The Propagation of Acoustic Gravity Waves
in the Atmosphere.

by

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CHAPTER 1 Introduction

1.1 Propagation of Gravity Waves in the Atmosphere

The Earth’s atmosphere is a medium which can support a large number of different types of wave motion. These consist of movements of ionised particles, of neutral particles or a combination of both. This thesis is concerned with the movements of neutral particles. Neutral waves exhibit periods ranging from a few seconds to many days. The higher frequency waves are acoustic waves, and form a family of longitudinal waves. At lower frequencies (periods 15 mins. to a few hours) a family of vertical transverse waves exist. These are termed gravity waves, as for this class of motion gravitational forces dominate pressure forces. Finally, a family of horizontal transverse waves, called Rossby waves exist, and these have even lower frequencies (periods greater than a few hours) than gravity waves. Coriolis forces have to be included in studies of this type of wave, but are unimportant for both acoustic and gravity waves. These types of atmospheric wave feature in figure 1.1.1. The bulk of this work is concerned with the propagation of gravity waves, but some attention is also given to acoustic waves.

In the ionised regions of the upper atmosphere, the so called ionosphere, the propagation of gravity waves produces disturbances in the ambient electron density distribution. These changes can be detected by ground based radio sounding techniques, and consequently information may be derived concerning the initial wave disturbances (Hunro, 1948, 1950). Since horizontal movements are involved, these fluctuations are termed
Three elementary atmospheric waves:
(a) longitudinal (acoustic);
(b) vertical transverse (gravity);
(c) horizontal transverse (Rossby);

Figure 1.1.1
(after Thompson, 1961)
travelling ionospheric disturbances, or TID's. Gravity waves and their associated TID's may be classified as either large scale or medium scale. Large scale TID's typically have horizontal phase speeds between 400 and 1000 m/s, periods from 30 mins. to 3 hours, and horizontal wavelengths exceeding 1000 km. Medium scale disturbances have horizontal phase speeds less than 300 m/s, periods from 15 mins. to more than an hour, and horizontal wavelengths of several hundred kilometres (Georges, 1968; Francis, 1975). The majority of this work is concerned with the propagation of medium scale disturbances.

The first theoretical interpretation of TID observations was in terms of cellular atmospheric waves (Martyn, 1950). This kind of wave is a standing, rather than a propagating wave, and would exist as a consequence of critical layers, which arise when the horizontal phase velocity of the wave, and the horizontal wind velocity are of the same order of magnitude. A discussion of critical layers appears later. The concept of cellular waves was extended and refined in subsequent analyses (Hines, 1955, 1956), but the frequencies and wavelengths for which the theory was valid were found to form only a small part of the spectrum observed in TID's.

An alternative interpretation of TID observations was put forward in terms of their being a manifestation of internal atmospheric gravity waves (Hines, 1959, 1960). These form a family of propagating waves, whose properties can account for the differing characteristics of TID's. An independent analysis of gravity wave
propagation was undertaken by Eckart,1960, but this was flawed by an incorrect application of WKB approximations (Heading,1962).

The advent of nuclear weapons testing led to several observational (e.g. Dieminger and Kohl,1962; Obayashi,1962; Stoffregen,1962) and theoretical (Weston,1961,1962; Pfeffer and Zarichny,1962) studies of TID's. Westons' analyses modelled the explosive source using Greens functions, and calculated the atmospheric far field response. Both his work and that of Pfeffer and Zarichny was based on the assumption of fully ducted modes of propagation, which are defined as modes whose propagation in the upper atmosphere is vertically evanescent, so that they suffer no energy leakage to infinite heights. These theoretical models were further refined by including height varying neutral winds (Weston and Van Hulsteyn,1962). This treatment was extended by Mackinnon,1968, who was able to explain all the observed features of nuclear explosions, except for the property of inverse dispersion for long period waves. (Normal dispersion implies that group velocity increases with wave period, and inverse dispersion implies the converse.) This feature was explained by Balachandran,1968 by the inclusion of winds of the order of 100 m/s around 100 km height. It is now well known that such winds exist in the atmosphere (Rishbeth,1972).

The gravity wave interpretation of the atmospheric response to nuclear explosions was challenged by Wickersham,1966, who provided an interpretation based on fully ducted acoustic modes. Hines,1967 refuted this analysis by showing that Wickershams' calculations of the
disturbances horizontal phase velocity were erroneous, but that an explanation in terms of obliquely propagating gravity waves was consistent with the observed data.

Obliquely propagating waves can be more realistically studied in terms of imperfectly ducted modes. These are modes for which there is always some energy leakage vertically, i.e. the vertical component of the wave number, $k_r$, is always complex. For these modes some energy can always escape through regions of strong ducting, and hence can be detected in the upper atmosphere. Such waves were first studied by Gossard, 1962 in the context of ducting by the temperature structure of the atmosphere at heights below 10 km. This type of analysis was extended to the upper atmosphere by Friedman, 1966, who modelled the atmosphere by a series of isothermal layers. Friedman also introduced the concept that the wave could attenuate horizontally by allowing the horizontal components of the wave number vector to be complex. It was subsequently found that the extensive spectrum of lower-speed modes predicted by Friedman was spurious, as the temperature layers employed were too coarse, leading to stronger upper atmospheric reflections than actually exist (Reddy, 1969; Francis, 1973). However, Friedman's paper is still instructive as to the mathematics used to study imperfectly ducted modes.

The validity of the multiple isothermal layer approximation in gravity wave analyses was questioned by Hines, 1965. It was justified by Pierce, 1966 for a simple model atmosphere, but some authors did not follow the criteria laid down by him (e.g. Kidgley and Liemohn, 1966). Hines, 1973 justified the use of multiple isothermal
layers for a more complex atmosphere than that of fierce, but with the proviso that the acceleration due to gravity was held constant. This has proved to be a valid approximation, the differences between using a realistic or a constant gravity field in a neutral wave analysis being negligible.

The precise nature of the upper boundary condition to be used in gravity wave analyses has received considerable attention. Studies by Press and Harkrider, 1962; Pfeffer and Zarichny, 1963; and Harkrider and Wells, 1968 have examined the effects of varying the upper boundary condition in a modal analysis. The conditions investigated were a free surface, a rigid surface and a radiation condition in an isothermal half space. The free surface condition was taken to have a physical significance by Tolstoy, 1967 in that such a surface could support a spectrum of surface waves. A later study (Tolstoy and Pan, 1970) using this condition derived a spectrum of modes qualitatively different from those found by Friedman. The question of which condition to use was resolved by Francis, 1973a, in a study which included the effects of energy dissipation. He showed that the presence of strong upper atmospheric dissipation prohibits waves from propagating from the regions of interest to the upper boundary and back. Consequently, there is no way in which information concerning the upper boundary can be communicated to the regions of interest. He therefore concluded that the nature of the upper boundary was irrelevant, except for the requirement that it must not act as a source, a condition which is met by all the boundary conditions mentioned.

Although the inclusion of dissipation resolved the
above argument, in general it has complicated the study of gravity wave propagation. Viscosity and thermal conduction were the first dissipative mechanisms proposed (Hines, 1960). It is now accepted that two types of viscosity must be included in any realistic analysis (Gossard and Hooke, 1975). In the lower atmosphere an eddy viscosity caused by turbulence dominates, whereas in the upper atmosphere ordinary molecular viscosity is dominant. The inclusion of viscosity leads to two extra families of waves, the 'ordinary' and 'extraordinary' viscosity waves (Volland, 1969). Similarly, the inclusion of thermal conductivity leads to the introduction of a family of thermal conduction waves (Francis, 1973a). Throughout most of the atmosphere these extra families of waves are damped much more strongly than gravity waves, but in the upper atmosphere conditions arise where this may no longer hold (Volland, 1969), and therefore coupling between the various families may occur. It has been demonstrated that the effects of viscosity and thermal conductivity on gravity wave propagation are similar (Hines, 1960; Volland, 1969; Klostermeyer, 1972), and this property may best be discussed on a molecular level (Hines, 1974). Superimposed on the organised motion due to the passage of a gravity wave are the random thermal motions of individual molecules, which serve to carry energy (mainly through thermal conduction) and momentum (mainly through viscosity) from one region to another in a partially chaotic way, so that characteristics of one portion of the wave are transferred to other portions where different characteristics should occur, resulting in a degradation of the organised patterns, and hence an
attenuation of the wave as it progresses. This degradation proceeds more readily the longer the mean free path of individual molecules, for the mismatched characteristics can then be carried farther through the wave system, and so can produce a greater mismatch, within a given time or a given fraction of the waves period of oscillation. The mean free path of molecules in the atmosphere increases with increasing height, as the ambient gas density decreases monotonically with height. In consequence, the effects under consideration grow in importance with height. Other noteworthy features are that the coefficient of thermal conduction can be calculated from the coefficient of dynamic viscosity (see appendix 1), and that both parameters only enter the equations used in subsequent calculations when divided by the ambient gas density.

Another form of dissipation is the phenomenon of ion drag, or magnetohydrodynamic absorption. The ions have a component of motion along the Earths' magnetic field lines, and thus, through collisions, neutral particles also acquire a velocity component along these paths. This tends to destroy the regular features of a neutral wave, and wave energy and momentum are consequently dissipated. This mechanism was first studied by Gershman and Grigor'yev, 1965, but their subsequent analysis was flawed by some unnecessarily restrictive mathematical assumptions. Rishbeth et al, 1965, associated a time constant with ion drag, roughly interpreted as the time that must pass before most neutral particles achieve at least one collision with an ion. This provides an indication of the importance of this type of dissipation.
The study of ion drag has been greatly extended by the analyses of Yeh and Liu and their co-workers (see, for example, Yeh and Liu, 1972), who have concluded that ion drag is a significant process for dissipating wave energy, in contradiction to the work of Francis, 1973, who has estimated the effect to be negligible, even when an atmospheric model maximising this effect was employed. This dichotomy is resolved in the current work by showing (Chapter 2) that if ion drag is the only dissipative mechanism considered, the effect is negligible, but when it is coupled with other forms of dissipation (e.g. viscosity) then its presence causes a significant change in wave propagation characteristics.

Dissipation of energy by ambipolar diffusion has also been proposed (Testud and François, 1971). However, all other authors have assumed this phenomenon to produce negligibly small effects, and have ignored it, an assumption also made in the present investigation.

Coriolis forces have also been ignored, because they only significantly affect waves with periods greater than a few hours (Francis, 1973a), and are therefore outside the scope of the present investigation which is limited to the maximum period of medium scale TID's, less than 2 hours. However, the effects of coriolis forces have been discussed by Tolstoy, 1972.

The most recent challenge to gravity wave theory has been in the analysis of Herron, 1973, who claimed to have observed disturbances with phase and group velocities inconsistent with a gravity wave interpretation. An explanation of the phenomenon in terms of hydromagnetic waves was proposed. However, these observations were
refuted by Hines, 1974a, 1974b, who showed Herrons' data analysis techniques to be suspect, and further that his calculations of phase velocity were erroneous.

In addition to gravity waves, this thesis is to a lesser extent concerned with the propagation of acoustic waves through the atmosphere. This problem was first seriously attacked by Pekeris, 1937, 1948, in the context of acoustic disturbances generated by the Siberian meteor explosion of 1908. As with gravity waves, the onset of nuclear weapons testing simulated more interest in acoustic wave propagation (Harkrider, 1964; Hines, 1967). Since both acoustic and gravity modes of propagation are derived from the same equations of motion (Hines, 1960), the study of both classes of disturbance have proceeded in parallel. However, there exist important distinctions between them, which are made clear in the subsequent analysis (Chapter 2).

No mention has yet been made of acoustic or gravity wave source mechanisms. This important topic is reviewed in Chapter 3.

1.2 Aims of the Present Work

The principal objective of the current analysis is to establish whether raytracing techniques can be applied for investigation of gravity wave propagation characteristics and for identifying possible source locations. A previous analysis (Georges, 1971) has developed a raytracing technique for use in a dissipationless atmosphere, and this forms the foundation of this study.

The equations of motion for atmospheric waves are formulated and discussed. Emphasis is laid on medium
scale gravity waves, but attention is also given to acoustic waves. The choice of coordinate system is important as acoustic and gravity waves are refracted differently by the Earth's gravitational field (Francis, 1972). The work already reviewed in this chapter has underlined the importance of dissipative processes, and this problem is addressed in detail using classical perturbation theory. The dispersion relation for acoustic gravity waves in a realistic atmosphere is derived, and is used to assess the effects of the various dissipative terms in detail. The ray equations are then obtained, and several raytracings performed in various model atmospheres, again emphasising the effects of dissipative terms. Source mechanism theory is then reviewed, and a simple theory is developed to calculate the atmospheric far-field response to a known source.

Neutral winds have an important influence on gravity wave propagation, and the raytracing technique is employed to calculate their effects, with particular emphasis on the much neglected vertical wind component, which is shown to produce significant changes in the wave propagation characteristics.

An attempt is made to identify gravity wave sources by means of reverse raytracing in an atmosphere including the effects of dissipation and winds. The source position is found to be critically dependent on 1) the neutral wind model assumed, and 2) the types of dissipative process included in the analysis.

The investigation has been extended to determine the atmospheric response to a known source. This is of particular importance in considering the wave propagation
from a distant energy input. The analysis has been
applied to the experimental observations of the waves
produced by the explosion of the chemical plant at
Flixborough in 1974.

Finally, a short discussion of coupling between
neutral atmospheric gravity waves and the resulting
modifications in electron density (TID's) has been
included. This relationship is important since radio
methods are widely used for the study of gravity wave
propagation in the upper atmosphere.
CHAPTER 2 Equations of Motion and Ray Theory

2.1 Introduction

This chapter begins by stating the equations of motion for acoustic gravity waves in an anisotropic atmosphere. For a derivation of these equations the reader is referred to standard hydrodynamical texts, e.g. Landau and Lifshitz, 1959. There follows a discussion on the forms of the various dissipative terms included (ion drag, viscosity and thermal conductivity). A brief section is included on the choice of coordinate systems, and the reasons for selecting a spherical polar system are made clear.

The dispersion relation for acoustic gravity waves is derived, and the relative importance of the various dissipative processes is assessed for a representative selection of wave parameters, by using dispersion diagrams. The ray equations including dissipative effects are then derived, and finally a comparison of raytracing results between an isotropic and various anisotropic atmospheres is given, for the same choice of wave parameters as featured in the dispersion diagrams.

The parameters of the various atmospheric models adopted are shown in appendix 1.

2.2 The Equations of Motion

The equations of motion used in a realistic, inhomogeneous atmosphere are the equation of continuity;

- \( \frac{\partial \rho}{\partial t} = \nabla \cdot (\rho \mathbf{U}) \)  

the equation of conservation of momentum;

- \( \frac{\partial (\rho \mathbf{U})}{\partial t} = \mathbf{F} - \nabla P + \mathbf{F} \)
the ideal gas law;
\[ p = \frac{fRT}{m} \]

and the heat conduction equation;
\[ \frac{JR}{(\gamma - 1)m} \frac{DT + p\nabla U}{-Q_o} = 0 \]

(Francis, 1973b).

where:
- \( p \) = pressure
- \( \rho \) = density
- \( T \) = temperature
- \( U \) = fluid velocity
- \( \mathbf{g} \) = acceleration due to gravity
- \( \gamma \) = ratio of specific heats, \( \frac{c_p}{c_v} \)
- \( m \) = mean molecular weight
- \( R \) = the gas constant
- \( Q_o \) = energy loss term, due to dissipative processes

\[ D = \frac{\partial}{\partial t} + (U \nabla) \] , the eulerian derivative

The form of the body force term \( \mathbf{f} \) used is
\[ \mathbf{f} = \nabla \cdot \mathbf{S} - \sigma_p \mathbf{B}^2 (U - \mathbf{b}) - 2 \mu \mathbf{S} \times U \]

The terms on the right hand side of 2.2.5 represent the effects of viscosity, ion drag and coriolis forces respectively. These are now discussed in turn.

1) The Viscous Term

The quantity \( \mathbf{S} \) is known as the viscous stress tensor, and is given in an orthogonal curvilinear coordinate system by:

\[ S_{ij} = \eta \left[ \frac{h_i}{h_j} \frac{\partial}{\partial x_i} \left( U_j \right) + \frac{h_j}{h_i} \frac{\partial}{\partial x_j} \left( U_i \right) \right] , \quad i \neq j \]
\[ S_{ii} = 2 \left[ \frac{1}{h_i} \frac{\partial U_i}{\partial x_i} + \frac{U_i}{h_i^2} \frac{\partial h_i}{\partial x_i} + \frac{U_k}{h_i h_k} \frac{\partial h_i}{\partial x_k} - \nabla \cdot \mathbf{U} \right] \]

(Goldstein, 1938).

where the \( h_i \) are the line elements of the coordinate
system, and \( \eta \) the coefficient of dynamic viscosity. \( \nabla \cdot \mathbf{S} \) is the form of the viscous term in the Navier-Stokes equations, and the reader is referred to standard hydrodynamical texts for its derivation (e.g. Landau and Lifshitz, 1959).

The off-diagonal elements of this tensor, given by 2.2.6, represent tangential stresses, i.e. \( S_{ij} \) represents a force per unit area acting in the \( j \)-direction on an element of surface perpendicular to the \( i \)-direction.

The diagonal elements, given by 2.2.7, represent the normal strain components, i.e. \( S_{ii} \) represents a force per unit area acting in the \( i \)-direction on an element of surface perpendicular to the \( i \)-direction.

2) The Ion Drag Term

The ion drag term used is given by \( -\sigma_p B^2 (\mathbf{U} - b U_i b_i) \), where \( \sigma_p \) is the Pederson conductivity, \( b \) a unit vector in the direction of the Earth's magnetic field, and \( B \) is the magnitude of the Earth's magnetic field. This is an approximation to the actual term, given by \( B b \cdot \mathbf{J} \), where \( \mathbf{J} \) represents the atmospheric current density, which was first used by Gershman and Grigor'yev, 1965. The approximations made are to assume that 1) the Earth's electric field is perpendicular to the Earth's magnetic field, and 2) that the Hall conductivity is very much smaller than the Pederson conductivity. These approximations have been justified by Hines, 1968 for a realistic atmosphere.

3) Coriolis Forces

The coriolis force term is given by \( -2\Omega \times \mathbf{U} \), where \( \Omega \) represents the Earth's angular velocity. It has been shown that this term will only have a noticeable effect
for waves with periods greater than a few hours (Tolstoy, 1972). Since a perturbation analysis is to be used in the subsequent sections of this work, waves with these periods fall outside the limits of the theory, and the neglect of coriolis forces is therefore justified.

The form of $Q_0$, the energy loss term employed in the heat conduction equation, is given by

$$Q_0 = S : \nabla U + \nabla \cdot (\lambda \nabla T)$$

The first term on the right hand side represents losses due to viscosity, and the latter losses due to thermal conduction.

1) Viscous Losses

The viscous loss term is given by $S : \nabla U$, and consists of the sum of the nine quantities $S_{ij} \partial U_i / \partial x_j$. These quantities are second order terms in velocity, and represent an energy flux due to processes of internal friction (Landau and Lifshitz, 1959).

2) Thermal Losses

Thermal energy losses are represented by the term $\nabla \cdot (\lambda \nabla T)$, where $\lambda$ is the coefficient of thermal conductivity. $\nabla T$ is the heat flux density due to thermal conduction. This term signifies the direct molecular transfer of energy from points where the temperature is high to those where it is low. It is important to remember that the process of thermal conduction does not involve macroscopic motion, and occurs even in a fluid at rest.

2.3 Coordinate Systems

Francis, 1972 has demonstrated that if only the gravity wave branch of the spectrum is considered, then the uses of a rectangular cartesian, and of a spherical
polar coordinate system are equivalent. This is so because for a spherical Earth, the radial gravitational field refracts gravity waves around the Earth in such a way that the raypath curvature induced is very nearly equal to the curvature of the Earth. However, Francis also demonstrated that acoustic waves do not exhibit this property, since they are comparatively unaffected by gravitational forces. As one of the aims of this work is to develop a theory valid for both acoustic and gravity waves, a spherical polar coordinate system will therefore be employed.

There is one further point concerning the use of coordinate systems which is worthwhile including at this point. Adopting a rectangular cartesian \((x,y,z)\) system, Hines, 1960 derived the acoustic gravity wave dispersion relation for an isotropic atmosphere.

\[
\omega^4 - \omega^2 (c^2 k \cdot k - ik_z \gamma g) + \gamma^2 (\gamma - 1) (k_x^2 + k_y^2) = 0 \tag{2.3.1}
\]

where \(\omega\) is the wave frequency, and \(k = (k_x, k_y, k_z)\) is the complex wave number vector. He assumed \(k_x, k_y\) to be purely real, and then writing \(k_z = k_{zr} + ik_{zi}\), split 2.3.1 into real and imaginary parts.

Real: \[
\omega^4 - \omega^2 (c^2 (k_{zr}^2 - k_{zi}^2 + k_x^2 + k_y^2) + k_{zi} \gamma g) + \gamma^2 (\gamma - 1) (k_x^2 + k_y^2) = 0 \tag{2.3.2}
\]

Imag: \[
k_{zr} (2c^2 k_{zi} - \gamma g) = 0 \tag{2.3.3}
\]

2.3.3 was then solved analytically for \(k_{zi}\) (assuming \(k_{zr} \neq 0\), i.e. ignoring surface waves), and the result

\[
k_{zi} = \frac{\gamma g}{2c^2} \tag{2.3.4}
\]

was substituted back into 2.3.2, yielding a modified, purely real dispersion relation

\[
\omega^4 - \omega^2 \left\{ c^2 k \cdot k + \frac{\gamma^2 g^2}{4c^2} \right\} + \gamma^2 (\gamma - 1) (k_x^2 + k_y^2) = 0 \tag{2.3.5}
\]
where now \( \mathbf{k} = (k_x, k_y, \text{Re}(k_z)) \).

The same analysis is now performed in a spherical polar \((r, \theta, \phi)\) system, with \( \mathbf{k} = (k_r, k_\theta, k_\phi) \). In this case the dispersion relation is given by

\[
\omega^2 - \frac{c^2}{r^2} \left\{ \frac{k^2 + ik_\phi \cot \theta + 2ik_r}{r} \right\} - \frac{ik_r \eta \varepsilon + 2\varepsilon(\varepsilon - 1)}{r} \\
+ \frac{\varepsilon^2(\varepsilon - 1)}{r^2} \left\{ \frac{k_\phi^2 + k_\phi^2 + ik_\phi \cot \theta}{r} \right\} = 0
\]

2.3.6

In the limit \( r \to \infty (\theta \neq 0, \theta \neq \pi) \) this reduces to 2.3.1.

This time \( k_\phi \) is assumed to be purely real, and \( k_r, k_\theta \) are written as \( k_r = k_{rr} + ik_{ri} \); \( k_\theta = k_{\theta r} + ik_{\theta i} \).

2.3.6 then has an imaginary part

\[
\omega^2 \left\{ \frac{c^2}{r^2} \left[ 2k_{rr} k_{ri} + 2k_{\theta r} k_{\theta i} + k_{\theta r} \cot \theta + 2k_{rr} \right] - \eta \varepsilon k_{rr} \right\} \\
+ \frac{\varepsilon^2(\varepsilon - 1)}{r^2} \left\{ 2k_{\theta r} k_{\theta i} + k_{\theta r} \cot \theta \right\} = 0
\]

2.3.7

Again ignoring surface waves, the analytic solution of 2.3.7 is

\[ k_{ri} = -\eta \varepsilon/2 \frac{c^2}{r^2} - 1/r \; ; \; k_{\theta i} = -\cot \theta/2 r \]

2.3.8

This reduces to 2.3.4 in the limit \( r \to \infty (\theta \neq 0, \theta \neq \pi) \). However, in general, \( k_{\theta i} \) is a function of \( \theta \), which is a physically unrealistic result, as different \( k_{\theta i} \) will be obtained for identical atmospheric conditions merely by rotating the coordinate system.

It is therefore evident that the above method of analysis breaks down if a non-cartesian coordinate system is used. The dispersion relation 2.3.6 must then be solved numerically for the various complex components of the wave number vector.

2.4 The Dispersion Relation

In this section the dispersion relation for acoustic
Gravity waves in an inhomogeneous atmosphere is derived, and employed to assess the effects of the various dissipative terms (thermal conductivity, ion drag, and viscosity) for a representative selection of waves. In this derivation the case of non-zero thermal conductivity is considered. If there is zero thermal conductivity, the equations of motion can be simplified, and the derivation of the dispersion relation in this case is given in appendix 4. Restating the equations of motion:

\[ p = \frac{\gamma RT}{m} \]

\[ \frac{\partial \rho}{\partial t} = \nabla \cdot (\rho \mathbf{u}) \]

\[ \frac{D (\rho \mathbf{u})}{Dt} = \rho \mathbf{g} - \nabla p + \nabla \cdot \mathbf{S} - \rho \frac{b^2}{\gamma} (\mathbf{U} - \mathbf{b}(\mathbf{U} \cdot \mathbf{b})) \]

\[ \frac{\rho R}{(\gamma - 1)m} \frac{DT}{Dt} + \rho \nabla \cdot \mathbf{U} - \nabla \cdot (\lambda \nabla T) - \mathbf{S} \cdot \nabla \mathbf{U} = 0 \]

A spherical polar coordinate system is chosen, and classical perturbation theory is applied in order to linearise the system 2.4.1. This theory is only valid for a medium which is slowly varying, i.e. one whose properties remain nearly constant over a wavelength. This means that disturbances with either a) long wavelengths, more than 500 km, or b) long periods, more than 2 hours, are excluded from the theory. The dynamic variables in 2.4.1 are the pressure, density, temperature and neutral fluid velocity. The perturbation assumptions used are:

\[ p(r, \theta, \phi, t) = p_0(r) + p_1(r, \theta, \phi, t) \]

\[ \rho(r, \theta, \phi, t) = \rho_0(r) + \rho_1(r, \theta, \phi, t) \]

\[ T(r, \theta, \phi, t) = T_0(r) + T_1(r, \theta, \phi, t) \]

\[ \mathbf{U}(r, \theta, \phi, t) = \mathbf{U}_1(r, \theta, \phi, t) \]
where the o-sufficed quantities are ambient atmospheric parameters, and are assumed to be functions of height only, and are taken to be much greater in magnitude than the 1-sufficed perturbation variables. No ambient wind profile is included at this stage. This implies that the analysis is performed in a coordinate system at rest with respect to the neutral wind. When this parameter is taken into account in the ray equations, it will be done by transforming the coordinate system into one at rest with respect to the Earth.

When the assumptions 2.4.2 are substituted into the equations of motion they yield the zeroth order equations (neglect terms of first order in the perturbation variables)

\[ \frac{\partial p_0}{\partial r} + \frac{\partial p_0}{\partial \theta} = 0 \]

\[ - \frac{\partial^2 T_0}{\partial r^2} + \frac{2 \partial T_0}{\partial r} \frac{\partial \theta}{\partial r} = 0 \]  \hspace{1cm} 2.4.3

\[ p_0 = \frac{p_0 R T_0}{m} \]

and the first order equations (neglect terms of second order in the perturbation variables, and use 2.4.3)

\[ p_1 = R \left( p_0 T_1 + p_1 T_0 \right) \frac{1}{m} \]

\[ \frac{\partial p_1}{\partial r} + \int_0^r \left[ \frac{\partial U_r}{\partial r} + \frac{1}{r} \frac{\partial U_r}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial U_\theta}{\partial \theta} \right] \frac{\partial p_1}{\partial \theta} \frac{\partial \theta}{\partial t} + \frac{U_r \partial p_0}{\partial r} = 0 \]  \hspace{1cm} 2.4.4

\[ \frac{\partial U_r}{\partial t} + \int_1^r \frac{\partial p_1}{\partial r} + \sigma B^2 (U_r - b_r U_\theta) - S_{rj, j} = 0 \]

\[ \frac{\partial U_\theta}{\partial t} + \frac{1}{r} \frac{\partial p_1}{\partial \theta} + \sigma B^2 (U_\theta - b_\theta U_r) - S_{\theta j, j} = 0 \]
\[ \frac{\partial}{\partial t} \left( \frac{\partial P}{\partial \sin \theta} \right) + \sigma_p B^2 (U - bU \cdot b) - \frac{\partial}{\partial \phi} \rho_j = 0 \]

\[ \sigma_p \frac{R}{(\delta - 1)m} \left( \frac{\partial T_1}{\partial t} + U \frac{\partial T_0}{\partial r} \right) + p_o \left( \frac{\partial U}{\partial r} + \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \right) \]

\[ + 2 \frac{U_r + \frac{U}{r} \cot \theta}{r} \left( \frac{\partial^2 T_1}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 T_1}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T_1}{\partial \phi^2} \right) \]

where terms of \( O(\delta / \partial r) \) have been ignored as \( \delta / \partial r \ll \lambda \) (see appendix 1). It is now assumed that the ambient pressure and density follow the barometric equations

\[ p_o, \gamma_o \propto \exp(-z/H) \]

where \( H \) is the atmospheric scale height, defined by

\[ H = c^2 / g = RT_0 / mg \]

and \( c \) is the atmospheric sound speed.

Combining 2.4.6 with 2.4.3 the relation

\[ \frac{\partial p}{\partial \phi} = c^2 \]

is obtained.

A time variation of \( \exp(i \omega t) \) is assumed throughout, such that \( \partial / \partial t = i \omega \). The system 2.4.4 then becomes

\[ \frac{gH \frac{\partial P}{\partial r} = \frac{R}{m} (T_1 + T_0 \frac{\partial P}{\partial r}) \]

\[ \frac{i \omega R + \frac{\partial U}{\partial r} + \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} + 2 \frac{U_r + \frac{U}{r} \cot \theta}{r} - U \frac{\partial U}{\partial r} + U \frac{\partial \cot \theta}{\partial \theta} - \frac{U_r}{r} = 0} \]

\[ \frac{i \omega U_r + \frac{\partial G}{\partial r} - \frac{\sigma_p}{p_o} \frac{\partial P}{\partial r} - \frac{\partial \cot \theta}{\partial \theta} - \frac{U}{r} = 0}{p_o \frac{\partial U}{\partial \theta} - \frac{\partial \cot \theta}{\partial \phi} \frac{U}{r} \sin \theta = 0} \]

\[ \frac{i \omega U_r + \frac{\partial G}{\partial \theta} - \frac{\partial \cot \theta}{\partial \phi} \frac{U}{r} \sin \theta = 0}{p_o \frac{\partial U}{\partial \phi} - \frac{\partial \cot \theta}{\partial \theta} \frac{U}{r} \sin \theta = 0} \]

\[ \frac{i \omega \frac{\partial U}{\partial \phi} + \frac{\partial G}{\partial \phi} - \frac{\partial \cot \theta}{\partial \theta} \frac{U}{r} \sin \theta = 0}{p_o \frac{\partial U}{\partial \theta} - \frac{\partial \cot \theta}{\partial \phi} \frac{U}{r} \sin \theta = 0} \]
\[ R \left\{ \frac{i \omega T_1 + U \frac{d T_0}{dr}}{(1-m)^2} \right\} + \frac{\partial}{\partial r} \left\{ \frac{\partial T_1}{\partial r} + \frac{1}{r} \frac{\partial U}{\partial \theta} + \frac{1}{r^2} \frac{\partial \Omega}{\partial \phi} + \frac{2U_U}{r} \right\} + \frac{U \cot \theta}{r} \ \frac{\partial T_1}{\partial r} \ \frac{\partial ^2 T_1}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial T_1}{\partial \phi} + \frac{1}{r^2} \frac{\partial ^2 T_1}{\partial \theta \partial \phi} + \frac{2}{r} \frac{\partial T_1}{\partial \phi} + \left( \frac{\partial ^2 T_1}{\partial \theta^2} + \frac{1}{r^2} \frac{\partial T_1}{\partial \phi} + \frac{1}{r^2} \frac{\partial ^2 T_1}{\partial \theta \partial \phi} + \frac{2}{r} \frac{\partial T_1}{\partial \phi} \right) \right\} = 0 \]

where \( \lambda = \frac{\lambda}{p_p} \); \( \rho = \frac{B^2}{p_o} \); and new variables \( P, R \) have been defined by \( P = \frac{p_1}{p_0} \); \( R = \frac{\gamma}{f} \).

The quantity \( 1/\rho \) has the dimensions of time, and has been interpreted by Rishbeth, Megill and Cahn, 1965 as the time that passes before most neutral particles achieve at least one collision with an ion.

To proceed further, the general Eikonal method (Weinberg, 1962) is used. The perturbation variables are assumed to be proportional to \( \exp(-iS) \), where \( S \) is a function such that \( \nabla S = k \), the wave number vector. Such a function is defined by Weinberg to be

\[ S(r,k) = \int k(\tau) \frac{\partial \tau(\tau)}{\partial \tau} d\tau \]

This implies that the identities \( \partial / \partial r = -ik_r \); \( \partial / \partial \theta = -ik_\theta \); \( \partial / \partial \phi = -ik_\phi \) \( \sin \theta \) hold.

The viscous stress tensor, \( S_{ij} \), is now expanded using 2.2.6 and 2.2.7. Terms of \( O(\eta / \partial r) \) are neglected, as \( \eta / \partial r < \eta \) (see appendix 1). The coefficient of kinematic viscosity, \( \nu \), is defined by \( \nu = \frac{\eta}{f} \). Finally, terms of \( O(1/r) \), and \( O(\cot \theta / r) \) are neglected. The latter assumption removes a mathematical singularity from the poles. Thus, strictly speaking, if it is required that a calculation near the poles should be performed, the coordinate system should be rotated through 90° in the \( \theta \) direction for accuracy to be maintained. Both assumptions
limit the wavelengths for which the theory is valid to less than around 500 km, an assumption which has already had to be made by invoking perturbation theory. 2.4.8 can now be written as

\[ g_{HP} = \frac{R(T_1 + T_o R)}{m} \]

\[ i\omega R - ik \cdot U - U = 0 \]

\[ i\omega U_r + Rg - ik \cdot g_{HP} + \nabla(3U_r \cdot k + k_r \cdot U \cdot k) + \beta(U_r - b_r U \cdot b) \]

\[ = 0 \quad (2.4.10) \]

\[ i\omega U_o - ik \cdot g_{HP} + \nabla(3U_o \cdot k + k_o \cdot U \cdot k) + \beta(U_o - b_o U \cdot b) = 0 \]

\[ i\omega U_f - ik \cdot g_{HP} + \nabla(3U_f \cdot k + k_f \cdot U \cdot k) + \beta(U_f - b_f U \cdot b) = 0 \]

\[ \frac{R}{m(\gamma - 1)} \left( i\omega T_1 + U_r \frac{\partial T_o}{\partial r} \right) - i\phi U \cdot k + \lambda T_1 \cdot k \cdot k = 0 \]

yielding a matrix equation of the form

\[ A \times = 0 \quad (2.4.11) \]

where \( \times \) is the column vector \((P, R, T_1, U_r, U_o, U_f)\) and \( A \) is a matrix with non-zero elements given by

\[ A_{11} = 1 \]
\[ A_{13} = -\frac{R}{m} \]
\[ A_{24} = -ik_r - \frac{1}{H} \]
\[ A_{26} = -ik_f \]
\[ A_{32} = g \]
\[ A_{35} = \nu k \cdot k_o / 3 - \beta r b \cdot b \]
\[ A_{41} = -ik_o \]
\[ A_{45} = i\nu [\gamma / 3](3k \cdot k + k^2) / b + \beta (1 - b^2) \]
\[ A_{12} = -\frac{R T_o}{m} \]
\[ A_{22} = i\nu \]
\[ A_{25} = -ik_o \]
\[ A_{31} = -ik_r \]
\[ A_{34} = i\nu [(\gamma / 3)(3k \cdot k + k^2) / b + \beta (1 - b^2) \]
\[ A_{36} = \nu k \cdot k_f / 3 - \beta b \cdot b \]
\[ A_{44} = \nu k \cdot k_o / 3 - \beta b \cdot b \]
\[ A_{46} = \nu k \cdot k_f / 3 - \beta b \cdot b \]
For 2.4.11 to have a non trivial solution the determinant of the coefficient matrix must vanish, i.e.

\[ |A| = 0 \]  

2.4.12 can be written

\[ D(\tau, k, \omega) = 0 \]  

The above relation is known as the acoustic gravity wave dispersion relation. It contains four unknowns, the three complex components of the wave number vector, \( \mathbf{k} \), and the wave frequency, \( \omega \). This relation will now be analysed to assess the effects of the various dissipative processes. Throughout the analysis \( k_\rho \) is taken to be zero, and \( k_\sigma \) is taken to be purely real. Thus, for a given atmospheric model the unknown quantities are now reduced to \( \omega \), \( k_\tau \), and \( \text{Re}(k_\sigma) \). 2.4.13 is solved for \( k_\tau \) for various values of \( \text{Re}(k_\sigma) \) and \( \omega \). These values are given in table 2.4.1. In full generality, the dispersion relation is an eighth order equation in \( k_\tau \). The root which is considered here is that corresponding to an upgoing, propagating gravity wave. It is easily seen that this root must be such that \( \text{Re}(k_\tau), \text{Im}(k_\tau) < 0 \) (Clark, Yeh and Liu, 1970).

<table>
<thead>
<tr>
<th>Horizontal Wavelength ( \lambda_\sigma, \text{km} )</th>
<th>( k_\sigma = 2\pi / \lambda_\sigma ), km(^{-1} )</th>
<th>wave period ( \tau ), mins</th>
<th>( \omega = 2\pi / 60\tau ), s(^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.063</td>
<td>20</td>
<td>0.005</td>
</tr>
<tr>
<td>300</td>
<td>0.021</td>
<td>20</td>
<td>0.005</td>
</tr>
<tr>
<td>500</td>
<td>0.013</td>
<td>40</td>
<td>0.003</td>
</tr>
</tbody>
</table>
The results of this analysis are presented in the form of graphs of $\log_{10}(-\text{Re}(k_r))$ against height (denoted by a letter $R$ on the graph), and of $\log_{10}(-\text{Im}(k_r)+1/2H)$ against height (denoted by a letter $I$). The real part, $\text{Re}(k_r)$, of the vertical wave number determines the phase propagation of the wave in the vertical direction, and the imaginary part, $\text{Im}(k_r)$, yields a measure of the wave attenuation. A factor of $1/2H$ has been subtracted from $\text{Im}(k_r)$ in the analysis, as this is the factor responsible for the exponential growth of the wave amplitude in a non-dissipative, isothermal atmosphere, and is independent of any dissipative process. Klostermeyer, 1972, 1972a has presented a similar analysis to this, but considers the atmosphere from heights of 150 to 600 km, whereas this work considers a height range from ground level to 500 km. Klostermeyer took his lower limit to be 150 km because he assumed dissipative processes to be unimportant below this height. This analysis is, however, extended to ground level, so that some indications of the heights of gravity wave sources may be obtained, in particular whether these sources are likely to be lower or upper atmospheric sources for the wave parameters considered. Also, ion drag is taken into account for daytime, sunspot maximum conditions, when this parameter is likely to have most effect on gravity wave propagation (Francis, 1973). The atmospheric model used in this section are given in appendix 1. Owing to limitations on the computing facilities available, it has not been possible to consider ion drag, thermal conductivity and viscosity together, and therefore the results are presented for the cases 1) no dissipation present, 2)
only one form of dissipation present, 3) two forms of dissipation present.

The first wave considered is one with a horizontal wavelength of 100 km, and a period of 20 minutes. This case is depicted in figs. 2.4.1 to 2.4.7, each graph being labelled to indicate the types of dissipation being considered. In figs. 2.4.2 to 2.4.4, only one type of dissipation is considered, and in each case the real part of the vertical wave number remains remarkably similar to that of the isotropic case, depicted in fig. 2.4.1. The imaginary part, whilst being of a similar magnitude in the upper atmosphere, exhibits significantly different effects in the lower atmosphere, below 150 km, when different types of dissipation are taken into account. In the case where thermal conductivity is considered, fig. 2.4.3, \text{Im}(k_r)\text{ is} several orders of magnitude greater than when either ion drag or viscosity is considered. Figs. 2.4.5 to 2.4.7 present the results obtained when two types of dissipative process are included. When thermal conductivity and ion drag are considered, fig. 2.4.5, at around 360 km \text{Re}(k_r)\ goes rapidly to zero as height is increased, implying strong reflection at this level. Above this level, it appears to recover its former magnitude rapidly, and then remains almost constant. However, if the numerical results of the computation are examined, it is seen that the propagating root above 360 km does not correspond to a propagating gravity wave, but to a propagating thermal conduction wave. The viscosity and ion drag model, fig. 2.4.6, also shows that the gravity wave exhibits strong upper atmospheric reflection, this time around the 450 km level. However,
HORIZ. WAVELENGTH = 1.000E+03KM, ISOTROPIC CASE.
LAT = 5.100E+02DEGS N., LONG = 1.000E+02DEGS EAST, T = 2.000E+02MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.1
HORIZ. WAVELENGTH = 1000E+03 KM. THERMAL CONDUCTIVITY.
LAT = 0.5100E+02 DEGS N, LONG = 1000E+02 DEGS EAST, T = 2000F+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.

FIG 2.4.3

HORIZ. WAVELENGTH = 1000E+03 KM. VISCOSITY.
LAT = 0.5100E+02 DEGS N, LONG = 1000E+02 DEGS EAST, T = 2000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.

FIG 2.4.4
HORIZ. WAVELENGTH = 1.000E+03 KM, THERMAL CONDUCTIVITY AND ION DRAG
LAT = 5.100E+02 DEGS N., LONG = 1.000E+02 DEGS EAST, T = 2.000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.5

HORIZ. WAVELENGTH = 1.000E+03 KM, VISCOSITY AND ION DRAG.
LAT = 5.100E+02 DEGS N., LONG = 1.000E+02 DEGS EAST, T = 2.000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.6
no upgoing propagating wave exists above this height. Finally, fig. 2.4.7 depicts results when viscosity and thermal conductivity are considered, and indicates that the net effect is roughly the sum of the individual contributions of viscosity and thermal conductivity.

The next case considered is that of a wave with 300 km horizontal wavelength and 20 minute period. In the isotropic case, fig. 2.4.8, it is seen that the wave is totally reflected at the mesopause. It appears that such a wave detected above this level would be more likely to have a source above this level. Correspondingly, a wave detected below the mesopause is more likely to have a source below the mesopause. However, since gravity waves are imperfectly ducted, it is possible that energy from a wave source below the mesopause could leak through the region of strong reflection and be detected in the thermosphere (Friedman, 1966). When ion drag alone is considered, fig. 2.4.9, similar results are obtained for Re(k_r), and Im(k_r) varies in a similar manner to the corresponding case for the previous wave considered, fig. 2.4.2. The cases of viscosity only, fig. 2.4.10, and thermal conductivity only, fig. 2.4.11, are similar in that no upgoing, propagating wave exists below the mesopause, thus indicating that it is unlikely that a source for this type of gravity wave exists below this level. The real parts of k_r are similar for these two cases throughout the atmosphere, but the imaginary parts, although similar above 300 km, exhibit marked differences below this level. This result is qualitatively different from the calculations of Klostermeyer, 1972 for a similar wave. The different result obtained can be accounted for
HORIZ. WAVELENGTH = 1.000E+03 KM, VISCOSITY AND THERMAL CONDUCTIVITY
LAT = 5.100E+02 DEGS N., LONG = 1.000E+02 DEGS EAST, T = 2.000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4. 7

HORIZ. WAVELENGTH = 3.000E+03 KM, ISOTROPIC CASE.
LAT = 5.100E+02 DEGS N., LONG = 1.000E+02 DEGS EAST, T = 2.000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4. 8
HORIZ. WAVELENGTH = 3000E+03 KM, ION DRAG.
LAT = 5100E+02 DEGS N, LONG = 1000E+02 DEGS EAST, T = 2000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.9
in terms of the different models adopted for the coefficients of viscosity and thermal conductivity. When two types of dissipation are considered, figs. 2.4.12 to 2.4.14, above the mesopause the behaviour of $k_T$ is similar to the case of the 100 km, 20 minute wave, figs. 2.4.5 to 2.4.7, with the exception that the regions of strong reflection in the ion drag and thermal conductivity, and the ion drag and viscosity cases are brought much closer together. Again, below the mesopause no upgoing, propagating waves exist, leading to the speculation that the source mechanism for disturbances of this wavelength is likely to be above this level.

Finally, a wave with a horizontal wavelength of 500 km, and a period of 40 minutes is discussed. The isotropic case, fig. 2.4.15, produces a continuous root throughout the atmosphere. When ion drag is included, fig. 2.4.16, a region of strong reflection occurs at the mesopause, but as this region is very narrow it is possible to envisage a tropospheric source of such a wave being able to leak energy upwards, to be detected in the thermosphere. When thermal conduction is included, fig. 2.4.17, the results are similar to those obtained for the previous wave, fig. 2.4.10. There is, however, a slight difference in that from 30 to 50 km a very weakly propagating wave can exist. Fig. 2.4.18 illustrates results obtained when viscosity alone is included. This again is similar to the corresponding case for the previous wave, fig. 2.4.11, but exhibits the interesting feature that two narrow waveguides appear to exist, one in the troposphere, and one around 50 km, these being regions of temperature minima. When two forms of
Fig 2.4.13

Horiz. Wavelength = .3000E+03 km, Viscosity and Ion Drag.
Lat = .5100E+02 degs N., Long = .1000E+02 degs E.
T = 2.000E+02 mins.
Plot of log (vertical wave number) against height.

Fig 2.4.14

Horiz. Wavelength = .3000E+03 km, Viscosity and Thermal Conductivity
Lat = .5100E+02 degs N., Long = .1000E+02 degs E.
T = 2.000E+02 mins.
Plot of log (vertical wave number) against height.
HORIZ. WAVELENGTH = 50000E+03KM, ISOTROPIC CASE.
LAT = 5100E+02DEGS N., LONG = 1000E+02DEGS EAST, T = 4000E+02MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.15

HORIZ. WAVELENGTH = 50000E+03KM, ION DRAG.
LAT = 5100E+02DEGS N., LONG = 1000E+02DEGS EAST, T = 4000E+02MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.16
HORIZ. WAVELENGTH = 5000E+03 KM. THERMAL CONDUCTIVITY.
LAT = 5100E+02 DEGS N., LONG = 1000E+02 DEGS EAST. T = 4000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.17

HORIZ. WAVELENGTH = 5000E+03 KM. VISCOSITY.
LAT = 5100E+02 DEGS N., LONG = 1000E+02 DEGS EAST. T = 4000E+02 MINS.
PLOT OF LOG (VERTICAL WAVE NUMBER) AGAINST HEIGHT.
FIG 2.4.18
dissipation are included, figs. 2.4.19 to 2.4.21, the following points are to be noted:— 1) thermal conduction effects tend to dominate in the lower atmosphere, below 100 km, and 2) ion drag effects have their greatest effect in the upper atmosphere, above 300 km. If thermal conduction is neglected the waveguides obtained when viscosity only is considered are retained, otherwise they disappear, leading one to the conclusion that they would not occur in a completely realistic atmosphere. When ion drag is included, no upper atmospheric reflection layer occurs, although Re(k^r) is strongly perturbed above 400 km. These last three graphs demonstrate the necessity of invoking an upper atmospheric source.

The results described above lead to several conclusions regarding gravity wave propagation. Firstly, it is evident that both upper atmospheric and tropospheric sources can exist, but that the former are more likely to be important for the larger, longer period waves, and the latter for smaller, shorter period disturbances. This is in qualitative agreement with the raytracing analysis of Bertin et al, 1975. Secondly, it has been shown that ion drag, when considered in isolation has an insignificant effect, but when coupled with another form of dissipation it becomes very important, especially in the upper atmosphere. It is worth reiterating that this parameter has been taken into account for conditions when it is likely to have a maximum effect, and thus this analysis places an upper bound on the perturbations to k^r caused by including this type of dissipation.
2.5 Group Velocity, and the Distinction between Acoustic and Gravity Modes of Propagation

In this section, the differences between acoustic and gravity wave propagation are examined by means of the dispersion relation. Group velocity is then defined, and together with a geometrical analysis a further difference between acoustic and gravity modes of propagation is made clear. For the sake of clarity, the two dimensional dispersion relation in an isotropic atmosphere is used (Hines, 1960). In a cartesian system this is

\[ \omega^4 - \omega^2 \left( \frac{c^2(k_x^2 + k_z^2) + \gamma^2 g^2}{4c^2} \right) + \frac{g^2(\gamma-1)k_x^2}{2c^2} = 0 \]  

2.5.1

or,

\[ \omega^4 - \omega^2 \left( c^2(k_x^2 + k_z^2) + \omega_a^2 \right) + \omega_b^2 c^2 k_x^2 = 0 \]  

2.5.2

where \( \omega_a = \gamma g/2c \); \( \omega_b = g(\gamma-1)^{1/2}/c \)

It is easily shown that both \( \omega_a \) and \( \omega_b \) have the dimensions of frequency. In the atmosphere, as \( 1.4 < \gamma < 1.7 \) (see appendix 1), \( \omega_a \) is always greater than \( \omega_b \).

2.5.2 can be recast in the form

\[ \frac{k_x^2}{L^2} + \frac{k_z^2}{b^2} = 1 \]  

2.5.4

where \( L^2 = \frac{\omega^2(\omega^2 - \omega_a^2)}{c^2(\omega^2 - b^2)} \); \( b^2 = \frac{(\omega^2 - \omega_a^2)}{c^2} \)

(Georges, 1967). 2.5.4 is the equation of a general conic in \( k_x - k_z \) space, and will have different interpretations according to the values of \( L^2 \) and \( b^2 \).

If \( \omega > \omega_a \), \( L^2 > 0 \) and \( b^2 > 0 \), then 2.5.4 represents a family of ellipses.

If \( \omega < \omega_a \), \( L^2 < 0 \) and \( b^2 < 0 \), then no real solutions of 2.5.4 exist.

If \( \omega < \omega_b \), \( L^2 > 0 \) and \( b^2 < 0 \), then 2.5.4 represents a family of hyperbolae.

Acoustic waves are defined as those waves corresponding
to the elliptic case, $\omega_\omega a$, and gravity waves as those corresponding to the hyperbolic case, $\omega_\omega b$. i.e. acoustic waves correspond to the high frequency branch of the spectrum and gravity waves to the low frequency branch. These cases are illustrates in figs. 2.5.1 and 2.5.2.

The phase velocity of an acoustic-gravity wave is defined as $\nu_p = \omega / k$. Thus it is seen that $\nu_p, k$ are parallel vectors. The group velocity is defined as $\nu_g = d\omega / dk$, and the group propagation direction is indicated by the outward normal vector to the propagation curve. Note that the vertical components of the phase and group velocities are oppositely directed for gravity waves, and codirectional for acoustic waves (see figs. 2.5.1 and 2.5.2). This illustrates another distinction between acoustic and gravity waves, and is one that must be borne in mind in raytracing theory, since the traces are along the group path. Thus, for an upgoing acoustic wave $Re(k_z)$ must be positive, and for an upgoing gravity wave it must be negative. Finally, it is seen that for acoustic waves, as frequency increases, the ellipses in fig. 2.5.1 approach a circular shape, because gravity exerts a smaller effect on waves with higher frequencies. Also, if $\omega_\omega \omega a$, it is seen that 2.5.2 reduces to $\omega^2 = c^2 k_x k_z$, the equation of a circle in $k_x - k_z$ space. For gravity waves, the hyperbolae in fig. 2.5.2 approach the horizontal axis as frequency tends to its upper limit, $\omega_b$. In this case phase propagation is nearly horizontal, whilst group propagation is nearly vertical.

The frequency $\omega_a$ is called the acoustic cut-off frequency (Hines, 1960), and as shown above represents the low frequency limit for acoustic wave propagation. $\omega_b$. 
**Decreasing ACOUSTIC WAVES**

Figure 2.5.1 (After Georges, 1967)

**Increasing INTERNAL GRAVITY WAVES**

Figure 2.5.2 (After Georges, 1967)
however, represents a high frequency cut-off for gravity wave propagation in an isothermal atmosphere (Georges, 1967), and has the physical interpretation of being the frequency with which an air parcel would oscillate about its equilibrium position after being slightly displaced (Vaisala, 1925; Brunt, 1927). It is commonly called the Brunt-Vaisala frequency.

2.6 The Ray Equations

The dispersion relation for acoustic-gravity waves in an inhomogeneous atmosphere has been derived in the form

$$D(r, k, \omega) = 0$$  \hspace{1cm} 2.6.1

Following the method of Weinberg, 1962 the ray equations can now be written in the form

$$\frac{dr}{dt} = -\frac{\partial D/\partial k}{\partial D/\partial \omega}$$ \hspace{1cm} \frac{dk}{dt} = \frac{\partial D/\partial r}{\partial D/\partial \omega}  \hspace{1cm} 2.6.2

These are valid in a coordinate system at rest with respect to the neutral wind. To take neutral winds into account, 2.6.2 has to be transformed into a coordinate system at rest with respect to the Earth, yielding

$$\frac{dr}{dt} = -\frac{\partial D/\partial k_r}{\partial D/\partial \omega} + U_r$$

$$\frac{d\phi}{dt} = \frac{1}{r} \left\{ \frac{\partial D/\partial k_\phi}{\partial D/\partial \omega} + U_\phi \right\}$$

$$\frac{dk_r}{dt} = \frac{\partial D/\partial r - k_\phi U + k_\omega d\omega + k \sin \theta d\phi}{\partial D/\partial \omega}$$

$$\frac{dk_\phi}{dt} = \frac{1}{r} \left\{ \frac{\partial D/\partial \phi - k_\omega U - k \omega d\omega + k \sin \theta d\phi}{\partial D/\partial \omega} \right\}$$

$$\frac{dk_\omega}{dt} = \frac{rsin\theta \left\{ \frac{\partial D/\partial \theta - k_\phi U - k_\omega d\omega + rk \cos \phi d\phi}{\partial D/\partial \omega} \right\}}{\partial D/\partial \omega}$$
(Georges, 1971), where \( \mathbf{u} = (U_r, U_\theta, U_\phi) \) represents the neutral wind.

These equations can then be integrated numerically to yield acoustic-gravity raypaths, and a description of the computer programme used to do this is given in appendix 3. The effects of viscosity, thermal conductivity and ion drag for the same wave parameters as in \( \S 2.4 \) have been determined from the analysis, and these are reproduced in table 2.6.1. The integration is started in each case from geographic coordinates 51°N, 0°E. Three different heights in the atmosphere are chosen to commence the raytrace; 10 km, 120 km, and 240 km. All rays are launched at an azimuth of 0°E of N, and the initial angle of elevation to the horizontal is given by

\[
\tan^{-1}\left[\frac{\omega^2}{(\omega^2 - \omega_0^2)^{\frac{1}{2}}}\right]\]

(Hines, 1967), and is taken to be upwards for the 10 and 120 km starting heights, and downwards for the 240 km start height. The integration is stopped when either:-

1) the ray reaches ground level.

2) the ray is ducted, and propagates horizontally.

3) the ray becomes evanescent, taken to be when the wave amplitude decreases by a factor of e in a height of less than 5 km.

4) the ray height exceeds 300 km above ground level, or the ground range exceeds 2500 km.

<table>
<thead>
<tr>
<th>Horizontal Wavelength, km</th>
<th>Period mins</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>300</td>
<td>20</td>
</tr>
<tr>
<td>500</td>
<td>40</td>
</tr>
</tbody>
</table>
or 5) the error in the integration becomes significant. Neutral winds are neglected throughout this analysis, and the atmospheric model used is described in appendix 1.

The results are presented as graphs of ray height against ground range. The case corresponding to a source height of 10 km is discussed first. Figs. 2.6.1 to 2.6.7 depict raypaths for the 100 km, 20 minute wave. All rays are reflected in the troposphere and then return to ground level. When the effects of thermal conductivity were ignored, figs. 2.6.1, .2, .4, and .6, the ray reaches ground level at a range of just over 2000 km, and the processes of viscosity and ion drag, which are considered both individually and collectively in these cases, are seen to have no noticeable effect. When thermal conductivity is included, however, fig. 2.6.3, the ground range for the ray is reduced to about 370 km.

The inclusion of ion drag, fig. 2.6.5, produces a similar result, whereas the inclusion of viscosity with thermal conductivity, fig. 2.6.7, produces reflection at a higher level, and ducting at a range of 2100 km, and a height of about 10 km.

Figs. 2.6.8 to .14 depict the 300 km, 20 minute wave. In this case all rays are reflected just above the launch height and ultimately propagate to ground level. When thermal conductivity is ignored, figs. 2.6.8, .9, .11 and .13, the ground range falls just short of 220 km, and the inclusion of viscosity and ion drag has little effect on the raypath. On the inclusion of thermal conductivity, figs. 2.6.10, .12 and .14, the ground range is reduced to about 80 km, being slightly greater than this value when viscosity is ignored, figs. 2.6.10 and .12, and slightly
FIG 2.6. 1: 100 KM., 20' MINS., H=10'KM., ISOTROPIC CASE.
AZ= 0.0000, GRAVITY

FIG 2.6. 2: 100 KM., 20' MINS., H=10'KM., ION DRAG.
AZ= 0.0000, GRAVITY
FIG 2.6. A 100 KM., 20 MINS., H=10 KM., THERMAL CONDUCTIVITY. 
AZ= 0.0000, GRAVITY

FIG 2.6. A 100 KM., 20 MINS., H=10 KM., VISCOSITY. 
AZ= 0.0000, GRAVITY
FIG 2.6, 5, 100 KM., 20 MINS., H=10 KM.; THERMAL CONDUCTIVITY & ION DRAG.
AZ= 0.0000, GRAVITY

FIG 2.6, 6, 100 KM., 20 MINS., H=10 KM., VISCOSITY & ION DRAG.
AZ= 0.0000, GRAVITY
FIG 2.6. 7. 100 KM., 20 MINS., H=10 KM., VISCOITY & THERMAL CONDUCTIVITY. AZ= 0.0000, GRAVITY

FIG 2.6. 8. 300 KM., 20 MINS., H=10 KM., ISOTROPIC CASE. AZ= 0.0000, GRAVITY
FIG 2.6. '9, 300 KM., 20 MINS., H=10 KM., ION DRAG.
AZ = 0.0000, GRAVITY

FIG 2.6. '10, 300 KM., 20 MINS., H=10 KM., THERMAL CONDUCTIVITY.
AZ = 0.0000, GRAVITY
FIG 2.6.13, 300 KM., 20 MINS., \( H=10 \) KM., 'VISCOSITY & ION' DRAG.'
\( \text{AZ}=0.0000, \text{GRAVITY} \)

FIG 2.6.14, 300 KM., 20 MINS., \( H=10 \) KM., 'VISCOSITY & THERMAL CONDUCTIVITY'
\( \text{AZ}=0.0000, \text{GRAVITY} \)
less when it is included, fig. 2.6.14.

The same type of behaviour is exhibited in the case of the 500 km, 40 minute wave, figs. 2.6.15 to 2.6.21. However, the ground range is about 500 km if thermal conductivity is ignored, and is about 200 km if it is included.

Thus, in the lower atmosphere thermal conductivity has a greater effect than the other two types of dissipation, although for the shortest wavelength considered the inclusion of viscosity with thermal conductivity produced a significant perturbation to the raypath. These results indicate that if a medium scale disturbance exists in the lower atmosphere, it is unlikely to propagate to ionospheric heights, where it can be detected using radio methods. In the lower atmosphere, neutral winds are unlikely to have a magnitude large enough to significantly affect the raypath.

The waves launched at 120 km are now considered. Figs. 2.6.22 to 2.6.25 and 2.6.27 depict the 100 km, 20 minute disturbance. When thermal conductivity is ignored, figs. 2.6.22, 2.6.23, 2.6.25 and 2.6.27, the presence of either viscosity or ion drag is seen to have only a small effect, the ray in each case being reflected at 130 km height, and reaching ground level at around 1800 km ground range from the launch point. When thermal conductivity alone is considered, fig. 2.6.24, the ray exhibits a quite different behaviour, and is reflected at a height just below 200 km, becoming ducted at the base of the thermosphere at around 2500 km ground range. If ion drag or viscosity are included with thermal conductivity, then...
FIG 2.6.15, 500 KM., 40 MINS., H=10 KM., ISOTROPIC CASE  
AZ = 0.0000, GRAVITY

FIG 2.6.16, 500 KM., 40 MINS., H=10 KM., ION DRAG  
AZ = 0.0000, GRAVITY
FIG 2.6.17 ' 500 KM., 40 MINS., H=10 KM., THERMAL CONDUCTIVITY
AZ= 0.0000, GRAVITY

FIG 2.6.18 ' 500 KM., 40 MINS., H=10 KM., VISCOSITY
AZ= 0.0000, GRAVITY
FIG 2.6.19, 1500 KM., 40 MINS., H=10 KM., THERMAL CONDUCTIVITY & ION DRAG
AZ = 0.0000, GRAVITY

FIG 2.6.20, 1500 KM., 40 MINS., H=10 KM., VISCOSITY & ION DRAG
AZ = 0.0000, GRAVITY
for all realistic integration step lengths (greater than or equal to \(0.1\) sec) numerical errors swiftly become significant and meaningful results cannot be obtained.

Figs. 2.6.29 to .32 and 2.6.34 depict the raypaths for the 300 km 20 minute wave. Once again, if thermal conductivity is neglected, figs. 2.6.29, .30, .32 and .34, the raypaths are almost identical, the ray being reflected at a height of about 160 km, and becoming ducted at the base of the thermosphere at a ground range of just over 1000 km. When the effects of thermal conductivity only are considered, fig. 2.6.31, the ray is propagated upwards throughout, and reaches the maximum height considered, 300 km, at a ground range of about 400 km. If other forms of dissipation are included with thermal conductivity, again error terms dominate and no meaningful results can be obtained.

The 500 km, 40 minute wave is considered in figs. 2.6.36 to .39 and 2.6.41. When thermal conductivity is ignored, figs. 2.6.36, .37, .39 and .41, the rays propagate upwards till they reach the upper limit at ranges between 2000 and 2500 km. The isotropic case, fig. 2.6.36, yields the smallest range and the largest range is obtained when both viscosity and ion drag are included. For thermal conductivity alone, fig. 2.6.38, then the same qualitative behaviour is observed, but the raypath is steeper, and the ray reaches the maximum height at a range of just over 1200 km. Once again, if other dissipative processes are included, error terms dominate and no realistic results can be presented.

For a middle atmospheric launch height viscosity and ion drag have thus been shown to have only a small effect
FIG 2.6.29, 300 KM, 20 MINS., H=120 KM., ISOTROPIC CASE.
AZ = 0.0000, GRAVITY

FIG 2.6.30, 300 KM, 20 MINS., H=120 KM., ION DRAG.
AZ = 0.0000, GRAVITY
FIG 2.6.31, 300 KM, 20 MINS., H=120 KM, THERMAL CONDUCTIVITY.
AZ = 0.0000, GRAVITY

FIG 2.6.32, 300 KM, 20 MINS., H=120 KM, VISCOSITY.
AZ = 0.0000, GRAVITY
FIG 2.6.34, 300 KM, 20 MINS., H=120 KM, VISCOSITY & ION DRAG.
AZ = 0.0000, GRAVITY

FIG 2.6.36, 500 KM, 40 MINS., H=120 KM, ISOTROPIC CASE
AZ = 0.0000, GRAVITY
FIG 2.6.39' 500 KM, 40 MINS., H=120 KM., VISCOSITY
AZ = 0.0000, GRAVITY

FIG 2.6.41' 500 KM, 40 MINS., H=120 KM., VISCOSITY & ION DRAG
AZ = 0.0000, GRAVITY
on the propagation characteristics of the shorter wavelengths. However, for the longest wavelength, which propagates into the upper atmosphere without being ducted, the inclusion of these parameters leads to differences in ground range of up to 25%, although the raypaths are qualitatively the same. For the longest wavelength thermal conductivity produces the same qualitative effect, but the raypath is far steeper, and the ray enters the upper atmosphere at a shorter range. For the shorter wavelengths, thermal conductivity is shown to produce a qualitatively different type of raypath. In all wavelengths considered, it was found impossible to include other dissipative effects with thermal conductivity as error terms rapidly dominated the integration even for small integration step lengths.

Finally, the waves launched from a height of 240 km are considered. Figs. 2.6.43 to .46 and 2.6.48 depict the raypaths of the 100 km, 20 minute wave. In the absence of thermal conductivity, fgs. 2.6.43, .44, .46 and .48, all rays propagate to ground level, which is reached at a range of about 700 km. If thermal conductivity alone is considered the ray is ducted at the base of the thermosphere at a range of about 1200 km. Once again, if either viscosity or ion drag are included with thermal conductivity numerical errors dominate and no meaningful results can be presented.

For the 300 km, 20 minute wave, fgs. 2.6.50 to .53 and 2.6.55, all rays exhibit the same qualitative behaviour, becoming ducted at the base of the thermosphere at ranges between 300 and 400 km. However, if thermal conductivity alone is considered, the raypath is shallower, and the ray becomes ducted at a greater
FIG 2.6.45, 100 KM, 20 MINS., H=240 KM, THERMAL CONDUCTIVITY.
AZ = 0.0000, GRAVITY

FIG 2.6.46, 100 KM, 20 MINS., H=240 KM, VISCOSITY.
AZ = 0.0000, GRAVITY
FIG 2.6.48, 100 KM., 20 MINS., H=240 KM., VISCOITY & ION DRAG.
AZ= 0.0000, GRAVITY

FIG 2.6.50, 300 KM., 20 MINS., H=240 KM., ISOTROPIC CASE.
AZ= 0.0000, GRAVITY
FIG 2.6.51, 300 KM., 20 MINS., H=240 KM., ION DRAG.

AZ = 0.0000, GRAVITY

FIG 2.6.52, 300 KM., 20 MINS., H=240 KM., THERMAL CONDUCTIVITY.

AZ = 0.0000, GRAVITY
height. The inclusion of ion drag or viscosity with thermal conductivity again leads to large numerical errors, and results for these cases are not presented.

The 500 km, 40 minute wave, figs. 2.6.57 to .62, is considered last. This has the same type of behaviour as the 300 km, 20 minute wave, except that now the numerical errors involved in the case where thermal conductivity and ion drag are considered are greatly reduced. The raypath in this case is horizontal throughout, indicating the wave to be a surface wave. However, even here the error terms are of the order of 10%, and this is probably enough to cause the raypath to behave in this manner.

For waves launched in the upper atmosphere, viscosity and ion drag are seen to have only a small effect on the raypath. Thermal conductivity produces for the larger wavelengths a qualitatively similar behaviour, but for the 100 km case significant differences are observed.

The results of this investigation may be summarised as follows. Thermal conductivity appears to be the most important dissipative effect. Viscosity and ion drag have only small effects in most cases, although for the tropospheric launch height viscosity produces significant perturbations when included with thermal conductivity. In most cases it has proved unrealistic to include ion drag or viscosity with thermal conductivity as significant numerical errors arise within a few integration steps, even with a steplength as small as 0.1 secs.

In view of the dichotomy in the results between including and ignoring thermal conductivity, it can be inferred that any attempt to identify gravity wave sources should prove difficult. It would be desirable to
FIG 2.6.61. 500 KM, 40 MINS., H=240 KM., THERMAL CONDUCTIVITY & ION DRAG
\( \alpha = 0.0000, \ \text{GRAVITY} \)

FIG 2.6.62. 500 KM, 40 MINS., H=240 KM., VISCOSITY & ION DRAG,
\( \alpha = 0.0000, \ \text{GRAVITY} \)
include all three dissipative processes together in the analysis, but this will only be possible once more efficient computing facilities, both larger and faster, are made available.
CHAPTER 3 Source Mechanism Theory

3.1 Introduction

The problem of identifying acoustic-gravity wave sources is one that has attracted much interest. The sources of acoustic waves and large scale gravity waves are in general well known, and a discussion of these sources forms the next section of this chapter. (The distinction between large scale, and the medium scale gravity waves with which this thesis is mainly concerned, has been made clear in chapter 1). Identifying medium scale gravity wave sources has, however, remained a problem. Many source mechanisms for these disturbances have been postulated, and some of these are discussed here. Once a source mechanism has been proposed, it is possible to model it mathematically, and to calculate the atmospheric far field response to it. The results of such an analysis can then be compared with observed data to see whether or not the proposed source mechanism is realistic. Such a model is derived here for a simple model atmosphere, and a way of extending the analysis to the case of a realistic, inhomogeneous atmosphere is indicated.

3.2 Acoustic Wave, and Large Scale Gravity Wave Sources

3.2.1 Acoustic Waves

The sources of acoustic waves fall into two classes, natural and man-made sources. Natural sources are discussed first.

One type of source which has been studied extensively is the emission of infrasound from weather disturbances (see, for example, the review by Georges, 1973). Three possible mechanisms for acoustic wave generation from
these disturbances have thus far been proposed.

1) Perturbation of the atmospheric mean flow by the penetration of convection cells generated in the weather disturbance through the tropopause has been proposed by Pierce and Coroniti, 1966; and Townsend, 1966, 1968. Georges and Young, 1972 have estimated that the kinetic energy in a typical cylindrical convection cell (10 km diameter, 10 km high, and with an updraft of 10 m/s) is of the order of 2.8×10^{13} Joules, and have, assuming energy conversion into acoustic energy to be uniform, found an energy conversion rate of 2.8×10^{10} watts. The resulting conversion efficiency, 0.1%, was pointed out by them to be about that estimated from measurements of the atmospheric response to explosive sources, and thus such a source mechanism was taken by them to be realistic. Gossard and Hooke, 1975, however, have estimated a much lower conversion efficiency for a convective source, because of the relatively low velocity of expansion of the convection cell compared with the speed of sound. They also calculated that the power spectrum for such a source would be expected to have most of its energy concentrated at wave periods of many minutes. The convection cell source was thus taken by them to be an unrealistic generator of acoustic waves.

2) Another possible source mechanism associated with weather disturbances is thunderstorms. The observed frequency range of typical severe weather acoustic emissions corresponds to wave periods of 10 to 60 seconds. Measurements by Uman, 1969 of source spectra of thunderstorms have shown that the amount of energy in infrasonic bands is entirely inadequate to account
directly for measured infrasound at great distances. However, it would still be possible for nonlinear effects over long paths to shift enough energy into the low frequency bands to account for the observed infrasonic spectra. Georges, 1974 has reported infrasonic spectra over short paths (about 60 km) similar to the long path spectra. These paths are too short for nonlinear effects to have any importance. Thus thunderstorms appear to be ruled out as the physical cause of severe weather emissions of infrasound.

3) The other source mechanism proposed is violent turbulence generated within the storm (Lighthill, 1952, 1954; Neecham, 1971; Georges and Young, 1972). Theoretical predictions indicate sharp onsets and cutoffs of energy emission for this mechanism. However, observational evidence has not yet found these cutoffs, and this is a major argument against this form of emission.

The most likely source of acoustic emissions in weather disturbances would thus appear to be perturbation of the atmospheric mean flow by the penetration of convection cells, despite the objections of Gossard and Hooke.

Another source mechanism proposed is the emission of acoustic waves from a moving sea surface. This kind of disturbance, termed microbaroms, typically has a period of about 6 seconds. Microbaroms were first reported by Benioff and Gutenberg, 1939, who noted their similarity to the atmospheric response to seismic disturbances. The relationship between sea state and microbaroms was made clear by the work of Saxer, 1945, 1954. The chief work in this field now lies in a more precise identification of
Acoustic emissions have been well correlated with seismic disturbances. One such source which has been much studied is the Alaskan earthquake of 1964 (Davies and Baker, 1965; Donn and Posmentier, 1964; Leonard and Barnes, 1965).

Airflow over irregular terrain is another proposed source of acoustic waves, and although a convincing correlation between airflow over mountains and infrasonic emission has been obtained (Greene and Howard, 1975), the precise mechanism of such emission remains unclear.

Other natural sources of acoustic waves include exploding meteors, such as the Siberian meteorite of 1908, and volcanic eruptions, such as the Krakatoa eruption of 1883. The two events mentioned above stimulated many of the early studies of acoustic emissions, e.g. Pekers, 1937, 1948; Scorer, 1950.

The last source region of natural acoustic disturbances to be discussed lies in the auroral zone. Auroral emissions were first proposed as a possible source mechanism by Chrzanowski et al, 1961; and a correlation between auroral activity and infrasonic emission was established by Campbell and Young, 1963. The theory of such emissions has been developed by Chimonas, 1970; and Chimonas and Peltier, 1970.

Man made sources of acoustic waves are now discussed. The type of source most studied has been that of nuclear explosions (e.g. Weston, 1961, 1962 for theory, and Dieminger and Kohl, 1962; Kanellakos, 1967; Obayashi, 1962
for observations). However, only a small part of the power spectrum of such sources corresponds to acoustic emissions. Care must be taken in analysing records of the atmospheric response to nuclear explosions as they can be interpreted as either acoustic (Wickersham, 1966) or gravity (Row, 1967) waves. Hines, 1967 has shown that Wickersham's interpretation could not account for several observed features of the disturbances, e.g. the absence of phase structures within the predicted group envelopes, and the forward tilting phase structure inferred from ionospheric observations.

Other man made explosions can generate acoustic waves, and one such disturbance, the Flixborough chemical works explosion, is studied in depth in chapter 6.

Balachandran and Donn, 1971 have investigated infrasound generated by rockets, and finally, the atmospheric response to sonic booms in aircraft has been studied by Goerke, 1975.

3.2.2 Large Scale Gravity Waves

Early observations of large scale gravity waves were made by Valverde, 1958, and Tveten, 1961. These disturbances were seen to travel from the auroral regions equatorwards, with horizontal phase speeds between 400 and 1000 m/s. Several papers suggested a correlation between the occurrence of large scale gravity waves and geomagnetic storm activity (Valverde, 1958; Wright, 1961; Chan and Villard, 1962). This correlation was confirmed by the work of Georges, 1968, and Davis and da Rosa, 1969, who found a one-to-one correspondence between large scale gravity waves and geomagnetic storms with a Kp index greater than 5. Davis and da Rosa found their
observations to be consistent with a source located in the evening section of the auroral oval.

A theoretical analysis of large scale gravity waves was undertaken by Thome, 1968, but this was flawed by his using an oversimplified atmospheric model, consisting of two isothermal layers. Such a model can only support one large scale gravity wave as a surface wave at the boundary between the layers, and this cannot explain the variation in speeds exhibited by large scale gravity waves. This analysis was extended by Francis, 1973a, who was able to explain the observed properties of the disturbances by invoking imperfectly ducted modes of propagation in a more realistic model atmosphere. Testud, 1973 provided an alternative explanation in terms of freely propagating waves, but Francis, 1975 has shown that this explanation can only be valid for disturbances with horizontal phase velocities around 400 m/s. An explanation in terms of imperfectly ducted modes is thus more likely to be a more realistic explanation of large scale gravity wave behaviour.

Most large scale gravity waves occur in wavetrains consisting of several oscillations (Georges, 1968). Such wavetrains have been observed in the atmospheric response to nuclear explosions (Hines, 1967), which are impulsive sources, and this indicates that the oscillatory nature of large scale gravity waves does not necessarily imply an oscillatory source. Two explanations for the observed oscillatory behaviour have been proposed. Davis, 1971, and Testud, 1973 have proposed that the observed periodicity is due to recurring source events, each event exciting only a single pulselike disturbance. On the other hand,
nuclear bomb tests have shown that an impulsive source can provide large scale gravity wave spectra of an oscillatory nature (Stoffregen, 1962). Francis, 1975 has pointed out that both explanations can be valid, and need not necessarily be contradictory.

3.3 Medium Scale Gravity Waves

The situation as regards medium scale gravity wave source mechanisms is much more confusing than for the acoustic or large scale gravity wave cases. Many sources have been proposed, and the most important of these have been listed by Hines, 1974 as follows:

"Winds blowing over hills and mountains (at relatively constant velocity to produce standing waves, or with variable velocity to produce propagating wave packets); thermal convective instabilities; thunderstorms; moving squall lines; moving fronts in general (but particularly cold fronts); instabilities in the general atmospheric circulation under distortion by planetary waves; major tropical cyclones (hurricanes and typhoons); general shear instabilities (including those of the jet streams) via Kelvin-Helmholtz waves; turbulence advected in a region of shear (as by the jet streams); instabilities caused by photochemical variations and water vapour changes of state; nonlinear interaction of other gravity waves (including the atmospheric tides, particularly at high altitudes); explosions (either natural, e.g. volcanoes, including as an extreme the case of Krakatoa, or man made, e.g. nuclear explosions); earthquakes; tsunamis; deposition of heat and/or momentum by extraterrestrial and higher energy particles; heating and Lorentz forces associated with auroral
current systems; and eclipses of the sun."

These sources can be split into two main categories:

1) ground/tropospheric sources, e.g. weather disturbances.
2) thermoclinic sources, e.g. Joule heating associated with auroral current systems.

Early observations indicated that medium scale waves could propagate relatively unattenuated for distances greater than 1000 km (Munro, 1950; Heisler, 1959). It was also found that these waves had a tendency to propagate equatorward (Heisler, 1959). This stimulated both experimental (Hunsucker and Tveten, 1967; Elkins and Slack, 1969) and theoretical (Chimonas and Hines, 1970) interest in auroral sources. However, Hunsucker and Tveten could only correlate a little more than half their observations with auroral disturbances, and other observers (Munro, 1958; Davis and Jones, 1971) noticed that propagation with a poleward component could occur. Therefore, sources other than high latitude ones were shown to be important. Flock and Hunsucker, 1968 have reported a medium scale disturbance associated with midlatitude particle precipitation, and Raitt and Clark, 1973 found a wave class whose occurrence was restricted to the neighbourhood of supersonic terminator motion.

Spizzichino, 1970 postulated that medium scale waves could be generated by nonlinear breaking of the diurnal tidal mode, but Lindzen and Blake, 1971 have shown that the diurnal mode becomes unstable before nonlinearities appear, thereby precluding this mechanism. However, breaking of the semi-diurnal or other tidal modes could generate medium scale disturbances.

The equatorial electrojet has been proposed as a
source mechanism in theoretical calculations by Chironas, 1970b, but because coupling of this source to gravity waves is likely to be weak, no experimental verification of this postulate has yet been sought.

Wind flow over rough terrain has been put forward by Bretherton, 1969 as a possible wave source, and Eliasson and Palm, 1960 have inferred that it is possible for these waves to reach the lower ionosphere. The jet stream has been proposed as a source region by many authors (e.g. Goe, 1971; Claerbout and Madden, 1968; Tolstoy and Herron, 1969; Bertin et al, 1975). Noctilucent clouds have been inferred by Hines, 1968b as indicating that medium scale gravity waves can be generated by weather fronts or jet streams.

Auroral current systems have also been proposed as a possible wave source. These could either generate disturbances through the momentum transfer from Lorentz forces, roughly proportional to $\sigma_p E_B$, where $\sigma_p$ is the Pederson conductivity, $E$ the Earth's electric field, and $B$ the Earth's magnetic field, or by displacement of the atmosphere by the Joule heating, roughly proportional to $\sigma_p E^2$, that these current systems would produce.

A major difficulty in identifying medium scale gravity wave sources is the lack of detailed knowledge about neutral winds. These can have speeds which reach 100 m/s in the lower atmosphere (Kantor and Cole, 1964) and which can be in excess of 200 m/s at thermospheric heights (Rishbeth, 1972). Francis, 1975 has pointed out that these speeds are of the same order as the horizontal phase speeds of medium scale gravity waves, and has inferred that winds must then have a significant
effect on their propagation. He also indicated that winds can also cause multipath propagation from the same wave source, different frequency components being refracted along different raypaths, and that waves from different sources can interfere with each other. This helps to make unambiguous identification of medium scale events difficult. Another difficulty is the continually changing waveform of many observed disturbances (Georges, 1968). Georges has pointed out that it might not be possible to identify a wave unambiguously at two stations, even if they are separated by less than 200 km.

Any theoretical explanation of medium scale gravity wave behaviour must account for their relatively unattenuated propagation for large distances at speeds between 75 m/s and the lower atmospheric sound speed, and with frequencies less than the Brunt frequency. The most realistic explanation to date has been that of Francis, 1974; who, using an impulsive source to generate freely propagating gravity waves, managed to model qualitatively the main features in medium scale gravity wave observations, including the fact that pulselike and oscillatory events appear to occur as often as each other (Titheridge, 1971).

A similar analysis to that of Francis will now be developed to model the atmospheric response to the passage of medium scale waves generated by an impulsive source. The effects of neutral winds will be neglected throughout the analysis. As gravity wave propagation only is considered, it is possible, without loss of generality to work in a cartesian system (Francis, 1972). To simplify the analysis, a momentum source only is used. This source
is taken to operate through the Lorentz force, which originates as a force on the charged particles, but is transferred to the neutral gas via collisions. In practice, heat sources such as Joule heating are also important. Dissipative effects are neglected until the latter stages of the analysis. The motion will be isotropic horizontally, and again without loss of generality a two dimensional coordinate system can be used. The atmosphere is assumed to consist of two isothermal layers, with a sound speed discontinuity at the base of the thermosphere, the source being taken to lie in the lower layer. The equations of motion to be satisfied in each layer are:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0
\]

\[
\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{U} = -\nabla p + \mathbf{F}(x, z, t)
\]

\[
\frac{\partial p}{\partial t} + (\mathbf{U} \cdot \nabla) p = 0
\]

being the equations of continuity, conservation of momentum and adiabatic state respectively. In these equations \( \rho \), \( p \) represent the atmospheric density and pressure respectively; \( \mathbf{U} \), the neutral fluid velocity; \( \mathbf{\ddot{F}} \), the acceleration due to gravity; \( \gamma \), the ratio of specific heats; and \( \mathbf{F} \) is a force/unit mass representing the source of the waves. The boundary conditions to be satisfied are

1) no source must exist at \( \infty \) (radiation condition)
2) \( U_z \) vanishes at \( z = 0 \)
3) \( U_z \) must be continuous across the discontinuity
4) \( p \) must be continuous across the discontinuity

The system 3.3.1 to 3.3.3 is solved using classical
perturbation theory. Perturbation variables $p'$, $j'$, $U'$ are defined by

\[
\begin{align*}
p(x,z,t) &= p_o(z)(1 + p'(x,z,t)) \\
def(x,z,t) &= f_o(z)(1 + f'(x,z,t)) \quad 3.3.4 \\
U(x,z,t) &= U'(x,z,t)
\end{align*}
\]

$p_o, f_o$ represent the ambient pressure and density respectively, and are assumed to be known. They are taken to obey the barometric law

\[
p_o, f_o \propto \exp(-z/H) \quad 3.3.5
\]

where $H = c^2/\gamma g$, the atmospheric scale height, a constant in each layer. Using 3.3.4 and 3.3.5 the first order equations

\[
\begin{align*}
\left\frac{\partial}{\partial t} - U'_z + \nabla \cdot U' &= 0 \\
\frac{\partial U'}{\partial t} + gH \nabla p' + (p' - f')g &= F \\
\frac{\partial p'}{\partial t} - \frac{U'}{z} + \gamma \nabla \cdot U' &= 0
\end{align*}
\]

are obtained from the equations of motion. 3.3.6 is now fourier transformed in time, yielding

\[
\begin{align*}
i\omega \frac{\partial}{\partial z} - U'_z + \nabla \cdot U' &= 0 \\
i\omega \frac{\partial U'}{\partial z} + \frac{\partial (p' - f')g}{\partial z} &= F_b \\
i\omega \frac{\partial p'}{\partial z} - \frac{U'}{z} + \gamma \nabla \cdot U' &= 0
\end{align*}
\]

Dropping the superscripts ' and $\omega$, and eliminating all variables except $p$, the equation

\[
\begin{align*}
\nabla^2 p + \frac{\omega^2}{(\omega^2 - \omega_b^2)} \frac{\partial^2 p}{\partial z^2} - \frac{\omega^2}{H(\omega^2 - \omega_b^2)} \frac{\partial p}{\partial z} + \frac{\omega^4 p}{(\omega^2 - \omega_b^2)c^2} = S_1(x,z,\omega)
\end{align*}
\]

is obtained where
\[ S_1(x,z,\omega) = \gamma \left\{ \gamma \frac{\partial \Psi}{\partial z} + \frac{\omega_z^2}{(\omega_z^2 - \omega_b^2)} \left( \frac{\partial \Phi_z}{\partial z} - \frac{\omega_z^2}{\omega_b^2 \gamma_{\Phi_z}} \right) \right\} \]

\[ \omega_b = \gamma (Y - 1)^{1/2}/c, \text{ is the Brunt cutoff frequency, a constant in each layer.} \]

\[ \omega_b = \gamma (Y - 1)^{1/2}/c, \text{ is the Brunt cutoff frequency, a constant in each layer.} \]

3.3.8 is now transformed by writing

\[ p(x,z,\omega) = \bar{p}(x,z,\omega) \cdot \exp(z/2H), \text{ giving} \]

\[ \left\{ \frac{\partial^2}{\partial x^2} - \frac{\omega_x^2}{(\omega_x^2 - \omega_b^2)} \frac{\partial^2}{\partial z^2} + \frac{2(\omega_x^2 - \omega_b^2)}{c^2(\omega_x^2 - \omega_b^2)} \right\} \bar{p}(x,z,\omega) = S(x,z,\omega) \]

\[ \text{where } S(x,z,\omega) = \exp(-z/2H)S_1(x,z,\omega) = \left\{ \frac{p_o(z)}{p_o(0)} \right\} \frac{1}{2} S_1(x,z,\omega) \]

3.3.10

and \( \omega_a \) is the acoustic cutoff frequency, \( \omega_a = \gamma g/2c, \) a constant in each layer.

The Greens function for the two layer atmosphere under consideration must satisfy

\[ \left\{ \frac{\partial^2}{\partial x^2} - \frac{\omega_x^2}{(\omega_x^2 - \omega_b^2)} \frac{\partial^2}{\partial z^2} + \frac{2(\omega_x^2 - \omega_b^2)}{c^2(\omega_x^2 - \omega_b^2)} \right\} G(x,z,k^2) = \delta(x) \delta(z-z_s) \]

3.3.12

where the source is taken to lie at \((0,z_s)\), and where \( \omega_a, \omega_b, c \) are taken to refer to the local values of these parameters. 3.3.12 is now Fourier transformed with respect to \( x \), yielding

\[ \left\{ \frac{-k_x^2}{(\omega_x^2 - \omega_b^2)} \frac{\partial^2}{\partial x^2} + \frac{\omega_x^2}{(\omega_x^2 - \omega_b^2)} \frac{\partial^2}{\partial z^2} \right\} G(k_x,z,\omega) = \delta(z-z_s) \]

3.3.13

The dispersion relation for acoustic gravity waves in an isothermal, isotropic atmosphere is \((\text{Hines, 1960})\)

\[ k_x^2 = \frac{(\omega_x^2 - \omega_b^2) k_x^2 - (\omega_x^2 - \omega_b^2)}{\omega_x^2} \]

3.3.14

Substituting this into 3.3.13 for \( k_x^2 \) yields

\[ \left\{ \frac{\partial^2}{\partial z^2} + \frac{k_x^2}{\omega_x^2} \right\} G(k_x,z,\omega) = -\frac{(\omega_x^2 - \omega_b^2)}{\omega_x^2} \delta(z-z_s) \]

3.3.15

which has the solution \((\text{see Roach, 1970, p152})\)

\[ G(k_x,z,\omega) = a_2 \exp(ik_x z_2 z) + b_2 \exp(-ik_x z_2 z) \]

3.3.16
\[ G(k, z, \omega) = a_1 \exp(ikz_1z) + b_1 \exp(-ikz_1z) + \frac{i(\omega^2 - \omega_1^2)\exp(ik_{z1}|z - z_s|), z \approx z_o}{2k_{z1}\omega^2} \]

where \( z_0 \) is the height of the sound speed discontinuity, and the branch of \( k_{z1} \) is taken to be (from 3.3.14)

\[ k_{z1} = \left\{ \begin{array}{l} \frac{(\omega_{b1}^2 - \omega^2)k_x^2}{\omega^2 - c_i^2} \left\{ \frac{\omega_{a1}^2 - \omega^2}{c_i^2} \right\}^{\frac{1}{2}} \right. \\
\left. \frac{k_x^2\omega_1^2(\omega_{a1}^2 - \omega^2)}{c_i^2(\omega_{b1}^2 - \omega^2)} \right\}^{\frac{1}{2}} + i\left\{ \frac{(\omega_{a1}^2 - \omega^2) - (\omega_{b1}^2 - \omega_1^2)k_x^2}{\omega^2 - c_i^2} \right\}^{\frac{1}{2}} \right) \]

where the subscripts \( i=1,2 \) refer to the lower \( (i=1) \) and the upper \( (i=2) \) layers respectively.

The boundary conditions 1) to 4) are now imposed to determine the constants \( a_j, b_j; j=1,2 \). The radiation condition implies that there can be no source in the upper layer, hence \( b_2 = 0 \).

Friedman, 1966 has shown that the continuity of pressure (boundary condition 4)) is equivalent to the continuity of \( \nabla \cdot \mathbf{U} \). From 3.3.7 it can be shown that

\[ \mathbf{U}_z(k, z, \omega) = -i\exp(z/2H)C(\partial/\partial z + B)G(k, z, \omega) \]

\[ \nabla \cdot \mathbf{U}(k, z, \omega) = -i\exp(z/2H)D(\partial/\partial z + A)G(k, z, \omega) \]

where \( A = \left( \frac{2\omega^2 - \omega_1^2 - \gamma \omega^2}{\gamma g} \right) \); \( C = c_i^2 \left( \frac{\omega}{\gamma(\omega_1^2 - \omega^2)} \right) \)

\[ B = \frac{g}{c_i^2} \left( 1 - \gamma \right); \quad D = \frac{\omega_1^2}{(\omega_1^2 - \omega^2)} \]

The remaining boundary conditions can now be used to obtain \( a_1, b_1, a_2 \). Substitution for \( a_2 \) into 3.3.16 yields the Greens function for the upper atmosphere.

\[ G(k, z, \omega) = \frac{1}{2k_{z1}} \left( \frac{\omega_{b1}^2 - \omega^2}{\omega} \right) e^{ik_{z1}(z_0 - z_s)} + R e^{ik_{z1}(z_0 + z_s)} \]

\[ = *\exp(ik_{z2}(z - z_0)) \]
53.

\[
R = \frac{-C_2D_1(ikz_2+B_2)(ikz_1+A_1)}{C_2D_1(ikz_2+B_2)(-ikz_1+A_1)} - \frac{C_1D_2(ikz_1+B_1)(ikz_2+A_2)}{C_1D_2(-ikz_1+B_1)(ikz_2+A_2)} \tag{3.3.21}
\]

\[
T = \frac{2C_1D_1ikz_1(A_1-B_1)}{C_2D_1(ikz_2+B_2)(-ikz_1+A_1)} - \frac{C_1D_2(-ikz_1+B_1)(ikz_2+A_2)}{C_1D_2(-ikz_1+B_1)(ikz_2+A_2)} \tag{3.3.22}
\]

and the suffices 1, 2 refer to the lower and upper layers respectively. \( R_0 \) is the reflection coefficient obtained by assuming a plane wave of unit amplitude to be incident on a rigid surface and solving for the amplitude \( R_0 \) of the reflected wave. Similarly, \( R \) and \( T \) are the reflection and transmission coefficients derived by assuming that a plane wave of unit amplitude is incident on an atmospheric temperature discontinuity and solving for the amplitudes of the reflected and transmitted waves (Francis, 1974).

Using the binomial theorem, the Greens function can be expanded as an infinite sum of multiply reflected waves.

\[
G(k_x, z, \omega) = \sum_{n=0}^{\infty} G_n(k_x, z, \omega)
\]

where \( G_n(k_x, z, \omega) = i \frac{(\omega^2 - k_x^2)(e^{ikz_1(z_o-z_s)})}{2kz1} + \frac{\omega^2}{e^{ikz_1(z_o-z_s)}} + R_0 e^{ikz_1(z_o+z_s)} * T e^{ikz_2(z-z_o)} (R_0 Re^{2ikz_1z_o})^n \tag{3.3.24}
\]

In \( G_0 \) for example, the term proportional to \( T e^{ikz_1(z_o-z_s)} * e^{ikz_2(z-z_o)} \) can be interpreted as a wave that propagates from the source \((0,z_s)\) up to the boundary, \(z=z_o\), is transmitted across the boundary with relative amplitude \( T \) and then propagates up to the field point \( z \). The other term can be interpreted as a wave that propagates from
the source region down to the ground, is reflected there with relative amplitude $R_0$, then propagates upwards, across the boundary (where it is transmitted with relative amplitude $T$) to the field point $z$. Similar reasoning can be applied to the other $G_i$, each of which will have been reflected $i$ times from the boundary. Since each of these reflections is imperfect, the dominant contribution will come from $G_0$, and in the following analysis this will be the only term considered, the other $G_i$ being neglected. The suffix $o$ is now dropped, hence $G = G_0$. $G(x,z,\omega)$ is now evaluated using the inverse Fourier transform

$$G(x,z,\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(-ik_x x)G(k_x,z,\omega)dk_x$$ 3.3.25

Having dropped the higher order $G_i$, the contribution to this integral comes solely from a branch line integral. (The higher order $G_i$ would, if included, yield a sum of residues at poles of the integrand) This appears because of the square root in the definition of $k_z$, and will be evaluated using the method of stationary phase, yielding a continuous spectrum of freely propagating waves as a solution.

When the source term $F(x,z,t)$ is included, the atmospheric pressure response is given by

$$p'(x,z,t) = \exp(z/2H) \int_0^\infty \int_0^\infty \int_0^\infty G(x-x',z-z',t-t')S(x',z',t') \, dx'dz'dt'$$ 3.3.26

where $S$ is defined by 3.3.11.

To proceed further with the analysis, a specific source mechanism is introduced. This is taken to be the Lorentz force associated with the auroral electrojet. The
current density of the electrojet is represented by
\[ j(x,z,t) = i \delta(x) \delta(z-z_s) \mu(t) \] 3.3.27

where \( I \) is the total integrated current; \( \mu(t) \) is the Heaviside step function; and \( \mathbf{j} \) is a unit vector in the \( y \) direction. (taken to be the East-West direction in this case, which implies that the \( x \) direction is also the North-South direction) The coordinate system has been extended to three dimensions in order to take account of the Earths' magnetic field.

The Lorentz force per unit mass is then given by
\[ F(x,z,t) = j(x,z,t) \mathbf{B} \] 3.3.28

where \( \mathbf{B} \) represents the Earths' magnetic field.

The assumption \( \omega \ll \omega_b \) is now made, which is true for most medium scale gravity waves, and 3.3.28 is substituted into 3.3.11, yielding:-
\[ S(x,z,t) = \exp(-z/2H) \frac{\delta j \mathbf{B}}{\int_0(z) c_1^2 \delta x} \] 3.3.29

and substitution of this into 3.3.26 gives
\[ \frac{p'(x,z,t)}{p_o(z)} = -\left\{ \frac{p_o(z)}{p_o(z)} \right\}^{1/2} \frac{\mathbf{IB}_{zs}}{p_o(z)} \left[ \int_0^\infty dk_x \int_0^\infty \omega_0 \frac{k_x \exp(\omega t - ikx)}{2\pi} \right] 2\pi \frac{\omega}{\omega_0} \frac{G(k_x, z, \omega)}{\mathbf{B}_z} \] 3.3.30

where \( \mathbf{B}_z \) is the \( z \) component of \( \mathbf{B} \) at the source.

For direct waves (those which propagate to the field point without being reflected), the atmospheric response is then
\[ \frac{p'(x,z,t)}{p_o(z)} = -\left\{ \frac{p_o(z)}{p_o(z)} \right\}^{1/2} \frac{\mathbf{IB}_{zs}}{p_o(z)} \left[ \int_0^\infty dk_x \int_0^\infty \omega_0 \frac{k_x \exp(\omega t - ikx)}{2\pi} \right] 2\pi \frac{\omega}{\omega_0} \frac{G(k_x, z, \omega)}{\mathbf{B}_z} \] 3.3.31

For Earth reflected waves, the response is
\[
\frac{p'(x,z,t)}{p_o(z)} = \left\{ \frac{p_o(z_s)}{p_o(z)} \right\}^{\frac{1}{2}} \frac{\gamma B_{zs}}{f_o(z_s)c_1^2} \int_0^\infty \int \frac{\omega}{2\pi} \frac{k_i T R_o}{2 k \omega^3} \exp(i\omega t - ik_x x + ik_z z_1 (z_0 + z_s) + ik_z^2 (z - z_0))
\]

where 3.3.24 has been substituted into 3.3.30.

The first integration performed is that with respect to \(k_x\), first substituting for \(k_{z1}, k_{z2}\) from 3.3.14. \(k_{z2}\) can be approximated by \(k_{z2} = k_x ((\omega_{b2}^2 - \omega^2)/\omega^2)^{\frac{1}{2}}\), which is a valid approximation for all medium scale gravity waves.

The principle of stationary phase now implies that for \(x > \eta_0\), i.e. distant from the source, the main contribution to the integral comes from the point of stationary phase \(k_x\), defined as the value of \(k_x\) for which

\[
\frac{d}{dk_x} \left\{ \frac{(-k_x + (\omega_{b1}^2 - \omega^2) k_x^2 - (\omega_{a1}^2 - \omega^2))}{\omega^2 c_1^2} \right\}^{\frac{1}{2}} (z_0 + z_s) + \left\{ \frac{\omega_{b2}^2 - \omega^2}{\omega^2} \right\}^{\frac{1}{2}}
\]

\[k_x (z - z_0)\] = 0

3.3.33

where \(\alpha = -1\) for the direct wave and +1 for the Earth reflected wave. The principle of stationary phase is used at this stage only to remove the factors \(k_x T, k_x T R_o\) from the integrals. \(T\) is defined by \(T = T_{k_x = k_x}\) for the direct wave, and \(T = (T R_o)_{k_x = k_x}\) for the Earth reflected wave and in this notation 3.3.31, 3.3.32 can be written

\[
\frac{p'(x,z,t)}{p_o(z)} = \left\{ \frac{p_o(z_s)}{p_o(z)} \right\}^{\frac{1}{2}} \frac{\gamma B_{zs}}{f_o(z_s)c_1^2} \int_0^\infty \int \frac{\omega}{2\pi} \frac{k_i T}{k \omega^3} \exp(i\omega t) \nonumber
\]

\[G_{oo}(x,z,\omega) = \frac{1}{4!} \frac{\omega_{b1}^2 - \omega^2}{\omega^2} H_0^{(2)}(2) \left\{ \left( \frac{\omega_{a1}^2 - \omega^2}{c_1^2} \right) \left( \frac{x^2 \omega^2 - z^2}{\omega_{b1}^2 - \omega^2} \right)^{\frac{1}{2}} \right\}
\]

3.3.35
$G_{oo}$ is the Greens function of a monochromatic source of acoustic gravity waves in an isothermal atmosphere (Chimonas and Hines, 1970). $H^{(2)}_o$ is a Hankel function of the zeroth order and the second kind.

To proceed further, the assumption $\omega \ll \omega_{b2}$ is made, thereby retaining only the low frequency response. Then becomes

$$p'(x,z,t) = \left\{ \frac{p_0(z)}{p_0(z)} \right\}^2 \gamma B \frac{ dz}{z_s} \frac{d \omega}{k_x} \frac{\bar{T}_{exp}(i \omega t)}{2 \pi \omega} \Phi_0(z) \left( \frac{\omega_{b1}}{\omega} \right)^\frac{1}{2}$$

where $t_L = \frac{\omega_{a1} x}{x} = \frac{\omega_{c1}}{\omega_{b1} (z + k z_s)}$; $\omega_{c2} = \frac{\omega_{b2} (z - z_o)}{c_1 \omega_{b1}}$ $c_L x$ $3.3.37$

Chimonas and Hines have shown that this response is causal, in that $p' = 0$ for $t < t_L$. Their argument uses the asymptotic form of $H^{(2)}_o$ as $\omega \rightarrow \infty$ to show that for $t < t_L$, the path of integration can be closed in the lower half $\omega$-plane, enclosing no singularities. $t_L$ is thus the earliest time at which low-frequency gravity waves can reach the field point, and therefore $c_L$ is the largest horizontal phase speed of low frequency gravity waves.

For $t > t_L$, the stationary phase method implies that the dominant contribution in the integral in 3.3.36 comes from the region of the stationary phase frequency, $\omega = \bar{\omega}$, where

$$\frac{d}{d \omega} \left\{ t_L \left( (\omega - \omega_{c2})^2 - \omega_{c1}^2 \right)^{\frac{1}{2}} \bar{T} - \omega t \right\} = 0$$

i.e. where $\bar{\omega} = \omega_{c2} + \omega_{c1} t (t^2 - t_L^2)^{-\frac{1}{2}}$

Evaluation of the integral by stationary phase yields the real response

$$p'(x,z,t) = \left\{ \frac{p_0(z)}{p_0(z)} \right\}^2 [T] [\bar{T}] \left( \frac{\omega_{b1} \bar{n} z_s}{\omega_{c2} t + \omega_{c1} (t^2 - t_L^2)^{\frac{1}{2}} + i} \right)$$

$$= \frac{2 \pi c_1^2 \phi_0(z)}{x (\omega_{c2} (t^2 - t_L^2)^{\frac{1}{2}} + \omega_{c1} t)}$$

3.3.40
where $T = T \exp(i \phi)$

$k_x$ is given by

$$k_x = \frac{\omega t_L (\omega - \omega_{c2})}{x ((\omega - \omega_{c2})^2 - \omega_{c1}^2)^{1/2}}$$

and 3.3.39, 3.3.42 give $\bar{u}/k_x = x/t$

which implies that the horizontal phase and group velocities of low frequency gravity waves are identical.

The expression for the horizontal velocity perturbation, $U_x$, is found to be (from 3.3.6, 3.3.43)

$$U_x(x, z, t) = \frac{c_k^2}{\bar{p}} \frac{p'(x, z, t)}{p_0(z)}$$

$$= \left\{ \begin{array}{l}
\rho_0(z) \left\{ 1 + \frac{2\pi}{\omega_{c2} c_1} \frac{(t^2 + \omega_{c1}^2 + \omega_{c1} t)}{2(t^2 + \omega_{c1}^2)} \right\} \\
\rho_0(z) \left\{ \frac{2\pi}{\omega_{c2} c_1} \frac{(t^2 + \omega_{c1}^2 + \omega_{c1} t)}{2(t^2 + \omega_{c1}^2)} \right\}
\end{array} \right.$$

The expression given for $T$ (eqn. 3.3.23) has been derived for an atmosphere with a large discontinuity in temperature, which would tend to reflect most of the energy of an incident gravity wave. Therefore the use of this expression for $T$ would greatly understate the gravity wave amplitude at thermospheric heights. In a realistic model atmosphere, however, temperature varies continuously with height, and thus propagating gravity waves are refracted continuously rather than reflected at any one level. More accurate results can thus be obtained if a realistic model for $T$ is adopted. This can be done by means of a multilayer analysis, and solving the dispersion relation 2.4.13 for an atmosphere including dissipative effects for $k_z$ in each layer, and using these values of $k_z$ to obtain $T$ from 3.3.23, again in each layer, values of the parameters $A_i, B_i, C_i, D_i$ also being calculated using a realistic model atmosphere.
Results are now presented depicting values of $U_x$, the horizontal neutral velocity fluctuation associated with Earth reflected (figs. 3.3.1 and 3.3.2) and direct (figs. 3.3.3 and 3.3.4) gravity waves at heights of 200 and 300 km, as graphs of amplitude against ground range at times from 1 to 5 hours after the action of the source.

The source model used is given by the following values:

- $I = -10^5$ A, strength of source current
- $z_s = 120$ km, altitude of source
- $B_{zs} = -6.65 \times 10^{-5}$ Wb/m$^2$, vertical component of $B$ at source
- $\rho_o(z_s) = 2.5 \times 10^{-8}$ kg/m$^3$, ambient density at source height

The ambient atmospheric model is given by the following values:

- $c_1 = 310$ m/s, sound speed in lower layer
- $c_2 = 900$ m/s, sound speed in upper layer
- $\gamma = 1.5$, ratio of specific heats
- $g = 8.83$ m/s$^2$, acceleration due to gravity
- $z_o = 150$ km, height of sound speed discontinuity

$T$ is calculated using the realistic atmospheric model given by Francis, 1973b.

For the Earth reflected wave, the average wavelength of each wave packet is indicated on the graphs. In this case it is seen that the longer wavelengths appear in the middle of the wave packet, and the shorter ones at each edge of it.

Another point of note is that the Earth reflected events occur in wavetrains, and the direct waves in less than two cycles. This is because the Earth reflected
Fig 3.3.1  (After Francis, 1974)

Fig 3.3.2  (After Francis, 1974)
Fig 3.3.3 (After Francis, 1974)

Fig 3.3.4 (After Francis, 1974)
waves disperse sufficiently before reaching the thermosphere so that in any region of the upper half space their response is governed by only a narrow range of frequencies and wavelengths and appears as a relatively monochromatic wave packet. The direct waves propagate to the thermosphere along a qualitatively different family of raypaths, there being less opportunity for dispersion to separate different spectral components into different regions of space. The result is a response that looks more like a single pulse than an elongated wave packet, as if the waves were nearly nondispersive (Francis, 1974). Titheridge, 1971 has produced some experimental results indicating that wavetrain events, and pulselike disturbances occur with about the same frequency, and this provides some indication that thermoclinic sources of medium scale gravity waves are more important than ground based or tropospheric sources.
CHAPTER 4 The Influence of Neutral Winds on the Propagation of Gravity Waves

4.1 Introduction

Neutral winds have an important effect on the propagation of gravity waves (Francis, 1975). Previous studies have considered the effect of the horizontal wind (e.g., Bertin et al., 1975), but the smaller vertical wind component has generally been neglected. The effects of horizontal winds are reviewed here, and then the influence of the vertical wind is investigated using the raytracing technique for a range of vertical wind shears.

The horizontal wind component can reach speeds of 100 m/s in the lower atmosphere (Kantor and Cole, 1964), and be in excess of 200 m/s in the thermosphere (Rishbeth, 1972). These speeds are of the order of the horizontal phase speeds of medium scale gravity waves, and this makes it likely that critical layers, where the wave frequency is doppler shifted to zero, will form (Hines, 1968a). When these layers occur, momentum is transferred between the wave and the neutral wind, and wave energy is thus dissipated (Booker and Bretherton, 1967).

Ray tracing analyses have shown that the horizontal wind component can produce widely varying effects on gravity waves (Cowling, Webb and Yeh, 1971). Figs. 4.1.1 to 4.1.3 depict the results of such an analysis, where the only parameters varying are the horizontal wind model, and the initial azimuth angle of propagation of the wave. The wind model employed is that calculated by Cho and Yeh, 1970, who solved the horizontal components of the momentum equation 2.4.1 for the horizontal wind components as functions of position and time of day,
using atmospheric models appropriate to the various seasons. The disturbance depicted in the graphs is one with an initial horizontal phase velocity of 100 m/s and an initial period of 30 minutes. In figs. 4.1.1 and 4.1.2 the wave is launched at an azimuth of 45° East of North, and in fig. 4.1.3 this value has changed to 225°. The wind model used is the summer wind model of Cho and Yeh, appropriate for a launch time of 0600 hours in fig. 4.1.1 and for midnight in figs. 4.1.2 and 4.1.3. Fig. 4.1.1 shows the wave propagating into the thermosphere, fig. 4.1.2 shows it being reflected in the region of the thermocline, and fig. 4.1.3 shows it reaching a critical level at the base of the thermocline.

The presence of horizontal winds may also lead to multipath propagation for disturbances originating at the same wave source (Francis, 1975). If the source is impulsive, a large spectrum of frequency components will be present in it, and winds will refract each component along a different group path. Occasionally, inverse dispersion has been deduced from TID observations. (This phenomenon occurs when group velocity increases with wave period. For normal dispersion group velocity would decrease with wave period.) This has been explained by Balachandran, 1968 by invoking horizontal winds of the order of 100 m/s at the base of the thermosphere.

The wind acts on a propagating wave of wavenumber \( k \) by doppler shifting the wave frequency according to the relationship

\[
\Omega = \omega - kU
\]

(Hines, 1968a), where \( U \) is the wind velocity, and \( \Omega \) the new frequency. The dispersion relation 2.4.13 now has to be solved employing \( \Omega \) instead of \( \omega \), and this obviously
SUMMER WIND MODEL
STARTING AZIMUTH 45°
STARTING TIME 0600
HORIZONTAL VELOCITY 100 m/s
PERIOD 30 MIN

GROUP ray trajectory showing penetration at F-region heights: (a) height versus horizontal distance; (b) vertical projection of the ray on the ground.

Figure 4.1.1 (After Cowling, Webb & Yeh, 1971)

GROUP ray trajectory showing reflection due to wind shear: (a) height versus horizontal distance; (b) vertical projection of the ray on the ground.

Figure 4.1.2 (After Cowling, Webb & Yeh, 1971)
SUMMER WIND MODEL
STARTING AZIMUTH 225°
STARTING TIME 0000
HORIZONTAL VELOCITY 100 m/s
PERIOD 30 MIN

Figure 4.1.3  (After Cowling, Webb & Yeh, 1971)
affects subsequent values of \( k \).

Winds can also act as wave sources, either by airflow over irregular terrain (Gossard and Hooke, 1975), or out of turbulence advected in a region of shear, e.g. the jet stream (Hines, 1974). In a raytracing study to identify gravity wave sources it is very difficult to accurately take into account the effects of neutral winds. Although several global, seasonal models of diurnal horizontal wind variations exist (Kantor and Cole, 1964; Kohl and King, 1967; Cho and Yeh, 1970), the actual wind profile may vary considerably from these values.

Even if wind data is available as a function of height at the point where a TID is detected, this data is likely to cover only a limited range of height. (e.g. incoherent scatter measurements will only give data at ionospheric heights; meteorological data will only extend up to about 30 km; meteor train decay measurements give data only in the range 80 to 115 km) The geographical coverage of wind data is extremely limited, and such data as is available cannot as yet be accurately extrapolated to give wind models far from the observation points. However, the horizontal component of the neutral wind has been successfully invoked to give a qualitative explanation of many features of gravity wave behaviour.

### 4.2 Vertical Wind Effects on Gravity Wave Propagation

The magnitude of the vertical component of the neutral wind is difficult to measure and very few observations have been reported in the literature. Incoherent scatter radar observations at Jicamarca (Harper and Woodman, 1977) suggest values of about 2 m/s at 80 km. This value is in general agreement with that
calculated by Rishbeth et al, 1969 from considerations of the winds likely to occur in model atmospheres. Vertical velocities have been measured by Anandaraao et al, 1977 who report zero velocity at 90 km, rising to 19±7 m/s at the 100 km level. Other ion cloud release experiments (Rees, 1969) indicate larger vertical velocities. The average values obtained from 17 experiments performed during the period 1959 to 1965 were 10 m/s at 60 km rising to 30 m/s at 130 km. Even higher velocities have been deduced by Vasseur, 1969 using the incoherent scatter technique, and for thermospheric heights values of 50 m/s were reported.

The raytracing technique is now used to assess the effects of vertical winds in an isotropic, temperature stratified atmosphere. Dissipative effects and horizontal winds are neglected, so that the effects of vertical winds alone are obtained. The dispersion relation for such an atmosphere is

\[ D(\omega, k, \omega) = \omega^4 - \omega^2 \left\{ c^2 k \cdot k + \frac{g^2 \epsilon^2}{4c^2} \right\} + \frac{\epsilon^2 (\delta - 1) (k_\epsilon^2 + k_\phi^2)}{4c^2} = 0 \]

4.2.1

The input to the raytracing programme used is through 1) the coordinates of the point of transmission, 2) the elevation and azimuth angles of transmission, and 3) the magnitude of the initial value of the wavelength. In the analysis which follows, the coordinates of the point of transmission and the initial azimuth angle are held constant for all raytraces, and only the elevation angle and the magnitude of the initial wavelength are varied. For a fixed initial wavelength however, the dispersion relation 4.2.1 will yield differing values of wave frequency for differing angles of elevation. It is found that waves launched at steep initial angles of elevation
with respect to the horizontal have lower frequencies, and ultimately propagate to greater ground ranges. These waves are incident on the ground with smaller angles of elevation than those waves launched initially with low elevation angles.

The temperature model adopted is the hyperbolic profile used by Georges, 1971, which is an analytic model based on the 1962 U.S. standard reference atmosphere. This profile represents average conditions in the atmosphere and is reproduced in appendix 1. The vertical wind, $U_r$, is assumed to vary as

$$U_r = Kh$$

where $h$ is the height above ground level, and $K$ a constant. $K$ is positive for upward, and negative for downward wind directions. The range of $K$ values taken is $-0.1$, $-0.025$, $0.025$, and $0.1$ m/s/km. The zero wind case is represented by $K = 0$, and this has been included as a reference with which the results obtained when including various vertical wind profiles can be compared. When $K = 0.1$ a vertical wind of $24$ m/s is obtained at $240$ km. These values are the maximum used in the present calculation and are approximately half those reported by Vasseur, 1969. The $K = 0.025$ values were chosen to produce a wind at $80$ km of $2$ m/s which corresponds to the value recently measured by Harper and Woodman, 1977, and to that predicted by Rishbeth et al, 1969. Thus, four wind profiles are available which lie within the range of values reported in the literature. Since the effects produced by the wind on gravity waves increases with increasing wind magnitude, it seemed appropriate to restrict the calculations to wind values somewhat smaller
than the maximum reported value. Thus a realistic evaluation of the likely influence of vertical wind shears on the propagation of gravity waves is obtained.

The raytracing is started at a height of 240 km, and the initial ray angle to the horizontal is varied from 0° to -80° in 10° steps. The ray is traced downwards through the atmosphere until either the ground is reached or the raypath becomes horizontal. As the atmospheric model is time independent, by applying the principle of reciprocity a gravity wave from a source located at either of these points is able to propagate to 240 km via the calculated raypath. (For a time dependent model atmosphere, time must be reversed for the principle of reciprocity to be applied.) This interpretation is sometimes known as reverse raytracing and can be used to locate possible sources of gravity waves (Bertin, Testud and Kersley, 1975). The present calculations demonstrate that vertical winds can significantly distort the raypath, and hence cause uncertainties in identifying possible sources in some circumstances. Fig. 4.2.1 illustrates the raypaths for an initial wavelength of 100 km, with no winds present. The ray launched horizontally is incident on the ground at a range of about 300 km, whereas for a launch angle of -80° this range is about 1800 km. Figs. 4.2.2 to 4.2.5 illustrate raypaths for the same initial wavelength for the four wind shears defined by $K = -0.1, -0.025, 0.025$ and 0.1 m/s/km respectively. The ground ranges are displaced from the zero wind situation by about 25% for the larger magnitude wind shears and by about 5% for the smaller ones. A noteworthy feature for the positive wind shear is the flattening of the raypaths
below 100 km and in fig. 4.2.5 one of the rays becomes ducted at 100 km.

In figs. 4.2.6 to 4.2.10 similar raytracings are presented, but for an initial wavelength of 200 km. Fig. 4.2.6 indicates that for zero vertical wind all the rays are ducted at around 100 km. When a negative wind shear is applied, some rays reach the ground. When this shear is increased to its maximum negative value, $K = -0.1$, five rays reach the ground, and the ground ranges are shorter than for the $K = -0.025$ case. For the positive wind shears all the rays are ducted. It should be noted that the height at which the rays become horizontal increases as the wind shear increases in the positive (upward) direction. Generally similar results are obtained for a wavelength of 300 km, and these are reproduced in figs. 4.2.11 to 4.2.15. Even when the maximum negative wind shear is applied, very few rays reach ground level. For the largest wavelength considered, 500 km, (figs. 4.2.16 to 4.2.20) all the rays are ducted except for one case when the maximum negative wind shear is applied. In this case the height at which the ray direction becomes horizontal varies appreciably with launch angle for every case considered, i.e. for positive, negative and zero vertical winds.

4.3 Discussion

The results of the raytracing calculations indicate that the presence of vertical winds can have a significant effect on the propagation paths of internal gravity waves in the atmosphere. The smaller the wavelength initially, the greater the influence of the vertical wind. However, even for large wavelengths (500
FIG 4.2, WAVELENGTH=200 KM., K=+0.025 M/S/KM., AZ=0.000, GRAVITY

FIG 4.2.10, WAVELENGTH=200 KM., K=0.1 M/S/KM., AZ=0.000, GRAVITY
FIG 4.2.11 WAVELENGTH=300 KM., ZERO WINDS.
AZ = 0.0000, GRAVITY

FIG 4.2.12 WAVELENGTH=300 KM., V=-0.1 M/S/KM.
AZ = 0.0000, GRAVITY
FIG 4.2.13, WAVELENGTH=300 KM, K=-0.025 M/S/KM.
$\Delta z = 0.0000, \text{GRAVITY}$

FIG 4.2.14, WAVELENGTH=300 KM, K=+0.025 M/S/KM.
$\Delta z = 0.0000, \text{GRAVITY}$
FIG 4.2.17. WAVELENGTH=500 KM, K=-0.1 M/S/KM, AZ= 0.0000, GRAVITY

FIG 4.2.18. WAVELENGTH=500 KM, K=-0.025 M/S/KM, AZ= 0.0000, GRAVITY
FIG 4.2.19: WAVELENGTH=500 KM., K=0.025 M/S/KM.  
AZ= 0.0000, GRAVITY

FIG 4.2.20: WAVELENGTH=500 KM., K=0.1 M/S/KM.  
AZ= 0.0000, GRAVITY
there exist a number of rays whose behaviour changes radically when the wind is included in the calculation. In many of the examples presented the vertical wind can result in rays reaching the ground instead of being ducted in the thermocline, and vice-versa.

These results have important implications when reverse raytracing is employed to locate the source of gravity waves. For example, a small vertical wind shear could lead to the identification of a tropospheric source (weather fronts, etc.) as opposed to a thermoclinic source, associated with an entirely different source mechanism, such as Joule heating. Moreover, even when there is no uncertainty that the ray does reach the ground, the ground range can be very considerably changed by the existence of a vertical wind, and thus an incorrect source location would be derived. When horizontal winds are included in the analysis as well, the uncertainty in identifying source regions is significantly increased. It is evident, therefore, that there is a great need to obtain accurate wind profiles in order that realistic attempts to locate gravity wave sources by raytracing techniques can be undertaken.
CHAPTER 5 Raytracing as a Tool for the Identification of Gravity Wave Sources

5.1 Introduction

Gravity waves have been observed at Leicester using the high frequency radio wave doppler technique for a number of years (Jones and Wand, 1965). Five events representative of medium scale disturbances, observed during the period 9/2/73 to 1/3/73 (Jones et al., 1973) are used in this analysis, and their propagation characteristics appear in Table 5.1.1. All of these disturbances are reverse raytraced in a variety of model atmospheres to attempt to locate their sources, remembering that for reverse raytracing to be undertaken successfully time must be reversed. All waves were observed at a height of about 200 km, and at geographic coordinates (52.5°N, -1.0°E). The atmospheric parameters which are most likely to affect the analysis are 1) the ambient temperature, 2) the ambient density, 3) neutral winds, and 4) variations in the dissipative terms.

Both the ambient temperature and density profiles are well documented as functions of season, time of day, etc.,

<table>
<thead>
<tr>
<th>Date and Time of Observation</th>
<th>Horizontal Wavelength (Km)</th>
<th>Horizontal Phase (°)</th>
<th>Horizontal Velocity (m/s)</th>
<th>Azimuth of Propagation of N</th>
<th>Period (Minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/3/73 12.45-13.15</td>
<td>95</td>
<td>105.6</td>
<td>154.7</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>21/2/73 10.30-11.30</td>
<td>150</td>
<td>125</td>
<td>127.9</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>9/2/73 14.45-15.15</td>
<td>235</td>
<td>195.7</td>
<td>115</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>17/2/73 13.45-14.15</td>
<td>306</td>
<td>255</td>
<td>111.8</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>20/2/73 11.00-11.30</td>
<td>400</td>
<td>333</td>
<td>209.2</td>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>
and will thus be held constant throughout this analysis. The density model used is an analytic model, based on the CIRA 1961 average conditions, medium solar activity model (Kallmann-Bijl et al., 1961), and the temperature model is the analytic model of Georges, 1971 based on the U.S. standard atmosphere, 1962. Both these profiles are illustrated in appendix 1. No data-based neutral wind model was available, and consequently a simple analytic model of the form

\[ U_r = 0 ; U_H = Kh \]

was used. \( U_r \) represents the radial wind, and \( U_H \) the horizontal wind component, \( h \) the atmospheric height, and \( K \) a constant. Ignoring radial winds is likely to lead to significant errors in the analysis, as shown in chapter 4, but as no adequate profile is currently available this assumption is the most reasonable one to make. Several analytic models of the horizontal wind component have been derived (Kohl and King, 1967; Kantor and Cole, 1964), but whilst these models give an understanding of average flow patterns, they may differ markedly from observed values (e.g. Bertin et al., 1975). It was therefore decided to use the simple model of equation 5.1.1. The wind direction was estimated from the analysis of Kohl and King, 1967, and appears in table 5.1.2. These directions

<table>
<thead>
<tr>
<th>Table 5.1.2</th>
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<tbody>
<tr>
<td>Horizontal Wind Azimuth</td>
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<tr>
<td>Wavelength degs. E of N</td>
</tr>
<tr>
<td>95 km</td>
</tr>
<tr>
<td>150 km</td>
</tr>
<tr>
<td>235 km</td>
</tr>
<tr>
<td>306 km</td>
</tr>
<tr>
<td>400 km</td>
</tr>
</tbody>
</table>
were varied through ± 10° in the analysis, to assess the effect of wind azimuth on the raytrace. Two values of K were employed, $\frac{1}{2}$ m/s/km and $\frac{1}{2}$ m/s/km; yielding horizontal wind speeds of 80 and 160 m/s at the 240 km level respectively. Both these values are typical of observed thermospheric wind speeds, and fall well within the upper limit of reported values (Rishbeth, 1972).

Two sets of raytraces are performed, to take into account dissipative effects. The first includes viscosity and ion drag, and the second thermal conductivity. The models for these effects are given in appendix 1. It has already been shown that the effects of ion drag at no time significantly distorts the raypath, and so the Pederson conductivity and magnetic field models employed are held constant throughout. The coefficients of thermal conductivity and viscosity are varied through ± 10°, so that errors due to variations in these parameters can be assessed. A more realistic model would include the effects of thermal conductivity and viscosity together. This was attempted, but in all cases numerical errors dominated after a few integration steps, and thus no confidence could be placed in the results obtained.

5.2 Results of the Ray Analysis

The results are presented as graphs of ray height against ground range (both measured in km). The first wave to be discussed is the 95 km case. Figs. 5.2.1 to .3 show the raypath for the small wind shear model, and include the effects of viscosity and ion drag. They depict the results of varying the azimuth of propagation of the wind. In each of these cases, the ray propagates to ground level, which it reaches in ranges between 500
and 650 km, a variation of about 25%. The raypaths are seen to be qualitatively the same, the most notable feature being the near vertical propagation as the ray approaches ground level. When thermal conductivity is included instead of viscosity and ion drag, although the ray still propagates to ground level, it now exhibits a qualitatively different raypath in the lower atmosphere, and reaches the ground at a range of about 850 km. The effect of varying the wind azimuth was found to be negligible in this case, and consequently only the results for the mean value of this parameter are depicted, this in fig. 5.2.4. For both the viscosity and ion drag, and the thermal conductivity cases, the effects of varying the dissipative terms were found to be negligible, and consequently no results including these perturbations are presented. If the larger wind shear is included, and the wind azimuth varied the results for the case of viscosity and ion drag (figs. 5.2.5 to .7) show an even greater variation than before. The rays still exhibit the same basic shape, but now reach ground level at ranges between 1250 and in excess of 2500 km, a variation of more than 100%. A similar result is obtained for when thermal conductivity is included, figs. 5.2.8 to .10, although the variation in ground range is not quite as pronounced, ranging from 1200 to 2100 km. The effects of varying the coefficients of viscosity and thermal conductivity were found to be negligible. All that can be said regarding the source of this wave is that it is likely to be a ground based/tropospheric source. Estimates of its ground range from the detection point varied from about 500 to more than 2500 km, a variation
Figure 5.2.1

Figure 5.2.2
95 KM, 15 MINS. VISCOSITY & ION DRAG, WIND AZ=30°, K=333 M/S/KM.
AZ= 205.3000, GRAVITY

Figure 5.2.3

95 KM, 15 MINS. THERMAL CONDUCTIVITY, WIND AZ=40°, K=333 M/S/KM.
AZ= 205.3000, GRAVITY

Figure 5.2.4
Figure 5.2.7

Figure 5.2.8
greater than 500 km. No source mechanism can thus be identified with this disturbance.

The 60 km, 20 minute wave is now considered, and the effects of the smaller wind shear are investigated first, both in the viscosity/ion drag and the thermal conductivity cases, the effects of varying the wind azimuth were small, and results depicting the effects of these variations are not presented. However, there is a cumulative difference in the ray path dependent on which type of dissipation are included. If viscosity and ion drag are included, Fig. 5.2.11, the ray propagates to ground level, which it reaches at a range slightly in excess of 700 km. The lower atmospheric ray path is shown to be much shorter. For winds of 95 km., 15 mins., thermal conductivity is included, Fig. 5.2.12, the ray is ducted at a height of about 110 km. It a range slightly greater than 600 km. has a thermocline or lower atmospheric source. When the wind shear is doubled, it appears to make little difference to the ground range, this being slightly increased if viscosity and ion drag are included, Fig. 5.2.13 and remaining essentially unchanged for the thermal conductivity case, Fig. 5.2.14. The effects of wind shear, altitude and azimuth and the coefficients of viscosity and thermal conductivity were found to be negligible. Once again, it has proved impossible to identify the source of the gravity wave under consideration, and the need for constructing a ray tracing procedure which can calculate accurate ray paths which including more dissipative effects has been underlined.

This inability to be possible will type of later...
greater than 500%. No source mechanism can thus be identified with this disturbance.

The 150 km, 20 minute wave is now considered, and the effects of the smaller wind shear are investigated first. Both in the viscosity/ion drag and the thermal conductivity cases, the effects of varying the wind azimuth were small, and results depicting the effects of these variations are not presented. However, there is a qualitative difference in the ray path dependent on which types of dissipation are included. If viscosity and ion drag are included, fig. 5.2.11, the ray propagates to ground level, which it reaches at a range slightly in excess of 700 km. The lower atmospheric ray path is seen to be far shallower than for the 95 km wave. If thermal conductivity is included, fig. 5.2.12, the ray is ducted at a height of about 110 km, at a range slightly greater than 600 km. Thus, it is unclear whether the disturbance has a thermoclinic or lower atmospheric source. When the wind shear is doubled, it appears to make little difference to the ground range, this being slightly increased if viscosity and ion drag are included, fig. 5.2.13, and remaining essentially unchanged for the thermal conductivity case, fig. 5.2.14. The effects of varying the wind azimuth and the coefficients of viscosity and thermal conductivity were found to be negligible. Once again, it has proved impossible to identify the source of the gravity wave under consideration, and the need for constructing a ray tracing programme which can calculate accurate ray paths whilst including more dissipative effects has been underlined. This is unlikely to be possible until larger and faster
Figure 5.2.11

Figure 5.2.12
150 KM, 20 MINS; VISCOSITY & ION DRAG, WIND AZ=-60., K=686 M/S/KM.
AZ= 232.1000, GRAVITY

Figure 5.2.13

150 KM, 20 MINS; THERMAL CONDUCTIVITY, WIND AZ=-60., K=686 M/S/KM.
AZ= 232.1000, GRAVITY

Figure 5.2.14
computing facilities are made available.

The results for the 235 km case are very similar to the 150 km wave. The effects of varying wind azimuth and dissipative term coefficients again prove to be negligible. The dichotomy between a possible ground/tropospheric or a thermoclinic source remains, however. The results for the small wind shear are presented in figs. 5.2.15 and 16, and for the larger shear in figs. 5.2.17 and 18. When viscosity and ion drag are included, the ray reaches ground level at a range of about 900 km, and when thermal conductivity is used the ray becomes ducted at a height of about 110 km, for the small wind shear at a range of about 420 km, and for the larger shear at about 500 km. No conclusion can thus be reached regarding the source of this disturbance, for reasons identical to those for the previous case.

The 306 km, 20 minute wave is considered next. For the small wind shear model, when viscosity and ion drag are included the variation of the wind azimuth proves important. For the mean value, and a variation of -10°, the ray is ducted at a height of 90 km, and a range between 400 and 500 km, figs. 5.2.19 and 20. If the variation is taken to be +10°, however, the ray propagates to ground level, which it reaches at a range slightly in excess of 1200 km, fig. 5.2.21. If thermal conductivity is considered, the ray is ducted at a height of about 120 km, and at a range of about 300 km, whatever the wind azimuth, fig. 5.2.22. This behaviour was retained for the larger wind shear case. When the effects of viscosity and ion drag are considered with the large wind shear, the ray behaviour differs from the small wind
Figure 5.2.15
235 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=0., K=.333 M/S/KM.
AZ=245.0000, GRAVITY

Figure 5.2.16
235 KM, 20 MINS., THERMAL CONDUCTIVITY, WIND AZ=0., K=.333 M/S/KM.
AZ=245.0000, GRAVITY
235 KM. 20 MINS., 'VISCOSITY & ION DRAG,' WIND AZ=0., K=0.666 M/S/KM.
AZ = 245.0000, GRAVITY

Figure 5.2.17

235 KM., 20 MINS., THERMAL CONDUCTIVITY, WIND AZ=0., K=0.666 M/S/KM.
AZ = 245.0000, GRAVITY

Figure 5.2.18
Figure 5.2.19

306 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=-20., K=333 M/S/KM.
AZ = 248.2000, GRAVITY

Figure 5.2.20

306 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=-30., K=333 M/S/KM.
AZ = 248.2000, GRAVITY
VISCOSITY & ION DRAG, WIND AZ= -10., K= 0.353 M/S/KM.

306 KM, 20 MINS, THERMAL CONDUCTIVITY, WIND AZ= -20., K= 0.333 M/S/KM.

Figure 5.2.21

Figure 5.2.22
shear case. Here, for the mean wind azimuth, fig. 5.2.23, and for a variation of $+10^\circ$, fig. 5.2.25, the ray propagates to ground level, which it reaches at ranges of about 1600 and about 1100 km respectively. With a variation of $-10^\circ$, fig. 5.2.24, the ray is again ducted, at a height slightly below 100 km, and a range of about 430 km. The results of varying the coefficients of viscosity and thermal conductivity was found to be negligible. The source of this disturbance is thus impossible to identify, the dichotomy between a thermoclinic and lower atmospheric source remaining. In this case the effect of varying both wind azimuth and magnitude was found to be of considerable importance for the viscosity/ion drag raytraces, and to be irrelevant when thermal conductivity was included.

The final disturbance considered is the 400 km horizontal wavelength case. Varying wind direction, and the coefficients of viscosity and thermal conductivity was shown to have negligible effect. Varying wind magnitude had no effect when thermal conductivity was included, the ray being ducted at a height of about 120 km, and a range of 210 km throughout, fig. 5.2.27. However, if viscosity and ion drag are included in the analysis, the variation in wind magnitude is seen to have a significant effect, the ray being ducted at a height of about 100 km, and a range of 350 km, fig. 5.2.26, for the smaller wind shear, and propagating to ground level at a range of 1100 km for the larger wind magnitude, fig. 5.2.28. Therefore, it is again impossible to identify a source for a gravity wave employing the raytracing technique.
Figure 5.2.23

Figure 5.2.24
306 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=-10., K=866 M/S/KM.
AZ= 248.2000, GRAVITY

Figure 5.2.25

400 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=-55., K=333 M/S/KM.
AZ= 150.8000, GRAVITY

Figure 5.2.26
400 KM, 20 MINS., THERMAL CONDUCTIVITY, WIND AZ=55., k=333 M/S/KM. 
AZ= 150.8000, GRAVITY

Figure 5.2.27

400 KM, 20 MINS., VISCOSITY & ION DRAG, WIND AZ=55., k=.666 M/S/KM. 
AZ= 150.8000, GRAVITY

Figure 5.2.28
5.3 Conclusions

In none of the waves considered has it been possible to identify a source. A few general conclusions can, however, be drawn.

1) The effects of varying the coefficients of viscosity and thermal conductivity through quite large values, \( \pm 10\% \), are seen to be negligible.

2) Wind variations are seen to have a small effect on waves of intermediate wavelength, the \( 150 \) km and \( 235 \) km cases, but to have a large effect when both their magnitude and direction are varied for the smallest, \( 95 \) km, and one of the larger, \( 306 \) km, events. For the two longest wavelengths, variations in wind magnitude proved unimportant if thermal conductivity was included but had a significant effect if viscosity/ion drag were used, namely the dichotomy between a thermoclinic and a lower atmospheric source.

3) Waves at the small wavelength end of the spectrum would appear to be more likely to have ground based/tropospheric sources. Other disturbances would appear to be more likely to have a thermoclinic source the greater their wavelengths.

As it has proved impossible to include all dissipative effects at once, some guidance must be sought as to which parameters can best be left out. The analysis of Klostermeyer, 1972 has indicated that thermal conductivity is the most important dissipative process, and thus it is possible that the calculations involving this parameter are the most realistic. If this is so, then neutral wind variations are likely to have a reduced importance.
It may be possible to avoid mistakes arising from the inclusion of erroneous wind profiles by a statistical type of analysis, i.e. if a sufficient number of gravity waves with similar wavelengths were observed under similar atmospheric conditions, then it might be possible to use an average wind model, such as that computed by Kohl and King, 1967. Then if a significant proportion of the disturbance were seen to emanate from the same source region, it may be possible to identify a physical source. This could then be modelled using an analysis similar to that of chapter 3, and the far field response to it computed, as a check on whether such a mechanism is likely.

It must be stressed that analysis of individual disturbances to identify their sources should be avoided if the ray technique as here developed is used.
6.1 Introduction

On June 1\textsuperscript{st} 1974 there was a large explosion at a chemical works at Flixborough, Lincolnshire. Jones and Spracklen, 1974 produced evidence indicating that the blast wave reached thermospheric heights where the wave was detected by means of the high frequency (HF) radio doppler sounding technique. In this method an HF radio wave is reflected from the ionosphere at steep incidence, and the frequency of the received signal is monitored. Acoustic gravity waves propagating through the atmosphere displace the reflection point and a doppler frequency shift is produced in the reflected signal.

The magnitude of the doppler shift, $\Delta f$, is proportional to the rate of change of phase path of the radio wave, according to the expression

$$\Delta f = -f \frac{\Delta \phi}{c_L \Delta t}$$

where $f$ is the radio frequency and $c_L$ the velocity of light. If three spaced reflection points are available then the horizontal phase speed and the direction of the disturbance may be determined from the time delays in the doppler frequency changes observed on the three transmissions.

At the time of the explosion three transmitters located at Gainsborough, Upwood and Stafford were operational. These were received at two sites, at Leicester (R1) and in Wiltshire (R2). Fig. 6.1.1 shows the locations of the transmitters and receivers and the six radio reflection points.

Recordings of frequency as a function of time were obtained from each receiver. The frequencies of the three
Figure 6.1.1
transmitters are offset by 3 Hz relative to each other so that they may be distinguished on the frequency-time recording. The Leicester record is reproduced in fig. 6.1.2 and the Wiltshire one in fig. 6.1.3. In both diagrams it is seen that there are two received signals from each transmitter. These are the 'ordinary' and 'extraordinary' radio waves, the Earth's magnetic field splitting the transmitted wave into two characteristic waves (see Ratcliffe, 1970, p75). The lower section of each trace is the extraordinary wave, and the upper the ordinary wave. It is evident from the Wiltshire recording that the ordinary wave signal only began to be received about 15 minutes before the Flixborough event appeared on the doppler recordings. This implies that at the time of the event, the radio frequency employed in the experiment was very close to the F-region ordinary wave penetration frequency, $f_o$, and it is evident from fig. 6.1.3 that the phase path of the ordinary component was changing rapidly at about this time thus accounting for the large frequency deviation of the received signals (cf. equation 6.1.1). Had the event occurred when the transmitted frequency was very much smaller than the critical frequency, the magnitude of the doppler shift might have been too small to appear on the records (note the magnitude of the extraordinary wave signals in figs. 6.1.2 and 6.1.3).

At 16.00.20 GMT a large disturbance was noted on the Gainsborough-Leicester path and later on the more southerly transmissions. The feature is a quasi-sinusoidal oscillation with a period of about 55 seconds. The maximum doppler frequency shift is 4.5 Hz, which corresponds to a rate of change of 142 m/s in the
reflection height. This deviation is a factor of about 5 greater than those produced by naturally occurring TJD's in the ionosphere. An interesting feature on the Leicester recording is the precursor event, marked P in fig. 6.1.2 which precedes the main event on all three transmissions. Jones and Spracklen, 1974 reported a second precursor, but on a closer inspection this was only evident on the Gainsborough-Leicester path.

The magnitudes of the disturbances recorded at Leicester are greater than those obtained in Wiltshire. This results from 1) the Wiltshire reflection points being farther from the wave source, thus the amplitude will be smaller due to spatial spreading, 2) the amplitude of acoustic waves increases exponentially with height (Hines, 1960), and as the Leicester reflection height (about 240 km) was greater than the Wiltshire one (about 225 km), the disturbance would be expected to have a greater amplitude at the former receiver, 3) the Wiltshire event occurred at a time when the wave frequency was further away from $f_0$ than it was for the Leicester event, i.e. at a time when $\Delta f/\Delta t$ was smaller (see eqn. 6.1.1) thus leading to a smaller doppler frequency shift at this receiver.

For both receivers, the Upwood transmitter produced a significantly fainter record. It is thought that this effect is a hardware, rather than an ionospheric effect.

Other noteworthy features are that 1) the precursor event on the Gainsborough-Leicester path was displaced from the main event by a shorter time than for the other Leicester propagation paths. 2) on the Leicester records, a slight discontinuity in the main event is observed (labelled D in fig. 6.1.2).
3) just after the commencement of the main event on the Upwood/Leicester path, a disturbance appears and tracks over and then appears to stabilise on the Gainsborough-Leicester record. This feature is labelled T on fig. 6.1.2.

In the following section, an attempt is made to account for the various features by means of the raytracing technique.

6.2 Analysis of the Event

A ray tracing analysis was performed to model the disturbance. Since the period of the wave (about 1 minute) is significantly less than the acoustic cutoff period at thermospheric heights, it is evident that the disturbance is of an acoustic, rather than a gravity type. A dissipationless model atmosphere is adopted, since for short path propagation nonlinearities introduced by dissipative effects are likely to be small. Neutral winds are neglected as they too are unlikely to have a large effect over a short distance of propagation. The ambient atmospheric model is therefore independent of latitude and longitude, and the raypaths will thus be independent of the azimuth of propagation, exhibiting cylindrical symmetry about the wave source. It is therefore necessary to perform raytraces at one angle of azimuth only, to obtain a complete three dimensional picture of the acoustic raypaths. The temperature model used was the analytic model of Georges, 1971 based on the 1962 U.S. standard atmosphere. This model is illustrated in appendix 1. The horizontal phase velocity and the horizontal wavelength of the disturbance was calculated using the method outlined by Jones, Kantarizis and Coward.
The dispersion relation 2.3.6 was then used to calculate the vertical component of the wavelength. An average value of 20 km was obtained for the wavelength at the points of observation. This value is less than the value reported by Jones and Spracklen, 1974, who estimated the wavelength assuming the phase velocity of the wave to be the same as the upper atmospheric sound speed. The actual phase velocity is smaller than this. A ray trace was performed varying the initial angle of elevation from 0° to 90° in 5° steps. The results were plotted as a graph of ray height against ground range (fig. 6.2.1).

Three distinct families of waves are evident in the graph.

1) one which propagates upwards into the thermosphere beyond the radio reflection height.
2) one which is reflected at heights between 120 and 180 km.
3) one which is reflected at heights below 120 km.

The first family is the one which gives rise to the main event, as this is the only one which reaches the radio reflection heights, which were calculated using the radio ray tracing programme of Jones and Stephenson, 1975, and an electron density profile obtained from an ionogram taken at the time of the explosion. Reflection heights of about 240 km for the Leicester receiver and 225 km for the Wiltshire receiver were obtained for the ordinary component of the radio wave, and about 220 km for all propagation paths for the extraordinary component. Calculations based on the ordinary wave reflection heights (the larger amplitude components in figs. 6.1.2 and 6.1.3) are used in the subsequent analysis, except where otherwise stated. The times after the explosion
Figure 6.2.1

FLIXBOROUGH, WAVELENGTH 20 KM., ZERO WINDS
AZ 180.0000, ACOUSTIC
at which the acoustic waves first reach the reflection points (and hence the times at which the main event would first be detected by the radio waves), were estimated from the acoustic and radio ray tracing analyses and are presented in table 6.2.1. The discrepancy of about 100 seconds between the calculated and measured times is probably due to the fact that only an estimate, and not an accurate measure of the time of the explosion was available (15.53 GMT, Dept. of Employment, 1974), and that the timing of the recordings was only accurate to about 30 secs. A modified table, table 6.2.2, was therefore constructed, subtracting the Gainsborough-Leicester estimated time from the first column, and the Gainsborough-Leicester measured time from the second. The estimated and measured time delays are in good agreement in that:

1) the Gainsborough event is seen to occur first for each receiver.

2) the time delays observed between the commencement of the Gainsborough event and the other two transmitters for both receivers are close to those calculated using the ray theory.

**Table 6.2.1**

<table>
<thead>
<tr>
<th>Reflection point</th>
<th>Estimated time of onset of main event (secs)</th>
<th>Measured time of onset of main event (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stafford-Leicester</td>
<td>636</td>
<td>531</td>
</tr>
<tr>
<td>Gainsborough-Leicester</td>
<td>600</td>
<td>504</td>
</tr>
<tr>
<td>Upwood-Leicester</td>
<td>636</td>
<td>537</td>
</tr>
<tr>
<td>Stafford-Wiltshire</td>
<td>688</td>
<td>573</td>
</tr>
<tr>
<td>Gainsborough-Wiltshire</td>
<td>632</td>
<td>528</td>
</tr>
<tr>
<td>Upwood-Wiltshire</td>
<td>684</td>
<td>573</td>
</tr>
</tbody>
</table>
3) the Stafford and Upwood records of the main event occur at practically the same time for each receiver.

Table 6.2.2

<table>
<thead>
<tr>
<th>Reflection point</th>
<th>Estimated time delay (secs)</th>
<th>Measured time delay (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stafford-Leicester</td>
<td>36</td>
<td>27</td>
</tr>
<tr>
<td>Gainsborough-Leicester</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Upwood-Leicester</td>
<td>36</td>
<td>33</td>
</tr>
<tr>
<td>Stafford-Wiltshire</td>
<td>88</td>
<td>69</td>
</tr>
<tr>
<td>Gainsborough-Wiltshire</td>
<td>32</td>
<td>24</td>
</tr>
<tr>
<td>Upwood-Wiltshire</td>
<td>84</td>
<td>69</td>
</tr>
</tbody>
</table>

It is seen from fig. 6.2.1 that the acoustic waves reflected below 120 km are focused around their reflection heights, and also along a line from the ground at the minimum acoustic skip distance to the reflection level, at a point closest in ground range to the wave source. These lines are the exterior and interior caustics of the rays, and meet at a cusp. They are illustrated in fig. 6.2.2. The rays are very strongly focused at the cusp, and more weakly focused the farther one moves out along the caustics. It is possible that the focused energy from the explosion enhances the electron density in the region of the caustics sufficiently to reflect the incident radio wave, and thus give rise to any precursor events. (The focusing of the pressure waves will lead to a larger temperature increase in the region of the caustics than in the rest of the atmosphere, and this temperature increase will cause an enhancement of the electron density.) This effect, should it exist, is illustrated in fig. 6.2.3. Tx denotes the radio transmitter, and Rx the receiver location.
Energy Flow: downwards and outwards along the caustic surfaces with time.

Radio wave yielding precursor event is given by dashed line.
This postulate can be checked simply by a geometrical analysis. The caustic surfaces are modelled analytically by polynomial fits, and to a first approximation assuming the incident radio wave to follow the laws of classical ray optics it can be shown whether it is possible for radio waves reflected by the caustics to be detected at the receivers. It was found that the exterior caustics could support such reflections, and using the results of the ray analysis it was possible to estimate the times after the explosion at which these would be detected at the two receivers for each transmitter. Calculated and measured time delays relative to the occurrence of the Gainsborough-Leicester main event (as in table 6.2.2) are presented in table 6.2.3. For some propagation paths, it was found that the caustic surface could support two reflections, and the time delays for both of these are presented.

<table>
<thead>
<tr>
<th>Propagation path</th>
<th>Estimated time delay (secs)</th>
<th>Measured time delay (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prec. 1</td>
<td>Prec. 2</td>
</tr>
<tr>
<td>Stafford-Leicester</td>
<td>6</td>
<td>none</td>
</tr>
<tr>
<td>Gainsborough-Leicester</td>
<td>-5</td>
<td>-2</td>
</tr>
<tr>
<td>Upwood-Leicester</td>
<td>6</td>
<td>none</td>
</tr>
<tr>
<td>Stafford-Wiltshire</td>
<td>120</td>
<td>165</td>
</tr>
<tr>
<td>Gainsborough-Wiltshire</td>
<td>50</td>
<td>80</td>
</tr>
<tr>
<td>Upwood-Wiltshire</td>
<td>108</td>
<td>220</td>
</tr>
</tbody>
</table>

The Wiltshire reflections occur towards the tail of the caustic, where the blast energy is more weakly focussed, and hence where it is less likely that the ambient electron density could be enhanced sufficiently to yield strong radio wave reflections. This is borne out
by fig. 6.1.3, as no precursor events can be unambiguously identified. A stronger agreement between calculated and measured values occur for the Leicester precursor events in that 1) the same number of precursor events are observed, and on the same propagation paths; 2) the Upwood and Stafford precursors occur at nearly the same time, and the Gainsborough ones slightly before these; 3) all precursors occur prior to the main event (compare table 6.2.2 with table 6.2.3).

The only discrepancy between measured and calculated values is that the measured precursor events occur around 30 seconds before the calculated precursor events. This is probably due to the fact that the calculated time delays have been estimated from reflections actually on the exterior caustic. However, it is likely that the electron density will be enhanced not only at the caustic, but in a band above and below it, and reflections below the caustic would yield precursor event times in advance of those presented, thus explaining the discrepancy. It is therefore evident that the precursor event can be adequately explained by invoking radio wave reflection from the exterior caustic.

The next feature to explain is the slight discontinuity on the Leicester recordings of the main event. A possible reason for this phenomenon is that the downgoing radio wave is refracted through the overdense regions caused by focussing at the caustics when these form. However, a simple geometrical analysis indicates that at no time does the reflected radio wave pass through, or even close to either of the caustics. An alternative explanation is that a weak reflection occurs from another part of the ionosphere, interfering with the
The obvious candidate for such a reflection lies in the extraordinary component of the radio wave, which has hitherto been neglected. Simple calculations indicate that this event would occur too early to cause the discontinuity, and thus this explanation is also excluded. The cause of the discontinuity thus remains unknown.

The final feature to be explained is labelled T in fig. 6.1.2. This feature appears to commence in the Upwood-Leicester record, and track across and settle on the Gainsborough record. The most probable explanation is that this trace begins as a precursor event on the Gainsborough record and is a discontinuous record of this event. Supporting evidence for this hypothesis lies in the fact that the precursor events are likely to be strongest for the Gainsborough-Leicester path, as the reflection causing these occur closest to the cusp, where the blast energy is most strongly focussed. No explanation has as yet been put forward as to why the signal should be unidentifiable before appearing on the Upwood trace.

6.3 Discussion

The ray theory is shown to provide a good explanation of the main features of this unusual event. Some minor features remain unexplained, and this is probably due to 1) the simplicity of the atmospheric model used and 2) the difficulty in identifying the very complicated traces on the doppler record.

In general, ray tracing provides a more satisfactory modelling for acoustic wave events than for gravity wave disturbances. This is because the acoustic propagation is over shorter paths than gravity wave propagation, and
hence non linear effects are unlikely to be significant. Similarly, winds are unlikely to have a large effect on acoustic waves, as the wind speeds are far less than acoustic wave phase velocities throughout the atmosphere.
CHAPTER 7  The Influence of Gravity Waves on the Ionised Atmosphere

7.1 Theory

The passage of a gravity wave through the upper atmosphere at ionospheric heights results in a perturbation of the local electron density $n_e$. These changes in ionisation density can be detected by ground based radio methods and are usually referred to as travelling ionospheric disturbances, or TID's. It is important, therefore, to relate gravity wave theory to the observed properties of TID's, and this problem is now considered.

The neutral atmosphere is coupled to the charged particles by collisions, with a collision period (less than 1 second) much shorter than gravity wave periods (about 6 minutes at ground level, about 15 minutes at thermospheric heights) at all altitudes of interest. The coupling is essentially instantaneous, and in the absence of non-linear effects the ions and electrons would have the same fluid velocity as the neutrals ($U_e = U_i = U$).

However, the motion of the charged particles is influenced by the presence of the Earth's magnetic field, $\mathbf{B}$. The ion gyrofrequency (defined by $\omega_c = eB/m c_L$, where $c_L$ is the speed of light, $m$ is the mass and $e$ the charge of the ion) is greater than $10^2 \text{ sec}^{-1}$, and is much larger than the ion-neutral collision frequency (less than 10 sec$^{-1}$) in the thermosphere, and thus the ions and electrons are constrained to move along the $\mathbf{B}$ field with velocity

$$U_e = \frac{U + \mathbf{B} \cdot \mathbf{U}}{B^2}$$

(7.1.1) (Francis, 1973). The electron density is governed by the
in which $Q$ is a production term due to photoionisation, and $L$ is a loss term \textit{(Francis, 1974)}. $Q$ is given by the expression

$$Q = \sigma_{i0} n(0) S \exp(-\sigma_{ao} \int L n(0) ds - \sigma_{aN} \int L n(N_2) ds)$$

where $n(0)$, $n(N_2)$ are the densities of atomic oxygen and molecular nitrogen respectively. $\sigma_{i0}$ is the effective ionisation cross section of the atomic oxygen. $\sigma_{ao}$ and $\sigma_{aN}$ are the effective absorption cross sections of the atomic oxygen and molecular nitrogen respectively. $S$ is the intensity of the solar ionising radiation at the top of the atmosphere. $L$ is the raypath of the solar radiation to the point where the production rate is investigated \textit{(Cho and Yeh, 1970)}. $L$ is given by the expression

$$L = \frac{\sigma_{i0}^2}{\sigma_{ao} n_h}$$

\textit{(Ratcliffe, 1956). This loss term is appropriate for an atmosphere in which the predominant electron loss mechanism is attachment}

$$N_2 + O^+ \rightarrow N + NO^+$$

with rate

$$\dot{n}_h = \eta n(N_2) n_h$$

where $\eta$ is an attachment rate constant; followed by dissociative recombination

$$NO^+ + e \rightarrow N + O$$

with rate $\sigma_{h}^2$.

These assumptions have been shown to be adequate by \textit{Clark, Yeh and Liu, 1970}, in the lower thermosphere, but
Francis, 1974 has indicated that they are not valid above the F-region peak in electron density, due to the neglect of ambipolar diffusion.

Perturbation theory is now used to simplify 7.1.2, using the assumptions

\[ n_h(x, z, t) = n_{ho}(z) + n_{h1}(x, z, t) \]  
\[ U_e(x, z, t) = U_{e1}(x, z, t) \]

where the 1-sufficed quantities are the perturbation variables, and \( n_{ho} \) is the ambient electron density profile. A two dimensional cartesian coordinate system is used to simplify the analysis. The perturbation assumptions yield the linearised equation of continuity,

\[ \frac{\partial n_{h1}}{\partial t} + \nabla \cdot (n_{ho} U_{e1}) = -\delta n_{h1} \]

where \( \delta = \frac{2\rho^2 \kappa_{n_{ho}} + \rho^2 n_{ho}^2}{(\kappa_{n_{ho}} + \rho)^2} = \frac{\partial L}{\partial n_{h}} \bigg|_{n_{h} = n_{ho}} \)

the Q term in 7.1.2 yielding only second order terms. The perturbations in Q and L induced by the passage of the gravity wave have been neglected. 7.1.10 is now solved by fourier transforming it with respect to \( x \) and \( t \), yielding

\[ \frac{n_{h1}(k_x, z, \omega)}{n_{ho}(z)} = \frac{1}{(i\omega + \delta)(H_e B \cdot B) B \cdot B B \cdot B} \left[ U_{e1}(k_x, z, \omega) \right] \]

where \( H_e = -n_{ho}/(\delta n_{ho} \partial z) \)

and where 7.1.2 has been used to substitute for \( U_e \) (Francis, 1974). To obtain the required electron density perturbation, 7.1.12 has to be inverse fourier transformed. If the neutral velocity perturbation induced by the source considered in chapter 3, the Lorentz force in the auroral electrojet is employed, an analytic solution can be obtained.
The velocity perturbations are given by

\[ U_{x1}(k, z, \omega) = \frac{c^2 k_x}{\gamma \omega} \frac{p'(k, z, \omega)}{p_0(z)} \]

\[ U_{z1}(k, z, \omega) = -\frac{i \omega c^2}{\gamma \omega_b^2} \frac{\partial}{\partial z} \frac{\gamma H}{\gamma B} \int \frac{p'(k, z, \omega)}{p_0(z)} \]

where \( p'(k, z, \omega) = -\left( \frac{p_0(z_s)}{p_0(z)} \right)^{1/2} \gamma IB G(k, z, \omega) \)

(\text{Francis, 1974}). \( G \) is the Greens function as derived in chapter 3, and the symbols in equations 7.1.14 and 7.1.15 are as defined in that chapter. 7.1.14 is now substituted into 7.1.12, which is inverse fourier transformed by phase integral techniques to yield

\[ n_{h1}(x, z, t) = -\left( \frac{p_0(z_s)}{p_0(z)} \right)^{1/2} \frac{1}{\gamma \omega_b^2} \int \frac{c^2 IB G(z_s, k_x) k_x^2}{\omega^2 + \gamma^2} (t^2 - t_L^2)^{1/2} \]

\[ \times \left\{ \frac{k_x \cos^2 d - k_z \sin 2d}{2} \left( \sin \gamma + \delta \cos \gamma \right) \right\} \]

\[ - \sin 2d \left\{ \frac{\gamma g}{2} \left( \frac{\omega_c^2 t + \omega_c^2 (t^2 - t_L^2)^{1/2}}{2 \gamma} \right)^{1/2} \right\} \]

\[ \frac{\sin \gamma}{\omega} \]

where \( \gamma = \omega_c^2 t + \omega_c^2 (t^2 - t_L^2)^{1/2} + \phi \)

and \( k_z = \omega_p^2 k_x / \omega \)

\( d \) is the dip angle of the magnetic field at the field point \((x, z)\). 7.1.16 gives the electron density perturbation caused by a gravity wave originating from the specific source mentioned above. In the following section, perturbations due to a selection of waves spanning the spectrum of medium scale disturbances are considered. The ambient electron density profile used in this analysis is an \( \alpha \)-Chapman layer of the form

\[ n_{ho} = N \exp(0.5(1-(z-S)/S1 - \exp(-(z-S)/S1))) \]

where \( N \) is the maximum electron density, \( S \) is the height at which \( N \) is attained, and \( S1 \) is the Chapman layer scale.
height. Values of $N = 2 \times 10^6 \text{ electrons/cm}^2$, $S = 300 \text{ km}$, $\text{Sh} = 50 \text{ km}$ are used throughout the analysis. Further, $n_{ho}$ is taken to be identically zero below 50 km height.

7.2 Results

In this section graphs depicting the quantity $n_{h1}/n_{ho}$ as a function of height are presented. The gravity wave source is taken to be at a ground range of a quarter of the Earth's circumference due North of the field point. The constants $\alpha$, the rate constant for dissociative recombination, and $\eta$ the rate constant for attachment are taken to be $10^{-7} \text{ cm}^3/\text{sec}$ and $2 \times 10^{-12} \text{ cm}^3/\text{sec}$ respectively. The number density of molecular nitrogen is taken from the CIRA 1965 mean atmosphere. The maximum velocity perturbation of the gravity wave is taken to be 40 m/s, which is in agreement with the calculations of Essex, 1976. The gravity waves considered are characterised by their period and horizontal phase speed. These are given in table 7.2.1. The dissipative parameters, viscosity, ion drag and thermal conductivity, are characterised by the models described in appendix 1. The results, figs. 7.2.1 to 7.7 have several features in common:

1) The maximum electron density perturbation occurs

<table>
<thead>
<tr>
<th>Period, mins</th>
<th>$v_{ph}, \text{m/s}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>20</td>
<td>200</td>
</tr>
<tr>
<td>30</td>
<td>200</td>
</tr>
<tr>
<td>30</td>
<td>250</td>
</tr>
<tr>
<td>30</td>
<td>300</td>
</tr>
<tr>
<td>40</td>
<td>300</td>
</tr>
</tbody>
</table>
between heights of 120 and 180 km, i.e. the region of the
greatest atmospheric temperature gradient, the
thermocline. This would imply that the best heights to
detect TID's and hence gravity waves by radio methods are
between these heights. The perturbation around 240 km,
where most of the TID's isolated by the Leicester HF
doppler technique have been observed, is roughly a factor
of half the maximum value. It is worthwhile stressing
that this analysis is valid only for medium scale
disturbances. Georges, 1967 has produced records of large
scale TID's which indicate that the amplitude of the
electron density perturbations induced by this class of
disturbance increases with height. These clearly show
that the above analysis is not valid for studying such
events.

2) There is a slight discontinuity at 50 km in each case.
This is so because the ambient electron density is
arbitrarily taken to be zero below this height.

3) Another noteworthy feature is the increase in the
variance of the perturbation profile as the horizontal
phase speed is increased. This results from numerical
errors arising when $v_{ph}$ approaches the lower atmospheric
sound speed of around 300 m/s. However, the gross
features of the TID still remain.

4) The TID amplitudes are seen to decrease with height
after reaching a maximum, due to the strong dissipation
in the upper atmosphere. This effect has been clearly
demonstrated in §2.4 already, in the analysis of the
dispersion relation; viz. the fact that the imaginary
part of the vertical wave number, $\text{Im}(k_z)$, is always of
the same magnitude as, but mainly greater than, the real
PERIOD: 20.0 MINS. SPEED=100.0 M/S.

Figure 7.2.1

RELATIVE ELECTRON DENSITY PERUBRATION \( \frac{n_e}{n_{eo}} \)

PERIOD: 20.0 MINS. SPEED=150.0 M/S.

Figure 7.2.2

RELATIVE ELECTRON DENSITY PERUBRATION \( \frac{n_e}{n_{eo}} \)
Figure 7.2.3
RELATIVE ELECTRON DENSITY PERIODIZATION $n_m/n_0$

Figure 7.2.4
RELATIVE ELECTRON DENSITY PERIODIZATION $n_m/n_0$
PERIOD: 30.0 MINS. SPEED=250.0 M/S.

Figure 7.2.5

PERIOD: 30.0 MINS. SPEED=300.0 M/S.

Figure 7.2.6
PERIOD = 40.0 MINS. SPEED = 300.0 M/S.

Figure 7.2.7

PERIOD = 20.0 MINS. SPEED = 100.0 M/S. PHASE = 45.0 DEGS.

Figure 7.2.8
5) It is noticeable that the maximum perturbations occur in the region of the thermocline.

Figs. 7.2.8 to 14 illustrate the relative electron density perturbation induced by one gravity wave, the 20 minute, 100 m/s horizontal phase speed case, as the phase of the wave is advanced through 360°. Fig. 7.2.1 gives the perturbation for phases of 0 and 360 degrees. The type of wave considered is an upgoing gravity wave, and these diagrams clearly indicate the downward phase progression of the TID.

Finally, fig. 7.2.15 is a plot of the relative electron density perturbation of the 20 minute, 100 m/s wave as a function of phase, calculated for a constant height of 240 km. The sinusoidal pattern obtained clearly reflects the simplicity of the analytic model employed, but such calculations are nevertheless useful in that they can be interpolated to give the change in reflection height of a radio wave, and hence to simulate the doppler records, some of which are illustrated in chapter 6.
Figure 7.2.9

Relative electron density perturbation $n_h / n_0$

PERIOD = 20.0 MINS, SPEED = 100.0 M/S, PHASE = 90.0 DEGS.

Figure 7.2.10

Relative electron density perturbation $n_h / n_0$

PERIOD = 20.0 MINS, SPEED = 100.0 M/S, PHASE = 135.0 DEGS.
Figure 7.2.11

PERIOD = 20.0 MINS. SPEED = 100.0 M/S. PHASE = 180.0 DEGS.

Relative electron density perturbation n_e/n_0

Figure 7.2.12

PERIOD = 20.0 MINS. SPEED = 100.0 M/S. PHASE = 225.0 DEGS.

Relative electron density perturbation n_e/n_0
PERIOD: 20.0 MINS. SPEED=100.0 M/S. PHASE=270.0 DEGS.

Figure 7.2.13

PERIOD: 20.0 MINS. SPEED=100.0 M/S. PHASE=315.0 DEGS.

Figure 7.2.14
PLOT OF RELATIVE ELECTRON DENSITY PERTURBATION AGAINST PHASE.

Figure 7.2.15
CHAPTER 8 Conclusions

This thesis has been concerned with the propagation of acoustic gravity waves in model atmospheres. In particular raytracing techniques have been developed and their validity assessed. The problem of identifying the sources of acoustic gravity waves has been considered and doubt cast on the use of reverse raytracing methods for this purpose.

Raytracing of the acoustic waves generated by the explosion at the Flixborough chemical works has been undertaken using a simple atmospheric model. No dissipative processes and no winds were considered since their influence is small for waves propagated over short paths. Also, the phase velocity of acoustic waves is significantly greater than the neutral wind velocity, and thus winds would be expected to have only a minor effect on each raypath. Raytracing is a useful technique for investigating acoustic propagation from wave sources of this type.

In raytracing medium scale gravity waves the following difficulties arise; 1) neutral winds have a significant effect on gravity wave propagation paths. It has been shown that even a very small variation in the magnitude of the vertical wind, a component which has hitherto been neglected as having an insignificant effect, can lead to a wave being ducted at the base of the thermosphere, instead of propagating to ground level, or vice-versa. Wind models currently available are wholly inadequate for one to be able to identify the source of an individual gravity wave. This problem may be partially overcome by back tracing of many waves observed under similar conditions, adopting an average wind model.
for the analysis. If a significant proportion of the rays appear to emanate from the same region, it may be possible to identify a wave source from within that region. 2) The ray equations themselves, although simple to derive, are in full generality too long to be included in a computer programme, using the facilities currently available. This has meant that a maximum of two dissipative processes can be included in any one analysis. Qualitatively different results are obtained depending on whether or not the process of thermal conductivity is included (chapter 2). This implies that the present analysis is unlikely to be useful in identifying TID sources when realistic atmospheric models are considered. 3) When the effects of thermal conductivity and ion drag, or those of thermal conductivity and viscosity are included in a raytrace, in many cases numerical errors swiftly dominate the solution unless the integration step is reduced to less than 0.1 seconds real time. If the integration step is taken below this limit then the computing time necessary to complete one ray integration becomes prohibitive. Thus meaningful results are difficult to obtain for these conditions.

Difficulties will occur in the use of raytracing for TID source identification unless the problems outlined above are overcome. An alternative to raytracing has been considered in chapters 3 and 7. This involves postulating a wave source and calculating the far field gravity wave response (chapter 3) and hence the TID induced by this wave (Chapter 7) by means of full wave techniques. This does not overcome the limitation imposed by lack of
knowledge of neutral wind profiles but is likely to
distinguish between ground/tropospheric and thermoclinic
sources of these disturbances.

An analysis of the acoustic gravity wave dispersion
relation has indicated that it is more likely that medium
scale events with shorter wavelengths (about 100 km)
have ground/tropospheric sources, and those with larger
wavelengths (more than 300 km) have thermoclinic
sources.

Large scale disturbances have been excluded from the
ray analysis because the atmosphere would change
significantly during one wavelength, thus violating the
assumptions of the ray equations. However, the sources of
these events are well known, and have been discussed in
chapter 3.

The analysis described in this thesis has extended
our knowledge of acoustic gravity wave propagation and
highlighted the considerable influence of dissipative
processes and of neutral winds, which are often
neglected. Provided an appropriate atmospheric model is
available raytracing can be a powerful tool for the study
of acoustic gravity wave sources and their characteristics.
It is suggested that the raytracing and full wave
techniques developed in the present study are applied to
the investigation of the many well defined wavelike
phenomena recorded by means of the doppler sounding
techniques both at mid and high latitudes.
Appendix 1  The Atmospheric Model

In this section the ambient atmospheric model is described, each parameter being depicted as a graph of its value against height. Figs. A1.1 and A1.2 show models of the atmospheric temperature and temperature gradients respectively. These are taken from the analytic profile of Georges, 1971 based on the U.S. Standard Atmosphere of 1962. The region of the atmosphere below the first temperature minimum is termed the troposphere, and the minimum itself is called the tropopause. The region between the tropopause and the first temperature maximum above it is called the stratosphere, the maximum being called the stratopause. The height range between the stratopause and the highest temperature minimum (the mesopause) is termed the mesosphere. Above the mesopause, the temperature increases very rapidly with height, and the height range in which it does so is termed the thermocline. The remainder of the atmosphere, above the thermocline is termed the thermosphere.

The ratio of specific heats \( \gamma = c_p/c_v \), and the mean molecular weight, \( m \), are taken from the models of Francis, 1973b and are presented in figs. A1.3 and A1.4.

The acceleration due to gravity is calculated from the formula \( g = 0.00980617 \times (1.0 - 3.14 \times 10^{-6}) \times h \) A1.1 where \( h \) is the height above ground level. This expression is the same as that given by Chapman and Lindzen, 1970, provided latitudinal variations are ignored. The graph of this quantity appears in fig. A1.5.

The quantity \( \log_{10}(\rho_o) \) where \( \rho_o \) is the ambient atmospheric density is presented in fig. A1.6. The value of \( \rho_o \) is taken from the CIRA 1961 average atmosphere.
FIG A1.1

TEMPERATURE, DEGS KELVIN

FIG A1.2

TEMPERATURE GRADIENT, DEGS KELVIN/KM
Fig A1.3

RATIO OF SPECIFIC HEATS, CP/CV

Fig A1.4

MEAN MOLECULAR WEIGHT, KG/KMOL
The atmospheric sound speed, \( c \), is given by
\[
c = \sqrt{RT/m}
\]
where \( R \) is the gas constant, \( 8.31432 \times 10^{-3} \text{ N.m/kmol/K} \), (U.S. Standard Atmosphere, 1976), and \( T \) the temperature taken from the model presented above. The variation of \( c \) with height is depicted in fig. A1.7.

The atmospheric scale height, \( H \), is calculated from the expression
\[
H = c^2 / g = RT/m / g
\]
and is given in fig. A1.8.

Figs. A1.9 and A1.10 depict the acoustic and Brunt-Vaisala cut off frequencies respectively. These are calculated from the formulae
\[
\omega_a = \gamma g / 2 c
\]
\[
\omega_b = g(\gamma-1)^{1/2} / c
\]
respectively.

Fig. A1.11 gives the pederson conductivity, \( \sigma_p \), and is taken from the data of Hanson, 1965 for daytime, sunspot maximum conditions.

Fig. A1.12 shows the 'Rishbeth' ion-neutral collision frequency,
\[
\beta = \sigma_p B^2 / \rho_0
\]
where \( B \) is the magnitude of the Earth's magnetic field, taken to be \( 3 \times 10^{-5} \text{ kg/amp/sec}^2 \) throughout.

Finally, figs. A1.13 and A1.14 show the logarithms of the coefficients of dynamic viscosity and thermal conductivity respectively. The dynamic viscosity \( \mu \) is given by
\[
\mu = \mu_m + \mu_e
\]
where \( \mu_m \) is the molecular and \( \mu_e \) the eddy component of this parameter. Values of \( \mu_m \) and \( \mu_e \) are taken from
Fig A1.7

Fig A1.8
ACOUSTIC CUT-OFF FREQUENCY, SEC^{-1}

Fig A1.9

BRUNT-VÄISÄLA FREQUENCY, SEC^{-1}

Fig A1.10
PEDERSON CONDUCTIVITY, KG**-1 *KM**-3 *SEC**3 *AMP**2

Fig A1.11

'RISHBETH' ION-NEUTRAL COLLISION FREQUENCY, SEC**-1

Fig A1.12
Fig A1.13

Fig A1.14
The coefficient of thermal conductivity is
\[ \lambda = \frac{\mu f R}{(\beta - 1) \kappa} \]  

where \( f \) is a numerical factor, related to the Prandtl number, and given by Francis, 1973b. It is evident from figs. A1.13 and A1.14 that the quantities \( \mu \) and \( \lambda \) are such that \( \frac{\mu}{\partial h} \ll \mu; \frac{\lambda}{\partial h} \ll \lambda \) throughout the atmosphere, thus justifying the neglect of the height derivatives of these parameters in this thesis.
The determinant used to evaluate the gravity wave dispersion relation is a complex mathematical entity. It would take a considerable amount of time to obtain the relation by hand, and such a process would be prone to simple arithmetic errors. Recourse was therefore made to algebraic computer packages. This appendix demonstrates the technique, employing the Reduce 2 algebraic language (Hearn, 1973).

Consider the statement \( z = (x+y)^2 \). In Reduce 2 this would be written

\[
Z := (X+Y)^2
\]

and the answer \( X^2 + 2XY + Y^2 \) would be output.

The package can also be used to partially differentiate expressions, i.e.

\[
\frac{\partial z}{\partial x} \quad \text{DZX := DF(Z,X,1)} \quad \text{yields} \quad 2(X+Y)
\]

\[
\frac{\partial^2 z}{\partial x^2} \quad \text{DZX2 := DF(Z,X,2)} \quad \text{yields} \quad 2
\]

\[
\frac{\partial^2 z}{\partial x \partial y} \quad \text{DZXY := DF(Z,X,1,Y,1)} \quad \text{yields} \quad 2
\]

Most algebraic packages have the reserved symbol \( i \) for \( (-1)^{1/2} \), and can deal with complex arithmetic. The Reduce 2 language has the facility for the user to define functions of his own, and can also perform matrix algebra. A limitation of algebraic packages is that they cannot, at the time of writing, factorise polynomials or divide one polynomial by another. Thus in the case of the ray equation

\[
\frac{dr}{dt} = - \left( \frac{\partial D/\partial k_r}{\partial D/\partial \omega} \right)
\]

\( \partial D/\partial k_r \) and \( \partial D/\partial \omega \) must be evaluated separately, and then merged in the fortran raytracing programme. Also, as the package is principally concerned with symbols, it does not hold any numerical constant to the same degree of
precision as a more conventional language, but this only has a significant effect when constants of widely differing magnitudes are multiplied together.

Algebraic packages provide a powerful tool which is of great help in simplifying and manipulating expressions which are too long to handle manually. In the case of the ray analysis with which this thesis is mainly concerned, it is possible to derive the ray equations for complex atmospheric models, the only limitation being that the fortran compilers available cannot readily handle equations of the length generated by the algebraic package.
Appendix 3  The Raytracing Programme

This programme is a development of the analysis of Georges, 1971; which considered a dissipationless atmosphere. The ray equations for this study are those given in chapter 2, equations 2.6.3. For such a programme to produce valid results, the medium through which the rays propagate must be slowly varying, i.e. it must not change significantly within one wavelength. To this end it is recommended that the programme should be restricted to a) waves with wavelengths less than 300 km, and b) waves with periods less than an hour. Because of these restrictions it is also recommended that the integration is stopped after a ray has been traced either a) to a ground range greater than 2500 km from the start point, or b) for a real time interval greater than 3 hours. One further restriction is that the upper boundary of the atmosphere is taken to be at a height of 500 km, so that any calculations involving raypaths above this level will be in error.

The main differences between this programme and that of Georges are as follows:-

a) The ray equations 2.6.3 are now complex equations, and thus the integration is in complex \((r, \theta, \phi)\) and \((k_r, k_\theta, k_\phi)\) space. This programme adopts the techniques of the radio wave raytracing programme of Jones and Stephenson, 1975, in taking \((r, \theta, \phi)\) to be purely real at all times.

b) The ambient atmospheric model adopted in the present work is more realistic. The acceleration due to gravity, \(g\); the ratio of specific heats, \(\gamma\); the acoustic and Brunt cutoff frequencies, \(\omega_a\) and \(\omega_b\); the mean molecular weight, \(m\); are taken as functions of height, instead of constant
quantities.

The input to the programme is through a 500 element array called W. The first data card, however, is always a data card containing text, to enable the user to describe the type of raytrace being performed. The W array is also used to transmit non-input control variables through the programme. The differences between this array, and the 400 element array of Georges are given below.

a) W(401) to W(494) inclusive are elements enabling the user to input a data-based temperature profile. If this facility is used, the temperature subroutine of the programme must be TC1965. W(401) to W(462) give the temperature from 0 to 120 km in 2 km steps, W(463) to W(480) give the temperature from 130 km to 300 km in 10 km steps, W(481) to W(494) give the temperature from 320 km to 600 km in 20 km steps.

b) W(496) to W(499) inclusive are variables specifying which forms of dissipative processes are to be included in the calculations. The values of these parameters for the various models are given in table A3.1. Any other

<table>
<thead>
<tr>
<th>W(496)</th>
<th>W(497)</th>
<th>W(498)</th>
<th>W(499)</th>
<th>processes included</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>none</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>ion drag</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>viscosity</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>viscosity and ion drag</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>thermal conductivity and ion drag</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>thermal conductivity and viscosity</td>
</tr>
</tbody>
</table>
combination is illegal, and the results given will be spurious.

c) new subroutines have been included to model the dissipative effects, DRAGON to model ion drag, and VISCOS to calculate the coefficients of kinematic viscosity and thermal conductivity. In addition to these, a subroutine DENST has been added to determine the ambient atmospheric density, $\rho_0$, and this routine is called by both VISCOS and DRAGON.

d) The ratio of specific heats, $\gamma$, and the mean molecular weight, $m$, are held in the W array as $W(321)$ and $W(320)$ respectively. These are calculated by a new subroutine, ATMODL, and are not input to the programme.

e) $W(309)$ now holds the magnitude of the initial horizontal component of the wavenumber vector, rather than the magnitude of the initial total wavenumber, as in Georges' programme.

f) The programme now uses the dispersion relation to calculate the initial angle of elevation of the wave, and thus it no longer loops on the angle of elevation. $W(256)$ and $W(257)$ are therefore obsolete. $W(255)$ is however, retained as a control variable, a value of +1 indicating that an initially upgoing ray, and a value of -1 indicating that an initially downgoing ray is required.

h) An additional loop has been included to enable a range of wave frequencies to be taken into account. Three new input parameters have therefore to be specified. $W(7)$ is the initial frequency, $W(8)$ the final frequency, and $W(9)$ the step in frequency, all in units of seconds$^{-1}$. If only one frequency is required, $W(7)$ alone is input.

i) The programme itself now calculates whether a ray is of an acoustic, gravity or evanescent type, and this
information is held in $W(305)$, which is no longer an input parameter. Values of $+1$, $-1$, $+2$, correspond to acoustic, gravity and evanescent modes of propagation respectively. When a wave becomes evanescent, the integration is stopped. A wave is defined to be evanescent if it attenuates by a factor of $1/e$ in a vertical distance of less than 5 km.

j) The graph plotting routines have had to be changed to be compatible with the Leicester graph plotting packages, and this means that $W(271)$, $W(286)$ are now obsolete. All other graph plotting variables retain the same meaning as in Georges programme.

k) Several changes have been made to the analytic neutral wind profiles. A routine WDATA, including the data based wind models of Balachandran, 1968 has been incorporated. If $W(370)$ holds the value 0, a model with a maximum speed of 110 m/s is used, and if it is greater than 0, a profile with a maximum speed of 190 m/s is employed. The model is a wind shear in the N-S direction. An alternative wind model is input through subroutine WCONST, which has been modified, and the variables within it to be specified are now:

- $W(370)$ height gradient of radial wind, m/s/km (+ve for upward, -ve for downward wind)
- $W(371)$ constant southward wind in m/s
- $W(372)$ constant eastward wind in m/s
- $W(375)$ height gradient of southward wind, m/s/km
- $W(376)$ height gradient of eastward wind, m/s/km

i.e. $U=(W(370)h,W(371)+W(375)h,W(372)+W(376)h)$, where $h$ is the height above ground level. A routine W2TIDE which is a composite of the above model, and the tidal wind
model WTIDE has been included, and the W-array variables to be specified are those listed above, plus the ones specified in WTIDE by Georges.

The above guide is designed to be used in conjunction with the manual of Georges, 1971 when performing a raytracing analysis. A listing of the raytracing programme follows.
PROGRAM RAYT(INPUT, OUTPUT, TAPES=INPUT, TAPE6=OUTPUT)
*** READS INPUT DATA, SETS INITIAL CONDITIONS ***
COMMON /SHARE/ N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART,SE
COMMON /MODELS/ MODEL(4)
COMMON R(12),T/W10D(10),DUM,W(500)
COMMON/CC/C,PCPR,PCPTH,PCPPH,PCPT,TK,DTDZ,DTDZ
COMMON /UI/ UR,UPH,PUPX(3,8),PURPT,PUTHPT,PUPHPT
COMMON /CONST/ PI,DEGS,P02,PIT2,RAD
COMMON/XV/UN,LA
COMMON/XION/BR,BTH,3PH,900TK,90KH,9,BXK2,BKH
DIMENSION FC(3),F0(5),F(3),AR(3),AC(3),AOZ(3),REZ(3),IMZ(3),AR(3)
J(5),AC2(5),REZ2(5),IMZ2(5),AR1(7),AC1(7),REZ1(7),IMZ1(7),AR(9),AC(9)
J(9),REZ(9),IMZ(9),V(9),X(10)
EQUIVALENT (LON,W(14)),(LAT,W(15)),(BETA,W(16)),(AZ1,W(17)),(EART
1,HR,W(18)),(XTMKR,W(20)),(INTYP,W(41)),(MAXERR,W(42)),(ERAT10,W(43
2)),(STEP1,W(44)),(STMPAX,W(45)),(STMPM,W(46)),(FACTR,W(47)),(RAY
3,W(67)),(MAXSTP,W(93)),(SKIP,W(180)),(LINES,W(181)),(NUXTST,W(291
4)),(STOP,W(253)),(FREQ,W(71)),(FRED,W(81)),(FRSTP,W(91)),
5,(AZDEG,W(265)),(AZEND,W(264)),(AZSTP,W(265)),(PLT,W(272)),(KTOT
6,W(303)),(RNEWW,W(291)),(TN,W(299)),(PNEWW,W(300)),(DORG,W(305))
C
REAL MAXERR,INTYP,MAXSTP,LON,LAT,KTOT,NDATE,LA,IMZ,IMZ1,IMZ2,IMZ3
COMPLEX K*,F,F,FB,FC,900TK,90KH,BXK2,RK,A2,A3,A4,A5,A6,A7,B1,84
J,85
DATA (MODEL=4(1H ))
DATA W/500*0.0,/>PUPX/9*0.0,/>V/9*0.0,/>VI/9*0.0/
*** SETS CONSTANTS ***
CP=1
NDATE = DATE(CP)
SECS = SECOND(CP)
PI = 3.1415926536
PI2 = PI/2
PI02 = PI/2
DEGS = 180/PI
RAD = PI/180.
IMP = 1
PCPR=PCPTH=PCPPH=PCPT=UR=UTH=UPH=PUTP=PUTHPT =PUPHPT=0.*
CALL READ
1 IF(WN *.LT. 6.) WN =6.
IF(SKIP *.LT. 0.) SKIP=MAXSTP
C *** LT-SUBROUTINES PRINR AND RAYPLT KNOW THERE IS A NEW W ARRA
PNEW=W=PNEW=1.
\begin{verbatim}
IF (RAY.EQ.1.) PRINT 11
IF (RAY.EQ.2.) PRINT 12
IF (RAY.EQ.3.) PRINT 13
PRINT 14, (I1) = 1, 10, NOATE, (MODEL(J1), J1 = 1, 4)
DO 4 I1 = 1, 100
CONTINUE

*** INITIALIZE PARAMETERS FOR RKAM ***
N = WN
MODE = INTYP
STEP = STEP1
ELMAX = MAXERR
ELMIN = MAXERR / ERAT10
EZMAX = MAXERR
EZMIN = MAXERR
FACT = FACTR

**** LOOP ON FREQUENCY ****
NBETA=1
IF (FRSTEP.NE.0.) NBETA = (FREND-FRBEG)/FRSTEP*1.5
DO 27 L = 1, NBETA
W(3) = FRBEG + (L-1)*FRSTEP

**** LOOP ON AZIMUTH ANGLE ****
NAZ = 1
IF (AZSTEP.NE.0.) NAZ = (AZEND-AZBEG)/AZSTEP + 1.5
DO 8 J = 1, NAZ
AZ1 = AZBEG + (J-1) * AZSTEP
AZA = AZ1 * DEGS
GAMMA = PI-AZ1
SGAMMA = SIN(GAMMA)
CGAMMA = COS(GAMMA)

*** INITIAL COMPONENTS OF RAY POINT AND WAVE VECTOR K ***
R(1) = CPLX(EARTH*XMTRH, 0)
CALL ATMOOL
CALL TEMP
CALL WIND
C2 = C
R(2) = CPLX(PID2-LAT, 0)
R(3) = CPLX(LON, 0)
R(4) = (0, 0)
R(5) = CPLX(SGAMMA*KTOT, 0)
R(6) = CPLX(SGAMMA*KTOT, 0)
I = (0, 1)
TOL = 1.0E-6
IFAIL = 0
COMEG = W(3)
\end{verbatim}
COMEG2=COMEG*COMEG
COMEG3=COMEG2*COMEG
COMEG4=COMEG3*COMEG
COMEG5=COMEG4*COMEG
CALL ATMODL
G=3.1432E-3/W(320)
C2=C*C
G=0.0980617*(1.+1.4E-4*(REAL(R(1))-W(19)))
WA2=G**2*N(321)**2/4./C2
WA=SQR(WA2)
WG2=G**2*(W(321)-1.)/C2
HK2=KTOT
E=COMEG2/C2*(WA2-COMEG2)/(W2-COMEG2)
EI=KTOT*(WG2-COMEG2)/COMEG2-(WA2-COMEG2)/C2
AORG=2
NTYPE=8*HEVSCNT
IF(COMEG2.LT.WG2) AORG=-1.
IF(COMEG2.GT.WA2) AORG=1.
IF(AORG.EQ.-1.) NTYPE=8*HAOUSTIC
IF(AORG.EQ.1.) GOTO 36
IF(W(499).NE.0.) GOTO 28
IF(AORG.EQ.-1.) AND.(KTOT.LE.E) GOTO 28
WRITE(6,400) COMEG2,KTOT,WA2,WG2,C2
400 FORMAT(1X,34*DATA UNREALISTIC, EVANESCENT WAVE., 5(3X,E10.4))
GOTO 8
28 RK=C*SQR(E1+(0.,0.))
RK=W(255).*AORG*RK
IF(W(499).NE.0.) RK=RK+W(A/C
IF(W(499).NE.0.) GOTO 26
W(497)=W(497)*W(498)*NE.0.) WRITE(6,35)
35 FORMAT(1X,43*WARNING- LA, B, UN CANNOT ALL BE NON-ZERO.)
IF(W(495).EQ.1. AND.W(495).EQ.0. AND.W(499).EQ.2.) GOTO 3
IF(W(495).EQ.0. AND.W(495).EQ.0. AND.W(499).EQ.4.) GOTO 95
IF(W(495).EQ.1. AND.W(495).EQ.0. AND.W(499).EQ.3.) GOTO 96
IF(W(495).EQ.0. AND.W(495).EQ.0. AND.W(499).EQ.3.) GOTO 96
CION DRAG
3 CALL DRAGON
B2=3*
A1=2.*B3H
A2=3R*B3K
A3=W/A/C
A4=W/2-HK2
A5=3*H*K**2
A5=A5+HK2
FC(1)=COMEG*C2*(-COMEG2+B*COMEG*B*A1+B2*BR**2)
FC(2)=2.*COMEG*C2*(I*COMEG2*A3+I*COMEG*B*(A2-I*A3*A1)+B2*(A2-I*A3)
J*BR**2))
FC(3)=COMEG5-2.*I*COMEG4*B-COMEG3*(B2+HK2*C2)+I*COMEG2*B*C2*(A6-2.*
GOTO 9
C THERMAL CONDUCTIVITY
94 CALL VISCOS
A1=C2*WG2/7G
A2=A1*LA
A3=A2/QG
A4=TK/LG
A5=Q/TK
A6=Q*TDHZ
A7=NA/G
A8=5*A6-2.*A5*A7
A9=C2*WG2*G*A6
B1=2.*A4*A7-1./Q
B2=A5+A1/2.*A7
FB(1)=*COMEG*4*A2
FB(2)=*COMEG*A2+31
FB(3)=I*COMEG3*A3-COMEG2*B2+COMEG*2.*I*A2*(A4*HK2+A7/Q)
FB(4)=I*COMEG2*A6-COMEG*HK2*A2*B1
FB(5)=COMEG4-2.*COMEG3*HK2*A3-COMEG2*(2.*G*A7+HK2*B2)+I*COMEG*HK2*A
J2*(2.*A7/3+HK2*A4)+HK2*A9
GOTO 10
C VISCOSITY
95 CALL VISCOS
UN=IN/3
UN2=UN*UN
FA(1)=9.*COMEG*UN2*(4.*I*COMEG*UN+C2)
FA(2)=-18.*I*COMEG*UN2+C*HA
FA(3)=3.*COMEG*UN*(-11.*COMEG2*UN2+2.*I*COMEG*(18.*UN2*HK2+C2)+9.*C
J2*UN*HK2)
FA(4)=13.*COMEG*UN*C*HA*(COMEG-3.*I*UN*HK2)
FA(5)=-10.*I*COMEG4*UN-COMEG3*(66.*UN2*HK2+C2)+12.*I*COMEG2*UN*HK2
J*(9.*UN2*HK2+C2)+7.*COMEGUN*HK2*2*C2+3.*I*UN*HK2*WG2*C2
FA(6)=2.*COMEG*C*HA*(I*COMEG*(COMEG-6.*I*UN*HK2)-9.*I*UN2*HK2)**2)
FA(7)=COMEG5-10.*I*COMEG4*UN*HK2-COMEG3*HK2*(33.*UN2*HK2+C2)+6.*I*
JUN-COMEG2*HK2**2*(6*UN2*HK2+C2)+COMEG*C2*HK2*(9*UN2*HK2**2+WG2)-3*
JUN-COMEG2*HK2**2
GOTO 91
C THERMAL CONDUCTIVITY AND ION DRAG
CALL VISCOS
CALL DRAGON
A1=C*WG2/G
A2=A1*LA
A3=A2/GQ
A4=TK
A5=Q
A6=Q*DTDZ
A7=NA/C
A8=Al/A7
A9=A5+A8/2
B1=3R*3DKH
B2=3
B3=2-3BH
B4=5MEG2-I*COME2+B*B3-B2*3R**2
B5=-HK2+B*3K2
B6=2*Al-A7-G-A6
B7=2*WG2+*A4
F3(1)=AAl*AI*34
F3(2)=AI*3H2*(-34/3+2.*A4*(A7*34+B*31*(COME2-I*B))
F3(3)=-1*COME2+*A3-COME2+(A9+*B3)*COME2*(I*B3*A9+2.*I*HK2*
J4*AI*3A**2**3*A7)++COME2+(B2*3R**2*A9+B*32*(I*A1+2.*A7*B3)/
J0*AI*3A**2**3*(B5+2,*HK*2**2**I*A1**8**1))++32*AI*3B**2*(B1-2,**I*A7**3R**2**2/2+2.*AI*3)
J5=2**A1*3A7)
F3(4)=I*COME2**86+CME2**86+(B*33**86+B1**2**89)+(HK2-A2**2**8,*A4+A7-1**J/)
J0**3H2**3H2**36+B2**3B1+A9+B*3H*K-A2**3B3/Q+2.*A4**(-I*+
J2(A3**33))**3H2**3B2**3A2**3B3**2**3+2.*A4**(-A7**3R**2**2-I*B1))
F3(5)=COME2+I*COME2**85*(HK2-A3**2**83-COME2**85*(B2+2,**G**A7+HK2**2**A5+A8/
J2***(B**2**8**LA**7**Q/G))**COME2**I*B**((3DKH**2**2**HK**2**A9**2**G**A
J1**B**2**8**1**B5+*HK2-A2**2**82**B2/G*3**2**A7+Q+A4**HK2)**COME2**2**B2**BDKH**2**2**
J1**A2**2**2**2**B2**3R**2**2**B1**B6**HK2**B2**A2**((I*A1**2**2**A7**3B**)/3+
J3**B7**95+B2**2**HK2**A2**((B1-2**I*A7**3R**2**2**3I*A7))**COME2**2**B7**3R**2**2**HK2**A2**((B1-2**I*A7**3R**2**2**3I*A7))
GOTO 10
C VISCOSITY AND ION DRAG
7 CALL VISCOS
CALL DRAGON
UN=UN/3
UN2=UN**3
UN3=UN2**UN
B2=3**3
A1=2**3BH
A2=C**WA
A3=3**3DKH
A4=IA/C
A5 = 3 * DH1 ** 2
A6 = A5 + HK2
A7 = B * HK2 - HK2
FA(1) = 9 * COMEG * UN2 * (4 * I * COMEG * UN + C2)
FA(2) = -18 * I * COMEG * UN2 * A2
FA(3) = 3 * COMEG * UN * (-11 * COMEG2 * UN + I * COMEG * (36 * UN2 + HK2 + B * UN * (8 * BB
JH) + 2 * C2) + C2 * (B * A1 + 9 * UN * HK2))
FA(4) = 6 * COMEG * UN * (COMEG * (2 * A2 + B * UN * I * A3) + B * C2 * (A3 - I * A4 * A1) - 6 * I*
JUN + HK2 * A2)
FA(5) = -15 * I * COMEG2 * C2 * UN + COMEG3 * (66 * UN2 + HK2 + B * UN * (14 * -BBH) + C2) + I
J * COMEG2 * (108 * UN3 * HK2 ** 2 + 3 * B * UN2 * (15 * HK2 - B * HK2) + UN * (B2 * (4 * -BBH) + 1
J2 * HK2 * C2) + 3 * C2 * A1) + COMEG * (27 * UN2 * HK2 ** 2 * C2 + 3 * B * UN * C2 * (3 * HK2 - B * HK2 -
J2 * I * A3 * A4) + 32 * C2 * BR ** 2) - 3 * I * UN * HK2 * W2 * C2
FA(6) = 2 * COMEG * (COMEG2 * (I * A2 + B * UN * A3) + COMEG3 * (I * B * A3 * (3 * UN2 + HK2 + B*
JUN + C2) * A2 * (6 * UN * HK2 + B * A1) - 9 * I * UN2 * HK2 ** 2 * A2 + 3 * B * UN * HK2 * C2 * (A3 -
J * A4 * A1) + 32 * C2 * (A3 - I * A4 * B + ** 2))
FA(7) = OMEG5 * 2 * I * COMEG4 * (3 * 5 + UN * HK2) - COMEG3 * (33 * UN2 + HK2 ** 2 * B + UN
J * (A5 + 13 * HK2) * B2 + HK2 * C2) + COMEG2 * (36 * I * UN3 + HK2 ** 3 + 3 * B * UN2 * HK2 * (J7 * -11 + HK2) + I * UN * (I * HK2 ** 2 * C2 + I2 * HK2) + 3 * HK2 * A1) + 3 * B * C2 * (A6 - 2 * I * A4 * A
J3) + COMEG6 * C2 * (9 * UN2 + HK2 ** 3 + 3 * B * UN * HK2 * (A6 - 2 * I * A4 * A3) + B2 * (A5 - 2 *
J * A4 * A3) + HK2 * W2) + I * W2 * C2 * (3 * A7 - 3 * UN * HK2 ** 2)
GOTO 91
C VISCOSITY AND THERMAL CONDUCTIVITY
98 CALL VISCONS
UN = UN / 3
U12 = UN * UN
U13 = UN * UN
HK22 = HK2 * HK2
HK23 = HK3 * HK2
A1 = 2 * W2 / G
A2 = A1 * LA
A3 = A2 / G
A4 = TK / G
A5 = I * TK
A6 = I * DT07
A7 = 12 * UN + 11 * A3
A8 = 33 * UN + 10 * A3
A9 = 10 * UN + A3
B1 = WA / C
B2 = 1 / Q - 2 * A4 * B1
B3 = G / A6
B4 = 2 * A5 * B1 - B3
B5 = A1 / B1
B6 = A5 / B5 / 2
B7 = 5 * A6 + C * W2
AR1(K)=REAL(FA(K))
AC1(K)=AIMAG(FA(K))
88 CONTINUE
M1=M-7
CALL CD2ADF(AR1,AC1,M,REZ1,IMZ1,TOL,IFAIL)
DO 19 K=1,6
V(K)=REZ1(K)
VI(K)=IMZ1(K)
19 CONTINUE
GOTO 83
92 DO 20 K=1,9
AR(K)=REAL(F(K))
AC(K)=AIMAG(F(K))
20 CONTINUE
M1=M-9
CALL CD2ADF(AR,AC,M,REZ,IMZ,TOL,IFAIL)
DO 21 K=1,3
V(K)=REZ(K)
VI(K)=IMZ(K)
21 CONTINUE
M=M-1
DO 22 K=1,M
WRITE(*,200) V(K),VI(K),K
200 FORMAT(1X,23X,E10.4),5X,I1)
22 CONTINUE
C *** SELECT ROOT CORRESPONDING TO PROPAGATING WAVE IN RIGHT DIRECTION.
DO 23 K=1,M
IF(REAL(RK)*V(K),LT,0.0) V(K)=VI(K)=(M+1.-K)*1.E5
23 CONTINUE
M1=M-1
DO 32 K=1,M1
IF(V(K),LT,V(K+1)) GOTO 33
32 CONTINUE
GOTO 34
XX=V(K+1)
YY=VI(K+1)
V(K+1)=V(K)
VI(K+1)=VI(K)
V(K)=XX
VI(K)=YY
GOTO 31
34 X(1)=1.E6
DO 2 K=1,M1
X(K+1)=AMOD(X(K),ABS(REAL(RK)-V(K)))
2 CONTINUE
DO 24 K=2,M
  IF(X(K+1) .EQ. X(K)) GOTO 25
24  CONTINUE
  GOTO 29
25  K=K-1
  GOTO 30
29  K=M
30  R(4) = CMPLX(Y(K), VI(K))
31  IF(W(499) .NE. 0.) R(4) = R(K)
32  WRITE(6,330) R(K), R(4)
33  FORMAT(1X,4(3X,E10.4))
34  BETA = ATAN2(R(REAL(R(4)), I(1))
35  GOTO 37
36  BETA = ATAN2(W(255), SQRT(WG2/COME62-1.))
37  R(4) = CMPLX(KTAN(BETA), 0.) * A006
38  IF(W(499) .NE. 0.) R(4) = R(4) + I * A/I/C
39  UDOTK = REAL(R(4) * UR + R(5) * UTH + R(6) * UPH)
40  W(I) = COMEG + UDOTK
41  T = 0.
42  RSTART = 1
43  EL = BETA * DEGS
44
*** INITIAL PRINTOUT ***
45  IF(I*NE.1., AND.*NPAGE,LT.3., AND.*LINES,LE.17) GO TO 5
46  NPAGE = LINES = 0
47  IF(RAY, E4-1.) PRINT 11
48  IF(RAY, E4-2.) PRINT 12
49  IF(RAY, E4-3.) PRINT 13
50  PRINT 14, (ID(I), I=1,10), NOATE, (MODEL(J1), J1=1,4)
51  FF = 2.*PI/ W(I)
52  XLAT = LAT * DEGS
53  XLONG = LON * DEGS
54  WRITE(0,16) FF, AZA, W(305), XLAT, XLONG
55  NPAGE = NPAGE + 1
56  PRINT 17, EL
57  DO 5 NN = 1, N
58  R(3*NN) = 0., 0., 0.
59  CONTINUE
60
61 CALL TRACE
62 OSEC = SECS
63 SECS = SECOND(CP)
64 DIFF = SECS - OSEC
65 PRINT 18, DIFF
66 WRITE(6,100) NTYP
100 FORMAT(33X,A8,5H WAVE)
8 CONTINUE
27 CONTINUE
IF(PLT,NE.,0.) CALL ENDPLT
GO TO 1

C 11 FORMAT(1H1,20X*ACOUSTIC WAVE WITH WINDS NO GRAV OR ABSORPTION*)
12 FORMAT(1H1,20X*ACOUSTIC-GRAV WAVE NO WINDS NO ABSORPTION*)
13 FORMAT(1H1,20X*ACOUSTIC-GRAV WAVE NO WINDS NO ABSORPTION*)
14 FORMAT(1X,10A6,20X,A10/1X,*MODELS- *4(1X,A7))
15 FORMAT(14,E14.6)
16 FORMAT(1X,3HPERIOD =,3X,F12.6,28HSEC, AZIMUTH OF TRANSMISSION =,F12.
J9,10HOEGS MODE =,F3.0,9H, XLAT =,E9.2,13H DIGES XLONG =,E9.2,5H DEG
JS)
17 FORMAT( /31X,33HELEVATION ANGLE OF TRANSMISSION =,F12.6,4HDEG ,)
18 FORMAT(9X,26HTHIS RAY CALCULATION TOOK ,F6.3,4HSEC.)
END

**********
SUBROUTINE READW
COMMON/CONST/PI, DEGS, PID2, PIT2, RAD
COMMON /W, W/ ID(10), DUM, W(500)
EQUIVALENCE (EARTH, W(13))
INTEGER DEG, CYCLE, FEET, SEC

C READ 1000, ID
1000 FORMAT(11A8)
1 READ 1000, K, W(K), DEG, KM, CYCLE, FEET, SEC
1100 FORMAT(13E14.7, 5I1)
IF (K.EQ.0) GO TO 10
IF (K.EQ.1 AND K.EQ.500) GOTO 5
PRINT 4000, K
4000 FORMAT(7H SUBSCR Nr.13 OUT OF BOUNDS)
IF (W(272).GT.0.) CALL GRENQ
CALL EXIT
5 IF (DEG.NE.0) W(K) = W(K)* RAD
IF (KM.NE.0) W(K) = W(K)/ EARTH
IF (CYCLE.NE.0) W(K) = W(K)*PIT2
IF (FEET.NE.0) W(K) = W(K)*3.048006096E-4
IF (SEC.NE.0) W(K) = PIT2 / W(K)
GO TO 3
10 RETURN
END

**********
SUBROUTINE TRACE
C *** CONTROLS RAYPATH INTEGRATION FOR A GIVEN RAY AND RETURN
TO INITIAL FOR NEW RAY CONDITIONS

COMMON/OLD,OLD(12),OROLD(12),TOLD
COMMON/GC,J,CC,AUX(7)
COMMON/SHARE,N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART,SSE
COMMON R(12),TSTP,DR(12),WH/IN/10),DUM,W(500)
COMMON/CONST,PI,DEG,PI0,PTT,RA
EQUIVALENCE(EARTH,W(13)),(GROUND,W(25)),(PERIGE,W(26)),(THERE,W(2
1
7)),(MINDIS,W(29)),(UNDER,W(33)),(HS,W(40)),(MAXSTP,W(93)),(SIP,
2
W(180)),(NUTEST,W(251)),(IHOP,W(253)),(HOP,W(254)),(TPOLAR,W(26
3)),(RPOLAR,W(263)),(HMAX,W(264)),(PLT,W(272)),(RAYBEG,W(292)),
4
(RANGE,W(301)),(KTOT,W(309))
REAL MAXSTP, KTOT
COMPLEX R, DRT, ROLD, DROLD
LOGICAL HOM, WASNT, PASSED, UNDROG, GROUND, PERIGE, THERE, MINDIS, UNDER

NHOP = HOP
MAX = MAXSTP
NSKIP = SKIP
RSTART = 1

CALL HASEL
H = REAL(R(1)) - EARTH
HOME = REAL(DRT(1)) * (H- HS) * GE. 0.
RAYBEG = 1.

CALL PRINTR (BHXMTR ,
IF (PLT,NE.0.) CALL RAYPLT

RAYBEG = 0.
*** LOOP ON NUMBER OF HOPS ***
DO 18 K = 1,NHOP
IHOP = K
NUTEST = 0
*** LOOP ON MAX NUMBER OF STEPS PER HOP ***
DO 10 J = 1,MAX
DO 1 L = 1,IN
ROLD(L) = RIL
DROLD(L) = DRT(L)
1 CONTINUE
TOLD = T

CALL RKAM
IF(W(493) .LT. 1.) OR(W(493)*W(496)*EQ. 2.) AND W(498).EQ. 0. AND H.LT.
J55.)) GOTO 21
IF(AIMAG(R(1)) .EQ. 0.) GOTO 21
IF(REAL(DRD(1)) * AIMAG(R(1)) * GT. 0. * AND N(395) * LT. -.5
J * AND ABS(1 / AIMAG(R(1))) * LT. 5) GOTO 19
21 H = REAL(R(1)) - EARTH
WASHT = NOT HOME
HOME = REAL(DRD(1)) * (H-HS) * GE. 0.
X = REAL(DRD(1) - DRD(1)) * (T - TOLD)
SMT = 0.
IF(K.HI. = 0.) SMT = 5. * (REAL(R(1) - ROLD(1))) ** 5 * X ** 2 / ABS(X)
UNDER = (H.LT. 0. OR REAL(DRD(1)) * GT. 0. * AND REAL(DRD(1)) * LT. 0. * AN
10 SMT.GT.H)
PASSED = (H-HS) * REAL(ROLD(1)) - EARTH-HS. * LT. 0.
IF(PASSED * AND (* NOT UARTGREO. OR. HS.GT. 0. 0.)) GOTO 13
IF(HS.EQ.REAL(ROLD(1)) - EARTH.AND REAL(DRD(1)) * REAL(DRD(1)) * LT. 10 * AND HOME) GOTO 16
IF(HOME * AND WASHT * AND (* NOT UARTGREO. OR. HS.GT. 0. 0.)) GOTO 2
IF(UNDER) GOTO 10
C GO TO 7
C *** RAY MAY HAVE MADE A CLOSEST APPROACH ***
2 IF(SMT.GT.ABS(H-HS)) GOTO 14
C NUTEST = 4
C CALL GRAZE(HS)
IF(UNDER) GOTO 4
IF(NUTEST EQ. 0.) GOTO 14
GOTO 12
C *** RAY WENT UNDERGROUND ***
3 IF(REAL(DRD(1)) * LT. 0.) GOTO 6
4 UNDER = .FALSE.
DO 5 L = I, N
R(L) = ROLD(L)
DRT(L) = DROD(L)
5 CONTINUE
T = TOLD
C 6 CALL BACKUP (0.)
R(1) = CHPLX (EARTH, 0.)
IF(K .EQ. NCHP) GOTO 20
DRT(1) = DRT(1)
R(4) = R(4)
RSTART = 1
C 20 CALL PRINTR(HGRND REF)
IF(HS.EQ. 0.) GOTO 17
H = 0.
GO TO 9

C 7 IF(REAL(DR(0,1)) .LT. 0., AND. REAL(DR(2,1)) .LT. 0.) CALL PRINTR(8HPER
11GE )
IF(REAL(DR(0,1)) .LT. 0., AND. REAL(DR(2,1)) .LT. 0.) CALL PRINTR(8HAPO
11GE )
IF(REAL(DR(0,2)) .LT. 0., AND. REAL(DR(2,2)) .LT. 0.) CALL PRINTR(8HMAX LAT )
IF(REAL(DR(0,3)) .LT. 0., AND. REAL(DR(2,3)) .LT. 0.) CALL PRINTR(8HMAX LONG)
DO 8 I = 4, 6
IF(REAL(R(0,L)) .LT. 0., AND. REAL(R(1)) .LT. 0.) CALL PRINTR(8HWA VE REV)
8 CONTINUE
9 IF(PLT .NE. 0.) CALL RAYPLT
IF(RANGE .GE. H(302)) GO TO 11
IF(MOD(J,NSTEP) .EQ. 0) CALL PRINTR(8H
GO TO 10
10 CONTINUE

C *** EXCEEDED MAXIMUM NUMBER OF STEPS ***
NTEST = 2
CALL PRINTR(8HMAX STEP)
RETURN
C ***** EVANESCENT WAVE *****
19 CALL PRINTR(8H EVNSCNT)
RETURN
C 30 CALL PRINTR(8HMAX HT )
RETURN
C *** RAY REACHED MAXIMUM GROUND RANGE ***
11 CALL PRINTR(8HMAX RNGE)
RETURN
C *** RAY MADE A CLOSEST APPROACH ***
12 NTEST = 4
DR(0,1) = 0
CALL PRINTR(8HMIN DIST)
IF(PLT .NE. 0,) CALL RAYPLT
GO TO 18
C *** RAY CROSSED RECEIVER HEIGHT ***
13 IF(HOME) GO TO 16
14 DO 15 L = 1, N
R(L) = ROLD(L)
DR(L) = DROLD(L)
15 CONTINUE
T = TOLD
RSTART = 1
CALL BACKUP (HS)
R(1)=CMPLX(EARTH+HS,0.)
CALL PRINTR (8HOMING)
IF (PLT, NE, 0.) CALL RAYPLT
HOM= TRUE.
RETURN
END

************  ************

C SUBROUTINE BACKUP(HS)

*** CONTROLS INTEGRATION WHEN RAY NEARS RECEIVER HEIGHT ***
COMMON/SHARE/N,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART,SSE
COMMON/CONST/P1,DEGS,PI02,PI12,PI2,PIAD
COMMON R(12),TS,DR(TS),HM/IO(10),DUM,W(500)
EQUIVALENCE (EARTH, W(1)), (UNDER, W(29)), (INTYP, W(41)), (STEP1, W(44)), (UTEST, W(251))

REAL INTYP
COMPLEX R, DR
LOGICAL UNDER
DO 1 I = 1,10
IF (REAL(DR(TS)), EQ, 0.) GOTO 5
STEP = -(REAL(R(1)) - EARTH+HS)/REAL(DR(TS))
STEP = SIGN(MIN(ABS(STEP),ABS(STEP)),STEP)
IF (ABS(REAL(R(1)) - EARTH+HS) LT .5E-4, AND, STEP LT .1.) GOTO 5

CALL PRINTR (8HOMING)
MODE = 1
RSTART = 1.
CALL RKAM

1 RSTART = 1.

ENTRY GRAZE
DO 2 I = 1,10
IF (REAL(DR(TS)), EQ, 0.) GOTO 5
STEP = -(REAL(R(TS)))/REAL(DR(TS))
STEP = SIGN(MIN(ABS(STEP),ABS(STEP)),STEP)
IF (ABS(STEP(1)) LE .1E-6, AND, STEP LT .1.) GOTO 5

CALL PRINTR (8HOMING)
MODE = 1
RSTART = 1.
CALL RKAM
RSTART = 1.
IF (REAL(R(1)) - EARTH+LT.0.) GOTO 4
IF ((REAL(R(1)) - EARTH+HS) *(ROLD - EARTH+HS) LT .0.) GOTO 3
2 CONTINUE
3 GO TO 5
4 NTEST = 0
5 GO TO 5
6 UNDER = *TRUE*
7 MODE = INTYP
8 STEP = STEPI
9 RETURN

END

******************************************************************************
** SUBROUTINE HASEL
**
** DIFFERENTIAL EQUATIONS FOR RAYPATH AND DOPPLER SHIFT
**
** $\frac{D\Theta}{D\lambda} = \frac{D\theta}{D\lambda}, \ldots \frac{D\Theta}{D\lambda} = \frac{D\theta}{D\lambda}$ **
** COMMON R(12), T, SP, DRT(12)/WW/1D(10), DUN, N(500)
** COMMON JENS/AHO
** COMMON/CONST/PI, DEGS, PI 02, PI 01, RAD
** COMMON $\gamma$ // U, U0, U, I, U, PUXT(3), PURPT, PUTHPT, PUPHPT
** COMMON $\gamma$ // C, PCPE, PCPHT, PCPPH, PCPT, TK, DTDZ, DTDZ
** COMMON/VISC10, BR, BRH, BOOTK, BOKH, BET4, 8X2, YB
** COMMON/VISUN, LA
** REAL OMEG, OMEG2, LA
** COMPLEX R, DRT, KSF, HKSQ, F, D, D2, D3, D4, BBOTK, BOKH, BOKX2, BDDW, DDQ, DCD, DDC
** J4, J1, J2, A, Z, H4, A, N, A, M, B, B3, B5, B6, B7, B8, F1, F2, F3, F4, F5, F6, F7, F8, F9
** DATA(W(67) = 3*)
** CALL ATMLD
** CALL TCMF
** CALL WIND
** R2 = REAL(R(2))
** R3 = REAL(R(3))
** SINT = SIN(R2)
** COST = COS(R2)
** I = (0, 0)
** C2 = 5.0
** G0 = 0.0006 0.617
** A = 3.1416 = 4
** G = G0 * (1.0 - A * (R1 - W(19)))
** W2 = G2 * H(321) **2 / 4 / G2
** W2 = G2 * H(321) **2 / G2
** WA = W2 * H(321) **2 / G2
** OMEG = W(3) * REAL((R4) **4 * UTH + R(5) * UTH + R(6) * UPH)
** OMEG2 = OMEG * OMEG
** KSQ = R(4) **2 + R(5) * R(6) + R(6) **2
** RK = REAL(R(4) **2 + R(5) **2 + REAL(R(5) **2 + REAL(R(6) **2)
** HKSQ = KSQ - R(4) **2 * R(4)
DENOM=2.*OMEG2-WA2-C2*WSQ
DIFF=(WA2-C2*WSQ)/DENOM*OMEG/C
IF(M(499),.GE.1,*) GOTO 1
DO 10 J=1,6
R(J)=REAL(R(J)),0.
10 CONTINUE
DRDT(1) = C2*R(4)*OMEG/DENOM+UR
DRDT(2) = (C2*R(5)*(OMEG2-WG2)/OMEG/DENOM+UTH)/R(1)
DRDT(3) = (C2*R(6)*(OMEG2-WG2)/OMEG/DENOM+UPH)/R(1)/SINTH
DRDT(4) = DIFF*PCPR-R(4)*PUPX(1,1)-R(5)*PUPX(2,1)-R(6)*PUPX(3,1)
1 +R(5)*DRDT(2)+R(6)*SINTH*DRDT(3)
DRDT(5) = (DIFF*PCPH-R(4)*PUPX(1,2)-R(5)*PUPX(2,2)-R(6)*PUPX(3,2)
J-R(5)*DRDT(1)+R(1)*R(6)*COSTH*DRDT(3))/R(1)
DRDT(6) = (DIFF*PCPH-R(4)*PUPX(1,3)-R(5)*PUPX(2,3)-R(6)*PUPX(3,3)-R
J(6)*SINTH*DRDT(1)-R(1)*R(6)*COSTH*DRDT(2))/R(1)/SINTH
C ***** DOPPLER SHIFT *****
DRDT(7)=R(4)*PURPT+R(5)*PUTHPT+R(6)*PUPHPT=DIFF*PCPT
DO 11 J=1,7
DRDT(J)=REAL(DRDT(J)),0.
11 CONTINUE
RETURN
1 A1=WA/C
E=R(4)
D=HKSQ+E**2
D2=0.*D
D3=0.*D
D4=0.*D
A2=I*E
A3=A1*A2
A4=0.-2.*A3
A5=D2=MG2/G
A6=A5/A1
IF(M(499),.EQ.2,*) GOTO 2
IF(M(499),.EQ.1.*AND.M(496),.EQ.0,*) GOTO 3
IF(M(498),.EQ.0.*AND.M(496),.EQ.0,*) GOTO 4
IF(M(498),.EQ.0.*AND.M(496),.EQ.1.*AND.R1,LT,6425,*) GOTO 5
GOTO 5
C ***** VISCOSITY AND ION DRAG *****
2 CALL DRAGON
CALL VISCOS
UN=UN/3
UN2=UN*UN
UN3=UN2*UN
B=BETA
B2=B*B
A7=2, -BBH
A6=-HKSQ+3XK2
A9=A8+2*XBDT*KBDK
B1=I*A1

DODT=5.*OMEG2**2-8.*I*OMEG2**3*(B+5.*UN*D)+3.*OMEG2*(33.*UN2*D2+B*
JUN)*(D*BBH-14.-A9)-B2-C2*A4)+2.*OMEG2*(36.*I*UN3*D3+3.*I*D*B*UN2*(
J0*(8.-3BH)+A9)+I*UN*(R2*(D*(4.-BBH)+A9))+6.*C2*D*A4)+I*BC*C2*(A7*A9-0*
J1*B2*(B*BR*KBDK+H*KBDK)+I*BDT*KBDK)

J2=8.*A2*UN*OMEG2**2-OMEG**3*(t-132.*E*UN2*D-2.*B*UN*E*(14.*J-5BBH)+BR*BDK)-2.*C2*(E-61))-OMEG2*(216.*A2*UN3*D2+6.*I*B*UN2*(E*
J2)*(8.-3BH)+A9)+BR*BDK)+6.*C2*(E+4.*BBH)+BR*BDK)+6.*C
J2*(15.*UN2*C2*D0=8.*I*(3.*E-61)+4.*HKSQ*B1)+6.*B*UN*C2*(E*(A7+
J6=*A2*UN*HKSQ+W2*2/2/DODW

DODT=2=((20.*E*R(5)*UN*OMEG2**2+2.*OMEG**3*(66.*UN2*R(5)+D*B*UN*(
J13*R(3)+1*B*BDT*K+2*C2*R(5))-OMEG2*(216.*I*R(5)+UN3*D2+2.*B*UN2*
J3*R(5)*(D*(15.-BBH)+A9))+2.*B*UN*E*(B*BDT*K2+2.*B*PH
J1*(BDT*K+2)+B*PH*(R(6)-BR*B1))/DODW/4

DODT=2=(DODT)+(-2.*OMEG)*(3.*E-61)+C2*UN2*R(5)+D(3.*D-4.*A3)+2.*B*UN*C2*
J3*R(5)*(D*(15.-BBH)+A9)+2.*B*PH*(R(6)+3.*D*BBH)+D*BDT*K+2.*B*PH
J1*(BDT*K+2)+B*PH*(R(6)-BR*B1))/DODW/4

DODT=3=((20.*E*R(6)+UN*OMEG2**2+2.*OMEG**3*(66.*UN2*R(6)+D*B*UN*(
J13*R(6)+1*B*BDT*K+2*C2*R(6))-OMEG2*(216.*I*R(6)+UN3*D2+2.*B*UN2*
J3*R(6)*(D*(15.-BBH)+A9))+2.*BBH)+D*BDT*K+2.*B*PH
J1*(BDT*K+2)+B*PH*(R(6)-BR*B1))/DODW/4

DO 6 K=1,3
CONTINUE

6 DODC=2C* (OMEG**3*(A-3)+OMEG2*(6.*I*UN*D-(D-A3)+I*B*(A9-B1*BR*BDK
J+(D-A3)+A7)+*OMEG{(3.*UN*D2*(D-A3)+3.*UN*D*(D-A3)+A7+A9-B1*BR*BD
JKH)+I*BDT*KBDK)+W2*HKSQ)+I*W2*(B*UN-3.*UN*D
J*KHK)))+2.*B2*C2
J*(BDT*K+2)+B*PH*(R(5)-BR*B1))/DODW/4

DO 1 A=2,C*(A2*OMEG**3+OMEG2*(6.*UN*D+B*(BR*BDK+E*A7))-OMEG**9.)
J*A2*UN2*D2+3.*IB*UN*D*(BR*DKH+E*A7)+I*B2*(BR*DKH+E*BR**2))
DDBMB=C*DFOMEG*HKSQ+I*(8*A6-2.3*UN*HKSQ*Q)
DDBCH=(DDDC-A1*DDDA-2.4*W62/C*DDDB2)/DDCW
DRO(T(4)=PCPH*DDCW-R(4)*PUPX(1,1)-R(5)*PUPX(2,1)-R(6)*PUPX(3,1)+R(5)
J*DRO(T(2)+R(6)*)SINTH*DRO(T(3)
DRO(T(5)=PCPH*DDCW-R(4)*PUPX(1,2)-R(5)*PUPX(2,2)-R(6)*PUPX(3,2)-R
J(DRO(T(1)+R(6)*)COSTH*DRO(T(3)))/R1
DRO(T(6)=PCPH*DDCW-R(4)*PUPX(1,3)-R(5)*PUPX(2,3)-R(6)*PUPX(3,3)-R
J(SINTH*DRO(T(1)-R(6)*)COSTH*DRO(T(2)))/R1/SINTH
DRO(T(7)=R(4)*PUPRT+R(5)*PUPHT+R(6)*PUPHT-DIFF*PCPT
RETURN
C ***** VISCOSITY AND THERMAL CONDUCTIVITY *****
3 CALL VISCOS
UN=UN/3,
UN2=UN*UN
Q=8.31432E-3/W(320)
A7=A*LA/4
A8=A7/UN+
A9=1.0*A7+3.3*UN
B1=A7+1.2*UN
B2=Q*DTDZ
B3=x*30
B4=0.3*TK
B5=TK/6
B6=2.*A1+E
B7=5.*A1-2.*HKSQ
B8=6.*D*HKSQ
B9=32.*W62*C2
F1=A5*A
F2=3*A4
F3=A1*B7+2.*A2*O
F4=7.*D-6.*HKSQ
F5=3.*D-4.*HKSQ
F6=A7*O
F7=0-A3
F8=3.*A4-A3
F9=A2/2
H1=3.*A3
H2=0-A3
H3=Q*DTDZ
H4=A5*H3*A2
DDBW5=OMEG2**2-4.*I7*OMEG**3*A8+3.*OMEG2*(-UN*92*A9-A2*B3-B4*A4
J=8+B1.*I*G*F3+3.*I0*D2*A6)+F1*D*(B6/4*I*B5*A4)))/36.*UN3*D4*A7+3.*UN
D2DT(7)=R(4)*PURPT+R(5)*PUTHPT+R(6)*PUPHPT-DIFF*PCPT
RETURN
C ****** THERMAL CONDUCTIVITY ******
4 CALL VISCOS
Q=8*314.152/9/(320)
A7=A5*LA/SQ
A8=Q+TDZ
A9=G+A8
B1=TK
B2=TK/2
B3=2.*I*A1-E
B4=G*A8+WG2*C2
B5=2.*I-2.*HKSQ
B6=A5*LA
B7=Q+MEG**3-A7+MEG2*A7+2.*MEG*(-(A2*A9-B1*A4-2.*G*A1-D*A6)
J/2.)*0.36+((3/3+I*B3*A4)
D2DT(1)=((2*A2*MEG**3*A7-OMEG2*(-I*A9-2.*B1*(E-I*A1)-E*A6)-OMEG*B
B7=2.*I*MEG**3*A7+OMEG2*(-(B3+B6)-2.*OMEG*B6*(B3/Q+2.*I*B2-(D-A3
J))-2.*J
D2DT(2)=(B7*R(5)/DDD+UTH)/R1
D2DT(3)=(B7*R(6)/DDD+UHP)/R1/SINH
DO 8 K=1,3
D2DT(4)=0.5XQX(REAL(D2DT(1)),0.0)
8 CONTINUE
DDD=2.*I*MEG**3*A7+OMEG2*(2*A1*(A-G-A2*B1)-1.5*A6*D)+OMEG*(2.*A1
J+2.*C2*WG2*KSQ)/C
J)*1/C
DDD2=I*MEG**3*A7/WG2-OMEG2*D/A6/2./WG2+OMEG2+B6/0/WG2*(B3/Q+I
J*2.*A4)+HKSQ*C2
DDD=DDD2-A1*DDD2A-2.*WG2/C*DDDNB2)/DDD
D2DT(4)=PCPR*DDCW*(OMEG2*(-A8*A4-A2*Q*DOTZ2)*I*OMEG*D*A7*A8*A4+G*H
J*KSQ*DOTZ2)/DDD+R(4)*PUPX(1,1)-R(5)*PUPX(2,2)-R(6)*PUPX(3,1)-R(5
J)*D2DT(2)-R(6)-SINTH*D2DT(3)
D2DT(5)=(PCPP*DDD+R(5)*PUPX(1,2)-R(5)*PUPX(2,2)-R(6)*PUPX(3,2)-R
J(5)*D2DT(1)-R(6)*SINTH*D2DT(3))/R1
D2DT(6)=(PCPP*DDD+R(5)*PUPX(1,3)-R(5)*PUPX(2,3)-R(6)*PUPX(3,3)-R
J(6)*SINTH*D2DT(1)-R(6)*SINTH*D2DT(2))/R1/SINH
D2DT(7)=R(4)*PURPT+R(5)*PUTHPT+R(6)*PUPHPT-DIFF*PCPT
RETURN
C ****** ION DRAG AND THERMAL CONDUCTIVITY ******
5 CALL VISCOS
CALL DRAGON
Please note the text contains a mix of symbols and notations that appear to be related to computer programming or mathematical expressions. Due to the nature of the content, the text is not naturally readable and may require specialized knowledge to interpret.
RETURN
END

SUBROUTINE RKAM
NUMERICAL INTEGRATION OF DIFFERENTIAL EQUATIONS

COMMON/CONST/PI,DEGS,PI2,PIT2,RAD
COMMON /SHARE/ NN,SPACE,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART,
COM, Y(12), T,STEP, DYDT(12)
DIMENSION DELY(4,12),BET(4),XV(5),FV(4,12),YUR(5,12),YUI(5,12)
COMPLEX Y,DYDT,FV,DELY,2,BET,J,EPS
DOUBLE PRECISION YUR,YUI
J=0,EQ.0) GO TO 2
LL=1
MH = 1
IF (MODE.EQ.1) MH = 4
ALPHA= T
EPS = 0.0
BET(1) = 0.5
BET(2) = 0.5
BET(3) = 0.5
BET(4) = 0.5
STEP = SPACE
R = 19.0/270.0
XV(MM) = T
IF (E1MIN.LE.0.0) E1MIN = E1MAX/55.
IF (FACT.LE.0.0) FACT = 0.5

CALL HASEL
DO I = 1,NN
FV(MM,I) = DYDT(I)
YUR(MM,I) = REAL(Y(I))
YUI(MM,I) = IMAG(Y(I))
RSTART = 0.
GOTO 3
2 IF(MODE.NE.1) GO TO 9
*** RUNGE KUTTA ***
3 DO K = 1,4
DO I = 1,NN
DELY(K,I) = STEP*FV(MM,I)
Z=YUR(MM,I)+J*YUI(MM,I)
Y(I) = Z+BET(K)*DELY(K,I)
Y(I) = BET(K)*STEP+XV(MM)
CALL HASEL
DO 5 I = 1,NN
5 FV(MM,I) = OYDT(I)
DO 6 I = 1,NN
DEL = (DELAY(1,I) + DELY(2,I) + 2*DELY(3,I) + DELY(4,I)) / 6.
YUR(MM+1,I) = YUR(MM,I) + REAL(DEL)
M4 = M4 + 1
XV(MM) = XV(MM-1) + STEP
DO 7 I = 1,NN
7 Y(I) = YUR(MM,I) + J*YUI(MM,I)
T = XV(MM)
CALL HASEL
IF (MODE.EQ.1) GO TO 15
DO 8 I = 1,NN
8 FV(MM,I) = OYDT(I)
IF (MM.LE.3) GOTO 3
** ADAMS MOULTON **
9 DO 10 I = 1,NN
9 DEL = STEP*55.*FV(4,I) - 59.*FV(3,I) + 37.*FV(2,I) - 9.*FV(1,I)) / 24.
Y(I) = YUR(4,I) + J*YUI(4,I) + DEL
10 DELY(1,I) = Y(I)
T = XV(4)*STEP
CALL HASEL
XV(5) = T
DO 11 I = 1,NN
11 DEL = STEP*19.*OYDT(I) + 19.*FV(4,I) - 5.*FV(3,I) + FV(2,I)) / 24.
YUR(5,I) = YUR(4,I) + REAL(DEL)
YUI(5,I) = YUI(4,I) + AIMAG(DEL)
11 Y(I) = YUR(5,I) + J*YUI(5,I)
CALL HASEL
IF (MODE.LE.2) GO TO 15
** ERROR ANALYSIS ****
C SSE = 0.
DO 12 I = 1,NN
12 EPS = EPS + Y(I) - DELY(1,I))
IF (MODE.EQ.3 .AND. CABS(Y(I)) .NE. 0.) EPS = EPS / Y(I)
EPSIL = CABS(EPS)
IF (SEE.EQ.1) EPSIL = SSE = EPSIL
12 CONTINUE
IF (F1 MAX .GT. SSE) GO TO 13
IF (ABS(STEP) .LE. E2MIN) GO TO 15
LL = I
MM = 1
STEP = STEP*FACT
13 GOTO 3
13 IF(LL.EQ.1.OR.SSE.GE.E1MIN.OR.E2MAX.LE.ABS(STEP)) GOTO 15
13 LL = 2
13 MM = 3
13 XV(2) = XV(3)
13 XV(3) = XV(5)
14 DO 14 I = 1,NN
14 FV(2,I) = FV(3,I)
14 FV(3,I) = DYDT(I)
14 YUR(2,I) = YUR(3,I)
14 YUR(3,I) = YUR(5,I)
15 STEP = STEP*STEP
15 GOTO 3
C
15 *** EXIT ROUTINE *****
C
15 LL = 2
15 MM = 4
16 DO 16 K = 1,3
16 XV(K) = XV(K+1)
16 DO 16 I = 1,NN
16 FV(K,I) = FV(K+1,I)
16 YUR(K,I) = YUR(K+1,I)
16 YUI(K,I) = YUI(K+1,I)
17 XV(4) = XV(5)
17 DO 17 I = 1,NN
17 FV(4,I) = DYDT(I)
17 YUR(4,I) = YUR(5,I)
17 YUI(4,I) = YUI(5,I)
18 IF(MODE.LE.2) RETURN
18 E = ABS(XV(4)-ALPHA)
18 IF(E.LE.EPH) GOTO 9
18 EPM = E
18 RETURN
C
C SUBROUTINE PRINT(NEW)
C CONTROLS PRINTOUT OF RAYPATH QUANTITIES
C DIMENSION G(3,3),G1(3,3)
C COMMON /SHER/ R,STEP,MODE,E1MAX,E1MIN,E2MAX,E2MIN,FACT,RSTART,
C SSE
C COMMON R (12),I,STP,DRDT (12)/MW/ID(10),DUM,W(500)
C COMMON/CC/C,CPPT,PCPT,PCPPT,PCPP,T,K,TDZ,DTDZ
C COMMON /U1/U2,U3,UPH,UPHX(3,3),PURPT,PUTHT,PUTHP
C COMMON/CONST/P1,DEGS,PIDZ,PITZ,RAD
C COMMON/PL/AZDEV
EQUIVALENCE (F,W(3)), (LON,W(14)), (LAT,W(16)), (BETA,W(17)),
1 (A1,W(11)), (EARTH,W(13)), (XMT, W(20)), (LINES,W(181)), (NUTEST,
2 W(25)), (LON,W(253)), (APHT,W(270)), (RAYBEG,W(292)), (PNW, W(300)), (RANGE,W(301))
REAL LON, LAT, SQK
COMPLEX R, RDRT
AZDEC=0
THETA=REAL(R(2))
PHI=REAL(R(3))
R1=REAL(R(4))
C
IF (PNW.EQ.0) GO TO 1
*** NEW W ARRAY - INITIALISE ***
PNW = 0
ABSORB = 0
*** MATRIX TO CONVERT TO RECTANGULAR COORDINATE SYSTEM ***
SPL = `-SIN(LON)
CPL = COS(LON)
SL = SIN(LAT)
CL = COS(LAT)
G(1,1) = CPL*CL
G(1,2) = SPL*CL
G(1,3) = `-SL*CPL
G(2,1) = `-SPL*CL
G(2,2) = CPL
G(2,3) = SL*SPL
G(3,1) = SL
G(3,2) = CPL
G(3,3) = CL
D = G(1,1)*G(2,2)*G(3,3) + G(1,2)*G(2,3)*G(3,1) + G(2,1)*G(3,2)*G(1,3) - 2*G(1,1)*G(2,2)*G(3,3) - G(1,2)*G(2,3)*G(3,1) + G(2,1)*G(3,2)*G(1,3)
1 = G(1,1)*G(2,2)*G(3,3) - G(1,2)*G(2,3)*G(3,1)
2 = G(1,1)*G(2,2)*G(3,3) - G(1,2)*G(2,3)*G(3,1)
3 = G(1,1)*G(2,2)*G(3,3) - G(1,2)*G(2,3)*G(3,1)
R0 = EARTH + XMT
C
** XMT LOCATION IN EARTH CENTRED COORDINATES
XR = R0*G(1,1)
YR = R0*G(2,1)
ZR = R0*G(3,1)
COSTHR = G(3,1)
SINTHR = SIN(ACOS(COSTHR))
PHIR = ATAN2(YR,XR)
ALPH = ATAN2(G(3,2),G(3,3))
1
HKS1 = REAL(R(5)**2 + REAL(R(6))**2
KSQ = HKSQ*REAL(R(4))**2
UDOTK = REAL(U*R(4) + UTH*R(5) + UPH*R(6))
COMEg2 = (W(3) - UDOTK)**2
C2 = G*C
G0 = 0.9880617
A = 3.1416-4
GG = 5.0*(1.0 - A*(R1-W(19)))
CALL AIMODL
WA2 = G**2*(W(321)**2/4.0)**2/C2/COMEg2
WG2 = G**2*(W(321)-1.0)/C2/COMEg2
V = 1.0 - WA2 - C2*KSQ/COMEg2 + WG2*G2*HKSQ/COMEg2
V = SSE
H = R1-W(19)
SINTH = SIN(THETA)
COSTH = COS(THETA)
A = 1.
IF(REAL(DRDT(1))*REAL(R(4)) < 0.) A = -1.
ANGDEG = A*ATAN(REAL(R(4))/SORT(REAL(R(5)))**2 + REAL(R(6))**2).*DEGS
TIME = T
IF(HKSQ < E-0.0) VPHERE = 0.
IF(HKSQ < E-0.1) VPHERE = SORT(COMEg2/HKSQ)*1.0E3
TAU = PIT2/SORT(COMEg2)
DENOM = COMEG2*HKSQ*C2
POLARR = REAL(R(4))*SORT(HKSQ)*C2/DENOM
POLARI = -0.3*DEDS*SORT(HKSQ)/DENOM
POLMAG = SORT(POLARR**2 + POLARI**2)
IF(POLARR < E-0.0) POLANG = ATAN(POLARI/POLARR)*DEGS
DOPPLER = REAL(R(7))/PIT2
IF(NWHY = EQ.8)XHMR  ) GOTO 10
A1 = REAL(DRDT(4))/REAL(R(4))**2
A2 = REAL(DRDT(5))/REAL(R(5))**2
A3 = REAL(DRDT(6))/REAL(R(6))**2
VG = 0.3*E3*ATAN1.0)*SORT(A1**2 + A2**2 + A3**2)
DOPPLER = VG
10
WADC=N(321)*GG/2.0/C2
ABSORB =-1.0/AIMAG(R(4))
IF(W(499) = E=8.) ABSORB = 1.0/WADC
IF(RAYBEQ.EQ.1.) GO TO 2
C
PRINT 6
CARTESIAN COORDINATES OF RAY POINT RELATIVE TO TX
2 XP=R1*SINTH*COS(PHI)-XR
YP=R1*SINTH*SIN(PHI)-YR
ZP=R1*COSTH-ZR
EPS = XP#G1(1,1)+YP#G1(1,2)+ZP#G1(1,3)
ETA = XP#G1(2,1)+YP#G1(2,2)+ZP#G1(2,3)
ZETA = XP#G1(3,1)+YP#G1(3,2)+ZP#G1(3,3)
RCE2 = ETA*ETA+ZETA*ZETA
RCE = SQRT(RCE2)
RANGE = EARTH*ATAN2(RCE,EARTH+EPS)
SR = SQRT(RCE2+EPS*EPS)
IF (SR#GE.1.E-6) GO TO 3
PRINT 7,V,NWHY,H,RANGE,TIME,ANGDEG,TAU,POLMAG,POLANG,VPHASE,
1 DOPPLER,ABSORB
GO TO 5
3 ANGE = ATAN2(EPS,RCE)
EL = ANGE-DEGS
IF (RCE#LE.3.E-6) GO TO 4
PRINT 8,V,NWHY,H,RANGE,TIME,EL,ANGDEG,TAU,POLMAG,POLANG,VPHASE,
1 DOPPLER,ABSORB
GO TO 5
4 ANGA = ATAN2(ETA,ZETA)
ANA = ANGA-ALPH
SINANA = SIN(ANA)
SINPHI = SINANA*SINHR/SINTH
COSPHI = COS(ANA)*COS(THI-THF)*COSTH
AZA=180.-AMOD(540.-ATAN2(SINPHI,COSPHI)-ATAN2(REAL(R(6)),REAL(R(5
1)))*DEGS,360.)
AZDEV = 180.-AMOD(540.- (AZA-ANGA)*DEGS,360.)
PRINT 9,V,NWHY,H,RANGE,TIME,AZDEV,AZA,EL,ANGDEG,TAU,POLMAG,POLANG,
1 VPHASE,DOPPLER,ABSORB
5 LINES = LINES+1
RETURN

C
6 FORMAT(1X,7H/2Z/38X,6HTRAVEL,6X9HELEVATION,9X9HELEVATION,
1 5X,9HTRAVGIC,3X,12HPOLARIZATION,5X,5H/2H/3X,5H/2H/3X,7H/2H/3X,5H/2H/3X
2,7H/2H/3X,5H/2H/3X,7H/2H/3X,5H/2H/3X,5H/2H/3X,7H/2H/3X,5H/2H/3X
3,4H/2H/3X,5H/2H/3X,4H/2H/3X,5H/2H/3X,4H/2H/3X,5H/2H/3X,4H/2H/3X
4,5H/2H/3X,5H/2H/3X,5H/2H/3X,5H/2H/3X,5H/2H/3X,5H/2H/3X,5H/2H/3X
+ 3H/2H/3X,5H/2H/3X,3H/2H/3X,5H/2H/3X,3H/2H/3X,5H/2H/3X,3H/2H/3X
5,5X,3H/2X,7X,3H/2X,5X,3H/2X,7X,3H/2X
6 5X,3H/2X,7X,3H/2X)
7 FORMAT(1X,1E9.1X,A3,3F9.2,27X,6F9.3,F8.2)
8 FORMAT(1X,1E9.1X,A3,3F9.2,18X,7F9.3,F8.2)
END
SUBROUTINE ATMOS
C THIS ROUTINE SETS UP VARIOUS AMBIENT ATMOSPHERIC PARAMETERS
COMMON R(12),T/WM,1D(10),DUM,W(500)
COMPLEX R
DIMENSION Z(41),G(41),WM(41)
DATA G/10*1.41.42*1.431.441.451.462*1.471.481.491.5
J/1.511.5211.5312*1.541.551.561.571.581.591.61.61.621.63.
J1/1.641.651.661.671.681.691.71.721.731.741.751.761.771.78.
J2/1.791.81.821.831.841.851.861.871.881.891.91.921.931.941.95.
J3/1.961.971.981.991.001.001.011.021.031.041.051.061.071.081.09.
J6/1.101.111.121.131.141.151.161.171.181.191.201.211.221.231.24.
J16/1.701.711.721.731.741.751.761.771.781.791.801.811.821.831.84.
Z(0)=0.
Z(1)=25.
Z(J)=300.*J.
1 CONTINUE
DO 2 J=1,10
Z(J+1)=300.+23.*J.
2 CONTINUE
H=REAL(R(1))-W(19)
IF(H.TEN.11) GOTO 3
IF(H.GE.11. AND H.LE.25.) GOTO 4
IF(H.GE.25. AND H.LE.30.) GOTO 5
IF(H.GE.30. AND H.LE.500.) GOTO 6
IF(H.GE.500.) GOTO 7
3 W(320)=W(1)+H/11.**(W(2)-W(1))
W(321)=G(1)+H/11.*G(2)-G(1)
RETURN
4 W(320)=W(2)+(H-11.)/14.**(W(3)-W(2))
W(321)=G(2)+(H-11.)/14.*G(3)-G(2)
RETURN
5 W(320)=W(3)+(H-25.)/5.**(W(4)-W(3))
W(321)=G(3)+(H-25.)/5.*G(4)-G(3)
RETURN
6 J=INT(H/10.)*1+
W(320)=W(J)+(H-Z(J))/10.**(W(J+1)-W(J))
W(321)=G(J)+(H-Z(J))/10.*G(J+1)-G(J)
RETURN
7 J=INT(H/20.)*16
W(320)=W(J)+(H-Z(J))/20.**(W(J+1)-W(J))
W(321)=G(J)+(H-Z(J))/20.*G(J+1)-G(J)
RETURN
END

SUBROUTINE DRAGON

C ***** CALCULATES HYDROMAGNETIC LOSSES IN THE ATMOSPHERE *****

C

BY USING AN EARTH CENTRED DIPOLE MAGNETIC FIELD

COMMON/DENS/RHO

COMMON R(12), TSTEP, ORD(12), WW/I0(10), DUM, W(500)

COMMON/CONST, PI, DEG, PI02, PI2, RAD

COMMON/C/C, PCPR, PCPT, PCPTM, PCPTL, RK, DTZ, DTOZ

COMMON/XION, BR, BTH, BMPH, BDKH, BETA, BK2, BBH

COMPLEX R, ORD, BDKK, BDKH, BK2, IJ

DIMENSION AT(10), B(3), C(2), D(3)

IJ = (0, 1)
DATA A/1/0.31500064, -39.6404341301, 0, 85360906766, -6.10061175E-3, -3
1.40623235E-6, 4.393065761E-7, 1, 350755196E-9, -3.61575256E-11, 1, 2.1273114
2.3E-13, B/1, 1.92393660241, -1.31989865E-2, -1.88508573E-5, 0, 1.06600428E-
36, -2.78126547E-93, 2.71228347E-115, 0.04820554E-14, 1, 1.98974656E-16, -2.1
4.4633216E-18, C/-1, 4.151538646153, 5, 23076923E-37, 0.0786269259171, 1.2
51979631.133993, -1.212054258E-2, 1.87884478E-5, 1, 1.428967E-8, -5, 2.6161805
6E-11, 5, 82316938E-14, 2, 32995353E-16, -1.69730651E-19/

R2=REAL(R(2))
R3=REAL(R(3))
H=REAL(R(1)) - H(19)
IF(H(496) .EQ. 0.) GOTO 0

6 BR=BTH=3PH=BETA=BBH=0.*
BDK=BDKH=3BK2=(0., 0., 0.)
RETURN

1 CALL DENS

T0=PI2/7.381*RAD
P0=31.4*RAD
SINH=SIN(R2)
COTH=COS(R2)
SINT=SIN(T0)
COST=COS(T0)
SINBH=SIN(R3-P0)
COSH=CO T(R3-P0)
Q=0.*
IF(H .GE. 118.) GOTO 2
DO 10 I=1, 9
10 Q=Q+AI(I)*H**J
10 CONTINUE

SIGMA=10.*Q*1.E5
GOTO 5

2 IF(H .GE. 213.) GOTO 3
DO 20 I=1,9
   J=I-1
   Q=Q+3*(I)*H**J
20 CONTINUE
SIGMA=10.*Q**1.**E5
GOTO 5
3 IF(H.**G.E.252.) GOTO 4
DO 30 I=1,2
   J=I-1
   Q=Q+C*(I)*H**J
30 CONTINUE
SIGMA=10.*Q**1.**E5
GOTO 5
4 DO 40 I=1,9
   J=I-1
   Q=Q+D*(I)*H**J
40 CONTINUE
SIGMA=10.*Q**1.**E5
5 T**3=-2.5
B0=SQRT(3.*COS*INT0+SINT0*COST0*COSPH)**2+1.
BETA=SIGMA/RH0**T1**1
B0=2.*COS*INT0+SINT0*COST0*COSPH)/B0
B0=2.*COST0+SINT0*COST0*COSPH)/B0
BPH=COST0*SINPH/B0
B0TK=4.*BTH**R(5)+BPH**R(6)
BKH=8.*BTH**R(6)
BKH2=(BTH**R(6)-BPH**R(6))**2
RETURN
END
C *** CALCULATES KINEMATIC VISCOSITY AS A FUNCTION OF HEIGHT ***
COMMON/XVIS/UN,LA
COMMON/COST0,PCH,PCPHT,PCPPH,PCPT,TK,DTDZ,DTDZ2
COMMON/DENS0,RHO
COMMON/R12,TSTEP,DRDT(12)/HM/ID(10),DUM,W(500)
COMMON/R,DRDT
DIMENSION Z(41),MU(41),F(41)
REAL MU,LA,W
DATA 70,-25,30,-49,50,-69,70,-89,90,-100,-110,-120,-130,-
140,-150,-160,-170,-180,-190,-200,-210,-220,-230,-240,-250,-260,-
270,-280,-290,-300,-310,-320,-330,-340,-350,-360,-370,-380,-390,-
400,-420,-440,-460,-480,-50
30,-MU/61,-180,-225,-144,-60,-23,225,-22,5,-12,01,643,214,056,-
128,-03,017,-032,-034,035,-0378,-0394,-0408,-042,-043,-044,-
045,-046,-046,-046,-047,-047,-047,-047,-047,-047,-047,-047,-047,-047,
C *** CALCULATES DENSITY PROFILES AS A FUNCTION OF HEIGHT ***

COMMON R(12),TSTEP,DRDT(12),/N/W/ID(10),DUM,W(508)
COMMON:DENS/RHO

COMPLEX R

DIMENSION A(9)

H=REAL(R(1))-W(19)

IF(H.LE.0) H=-0.1

A(1)=20.51443734414
A(2) = -5.77581396E-2
A(3) = -2.501558286E-3
A(4) = 2.46967087E-5
A(5) = -1.04986535E-7
A(6) = 2.40239968E-10
A(7) = 3.1419355E-13
A(8) = 2.16048719E-16
A(9) = -6.13262986E-20
Q=0  
DO 1 I=1,9
Q=Q*A(I)  
1 CONTINUE
RHO=EXP(Q)
RETURN
END

SUBROUTINE TLINEAR
COMMON/MODELS/MODEL(4)
COMMON R(6)/WW/ID(10)/UM,W(500)
COMMON/GC/G,PCPR,PCPT,PCPH,PCPT,TK,DTOZ,DTOZ2
EQUIVALENCE(TGND,W(340)),(A,W(341)),(MW,W(320))
REAL *W
COMPLEX R
DATA (MODEL(1) = 6HTLINEAR),(TGND = 0.), (A = 0.)
ENTRY TEMP
H=REAL(R(1)) - W(19)
TK=TGND + A*H
DTOZ=A
DTOZ2=0
C=SQRT(TK/MW)*0.108
POPK = A/2./MW/C*108.*108.E-6
RETURN
END

SUBROUTINE TANTH
DIMENSION D(5),Z(4),B(4)
COMMON/GC/G,PCPR,PCPT,PCPH,PCPT,TK,DTOZ,DTOZ2
COMMON R(6)/WW/ID(10)/UM,W(500)
COMMON/MODELS/MODEL(4)
EQUIVALENCE(MW,W(320))
REAL *W
COMPLEX R
DATA (O(1)= -6.5), (O(2)= 3.5), (O(3)= -3.5), (O(4)= 18.5), (O(5)= 1.),
1 (Z(1)= 15.), (Z(2)= 52.), (Z(3)= 95.), (Z(4)= 165.), (B(1)= 10.), (B(2)= 7.5
2 ) (B(3)= 30.), (B(4)= 50.), (TO= 38.),
3 (SUM=0.), (MODEL(1)= 6HTTANNH)
COSH(X) = (EXP(X) + 1.0 / (EXP(X))) / 2.
ENTRY TEMP
H = REAL(R(1)) - W(19)
SUM = 0.
DO 1 I = 1, 4
SUM = SUM + B(I) * (D(I+1) - D(I)) / 2.0 * ALOG(COSH((H-Z(I))/B(I)) / COSH(Z(I)) / B(I)))
1 CONTINUE
TK = TO + SUM + D(I) * H/2.
SUM = 0.
DO 2 I = 1, 4
SUM = SUM + (D(I+1) - D(I)) / 2.0 * TANH((H-Z(I))/B(I)))
2 CONTINUE
DT0Z = D(I) / 2.0 + SUM
DT0Z2 = 0.
DO 3 I = 1, 4
DT0Z2 = DT0Z2 + (D(I+1) - D(I)) / 2.0 * B(I) / (COSH((H-Z(I))/B(I)))**2)
3 CONTINUE
G = SQRT(TK/MW) * 0.108
Q = 0.108 * 100.0 / 2.0 / C / MW
PCPR = DT0Z * Q * 1.0 * E - 6
RETURN
END
SUBROUTINE TC1965
COMMON/CC/C, PCPR, PCPRH, PCPPH, PCPT, TK, DT0Z, DT0Z2
COMMON R(6), MW/ID(I), DHM, W(500)
COMMON/MODELS/MODEL(4)
EQUIVALENCE (MW, W(320))
REAL MW
COMPLEX R
DIMENSION TT(94)
ENTRY TEMP
DO 5 I = 1, 94
TT(I) = W(430) + I
5 CONTINUE
H = REAL(R(1)) - W(19)
IF (H <= 0.0) H = 0.
IF (H <= 120.0) GOTO 1
IF (H <= 300.0) GOTO 2
IF (H <= 600.0) GOTO 3
IF (H > 600.0) TK = TT(94)
A = 0.
GOTO 4
1 II = INT(H/2.0)
TK=TT(II+1)+(TT(II+2)-TT(II+1))*(H-2.*II)/2.
D02Z=TT(II+2)-TT(II+1)/2.*
IF(JJ.EQ.0) D02Z=(TT(II)-TT(II))/4.*
IF(JJ.GT.0) D02Z=(TT(II+2)-2.*TT(II+1)+TT(II))/4.*
A=(TT(II+2)-TT(II+1))/2.*0
GOTO 4
2 JI=INT(H/10.*0)
TK=TT(JJ+49)+(TT(JJ+50)-TT(JJ+49))*H-10.*JJ)/10.*
D02Z=(TT(JJ+50)-TT(JJ+49))/10.*
IF(JJ.EQ.0) D02Z=(TT(JJ+50)-TT(JJ+49))/10.*
IF(JJ.GT.0) D02Z=(TT(JJ+50)-2.*TT(JJ+49)+TT(JJ+48))/100.*
A=(TT(JJ+50)-TT(JJ+49))/10.*0
GOTO 4
3 KK=INT(H/20.*0)
TK=TT(KK+64)+(TT(KK+65)-TT(KK+64))*H-20.*KK)/20.*
D02Z=(TT(KK+65)-TT(KK+64))/20.*
IF(KK.EQ.0) D02Z=(TT(KK+65)-TT(KK+64))/400.*
IF(KK.GT.0) D02Z=(TT(KK+65)-2.*TT(KK+64)+TT(KK+63))/400.*
A=(TT(KK+65)-TT(KK+64))/20.*0
4 C=SQR(TK/MW)*0.108
PCPR=A/2.0/MW/C*0.108*0.108
RETURN
END

******************************************************************************
C SUBROUTINE WCONST
COMMON MODELS/MODEL(4)
COMMON R(6)/HWID(10) DUM/W(500)
COMMON / DUM/UR,UTHI,UPH,PUPX(3,3),PURPT,PUTHPT,PUPHTPT
EQUIVALENCE (UGRAD,W(370)),(UTHO,W(371)),(UPHO,W(372)),(VGRAD,W(371))
15)/WGRAD,W(376))
COMPLEX R
DATA (MODEL(2) = 6HWCONST)
ENTRY WIND
H=REAL(R(1))=W(19)
UR=UGRAD*1.E-3
UTH=UTHO+VGRAD*H)*1.E-3
UPH=(UPHO+WGRAD*H)*1.E-3
PUPX(1,1)=UGRAD*1.E-3
PUPX(2,1)=VGRAD*1.E-3
PUPX(3,1)=WGRAD*1.E-3
RETURN
C SUBROUTINE WDATA
C THIS SUBROUTINE GIVES DATA WIND MODELS BASED ON THE DATA OF BALACHANDR
COMMON/MODELS/MODEL(4)
COMMON R(10), DUM, W(500)
COMMON /R/ U/R, UTH, UPH, PUPX(3, 3), PURPT, PUTHPT, PUPHPT
COMPLEX R
DIMENSION W(67, 2)
DATA W(5*10, 90) = [List of complex numbers]
J2(-5), J100, J100, J100, J100, J100, J100, J100
J = [List of complex numbers]
DATA (MODEL(2) = 6H, DATA)
ENTRY WIND
H = REAL(R(1)) - W(19)
I = 1
IF (H(370) .GT. 0.) I = 2
IF (H .LE. 90.) GOTO 1
IF (H .LT. 30.) GOTO 2
UTH = W(67, I) * 1. E-3
RETURN
1 J = INT(W(2/4)
UTH = (W(J+1, I) + (W(J+2, I) - W(J+1, I)) * (H - 2 * J) / 2.) * 1. E-3
PUPX(2, 1) = (W(J+2, I) - W(J+1, I)) / 2. * 1. E-3
RETURN
2 J = INT(W(1/4)) + 36
UTH = (W(J+1, I) + (W(J+2, I) - W(J+1, I)) * (H - 10 * J + 360.) / 10.) * 1. E-3
PUPX(2, 1) = (W(J+2, I) - W(J+1, I)) / 10. * 1. E-3
RETURN
END
SUBROUTINE WGAUSS
COMMON /R/ U/R, UTH, UPH, PUPX(3, 3), PURPT, PUTHPT, PUPHPT
COMMON R(10), W(500)
COMMON MODELS/MODEL(4)
EQUVALENCE (UPHR, W(381)), (UTH, W(382)), (UPH, W(383)), (PH, W(384)),
1 (TH, W(385)), (TH0, W(386)), (PH0, W(387))
COMPLEX R
DIMENSION W(385)
DATA (MODEL(2) = 6H, WGAUSS)
ENTRY WIND
DO 13 I = 1, 3
Y(I) = REAL(R(I))
13 CONTINUE
H = Y(1) - W(19)
IF(WH*NE.0) GOTO 1
EXR=1.
GOTO 2
1 EXR=EXP(-(((H-H)/WH)**2))
2 IF(WTH*NE.0) GOTO 3
EXTH=1.
GOTO 4
3 EXTH=EXP(-(((THO-Y(2))/WTH)**2))
4 IF(WPH*NE.0) GOTO 5
EXPH=1.
GOTO 6
5 EXPH=EXP(-(((PHO-Y(3))/WPH)**2))
6 UPH=UPHO*EXR*EXTH*EXPH*1.E-3
   IF(WH*NE.0) GOTO 7
   PUPX(3,1)=0.
   GOTO 8
7 PUPX(3,1)=-2.*UPH*(H-HO)/WH/WH
8 IF(WTH*NE.0) GOTO 9
   PUPX(3,2)=0.
   GOTO 10
9 PUPX(3,2)=-2.*UPH*(Y(2)-THO)/WTH/WTH+UPH
10 IF(WPH*NE.0) GOTO 11
   PUPX(3,3)=0.
   GOTO 12
11 PUPX(3,3)=-2.*UPH*(Y(3)-PHO)/WPH/WPH
12 RETURN
END

******************************************************************************
SUBROUTINE VVORTEX
COMMON/MODELS/MODEL(4)
COMMON/R(5)/WM,WD,1D(10),DUM,W(500)
      EQUIVALENCE (RO,W(331)),(THO,W(332)),(PHO,W(333)),(WR,W(334)),
1 (UO,W(335)),(RE,W(19))
C
COMPLEX R
DATA (MODEL(2) = 6HVVORTEX)
ENTRY WIND
DTH=REAL(R(2))-THO
DPH=REAL(R(3))-PHO
RAD = RE*SQR(DTH*DTH+DPH*DPH)
A = 1.397
B = -1.26
EXB = EXP((B*RAD*RAD/(RO*RO))
   DUM = A*RE*UO*RO/(RAD*RAD)**1.E-3
   DUX = (1.-EXB)/RAD*RAD*3.*EXB/(RO*RO)
C
UTH = -DUM*(1.0-EXB)*DPH
UPH = DUM * (1.0-EXB)*DTH
PUPX (2,2) = 2.*DUM*RE*RE*DTH*DPH/RAD*DUX
PUPX (3,3) = PUPX (2,2)
PUPX (3,4) = 2.*DTH*DTH*DUM*RE*RE/RAD*DUX+DUM*(1.0-EXB)
PUPX (2,3) = 2.0*DPH*DPH*DUM*RE*RE/RAD*DUX-DUM*(1.0-EXB)
RETURN
END

******************************************************************************************

SUBROUTINE ULOGZ
COMMON /MODELS/MODEL(4)
COMMON R(6)/WWID(10), DUM,W(500)
COMMON /UU/UR,UTH,UPH,PUPX(3,3),PURPT,PUTHPT,PUTHPD
EQUIVALENCE (UU,W(372))
COMPLEX R
DATA (MODEL(2)=6HULOGZ)
ENTRY WIND
H=REAL(R(1))=W(19)
IF (H .LE. 0.) H=0.
UPH = UU/ALOG(2.)*ALOG(H+1.)*1.0*E-3
PUPX (3,1) = UU/ALOG(2.)/(H+1.)*1.0*E-3
RETURN
END

******************************************************************************************

SUBROUTINE WTIDE
COMMON /MODELS/MODEL(4)
COMMON /CONST/PIT2,PIT2,PIT2,RAD
COMMON R(6)/WWID(10), DUM,W(500)
COMMON /UU/UR,UTH,UPH,PUPX(3,3),PURPT,PUTHPT,PUTHPD
EQUIVALENCE (UPH0,W(390)),(UTH0,W(391)),(LAMZ,W(392)),(TP,W(393)),
1(TAU,W(394))
COMPLEX R
REAL LAMZ
DATA (MODEL(2)=6HWTIDE)
ENTRY WIND
H=REAL(R(1))=W(19)
ARG=PIT2*(W/LAMZ+TP)
UPH=UPH0*COS(ARG)*1.0*E-3
UTH=UTH0*SIN(ARG)*1.0*E-3
Q=PIT2/LAMZ
PUPX (3,1) = -SIN(ARG)*Q*UTH0*1.0*E-3
PUPX (2,1) = COS(ARG)*Q*UTH0*1.0*E-3
S=PIT2/TAU
PUTHPT=CO3(ARG)*S*UTH0*1.0*E-3
PUTHPD=-SIN(ARG)*S*UPH0*1.0*E-3

******************************************************************************************

C
RETURN
END

***************
**************

SUBROUTINE 42TIDE

CALCULATES A WIND PROFILE CONSISTING OF A CONSTANT SHEAR

AND A TIDAL COMPONENT

COMMON/MODELS/MODEL(4)
COMMON/CONST/P1,DEGS,P102,PIT2,RAD
COMMON R(5),W(500),DUM,W(500)
COMMON/U(1),UTH,UPH,PUPX(3,3),PURP,PUPMTP,PUPHTP
EQUIVALENCE (UGRAD,W(371)),(UTHO,W(371)),(UPHO,W(372)),(VGRAD,W(371))
REAL LAMZ
COMPLEX R
DATA (MODEL(2)=64W2TIDE)
ENTRY WIND
H=REAL(31)-W(19)
ARG=PIT2*(H/LAMZ+TP)
Q=PIT2/LAMZ
S=PIT2/TAU
UR=UGRAD*H*1.E-3
UTH=(UTHO+VGRAD*H*1.E-3)*(1.+SIN(ARG))
UPH=(UPHO+VGRAD*H*1.E-3)*(1.+COS(ARG))
PUPX(1,1)=UGRAD*H*1.E-3
PUPX(2,1)=VGRAD*H*1.E-3*(1.+SIN(ARG))*Q*COS(ARG)+UTHO+VGRAD*H*1.E-3
PUPX(3,1)=UGRAD*H*1.E-3*(1.+COS(ARG))=S*SIN(ARG)*(UPHO+VGRAD*H*1.E-3)
PUPHTP=S*COS(ARG)*((UTHO+VGRAD*H*1.E-3)
\+PUPHT=S*SIN(ARG)*(UPHO+VGRAD*H*1.E-3)
RETURN
END

SUBROUTINE RAYPLT

COMMON W(500),DUM,W(500)
ENTRY ENDPPLT
W(272)=0.
RETURN
END

SUBROUTINE RAYPLT

W(272)=1. PLOTS PROJECTION OF RAYPATH ON VERTICAL PLANE
W(273)=2. PLOTS PROJECTION OF RAYPATH ON GROUND
COMMON/PLT/PLT,XR,YB,YT,RESET
COMMON/PLT/PLT
COMMON R(5),W(500)
EQUIVALENCE (PLT,3(272)),(RNEW,W(291)),(RAYBE,W(292))
COMPLEX R
DIMENSION Z(6)
DO 2 I=1,6
Z(I)=REAL(R(I))
2 CONTINUE
IF(RNEW.EQ.0.) GOTO 1
RNEW=0.
RESET=1
1 RADS=ATAN(1.)/45.
RANGE=W(301)
NEW=0.
IF(RAYBEG.EQ.1.) NEW=1
IF(PLT.EQ.2.) GOTO 3
XL=W(274)
XR=W(276)
YB=W(287)
YT=W(303)
CALL PLOT(RANGE,Z(1)-W(19),NEW)
RETURN
3 YB=W(275)
YT=W(277)
XL=W(274)
XR=W(276)
X=W(301)*SIN(AZDEV*RADS)
Y=W(301)*COS(AZDEV*RADS)
CALL PLOT(X,Y,NEW)
RETURN
C DRAW AXES AND CALL FOR LABELING AND TERMINATION OF THIS PLOT
ENTRY ENDPLT
CALL PLTEND
RETURN
END
SUBROUTINE PLOT(X,Y,NEW)
COMMON/PLT/XMINO,XMAXO,YMNO,YMAXO,RESET
COMMON/WW/ ID(10),WO,W(500)
EQUIVALENCE(PLT,W(272))
DATA(INITIAL)=1
C INITIALIZE LIBRARY PLOTTING ROUTINES
IF(INITIAL.EQ.0) GOTO 1
INITIAL=0
CALL GPSTOP(25)
CALL PAPER(1)
CALL DENSIT(2)
C COMPUTE SCALE FACTORS
1 IF(RESET.EQ.0.) GOTO 5
RESET=0.
CALL PSPACE(1,5,5,9)
CALL MAP(XMINO,XMAXO,YMINO,YMAXO)

C

START A NEW LINE

5 IF(NEW.EQ.0) GOTO 10
CALL POSITN(X,Y)
RETURN

10 CALL JOIN(X,Y)
RETURN

C

TERMITE THE CURRENT PLOT
ENTRY PLTEND
CALL LABELT
DX=100*INT((W(276)-W(274))/500.)
CALL SCALSI(DX,50.)
CALL FRAME
RETURN

END

SUBROUTINE LABPLT
LABEL THE CURRENT PLOT
DIMENSION IHEAD(8),TYPE(3),LABEL(8)
COMMON/WW/ID(10),NO,W(530)
EQUIVALENCE (F,W(3)),(AZ,W(18)),(EARTH,W(19)),(PLT,W(272))
DATA(DEGS=57.295779513),(TYPE=8HGRAVITY ,8H ,8HACOUSTIC)
NTYP=W(305)+2,
ENCOD(80,10,1HEAD) ID

10 FORMAT(10A8)
CALL CRTMAG(7)
CALL PLACE(20,11)
CALL TYPECS(IHEAD,80)
AZA=AZ+DEGS
ENCOD(80,1000,1LABEL) AZA,TYPE(NTYP)

1000 FORMAT(1/AHZ= ,F3.4,2H ,A10)
CALL PLACE(20,12)
CALL TYPECS(LABEL,80)
RETURN

END
Appendix 4  The Dispersion Relation in the Case of Zero Thermal Conductivity

The equations of motion employed in this case are

\[ p = \frac{fRT}{m} \]

\[ - \frac{\partial p}{\partial t} = \nabla \cdot (p \mathbf{U}) \]

\[ \frac{\partial (p \mathbf{U})}{\partial t} = \mathbf{F} - \nabla p + \nabla \cdot \mathbf{S} - \sigma p B^2 (\mathbf{U} - \mathbf{B}(\mathbf{U}, \mathbf{b})) \]

\[ \frac{\partial P}{\partial t} + p \nabla \cdot \mathbf{U} - \frac{\alpha}{c} \mathbf{U} \cdot \nabla \mathbf{U} = 0 \]

where all symbols are as defined in §2.4.

Temperature can be eliminated between the first and last of these equations, yielding the system

\[ - \frac{\partial p}{\partial t} = \nabla \cdot (p \mathbf{U}) \]

\[ \frac{\partial (p \mathbf{U})}{\partial t} = \mathbf{F} - \nabla p + \nabla \cdot \mathbf{S} - \sigma p B^2 (\mathbf{U} - \mathbf{B}(\mathbf{U}, \mathbf{b})) \]

\[ \frac{\partial P}{\partial t} + p \nabla \cdot \mathbf{U} - \frac{\alpha}{c} \mathbf{U} \cdot \nabla \mathbf{U} = 0 \]

The perturbation assumptions made in this case are

\[ p(r, \phi, \rho, t) = p_0 (r)(1 + p_1 (r, \phi, \rho, t)) \]

\[ \mathbf{J}(r, \phi, \rho, t) = \mathbf{J}_0 (r)(1 + \mathbf{J}_1 (r, \phi, \rho, t)) \]

\[ \mathbf{U}(r, \phi, \rho, t) = \mathbf{U}_1 (r, \phi, \rho, t) \]

These are different in form to the perturbation assumptions made in §2.4, but the only effect this has is to simplify the algebra involved. Following the same method as in 2.4 the set of linear equations

\[ i \omega p_1 - ik \cdot \mathbf{U} - \mathbf{U}_r / H = 0 \]

\[ i \omega \mathbf{U}_r + \mathbf{J}_1 \mathbf{E} - ik \cdot \mathbf{G} \mathbf{p}_1 - \mathbf{p}_1 \mathbf{E} + \mathbf{P}(\mathbf{U}_r - \mathbf{B}(\mathbf{U}, \mathbf{b})) \]

\[ + \mathbf{V}(3 \mathbf{U}_r \cdot \mathbf{k} \cdot \mathbf{k} + \mathbf{k} \cdot \mathbf{U}_r \cdot \mathbf{k}) / 3 = 0 \]
\[ \begin{align*}
&\iota \omega \mathbf{U}_e - \iota k \mathbf{E}_0 \mathbf{p}_1 + \beta (\mathbf{U}_e - \mathbf{b}_e (\mathbf{U} \cdot \mathbf{b})) + \nu (3 \mathbf{U}_e \cdot \mathbf{k} + k \mathbf{U}_e \cdot \mathbf{k})/3 = 0 \\
&\iota \omega \mathbf{U}_f - \iota k \mathbf{E}_0 \mathbf{p}_1 + \beta (\mathbf{U}_f - \mathbf{b}_f (\mathbf{U} \cdot \mathbf{b})) + \nu (3 \mathbf{U}_f \cdot \mathbf{k} + k \mathbf{U}_f \cdot \mathbf{k})/3 = 0 \\
&\iota \omega \mathbf{P}_1 - \mathbf{U}_r/H - \iota \omega \mathbf{S}_1 + \mathbf{U}_r \mathbf{S}/H = 0
\end{align*} \]

is obtained, and this yields the matrix equation \( \mathbf{A} \mathbf{x} = \mathbf{0} \), where \( \mathbf{x} \) is the column vector \((\mathbf{p}_1, \mathbf{S}_1, \mathbf{U}_r, \mathbf{U}_e, \mathbf{U}_f)\). As before, for a nontrivial solution of the system \( \mathbf{A} \mathbf{x} = \mathbf{0} \), the determinant of the coefficient matrix \( \mathbf{A} \) must vanish, and this is the dispersion relation

\[ D(\mathbf{r}, \mathbf{k}, \omega) = 0 \]
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