{align*}
\]

Alkhazov has solved equations (3.36) and (3.37) and obtained:
\[
\begin{align*}
\bar{k}(y) &= \exp(\alpha y) \\
f(\bar{k}) &= f_0 \left[ 1 - 1/\bar{k} \right]
\end{align*}
\]

where \( \alpha \) is the Townsend coefficient

- \( f_0 \) is the value that \( f \) would take if the avalanche were of infinite size.

In figure 3.28 \( f/f_0 \) is plotted as a function of \( \bar{k} \). \( (f/f_0)^{1/3} \) is also plotted as the resolution of a proportional counter is proportional to \( f^{1/3} \). It can be seen that operation with low values of gas gain is desirable. In practice, of course, the ratio of the final avalanche charge to the noise charge of the preamplifier may become so low that the overall resolution deteriorates. This will be the case at low X-ray energies. If the electron avalanche develops under a constant electric field then the theoretical values of \( f \) can be calculated directly using equations (3.28) to (3.33) together with equation (3.39).

With a cylindrical counter one must take account of the varying electric field between cathode and anode. From equations (3.38) and (3.39) one can write down the following expressions concerned with the rate of change of \( \bar{k} \) and \( f(\bar{k}) \):
\[
\begin{align*}
\frac{d}{dy} \bar{k} &= \alpha \bar{k} \\
\frac{d}{d\bar{k}} f(\bar{k}) &= \frac{1}{\bar{k}^\tau} f_0
\end{align*}
\]
$f/f_o$ as a function of $\bar{K}$ as governed by equation (3.39). $(f/f_o)^{\frac{3}{2}}$ is also plotted as a function of $\bar{K}$.  

Figure 3.28
Using these equations Alkhazov has obtained the following expression for $f$ for the case of a cylindrical counter:

$$f = \int_{r_a}^{r_c} \frac{\alpha f_0}{\exp \left( \frac{r_c}{r} \right) \alpha d r} \ dr$$

where $\alpha$ and $f_0$ vary with $r$ as determined by the $\alpha/E$ versus $E/p$ characteristic of the gas and the chosen avalanche model.

### 3.6 Avalanche Fluctuations in Penning Mixtures

Using the analysis presented in section 3.5 we can now evaluate theoretical values of $f$ corresponding to the high values of $\chi$ that can be obtained with Penning mixtures (as described in chapter 4).

Table 3.2 gives theoretical values of $f$ and resolution (at 5.89 keV) for the Penning mixture $99.9\%$ Ne + $0.1\%$ Ar at $E/p = 3\$v/cm/torr$. These are the conditions that give the highest possible value of $\chi$ (0.61) with neon-argon. Model numbers 1 to 6 are those of Alkhazov (1970). Model 7 is Snyder's model. With model 3 $f_0 = 0$ at $\chi = 0.545$ so that $f_0$ is not strictly defined at $\chi = 0.61$. It can be seen that models 1 and 2 predict very low values of resolution under these conditions.

In table 3.3 similar data is given for the Penning mixture $99.7\%$ Ar + $0.3\%$ C$_2$H$_4$ at $E/p$ of 20 $v/cm/torr$ - the conditions that give the highest value of $\chi$ (0.46). Very low values of $f$ and resolution are predicted by models 1 to 3.

The behaviour of these gas mixtures is investigated experimentally using a uniform field chamber described in chapter 7. The results obtained from the chamber are presented and discussed in chapter 9. The use of Penning mixtures in a cylindrical counter is examined in chapter 6.
Theoretical values of $f$ and resolution (with an X-ray energy of 5.89keV) predicted by various avalanche models with the Penning mixture 99.9%Ne + 0.1%Ar at $E/p = 3$ V/cm/torr, i.e. where $\chi = 0.61$. $K$ has been set at 100. Model 7 is Snyder's model.

**Table 3.2**

<table>
<thead>
<tr>
<th>Model No</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{th}$</td>
<td>0.09</td>
<td>0.26</td>
<td>0.23</td>
<td>0.49</td>
<td>0.46</td>
<td>0.79</td>
<td>1</td>
</tr>
<tr>
<td>$R_{th}$%</td>
<td>5.7</td>
<td>8.0</td>
<td>7.7</td>
<td>10.4</td>
<td>10.1</td>
<td>12.9</td>
<td>14.4</td>
</tr>
</tbody>
</table>

Theoretical values of $f$ and resolution (with an X-ray energy of 5.89keV) predicted by various avalanche models with the Penning mixture 99.7%Ar + 0.3%$\text{C}_2\text{H}_2$ at $E/p = 20$ V/cm/torr, i.e. where $\chi = 0.46$. $K$ has been set at 100.

**Table 3.3**
3.7 Overall Discussion

In this section the various models are examined and compared. The modelling of ionization in Penning mixtures is also considered.

The first point to note is that the validity of a given model may vary with the value of $\chi$ and the distance into the gas, i.e. as the electron energy distribution varies.

For low values of $\chi$ (i.e. $\frac{aU_i}{E} << 1$) Snyder's model seems suitable. Under these conditions the mean distance travelled by an electron in producing an ionization, $\frac{1}{\alpha}$, is considerably greater than $U_i/E$, the minimum distance over which an electron can gain an energy equal to the ionization potential. The instantaneous probability of ionization does not therefore vary with distance from the previous ionization. The resulting Furry distribution is in good agreement with the experimental data.

At high values of $\chi$ ($0.1 < \chi < 1$) however the situation is more complicated. The mean ionization distance is now comparable to $U_i/E$. The local ionization coefficient will now vary with distance from the previous ionization. Calculation of the relative variance of the avalanche size requires an accurate knowledge of the form of this variation.

Model 2 would seem to be applicable to situations where the ionization efficiency is very high ($aU_i/E = 1$). Ionization is only allowed at exact multiples of $U_i/E$. This will clearly not be the case at more moderate values of $\chi$.

The step function form of Legler's model (models 1, 3, 5) would appear to be an oversimplification of the ionization process. Whilst an electron must clearly attain the energy $U_i$ before ionization is possible, it is not apparent why the ionization coefficient should then immediately become constant as in Snyder's model. Model 3 does not seem to fully allow for changes in the electron energy distribution.
Model 6 (Byrne's model) was discussed in some detail in section 3.4.3. Using equation 3.33 (plotted in figure 3.26) however, we can now examine the model further. According to model 6, $f_0$ can never be less than 0.78. Sipila (1978) however has obtained an $f$ value of 0.46 with a cylindrical counter filled with neon-argon and operated at a gas gain of 50 (where $f = f_0$). Another difficulty with model 6 is the tendency to larger values of $f_0$ at $\chi$ greater than 0.33. As the ionization efficiency increases one would expect $f_0$ to continue to fall.

The same problems apply to model 4 which allows $f_0$ to be as low as 0.49. Models 4 and 6 also suffer from the difficulty of allowing ionization to take place even when the electron energy is less than $U_1$.

To examine the validity of the various models one can attempt to make comparisons between theoretical and experimental forms of both $f$ and $a_1$. In figure 3.26 data obtained by Schlumbohm (1958) and Stumpel et al (1973) is shown. For all the models, $f_0$ takes the value unity when $\chi = 0$. Data obtained at low values of $\chi$ is thus not very helpful as an indication of the validity of a particular model as all models behave in a similar fashion in this region. The results obtained by Schlumbohm in the region $0.1 \leq \chi \leq 0.25$ do not seem to closely follow any particular model. No data is available at values of $\chi$ that can be obtained with Penning mixtures.

The oscillatory behaviour of $a_1$ suggested by the measurements of Chanin and Rork supports a form of model 2 in which the delta functions are replaced by broader curves of the form measured. We will not consider the decrease of $a_1/p$ measured by Chanin and Rork as this may have been due to problems with the electrode system (Haydon and Stock (1966)).
To obtain a reasonable estimate of the value of $f_0$ likely to be observed with the Penning mixture neon-argon, at conditions of high ionization efficiency, one can make use of experimental measurements of ionization probability. Kruihof and Druyvesteyn (1937) have measured the variation of photoelectric current with voltage in a parallel plate chamber filled with neon-argon mixtures (this work is discussed more fully in chapter 4 - see figure 4.9 in section 4.2.1). By measuring the rate of change of appropriate curves Kruihof and Druyvesteyn have derived the probability functions shown in figure 4.8. An electron released from the cathode with zero velocity, having travelled through a potential difference $V$ will excite (with probability $f(V)dV$) or ionize (with probability $g(V)dV$) a neon atom between $V$ and $V+dV$.

From figure 4.8 it can be seen that the probability of neon excitation is very high. The area under the $f(V)$ curve is 0.98 (the area under the $g(V)$ curve is 0.02). With an $E/p$ of 3 V/cm/torr and a mixture of 99.9% Ne + 0.1% Ar the probability that an excited neon atom ionizes an argon atom is about unity per collision (Alkhazov (1967)).

The $f(V)$ function was taken to represent $\rho_e(\lambda)$ for the purpose of evaluating $J_1(n)$ for equation (3.27). The horizontal axis can easily be changed to distance by dividing by the electric field strength $E$ ($\lambda=V/E$).

To calculate $f_0$ we set $n=2$ in equation (3.27) ($f_0=A-1$). $J_1(2)$ was evaluated numerically by applying the trapezium rule with $\frac{1}{2}$ volt intervals.

Using the above technique $J_1(2)$ was calculated to be $0.29 \pm 0.01$ under the conditions specified above for neon-argon. The error limits given are those which arise from the use of the trapezium rule (Jeffrey (1969)). Substituting the value of $J_1(2)$ into equation (3.27) we obtain $A_2 = 1.38 \pm 0.12$ and thus $f_0 = 0.38 \pm 0.12$. This corresponds to a resolution of $10.2\% \pm 1.3\%$ at 5.89 KeV (it is assumed that $K$ is large enough so that $f = f_0$).
Note that disruptive effects (due, for example, to impurities) will tend to degrade the resolution actually measured in an experimental system. These values can therefore be regarded as lower limits.

It is interesting to derive from figure 4.8 the variation of the ionization coefficient $a_i$ with distance ($\xi$) travelled by an electron from its point of origin. Alkhazov (1970) has given the following equation relating $a_i(\xi)$ to $\rho_i(\xi)$:

$$a_i(\xi) = \rho_i(\xi) \left[ 1 - \int_{-\infty}^{\xi} \rho_i(\lambda) \, d\lambda \right]^{-1} \tag{3.43}$$

Equation (3.43) was used to evaluate $a_i(\xi)$ from figure 4.8. The calculation was performed numerically using the trapezium rule. The resulting function is shown in figure 3.29. The axes are labelled as in figure 3.25 to allow direct comparison.

The function shown in figure 3.29 is like a modified form of model 2 in which the distribution is considerably broader. It is also similar in form to the results obtained by Chanin and Rork (1963) for neon and hydrogen.

† This expression is derived in Appendix 2.
The variation of $a_i$ with $\xi$. The function was derived from figure 4.8 using equation (3.43). The figure applies to the Penning mixture 99.9% neon + 0.1% argon at $E/p = 3$ v/cm/torr.

Figure 3.29
CHAPTER 4 : THE PENNING EFFECT

4.1 Introduction

4.2 Neon Based Penning Mixtures
   4.2.1 Neon-Argon
   4.2.2 Other Neon based Penning mixtures

4.3 Argon Based Penning Mixtures
   4.3.1 Argon-Hydrocarbon Penning mixtures
   4.3.2 Argon-rare gas mixtures: the non-metastable Penning effect
4 The Penning Effect

4.1 Introduction

In the preceding chapters the reduction of ionization and avalanche fluctuations via the Penning effect has been outlined. In this chapter studies of the behaviour of Penning mixtures are described. Particular attention is paid to the gas mixtures used in this study but some other Penning mixtures are considered briefly. The behaviour of the gases at the mixture ratios and E/p values which give highest ionization efficiency is of greatest interest.

Before considering particular gas mixtures, we will review, in a more general way, the processes involved in the Penning effect. In both the primary ionization and the subsequent avalanche, noble gas atoms are excited to long lived metastable states. If a second gas is present, then it may be ionized, if its ionization energy is less than the metastable level of the main gas. The process can take place via collision in the following way:

\[ A^* + B \rightarrow A + B^+ + e^- \]  \hspace{3cm} (4.1)

where \( A^* \) is a main gas atom in a metastable state

\( B \) is a foreign gas atom

The process can also be photon mediated i.e. the metastable main gas atom de-excites with the emission of a photon which may ionize a foreign gas atom:

\[ A^* \rightarrow A + \nu \nu \]  \hspace{3cm} (4.2)

\[ \nu \nu + B \rightarrow B^+ + e^- \]

In the pressure range in which proportional counters are normally operated (of the order of an atmosphere) reaction (4.1) is the more dominant, Koch (1959). The long lifetime of the metastable states
implies that a collision will take place before de-excitation. In addition the atoms of the main gas will tend to absorb any photons emitted.

If the admixture gas is polyatomic, then as well as reaction (4.1), reactions involving the dissociation of the admixture molecule can take place. In some cases the molecule dissociates into neutral components and so the increase in ionization is not so strong.

The variation in efficiency of ionization with admixture content is discussed later in the chapter.

The probability of the Penning effect taking place has been considered by Massey (1971) for the case of neon-argon:

\[
\text{Ne}^* + \text{Ar} \rightarrow \text{Ne} + \text{Ar}^+ + e^- \quad (4.3)
\]

By studying the variation of \( \alpha/E \) with argon concentration one can obtain values for the probability that an excited neon atom passes into a metastable state and the ratio of the probability of destruction of a metastable neon atom by collision with, respectively, a neon atom and an argon atom.

According to Massey the latter ratio is \( 2.4 \times 10^{-5} \) and the probability of destruction in a neon collision is of the order of \( 10^{-5} \).

This means that the probability of reaction (4.3) taking place is

\[
\approx 10^{-5} \times \frac{1}{2.4 \times 10^{-5}} \quad \text{which is of the order of unity per collision.}
\]

The high probability of ionization via the Penning effect in neon-argon is to be expected when one considers the similarity between the metastable level of neon (16.53 eV) and the ionization energy of argon (15.76 eV). The reaction can be regarded as being one of exact resonance as far as the relative translational energy of the atoms is
concerned. The small excess energy (0.77 eV) is taken up by the electron produced in the ionization.

Table 4.1 gives a summary of data most relevant to this study. A number of neon and argon based Penning mixtures are listed and a comparison is made with pure neon and pure argon. The admixture percentage and E/p value required to give the highest value of α/E are indicated. The corresponding maximum value of χ is also shown. For the pure gases αU₁/E is given, whilst for the Penning mixtures αU₁/E is given. This has been done because, in a Penning mixture, it is the excitation of the main gas which leads to ionization.

The quantity ΔE = U₁ - U₂ is shown because the difference between the levels is an indication of closeness to resonance in the system.

Column 9 of the table shows the potential difference Vₙ through which an electron must pass in order to produce a new ion pair — under conditions of maximum α/E, and is given by the following equation:

\[ \exp \left( \frac{(α/E)_{\text{max}} \times Vₙ}{2} \right) = 2 \]

So \[ Vₙ = \frac{\ln 2 \ln E}{(α/E)_{\text{max}}} \]  \hspace{1cm} (4.4)

The ratio of the ionization potential of the added gas U₁, to the potential difference Vₙ is a measure of the efficiency of charge multiplication in the avalanche. This quantity is shown in the final column.

It is interesting to compare the behaviour of the pure gases and the Penning mixtures. It can be seen from the table that, for a pure gas, a much higher value of E/p is required to achieve the maximum value of α/E. The highest value of α/E obtained with a pure gas, though, is lower than with a Penning mixture.
<table>
<thead>
<tr>
<th>Main gas</th>
<th>Admixture gas (% for max α/E)</th>
<th>Main gas U₁ (eV)</th>
<th>Admixture gas U₁ (eV)</th>
<th>U₁ - U₁</th>
<th>ΔE (eV)</th>
<th>E/p for maximum α/E</th>
<th>(α/E) max</th>
<th>X max</th>
<th>Vₙ = ln₂ (α/E) (volts)</th>
<th>U₁ x 100% VN</th>
<th>Data Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ne</td>
<td>0.1% Ar</td>
<td>16.54</td>
<td>15.76</td>
<td>0.78</td>
<td>3</td>
<td>0.037 (V⁻¹)</td>
<td>0.61</td>
<td>18.7</td>
<td>84%</td>
<td></td>
<td>Kruijthof and Penning (1937)</td>
</tr>
<tr>
<td>Ne</td>
<td>0.01% Xe</td>
<td>16.54</td>
<td>12.08</td>
<td>4.46</td>
<td>2.5</td>
<td>0.04</td>
<td>0.66</td>
<td>17.3</td>
<td>70</td>
<td></td>
<td>Bhattacharya (1976)</td>
</tr>
<tr>
<td>Ne</td>
<td>Krypton</td>
<td>16.54</td>
<td>13.94</td>
<td>2.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>0.1% H₂</td>
<td>16.54</td>
<td>15.45</td>
<td>1.1</td>
<td>3</td>
<td>0.0285</td>
<td>0.47</td>
<td>24.3</td>
<td>63.6</td>
<td></td>
<td>Chanin and Rork (1964)</td>
</tr>
<tr>
<td>Pure Ne</td>
<td></td>
<td>16.54</td>
<td>21.56</td>
<td>na</td>
<td>70</td>
<td>0.015</td>
<td>0.32</td>
<td>46.2</td>
<td>46.7</td>
<td></td>
<td>Kruijthof and Penning (1937)</td>
</tr>
<tr>
<td>Ar</td>
<td>0.3% C₂H₂</td>
<td>11.49</td>
<td>11.45</td>
<td>0.04</td>
<td>20</td>
<td>0.04</td>
<td>0.46</td>
<td>17.3</td>
<td>66</td>
<td></td>
<td>Heylen (1970)</td>
</tr>
<tr>
<td>Ar</td>
<td>2% Xe</td>
<td>11.49</td>
<td>12.08</td>
<td>na</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pure Ar</td>
<td></td>
<td>11.49</td>
<td>15.76</td>
<td>na</td>
<td>200</td>
<td>0.022</td>
<td>0.35</td>
<td>31.5</td>
<td>50.0</td>
<td></td>
<td>Kruijthof and Penning (1937)</td>
</tr>
</tbody>
</table>

Table 4.1 Comparison of data relating to the behaviour of pure gases and Penning mixtures at conditions of highest ionization efficiency. The Table is explained more fully in the text.

U₁ is first ionization energy. U₁ is first metastable level.
Energy level diagram for neon.

*Figure 4.1* (From Massey *(1971)*
4.2 Neon based Penning mixtures

4.2.1 Neon-Argon

Neon-argon is commonly regarded as being the "classic" Penning mixture. This is due to the very high ionization efficiency obtained and also because it was the mixture used by Penning in the 1930's in the first quantitative analysis of the effect.

Penning and co-workers measured Townsend's first and second ionization coefficients for a variety of mixture ratios and E/p values. The measurements were made in a parallel plate discharge tube. A u.v. light source was used with a photo-electric cathode to provide an electron supply. The distance between the anode and cathode could be varied so that measurements could be made for a variety of voltages but with a fixed value of E/p. We will consider aspects of Penning's work most relevant to this study.

In Figure 4.2 the photo-electric current gain obtained as a function of voltage at a fixed value of E/p of 3.36 v/cm/torr is shown. The mixture of Ne + 0.126% Ar clearly gives the highest gain at this value of E/p. The "step-like" increase in the current is also most enhanced for this mixture. Kruithof and Penning discuss the reasons for the step-like increase. The electrons leave the cathode with small velocities, and after traversing approximately the same potential difference, lose the energy $eU_1$ needed for the excitation of the main gas (Figure 4.1). They are then accelerated once more and the process is repeated. In the absence of other energy losses (e.g. in-elastic collisions, excitation of Ar-atoms, direct ionization of Neon or Argon) the fluctuations in $\alpha/E$ die out rather slowly. This decaying effect, observed at high values of gas gain, was discussed in Chapter 3.
Photoelectric current \( i \) in mixtures of Ne and Ar as a function of the anode potential \( V_A \).

**Figure 4.2** From Kruithof and Penning (1937)

Example for the calculation of \( \eta \).

\[ \eta = \text{measured value of photoelectric current} \]
\[ \eta_r = \text{reduced value of photoelectric current} \] (action of positive ions eliminated)
\[ \eta_i = \text{line, corresponding to } \eta_i = \frac{\eta}{(1 + \beta)} \]

**Figure 4.3** From Kruithof and Penning (1937)
The step-like growth of current is very marked for mixtures containing \(10^{-3}\) to 0.1% of argon. Over this mixture range the excitation of neon results (with very high probability) in the ionization of argon. The excitation of argon and the direct ionization of neon and argon are negligible. For the Ne + 0.126% Ar curve, Kruithof and Penning found that the current first started to increase at an anode potential of about 19V. The voltage difference between the minima of the gain curve was found to be about 17.3 V. These values can be compared with the first metastable level of neon of 16.54 eV.

Figure 4.3 shows how Kruithof and Penning derived the quantity \(\eta = \alpha/E\) from the photo-electric current. \(V_\delta\) is the value at which a line, passing through the reduced photo-electric current data points, intercepts the voltage axis. (The reduction is to allow for the action of positive ions.) Physically \(V_\delta\) is the voltage that the electrons have to pass through before they can be considered to have reached equilibrium under the action of the electric field (i.e. are no longer affected by the positive ions). It is typically about 20 V.

The mean value of the gas gain, \(\overline{K}\), at a distance \(x\), or a voltage \(V\) into the gas, is given by the following expression (it is assumed that the electric field is uniform):

\[
\overline{K} = \exp \alpha(x - \delta) \quad (4.5a)
\]

\[
= \exp \alpha/E (V - V_\delta) \quad (4.5b)
\]

where \(\delta\) is distance into the gas at which the voltage is equal to \(V_\delta\).

Allowing for secondary avalanches the gas gain is given by a modified form of equation (3.11):

\[
\overline{K} = \frac{\exp \alpha(x - \delta)}{1 - \gamma (\exp \alpha(x - \delta) - 1)} \quad (4.6a)
\]

(From Kruithof and Penning (1937))

\[
= \frac{\exp(\alpha/E(V - V_\delta))}{1 - \gamma (\exp(\alpha/E(V - V_\delta)) - 1)} \quad (4.6b)
\]

(From Kruithof and Penning((1937)))

Figure 4.4 shows \(\alpha/E\) against \(E/p\) for a range of mixture ratios. The maximum value of \(\alpha/E\) achieved is 0.037 (\(V^{-1}\)) at an \(E/p\) of 3 v/cm/torr with an argon percentage of 0.1% (see Table 4.1).

In setting up a practical Townsend discharge region (for example in a uniform field proportional counter) it is important to know the breakdown characteristics of the gas. Figure 4.5 shows a set of Paschen
The ionization coefficient $\gamma$ as a function of $E/p_0$. $100a = \text{argon percentage}$. 

**Figure 4.4** From Kruithof and Penning (1937)

Paschen curves for breakdown between flat iron plates in various gases and gas mixtures; the breakdown voltage $V_b$ is plotted against $p_0d$ (gas concentration $\times$ distance between electrodes) on a double logarithmic scale. The similarity between these curves and those of fig. 13 can be brought out by investigating $V_b$ for different gases at the same value of $E/p_0$; the lines for constant $E/p_0$ (shown broken) lie at an angle of 45° to the axes of the graph.

**Figure 4.5** From Penning (1957)
curves (breakdown voltage plotted against the product of gas pressure and gap between parallel plates) obtained by Penning (1957) for some neon-argon mixtures. Nasser (1971) considered the underlying reasons for the variation in efficiency of the Penning effect with argon concentration. At voltages less than 21V (the ionization potential of neon) the ionization results from neon metastables ionizing the argon atoms by collision. When enough argon atoms are available for such ionization, the highest electron yield is observed. With greater concentration of argon atoms, inelastic collisions between the electrons and argon atoms will result - a process which is quite improbable at small argon concentrations. With a high argon concentration, argon atoms will be excited ($U_i(Ar) = 11.5$ eV) by inelastic collisions with electrons which, in turn, would have ionized some argon atoms. The electrons colliding inelastically with argon atoms will not produce any further electrons since there is no other gas in the mixture which can be ionized at this low potential. Thus the electrons which collide with the argon atoms do not contribute to the ionization and so the value of $a/E$ obtained is reduced. In Figure 4.6 the above principles are illustrated. The relatively rare photon mediated Penning effect is shown but the above considerations apply in the same way. To give an impression of how the Penning effect alters the proportion of energy lost by electrons to various processes, some curves derived by Druyvesteyn and Penning (1940) are presented in Figure 4.7. Because of some original data being uncertain Loeb (1955) has questioned the accuracy of these curves. They are, however, of qualitative interest.

Kruithof and Druyvesteyn (1937) have derived (from discharge current vs voltage characteristics (Figure 4.9) the probability that an electron (assumed to leave the cathode with zero velocity) will excite $(f(V))$ or ionize $(g(V))$ a neon atom - see Figure 4.8. Clearly the former is much more likely.
The Penning effect at different concentrations of argon in neon: (a) very small admixture—only few photons from Ne metastables find A-atoms to ionize (some increase in the electron number); (b) optimum amount admixture—all photons from Ne metastables find A-atoms to ionize and produce a maximum number of additional electrons; (c) large admixture—total number of Ne metastables decreases because of electron collisions with A-atoms. Number of photoionizations of A decreases, lowering the total number of electrons.

Figure 4.6 (From Nasser (1970))

Approximate computed curves showing the percentage of electron energy at any given $E/p$ for A, N$_2$, and H$_2$ gases, going to various actions. $E_I =$ loss to elastic impacts, $EE =$ excitation of electronic levels, leading to light emission and metastable states, $I =$ ionization by direct impact. $S$ represents the average kinetic energy of the electrons, viz., their "temperature." $EV =$ energy going to excitation of vibrational levels. The fraction of energy is given in per cent. $S$ comes from Townsend's studies, and is reasonably good. The other computed values can be off by as much as a factor of 2.

Figure 4.7 (From Loeb (1955))
The functions $f(V)$ and $g(V)$ for $E/p = 3.36 \text{ V/cm} \times \text{mm}$, calculated from the experimental curves I and VII Fig. 4.8.

**Figure 4.8** From Kruithof and Druyvesteyn (1937)

The photoelectric current $n$ as a function of the anode potential:

- I for pure Neon.
- II for Ne + $0.82 \times 10^{-7}$ % Ar.
- III for Ne + $2.93 \times 10^{-7}$ % Ar.
- IV for Ne + $9.6 \times 10^{-7}$ % Ar.
- V for Ne + $28.1 \times 10^{-7}$ % Ar.
- VI for Ne + $97 \times 10^{-7}$ % Ar.
- VII for Ne + 1 % Ar.

**Figure 4.9** From Kruithof and Druyvesteyn (1937)
4.2.2 Other Neon based Penning mixtures

Since the work of Penning and colleagues in the late 1930's some measurements have been made of Townsend's first ionization coefficient in other neon based Penning mixtures. Chanin and Rork (1964) have published the variation of $\alpha/E$ with $E/p$ for various mixtures of Ne-H$_2$. Figure 4.10 and Table 4.1 provide a summary of their results. The maximum value of $\alpha/E$ obtained is rather lower than for Ne-Ar.

The Penning effect is also possible in the mixtures Ne-Xe and Ne-Kr. Townsend coefficient measurements, however, have only been reported for the former, by Bhattacharya (1976). The method used was rather more direct than that of Penning. Instead of measuring the current flowing across a parallel plate gap, the radiation emanating from the discharge (as a result of excited atoms reverting to a lower excited state) was measured, with a photomultiplier tube, as a function of distance from cathode to anode. It was assumed that the radiation flux emanating from an elementary slab of the discharge is proportional to the local electron density. Thus it was possible to measure $\alpha$ directly. Values of $\alpha/E$ obtained by Bhattacharya are given in Figure 4.11 and Table 4.1. The maximum value of $\alpha/E$ obtained is slightly higher than for Ne-Ar but the $E/p$ value needed to achieve it is lower. Paschen curves for Ne-Xe are given in Figure 4.12.
The variation of $\eta$ with $E/p_0$ in neon-hydrogen mixtures.

**Figure 4.10** From Channin and Rork (1964)

Ionization efficiency function $\eta$ vs $E/p_0$ for neon containing 0.01 and 0.1% of xenon and argon.

**Figure 4.11** (From Bhatacharya (1976))

Breakdown potential $V_b$ vs $p_d$ for Ne, Xe, and Ne-Xe mixtures.

**Figure 4.12** (From Bhatacharya (1976))
4.3 Argon based Penning mixtures

4.3.1 Argon-hydrocarbon Penning mixtures

The Penning effect can take place in a variety of argon-hydrocarbon mixtures - Table 4.2. The first measurements of Townsend coefficients in such mixtures were made by Yamane (1960). Unfortunately the E/p range was rather limited and the measurements were carried out with a non-uniform electric field. Yamane did find, however, that the ionization was most enhanced with the mixture argon-acetylene. Heylen (1968a,b and 1970) later made similar studies but over a wider E/p range and with a uniform electric field. As mentioned elsewhere in this thesis an interesting aspect of Penning mixtures with a hydrocarbon additive gas is the quenching effect which may result. Heylen's work throws some light on this.

Heylen, like Yamane, found argon-acetylene to be the most efficient argon-hydrocarbon Penning mixture. Table 4.1 and Figure 4.13 show the primary ionization coefficient data obtained by Heylen for such a composition. The highest value of \( \alpha / E \) obtained is about \( 4 \times 10^{-2} \) electrons/volt with 99.7% Ar + 0.3% \( C_2H_2 \) at an E/p of about 30 v/cm/torr. Note that the value of \( \frac{\alpha U_1}{E} \) obtained is less than with Ne-Ar under the optimum conditions because \( U_1 \) for argon is only 11.55 eV. The value of E/p required is about seven times higher than with Ne-Ar but the breakdown voltages do not increase in as great a proportion. Figures 4.14 and 4.15 show respectively the secondary ionization coefficient and sparking voltage characteristics obtained by Heylen. The sparking voltages have only increased by a factor of about two. As we shall see later it is thus more difficult to set up a practical uniform field avalanche region with Ar-\( C_2H_2 \) at the condition of optimum ionization efficiency.
Metastable and ionization potential and molecular masses of constituent gases

<table>
<thead>
<tr>
<th>Gas</th>
<th>Ionization potential $U_i$ (eV)</th>
<th>Molecular mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH4</td>
<td>13-0</td>
<td>18</td>
</tr>
<tr>
<td>C2H6</td>
<td>11-75</td>
<td>34</td>
</tr>
<tr>
<td>C3H8</td>
<td>11-20</td>
<td>50</td>
</tr>
<tr>
<td>C5H12</td>
<td>10-55</td>
<td>82</td>
</tr>
<tr>
<td>C2H4</td>
<td>10-60</td>
<td>32</td>
</tr>
<tr>
<td>C2H2</td>
<td>11-45</td>
<td>30</td>
</tr>
</tbody>
</table>

Argon: $U_i = 11-55$ eV; Mass = 40

Energies of metastable levels and ionization potential of the inert gases in eV.

**Key:**
- Methane - CH$_4$
- Ethane - C$_2$H$_6$
- Propane - C$_3$H$_8$
- Pentane - C$_5$H$_{12}$
- Ethylene - C$_2$H$_4$
- Acetylene - C$_2$H$_2$

Table 4.2 showing how the Penning effect can occur with a variety of Argon-Hydrocarbon mixtures. From Heylen (1970)

![Graph showing the primary ionization coefficient $\eta$ as a function of the reduced field $E/p_0$.](image)

$\eta$ as a function of the reduced field $E/p_0$. 

Primary ionization coefficient $\eta$ as a function of the reduced field $E/p_0$. 

Figure 4.13 From Heylen (1970)
Secondary ionization coefficient $\omega/a$ as a function of the reduced field $E/p_0$. Chain line: 100% $C_2H_2$; $\cdots$ - 100% Ar; broken lines: ethylene, ethane; $\bigcirc$ mixtures of Ar with 3% $C_2H_2$; $\times$ 0.3% $C_2H_2$; $\bigtriangleup$ 0.003% $C_2H_2$ from three-point method; $\bigcirc$ mixtures of Ar with 3% $C_2H_2$; * 0.3% $C_2H_2$; $\bigtriangledown$ 0.003% from sparking criterion exp (ads).

Figure 4.14  From Heylen (1970)

Sparking voltage as a function of the product of pressure at 0°C and gap width for $C_2H_2$, Ar and Ar mixed with various percentages of $C_2H_2$ as indicated.

Figure 4.15  From Heylen (1970)
4.3.2 Argon-rare gas mixtures: the non-metastable Penning effect

Jesse and Sadauskis (1952) found that the ionization yield by charged particles in helium was increased markedly by the addition of a small amount (about 0.01%) of any gas other than neon. They showed that the effect is interpreted well in terms of the Penning process:

$$\text{He}^m + Y \rightarrow \text{He} + Y^+ + e^- \quad (4.7)$$

where $\text{He}^m$ is a metastable helium atom

$Y$ is an added gas atom whose ionization potential is lower than the excitation energy of a metastable helium atom.

Jesse and Sadauskis considered that the life-time of a non-metastable excited state was so short compared with the mean time between collisions with the $Y$ atoms that the Penning ionization by a non-metastable helium atom was unlikely to occur, though the process was energetically possible.

Melton, Hurst and Bortner (1954), however, found that the addition of various gases to argon resulted in increased ionization yield even when the ionization potential $U_i$ of the added gas was higher than the excitation energy of the metastable states of argon. It was found that several percent of the added gas was needed to make the effect measurable - significantly more than for gas mixtures which meet the previously established conditions for the Penning effect. Hurst, Bortner and Glick (1965) suggested that the observed increase in ionization yield was due to argon atoms in non-metastable states higher than $U_i$ and that these states persist long enough to allow "non-metastable Penning ionization" to occur.

Considering rare gas mixtures, Yamane (1960), found that the first Townsend coefficient of argon could be increased by the addition of small amounts of xenon or krypton (Figure 4.16). Kubota (1970) found
that $\alpha$-particle ionization was enhanced in such mixtures (Figure 4.17) and also in the mixtures helium-neon and krypton-xenon. As Table 4.3 shows, metastable Penning ionization is not possible for such mixtures.

<table>
<thead>
<tr>
<th>Element</th>
<th>$^3P_2$</th>
<th>$^3P_0$</th>
<th>$^1S_0$</th>
<th>$^3S_1$</th>
<th>Ionization potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>He</td>
<td>20.96</td>
<td>20.96</td>
<td>20.62</td>
<td>19.81</td>
<td>24.58</td>
</tr>
<tr>
<td>Ne</td>
<td>16.62</td>
<td>16.72</td>
<td></td>
<td></td>
<td>21.56</td>
</tr>
<tr>
<td>A</td>
<td>11.53</td>
<td>11.72</td>
<td></td>
<td></td>
<td>15.76</td>
</tr>
<tr>
<td>Kr</td>
<td>9.82</td>
<td>10.51</td>
<td></td>
<td></td>
<td>14.00</td>
</tr>
<tr>
<td>Xe</td>
<td>8.32</td>
<td>9.41</td>
<td></td>
<td></td>
<td>12.13</td>
</tr>
</tbody>
</table>

Table 4.3 From Kubota (1970)

Note that the use of krypton in proportional counters has been made inconvenient by the nuclear tests that have been conducted in recent years. The tests have led to the formation of radio-active isotopes of the gas which is extracted from the atmosphere. Background problems thus arise.
Gas multiplication $i_0$ in argon-krypton and argon-xenon mixtures at $V=700$ as a function of the concentration of krypton and xenon.

**Figure 4.16** From Yamane (1960)

Ionization yields for argon-krypton and argon-xenon mixtures. Straight lines are the ionization yields $I_i(X, Y)$ to be expected from the primary process.

**Figure 4.17** From Kubota (1970)

$$z = \frac{1}{1 + (S_y P_y/ S_{Ar} P_{Ar})}$$

where: $S_{Ar}$ and $P_{Ar}$ are the stopping power and partial pressure of argon.

$S_y$ and $P_y$ are the stopping power and partial pressure of krypton or xenon as appropriate.
CHAPTER 5: EXPERIMENTAL TECHNIQUES

5.1 Introduction

5.2 Gas and Vacuum Techniques
   5.2.1 Gas filling system
   5.2.2 Leak detection
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5.3 Electronic Techniques
   5.3.1 Measurement of gas gain
   5.3.2 Reduction of pre-amplifier noise
   5.3.3 Description of pre-amplifier used in this study
   5.3.4 Description of complete electronic system
5 Experimental Techniques

5.1 Introduction

In this chapter the main experimental techniques involved in the study are outlined. In any high purity gas physics work there is a clear requirement for the use of such procedures as leak detection, gas purification, evacuation and filling. This was particularly the case in this study as various modifications to the research chamber had to be made - these involved opening the device up to air and hence subsequent leak testing and baking were required. The various procedures involved are briefly described here.

In addition, some of the electronic techniques are presented. One of the main parameters of interest in these measurements is the charge multiplication of the avalanche - the gas gain. The usual methods of measuring gas gain did not seem appropriate here so a pulse simulation technique was developed - this is described. The low values of gas gain allowed with Penning mixtures require the use of low noise analysis electronics. The main noise source is usually the preamplifier. The principles involved in the reduction of preamplifier noise are considered. The preamplifier used in the measurements is described. Finally, a brief description is provided of the complete electronic system.
5.2 Gas and Vacuum Techniques

5.2.1 Gas Filling System

A block diagram of the gas filling system is shown in Figure 5.1. The rotary pump was used to bring the system pressure down to about 0.1 torr and to provide a suitable backing pressure for the diffusion pump. The diffusion pump was used down to about $10^{-6}$ torr, at which point the ion pump was operated. Electrical heaters were provided to increase the rate of outgassing from both the chamber being evacuated and the system itself. It was found that with a fully outgassed and leak free system, the pressure, as measured at the ion pump, would fall to about $10^{-9}$ torr. The pressure at the ion pump is inferred from the current drawn across the pump plates.

The fairly small cylindrical counter, described in Chapter 6, could be pumped and baked out (if necessary) over a couple of days. The comparatively large research chamber, described in Chapter 7, needed to be baked at about 100° C, for about two weeks (if it had been opened up to air) to achieve a final pressure approaching $10^{-9}$ torr.

The titanium sublimation pump is discussed in Section 5.2.3.

The materials used in the filling system and detectors were chosen with high gas purity in mind. To reduce outgassing, metals were used wherever possible. The gas purity requirements of the research chamber are described in Chapter 7.

5.2.2 Leak Detection

The main method employed for leak detection involved the use of a mass spectrometer with a separate pumping system. The chamber under test was connected to the pumping system and mass spectrometer. The whole arrangement was evacuated to a pressure low enough to allow
Figure 5.1

BLOCK DIAGRAM OF FILLING SYSTEM

- Detector
- Heaters
- Pirani Gauge
- Absolute Pressure Gauge
- Titanium Sublimation Pump
- Diffusion Pump + Penning Gauge
- Ion Pump + Gauge
- Pirani Gauge
- Rotary Pump
- 1000 PSI Bottle Gauge
- Gas
- Bottles
the operation of the mass spectrometer filament. With the spectrometer operating, the probe gas (helium in this case) was sprayed around the outside of the chamber under test. The helium would then enter the chamber through any leak and subsequently be detected by the mass spectrometer (tuned to helium). The system was first calibrated by means of a standard leak. With the system used it was possible to measure leaks as small as about $10^{-8}$ torr litres/sec.

5.2.3 Gas Purification

The gases used in the measurements were of British Oxygen Company "Research Grade" type. The total impurity level of such gas is about 10 vppm. Table 5.1 gives a breakdown of the impurity levels of various gases in neon and argon. These are the stated impurity levels of the gas before it is fed to the cylinders. Account is not taken, therefore, of cylinder contamination, valve and seal leaks etc.

In the measurements with the research chamber the gas purity was one of the experimental variables involved. In this section we will briefly consider the principles involved in the techniques used to purify the gas.

Titanium Sublimation Pump

In this device a material able to chemically absorb contaminant gases is sublimed onto the inside surface of a vessel. The device will thus act as a vacuum pump to the contaminant gases if the system is under vacuum, or as a rare gas purifier when so filled.

In this work titanium rods were heated by the passage of a high electric current. The rods are first heated under vacuum to about 800°C for a few minutes to promote outgassing. After the surrounding vessel has also been outgassed the rods are reheated so that some of the titanium sublimes onto the vessel wall. The rare gas mixture can then
### ARGON

**Research Grade**

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>99.9995%</td>
</tr>
<tr>
<td>Oxygen</td>
<td>less than 2vpm</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>less than 5vpm</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Carbon Dioxide</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Hydrocarbons</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Moisture</td>
<td>less than 2vpm</td>
</tr>
</tbody>
</table>

### NEON

**Research Grade**

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neon</td>
<td>99.999%</td>
</tr>
<tr>
<td>Helium</td>
<td>less than 5vpm</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Oxygen</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Carbon Dioxide</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Hydrocarbons</td>
<td>less than 1vpm</td>
</tr>
<tr>
<td>Moisture</td>
<td>less than 1vpm</td>
</tr>
</tbody>
</table>

Table 5.1

Claimed impurity levels in B.O.C. Research Grade Gas
be admitted to the vessel for purification. A diagram of the apparatus used here and a table of pumping speeds is provided in Figure 5.2. Due to the nature of operation of the device, inert gases are not pumped.

Note that this method is similar to that used by Kruithof and Penning (1937). They, however, used barium as the absorbing material.

**S.A.E.S. Getter Device**

One of the methods used to try and improve gas purity involved the installation inside the research chamber of a S.A.E.S. non-evaporable getter. The active material used is zirconium. Heating is provided by means of an externally supplied resistance wire - Figure 5.3. The device is activated by being heated to around 1000°C under vacuum. The rare gas mixture is then admitted to the chamber. The porous construction of the getter gives it an extremely high effective surface area. The device used here, for example, has a geometric surface area of 2 cm² but an effective area for gettering of 3300 cm². The getter has a large pumping capacity at room temperature for chemically active gases such as CO, CO₂, O₂, N₂, H₂O and H₂. Methane and other hydrocarbons can be pumped at temperatures of about 400°C.
Pumping speeds (ls⁻¹ per square inch of film) for a fresh titanium layer at room temperature are:

- Carbon dioxide: 30
- Oxygen: 11
- Carbon monoxide: 66
- Hydrogen: 20
- Nitrogen: 15
- Deuterium: 8

**Figure 5.2** Vacuum Generators T.S.P.

**Figure 5.3** S.A.E.S. Getter

All dimensions are in millimeters.
5.3 **Electronic Techniques**

5.3.1 **Measurement of Gas Gain**

With most of the measurements it was necessary to have a fairly accurate knowledge of the gas multiplication taking place. Because the measurements were carried out at fairly low gas gains and count rates it was not possible to use an electrometer system. A pulse simulation technique was used as described below.

The calibration of a charge sensitive system (preamplifier, amplifier and multi-channel analyser) is commonly carried out by feeding a known "test charge" into the system and observing the response. The test charge is typically provided by means of a precision step function generator and an accurately known test capacitor. Figure 5.4 shows a typical arrangement.

![Figure 5.4](image)

*Figure 5.4 Typical arrangement for feeding a test pulse to a charge sensitive system.*

\[
R_g = \text{output resistance of generator} \\
C_T = \text{test capacitance}
\]
Using such an arrangement one can successfully calibrate the system for step function pulses. The pulse from a proportional counter though does not take the form of a step function. The shaping time constants of the system will attenuate the relatively slow rising proportional counter pulse more severely than the step function. This will lead to an error in the gas gain measurement. With the normal range of time constants used with a proportional counter (\( \approx 1 \) to \( 5 \mu \) sec) the detector pulse may be attenuated by a factor of, say 10, (Charles (1972)). The step function, however, is only attenuated by the factor \( e ( = 2.7183) \), assuming that the system has equal integrating and differentiating time constants.

Mathieson and Charles (1969) have calculated the attenuation of a proportional counter pulse and have presented their results in a form which allows the estimation of gas gain in a particular system. It is necessary however to have a knowledge of the positive ion mobility. In a Penning mixture the positive ion population is largely made up of minority gas ions, so we need to know the mobility of, for example, Ar ions in Ne. Such measurements have not been reported. The work of Gott and Charles (1969) indicates though how the effective positive ion mobility may be estimated. It was decided however to investigate the possibility of simulating the proportional counter pulse so that the system would then respond in a similar way to both the test and detector pulses. It was found that a simple integrating circuit with correctly chosen component values was able to simulate the detector pulses to a good approximation (see Figures 5.5 and 5.6).
Figure 5.5 System used to simulate the charge pulse from a proportional counter. R and C are chosen so as to give a suitable rise time.

The method used involved observation of the detector pulses on an oscilloscope with the shaping circuitry switched out. The pulse rise time could thus be estimated. By choosing the values of R and C so that \(3RC \approx \) pulse rise time a good simulation of the pulse should result. The method was tested by adjusting the output voltage of the generator so that the unshaped generator and detector pulses were the same height. The shaping was then switched in and the m.c.a. operated. It was found that the two signals, as measured on the m.c.a., were still the same height.

It is necessary to know the relationship between the voltage measured by the meter and the voltage which is actually applied across the test capacitor. The output resistance of the generator and the matching resistance of the network will, by themselves, attenuate the signal by a factor of two. The RC network though will attenuate the signal further. The overall attenuation of the circuit was determined by measuring the signal generator output voltage, with a digital voltmeter, and comparing it with the output voltage of the shaping circuit, measured
Integrating circuit.

\[ V_{\text{OUT}} = V_0 \left( 1 - \exp\left( -\frac{t}{RC} \right) \right) \]

\textbf{Figure 5.6} (From Delaney (1969))
by means of a calibrated oscilloscope.

It should be borne in mind when using this method that the rise time of the detector pulse may alter with anode voltage. This is especially the case close to breakdown where secondary avalanches tend to increase the rise time. When measuring the rise time with the shaping switched out acoustic noise tends to determine the minimum gas gain at which the measurement can be made. It thus has to be assumed that the rise time does not alter significantly at lower gas gains.

Unlike electrometer studies, the method presented here assumes a knowledge of the mean ionization energy of the gas.

5.3.2 Reduction of Preamplifier Noise

Due to the lack of a quenching agent in a Penning mixture, the gas gain operating range of a proportional counter with such a filling is typically 2 to 3 orders of magnitude below that allowed with a more conventional mixture such as 90% Ar + 10% CH₄. Further, it can be concluded from Chapter 3 that the gas gain should be kept low to reduce avalanche fluctuations. Under these conditions the electronic noise may become significant, and will of course dictate, for a given X-ray energy, the minimum gas gain that can be used. It is helpful to quantify some of the parameters.

Consider the case of the Penning mixture 99.9% Ne + 0.1% Ar. The mean ionization energy of such a mixture is 25.3 eV. This means that a 5.9 KeV X-ray produces an average of 233 ion pairs. The amount of charge produced is, in fact, around the noise level of what would be considered to be quite a good modern preamplifier. With such an amplifier it would be necessary to operate at a gas gain of around 100 if a resolution of about 14% is not to be significantly degraded, (see
Loss of resolution due to pre-amplifier noise. The figure shows how an intrinsic resolution of 14% is degraded by a pre-amplifier with an rms noise level of 200 electrons. A gas gain of about 100 is needed if the observed resolution is to be unaffected by noise.

Figure 5.7
Figure 5.7). The resolution actually "observed" when taking a proportional counter spectrum, the real "intrinsic" resolution of the device and the electronic noise are related by the following expression:

\[ R_o^2 = R_i^2 + \frac{\text{FWHM}^2_{\text{noise}}}{C_p^2} \]  

where

- \( R_o \) is the observed resolution
- \( R_i \) is the intrinsic resolution
- \( C_p \) is the peak channel of the spectrum
- \( \text{FWHM}_{\text{noise}} \) is the full width at half the maximum of the noise spectrum (in channels).

The noise spectrum is typically measured by feeding a noise free pulse into the system operating at high gain so that the number of channels involved, and hence the accuracy, is high. The noise can also be measured using an r.m.s. voltmeter at the main amplifier output. It is important that the meter is a "true r.m.s." device and that signal components (such as low frequency ripple) which are "read" by the meter but not by the multi-channel analyser, are not present. If the above conditions are not met then a false impression of the noise will result.

A system for measuring preamplifier noise and the noise broadening of an ideal line spectrum are shown in Figures 5.8 and 5.9 respectively.

The components of the electronic system other than the preamplifier i.e. the main amplifier and m.c.a. will contribute to the noise but the main source is usually the preamplifier itself. In the following section the noise is analysed from a fundamental point of view. Pathways leading to noise reduction are indicated. The practical construction of the device is also considered.
System for Measuring Charge-Sensitive-Preamplifier Noise.

Figure 5.8 (From Ortec Instrumentation Handbook (1976))

Noise broadening of an ideal line spectrum.

Figure 5.9 (From Delaney (1969))
A Charge-sensitive amplifier (schematic).

**Figure 5.10** (From Delaney (1969))

A Simplified schematic of most currently used low-noise preamplifiers.

**Figure 5.11** (From Ferrari and Fairstein (1968))
Analysis of Preamplifier Noise

It is useful to first consider briefly the operation of a charge sensitive preamplifier. The device is essentially a current integrator (see Figure 5.10). The output voltage $V_{\text{OUT}}$ is related to the input charge $Q$ by the following expression:

$$V_{\text{OUT}} = \frac{Q}{C_f}$$ (5.2)

where $C_f$ is the value of the feedback capacitor. It is usual to have some voltage amplification after the first stage. A more complete form of the amplifier is shown in Figure 5.11.

The noise of a preamplifier results from various sources related to the input circuit and can be conveniently expressed as an equivalent signal charge. The noise sources have been identified by various authors (Radeka (1965), Smith and Cline (1966) and Delaney (1969)). A brief summary is given below:

Sources relating to input f.e.t.

1. Thermal noise in the gate resistor $R_g$, see Figure 5.12.

![Detector and input circuit of f.e.t.](From Delaney (1969))
2. Shot noise from the gate current $I_g$. This is the small current which flows in a reverse biased junction.

3. Thermal noise in the f.e.t. channel.

4. Flicker noise in the f.e.t. channel.

The various sources are described in detail in the above references. This group of noise sources can be reduced by careful selection of the f.e.t. The main requirements of the f.e.t. are high gain, low input capacitance and low gate leakage current.

Noise sources relating to connection to detector

The HT feeding resistor $R_F$ (Figure 5.13) contributes a noise current given by

$$I_{rms} = \sqrt{\frac{4kTB}{R_F}}$$  \hspace{1cm} (5.3)

where $k$ = Boltzmann's constant

$B$ = Bandwidth of circuit seen by resistor

$T$ = Absolute temperature

To minimise this contribution to the noise the value of $R_F$ should be very high - preferably $> 10^9 \, \Omega$.

The isolation capacitor $C_{IS}$ contributes to the noise by way of its stray capacitance. If the capacitor is piezo-electric then acoustic noise may arise. The isolation capacitor should therefore be carefully chosen and mounted to reduce these effects.
The isolation capacitor and feeding resistor may be eliminated by operating the anode at zero volts and applying negative H.T. to the cathode. For reasons relating to screening, connections to other equipment and safety, it may however be necessary to have an earthed casing around the detector.

**Noise sources relating to actual construction**

In the physical realisation of the device, the input capacitance should be minimized. The f.e.t. should be mounted with heat transfer to the board and casing in mind. It may be advisable not to connect the gate directly to the printed circuit board as a low current across the insulation could be set up. A well insulated stand-off may be better.
A partial schematic of the test preamplifier.

**Figure 5.14**

Schematic diagram of preamplifier.

**Figure 5.15**
5.3.3 Preamplifier used in this study

The preamplifier circuit selected for use in this study was originally developed by Smith and Cline (1966). Figure 5.14 shows the separate current integrating and voltage amplification stages. In Figure 5.15 a complete circuit of the preamplifier is given.

The amplifier is operated in a charge sensitive mode with feedback provided by a 0.5 pF capacitor. The first stage is connected as a grounded source voltage amplifier with a 0.5 henry coil as the load.

A $10^9 \, \Omega$ resistor provides biasing by holding the gate-to-source voltage near zero. The second, source follower stage, acts as a buffer between the common source amplifier and the voltage gain section of the circuit. The low output impedance of the buffer allows the charge sensitive and voltage amplification stages to be physically separated by several feet. It is thus necessary to mount only the first stage of the preamplifier at the detector.

Noise performance of preamplifier

Smith and Cline have considered the factors listed in Section 5.3.2 and have predicted the noise figure of the preamplifier. Figures 5.16 and 5.17 give a summary of their results. By putting two or more first stage f.e.t.s. in parallel the effective value of $g_m$ ($g_m$ is the "gain" of a f.e.t. = rate of change of drain current with gate to source voltage) is increased in proportion. Smith and Cline have shown that this reduces the rate of increase of noise with input capacitance loading. Significant reductions in electronic noise at high input capacitances are thus possible. Smith and Cline found that the measured performance of the preamplifier agreed quite well with their predictions - Figures 5.18 to 5.20.
Theoretical amplifier noise vs time constant for 1, 2, and 4 paralleled field-effect transistors at $T = 298^\circ K$ and $140^\circ K$ with $C_{\text{Total}} = 100 \text{ pF}$ and $n\sqrt{C_{gs}}$ at single RC integrating and differentiating time constants.

**Figure 5.16**

---

Theoretical amplifier noise vs $C_{\text{Total}}$ for 1, 2, and 4 paralleled field-effect-transistors at $T = 298^\circ K$ and $140^\circ K$ with $t_0 = 1.6 \mu\text{sec}$ and single RC integrating and differentiating time constants.

**Figure 5.17**
Measured amplifier noise vs time constant for 1, 2, 3, and 4 paralleled field-effect-transistors at $T = 298^\circ K$ and $140^\circ K$ with $C_{\text{external}} = 101 \text{ pF}$ and $C_{\text{external}} = 0 \text{ pF}$.

Figure 5.18

Measured amplifier noise vs external capacitance for 1, 2, 3, and 4 paralleled field-effect-transistors at $T = 298^\circ K$ and $140^\circ K$ with $\tau = 1.6 \mu\text{sec}$.

Figure 5.19

Measured amplifier noise vs case temperature for 1 and 4 paralleled field-effect-transistors with $\tau_0 = 1.6 \mu\text{sec}$.

Figure 5.20

Figures 5.14 to 5.20 are from Smith and Cline (1966)
Figure 5.21 shows a typical noise characteristic obtained by the author. The curve obtained with 100 pF input capacitance loading is in quite good agreement with the prediction of Smith and Cline. The curve obtained with about 1 pF input capacitance, however, indicates a value of noise approximately twice that predicted by Smith and Cline. It is suggested that the rather high input capacitance of the f.e.t. (about 10 pF for the BF817) and the stray capacitance (about 5 pF) are the reasons for this.
Typical noise characteristic of Smith and Cline pre-amplifier. A EP617 was used as a first stage f.e.t. The capacitance of the cylindrical counter is about 1pF. The capacitance of the research chamber is about 100pF.

Figure 5.21
5.3.4 Description of complete electronic system

A block diagram of the electronic system is given in Figure 5.22.
The individual components are identified more fully below.

Preamplifier  Smith and Cline (1966) design.
Amplifier     Ortec 450 Research Amplifier
Multi-channel Analyser  Harwell 6000 series
Cathode-Ray Oscilloscope  Tektronix 531A
Signal Generator     Ortec Precision Pulse Generator
Pulse Shaper         As described in Section 5.3.1
Digital Voltmeter    Solartron LM1240
E.H.T. Supply Brandenburg Module to fit Harwell 6000. Voltage
variable in 1 volt steps from 0 to 3 KV.

Plate 5.1 shows the experimental system used in the measurements.
On the left hand side a sound attenuating enclosure used to house the
research chamber, described in Chapter 7, is shown.
Figure 5.22 Block diagram of complete electronic system
CHAPTER 6: CYLINDRICAL COUNTER STUDIES

6.1 Introduction

6.2 Experimental Arrangement

6.3 Resolution Measurements
   6.3.1 Operation with Argon-Methane
   6.3.2 Operation with Neon-Argon
   6.3.3 Operation with Neon-Xenon
   6.3.4 Operation with Argon-Acetylene
   6.3.5 Operation with Argon-Xenon
   6.3.6 Discussion

6.4 Quenching of Penning Mixtures
   6.4.1 Introduction
   6.4.2 Previous work
   6.4.3 Results and discussion

6.5 Use of Neon-Argon in a Large Area Proportional Counter
6 Cylindrical Counter Studies

6.1 Introduction

The use of Penning mixtures in proportional counters has been studied previously by Pawlowski (1970) and Sipila (1976 to 1979). Pawlowski found that the Penning mixture argon-acetylene gave improved resolution over the more conventional mixture of argon-methane. Quantitative data was not presented however. Sipila has investigated a number of Penning mixtures and has obtained an observed resolution (i.e. the resolution obtained without subtracting the effect of electronic noise) of about 11.5% at 5.89 keV with neon-argon.

In this chapter a range of Penning mixtures is investigated and, where possible, the results are compared with previous work. Measurements are made at 5.89 keV and also at 22.1 keV to improve the signal to noise ratio at low values of gas gain. The resolution measurements are also compared with those obtained using the more conventional gas filling of argon-methane.

Using the analysis presented in chapter 3, theoretical values of resolution for the various avalanche models are calculated where suitable data is available. The experimental data is compared with the theoretical models.

The non-uniform electric field distribution encountered in a cylindrical counter tends, however, to reduce the clarity of the comparison (as opposed to the case where measurements are made with a uniform electric field). There are a number of reasons for this:

(i) The theoretical evaluation of $f$ in a cylindrical counter using equation (3.42) requires, ideally, an exact knowledge of the variation of $\alpha/E$ with $E/p$ over the entire avalanche path. Empirical equations for such
variations have not been reported for Penning mixtures. It is thus necessary to take data directly from the published graphs and perform the calculation numerically.

(ii) Close to the anode the electric field increases so rapidly that it may vary along the mean free path travelled by the electron. The value of $q'E$ will therefore tend to vary as well and this will lead to an error in the calculations. (For a review of the above considerations see Shallal and Harrison (1970).)

(iii) It is possible that some of the avalanche models are only valid over a limited range of $X$. Measurements made under conditions involving a varying value of $X$ may therefore give rise to misleading results.

Despite these objections some attempt is made to relate the resolution measurements to the various theories.

Cylindrical counters have the advantage of being easier to set up and operate than uniform field counters. The high electric field near the anode causes the positive ions developed in the avalanche to move away very rapidly. The resulting signal thus has a very short rise time and so shorter main amplifier time constants can be used. Acoustic noise problems are thus reduced. The capacitance of a cylindrical counter is much less than that of a uniform field device and so the electronic noise of the pre-amplifier is reduced accordingly. With a cylindrical counter there is, of course, no requirement for a grid electrode.

Investigations are also carried out into possible ways of operating a Penning mixture at high values of gas gain (to improve the signal to noise ratio at the pre-amplifier input)
without the presence of disruptive secondary avalanches. Secondary avalanches tend to degrade proportional counter resolution because the gas amplification is affected in a rather uncertain way by positive ions and ultra-violet radiation striking the cathode and initiating new avalanche chains. With conventional main gas fillings such as argon the use of a quenching agent such as methane reduces secondary avalanches considerably and allows operation at very high gas gains - typically up to about $10^5$.

With a Penning mixture one has to be careful not to affect the ionization process in attempting to quench the discharge. Various possible schemes are investigated.

The use of Penning mixtures in a large area multi-wire proportional counter is also examined briefly.
6.2 Experimental Arrangement

The measurements were made with a proportional counter manufactured by A.E.R.E. (Harwell). The cathode was formed from a cylindrical section of 254 micron thick beryllium with mild-steel end fittings. The 25.4 micron diameter anode wire was supported by ceramic insulators. Inconel wire was used to ensure a smooth surface (inconel is an alloy of nickel and iron and can be produced with a smoother surface than tungsten). To help ensure uniformity of wire radius, new dies were used for the drawing process. A general external view of the counter is shown on plate 6.1.

The internal diameter of the cathode is 2.34 cm and the effective length of the counter is 10 cm.

When using Penning mixtures in proportional counters it is particularly important that a high level of gas purity is maintained. Processes such as ionization of impurity atoms by direct electron impact and de-excitation of metastable main gas atoms by impurities will interfere with the growth of the avalanche and tend to increase its variation. Particular care was thus taken to eliminate leaks and reduce outgassing. B.O.C. Research Grade gas was used for all the measurements.

The counter was connected to the gas filling system via a Cajon type coupling and sealed with a Nupro valve. Both these fittings are of all metal construction to reduce outgassing. Prior to filling, the counter was ion pumped for a few days. A bake-out (i.e. heating under vacuum) was required if the counter had been opened up to air for any reason.

All the measurements were carried out with a pressure of 1.1 atmospheres. The gas gain and resolution of the counter were measured occasionally, at a certain fixed anode voltage. It was thus possible to determine whether the gas was deteriorating as a result of leaks or outgassing during the measurements.
The pre-amplifier used was of the low noise Smith and Cline type described in section 5.3.3. The actual unit used is shown on plate 6.1. A pre-amplifier may be damaged by transients arising from sudden changes to the HT applied to the counter anode. A simple switched input grounding system was employed for protection. The HT to the anode was fed via a low pass filter to reduce interference. A high value feed resistance \((10^3 \Omega)\) was used to keep the thermal noise low.

To reduce microphonic pick-up the counter body was insulated from the main casing by rubber matting. The casing itself was mounted on a block of sound absorbing material to reduce vibration transmitted along the floor and walls of the laboratory. A quiet environment was chosen for making the measurements. The presence of acoustic noise was monitored by operating the main amplifier and oscilloscope at high gain.

The gas gain of the counter and the electronic noise level were measured as described in sections 5.3.1 and 5.3.2 respectively. The electronic system used to analyse the detector pulses was outlined in section 5.3.4. In operating the system the main amplifier shaping constants were chosen for optimum signal to noise ratio. The optimum time constants are influenced by the rise-time of the counter pulse and the characteristics of the pre-amplifier. The most suitable time constants were determined experimentally.

The radio-active sources used in the resolution measurements were \(^{55}\text{Fe}\) and \(^{109}\text{Cd}\) which have their X-ray emissions at \(5.89\) keV and \(22.1\) keV respectively. These sources were used because

1. they produce sufficient charge in the gas to give a reasonable signal to electronic noise ratio;
(2) the photo-electron track length is less than the counter dimensions; and
(3) the half-lives of the sources are conveniently long (2.9 and 1 years, respectively).

The sources were collimated to give a beam of a few mm diameter with narrow dispersion. The count rate was adjusted by exposing different areas of the radio-active source to the collimator. The rate was chosen so that the measurements could be made within a reasonable time but was kept low enough to avoid pulse pile-up problems.
6.3 Resolution Measurements

6.3.1 Operation with Argon-methane

To confirm the suitability of the proportional counter and associated electronic system for the proposed measurements, and to provide a standard of comparison, the counter was filled with the standard counter gas mixture of 90\% argon + 10\% methane (commonly known as P-10).

Theoretical Resolution

Alkhazov (1969) has evaluated \( \varepsilon \) using equation 3.42 for Legler's model (with \( U_0 = U_1 \)) for pure argon (Townsend data for 90\% argon + 10\% methane has not been reported but the work of Yamane (1960) suggests that argon behaves in a similar way to 90\% argon + 10\% methane). Alkhazov has calculated \( \varepsilon \) as a function of \( pr_a \) (the product of the gas pressure and the anode radius) and as a function of \( K \) (the gas gain). The results of his calculations are shown in figures 6.1 and 6.2. Similar results for methane are also shown. The Townsend data used in the evaluation was that obtained by Kruithof and Penning (1936) and Cookson, Ward and Lewis (1966) respectively. The weak dependence of \( \varepsilon \) on \( K \) for \( K \) greater than about 30 can be seen.

Figure 6.1 was used to estimate \( \varepsilon \) for the conditions of the experimental study (i.e. \( p = 1.1 \) atm, \( r_a = 1.27 \times 10^{-3} \) cm, \( pr_a = 1.06 \) torr cm).

Alkhazov (1970) and Sipila (1979) have shown that the value of \( \varepsilon \) obtained in a cylindrical counter is effectively governed by the early stages of the avalanche and in particular the first multiplication. More specifically they have shown that:

\[
\left[ \varepsilon \right] (K >> 1) \approx \left[ \varepsilon_0 \right] (K = 2) \quad (6.1)
\]
Theoretical variation of $f$ with $p_{r_{3}}$ for Legler's Model. The calculation has been made for argon and methane. The gas gain is set at 100.

Figure 6.1
(From Alkhazov (1969))

Theoretical variation of $f$ with $K$ for Legler's Model. Experimental points are taken from the works of Curran et al (1949) (50% Ar + 50% CH$_4$; $p_{r_{3}} = 3.8$ Torr·cm), Gold and Bennet (1966) (95% Ar + 5% H$_2$; $p_{r_{3}} = 0.51$ Torr·cm) and Campbell and Ledingham (1966) (75% Ar + 25% C$_3$H$_6$; $p_{r_{3}} = 0.95$ Torr·cm).

Figure 6.2
(From Alkhazov (1969))
where \([\hat{f}] (\bar{K} \gg 1)\) is the relative variance of \(K\) for the entire avalanche (assumed to be \(\gg 1\) in magnitude).

\([f_{o}] (\bar{K} = 2)\) is the value of \(f_{o}\) at the first multiplication, i.e. when \(\bar{K} = 2\).

The value of \(\chi\) appropriate to the value of \(E/p\) obtained where \(\bar{K} = 2\) can be taken as a guide to the overall avalanche fluctuation. This means that whilst the curve shown in figure 6.1 was calculated for the complete avalanche the values of \(f\) obtained can be regarded as being applicable to the first multiplication of the avalanche. The value of \(E/p\) will be approximately constant over this region. We can therefore directly relate \(f_{o}\) to the value of \(\chi\) and hence to the value of \(E/p\) at the start of the avalanche (using equation 3.28).

Having derived the appropriate value of \(\chi\) one can obtain values of \(f_{o}\) predicted by the other avalanche models using equations 3.29 to 3.33. Theoretical values of \(f\) and resolution \((R_{th})\) can be obtained using equation (6.1) and the resolution equation (1.8).

Table 6.1 gives theoretical values of \(f\) and resolution for the cylindrical counter filled with argon. The theoretical resolution is given for energies of 5.89 KeV and 22.1 KeV.

<table>
<thead>
<tr>
<th>Model No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_{th})</td>
<td>0.57</td>
<td>0.74</td>
<td>0.65</td>
<td>0.65</td>
<td>0.76</td>
<td>0.83</td>
<td>1</td>
</tr>
<tr>
<td>(R_{th}) (5.89 KeV)</td>
<td>13.5</td>
<td>15.0</td>
<td>14.3</td>
<td>14.3</td>
<td>15.2</td>
<td>15.7</td>
<td>17.0</td>
</tr>
<tr>
<td>(R_{th}) (22.1 KeV)</td>
<td>7.0</td>
<td>7.8</td>
<td>7.4</td>
<td>7.4</td>
<td>7.9</td>
<td>8.1</td>
<td>8.8</td>
</tr>
</tbody>
</table>

Theoretical values of \(f\) and resolution for the cylindrical counter filled with argon. The \(p_{atr}\) product is 1.06 torr cm and \(\bar{K} = 100\). The model numbers correspond to those of chapter 3. Values of \(f\) and \(w\) required for the resolution calculations were given in chapter 2.

Table 6.1
Results and Discussion

Figure 6.3 shows the gas gain obtained as a function of voltage. The rather slow departure from linearity at high values of gas gain indicates the effectiveness of methane as a quenching agent.

In figures 6.4 and 6.5 the resolutions obtained with 5.89 keV and 22.1 keV X-ray sources are shown. The resolution at a gas gain of 100 is rather higher than most of the theoretical values.

As indicated by equation (1.8) the observed resolution $R_0$ of a proportional counter should be proportional to $E^{-\frac{1}{2}}$ (where $E$ is the X-ray energy). The quantity $R_0 \times E^{\frac{1}{2}}$ should thus remain constant. The experimental values presented in table 6.2 are in good agreement with the theory.

<table>
<thead>
<tr>
<th>X-ray Energy, $E$ (keV)</th>
<th>Measured Resolution at $K=100$, $R_0(%)$</th>
<th>$R_0 \times E^{\frac{1}{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.89</td>
<td>16.1</td>
<td>39.1</td>
</tr>
<tr>
<td>22.1</td>
<td>9.1</td>
<td>42.8</td>
</tr>
</tbody>
</table>

Experimental values of $R_0 \times E^{\frac{1}{2}}$ obtained at X-ray energies of 5.89 and 22.1 keV with Ar-CH₄.

Table 6.2

The resolution results obtained here can be compared with those reported in the literature. Mulvey and Campbell (1958) obtained a resolution of 18.9% at 5.89 keV and suggested that anode wire non-uniformity had lead to some degradation.

Chârles and Cooke (1968) found that by reducing the variation of the anode wire diameter to $\Delta$% per mm along its length (from 1% per mm) the resolution at 5.89 keV improved from 17.3% to 14.2%.
Experimental variation of gas gain with anode voltage for the cylindrical counter filled with 90% argon + 10% methane. The measurements were made at 5.89 KeV. The error limits correspond to the FWHM of the spectra obtained. All cylindrical counter measurements were made at a pressure of 1.1 atmospheres.

Figure 6.3
Experimental variation of resolution with gas gain for the cylindrical counter filled with 90% argon + 10% methane. The measurements were made at 5.89 KeV. The error limits arise from the finite number of channels which the pulse-height-analyzer operates with. The directly observed and intrinsic resolutions are shown. These are denoted by $R_o$ and $R_I$ respectively. The intrinsic resolution is obtained by subtracting the disturbing effects of electronic noise. All the cylindrical counter measurements were made with main amplifier integration and differentiation time-constants of 3 micro-seconds as the best signal to noise ratio was obtained with these conditions.

Figure 6.4
Experimental variation of resolution with gas gain for the cylindrical counter filled with 90% argon + 10% methane. The measurements were made at 22.1 KeV.

Figure 6.5
The lowest values of resolution measured with argon-methane have been reported by Sipila (1976a, 1976b) who has obtained 13% at a gas gain of 100. In making this investigation gas impurities were kept to a minimum (oxygen and water vapour levels in the argon were reduced to <0.1 ppm). In addition an extremely uniform anode wire was used.

It can be seen from table 6.1 that all the models predict fairly similar values of resolution. It is therefore difficult to make a meaningful comparison between the experimental results and particular models.

The results given here do, however, provide a basis which can be used for comparison with other gas fillings.
6.3.2 Operation with Neon-Argon

The values of $\chi$ that can be obtained with the Penning mixture neon-argon should lead to improved energy resolution over the mixture argon-methane. In this section the energy resolution of a neon-argon filled proportional counter is investigated.

Theoretical Resolution

Sipila (1976b) has evaluated $f$ for Legler's model for a variety of neon-argon mixture ratios - figure 6.6. The optimum theoretical resolution is predicted for an argon concentration of 0.1%. Sipila (1976a, 1976b and 1978) has found, however, that the best experimental resolution is obtained with 0.5% argon. The greater concentration of argon atoms may lead to the Penning ionization process taking place closer to the point at which the metastable neon atom is formed. This will reduce local variations in the Townsend coefficient and thus reduce fluctuations in the avalanche size. With these considerations in mind it was decided to operate the counter with 99.5% Ne + 0.5% Ar.

In table 6.3 theoretical values of $f$ and resolution for the other six avalanche models are shown. The calculations were made in the same way as for argon-methane.

<table>
<thead>
<tr>
<th>Model No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{th}$</td>
<td>0.17</td>
<td>0.38</td>
<td>0.23</td>
<td>0.49</td>
<td>0.52</td>
<td>0.78</td>
<td>1</td>
</tr>
<tr>
<td>$R_{th}$ (5.89 KeV)</td>
<td>7.3</td>
<td>10.1</td>
<td>8.2</td>
<td>11.4</td>
<td>11.7</td>
<td>14.1</td>
<td>15.9</td>
</tr>
<tr>
<td>$R_{th}$ (22.1 KeV)</td>
<td>3.8</td>
<td>5.3</td>
<td>4.3</td>
<td>5.9</td>
<td>6.1</td>
<td>7.3</td>
<td>8.2</td>
</tr>
</tbody>
</table>

Theoretical values of $f$ and resolution for the cylindrical counter filled with 99.5% Ne + 0.5% Ar. The other conditions are as for table 6.1.

Table 6.3
The theoretical variation of $f$ with percentage of argon in neon - for Legler's Model.

Figure 6.6
(From Sipila (1976b))
Results and Discussion

In figure 6.7 the gas gain is plotted as a function of anode voltage. The lack of a quenching agent is readily apparent from the nature of the curve. Departure from linearity takes place at low values of gas gain. The corresponding curve for argon-methane was still linear at gas gains of $\approx 10^4$.

The resolution is plotted as a function of gas gain in figures 6.8 and 6.9. In making a comparison with the argon-methane measurements it is perhaps easier to consider the $\text{Cd}^{109}$ curves as the effect of electronic noise is considerably less than for $\text{Fe}^{55}$. The optimum observed $\text{Cd}^{109}$ resolution for neon-argon is $\approx 6.5\%$ (see plate 6.2) as compared to $\approx 9\%$ for argon-methane. The corresponding intrinsic resolutions (noise subtracted) for $\text{Fe}^{55}$ are $\approx 16\%$ and $\approx 11.5\%$.

The resolution results seem to be in closest agreement with models 4 and 5. A discussion of the applicability of the various models is provided in section 6.3.6.

To examine the behaviour of the counter at low values of gas gain, the anode voltage was increased in 1 volt steps from 176 to 203 volts. The corresponding gas gain range was from 3 to 8. Over this range the mean value of intrinsic resolution was found to be $10.9\% \pm 1.55$ at 5.89 KeV. Sipila has obtained slightly lower values of intrinsic resolution in this gas gain region ($\approx 9\%$) but with the higher level of gas purity mentioned in section 6.3.
Experimental variation of gas gain with anode voltage for the gas mixture 99.5% neon + 0.5% argon. The measurements were made at 22.1 KeV.

Figure 6.7
Experimental variation of resolution with gas gain for 99.5% neon + 0.5% argon. The measurements were made at 5.89 keV.

Figure 6.8
Experimental variation of resolution with gas gain for 99.5% neon + 0.5% argon. The measurements were made at 22.1 keV.

Figure 6.9
Spectrum obtained with 99.5% neon + 0.5% argon at 22.1 KeV. The spectrum was obtained at the value of gas gain corresponding to optimum observed resolution: $K = 25$, $R_o = 6.5\%$.

Plate 6.2
<table>
<thead>
<tr>
<th>X-ray Energy E (keV)</th>
<th>Intrinsic Resolution at $K = 15$ $R_I(%)$</th>
<th>$R_I \times E^{3/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.89</td>
<td>11.5</td>
<td>27.9</td>
</tr>
<tr>
<td>22.1</td>
<td>6</td>
<td>28.2</td>
</tr>
</tbody>
</table>

Experimental values of $R_I \times E^{3/2}$ obtained at X-ray energies of 5.89 and 22.1 keV with Ne-Ar.

Table 6.4 shows that the counter was operating with a constant value of $R_I \times E^{3/2}$ at the energies under investigation.

It is interesting to consider why the intrinsic resolution, measured at 22.1 keV, deteriorates at very low values of gas gain (i.e. the opposite effect to that predicted by equation (3.39)). The low values of gas gain are obtained with low values of voltage and thus with a lower range of $E/p$. The values of $\alpha/E$ are also reduced leading to increased theoretical values of $f$.

In addition the values of $1/\alpha$, the mean free path for ionization, became larger. With high values of $1/\alpha$ the electric field variation over a single ionization mean free path will increase and so the avalanche variation will be higher.

It is also possible that recombination or charge loss to the cathode wall may take place in the weak electric field area near the cathode. With an anode voltage of 100V the value of $E/p$ at the cathode is about 0.015 v/cm/torr. The photo-electron track length in neon at this pressure is 13 mm at 22.1 keV. Charge loss to the wall will, of course, affect the final avalanche charge and thus degrade the resolution.
6.3.3 Operation with Neon-Xenon

As was discussed in section 4.2.2, the Penning mixture neon-xenon has been shown by Bhattacharya (1976) to give values of ionization efficiency that are as high as for neon-argon. The use of neon-xenon as a counter gas is thus of interest. The mixture composition which gives the highest value of $X$ is

$$99.99\% \text{Ne} + 0.01\% \text{Xe}.$$  

Bhattacharya has only measured $\alpha/E$ over a limited range of $E/p$ (see figure 4.11) hence exact calculations of $f$ for a cylindrical counter do not seem possible. Values of $F$ for neon-xenon have not been reported either. The behaviour of $99.99\% \text{Ne} + 0.01\% \text{Xe}$ is, however, similar to $99.9\% \text{Ne} + 0.1\% \text{Ar}$. One can therefore estimate, to a first approximation, that the theoretical resolutions of the two mixtures should be similar (i.e. assume that $f$, $F$ and $W$ are approximately the same).

With these assumptions table 6.3 can be taken as a guide to theoretical resolution.

The cylindrical counter was filled with $99.97\% \text{Ne}$ $0.03\% \text{Xe}$. The gases were mixed on the filling system. The pressure instrumentation used was not suitable for direct filling of gases at pressures of $0.01\%$ of 1.1 atmospheres (i.e. $8.34 \times 10^{-2}$ torr). Using the gas expansion technique described later in section 6.4, however, it was possible to "fill" the counter with $0.03\%$ xenon (i.e. to a pressure of 0.25 torr).

The experimental results obtained are shown in figures 6.10 to 6.12. Because of the lack of an explicit value of $W$ for neon-xenon the measured charge is shown rather than the gas gain. The resolution obtained is not as good as for $99.5\%$ Neon $+ 0.5\%$ Argon.
Experimental variation of measured charge with anode voltage for the gas mixture 99.97% neon + 0.03% xenon. The measurements were made at 22.1 keV.

Figure 6.10
Experimental variation of resolution with measured charge for 99.97% neon + 0.03% xenon. The measurements were made at 5.89 KeV.

Figure 6.11
Experimental variation of resolution with measured charge for 99.97% neon + 0.03% xenon. The measurements were made at 22.1 KeV.

Figure 6.12
It is interesting to consider why the avalanche fluctuation should be higher for the neon-xenon mixture. It is suggested that the reason for this is related to the fact that the quantity of xenon required for maximum ionization efficiency is ten times less than the corresponding amount of argon. Because the number of xenon atoms per cm$^3$ is less, the degree of spatial localization involved in the de-excitation of a metastable neon atom by an argon atom may be reduced. The neon atom may have to travel further before encountering a xenon atom. At the same time photon mediated (and thus less localized) Penning ionization may become more prominent (Buursen and Van Montfort (1973) have indicated the involvement of such a process in mixtures of neon + 0.001% argon). Lack of localization of the charge multiplication leads to increased avalanche fluctuation.

The electron-neutral collision cross-section of xenon is much higher than for argon (Brown (1959)) and so the xenon concentration must be kept lower to avoid direct excitation and ionization of xenon which reduces the average energy of electrons and leads to fewer neon metastables.
6.3.4 Operation with Argon-Acetylene

The Penning mixture argon-acetylene is an interesting counter gas filling for a number of reasons. With a mixture of 99.7\% Ar + 0.3\% C_2H_2 it is possible to obtain a value of X of 0.46 at an E/p of 20 v/cm/torr (figure 4.13). This is only slightly lower than the maximum X value of 0.61 that can be obtained with neon-argon (figure 4.4). The FW products of the two mixtures are also similar (table 2.2). The theoretical resolutions should therefore be comparable.

Another aspect of argon-acetylene is the possible role of the organic admixture gas as a quenching agent. This was first suggested by Heylen (1970). The reduction of secondary avalanches may allow the use of higher gas gains and thus improve the signal to noise ratio. This is especially important with low X-ray energies as only a small number of ion-pairs are created in the gas.

Like other organic quench gases acetylene will eventually become degraded by the action of the electron avalanche.

An advantage of using argon as opposed to neon as the majority gas is the much higher absorption efficiency which results. In the energy range 6-30 KeV for example, the absorption efficiency of 1\% atmosphere-cm of argon is 10 times greater than for neon.
Theoretical Resolution

The theoretical resolution for Ar-C₂H₂ was calculated for the seven avalanche models using the equations for $f$ obtained in chapter three (3.28 to 3.33) and the data of Heylen (1970) presented in figure 4.13. The calculation was simplified by evaluating $f_0$ at $K = 2$ and using the approximation (6.1). The value of $K$ was set at 20 — where the disturbing effects of secondary avalanches are low.

The results of the calculations are given in table 6.5.

<table>
<thead>
<tr>
<th>Model No.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{th}$</td>
<td>0.27</td>
<td>0.48</td>
<td>0.35</td>
<td>0.51</td>
<td>0.59</td>
<td>0.78</td>
<td>1</td>
</tr>
<tr>
<td>$R_{th}^C$ (5.89 KeV)</td>
<td>8.1</td>
<td>10.3</td>
<td>9.0</td>
<td>10.6</td>
<td>11.3</td>
<td>12.8</td>
<td>14.4</td>
</tr>
<tr>
<td>$R_{th}^C$ (22.1 KeV)</td>
<td>4.2</td>
<td>5.3</td>
<td>4.7</td>
<td>5.5</td>
<td>5.8</td>
<td>6.6</td>
<td>7.4</td>
</tr>
</tbody>
</table>

Theoretical values of $f$ and resolution for the cylindrical counter filled with 99.7% Ar + 0.3% C₂H₂. The calculations were made for $K = 20$.

Table 6.5

The counter was filled with 99.5% Ar + 0.5% C₂H₂ as such a mixture was readily available.
Results and Discussion

In figure 6.13 the gas gain is shown as a function of anode voltage. By comparing the linearity of this curve with the corresponding curves for argon-methane and neon-argon one can see that, with this concentration of acetylene, there does not appear to be a significant amount of quenching.

The intrinsic resolution obtained is an improvement over that for argon-methane, but not as good as for neon-argon (see figures 6.14 and 6.15 and plate 6.3). One cannot easily purify an organic admixture gas and so impurities present in the acetylene may have degraded the resolution.

The values of resolution obtained are in good agreement with those of Sipila (1976 and 1979a) who has obtained an optimum intrinsic resolution of about 11.5% at 5.89 KeV.

The resolution results are discussed further in section 6.3.6.

Table 6.6 shows that the results obtained agree with the theoretical prediction that the $R_1 E^3$ product should remain constant as $E$ is varied.

<table>
<thead>
<tr>
<th>X-ray Energy E (KeV)</th>
<th>Intrinsic Resolution at $\bar{K}=10$ $R_1$(%)</th>
<th>$R_1 \times E^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.89</td>
<td>13</td>
<td>31.6</td>
</tr>
<tr>
<td>22.1</td>
<td>6.2</td>
<td>29.2</td>
</tr>
</tbody>
</table>

Experimental values of $R_1 \times E^3$ obtained at X-ray energies of 5.89 and 22.1 KeV with Ar-C$_2$H$_2$.

Table 6.6
Experimental variation of gas gain with anode voltage for the gas mixture 99.5% argon + 0.5% acetylene. The measurements were made at 22.1 KeV.

Figure 6.13
Experimental variation of resolution with gas gain for 99.5% argon + 0.5% acetylene. The measurements were made at 5.89 KeV.

Figure 6.14
Experimental variation of resolution with gas gain for 99.5% argon + 0.5% acetylene. The measurements were made at 22.1 KeV.

Figure 6.15
Spectrum obtained with 99.5% argon + 0.5% acetylene at 22.1 keV. The spectrum was obtained at the value of gas gain corresponding to optimum observed resolution: $K = 18$, $R_o = 7.1\%$.

Plate 6.3
6.3.5 Operation with Argon-Xenon

The Penning mixture argon-xenon was also investigated as a proportional counter filling. As discussed in section 4.3.2 argon-xenon is the only argon-rare gas Penning mixture suitable for use in a proportional counter.

From figure 4.16 the mixture ratio which gives optimum enhancement of charge multiplication in the avalanche (and hence highest ionization efficiency) appears to be $98\% \text{Ar} + 2\% \text{Xe}$. This mixture was chosen as a gas filling. Sipila (1977) however has obtained good values of resolution ($6.2\%$ at $22.1\text{ keV}$ with $K=100$) using the mixture ratio which gives the greatest increase in primary ionization yield, i.e. $80\% \text{Ar} + 20\% \text{Xe}$. With such a mixture the stopping power is improved considerably. In the range between $10\text{ keV}$ and $30\text{ keV}$ the absorption efficiency goes up by about a factor of three.

Due to the lack of experimental measurements of $\alpha/E$ and $F$ it is not possible to calculate theoretical values of resolution for Ar-Xe mixtures.

Figures 6.16 to 6.18 show the gas gain and resolution curves obtained with the counter. To achieve the same gas gain a considerably higher voltage must be applied to the anode than was the case with the other Penning mixtures. The presence of the Penning effect in argon-xenon is indicated though by the fact that even higher voltages had to be applied to argon-methane.

It can also be observed that the linearity of the gas gain curve for argon-xenon extends over a much greater range of $K$ than was observed with the other Penning mixtures. Secondary avalanches would thus appear to have been suppressed. This is also borne out by the way that the resolution does not become
degradated until very high values of $\bar{K}$ are reached, i.e. greater than about 500.

Sipila (1979b) has suggested that this is because uv radiation from xenon is of relatively low energy. In addition the xenon will tend to absorb any uv emitted by the argon.

The resolution measurements obtained are similar to those for Ar-C$_2$H$_2$. The use of argon-xenon in a proportional counter is discussed further in the next section.
Experimental variation of gas gain with anode voltage for 98% Ar + 2% Xe. The measurements were made at 22.1 KeV.
Experimental variation of resolution with gas gain for 98% argon + 2% xenon. The measurements were made at 22.1 KeV.

Figure 6.18
6.3.6 Discussion

In the light of the experimental measurements reported in this chapter it is interesting to compare the values of resolution obtained with those predicted by the various avalanche models.

As was discussed in section 5.7 such a comparison is not easily made at the low values of $\chi$ that are obtained with conventional proportional counter gas mixtures such as argon-methane. It is thus difficult to correlate the data presented in figures 6.4 and 6.5 with any particular model listed in table 6.1.

Such a correlation is easier to make though with Penning mixtures - at high values of $\chi$ the models predict a wider range of resolution values. With neon-argon the results obtained seem to be in closest agreement with models 4 and 5. The comparison was made at fairly low values of gas gain where secondary avalanche effects are negligible. The values given in table 6.3 should still be reasonably accurate at gas gains of around 20 as the range of $E/p$ across the cathode-anode gap is only slightly lower (figure 6.7); in addition the value of $\bar{K}$ is still too high for $f$ to be affected by $\bar{K}$ as predicted by equation (3.39). The theoretical justification for models 4 and 5 (which is a second case form of Legler's model) was however questioned in section 3.7.

The results obtained with argon-acetylene seem to be in closest agreement with model 6. As was discussed in section 6.6, however, the results may have been influenced by impurities.

The modelling of the ionization process in Penning mixtures is analyzed more quantitatively in chapter 9 where the measurements are made with a uniform electric field.
From the results presented in this section the most appropriate Penning mixture for use in a cylindrical counter appears to be argon-xenon. The xenon acts as a quenching agent and so allows operation at high $K$ and thus with a good signal to noise ratio. The absorption efficiency of such a mixture is good and the lack of an organic admixture gas prevents any deterioration with use. With a view to optimizing the resolution it is suggested that the effects of gas mixture ratio and gas purity are investigated.

Due to the lack of measurements of $\alpha/E$ and $F$ it is not possible to correlate resolution measurements with the theoretical models. Lack of $\alpha/E$ and breakdown data prevents the design of a uniform field system for Ar-Xe.
6.4 Quenching of Penning Mixtures

6.4.1 Introduction

In this section the use of Penning mixtures at high values of gas gain is considered further. It was shown in section 6.3.2 that, with neon-argon, the resolution becomes degraded (due to secondary avalanches) at values of gas gain above about 50. With argon-methane the counter was stable up to gas gains of around 1000.

With the gas filling of 99.5% Ar + 0.5% C₂H₂ the organic admixture gas does not seem to be present in a sufficient concentration to give significant quenching. The use of higher levels of acetylene should result in more effective quenching but at the same time will also lead to reduced ionization efficiency (figure 4.13) and thus degraded resolution.

As has already been noted the addition of xenon to argon appears to increase the ionization efficiency and act as a quenching agent.

In this section the problem is considered in a rather different way. Small quantities of methane are added to neon-argon. The subsequent effect on ionization efficiency and secondary avalanches is then assessed.

The presence of small quantities of methane also tends to increase electron mobility. Because the electrons travel through the gas more quickly they are less likely to attach to impurity atoms.
6.4.2 Previous Work

The use of a quench gas with a Penning mixture in a proportional counter has only been investigated by a small number of researchers. Pawlowski (1970) found experimentally that a 5% addition of quench gases (such as methane, nitrogen and butane) to the Penning mixture 99.2% Ar + 0.8% C$_2$H$_2$ did not affect the mean ionization energy W by more than ±0.3 eV. Because the number of ion pairs produced per unit X-ray energy remained the same, Pawlowski suggested that the Pano factor should also be unaltered.

Fuzesy et al. (1972) used mixtures of Ar-Xe-CH$_4$ and Ar-Xe-CO$_2$ in a multi-wire proportional chamber. It was found that such mixtures allowed proportional operation at gas gains of about $6 \times 10^6$ whereas with Ar-Xe the maximum gas gain was only about $10^6$. Some of the quenching effect may have been due to the xenon (as in section 6.3.5). The voltage required to operate the chamber with an argon-xenon-7% quench gas mixture was considerably lower than with a basic argon-quench gas mixture. One may therefore conclude that a considerable amount of Penning ionization was still taking place.
6.4.3 Results and Discussion

The cylindrical counter was filled to 1.1 atmospheres with 99.5% Ne + 0.5% Ar plus various small admixtures of methane. The methane pressure ranged from 0.41 to 2 torr.

The absolute pressure gauge shown in figure 5.1 was only graduated at intervals of 2 torr and so direct filling of the methane was not possible. Instead, the counter was evacuated and filled to a much higher pressure (which was therefore more easily measured). The counter valve was then closed and the relatively large volume of piping adjacent to the counter was evacuated. On re-opening the counter valve, the methane pressure then fell significantly as the gas expanded out to the larger volume. The ratio of the two volumes was previously determined by a number of calibration runs. Once the desired pressure of methane had been passed to the counter the pre-mixed neon-argon could be added directly.

It seems reasonable to expect that the addition of methane to neon-argon may reduce the predominance of the Penning effect. The counter was also filled with pure neon to provide a standard of comparison.

In figure 6.19 the effect of methane concentration on gas gain is indicated. The addition of methane has the effect of extending the range of linearity of the gas gain curves.

As more methane is added though, the gas gain, at the same anode voltage, decreases (in deriving the values of gas gain it was assumed that the methane does not alter the value of W - Pawlowski (1970)). This indicates that the ionization efficiency is being reduced by the methane. It is suggested that some of the metastable neon atoms are de-exciting by collision with,
Values of gas gain obtained as a function of anode voltage for 99.5% neon + 0.5% argon with various admixture concentrations of methane. In all cases the total gas pressure is 1.1 atmospheres. The curves are identified by the corresponding methane pressure. The curve labelled 0 torr CH₄ refers to the basic mixture of 99.5% neon + 0.5% argon at 1.1 atmospheres (i.e. 832 torr Ne + 4 torr Ar). Results obtained with pure neon are also shown for comparison. All the results were obtained at 5.89 KeV.

Figure 6.19
methane molecules. The 3.5 eV energy difference between the metastable level of neon and the ionization level of methane would appear to be too large for the efficient production of methane ions. Figure 6.19 also shows how the addition of methane tends to make the mixture behave more like pure neon. The presence of methane will tend to reduce the energy gained by an electron per cm (due to inelastic collisions with methane molecules). The overall ionization efficiency can be expected to fall.

The quenching effect of the methane can be clearly seen by reference to figures 6.20 and 6.21. As the methane level is increased the resolution obtained at high values of gas gain improves. Unfortunately though, the resolution does not improve to the value obtained at very low values of gas gain with the basic neon-argon mixture. This is presumably due to the reduced ionization efficiency and only partial quenching arising from the addition of methane. The good intrinsic resolution observed with neon-argon at low values of gas gain arises partially because \( f \) decreases with \( \overline{K} \) as governed by equation (3.39). It is inevitable therefore that this contribution to improved resolution will be lost at higher values of gas gain.

In figures 6.20 and 6.21 values of intrinsic resolution, at low values of gas gain, are only shown for the basic neon-argon mixture. This is to improve graphical clarity.

Table 6.7 shows the average resolutions obtained for values of \( \overline{K} \) between 10 and 30 for the various methane levels.
Values of intrinsic resolution obtained as a function of gas gain for 99.5% neon + 0.5% argon with various admixture concentrations of methane. Results obtained with pure neon are also shown. The measurements were made at 5.89 KeV.

- 99.5% Ne + 0.5% Ar (832 torr Ne + 4 torr Ar)
- (99.5% Ne + 0.5% Ar) + 0.41 torr CH₄
- (99.5% Ne + 0.5% Ar) + 1.14 torr CH₄
- (99.5% Ne + 0.5% Ar) + 2 torr CH₄

Figure 6.20
Values of intrinsic resolution obtained as a function of gas gain for 99.5% neon + 0.5% argon with various admixture concentrations of methane. The measurements were made at 22.1 KeV. Pure neon was found to be very unstable at this energy. The various gas fillings are denoted by the same symbols as figure 6.20.

Figure 6.21
<table>
<thead>
<tr>
<th>Methane level (torr)</th>
<th>Intrinsic Resolution (%) at 5.89 KeV</th>
<th>Intrinsic Resolution (%) at 22.1 KeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11.4 ± 0.07</td>
<td>5.92 ± 0.55</td>
</tr>
<tr>
<td>0.41</td>
<td>11.86 ± 0.62</td>
<td>5.80 ± 0.28</td>
</tr>
<tr>
<td>1.14</td>
<td>11.86 ± 0.34</td>
<td>5.86 ± 0.06</td>
</tr>
<tr>
<td>2.0</td>
<td>13.13 ± 0.41</td>
<td>6.38 ± 0.46</td>
</tr>
</tbody>
</table>

Values of average intrinsic resolution obtained at 10 ≤ K ≤ 30 for various methane pressures. The standard deviation limits are given.

Table 6.7

It can be seen from table 6.7 that methane levels greater than 1.14 torr are needed before the intrinsic resolution at low values of gas gain is degraded significantly.

The overall conclusion of this investigation is that it does not appear possible to quench neon-argon with methane without increasing the variation of the primary electron avalanche at high values of $\bar{K}$.

As a practical counter gas argon-xenon seems to be more suitable. Much larger values of gas gain can be used and so signal to noise ratio problems are less important.
6.5 Use of Neon-Argon in a large area proportional counter

The use of Penning mixtures in large area proportional counters has not been previously reported. In this section a brief account is given of some measurements made with a prototype EXOSAT, low energy proportional counter filled with neon-argon.

A schematic diagram of the counter is given in figure 6.22. The anode consists of a matrix of wires running across the counter as shown. A cathode guard strip is provided for picking up signals from events taking place near the ends of the anode wires. These signals are used for obtaining good background rejection (Bailey et al (1978)). The effective area of the counter is about 500 cm².

The counter was filled to 1.1 atmospheres with 99.5% Ne + 0.5% Ar. The rather high capacitance of the electrode systems lead to high pre-amplifier noise levels. Table 6.8 shows values of intrinsic resolution obtained at 5.89 KeV and 22.1 KeV. The counter was operated in two modes - with HT applied to the anode and HT applied to two side mounted guard wires. These wires have lower capacitance and lead to reduced electronic noise.

In table 6.9 values of observed resolution ($R_O = R_T$ at high values of $K$) obtained by Bailey et al with conventional gas mixtures are shown for comparison. One can compare the results shown in table 6.8 with those reported in section 6.3.2 for the single wire cylindrical counter filled with neon-argon. Rather better resolution was obtained with the small counter. Because the large area counter has much higher capacitance the counter has to be operated at higher gas gain (for the same signal to noise ratio). At higher gas gains the disrupting effect of secondary avalanches becomes more prominent and the resolution is degraded. The resolution obtained is similar to that observed with more conventional fillings - table 6.9.
Schematic Diagram of EXOSAT Low Energy Counter

Figure 6.22

(From Bailey et al (1978))
Values of optimum intrinsic resolution obtained from EXOSAT low energy detector filled to 1.1 atmospheres with 99.5% Ne + 0.5% Ar.

<table>
<thead>
<tr>
<th>Mode of Operation</th>
<th>Optimum $R_I$ at 5.89 KeV (%)</th>
<th>Optimum $R_I$ at 22.1 KeV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HT applied to anode</td>
<td>$16.2 \pm 0.4$ ($K=86$)</td>
<td>$9.4 \pm 0.3$ ($K=52$)</td>
</tr>
<tr>
<td>HT applied to guard</td>
<td>$14.6 \pm 1.0$ ($K=60$)</td>
<td>$9.8 \pm 0.2$ ($K=57$)</td>
</tr>
</tbody>
</table>

Values of optimum observed resolution obtained from EXOSAT low energy detector with two conventional gas fillings. Similar results were obtained with HT applied to the anode or guard. These results were obtained by Bailey et al (1978).

<table>
<thead>
<tr>
<th>Gas Filling (2 atm)</th>
<th>Optimum $R_0$ at 5.89 KeV (%)</th>
<th>Optimum $R_0$ at 22.1 KeV (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95% Ar + 5% CO₂</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td>95% Xe + 5% CH₄</td>
<td>not measured</td>
<td>9</td>
</tr>
</tbody>
</table>
It may be possible to reduce the pre-amplifier noise by using a number of input stage F.E.T.'s in parallel (as reported by Smith and Cline (1966) - section 5.3.3). From the results obtained in section 6.3 it is suggested that argon-xenon is investigated as a potential gas filling for large area counters.
CHAPTER 7: DESIGN AND CONSTRUCTION OF RESEARCH CHAMBER

7.1 Introduction

7.2 Design Requirements

7.3 Construction of Research Chamber
   7.3.1 Window arrangement
   7.3.2 Connection to pumping system and sealing
   7.3.3 Electrode arrangement
   7.3.4 Electrical system
   7.3.5 Preparation of components
7 Design and Construction of Research Chamber

7.1 Introduction

The underlying philosophy relating to the measurement of the behaviour of Penning mixtures in a uniform field proportional chamber was outlined in chapter one. In this chapter the physical realization of such a chamber is described. The general design requirements are summarized and then translated into a practical engineering device. The constructional procedures involved in producing the device are outlined.

7.2 Design Requirements

The essential requirements of the chamber are that separate uniform field regions are provided for absorption of the X-rays and gas amplification of the original charge. Such requirements can be met by the use of planar electrodes - figure 1.6 showed the general arrangement. Part of the cathode must be transparent to X-rays in the energy range of interest - 6KeV was chosen for use in this study for reasons relating to ease of absorption by the gas and pre-amplifier noise level.

The essential requirements of the grid relate to shielding between the two regions, transmission of electrons and field uniformity. It is also important that the grid to anode distance is kept constant.

To operate with the highest possible value of $\alpha/E$, with Ne-Ar, it is necessary to have an electric field in the
avalanche region of 3 v/cm/torr and a mixture ratio composition of 99.9% Ne + 0.1% Ar (figure 4.4). To keep secondary avalanches to a minimum it is also necessary to operate at a voltage significantly below the breakdown voltage corresponding to the p.d (pressure x distance product) of the gap (figure 4.5). In practice, these two requirements dictate that the grid-anode gap must be rather small. For example, if the chosen pressure is 1 atmosphere then E = 2280 v/cm. To obtain a reasonable gas gain (say 50) equation (4.5) dictates that the gap length must be about 0.5 mm. The voltage required - 114v - is then well below the breakdown voltage of about 170v.

Owing to the mechanical tolerance difficulties involved in constructing such a shallow grid-anode region it was decided to first operate at a lower pressure and with a correspondingly higher gap. With a pressure of 1 atmosphere the electric field required is only 570 v/cm. A suitable operating voltage, of about 120v, can now be applied across a gap of 2 mm. Following experience of operating the chamber in this configuration, modifications were subsequently made to allow operation at one atmosphere (thus reducing electron cloud diffusion in the drift region). This, and other development work, is fully described in chapter eight.

In designing the electrode system so that there is 100% transparency to electrons it is necessary to consider the voltage which will be applied to the grid to ensure that the electrons are transported across the drift region without recombination or
Drift velocity of electrons in neon and neon-carbon dioxide mixtures.

Figure 7.1
(From English and Hanna (1955))
attachment taking place. The electron drift data for pure neon obtained by English and Hanna (1955) indicates that the E/p value required for the drift region is about 0.02 v/cm/torr (figure 7.1). Working at a pressure of $\frac{1}{2}$ atmosphere and with a drift distance of 2 cm (see section 7.3.3) it is thus necessary to have about $7\frac{1}{2}$ volts applied to the grid. Such a value was used for the transparency calculations.

The other main requirements of the chamber relate to evacuation and gas purity. It is important that the chamber can be pumped down to very low pressures so that the quantity of gas remaining in the chamber prior to fill is significantly less than the impurity content of the gas filling. The device may have to operate for a period of several weeks without significant deterioration of gas purity. To meet these requirements low outgassing materials and very low leak rate sealing and valving techniques were used. Any potential "trapped volumes" were vented.

In the next section the realization of these requirements is outlined.

7.3 Construction of Research Chamber

A cross-sectional view of the research chamber is provided in figure 7.2. The upper enclosure, which houses the grid and anode, is formed from a single section of stainless steel. This is bolted down onto a stainless steel base plate. Gas sealing is provided by means of an indium wire gasket.
Cross-sectional view of upper chamber assembly showing detail of window arrangement. The beryllium disc is brazed to a stainless steel ring. The ring is welded to the cathode. A second ring is used to keep the cathode plane flat. A detail is also shown of the \( \frac{1}{4} \) inch pipe used for evacuation and filling.

Figure 7.3
7.3.1 Window Arrangement

A detailed drawing of the window arrangement is shown in figure 7.3. The window itself is constructed from a 30 mm diameter section of 0.01 inch thick beryllium sheet. This will reduce the flux of 6 KeV X-rays by 10%. The beryllium disc was brazed to a supporting stainless steel ring. This assembly was then welded to the cathode plane as shown in figure 7.3. A ring was provided to keep the surface of the cathode plane flat over the welded surfaces.

The window was designed to allow the chamber to be evacuated with atmospheric pressure on the outside. Prior to welding to the cathode the window was pressure tested with a pressure difference of 22 p.s.i. applied both ways across the window.

7.3.2 Connection to Pumping System and Sealing

To provide for evacuation and filling, a length of ½ inch outer diameter stainless steel tubing was welded to the chamber side wall as shown in plate 7.1. For sealing and connection to the gas handling system a "Nupro" valve and "Gajon" coupling were used. Both these fittings are of all metal construction to reduce outgassing.

When fully assembled the research chamber was tested for possible gas leakage using the method described in section 5.2.2. The measurement indicated that the leak rate was $< 10^{-8}$ torr.litres/second - the sensitivity limit of the instrument used. The lack of significant leakage was also indicated by the stable operation of the chamber subsequently observed.
The techniques used to purify the gas were outlined in chapter five. The influence of gas purity on the operation of the chamber is examined in chapter eight.

7.3.3 Electrode Arrangement

The general arrangement of the electrodes can be seen from figure 7.2 and plate 7.2. Figure 7.4 is a plan view looking down from below the window. Between the window plane and grid, a guard electrode is provided. This simply serves to help keep the electric field uniform in the drift region. It is located three quarters of the way down the drift region and so is held at three quarters the grid potential. The central hole was made 3 cm in diameter to allow the diffusing electron cloud to drift through. The calculation of electron cloud diffusion is considered in chapter eight.

The construction of the grid-anode system can be inferred from figures 7.5 and 7.6. The grid was wound round a stainless steel frame. A lathe was used to rotate the frame and tensioning was provided by means of a feed spool with compressed air braking.

The anode was formed from a gold evaporated ceramic block - figure 7.5. This construction allowed the anode itself to have a smooth finish and, in addition, the distance between grid and wires could be accurately defined by ridges running along the side of the block.

The ceramic block was carefully inserted between the wires and grid frame and tensioned against the grid by bolts threaded from underneath the frame and fitting into recesses in the block. Electrical connection to the anode was provided by means of a gold coated wire clip which was located by a recess in the ceramic.
Full-size plan view of research chamber with top cover removed. The grid and guard-ring are shown. The detail shows the arrangement for supplying HT to the anode.

Figure 7.4
Full-size drawing of ceramic block. The gold coated surface forms the anode. The two ridges are used to accurately define the distance between grid and anode. The tolerance on the grid-anode distance was chosen so that gain variations across the gap were less than ± 2%. 

Figure 7.5
Full-size drawing of grid frame. Tensioning bolts are used to hold the ceramic block against the grid wires.

Figure 7.6
Feed-throughs to the screened lower enclosure containing the preamplifier were provided as shown in figure 7.4.

To ensure that significantly more radiation would be absorbed in the drift region rather than the avalanche region the former was made ten times deeper than the latter. A drift distance of 2 cm was thus used. The chosen area of the electrodes was dictated by requirements relating to field uniformity and ease of grid winding.

The choice of grid wire diameter and pitch was made by reference to the work of Buneman, Cranshaw and Harvey (1949). They have examined theoretically aspects of a parallel plate electrode system forming a grided ionization chamber - figure 7.7. In particular they have considered the ability of the grid to screen the drift and avalanche regions from each other, the transparency of the grid to electrons and the uniformity of the electric field.

The inefficiency of grid shielding (i.e. the extent to which the grid fails to shield the two regions from each other) is denoted by $Y$ and is defined in the following way

$$Y = \frac{dX_B}{dX_Q} \quad (7.1)$$

where $X_B$ is the field between grid and anode

$$(X_B = (V_B - V_G)/b_o - \text{figure 7.5})$$

$X_Q$ is the field between the ion cloud at $Q$ and the grid wires (figure 7.5).

In practice, $X_Q = X_A = (V_G - V_A)/a_o$ if the charge density due to the ionizing radiation is low. Buneman et al have shown that $Y$ can be expressed

$$Y = \left(\frac{e}{2\pi b}\right) \ln \left(\frac{e}{2\pi r}\right) \quad (7.2)$$
Diagram showing the geometrical arrangements of electrodes in a parallel plate chamber. The symbols are explained in the text.

Figure 7.7
(From Buneman, Cranshaw and Harvey (1949))

The relation between the ratio of grid radius to grid-wire separation ($r/g$) and the ratio of grid-wire separation to grid-collector spacing ($g/b$) for various values of constant inefficiency $Y$.

Figure 7.8
(From Buneman, Cranshaw and Harvey (1949))
where $g$ is the wire pitch

$r$ is the wire radius

$$b = b_0 - \pi(r^2/g)$$

Figure 7.8 shows the relation between the ratio $(r/g)$ and $(g/b)$ for constant inefficiency. Using equation (7.2) appropriate values of $r$, $g$ and $b$ may be selected to give the desired value of $Y$.

For the grid to be fully transparent to electrons $X_A$, $X_B$, $r$ and $g$ have to satisfy the following inequality:

$$\frac{X_B}{X_A} > \frac{1 + 2\pi r/g}{1 - 2\pi r/g} \quad (7.3)$$

Figure 7.9 shows the proportion, $1 - \omega$, of lines of force that end on the anode rather than the grid (where $\omega$ is the fractional "grid loss") plotted as a function of $X_B/X_Q$ for different values of the parameter $R = 2\pi r/g$. If $X_B/X_Q$ is chosen such that $1 - \omega = 1$, i.e. $\omega = 0$, for the appropriate value of $R$ then the grid should be fully transparent.

The analysis of Buneman et al, and others (Maxwell (1904), Abraham (1919) and Von Laue (1919)) reveals that the electric field between grid and anode becomes uniform very close to the grid. It can be taken as being uniform at a distance from the grid of about half the grid pitch ($g/2$).

Buneman et al have made experimental measurements of grid shielding and transmission and have found good agreement with their theory.

Using the above analysis the grid was designed to have 25 micron diameter wires at a pitch of 100 microns. Such a configuration gives virtually complete transparency to electrons whilst providing effective shielding between the drift and avalanche regions. The electric field will be uniform from about 50 microns below the wire.
The relation between the proportion of lines of force that end on the grid rather than on the collector \((1-\omega)\) and the ratio of the electric fields \(X_B\) and \(X_Q\) for different values of \(R = 2mr/g\).

**Figure 7.9**
(From Buneman, Cranshaw and Harvey (1949))
This distance is considerably less than the distance to the first multiplication in the gap. Using equation (4.5) one can show for example that with a gas gain of 45 the first multiplication takes place at a distance of 660 microns from the grid.

Because of the choice of such a low value of grid wire diameter, tungsten wire was used to give adequate strength.

7.3.4 Electrical System

The preamplifier used was of the type developed by Smith and Cline (1966) and was described in section 5.3.3. The preamplifier was mounted so that a short direct connection to the anode feedthrough was possible - thus reducing stray capacitance and electronic noise.

The anode was supplied via a $10^6 \Omega$ resistor. The thermal noise of a resistance R is proportional to $R^{-\frac{1}{2}}$ so that the choice of such a value results in a small contribution to noise.

The guard ring and grid were supplied via suitable low pass filters to reduce interference.

A suitable capacitor was connected between the grid and ground to provide a return path for the signal developed by the positive ions moving between anode and grid. This can be appreciated more easily by reference to figure 7.10.
Current flow in electrode system. A capacitor is connected between grid and ground to provide a return path. The value of the capacitor is chosen so that there is negligible impedance to the signal waveform.

Figure 7.10
7.3.5 **Preparation of Components**

Prior to final assembly the various internal components of the research chamber were extensively cleaned and baked to reduce the likelihood of gas contamination. The procedure involved the use of detergents and ultrasonic cleaning with solvents. Following rinsing, the components were baked at about 100°C for several hours. This procedure reduces outgassing during subsequent evacuation and baking of the chamber prior to filling with gas.

The chamber described above (and used in conjunction with the electronic system outlined in section 5.3.4) formed the basic instrument used in the study of avalanche fluctuations in Penning mixtures under the condition of a uniform electric field. In chapters 8 and 9 the experimental programme carried out with the chamber is described.
CHAPTER 8 : RESEARCH CHAMBER DEVELOPMENT

8.1 Introduction
8.2 Initial Testing
8.3 Effect of Gas Purity
8.4 Effect of Argon Concentration
8.5 Effect of Gas Pressure
8.6 Development of Chamber Grid
8.7 Drift Characteristics
8 Development of Chamber

8.1 Introduction

In this chapter the development of the research chamber is described. The initial testing of the chamber (outlined in the next section) suggested that spurious effects were taking place. Such effects tend to increase the variation in the avalanche size and will therefore give rise to misleading results.

In the following pages the development programme is described. Disturbing effects are investigated and eliminated as far as possible.
8.2 Initial Testing

In its preliminary form the chamber was operated in the mode summarized by the following table:

<table>
<thead>
<tr>
<th>ft. length</th>
<th>Avalanche Length</th>
<th>Grid Wire Diameter</th>
<th>Grid Wire Pitch</th>
<th>Shield Hole Diameter</th>
<th>Gas Mixture</th>
<th>Gas Pressure</th>
<th>Additional Purification</th>
<th>Pumping Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>2.12 mm</td>
<td>25 μ</td>
<td>100 μ</td>
<td>3 cm</td>
<td>Ne/Ar 99.9/0.1</td>
<td>190 torr</td>
<td>None †</td>
<td>3 days at 100°C</td>
</tr>
</tbody>
</table>

† B.O.C. "Research Grade" gas was used.

Table 8.1

The chamber was initially tested with a few volts applied to the grid and about 135 V across the grid-anode gap. Using a 5.89 KeV Fe55 source (collimated to give a 2-3 mm radius beam) "proportional counter type" pulses with a rise time of about 25 μsec were observed.

The first experimental requirement was to ascertain (with reference to chapter 7) the characteristics of the drift region of the chamber. It is important to ensure that the electrons produced in the initial ionization are transported to the avalanche region without loss due to recombination etc. The drift characteristics, however, are somewhat peripheral to the main objective of the work and so are only summarized here. A fuller discussion is provided later in section 8.7. It was found that the correct avalanche charge (as predicted by Kruithof and Penning (1937)) was obtained with a grid voltage of about 2 V (figure 8.1).

The general shape of the pulse height analyser spectrum was found to be quite dependent on the drift and avalanche fields. For a fixed drift field, the behaviour of the spectrum as a function of the anode voltage was found to be similar to that of a conventional...
Theoretical (●) and experimental (○) variation of avalanche charge with grid-anode voltage ($V_{ga}$). The theoretical variation shown does not take account of secondary avalanches and so is only accurate at low values of $V_{ga}$. A grid voltage of about 2 V gives the correct final charge.

Figure 8.1
cylindrical counter. The spectrum broadens as the voltage is increased towards the breakdown region. At lower voltages the resolution improves but the spectra become considerably broadened by electronic noise at low values of gas gain.

For a fixed field in the avalanche region, the shape of the spectrum was found to vary with grid voltage. The spectra produced tended to be asymmetrical about an axis passing vertically through the peak channel. The section of the spectrum to the right of the peak tended to be steeper than that on the left - see figure 8.2. This asymmetry was more prominent at low grid voltages.

If one assumes that the asymmetrical nature of the spectra is due to some charge loss effect in the drift region of the chamber then it seems reasonable to use the steeper upper side when considering what is actually happening in the avalanche region. The resolution obtained by assuming that the spectra should be symmetrical in the above way is denoted by \( R_F \) - the "forward resolution". In table 8.2 the best forward intrinsic resolution \( R_{FI} \) (i.e. with electronic noise subtracted) obtained with a 5.89 KeV Fe\(^{55}\) source is shown for a variety of grid voltages. Optimization of resolution clearly occurs with a grid voltage of about 2V.

<table>
<thead>
<tr>
<th>Grid Voltage ( V_g ) (volts)</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum ( R_{FI} ) (%)</td>
<td>17.7</td>
<td>16.9</td>
<td>21.9</td>
<td>23.9</td>
<td>24.5</td>
<td>32.2</td>
</tr>
</tbody>
</table>

Optimum values of forward intrinsic resolution obtained at 5.89 KeV for a number of grid voltages.

Table 8.2
Typical shape of spectrum observed under conditions of initial testing for $V_g = 2\, \text{V}$, $V_{ga} = 130\, \text{V}$.

*Figure 8.2*
In an experimental study of this nature it is important that any disturbing effects are eliminated or adequately accounted for. This is particularly the case here as the investigation is concerned with measuring fluctuation phenomena. Any disturbing effects will tend to increase the observed variation and so will lead to an over-estimate of the true situation.

The initial testing of the chamber indicated that disturbing effects were taking place. The poor energy resolution (greater than predicted by any of the models) and the non-symmetrical nature of the charge pulse height spectra suggested that further investigation was needed.
8.3 Effect of Gas Purity

The presence of a significant quantity of impurities will affect the operation of both the drift and avalanche regions of the chamber. In the drift region, electron attachment to impurities will, of course, result in a proportionally reduced charge being formed in the subsequent avalanche. Such behaviour would explain the asymmetrical m.c.a. spectra produced.

In the avalanche region impurities will again increase the variation in the final charge. This is because the impurities have different ionization energies from the admixture gas, argon. The distance an electron has to travel before ionization can occur will therefore vary. The local variation in the value of \( \alpha \) will lead to increased variation in the overall gas amplification.

The impurity level of the gas in the chamber will be increased above the supply value by outgassing from the chamber walls and components, and by gas flow from any trapped volumes present. To ascertain whether effects of this nature were significantly affecting the results, the chamber was ion pumped and baked for ten days (more than three times as long as for the previous fill). A comparison of the resolution obtained with 2 V applied to the grid shows little change from the first fill (see table 8.3).

Having demonstrated that the chamber itself was not contributing significantly to the impurity level, the next step was to further purify the "Research Grade" gas supplied by B.O.C. This purification was achieved using a titanium sublimation pump (the procedure involved was described in section 5.2.3).

The improved gas purity resulted in a considerable reduction in the low energy part of the spectra. The spectra therefore became much more symmetrical. Table 8.3 shows the resolution obtained using
<table>
<thead>
<tr>
<th>Fill Key</th>
<th>ANODE VOLTAGE (VOLTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>122</td>
</tr>
<tr>
<td>1</td>
<td>(R_{NO}(%))</td>
</tr>
<tr>
<td></td>
<td>(R_{FO}(%))</td>
</tr>
<tr>
<td>2</td>
<td>(R_{NO}(%))</td>
</tr>
<tr>
<td></td>
<td>(R_{FO}(%))</td>
</tr>
<tr>
<td>3</td>
<td>(R_{NO}(%))</td>
</tr>
<tr>
<td></td>
<td>(R_{FO}(%))</td>
</tr>
<tr>
<td>4</td>
<td>(R_{NO}(%))</td>
</tr>
<tr>
<td></td>
<td>(R_{FO}(%))</td>
</tr>
<tr>
<td>5</td>
<td>(R_{NO}(%))</td>
</tr>
<tr>
<td></td>
<td>(R_{FO}(%))</td>
</tr>
</tbody>
</table>

Forward and Normal observed resolutions (\(R_{FO}\) and \(R_{NO}\)) obtained at a variety of anode voltages with 5.89 KeV radiation \(V_g = 2V\).

**Conditions**

1. Ion pump and bake chamber for 3 days.
2. Ion pump and bake chamber for 10 days.
3. Purify gas using T.S.P. at \(\frac{1}{2}\) rated current.
4. Purify gas using T.S.P. at full rated current. Gas was left in purifier for 24 hours.
5. Purify gas using T.S.P. at full rated current. Gas was left in purifier for 72 hours.
the T.S.P. in three different modes. It can be seen that $R_{PO}$ and $R_{NO}$ are now similar ($R_{NO}$ is the normally defined observed resolution). These results seem to indicate that some form of attachment process was taking place in the previously unpurified gas.

Attempts to purify the gas using an internally mounted getter were less successful. A S.A.E.S. ST171/HI/7-6/150 getter was fitted inside the chamber and electrically connected to the outside via a spare feed-through (gettering devices of this type were described in section 5.2.3). The getter was first fired with the chamber at high vacuum ($<10^{-8}$ Torr). The research-grade gas was then admitted to the chamber. The results obtained were very similar to those of the initial fills made with unpurified research grade gas. With 2 V on the grid and 140 V on the anode values of $R_{NO}$ and $R_{PO}$ of 43% and 30% were obtained.

It was found that firing the getter with the chamber operating caused the charge pulses to decrease in magnitude - eventually to below the preamplifier noise level. During firing, gases are emitted from the getter and it is suggested that electrons are lost by attachment. Eventually, however, the gases return to the getter and the chamber behaves as before. It seems however that over a timescale of a few days the getter was unable to lower the intrinsic impurity level significantly. Sipila (1979b) found that the resolution of a small cylindrical counter improved over a period of about a month when fitted with a similar gettering device.
8.4 **Effect of Argon Concentration**

It has been suggested (Sipila (1977)) that with a Ne-Ar filling in a cylindrical counter the best energy resolution is obtained with a mixture ratio of 996:4. To ascertain as to whether this might be the case for a uniform field device working at 1 atm pressure, the chamber was filled with 99.5% Ne + 0.5% Ar at that pressure. Measurements of $\alpha/E$ against $E/p$ and breakdown characteristics have not been reported for such a mixture. However, the measurements made for argon percentages of 0.1 and 1.0 may give an indication of behaviour for the case of 0.5%. The resolution obtained for the filling 99.5% Ne + 0.5% Ar was found to be very similar to that obtained with 99.9% Ne + 0.1% Ar (table 8.4).

<table>
<thead>
<tr>
<th>ANODE VOLTAGE (VOLTS)</th>
<th>140</th>
<th>145</th>
<th>150</th>
<th>155</th>
<th>160</th>
<th>170</th>
<th>180</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{NO}(%)$</td>
<td>28.7</td>
<td>24.9</td>
<td>23.4</td>
<td>23.2</td>
<td>27.7</td>
<td>24.4</td>
<td>31.8</td>
</tr>
<tr>
<td>$R_{FO}(%)$</td>
<td>26.4</td>
<td>24.5</td>
<td>21.4</td>
<td>22.7</td>
<td>19.1</td>
<td>22.6</td>
<td>29.8</td>
</tr>
</tbody>
</table>

Forward and Normal observed resolutions obtained with a variety of anode voltages with a $^{55}$Fe source. Grid voltage = 2 V. Gas filling of 99.5% Ne + 0.5% Ar purified with T.S.P. at 190 torr.

Table 8.4

The effect of argon concentration is discussed further in chapter 9.
8.5 Effect of Gas Pressure

In order to get a clearer picture of the behaviour of 99.9% Ne + 0.1% Ar at \( \frac{1}{2} \) atmosphere, the cylindrical counter (as used in chapter 6) was filled with the above gas at pressures of \( \frac{1}{2} \) atmosphere and 1.1 atmospheres. It was found that the resolution was not as good at the lower pressure — see table 8.5. Possible reasons for this include increased photo-electron track length and electron cloud diffusion — both of which may result in electron loss to the counter walls. The results obtained in this comparison indicated operation of the chamber at 1 atmosphere. The main advantages which arise from the operation of a proportional counter at a higher pressure are as follows:

1. Increased absorption of ionizing radiation.
2. Reduced photo-electron track length.
3. Reduced electron cloud diffusion.
4. Reduced pulse rise-time (if the avalanche region depth is reduced in inverse proportion to the pressure increase).

These factors are quantified below.
Average Intrinsic Resolution obtained with cylindrical counter filled with 99.9% Ne + 0.1% Ar at pressures of 0.25 and 1.1 atmospheres. The values shown are averages for the gas gain range 17.5 < \bar{K} < 53 (i.e. the range where secondary avalanche effects are small). The best intrinsic resolution obtained is also given.

Table 8.5

<table>
<thead>
<tr>
<th>Gas Pressure (atm)</th>
<th>Average Intrinsic Resolution (%)</th>
<th>Optimum Intrinsic Resolution (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>18.2</td>
<td>15.3 ± 0.8</td>
</tr>
<tr>
<td>1.1</td>
<td>15.9</td>
<td>13.1 ± 1.0</td>
</tr>
</tbody>
</table>
X-ray absorption

With a drift length of 2 cm and a pressure of 1 atmosphere the absorption efficiencies at 6 KeV and 20 KeV are 2.2% and 0.06% respectively. Increasing the pressure to 1 atmosphere improves these by a factor of about 4 to give 8.5% and 0.24% respectively.

Photo-electron track length

It was not possible to resolve Cd$^{109}$ 22 KeV X-rays with the avalanche chamber. The reason for this may be connected with the rather long photo-electron track lengths involved. The table below (obtained using data derived from Charpak and Sauli (1978)) shows values of photo-electron range in Neon at 6 KeV and 22 KeV for a number of gas pressures.

<table>
<thead>
<tr>
<th></th>
<th>1/4</th>
<th>1/2</th>
<th>3/4</th>
<th>1</th>
<th>PRESSURE (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 KeV</td>
<td>4.3</td>
<td>2.1</td>
<td>1.4</td>
<td>1.1</td>
<td></td>
</tr>
<tr>
<td>22 KeV</td>
<td>51.0</td>
<td>25.5</td>
<td>17.0</td>
<td>12.8</td>
<td></td>
</tr>
</tbody>
</table>

Photo-electron range in Neon (units are mm).

Table 8.6
Electron Cloud Diffusion

As the electron cloud drifts towards the grid it diffuses outwards radially, forming a sphere of increasing size. This would lead to difficulties if sections of the electron cloud were to diffuse away from the grid-anode region and out to the chamber walls.

Einstein's analysis of Brownian motion leads to the following equation for $\bar{x}$ the average distance diffused (English and Hanna (1953)):

$$\bar{x} = 0.17 \sqrt{\frac{LR}{E}}$$

(8.1)

where $L$ is the length of the drift space.

$R$ is the ratio of energy of agitation of free electrons to energy of agitation of gas molecules.

$E$ is the electric field strength.

Note that $\bar{x}$ is independent of the drift velocity and for fixed values of $L$ and $E$ depends only on $\sqrt{R}$.

Using published values of $R$ (Healey and Reed (1941), see table 8.7) and the knowledge that $R=1$ for $E/p = 0$ one can estimate values of $R$ corresponding to the low values of $E/p$ used in the drift region of the chamber. Table 8.8 shows values of $\bar{x}$ obtained for the drift configurations $p = \frac{1}{2}$ atm, $L = 2$ cm and $p = 1$ atm, $L = 0.5$ cm for a variety of grid voltages.

<table>
<thead>
<tr>
<th>$E/p$</th>
<th>$R$</th>
<th>R values in Neon</th>
<th>E/p values are in v/cm/torr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>23.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>34.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>158</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 8.7

(From Healey and Reed (1941))
<table>
<thead>
<tr>
<th>GRID VOLTAGE (VOLTS)</th>
<th>$\bar{x}$ [for $p = \frac{1}{2}$ atm, $L = 2$ cm] (cm)</th>
<th>$\bar{x}$ [for $p = 1$ atm, $L = 0.5$ cm] (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.63</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.44</td>
<td>0.11</td>
</tr>
<tr>
<td>10</td>
<td>0.11</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Calculated values of mean drift distance $\bar{x}$ (in cm) for neon for the two drift region configurations shown.

Table 8.8
Due to the factor of four reduction of X obtained by using the second configuration, it was decided to adopt this for the work at 1 atmosphere.

**Pulse Rise Time**

In order to operate the avalanche region at the optimum value of electric field (3 v/cm/torr) at a pressure of 1 atmosphere it is necessary to reduce the grid-anode distance by a factor of four. This is necessary because the Paschen curve for 99.9% Ne + 0.1% Ar increases only very slowly with p.d in the region of interest (figure 4.5).

By reducing the avalanche distance by a factor of four the same voltage can be used as before and the operating point is on the lower side of the Paschen curve.

Because we are working at the same E/p value, but with a gap four times shorter, the pulse rise-time should be similarly reduced. This allows the use of shorter main amplifier time constants and helps to reduce acoustic noise.

Due to the above considerations it was decided to reduce the cathode to grid, and grid to anode distances by a factor of four and to operate the chamber at 1 atmosphere. Operation at pressures above 1 atmosphere was not considered as such operation would result in an outward force on the beryllium window.

Before modifying the electrode system an attempt was made to operate the chamber at 1 atmosphere with the original spacings. The Townsend data (figure 4.4) indicated that the device would still function as a proportional counter but with rather low gas gain and poorer resolution (due to the weaker electric field that had to be used to keep below the breakdown voltage).

In practice it was found that the weaker electric field resulted in pulses with very long rise times - of the order of 120 μsec. These were much too long for the multi-channel analyser.
To overcome this problem the electronic system shown in figure 8.3 was set up. A single-channel analyser measures the point where the signal crosses zero (goes just above the noise) and immediately produces a square-wave pulse. This pulse is sent to a delay unit which is set to produce a pulse after a time interval equal to the rise time of the signal. This pulse triggers a gate which feeds what should now be the peak value of the signal to the m.c.a.

Due to the rather low signal to noise ratio, and some pulse rise time variation, the system was not really able to give a clear picture of the signal variation and so it was not possible to make accurate resolution measurements.
Alterations to Experimental Chamber

To reduce the cathode to grid distance down to 0.5 cm, the insulators supporting the grid assembly away from the base plate were increased in length by 1.5 cm. With the aim of reducing outgassing, the material used was changed from pyropholite to ceramic. Because the window to grid distance was now only 0.5 cm the guard ring was removed. The shallower avalanche region was fabricated from a new block of ceramic using the same techniques as before. The new grid-anode distance was 0.48 mm. Initially the original grid was used.

The chamber was filled to 1 atmosphere with 99.9% Ne + 0.1% Ar after being ion pumped for 3 days at about 100°C. The gas was fed to the T.S.P. for 1 day before being admitted to the chamber.

On initial testing the chamber was found to give much shorter pulses (about 3 μsec at low anode voltages) and improved resolution.

Drift Characteristic

As before it was first necessary to examine the drift characteristics of the chamber. As described in section 8.7 the chamber was found to operate satisfactorily with a grid voltage of 5-10 V.

Pulse Rise Time

It is necessary to measure the pulse rise time in order to use the gas gain measuring technique outlined in section 5.3.1. The rise time was measured by observing the un-shaped pulses on an oscilloscope. The variation of pulse rise time with grid-anode voltage (V_{ga}) is shown in figure 8.4. The long rise times observed close to the breakdown voltage are caused by the positive ions drifting back to the cathode to initiate secondary avalanches.
Experimental variation of pulse rise-time with $V_g$ for 99.9% Ne + 0.1% Ar.
$E/p \approx 3$ V/cm/torr, avalanche gap = 0.48 mm. The breakdown voltage = 165 V.

Figure 8.4
Gas Gain

A typical gas gain curve is shown in figure 8.5. The curve is in good agreement with the theoretical prediction based on the data of Kruithof and Penning (1937) (figure 4.4). The sinusoidal fluctuation in the gain (discussed in section 4.2.1) can be seen. This is considered in more detail in chapter 9.

Resolution

Operation at the higher pressure of 1 atmosphere resulted in a considerable improvement in resolution - figure 8.6. An optimum intrinsic resolution of about 12-13% was obtained.
Theoretical and experimental variation of gas gain with grid-anode voltage for 99.9% Ne + 0.1% Ar. The theoretical curve was obtained using Townsend's first ionization coefficient and so is only valid at low values of voltage. The grid voltage = 10V.

Figure 8.5
Experimental variation of observed and intrinsic resolution with gas gain for 99.9% Ne + 0.1% Ar at 1 atmosphere. The measurements were made at 5.89 keV.

Figure 8.6
8.6 Development of Chamber Grid

The original grid assembly consisted of 25 micron diameter wires spaced at 100 microns. As discussed in section 7.3.3 the electric field becomes uniform at a distance of about 50 microns below the grid plane. This is considerably less than the average distance of 660 microns to the first ionization when the gas pressure is 1 atmosphere and $K = 45$. At the higher pressure of 1 atmosphere, however, the first ionization takes place at an average distance of 165 microns. This means that, with the original grid, the electric field will be uniform for 70% of the first ionization mean free path.

To confirm that the electric field was sufficiently uniform, a second grid was constructed with 8 micron diameter wires at a pitch of 25.4 microns. The electric field now becomes uniform at about 13 microns below the wire plane and so is uniform for 92% of the first ionization mean free path.

The thinner wire required rather more delicate handling and a different technique had to be adopted to wind the grid. To eliminate the fluctuation in wire tension which arose when winding around the grid frame, the wire was first wound round a stainless steel cylinder (figure 8.7). A trough cut along the length of the cylinder was used to hold two stainless steel bars. Once the wire had been wound it was bonded to the tops of the two bars. The wire between the bars could then be cut and the grid assembly removed from the cylinder. An arrangement was provided on the grid frame to mount and tension the wires. The pitch of the completed grid was found to be constant to within a few microns.

It was found that the use of the new grid did not lead to any improvement in resolution. It was therefore concluded that the electric field distribution obtained with the original grid was sufficiently uniform.
Figure 8.7

Arrangement for winding chamber grid

GRID WIRE

SIDE VIEW

STAINLESS STEEL CYLINDER

END VIEW

BARS
By this stage the main experimental conditions of the experiment i.e. gas purity, gas pressure and grid configuration had been examined. The conditions were arranged so that, as far as possible, the results were not disrupted by secondary effects. The main results, obtained under these conditions, are presented and discussed in chapter 9.
8.7 Drift Characteristics

In this section the drift characteristics of the chamber are briefly outlined. As indicated in section 8.2 the essential requirement from the drift region is that the electrons produced in the initial ionization are transported, without loss, to the avalanche region. Possible loss mechanisms include recombination and attachment to impurities, grid and chamber walls. Attachment to impurities can be prevalent with pure inert gases as the electron drift velocities tend to be rather low. The chamber was operated with a range of grid voltages with a view to finding a suitable operating region.

The work of Kruithof and Penning (1937) (reviewed in chapter 4) allows the gas amplification factor in the avalanche region to be predicted. It is thus possible to predict the final charge of the fully developed avalanche. If this charge is obtained experimentally then it may be concluded that the drift and multiplication regions are operating correctly. The drift characteristics were investigated separately for the various modes in which the chamber was operated.

The operation of the chamber in its "initial testing" mode was outlined in section 8.2. From figure 8.1 and table 8.2 it can be deduced that the correct charge and optimum resolution are obtained with a grid voltage of about 2V.

According to the theory of Buneman, Cranshaw and Harvey (1949) the grid (used in the above configuration with about 100 V across the grid-anode gap) should be fully transparent for grid voltages of up to about 115 V. However the above analysis does not take into account the effects of electron diffusion - which may allow the electrons to strike the grid. Another possible mechanism for charge loss is electron attachment to impurities - the most common contaminant is oxygen.
(present here at \(\pm 1\) ppm). The mean agitation energy of the electrons at these low values of \(E/p\) is of the order of 0.05 eV. Unfortunately attachment cross-section data is uncertain in this region - see figure 8.8.

It is possible that as the electric field in the drift region is increased more attachment takes place as the mean agitation energy and hence cross-section for attachment also increase (moving up the left-hand side of the peak). The electrons have a mean electron energy of 2.5 eV at a value of \(E/p\) corresponding to a grid voltage of \(\approx 95\) V (Healey and Reed (1941)). As reported in section 8.2.2 improvements to the gas purity largely removed the low charge region of the spectrum. This suggests that charge was being lost by attachment to impurities.

The general shape of the drift characteristic (of the form indicated by figure 8.1) did not however vary with gas purity. With the various modes of operation outlined in sections 8.2 to 8.6 the best resolution tended to be obtained with a grid voltage of a few volts. Another typical characteristic is shown in figure 8.9.

The loss of charge at high values of grid voltage may be caused by electron attachment to the grid. This could be investigated experimentally by connecting a suitably sensitive electrometer to the grid.

In conclusion it would appear that electron loss to impurities and grid have to be carefully considered in a study of this nature. Steps must be taken to ensure that a symmetrical spectrum of the correct amplitude is obtained.
Cross section for electron capture by oxygen. Curve a, Bradbury; curve b, Burch and Geballe; curve c, Chanin and Biondi; curve d, Doehring; curve e, Harrison and Geballe; curve f, Herreng; curve g, Healey and Kirkpatrick.

Figure 8.8
(From Snell (1962) to which reference numbers refer)
Experimental variation of measured charge (●), and intrinsic resolution (●) as a function of grid voltage for $V_{ga} = 130$ volts. The results were obtained with 99.9% Ne + 0.1% Ar at 1 atmosphere with a grid-anode gap of 0.48 mm. The original grid was used. The final charge is greatest and the resolution optimized with a few volts applied to the grid.

Figure 8.9
CHAPTER 9 : RESOLUTION RESULTS

9.1 Introduction

9.2 Presentation of Resolution Results
   9.2.1 Operation with 99.9% Ne + 0.1% Ar at 1 atmosphere
   9.2.2 Operation with 99.5% Ar + 0.5% C₂H₂ at 1/4 atmosphere

9.3 Discussion of Results
   9.3.1 Discussion of Neon-Argon results
   9.3.2 Discussion of Argon-Acetylene results
   9.3.3 Localization of the ionization process
   9.3.4 Proposal for a multi-stage chamber
9 Resolution Results

9.1 Introduction

Avalanche fluctuations were considered theoretically in chapter 3. Some of the avalanche models predict that low values of $f$ (the relative variance of the avalanche size) can be obtained with high values of $\lambda$ ($\alpha U / E$). Low values of $f$ lead to improved energy resolution (tables 3.2 and 3.3).

It was shown in chapter 4 that high values of $\lambda$ can be obtained using Penning mixtures such as neon-argon. As reported in chapter 6, Penning mixtures have only been used previously with cylindrical rather than uniform field proportional counters. In a cylindrical counter $\alpha / E$ will vary from cathode to anode (due to $E / p$ varying) and, very close to the anode, $\alpha / E$ can vary over a single ionization mean free path. The direct correlation of $f$ to $\lambda$ is thus less clear than with a uniform field where $\alpha / E$ is constant.

With a uniform electric field it is possible to operate a Penning mixture at the $E / p$ value which corresponds to the maximum obtainable value of $\alpha / E$. In this chapter an account is given of resolution measurements made under such conditions with the apparatus described in chapter 7. Using published values of the Fano factor $F$ and mean ionization energy $W$ (given in chapter 2) one can derive experimental values of $f$ from the measurements. Such values are compared with the theoretical predictions given in section 3.6 and the predictions based on experimental ionization probability data obtained by Kruithof and Penning (outlined in section 3.7).
9.2 Presentation of Resolution Results

9.2.1 Operation with 99.9% Ne + 0.1% Ar at 1 atmosphere

In this section an account is given of results obtained from the chamber operating with the modified grid (section 8.6). Before filling the gas was purified with the T.S.P. as before. The chamber was vacuum baked for 4 days.

Gas Gain

Figure 9.1 shows the gas gain as a function of $V_{ga}$ for a grid voltage of 2V. The anode voltage was varied in 1V steps to give a detailed impression of small scale variations in the multiplication process. The sinusoidal variations observed by Kruithof and Penning (1937) can be very clearly seen in these results.

In figure 9.2 the quantity $\alpha U_1/E$ ($U_1$ is the first metastable level of neon = 16.53eV) is plotted against $V_{ga}$. The experimental value of $\alpha U_1/E$ was derived from the gas gain measurements using equation (4.6). The second ionization coefficient was obtained experimentally by measuring the breakdown voltage ($V_{BD}$). At breakdown the denominator term of equation (4.6) becomes zero so that $\gamma$ is given by

$$\gamma = \frac{1}{\exp \frac{\alpha}{E} (V_{BD} - V_F) - 1} \tag{9.1}$$

To derive $\gamma$ it was necessary to use an appropriate experimental value of $\alpha/E$ obtained by Kruithof and Penning. An experimental value of $4 \times 10^{-3}$ was obtained. This compares with the value of $3.6 \times 10^{-3}$ reported by Kruithof and Penning (1937) for an iron plate system.

The voltage between successive minima of the curve is about 17V which is in good agreement with the observations of Kruithof and Penning (1937).
Theoretical and experimental variation of gas gain with grid-anode voltage ($V_{ga}$) for 99.9% Ne + 0.1% Ar at 1 atm. All measurements reported in this chapter were made at 5.89 KeV with the fully developed apparatus and with $V_g = 2V$. The theoretical gas gain was calculated using the data of Kruithof and Penning (1937). Secondary avalanche multiplication was not considered.

Figure 9.1
Resolution

In figure 9.3 the observed and intrinsic resolutions obtained with a grid voltage of 2V are shown. The optimum intrinsic resolution is about 13% similar to that obtained with the original grid configuration (section 8.5). The resolution results are fully discussed in section 9.3.
Experimental variation of observed ($R_o$) and intrinsic ($R_I$) resolution obtained at 5.89 keV for 99.9% Ne + 0.1% Ar. The experimental conditions are the same as for figure 9.2.

Figure 9.3
9.2.2 Operation with 99.5% Ar + 0.5% C₂H₂ at ½ atmosphere.

The 99.9% Ne + 0.1% Ar measurements were made at the peak value of the α/E - E/p curve. The value of X was thus essentially set at about 0.6. To provide additional data on the applicability of the various models it was decided to examine the behaviour of a different Penning mixture. The highest value of X available with argon-acetylene is 0.46. This Penning mixture was thus chosen for the study.

As described in chapter 4 the Penning mixture argon-acetylene requires higher values of E/p than Ne-Ar to give high values of ionization efficiency. For example the highest possible value of α/E, obtainable with Ar-C₂H₂, occurs with the mixture ratio 99.7:0.3 at an E/p value of about 20 V/cm/torr. This is nearly seven times the E/p value required for most efficient ionization in Ne-Ar mixtures. The breakdown potentials, however, do not increase in the same proportion. It is therefore more difficult to set up a practical avalanche region. With a grid-anode gap of 0.48 mm, however, the results of Heylen (1970) show that it should be possible to operate quite close to the maximum ionization efficiency provided a pressure of ½ atmosphere is used. The range of voltage operation could not be exactly predicted because it is not possible to use Heylen's Paschen curves very accurately over this range of pressure x gap distance (9.12 torr cm) - see figure 4.15. Once the chamber was filled, however, the operating range was assessed experimentally. The gas had to be fed to the chamber without use of the T.S.P. as this would of course tend to pump away the acetylene. The chamber was filled to 190 torr with 99.5% Ar + 0.5% C₂H₂ as this was readily available. The mean diffusion distance of the electron cloud in the 0.5 cm deep drift region is about 0.26 cm with a grid voltage of 2V. This is fairly similar to the mean drift distance obtained with neon at 1 atmosphere (table 8.8).
Pulse rise-time and calibration

As pointed out in chapter 6 an advantage of having a hydrocarbon gas as the minority component in a Penning mixture is that the gas may act as a quenching agent and so reduce the number of secondary avalanches. Table 9.1 shows how the pulse rise-time varied with voltage. Close to break-down the pulses do not lengthen greatly as they did with neon-argon (see figure 8.4). This suggests that the secondary avalanches are mainly generated by U-V photons (as opposed to positive ions) produced in the initial avalanche.

<table>
<thead>
<tr>
<th>( V_{ga} ) (Volts)</th>
<th>125</th>
<th>135</th>
<th>145</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulse R/T (( \mu )sec)</td>
<td>( \approx 3 )</td>
<td>2.5</td>
<td>2.5</td>
<td>( \approx 4 )</td>
</tr>
</tbody>
</table>

Pulse rise-time (\( \mu \)sec) for various values of grid-anode voltage (volts) for 99.5% Ar + 0.5% \( \text{C}_2\text{H}_2 \) at \( \frac{1}{2} \) atmosphere with 0.48 mm avalanche gap. \( V_{BD} = 154 \) Volts.

Table 9.1

Gas Gain

As before a detailed study was made of the variation of gas gain with anode voltage. Figure 9.4 shows the data obtained with \( V_g = 2V \) for single volt. increments in \( V_{ga} \). (Such a detailed study has not been previously reported.) To give a clearer picture of variations in ionization efficiency the parameter \( \alpha U_1/E \) is also plotted (see figure 9.5).
Theoretical (●) and experimental (○) variation of gas gain with $V_{ga}$ for 99.5% Ar + 0.5% $C_2H_2$ at $\frac{1}{2}$ atm. The theoretical gas gain was calculated using the data of Heylen (1970). Secondary avalanche multiplication was not considered.

Figure 9.4
Experimental variation of $\alpha U_1/E$ with $V_{ga}$ for 99.5% Ar + 0.5% $C_2H_2$. The experimental conditions are the same as for figure 9.4. As $V_{ga}$ is increased from 97 to 150V, $E/p$ increases from 10.7 to 16.5 V/cm/torr.

Figure 9.5
There does appear to be some periodic variation in $\alpha U_1/E$ but it is not as marked as with Ne-Ar. With regard to secondary avalanches it can be seen from figure 9.4 that the gas gain curve breaks away significantly from linearity at about $\bar{K} = 100$. This is very similar to the behaviour of Ne-Ar.

The value of $\gamma$ was obtained experimentally as before. The measured value of $\gamma = 4 \times 10^{-3}$ for this experimental arrangement compares with Heylen's value of about $2 \times 10^{-3}$ for 99.7% Ar + 0.3% C$_2$H$_2$ obtained with a solid gold electrode system (Heylen (1960, 1962, 1968a, 1968b)).

Resolution

The observed and intrinsic resolutions obtained with $V_g = 2V$ are plotted in figure 9.6. The optimum intrinsic resolution is about 15%. The results are discussed in section 9.3.
Experimental variation of observed \( (R_o) \) and intrinsic \( (R_i) \) resolution obtained at 5.89 KeV for 99.5% Ar + 0.5% \( \text{C}_2\text{H}_2 \). The experimental conditions are the same as for figure 9.4.

Figure 9.6
9.3 Discussion of Results

9.3.1 Discussion of Neon-Argon Results

In chapter 8 the influence of disturbing effects on the experimental measurements was examined. Experimental conditions such as gas purity, gas pressure and field uniformity were considered. The experimental system was arranged so that, as far as possible, disturbing influences were minimized. In this section the results obtained with Neon-Argon are discussed in relation to the theoretical predictions based on the seven avalanche models and the ionization probability measurements made by Kruithof and Druyvesteyn (1937) (outlined in sections 3.6 and 3.7, respectively).

As an aid to comparison, table 9.2 shows predicted and measured values of $f$ and resolution at 5.89 keV. From table 9.2 one can see that models 4 (figure 3.25) and 6 (Byrne's Model) are in reasonable agreement with the data. The variations of $a_i$ (the local value of Townsend's first coefficient) with $\xi$ (the distance travelled by an electron from the previous ionization) assumed by the other models (figure 3.25) would seem to be unrepresentative of the ionization process taking place under the conditions of the present work. The step-function form of Legler's model (nos.1,3,5), the delta-function form of model 2 and the constant ionization probability function suggested by Snyder (model 7) seem to be oversimplifications of the true situation and give inaccurate predictions of resolution.

As discussed in section 3.7, models 4 and 6 are difficult to justify from a theoretical point of view. For both models $f_o$ rises at large values of $X$. One would tend to expect $f_o$ to be low at high values of ionization efficiency. Models 4 and 6 both allow ionization to take place when the electron energy is less than the ionization energy.
Values of $f$ and resolution ($R$) at 5.89 keV predicted by various avalanche models and ionization probability data obtained by Kruithof and Druyvesteyn (1937) for 99.9% Ne + 0.1% Ar at $E/p = 3$ V/cm/torr. Optimum values of measured intrinsic resolution and $f$ are also shown.

Table 9.2

<table>
<thead>
<tr>
<th>Theoretical Models</th>
<th>Prediction based on Kruithof's data</th>
<th>Experimental Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0.01, 0.08, 0.54, 0.37, 0.83, 1</td>
<td>$0.38 \pm 0.12$</td>
</tr>
<tr>
<td>$R(%)$</td>
<td>3.8, 5.6, 11.8, 10.4, 15.9</td>
<td>$10.2 \pm 1.3$</td>
</tr>
</tbody>
</table>

242.
The models are also difficult to justify experimentally as values of \( f_0 \) have been measured which are below that allowed by either model (Sipila (1976)).

It would seem from the above discussion that it is not reasonable to use the models in table 9.2, in their present form, to represent the ionization process taking place under the conditions of the study.

It is interesting to compare the experimental results with the predicted values of \( f \) and resolution obtained using the ionization probability measurements made with neon-argon by Kruithof and Druyvesteyn (1937) (section 3.7). As discussed in section 3.7, these values are lower limits to what may be expected. This is because any remaining secondary effects, however small, will tend to increase the signal variation. With these considerations in mind the results obtained seem to relate fairly well to the predictions - table 9.2.

The relationship of \( a_i \) to \( \xi \) shown in figure 3.29 indicates that the ionization process could be represented by a form of model 2 which has the delta functions replaced by broader distributions. The exact shape of the distribution has, it seems, to be obtained experimentally. As the ionizing electron moves further into the gas the distributions for successive ionizations will become broader and more shallow (as discussed in section 3.4.3). Eventually the ionization probability will become constant as in Snyder's model. The shape of subsequent distributions can be obtained experimentally using the technique adopted by Kruithof and Penning (1937).

The cyclic nature of the variation of \( X \) reported here is also in agreement with the work of Penning, Kruithof and Druyvesteyn (reviewed in chapter 4). The measurements made by Chanin and Rork (1963) for neon and hydrogen (reviewed in chapter 3) are also cyclic in nature and support the same type of ionization model.
9.3.2 Discussion of Argon-Acetylene Results

The resolution obtained with Ar-C$_2$H$_2$ does not agree exactly with any of the seven models - but is closest to Snyder's model. With the pre-mixed gas used though, it was not possible to carry out further purification as this would have tended to remove the acetylene. The presence of impurities will of course lead to increased avalanche fluctuation (due to electron attachment and ionization by collision - such ionization of the impurities requires the electrons to travel distances different from that involved in exciting the main gas and so the first ionization coefficient is altered locally). The results obtained with argon-acetylene are thus a less accurate guide to the multiplication process. The behaviour of argon-acetylene could be investigated more fully by separate purification of the argon and then mixing with the acetylene.

The poorer resolution obtained with Ar-C$_2$H$_2$ is compatible with the less marked variation in $\alpha/E$ with voltage. The ionization probability varies more gradually than with Ne-Ar and so the variation in the avalanche size can be expected to be greater.
9.3.3 Localization of the ionization process

In the above comparison of results with the theoretical models it is assumed that the excitation of the main gas and subsequent ionization of the added gas take place at the same point. It is interesting to consider how the two processes could become physically separated.

Sipila (1979a) has examined the diffusion of the excited states away from their point of creation. He has shown, for neon-argon at low values of E/p, that the mean diffusion distance is small compared to the mean doubling distance (see figure 9.7) and hence that diffusion of the metastable atoms does not affect the statistics of the multiplication.

At high values of E/p, however, the distances become more comparable. In a cylindrical counter diffusion may affect the statistics of the avalanche in the region close to the anode wire.

The results obtained in section 8.4 suggested that admixture gas concentration could be varied without affecting the resolution. These measurements, however, were made at a pressure of $\frac{1}{4}$ atmosphere. Later development work showed that spurious effects were taking place at this pressure. At higher pressures the admixture concentration may be more significant. Sipila (1976) has found this to be the case at a pressure of 1 atmosphere with a cylindrical counter.

From a theoretical point of view the degree of localization of the ionization process can be expected to vary with the concentration of admixture gas. For example with a very low concentration of admixture gas one would expect reduced localization. This is because a metastable main gas atom will have to travel further before colliding with an admixture gas atom.
For future study it is suggested that resolution measurements are made with a range of admixture concentrations to examine whether any improvement can be obtained. A suitable range would be 0.1% Ar to 1% Ar in neon.

Photon mediated Penning ionization will tend to be less localized than the collision mediated form. With the pressures and gas concentrations involved in this study, though, collision mediated Penning ionization is regarded as being the most important (see section 4.1 and Buursen and Van Montfort (1973)).

![The ratio of the mean diffusion distance \(1/2k\) to the mean doubling distance \(1/a\).](image)

**Figure 9.7**

(From Sipila (1979a))
9.3.4 Proposal for a multi-stage chamber

For reasons discussed in chapter 2 a single electron study of avalanche fluctuations in Penning mixtures cannot be carried out with a conventional single stage proportional chamber. An X-ray with an energy of a few KeV has to be used to provide a large number of primary ion-pairs. The final avalanche charge is then sufficiently greater than the noise charge of the pre-amplifier and associated electronics.

The work of Alkhazov (1970) permits the calculation of the theoretical form of the single electron spectrum for the various avalanche models and for ionization probability data of the kind obtained by Kruithof and Druyvesteyn. By measuring the single electron spectrum one could make a rather more quantitative comparison than is possible with the multi-electron experiment.

For future study it is suggested that a multi-stage avalanche chamber is designed and constructed for use with Penning mixtures. A schematic diagram of such a chamber is shown in figure 9.8. For a general discussion of multi-stage avalanche chambers see for example Imrie (1978) and Breskin et al (1979).

\[ \text{U-V RADIATION} \quad \downarrow \quad \text{CATHODE WITH PHOTO-ELECTRIC AREA} \]

\[ D_1 \]

\[ A_1 \quad \text{GRIDS} \]

\[ D_2 \]

\[ A_2 \quad \text{GRID} \quad \downarrow \quad \text{ANODE} \]

\[ \text{PRE-AMPLIFIER (ETC.)} \]

\[ D_1, D_2 \text{ are the first and second drift regions.} \]

\[ A_1, A_2 \text{ are the first and second avalanche regions.} \]

Two stage avalanche chamber

\textbf{Figure 9.8}
The device shown is effectively two conventional chambers operating "in series". An electron is released in region $D_1$ and drifts to the first grid. Multiplication takes place in region $A_1$. The electrons produced in the first avalanche then drift to region $A_2$ and are multiplied further. Because there are two amplification regions the gain of each region needs to be only the square-root of the total gain. The separate amplification regions can thus be operated at gas gains which are low enough for secondary avalanche effects to be negligible. The two drift regions can be set up so that positive ions and U.V. produced in one avalanche region do not affect the other region.
CHAPTER 10 : CONCLUSIONS
10 Conclusions

In this final section the main results and conclusions arising from the study are presented and suggestions for future work are summarized.

A cylindrical counter was operated with a number of Penning mixtures. The measured resolution was found to be an improvement over that obtained with argon-methane. This was especially the case with neon-argon. Most Penning mixtures are unquenched and have to be operated at low values of gas gain with subsequent signal to noise ratio problems. It was shown that the addition of a quenching agent such as methane to neon-argon does not lead to a satisfactory situation. The Penning mixture argon-xenon, however, gives improved resolution over argon-methane and, at the same time, allows operation at fairly large values of gas gain. The xenon appears to act as a quenching agent.

Improvements to gas purity and optimization of the mixture ratio may lead to improved energy resolution. The presence of xenon increases the absorption efficiency and, in addition, xenon does not suffer the degradation problems associated with the use of organic admixture gases. The Penning mixture argon-xenon seems to merit further study.

The modelling of the ionization process in Penning mixtures was considered in a more fundamental way using a constant electric field. Various models predicted a range of values of \( f \) and resolution \( (R) \). The step function and delta function models suggested low values of \( f \) and \( R \). Results obtained are in approximate agreement with two models which predicted larger values of \( f \) and \( R \). These models, however, seem unsatisfactory from a theoretical point of view.

The results are also fairly close to those predicted by experimental measurements of ionization probability made by Kruithof and Druyvesteyn (1937). The detailed variation of the quantity \( \alpha U_1 / E \) with grid-anode voltage was found to be cyclical in nature. This is in
agreement with previous workers.

Reduced variation in the multiplication may be obtained by varying the argon admixture concentration in neon.

Avalanche fluctuations in Penning mixtures could also be investigated further with a multi-stage chamber.
APPENDIX 1 : Summary of Moments

In section 3.5.1 expressions were given for the moments of the avalanche size distribution. In this appendix the theory of moments is briefly introduced. A fuller review can be found in works such as Paradine and Rivett (1964).

Mean of a Frequency Distribution

If the observations $x_1, x_2, ..., x_n$ occur respectively with frequencies $f_1, f_2, ..., f_n$ then the mean of the frequency distribution, $\bar{x}$, is given by:

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + ... + f_n x_n}{f_1 + f_2 + ... + f_n}$$

$$= \frac{\sum fx}{\sum f} \quad (A1.1)$$

First Moment

From equation (A1.1) it can be seen that $\bar{x}$ is analogous to the $x$ co-ordinate of the centre of mass of a system of particles. Using the mechanical analogy, $\bar{x}$ is known as the first moment of the distribution about the origin. Moments can be taken about points other than the origin and $\Sigma f(x-a)/\Sigma f$ is called the first moment of the distribution about $x=a$. The first moment of a distribution about its mean is zero; i.e. $\Sigma f(x-\bar{x})/\Sigma f = 0$.

Variance and standard deviation of a frequency distribution

The variance, $s^2$, of the frequency distribution is given by:

$$s^2 = \frac{f_1(x_1-\bar{x})^2 + f_2(x_2-\bar{x})^2 + ... + f_n(x_n-\bar{x})^2}{f_1 + f_2 + ... + f_n}$$

$$\quad = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} \quad (A1.2)$$
The standard deviation \( s \) is simply the square-root of the variance.

**Higher Moments**

The second moment of the distribution about the origin is given by \( \frac{\Sigma f x^2}{\Sigma f} \). The second moment of the distribution about \( x=a \) is given by \( \frac{\Sigma f(x-a)^2}{\Sigma f} \). It can be seen that the second moment about the mean \( \frac{\Sigma f(x-x)^2}{\Sigma f} \) is the variance.

Higher moments are obtained in a similar way to the first two. With the third moment the asymmetry (or skewness) of a distribution can be assessed.
APPENDIX 2 : The Relationship between $a_i(\xi)$ and $\rho_i(\ell)$

In this appendix equation (3.43) given in section 3.5.1 is derived. It may be useful to first restate the definitions of $a_i(\xi)$ and $\rho_i(\ell)$:

$a_i(\xi)$ is the ionization probability per unit path length at $\xi$ i.e. the probability that an electron which has reached $\xi$ will be duplicated between $\xi$ and $\xi + d\xi$. $\xi$ is the distance travelled by an electron from its point of emergence.

$\rho_i(\ell)$ is the probability distribution of the path $\ell$ at which one electron is duplicated. Such a distribution can be obtained experimentally by releasing an electron stream from the cathode and measuring the variation of current increase with distance of anode from cathode (Kruithof and Druyvesteyn (1937)).

The "Survival Equation"

Let us assume that we have $N_0$ electrons setting out from the cathode in a uniform field system - Figure A2.1.

Consider a slab of thickness $d\xi$ at $\xi$. The number of electrons undergoing duplication by ionizing collision in the slab is given by

$$dN_\xi = -a_i(\xi)N_\xi d\xi \quad (A2.1)$$
where \( N_\xi \) is the number of electrons which have "survived" to \( \xi \) without being duplicated. From this expression a survival equation results:

\[
\frac{dN_\xi}{N_\xi} = -a_1(\xi)
\]

\[
\Rightarrow \ln \left( \frac{N_\xi}{N_0} \right) = - \int_0^\xi a_1(\xi) \, d\xi
\]

\[
\Rightarrow N_\xi = N_0 \exp \left( - \int_0^\xi a_1(\xi) \, d\xi \right) \quad (A2.2)
\]

We thus have an expression for the number of electrons which "survive" to \( \xi \) without being duplicated by an ionizing collision.

Equation (3.43) may be derived by writing down and equating two expressions (one in terms of \( \rho_1 \) and one in terms of \( a_1 \)) for the number of electrons duplicated between \( \xi \) and \( \xi + d\xi \):

\[
N_\xi \rho_1(\xi) \, d\xi = N_\xi a_1(\xi) \, d\xi \quad (A2.3)
\]

We can re-express \( N_\xi \) in the following way:

\[
N_\xi = N_0 - \int_0^\xi N_0 \rho_1(\lambda) \, d\lambda
\]

The right-hand side of equation (A2.3) can thus be re-written:

\[
N_\xi a_1(\xi) \, d\xi = N_0 \left[ 1 - \int_0^\xi \rho_1(\lambda) \, d\lambda \right] a_1(\xi) \, d\xi
\]

With this substitution equation (A2.3) leads to:

\[
\rho_1(\xi) = a_1(\xi) \left[ 1 - \int_0^\xi \rho_1(\lambda) \, d\lambda \right]
\]

which can be re-arranged to give

\[
a_1(\xi) = \rho_1(\xi) \left[ 1 - \int_0^\xi \rho_1(\lambda) \, d\lambda \right]^{-1} \quad (3.43)
\]
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ABSTRACT

Fluctuation Phenomena in the Proportional Counter

J P Sephton

The energy resolution of a proportional counter depends fundamentally on (i) fluctuations in the number of electron-ion pairs produced in the primary ionization and (ii) fluctuations in the size of the subsequent electron avalanche. The avalanche fluctuation is typically 3-4 times larger than the ionization fluctuation. Previous theoretical and experimental work has indicated that fluctuations in both processes can be reduced by improving the efficiency of the ionization mechanisms involved. The ionization efficiency can be increased by means of the Penning effect. Energy spent exciting metastable states is used to increase the ionization yield. This is achieved by introducing a second gas whose first ionization potential is slightly lower than the first metastable level of the main gas. Cross ionization of the admixture gas by metastable states in the main gas can thus occur. With appropriate conditions, the Penning effect becomes the dominant ionization process.

The operation of a proportional counter filled with a range of Penning mixtures is investigated experimentally. Penning mixtures are shown to give improved resolution over the more conventional filling of argon-methane. The Penning mixture, argon-xenon, seems to be a practical alternative filling.

In order to make rather more fundamental studies of avalanche fluctuations in Penning mixtures, a uniform field chamber is constructed. With this chamber it is possible to directly compare experimental measurements of $f$ (the relative variance of the avalanche size) with values predicted by various avalanche models. (With a cylindrical counter the ionization efficiency varies from cathode to anode and such a comparison is less clear.) The models assume different variations of the local value of Townsend's first ionization coefficient $a_1$ with the distance travelled by an electron from the previous ionization. The results obtained are in reasonable agreement with two of the models but these are unsatisfactory from a theoretical point of view. The results are, however, reasonably close to predictions based on experimental measurements of ionization probability.