X-RAY AND ULTRA-VIOLET OBSERVATIONS OF BL LACERTAE TYPE OBJECTS

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Thesis submitted to the University of Leicester
for the degree of Doctor of Philosophy

1988

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Abstract

We present the results from EXOSAT and IUE observations of five BL Lac type objects, Mrk 421, Mrk 501, 1218+304, Mrk 180 and 0414+009. We find that at least for the relatively well studied sources Mrk 421 and Mrk 501, the behaviour in the X-ray and ultra-violet differs significantly. In the majority of cases, the X-ray spectra are well represented by a simple power-law model over the 1-6 keV band. In the case of the high state observations of Mrk 421, however, a marked downward curvature of the spectrum is required above ~3 keV, which can be modelled as an exponential decrease. Similar convex spectra are also suggested, although not statistically required, in a number of other cases. We found no evidence in any of the observations for a hardening of the X-ray spectra above a few keV as has been previously reported. In all cases we find low energy cut-offs in the X-ray spectra consistent with absorption in the line-of-sight gas column density through our own galaxy implying that the intrinsic column density of cold gas in these objects is small (< 1 x 10^{20} cm^{-2}). All five sources exhibit significant X-ray variability with minimum variability timescales in the range ~3 hr to ~7 days. Furthermore, the X-ray flux and spectral index for each object appears to be correlated in the sense that the X-ray spectrum hardens as the source brightens. In contrast, the ultra-violet spectra are generally consistent with a simple power-law of index ~1, and exhibit only slow drifts in flux on a timescale ~weeks. The ultra-violet to X-ray continuum can therefore be modelled as a power-law of index ~1 below about 0.1 keV, above which the source steepens. The spectral variability in the X-ray band can then be described in terms of a 'pivoting' of the high-energy continuum about the break-point.

We have investigated two specific SSC models for the continuum emission. It was found that whilst a simplistic homogeneous disk-shaped emission region can provide an acceptable explanation of single epoch snap-shots of the multi-waveband spectrum, it experiences some difficulty accounting for the high energy spectral variability. In contrast, by allowing a variable radial dependence in the upper cut-off in the synchrotron emitting electron population in an inhomogeneous relativistic jet model, we have demonstrated that the multi-waveband spectrum of the most demanding source, Mrk 421, at all epochs can be reproduced.

Thus we conclude that a jet model, broadly following the current paradigm for BL Lac type objects can give an acceptable explanation of present multi-waveband measurements.
Declaration

I hereby declare that no part of this thesis has been previously submitted to this or any other university as part of the requirement for a higher degree. The work described herein was conducted by the undersigned except for the contributions from colleagues and other workers who are acknowledged in the text.

Ian M George

September 1988
Publications

Some of the work reported in this thesis has either been published or submitted for publication elsewhere, as follows:

*The X-ray spectrum and variability of Markarian 421.*

*X-ray and ultra-violet observations of Markarian 421.*

*Uncertainties in the UV continua of weak sources: a demonstration of systematics in the analysis of IUE images.*
George, I.M., 1988, in *A Decade of UV Astronomy with the IUE satellite*, ESA SP-281, 383.

*X-ray and ultra-violet observations of Markarian 421.*

*UV and X-ray observations of BL Lac type objects.*

*The ultra-violet to X-ray continua of BL Lac objects.*
To Mum, Dad, Elaine

and all my family

and also to Kerry
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Chapter 1

Introduction

Overview

In first part of this chapter we present a brief review of the AGN phenomenon and the discuss the context of BL Lac type objects therein. We then summarize the general properties of BL Lac type objects and review previous studies. Finally we outline the motivation behind the present work.

1.1 The AGN Phenomenon

Approximately 10% of the galaxies in the observable universe are prolific sources of non-thermal radiation that cannot be explained in terms of normal stellar processes. This radiation is invariably found to emerge from a very small region at the centre of the galaxy, and often exhibits variability on a variety of timescales. These properties have led to the wide-spread use of the term Active Galactic Nuclei (AGN) to collectively describe such objects.

Over the past decade, a substantial effort has been devoted to improving our understanding of the AGN phenomenon, much of which has been prompted by...
recently opened up regions of the electromagnetic spectrum.

1.1.1 Classification Schemes

As has been pointed out by Lawrence (1987) in an excellent recent review, although classification is not the aim of science, it is necessary in a field as young as the study of AGN since as yet we have no stable conceptual framework to account for the variety of phenomena observed. We are therefore unable to make precise conjectures necessary for any falsificationist orthodoxy. Rather, we are forced to collect and study objects making as few assumptions as possible, and to crudely classify them according to their properties. Unfortunately however, the list of known properties of AGN has recently become frighteningly long and the number of AGN classifications have multiplied until the subject can justifiably be referred to as a zoo. Indeed, a recent count gave over 20 different name conventions (Lawrence 1987). A large observational and theoretical effort is currently underway to explain and understand both the individual AGN classes and their relationships to each other. In many respects, analogies can be made with elemental particle physics in the 1950s: a search for the fundamental constituents, and for the secondary variables determining the observational consequences. Below the traditionally recognized major classifications are briefly described:

**Quasi-stellar objects (QSOs/Quasars)** were first discovered, as their name implies, as star-like optical counterparts to compact sources in radio surveys (Hazard, Mackay & Skimmins 1963; Schmidt 1963; Greenstein & Mathews 1963). Optical spectroscopy showed QSOs to possess excessively blue spectra with highly red-shifted broad emission lines. Despite some controversy (*e.g.* Terrell 1964; Hoyle, Burbidge & Sargent 1966; Arp 1966, 1967), it is now generally accepted that QSOs are extra-galactic. A taxonomical distinction is commonly made on the basis of the observed radio emission, with the radio loud objects (amounting to ~30% of the total) referred to as Quasars.
Blazars represent the union of three historical classes of AGN, BL Lac type objects, Optically Violently Variables (OVVs) and High Polarization Quasars (HPQs). All three are flat spectrum radio sources with highly variable, highly polarized continua. OVVs and HPQs both show emission lines, whilst the continua of BL Lac type objects are by definition featureless (see below). It is now reasonably well accepted that OVVs and HPQs are basically identical, whilst any fundamental distinction between BL Lac type objects and OVVs remains a matter of contention (e.g. see Antonucci & Ulvestad 1985 and references therein). Blazars are a relatively rare species of AGN accounting for only ~2% of currently known sources.

Radio Galaxies are traditionally defined as those objects emitting significantly more power in the radio band than 'normal' galaxies. A compact, flat spectrum 'core' is often observed coincident with the nucleus of the host galaxy, whilst many sources also contain extended, steep spectrum radio structure with a variety of morphologies and scales (e.g. Bridle & Perley 1984). In many cases jet-like structures are either directly observed or implied, and a number of objects contain superluminal components (e.g. Porcas 1987). Radio Galaxies are often divided into broad and narrow line radio galaxies (BLRGs and NLRGs respectively) and almost without exception, reside in elliptical galaxies.

Seyfert Galaxies were first discovered in 1943 by Carl Seyfert (Seyfert 1943). Spectroscopy indicates a wide range of high excitation emission lines in their nuclear spectrum covering a range of ionization. Historically Seyferts were divided into two subclasses on the basis of their optical emission lines (Khachikan & Weedman 1971). Seyfert 1 type nuclei contain strong, apparently non-thermal continua from infra-red to X-ray energies, with broad (widths \(\sim 10^4 \text{ km s}^{-1}\)) permitted and narrow (\(\sim\text{few}\times 10^2 \text{ km s}^{-1}\)) forbidden lines; whilst Seyfert 2 type nuclei contain weak continuum emission (except in the infra-red) and narrow (\(\sim\text{few}\times 10^2 \text{ km s}^{-1}\)) permitted and forbidden lines. It has now been discovered however that large number of Seyferts fall between these two extremes, thus giving rise to the Seyfert 1.5–1.9 subclasses, based upon the relative strengths of
broad and narrow lines. Some objects have even been seen to change subclass on timescales of several years (e.g. NGC 7603, Osterbrock 1978; Mrk 6 Khachikian & Weedman 1971). The majority of Seyferts have been found in the nuclei of spiral galaxies, although recently at least one Seyfert has been found in an elliptical galaxy (NGC 6212, Halpern & Filippenko 1986).

A number of additional classes of AGN and related objects are also often discussed, although of little relevance to the work reported here. Definitions and details of such objects can be found in Lawrence (1987) and elsewhere.

1.1.2 AGN Unification

It is important to note that in practice the traditional classification criteria are often found to be too rigid. For example, it has long been accepted that there is no real fundamental difference between QSOs and Seyfert 1s (see e.g. Weedman 1976). The distinction is primarily a question of the ratio of the luminosity of the nucleus to the host galaxy. The exact relationship between Seyfert 1s and Seyfert 2s also remains somewhat controversial. At least three schemes have been proposed in which i) Seyfert 2s are simply Seyfert 1s in which the line-of-sight to the inner regions is obscured (e.g. Lawrence & Elvis 1982; Antonucci & Miller 1985), ii) the two types are different evolutionary stages of the same phenomenon, and iii) different physical conditions prevail close to the central engine leading to Seyfert 2s lacking certain Seyfert 1 properties.

There is growing evidence, however, that all AGN are fundamentally similar and a theoretical consensus is forming concerning the nature of the common powerhouse. The favourite contender for a complete physical theory is that the ultimate energy source is due to the accretion of matter onto a supermassive black hole (SMBH) with a mass > 10^6M☉, where M☉ is a solar mass (e.g. see Rees 1984; Begelman 1985; Blandford 1985; Wiita 1985). The fundamental parameters are then the mass of the black hole, Mbh, and the accretion rate, $\dot{m}_{\text{acc}}$. For emission line AGN, photoionization theory is developing fast with increasing levels
of sophistication, relating the observables to the physical state of the gas in the nucleus (see e.g. Ferland & Shields 1985) although there remain a number of potentially pernicious problems (e.g. Collin-Souffrin 1987; Netzer 1987). An intermediate theory has emerged with three building blocks, i) a central continuum source - the underlying powerhouse, ii) a compact region of dense high velocity gas known as the Broad Line Region (BLR), and iii) a larger region of low density, low velocity gas known as the Narrow Line Region (NLR).

1.1.3 The Relevance of BL Lac Type Objects

The general absence of emission features and presence of substantial polarization in BL Lac type objects was historically interpreted to imply that the radiation from the innermost regions was not being reprocessed in some surrounding region. Indeed, it was often suggested that BL Lac type objects were devoid of cold gas and hence 'naked' AGN. It was therefore anticipated that the continuum emission in these sources would provide vital information on both the fundamental emission processes, and on the physical conditions at or very near to the ultimate energy source in all classes of AGN. Recently however, this interpretation has been shown to be unlikely (see below) and the relationship of BL Lac type objects to emission line AGN has become less clear-cut. Ostriker & Vietri (1985), for example, have suggested that BL Lac type objects are gravitationally lensed OVVVs, whilst Weiler & Johnson (1980) have suggested that all Quasars go through a 'BL Lac phase' with a temporarily violently active continuum source. The latter scenario is particularly intriguing in view of the parallels between the flat and steep X-ray spectra of Seyferts and BL Lac type objects with the low and high states of the galactic X-ray black hole candidate, Cyg X-1 (White, Fabian & Mushotzky 1984). Currently, the most fashionable hypothesis is that BL Lac type objects are dominated by emission from a relativistic jet (see below). The observed emission therefore reveals little or nothing about either the photoionizing continuum in emission line AGN or the central powerhouse. In the 'unified schemes' of Blandford & Rees (1978) and Orr & Browne (1982) however,
BL Lac type objects are classical radio sources in which the relativistic jet is viewed almost end-on (see also Chapter 7). The growing evidence in support of this scenario, and the possibility that jets may also be common in emission line AGN implies a more detailed understanding of BL Lac type objects may have important consequences for AGN studies in general.

1.2 BL Lac Type Objects

1.2.1 An Historical Perspective

BL Lac type objects were initially discovered about 20 years ago with the identification by Schmitt (1968) of the prototype object, the variable ‘star’ BL Lacertæ¹, as the optical counterpart to the variable radio source VRO 42.22.01 (MacLeod & Andrew 1968). Subsequent radio observations confirmed the variability of this source (Biraud & Veron 1968; Andrew et al. 1969) and detected linear polarization (Olsen 1969). Furthermore, optical observations revealed a non-stellar continuum spectrum devoid of any emission or absorption lines (DuPuy et al. 1969) and substantial linear polarization (Visvanathan 1969). Spectroscopic analysis of the faint nebulosity surrounding the central point source strongly suggested that the nucleus of BL Lacertæ was embedded in an elliptical galaxy of redshift $z = 0.07$ (Oke & Gunn 1974). By 1972, the discovery of a number of other objects with similar properties led Strittmatter et al. (1972) to propose that they constituted a new class of AGN, perhaps related to Quasars.

The ‘classical’ defining characteristics of BL Lac type objects were first listed in the review of Stein, O'Dell & Strittmatter (1976). These are

- a stellar or almost stellar appearance on optical plates,
- a strong, featureless power-law continuum with most of the luminosity ra-

¹Zwicky (1966) had in fact discovered the first galactic nucleus with a strong featureless continuum (I ZW 187) before the discovery of eponymous BL Lacertæ.
diated at infra-red wavelengths,

- large amplitude, rapid variability in all wavebands, and
- substantial, variable polarization in the optical and radio bands.

In 1976, eight years after their discovery, approximately 30 BL Lac type objects were known and it was anticipated that many more were likely soon to be identified (Stein, O'Dell and Strittmatter 1976). However, in contrast to the rapid increase in the number of known Seyferts and Quasars in recent years (Hewitt & Burbidge 1987; Veron-Setty & Veron 1985), the rate of discovery of BL Lacs has been relatively slow. In the most recent compilation of Burbidge & Hewitt (1987), 87 objects are listed compared to several thousand known Quasars.

The lack of strong features in the optical spectra of BL Lac type objects clearly makes the determination of their redshift and hence distance somewhat problematic. Only approximately a third of the objects in the compilation of Burbidge & Hewitt (1987) have measured redshifts. In a minority (~30%) of cases, the derived value is based upon the detection of either broad emission lines of the type seen in Quasars and/or absorption redshift systems of the type seen in high-redshift Quasars. The remainder contain absorption features typically seen in elliptical galaxies (e.g. Ca II, the G band, the 4000Å break etc), and/or weak narrow emission lines including the Balmer series, [O II] λ3727 and [O III] λ5007 which probably originate in the outer parts of the galaxy, possibly excited by the non-thermal central source of optical and ultra-violet radiation. In only a handful of cases have attempts been made to separate the underlying galactic emission from a non-thermal variable nuclear source (e.g. Mrk 501, Ulrich et al. 1975; BL Lacertae, Miller, French & Hawley 1978). However, Burbidge & Hewitt (1987) have shown that those objects not showing Quasar-like spectra have apparent optical magnitudes well correlated with redshift, indicating that the presence of a dominant underlying galaxy. They estimate the average value of the absolute magnitude of the host galaxies is $M_V = -22.5$, which is a typical value for the brighter elliptical galaxies (Frogel et al. 1978). They also show that the BL Lac
type objects with Quasar-like spectra all lie far above the Hubble relationship and point out that these objects are really indistinguishable from Quasars in all their observed properties except for the weakness or apparent absence of emission lines.

Numerous reviews of BL Lac type objects have appeared in the literature, the most significant being those of Stein, O'Dell & Strittmatter (1976), Angel & Stockman (1980), Weiler & Johnston (1980), Cruz-Gonzalez & Huchra (1984), Ledden & O'Dell (1985), Veron-Cetty & Veron (1985).

1.2.2 Continuum Emission

The non-thermal spectrum of BL Lac type objects can be well represented over at least a decade in frequency in any band by a power-law of the form $S_\nu \propto \nu^{-\alpha}$ (where $S_\nu$ is the flux received per unit frequency, $\nu$, and $\alpha$ is the spectral index). The presence of significant variability has emphasized the need for simultaneous multi-waveband observations. Reliable, single 'snap-shots' of the radio to X-ray continuum currently exist for $\sim$25 BL Lac type objects and OVV$s$ (e.g. Urry 1988 and references therein). It is found that the spectra are typically flat in the radio band with $-0.5 \leq \alpha_r \leq 0.5$, but progressively steepen towards higher frequencies such that $1 \leq \alpha_{opt} \leq 2$ in the optical and $1 \leq \alpha_{uv} \leq 3$ in the ultraviolet (Ghisellini et al. 1986; Maraschi et al. 1986; but see also Landau et al. 1986). The smooth steepening of the spectrum is often continued into the X-ray band, although a number of sources have X-ray spectra that lie significantly above the extrapolated infra-red to ultra-violet continuum and/or have flatter spectral indices.

The difficulties of obtaining adequate simultaneous multi-waveband coverage has so far prevented detailed studies of the relationship between the variability exhibited in the various wavebands for all but a handful of sources (e.g. OJ 287, Pumphrey et al. 1976; Mrk 421, Makino et al. 1987a). Nevertheless, there is evidence that the timescale of any variability decreases as one moves to higher
frequencies.

1.2.3 Proposed Emission Mechanisms

The smoothness of the multi-waveband spectra over a wide frequency range observed in BL Lac type objects suggests that the emission in the different bands is likely to arise by a common process. This spectral form, together with the measurement of substantial radio, infra-red and optical polarization provides the observational basis for the description of BL Lac type objects in terms of synchrotron emission (although see also Sikora & Begelmann 1988). The detection of variability on very short timescales (<1 year) implies the synchrotron emitting region is extremely small. An important constraint on the conditions within the emission region can therefore be obtained if the implied radiation energy density greatly exceeds that of the magnetic field. Under these circumstances, the synchrotron emitting electrons are able to interact with the dense photon field via the inverse-Compton process. Such 'synchrotron self-Compton' (SSC) emission can lead to prodigious amounts of high energy photons many orders of magnitude in excess of observations. As first pointed out by Shklovsky (1963, 1965), however, an 'inverse-Compton catastrophe' can be avoided if the emission is anisotropic. Specifically, if the emission region is moving at relativistic speeds towards the observer, then the observed emission will be substantially Doppler enhanced. The co-moving radiation and electron densities will then be significantly lower than those inferred by direct observations and hence the predicted inverse-Compton emission is dramatically reduced (Chapter 6). The detection of superluminal motion in at least one BL Lac type object (BL Lacertæ, Mutel & Phillips 1987) clearly strongly supports such a hypothesis. Furthermore, the enhancement of the continuum may swamp line emission from any stationary surrounding region and hence offer an explanation of the featureless spectra in these objects. Following the seminal work of Jones, O'Dell & Stein (1974a,b) a number of SSC models of varying degrees of complexity have been successfully applied to multi-waveband observations of BL Lac type objects. In the more simple mod-
els, the usual approach is to estimate a lower-limit to the relativistic boosting required to match the predicted X-ray flux with that observed. Recently there has been a shift in attention towards inhomogeneous relativistic jet models. Such models are certainly able to provide good fits to the multi-waveband observations, but tend to have too many free parameters to be well constrained. A number of *ad hoc* assumptions concerning the geometry and physical conditions within the emission region must be made (*e.g.* George, Warwick & Bromage 1988a).

More detailed study of the spectral variations exhibited by BL Lac type objects, particularly in the high-energy regime, may help to further constrain such SSC and other models.

### 1.2.4 The Scarcity of Emission Lines

By definition, BL Lac type objects are sources devoid of strong spectral features. The lack of emission lines however, raises a number of questions concerning why such an object should exist at all, and hence what the lack of lines implies about the physical conditions.

The region of the spectrum of most interest for the ionization of emission lines in AGN is the ultra-violet to soft X-ray band. Inevitably this regime is *ipso facto* almost totally unobservable due to absorption. However, we are able to make a number of general conjectures. Firstly, it is clear from the strong ultra-violet continua observed in these sources that BL Lac type objects are unlikely to possess a deficiency of ionizing radiation. Guilbert, Fabian & McCray (1983) have suggested that the steep X-ray spectra observed (compared to say Seyferts) will not be able to heat the accreting gas sufficiently for a Seyfert-type BLR to form. An alternative explanation that the lack of lines is due to a lack of gas, seems to be unlikely for BL Lac type objects as a class since a number of sources (*e.g.* OJ 287, 0846+51, 3C 446) have shown featureless continua at maximum light, but emission lines at minimum light (Sitko & Junkkarinen 1985; Arp *et al.* 1979; Wills *et al.* 1983). Furthermore, there is growing evidence that BL Lac
type objects are intimately related to OVVs and HPQs both of which show line emission. Thus, it seems most plausible the lack of lines is simply a selection effect. The classical BL Lac characteristics then represent an extreme end of a distribution of possible emission line strengths, although the ultimate cause of such a distribution is currently far from clear. The lack of emission lines is not of primary interest to the work presented here, however we shall take the view that it gives credence to the beaming scenario outlined above.

1.2.5 The Importance of High Energy Observations

The importance of observations in the X-ray and ultra-violet regimes has long been recognized for a more complete understanding of BL Lac type objects. Firstly of course, the production of photons at these wavelengths inevitably involves high energy phenomena. Clearly any proposed emission mechanism must be able to provide a viable explanation for such high energy radiation. Secondly, SSC models of the emission region make specific predictions concerning the X-ray and ultra-violet emission. Thus observations in these bands provide an important diagnostic tool in choosing between rival models. In particular, strong constraints can be imposed by the detection of (or upper limit upon) inverse-Compton emission in the hard X-ray band.

Also of importance is the form of any observed variability. Since the first results were obtained with the Ariel V and HEAO 1 satellites, it has been evident that substantial variability on short timescales is a common feature of BL Lac type objects (e.g. Ricketts, Cooke & Pounds 1976; Synder et al. 1980; Schwartz, Madejski & Ku 1982; Giommi et al. 1987b). As we shall discuss further in Chapter 6, the observed variability timescales potentially carry vital information on the geometry and mechanisms operating in the emission region.

It may be prudent to add a few words of caution at this point. By far the majority of BL Lac type objects have been identified as the counterparts of strong, flat-spectrum radio sources (and hence are commonly referred to as 'radio selected').
Due to the difficulties associated with a systematic search for complete optical samples in the absence of characteristic features, only a handful of objects have been optically selected. A significant fraction have been identified as a result of their X-ray emission (or would have been had the instrumentation flown earlier). It is currently uncertain how representative such X-ray selected BL Lac type objects (XBLs) are of the class as a whole. It has been established for example, that XBLs differ from the radio selected objects in their radio properties (Maraschi et al. 1986; Giommi et al. 1987a). It has been argued however that both can still be considered members of a single population (Stocke et al. 1985). The problem is of relevance since the inferences drawn from the five brightest, best studied X-ray bright objects (namely PKS 0548-322, Mrk 421, 1218+304, Mrk 501 and PKS 2155-304) are often applied to BL Lac type objects in general. The potential temerity of this is clear.

1.3 Thesis Overview

In this thesis we present the results from a study of the spectral variability of five BL Lac type objects in the X-ray and ultra-violet energy bands. The motivation came primarily from the relatively wide-band coverage and high sensitivity of the European X-ray Observatory Satellite, EXOSAT, which offered the opportunity of measuring the spectral parameters of many of the brighter BL Lac type objects with unparalleled accuracy. The inclusion of IUE measurements and the possibility of regularly monitoring such sources meant that the variability of the ultra-violet to X-ray continuum as a whole could be considered.

In Chapter 2 we briefly review the design and performance of the instrumentation used to make the X-ray and ultra-violet observations. The data reduction and analysis techniques are also discussed. Then in Chapters 3 and 4 the results in the X-ray and ultra-violet bands (respectively) are presented and discussed. Chapter 4 also discusses a number of systematic uncertainties encountered during the analysis of IUE data. In Chapter 5 we further discuss the form and variability
of the X-ray to ultra-violet continuum and review previous observations at longer wavelengths. The form of the multi-waveband spectrum for each object is presented. Chapter 6 contains the main bulk of our theoretical interpretation and evaluation of the observational results. We discuss the implications of the observed spectral variations for the standard synchrotron self-Compton model of BL Lac objects, and argue that inhomogeneous models are likely to be more appropriate. Finally in Chapter 7 we present the general conclusions from this work and briefly review future prospects.
Chapter 2

The EXOSAT and IUE Observatories

Overview

In this chapter, we discuss the design and performance of the instrumentation used to obtain the data presented in this thesis. The data reduction and analysis techniques employed are also described. Emphasis is placed on those aspects most relevant to the present work.

2.1 The European X-ray Observatory Satellite

The origins of a European X-ray astronomy satellite can be traced back to a mission conceived in the late 1960s to determine the location of bright X-ray sources using lunar occultation techniques. After numerous delays, the European X-ray Observatory Satellite (EXOSAT) was finally launched from the Vandenberg Air Force Base, California, USA on 1983 May 26 onboard a Delta 3914 rocket. Although the mission was funded by the European Space Agency (ESA), EXOSAT was operated as an international observatory with a series of Announcements of
Opportunity (AOs) for observing proposals and selection based on peer review. The Ground station was at Villafranca del Castills (VILSPA), Spain, whilst operational control was provided by European Space Operations Centre (ESOC), Darmstadt, BRD. The highly eccentric orbit of the satellite (see Table 2.1), primarily chosen to enable the occultation of sources over as large a part of the celestial sphere as possible, had the additional advantage of giving long uninterrupted periods above the radiation belts allowing continuous observations of up to ~76 hr to be made. It was this facility to make extended observations ('long-looks') which probably proved to be the most scientifically rewarding in both the galactic and extragalactic fields. The design lifetime of the satellite was limited by orbital decay which was predicted to occur in ~3 years. During the early months of 1986, serious problems were encountered with the attitude and orbit control system, these progressively worsened and the mission ended on 1986 April 9 following a thruster problem resulting in the spacecraft tumbling out of control and total loss of telemetry. Subsequent efforts to re-establish contact failed and EXOSAT finally re-entered the atmosphere above the Pacific Ocean south of New Zealand approximately 1 month later. In any event, there was only sufficient propane required to maintain stable pointing for another three months at the most.

The scientific payload (see Fig. 2.1(a) and Table 2.1) consisted of two Low Energy (LE) imaging telescopes, an array of 8 Medium Energy (ME) proportional counters, and a Gas Scintillation Proportional Counter (GSPC). The sensitivity of the latter instrument made it of limited use for most extragalactic sources; indeed, none of the sources reported in this thesis were bright enough to be usefully detected in the GSPC and hence this instrument is not considered further. A full description of the scientific instrumentation can be found in de Korte et al. (1981), Turner, Smith & Zimmermann (1981) and Peacock et al. (1981). The spacecraft was 3-axis stabilized to a few arcseconds with a pointing accuracy of ~10 arcseconds and the optical axes of all three instruments co-aligned. Sun angles were normally constrained to lie in the range 90°–130° to avoid the scattering of solar radiation from the LE baffles and to keep the ME detectors within their
operational temperature range. The telemetry rate of only 8 kbit s\(^{-1}\) did not al-
low all the raw data to be downlinked in real time. However the flexibility of the
onboard computer (OBC) enabled the transmission of preprocessed data with a
variety of selectable time and spectral resolutions to suit particular observational
requirements.

During the 3 years of successful operation 1780 observations (\(\sim 1.5 \times 10^4\) on-
source hours) were made covering almost all categories of astronomical object,
with approximately 31% of the time dedicated to extragalactic targets. Below we
give a brief overview of the LE and ME instruments and discuss their associated
data reduction and analysis techniques.

2.1.1 The Low Energy (LE) Telescopes

The LE instrument consisted of two identical, fully independent Wolter-I type
imaging telescopes (LE1 and LE2). The grazing incidence optics (with angles
\(\sim 1^\circ\)) in this design is achieved using a gold coated paraboloid followed by a
co-axial hyperboloid, the latter reducing the focal length (to 109 cm) and pro-
viding a degree of compensation for off-axis aberrations. This design provides an
operational energy range of 0.05–2.0 keV. Each telescope consisted of a nest of
two such mirrors increasing the total collecting area to 10 cm\(^2\).

Both telescopes could be used in either an imaging or spectroscopic mode. When
in imaging mode either a position sensitive proportional counter (PSD) or a
channel multiplier array (CMA) could be placed in the focal plane; when in
spectroscopic mode a dispersive element was provided by either a 500 lines mm\(^{-1}\)
or a 1000 lines mm\(^{-1}\) transmission grating with the CMA in focal plane. Due
to the early failure of the PSDs, all the observations reported here were made in
imaging mode with the CMA in the focal plane.

The CMA consisted of a pair of microchannel plates (MCPs) mounted in series
in a chevron configuration. Each MCP was 40 mm in diameter and had ap-

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Figure 2.1 Schematic view of the EXOSAT scientific payload showing a) the whole satellite, b) the configuration of a single LE telescope, and c) a cut-away view of half the ME detector array.
Table 2.1 EXOSAT summary

Orbital Characteristics

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<td>Time above</td>
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</tr>
</tbody>
</table>

Scientific Payload

**LE (CMA)**
- Two double nested Wolter-I telescopes
- Energy Range: 0.04-2.0 keV (6-300Å)
- Field of view: 2.2 degrees
- Plate scale: 5.3 μm/arcsec
- Spatial Resolution: 10 arcsec (on axis), 300 arcsec (off axis)
- Energy Resolution: Broad band spectroscopy available with the use of 5 filters
- Area: 10 cm$^2$

**ME**
- Array of 8 double celled (Ar & Xe) proportional counters
- Energy Range: 1-15 keV (Ar), 5-50 keV (Xe)
- Field of view: 45 arcmin FWHM triangular response
- Energy Resolution: $\Delta E/E = 51E_{keV}^{-0.8}$ % (FWHM) for Ar
- Effective Area: 1600 cm$^2$ (all detectors co-aligned).

**GSPC**
- Single Xe gas scintillation proportional counter
- Energy Range: 2-40 keV
- Energy Resolution: $\Delta E/E = 27E_{keV}^{-0.8}$ % (FWHM)
- Field of View: 45 arcmin FWHM, triangular response
- Effective Area: 100 cm$^2$
proximately a million channels, each 2 μm in diameter. Incident X-rays photons created an initial photo-electron on collision with the channel wall. This electron is then accelerated by the high potential difference (~1000 V) maintained across each plate, to finally produce a cascade (~5 × 10^7) of secondary electrons. The MCPs were operated in saturated mode to prevent variations in the output voltages from small changes in bias. The amplified charge was dumped onto a resistive disc with four symmetrically spaced read-out electrodes, providing positional information using the relative delays between the electrodes with a final pixel size of 4 × 4 arcsec. A small angle (12°) between the inclination of the channels in the top and bottom plates prevented positive ions drifting back to the front surface of the plate and inducing a large background count rate. The quantum efficiency was increased by a MgF_2 coating on the top plate. The detector region is surrounded by guard cells to monitor the non X-ray background. Risetime and energy information provides additional background rejection criteria. The count rates were generally sufficiently low that the time, position and 'sum-signal' of each event could be telemetered via the OBC. For count rates of the order reported here, deadtime losses were ~3.5%. The in-flight calibration of the LE instrument is discussed by Giommi (1985) and Giommi & Angelini (1987).

The LE + CMA platescale was 5.3 μm arcsec^{-1} giving a field of view of ~ 2.2°. As an example, a low energy image is shown in Fig. 2.2 (where there are 16 CMA pixels per image pixel). The sensitivity of the CMA changed by ≤ 1% over the duration of the mission (Giommi & Angelini 1987).

The position of an on-axis source could be estimated to an accuracy of ~8 arcsec (at 90% confidence; Osbourne & Angelini 1986). The CMA had no intrinsic energy resolution, but crude spectroscopic information (equivalent to X-ray colours) was provided by a series of filters mounted on a wheel just above the focal plane (Fig. 2.1). Indeed, the high ultra-violet sky background due to He I (58.4 nm) and H Ly_α (121.6 nm) emission lines necessitates the use of a filter with the CMA. The filters were plastic polymer sheets of varying energy dependent absorption
Figure 2.2 Grey scale image of the LE field of view using the CMA and 3lx filter. The target source (1218+304) is indicated close to centre, as is the CMA 'hot-spot' (diamond). Also evident is the serendipitous detection of two additional sources (Mrk 766 and ON 325) as discussed in Chapter 3. Note the greatly broadened point response function for these off-axis sources. The box indicates the position and orientation of the field of view of the ME detector array for this observation (005/1985).
Figure 2.3 The effective area of a single LE telescope, for three CMA/filter combinations as a function of photon energy. The curves for the 3lx, Al/P and Bor filters are shown as solid, dashed and dotted curves respectively.
depths and in Fig. 2.3 we show the effective area against photon energy of the telescope/filter/CMA combination for the three most commonly used filters for AGN observations, namely 300nm lexan (3lx), 100nm Aluminium plus 100nm Parylene (AL/P), and 100nm boron plus 50nm polypropylene (Bor) which generally give a progressively harder response. As can be seen from Fig. 2.3, the 3lx filter had the largest integrated effective area and provided the highest sensitivity, but unfortunately suffered from ultra-violet leakage below ~ 0.01 keV (not shown in Fig. 2.3). Although the CMA has no useful intrinsic X-ray energy resolution, each event was assigned a value proportional to total charge. If many such events are recorded, the resultant ‘sum-signal’ distribution allowed ultra-violet sources to be easily identified (see Chiappetti 1984; Chiappetti & Giommi 1985). Alternatively, discrimination between ultra-violet and X-ray sources could also be achieved by use of a second filter.

The spatial point spread function (psf) is very strongly dependent upon the off-axis distance, filter and energy. In Fig. 2.4 we show the derived (on-axis) psf from the 22 hour 3lx observation of Mrk 501 reported in Chapter 3. Annuli of width $1 \times 10^{-3}$ degrees were constructed about the centroid of peak intensity and the sum of the observed counts from pixels whose centre lay within each annulus calculated and divided by the total area of the included pixels. The resulting radial profile was then normalized using the innermost bin and reflected about the y-axis. As can be seen from Fig. 2.4, for small angles the 3lx psf is a rapidly decreasing function of radius with a FWHM ~23 arcsec, however large wings encompassing ~1% of the total counts lie outside a radius of ~41 arcsec and 0.1% outside ~85 arcsec. The 3lx psf has been found to broaden approximately linearly with off-axis angle for angles greater than ~0.22° such that 1° off-axis, the FWHM ~4 arcmin (Giommi 1985). The Al/P psf is similar to that found with the 3lx filter, however in the case of the Bor filter, the psf is significantly broadened at low energies as a result of energy dependent scattering of photons in the filter (Davelaar & Giommi 1985).

The background in the CMA is dominated by dark current and the ultra-violet
Figure 2.4 The on-axis point response function of the LE CMA with the 3lx filter (from the 22 hr observation of Mrk 501 1986/074 reported in Chapter 3). The curve has been reflected about the y-axis.
sky background (due to resonant scattering of He I & H Lyα) and is therefore relatively uniform over the whole field of view, peaking slightly at centre. In the case of the Mrk 501 observation, the mean background count rate was \( \sim 2.5 \text{ count s}^{-1} \text{ cm}^{-2} \) (\( \sim 9 \text{ count s}^{-1} \text{ deg}^{-2} \) or \( \sim 1.1 \times 10^{-5} \text{ count s}^{-1} \text{ pixel}^{-1} \)) a fairly typical value, although flaring at several times this value was occasionally observed during periods of increased solar activity. Vignetting in the telescope reduces the effective area 1° off-axis to \( \sim 45\% \) of the peak value.

### 2.1.2 The Medium Energy (ME) Experiment

The ME instrument consisted of a large area array of 8 double cell multi-wire proportional counters grouped in four pairs. The upper cells (Fig. 2.1(c)) contain an Argon (Ar) CO\(_2\) gas mixture and are sensitive over the energy range 1–15 keV; the lower cells are filled with a Xenon (Xe) CO\(_2\) mixture and sensitive to 5–55 keV X-rays. Both cells were maintained at a pressure of \( \sim 2 \) atmospheres. The collimators are made of lead glass with Beryllium front windows; six with a thickness of 32\( \mu \)m, two with a thickness of 62\( \mu \)m. The individual detector responses are slightly different, but the total array has a square truncated pyramidal response (defined by the collimators) with a \( \sim 4 \) arcmin flat top and 45 arcmin FWHM. The effective area of the total ME array was 1600 cm\(^2\).

The photo-electric absorption of an X-ray in the filling gas causes, in most cases, an electron to be ejected with enough kinetic energy to ionize additional gas atoms, leading to an avalanche of secondary electrons onto the anode. Inert gases of high atomic number were employed since the mean energy required to produce an ion pair is low, the photo-absorption cross-sections are high and they exhibit zero electron affinity. A high degree of electron multiplication can therefore be obtained (see e.g. Thomas 1984). In the case of Ar, one electron–ion pair was produced for every \( \sim 26 \) eV of incident X-ray energy leading to a typical gain of the order of \( 10^2–10^3 \). The resultant pulse was amplified and measured by the pulse height analyser (PHA). The presence of the CO\(_2\) quench gas ensures that the
Figure 2.5 The effective area of half the ME array as a function of energy for the Argon cells.

The amplified pulse is approximately proportional to the energy of the incident photon (but see below). The Ar chambers proved the most useful for the observation of faint sources such as those reported here. All the sources reported in Chapter 3 were too weak to produce useful data in the Xe cells. The effective area of the Ar cells is shown in Fig. 2.5 as a function of photon energy for half the ME array. The overall shape is primarily governed by the front window at low energies and the density and depth of the absorbing gas at high energies.

The energy resolution was limited primarily by wire-to-wire gain variations caused partly by small diameter variations and end position errors and partly by gas effects. The resolution obtained was $51E_{keV}^{-0.5\%}$ FWHM with slight differences between the individual array elements. In-orbit calibration was provided by an $^{55}$Fe source and 9 observations of the Crab nebula and has been discussed by Palmar & Smith (1985) and Smith & Palmar (1987).
The eccentric orbit exposes the ME instrument to a high, variable cosmic ray induced background and hence necessitates highly efficient (>99%) background rejection. The techniques employed were the traditional 3 sided guard for each cell using the outer wires of each anode layer and the rear layer, the end guard cathodes giving efficient end anti-coincidence, and rise-time discrimination (cosmic rays producing a spatially dispersed trail of ionization within the detector, thus having a much longer rise-time).

Each pair of detectors (i.e. array quadrant) pivoted around the line joining opposite corners of the quadrant. Each could therefore be off-set relative to the spacecraft pointing position (typically by ~2°) to allow simultaneous background monitoring of a source free region of the sky during an observation. Unfortunately off-setting a detector slightly modified the background spectrum, making background subtraction more complex than anticipated (see Smith 1984). After the first 12 months of the mission, the observational procedure for weak sources was slightly altered such that the on-source and off-set halves were interchanged (or ‘nodded’) typically every $10^4$ seconds throughout an observation. In the normal case where the background count rate was constant, this enabled background subtraction to be performed using the (non-simultaneous) background recorded by the same half. The correction to the background spectrum as a result of the off-set (known as the ‘difference spectrum’) was found to be dependent upon the off-set angle and in particular on the direction of the off-set (see Parmar & Izzo 1986). In practice such corrections were found to be a second order effect and a subjective decision was made in each case whether to include corrections for the difference spectra on a trial and error basis. An alternative method of ME background subtraction is provided by making use of the data recorded during the prior and post on-source slew manoeuvres. These are usually made in the same detector configuration as the adjoining on-source data (i.e. with the same off-set half, angle etc) and hence are directly compatible.

On purely statistical grounds, a 5σ detection of a 1 mCrab source$^1$ was obtain-

$^1$ 1 mCrab = $1.7 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, 2–6 keV
able in ~500 s with a reasonable estimation of its spectrum within ~ $10^4$ s (1 'unit'). However, several factors such as variations in the background between array elements and with time lead to a requirement for longer exposure times in practice. A variety of OBC modes were available for the ME, however all the work presented here was recorded using HER4 mode giving a 256 channel PHA spectrum every 10 s for each of the 8 array elements. Deadtime losses of 10% have been assumed throughout.

2.1.3 In-flight Performance

The satellite generally functioned well throughout its 3 year lifetime with only minor failures. Both PSDs exhibited anomalous behaviour soon after launch and were not used. The CMA in LE2 also failed, whilst the grating mechanism in LE1 jammed and had to be placed permanently in the out position. Furthermore, the shutter used to protect the ME instrumentation during launch over-opened to an angle of ~95°. It is estimated that this resulted in a reduction of the observed count rate by 28% (Aschenbach 1985); this has been corrected for in spectral analysis reported in Chapter 3, but not in any of the quoted LE count rates. Due to a minor hardware bug, a spurious count was recorded in all LE images in a fixed pixel ('hot spot') every 64 s (shown as diamond in Fig. 2.2). Minor gas leakages from the ME cells lead to slight gain variations which were compensated for by regular adjustments in amplifier gain (Parmar & Smith 1985). In addition, occasional temporary reductions in gain were experienced for an individual detector (probably indicating a small breakdown), however this was cured by turning the detector off for a few hours. On 1985 August 20 a dramatic decrease was experienced in the anode gains in both cells of array element C which were subsequently switched off and remained unoperational throughout the remainder of the mission.

The detector, housekeeping and telemetry data was preprocessed at ESOC to produce a Final Observation Tape (FOT) for each instrument. These were dis-
patched to the Principle Investigator (PI) within typically a month. All scientific data entered the public domain after one year. The majority of the observations reported in this thesis were obtained as result of proposal in which the author was directly involved\(^2\), however 4 observations of the source Mrk 421 were obtained from the archive.

### 2.1.4 Data Analysis Procedures

The data from the individual ME detectors is recorded separately on the FOT and hence can be analysed independently. However, in all but a few problematic cases (noted in Chapter 3), the data from all the detectors in each half were combined to increase the signal to noise.

In the analysis of both time-series and spectral data, we follow the standard falsificationist orthodoxy whereby the probability is calculated of the observed sample being consistent with a 'null hypothesis' for an assumed parent distribution. In all cases the Chi-square statistic, \(\chi^2\) is used to test the null hypotheses. Generally, if \(x_i\) is the observed number of events in bin \(i\), \(\sigma_i\) the error associated with \(x_i\) and \(n_i\) is the number expected according to the assumed distribution, then

\[
\chi^2 = \sum_{i=1}^{N} \frac{(x_i - n_i)^2}{\sigma_i^2}
\]  

[2.1]

where \(N\) is the number of bins. A value of \(\chi^2 \gg N\) indicates that the null hypothesis (that the \(x_i\)'s are drawn from the population represented by the \(n_i\)'s) is rather unlikely. More explicitly, the Chi-square probability function, \(Q(\chi^2_{\text{obs}} \mid \nu)\), gives the probability that, assuming the null hypothesis to be correct, the observed value of Chi-square, \(\chi^2_{\text{obs}}\), is due to chance alone, where \(\nu\) is the number of degrees of freedom.

We now discuss the statistical analysis of time-series and spectral data in more

\(^2\)with PIs R.S. Warwick and I.M. McHardy
detail. All the EXOSAT analysis was performed using the software developed by members of the Leicester X-ray Astronomy Group.

**Time-series analysis**

In the analysis of any observation, one of the first tests that must be carried out is to determine whether the source varied on a timescale shorter than or comparable to the observation, and if so, some attempt at the characterization of this variability should be made.

For counting experiments, generally the Poisson distribution is appropriate. However in most cases (X-ray astronomy included), the total number of events (photons) recorded is large enough (i.e. >20) such that the Poisson distribution is well approximated by a Normal distribution. If we make the null hypothesis that the signal did not vary with time, we are then able to determine whether there are any differences between individual members of the sample in excess of those expected (at some level of confidence) from a Normal distribution with the calculated mean.

There are two complications however. Firstly, the ME source signals reported here are generally much weaker than those due to the X-ray and solar backgrounds. Furthermore, since the latter in particular, may be variable on a variety of timescales, careful analysis of the background is required before any apparent source variability can be considered to be real. A second complication is that the true source count rate of $C$ is unknown. One is therefore forced to estimate it from the data. This can be achieved in a fairly straightforward way by calculating the weighted mean, $C_W$. However, counting statistics will introduce an uncertainty $\sigma_p = (C_W \Delta t)^{1/2}$ in the total number of photons, $C_W \Delta t$, recorded. Hence increasing the bin size will decrease the fractional error on any estimate of $C_W$ and hence $C$. However, such a procedure stands the risk of averaging out any variability on short timescales. Conversely, any variability on timescales of the same order as the exposure time may be masked if small time bins are used. It is therefore necessary to analyse the signal employing a range of bin sizes,
and compare each with that expected from counting statistics alone.

Most of the LE exposures were spent with the Bor filter in position, resulting in poor signal-to-noise and hence little chance of seeing variability. The standard ME analysis technique employed for the work reported here is as follows. Firstly light curves for the whole observation for each array half were constructed (typically using the most sensitive 2–6 keV band; PHA channels 8–25). The data was then inspected visually to identify the prior and post slews, the on-source observation times, the nods, and any other anomalous features in either the on-source or the background halves. On-source data windows were then defined. For those weak sources where there was no noticeable increase in the on-source count rate above the slew (background) count rates, the nominal stable pointing time window as determined by the ESOC observatory controller at the time of the observation was used. The light curves from the on-source and off-set halves (i.e. containing respectively source plus background and background only) during the good data windows were then extracted and binned up on timescales of 10, 100, 500, 1000 and 3000 seconds. Each off-set time series was then tested against the null hypothesis that the observed sample is consistent with counting statistics. The Chi-square test (equation [2.1]) was used to test the goodness-of-fit, with the weighted mean count rate, \( \bar{\nu} \) substituted for \( n_i \) giving \( \chi^2_{o,B} \).

In cases where \( Q(\chi^2_{o,B} | \nu) > 0.1 \), the background count rate was deemed to be constant with time and the weighted mean count rate (with its associated error) subtracted form the on-source half to produce the background subtracted light curve. When \( Q(\chi^2_{o,obs} | \nu) < 0.1 \), it was deemed that there was a high enough probability of excess variations in the off-set half that the background light curve was subtracted point-for-point from the on-source half to produce the background subtracted light curve with an inevitable decrease in signal-to-noise. For nodded observations, off-set light curves from each configuration were tested both independently and combined after rescaling by the appropriate H1/H2 ratio (see below) in order to increase the signal-to-noise.
Once a background subtracted light curve had been obtained, each source time series was then tested against the null hypothesis that the observed sample is consistent with counting statistics using the Chi-square test and the likelihood that the variance in the observed signal is due to chance alone calculated. The results from this analysis are presented in Chapter 3. The criteria for source variability employed (i.e. for falsification of the null hypothesis) was that \( Q(\chi^2_{\nu, S} | \nu) < 0.001 \).

In cases where the data was inconsistent with the null hypothesis further analysis is possible. Unfortunately, the sources reported here do not have the signal-to-noise for application of the powerful Fourier or Fractal analysis methods which have been applied to stronger sources. However as will be discussed in Chapter 5, we are primarily interested in large amplitude variability (>30%) and hence a simple doubling timescale, \( t_{\text{var}} \), as defined by

\[
t_{\text{var}} = C_{\text{min}} \left( \frac{C_{\text{max}} - C_{\text{min}}}{\Delta t} \right)^{-1}
\]

was deemed to be more appropriate. For variations <30% use of equation [2.2] was considered inappropriate.

Spectral Analysis

All the primary targets reported in this thesis are close to the centre of the LE field of view and not obviously extended. This greatly facilitates background subtraction. The LE count rates were calculated by selecting a 100 x 100 arcsec box centred on the peak count response, the background count rate was determined from the concentric 100–300 arcsec (square) annulus.

After particle induced background has been rejected, the main source of background in the ME array are X-rays both due to the in-orbit environment and the cosmic X-ray background. All possible methods of background subtraction were employed and the most satisfactory selected. Although the sources reported here are typically too weak to be significantly detected in the Xe detectors, the Xe
channels do facilitate the decisions to be made concerning the best method of background subtraction for the Ar detectors. An example of a background subtracted PHA spectrum is shown in Fig. 2.6 from the 1986 'long-look' observation of Mrk 501. The data from the Ar and Xe cells is shown in PHA channels 1–128 and 129–256 respectively. The source spectrum is clearly visible in channels 5–30.

Having obtained an acceptable background subtracted PHA spectrum, a suitable range of ME channels was selected for spectral analysis. The precise range was somewhat dependent upon the source strength but typically channels 5–35 were chosen, the higher channels often being binned up to increase the signal-to-noise. Unfortunately, a simple deconvolution of the observed spectrum with the detector response is inappropriate in the case of the ME since it is unable to yield a unique solution. This is due to the fact that following the absorption of the incident X-ray photon by an atom of the filling gas, de-excitation takes place either by the re-emission of an X-ray photon via (K-shell) fluorescence, or by the expulsion of an electron. The fluorescence photons can either be re-absorbed elsewhere within
the detector cell leading to an avalanche of secondary electrons at the same or
different anode, or may escape from the system completely reducing the total
detected energy by a fixed amount. In the case of the Ar cells, \(~4\%\) of the
incident photons are effected in this way, whilst for the Xe cells the proportion
is considerably higher. An additional source of uncertainty is also introduced by
statistical noise.

The standard technique used throughout X-ray astronomy is therefore to express
the detector response in the form of a matrix in which each row corresponds
to the response to a mono-energetic beam as a function of PHA channel. The
analysis of spectral data is then iterative whereby an initial spectral form is
assumed, convolved with the instrument response and compared to the observed
PHA spectrum using the Chi-square test. The model parameters are then varied
and the process repeated until the Chi-square value is minimized. The minimum
Chi-square can then be used to determine the likelihood of the best fitting model
being an acceptable representation of the data. The fits described in Chapter
3 were performed simultaneously using both the ME and LE data, a separate
response matrix being required for each. Also quoted are the 90% confidence
limits on each parameter using the procedure detailed by Lampton et al. (1976)
using multi-parameter errors.

In Fig. 2.7 we show an example of an observed photon spectrum and model before
and after deconvolution with instrumental response. The LE and ME data are
represented as diamonds and crosses respectively. In the case shown (Mrk 421
340/1984), to be further discussed in Chapter 3, the assumed model spectrum
is a power-law with low energy absorption. As can be seen from Fig. 2.7, the
best fitting model spectrum (shown as a dotted line) is in this case not a good
representation of the data, being higher than the low energy ME data and higher
than the high energy data. The chi-square test on the data and model gives a
reduced chi-squared, \(\chi^2\) of 5.5 for \(\nu = 30\). This corresponds to a probability of
\(Q(\chi^2_{\text{obs}}, \nu) \ll 0.0001\) that the best fitting model is a good representation of the
data, hence in this case the model is deemed to be statistically unacceptable.
Figure 2.7 An example of a) the best fitting observed count rate spectrum, and b) the derived incident photon spectrum for a high state observation of Mrk 421 (1984/338 – see Chapter 3) assuming a simple power-law model (dotted) of the form given by equation [3.1]. It can be seen that in this case, such a model does not adequately reproduce the data in view of the notable deficit of high energy photons.
2.2 The International Ultra-violet Explorer

2.2.1 Overview

Launched on 1978 January 26 from the John F. Kennedy Space Center, the International Ultra-violet Explorer (IUE) is now in its tenth year of operation, with planning for an eleventh year underway. In 1986 IUE superceded Copernicus to become the longest lived astronomical satellite with its scientific performance essentially unimpaired. The project is a joint undertaking between the US National Aeronautics and Space Administration (NASA), the UK Science and Engineering Research Council (SERC) and the European Space Agency (ESA). The satellite is in a (nearly) geosynchronous elliptical orbit over the Atlantic Ocean ensuring uninterrupted ground contact with the NASA ground station at the Goddard Space Flight Center, Greenbelt, Maryland and the ESA ground station at VILSPA.

The IUE observatory has been operated much like a ground-based observatory, controlled for 16 hours each day from Greenbelt for NASA sponsored observers and for the remaining 8 hours from VILSPA for the European observers. The satellite is 3 axis stabilized and provides a general facility for procurement of astronomical spectra in the ultra-violet band. The instrumentation is sensitive to wavelengths in the range 1150–3200 Å. A complete description of the spacecraft and instrumentation design specifications is given in Boggess et al. (1978a). The early performance and data preprocessing procedures are discussed by Boggess et al. (1978b). These articles have been periodically supplemented by NASA and ESA IUE Newsletters; references to specific newsletters and other articles will be mentioned in context. An early review of IUE observations of extragalactic objects, including the BL Lac type object Markarian 421, is given in Boksenburg et al. (1978). Further details on the history, spacecraft subsystems and operations of the satellite can be found in Boggess & Wilson (1987) and Fälker, Gordon & Sandford (1987). The remaining sections of this chapter discuss the relevant features of the spacecraft instrumentation and data analysis procedures necessary to
2.2.2 Scientific Instrumentation

A 0.45 m diameter Cassegrain telescope with an effective focal length of 6.75 m collects and focuses the radiation onto the spectrograph aperture which is inclined at 45° to the telescope optical axis (see Fig. 2.8). The aperture plate is mirrored so
Table 2.2 IUE summary

**Orbital Characteristics**

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**Scientific Payload**

**Telescope**
- **Richey-Chretien figure**
  - Aperture: 45 cm
  - Primary Focal Ratio: f/2.8
  - Effective Focal Ratio: f/15
  - Plate scale: 30.6 arcsec mm⁻¹
  - Acquisition Field: 16 arcsec diameter

**Spectrographs**
- **Echelle**
  - Entrance Aperture
    - Small: 3 arcsec circle
    - Large: 10 x 20 arcsec oval
  - Detectors: 4 SEC Vidicon Cameras

<table>
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<th>High dispersion</th>
<th>Range(Å)</th>
<th>Max Resolution</th>
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<td>13000 (at 1300Å)</td>
</tr>
<tr>
<td>Long</td>
<td>1845-3230</td>
<td>16000 (at 3100Å)</td>
</tr>
<tr>
<td>Low dispersion</td>
<td>Range(Å)</td>
<td>Max Resolution</td>
</tr>
<tr>
<td>Short</td>
<td>1150-2000</td>
<td>333 (at 1350Å)</td>
</tr>
<tr>
<td>Long</td>
<td>1825-3300</td>
<td>420 (at 2400Å)</td>
</tr>
</tbody>
</table>
as to reflect the telescope field surrounding the aperture into duplicate Fine Error Sensors (FES). The FES are sensitive to optical wavelengths with a 16 arcmin field of view and are used as off set star trackers and for pseudo-simultaneous photometry. The limiting magnitude of the FES depends upon its operating mode but can track stars as faint as 14th magnitude. Only one FES is used at a time and since March 1978, FES 2 has been operated due to its higher sensitivity to faint objects.

After passing through the aperture plate, the radiation is fed into one of two separate echelle spectrographs sensitive to either short (1150–1950 Å) or long wavelengths (1900–3200 Å). Each spectrograph has a pair of entrance apertures consisting of a hole 3 arcsec in diameter and a closeable oval slot 10×20 arcsec, known as the small and large apertures respectively. Light passing through the aperture plate into either the short-wavelength or long-wavelength spectrograph is reflected from a parabolic collimating mirror onto an echelle grating (for high dispersion spectra) or a plane mirror (for low dispersion spectra), then again onto a spherical grating to cross-disperse the high dispersion spectrum or create the low dispersion spectrum. The light is then brought to a focus again at the entrance aperture of one of two cameras (i.e. there is a total of four cameras on board, known as the short- and long-wavelength prime and redundant cameras, SWP, LWP, SWR and LWR respectively). All the observations reported here were made using the large aperture and low dispersion mode.

There are also a number of light sources within the spectrograph assembly. A platinum-neon hollow cathode lamp is used as a wavelength standard. Tungsten filament lamps are mounted in front of the cameras in order to flood the camera faceplate with light as part of the camera preparation procedure before each new image. The previous image is erased by reading out this now overexposed image with a defocused vidicon beam, then repeating the process. A mercury lamp is also provided to illuminate the cameras with 2537 Å radiation to produce sets of flat-field images for different exposure times in order to derive the intensity transfer functions (ITF) for each camera pixel (see below). Further details can be
found in an introduction to the use of IUE data by Harris & Sonneborn (1987).

The Short- and Long-Wavelength Cameras

Each camera consists of an ultra-violet to visible image converter (UVC) coupled to a secondary electron conduction (SEC) television camera sensitive to visible light. The photoelectrons generated at the SEC photocathode are accelerated and focused onto a 10μm thick potassium chloride target where they produce secondary electrons. These are swept away by a small electric field, leaving an amplified positive charge image stored on the target. On completion of an exposure the image is read out, the charge in each pixel converted to digital units called Data Numbers (DN) running from 0–255 and telemetered in real time to the ground station. Optimum exposures have DN values ~200; at DN levels close to 255 the exposures are highly nonlinear and unreliable. DN values >255 correspond to over-exposed pixels. Observing time is maximized by beginning an observation with the second spectrograph before reading out and erasing the first.

Perpendicular to the dispersion direction, Cassatella, Barbero & Benvenuti (1985) have found that the point spread function (psf) is strongly dependent on wavelength and telescope focussing conditions. For the LWR and LWP cameras the psf is well represented by a central Gaussian with Lorentzian wings containing ~85% and ~15% of the total source counts respectively. In the case of the SWP camera, the psf is asymmetric and better represented by a skewed Gaussian.

2.2.3 Data Preprocessing

Each raw IUE image consists of a 768×768 array of pixels, each of which has an associated DN value approximately proportional to the integrated charge read out from the SEC vidicon target. The raw images are first corrected by the IUE Spectral Image Processing Software (IUESIPS) to produce either a photometrically
corrected image (PHOT) or a geometrically and photometrically corrected image (GPHOT). The geometrical corrections make use of 169 fiducial ('reseaux') marks on the target to remove distortions introduced by the SEC tube. In the geometrically corrected frame, the plate scale is $1.53 \pm 0.01$ arcsec pixel$^{-1}$. Cassatella, Barbero & Benvenuti (1985) have estimated that the maximum resolution, $R$, for low dispersion images is $\sim 330$ for the SWP at 1350Å and $\sim 420$ for the LWR at 2400Å. The photometric correction takes into account the nonuniform, nonlinear vidicon response using a specific ITF for each pixel. The input DN value for each pixel is transformed to a normalised flux number (FN) which is linearly related to the incident flux on the vidicon faceplate at the location corresponding to the particular pixel in question. The ITF is actually generated by an interpolation of a series of 12 (11 for SWP) graduated geometrically corrected mercury flood-lamp flat field images. Pixels with DN values out of the range calibration exposures for the ITF are somewhat problematic. Below-range pixels (i.e. with $DN_{\text{image}} < DN$ of the lowest level of the ITF) and above-range pixels are assigned FN values based on a linear extrapolation of the first and last two points on the ITF respectively. Saturated pixels ($DN_{\text{image}} > 255$) are assigned an artificially large value as a simple flagging mechanism. Since 1981 March 1, IUESIPS has performed the photometric correction without prior geometric correction. Each raw data pixel in the (distorted) readout frame will have a position between four ITF curves. Each ITF is applied to the pixel DN and a bilinear interpolation of the four values used to calculate the FN.

The wavelength scale is calibrated using the position of the platinum emission lines. High and Low-dispersion images are taken with both active cameras every 2 weeks using the small aperture. To date, no secular drifts have been detected. Short timescale variations do however exist in the low-dispersion wavelength calibration (possibly thermally induced), but an average wavelength calibration has been derived. The nominal accuracy of the SWP and LWP is $\sim 3$Å (Harvel, Turnrose & Bohlin 1979). Further details of the corrections and accuracies can be found in the IUE Users Guide (Volume II).
An absolute energy calibration is then provided by reference to the so-called inverse sensitivity curve, $S^{-1}$ generated from the observation of standard stars.

2.2.4 In-flight Performance

IUE has generally operated exceptionally well during its entire in-orbit life. Despite the loss of four of the six gyros the spacecraft continues to be be operated as intended and a control system using only one gyro and the FES is currently under test. The solar panels have suffered some degradation but a satisfactory spacecraft power budget can still be supplied. All other systems are operating within design constraints.

From the start of the guest observer operations until 1983, the SWP and LWR cameras were the standard (default) instruments. However after 1983 April, the LWR experienced a progressive deterioration in image quality due to a flare discharge in the UVC, hence in 1983 October the LWP was promoted to default long wavelength camera.

2.2.5 Data Analysis Procedures

The photometrically corrected image files are written, along with some IUESIPS automatic analysis, line-by-line spectra etc to a Guest Observer Tape (GOT) which is normally available to the guest observer within a day or so. All scientific data enters the public domain after 6 months. Some of the images reported here were obtained as a result of proposals in which the author was directly involved\(^3\), the rest were retrieved from the data archive.

The IUE images were reduced from the GPHOT or PHOT in a uniform manner using the IUEDR software package versions 1.3 & 1.4 available on the UK STARLINK network (Giddings & Hook 1985). Each image was first examined

\(^3\)with GO G.E. Bromage
to identify and flag bad data points (due to radioactive decays, cosmic ray hits etc) which are neglected in the subsequent analysis. Scans perpendicular to the direction of dispersion were then performed in order to determine the offset of the source spectrum from the centre of the slit. The gross spectrum was then extracted using a slit of width $W_S$ pixels wide located about the centroid of the source response. The simultaneous background spectrum was determined from the mean spectrum extracted from two regions each with a triangular profile of FWHM $W_B$ pixels situated 10.0 pixels off-axis on either side of the source spectrum. The background was smoothed with a running mean filter (with a triangular weighting over 30 pixels FWHM along the spectrum) after an initial iteration to reject those pixels more than $2\sigma$ from the running mean.

In Chapter 4 it will be shown that the value of $W_S$ chosen has a significant effect on the derived spectral parameters. This is likely to be due to a change in focus along the direction of dispersion of the cameras. In contrast we find no dependence of the derived spectra parameters on $W_B$. In Chapter 4 we argue that a source width of 12.8 pixels is likely to give the best results. A value of $W_B=2$ was found to give an adequate estimation of the background.

The background subtracted source spectra were converted to flux densities, then averaged over 100 Å intervals in the bands 1230–1900 Å (SWP) and 2100–3200 Å (LWR and LWP). The error associated with each bin was determined from the root mean square (rms) scatter on the data. A weighted least-squares fit of a power-law function of the form

$$\log F'(\text{mJy}) = \log F_o - \alpha (\log \nu - \log \nu_o)$$

was then applied both to the individual spectra from each camera and, for observations made within the period of a day, to the combined data set. The goodness-of-fit was again estimated using the $\chi^2$ statistic (equation [2.1]).

\footnote{where 1 (geometric) pixel = $1/\sqrt{2}$ pixel diagonals}
In our experience, the most reliable estimate of the ultra-violet continuum in BL Lac type objects is provided by the spectrum derived from the combined long- and short-wavelength data. This procedure, of course, ignores the possibility of either spectral curvature or rapid variability (on timescales < 1 day), but since there is no evidence of either of these effects in any of the objects presented here, we believe this assumption to be valid. Within the analysis procedure outlined above there are several possible and known sources of error. These can broadly divided between 1) those known sources of error associated with the calibrations, and 2) those uncertainties arising either as a result of unwise choices within the data analysis procedure or as a result of suspected calibration errors. Examples of the latter are the effects of the source extraction width, $W_S$, on the best fitting spectral parameters and the discrepancies between the individual camera and combined best fitting parameters. We shall postpone a detailed discussion of such systematics until Chapter 4.

The most fundamental changes to the calibration of IUE data has been due to the several revisions to the individual camera ITFs throughout the mission, the latter ones usually refinements of earlier ones. As described above, the data presented in this thesis has been extracted from the photometrically corrected images (i.e. GPHOTs or PHOTs which are FN images) to which an ITF has already been employed. In addition, it is important to note that any update to a camera ITF table requires modification of the corresponding inverse sensitivity curve, $S^{-1}$, used to absolutely calibrate the background subtracted source ultra-violet spectrum. In Table 2.3 we list the various updates to the ITFs and $S^{-1}$ curves for each camera. The initial LWR and SWP ITFs were only very preliminary and rather inaccurate. Very few images were processed using them and no correction algorithms exist. Unfortunately the SWP ITF1 contains a gross error (~25%) at the 20% exposure intensity level and smaller errors at the 10% and 40% intensity levels. The LWR ITF1 also contains minor photometric errors. For both the above, correction algorithms are available (Cassatella et al. 1980) within IUEDR.

Recently it has become evident that there are several errors within the LWP
Table 2.3 IUE ITF and $S^{-1}$ updates

<table>
<thead>
<tr>
<th>Camera</th>
<th>ITF</th>
<th>Notes</th>
<th>$S^{-1}$</th>
<th>Dates in use$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LWR</td>
<td>ITF</td>
<td>Prelimin</td>
<td>Bohlin &amp; Holm (1980)</td>
<td>07-04-78 –  14-06-78</td>
</tr>
<tr>
<td></td>
<td>ITF1</td>
<td>(slightly) faulty$^b$</td>
<td>Clavel et al. (1986)</td>
<td>14-05-78 –</td>
</tr>
<tr>
<td></td>
<td>ITF2</td>
<td>New</td>
<td>Oliversen &amp; Gerhart, in prep</td>
<td></td>
</tr>
<tr>
<td>SWP</td>
<td>ITF</td>
<td>Prelimin</td>
<td>Bohlin &amp; Holm (1980)</td>
<td>07-04-78 –  14-06-78</td>
</tr>
<tr>
<td></td>
<td>ITF1</td>
<td>Faulty$^b$</td>
<td>none</td>
<td>14-06-78 –  07-08-79</td>
</tr>
<tr>
<td></td>
<td>ITF2</td>
<td>Corrected</td>
<td>Bohlin (1986)</td>
<td>07-08-79 –</td>
</tr>
<tr>
<td></td>
<td>ITF3</td>
<td>in preparation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LWP</td>
<td>ITF1</td>
<td>Initial</td>
<td>Cassatella &amp; Harris (1982)</td>
<td>07-04-78 –</td>
</tr>
<tr>
<td></td>
<td>ITF2</td>
<td>New</td>
<td>Cassatella, Lloyd</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&amp; Gonzalez Riestra, in prep</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ At VILSPA.
$^b$ Correction algorithm available (Cassatella et al. 1989).

ITF1 and SWP ITF2 and new ITFs have been produced. Other known sources of inaccuracy within the ITF calibrations include the assumed linear extrapolations of the ITFs for Low and High DN numbers below and above the lowest ITF levels, the linearity of the ITF and drifts in the sensitivity of the cameras with time. Although little quantitative information exists, the first two are likely to be unimportant in the work reported here since all images are well exposed.

The accuracy of the absolute energy calibrations has also recently been discovered to be somewhat questionable. The slope of the $S^{-1}$ curves were determined using trailed spectra of bright OAO-2 standards, whilst the absolute levels were found using point spectra of the fainter TD1 standards. As pointed out by Harris & Cassatella (1985) this calibration philosophy implicitly assumes that a unique absolute curve holds for both point and trailed spectra. This assumption is only valid if the ITF is accurate and the camera characteristics are invariant. It has however, been known for several years, that a discrepancy exists in the region of overlap of the SWP and LWP spectral ranges (1900–1950 Å), low dispersion
absolutely calibrated LWP spectra giving on average ∼15% higher fluxes than the SWP. Observations of standard stars have revealed discrepancies up to 30% (Harris & Cassatella 1985). Furthermore it was found that a better agreement between LWP and SWP fluxes in the overlap region was obtained if the LWP images were overexposed (in the region of highest sensitivity). In a preliminary study (Cassatella 1984) excluded some of the possible causes of this discrepancy such as a non-linearity in the LWP and an incorrect scaling of the LWP sensitivity curve. More recent investigations have shown that at least part of the problem is caused by inconsistencies between point and trailed spectra (Harris & Cassatella 1985). These authors show that the ratio of point/trailed fluxes for the object BD + 28°4211 rises by 20–30% for the LWP at wavelengths <2000 Å. They claim similar trends in other objects. However the opposite appears to be the case in a later study of BD + 28°4211 by Cassatella & Lloyd (1987) where a drop (<14%) in the point/trailed ratio at low wavelengths is found. Nevertheless, both studies show that the flux ratio of point to trailed spectra is not grey with wavelength when LWP spectra are processed with ITF1 (Cassatella & Lloyd have also shown the same is true for LWP spectra processed with ITF2). Harris & Cassatella (1985) claim that no deviations of the point/trailed ratio which could affect the SWP-LWP overlap discrepancy are found for SWP spectra, but their Fig. 8 appears to show a 5–10% deficit in the overlap region for BD + 28°4211. They find that by renormalizing the trailed LWP, presumably by the point/trailed ratio reduces the overlap discrepancy to 5–10% and postulate that at least part of this may be due to the drop in sensitivity of the SWP with time. In addition, Finley, Basri & Bowyer (1988) have recently used the smooth, well-defined theoretical continua of 7 white dwarfs to independently estimate corrections to the combined SWP/LWR flux calibration. They find that the average correction over the IUE wavelength range is ∼5%, but can be as large as 20% in individual 5 Å wavelength bins close to the overlap range.

Long exposures of faint sources can also often reveal a number of residual camera artifacts. These are further discussed in Chapter 4.
Chapter 3

EXOSAT Observations and Results

Overview

In this chapter we report the results from a total of 42 EXOSAT observations of the five X-ray bright BL Lac objects Mrk 421, Mrk 501, 1218+304, Mrk 180 and 0414+009 made during the 3 year operational lifetime of the satellite. Preliminary results have been previously presented by ourselves (Warwick, McHardy & Pounds 1984; Warwick et al. 1986; George, Warwick & Bromage 1987; George, Warwick & Bromage 1988a; George, Warwick & McHardy 1988) and others (Brodie, Bowyer & Tennant 1987).

3.1 Introduction

Measurements were made as described in Chapter 2, in the 0.03–2.0 keV energy range using the EXOSAT LE telescope in association with the CMA and typically three filters, and in the 1–10 keV range using the ME detector array. The standard ME observing mode for AGN was generally employed, with one half of ME
detector array offset by ~2° from the source position to monitor the background; the detector halves being interchanged every ~10⁴ s throughout the observation to facilitate background subtraction. In the few observations where the detectors were not interchanged, the ME background subtraction was performed using data recorded during slew manoeuvres both before and after the observation.

The results from the individual sources are discussed and compared to previous X-ray observations in Sections 3.2–3.6 and the general findings summarized in Section 3.7

### 3.2 Markarian 421 (Mrk 421)

Over the period January 1984 to June 1985 EXOSAT observed Mrk 421 on a total of 14 occasions. Table 3.1 provides a log of the observations and a summary of the average count rates obtained (with 1σ statistical errors). The values for the LE correspond to the total count rate (in count s⁻¹) within a 100 x 100 arcsec region centred on the peak source response after applying background, point response, scattering and dead-time corrections (see Chapter 2). The ME count rates are the averages per detector for each observation in channels 8-24 corresponding to an energy range 2–6 keV; again after background and dead-time corrections have been applied.

The most obvious feature from Table 3.1 is the dramatic brightening of Mrk 421 in 1984 December in both the LE and ME bands. This X-ray outburst had completely decayed by the time of the next observation in 1985 January. During the subsequent observations in 1985 April/May the source remained at a level similar to that observed in 1984 February which we will refer to as the low or quiescent state of the source. It should be noted that the outburst was stronger in the ME (~10x increase) than in the LE (~3x increase). Lower amplitude variability is also present between the low state observations.
In Fig. 3.1 we show that variability is also apparent within the individual observations. The lower panel in each plot shows the background subtracted count rate (binned up on 3000 s timescale), whilst the upper panel shows the simultaneous background data from the off-set half (on the same scale, but with an arbitrary absolute level) for direct comparison. It should be noted that the source data for this and the other sources below have not been corrected for deadtime and difference spectra etc, hence the slight discrepancy between the mean plotted count rate and those given in the observing logs; in no case does this correction exceed ±10%. In Chapter 5 we are primarily interested in large amplitude variability (> 30% say). The light curves were searched for such variability by binning the individual data points (with an intrinsic 10 s resolution) up employing a variety of binsizes in the range 10–3000 s, and testing each against a constant flux hypothesis. We found no evidence for isolated events which deviated from Gaussian statistics by > 5σ. More common, as is apparent from Fig. 3.1, are longer term trends throughout an observation. In Table 3.2 we summarize the variabili-

### Table 3.1 EXOSAT observing log and count rates for Mrk 421

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Time(UT)</th>
<th>3Lx</th>
<th>AI/P</th>
<th>B</th>
<th>ME chan(8-25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>032/1984</td>
<td>1047-1524</td>
<td>0.450±0.013</td>
<td>0.162±0.004</td>
<td>–</td>
<td>0.240±0.014</td>
</tr>
<tr>
<td>033/1984</td>
<td>1247-1430</td>
<td>0.292±0.010</td>
<td>0.098±0.007</td>
<td>0.028±0.007</td>
<td>0.135±0.011</td>
</tr>
<tr>
<td>035/1984</td>
<td>1230-1610</td>
<td>0.382±0.017</td>
<td>0.140±0.007</td>
<td>0.031±0.005</td>
<td>0.256±0.007</td>
</tr>
<tr>
<td>037/1984</td>
<td>0031-0344</td>
<td>0.441±0.012</td>
<td>0.154±0.007</td>
<td>0.030±0.005</td>
<td>0.251±0.009</td>
</tr>
<tr>
<td>337/1984</td>
<td>1644-2103</td>
<td>0.781±0.039</td>
<td>0.336±0.017</td>
<td>0.091±0.006</td>
<td>1.982±0.010</td>
</tr>
<tr>
<td>338/1984</td>
<td>1849-0250</td>
<td>0.996±0.050</td>
<td>0.424±0.021</td>
<td>0.100±0.006</td>
<td>2.512±0.008</td>
</tr>
<tr>
<td>340/1984</td>
<td>1858-2323</td>
<td>1.011±0.051</td>
<td>0.455±0.023</td>
<td>0.110±0.006</td>
<td>2.505±0.011b</td>
</tr>
<tr>
<td>112/1985</td>
<td>0502-1105</td>
<td>0.515±0.010</td>
<td>0.218±0.007</td>
<td>0.038±0.006</td>
<td>0.537±0.008</td>
</tr>
<tr>
<td>118/1985</td>
<td>1854-0132</td>
<td>0.403±0.009</td>
<td>0.162±0.006</td>
<td>0.030±0.004</td>
<td>0.528±0.007</td>
</tr>
<tr>
<td>126/1985</td>
<td>2150-0512</td>
<td>0.399±0.008</td>
<td>0.162±0.004</td>
<td>0.028±0.004</td>
<td>0.322±0.007</td>
</tr>
<tr>
<td>131/1985</td>
<td>2036-0310</td>
<td>0.453±0.009</td>
<td>0.173±0.006</td>
<td>0.030±0.003</td>
<td>0.481±0.007</td>
</tr>
<tr>
<td>132/1985</td>
<td>1606-1729</td>
<td>0.434±0.012</td>
<td>–</td>
<td>–</td>
<td>0.345±0.030</td>
</tr>
<tr>
<td>141/1985</td>
<td>1326-1932</td>
<td>0.361±0.018</td>
<td>0.140±0.007</td>
<td>0.028±0.003</td>
<td>0.401±0.009b</td>
</tr>
</tbody>
</table>

b. Using corner detectors only.
Figure 3.1 Montage of 2–6 keV ME light curves for Mrk 421 showing in the lower panel (filled circles), the background subtracted count rate, and in the upper panel (open circles), on the same scale the simultaneous background data. The large tickmarks on the time axis are every $10^4$ s. The plots are in chronological order and identified by their day number. A binsize of 3000 s is used in all cases except for days 337, 338 & 340 in which 1000 s is used (it should be noted that these epochs also have a different vertical scale).
Table 3.2 Summary of short timescale variability in Mrk 421

<table>
<thead>
<tr>
<th>Date</th>
<th>ν</th>
<th>$\chi^2_{\nu,B}$</th>
<th>$\chi^2_{\nu,S}$</th>
<th>$Q(\chi^2_{\nu,S} \mid \nu)$</th>
<th>$t_{var}$ (hours)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>032/1984</td>
<td>5</td>
<td>1.315</td>
<td>1.980*</td>
<td>0.078</td>
<td></td>
<td>1.2.</td>
</tr>
<tr>
<td>033/1984</td>
<td>2</td>
<td>2.465</td>
<td>0.959*</td>
<td>&gt;0.500</td>
<td></td>
<td>2.</td>
</tr>
<tr>
<td>035/1984</td>
<td>4</td>
<td>5.157</td>
<td>0.885*</td>
<td>&gt;0.500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>037/1984</td>
<td>3</td>
<td>2.652</td>
<td>1.094</td>
<td>&lt;0.001</td>
<td>6.3</td>
<td>2.</td>
</tr>
<tr>
<td>337/1984</td>
<td>5</td>
<td>2.180</td>
<td>21.385*</td>
<td>&lt;0.001</td>
<td>5.4</td>
<td></td>
</tr>
<tr>
<td>338/1984</td>
<td>9</td>
<td>1.837</td>
<td>40.784*</td>
<td>&lt;0.001</td>
<td>7.1</td>
<td></td>
</tr>
<tr>
<td>340/1984</td>
<td>5</td>
<td>1.358</td>
<td>11.733</td>
<td>&lt;0.001</td>
<td>17.5</td>
<td></td>
</tr>
<tr>
<td>004/1985</td>
<td>5</td>
<td>0.898</td>
<td>4.094</td>
<td>0.001</td>
<td>5.7</td>
<td>2.</td>
</tr>
<tr>
<td>112/1985</td>
<td>7</td>
<td>1.525</td>
<td>2.140</td>
<td>0.036</td>
<td></td>
<td></td>
</tr>
<tr>
<td>118/1985</td>
<td>8</td>
<td>1.049</td>
<td>16.986</td>
<td>&lt;0.001</td>
<td>20.4</td>
<td></td>
</tr>
<tr>
<td>126/1985</td>
<td>8</td>
<td>0.585</td>
<td>1.429</td>
<td>0.178</td>
<td></td>
<td></td>
</tr>
<tr>
<td>131/1985</td>
<td>7</td>
<td>1.262</td>
<td>8.413</td>
<td>&lt;0.001</td>
<td>8.9</td>
<td></td>
</tr>
<tr>
<td>132/1985</td>
<td>1</td>
<td>4.502</td>
<td>0.281</td>
<td>&gt;0.500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>141/1985</td>
<td>8</td>
<td>1.064</td>
<td>2.332</td>
<td>0.017</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
* Straight background subtraction.
1. Excluding detector 5.
2. Correlation between source and background data.

As can be seen from Fig. 3.1 and Table 3.2, there is evidence for large amplitude flux variations in ~50% the EXOSAT ME observations of Mrk 421. It should be noted that there seems to be no difference between the variability observed...
### Table 3.3 Parameters of the best fitting power-law models for Mrk 421

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$F_{2-8}$</th>
<th>$\alpha_s$</th>
<th>$n_H$ free</th>
<th>$\chi^2$/dof</th>
<th>$\alpha_s$</th>
<th>$n_H$ fixed*</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>032/1984</td>
<td>0.99±0.06</td>
<td>1.70±0.15</td>
<td>1.64±0.67</td>
<td>1.62/23</td>
<td>1.73±0.06</td>
<td>1.57/24</td>
<td></td>
</tr>
<tr>
<td>033/1984</td>
<td>0.52±0.04</td>
<td>1.95±0.27</td>
<td>2.27±0.38</td>
<td>1.58/12</td>
<td>1.82±0.04</td>
<td>1.72/13</td>
<td></td>
</tr>
<tr>
<td>035/1984</td>
<td>0.91±0.05</td>
<td>1.67±0.12</td>
<td>1.65±0.41</td>
<td>0.94/24</td>
<td>1.70±0.03</td>
<td>0.92/25</td>
<td></td>
</tr>
<tr>
<td>037/1984</td>
<td>0.93±0.05</td>
<td>1.81±0.15</td>
<td>2.16±0.63</td>
<td>1.31/13</td>
<td>1.74±0.03</td>
<td>2.59/14</td>
<td></td>
</tr>
<tr>
<td>337/1984</td>
<td>8.2±0.41</td>
<td>1.14±0.03</td>
<td>2.13±0.90</td>
<td>1.76/28</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>338/1984</td>
<td>10.2±0.51</td>
<td>1.15±0.02</td>
<td>2.19±0.11</td>
<td>6.04/28</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>340/1984</td>
<td>10.3±0.52</td>
<td>1.25±0.03</td>
<td>2.96±0.41</td>
<td>5.52/28</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>005/1984</td>
<td>1.04±0.05</td>
<td>1.64±0.18</td>
<td>1.61±0.49</td>
<td>0.89/30</td>
<td>1.67±0.04</td>
<td>0.85/31</td>
<td></td>
</tr>
<tr>
<td>112/1985</td>
<td>2.20±0.11</td>
<td>1.55±0.07</td>
<td>2.12±0.36</td>
<td>0.73/27</td>
<td>1.47±0.02</td>
<td>0.94/28</td>
<td></td>
</tr>
<tr>
<td>118/1985</td>
<td>2.16±0.11</td>
<td>1.35±0.07</td>
<td>1.70±0.48</td>
<td>0.68/25</td>
<td>1.26±0.09</td>
<td>0.84/26</td>
<td></td>
</tr>
<tr>
<td>126/1985</td>
<td>1.32±0.07</td>
<td>1.61±0.07</td>
<td>1.90±0.30</td>
<td>0.89/24</td>
<td>1.58±0.07</td>
<td>0.90/25</td>
<td></td>
</tr>
<tr>
<td>131/1985</td>
<td>1.98±0.10</td>
<td>1.37±0.07</td>
<td>1.43±0.42</td>
<td>0.84/29</td>
<td>1.44±0.05</td>
<td>1.35/30</td>
<td></td>
</tr>
<tr>
<td>132/1985</td>
<td>1.37±0.07</td>
<td>1.69±0.33</td>
<td>2.30±1.17</td>
<td>0.42/16</td>
<td>1.58±0.07</td>
<td>0.41/17</td>
<td></td>
</tr>
<tr>
<td>141/1985a</td>
<td>1.66±0.08</td>
<td>1.44±0.03</td>
<td>1.75±0.41</td>
<td>1.29/40</td>
<td>1.43±0.03</td>
<td>1.26/41</td>
<td></td>
</tr>
</tbody>
</table>

a. at a value of $1.73 \times 10^{50} \text{ cm}^{-2}$.
b. 2-6 keV flux in units of $10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$.
c. Simultaneous 4 corner fit.

in the low and high states. Specifically we find that the shortest timescale for significant variability is $t_{var} \sim 5$ hours.

The results from a spectral fitting analysis on the combined LE and ME data for each of the observations is given in Tables 3.3 and 3.4. To allow for variability during an observation, the ratio of the mean ME flux for the whole observation to the mean ME flux measured during the actual LE filter exposure was applied as a scaling factor to each LE count rate. Similar corrections are applied to all other observations reported in this thesis in which we have evidence for significant source variability. On no occasion did this correction exceed 11%. Table 3.3 gives the results of fitting a single power-law spectrum of the form

$$F(E) = C E^{-\alpha_s} e^{-(\sigma(E)n_H)} \text{ keV cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1}$$

[3.1]

to the X-ray data. Here, the effective absorbing column of cold gas is char-
characterized by an equivalent hydrogen column density, $n_H$, assuming elemental abundances and absorption cross-sections, $\sigma(E)$, from Morrison & McCammon (1983). In column 2 of Table 3.3 we list the 2–6 keV flux density, $F_{2-6}$, for each observation and in columns 3–4 the best fitting parameters (with errors at 90% confidence) for a model in which $n_H$ is a free parameter. For the majority of the observations this simple three parameter model gives an adequate fit to the data (the relatively high reduced $\chi^2$ for two of the early observations is most likely due to background subtraction limitations). The three observations made when the source was in its bright state are exceptions however. The discrepancy appears to be due to a deficiency of photons above $\sim$3 keV (see Fig. 3.2 below). Significantly improved fits were obtained by including in the power-law model a high energy exponential cut-off above an energy $E_c$ with a rate $E_r$ such that

$$F(E) = C E^{-\alpha} e^{-(\sigma(E)n_H)E} e^{-\left(E - E_c\right) E_r} \text{ keV cm}^{-2} \text{ s}^{-1} \text{ keV}^{-1} \quad [3.2]$$

for $E \geq E_c$ and as given by equation [3.1] for $E < E_c$. The results of a spectral fitting analysis using this five parameter model are given in Table 3.4, where both $E_c$ and $E_r$ are in keV. A similar model was applied to the high state observations by Brodie, Bowyer & Tennant (1987), but with $E_c = 0$. We find that the inclusion of this extra parameter significantly improves the fits. The individual best fitting incident spectra are shown in Fig. 3.2.

The weighted mean value of $n_H$ determined from the X-ray data (using the values from Table 3.4 for the bright state observations) is $(1.73^{+0.27}_{-0.25}) \times 10^{20}$ atom cm$^{-2}$, which is in excellent agreement with the line-of-sight column density through our own galaxy in the direction of Mrk 421 from the 21 cm measurements of Stark et al. (1988). There is no evidence for any variation of $n_H$ in the EXOSAT observations.

We have repeated the spectral analysis with $n_H$ fixed at the weighted mean value. The results are listed in columns 6-7 of Table 3.3 for the low states and in Table 3.4 in the case of the high state observations. As can be seen, the fits were found to
Figure 3.2 Derived photon spectra for Mrk 421 assuming best fit power-law models. The crosses correspond to the ME data whilst the diamonds are representative of the LE filter measurements. The best-fitting model spectrum is shown dotted.
Figure 3.2 continued.
Figure 3.2 continued.
Table 3.4 Parameters of a power-law plus high energy cut-off model for Mrk 421

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$F_{2-6}$</th>
<th>$\alpha$</th>
<th>$n_H$</th>
<th>$E_c$</th>
<th>$E_r$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>337/1984</td>
<td>8.2$^{+0.4}_{-0.4}$</td>
<td>1.0$^{+0.09}_{-0.15}$</td>
<td>1.5$^{+0.6}_{-0.70}$</td>
<td>3.0$^{+1.56}_{-2.47}$</td>
<td>22.5$^{+29.1}_{-18.0}$</td>
<td>1.27/27</td>
</tr>
<tr>
<td>338/1984</td>
<td>10.4$^{+0.5}_{-0.5}$</td>
<td>0.89$^{+0.12}_{-0.09}$</td>
<td>1.2$^{+0.56}_{-0.58}$</td>
<td>2.7$^{+0.70}_{-0.75}$</td>
<td>11.0$^{+3.5}_{-2.6}$</td>
<td>1.19/27</td>
</tr>
<tr>
<td>340/1984</td>
<td>10.5$^{+0.5}_{-0.5}$</td>
<td>0.91$^{+0.14}_{-0.09}$</td>
<td>1.1$^{+0.70}_{-0.75}$</td>
<td>2.7$^{+0.70}_{-0.75}$</td>
<td>8.2$^{+3.0}_{-2.8}$</td>
<td>1.19/27</td>
</tr>
</tbody>
</table>

a. 2-6 keV flux in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$

b. fixed.
be statistically acceptable in all cases. In Fig. 3.3 we show that there is a close correlation between the best fitting spectral index and the 2–6 keV flux for both those fits with $n_H$ as a free parameter and when fixed at the mean value. Despite a degree of scatter, a rapid decrease in index as the source brightens is apparent when the source is in the low state. In contrast, the index appears to flatten off at $\alpha_x \sim 1$ during the high state observations. The dotted line in Fig. 3.3 shows the best fitting correlation expected assuming the ‘pivot-point’ model discussed in Chapter 5.

**Comparison with previous results.**

Since the detection of a dramatic brightening of Mrk 421 by the Ariel V satellite (Ricketts, Cooke & Pounds 1976), this source has been observed on numerous occasions by the majority of the subsequent X-ray astronomy missions. The results from these observations are summarized in Appendix A (Table A.2). The quality of the data and the information that can be drawn from it varies tremendously
due to the use of different instrumentation covering different (although often overlapping) energy bands and the use of different observing strategies. Furthermore, the substantial systematic uncertainties between the results obtained makes a detailed comparison of these results difficult. Nevertheless, several general conclusions can be reached.

It is clear that Mrk 421 is extremely variable in the X-ray band. The most dramatic individual event remains the 1 day flare superimposed on a longer term trend that led to its detection (Ricketts, Cooke & Pounds 1976). The Ariel V SSI count rate was seen to double in approximately 12 hr and at the peak reached an estimated flux of $5-7 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ (assuming spectral indices of 1 and 2 respectively) in the 2-6 keV band. These values are comparable to or even brighter than the Perseus Cluster, the brightest steady extragalactic object in the 2-6 keV X-ray sky. Subsequent Ariel V SSI observations over a 5 year period revealed a second, weaker flare lasting ~3 days in 1977 December (Marshall, Warwick & Pounds 1981). At the peak this event reached a 2-6 keV flux of $\sim 1.5 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ corresponding to a factor $\sim 4$ increase in the quiescent flux. A factor 2 brightening lasting ~2 days was also observed by the EINSTEIN MPC with a peak flux of $5.6 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, and finally of course, there are the EXOSAT high state observations, lasting at least 3 days with a maximum 2-6 keV flux of $1 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$.

Flares excluded, the data in Table A.2 show no obvious long term trends. Indeed, although there are major problems due to uneven sampling and the use of different instrumentation, the general appearance of the X-ray light curve for Mrk 421 is chaotic and it is difficult to find general trends on any timescales longer than a few days (see Fig. 5.2). This is hardly surprising if significant variability on the timescales reported above is common as indeed seems to be the case. For example, Mrk 421 was seen to vary by as much as a factor 20 by the EINSTEIN MPC (Madejski 1985) with a fastest observed change in count rate of a factor of 2 in 14 hr. However it is interesting that a weighted average flux in the 2-6 keV band of all the SSI data presented in Marshall, Warwick & Pounds (1981) gives a value
3.8 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ which is comparable to a similar average of all our EXOSAT observations (F}_{2-6} \approx 3.6 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}). \text{ Furthermore, a weighted average in the same energy range of all the SSI observations excluding the two flares discussed above gives a quiescent flux of } 1.5 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ which is comparable to a weighted average of the MPC data, } (1.8 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}), \text{ excluding the 1980 brightening, and to a weighted average flux for the EXOSAT low-state observations } (1.3 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}). \text{ Thus the average behaviour of Mrk 421 in the X-ray band seems to be similar during the operation of all three satellites. A useful scenario may therefore be that this object has a quiescent state, yet shows significant variability by up to a factor } \sim 10 \text{ with the occasional dramatic short term flare superimposed. In all, the flux in the } 2-6 \text{ keV band has been seen to vary by at least 2 orders of magnitude and the quiescent flux is } \sim 2\% \text{ that of the peak of the 1975 May flare. From the MPC and EXOSAT observations we estimate that Mrk 421 spends } \sim 30\% \text{ of its time brighter than } 3 \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} .

\text{Significant spectral changes had also been seen prior to EXOSAT (with a variety of spectral indices between 0 and 3 reported), although at least part of this maybe due to the disparate energy ranges of the X-ray detectors used. In the majority of cases the form of the spectrum is consistent with a single power-law, however a more complex two power-law model has occasionally been implied (Mushotzky et al. 1978; Hall et al. 1981; Makino et al. 1987a). Unfortunately the early fits to the spectra were unable to constrain the spectral index very well and it was not clear how the apparent spectral variations observed were related (if at all) to the flux variations. EXOSAT was the first satellite with adequate sensitivity over a broad band of X-ray energies to enable this to be unambiguously attempted.}

3.3 Markarian 501 (Mrk 501)

Mrk 501 was observed on 11 occasions during the EXOSAT mission. Here we report the results from 10 of these observations. A further observation (on 1985,
Table 3.5 EXOSAT observing log and count rates for Mrk 501

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Time(UT)</th>
<th>3Lx</th>
<th>AI/P</th>
<th>B</th>
<th>ME chan(8–25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>032/1984</td>
<td>1717–2221</td>
<td>0.390±0.020</td>
<td>0.171±0.009</td>
<td>0.038±0.006</td>
<td>–</td>
</tr>
<tr>
<td>034/1984</td>
<td>1141–1432</td>
<td>0.373±0.019</td>
<td>0.175±0.010</td>
<td>0.044±0.012</td>
<td>0.782±0.013b</td>
</tr>
<tr>
<td>036/1984</td>
<td>1743–2233</td>
<td>0.376±0.019</td>
<td>0.173±0.007</td>
<td>0.038±0.006</td>
<td>0.761±0.019</td>
</tr>
<tr>
<td>038/1984</td>
<td>1219–1429</td>
<td>0.332±0.017</td>
<td>0.138±0.014</td>
<td>0.036±0.006</td>
<td>0.716±0.014c</td>
</tr>
<tr>
<td>183/1984</td>
<td>2000–2424</td>
<td>0.351±0.018</td>
<td>0.150±0.007</td>
<td>0.031±0.010</td>
<td>0.492±0.020d</td>
</tr>
<tr>
<td>191/1984</td>
<td>0623–0955</td>
<td>0.333±0.017</td>
<td>0.142±0.009</td>
<td>0.033±0.007</td>
<td>0.467±0.017</td>
</tr>
<tr>
<td>201/1984</td>
<td>0609–0954</td>
<td>0.320±0.016</td>
<td>0.132±0.008</td>
<td>0.028±0.005</td>
<td>0.483±0.015</td>
</tr>
<tr>
<td>209/1984</td>
<td>1953–2358</td>
<td>0.322±0.016</td>
<td>0.150±0.008</td>
<td>0.034±0.004</td>
<td>0.553±0.014</td>
</tr>
<tr>
<td>099/1985</td>
<td>0942–1545</td>
<td>0.335±0.017</td>
<td>0.154±0.008</td>
<td>0.030±0.004</td>
<td>0.410±0.008</td>
</tr>
<tr>
<td>074/1986</td>
<td>1317–3902</td>
<td>0.355±0.015</td>
<td>–</td>
<td>–</td>
<td>0.800±0.004*</td>
</tr>
</tbody>
</table>

a. Intense solar flaring: No useful ME data.
b. Xe detectors off during first half of observation.
c. Initial inaccurate pointing.
d. Flaring in detector 2.
e. Flaring in detector 5.

day 207) has been previously reported by Staubert et al. (1986) Table 3.5 provides a log of the observations and a summary of the average count rates in the same format as Table 3.1 and in Fig. 3.4 we show the individual ME (2–6 keV) light curves with a 3000 s binsize.

Intense flaring is apparent in all ME detectors throughout the 032/1984 observation, probably due to intense solar activity. There is therefore no useful ME data. The three LE filter measurements are however unaffected and meaningful count rates were obtained. The Xe ME detectors were switched off for the approximately the first half of the 034/1984 observation. This is apparent from the distinctive step (down, likely to be due to the increase in anti-coincidence rejection rate) evident in the light curves from both the on-source and off-set halves when the Xe detectors were switched back on. The effect on the count rate in channels 8–25 was ~ 5% in both halves. The most satisfactory background subtracted PHA spectrum at this epoch was achieved by use of the second half of the observation (only). From Table 3.5 and Fig.3.4 it is clear that contrary to as has
Figure 3.4 Montage of 2–6 keV light curves for Mrk 501 in the same format as Fig. 3.1.
Table 3.6 Summary of short timescale variability in Mrk 501

<table>
<thead>
<tr>
<th>Date</th>
<th>ν</th>
<th>$x^2_{ν,B}$</th>
<th>$x^2_{ν,S}$</th>
<th>$Q(x^2_{ν,S} \mid ν)$</th>
<th>$t_{var}$ (hours)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>034/1984</td>
<td>3</td>
<td>0.235</td>
<td>0.320</td>
<td>&gt;0.500</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>036/1984</td>
<td>5</td>
<td>0.841</td>
<td>1.675</td>
<td>0.137</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>086/1984</td>
<td>2</td>
<td>0.555</td>
<td>0.191</td>
<td>&gt;0.500</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>183/1984</td>
<td>5</td>
<td>1.532</td>
<td>1.795</td>
<td>0.110</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>191/1984</td>
<td>3</td>
<td>0.174</td>
<td>0.214</td>
<td>&gt;0.500</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>201/1984</td>
<td>3</td>
<td>0.413</td>
<td>0.577</td>
<td>&gt;0.500</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>209/1984</td>
<td>4</td>
<td>0.464</td>
<td>0.225</td>
<td>&gt;0.500</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>099/1985</td>
<td>6</td>
<td>0.931</td>
<td>4.529</td>
<td>&lt;0.001</td>
<td>6.6</td>
<td></td>
</tr>
<tr>
<td>074/1986</td>
<td>24</td>
<td>1.214</td>
<td>2.732*</td>
<td>&lt;0.001</td>
<td>56.9</td>
<td>1.</td>
</tr>
</tbody>
</table>

Notes:
* Straight background subtraction.
1. Rise during first $\sim 2 \times 10^4$ s, consistent with constant count rate thereafter.

been implied previously (Staubert et al. 1986), Mrk 501 does exhibit significant variability on a timescale $\sim$months in both the medium and low energy X-ray bands.

The results from an analysis of the 3000 s ME light curves are given in Table 3.6 in the same format as Table 3.2. Significant variability was detected within only two of the observations. A slow upward drift was observed at the beginning of the 'long look' (074/1986) with a corresponding doubling timescale $\sim$2 days, although after the first $2 \times 10^4$ s, this observation was consistent with a constant flux hypothesis with a reduced Chi-square, $\chi^2 = 1.15$ for 24 degrees of freedom. Statistically a $\sim$7 hr doubling timescale was derived for the 099/1985 observation, although it should be pointed out that this value is based upon an extrapolation of low amplitude ( 25%) variability and hence is possibly misleading.

A number of the LE images of Mrk 501 also show the serendipitous detection of a recently discovered weak source EXO 165228+3930.3 (Bassani et al. 1987), approximately 20 arcmin S of the BL Lac type object. Using the 074/1986 3lx image, we find the centroid of the X-ray emission at RA = 16h 52m 27.0s,
Table 3.7 Parameters of the best fitting power-law models for Mrk 501

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$P_{2-6}$</th>
<th>$\alpha_x$</th>
<th>$n_H$ free</th>
<th>$n_H$ fixed</th>
<th>$\chi^2$/dof</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>034/1984</td>
<td>3.07$^{+0.05}_{-0.05}$</td>
<td>1.40$^{+0.09}_{-0.09}$</td>
<td>3.02$^{+0.49}_{-0.49}$</td>
<td>1.26/24</td>
<td>1.30$^{+0.02}_{-0.02}$</td>
<td>1.48/25</td>
</tr>
<tr>
<td>036/1984</td>
<td>3.02$^{+0.08}_{-0.08}$</td>
<td>1.22$^{+0.11}_{-0.11}$</td>
<td>1.98$^{+0.47}_{-0.47}$</td>
<td>1.08/21</td>
<td>1.28$^{+0.03}_{-0.03}$</td>
<td>1.05/22</td>
</tr>
<tr>
<td>086/1984</td>
<td>2.85$^{+0.06}_{-0.06}$</td>
<td>1.16$^{+0.10}_{-0.10}$</td>
<td>1.69$^{+0.43}_{-0.43}$</td>
<td>1.15/23</td>
<td>1.20$^{+0.03}_{-0.03}$</td>
<td>1.20/24</td>
</tr>
<tr>
<td>183/1984</td>
<td>1.87$^{+0.08}_{-0.08}$</td>
<td>1.43$^{+0.13}_{-0.13}$</td>
<td>2.05$^{+1.00}_{-0.44}$</td>
<td>1.22/76</td>
<td>1.45$^{+0.04}_{-0.04}$</td>
<td>1.21/77</td>
</tr>
<tr>
<td>191/1984</td>
<td>1.81$^{+0.07}_{-0.07}$</td>
<td>1.35$^{+0.13}_{-0.13}$</td>
<td>1.71$^{+0.42}_{-0.53}$</td>
<td>1.17/18</td>
<td>1.44$^{+0.04}_{-0.04}$</td>
<td>1.22/19</td>
</tr>
<tr>
<td>201/1984</td>
<td>1.92$^{+0.06}_{-0.06}$</td>
<td>1.42$^{+0.15}_{-0.15}$</td>
<td>2.31$^{+0.37}_{-0.37}$</td>
<td>0.88/19</td>
<td>1.40$^{+0.05}_{-0.05}$</td>
<td>0.84/20</td>
</tr>
<tr>
<td>209/1984</td>
<td>2.26$^{+0.06}_{-0.06}$</td>
<td>1.38$^{+0.12}_{-0.12}$</td>
<td>2.31$^{+0.37}_{-0.49}$</td>
<td>0.81/22</td>
<td>1.36$^{+0.03}_{-0.03}$</td>
<td>0.78/23</td>
</tr>
<tr>
<td>099/1985</td>
<td>1.68$^{+0.03}_{-0.03}$</td>
<td>1.57$^{+0.11}_{-0.04}$</td>
<td>2.62$^{+0.79}_{-0.34}$</td>
<td>1.10/19</td>
<td>1.51$^{+0.03}_{-0.04}$</td>
<td>1.15/20</td>
</tr>
<tr>
<td>074/1986</td>
<td>3.37$^{+0.02}_{-0.02}$</td>
<td>1.25$^{+0.04}_{-0.03}$</td>
<td>2.34$^{+0.48}_{-0.31}$</td>
<td>1.38/64</td>
<td>1.24$^{+0.02}_{-0.03}$</td>
<td>1.36/65</td>
</tr>
</tbody>
</table>

a. at a value of $2.21 \times 10^{20}$ cm$^{-2}$.
b. 2-6 keV flux in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$
c. Simultaneous 4 corner fit.
d. Simultaneous 3 corner fit.

dec = +39d 30m 33.7s with an error radius of $\sim 20$ arcsec, consistent with that reported by Bassani et al. from the independent analysis of the EXOSAT AO-1 data. Inspection of the sum-signal distribution indicates no significant ultra-violet excess, thus we conclude this source is unlikely to stellar in origin. Bassani et al. note that the same object was also detected serendipitously by the EINSTEIN IPC in 1980 January and 1980 August, although never reported in the literature. The IPC observations show no evidence for variability in a 7 month timescale. We found a 3lx count rate of $0.005^{+0.002}_{-0.002}$ count s$^{-1}$, thus although this source is within the ME field-of-view, it unlikely that it significantly contributes to the observed ME flux. It is possible that this source maybe associated with the far infra-red source IRAS 16524+39.5 (Bassani et al. 1986).

The results from a spectral fitting analysis on the combined LE and ME data for each observation is given in Table 3.7 and the individual fits are shown in Fig. 3.5. In all cases a simple power-law model as described by equation [3.1] was found to be statistically acceptable. The weighted mean value of the equivalent hydrogen column density, $n_H$, is $(2.21^{+0.39}_{-0.39}) \times 10^{20}$ atom cm$^{-2}$. This is in good
agreement with the line-of-sight column density through our own galaxy from 21 cm measurements (Stark et al. 1988).

In Table 3.7, columns 6–7 we list the results from repeating the spectral fitting analysis with $n_H$ fixed at the weighted mean value. In all cases the fits were again found to be statistically acceptable although the 074/1986 observation shows evidence for a deficit of high energy photons above ~6 keV (c.f. the high state observations of Mrk 421 above).

From the spectral analysis in which $n_H$ is a free parameter there is marginal evidence for a correlation between $\alpha_x$ and the 2–6 keV flux (Fig. 3.6a), which is strengthened when $n_H$ is fixed (Fig. 3.6b).

The results reported here for Mrk 501 in 1984 June/July are in fair agreement with those reported by Staubert et al. (1986) for a further EXOSAT observation of this source made on July 25, 1984 (day 207). Staubert et al. found a 2–6 keV flux of $(2.2\pm0.2) \times 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1}$ and spectral analysis of the combined LE and ME data set obtained a best fitting power-law index of $1.53\pm0.11$ with an assumed equivalent column density of $3.1\pm0.9 \times 10^{20} \text{ cm}^{-2}$.

Comparison with previous results.

Following a cross-correlation between the Fourth UHURU Catalogue of X-ray sources and existing catalogues suggesting that the newly discovered source may be associated with the radio loud BL Lac type object Mrk 501 (Forman et al. 1978), the identification was finally confirmed using the HEAO-1 SMC in 1977 August (Schwartz et al. 1978). Numerous subsequent observations (Appendix A, Table A.3) have shown 2–6 keV flux densities in the range $1.7–6.5 \times 10^{-11} \text{ erg s}^{-1} \text{ cm}^{-2}$ (corresponding to a total dynamic range of ~2–3), with no obvious long-term trends. From a study of EINSTEIN IPC data, Schwartz, Madejski & Ku (1983) concluded that at the 96% confidence level, the X-ray emission from Mrk 501 was variable, however it should be noted that this has now been ascribed to an IPC gain fluctuation and is no longer considered real (Madejski
Figure 3.5 Derived photon spectra for Mrk 501 in same format as Fig. 3.2.
Figure 3.5 continued
Figure 3.5 continued

Figure 3.6 $F_{2-6}$ (in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$) versus $\alpha_s$ for Mrk 501 in the same format as Fig. 3.3 where $n_H$ is a) a free parameter, and b) fixed at $2.21 \times 10^{20}$ cm$^{-2}$. 

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1985). Prior to EXOSAT, the only reported instance of inter-observational variability was the factor 2 decrease in flux in the 0.15–0.5 keV band on a timescale of two days observed with the HEAO-1 LED (Singh & Garmire 1985) The spectral data in most cases is well represented by a single power-law of index ~1.3–1.6. However on three occasions a more complex spectrum has been implied. In 1977 August the combined HEAO-1 A-2 and A-3 observations implied an extremely steep spectrum (with $\alpha \sim 2.4$) at low energies (Singh & Garmire 1985), flattening to $\alpha \sim 0.2–0.8$ above 2 keV (Mushotzky et al. 1978 and Schwartz et al. 1978 – but see also Snijders et al. 1979); and a very flat low energy spectrum was implied by the EINSTEIN SSS and IPC observations of 1979 March (Urry, Mushotzky & Holt 1986) and 1980 August (Mufson et al. 1984) respectively. Nevertheless the general impression is that Mrk 501 is a relatively sedate BL Lac type object in the X-ray band.

### 3.4 1218+304 ($\equiv$2A 1219+305)

EXOSAT observed 1218+304 on 9 occasions as listed in Table 3.8. In Fig. 3.7 we show the individual ME (2–6 keV) light curves binned up on 3000 s timescale and the results from an analysis of the individual light curves presented in Table 3.9.

It is clear that whilst 1218+304 exhibits variability between individual observations, statistically significant intra-observational variations are relatively rare. Indeed there are only two clear examples of large amplitude variability within the EXOSAT observations. During the 031/1984 observation the 2–6 keV flux was seen to increase up a factor 25% in ~50 minutes corresponding to a doubling timescale of ~3 hr, and variability on a similar timescale was also observed towards the end of the 014/1986 observation. The gradual rise apparent during the AO1 observations (50% over 5 days) and the 50% decrease between the 020/1985 and 029/1985 epochs suggests that a timescale of the order of days-weeks may be more common.
Table 3.8 EXOSAT observing log and count rates for 1218+304

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Time(UT)</th>
<th>3Lx</th>
<th>AI/P</th>
<th>B</th>
<th>ME chans(8-25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>031/1984</td>
<td>2324-2720</td>
<td>0.183±0.009</td>
<td>0.086±0.006</td>
<td>0.018±0.005</td>
<td>0.606±0.008</td>
</tr>
<tr>
<td>033/1984</td>
<td>1656-2042</td>
<td>0.207±0.014</td>
<td>0.094±0.012</td>
<td>0.024±0.004</td>
<td>0.696±0.008</td>
</tr>
<tr>
<td>035/1984</td>
<td>1747-2153</td>
<td>0.212±0.013</td>
<td>0.112±0.007</td>
<td>0.031±0.005</td>
<td>0.783±0.008</td>
</tr>
<tr>
<td>037/1984</td>
<td>0610-1026</td>
<td>0.233±0.008</td>
<td>0.120±0.007</td>
<td>0.028±0.005</td>
<td>0.773±0.009</td>
</tr>
<tr>
<td>356/1984</td>
<td>1820-2215</td>
<td>0.193±0.008</td>
<td>0.093±0.004</td>
<td>0.017±0.004</td>
<td>0.566±0.011</td>
</tr>
<tr>
<td>005/1985</td>
<td>1423-1725</td>
<td>0.183±0.009</td>
<td>0.093±0.007</td>
<td>0.027±0.005</td>
<td>0.522±0.009</td>
</tr>
<tr>
<td>020/1985</td>
<td>0244-0556</td>
<td>0.192±0.011</td>
<td>0.105±0.007</td>
<td>0.027±0.005</td>
<td>0.602±0.011</td>
</tr>
<tr>
<td>029/1985</td>
<td>0956-1329</td>
<td>0.165±0.007</td>
<td>0.091±0.006</td>
<td>0.015±0.005</td>
<td>0.380±0.011</td>
</tr>
<tr>
<td>014/1986</td>
<td>1016-2239</td>
<td>0.123±0.003</td>
<td>0.062±0.005</td>
<td>0.012±0.002</td>
<td>0.370±0.010</td>
</tr>
</tbody>
</table>

a. Flaring in detector 5.

Table 3.9 Summary of short timescale variability in 1218+304

| Date     | \(\nu\) | \(x_{\nu,B}^2\) | \(x_{\nu,S}^2\) | \(Q(x_{\nu,S}^2 | \nu)\) | \(t_{var}\) (hours) | Notes |
|----------|--------|----------------|----------------|-----------------|------------------|--------|
| 031/1984 | 4      | 3.788          | 7.682*         | <0.001          | 3.3              | 1.     |
| 033/1984 | 4      | 0.648          | 0.459          | >0.500          | -                | -      |
| 035/1984 | 4      | 2.268          | 1.057*         | 0.376           | -                | 1.     |
| 037/1984 | 4      | 0.380          | 1.543          | 0.187           | -                | -      |
| 356/1984 | 5      | 0.860          | 0.714          | >0.500          | -                | -      |
| 005/1985 | 5      | 0.864          | 3.007          | 0.010           | -                | -      |
| 020/1985 | 3      | 1.357          | 1.010*         | 0.387           | -                | -      |
| 029/1985 | 4      | 1.171          | 3.351          | 0.009           | -                | -      |
| 014/1986 | 14     | 0.616          | 3.174          | <0.001          | 3.8              | -      |

Notes:
* Straight background subtraction.
1. Degree of anti-correlation between source and background data.
Figure 3.7 Montage of 2–6 keV light curves for 1218+304 in the same format as Fig. 3.1.
Table 3.10 Parameters of the best fitting power-law models for 1218+304

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$F_{2-6}^e$</th>
<th>$\alpha_s$</th>
<th>$n_H$ free</th>
<th>$x_1^2$/dof</th>
<th>$n_H$ fixed$^a$</th>
<th>$x_2^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>031/1984</td>
<td>2.38±0.03</td>
<td>1.26±0.08</td>
<td>3.79±0.94</td>
<td>2.02/11</td>
<td>3.08±0.04</td>
<td>1.95/22</td>
</tr>
<tr>
<td>033/1984</td>
<td>2.73±0.03</td>
<td>1.20±0.07</td>
<td>3.20±0.83</td>
<td>1.73/32</td>
<td>3.08±0.04</td>
<td>1.95/22</td>
</tr>
<tr>
<td>035/1984</td>
<td>3.09±0.03</td>
<td>1.14±0.06</td>
<td>3.02±0.76</td>
<td>0.94/25</td>
<td>3.08±0.04</td>
<td>1.95/22</td>
</tr>
<tr>
<td>037/1984</td>
<td>3.08±0.04</td>
<td>1.15±0.05</td>
<td>2.72±0.42</td>
<td>1.19/32</td>
<td>3.08±0.04</td>
<td>1.95/22</td>
</tr>
<tr>
<td>356/1984</td>
<td>2.32±0.03</td>
<td>1.33±0.11</td>
<td>4.14±1.28</td>
<td>1.97/22</td>
<td>3.29±0.04</td>
<td>2.33/23</td>
</tr>
<tr>
<td>005/1985</td>
<td>2.11±0.04</td>
<td>1.30±0.10</td>
<td>3.37±1.01</td>
<td>1.37/18</td>
<td>3.29±0.04</td>
<td>2.33/23</td>
</tr>
<tr>
<td>020/1985</td>
<td>2.43±0.04</td>
<td>1.29±0.10</td>
<td>3.50±1.18</td>
<td>0.54/22</td>
<td>3.29±0.04</td>
<td>2.33/23</td>
</tr>
<tr>
<td>029/1985</td>
<td>1.53±0.04</td>
<td>1.44±0.14</td>
<td>3.56±1.37</td>
<td>0.76/21</td>
<td>3.29±0.04</td>
<td>2.33/23</td>
</tr>
<tr>
<td>014/1986$^d$</td>
<td>1.38±0.03</td>
<td>1.16±0.13</td>
<td>2.34±0.42</td>
<td>2.12/27</td>
<td>1.28±0.04</td>
<td>2.19/28</td>
</tr>
</tbody>
</table>

a. at a value of $3.11 \times 10^{20}$ cm$^{-2}$.
b. 2–6 keV flux in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$.
c. Possible significant contribution from Mrk 766.
d. Simultaneous 3 corner fit.

The results from a spectral fitting analysis on the combined LE and ME data for each observation is given in Table 3.10 and the individual fits are shown in Fig. 3.8. In the majority of cases a simple power-law model as described by equation [3.1] was found to provide a statistically acceptable fit to the combined LE and ME data. However on four occasions such a model resulted in a reduced Chi-square values of $\chi^2 \sim 2$ or more. These less satisfactory fits will be discussed further below.

The weighted mean value of the equivalent hydrogen column density, $n_H$, is $(3.11\pm0.50) \times 10^{20}$ atom cm$^{-2}$. As will be discussed further in Section 3.7.3, this is slightly higher than the line-of-sight column density through our own galaxy calculated from the 21 cm measurements of Stark et al. (1988). In Table 3.10 (columns 6–7), we also list the results from repeating the spectral fitting analysis with $n_H$ fixed at the weighted mean of the EXOSAT values. In the majority of cases the fits were again found to be statistically acceptable. The derived spectral indices are plotted in Fig. 3.9 against 2–6 keV flux. Again we find some indication of a degree of correlation between these parameters, particularly if the lowest flux
point from the 1986/014 epoch is neglected (see below).

3.4.1 Serendipitous detection of Mrk 766 and ON 325

A difficulty arose in the analysis of the ME data from 1218+304 due to the proximity of the Seyfert I galaxy Mrk 766 (Weedman, 1978) and the BL Lac object ON 325 (Craine, 1977), both of which are known X-ray sources (Kriss, Canizares & Ricker, 1980; Maccagni et al., 1984). Mrk 766 lies 49 arcmin from 1218+304 and clearly visible in all the LE images, whilst ON 325 lies 53 arcmin from 1218+304 and can be detected in several of the images (see Fig. 2.2). Unfortunately, since both objects lie towards the edge of the LE field of view, in regions where the point spread function is wide and ill-determined, only relatively crude estimates of their count rates were possible. These are listed in Table 3.11. We find that the corrected 3lx count rate of Mrk 766 varies from 0.1 to 0.3 count s\(^{-1}\) between individual observations. Similarly ON 325 is also found to be variable, but with a significantly weaker 3lx count rate ranging from <0.02 to 0.08 count s\(^{-1}\). Photon statistics generally prevented any search for variability on shorter timescales, although there is marginal evidence for variations on a timescale of \(\sim 10^3\) s in Mrk 766 during the 1986 observation. Preliminary analysis of a 56 hr pointed observation of Mrk 766 in late 1985 carried out by the EXOSAT observatory team indicates that this source is indeed variable on a timescale of \(\sim 10^3\) s in the 3lx band within a range of count rates consistent with our findings (P.Barr, private communication). During the the 1218+304 observations Mrk 766, and on one occasion ON 325, will be within the the ME field-of-view and may therefore contribute to the observed ME flux. After correcting for the ME beam response and assuming a steep power-law spectrum with an energy index of \(\sim 2\) (Maccagni et al., 1984) and a column density of \(3.1 \times 10^{20}\) cm\(^{-2}\) as derived above, we estimate (Table 3.11) that ON 325 contributes less than 2\% to the observed 2–6 keV flux. In the case of Mrk 766, if we assume a single power-law spectral form with an energy index \(\sim 1\) and a column \(\sim 2 \times 10^{20}\) cm\(^{-2}\) (P.Barr, private communication), we estimate that in the worse case (days 031/1984 and
Figure 3.8 Derived photon spectra for 1218+304 in same format as Fig. 3.2.
Figure 3.8 continued
Figure 3.8 continued

Figure 3.9 $F_{2-6}$ (in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$) versus $\alpha_*$ for 1218+304 in the same format as Fig. 3.3 where $n_H$ is a) a free parameter, and b) fixed at $3.11 \times 10^{20}$ cm$^{-2}$.
Table 3.11 Mrk 766 and ON 325 count rates and fluxes

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Mrk 766</th>
<th>ON 325</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3Lx&lt;sup&gt;a&lt;/sup&gt;</td>
<td>3Lx&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>count s&lt;sup&gt;-1&lt;/sup&gt;</td>
<td>F&lt;sub&gt;2−6&lt;/sub&gt;</td>
</tr>
<tr>
<td>031/1984</td>
<td>0.23±0.04</td>
<td>0.72</td>
</tr>
<tr>
<td>033/1984</td>
<td>0.28±0.04</td>
<td>0.64</td>
</tr>
<tr>
<td>035/1984</td>
<td>0.28±0.04</td>
<td>0.62</td>
</tr>
<tr>
<td>037/1984</td>
<td>0.16±0.04</td>
<td>0.38</td>
</tr>
<tr>
<td>036/1984</td>
<td>0.22±0.04</td>
<td>0.21</td>
</tr>
<tr>
<td>005/1985</td>
<td>0.22±0.04</td>
<td>0.44</td>
</tr>
<tr>
<td>020/1985</td>
<td>0.30±0.04</td>
<td>0.33</td>
</tr>
<tr>
<td>029/1985</td>
<td>0.12±0.02</td>
<td>0.15</td>
</tr>
<tr>
<td>014/1986</td>
<td>0.32±0.04</td>
<td>0.41</td>
</tr>
</tbody>
</table>

<sup>a</sup> observed count rate in LE.  
<sup>b</sup> expected 2–6 keV flux in units of 10<sup>-11</sup> erg cm<sup>-2</sup> s<sup>-1</sup>  
<sup>c</sup> outside ME field of view.
014/1986), Mrk 766 could have contributed ~30% of the total observed 2–6 keV flux. We note that at these epochs statistically poorer single power-law spectral fits were obtained and have investigated the possibility that high values of $\chi^2$ are a result of a significant contribution from Mrk 766. We find however, that repeat spectral analysis of the data with the inclusion of a second power-law component does not lead to a statistically significant improved spectral fits (although on day 014/1986, a steepening of the spectrum of 1218+304 by $\Delta \alpha \leq 0.1$ and a 30% reduction in the 2–6 keV flux was required, which we note does improve the correlation between 2–6 keV flux and spectral index for 1218+304 shown in Fig. 3.9).

In the absence of a more definitive spectrum for Mrk 766 and ON 325, we have neglected these sources in the further analysis and assumed that the observed ME spectra (as listed in Tables 3.8 and 3.10) are due to 1218+304 alone. Such an approach is supported by the fact that the observed variations in 2–6 keV flux are uncorrelated with the predicted variations from Mrk 766 (based upon its LE count rate) and by the lack of a flattening of the ME spectrum at high energies.

Visual inspection of the data reveals that a more likely cause of the less satisfactory fits is that due to background subtraction limitations. In the case of the 356/1984 observation, the $\chi^2$ for a simple power-law fit can be greatly improved if Al/P and Bor filter count rates are neglected in the spectral fitting, giving best fitting parameters of $\alpha_x = 1.44^{+0.11}_{-0.05}$, $n_H = 4.59^{+1.25}_{-0.88} \times 10^{20} \text{ cm}^{-2}$ with $\chi^2 = 1.162$ for 20 degrees of freedom. In addition there is slight evidence for a steepening of the spectrum above ~5 keV at this epoch.

Comparison with previous results.

The detection of a unidentified variable source designated 2A 1219+305 in the second Ariel V catalogue (Cooke et al. 1978) led to an optical and radio search of the error box (Wilson et al. 1979). The favoured candidate (1218+304) was a weak, flat spectrum radio source lying just outside the 90% confidence error box with an intense blue power-law optical continuum and no emission or absorption
lines – the classical hallmarks of a BL Lac type object. Firm identification was provided in 1978 May by Schwartz et al. (1979) using the Scanning Modulation Collimator (SMC) on board the HEAO-1 satellite, thus 1218+304 was the first example of a BL Lac type object discovered through its X-ray properties. A history of these and subsequent X-ray observations is given in Appendix A, Table A.4.

It is clear that 1218+304 exhibits significant variability in the 2–6 keV band. Indeed the historic 2–6 keV light curve of 1218+304 is somewhat similar to that of Mrk 421. The majority of observations indicate a flux < $5 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$, however Ariel V detected 3 outbursts each lasting 2–3 days and rising to ~3–4 times the quiescent flux, the largest peaking at ~$2 \times 10^{-10}$ erg cm$^{-2}$ s$^{-1}$ in the 2–6 keV band (Marshall, Warwick & Pounds 1981). At the beginning of 1976 the source suddenly appeared to enter a 1.5 year low state with a 2–6 keV flux ~ $1 \times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$. Following a period of renewed activity during 1978-9, the source again appeared to be in the low state during the (relatively few) observations carried out using the EINSTEIN MPC. Despite inter-observational variations, the EXOSAT data has the general appearance of a gradual decline in flux. The total dynamic range of the 2–6 keV flux from 1218+304 is ~15.

Prior to EXOSAT dramatic variations spectral index had been reported, although their relationship with the observed changes in flux were very unclear: on some occasions significant spectral variability with minimal changes in flux have been observed, whilst between other epochs the reverse appears to be true. Interestingly, the wide range of indices reported in previous observations ($\alpha \sim 0.4$–3.0) contrasts with the relatively small range of indices seen with EXOSAT. It is difficult to estimate the likely effects of contamination from Mrk 766 and ON 325 on the earlier measurements, however they are likely to be minimal for scanning observations.
Table 3.12 EXOSAT observing log and count rates for Mrk 180

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Time(UT)</th>
<th>3Lx</th>
<th>Al/P</th>
<th>B</th>
<th>ME chans(8-25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>308/1984</td>
<td>0148-0530</td>
<td>0.051±0.004</td>
<td>0.021±0.003</td>
<td>0.003±0.002</td>
<td>0.034±0.009</td>
</tr>
<tr>
<td>317/1984</td>
<td>2050-2330</td>
<td>0.057±0.006</td>
<td>0.018±0.003</td>
<td>0.003±0.002</td>
<td>&lt; 0.030</td>
</tr>
<tr>
<td>324/1984</td>
<td>0631-0928</td>
<td>0.061±0.003</td>
<td>-</td>
<td>-</td>
<td>&lt; 0.030</td>
</tr>
<tr>
<td>333/1984</td>
<td>0444-0920</td>
<td>0.088±0.006</td>
<td>0.042±0.004</td>
<td>0.006±0.002</td>
<td>0.067±0.008</td>
</tr>
<tr>
<td>093/1985</td>
<td>1635-2006</td>
<td>0.169±0.011</td>
<td>0.068±0.006</td>
<td>0.012±0.003</td>
<td>0.204±0.010</td>
</tr>
</tbody>
</table>

Table 3.1 Summary of short timescale variability in Mrk 180

| Date    | ν  | $\chi^2_B$ | $\chi^2_S$ | $Q(\chi^2_{S,B} | \nu)$ | $t_{var}$ (hours) | Notes |
|---------|----|------------|------------|-----------------|-------------------|-------|
| 308/1984| 4  | 0.704      | 0.510      | >0.500          | -                 | -     |
| 333/1984| 5  | 0.607      | 1.313      | 0.255           | -                 | -     |
| 093/1985| 4  | 1.646      | 3.245      | 0.007           | -                 | -     |

Notes:
* Straight background subtraction.

3.5 Markarian 180 (Mrk 180)

Mrk 180 was observed at 5 epochs, however only significantly detected in the ME on 3 occasions. From the log of the observations listed in Table 3.12 it is clear that Mrk 180 exhibited dramatic variability between observations in both the LE and ME bands. However, as summarized in Table 3.13, with the signal-to-noise obtainable with the ME, we were unable to detect significant intra-observational variability on timescales $\lesssim 10^4$ s.

In Table 3.14 the results from the spectral analysis of the observations are pre-
Table 3.14 Parameters of the best fitting power-law models for Mrk 180

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>$F_{2-6}$</th>
<th>$\alpha_s$</th>
<th>$n_H$ free $\times 10^{20}\text{cm}^{-2}$</th>
<th>$x^2$/dof</th>
<th>$n_H$ fixed$^a$ $\alpha_s$</th>
<th>$x^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>308/1984</td>
<td>0.13$^{+0.04}_{-0.03}$</td>
<td>1.86$^{+0.56}_{-0.56}$</td>
<td>2.33$^{+2.40}_{-1.77}$</td>
<td>0.56/3</td>
<td>1.84$^{+0.32}_{-0.14}$</td>
<td>0.42/4</td>
</tr>
<tr>
<td>317/1984</td>
<td>$&lt; 0.2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$&gt; 1.8$</td>
<td>-</td>
</tr>
<tr>
<td>324/1984</td>
<td>$&lt; 0.2$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$&gt; 1.8$</td>
<td>-</td>
</tr>
<tr>
<td>333/1984</td>
<td>0.29$^{+0.04}_{-0.03}$</td>
<td>1.48$^{+0.39}_{-0.38}$</td>
<td>2.33$^{+2.22}_{-1.49}$</td>
<td>0.62/13</td>
<td>1.69$^{+0.11}_{-0.08}$</td>
<td>0.48/14</td>
</tr>
<tr>
<td>093/1985</td>
<td>0.84$^{+0.04}_{-0.07}$</td>
<td>1.44$^{+0.22}_{-0.07}$</td>
<td>2.00$^{+1.15}_{-0.79}$</td>
<td>1.02/16</td>
<td>1.46$^{+0.05}_{-0.06}$</td>
<td>0.88/18</td>
</tr>
</tbody>
</table>

$^a$ at a value of $2.11 \times 10^{20} \text{cm}^{-2}$.

$^b$ 2–6 keV flux in units of $10^{-11} \text{erg cm}^{-2} \text{s}^{-1}$

sent and in Fig. 3.10 we display the best fitting model and data for each observation. For the epochs when Mrk 180 was not detected in the ME instrument only upper limits on the spectral index are possible. In all cases a simple power-law of the form given by equation \([3.1]\) proved to be statistically acceptable.

The best fitting values for the column density, $n_H$, in Table 3.14 are relatively ill-determined, however we find a weighted mean value of $2.11 \times 10^{20} \text{cm}^{-2}$. Repeated spectral analysis assuming a simple power-law model with the column fixed at this value also provide statistically acceptable fits. In Fig. 3.11 we plot the derived spectral indices versus 2–6 keV flux. It can seen when $n_H$ is fixed there is evidence for spectral variability consistent with a flattening of the spectrum as the source brightens.

Comparison with previous results.

X-ray emission within 30 arcsec of optical nucleus of the known BL Lac type object Mrk 180 was first discovered from an EINSTEIN IPC image taken in 1979 November. An earlier detection in 1977 November was then identified in the HEAO-1 A-1 archival records (Hutter & Mufson 1981). The best fitting IPC (0.5–2.8 keV) spectrum (see Appendix A, Table A.5) had a relatively flat index of $1.3^{+0.3}_{-0.2}$ and a column density of $2.5^{+0.7}_{-0.5} \times 10^{20} \text{cm}^{-2}$ in good agreement with
Figure 3.10 Derived photon spectra for Mrk 180 in same format as Fig. 3.2.
the average EXOSAT value. Simultaneous MPC data (1.2–6.6 keV) however implied a much steeper slope of ~3 (Mufson & Hutter 1981; Mufson et al. 1984; Madejski 1985). These MPC data also revealed that Mrk 180 was at a similar intensity level as 2 years previously. A further EINSTEIN IPC observation in 1980 December indicated that the source had weakened by a factor ~2 in the low energy band. The best fitting spectrum was fairly ill-determined with an index of $0.7^{+1.0}_{-0.2}$ and a column roughly consistent with the EXOSAT value. MPC data indicated that the source had weakened by a similar factor at higher energies also, but with little change in spectral index. Hence both EINSTEIN observations imply a substantial steepening of the spectrum with increasing energy from ~1 to ~3–4. In contrast the EXOSAT data presented above are all adequately satisfied by a simple power-law model over a similar energy range with a variable spectral index of ~1.5–2.
Table 3.15 EXOSAT observing log and count rates for 0414+009

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Time(UT)</th>
<th>3Lx</th>
<th>AI/P</th>
<th>B</th>
<th>ME chan(6-25)</th>
</tr>
</thead>
<tbody>
<tr>
<td>253/1984</td>
<td>0923-1331</td>
<td>0.028±0.004</td>
<td>0.019±0.003</td>
<td>0.015±0.003</td>
<td>0.409±0.013</td>
</tr>
<tr>
<td>258/1984</td>
<td>0734-1148</td>
<td>0.031±0.003</td>
<td>0.024±0.002</td>
<td>-</td>
<td>0.513±0.011</td>
</tr>
<tr>
<td>266/1984</td>
<td>1007-1402</td>
<td>0.034±0.004</td>
<td>0.013±0.002</td>
<td>-</td>
<td>0.247±0.014</td>
</tr>
<tr>
<td>274/1984</td>
<td>0843-1216</td>
<td>0.032±0.003</td>
<td>0.023±0.003</td>
<td>0.004±0.003</td>
<td>0.249±0.011</td>
</tr>
</tbody>
</table>

Table 3.16 Summary of short timescale variability in 0414+009

| Date     | ν | $x^2_{\nu,B}$ | $x^2_{\nu,S}$ | $Q(x^2_{\nu,S} | \nu)$ | $t_{var}$ (hours) | Notes |
|----------|---|---------------|---------------|----------------|-------------------|-------|
| 253/1984 | 5 | 0.082         | 4.896         | <0.001         | 3.2               |
| 258/1984 | 5 | 1.241         | 3.058         | 0.009          | -                 |
| 266/1984 | 4 | 2.285         | 3.068*        | 0.011          | -                 |
| 274/1984 | 4 | 0.200         | 2.681         | 0.024          | -                 |

Notes:
* Straight background subtraction.

3.6 0414+009

The source 0414+009 was observed by EXOSAT on four occasions separated by about a week in 1984 September. Each observation lasted ~10^4 s and the observing log is given in Table 3.15.

Significant intra-observational ME variability was detected during only the first observation (Table 3.16), although the individual light curves (Fig. 3.12) do seem to indicate the likely presence of slow trends. Specifically we find a minimum doubling timescale ~3 hours.
Figure 3.12 Montage of 2-6 keV light curves for 0414+009 in the same format as Fig. 3.1.

The results from a spectral fitting analysis of the combined LE and ME data for each of the observations is shown in Table 3.17. In all cases a simple power-law spectrum of the form given by equation [3.1] was found to be statistically acceptable both when \( n_H \) was a free parameter (columns 3–5) and when \( n_H \) was fixed at the mean value of \( (1.10 \pm 0.48) \times 10^{21} \) atom cm\(^{-2} \). The data and best fitting spectra are shown in Fig. 3.13.

Unfortunately the short LE observations resulted in relatively ill determined col-

Table 3.17 Parameters of the best fitting power-law models for 0414+009

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>( F_{2-6} )</th>
<th>( \alpha_x )</th>
<th>( n_H ) free ( \times 10^{21} \text{ cm}^{-2} )</th>
<th>( \chi^2/\text{dof} )</th>
<th>( n_H ) fixed ( ^a )</th>
<th>( \alpha_x )</th>
<th>( \chi^2/\text{dof} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>253/1984</td>
<td>1.67( ^{+0.05}_{-0.10} )</td>
<td>0.90( ^{+0.19}_{-0.27} )</td>
<td>1.05( ^{+0.61}_{-0.76} )</td>
<td>0.88/19</td>
<td>0.91( ^{+0.08}_{-0.08} )</td>
<td>0.33/20</td>
<td></td>
</tr>
<tr>
<td>258/1984</td>
<td>1.97( ^{+0.04}_{-0.13} )</td>
<td>0.87( ^{+0.08}_{-0.21} )</td>
<td>0.98( ^{+0.25}_{-0.29} )</td>
<td>0.64/21</td>
<td>0.90( ^{+0.06}_{-0.06} )</td>
<td>0.85/22</td>
<td></td>
</tr>
<tr>
<td>266/1984</td>
<td>1.33( ^{+0.05}_{-0.23} )</td>
<td>1.26( ^{+0.30}_{-0.28} )</td>
<td>1.49( ^{+0.32}_{-0.25} )</td>
<td>0.93/20</td>
<td>1.13( ^{+0.47}_{-0.19} )</td>
<td>0.96/18</td>
<td></td>
</tr>
<tr>
<td>274/1984</td>
<td>0.92( ^{+0.04}_{-0.23} )</td>
<td>1.49( ^{+0.30}_{-0.13} )</td>
<td>1.29( ^{+0.52}_{-0.44} )</td>
<td>1.25/18</td>
<td>1.41( ^{+0.08}_{-0.08} )</td>
<td>1.28/13</td>
<td></td>
</tr>
</tbody>
</table>

\( a. \) at a value of \( 1.10 \times 10^{21} \) cm\(^{-2} \).
\( b. \) 2-6 keV flux in units of \( 10^{-11} \text{ erg cm}^{-2} \text{ s}^{-1} \)
Figure 3.13 Derived photon spectra for 0414+009 in same format as Fig. 3.2.
It is nevertheless in good agreement with the line of sight column density through our own galaxy in the direction of 0414+009 as determined from 21 cm measurements (Stark et al. 1988; Heiles 1975), although this object lies near the edge of spiral arm in a region where there are likely to significant changes in the galactic column on relatively small angular sizes.

It should be noted that a spectral index $\leq 1$ over the low and medium energy X-ray bands, as is the case of the first two observations, is relatively rare for BL Lac type objects. As can be seen from Fig. 3.14, in common with the other objects reported here there appears to be a correlation between $\alpha$ and $F_{2-6}$ in the sense that the spectrum hardens as the source brightens.

Figure 3.14 $F_{2-6}$ (in units of $10^{-11}$ erg cm$^{-2}$ s$^{-1}$) versus $\alpha_*$ for 0414+009 in the same format as Fig. 3.3 where $n_H$ is $a)$ a free parameter, and $b)$ fixed at $1.10 \times 10^{21}$ cm$^{-2}$. 
3.6.1 Search for extended X-ray emission from 0414+009

Since 0414+009 is thought to lie in a cluster of galaxies (McHardy et al. 1988), it is conceivable that some fraction of the X-ray emission emanates from an extended region such as a cooling flow (see e.g. Stewart et al. 1984). In the case of the observations reported here, the search for any such extended emission is greatly facilitated by the proximity of the target to the centre of the LE field of view, in a region where the point response function (psf) is well determined. As the response to a point source varies with energy, and hence with the source spectrum, the most appropriate psf for this task is clearly that derived from another bright isolated BL Lac type object. We have therefore used the 3Lx psf estimated from the 22 hr observation of Mrk 501 in 1986 (see Chapter 2). The four 0414+009 3Lx images were therefore rotated to the same roll angle (only a few degrees in all cases) and co-added. We found no evidence of any asymmetry in the stacked image and therefore following background subtraction, a radial profile was derived as described in Chapter 2. This is shown in Fig. 3.15. The (Mrk 501) psf was then scaled in intensity until the best fit was obtained between it and the 0414+009 radial profile. As we are interested in searching for possible extended emission surrounding 0414+009 we restricted the fitting procedure to the inner 22 arcsec of the radial profile. The best fit produced a reduced $\chi^2$ of 0.64 and hence considered statistically acceptable. However, as can be seen from Fig. 3.15, the scaled psf is also in excellent agreement with that of 0414+009 at radii >22 arcsec. From consideration of the errors we estimate that an upper limit to any extended emission in a region of 70 arcsec radius surrounding 0414+009 is $5 \times 10^{-3}$ count s$^{-1}$. We found no other point source brighter than $3.6 \times 10^{-3}$ count s$^{-1}$ (10$\sigma$) in the LE field of view.

Comparison with previous results.

X-rays from the source 1H 0414+009 were first detected in 1977 August by the HEAO-1 A-1 instrument during a survey of Abell clusters of galaxies (Ulmer et al. 1980). Although originally mis-identified as being associated with Abell 480,
subsequent follow-up observations with the EINSTEIN IPC and HRI separated by ~1 year led to a refined X-ray position, and optical and radio measurements led to its re-classification as a BL Lac object (Ulmer et al. 1983). Simultaneous MPC observations revealed a simple power-law spectrum with an index, $\alpha \sim 1.5^{+0.3}_{-0.3}$ was acceptable at both epochs despite a factor ~2 decrease in flux (see Appendix A, Table A.6). Both the range of spectral indices and range of 2–6keV flux observed by EXOSAT encompasses the HEAO 1 and EINSTEIN measurements reported by Ulmer implying the behaviour of 1H0414+009 we report here is fairly typical.

3.7 Summary of EXOSAT Results

3.7.1 Variability

The amplitude and timescale of any variability provides a potentially powerful means of investigating both the emission processes in AGN and the region in which they occur. BL Lac type objects generally tend to vary on the fastest timescales in the X-ray band making observations in this regime of prime impor-
tance. In addition, the production of X-rays inevitably involves relatively high energy phenomena and is therefore likely to take place in the innermost regions of an emission region where the physical conditions are the most extreme. As shall be discussed further below, the minimum timescale for significant variability is of particular importance in studies of the kind.

We have detected statistically significant variability in the 2–6 keV band for all five objects under study. In Table 3.18 we list the minimum observed doubling timescale, $t_{\text{min}}$, from the EXOSAT observations of each object. Specifically, we find $t_{\text{min}}$ in the range $\sim 3$ hr (1218+304) to $\sim 7$ days (Mrk 180). With the signal-to-noise available with EXOSAT, in no case did we find evidence for statistically significant, large amplitude variability on timescales $<1000$ s. Also listed in Table 3.18 are the full dynamic range of the variability, $R_X$ (as defined by $F_{\text{max}}/F_{\text{min}}$), and the maximum observed rate of change of 2–6 keV flux for each object $(\Delta F/\Delta t)$ in cgs units. We find $R_X$ lies between $\sim 2$ (Mrk 501, 1218+304, 0414+009) and $\sim 19$ (Mrk 421), and $\Delta F/\Delta t$ between $10^{-17}$ erg s$^{-2}$ cm$^{-2}$ and few $\times 10^{-15}$ erg s$^{-2}$ cm$^{-2}$.

These results are consistent with previous studies of the X-ray variability of BL Lacs (Schwartz, Madejski & Ku 1983; Madejski 1985). Statistically significant variability is very common on intermediate (few hours – day) and long (weeks – months) timescales, yet large amplitude, rapid events (<1000 s) seem extremely rare. Indeed, in only one case has such an event been seen (0323+022; Fiegelson et al. 1986), although lack of detection may be in some part due to the limited sensitivity of the instrumentation.

The minimum variability timescale

Below we apply a number of well known arguments to the observed minimum doubling timescale and draw some general conclusions concerning the sources under study. It should be noted that all the following deductions are to some extent model dependent – all, for example, can be violated in the presence of
Table 3.18 Inferences from observed minimum doubling timescales

<table>
<thead>
<tr>
<th></th>
<th>Mrk 421</th>
<th>Mrk 501</th>
<th>1218+304</th>
<th>Mrk 180</th>
<th>0414+009</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{\text{min}}$ ($10^4$ s)</td>
<td>1.9</td>
<td>2.4</td>
<td>1.2</td>
<td>63</td>
<td>1.2</td>
</tr>
<tr>
<td>$R_X$</td>
<td>19.8</td>
<td>2.0</td>
<td>2.2</td>
<td>&gt;4.2</td>
<td>2.1</td>
</tr>
<tr>
<td>$\Delta F/\Delta t$</td>
<td>$3.9 \times 10^{-14}$</td>
<td>$5.8 \times 10^{-16}$</td>
<td>$4.3 \times 10^{-15}$</td>
<td>&gt;$1.0 \times 10^{-17}$</td>
<td>1.3 $\times 10^{-15}$</td>
</tr>
<tr>
<td>$M_{\text{bh}}$ ($M_\odot$)</td>
<td>$\leq 2.0 \times 10^8$</td>
<td>$\leq 2.4 \times 10^8$</td>
<td>$\leq 1.2 \times 10^8$</td>
<td>6.4 $\times 10^8$</td>
<td>$\leq 1.2 \times 10^8$</td>
</tr>
<tr>
<td>$L_{\text{bol}}$ (erg s$^{-1}$)</td>
<td>$\geq 1.3 \times 10^{45}$</td>
<td>$\geq 7.5 \times 10^{44}$</td>
<td>$\geq 1.1 \times 10^{46}$</td>
<td>2.0 $\times 10^{45}$</td>
<td>$\geq 8.8 \times 10^{44}$</td>
</tr>
<tr>
<td>$L_{\text{bol}}/L_{\text{Edd}}$</td>
<td>$\geq 5.4 \times 10^{-3}$</td>
<td>$\geq 2.4 \times 10^{-3}$</td>
<td>$\geq 7.2 \times 10^{-2}$</td>
<td>$\geq 2.5 \times 10^{-4}$</td>
<td>$\geq 5.7 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\Delta L/\Delta t$ (erg s$^{-2}$)</td>
<td>$7.2 \times 10^{39}$</td>
<td>$1.3 \times 10^{39}$</td>
<td>$1.5 \times 10^{40}$</td>
<td>4.1 $\times 10^{37}$</td>
<td>1.1 $\times 10^{41}$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$3.5 \times 10^{-3}$</td>
<td>$6.2 \times 10^{-3}$</td>
<td>$7.2 \times 10^{-2}$</td>
<td>2.0 $\times 10^{-3}$</td>
<td>5.3 $\times 10^{-2}$</td>
</tr>
<tr>
<td>$\dot{m}$ ($M_\odot$ yr$^{-1}$)</td>
<td>6.5</td>
<td>21.3</td>
<td>2.7</td>
<td>1.76 $\times 10^3$</td>
<td>2.9 $\times 10^{-1}$</td>
</tr>
<tr>
<td>$a^b$</td>
<td>0.9-2.0</td>
<td>1.2-1.6</td>
<td>1.1-1.4</td>
<td>1.4-1.9</td>
<td>0.9-1.5</td>
</tr>
<tr>
<td>$s^{b}$</td>
<td>0.24</td>
<td>0.13</td>
<td>0.12</td>
<td>0.31</td>
<td>0.58</td>
</tr>
</tbody>
</table>

a. In cgs units unless otherwise stated.

b. For data in which $n_H$ is a free parameter.

$M_\odot = 1.99 \times 10^{30}$ g

bulk relativistic motion or an anisotropic emission region. They can therefore be used to provide circumstantial evidence in favour of such scenarios.

Probably the least model dependent deduction is based upon light travel-time arguments whereby a source of size $R$ is assumed not to be able to show significant variations on a timescale, $t_\nu$, shorter than that it takes light to travel across it$^1$. Hence

$$t_\nu = \frac{R}{c}$$

[3.3]

As noted in Chapter 5, even inter-continental VLBI observations are only able to resolve components on angular sizes $\geq 0.3$ mas, corresponding to linear sizes typically $\sim$ several light years. Thus in the absence of bulk relativistic motion

$^1$It is conceivable that even this limit does not hold. Consider for example a model in which there is an outflow of the emitting material; an instantaneous reduction in the central supply will result some time later, in the simultaneous extinguishing of greatly separated regions on a very short timescale.
the above expression, with \( t_v = t_{\text{min}} \), indicates that the X-ray emitting region is likely to be several orders of magnitude smaller than that resolvable at radio wavelengths.

It is now generally accepted that the ultimate source of the copious amounts of energy radiated by AGN is the loss of gravitational energy by matter accreting onto a compact object – possibly a supermassive black hole (SMBH). If we make the explicit assumption that the observed variations occur in plasma very close to the event horizon of the putative SMBH (say within 10 Schwarzschild radii, \( R_S \), where \( R_S = 2GM_{bh}/c^2 \)), then from above we have \( t_v \geq 10R_S/c \) giving a direct relationship between \( t_v \) and the mass of the black hole, \( M_{bh} \), namely

\[
t_v \geq 9.8 \times 10^{-5} \left( \frac{M_{bh}}{M_\odot} \right) \text{ s} \tag{3.4}
\]

where \( M_\odot \) is a solar mass \( (1.99 \times 10^{33} \text{ g}) \). In Table 3.18 we list the upper limits on \( M_{bh} \) for each source calculated from equation [3.4]. It can be seen that typically \( M_{bh} < \text{few} \times 10^8 M_\odot \). In addition, assuming spherical symmetry, we can relate the Eddington luminosity, \( L_{Edd} \), (calculated by assuming the central mass is sufficiently large such that the inward pull of gravity on the plasma exactly balances the outward radiation pressure) to the light crossing time, \( t_{lc} \), at the event horizon of the SMBH by

\[
t_{lc} = 7.8 \times 10^{-44} L_{Edd} \text{ s} \tag{3.5}
\]

Setting \( t_{lc} \leq t_{\text{min}} \) leads to an upper limit on \( L_{Edd} \) which can then be compared to the observed bolometric luminosity, \( L_{bol} \). As can be seen from Table 3.18, for all five sources reported here, \( L_{bol} \) is only a small fraction of \( L_{Edd} \). Indeed, inspection of equation [3.5] shows that for sources with luminosities of the order of those reported here, significant variability on a timescale < \( 10^2 \) s is required in order for the Eddington limit to be exceeded.

Finally, if one assumes that an increase in luminosity, \( \Delta L \), is the result of the conversion of matter to radiation with an efficiency \( \eta \) (\( \leq 1 \)), then an observer
is unable to see this increase on a timescale shorter than that necessary for a photon to diffuse out of the source. Using the expression for the density of a spherical source given in Cavallo & Rees (1979), it can easily be shown (Fabian & Rees 1979) that for electron scattering optical depth $\tau_{es}$, the minimum variability timescale is related to $\Delta L$ and $\eta$ via

$$t_v = \frac{3\sigma_T(\tau_{es} + 1)^2}{4\pi\eta\tau_{es}m_pc^4} \Delta L$$  \[3.6\]

where $\sigma_T = 6.66 \times 10^{-25}$ cm$^2$ is the Thomson cross-section and $m_p$ is the mass of a proton. Minimizing for $\tau_{es}$ (at $\tau_{es}=1$) gives

$$t_v \geq 4.8 \times 10^{-43}\eta^{-1}\Delta L \quad \text{s}$$  \[3.7\]

the so-called Fabian–Rees relation, which is often more restrictive than the Eddington limit above. The maximum efficiency of nuclear fusion (for $\text{H} \rightarrow \text{He}$) is $\eta = 0.007$, thus efficiencies in excess of this value imply an alternative energy release mechanism, and are usually taken as direct evidence for the conversion of gravitational potential energy via accretion onto a compact object. The maximum rate of change of luminosity, $\Delta L/\Delta t$, and the corresponding efficiencies implied by equation [3.7] are listed for each source in Table 3.18. As can be seen, we have (marginal) evidence for accretion in only two cases (1218+304 and 0414+009). However, in view of the large luminosities implied in AGN, alternate processes are considered unlikely. Using the derived value of $\eta$ we also list in Table 3.18 the rate, $\dot{m}$, at which matter must be converted into radiation to account for the observed bolometric luminosity. It can be seen that with the exception of Mrk 180, the values of $\dot{m}$ do not seem unreasonable.

In systems in which there is some observational evidence for an accretion disk (e.g. Arnaud et al. 1985; Pounds et al. 1987), further information can be gained by associating the observed variability timescale with the dynamical or viscous timescales of the disk. However in the case of BL Lac type objects, observations to date show no evidence for an accretion disk and such timescales are not considered
3.7.2 Spectra

We have found that with the notable exception of the high state observations of Mrk 421 (see below), a single power-law model with low energy absorption gives a statistically acceptable fit to the data. We found no evidence in any of the EXOSAT observations for a hardening of the X-ray spectrum above a few keV as has been reported previously for Mrk 421 (Mushotzky et al. 1978; Hall 1981), Mrk 501 (Mushotzky et al. 1978) and several other X-ray bright BL Lacs (e.g. PKS 2155-304, Urry & Mushotzky 1982; PKS 0548-322, Riegler, Agrawal & Mushotzky 1979).

The derived spectral indices, $\alpha_x$, of all 5 objects were found to show variations between the individual observations. As can be seen from the range of indices are listed in Table 3.18, this corresponded to $\Delta \alpha_x = 0.3$ (1218+304) to $\sim 1$ (Mrk 421). Furthermore, in the cases of Mrk 421, Mrk 501 and 0414+009, we found strong evidence for a correlation between $\alpha_x$ and the 2–6 keV flux, $F_{2-6}$, in the sense that the spectrum flattened as the source brightened. Interestingly, the behaviour of the other two sources reported here are also consistent with this picture. Some indication of the strength of the dependency of $\alpha_x$ on $F_{2-6}$ is given in Table 3.18 by the slope, $s$, of the best (weighted least squares) straight line fit to the $F_{2-6}$ versus $\alpha_x$ correlation, where $s$ is given by $-d\alpha_x/df_{2-6}$. In all cases a straight line is statistically acceptable (excluding the high state observations of Mrk 421), although in Chapter 5 we use the form of the ultra-violet to X-ray continuum to argue that a more complex model maybe more appropriate.

During the outburst in 1984 December, the spectrum of Mrk 421 shows a steepening above $\sim 3$ keV. We have modeled this with a power-law model plus an exponential cut-off, although this is by no means a unique solution. Interestingly, less pronounced deficits of high energy photons are also apparent on a number of other occasions (see e.g. Fig. 3.5). Although these are not formally...
required and may be due to background subtraction problems, they do raise the possibility that single power-law may not be appropriate over very wide energy bands. A similar downward curving spectrum has also been detected in the EXOSAT observations the BL Lac object PKS 0548-322 (Barr, Giommi & Maccagni 1988; see also Barr et al. 1988). From the X-ray data presently available, it is not clear whether there is a real distinction between the 'High' and 'Low' X-ray states, or whether they are simply opposite ends of a continuum of possible states of Mrk 421. The analysis to be presented in Chapter 6 implies the former.

3.7.3 Line-of-sight column densities

The wide waveband coverage provided by the EXOSAT instrumentation has enabled the determination of the effective hydrogen column densities, $n_H$, of cold gas along the line-of-sight to these objects with un-paralled accuracy. In Table 3.19 we list the RA and dec of the mean centroid of the X-ray emission from each object as determined from the EXOSAT (3lx) LE images. The weighted mean values of $n_H$, as determined from the X-ray spectral fitting analysis are also given (column 4) and compared to that determined from the 21 cm measurements of Stark et al. (1988). The latter were calculated by performing a bi-linear interpolation between the 4 closest grid points in the survey. The errors quoted for the Stark points are the maximum and minimum grid values.

In all cases the position of the X-ray centroid is consistent with the centre of optical emission as determined from the Palomar Observatory Sky Survey (POSS) and the strongest unresolved radio source (see Chapter 5). It can be seen that for 4 of the objects, the X-ray column density is in excellent agreement with the Stark et al. values. In the case of 1218+304, the X-ray column is marginally in excess of that from the radio data, however the earlier 21 cm maps of Heiles (1975) imply a column of $(3.4 \pm 0.6) \times 10^{20}$ cm$^{-2}$ (a factor ~2 higher than Stark et al., although the source does lie in a region where the Heiles maps are quoted as being inaccurate). We therefore conclude that the intrinsic column of cold gas in these
Table 3.19 X-ray centroids and column densities

<table>
<thead>
<tr>
<th>Object</th>
<th>R.A. (1950)</th>
<th>dec (1950)</th>
<th>( n_H \times 10^{20} \text{ cm}^{-2} )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mkn 421</td>
<td>11 01 40.7±0.2</td>
<td>+38 28 45±4</td>
<td>1.73±0.27</td>
<td>1.75±0.27</td>
</tr>
<tr>
<td>Mkn 501</td>
<td>16 52 11.1±0.2</td>
<td>+39 50 28±4</td>
<td>2.21±0.39</td>
<td>1.6±0.2</td>
</tr>
<tr>
<td>1218+304</td>
<td>12 18 51.2±0.2</td>
<td>+30 27 11±7</td>
<td>3.11±0.50</td>
<td>1.65±0.3</td>
</tr>
<tr>
<td>Mkn 180</td>
<td>11 33 01.0±1.0</td>
<td>+70 25 59±7</td>
<td>2.11±1.59</td>
<td>1.4±1.4</td>
</tr>
<tr>
<td>0414+009</td>
<td>04 14 16.4±0.5</td>
<td>+00 58 00±7</td>
<td>10.90±4.80</td>
<td>8.56±3.3</td>
</tr>
</tbody>
</table>

a. Weighted mean of individual X-ray columns.
b. From Stark et al (1988); errors are max/min grid values.
c. Heiles (1975) gives \( n_H = 3.4±0.6 \times 10^{20} \text{ cm}^{-2} \).
d. Close to edge of spiral arm.

dfive BL Lac objects is small \(< 1 \times 10^{20} \text{ cm}^{-2}\) and for at least 3 of the sources maybe \(< 5 \times 10^{19} \text{ cm}^{-2}\). Urry, Mushotzky & Holt (1986) have interpreted data from the Solid State Spectrometer onboard the EINSTEIN satellite, as giving evidence for a substantial absorption from a hot, ionized gaseous component in Mrk 421, Mrk 501 and 1218+304. Unfortunately the relatively crude X-ray photometry obtainable with the EXOSAT LE filters does not allow us to test this hypothesis.

A rough estimation of the total HI mass in the elliptical galaxy can be obtained if we assume a uniform distribution of cold gas with solar abundances and a radial extent \( R \) (kpc). An intrinsic column \(< 10^{20} \text{ cm}^{-2}\) then implies a total HI mass of \(< 3 \times 10^8 R^2 M_\odot\), a value significantly lower than the HI mass detected in some elliptical galaxies with nuclear radio sources (e.g. Dressel, Bania & O'Connell 1982).
Chapter 4

IUE Observations and Results

Overview

Results are presented from a detailed analysis of the five BL Lac type objects in the ultra-violet band using the International Ultra-violet Explorer (IUE) satellite. Some of the images were obtained from our own observations, the rest were retrieved from the archives. Following a brief introduction, in Section 4.2 we discuss a number of systematic uncertainties encountered during the analysis. In Section 4.3 we present the results from the individual targets and finally in Section 4.4 we give a brief summary.

4.1 Introduction

Since the launch of IUE, more than 400 low dispersion spectra of BL Lac type objects have been obtained. However, due to the relative faintness of these objects, ~50% of these images are of three of the brightest objects namely Mrk 421, PKS 2155-304 and OJ 287. The importance of the ultra-violet band to the study of BL Lac type objects (as will be discussed more fully in Chapter 5) is that there is negligible contribution from any galactic starlight. Furthermore, it is
in or around the ultra-violet regime where the underlying non-thermal continuum in these sources is expected to show evidence of radiation losses leading to independent information about the magnetic field and energy densities within the emission region. Unfortunately only Mrk 421 and PKS 2155-304 have been consistently bright enough in both the ultra-violet and X-ray bands for detailed study. Nevertheless, as will be shown below, despite the low signal-to-noise, useful insights into the physical conditions can be gained from IUE observations even of extremely weak sources. Results for PKS 2155-304 and OJ 287 based on an analysis of an almost complete set of IUE data, have been presented by Hanson & Coe (1986) and Urry (1986b). In addition, general results from eight years IUE observations of Blazars have recently been reviewed by Bregman, Maraschi & Urry (1987).

All the images were reduced in a uniform manner as described in Chapter 2. Preliminary reports of this work have been presented elsewhere (George 1988; George, Warwick & Bromage 1988a; George, Warwick & Bromage 1988b).

4.2 Systematic Uncertainties

During long exposures of weak sources such as those reported here, background signals of up to ~50% of the gross spectrum can be accumulated. Under these circumstances, particularly when we wish to extrapolate the ultra-violet spectrum over more than a decade in frequency into the X-ray regime as in Chapter 5, serious consideration must be taken of the various sources of uncertainty.

Several authors have commented on the presence of systematic effects in low-resolution IUE spectra of weak sources (e.g. Hanson & Coe 1986; Urry 1986b) The characteristic featureless, power-law continua of BL Lac type objects provides a useful probe of such effects; conversely of course, the study of these sources in the ultra-violet is somewhat hindered by such systematics. A number of such effects were encountered and investigated during the course of the work presented here.
In this section we illustrate the effects we have encountered in the analysis of IUE images of the BL Lac type object Mrk 421 and quantitatively discuss their consequences. Although the results presented here refer explicitly to Mrk 421, similar effects were often seen in the other (weaker) sources.

### 4.2.1 Analysis technique

The images were reduced as described in Chapter 2. Following the flagging of bad data points and determination of the centroid of the source spectrum, the gross spectrum was extracted using a slit of width $W_S$ pixels.

The background subtracted source spectra were converted to flux densities and averaged over 100Å intervals in the bands 1230–1900Å (SWP) and 2100–3200Å (LWR and LWP). A weighted least-squares fit of a power-law function of the form given by equation [2.3] was then applied both to the individual spectra from each camera and, for observations made within the period of a day, to the combined data set as detailed below.

### 4.2.2 Slit width

In Fig. 4.1 we show the effect on the best fitting spectra of varying the source extraction slit, $W_S$, whilst maintaining the same background slits. Crivellari & Morossi (1980) have shown that the profile perpendicular to the direction of dispersion is well represented by a central Gaussian with Lorentzian wings containing ~85% and 15% of the total source counts respectively. As expected, the derived fluxes at 1500, 2000 and 2500Å all show a systematic decrease as $W_S$ is reduced (Fig. 4.1a). In Fig. 4.1b we show, however, that there are also accompanying changes in the derived spectral indices; the LWP spectrum is dramatically steepened (by as much as $\Delta \alpha \sim 1$ in the case considered), whilst the SWP spectrum is gradually flattened as $W_S$ is decreased. Such an effect is likely to be a result of a change in focus along the dispersion direction of the respective cameras.
and may account for a number of the discrepancies between the published best fitting spectra for the same data set presented by different authors. It is noted that there appears to be little change in the combined index for $W_S > 5$ pixels.

We found no dependency of either the derived flux or spectral index on $W_B$. A value of $W_B = 2$ gave an adequate estimation of the background and was therefore assumed throughout this work.

### 4.2.3 Individual camera vs combined fits

In Fig. 4.2 we demonstrate the well known discrepancy between the best fitting spectra to the individual cameras with that of the combined data set. Prior to 1984, whilst the LWR camera was in use, the derived index for the LWR camera only ($\alpha_{LWR}$) is generally in good agreement with that of the combined data set ($\alpha_c$). However, the derived index from the SWP camera only, ($\alpha_{SWP}$) over the same period is seen to be flatter than $\alpha_c$ by 0.1–0.4. Formally we find the weighted mean of $\alpha_c - \alpha_{SWP} = +0.23^{+0.03}_{-0.04}$. After 1984, when the LWP became the
Figure 4.2 The discrepancy between the spectral indices from power-law fits to the individual cameras and those from the whole data set. Epochs during which the LWR and LWP cameras were in use are shown as circles and squares respectively.

standard long wavelength camera, $\alpha_c$ exceeds $\alpha_{LWP}$ by typically $\sim 0.4$ (although it should be noted that a number of these spectra are significantly flattened by an excess at short wavelengths in the LWP as discussed below). Similarly, for this period we find $\alpha_C - \alpha_{SWP} = +0.39^{+0.03}$.

This effect has also been seen using a different extraction procedure (Urry 1986b) and it has been suggested that it may be the result of a miscalibration of the LWP camera leading to LWP flux densities $\sim 10-20\%$ too high. Indeed a study of a subset of the Mrk 421 data used in Fig. 4.2 has shown that a 22\% reduction in the LWP flux leads to a far better agreement between the individual spectra and the combined spectra at each epoch (George, Warwick & Bromage 1988a). However, as has been pointed out, the uncorrected LWP data at the long wavelength end do seem to extrapolate well onto (pseudo) simultaneous optical and infra-red data for Mrk 421. Furthermore, it is highly unlikely that any calibration errors are wavelength independent (see Chapter 2). Indeed Cassatella & Lloyd (1987) have recently reported wavelength dependent inconsistencies between the absolutely calibrated point and trailed sources contrary to that implicitly assumed in the
calibration philosophy.

We find no significant differences between the combined SWP-LWP spectra compared with the SWP-LWR spectra contrary to that reported by Urry (1986b) for the BL Lac type object PKS 2155-304.

4.2.4 Sporadic excess in LWP spectra

In the course of our analysis we have encountered a number of the LWP spectra with an apparent systematic excess of flux at short wavelengths (< 2400Å). An example is shown in Fig. 4.3. This sporadic feature is at the noisy, low sensitivity end of the camera and does not appear to be related to exposure time or gross FN. As can be seen from Fig. 4.3, although the excess considerably flattens the derived spectrum for the LWP camera alone, it has little effect on the best fitting combined spectrum. The most natural explanation is that this excess is caused by inaccuracies in the ITF and/or absolute calibration, and it will therefore be interesting to see whether it is also present in images processed using LWP ITF2.

4.2.5 The ITFs and absolute calibrations

As is well known, any update of a camera ITF table (see Chapter 2, Table 2.3) requires modification of the corresponding inverse sensitivity curve, $S^{-1}$, used to calibrate absolutely the background subtracted spectrum. Unfortunately IUEDR Version 1.3 only had one $S^{-1}$ available for each camera - namely those of Bohlin & Holm (1980) and Cassatella & Harris (1983). However, as shown in Fig. 4.4 for the LWR camera, this leads to only very small systematic errors between the derived best fitting spectra. The errors in both flux and spectral index resulting from the use of an incorrect $S^{-1}$ for the other cameras are equally negligible, hence previously reported power-law spectra for similar weak sources analysed using IUEDR 1.3 ought to be correct within the statistical errors.
Figure 4.3 An example of an excess at short wavelengths in LWP spectra. Both the extracted spectrum (top) and rebinned spectra (×0.1, bottom) are shown as are the best-fitting power-laws to the individual cameras (dotted) and combined data set (solid).
The effects of, and correction algorithms for, the known errors in the LWR (slightly faulty) and SWP (faulty) ITFs have been discussed in detail and well documented elsewhere (e.g. Holm 1979; Snijders 1980).

### 4.2.6 Residual camera artifacts

All three operational cameras are contaminated by narrow bands of periodic microphonic noise cutting horizontally across the images. The cause is thought to be due to spacecraft mechanical activity during image readout. In the LWP and SWP cameras, this microphonic noise is generally of low amplitude (1–3 DN) and insignificant compared to the random background. In the LWR however, the noise maybe as high as 100 DN and greatly exceed the background (although localized in a small number of image lines and therefore immediately identifiable on inspection of the image). The cameras are also known to contain a number of permanent ‘hot’ pixels, the locations of which are given in Ponz (1980), Turnrose & Thompson (1984) and Settle, Shuttleworth & Sandford (1981). In addition,
low-dispersion, large aperture SWP spectra are impressed with a fixed pattern noise which is thought to be somewhat dependent on exposure time (Hackney, Hackney & Kondo 1982). This effect can be seen in the SWP spectrum presented in Fig. 4.3. Generally however, none of the above artifacts have any significant effect on the derived spectra for the weak, low signal-to-noise sources under consideration here and can be neglected.

4.2.7 Discussion of systematics

From the analysis presented above, we conclude that a number of systematic uncertainties are likely to be present in low-resolution IUE spectra of weak sources. These may lead to substantial discrepancies between different estimations of the ultra-violet continuum for the same data set.

In the specific case of the study of BL Lac type objects, we have generally adopted a slit width of 12.8 pixels in order to integrate over the maximum amount of the source signal (admittedly at the expense of some reduction in signal-to-noise) and to coincide with that used in most calibration derivations. In the absence of spectral curvature or rapid variability, we consider a power-law fit to the combined long- and short-wavelength data sets to give the most reliable estimate of the spectral shape in the ultra-violet.

4.3 Results from Individual Sources

We now present the results from the individual sources taking into account the various systematic effects discussed above.
4.3.1 Mrk 421

Mrk 421 has been observed on ~23 epochs since the 1978, with over 80 low-resolution images available in the archive. Table 4.1 lists in columns 1-3 the observation date, the image number and exposure time of each spectrum. The best fitting parameters are listed in columns 4-6 for the individual cameras and in columns 7-9 for the combined data set at each epoch.

It can be seen that all the spectra are generally well represented by a power-law model. However, a number of the LWP spectra were seriously affected by excess of counts at short wavelengths (identifiable by high $\chi^2$ values); furthermore spectral fitting of the combined long- and short-wavelength data revealed a discrepancy in the sense that the spectral indices derived from the individual cameras were found to be consistently lower than those from the combined data set (see above).

As illustrated in Fig. 4.5, the ultra-violet intensity of Mrk 421 is variable on a timescale of weeks-months, with a dramatic brightening of the source by a factor $\sim 3-4$ during the 1981-1982 epoch (see also Ulrich et al. 1984). The dotted lines are meant only as a guide to the eye. As pointed out by Warwick et al. (1986; see also Chapter 5), the long-term trends in the ultra-violet for this source seem to be correlated with similar trends the radio and infra-red bands indicating a single, or several closely related emission mechanisms may be appropriate.

Unfortunately, it is not possible to study ultra-violet variability on timescales of less than $\sim$ day for such a weak source. Fig. 4.6 shows that the best fitting spectral indices were remarkably similar for the majority of the observations with $\alpha_C \sim 1$. During the 1981-1982 outburst, however, a significant flattening (by $\Delta \alpha_C \sim 0.2$) was apparent as first noted by Ulrich et al. (1984). Similar behaviour is also apparent in the SWP data alone and is therefore likely to be real.

The featureless ultra-violet continuum of BL Lac objects provides a useful probe of the structure of the Galactic halo. In Fig. 4.7 we show the detection of the Mg
Table 4.1 IUE observing log and results for Mrk 421

<table>
<thead>
<tr>
<th>Day/Year</th>
<th>Image</th>
<th>Exposure (sec)</th>
<th>Flux (mJy)</th>
<th>Individual log $F^\alpha$</th>
<th>Combined log $F^\alpha$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/1/1978</td>
<td>LWR2745</td>
<td>25200</td>
<td>1.45±0.15</td>
<td>0.592±0.009</td>
<td>2.04/4</td>
<td></td>
</tr>
<tr>
<td>09/17/1979</td>
<td>LWR4492</td>
<td>5400</td>
<td>1.20±0.14</td>
<td>0.685±0.008</td>
<td>11.31/9</td>
<td></td>
</tr>
<tr>
<td>130/1979</td>
<td>SWP3389</td>
<td>12600</td>
<td>1.03±0.08</td>
<td>0.442±0.005</td>
<td>10.26/5</td>
<td></td>
</tr>
<tr>
<td>136/1979</td>
<td>LWR4267</td>
<td>7199</td>
<td>1.17±0.16</td>
<td>0.672±0.008</td>
<td>8.35/9</td>
<td></td>
</tr>
<tr>
<td>137/1979</td>
<td>LWR4530</td>
<td>8399</td>
<td>1.15±0.14</td>
<td>0.662±0.008</td>
<td>5.12/9</td>
<td></td>
</tr>
<tr>
<td>320-1/1979</td>
<td>LWR4530</td>
<td>7199</td>
<td>1.17±0.15</td>
<td>0.672±0.008</td>
<td>8.35/9</td>
<td></td>
</tr>
<tr>
<td>320/1979</td>
<td>LWR4530</td>
<td>8399</td>
<td>1.15±0.14</td>
<td>0.662±0.008</td>
<td>5.12/9</td>
<td></td>
</tr>
<tr>
<td>138/1980</td>
<td>LWR7779</td>
<td>5400</td>
<td>0.93±0.04</td>
<td>0.479±0.004</td>
<td>4.91/6</td>
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<td>07/1981</td>
<td>SWP3952</td>
<td>12600</td>
<td>0.61±0.07</td>
<td>0.566±0.005</td>
<td>5.01/5</td>
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</tr>
<tr>
<td>337/1981</td>
<td>SWP4254</td>
<td>16800</td>
<td>0.61±0.06</td>
<td>0.542±0.004</td>
<td>14.25/5</td>
<td></td>
</tr>
<tr>
<td>07/1982</td>
<td>LWR12803-4</td>
<td>7200</td>
<td>0.82±0.12</td>
<td>0.911±0.009</td>
<td>3.67/9</td>
<td></td>
</tr>
<tr>
<td>07/1983</td>
<td>LWR11517-8</td>
<td>9900</td>
<td>0.89±0.16</td>
<td>0.913±0.008</td>
<td>5.19/5</td>
<td></td>
</tr>
<tr>
<td>02/1984</td>
<td>SWP19088</td>
<td>9000</td>
<td>0.48±0.08</td>
<td>0.728±0.005</td>
<td>4.29/5</td>
<td></td>
</tr>
<tr>
<td>02/1985</td>
<td>SWP2699</td>
<td>4800</td>
<td>0.75±0.15</td>
<td>0.781±0.009</td>
<td>3.32/9</td>
<td></td>
</tr>
<tr>
<td>02/1988</td>
<td>LWP2271</td>
<td>4800</td>
<td>0.77±0.15</td>
<td>0.776±0.009</td>
<td>1.98/8</td>
<td></td>
</tr>
<tr>
<td>03/1984</td>
<td>SWP22128</td>
<td>12600</td>
<td>0.65±0.08</td>
<td>0.531±0.005</td>
<td>7.01/5</td>
<td></td>
</tr>
<tr>
<td>06/1984</td>
<td>LWP2732</td>
<td>5400</td>
<td>1.00±0.25</td>
<td>0.72±0.014</td>
<td>7.50/9</td>
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</tr>
<tr>
<td>06/1984</td>
<td>LWP2891-2</td>
<td>8400</td>
<td>0.61±0.10</td>
<td>0.70±0.010</td>
<td>1.31/9</td>
<td></td>
</tr>
<tr>
<td>06/1984</td>
<td>SWP22398</td>
<td>14400</td>
<td>0.54±0.10</td>
<td>0.449±0.006</td>
<td>0.94/5</td>
<td></td>
</tr>
<tr>
<td>338/1984</td>
<td>SWP4917</td>
<td>1800</td>
<td>0.36±0.32</td>
<td>0.773±0.019</td>
<td>8.23/11</td>
<td></td>
</tr>
<tr>
<td>340/1984</td>
<td>SWP4928</td>
<td>4800</td>
<td>0.72±0.17</td>
<td>0.755±0.010</td>
<td>11.66/9</td>
<td></td>
</tr>
<tr>
<td>004/1985</td>
<td>SWP4939</td>
<td>4800</td>
<td>0.24±0.30</td>
<td>0.78±0.017</td>
<td>5.92/9</td>
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</tr>
<tr>
<td>112/1985</td>
<td>SWP4939</td>
<td>4800</td>
<td>0.56±0.10</td>
<td>0.470±0.006</td>
<td>12.36/5</td>
<td></td>
</tr>
<tr>
<td>126/1985</td>
<td>SWP4928</td>
<td>4800</td>
<td>0.36±0.21</td>
<td>0.619±0.011</td>
<td>3.44/9</td>
<td></td>
</tr>
<tr>
<td>141/1985</td>
<td>SWP6027-8</td>
<td>11428</td>
<td>0.33±0.20</td>
<td>0.593±0.011</td>
<td>6.10/9</td>
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<tr>
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<td>SWP6059-62</td>
<td>7200</td>
<td>0.47±0.23</td>
<td>0.58±0.020</td>
<td>13.80/9</td>
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</tr>
<tr>
<td>136/1985</td>
<td>SWP6061-26</td>
<td>12600</td>
<td>0.73±0.16</td>
<td>0.309±0.012</td>
<td>12.00/5</td>
<td></td>
</tr>
</tbody>
</table>

$^a$ Flux (in mJy) at 2500Å or 1500Å for LWR/P or SWP respectively.

$^b$ Flux (in mJy) at 2000Å.
Figure 4.5 Ultra-violet light curve for Mrk 421 showing fluxes at 2500Å (○), 2000Å (●) and 1500Å (△). The dotted lines are meant to guide the eye only.

Figure 4.6 Spectral index of the best fitting power-law to the combined data set versus flux at 200 nm, $F_{200}$, for Mrk 421.
II (λ\lambda2795.4, 2803.5) and possibly Fe II (λ\lambda2586.7, 2600.2) absorption doublets following the renormalization (using the 2650–2750Å band) and co-addition of the 22 highest signal-to-noise LWR and LWP spectra of Mrk 421. Unfortunately the Fe II feature lies close to a reseau mark in most of the spectra. We also find marginal evidence for additional absorption features in the co-added SWP spectra. Assuming a local continuum level based on a Chebyshev polynomial fit, we have made preliminary estimates of the equivalent widths, \( W_\lambda \). These are listed in Table 4.2. A more detailed discussion of the galactic absorption features present in the spectrum of Mrk 421 will be presented elsewhere (George & Bromage, in preparation).

### 4.3.2 Mrk 501, Mrk 180 & 1218+304

Unfortunately, these three X-ray bright BL Lac type objects are generally too weak and have been too sporadically monitored to enable detailed statements to
Table 4.2 Equivalent widths for Mrk 421

<table>
<thead>
<tr>
<th>Species</th>
<th>$\lambda$ ($\AA$)</th>
<th>$W_{\lambda}$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si II</td>
<td>1260</td>
<td>700$^{+200}_{-200}$</td>
</tr>
<tr>
<td>Si II</td>
<td>1304</td>
<td>1200$^{+300}_{-300}$</td>
</tr>
<tr>
<td>Si IV</td>
<td>1394</td>
<td>200$^{+100}_{-100}$</td>
</tr>
<tr>
<td>Si IV</td>
<td>1403</td>
<td>100$^{+100}_{-100}$</td>
</tr>
<tr>
<td>Fe II</td>
<td>2586.7</td>
<td>900$^{+700}_{-700}$</td>
</tr>
<tr>
<td></td>
<td>2600.2</td>
<td>1300$^{+700}_{-700}$</td>
</tr>
<tr>
<td>blend</td>
<td></td>
<td>2100$^{+900}_{-900}$</td>
</tr>
<tr>
<td>Mg II</td>
<td>blend</td>
<td>2300$^{+500}_{-500}$</td>
</tr>
</tbody>
</table>

Figure 4.8 SWP spectral index versus flux for Mrk 180 (□) and Mrk 501 (●).
Table 4.3 IUE observing log and results for Mrk 501, Mrk 180, 1218+304 and 0414+009

| Day/Year | Image  | Exposure (sec) | $\alpha$ | $\sigma$ | $\log F_{\alpha}$ | $\chi^2$/dof | | Day/Year | Image  | Exposure (sec) | $\alpha$ | $\sigma$ | $\log F_{\alpha}$ | $\chi^2$/dof |
|----------|--------|----------------|---------|---------|-----------------|-------------|----------|----------|--------|---------|-----------------|-------------|
| 237/1978 | SWP2394| 22800          | 0.74±0.16| 0.154±0.010 | 4.56/5      | Mrk 501     | 138/1980 | LWR7780 | 10800       | 0.61±0.93   | 0.29±0.045 | 4.16/9        |               |
| 297/1978 | SWP3123| 21600          | 0.52±0.11 | 0.187±0.007 | 4.97/5      | Mrk 501     | 142/1980 | LWR7834 | 10680       | 0.56±0.36   | 0.36±0.029 | 7.12/9        |               |
| 300/1978 | LWR2728| 19200          | 1.12±0.15 | 0.367±0.008 | 10.21/9     | Mrk 501     | 312/1980 | SWP10563| 23200       | 0.86±0.17   | 0.172±0.012 | 2.69/5        |               |
| 941/1979 | SWP4454| 28799          | 0.78±0.09 | 0.210±0.006 | 12.31/5     | Mrk 501     | 153/1980 | LWR2977 | 15600       | -          | -0.139±0.029 | 33.96/4       |               |
| 318/1979 | SWP7147| 23400          | 0.80±0.13 | 0.151±0.007 | 2.00/5      | Mrk 501     | 270/1982 | SWP18123| 21299       | 0.63±0.20   | -0.118±0.013 | 3.23/5        |               |
| 426/1980 | SWP9984| 25200          | 0.87±0.11 | 0.063±0.007 | 6.14/5      | Mrk 501     | 315/1980 | SWP24412| 23699       | 0.60±0.35   | -0.403±0.025 | 4.18/5        |               |
| 086/1981 | SWP13427| 18000        | 0.81±0.12 | 0.134±0.008 | 6.63/5      | Mrk 501     | 020/1985 | LW P5238 | 2760        | -          | -0.060±0.033 | 6.03/4        | 1.02±0.16 -0.169±0.019 | 7.51/11 |
| 078/1982 | SWP16567| 26679         | 0.40±0.12 | 0.161±0.008 | 2.45/5      | Mrk 501     | 020/1985 | SWP24936| 17400       | 0.63±0.29   | -0.293±0.019 | 2.07/5        |               |
| 274/1983 | SWP21211| 12122         | 0.81±0.18 | 0.070±0.012 | 3.13/5      | Mrk 501     | 303/1984 | SWP22187| 23700       | 0.79±0.31   | -0.34±0.02  | 0.38/5        | 1.02±0.13 -0.22±0.01 | 6.36/11 |
| 024/1985 | LW P5237 | 4500          | 1.17±0.57 | 0.392±0.028 | 5.39/9      | Mrk 501     | 034/1984 | LW P2733 | 12600       | 1.08±0.16   | -0.17±0.02  | 5.96/4        |               |
| 271/1986 | LW P9209 | 5400          | 0.36±0.34 | 0.381±0.018 | 10.23/9     | Mrk 501     | 096/1985 | LW P5680 | 5400        | 0.82±0.59   | 0.459±0.030 | 5.86/9        |               |
| 272/1986 | SWP29326| 7200          | 0.60±0.22 | 0.131±0.015 | 8.49/5      | Mrk 180     | 110      |          |             |            |              |               |               |
| 138/1980 | LWR7780 | 10800         | 0.61±0.93 | 0.29±0.045 | 4.16/9      | Mrk 180     |          |          |             |            |              |               |               |
| 142/1980 | LWR7834 | 10680         | 0.56±0.36 | 0.36±0.029 | 7.12/9      | Mrk 180     |          |          |             |            |              |               |               |
| 313/1980 | SWP10563| 25200         | 0.86±0.17 | -0.172±0.012 | 2.69/5      | Mrk 180     |          |          |             |            |              |               |               |
| 315/1980 | LWR2977 | 15600         | -        | -0.139±0.029 | 33.96/4     | Mrk 180     |          |          |             |            |              |               |               |
| 270/1982 | SWP18123| 21299         | 0.63±0.20 | -0.118±0.013 | 3.23/5      | Mrk 180     |          |          |             |            |              |               |               |
| 312/1984 | SWP24412| 23699         | 0.60±0.35 | -0.403±0.025 | 4.18/5      | Mrk 180     |          |          |             |            |              |               |               |
| 020/1985 | LW P5238 | 2760          | -        | -0.060±0.033 | 6.03/4      | Mrk 180     |          |          |             |            |              |               |               |
| 020/1985 | SWP24936| 17400         | 0.63±0.29 | -0.293±0.019 | 2.07/5      | Mrk 180     |          |          |             |            |              |               |               |
| 1218+304 |          |               |          |              |              | Mrk 180     |          |          |             |            |              |               |               |
| 033/1984 | SWP22187| 23700         | 0.79±0.31 | -0.34±0.02  | 0.38/5      | 1.02±0.13  -0.22±0.01 | 6.36/11     |               |
| 034/1984 | LW P2733 | 12600         | 1.08±0.16 | -0.17±0.02  | 5.96/4      | 1.02±0.13  -0.22±0.01 | 6.36/11     |               |
| 063/1987 | LW P10266| 4620          | 0±4      | -0.50±0.08  |            | 0414+009    |               |               |
| 069/1987 | LW P10310| 22200         | 3±4      | -0.73±0.07  |            | 0414+009    |               |               |

a Flux (in mJy) at 2500Å or 1500Å for LWR/P or SWP respectively.
b Flux (in mJy) at 2000Å.
c Log of mean flux in 2600–3100Å band.
be made concerning variations in either the ultra-violet flux or spectral index. The results are listed in Table 4.3 in the same format as Table 4.1. It is clear however that no dramatic variability has been observed in these sources. Formally, we find weighted mean indices of $\bar{\alpha}_{LWR/P} = 1.00 \pm 0.13$ and $\bar{\alpha}_{SWP} = 0.68 \pm 0.04$ for Mrk 501, and $\bar{\alpha}_{LWR/P} = 0.6 \pm 0.5$ and $\bar{\alpha}_{SWP} = 0.74 \pm 0.11$ for Mrk 180. In no case did we find any obvious correlation between spectral index and flux (Fig. 4.8). We note, however, that in all three objects, when pseudo simultaneous long- and short-wavelength spectra are available, the combined spectral index $\alpha_C$ was $\sim 1$.

4.3.3 H0414+009

The IUE observations presented here represent the first published ultra-violet observations of this source. The two LWP spectra were obtained in 1987 March, the source being considered to be too weak for any attempt at short wavelength images to be made. The images were reduced as described above, but the instrumental profile and source spectrum restricted the useful data to the 2600-3100Å wavelength band. Significant excess counts were observed below 2400Å. The running mean of the background was determined using a 100 pixel triangular weighting function.

The mean fluxes in this band are given in Table 4.3. It can been seen that there is marginal evidence for variability with a decrease of $\sim 30\%$ in one week. Extremely limited spectral information was obtained from these observations with $\alpha_{LWP} = 0.0 \pm 4$ for the brighter observation, and consistent results for the latter. We have attempted to increase the signal to noise by reducing the width, $W_S$, of the source extraction slit to as little as 4 pixels (with a corresponding decrease in observed flux), however this did not lead to any more stringent constraints on $\alpha$. Furthermore, we found little improvement when the two data sets were co-added.
4.4 Summary of IUE Observations

From a detailed analysis of the available data from Mrk 421 we have found a number of significant systematic uncertainties are often present within low-resolution IUE spectra from weak sources. These may lead to substantial discrepancies between different estimations of the ultra-violet continuum from the same data set. A number of these effects (such as the dependency of the derived slope and fluxes on $W_5$) are purely the result of the analysis technique – although often overlooked. However, potentially more serious (intrinsic) systematics were also found to be present (such as the discrepancy between the individual camera and combined indices), the causes of and solutions to which are currently unclear. We have illustrated and quantitatively described a number of such effects and our favoured analysis technique.

For the five sources under study, we found in all cases that the ultra-violet data could be adequately described by a simple power-law model. In no case did we find any obvious correlation between spectral index and flux. Indeed, only during the 1981–82 ultra-violet outburst of Mrk 421 do we have strong evidence of spectral variability in any of the sources. Furthermore, it is intriguing that in all other cases, the spectral index, $\alpha_c$, of the combined long- and short-wavelength data set is $\sim 1$. The relatively slow, small amplitude variability seen in the ultra-violet contrasts with the more erratic behaviour of these sources in the X-ray band (Chapter 3).

In addition, in the case of the relatively bright source Mrk 421, we have shown that the co-addition of the featureless ultra-violet continuum in this source can provide as useful back-drop for the detection of spectral features due to the Galactic halo.
Chapter 5

The Multi-waveband Spectra

Overview

In the first part of this chapter we discuss the general form and variability of the ultra-violet to X-ray continuum of the five objects under consideration. Then in section 5.2, we briefly review the information that can be gained from optical and radio observations of BL Lac type objects. Finally in Section 5.3 we discuss the general form of the multi-waveband continua of the individual objects with particular emphasis on those facets most relevant to the modelling of the multi-waveband non-thermal continuum to be presented in Chapter 6.

5.1 The ultra-violet to X-ray Continua

From the results presented in the previous two chapters, it is clear that, at least for the relatively well studied sources Mrk 421 and Mrk 501, the behaviour in the X-ray and ultra-violet bands seems to differ significantly. In Fig. 5.1 we plot the highest and lowest best-fitting spectral models to the EXOSAT data (with the effects of low energy galactic absorption removed) of the five objects under consideration. We also show the highest and lowest best-fitting power-law models
to the IUE data obtained during the years EXOSAT was operational. It should be remembered however, that as discussed in Chapter 4, there are a number of calibration uncertainties and other difficulties associated with the analysis of IUE spectra of very weak sources such that the derived ultra-violet spectral indices are probably not accurate to within $\sim 0.3$. Nevertheless it is clear from Fig. 5.1 that for all five objects, there is a general steepening of the continuum between the ultra-violet and X-ray bands, with an increase of spectral index ranging from $\Delta \alpha \sim 0.1$ to 1.3.

The X-ray index-flux correlation reported in Chapter 3 along with the absence of marked spectral variability in the ultra-violet band (Chapter 4) suggests that the variability of the ultra-violet to X-ray continuum may be described as a 'pivoting' of the X-ray spectrum about a break-point. We have followed such a scenario and found the best fitting break-point, $E_p$, and flux density, $F_p$, to the observed correlation between $F_{2-6}$ and $\alpha_x$ for each object using the results from the spectral analysis in which $n_H$ is fixed at the respective weighted mean value. The results for a model in which $E_p$ and $F_p$ are free parameters are given in Table 5.1 (columns 2–4). The resulting best fitting curves are shown (dotted) in Figs. 3.3, 3.6, 3.9, 3.11 and 3.14, and the corresponding pivot points (as filled circles) in Fig. 5.1. It can be seen from the reduced Chi-square values, $\chi^2_r$, listed in Table 5.1, that such a model provides a satisfactory description of the data for 4 of the 5 objects. (It should be noted however, that in the case of Mrk 180 $E_p$ and $F_p$ are not well constrained due to the lack of X-ray observations.) The exception, Mrk 421, will be discussed further below. As is to be expected from Fig. 3.9, we find a vastly superior fit to the 1218+304 data is obtained if the 074/1986 epoch is excluded (when there may be a significant contribution in the ME from Mrk 766 – see Chapter 3).

Also listed in Table 5.1 (columns 5–6) are the results from a model where $E_p$ is fixed at 0.1 keV as first suggested by George, Warwick & McHardy (1988). Satisfactory fits are again obtained in all cases except Mrk 421. The corresponding pivot-points are shown for comparison (open circles) in Fig. 5.1. For Mrk 180
Figure 5.1 The spectral flux distribution (in units of keV cm$^{-2}$ s$^{-1}$ keV$^{-1}$) of the ultra-violet to X-ray continuum in the five sources showing the highest and lowest EXOSAT spectra, and also the highest and lowest IUE spectra obtained during the operational lifetime of EXOSAT. The best-fitting pivot points to the X-ray data alone in which $E_p$ and $F_p$ are both free parameters, and in which $E_p$ is fixed at 0.1 keV are shown as filled and open circles respectively (see text).
Table 5.1 Best fitting 'Pivot' parameters

<table>
<thead>
<tr>
<th>Object</th>
<th>$E_p$</th>
<th>$F_p$</th>
<th>$\chi^2$/dof</th>
<th>$E_p$</th>
<th>$F_p$</th>
<th>$\chi^2$/dof</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mrk 421</td>
<td>0.14</td>
<td>0.20</td>
<td>4.68/12</td>
<td>0.35</td>
<td>4.57/13</td>
<td>all data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.21</td>
<td>0.11</td>
<td>4.87/9</td>
<td>0.35</td>
<td>5.80/10</td>
<td>low states only</td>
<td></td>
</tr>
<tr>
<td>Mrk 501</td>
<td>0.19</td>
<td>0.13</td>
<td>1.36/7</td>
<td>0.31</td>
<td>1.44/8</td>
<td>all data</td>
<td></td>
</tr>
<tr>
<td>1218+304</td>
<td>0.01</td>
<td>0.25</td>
<td>1.12/7</td>
<td>0.20</td>
<td>1.80/8</td>
<td>all data</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.13</td>
<td>0.15</td>
<td>0.38/6</td>
<td>0.20</td>
<td>0.27/7</td>
<td>excluding 014/1986</td>
<td></td>
</tr>
<tr>
<td>0414+009</td>
<td>1.10</td>
<td>0.0050</td>
<td>0.24/2</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mrk 180</td>
<td>0.03</td>
<td>1.05</td>
<td>0.01/1</td>
<td>0.14</td>
<td>0.67/2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

the best fitting curve to the $F_{2-6}$ vs $\alpha_x$ data where $E_p = 0.1$ keV is also shown (dashed) in Fig.3.11.

Interestingly, the resulting pivot points generally lie on an extrapolation of the ultra-violet spectra (within the limitations of the IUE measurements as noted above). It is therefore intriguing that both the X-ray data alone and the ultra-violet to X-ray continua are therefore broadly consistent with such a scenario.

A correlation between X-ray flux and spectral index has also been noted in the case of PKS 2005-489 (Wall et al. 1986), suggesting it may be a common property of X-ray bright BL Lac type objects. The highly variable source PKS 2155-304 may initially appear an exception to this hypothesis. Although large spectral variations in the sense that the source hardens as it brightens have been seen in the X-ray band on timescales as short as $\sim 2200$ s (Morini et al. 1986), there seems to be no clear relationship in this object between X-ray flux and spectral index over the full set of EXOSAT observations. However, since PKS 2155-304 also exhibits significant flux variations in the ultra-violet on a timescale of $\sim 10$ days with little change in spectral index (Urry 1986b), it is plausible that an underlying correlation between X-ray flux and spectral index for the whole EXOSAT data set is masked by changes in the overall normalisation of the ultra-violet to X-ray continuum on timescales of a few days. A similar explanation
may also be relevant in the case of Mrk 421 where the high values of reduced Chi-square in Table 5.1 for this object are mainly due to scatter in the low state data. Since variability is evident in the ultra-violet data (on a timescale of weeks - Chapter 4), there may also be some scatter induced by changes in the overall normalisation of the continuum. It should be noted (Fig. 3.3) that the high-state observations of Mrk 421 are also consistent with a break at \( \sim 0.1 \) keV. Hence the spectral steepening above a few keV during these epochs need not necessarily be due to a shift in the pivot-point.

5.2 Long Wavelength Continua

5.2.1 Optical and infra-red observations

Optical images of BL Lac type objects often reveal a bright, unresolved central source surrounded by a faint nebulosity. Spectroscopic measurements of these nebulosities have shown they display spectral energy distributions and absorption features typical of giant elliptical galaxies (e.g. Miller, French & Hawley 1978). In a number of nearby objects, such a view is supported by radial profile measurements (e.g. Weistrop et al. 1979, 1981, 1983). Burbidge & Hewitt (1987) have recently suggested that all BL Lac type objects of known redshift may lie in a galaxy with an average absolute V-band magnitude \( M_V = -22.5 \). Elliptical galaxies generally have \( M_V \) in the range \(-20 \geq M_V \geq -23\) (Frogel et al. 1978), thus the host galaxies of BL Lac type objects maybe fairly typical of the brighter members, such as the first-ranked cluster elliptical NGC 4489 (Mihalas & Binney 1981). The optical to infra-red continuum in these objects is therefore likely to be composed of two components: nonthermal, nuclear emission (the 'BL Lac' itself), and (thermal) galactic starlight. If the host galaxies are indeed giant ellipticals, then the integrated stellar spectrum is likely to be dominated by late-type stars (predominately K, M and G stars contributing \( \sim 50\%\), 40\% and 10\% of the

\[1\text{assuming } H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}.\]
thermal flux respectively) peaking at \( \sim 0.5-1 \) microns (see *e.g.* Whitford 1975). Clearly, in order to study the form of the underlying non-thermal component, the stellar contribution must be estimated and subtracted. This can be attempted in one of several ways as briefly outlined below.

i) The integrated stellar spectrum of a typical giant elliptical is assumed, renormalized and subtracted from the observed optical to infra-red continuum until the derived nonthermal continuum is considered reasonable (*e.g.* Cruz-Gonzalez & Huchra 1984).

ii) The luminosity profile is measured, directly using CCD imaging, or indirectly from multi-aperture photometry (resulting in a ‘curve-of-growth’) and compared to a typical galactic profile and subtracted to leave only the non-thermal flux (assuming it to emanate from an unresolved point source). In this case also, it is invariably necessary to assume an integrated stellar spectrum in order to extend the method to other wavelengths (*e.g.* Hutter 1983).

iii) Comparison of the observed colour indices with those of a typical elliptical on a colour–colour diagram and the estimation of the mixture required of stellar and non-stellar components directly from difference (the ‘Colour-given’ method; *e.g.* Sandage 1973).

iv) Use of polarization data presupposing the intrinsic degree of polarization of the non-thermal source is independent of wavelength, whilst the galactic starlight component is unpolarized. The wavelength dependence of observed polarization for the composite object can then be used to estimate the dilution of the non-thermal continuum by the stellar component (*e.g.* Maza, Martin & Angel 1978).

v) Extrapolation of far infra-red data assuming no spectral curvature: at frequencies less than \( \sim 2.9 \times 10^{13} \) Hz (the \( N \) band), the contribution from galactic starlight is generally considered to be insignificant compared to the non-thermal component.
The choice of the method employed is clearly dependent upon the type and quality of data available, however the faintness of the extended emission surrounding BL Lac type objects leads to some uncertainty even in the brightest sources. An additional complication to the deconvolution of the two spectral components in these sources is created by the variability of the non-thermal emission. Clearly single epoch, 'snap-shot' observations provide the greatest hope of the unambiguous determination of the two components.

Polarization measurements potentially provide one of the most powerful diagnostic tools for the study of the physical conditions in either the emission region itself, or any region in which incident radiation becomes polarized. The optical polarization of Blazars (BL Lac type objects and OVVs) has been studied by numerous authors (Brindle et al. 1986 and references therein). It is found that Blazars generally show a variety of polarization behaviours, with substantial variability in the percentage linear polarization, $P$, often occurring on timescales of $\sim 1$ day. As will be discussed further in Chapter 6, the high polarization and rapid variability have led to the conclusion that optically thin incoherent synchrotron radiation is the most likely emission mechanism in these sources. Since the polarization from a power-law electron distribution is frequency independent (e.g. Ginzburg & Syrovatskii 1965), it is hoped that a study of any wavelength dependence of the polarization beyond that due to dilution by galaxy starlight, will provide independent information on the emission region (see Björnsson 1985).

Unfortunately however, except in the case of a few well studied sources such as OJ 287 (Holmes et al. 1984), the wavelength dependence of the polarization is still generally poorly determined. Brindle et al. (1986) found wavelength dependence of polarization in approximately a third of the 42 Blazars studied indicating it may be common feature. The levels of polarization in the infra-red are generally equal to or lower than those in the optical (by up to a factor $\sim 2$, but more typically $10-20\%$). Percentage polarizations as high as $30\%$ have been observed (Impey et al. 1982) strongly supporting the synchrotron origin of the emission and a direct indication of a well-ordered magnetic field. Brindle et al. (1986)
also find a general constancy of the position angle of the linear polarization, $PA$, with frequency.

### 5.2.2 Radio observations

The morphology of Blazars have been studied by a variety of workers (Weiler & Johnston 1980; Stannard & McIlwrath 1982; Browne *et al.* 1982; Schilizzi & de Bruyn 1983; Ulvestad, Johnston & Weiler 1984; Wardle, Moore & Angle 1984; Antonucci & Ulvestad 1985). The majority of the known Blazars now have maps published in the literature with resolutions of a few arcseconds, mainly from the VLA. Many show a complex morphology on a variety of scales, although it should be remembered that morphological classifications are somewhat subjective and highly dependent on map quality.

Antonucci & Ulvestad (1985) have shown that extended emission is common in blazars at 1.5 GHz (present in 49 out of 54 objects), often with a core–halo morphology, although one-sided and double-sided jets are also seen. The jets typically contain substantial power consistent with both Fanaroff–Riley classes (Fanaroff & Riley 1974). There is some evidence that the ratio of core-to-halo flux is correlated with optical polarization, with optical and radio core variability and with one-sided radio morphology. Wardle, Moore & Angle (1984) have noted that the preferred $PA$ of the optical polarization is within $15^\circ$ of the radio axis in a number of sources, however Antonucci & Ulvestad (1985) have shown that there is only a slight tendency for such an effect using a larger sample.

On a smaller scale, Very Long Baseline Interferometry (VLBI) has revealed milliarcsecond (mas) jets and highly relativistic proper motions in number of cases (Mutel & Phillips 1984; Walker 1984).

Below we briefly review the optical, infra-red and radio observations of the individual sources of interest here.
5.3 The Multi-waveband Continua of the Individual Sources

Mrk 421

With an apparent magnitude $m_v \sim 13.5$, Mrk 421 is one of the brightest known BL Lac type objects in the optical and infra-red bands. It has long been established that the central nucleus in this source is surrounded by a recognizable elliptical host galaxy (Ulrich et al. 1975) with more recent CCD photometry revealing a modest ellipticity, $\epsilon = 0.2$ (Hickson et al. 1982). Optical and infra-red measurements by numerous workers have shown that this object possesses a few percent linear polarization in the range $P \sim 0$–7%, with evidence of some intrinsic wavelength dependency in the sense $P_{IR} < P_{opt}$ (Maza, Martin & Angel 1978; Bailey et al. 1981; Gagen-Torn, Marchenko & Smekhacheva 1983; Sitko, Schmitt & Stein 1985; Kikuchi & Mikami 1986). Significant changes in $P$ have been seen over a period of ~ days and weeks (Bailey et al. 1981; Kikuchi & Mikami 1986), whilst the magnetic field seems to possess a short and long term geometric memory since the position angle is restricted to a small portion of the compass ($PA \sim 175^\circ \pm 20^\circ$). Furthermore, $PA$ seems to be independent of frequency and $P$, implying the non-thermal source is the only significant contributor to the polarization. It has been proposed that dilution from the galactic starlight is totally responsible for the frequency dependence of $P$ (Maza, Angel & Martin 1978) but this has been challenged by Bailey et al. (1981) who favour some intrinsic dependency.

Significant variability has been observed in the total optical and infra-red flux on a variety of timescales in Mrk 421. Archival measurements dating back to the turn of the century have shown that this source exhibits large amplitude variations with a total range $\Delta B \geq 4.7$ mag (Miller 1975). Inspection of the historic light curves suggests erratic short term variability superimposed on longer term trends (~ several years). There is no evidence for any periodicity in the long-term light
Figure 5.2 Composite light curves for Mrk 421 in the period 1974 – 1987. (a) Radio flux at 4 cm (in Jy) from Aller et al. (1985). (b) Monthly average V band fluxes (in mJy) adapted from Gagen-Torn, Marchenko & Smekhacheva (1983) (*), and as derived from IUE FES count rates (o; this work); the dotted line indicates the estimated galactic contribution in this band for a 26 arcsec aperture (using Makino et al. 1987a). (c) Ultra-violet flux at 250nm (this work; as in Fig. 4.5). (d) X-ray flux in the 2–6 keV band ($\times 10^{-11}$ erg cm$^{-2}$ s$^{-1}$) from this work, Rickets, Cooke & Pounds (1976), Marshall, Warwick & Pounds (1981), Mushotzky et al. (1978), Hearn, Marshall & Jernigan (1979), Mushotzky et al. (1979), Urry, Mushotzky & Holt (1986), Madejski (1985), Hall (1981) and Makino et al. (1987a).
Figure 5.3 Multi-waveband spectrum of Mrk 421 compiled from our own observations and from the literature. The X-ray and ultra-violet data are as in Fig. 5.1. The optical and infra-red data are from Makino et al. (1987a) after subtraction of the galactic starlight. The radio data are from the low resolution observations of Angel & Stockman (1980), Landau, Epstein & Rather (1980), Owen, Spangler & Cotton (1980) and Weiler & Johnston (1980), and the VLBI observations of Bäath et al. (1981). The open circles refer to data where there is likely to be contamination from extended emission.
curve. On at least one occasion a rapid decrease by 1.2 mag has been observed on a timescale of 3 days. A more recent long term data set has been reported by Gagen-Torn, Marchenko & Smekhacheva (1983). These workers find $\Delta B \sim 1.3$ mag over a 7 year period again with long term trends ($\sim$years). Furthermore, they find some indication of a reddening of the spectrum as the spectrum becomes fainter. In Fig. 5.2 we show the monthly average $V$ band flux derived from this work along with the $V$ band flux derived from the IUE FES count rates (Chapter 2). Using the curve-of-growth derived from Makino et al. (1987a), we estimate that the galaxy contributes $\sim 9$ mJy to the total $V$ band flux observed with a 26 arcsec aperture (as used by Gagen-Torn, Marchenko & Smekhacheva). This is shown dotted in Fig. 5.2. Also shown are the corresponding X-ray, ultraviolet and radio light curves for direct comparison. It can be seen that the radio, optical and ultra-violet fluxes show some degree of correlation on long timescales.

The use of infra-red colours have led Impey (1983) to estimate that the non-thermal component was flat in the infra-red with a spectral index, $\alpha_{IR} \sim 0.25$. Comparison with the optical measurements of Maza, Martin & Angel (1978), which indicate an optical index $\alpha_{opt} \sim 1.0$, led Impey to conclude that there must be a break between two bands. A similar conclusion has also been reached independently by Cruz-Gonzalez & Huchra (1984). The surface brightness distribution of Mrk 421 has been studied by Kinman (1978), Mufson et al. (1984) and Hickson et al. (1982) but there are some discrepancies between their results. Hickson et al. found both Mrk 421 and its small companion to be anomalously red leading to a flat non-thermal spectral index $\sim 0.2$. However they point out that this is very likely due to a systematic error. The luminosity profile of Mrk 421 was found to be consistent with a composite profile consisting of an unresolved point source and an elliptical galaxy which obeys the de Vaucouleurs surface brightness law (de Vaucouleurs 1958). Recently, Kikuchi & Mikami (1986) and Makino et al (1987a) have performed pseudo simultaneous multi-aperture photometry in the optical and infra-red bands respectively. Combining these results, it has been shown that a single power-law of index $\sim 0.5–0.7$ is an acceptable representation of the non-thermal spectral component, with only marginal evi-
idence for a steepening between the two bands. The latter estimation is favoured in this work, particularly since the observations were made at a similar epoch to the (AO-1) EXOSAT observations. Wills et al. (1980) have measured a redshift of $z = 0.0308$ for Mrk 421.

Mrk 421 shows a complex, double-lobe radio structure with an angular scale $\sim 10$ arcsec at 20 cm (Antonucci & Ulvestad 1985). Weiler & Johnson (1980) have modelled this large scale structure as a Gaussian with a Full Width Half Maximum (FWHM) diameter, $\theta_{\text{rad}} \sim 30$ arcsec, and containing a flux $\sim 0.1$ Jy. Centimetric VLBI observations (e.g. Weiler & Johnson 1980; Bååth et al. 1981; Wehrle et al. 1984) indicate a (partially) resolved component with $\theta_{\text{rad}} \sim 0.9$–few milliarcsec (mas) containing $\sim 0.3$ Jy. In addition, more recent observations of Bååth (1984) show a component $< 0.1$ mas in diameter and a thin 5 mas long jet which seems to be fairly well aligned with one of the large scale structures (Antonucci & Ulvestad 1985).

Aller et al. (1985) have found that the percentage polarization in Mrk 421 is $P_{\text{rad}} \sim 0$–4% in the radio band with some evidence of a decrease towards longer wavelengths. Furthermore, the position angle rotates from $\sim 150^\circ$ at 6 cm through $\sim 0^\circ$ at 4 cm (an angle similar to the $PA$ in the optical and infra-red bands) to $\sim 90^\circ$ at 2 cm.

In Fig. 5.3 we show the composite multi-waveband spectrum of the non-thermal component in Mrk 421. The references for the various data are given in the figure caption. As can be seen, the non-thermal spectrum of Mrk 421 is flat in the radio band (with $\alpha_{\text{rad}} \sim 0.1$), but progressively steepens at higher energies. The excellent agreement between the measured spectrum in the far infra-red (where the contribution from galactic starlight is negligible) and the calculated spectrum in the optical and (near) infra-red bands following the subtraction of the contribution due to starlight (see above) indicates that the latter contribution has been reasonably well estimated. For the purposes of the modelling to be presented in Chapter 6, we assume a single power-law of index $\alpha = 0.65$ adequately describes
the non-thermal continuum throughout the infra-red and optical regimes.

Mrk 501

The nebulousity surrounding Mrk 501 is clearly visible on POSS plates. CCD photometry reveals a radial profile consistent with the de Vaucouleurs surface brightness law (de Vaucouleurs 1958) with a modest degree of ellipticity, $\epsilon \sim 0.3$ with a major axis along $PA \sim 0^\circ$ and a maximum angular extent $\sim 80$ arcsec (Hickson et al. 1982; van Breugel & Schilizzi 1986). Significant linear polarization ($P \sim 2$–7%) is found in the optical and infra-red bands with $P_{opt} > P_{IR}$ (Maza, Martin & Angel 1978; Puschell & Stein 1980). The preferred optical polarization angle, $PA \sim 149^\circ$ appears to be aligned with the VLBI radio jet (below)

Mrk 501 seems to relatively stable at optical frequencies with $m_v \sim 13.8$ (Bardieri & Romano 1977; McGimsey & Miller 1978; Mufson et al 1984). The historical optical lightcurve in Barbieri & Romano (1977) revealed slow drifts on a timescale $\sim$ few years with no indication of rapid variations. In contrast, Impey et al. (1982) have reported a 35% increase in infra-red flux on a timescale of 3 days.

The separation on the non-thermal and stellar components has proved to be somewhat troublesome in Mrk 501. Following subtraction of the stellar component, there is a tremendous range in the quoted non-thermal spectra. For example Cruz-Gonzalez & Huchra (1984) find a steepening ($\alpha = 0.3 \rightarrow 1.4$) between the infra-red and optical bands whilst Kondo et al. (1981) find a similar break ($\alpha = 0.0 \rightarrow 1.0$) at a different epoch but between the optical and ultra-violet bands. Recently however, IRAS observations have revealed that the spectral slope in the far infra-red is $\alpha = 0.6^{+0.1}_{-0.1}$ (Sembay et al. 1985; Impey & Neugebauer 1988). This value is consistent estimations of the underlying non-thermal spectrum in the optical and infra-red bands made by several workers (Maccagni et al. 1983; Maza, Martin & Angel 1978; Impey 1983; Hickson et al. 1982) and with that of X-ray selected BL Lacs in general (Ghisellini, Maraschi & Treves 1986), thus is likely to be reasonable. Furthermore, as shown in Fig. 5.4 the best
Figure 5.4 Multi-waveband spectrum of Mrk 501 in the same format as Fig. 5.3. The X-ray and ultra-violet data are as in Fig. 5.1. The optical and infra-red data are from Puschell & Stein (1980) and Impey & Neugebauer (1988); the radio data are from the low resolution observations of Angel & Stockman (1980), Kuhr et al. (1981), Landau, Epstein & Rather (1980) and Owen, Spangler & Cotton (1980), and the VLBI observations of van Breugel & Schilizzi (1986).
fit to the IRAS data extrapolates well onto the ultra-violet. We have therefore adopted such a non-thermal spectrum in all further analysis. The redshift of Mrk 501 is \( z = 0.034 \) (Ulrich et al. 1975).

The radio morphology of Mrk 501 is one of the best studied and most intriguing of all BL Lac type objects. VLBI observations at 5 GHz have revealed a one-sided jet of total length \( \sim 23 \) mas emerging from the nucleus. (van Breugel et al. 1986). The position angle of the jet is \( PA \sim 133^\circ \) with a possible curvature to the NE (towards \( PA \sim 104^\circ \)). The total flux density of this component is \( \sim 0.39 \) Jy consistent with the results from earlier (shorter) VLBI observations (Weiler & Johnson 1980; Pearson & Readhead 1981). This small scale jet is however, grossly misaligned (by \( \sim 80^\circ \)) with respect to the elongation (\( PA \sim 55^\circ \)) seen in 2.3 GHz and 5 GHz VLA observations and with the more extended, diffuse (60 arcsec) emission (\( PA \sim 45^\circ \); Ulvestad, Johnston & Weiler 1983). Aller et al. (1985) have shown the total radio flux in Mrk 501 shows slow drifts on a timescale \( \sim \) years with some indication of more erratic behaviour on shorter timescales.

The observed multi-waveband spectrum of Mrk 501 is shown in Fig. 5.4. Despite some evidence for spectral curvature, the radio spectrum can reasonably well approximated by a single power-law of index \( \alpha_{\text{rad}} \sim 0.2 \). In the optical and infrared bands, the contribution from galactic starlight is clearly visible in Fig. 5.4. However, from the far infra-red (IRAS) and ultra-violet observations we assume an underlying non-thermal spectrum with \( \alpha \sim 0.7 \).

1218+304

Direct CCD imaging of 1218+304 has revealed evidence for a faint surrounding nebulosity consistent with an elliptical galaxy (Weistrop et al. 1981). Linear polarization measurements have shown \( P_{\text{opt}} \sim 5-8\% \) with \( PA \sim 70^\circ \pm 10^\circ \), however the weakness of the source (\( m_v \sim 16.5 \)) has resulted in the existence of very little information on either time or wavelength variability of the polarization (Sitko, Schmidt & Stein 1985). 1218+304 does not seem to be strongly variable at optical
Figure 5.5 Multi-waveband spectrum of 1218+304 in the same format as Fig. 5.3. The X-ray and ultra-violet data are as in Fig. 5.1. The optical and infra-red data are from Ledden et al. (1981) and Weistrop et al. (1981); IRAS upper limits from Impey & Neugebauer (1988). The low resolution radio data are from Wilson et al. (1979), Stocke et al. (1985) and Unger et al. (1987).
and infra-red wavelengths.

The determination of the underlying non-thermal continuum has proved somewhat problematic in this source also. Extremely steep optical spectra and spectral breaks between the infra-red and optical bands have been proposed (Wilson et al. 1979; Ledden et al. 1981; Cruz-Gonzalez & Huchra 1984). However, following the direct CCD imaging of galaxy (above), the most likely underlying non-thermal spectrum seems to be a single power-law with $\alpha = 0.9^{+0.2}_{-0.1}$ over the complete optical to infra-red regime (Weistrop et al. 1981).

Despite the detection of some weak features in the spectrum, Wilson et al. (1979) were unable to determine the redshift of this source. However, by assuming a giant elliptical galaxy with a fixed absolute magnitude of $M_V = -22.4$ mag (c.f. Burbidge & Hewitt 1987), and adjusting the redshift and the intensity of the point source to obtain the best fit to the spatial intensity distribution, Weistrop et al. (1981) estimate the redshift of 1218+304 to be $z = 0.13^{+0.03}_{-0.01}$.

The radio flux of 1218+304 is $\sim 40$–60 mJy in the 1.5–15 GHz band with no evidence of significant variability (Wilson et al. 1979; Stocke et al. 1985; Unger et al. 1987). The radio polarization is $\leq 3\%$ at 5 GHz (Stocke et al. 1985). VLA observations have shown source is unresolved with angular size $<1$ arcsec (Unger et al. 1987). Unfortunately no VLBI observations exist for this source.

In Fig. 5.5 we show the observed multi-waveband spectrum of 1218+304. We estimate that the spectral index in the radio is $\alpha_{rad} \sim 0.18$. In the infra-red band a slight excess is clearly visible, presumably due to a contribution from galactic starlight. Making use of the ultra-violet and X-ray data, we will assume a power-law of index $\alpha \sim 0.5$ provides a reasonable estimation of the underlying non-thermal continuum.
Figure 5.6 Multi-waveband spectrum of Mrk 180 in the same format as Fig. 5.3. The X-ray and ultra-violet data are as in Fig. 5.1. The optical data are from Mufson et al. (1984), and the radio data from Mufson et al. (1984) and Antonucci & Ulvestad (1985).
Mrk 180

Mrk 180 is relatively faint at optical to infra-red wavelengths ($m_V \sim 14.5$) leading to lack of detailed knowledge of either the spectrum or variations with time\(^2\). However, it appears that Mrk 180 is relatively stable in the optical with no significant variations observed over a 7 month period (Mufson & Hutter 1981: Mufson et al. 1984). The composition of the continuum in these bands is highly uncertain. Whilst it is likely that the component due to galactic starlight far outshines the non-thermal component, no deconvolution of these two components has, as yet, been achieved. Mufson et al. (1984), however, have estimated the ratio of non-thermal to galactic flux to be 0.67 in the V band corresponding to an absolute magnitude of $M_V = -21.9$ by working backwards from a specific SSC model. The presence of weak absorption lines in the optical presumably due to the galaxy led Markarian (1969) to estimate a redshift of $z = 0.0458$.

The VLA map of Ulvestad & Antonucci (1986) shows the highly complex core-halo of this source. No VLBI observations of this source are yet available.

The overall energy distribution of Mrk 180 is shown in Fig. 5.6. Interesting, the radio spectrum is a rising function of frequency ('inverted') with an index $\alpha_{\text{rad}} \sim -0.5$. The steep optical spectrum implies a significant contribution from the galaxy. In the absence of further information, we assume that steep optical spectrum is dominated by galactic starlight and the underlying non-thermal component as a spectral index of $\alpha \sim 0.6$ consistent with the ultra-violet.

0414+009

This source is very weak in the optical and infra-red bands with $m_V \sim 16.4$ and as in the case of Mrk 180, very little spectral or temporal information has been published. From a search of archival plates, Ulmer et al. (1983) found no evidence for optical variability over the last century although the majority of the plates examined had faint limits above the present brightness of 0414+009 and

\(^2\)In addition, optical measurements of Mrk 180 are somewhat complicated by the presence of a foreground star 7.2 arcsec from the nucleus (Mufson & Hutter 1981).
Figure 5.7 Multi-waveband spectrum of 0414+009 in the same format as Fig. 5.3. The X-ray and ultra-violet data are as in Fig. 5.1. The optical and infra-red data are from Ulmer et al. (1983) and our own (unpublished) UKIRT observations. The radio data are from Ulmer et al. (1983) and McHardy et al. (1988).
hence the object was invisible.

Recent VLA observations have shown that large scale morphology of 0414+009 is very similar to that of a standard radio-trail source (McHardy et al. 1988). An unresolved core, coincident with the centroid of the X-ray emission (Chapter 3) and optical counterpart, is apparent at 6, 20 and 90 cm with a flux density $\sim$60–80 mJy, similar to those observed by Ulmer et al. (1983). The spectral index of this component between 6 and 20 cm is $\alpha_{\text{rad}} \sim 0.2$. A moderate degree of linear polarization is also apparent ($P_{\text{rad}} \sim 3\%$). Most interestingly though, a large one-sided jet of total extent $\sim$ 45 arcsec emerges at $PA \sim 50^\circ$ curving to the SE towards the end. The jet contains a total flux density $\sim$10 mJy at 20 cm and has a spectral index $\alpha \geq 1.5$. Unfortunately no VLBI observations of this source have as yet been reported.

As shown in Fig. 5.7, the optical observations reported by Ulmer et al. (1983) and our own (unpublished) UKIRT observations are reasonably consistent with a single power-law spectrum of index $\alpha \sim 0.8$ throughout the infra-red and optical bands. Hence the contribution to the spectrum from any galactic starlight in 0414+009 is likely to be negligible. Such a scenario is supported by recent near infra-red CCD imaging which reveal only a small degree of extended emission (McHardy et al. 1988).

No features have been found in the continuum of 0414+009 hence the redshift has not been determined. However, based upon the similarity of the surrounding cluster with other groups and clusters, a crude estimate of $z = 0.2$ can be made (McHardy, p.communication).
Chapter 6

Synchrotron Self-Compton Models for the Continuum Emission in BL Lac Type Objects

Overview

In this chapter we address the fundamental question of the origin of the electromagnetic radiation from BL Lac type objects. Following some introductory remarks, in Section 6.2 we briefly review synchrotron and inverse-Compton emission. Then in Sections 6.3 and 6.4 we apply two specific synchrotron self-Compton models to the multi-waveband spectra of the five sources under consideration paying particular attention to their ability to account for the observed spectral variability in the X-ray band.
6.1 Introduction

There are two principle means by which observational data are used to infer the physical conditions within an emission region:— time variability studies, and examination of the multi-waveband spectrum. Clearly a complete understanding of the underlying physics requires the simultaneous utilization of both types of information. To date however, a general lack of detailed observational data has prevented the construction of fully time dependent physical models in any class of AGN. Nevertheless, from the observations reported in the previous chapters, it seems likely that the study of the variability exhibited in the X-ray and ultra-violet bands will provide significant insights into the emission regions of the five sources under consideration here. In Chapter 3 (Section 3.7.1) a number of well known arguments were applied to the minimum observed X-ray timescale. However it was emphasized that the inferences that can be drawn from such studies are highly model dependent. In the remainder of this chapter we will try and account for the observed multi-waveband spectral variability paying particular attention to the high energy regime.

6.2 Synchrotron and Inverse-Compton Emission

As discussed in the previous chapter, the multi-waveband spectra of BL Lac objects are reasonably well represented by a succession of simple power-laws. The smoothness of the multi-waveband spectra and the correlation between variations in intensity in the radio, optical and ultra-violet bands observed in some sources imply a single, or at least several closely related emission mechanisms are responsible. The presence of significant polarization in the optical and radio has led many authors (e.g. Ginzburg & Syrivatskii 1965; Kellermann 1966) to conclude that the mechanism responsible for the radio emission is likely to be inhomogeneous synchrotron radiation. Alternatives have been suggested which can mimic a power-law over a limited bandwidth, (e.g. thermal photon/Compton scattering
models of Sharpiro, Lightman & Eardley 1975; Katz 1976), however these are likely to produce a Wein peak somewhere in the spectrum and none claims to explain all the observed emission with a single process. Thus the synchrotron nature of the radio to ultra-violet spectra in these objects is fairly well established. As yet there is no general consensus on the origin of the X-ray emission from BL Lac type objects as a class although for all the objects reported here, particularly in the absence of any flattening of the spectra above a few keV, we argue that the X-rays are produced by the synchrotron process also.

Synchrotron emission occurs when relativistic electrons spiral around magnetic field lines, radiating as a result of their acceleration. The emission characteristics for a single electron and for an ensemble of relativistically moving particles in a magnetic field have been derived by many workers including Ginzburg & Syrovatskii (1965, 1969) and Blumenthal & Gould (1970) and has been successfully applied to extended extra-galactic radio sources (e.g. Pauliny-Toth & Kellermann 1966, Kellermann 1966). Most significantly for the work discussed here, it is found that in a region of uniform magnetic field, a power-law distribution of synchrotron emitting electrons leads to an emergent radiation spectrum which is also a power-law over a wide range of frequencies.

If the density of high energy electrons and radiation is very high, the electrons are able to interact with the ambient synchrotron photons via the inverse-Compton effect\(^1\). The latter is commonly referred to as the synchrotron self-Compton (SSC) process and can lead to prodigious amounts of high energy photons. The emissivity due to a single inverse-Compton scattering of synchrotron photons by relativistic electrons can be estimated using the so-called \(\delta\)-approximation in which the final frequency, \(\nu_c\), of a scattered photon is related to the initial (synchrotron) frequency, \(\nu_s\), by \(\nu_c = \gamma_s^2 \nu_s\), where \(\gamma_s (=E/m_e c^2)\) is the Lorentz factor of the electron (but see Appendix C, section C-3). The resultant SSC spectrum therefore mimics the synchrotron spectrum but is generally weaker and

\(^1\)c.f. the usual Compton process observed in the laboratory in which the net transfer of energy is from the photon to the particle.
blue shifted. The detection of a flattening of the spectra of several X-ray bright BL Lac type objects above a few keV (see Appendix A) and in other objects (e.g. 3C 273, Bezler et al. 1984; Courvoisier et al. 1987) has been interpreted as the result of SSC emission and hence as evidence in favour of SSC models. Higher order inverse-Compton spectra are also possible but generally lie many orders of magnitude below and beyond the frequency range of interest here. The seminal works of Jones and collaborators (Jones, O'Dell & Stein 1974a,b and Burbidge, Jones & O'Dell 1974) first established a framework within which compact non-thermal sources observed in both the radio and X-ray bands have been analysed in terms of SSC emission. In these papers the canonical source model for the emission spectra is developed.

The canonical source is assumed to be a spherically symmetric volume of constant density containing a highly disordered magnetic field and a power-law distribution of relativistic electrons of the form

\[
N(\gamma_e) = \begin{cases} 
K\gamma_e^p & \text{for } \gamma_e,\text{min} \leq \gamma_e \leq \gamma_e,\text{max} \\
0 & \text{otherwise}
\end{cases}
\]  

[6.1]

where \( N(\gamma_e) \) is the number density of electrons with a Lorentz factor \( \gamma_e \).

Numerous workers have applied such a model to the multi-waveband spectra of BL Lac type objects, with varying degrees of success (see Urry 1988 and references therein). The level of the inverse Compton emission can be predicted directly from a small number of observable parameters (see below) and compared to the observed SSC emission or an upper limit thereon. It is often found however, that the predicted inverse Compton emission is many orders of magnitude higher than actually observed (Madejski & Schwartz 1983; Madau, Ghisellini & Persic 1987) — a discrepancy often referred to as the 'inverse Compton catastrophe' or 'brightness temperature problem'. The most common resolution of this discrepancy was first suggested by Shklovsky (1963, 1965) and further developed by Rees (1966), Rees & Simon (1968) and Burbidge, Jones & O'Dell (1974). These workers pointed out that bulk relativistic motion towards the observer would lead to an enhancement.
of the observed source flux as compared to that in the co-moving frame of the emitter. Specifically, if the source is moving relativistically with a velocity $v = \beta c$ (where $c$ is the speed of light) at an angle $\theta$ to the line of sight, then the degree of relativistic beaming observed is determined by the kinematic Doppler factor $D$ where

$$D = \left[\Gamma(1 - \beta \cos \theta)\right]^{-1}$$  \hspace{1cm} \text{[6.2]}

where $\Gamma$ is the bulk Lorentz factor given by

$$\Gamma = (1 - \beta^2)^{-1/2}$$  \hspace{1cm} \text{[6.3]}

The observed luminosity may therefore be considerably brighter or considerably dimmer than the luminosity emitted in the co-moving frame depending on the $\theta$. For angles smaller than

$$\theta_{\text{crit}} = \arccos \left(\frac{\Gamma - 1}{\Gamma + 1}\right)^{1/2}$$  \hspace{1cm} \text{[6.4]}

$D > 1$ and hence the co-moving (emitted) synchrotron flux density is fainter than that implied in the observer's frame. As will be discussed further below, additional effects of relativistic motion lead to the predicted SSC emission being a strong function of $D$, thus often only a modest degree of relativistic beaming is required to prevent over-production of inverse Compton photons. In addition, Doppler enhancement of the synchrotron continuum has also been proposed as an explanation for the featureless spectra observed in BL Lac type objects due to the swamping of any line emission from any stationary surrounding region.

### 6.3 The Homogeneous Disk Model

The basic homogeneous disk model comprises of a stationary volume filled with a uniform density of relativistic electrons radiating isotropically. The electron
number density distribution, $N(\gamma)$ is assumed to be a power-law as given in equation \[6.1\], leading to a resultant synchrotron spectrum of the form $S_\nu \propto \nu^{-\alpha}$ where $\alpha = (p-1)/2$ over a wide range of frequency, $\nu_*$ in the observer's frame. At high frequencies there will be a steepening of the spectrum due to synchrotron losses (discussed in more detail below), whilst at low frequencies, the emitted synchrotron photons are absorbed by the electron population and the source becomes optically thick. There will therefore be a sharp break in the emergent radiation spectrum close to the synchrotron self-absorption frequency, $\nu_{sm}$ (where the optical depth, $\tau = 1$). For a power-law distribution of electrons with $p > 1$, the spectrum at frequencies below $\nu_{sm}$ will be a power-law of index $\alpha_{abs} = 5/2$ independent of $p$. As discussed in Urry (1984), there are slight differences in the literature between the precise definitions of the fiducial frequency – some workers choosing $\nu_{sm}$, whilst others use the turn-over frequency where the gradient on a log $S_\nu$ – log $\nu_*$ plot is zero. The former has the advantage that it is invariably better determined observationally and is used throughout this work.

### 6.3.1 Derivation of the physical conditions

The standard procedure for the determination of the physical conditions within such a model is as follows. After making a number of simplifying assumptions concerning the electron distribution in the co-moving frame, expressions for the self-absorption frequency, $\nu_{sm}$, and the observed flux at this frequency, $S_\nu(\nu_{sm})$ ($\equiv S_{sm}$), can be derived (Appendix C)

$$\nu_{sm} = \left(\frac{3.086 \times 10^{18}\theta_\phi D_L C_2(\alpha)KB^{a+3/2}}{2}\right)^{1/(\alpha+5/2)}$$

$$\times D(1+z)^{-\frac{\alpha+2}{3+\alpha}} \text{ Hz} \quad [6.5]$$

$$S_{sm} = \frac{3.086 \times 10^{18}\theta_\phi^3 D_L C_1(\alpha)KB^{1+\alpha}\nu_{sm}^{-\alpha}}{32}$$

$$\times D^{3+\alpha}(1+z)^{-(5+\alpha)} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \quad [6.6]$$

where $C_1(\alpha)$ and $C_2(\alpha)$ are functions of $\alpha$ given by equations \[C.6\] and \[C.10\] respectively and tabulated in c.g.s. units for various values of $\alpha$ in Table C.1, $\theta_\phi$ is
the angular diameter of the source in radians, and \( D_L \) is the luminosity distance of the source as defined by equation [B.10]. Here we have explicitly assumed a disk-like geometry where the thickness of the disk is equal to its radius (see Fig. C.1).

As can be seen, equations [6.5] and [6.6] are both functions of three parameters, \( K, B \) and \( D \) which cannot be directly determined observationally. However, the addition of a 6th observable parameter, the upper cut-off in the observed synchrotron spectrum, \( \nu_{\text{ub}} \), can lead to either independent information on \( B \) through consideration of synchrotron loss timescales, or (more usually) an estimation of the predicted synchrotron self-Compton flux, \( S_{\text{sc}}^{\text{pred}} \), at some frequency \( \nu_c \). Following a series of simplifying assumptions, it can be shown that for a homogeneous disk model (Appendix C)

\[
S_{\text{sc}}^{\text{pred}}(\nu_c) = \frac{3\sigma_T^{29+8\alpha} A(\alpha) C_1(\alpha)^{-3(3+2\alpha)} C_2(\alpha)^{(2+2\alpha)} \ln(\nu_{\text{ub}}/\nu_{\text{sm}})}{\nu_c^{-(3+2\alpha)} S_{\text{sm}}^{2(2+\alpha)} \nu_{\text{sm}}^{-(5+3\alpha)} D^{-2(2+\alpha)}}
\times (1 + z)^{(2+\alpha)} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}
\]

over the frequency range \( \gamma_{\text{sm}}^2 \nu_{\text{sm}} \leq \nu_c \leq \gamma_{\text{ub}}^2 \nu_{\text{ub}} \), where \( A(\alpha) \) is a function of \( \alpha \) given by equation [C.62] and tabulated in Table C.1, and \( \gamma_{\text{sm}} \) is the Lorentz factor of electrons which radiate synchrotron radiation peaking at \( \nu_{\text{sm}} \) (equation [C.24]). Outside this frequency range we consider \( S_{\text{sc}}^{\text{pred}}(\nu_c) \) to be negligible.

Thus, by equating the predicted SSC flux with that observed, \( S_{\text{sc}}^{\text{obs}}(\nu_c) \), we can simultaneously solve equations [6.5], [6.6] and [6.7] for \( B, K \) and \( D \) to give

\[
B = \left( \frac{C_1(\alpha)}{C_2(\alpha)} \right)^2 \theta_d^4 S_{\text{sm}}^{-2} \nu_{\text{sm}}^5 \left( \frac{D}{1 + z} \right) \text{ G}
\]

\[
K = \frac{C_2(\alpha)^{(2+2\alpha)} 2^{(13+8\alpha)} S_{\text{sm}}^{(3+2\alpha)}}{C_1(\alpha)^{(3+2\alpha)} 3.086 \times 10^{18} D_L}
\times \theta_d^{-(7+4\alpha)} \nu_{\text{sm}}^{-(5+4\alpha)} D^{-(4+2\alpha)} (1 + z)^{(6+2\alpha)} \text{ cm}^{-3}
\]

and

\[
D \geq \left\{ 3\sigma_T^{29+8\alpha} A(\alpha) \frac{C_2(\alpha)^{(2+2\alpha)}}{C_1(\alpha)^{(3+2\alpha)}} (1 + z)^{(2+\alpha)} \right\}
\]
A number of authors have considered the derivation of similar formulae for B, K and D. The reported results vary widely due to the use of different assumptions, approximations and definitions (Urry 1984). The above expressions are in good agreement with those derived using the formulism of Jones, O'Dell & Stein (1974a,b) despite the use of a slightly different emission region geometry. Unfortunately, however, there are a number of observational problems associated with the practical application of this procedure to BL Lac type objects. These are due mainly to inadequate simultaneous multi-waveband coverage and ambiguities in the interpretation of the spectra. It is clear from equations [6.8], [6.9] and [6.10] that assuming the redshift of a given BL Lac has been determined, the key parameters that must be determined are α, ν_{sm}, S_{sm} and S_{sc}^{obs}(ν_c) from the multi-waveband spectrum and θ_d from VLBI radio observations or elsewhere (see below). However in general none of these are well determined observationally, a problem which is amplified by the high powers to which these parameters are raised in the above expressions.

Observational uncertainties

α and ν_{sm}. The simplistic model described above predicts a rising spectrum of the form S ∝ ν^5/2 at low energies. However such a steep spectrum is never observed in practice. Indeed as discussed in Chapter 5, the radio spectra of BL Lac type objects are often flat. The optically thick regime is therefore never unambiguously identified. Thus we are left with a choice of assuming the source becomes optically thick at a frequency i) below the lowest observable radio frequency and hence that the optically thin spectral index is very flat; or ii) between the radio and infra-red bands, forcing us to explain the flat radio spectrum by other means. The latter is the most popular choice since the optically thin index of the non-thermal continuum is then ~0.5–1.0 consistent with theoretical electron acceleration mechanisms. The so-called ‘cosmic conspiracy’ of a series of
synchrotron regions of increasing angular size and decreasing $\nu_{sm}$ is then usually invoked to reproduce the observed flat radio spectrum. Such a picture is supported by multi-frequency VLBI maps of spatially resolved flat-spectrum radio sources (although admitted not BL Lac type objects) which reveal a number of separate clumps, each with different synchrotron characteristics

$\theta_d$ and $S_{sm}$. Despite the detection of weak extended structure on scales ~few mas, VLBI radio observations invariably reveal unresolved cores from which the majority of the emission emanates. Indeed as discussed in section 3.7.1, the application of light travel-time arguments to the X-ray variability timescale observed in these sources imply scale sizes $\sim 10^{-4}$–$10^{-3}$ mas. Due to the possible presence of radio components on intermediate scales ($10^{-3}$–$10^{0}$ mas), the central SSC source may be responsible for only a small fraction of the total observed flux from the VLBI core and hence $S_{sm}$ may be overestimated. In addition, VLBI observations are usually not available at $\nu_{sm}$ itself, hence a radio spectral index must be assumed and the measured flux extrapolated to give an estimation of $S_{sm}$.

$S_{c}^{obs}(\nu_c)$. As mentioned above, a flattening in hard X-rays that can be ascribed to SSC emission has only been observed in a handful of occasions in the five brightest objects. We have found no evidence for such a hard-tail in the EXOSAT data presented in Chapter 3. It is therefore usually necessary to use an upper-limit on $S_{c}^{obs}(\nu_c)$ leading to lower and upper limits on $B$ and $K$ respectively.

### 6.3.2 Application of the disk model

In Fig. 6.1 we illustrate the model synchrotron and SSC spectra using the data for Mrk 421. For frequencies in the range $\nu_{sm} < \nu_s < \nu_{sb}$, the synchrotron spectrum has an injected spectral index $\alpha$. For $\nu_s < \nu_{sm}$, the principal synchrotron component becomes self-absorbed and the flat radio spectrum must to explained by a conspiracy of additional synchrotron components as illustrated, or by other means. In the example shown we have assumed that for $\nu_s > \nu_{sb}$, the rate at which electrons lose energy due to radiation losses is exactly balanced by the
Figure 6.1 An example of the synchrotron (S) and inverse-Compton (IC) spectra from a homogeneous disk. The synchrotron self-absorption frequency, $\nu_{sm}$, and break due to radiation losses, $\nu_{sb}$, are indicated (see text). The steepening of the spectrum above $\nu_{sb}$ is clearly visible. The conspiracy of additional synchrotron components required to reproduce the flat radio spectrum is also illustrated. The observational data are for Mrk 421 as in Fig. 5.3.

Continuous injection of fresh electrons into the emission region. Hence the synchrotron spectral index is steepened to $\alpha + 1/2$ (see below) leading to a moderately satisfactory fit to the power-law form of the low energy X-ray spectrum when the source is in the high state (Chapter 3). The SSC spectrum, determined from the lowest EXOSAT observation is also shown.

In view of the observational uncertainties on $\theta_d$ in particular, we have chosen to use the formalism described above to place constraints on the $\theta_d$ versus $D$ plane. We have used equations [6.8], [6.9] and [6.10] to derive the physical conditions within the emission regions of the five sources presented here assuming a homo-
Table 6.1 SSC model input parameters

<table>
<thead>
<tr>
<th>Object</th>
<th>Mrk 421</th>
<th>Mrk 501</th>
<th>1218+304</th>
<th>Mrk 180</th>
<th>0414+009</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>0.0308</td>
<td>0.0337</td>
<td>0.13</td>
<td>0.0458</td>
<td>0.2</td>
</tr>
<tr>
<td>$D_L$ (Mpc)</td>
<td>124.1</td>
<td>135.9</td>
<td>536.9</td>
<td>185.3</td>
<td>840</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.65</td>
<td>0.73</td>
<td>0.51</td>
<td>0.58</td>
<td>0.80</td>
</tr>
<tr>
<td>$\nu_{sm}$ (GHz)</td>
<td>$1.8 \times 10^3$</td>
<td>$10^3$</td>
<td>$4.0 \times 10^3$</td>
<td>15</td>
<td>$6.9 \times 10^3$</td>
</tr>
<tr>
<td>$S_{sm}$ (Jy)</td>
<td>0.3</td>
<td>0.39</td>
<td>0.02</td>
<td>0.49</td>
<td>0.026</td>
</tr>
<tr>
<td>$\nu_b$ ($10^{15}$Hz)</td>
<td>3.5</td>
<td>4.6</td>
<td>3.2</td>
<td>2.4</td>
<td>26</td>
</tr>
<tr>
<td>$\nu_c$ ($10^{17}$Hz)</td>
<td>12.1</td>
<td>12.1</td>
<td>12.1</td>
<td>7.23</td>
<td>12.1</td>
</tr>
<tr>
<td>$S_{\nu}^{obs}$ (\mu Jy)</td>
<td>&lt; 0.30</td>
<td>&lt; 0.06</td>
<td>&lt; 1.7</td>
<td>&lt; 0.1</td>
<td>&lt; 0.4</td>
</tr>
</tbody>
</table>

\(a\) Uncertain.  
\(b\) Assumed.  
\(c\) Assuming $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0.5$.  

The results from this analysis are shown in the form of a plot of the $\theta_d - D$ plane for each source in Fig. 6.2. The shaded lines show the restrictions imposed by the requirements that (a) the predicted SSC flux at a frequency $\nu_c$ is less than or equal to that observed, (b) the Lorentz factor of electrons radiating at $\nu_{sm}$ is greater than or equal to 1.0, and (c) that the light crossing timescale of the implied region is shorter than the observed minimum variability timescale. The dashed lines (c) and (d) delineate the region of the plane in which significant SSC radiation is expected at the frequency $\nu_c$ (i.e. in which $\gamma_{sm}^2 \nu_{sm} \leq \nu_c \leq \gamma_{sm}^2 \nu_{ab}$; see above). Values of $\theta_d$ and $D$ outside this region clearly invalidates constraint (a) above. Finally the dotted line in Fig. 6.2. indicates the values of $\theta_d$ and $D$ such that the magnetic field and electron energy densities are in equipartition.

It is clear that for all five sources, a large region of the $\theta_d, D$ plane is consistent with the observations. However a number of general conclusions can be made. We find that in all cases for example, $\theta_d \leq 10^{-2}$ mas, at least an order of magni-
Table 6.2 Summary of the constraints on $D$ assuming a homogeneous disk SSC model

(a.) $S_{\text{red}}^{\text{te}}(\nu_c) \leq S_{\text{obs}}^{\text{te}}(\nu_c)$

$$D \geq \left\{ 3 \sigma_T^{2\alpha+\alpha} A(\alpha) \frac{C_2(\alpha)}{C_1(\alpha)} \frac{(1+z)^{2+\alpha}}{(1+z)^{1+\alpha}} \right\} \ln \left( \frac{\nu_{\text{sm}}^2}{\nu_c^2} \right) g_d^{-(2+2\alpha)} \frac{\nu_{\text{sm}}^2}{\nu_c^2} S_{\text{sm}} S_{\text{et}}^2(\nu_c)^{-1} \ln \left( \frac{1+z}{\nu_{\text{sm}}^2} \right)$$

(b.) $\tau_{\text{sm}} > 1$

$$D \leq 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} g_d^{-2} S_{\text{sm}} \nu_{\text{sm}}^2 (1+z)$$

(c.) $\nu_{\text{min}}^{\nu_{\text{sm}}} \leq \nu_c$

$$D \geq 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} (1+z) S_{\text{sm}} \nu_{\text{sm}}^{-2/2} \nu_c^{-1/2} g_d^{-2}$$

(d.) $\nu_{\text{max}}^{\nu_{\text{sm}}} \geq \nu_c$

$$D \leq 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} (1+z) S_{\text{sm}} \nu_{\text{sm}}^{-2/2} \nu_c^{-1/2} g_d^{-2}$$

(e.) $t_{\text{ear}} \geq t_{\text{c}}$

$$D \geq \frac{\theta_d D_L}{\text{c}_{\text{var}}} (1+z)^{-1}$$

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Figure 6.2 Constraints imposed on the angular diameter ($\theta_d$) and kinematic Doppler factor ($D$) for Mrk 421, Mrk 501, 1218+304, Mrk 180 and 0414+009 assuming a canonical homogeneous disk synchrotron self-Compton model. The input parameters are listed in Table 6.1. The shaded lines show the restrictions imposed by the requirements that (a) $S_{\nu_{\infty}}^{\text{obs}} \leq S_{\nu_{\infty}}^{\text{inj}}$, (b) $\gamma_{\text{em}} \geq 1$ and (c) $t_{\text{ion}} \leq t_{\text{ref}}$ — see text and Table 6.2. Also shown is the region of the plane such that $\gamma_{\text{em}}^2 \nu_{\text{em}} \leq \nu_c \leq \gamma_{\text{em}}^2 \nu_{\text{sh}}$, between the unshaded lines (c) and (d), and (as a dotted line) the values of $\theta_d$ and $D$ such that the magnetic field and electron energy densities are in equipartition. The vertical lines show the size of any unresolved VLBI core.
Figure 6.2 Continued
tude smaller than resolvable with current VLBI baselines. At large values, \( D \) is essentially unbound, although \( D \geq 50 \) (say) requires velocities in excess of 0.999c at angles \( \theta_v < 0.5^\circ \) to the line of sight. The minimum value of \( D \) consistent with the observational data for each source is listed in Table 6.3 (as \( D_{\text{min}} \)) with the corresponding value of \( \theta_d \). In all cases, \( D_{\text{min}} \) is determined from the intersection of the constraints imposed by the requirements \( S_{\text{sc}}^{\text{obs}} \geq S_{\text{sc}}^{\text{pred}} \) and \( t_{\text{min}} \geq t_c \). Clearly the detection of significant variability for any object on a shorter timescale than that listed in Table 6.3 will increase \( D_{\text{min}} \). It can be seen that for all sources except Mrk 180 there is a requirement for a modest degree of relativistic motion (\( D_{\text{min}} \sim 2-3 \)). As shown in Fig. 6.3 such values of \( D \) require bulk velocities in excess of 0.6c at angles, \( \theta_v \leq 25^\circ \) to the line of sight. In the case of Mrk 180, the observations merely restrict \( D \geq 0.6 \) and hence, for \( D = 1 \) impose no restrictions on either \( \beta \) or \( \theta_v \).

Also listed in Table 6.3 are a number of derived physical conditions within the emission region (assuming \( D_{\text{min}} \) and corresponding \( \theta_d \)). We find magnetic fields, \( B \sim 10-10^3 \) G and values of \( K \sim 10^2-10^8 \) cm\(^{-3} \), both of which seem physically
Figure 6.3 Velocity, $v$, versus the viewing angle, $\theta_v$, for various values of the kinematic Doppler factor, $D$.

Table 6.3 Derived physical conditions assuming homogeneous disk model

<table>
<thead>
<tr>
<th>Object</th>
<th>Mrk 421</th>
<th>Mrk 501</th>
<th>1218+304</th>
<th>Mrk 180</th>
<th>0414+009</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{\text{min}}$</td>
<td>2.4</td>
<td>2.7</td>
<td>2.0</td>
<td>0.63</td>
<td>2.7</td>
</tr>
<tr>
<td>$\theta_d$ (mas)$^a$</td>
<td>$8.9 \times 10^{-4}$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$9.0 \times 10^{-4}$</td>
<td>$4.2 \times 10^{-3}$</td>
<td>$7.8 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B$</td>
<td>$8.7 \times 10^4$</td>
<td>$1.8 \times 10^4$</td>
<td>$9.6 \times 10^4$</td>
<td>$2.0 \times 10^4$</td>
<td>$3.5 \times 10^2$</td>
</tr>
<tr>
<td>$K$</td>
<td>$2.4 \times 10^6$</td>
<td>$8.6 \times 10^6$</td>
<td>$6.0 \times 10^7$</td>
<td>$3.9 \times 10^2$</td>
<td>$1.2 \times 10^8$</td>
</tr>
<tr>
<td>$U_{\text{mag}}$</td>
<td>$3.0 \times 10^2$</td>
<td>9.9</td>
<td>$3.7 \times 10^2$</td>
<td>$1.6 \times 10^5$</td>
<td>$4.9 \times 10^3$</td>
</tr>
<tr>
<td>$U_{\text{elec}}$</td>
<td>1.3</td>
<td>1.4</td>
<td>$1.9 \times 10^2$</td>
<td>$6.1 \times 10^{-4}$</td>
<td>$1.1 \times 10^1$</td>
</tr>
<tr>
<td>$U_{\text{phot}}$</td>
<td>$6.8 \times 10^{-2}$</td>
<td>$6.8 \times 10^{-1}$</td>
<td>8.0</td>
<td>$7.0 \times 10^{-2}$</td>
<td>$6.3 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\ell_e(1 \text{ keV})$</td>
<td>$2.2$</td>
<td>$2.9 \times 10^6$</td>
<td>1.6</td>
<td>$9.7 \times 10^{-3}$</td>
<td>$2.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\ell_e(1 \text{ keV})$</td>
<td>$9.6 \times 10^4$</td>
<td>$1.1 \times 10^4$</td>
<td>$7.4 \times 10^4$</td>
<td>$2.2 \times 10^4$</td>
<td>$2.0 \times 10^4$</td>
</tr>
<tr>
<td>$l$</td>
<td>$7.7 \times 10^{14}$</td>
<td>$1.5 \times 10^{15}$</td>
<td>$2.8 \times 10^{14}$</td>
<td>$5.3 \times 10^{15}$</td>
<td>$3.4 \times 10^{14}$</td>
</tr>
<tr>
<td>$t_\text{le}$</td>
<td>$5.2 \times 10^4$</td>
<td>$9.7 \times 10^4$</td>
<td>$1.9 \times 10^4$</td>
<td>$3.5 \times 10^5$</td>
<td>$2.3 \times 10^6$</td>
</tr>
<tr>
<td>$N_{\text{elec}}$</td>
<td>$1.6 \times 10^8$</td>
<td>$1.9 \times 10^8$</td>
<td>$2.4 \times 10^8$</td>
<td>$7.5 \times 10^2$</td>
<td>$1.4 \times 10^7$</td>
</tr>
<tr>
<td>Mass</td>
<td>$3.9 \times 10^{27}$</td>
<td>$3.1 \times 10^{28}$</td>
<td>$2.8 \times 10^{28}$</td>
<td>$5.9 \times 10^{26}$</td>
<td>$2.8 \times 10^{27}$</td>
</tr>
</tbody>
</table>

All unit cgs unless stated.

$^a$ At $D_{\text{min}}$. 

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plausible. The implied energy densities of the magnetic field, relativistic electrons and synchrotron photons \( U_{\text{mag}}, U_{\text{elec}} \) and \( U_{\text{phot}} \) also do not seem unreasonable. Generally we find \( U_{\text{mag}} > U_{\text{elec}} > U_{\text{phot}} \), although as is also apparent from Fig. 6.2, in the cases of Mrk 501 and 1218+304 there is approximate equipartition between \( U_{\text{mag}} \) and \( U_{\text{elec}} \) at \( D_{\text{min}} \). We find that the implied radii of the disks, \( l \), were \( \sim 10^{14} \) – few \( 10^{15} \) cm giving light crossing timescales, \( t_{\text{lc}} \sim \) several hours. In all cases we found that in the co-moving frame, the timescales for both the synchrotron and Compton losses \( (t_{s} \) and \( t_{c} \) respectively) at X-ray energies are much smaller than either \( t_{s} \) or the observed minimum variability timescale, \( t_{\text{min}} \). Some form of \textit{in situ} replenishment of the relativistic electron population throughout the emission region is therefore required in order to maintain the observed X-ray emission. Since \( t_{s} < t_{c} \), the dominate loss mechanism is clearly via synchrotron emission, consistent with the lack of inverse Compton emission observed.

Finally in Table 6.3 we list a crude lower limit on the total mass in the emission region by assuming the number density of protons is equal to that of relativistic (synchrotron emitting) electrons, \( N_{\text{elec}} \). Clearly, however it is highly probable (particularly in view of the requirement for continuous reacceleration), that there exists a vast reservoir of undetectable, non-relativistic electrons and hence presumably an equal number of protons.

The findings reported here are broadly consistent with those of numerous workers who have modeled single epoch snap-shots of the multi-waveband spectra of BL Lac type objects. Madejski & Schwartz (1983) and more recently Madau, Ghisellini & Persic (1987) have applied a homogeneous SSC model very similar to that described above to a number of Blazars. Although the samples used were neither complete, homogeneous and some were of unknown \( z \), these workers found \( D \) in the range \( 10^{-2} \) to 15 with approximately half the sources requiring \( D > 1 \). In addition loss timescales significantly shorter than light crossing timescales are often implied and thus a requirement for \textit{in situ} acceleration not uncommon.

Such models have, however, been less successful at explaining the evolution of the
spectrum with time. For example, Makino et al. (1987a) were forced to invoke adiabatic expansion of the source with a simultaneous increase in the relativistic bulk expansion of the source in order to explain the spectral variations seen between two observations of Mrk 421 separated by 5 weeks.

6.3.3 Spectral variability and the continuity equation

A full description of the time evolution of the spectrum of an ensemble of synchrotron emitting electrons requires a knowledge of the distribution of electron energies, \( N(\gamma_e) \), at a given point in the source at some initial time, \( t_0 \), and the rate of electron energy losses and gains, \( \frac{d\gamma_e}{dt} \). In addition any sources, \( q \), and sinks, \( p \), of relativistic electrons must be known.

In general all the functions mentioned above are dependent on spatial coordinates, direction and time. However, for simplicity it is usually assumed that the problem is uniform and isotropic. Hence the time dependence of the electron energy distribution is governed by the continuity equation (Kardashev 1962; Blumenthal & Gould 1970)

\[
\frac{\partial N(\gamma_e, t)}{\partial t} + \nabla \gamma \left[ N(\gamma_e, t) \frac{d\gamma_e}{dt} \right] = q(\gamma_e, t) - p(\gamma_e, t) \tag{6.11}
\]

For both synchrotron and inverse-Compton radiation, the rate of energy loss suffered by the electrons is proportional to the square of the electron energy. Specifically, in the case of synchrotron radiation

\[
\frac{d\gamma_e}{dt} = -A\gamma_e^2 \tag{6.12}
\]

where \( A = 1.93 \times 10^{-9} \ B^2 \ \text{s}^{-1} \) (Appendix C, equation [C.17]) and \( B \) is the magnetic field density in Gauss.

In order to infer the source function, \( q(\gamma_e, t) \) from the observed spectrum it is necessary to solve equation [6.11]. The solution is obviously dependent upon the
initial conditions and several authors have considered a number of simple cases of interest here (Kellermann 1966; Blumenthal & Gould 1970; Pacholczyk 1970).

Firstly for example, let us consider the case of a single instantaneous injection. If a population of relativistic electrons with a power-law distribution of the form given by equation [6.1] is initially injected into a homogeneous source as a short burst, then in the absence of additional sources and sinks, after a time \( t \) synchrotron radiation losses will produce a distribution of the form

\[
N(\gamma_e, t) d\gamma = \begin{cases} 
\frac{K \gamma_e^{-p} d\gamma}{1 - A \gamma_e t^{-p}} & \text{for } \gamma_{e,min} \leq \gamma_e \leq \gamma_{e,max} \\
0 & \text{otherwise}
\end{cases}
\]

where

\[
\gamma_e' = \frac{\gamma_e}{1 + A \gamma_e t}
\]

and \( A \) is given by equation [C.17]. Thus the resulting radiation spectrum changes accordingly. Above a frequency \( \nu'_{\text{max}} \), corresponding to the peak emitted frequency of electrons with \( \gamma_{e,max} \), the flux density falls rapidly as \((\nu / \nu'_{\text{max}})^{0.5 e(\nu / \nu'_{\text{max}})}\). As pointed out by Kellermann (1966), it should be noted that if \( p < 2 \), \( N(\gamma_e, t) d\gamma \) will increase with increasing energy just below the upper cut-off energy \( \nu'_{\text{max}} \) and then drop sharply to zero above this energy. If there are a sufficient number of high energy electrons, the corresponding synchrotron radiation spectrum will then show an up-turn just below this energy due to the large number of electrons with appropriate critical frequencies. If the pitch angle distribution is isotropic the spectrum above \( \nu'_{\text{max}} \) will be a power-law with an index of \( \alpha' = (2p+1)/3 = 4/3 + 1 \) or steeper rather than being sharply cut off (Kardashev 1962).

Alternatively, if the electrons are continually replenished by continuous injection of fresh electrons into the emission region with a source function \( q(\gamma_e, t) \), then equation [6.11] has a steady state solution given by (Ginzburg & Syrovatskii 1965; Blumenthal & Gould 1970)

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\[ N(\gamma_e) = \frac{1}{(d\gamma_e/dt)} \int_\gamma g(\gamma_e')d\gamma' \]
giving

\[ N(\gamma_e, t) = \begin{cases} 
Kt\gamma_e^{-p} & \text{for } \gamma_e \ll (At)^{-1} \\
\frac{K\gamma_e^{-(p+1)}}{A(p+1)} & \text{for } \gamma_e \gg (At)^{-1}
\end{cases} \tag{6.13} \]

For \( \gamma_e < At^{-1} \) radiation losses are unimportant and the source brightens linearly with time with a spectral index equal to its initial value \( \alpha \) (in the absence of alternative loss mechanisms). Above \( \gamma_e = At^{-1} \) (corresponding to a frequency \( \nu_{sb} = 1.12 \times 10^{24} B^{-3} t^{-2} \) Hz; see Appendix C, equation [C.21]) the rate of energy lost by radiation is exactly balanced by the addition of new electrons and the spectral index of the emergent synchrotron radiation steepens to \( \dot{\alpha} = \alpha + 1/2 \).

Thus the time evolution of the source spectrum may be expected to proceed from an initial power-law spectrum of a spectral index \( \alpha \) to a curved spectrum and finally to a new power-law of index \( \alpha + 1/2 \) or \( 4/3\alpha + 1 \) depending on whether the electrons are continually injected into the magnetic field or whether the injection occurs almost instantaneously.

At least in qualitative terms both the relative stability of the ultra-violet spectrum (Chapter 3) and the high degree of spectral variability observed in the X-ray band (Chapter 4) can be explained if the injection of relativistic electrons into the source region occurs in a series of recurring bursts. The variations in the synchrotron spectrum which are predicted in such a model are described, for example, by Kellermann (1966). If we attribute the ultra-violet continuum to the regime in which the synchrotron lifetime of the electrons is significantly longer than the time between bursts and where the loss and replenishment rates are in rough equilibrium, then a fairly stable the ultra-violet flux would be expected. The ultra-violet spectral index in these sources is typically \( \sim 1 \), which implies \( \alpha \sim 0.5 \) broadly consistent in most cases with the optical/infra-red index listed in Table 6.1. The relativistic electrons which produce the X-ray photons have
significantly shorter lifetimes ($\tau \propto \nu^{-0.5}$) and hence the X-ray spectrum will be much more sensitive to individual injection bursts. Immediately following a burst the source would brighten in X-rays and the X-ray spectrum would flatten (to $\alpha$) but then radiation losses soon deplete the freshly injected high energy electrons, the source fades and the spectrum steepens (to at least $4/3\alpha + 1$). For the sources reported here, the predicted range for the X-ray spectral index is 0.5–2.7, which is somewhat wider than actually observed. Kellermann (1966) notes that the boundary between the stable (lower energy) region of the spectrum and the more variable higher energy regime in the recurrent injection model is only a weak function of time provided the interval between bursts remains fairly constant. However, a difficulty with this description is that as shown above, the lifetime for the X-ray emitting electrons is generally found to be very short, i.e. the order of seconds, so that the model predicts the spectral index/flux correlation will be apparent in rapid X-ray flickering rather than in spectral comparisons over extended periods (days to years) as reported in Chapter 3. Thus we conclude that such a simple model is unable to adequately describe the observed multi-waveband spectral variations seen in these sources.

6.4 An Inhomogeneous Jet Model

The homogeneous disk model discussed above is clearly somewhat simplistic and at some level physically unrealistic. It is hardly surprising therefore that such a model experiences some difficulty accounting for the spectral evolution of the sources reported above. Now let us consider the alternatives. Models in which the assumption of a homogeneous emission region is retained but with a different geometry (invariably spherical) have been widely applied to single snap-shot multi-waveband spectra of BL Lac type objects in the past. However, since the geometrical assumptions in homogeneous models result in only minor differences between the implied physical conditions within the emission region (see e.g. Band & Grindlay 1985), all such models are likely to suffer the similar difficulties as a
The only alternative therefore seems to be inhomogeneous models in which the physical conditions vary with position throughout the source. (Although a number of workers have also considered models containing two or more physically distinct emission homogeneous regions.) Such models have the advantage of being likely to be more physically realistic, but the disadvantage of having a larger number of parameters to be specified. Not only do we have a choice of geometry, but also a choice in the specification of the spatial dependency for the various parameters. In view of the lack of detailed ‘physical’ models, the general approach is to make a simple parameterization of the emission region, thus power-law functions are invariably chosen. By far the most popular inhomogeneous models employ jet-type geometries in which the synchrotron emitting electrons stream outwards with relativistic bulk velocities relative to the host galaxy. The attraction of such relativistic jet models compared to alternative inhomogeneous geometries such as, for example, ‘power-law’ spheres, for BL Lac type objects is primarily due to reasons beyond the bare modeling of the multi-waveband spectrum. Firstly, jet-like structures have been detected in a number of BL Lac type objects, including Mrk 421, Mrk 501 and 0414+009 (Chapter 5). Secondly, any small variations in the direction of a relativistic jet can produce large apparent changes in position angle of polarization if viewed end-on (Blandford & Königl 1979). Finally, as discussed above, the Doppler enhancement of the continuum as a result of relativistic motion of the synchrotron emitting region towards the observer offers an explanation for the featureless spectra in BL Lacs due to the swamping of any line emission from any stationary surrounding region. Jet-like structures appear to be a common feature in extended radio sources (Kellermann & Pauliny-Toth 1981) and are regarded as the mass and energy supply route for the extended radio lobes (de Young 1976). This has led many workers to speculate that BL Lac type objects and classical radio sources might not fundamentally differ, rather be the same class of object viewed from different angles. In this scenario, BL Lac type objects correspond to the sources in which the jet is seen almost end-on, whilst other radio sources are those seen at larger viewing angles.
The consequences of such 'unified schemes' are further discussed in Chapter 7.

A variety of relativistic jet models have been proposed in the literature (Blandford & Königl 1979; Marscher 1980; Königl 1981; Reynolds 1982a,b; Ghisellini, Maraschi & Treves 1985; Hutter & Mufson 1986). Generally, the observed flux in different energy bands is dominated by emission from physically different regions of the jet, the X-rays usually originating from the innermost regions, the optical and radio from progressively further out. Such a scenario seems to be supported by measurements of the minimum variability timescale in each band which are generally considered to increase as one moves to longer wavelengths. However, as discussed in Chapter 3, the problems associated with measuring such a timescale in these sources are considerable and the situation is far from clear. To the first-order, the X-ray emitting region will presumably have similar physical conditions to those predicted by the homogeneous disk SSC model mentioned above, namely very short cooling timescales and a modest requirement for relativistic bulk motion. This reasoning led us to consider a relativistic jet model very similar to that developed by Hutter (1983) and Hutter & Mufson (1986) in which the relativistic electron distribution is continually re-accelerated throughout the jet, offsetting expansion and radiation losses. In view of the observed spectral variations reported here we pay particular attention to that part of the jet responsible for the high energy emission. We concentrate on Mrk 421 since of the sources reported here this object provides the most stern test for such a model.

6.4.1 Parameterization

The Hutter & Mufson (1986) model is an adaption of the earlier models of Blandford & Königl (1979) and Königl (1981). A 'free' conical jet of opening angle $2\phi$ is viewed at an angle $\theta_v$ to its axis (where $\theta_v \geq \phi$). The outflowing material is assumed to move outwards with constant velocity, $v_j(= \beta_j c)$ with a Lorentz factor, $\gamma_j = (1 - \beta_j^2)^{-1/2}$ given by $\gamma_j = 1/\phi$. Unless otherwise stated we use the same notation as Hutter & Mufson. The injected electron spectrum is assumed to
be a power-law function of electron Lorentz factor $\gamma_e$, as given by equation [6.1] above. The radial dependencies of the particle number density and the magnetic field strength are assumed to be power-laws represented by

$$K = K_1 r^{-n} \quad \text{and} \quad B = B_1 r^{-m} \quad \text{[6.14]}$$

where $r$ is measured in parsecs from the jet apex. The parameters $n$ and $m$ are assumed to be positive. Unlike Hutter & Mufson, we also allow a radial variation in the upper termination of the electron population parameterized by

$$\gamma_{eu} = \gamma_1 r^{-g} \quad \text{[6.15]}$$

where $g$ can take on any value.

### 6.4.2 Derivation of physical parameters.

Since the magnetic field and electron distribution both vary with distance, $r$, along the jet, the local (i.e. co-moving frame) synchrotron spectrum is also a function of $r$. For the specific model under consideration, the local synchrotron spectrum emitted at a given $r$ is very similar to that from a homogeneous disk model. Hence in the observer's frame, below a frequency, $\nu_{sm}(r)$, the source is optically thick due to synchrotron self-absorption and the flux density rises as $\nu_{sm}^{5/2}$; above $\nu_{sm}(r)$ the spectrum is optically thin with an index, $\alpha$, up to a frequency $\nu_{ab}(r)$, where the spectral index is assumed to abruptly increase to $\alpha + 1/2$ as a result of a balance between re-injection and radiation losses; above $\nu_{su}(r)$, the peak frequency of electrons with the maximum Lorentz factor, $\gamma_{eu}(r)$, the spectrum drops off exponentially. An expression for $\nu_{sm}(r)$ can be found by setting the optical depth of a line of sight through the jet at $r$ to unity, giving (Appendix C)

$$\nu_{sm}(r) = (1 + z)^{-1}[T_{VII}K_1 B_1^{\alpha+3/2}]^{1/(\alpha+5/2)} r^{-kn} \quad \text{Hz} \quad \text{[6.16]}$$
where

\[ T_{\text{VII}} = 6.2 \times 10^{18} C_2(\alpha) D^{\alpha+3/2} \phi \csc \theta \]  

[6.17]

\[ k_m = [(3 + 2\alpha)m + 2n - 2]/(5 + 2\alpha) \]  

[6.18]

(A summary of the expressions for these and all other \( k \) and \( T \) indices and coefficients used throughout this work is given in Table 6.4). The break frequency \( \nu_{\text{sb}}(r) \) due to synchrotron losses is found by equating the synchrotron loss timescale with the travel time to that radius \( r \), giving

\[ \nu_{\text{sb}}(r) = 1.1 \times 10^8 (1 + z)^{-1} D \gamma^2 \beta^2 B r^{k_b} \text{ Hz} \]  

[6.19]

where

\[ k_b = (3m - 2) \]  

[6.20]

Finally, the maximum frequency, \( \nu_{\text{su}}(r) \) of the observed synchrotron spectrum can be estimated from the peak emitted frequency of electrons with \( \gamma_{\text{eu}}(r) \). Thus we find

\[ \nu_{\text{su}}(r) = 1.22 \times 10^6 (1 + z)^{-1} D B_1 \gamma^2 r^{-k_u} \text{ Hz} \]  

[6.21]

where

\[ k_u = (2g + m) \]  

[6.22]

The expressions for \( \nu_{\text{sm}}(r) \), \( \nu_{\text{sb}}(r) \) and \( \nu_{\text{su}}(r) \) define two regions on the frequency versus radius plane which are responsible for optically thin synchrotron emission. These are indicated in Fig. 6.4. In Region I the local spectral index is \( \alpha \), whilst in Region II synchrotron losses result in a steepening of the local index to \( \alpha + 1/2 \). Emission outside these regions does not significantly contribute to the total observed flux density and is neglected in our calculations. It should be noted that due to the decrease in the magnetic field, \( B \), with \( r \) and the continual re-injection of fresh electron along the jet, the synchrotron cooling timescale, \( \tau_c(r) \), and hence \( \nu_{\text{sb}}(r) \) are both increasing functions of \( r \) (see below).
Table 6.4 Summary of Indices and Coefficients used in Section 6.4

\[
\begin{align*}
  k_m & = \frac{(3+2\alpha)m + 2n - 2}{(5+2\alpha)} & k_b & = (3m - 2) \\
  k_a & = 2g - m & k_s & = (1 + \alpha)m + n - 3 \\
  T_I & = (5 + 2\alpha) & T_{II} & = (3 + 2\alpha) \\
  T_{III} & = 2(\alpha_s - \alpha) & T_{IV} & = 2(1 + \alpha)
\end{align*}
\]

\[
\begin{align*}
  T_V & = 6.2 \times 10^4 (6.9 \times 10^7)^{-\langle \alpha + 5/2 \rangle} C(\alpha)D^{-1}(\gamma) - 2(\alpha + 5/2)\phi \csc \theta \\
  T_{VI} & = 1/(3m - 2)(\alpha + 5/2) + (\alpha + 3/2)m + n - 1 \\
  T_{VII} & = 6.2 \times 10^4 C(\alpha)D^{(\alpha + 3/2)}\phi \csc \theta \\
  T_{VIII} & = S_m(3.333 \times 10^{-3})[C1(0.50/C1(\alpha))((1 + z)^{\alpha - 1}D_1^2 \phi D^{-2(\alpha + 2)}\phi \csc \theta \\
  T_{IX} & = (9 + 4\alpha)T_{V} - (1 + \alpha)/k_s \\
  T_X & = T_V^{-1}T_{XI}^{-1}(k_s - T_{X})/T_{VIII}^{-1}(k_s - T_{X}) \\
  T_{XII} & = (3m - 2)(\alpha + 5/2) + (9 + 4\alpha)T_{VI}k_m \\
  T_{XIII} & = (1 + z)/T_{VII}k_m T_{VIII}^{-1}(\alpha + 3/2)T_{X}^{-1}T_{X} \\
  T_{XIV} & = 1/(\alpha + 5/2) - T_{VI}k_m + [(1/k_s - T_{VI})T_{XII}]T_{IX} \\
  D_L & = 10^{-9}D_L \\
  m & = [2T_I + T_{III}(2\alpha_s + 5) - 4\alpha_s - T_I - T_{II}(\alpha_s + 5/2) + T_{III}(3\alpha_s + 15/2) \\
  n & = (\alpha_s - \alpha)(3m - 2) - (1 + \alpha)m + 3 \\
  r_M & = [T_V k_1 B_1^{\alpha + 4/2}]T_{VI} \\
  \nu_sM & = (1 + z)^{-1}[T_{VIII}k_1 B_1^{\alpha + 3/2}]T_{VII}^{-1}[\alpha + 5/2] - [T_V k_1 B_1^{\alpha + 4/2}] - \nu_mT_{VI} \\
  k_1 & = (1/k_s - T_{VI})T_{XII}^{-1} \\
  B_f & = T_{XII}k_s^{-1}(1/k_s - T_{VI})T_{IX}
\end{align*}
\]
Figure 6.4 Regions of the frequency ($\nu_s$) versus radius ($r$) plane responsible for optically thin synchrotron emission produced by the relativistic jet of Mrk 421. Region I emits radiation with a local spectral index, $\alpha$, whilst in Region II synchrotron losses result in a local spectral index, $\alpha - \frac{1}{2}$. The two regions are defined by the three solid lines which represent the radial dependences of the synchrotron self-absorption frequency ($\nu_{sm}(r)$), the break frequency at which the local spectral index steepens due to synchrotron losses ($\nu_{br}(r)$) and the upper frequency cut-off in the synchrotron spectrum ($\nu_{cu}(r)$) for a model with $\gamma_{cu}$ independent of $r$. The dotted lines define the range of $\nu_{cu}$ which give good fits to the observed low-state X-ray spectra when a radial dependence of $\gamma_{cu}$ is included of the form $\gamma_{cu} = \gamma_1 r^{-\frac{9}{7}}$ and correspond to $g = 0.25$ to $+1.00$. The dashed line is the form of $\nu_{cu}$ required to match the high-state X-ray spectra of Mrk 421 (see Section 6.4.4).
The smallest radius, $r_M$, from which optically thin synchrotron emission with spectral index $\alpha$ emanates can be obtained by setting $\nu_{sm}(r) = \nu_{ab}(r)$ (equations [6.16] and [6.19]) and solving for $r$ to give

$$r_M = \left[ T_V K_1 B_1^{\alpha+4\alpha} \right]^{1/(\alpha+5/2)} \text{ pc} \quad [6.23]$$

An expression for the fiducial frequency $\nu_{sm}(r_M)$, usually referred to as $\nu_{sM}$, (at which the observed spectrum steepens) can then be found by substituting for $r_M$ (equation [6.23]) into the expression for $\nu_{sm}(r)$ (equation [6.16]) to give

$$\nu_{sM} = (1 + z)^{-1}\left[ T_{III} K_1 B_1^{\alpha+3/2} \right]^{1/(\alpha+5/2)} \left[ T_V K_1 B_1^{\alpha+4\alpha} \right]^{-k_m T_V} \text{ Hz} \quad [6.24]$$

Similarly, expressions for $r_U$ and the frequency of the corresponding spectral break, $\nu_{sU}$, can be found from equations [6.19] and [6.21] (using $\nu_{ab}(r) = \nu_{su}(r)$).

**Observed Flux density**

The local synchrotron luminosity at a given frequency $\nu_s$, produced at a radius $r$ can be calculated by multiplying the local emissivity by the local solid angle into which the radiation is emitted (assumed in all cases to be $4\pi$) and by the elemental volume element, $\pi(r \phi)^2 dr$. The total flux density, $S_\nu(\nu_s)$, observed at the Earth at the frequency $\nu_s$, can then be found by transforming to the observer's frame and integrating over all contributing radii. From Fig. 6.4 it is clear that for frequencies in the range $\nu_{sM} \leq \nu_s \leq \nu_{sU}$, both regions contribute to the integrated flux. Since the expressions for the local emissivity are different in these two regions it is necessary to perform the integration in two parts (Appendix C). Unlike Hutter & Mufson we do not arbitrarily curtail the jet at a radius $r_u$ where $\nu_{sm}(r_u)$ is the minimum observed frequency of optically thin synchrotron emission. Likewise we allowed emission at radii smaller than $r_M$.

At $\nu_{sM}$ it can be shown that $S_\nu(\nu_{sM})$, usually referred to as $S_{sM}$ is given by
The superposition of many different synchrotron spectra from regions of differing \( \nu_{\text{sm}}, \nu_{\text{sb}} \) and \( \nu_{\text{su}} \) produces a composite spectrum with a gradual variation in spectral index. However as pointed out by Hutter & Mufson, on the log \( \nu \) - log \( \tau \) plane the integrated flux density is dominated by the inner boundary of Region I for frequencies \( \nu_{\tau} < \nu_{\text{M}} \), the Region I/Region II boundary for \( \nu_{\text{M}} < \nu_{\tau} < \nu_{\text{U}} \) and the outer boundary of Region II for \( \nu_{\tau} > \nu_{\text{U}} \). Hence it can shown that the overall spectrum may usefully be approximated to three power-laws with indices \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) respectively over these three frequency ranges. Specifically

\[
\alpha_1 = \frac{4 + m - 5k_m}{2k_m} \tag{6.26}
\]

\[
\alpha_2 = \alpha + \frac{k_s}{k_b} \tag{6.27}
\]

\[
\alpha_3 = \frac{k_u + 2 - n}{k_u} \tag{6.28}
\]

where

\[
k_s = (1 + \alpha)m + n - 3 \tag{6.29}
\]

\[
k_b = (3m - 2) \tag{6.30}
\]

and \( k_m \) and \( k_u \) are given by equations [6.18] and [6.22] above.

### 6.4.3 Calculation of integrated spectrum.

To calculate the integrated spectrum from such a jet, a number of input parameters must first be specified. Firstly, the expressions for \( \alpha_1 \) (equation [6.26]),
\( \alpha_s^2 \) (equation [6.27]) and \( \alpha_s^3 \) (equation [6.28]) are functions of 4 parameters, namely \( m, n, \alpha \) and \( g \). Thus interpreting the observed spectral indices in the radio, optical/infra-red and X-ray bands in terms of \( \alpha_{s1}, \alpha_{s2} \) and \( \alpha_{s3} \) respectively, the assumption of one of these parameters allows the simultaneous solution of the above equations to give the remaining three. A natural explanation of the observed spectral variability in the X-ray band is provided by an evolution of either the magnetic field density or synchrotron emitting electron population with time via the parameters \( m, n \) or \( g \). In view of the large number of electron re-acceleration sites in the region of the jet responsible for the X-ray emission (see below), we believe the most likely explanation is in terms changes in the high energy cut-off in the electron population (i.e. in \( g \)). This is further supported by the insensitivity of \( \alpha_{s1} \) and \( \alpha_{s2} \) to variations in \( g \), consistent with the lack of dramatic spectral variability observed in these sources in either the radio or optical/infra-red bands (Chapters 4 and 5). In the rest of this work we follow such a scenario. We speculate that the most plausible cause of such variations in \( \gamma_{eu} \) with radius and time is provided by time variations in the efficiency of the re-acceleration process at the numerous re-acceleration sites within the X-ray emitting region of the jet. We have therefore chosen to assume a value for the local synchrotron spectral index and derive values for \( m \) and \( n \) from \( \alpha_{s1} \) and \( \alpha_{s2} \).

The expressions are given in Table 6.4. The constraints upon \( \alpha \) and consequences of our choice are discussed below.

Once \( \alpha, m \) and \( n \) have been determined, then in a manner similar to that for a homogeneous disk, by assuming values for \( \theta \) and \( \phi \) (the latter characterizing the bulk behaviour of the jet via \( \gamma_j, \beta_j \) and \( D \)), the expressions for \( \nu_{sM} \) (equation [6.24]) and \( S_s(\nu_{sM}) \) (equation [6.25]) are functions of only two unknowns, \( K_1 \) and \( B_1 \) and can therefore be simultaneously solved. The relevant expressions are given in Table 6.4. Once the parameters \( K_1 \) and \( B_1 \) (and hence also \( \tau_M \) using equation [6.23]) had been determined for a given object, they are assumed fixed for the rest of the calculations.

One is now in a position to perform the actual calculation of the composite syn-
chrotron spectrum using the expressions given in Appendix C (equations [C.44] and [C.47]). We have used a step of 0.1 in log $\nu_s$ over the range $10^{-10}$–$10^{20}$ Hz. The resultant model spectrum is then compared to the observed multi-waveband spectrum at each epoch and the parameters $\gamma_1$ and $g$ were varied until satisfactory fits were obtained.

The local synchrotron self-Compton emissivity in an inhomogeneous jet is a complex function of $r$. Both the electron and photon distributions are broken power-laws due to the effects of radiation losses, and whilst the electron electron density at a given point is simply defined by $K$ (equation [6.14]), the calculation of the synchrotron photon energy density and spectrum requires an integration over the entire emission region. In practice however, it is usually assumed that the SSC emission at a radius $r$ is mainly due to the scattering of photons produced within a distance $r\phi$ of the scattering site. Such a 'local approximation' is generally considered valid for a jet with $\phi \ll 1$ and certainly sufficient for the present work in which we have very little information on the SSC spectrum. A full treatment of the SSC calculation for a jet model in which full consideration is taken of the two component electron and photon distributions is presented in Hutter (1983). However, since the results obtained are very similar (within a factor $\leq 2$ – at least for the case tested) to those obtained using the very much simpler expression of Königl (1981, equation [13]), we have chosen to use the latter.

In Fig. 6.5 we show an example of model synchrotron and SSC spectra from our calculations (solid line). The input parameters are as as given in the example of Königl (1981) to enable direct comparison. The two calculations are generally in good agreement. For the purpose of this example we have followed Königl and restricted the jet to radii between $r_M$ and $370 \times r_M$. This gives rise to the downward curvature of the synchrotron spectra below $10^{10}$ Hz and above $10^{20.5}$ Hz evident in Fig. 6.5. In all following model calculations however, we allow the jet to extend to all $r$.

In summary, there are 11 parameters that must be specified for the calculation.
Figure 6.5 An example of the synchrotron (S) and inverse-Compton (IC) spectra for an unresolved inhomogeneous jet. The parameters are as given in Königl (1981)) to enable direct comparison. The frequencies $\nu_{sM}$, $\nu_{sU}$ and $\nu_{cM}$ are indicated and explained in the text. Note that the downward curvature of the synchrotron spectrum below $10^{19}$ Hz and above $10^{20.5}$ Hz is primarily due to the curtailment of the specific jet chosen by Königl below $r_M$ and above $370 \times r_M$. No such restriction was placed on the physical extent of any other of the jets presented in this work.
of the synchrotron and SSC spectra from the chosen jet model, namely $z$, $\alpha_{s1}$, $\alpha_{s2}$, $\alpha_{s3}$, $\nu_{sM}$, $S_{sM}$, $S_{sU}$, $S_{sc}^{obs}(\nu_c)$, $\alpha$, $\theta$ and $\phi$. The majority of these can be measured or estimated from the observed multi-waveband spectrum. However, as mentioned above, there is no direct way in which $\alpha$, $\theta$ and $\phi$ can be determined observationally. Instead we must use indirect arguments to help constrain them whilst bearing in mind the effects of our choice on the results. These three parameters clearly play a major role in the parameterization of the jet and the effect of our assumptions are considered further below for the specific example of Mrk 421.

6.4.4 Application to Mrk 421

The multi-waveband observations of Mrk 421 reported in previous Chapters are the most constraining. In particular we wish to account for the observed spectral variability in the X-ray band.

The effect of the assumed values

The first step in the determination of the physical parameters within the jet model is the use of the observed spectral indices to derive the indices $m$ and $n$. In Fig. 6.6(a) we plot $m$ and $n$ as a function of $\alpha$ assuming $\alpha_{s1} = 0.0$ and $\alpha_{s2} = 0.65$ as governed by the observed radio and optical/infra-red indices of this source. As stated above, $\alpha$ cannot be directly determined however the following arguments can be used to place general constraints upon it. Firstly, since both parameters are required to be positive (otherwise the magnetic field or electron distribution will increase with radius along the jet - a physically unlikely situation) it can be seen that in this specific case we can exclude $\alpha > 0.5$. Also, as pointed out by Hutter (1983), the constraint that the spectral index in Region I does not exceed that in Region II restricts $\alpha > \alpha_{s2} - 0.5$, thus here we find $\alpha > 0.15$. Finally, it should be noted that $m = 2$ and $n = 2$ corresponds to the conservation of magnetic flux and particle number respectively along the jet,
thus we should be a little surprised if \( m, n > 2 \) and additional sinks are required. Hence for Mrk 421 we find \( 0.33 \leq \alpha \leq 0.5 \). A value of \( \alpha = 0.5 \) was therefore adopted for this source, consistent with observations of radio jets (Bridle, 1982) and theoretical acceleration models (e.g. Bell 1978; see also Zdziarski 1986 and references therein). Interestingly the assumption of \( \alpha = 0.5 \) results in \( m = 2.0 \), \( n = 0.6 \) and, for \( g = 0 \), a predicted X-ray spectral index \( \alpha_{X} = 1.7 \) which is very similar to the average low state EXOSAT spectral index reported in Chapter 3.

Having estimated \( \nu_{M} \) and \( S_{M} \) from the multi-waveband spectrum we can now calculate \( B_{1}, K_{1} \) and \( r_{M} \). In Fig. 6.6(b) it is shown that \( B_{1} \) and \( K_{1} \) in particular are strong functions of \( \alpha \) (although none give unpalatable values for \( 0.0 \leq \alpha \leq 0.5 \)).

Now let us consider \( \theta \) and \( \phi \). In Fig. 6.7 we show the effect of the assumed values of \( \theta \) and \( \phi \) on the predicted SSC flux, \( S_{\nu}^{pred}(\nu_{s}) \) (with \( \alpha = 0.5 \)). Contours of \( S_{\nu}^{pred}(\nu_{s}) \) at 5 keV are plotted. As is to be expected, \( S_{\nu}^{pred}(\nu_{s}) \) decreases as one moves to smaller \( \theta \) and \( \phi \) (small values of \( \theta \) and \( \phi \) result in a large Doppler enhancement of the spectrum in the observer's frame and hence less extreme physical conditions.
Figure 6.7 The $\theta$ versus $\phi$ plane for the Mrk 421 relativistic jet. Shown are the contours of $S_{\text{spec}}$ at 5 keV given in Jy. The shaded line shows the constraint imposed by $S_{\text{spec}}(5\text{keV}) < 10^{-8}$ Jy as required observationally. Also shown is the condition $\theta \geq \phi$ as required by the specific model considered.
within the emission region). In the specific case of Mrk 421 where \( S_{\text{obs}}^{\text{obs}}(5 \text{ keV}) \sim 10^{-6} \text{ Jy} \), \( \theta \) and \( \phi \) are therefore confined to lie below the shaded contour, with \( \theta \leq 0.25 \text{ rad} (\sim 14^\circ) \). The additional constraint that \( \theta > \phi \) is imposed by a number of assumptions that have been made during the development of the specific model used here. These primarily involve the distance travelled through the jet by photons before escape. Clearly if \( \theta < \phi \), all the observed photons have to travel along the remainder of the jet with the associated increase in the probability of absorption or scattering. Whilst a full numerical treatment of such a jet is clearly possible, it has not been attempted here. In the following an opening half-angle, \( \phi \) is assumed to be 0.1 radians, a value typical of resolved jets (Begelman, Blandford and Rees 1984). The viewing angle, \( \theta \) was then also set to 0.1 radians maximizing the relativistic effects to the observer. We note that the assumed value of \( \phi \) results in a jet Lorentz factor, \( \gamma_j = 10 \). This is somewhat higher than the universal quasar jet velocities required in the 'unified scheme' studies of Orr & Browne (1982) and Morisawa & Takahara (1987).

Finally, we note that since the ratio of synchrotron to SSC emissivity is proportional to \( K_i \) and independent of \( B_i \) and \( r_M \), the predicted SSC flux decreases as \( \alpha \) is reduced. Hence the condition \( S_{\text{ssc}}^{\text{pred}} \leq S_{\text{ssc}}^{\text{obs}} \) is less restrictive on \( \theta \) and \( \phi \) for small \( \alpha \).

### 6.4.5 Physical conditions within the Mrk 421 jet

Using the fixed input parameters listed in Table 6.5 we have varied \( \gamma_1 \) and \( g \) until satisfactory fits were found to the multi-waveband spectrum of Mrk 421 at each epoch. We now discuss the derived physical conditions within the proposed Mrk 421 jet.

We first consider the X-ray low-state of the source. In this case the observed X-ray spectral index ranges from 1.95 to 1.35 corresponding to values for \( g \) in the range -0.25 to 1.0. In general model multi-waveband spectra were obtained which were in excellent agreement with the observations (see Fig. 6.8). The actual forms
Figure 6.8 The multi-waveband spectrum of Mrk 421 (Fig. 5.3) with example model calculations superimposed (dotted).
Table 6.5 Fixed model parameters for Mrk 421 jet

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
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<tr>
<td>$z$</td>
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</tr>
<tr>
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<tr>
<td>$\theta$ (rad)</td>
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</tr>
<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$m$</td>
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</tr>
<tr>
<td>$n$</td>
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</tr>
<tr>
<td>$\nu_{\ell M}$ (Hz)</td>
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</tr>
<tr>
<td>$S_{\nu(eM)}$ (Jy)</td>
<td>0.3</td>
</tr>
<tr>
<td>$B_1$ (cgs)</td>
<td>$1.2 \times 10^5$</td>
</tr>
<tr>
<td>$r_M$ (pc)</td>
<td>$7.7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

of $\gamma_{\ell u}(r)$ required to match the low-state observations are illustrated in Fig. 6.9. Also, in Fig. 6.4 we show (as dotted lines) the range of $\nu_{su}(r)$ on the $\log r - \log \nu_s$ plane which reproduce the observed range of spectral index in the X-ray band, although it should be noted that since emission at $\nu_{su}(r)$ in Region I makes a negligible contribution to the total flux, we have no information on the form of $\nu_{su}(r)$ at radii $r > 10^{-2}$ pc. Finally, Table 6.6 lists values of several physical parameters derived from the low-state models.

The high-state observations are somewhat more complicated to model. EXOSAT measured a soft X-ray spectral slope of $\alpha_s \sim 1$ requiring $g \gg 1$. However, the observations also reveal a marked downward curvature of the spectrum above a few keV. It is possible to accommodate this additional spectral feature by assuming a discontinuous variation of $\gamma_{\ell u}(r)$ with radius as illustrated by the dashed line in Fig. 6.9 (with the corresponding form of $\nu_{su}(r)$ also shown as a dashed line in Fig. 6.4). The form of the multi-waveband spectrum obtained for this high-state case is illustrated in Fig. 6.8 and corresponding physical parameters are given in Table 6.6. In this description the excess X-ray emission during outburst emanates predominantly from a very restricted region of the jet (specifically $10^{-2}$ pc in the present model). The X-ray outburst may thus be interpreted as due to the development of an X-ray knot or hot-spot within the jet, due to a perturbation
Figure 6.9 The upper termination of electron population ($\gamma_{eu}$) as a function of distance from jet apex ($r$) required by the models. The dotted lines define the range of $\gamma_{eu}(r)$ required during the low-state epochs. The dashed line shows the form of $\gamma_{eu}(r)$ which produces a good representation of the high state X-ray spectra of Mrk 421. The specific form of $\gamma_{eu}$ assumed within the 2-6 keV X-ray emitting region for each epoch are represented by the short solid lines.

Table 6.6 Derived physical conditions within Mrk 421 jet

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<tr>
<td>$U_{\text{phot}}$</td>
<td>$10^{-10}$ - $10^{-9}$</td>
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* Assumed
Figure 6.10 The synchrotron cooling timescale, $t_c$, as a function of frequency, $\nu_s$, for the epochs of lowest (solid line) and highest (dashed) X-ray flux.

in the electron re-acceleration process.

It is apparent from Fig. 6.4 that in a model of this kind, the bulk of the X-ray flux originates from different regions of the jet from epoch to epoch. For the low state observations we find that the region of the jet dominating the emission in the 2–6 keV energy band is at distance, $r_{dom} \sim 10^{-3.0}$ to $10^{-2.3}$ pc from the apex. The time taken, $t_{trav}$, for the relativistically moving material to pass through the region is $\sim 25–40$ hr in the observer’s frame, whilst the width of the jet at these radii is typically $\sim 10^{-3.6}–10^{-3.1}$ pc ($\sim 10–25$ light hr). Both timescales are therefore slightly longer than the observed minimum timescale, $t_{\text{min}} \sim 5.3$ hr referred to in Chapter 3. We have calculated the radiative cooling timescale in all bands (at the dominating radius; Hutter & Mufson 1986, equation [8]) and the results are shown in Fig. 6.10. It can be seen that the timescales are extremely short in the X-ray band ($\sim$ few seconds – several minutes), that is much shorter than either $t_{trav}$ or the light crossing time of the jet at contributing radii. Thus there is a requirement for continuous re-injection of electrons consistent with the assumption of spatial and temporal variability of $\gamma_{eu}$. At other wavelengths we
typically find timescales ~1 hour in the ultra-violet/optical/infra-red and ~ few
months (at 100 GHz) to very many years (1 GHz) in the radio.

The mean upper cut-off in the electron energy distribution, $\gamma_{eu}$ required for the
low state epochs ranges from $4.0 \times 10^4$ to $1.3 \times 10^5$ in the X-ray emitting region
(Fig. 6.9). During outburst an increase of ~ few in $\gamma_{eu}$ is required. Such large
values are thought to be achievable through various acceleration mechanisms such
as strong shocks and magnetic reconnection. In all cases the required values of $\gamma_{eu}$
are still several orders of magnitude below the maximum attainable by such
processes ($\gamma_{max} \sim 10^8 B^{-1/2}$) obtained when the electron energy losses balance
the energy gains in the acceleration sites (Cavaliere & Morrison 1980; Rees 1984;
Schlickeiser 1984). Thus our values of $\gamma_{eu}$ are physically plausible.

The small range of $\nu a U$ (and hence $\tau_U$) listed in Table 6.6 is consistent with the
apparent pivoting of the X-ray spectrum suggested in Chapter 5. Indeed, for
the low state observations $\nu a U \sim 0.2$ keV, in good agreement with $E_p$ listed in
Table 5.1.

Values for the mean magnetic field, $\overline{B}$ and mean electron number density, $\overline{n}_e$
in the X-ray emitting region are listed in Table 6.6, as are the mean magnetic field,
electron and photon energy densities ($\overline{U}_{mag}$, $\overline{U}_{elec}$ and $\overline{U}_{phot}$ respectively). For
the low state observations we find that typically $\overline{U}_{elec}$ is of the same order as
$\overline{U}_{mag}$, whilst for the high states $\overline{U}_{elec} \sim 100 \times \overline{U}_{mag}$. In all cases $\overline{U}_{elec}$ and $\overline{U}_{mag}$
are several orders of magnitude higher than $\overline{U}_{phot}$ thus confirming the lack of
SSC emission expected from the X-ray emitting region.

Finally we note that the predicted angular diameter of the jet considered here is
~0.1 mas at 6 cm, consistent with the unresolved (<0.3 mas) component reported
(Chapter 5).
6.5 Summary

In summary, we have shown that a homogeneous disk-shaped emission region can account for individual single epoch snap-shots of the multi-waveband continuum observed in the five X-ray bright BL Lac type objects under consideration. We find that generally a modest degree of relativistic beaming is required ($D \sim 2-3$) in order to suppress the predicted inverse-Compton emission below that observed. The implied physical conditions within such an emission region are generally found to be reasonable. However, a difficulty with this description is that the lifetime for the X-ray emitting electrons is predicted to be very short (i.e. $\sim$seconds). Thus, there is a requirement for a large degree of \textit{in situ} re-acceleration, but more significantly predicting a spectral index/flux correlation will be apparent in rapid X-ray flickering, rather than in spectral comparisons over extended periods (days to years) as observed.

We have shown however, that a specific inhomogeneous jet model in which there is a continuous injection of relativistic electrons through the emission region can explain the observed spectra at all epochs. If the upper cut-off in the synchrotron emitting electron population has a radial dependence, then the maximum emitted frequency will also be a function of radius and the observed X-ray spectrum may be interpreted as being due to the superposition of the highest observed synchrotron frequencies from the various contributing regions. Radial and/or temporal variations in the upper termination of the electron population can then lead to dramatic spectral variability within the X-ray band with little or no effect at longer wavelengths. Such effects might reflect temporal changes in the electron re-acceleration mechanism due, for example, to the formation of shockfronts within the jet.
Chapter 7

Conclusions and Future Prospects

In the first part of this Chapter we summarize the observational results and interpretation presented in earlier chapters. In the second part we briefly review the future prospects for the study of BL Lac type objects.

7.1 General Summary

7.1.1 The EXOSAT and IUE data

We have presented a large number of observations of five BL Lac type objects in the X-ray and ultra-violet energy bands. We have found that the X-ray spectra generally have soft power-law spectra with low-energy cut-offs consistent with absorption in the line-of-sight gas column density through our own galaxy. In a number of cases however (most notably the high state observations of Mrk 421), a downward curvature of the spectrum is implied above $\sim$3 keV. All five sources exhibit significant variability with minimum variability timescales in the range $\sim$3 hr to $\sim$7 days. Furthermore, the X-ray flux and spectral index for each object
appears to be correlated in the sense that the X-ray spectrum hardens as the source brightens. In contrast, at ultra-violet wavelengths these objects exhibit only slow drifts in flux on a timescale ~weeks with little evidence for spectral variability. The spectra are generally consistent with a simple power-law of index ~1, and seem to extrapolate well into the optical and infra-red bands. The ultra-violet to X-ray continuum can therefore be modelled as a power-law of index ~1 below about 0.1 keV, above which the source steepens. The spectral variability in the X-ray band can then be described in terms of a 'pivoting' of the high-energy continuum about the break-point.

7.1.2 Synchrotron models

The observed spectral variability in the ultra-violet to X-ray continuum provides a stringent test for physical models of emission region. The smoothness of the multi-waveband spectra suggests a single process is responsible for the entire radio to X-ray continuum. The characteristic power-law forms observed tend to confirm the hypothesis that the emission is the result of the synchrotron process.

We have investigated two specific SSC models for the continuum emission. It was found that whilst a simplistic homogeneous disk-shaped emission region can provide an acceptable explanation of single epoch snap-shots of the multi-waveband spectrum, it experiences some difficulty accounting for the high energy spectral variability observed. Specifically, such a model predicts a spectral index/flux correlation will be apparent in rapid X-ray flickering, rather than in spectral comparisons over extended periods (days to years) as observed. In contrast, by allowing a variable radial dependence in the upper cut-off in the synchrotron emitting electron population in an inhomogeneous relativistic jet model, we have demonstrated that the multi-waveband spectrum of the most demanding source, Mrk 421, at all epochs can be reproduced.

The immediately obvious feature of the application of SSC models presented in Chapter 6, however, is that almost all the key observational parameters are
ill-determined. Indeed, in the case of the relativistic jet model the number of free parameters outnumbered the observables. A number of somewhat *ad hoc* assumptions had to be made and the derived model parameters are therefore not unique solutions. Nevertheless, we have shown the results to be reasonable. Thus the present work demonstrates that a jet model, broadly following the current paradigm for BL Lac type objects can give an acceptable explanation of present multi-waveband measurements.

### 7.2 The Implications of Beaming

The interpretation of BL Lac type objects as sources in which relativistic bulk motion plays a significant role has a number of observational and theoretical consequences which can be used to both give credence to the general scenario, and possibly to help constrain physical models.

#### 7.2.1 The parent population and unification

A fundamental corollary to the beaming scenario is that assuming the direction of relativistic motion is randomly orientated on the sky, then BL Lac type objects are likely to be a subset of a much larger parent population. The BL Lac type objects are then those sources with emission regions moving at velocities close to the line-of-sight and therefore with enhanced apparent luminosities, whilst the remaining sources in the parent population (forming the majority) are viewed at much larger angles and hence essentially unbeamed. The firm identification of this parent population is clearly of prime importance.

**Space Densities**

Clearly, comparison of the relative space densities of BL Lac type objects and potential parent populations ought to be particularly illuminating. Ideally, de-
termination of the space density requires a complete sample in a well defined volume of space with an observed flux greater than some well determined limit. In practice however, incomplete samples, variability and indeterminable selection effects add a significant degree of uncertainty to the results.

In the X-ray band, the space density of BL Lac type objects with an X-ray luminosity, $L_X > 10^{45} \text{ erg s}^{-1}$ (in the 0.5–4.5 keV band) is $\sim \text{few} \times 10^{-9} - 10^{-7} \text{ Mpc}^{-1}$ (Schwartz & Ku 1983; Urry 1984). It is argued that since the space density of both Seyfert Galaxies and optically selected Quasars with the same X-ray luminosities is comparable, the BL Lac parent population is unlikely to be either of these classes of AGN. Assuming that the X-ray emission is relativistically beamed with a Lorentz factor, $\gamma_j = 10$ (as used in Chapter 6), it is estimated that the space density of the BL Lac parent population with $L_X \geq 10^{45} \text{ erg s}^{-1}$ is $\sim 10^{-6} \text{ Mpc}^{-1}$.

Unfortunately, accepting the argument against Seyferts and Quasars being suitable candidates, there are no alternative X-ray sources which have been detected in sufficient numbers to unequivocally identify the BL Lac parent population. However, it should be noted that the spectral variability reported in this work and in particular our interpretation in terms of temporal variations in the synchrotron emitting electron population make the X-ray space density of limited use in studies of this kind. Similarly in the optical band, short timescale variability, substantial contributions from galactic starlight and inevitable selection effects make similar studies equally uncertain.

The most promising possibility of progress is likely to be provided by studies in the radio band. Relativistic motion and jet-like structures are relatively common in Radio Galaxies and Quasars, and it has long been suggested that Blazars (BL Lac type objects and OVVs) are normal radio galaxies in which the jet is viewed almost end-on (e.g. Blandford & Rees 1978); a scenario we shall refer to as the 'BLazar Unified Scheme'. Indeed, Blandford & Königl (1979) and Orr & Browne (1982) have even argued in favour of a 'Generalized Unified Scheme' in which flat spectrum, core-dominated radio sources are in general lobe-dominated sources viewed from close to the jet direction.
The Blazar Unified Scheme

Under the Blazar Unified Scheme it is assumed that the emission from the radio core flux is highly anisotropic (i.e. from the relativistic jet), whilst any extended (off-nuclear) emission is unbeamed and hence isotropic. Thus, the existence of and study of the extended radio emission in BL Lac type objects provides strong test of such a hypothesis. Antonucci & Ulvestad (1985) have recently performed a detailed comparison between the structure of Blazars and Radio Galaxies and confirmed that the extended radio power and morphology in Blazars is indeed consistent with the prediction of the Blazar Unified Scheme. Using a similar sample to Schwartz & Ku (1983), Browne (1983) has found that the space density of BL Lac type objects with extended emission with a luminosity, $L_R \geq 10^{22}$ W Hz$^{-1}$ ster$^{-1}$ at 1.4 GHz is $\sim 10^{-7}$ Mpc$^{-1}$, a factor of 60 lower than that for Radio Galaxies of a similar luminosity. Assuming the Blazar Unified Scheme, Browne then used this ratio of space densities to estimates that $\gamma_j \leq 5.4$. It should be re-emphasized however that the ratio of space densities is somewhat uncertain and only a factor of 3 decrease in the ratio is consistent with $\gamma_j = 10$. Thus the Blazar Unified Scheme seems to be at least qualitatively reasonable (see also Antonucci 1988), and in fair agreement with our findings.

7.2.2 The BL Lac luminosity function

An intriguing result to come from the study of samples of BL Lac type objects is that in all frequency bands the luminosity function (or equivalently the number-flux relationship, $\log N(>S) - \log S$, where $N(>S)$ is the number of sources observed with a flux $> S$) of these objects seems to be flatter than for other classes of AGN (Veron 1979; Impey & Brand 1982; Borra & Corriveau 1982; Schwartz & Ku 1983, Urry 1984). In the X-ray band, the slope of the luminosity function is $\sim 1$ (Urry 1984, Maccacaro et al. 1984) which can be compared with a slope of 1.75 for Seyfert galaxies (Piccinotti et al. 1982), although it should be pointed out that the X-ray sample is extremely small and hence subject to large system-
atic errors. Furthermore there is some indication of an observed deficit of weaker sources (Maccacaro et al. 1984; Giommi et al. 1988b). It has been suggested that this behaviour may reflect very little, or even 'negative' (i.e. the opposite for Quasars) cosmological evolution (Maccacaro et al. 1984; Stocke et al. 1988). However, an alternative explanation of more relevance in the context of the current work has been offered by Urry & Shafer (1984) and Cavaliere, Giallolongo & Vagnetti (1986). These workers point out that for relativistically beamed sources with an intrinsic low luminosity cut-off, the observed luminosity function can be described by a double power-law, much flatter than the intrinsic function at low luminosities, and with the intrinsic slope at higher luminosities. The slope at the low luminosity end depends weakly on the spectral index of the emission, whilst the break point depends upon the intrinsic low luminosity cut-off, the spectral index of the emission and the Lorentz factor of the bulk motion. The large number of new sources likely to be discovered by the ROSAT sky survey (below) are likely to lead to a vastly better determined X-ray luminosity function for BL Lac type objects and hence offer some hope of constraining physical models.

7.2.3 Blazar sub-classes

The presence of a strong, non-thermal continuum as a common property of BL Lac type objects, Optically Violently Variables (OVVs) and Highly Polarized Quasars (HPQs) first led to the grouping together of these classes into Blazars at the Pittsburgh Conference in 1977. The study of these objects may also help to determine the similarities and differences between these objects and hence the physical and give valuable insights into the physical conditions within the nuclear regions.

Whilst BL Lac type objects and OVVs can usefully taken together in regard to their continua, it has been suggested that OVVs may represent a link to the 'classical' low-polarization Quasars through their emission line spectra. Moore & Stockman (1984), in a summary of the polarization properties of Blazars, have
concluded that virtually the only properties of HPQs which differ from those of BL Lac type objects are the overall luminosity and the equivalent width of the Mg II λ2798 emission line. Since the wavelength dependency of the degree and position angle of polarization is not as model dependent as the interpretation of the multi-waveband flux distribution, a detailed comparison of the polarization characteristics and behaviour of OVVs and BL Lac type objects has the potential of revealing independent information on the geometry of these sources (and perhaps AGN in general). Obviously there is the major problem of correcting for any wavelength dependency introduced by the dilution of the non-thermal emission by (unpolarized) starlight. However, with the slow collection of more accurate data this is slowly becoming more feasible.

7.3 Future Observational Prospects

Clearly the work presented here would benefit from improved data. Specifically, adequate X-ray and ultra-violet observations before, during and after a dramatic X-ray flare such as that observed in Mrk 421 in 1984 December will provide invaluable information on the evolution of the X-ray emitting electron population. The third Japanese X-ray astronomy satellite, GINGA, has now been in orbit for approximately 18 months (e.g. Makino et al. 1987b) and reliable results should be available in the near future. The scientific payload includes a Large Area proportional Counter (LAC) with an effective bandwidth of 2–30 keV and a detection limit of $\sim 2 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ in the 2–10 keV band. Preliminary analysis of a number of BL Lac type objects (including Mrk 421) has confirmed the soft power-law form of the continuum and the lack of any significant spectral flattening towards high energies. The results from a more detailed analysis are awaited with great interest. A number of further missions suitable for such work are either in preparation or have been proposed. Of particular interest in the near future are the complementary Röntgensatellit (ROSAT) and Extreme Ultra-Violet Explorer (EUVE) missions. The primary objective of both missions
is to perform imaging all-sky surveys, however a number of pointed observations will also be performed. The ROSAT payload consists of a large X-Ray Telescope (XRT) and a smaller Wide Field Camera (WFC). In the survey mode the sensitivity at 2 keV will be $\sim 3.5 \times 10^{-5}$ keV cm$^{-2}$ s$^{-1}$ keV$^{-1}$ and is expected to reveal $\sim 10^5$ extragalactic sources, a large number of which ($\sim 10\%$) are likely to be BL Lac type objects (e.g. Trümper 1984) and hence will be invaluable in the determination of the luminosity function. In the WFC energy band, the low intrinsic column densities in BL Lac type objects may result in these objects being the only class of AGN detected in significant numbers. Assuming 10% of the sky has a column density $n_H \leq 4 \times 10^{19}$ cm$^{-2}$ due to our own galaxy, $\sim 40$ BL Lac type objects are expected to be detected. The EUVE mission will cover a similar energy range as the ROSAT WFC with a higher sensitivity ($\sim$ factor two) although somewhat inferior source location accuracy (Bowyer & Malina 1980). The launch date of this and all other NASA funded missions are, however, uncertain following the Challenger accident.

In the medium term, improved spectral and variability studies will be possible over a broad band of X-ray energies (0.1–200 keV) following the launch of several instruments, namely the Satellite for Astronomy in X-rays (SAX), the X-ray Timing Explorer (XTE), Spectrum-X, and ASTRO-D. It is likely that these experiments will greatly aide the determination of the form of the X-ray spectra in BL Lac type objects above a few keV. Specifically, the existence of downward spectral curvature (as seen in the high state observations of Mrk 421) and/or a spectral flattening attributable to inverse-Compton emission will further help constrain SSC models of the emission region.

In the long term, dramatically improved spectral and variability information in the low to medium X-ray bands will be provided by two major missions, the Advanced X-ray Astronomy Facility (AXAF) and the X-ray Multi-Mirror observatory (XMM). Furthermore, if inverse-Compton tails are present in BL Lac type objects with strengths close to those implied by previous medium energy experiments, then important spectral and variability information may be obtained.
for a handful of the brightest objects in the hard X-ray to gamma-ray regime (15–10⁵ keV) by the Gamma-Ray Astronomy with Spectroscopy and Positioning (GRASP) mission. GRASP will employ a coded-mask aperture with a sensitivity of ~ 10⁻⁵ keV cm⁻² s⁻¹ kev⁻¹ for a 12 day exposure.

The near to medium term future also holds good prospects of observational progress in other wavebands. Developments in ground-based sub-millimeter and far infra-red observations will enable the determination of the self-absorption frequency as well as a better determination of the optically thin synchrotron spectrum without the complications of significant contributions from galactic starlight and synchrotron losses. In the ultra-violet, LYMAN, although primarily a high resolution spectroscopic mission will allow better determination of the 90–200 nm continuum in BL Lac type sources. LYMAN is also likely provide more stringent limits on intrinsic spectral features in the continua of BL Lac type objects, and will allow a much larger sample of these sources to be observed in the ultra-violet. Finally in the radio band, a quantum leap in resolution may be offered by an Earth-orbiting antenna (QUASAT¹) to be used in conjuncture with the ground-based VLBI networks. Baselines of up to 5 x 10⁷ m will lead to angular resolutions ~ micro-arcsec (µas), thus allowing structure on typical linear scales of ~few x 10⁻⁴ pc (~ 10 light hr) to be probed and hence offering the possibility of being able to resolve the central cores and to investigate radio jets in substantially more detail. In the context of the work reported here, the relevance of such a mission to the study of BL Lac type objects is clear.

¹Although it should be noted that LYMAN, GRASP, QUASAT and two other missions are competing candidates for selection as the next flexible component to ESA’s Horizon 2000 programme, and hence only one is likely to fly. Final selection is currently scheduled for 1988 November.
7.4 Final Remarks

In the decade since the first detection of BL Lac type objects in the X-ray and ultra-violet regimes, substantial progress in the understanding of this intriguing class of AGN has been made. However, BL Lac type objects currently remain frustratingly far from being fully understood, although a general consensus does appear to be forming in favour of relativistic jet-type models. Such models are difficult to adequately constrain with presently available data and a number of fundamental assumptions must be made. Future observational and theoretical advances however offer the opportunity of a far better understanding of jets in general, and hence for the construction of physically more realistic models. It is therefore anticipated that the next decade will be a challenging and exciting period for the study of the BL Lac phenomenon.
Appendix A

Previous X-ray Observations of BL Lac Type Objects

In the past 20 years, of the order of 15 satellite missions bearing instrumentation specifically designed for cosmic X-ray astronomy have been launched. These have, not unnaturally, been of progressively increasing sophistication and success. The early missions were primarily dedicated to the production of all-sky surveys of progressively higher sensitivity in the various energy bands, and were thus responsible for detecting a large fraction of the X-ray sources known today. The later missions were true observatories in which single specific objects were targeted.

It was arguably with the HEAO-1 and EINSTEIN satellites that X-ray astronomy truly came of age and 'real' science started being attempted. EXOSAT, with its broad band capability and highly eccentric orbit allowed the spectral parameters of many of the brighter AGN to be determined with unparalleled accuracy.

In Table A.1 we briefly summarize the missions and on-board instrumentation that have proved most useful for the study of BL Lac type objects. In Tables A.2 to A.6 we summarize and reference the published results from earlier observations of the five sources considered in the present work.
Table A.1 Summary of previous X-ray astronomy missions

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<td>TENMA (Japan)</td>
<td>prop. counter (GSPC)</td>
<td>2-60</td>
<td>T84, K84</td>
</tr>
<tr>
<td>1983 Feb</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GINGA (Japan)</td>
<td>prop. counter (LAC)</td>
<td>1.5-37</td>
<td>M87, T88</td>
</tr>
<tr>
<td>1987 Feb</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Seriously effected by build up of water ice - see Urry, Mushotzky & Holt (1986).
2. Error recently detected in all calibrations prior to 1987

References to Table A.1

D77 - Doxsey et al. (1977); D78 - Donaghy & Canizares (1978); dK81 - deKorte et al. (1981);
F81 - Friedmann et al. (1981); G71 - Giacconi et al. (1971); G78 - Gursky et al. (1978);
G79 - Giacconi et al. (1979); G78 - Gaillardetz et al. (1978); H76 - Hearn et al. (1976);
H76 - Humphrey et al. (1978); J78 - Joyce et al. (1978); K78 - Kubierschky et al. (1978);
K84 - Koyama et al. (1984); M87 - Makino et al. (1987b); P81 - Peacock et al. (1981);
R79 - Rothschild et al. (1979); S76 - Sanford & Ives (1976); T81 - Turner et al. (1981);
T84 - Tanaka et al. (1984); T88 - Turner et al. (1988); V76 - Villa et al. (1976);
### Table A.2 Previous X-ray observations of Mrk 421

<table>
<thead>
<tr>
<th>Date</th>
<th>Energy Range (keV)</th>
<th>Index $\alpha_x$</th>
<th>$n_H \times 10^{20}$ cm$^{-2}$</th>
<th>Flux$^a$ (2-6keV)</th>
<th>Notes &amp; References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975 May–Jul</td>
<td>2-10</td>
<td>–</td>
<td>–</td>
<td>58-74</td>
<td>Ariel V (SSI),$s$,1, R76</td>
</tr>
<tr>
<td>1975 Mar-1979 Sep</td>
<td>2-10</td>
<td>–</td>
<td>1.5$^{+0.7}_{-0.3}$</td>
<td>–</td>
<td>Ariel V (SSI),$s$,2, M81</td>
</tr>
<tr>
<td>1976 Apr 25-26</td>
<td>0.15-6</td>
<td>1.1$^{+0.4}_{-0.2}$</td>
<td>&lt; 3</td>
<td>8.5$^{+1.1}_{-0.7}$</td>
<td>SAS-3 LE,$s$,3, H79</td>
</tr>
<tr>
<td>1977 May 18-20</td>
<td>2-30</td>
<td>–</td>
<td>0.1$^{+0.5}_{-0.2}$</td>
<td>&lt; 3</td>
<td>OSO-8,$p$,4, M78</td>
</tr>
<tr>
<td>1977 Nov 20-27</td>
<td>0.15-2.5</td>
<td>2.5$^{+1.7}_{-1.3}$</td>
<td>2.2$^{+2.5}_{-0.1}$</td>
<td>–</td>
<td>HEAO-1 A-2 (LED),$s$, S85</td>
</tr>
<tr>
<td>1977 Oct 27-Nov 1</td>
<td>2-10</td>
<td>–</td>
<td>–</td>
<td>&lt; 0.8</td>
<td>HEAO-1 A-2 (MED),$s$, 5, M78</td>
</tr>
<tr>
<td>1978 May 20-24</td>
<td>0.15-2.5</td>
<td>&gt; 3</td>
<td>–</td>
<td>–</td>
<td>SAS-3 (SMC),$p$, H79</td>
</tr>
<tr>
<td>1978 May 28</td>
<td>2-10</td>
<td>2.9$^{+0.7}_{-0.4}$</td>
<td>&lt; 70</td>
<td>2.6$^{+0.3}_{-0.2}$</td>
<td>HEAO-1 A-2 (MED),$p$, M79</td>
</tr>
<tr>
<td>1978 Apr 27</td>
<td>0.15-2.5</td>
<td>&gt; 3</td>
<td>–</td>
<td>–</td>
<td>HEAO-1 A-2 (LED),$s$,5, S85</td>
</tr>
<tr>
<td>1978 Nov 20-24</td>
<td>0.2-4.5</td>
<td>3$^{+1.0}_{-0.2}$</td>
<td>0.9$^{+0.2}_{-0.1}$</td>
<td>–</td>
<td>HEAO-1 A-2 (MED),$p$, M85</td>
</tr>
<tr>
<td>1978 Dec 5</td>
<td>0.15-2.5</td>
<td>1.7$^{+0.4}_{-0.3}$</td>
<td>1.1$^{+0.5}_{-0.3}$</td>
<td>–</td>
<td>HEAO-1 A-2 (LED),$p$, S85</td>
</tr>
<tr>
<td>1979 May 10</td>
<td>0.6-4.5</td>
<td>2.3$^{+1.0}_{-0.2}$</td>
<td>26$^{+18}_{-12}$</td>
<td>1.2$^{+0.2}_{-0.1}$</td>
<td>EINSTEIN SSS,$e$, 7, U86</td>
</tr>
<tr>
<td>1979 May 10-17</td>
<td>1.2-10</td>
<td>–</td>
<td>–</td>
<td>2.5$^{+0.5}_{-0.2}$</td>
<td>EINSTEIN MPC,$p$, 8, M85</td>
</tr>
<tr>
<td>1979 Nov 20-24</td>
<td>0.2-4.5</td>
<td>1.7$^{+0.7}_{-0.3}$</td>
<td>3$^{+2}_{-1}$</td>
<td>0.9$^{+0.1}_{-0.1}$</td>
<td>EINSTEIN IPC,$e$, H83, M85</td>
</tr>
<tr>
<td>1979 Dec 8-9</td>
<td>1.2-10</td>
<td>2.4$^{+0.4}_{-0.2}$</td>
<td>1.6$^{+0.3}_{-0.2}$</td>
<td>–</td>
<td>EINSTEIN MPC,$p$, M85</td>
</tr>
<tr>
<td>1980 May 12-17</td>
<td>0.2-4.5</td>
<td>2.3$^{+1.0}_{-0.2}$</td>
<td>5$^{+3}_{-2}$</td>
<td>1.2$^{+0.6}_{-0.3}$</td>
<td>EINSTEIN IPC,$e$,10, H83, M85</td>
</tr>
<tr>
<td>1981 Jan 1-31</td>
<td>1.2-10</td>
<td>1.7$^{+0.2}_{-0.1}$</td>
<td>–</td>
<td>4.7$^{+0.9}_{-0.6}$</td>
<td>EINSTEIN MPC,$p$,8,11, M85, H83</td>
</tr>
<tr>
<td>1984 Jan 23-27</td>
<td>1.5-10</td>
<td>1.1$^{+0.3}_{-0.1}$</td>
<td>–</td>
<td>1.6$^{+0.3}_{-0.2}$</td>
<td>TENMA,$p$, 13, M87</td>
</tr>
<tr>
<td>1984 Mar 2-9</td>
<td>1.5-10</td>
<td>1.8$^{+0.3}_{-0.2}$</td>
<td>–</td>
<td>0.6$^{+0.2}_{-0.1}$</td>
<td>TENMA,$p$, 14, M87</td>
</tr>
</tbody>
</table>

Notes:

- Flux in units of $10^{-11}$ erg s$^{-1}$ cm$^{-2}$.
- $s$: Scanning observation.
- $p$: Pointed observation.
- $e$: Extrapolated flux.

1. Detection, tentative ident. Bright 1 day flare superimposed on longer term trend; flux quoted at peak.
2. Second flare lasting ~5 days ob's'd in 1977 Dec reaching peak of $1.5 \times 10^{-10}$ erg s$^{-1}$ cm$^{-2}$; quoted flux weighted mean flux of all SSI non-outburst observations 1975–79.
3. Association with Mrk 421 confirmed; factor 3 variability seen in 0.15–0.28 keV band.
4. Evidences for a soft excess below 3.2 keV; detection of hard-tail ?
6. 0.15–0.5 keV count rate similar to 1977 Nov obs., whilst no flux obs'd above 1 keV.
7. No evidence of variability over 3 days. Excess absorp. due to highly ionized plasma ? (but $n_H$(ice) $\sim 130 \times 10^{-20}$ cm$^{-2}$).
9. Excess absorp., but $n_H$(ice) $\sim 42 \times 10^{-20}$ cm$^{-2}$.
10. Fit statistically unacceptable ($\chi^2 = 54.1$ for 10 dof).
11. Factor 2 variability seen in 14 hrs.
12. Two comp. spectrum also acceptable with $\alpha_x = 2.4^{+0.4}_{-0.2}$, $E < 3$ keV; $\alpha_x = 0.0^{+0.8}_{-0.1}$, $E > 3$ keV.
13. Possible two comp. spectrum with $\alpha_x = 1.6^{+0.2}_{-0.1}$, $E < 4$ keV; flatter for $E > 4$ keV.
14. Possible two comp. spectrum with $\alpha_x = 1.7^{+0.3}_{-0.2}$, $E < 4$ keV; steeper for $E > 3$ keV.
### Table A.3 Previous X-ray observations of Mrk 501

<table>
<thead>
<tr>
<th>Date</th>
<th>Energy Range (keV)</th>
<th>Index</th>
<th>$n_H$ x10$^{20}$ cm$^{-2}$</th>
<th>Flux$^a$ (2-6keV)</th>
<th>Notes &amp; References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1971 Jan–1973 Mar</td>
<td>2–6</td>
<td>–</td>
<td>–</td>
<td>2.7$^{+0.5}$ to 3.9$^{+0.5}$</td>
<td>UHURU,a,1,F78</td>
</tr>
<tr>
<td>1975 Mar 15–18</td>
<td>2–10</td>
<td>0.8$^{+0.5}$</td>
<td>&lt; 10</td>
<td>3.6$^{+1.8}$</td>
<td>Ariel V (PCS),s,Sn79</td>
</tr>
<tr>
<td>1977 Aug 19–30</td>
<td>0.15–2.5</td>
<td>2.4$^{+0.4}$</td>
<td>1.2$^{+0.5}$</td>
<td>6.5$^{+1.3}$</td>
<td>HEAO-1 A-2 (LED),a,2,3,S85</td>
</tr>
<tr>
<td></td>
<td>2–30</td>
<td>0.2$^{+0.2}$</td>
<td>–</td>
<td>3.6$^{+1.2}$</td>
<td>HEAO-1 A-2 (MED,HED),a,3,M78</td>
</tr>
<tr>
<td></td>
<td>2–30</td>
<td>–</td>
<td>–</td>
<td>3.6$^{+1.2}$</td>
<td>HEAO-1 A-3 (SMC),a,3,4,S78</td>
</tr>
<tr>
<td>1977 Aug–1978 Feb</td>
<td>0.5–25</td>
<td>–</td>
<td>–</td>
<td>2.8$^{+0.13}$</td>
<td>HEAO-1 A-1,a,W85</td>
</tr>
<tr>
<td>1978 Sep 8–9</td>
<td>2–30</td>
<td>1.5$^{+0.5}$</td>
<td>0.8$^{+0.2}$</td>
<td>2.9$^{+1.0}$</td>
<td>HEAO-1 A-2 (MED),a,K81</td>
</tr>
<tr>
<td>1979 Jan 25–26</td>
<td>0.6–4.5</td>
<td>1.2$^{+0.5}$</td>
<td>20$^{+8}$</td>
<td>3.1$^{+1.6}$</td>
<td>EINSTEIN SSS,e,5,U86</td>
</tr>
<tr>
<td></td>
<td>1.2–10</td>
<td>1.1$^{+0.1}$</td>
<td>–</td>
<td>3.7$^{+0.2}$</td>
<td>EINSTEIN MPC,e,M85</td>
</tr>
<tr>
<td>1979 Mar 1–2</td>
<td>0.6–4.5</td>
<td>0.7$^{+0.5}$</td>
<td>12$^{+5}$</td>
<td>4.4$^{+1.2}$</td>
<td>EINSTEIN SSS,e,5,U86</td>
</tr>
<tr>
<td>1979 Aug 22</td>
<td>0.6–4.5</td>
<td>1.6$^{+0.2}$</td>
<td>18$^{+2}$</td>
<td>3.2$^{+0.3}$</td>
<td>EINSTEIN SSS,e,5,U86</td>
</tr>
<tr>
<td></td>
<td>1.2–10</td>
<td>1.6$^{+0.2}$</td>
<td>–</td>
<td>4.0$^{+0.3}$</td>
<td>EINSTEIN MPC,e,M85</td>
</tr>
<tr>
<td>1980 Jan 19–20</td>
<td>0.3–3.4</td>
<td>1.4$^{+0.2}$</td>
<td>3.2$^{+1.9}$</td>
<td>–</td>
<td>EINSTEIN IPC,M84</td>
</tr>
<tr>
<td></td>
<td>1.2–10</td>
<td>1.5$^{+0.3}$</td>
<td>10$^{+2.3}$</td>
<td>2.4$^{+0.3}$</td>
<td>EINSTEIN MPC,e,M84,M85</td>
</tr>
<tr>
<td>1980 Aug 15</td>
<td>0.3–5.3</td>
<td>0.8$^{+0.5}$</td>
<td>2.0$^{+1.2}$</td>
<td>–</td>
<td>EINSTEIN IPC,M84</td>
</tr>
<tr>
<td></td>
<td>1.2–10</td>
<td>1.7$^{+0.3}$</td>
<td>20$^{+12}$</td>
<td>4.1$^{+0.7}$</td>
<td>EINSTEIN MPC,e,M84,M85</td>
</tr>
</tbody>
</table>

**Notes:**

- Flux in units of 10$^{-11}$ erg s$^{-1}$ cm$^{-2}$.
- Scanning observation.
- Pointed observation.
- Extrapolated flux.
- Tentative ident., 30% variability obs'd 1971–73.
- Prelim. analysis reported by W81 and K81 used less satisfactory bkgd sub., factor 2 decrease in flux seen over 2 days.
- Implied 2 component spectrum, but see S79.
- Identified. confirmed.
- Excess absorpt. due to hot gas? (but $n_H$(ice) $\sim$ 85 x 10$^{20}$ cm$^{-2}$, 54 x 10$^{20}$ cm$^{-2}$ and 21 x 10$^{20}$ cm$^{-2}$ for 1979 Jan, Mar and Aug respectively.
- Using pre-1987 calibration.
Table A.4 Previous X-ray observations of 1218+304

<table>
<thead>
<tr>
<th>Date</th>
<th>Energy Range (keV)</th>
<th>Index ( \alpha_x )</th>
<th>( n_H ) ( \times 10^{20} ) cm(^{-2} )</th>
<th>Flux* (2-6keV)</th>
<th>Notes &amp; References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1975 Dec-1977 Nov</td>
<td>2-10</td>
<td>-</td>
<td>-</td>
<td>1.1-18.0</td>
<td>Ariel V,s,l,W79, Ma81</td>
</tr>
<tr>
<td>1977 Dec 11-16</td>
<td>2-20</td>
<td>1.1±0.9</td>
<td>-</td>
<td>3.1±2.2</td>
<td>HEAO-1 A-2 (MED),s,W81</td>
</tr>
<tr>
<td>1977 Aug-1978 Feb</td>
<td>0.5-25</td>
<td>-</td>
<td>-</td>
<td>3.5±0.3</td>
<td>HEAO-1 A-1,s,W85</td>
</tr>
<tr>
<td>1978 May 31</td>
<td>2-20</td>
<td>1.0±0.1</td>
<td>&lt; 86</td>
<td>2.9±0.5</td>
<td>HEAO-1 A-2 (MED),p,W81</td>
</tr>
<tr>
<td>1977 Aug-1978 Feb</td>
<td>2-6</td>
<td>0.5±0.5</td>
<td>&lt; 300</td>
<td>1.5±0.7</td>
<td>HEAO-1 A-3 (SMC),p,2,Sc79</td>
</tr>
<tr>
<td>1978 May 31</td>
<td>0.5-4.5</td>
<td>0.4±0.3</td>
<td>&lt; 16</td>
<td>2.0±1.3</td>
<td>EINSTEIN SSS,e,3,U86</td>
</tr>
<tr>
<td>1978 Aug-1978 Feb</td>
<td>1.2-10</td>
<td>1.1±0.2</td>
<td>&lt; 100</td>
<td>1.0±0.2</td>
<td>EINSTEIN MPC,p,2,3,M85</td>
</tr>
<tr>
<td>1978 Dec 11-16</td>
<td>2-20</td>
<td>3.0±1.8</td>
<td>-</td>
<td>1.5±1.2</td>
<td>HEAO-1 A-2 (MED),s,W81</td>
</tr>
<tr>
<td>1979 Jun 5</td>
<td>0.5-4.5</td>
<td>2.6±0.5</td>
<td>17±8</td>
<td>0.5±0.1</td>
<td>EINSTEIN SSS,e,5,U86</td>
</tr>
<tr>
<td></td>
<td>1.2-10</td>
<td>1.8±0.6</td>
<td>&lt; 60</td>
<td>1.3±0.2</td>
<td>EINSTEIN MPC,p,2,3,M85</td>
</tr>
</tbody>
</table>

Notes:
1. Flux in units of \( 10^{-11} \) erg s\(^{-1} \) cm\(^{-2} \).
2. Scanning observation.
3. Pointed observation.
4. Extrapolated flux.
5. Detection. Highly variable; 3 flares each lasting 2-3 days superimposed on possible long term trend.
6. Ident. confirmed.
3. High ice \((n_H) \sim 150 \times 10^{20} \) cm\(^{-2} \).
5. Excess absorp. \((n_H) \sim 33 \times 10^{20} \) cm\(^{-2} \).

Table A.5 Previous X-ray observations of Mrk 180

<table>
<thead>
<tr>
<th>Date</th>
<th>Energy Range (keV)</th>
<th>Index ( \alpha_x )</th>
<th>( n_H ) ( \times 10^{20} ) cm(^{-2} )</th>
<th>Flux* (2-6keV)</th>
<th>Notes &amp; References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977 Nov 2-3</td>
<td>0.5-20</td>
<td>-</td>
<td>-</td>
<td>1.4±0.2</td>
<td>HEAO-1 A-1,s,l,Ma81, W84</td>
</tr>
<tr>
<td>1979 Nov 13</td>
<td>0.5-2.8</td>
<td>1.3±0.3</td>
<td>2.5±0.7</td>
<td>-</td>
<td>EINSTEIN IPC,p,2,M84</td>
</tr>
<tr>
<td></td>
<td>1.2-6.6</td>
<td>3.1±0.3</td>
<td>158±78</td>
<td>1.3±0.2</td>
<td>EINSTEIN MPC,p,2,3,M84,M85</td>
</tr>
<tr>
<td>1980 Dec 28</td>
<td>0.5-4.6</td>
<td>0.7±1.0</td>
<td>0.4±0.5</td>
<td>3.5±0.4</td>
<td>EINSTEIN IPC,p,2,3,M84</td>
</tr>
<tr>
<td></td>
<td>1.2-6.6</td>
<td>3.7±0.3</td>
<td>-</td>
<td>0.5±0.2</td>
<td>EINSTEIN MPC,p,2,3,M85</td>
</tr>
</tbody>
</table>

Notes:
1. Flux in units of \( 10^{-11} \) erg s\(^{-1} \) cm\(^{-2} \).
2. Scanning observation.
3. Pointed observation.
4. Extrapolated flux.
5. Re-analysis of archival scans following 1979 Nov detection.
6. Substantial steepening of spectrum towards higher energies implied.
### Table A.6 Previous X-ray observations of 0414+009

<table>
<thead>
<tr>
<th>Date</th>
<th>Energy Range (keV)</th>
<th>Index $\alpha^*$</th>
<th>$n_H \times 10^{20}$ cm$^{-2}$</th>
<th>Flux$^a$ (2-6keV)</th>
<th>Notes &amp; References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977 Aug 23–27</td>
<td>0.5–20</td>
<td>-</td>
<td>-</td>
<td>$1.16_{-0.14}^{+0.14}$</td>
<td>HEAO-1 A1,s,1,U80,U81</td>
</tr>
<tr>
<td>1980 Feb 9</td>
<td>1.2–10</td>
<td>$1.4_{-0.3}^{+0.3}$</td>
<td>-</td>
<td>$1.67_{-0.14}^{+0.14}$</td>
<td>EINSTEIN MPC,p,2,3,U83</td>
</tr>
<tr>
<td>1981 Jan 23</td>
<td>1.2–10</td>
<td>$1.6_{-0.2}^{+0.2}$</td>
<td>-</td>
<td>$0.94_{-0.10}^{+0.10}$</td>
<td>EINSTEIN MPC,p,2,3,U83</td>
</tr>
</tbody>
</table>

**Notes:**

1. Detection. Also detected at 3.3σ in 0.9–2.5 keV band with HEAO-1 A-3 experiment.
2. Assuming a model of the form $F(E) \propto E^{-\alpha} \exp(E^{-8/3})$.

**References to Tables A.2–A.6**

- **F78** - Forman et al. (1978);
- **H83** - Hutter (1983);
- **M79** - Mushotzky et al. (1979);
- **M84** - Mufson et al. (1984);
- **R76** - Ricketts et al. (1976);
- **Sc79** - Schwartz et al. (1979);
- **U81** - Ulmer et al. (1981);
- **W79** - Wilson et al. (1979);
- **H79** - Hearn et al. (1979);
- **K81** - Kondo et al. (1981);
- **Ma81** - Marshall et al. (1981);
- **M85** - Madejaki (1985);
- **S78** - Schwartz et al. (1978);
- **S85** - Singh & Garmire (1985);
- **U83** - Ulmer et al. (1983);
- **W81** - Worrall et al. (1981);
- **H81** - Hall et al. (1981);
- **M78** - Mushotzky et al. (1978);
- **M81** - Mufson & Hutter (1981);
- **M87** - Makino et al. (1987a);
- **Sn79** - Snijders et al. (1979);
- **U80** - Ulmer et al. (1980);
- **U86** - Urry et al. (1986);
- **W84** - Wood et al. (1984);
Appendix B

Relativistic Effects

Overview

Due to special relativistic effects, if an object is moving relative to an observer with a bulk velocity close to the speed of light, $c$, then the observed characteristics of any emitted signal may be very different to the characteristics in the co-moving frame. Below we briefly review the well known expressions needed within this thesis.

B-1 Kinematic Doppler Factor $\mathcal{D}$

If a source is moving with a velocity $v = \beta c$ at an angle $\theta_v$ to the line of sight, then we may define the kinematic Doppler factor, $\mathcal{D}$ as

$$\mathcal{D} = \left[\Gamma(1 - \beta \cos \theta_v)\right]^{-1}$$

[6.2]

where $\Gamma$ is the Lorentz factor corresponding to this velocity given by

$$\Gamma = (1 - \beta^2)^{-1/2}$$

[6.3]

As shown in for example Urry (1984, Appendix B), for a fixed $\Gamma$ and small $\theta_v$, $\mathcal{D}$ can be large, tending towards $2\Gamma$ as $\theta_v$ approaches zero. However, as $\theta_v$ increases, $\mathcal{D}$ quickly declines tending to $1/(2\Gamma)$ for $\theta_v = 180^\circ$. From equations [6.2] and [6.3] it can easily be shown that $\mathcal{D} > 1$ for $\theta_v < \theta_{crit}$ where

$$\theta_{crit} = \arccos \left(\frac{\Gamma - 1}{\Gamma + 1}\right)^{1/2}$$

[6.4]

The effects due to the redshift, $z$ of the source can be directly obtained from equations [6.2] and [6.3] by letting $\beta$ refer to the velocity of recession and setting $\theta_v = 180^\circ$. Then (Urry 1984)
\[ D_z = \left\{ \frac{(1 - \beta)}{(1 + \beta)} \right\}^{1/2} = (1 + z)^{-1} \]  \[ \text{[B.4]} \]

Thus by de-coupling the cosmological velocity (with \( \theta_v = 180^\circ \) and \( D_z \)) from any non-cosmological velocity (in an arbitrary direction and with kinematic Doppler factor \( D \)) we have
\[ D_{\text{net}} = \frac{D}{(1 + z)} \]  \[ \text{[B.5]} \]

### B-2 Lorentz Transforms

The observed fluxes and frequencies will differ systematically from those in the co-moving (emitter) frame. Specifically, if primes indicate the co-moving frame\(^2\), we have (e.g. Urry 1984)
\[ \Delta t = \left\{ \frac{D}{(1 + z)} \right\}^{-1} \Delta \hat{t} \quad \text{and} \quad \nu = \left\{ \frac{D}{(1 + z)} \right\} \hat{\nu} \]  \[ \text{[B.6]} \]
\[ I(\nu, \Omega) = \left\{ \frac{D}{(1 + z)} \right\}^{3} \hat{I}(\hat{\nu}, \hat{\Omega}) \]  \[ \text{[B.7]} \]
and
\[ F(\nu, \Omega) = \left\{ \frac{D}{(1 + z)} \right\}^{3} \hat{F}(\hat{\nu}, \hat{\Omega}) \]  \[ \text{[B.8]} \]
for the Lorentz transforms for time (\( \Delta t \)), frequency (\( \nu \)), specific intensity (\( I \)) and flux density (\( F \)) where \( \Omega \) is the solid angle.

### B-3 Observed Flux and Luminosity Distance

Since the flux per unit frequency, \( S(\nu) \), received in the observer's frame will have been emitted at a higher frequency and within a broader band, the relationship between \( S(\nu) \) and the luminosity, \( L(\nu) \), of a source with \( L(\nu) \propto \nu^{-\alpha} \), is given by the standard relation (von Hoerner 1974, equations [13.23] and [13.25])
\[ S(\nu) = \frac{L(\nu)}{4\pi D_L^2 (1 + z)^{1-\alpha}} \]  \[ \text{[B.9]} \]

\(^2\)This notation is maintained throughout the thesis
where $D_L$ is the luminosity distance defined as (e.g. Weinberg 1972)

$$D_L = cH_0^{-1}(z + 1/2(1 - q_0)z^2 + \cdots) \quad [B.10]$$

and $H_0$ and $q_0$ are the Hubble and deceleration parameters. Throughout this work we assume $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $q_0 = 0.5$.

If the emitting region is in addition moving with a kinematic Doppler factor, $\mathcal{D}$, then equation [B.9] becomes

$$S(\nu) = \frac{L(\nu)}{4\pi D_L^2} (1 + z)^{1-\alpha}\mathcal{D}^{3+\alpha} \quad [B.11]$$

The corresponding relationship between the total flux, $S_{bol}$, and the bolometric luminosity, $L_{bol}$, is given by

$$S_{bol} = \frac{L_{bol}}{4\pi D_L^2}\mathcal{D}^{3+\alpha} \quad [B.12]$$

Finally, the observed angular size, $\theta_d$, is related to the true linear size, $x$, by (von Hoerner 1974, equation [13.28])

$$\theta_d = \frac{x}{D_L} (1 + z)^2 \quad [B.13]$$
Appendix C

Derivation and comparison of synchrotron self-Compton (SSC) Formulae

Overview

In this appendix we summarize the SSC formulism. Although many of the derivations have been presented elsewhere, particularly in Hutter (1983), they are repeated here for completeness and to enable a direct comparison with other workers.

C-1 Synchrotron Radiation

The emission of synchrotron radiation by relativistic electrons gyrating in a magnetic field has been considered by numerous authors (e.g. Ginzburg & Syrovatskii 1965, 1969; Blumenthal & Gould 1970). Most workers assume an isotropic distribution of electrons relative to the local magnetic field (not least for simplicity). Such an assumption may seem strange in view of the strength of the magnetic fields implied by astrophysical sources, however, Jones, O'Dell & Stein (1974a,b) argue this approximation is probably valid since the production and maintenance of anisotropic electron distributions is difficult in such environments, and any effects of such an anisotropy are likely to be unobservable. Furthermore, when the particles are energetically important, the magnetic field itself is likely to be disordered on small scales due to plasma phenomena.
C-1.1 Synchrotron Emissivity

An electron of mass, \( m_e \) and charge, \( e \), moving through a magnetic field, \( B \), with a pitch angle, \( \Psi \), between the field and velocity vectors, will be accelerated onto a helical trajectory by the Lorentz force. Such an accelerated charge will emit cyclotron radiation at the gyro-frequency, \( \nu_g \) given by

\[
\nu_g = \frac{eB}{2\pi m_e c}
\]  

[C.1]

In the relativistic case, electrons with an energy \( E = \gamma_e m_e c^2 \) will have a gyro-frequency lower than in the non-relativistic case by a factor \( \frac{1}{\gamma_e} \). Furthermore, due to the effects of relativistic beaming, the emitted radiation does not appear to a stationary observer at a single frequency, but is instead composed of a power spectrum peaking at a frequency, \( \nu = 0.29\nu_{\text{crit}} \) where \( \nu_{\text{crit}} \), the ‘critical frequency’ is given by

\[
\nu_{\text{crit}} = \frac{3}{2} \gamma_e^2 \nu_g \sin \Psi
\]  

[C.2]

For \( \nu \ll \nu_{\text{crit}} \) the spectrum rises as \( (\nu/2\nu_{\text{crit}})^{1/3} \) whilst for \( \nu \gg \nu_{\text{crit}} \) the spectrum falls as \( \exp(-\nu/\nu_{\text{crit}}) \). Thus the radiation from an individual electron peaks at a frequency, \( \dot{\nu} \), given by

\[
\dot{\nu} = 0.29 \frac{3eB}{2\pi m_e c} \gamma_e^2 = 1.22 \times 10^6 B \gamma_e^2 \text{ Hz}
\]  

[C.3]

for \( \sin \Phi \sim 1 \).

The current discussion follows the assumption that there is an ensemble of electrons within the emission region described by a power-law distribution in energy,

\[
N(\gamma_e) = \begin{cases} 
K \gamma_e^{-p} & \text{for } \gamma_{e,\text{min}} \leq \gamma_e \leq \gamma_{e,\text{max}} \\
0 & \text{otherwise}
\end{cases}
\]  

[6.1]

where \( N(\gamma_e) \) is the number density of electrons with a Lorentz factor, \( \gamma_e \). The total power emitted per unit volume per unit frequency in the rest frame from such a distribution of electrons in a well tangled magnetic field is given by (Ginzburg & Syrovatskii 1965, equation [3.31]; Blumenthal & Gould 1970, equation [4.59])

\[
\frac{dW}{d\nu_s dt} = \frac{4\pi K e^3 B^{p+1}}{m_e c^2} \left( \frac{3e}{4\pi m_e c} \right)^{p-1} a(p) \nu_s^{-(p+1)}
\]  

[C.4]
where

\[
a(p) = \frac{2^{(\frac{p-1}{2})} \Gamma\left[\frac{3p-1}{12}\right] \Gamma\left[\frac{3p+19}{12}\right] \Gamma\left[\frac{p+5}{4}\right]}{8\pi^{1/2}(p+1) \Gamma\left[\frac{p+7}{4}\right]}
\]

\[\text{[C.5]}\]

\(a(p)\) is tabulated for various values of \(p\) in Table C.1. It can be seen that for 
\(1.5 < p < 5.0, a(p) \sim 0.1\). It should be noted that equation [C.4] is strictly 
only valid when the mean pitch of the synchrotron emitting electrons is large 
(i.e. \(\cos^2 \Psi \ll 1\)). However, Jones, O'Dell & Stein (1974a,b, Appendix A) argue 
that the complete lack of detailed knowledge about either the fine- or the coarse-

scale average geometry of the electron–magnetic field interactions and the fact 
that the differences amount to only a few percent generally make the isotropic 
approximations justified.

Following Hutter (1983) by defining

\[
C1(\alpha_\epsilon) = \frac{4\pi e^3}{m_\epsilon c^2} \left(\frac{3e}{4\pi m_\epsilon c}\right)^{\frac{(p-1)}{2}} a(p)
\]

\[\text{[C.6]}\]

(tabulated for various \(\alpha\) in Table C.1) and the synchrotron emissivity, \(\dot{\varepsilon}_s\), as the 
power generated per unit volume per unit frequency per unit solid angle (e.g.

Jones, O'Dell & Stein, 1974; Hutter, 1983; Urry 1984), we therefore find

\[
\dot{\varepsilon}_s(\nu_s) = \frac{C1(\alpha_K)KB^{1+\alpha}}{4\pi} \nu_s^{-\alpha} \text{ erg s}^{-1} \text{ cm}^{-3} \text{ Hz}^{-1} \text{ ster}^{-1}
\]

\[\text{[C.7]}\]

where \(\alpha = (p-1)/2\). It should be noted that a number of authors (e.g. Ginzburg 
& Syrovatskii, 1965; Tucker, 1974; Rybicki & Lightman, 1979; Königl 1981) define 
\(dW/d\nu_s dt\) (equation [C.4]) as \(\dot{\varepsilon}_s\) creating some confusion and making errors of \(4\pi\) easy.

**C.1.2 Synchrotron absorption coefficient**

The synchrotron self absorption coefficient per unit length, \(\kappa_s\), is given by

\[
\kappa_s(\nu_s) = \frac{g(p)e^3}{2\pi m_\epsilon} \left(\frac{3e}{2\pi m_\epsilon c^2}\right)^{p/2} (m_\epsilon c^2)^{p-1} KB^{\frac{p+2}{2}} \nu_s^{-\frac{p+4}{2}}
\]

\[\text{[C.8]}\]

(Ginzburg & Syrovatskii 1965, 1969) where

\[
g(p) = \frac{3^{1/2}}{4} \Gamma\left[\frac{2p+2}{12}\right] \Gamma\left[\frac{3p+22}{12}\right] \times \frac{\pi^{1/2}}{2\Gamma\left[\frac{p+9}{4}\right]}
\]

\[\text{[C.9]}\]
Table C.1 Functions of spectral index in SSC formulae

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( p )</th>
<th>( a(p) )</th>
<th>( g(p) )</th>
<th>( C1(\alpha) )</th>
<th>( C2(\alpha) )</th>
<th>( A(\alpha) )</th>
<th>( G(\alpha) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.00</td>
<td>0.283</td>
<td>0.69</td>
<td>1.333</td>
<td>1.577</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.25</td>
<td>1.50</td>
<td>0.147</td>
<td>( 1.14 \times 10^{-20} )</td>
<td>2.00 ( \times 10^{-9} )</td>
<td>1.324</td>
<td>2.053</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>2.00</td>
<td>0.103</td>
<td>( 3.58 \times 10^{-19} )</td>
<td>9.24 ( \times 10^{-10} )</td>
<td>1.402</td>
<td>2.536</td>
<td></td>
</tr>
<tr>
<td>0.75</td>
<td>2.50</td>
<td>0.0852</td>
<td>( 1.31 \times 10^{-17} )</td>
<td>4.52 ( \times 10^{-12} )</td>
<td>1.553</td>
<td>3.023</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>3.00</td>
<td>0.0742</td>
<td>( 5.27 \times 10^{-16} )</td>
<td>2.32 ( \times 10^{-14} )</td>
<td>1.778</td>
<td>3.510</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that the last term in equation [C.9] arises from an averaging over all pitch angles assuming a randomly tangled magnetic field. In their earlier paper, Ginzburg & Syrovatskii do not perform such an averaging and hence this term is not present in their equation [4.19]. However in the later work, the averaging is performed and their equation [3.33] agrees with equation [C.8]. Values of \( g(p) \) are tabulated in Table C.1.

Defining \( C2(\alpha) \) by

\[
C2(\alpha) = \frac{g(p)e^3}{2\pi m_e} \left( \frac{3e}{2\pi m_e^2 c^5} \right)^{\nu/2} \left( m_e c^2 \right)^{p-1}
\]

equation [C.8] becomes

\[
\kappa_s(\nu_s) = C2(\alpha) K B^{\alpha+3/2} \nu_s^{-(\alpha+3/2)}
\]

For a photon travelling through a region of length \( l \) cm, the optical depth, \( \tau \), at a frequency \( \nu_s \) is therefore defined as \( \tau = \kappa_s(\nu_s)l \).

### C.1.3 The Observed Spectrum

From our definition of the volume emissivity, \( \dot{\epsilon}(\nu) \), the luminosity, \( \dot{L}(\nu) \), of the source at some frequency, \( \nu \) (with \( \tau \ll 1 \); see below) is given by

\[
\dot{L}(\nu) = 4\pi \int_V \dot{\epsilon}(\nu) \, dV
\]

where the integral is over the volume of the source. Hence from equation [B.11], the observed flux at this frequency at Earth is given by

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\[ S(\nu) = \frac{(1 + z)^{1-\alpha} D_L^{3\alpha}}{D_L^2} \int V \epsilon_{\nu}(\nu) \, dV \]  

[C.13]

where \( D_L \) is the luminosity distance (equation [B.10]). Thus, the resultant synchrotron spectrum is a power-law function of frequency (with an index of \( \alpha \)) as required observationally.

**Synchrotron self-absorption**

The increase in the absorption coefficient, \( \kappa_s \) (equation [C.11]), towards low frequencies can result in the reabsorption of the emitted synchrotron photons by the electron population, and hence a deficit of low frequency photons in the emergent synchrotron spectrum. In the case of a homogeneous disk-shaped geometry, the precise form of the flux density distribution, \( \hat{S}_s(\nu_s) \), with frequency is given by (Pacholczyk 1970, equation [3.53])

\[ \hat{S}_s(\nu_s) = \hat{S}_{sm} \left( \frac{\nu_s}{\nu_{sm}} \right)^{5/2} \left[ 1 - \exp \left( -\frac{\nu_s}{\nu_{sm}} \right)^{-\alpha+5/2} \right] \]  

[C.14]

where \( \hat{S}_{sm} \) and \( \nu_{sm} \) are defined below, and \( \alpha \) is the spectral index of the synchrotron spectrum in the optically thin regime. In the limiting cases \( \nu_s \ll \nu_{sm} \) and \( \nu_s \gg \nu_{sm} \), equation [C.14] reduces to the well known expressions \( \hat{S}_s(\nu_s) \propto \nu_s^{5/2} \) and \( \nu_s^{-\alpha} \) respectively. For a \( \nu > 1 \) there will therefore be a maximum in the emergent radiation spectrum around \( \tau \sim 1 \). However, as discussed in detail by Urry (1984), there are slight differences in the literature between the precise definitions of the 'self-absorption' break in the spectrum. We shall define a fiducial frequency, \( \nu_{sm} \), and a corresponding flux density, \( S_{sm} \), as the intersection of the extrapolated optically thin (\( \tau \ll 1 \)) and optically thick (\( \tau \gg 1 \)) spectra. This has the advantage of being the most well determined observationally.

**Synchrotron Losses**

The total power, \( \hat{P}_s \), emitted in the form of synchrotron photons by an individual electron of Lorentz factor \( \gamma_e \) gyrating in a magnetic field is given by the well known relativistic Larmor formula.
The rate of loss of kinetic energy due to synchrotron radiation suffered by the electron is therefore given by

$$\frac{d\gamma_e}{dt} = -\frac{\dot{p}_e}{m_e c^2} = -A\gamma_e^2 \beta^2$$ \[C.16\]

where

$$A = \left(\frac{2e^4 B_1^2}{3m_e^2 c^5}\right) = 1.93 \times 10^{-9} B_1^2 \text{ s}^{-1}$$ \[C.17\]

which assuming $\beta \approx 1$ has the solution

$$\gamma_e(t) = \gamma_e(0) \times \left[1 + \gamma_e(0) A t\right]^{-1}$$ \[C.18\]

where $\gamma_e(0)$ is the initial electron Lorentz factor. One half of the electron's energy will therefore be lost in a time, $t_s$, given by

$$t_s(\gamma_e(0)) = (A\gamma_e(0))^{-1} \text{ s}$$ \[C.19\]

$$= 2.0 \times 10^7 \gamma_e^{-1} U_{mag}^{-1} \text{ s}$$ \[C.20\]

where $U_{mag}$ is given by equation [C.26] below. With the use of equation [C.3], it can easily be shown that after a time $t_s$, electrons initially radiating at a frequency $\nu_{ab}$ will have lost half their initial energy, where

$$\nu_{ab} = 1.12 \times 10^{24} B^{-3} t_s^{-2} \text{ Hz}$$ \[C.21\]

Thus, in the absence of a substantial injection of fresh electrons, a deficit of high energy electrons will develop with a corresponding break in the synchrotron spectrum close to $\nu_{sb}$.

**C-2 Synchrotron self-Compton Models**

When attempting to model almost any physical environment, a fundamental decision must first be made concerning the assumed geometry of the region and the behaviour of the physical parameters within that region. In the application
of SSC models to spectra of BL Lac type objects several basic geometries and parameterizations have been considered to date. The simplest are the homogeneous disk and sphere in which the physical parameters are assumed to be constant within the emitting region. Alternatively, inhomogeneous spheres and jets, in which the physical parameters are assumed to be simply power-law functions of position, have been considered. A number of the apparent discrepancies between the various derived formulae presented in the literature are a result of different geometrical assumptions. However, particularly in the case of homogeneous models, such differences are usually negligible in comparison to those due to observational uncertainties. In the remainder of this appendix we derive expressions for the physical characteristics of both a homogeneous disk SSC model and a particular inhomogeneous jet model that are required in Chapter 6. The derivations closely follow Hutter (1983). For simplicity we have chosen to assume the continuous re-acceleration of the relativistic electron population throughout the emission region exactly balancing the energy lost to radiated photons. Thus the local spectral index above $\nu_{ab}$ is steepened from $\alpha$ to $\alpha + 1/2$ (see Chapter 6, section 6.3.3). All expressions include the possibility of relativistic bulk motion of the emission region (see Appendix B).

C-2.1 Geometries

Homogeneous disk We assume that the thickness of the disk is equal to its radius, $l$, and hence the volume is given by $\pi l^3$ (see Fig. C.1). The observed angular diameter of the disk, $\theta_d$, is then related to $l$ via

$$\frac{\theta_d}{2} = \frac{l}{3.086 \times 10^{18} D_L} (1 + z)^2$$  \hspace{1cm} \text{[C.22]}$$

where $\theta_d$ is in radians, $l$ in cm and $D_L$ in pc. Although such a geometry may be less physically plausible than (for example) a sphere, it does have the advantage of greatly simplifying a number of the derivations. Indeed many authors implicitly or explicitly make 'slab approximations' when considering more complex geometries.
For the homogeneous disk (a), $l$ is the radius of the disk (≡ thickness), $\theta_d$ is the angular diameter of the region and $D_L$ is the luminosity distance. Also shown is the angle $\theta_v$ between the line of sight and any relativistic bulk motion. For the inhomogeneous jet (b), $r$ is the radial distance from the jet apex, $\phi$ is the opening half-angle of the conical jet and $\theta_v$ the viewing angle. In the model developed it is required that $\theta_v \geq \phi$. 

Figure C.1 Schematic showing model geometries assumed throughout this work.
Inhomogeneous jet. We assume that the jet has a conical geometry with an opening angle $2\phi$ and is viewed at an angle $\theta_0 \geq \phi$ to its axis (Fig. C.1). The outflowing material is assumed to move outwards with constant velocity, $v_j = \beta_j c$ with a Lorentz factor, $\gamma_j = (1 - \beta_j^2)^{-1/2}$ given by $\gamma_j = 1/\phi$. The number density of electrons is assumed to be a power-law as given by equation [6.1]. The radial dependencies of the particle number density, magnetic field strength and upper termination of the electron population are also assumed to be power-laws represented by

$$K = K_1 r^{-n}, \quad B = B_1 r^{-m} \quad [6.14]$$

and

$$\gamma_e = \gamma_1 r^{-g} \quad [6.15]$$

where $r$ is measured in parsecs from the jet apex (see also Chapter 6, section 6.4.1).

C-2.2 Derivations for the Homogeneous Disk Model

Self-absorption frequency, $\nu_{sm}$

From the expressions for $\kappa_s(\nu_s)$ (equation [C.11]) and $I$ (equation [C.22]), and the definition of the self-absorption frequency, $\nu_{sm}$ (above), it can be shown that

$$\nu_{sm} = \left(\frac{3.086 \times 10^{18} \theta_d D_e C_2(\alpha) K B^{\alpha+3/2}}{2}\right)^{1/(\alpha+5/2)}$$

$$\times D (1 + z)^{-\frac{\alpha+2}{\alpha+5}} \quad [6.5]$$

where unprimed quantities refer to the observer's frame.

The flux, $S_{sm}$ at the self-absorption frequency

For a homogeneous disk in which neither $K$ or $B$ are functions of position within the source and volume $\pi r^2$, the integral over the source volume in equation [C.13] is trivial. Thus substituting for $\nu_s$ (equation [C.7]), $I$ (equation [C.22]) and $\nu_{sm}$
(equation [6.5]), we find that the observed flux, \(S_{\nu_{sm}}\), at \(\nu_{sm}\) (usually referred to as \(S_{sm}\)) is given by

\[
S_{sm} = \frac{3.086 \times 10^{18} \theta_d^3 D_L C_1(\alpha) K B^{1+\alpha} \nu_{sm}^{-\alpha}}{32} \times D^{3+\alpha} (1 + z)^{-(5+\alpha)} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}
\] [6.6]

The magnetic field, \(B\), and electron normalization, \(K\)

Equations [6.5] and [6.6] are both functions of \(B\) and \(K\) and hence can solved simultaneously to give

\[
B = \left( \frac{C_1(\alpha)}{C_2(\alpha)} \right)^2 \frac{\theta_d^4 S_{sm}^{-2} \nu_{sm}^5}{2^8} \left( \frac{D}{1 + z} \right) \text{ G}
\] [6.8]

and

\[
K = \frac{C_2(\alpha)^{(2+2\alpha)} 2^{(13+8\alpha)} S_{sm}^{(3+2\alpha)}}{C_1(\alpha)^{(3+2\alpha)} 3.086 \times 10^{18} D_L} \times \theta_d^{-(7+4\alpha)} \nu_{sm}^{-(5+4\alpha)} D^{-(4+2\alpha)} (1 + z)^{(6+2\alpha)} \text{ cm}^{-3}
\] [6.9]

These can be compared to other previously published derivations.

Many authors (Urry 1984 and references therein) have considered the derivation of an expression for the magnetic field, \(B\), and the reported results vary widely. A thorough review of the different approaches is given in Urry (1984).

The Lorentz factor of electrons radiating at a frequency \(\nu_s\)

Transforming the expression for the peak frequency, \(\nu\), of synchrotron radiation from an electron with a Lorentz factor \(\gamma_e\) (equation [C.3]) to the observer’s frame, substituting for \(B\) (equation [6.8]) and rearranging we find

\[
\gamma_e(\nu_s) = 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} \theta_d^{-2} S_{sm} \nu_{sm}^{-5/2} \nu_s^{1/2} \left( \frac{D}{1 + z} \right)^{-1}
\] [C.23]

In the particular case where \(\nu_s = \nu_{sm}\) we have

\[
\gamma_{sm} = \gamma_e(\nu_{sm}) = 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} \theta_d^{-2} S_{sm} \nu_{sm}^{-2} \left( \frac{D}{1 + z} \right)^{-1}
\] [C.24]

The requirement that \(\gamma_e > 1\) for synchrotron emission enables equation [C.24] to be rearranged and an upper limit placed on \(D\) (constraint (b) in Table 6.2 and Fig. 6.2).
Energy densities $U_{mag}$, $U_{elec}$ and $U_{phot}$.

The magnetic energy density, $U_{mag}$, is simply given by

$$U_{mag} = \frac{B^2}{8\pi}$$  \hspace{1cm} \text{[C.25]}

hence from equation [6.8] we have

$$U_{mag} = \left(\frac{C1(\alpha)}{C2(\alpha)}\right)^4 \frac{\theta_d^2 S_{sm} \nu_{sb}^{10}}{2^{19}\pi} \left(\frac{D}{1+z}\right)^2 \text{erg cm}^{-3}$$  \hspace{1cm} \text{[C.26]}

The electron energy density, $U_{elec}$, can be calculated by integrating $N(\gamma_e) \times \gamma_e m_e c^2$ over the electron distribution, i.e.

$$U_{elec} = \int_{\gamma_{min}}^{\gamma_{max}} K \gamma_e^{-p} m_e c^2 d\gamma \text{ erg cm}^{-3}$$  \hspace{1cm} \text{[C.27]}

where $K$ is given by equation [6.9]. The upper limit of integration, $\gamma_{max}$, can be estimated if we assume that radiation losses have significantly depleted the electron distribution above $\gamma_e(\nu_{eb})$ (using equation [C.23]); the lower limit, $\gamma_{min}$, is given by the lower limit on the electron distribution, $\gamma_{el}$ (see equation [6.1]), however this is generally not well determined observationally due to synchrotron self-absorption (although $1 \leq \gamma_{el} \leq \gamma_{sm}$). Assuming $\gamma_{min} = \gamma_{sm}$ (equation [C.24]), a conservative lower limit on $U_{elec}$ is given (for $\alpha \neq 0.5$) by

$$U_{elec}^{\text{min}} = K m_e c^2 \times \frac{\gamma_e(\nu_b)^{(1-2\alpha)} - \gamma_{sm}^{(1-2\alpha)}}{(1-2\alpha)}$$  \hspace{1cm} \text{[C.28]}

For $\alpha = 0.5$, the last term in equation [C.28] should be replaced by $\ln(\frac{\gamma_{max}}{\gamma_{sm}})$.

The co-moving synchrotron photon energy density, $\dot{U}_{phot}$, is given by

$$\dot{U}_{phot} = \frac{\dot{L}_{bol}}{\pi l^2} \times \frac{1}{c}$$  \hspace{1cm} \text{[C.29]}

where $\dot{L}_{bol}$ is the bolometric luminosity, i.e.

$$\dot{L}_{bol} = 4\pi D_L^2 \int_{\nu_{min}}^{\nu_{max}} \hat{S}(\nu_s) d\nu_s$$  \hspace{1cm} \text{[C.30]}

Thus, from the expression for $l$ (equation [C.22]), transforming to the observer's frame and using $S(\nu_s) = S_{sm}(\nu_s/\nu_{sm})^{-\alpha}$, it can be shown that

$$\dot{U}_{phot} = \frac{2\nu_d^4}{c} D^{-3} (1+z)^4 S_{sm} \theta_d^{-2}$$

$$\times \left[ \frac{\nu_{sm}}{(1-\alpha)} \left\{ \left(\frac{\nu_{sb}}{\nu_{sm}}\right)^{1-\alpha} - 1 \right\} \right] \text{erg cm}^{-3}$$  \hspace{1cm} \text{[C.31]}

For $\alpha = 1$, the last term in equation [C.31] should be replaced by $[\nu_{sm}^2 \ln(\nu_b/\nu_{sm})]$. 

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Synchrotron loss timescale, $t_s$

The synchrotron loss timescale is given by equation [C.21]. Transforming to the observer’s frame and rearranging we find

$$t_s(\nu_s) = \left( \frac{D}{(1+z)} \right)^{-1/2} 1.06 \times 10^{12} B^{-3/2} \nu_s^{-1/2} \ s \quad [C.32]$$

C-2.3 Derivations for the Inhomogeneous Jet Model

To the first order a jet can usefully be represented by a series of co-axial disks. A number of the following derivations therefore closely resemble those for a homogeneous disk as outlined above.

Self-absorption frequency, $\nu_{sm}(r)$

Since the emitting frame is moving relativistically with a kinematic Doppler factor $\mathcal{D}$ with respect to the geometry of the jet, the distances in the co-moving frame will appear Lorentz contracted by a factor $\mathcal{D}^{-1}$. Therefore, for a jet of opening angle $\phi$ and viewing angle $\theta$, the distance $l$ travelled by a photon through the jet along the line of sight if emitted a distance $r$ pc down the jet is given by

$$l = 2(3.086 \times 10^{18}) \phi \csc \theta \mathcal{D}^{-1} \ cm \quad [C.33]$$

Thus using this expression for $l$ and those for $k_s$, $B$ and $K$ (equations [C.11] and [6.14]), the self-absorption frequency, $\nu_{sm}(r)$, (where $\tau = 1$) is given by

$$\nu_{sm} = (1+z)^{-1} \left( T_{VII} K_1 B_1^{a+3/2} \right)^{1/(a+5/2)} \times r^{-k_m} \quad [6.16]$$

in the observer’s frame, where

$$T_{VII} = 6.2 \times 10^{18} D^{a+3/2} C_2(\alpha_e) \phi \csc \theta \quad [6.17]$$

$$k_m = \frac{n + m(\alpha_e + 3/2) - 1}{(\alpha_e + 5/2)} \quad [C.34]$$

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Synchrotron losses

Transforming equation [C.32] to the co-moving frame of the jet outflow, we find that the time taken, $t_s$, for an electron radiating at a frequency $\nu_s$ to lose half its initial energy is given by

$$t_s(\nu_s) = 1.06 \times 10^{12} \left( \frac{D}{1+z} \right)^{1/2} B^{-3/2}_\perp \nu^{-1/2}_s \text{ s} \quad [C.35]$$

The time taken (in the co-moving frame), $t_{\text{trav}}$, for a photon to travel a distance $r$ along the jet is given by

$$t_{\text{trav}} = 3.086 \times 10^{18} r \beta^{-1} c^{-1} \gamma_j^{-1} \text{ s} \quad [C.36]$$

Thus equating these two, we can derive an expression for the frequency, $\nu_{sb}$, at which synchrotron losses will steepen the observed spectrum from an index of $\alpha$ to $\alpha + 1/2$ as a function of $r$ assuming the continuous re-injection of fresh electrons. Substituting for $K$ and $B$ (eqn. [6.14]) we find

$$\nu_{sb}(r) = 1.1 \times 10^8 \left( \frac{D}{1+z} \right) \beta^2 B^{-3}_\perp \gamma_j^{-1} r^{k_b} \text{ Hz} \quad [C.37]$$

where

$$k_b = 3m - 2 \quad [6.20]$$

Maximum frequency, $\nu_{su}$

Using equation [C.3]) to find the peak emitted frequency, $\nu_{su}$, of the synchrotron radiation from an electron with the maximum lorentz factor, $\gamma_{eu}$, transforming to the observer's frame and substituting for $B$ (equation [6.14]) and $\gamma_{eu}$ (equation [6.15]), we find

$$\nu_{su}(r) = 1.22 \times 10^6 B_\perp \gamma_j^2 \left( \frac{D}{1+z} \right) r^{k_u} \text{ Hz} \quad [C.38]$$

where

$$k_u = 2g - m \quad [C.39]$$
The Fiducial $r_M$ and $\nu_s M$

The smallest radius, $r_M$, from which significant optically thin synchrotron emission with a spectral index $\alpha$ is observed can be obtained by setting $\nu_{sm}(r) = \nu_{sh}(r)$ (equations [6.16] and [C.37]) and solving for $r$. Hence

$$r_M = \left( T_V K_1 B_1^{(9+4\alpha)} \right)^{TV \phi PC[6.23]}$$

where

$$T_V = 6.2 \times 10^{18}(6.9 \times 10^{-7})^{-(2.5+\alpha)} C 2(\alpha) D^{-1}(\gamma_j \beta_j)^{-2(2.5+\alpha)} \phi \csc \theta \hspace{1cm} [C.40]$$

$$T_{VI} = 1/[(3m - 2)(2.5 + \alpha) + (1.5 + \alpha)m + n - 1] \hspace{1cm} [C.41]$$

Substituting $r_M$ (equation [6.23]) into the expression for $\nu_{sm}(r)$ (equation [6.16]) we find the self-absorption frequency at this radius is given by

$$\nu_s M = (1 + z)^{-1} \left[ T_{VI} K_1 - 1 B_1^{(\alpha+3/2)} \right]^{1/(\alpha+5/2)}$$

$$\times \left[ T_V K_1 B_1^{(9+4\alpha)} \right]^{-T_{VI} \nu m} \hspace{1cm} [6.24]$$

Observed flux densities

As in the case of the homogeneous disk model, the total observed flux at a given frequency can be found by integrating the local emissivity, $\epsilon_s(\nu_s)$, over the source volume. In the case of the specific jet model under consideration, both $\epsilon_s(\nu_s)$ and elemental volume element, $\pi(\nu)^2 dr$, are functions of $r$. Hence (c.f. equation [C.13])

$$S_s(\nu_s) = 3.086 \times 10^{18} \frac{(1 + z)^{1-\alpha} D^{2+\alpha}}{D_L^2} \int_{r_{\min}}^{r_{\max}} \epsilon_s(r) \pi(\nu)^2 dr$$

$$\text{erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \hspace{1cm} [C.43]$$

where $r_{\min}$ and $r_{\max}$ are the inner and outer portions of the jet which emits optically thin synchrotron radiation; both of which are functions of $\nu_s$ (see below).

In Region I, $\epsilon_s(\nu_s)$ is simply given by equation [C.7], hence substituting for $K$ and $B$ (equation [6.14]), equation [C.43] gives

$$S_{RI}(\nu_s) = 3.086 \times 10^{18} \frac{1}{4 D_L^2} (1 + z)^{1-\alpha} D^{2+\alpha} C 1(\alpha) K_1 B_1^{1+\alpha}$$

$$\phi^2 \nu_s^{-\alpha} \frac{r-k_s}{(-k_s)} \left[ \frac{r_{\max}}{r_{\min}} \right] \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \hspace{1cm} [C.44]$$
Table C.2 Constraints on $r_{\text{min}}$ and $r_{\text{max}}$

<table>
<thead>
<tr>
<th>Frequency Interval</th>
<th>$r_{\text{min}}$</th>
<th>Region I</th>
<th>$r_{\text{max}}$</th>
<th>Region II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_s \leq \nu_{\text{min}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\nu_{\text{min}} &lt; \nu_s \leq \nu_{M}$</td>
<td>$(\nu_s/\nu_{M})^{-1/k_m}$</td>
<td>$(\nu_s/X)^{1/k_s}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\nu_M &lt; \nu_s &lt; \nu_{M_2}$</td>
<td>$(\nu_s/\nu_{M_2})^{1/k_s}$</td>
<td>$(\nu_s/X)^{1/k_s}$</td>
<td>$(\nu_s/\nu_{M_2})^{-1/k_m}$</td>
<td>$(\nu_s/X)^{1/k_s}$</td>
</tr>
<tr>
<td>$\nu_{M_2} &lt; \nu_s &lt; \nu_{\text{max}}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\nu_{\text{max}} \leq \nu_s$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

where $X = 1.22 \times 10^6 (D/(1+z)) B_1 \gamma_1$

where

$$k_s = [3 - n - m(1 + \alpha)] \quad [C.45]$$

The behaviour of the limits $r_{\text{min}}$ and $r_{\text{max}}$ in the various frequency intervals can most easily be obtained from inspection of the frequency versus radius plane (e.g. Fig. 6.4) and are summarized in Table C.2.

In Region II, where the local spectral index is $\alpha + 0.5$, the local emissivity, $\epsilon_{II}(\nu_s)$, is given by

$$\epsilon_{II}(\nu_s) = \epsilon_s(\nu_{ab}) \left(\frac{\nu_s}{\nu_{ab}}\right)^{-(\alpha+1/2)} \text{erg s}^{-1} \text{cm}^{-3} \text{Hz}^{-1} \text{ster}^{-1} \quad [C.46]$$

thus substituting for $\nu_{ab}$ (equation [C.37]) we get

$$S_{RII}(\nu_s) = 2.564 \times 10^{22} \frac{1}{4D_L^2} (1 + z)^{1/2-\alpha} D^{5/2+\alpha} \times C 1(\alpha) \gamma j \beta j K_1 B_1^{1-1/2} \nu_s^{-(\alpha-1)/2} \times \left(\frac{r_{\text{max}}}{r_{\text{min}}}\right)^{-1} \text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \quad [C.47]$$

where the limits $r_{\text{min}}$ and $r_{\text{max}}$ are different to those in equation [C.44] and listed in Table C.2.
The integrated flux, $S_{sM}$, at $\nu_{sM}$

The observed integrated flux, $S_{sM}$, from Region I at the frequency $\nu_{sM}$ is given directly by equation [C.44] above with $r_{\text{min}} = r_M$ (equation [6.23]). Now, assuming $r_{\text{max}} \gg r_M$, the last term in equation [C.44] becomes $k_s^{-1}r_M^{-k_s}$, hence

$$S_{sM} = 3.086 \times 10^{18} \frac{1}{4D_L^2} (1 + z)^{1 - \alpha} \rho^2 \nu_{sM}^{-\alpha} \frac{r_M^{1-k_s}}{(-k_s)} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$$

[6.25]

The Coefficients $K_1$ and $B_1$

Solving equations [6.24] and [6.25] simultaneously for $K_1$ and $B_1$ we find

$$K_1 = T_{XIII}^{-1/T_{IX}}$$

[C.48]

$$B_1 = T_X K_1^{(1/k_s-T_{VI})/T_{IX}}$$

[C.49]

where

$$T_V = 6.2 \times 10^{18} (6.9 \times 10^7)^{-(\alpha+5/2)} C_2(\alpha) D^{-1} (\gamma \beta)^{-2(\alpha+5/2)} \rho \csc \theta$$

[C.50]

$$T_{VI} = 1/[(3m - 2)(\alpha + 5/2) + (\alpha + 3/2)m + n - 1]$$

[C.51]

$$T_{VII} = 6.2 \times 10^{18} C_2(\alpha) D^{\alpha+3/2} \rho \csc \theta$$

[C.52]

$$T_{VIII} = S_{sM} (3.333 \times 10^{-5}) \left[ \frac{C_1(0.50)}{C_2(\alpha)} \right] (1 + z)^{\alpha-1} D_L^2 D^{-2(\alpha+5/2)} \rho^{-2k_s}$$

[C.53]

$$T_{IX} = (9 + 4\alpha)T_{VI} - (1 + \alpha)/k_s$$

[C.54]

$$T_X = \frac{T_{V}^{-1/T_{IX}}}{(k_s T_{IX})^{\alpha/(k_s T_{IX})}} \nu_{sM}^{-\alpha/(k_s T_{IX})}$$

[C.55]

$$T_{XI} = (\alpha + 3/2)/((\alpha + 5/2) - (9 + 4\alpha) T_{VI} k_m$$

[C.56]

$$T_{XII} = \nu_{sM} (1 + z) T_{V}^{T_{VI} k_m T_{VIII}^{-1/(\alpha+5/2)}} T_X^{-T_{IX}}$$

[C.57]

$$T_{XIII} = 1/(\alpha + 5/2) - T_{VI} k_m + [(1/k_s - T_{VI}) T_{IX}]/T_{IX}$$

[C.58]
Energy Densities

The local energy densities in a jet model can be obtained directly from equations [C.25], [C.27] and [C.29] above, paying due consideration of the radial dependences of $B$, $K$ and $\gamma_{\text{max}}$. The mean energy densities ($\langle U_{\text{mag}} \rangle$ etc) referred to in Chapter 6 within a specific portion of the jet are defined as the total energy within the region divided by the volume.

C-3 Inverse-Compton Emission

So far we have considered the interaction of relativistic electrons with a magnetic field (in a process which is essentially the Compton scattering of the virtual photons of the static magnetic field) resulting in synchrotron photons. However, if there is in addition a significant radiation field in the vicinity of the electrons, then the electrons may also undergo Compton scattering with the photon field. Since the emitted synchrotron photons themselves constitute an ambient radiation field this process is termed synchrotron self-Compton (SSC) emission. In addition, since the electrons are of relatively high energy (relative to the energies attainable in the laboratory) the net transfer of energy is from the electrons to the photons. Hence the scattering is also referred to as the inverse-Compton process.

Of interest here therefore, is the interaction of a power-law distribution of electrons with a power-law distribution of photons. Even in the homogeneous disk geometry the inverse Compton calculations are fairly complex and require a number of simplifying assumptions. Firstly, since we are primarily interested in photon energies $< 10$ keV, we need only consider the problem in the Thomson limit in which the energy, $E_*$, of the incident photon in the rest frame of the electron is small compared to $m_e c^2$. In this case, the cross-section, $\sigma_T$, is independent of energy and given by

$$\sigma_T = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2 = 6.66 \times 10^{-25} \text{ cm}^2$$

[C.59]

Since the electrons are highly relativistic in the source rest frame with $\gamma_e \gg 1$, 

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an isotropic photon distribution becomes highly anisotropic in the electron rest frame. Thus in the electron rest frame, all scatterings can be considered to be 'head-on'. Although such an assumption does slightly distort the distribution of scattered photon directions, the effect will not be large (Jones 1968; Blumenthal & Gould 1970). Indeed Reynolds (1982b) has shown that the head-on approximation is reasonable even for highly anisotropic incident photon distributions in the source frame.

In addition, the calculation is greatly simplified since only a fraction of its total interaction energy is lost by the electron per collision. In the electron rest frame, the photon therefore bounces off the electron with a final energy approximately equal to its incident energy, but with a distribution in angle. Transforming back to the source frame, this distribution in angle results in a broad distribution in energies, $\mathcal{E}_c$, up to a maximum (in the Thomson limit) given by

$$\mathcal{E}_c \simeq 4\gamma_e^2\mathcal{E}_s$$  \[C.60\]

but favouring lower energies (c.f. Blumenthal & Gould 1970, equation [2.45]). Since the average boost is of the order of $\gamma_e^2$, many authors approximate the inverse-Compton scattering by a $\delta$-function boost of this value (e.g. Königl 1981). However, as pointed out by Band & Grindlay (1985), the probability of a photon being upscattered to a given frequency per log frequency interval is in fact fairly constant up to a boost $\sim \gamma_e^2$. The $\delta$-function approximation therefore yields an overestimate of the true inverse-Compton emissivity, $\dot{\mathcal{E}}_c(\nu_c)$, since the scattered photons are all assumed to possess close to the maximum possible energy. Nevertheless, Blumenthal & Gould (1970) have shown that the fractional overestimation is $\lesssim 2$ over a large part of the scattered spectrum for $0 \leq \alpha \leq 1$. Thus we consider such an approximation to be sufficiently accurate for the application discussed here.

Assuming no external sources of incident photons, the inverse-Compton emissivity is given by (c.f. Blumenthal & Gould 1970, equation [2.76])

$$\dot{\mathcal{E}}_c(\nu_c) = \frac{3}{8}\sigma_T K A(\alpha)\dot{\mathcal{E}}_s(\nu_s)(\nu_c/\nu_s)^{-\alpha}$$
where the integration is over the co-moving synchrotron spectrum,

\[ A(\alpha) = 2^{2(\alpha+1)} \frac{a^2 + 3\alpha + 4}{(\alpha + 1)(\alpha + 2)(\alpha + 3)} \]

\[ G(\alpha) = 2^2 \frac{4\alpha^2 + 16\alpha + 23}{(2\alpha + 5)^2(2\alpha + 7)(\alpha^2 + 3\alpha + 4)} \]

\[ \nu_{KN} = \frac{m_e c^2}{h} \]

and \( \epsilon_s \) is given by equation [C.7]. It can be seen that as is to be expected, \( \epsilon_s(\nu_c) \)

is proportional to the product of the electron and photon number densities. \( A(\alpha) \)

and \( G(\alpha) \) are given for several values of \( \alpha \) in Table C.1.

For \( \nu_c \ll \nu_{KN} \) (as is the case of interest here), the term including \( G(\alpha) \) in eqn C.61

is \( \ll 1 \) hence the integrand reduces to \( \dot{\nu}_s^{-1} \). Thus the integral contributes only the

'Compton-synchrotron logarithm' containing the ratio of the effective maximum and minimum frequencies of the synchrotron spectrum (see Gould 1979 for a more thorough discussion of this term).

Compton loss timescale, \( t_c \)

An electron of Lorentz factor \( \gamma_e \) undergoing successive inverse-Compton scatterings losses half its initial energy in a time

\[ t_c = 2.0 \times 10^7 \gamma_e^{-1} \dot{U}_{phot}^{-1} \]

[Rybicki & Lightman 1979] where \( \dot{U}_{phot} \) is the total energy density of the synchrotron photons (c.f. equation [C.20]).

C-3.1 Constraints on SSC models provided by the inverse-Compton spectrum

In the case of the disk model, equations [C.13], [C.61], [C.22] and [C.7] can be used to shown that the predicted SSC flux in the observer's frame, \( S_{SC}^{pred}(\nu_c) \), at
a frequency $\nu_c$ is given by

$$S_{sc}^{pred}(\nu_c) = 3\sigma_T 2^{\delta + 8\alpha} A(\alpha) C_1(\alpha)^{-\delta(3+2\alpha)} C_2(\alpha)^{(3+2\alpha)}$$

$$\times \ln(\nu_b/\nu_{sm}) \nu_c^{-\alpha} \theta_d^{-2(3+2\alpha)} \nu_{sm}^{2(2+\alpha)} \nu_{sm}^{-5+3\alpha}$$

$$\times D^{-2(2+\alpha)} (1+z)^{2(2+\alpha)}$$

$$\text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1} \quad [6.7]$$

between the limits $\nu_c^{\text{min}}$ and $\nu_c^{\text{max}}$, conservatively estimated (Urry 1984, Appendix C) to be $\gamma_{sm}^2 \nu_{sm}$ and $\gamma_{sm}^2 \nu_{sb}$. Therefore, as can be seen from equation [6.7], the Compton scattered spectrum is a power-law of the same spectral index as the incident synchrotron spectrum over a large range of frequency.

Equating $S_{sc}^{pred}(\nu_c)$ with the observed flux, $S_{sc}^{obs}(\nu_c)$, we can rearrange equation [6.7] to give the kinematic Doppler factor, $D$.

$$D = \left\{ 3\sigma_T 2^{\delta + 8\alpha} A(\alpha) \frac{C_2(\alpha)^{(2+2\alpha)}}{C_1(\alpha)^{(3+2\alpha)}} (1+z)^{2(2+\alpha)} \right.$$  

$$\times \ln(\nu_b/\nu_{sm}) \nu_c^{-\alpha} \theta_d^{-2(3+2\alpha)} \nu_{sm}^{2(2+\alpha)} \nu_{sm}^{-5+3\alpha} S_{sc}^{obs}(\nu_c)^{-1} \right\}^{\frac{1}{3+2\alpha}} \quad [6.10]$$

Clearly when we only have an upper limit on $S_{sc}^{obs}(\nu_c)$, only a lower limit on $D$ can be found. Additional constraints can be placed on $D$ from the requirement that $\nu_c$ lies between $\nu_c^{\text{min}}$ and $\nu_c^{\text{max}}$. From equation [C.24] it can be shown that

$$D \geq 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} (1+z) S_{sm} \nu_{sm}^{3/2} \nu_c^{-1/2} \theta_d^{-2} \quad [C.63]$$

and

$$D \leq 1.45 \times 10^{-2} \frac{C_2(\alpha)}{C_1(\alpha)} (1+z) S_{sm} \nu_{sm}^{2} \nu_{sb} \nu_c^{-1/2} \theta_d^{-2} \quad [C.64]$$

Equations [6.10], [C.63] and [C.64] are used in Chapter 6 to place constraints (a), (c) and (d) on $D$ respectively (Table 6.2 and Fig. 6.2).

In the case of an inhomogeneous jet model in which both the electron distribution and the magnetic field are functions of $r$, the observed inverse-Compton spectrum is the result of the co-addition of all the spectra from contributing radii. Using equations [C.13], [C.61] and [C.7], and assuming the photons travel a distance $l = 2r \phi D^{-1}$ on average before escaping from the jet, it can be shown that the
predicted SSC flux from Region I (in the observer’s frame) is given by

\[
S_c(\nu_c) = 3.08 \times 10^8 \frac{(1+z)^{1-\alpha}D^{2+\alpha}}{3.08 \times 10^{18} D_L^2} \frac{C_1(\alpha)K_1^2 B_1^{1+\alpha} A(\alpha) \phi^3}{\sigma T} \int_{r_{min}}^{r_{max}} \ln \left( \frac{\nu_{sb}}{\nu_{sm}} \right) r^{-k_c-1} \, dr \quad \text{erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} \quad [C.65]
\]

where

\[
k_c = k_s + n - 1 \quad [C.66]
\]

The limits \( r_{min} \) and \( r_{max} \), and the integrand itself are rather complex functions of \( r \). With the precondition that \( r \geq r_M \), \( r_{min} \) and \( r_{max} \) can be found assuming that the SSC spectrum at a given \( r \) extends over the range \( \gamma_{sm} \nu_{sm} \geq \nu_c \geq \gamma_{sm} \nu_{sb} \) (using equations [C.3], [6.14], [6.16], [C.37] and [C.44]). A similar expression can be derived for Region II. A full treatment taking into full consideration of the two component electron and photon distributions and their radial dependences is given by Hutter (1983). However, for the purposes of this work we have chosen to use the very much simpler expression of Königl (1981, equation [13]).
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Acknowledgements

Most of the work described in this thesis is as a result of data obtained using the EXOSAT and IUE satellites. I'd therefore firstly like to acknowledge my debt to the numerous people who made these missions such a success. Also, of course, to all those who contributed to the analysis software, in particular those involved in the creation of the EXOSAT analysis system (in all its many guises) in Leicester. Special thanks are due to Dick Willingale, Mike Watson and Gordon Stewart. Thanks also to Gordon Bromage for teaching me the wonders of IUEDR and DIPSO.

My work has undoubtedly benefitted from discussions with many other members of the XRA Group and Astronomy Dept., notably my supervisor Bob Warwick, for being my guide and mentor, and for fatherly advice on the Foreign Legion; Ian McHardy for taking me to America and actually believing BL Lacs are worth looking at; Ken Pounds for allowing me to pursue my research within the group and for not believing BL Lacs are worth looking at; and Derek Raine for believing in relativistic jets.

I also owe an enormous debt to my fellow students and friends: Tracey (Jane) Turner for being a cell-mate and confidant, help with The List and generally for being as bad at astronomy as myself (...and for copying all my clothes); Loz Breedon for taking even longer to write up than me, for her vicious wit and smelly sports kit; Tahir Yaqoob for endless discussions about (his) philosophy; Gordon Stewart for basically being a high energy astrophysicist and an excellent role model; Paul Nandra for being the Strange Science king and his unique home-brew; Richard Saxton for lots of pints of beer and Rees Williams for lots more. Thanks also to all the other members of the 2am Coffee Club (Alastair Edge, Andy Norton, Mark Jones ...) and to Mark E Smith, Ian Curtis, Morrissey and especially Robert Smith for supplying the music.

Finally, I would like to thank my family for their support and confidence in me, and Kerry for being well wacky.

I acknowledge receipt of SERC and DHSS studentships.
X-RAY AND ULTRA-VIOLET OBSERVATIONS OF BL LACERTAE TYPE OBJECTS

Ian Michael George

Abstract

We present the results from EXOSAT and IUE observations of five BL Lac type objects, Mrk 421, Mrk 501, 1218+304, Mrk 180 and 0414+009. We find that at least for the relatively well studied sources Mrk 421 and Mrk 501, the behaviour in the X-ray and ultra-violet differs significantly. In the majority of cases, the X-ray spectra are well represented by a simple power-law model over the 1-6 keV band. In the case of the high state observations of Mrk 421, however, a marked downward curvature of the spectrum is required above ~3 keV, which can be modelled as an exponential decrease. Similar convex spectra are also suggested, although not statistically required, in a number of other cases. We found no evidence in any of the observations for a hardening of the X-ray spectra above a few keV as has been previously reported. In all cases we find low energy cut-offs in the X-ray spectra consistent with absorption in the line-of-sight gas column density through out own galaxy implying that the intrinsic column density of cold gas in these objects is small (< 1 x 10^{20} cm^{-2}). All five sources exhibit significant X-ray variability with minimum variability timescales in the range ~3 hr to ~7 days. Furthermore, the X-ray flux and spectral index for each object appears to be correlated in the sense that the X-ray spectrum hardens as the source brightens. In contrast, the ultra-violet spectra are generally consistent with a simple power-law of index ~1, and exhibit only slow drifts in flux on a timescale ~weeks. The ultra-violet to X-ray continuum can therefore be modelled as a power-law of index ~1 below about 0.1 keV, above which the source steepens. The spectral variability in the X-ray band can then be described in terms of a ‘pivoting’ of the high-energy continuum about the break-point.

We have investigated two specific SSC models for the continuum emission. It was found that whilst a simplistic homogeneous disk-shaped emission region can provide an acceptable explanation of single epoch snap-shots of the multi-waveband spectrum, it experiences some difficulty accounting for the high energy spectral variability. In contrast, by allowing a variable radial dependence in the upper cut-off in the synchrotron emitting electron population in an inhomogeneous relativistic jet model, we have demonstrated that the multi-waveband spectrum of the most demanding source, Mrk 421, at all epochs can be reproduced.

Thus we conclude that a jet model, broadly following the current paradigm for BL Lac type objects can give an acceptable explanation of present multi-waveband measurements.