X-RAY SPECTRAL VARIABILITY AS A PROBE OF ACTIVE GALACTIC NUCLEI

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Abstract

This thesis is concerned with how studies of variability in the X-ray spectrum of Active Galactic Nuclei (AGN) can place constraints on physical models of the X-ray emission and absorption mechanisms in such objects. An underlying theme of the thesis is to place emphasis on modelling individual objects as well as a class of objects and to this end the Seyfert I galaxy, NGC 4151, has been targeted for an individual case study. NGC 4151 is ideally suited for the present purpose as it has been extensively studied in all wavebands, from radio to $\gamma$-ray energies and has long been known to exhibit X-ray spectral variability.

We first review the diagnostics of AGN which are of contemporary importance, mainly due to studies of the optical/UV emission-line spectrum, and outline how the study of X-ray spectral variability may be used as an additional diagnostic tool. We then present the results of an extensive (~5.5 year) monitoring campaign of NGC 4151 with EXOSAT and Ginga. The medium energy X-ray flux exhibited large amplitude variability accompanied by significant spectral variations. We show that these spectral changes can be attributed to both changes in the configuration of the X-ray absorbing material in the nucleus (independent of the continuum level) and in the spectral index of X-ray power law which is correlated with the 2-10 keV flux. We then investigate some theoretical models of spectral index variability and of complex and variable absorption. The implications of these models for AGN in general and NGC 4151 in particular are discussed. We also present X-ray observations of two other AGN, namely ESO 103-G35 and QSO 1821+643, both of which exhibit significant X-ray spectral variability. The results are discussed in terms of contemporary models.
Declaration

I hereby declare that no part of this thesis has been previously submitted to this or any other university as part of the requirement for a higher degree. The work described herein was conducted by the undersigned except for contributions from colleagues and other workers who are acknowledged in the text.

Tahir Yaqoob

April 1990
Some of the work reported in this thesis has either been published or is to be submitted for publication elsewhere, as follows:

**Variable X-ray Absorption in NGC 4151.**

**The X-ray Spectrum of QSO 1821+643.**

**A Ginga observation of NGC 4151.**

**X-ray spectral index variability in NGC 4151,**

**X-ray spectral variability in NGC 4151.**

**X-ray properties of active galaxies with high intrinsic absorption.**
Dedication

This thesis is dedicated to my family, especially my Mother and Father who have both endured much to have made this work possible.
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Chapter 1

Diagnostics of Active Galactic Nuclei

Overview

The physical conditions in Active Galactic Nuclei (AGN) which can be probed by studies of the emission-line spectrum are reviewed. Ways in which such studies can be complemented by measurements of the X-ray spectrum, and in particular X-ray spectral variability, are discussed. Emphasis is placed on the successes and failures of currently favoured models and how studies of X-ray spectral variability may affect these models.

"Now there's a look in your eyes,
like black holes in the sky ..."

(Roger Waters, 1975. From 'Wish you Were Here'.)
1.1 Introduction

This thesis is an investigation into how variability of the X-ray spectrum of Active Galactic Nuclei (AGN; see §1.1.1) can be used as a diagnostic tool, in conjunction with diagnostics at our disposal from other wavebands, to probe the physics of the compact central energy source and its environment. Our aim will be to constrain physical models of the active nucleus using X-ray data presented in this work and that published in the literature. However, at present, few AGN have been observed with sufficient frequency and sensitivity to quantify (or even detect) X-ray spectral variability. This has meant that the objects which do exhibit X-ray spectral variability cannot always be incorporated into currently favoured schemes designed to model AGN as a whole. One of the underlying themes throughout this thesis will be to emphasise the importance of modelling individual objects alongside modelling the general properties of a class of objects. Both approaches are equally necessary, mainly because modelling the 'peculiarities' in a given object can often give vital clues as to how the general scheme should proceed and obviously provides a taxing test for the general models. Of course, ideally the ultimate aim would be to finally end up with just one model which can account for all objects. Such an attempt at 'Grand Unification' would, arguably, be premature, as the study of AGN is still very much a phenomenological science (see also §1.3.13).

In this thesis, the object that has been targeted for an individual case study is the active galaxy NGC 4151. NGC 4151 is well suited for the present purpose because (i) it happens to be one of the brightest AGN in the sky, (ii) a large amount of data from radio to γ-ray energies is available and (iii) it has long been known to exhibit X-ray spectral variability (see §3.1).

1.1.1 The AGN 'zoo'

In this thesis we shall use the term AGN to include all classifications (e.g. QSOs, quasars, Seyferts, NELGs, LINERs etc.) except the BL Lac type objects. The reason for this exclusion will be given shortly. Due to the phenomenological nature of the study of AGN, the number of different classifications has become large and bewildering over recent years. The reader is referred to the review by Lawrence (1987) for a thorough discussion of the empirical inter-relations between the different types of AGN. Here we simply note that the most important factors which determine the classification of an AGN are the relative proportions of the broad and narrow optical/UV emission-line components and the strength of the non-thermal continuum emanating from the central source. Thus, as one goes from quasars to Seyfert type I to Seyfert type II, the broad emission-line components become less and less dominant while the broadband continuum becomes weaker and steeper, with the infrared luminosity becoming more dominant and thermal in nature (see §4.1.1). From the foregoing comments it should be clear that the different AGN classifications
are far from rigid and the use of a single term to cover all of them helps to emphasise the widely held belief that they are all different manifestations the same phenomenon (see §3.1.13). Indeed, in addition to the fact that it has been found necessary to introduce Seyfert types intermediate between I and II (see Osterbrock & Koski, 1976), a few objects have been observed to change their Seyfert classification over a timescale of years (e.g. see Lawrence, 1987). For instance, NGC 4151, normally classified as a Seyfert type 1–1.5 has been observed to change temporarily to something more akin to a Seyfert type II (Penston & Pérez, 1984).

BL Lac objects, characterized mainly by a different broadband continuum shape and lack of emission lines, have traditionally been treated as a separate class of object to the emission-line AGN discussed above. More recently however, several plausible scenarios have been proposed which place BL Lac objects on a par with emission-line AGN (e.g. see Lawrence, 1987). Nevertheless, we have chosen to exclude BL Lac objects from the present work, principally because a study of X-ray spectral variability in BL Lac objects has already been carried out by George (1988).

1.2 The ‘standard model’

We know empirically what the essential ingredients of an AGN model should be; the problem lies in piecing them all together in a self-consistent way. Such an attempt at reproducing the properties of the emission-line spectrum of AGN led to what has been dubbed as the ‘standard model’. The standard model played an important role in AGN research during the 70’s and early 80’s as it was able to successfully account for many of the observed emission-line ratios with the simplest of assumptions. Recently, however, many of the fundamental assumptions have been thrown open to question due to the increasing number of discrepancies between theory and observation and the failure of the standard model to explain certain observational details at all. Below we outline the salient features of the standard model. We then discuss how the study of the emission-line spectrum of AGN led to the standard model and how we might expect the study of X-ray spectral variability to improve upon it. The greatest value of the standard model in the present context is that it provides a basic framework to build on.

In its most simplistic form, the standard model can be summarized as follows (see, for example, Ferland (1987) for more details).

- There is a central ionizing continuum but the standard model does not specify how the different continuum components are generated. The present consensus is that the EUV/UV part of the continuum is thermal emission from the accretion of matter onto a Massive Black Hole (MBH) (see §1.3.2 and §4.1.3 - §4.1.5). The X-rays are thought to originate from a
compact region near the centre of the system, close to the hypothesised accretion disc (e.g. see George & Fabian, 1990b). Separate theories have evolved to explain how the X-ray (and \( \gamma \)-ray) continuum is generated (see §4.2). The shape of the unseen EUV ionizing continuum is one of the major uncertainties in the standard model and in the most simplistic models of the emission-line spectrum, the entire ionizing continuum has often been approximated by two power laws (e.g. Ferland & Mushotzky, 1982). Finally, we note that the standard model does not specify the source of the accreting material.

- Further out from the source of the continuum are a large number of filaments, or ‘clouds’ of cool \( \sim 10^4 K \) gas with a volume filling factor of \( \sim 10^{-5} \). This is the Broad Line Region (BLR) which is thought to have a typical radius of \( \sim 10^{16} - 10^{18} \) cm (see §1.3.4). The clouds are in thermal and photoionization balance with the central continuum and are the sites where the broad optical/UV emission lines are generated. Typical column densities of individual clouds are in the region of \( \sim 10^{22} - 10^{23} \) cm\(^{-2} \) with particle densities of \( \sim 10^{9.5} \) cm\(^{-3} \), implying a typical cloud thickness of \( \sim 10^{12.5} - 10^{13.5} \) cm. In the most simplistic versions of the standard model, all BLR clouds are assumed to have the same physical parameters. The origins of these particular ‘canonical’ BLR parameters are given below. The widths of the broad lines imply cloud velocities as high as thousands to tens of thousands of km s\(^{-1} \). However, the standard model does not specify the cloud kinematics as there is no unique solution which is consistent with observation (see §1.3.7).

- The BLR clouds are embedded in, and in pressure equilibrium with, a Hot Inter-cloud Medium (HIM) at \( \sim 10^8 K \) which fills the BLR. The HIM is responsible for confining the BLR clouds (or else they would evaporate) and preventing dynamic instabilities disrupting them. The initial success of this 2-phase theory, originally proposed by Krolik, McKee & Tarter (1981) (hereafter KMT), has since been recognised as premature (see §1.3.8).

- The photoionization equilibrium of the BLR clouds is characterized by the dimensionless ratio of ionizing photons to the particle density at the inner face of the clouds (i.e. the ionization parameter). Many different definitions of the ionization parameter are currently in use (see Collin-Souffrin (1987) for a list). In the present work we shall use the following definition:

\[
U = \frac{Q}{4\pi r^2 c N_H}.
\]  

Here \( Q \) is the number of ionizing photons emanating from the source per unit time, \( N_H \) is the neutral hydrogen density of the cloud and \( r \) is the distance of the cloud from the continuum source. Now, the general similarity of the broad emission-line spectrum amongst AGN which cover several decades in luminosity translates into a ‘universal’ or ‘canonical’ value for \( U \) of \( \sim 0.01 \) for the BLR (also see §1.3.4).
• The Narrow Line Region (NLR), the site where the narrow emission lines are produced, is a factor of $\sim 10^3$ larger than the BLR. Densities are much lower in the NLR (typically $\sim 10^4$ cm$^{-3}$) and the line widths correspond to velocities of the order of hundreds of km s$^{-1}$. Whereas the dominant role played by photoionization of BLR material is accepted from the general agreement between predicted and observed line ratios, the same cannot be said of the NLR. The NLR is poorly understood and other sources of ionization, such as shocks, may be important.

• Finally, AGN that do not have any broad emission lines (e.g. Seyfert IIs) can be accommodated into the standard model if (i) they do not have a BLR, (ii) the BLR is obscured from the line of sight (see §1.3.13) or (iii) the continuum is strongly beamed so that any line features are ‘swamped’.

THE 'CANONICAL' PHOTOIONIZATION MODEL

There is now ample evidence of the dominating role played by photoionization in forming the broad emission lines in AGN (e.g. see Weedman (1986) for a summary). More details of the theory of AGN photoionization models can be found in, for example, Davidson & Netzer, 1979; Kwan & Krolik, 1981; Weisheit, Shields & Tarter, 1981 and Kwan, 1984. The broad emission lines can be roughly divided into two groups: the high ionization lines formed in the HII region of a BLR cloud and the low ionization lines formed in the relatively neutral region behind the HII zone. The X-ray continuum is important in this respect since it is the main energy source for the neutral zone (Weisheit, Shields & Tarter, 1981). A summary of the range of relative intensities of prominent broad emission lines observed in AGN can be found in Collin-Souffrin (1987) while typical line widths can be found in Joly (1987).

The canonical value of $10^{9.5}$ cm$^{-3}$ for the density of BLR clouds traditionally comes from the presence of broad CIII] $\lambda$1909 emission and the absence of broad [O III] $\lambda$5007 emission (collisional de-excitation increases relative to radiative decay at higher densities; e.g. see Davidson & Netzer, 1979). The canonical value of $U \sim 0.01$ is determined mainly by certain line ratios which are sensitive to $U$ (such as C IV $\lambda$1549 / C III] $\lambda$1909). The remaining parameters of the standard photoionization model mentioned above (temperature, column density and filling factor) then come from matching the overall emission-line spectrum to that observed.

1.2.1 NGC 4151 and the standard model

Detailed studies of the (highly variable) emission-line spectrum of NGC 4151 have been performed by several workers (e.g. Osterbrock and Koski, 1976; Penston et al., 1981; Antonucci & Cohen, 1983; Ulrich et al., 1984). Ferland & Mushotsky (1982) applied the standard model to NGC 4151.
in order to account for the observed broad and narrow emission-line spectrum. They found that a value of $U = 10^{-0.78}$ for the BLR gave good overall agreement for the broad line spectrum. This is $\sim 18$ times higher than the canonical value of 0.01. The same model could not reproduce the narrow line spectrum, however, unless ad hoc elemental abundances were assumed (see §1.3.10). The other major drawback of the model was that it assumed a simple two power-law continuum and neglected the large amplitude and spectral variability which occurs from UV to $\gamma$-ray energies. The main reason for this, apart from simplicity, was that the continuum was poorly understood. In §4.1 we review the properties and behaviour of the entire radio to $\gamma$-ray continuum in NGC 4151 and characterize its spectral variability more precisely than has been possible before, especially that of the X-ray continuum. However, it is beyond the scope of the present work to use the derived continuum to calculate an emission-line model for NGC 4151.

1.3 What can be probed?

Generally speaking, the intrinsic X-ray spectrum of an AGN is thought to consist of a power-law continuum with a photon spectral index in the range $\sim 1.0 - 2.0$, at least over the energy range $\sim 0.3 - 40$ keV. In some cases there is evidence for a second, steeper, possibly thermal, continuum component dominating below $\sim 1$ keV. See §4.1.5 for a more detailed discussion of the X-ray continuum. The X-ray spectrum may also have one or both of the following features: (i) a low energy cut-off below $\sim 4$ keV in some objects, due to photo-electric absorption in the line-of-sight (see §5.1) which also gives rise to absorption edges at various energies (see §5.4.1.3) and (ii) an iron $K_{\alpha}$ emission line at $\sim 6.4 - 6.9$ keV (see Makishima, 1986). The following can then be said to constitute X-ray spectral variability:

- variability of the spectral index of the power-law continuum,
- variability in the relative magnitudes of the soft and hard X-ray continua (if indeed both components are present),
- variability in the amount and/or nature of the low energy cut-off (see §5.1) which may or may not be accompanied by variability in the absorption edge energies,
- variability in the iron emission line. The latter includes any combination of intensity, equivalent width, energy and profile variations.

Note that iron emission lines have only recently been recognized as a common feature of AGN; e.g. see Pounds et al., 1990a. Also, a hard tail to the power law above 10 keV has been postulated by several workers and some evidence of it has been found in the combined X-ray spectrum of several AGN (Pounds et al., 1990b).
Below we outline the ways in which the measurement of X-ray spectral variability (or lack of it) can contribute to our understanding of AGN, side by side with studies of the optical/UV emission-line spectrum.

1.3.1 Non-thermal compact emission regions

From observed X-ray variability timescales (which can be as short as hundreds of seconds; e.g. see Pounds & McHardy, 1989) and simple light-crossing time arguments, it has long been known that the X-rays must originate in a comparatively small region $\sim 10^{12} - 10^{15}$ cm. In some models the underlying non-thermal radio to optical continuum is also thought to originate in the same region (see §4.1). One of the major objectives of AGN research has been, and still is, to explain how such copious amounts of energy ($\sim 10^{42} - 10^{47}$ erg s$^{-1}$) can be generated in such a compact region. Variability in the slope of the hard X-ray power law in AGN can potentially place important constraints on the nature of the non-thermal emission mechanism and possibly the location of the X-ray source and how it interacts with its environment (e.g. an accretion disc). These aspects of X-ray spectral variability are investigated in §4.

1.3.2 Thermal emission: The putative accretion disc and Massive Black Hole

Accretion of matter onto a Massive Black Hole (MBH) has always been a favourite contender for the source of the energy of the central engine in AGN. This is mainly because it is the most plausible scenario amongst competing ideas (e.g. see Frank, King & Raine, 1985; hereafter FKR). However, there is still no direct, conclusive evidence for the existence of an accretion disc. There is even no consensus on the source of the accreting material. Some of the possibilities are the BLR clouds, gas from the ISM or directly from stars, stars themselves or gas associated with the original formation of the MBH. If the accretion disc exists it is also possible that there may be a significant contribution to the observed flux in some of the broad optical/UV emission lines formed in the atmosphere of the disc (e.g. see FKR).

Below we outline the ways in which X-ray spectral variability might be expected to yield information about the properties of the disc (if it exists).

- In some AGN the tail-end of the so called ‘big blue bump’ (see §4.1.5) dominates the flux at soft X-ray energies, below $\sim 1$ keV (e.g. Mkn 841, Arnaud et al., 1985; Mkn 335, Pounds et al., 1986b). This ‘soft excess’ is widely believed to be thermal emission from the hot inner regions of an accretion disc. Comparison of the observed X-ray spectrum with predicted
accretion disc spectra can provide indirect information on the luminosity of the disc and the mass of the central object (e.g. Pounds et al., 1986b). Simple arguments concerning the variability of the soft X-ray component can provide an independent estimate of the mass of the proposed MBH. Testing for a correlation between the variability of the soft and hard flux may provide clues as to the relationship between the two types of emission component. However, interpretation of such an analysis may be complicated by the fact that the disc origin of the thermal emission is not the only possibility. Reprocessing of the non-thermal continuum in optically thick material can also give rise to a similar thermal component (Guilbert & Rees, 1988).

• The iron Kα emission line, when detected, can also potentially provide further information on the accretion disc. George & Fabian (1990b) have performed Monte Carlo calculations of the expected X-ray spectrum from a model in which an optically thick, cool disc is illuminated by a compact X-ray source. Measurement of the intensity, equivalent width, energy, profile and broadening of the observed line can then give indirect (and model dependent) information on the disc inclination, ionization state of the disc material, amount of gravitational redshift and Doppler boosting and kinematics of the disc. In particular, Doppler boosting enhances the blue wing of the line so that the resulting asymmetry may cause the apparent energy of the line to increase. However, there is a problem here because, in a given case, it may not be certain how much (if any) of the observed line emission is produced in the disc and how much in BLR material surrounding the X-ray source (see §1.3.6). Also, the line energy and profile cannot be well constrained with the sensitivity and energy resolution of present instrumentation (see §2). From the data presented in this thesis, we shall not be able to make any strong statements concerning the line energy and profile.

• The above disc model of George & Fabian (1990b) also predicts changes in the apparent spectral index of the observed X-ray continuum, the magnitude of the changes depending on the relative sizes of the X-ray source and disc and the inclination of the disc relative to the observer's line-of-sight. We investigate this type of spectral variability in §4.4. However, the disc model is not a unique explanation for such behaviour; other models of the X-ray continuum variability turn out to be equally valid (see §4.2).

1.3.2.1 MASS OF THE BLACK HOLE

Some very simple arguments lead to the inescapable conclusion that the mass at the heart of the AGN machinery (i.e. the mass of the putative MBH) must be at least ~ 10^6 M⊙ (see FKR). Some estimates suggest a mass as high ~ 10^{11} M⊙ for the most luminous AGN (see below).

One example of a simple estimate of the central mass relates to the Eddington luminosity L_{EDD}, the maximum luminosity generated by spherical accretion on to a mass M (see FKR):
AGN having bolometric luminosities in the approximate range $10^{43} - 10^{48}$ erg s$^{-1}$ then require masses in the range $\sim 10^5 - 10^{11}$ $M_\odot$ according to this argument.

More sophisticated arguments have been proposed but unfortunately do not lead to any tighter constraints on $M$ than the simple ones. For instance, the widths of broad emission lines, combined with the assumption of a Keplerian velocity field, leads to a mass estimate of

$$M = \frac{3 \times 10^4}{G} \left[ \frac{V(\text{km} \text{ s}^{-1})}{2} \right]^2 r (\text{cm}) \text{ kg}$$

$$\sim 5.6 \times 10^{11} V_4^2 r_{18} M_\odot$$

(see Joly, 1985 and 1987). Here, $V_4$ is the Full Width at Half Maximum (FWHM) of the relevant line, in units of $10^4$ km s$^{-1}$, and $r_{18}$ is the distance of the BLR clouds to the source, in units of $10^{18}$ cm. $r_{18}$ can be estimated from equation 1.4. From a large sample of AGN, Joly (1987) finds that masses in the range $10^7 - 10^{11}$ $M_\odot$ are required for bolometric luminosities, $L$, (assumed here to be $\sim 10L_{\text{optical}}$) in the range $10^{43} - 10^{48}$ erg s$^{-1}$. The upper limit to the range in mass may be too high since it is not clear how such massive objects could form in the required time. Moreover, the above analysis implies a rather low value for the ratio $L/L_{\text{edd}}$ of $\sim 0.002$ (Joly, 1987). However, the method is subject to large uncertainties mainly in the value of the product $UN_H$ (see equation 1.4) and the estimate of bolometric luminosity. Also, slightly different results are obtained for different emission lines.

We can envisage extending the above ideas to the iron emission line, if it is present. A model of the kinematics of the material in which it is formed (either a disc or BLR clouds) may then provide an estimate of $M$.

Some models of variable X-ray absorption involve the motion of high velocity clouds across the face of the source (see §5.1). In cases where this velocity can be estimated from the data and $U$ can be estimated by measuring, say, the iron K-edge energy, an independent check on $M$ can be obtained.
1.3.2.2 MASS OF THE BLACK HOLE IN NGC 4151

Several independent estimates of the mass of the black hole in NGC 4151 have been published. Below we give three of the more recent estimates.

- From measurements of line widths, Clavel et al. (1987) have derived a velocity versus radius profile for line-emitting gas in NGC 4151 (see §1.3.7). Simple dynamical arguments then lead to an estimate of the central mass of $3.7 \pm 0.5 \times 10^7 \, M_\odot$. An implicit assumption in this calculation is that macroturbulence (and not radial motion) is the main source of the line broadening.

- Gaskell (1988), on the other hand deduces that BLR clouds are probably inflowing (see 1.3.7) and gets $M \sim 5 \times 10^7 \, M_\odot$.

- Smith & Raine (1988) deduce, on the basis of a comparison of observables predicted by their duelling wind theory for AGN with the data, that $M \sim 2 \times 10^8 \, M_\odot$ for NGC 4151.

In summary, it is, at present, only possible to place a loose constraint of $\sim 10^6 - 10^8 \, M_\odot$ for the mass of the central object in NGC 4151.

1.3.3 Extended emission regions

So far we have talked about continuum emission in AGN which arises from a relatively compact region close to the centre. In some objects, there exist extended emission components such as re-radiation by dust in the infrared and extended radio emission, often in the form of jets. Of particular interest in the present context is extended X-ray emission. Good evidence for such an extended X-ray emission component has been found in at least one object, namely NGC 4151 (see Pounds et al., 1986a and §3.2) and in this particular case the emission peaks below $\sim 1 \, \text{keV}$. It was first discovered by the low energy telescope aboard EXOSAT (see §2.1). The lack of variability compared with the large amplitude variability of the hard X-ray continuum and observations by the Einstein HRI (Elvis, Briel & Henry, 1983) provide ample evidence of the extended nature of the soft emission. The origin of this soft component is believed to be thermal emission from inter-cloud gas in the NLR of NGC 4151. However, the spectrum of the soft emission has not yet been measured with sufficient resolution to qualify this interpretation, although Pounds et al. (1986a) estimate a luminosity of $\sim 3 \times 10^{41} \, \text{erg cm}^{-2} \, \text{s}^{-1}$ and a temperature of $\sim 3 \times 10^6 \, K$ based on a thermal bremsstrahlung spectrum. Better measurements of the spectrum must await improved instrumentation (see §7.3) so we shall not have much more to say about extended X-ray emission in AGN.
1.3.4 Size of the Broad Line Region

The finding of a ‘universal’ value of $U$ (see §1.2) for the material in which the broad emission lines are formed implies that the size of the BLR scales with luminosity. Rearranging equation 1.1 and expressing $Q$ in terms of the ionizing luminosity, $L_{\text{ion},46}$ (in units $10^{46}$ erg s$^{-1}$) gives a characteristic BLR radius of

$$r_{\text{BLR}} \sim 7.2 \times 10^{16} [N_{H,0.5} U < \epsilon >]^{-\frac{1}{2}} L_{\text{ion},46}^{\frac{1}{2}} \text{ cm.}$$

Here, $N_{H,0.5}$ is the neutral hydrogen density in units $10^{0.5}$ cm$^{-3}$, $< \epsilon >$ is the mean photon energy of the continuum in keV. Generally speaking, equation 1.4 gives typical values for Seyfert I nuclei of $r_{\text{BLR}} \sim 10^{16}$ cm while for the higher luminosity quasars, $r_{\text{BLR}} \sim 10^{18}$ cm.

The size of the emission region can also be estimated from studies of the time lag between variability of the emission lines and the continuum (e.g. see Gaskell, 1988; Gondhalekar, 1987; Peterson, 1987). Before many observations of emission-line variability in high luminosity AGN existed it was believed that the few observations that did exist were consistent with the scaling law implied by equation 1.4. A recent study by Gondhalekar (1987), however, has shown evidence for emission line variability in high luminosity AGN on a timescale of days to weeks. This implies a BLR size which is ‘too small’ and remains a problem for the standard model.

In principle, a study of the variability of the iron line intensity relative to the continuum can also be used to estimate the size of the BLR, in cases where much of the line flux is thought to be due to fluorescence in BLR material. However, in practice it may not be easy to determine how much of the line flux is formed elsewhere (e.g. in an accretion disc). At present, the low sampling frequency of measurements of the iron line in AGN is also a problem.

1.3.5 Stratification and the density profile in the BLR

The assumptions that the BLR occurs at a localized radius and that all BLR clouds have the same density represent major inadequacies of the standard model. In particular, there is no reason why material much denser than $10^{9.5}$ cm$^{-3}$ should not exist in the BLR as long as there is enough lower density material elsewhere to produce C III] $\lambda 1909$ (see §1.2). In reality, the BLR is likely to extend over a large distance with a continuous gradation in physical properties (e.g. density). There may be no real gap between the BLR and NLR.

Indeed, a detailed study by Clavel et al. (1987) of the variability and profiles of some of the UV
emission lines in NGC 4151 has led to a picture of the BLR in this object in which a range in the physical properties of the emitting material is required. The broadest components of the lines are produced in material which is located at \( \sim 5 \) light days (\( \sim 10^{16} \) cm) from the centre with a density which may be as high as \( \sim 10^{11} \text{ cm}^{-3} \). Further out is material of decreasing density, producing lines with decreasing widths. At \( \sim 250 \) light days (\( \sim 6.5 \times 10^{17} \) cm) the density has decreased to \( \sim 10^{7} \text{ cm}^{-3} \) and this region has been dubbed as the ILR (intermediate line region). Presumably material of lower densities exists right out to the NLR (\( > 10^{20} \) cm). This run of density with radius appears to be faster than \( r^{-\frac{3}{2}} \) which would be expected from mass conservation in a spherically symmetric and homogeneous, Keplerian flow (see Clavel et al., 1987). An interesting possibility is if the density law is close to \( r^{-2} \) in which case all material (at least in the BLR-ILR) would have a similar ionization parameter (see equation 1.1).

X-ray spectral measurements cannot by themselves yield independent information on the density of BLR material. They may, however, indicate whether very dense material (\( > 10^{15} \text{ cm}^{-3} \)) is present. The possibility that cool (\( \sim 10^{4} \text{ K} \)) material with such a high density may exist very close to the X-ray source (\(< 10 \) Schwarzschild radii) has recently been suggested by Guilbert & Rees (1988). Unfortunately, present instrumentation cannot unambiguously identify the expected spectral signatures expected from these ultra-dense clouds. In particular, the thermal ‘bump’ at UV/soft X-ray energies (due to reprocessing of the primary non-thermal continuum) would not be distinguished from say, thermal emission from an accretion disc (e.g. see §1.3.2 and §4.1.5).

### 1.3.6 Covering factor and geometry

It is generally believed that the fraction of the sky covered by clouds in high luminosity AGN is of the order of 10%. This is based on an energy budget argument concerning the ratio of the number of Ly\(_{\alpha}\) photons observed in the spectrum to the number of Lyman photons inferred for the ionizing continuum (e.g. Smith et al., 1981; Weedman, 1986). However, since X-ray absorption appears to be more common in lower luminosity AGN (see Turner & Pounds, 1989) it follows that the covering factor is probably larger in these objects.

The geometry of the distribution of BLR clouds about the central source is very uncertain. In particular, it is not known whether the cloud distribution is closer to being spherically symmetric or disc-like and how the covering factor changes with distance from the source (if at all). Nor is it certain how (if at all) geometry and covering fraction are related to the luminosity of the source. The geometry of the BLR has important consequences for photoionization models. For instance, in a spherical geometry photons emitted from the back of a cloud towards the source following recombinations of hydrogen directly to the ground state can ionize hydrogen atoms in the clouds on the far-side of the source. This contrasts with the case of a disc-like geometry when the far-side
clouds may be obscured by the source.

Most photoionization models make the simplest assumptions of a spherically symmetric BLR and ionizing continuum. Notable exceptions are the angle-dependent photoionization models of Netzer (1987a,b). Here, the X-ray source is isotropic but the UV continuum (assumed to be thermal emission from an accretion disc) is anisotropic. Clouds directly above the disc 'see' a stronger UV continuum than clouds closer to the plane of the disc so that a range of ionization parameters is allowed. Also, the covering fraction in these models is angle-dependent, it being greater at low cloud latitudes. The extra degree of freedom in this type of model (the inclination of the disc) means that some, but not all, of the problems with the standard model can be solved (see Collin-Souffrin, 1987 and §1.3.12).

Some emission-line models have been published which are based on specific dynamics and geometries and give acceptable general agreement with observation but need to be tested on individual objects (e.g. the duelling wind model; Mobasher & Raine, 1989). Other alternative geometries have also been discussed in the literature; formation of the broad lines in an accretion disc (Shields, 1978; Collin-Souffrin, 1987), a comet-tail (Edwards, 1980) or the shock models of Perry & Dyson (1985). However, no detailed spectroscopic line predictions have so far been made.

X-ray spectral measurements can only give limited information on the geometry and covering factor of the BLR. Measurements of the iron line intensity and equivalent width can potentially provide estimates of the covering factor with a large uncertainty due to a possible contribution from a disc. When present, low energy absorption (particularly if it is variable) in the X-ray spectrum may be modelled to give indirect information about the configuration of material in the line-of-sight. It is often found that the absorption cannot be attributed to a uniform screen of cold (\(\sim 10^4 \text{ K}\)), solar abundance gas (see §5.1) and this is referred to as complex absorption. However, deductions about the geometry of the BLR from these measurements cannot easily be made without a detailed model. Complex and variable X-ray absorption is fully discussed in §5.

Indirect information concerning the geometry of the BLR may also be obtained from the relative magnitude of the UV/soft X-ray 'bump'. In the classical 2-phase theory of KMT, BLR clouds cannot exist if they are exposed to a continuum with a 'bump' as large that seen in objects like Mkn 841 (see Fabian et al., 1986b). In such a scenario the clouds could exist in a toroidal distribution for which the relative amplitude of the 'bump' to the X-ray continuum is smaller. Large amplitude variability of the 'bump' relative to the X-ray continuum could have important implications for the thermal stability of BLR clouds (see §5.5).
1.3.6.1 COVERING FACTOR IN NGC 4151

Using the method of comparing the flux in \( L_{\text{Ly}\alpha} \) to that in the ionizing continuum (see discussion above), Penston et al. (1979) estimate the covering factor of BLR clouds in NGC 4151 to be \( \sim 80\% \). The observation that broad UV absorption lines only occur in the outer regions of the BLR (Bromage et al., 1985) lend support to the view that the covering factor in NGC 4151 increases radially. However, there is a discrepancy between the apparent line-of-sight column density inferred from UV absorption (< \( 10^{22} \text{ cm}^{-2} \); Bromage et al., 1985) and that inferred from X-ray absorption (\( \sim 10^{23} \text{ cm}^{-2} \); §3). This may be due to the fact that some of the UV continuum originates in a region much larger than the X-ray source and X-ray absorbing region (e.g. a disc or reprocessing in inner BLR clouds) so that the X-rays ‘see’ a larger column.

1.3.7 The velocity field: inflow or outflow?

At present, virtually all our knowledge of the velocity field in the BLR comes from studies of the broad emission-line profiles (and absorption lines, if present) (see Wilkes, 1987 for a review). Some very general deductions made from observations of line profiles are listed below.

- The higher ionization lines are generally broader than the low ionization lines (Osterbrock & Shuder, 1982; Wilkes, 1986). This, combined with evidence for faster variability in the wings of lines in some objects (e.g. NGC 4151; Clavel et al., 1987), suggests that velocities are higher closer to the central source.

- The shape of the profiles rules out uniform spherical expansion (which would give flat-topped profiles) and an ordered accretion disc (which would give double-peaked profiles). A discussion of the expected profile shapes for different assumptions about dynamics may be found in Mathews & Caprotti (1985) and Mobasher & Raine (1989) and references therein.

- The high ionization lines are consistently observed to be blueshifted with respect to the system rest frame (Wilkes, 1987). Shifted lines can only be explained by radial motion combined with obscuration. If the clouds are inflowing then the emission from the clouds on the far-side of the source must dominate, requiring the emission from near-side clouds to be obscured. On the other hand, if the clouds are outflowing, emission from the far-side clouds must be obscured. For the inflow scenario, the obscuration could be produced by a small amount of dust embedded in the relatively cool, neutral regions of a BLR cloud (Rudy & Puetter, 1982). In an outflow model, obscuration by dust in an HIM (if it can survive; see Ferland & Mushotzky, 1982) is a possibility, as is the presence of an accretion disc blocking off some of the far-side emission.
It is not clear, however, why the observed profiles are largely symmetric. Shifting and asymmetry are generally strongly linked.

- An important case with respect to the above discussion is the $\text{Ly}_\alpha$ line which is observed to be symmetric in most AGN. However, the standard model predicts that $\text{Ly}_\alpha$ should be asymmetric because we should preferentially see emission from the back of clouds on the far-side due to the large optical depth in $\text{Ly}_\alpha$. Several ways to resolve the problem have been proposed (see Wilkes, 1987) which include invoking some orbital motion of the high ionization clouds, scattering in an HIM, questioning the large $\text{Ly}_\alpha$ optical depth required by the standard model and invoking angle-dependent ionization (Netzer, 1987b).

The inescapable fact from the above discussion is that it is still unknown whether the motion of BLR clouds is predominantly radial, orbital or chaotic or whether the clouds are infalling or outflowing.

Observations of the $K_\alpha$ iron line in AGN may provide supplementary information on the velocity field. However, there is confusion due to the uncertainty in the relative contribution of the line flux from an accretion disc. AGN that exhibit variable X-ray absorption which can confidently be attributed to clouds crossing the line-of-sight (see §5.1–5.3) may also tell us something about the velocity field.

1.3.7.1 THE VELOCITY FIELD IN NGC 4151

The velocity field in NGC 4151 has been studied in considerable detail (Penston et al., 1979; Blanford & McKee, 1982; Ulrich et al., 1984; Bromage et al., 1985; Leech et al., 1987; Clavel et al., 1987). NGC 4151 is somewhat unusual in the sense that there are significant differences in the emission and absorption lines from line to line. Also, virtually the entire blue half of $\text{Ly}_\alpha$ is missing (which may be due to absorption) and the line is much narrower than in other AGN (Penston et al., 1979). The presence of broad, variable absorption lines is also unusual (see Bromage et al., 1985).

Early work based on the response of the emission-line profiles to a continuum outburst in NGC 4151 suggested that the velocity field was essentially chaotic with a possible outflow superimposed but no evidence for transverse orbital motion (Ulrich et al., 1984). Clavel et al. (1987) confirm this picture from a study of the C IV $\lambda1549$ profile which has a small blue asymmetry that diminishes as the continuum level decreases. Clavel et al. (1987) favour a model in which a decelerated outflow is superimposed upon chaotic motion in the form of random orientations of parabolic orbits about the central mass. The radial motions are not the main source of the line broadening. The required obscuration of the far-side emission is then provided by an accretion disc viewed close to edge-on. The main argument forwarded by Clavel et al. (1987) against inflow is that dust could not survive
in the BLR. However, as pointed out above, Rudy & Puetter (1982) demonstrate that a small amount of dust actually imbedded in the cool region of BLR clouds (as opposed to an HIM) would be sufficient. Indeed, Gaskell (1988) has more recently come to a completely different conclusion about the nature of the motion of BLR clouds in NGC 4151. It is claimed, from a study of the variability of the C IV $\lambda 1549$ and Mg II $\lambda 2798$ lines, that changes in the red wings of both lines lead the blue wings, with the inference that outflow at small radii is excluded at the 99.5% confidence level. Totally random or chaotic motions are excluded only at the 93% confidence level, however. The bottom line is that we still don’t know the precise nature of the motion of clouds in the BLR. Despite this uncertainty, the run of absolute velocity with radius deduced by Clavel et al. (1987) (from the FWHM of a number of emission lines) still stands. That is, the velocity field in NGC 4151 is Keplerian up to a distance of at least 1 light year from the source ($\sim 10^{18}$ cm). Beyond this, the velocity falls off more slowly than $r^{-\frac{1}{2}}$, implying that gas in the NLR ‘feels’ a larger central mass than the BLR.

1.3.8 The cloud confinement problem

If BLR clouds were not confined, they would expand rapidly on a dynamical timescale and/or become physically disrupted. The alternative to confinement is continuous production of BLR clouds. The 2-phase theory of KMT was successful in that it was a very economical way of producing BLR clouds at $\sim 10^4$ K which were pressure-confined by an HIM at $\sim 10^8$ K for a narrow range in the ionization parameter which happened to include the ‘canonical’ observed value in most AGN (see §1.2). However, in recent years some serious objections to the theory have been raised and these are summarized below.

- The continuum used by KMT was based on 3C 273 which is an anomalously strong $\gamma$-ray emitter (see §4.1.6). Since the temperature of the hot phase is determined by a balance between Compton heating and cooling, a more ‘typical’ AGN continuum, having a lower Compton temperature, would give rise to a hot phase with a temperature as low as $10^6 - 10^7$ K. The problems with such a ‘cool’ hot phase have been pointed out by Mathews & Ferland (1987). Among them are a large optical depth to UV/ X-ray photons and the likelihood of disruptive drag forces on the BLR clouds as they attempt to move through the HIM.

- The 2-phase equilibrium is absent if a large UV/ soft X-ray ‘bump’ is present (see Fabian et al., 1986b).

The fact that the properties of the BLR are fairly similar for most AGN while they may have very different continuua strongly suggests that physical processes other than radiative heating and
cooling are important in determining the thermal state of BLR material and any inter-cloud gas that may be present (see Mathews and Ferland, 1987 and §5.5). Indeed, Mathews & Ferland (1987) showed that dynamical models in which cool BLR clouds condense out of a co-moving HIM which is heated by known mechanisms and a realistic continuum are not satisfactory. However, there is no shortage of alternative solutions to the cloud confinement problem (see §5.5.2) but there is as yet no consensus as to what the correct solution should be.

The cloud confinement problem is important in the present context of understanding X-ray spectral variability because (i) the validity of any explanation to account for variable X-ray absorption in a particular AGN may depend on the confinement mechanism (see §5) and (ii) the effect on BLR material exposed to a continuum whose shape at UV and/or X-ray energies is variable may have significant observational consequences.

1.3.9 Dust and obscuration

The importance of obscuration by dust with respect to understanding emission-line profiles has already been pointed out in §1.3.7. A knowledge of the dust content or intrinsic reddening of the BLR is also important for comparing observed emission-line intensities with those predicted from photoionization models since the whole spectrum may be significantly distorted. The ratio of $\frac{H_\alpha}{H_\beta}$ (i.e. the Balmer decrement) is normally used as a reddening indicator. The case B recombination value is 2.87 and since dust preferentially absorbs blue photons to red ones, the Balmer decrement will be larger than this if it is affected by dust. The fact that the observed $\frac{H_\alpha}{H_\beta}$ is generally observed to be not much larger than 2.87 in most Seyfert Is and quasars ($\sim 3 - 10$) is take as evidence that the intrinsic reddening of the BLR is small (e.g. see Collin-Souffrin, 1987 and O'Brien, 1987). In this respect NGC 4151 is no exception; Perola et al. (1982) estimate $E(B - V) \sim 0.05$. On the other hand, dust may play a much more important role in Seyfert type II nuclei (see §1.3.13).

1.3.10 Metal abundances

The relative proportions of elements present in the AGN environment is completely unknown. Standard photoionization models normally assume solar abundances (§5.4.1.1). Obviously, significant deviations from this assumption can have important consequences for the predicted emission-line spectrum and the X-ray opacity of absorbing material in AGN.

It is possible to determine the abundance of iron relative to lighter elements such as O, Ne, Si and S, from the measurement of the the iron K-edge optical depth in the X-ray spectra of AGN.
which are significantly absorbed (e.g. see §3.3.3). Determination of the iron abundance in this way requires a knowledge of the ionization state of the gas, as indicated, for example by the iron K-edge energy (see §5.4.1.3). Measurements of the iron abundance and iron K-edge energy have hitherto been poorly constrained until the launch of the Large Area Counter aboard Ginga (see §2.2). Some of the implications of an iron abundance which is different to the solar value adopted in most AGN models are listed below.

- An enhanced iron abundance may help to relieve the so called 'Fe II' problem somewhat (e.g. see Wills, Netzer & Wills, 1985 and Collin-Souffrin et al., 1986). That is, assuming a solar abundance for iron, the strengths of the myriad of Fe II emission lines predicted by current photoionization models are at least a factor of 3 below the observed values. This is an important problem with the standard model since the Fe II lines represent ~ 1/4 of the total energy emitted by the BLR and it cannot be solved by invoking a larger amount of reddening (Collin-Souffrin, 1987). An enhancement of the iron abundance relative to solar of at least an order of magnitude is required to match the observed spectra. However, even if a more modest over-abundance were found from the X-ray spectra it must still be incorporated into new photoionization models (see §7.2). Another point of relevance here is that the production of the Fe II lines is sensitive to the hardness of the X-ray spectrum (Collin-Souffrin, 1987). Hence, variability of the X-ray spectrum may have important observational consequences for the Fe II emission which may be used to constrain future models.

- The iron abundance is obviously a crucial input parameter for models of the formation of the iron Kα emission line, whether it is produced in an accretion disc (George & Fabian, 1990b) or in BLR material (Makishima, 1986). Some of the models predict a maximum equivalent width of the iron line which falls short of that observed in some AGN (e.g. see Pounds et al., 1990a) if a solar abundance of iron is assumed.

- Measurement of the iron abundance in AGN may someday tell us something about their evolutionary history.

Future instrumentation should enable measurement of the relative abundances of some of the other elements from the X-ray spectrum (see §5.4.4.2 & §7).
1.3.11 The Narrow Line Region

In the present work we shall not be too concerned with the narrow line region (NLR) since the only effects on the X-ray spectrum are expected to be (i) soft thermal emission (< 1 keV) of the type mentioned in §1.3.3 which is not expected to vary on timescales less than ~ 1 year and (ii) a small amount (if any) of low-energy X-ray absorption which again is not expected to be variable. Another point of relevance is that the kinematics of the BLR and NLR appear to be linked (Heckman et al., 1984) and that radial motion dominates in the NLR. The intensity and profiles of the narrow emission lines have been modelled with a fair amount of success with different dynamical assumptions (e.g. see Vrtilek, 1985 and Mobasher & Raine, 1987). However, the models are not unique but future X-ray mapping of the NLR may help to distinguish between them (see §7.3). Mapping in the optical has recently revealed some new features in the NLR of NGC 4151 which have not lent themselves to a simple interpretation (Pérez et al., 1989) and future X-ray imaging may complement such studies.

1.3.12 Outstanding problems with the standard model

At this point we summarize the principal unresolved problems with the standard model which have arisen from studying the emission-line spectrum of AGN.

- The $L_{\text{Ly}\alpha}/H_\beta$ problem. This line ratio is always observed to be an order of magnitude weaker than that predicted by photoionization models unless significant reddening is invoked (Baldwin, 1977; Weedman, 1986).
- The $L_{\text{Ly}\alpha}$ asymmetry problem (see §1.3.7).
- The Fe II problem (see §1.3.10).
- The nature of the motion of BLR clouds (and inter-cloud gas, if it exists) (§1.3.7) is unknown. In particular it is not known whether the radial component corresponds to inflow or outflow.
- The contribution (if any) of an accretion disc to the line emission is unknown (e.g Collin-Souffrin, 1987).
- The cloud confinement problem (§1.3.8).
- The elemental abundances in the AGN environment is unknown (§1.3.10).
1.3.13 AGN connections

To complete the discussion of AGN diagnostics we give a brief resumé of current ideas about the connections between different AGN types (§1.1.1) since it is, after all, one of the ultimate aims of AGN research. Some of the points are necessarily vague. Arguably, one of the key issues is to first understand why Seyfert type IIs on the one hand and BL Lac objects on the other, appear to be so different to all other AGN classifications.

- Radio power has come to be recognized as a potentially important clue. The tantalizing correlations between radio power and other properties of AGN such as soft X-ray spectral index (Wilkes & Elvis, 1987; §4.1.5) and the links with optical and X-ray luminosity (e.g. Browne, 1987; Worrall et al., 1987) and NLR kinematics (Heckman et al., 1984) have not yet led to any firm conclusions, however.

- Relativistic beaming of the continuum and the observer's orientation relative to beaming direction are believed to be responsible for many of the diversities amongst AGN (see Browne, 1987 for a review). In particular, beaming is a popular hypothesis to explain the lack of emission lines in BL Lac objects.

- Whether beaming is important or not, it is generally accepted that the orientation of the observer relative to the AGN system must be very important, especially in view of the possible presence of an accretion disc and the inadequacy of spherically symmetric models of the BLR (e.g. Netzer, 1987a).

- In general, along the sequence Seyfert IIIs, Seyfert Is, quasars and BL Lacs there appears to be less and less 'junk' surrounding the central emission region, as indicated by less and less obscuration of the central continuum. It is not clear whether this is telling us something about the thermodynamics in the AGN environment or something completely different such as age (BLR material may accumulate over a period of time). Also, it is uncertain whether the presence or absence of significant X-ray absorption in the low luminosity AGN is telling us about geometry or thermodynamics or both. Observations of variable X-ray absorption may help to resolve this (see §5).

- The currently popular view of Seyfert type II AGN is that they are heavily obscured type I AGN (e.g. Lawrence, 1987). Several arguments favour this hypothesis. These are (i) the detection of hard X-ray emission in at least two Seyfert IIIs (NGC 1068, Monier & Halpern, 1987; Mkn 348, Warwick et al., 1989b), (ii) the large equivalent width of the iron Kα emission line in NGC 1068 (Elvis & Lawrence, 1987) suggests that the direct continuum is hidden from view, (iii) significant polarization of the optical continuum in NGC 1068 (Antonucci & Miller, 1985) again suggests that we see mainly the scattered continuum and (iv) the dust
content appears to be systematically larger in Seyfert IIIs than Seyfert Is as indicated by the large Balmer decrements (§1.3.9) and the thermal nature of the infrared continuum (§4.1.1), thought to be re-radiation by dust. The obscuration in Seyfert type IIIs could take the form of an accretion disc or a dusty scattering torus (Krolik & Kallman, 1987). Both may have some relevance to the presence of a substantial line-of-sight X-ray absorbing column in some Seyfert Is, depending on the observer's viewing angle.

1.4 Thesis outline

Below is an outline of the remainder of the thesis.

- **Chapter 2** The instrumentation aboard the EXOSAT and Ginga satellites which was used to obtain the X-ray data used in this thesis is described. X-ray data was obtained for three AGN, namely NGC 4151, QSO 1821+643 and ESO 103-G35 (see below). The techniques used to reduce the raw data and subtract the background are discussed. Finally the general methods used for spectral analysis are described.

- **Chapter 3** Results are presented of an extensive monitoring campaign of NGC 4151 with EXOSAT and Ginga covering a period of ~5.5 years. A preliminary spectral analysis is performed on the data which reveals X-ray spectral variability attributable to changes in both the slope of the X-ray continuum (correlated with 2–10 keV flux) and in the line-of-sight absorption (independent of the continuum level). A detailed interpretation of the complex and variable X-ray spectrum is deferred until chapters 4 and 5. The results of this preliminary analysis are discussed in the context of observations of NGC 4151 with previous X-ray astronomy missions.

- **Chapter 4** The nature of the broadband continuum emission in AGN is reviewed, with NGC 4151 as a particular case study. The evidence for X-ray spectral index variability in AGN is assessed, including recent reports of a correlation between the index and 2–10 keV flux. Two particular types of model of X-ray spectral index variability are investigated, with a view to accounting for the flux-index correlation reported for NGC 4151 in chapter 3.

- **Chapter 5** The properties of several models of complex and variable X-ray absorption are investigated. In particular, a spectral fitting code is developed which is suitable for testing X-ray spectra against a model in which the opacity of X-ray absorbing material is controlled by photoionization and heating by the continuum source (the 'warm absorber model'). The above models are applied to NGC 4151 in order to account for the spectral variability below ~4 keV. Finally, the results of this chapter are discussed in terms of possible implications for the physics of the broad line region.
- **Chapter 6** The results of X-ray observations of the high luminosity QSO 1821+043 and the low luminosity Seyfert 1.9 galaxy ESO 103-G35 are presented. Both objects exhibit marked X-ray spectral variability. The results are discussed in terms of the X-ray continuum and/or absorption variability models of chapters 4 and 5 respectively.

- **Chapter 7** The work presented in this thesis is summarized, emphasising what has been learnt about the individual objects studied as well as the implications of the models in chapters 4 and 5 for AGN in general. Possible directions of future theoretical work are outlined, followed by a review of the observational prospects for AGN in the next decade.
Chapter 2

Reduction and Analysis of Data from EXOSAT and Ginga

Overview

In this chapter we discuss the instrumentation aboard the EXOSAT and Ginga satellites which was used to obtain the X-ray data used in this thesis. We concentrate only on aspects which are directly relevant to the present work. We then discuss how the raw data is reduced and describe the techniques used to subtract the X-ray background in order to obtain source light curves and pulse height spectra. Finally, we briefly describe the techniques used for spectral analysis.

"Welcome my son, welcome to the machine ..."

(Roger Waters, 1975. From 'Wish you Were Here'.)
2.1 EXOSAT

The European X-ray Observatory Satellite, (EXOSAT), was launched on 26th May, 1983. The major objectives of the mission are listed below.

- Precise location of X-ray sources in the 0.04 - 2 keV (6 - 300 Å) band with the low energy (LE) imaging telescopes (see §2.1.1).
- Broadband spectroscopy in the energy range 0.04 - 50 keV.
- Study of the temporal behaviour of X-ray sources down to a resolution of $10^{-6}$ s.
- Mapping of diffuse X-ray sources with the LE telescopes.

Some details of the satellite and its orbit are given in Table 2.1; further details can be found in Taylor et al. (1981). As can be seen from Table 2.1, one of the unique features of the EXOSAT orbit was the very high eccentricity which was primarily intended to maximize the number of lunar occultations for the precise location of X-ray sources. This orbit had the additional advantage of providing as much as $\sim 76$ hours of uninterrupted data accumulation above the Earth's radiation belts.

The telemetry rate of 8 kbits/s did not allow transmission of all the raw data in real time. However, the flexibility of the On-Board Computer (OBC) allowed the transmission of pre-processed data with a variety of different combinations of time and spectral resolution. All the EXOSAT data in this thesis was obtained in HER4 mode which gives a maximum time resolution down to 10 s and 128 pulse height channels for both Argon and Xenon with full detector identification.

The scientific payload aboard EXOSAT consisted of four co-aligned instruments, namely the Gas Scintillation Proportional Counter (GSPC), two low energy (LE) imaging telescopes & an array of 8 medium energy (ME) proportional counters. The low sensitivity and small effective area of the GSPC limited its usefulness for extragalactic sources. For this reason, no GSPC data has been used in the present work and we do not comment further on this instrument. However, further details of the GSPC can be found in Peacock et al. (1981).

Use of the LE telescopes and the ME array is discussed in §2.1.1 and §2.1.2 respectively.
Table 2.1 The *EXOSAT* orbit

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</tr>
<tr>
<td>Re-entry</td>
<td>6th May 1986</td>
</tr>
</tbody>
</table>
2.1.1 The LE Telescopes

Some technical details of the LE imaging telescopes aboard EXOSAT are given in Table 2.2. Further details can be found in de Korte et al. (1981). Each telescope had two detectors which could be placed in the focal plane. One of these detectors was a Channel Multiplier Array (CMA) while the other was a Position Sensitive Detector (PSD). A transmission grating was also available which could provide the telescopes with some spectroscopic capability in the operational energy range of 0.04 – 2 keV. A much cruder spectral capability was afforded by a series of filters which could be placed in front of the focal plane detectors (see Table 2.2). Fig. 2.1 shows the on-axis effective area, as a function of energy, of the CMA plus LE telescope for three different filters (3Lx, Al/P and Boron).

Unfortunately, both PSDs, the CMA on one telescope (LE2) and the transmission grating for the remaining telescope failed early on in the mission. As a result, all the data reported in this thesis was obtained from LE1 with the CMA in the focal plane with the use of one or more filters.

2.1.1.1 LE BACKGROUND SUBTRACTION

In this section we outline the methods we have used to extract background-subtracted count rates from the LE filter measurements. First, the total count rate is measured from a small square box (usually of size 100 arcsec) centered on the source and the background contribution from this is estimated from an annulus surrounding the source box (usually a square whose sides are three times the sides of the source box). In cases where there is another source close to the one of interest, the background contribution must be estimated from the average count rates from several regions nearby the source.

Once the background component has been subtracted, several corrections must be applied to the residual source counts and these are listed below.

- A fraction of the source counts are scattered outside the source box. This must be corrected for by using the point spread function (p.s.f.) appropriate to the particular filter and box size. For the Boron filter the p.s.f. also depends on the shape of the source spectrum. The p.s.f.'s we have used are taken from the EXOSAT Express (No. 13, 1985).
- A deadtime correction must be applied. For the sources in this thesis it is of the order of 3.5%.
- Vignetting of the LE telescope by the ME sunshield must also be corrected. This is normally incorporated into the instrument effective area calculation.
Table 2.2 The EXOSAT LE Telescopes

<table>
<thead>
<tr>
<th>Description</th>
<th>Two double-nested Wolter-I type telescopes †</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total collecting area of two telescopes</td>
<td>10 cm²</td>
</tr>
<tr>
<td>Platescale</td>
<td>5.3 μm arcsec⁻¹</td>
</tr>
<tr>
<td>Pixel size</td>
<td>4 × 4 arcsec</td>
</tr>
<tr>
<td>Field of view</td>
<td>2.2° (FWHM)</td>
</tr>
<tr>
<td>Source location accuracy</td>
<td>8&quot;</td>
</tr>
<tr>
<td>Spatial resolution (on axis)</td>
<td>18&quot;</td>
</tr>
<tr>
<td>Spatial resolution (20' off axis)</td>
<td>40&quot;</td>
</tr>
<tr>
<td>Energy range</td>
<td>0.04 – 2 keV</td>
</tr>
<tr>
<td>Spectroscopic capability:</td>
<td></td>
</tr>
<tr>
<td>Transmission grating</td>
<td>500 lines mm⁻¹ or 1000 lines mm⁻¹</td>
</tr>
<tr>
<td>Filters</td>
<td>3000 Å Thin Lexan (3Lx)</td>
</tr>
<tr>
<td></td>
<td>4000 Å Thick Lexan (4Lx)</td>
</tr>
<tr>
<td></td>
<td>1000 Å Al plus 1000 Å P (Al/P)</td>
</tr>
<tr>
<td></td>
<td>1000 Å B + 500 Å polypropylene (Bor)</td>
</tr>
</tbody>
</table>

† e.g. see Kitchin (1984).
Figure 2.1 The effective area of a single LE telescope for three CMA/filter combinations as a function of photon energy. The 3Lx, Al/P, and Bor filters correspond to the solid, dashed and dotted curves respectively.
2.1.2 The Medium Energy experiment

The medium energy (ME) experiment aboard EXOSAT consisted of an array of 8 multi-wire proportional counters grouped in pairs into a rectangular format in which all the detectors faced the same direction. Simultaneous monitoring of the X-ray background was provided by one pair of quadrants (half-array) pointing at the source while the other half-array was offset by ~ 2°, pointing at blank sky. Each detector was divided into two parts, one filled with Argon and the other with Xenon. The Argon detectors had an energy range of 1 – 20 keV while the Xenon detectors had a range of 5 – 50 keV. The sensitivity and calibration of the Xenon detectors was much poorer than the Argon ones. Table 2.3 gives a summary of the specifications of the ME array. Further details can be found in Turner et al. (1981). For reference, Fig. 2.2 shows the effective area, as a function of energy, of the Argon detectors.

2.1.2.1 NATURE OF THE ME BACKGROUND

The X-ray background in the ME detector array is dominated by particle-induced events and cosmic rays. The contribution from the diffuse X-ray background is less important since the field of view of the ME is small and the bodies of the detectors are protected by lead shielding. The highly eccentric orbit of EXOSAT meant that for most of the time the ME detectors were able to avoid the charged particles trapped in Earth’s magnetic field (the Van Allen radiation belts).

Even though the ME has an intrinsic capability of rejecting anomalous background events, using methods such as rise-time discrimination and 5-sided anti-coincidence, some events still escape detection. The light curves from each ME detector (for both source and background pointing) for each of the observations presented in this thesis were individually examined at the earliest possible stage in the analysis. Anomalous events and periods of variable background were then manually rejected using time discriminators.
Table 2.3 The EXOSAT ME experiment

<table>
<thead>
<tr>
<th>Description</th>
<th>8 Ar + Xe proportional counters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total effective area</td>
<td>1600 cm²</td>
</tr>
<tr>
<td>Energy range (Ar)</td>
<td>1 – 20 keV</td>
</tr>
<tr>
<td>Energy range (Xe)</td>
<td>5 – 50 keV</td>
</tr>
<tr>
<td>Energy resolution (Ar) †</td>
<td>$51E_{keV}^{-0.5}$ % (FWHM)</td>
</tr>
<tr>
<td>Energy resolution (Xe) †</td>
<td>~18 % for 10 &lt; E &lt; 30 keV (FWHM)</td>
</tr>
<tr>
<td>Entrance window</td>
<td>37 µm, Beryllium</td>
</tr>
<tr>
<td>Pressure in all chambers (Ar &amp; Xe)</td>
<td>2 atm</td>
</tr>
<tr>
<td>Time reqd for 5σ detection of a 1 mCrab source ††</td>
<td>~100 s. (10⁴ s for measurement of spectrum).</td>
</tr>
<tr>
<td>Field of view</td>
<td>Square, 45° (FWHM)</td>
</tr>
<tr>
<td>Response</td>
<td>Triangular (45° FWHM) with a ~3° flat top</td>
</tr>
</tbody>
</table>

† $\Delta E/E$

†† 1 mCrab = $10^{-11}$ erg cm⁻² s⁻¹ (2–6 keV).
Figure 2.2 The effective area of one Argon half-array of the EXOSAT ME as a function of energy (taken from Turner et al., 1981).
2.1.2.2 ME BACKGROUND SUBTRACTION

Unfortunately, it turned out that the responses of the two array-halves (H1 and H2) of the ME were sufficiently different from each other that the simultaneous background from the offset half could not simply be subtracted from data from the on-source half. Instead, the aligned and offset halves had to be swapped part-way through an observation (referred to as a 'nod') so that the non-simultaneous background had to be used for the on-source data from the same half. In general, a nod was performed every ~ $10^4$ seconds.

Further background subtraction problems arose from the fact that a given detector had a different response in the offset modes (negative or positive) to that in the aligned mode. A 'difference spectrum' then had to be used to correct for this effect (see Izzo & Parmar, 1986). However, it was usually not possible to construct difference spectra for the majority of observations and a 'standard' set had to be used for most cases. The uncertainty in this procedure is large since the required difference correction depends on a large number of factors, including the date of the observation.

An alternative approach is to make the reasonable assumption that the difference correction for oppositely offset halves is equal and opposite (e.g. diff(H1+) ~ diff(H2-)). Hence, by taking the average of two background-subtracted spectra from a single nod it is possible to cancel out the difference correction to a large extent. Due to the more efficient shielding of the four outer (corner) detectors of the ME array, the magnitude of the difference correction for these detectors happens to be much less (almost negligible) than that for the four inner detectors. Hence, the use of data from the corner detectors only may provide a better background-subtracted spectrum, provided that the corresponding decrease in signal-to-noise ratio is acceptable.

Some of the EXOSAT observations reported in this thesis were performed without a nod. In such cases offset data from the same (on-source) half-array had to be used from periods when the satellite was slewing onto and/or off the source.
In summary, we have effectively described six methods of obtaining a background-subtracted source spectrum from the ME, which we list below.

- **Method I.** Subtract offset data from one 'side' of a nod from on-source data from the other side of the nod. Offset and on-source data is from the same half-array. Apply appropriate difference corrections. Repeat using on-source and offset data from the other half-array. The two spectra obtained in this way may be averaged.

- **Method II.** As method I but no difference correction is applied. The two spectra obtained must then be averaged as this method assumes that the difference corrections for the two offset halves are roughly equal and opposite.

- **Method III.** As method I but only the corner detectors are used.

- **Method IV.** As method II but only the corner detectors are used.

- **Method V.** Blank sky pointing data obtained from the offset half-array during slew manoeuvres is subtracted from data obtained from the same half aligned on-source. The appropriate difference correction is applied to the resulting spectrum.

- **Method VI.** As method V but only the corner detectors are used. No difference correction is applied as it is assumed that the magnitude of the correction is negligible for the corner detectors.

As the ME spectral measurements of weak sources are predominantly limited by systematic errors associated with the background-subtraction process, it is important to choose carefully which of the above methods to use. Since, in the present work, we shall be concerned with variation of spectral parameters for a given source, we will use the most conservative approaches at the expense of a decrease in signal-to-noise by a factor $\sqrt{2}$ (i.e. methods IV and VI).

We illustrate typical results using methods IV and VI in Fig. 2.3. Figs. 2.3a and 2.3b show background subtracted Argon and Xenon pulse height spectra respectively, for an EXOSAT observation of NGC 4151 (day 002/1985; see §3.2) using method IV. Fig. 2.3c shows the background subtracted Argon pulse height spectrum from another EXOSAT observation of NGC 4151 (day 351/1984; see §3.2) using method VI. In general, the quality of the Xenon spectra obtained by method VI was insufficient for spectral analysis.
Figure 2.3 Examples of background-subtracted pulse height spectra from the EXOSATME experiment. (a) The Argon spectrum for an observation of NGC 4151 (day 002/1985 as described in §3.2). The Xenon spectrum corresponding to (a). Both of the spectra (a) and (b) were obtained using method IV of background subtraction described in §2.1.1.2. (c) The Argon spectrum for another observation of NGC 4151 (day 351/1984 as described in §3.2). This spectrum was obtained using method VI of background subtraction described in §2.1.1.2.
2.2 *Ginga*

*Ginga* (originally named ASTRO-C) was launched on 5th February, 1987 from the Kagoshima Space Centre in Japan. Table 2.4 gives some details of the (low-Earth) orbit of *Ginga* (*cf.* Table 2.1 for *EXOSAT*). Further details may be found in Makino *et al.* (1987). The main components of the scientific payload are the Large Area Counter (LAC), the All Sky Monitor (ASM) and the Gamma-ray Burst Detector (GBD). Since all the *Ginga* data in this thesis comes from the LAC, we shall not comment further on the other two instruments. The LAC provided a natural follow up to the ME array aboard *EXOSAT*, with an improved sensitivity and an energy range of 1–37 keV.

<table>
<thead>
<tr>
<th><strong>Table 2.4 The <em>Ginga</em> orbit</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Launch date</strong></td>
</tr>
<tr>
<td><strong>Initial apogee</strong></td>
</tr>
<tr>
<td><strong>Initial perigee</strong></td>
</tr>
<tr>
<td><strong>Eccentricity</strong></td>
</tr>
<tr>
<td><strong>Inclination</strong></td>
</tr>
<tr>
<td><strong>Period</strong></td>
</tr>
<tr>
<td><strong>Attitude system</strong></td>
</tr>
<tr>
<td><strong>Pointing accuracy</strong></td>
</tr>
<tr>
<td><strong>Sun angle</strong></td>
</tr>
<tr>
<td><strong>Re-entry</strong></td>
</tr>
</tbody>
</table>
2.2.1 The Large Area Counter

The LAC consists of 8 proportional counters, each with its own anti-coincidence and internal guard monitors. The total effective area is $\sim 4000 \text{ cm}^2$. Each of the 8 detectors is separated into two layers (identified as 'top' and 'mid'). The top layer is sensitive to X-rays in the 1–37 keV band while the mid layer is sensitive to X-rays in the 8–37 keV band and has twice the stopping power of the top layer. A summary of the specification of the LAC is given in Table 2.5. Further details can be found in Turner et al. (1989a).

Data from the LAC can be accumulated in a number of different operating modes, each with its own combination of time and spectral resolution (see Turner et al., 1989a). All the Ginga data in this thesis was obtained with the LAC in MPC1 mode which gives a maximum time resolution down to 16 s, 48 pulse height channels and full detector identification. Separation of data from the top and mid layers of the LAC is important for weak sources in improving the signal-to-noise ratio.

<table>
<thead>
<tr>
<th>Description</th>
<th>8 proportional counters (75% Ar, 20% Xe, 5% CO$_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total effective area</td>
<td>4000 cm$^2$</td>
</tr>
<tr>
<td>Energy range (Top)</td>
<td>1 – 37 keV</td>
</tr>
<tr>
<td>Energy range (Mid)</td>
<td>8 – 37 keV</td>
</tr>
<tr>
<td>Energy resolution</td>
<td>$\Delta E/E \sim 4.4E^{-0.5}$ %</td>
</tr>
<tr>
<td>Entrance window</td>
<td>62$\mu$m, Beryllium</td>
</tr>
<tr>
<td>Pressure in all chambers</td>
<td>1.8 atm</td>
</tr>
<tr>
<td>Sensitivity limit</td>
<td>$2.6 \text{ ct s}^{-1}$ (2–10 keV) for a $3\sigma$ detection $\equiv 6 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ for a Crab–like spectrum</td>
</tr>
<tr>
<td>Field of view</td>
<td>$1^\circ \times 2^\circ$ (FWHM)</td>
</tr>
</tbody>
</table>
2.2.1.1 AUXILIARY DETECTORS ABOARD GINGA

The LAC has no provision for simultaneous monitoring of the X-ray background. Instead, there are a number House-Keeping (HK) count rates from the LAC and other instruments aboard Ginga which provide essential indirect information on the simultaneous radiation environment. The most important of these HK count rates are listed below.

- **SUD (Surplus Upper Discriminator)**. This is the count rate of X-ray like events whose energy deposit in the LAC exceeds some upper value, normally set at 24 keV. To first order, the SUD rate scales linearly with the LAC internal X-ray background.
- **S0L2**. This is the count rate from the solid state electron monitor.
- **PIM**. This is the count rate from the LAC sense wires which are screened from direct illumination. PIM normally scales linearly with SUD but some events cause this scaling to breakdown.
- **COR (Cut-Off Rigidity)**. This is a measure of the charged particle background. It is the momentum per elementary unit charge which a particle must have to penetrate from infinity to a given point in the Earth’s magnetosphere.

2.2.1.2 NATURE OF THE LAC BACKGROUND

The LAC internal background varies with time in a complex fashion. There appears to be a 24 hour period which is modulated by a 37 day period (see Hayashida et al., 1989). A typical Ginga observation consists of ~1 day pointing at the source and ~1 day pointing at blank sky in order that the X-ray background during the source pointing may be reconstructed. In order to illustrate the 24 hour periodicity in the raw data, Figs. 2.4a and 2.4b show the 2–10 keV LAC count rates, from the top and mid layers respectively, for a typical Ginga observation. The particular example in Fig. 2.4 is observation 6 of NGC 4151 described in §3.3 (see Table 3.4). Fig. 2.4c shows the corresponding SUD rate. The periods of high background every 24 hours can clearly be seen in the data. They correspond to orbits which pass through the South Atlantic Anomaly (SAA). Due to the location of the ground station, contact with the satellite always occurs during the orbits with high background and so are called ‘contact orbits’. In this thesis spectra accumulated during periods of low background (remote orbits) have been used whenever possible.
Figure 2.4 LAC count rates during a typical Ginga observation (observation 6 of NGC 4151 described in §3.3). In this observation the source is observed between \( \sim 0 - 20 \) hours while the remainder of the time is devoted to blank sky pointing. *All* the raw data is shown, including anomalous background events. (a) The 2–10 keV LAC count rates from the top layer. (b) The corresponding LAC count rate from the mid layer. (c) The SUD rate corresponding to (a) and (b).
Hayashida et al. (1989) give a detailed discussion of all the different contributions to the LAC background. Here we simply summarize these contributions below.

- **Diffuse X-ray background.** This component represents a significant contribution, due mainly to the large field of view of the LAC. It amounts to \( \sim 18 \text{ ct s}^{-1} \) (summed over top and mid layers, between 1–37 keV) out of a total background count rate of \( \sim 70–150 \text{ ct s}^{-1} \). The flux from the diffuse X-ray background varies from place to place in the sky but is constant with time and remains an ultimate source of uncertainty in spectral measurements from the LAC.

- **Cosmic rays.** Although charged particles entering the LAC can be rejected, Compton interactions of \( \gamma \)-rays produced in the material outside the detectors can still contribute to the internal background. This component of the background is strongly dependent on COR.

- **Geomagnetically trapped charged particles.** These can contribute to the LAC internal background in a variety of ways (see Hayashida et al., 1989) but cannot be adequately modelled. Such events must therefore be rejected at an early stage in the analysis. The most prominent signatures are usually a substantial increase in the SOL2 rate and a breakdown of the linearity of the SUD–PIM relation.

- **Fluorescence.** Several line features occur due to fluorescence of material in various parts of the LAC.

- **Radioactive decays.** The characteristic exponential decay of the X-ray background after a passage through the SAA (see Fig. 2.4) is due to the creation of radioactive isotopes in the material of the spacecraft and the LAC. At least four half-lives have been identified in the data (see Hayashida et al., 1989). The longest of these is of the order of tens of days and so is not important to model for our purpose.

- **Solar contamination.** Occasionally, the angle between the LAC pointing direction and the sun may be less than \( 90^\circ \), in which case it is possible that solar X-rays may enter the detectors. The main effect is then to produce excess flux in the spectrum between \( \sim 1–3 \text{ keV} \). Solar contamination may be identified by comparing spectra obtained when the satellite is in sunshine to those when the satellite is in the Earth's shadow. In such cases only data which is sunshine-free or which is from LAC detectors in the shadow of the solar panels may be used. If neither of these options is possible then only data above \( \sim 4 \text{ keV} \) may be used as a last resort.

- **Contamination from the bright limb of the Earth.** This can occur when the angle between the LAC pointing direction and the Earth's horizon angle lies between \( -90^\circ \) and \( \sim 6^\circ \).
2.2.1.3 LAC BACKGROUND SUBTRACTION

We now outline the procedure that we have used to obtain background-subtracted light curves and spectra from the Ginga data presented in this thesis.

The raw data (both source and blank sky observations) is first inspected and data is rejected according to the criteria listed in Table 2.6. Any other suspect data points which are not removed by this process are removed manually. Next, the LAC count rate (during the blank sky observation) in pulse height channel $j$ is parameterized as a linear function of six quantities, $b_i$, which are related to the X-ray background. A series of least-squares fits for each pulse height channel then yields $48 \times 6 = 288$ best-fit coefficients, $a_{ji}$. These can then be used to predict the LAC count rate, $C_j$, in pulse height channel $j$, due to the X-ray background, at any other time within $\sim 2$ days of the blank sky observation (see equation 2.1 below).

$$C_j = \sum_{i=1}^{i=6} a_{ji}b_i$$  \hspace{1cm} (2.1)

The $b_i$ correspond to (i) a constant component (i.e. $b_1 = 1$), (ii) SUD, (iii) COR and (iv) three orbital history parameters (called SAA1, SAA2 and SAA3) which model the contribution due to radioactive isotopes formed in the SAA. The half-lives we used throughout the present work are 0.5, 1.0 and 12.5 hours.

Equation 2.1 can then be used to predict the X-ray background throughout the source observation. This background is then subtracted from the on-source data. Finally, the data must be attitude-corrected if the source does not lie on the axis of the LAC. The required renormalization of the count rates is computed from data obtained from many off-axis observations of the Crab Nebula. The small energy dependent corrections due to reflection in the collimator have been neglected in this work. Final light curves of the source may then be obtained by summing count rates over the desired energy range for each time bin. Likewise, spectra may be obtained by summing count rates for each pulse height channel over the desired time bins.

The method of background subtraction described above is used throughout this thesis and corresponds to method I in Hayashida et al. (1989). The alternative method of constructing a 'universal background' on the basis of many blank sky observations, covering a period of many months, has not been used. Ideally, the blank sky observation should cover at least a 24 hour period in order that the $a_{ji}$ are well constrained. However, the blank sky observation is quite often shorter than 24 hours. In such cases it is sometimes possible to utilize two (or more) independent blank sky observations which are close in time to the source observation but at different locations in the sky.
Fig. 2.5 shows an example of a background-subtracted spectrum from the top and mid layers of the LAC. The example corresponds to spectrum 6b for NGC 4151 described in §3.3.

Table 2.6 Summary of Ginga data selection procedure

1. Reject data contaminated by solar X-rays (see text).

2. Reject data points which lie further than 3σ from the best-fitting straight line in the SUD–PIM plane (see text).

3. Reject data for which the SUD rate lies outside the range 0–15 ct s⁻¹.

4. Reject data for which the Cut-Off Rigidity (COR) lies outside the range 7–20 GeV/c / count.

5. Restrict the Earth–horizon elevation angle to values > 6°.

6. Reject data when the soft electron monitor (SOL2) count rate is > 15.
Figure 2.5 An example of a background-subtracted pulse height spectrum from the LAC. The example corresponds to spectrum 6b for NGC 4151 described in §3.3. The upper panel shows the count spectrum from the top layer of the LAC while the lower panel shows the count spectrum from the mid layer.
2.3 Spectral Analysis

Once a background-subtracted spectrum of the source has been obtained (either from EXOSAT or Ginga), the range of pulse height channels over which spectral analysis is to be performed is selected. The upper limit is usually determined by the decreasing signal to noise ratio. It is not possible to infer the form of the incident photon spectrum by deconvolving the pulse height spectrum with the instrument response since there is no unique solution. Instead, a trial spectrum characterized by a number of free parameters is folded with the instrument response to produce a predicted pulse height count spectrum. This is compared with the actual data by means of the reduced chi-square statistic (e.g. see Bevington, 1969) and the parameters of the trial model spectrum are adjusted in an iterative manner until the value of the reduced chi-square is minimized.

If the spectral fit has an acceptable value of the reduced chi-square (see Bevington, 1969) and the residuals (data – model) are not too large, the trial model with the best-fitting values of the parameters may be said to be a good description of the data. The errors on the best-fitting parameters in all the spectral fitting performed in the present work correspond to 90% confidence limits. The number of interesting parameters will be stated in each case as it arises (see Lampton, Margon & Bower, 1976).

When the trial model cannot be expressed in an analytical form, the model photon flux is computed at a finite number of discrete energies. The photon flux at any other energy within the range is then obtained by linear interpolation in log-log space. This is the case with the ‘warm absorber’ model described in §5.4.
Chapter 3

EXOSAT and Ginga observations of NGC 4151

Overview

In this chapter we report on the results of an extensive monitoring programme of the archetypal Seyfert galaxy NGC 4151 with EXOSAT and Ginga covering a period of \( \sim 5.5 \) years. A simple spectral analysis of the data is presented, while detailed interpretation of the behaviour of the X-ray continuum and X-ray absorption is deferred until chapters 4 and 5 respectively. The results of this preliminary analysis are discussed in the context of observations of NGC 4151 by previous X-ray astronomy missions.

"There's no dark side of the moon really ...
As a matter of fact, it's all dark."

(Roger the Hat, 1972. From 'The Dark Side of the Moon.')
3.1 Historical perspective

As stated in §1.1, the Seyfert galaxy NGC 4151, being one of the brightest objects of its class, has been extensively monitored in all wavebands. In §4.1 we will need to review the properties of the radio, infrared, optical, ultraviolet and γ-ray continua of NGC 4151 in order to interpret the results of the study of the X-ray spectra. In the present chapter we first review our understanding of the X-ray spectrum of NGC 4151 prior to the launch of EXOSAT. Then we report on the results of an extensive ~ 5.5 year monitoring of the X-ray emission from NGC 4151 by EXOSAT and Ginga, covering the period 1983 July to 1989 January.

3.1.1 Previous observations and spectral parameters

NGC 4151 was first detected as an X-ray source by the UHURU satellite in December 1970 (Gursky et al., 1971). The source has been observed by all subsequent X-ray astronomy missions since UHURU. The temporal behaviour of the X-ray emission is characterized by sluggish variability with a typical flux doubling timescale of ~ 5 × 10^4 – 10^5 s, accompanied by frequent flaring events occurring on a timescale of days (Lawrence, 1980; Perola et al., 1986). The spectrum of NGC 4151 above 1 keV is characterised by a hard power-law continuum, which may extend out to 1 MeV (Baity et al., 1984). The X-ray spectrum also exhibits a low energy cut-off corresponding to an absorbing column of apparently cold gas of ~ 5 – 15 × 10^22 cm^-2 (Barr et al., 1977; Mushotsky, Holt & Serlemitsos 1978; Hall et al., 1981) and an intense 6.4 keV iron line probably due to the fluorescence of cold gas illuminated by the continuum X-rays (Matsuoka et al., 1986). An observation by the solid-state spectrometer (SSS) on the Einstein observatory showed the attenuation in the spectrum below 2 keV to be incompatible with a uniform screen of cold absorbing gas, a result which has been interpreted by Holt et al. (1980) in terms of a partial covering model. In this model the SSS observations can be explained if ~ 90% of the central X-ray source in NGC 4151 is obscured by an absorbing screen with a column density of ~ 6 × 10^22 cm^-2, the remaining fraction being uncovered and hence giving rise to the observed soft X-ray excess. More recently EXOSAT observations have shown that there is a separate non-varying spectral component present below 1 keV, possibly attributable to thermal emission from within the narrow line region of this galaxy (see §1.3.3). The EXOSAT spectra above 1 keV confirm, however, the presence of a soft X-ray flux in excess of that expected from a source which is covered by a uniform distribution of cold gas (Pounds et al., 1986a; Perola et al., 1986). Spectral changes appear to accompany variability in the 2–10 keV flux from NGC 4151 and in this context Perola et al. (1986) report a correlation between spectral index and unabsorbed flux in the sense that the source spectrum steepens as its X-ray luminosity increases.
### Table 3.1 NGC 4151 X-ray spectral measurements 1977-1986

<table>
<thead>
<tr>
<th>Epoch</th>
<th>Satellite</th>
<th>$\Gamma$</th>
<th>$n_H$</th>
<th>$n_{Fe}$</th>
<th>$I_{Fe}$</th>
<th>$F_X$</th>
<th>Refs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977/May</td>
<td>OSO-8</td>
<td>1.42 ± 0.06</td>
<td>75 ± 5</td>
<td>—</td>
<td>4.1±4.6</td>
<td>15.2</td>
<td>1</td>
</tr>
<tr>
<td>1977/Dec</td>
<td>HEAO 1/A2</td>
<td>1.43 ± 0.08</td>
<td>100 ± 20</td>
<td>—</td>
<td>3.7 ± 1.7</td>
<td>15.0</td>
<td>2</td>
</tr>
<tr>
<td>1978/June</td>
<td>HEAO 1/A2</td>
<td>1.55±0.05</td>
<td>58±3</td>
<td>250 ± 70</td>
<td>6.8 ± 3.2</td>
<td>27.9</td>
<td>3</td>
</tr>
<tr>
<td>1980/Jan</td>
<td>ARIEL VI</td>
<td>1.51 ± 0.15</td>
<td>80 ± 18</td>
<td>—</td>
<td>7.5 ± 3.5</td>
<td>20.8</td>
<td>4</td>
</tr>
<tr>
<td>1984/Jan</td>
<td>TENMA</td>
<td>1.34 ± 0.11</td>
<td>80 ± 18</td>
<td>140 ± 40</td>
<td>2.4 ± 0.5</td>
<td>6.6</td>
<td>5</td>
</tr>
<tr>
<td>1984/Mar</td>
<td>TENMA</td>
<td>1.44 ± 0.09</td>
<td>68 ± 6</td>
<td>160 ± 30</td>
<td>2.4 ± 0.6</td>
<td>6.9</td>
<td>5</td>
</tr>
<tr>
<td>1983/Nov-Dec</td>
<td>EXOSAT</td>
<td>1.67 ± 0.13</td>
<td>94 ± 15</td>
<td>89 ± 76</td>
<td>2.8 ± 0.8</td>
<td>8.3 - 14.8</td>
<td>6,7</td>
</tr>
<tr>
<td>1984-5/Dec-Jan</td>
<td>EXOSAT</td>
<td>1.60 ± 0.03</td>
<td>54 ± 3</td>
<td>112 ± 21</td>
<td>1.7 ± 0.5</td>
<td>17.8 - 40.1</td>
<td>6,7</td>
</tr>
<tr>
<td>1985/Apr-Jun</td>
<td>EXOSAT</td>
<td>1.60 ± 0.07</td>
<td>72 ± 8</td>
<td>56 ± 46</td>
<td>1.8 ± 0.7</td>
<td>8.1 - 23.6</td>
<td>7</td>
</tr>
<tr>
<td>1986/Mar</td>
<td>EXOSAT</td>
<td>1.54 ± 0.11</td>
<td>80 ± 20</td>
<td>27±23</td>
<td>3.8 ± 1.1</td>
<td>8.0 - 13.0</td>
<td>7</td>
</tr>
</tbody>
</table>

**Footnotes:**

- $a$ Equivalent Hydrogen column in units of $10^{21}$ cm$^{-2}$.
- $b$ Iron column in units of $10^{16.5}$ cm$^{-6}$.
- $c$ Iron emission line intensity in units of $10^{-4}$ photons cm$^{-2}$ s$^{-1}$.
- $d$ 2–10 keV absorption corrected flux in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$.

**References:**

7. §3.2, this work.
Table 3.1 (taken from Warwick et al., 1989a) shows measurements of the spectral parameters of NGC 4151 from various X-ray astronomy missions, during the period 1977 - 1986, for a simple power-law model with low-energy cut-off. \( \Gamma \) is the photon spectral index, \( n_H \) is the equivalent Hydrogen column density, \( n_{Fe} \) is the column density of iron normalized to the solar abundance, \( I_{Fe} \) is the intensity of the \( K_\alpha \) iron emission line and \( F_{2-10}^e \) is the 2-10 keV flux corrected for absorption. Hereafter we use the term \( I_{Fe,4} \) for the iron line intensity when the line energy is specifically fixed at 6.4 keV and \( I_{Fe} \) otherwise (the line energy was floating in some of the fits in Table 3.1). We discuss the implications of the range of the quantities in Table 3.1 in the light of the most recent X-ray observations of NGC 4151 in §3.4 below. The entries for EXOSAT in Table 3.1 are averages taken over different epochs (see §3.2.3).

3.2 The EXOSAT observations

EXOSAT observed NGC 4151 on a total of 24 occasions in the period 1983 July to 1986 March as summarized in Table 3.2 (for convenience in the later discussion, Table 3.2 shows the observations divided into six groups). The observations were typically of 4 to 11 hours duration, the exceptions being those on days 193/1983, 027/1985 and 060/1986, which spanned over 16 hours. In all the observations, pulse height spectra covering an energy range 1.5–25 keV were recorded by the medium energy (ME) detector array (see §2.1.2). Additional flux measurements were obtained in the 0.15–2.0 keV band from the low energy (LE) telescope plus channel multiplier array detector (see §2.1.1). Note that some results of various sub-sets of the EXOSAT observations listed in Table 3.2 have already been published (Pounds et al., 1986a; Perola et al., 1986; Fiore et al., 1989 and 1990). All the work in §3.2 has been published in Yaqoob, Warwick & Pounds (1989).

3.2.1 LE data analysis and results

The LE observations were made through either the thin Lexan (3Lx), Aluminium/ Parylene (Al/P) or Boron (B) X-ray filters. In some of the observations all three filters were deployed in sequence, but in other instances the filter measurements were more limited. The count rates measured in the 3Lx and Al/P filters, after background subtraction, are given in Table 3.2. The quoted values include corrections for scattering in the filters, the instrumental response function and dead-time (see §2.1.1.1). The B filter measurements (for the observations indicated in Table 3.2) were all consistent with a marginal detection at the level of \( 2.5 \pm 0.3 \) ct/1000 s.
<table>
<thead>
<tr>
<th>Day/Year</th>
<th>ME cts/s</th>
<th>Hardness ratio</th>
<th>Softness ratio</th>
<th>3Lx ct/10^3s</th>
<th>Al/P ct/10^3s</th>
</tr>
</thead>
<tbody>
<tr>
<td>193/1983*</td>
<td>3.89 ± 0.02</td>
<td>0.68 ± 0.01</td>
<td>0.52 ± 0.01</td>
<td>22.8 ± 1.2</td>
<td>11.2 ± 1.0</td>
</tr>
<tr>
<td>311/1983</td>
<td>2.64 ± 0.04</td>
<td>0.73 ± 0.02</td>
<td>0.50 ± 0.02</td>
<td>25.4 ± 3.1</td>
<td>11.1 ± 1.4</td>
</tr>
<tr>
<td>315/1983</td>
<td>2.37 ± 0.04</td>
<td>0.70 ± 0.03</td>
<td>0.47 ± 0.02</td>
<td>18.6 ± 3.0</td>
<td>9.6 ± 1.9</td>
</tr>
<tr>
<td>319/1983*</td>
<td>1.65 ± 0.03</td>
<td>0.61 ± 0.02</td>
<td>0.52 ± 0.02</td>
<td>27.8 ± 3.0</td>
<td>8.6 ± 1.0</td>
</tr>
<tr>
<td>323/1983*</td>
<td>1.93 ± 0.05</td>
<td>0.65 ± 0.04</td>
<td>0.46 ± 0.03</td>
<td>17.1 ± 2.7</td>
<td>10.6 ± 2.5</td>
</tr>
<tr>
<td>351/1983*</td>
<td>2.59 ± 0.04</td>
<td>0.59 ± 0.02</td>
<td>0.54 ± 0.02</td>
<td>21.1 ± 1.4</td>
<td>-</td>
</tr>
<tr>
<td>098/1984*</td>
<td>1.07 ± 0.04</td>
<td>0.62 ± 0.06</td>
<td>0.69 ± 0.05</td>
<td>21.9 ± 3.0</td>
<td>8.5 ± 1.8</td>
</tr>
<tr>
<td>109/1984*</td>
<td>1.06 ± 0.06</td>
<td>0.49 ± 0.05</td>
<td>0.54 ± 0.05</td>
<td>23.1 ± 2.7</td>
<td>10.6 ± 1.9</td>
</tr>
<tr>
<td>154/1984*</td>
<td>1.63 ± 0.06</td>
<td>0.71 ± 0.06</td>
<td>0.62 ± 0.05</td>
<td>19.4 ± 4.3</td>
<td>9.2 ± 2.6</td>
</tr>
<tr>
<td>351/1984</td>
<td>4.56 ± 0.10</td>
<td>0.64 ± 0.03</td>
<td>0.51 ± 0.02</td>
<td>23.4 ± 3.4</td>
<td>10.0 ± 1.7</td>
</tr>
<tr>
<td>354/1984</td>
<td>6.04 ± 0.04</td>
<td>0.63 ± 0.01</td>
<td>0.52 ± 0.01</td>
<td>23.8 ± 2.7</td>
<td>11.2 ± 1.7</td>
</tr>
<tr>
<td>357/1984*</td>
<td>7.42 ± 0.04</td>
<td>0.58 ± 0.01</td>
<td>0.59 ± 0.01</td>
<td>-</td>
<td>12.2 ± 2.0</td>
</tr>
<tr>
<td>359/1984*</td>
<td>6.47 ± 0.05</td>
<td>0.61 ± 0.01</td>
<td>0.59 ± 0.01</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>363/1984*</td>
<td>3.49 ± 0.05</td>
<td>0.63 ± 0.02</td>
<td>0.54 ± 0.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>002/1985*</td>
<td>9.30 ± 0.03</td>
<td>0.55 ± 0.01</td>
<td>0.67 ± 0.01</td>
<td>23.9 ± 2.9</td>
<td>11.6 ± 1.7</td>
</tr>
<tr>
<td>027/1985</td>
<td>4.10 ± 0.02</td>
<td>0.60 ± 0.01</td>
<td>0.63 ± 0.01</td>
<td>23.3 ± 1.5</td>
<td>12.2 ± 1.7</td>
</tr>
<tr>
<td>111/1985</td>
<td>1.76 ± 0.03</td>
<td>0.66 ± 0.02</td>
<td>0.52 ± 0.02</td>
<td>18.4 ± 2.3</td>
<td>9.3 ± 1.9</td>
</tr>
<tr>
<td>118/1985</td>
<td>2.81 ± 0.03</td>
<td>0.68 ± 0.02</td>
<td>0.52 ± 0.02</td>
<td>19.2 ± 2.0</td>
<td>13.2 ± 2.1</td>
</tr>
<tr>
<td>128/1985</td>
<td>3.94 ± 0.05</td>
<td>0.62 ± 0.02</td>
<td>0.56 ± 0.02</td>
<td>20.4 ± 2.6</td>
<td>8.3 ± 1.9</td>
</tr>
<tr>
<td>135/1985*</td>
<td>4.58 ± 0.03</td>
<td>0.61 ± 0.01</td>
<td>0.59 ± 0.01</td>
<td>21.9 ± 1.9</td>
<td>8.4 ± 1.8</td>
</tr>
<tr>
<td>143/1985*</td>
<td>4.37 ± 0.08</td>
<td>0.58 ± 0.02</td>
<td>0.55 ± 0.02</td>
<td>18.2 ± 3.3</td>
<td>-</td>
</tr>
<tr>
<td>150/1985</td>
<td>2.87 ± 0.04</td>
<td>0.66 ± 0.02</td>
<td>0.50 ± 0.02</td>
<td>18.9 ± 2.3</td>
<td>7.5 ± 2.2</td>
</tr>
<tr>
<td>060/1986</td>
<td>2.51 ± 0.04</td>
<td>0.68 ± 0.02</td>
<td>0.55 ± 0.02</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>062/1986*</td>
<td>2.00 ± 0.06</td>
<td>0.75 ± 0.05</td>
<td>0.47 ± 0.04</td>
<td>22.7 ± 3.3</td>
<td>-</td>
</tr>
</tbody>
</table>

Footnotes:

a 3.5-6.0 keV energy range.
b 6-10 keV : 3.5-6.0 keV hardness ratio.
c 1.5-3.5 keV : 3.5-6.0 keV softness ratio.
d Detectors 5,8 only. Argon spectra only.
e Detectors 1,4 only. Argon spectra only.
f Boron filter data available.
Fig. 3.1a shows the light curve obtained in the 0.15-1.0 keV band of the 3Lx filter. The soft X-ray flux of NGC 4151 remained essentially constant over the 32 month period of the EXOSAT monitoring. This is confirmed by a chi-square test on the 3Lx count-rates, which gives a reduced chi-square of 1.13 (19 degrees of freedom) for a constant flux hypothesis; conversely the best-fitting linear drift rate is $-3.2 \pm 3.1 \%$ per year. The Al/P filter measurements are also consistent with a constant count-rate, giving a reduced chi-square of 0.90 (17 degrees of freedom) and a best-fitting linear drift rate of $-0.5 \pm 4.8 \%$ per year. Similar results have been obtained in earlier analyses of sub-sets of the EXOSAT data (Pounds et al., 1986a; Perola et al., 1986).

![Figure 3.1](image)

**Figure 3.1** Long term EXOSAT light curves for NGC 4151. (a) The LE light curve obtained in the 0.15 - 1.0 keV band of the 3Lx filter. (b) The corresponding ME light curve in the 3.5 - 6.0 keV band.
3.2.2 ME data analysis and results

In the majority of the observations a standard observing mode was employed whereby one half of the ME array was offset from the source position to monitor the background count-rate, with the array halves being inter-changed part-way through the observation to facilitate background subtraction (see §2.1.2). In the longer observations this 'nodding' procedure was repeated more than once (except for day 193/1983). Background-subtracted source spectra for each of the eight ME detectors were then obtained as the difference between pulse-height spectra taken in the on-source and offset configurations. In order to minimize the systematic error, in the present analysis we have specifically excluded data from the inner detectors, and restricted our attention to data from the corner detectors (detectors 1,4,5,8) taken during periods when the background count-rate in the offset detectors was stable (i.e. we have used method IV for ME background subtraction described in 2.1.2.2). Unfortunately, in a number of observations the inter-changing of the offset detectors was not performed. In these cases we have used corner detector data taken during slew manoeuvres both before and after the source observation to perform a background subtraction (i.e. method VI described in §2.1.2.2).

Our analysis is based on the pulse height spectra from both the Argon (1.5–15 keV) and Xenon (8–25 keV) cells of the ME detectors except for the slew-subtracted data sets (method VI, §2.1.2.2) where only the Argon spectra proved to be of sufficient quality to be useful.

Table 3.2 gives the average ME count rate (summed over four detectors) in the 3.5–6.0 keV range for each of the observations. This narrow energy range avoids the region of the spectrum strongly influenced by absorption and hence provides a useful measure of the level of the X-ray continuum independent of column changes. The corresponding light-curve is plotted in Fig. 3.1b. In contrast to the stability of the soft X-ray emission, the 3.5–6.0 keV flux from NGC 4151 varied by a factor of ~ 10 over the period of the EXOSAT monitoring, with the most sudden change being a factor of ~ 2.5 increase over 5 days in December 1984/January 1985.

Significant spectral variations appear to accompany the flux changes observed in NGC 4151 as is apparent from the 6-10 keV/3.5-6.0 keV hardness ratio and 1.5-3.5 keV/3.5-6.0 keV softness ratio measurements listed in Table 3.2 for each observation. If we first consider the restricted period covered by the fourth and fifth groups of observations (i.e. the six month interval from late 1984 to mid 1985) then both the hardness and softness ratio values appear to be related to the level of the continuum flux (as measured by the 3.5–6.0 keV count rate) as shown in Fig. 3.2. The linear correlation coefficient for the data in Fig. 3.2a (hardness ratio versus ME flux) is -0.90 and for Fig. 3.2b (softness ratio versus ME flux) is 0.65, which correspond to correlations significant at the 99.99% and 97.00% confidence level respectively. Both effects are in the sense that the X-ray spectrum softens as the X-ray flux increases. These correlations are more masked if the full data set
is considered suggesting that the presence of relatively long term random variations superimposed on an underlying process which gives rise to the observed spectral softening versus flux effect; we shall have more to say about this below. The softening of the spectrum as the X-ray flux increases is at first sight consistent with the changes in the continuum slope with flux, reported by Perola et al. (1986) from an analysis of a sub-set of the EXOSAT data. In the following sections, however, we show that the quality of the EXOSAT data is insufficient to confirm this effect.

![Figure 3.2](image_url)  
**Figure 3.2** (a) The 6.0 - 10.0/3.5 - 6.0 keV hardness ratio plotted against the 3.5 - 6.0 keV ME count rate for the fourth and fifth groups of EXOSAT observations (see Table 3.2). (b) The 1.5 - 3.5/3.5 - 6.0 keV softness ratio plotted as in (a).
TEMPORAL BEHAVIOUR OF THE EXOSAT LIGHT CURVES

We have also investigated the characteristics of the relatively short-timescale variability exhibited by NGC 4151. Fig. 3.3 shows the 3.5–6.0 keV light curves obtained in the three longest EXOSAT observations. Slow drifts in the X-ray flux, with a flux doubling timescale of $\sim 5 \times 10^4 - 10^5$ s, are apparent, with some evidence for more rapid variations at a much lower level. Perola et al. (1986) and Warwick (1986) have reported similar drift rates in some of the shorter EXOSAT observations. These slow, almost continuous, changes in the level of the X-ray continuum of NGC 4151 represent rather atypical behaviour, at least amongst the present set of well studied low luminosity active galaxies (Warwick, 1986). We have searched for spectral changes on a timescale of less than a day by dividing the observations shown in Fig. 3.3 into sections and examining the hardness and softness ratio (as defined earlier) for these periods. This analysis revealed no evidence for significant spectral changes (i.e. hardness and softness ratio variations $< 0.05$) within the individual data sets.

More detailed studies of the short and long-term temporal behaviour and power spectrum of some of the EXOSAT observations reported here have been carried out by Fiore et al. (1989), Lehto (1989) and McHardy (1989) and references therein. In the present work we are concerned mainly with the spectral behaviour.
Figure 3.3.5 - 6.0 keV light curves for the EXOSAT observations on days 193/1983, 027/1985, 060/1986. The data are binned at 500 s. The gaps in the light curves correspond to periods of relatively high background variability.
3.2.3 Spectral fitting above 3.5 keV

As stated in §3.1.1, the X-ray spectrum of NGC 4151 below a few keV cannot be described adequately as a power-law continuum with a low-energy cut-off due to absorption in a uniform distribution of cold gas. Due to the potential complexity of the low energy spectrum of NGC 4151, we therefore follow the procedure adopted in the earlier studies of first examining the spectrum above 3.5 keV.

We have performed a standard spectral fitting analysis for each of the observations (excluding the third group of observations for which the statistical errors were large) assuming a model consisting of a power-law continuum of photon spectral index, $\Gamma$, plus low energy absorption due to a uniform column, $n_H$, of cold gas with cross-sections and abundances as in Morrison & McCammon (1983). The one exception was that the elemental abundance of iron was allowed to be a free parameter by independently fitting to the column density of cold iron, $n_{Fe}$. An iron emission line at 6.4 keV with intensity, $I_{6.4}$, was also included giving a final model with five free parameters. Table 3.3 lists the best-fitting values of the parameters obtained for each of the data sets, including the integrated model flux in the 2-10 keV band, $F_x$, and the corresponding flux after correcting for absorption, $F_x^c$. The quoted errors correspond to 90% confidence limits assuming four interesting parameters (see §2.3). The total number of degrees of freedom in the fitting was 68 in the case of the Argon plus Xenon spectra and 32 for the Argon data alone. Figs. 3.4a - 3.4c show the pulse height spectrum, best-fit model extrapolated down to 2 keV and residuals for the observations on days 002/1985, 111/185 and 135/1985 respectively. The 2 - 4 keV soft excess is evident from the residuals in each case. Fig. 3.4d shows the corresponding incident photon spectra for these three observations. The three spectra illustrate how the spectral shape changes as NGC 4151 progresses from a low to a high flux state.
Table 3.3 EXOSAT spectral fitting results above 3.5 keV

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$F_x$ a</th>
<th>$F_H$ a</th>
<th>$\Gamma$</th>
<th>$n_H$ b</th>
<th>$n_F_x$ c</th>
<th>$I_{6.4}$ d</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>193/1983</td>
<td>12.9</td>
<td>20.2</td>
<td>1.23±0.11</td>
<td>52±17</td>
<td>223±98</td>
<td>1.4±1.8</td>
<td>0.95</td>
</tr>
<tr>
<td>311/1983</td>
<td>8.9</td>
<td>15.2</td>
<td>1.50±0.35</td>
<td>91±47</td>
<td>53±243</td>
<td>3.8±3.1</td>
<td>1.34</td>
</tr>
<tr>
<td>315/1983</td>
<td>7.8</td>
<td>12.6</td>
<td>1.80±0.50</td>
<td>73±55</td>
<td>74±503</td>
<td>3.5±3.3</td>
<td>0.99</td>
</tr>
<tr>
<td>319/1983</td>
<td>5.4</td>
<td>7.5</td>
<td>1.76±0.48</td>
<td>81±56</td>
<td>175±341</td>
<td>2.4±2.2</td>
<td>1.41</td>
</tr>
<tr>
<td>323/1983</td>
<td>6.2</td>
<td>9.0</td>
<td>1.80±0.61</td>
<td>90±54</td>
<td>0.0±0.0</td>
<td>1.6±1.6</td>
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</tr>
<tr>
<td>351/1983</td>
<td>8.6</td>
<td>12.4</td>
<td>1.35±0.69</td>
<td>96±74</td>
<td>223±534</td>
<td>2.7±2.4</td>
<td>1.06</td>
</tr>
<tr>
<td>351/1984</td>
<td>14.9</td>
<td>22.2</td>
<td>1.32±0.43</td>
<td>51±35</td>
<td>327±387</td>
<td>3.3±6.5</td>
<td>1.22</td>
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<tr>
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<td>19.5</td>
<td>28.9</td>
<td>1.35±0.25</td>
<td>92±49</td>
<td>81±211</td>
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<td>1.11</td>
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<td>23.7</td>
<td>32.7</td>
<td>1.63±0.12</td>
<td>62±13</td>
<td>125±75</td>
<td>0.0±2.0</td>
<td>1.20</td>
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<td>359/1984</td>
<td>20.8</td>
<td>29.7</td>
<td>1.54±0.18</td>
<td>56±16</td>
<td>64±110</td>
<td>0.0±3.1</td>
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<tr>
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<td>11.2</td>
<td>16.6</td>
<td>1.55±0.27</td>
<td>73±44</td>
<td>238±259</td>
<td>0.0±2.9</td>
<td>0.91</td>
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<tr>
<td>002/1985</td>
<td>29.9</td>
<td>37.6</td>
<td>1.64±0.09</td>
<td>44±20</td>
<td>144±52</td>
<td>1.4±1.4</td>
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<tr>
<td>027/1985</td>
<td>13.2</td>
<td>18.3</td>
<td>1.53±0.10</td>
<td>47±12</td>
<td>80±47</td>
<td>2.6±1.3</td>
<td>1.47</td>
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<td>111/1985</td>
<td>5.8</td>
<td>8.5</td>
<td>1.44±0.40</td>
<td>57±50</td>
<td>34±273</td>
<td>1.7±2.2</td>
<td>1.13</td>
</tr>
<tr>
<td>118/1985</td>
<td>9.2</td>
<td>14.2</td>
<td>1.55±0.30</td>
<td>73±35</td>
<td>134±194</td>
<td>1.7±3.7</td>
<td>1.19</td>
</tr>
<tr>
<td>128/1985</td>
<td>12.7</td>
<td>18.8</td>
<td>1.75±0.29</td>
<td>86±40</td>
<td>324±268</td>
<td>1.9±1.9</td>
<td>1.06</td>
</tr>
<tr>
<td>135/1985</td>
<td>14.7</td>
<td>21.2</td>
<td>1.56±0.18</td>
<td>60±33</td>
<td>75±122</td>
<td>1.9±3.5</td>
<td>1.04</td>
</tr>
<tr>
<td>143/1985</td>
<td>13.5</td>
<td>19.3</td>
<td>1.94±0.46</td>
<td>99±56</td>
<td>214±21</td>
<td>1.4±3.9</td>
<td>0.99</td>
</tr>
<tr>
<td>150/1985</td>
<td>9.4</td>
<td>14.7</td>
<td>1.55±0.38</td>
<td>84±56</td>
<td>121±238</td>
<td>1.9±3.6</td>
<td>0.73</td>
</tr>
<tr>
<td>060/1986</td>
<td>8.6</td>
<td>13.8</td>
<td>1.55±0.24</td>
<td>81±24</td>
<td>23±163</td>
<td>3.4±1.8</td>
<td>0.76</td>
</tr>
<tr>
<td>062/1986</td>
<td>6.6</td>
<td>11.6</td>
<td>1.29±1.00</td>
<td>49±49</td>
<td>0.0±0.0</td>
<td>6.3±5.2</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Footnotes:

a In units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$.

b In units of $10^{21}$ cm$^{-2}$.

c Column density of cold iron in units of $10^{16.52}$ cm$^{-2}$.

d 6.4 keV emission line flux in units of $10^{-4}$ photon cm$^{-2}$ s$^{-1}$. 

55
Figure 3.4 2-25 keV spectra for the EXOSAT observations on days 002/1985, 135/1985 and 111/1985. (a) - (c) The pulse height spectrum, and best-fitting model for the five parameter fits above 3.5 keV (extrapolated down to 2 keV). A soft excess is apparent from the residuals in the lowest pulse height channels. (d) The incident photon spectra corresponding to observations 111/1985 (lower spectrum), 135/1985 (middle spectrum) and 002/1985 (upper spectrum).
SPECTRAL INDEX

There is only marginal evidence for changes in the slope of the X-ray continuum in NGC 4151 from the EXOSAT data. The weighted mean value of $\Gamma$ for all the observations is $1.55 \pm 0.04$, with only the first observation (day 193/1983) indicating a spectral slope significantly different to this value. Since this observation was made during the performance verification phase of the mission (prior to the adoption of the 'standard' weak source observing mode) and at a time when flaring was occasionally visible in the detectors monitoring the background, the derived X-ray spectrum may be subject to larger than normal systematic error. We do not confirm (see Fig. 3.5) the spectral slope versus flux correlation reported by Perola et al. (1986) in an independent analysis of a subset of the current data. Our measurements of $\Gamma$ are in good agreement with those of Perola et al. (1986) for the observations when the source was relatively bright but are inconsistent for the low flux states. A possible explanation of this discrepancy is that Perola et al. (1986) included data from the inner EXOSAT detectors in their analysis which, through the 'difference' correction (see §2.1.2), may have introduced significant systematic errors. Such errors would have most effect on the derived spectral parameters when the source flux is relatively low and, we estimate, would be of sufficient magnitude to produce the observed discrepancies. We note, however, that the analysis of the Ginga data to be reported in §3.3 does reveal significant spectral index variability which may be masked by the lower signal-to-noise level in the EXOSAT data. From Table 3.3 it can be seen that the errors associated with the remaining parameters are relatively large (due to the use of five free parameters in the spectral fitting and a relatively conservative error estimation criterion).

ABSORBING COLUMN

There is some evidence for changes in $n_H$, (see Table 3.3) which we shall discuss in §3.3.2 in the light of an analysis of Ginga data. The iron features, namely the 7.11 keV absorption edge and the 6.4 keV emission line, were only weakly constrained in most of the individual observations. Over the set of observations, the weighted mean values of $n_{Fe}$ and $I_{\text{Fe}}$ are $116 \pm 29 \times 10^{16.52}$ cm$^{-2}$ and $2.0 \pm 0.5 \times 10^{-4}$ photon cm$^{-2}$ s$^{-1}$ respectively; all the observations are, in fact, consistent with these two estimates. The corresponding weighted mean value of $n_H$ is $57 \pm 5 \times 10^{21}$ cm$^{-2}$, which implies an iron abundance with respect to the solar value of $2.0 \pm 0.5$ (90% errors). However, this may be an over-estimate of the true iron abundance if the light elements such Mg, Si and S, which give rise to the bulk of the observed absorption in the 1.5–3.5 keV range, are either themselves under-abundant or significantly ionized (see §5.4). These results are in general agreement with those obtained by Perola et al. (1986).
Figure 3.5 The photon spectral index, $\Gamma$, plotted against the absorption corrected flux, $F'_x$, (in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$). $\Gamma$ is measured above 3.5 keV from the EXOSAT data. Data points from the fourth and fifth groups of observations (see Table 3.2) are shown as filled circles and all other points as open circles.
3.2.4 The spectrum below 6 keV

Detailed modelling of the low energy spectrum down to 2 keV will be deferred until §5 where we investigate a variety of physical models of the X-ray absorbing system in NGC 4151 and other AGN. However, an important point concerning the low energy spectrum of NGC 4151 is that we must consider the possible contribution to the ME flux, particularly in the lowest energy channels, of sources other than the active nucleus of NGC 4151. The BL Lac object E1207.9+3945 is situated just ~ 5 arcminutes from NGC 4151 (Stocke et al., 1985) and therefore is always present as a confusing source within the 45 arcminute field of view of the ME detectors. Giommi et al. (1988) note that this source does not show significant X-ray variability and quote a flux of 1.4 \times 10^{-12} \text{ erg cm}^{-2}\text{s}^{-1} (0.3-3.5 \text{ keV}). Assuming a power-law spectrum with a photon index of 2 and low intrinsic absorption (properties which are consistent with Einstein measurements for this source and also typical of BL Lac objects), we can predict the ME count rate by folding the incident spectrum through the instrument response function. We find that the contribution to the count rates in the lowest ME channels corresponds to between 2%-15% of the signal from NGC 4151 (the 1.5–2.5 keV range is the worst affected given the relative spectral forms of the BL Lac and NGC 4151). A further possible contribution is from the non-varying soft X-ray source detected in the LE observations. However, using spectral parameters which provide a good fit to the LE filter measurements (see Pounds et al., 1986a), we find that this component never exceeds 2% of the NGC 4151 flux in the lowest ME channels. As a check of these estimates we have plotted the observed count rates in several of the lowest ME channels against the corresponding 3.5–6.0 keV count rate and used a parabolic extrapolation to determine the intercept at zero 3.5–6.0 keV flux. This analysis indicated a 'non-varying' count rate in the individual channels generally consistent with the predicted signal from the BL Lac object.

3.3 The Ginga observations

Ginga observed NGC 4151 on seven occasions in the period 1987 May/June to 1989 January, with the Large Area Counter (LAC). All pointings employed MPC1 mode which allows complete detector identification and full spectral resolution in 48 pulse height channels covering the full 1.2–37 keV energy range of the LAC (see §2.2.1). Separation of the top and middle layers of the LAC detectors gives an improved signal to noise ratio for weak sources. Table 3.4 gives an observing log of the observations along with a corresponding average 4-10 keV count rate in the top layer of the LAC after background subtraction. Note that the count rates in Table 3.4 have been normalized to eight detectors. The first observation was performed during the performance verification phase of the satellite and lasted approximately 3.6 days. Background subtraction was performed in the manner described in §2.2.1.3. Observations 1,2 and 3 in the present data set were affected by solar
contamination. In these cases only data from those detectors which were most shielded from the sun was used. Also, data obtained during periods of high solar activity was rejected completely (see also §2.2.1.2).

Table 3.4 Log of the Ginga observations.

<table>
<thead>
<tr>
<th>Obs</th>
<th>Date</th>
<th>UT</th>
<th>4-10 keV ct/s $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29/5/87</td>
<td>21:08 to</td>
<td>15.57 ± 0.03</td>
</tr>
<tr>
<td></td>
<td>31/5/87</td>
<td>14:42</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>21/11/87</td>
<td>16:11 to</td>
<td>21.36 ± 0.12</td>
</tr>
<tr>
<td></td>
<td>23/11/87</td>
<td>09:38</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>28/11/87</td>
<td>11:22 to</td>
<td>18.54 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>28/11/87</td>
<td>22:38</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7/12/87</td>
<td>04:12 to</td>
<td>51.72 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>7/12/87</td>
<td>23:18</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>17/12/87</td>
<td>20:38 to</td>
<td>44.95 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>18/12/87</td>
<td>11:54</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>13/12/88</td>
<td>12:43 to</td>
<td>30.74 ± 0.07</td>
</tr>
<tr>
<td></td>
<td>13/12/88</td>
<td>05:30</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7/1/89</td>
<td>00:06 to</td>
<td>58.22 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>7/1/89</td>
<td>21:17</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Attitude corrected count rates per 8 detectors.
3.3.1 Results

Background-subtracted light curves for all seven observations are presented in Fig. 3.6. The data has been binned up over periods corresponding to approximately 20-40 minutes integration time. The slow drifting behaviour of the continuum level of NGC4151 which is typical of this source (see §3.2.2), is again evident from the *Ginga* observations with the same flux doubling timescale of $\sim 5 \times 10^4 - 10^5$ s. A representative pulse height spectrum was extracted for each observation with the criteria of (i) relatively low background, (ii) a stable attitude of the LAC and (iii) an integration time which was long enough to obtain good statistics but not so long that significant spectral changes may have occurred. These criteria allowed two spectra each to be extracted for observations 1 and 6. Note that only the top layer of the LAC was utilized. The integration periods of the final nine spectra are shown by the arrows in Fig. 3.6.

3.3.2 Spectral fitting above 4 keV

In order to determine the spectral index of the power-law continuum we again adopt the policy of spectral fitting over a range which excludes the soft excess. The actual energy range we use here is 4-23 keV as opposed to the 3.5-25 keV range used with the *EXOSAT* data. The different choices of lower limits reflect the lowest pulse height channels in which the soft excess is thought to set in, while the upper limits correspond to the highest pulse height channel which is not dominated by noise. Again, a standard spectral fitting analysis has been performed for each of the spectra from the seven *Ginga* observations. We model the spectrum of NGC4151 above 4 keV as in §3.2.3, namely a power law with photon spectral index $\Gamma$, plus low energy absorption due to a uniform column, $n_H$, of cold gas with cross-sections and abundances taken from Morrison and McCammon (1983), except that the abundance of iron was allowed to float. This was done by independently fitting to the column density of cold iron, $n_{Fe}$, with the energy of the iron K-edge fixed at 7.11 keV (the value appropriate for neutral iron). An iron emission line at 6.4 keV with intensity, $I_{6.4}$, was also included giving a final model with five free parameters. The assumption that the iron (and therefore presumably all the absorbing material) is essentially neutral will be justified in §3.3.3.
Figure 3.6 The 4 - 10 keV light curves for the seven *Ginga* observations of NGC 4151. The time axes are calibrated to the start times for each observation given in Table 3.4. The arrows indicate the integration periods for which useful spectra were extracted.
Table 3.5 Ginga spectral fitting results above 4 keV

<table>
<thead>
<tr>
<th>Obs</th>
<th>$\Gamma$</th>
<th>$n_H$ a</th>
<th>$n_{Fe}$ b</th>
<th>$I_{0,4}$ c</th>
<th>$F_\gamma^d$</th>
<th>$\chi^2_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>$1.44^{+0.12}_{-0.10}$</td>
<td>$112^{+32}_{-26}$</td>
<td>$165^{+74}_{-67}$</td>
<td>$1.8^{+0.6}_{-0.6}$</td>
<td>9.4</td>
<td>1.06</td>
</tr>
<tr>
<td>1b</td>
<td>$1.44^{+0.10}_{-0.10}$</td>
<td>$108^{+24}_{-23}$</td>
<td>$189^{+60}_{-62}$</td>
<td>$2.0^{+0.5}_{-0.5}$</td>
<td>8.6</td>
<td>1.21</td>
</tr>
<tr>
<td>2</td>
<td>$1.39^{+0.12}_{-0.12}$</td>
<td>$84^{+29}_{-27}$</td>
<td>$154^{+76}_{-76}$</td>
<td>$2.9^{+0.8}_{-0.8}$</td>
<td>10.1</td>
<td>1.29</td>
</tr>
<tr>
<td>3</td>
<td>$1.48^{+0.20}_{-0.13}$</td>
<td>$101^{+46}_{-30}$</td>
<td>$168^{+21}_{-25}$</td>
<td>$2.5^{+1.0}_{-1.0}$</td>
<td>10.6</td>
<td>0.42</td>
</tr>
<tr>
<td>4</td>
<td>$1.68^{+0.05}_{-0.05}$</td>
<td>$96^{+11}_{-10}$</td>
<td>$179^{+29}_{-32}$</td>
<td>$1.4^{+0.6}_{-0.9}$</td>
<td>31.9</td>
<td>1.06</td>
</tr>
<tr>
<td>5</td>
<td>$1.67^{+0.06}_{-0.03}$</td>
<td>$89^{+12}_{-8}$</td>
<td>$165^{+33}_{-28}$</td>
<td>$2.0^{+0.7}_{-0.8}$</td>
<td>26.4</td>
<td>1.10</td>
</tr>
<tr>
<td>6a</td>
<td>$1.44^{+0.09}_{-0.09}$</td>
<td>$44^{+19}_{-18}$</td>
<td>$133^{+56}_{-56}$</td>
<td>$3.1^{+0.7}_{-0.8}$</td>
<td>12.6</td>
<td>1.43</td>
</tr>
<tr>
<td>6b</td>
<td>$1.53^{+0.07}_{-0.04}$</td>
<td>$49^{+16}_{-9}$</td>
<td>$118^{+48}_{-38}$</td>
<td>$2.4^{+0.6}_{-0.8}$</td>
<td>16.8</td>
<td>0.90</td>
</tr>
<tr>
<td>7</td>
<td>$1.70^{+0.04}_{-0.05}$</td>
<td>$35^{+9}_{-9}$</td>
<td>$88^{+49}_{-28}$</td>
<td>$1.4^{+0.9}_{-0.9}$</td>
<td>31.3</td>
<td>1.03</td>
</tr>
</tbody>
</table>

a $10^{21}$ cm$^{-2}$.
b $10^{16.52}$ cm$^{-2}$.
c $10^{-4}$ photons cm$^{-2}$ s$^{-1}$.
d $10^{-11}$ erg cm$^{-2}$ s$^{-1}$.
e 24 degrees of freedom.
Figure 3.7 2-23 keV spectra for the *Ginga* observations 2, 4 and 7. (a) - (c) The pulse height spectra, best-fitting models for the five parameter fits above 4 keV (extrapolated down to 2 keV) and residuals. (d) Corresponding incident photon spectra for obs 2 (lower spectrum), obs 4 (middle spectrum) and obs 7 (upper spectrum).
Table 3.5 lists the best-fitting parameters obtained from each observation, along with the 2-10 keV integrated model flux, $F_\gamma^c$, after correcting for absorption. The quoted errors correspond to 90% confidence limits assuming four interesting parameters (see §2.3). An acceptable value for the reduced $\chi^2$ is obtained in every case (but note that the errors on the parameters for observation 3 are relatively large because the amount of useful data present allowed only a short integration time). Figs. 3.7a - 3.7c show the pulse height spectrum, best-fit model extrapolated down to 2 keV and residuals for observations 2, 4 and 7 respectively. The corresponding incident photon spectrum for each of these observations is shown in Fig. 3.7d. The now familiar 2 - 4 keV soft excess is again evident from Fig. 3.7. In §3.3.4 we investigate the behaviour of the soft excess with continuum level.

SPECTRAL INDEX

The most striking result evident from Table 3.5 is the clear variability of both the photon index, $\Gamma$, and $n_H$. This somewhat contrasts with the EXOSAT data, where $\Gamma$ was consistent with a constant value of 1.55 and changes in $n_H$ were not obvious. $\Gamma$ is plotted in Fig. 3.8 against $F_\gamma^c$. It can be seen from Fig. 3.8 that $\Gamma$ does not appear to follow the linear correlation with $F_\gamma^c$ found by Perola et al. (1986), which is indicated by the dashed line in Fig. 3.8. For comparison, some measurements of the photon index from previous X-ray astronomy missions have also been included in Fig. 3.8 (see Table 3.1). A Spearman Rank correlation test on the Ginga points only, gives a correlation coefficient of 0.93 at a confidence level > 99.9%. The best-fit straight line to the Ginga points in Fig. 3.8 is $\Gamma = (1.35 \pm 0.07) + (0.010 \pm 0.003)F_\gamma^c$. The correlation coefficient is 0.93, at the 99.2% confidence level. However, despite the good fit to a linear relation, it appears that $\Gamma$ does not increase indefinitely with $F_\gamma^c$ but ‘saturates’ at a value of $\sim 1.7$. Indeed, all the models of the X-ray spectral variability that we consider in §4 predict that $\Gamma$ does not increase indefinitely with $F_\gamma^c$.

Note that since the spectral parameters for the two parts of observation 1 (see Table 3.5) show no significant difference, Fig. 3.8 shows the average values of the spectral index and $F_\gamma^c$ for the two parts of this observation. This contrasts to the spectral parameters obtained for the two parts of observation 6 during which the continuum flux underwent a ‘step’ increase over a few hours. If the spectral fit to the second part of observation 6 is repeated with $\Gamma$ fixed at 1.44 (the best-fit value for the first part), $\chi^2$ increases from 0.90 to 1.76. Pre-Ginga studies of NGC 4151 have been unable to ascertain whether significant spectral changes occur on timescales of a few hours.
Figure 3.8 The photon spectral index, $\Gamma$, plotted against the absorption corrected flux, $F_x^c$, (in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$). $\Gamma$ is measured above 4 keV from the Ginga data. Measurements of $\Gamma$ from previous satellites have also been included as indicated, along with some of the EXOSAT measurements from §3.2. The dashed line is the flux-index correlation obtained by Perola et al. (1986) from an analysis of a subset of the EXOSAT data.
ABSORBING COLUMN

As well as the variation in spectral index the present modelling also reveals clear changes in \( n_H \), the apparent column density in the line-of-sight, which are independent of the continuum level, as can be seen from columns 3 and 7 of Table 3.5. Measurements of \( n_H \) from November 1983 up to January 1989 are plotted in Fig. 3.9b as a function of time, along with the 2-10 keV flux level over the \(~ 5.5 \) year period in Fig. 3.9a. Values of \( n_H \) from the EXOSAT observations (Table 3.3) and the Ginga observations (Table 3.5) have been averaged over a period of 1-4 weeks. It is apparent from Fig. 3.9 that \( n_H \) varies on a timescale of months to years (over a range \(~ 35 - 120 \times 10^{21} \text{ cm}^{-2} \) independently of the continuum level and therefore also of variations in the spectral index. Fig. 3.7d contrasts how the spectral shape changes with variations in spectral index and column density. The incident photon spectra, corresponding to observations 2, 4 and 7, are representative of (i) high \( n_H \) and small \( \Gamma \), (ii) high \( n_H \) and high \( \Gamma \) and (iii) low \( n_H \) and high \( \Gamma \) respectively.

![Figure 3.9](image)

**Figure 3.9** (a) 2-10 keV absorption corrected flux, \( F_x^c \), (in units of \( 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \)) during the \(~ 5.5 \) year EXOSAT and Ginga monitoring period of NGC 4151. (b) The apparent column density measured above \(~ 4 \) keV, \( n_H \), during the same period. Note that some of the data points represent averages over a period of 1-4 weeks.
3.3.3 The iron K-edge and fluorescence line

EDGE AND LINE ENERGIES

We now test our original assumption made in the analysis of §3.3.2, that the iron is indeed essentially neutral, by repeating the spectral fitting with the iron edge and line energies floating. However, we cannot allow both the energy of the line and the edge to float simultaneously as this would result in an unstable fit and poor parameter definition. Therefore, the spectral fitting above 4 keV has been repeated twice, once with the line energy floating and the edge fixed (at 7.11 keV) and then with the edge energy free while the line was held fixed (at 6.4 keV). Both sets of fits involve six free parameters in all. For the spectral fitting with the edge energy free it was assumed that the K-shell absorption cross-section at threshold remains constant at the value for neutral iron. In reality the cross-section at threshold is a slowly varying function of the K-edge energy so the measured column density, $n_{Fe}$, is actually the equivalent column density for neutral iron. The measured edge and line energies are presented in Table 3.6, along with the new values of $n_{Fe}$ and $I_{6.4}$ respectively. The errors quoted correspond to 90% confidence limits for 2 interesting parameters. The remaining parameters did not change significantly with respect to their values in Table 3.5 and therefore are not shown in Table 3.6. In fact, both the edge and line energies are consistent with the ‘cold’ values for every observation, justifying our original assumption. The weighted mean edge and line energies are $7.17 \pm 0.08$ keV and $6.48 \pm 0.06$ keV respectively. These values place an upper limit of Fe V to the ionization state of the iron (Makishima, 1986). However, this statement is model dependent to a certain extent (see §5.4). Note that the observed line energy may appear to be higher than it actually is if most of the emission comes from fluorescence in an accretion disc (which is essentially cold), viewed close to edge-on, in which case the blue wing is Doppler boosted (see George & Fabian (1990b) and §1.3.2).
Table 3.6 NGC 4151 *Ginga* iron edge and line parameters

<table>
<thead>
<tr>
<th>Obs</th>
<th>$E_{\text{edge}}$</th>
<th>$n_{\text{Fe}}$</th>
<th>$\chi^2$</th>
<th>$E_{\text{line}}$</th>
<th>$I_{6.4}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>7.59$^{+0.28}_{-0.85}$</td>
<td>199$^{+70}_{-60}$</td>
<td>0.88</td>
<td>6.61$^{+0.20}_{-0.16}$</td>
<td>1.6$^{+0.4}_{-0.4}$</td>
<td>0.86</td>
</tr>
<tr>
<td>1b</td>
<td>7.21$^{+0.26}_{-0.17}$</td>
<td>239$^{+46}_{-36}$</td>
<td>0.87</td>
<td>6.57$^{+0.13}_{-0.14}$</td>
<td>1.8$^{+0.3}_{-0.4}$</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>7.28$^{+0.31}_{-0.56}$</td>
<td>172$^{+66}_{-79}$</td>
<td>1.33</td>
<td>6.41$^{+0.13}_{-0.10}$</td>
<td>2.9$^{+0.6}_{-0.7}$</td>
<td>1.34</td>
</tr>
<tr>
<td>3</td>
<td>6.99$^{+0.40}_{-0.71}$</td>
<td>183$^{+105}_{-111}$</td>
<td>0.43</td>
<td>6.35$^{+0.20}_{-0.17}$</td>
<td>2.6$^{+0.8}_{-1.0}$</td>
<td>0.41</td>
</tr>
<tr>
<td>4</td>
<td>7.07$^{+0.09}_{-0.17}$</td>
<td>180$^{+52}_{-9}$</td>
<td>1.11</td>
<td>6.56$^{+0.39}_{-0.29}$</td>
<td>1.3$^{+1.0}_{-0.8}$</td>
<td>1.05</td>
</tr>
<tr>
<td>5</td>
<td>7.08$^{+0.08}_{-0.21}$</td>
<td>169$^{+50}_{-12}$</td>
<td>1.14</td>
<td>6.43$^{+0.21}_{-0.13}$</td>
<td>2.0$^{+0.5}_{-0.6}$</td>
<td>1.13</td>
</tr>
<tr>
<td>6a</td>
<td>7.34$^{+0.36}_{-1.11}$</td>
<td>149$^{+56}_{-47}$</td>
<td>1.38</td>
<td>6.43$^{+0.12}_{-0.10}$</td>
<td>3.0$^{+0.6}_{-0.6}$</td>
<td>1.48</td>
</tr>
<tr>
<td>6b</td>
<td>6.98$^{+0.34}_{-0.25}$</td>
<td>130$^{+41}_{-28}$</td>
<td>0.92</td>
<td>6.42$^{+0.15}_{-0.12}$</td>
<td>2.3$^{+0.5}_{-0.6}$</td>
<td>0.91</td>
</tr>
<tr>
<td>7</td>
<td>7.01$^{+0.32}_{-0.28}$</td>
<td>92$^{+23}_{-22}$</td>
<td>1.08</td>
<td>6.58$^{+0.35}_{-0.30}$</td>
<td>1.3$^{+0.6}_{-0.6}$</td>
<td>1.01</td>
</tr>
</tbody>
</table>

* Energy in keV.
* $10^{16.52}$ cm$^{-2}$.
* 23 degrees of freedom.
* $10^{-4}$ photons cm$^{-2}$ s$^{-1}$. 
IRON ABUNDANCE

Column 4 of Table 3.5 shows that the apparent line-of-sight column of iron, \( n_{Fe} \), is variable and appears to be correlated with \( n_H \). The correlation is shown in Fig. 3.10, for which the linear correlation coefficient is 0.96, significant at the 99.7% confidence level. Values of \( n_{Fe} \) and \( n_H \) from previous X-ray astronomy missions have not been included since \( n_{Fe} \) has always been poorly constrained prior to Ginga. The fact that the best linear fit to the correlation between \( n_{Fe} \) and \( n_u \) has a non-zero intercept (\( n_{Fe} = 57 \pm 46 \) when \( n_H = 0 \)) suggests that there may be a small residual quantity of iron in the line-of-sight not associated with the low energy absorption. Although the correlation is improved somewhat if the origin is included in the regression, in §5.2.1 and 5.3.2 we present evidence, from modelling of the low energy absorption and soft excess, that part of the iron edge is formed in a region distinct from the site of the low energy absorption. One possibility is that the observed spectrum is a composite of the direct continuum and a reflected component from an accretion disc or optically thick matter out of the line-of-sight (see §1.3.2). The depth of the iron edge from the reflected component may be constant while the line-of-sight absorption varied (see §4.4). The slope of the correlation which excludes the origin implies an over-abundance of iron of \( 1.21 \pm 0.57 \), while the slope of the correlation which includes the origin implies an apparent over-abundance of \( \sim 1.88 \pm 0.17 \), relative to the solar value. However, the actual value of the apparent over-abundance is model dependent, since the absorbing material may only partially cover the source. Partial photoionization of the absorbing material will also affect estimation of the iron abundance. The question of the actual iron abundance in NGC 4151 will be taken up again in §5.

IRON LINE INTENSITY AND EQUIVALENT WIDTH

We now turn our attention to the \( K_\alpha \) fluorescence line. Column 5 of Table 3.5 indicates that the intensity, \( I_{6.4} \), remained roughly constant despite a factor \( \sim 3.5 \) increase in continuum flux. The weighted mean intensity of the line from Table 3.5 is \( (2.2 \pm 0.2) \times 10^{-4} \) photons cm\(^{-2}\) s\(^{-1}\) and is consistent with the mean line intensity derived from the EXOSAT data (§3.2.3). The equivalent width of the line varies from \( \sim 200 \) eV to \( \sim 50 \) eV as the source brightens. Interpretation of this latter result is complicated by the fact that there may be a large time-lag between continuum variation and fluorescence since the distance between the source and fluorescing material is unknown. Also electron scattering of the line would 'smear' out any line variability. The same effect may also be responsible for the lack of rapid variability of the X-ray continuum.
Figure 3.10 Values of the apparent column density of iron, $n_{Fe}$, plotted against the equivalent column density of Hydrogen, $n_H$. Both $n_{Fe}$ and $n_H$ were obtained from spectral fitting to the Ginga data above 4 keV (see Table 3.5). The best-fit straight line which includes the origin (dashed curve) is $n_{Fe} = (1.88 \pm 0.17) n_H + (57 \pm 46)$ and the best-fit straight line which excludes the origin (dotted curve) is $n_{Fe} = (1.21 \pm 0.57) n_H$. 
Although we cannot derive a meaningful source-to-fluorescence distance from the present data set, we can make some general deductions about the configuration of the fluorescing material. Firstly, we can compute a 'mean' equivalent width by assuming a 'mean' flux state corresponding to $20 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$ and a photon spectral index of 1.55. This gives a value of $\sim 100$ eV, which is about 2/3 of the maximum possible (i.e. when viewed face-on) from a geometrically thin accretion disc (George & Fabian, 1990b). It is difficult ascertain how much of the iron line emission comes from a disc (if at all) and how much from fluorescing material surrounding the source. If the disc is viewed close to edge-on, then the fluorescing material would have to cover a large fraction of the sky to account for the observed 'mean' equivalent width, assuming that the column density of the material is as high as that in the line-of-sight (Makishima, 1986). Clavel et al. (1987) suggest that the accretion disc is indeed viewed close to edge-on on the grounds that it is perpendicular to the radio jet (inclination 75°) in NGC 4151. Also, Ferland & Mushotsky (1982) argue, from observed and predicted emission line intensities, that BLR clouds cover at least $\sim 80\%$ of the sky (see §1.3.6). Hence it is likely that most of the line emission originates in BLR material. We shall have more to say about the probable configuration of BLR material in NGC 4151 in §5.5.3.

LINE BROADENING

We have also investigated the broadening of the $K_\alpha$ iron emission line in NGC 4151. The approach is simply to fit a Gaussian profile to the line; the data does not warrant a more sophisticated procedure. We have repeated the fits above 4 keV (see §3.3.2) for each of the Ginga observations, with the iron K-edge and line energies fixed at 7.11 and 6.4 keV respectively, with the addition of the r.m.s. width of the line as an extra free parameter. We obtain a mean value of $0.36 \pm 0.07$ keV for the r.m.s. width, where the quoted error correspond to 90% confidence limits for 1 interesting parameter. This line width corresponds to a FWHM of $\sim 38000$ km s$^{-1}$.

3.3.4 Nature of the 2-4 keV soft excess

As with the EXOSAT spectra, detailed modelling of the low energy Ginga spectra will be deferred until §5. It can be seen from Figs. 3.3 and 3.7 that the soft excess flux is localized between 2 and 4 keV. Here we investigate the nature of the excess from the Ginga data by considering the quantity, $\Delta q$, defined by

$$\Delta q = f_{2-4} - f_{2-4}^X.$$

Here $f_{2-4}$ is the 2-4 keV count rate and $f_{2-4}^X$ is the predicted 2-4 keV count rate when the best fits
above 4 keV, described in §3.3.2, are extrapolated down to 2 keV. Hence, $\Delta q$ represents the excess count rate, above that expected from cold, uniform, solar abundance absorption. $\Delta q$ is plotted against the 4-10 keV LAC count rate in Fig. 3.11. The excess count rate, $\Delta q$ is well correlated with the continuum flux (linear correlation coefficient of 0.99 at confidence level of better than 99.99%) and the best fit straight line passes through the origin. This is exactly what is to be expected if the soft excess is associated with the main continuum source and is not due to a separate source. Unfortunately, the limited spectral resolution in the energy range 2–4 keV means that any model of the low-energy absorption and soft excess (§5) will be poorly constrained.

![Figure 3.11](image)

**Figure 3.11** The 2-4 keV soft excess, $\Delta q$, (as defined in equation 3.1), plotted against the 4-10 keV LAC count rate. The dashed line corresponds to the linear fit to the data points and passes through the origin. The correlation coefficient is 0.99 (99.99% confidence), strongly suggesting that this soft excess component in NGC 4151 is associated with the main (medium energy) X-ray source.
3.4 Chapter summary

In this chapter we have reported on the results of an extensive monitoring program of the X-ray spectrum of NGC 4151 covering the ~5.5 year period from 1983 July to 1989 January. We have performed a simple analysis of the spectrum above 3.5-4 keV, deferring a detailed interpretation of the behaviour of the X-ray continuum until §4, and of the low-energy spectrum until §5. This preliminary analysis has led to the following conclusions listed below.

Temporal behaviour

The 2-10 keV flux varied by a factor of ~10 over the EXOSAT and Ginga monitoring period, over the range ~3.6 - 38 x 10^-11 erg cm^-2 s^-1. The medium energy flux exhibits a slow drifting behaviour with a flux doubling timescale of ~5 x 10^4 - 10^5 s. NGC 4151 is frequently observed to flare in the X-ray band, during which the 2-10 keV flux rises and then falls by a factor ~3 - 4 over a few days. This behaviour contrasts with many other Seyfert type I's in which rapid variability is observed on a timescale of hours or less. The unusual temporal behaviour of NGC 4151 may be due to an electron scattering region present in the nucleus of the galaxy. We shall have more to say about this in §4 in the context of models of the X-ray emission.

Spectral index

Although measurements of the photon spectral index, \( \Gamma \), by EXOSAT were all consistent with a constant value of 1.55, the much improved sensitivity of the LAC revealed clear changes in the spectral index which appear to be correlated with the 2-10 keV continuum flux. The value of \( \Gamma \) ranged from ~1.2 - 1.7 over the EXOSAT and Ginga monitoring periods. This range also covers all previous measurements of \( \Gamma \). In §4 we explore the implications of this important result for the physics of the emission region and the properties of some models which can account for the flux-index correlation observed in NGC 4151.

Column density

The apparent column density measured above ~4 keV, \( n_H \), varied on a timescale of months to years over the range ~35 - 150 x 10^{21} cm^-2, independent of the continuum level. This range also covers all previous measurements of \( n_H \). The physical implications of variable X-ray absorption in the active nucleus of NGC 4151 and other Seyferts will be discussed in §5.
Iron absorption

The *EXOSAT* measurements of the line-of-sight iron absorption column for individual observations were poorly constrained but using the whole *EXOSAT* data set, we obtain an average value of the ratio $n_{Fe}/n_H$ of $2.0 \pm 0.5$. This is consistent with the value of $1.88 \pm 0.17$ obtained from the *Ginga* data set. Moreover, the values obtained from both *EXOSAT* and *Ginga* also happen to be consistent with those obtained from observations by previous X-ray astronomy missions (Table 3.1). However, the ratio $n_{Fe}/n_H$ does not directly translate into a value for the actual abundance of iron (relative to the solar value).

The *Ginga* measurements also reveal a strong correlation between $n_{Fe}$ and $n_H$. The non-zero intercept of the correlation suggests that some iron absorption occurs in material which is distinct from the variable column responsible for the low-energy absorption. This may be evidence that the observed spectrum contains a component of the continuum which has been reflected in an accretion disc (see also §4.4). In §5.2.1 and §5.3.2 we present further evidence that there is a 'residual' amount of iron absorption which is not associated with the material responsible for the low-energy absorption and argue that the iron abundance (relative to solar) in the low-energy absorber is in fact $\sim 2$.

The iron K-edge energy measurements from the *Ginga* data give a mean value of $7.17 \pm 0.08$ keV, indicating that the absorbing material is in a fairly low state of ionization. We will investigate further the thermal and ionization state of the X-ray absorbing material in NGC 4151 in §5.4.

$K_{\alpha}$ fluorescence line

The intensity, $I_{6.4}$, of the 6.4 keV $K_{\alpha}$ iron emission line does not follow continuum variations in either the *EXOSAT* or *Ginga* set of observations. Moreover, $I_{6.4}$ does not correlate with the continuum level for the set of historical measurements presented in Table 3.1. The mean intensities of the iron line from the *EXOSAT* and *Ginga* measurements are $2.5 \pm 0.5$ and $2.2 \pm 0.2 \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$ respectively. Uncertainty in the distance from the source to fluorescing material and the scattering depth in the nucleus makes it difficult to ascertain the true equivalent width but we estimate a 'mean' value of $\sim 100$ eV (see §3.3.3). The line flux may contain contributions from fluorescence both in BLR clouds and in an accretion disc.

The measurements of the line energy from the *Ginga* data give a mean value of $6.48 \pm 0.06$ keV. Doppler boosting of the line flux formed in an accretion disc viewed close to edge-on can increase the line energy above 6.4 keV (as well as fluorescence in ionized material). The *Ginga* data also reveal evidence of broadening of the iron emission line, the mean r.m.s. width being $\sim 0.36 \pm 0.07$
keV. The implied velocities are somewhat higher than those of UV/ optical emission line material in the BLR (see §1.3.7).

2-4 keV soft excess

We have shown that the excess flux above that expected from cold, uniform, solar abundance material is only present in the narrow 2-4 keV band (possibly extending down to 1 keV) and correlates with the continuum level. The implication is that the soft component is intimately related to the continuum emission and does not arise from a separate or extended region.

The LE soft excess

In contrast to the 2-4 keV excess, the soft flux detected by the LE does not correlate with the continuum level, a result first reported by Pounds et al. (1986a) on the basis of a small subset of the EXOSAT data. The analysis of the complete EXOSAT database has shown the LE soft component to be constant over the entire 32 month EXOSAT monitoring period, confirming its separate origin from an extended emission region, possibly from hot gas in the NLR. Measurement of the spectral form of this soft component will have to await better instrumentation (§7.3).
Chapter 4

Models of continuum X-ray spectral variability

Overview

The nature of the broadband continuum emission in AGN and in NGC 4151 as a particular case study is reviewed. The spectral variability properties of some models of the X-ray emission region, which are of contemporary importance, are investigated. The results are discussed in the light of recent reports of spectral index variability in a number of AGN. In particular, the ranges of model parameters are found which can account for the flux-index correlation reported for NGC 4151 in chapter 3.

"So, so you think you can tell
Heaven from Hell?"

(Roger Waters, 1975. From 'Wish you Were Here'.)
4.1 Continuum emission in AGN

It is now well established that the continuum emission from active galaxies is characterized by approximately equal luminosities per logarithmic bandwidth, from the far infrared, right up to $\gamma$-ray energies (in some cases). The motivation for studying the broadband continuum in the present context of understanding the X-ray spectral variability is twofold: (i) the X-ray emission is thought to be due to Compton scattering of either UV photons or synchrotron photons at lower energies and (ii) the thermodynamic and ionization equilibrium of gas which may be present in the nucleus (which is a potential source of low energy X-ray absorption), depends on the shape of the entire continuum, from radio to $\gamma$-ray energies. In this section a 'mean' AGN continuum is compiled from the literature, as well as a broadband continuum for NGC 4151. These continuums will be used in this chapter as a guide in constraining models of the X-ray emission in AGN and in chapter 5 to compute the ionization and thermal structure of gas in the AGN environment.

Although a lot of data has now been accumulated for large numbers of AGN in the radio, infrared, optical and X-ray wavebands, simultaneous, multi-waveband observations of continuum emission from active galaxies is still quite rare. Apart from NGC 4151, some notable exceptions are NGC 4051 (Done et al., 1990) and NGC 5548 (for which the monitoring campaign has only recently been completed). Below, we outline the salient features of the the continuum emission from AGN as a whole and NGC 4151 in particular, emphasising that the gross features of the continuum from active galaxies are remarkably similar (but different in detail) even across different subclasses of object. Note that the observed continuum from AGN does not necessarily correspond to that directly emanating from the active nucleus itself. Secondary emission components include contributions from extended radio structure, thermal emission from dust in the infrared, and Balmer continuum emission in the optical, to name a few. Also, it is far from clear how the geometry, angular distribution of radiation and viewing angle of the emitting system(s) affects the relative magnitudes of the observed fluxes in various wavebands. This is an important point to bear in mind throughout this chapter. In the present work we shall be interested mainly in the primary core emission from the radio to $\gamma$-ray bands, with a view to gaining insight into the mechanisms responsible for the emission and absorption of the primary X-ray component. Note that in subsequent discussions all frequencies, wavelengths and energies are referred to the rest frame of the objects under consideration.

4.1.1 Infrared to optical emission

The infrared continuum is treated before the radio portion of the spectrum for reasons which will become apparent. Recent studies based on IRAS data and ground based observations (Edelson
& Malkan, 1986; Edelson, Malkan & Rieke, 1987; Edelson & Malkan, 1987; Ward et al., 1987; Carleton et al., 1987) have unveiled a consistent picture of the infrared to optical continuum of AGN. The underlying continuum is thought to be non-thermal in origin and can be well described by a flat power law (in $\log \nu F_\nu - \log \nu$ space). Superimposed on this (in some objects) is a thermal emission component arising from dust intrinsic to the source, a possible contribution from stars in the host galaxy, and Balmer and Paschen continuum emission. Hence, as one goes from quasars to Seyfert I's to Seyfert II's (dust-free to very dusty) the observed infrared continuum becomes much steeper relative to the flat quasar-like form. In fact the observed infrared continuum from Seyfert II's is not well fit by a power law and is predominantly thermal but the non-thermal flat power law is still thought to be present. Edelson & Malkan (1986) find the mean energy index of the non-thermal component to be 1.36 from their sample which is the value we adopt. Another remarkable feature of the infrared continuum is that a low energy turnover between $1.0 \times 10^{12}$ to $\sim 7.0 \times 10^{12}$ Hz is observed or inferred in every object which has been observed. Moreover, in the quasar-like objects the infrared and optical luminosity is found to be well correlated with the X-ray flux at 2 keV for a given sample of objects (Edelson & Malkan, 1986) and in many cases the non-thermal infrared power law extrapolated to X-ray energies usually predicts the correct flux at 2 keV within a factor of 2. These latter points have usually been interpreted as evidence for the emission mechanism of the infrared, optical and X-ray components being one and the same, with the low-frequency turnover being attributed to synchrotron self-absorption (rather than free-free absorption). More recently, however, Sanders et al. (1989) find no correlation between the infrared/ optical and X-ray luminosities. Also, in most objects the infrared/ optical continuum generally shows very little variability whilst the X-ray flux may exhibit large amplitude, rapid variability. The recent simultaneous multi-waveband monitoring campaign of NGC 4051 (Done et al., 1990) has shown that the ratio of the amplitude of variability in the X-ray and optical bands is at least 50:1 in this object. This means that the infrared/ optical source must be at least an order of magnitude larger than, or completely separate from, the X-ray source. See §4.2.2.3 for further comment on this point.

4.1.1.1 INFRARED TO OPTICAL EMISSION IN NGC 4151

For NGC 4151 there is a wealth of literature on measurements for the infrared and optical continuum (McAlary & McLaren, 1981; Rieke & Lebofsky, 1981; Beall & Rose, 1981; McAlary et al., 1983; Bassani et al., 1986; Edelson & Malkan, 1986; Edelson & Malkan, 1987; Edelson, Malkan & Rieke, 1987). The result of all these studies is that NGC 4151 has a relatively dust-free quasar-like infrared continuum which, like all other AGN, does not show any large amplitude variability shortward of 2.2 μm (lower limit $\sim 1$ year). Edelson & Malkan (1986) estimate the slope of the underlying power law to be 1.37, virtually identical to the mean value derived from the sample used by the same authors.
The turnover frequency in NGC 4151 (between ~ 1 and $3 \times 10^{12}$ Hz) happens to be at just the value expected from the free-free opacity of typical BLR clouds (see Ferland & Mushotsky, 1982). However, this would require virtually 100% coverage of the source, which, from UV emission line intensity (§1.3.6) and X-ray absorption (§3 & §5) data, is usually not the case. Even a 'leak' in the absorbing material as small as 0.01% would be inconsistent with the radio measurements of NGC 4151 (see below).

### 4.1.2 Radio emission

The drop in flux due to the low frequency turnover in the infrared discussed above can be as much as 6 decades before the continuum resumes a flat power law form once again. Objects in which the drop is only ~ 2 decades are generally known as 'radio-loud' while objects at the other extreme are 'radio-quiet'. The majority of AGN tend to be radio-quiet. In many cases the cm emission from radio loud objects extends well beyond the active nucleus and it is often not clear how much of it is due to primary processes in the compact core (but see Preuss & Fosbury (1983) who attempt to separate core from extended emission). Also, it is not clear what role free-free absorption due to ionized material in the line of sight plays in determining the observed radio flux and more importantly, whether any of the radio emission is related to the X-ray emission (and therefore also emission in the infrared and optical), although there is some evidence of a correlation of radio luminosity with X-ray luminosity (Worrall et al., 1987, Owen et al., 1981). Also, Elvis & Wilkes (1987) find an anti-correlation between radio-loudness (which they define as $f(5\text{GHz}, \text{mJy})/B$ magnitude flux) and the 0.2-3.5 keV soft X-ray slope from a sample of 33 quasars. That is, radio-quiet objects in their sample have steeper soft X-ray spectra than radio loud objects. However measurements of the X-ray slope for a different sample of objects, which includes Seyfert galaxies, over the 2-10 keV energy range (Turner & Pounds, 1989) do not confirm such a correlation.

#### 4.1.2.1 Radio emission in NGC 4151

The drop in flux from the infrared to radio frequencies in NGC 4151 is about 3 decades and the value of the power-law energy index from 20 cm to 6 cm is ~ 0.88 (Bassani et al., 1986). Moreover, the level of radio emission from NGC 4151 has been established to be fairly constant over a timescale of years (Beall & Rose, 1981, Bassani et al., 1986). The emission arises from both the compact core and an extended region (Preuss & Fosbury, 1983). Mapping of the radio structure has also revealed an elongated jet-like component (Booler et al., 1982) which may have important implications for the dynamics and excitation of emission line gas.
4.1.3 Ultraviolet emission

An assessment of the UV continuum poses a more difficult problem as the sampling frequency range is not heterogeneous for the higher redshift quasars. Also, separating the underlying continuum from the emission lines and recombination continua is not a trivial task. What is clear is that the UV continuum requires a thermal component (normally referred to as the 'blue bump') above the extrapolated infrared power law which Edelson & Malkan (1986) modeled as a blackbody with temperature ~ 25000 K (although further work by the same authors (1987) showed that a single temperature blackbody was not adequate; see, e.g., Sun & Malkan (1989) who fit the bump with accretion disc spectra). Superimposed on this is a 'small blue bump' or 'UV excess', at 3000Å due mainly to Fe II lines and the Balmer continuum. A convenient summary of the 'mean' AGN UV spectrum is provided by O'Brien (1987). Based on a large sample, the continuum between 1900Å and 912Å (this range avoids the 3000Å feature) can be characterized by a broken power law with the break at Lyα (1215Å). The mean energy slopes longward and shortward of Lyα respectively are found to be 0.87 and 1.38. Neither of these slopes is correlated with radio loudness. The situation regarding spectral variability of the UV continuum is as yet rather unclear (e.g. see Chapman et al., 1985) but temporal variability is certainly common, but not on timescales of less than a few days (usually months).

4.1.3.1 ULTRAVIOLET EMISSION IN NGC 4151

For NGC 4151, the most extensive analysis of the UV continuum has been presented by Perola et al. (1982) using the large archive of IUE data available for this object. In summary, the flux longward of ~ 4286Å remains roughly constant but at shorter wavelengths changes in amplitude of up to a factor ~ 4 are observed with flux doubling timescales of 5-30 days. These large amplitude changes are accompanied by spectral changes in the IUE wavelength range (~ 3020Å to ~ 1450Å) in the sense that the spectrum becomes harder as the source brightens. More specifically, the flux longward of 2200Å can be characterized by a power law with an energy index which is linearly correlated with the flux at 2500Å. The extrapolation of this power law shortward of 2200Å leaves an excess flux which Perola et al. (1982) characterize by the excess at 1455Å (Δf_{1455}). Hence at the least, a two component UV continuum is implied for NGC 4151. Although f_{2500} is not correlated with f_{1455}, it is anti-correlated with Δf_{1455}, but in the so called 'anomalous epochs' (Ulrich et al., 1984), Δf_{1455} always seems to have the constant value of ~ 1 mJy, the lowest observed. When NGC 4151 is in this state the entire UV continuum in the IUE band can be represented by a single power law. The implications are that the short wavelength excess is probably due to a thermal component having a separate origin to the longer wavelength component but the similar timescales of variability suggest that both sources are located within the BLR. (The 'thermal' component is usually associated with emission from an accretion disk). Studies of the emission and absorption line variability indicate that it is the non-thermal part of the UV continuum which is responsible
for ionization of the emission-line gas (Bromage et al., 1985; Clavel et al., 1987).

Figure 4.1 illustrates how the shape of the continuum between $\sim 3 \times 10^{14}$ Hz and $\sim 2 \times 10^{15}$ Hz varies for different values of $f_{2500}$. The pivot point has been taken at $10^{14.6}$ Hz and the three continuum levels correspond to $f_{2500} = 15, 20$ and $25$ mJy. The dotted lines show the same continua with the Balmer continuum subtracted off (see Edelson & Malkan, 1986). Note that the mean level of $f_{2500}$ relative to the 2-10 keV flux can change with epoch (Perola et al., 1986) but this could be due to variable absorption in the line-of-sight. Also, the shape of the UV spectrum is subject to error due to uncertainties in the amount of reddening. However Perola et al. (1982) concluded that intrinsic reddening in NGC 4151 is probably negligible, at worst $E(B - V) = 0.05$.

Figure 4.1 Spectral variability of the UV spectrum of NGC 4151 (§4.1.3.1). The arrows mark the wavelengths 2500Å, 2200Å and 1455Å as indicated. The three solid curves correspond (from bottom to top) to flux levels at 2500Å of 15, 20 and 25 mJy. The dotted curves are the corresponding spectra after the Balmer continuum has been subtracted.
4.1.4 The Extreme UV continuum

The portion of the AGN spectrum from the hydrogen ionization edge (13.6 eV) up to ~ 0.2 keV is and always will be the most enigmatic. We can never observe the EUV emission as the absorption which prevents us 'seeing' it is an unknown quantity. This is rather unfortunate since a crucial input to photoionization models of the broad emission line region (BLR) in AGN is the shape of the ionizing continuum above 1 Rydberg. Also, there is good evidence now that the energy output from AGN peaks in the EUV, as might be expected if accretion onto a compact object was the main source of energy. One would think that it may be possible to deduce the shape of the EUV by comparing predictions from photoionization models of the BLR with observations. However, such models are subject to a large number of additional uncertainties and physical unknowns (see §5.4).

In the present work the EUV is important because photons in this energy band (i) may be the 'seed' photons for Comptonization models of the X-ray emission (§4.2) and (ii) are important in determining the thermal and ionization structure of X-ray absorbing material (see §5.5). However, as so little is known about the EUV, the best that can be done is to build up a picture of the EUV gap which we hope is not too far off the truth and then test the sensitivity of our conclusions to the assumed model continuum. To this end there follows a list of currently accepted 'knowledge' of the EUV:

- The turn up in the UV and soft X-ray region seen in many objects implies that any smooth extrapolation cannot avoid a fairly sizable peak of the energy distribution somewhere between 13.6 eV and 0.2 keV (the lower limit of the EINSTEIN IPC).
- Several high redshift quasars have been observed down to 600 Å in their rest frame (Bechtold et al., 1984; O'Brien, 1987) and such observations suggest that the energy index from the far UV to 2 Rydberg is ~ 1.5.
- The energy index between 2-4 Rydberg must be close to 1, a result based on energy budget arguments (Netzer, 1985).
- The equivalent width of He II λ1640 is very sensitive to the spectral index above 4 Rydberg (McAlpine, 1985). Observations imply a very steep value of ~ 3 (see Mathews & Ferland, 1987), which is consistent with a straight extrapolation from the He II ionization edge down to the X-ray continuum.

4.1.4.1 THE EUV CONTINUUM IN NGC 4151

The UV to X-ray gap in NGC 4151 is much wider than usual due to the high intrinsic column density in this object so that we are forced to use the above model. However, although we shall never be able to see the underlying continuum in this region, it may be possible to quantify the
nature of the constant, extended soft emission component discovered by the EXOSAT LE (see §1.3.11 and §3.2) with future X-ray astronomy missions (see §7.3).

4.1.5 X-ray emission

Below is a brief summary of current knowledge of the temporal and spectral behaviour of the X-ray emission from AGN in general, which is to be compared with that from NGC 4151 (already described in §3).

4.1.5.1 X-RAY TEMPORAL CHARACTERISTICS AND VARIABILITY

Prior to the launch of EXOSAT it was generally thought that > 10% variability of the medium energy (~ 2 — 6 keV) continuum on timescales of less than a day was not common (Tennant, 1983) and the few objects that were observed to vary (e.g. NGC 4151, NGC 3227 and NGC 6814) all tended to be low X-ray luminosity Seyfert I's. Here, low luminosity is generally taken to mean \( L_x < 3 \times 10^{42} \text{ erg s}^{-1} \). However, since the launch of EXOSAT, not only has the list of variable objects steadily grown but the range of observed timescales for the variability has been extended from right down to the EXOSAT faint source detection limit of 100 s up to years (Turner & Pounds, 1989 and Pounds & McHardy, 1988). However, very rapid (< few \( \times 10^3 \text{ s} \)) large amplitude X-ray variability is still quite rare. Examples of very rapid variability include NGC 4051 (Marshall, Warwick & Pounds, 1981; Lawrence et al., 1985; Done et al., 1990), NGC 6814, (Tennant, 1983; Beall et al., 1986), MCG-6-30-15, (Nandra, 1989a), NGC 7314, Akn 120, Mkn 335, (Turner & Pounds, 1989), and NGC 5506 (McHardy & Czerny, 1987). In most of these objects, which were observed by EXOSAT, the very soft X-ray flux (as measured by the LE) was seen to crudely correlate with the medium energy flux (Pounds & McHardy, 1988). This is a very important point which must be taken into account by any model which attempts to link the UV/EUV/X-ray emission. In general the temporal X-ray variability in AGN is erratic in nature; there have only been two reports of the detection of a periodicity in the light curve of an active galaxy, namely NGC 6814 (Mittaz & Branduardi-Raymont, 1989), and NGC 4151 (Fiore et al., 1989). The period for NGC 6814 is claimed to be \( \sim 12000 \text{ s} \) and that for NGC 4151 \( \sim 6000 \text{ s} \). Obviously the truth of either of these claims would have profound implications for the physical conditions in the active nucleus. Unfortunately, the result for NGC 4151 is based on poor quality data and it is not possible to test the result on the Ginga data (see §3.3) due to the gaps in the light curves. We shall not go into the temporal behaviour of X-ray emission in any more detail, as we are interested mainly in the spectral behaviour. Much work has already been published on the short and long term temporal variability of the EXOSAT observations of NGC 4151 reported in §3.2 (e.g. see Pounds & McHardy, 1988; McHardy, 1989; Lehto, 1989 and references therein).
4.1.5.2 X-RAY SPECTRAL CHARACTERISTICS

It is now generally accepted that the underlying X-ray spectrum of AGN (neglecting absorption and local emission features) above ~ 1 keV can be characterized by a hard power-law component and a steeper soft (possibly thermal) component (cf. discussion of the EUV continuum, §4.1.4). The soft component is thought to dominate below 0.3 keV (Wilkes & Elvis, 1987) but in several objects has been observed to dominate the flux up to ~ 1 keV (Elvis, 1987; Arnaud et al., 1985; Pounds et al., 1986b). The photon spectral index, $\Gamma$, of the hard component can have a value anywhere in the range ~ 1.0 – 2.0 but $\Gamma$ tends to cluster around the so called ‘canonical’ value of 1.7. For instance, Petre et al., (1984) find a mean value of $\Gamma = 1.65 \pm 0.15$ from their sample of ~ 30 objects, while Turner & Pounds (1989) found that $\Gamma = 1.7 \pm 0.17$ in their sample of 48 and Turner et al., (1989b) found $\Gamma = 1.69 \pm 0.29$ from a sample of 10 high luminosity quasars observed by Ginga. Wilkes & Elvis (1987) have also completed a survey of 33 quasars and measured the soft X-ray slopes from *EINSTEIN* IPC spectra in the 0.2-3.5 keV energy band. They find that $\Gamma$ measured in this energy band ranges from 0.8 - 2.8. This result, based on a group of quasars (no Seyfert galaxies) biased against high intrinsic absorption, is not inconsistent with the measurements reported above. Wilkes & Elvis suggest that since the radio-quiet objects in their sample have values of $\Gamma$ strongly grouped around 2.5 while radio-loud objects are grouped around $\Gamma \sim 1.5$, the composite spectrum is in general a mixture of these two components in a given object.

It is important to realize that, despite the persistent finding of a canonical X-ray slope, there are a sizable fraction of objects that have an X-ray slope which is inconsistent with a canonical value. In particular, a significant number of AGN are found to have a ‘flat’ X-ray slope ($\Gamma$ less than ~ 1.55). For instance, > 25% of the objects in the *EXOSAT* survey (Turner & Pounds, 1989) fit into this category and 2 out of 10 of the high luminosity quasars in Turner et al. (1989b). Removal of these ‘flat’ objects would then raise the mean X-ray slope of the remaining objects.

4.1.5.3 SOFT EXCESSES

Before proceeding, it is worth pointing out that the term ‘soft-excess’ has become rather a confused quantity over recent years and since we shall constantly need to refer to various types of soft excess we define these below once and for all.

- **Type I** The ‘small blue bump’ at 3000Å due to recombination continua and Fe II emission.
- **Type II** The ‘blue bump ’ in the far UV, which is the excess flux above the extrapolated infrared power law and is usually ascribed to thermal emission from an accretion disk.
- **Type IIIa** The ‘big blue bump’ in the soft X-ray (below ~ 0.3 keV ) region which is presumed to be the tail end of the peak in the EUV.
• Type IIIb This refers to the same turn up in the X-ray spectrum to meet the EUV but refers to cases where the energy of the turn-up dominates at energies larger than 0.3 keV (e.g. Mkn 841, Arnaud et al., 1985; Mkn 335, Pounds et al., 1986b).

• Type IV The excess flux below ~ 4 keV above that expected from photo-electric absorption by a uniform screen of cold, solar abundance gas with the assumed input spectrum.

• Type V Extended soft X-ray emission which is not related to the central X-ray continuum (see §1.3.3).

Type II and III soft excess are likely to have the same origin; this type of thermal emission may just appear at different energies in different objects. Recently, Wandel (1987) has found that the ratio of UV bump to power-law luminosity (i.e. the magnitude of the excess) is well correlated with total luminosity in a sample of 114 bright quasars and Seyfert galaxies. This is interpreted by Wandel (1987) as a change in the accretion disc spectrum with luminosity, the more luminous objects having cooler, more massive discs. Madau (1988) has found a striking correlation between the bump temperature and inclination angle of the hypothesised accretion disc. In this model the accretion disc spectrum would appear to peak at higher energies for decreasing viewing angles ($\theta = 0$ for a face-on disc). Hence a continuous gradation from a type II to type III soft excess is implied.

Note that NGC 4151 is known to possess all except type III (although this soft component may exist behind the large column).

4.1.5.4 X-RAY SPECTRAL VARIABILITY

In §1.3 we outlined the ways in which X-ray spectral variability could manifest itself. In this chapter we shall be interested in modelling the variability of the spectral index of the hard X-ray power law. Note that a variable type III soft excess may sometimes mimic variations in the slope of the hard component (see §4.3). Also, if the X-ray spectrum has a low-energy cut-off due to photo-electric absorption (as is the case for NGC 4151), it is often very difficult to unequivocally distinguish variability of the X-ray slope from variable absorption.

Given that a 'universal' X-ray slope exists for objects which span over four decades of luminosity, spectral index variability (whether it is intrinsic or extrinsic to the X-ray source) in individual objects has important implications for the physics of the innermost regions of AGN. It is therefore important be certain that the spectral index variability is genuine. In Table 4.1 we have compiled a list of eight AGN, for which spectral index variability has been claimed, and the approximate range in $\Gamma$ and 2–10 keV flux. The statistical significance of spectral index variability claimed for other AGN (e.g. see Matsuoka, 1989) makes them unworthy for inclusion in Table 4.1. In fact, for the first four objects (NGC 7314, NGC 2992, NGC 4051 and MCG-6-30-15) the column density
was held fixed, at the spectral fitting stage, at an assumed value and the errors do not exclude a single value of $\Gamma$. For NGC 3227, it is difficult to assess the validity of the index variability since 5 of the 6 observations (with $\text{EXOSAT}$) all gave $\Gamma \sim 1.45 - 1.55$ and the value of 1.7 was obtained from just one $\text{Ginga}$ observation. Confirmation will have to await further $\text{Ginga}$ observations. The last three objects in Table 4.1 are the only ones in which no parameters were fixed in fitting the continuum (excluding the iron line energy) in order to obtain the spectral index variability result. We conclude that 3C 273, NGC 5548 and NGC 4151 are the only objects in which statistically significant changes in the apparent slope of the hard X-ray power law have been measured. Note that in the case of 3C 273, although the changes in $\Gamma$ are very small, the errors on each measurement are of order $\sim 0.01 - 0.02$ (see Turner et al., 1989b, 1990). Table 4.1 also indicates whether $\Gamma$ is correlated with the 2-10 keV continuum flux, from which it can be seen that there is no such correlation for 3C 273 and NGC 5548. The flux-index correlation for NGC 4151 has been reported in §3.3.2. Finally, we note that 3C 273, NGC 5548 and NGC 4151 can all have a value of $\Gamma$ less than 1.5, a point which has particular significance for non-thermal models of the X-ray emission (see §4.2).
Table 4.1 AGN exhibiting significant spectral variability

<table>
<thead>
<tr>
<th>Object</th>
<th>Range in $\Gamma$</th>
<th>$\Delta \Gamma$</th>
<th>No. of $^b$</th>
<th>$F_{\text{max}}/F_{\text{min}}$</th>
<th>$F - \Gamma$ correlation</th>
<th>Satellite$^d$</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 7314</td>
<td>1.83–2.02</td>
<td>0.21</td>
<td>6</td>
<td>No</td>
<td>E</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>NGC 2992</td>
<td>1.46–1.68</td>
<td>0.22</td>
<td>3.6</td>
<td>No</td>
<td>E</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>NGC 4051</td>
<td>1.40–2.10</td>
<td>0.70</td>
<td>3.2</td>
<td>Yes</td>
<td>G</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>MCG-6-30-15</td>
<td>1.50–1.80</td>
<td>0.30</td>
<td>2.1</td>
<td>Yes</td>
<td>G</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>NGC 3227</td>
<td>1.45–1.70</td>
<td>0.25</td>
<td>6</td>
<td>6.5</td>
<td>No</td>
<td>E,G</td>
<td>2,5</td>
</tr>
<tr>
<td>3C 273 *</td>
<td>1.42–1.53</td>
<td>0.11</td>
<td>5</td>
<td>3.4</td>
<td>No</td>
<td>E,G</td>
<td>4</td>
</tr>
<tr>
<td>NGC 5548 *</td>
<td>1.43–1.75</td>
<td>0.32</td>
<td>5</td>
<td>1.8</td>
<td>No</td>
<td>E,G</td>
<td>5,6</td>
</tr>
<tr>
<td>NGC 4151 *</td>
<td>1.23–1.70</td>
<td>0.49</td>
<td>31</td>
<td>10</td>
<td>Yes</td>
<td>E,G</td>
<td>7,8</td>
</tr>
</tbody>
</table>

$^a$ Some of these are approximate values only. See references for details.

$^b$ Number of observations on which the variability result is based on.

$^c$ Ratio of maximum to minimum observed flux.

$^d$ E = EXOSAT, G = Ginga.

$^*$ These are the only objects in which we consider the spectral index variability to be statistically significant (see text).

References.

8. This work (§3).
4.1.6 \(\gamma\)-ray emission

Second to the EUV, the \(\gamma\)-ray region in the AGN spectrum (above \(\sim 100\) keV) is the least understood and, at present, most poorly observed. Only three AGN have so far been established as \(\gamma\)-ray sources (see Bassani & Dean (1983) for a review), namely NGC 4151, Cen A and 3C 273. One thing for certain is that the hard X-ray power law in all AGN must steepen dramatically below a few MeV to be meet the constraints imposed by observations of the diffuse \(\gamma\)-ray background and currently assumed space densities of AGN (see Bignami et al., 1979).

\(\gamma\)-RAY EMISSION IN NGC 4151

NGC 4151 has been detected out to \(\sim 16\) MeV (Baity et al., 1984, Bassani et al., 1986) and the X-ray power law is thought to extend out to at least 1 MeV. The energy of the spectral break is uncertain but is thought to be \(\sim 5\) MeV. In the present work the \(\gamma\)-ray part of the AGN spectrum will be important for two reasons (i) constraining models of the BLR (§5) since Compton heating by high energy \(\gamma\)-ray photons can be the dominant heat source for BLR gas and (ii) constraining models of the X-ray emission and spectral variability.

4.1.7 The ‘mean’ AGN spectrum

The idea of constructing a ‘mean’ AGN spectrum is not new (see for instance Krolik, McKee & Tarter, 1981; O'Brien, 1987; Mathews & Ferland, 1987). The continuum used by Krolik, McKee & Tarter (1981) to investigate the thermodynamics of the BLR and HIM suffered from the fact that it was based on 3C 273 which, apart from being an anomalously strong \(\gamma\)-ray emitter has an X-ray slope which is much flatter than the ‘typical’ AGN. From the remarks in the preceding sections we have constructed an up-to-date mean AGN spectrum which is shown in Fig. 4.2a and also in \(\nu F_{\nu}\) space in Fig. 4.2b. Below we list some points which were important in the construction of the spectrum in Fig. 4.2.

- The radio continuum is a power law with an energy index of 1. The flux at 20 GHz for the radio-loud (radio-quiet) version is 2 (6) decades below the the flux at \(10^{12}\) Hz. The low frequency turnover is at \(\sim 2 \times 10^{12}\) Hz.
- The EUV continuum was constructed using the prescription given by Mathews & Ferland (1987) (see §4.1.4).
- The flux at 2 keV is related to the flux at 2500\(\AA\) by adopting a value of \(\alpha_{ex} = 1.4\), regardless of radio power.
- The X-ray slope is independent of radio power.
• The γ-ray continuum is simply the extrapolated X-ray power law with an exponential cutoff at 100 keV. However, the cut-off energy is highly uncertain.

For reference, the spectral indices in the different frequency bands are given in Table A.1 (appendix A) for an X-ray slope of 1.7 and 1.5.

4.1.8 The broadband continuum of NGC 4151

Fig 4.3 shows the continuum of NGC 4151 constructed in a similar manner to the mean AGN continuum. The UV continuum is not reproduced in the detail described in §4.1.3.1 For our purpose it is sufficient to approximate this part of the continuum as a single (variable) power law. Since the continuum above ~ 10^{14.6} Hz is highly variable, two representative states have been shown in Fig. 4.3. These correspond to $\Gamma = 1.45$ and $\Gamma = 1.7$, the normalization of the X-ray continuum in each case follows from the flux-index correlation in §3.3.2 (Fig. 3.8). The normalization of the UV spectrum relative to the X-ray spectrum has been deduced using the correlation between $f_{1455}$ and the 2–10 keV flux found by Perola et al., (1986). The EUV continuum has been constructed using the prescription described in §4.1.4 and the γ-ray cut-off is placed at the very conservative value of 1 MeV. It can be seen that the broadband continuum of NGC 4151 is not markedly different to the mean AGN continuum; the most noticeable difference is the relative smallness of the EUV bump and the larger γ-ray luminosity. For reference, the spectral indices in the different frequency bands are given in Table A.1 (appendix A) for an X-ray slope of 1.45 and 1.7. Also given in appendix A (Table A.2) are the luminosities, in various wavebands, of the spectra in Fig. 4.3. Note, however, that luminosities derived from the spectrum in Fig. 4.3 above ~ 100 keV should be treated with caution as the form of the assumed γ-ray spectrum is rather arbitrary.
Figure 4.2 (a) Flux per unit frequency for the 'mean' AGN continuum (see §4.1.7 for details). The X-ray photon spectral index is 1.7 and the γ-ray cut-off is 100 keV. (b) As (a) with the spectrum plotted in $\nu F_\nu$ space (equal areas corresponding to equal luminosities). The dotted curves correspond to a 'radio quiet' version (see text).
Figure 4.3 (a) The broadband spectra of NGC 4151 (see §4.1.8) corresponding to values of the X-ray photon spectral index of 1.45 (lower spectrum) and 1.7 (upper spectrum). The $\gamma$-ray cut-off is at 1 MeV in both cases. (b) As (a) with the spectrum plotted in $\nu F_\nu$ space.
4.2 Non-thermal models

Much effort has been expended in recent years to account for the narrow range of \( \sim 1.0 - 2.0 \) in the X-ray photon spectral index, \( \Gamma \), found in large samples of emission-line AGN (Rothschild et al., 1983; Petre et al., 1984; Mushotsky, 1984; Turner & Pounds, 1989). As already mentioned, \( \Gamma \) appears to cluster around the so called 'canonical' value of 1.7. There are many radiation mechanisms which have formed the basis of models that can quite easily reproduce the range in \( \Gamma \) for observed compactness parameters covering several orders of magnitude (Zdziarski & Lightman, 1985; Zdziarski, 1986a; Band & Grindlay, 1986; Fabian et al., 1986a (F86); Svensson, 1987 (S87); Lightman & Zdziarski, 1987 (LZ87); Done & Fabian, 1989 (DF89); Ghisellini, 1989a). The type of model which has received the most attention involves Compton scattering of soft photons (either 'virtual' synchrotron photons or UV photons) on a non-thermal steady distribution of relativistic particles (electrons and/or positrons). Thermal models of the X-ray continuum and its variability (e.g. see Zdziarski, 1985 and 1986b; Dermer, 1988 and 1989) will not be considered as such an investigation is beyond the scope of this work. However, we note that they can just as well reproduce the range of X-ray power-law slopes in AGN as well as spectral variability (Dermer, 1988 and 1989) but suffer from their own drawbacks (as do non-thermal models). One of these is the question of whether the thermal plasma can be maintained at the required high temperatures.

None of the above types of model can provide a reason for why AGN should fall into the parameter range which gives rise to the apparently 'universal' X-ray slope. Models which force a universal spectrum by say, assuming a given electron distribution (e.g. see Kazanas, 1984 and Zdziarski, 1986a), cannot account for the growing number of AGN which are found to have X-ray slopes significantly different from 1.7, especially those with a 'flat' X-ray slope with \( \Gamma \) about 1.5 or less (see §4.2.1.2). It appears that the data do not require a universal X-ray slope, but a value of 1.7 seems to be preferred (see LZ87 and DF89).

Another problem which is inherent in models which attempt to account for a universal spectrum is that they cannot easily explain significant spectral index variability in individual sources (see §4.1.5.4). The purpose of this section is to investigate the spectral variability properties of the particular type of model in which the seed photons are thermal and have UV energies (as opposed to synchrotron photons which have lower energies) and photon-photon absorption which produces \( e^+e^- \) pairs is self-consistently taken into account. The main motivation for specializing to this case is that it has recently become one of the best studied (see references above) and is (arguably) the most successful so far, although contemporary models of this type do not give rise to a universal spectrum (but see §4.2.4). DF89 have investigated in detail the temporal behaviour of the continuum flux in the above type of model but not the spectral variability.

The primary objective of the present spectral variability study is to account for the flux-index
correlation reported for NGC 4151 in §3.3.2. The results will be presented in a general form (§4.2.1.4) so that they may be applied to sources other than NGC 4151. We then discuss in §4.2.4 how the results and data impose constraints on future models of the X-ray emission in AGN. First, however, we discuss the likely importance of the alternative case in which the source of soft photons is synchrotron radiation produced by relativistic particles in a magnetic field (synchrotron self-Compton models), instead of UV photons from, say, an accretion disc.

SYNCHROTRON SELF-COMPTON (SSC) MODELS

SSC models of the continuum emission in AGN have been investigated by many authors (e.g. Zdziarski, 1986a; Band & Grindlay, 1985 and 1986; de Kool et al., 1989, Ghisellini, 1989a,b; Ghisellini et al., 1989a; Band & Malkan, 1989). Each have their own different physical assumptions and approximations and have been applied to AGN continua with varying degrees of success. Some can account for the entire infrared to X-ray/ $\gamma$-ray continuum. For instance Zdziarski (1986a) proposes a model where the relativistic electron distribution is a carefully designed broken power law. However, this model, like most others of its type, suffers from the fact that simultaneous variability of the infrared and X-ray components is predicted, in direct conflict with observation (see §4.1.1). However, we note that a sub-class of some models calculated by Ghisellini et al. (1989a) can actually produce a steady infrared component and highly variable X-ray component, both originating in the same region. However, this class of model requires that $\Gamma \sim 2$ (so cannot apply to NGC 4151) and that the energy density in the magnetic field is much larger than that in the radiation field. The latter condition implies that the X-ray luminosity is much less than the infrared luminosity and this has led Done et al. (1990) to reject this model for NGC 4051 in particular, for which a simultaneous, multi-waveband observing campaign has recently been carried out. Moreover, current SSC models, like ‘pure’ Compton models, cannot really account for the prevalence of the canonical X-ray spectral index.

One of the attractions of SSC (apart from producing the infrared continuum and not just the X-ray/ $\gamma$-ray continuum) is that the $\gamma$-ray spectra in SSC models do not, in general, violate the constraints imposed by the $\gamma$-ray background. This is because the seed photons are much softer than the UV photons in ‘pure’ Compton models. So far an investigation into the X-ray spectral variability in SSC models, along similar lines to those in the present work for ‘pure’ Compton models, has not been carried out. Such an investigation would give further insight into the possible emission mechanisms in AGN, but is beyond the scope of the present work. Of course, in reality the observed broadband spectrum in AGN may arise from Compton scattering of both synchrotron and UV photons, with different relative contributions in different AGN (or even in the same object). SSC is important when the energy density in the magnetic field ($U_B$) is comparable to or larger than that in the X-ray/ $\gamma$-ray radiation field ($U_r$). The ratio $U_B/U_r$ is approximately
where $R_{14}$ is the radius of the emission region in units of $10^{14}$ cm and $L_{44}$ is the luminosity in units of $10^{44}$ erg s$^{-1}$. Now, from the broadband emission spectrum for NGC 4151 (§4.1.8 and Table A.2 in appendix A) we find that the ratio of the infrared / optical luminosity to the X-ray / $\gamma$-ray luminosity is approximately unity, implying that $U_B/U_r \sim 1$. However, in §4.2.2.3 we present arguments for (i) the infrared/ optical emission originating in a different region to that in which the X-ray emission is produced and (ii) in fact $U_B/U_r \ll 1$ or $U_B/U_r \gg 1$ in the X-ray emission region, implying that if there is an SSC component, it cannot be the dominant source of X-rays in NGC 4151. Hence we choose to ignore the SSC process in our models.

\[
\frac{U_B}{U_r} \sim \left( \frac{B^2}{8\pi L} \right) \left( \frac{4\pi R^2 c}{L} \right) \sim 1.5 \times 10^{-6} \frac{B^2 R_{14}^2}{L_{44}} \tag{4.1}
\]

"STATE OF THE ART" OF PAIR MODELS

The reader is referred to the references cited above for a full briefing on the current status of steady state pair models of the X-ray and $\gamma$-ray continuum emission in AGN. In the present work, attention must be drawn to two major failings of contemporary pair models and these are given below.

- No part of the parameter space in the theory is preferred which makes the 2-10 keV photon spectral indices of the steady state models cluster around the value $\sim 1.7$.

- More serious is the fact that the physically relevant models in general have $\gamma$-ray spectra in which the flux exceeds the upper limits set by the $\gamma$-ray background ($\gamma$RB), unless evolution is invoked. Essentially, the problem is that there is not enough pair production in the parameter range of interest to sufficiently deplete the $\gamma$-ray flux. Models with large pair production rates predict a luminosity of the annihilation line which may be in conflict with observation (see DF89). One way to overcome the $\gamma$-ray problem is to inject the primary electrons with very low Lorentz factors ($\sim 100$ or less as opposed to $\sim 1000$ or more) or to invoke the idea of 'pair-loading' (see Ghisellini et al., 1989b) but the X-ray slope in this type of model can never be much greater than $\sim 1.5 - 1.6$ as the pair production rate at low Lorentz factors is insufficient to steepen it. We shall use Lorentz factors which actually lie within the above range (i.e. between 100 and 1000). In order to keep the $\gamma$-ray spectra of the models that we calculate 'in check' the 'gamma excess' parameter, $G_{EX}$, as defined in DF89, has been adopted. DF89 define $G_{EX}$ as the ratio of $(1-100$ MeV / 2-10 keV) model luminosity ratio to the maximum $(1-100$ MeV / 2-10 keV) luminosity ratio allowed by the $\gamma$RB:
\[ G_{EX} = 10^{[2.67 \Gamma - 5.24]} \left( \frac{L_{1-100 \text{ MeV}}}{L_{2-10}} \right)_{\text{model}} \] (4.2)

The factor \(10^{-2.67 \Gamma - 5.24}\) is a fit to the integral in equation 3.4 in DF89.

### 4.2.1 Spectral index variability in pair models

**MODEL PARAMETERS**

Here we briefly review the physical basis of the models we shall calculate. Our approach follows closely that of LZ87, except that we treat thermal Comptonization as in F86 instead of solving the full Kompaneets equation as in LZ87.

We use the dimensionless energy \( x = h \nu / m_e c^2 \), where \( m_e \) is the electron rest mass. Soft photons are injected into a spherical, homogeneous source region of radius \( R \) at a rate \( n_{x,t}^S \) cm\(^{-3}\) s\(^{-1}\) per unit energy. We use the notation \( n_{x,t} \) to denote the partial derivative \( \left[ \frac{\partial n_{x,t}}{\partial t} \right]_x \). The internal soft photon density is then \( n_{x}^S = \left( R/c \right) n_{x,t}^S \). We follow LZ87 and DF89, taking \( n_{x,t}^S \) to be a blackbody peaking at an energy \( x_\nu \equiv 2.8 kT / m_e c^2 \). Relativistic electrons of Lorentz factor \( \gamma \) are continuously injected into the source region at a rate \( Q(\gamma) = Q_0 \gamma^{-n} \) cm\(^{-3}\) s\(^{-1}\) per unit energy. The electrons cool on the soft photons via inverse Compton scattering and freshly injected electrons replenish the energy carried off by the escaping radiation. The injected electrons have Lorentz factors extending from \( \gamma_{\text{min}} \) to \( \gamma_{\text{max}} \). The dimensionless soft photon and electron compactness parameters, \( l_s \) and \( l_e \) respectively, are defined as in LZ87 to be

\[
\begin{align*}
  l_s &= \frac{L_s \sigma_T}{R \ m_e c^3} \\
  l_e &= \frac{L_e \sigma_T}{R \ m_e c^3} 
\end{align*}
\] (4.3)

where \( L_s = \int n_{x,t}^S x dx \) and \( L_e = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} Q(\gamma)(\gamma - 1) d\gamma \) are the injected luminosities in the form of soft photons and electrons respectively. If the primary electron energy density is sufficiently large (as measured by \( l_s \)) then upscattered photons with \( x > 1 \) interact with photons of energy \( 1/x \) (in our approximation) to produce electron-positron pairs at a rate \( P(\gamma) \) cm\(^{-3}\) s\(^{-1}\) per unit energy (equation 13 in LZ87). These pairs also cool on the soft photons via inverse Compton scattering (and hence reprocess the primary radiation field) and are assumed to thermalize before annihilating. Compton scattering on this thermal population further modifies the radiation field (see below). Since the inverse Compton cooling time of an electron or positron with a Lorentz factor
\( \gamma (\sim 1/\gamma l_e \text{ light-crossing times}) \) is much shorter than the escape time, the rate of production and cooling (to thermal energies) of particles balance at all times. This gives rise to a steady particle distribution \( N(\gamma) \) (equation 6 in LZ87). Hereafter, we specialize to the case where the relativistic electrons are injected monoenergetically at a Lorentz factor \( \gamma_0 \) and discuss how our results would be modified for power law injection in §4.2.1.4.

In summary, the input parameters of the models are \( x, \gamma, \gamma_0 \) and \( \gamma_0 \). Hereafter we shall refer to a particular model with a given \( x \), as \([\gamma, \gamma_0, \gamma_0] \) (with appropriate parameter values inserted).

**Output parameters**

Four of the most important output parameters of the models which we shall use are (i) the 2-10 keV photon spectral index, \( \Gamma \), (defined as a least-squares fit to the spectrum with a power law in the 2-10 keV band), (ii) \( l_{2-10} \), defined as \( L_{2-10}/R^2 \) where \( L_{2-10} \) is the 2-10 keV luminosity, (iii) the break energy, \( x_B \), of the emergent spectrum and (iv) the \( \gamma \)-ray excess, \( G_{EX} \) (equation 4.2). Note that we prefer to use \( l_{2-10} \) rather than the usual X-ray compactness, \( l_x \), since \( l_{2-10} \) is more practical for normalizing observed luminosities to model luminosities. In our models \( l_x \sim 0.27 l_{2-10} \).

Three other output parameters are of importance, namely the pair yield, \( \omega \), (i.e. the fraction of injected energy in electrons converted to pairs), the Thomson scattering depth of thermal pairs, \( \tau_T \), and the dimensionless temperature of the thermal pair population, \( \Theta \equiv kT/m_e c^2 \). \( \Theta \) is determined by requiring that no net energy is exchanged between particles and photons and is therefore equivalent to the Compton temperature of the radiation field at any instant.

The pair yield indicates the relative importance of pair reprocessing and in particular gives the fraction of the total luminosity in the annihilation line, while \( \tau_T \) (\( \propto \sqrt{\omega} \)) indicates the relative importance of thermal Comptonization. The thermal pair plasma amplifies soft photon energies by a factor \( \sim 1 + 4\Theta \) while high energy photons are down-scattered to energies \( \sim 1/\tau_T^2 \), causing a depletion of photons in the energy range \( \sim 1/\tau_T^2 \) to 1. If \( \tau_T \gg 1 \), the spectrum breaks at \( \sim 1/\tau_T^2 \), causing significant deviation from a true power law. Since we shall be dealing with models with flat X-ray spectra, \( \tau_T \) will never be much larger than unity and thermal Comptonization is expected to have negligible effect on the X-ray spectra. However, reprocessing by non-thermal pairs can also potentially cause significant deviation from a true power law in the X-ray band. To this end we shall use the quantity \( \Gamma_{4-23} \), defined as the 4–23 keV least-squares photon index, as a check on the X-ray spectra. The range 4–23 keV happens to be the effective energy range of the *Ginga* spectra of NGC 4151 (see §3.3).
4.2.1.1 NUMERICAL METHOD

The equation describing the time evolution of the internal photon density at an energy \( x \), \( n_x \), is

\[
n_x,t = n_x^{\text{S}} + n_x^{\text{NT}} + n_x^{\text{P}} + n_x^{\text{A}} - n_x \frac{c}{R} \left( t_x^{\text{NT}} + t_x^{\text{PP}} \right) - n_x^{\text{ac}}
\]  

(4.4)

Successive terms on the RHS of equation 4.4 describe the time evolution at energy \( x \) of (i) the soft input photons, (ii) photons appearing at an energy \( x \) from a lower energy, \( \tilde{x} \), due to Compton scattering on the non-thermal particle where \( x = \frac{3}{4} \gamma^2 \tilde{x} \), (iii) photons due to Compton scattering on thermalized pairs, (iv) photons from the annihilation of thermal pairs, (v) photons disappearing due to Compton scattering on the non-thermal particles to reappear at some higher energy, (vi) photon-photon absorption (creating \( e^+e^- \) pairs) and (vii) photons escaping from the source. The physical assumptions and approximations for all except the thermal Comptonization term follow closely those of LZ87. Thermal Comptonization is treated in a cruder manner, as in F86. Note that \( n_x^{\text{ac}} \) includes both creation and destruction (i.e., scattering to a new energy) of photons at energy \( x \). Equation 4.4 is solved numerically for the internal photon density, \( n_x,t \), and hence the emergent spectrum, as a function of time \( n_x^{\text{eff}} \); see equation 21 in LZ87) using a typical timestep of 0.05R/c.

We do not include the annihilation term in our solution of equation 4.4, however, since there are many unknown factors which would affect the strength and equivalent width of the observed line, such as geometry and scattering. In any case, we are primarily interested in the X-ray spectrum and the omission of the annihilation term will not affect our results since down-scattering of the line photons to the 2–10 keV X-ray band is negligible, especially for the low values of \( \tau_T \) that we shall be dealing with.

We have checked our numerical code by computing steady state solutions corresponding to various models computed by F86, LZ87 and DF89 and find good agreement, considering the different physical assumptions and approximations. Fig. 4.4. shows four such models in which the broken curves correspond to models calculated by LZ87 and shown in their Fig. 1 and the solid curves correspond to our solutions of equation 4.4. The models have \( l_s/l_e = 2.5 \), \( \gamma_0 = 1200 \) and \( l_e = 1, 10, 100 \) and 1000. It can be seen that our \( \gamma \)-ray spectra are slightly below and the X-ray spectra slightly above those of LZ87. The difference can be explained by our different treatment of Comptonization and not different pair production rates since our calculation of the photon-photon absorption depth is identical to that of LZ87. Note that the sharp features in the solid curves with large \( l_e \) are a result of the lack of dispersion in our treatment of thermal Comptonization and the use of a fine energy mesh (Done, private communication).
Figure 4.4 A comparison of four steady state pair models computed from equation 4.4 (solid curves) with those computed by Lightman and Zdziarski (1987) in their Fig. 1 (broken curves). All models have monoenergetic electron injection at a Lorentz factor of 1200 and a value of $l_e/l_a = 2.5$. The $l_e = 1000$ model is shown with a dashed curve for clarity. See text for more comments on the comparison.
FUNCTIONAL DEPENDENCE OF $\Gamma$

Before investigating time-dependent behaviour of $\Gamma$ and $l_2-10$ when the input parameters vary we must first review the functional dependence of $\Gamma$ on the input parameters for stationary solutions of equation 4.4 (see S87, LZ87 and DF89 for more details).

Dependence on $l_s/l_e$

In the limit of $l_s \ll 1$, when pair production is negligible, $\Gamma$ is actually independent of $l_e$. There is a strong dependence on $l_s/l_e$, however. When $l_s/l_e \ll 1$, the electrons can cool on photons which have already been upscattered and the superposition of successive orders of upscattered spectra flattens the X-ray slope so that $\Gamma \rightarrow 1$ as $l_s/l_e \rightarrow 0$. On the other hand, for $l_s/l_e \gg 1$, the first order spectrum dominates so that $\Gamma$ saturates at the value of 1.5 (for monoenergetic electron injection). Fig. 4.5 illustrates the dependence of $\Gamma$ on $l_s/l_e$ for $\gamma_0 = 1000$ and $x_s = 3 \times 10^{-5}$, in the limit of small $l_s$. Also included in Fig. 4.5 (for the purpose of later discussion) are similar curves for power-law electron injection corresponding to a range of values of the injection index, $\eta$, but all models have $\gamma_{\text{min}} = 1$ and $\gamma_{\text{max}} = 1000$. In general, $\Gamma = \frac{1}{\beta+1}$, where $\beta$ is the power law index of the steady state particle distribution, $N(\gamma)$. In the limit $l_s/l_e \ll 1$, $\beta = \eta + \Gamma - 1$ and $\Gamma \rightarrow 1$ (e.g. see Rybicki & Lightman, 1974). However, if $l_s/l_e \gg 1$, $\beta = \eta + 1$ if $\eta > 2$ or $\beta \rightarrow 2$ otherwise. Hence the saturation values of $\Gamma$ are given by $\frac{1}{2}\eta + 1$ for $\eta > 2$ and 1.5 for $\eta \rightarrow 0$. There are no analytical solutions for intermediate values of $\eta$.

Even if pair production is not negligible, the behaviour of $\Gamma$ with $l_s/l_e$ is qualitatively similar to that depicted in Fig. 4.5 (except for low Lorentz factors; see §4.2.4). That is, when $l_s/l_e$ is small, repeated scatterings tend to flatten the X-ray slope, in direct opposition to any steepening produced by pairs. When $l_s/l_e$ is large (and $l_s \ll 1$), the X-ray slope is given by the saturation values in Fig. 4.5, but is modified by pairs for larger $l_s$ (see below). Note that all the models of S87 and DF89 only included first order Compton cascades ($l_s/l_e \gg 1$) and therefore could not be applied to AGN with $\Gamma < 1.5$, such as NGC 4151. This case $l_s/l_e \gg 1$ has also been studied analytically by Zdziarski & Lamb (1986) in the limit when pair production can be neglected.
Figure 4.5 The 2–10 keV photon spectral index, $\Gamma$, as a function of $I_e/I_0$ when there is no pair production (i.e. $I_e \ll 1$) for a range of values of the power-law electron injection index, $\eta$ (as indicated) and also for monoenergetic injection. The Lorentz factor of the electrons is fixed at 1000.
Dependence on \( l_e \)

As \( l_e \) increases, the pair production rate increases, steepening the steady state particle distribution and hence the X-ray slope. Successive generations of pair cascades may follow and further steepen the X-ray slope (see S87 and LZ87). As \( l_e \) becomes larger, the spectral break at \( \sim 1/\tau_x^2 \) caused by Comptonization of the radiation field on the thermal population of cooled pairs steepens the X-ray slope even further. However, if \( l_e \) is larger still and \( \Theta \) is large, a Wien peak can develop above the X-ray band at \( 4\Theta \), flattening the X-ray slope again. However, this effect only becomes important when \( 4\Theta \tau_T \gg 1 \) and so is not relevant for the present work. See LZ87 and DF89 for a thorough discussion of all these effects.

In the limit of small \( l_e \), \( l_{2-10} \) scales directly with \( l_e \). However, as \( l_e \) increases the increasing importance of pair reprocessing means that \( l_{2-10} \) increases faster than \( l_e \), the greatest ‘amplification’ occurring for values of \( l_e \) between 1 and 10, approximately (see LZ87). When the pair yield saturates \( (\omega \sim 0.1) \) \( l_{2-10} \) scales with \( l_e \) again.

Dependence on \( \gamma_0 \)

In the limit of small \( l_e \), \( \Gamma \) increases as \( \gamma_0 \) increases. This is essentially due to the form of the non-thermal Compton scattering cross-section (see LZ87), namely that the maximum energy of a photon before scattering is \( 3/(4\gamma_0) \), otherwise the cross-section is zero in this approximation. Hence, repeated scatterings become more important as \( \gamma_0 \) decreases and this flattens the X-ray slope. The effect is illustrated in Fig. 4.6 which shows \( \Gamma \) as a function of \( \gamma_0 \) for several values of \( l_e/l_e \), for the case of no pair production and monoenergetic electron injection.

Note that if all the other parameters are fixed, \( l_{2-10} \) must also decrease as \( \gamma_0 \) increases since the total luminosity is distributed over a larger range of energy.

Dependence on \( \tau_1 \)

In the limit of small \( l_e \), \( \Gamma \) is virtually independent of \( \tau_1 \), unless of course, \( \tau_1 \) is large enough for the soft photons to enter the 2-10 keV band. However, when pairs are important, more of the pair reprocessed part of the spectrum enters the lower end of the 2-10 keV band as \( \tau_1 \) increases, thus increasing \( \Gamma \). Note that the reprocessed part of the spectrum due to the first pair generation extends up to an energy

\[
x_{P1} = \frac{16}{27} \frac{3}{\gamma_0^4} \quad (4.5)
\]

(see LZ87).
Figure 4.6 The 2–10 keV photon spectral index, $\Gamma$, as a function of the Lorentz factor, $\gamma_0$, of monoenergetically injected electrons when there is no pair production. The curves correspond to different values of $l_\gamma/l_e$ as indicated.
4.2.1.2 STEADY STATE SOLUTIONS FOR FLAT-SPECTRUM SOURCES

We have computed a grid of steady state models in the range $0.3 \leq l_e \leq 30$, $10^{-2} \leq l_x/l_e \leq 10$ and $100 \leq \gamma_0 \leq 1000$. The reasons for the choice of these parameter ranges will become apparent shortly. In all our stationary models we fix $\alpha_x$ at the fiducial value of $3 \times 10^{-5}$, corresponding to a temperature of $\sim 6 \times 10^4$ K, a value typical for the UV 'bump' in AGN (see §4.1.3 - 4.1.5). We then use the grid to find steady state solutions of equation 4.4 which have $\Gamma = 1.45$. This is the X-ray slope NGC 4151 appears to have near a low state (see §3) and is also a value which is typical of AGN having flat X-ray spectra. As mentioned above, there is a significant number of 'flat' X-ray AGN ($\sim 25\%$ in the EXOSAT survey sample; Turner & Pounds. 1989) and more recent Ginga results (Nandra, private communication) have revealed more Seyferts and QSOs with flat X-ray spectra. Note that, hereafter, we shall use the term 'flat' to specifically mean $\Gamma \leq 1.55$.

We will then investigate the spectral variability properties of the steady models under a variety of different physical assumptions (§4.2.1.4). Our general conclusions will be insensitive to the actual value of $\Gamma = 1.45$ used for the steady state models.

The parameters of 21 steady state models (labelled a–u) with $\Gamma = 1.45$ are shown in Table 4.2 and each of the models a–u has been marked on the $\gamma_0 - l_e$ plane in Fig. 4.7. The spectra corresponding to models a–u are shown in Fig. 4.8 and are plotted in units $dl(x)/d \ln x = \frac{z^2 n_{\gamma,0}(4\pi R^2 \sigma_T/3c)}{e^{(4\pi R^2 \sigma_T/3c)}}$. When comparing these with observed AGN spectra (such as those in Figs. 4.2 and 4.3), it must be remembered that the spectrum emerging directly from the source may be modified by the geometry of the system. In other words, the relative strengths of the UV and X-ray continuum may be different to that which is observed due to anisotropy.

Referring to Fig. 4.7, the region above the dashed curve has $l_x/l_e < 10^{-2}$ and this region is excluded by observational constraints, as described in §4.2.1.3. $l_x/l_e$ increases towards the bottom-left of Fig. 4.7 since large values of $l_e$ and $\gamma_0$ favour a steepening of the X-ray slope and thus require a small value of $l_x/l_e$ to compensate in order to bring $\Gamma$ down to 1.45. If Fig. 4.7 represented the parameter space for solutions with $\Gamma < 1.45$ the $l_x/l_e = 10^{-2}$ curve would move down towards the bottom-left of Fig. 4.7. Models with smaller values of $l_x/l_e$ have correspondingly larger values of $\Theta$ as can be seen from Table 4.2. This is because the Compton temperature is proportional to the mean photon energy and for small values of $l_x/l_e$, more soft photons are upscattered to high energies. The effect can be seen in Fig. 4.8.
Table 4.2 Steady state pair models with $\Gamma = 1.45$

<table>
<thead>
<tr>
<th>$\gamma_0$</th>
<th>$l_e$</th>
<th>$l_s/l_e$</th>
<th>$\tau_T$</th>
<th>$\omega$</th>
<th>$\Theta$</th>
<th>$l_{2-10}$</th>
<th>$G_{EX}$</th>
<th>Model</th>
</tr>
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<tr>
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<td>0.016</td>
<td>$7.0 \times 10^{-4}$</td>
<td>$1.4 \times 10^{-2}$</td>
<td>0.117</td>
<td>0.044</td>
<td>a</td>
</tr>
<tr>
<td>100</td>
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<td>0.750</td>
<td>0.059</td>
<td>$2.7 \times 10^{-3}$</td>
<td>$1.7 \times 10^{-2}$</td>
<td>0.376</td>
<td>0.053</td>
<td>b</td>
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<tr>
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<td>0.300</td>
<td>0.21</td>
<td>$1.2 \times 10^{-2}$</td>
<td>$2.6 \times 10^{-2}$</td>
<td>0.968</td>
<td>0.095</td>
<td>c</td>
</tr>
<tr>
<td>100</td>
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<td>0.050</td>
<td>0.54</td>
<td>$3.7 \times 10^{-2}$</td>
<td>$4.7 \times 10^{-2}$</td>
<td>1.249</td>
<td>0.27</td>
<td>d</td>
</tr>
<tr>
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<td>0.96</td>
<td>$7.1 \times 10^{-2}$</td>
<td>$6.1 \times 10^{-2}$</td>
<td>1.450</td>
<td>0.50</td>
<td>e</td>
</tr>
<tr>
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<td>0.500</td>
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<td>0.31</td>
<td>f</td>
</tr>
<tr>
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<td>0.10</td>
<td>$8.3 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-2}$</td>
<td>0.158</td>
<td>0.31</td>
<td>g</td>
</tr>
<tr>
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<td>0.30</td>
<td>$2.3 \times 10^{-2}$</td>
<td>$5.4 \times 10^{-2}$</td>
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<tr>
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</tr>
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<td>0.60</td>
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<td>$9.5 \times 10^{-2}$</td>
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<td>2.22</td>
<td>n</td>
</tr>
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<td>$0.11$</td>
<td>0.182</td>
<td>2.53</td>
<td>o</td>
</tr>
<tr>
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<td>0.019</td>
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<td>$9.4 \times 10^{-2}$</td>
<td>0.014</td>
<td>2.68</td>
<td>p</td>
</tr>
<tr>
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<td>0.059</td>
<td>$2.5 \times 10^{-3}$</td>
<td>$0.11$</td>
<td>0.040</td>
<td>3.35</td>
<td>q</td>
</tr>
<tr>
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<td>0.010</td>
<td>0.12</td>
<td>$4.6 \times 10^{-3}$</td>
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<td>0.073</td>
<td>4.15</td>
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<td>$0.13$</td>
<td>0.045</td>
<td>5.11</td>
<td>u</td>
</tr>
</tbody>
</table>
Figure 4.7 The parameter space for pair models having a 2–10 keV photon spectral index, \( \Gamma \), of 1.45. The region above the dashed curve has \( I_s/I_\gamma < 10^{-2} \) and is not likely to be observationally relevant. Models to the left of the dotted line have a \( \gamma \)-ray excess less than unity. The letters a–u mark the positions of steady state models which have actually been computed from equation 4.4. Details of these models can be found in Table 4.2.
Figure 4.8 (a) Spectra corresponding to steady state models with $\Gamma = 1.45$ (models a–e in Table 4.2). (b) As (a) but for models f–j.
Figure 4.8 - continued (c) As (a) but for models k-o.
Figure 4.8 - continued (d) As (a) but for models p-r. (e) As (a) but for models s-u.
The steady state spectra first break at an energy corresponding to the maximum energy, $x_B$, a photon of initial energy $x$, can attain in a single (first order) scattering and is given by

$$x_B = \frac{4}{3} \gamma_0^2 x,$$

$$\sim 0.2(\gamma_0/100)^2 \text{MeV.} \quad (4.6)$$

However, when repeated scattering is important (small $l_s/l_e$) this break is less prominent but there is still a significant steepening at the energy given by equation 4.6. The spectrum then breaks at $\gamma_0$. We note that nowhere in the parameter space of Fig. 4.7 does the photon-photon absorption optical depth reach unity so that pair production does not cause significant steepening of the $\gamma$-ray spectra of the steady state models. Table 4.2 shows, not surprisingly, that the Thomson depth of all the calculated models is less than unity and that $\omega$ is in most cases much less than the saturation value of $\sim 0.1$. However, we shall see that when the input parameters are variable, there is the possibility that pairs can steepen the $\gamma$-ray spectra for models with an initial $\gamma T$ close to unity. Finally we note that $\Gamma_{4-23}$ never differs by more than 0.005 for all the calculated steady state models, indicating that a power law is indeed a good approximation to the 4–23 keV spectra.

### 4.2.1.3 OBSERVATIONAL CONSTRAINTS ON THE PARAMETER SPACE

Not all of the parameter space for the steady state models with $\Gamma = 1.45$ in Fig. 4.7 is allowed by observational constraints. The first of these is the value of $l_s/l_e$. The observed/implied EUV to X-ray luminosity ratio ($l_s/l_e$) in AGN tends to fall largely in the range $1 - 10$ (see DF89). The implied range in $l_s/l_e$ for a given set of models depends on the range in $l_s$ and $\gamma_0$. DF89 required $0.1 \leq l_s/l_e \leq 1$ but we require a lower limit on $l_s/l_e$ since our models have, on average, less pair production ($l_s/l_e = (l_s/l_e)(l_s/l_e)$. We point out that $l_s/l_e$ could be even lower since we do not know how much of the X-ray source intercepts the soft photon source. Also, there is considerable uncertainty in calculating the EUV luminosity (see §4.1.4) and it is not known how the geometry of the system affects the fraction of the EUV or X-ray / $\gamma$-ray emission we observe. For instance smaller values of $l_s/l_e$ would be allowed if the X-ray / $\gamma$-ray continuum was strongly beamed away from our line-of-sight. However, the recent discovery of iron lines in many AGN (e.g. Pounds et al., 1990a), argues against this. We find that a lower limit of $10^{-2}$ on $l_s/l_e$ is reasonable for the ranges in $l_s$ and $\gamma_0$ that we use (see also §4.2.4). Hence, the parameter space above the dashed curve in Fig. 4.7 is considered not to be ‘observationally relevant’. The upper limit of $l_s/l_e = 10$ in the grid of models is actually larger than necessary for the immediate purpose but these models will be required in §4.2.4.

Another observational constraint comes from the requirement that the $\gamma$-ray excess parameter, $G_{EX}$, (equation 4.2) is less than unity. Models to the left of the dotted line in Fig. 4.7 satisfy
this requirement. However, we must concede that individual AGN which are known to be strong γ-ray sources (such as NGC 4151 and 3C 273) are allowed to lie to the right of the $G_{EX} = 1$ curve. Further constraints on the parameter space could be placed by the break energy of the spectrum but this has so far not been measured with sufficient confidence for any AGN. Note that if Fig. 4.7 represented the parameter space for models with a $\Gamma$ smaller than 1.45 the $G_{EX} = 1$ curve would move to the right.

### 4.2.1.4 EFFECTS OF PAIRS ON SPECTRAL INDEX VARIABILITY

We now investigate the behaviour of the steady state flat-spectrum models computed above when the input parameters vary with time. DF89 have investigated the temporal behaviour of the X-ray flux but not the explicit behaviour of $\Gamma$ with $l_{2-10}$, which is our primary concern. We consider seven types of variability, defined as follows:

- I $z_s$, $l_s/l_e$, $\gamma_0$ fixed, $l_s$ and $l_e$ vary,
- II $z_s$, $l_s$, $\gamma_0$ fixed, $l_e$ varies,
- III $l_s$, $l_e$, $\gamma_0$ fixed, $z_s$ varies,
- IIIa $z_s$, $l_s$, $\gamma_0$ fixed, $l_e$ varies,
- IV $z_s$, $l_s$, $l_e$ fixed, $\gamma_0$ varies,
- V $z_s$, $l_s/l_e$ fixed while $\gamma_0$ and $l_e$ vary in such a way that the electron injection rate, $\sim 10^6 l_e/\gamma_0 R_{14}^4$ cm$^{-3}$ s$^{-1}$, is constant and
- VI as V but $l_e$ is fixed instead of $l_s/l_e$.

The physical meaning of each of these will be elaborated below. When $l_e \ll 1$ (pairs not important) the response of $\Gamma$ and $l_{2-10}$ to all seven types of variability may be deduced from the discussion in §4.2.1.1, but when pairs cannot be neglected the variability of the output parameters may delayed and/or amplified (see F86 and DF89). The response of $\Gamma$ and $l_{2-10}$ must then be computed from equation 4.4. Our approach is to apply a step increase to the relevant input parameters by a factor of 3, for each of the flat-spectrum steady state models computed in §4.2.1.2 (see Table 4.2), according to each of the variability types I-VI. The new model is allowed to come to equilibrium, after which the input parameters are then returned to their original values. We will then be able to draw some very general conclusions about the behaviour of $\Gamma$ with $l_{2-10}$.
VARIABILITY RESULTS

We present our results in Tables 4.3-4.9 (corresponding to variability types I-VI respectively) which give the Thomson depth of the new steady state model ($\tau_T(v)$), the range in $\Gamma$ during the variability, the ratio of $l_{2-10}$ for the new steady state model to that of the initial model ($f_l$), the ratio of the fluxes at 100 keV and 1 MeV of the new steady state model to that of the initial model ($f_{11}$ and $f_{12}$ respectively) and finally, the $\gamma$-ray excess of the new steady state model ($G_{EX}(v)$).

We also present diagrams of $\Gamma$ versus $l_{2-10}(t)/l_{2-10}(0)$ (i.e. the 2-10 keV luminosity relative to the initial value at $t = 0$) during the variability for each model and variability type. These results will be discussed below, for each variability type.

However, before considering the variability results in detail, we make some general points which will help to interpret them. First, it must be borne in mind that the single most important factor in determining whether there is a large change in $\Gamma$ is whether the source makes a transition from a relatively 'pair-free' state to one which is pair-dominated. Apart from $f_l$ controlling the pair production rate, the absolute initial values of $z_s$ and $\gamma_0$ may be important in determining how sensitive $\Gamma$ is to the input parameters for. One of the reasons for this is that a change in $z_s$ or $\gamma_0$ may take the break energy, $x_B$, from below the pair production threshold (at $x = 1$), to a value which is far enough beyond this threshold to significantly increase the pair production rate and thus steepen the X-ray spectrum. Suppose that $\gamma_0$ changes by a factor $f_{\gamma}$ and/or $z_s$ by a factor $f_z$. Then for a given initial $z_s$, equation 4.6 defines a (minimum) critical initial $\gamma_0$ above which the break energy, $x_B$, moves beyond $x = 1$. We shall call this critical initial value of the Lorentz factor $\gamma_L$, where

$$\gamma_L \approx 158 f_{\gamma}^{-1} f_z^{-\frac{1}{2}} \left( \frac{3 \times 10^{-8}}{x_s} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4.7)

As $\gamma_0$ increases further, the rate of increase in luminosity beyond $x = 1$ is reduced as the $\gamma$-ray 'hump', which contains much of the total luminosity, has moved past $x = 1$. Also, another effect comes into play which actually tends to decrease $\Gamma$, especially if $f_z$ increases simultaneously. That is, since $\gamma$-ray photons with energy $x$ interact with X-ray photons of energy $1/x$, as more $\gamma$-ray photons appear at higher energies there is a progressive depletion of photons at lower X-ray energies. The effect on $\Gamma$ will begin to be important when photons in the $\gamma$-ray 'hump' interact with 10 keV photons and the same effect will dominate when $\gamma_0$ is large enough for X-ray photons with energies less than 2 keV to be significantly absorbed. Hence there is some upper value of initial $\gamma_0$, $\gamma_U$, above which the flattening of the X-ray slope due to increased photon-photon absorption dominates. Using the same notation as for equation 4.7, equation 4.6 gives

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where $E_{\text{keV}}$ is the energy of X-ray photons (in keV) which interact with photons at $z_B$. Hence for a fixed $z_s$ and $f_{\gamma} = 3$, flattening of the X-ray spectrum begins when the initial $\gamma_0 \sim 270$ ($E = 10$ keV) and dominates when $\gamma_0 > 600$ ($E < 2$ keV). Hereafter, we refer to the range in Lorentz factor from $\gamma_L$ to $\gamma_U$ as $\gamma_{LU}$.

Another effect which can cause the source to be sensitive to a transition of the 2–10 keV spectrum from being pair-free to pair-dominated is for the pair reprocessed spectrum to extend into the lower end of the 2–10 keV band in the new state (see equation 4.5). Again, using the same notation as above, this effect will be most pronounced when

$$\gamma_0 \sim 703 f_{\gamma}^{-1} f_{s}^{-1} \left( \frac{3 \times 10^{-5}}{x_s} \right)^{\frac{1}{2}} . \quad (4.9)$$

Finally, if $\tau_\gamma > 1$, 'pair-trapping' can be important in determining the variation of $\Gamma$ with 2-10 keV flux, whereby X-rays which have previously been 'trapped' through interaction with $\gamma$-rays to produce pairs, can suddenly be released if the injected energy is decreased (and vice versa; see F86). The effect is illustrated for type V variability for the initial model [1.0, 0.1, 500], in Fig. 4.9 and the initial model [5.0, 0.15, 250], in Fig. 4.10. Both Figs. 4.9 and 4.10 show $\Gamma$, $G_{EX}$, the 2–10 keV, 100 keV and 1 MeV fluxes (relative to initial values) as a function of time. In the model [1.0, 0.1, 500] (Fig. 4.9) pair-trapping is not important, although the initial large increase in $G_{EX}$ can be accounted for by the finite time for pairs to establish themselves. For the model [5.0, 0.15, 250] (Fig. 4.10), however, pair-trapping causes $\Gamma$ to temporarily increase when the injected energy is decreased and the same effect causes the $\gamma$-ray excess of the perturbed model to be less than that of the initial model.
Figure 4.9 Variation of (a) $\Gamma$, (b) the $\gamma$-ray excess, $G_{EX}$, (c) $l_{2-10}$ and (d) the 100 keV and 1 MeV fluxes (solid and dashed curves respectively), as a function of time when the parameters of the model $[1.0, 0.1, 500]$ are changed according to type V variability. At $t = 0$ the initial values of $l_3$, $l_e$ and $\gamma_0$ are increased by a factor 3 and returned to their original values at a time $10R/c$. In this case even the perturbed model is not heavily pair-dominated ($\gamma_T = 0.17$) so that the $\gamma$-ray excess of the new model is significantly higher than that of the initial model.
Figure 4.10 Variation of (a) $\Gamma$, (b) the $\gamma$-ray excess, $G_{EX}$, (c) $l_{2-10}$ and (d) the 100 keV and 1 MeV fluxes (solid and dashed curves respectively), as a function of time when the parameters of the model [5.0, 0.03, 250] are changed according to type V variability. At $t = 0$ the initial values of $l_e$, $l_\gamma$ and $\gamma_0$ are increased by a factor 3 and returned to their original values at a time $15R/c$. In this case the perturbed model is moderately pair-dominated ($\tau_T = 1.44$) so that the $\gamma$-ray excess of the new model is actually lower than that of the initial model.
Type I variability results

Type I variability corresponds to an increase in the rate of injection of primary electrons, while the energy of each electron remains constant. In order to keep $l_e/l_s$ constant, the EUV flux must change simultaneously in the same sense or the X-ray source must intercept a fraction of the EUV source which is controlled by $l_s$. The latter situation might arise if the size of the X-ray source changes with $l_s$ and this requires that the X-ray source is smaller than the EUV source (which is very likely).

The results for type I variability are presented in Table 4.3 and Fig. 4.11. The points on the flux-index curves in Fig. 4.11 represent equal intervals of time so that the density of points is indicative of the relative proportion of time spent on a particular part of the curves. It is also possible to deduce the direction traversed by the model for each curve from the density of points. The considerable amount of hysteresis present is due to the fact that the pair annihilation timescale $(\sim R/(c\tau_p))$ is in general different from the escape time $(\sim (1+\frac{1}{2}\tau_p)R/c)$. No flux-index correlation would be apparent for a source exhibiting this type of variability unless it was frequently sampled during a flare.

It can be seen that for a given $\gamma_0$ there is some value of $l_\delta$ for which there is a maximum increase in $\Gamma$. Similarly, for a given $l_\delta$, there is some value of $\gamma_0$ for which there is a maximum increase in $\Gamma$. This is because models with larger $l_\delta$ and/or $\gamma_0$ have smaller $l_s/l_e$ so that steepening produced by increased pair reprocessing is counteracted by flattening due to repeated scatterings in these models. The increase in $l_{2-10}$, however, always increases with increased pair reprocessing. The change in $l_{2-10}$ for type I variability can be as much as a factor 10 for a factor 3 increase in $l_\delta$.

These effects are illustrated in Fig. 4.12 for the two models m and o which have initial parameters $[2.0, 0.04, 500]$ and $[4.0, 0.01, 500]$ respectively. The dominance of repeated scattering in the high state of model o can clearly be seen.

Table 4.3 shows that in general the changes in the 100 keV flux mimic those of $l_{2-10}$ but on a smaller scale and the changes in the 1 MeV flux simply reflect the factor change in $l_s$ (i.e. 3 in this case). Also, $G_{EX}$ does not greatly exceed unity for models with an initial $\gamma_0$ of up to 500.

Finally, we note that for type I variability $\Gamma$ can be greater than $\Gamma_{4-23}$ by as much as $\sim 0.05$ for the models which have a high state with $\tau_p > 1$. This would be apparent as a variable hard tail to the 2–10 keV power law which could be detected with future instrumentation (see §7.3).
Table 4.3 Type I variability results

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$^a \gamma_0(s)$ = Lorentz factor of steady state model; $^b l_e(s)$ = electron compactness of steady state model; $^c \tau_T(v)$ = Thomson depth of new steady state model when the input parameters are changed; $^d$ range in $\Gamma$ when the input parameters are varied and returned to their original values (see text); $^e f_1 = \text{ratio of } 2-10 \text{ keV luminosity for the new steady state model to that of the initial model}$; $^f f_{\tau_1} = \text{ratio of the flux at 100 keV for the new steady state model to that of the initial model}$; $^g f_{\tau_2} = \text{ratio of the flux at 1 MeV for the new steady state model to that of the initial model}$; $^h G_{EX}(v) = \gamma\text{-ray excess of new steady state model.}$
Figure 4.11 Flux-index diagrams for models a–u when the input parameters change according to type I variability (i.e. only $l_x$ and $l_x$ vary such that $l_x/l_x$ is constant). The values of $l_x$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
Figure 4.12 Example spectra for type I variability for a case when there is a large increase in $\Gamma$ (solid curves) and a small increase in $\Gamma$ (dashed curves). The lower spectra are the initial models (parameters shown) and the upper spectra are the steady state models with the input parameters increased according to type I variability. See text for more details.
Type II variability results

Type II variability corresponds to the primary electron injection rate varying while $\gamma_0$ remains constant (as does type I) but without correlated changes in the EUV input. The results for type II variability are presented in Table 4.4 and Fig. 4.13. It can be seen that the increase in $\Gamma$ is less than the corresponding increase for type I variability because $l_s/l_e$ decreases when $l_e$ increases (see Fig. 4.5). In fact, $\Gamma$ can actually decrease when the increase in pair production is insufficient to balance the decrease in $l_s/l_e$ (i.e. when the initial $l_e$ is small). The increase in $l_{2-10}$ is also less than the corresponding case for type I variability, since there are relatively less soft seed photons to scatter up to high energies. The behaviour of the fluxes at 100 keV and 1 MeV are in general similar to the type I cases except that the factor increase relative to the initial values are less. The $\gamma$-ray excess, however, can be slightly higher than the corresponding type I models due to the shallower X-ray slopes of the type II models. As with type I variability, a hard tail above 10 keV can be present for models which have a high Thomson depth in the high state.
## Table 4.4 Type II variability results

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$a$ $\gamma_0(s)$ = Lorentz factor of steady state model; $b$ $l_e(s)$ = electron compactness of steady state model; $c$ $\tau_T(v)$ = Thomson depth of new steady state model when the input parameters are changed; $d$ range in $\Gamma$ when the input parameters are varied and returned to their original values (see text); $e$ $f_l$ = ratio of 2-10 keV luminosity for the new steady state model to that of the initial model; $f$ $f_{\gamma_1}$ = ratio of the flux at 100 keV for the new steady state model to that of the initial model; $g$ $f_{\gamma_2}$ = ratio of the flux at 1 MeV for the new steady state model to that of the initial model; $h$ $G_{EX}(v)$ = $\gamma$-ray excess of new steady state model.
Figure 4.13 Flux-index diagrams for models a–u when the input parameters change according to type II variability (i.e. only $L_x$ varies). The values of $L_x$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
Type IIIa variability results

Type IIIa variability corresponds to fluctuations in the temperature (or peak energy) of the EUV source with no changes in the high energy electron parameters. The results are presented in Table 4.5 and Fig. 4.14. Now $l_{2-10}$ always decreases when $z$, increases because for a fixed $l_{2}/l_{e}$ the extrapolated X-ray power law always lies below the peak at $z$, by a fixed amount (in other words, the peak energy is shifted to higher values). From equation 4.9 (putting $f_{\gamma} = 1$ and $f_{s} = 3$), the increase in $\Gamma$ is largest when $\gamma_{0} \sim 308$. This is why the largest increase in $\Gamma$ occurs for the $\gamma_{0} = 250$ models (up to $\Gamma \sim 1.6$ for model $h$; see Fig. 4.14). However, for a given $\gamma_{0}$ the increase in $\Gamma$ is smaller if $l_{2}$ is too small (not enough pair reprocessing) or too large (mainly due to the increasing dominance of repeated scattering). Also, if $\gamma_{0}$ is large, $\Gamma$ can actually decrease due to an increase in photon absorption (to create pairs) below 10 keV. Putting $f_{\gamma} = 1$, $f_{s} = 3$ and $E_{\text{keV}} = 10$ into equation 4.8 implies that this effect becomes important when $\gamma_{0} > 460$. The effect is illustrated in Fig. 4.15 which shows the spectra corresponding to the high and low states of the models $h$ and $o$ which have initial parameters $[3.0, 0.15, 250]$ and $[1.7, 0.01, 1000]$ respectively. We note that models which show a large increase in $\Gamma$ have an X-ray slope which flattens slightly above 10 keV.

For type IIIa variability the factor change in 100 keV flux is in general similar to the factor change in $l_{2-10}$ while the factor change in the 1 MeV flux is somewhat higher than the factor change in $l_{2-10}$ for the $\gamma_{0} = 100$ and 250 models (see Table 4.4). Note that the $\gamma$-ray excess can be larger than unity even for the $\gamma_{0} = 250$ models.
Table 4.5 Type IIIa variability results

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</table>

$^a \gamma_0(s) = \text{Lorentz factor of steady state model}; \ ^b \Gamma_0(s) = \text{electron compactness of steady state model}; \ ^c \tau_T(v) = \text{Thomson depth of new steady state model when the input parameters are changed}; \ ^d \Gamma = \text{range in } \Gamma \text{ when the input parameters are varied and returned to their original values (see text)}; \ ^e f_1 = \text{ratio of 2–10 keV luminosity for the new steady state model to that of the initial model}; \ ^f f_{\gamma_1} = \text{ratio of the flux at 100 keV for the new steady state model to that of the initial model}; \ ^g f_{\gamma_2} = \text{ratio of the flux at 1 MeV for the new steady state model to that of the initial model}; \ ^h G_{\text{EX}}(v) = \gamma\text{-ray excess of new steady state model.}$
Figure 4.14 Flux-index diagrams for models a–u when the input parameters change according to type IIIa variability (i.e. only $x_s$ varies). Note that models b and c are not shown for the sake of clarity. The values of $l_s$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
Figure 4.15 Example spectra for type IIIa variability for a case when there is a large increase in $\Gamma$ (solid curves) and when $\Gamma$ actually decreases (dashed curves). The lower spectra are the initial models (parameters shown) and the upper spectra are the steady state models with the input parameters increased according to type IIIa variability. See text for more details.
Type IIIb variability results

Type IIIb variability corresponds to changes in the luminosity of the soft photon source without corresponding changes in its temperature or the parameters of the relativistic electrons. The results are shown in Table 4.6 and Fig. 4.16. It can be seen that both $\Gamma$ and $l_{2-10}$ always increase when $l_s$ is increased. For a factor 3 change in $l_s$, the maximum increase in $l_{2-10}$ is by a factor $\sim 1.6$ while the largest value of $\Gamma$ attained is $\sim 1.56$. The increase in both $\Gamma$ and $l_{2-10}$ is larger for larger values of $l_s$ and $\gamma_0$, due to the increased role of pair reprocessing, as can be seen from the greater hysteresis in the curves of Fig. 4.16 with large $l_s$ and/or $\gamma_0$. However, when there is little pair production the increase in $l_s/l_s$ in type IIIb variability is important in increasing $\Gamma$ (see Fig. 4.5). These effects can also be seen in Fig. 4.17 which shows spectra corresponding to high and low states of models a and r which have parameters $[0.3, 1.0, 100]$ and $[2.3, 0.01, 750]$ respectively.

As might be expected for type IIIb variability, the changes in the fluxes at 100 keV and 1 Mev are negligible for most cases, except if $\gamma_0$ is small (see Table 4.6). Also, the $\gamma$-ray excess for the $\gamma_0 = 100$ and 250 models actually decreases relative to the initial value. The reason for both these effects is that the increase in $l_s/l_s$ means that the first order spectrum contains relatively more luminosity in the high state than in the low state. Since, for small $\gamma_0$ the very high energy $\gamma$-ray spectrum is due to repeated scatterings, the decrease in flux at these energies can be significant as $l_s$ is constant. Finally, $\Gamma_{4-23}$ does not differ significantly from $\Gamma$ for type IIIb variability.

In reality of course, both the temperature and the luminosity of the source may vary simultaneously, in which case the opposite senses of change in $l_{2-10}$ for types IIIa and IIIb variability may cancel. For large $\gamma_0$ the changes in $\Gamma$ are also in opposite directions so there may be very little net change in $\Gamma$. For smaller values of $\gamma_0$ the net change in $\Gamma$ may be larger than that for either type IIIa or IIIb alone.
Table 4.6 Type IIIb variability results

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<th>( \Gamma )</th>
<th>( f_l )</th>
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\( \gamma_0(s) \) = Lorentz factor of steady state model; \( L_e(s) \) = electron compactness of steady state model; \( \tau_T(v) \) = Thomson depth of new steady state model when the input parameters are changed; \( \Gamma \) range in \( \Gamma \) when the input parameters are varied and returned to their original values (see text); \( f_l \) = ratio of 2-10 keV luminosity for the new steady state model to that of the initial model; \( f_{\gamma 1} \) = ratio of the flux at 100 keV for the new steady state model to that of the initial model; \( f_{\gamma 2} \) = ratio of the flux at 1 MeV for the new steady state model to that of the initial model; \( G_{\gamma E X}(v) \) = \( \gamma \)-ray excess of new steady state model.
Figure 4.16 Flux-index diagrams for models a–u when the input parameters change according to type IIIb variability (i.e. only $l_y$ varies). The values of $l_y$ and $\gamma_0$ of the initial models are indicated. Note that models g and k are not shown for the sake of clarity. The 2–10 keV luminosity is shown relative to the initial value.
Figure 4.17 Example spectra for type IIIb variability for a case when there is a moderate increase in $\Gamma$ (dashed curves) and when the change in $\Gamma$ is small (solid curves). The lower spectra are the initial models (parameters shown) and the upper spectra are the steady state models with the input parameters increased according to type IIIb variability. See text for more details.
Type IV variability results

Type IV variability corresponds to fluctuations in the Lorentz factor of the primary injected electrons with no changes in the EUV input. Alternatively, it can be interpreted as fluctuations in the electron injection rate while the total energy density injected per unit time remains constant.

The results for type IV variability are presented in Table 4.7 and Fig. 4.18. For an increase in \( \gamma_0 \), \( I_{2-10} \) must decrease since the total energy density is fixed. However, Fig. 4.18 shows that for a given \( \gamma_0 \), the decrease in \( I_{2-10} \) is least when \( \gamma_0 \) is largest, since more luminosity is 'pumped' into the X-ray band by pairs for larger \( \gamma_0 \). For a given \( \gamma_0 \) the decrease in \( I_{2-10} \) is least when \( \gamma_0 \) is larger, for the same reason. By putting \( f_\gamma = 3 \) and \( f_s = 1 \) into equation 4.9 we can deduce that \( \Gamma \) will be most sensitive to changes in \( \gamma_0 \) when \( \gamma_0 \sim 234 \). This is why the models with \( \gamma_0 = 250 \) can show the largest increase in \( \Gamma \) (e.g. for model h, \( \Gamma \) can actually reach 1.75; see Fig. 4.18). However, the steepening of the X-ray slope due to pair reprocessing can be counteracted by increased photon absorption below 10 keV if \( \gamma_0 \) is sufficiently large. The effect will become important when \( \gamma_0 > 266 \) (putting \( E_{\text{keV}} = 10 \) into equation 4.8) and will dominate when \( \gamma_0 > 596 \) (putting \( E_{\text{keV}} = 2 \) into equation 4.8). This is why \( \Gamma \) always at first increases with \( \gamma_0 \), but is less for larger values of \( \gamma_0 \), unless \( \gamma_0 \) is small (see Fig. 4.18). Example spectra are shown in Fig. 4.19 for the high and low states of models h (large increase in \( \Gamma \)) and u (slight decrease in \( \Gamma \)) which have initial parameters \([3.0, 0.15, 250]\) and \([1.7, 0.01, 1000]\) respectively. \( \Gamma_{4-23} \) can be less than \( \Gamma \) by as much as \( \sim 0.05 \) (when the change in \( \Gamma \) is large) for type IV variability and this can also be seen for model h in Fig. 4.19.

Table 4.7 shows that the fluxes at 100 keV and 1 MeV generally follow the same trends as \( I_{2-10} \) except for the \( \gamma_0 = 100 \) models, in which the flux at 1 MeV can actually increase significantly. This is because the peak in the first order spectrum moves past 1 MeV when \( \gamma_0 \) increases from the initial value of 100. Also evident from Table 4.7 is the fact that the \( \gamma \)-ray excess can exceed unity even for the \( \gamma_0 = 100 \) models. Hence, type IV variability is unlikely to be common in AGN.
<table>
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* \( \gamma_0(s) \) = Lorentz factor of steady state model; \( l_e(s) \) = electron compactness of steady state model; \( \tau_T(v) \) = Thomson depth of new steady state model when the input parameters are changed; \( \Gamma \) = range in \( \gamma \) when the input parameters are varied and returned to their original values (see text); \( f_1 \) = ratio of 2–10 keV luminosity for the new steady state model to that of the initial model; \( f_{T1} \) = ratio of the flux at 100 keV for the new steady state model to that of the initial model; \( f_{T2} \) = ratio of the flux at 1 MeV for the new steady state model to that of the initial model; \( G_{EX}(v) \) = \( \gamma \)-ray excess of new steady state model.
Figure 4.18 Flux-index diagrams for models a–u when the input parameters change according to type IV variability (i.e. only $\gamma_0$ varies). Note that model b is not shown for the sake of clarity. The values of $l_e$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
Figure 4.19 Example spectra for type IV variability for a case when there is a large increase in $\Gamma$ (solid curves) and when the change in $\Gamma$ is small (dashed curves). The lower spectra are the initial models (parameters shown) and the upper spectra are the steady state models with the input parameters increased according to type IV variability. See text for more details.
Type V and VI variability results

Type V variability corresponds to the acceleration mechanism causing fluctuations in the Lorentz factor of the injected primary electrons while the injection rate remains constant. A correlated change in the EUV input is also required to keep \( l_s/l_e \) constant. Now, the composite effect of an increase in \( l_s \) and \( \gamma_0 \) is to combine the properties of type I and type IV variability.

The results for type V variability are presented in Table 4.8 and Fig. 4.20. It can be seen that large changes in both \( l_{2-10} \) and \( \Gamma \) are now possible, both in the same sense as the change in input parameters. Fig. 4.20 shows that for a given \( \gamma_0 \), there is some value of (initial) \( l_s \) for which the increase in \( \Gamma \) is largest. This is because models with high initial \( l_s \) have small \( l_s/l_e \) so that the increase in \( \Gamma \) due to pair reprocessing cannot compensate for the decrease in \( \Gamma \) due to repeated scattering. For a given \( l_s \), the increase in \( \Gamma \) is less for larger values of \( \gamma_0 \) due to an increased amount of pair absorption below 10 keV for large values of \( \gamma_0 \). The effect dominates above \( \gamma_0 \sim 600 \) for the parameters that we have used (i.e., putting \( f_\gamma = 3, f_s = 1 \) and \( E_{keV} = 2 \) in equation 4.8). \( \Gamma \) is most sensitive to changes in the input parameters when \( \gamma_0 \sim 234 \) (from equation 4.9) and this is why the largest values of \( \Gamma \) are obtained for some of the \( \gamma_0 = 250 \) models. For instance, \( \Gamma \) can be as high as 1.86 for model h (see Fig. 4.20). It is evident that a strong, positive flux-index correlation may be obtained for type V variability (unless \( \gamma_0 \) is large), especially when the hysteresis effect is small, as in the models with \( \gamma_0 = 500 \). Note that no significant changes in \( l_{2-10} \) or \( \Gamma \) are expected if \( \gamma_0 < 50 \) (putting \( f_\gamma = 3 \) in equation 4.7). Example spectra are shown in Fig. 4.21 which correspond to the high and low states of models 1 and 2 which have initial parameters [1.0, 0.1, 500] and [1.7, 0.01, 1000] respectively.

Table 4.8 shows that the 100 keV flux follows the same trends as \( l_{2-10} \) but the 1 MeV flux shows comparatively little variability except for the \( \gamma_0 = 100 \) models. It is interesting to note that for a given \( \gamma_0 \) models with the highest initial values of \( l_e \) give the smallest \( \gamma \)-ray excess in the high state, indicating that pairs can cause a significant steepening of the \( \gamma \)-ray spectrum above \( x = 1 \) for type V variability.

The results for type VI variability are presented in Table 4.9 and Fig. 4.22 and can be interpreted closely along the lines of type V variability. In this case however, there is no correlated change in the EUV input so that the decrease in the ratio \( l_s/l_e \) causes the change in \( \Gamma \) and \( l_{2-10} \) to be less than the corresponding case for type V variability. In fact, \( \Gamma \) can actually decrease in some cases (see Fig. 4.22). Another point of interest is that \( \Gamma_{4-23} \) is always close to \( \Gamma \) for all models in the parameter space for both type V and VI variability, so that no hard tail above 10 keV is expected for this type of variability.
Table 4.8 Type V variability results

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<th>$f_2$</th>
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$^{a} \gamma_0(s) =$ Lorentz factor of steady state model; $^{b} I_e(s) =$ electron compactness of steady state model; $^{c} \tau_T(v) =$ Thomson depth of new steady state model when the input parameters are changed; $^{d} \Gamma =$ range in $\Gamma$ when the input parameters are varied and returned to their original values (see text); $^{e} f_1 =$ ratio of 2–10 keV luminosity for the new steady state model to that of the initial model; $^{f} f_2 =$ ratio of the flux at 100 keV for the new steady state model to that of the initial model; $^{g} f_{250} =$ ratio of the flux at 1 MeV for the new steady state model to that of the initial model; $^{h} G_{EX}(v) =$ $\gamma$-ray excess of new steady state model.
Figure 4.20 Flux-index diagrams for models a–u when the input parameters change according to type V variability (i.e. $l_s$, $l_e$ and $\gamma_0$ vary such that $l_s/l_e$ and $l_e/\gamma_0$ are constant). Note that model a is not shown for the sake of clarity. The values of $l_e$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
Figure 4.21 Example spectra for type V variability for a case when there is a large increase in $\Gamma$ (solid curves) and when there is very little change in $\Gamma$ (dashed curves). The lower spectra are the initial models (parameters shown) and the upper spectra are the steady state models with the input parameters increased according to type V variability. See text for more details.
Table 4.9 Type VI variability results

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<td>1.45-1.48</td>
<td>1.9</td>
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<td>1.1</td>
<td>5.47</td>
<td>q</td>
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<tr>
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<td>1.43-1.46</td>
<td>2.3</td>
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<td>1.3</td>
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<td>1.45-1.45</td>
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</tr>
<tr>
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<td>1.40-1.45</td>
<td>1.8</td>
<td>1.7</td>
<td>1.3</td>
<td>4.44</td>
<td>u</td>
</tr>
</tbody>
</table>

*a* $\gamma_0(s)$ = Lorentz factor of steady state model; *b* $l_e(s)$ = electron compactness of steady state model; *c* $\tau_T(v)$ = Thomson depth of new steady state model when the input parameters are changed; *d* range in $\Gamma$ when the input parameters are varied and returned to their original values (see text); *f* $f_1$ = ratio of 2–10 keV luminosity for the new steady state model to that of the initial model; *f* $f_{71}$ = ratio of the flux at 100 keV for the new steady state model to that of the initial model; *f* $f_{72}$ = ratio of the flux at 1 MeV for the new steady state model to that of the initial model; *g* $G_{EX}(v)$ = γ-ray excess of new steady state model.

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Figure 4.22 Flux-index diagrams for models a-u when the input parameters change according to type VI variability (i.e. $l_s$ and $\gamma_0$ vary such that $l_s / \gamma_0$ is constant). Note that model a is not shown for the sake of clarity. The values of $l_s$ and $\gamma_0$ of the initial models are indicated. The 2–10 keV luminosity is shown relative to the initial value.
SUMMARY OF THE EFFECTS OF DIFFERENT TYPES OF VARIABILITY

We now express the results above in a very general form below.

- If there are fluctuations in the injection rate of primary electrons at a constant Lorentz factor then no flux-index correlation is expected (unless the source is sampled continuously or frequently during a flare). Instead, a spread in $\Gamma$ will be apparent for a range in 2–10 keV flux. The spread in $\Gamma$ is of the order $\sim -0.05$ to $\sim 0.10$ for small and large values of the electron compactness and can be as much as $\sim -0.05$ to $\sim 0.20$ for intermediate values.

- Variations in the temperature and/or luminosity of the soft photon source (while the relativistic electron parameters remain constant) can only produce small changes in both $\Gamma$ (in most cases) and the 2–10 keV flux. A flux-index correlation (or anti-correlation) may be apparent, depending on model parameters and the sensitivity of instrumentation.

- Fluctuations in the Lorentz factor of the injected electrons can give rise to
  
  (i) a flux-index anti-correlation if the total rate at which energy is injected into the source region remains constant, or

  (ii) a strong positive flux-index correlation if the number of electrons injected per unit time remains constant.

  However, both cases require that the initial Lorentz factor, $\gamma_0$, lie in the approximate range $\sim 50 (6 \times 10^4)^{1/2} - \sim 600 (6 \times 10^4)^{1/3}$ (see equations 4.7 and 4.8 respectively) and that the electron compactness lies within a certain range, depending on the actual value of $\gamma_0$. If these conditions are not satisfied a small range in $\Gamma$ would be apparent for a range in 2–10 keV flux.

In other words, if the X-ray and $\gamma$-ray spectrum of flat-spectrum AGN is indeed produced by Compton scattering of UV photons on monoenergetically injected electrons then spectral index variability, uncorrelated with the 2–10 keV continuum level, is expected to be quite common. Spectral index variability which is more tightly correlated or (anti-correlated) with the continuum flux is expected to be less common since this requires more specific physical conditions. In general, the changes in both $\Gamma$ and 2–10 keV flux are exaggerated if the injected energy in soft photons is correlated with changes in the injected electron parameters.

The different types of variability that we have considered make specific predictions about the variability of the $\gamma$-ray flux and the presence/absence of a hard tail above 10 keV and comparison of these with observation may be important in constraining the parameter space of possible models for individual objects.
Tests show that if the factor increase in the input parameters were 4 instead 3, as used for the present variability models, the change in $\Gamma$ is not significantly different (and is often less), unless the initial model has $\tau_T \ll 1$. This can usually be explained in terms of either the saturation of the pair yield or an increase in trapped X-rays at lower energies to produce pairs. The same principles described in the present study may be used to deduce how the variability results would differ if the initial models had $\Gamma > 1.5$ but is beyond the scope of this work. Such models may already be pair-dominated since higher values of $I_e$ are allowed and also thermal Comptonization and 'pair-trapping' would play a more important role. Without detailed calculation we can only say that spectral index variability would still be quite common.

**POWER-LAW ELECTRON INJECTION**

In the spectral variability study above our attention has been restricted to the case when all the primary electrons are injected into the source region at the same Lorentz factor. If the electrons were injected with a range of Lorentz factors from 1, say, to some cutoff, $\gamma_{\text{max}}$, at a rate $\propto \gamma^{-\eta}$, then we can make some general statements about how our results would differ if $\eta > 2$ (i.e. when most of the energy is injected in electrons with a small Lorentz factor). Firstly, the parameter space for flat-spectrum sources would be more restricted since the $I_e/I_\gamma = 10^{-2}$ curve in Fig. 4.7 would move towards the bottom-left of the diagram by an amount which depends on the value of $\eta$ (see Fig. 4.5). Secondly, for $\eta > 2$, electrons with a low Lorentz factor dominate. Although this would somewhat ease the $\gamma$-ray excess problem, pair production becomes less important so that all the spectral variability effects due to pairs would be diminished. Hence, if $\eta$ remained constant, the variations in spectral index would, in most cases, be small for the modest changes in input parameters ($\sim 3-4$) that we have considered (see LZ87, who have computed several stationary pair models for $\eta > 2$). On the other hand, the spectral variability properties for models with $\eta < 2$ are less obvious and must be investigated by solving equation 4.4. as we have done for the case of monoenergetic electron injection. Models with variable $\eta$ can also produce significant spectral changes but this case needs further investigation. In the case of NGC 4151, we will present arguments in §4.2.2.3 which actually favour very flat or monoenergetic injection.

### 4.2.2 A class of models for NGC 4151

As described in §3.3.2, the recent *Ginga* measurements of the X-ray photon spectral index, $\Gamma$, of NGC 4151 have revealed that $\Gamma$ is correlated with the 2-10 keV continuum flux, $F_\gamma$, possibly down to a timescale of several hours (see Fig. 3.8). In this section we investigate whether any of the above variability models can account for the observed flux-index correlation and if so, whether the parameter space can be constrained at all. The reader is reminded that the flat-spectrum stationary models computed in §4.2.1.2, specifically had $\Gamma = 1.45$ so that they could be applied
directly to NGC 4151. This is the value of $\Gamma$ that NGC 4151 appears to have near its lowest state (see §3), for which we take $F_\gamma^\ast$ to be 10 (in units $10^{-11}$ erg cm$^{-2}$ s$^{-1}$).

### 4.2.2.1 OBSERVATIONAL CONSTRAINTS FOR NGC 4151

Now that we are considering a specific source we can be more precise about observational constraints. First, we can place a lower limit on $l_e$ since the observed luminosity from X-ray to $\gamma$-ray energies, $L_{\gamma\gamma}$ (in units $10^{42}$ erg s$^{-1}$), cannot be greater than the injected electron luminosity, $L_e$ (see equation 4.3). Hence

$$l_e > \frac{0.27 L_{\gamma\gamma}}{R_{14}}$$

Baity et al. (1984) estimate $L_{\gamma\gamma} = 30$ (5 keV - 1 MeV) in the low state and 120 in a high state. This is a lower limit since it has been suggested that the luminosity in NGC 4151 peaks at $\sim 500 - 2000 \times 10^{42}$ erg s$^{-1}$ in the 100 keV - 10 MeV range (Perotti et al., 1981). An upper limit of 1 light day on $R_{14}$ then gives $l_e > 0.3$. The source radius is likely to be somewhat less than 1 light day since, as pointed out by DF89 and confirmed in the present work, time lags introduced by pairs can make distances deduced from light-crossing time arguments appear to be a factor $\sim 3 - 4$ larger than they actually are.

It is difficult to tighten the constraints on $l_s/l_e$ since the fraction of the observed soft photons that the electrons 'see' is unknown (§4.2.1.3). However, we note that from the compiled broadband spectrum of NGC 4151 (§4.1.8) the ratio of UV to X-ray/ $\gamma$-ray luminosity is $\sim 0.14$ in the low state and $\sim 0.25$ in the high state. However, these values are subject to the uncertainties in the compilation of the spectrum in §4.1.8. We already have an upper limit on $l_s/l_e$ of $\sim 1$ simply from the fact that an X-ray slope of less than 1.5 is not possible for larger values of $l_s/l_e$ (see Fig. 4.5). Whether $l_s/l_e$ is constant as the non-thermal continuum level varies is also uncertain. Recent simultaneous measurements by IUE and EXOSAT showed that the UV and X-ray fluxes are well correlated (Perola et al., 1986) whereas older measurements (Ulrich et al., 1984) suggest that they are independent. Finally, as discussed in §4.1.6, the spectral form above 100 keV and the location of the high energy break are poorly determined, although the X-ray power law is believed to extend out to at least $\sim 1$ MeV (Baity et al., 1984). The $\gamma$-ray excess, as defined by equation 4.2, can therefore not be estimated with any certainty but is likely to be greater than unity.
4.2.2.2 THE FLUX-INDEX CORRELATION IN NGC 4151

From the results obtained in §4.2.1.4 we find that only variability of types V and VI can reproduce the form of the flux-index correlation in NGC 4151 (see Fig. 3.8), provided that $\gamma_0$ lies in the range $\gamma_L - \gamma_U$ (equations 4.7 and 4.8 respectively) (i.e. $\sim 50$ to $\sim 600$ if the peak energy of the soft photon input is taken to be $x_\gamma = 3 \times 10^{-5}$). From Fig. 4.9 it would appear that some type I models could also reproduce the form of the flux-index correlation in NGC 4151 (e.g. the model with $l_\gamma = 2$ and $\gamma_0 = 500$). However, the amount of hysteresis is so large for type I variability that it is likely that any flux-index correlation would be masked for data which has been accumulated for isolated observations over a number of years, as is the case for NGC 4151.

Obviously, there are an infinite number of combinations of $l_\gamma$ and $\gamma_0$ which will fit the data. Of the models that we have calculated, the ones with the following initial parameters appear to be able to account for the flux-index correlation in NGC 4151: $[6.0, 0.05, 100]$ (type V), $[3.0, 0.15, 250]$ (type VI) and $[1.0, 0.1, 500]$ (type V). The $\gamma$-ray excess in the high state of these models is 0.93, 2.06 and 4.56 respectively. Considering the relatively short length of time occupied by the high state, all these are acceptable for NGC 4151 (provided of course, that the same type of behaviour is not common in most AGN). The relevant flux-index curves for the above three models (Figs. 4.20 and 4.22) have been plotted in Fig. 4.23 against the flux-index data for NGC 4151 (Fig. 3.8) after re-scaling so that $F_{\gamma}^Z = 10$ for $\Gamma = 1.45$. The required normalizations give $R_{14} \sim 1.6, 2.2$ and 4.1 respectively for the models $[6.0, 0.05, 100]$, $[3.0, 0.15, 250]$ and $[1.0, 0.1, 500]$. It can be seen that for the $[3.0, 0.15, 250]$ model $\Gamma$ increases 'too fast' with $F_{\gamma}^Z$ but the $[6.0, 0.05, 100]$ and $[1.0, 0.1, 500]$ models provide acceptable fits to the data. For the $[1.0, 0.1, 500]$ model, values of $\Gamma$ for $F_{\gamma}^Z < 10$ in Fig. 4.23 were generated by decreasing the relevant parameters of the same steady state model according to type V variability.
Figure 4.23 The X-ray photon spectral index measurements for NGC 4151, $\Gamma$, (as in Fig. 3.8) plotted against the 2–10 keV absorption corrected flux, $F_x^\circ$. The solid curve corresponds to the flux-index relation obtained when the input parameters of the model [1.0, 0.1, 500] are changed by a factor of 3 for a time $10R/c$, according to type V variability. The dashed curve corresponds to the parameters of the same steady state model changing by a factor 4 for a time $2.5R/c$, shorter than that required for the new model to come to equilibrium. The dotted curve corresponds to a type V model with initial parameters [6.0, 0.05, 100] and the dot-dashed curve to a type VI model with initial parameters [3.0, 0.15, 250] (see text for details).
It is difficult to be more precise about the parameter range which may be relevant due to the obvious freedom in the factor by which the input parameters may change and the duration of the perturbation. The data points lying below the curves in Fig. 4.23 can easily be accommodated by perturbing the steady state model by different amounts and for different lengths of time. For example, the dashed curve in Fig. 4.23 shows the predicted flux-index correlation when the input parameters for the model $[1.0, 0.1, 500]$ are changed by a factor 4 (according to type V variability) for a time $2.5R/c$. The 2-10 keV light curve for this case is shown in Fig. 4.24a. The flux doubling timescale for this particular model is $\sim 3 \times 10^4$ s, somewhat faster than is typical of NGC 4151 (see §3).

γ-ray variability

As mentioned above, simultaneous monitoring of the X-ray and γ-ray fluxes in the 100 keV to MeV band with future γ-ray missions (see also §7.3), should provide important tests and further constraints on the models presented here. For the simulated flare-like event for the $[1.0,0.1,500]$ model (Fig. 4.24a), the 0.3–3 MeV flux actually remains steady while the flux below and above this range is highly variable. The effect is illustrated in Fig. 4.24b, where the 100 keV and 1 MeV fluxes are shown relative to their initial values. In fact, as can be seen from Tables 4.8 and 4.9, the behaviour of the γ-ray flux is slightly different for different models in the allowed parameter space but more sensitive measurements at MeV energies are required for this to be a useful diagnostic. Comparison of the model γ-ray spectra and luminosities with the data would then provide much more stringent constraints on the parameter space. At present there is little point in fine-tuning the models any further.
Figure 4.24 (a) Variation of the 2–10 keV model flux (relative to the initial value) with time when the parameters of the steady state model [1.0, 0.1, 500] are increased by a factor 4 for a time 2.5R/c. (b) The corresponding variation of the 100 keV (solid curve) and 1 MeV (dashed curve) fluxes, relative to their initial values. (c) The corresponding variation of the $\gamma$-ray excess.
SUMMARY OF SPECTRAL INDEX VARIABILITY MODELS FOR NGC 4151

We have modelled the X-ray to \( \gamma \)-ray emission in NGC 4151 by a simple spherical homogeneous source region injected with monoenergetic relativistic electrons which cool on EUV photons intercepting the source region. The soft photons are presumed to be produced by accretion processes and their distribution in energy was assumed to peak at a temperature, \( T \), of \( 6 \times 10^4 \) K. We have found that the observed variation of \( F \) with \( \Gamma \) in NGC 4151 can be reproduced for a modest change (by a factor \( \sim 3 - 4 \)) in the injected luminosity only if the rate of electron injection is roughly constant, provided that the constraints on the input parameters (in a low flux state) listed below are satisfied.

(i) The electron Lorentz factor, \( \gamma_0 \), lies in the range \( \sim 50 \left( \frac{6 \times 10^4}{E} \right)^{\frac{1}{2}} \) to \( \sim 600 \left( \frac{6 \times 10^4}{E} \right)^{\frac{1}{2}} \).

(ii) the electron compactness lies in the range \( \sim 1 - 10 \) and

(iii) \( l_e/l_s \sim 0.05 - 0.15 \).

Only one of the parameters \( l_s, l_e/l_s \) and \( \gamma_0 \) is independent since for a given \( l_s \) and \( \gamma_0 \), \( l_s/l_e \) is determined by \( \Gamma \) in the low state and only certain combinations of \( l_e \) and \( \gamma_0 \) can provide acceptable fits to the data.

4.2.2.3 CONSTRAINTS ON THE BROADBAND CONTINUUM EMISSION IN NGC 4151

We now justify our assumption in §4.2. that the dominant source of X-rays in NGC 4151 is not Compton scattering of synchrotron photons. The arguments, and the implications for the broadband emission in NGC 4151 are as follows.

- Since the X-ray emission is highly variable and the infrared/ optical emission is not, the two components cannot share the same relativistic electron distribution, unless that distribution is highly contrived. In the case of NGC 4151, even a carefully designed electron distribution cannot work because X-ray spectral variability implies that the shape of the distribution must change and this would produce notable variability in the infrared/ optical spectrum. Hence, the infrared/ optical emission and X-ray emission must come from different regions.

- If synchrotron emission is responsible for the infrared/ optical spectrum, the relativistic electron distribution must be steep, with a steady state power law index of 3.74 (for an infrared energy index of 1.37). The SSC spectrum from this region cannot be the dominant source of X-rays in NGC 4151 otherwise the X-ray flux would be much less variable than is observed. Hence, the ratio of the energy densities of the magnetic field and the relativistic electrons, \( U_B/U_e \), must be large in this region.

Even if the SSC spectrum from the infrared/ optical region were not negligible and was
important in the low state of NGC 4151 it would have to be flat (to account for measurements of $\Gamma < 1.5$ in the low state). The only way to obtain a flat SSC spectrum is to superpose spectra from multiple orders of scattering. Under such conditions, the infrared/optical spectrum is also flat, with no break between it and the X-ray spectrum. This is clearly inconsistent with observation.

- The above point strongly suggests that the dominant source of X-rays in NGC 4151 is likely to be scattering of UV photons on relativistic electrons (rather than SSC). The size of the X-ray emission region is of the order $\sim 10^{14} - 10^{15}$ cm and $U_B/U_r$ must be either very small here (otherwise there would be a significant (variable) contribution to the infrared/optical flux – unless of course the synchrotron emission is highly directional) or very large (so that the synchrotron spectrum is heavily self-absorbed).

- The fact that the X-ray spectrum extends out to 1 MeV in NGC 4151 implies that Lorentz factors of at least $\sim 200$ are required for the first order spectrum (equation 4.6). In any case, the spectral variability models in §4.2.2.2 which fit the data require Lorentz factors of at least $\sim 50$ to work. This means that the electron injection into the X-ray emission region must be either very flat or monoenergetic.

- Hence, two different electron distributions are required to explain the broadband emission in NGC 4151 and these may originate from two different acceleration mechanisms, or the same mechanism giving rise to different electron populations at different distances from the central object. However, the nature of the acceleration mechanism(s), and the distance of the infrared/optical emission region from the central object is highly uncertain.

### 4.2.3 Application to other AGN

The spectral variability of the pair models discussed so far can also qualitatively account for the remaining seven objects in Table 4.1. It is interesting to note that seven out of the eight AGN in Table 4.1 have an X-ray slope which can be less than 1.5 (the exception being NGC 7314). Of course, it may just be that NGC 7314 has not been 'caught' with a flat X-ray slope, although there have been two other observations of this object with Ginga, both of which had a 'near canonical' X-ray slope (Turner, T. J., private communication).

Obviously, a more detailed comparison, than can be presented here, with the models and data for the objects in Table 4.1 is necessary. Here, we simply make some general comments regarding the flux-index diagrams presented for each object in the references listed with Table 4.1.

For NGC 4051 and MCG-6-30-15 types V and VI variability (constant injection rate, variable energy input) are probably most appropriate with $\gamma_L \leq \gamma_0 \leq \gamma_U$, very similar to NGC 4151;
provided that the flux-index correlation in the two AGN is real. For 3C 273 types I and II variability (variable injection rate, constant energy) provide an adequate description of the spectral index variability. For the remaining objects (NGC 2992, NGC 3227 and NGC 5548) a mixture of types I, II, V and VI variability could be at work. It is difficult to constrain the parameter space further without better γ-ray observations.

Of course, all the comments in this section are based on the implicit assumption that the X-ray emission is indeed due to the proposed mechanism.

4.2.4 The Canonical AGN?

We have shown that the philosophy behind contemporary models of the X-ray emission should be modified from attempting to account for a ‘canonical’ X-ray spectral index of \( \sim 1.7 \) to something more sophisticated. Any future models of the X-ray emission in AGN should not only account for the preference of \( \Gamma \sim 1.7 \) but also explain (i) the sizable fraction (> 25%) of objects which have flat X-ray spectra and (ii) flux-correlated spectral index variability of the type seen in NGC 4151. Of course, all models must also satisfy the constraints imposed by the γ-ray background.

Standard pair models with monoenergetic electron injection normally have Lorentz factors which are of the order of \( \sim 1000 \) or greater. However, if the Lorentz factor is restricted to values of a few hundred or less, for a given \( l_e \), \( \Gamma \) can actually be a decreasing function of \( l_e/l_e \) for large values of \( l_e/l_e \). This is because for low Lorentz factors, much of the γ-ray flux is due to repeated scatterings of the soft photon input so that the γ-ray flux available to produce pairs is less for larger values of \( l_e/l_e \). This is contrary to the situation for large Lorentz factors where the first order spectrum always extends well beyond \( z = 1 \) so that \( \Gamma \) is always an increasing function of \( l_e/l_e \) in these models. If we impose the condition that the γ-ray flux from multiple orders is the main source of high energy photons for pair production if the first order spectrum does not extend beyond \( z \sim 10 \), the maximum Lorentz factor of injected electrons turns out to be \( \sim 500 \) (see equation 4.6). Hence, there is the possibility that a simple upper limit on the maximum injected Lorentz factor in pair models may give rise to a preferred range in \( \Gamma \) since the value of \( \Gamma \) for large \( l_e/l_e \) and \( l_e \) is forced to be lower than the value \( \sim 2.0 \) which would pertain without a restriction on the Lorentz factor.

We have tested the above hypothesis by randomly selecting the parameters \( [l_e, l_e/l_e, \gamma_0] \) for a 1000 pair models, where the range in \( l_e \) is always 0.3 – 30. Some real sources are likely to have values outside this range but the relative numbers involved are likely to be small and will not affect our conclusions. Given that all three variables are likely to be variable in at least some sources, the parameter sets for the models then represent some average values in a ‘snap shot’. Fig. 4.25a shows the resulting histogram of \( N \) (number of models) against \( \Gamma \) when values of \( l_e/l_e \) are randomly
selected from the range 0.01 - 1.0 and $\gamma_0$ from 100 - 1000. As expected, the resulting distribution of $N$ is approximately uniform in the range $\Gamma \sim 1.5 - 2.0$. On the other hand Fig. 4.25b shows the distribution when values of $l_s/l_e$ are selected from the range 0.01 - 1.0 and $\gamma_0$ from 100 - 500. The distribution now peaks at $\sim 1.60 - 1.65$ so that there is now definitely a 'preferred' value of $\Gamma$ and the large width of the distribution still allows significant deviations of $\Gamma$ from the preferred value. Note that the fraction of flat-spectrum models ($\Gamma \leq 1.55$) in Fig. 4.25b is $\sim 0.22$. We find, however, that this fraction is very sensitive to the lower and upper limits on the range in $l_s/l_e$. Increasing the lower or upper limit on $l_s/l_e$ increases the relative abundance of flat-spectrum sources. This is illustrated in Figs. 4.25c and 4.25d, both samples having $\gamma_0$ in the range 100 - 500. In Fig. 4.25c the range in $l_s/l_e$ is 0.01 - 10 and the fraction of flat-spectrum models is $\sim 0.41$ and the distribution actually peaks around $\Gamma \sim 1.50 - 1.55$. In Fig. 4.25d the range in $l_s/l_e$ is 0.01 - 0.5 and the fraction of flat-spectrum models is $\sim 0.19$ (still not far off the value of $\sim 0.25$ found by Turner & Pounds, 1989) and the distribution actually peaks around $\Gamma \sim 1.65 - 1.70$, i.e. the observed 'canonical' value for AGN. Of course, despite these two coincidences the question arises as to why the above upper limits on the average $l_s/l_e$ and $\gamma_0$ should pertain. Also, we have said nothing of possible acceleration mechanisms which might produce the required relativistic electrons.

Regarding the 'observational relevance' of the models in Fig. 4.25d, we note the following points (see §4.2.1.3). The fraction of models which have a $\gamma$-ray excess less than or equal to 1, 2 and 3 are $\sim 0.66, 0.90$ and 0.97 respectively (compared with 0.34, 0.50 and 0.61 for the models in Fig. 4.25a which have $\gamma_0$ up to 1000). The models in Fig. 4.25d cover over three orders of magnitude in 2-10 keV compactness. We find that the ratio of soft photon to 2-10 keV compactness ($l_s/l_e$) is $< 1$ for $\sim 12\%$ of the models in Fig. 4.25d and $> 10$ for $\sim 5\%$ of those models (see §4.2.1.3). This is to be compared with $\sim 9\%$ of the models in Fig. 4.25a having $l_s/l_e < 1$ and $\sim 27\%$ having $l_s/l_e > 10$. In other words, the models in Fig. 4.25d are in good agreement with observation.
Figure 4.25 Histograms of the 2–10 keV photon index, $\Gamma$, of a sample of 1000 pair models (where $N$ is the number of objects with $\Gamma$ lying in the appropriate bin), with parameters randomly chosen from the following ranges, (a): $l_e : 0.3 - 30$, $l_e/l_e : 0.01 - 1.0$, $\gamma_0 : 100 - 1000$, (b) $l_e : 0.3 - 30$, $l_e/l_e : 0.01 - 1.0$, $\gamma_0 : 100 - 500$, (c) $l_e : 0.3 - 30$, $l_e/l_e : 0.01 - 10$, $\gamma_0 : 100 - 500$ and (d) $l_e : 0.3 - 30$, $l_e/l_e : 0.01 - 0.5$, $\gamma_0 : 100 - 500$. Plot (a) shows that for 'standard' pair models with monoenergetic injection at Lorentz factors of up to $\sim 1000$, there is no preferred value of $\Gamma$. Plot (b) shows that this is not the case if the Lorentz factor is restricted to values below $\sim 500$. Plots (c) and (d) show the sensitivity of the proportion of flat-spectrum sources to the upper limit on $l_e/l_e$ in the sample. The dotted line in each plot marks the 'canonical' value of $\Gamma = 1.7$. 
4.3 Two-component power law models

When there is virtually no low energy absorption in the X-ray spectrum of an AGN (less than \( \sim 10^{21} \text{ cm}^{-2} \)) the LAC aboard *Ginga* is sensitive enough to detect significant deviations from a true power law (e.g. see Nandra et al., 1990a). However, this is not the case for objects like NGC 4151 in which the low-energy absorption can exceed \( 10^{23} \text{ cm}^{-2} \). In this section we wish to investigate the hypothesis that the X-ray spectral index variability in NGC 4151 is actually caused by a variable amount of mixing of two power law components, each having a fixed spectral index.

We cannot meaningfully perform a direct spectral fitting analysis on the data as the number of free parameters used to fit the complex spectrum is already large (see §3).

Instead, we take a model consisting of a pair of power laws, an absorbing column of \( 90 \times 10^{21} \text{ cm}^{-2} \), an iron abundance of 2 relative to solar and, for a range of relative normalizations of the power laws, we compute simulated *Ginga* pulse-height spectra. These are computed using an exposure of 15000 s with random noise added with a Gaussian distribution. We then fit the simulated spectra above 4 keV with a single power law and low-energy absorption (but no iron emission line) in a way which is entirely analogous to that used to fit the actual data in §3. What we are looking for here is (i) the relation between the spectral index, \( \Gamma \), and the 2–10 keV absorption corrected flux, \( F_2^c \), from the single power law fits and (ii) whether the residuals from these spectral fits are likely to reveal the real two-component identity of the original continuum if the simulated data were real.
Figure 4.26 The photon spectral index, $\Gamma'$, obtained when a two-component power law simulated *Ginga* spectrum is fitted with a single power law, plotted against the normalization of the steeper power law relative to the harder one. The solid curve corresponds to mixing of two power laws with $\Gamma = 1.3$ and $1.8$, while the dashed curve corresponds to the mixing of two power laws with $\Gamma = 1.4$ and $1.7$. 

**Figure 4.26**
Figure 4.27 (a)–(d) The residuals obtained from spectral fitting to the two power law ($\Gamma = 1.3$ and 1.8) simulated *Ginga* spectra with a single power law for the cases when the normalization of the steep power law relative to the harder one is 1, 2, 5 and 8 respectively. The corresponding single power-law photon indices obtained for these cases are 1.43, 1.49, 1.60 and 1.65 respectively.
In general we obtain very good fits and 'flat' residuals for a variety of pairs of input slopes, $\Gamma_1$ and $\Gamma_2$, for a large range in relative normalizations. Fig. 4.26 illustrates how the apparent single power law slope, $\Gamma$, varies with the relative normalizations of the two input power laws for the two cases $\Gamma_1 = 1.4$, $\Gamma_2 = 1.7$ (dashed curve) and $\Gamma_1 = 1.3$, $\Gamma_2 = 1.8$ (solid curve). Figs. 4.27a to 4.27d show the residuals obtained for the latter pair of power laws when the normalization of the steeper continuum relative to the harder one is 1, 2, 5, and 8 respectively. We have also plotted the values of $\Gamma$ obtained from $\Gamma_1 = 1.3$, $\Gamma_2 = 1.8$ against $F_\nu^c$, in Fig. 4.28, along with the measurements of $\Gamma$ for NGC 4151 originally presented in Fig. 3.8. The actual normalization of the $\Gamma_1 = 1.3$ power law was chosen so that $F_\nu^c = 6.0 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ for this component alone. It can seen from Fig. 4.28 that the hypothesised two-component model is consistent with the data. Implicit in the calculation of the model curve in Fig. 4.28 is the fact that the normalization of the 1.3 power law remains constant while that of the 1.8 power law varies. The scatter in the data can easily be reproduced if the 1.3 power law normalization is allowed to vary as well. However, very large, random variations in both power laws would tend to remove the flux-index correlation altogether.

This two-component power law model of the spectral index variability in NGC 4151 does not, of course, explain how the power laws are generated, whether there are two emitting regions and if so, the relation between them. Below (§4.4), we suggest a possible physical basis for the two-component model which in fact requires only one emitting region producing a single power law with a fixed spectral index.
Figure 4.28 The X-ray photon spectral index measurements for NGC 4151 (as in Fig. 3.8) plotted against the 2–10 keV absorption corrected flux, $F_x$. The dashed curve corresponds to the apparent flux-index correlation obtained when the spectrum actually consists of two power laws with photon indices 1.3 and 1.8 and the relative normalization of the steeper one varies while that of the harder one remains constant.
4.4 Reflection in an accretion disc

George & Fabian (1990b) have recently performed Monte Carlo calculations of the expected X-ray spectrum (above 1 keV) reflected from a 'cold', optically thick accretion disc when the incident spectrum illuminating the disc is a power law. They find that the reflected spectrum is much flatter than the incident power law and the total spectrum (direct plus reflected) is therefore also flatter, in general. The degree of flattening depends on the proportion of reflected to direct flux that is observed, which in turn depends, in a steady state situation, on the viewing angle, θ, (here, a face-on disc has θ = 0) and the albedo, λ. If the size of the disc is much larger than the X-ray source then the time lag due to the disc responding to flux variations in the X-ray source makes it possible for the proportion of reflected to direct continuum to vary with time. Hence the observed continuum will exhibit apparent spectral variations. In this section we investigate the nature of these spectral variations and in particular, whether or not this scenario can account for the flux-index correlation in NGC 4151 (see §3.3.2).

Suppose that the source flux varies as

\[ N_S(\epsilon, t) = A(t) \epsilon^{-\Gamma_i} \text{ photons cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \]  \hspace{1cm} (4.11)

and that the reflected spectrum varies as

\[ N_D(\epsilon, t) = A'(t) D(\epsilon) \text{ photons cm}^{-2} \text{s}^{-1} \text{keV}^{-1} \]  \hspace{1cm} (4.12)

From the Monte Carlo results of George & Fabian (1990b) we find that the reflected continuum, neglecting all absorption edges below the iron K-edge and the iron emission line, can be well fitted by the formula

\[ D(\epsilon) = 0.127(0.142)^{\Gamma_i} F(\lambda, \cos \theta) \left( \frac{\epsilon}{7.11} \right)^{2.55-\Gamma_i}, \quad \epsilon \leq 7.11 \text{ keV} \]

\[ D(\epsilon) = 0.077(0.142)^{\Gamma_i} F(\lambda, \cos \theta) \exp \left[ (25 - \epsilon)(\epsilon - 7.11)/372 \right], \quad \epsilon > 7.11 \text{ keV}. \]  \hspace{1cm} (4.13)

Equation 4.13 is valid for 1.3 \leq \Gamma_i \leq 2.0 and 1 \leq \epsilon \leq 25 \text{ keV}. Here, \( F(\lambda, \cos \theta) \) is a function valued between 0 and 1 which parameterizes the degenerate dependency of the reflected spectrum on the albedo and viewing angle of a disc.
Figure 4.29 An example of a simulated accretion disc spectrum (dashed curve) computed using equation 4.13. The incident power law with photon index 1.7 which illuminates the disc is shown as a dotted line. The solid curve is the resultant (direct plus reflected) continuum. The value of $F(\lambda, \cos \theta)$ in this example is 0.9.
For a given AGN, we assume that the value of $F(\lambda, \cos \theta)$ is fixed. The energy dependence of the reflected and observed continuum is illustrated in Fig. 4.29 for a case where the incident spectrum has a photon spectral index of 1.7 and the disc has $F(\lambda, \cos \theta) = 0.9$ and $A(t) = A'(t) = 1.0$. Note that, in general, the total, observed continuum is not a power law. The deviation from a true power law may appear as a high-energy tail which Pounds et al. (1990b) claim to have detected from the combined Ginga spectra of several AGN. However, if the total continuum suffers further absorption due to matter in the line-of-sight, the deviation from a power law may appear as a low-energy soft excess (type IV as defined in §4.1.5.3) and the deviation may not even be detectable above 4 keV (see §4.3). Assuming that above ~4 keV, the underlying total spectrum is approximately a power law we define $\Gamma$ in this section to be the 4-23 keV slope of the total spectrum. This will allow direct comparison of theoretical predictions with the data for NGC 4151 (see §3.3.2) as well as providing a general indicator of the X-ray slope. The value of $\Gamma$ for the particular example in Fig. 4.29 is 1.45.

The total (observed) spectrum, when $A(t)$ varies with time can be written as

$$N(\epsilon, t) = A(t)\epsilon^{-\Gamma} + D(\epsilon) \int_{t-(t_D-t_S)}^{t} S(t, t')A(t') \, dt'.$$

(4.14)

In equation 4.14 $t_S$ and $t_D$ are the source and disc radial light-crossing times respectively and $S(t, t')$ is a 'smearing' function which describes the way that different parts of the disc respond to changes in the continuum level of the X-ray source at different times. As the disc size approaches the source size $S(t, t')$ tends to a delta-function and no spectral variation should be seen. As $t_D$ increases relative to $t_S$ the smearing function becomes more elongated and flatter whilst $\int S(t, t') \, dt' = 1$, since the total response from the disc is spread over a longer time. Also, as the disc size increases relative to the source, another timescale comes into play, namely the actual typical variability timescale of large amplitude variations, $t_{\text{var}}$. $t_S$ merely represents the minimum variability timescale of the source. In cases where $t_D \gg t_{\text{var}}$ we might expect the amplitude of the reflected component to always be at some 'average level', whether the continuum flux is varying erratically or is fairly steady with occasional flaring. Spectral variability will then be most pronounced and may even be loosely correlated with the continuum level. However, as $t_D$ approaches $t_{\text{var}}$, the properties of the spectral behaviour are less obvious and depend on the details of the nature of the variability of the X-ray source. We investigate this point in §4.4.1 with specific reference to NGC 4151 but our general conclusions will apply to other AGN.
Figure 4.30 (a) A one-year simulated light curve for NGC 4151. The normalization, $A(t)$ (in arbitrary units), of a power law with photon spectral index 1.7 is plotted against time. (b) The normalization of the disc-reflected continuum, $A'(t)$, obtained when $A(t)$ is 'smeared' with the function $S(t, t')$ in Figure 4.31 (see §4.4.1 and equation 4.15). The parameters disc spectrum in this example is calculated using $t_s = 0.15$ lt-day, $t_D = 5$ lt-day and $F(\lambda, \cos \theta) = 0.9$; see text for details. (c) The ratio of the normalization of the direct continuum to that of the reflected continuum, as a function of time.
Figure 4.31 The form of the smearing function, $S(t, t')$ (equation 4.15) used to calculate the disc-reflected spectrum from NGC 4151 in §4.4.1.
4.4.1 Application to NGC 4151 and other AGN

In order to investigate whether the 'disc model' described above can account for the X-ray spectral index variability in NGC 4151, we have constructed a one-year simulated light curve for the continuum level of the X-ray source. Fig. 4.30a shows the normalization, $A(t)$, of the incident (direct) power law with photon index $\Gamma_i = 1.7$. In constructing the light curve, we have used the results of the observational data presented in §3. The most important features being (i) slow drifting behaviour with a flux-doubling timescale of $\sim 10^5$ s and (ii) occasional flaring, the flares lasting typically $\sim 5$ days (we use an estimated 'flaring' probability of $\sim 1.25\%$ per day). However, these kind of details are not important as they will not affect our general conclusions. Also, we realize that the actual source flux may be varying faster than is observed and may already be 'smeared' due to electron scattering. However, this will not affect our conclusions either since it is only the relative magnitudes of $t_s$, $t_D$ and $t_{\text{var}}$ which are important. We can simply substitute $t_s$ for $t_{\text{var}}$ in the discussion below. We compute the observed spectrum (direct plus reflected) numerically from equations 4.13 and 4.14, adopting a simple parabolic form for the smearing function:

$$S(t, t') = \frac{6}{(t_D - t_s)^2} [t(t_D - t_s - t) + (2t - [t_D - t_s])t' - t'^2].$$

(4.15)

This function is illustrated in Fig. 4.31 and has the basic desired properties, as discussed in §4.4. However, equation 4.15 is a very crude approximation to the actual emissivity of the disc as a function of radius in the George & Fabian (1990b) model. The emissivity function should be flat at small radii, peaking at $\sim 15 - 30\ r_{\text{SW}}$ (Schwarzschild radii) and subsequently falling off as a power-law function of radius with index $\sim 3$. If we set an upper limit of $10^{17} \text{ cm}$ for the radius of a hypothesised disc in NGC 4151, a central mass of $5 \times 10^7 \text{ M}_\odot$ (see §1.3.2.2) corresponds to a maximum disc radius of $\sim 67\ r_{\text{SW}}$. Hence the emissivity peaks between $\sim 0.25 - 0.50$ of the disc radius, so that our approximation for the smearing function is adequate for the present purpose.

We fix $t_s$ at 0.15 lt-day ($\sim 4 \times 10^{14} \text{ cm}$) and compute the spectral index, $\Gamma$, and the 2-10 keV flux of the observed spectrum for a range of values of $t_D$ and $F(\lambda, \cos \theta)$. Figs. 4.32a-f show the expected flux-index relation for six particular examples and Figs. 4.30b and 4.30c respectively show the smeared light curve $A'(t)$ and the ratio $A'(t)/A(t)$ for a model with parameters shown in Fig. 4.32b. The results depicted in Fig. 4.32 are indicative of some very general conclusions which depend on whether $t_D \gg t_{\text{var}}$ or $t_D \sim t_{\text{var}}$, as follows.
Figure 4.32 (a)-(f) The expected relation between the photon index, $\Gamma$, of the direct plus reflected continuum corresponding to the light curve in Fig. 4.30, plotted against the 2–10 keV flux (in arbitrary units). The photon index of the incident power law is 1.7 and $t_p = 0.15$ lt-day. The values of $t_p$ and $F(\lambda, \cos\theta)$ used for the models (a) to (f) are indicated in the Figure. See text for more details.
(i) $t_D \gg t_{\text{var}}$. A flux-index correlation may be observed for values of $F(\lambda, \cos \theta)$ as low as $\sim 0.3 - 0.4$. However, a large number of observations are required to confirm such a correlation as the dispersion is expected to be large (e.g. see Figs. 4.32c and 4.32e). For the case of NGC 4151, a model with $t_D \sim 20$ lt-day ($\sim 6 \times 10^{16}$ cm) and $F(\lambda, \cos \theta) \sim 0.4$ (see Fig. 4.32e) appears to be qualitatively consistent with the data (see Fig. 4.28). However, the simulated light curve implicitly assumes $t_{\text{var}} \sim 3 - 5$ days so that if the continuum actually varies on a shorter timescale (which is very likely) then $t_D$ scales down accordingly. If the 'disc model' is indeed relevant, then a disc of radius of $\sim 10^{16}$ cm is probably more appropriate. If the source light-crossing time, $t_{S}$, were larger than the 0.15 lt-days we have assumed then the smearing function would be taller and narrower, leading to a smaller range in $\Gamma$ and hence requiring a larger $t_D$ and/or $F(\lambda, \cos \theta)$ to compensate.

Clavel et al. (1987) argue that the disc in NGC 4151 is viewed at an inclination of $\sim 75^\circ$ and that $t_D$ is at least 5 lt-day. A disc radius of 5-20 lt-day is actually consistent with the UV variability (see §4.1.3.1). If we take $t_D \sim 20$ lt-day, the range in $\Gamma$ can be produced for $F(\lambda, \cos \theta) \sim 0.2 - 0.4$ which implies that the albedo must be at least $\sim 0.7$ (using the Monte Carlo results of George & Fabian, 1990b). We conclude that the flux-index correlation in NGC 4151 can be explained by the disc model alone, with an incident power law having a fixed spectral index. However, we note that if the spectral index of the incident power law is not fixed but varies in a manner uncorrelated with continuum flux, then obviously no flux-index correlation in the disc model is expected at all. Also, tests show that the same disc model parameters which give a flux-index correlation when the incident power law has a fixed spectral index tend to mask any correlation if the spectral index of the incident continuum is variable and correlated with flux. Now, we have presented arguments in §4.2.2.3 which strongly suggest that the X-ray emission in NGC 4151 is due to Compton scattering of UV photons on relativistic electrons with Lorentz factors of the order of several hundreds. From the results of §4.2.1.4 it is difficult to conceive how such a mechanism can generate a spectrum with a fixed X-ray spectral index, given the large amplitude variability of the X-ray flux in NGC 4151. The fact that a flux-index correlation is observed then implies that the disc contribution to the spectral index variability cannot be important in NGC 4151. The disc can only degrade any intrinsic flux-index correlation.

The particular case of the disc model with $t_D \gg t_{\text{var}}$ has, however, been used to account for the apparent flux-correlated spectral variability in MCG-6-30-15 by Nandra et al. (1990a).

(ii) $t_D \sim t_{\text{var}}$. Fairly large variability in the spectral index may be observed, uncorrelated with flux for any value of $F(\lambda, \cos \theta)$. However, a spurious correlation may be obtained if there are only a few data points, especially if they correspond to a large amplitude flare. As $t_D$ approaches the true source size, $t_S$, the spectral index variability vanishes. This version of the disc model has been successfully applied to the spectral index variability in NGC 5548 (Nandra et al., 1990b) and
may also be relevant to the spectral index variability in 3C 273 (Turner et al., 1989b and 1990).

We also note that if the X-ray source is partially covered by 'cold', optically thick 'clouds' out of the line-of-sight, they may also produce a reflected component which would be very similar in nature to that produced by an accretion disc (see also §5).

4.5 Chapter summary

- We have reviewed the nature of the broadband continuum emission in AGN and in NGC 4151 as a particular case study. We have also assessed the evidence for X-ray spectral index variability in AGN and the associated flux-index correlation, in cases where it has been claimed (see Table 4.1).

- Regarding the origin of the broadband emission in NGC 4151, we have presented arguments (§4.2.2.3) which lead to the following conclusions: (i) the dominant source of X-rays cannot be due to the synchrotron self-Compton process, (ii) the infrared/optical emission originates in a different region to that in which the X-rays are produced, (iii) if the non-thermal broadband emission is produced by inverse Compton scattering of soft photons on relativistic electrons then the infrared/optical and X-ray emission processes cannot share the same relativistic electron population and (iv) the injection of relativistic electrons into the X-ray emission region must be very flat or monoenergetic. An implication of (i) and (ii) is that the gradient of the magnetic field density from the X-ray to the infrared/optical emission region is large.

- We have investigated the time-dependent spectral behaviour of the class of non-thermal models of the X-ray emission in which UV photons are upscattered by relativistic electrons which are monoenergetically injected into a homogeneous, spherical source region, with the process of photon-photon absorption producing $e^+e^-$ pairs self-consistently taken into account. We find that measurable changes in the X-ray photon spectral index, $\Gamma$, are produced for modest (factor $\sim 3 - 4$) fluctuations in either the Lorentz factors or injection rates of the primary electrons. However, a clear flux-index correlation is only possible if the injection rate remains roughly constant and the model input parameters lie in a restricted range. The changes in $\Gamma$ are larger when there are fluctuations in the soft photon input which are correlated with the fluctuations in the relativistic electron parameters. The largest changes in $\Gamma$ occur when the source makes a transition from a relatively pair-free state to a pair-dominated one. We have deduced the approximate parameter ranges of models which can account for the flux-index correlation in NGC 4151. However, simultaneous measurements at X-ray and $\gamma$-ray energies with improved sensitivity and resolution are required to further constrain the parameter space.
• Previous models of the above type have suffered from the fact that their $\gamma$-ray spectra violate the constraints imposed by observations of the diffuse $\gamma$-ray background. We have overcome this problem by using lower minimum Lorentz factors. Further X-ray and $\gamma$-ray observations of AGN are required to test the relevance of such models. The same emission mechanism may not be operating in all AGN. Compton scattering of synchrotron photons (instead of or in addition to UV photons) may also be important.

• We have also investigated the nature of the spectral index variability expected if the observed X-ray continuum also has a significant contribution from reflection in a cold, optically thick accretion disc. This scenario implicitly assumes that there is a mechanism which produces an X-ray power law with a fixed spectral index.

We find that measurable changes in the spectral index can be expected even when the disc is viewed close to edge-on. However, a flux-index correlation will be observed only when the radial light-crossing time of the disc is much longer than the longest timescale of variability of the source flux.

• The above type of model can, in principle, account for the flux-index correlation in NGC 4151. However, we have presented arguments in §4.4.1 suggesting that the spectral index variability in this object is largely intrinsic to the emission mechanism.

• We have emphasised that the next generation of models of the X-ray emission in AGN must be able to account for the large fraction of objects which have an X-ray spectral index significantly different to the 'canonical' value of 1.7 (particularly those with $\Gamma < 1.5$). Spectral index variability within individual AGN, of the types discussed in this chapter, must also be an integral part of any future models of the X-ray emission.

• We find that the type of pair models that we have considered can give rise to a distribution of X-ray slopes which peaks between $\Gamma = 1.5$ and 2.0, provided that the average Lorentz factor of injected electrons for a given set of models is restricted to a range with an upper limit of $\sim 500$ (for a UV seed photon source whose energy peaks at $\sim 6 \times 10^4$ $K$). The actual 'preferred' value of $\Gamma$ also depends on the range of $I_{y}/I_{x}$. If the range in $\gamma_0$ is $\sim 100 - 500$ and that in $I_{y}/I_{x}$ is $\sim 0.01 - 0.5$ (corresponding to UV to X-ray luminosity ratios of $\sim 1 - 10$), the model distribution is generally consistent with observational constraints and has a 'canonical' X-ray slope of $\sim 1.65 - 1.7$. The width of the distribution is also consistent with observation. However, it is not known why such restrictions on the input parameters should pertain or what acceleration mechanisms could give rise to the required range in Lorentz factors.
Chapter 5

Models of Complex and Variable X-ray Absorption

Overview

In this chapter we investigate the properties of some models of complex and variable X-ray absorption in AGN. In particular we develop a photoionization code which is suitable for spectral fitting to a 'warm absorber' model. We also investigate which of the above models can best account for the spectral variability below 4 keV and the 2–4 keV soft excess reported for NGC 4151 in §3. Finally, we discuss our results in terms of the physics of the broad line region.

"The sun is in tune ...
But the sun is eclipsed by the moon."

(Roger Waters, 1972. From 'The Dark Side of the Moon'.)
5.1 Complex and variable X-ray absorption in AGN

Although significant intrinsic X-ray absorption is certainly more common in the low luminosity AGN \((L_x < 3 \times 10^{43} \text{ erg s}^{-1})\), it also appears to be present in some high luminosity AGN (e.g. see Turner & Pounds, 1989; Pounds, 1990). However, very high column densities (of the order \(~10^{23} \text{ cm}^{-2}\)) still appear to be restricted to the low luminosity category. Recently, it has been found that, in many of the objects in which there is a significant low-energy cut-off,

- the X-ray absorption cannot be explained by a uniform screen of cold, solar abundance gas fully covering the source (using standard atomic cross-sections) and/or
- the amount of low-energy cut-off is often variable (see references below).

It is these two properties of the X-ray spectrum of AGN which we wish to model in this chapter and the two are usually intimately linked. The first of these properties (which gives rise to a type IV soft excess (see §4.1.5.3)) we refer to as complex absorption. As mentioned in §4.1.5.4, variable X-ray absorption can often be mistaken for spectral index variability (or vice-versa) especially if the quality of the data is poor. However, when variable absorption can be genuinely distinguished from spectral index variability, we can hope to learn something about the distribution, dynamics and physical state of matter surrounding the emission region in AGN (see §1.3).

Complex absorption has been reported for a number of AGN, the classic example being NGC 4151 (see §3 and references therein); others include NGC 2992, NGC 3227, NGC 3783, (Riechert et al., 1985); MR 2251 (Halpern, 1980); MCG-6-30-15 and NGC 4051 (Matsuoka et al., 1989); NGC 2110 (Turner & Pounds, 1989); and ESO103-G35 (Warwick et al., 1988). Establishing the existence of variable absorption in a particular object is more difficult since only a handful of objects with substantial intrinsic columns have been observed with sufficient frequency. Even then, it may not be possible to determine whether the spectral variability is indeed due to variable absorption. The only objects known to exhibit variable X-ray absorption, which we consider to be statistically significant, are NGC 4151 (§3), NGC 3227 (Turner & Pounds, 1989), ESO103-G35 (Warwick et al., 1988 and §6.1) and the QSO MR2251 (Pan, Stewart & Pounds, 1990). Other, more marginal results include NGC 2992 (Turner & Pounds, 1989) and MCG-6-30-15 (Nandra et al., 1990a). However, we note that the variable absorption in MCG-6-30-15 has been interpreted by Matsuoka et al. (1989) as spectral index variability (see also §4.1.5.4), a conclusion based on the same data. Of the above objects, only in MR 2251 and MCG-6-30-15 is the variable absorption clearly (anti-) correlated with the X-ray continuum level. Amongst AGN as a whole, Turner & Pounds (1989) found that the column density of absorbing material in the line-of-sight was not correlated with luminosity or for that matter, with axial ratio.
In the remainder of this chapter we investigate the properties of some models of complex and variable X-ray absorption in AGN and discuss how comparison of these models with X-ray data can place physical constraints on the nature of the AGN environment beyond the emission region. In particular, we wish to account for the spectral variability below ~4 keV reported for NGC 4151 in §3.

5.2 Partial covering

In its simplest form, the partial covering model involves a uniform column of absorbing material, \( n_{H_1} \), covering a fraction, \( C_F \), of the source. Flux from the uncovered part of the source then gives rise to the soft excess (type IV; §4.1.5.3). Variability of the low-energy spectrum may then be produced by variations in either \( C_F \) or \( n_{H_1} \). Physically, changes in the apparent covered fraction could be caused by changes in the relative sizes of source and absorber or by bulk motion of absorbing material into or out of the line-of-sight. In models where absorbing material is continuously created and destroyed (e.g. see §5.5.3) changes in \( n_{H_1} \) and/or \( C_F \) could be produced by changes in the production/destruction rate.

In its most general form, the fraction of source flux transmitted in the partial covering model, as a function of energy, \( \epsilon \), is

\[
T_{PC}(\epsilon) = C_F e^{-\sigma_\epsilon(n_{H_1})} + (1 - C_F) e^{-\sigma_\epsilon(n_{H_2})}.
\]

Here \( \sigma_\epsilon(\epsilon) \) is the photoelectric cross-section per hydrogen atom, as a function of energy, of cold solar abundance material. We use the \( \sigma_\epsilon(\epsilon) \) given by Morrison and McCammon (1983). \( n_{H_2} (< n_{H_1}) \) is a uniform absorbing screen covering the remainder, \( 1 - C_F \), of the source and represents material either intrinsic to the AGN or (Galactic) material in the line-of-sight. The notation used in equation 5.1 is equivalent to a column \( n_{H_1} - n_{H_2} \) covering a fraction \( C_F \) of the source and a column \( n_{H_2} \) fully covering it.

The form of the transmission function in equation 5.1 (to be compared with the same for other complex absorber models below) is shown in Fig. 5.1, with \( n_{H_2} = 0 \). The solid curves correspond to \( n_{H_1} = 100 \times 10^{21} \text{ cm}^{-2} \) and \( C_F = 0.2, 0.4, 0.5, 0.6, 0.8, \) and 0.99, while the dotted curves correspond to \( C_F = 0.99 \) and \( n_{H_1} = 25, 50, 75, 100, 125 \) and \( 150 \times 10^{21} \text{ cm}^{-2} \). The dashed curve illustrates, for comparison, the transmission of a uniform, fully covering column of \( 100 \times 10^{21} \text{ cm}^{-2} \).
Figure 5.1 The fraction of the flux transmitted by a partial covering absorber, $T_{PC}(\varepsilon)$, as a function of energy (see equation 5.1). Solid lines correspond to a fixed column density of $n_{H_1} = 100 \times 10^{21} \text{ cm}^{-2}$, for values of the covered fraction corresponding to (top to bottom) 0.2, 0.4, 0.5, 0.6, 0.8 and 0.99. Dotted lines correspond to a fixed covered fraction of 0.99, for values of the column density corresponding to (left to right) 25, 50, 75, 100, 125 and $150 \times 10^{21} \text{ cm}^{-2}$. The dashed curve corresponds to the transmission due to a cold, uniform absorber with a column density of $100 \times 10^{21} \text{ cm}^{-2}$.
The partial covering model has been applied to a number of AGN (e.g. see Petre et al., 1984; Riechert et al., 1985; Turner & Pounds, 1989; Matsuoka et al., 1989). Partial covering provided a better fit than uniform absorption for three out of the eight significantly absorbed objects in the low luminosity EINSTEIN SSS sample of Riechert et al. (1985) and for only one object (NGC 2110) in the sample of Turner & Pounds (1989) (excluding NGC 4151 and MR 2251). For the high luminosity sample of Petre et al. (1984) (in which no significant intrinsic absorption was detected in any of the 15 objects) only loose upper limits to the covered fraction (a mean upper limit of \( \sim 0.5 \)) could be placed. Complex absorption is likely to be present in many of the other objects in these samples but the quality of the data prevents its detection (especially if the covered fraction is small).

### 5.2.1 Partial covering in NGC 4151

Partial covering was first invoked to account for the complex absorption in NGC 4151 by Holt et al. (1980), on the basis of an EINSTEIN SSS observation. In that model, \( \sim 90\% \) of the source was covered by a column of \( \sim 6 \times 10^{21} \) cm\(^{-2} \). However, at that time the separate, extended soft emission component (see §1.3.3 and §3.2.1) had not been discovered. Fiore et al. (1990) have applied the partial covering model to some of the EXOSAT observations of NGC 4151 reported in §3.2. They find that the data is well described by the model, with \( C_F \) varying between \( \sim 0.7 - 0.9 \) and \( n_{H_1} \) varying between \( \sim 60 - 110 \times 10^{21} \) cm\(^{-2} \). \( n_{H_2} \) was fixed at \( 10^{22} \) cm\(^{-2} \); this latter column is required to extinguish the continuum flux below 1 keV in order to be consistent with the lack of variability of the extended soft component seen by the EXOSAT LE (see §3.2.1).

We have applied the partial covering model to the Ginga observations of NGC 4151 (§3.3) over the energy range 2–23 keV. The spectral fitting involves six free parameters, namely the normalization of the continuum, the X-ray photon spectral index (\( \Gamma \)), \( n_{H_1} \), \( C_F \), the abundance of iron relative to solar (\( A_{Fe} \)) and the intensity of an iron emission line at 6.4 keV. \( n_{H_2} \) was fixed at \( 10^{22} \) cm\(^{-2} \).

The results are shown in Table 5.1, in which the errors correspond to 90% confidence limits for 2 interesting parameters. Only observations 4 and 5 give statistically unacceptable values of \( \chi^2 \). The pulse height spectrum, best-fitting model, residuals and corresponding incident photon spectrum for observation 4 are shown in Fig. 5.2. It can be seen that the worst residuals in fact lie in the 2–4 keV band (also true of observation 5). It appears that in this model, \( C_F \) varies between \( \sim 0.55 - 0.9 \) over the set of observations while \( n_{H_1} \) varies between \( \sim 40 - 125 \times 10^{21} \) cm\(^{-2} \). Fig. 5.3 shows a plot of \( C_F \) against \( n_{H_1} \) from which it can be seen that \( C_F \) is correlated with \( n_{H_1} \). Below (§5.3) we discuss a particular type of model in which such a correlation might be expected.
Table 5.1 Spectral fits to the Ginga data for NGC 4151 with a partial covering model

<table>
<thead>
<tr>
<th>Obs</th>
<th>( n_{H_1} ) (^a)</th>
<th>( C_P )</th>
<th>( A_{Fe} ) (^b)</th>
<th>( F_2 ) (^c)</th>
<th>( \chi^2 ) (^d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>( 103^{+27}_{-23} )</td>
<td>( 0.90^{+0.06}_{-0.04} )</td>
<td>( 1.45^{+1.21}_{-1.06} )</td>
<td>8.5</td>
<td>1.15</td>
</tr>
<tr>
<td>1b</td>
<td>( 97^{+25}_{-18} )</td>
<td>( 0.91^{+0.03}_{-0.04} )</td>
<td>( 2.59^{+1.44}_{-1.07} )</td>
<td>8.5</td>
<td>0.76</td>
</tr>
<tr>
<td>2</td>
<td>( 124^{+20}_{-20} )</td>
<td>( 0.84^{+0.02}_{-0.02} )</td>
<td>( 2.40^{+0.90}_{-0.80} )</td>
<td>12.0</td>
<td>1.36</td>
</tr>
<tr>
<td>3</td>
<td>( 125^{+45}_{-31} )</td>
<td>( 0.83^{+0.04}_{-0.03} )</td>
<td>( 2.32^{+2.03}_{-1.36} )</td>
<td>11.5</td>
<td>0.56</td>
</tr>
<tr>
<td>4</td>
<td>( 81^{+7}_{-5} )</td>
<td>( 0.91^{+0.01}_{-0.01} )</td>
<td>( 2.67^{+0.48}_{-0.43} )</td>
<td>31.2</td>
<td>1.86</td>
</tr>
<tr>
<td>5</td>
<td>( 91^{+7}_{-5} )</td>
<td>( 0.85^{+0.01}_{-0.01} )</td>
<td>( 2.49^{+0.50}_{-0.42} )</td>
<td>26.9</td>
<td>1.49</td>
</tr>
<tr>
<td>6a</td>
<td>( 44^{+18}_{-15} )</td>
<td>( 0.54^{+0.11}_{-0.13} )</td>
<td>( 6.02^{+4.61}_{-2.56} )</td>
<td>13.0</td>
<td>1.28</td>
</tr>
<tr>
<td>6b</td>
<td>( 45^{+10}_{-9} )</td>
<td>( 0.74^{+0.05}_{-0.07} )</td>
<td>( 3.85^{+2.10}_{-1.29} )</td>
<td>16.6</td>
<td>0.94</td>
</tr>
<tr>
<td>7</td>
<td>( 37^{+13}_{-7} )</td>
<td>( 0.55^{+0.07}_{-0.08} )</td>
<td>( 4.45^{+2.09}_{-1.54} )</td>
<td>32.0</td>
<td>1.06</td>
</tr>
</tbody>
</table>

\(^a\) \( 10^{21} \) cm\(^{-2}\).
\(^b\) Iron abundance relative to Solar.
\(^c\) \( 10^{-11} \) erg cm\(^{-2}\) s\(^{-1}\).
\(^d\) 27 degrees of freedom.
Figure 5.2 Spectral fit to observation 4 of the Ginga data set for NGC 4151 with a partial covering model. (a) Pulse height spectrum, best fit model and residuals. (b) Incident photon spectrum corresponding to (a).
Figure 5.3 The covered fraction, $C_F$, plotted against the column density, $n_{HI}$, from the partial covering spectral fits to the *Giαs* data for NGC 4151 (see Table 5.1). The solid curves correspond to the expected relation between $C_F$ and $n_{HI}$ from a Poissonian absorber, for cloud column densities (from left to right) of 20, 30, 40, 50, 60 and $70 \times 10^{21}$ cm$^{-2}$, as $\mu$ varies (see equation 5.9).
Note that the values of $\Gamma$ obtained from all the fits (not shown in Table 5.1) are very similar to those obtained from spectral fitting above 4 keV only (see Table 3.5). In other words, spectral index variability is still required. Another important point is that the best-fit values of the iron abundance are systematically higher for observations 6 and 7, in which $n_{H_1}$ is significantly lower than in the remaining observations. For observations 1–5 the weighted mean value of the relative iron abundance is $2.5 \pm 0.3$ while the corresponding value for observations 6–7 is $4.6 \pm 1.3$ (90% errors). This confirms our suspicion in §3.3.3, that the iron K–edge feature in the observed spectrum contains a contribution from some iron which is not associated with the material responsible for the low energy absorption. This could be evidence for the presence of a reflected continuum component from an accretion disc or optically thick matter out of the line-of-sight (see also §4.4). In §5.4.4 we show that the apparent over-abundance of iron cannot be explained entirely by photoionization.

5.3 The Poissonian absorber

The Poissonian absorber is based on a model in which the face of the X-ray source is covered by a large number of discrete absorbing clouds with random motions. In addition, the clouds are assumed to be small compared to the size of the source so that their distribution follows Poissonian statistics. If $\mu$ is the mean number of clouds in the line-of-sight then the probability, $p(k)$, that there are $k$ clouds in a particular line-of-sight is

$$p(k) = \frac{\mu^k e^{-\mu}}{k!}. \quad (5.2)$$

Hence, approximately a fraction $p(k)$ of the projected area of the source has $k$ clouds in the line-of-sight. We will assume that all the clouds have the same column density, $n_e$. It follows that the transmission function for the Poissonian absorber, $T_{PA}(\epsilon)$, is then given by

$$T_{PA}(\epsilon) = e^{-\sigma_e(\epsilon)n_{H_2}} \sum_{k=0}^{\infty} p(k)e^{-\sigma_e(\epsilon)kn_e}$$

$$= e^{-\sigma_e(\epsilon)n_{H_2}} e^{-\mu} \sum_{k=0}^{\infty} \frac{(\mu e^{-\sigma_e(\epsilon)n_e})^k}{k!}$$

$$= e^{-\sigma_e(\epsilon)n_{H_2}} \exp \left[\mu(e^{-\sigma_e(\epsilon)n_e} - 1)\right]. \quad (5.3)$$
Figure 5.4 The fraction of the flux transmitted by a Poissonian absorber, $T_{\nu,\lambda}(\epsilon)$, as a function of energy (see equation 5.3). Solid lines correspond to a fixed cloud column density of $n_c = 25 \times 10^{21} \text{ cm}^{-2}$, for values of $\mu$, (the mean number of clouds in the line-of-sight), corresponding to (top to bottom) 0.5, 1.0, 2.0, 3.0, 4.0 and 5.0. Dotted lines correspond to $\mu$ fixed at 4.0, for values of the cloud column density corresponding to (from left to right) 5, 10, 15, 20, 25 and $30 \times 10^{21} \text{ cm}^{-2}$.
In equation 5.3 \( n_{H_2} \) is a uniform column fully covering the source (note that this is different to the definition of \( n_{H_2} \) in equation 5.1). The form of this transmission function (with \( n_{H_2} = 0 \)) is shown in Fig. 5.4 where we have plotted \( T_{PA}(\epsilon) \) for \( n_c = 25 \times 10^{21} \text{ cm}^{-2} \) and \( \mu = 0.5, 1.0, 2.0, 3.0, 4.0 \) and 5.0 (solid curves). The dotted curves correspond to \( \mu = 4.0 \) and \( n_c = 5, 10, 15, 20, 25 \) and \( 30 \times 10^{21} \text{ cm}^{-2} \). It can be seen that shape of the transmission curves are qualitatively similar to those for partial covering. However, they are different in detail (see below).

5.3.1 Poissonian absorber versus Partial covering

In the Poissonian absorber the fraction of the source covered by clouds is

\[
C_F = 1 - p(0) = 1 - e^{-\mu}. \tag{5.4}
\]

A simplistic correspondence between the Poissonian absorber and partial covering is then obtained by naively assuming that \( \mu = \left( \frac{n_{H_1} - n_{H_2}}{n_c} \right) \), so that

\[
C_F = 1 - \exp \left[ -\left( \frac{n_{H_1} - n_{H_2}}{n_c} \right) \right]. \tag{5.5}
\]

However, we can find a more realistic expression by equating equations 5.1 and 5.3 in the limits of small and large \( \epsilon \). In the limit of small \( \epsilon \) we simply get equation 5.4. In the limit of large \( \epsilon \) we get, expanding all exponentials to first order,

\[
\mu = \left( \frac{n_{H_1} - n_{H_2}}{n_c} \right) C_F. \tag{5.6}
\]

Hence, for a given partial covering model, the equivalent Poissonian absorber in which the transmission coincides at low and high energies is given by

\[
\mu = \ln \left[ 1 - C_F \right]^{-1} \tag{5.7}
\]

\[
n_c = \left( \frac{n_{H_1} - n_{H_2}}{\ln \left[ 1 - C_F \right]^{-1}} \right) C_F. \tag{5.8}
\]
Equations 5.7 and 5.8 have been used to compute the transmission functions of the equivalent Poissonian absorbers for the partial covering models shown in Fig. 5.1 with \( n_{H1} = 100 \times 10^{21} \text{ cm}^{-2} \). These are shown in Fig. 5.5 (dotted curves). It can be seen that the transmission function for the Poissonian absorber, at a given energy, is always greater than or equal to that for the equivalent partial covering model. This can be an important difference between 2–4 keV, the energy range of the type IV (see §4.1.5.3) soft excess in NGC 4151.

Substituting equation 5.6 into equation 5.4 gives the result that \( C_F \) and \( n_{H1} \), in the equivalent partial covering model are related through the equation

\[
1 - \exp \left[ \frac{-C_F(n_{H1} - n_{H2})}{n_C} \right] - C_F = 0 \quad (5.9)
\]

Some solutions of equation 5.9 with \( n_{H2} = 10^{22} \text{ cm}^{-2} \) are shown in Fig. 5.3, along with the measurements of \( C_F \) and \( n_{H1} \) obtained from spectral fitting to the Ginga data for NGC 4151 with the partial covering model (see Table 5.1). The curves correspond to values of \( n_e \) equal to 20, 30, 40, 50, 60 and 70 \( \times 10^{21} \text{ cm}^{-2} \). It can be seen that for small values of \( C_F \), the curves obtained from equation 5.9 differ significantly from those that would obtained from the more simplistic equation 5.5.
Figure 5.5 A comparison of the transmission function for the partial covering and Poissonian absorber models. Solid lines correspond the partial covering model with the column density, $n_{H_1}$, fixed at $100 \times 10^{21}$ cm$^{-2}$ for values of the covered fraction of (top to bottom) 0.2, 0.4, 0.5, 0.6, 0.8, and 0.99. The dotted lines correspond to the Poissonian absorber with values of $\mu$ and $n_c$ calculated from equations 5.7 and 5.8.
5.3.2 Application to NGC 4151

We now perform a direct spectral fitting analysis on the *Ginga* data for NGC 4151 (see §3.3) with the Poissonian absorber model (equation 5.3). The model involves six free parameters (the same as in the partial covering model), namely the continuum normalization, the photon spectral index ($\Gamma$), $n_e$, $\mu$, the abundance of iron relative to solar ($A_{Fe}$) and the intensity of an iron emission line at 6.4 keV. We fix $n_{H_2}$ at $10^{22}$ cm$^{-2}$ as in §5.2.1. The results are shown in Table 5.2, in which the errors correspond to 90% confidence limits for 2 interesting parameters. Statistically acceptable values of $\chi^2$ are now obtained for all except observation 2. Apart from this observation all the remaining fits are better, or as good as, the corresponding partial covering fits (§5.2.1). Fig. 5.6 shows the pulse height spectrum, best-fit model, residuals and corresponding incident photon spectrum for observation 4.

In this model the low energy spectral variations are caused by $\mu$ varying between ~ 0.9 — 3.3 and $n_e$ varying between ~ 20 — 70 x10$^{21}$ cm$^{-2}$. However, in a realistic model, all clouds need not have the same column density. Alternatively, the clouds may not be small compared to the source.

As in the partial covering fits, the apparent over-abundance of iron is systematically lower for observations 1–5 (weighted mean value of 2.3 ± 0.3) than that for observations 6–7 (weighted mean value of 4.5 ± 1.1). The best-fit values of $\Gamma$ (not shown in Table 5.2) are again similar to the values in Table 3.5, obtained from spectral fitting above 4 keV.
Table 5.2 Spectral fits to the Ginga data for NGC 4151 with a Poissonian absorber model

<table>
<thead>
<tr>
<th>Obs</th>
<th>$n_e$</th>
<th>$\mu$</th>
<th>$A_{Fe}$</th>
<th>$F_Z$</th>
<th>$\chi^2_{\nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>$35_{-20}^{+21}$</td>
<td>$2.96_{-0.60}^{+1.04}$</td>
<td>$1.38_{-0.89}^{+1.09}$</td>
<td>9.2</td>
<td>1.05</td>
</tr>
<tr>
<td>1b</td>
<td>$35_{-18}^{+15}$</td>
<td>$3.03_{-0.49}^{+1.26}$</td>
<td>$2.41_{-0.67}^{+1.11}$</td>
<td>9.3</td>
<td>0.69</td>
</tr>
<tr>
<td>2</td>
<td>$58_{-11}^{+15}$</td>
<td>$2.17_{-0.18}^{+0.20}$</td>
<td>$2.17_{-0.66}^{+0.82}$</td>
<td>12.6</td>
<td>1.52</td>
</tr>
<tr>
<td>3</td>
<td>$66_{-25}^{+26}$</td>
<td>$2.15_{-0.29}^{+0.35}$</td>
<td>$2.19_{-1.14}^{+1.44}$</td>
<td>13.3</td>
<td>0.58</td>
</tr>
<tr>
<td>4</td>
<td>$24_{-4}^{+4}$</td>
<td>$3.31_{-0.27}^{+0.40}$</td>
<td>$2.48_{-0.41}^{+0.44}$</td>
<td>33.0</td>
<td>1.27</td>
</tr>
<tr>
<td>5</td>
<td>$38_{-4}^{+4}$</td>
<td>$2.33_{-0.10}^{+0.11}$</td>
<td>$2.33_{-0.37}^{+0.41}$</td>
<td>28.8</td>
<td>1.28</td>
</tr>
<tr>
<td>6a</td>
<td>$23_{-11}^{+15}$</td>
<td>$0.87_{-0.30}^{+0.42}$</td>
<td>$6.15_{-2.28}^{+3.35}$</td>
<td>13.1</td>
<td>1.27</td>
</tr>
<tr>
<td>6b</td>
<td>$17_{-6}^{+9}$</td>
<td>$1.67_{-0.33}^{+0.55}$</td>
<td>$3.79_{-1.24}^{+1.63}$</td>
<td>16.9</td>
<td>0.90</td>
</tr>
<tr>
<td>7</td>
<td>$17_{-5}^{+7}$</td>
<td>$0.93_{-0.22}^{+0.28}$</td>
<td>$4.32_{-1.43}^{+1.83}$</td>
<td>31.8</td>
<td>1.02</td>
</tr>
</tbody>
</table>

$^a$ 10$^{21}$ cm$^{-2}$.

$^b$ Iron abundance relative to Solar.

$^c$ 10$^{-11}$ erg cm$^{-2}$ s$^{-1}$.

$^d$ 27 degrees of freedom.
Figure 5.6 Spectral fit to observation 4 of the Ginga data set for NGC 4151 with a Poissonan absorber model. (a) Pulse height spectrum, best fit model and residuals. (b) Incident photon spectrum corresponding to (a).
5.3.3 Monte Carlo simulations of the Poissonian absorber

We have investigated how changes in $\mu$ might be related to the total number of clouds surrounding the source and to the relative size of clouds compared to the source by performing some simple Monte Carlo simulations. We assume spherical clouds (radius $r_c$) and a spherical source (radius $r_s$) surrounded by a spherically symmetric random distribution of $N_T$ clouds extending between a distance $100r_s$ from the source out to a distance $10^4r_s$.

The results are shown in Fig. 5.7. Fig. 5.7a shows how the mean number of clouds in the line-of-sight, $\mu$, is related to the total number of clouds in $4\pi$, $N_T$, for values of $r_c/r_s$ equal to 0.01, 0.02, 0.05, 0.08, 0.10 and 0.20. Fig. 5.7b shows how the covered fraction, $C_F$, is related to $N_T$ for the same values of $r_c/r_s$ as in Fig. 5.7a. Values of $C_F$ and $\mu$ obtained from the Monte Carlo runs give excellent agreement with the theoretical relation $C_F = 1 - e^{-\mu}$ (equation 5.4) and this shows that Poissonian statistics are indeed obeyed over all the parameter ranges that we have considered.

We find that our Monte Carlo calculations can be well fit by the formula

$$\mu \sim 1.4 \times 10^{-4} \left(\frac{r_c}{r_s}\right)^2 N_T. \quad (5.10)$$

In other words, changes in $\mu$ of a factor $f$ can be produced by a factor $f$ change in the total number of clouds in $4\pi$ or a factor $f^{1/2}$ change in $r_c/r_s$. Of course a combination of both effects is possible. If the cloud distribution was anisotropic, the dependence of $\mu$ on $r_c/r_s$ and $N_T$ could be weaker or stronger than in the spherically symmetric case, depending on the observer's viewing angle. More sophisticated calculations, which are beyond the scope of this work, need to be performed to investigate the effects of departures from spherical symmetry. In any case, it is clear that the changes in $\mu$ required to account for the low energy spectral variability in NGC 4151 (see Table 5.2) can easily be produced by modest changes in, say, the total cloud number or production rate and/or in the cloud or source radii.
Figure 5.7 Monte Carlo simulations of a spherically symmetric Poissonian absorber. (a) The mean number of clouds in the line-of-sight, $\mu$, as a function of the total number of clouds in 4π steradians, $N_T$, for a given cloud to source radius ratio, $r_c/r_s$. From bottom to top, the curves correspond to values of $r_c/r_s$ of 0.01, 0.02, 0.05, 0.08, 0.10 and 0.20. (b) The covered fraction, $C_F$, for the same models as (a).
5.4 Photoionization and the ‘warm absorber’

In this section we consider the consequences of X-ray absorbing material in AGN being photoionized by the UV/X-ray continuum of the central source. The X-ray opacity below ~ 4 keV of gas in which the lighter elements have been stripped of electrons can be orders of magnitude less than that of a gas consisting of neutral atoms while the higher energy opacity is hardly affected at all. Hence photoionization can qualitatively account for soft X-ray excesses of type IV (see §4.1.5.3). The soft X-ray opacity of such ‘warm’ gas will then be sensitive to the level of the incident UV/X-ray continuum. The apparent column density, as measured by say, assuming opacities relevant to cold material (cf. §3 for NGC 4151), can then vary and may be anti-correlated with the continuum level. Indeed, this has been argued to be the case for the QSO MR2251 (Halpern, 1980; Pan, Stewart & Pounds, 1990) and MCG-6-30-15 (Nandra et al., 1990a).

Unlike the complex absorber models considered in §5.2 – §5.3, it is not a trivial exercise to perform a spectral fitting analysis on X-ray data with a warm absorber model. This is primarily because the opacity of photoionized gas depends on the shape of the ionizing continuum, which is one of the unknowns in the problem. This is aside from the fact that the number of physical parameters specifying a particular model is very large. Standard photoionization codes exist which are able to compute the thermal and ionization structure (and the emergent continuum and emission line spectrum) of gas exposed to a given continuum but are too slow to use in an iterative minimization procedure. This is the case even if parts of the code not affecting the X-ray opacity are removed since the calculation of thermal and ionization equilibrium is sufficiently complex to make it unsuitable for spectral fitting. Yet another problem with many photoionization codes is that the energy resolution in the X-ray regime, above ~ 3 keV is poor and the treatment of L- and K-shell ionization for some elements inadequate. This is particularly important since the energies of the L- and K-shell ionization edges are a function of ionization state, and as we have already seen in §3.3.3, the K-edge energy of iron can be measured, with present instrumentation, to a fair degree of accuracy. Future instrumentation (see §7.3) will have an even greater capability for accurate measurement of absorption edges.

In the light of the above comments, the need for a photoionization spectral fitting code cannot be over-emphasised. It is important to be able to test a model against actual data as opposed to merely comparing measurements with a grid of models in which the limited dimensions of the grid force one to make arbitrary assumptions. The construction of such a spectral fitting code will be described in §5.4.2, along with the necessary physical assumptions and approximations involved. First, however, we must look into the warm absorber model in more detail before considering any simplifications of it.
5.4.1 X-ray opacity of photoionized gas

The ionization and thermal equilibrium of gas exposed to a source of radiation can be characterized by a minimum of two parameters; namely the so-called 'ionization parameter', $U$, and the temperature of the gas, $T$. As pointed out in §1.2, many different definitions of $U$ are currently in operation and in the present work we use the definition in equation 1.1: 

The temperature of the gas is then determined in an iterative manner by balancing all heating and cooling processes until both $T$ and $N_e$ (the electron number density) converge. If the gas is optically thick, the temperature of the gas falls with optical depth and the temperature profile must then be computed by numerically solving the radiative transfer problem. This is achieved by dividing the gas into radial zones and balancing all heating and cooling processes in each zone. This process is illustrated in Fig. 5.8 which shows an extremely simplified view of a 'standard' photoionization code. Here we make the distinction that $T_g$ is the temperature within a particular zone while $T$ is a representative average over the whole cloud. For most of the physical situations that we shall be concerned with, the temperature gradient from the face of a cloud to the back of a cloud is small.

The 'inputs' to the photoionization calculation shown in Fig 5.8 are the bare minimum required; see Ferland & Rees (1988) and Ferland & Truran (1981) for more details.

5.4.1.1 Optical Depth

The ionization structure (and therefore X-ray opacity) of optically thin gas scales with $U$. In other words, the ionization structure of optically thin gas with a given $r$ and $N_H$ (see equation 1.1) is identical to that of a gas with the same $U$ but different $r$ and/or $N_H$. This is not the case for optically thick material such as the absorbing column found in NGC 4151 and other highly cut-off AGN. In that case there is no simple scaling law. Of particular interest, as it is a contemporary issue with regard to reprocessing of the primary non-thermal continuum in AGN, is the case of a very high particle density, $N_H > 10^{15}$ cm$^{-3}$. Such high densities force the gas towards LTE and enhance the bremsstrahlung cooling rate above the Compton heating rate. Hence, if the cooling time is less than the free fall time, cold ($T \sim 10^4$ K) material may exist very close to the central continuum. In our detailed modelling we shall only consider cloud densities of order $\sim 10^{10}$ cm$^{-3}$ or less, as the physics becomes much more complex and uncertain at higher densities.

The main contributions to the optical depth are due to valence and L- and K-shell photoionization and electron scattering. The electron scattering opacity is $	au_s = \sigma_T R N_e \sim 1.2 \sigma_T n_H$, where $R$ and $n_H$ are respectively, the thickness and column density of the cloud. In all the calculations in this section, unless otherwise stated, we shall include 13 elements with solar abundances relative to hydrogen, and these are listed in Table B.1 (appendix B) for reference.
Figure 5.8 An extremely simplified schematic diagram of the action of a 'standard' photoionization code such as CLOUDY (see text). $T_e$ and $N_e$ are the electron temperature and number density, respectively, in a zone.
5.4.1.2 TRANSMISSION FUNCTION

The transmission function of a photoionized absorber, $T_{PI}(\epsilon)$, as indicated above, involves a vast number of parameters in its calculation, unlike that for the partial covering and Poissonian absorbers (excluding atomic cross-sections of course). The transmission profile as a function of energy is complex, with a large number of ionization edges. However, we can make some very general statements which are important in the present context. The first is that dramatic changes in the soft X-ray opacity occur for a relatively narrow range in $U$. This is because elements which dominate the $\sim 0.5 - 5$ keV opacity (namely O, Ne, Si and S) are hardly photoionized (for the moderately high densities that we shall be interested in) until $U$ is large enough to heat the gas sufficiently above $\sim 5 \times 10^4 K$. After this point, the temperature increases steeply as a function of $U$ (see §5.4.3) and ions are rapidly stripped of more and more electrons due to the inverse temperature dependence of the recombination coefficients and the increased photo- and collisional ionization rates. The effect is illustrated in Fig. 5.9 which shows the transmission of a cloud with column density $50 \times 10^{21}$ cm$^{-2}$ photoionized by the NGC 4151 continuum (see §4.1.8) for $U = 0.01$ (almost 'cold'), 0.1, 0.5 and 1.0. Fig. 5.9 also illustrates how the transmission can vary by orders of magnitude across some of the ionization edges, the oxygen K-edge being one of the most prominent. This is important because this can potentially distinguish a warm absorber situation from complex, cold absorption (§5.2 - §5.3). However, good energy resolution and sensitivity below 1 keV is required to achieve this.

Note that in the remainder of this section we shall consider only single-cloud models. In reality, there may be several clouds in the line-of-sight (cf. §5.3). In that case, the ionization state, and hence transmission, of a given cloud depends on the ionization state of all other material between it and the source. Hence, a detailed geometrical model is required to compute the total transmission. This is beyond the scope of the present work.
Figure 5.9 The fraction of flux transmitted by a single-cloud warm absorber model, as a function of energy. The cloud has a column density of $50 \times 10^{21}$ cm$^{-2}$, is photoionized by the continuum of NGC 4151 (with $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 1 MeV) and is assumed to be in radiative thermal equilibrium. The solid curves correspond to values of the ionization parameter $U$, of 0.1, 0.5 and 1.0 (as indicated). The dashed curve corresponds to an ionization parameter of 0.01 (i.e. almost 'cold'). Note that, due to electron scattering, the transmission is never 1.0.
5.4.1.3 L- AND K-SHELL ENERGIES

As an ion loses more and more electrons, the threshold energy for a photon to remove an electron from an L- or K-shell increases as the electron then experiences a stronger nuclear attraction. The observed 'mean' edge energy from an ionized gas will then be some function of $U$. The measurement of these edge energies can be vital in distinguishing a warm absorber from a complex cold absorber. For instance, Fig. 5.9 shows that if the spectrum of a source is measured above 1 keV with poor resolution, the apparent variable absorption and/or soft excess may be explained by partial covering or Poissonian absorption (cf. Figs. 5.1 and 5.4). The only distinguishing feature is then the iron K-edge energy (and possibly the dependence of the soft excess with continuum flux).

An important property of the 'mean' L- and K-edge energies of ions in a photoionized cloud is that they depend mainly on $U$, the shape of the UV/ X-ray continuum and the cloud column density. They are relatively insensitive to details such as the strength of the radio continuum and the $\gamma$-ray cut-off energy. These points are illustrated in Figs. 5.10 and 5.11. Fig. 5.10 shows the K-edge energies of O, Ne, Si, S, Fe and the L-edge energy of Fe as function of $U$, calculated using Ferland's ionization code 'CLOUDY' (see Ferland & Rees, 1988) with the continuum of NGC 4151. The solid curves correspond to cloud column densities of 25, 50 and $100 \times 10^{21}$ cm$^{-2}$, an X-ray slope of $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 1 MeV. The dashed curves show the effect of increasing the $\gamma$-ray cut-off energy. They correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 10 MeV. The dotted curves show the effect of changing the slope of the X-ray spectrum. They correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, a $\gamma$-ray cut-off of 1 MeV and an X-ray slope of $\Gamma = 1.45$.

Fig. 5.11 shows similar calculations to those in Fig. 5.10 but using the 'mean' radio-quiet AGN continuum (see §4.1.7). The solid curves correspond to cloud column densities of 25, 50, and $100 \times 10^{21}$ cm$^{-2}$, an X-ray slope of $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 100 keV. The dashed curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 500 keV. The dotted curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, $\Gamma = 1.5$ and a $\gamma$-ray cut-off of 100 keV.
Figure 5.10 The K-edge energies due to Oxygen, Neon, Silicon, Sulphur and iron, as a function of ionization parameter, in the spectrum emerging from a cloud photoionized by the continuum of NGC 4151. Also shown is the L-edge energy of iron. Calculations have been performed assuming radiative equilibrium. The solid curves correspond (from top to bottom) to cloud column densities of 25, 50 and $100 \times 10^{21}$ cm$^{-2}$, an X-ray photon spectral index of 1.7 and a $\gamma$-ray cut-off of 1 MeV. The dotted curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, an X-ray photon index of 1.45 and a $\gamma$-ray cut-off of 1 MeV. The dashed curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, an X-ray photon index of 1.7 and a $\gamma$-ray cut-off of 10 MeV. All models have a neutral Hydrogen density of $10^{9.5}$ cm$^{-3}$. 
Figure 5.11 The K-edge energies due to Oxygen, Neon, Silicon, Sulphur and iron, as a function of ionization parameter, in the spectrum emerging from a cloud photoionized by the 'mean' AGN continuum. Also shown is the L-edge energy of iron. Calculations have been performed assuming radiative equilibrium. The solid curves correspond (from top to bottom) to cloud column densities of 25, 50 and $100 \times 10^{21}$ cm$^{-2}$, an X-ray photon spectral index of 1.7 and a $\gamma$-ray cut-off of 100 keV. The dotted curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, an X-ray photon index of 1.5 and a $\gamma$-ray cut-off of 100 keV. The dashed curves correspond to a cloud column density of $50 \times 10^{21}$ cm$^{-2}$, an X-ray photon index of 1.7 and a $\gamma$-ray cut-off of 500 keV. All models have a neutral Hydrogen density of $10^{9.5}$ cm$^{-3}$.
The most noticeable difference between the curves in Figs. 5.10 and 5.11 is the slower dependence of the edge energies on $U$ for the mean AGN continuum. This is mainly due to the different shapes of the UV spectrum for NGC 4151 and the mean AGN continuum (cf. Figs. 4.2 and 4.3). Obviously, any real situation may be somewhere in between that indicated in Figs. 5.10 and 5.11. Again, this emphasises the importance of modelling individual sources. In as much as the 'mean' edge energies shown in Figs. 5.10 and 5.11 are indicative of the ionization state of the absorbing material, they are also indicative of the sensitivity of the X-ray transmission to $U$, cloud column density, and continuum shape since the cross-sections (at the threshold energy) of the various ionic species are only slowly varying functions of ionization state. Note that the minimum and maximum K- and L-edge energies of all 13 elements are listed for reference in Table B.1 (appendix B).

5.4.2 Reduction of the photoionization problem

This section is concerned with a critical assessment of the photoionization problem (i.e. in the sense of modelling X-ray data) with a view to simplifying the calculation of X-ray opacity sufficiently so that a satisfactory computation speed is obtained. Obviously, the level of the approximations that are involved will be a compromise between speed and precision. In the present case that compromise is determined by the quality of X-ray spectra obtainable from current instrumentation. Our primary concern is to obtain sufficient precision for the X-ray opacity in the 1-40 keV band (which covers the range of the EXOSAT ME (§2.1.2) and the Ginga LAC (§2.2.1)). The principles outlined below can be used to extend the accuracy below this range, with the sacrifice of some speed. In any case, the positions of the edge energies are more important to model than the details of the opacity profile. It is trivial to fit the opacity profile to data since it is sensitive to so many input parameters. It is the ionization edge energies which will provide the real physical constraints.

Below we discuss the physical approximations that we can make for the present purpose of constructing a spectral fitting photoionization code (refer to Fig. 5.8).

5.4.2.1 THE CONTINUUM

Normally, the form of the entire radio to $\gamma$-ray continuum is required in order to compute the self-consistent thermal and ionization structure (see Ferland & Rees 1988 and §5.4.3). The computation of the radiative equilibrium temperature and ionization state involves many iterations before convergence is obtained and this process represents a major overhead in computation time. For the present purpose, the temperature does not need to be computed to this sort of accuracy ($\sim 0.5\%$) since the effect on the X-ray opacity of the gas is small. In fact in §5.4.3 it is shown that the temperature should actually be a free parameter so that it does not even need to be computed. Hence, only the ionizing continuum is required and in the present case is treated by numerical
binning, extending from 2.5 eV to 80 keV in 300 bins. This number of bins was found to be a good compromise between speed and energy resolution. The bin widths vary across the energy range in a manner which gives accurate integration over photoionization cross-sections in the UV and sufficient energy resolution in the crucial 1-10 keV band. Table 5.3 below compares the energy resolution, \( E/\Delta E \), of the spectral fitting code (ZAPPER) and CLOUDY, at a few representative energies.

**Table 5.3 Comparison of the energy resolution, \( E/\Delta E \), of CLOUDY and ZAPPER**

<table>
<thead>
<tr>
<th>ENERGY (keV)</th>
<th>CLOUDY</th>
<th>ZAPPER</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>3.0</td>
<td>12</td>
<td>75</td>
</tr>
<tr>
<td>7.0</td>
<td>14</td>
<td>70</td>
</tr>
<tr>
<td>40.0</td>
<td>11</td>
<td>20</td>
</tr>
</tbody>
</table>
5.4.2.2 GEOMETRY

Details of the geometry of absorbing material about the central source affects line transfer in a complex manner, but the only aspect of the geometry which significantly affects the emergent continuum in the energy range of interest is the covering fraction of the source. The way the geometry affects the transfer of Lyα affects the effective ground state recombination rates of ions of H and He (see Osterbrock, 1974 and also §1.3.6). However, the ionization fractions of H and He need only be computed approximately (to obtain a value of the electron density, \( N_e \)) since these elements do not make a significant contribution to the opacity in the 1-40 keV range. In the present context, the only emission line we are interested in is the iron \( K_α \) fluorescence line. However, in view of our ignorance of the geometry of X-ray absorbing systems in AGN, the iron emission line will be simply added post-hoc to the emergent continuum, its intensity being a free parameter but its energy tied to the ionization equilibrium of iron (see Makishima, 1986). The above comments allow the use of a simple plane slab geometry. If the true geometry were more akin to a spherical shell, the \( 1/r^2 \) dilution factor would be negligible for the column densities of interest.

5.4.2.3 RADIATION TRANSFER

The equation of transfer for the passage of the continuum through the slab is normally solved numerically, dividing the slab into radial zones whose length is calculated so that the ionization structure does not change significantly within each zone. All emission and absorption processes are then balanced in each zone (once the self-consistent ionization structure has been computed) and the diffuse recombination fields (mainly due to hydrogen and helium) are then taken into account on a second iteration through the slab. The entire process represents a large overhead in computation time.

In the spectral fitting code we use a zone length which is a fixed fraction of the slab thickness and neglect all diffuse fields, which have a negligible effect on the X-ray opacity in the energy range of interest. Tests show that these approximations are good for \( U < 0.1 \) and \( U > 1 \), but for intermediate values of \( U \) the X-ray opacity can be in error if the zone length is too small or too large. The intermediate range happens to be the one of most interest in the case of NGC 4151 and the problem has been overcome by using the full ionization code (e.g. CLOUDY) as a guidance for the correct zone length appropriate to the particular problem.
5.4.2.4 IONIZATION BALANCE

The equilibrium fraction of the \( j \)th ionization stage of element \( i \), \( X_{i,j} \), is found by invoking the conservation of charge and equating all ionization rates from stage \( X_{i,j} \) to all recombinations from stage \( X_{i,j+1} \). This is a standard calculation (e.g. see Osterbrock, 1974) but is shown in full below in equations 5.11 - 5.13 to illustrate the physical processes which have been included and also the explicit dependence on the electron density, \( N_e \):

\[
X_{i,1} = \left[ 1 + \sum_{k=2}^{k=N_{\text{atomic}}+1} \prod_{m=2}^{m=k} \left( \frac{\eta_{i,m-1}}{\rho_{i,m-1}} \right) \right]^{-1} \tag{5.11}
\]

and

\[
X_{i,j>1} = X_{i,1} \prod_{m=2}^{m=j} \left( \frac{\eta_{i,m-1}}{\rho_{i,m-1}} \right) \tag{5.12}
\]

where

\[
\frac{\eta_{i,m}}{\rho_{i,m}} = \frac{N_e^{-1} \int_{\epsilon_0,i,m}^{\epsilon} \sigma_{i,m}(\epsilon)n(\epsilon)d\epsilon + \Gamma_L + \Gamma_K + C_{i,m}(T_e)}{\alpha_{i,m}(T_e) + \beta_{i,m}(T_e)D_n(\epsilon) + \beta_{i,m}(T_e, N_e)} \tag{5.13}
\]

is the ratio of ionization rates from stage \( m \) to recombinations to stage \( m \). \( N_e \) is an unknown in the problem and is calculated by numerically solving the ionization balance equations for H and He until \( N_e \) converges to some desired accuracy, neglecting the contribution from other elements. The ionization balance of H and He is treated in the crudest manner, neglecting ionization by diffuse fields and assuming case B recombination (e.g. see Osterbrock, 1974). The remaining terms in equation 5.13 are now described below.

- The first term in brackets is the photoionization rate (s\(^{-1}\)) of stage \( m \) to \( m+1 \) and is the integral of the photon flux, \( n(\epsilon) \), over the photoionization cross section, \( \sigma_{i,m}(\epsilon) \), from the threshold energy, \( \epsilon_0,i,m \).
- \( \Gamma_L \) and \( \Gamma_K \) are the ionization rates (s\(^{-1}\)) from the L- and K-shells (for ionic species which have them) respectively and involve a rather more complex integral over the photon spectrum than the valance photoionization rates. The formalism described by Weisheit & Dalgrano (1972) and Weisheit (1974) was used, modified for iron to take account of the non-negligible fluorescence yield.
- $C_{i,m}(T_e)$ is the collisional ionization rate (cm$^3$ s$^{-1}$) from stage $m$ to stage $m+1$.
- $\alpha_{i,m}(T_e)$ is the net radiative recombination rate (cm$^3$ s$^{-1}$) from stage $m+1$ to stage $m$.
- $\beta_{i,m}(T_e)$ is the dielectronic recombination rate (cm$^3$ s$^{-1}$) from stage $m+1$ to stage $m$. $f_d(N_e)$ is a density dependant suppression factor (higher densities favour larger recombination rates).
- $H_{i,m}(T_e, N_e)$ are three-body recombination rates (cm$^3$ s$^{-1}$), the inverse process of collisional ionization.

Note that ionization due to suprathermal particles (e.g. see Krolik, Mckee & Tarter, 1981; hereafter KMT) and charge exchange reactions (see Osterbrock, 1974) have not been included. The atomic database that we have used is largely similar to that used by CLOUDY (see Ferland & Rees (1988) and references therein).

The calculation of the photoionization cross-sections at a given energy, or any of the recombination or collision rates at a given temperature, each time they are needed, represents a considerable overhead in computation time and is actually unnecessary for the present purpose. This redundancy is overcome in our spectral fitting code by performing the computations once and for all, storing the cross-sections as functions of energy (using the same binning as the continuum) and storing all the recombination and collision rates as functions of temperature in 250 bins from $T_e = 10^3$ K to $10^8$ K, equally spaced in log$T$. The three-body rates must be stored in two dimensions due to the dependence on $N_e$. This procedure represents a major contribution to speed and is accurate enough for the present purpose. The practice of storing all the atomic data in a separate file also makes it simple to update the atomic database.
5.4.3 Temperature

So far, the temperature of the model warm absorber has not been computed. KMT have investigated in detail the dependence of the temperature of gas, exposed to a broadband continuum, as a function of ionization parameter, assuming radiative equilibrium. Apart from heating and cooling due to processes associated with the terms in equation 5.13 and line cooling, the other main processes which are usually taken into account in such calculations are (i) free-free heating by the radio and infrared continuum, (ii) free-free cooling (bremsstrahlung), (iii) Compton heating and (iv) Compton cooling. See Ferland & Rees (1988) for a detailed discussion. The results of KMT, where the temperature versus ionization parameter relation had the now familiar ‘S’-shaped form, led to the classical 2-phase theory of the broad line region (see §1.3.8). Unfortunately KMT used a continuum based on 3C 273 which has a flat X-ray spectrum (energy index ~ 0.5) and is an anomalously strong γ-ray emitter. Both these properties give rise to a high Compton temperature, the equilibrium temperature of the hot phase, which is of the order 10^8 – 10^9 K. However, Mathews & Ferland (1987) have recently pointed out that using a more realistic continuum, no 2-phase instability occurs (a result which is sensitive to details of the atomic data base, however) and the Compton temperature can be as low as 2 x 10^6 K. Also, Fabian et al. (1986b) have shown that the 2-phase instability disappears if the input continuum includes a steep soft excess (type III, §4.1.5.3), of the type seen in Mkn 841 by EXOSAT (Arnaud et al., 1985).

We have performed similar calculations of the radiative equilibrium temperature as a function of \( U \), using the continuum of NGC 4151 (§4.1.8) and the mean (radio quiet) AGN continuum (§4.1.7), using CLOUDY. Examples of the heating and cooling curves for NGC 4151 are shown in Fig. 5.12 for three values of \( U \), a cloud column density of 50 x 10^{21} \text{ cm}^{-2} , \( \Gamma = 1.7 \) and a γ-ray cut-off of 1 MeV. See KMT for a discussion of the general shape of the cooling curves. It can be seen that in general, there is only one value of \( T \) for which heating balances cooling. These equilibrium values of \( T \) (at the inner face of the cloud) are plotted against \( U \) (solid curves) in Fig. 5.13a for the NGC 4151 continuum and in Fig. 5.13b for the mean AGN continuum (\( \Gamma = 1.7 \) and a γ-ray cut-off of 100 keV). It can be seen that the Compton temperature of the NGC 4151 continuum (\( \sim 10^8 \) K) is much higher than that for the mean AGN continuum (\( \sim 2 \times 10^6 \) K).

The dotted curves in Figs. 5.13a and 5.13b show the effect of flattening the X-ray slope; \( \Gamma = 1.45 \) for the NGC 4151 continuum and \( \Gamma = 1.5 \) for the mean AGN continuum. The dashed curves show the effect of increasing the γ-ray cut-off energy; 10 MeV for NGC 4151 and 500 keV for the mean AGN. As might be expected, the Compton temperature is higher for all these cases. Figs. 5.13a and 5.13b again show the importance of modelling individual objects, since the mean AGN continuum really represents a ‘worst case’ as far as the Compton temperature is concerned. In reality, the Compton temperature may lie anywhere between \( \sim 10^6 \) and \( \sim 10^8 \) K in a particular object.
Figure 5.12 The net cooling (solid curves) and heating (dashed curves) rates (for three values of the ionization parameter, $U$) of a cloud with column density $50 \times 10^{21}$ cm$^{-2}$, photoionized by the continuum of NGC 4151 ($\Gamma = 1.7$ and a $\gamma$-ray cut-off of 1 MeV), in units $10^{-24}$ erg cm$^3$ s$^{-1}$. The dashed curve actually represents net heating less three-body cooling.
Figure 5.13 (a) The radiative equilibrium temperature, as a function of the ionization parameter, $U$, at the inner face of a cloud photoionized by the continuum of NGC 4151. Solid curve corresponds to a photon spectral index, $\Gamma$, of 1.7 and a $\gamma$-ray cut-off of 1 MeV. Dotted curve corresponds to $\Gamma = 1.45$ and a $\gamma$-ray cut-off of 1 MeV. Dashed curve corresponds to $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 10 MeV. (b) The radiative equilibrium temperature at the inner face of a cloud photoionized by the 'mean' AGN continuum. Solid curve corresponds to $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 100 keV. Dotted curve corresponds to $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 100 keV. Dashed curve corresponds to $\Gamma = 1.5$ and a $\gamma$-ray cut-off of 500 keV. In all cases, the temperature gradient through the cloud depends on the ionization parameter and the cloud column density.
Although no 2-phase instability occurs with either the NGC 4151 or mean AGN continuum, it is still possible for ‘cold’ clouds to be in pressure equilibrium with a less dense hot medium at the Compton temperature. However, Mathews & Ferland (1987) conclude, on the grounds of dynamics and the large optical depth of a ‘cool’ inter-cloud medium, that the 2-phase theory should be discarded. This question will be addressed again in §5.5.2, especially in view of the high Compton temperature for NGC 4151. The main conclusion from the discussion above is that for modelling X-ray spectra with a warm absorber there are two choices for the treatment of the temperature of the absorber:

(i) If radiative equilibrium is to be assumed, then the temperature, $T_{\text{rad}}$, can be tied to $U$ using broken power law fits to an appropriate version of curves of the type shown in Fig 5.13.

(ii) If radiative equilibrium is not assumed then the temperature should be a free parameter. It is shown in §5.5.1 that other heating effects, not normally considered in the calculation of thermal equilibrium can have profound effects on the behaviour of temperature as function of $U$.

The effect on the X-ray opacity of using a temperature which is significantly different to $T_{\text{rad}}$ can be quite large. This is illustrated in Fig 5.14 which shows the transmission of a cloud of column density $50 \times 10^{21}$ cm$^{-2}$, photoionized by the NGC 4151 continuum with $U = 0.1$ for four different temperatures, including the radiative equilibrium value.
Figure 5.14 The effect on the X-ray transmission of a photoionized gas when the temperature is increased above its radiative equilibrium value (by extra heating) when the ionization parameter, U, is held constant. In this case U = 0.1 and the dashed curve shows the transmission when the gas is in radiative equilibrium, at a temperature of $3 \times 10^4$ K, photoionized by the continuum of NGC 4151 (with $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 1 MeV). The solid curves I–III correspond to gas temperatures of $10^5$, $5 \times 10^5$ and $10^6$ K respectively.
5.4.4 Results and Application to NGC 4151

It is difficult to directly compare the X-ray opacity computed using all the approximations in the preceding sections with the full calculation using CLOUDY. This is because the latter code has some K-edge energies which are fixed and some of the physics is different. However, the weighted mean ionization stage, $\kappa_i$, defined by

$$
\kappa_i = \sum_{j=1}^{j=N_{\text{atomic}}+1} j \chi_{i,j}
$$

(5.14)

can be compared directly. The $\kappa_i$'s are not only a convenient measure of the most abundant ionization stages present but also indicative of the mean L- and K-shell energies which might be measured. Tests show that the agreement of values of $\kappa_i$ obtained from CLOUDY and ZAPPER is good for a wide range of physical conditions.

5.4.4.1 SPECTRAL FITTING

In this section we investigate whether the complex absorption in NGC 4151 can be accounted for by the simple single-cloud warm absorber model. We shall not use the EXOSAT data in this analysis as the quality of the data is poor. In §3.3.3 we measured a mean iron K-edge energy of $7.17 \pm 0.08$ from the Ginga data and it can be seen from Fig. 5.10 that this constraint still allows values of $U$ up to $\sim 0.3$. Moreover, taking instead, the 90% upper limits for individual observations as the constraint, allows values of $U$ up to $\sim 1.2$. Although, in §3, we established that temporal changes in the apparent column density occur independent of the continuum level, this does not, in itself, rule out the warm absorber. This is because (i) $U$ does not depend only on the continuum level (see equation 1.1); the configuration of the absorbing system may be changing and (ii) the warm absorber might still explain the 2-4 keV soft excess even if it cannot account for the apparent column variations.

We have performed a spectral fitting analysis over the energy range 2-23 keV, using the code described above, to the seven Ginga observations of NGC 4151 (see §3.3). The continuum compiled in §4.1.8 has been used, with the X-ray photon index fixed at the value obtained from spectral fitting above 4 keV (Table 3.5) but with the X-ray normalization floating (the shape of the UV continuum is then defined by these two parameters; see §4.1.3). Four other free parameters (making five in total) were included in the fits, namely the neutral column density, $n_H$, the ionization parameter, $U$, the iron abundance relative to solar ($A_{Fe}$) and the intensity of the iron emission line. In order to reduce the number of free parameters, radiative equilibrium was assumed so that the temperature is pre-determined by the other parameters. The neutral hydrogen density was fixed at $10^{9.5}$ cm$^{-3}$ (see §1.3.5).
Table 5.4 Spectral fits to the Ginga data for NGC 4151 with a warm absorber model

<table>
<thead>
<tr>
<th>Obs</th>
<th>U</th>
<th>$n_H$ $^a$</th>
<th>$A_{Fe}$ $^b$</th>
<th>$\chi^2_r$ $^c$</th>
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<tbody>
<tr>
<td>1 †</td>
<td>1.58</td>
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<td>1.31</td>
<td>43</td>
<td>3.74</td>
<td>1.66</td>
</tr>
</tbody>
</table>

$^a$ $10^{21}$ cm$^{-2}$  
$^b$ Iron abundance relative to Solar  
$^c$ 28 degrees of freedom  
† These represent the fits to the spectra 1b and 6b respectively.  
The best fitting parameters for the spectra 1a and 6a are very similar.
Figure 5.15 Spectral fit to observation 4 of the Ginga data set for NGC 4151 with a warm absorber model. (a) Pulse height spectrum, best fit model and residuals. (b) Incident photon spectrum corresponding to (a).
The results are shown in Table 5.4. Errors on the best-fit parameters are not included as their calculation requires a large overhead in computation. It can be seen that $U$ varies between ~ $1 - 2$ and that significant variations in the column density, $n_H$, are still required. Moreover, the iron abundance is still required to be greater than solar and some of the reduced chi-squares are very poor. This is essentially because for a given $n_H$ the ionization parameter required to fit the soft excess always gives an iron K-edge energy which is too high, thus giving poor residuals around 6–9 keV. This is illustrated in Fig. 5.15 which shows the pulse height spectrum for observation 4 (see Table 3.4), best-fitting model, residuals and the corresponding incident photon spectrum.

It seems, then, that the simple single-cloud warm absorber which fully covers the source cannot account for either the variable absorption, soft excess (type IV - §4.1.5.3) or the apparent iron over-abundance. This conclusion is independent of the many detailed assumptions that go into the warm absorber model and even the fact that we have only a few pulse height channels in the 2–4 keV range. It is based purely on the fact that the ionization state required to account for the magnitude of the soft excess predicts an iron edge energy which is too high to be consistent with observation. Test runs of the photoionization code CLOUDY show that the situation would not be improved with a multi-cloud model. This is essentially because all clouds in the line-of-sight would have to be in a fairly high state of ionization in order to account for the magnitude of the soft excess, which again pushes up the iron edge energy. A mixed solution, whereby the edge is composed of a 'cold' and 'warm' component (as proposed for MCG-6-30-15; Nandra et al., 1990a) is not excluded but the present data does not warrant such a refinement.

We can also compute the expected ionization state of X-ray absorbing material in NGC 4151. From the broadband continuum of NGC 4151 compiled in §4.1.8, we find that the number of photons emanating from the nucleus per second in the low state ($\Gamma = 1.45$) is

$$Q_L \sim 1.335 \times 10^{53} \text{ photons s}^{-1}$$

and

$$Q_H \sim 4.256 \times 10^{53} \text{ photons s}^{-1}$$

in the high state ($\Gamma = 1.7$). Now, from the work of Clavel et al. (1987), we can approximate the density profile of clouds across the BLR in NGC 4151 by

$$n \approx 10^{10.7} \left( \frac{r}{r_0} \right)^{-2} \text{ cm}^{-3}$$
where $r_0 \sim 10^{16}$ cm (see §1.3.5). Substituting equation 5.17 into equation 1.1 then gives $U \sim 0.16Q_{53}$ where $Q_{53}$ is the number of photons per unit time in units $10^{53}$ s$^{-1}$. Hence $U$ varies between the low and high states (corresponding to equations 5.15 and 5.16 respectively) from $\sim 0.21$ to $\sim 0.68$. Fig. 5.13a shows that this range of $U$ corresponds a temperature range of $\sim 3 \times 10^4 K$ (assuming radiative equilibrium). The lower limit of the range in $U$ is very close to the value of $10^{-0.75}$ deduced by Ferland & Mushotsky (1982) for BLR material in NGC 4151. This result was based on modelling optical and UV emission lines in NGC 4151. Similar modelling of other AGN normally yields a much lower ionization parameter of $\sim 0.01$ (see §1.2). Note that the $r^{-2}$ dependence of the cloud density (equation 5.17) means that the ionization parameter is independent of the distance from the source.

### 5.4.4.2 METAL ABUNDANCES

As pointed out in §1.3.10, one of the major uncertainties pertaining to absorbing material in AGN is the abundance of the elements relative to hydrogen. In fact the relative abundances of all metals in the AGN environment (including helium) are completely unknown and cannot be deduced in a model independent way. It is also likely that, in objects that radiate significant energy above $\sim 10$ MeV, such as NGC 4151, the metal abundances may actually vary with time due to photo-erosion of nuclei. Calculations for NGC 4151 (Boyd & Ferland, 1987) suggest that order of magnitude changes in the metal abundances can occur on timescales of less than $10^3$ yr. The modelling of the photo-electric absorption in NGC 4151 so far has been based on the ad-hoc assumption that the metal abundances (apart from iron, of course) relative to hydrogen have solar values (Morrison & McCammon, 1983 or Table B.1). It is possible to test whether significant deviations from solar abundances can account for the 2–4 keV soft excess in NGC 4151, using the spectral fitting code described above, by simply disabling the photoionization routines. A spectral fitting analysis has been performed on the Ginga data set (see §3.3), over the 2–23 keV range, allowing the abundances of oxygen, silicon, sulphur and iron to float. The other free parameters in the fit were the normalization of the X-ray power law, the photon spectral index (Γ) and the equivalent hydrogen column density, $n_H$, making seven free parameters in total. The iron line energy was fixed at 6.4 keV and its intensity fixed at the value in Table 3.5. The results for observation 4 (see Table 3.4) are shown in Table 5.5; the X-ray slope turns out to be similar to that in Table 3.5. The fit is quite poor ($\chi^2 = 2.1$), as can be seen from Fig. 5.16, which shows the pulse height spectrum, best fit model and residuals and corresponding incident photon spectrum. Similar results for the remaining Ginga observations indicate that abundance anomalies alone cannot account for the soft excess in NGC 4151.
Figure 5.16 Spectral fit to observation 4 of the Ginga data set for NGC 4151 with a cold, uniform absorber, but with the abundances of Oxygen, Silicon, Sulphur and iron floating. (a) Pulse height spectrum, best fit model and residuals. (b) Incident photon spectrum corresponding to (a).

Table 5.5 Spectral fit for observation 4 of the Ginga data set for NGC 4151 with four metal abundances free (see text)

<table>
<thead>
<tr>
<th>Abundances $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_H$ $^b$</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>80</td>
</tr>
</tbody>
</table>

$^a$ All abundances relative to Solar.

$^b$ $10^{21}$ cm$^{-2}$.

$^c$ 26 degrees of freedom.
5.5 Implications for models of the Broad Line Region

5.5.1 Effects of extra heating/cooling processes

In this section some consequences of our ignorance of other heating/cooling processes in the BLR is illustrated by considering a form of heating which is proportional to the gas temperature. Such a temperature dependence could arise from compressional heating of inflowing gas (see KMT for some other sources of extra heating and cooling). We have computed the thermal and ionization equilibrium as a function of $U$, of gas exposed to the continuum of NGC 4151 and the mean AGN continuum (using CLOUDY) with an additional heating term of the form

$$T = \lambda T (10^{-24} \text{ erg cm}^3 \text{ s}^{-1}).$$

To illustrate the qualitative effect of this extra term, the heating/cooling curves shown in Fig. 5.12 are shown again in Fig. 5.17 with the inclusion of the extra heating term with $\lambda = 10^{-5}$. It can be seen that, in general, more than one solution for the equilibrium temperature is now possible. The solutions for the equilibrium temperature at the inner face of a cloud are shown in Fig. 5.18a for the NGC 4151 continuum ($\Gamma = 1.7$, $\gamma$-ray cut-off = 1 MeV) and in Fig. 5.18b for the mean AGN continuum ($\Gamma = 1.7$, $\gamma$-ray cut-off = 100 keV). Curves I-IV correspond to values of $\lambda$ equal to $5 \times 10^{-7}$, $10^{-5}$, $10^{-4}$ and $10^{-3}$ respectively; we shall investigate below whether these values are physically realistic at all.

Curves I ($\lambda = 5 \times 10^{-7}$) illustrate the fact that, for small values of $\lambda$, there are two distinct branches to the allowed equilibrium temperature curves. For the mean AGN continuum (Fig. 5.18b), there is now the interesting possibility that a hot phase at $T > 10^8$ K can exist despite the low Compton temperature of the continuum. However, for this particular value of $\lambda$, the hot phase may be thermally unstable (i.e. the slope of $T$ versus $U$ is negative) unless $0.5 < \log (U_{\text{HIM}}) < 1.5$. If pressure equilibrium prevails then the maximum ionization of cool clouds is restricted to $\sim 10^{-2.5}$. Another interesting feature is that if $U_{\text{HIM}}$ happens to lie at the point where the upper branch terminates, a very small increase in $U_{\text{HIM}}$ can force the temperature of the hot phase to drop by two orders of magnitude. This may give rise to the possibility of variable X-ray absorption.

For larger values of $\lambda$ only a cold phase can exist, thus requiring some other mechanism for cloud confinement. Moreover, a maximum value of $U$ is implied, which, as can be seen from Figs. 5.18a and 5.18b, can be less than 0.1 for $\lambda > 10^{-3}$. The upper limit on $U$ implies that at a given distance to the source, gas cannot exist below a certain critical density.
Figure 5.17 The effect of additional heating, proportional to $T$, on the thermal equilibrium of a cloud photoionized by the NGC 4151 continuum. The models shown have $\lambda = 10^{-5}$ (see equation 5.18) and the other parameters of the models are the same as those in Figure 5.12, in which radiative equilibrium was assumed. The heating and cooling rates are plotted in units $10^{-24}$ erg cm$^{-2}$ s$^{-1}$. It can be seen that multiple solutions for the equilibrium temperature are now possible.
Figure 5.18 (a) The equilibrium temperature at the inner face of a cloud photoionized by the continuum of NGC 4151 ($\Gamma = 1.7$ and a $\gamma$-ray cut-off of 1 MeV) when the effects of additional heating, proportional to $T$, are included. Curves I-IV correspond to values of $\lambda$ (see equation 5.18) of $5 \times 10^{-7}$, $10^{-5}$, $10^{-4}$ and $10^{-3}$ respectively. (b) As (a) but the 'mean' AGN continuum has been used, with $\Gamma = 1.7$ and a $\gamma$-ray cut-off of 100 keV.
In this section values of $\lambda$ are calculated under the specific assumption that the extra heating, $\mathcal{Y} = \lambda T$, is due to compressional heating of inflowing gas in the BLR, using physical parameters relevant to NGC 4151. Whether gas in the BLR of AGN is inflowing or outflowing is still an unsettled, controversial question (see §1.3.7) but the present example simply serves to illustrate the order of magnitude of $\lambda$ which might be expected under specific assumptions. Suppose the density and velocity profiles across the BLR vary as a function of radius as $n = n_0 (r/r_0)^{-\alpha}$ and $u = u_0 (r/r_0)^{-\beta}$ respectively, quantities with a subscript 0 being the values at $r_0$. The compressional heating, in the same units as in Figs. 5.12 & 5.17 is then

$$\mathcal{Y} = -\frac{10^{24}}{n^2} \left[ \frac{P}{V} \right] \frac{dt}{dV} = 10^{24} \frac{kT}{n^2} \frac{dn}{dr} \left( \frac{dr}{dt} \right)$$

$$\equiv \lambda T \quad (10^{-24} \text{ erg cm}^3 \text{ s}^{-1}). \quad (5.19)$$

Hence

$$\lambda = \frac{10^{24} \alpha k u_0}{r_0 n_0} \left( \frac{r}{r_0} \right)^{\alpha-\beta-1}. \quad (5.20)$$

If the BLR clouds (which have $T \sim 10^4 K$) are in pressure equilibrium with an HIM at $\sim 10^8 K$ then the density profile of the HIM is $n_{HIM} = 10^{-4} n$ so that $\lambda_{HIM} = 10^4 \lambda$. Taking $u_0 \sim 10^9 \text{ cm s}^{-1}$, consistent with the broadest velocities of the CIV line profile (see Clavel et al., 1987) and $\beta = 0.5$, a value appropriate to Keplerian motion (see §1.3.7), gives $\lambda_{HIM} = 5.5 \times 10^{-6} (r/r_0)^{0.5}$ (using equation 5.17 for the density profile). $\lambda_{HIM}$ is tabulated for various values of $r$ in Table 5.6, from which it can be seen that compressional heating of the HIM, if it exists and is co-moving with the cool clouds, can be important throughout the BLR. However, $\lambda$ is four orders of magnitude smaller than $\lambda_{HIM}$ so that compressional heating of BLR clouds in NGC 4151 is not significant.

The implications of this are that an HIM may exist in the outer BLR but low density gas in the inner regions of the BLR may be thermally unstable. Gas would then condense out into the cool phase. An alternative confinement mechanism for the inner BLR is then required. Variations in the flow conditions might then lead to a variable amount of matter which is in the cold phase at any time, possibly giving rise to variable X-ray absorption.
Table 5.6 Values of the heating constant, $\lambda_{HIM}$ (see equation 5.18), at various radii from the nucleus of NGC 4151 (see text)

<table>
<thead>
<tr>
<th>$r$ (cm)</th>
<th>$\lambda_{HIM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 \times 10^{14}$</td>
<td>$3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^{16}$</td>
<td>$6 \times 10^{-6}$</td>
</tr>
<tr>
<td>$10^{17}$</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>$10^{18}$</td>
<td>$6 \times 10^{-7}$</td>
</tr>
<tr>
<td>$10^{19}$</td>
<td>$2 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
5.5.2 The Inter-Cloud Medium and cloud confinement

We have shown in §5.5.1 that BLR gas exposed to the 'mean' AGN continuum (which represents a 'worst case' as far as low Compton temperature of the continuum is concerned), can be heated to temperatures above $\sim 10^8$ K if extra heating processes are important. Hence, it may still be possible for BLR clouds to be confined by a Hot Inter-cloud Medium (HIM) as originally proposed by KMT (see §1.3.8). In the case of NGC 4151, an HIM could form from radiative heating and cooling alone. However, it is clear that any cloud-confinement theory must take into account the dynamics of the BLR. Mathews & Ferland (1987) have investigated models in which a co-moving BLR forms out of either an inflowing or outflowing HIM, taking into account compressional heating/cooling. Using a continuum similar to the 'mean' AGN continuum in §4.1.7. They find that a BLR which has physical parameters consistent with observation does not form unless specific initial conditions are imposed on the flow of the HIM. These models assume spherical symmetry, but clearly, if the UV source is anisotropic, the cloud distribution itself may be anisotropic, since the existence of an HIM also depends on the relative strength of the UV and X-ray continuua.

Alternative mechanisms to simple pressure balance have also been proposed for cloud-confinement. For example, confinement by ram pressure in a wind (Perry & Dyson, 1985; Smith & Raine, 1988) or by a magnetic field (Rees, 1987). In summary, whatever it is that is responsible for confining the clouds, an HIM may or may not exist in different AGN and may not necessarily be in pressure equilibrium with the cool clouds. Clearly, it is important to consider the properties of more detailed geometrical and dynamical models.

5.5.3 The variable X-ray absorption in NGC 4151

The X-ray data for NGC 4151 still leaves open the question of the origin of the X-ray absorbing material, its relationship to the broad line-emitting material and the cause of the variability of the absorption. We know that the covering factor of 'traditional' BLR material in NGC 4151 is large (see §1.3.6) but the column density cannot be as large as $\sim 10^{23}$ cm$^{-2}$ (a typical value inferred from X-ray data) in all directions as there would be insufficient ionizing flux in the outer BLR (unless the UV source lay beyond the X-ray absorbing region). Hence the implied geometry is one in which the X-ray source is surrounded by a system of absorbing clouds which has a low column density (a few $\sim 10^{22}$ cm$^{-2}$) in most directions but a larger column density $\sim 10^{23}$ cm$^{-2}$ in a direction which happens to lie along our line-of-sight. This latter column is significantly variable (independent of the continuum level) on a timescale of months to years. However, the total amount of material surrounding the source cannot, on average, vary significantly since the iron $K_{\alpha}$ line intensity is not variable (see §3.3.3).
If variability of the line-of-sight column in NGC 4151 is due to random motions of individual motions of absorbing clouds then the size of the clouds needs to be comparable to that of the X-ray source in order to produce significant changes in the low-energy spectrum. If the clouds are small compared to the source then systematic changes in the cloud distribution are required. In either case the thermal/dynamical stability and confinement mechanism of the clouds is uncertain and needs further investigation. In particular, the effects of (i) spectral variability of the continuum (see §3.3.2 and 4.1.8) and (ii) non-radiative thermal equilibrium, may have important consequences for the physical state of absorbing material in the BLR. An alternative origin for the X-ray absorption is the material in or close to a possible accretion disc. The fact that the proposed disc in NGC 4151 is probably viewed close to edge-on (Clavel et al., 1987) is favourable for this scenario.

A sophisticated implementation of the above disc-origin for the X-ray absorption is the the duelling wind model, proposed by Smith & Raine (1988). In this model an out-flowing cloud system forms from the interaction of a nuclear wind (originating close to the putative black hole) with a disc wind which is driven by Compton heating by the X-ray source. In the duelling wind model the clouds are confined by the ram pressure of the nuclear wind so that an HIM is not required. Varying physical conditions, not necessarily related to the luminosity of the X-ray source, can then lead to changes in the cloud formation rate, cloud column density, mean number of clouds in the line-of-sight and the covering factor. This is a plausible physical interpretation of the spectral fits to the Poissonian absorber model for NGC 4151 described in §5.3. It is interesting to note that the radial dependence of the density profile of clouds in the duelling wind model is $r^{-2}$, consistent with observation (cf. equation 5.17). A problem with the duelling wind model which prevents its application to AGN in general is that the cloud system forms close to the plane of the disc and clouds are disrupted (in a sound-crossing time) before they can reach high altitudes.

5.6 Chapter summary

Below is a summary of the work in this chapter.

- We have investigated the properties of some models of complex and variable X-ray absorption in AGN. In particular we have developed a photoionization code which is suitable for use in spectral fitting to a warm absorber model.
- The warm absorber model cannot account for either the variable X-ray absorption, the 2–4 keV soft excess or the apparent over-abundance of iron in NGC 4151. This is essentially because the ionization state of material required to produce the soft excess is too high to be consistent with the iron K-edge energy measurements in §3.3.3.
We find that the variable X-ray absorption and 2–4 keV soft excess in NGC 4151 is well described by the partial covering and Poissonian absorber models (based on spectral fitting to the Ginga data set; see §3.3). In the partial covering model the covered fraction, $C_F$, varies between $\sim 0.55 - 0.90$, while the column density, $n_{H_1}$, varies between $\sim 40 - 125 \times 10^{21} \text{ cm}^{-2}$. In the Poissonian absorber model the data require the mean number of clouds in the line-of-sight, $\mu$, to vary between $\sim 0.9 - 3.3$ and the cloud column density, $n_c$, to vary between $\sim 20 - 70 \times 10^{21} \text{ cm}^{-2}$. The Poissonian absorber provides a marginally better description of the data than partial covering. It also provides a natural explanation for the correlation of $C_F$ with $n_{H_1}$ that is implied from spectral fitting to the partial covering model.

We have shown that, using the simplest geometry, the changes in $n$ required by the Poissonian absorber can be provided by modest changes in the ratio of cloud to source size (a factor $\sim \sqrt{3}$) and/or in the total number of clouds surrounding the source (a factor $\sim 3$). The changes in $n_c$ could be produced by changes in the cloud thickness. However, our implementation of the Poissonian absorber is based on the assumptions that individual clouds are small compared to the X-ray source and that all clouds have the same column density. In reality, neither of these assumptions may be justified so that the large changes in cloud column density implied from spectral fitting to the Poissonian absorber may not be necessary.

The above modelling suggests that there is an over-abundance of iron in the absorbing material, but also an additional amount of iron which is not associated with the material responsible for the low-energy absorption. Due to the uncertainty in the quantity of this residual iron, it is not possible to derive a value for the iron over-abundance, but we estimate that it is $\sim 2$ relative to solar.

It is important to note that whatever model is adopted for the complex absorption in NGC 4151, we find that the spectral index variability reported in §3.3.2 is always required. The values obtained for the spectral index in each case are similar to those obtained from the spectral fitting above 4 keV (see §3.3.2).

We deduce that the ionization parameter, $U$, of absorbing material in NGC 4151 varies from $\sim 0.2$ to $\sim 0.7$ between low and high flux states corresponding to photon fluxes in the range $\sim 1.3 - 4.3 \times 10^{53} \text{ s}^{-1}$. This corresponds to radiative equilibrium temperatures in the range $\sim 3 - 10 \times 10^4 \text{ K}$. The corresponding range in the iron K-edge energy is consistent with the 90% confidence limits on the individual measurements in §3.3.3. The lower value of $U \sim 0.2$ is consistent with the value of $10^{-0.75}$ independently deduced by Ferland and Mushotsky (1982). It is much higher than the value of $\sim 0.01$ which is typical of BLR material in most AGN.

Despite the low Compton temperature of the 'mean' AGN continuum, it is still possible for an optically thin Hot Inter-cloud Medium (HIM) to form if additional heating processes are
important. The Compton temperature of the broadband continuum for NGC 4151 is high enough for the HIM to form from radiative heating alone. However, whether the HIM, if it exists, is responsible for confining the cool clouds remains an open question.

• The origin of the X-ray absorbing material in NGC 4151, the cause of its variability and its relationship to 'traditional' BLR material is still unclear. More detailed geometric and dynamical models need to be investigated. In particular, the effects of the variable X-ray continuum and additional heating/cooling processes may be important in determining the physical state of the absorbing material and the HIM, if it exists.
Chapter 6

X-ray Observations of ESO 103-G35 and the QSO 1821+643

Overview

We report on the results of a Ginga observation of the low luminosity Seyfert 1.9 galaxy ESO 103-G35 and seven EXOSAT observations of the high luminosity quasar 1821+643. Both sources exhibit marked X-ray spectral variability. We discuss to what extent the present observations can constrain physical models of the active nuclei in each case.

"We're just two lost souls, swimming in a fish bowl..."

(Roger Waters, 1975. From 'Wish you Were Here'.)
6.1 A *Ginga* observation of ESO 103-G35

6.1.1 Introduction

ESO 103-G35 was identified by Phillips *et al.* (1979) as an emission-line galaxy within the error box of the HEAO-1/A2 X-ray source H1834-653 (Piccinotti *et al.*, 1982). Optical spectroscopy subsequently revealed a high-excitation forbidden-line spectrum with weak broad emission wings to Hα, a steep Balmer decrement and other characteristics consistent with a Seyfert 1.9 classification (Phillips *et al.* 1979; de Zotti & Gaskell, 1985). The first detailed X-ray measurements of this source were performed by *EXOSAT* in 1983/84 and revealed a heavily cut-off low-energy spectrum indicative of a line-of-sight column density in excess of $10^{23}$ cm$^{-2}$ (Pounds *et al.*, 1984). This is in accord with the previously established tendency for substantial columns to be found generally in low luminosity X-ray sources, including many Seyfert 1.9 galaxies (e.g. NGC 526a, NGC 5506, NGC 7582; see also §5.1). Further *EXOSAT* observations of ESO 103-G35 in 1985 revealed evidence for changes in the X-ray absorbing column, with the inferred column density decreasing from $\sim 170 \times 10^{21}$ cm$^{-2}$ to $\sim 100 \times 10^{21}$ cm$^{-2}$ over a 90-day period (Warwick, Pounds & Turner, 1988).

Here, we report on a recent *Ginga* observation of ESO 103-G35 which has provided further information on the configuration of the absorbing medium in the nucleus of this galaxy.

6.1.2 The *Ginga* results

The *Ginga* observation of ESO103-G35 was performed on September 24-25, 1988 (UT 1:30). We have applied standard analysis techniques (see §2.2.1), using data from all eight counters of the LAC (top layer only).

The background-subtracted 4–10 keV count rate recorded in the observation is shown in Fig. 6.1a while Fig. 6.1b shows the simultaneous SUD count rate. The count rate from the source exhibited a sharp increase $\sim 10$ hours into the observation and then showed a gradual decline to its original value. The spectral characteristics of this event are shown in Figs 6.1c – 6.1f, where we have plotted the LAC count rate in the energy bands 2–4, 4–7, 7–10 and 10–18 keV. The 2–4 keV flux undergoes the most dramatic increase by a factor $\sim 5$ in about an hour, whilst the 4–7 keV flux increases by a factor $\sim 2$ on a similar timescale. In contrast, both the 7–10 and 10–18 keV fluxes exhibit relatively little variability.
Figure 6.1 LAC count rates during the *Ginga* observation of ESO 103-G35. (a) 4–10 keV LAC count rate from the top layer only. (b) The corresponding SUD count rate. (c)–(f) The LAC counts rates (top layer only) in the energy bands 2–4, 4–7, 7–10 and 10–18 keV respectively.
The soft X-ray flare described above is present in all eight detectors with a similar relative magnitude in each. The event is not due to inadequate background subtraction since it is present in the original uncorrected light curve. It is also not due to solar contamination since the LAC was shielded from direct sunlight throughout both the source and background observations. Although we cannot completely rule out the possibility that the flare was caused by a background phenomenon, we have no evidence for this hypothesis. Similarly it is possible that the soft flare was from a confusing source in the field of view of the LAC, although we know of no likely candidate source. However, we note the case of the active galaxy III Zw2 in which a similar soft X-ray flare was observed by EXOSAT and the event was shown to be due to a confusing source in the field of view (see Tagliaferri et al., 1988). In the present work we will consider the implications of the soft X-ray flare in ESO 103-G35 being intrinsic to the source but do not reject the possibility of the event being due to a confusing source.

6.1.3 Spectral fitting

We have extracted three spectra from time intervals before, during and after the flare (hereafter referred to as A, B and C respectively) (see Fig. 6.1a). We have also derived a spectrum of the flare by subtracting the mean of A and C from B. A preliminary analysis of the Ginga spectra of ESO 103-G35 revealed substantial low-energy absorption consistent with the earlier EXOSAT results but also gave evidence for a soft excess below ~ 4 keV similar to that seen in NGC 4151 (see §3). We first consider the 4-18 keV spectrum, thus avoiding the complication of the soft excess component.

6.1.3.1 THE SPECTRUM ABOVE 4 KEV

For each spectrum (A - C) we have performed a standard model fitting analysis using a spectral model with six free parameters namely, the normalization of the continuum, the photon spectral index of the power-law continuum, \( \Gamma \), the effective hydrogen column density, \( n_H \), the column density of iron, \( n_{Fe} \), the iron edge energy, \( E_{\text{edge}} \), and the intensity, \( I_{6.4} \), of an iron K\( \alpha \) emission line. The line energy was fixed at 6.4 keV (a value appropriate for ionization states up to about iron XVII; see Makishima, 1986). Table 6.1 gives the results of the spectral fits with the errors corresponding to 90% confidence limits for 5 interesting parameters. The values for \( F_{2-10} \) in Table 6.1 refer to the 2-10 keV absorption corrected flux in units \( 10^{-11} \) erg cm\(^{-2} \) s\(^{-1} \). The pulse height spectra, best-fitting models extrapolated down to 2 keV and residuals for A - C are shown in Figs. 6.2a - 6.2c respectively. Fig. 6.2d shows the corresponding incident photon spectra. It can be seen from Fig. 6.2 that the form of the soft excess is strikingly similar to that in NGC 4151 (cf. Fig. 3.7)
The photon index, $\Gamma$, was poorly constrained in the earlier EXOSAT observations (Warwick, Pounds & Turner, 1988) but Table 6.1 shows a consistent value for all three spectra, close to the canonical value of 1.7. The effective column density, $n_H$, of $\sim 140 \times 10^{21}$ cm$^{-2}$, appears to decline by a factor of 2 during the flare. The derived values of $n_{Fe}$ imply an apparent iron over-abundance of $3.1 \pm 1.5$. The weighted mean value of the three measurements of the iron edge-energy is $7.54 \pm 0.24$ keV, implying a mean ionization state for the iron of Fe XV (Makishima, 1986). The average equivalent width of the iron $K_\alpha$ line was $\sim 80$ eV with marginal evidence for a change in the line flux and equivalent width during the flare.

### 6.1.3.2 THE SPECTRUM OF THE FLARE

We have also performed a spectral fitting analysis on the spectrum of the flare itself, that is spectrum B minus the mean of spectra A and C. We find that either a thermal bremsstrahlung or power-law model with cold, uniform low-energy absorption and an iron $K_\alpha$ emission line at 6.4 keV give acceptable fits. The power law fit is marginally better, however. In both cases the iron line is statistically required by the $F$-test. The results for the power-law fit are given in Table 6.2 (here the errors correspond to 90% confidence limits for 1 interesting parameter). Fig. 6.3 shows the inferred incident photon spectrum along with the data.

It can be seen from Table 6.2 and Fig. 6.3 that the form of the difference spectrum is that of a typical AGN. The implication is that if the flare is intrinsic to the source, more of the unabsorbed continuum of ESO 103-G35 is revealed. If not, then the serendipitous source is likely to be an AGN which underwent a substantial increase in X-ray luminosity.

#### Table 6.1 4–18 keV spectral fits for ESO 103-G35

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma$</th>
<th>$n_H$</th>
<th>$n_{Fe}$</th>
<th>$E_{edge}$</th>
<th>$I_{6.4}$</th>
<th>$\chi^2/\nu$</th>
<th>$F_{00}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.69$^{+0.36}_{-0.24}$</td>
<td>148$^{+88}_{-51}$</td>
<td>405$^{+246}_{-198}$</td>
<td>7.38$^{+0.39}_{-0.27}$</td>
<td>0.5$^{+0.3}_{-0.2}$</td>
<td>0.78</td>
<td>6.1</td>
</tr>
<tr>
<td>B</td>
<td>1.69$^{+0.36}_{-0.24}$</td>
<td>63$^{+43}_{-33}$</td>
<td>245$^{+165}_{-197}$</td>
<td>7.71$^{+0.97}_{-0.81}$</td>
<td>1.2$^{+0.9}_{-0.9}$</td>
<td>0.70</td>
<td>6.5</td>
</tr>
<tr>
<td>C</td>
<td>1.62$^{+0.26}_{-0.24}$</td>
<td>134$^{+54}_{-67}$</td>
<td>373$^{+156}_{-207}$</td>
<td>7.55$^{+0.29}_{-0.45}$</td>
<td>0.4$^{+0.7}_{-0.4}$</td>
<td>0.64</td>
<td>4.8</td>
</tr>
</tbody>
</table>

a $10^{21}$ cm$^{-2}$.
b $10^{16.52}$cm$^{-2}$.
c Iron edge energy in keV.
d $10^{-4}$ photons cm$^{-2}$ s$^{-1}$.
e 19 degrees of freedom.
f $10^{-11}$ erg cm$^{-2}$ s$^{-1}$. 
Figure 6.2 The incident photon spectrum corresponding to the best-fitting power law plus iron emission line model of the spectrum of the soft X-ray flare in ESO 103-G35 (see Table 6.2).

Table 6.2 Spectral fit to the spectrum of the flare in ESO 103-G35

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$n_H^a$</th>
<th>$I_{6.4}^b$</th>
<th>$\chi^2_c$</th>
<th>$F_{\nu}^d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.04$^{+0.16}_{-0.11}$</td>
<td>1.9$^{+4.2}_{-1.9}$</td>
<td>0.6$^{+0.4}_{-0.3}$</td>
<td>0.91</td>
<td>2.0</td>
</tr>
</tbody>
</table>

$^a$ $10^{21}$ cm$^{-2}$.
$^b$ $10^{-4}$ photons cm$^{-2}$ s$^{-1}$.
$^c$ 16 degrees of freedom.
$^d$ $10^{-11}$ erg cm$^{-2}$ s$^{-1}$.
Figure 6.3 Spectral fits above 4 keV for ESO 103-G35 with the best-fitting models extrapolated down to 2 keV. (a) – (c) Pulse height spectra A – C (see text and Fig. 6.1) and the corresponding best-fitting models and residuals. (d) The incident photon spectra corresponding to A (dotted curve), B (solid curve) and C (dashed curve).
6.1.3.3 WARM ABSORBER

The measured iron K-edge energy of 7.54 ± 0.24 keV from the Ginga spectra of ESO 103-G35 strongly suggests that photoionization of X-ray absorbing material in the nucleus may account for the soft excess as well as the apparent column density variations. This is despite the fact that the continuum level changes very little during the flare (most of the increase in flux occurs below ~ 4 keV); it is possible for the ionization state of material in the nucleus to change without corresponding changes in the photoionizing continuum (see §5.4). We have tested the Ginga spectra (over the energy range 2-18 keV) with such a warm absorber model, using the spectral fitting code described in §5.4. The model involves six free parameters, namely the normalization of the X-ray continuum, the photon spectral index of the X-ray continuum, \( \Gamma \), the neutral column density of material fully covering the source, \( n_H \), the ionization parameter, \( U \), the abundance of iron relative to solar, \( A_{Fe} \) and the intensity of the iron \( K_\alpha \) emission line, \( I_{Fe} \). We have used the 'mean' AGN continuum described in §4.1.7 with the X-ray slope floating. The temperature of the warm absorber has been determined by assuming radiative equilibrium (see §5.4.3).

The results are shown in Table 6.3. No error bounds were computed for the best-fitting parameters as the computational overhead is large. Figs. 6.4a - 6.4c show the pulse height spectra, best-fitting models and residuals for A - C respectively. Fig 6.4d shows the corresponding incident photon spectra. It can be seen from the values of the reduced chi-square (\( \chi^2 \)) and the poor residuals in Figs. 6.4a - 6.4c that the warm absorber provides a very poor fit to the data in every case. The large values of \( U \) (which are probably physically unrealistic) in Table 6.3 are required to fit the magnitude of the soft excess, but even then it appears that the warm absorber model cannot reproduce the shape of the spectrum below 4 keV. For instance, in fitting the warm absorber model to the 2-18 keV spectrum B, the iron abundance has been forced to zero (see Table 6.3).

<table>
<thead>
<tr>
<th>( \Gamma )</th>
<th>( n_H ) (^a)</th>
<th>( U )</th>
<th>( A_{Fe} ) (^b)</th>
<th>( I_{Fe} ) (^c)</th>
<th>( \chi^2 ) (^d)</th>
<th>( F_x ) (^e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.65</td>
<td>211</td>
<td>9.2</td>
<td>1.2</td>
<td>1.3</td>
<td>3.04</td>
</tr>
<tr>
<td>B</td>
<td>1.83</td>
<td>183</td>
<td>39.2</td>
<td>0.0</td>
<td>1.8</td>
<td>1.64</td>
</tr>
<tr>
<td>C</td>
<td>1.74</td>
<td>318</td>
<td>28.2</td>
<td>0.5</td>
<td>1.0</td>
<td>3.02</td>
</tr>
</tbody>
</table>

\(^a\) \( 10^{21} \text{ cm}^{-2} \).
\(^b\) Iron abundance relative to solar.
\(^c\) Iron line intensity in units \( 10^{-4} \text{ photons cm}^{-2} \text{s}^{-1} \).
\(^d\) 23 degrees of freedom.
\(^e\) \( 10^{-11} \text{ erg cm}^{-2} \text{s}^{-1} \).
Figure 6.4 Spectral fits for ESO 103-G35 with a warm absorber model. (a)–(c) Pulse height spectra A–C (see text and Fig. 6.1) and the corresponding best-fitting models and residuals. (d) The incident photon spectra corresponding to A (dotted curve), B (solid curve) and C (dashed curve).
Hence, we conclude that neither the soft excess nor the spectral variability in ESO 103-G35 can be explained by the changing ionization state of absorbing material which fully covers the X-ray source. However, the iron K-edge energy indicates that the absorbing material may still be in a fairly high state of ionization but need not fully cover the source. The measured energy range of the iron K-edge of 7.54 ± 0.24 keV implies a range in \( U \) of \( \sim 1 - 40 \) (see §5.4.1.3 and Fig. 5.11) although this is dependent on how closely the 'mean' AGN spectrum of §4.1.7 resembles the broadband spectrum of ESO 103-G35.

### 6.1.3.4 PARTIAL COVERING

In this section we consider whether the partial covering model (see §5.2) can account for the soft excess and X-ray spectral variability in ESO 103-G35. The model involves six free parameters, the normalization and spectral index of the power-law continuum, an absorbing column, \( n_{H_1} \), covering a fraction \( C_F \) of the source, the abundance of iron relative to solar, \( A_{Fe} \), and the intensity, \( I_{6.4} \), of an iron emission line at 6.4 keV. We use opacities for cold material, in order to keep the number of free parameters down to a minimum, but fix the iron K-edge energy at 7.54 keV. An absorbing screen of \( 5 \times 10^{21} \) cm\(^{-2}\) fully covering the remainder of the source is also included in the fit since ESO103-G35 was not detected by the low-energy telescope on EXOSAT (cf. NGC 4151, §5.2.1).

Table 6.4 gives the best fit parameters derived from this analysis, from which it can be seen that excellent fits are obtained for all three spectra. The errors correspond to 90% confidence limits for 3 interesting parameters (see §2.3). Figs. 6.5a – 6.5c show the pulse height spectra, best-fitting models and residuals for spectra A – C respectively. Fig. 6.5d shows the corresponding incident photon spectra.

<table>
<thead>
<tr>
<th></th>
<th>( \Gamma )</th>
<th>( n_{H_1} ) ( ^a )</th>
<th>( C_F )</th>
<th>( A_{Fe} ) ( ^b )</th>
<th>( I_{6.4} ) ( ^c )</th>
<th>( \chi^2 ) ( ^d )</th>
<th>( F_\gamma ) ( ^e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.93( ^{+0.18}_{-0.21} )</td>
<td>167( ^{+52}_{-52} )</td>
<td>0.90( ^{+0.02}_{-0.03} )</td>
<td>4.18( ^{+3.52}_{-1.95} )</td>
<td>0.1( ^{+0.7}_{-0.1} )</td>
<td>0.81</td>
<td>8.2</td>
</tr>
<tr>
<td>B</td>
<td>2.05( ^{+0.14}_{-0.16} )</td>
<td>216( ^{+40}_{-44} )</td>
<td>0.70( ^{+0.06}_{-0.10} )</td>
<td>3.12( ^{+2.17}_{-1.58} )</td>
<td>0.6( ^{+0.8}_{-0.6} )</td>
<td>0.95</td>
<td>10.1</td>
</tr>
<tr>
<td>C</td>
<td>1.90( ^{+0.16}_{-0.17} )</td>
<td>187( ^{+45}_{-41} )</td>
<td>0.88( ^{+0.02}_{-0.03} )</td>
<td>3.62( ^{+1.90}_{-1.37} )</td>
<td>0.1( ^{+0.5}_{-0.1} )</td>
<td>1.06</td>
<td>6.8</td>
</tr>
</tbody>
</table>

\( ^a \) \( 10^{21} \) cm\(^{-2}\).
\( ^b \) Iron abundance relative to solar.
\( ^c \) \( 10^{-4} \) photons cm\(^{-2}\) s\(^{-1}\).
\( ^d \) 23 degrees of freedom.
\( ^e \) \( 10^{-11} \) erg cm\(^{-2}\) s\(^{-1}\).
Figure 6.5 Spectral fits for ESO 103-G35 with a partial covering model. (a) – (c) Pulse height spectra A – C (see text and Fig. 6.1) and the corresponding best-fitting models and residuals. (d) The incident photon spectra corresponding to A (dotted curve), B (solid curve) and C (dashed curve).
The values of $\Gamma$ are slightly steeper than those obtained from spectral fitting above 4 keV (Table 6.1), the weighted mean value of $\Gamma$ being $1.98 \pm 0.10$. The column density, $n_{H_1}$, is consistent with a constant value of $194 \pm 26 \times 10^{21}$ cm$^{-2}$ for all three spectra. The softening of spectrum B relative to A and C is explained in this model as a decrease in $C_F$ from $\sim 0.9$ to $\sim 0.7$ while the column density, $n_{H_1}$, remains relatively constant. The derived values of $A_{Fe}$ indicate an apparent over-abundance of iron of $3.7 \pm 1.2$, consistent with the value derived previously in §6.1.3.1.

The iron line intensity, $J_{6.4}$, is poorly constrained. The values in Table 6.4 imply a mean equivalent width of $\sim 35$ eV, a lower limit of zero and an upper limit of $\sim 150$ eV.

6.1.3.5 POISSONIAN ABSORBER

In this section we investigate whether the Poissonian absorber model (see §5.3) can also account for the soft excess and spectral variability in ESO 103-G35. As with the partial covering model, six free parameters are involved, namely the normalization and spectral index of the continuum, the mean number of clouds in the line-of-sight, $\mu$, the column density of each cloud, $n_c$, the abundance of iron relative to solar, $A_{Fe}$ and the intensity of an iron emission line at 6.4 keV, $J_{6.4}$. Again, as in the partial covering model we use opacities for cold material, fix the iron K−edge energy at 7.54 keV and include an additional uniform screen fully covering the source with a column density of $5 \times 10^{21}$ cm$^{-2}$. The results are shown in Table 6.5, where the errors correspond to 90% limits for 3 interesting parameters (see §2.3).

It can be seen from Table 6.5 that statistically acceptable fits are obtained for all three spectra, although for spectra A and C they are worse than the corresponding partial covering spectral fits. It can also be seen that the Poissonian absorber model requires substantial changes in the cloud column density (from $\sim 90$ to $\sim 160$) as well as changes in $\mu$ (from $\sim 1.4$ to $\sim 2.7$) to account for the spectral variability. These points, coupled with the rather high column density implied for individual clouds, prompts us to accept the simpler partial covering model as an adequate description of the shape and variability of the X-ray spectrum of ESO 103-G35. Arguments based on the dynamics of the absorbing system also favour this interpretation since it is not clear how significant changes in the total number of clouds and/or the cloud sizes implied by the required changes in $\mu$ and $n_c$ (see §5.3) can occur in $\sim 2$ hours (the rise time of the soft X-ray flare). This of course assumes that the flare is indeed intrinsic to the source. Moreover, the implied parameters of the cloud system appear to return to their original values after the flare.
Table 6.5 Spectral fits for ESO 103-G35 with a Poissonian absorber model

<table>
<thead>
<tr>
<th></th>
<th>$\Gamma$</th>
<th>$\mu$</th>
<th>$n_H$</th>
<th>$A_F$</th>
<th>$I_{6.4}$</th>
<th>$\chi^2_\nu$</th>
<th>$F_\nu^{*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.05$^{+0.26}_{-0.21}$</td>
<td>2.78$^{+0.39}_{-0.34}$</td>
<td>87$^{+29}_{-24}$</td>
<td>3.23$^{+2.31}_{-1.35}$</td>
<td>0.0$^{+0.6}_{-0.0}$</td>
<td>1.04</td>
<td>10.6</td>
</tr>
<tr>
<td>B</td>
<td>2.15$^{+0.19}_{-0.20}$</td>
<td>1.44$^{+0.34}_{-0.36}$</td>
<td>161$^{+27}_{-27}$</td>
<td>2.57$^{+1.82}_{-1.28}$</td>
<td>0.5$^{+0.7}_{-0.5}$</td>
<td>0.89</td>
<td>11.7</td>
</tr>
<tr>
<td>C</td>
<td>2.07$^{+0.18}_{-0.21}$</td>
<td>2.60$^{+0.26}_{-0.34}$</td>
<td>109$^{+21}_{-23}$</td>
<td>2.72$^{+1.23}_{-0.93}$</td>
<td>0.0$^{+0.4}_{-0.0}$</td>
<td>1.37</td>
<td>8.7</td>
</tr>
</tbody>
</table>

$^a$ 10$^{21}$ cm$^{-2}$.
$^b$ Iron abundance relative to solar.
$^c$ 10$^{-4}$ photons cm$^{-2}$ s$^{-1}$.
$^d$ 23 degrees of freedom.
$^e$ 10$^{-11}$ erg cm$^{-2}$ s$^{-1}$.
6.1.4 The X-ray absorbing system in ESO 103-G35

From the above modelling we have obtained a clearer picture of the nature of the X-ray absorbing system in ESO 103-G35 than was possible from the previous EXOSAT observations (Warwick, Pounds & Turner, 1988). It appears that the line-of-sight to the X-ray source is obscured by a thick absorbing screen which is in a fairly high state of ionization \((U > 1)\), nominally covering \(\sim 90\%\) of the source. It is not possible to distinguish whether the absorbing screen consists of a dense population of discrete clouds or a uniform column of gas. The soft X-ray flare in the present Ginga observation indicates that the fraction of the source covered by the absorbing screen can change substantially (down to 70\% in this case) on a timescale of a few hours. Variability in the low-energy absorption on such short timescales has been observed in the X-ray spectrum of only two other AGN, namely MR 2251 (Pan, Stewart & Pounds, 1990) and MCG-6-30-15 (Nandra et al., 1990a) (see also §5.1). In both these objects the variable absorption could be explained by changes in the level of photoionization of the absorber by the continuum. We have shown that this is not the case for ESO 103-G35.

One possible physical interpretation of the rapidly variable low-energy cut-off in terms of the partial covering model is that the absorbing screen is not uniform everywhere but has large ‘patches’ or ‘holes’ in places. If the screen then has a substantial transverse velocity relative to the source then the temporary change in covered fraction can be explained by a ‘hole’ passing over the source, crossing the line-of-sight. Taking a velocity of 15000 km s\(^{-1}\) (a value typical of traditional inner BLR clouds), the duration of the soft X-ray flare of \(\sim 15\) hours then implies a source size of \(\sim 10^{14}\) cm. This is physically plausible. An alternative possibility is that the flare was not caused by changes in the absorbing system but by a violent, localized event on the uncovered part of the source. This would be indistinguishable from a genuine change in the covered fraction.

It is difficult to place constraints on the three-dimensional geometry of the X-ray absorbing system from the present data. However, we can make some general inferences. The first is that the column density cannot be as large as that in the line-of-sight (\(\sim 200 \times 10^{21} \text{ cm}^{-2}\)) in all directions as there would be insufficient ionizing flux for broad line-emitting material in the outer BLR. The second inference depends on whether the equivalent width of the iron \(K_a\) is in fact as small as the mean value of \(\sim 35\) eV implied by the results in Table 6.4. If this is the case then, given that the iron abundance (relative to solar) is enhanced by almost a factor 4, the configuration of the absorbing system must be more disc-like than spherical. We then observe the system close to edge-on.

Finally, we note that the long term variable absorption in ESO 103-G35 reported by Warwick, Pounds & Turner (1988) on the basis of the EXOSAT observations may in fact have been due to changes in the covered fraction only, while the column density of the absorbing screen remained
steady. The sensitivity of EXOSAT was insufficient to make the distinction. Further observations are required to clarify this point.

6.1.5 Conclusions

The present Ginga observation of ESO103-G35 has revealed new features in the X-ray spectrum namely an excess of flux, in the 2-4 keV band, above that expected from absorption in cold, uniform, solar abundance material, the marginal detection of an iron emission line of mean equivalent width $\sim 35$ eV and an apparent iron over-abundance of $\sim 3.7$ with the energy of the iron K-edge at $\sim 7.5$ keV. The energy of the iron K-edge implies a state of ionization ($U > 1$) of the X-ray absorbing material which is higher than is typical of traditional BLR clouds. The presence of substantial line-of-sight absorption in ESO 103-G35, coupled with a soft excess is reminiscent of the situation pertaining in NGC 4151 (see §3).

We find that neither the soft excess, iron over-abundance nor the soft X-ray flare in the present Ginga observation can be explained by the warm absorber model (§5.4). The spectral variability and soft excess is best described in terms of the partial covering model. Both the pre- and post-flare spectra are consistent with $\sim 90\%$ of the source covered by a column $\sim 200 \times 10^{21}$ cm$^{-2}$. The soft X-ray flare (if it is intrinsic to the source) can then be explained by the covered fraction decreasing to $\sim 70\%$ whilst both the column density and the level of the underlying X-ray continuum remain relatively constant. In addition, there is no evidence for variability of the spectral index of the X-ray continuum. We obtain a mean value for $\Gamma$ of $1.98 \pm 0.10$ (see Table 6.4), somewhat steeper than canonical.

We have made tentative suggestions of a physical interpretation of the above partial covering model and short timescale (hours) variability of the covered fraction that it implies, but caution that this variability may be due to a serendipitous source.
6.2 EXOSAT observations of the QSO 1821+643

6.2.1 Introduction

The QSO 1821+643 is one of the most X-ray luminous radio-quiet quasars. It has an optical magnitude of 14.1 and also happens to be one of the most luminous objects in the far IR (the flux at 60 μm in QSO 1821+643 exceeds that of 3C 273).

Most of the available information concerning the X-ray spectra of QSOs has come from Einstein IPC observations in the energy range ~ 0.1—4 keV with very limited spectral resolution (e.g. ΔE/E ~ 1 at 1 keV) (Elvis, Wilkes & Tananbaum, 1985; Elvis et al., 1986; Wilkes & Elvis, 1987). X-ray spectra extending into the medium energy (2-10 keV) regime from collimated proportional counter experiments are also available, but only for a limited number of X-ray bright QSOs. QSO 1821+643 is one such source and was first identified as the optical counterpart to a serendipitous Einstein X-ray source, but subsequently associated by Pravdo & Marshall (1984) with the unidentified soft X-ray source, H1814+63, listed in the HEAO-1/A2 low energy catalogue (Nugent et al., 1983), and also with an unidentified hard X-ray source, H1824+644, listed in a compilation of sources detected by the HEAO-1/A2 high energy experiment (Marshall et al., 1979). This object is of particular interest both because of its high X-ray luminosity (~ 10^{46} erg s^{-1}) and its unusually soft X-ray spectrum. From the combined HEAO-1/A2 low energy detector (LED) and high energy detector (HED) data, Pravdo & Marshall (1984) find the X-ray spectrum of 1821+642 to be a power law with a photon spectral index of 2.31 ± 0.30, considerably steeper than the canonical value of 1.7 found in lower luminosity samples comprised mainly of Seyfert galaxies (see §4.1.5).

Here, we report on a series of EXOSAT observations which reveal an X-ray spectral slope for QSO 1821+643 which is more typical of lower luminosity active galaxies. We also discuss possible explanations for the apparently large spectral changes which have occurred in this object.

6.2.2 The EXOSAT observations and results

EXOSAT observed QSO 1821+643 on a total of 7 occasions from July 7, 1984 to October 23, 1985, details of which are given in Table 6.6. The prime target for the first two observations was in fact Kohoutek 1-16 (K1-16), a planetary nebula excited by a 15th magnitude central star with an effective temperature, $T_{\text{eff}}$, in excess of 10^5 K, which lies only 1.5 arcmin from the QSO position. We will have more to say about this possible confusing source below (§6.2.4.2). The present analysis is based on data from the medium energy (ME) detector array (§2.1.2) and the low energy (LE) telescope on EXOSAT (§2.1.1).
Background subtraction was performed using method IV described in §2.1.2.2 for all except the first observation. For this latter observation a 'nod' (see §2.1.2.2) of the ME array halves was not performed so method VI in §2.1.2.2 was used in order to subtract the background. Recall that both methods IV and VI utilize only the corner detectors of the ME detector array. In the present analysis we have used spectra only from the Argon chambers of the corner ME detectors so as to minimize systematic uncertainties. The effective energy range of the ME spectra was 1.5–10 keV.

Table 6.6 lists the average count rates (per 4 ME detectors) measured in the 2–6 keV band during each observation. Relatively low-amplitude (i.e. ~ 20% peak-to-peak) variability is evident over the set of observations. We have also searched for shorter-term X-ray variability within the individual observations but find no evidence for changes greater than ~ 20% on timescales from 1000–60000 s.

Broadband spectral measurements were also made with the EXOSAT LE telescope with the CMA in the focal plane in combination with an X-ray filter (see §2.1.1). Three filters, namely the thin Lexan (3Lx), the aluminium/parylene (Al/P) and Boron (B) filters, which provide a progressively harder response in the 0.03–2 keV band, were generally used consecutively in each observation, although only the 3Lx and Al/P filters were employed in the first and second observations. Table 6.6 gives the count rates (per 100 s) measured within a 75 x 75 arcsec$^2$ region centered on the position of the QSO and also (for the 3Lx observations) at the position of K1-16, after applying background, point-response, scattering and dead-time corrections (see §2.1.1.1). There is again evidence for variability although we note that the ME and 3Lx light curves (Fig. 6.6) are not closely correlated suggesting the occurrence of significant spectral changes from observation to observation. There was insufficient signal-to-noise ratio in the LE observations to investigate possible short-timescale variability in the soft X-ray band.
Table 6.6  Observing log and count-rates for QSO 1821+643

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>ME ct/s</th>
<th>Al/P</th>
<th>3Lx</th>
<th>BOR</th>
<th>3Lx †</th>
</tr>
</thead>
<tbody>
<tr>
<td>189/1984</td>
<td>1.52 ± 0.07</td>
<td>2.71 ± 0.46</td>
<td>5.13 ± 0.97</td>
<td>–</td>
<td>2.02 ± 0.68</td>
</tr>
<tr>
<td>136/1985</td>
<td>1.25 ± 0.03</td>
<td>2.88 ± 0.44</td>
<td>4.71 ± 0.11</td>
<td>–</td>
<td>2.05 ± 0.08</td>
</tr>
<tr>
<td>160/1985</td>
<td>1.43 ± 0.04</td>
<td>2.58 ± 0.28</td>
<td>4.86 ± 0.27</td>
<td>0.89 ± 0.21</td>
<td>2.13 ± 0.20</td>
</tr>
<tr>
<td>187/1985</td>
<td>1.42 ± 0.05</td>
<td>2.73 ± 0.33</td>
<td>4.45 ± 0.38</td>
<td>0.86 ± 0.22</td>
<td>1.77 ± 0.27</td>
</tr>
<tr>
<td>223/1985</td>
<td>1.35 ± 0.04</td>
<td>3.00 ± 0.32</td>
<td>4.40 ± 0.28</td>
<td>0.72 ± 0.20</td>
<td>2.40 ± 0.22</td>
</tr>
<tr>
<td>280/1985</td>
<td>1.37 ± 0.04</td>
<td>2.60 ± 0.36</td>
<td>3.82 ± 0.30</td>
<td>0.76 ± 0.21</td>
<td>1.72 ± 0.22</td>
</tr>
<tr>
<td>286/1985</td>
<td>1.36 ± 0.04</td>
<td>2.16 ± 0.35</td>
<td>3.48 ± 0.25</td>
<td>0.75 ± 0.22</td>
<td>1.76 ± 0.19</td>
</tr>
</tbody>
</table>

† Count rates with the 3Lx filter for K1-16.
Figure 6.6 The results of EXOSAT monitoring of QSO 1821+643. (a) The 2–6 keV ME light curve; (b) the LE (3Lx) light curve; (c) the 3Lx/ME softness ratio.
6.2.3 Spectral fitting

The X-ray spectrum of 1821+643 was investigated using a standard spectral fitting analysis. This was performed on the combined ME and LE (QSO) data for each observation (the ME data alone gave consistent but less well constrained results). The results of fitting a simple power-law model are given in Table 6.7. This model requires three free parameters, namely the normalization of the power-law continuum (in Table 6.7 we quote the observed X-ray flux integrated between 2–10 keV, \( F_x \)), the photon spectral index, \( \Gamma \), and the gas column density, \( n_H \). We have assumed that any low-energy absorption present in the X-ray spectrum of the QSO is due to cold gas local to our galaxy with solar abundances and absorption cross-sections as defined by Morrison & McCammon (1983).

It is evident from the \( \chi^2 \) values (Table 6.7) that this power-law model provides an excellent fit to all the measured spectra. Figs. 6.7a and 6.7c show the pulse height spectra, best-fitting models and residuals for the observations on days 136/1985 and 296/1985 respectively. Figs. 6.7b and 6.7d show the inferred incident photon spectra corresponding to the observations on days 136/1985 and 296/1985 respectively.

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>( \Gamma )</th>
<th>( n_H ) (^a)</th>
<th>( \chi^2 ) (^b)</th>
<th>( F_x ) (^c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>189/1984</td>
<td>1.84(^{+0.37}_{-0.16})</td>
<td>3.0(^{+3.6}_{-1.7})</td>
<td>0.90</td>
<td>2.1</td>
</tr>
<tr>
<td>136/1985</td>
<td>1.74(^{+0.13}_{-0.04})</td>
<td>1.8(^{+0.9}_{-0.6})</td>
<td>0.96</td>
<td>1.8</td>
</tr>
<tr>
<td>160/1985</td>
<td>1.82(^{+0.16}_{-0.06})</td>
<td>2.7(^{+1.5}_{-1.0})</td>
<td>0.90</td>
<td>2.0</td>
</tr>
<tr>
<td>187/1985</td>
<td>1.74(^{+0.18}_{-0.08})</td>
<td>2.4(^{+1.9}_{-1.1})</td>
<td>1.05</td>
<td>2.1</td>
</tr>
<tr>
<td>223/1985</td>
<td>1.80(^{+0.15}_{-0.07})</td>
<td>2.7(^{+1.4}_{-1.1})</td>
<td>1.02</td>
<td>1.9</td>
</tr>
<tr>
<td>280/1985</td>
<td>1.84(^{+0.16}_{-0.09})</td>
<td>3.7(^{+2.1}_{-1.6})</td>
<td>0.87</td>
<td>1.9</td>
</tr>
<tr>
<td>296/1985</td>
<td>1.87(^{+0.19}_{-0.09})</td>
<td>4.6(^{+3.1}_{-1.9})</td>
<td>0.80</td>
<td>1.9</td>
</tr>
</tbody>
</table>

\(^a\) In units of \(10^{20}\) cm\(^{-2}\).

\(^b\) 29 degrees of freedom.

\(^c\) In units of \(10^{-11}\) erg cm\(^{-2}\) s\(^{-1}\).
Figure 6.7 Spectral fits for the EXOSAT observations of QSO 1821+643 on days 136/1985 and 296/1985. (a) Pulse height spectrum, best-fitting model and residuals for the observation on day 136/1985. (b) The incident photon spectrum corresponding to (a). (c) As (a) but for the observation on day 296/1985. The incident photon spectrum corresponding to (c).
The spectral index measurements are all consistent with $\Gamma = 1.8$, whereas there is the suggestion of an increase in $n_H$ over the six observations performed in 1985 corresponding to the observed decrease in the $3L_x/ME$ softness ratio over the same period (Fig. 6.6). 21 cm measurements give the line-of-sight column density through our own galaxy as $4 \times 10^{20}$ cm$^{-2}$ (Stark et al., 1988) with a typical error of $1 \times 10^{20}$ cm$^{-2}$ (Elvis et al., 1986), which is significantly higher than the minimum observed $n_H$ value. For day 136/1985 if $n_H$ is fixed at the galactic value then the reduced $\chi^2$ for the best fitting power-law model increases to 1.58 (with 30 degrees of freedom). We interpret this as indicative of the presence of a variable soft component in the X-ray spectrum of 1821+643 in addition to the power-law continuum which dominates the medium energy band. Unfortunately the lack of spectral resolution below 1 keV means that very little can be said directly about the spectral form of the excess soft flux.

Fig. 6.8 illustrates the incident spectrum derived from the EXOSAT observations on day 136/1985 in comparison with the spectrum obtained by Pravdo & Marshall (1984) from the HEAO-1 observations. The spectral slope in the 2–10 keV band in both sets of observations appears to be similar and consistent with $\Gamma \sim 1.8$, although the 2-10 keV X-ray fluxes differ by a factor $\sim 2.0$ (the source was fainter in the EXOSAT observation with an incident 2-10 keV flux of $1.8 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$ which corresponds to an intrinsic 2-10 keV X-ray luminosity of $2.1 \times 10^{45}$ erg s$^{-1}$ for a source at a redshift of 0.297 and for $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$, $q_0 = 0$). The soft flux detected by the LED can then be described in terms of a separate soft component of spectral slope $\Gamma \sim 2.5$ (whether this soft flux originates in the QSO is discussed below). Given this description of the HEAO-1 spectrum in terms of two separate power-laws it is reasonable to consider whether the soft excess noted in the EXOSAT observations may be due to the continued presence of the same soft component in the QSO spectrum, albeit at a much lower level. We have quantified this possibility by performing a spectral fitting analysis on the day 136/1985 data using a model comprised of two power-law components (i.e. a hard component with $\Gamma$ fixed at 1.8 and a soft component with $\Gamma$ fixed at 2.5) and with $n_H$ fixed at the galactic value. This model produced a reasonable fit to the EXOSAT spectrum (i.e. a reduced $\chi^2 = 1.23$ for 29 degrees of freedom), in which the absorption corrected 0.2–2.0 keV flux in the soft component was $6.2 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$, a factor $\sim 10$ below that obtained for the soft component from the HEAO-1 observations (the corresponding 0.2–2 keV intrinsic luminosity in the EXOSAT observation was $8.9 \times 10^{44}$ erg s$^{-1}$).

To extend this investigation the spectral fitting analysis was repeated with the slope of the hard power-law as a free parameter. In this case a reduced $\chi^2$ of 1.00 was obtained with the 0.2–2 keV flux in the soft component a factor 1.7 higher than noted above and with $\Gamma$ for the hard component equal to $1.43 \pm 0.27$. This serves to illustrate the difficulty in distinguishing between spectral models involving more than one power-law component with the limited spectral resolution of the presently available X-ray data (cf. Wilkes & Elvis, 1987).
Finally we have performed a spectral fitting analysis to derive constraints on a possible redshifted iron-line in the QSO spectrum (corresponding to a line at 6.4 keV in the QSO rest frame). Again for day 136/1985, using a single power-law model for the continuum, we obtained a line-intensity of $0.5^{+0.6}_{-0.5} \times 10^{-4}$ photons cm$^{-2}$ s$^{-1}$ and a corresponding equivalent width of $170^{+204}_{-170}$ eV.

![Figure 6.8](image)

**Figure 6.8** A comparison of the derived incident spectra for QSO 1821+643 from the *HEAO-1/A2* experiment (filled triangles) and from the *EXOSAT* LE and ME measurements on day 136/1985 (filled circles). The *EXOSAT* points below 1 keV correspond to the LE (3Lx and Al/P) filter measurements plotted at representative energies.
6.2.4 Discussion

The results of the EXOSAT monitoring demonstrate that QSO 1821+643 has a 2–10 keV spectrum consistent with that generally observed in much lower luminosity active galaxies (see §4.1.5). In §4 we discussed at length the possible implications of the similarity of the X-ray spectra of AGN which span several decades in luminosity. QSO 1821+643 then joins the long list of objects in the ‘canonical’ category. Before EXOSAT there was a distinct lack of X-ray spectral measurements for very high luminosity quasars. More recently, Ginga has provided observations of some more of these objects. The most recent survey (Turner et al., 1989b), in which QSO 1821+643 was included (see §6.2.4.3) involved 8 objects all with $L_x > 10^{45}$ erg s$^{-1}$. The mean X-ray photon index of the sample was 1.69 ± 0.29. Two of the objects were much flatter than canonical, however, and the remainder somewhat steeper than canonical.

Our prime interest in the EXOSAT observations of 1821+643 is the implication that, given the earlier results of Pravdo & Marshall (1983), large spectral changes may have occurred in this source. It is evident from the comparison in Fig. 6.8 that the change of spectral slope between the HEAO-1 and EXOSAT observations is the result of very different X-ray flux measurements below 2 keV. Hence we can rule out the pair models of §4.2 as a cause of the variability (although of course pairs may still play a role in the emission of the hard X-ray power law with a fixed spectral index).

We can also rule out the possibility that the spectral variability is caused by a varying mixture of a reflected continuum (e.g. from an accretion disc) and the direct continuum, as described in §4.4. This conclusion is based on the fact that the hard power law slope of the HEAO-1 spectrum (dominated by the reflected component) would have to be much flatter than that of all the EXOSAT spectra (dominated by the direct continuum).

Below we consider two possible explanations of the observed soft X-ray variability, namely changes inherent to QSO 1821+643 or effects due to confusion by soft X-ray sources in the vicinity of the QSO.

6.2.4.1 SOFT X-RAY VARIABILITY IN QSO 1821+643

There is now considerable evidence for the presence of at least two components in the X-ray continuum spectra of Seyfert galaxies and QSOs (see §4.1.5). For example, recent EXOSAT observations have revealed a soft X-ray component, in addition to the canonical medium-energy power-law, in a number of Seyfert I galaxies (e.g. Mrk 841, Arnaud et al., 1985; Mrk 335, Pounds et al., 1986b). Variability measurements have established these as separate spectral components originating within the innermost region of the active nucleus and prompting the suggestion that the soft X-ray emission arises at the inner edge of a putative accretion disc.
As noted in the previous section the present results for QSO 1821+643 can be interpreted in terms of a soft spectral component which exhibited an order of magnitude decrease between 1977 and 1985 compared to a factor of 2 decline in the medium energy continuum over the same period. There is also evidence within the EXOSAT observations of uncorrelated variability in both the soft and hard soft spectral components on a timescale of weeks to months. Large amplitude variations in the soft components in AGN X-ray spectra appear to be a fairly common phenomenon. For example, Pounds et al. (1986b) reported a factor ~ 6 increase in strength of both the soft and medium energy components in Mkn 335 between November 1983 and December 1984. An interesting parallel to the present study is provided by the Seyfert I galaxy E1615+061 for which HEAO-1 LED measurements have also indicated a very steep power-law index, i.e. $\Gamma \geq 3$ (Pravdo et al., 1981). In comparing the HEAO-1 result with EXOSAT observations, Piro et al. (1988) found that the soft X-ray flux from this galaxy had decreased by two orders of magnitude and that the medium energy spectrum had reverted to the canonical form. This again suggests that the X-ray spectrum can be modelled as two components with the softer continuum exhibiting the more pronounced variability.

6.2.4.2 CONFUSED SOURCES IN THE VICINITY OF QSO 1821+643

An alternative to invoking large changes in the soft X-ray flux from QSO 1821+643 is to question whether the soft X-ray flux measured by the LED did in fact arise from the QSO rather than a confusing source in the 1.5° x 3.0° (FWHM) field-of-view of the LED.

One known soft X-ray source close to the QSO position is Kohoutek 1-16 (K1-16), a high excitation planetary nebula containing a central star with $m_V = 15.09$ and $T_{\text{eff}} > 10^5$K, which shows strong similarities to the group of hot helium-rich pulsating white dwarfs characterised by PG1159-035 (Grauer & Bond, 1984). The results of the EXOSAT observations of K1-16 have been discussed in detail elsewhere (Barstow, 1988; Holberg & Barstow, 1988) and here we simply note a few points relevant to the confusion question. As might be expected from its high temperature K1-16 is a soft X-ray source yielding a count rate in the 3Lx filter approximately half that for the QSO. No pulsational activity was found and also there is little evidence for significant long term variability over the seven EXOSAT observations (Table 6.6). A comparison of the observed X-ray count rates (in the LE filters) with those predicted by He-rich model atmospheres indicates that $T_{\text{eff}}$ is in the range $1.25 - 1.8 \times 10^5$K. The EXOSAT data also constrain the abundance of C, N and O with respect to He to be within a factor of 0.02-20 times the solar value. In attempting to estimate the possible contribution of K 1-16 to the observed LED spectrum we have considered two models near the extremes of the CNO abundance range, in each case taking the maximum allowed $T_{\text{eff}}$ value. At 1 keV the flux density predicted by either model is several orders of magnitude below that observed by the LED. Similarly the estimated flux from K1-16 in the 0.2-2 keV band is always less than $\sim 3 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$. We conclude that K 1-16 would have to have been at least
an order of magnitude brighter to account for the ‘excess’ flux in the HEAO-1 spectrum of QSO 1821+643. There is no evidence for flux changes of this magnitude in planetary nebulae nuclei.

An alternative possibility is that there are one or more additional confusing soft X-ray sources in the vicinity of the QSO. A rough estimate of the probability that the soft X-ray source detected by the LED experiment is not associated with the QSO can be obtained by noting that the LED error box for H1814+63 has an area of 1.5 square degrees and that there are 9 1-keV sources listed in the HEAO A-2 soft X-ray source catalogue in the region $|b| > 30^\circ$ (excluding the cluster of sources associated with the LMC) (Nugent et al., 1983). The probability of the QSO falling by chance within an LED error box is then $\sim 7 \times 10^{-4}$. This is course an *a posteriori* argument and may be misleading.

### 6.2.4.3 RECENT GINGA OBSERVATIONS OF QSO 1821+643

Since the EXOSAT observations reported above, QSO 1821+643 has been observed twice by Ginga, once on 12 – 14 May, 1987 and again on 27 September, 1988 (see Turner et al., 1989b and Kii et al., 1990). The 2–10 keV flux level of the QSO in both observations was very similar to that in all the EXOSAT observations (i.e. $\sim 2.2 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$). Moreover, the photon spectral index in both the Ginga observations is consistent with a value $\sim 1.9$ which is also consistent with all the EXOSAT observations. Ginga has no sensitivity below 2 keV and so was unable to quantify the soft excess measured by HEAO-1 and EXOSAT.

Measurement of the equivalent width of the iron line in the QSO spectrum from the Ginga observations was complicated by the 6.7 keV iron emission line from intra-cluster gas in which the QSO is situated. Kii et al. (1990) place loose constraints of 50–200 eV for the equivalent width of the 6.4 keV emission line from QSO 1821+643. In addition, no significant iron K–edge was detected from either of the Ginga observations, implying that most of the line emission from the QSO must arise in material out of the line-of-sight. Incidentally, we note that the contribution of flux to the EXOSAT spectra of QSO 1821+643 from the associated cluster is not expected to be significant. This conclusion is based on the correlation between cluster luminosity and temperature (e.g. see Edge, 1989) using an estimated temperature of $\sim 3 – 5$ keV.

### 6.2.5 Conclusions

The most plausible interpretation of the available data on the X-ray spectrum of QSO 1821+643 is that two continuum components are present; a soft component which declined by a factor of $\sim 10$ over the period 1977 to 1985 and a somewhat steeper than canonical medium energy component which showed only a factor $\sim 2$ decrease over the same period. Several authors have argued that these soft X-ray components originate in the inner regions of an accretion disc and correspond to
the high energy tail of the thermal UV excess identified in many AGN spectra (Malkan & Sargent, 1982; Arnaud et al., 1985; Pounds et al., 1986b; Czerny & Elvis 1987; §4.1.3 – §4.1.5). Further work is required on QSO 1821+643 to establish whether a 'big bump' feature is present in the optical/UV spectrum of this object and whether the observed large amplitude changes in the soft X-ray flux are compatible with models of the accretion disc in this high luminosity source.

The EXOSAT observations were not able to constrain the iron $K_{\alpha}$ emission line equivalent width but more recent Ginga observations indicate a value in the range 50 – 200 eV. The lack of a detectable iron K-edge indicates that if there is any absorbing material in the line-of-sight the covering factor must be small.
Chapter 7

Prospects for AGN in the next decade

Overview

It will have become evident that most of the objectives of the study of X-ray spectral variability outlined in chapter 1 have not been realised from the analysis of data for the three AGN presented in this thesis. The purpose of this chapter is to (i) summarize what has been learnt of the three objects (§7.1), (ii) summarize the general conclusions from the theoretical investigations presented in this thesis and indicate some of the directions which future models of AGN should take and (iii) outline the ways in which the proposed X-ray astronomy missions in the next decade will help to answer many of the questions concerning AGN posed in chapter 1 (§7.3).

“All in all, it was just bricks in the Wall ... ”

(Roger Waters, 1979. From ‘The Wall’.)
7.1 Summary of results

Below we present a summary of the results for the individual objects studied in this thesis.

7.1.1 NGC 4151

TEMPORAL BEHAVIOUR

The medium energy X-ray continuum in NGC 4151 > 2 keV is highly variable, characterized by slow drifting behaviour with a typical flux-doubling timescale of $\sim 0.5 - 1.0 \times 10^5$ s, with frequent large amplitude flares that typically last for a few days. During the $\sim 5.5$ year monitoring campaign with EXOSAT and Ginga, the 2-10 keV flux varied by a factor of 10 (see Tables 3.3 and 3.5). The absence of very rapid X-ray variability suggests that either the size of the X-ray source is large ($\sim 10^{14}$ cm) or that rapid variability is 'smeared' due to Thomson scattering. If the X-rays are in fact generated in a compact region, the scattering opacity may be identified with thermalized pairs (see below). The source size may then be of the order of $\sim 1 - 5 \times 10^{14}$ cm.

THE BROADBAND CONTINUUM OF NGC 4151

We have presented arguments in §4.2.2.3 which lead to the following conclusions: (i) the dominant source of X-rays in NGC 4151 cannot be due to the synchrotron self-Compton process, (ii) the infrared/optical emission originates in a different region to that in which the X-rays are produced, (iii) if the non-thermal emission is produced by Compton scattering of soft photons on relativistic electrons then the infrared/optical and X-ray emission processes cannot share the same relativistic electron population (two acceleration mechanisms may be required) and (iv) the injection of relativistic electrons into the X-ray emission region must be very flat or monoenergetic. An implication of (i) and (ii) is that the gradient of the magnetic field density from the X-ray to the infrared/optical emission region is large.

The broadband continuum and spectral variability for NGC 4151 has been quantified in §4.1. In particular, the power-law photon spectral index of the X-ray continuum, $\Gamma$, varies between $\sim 1.2 - 1.7$ and appears to be correlated with the 2-10 keV flux (see Fig. 3.8, §3.3.3). This result is independent of the particular model of X-ray absorption adopted to describe the continuum below 4 keV (§5). These results should be used to calculate new emission-line models for this object since previous models have simply assumed a fixed over-simplified continuum. The X-ray spectral index variability in particular may have important observational consequences since the X-ray spectrum is responsible for the heating of the warm, neutral zone behind the ionization front in a photoionized cloud (Weisheit, Shields & Tarter, 1981). Such modelling supplemented with
more frequent observations of the emission-line spectrum and continuum will help to elucidate the geometry of material in the BLR and indicate to what extent the continuum is actually responsible for determining the thermal and ionization state of the emitting clouds. Similar work is required for other AGN as the quality and quantity of data for individual objects improves.

SPECTRAL INDEX VARIABILITY

We have discussed two types of model which can qualitatively reproduce the observed flux-index correlation in NGC 4151. In the first of these the spectral index variability is intrinsic to the emission mechanism which involves Compton scattering of UV photons on relativistic electrons which are monoenergetically injected into the source region. The spectral index and flux variability is then due to fluctuations in the model input parameters, as described in §4.2. The second type of model assumes that there is some mechanism which produces an X-ray continuum with a fixed spectral index which is incident on optically thick material (e.g. an accretion disc) giving rise to a reflected component in the observed spectrum which is much flatter than the direct continuum. Under some circumstances light travel-time effects can introduce significant variability of the relative proportion of the direct and reflected components in the observed continuum which can then give rise to apparent spectral index variability (§4.4). We find that both types of model alone can, for a restricted range in respective model parameters, account for the flux-index correlation of the type reported for NGC 4151. In the first type of model the parameter range appropriate to NGC 4151 is such that $e^+e^-$ pairs, created by photon-photon absorption, play an important role in the variability.

Given that the hypothesised accretion disc in NGC 4151 is likely to be viewed close to edge-on (Clavel et al., 1987) and that spectral variability of the continuum incident on the disc would tend to mask any flux-index correlation due to the presence of the disc alone, we conclude that the X-ray spectral index variability in NGC 4151 is likely to be intrinsic to the emission mechanism (see §4.4.1). However, more sensitive spectral measurements above 10 keV and at γ-ray energies are required to make further progress in understanding the spectral index variability.

VARIABLE X-RAY ABSORPTION

The low-energy cut-off in the X-ray spectrum of NGC 4151 appears to be variable on a timescale of months to years, independent of the continuum level. However, the variable soft excess below 4 keV is well correlated with the continuum, evidence that it does not have a separate origin to the main X-ray source (in contrast to the soft excess detected by the EXOSAT LE which remained steady over a period ~ 32 months). We find that the ‘warm absorber’ model (§5.4) cannot account for the soft excess, variable absorption or apparent over-abundance of iron. We estimate that the probable (variable) ionization state of the absorbing material corresponds to an ionization parameter in the range $U \sim 0.1 - 1.0$, somewhat higher than the canonical value of 0.01 for traditional BLR clouds.
The complex and variable X-ray absorption can be adequately described by a partial covering model (§5.2) or a Poissonian absorber model (§5.3). We prefer the latter as it provides a natural physical interpretation. In this model the Ginga data require μ to vary between ~ 0.9 — 3.3 and n_c between ~ 20 — 70 x 10^{21} cm^{-2}. However, even the Poissonian absorber is too simplistic since all clouds are assumed to have the same physical parameters and a spherically symmetric geometry is assumed. The origin of the X-ray absorbing material in NGC 4151, its relationship to ‘traditional’ BLR material and the cause of the variability is still uncertain. However, it appears that the implied geometry of the distribution of absorbing material in the BLR is one in which the column density is low in most directions (up to a few ~ 10^{22} cm^{-2}) but high (~ 10^{23} cm^{-2}) in a direction which happens to lie along our line-of-sight (see §5.5.3).

**IRON ABUNDANCE**

We find evidence of some iron absorption in NGC 4151 which is not associated with the material responsible for the line-of-sight X-ray absorption. This could be due to the presence of a continuum component in the observed spectrum arising from reflection in optically thick matter out of the line-of-sight (e.g. an accretion disc). This residual iron column is difficult to quantify with the sensitivity of the present data (see §3.3.3, §5.2 and §5.3).

There is also evidence that the iron abundance of the material in the line-of-sight is significantly higher than the solar value (§5.2 & §5.3). However, the over-abundance is again difficult to quantify due to the uncertainty in the residual iron absorption mentioned above, but is likely to be ~ 2 (relative to solar).

**IRON EMISSION LINE**

The iron Ka emission line in NGC 4151 has so far not been a very useful diagnostic in NGC 4151. This is mainly due to the limited sensitivity and spectral resolution of present instrumentation combined with the possibility of significant time delays between continuum variations and fluorescence and ‘smearing’ of the line flux.

The intensity of the line is consistent with a constant value of 2.2 ± 0.2 x 10^{-4} photons cm^{-2} s^{-1} despite large continuum variations (§3.3.3). The mean equivalent width is ~ 100 eV. A preliminary measurement of the line broadening corresponds to a FWHM of ~ 38000 km s^{-1} but confirmation of this must await improved instrumentation (e.g. BBXRT; see §7.3).
7.1.2 ESO 103-G35

A single *Ginga* observation of the Seyfert 1.9 galaxy has revealed much new information about its X-ray spectrum. Previous X-ray observations with *EXOSAT* were unable to constrain the power-law slope of the continuum but showed a significantly variable low-energy cut-off. From the present *Ginga* observation we find

- a photon spectral index of $1.98 \pm 0.10$ with no evidence of significant variability,
- complex absorption in the form of an excess of flux below 4 keV above that expected from cold, uniform solar abundance material, as in NGC 4151 and
- an over-abundance of iron of $\sim 3.7$ relative to solar.

A soft X-ray flare (below $\sim 4$ keV) lasting for $\sim 10$ hours was also present in the *Ginga* observation. A partial covering model adequately describes the observed spectral variability and soft excess, with the covering fraction, of an absorbing column of $\sim 200 \times 10^{21}$ cm$^{-2}$, changing from $\sim 90\%$ to $\sim 70\%$ during the flare. However, we caution that the flare may have been due to a serendipitous source in the field of view of the LAC. On the other hand, if the flare is intrinsic to the source a physical interpretation of it will have to await further, more frequent measurements of the X-ray spectrum, with improved instrumentation.

The measured energy of the iron K-edge of $7.54 \pm 0.24$ keV indicates a fairly high ionization state for the X-ray absorbing material. The implied ionization parameter depends on the unknown shape the continuum at UV and $\gamma$-ray energies. However, the above range in the edge energy implies a mean ionization state for iron of VII – XVII (Makishima, 1986). Also, there is marginal evidence for an iron $K_\alpha$ emission line. Again, measurements of both the edge and line parameters must be confirmed with improved instrumentation in order to provide an interpretation for the physical state and geometry of the absorbing material in ESO 103-G35.

7.1.3 QSO 1821+643

The results of seven *EXOSAT* observations in 1984/5 of the high luminosity QSO 1821+643 show that the soft flux below 2 keV declined by a factor $\sim 10$ since the *HEAO-1* observation in 1977. There is no evidence for spectral variability in the *EXOSAT* spectra above 2 keV (the 2–10 keV photon index was consistent with a constant value of $\sim 1.8$). However, there is evidence for a gradual decline (by a factor of $\sim 1.4$) of the soft flux over a period of $\sim 1$ year, uncorrelated with the medium energy flux. A likely contender for the soft component of the X-ray spectrum is thermal emission from an accretion disc but more observations with greater sensitivity and
resolution are required to quantify such an hypothesis. The two Giag observations following the EXOSAT campaign have confirmed the EXOSAT measurements of the slope of the hard X-ray power law but are unable to constrain the intensity of the iron \( K_a \) emission line and the soft excess.

7.2 General conclusions and future theoretical work

7.2.1 Spectral index variability

Present models of the X-ray emission in AGN concentrate on reproducing the 'canonical' photon spectral index of \( \sim 1.7 \) for a wide range in compactness parameter. These models have largely ignored the necessity to account for spectral index variability within individual objects and the fact that a significant number of AGN actually have a photon spectral index \( < 1.5 \). It has been shown (§4.2) that models in which the X-ray emission arises from Compton scattering of UV photons on monoenergetically injected relativistic electrons can yield significant spectral index variability (under certain physical conditions) if (i) the parameters of the relativistic electrons and/or UV source are variable and/or (ii) there is an extended (relative to the size of the source), optically thick reflecting medium present (e.g. an accretion disc). If either (i) or (ii) or both are relevant to AGN in general, then spectral index variability of the order \( \Delta \Gamma \sim 0.05 - 0.2 \) is expected to be quite common, but large changes in \( \Gamma \) which are correlated with continuum flux are expected to be less common as the required model parameter range is restricted. If both (i) and (ii) above are important then a flux-index correlation is unlikely (see §4.4.1).

THE CANONICAL AGN

We have also shown that non-thermal 'pair models' of the X-ray emission in AGN, of the type investigated in this thesis, can give rise to an a 'preferred' or 'canonical' value of the X-ray spectral index for a wide range in compactness, while still allowing significant deviation from the preferred value, as is required by the data. The condition for this is an upper limit on the Lorentz factor, in a given set of models, of the injected electrons. Contemporary models of this type do not usually have such a restriction. However, the problem requires further investigation. In particular, more attention must be given to possible acceleration mechanisms which might produce the relativistic electrons.

Future models must include the possibility of the scattering of both synchrotron and UV photons within a more realistic geometry. Perhaps the putative accretion disc could be an integral part of this next generation of models, with the inclination of the disc as an additional parameter.
7.2.2 The Broad Line Region

The relative simplicity of present models of complex and variable X-ray absorption in AGN (§5) is adequate for the quality of data currently available. However, as the sensitivity and spectral resolution of the data improves, more sophisticated models will need to be developed. In particular, future models need to include more information about the geometry, dynamics and the thermal and ionization state of the absorbing material and should also be applied in conjunction with models of the emission-line spectrum. We have emphasised that heating and cooling from sources other than the continuum may be important in determining the physical state of BLR material. Such considerations may help to answer the question of whether an HIM exists in a given object which may in turn throw some light on the cloud-confinement problem.

METAL ABUNDANCES

The abundance of iron in both of the Seyfert galaxies studied in the present work has been found to be significantly greater than solar. Previous studies of NGC 4151 hinted that this was the case but it was not possible to eliminate the possibility of an apparent over-abundance due to, say, photoionization. Similar reports of a genuine over-abundance of iron in other Seyfert galaxies observed by Ginga are appearing (e.g. MCG-6-30-15, Nandra, 1990a; NGC 5548, Nandra, 1990b; NGC 7172, Sembay, private communication; NGC 7582, Warwick, private communication). If such an over-abundance of iron (at least 2 relative to solar in most cases) is common, it could have important consequences for models of the BLR and future evolutionary scenarios of AGN. The immediate task is to routinely incorporate the result into photoionization models of the emission-line spectrum for these objects.

Improved instrumentation can soon be expected to yield similar information on the relative abundance of some of the other elements.

7.3 Observational prospects for AGN

The coming decade will see the launch of many major X-ray astronomy missions, climaxing with AXAF and XMM towards the end of the century. Table 7.1 gives a summary of the more important missions, their expected launch dates, energy range and energy resolution of some of the experiments. See Mushotzky (1989) for a more detailed discussion of the capabilities and potential of each mission. An important feature of the X-ray astronomy missions in Table 7.1 is that all except XTE have imaging capabilities (up to $\sim 10$ keV). Fig. 7.1 shows the effective area as a function of energy for some of these imaging experiments.
Table 7.1 Future X-ray astronomy experiments

<table>
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<th>Project</th>
<th>Launch</th>
<th>Detectors a</th>
<th>Energy range (keV) b</th>
<th>Resolution E/ΔE b</th>
</tr>
</thead>
<tbody>
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<td>1990</td>
<td>BBXRT</td>
<td>0.3 - 10</td>
<td>5 - 50</td>
</tr>
<tr>
<td>ROSAT</td>
<td>1990</td>
<td>PSPC</td>
<td>0.1 - 2 keV</td>
<td>~ 2 - 4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HRI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ASTRO-D</td>
<td>1993</td>
<td>CCD</td>
<td>0.1 - 10</td>
<td>10 - 70</td>
</tr>
<tr>
<td>SAX</td>
<td>1993</td>
<td>IGSPC</td>
<td>0.1 - 200</td>
<td>2 - 15</td>
</tr>
<tr>
<td>SPECTRUM-X</td>
<td>1994</td>
<td>CCD</td>
<td>0.3 - 10</td>
<td>10 - 70</td>
</tr>
<tr>
<td>(JET-X)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>XTE</td>
<td>1994</td>
<td>PCA</td>
<td>2 - 60</td>
<td>&lt; 10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HEXTE</td>
<td>15 - 200</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>AXAF †</td>
<td>1996</td>
<td>CCD</td>
<td>0.1 - 10</td>
<td>10 - 70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>HRC</td>
<td>low</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>CAL</td>
<td>&gt; 500</td>
<td></td>
</tr>
<tr>
<td>XMM</td>
<td>1998</td>
<td>CCD</td>
<td>0.1 - 10</td>
<td>10 - 70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>RGS</td>
<td>0.2 - 2.5</td>
<td>~ 500</td>
</tr>
</tbody>
</table>

† An energy resolution of ~2000 is possible with the use of gratings.

a Not all the detectors aboard the relevant satellite are necessarily listed. The abbreviations correspond to: PSPC = position sensitive proportional counter; HRI = high resolution imager; IGSPC = imaging gas scintillation proportional counter; CCD = X-ray charge coupled device; SSS = solid state spectrometer; PCA = proportional counter array; HEXTE = high energy X-ray timing experiment; HRC = high resolution camera; CAL = X-ray calorimeter; RGS = reflection grating spectrometer.

b Approximate values only.
Figure 7.1 Effective area of various X-ray telescopes, taken from Hasinger & Trümper (1989).
The superior energy resolution of some of the future instrumentation will enable measurements of some of the features in the X-ray spectrum of AGN which have hitherto not been possible. The most important of these will be the ionization edge energies of elements other than iron, the iron line profile and the spectra of soft excesses. Studies of the iron line profile and variability may eventually tell us something about where different parts of the line are formed, the relative contribution (if any) from an accretion disc (see §1.3.2), the geometry of the fluorescing material and the dynamics of that material. It may be the iron line which finally resolves the question of whether BLR material is inflowing or outflowing (§1.3.7). The improved sensitivity of future instrumentation, apart from detecting a huge number of new AGN and possibly a new type of object, will enable much smaller variations in the hard X-ray spectral index to be detected. An important constraint on models of the hard X-ray emission and spectral index variability will come from sensitive measurements above 10 keV which are capable of detecting a hard tail or other deviations from a true power-law.

Improved instrumentation in other wavebands, to become operational in the near future, will be just as important as the above X-ray astronomy missions. Notably, the program of γ-ray missions, the UV/EUV projects LYMAN and EUVE and the Hubble Space Telescope (HST) will be important in this respect. The γ-ray observations will be crucial in constraining models of the hard X-ray emission, as pointed out in §4.2. Rapid progress in measuring the γ-ray spectrum of AGN can be expected very soon since the SIGMA experiment (30 keV – 2 MeV) is already operational and the Gamma Ray Observatory (GRO) (sensitive up to 30 GeV) is expected to be launched in 1990. Broadband, simultaneous, intensive observing campaigns of selected AGN, similar to those for NGC 4151 will be essential. Polarimetry with future instrumentation will also play an important role in constraining models of AGN.

The immediate future for NGC 4151 (next ~ 2 – 3 years) is likely to be particularly prosperous since, not surprisingly, it is one of the select AGN to be targeted by BBXRT and is likely to be one of the first AGN to be frequently observed with the new γ-ray detectors. BBXRT will be especially useful for providing higher resolution measurements of the iron line in NGC 4151 than has been possible before. Also in the immediate future are observations with ROSAT. These will provide measurements of the spectrum of the extended soft emission component (§1.3.3) for the first time and maybe elucidate its nature and origin.

In summary, provided that theoretical models can keep up with the expected enormous wealth of data for AGN in X-rays and other wavebands, many of the questions that are being posed at present will have been answered by the turn of the century. We may also expect to understand the connection between the X-ray spectrum of AGN and the diffuse X-ray background and thereby resolve the present enigma (see Mushotzky, 1989). Of course, despite this revolution in our understanding of AGN, many new questions will have arisen.
Appendix A

Table A.1 The energy spectral indices, in various wavebands, of the broadband spectra for the 'mean' AGN (§4.1.7) and NGC 4151 (§4.1.8), compiled in §4. In both cases the energy index of the X-ray power law is 0.7. The footnotes below Table A.1 indicate which energy indices change if the X-ray spectral index is changed. Specifically, the effect of changing the 'mean' AGN X-ray index to 0.5 and the X-ray index for NGC 4151 to 0.45 is shown.

<table>
<thead>
<tr>
<th>Interval ( ^\dagger )</th>
<th>'mean' AGN</th>
<th>NGC 4151</th>
</tr>
</thead>
<tbody>
<tr>
<td>(&lt; 2 \times 10^{10} \text{ Hz})</td>
<td>1.000</td>
<td>0.880</td>
</tr>
<tr>
<td>(2 \times 10^{10} \rightarrow 10^{11} \text{ Hz})</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>(10^{11} \rightarrow 4 \times 10^{11} \text{ Hz})</td>
<td>-8.966</td>
<td>2.736</td>
</tr>
<tr>
<td>(4 \times 10^{11} \rightarrow 2 \times 10^{12} \text{ Hz})</td>
<td>(-3.320) (^a)</td>
<td>0.000</td>
</tr>
<tr>
<td>(2 \times 10^{12} \rightarrow 4 \times 10^{14} \text{ Hz})</td>
<td>1.360</td>
<td>1.370</td>
</tr>
<tr>
<td>(4 \times 10^{14} \text{ Hz} \rightarrow 4.2 \text{ eV})</td>
<td>ditto</td>
<td>1.080</td>
</tr>
<tr>
<td>(4.2 \text{ eV} \rightarrow 24.6 \text{ eV})</td>
<td>0.500</td>
<td>0.500</td>
</tr>
<tr>
<td>(24.6 \text{ eV} \rightarrow 54.4 \text{ eV})</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(54.4 \text{ eV} \rightarrow 0.3 \text{ keV})</td>
<td>3.206</td>
<td>1.448</td>
</tr>
<tr>
<td>(&gt; 0.3 \text{ keV} \dagger)</td>
<td>0.700</td>
<td>0.700</td>
</tr>
<tr>
<td>(\dagger)</td>
<td>(0.500) (^c)</td>
<td>(0.450) (^b)</td>
</tr>
</tbody>
</table>

\(^\dagger\) Frequency/energy range. Note the use of mixed units.
\(^\dagger\) Recall that the X-ray power laws have an exponential high energy cut-off.
\(^a\) This value refers to the radio-loud version of the 'mean' AGN continuum (see §4.1.7).
\(^b\) These values refer to energy indices appropriate to an X-ray power law with an energy index of 0.45.
\(^c\) These values refer to energy indices appropriate to an X-ray power law with an energy index of 0.5.
Table A.2 The luminosities, in various wavebands, of the continuum of NGC 4151 from the compilation in §4.1.8, for values of the X-ray energy spectral index, $\alpha$, equal to 0.45 and 0.7. These correspond to typical 'low' and 'high' flux states of NGC 4151. An exponential cut-off energy of 1 MeV has been assumed for the X-ray/\gamma-ray spectrum. The luminosities are given in units $10^{42}$ erg s$^{-1}$ and a distance to the source of 20 Mpc has been assumed.

<table>
<thead>
<tr>
<th>Interval $^\dagger$</th>
<th>$\alpha = 0.45$</th>
<th>$\alpha = 0.7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12} \rightarrow 4 \times 10^{14}$ Hz (IR)</td>
<td>101.7</td>
<td>101.7</td>
</tr>
<tr>
<td>$4 \times 10^{14}$ Hz $\rightarrow 7.3 \times 10^{16}$ Hz (UV)</td>
<td>13.5</td>
<td>41.0</td>
</tr>
<tr>
<td>0.3 keV $\rightarrow 100$ MeV (X-ray/\gamma-ray)</td>
<td>98.5</td>
<td>164.3</td>
</tr>
<tr>
<td>$10^8$ Hz $\rightarrow 100$ MeV (bolometric)</td>
<td>225.6</td>
<td>318.8</td>
</tr>
<tr>
<td>IR/ X-ray ratio</td>
<td>1.03</td>
<td>0.62</td>
</tr>
<tr>
<td>UV/ X-ray ratio</td>
<td>0.14</td>
<td>0.25</td>
</tr>
</tbody>
</table>

$^\dagger$ Frequency/energy range. Note the use of mixed units.
Appendix B

Table B.1 The default solar abundances of the elements (relative to Hydrogen) used in the photoionization spectral fitting code, ZAPPER, described in §5.4. Also shown is the range of K- and L-shell energies from the neutral atom to the Hydrogen-like ion of each element.

<table>
<thead>
<tr>
<th>Element</th>
<th>Solar (^a) Abundance</th>
<th>K-edge (^b) energy range</th>
<th>L-edge (^b) energy range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Carbon</td>
<td>(4.7 \times 10^{-4})</td>
<td>0.28-0.67</td>
<td>-</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>(9.8 \times 10^{-5})</td>
<td>0.39-0.67</td>
<td>-</td>
</tr>
<tr>
<td>Oxygen</td>
<td>(8.3 \times 10^{-4})</td>
<td>0.53-0.87</td>
<td>-</td>
</tr>
<tr>
<td>Neon</td>
<td>(10^{-4})</td>
<td>0.88-1.36</td>
<td>-</td>
</tr>
<tr>
<td>Magnesium</td>
<td>(4.2 \times 10^{-5})</td>
<td>1.32-1.96</td>
<td>0.069-0.074</td>
</tr>
<tr>
<td>Aluminium</td>
<td>(1.7 \times 10^{-6})</td>
<td>1.95 (^\dagger)</td>
<td>0.080-0.103</td>
</tr>
<tr>
<td>Silicon</td>
<td>(4.3 \times 10^{-5})</td>
<td>1.87-2.67</td>
<td>0.116-0.152</td>
</tr>
<tr>
<td>Sulphur</td>
<td>(1.7 \times 10^{-5})</td>
<td>2.52-3.50</td>
<td>0.182-0.261</td>
</tr>
<tr>
<td>Argon</td>
<td>(3.8 \times 10^{-6})</td>
<td>3.28-4.43</td>
<td>0.260-0.410</td>
</tr>
<tr>
<td>Calcium</td>
<td>(2.3 \times 10^{-6})</td>
<td>4.20 (^\dagger)</td>
<td>0.356-0.561</td>
</tr>
<tr>
<td>Iron</td>
<td>(3.3 \times 10^{-5})</td>
<td>7.11-9.28</td>
<td>0.731-1.250</td>
</tr>
</tbody>
</table>

\(^a\) Relative to Hydrogen.

\(^b\) Energies in keV.

\(^\dagger\) A fixed, 'mean' K-edge energy was used in these cases to improve the speed of calculation (note relatively low abundance).
References

The following abbreviations have been used:

*Astron. Astrophys.* Astronomy and Astrophysics


*Space Sci. Rev.* Space Science Reviews


I am indebted to myriads of people for enabling this work to be pulled out of the dark depths of night into the glaring light of day. The following list is not exhaustive but indicates the main contributors at the centre of the web. I thank my supervisor, Bob Warwick, for guidance and endless discussions and arguments, usually about why NGC 4151 has been misbehaving (yet again). Ken Pounds for giving me the opportunity to embark on a Phd with the X-ray Astronomy Group at Leicester and also for many stimulating discussions. Gordon Stewart for reading evolving versions of this thesis and helping to knock it into the right shape. To this end I also thank Derek Raine, Rees Williams and Steve Sembay. Much credit is due to everyone who has contributed to the data analysis system at Leicester, especially Dick Willingale who has been helpful when the system has been at war. I also thank Gary Ferland for the use of his photoionization code, ‘CLOUDY’, which has been the source of many hours of amusement (and even more of distress) and Ian George for the use of his Monte Carlo calculations for the work in §4.4. I owe a huge amount to Chris Done for her helpful advice and criticism, without which much of the work in chapter 4 would not have been possible.

The culmination of this work is partly a result of many valuable friendships, not just during my time at Leicester, which have influenced my thinking and provided encouragement, support and insight. Some of the individuals due for such a gruesome credit are as follows. Lorraine Breedon for lots of stimulation, inspiration, gratuitous arguments and café conversations but most of all for being completely loopy. Jas Flora for ‘sussing out’ Leicester with me in the early days and then enduring it. Jane Gilbert for not falling asleep while watching ‘Live at Pompeii’ (most of the time), Rachel Drake for ‘raring to go’ (most of the time), T. Jane Turner and Ian George for depriving the top corridor of all sanity, Paul Nandra for teaching me the rules of Golf and Richard Saxton for ‘clubbing’ and Rukhsana for some good times. There are many others I’d like to mention but space prohibits. Many a session has been had with different permutations of the above and sometimes the nightmares of chapters 4 and 5 would fade away. Sarah Tucker for being one of the few ‘real’ people during my time at Oxford and with Rob Kyppta and Anton Wasilewski, partaking in sessions of quite a different variety. Then there’s Richard Keymes ... shine on, you crazy diamond!

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