POSITION SENSING IN A MULTIWIRE PROPORTIONAL CHAMBER

by

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The work reported in this thesis divides naturally into two parts: first, a study of the effects of angular localisation of the avalanche in a multiwire proportional chamber, and of a drift region (such as is frequently employed in such chambers), carried out with a view to improving position linearity in the direction perpendicular to the anode wires; and second, a study of two position encoding techniques not previously employed in this type of detector. The second of these new techniques is considered to be of some significance in application to large-area detectors.

In view of this natural separation, the experimental work will be presented in two sections: Part II of the thesis deals with the drift field and angular localisation results, while Part III describes the two novel position encoding methods (the capacitance-resistance line and the graded-density cathode). To provide a background for these two accounts, Part I follows the development of the imaging multiwire proportional chamber, with particular reference to the design employed in this work; an account of the operation of a MWPC is presented, with an outline of the relevant theory; and the experimental apparatus available for these investigations is described. Part I thus provides a common reference point for both the experimental sections, which otherwise are largely independent, and contains information required in each. A single concluding chapter attempts to summarise the major findings of the present investigations and to point out possible areas where further work is required.

Finally, the two appendices, due to E. Mathieson, are theoretical descriptions of the operation of the CR line (see Chapter 8) and of the noise performance of a charge-sensitive preamplifier loaded by a shunt capacity and shunt resistance, and are of use throughout Part III.
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LIST OF PUBLICATIONS

A substantial number of the experimental results reported in this thesis have already appeared in the following papers. The corresponding Chapter of this thesis is given in each case.


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PART I:

HISTORICAL AND THEORETICAL ASPECTS
CHAPTER 1

X-RAY IMAGING AND THE MULTIWIREF PROPORTIONAL CHAMBER

i) Applications of X-ray Imaging

Technological discoveries made over the last thirty years have provided many uses and applications for X-ray and other ionising radiations, of great diversity and sophistication. Amongst the applications of x-rays, the most important are in the fields of medicine, biochemistry and astronomy, where the production of X-ray images is of importance. In particular, the multiwire proportional chamber (MWPC) is employed as a position-sensitive detector in each of these fields, illustrating its characteristic of great adaptability.

Medicine. The X-rays used in medical diagnostic techniques are typically of energies greater than \( \sim 20 \text{ keV} \), so that devices having solid X-ray absorbers are most efficient; probably the commonest type of detector is the Anger-type gamma camera \(^1\), which uses a sodium iodide scintillator to convert the X-rays into optical photons, for imaging with photomultiplier arrays.

Much "in vivo" imaging is at present carried out by administering to the patient a small dose of \( \gamma \)-emitting radionuclide, which is usually one which produces radiation of around 140 keV. In recent years, however, the use of \( ^{125}I \) has increased in popularity, because of its convenient half-life of about 60 days, and because many biological molecules may be easily labelled with it. The radiation produced by this isotope is of energy 27 keV, and it is quite feasible to image organs doped with it by means of a gaseous detector, such as the MPWC \(^2\). Another application of gaseous detectors is in the field of absorptiometry; Bateman and Connolly \(^3\) have described a MWPC, developed for bone mass measurement, with a spatial resolution (FWHM) of better than 1 mm at 42 keV.
The MWPC, then, is finding limited use in diagnostic medicine, either for direct imaging of hard X-rays, or for imaging higher-energy radiations or particles by means of some type of dense converter\textsuperscript{4}. In all fields, of course, an imaging detector is required to give good spatial resolution, but medical applications also call for ruggedness, longevity, simplicity, and, rather importantly as regards patient dosage, high efficiency. It is possible to construct MWPC systems with all of these features.

Biochemistry. In biochemical application, X-ray diffraction techniques are widely used for determining molecular arrangements and structures. The optimum X-ray energy for such work is close to 8 keV\textsuperscript{5}, which corresponds to the characteristic $K_a$ radiation of copper (frequently used as the target of X-ray generators), and which is also convenient for use with MWPCs. Faruqi\textsuperscript{5} has pointed out the advantages of the MWPC over film, and has more recently\textsuperscript{6} described a system designed for use with a synchrotron source. The MWPC of this system must be able to cope with counting rates of 5-10 MHz. Hendrix and Fürst\textsuperscript{7} have designed a detector of different geometry to cope with the high rates available from such a source, while Helliwell et al.\textsuperscript{8} describe a MWPC system currently being developed for use in small-angle scattering experiments on biological systems with the Daresbury Laboratory synchrotron source.

X-ray Astronomy. In the field of astronomy, the X-ray region of the electromagnetic spectrum has recently commanded much interest, because of the large number of sources discovered, and because of the light thrown on the subject of cosmology by observations of distant galaxies and the diffuse X-ray background. X-ray sources may be roughly sorted into two categories, from the point of view of imaging requirements: extended sources, such as clusters of galaxies and supernova remnants; and point-sources, whose positions need to be accurately known, for identification with optical and radio counterparts. MWPC's (of the class of device called Imaging Proportional
Counters, or IPC's, in the literature) are very well suited to observations of the former category because of their large active areas, and are moderately well suited to the latter, because of their good background rejection efficiency. Generally speaking, the detectors employed need not have great rate capability, and an energy range of 0.1 to 6 keV is quite adequate; the requirement for positional accuracy is however, quite stringent. The MWPC as an astronomical instrument will be further discussed later in this chapter.

ii) History and Development of the MWPC

This section will briefly describe the configuration of the multiwire proportional chamber, and will review some of the stages of its development into present forms.

Operation of the MWPC. In its basic form, the MWPC consists of a planar array of thin and parallel anode wires set between two planar cathodes, which may be solid, of mesh, or of parallel wires. The areas of these planes are usually equal, and are typically in the range of a few to a few thousand cm². Usually the anode plane lies halfway between the cathode planes, and the anode-cathode separation is not more than 1 cm or so. The three electrodes are enclosed in a gas envelope containing, typically, a noble gas, such as argon, and a simple molecular gas, such as carbon dioxide, the second gas serving to absorb ultraviolet photons, which can be detrimental to counter operation. As in a cylindrical (single-wire) proportional counter, a large potential difference exists between the anode and cathodes, so that when an ionising event occurs, the liberated electrons drift towards the anode; these acquire more energy as they approach the anode, and as the potential gradient increases more and more quickly, they reach a point where collision with gas atoms deposits enough energy to ionise them. The resultant process is cumulative, and an avalanche
develops which deposits a detectable charge (see figure 1). For a fuller description of the avalanche process and pulse development, see references 9 and 10; some further discussion will also be included in Chapter 2.

Historical Perspective. Although the MWPC had been in existence for some time, it was not until 1968 that its usefulness became recognised by a group at CERN. Each anode wire, they found, would behave as an independent proportional counter; and if a small solid-state amplifier were connected to each, the wire upon which the avalanche occurred could be identified. This was possible because the charge observed on the avalanche wire is of opposite polarity to the charge observed on all other anode wires, contrary to previous assumptions: evidently, it was imagined that a pulse of the same polarity would appear on all the anode wires, due to capacitive coupling, and Charpak suggests that this is a reason for the neglect of MWPC's up to 1968. Properties which recommended the MWPC for further study also included its good detection efficiency, its moderate

X-ray photon

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window

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absorption

cathode

diffusion

anode wires

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avalanche at wire

cathode

Figure 1 Schematic diagram illustrating the operation of a MWPC (not to scale).
energy resolution (useful for discrimination and background particle rejection), its good rate capability, a certain degree of geometrical adaptability, and the possibility of temporal resolution.

The chief rôle of the MWPC is as a position-sensitive device, and it was in the area of position encoding that much research took place. It was soon realised that the use of one amplifier per anode wire for position determination would be, in many cases, inconveniently complicated and expensive, and a number of other methods were examined by the CERN group. Separating the anode wires by inductances converts the electrode into a delay line having charge-injection points at each wire position, so that the time difference between the arrival of pulses at each end of the line is uniquely related to position; if, on the other hand, the wires are separated by resistances, pulse division occurs, and a ratio of pulse heights may be used to determine position. Both of these methods had previously been used with spark chambers and solid-state detectors (see, for example, references 15 and 16). Meanwhile, Borkowski and Kopp had successfully developed single-wire counters using an anode wire of high resistance to form an RC line for position detection, and employing timing methods of encoding, for which the relevant circuitry was simpler than for ratio methods.

Further work at CERN led to a description of field shapes in MWPC's and to a greater understanding of the avalanche process and pulse development; in particular, the usefulness of pulses induced on electrodes close to the avalanche anode was recognised. These pulses can have quite a large spatial extent (in a typical chamber, the induced charge FWHM is of the same order as the anode-cathode separation), and the centroid of the distribution corresponds to the location of the positive ions liberated during the avalanche. The major advantage of studying the cathode induced charges is that two-dimensional information then becomes available.

Other studies were made of efficiency, spatial resolution and gas
mixture. Surprising results were obtained with some electronegative gases; in particular, a certain mixture of argon, isobutane and freon-13-B1 (CF$_3$Br) was found to allow a greater increase in gain, before entry to the Gerger mode, than was possible with more conventional mixtures not containing the freon$^{20}$. This particular mixture acquired the name "magic gas", and much work has been carried out since which makes use of its properties.

The use of induced charges to determine position was soon explored in the work of Borkowski and Kopp$^{21}$, who used the charge induced on resistive cathodes in a further development of their risetime technique, and in the work of Charpak et al$^{22,23}$, who took advantage of the large spatial extent of the induced charge, in a centroid-finding sampling technique. The cathode wires were connected together in groups of five or six, each of which was connected to an amplifier; because the charge was induced on several groups, the sampling so achieved enabled the computation of the centroid to an accuracy not limited by the readout method. A prime advantage of this method is that to a first order, its accuracy does not depend on the size of the cathode concerned, a fact which is especially beneficial in the case of large-area detectors. Recently, Gatti et al$^{24}$ have shown how to determine the correct group (or strip) width for optimum resolution.

Position encoding techniques will be discussed in more detail in Part III, but some idea of the present state of development of MWPC's, and results gained from this development, may be obtained from the review articles by Charpak$^{25}$ and Charpak and Sauli$^{26}$. It will be seen that the "centroid" method described above has yet found no equal in accuracy, and has much aided understanding of MWPC operation; for instance, it has helped to show that an avalanche will often only partially surround the anode wire$^{22}$. The MWPC now has very great advantages for experimental use: besides the attractive features mentioned above, the experimenter now has choices of position readout method and gas filling which can be selected according to his requirements, not to mention the adaptations in which the MWPC is used.
in conjunction with other devices (some of these are mentioned later in this chapter). It is not surprising that the MWPC is so widely used.

**Drift Chambers.** No mention has yet been made in this short review section of a device very similar in design and application to the MWPC, namely the drift chamber, which also frequently appears in multiwire form. This is because this thesis does not deal with timing information or particle identification; however, one of the review articles just mentioned deals with the present state of development of such devices, indicating how closely the two are related.

iii) Position-Sensitive Detectors for X-ray Astronomy

Besides the MWPC, a number of other types of imaging detector have been developed for use in X-ray astronomy. Such detectors require good spectral (energy) resolution, with large active areas and wide dynamic ranges, though high rate capabilities are not a great necessity. The following description of the principles of some of these devices will be necessarily brief and incomplete, but will serve to give an idea of their characteristics. A comparison of some example performance figures, compiled from the given references, is shown in Table 1. The detectors discussed are designed for operation with soft X-rays (<10 keV), and Table 1 refers to resolutions and efficiencies at 1 keV.

<table>
<thead>
<tr>
<th>Device</th>
<th>Energy Resolution (FWHM)</th>
<th>Spatial Resolution $\mu$m (FWHM)</th>
<th>Useful area $\text{cm}^2$</th>
<th>Dynamic Range keV</th>
<th>Detection Efficiency %</th>
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<tr>
<td>MCP</td>
<td>Small area</td>
<td>$\infty$</td>
<td>25</td>
<td>0.01 - 6</td>
<td>&lt;10</td>
</tr>
<tr>
<td></td>
<td>High Resolution</td>
<td>250 eV</td>
<td>2</td>
<td>0.3 - 8</td>
<td>90</td>
</tr>
<tr>
<td>CCD</td>
<td></td>
<td>&lt;20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GSPC</td>
<td>Large area</td>
<td>&lt;20%</td>
<td>1100</td>
<td>1 - 10</td>
<td>85</td>
</tr>
<tr>
<td>PPPC</td>
<td>Moderate Resolution</td>
<td>$\sim$ 30%</td>
<td>300</td>
<td>0.1 - 4</td>
<td>90</td>
</tr>
<tr>
<td>MWPC</td>
<td>Resolution</td>
<td>$\sim$ 30%</td>
<td>300</td>
<td>0.1 - 4</td>
<td>90</td>
</tr>
</tbody>
</table>

Table 1. Example performance figures for various X-ray astronomy imaging devices (see text)
Two detectors which have been used in astronomical experiments to date are the parallel-plate proportional counter (PPPC) and the microchannel plate (MCP) camera. The other two devices discussed are the gas scintillation proportional counter (GSPC) and the charge-coupled device (CCD), a solid-state instrument, and are still under development. Figure 2 illustrates the principles of operation of these devices.

The Parallel-Plate Proportional Counter. The PPCP (figure 2a) works on the same principle as the MWPC, namely that of gas electron multiplication, except that in this case the uniform field between parallel plates is maintained at a high enough value for gas multiplication and avalanching to occur. The plane electrodes can be wire grids, with position readout as in a MWPC, or the lower one (the anode) can be a resistive sheet, furnishing two-dimensional readout. In general, the performance of such a detector is similar to that of a MWPC.

Microchannel Plates. The MCP (see, for example, references 29 and 30) is essentially a thin and circular piece of flat glass, with a honeycomb of very fine holes, or channels, passing through it. Each cylindrical channel is typically 12 μm in bore, and the separation of channel centres is usually about 15 μm. In operation, a potential difference exists across the faces of the plate, so that when an X-ray strikes the wall of a particular channel, this will act as a continuous-dynode photomultiplier. The electron cascade issuing from the back face of the plate is then located by, for example, a resistive sheet anode. The chief characteristic of this device is its very high spatial resolution (of the same order as the channel separation). It has, however, several disadvantages: firstly, its detection efficiency is low, due chiefly to the inefficiency of the photoelectric process in the glass; secondly, because of the statistics of the multiplication process, it possesses no energy discrimination; and thirdly, it is difficult to fabricate plates whose diameter exceeds about 6 cm, making these devices of limited area.
Figure 2 Principles of operation of (a) PPC; (b) MCP camera (two MCPs are shown, operating in tandem); (c) GSPC; and (d) CCD. Figures 2a-2c from references 28, 29 and 31 respectively; figure 2d is adapted from reference 33.
The Gas Scintillation Proportional Counter. This has been for some time a device which yields energy resolutions superior to those of ordinary proportional counters, and therefore has been a useful tool for spectroscopic work. Its superior resolution results from a statistically more accurate method of ion pair counting. Although the number of ion pairs produced in an X-ray absorption event is the same as for a proportional counter, this number is more accurately counted by photon multiplication than by electron multiplication, where fluctuations in gain introduce errors. Taylor et al.\textsuperscript{31} have developed an imaging system based on the design of Anger cameras, in an effort to adapt GSPC's for position sensing; in this, the photons produced in one scintillation activate seven photomultiplier tubes, and their centroid is calculated from the charge outputs of these. Although the energy resolution of this device improves on that of the MWPC and PPPC, its spatial resolution is, as yet, poorer, and it is also bulkier and heavier, requiring also more complex image decoding.

The Charge-Coupled Device. A newcomer to X-ray astronomy is the CCD\textsuperscript{32,33}, whose principle is best described with reference to figure 2(d). It is essentially a two-dimensional array of MOS capacitors, consisting of a large crystal of silicon, having a layer of silicon oxide, $\text{SiO}_2$, deposited on its surface; narrow strips of conductor (e.g. aluminium) are in turn deposited onto the $\text{SiO}_2$ layer. The crystal is also divided by "walls" of $\text{SiO}_2$ running perpendicular to the conducting strips. Every third metal electrode is connected together and the application of a voltage (e.g. 10 V) to one of the three sets produces a depletion layer in the crystal; so if an X-ray is absorbed in this depletion layer, the charge carriers thereby liberated will be confined within an area defined by the nearest electrodes at non-zero potential and the $\text{SiO}_2$ "walls". This area forms one pixel, and is usually of the order of 20 $\mu$m x 20 $\mu$m. The charge so stored is accumulated for -1 sec., and then, by manipulation of the electrode voltages, the carriers are physically transported to an on-chip amplifier at
one corner of the crystal, forming a train of output pulses. The location of a pulse along this train gives position information, and its size gives energy information. Because of the large numbers of charge carriers involved, the energy resolution can be very good indeed. The CCD is potentially the ultimate position-sensitive detector, with excellent energy and spatial resolution, its chief drawback being the present limitation on crystal size (2 - 3 cm$^2$). Processing of the data (typically from $10^5$ pixels) takes much computing time, and development is by no means complete in terms of optimisation of dopant concentration, operating temperature (low temperatures are required), and readout time ($10^5$ pixels take ~ 0.1 s to be read out).

iv) The Multiwire Proportional Chamber in X-ray Astronomy

From the above table it may be seen that none of the devices considered has a clear advantage which singles it out for use (given the present states of development), and for this reason, both the small-area/high resolution and large-area/moderate resolution types of device have been used for particular observations. The MWPC falls into the latter category, and is therefore useful as a survey instrument or for studying extended sources. It has the practical advantage over three of the other instruments that its development can be largely carried out "in house", whereas the MCP and CCD must be obtained commercially, and the GSPC requires extensive supporting research, as well as some sophisticated and expensive hardware. Recent developments will no doubt ensure the competitiveness of the MWPC for some time.

In a typical imaging payload, the position-sensitive detector is mounted at the focus of a grazing-incidence telescope (see reference 34, for example), so that an image forms on the sensitive area of the detector. The image is then coded, for transmission and subsequent decoding and
processing. An alternative to the grazing-incidence telescope is the coded mask\textsuperscript{35,36}, which essentially forms a many-pinhole camera: from a knowledge of the mask pattern and the shadow detected by the imaging device, the position of a point-source can be recovered by deconvolution. The method can be very complicated, but has the advantages, over the use of mirrors, of a larger field of view (several tens of degrees instead of less than ten) and of usefulness at higher energies (the effective area of a grazing-incidence telescope falls sharply above about 4 keV\textsuperscript{37}). The "shadow camera" is thus suitable for hard energy and sky survey work, though not for observations of extended objects.

The particular MWPC described and developed in this thesis was of identical design to the instruments employed in two recent sounding rocket flights\textsuperscript{38,39}, and an example of the images obtainable from such devices is the X-ray image of the Cygnus Loop\textsuperscript{38} (in the energy range 0.15 - 1 keV) shown in figure 3. A similar device has been operating successfully since November 1978 on the Einstein Observatory (HEAO-B) satellite\textsuperscript{34,40}.

As used on the rocket flights, then, the MWPC left room for development in several ways, for applications using both telescopes and coded masks. Firstly, a smaller absorption depth is required for use with high energies and large angles of incidence (coded mask operation); secondly, for survey purposes, an increase in active area is desirable; and thirdly, an improvement in position resolution is always desirable, especially in view of the construction of improved mirrors (see reference 37). Finally, the problem of position linearity in the direction perpendicular to the anode wires requires some study (see Chapters 2 and 6). The immediate development goal is to meet the IPC performance requirements for the forthcoming AXAF\textsuperscript{37} and LAMAR\textsuperscript{41} missions. Work in the first and second of the above areas is in progress at Leicester\textsuperscript{42,43}, while the present work concerns the third (which is associated with the second) and fourth areas.
Figure 3  X-ray image of the Cygnus Loop supernova remnant, in the energy range 0.15 - 1 keV, taken by the particular type of MWPC used in the present work.  From reference 38.
v) Two Recent Adaptations of the MWPC

To close this introduction, we shall examine a new development which promises to improve imaging capability for astronomical and other applications. This development is the recognition of the usefulness of non-ionising excitation of noble gas atoms. When such an excitation occurs, the uv photon arising from the subsequent de-excitation may have sufficient energy to ionise a second gas, such as ethanol.

Recently, two devices have been developed which make use of this effect. The first is the Multi-Step Avalanche Chamber (MSC)\textsuperscript{44}, which uses the effect to "transfer" electrons from a parallel-plate avalanche region to a drift region and thence to a MWPC, giving two stages of amplification (figure 4(a)). Other chamber configurations are also possible. The consequences of this ability to "transfer" electrons from one field region to another include the facility to gate and delay the electrons, allowing

![Diagram](image)

Figure 4  
(a) Example of a multistep avalanche chamber (MSC). The labelled regions are: absorption and drift (A & D), preamplification (PA), transfer or second drift (T) and the MWPC.  
(b) Example of a PIPS counter. Figures adapted from references 46(a) and 47(b).
high rates, while the high gain available is of obvious advantage for low-energy photons and particles. The instrument is more fully described in reference 44.

The second device\textsuperscript{45} has several names: its developers at CERN\textsuperscript{46} propose the name Photoionisation Proportional Scintillation Counter (PIPS). It consists of an absorption and re-emission region in which the X-ray is absorbed and a number of uv photons (whose energy is the appropriate excitation energy for the noble gas concerned) are emitted, separated by a uv-transparent window (e.g. LiF\textsubscript{2}) from a MWPC containing a gas which is readily ionised by the uv photons (figure 4(b)).

The latter device is another which combines the excellent energy resolution of the GSPC with an imaging facility. This configuration is certainly more compact than that of Taylor et al\textsuperscript{31}; however, it is probably too early to make any meaningful comparison of their capabilities. The PIPS instrument, if proven viable, may well succeed the plain MWPC in soft X-ray astronomy work\textsuperscript{47,48}.

The development of these two adaptations of the MWPC only serves to highlight the importance of further work on that instrument.
 CHAPTER 2
SOME DETAILS OF MWPC OPERATION

This chapter provides a summary of some of the more important operating characteristics of the MWPC, including its design, mode of operation, and behaviour. It is a general description, and may be applied to all planar MWPC geometries, although applications to soft X-ray detection will be given emphasis.

i) A General Description of the MWPC

Operation of a Proportional Counter. The proportional chamber is one of many devices which, in order to detect ionization, allow electrons to multiply and hence increase the charge collected up to a size which can be measured. The steps in the amplification process will now be briefly described.

An X-ray of energy $E_x$ entering the chamber will, within some distance, make an ionizing collision with a gas atom, ejecting a photoelectron of energy $E_p = E_x - E_B$, where $E_B$ is the binding energy for the particular atomic shell concerned. If $E_B$ is small, $E_p = E_x$; otherwise the remaining energy, $E_B$, is released when the atom de-excites, by emission of either an Auger electron or a fluorescent X-ray. Sometimes the latter is not re-absorbed within the chamber volume, in which case some of the X-ray energy escapes.

The X-ray energy has now been converted into the photoelectron kinetic energy, plus that of the Auger electron or a fluorescent photon. Each of these make further ionizing collisions until all their energy is transformed into the potential energy of the released electrons, which are punctually defined in a volume of typically less than $10^{-2}$ mm$^3$. The number of electrons in this cloud is proportional to the energy of the
incident X-ray. Proportionality will be preserved if the avalanche amplification, which subsequently occurs at an anode wire, is also proportional; in fact, it is now well established (see, for example, reference 49) that for voltages well below the Geiger threshold, the avalanche size is indeed proportional to the initial charge deposited. The amplification process itself is already well documented, and for a full discussion the reader should refer to the works by Rose and Korff\textsuperscript{10}, Wilkinson\textsuperscript{50} (Chapter 6) or Rice-Evans\textsuperscript{9} (Chapter 7). Following the avalanche, the liberated positive ions move away from the anode, inducing charges on all electrodes as they separate from the electrons, and are eventually collected at the cathode (usually in a time less than 100 $\mu$s after the avalanche).

The Gain of a MWPC. Besides the choice of counter gas and the anode potential, there are several other parameters which influence the gain of a MWPC. All of these describe its geometry, and are indicated in figure 5. Increasing $h$, the anode-cathode separation, or $r_a$, the anode wire radius, causes the gain $M$ to fall, while increasing $s$, the anode wire spacing or pitch, causes $M$ to increase. These variations can be predicted from expressions developed by Matheieson and Harris\textsuperscript{51} and given in Chapter 4.

The ranges of values of these geometric parameters are limited by both operational and constructional considerations. The anode wire radius, $r_a$, usually lies between 5 and 15 $\mu$m, where the wires are still strong but require only moderate voltages to produce an operating field; the value of $s$ is usually 1 or 2 mm, because smaller pitches require dangerously high voltages, and larger pitches degrade spatial resolution. The anode-cathode separation, $h$, is seldom less than 2 mm or more than 1 cm (note that the case of unsymmetric chambers, with unequal values of $h$, is not often of interest and will not be discussed here). For the cathodes, thick wires and small pitches are desirable in order to minimise the field around them, though the choices of diameter and pitch must not be such that
Figure 5  Various geometric parameters and operating regions of the MWPC. Discussion of the functions of the various regions and electrodes is included in the text.
the X-ray transparency of the cathode is seriously reduced; a typical combination is \( s_c = 1 \text{ mm} \) and \( r_c = 62.5 \text{ \textmu m} \). Wire cathodes are often used in preference to planar ones, principally because electrons may then be drifted through the cathode from an external absorption region (see below).

Despite these limitations, the MWPC remains an instrument of adaptable geometry: its area may be varied quite considerably, while in some applications the wire "planes" are curved (see reference 7, for example).

Gas Mixture. The gas mixture used in proportional counters almost always contains a noble gas, such as argon or xenon, because the mean energy required to produce one ion pair in such gases is low. This maximises the number of primary electrons released by an X-ray. Another common component of counting gas mixtures is the polyatomic "quenching" gas, which has two chief functions. The first is to prevent a large number of noble gas ions from colliding with the cathode wires, liberating some electrons and causing the counter to recycle. How this is effected is not clear, but it is thought that charge exchange takes place during ion transit, and that dissociation of the molecule, rather than electron ejection, occurs at the cathode. The second function is to absorb ultra-violet photons produced during the avalanche, which could otherwise eject electrons from the cathodes; again, the molecule is thought to dissociate. The quenching process is discussed by Rice-Evans (p.42), while ultra-violet photons will be mentioned again in section (iv).

The Drift and Absorption Region. The efficiency of a gaseous detector depends, of course, upon the gas density and the X-ray energy; but the thickness of absorber (the gas) is also important. A deep absorption region (see figure 5) is often used to improve efficiency, and can, in principle, be made as deep as necessary, although a more effective measure is simply to increase the gas pressure. In practice, however, the falling
efficiency of gaseous detectors with increasing X-ray energy provides an upper limit to their energy range, beyond which other detectors become more suitable.

The absorption region is also used to attempt to overcome one of the more serious drawbacks of MWPCs, namely their position non-linearity in the direction perpendicular to the anode wires. The fact that avalanches may only occur in specific positions (i.e. on a wire) results in there being only one avalanche position for a range of X-ray locations (see figure 6), quantising the output position in integer multiples of $s$.

One way of overcoming this problem is to allow the primary electron cloud to drift and diffuse in a low-field region, so that when it reaches the anode it is broad enough to cause avalanches on several wires, allowing an averaging effect to take place. The cost of this improvement is in position resolution, in both sensing directions, because the uncertainty in the

![Diagram of position signal quantisation](image)

Figure 6 Quantisation of position output signal by the anode wire spacing. Compare the cases of sensing along the anode wire direction (a), and perpendicular to it (b): in the latter case the position output is constant for a range of source positions.
location of the charge-cloud centroid is increased (see also section (iii)). This subject forms the central theme of Part II of this thesis, and, in particular, investigations concerning the drift region are reported in Chapter 6.

Another disadvantage of employing a deep absorption region is in the observation of an inclined X-ray beam: absorption events may occur over a range of positions (because the absorption depth varies), again producing an uncertainty in X-ray location, when averaged over many events. Such a consideration is important in X-ray astronomy, where use is often made of grazing-incidence mirrors and coded masks; however, the problem may be alleviated, for instance, by increasing the gas pressure.

**The Chamber Window.** A lower limit to the useful energy range of a MWPC is often provided by the X-ray window. This must be thick enough and strong enough to withstand a pressure differential and prevent leaks, and yet have a large transmission for the energies concerned. For most soft X-ray work, windows such as stretched polypropylene (~1 μm thick) coated with colloidal carbon can be used, with a lower energy limit of about 0.1 keV; however, recent interest in the extreme ultra-violet (XUV) region of the spectrum has led to the development of windows of thicknesses between 0.1 and 0.2 μm, which extend the energy range down to about 0.05 keV.

**Background Rejection.** One major source of error in X-ray astronomy is the background cosmic ray radiation, which deposits large amounts of ionization in a detector, usually along an inclined track (see figure 5), causing many spurious events to be registered in an X-ray image. A very effective way of eliminating this is to locate another anode plane (for which a larger pitch is adequate) a short distance below the active volume, so that if a particle passes through the active volume and into the second "coincidence" volume, avalanches will occur almost simultaneously on both anodes, and it will be possible to veto the signal from the primary anode.
The extra grid is called the anticoincidence anode.

**Readout.** Finally in this section, it must be pointed out how the information is extracted from a chamber. The changed induced on all electrodes are measured by means of a charge-sensitive preamplifier, which has the advantage (over the voltage-sensitive type) of an output which is independent of detector capacity. This is followed by a second stage of processing, such as a shaper amplifier or an analog-to-digital converter (ADC). The methods by which information is coded are discussed in Part III.

**ii) Induced Charges in Proportional Chambers**

The sections (ii) and (iii) show how it is possible to calculate the way in which the size and spatial extent of the charges induced on the electrodes of an MWPC develop with time, following an avalanche.

**Maxwell's Theorem.** The case of interest, namely of ions in a proportional chamber, can be represented by the general case of three conductors (figure 7(a)): we can say

\[
q_1 = c_{11}v_1 + c_{12}v_2 + c_{13}v_3 \\
q_2 = c_{21}v_1 + c_{22}v_2 + c_{23}v_3 \\
q_3 = c_{31}v_1 + c_{32}v_2 + c_{33}v_3
\]

where the \( c_{ij} \) represent the capacities existing between any pair of conductors, with \( c_{ij} = c_{ji} \) (a form of Green's reciprocation theorem: see reference 53, Art.85a), and the \( v_i \) are the potentials of the various conductors. Let us now consider two cases:

**Case (a):**

\[
v_1 = v_2 = 0 \quad \text{(electrodes 1, 2 earthed)}
\]

\[
q_3 = q_0
\]

Then we have:
Figure 7  (a) Three conductors, general case, for proof of Maxwell's theorem; (b) polarity of the output of a non-inverting amplifier, compared with that of the inducing charge; (c) single-wire (cylindrical) proportional counter.
\[ q_1 = c_{13} v_3 \quad \text{and} \quad q_o = c_{33} v_3, \]
so that
\[ q_1 = q_o \left( \frac{c_{13}}{c_{33}} \right) \]

**Case (b):** \[ v_1 = 1 \quad \text{(unit potential),} \quad v_2 = 0 \quad \text{and} \quad q_3 = 0. \]

Then:
\[ 0 = c_{31} + c_{33} v_3 \neq 0 \quad \text{or} \quad \frac{1}{-c_{33}} = \frac{c_{13}}{c_{33}}; \]
combining both results,
\[ q_1 = -v_3 q_o \quad (1) \]

Equation (1) embodies a theorem due to Maxwell (see Art. 86 of reference 53), which allows the calculation of induced charges in many-electrode systems. Provided that \( q_o \) is small (so that it causes negligible field distortion), the third conductor can represent an ion in a proportional chamber; a knowledge of \( v_3 \) (the potential at conductor 3) then allows calculation of the induced charge.

The negative sign shows that the charge induced on a particular electrode is of opposite polarity to the inducing charge. However, the output of an inverting amplifier connected to that electrode will be of opposite polarity to the inducing charge, as may be demonstrated with reference to figure 7(b). As the inducing ion charge \( q_o \) approaches the electrode A (which has a shunt capacity, \( C \), and a shunt resistance, \( R \), to ground), provided \( RC_T \) is very long in comparison with the ion drift time, the induced charge \( q_i \) can only be provided by leaving an equal and opposite charge on the capacity \( C_T \). The voltage across \( C_T \) is then \( -q_i/C_T \), and if the amplifier (charge-sensitive) has a conversion gain of \(-1/C_o\), its output is \( q_i/C_o \).

Note that in real cases ions will occur in pairs, so that an induced charge only becomes apparent as the ions move apart under the influence of the field.
Example: the SWPC. Consider a single-wire proportional chamber with its anode wire at potential \( V_a \), and suppose an avalanche occurs: the total charge generated is \( q_e \) (electrons) and \( q_p (-q_e) \) (positive ions). From equation (1), the charge induced on the anode wire is:

\[
q_i = -q_e \frac{v_e}{V_a} - q_p \frac{v_p}{V_a},
\]

so that the charge delivered to the preamplifier input (say \( q_a \)) is

\[
q_a = \frac{q_e}{V_a} (v_e - v_p) = -q_i
\]

Now in an avalanche, the ion pairs are produced close to the anode wire, so that the potential then experienced is almost \( V_a \). The electrons move quickly towards the wire, and therefore do not experience a significant change in potential; that is, \( v_e = V_a \) for all times \( t \), so that approximately

\[
q_a = q_e \left( 1 - \frac{v_p}{V_a} \right)
\]

The positive ions, on the other hand, move through almost the whole potential difference, so that it is the variation of \( v_p \) as the positive ions travel towards the cathode which provides the pulse in a proportional counter.

To examine the time-variation of \( q_a \), we need to know the potential experienced by the positive ions at any particular \( t \). This is found from field equations and a knowledge of positive ion mobility: we find the dependence of \( v_p \) on position, and use the mobility to find out how the location of the positive ions varies with time.

For a cylindrical counter (figure 7(c)), we know that

\[
E = \frac{2CV_a}{r},
\]

where \( r \) is the (radial) position of the positive ions and
\[
C = \frac{1}{\ln(r_c/r_a)^2} = \text{constant}; \text{ hence}
\]

\[
V(r) = v_a \left( 1 - C \ln(r/r_a)^2 \right)
\]  (3)

Now, for positive ions, the mobility is not a very strong function of field, so we can say

\[
\omega = \mu E
\]  (4)

where \( \mu \) and \( \omega \) are respectively the mobility and velocity of the positive ions. Hence, substituting for E,

\[
\omega = \frac{dr}{dt} = \frac{2C\mu v_a}{r}
\]

and, integrating from \( t = 0 \), \( r = r_a \) to a general \( t \) and \( r \), we find

\[
\left( \frac{r}{r_a} \right)^2 = 1 + t/t_o
\]  (5)

where \( t_o = r_a^2/4C\mu v_a \). Now that we have both the desired relationships, (3) and (5), we can combine them with equation (2) to find that:

\[
q_a = q_e C \ln(1 + t/t_o)
\]  (6)

This relation describes the time development of the anode induced charge in a SWPC (see figure 8). The development stops when the positive ions are collected at the cathode: by then \( q_a = q_e \), and we can say that, if \( t_c \) is the time of arrival of the positive ions,

\[
t_c = t_o \left( [r_c/r_a]^2 - 1 \right).
\]

### iii) Induced Charges in a MWPC

**Procedure.** We now wish to extend this method to examine the time-development of induced charges in MWPC's. The steps in the process will be the same: firstly, a derivation of expressions for the field and potential; secondly, a description of the motion of the positive ions in the
Figure 8  Time development of the induced charge at the anode of a typical SWPC.

Figure 9  MWPC, showing parameters used in calculations of induced charges. The terminology of references 54 and 55 has been adopted.
field; and thirdly, a combining of this knowledge with equation (1) above, to find the time-development of the induced charge. The case of the MWPC is more complicated than that of the SWPC, for in addition to the increased asymmetry of the electrode arrangement, the presence of the other anode wires must be considered. The analysis is that of Mathieson and Harris, and employs the concept of complex potential. For a fuller description than given here, these references should be consulted.

The parameters of interest are indicated in figure 9; the cathodes are assumed to be infinite planes, and the anode an infinite array of parallel wires. Because the field varies with the angle $\alpha$, any description of how the pulse develops will refer to one particular field line only. As it happens, the actual pulse development corresponds more closely to this case than to one in which positive ions leave the anode along all field lines, which would be more difficult to examine (see Chapters 4 and 5).

The use of the complex potential function $W$ allows description of both potential (from the real part of $W$) and field configuration (from the imaginary part). The following treatment firstly derives the induced charge on the anode and the field configuration in a MWPC; secondly, the time-development of the anode charge is derived; and thirdly, the induced charge on the cathode is examined. Some modification to this model is necessary to deal with the subject of lateral distribution of the induced charge.

A separate description of the field in a MWPC has been given by Erskine, who examines the effects of displaced wires and unsymmetrically-placed grids, but does not go on to calculate induced charges. Nonetheless, the agreement with the present treatment is satisfying.

**Charge Induced on the Anode.** Weber has considered the case of the geometry shown in figure 9, and arrives at an expression for the complex
potential function which, provided \( s \gg r_a \) and \( h \gg s \), may be written:

\[
W = -K \ln(\sin(\pi z/s)) + V_o
\]  

(7)

where \( K \) and \( V_o \) are real constants dependent upon the chamber geometry and electrode potentials, and \( z = x + jy \), where \( j = \sqrt{-1} \) (see figure 9). As an aside, it may be remarked that the limitation \( h \gg s \) is quite severe; according to Weber, \( h \gg s/2 \) is sufficient.

The potential \( P \) experienced by a positive ion (or localised ion cloud, of radius \( r_{ion} \ll s \)) is given by the real part of \( W \) when the cathodes are grounded and all the anode wires are at the same potential (taken to be unity, for convenience), which allows the use of equation (1). There are thus two constraints to be imposed on \( W \):

\[
W = 0 \quad \text{when} \quad y = h, \gg s
\]

and

\[
W = 1 \quad \text{when} \quad x^2 + y^2 = r_a^2, \quad \text{so that} \quad x, y \ll s.
\]

These two conditions may be applied to equation (7) to find simultaneous equations in \( K \) and \( V_o \):

\[
0 = -K (\pi h/s - \pi n^2) + V_o
\]

\[
0 = -K \ln(\pi r_a/s) + V_o.
\]

The values of \( K \) and \( V_o \) thus found can be substituted into equation (7), and, following some algebraic manipulation, the real part of \( W \) can be found to be:

\[
P = \frac{\pi h/s - \ln(2(\cosh 2\pi y/s - \cos 2\pi x/s))^{1/2}}{\pi h/s - \ln(2\pi r_a/s)}
\]

(8)

Note that this expression contains a term which is suitably periodic in \( x \), and one which indicates a fall in potential as the ion cloud leaves the anode plane. If the ion charge is \( q_o \), situated at \( (x,y) \), then the product \( q_o P \) gives the charge induced on the anode.
Field Line Equation. This may be found using a single condition, that the field line leaves the anode surface at an angle \( \alpha \). The imaginary part of \( W \) is then simply

\[
\theta = \tan^{-1}\left(\frac{r_a \sin \alpha}{r_a \cos \alpha}\right) = \alpha ,
\]

if \( z \) is expressed as \( z = re^{i\theta} \). If the sine term in equation (7) is expanded, it can be seen that the imaginary part of \( W \) can be written

\[
\tan \theta = \cot \frac{\pi x}{s} \tanh \frac{\pi y}{s} ,
\]

so that

\[
\tan \frac{\pi y}{s} = \tan \alpha \cdot \tan \frac{\pi x}{s} . \quad (9)
\]

This is the equation of a field line which leaves the anode wire at an angle \( \alpha \): as \( y \) increases, \( x \) approaches a limit of \( s/2 \) or less, depending on \( \alpha \). For the full range of \( \alpha \), the field lines will terminate on the cathode over a position range \( -s/2 \leq x \leq s/2 \). This observation is embodied by setting \( y \gg s \) in equation (9), so that

\[
\alpha = \frac{\pi}{2} \left(1 - 2x/s\right) \quad (10)
\]

which shows the linear relationship between \( x \) and \( \alpha \).

Figure 10 shows the field and equipotential configurations in a typical MWPC.

Time Development of the Induced Charge. We have now arrived at expressions which allow calculation of the induced anode charge for all possible locations of the positive ion cloud. The time-development of the induced charge is arrived at by finding the location of the ion cloud, as a function of time.

In order to do this, it is necessary to perform a numerical integration, starting from a set of given conditions. From a starting point at the surface of an anode wire, the position reached by the positive ions after a small time interval may be calculated if its velocity and direction
Figure 10  Field lines and equipotentials, in a MWPC having h/s = 2. The field lines are equally spaced in $\alpha$, at $\alpha = \pm 15^\circ, 45^\circ, 75^\circ, 105^\circ, 135^\circ$ and $165^\circ$, and the equipotentials are spaced at intervals of $\nu_a/11$. 
of motion are known. The direction of motion is given by:

$$\frac{dy}{dx} = \tan \alpha \cdot \frac{\cosh \frac{\pi y}{s}}{\cos \frac{\pi x}{s}}$$

(11)

which is obtained by differentiating (9); the velocity is found from equation (4):

$$\omega = \frac{1}{s} \left\{ \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 \right\} = \frac{\mu E}{s},$$

(12)

where s has been adopted as the unit of distance, for convenience in the numerical calculations. Equations (11) and (12) are solved simultaneously to give x and y following a short time interval, using specified initial values of x and y. A natural unit of time, $T_o = s^2/\nu_a$ (where $\nu_a$ is the anode potential), emerges from these calculations, and it is convenient to normalise all times to this quantity. The value of $E(x,y)$ appropriate to equation (12) is found by differentiating equation (8):

$$E = \frac{v_a}{\hbar} (1 - \frac{s}{\pi \hbar}) \ln (2\pi r_a/s)^{-1} \left( \frac{\cosh \frac{\pi y}{s} + \cos \frac{\pi x}{s}}{\cosh \frac{\pi y}{s} - \cos \frac{\pi x}{s}} \right)^{\frac{1}{2}}.$$

(13)

Having found the new coordinate $(x,y)$, the process can be repeated and the next position found. If P is calculated, using equation (8), at each position in this sequence, the value of the induced charge at each time can be found. An example result of such a calculation is shown in figure 11, for comparison with figure 8.

**Effective Cathode Radius.** During the early stages of pulse development, the charge induced on the anode behaves similarly to the same quantity in a SWPC. It is sometimes useful to express this by imagining the plane cathodes to be equivalent to a cylinder of radius $r_{eff}$, the "effective cathode radius". Equation (13) is

$$E = K \frac{v_a}{\hbar} \left( \frac{\cosh 2\pi y/s + \cos 2\pi x/s}{\cosh 2\pi y/s - \cos 2\pi x/s} \right)^{\frac{1}{2}}.$$

where

$$K = (1 - \frac{s}{\pi \hbar}) \ln(2\pi r_a/s)^{-1}.$$
Figure 11  Time development of the charge on the anode of a MWPC. The ordinate is given as a fraction of the total charge developed, and the abscissa is given in units of $T = s^2/\mu v$. Profiles correspond to $\alpha = \pm (n\pi/16)$ rad, where $n = 1, 3, 5, 7$. 
Near the anode wire, \( y \ll s/2\pi \) and \( x \ll s/2\pi \), so that (13) reduces to

\[
E = K \frac{v}{h} \frac{s}{\pi r},
\]

which is in the same form as for the SWPC. In fact we can rewrite this as

\[
E = \frac{2C_{\text{eff}} v}{r},
\]

where \( C_{\text{eff}} = \frac{1}{\ln(r_{\text{eff}}/r_a)^2} \), by analogy with the case of the SWPC. In other words,

\[
C_{\text{eff}} = \frac{1}{\ln(r_{\text{eff}}/r_a)^2} = \frac{K_s}{2\pi h},
\]

which enables us to find an expression for the effective cathode radius:

\[
R_{\text{eff}} = \frac{s}{2\pi} e^{\pi h/s}.
\]

**Charge Induced on a Cathode.** This is calculated in the same way as the anode induced charge; but in this case the slightly more complicated situation means that the expression

\[
W = -K \ln(\sin[vz/s]) - jE_0 z + V_0
\]

is a more suitable form for \( W \). This time, \( K, E_0 \) and \( V_0 \) have to be determined in order to find \( W \), and the boundary conditions are found by setting the potential of the anode and one cathode to zero, while the cathode of interest is raised to unit potential:

\[
0 = -K \ln\left(\frac{\pi r_a}{s}\right) + V_0 \quad \text{(when } x^2 + y^2 = r_a^2 \text{)}
\]
\[
0 = -K \left(\frac{\pi h}{s} - \ln 2\right) - E_0 h + V_0 \quad \text{(when } y = -h \text{)}
\]
\[
1 = -K \left(\frac{\pi h}{s} - \ln 2\right) + E_0 h + V_0 \quad \text{(when } y = h \text{)}.
\]

The first of these holds if \( E_0 r_a \ll V_0 \), a condition which can be shown to be equivalent to \( r_a/h \ll V_0 \), and which is therefore normally satisfied. Again, a certain amount of manipulation is required before arriving at
\[ P = \frac{y}{2h} - \frac{\ln(2\pi r_a/s) - \ln(2(\cosh 2\pi y/s - \cos 2\pi x/s))}{2(\pi h/s - \ln(2\pi r_a/s))} \]  \hspace{1cm} (16)

This expression can now be used to calculate the time development of the charge induced on the cathode (situated at \( y = +h \)), using the locations \((x,y)\) found by the process described earlier.

**Spatial Distribution of the Cathode Induced Charge.** The remainder of this section deals with the spatial distribution of the charge induced on the cathode, and how this develops with time; the simple model used so far is inadequate, and the plane cathode must be replaced by an array of parallel wires. The treatment given here is that of Mathieson\(^5\), in which the realistic assumption is made that the cathode of interest consists of infinite parallel wires, of radius \( r_c \) and separations \( s_c \) such that \( r_c < s_c \), while the other cathode is an infinite earthed plane. Once again, Maxwell's theorem (equation (1)) is used, but this time the charge induced on individual cathode wires is calculated. The potential function for this case is found by the following procedure.

First we imagine that \( n^{th} \) cathode wire and the other cathode exist alone; if a line charge is placed at this wire position, the potential at a point \((x,y)\) due to that charge is given by image theory as \( D_n M_n \), where

\[ M_n = \ln \left( \frac{(x - ns_c)^2 + (y + 3h)^2}{(x - ns_c)^2 + (y - h)^2} \right)^{\frac{1}{2}} \]

and \( D_n \) is to be determined from the boundary conditions. Similarly, if we imagine instead a line charge placed at an anode wire position, the potential at \((x,y)\) is \( C_k L_k \), where

\[ L_k = \ln \left( \frac{(x - ks)^2 + (y + 2h)^2}{(x - ks)^2 + y^2} \right)^{\frac{1}{2}} \]

and the \( C_k \) are found from the boundary conditions. The real situation involves \((2a + 1)\) cathode wires and \((2b + 1)\) anode wires, so that the overall potential is the superposition of potentials from all these line
charges:

\[ P = \sum_{n}^{a} D_{n} M_{n} + \sum_{k}^{b} C_{k} L_{k}, \quad (17) \]

where the \( D_{n} \) and \( C_{k} \) are determined by the boundary conditions that the wire surfaces are equipotentials. As before, this is carried out by setting equal to unity the potential of the conductor for which the induced charge is being calculated, and that of all others to zero. The following equations are then applied:

\[
P_{m} = \sum_{n}^{a} D_{n} M_{nm} + \sum_{k}^{b} C_{k} L_{km}
\]

for the cathode wire potentials, and

\[
P_{i} = \sum_{n}^{a} D_{n} M_{ni} + \sum_{k}^{b} C_{k} L_{ki}
\]

for the anode wire potentials. The matrix elements are given by:

\[
M_{nm} = \xi n \left\{ \frac{(m-n)^2 s_{c}^2 + (4h)^2}{(m-n)^2 s_{c}^2} \right\} \quad (n \neq m); \quad M_{mm} = \xi n \left( 4h/r_{c} \right)
\]

\[
L_{km} = \xi n \left\{ \frac{\left( m_{s} - k s \right)^2 + (3h)^2}{\left( m_{s} - k s \right)^2 + h^2} \right\}
\]

\[
M_{ni} = \xi n \left\{ \frac{(i-s - n s_{c})^2 + (3h)^2}{(i-s - n s_{c})^2 + h^2} \right\}
\]

and

\[
L_{ki} = \xi n \left\{ \frac{(i-k)^2 s_{c}^2 + (2h)^2}{(i-k)^2 s_{c}^2} \right\} \quad (i \neq k); \quad L_{ii} = \xi n \left( 2h/r_{a} \right)
\]

These \((2a + 2b + 2)^2\) elements may be arranged into a symmetric positive definite matrix, so that the \( D_{n} \) and \( C_{k} \) may be calculated by numerical methods. Again, having already determined the location \((x, y)\) of the positive ions, at a given time, the charge they induce on any particular wire may be found by use of equations (1) and (17). The charge distribution may then be exhibited by comparing the charge induced on all wires, or
iv) Some Limitations of the MWPC

This section looks at some of the characteristics of the MWPC which limit its usefulness: energy proportionality is briefly discussed and factors affecting position and energy resolutions are mentioned, but firstly a problem associated with measurement of the avalanche charge is described.

Avalanche Charge Measurement. The effect which prohibits the simple measurement of the charge produced in an avalanche is common to all devices in which multiplication occurs at a wire, and is a result of the long time taken for the positive ions to reach the cathodes: only when they do so has the full avalanche charge, \( q^* \), been acquired (a typical value for this time, \( t_c \), in a MWPC, is 60 \( \mu s \)). Although in X-ray astronomy event rates which could cause pulse pile-up are rare, other considerations, such as minimisation of electronic noise, have meant that shaper-amplifiers with time constants of only a few microseconds have been most often used, so that the shaping occurs during the early stages of the pulse development. This consideration means that the widely-used calibration procedure, of injecting a step-function of charge using an external capacitor and matching the heights of the shaped pulses, is invalid.

This particular problem has been examined, for the case of a SWPC, with unipolar shaping (single integration and single differentiation) by Mathieson and Charles. In this treatment, the preamplifier fall-time is considered to be infinite: in fact, a typical 1/e fall time is 50 \( \mu s \), longer than the shaping time constants considered. The known response of the filter to a step-function input is used to predict the charge needed to be applied, in proportional counter pulse form, to produce a particular response. It just so happens that the resultant calibration may be
expressed as a simple function of $T_a$, the shaping time constant:

$$q_o = N_o q_T \{N_T e C (a + b \log (T_a/t_o))\}^{-1}, \quad (18)$$

where:
- $q_o$ = avalanche charge
- $q_T$ = calibration (step) input
- $C$ = chamber capacity
- $e = 2.718$, $a = 0.087$, $b = 0.797$
- $N_o$ = system response to avalanche $q_o$
- $N_T$ = system response to calibration pulse

and $t_o$ is as defined for equation (5). This simple function gives a good approximation to the more precise numerical calculations; and it turns out that by replacing $C$ by $C_{eff}$, as defined by $r_{eff}$ (equation (14)), the good approximation of this simple relationship is maintained in the case of the MWPC. For example then, suppose we have a MWPC with $h = 6 \text{ mm}$, $s = 2 \text{ mm}$, $r_a = 7.5 \mu m$, and $v_a = 4 \text{ kV}$, filled with argon/10% methane ($\mu = 160 \text{ mm}^2 \text{ V}^{-1} \text{ s}^{-1}$), and $T_a = 1 \mu s$. If we then follow the common procedure of setting $N_T = N_o$ by adjusting $q_T$, equation (18) tells us that

$$q_o \approx 3.6 q_T$$

the avalanche charge is more than three times the test charge, and this is seen to be no small effect.

Proportionality. This subject has been briefly mentioned in section (i), but it will be discussed here with reference to a little-studied effect: the production of secondary avalanches, initiated by electrons removed from the cathodes or window by ultra-violet photons. Limitations on proportional operation, such as space-charge effects, are already documented, and will not be discussed here (see, for example, references 9 and 50).
Proportionality implies a linear increase of avalanche charge with \( n \), the number of primary electrons, which is itself proportional to X-ray energy. It is usually far more convenient, however, to vary avalanche size by changing the chamber voltage, a procedure which, besides allowing observation of effects which depend on X-ray energy, also exposes effects which are a function of gain. Strictly speaking, the latter type are not associated with proportionality, and one such effect is that described above (see reference 50, p.149), which will also be mentioned in later chapters.

Because the electron drift velocity in the argon/methane mixtures used in this work is quite high, the secondary avalanches can be thought of as occurring at the same time as the primary one, so that from the point of view of the processing electronics they are incorporated in the same event. This is an important consideration. If \( N \) is the total number of electrons in the primary avalanche, and secondary avalanches do not occur, then

\[
N = nM ,
\]

where \( n \) is the number of primary electrons and \( M \) is the gas gain.

Suppose now, however, that secondary avalanches do occur, and that the probability, per electron in the primary avalanche, of a secondary electron being ejected from a cathode is \( \gamma \). Then

\[
N = nM \left(1 + \gamma M + (\gamma M)^2 + (\gamma M)^3 + \ldots\right)
\]

or, if \( \gamma M < 1 \),

\[
N = nM \left(1 - \gamma M\right)^{-1} , \quad (19)
\]

because there is obviously a probability that secondary avalanches will initiate "tertiaries", and so on. Notice that now \( N \) is no longer proportional to \( M \), although it is still proportional to \( n \): energy proportionality is preserved.
The size of $\gamma$ must be quite small, for unless $\gamma M$ is less than unity, the avalanche will be self-regenerating. Thus, for stable operation we have $\gamma \leq 1/M$, and since this is achievable up to gains in excess of $3 \times 10^5$, it must be that $\gamma \leq 3 \times 10^{-6}$.

**Useful Dynamic Range.** It is pertinent at this point to list the factors which affect energy and position resolution in a MWPC. The two chief influences which determine the energy resolution are the statistical fluctuations in $n$, the number of primary electrons, and in $M$, the gas gain. For a MWPC, the minor influences include electron attachment, avalanche angular localisation (see Chapters 4 and 5), and non-uniformity in anode pitch or wire diameter. Energy and spatial resolutions also degrade for events close to the edge of the grids, where the field departs from its normal value, while secondary avalanche effects may also increase the fluctuations in $M$.

Factors affecting position (spatial) resolution will be discussed in more detail subsequently, but they may be briefly introduced here. One factor is the system electronic noise, which actually contributes to energy resolution as well, although it is usually negligible, in that case, in comparison with other factors. For position resolution, it is an important consideration, and depends upon the encoding method used (see Part III of this thesis). The number of primary electrons is also important: as well as affecting the system signal-to-noise ratio, its finiteness means that the position of the centroid cannot be statistically well defined. Furthermore, the spatial extent of the primary electron cloud also matters, in determining its centroid, and therefore the amount of diffusion undergone by the electrons is of importance. The initial extent of the cloud (before any diffusion occurs) depends upon the range of the initial photoelectron, which increases with the energy of the incident photon.
As previously mentioned, if the incoming X-ray beam is not parallel to the drift field direction, the distribution of absorption depths will be transformed into a distribution of lateral positions, and will thus also contribute to spatial uncertainty. Finally, it is to be noted that secondary avalanches may increase the measured uncertainty. Despite the small size of such avalanches, if they occur a large distance away from the primary one, they will have a large moment and may therefore displace the measured centroid (remember that the secondary and primary avalanches are processed together by the electronic system).

We are now in a position to assess the limits of the useful energy range of a MWPC. As the photon energy is increased, the counter will eventually operate non-proportionally, and energy information will be lost. Efficiency falls as the energy increases (this is most often the prime consideration), and position resolution deteriorates as the X-rays penetrate further and the photoelectron range increases. At the low-energy end of the range, the spatial resolution worsens as the signal-to-noise ratio falls and the "centroid jitter" increases, and the energy resolution degrades because the number of primary electrons falls. The useful dynamic range, then, is that over which all these considerations allow the experimental requirements to be met.
CHAPTER 3

THE 10 cm GAS CAMERA

This chapter describes the experimental apparatus used for the work reported in Parts II and III. The detector itself, as has already been pointed out, is a MWPC identical in design to two chambers employed in recent sounding rocket observations of supernova remnants\(^\text{38,39}\), and as such can be assumed to be of suitable mechanical structure for experiments of that type. The extreme flexibility of the general design has enabled a multitude of experiments to be carried out, and the choice of a gas flow system, with methane as one of the components of the gas filling, has allowed rearrangements of the internal components of the chamber to be made rapidly, with little waste of time. In addition, the variety of NIM modules available for the processing electronics has been sufficient for most of the experiments carried out.

1) Description of Chamber.

Figure 12 shows the basic configuration of the MWPC. This is almost exactly the same as that of the devices flown, except that for simplicity the electrodes used to define the drift field (drift electrodes) have been omitted. Indeed, many experiments were carried out with such a short drift region that drift electrodes were unnecessary.

The detector body was entirely of aluminium, and comprised a gas volume separated by walls of about 4 mm thickness from a preamplifier and biasing network housing. Contact was made to the grids by feedthroughs passing through isolating glass-to-metal seals of 6 kV tolerance, and having internal standoffs of about 7 mm length to reduce the possibility of breakdown or sparking. Short leads were soldered between the grids and the feedthroughs. Access to the preamplifier and biasing network
Figure 12  Basic MWPC design, as used on rocket flights. Key:
1 - preamplifier housing; 2 - standoffs; 3 - "O"-ring;
4 - gas ports; 5 - support pillars; 6 - grid frames;
7 - grid wires; 8 - grid substrates; 9 - glass-to-metal seals;
10 - lid, with slot for X-ray entry; 11 - window; 12 - spacers.
housing was possible from outside, allowing great flexibility in the use of the six available feedthroughs (in the original design, two had been used per cathode and one per anode); for instance, because the full two-dimensional capability was seldom used, one cathode was often "plain" - a simple grid, like the anodes, and requiring only one contact. The other feedthrough could then be used for another purpose.

The grids themselves were supported by four pillars, each comprising a central column of 4BA stainless steel studding, covered by a ceramic collar of 6 mm external diameter. These pillars were arranged at the corners of a 10.8 cm square, corresponding to locating holes in the grid frames, and vertical support was supplied by stacking some spacers, which also fitted accurately over the pillars, up to the desired height; the whole stack was retained by means of washers and nuts at the top of the pillars. A number of spacers of several thicknesses (6, 3 and 1 mm) were available, having external diameters of 12.5 mm and being composed of the same ceramic material (aluminous porcelain) as the collars. In addition, several "blank" grid frames, without wires or substrates and of 4 mm thickness, could be used as spacers. With this selection, it was possible to vary the location and spacing of the grids quite conveniently.

Several lids of differing configuration were available for this laboratory programme. They were each of aluminium and approximately 18 cm square, and were fitted with X-ray transmitting windows, some of which were recessed. Twelve 4BA screws secured the lid to the detector body, and a moderately gas-tight seal was achieved by means of a compressible O-ring (no more is necessary in a flow-type system using cheap gases). The thin polypropylene used as the window material for the rocket flights was replaced by the sturdier aluminium, principally because polypropylene windows are fragile and difficult to construct. The ability of polypropylene to transmit low energy X-rays was not required in this case,
because a source of comparatively high energy (1.5 keV) was used throughout the work. Two thicknesses of aluminium were used: 25 μm, and, later, 4 μm; and if the exposed window area was large, additional support could be provided by a stainless steel mesh in order to prevent "bowing" and consequent distortions of the field within the chamber. All lids had open window areas which allowed traversal over a wide range of position, and one was available which enabled the entire grid widths (9 or 9.5 cm) to be examined.

Two gas ports at the rear of the chamber provided an inlet and an outlet, via needle-valves and "Swagelok" fittings, and the gas leaving the chamber was bled into the atmosphere via an oil bottle, providing an approximate means of monitoring the flow rate.

ii) Electrodes

The wire grids were assembled on individual frames which located accurately on the chamber support pillars, and could be removed or replaced with ease. Figure 13 shows examples of an anode grid (a) and a cathode grid (b).

The basic configuration of all grids was the same: the square support frame, of "G10" fibreglass epoxy, was 4 mm thick, of internal width 10.1 cm and of external width 11.6 cm. Two sides of the frame had 9.5 cm long and 2 mm-deep recesses, into which were araldited the 1 mm-thick "substrates". To these the wires were attached, so that the wire planes lay 1 mm below the top of the frame and 3 mm above the bottom, affording a degree of protection. The substrates were 9.5 cm x 1.5 cm x 1 mm, and were composed of alumina; conducting "fingers" of a palladium/gold alloy, accurately spaced at 1 mm intervals, were deposited onto the substrates to allow grid wires and signal leads to be soldered onto them. For the anode substrates, all the fingers were connected together at one
Figure 13 Anode (2 mm pitch) and cathode (R = 50 kΩ) grids.
end and fed to a signal lead, but the cathode substrates differed in that a resistive strip, forming part of the RC line used for readout, was incorporated. These substrates were manufactured by EMI*, and have a 1% tolerance on the value of the resistance between any two fingers (this is achieved by finding the largest value of resistance and adjusting all the others to the same value by eroding the resistive material away, using a type of bead-blasting technique). The value of the total resistance of the strips was $50 \, \text{k}\ksi$, forming, in conjunction with the capacity of the grid to ground, an RC transmission line of time constant $(RC) \approx 0.15 \, \mu\text{s}$ (a $250 \, \text{k}\ksi$ strip was available for later work). The other ends of the cathode wires were attached to a substrate with isolated and individual fingers.

The grid wires themselves also differed between anodes and cathodes. The cathode wires were of a copper - (2%) beryllium alloy and were 125 \, \mu\text{m} in diameter, but the anode wires were of tungsten, plated with gold in order to facilitate soldering; these were 15 \, \mu\text{m} in diameter. The edge wires of the anode grids were of diameters which increased towards the edge, in order to prevent very strong fields and consequent discharge problems; for instance, the 2 \, \text{mm} pitch anode (which was used most often) had three edge wires on each side which were of 50, 125 and 250 \, \mu\text{m} diameters.

The wires were attached to the substrates by the following process. A spool of wire was played out, and the wire wound continuously around an array of vertical pins spaced at 1 or 2 \, \text{mm} intervals, so that a series of parallel lengths was formed. The pins at one end were mounted on horizontal pneumatic rams, to which a small amount of gas pressure (e.g. from a nitrogen bottle) could be applied to tension the wires. This tension was about 25 \, \text{g}, a value far in excess of the critical tension for

* EMI Microelectronics, Hayes, Middx.
prevention of electrostatic deflection, of $3.5 \times 10^6$. The frame, with substrates, was then raised from below the plane of wires on an adjustable table, so that the wires made contact with the appropriate fingers. The wires were soldered to the fingers using a low-temperature solder cream, the completed grid was released and excess wires removed with wire cutters. This process was carried out in this Department, and several anodes and cathodes had been produced in this way: the cathodes were all of $1 \text{ mm}$ pitch, while the anodes had pitches of $2 \text{ mm}$, $1 \text{ mm}$ or $9 \text{ mm}$ (for anticoincidence electrodes). The special cathodes discussed in Part III were fabricated in a different manner, which will be described there.

Drift electrodes were not employed frequently in this work, although their use in real applications is often quite important. In form they are thin square conductors which pass around the edges of the drift region, and are separated equally, both in potential and spatially. Their mention conveniently raises the subject of biasing, and a typical biasing network is shown in figure 14. In practice, the outer case of the chamber is

![Figure 14 Biasing network. Only the electrodes and drift field dividing resistors are housed within the chamber.](image)
grounded (0 V) and the cathodes held at a few hundred volts, so that the anode operating voltage is slightly higher than it would be if the cathodes were grounded; the potential division for the drift electrodes is then effected by a chain of resistors within the gas volume. Potential is delivered to the electrodes via the system of two large resistances and a large capacity as shown: the first resistor (from the power supply) ensures that no large current can be drawn, the large capacity "clamps" the grids at their d.c. voltages, and the second resistor prevents signal from being lost to ground through the capacitor. This network is situated outside the gas envelope.

iii) Accessories.

X-ray Source. For most of the work reported here, the chamber was examined by means of a traversable Al-K X-ray source, producing radiation of 1.5 keV energy. The source*, shown in figure 15, was of Henke design, with an aluminium target separated by 3 mm from a tungsten filament, and was operated at 1.9 kV. It was enclosed in a lightweight housing of aluminium and polystyrene, including a cylindrical enclosure for the lower part of the tube, and the exit window was 3 mm in diameter, consisting of 75 μm-thick beryllium. The cylindrical enclosure possessed a small gas port, so that when it was fitted into the source holder on the traversing table, helium could be infused into both, in order to improve transmission. The essential features of the source collimation are shown in figure 16: to the end of the brass source-holder was fixed a brass disc, with a long collimating slit (~6 mm x ~50 μm) at its centre. A second disc, with a similar slit, was attached to the end of the cylindrical part of the source housing (for some work only), therefore being about

*Supplied by Centronic(20th Century) Ltd., of Croydon.
4 cm from the source window. The total collimation achieved in this way thus comprised two 50 μm slits approximately 4 cm apart.

The brass source holder formed part of the traversing-table arrangement: it was firmly secured to a brass block through which a screw thread of constant 1 mm pitch passed, so that one revolution of the screw would move the source 1 mm. The ends of the screw were retained in a baseplate of aluminium which could be bolted to the detector body, and one of the ends was also fixed to a graduated dial which could be used to turn the screw by specified amounts. The smallest graduations were 1/40 of a revolution, or 25 μm, though such accuracy was seldom used. The assembled detector, with the source and traversing-table, is shown in figure 17.
Figure 17  The assembled detector, showing source, traversing table, preamplifiers and bubble bottle.
Preamplifiers. The same picture also shows two special low-noise preamplifiers used in the later stages of the work. They replaced those used in the rocket flights, adapted from a Pye design, which were also employed for some of the present work. Once again, the flexibility of the overall system is demonstrated, in that it was quite simple to replace the preamplifiers in this manner, and it should now be possible to manufacture preamplifiers which combine the small size of the flight models with the superior performance of the other types employed here. These other preamplifiers were the Canberra 2001A model, and an adaptation of the Ortec 109 PC. A useful way of characterising the noise performance of a preamplifier is by means of the "load line", which describes how the noise varies with load capacity. For small loads, a plot of these quantities is a straight line whose slope and intercept characterise the noise performance of the device; however it must be stressed that it gives no information about performance with respect to resistive load. Figure 18, then, shows the load lines for each of the three preamplifiers, for shaping time constants of 1 μs.

Counter Gases. The counter gases used here were almost exclusively argon-methane mixtures, though some attempts were also made to observe the performance of argon-carbon dioxide. Argon-methane mixtures have the great advantage of being "fast" gases: the positive ion mobility is high, so that the pulses develop quickly, and the electron mobility is high, so that electrons drifting through the gas have less time for attachment to other atoms. Probably for the latter reason, methane mixtures are also far less sensitive to impurities than others, allowing hygiene requirements to be relaxed, while the use of argon permits, in view of its cheapness, the use of the inefficient flow-type of system.
Figure 18  Load lines. Noise charge (electrons rms) as a function of load capacity (pF) for various preamplifiers, assuming zero resistive load and using processing time constants of 1 μs.
P - Pye design; O - Ortec 109 PC copy; c - Canberra 2001 A (each with bipolar shaping); u - Canberra 2001 A (with unipolar shaping).
The gases used for the flight detectors were Ar/75% CH₄ and Ar/50% CH₄; however, for this work the performance of widely-used "proportional" gas, "P10" (Ar/10% CH₄), was examined, as well as the 50% mixture ("P50"). Some experiments were also carried out with pure methane. All gases were of CP grade purity.

One reason for wishing to examine P10 is that argon has a larger X-ray absorption coefficient than methane, and that a more argon-rich mixture should therefore improve detection efficiency; another is that electron diffusion is greater in argon, a fact which, it was hoped, would improve linearity in the sensing direction perpendicular to the anode wires.

Table 2 gives values of the linear absorption coefficient, $\lambda$, for 1.5 keV radiation, and $D/\mu$ values for the three gases P10, P50 and CH₄ ($D$ is the lateral diffusion coefficient and $\mu$ is the electron mobility). A disadvantage of reducing the methane concentration in the mixture, as will be seen, is that the efficiency of absorption of ultra-violet photons is reduced, to the detriment of position and energy resolutions.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\lambda$ (mm⁻¹)</th>
<th>$D/\mu$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10</td>
<td>0.189</td>
<td>0.240</td>
</tr>
<tr>
<td>P50</td>
<td>0.122</td>
<td>0.067</td>
</tr>
<tr>
<td>CH₄</td>
<td>0.038</td>
<td>0.057</td>
</tr>
</tbody>
</table>

Table 2. Comparison of absorption coefficients and $D/\mu$ values for the three gases chiefly used in the present work. $\lambda$=mean absorption coefficient for 1.5 keV X-rays; $D$=lateral diffusion coefficient in cm².s⁻¹; and $\mu$= electron mobility of cm².V⁻¹.s⁻¹. The $D/\mu$ values are for a reduced field value of 0.22 V.cm⁻¹.torr⁻¹. Data from references 62 and 63.
iv) The Electronic System

Processing Units. The preamplifier outputs were processed, for much of the work, by an analog system based on a wide variety of NIM modules which allowed great flexibility of experimental technique. The power supplies delivering the high voltages to the chamber electrodes were also of NIM type, with reversible polarity and ranges of 0 to 5 or 6 kV, and a particularly stable unit (Nuclear Enterprises NE 4519, 0-6 kV) was used to supply the anode.

Several tail-pulse generators were available for examining the processing electronics (including a unit with adjustable pulse rise and fall times, the BNC model BH-1), while externally-triggerable square pulse generators, such as the Marconi TF2010, could be used to provide interfaces in the post-amplification stages of the processing. A high-quality fast-sweep oscilloscope (Tektronix 7704) was used throughout the work, and different multi-channel analysers (MCAs) were employed at different stages, the most often-used being the Camberra Series 30 model.

The individual NIM units were either the manufactured model or built by the electronics workshop in this Department. "Home-built" shaping amplifiers, with gains adjustable by factors of 1 up to 320, time constants of 0.2, 0.5, 1, 2, 5 or 10 μs, unipolar or bipolar shaping, and reversible polarity were used throughout the work, though a pair of Ortec 471 spectroscopy amplifiers, of higher quality, were used additionally for precision work. These had a greater range of gain, but only three available time constants (0.5, 1 and 2 μs). For the remainder of the processing, several Ortec single-channel analysers (SCAs), two dual zero-cross (crossover) detectors with delays, linear sample-and-hold units ("stretchers"), a pulse divider, pulse height summing units, time-to-amplitude converters (TACs), a coincidence unit, a digital counter and a timer-counter could be used in a variety of different arrangements, to
suit the experiments.

For later work, some digital equipment was loaned by AERE Harwell (the Harwell 6000 series models). All the post-preamplifier processing could then be carried out in a Harwell 3769 Position-Sensing Detector System which included amplifiers, analog-to-digital converters (ADCs), and a digital divider, whose output could then be fed to a 3687 MCA.

**Position Decoding.** Two position-decoding schemes were used: rise-time comparison, based on the method of Borkowski and Kopp\(^{17}\), as used on the rocket flights, and the pulse height ratio method. Block diagrams of the processing required for these methods are shown respectively in figures 19 and 20. In the risetime technique, the pulse is given in a bipolar shape by the amplifier (single integration and double differentiation), and a standard pulse is output from the crossover detector (COD) when the bipolar signal crosses zero. Pulses from the two CODs provide the "start" and "stop" inputs for the TAC, and a delay is incorporated in one signal line to ensure that the "start" signal never arrives later than the "stop". In the ratio method, unipolar shaping (single integration and single differentiation) is used, for its better signal-to-noise characteristics. Because pulses to be compared do not necessarily peak at the same time, a stretcher is employed to sense the peak value of the unipolar pulse and hold it until such time as the division has been accomplished. The position signals are the outputs of the TAC (the height of this output is proportional to the time difference of the inputs), or the pulse divider. Because only one sensing axis was under study at any one time, it was preferred to feed the position signal into a MCA, rather than a non-quantitative but two-dimensionally imaging device, such as a storage oscilloscope.

Eccentricities of certain units caused problems from time to time, usually affecting position resolution in some way. For instance, the most inconvenient effect was what was termed the "denominator-dependence" of
Figure 19  Position decoding by the risetime technique, for one dimension. Key: PA - preamplifier; SA - shaping amplifier; COD - crossover detector; TAC - time-to-amplitude converter; PHA - pulse height analyser.

Figure 20  Position decoding by pulse ratio (analog method). Key as for figure 19; S & H - sample and hold units. Digital division is carried out with summation before shaping, with simultaneous peak sampling and digitisation, and with an 11-bit digital divider.
the analog divider, which, for a fixed-ratio input, showed a variation of output with input pulse height. The effect was minimised by limiting the inputs to the 4-10 V range. Other problems included the inequality of the time constants of the home-made amplifiers (overcome by replacing them with the Ortec 471s), and the necessity of maintaining zero d.c. offset on all units.
PART II:
SOME PHYSICAL ASPECTS OF
MWPC OPERATION
CHAPTER 4
ANGULAR LOCALISATION OF THE AVALANCHE

i) Position Sensing Perpendicular to the Anode Wires

As has already been described in Chapter 2, one of the major drawbacks of using a MWPC for two-dimensional imaging is the intrinsic limitation to resolution and linearity caused by the finite anode wire pitch. Without steps being taken to alleviate this, a two-dimensional image will appear as a series of parallel lines, corresponding to the anode wires, while a one-dimensional image will appear as a series of peaks (figure 21). (In order to conform to published conventions, the axis perpendicular to the anode wires will be referred to in subsequent discussions as the x-axis, and that parallel to the wires will be the y-axis.)

By far the most popular method of alleviating the problem has been by use of a drift region\(^{30,64}\), initially employed in order to improve detection efficiency for X-rays, but having the additional advantage of causing the primary charge cloud to become so extended as to produce avalanches on more than one wire, so allowing a statistical "averaging" of the position. The disadvantage of this method lies in the fact that position resolution in both sensing directions is sacrificed: in the y-axis, because of the greater extent of the primary cloud by the time it reaches the anode, and in the x-axis, for the same reason, and also because of the presence of a partition effect (the inaccuracy in the averaging, introduced by the finiteness of the number of primary electrons). It is therefore desirable to find an alternative means of interpolating the x-axis position to an accuracy better than \(s\), the wire spacing, although it must be said that because an absorption region may still be necessary to improve efficiency, any new method may only reduce these effects, rather than eliminate them.
Figure 21  Quantization of x-axis position output by the finite anode wire pitch. Detector response to uniform irradiation, with negligible diffusion occurring: the peaks correspond to the wire positions.
One effect which promised to resolve this difficulty was that of angular localisation of the avalanche. In the early stages of work on MWPCs, a misapprehension concerning induced charge polarity led to the erroneous conclusion that an avalanche would be essentially symmetric about the anode wire, a result which was surprising even at that stage, in view of the fact that under proportional operating conditions, avalanching commences only a few wire radii away from the wire surface. Subsequent investigations, however, refuted this idea. During work at CERN on charges induced on electrodes placed near an anode wire, a clear distinction was observed between pulse ratios obtained for source positions to the left and right of the wire, while a comparison of currents induced on the lower and upper cathodes of a MWPC led Borkowski and Kopp to a similar result: namely, that the proportional chamber avalanche exhibits an asymmetry. The extent of this asymmetry remained to be seen, but the deduction could be made that if the avalanche were confined to a range of angles \( \alpha \) (see figure 9, Chapter 2), the liberated positive ions would be confined to the same angles when leaving the wire. Making the obvious remark that the most intense part of the avalanche must occur at the angle at which the greatest number of electrons arrived, it can be seen that the positive ions pass along the same field lines on leaving the anode as the electrons did when approaching. How well the initial distribution of electrons is reflected in the final distribution of the positive ions depends on how much extra spreading occurs during the avalanche.

The preservation of information in this way suggested that the effect could be exploited, as an alternative means to deliberate diffusion of the electron cloud, in order to overcome the problem described above. Two questions, then, arose: firstly, could position sensitivity due to this effect be observed, and secondly, what was the magnitude of the effect (i.e. how well was the avalanche localised)?
ii) The Extent of Localisation

Following the initial experimental indications of avalanche asymmetry, the charges induced on various electrodes were examined in several different ways, with a view to answering the two questions posed above. Workers at Brookhaven\textsuperscript{66,67} further demonstrated avalanche localisation, expressing its magnitude as a ratio of pulses induced on sense wires parallel to the avalanche wire, but did not convert this ratio to a measurement of $\sigma_{\alpha}$, the angular standard deviation of the avalanche spread. It was, however, quite clear that the avalanche did not appreciably spread further than halfway around the wire. Using a cylindrical arrangement of sense wires, the mean angle, $\bar{\alpha}$, at which the avalanche occurred was recoverable. In separate work, Charpak et al.\textsuperscript{68} used both the left-right and "up-down" effects by comparing the ratio of the signals from the two plane cathodes of a MWPC with that of the signals from the two anode wires, immediately adjacent to the avalanche wire, again providing a measurement of $\bar{\alpha}$. Using the latter ratio to define $x$-position, the resolution obtainable was better than 150 $\mu$m rms (for $s = 2$ mm); and on the subject of avalanche spread, an estimate of $\sigma_{\alpha}$ was made from diffusion considerations only. This treatment yielded $\sigma_{\alpha} = 30^\circ$, and a subsequent measurement\textsuperscript{69}, using strips arranged in a cylinder around a single anode wire, gave a distribution of rms spread $\sigma_{\alpha} = 36^\circ$, indicating that diffusion was the major contributor to the spread in $\alpha$. The distribution found here was markedly non-gaussian (FWHM = 1.6 $\sigma$), but subsequent measurements made at Brookhaven\textsuperscript{70} indicated that the distribution was in fact quite close to gaussian. The latter work (using, again, a cylindrical counter with segmented cathodes) gave a FWHM in $\alpha$ of less than 100$^\circ$ (i.e. $\sigma_{\alpha} < 42^\circ$, if a gaussian distribution is assumed), and also examined the processes contributing to the spread of the avalanche. In agreement with the results of Charpak et al., diffusion was found to be the chief factor in
the proportional region, while it was also discovered that varying the concentration of quench gas affected $\sigma_\alpha$ when operating in the semi-proportional mode. The latter observation presumably indicates that uv photons are responsible for the increasing spread as charge level increases (but note that this was not considered the major factor when examining with charged particles). In comparing estimates of $\sigma_\alpha$, then, note must be taken of the operating gain and gas mixture. Another point to appreciate is that this uv photon effect is different from the secondary avalanches previously mentioned: here some photons are re-absorbed in the gas, close to the avalanche, and initiate (smaller) avalanches at points further around the wire. It must lastly be mentioned that Breskin et al.\textsuperscript{71}, during work on angular localisation in a drift chamber, confirmed that the induced currents on various electrodes may have different polarities (compare the results of Borkowski and Kopp\textsuperscript{65}).

Independent investigations at Leicester\textsuperscript{54,72}, again employing two wires parallel to the avalanche wire, confirmed that position sensitivity due to angular localisation was available. The position resolution obtained was 260 $\mu$m FWHM (110 $\mu$m rms), in agreement with the result of Charpak et al. quoted above, while confirmation was also obtained of the increase of spread with avalanche size (though this increase was slow, the avalanche remaining well localised even up to seriously non-proportional gains). Theoretical predictions, utilising the methods described in Chapter 2 for calculating induced charges, showed that the avalanche must be localised to a very high degree, the angular spread being taken into account in the following way. The calculations just mentioned are strictly applicable to a single path, i.e., a single value of $\alpha$, but by assuming a (gaussian) distribution of the avalanche in $\alpha$, weighting several trajectories accordingly and summing the individual effects, it was possible to extend them to take into account a specific distribution. The model was very insensitive to the radial distribution of the positive ions.
Experimental results for comparison with this model were obtained as the ratio of the charges induced on the cathodes of a MWPC, as a function of distance from the anode wire: this ratio was a maximum with the source over a wire, and a minimum with the source halfway between wires (provided that the cathode closest to the absorption event is used as the numerator). Excellent general agreement between experiment and theory was obtained, and in particular the experimental results fitted exactly the predictions for $\sigma = 33^\circ 70$; this time, however, it was thought that the extent of the primary electron cloud could only account for $18^\circ$ of this.

Thus, angular localisation had been shown to be capable of providing position sensitivity in MWPCs, although its effect in this respect had only been studied using rather specialised techniques, and it remained to be seen whether standard readout methods could be used to exploit it. The chief factors determining the angular spread of the avalanche were the charge level and the extent of the primary electron cloud (and hence the choice of gas filling), and it seemed that the latter could account for a large part of the observed spread. The value of this spread was experimentally determined to be between $30^\circ$ and $45^\circ$ (rms).

iii) Application of Angular Localisation

As noted above, angular localisation was yet to be examined in standard systems. It was hoped that the sensitivity observed already could be applied to all readout systems by use of the correct methods; for instance, as the positive ions leave the anode wire, they reflect more and more the original position of the absorption event, and therefore, as demonstrated by Harris and Mathieson$^{54}$, increasing the processing time constant enhances the effect of the localisation. Ultimately, then, the original absorption location should be completely recoverable. The work reported in Chapter 5 is directed to such an end, in connection with the use of the 10 cm gas
camera RC line readout, but before that is described, a few observations need to be made.

Firstly, it is quite possible to recover completely the value of $x$, the position of an incident X-ray, by simple calibration of source position vs. position output signal, if a moderate amount of angular localisation exists (see figure 22(a)): to each position there is a different position signal. To include such a procedure in the computer deconvolution of X-ray image data, however, would make an already complex process more difficult and time-consuming, and it is therefore desirable to render this sort of calibration unnecessary.

Another justification for attempting to exploit angular localisation for position sensing concerns spatial resolution at low signal charges, where the electronic contribution is important. The electronic contribution to spatial resolution, unlike other factors, is dependent upon the local sensitivity, because the corresponding fluctuation on the position output signal $Q$ is constant for a given system, irrespective of the range of $Q$. If the sensitivity is low, a given fluctuation in $Q$ produces a greater uncertainty in $x$, as demonstrated in figure 22(b).

Note that only perfect localisation ($\sigma_\alpha = 0^\circ$) can produce a sensitivity in the $x$-direction equal to that in the $y$; however, it is worthwhile to examine how the effects of the present localisation ($\sigma_\alpha = 35^\circ$) can be enhanced.

A noiseless readout would not suffer from degraded resolution in the manner just described, of course, and this is demonstrated by Charpak et al. 26,74, whose readout technique (see Chapter 7) provides such low noise that the resolution is little affected (the rms value is about 200 $\mu$m for $s = 2.54$ mm, comparing with 60 $\mu$m in the $y$-direction). Position recovery is achieved by computer deconvolution, and an excellent example is shown in figure 23.
Figure 22  (a) Modification of completely quantised output Q (i) by a small amount of angular localisation (ii). In case (ii), the value of Q uniquely defines x.
(b) The fixed (electronic) uncertainty ΔQ in Q produces a greater uncertainty in inferred position x if the local sensitivity is smaller: Δx₁ > Δx₂ because S₁ < S₂.
Figure 23. Removal of anode wire position modulation by measurement of azimuthal angle, using the "centroid" method of readout (see Chapter 7). In (a) are represented the directly-computed centres of charge for an image of a simple object (a clip holding a ring), and the anode wire structure is clear. In (b) however, a knowledge of the azimuthal location of the positive ions near the wire has been used to obtain a continuous response. (Taken from reference 26).
Such a system, it might be felt, could be of great advantage in the X-ray astronomy application with which the present work is concerned. There are several reasons why it has not been employed: one is that the method requires a large number of preamplifiers, which must have equal and stable gains, necessitating careful adjustment before use, and which also add to the detector bulk. The second is the large amount of computing time required to reconstruct an image by this method, and another is that the accuracies obtainable by the existing (RC line) method have not hitherto been the limiting factor in spatial resolution (other factors include, for example, image blurring due to the mirror focal plane curvature). Presumably, however, one may anticipate developments in mirror technology, computer sophistication (leading to more rapid data analysis) and hardware miniaturisation, which will bring the "centroid" methods more into favour. They are particularly suitable, too, for the larger-area detectors currently under development.

It has already been pointed out that an absorption region may still be necessary to achieve the desired X-ray detection efficiency, despite the fact that a new means of position interpolation may well supersede the diffusion technique. If angular localisation is this new means, the unavoidable diffusion occurring in the absorption region will reduce its effectiveness by increasing $\sigma_\alpha$; in other words, the use of an absorption region while exploiting angular localisation may not be feasible.

iv) Modulation of the Anode Signal

One further caution about angular localisation concerns the energy resolution of a MWPC: marked angular localisation is likely to cause this to deteriorate, because of a variation of anode signal with $\alpha$. Charpak et al. have identified a modulation in gain due to the field variation around a wire, while Mathieson and Harris have estimated the magnitudes
both of this effect and of a second one, namely a variation with $\alpha$ of the induced charge at any given time. In this discussion, we shall consider first the gain variation, reverting temporarily to the previous definition of $y$.

**Gain Modulation.** Equation (13) gives the field in a MWPC:

$$E = \frac{KV_a}{h} \left( \frac{\cosh 2\pi y/s + \cos 2\pi x/s}{\cosh 2\pi y/s - \cos 2\pi x/s} \right)^{1/2},$$

where, as before, $K = (1 - s/\pi \ln 2\pi r_a/s)^{-1}$, and if we now consider the situation close to the wire by making $x$ and $y$ small and taking the first three terms in the expansions of $\cosh$ and $\cos$, we can arrive at the approximation

$$E = \frac{2 C_{\text{eff}} V_a}{r} \left( 1 - \frac{\pi^2}{3} \left( \frac{r}{r_a} \right)^2 \cos 2\alpha \right), \quad (20)$$

where $r^2 = x^2 + y^2$ and $\alpha = \tan^{-1} (y/x)$ as usual, and $C_{\text{eff}}$ is the effective capacity $C_{\text{eff}} = 1/(\ln(r_{\text{eff}}/r_a)^2)$. Notice that by considering only the first two terms in the expansions, we would have arrived at the radial expression $E = 2 C_{\text{eff}} V_a/r$ encountered in the derivation of the expression (14) for $r_{\text{eff}}$.

Now the gas gain in a proportional counter may be written

$$\ln M = \int_{r_a}^{r_0} K(r)dr,$$

where $K(r)$ is the mean number of ionising collisions per unit distance and $r_0$ is the critical radius at which amplification begins. The latter quantity is not directly determinable from experiment, but is related to another quantity, the critical voltage $V_0$, which it is possible to measure. If we say that a critical field exists, then $r_0 = r_a$ at $V_0$, and $r_0 = r_0$ at some higher voltage $V_a$, so that
\[ E = \frac{2 C_{\text{eff}} V_o}{r_a} = \frac{2 C_{\text{eff}} V_a}{r_o} \left\{ 1 - \frac{\pi^2}{3} \left( \frac{r_a}{s} \right)^2 \cos \alpha \right\} , \]

which gives for \( r_o \):

\[ r_o = \frac{V_a}{V_o} \left\{ 1 - \frac{\pi^2}{3} \left( \frac{r_a}{s} \right)^2 \left( \frac{V_a}{V_o} \right)^2 \cos \alpha \right\} , \]

because \( r_a << s \). If we now assume that \( K(r) = (BE)^{\frac{1}{2}} \) (see reference 10), where \( B \) is a constant, we can perform the above integration by making the approximations allowed by \( (r/s)^2 << 1 \) (for \( r \ll r_o \)). The result is

\[ \ln M = \ln M_o - \ln M_o \cdot \left( \frac{V_a}{V_o} \right)^{\frac{3}{2}} - 1 \right) ^{-1} \pi^2 \left( \frac{r_a}{s} \right)^2 \left[ 6 \left( \frac{V_a}{V_o} \right)^{\frac{5}{2}} - 1 \right] \cos \alpha , \]

(21)

where \( \ln M_o = 2(2 BC_{\text{eff}} r_a V_a)^{\frac{3}{2}} \left( \frac{V_a}{V_o} \right)^{\frac{3}{2}} - 1 \right) \), expressed in the same form as given by Rose and Korff\(^{10} \). The fractional gain change is then

\[ \Delta M = - \frac{\ln M_o}{\left( \frac{V_a}{V_o} \right)^{\frac{3}{2}} - 1} \cdot \left( \frac{V_a}{V_o} \right)^{\frac{5}{2}} \cdot \pi^2 \left( \frac{r_a}{s} \right)^2 \cos \alpha , \]

(22)

which expression demonstrates the form of \( \Delta M \), and may be used to find the gain variation expected from a particular distribution in \( \alpha \).

**Induced Charge Variation.** The second effect, the variation with \( \alpha \) of induced charge, may be of magnitude equal to or greater than the first. This depends on the type of signal processing used. If an ideal preamplifier of conversion gain \( 1/c \) is used, and \( q \) is the avalanche charge, the total charge delivered to the input of the anode preamplifier is the sum of the collected and induced charges:

\[ v(x,y) = - \frac{q}{c} \left\{ 1 - 2 C_{\text{eff}} \frac{wh}{s} \left[ 1 - s/wh \ln(2 \cosh 2\pi y/s - 2 \cos 2\pi x/s)^{\frac{3}{2}} \right] \right\} , \]

(23)

using equation (8), Chapter 2. The time variation of \( v \) can be calculated numerically for any field line, from a knowledge of the field and mobility, as before, and by again assuming some distribution in \( \alpha \) and
weighting several paths accordingly, an initial angular spread may be taken into account. The output of a shaping stage can then be calculated, also numerically, from the convolution integral, provided the shaping stage impulse response is known.

The magnitude of this effect, according to reference 51, is small if $T_a/T_o$ is small. (As an aside, it may be noted that as the time of collection is approached the induced charge will depend less and less upon the initial $\alpha$; therefore it must be that at some particular time this second effect is maximised.) This is what tends to suggest that the signal modulation of 11% observed by Charpak et al. was due chiefly to the first effect, in view of the fast shaping employed.

To summarise, then, monitoring of the signal modulation

$$M = \frac{(V_{\text{max}} - V_{\text{min}})}{(V_{\text{max}} + V_{\text{min}})}$$

can give information on angular localisation. The effect should be enhanced if we observe at larger time constants (because of the second effect above), or at low gains (because the avalanche spread has not begun to increase), or in methane-rich gas mixtures (because the diffusion will be small). These predictions are compared with experiment in the next chapter.
CHAPTER 5
THE EFFECTS OF ANGULAR LOCALISATION

This chapter describes experimental investigations into angular localisation of the avalanche, dividing into two categories: first, a confirmation of some predictions made from the mathematical models already described, and second, an investigation of the possibility of exploiting the effect for position-sensing, particularly with regard to an RC line readout system.

i) Confirmatory Experiments.

Anode Signal Modulation. Experimental verification of the presence of angular localisation was to be obtained by examining the modulation of the anode signal described in the previous chapter, and it was expected that the magnitude of this effect would vary, as predicted, according to charge level, gas mixture and processing time constant. For these measurements the chamber electrodes were arranged as described in Chapter 3: the anode-cathode gap, $h$, was 4 mm and the anode-window distance was 17 mm, while the grids themselves were exactly as described (the anode pitch was 2 $\mu$m, as usual). The chamber was irradiated with the Al-K source, using only the lower collimation slit (see figure 16), which resulted in a beam width of approximately 65 $\mu$m rms at absorption.

Once again, a brief mention must be made of anode charge, and what is meant by the term "charge level". This will be taken to mean the value of the charge, input in step form to the preamplifier, which gives a shaped output that matches the average shaped output from the X-ray pulses; so in this case, the "charge level" is actually about one third of the avalanche charge (1 $\mu$s bipolar shaping). This definition will apply to all subsequent results.
To perform the measurements, the X-ray beam was traversed across the chamber in a direction perpendicular to that of the anode wires in intervals of 200 \( \mu m \). The pulse height spectrum was accumulated on a PHA (e.g. the Northern NS-200 MCA) and the channel number of the spectral average recorded as a function of position (the energy resolution was about 31% FWHM, but it was felt that the error on the mean pulse height could be no more than \( \pm 2\% \)). To emphasise any effects of angular localisation, the first runs were carried out using a long time constant \( T_a = 10 \mu s \), bipolar shaping) and at a low charge level (0.05 pC). Figure 24 shows a typical modulation, for the gas P50, obtained under these conditions; the signal modulation \( m = \frac{[V_{\text{max}} - V_{\text{min}}]}{[V_{\text{max}} + V_{\text{min}}]} \) is about 3.9\%.

The value of \( T_o (= s^2/\mu W_a) \) was approximately 8.3 \( \mu s \) in this case, so that the quantity \( T_a/T_o \) was 1.2. According to Mathieson and Harris\(^5\), then, a perfectly localised avalanche would lead to a modulation of about 5.5\%, due to the time variation of the induced charge (figure 25); however, perfect localisation is by no means the case here, because the expected diffusion for this drift region produces a spread, \( \sigma_x \), of the charge cloud, of about 0.36 \( \mu m \). The spread in angle \( \alpha \) can thus be estimated from equation (10) (Chapter 2) to be \( \sigma_\alpha = 33^\circ \), and one is led to expect a smaller modulation. Whether this fully accounts for the difference between the two figures is not known, but it does not seem that the mechanism of gain variation with \( \alpha \) can account for much of the observed modulation: inserting values of \( M_o = 5 \times 10^3 \) and \( (V_a/V_o) = 2 \) into equation (22) leads us to expect less than half a percent modulation from this effect. In view of the fact that Charpak et al.\(^6\) used a very similar chamber, it remains something of a mystery how their observed modulation occurred, for, as already pointed out, the shaping employed was very fast. This consideration aside, the general quantitative agreement between predictions and results is fair.
Figure 24 Anode signal modulation, at a charge level of 0.05 pC, $T_a = 1 \mu$s and with P50 gas. The vertical arrows correspond to the anode wire positions.

Figure 25 Signal variation due to dependence of risetime on angle $\alpha$, for a perfectly localised avalanche ($\sigma_\alpha = 0^\circ$); taken from reference 51.
The effects of variation in processing time constant, gas mixture and charge level were next examined. Table 3 indicates how the first two of these factors influenced \( m \), giving data taken with other gases and at one shorter time constant, but all at the same charge level (0.05 pC). Qualitatively, the modulation behaves as expected: it increases as the diffusion is reduced, and falls as the processing time constant is reduced; however, quantitative comparison shows that the observed modulation at 1\( \mu \text{s} \) is too large to be accounted for by either of the expected effects.

Further examination of signal modulation to explain these anomalies was not made, because the major objective of the experiment had been a qualitative one.

It still remains to mention the variation of \( m \) with charge level. According to equation (22), \( m \) should increase slowly with gain; but it turns out that this effect is not observed, at the charge levels dealt with here, because the amplification is in fact becoming appreciably non-proportional. In any case, as pointed out above, the value of \( m \) due solely to gain variation is expected to be very small over this range.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>( m ) (%)</th>
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</thead>
<tbody>
<tr>
<td>P10, 10 ( \mu \text{s} )</td>
<td>1.9</td>
</tr>
<tr>
<td>P50, 10 ( \mu \text{s} )</td>
<td>3.9</td>
</tr>
<tr>
<td>( \text{CH}_4 ), 10 ( \mu \text{s} )</td>
<td>4.4</td>
</tr>
<tr>
<td>P50, 1 ( \mu \text{s} )</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Table 3. The influence of gas choice (diffusion) and processing time constant on angular localisation, as indicated by the anode signal modulation, \( m \).
Figure 26  Reversal of the modulation pattern with increasing non-proportionality, with $T_a = 10 \mu s$ and P50 gas. Charge levels: (a) 0.05 pC; (b) 0.5 pC; (c) 1.6 pC.
Measurements over the range 0.01 to 0.1 pC showed no marked variation in \( m \), and above this any effects of either increasing gain variation or increasing angular spread were masked by a rather spectacular manifestation of increasing non-proportionality: figure 26 compares the data (a) of figure 24 with data taken over the same range of positions and at the same time constant (10 \( \mu s \)), but at charge levels of 0.5 pC (b) and 1.6 pC (c). The signal variation has completely changed phase at the highest charge level, with the maximum signal now occurring halfway between wire positions! The most likely explanation of this seems to be that serious non-proportionality is occurring, so that a reduced initial number of electrons experiences an increased gain. Halfway between wires, the charge-cloud may be expected to divide, so that exactly this situation exists; the two halves of the cloud, avalanching on different wires, are now able to experience a greater gain, with the result that the total signal is increased.

What these studies of anode signal modulation demonstrate, then, is that angular localisation is present to roughly the expected degree, and that this degree varies as expected with the particular parameters. Energy resolutions of the sort obtained for 1.5 keV X-rays (~30%) ought not to be much affected by angular localisation, but the effect of non-proportionality at charge levels greater than 1 pC may increase \( \Delta E/E \) by several percent.

**Induced Pulse Shapes.** Another simple way of demonstrating the presence of angular localisation is by observation of the induced pulse shape. If the preamplifier fall time is long, this shape is approximately the same as that of the preamplifier output pulse, which may be conveniently studied on an oscilloscope. For this observation, a plain cathode grid was employed, in order that the pulse shapes should not be subject to any delay or attenuation. The charge level was 0.2 pC, and the gas P50. Figure 27 shows the results of calculations carried out for this particular chamber by the method described in Chapter 2, which also take into account a gaussian
Figure 27 Cathode induced pulse development. Calculations\textsuperscript{75} take into account a gaussian spread in $\alpha$ of $\sigma_\alpha = 30^\circ$. See figure 11 for other conditions.
Figure 28 Cathode induced pulse profiles observed at the output of an inverting preamplifier. (a) $\alpha = \pi/2$; (b) $\alpha = 0$; (c) $\alpha = \pm \pi/2$. 
spread in $\alpha$, of $\sigma_\alpha = 30^\circ$. This is to be compared with the photographs 28 a-c (unfortunately the polarity is reversed), which show the pulse shapes corresponding to $\alpha = \pi/2$ (a: source above a wire), $\alpha = 0$ (b: source halfway between wires), and $\alpha = \pm \pi/2$ (c: source above a wire, with no drift region). Once again, the general agreement is rather satisfactory.

**Induced Charge Distributions.** For a more quantitative examination of the model of Mathieson and Harris$^{55,58}$, a special cathode was constructed (using a technique described in Part III) without a resistive element, and was connected into two halves. Each was independently supplied with a drift voltage, and each fed its own preamplifier. If one half is designated A and the other B, a crude position signal $B/(A+B)$ may be obtained from a pulse ratio system, figure 20; this signal is the fraction of charge induced on one half of the cathode. The fraction varies as the X-ray source is traversed across the division, as shown in figure 29, where $y_0$ is the location of the centroid of the charge distribution, and the coordinate origin coincides with the cathode division. If the induced charge distribution is the function $q(y - y_0)$, then the position signal $Q = B/(A+B)$ is

$$Q(y_0) = \int_{y_0}^{\infty} q(y - y_0)dy / \int_{-\infty}^{\infty} q(y - y_0)dy .$$

Thus, if $Q(y_0)$ can be found (by a simple scan of the source over the sensitive region) its differential will give the induced charge distribution.

In reality, it is impractical to scan the source continuously, so that $Q(y_0)$ must necessarily be a histogram, and $q(y - y_0)$ a histogram of differences. Nevertheless, this technique yields a histogram through which a smooth curve, showing very good agreement with theoretical predictions$^{58}$, may be drawn. Another practical point is that while the calculations refer to the case of parallel anode and cathode wires, the measurements were more conveniently carried out with the anode wires perpendicular
Figure 29. Scanning the induced charge distribution (a) over the division between the two halves of the divided cathode yields a position calibration curve (b) which is the integral of the charge distribution (c).
to the cathode wires (although one measurement was made "across the wires", in which the source was moved in 2 mm intervals, so as to be above an anode wire for each reading).

Figure 30 shows a typical distribution obtained in this way (a), for h = 4 mm, together with a theoretical distribution for the same conditions (b), and table 4 summarises all such measurements: two charge levels and three values of h (4, 6 and 8 mm) were employed. The distributions are not expected to be influenced much by the amount of localisation unless $\sigma_a$ becomes quite large, but because the avalanche is localised, the charge corresponding to absorption events on the far side of the anode from the sensing cathode should develop differently from that corresponding to "nearside" events. Discrimination between the two types of event indeed revealed the expected slight difference in spread, while the similarity of results for 0.1 and 1 pC suggests that the avalanche spread has not changed drastically over this range.

The agreement between model and results is rather good, and indeed one can summarise this section by saying that the models available seem quite adequate for describing induced charge, angular localisation and associated effects for this chamber.

<table>
<thead>
<tr>
<th>(FWHM)/h</th>
<th>0.1 pC</th>
<th>1 pC</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (mm)</td>
<td>NEARISE</td>
<td>FARSIDE</td>
</tr>
<tr>
<td>4</td>
<td>1.44</td>
<td>1.53</td>
</tr>
<tr>
<td>6</td>
<td>1.37</td>
<td>1.46</td>
</tr>
<tr>
<td>8</td>
<td>1.37</td>
<td>1.37</td>
</tr>
</tbody>
</table>

Table 4. Induced charge spread as a function of $h$ and charge level.
Figure 30 Spatial distribution of the cathode induced charge, for $h = 4$ mm and with $T_a = 1$ μs. (a) measured; (b) predicted.
ii) The Effect of Angular Localisation on RC Line Position Sensing

Processing Time Constant. This section describes an investigation directed towards an improvement of x-direction linearity by exploitation of angular localisation, using the existing chamber RC cathodes.

Figure 31 illustrates a typically "stepped" position calibration curve for the x-position output: large portions of the curve are quite flat. This particular data was taken using P50 gas and 1 μs shaping time constant, and it should be pointed out that the use of P10 gas instead would considerably smooth out this curve, because of increased lateral diffusion. However, the use of P50 both emphasises any angular localisation effects and disengages them from diffusion effects, so that if any smoothing is seen, it will probably be due to angular localisation. Also, similar data for pure methane was not noticeably different from that of P50, presumably because of their similar diffusion coefficients; thus the gas choice for these experiments was not difficult to make. There was also little reason for examining the effect of variation in charge level, because great reduction of this quantity would make the signal-to-noise ratio inconveniently large. A charge level of 1 pC was therefore employed throughout.

These considerations meant that the only remaining way of exploiting angular localisation was by optimising the processing time constant; that is, by effectively observing the positive ions at a time late enough for them to have travelled back along the field lines to the location of the original absorption. Accordingly, an attempt was made to observe how the "steppedness" of the position signal changed with time constant, by effectively measuring the slope of the steep part of the curve. Over the range of time constants 0.2 to 2 μs, little change was observed, although this was not very surprising in view of the fact that the collection time of the positive ions was probably in excess of 50 μs (estimated from observations of preamplifier output), so that by 2 μs they were unlikely to have moved
Figure 31  Position calibration curves for the x-direction, at a charge level of 0.05 pC, with P50 gas and using RC line risetime position encoding: (a) with source near centre of line, and $T_a = 1 \mu$s; (b) with source away from centre, and $T_a = 5 \mu$s.
very far from the wire. Better results were expected at 5 and 10 μs, but the reality of the situation proved rather surprising: a marked turnback of the position curve was observed, which completely swamped the normal stepped pattern. The position curve for 5 μs (b) is compared with that for 1 μs (a) in figure 31: note that the position sensitivity of the RC line alone (measured as the difference between two points 2 mm apart) is about the same in each case. If the sensitivity at 5 μs is 15 CH/s (= 7.5 CH/mm), the measured position of the source moves backwards by more than 3 mm before advancing to the required 2 mm position!

Crossover Time Modulation. Observation of the shaping amplifier outputs by visual measurement of the bipolar pulse crossover times showed that the latter varied markedly, in accordance with the observed variation in position signal. It was realised, therefore, that the same similar variation must occur in the crossover time of the anode pulse, and accordingly this quantity was observed as a function of source position, again at 5 μs. A pattern similar to that of the pulse height modulation was observed, and it was realised that the two effects were due to the same cause: the pulse risetime variation around the wire. Ions leaving along different trajectories will produce shaped pulses differing in height and crossover time because their initial profiles are different. A discussion of the implications of this for position encoding and an explanation of the turnback effect follows, but it is first of all worth noting a few points concerning the crossover time modulation.

As a demonstration of angular localisation, this quantity has two clear advantages over signal modulation: firstly, measurements are not obscured by wire-to-wire gain variations (a scan over many wire pitches produces a level sinusoidal pattern whose peaks and troughs are always at the same heights); and secondly, no reversal of this modulation pattern occurs at larger charges, because it does not depend on proportional operation.
These considerations prompted a series of observations to complement those of signal modulation (table 3), with an additional measurement at a higher charge level to demonstrate that angular localisation was indeed reduced in that case. The results are given in table 5, and a typical crossover time modulation pattern is shown in figure 32. It can be seen that once again the modulation varies as expected, in all respects. One further advantage of this characteristic is that locating the maxima of this modulation pattern provides a simple way of determining the wire positions.

Observations were next made of cathode signal crossover times, and as a preliminary and simplified arrangement, two plain cathodes were employed. In order to examine the pulses induced on one cathode by positive ions travelling towards the other, the intensities of events occurring above and below were made more equal by eliminating the drift region: the drift voltage was set to - 100 V, and to increase overall intensity, the active volume was raised so as to be 6 mm from the window. The same collimation was employed as for the gain modulation measurements, resulting in a slightly greater beam width here because of the divergence of the beam. The measurements were all carried out at a charge level of 0.05 pC.

<table>
<thead>
<tr>
<th>Conditions</th>
<th>m_t (%)</th>
<th>Charge level 0.05 pC unless otherwise stated.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10, 5 µs</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>P50, 5 µs</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>CH₄, 5 µs</td>
<td>4.7</td>
<td></td>
</tr>
<tr>
<td>P50, 1 µs</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>P50, 5 µs, 1 pC</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>
Figure 32  Typical anode signal crossover time modulation pattern. Conditions: P50 gas; T = 5 μs; charge level = 0.05 pC. The value of $m_t$ is 3%.

Figure 33  Normalised cathode pulse crossover time as a function of position, for P50 gas, 1 μs time constant and 0.05 pC charge level. The full curves are theoretical predictions based on $\sigma_\alpha = 60^\circ$. 
Avalanche Angular Spread. In addition, calculations were made predicting the variation of these crossover times\(^7\). The current waveform expected from induced charge calculations, using a given initial angular spread, was convolved with the system transfer function and the response predicted as a function of the angle \(\alpha\) (and hence beam position). Comparison of the theoretical and experimental results is given in figure 33, where the crossover time, \(t_c\), has been normalised to units of the processing time constant, \(T_a\) (\(T_a\) was nominally 1 \(\mu\)s but was found to be really about 1.1 \(\mu\)s). The upper and lower branches refer respectively to events on the side of the anode nearer to the sensing cathode, and to those on the far side. In simple terms, the crossover time falls as the slope of the induced pulse profile (see figure 27) becomes more and more negative; increasing angular localisation causes the branches to separate further. At first sight the agreement is rather good, but unfortunately, these theoretical predictions pertain to the case where the angular spread is \(\sigma_\alpha = 60^\circ\), in disagreement both with previous estimates and with some measurements, which were also made, of the amplitude ratio of the pulses induced on the cathode by the two types of event (for a fixed \(\alpha\) of \(\pi/2\)). Table 6 compares pulse height ratios and crossover times with predictions, for the two gases P50 and CH\(_4\). The disagreement is somewhat disappointing, and as yet it is only possible to speculate as to what the cause might be.

<table>
<thead>
<tr>
<th></th>
<th>Predictions for (\sigma_\alpha)</th>
<th>Experimental Result</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gas 0° 30° 60°</td>
<td></td>
</tr>
<tr>
<td>Ratio V(<em>\text{near}/V</em>{\text{far}})</td>
<td>P50 1.47 1.41 1.27</td>
<td>1.39 ((-30^\circ))</td>
</tr>
<tr>
<td></td>
<td>CH(_4) 1.53 1.46 1.30</td>
<td>1.48 ((-30^\circ))</td>
</tr>
<tr>
<td>Crossover Times (t_c/T_a)</td>
<td>P50 { 2.81 2.78 2.71 }</td>
<td>2.75 { (-60^\circ) }</td>
</tr>
<tr>
<td></td>
<td>CH(_4) { 2.34 2.35 2.41 }</td>
<td>2.43 }</td>
</tr>
<tr>
<td></td>
<td>P50 { 2.85 2.82 2.73 }</td>
<td>2.79 { (-60^\circ) }</td>
</tr>
<tr>
<td></td>
<td>CH(_4) { 2.31 2.32 2.38 }</td>
<td>2.42 }</td>
</tr>
</tbody>
</table>

Table 6. Comparison of experimental measurements of cathode pulse crossover times and ratio with predictions for three values of \(\sigma_\alpha\).
For a time, it was not possible to find symmetric behaviour from the two cathodes: for the two types of event, differing pulse height ratios and crossover times were found. This appeared to be associated with the large dielectric constant (≈ 9) of the alumina substrates of the anode grid, which extend some distance into the active volume, and which perhaps cause a "bunching" of the field lines on their side of the anode. By laying some aluminium strips of similar dimensions on top of the anode wires, the symmetry of the results was restored; but of course such effects cannot be taken into account in the calculations.

Combination of RC Line Properties with Angular Localisation Effects.

Now is it possible to explain the turnback phenomenon. It arises because of this variation in crossover time with the angle $\alpha$, combined with the characteristic of the RC line that the delay imparted to a pulse is dependent upon its risetime. So it is that unless the pulse is injected at the centre of the line, the time difference between the pulses arriving at the two ends of the line is greater for pulses due to ions travelling along the trajectories near $\alpha = \pm \pi/2$ (over a wire) than for those corresponding to values of $\alpha$ near 0 (between wires). This discrepancy increases as the pulse is injected further and further away from the line centre. The behaviour is illustrated in figure 34, for a source position one-quarter of the way along the line (i.e. $x_L = 0.25$, if $x_L$ is the fractional distance along the line). The RC line termination for those measurements was $C_d = 20$ pF, and line itself was that of 50 kΩ previously described. It is possible to combine the calculations described above with a mathematical model of the RC line (rather than employ a delta-function of current, $i_c = \delta(t)$, as used in a previous work\textsuperscript{77}) to again predict the behaviour of the crossovers. The result of such calculations, also shown in figure 34, is in reasonable agreement with experiment, given that $\sigma_0 = 60^\circ$ has been used.
Figure 34 Normalised pulse crossover time from the two ends of the cathode RC line, for charge injection at $x_L = 0.25$. Conditions were as for figure 33.
We have now seen that the combination of an angular localisation risetime effect with the properties of an RC line will cause a distortion of the position signal (the crossover time difference). It is important to appreciate that this distortion takes place no matter along which direction the RC line lies: that a movement of the source across the anode wires causes a marked modulation of position signal even if the RC line is ordinarily insensitive to motion in that direction (e.g. the y-position sensing cathode, whose RC line is parallel to the anode wires). Figure 35 shows the measured and predicted variations in the x-position signal (again at 0.05 pC, 1 μs and in P50). The discontinuity in the theoretical curve represents the switching of the avalanche from one wire to the next, and has been rather smoothed over in the experimental data by the large beam width. Figure 36(a), on the other hand, shows what happens to the y-position signal as the source is moved in the x-direction: the measured position oscillates, even though the true position does not change! Of course, the magnitude of the effect as demonstrated here is artificially high, because in normal operation the avalanche will be spread by diffusion and by the higher charge level. This is exemplified in figure 36(b), taken at 1 pC and with a drift field of 400 V/2.5 cm. The peak-to-peak variation of the y-position signal due to motion in the x-direction has been reduced, but it still corresponds to about 0.3 mm, certainly not a negligible quantity.

It seems, then, that far from improving position linearity with an RC line readout, the effect of angular localisation is actually detrimental to it. Also, a similar effect is seen when using pulse ratio position encoding, because the pulse height at the end of the RC line is also dependent on the initial risetime: it must therefore be remarked that all previous work involving this type of encoding must have suffered from the effects reported here, to a greater or lesser extent. How important this depends on the experiment concerned, but caution is to be recommended
Figure 35  Measured and predicted position signal, showing the effect of angular localisation. Conditions were as for figure 34.

Figure 36  The "cross-modulation" effect: y-position output as a function of source x-position. (a) under the conditions of figure 34;
in choosing RC lines for a MWPC readout system.

iii) Use of a Non-Line Method

As a further brief illustration of the position-sensitivity available from angular localisation, an experiment was carried out using the plain divided cathode described in section (i), which has the advantage of much higher sensitivity (over a limited range) than the RC line. This position sensitivity is exactly what is plotted in figure 30a (it is the derivative of the position calibration curve), where it can be seen that beyond a few centimetres from the centre of the cathode the sensitivity has fallen to zero. Using a graph such as that shown in figure 37 (the integral of the data of figure 30a), it was possible to infer the real position from the position signal, for the case of position sensing along the anode wire direction. For the present experiment, the cathode sensed position in

Figure 37  Position calibration curve for the plain divided cathode, taken at 1 μs and 1 pC, with h = 4 mm.
the direction perpendicular to this, and two scans over the most sensitive region were made, measuring position signal at intervals of 100 μm: one was carried out at 0.1 pC charge level, and the other at 1 pC, but both were made using P50 gas and 1 μs unipolar shaping, with no drift region. The X-ray source was fully collimated, so that the beam width was approximately 40 μm r.m.s. The results of these scans are shown in figures 38(a) and (b) respectively, the typical form of curve observed by Mathieson, Harris and Smith\textsuperscript{54,72} being at once apparent. Further indication of increasing spread with increasing avalanche charge is provided by comparing the mean position sensitivities for the two cases, over the range of influence of angular localisation; these values are 0.60 mm/mm at 0.1 pC and 0.41 mm/mm at 1 pC.

For comparison with these measurements, calculations were carried out\textsuperscript{78} which predicted the position signal for a given beam position. These summed the outputs from the respective shaping amplifiers resulting from charges induced on all wires left of centre, and from those induced on all wires right of centre; the ratio $B/(A+B)$ was then found. Because the computing time required for such calculations was great, it was not possible to incorporate a range of trajectories $\alpha$, and consequentially the result of the computations refers to zero angular spread ($\sigma_\alpha = 0^\circ$). Furthermore, comparison between experimental results and calculations is made more difficult because the latter refer to the case in which an anode wire lies at the centre of the system, whereas in reality the centre of the system lies halfway between two wires. Nonetheless, a comparison of figure 39, showing the result of these calculations, with figure 40 (an "unlinearised" version of figure 36b) shows that the true sensitivity certainly approaches the theoretical one, although it is smaller, indicating once more a small but non-zero angular spread. One other notable feature of the calculated output is that full position interpolation is impossible, with this system and
Figure 38  Position sensitivity due to angular localisation, demonstrated by use of the plain divided cathode: (a) at 0.1 pC; (b) at 1 pC. Both obtained with P50 gas, $T_a = 1 \mu s$ (unipolar shaping) and $h = 4 \text{ mm}$.
Figure 39  Calculations of position output as a function of source position, showing sensitivity due to angular localisation, for P50 gas, 1 μs unipolar shaping and h = 4 mm.

Figure 40  "Unlinearised" version of figure 38(b), for comparison with figure 39.
under these conditions (the segments of the curve cannot be joined), a fact which can be explained if, at this processing time constant, the positive ions have not yet passed far enough along their trajectories to restore completely the location of the X-ray absorption.

One obvious course for future work to take, then, is to examine angular localisation at longer time constants, using this or any of the readout methods studied in Part III, none of which exhibit the peculiar behaviour of the RC line described in the previous section.
CHAPTER 6
POSITION INTERPOLATION BETWEEN WIRES

1) The Amount of Diffusion

Previous Work. Although drift-and-absorption regions have been employed in several X-ray astronomy IPC's, little work has been done to determine criteria by which the depth of such regions should be chosen. In view of the direct relationship between position resolution and the amount of lateral diffusion, this may be thought to be somewhat surprising; however, it is probably true to say that the performance of the IPCs used to date has not been the limiting factor in position resolution for such experiments, and that the rather cavalier attitude adopted hitherto has thus been understandable. Obviously, the precedence of linearity or resolution depends upon the particular experiment, but there is certainly no need for a drift depth to exceed the value at which perfect linearity (in the x-direction, across the anode wires) is achieved. For instance, Rappaport et al.\textsuperscript{38} describe an arrangement in which the diffusion is probably insufficient to provide full linearity, while the Einstein Observatory IPC\textsuperscript{34} employs what appears to be an excessively deep region, for the gas concerned. On the other hand, the arrangement of Levine et al.\textsuperscript{39} probably approaches the best case, using P50 gas, an anode pitch $s$ of 1 mm and a drift depth of about 2 cm.

Some steps have been taken fairly recently by Reid et al.\textsuperscript{64} in assessing the effects of diffusion on resolution in both sensing directions of a proportional counter, although linearity was not studied. In this work, Monte Carlo simulations were carried out to predict these contributions to position resolution, presumably using values of $(D/\mu)/E$ taken from the literature available. One justification for a new study of the use of a drift region lies in the comparative scarcity of $(D/\mu)/E$ data for the
present gases, for agreement between new values inferred from such a study and existing ones would be welcome; and of course the chief aim of the investigation would be to find an experimental relationship between x-direction nonlinearity and position resolution.

Experiment. Once more the adaptibility of the counter proved invaluable, as this series of experiments required repeated dismantling and re-assembly of the counter, varying the drift region depth on each occasion. One set of measurements gave the differential nonlinearity $\epsilon_a$ due to the anode wire spacing (see below), while the other gave the y-direction rms position uncertainty $\sigma_y$, each as a function of the depth of the drift region, $d-h$ (where $d$ is the anode-window separation and $h$ is the anode-cathode separation). Both sets were repeated, to yield data for the two gases P10 and P50.

The resolution measurements were carried out using full source collimation, resulting in an rms beam width of $\sim 40 \mu m$ at absorption. The FWHM of a position distribution at the centre of the counter was converted to a real distance using the sensitivity inferred from two points, 1 mm either side of centre, and the resolution $\sigma_y$ was found by assuming the distribution to be gaussian. For optimal resolution and good linearity, an RC cathode of $R = 250 \, k\Omega$ was employed$^{79,80}$, with $h = 4 \, mm$, $s = 2 \, mm$, and using the $1 \mu s$ semi-gaussian shaping of the Harwell 3769 pulse ratio system. The nonlinearity measurements were carried out using the same system, but with no source collimation, so that the beam width was of FWHM $\sim 2 \, mm$. To simulate uniform irradiation, the source was shifted in 1 mm intervals and the position spectrum allowed to accumulate on the PHA for the same number of counts at each location, a procedure which resulted in a nonlinearity below the detection limit of the experiment. This limit was determined by the electronic noise, and was about $1\%$, for 5000 counts in each channel.
The drift region depth was varied by moving the active volume of the chamber away from the window in steps of 3 or 6 mm, and values of $\sigma_y$ and $\varepsilon_a$ were found at each step. For drift region depths of more than 9 mm, two drift electrodes were employed to define the field, separated by $(d-h)/3$, and for $(d-h) = 24$ mm, a third electrode was introduced, so that the separations were then 6 mm. For all measurements the drift field was ~167 V/cm, so that $E/\rho$ was $0.22 \text{ V} . \text{cm}^{-1} . \text{torr}^{-1}$, except for those at $(d-h) = 21$ and 24 mm, for which the drift voltage was maintained at 300 V (to avoid discharge phenomena at the anode).

The measured values of $\sigma_y$ and $\varepsilon_a$ were each used to infer a value of $\sigma_d$, the rms width of the (gaussian) distribution of the primary electron cloud upon reaching the anode, to be compared with the lateral diffusion data of Mathieson et al. by means of the expression

$$ \sigma_a^2 = 2\bar{z}(D/\mu)/E $$

(25)

where $\bar{z}$ is the mean electron drift distance (see, for example, reference 51). It turns out that for several counter gases, including those concerned here, the quantity $(D/\mu)/E$ is roughly constant with field, having values of ~15 $\mu$m for P10 and ~5 $\mu$m for P50; thus $\sigma_d^2 \approx \bar{z}$. The value of $\bar{z}$ can be found from a knowledge of the relevant X-ray absorption coefficients, which, for the present work, have been given in table 2 (Chapter 3).

ii) Linearity

A perfectly linear system will respond to uniform irradiation with an equal number of counts, $N$, in each output channel, and the departure from this case can be quantified by the parameter $\varepsilon_a = (N_{\text{max}} - N_{\text{min}})/(N_{\text{max}} + N_{\text{min}})$, called the differential nonlinearity (cf. the definition of $m$ in the previous chapter). The "quantisation" of position due to the
anode wire spacing results in an approximately sinusoidal position distribution instead of a flat one, and $\varepsilon_a$ is thus simple to determine. Mathieson has shown how this quantity may be predicted as a function of $\sigma_d$, assuming the distribution of the primary electrons to be gaussian. A simple averaging of the relative numbers of electrons arriving at each wire yields a position calibration curve given by:

$$\bar{x} = \frac{1}{2} \sum_{n=-\infty}^{\infty} n s \left\{ \text{erf} \left( \frac{s(n+\frac{1}{2}) - x_0}{\sigma_d \sqrt{2}} \right) - \text{erf} \left( \frac{s(n-\frac{1}{2}) - x_0}{\sigma_d \sqrt{2}} \right) \right\}$$

(26)

where $\bar{x}$ is the recorded position, $x_0$ is the true location of the electron cloud centroid, $n$ is the (integer) wire number, and $\text{erf}(y)$ is the probability integral:

$$\text{erf}(y) = \frac{2}{\sqrt{\pi}} \int_{0}^{y} e^{-z^2} dz.$$

This calibration, and its differential (which will be called here the local sensitivity function), can be numerically calculated for given $s$ and $\sigma_d$ values. The maxima and minima of the local sensitivity function give $N_{\text{max}}$ and $N_{\text{min}}$ for the evaluation of $\varepsilon_a$. Now, assuming that the local sensitivity function can be represented by a sinusoidal one, the effect of the system electronic noise in reducing the value of $\varepsilon_a$ can be expressed as:

$$\frac{\varepsilon_a^{\text{obs}}}{\varepsilon_a} = \exp[-2\pi^2(\Delta/s)^2]$$

(27)

where $\Delta$ is the rms position uncertainty due to electronic noise alone. Equations (26) and (27) can thus be used to obtain $\sigma_d$ from the measured $\varepsilon_a$ and $\Delta$ values, and to indicate that this measurement refers to position-sensing in the x-direction, we shall write $\sigma_{dx}$ for $\sigma_d$.

Figure 41 shows the experimental results for PIO (carried out at a charge level of 0.5 pC) and P50 (at 1.0 pC), where the values of $\sigma^2_{dx}$
Figure 41 Squared rms spread of the primary charge cloud, $\sigma_{\text{ax}}^2$, plotted as a function of the mean drift distance $z$, for the gases PIO and P50. The values of $\sigma_{\text{ax}}^2$ are inferred from measurements of differential nonlinearity.
deduced from measurements of $\varepsilon_a$ are plotted as a function of $\tilde{z}$. The straight lines predicted by equation (25) indeed appear, and their slopes are 13.2 $\mu$m (P10) and 7.5 $\mu$m (P50), in good agreement with predictions; however, there is a large and completely unaccounted-for offset, of $\sigma^2_{dx} = 0.17 \text{ mm}^2$. It seems that an effect is taking place which is independent of the amount of diffusion occurring (note: this additive constant is not, of course, the photoelectron range for 1.5 keV; this contributes only about 30 $\mu$m rms $\approx 0.001 \text{ mm}^2$).

iii) Resolution

**y-direction.** A rather more direct measurement of the size of the electron cloud is provided by the position resolution in either direction. This is more simple to deal with in the y-direction, where experimental determination of the position uncertainty is not complicated by the variations in position sensitivity which are otherwise present; for instance, when sensing in the x-direction, the sensitivity is very low at a wire position (if little interpolation occurs), so that the dominant contribution to the spread of the position spectrum is electronic noise.

Suppose that because of diffusion the $N_0$ electrons in the charge-cloud are deposited in $K$ position "bins", the $i^{\text{th}}$ of which, located at position $x_i$, receives a fraction $f_i$ of the total; and suppose also that the variance of the gain experienced by the electrons deposited in any bin is $\sigma^2_M$, with a mean value $M$. Then if $M$ is not dependent upon $x_i$ (this is only approximately true), the variance in the position of the centroid of the distribution can be written:

$$\sigma^2_x = \frac{1}{N_0} \left\{ 1 + (\sigma_M/M)^2 \right\} \left\{ \frac{\sum_{i=1}^{K} f_i x_i^2}{\sum_{i=1}^{K} f_i x_i^2} - \left( \frac{\sum_{i=1}^{K} f_i x_i}{\sum_{i=1}^{K} f_i x_i} \right)^2 \right\}, \quad (28)$$

where the factor $1/N_0$ arises because position spectra are always built up from a number of events: $N_0$ is not a constant, but from event to event.
has a fluctuation which may be assumed to be Poissonian.

In this form, expression (28) is particularly applicable to the x-direction, where the "bins" are regions 5 mm broad and centred on the anode wires, whose positions are the \( x_i \) values. However, if one allows \( K \rightarrow \infty \) and the \( x_i \) values to become infinitely close together, the final factor in the expression becomes, quite simply, \( \sigma_d^2 \). Thus

\[
\sigma_y^2 = \frac{1}{N_0} \left( 1 + \frac{(\sigma_{\text{M}}/M)^2}{(\sigma_{\text{M}}/M)^2} \right) \sigma_d^2
\]

(29)

the case for position sensing in the y-direction. The value of \( \sigma_{\text{M}}/M \) for cylindrical counters is about 0.7\(^9\), but in this case it will be assumed to be about unity, to allow for gain variations from one anode wire to the next, as well as around the anode wires (see Chapter 5).

Hence, by deciding how much of the uncertainty in the position spectrum is due to this "centroid jitter", it is possible to obtain another estimate of \( \sigma_d^2 \). Measurements were carried out as described above, and values of the total uncertainty, \( \sigma_y^2 \), derived. Other contributions (photo-electron range, beam width and electronic noise) totalled about 55 \( \mu \)m rms, an important portion of \( \sigma_y^2 \) for small values of \( \sigma_d^2 \), and the value of \( \sigma_y^2 \) was found by subtracting this figure quadratically from \( \sigma_y^2 \). The results are plotted as a function of \( \tilde{z} \) in figure 42, where the two straight lines indicate the results expected from the values of \( (D/\mu)/E \) given above.

The agreement this time is rather good, with no hint of an offset; at higher \( \tilde{z} \)-values there is some distortion of the pattern, but this may be due to factors such as a nonuniform field (caused by incorrect potential division or by unequal electrode spacings). It is now seen that whatever is causing the large offset of the \( \sigma_{dx}^2 \) values inferred from nonlinearity measurements does not affect measurements in the y-direction, which is a rather unexpected result.
Figure 42 Squared rms spread of the primary charge cloud, $\sigma_{d}^{2}$, as a function of $\bar{z}$. These values are inferred from y-direction position resolution measurements, the two straight lines giving predictions based on existing diffusion data.
x-direction. Resolution in the x-direction was next considered. The problem of low sensitivity at an anode wire was overcome by use of the divided cathode described in the previous chapter, employing the position sensitivity available through angular localisation; resolution measurements were made at an anode wire position and a single value of d-h (zero, so that $\tilde{z}$ was 2.04 mm) for a range of charge levels, using P50 gas. These results are given in table 7, and can be seen to agree better with the values of $\sigma_{dx}^2$ inferred from nonlinearity measurements than with those of $\sigma_d^2$ taken from y-direction resolution measurements. Note that $\sigma_{dx}^2$ has here been estimated by assuming that equation (29) applies.

<table>
<thead>
<tr>
<th>Charge level (pC)</th>
<th>$\sigma_{dx}^2$ (mm$^2$)</th>
<th>Approx. Sensitivity (CH mm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.182</td>
<td>115 $\sigma_{dx}^2 (\varepsilon_a) = 0.206$ mm$^2$</td>
</tr>
<tr>
<td>0.5</td>
<td>0.102</td>
<td>140 $\sigma_d^2 (\varepsilon_y) = 0.013$ mm$^2$</td>
</tr>
<tr>
<td>0.2</td>
<td>0.078</td>
<td>180</td>
</tr>
<tr>
<td>0.1</td>
<td>0.160</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 7. Electron cloud size, inferred from measurements of x-direction position resolution, vs. charge level.

Discussion. The deduction made above is borne out: effectively, $\sigma_{dx} > \sigma_d$, i.e. the electron cloud appears to extend further in the x-direction than in the y, by an additive amount. The large value of $\sigma_{dx}$ must be responsible for the low value of $\varepsilon_a$; and the effect must be due to one of the following causes. Firstly, it could really be that $\sigma_{dx} > \sigma_d$; however, no reason for this can be proposed, and this explanation may be rejected as very unlikely. Secondly, angular localisa-
tion may be producing the interpolation: but graphs such as figure 38 show that although interpolation does occur, it is insufficient to account for any but a small fraction of the discrepancy observed. Furthermore, the discrepancy is no less for P10 than for P50, which would not be so if angular localisation were the culprit. Thirdly, the distribution of the primary electrons in the charge cloud could be significantly non-gaussian, such that the resolution would be worse in the x-direction as a result of the last factor in expression (28). Surely, though, the discrepancy would then depend upon $z$, and in any case, past work on diffusion indicates that the supposition is unlikely to be valid. Lastly, another effect may be taking place which contributes to the resolution, and depends upon the anode wire direction. In the next section, evidence is presented which indicates that the real explanation is probably one such, but before passing on to that, a few points may be made concerning table 7, and concerning division of the electron cloud.

Considering first the sensitivity figures (which arise solely from the influence of angular localisation), it can be seen that as the charge level falls, the sensitivity increases (presumably as the avalanche angular spread $\sigma_a$ decreases) and then falls. Is this single measurement an indication that angular localisation effects are indeed enhanced as the operating charge level falls (see equation (22))? On the other hand, if this last value of $\sigma_{dx}^2$ is in error, these measurements may indicate that the effect which raises $\sigma_{dx}^2$ to its anomalous value is less pronounced at lower gains, giving some information about this peculiar phenomenon.

An interesting illustration of the effect of the final (partition) factor in expression (28) was observed during measurements of angular localisation made using the high-sensitivity cathode. For a small $\sigma_d$, when the primary electron cloud arrives halfway between two wires, each wire receives half of the cloud, so that the partition term becomes $s^2/4$;
and, for \( N_0 = 50 \) and \( \sigma_{N/M} = 1 \), we have \( \sigma_x = s/10 \). This was experimentally confirmed by the large increase of the position FWHM as the avalanche switched from one wire to the next; for instance, at 0.1 pC, an rms value of 0.204 mm, or about \( s/10 \) was recorded. Note that \( \sigma_d \) is, in fact, immaterial at this particular (halfway) position.

iv) Secondary Avalanches

**Detection.** During certain experiments in which y-position resolution was measured at quite a high charge level, a discrepancy was noticed between the values of \( \sigma'_y \) predicted (from centroid jitter, beam width, electronic noise and photoelectron range considerations) and measured. It was suggested\(^{43,83} \) that ultraviolet photons, initiating prompt secondary avalanches, were responsible for this, by means of the "centroid weighting" effect already described; and accordingly some evidence was sought to support this theory. An amplifier was modified by inserting a 560 pF capacitor in series at the input, which, in combination with the input impedance \( Z_{in} = 50 \Omega \) of the amplifier, produced a differentiator of very short (~30 ns) time constant. The electron drift velocity for pure argon at the relevant reduced field \( (E/p \sim 7 \text{ V/cm} \cdot \text{torr}^{-1}) \) must be\(^{60,84} \) around 20 mm \( \mu \text{s}^{-1} \), so that in the present case the time taken to traverse the chamber gap of 6 mm (=h) must be about 300 ns. The differentiation was thus fast enough to allow secondary avalanches to be distinguished; also, the production of these was encouraged by employing P10 gas and a high charge level. Figure 43a shows a typical fast amplifier output for a charge level of ~2 pC, with h=6 mm and a negative drift voltage (to prevent window-initiated electrons from arriving), and a set of peaks occurring about 300 ns later can be seen quite clearly. Their sizes are comparable to the initial ones, and about 25% of all events are affected in this way. Figure 43b shows what happens when electrons ejected by
Figure 43 Fast amplifier output pulses, showing secondary avalanches. All are taken at a charge level of 2 pC: (a) P10, negative drift voltage; (b) P10, positive drift voltage; (c) P50, negative drift voltage. Figure 43(d) shows the effect of secondary avalanches on a typical energy spectrum (Al-K).
photons striking the chamber window are admitted: a variety of drift times is observed and events having attendant tertiary avalanches or more also take place (these may just be events in which electrons are ejected from both window and cathode). The effect of increasing the concentration of a quench gas is illustrated in figure 43c, where the conditions were as for figure 43a, except that P50 gas was used. No events having secondary avalanches appear in this sample.

Figure 43d is a sketch of the energy spectrum at this charge level, for the case of figure 43b: a long high-energy tail has appeared, corresponding to avalanches having attendant secondaries, and it can be seen that it becomes difficult to talk in terms of "charge level" and "energy resolution".

Distribution of the Secondary Avalanches. A second experiment was carried out to examine in more detail the effect of the secondary avalanches on position resolution. A single-channel analyser, fed from the anode amplifier, was employed to gate out "ordinary" events, so that only the positions of events much affected by secondary avalanches were recorded. The resulting position spectra for the x- and y-sensing directions, shown in figures 44(a) and (b) respectively, demonstrate that a "moment" effect is indeed taking place, and that the distribution of secondary avalanches may just be of the correct shape to explain the discrepancy between the values of $\sigma_{dx}$ and $\sigma_d$. This distribution must be of the same shape as shown in figure 44b, but must be much broader, for the points in this spectrum are the weighted centroids for each event. The shape is suitable to explain the $\sigma_{dx} - \sigma_d$ discrepancy if the large and narrow central peak is assumed to contain chiefly events which are large enough to exceed the gate threshold but are unaffected by secondaries. The discrepancy is then simply an effect of the increased "binning" in the x-direction (cf. the third factor in expression (28)); but note that this explanation is purely speculative as yet, for no quantitative work has been carried out. Should
Figure 44 Position spectra, for events much affected by secondary avalanches only. P10 gas, ~ 3 pC charge level. (a) sensing in x-direction (across anode wires); (b) sensing in y-direction (along anode wires).
it prove that the shape of the distribution deduced here is not suitable, it may still be possible to explain the discrepancy if the distribution in the x-direction is modified at its centre by a shadowing effect of the anode wire.

Figure 44a shows the interesting feature of two minima, adjacent to the presumed wire position, which are believed to arise as follows. The energy spectra of the sets of events affected and unaffected by secondaries must overlap quite considerably: thus it is probably the case that large unaffected pulses are allowed through the SCA gate, while some small affected ones are missed. Events only affected a little by secondaries will be "pulled" only a short way from the anode wire, and whereas small moments can be achieved in the y-direction by secondary avalanches occurring close to the primary, they cannot appear in the x-direction because the secondary avalanches will occur either on the avalanche wire or at least 9 mm away.

Some objections to the theory may be raised. It may be asked why, in figure 41, the intercepts for PIO and P50 are the same, if secondary avalanches are less likely in P50. Remember, though, that the PIO measurements were carried out at half the charge level used for P50, so that it is just possible that the frequency of secondaries is about the same in each case. A second and more serious objection arises from some separate linearity measurements, which effectively measured the "stepped-ness" of the position calibration, quantified by the rms deviation from a straight line. These gave agreement with the values of \( \sigma_{dx} \) from the other methods, but it is not yet understood how this can be the case: surely a position peak (large number of events) is not shifted by secondaries, as well as broadened? These measurements are admittedly few, but all the same it must be stressed that it is at present unsafe to conclude that secondary avalanches are completely responsible for the \( \sigma_{dx} - \sigma_d \).
discrepancy. It does, however, seem likely that they adversely affect the position and energy resolutions; although if they do affect linearity, they are not completely unwelcome, because this improvement is gained at little cost to $\sigma_y$ (see figure 45).

Further observations concerning the effects of ultraviolet photons have been made by Rose and Korff\textsuperscript{10}, Campion\textsuperscript{85} and Romero and Campos\textsuperscript{86}, and for a MWPC by Sims et al.\textsuperscript{43}.

![Figure 45](image_url) Charge cloud size required to produce a given linearity. Differential nonlinearity $\varepsilon_a$ as a function of $(\sigma_d/s)^2$ for $s = 2$ mm: (a) predicted; (b) observed.
PART III:
NEW POSITION READOUT METHODS
CHAPTER 7
SPATIAL RESOLUTION AND READOUT METHOD

i) Introduction

This chapter deals with the subject of position resolution in MWPC's, with particular reference to the various methods available for position encoding. The choice of method determines the "electronic" contribution to spatial uncertainty, as well as how this varies with detector size (an important consideration in view of present requirements). Detectors proposed for future missions are of larger active areas than previous models, and a knowledge of scaling properties would be of value.

Hitherto, all MWPC's for X-ray astronomy have employed RC lines for position encoding, with either risetime or ratio decoding systems; recently, however, some devices using other types of encoding have been proposed\(^4\),\(^3\),\(^6\), and it would be useful at this point to compare the various methods available. The following section is concerned with this, while the subsequent sections illustrate some of the more important "electronic" resolution considerations (with reference to the RC line), and examine the other contributions to spatial uncertainty.

ii) Available Position Encoding Methods

The position encoding methods used in MWPCs fall roughly into two categories, which may be labelled "global" and "local"\(^8\). In the first, a position signal is formed from the outputs of two preamplifiers, connected to either end of the cathode, while in the second, some form of selection is used to indicate upon which of many smaller electrodes the induced charges occur; the signals from these electrodes only are then used in a centroid computation.
Global Methods. The "line" methods fall into the global category. Besides the RC line, which has received much study to date\textsuperscript{16,17,88,89}, two other such methods exist, namely the delay line, which delays the arrival of a pulse passing along it and so allows location by zero-cross timing, and the series capacitance line, which attenuates a pulse travelling along it and affords a position signal by ratio methods. Of these, the delay line has been extensively used for ground-based work\textsuperscript{90,91}, but has not been employed for X-ray astronomy, probably in view of its more complex design\textsuperscript{92} (the cathode wires, for instance, are usually coupled to a delay line external to the chamber) and greater cost. The advantage of delay lines over RC lines is that the line itself contributes no noise (this only arises from the preamplifiers and line terminations\textsuperscript{93}), a property which allows better spatial resolution than possible with an RC line. The series capacitance line has not received extensive study for use in MWPCs, although it has been used by several groups in application to microchannel plate readout\textsuperscript{30,94}; once again, this is probably because a physical realisation of such a line would be rather clumsy. This type of line, however, should also show a smaller noise contribution to resolution than the RC line, and an investigation employing a MWPC cathode equipped with a capacitive line is reported in Chapter 8. Note that all these line methods require a high degree of parameter uniformity along their length, if good linearity is to be achieved.

Another member of the global category is the "progressive geometry" class of method, in which the charge induced on electrode segments whose area changes progressively across the cathode gives a measure of position, as in the "backgammon" cathode design of Allemand and Thomas\textsuperscript{95}. Cathodes of this type are examined in Chapters 9 and 10.
Local Methods. There are two types of "local" method. The first was developed at CERN\textsuperscript{23,68,96} and involves the division of the cathode grid into "strips", comprising five or six wires connected together, with each strip connected to an amplifier. The strips upon which most of the charge is induced are selected by a threshold setting on the amplifiers, and only the signals from these strips are used in the subsequent centroid computation (see figure 46). The justification for this procedure is that while each preamplifier gives a particular equivalent noise charge, only those receiving the induced charge contribute to the computation; and because the loading of the amplifier by a single strip is small, a large signal-to-noise ratio may be attained. Obviously, it is desirable to use as few separate signals as possible to maintain the best resolution, and a strip width corresponding to about one anode-cathode separation appears to be optimum\textsuperscript{74}. The selection of particular strips in this way means that if the number of strips is increased in order to make the cathode wider, no increase in noise experienced: this property is common to all local methods and contrasts with the situation existing for global methods. Because this "centroid" method has the disadvantage that it requires a large amount of data handling, its application has been somewhat limited\textsuperscript{74}; nevertheless, Reid et al. have proposed a chamber which employs the technique for cosmic soft x-ray imaging\textsuperscript{64}.

The second local method is in fact a special case of a subdivided line method (see below). Strips are again employed, but in this case the outputs from these strips are sequentially switched through a "centroid-finding filter"\textsuperscript{87,97}. Because of the sequential nature of the processing, this is essentially a timing method: the position of the charge centroid is given by the zero-cross time of an output signal, which is itself a result of convolving the special filter function with the output signal sequence. In terms of processing, this is somewhat simpler than the
Principle of the centroid method. The avalanche at a induces positive pulses on the cathode strips, b; the centre of gravity of these pulses gives the avalanche location. After Charpak et al.\textsuperscript{23}. 

\textbf{Figure 46}
"centroid" method described above, and again, noise from only three outputs enters the result; however, the global methods still remain the simplest. The latter are also superior in terms of speed, for the local methods require longer processing times.

**Subdivided Lines.** In the "centroid-finding filter" method of Radeka and Boie, the strips are separated by equal resistances, and to this extent the method can be regarded as that of a subdivided resistive line. This approach has been employed by several authors in an attempt to improve resolution over that of an undivided line, and is effectively an improvement in the sampling of the induced charge distribution. Preamplifiers are spaced at equal intervals (which need not be small), and their outputs, containing spatial information from the local resistive line elements, are weighted according to their position along the cathode. Centroid computation is carried out using all the preamplifier outputs, and the spatial resolution improves as the number of divisions, \( N \), increases.

The general case of the method of Radeka and Boie described above is a subdivided resistive readout system, in which several of the strips may be connected together (figure 47). The improvement over other such methods lies in the elimination of the redundant preamplifier outputs by the filter. The case described above is the limit as \( N \) becomes very large (once more, the resolution improves as \( N \) increases, so that this is a logical extension), so that a preamplifier is connected to each strip. The general centroid-finding-filter method may thus correspond to global (\( N=1 \)), local (one strip per amplifier) or an intermediate type of encoding. It should be remarked that if \( N \) is increased so that the strips are narrower than the optimum mentioned above, the resolution begins to degrade: resolution improves as \( N \) increases up to a limit.

The subdivided systems, then, do offer position resolution superior to that offered by undivided ones, but at the cost of increased complexity, and may therefore prove unsuitable for astronomical applications. Thus,
Figure 47  The "centroid-finding filter" approach (from reference 87). The zero-cross time \( t_{fo} \) of the filter output is a continuous and linear function \( t_{fo} \) of the avalanche position, \( x \).
while systems superior in performance to RC line readout do exist, the latter has been recommended for X-ray astronomy by its simplicity.

Table 8 summarises how the fractional position resolution varies with detector size (for square detectors) and, where applicable, number of subdivisions.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\Delta x$ increases as</th>
<th>Table 8. Resolution, with various readout methods, vs. size (assuming $C = L^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delay</td>
<td>$L^{1+2}$ (then $L^2$)*</td>
<td>* when preamplifier noise becomes dominant</td>
</tr>
<tr>
<td>RC</td>
<td>$L$</td>
<td></td>
</tr>
<tr>
<td>Subdivided RC</td>
<td>$L, N^{-1/2}$</td>
<td></td>
</tr>
<tr>
<td>Centroid-Finding Filter</td>
<td>$L, N^{-3/2}$ $^{1/2*}$</td>
<td>** strip width less than optimum.  (Taken from references 23, 93 and 99)</td>
</tr>
<tr>
<td>Centroid</td>
<td>$N^{1/2*}$</td>
<td></td>
</tr>
</tbody>
</table>

iii) Spatial Resolution

This section introduces the quantities which are important in discussion of the spatial resolution obtainable by the global class of readout methods, of which the systems described in the following chapters are members. The particular example of RC line readout is also examined, as a guide to the performances previously available.

Resolution Expressions. The limiting spatial resolution obtainable with a particular readout system depends upon the total electronic noise associated with the delivery of the signal to the preamplifier output (this includes, for example, preamplifier noise and line thermal noise). This "front-end" noise may be described quantitatively by an equivalent noise charge $q_n$. Further, any expressions of spatial resolution must also account for the noise correlation in the system, and its dependence on...
position, $x$ (where $x$ is the position along the readout electrode, expressed as a fraction of the electrode width: $0 \leq x \leq 1$). Thus the rms spatial uncertainty $\Delta x$ is written as:

$$\Delta x = D \left( \frac{q_n}{q_D} \right)$$

(30)

where $q_D$ is the total signal charge input to the (two-channel) system and $D$ is a dimensionless resolution parameter, which must take into account the noise correlation and the form of the position algorithm.

Noise correlation can be defined as the extent to which the signal fluctuations occurring at one preamplifier input influence those occurring at the other, and is expressed quantitatively as the noise correlation coefficient, $r_{ab}$:

$$r_{ab} = \frac{\langle dq_a dq_b \rangle}{q_n^2}$$

(31)

where $q_a$ and $q_b$ are the preamplifier input signal charges, having fluctuations $dq_a$ and $dq_b$ respectively. The bars denote mean values, and it is assumed that $\overline{dq_a^2} = \overline{dq_b^2} = q_n^2$. The value of $r_{ab}$ is zero if the signal fluctuations are independent, unity if they are identical (complete correlation) and $-1$ if they are equal and opposite (complete anticorrelation). In all two-terminal readout systems, $0 \leq r_{ab} \leq 1$.

If the position algorithm is $Q = q_b/(q_a + q_b)$, as employed for all the experimental work reported in Part III, then because $q_a + q_b = q_D$,

$$D(x) = \frac{1}{S(x)} \cdot \left[ 1 - 2(1 + r_{ab}) Q(1 - Q) \right]^{1/2}$$

(32)

where $S(x) = dQ(x)/dx$, the system sensitivity. If $S = 1$, then $Q = x$, so that $D(0) = 1$ and $D(0.5) = \left[ 1 - (1 + r_{ab})/2 \right]^{1/2}$.

A knowledge of $q_n$, $r_{ab}$ and $S$, then, allows evaluation of the rms electronic spatial uncertainty $\Delta x$ at any location.
Experimentally, the noise may be measured by injecting a step function of charge (using a pulse generator and standard capacity) into the input of the loaded preamplifier; because the equivalent noise charge may also be regarded as a step function from the point of view of the filter network, the output voltage signal-to-noise ratio is the same as the input charge signal-to-noise ratio. Thus, from a determination of this ratio and a knowledge of the input charge, the value of $q_n$ may be found.

Experimental determination of the correlation coefficient is carried out as follows. The mean square noise in the denominator signal

$$q_D = q_a + q_b$$

is

$$\overline{dq_D^2} = 2q_n^2 + 2r_{ab}q_n^2,$$

so that

$$2(1 + r_{ab}) = \frac{\overline{dq_D^2}}{q_n^2}$$

(33)

Thus, if the equivalent noise charge $\sqrt{\overline{dq_D^2}}$ is measured at the output of the summing unit which produces the denominator signal, $r_{ab}$ can be found. If this is carried out by injection of a known charge into one of the preamplifiers $r_{ab}(x)^8$ takes the value corresponding to $x = 0$ or 1.

To summarise, then, the value of the "electronic" position uncertainty is conveniently found, provided the induced charge is known, by measurement of the rms equivalent noise charge and of the correlation coefficient.

**Example: RC Line Performance.** Several theoretical descriptions of RC line behaviour have appeared in the literature to date\textsuperscript{16,88,89}, but a most comprehensive and accessible treatment is that of Fraser et al.\textsuperscript{79,80,100}, carried out in this Department and extending the earlier work of Mathieson et al.\textsuperscript{77,101,102,103}. The general procedure is as follows. Firstly, the voltages at the two ends of the RC line are calculated, as a function of time, $t$, and injection position, $x$, given an initial excitation by a delta-function of charge (of magnitude $q_D$) at $t = 0$. (Remember that $0 \leq x \leq 1$, i.e. $x$ is expressed as a fraction of the line length, $L$.) The form of $V(x,t)$ depends upon the particular line model chosen to
represent an element $dx$ of the line, and any combination of resistances and reactances, in series or parallel, may be chosen. For simplicity, Fraser et al. selected the model shown in figure 48a, which assumes negligible stray capacitance across $R$ and negligible conductance across $C$, normally quite a realistic approach.

**Figure 48** (a) Model element of RC line for analysis; (b) RC transmission line and processing electronics, after Fraser et al.

Other models had previously been examined by Mathieson et al., who found some evidence, in that particular case, that a "longitudinal" capacity existed in parallel with $R$, but that any shunt conductance to ground was probably small. Some examination of inductance was also made, and for the particular lines concerned, its effect was also found to be small. The flexible and more general model used in that paper will be mentioned during discussions in the following chapter.
Having calculated $V(x,t)$ (taking into account the line terminations $C_d$ - see figure 48b), this waveform may be injected into a filter network of known response, and the resulting output, $V_2(x,t)$, found by standard transform methods. The position signal (be it difference zero-cross time or amplitude ratio) may then be found, because $V_2(x,t)$ is calculable for all $x$ and all necessary values of $t$. Knowledge of the position signal $Q = Q(x)$ completely describes the system behaviour, for, provided the equivalent noise charge is known, all subsidiary parameters may be calculated from it. By comparison of the position signal at adjacent values of $x$ the local sensitivity $S(x)$, and hence its mean value $S_m$ over a particular length of line, can be found.

Position uncertainty may be found by considering the line thermal noise, quantified as an equivalent noise charge. If $q_n$ is known, it may be used with equation (30) to find the resultant spatial resolution, given a value of $r_{ab}$. If the preamplifier noise is not negligible, it must be added quadratically to the line thermal noise in order to form $q_n$.

The thermal noise generated by a resistive device of capacity $C$ to ground is given generally by

$$q_n = [kT_e C]^{1/2} \mu,$$

where $k$ is the Boltzmann's constant, $T_e$ is the temperature of the device and $\mu$ is a dimensionless parameter which depends upon the details of the particular experimental configuration. $\mu$ is a function of $R$, the total line resistance, $C_d$, the termination capacity, $T_a$, the processing time constant of the shaper-amplifier, and $C$. The calculations of Fraser et al. are founded on analytical predictions of $\mu$ and $r_{ab}$ (note that they apply to the particular case of bipolar shaping (single integration and double differentiation).
iv) Other Contributions to Position Uncertainty

In addition to consideration of the signal-to-noise characteristics of the processing electronics, several "non-electronic" factors influence the position resolution of a MWPC. These may be listed as: the range of the initial photoelectron in the gas, the "centroid jitter" of the primary electron cloud, the effect of secondary avalanches, and the effect of an inclined X-ray beam. Of these, the subject of centroid jitter has been dealt with, in some detail, in Chapter 6; its effect may be assessed by use of equations (25) and (29). (Of course, the number of primary electrons, appearing in equation (29), affects the resolution in another way, because the signal size is proportional to this number, thus determining the electronic contribution. The subject of secondary avalanches was also considered in Chapter 6, and the effect of an inclined X-ray beam has already been described (Chapter 2; see also reference 43). Some discussion is now given of the remaining factor, the photoelectron range.

**Photoelectron Range.** When an X-ray, of energy $E_X$, say, is absorbed by a gas atom, a photoelectron is ejected from the atom with a kinetic energy $E_p = E_X - E_B$, where $E_B$ is the binding energy for the atomic shell concerned. If $E_B$ is large, $E_p$ is small, and the remaining energy appears as the atom de-excites, by emission of an Auger electron or fluorescent photon. Fortunately, in the case dealt with in these experiments (Al-K X-rays and argon gas mixtures), the highest energy level (the L-shell) is concerned, with the result that 83% of the X-ray energy is converted into kinetic energy of the photoelectron. Description is thus simpler than for the case where $E_B$ is large (see below).

In assessing this particular contribution to resolution, two questions must be answered: firstly, what is the range-energy relationship for the electrons, and secondly, how does this affect the position resolution?
Data is scarce with regard to the first of these: cloud-chamber measurements by Williams\textsuperscript{104} refer to a higher energy than concerned here, while measurements using thin foils have been carried out by other workers\textsuperscript{105,106,107}.

Cloud chamber measurements\textsuperscript{104} show that the path of a photoelectron in the gas is quite tortuous, with energy loss and change of direction occurring at each collision, and that this process may be usefully represented by assuming an "extrapolated range" $R_{\text{ex}}$, equivalent to the furthest point of travel from the original absorption position. The rate of energy loss with distance may be assumed to be constant, so that a uniform line of charges may be considered to have been deposited along $R_{\text{ex}}$; the mean distance from the origin is then $R_{\text{ex}}/2$. The quantity $R_{\text{ex}}$ is suitably represented by a semiempirical range-energy relationship of the form

$$R_{\text{ex}} = C E^\frac{n}{p},$$

where $n$ and $C$ depend upon the particular model chosen (note that different values of $n$ and $C$ are necessary for different energy ranges).

It remains to be seen how a particular $R_{\text{ex}}$ affects the position resolution. Sauli\textsuperscript{108} points out that the randomising effect of multiple collisions justifies the assumption that all directions of $R_{\text{ex}}$ are equally likely; that is, for a large number of absorption events, the centroids of the ionisation tracks are distributed spherically. This case is represented in figure 49, where the spherical surface (of radius $R_{\text{ex}}/2$) is the locus of the track centroids, assuming that all events occur at the point $p$ (this is not, of course, a realistic assumption, but variation in the $y$ or $z$ directions makes no difference to location in the $x$-direction). What we are now interested in is the variance in the location of $p$ caused by this spherical (rather than punctual) distribution. Let the fraction of all centroids having coordinates between $x$ and $x\,\text{dx}$ be $n(x)\,\text{dx}$; this is the same as the fraction lying on the annulus defined by $\theta$ and $\theta + \text{d}\theta$.
Figure 49 Calculation of the effect of a spherical distribution of charge centroids, for position sensing in the x-direction (the smaller diagram of the MWPC indicates the coordinate axes).
(these symbols are defined in the diagram). Thus

\[ n(x) = \frac{1}{2} \sin \theta \ d\theta . \]

Now since \( x = \left( \frac{R_{ex}}{2} \right) \cos \theta \), we also have

\[ \frac{1}{2} \sin \theta \ d\theta = -\frac{dx}{R_{ex}} , \]

where the negative sign merely embodies the decrease in \( x \) with increasing \( \theta \). Thus,

\[ n(x) = \frac{1}{R_{ex}} , \]

that is, the distribution is a rectangular one, of width \( R_{ex} \), and the standard deviation is thus \( \sigma_p = \frac{R_{ex}}{2\sqrt{3}} \). Sauli\(^{108}\) quotes \( \frac{1}{2} \sqrt{3} R_{ex} \), but does not justify this: further, our own experimental results cannot agree with such a figure, and tally more closely with the expression derived above.

The relationship between energy and the contribution of the photo-electron range to position uncertainty may now be written:

\[ \sigma_p = a \frac{E_p^n}{\rho} , \quad (34) \]

where \( \rho \) is the gas density in g/l, \( \sigma_p \) is given in \( \mu \)m and \( E_p \) in keV, and the values of \( a \) and \( n \) are found from the available literature (table 9a). For the case of Al-K (1.5 keV) X-rays, the resulting values

<table>
<thead>
<tr>
<th>Energy Range (keV)</th>
<th>n</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Williams</td>
<td>7.5 - 20</td>
<td>1.8</td>
</tr>
<tr>
<td>Kanter</td>
<td>0.1 - 10</td>
<td>1.54</td>
</tr>
<tr>
<td>Tabata et al. (i)</td>
<td>1 - 5</td>
<td>1.3</td>
</tr>
<tr>
<td>Tabata et al. (ii)</td>
<td>5 - 20</td>
<td>1.64</td>
</tr>
</tbody>
</table>

| Gas          | p(g/l) | | | | | | |
|--------------|--------|---|---|---|---|
| P10 | 1.78 | 1.22 | 0.72 |
| P50 | | | |
| CH\(_4\) | | | |

| Williams | 12 | 17 | 30 |
| Kanter | 33 | 47 | 80 |
| Tabata et al. (i) | 29 | 42 | 71 |
| Experiment | 21 | 29 | - |
of $\sigma_p$ for the gas mixtures PIO, P50 and CH₄ are shown in table 9b, again for the various models. Corresponding experimental values of $\sigma_p$, obtained with the high-sensitivity electrode and therefore effectively free from the effects of noise, are also given for PIO and P50: they fall within the range covered by the various estimates. It must be stressed, however, that the accuracy of the experimental values is not great, because the effect of photoelectron range was not the dominant contributor to the uncertainty.

Above the argon K-edge, it is thought that the value of $\sigma_p$ actually falls, and passes through a minimum before continuing to increase. This is expected because for most X-rays (88%)⁶³ having energies $E_X$ in excess of the K-shell binding energy $E_K$ (= 3.2 keV for argon), two electrons are produced, whose total energy is approximately $E_X$. One of these is a photoelectron of energy $E_X - E_K$, ejected from the K-shell by the X-ray interaction, while the other is an Auger electron, of approximate energy $E_K$, liberated as the absorbing atom de-excites. Each of these electrons makes a contribution to spatial uncertainty according to equation (34); if we write these as $\sigma_a$ for the Auger electron and $\sigma_b$ for the K-photoelectron, the resultant uncertainty is given by¹⁰⁹:

$$\sigma_p^2 = \left[\sigma_a^2 E_K + \sigma_b^2 (E_X - E_K)^2\right]/E_X^2,$$

a function which minimises when $E_X = 2E_K$, i.e. $\sigma_a = \sigma_b$. In that case, $\sigma_p$ has fallen to $1/\sqrt{2}$ of its value at $E_X = E_K$.

Beam Width. All factors so far discussed are those which cause uncertainty in the location of an infinitesimally narrow beam of X-rays; but it is, of course, impossible to produce such a beam in practice. To maintain useful intensities, and because of mechanical limitations, it was not possible in this work to employ a collimation finer than that described in Chapter 3 (two slits, -50 μm wide, separated by ~4 cm).
A very rough estimate of the \( \text{rms} \) contribution of beam width to the position uncertainty may be arrived at by examining the beam intensity profile. This was carried out by first allowing the X-ray source to "warm up" and achieve roughly constant intensity, and then traversing the source in steps of \( -12.5 \mu m \) (half the smallest graduation on the traversal control) across a straight-edge provided by the design of one of the lids. At each position, the intensity (averaged over one minute) was measured and the results used to obtain an approximate beam profile, of \( \text{rms} \) width about \( -30 \mu m \). To account for the slight beam divergence, and allowing that the above method is rather inaccurate, a value of \( 40 \mu m \) (\( = \sigma_W \), say) was assumed in subsequent calculations.

Divider Unit. In addition to the above factors, imperfections of the post-preamplifier processing electronics also contributed to the spatial uncertainty: in particular the denominator-dependence of the divider unit output (mentioned in Chapter 3) was of importance. If this effect contributes \( \sigma_r \) to the uncertainty in the output ratio, the contribution to position uncertainty is \( \sigma_r / S \), where \( S \) is the system sensitivity. An estimate of \( \sigma_r \) could be made by examination with a pulse generator, and for the present system, this varied between \( 110 \mu m \) and \( 30 \mu m \) (\( \text{rms} \)).

iv) Summary

This chapter has attempted to compare existing available readout methods for MWPCs in terms of their position resolution, indicating the need for a readout system equal in simplicity but superior in resolution to the RC line method. This is important in view of the present development of larger-area detectors. To aid discussion of the subject of position resolution in subsequent chapters, the electronic contribution to spatial uncertainty has been examined briefly, with particular reference to the RC line method of readout, and the other important contributing factors have
been described. The following chapters will deal with some readout methods which may fulfil the requirement for a superior but simple readout technique.
CHAPTER 8
THE CAPACITANCE - RESISTANCE LINE

i) Introduction

One particular position readout method which may fulfil the requirement of superior performance to that of an RC line, with equal simplicity, is the use of a series-capacitance ("CR") line (figure 50). It has already been pointed out that present practical CR lines are rather clumsy; however, should it prove that much advantage may be gained by use of a CR line, it is possible that with further development this limitation may be overcome.

Figure 50  The series capacitance line, showing the various important parameters.
Previous realisations of such lines$^{30,94}$ were restricted to applications with microchannel plate detectors, although investigations showed that at least moderate resolution and linearity could be obtained. The present work appears to be the first attempt to examine the performance of a CR line in a MWPC, with a view to exploiting its major advantage over the RC type of line, namely the absence of significant thermal noise. A second advantage concerns the peculiar behaviour of the RC line with respect to angular localisation, as described in Chapter 5: Fraser$^{100}$ points out that this occurs because the natural line unit of time, $RC/\pi^2$, is short in comparison with the pulse development time. With a reasonable choice of parameters for the CR line, the corresponding time unit should be much longer.

The theory of CR line operation is summarised separately in Appendix 1, and it is also convenient at this point to refer to Appendix 2, in which the effects of resistive and capacitive loads on preamplifier noise are dealt with.

ii) Characteristics of the CR Line

**Line Model.** An ideal CR line consists of a large number, $N$, of discrete capacities, of value $C_1$, connected in series, with large resistances, $R_1$, providing charge leakage paths between each node and ground. The resultant series capacity is denoted by $C_s (= C_1/N)$ and the total shunt conductance by $G (= N/R_1)$. In practice, the situation is complicated by the presence of the grid capacity to ground, $C$, and the representation becomes that shown in figure 50. Provided $N$ is large enough that the various parameters may be considered as continuously-distributed, the generalised model reported by Mathieson et al.$^{77}$ may be employed to predict the line behaviour (see reference 110); however, it is possible to arrive at the same results by the simpler and more rigorous method (considering the line as a cascade of $\pi$-section filters) given in Appendix 1.
Linearity. If $T_a$ is the time constant of the electronic processing system, then provided that $T_a \ll C/G$, it can be shown (see Appendix 1) that the charge $q_B$ appearing at one of the line terminations can be approximated by a temporal step function of size:

$$q_B = q_D \sinh ax / \sinh a, \quad (35)$$

where $a = (C/C_\|)^{1/2}$, $q_D$ is the charge injected into the line and $x$ is the position of injection (as before, $0 \leq x \leq 1$). Because no information is contained in the risetimes of the signals reaching the preamplifiers, pulse amplitude ratio encoding has to be employed, and the position signal is $Q(x) = q_B/(q_A + q_B)$.

The cause of nonlinearity in a CR line system is the finite value of $a$, i.e. the presence of the shunt capacity to ground, $C$, which is uniformly distributed along the cathode: if $C$ is reduced in relation to $C_\|$, the nonlinearity is also reduced. Readout nonlinearity may be quantified by means of the rms deviation, $\delta$, of a set of data points from the best-fitting straight line (note that this is, in practical cases, only available from discrete data points, rather than a continuous curve). Obviously, the value of $\delta$ depends on which data points are included; for instance, the nonlinearity may be smaller over the central half of the line than over its whole length. Figure 51 indicates how the rms (or "integral") nonlinearity falls off as $C_\|/C$ increases, showing the results of numerical calculations in which either the central two-thirds of the line, or all of the line, are considered. For comparison, the rms nonlinearity obtainable with a 50 kΩ RC cathode ($C = 30 \text{ pF}$), at $T_a = 1 \mu s$, is about 0.05%; with a 250 kΩ line, this figure would be about 0.6%.

Resolution. A discussion of resolution performance is basically one of electronic noise. The treatment here is not as simple as that outlined in the previous chapter, because in this case the noise generated by the loaded preamplifier is dominant. It may be shown (see Appendix 1)
Figure 51  System rms nonlinearity, expressed as a percentage of the total line length, as a function of $C_l/C$. 
that the present line may be approximated by a resistance \( R_e \) in parallel with a capacity \( C_e \), where:

\[
R_e = \frac{G}{4} \left( \frac{\sinh^2 \alpha}{\sinh 2\alpha - 2\alpha} \right) \quad \text{and} \quad C_e = C_e \cdot \frac{\alpha}{\tanh \alpha}. \tag{36}
\]

These quantities may be used in conjunction with the expressions A2.3 and A2.4 given in Appendix 2 to evaluate the effect of the various parameters on the electronic noise of the system. Note that for a fixed \( C \), \( R_e \) increases with \( \alpha \), but \( C_e \) decreases as \( \alpha \) increases; thus, if we try to improve linearity by increasing \( \alpha \), the value of \( \alpha \) falls, and both \( R_e \) and \( C_e \) change so as to increase the equivalent noise charge. A compromise is thus necessary: for instance, at a \( C_e/C \) value close to unity, the rms nonlinearity over the central 67% of the line should only be about 0.2%, with \( C_e \) being approximately the same as \( C \). In such a case, the resolution obtainable should be superior to that of any RC lines giving comparable linearity (e.g. \( \delta < 0.5\% \)). Note that because preamplifier noise is now a major consideration, the performance of the particular type chosen becomes important. It is desirable to use preamplifiers whose load lines (see Chapter 3) are not steep, and for this reason the Canberra 2001 A model was selected for experimental work.

On the basis of the calculations described above, then, the CR line seemed to have some potential as a rival to the RC line, and experiments were therefore carried out to examine the performance of a real line.

iii) Fabrication of the CR Line

Physical Structure. A prototype system was produced by grouping the wires of a normal 1 mm -pitch cathode in fours, and connecting each group to a small and isolated copper contact, located on a second G10 frame. Twenty-four such contacts existed on the second frame (placed immediately below the grid frame), and successive contacts were linked by physically
small capacitors, each of value 1250 pF. Each contact was also connected via a 22 MΩ resistor to a common rail, which itself ran to the drift voltage supply. There were twenty-two groups of wires in all, each of four wires, except for the two outermost groups, which were of five wires each. The remaining two contacts allowed for the addition of terminating capacities at each end. The complete assembly is shown in figure 52, where the bulkiness compared to an RC-type grid is noticeable. As already mentioned, however, present technology may well allow the solution of this problem.

Noise Discrepancy. The values \( N = 22, C_1 = 1250 \text{ pF}, R_1 = 22 \text{ MΩ} \) and \( C = 30 \text{ pF} \) lead one to expect values of \( R_e \) and \( C_e \) of \( -3.4 \text{ MΩ} \) and 62 pF respectively. For the Ortec copy preamplifiers used in the initial testing, the equivalent noise charge was therefore expected to be about 1650 electrons rms; in fact, a value nearer to 2000 was obtained. This was resolved by measurement of the resistance and capacitance values using a Q-meter* operating at an angular frequency \( \omega = 10^6 \text{ rad/s} \) (\( f = 160 \text{ kHz} \)), which corresponded to a filter time constant \( T_a = 1 \mu s \). The following table compares the a.c. and d.c. values. The noise discrepancy is

<table>
<thead>
<tr>
<th>( C_e ) (pF)</th>
<th>( G ) (( R^{-1} ))</th>
<th>( R_e ) (kΩ)</th>
<th>d.c</th>
<th>a.c. (160 kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-66</td>
<td>( -10^{-6} )</td>
<td>-∞</td>
<td>-66</td>
<td>-66</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>( -1.67 \times 10^{-6} )</td>
<td>600</td>
</tr>
</tbody>
</table>

Table 10. Comparison between certain line impedance parameters, as measured at low and radio-frequencies.

* This was a Marconi model WY0282 Q-meter, which gave values of \( C \) and \( Q \) (here \( Q = fR_C \)) for the particular frequency.
Figure 52  The CR cathode, showing the arrangement of capacitors and resistors on a separate support frame.
thus seen to arise from the non-infinite and frequency-dependent leakage resistances across the resistors and capacitors employed, which not only alter the value of the shunt conductance $G$, but also produce a non-infinite value of the series resistance, $R_x$.

Accordingly, the 22 MΩ resistors ($R_1$) were each replaced by one of 100 MΩ, and the 1250 pF capacitors by some of higher quality (these were each 660 pF, giving a smaller value of $C_x/C$), with the results that the value of $G$ was $-2.2 \times 10^{-7}$, and that of $R_x$ exceeded 1 MΩ. The other line parameters, measured at 160 kHz, now became $C_x = 34$ pF, $C_e = 47$ pF, $R_e = 3.7$ MΩ and $C = 41$ pF, so that with the superior Canberra 2001A preamplifier, the expected noise charge was about 500 electrons rms (with no allowance for the non-infinite value of $R_x$). The noise charge then measured was about 600 electrons, the discrepancy presumably being attributable to the non-infinite $R_x$ value.

iv) Performance

Resolution. The Al-K X-ray beam was fully collimated for these measurements, and figures for the spatial resolution at various points along the cathode length were determined by allowing a one-dimensional image to build up on the multi-channel analyser at points 5 mm apart, between 1.5 and 7.5 cm from one edge of the cathode. The FWHM of each peak could be easily measured using the digital display facility of the Canberra Series 30 MCA, and converted to a real distance using the system sensitivity, in channels per millimetre (CH/mm), inferred either from all data points (mean sensitivity) or from the two adjacent points (local sensitivity).

A method was devised of ensuring beam verticality, to within a few degrees. It depends on the sensing cathode being the lower one, with its wires running perpendicular to those of the anticoincidence electrode, as well as to those of the anode. When the beam is vertical, avalanches
should occur at the same coordinate (in the sensing direction, say \( x \)) on both grids, provided that some absorption events do occur below the lower cathode. Both sets of avalanches cause charges to be induced on the sensing cathode, so that by appropriate gating of the system, the position of either set can be found, any discrepancy indicating an inclined beam. The method of correction is rather approximate: it is possible to place small pieces of melinex sheet (~0.1 mm thick) under part of the traversing-table assembly before bolting it together, in such a way that the source-holder is tilted with respect to its previous alignment. The amount of inclination achievable on this way is small, but the deviation from verticality is not usually large to begin with. One requirement for this method is that the position readout should not be locally nonlinear to any large extent; for instance, an abnormally low sensitivity makes accurate alignment difficult.

This precaution meant that the remaining contributions to spatial uncertainty were photoelectron range (\( \sigma_p \sim 30 \mu m \)), beam width (\( \sigma_w \sim 40 \mu m \)), centroid jitter and electronic contributions. Of the latter type, the front-end contribution expected from the measured noise charge of 600 electrons was \( \sigma_e = 12.2/q_0 \mu m \), where \( q_0 \) is the measured anode charge (i.e. twice the cathode charge), expressed in picocoulombs. This estimate is based on the predicted value for the correlation coefficient, \( r_{ab} \), of zero (see Appendix 1), although this figure has yet to be confirmed experimentally. The value of \( D \) (see equation (30)) for \( x = 0.5 \) has been used, and the total line length is 9 cm, as for the RC line.

Measurements already reported\(^\text{110}\) have indicated an average rms position uncertainty of 170 \( \mu m \) rms, but were carried out using a 6 mm drift region and a poorly-adjusted divider unit. These effects contributed approximately 60 \( \mu m \) (centroid jitter, \( \sigma_D \)) and 100 \( \mu m \) (denominator-dependence, \( \sigma_T \)) respectively to the uncertainty. Subsequent measurements, however, improved upon these, by restricting the range of pulse heights
input to the divider to about 10\% of the average (using a SCA to provide
a gate signal) and by eliminating the drift region. The total uncertainty
in this case was only about 90 μm, and subtracting (quadratically) the
contributions of beam width, photoelectron range, centroid jitter (now about
30 μm) and front-end electronic noise, the contribution of the denominator-
dependence of the ratio unit was estimated to have been reduced to approxi-
mately 65 μm.

For these measurements, P50 gas was used, the charge level was 1.5 pC,
and the signal shaping was bipolar, with time constant 1 μs.

Figure 53 indicates the resolution to be expected at a particular anode
charge level, based on the above figures. At large charges, the combined
constant contributions from effects other than front-end noise dominate,
while the latter is more important at small charge levels. The broken line
gives the value of the front-end contribution alone.

Event Rate. One possible drawback to use of the CR line is the long
charge leakage time, -C/G, which is necessary for a charge deposited on
the cathode to leak away to ground. If the event rate is high, the charge
will tend to "pile up" and cause small fluctuations in potential, and
possible distortion of readout. With the present source, using only the
lower collimation slit, an event rate of \(-2.5 \times 10^3\) s\(^{-1}\) was attainable,
which is less than the value of G/C for this particular line (G/C \(-5.4 \times
10^3\) s\(^{-1}\)). No displacement or broadening of the position output was
observed up to this rate, although of course further testing is desirable.

Linearity: Integral and Differential. With the improved line, the value
of \(C_a/C\) was 0.83 (\(a = 1.45\)), so that according to figure 51 the rms
nonlinearity should be about 0.76\% over the whole line, and about 0.26\%
over the central two-thirds. Because of signal loss at the edges of the
cathode it was not possible to measure the former quantity, but a scan over
the central 6 cm in 5 mm intervals allowed measurement of the latter,
with a result, 0.3\%, which agreed well with prediction.
Figure 53 Expected rms position uncertainty (μm) as a function of anode charge level, based on the data given in the text.
A more demanding test of readout linearity is the examination of the differential nonlinearity $\varepsilon$. This quantity has already been introduced in connection with the position "binning" caused by the anode wire spacing (see Chapter 6), and may be used instead to highlight the nonlinearities of the readout system (this time sensing in the direction parallel to the anode wires). The value of $\varepsilon$ may be determined as before, as

$$\varepsilon = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{max}} + N_{\text{min}}}$$

where $N$ is the number of counts in a given PHA output channel, in the system response to uniform irradiation.

It is important to remember that what appears on the screen is

$$\frac{1}{S(Q)} = \frac{dx(Q)}{dQ}.$$  

The sensitive nature of the parameter $\varepsilon$ will be fully demonstrated in later chapters, but it is instructive to examine the present situation.

Figure 54 shows the PHA spectrum resulting from illumination of the detector

![Figure 54](image)

**Figure 54** Local sensitivity function $dx/dQ$, for the present CR line, compared with that of a 250 kΩ RC line. These are the respective responses to uniform irradiation.
(with CR line) by the "quasi-uniform" method described in Chapter 6, and indicates that the sensitivity \( S(x) \) falls off towards the edges, with a maximum value in the centre, as expected. In this case, \( N_{\text{max}} = 31650 \) and \( N_{\text{min}} = 26320 \), so that \( \varepsilon = 9.2\% \).

v) **Comparison with the RC Line.**

The object of this examination has been to discover whether or not the CR line is a suitable alternative to the RC line. It certainly does seem advantageous, from several points of view, to employ the CR line in the present detector. Table 11, for instance, shows that while the CR line nonlinearity is rather greater than that of the RC line, the performances are more importantly different in terms of electronic noise. The spatial resolution is given for a low anode charge level (0.01 pC) in order to emphasise the contribution of electronic noise alone.

One further advantage of the CR line is that it does not suffer from the same effects of angular localisation risetime variation as the RC line, as

<table>
<thead>
<tr>
<th></th>
<th>RC</th>
<th>CR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise (electrons, rms)</td>
<td>1750</td>
<td>600</td>
</tr>
<tr>
<td>Spatial uncertainty (( \mu \text{m}, \text{rms at 0.01 pC} ))</td>
<td>4840</td>
<td>1220</td>
</tr>
<tr>
<td>Linearity: integral (( \delta ))</td>
<td>&lt;0.01%*</td>
<td>0.3%</td>
</tr>
<tr>
<td>differential (( \varepsilon ))</td>
<td>2.1% **</td>
<td>9.2%</td>
</tr>
</tbody>
</table>

Table 11. Noise and linearity of the present CR line, compared with optimal RC line figures. RC line data is from the work of Fraser et al.\(^{79, 80}\) except **, an experimental result; all apply for \( R = 250 \text{ k}\Omega \) except *, which is for \(-30 \text{ k}\Omega < R < -1 \text{ k}\Omega \). Both sets refer to bipolar shaping, with a time constant of 1 \( \mu \text{s} \).
is demonstrated by comparison of figure 55 with figure 35 (Chapter 5). Both sets of readings were taken under the same conditions: a charge level of 0.05 pC, processing time constants of 1 μs and a fractional line position of \( x = 0.75 \), with P50 gas and no drift region. The "turnback" of the position signal, evident in the case of the RC line, is absent from figure 55, indicating that it may be possible to use the CR line to exploit angular localisation information, without the ambiguities which arise from RC line encoding. As already noted, the problems arise in the case of the RC line because the pulse development time is long in comparison with \( RC/\pi^2 \) (\( RC/\pi^2 = 0.15 \) μs for \( R = 50 \) kΩ and \( C = 30 \) pF, but much of the pulse development occurs within the first 5 μs). In the case of the CR line the corresponding time unit is \( C/G \), or about 180 μs for the present values.

![Figure 55](image)

**Figure 55** Typical "stepped" position calibration curve for the x-sensing direction (perpendicular to the anode wires), taken with the CR line under the same conditions as figure 35.
vi) Scaling Properties

The advantages, then, of the CR line are its noise performance and its immunity to the risetime effects of angular localisation which afflict RC line systems, while its disadvantages are its poorer linearity and physical unwieldiness. It remains, in view of the general aims of the present work (namely, to develop readout systems for larger detectors), to discuss the scaling properties of the CR and RC systems. Once more it will be assumed that \( C = L^2 \) where \( L \) is the detector length (of a square detector) and \( C \) is, as usual, the grid capacity to ground. Of course, above a certain size, the two systems behave identically in terms of noise, as the preamplifier capacitive loading becomes dominant (see Appendix 2). In that case, both systems also begin to suffer from large nonlinearities, and operation becomes increasingly difficult. In the intermediate regime, the CR line parameters can always be chosen to produce superior resolution, because the RC line always suffers from the additional effect of thermal (line) noise. The preamplifier noise is likely to be similar for the two cases. Note that in either case, the values of the various parameters can be adjusted so as to reduce noise, although such adjustments must affect linearity. It is in this last respect that the RC line may be preferred, for instance in an application where linearity is important.

In conclusion, we can say that if the CR line can be made physically more compact, it is very likely to be preferred to the RC line in applications such as the present one. Some reservations must be made, however, concerning event rate and linearity, and further work on these topics is to be recommended.
The novel MWPC readout methods described in this and the following chapters constitute another step in the search for a compact, simple and low-noise system. The "graded-density" (GD) cathodes can provide the lowest noise charge values of any available global methods (see next chapter), but, as in the case of the CR line, the cost is once more a reduction in linearity. The GD cathodes also suffer from no detrimental effects of angular localisation.

i) Principles of Operation

The graded-density method is one of the group of progressive geometry methods mentioned in Chapter 7. The principle of such methods is that the charges on the active electrodes are varied simply by varying the area presented to the received charge. The ratio of area densities of each electrode varies with position, and the ratio of the charges received on each electrode, which varies in the same way, can give unique position information.

Allemand and Thomas\textsuperscript{95} have achieved this by wedge-shaped segmentation of a solid electrode (the "backgammon device"), but in the present case, of a constant-pitch wire cathode, the wires are arranged in two groups, such that the linear density (number per unit distance of one group) falls with position across the cathode, while that of the other group increases (see figure 56). For the present application, this structure has two advantages over that of Allemand and Thomas. Firstly, it is electrostatically and physically equivalent to a normal wire cathode, so that a drift region may be employed as usual, and secondly, it is very simple to construct (see section iii).
The attraction of progressive geometry methods is that no resistive element is necessary, while the preamplifier loading remains small, resulting in reduced front-end noise and, therefore, improved spatial resolution. All such methods, however, require that the spread of the received charge should exceed the width of the widest part of the electrodes; otherwise, the position signal does not uniquely define position. In the case of MWPCs, the induced charge spread is usually quite a large fraction of the cathode width, so that insensitive regions are not a serious problem.

A more important consideration here is that it is impossible to build a GD cathode with perfectly continuous grading, because each wire forms a finite and discrete sensor. To approximate to continuous grading, the induced charge must cover a large number of wires, but the presence of this imperfection will always cause local nonlinearities which can only be reduced by reducing the wire spacing $s_c$, or by increasing the charge spread.

![Diagram of graded-density cathode](image-url)
Note that the concept of the GD cathode allows of nonlinear gradings; however, no such grading has been found which is expected to produce the resolution performance obtainable with linear gradings (analysis was made in the same way as the following treatment).


Sensitivity. Assuming for the moment that perfectly linear grading is possible, then if the cathode wires are divided into groups A (whose linear number density, $v_A$, falls with position, $x$, across the cathode) and B (whose linear number density, $v_B$, increases), we can say that

$$v_A = N[\frac{1}{2} - b(x - \frac{1}{2})]$$

and

$$v_B = N[\frac{1}{2} + b(x - \frac{1}{2})]$$

(37)

where $Nb$ is the rate of change of the linear density of each group, $N$ is the total number of wires, and $x$ is the distance across the cathode, again normalised to the cathode width. Note that $v_A + v_B = N = \text{constant}$.

Now suppose a charge to be induced on the cathode, covering many wires. The charges, $q_A$ and $q_B$, received by each electrode, are then proportional to $v_A$ and $v_B$ respectively, and a position signal may be formed as, say,

$$Q = q_B/(q_A + q_B) = bx + \frac{1}{2}(1 - b)$$

(38)

a linear function of position. The system sensitivity is then

$$S \equiv dQ/dx = b$$

(39)

so that if $b = 1$, then $Q = x$ and $S = 1$. In reality, however, the sensitivity will be modified by the inter-component capacitance, if this is not negligible in comparison with the dynamic input capacities, $C_{\text{IN}}$, of
the preamplifiers. Consider, in figure 57a, a charge \( q \) being deposited at termination B (one end of the cathode). If \( C_c \), the inter-component capacity, is negligible, the signal developed across the input capacity at B will correspond to \( q \). If it is not, then \( q \) sees two paths to ground and the charge measured at B is:

\[
q_B = \frac{q \cdot C_{IN}}{C_{IN} + C_{IN}^2/(C_{IN}^2 + C_c^2)}
\]

so that an amount \( q - q_B \) has been lost. By symmetry, a similar loss occurs when \( q \) is injected at A, and the available range of charge variation is reduced to

\[
q - 2(q - q_B)
\]

i.e. the sensitivity has been reduced by a factor of \( (2q_B/q - 1) \). Substituting for \( q_B \), then,

\[
\frac{A}{C_c} \quad \frac{B}{C_{IN}} \quad \frac{C_{IN}}{C_c}
\]

\[
\frac{C_c}{C/2} \quad \frac{C/2}{C_c}
\]

Figure 57 Capacities of a graded-density cathode: (a) for calculation of sensitivity; (b) for calculation of capacitive load. \( C, C_c \) and \( C_{IN} \) are defined in the text.
Note that this is an approximate result, for the assumption has been made that \( C_{\text{IN}} \gg C/2 \), where \( C \) is the total grid capacity to ground. The effect of this term is not often more than about 10%.

**Spatial Resolution.** The noise charge developed in this case arises almost entirely from capacitive loading of the preamplifiers, with a small contribution from the resistive loading (e.g. the biasing resistors, typically \( 10^7 - 10^8 \Omega \)). The expected noise, therefore, once again depends upon the particular design of preamplifier used, according to the expressions given in Appendix 2; the equivalent capacitive load may be found by reference to figure 57b, which shows the capacities by which a GD cathode can be represented. This gives

\[
C_e = C_c + \frac{C}{2}
\]

which, for the particular cathodes employed here, is \( \sim 80 \text{ pF} \), somewhat larger than the same quantity for a CR grid. Thus, although the GD cathode is a simpler device than the CR line, its noise performance is not quite as good.

The spatial resolution of a GD cathode is also poorer than that of a CR line for another reason; namely, that the correlation coefficient is non-zero. Examination of equations (30) and (32) (Chapter 7) shows that if the correlation coefficient is \( r_{ab} = -1 \), the spatial resolution, \( \Delta x \), is equal to \( q_n/q_D \) (where \( q_D \) is the total charge received by the cathode), and is independent of \( x \); whereas if \( r_{ab} = 0 \), the value of \( \Delta x \) is \( q_n/q_D \) at each edge and \( q_n/q_D \sqrt{2} \) at the centre. The value of \( r_{ab} \) depends upon the capacities \( C \) and \( C_c \); here

\[
r_{ab} = -\frac{2(1 + C/2C_c)}{(1 + C/2C_c^2 + 1)} \quad (41)
\]
(see Appendix 1), which, for the present real detector, gives a value of $r_{ab}$ very close to -1.

The Effect of Low Sensitivity on Spatial Resolution. The general case, of $0 \leq b \leq 1$, is of enough interest to justify a brief examination of the relevant theory. It is required to find the RMS position uncertainty, $\Delta x$, expected from a system having preamplifiers giving an equivalent noise charge $q_n$, and a graded-density cathode of sensitivity $b$. Since $q_D$ is the total charge received, we can write $q_D = q_A + q_B$. The position signal is

$$Q = q_B/(q_A + q_B)$$

Differentiating, squaring and averaging, we find that

$$dQ^2 = \left[ q_A^2 dq_B^2 + q_B^2 dq_A^2 - 2q_Aq_B dq_A dq_B \right]/q_D^4.$$  

If we assume that the RMS noise in each channel is the same, i.e. $q_A^2 = dq_A^2 = q_n^2$, say, then we can write

$$dQ^2 = q_n^2 \left\{ q_A^2 + q_B^2 - 2q_Aq_B r_{AB} \right\}/q_D^4,$$  

(42)

where, by definition, $r_{ab} = dq_A dq_B / q_n^2$. Now since $q_A/q_D = v_A/N$ and $q_B/q_D = v_B/N$, we can say

$$q_A/q_D = \frac{1}{2} - b(x - \frac{1}{2})$$

and

$$q_B/q_D = \frac{1}{2} + b(x - \frac{1}{2}),$$

so that, by substituting these expressions into equation (42) and making some algebraic rearrangement, we find

$$\Delta x = \sqrt{dQ^2}/b = q_n \left\{ 1 - 2x(1 - x)(1 + r_{AB}) + \frac{1}{2}(1 - r_{AB})(1/b^2 - 1) \right\}^{\frac{1}{2}},$$  

(43)

using expression (39) above. Equation (43) shows that reducing the grading sensitivity, $b$, increases the position uncertainty, as is
intuitively apparent; note that setting $b = 1$ returns us to the case of equations (30) and (32) (Chapter 7).

iii) Construction of Graded-Density Cathodes

**Prototype Grid.** The prototype GD cathode, unlike its successors, was produced by the same process as all previous grid electrodes, a process which was described in Chapter 3. As for the RC and CR cathodes, the wires were spaced at 1 mm intervals and soldered to ceramic substrates. For this grid, however, both substrates had isolated finger-contacts, so that each wire was separate; the laborious process of taking some fine wire and soldering it to the desired connections (one group to each substrate) was used to produce the grading. The other difference between this and the previous cathodes was that the wire diameter was reduced to 50 μm, in an effort to keep the inter-component capacity small (see below).

**Small Pitch Grids.** All other GD cathodes referred to in this and the following chapter were produced by a special process, developed in this Department. A small and manually-operated lathe was fitted with a mandrel having a screw-thread of pitch 0.5 mm, and having two slots, for the purpose of locating the end-members of the cathode frame. These slots had dimensions 11 mm x 9.5 cm, and were separated by a distance of 10.3 cm around the mandrel circumference (see figure 58).

Once the G10 end-members had been located in their slots, the wire (of diameter 20 μm) was wound onto the screw thread, from a supply bobbin, by turning the mandrel. When the required number of wires had been wound (at one turn per wire) onto the mandrel, embracing the slots containing the end-members, all the wires were araldited to both end-members, and the araldite left to set. Next, the required gradings (see below) were produced by means of a traversing punch. As the mandrel rotated by one revolution, the punch moved along by one wire pitch, so that by counting
Figure 58. Mandrel for GD cathode construction, showing screw-thread and slots for end-members (schematic).
the number of revolutions, the wire number could be monitored. At the appropriate places, the punch was lowered so as to pierce the araldite, and break the wire. The end-members were then released from the mandrel by cutting through the unwanted portions of the wires, leaving the two end-cards linked by 190 wires alone. All wires protruding from the araldite were connected together by sandwiching them between two copper strips situated at the rear of the end-members, with adhesion and electrical contact being provided by a layer of conducting epoxy.

The end-members were in fact T-shaped (see figure 59), the extra bars locating into corresponding recesses in the two side-members. Tensioning was achieved by means of grub-screws which passed through the bars of the end-members and pushed against the ends of the side-members. When this was done, the joints were araldited together and left to set, completing the production process. Electrical isolation of the two segments could be checked by use of an avometer.

Approximation to Linear Grading. Apart from mechanical restrictions, the pitch of a GD cathode must be finite and non-zero to maintain a low value of inter-component capacity, and must be constant in order to maintain field uniformity. These constraints mean that the real grading can only be an approximation to the ideal case; how the real grading is derived is described here.

As we move from, say, the left-hand edge of the cathode, we will need to know how many wires of types A and B we have passed. This is found by integrating expressions (37), and we find that

\[ n_A = \frac{N}{2} [(1 + b)x - bx^2] \quad \text{and} \quad n_B = \frac{N}{2} [(1 - b)x + bx^2], \tag{44} \]

which both give \( N/2 \) when \( x = 1 \), as desired (here \( n_A \) and \( n_B \) represent the number of wires between 0 and \( x \)). The inverse functions are:
Figure 59 Assembly of a graded-density cathode.
\[ x_a = \frac{1}{2b} \left( (1 + b) - \left[ (1 + b)^2 - \frac{8n_A}{N} \right]^{1/2} \right) \]

and

\[ x_b = \frac{1}{2b} \left\{ \left[ (1 - b)^2 + \frac{8n_B}{N} \right]^{1/2} - (1 - b) \right\} \]

and if we arrange that \( n_A \) and \( n_B \) are integers, the positions \( x_A \) and \( x_B \) may be taken to represent provisional wire positions, where the number of wires between 0 and \( x \), of either type, changes to the next integer value. In the case of the cathodes employed here, the value of \( b \) was set to unity, simplifying the above expressions to:

\[ x_A = 1 - \left( 1 - \frac{2n_A}{N} \right)^{1/2} \]

\[ x_B = \frac{1}{2} \left( \frac{2n_B}{N} \right)^{1/2} \quad (45) \]

These positions do not satisfy the constant pitch constraint; the actual grading is found by setting the wires nearest to the positions \( x_A \) as A type, and those nearest to \( x_B \) as B type.

Table 12 gives the actual gradings for the 90-wire (prototype) and 190-wire GD cathodes.

| 0010 0 0 0 0 0 0 0 0 0 | 0030 0 0 0 0 0 0 0 0 0 |
| 0020 0 0 0 0 0 0 0 0 0 | 0040 1 0 0 0 0 0 0 0 0 |
| 0050 1 0 0 0 0 0 0 0 0 | 0060 1 1 0 0 0 0 0 0 0 |
| 0070 1 1 0 0 0 0 0 0 0 | 0080 1 1 0 0 0 0 0 0 0 |
| 0090 1 1 0 0 0 0 0 0 0 | 0100 0 1 0 0 0 0 0 0 0 |
| 0110 0 1 0 0 0 0 0 0 0 | 0120 0 1 0 0 0 0 0 0 0 |
| 0130 0 1 0 0 0 0 0 0 0 | 0140 0 1 0 0 0 0 0 0 0 |
| 0150 0 1 0 0 0 0 0 0 0 | 0160 0 1 0 0 0 0 0 0 0 |
| 0170 0 1 0 0 0 0 0 0 0 | 0180 0 1 0 0 0 0 0 0 0 |
| 0190 0 1 0 0 0 0 0 0 0 | 0200 0 1 0 0 0 0 0 0 0 |

Table 12. Wire connections for the 90-wire and 190-wire GD cathodes. The 0's represent A-type wires, and the 1's represent B-type wires (see text). The number in the left-hand column is the number of the rightmost wire in each row.
**Inter-Component Capacity.** It is possible to estimate this quantity, by considering the case of alternating wires of type A and type B (the GD cathode is only of this form at its centre, of course). The potential function pertaining to this particular situation is given in reference 57, p.296:

\[
V(x,y) = -\frac{\lambda}{2\pi\varepsilon_0} \ln \left( \frac{\cosh \frac{\pi y}{s_c} - \cos \frac{\pi x}{s_c}}{\cosh \frac{\pi y}{s_c} + \cos \frac{\pi x}{s_c}} \right), \tag{46}
\]

where the coordinate system \((x,y)\) is centred on one wire from the set possessing a charge \(\lambda\) per unit length, the other set of wires having no charge. Here \(s_c\) is the wire spacing, as usual. If we consider a point on the surface of the wire which is at the origin, we can say that \(x,y << s_c\), so that expression (46) becomes:

\[
V = -\frac{\lambda}{2\pi\varepsilon_0} \ln \left( \frac{\pi r_c/2s_c}{2s_c} \right)^2,
\]

where \(r_c\) is the cathode wire radius and \(V\) is the potential at the wire surface. The capacity per unit length between adjacent wires is then \((\lambda/V)\), and since there are \((1/2s_c)\) pairs of wires in a unit width, the capacity per unit area of this grid of alternating wires is:

\[
C_c = \frac{\pi\varepsilon_0}{2s_c \ln(2s_c/r_c)} \tag{47}
\]

For the prototype grid (9 cm x 9 cm), having \(r_c = 25 \mu m\) and \(s_c = 1 mm\), this yields a value for the total inter-component capacity, \(C_c\), of -35 pF, while for the remaining grids, of area 9.5 cm x 10.3 cm and having \(r_c = 10 \mu m\) and \(s_c = 0.5 mm\), we find that \(C_c = 79 pF\). Strangely, these figures are quite close to the measured values, 44 pF and 66 pF respectively, and it remains something of a mystery why this should be so, when much of the cathode is not of the alternating form.
iv) **Performance**

**Integral Nonlinearity.** The first concern of the investigation was to assess the extent of the nonlinearity, and to attempt minimisation; for preliminary observation, the integral nonlinearity was measured (see previous chapter). The fully-collimated X-ray beam was moved in 5 mm intervals across the cathode, and the position signal noted at each position. All measurements were carried out using P50 gas and a filter network with 1 µs bipolar shaping, at a charge level of $\sim 1.5$ pC.

The two cathode structures (the prototype, with $s_c = 1$ mm and $N = 90$, and the more advanced version, with $s_c = 0.5$ mm and $N = 190$) were examined, and the integral nonlinearity, $\delta$, was measured as a function of the anode-cathode separation $h$, in order to observe the effect of varying the spread of the induced charge$^{58}$. In addition, the cases of "nearside" and "farside" events were examined. Table 13 summarises the results of this survey, which were calculated for the central two-thirds of the cathode width.

<table>
<thead>
<tr>
<th>$s_c$ (mm)</th>
<th>events</th>
<th>$h$(mm)</th>
<th>$\delta$(%)</th>
<th>$s_c$ (mm)</th>
<th>events</th>
<th>$h$(mm)</th>
<th>$\delta$(%)</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>4</td>
<td>1.46</td>
<td>1</td>
<td>f</td>
<td>6</td>
<td>0.76</td>
</tr>
<tr>
<td>1</td>
<td>n</td>
<td>6</td>
<td>0.83</td>
<td>0.5</td>
<td>n</td>
<td>6</td>
<td>0.30</td>
</tr>
<tr>
<td>&quot;</td>
<td>n</td>
<td>8</td>
<td>0.57</td>
<td>0.5</td>
<td>f</td>
<td>6</td>
<td>0.33</td>
</tr>
<tr>
<td>&quot;</td>
<td>f</td>
<td>4</td>
<td>1.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 13. Integral Nonlinearity, for various values of $s_c$ and $h$. The event locations are designated nearside (n) and farside (f).

The results for $s_c = 1$ mm demonstrate well the effect of changing the induced charge spread by varying $h$, and even the difference between nearside and farside events is noticeable. The further improvement available through the reduction of $s_c$ is apparent in the last two results, for which $s_c = 0.5$ mm.
Figures 60(a) and (b) show graphically the behaviour of the position signal for each cathode, having been recorded with \( h = 6 \text{ mm} \), under the same conditions as above. Graph (a) shows the response for the prototype grid, and comparison with graph (b), for the advanced type, again shows the improvement achieved by reducing \( s_c \). Note that the shape of these curves is not the "S"-shape typical of line systems (RC and CR) but is more oscillatory.

Two further experiments which examined integral nonlinearity were carried out. The first was an extension of the previous measurements to 84% of the cathode width, and yielded a value for \( \delta \) of 0.43%, still satisfyingly small. The second was an investigation of local nonlinearity, the source being moved in steps of only 0.5 mm over a distance of 5 mm, beginning at a position 4.95 cm from the cathode edge (\( x = 0.521 \)). These results are plotted in figure 61, for \( h = 4, 5 \) and \( 6 \text{ mm} \) (the curves have been deliberately displaced), and the increase in curvature with decreasing \( h \) is apparent, once again indicating how the induced charge spread affects linearity.

**Sensitivity.** The dynamic input capacity, \( C_{IN} \), of the Ortec copy preamplifiers used in this work was measured by observing the depression of the output pulse height caused by the attachment of a known load capacity to the input. Assuming that the output pulse height is inversely proportional to the total capacity at the input, the factor by which this quantity falls when a load \( C_e \) is added is \((1 + C_{IN}/C_e)^{-1}\). With a processing time constant of 1 \( \mu \text{s} \), the addition of a 100 pF load caused the output pulse height from a fixed pulse-generator input to fall from 5.7 V to 5.2 V, indicating a dynamic input capacity of 1040 pF. This value is in effect a measure of the input decoupling capacity, of nominal value 1000 pF; according to specifications, the value of \( C_{IN} \) in direct-coupled operation would be more than ten times as large.

It is possible to predict, by use of expression (40), the value of \( S \),
Figure 60  Position calibration curves for (a) prototype and (b) advanced GD cathodes, recorded with h = 6 mm.
Figure 61 Small-scale position calibration curves for the 190-wire, 0.5 mm-pitch cathode h = 4, 5 or 6 mm, as indicated. The curves have been deliberately displaced, for clarity.
the system sensitivity, for the two values of $C_c$ concerned. For the prototype grid, $C_c = 44 \text{ pF}$, so that $S = 0.925$, while for the more advanced model, $C_c = 66 \text{ pF}$, with the result that $S = 0.895$. Figures 60(a) and (b) show immediately that there is approximate agreement between theory and experiment, in this respect.

**Differential Nonlinearity.** It is this rather demanding test which reveals the major drawback of graded-density electrode operation, namely, its poor local linearity. Recall that an ideal detector will respond to uniform irradiation such that the output position spectrum is flat-topped. The uniform irradiation in this case was provided by a $^{55}\text{Fe}(5.9 \text{ keV})$ source, supported atop a plastic tubular column of $\sim 70 \text{ cm}$ length and $11 \text{ cm}$ diameter, sealed by melinex windows at each end and filled with helium to increase transmission. A position spectrum was allowed to accumulate until the average number of counts per PHA channel was at least 5000, for each of three values of $h$ (4, 6 and 8 mm), and the final PHA display photographed. The cathode under test was that having $0.5 \text{ mm}$ pitch and 190 wires, and all measurements were carried out using P50 gas and a charge level of 1 pC; for signal processing, the Harwell 3769 digital pulse ratio system ($T_a = 1 \mu s$) again replaced the analog NIM-module system.

For comparison with these results, it should be recalled that the values of $\varepsilon$ for the RC and CR lines were $\sim 2\%$ and $\sim 9\%$ respectively. Figure 62 shows the system response for the three values of $h$. The marked local variations in sensitivity are exposed, particularly in the case where $h = 4 \text{ mm}$, and the poor comparison with the line systems (RC and CR) is obvious. The values of $\varepsilon$, for the cases $h = 4, 6$ and $8 \text{ mm}$, are $26\%$, $11\%$ and $9\%$ respectively. Although this last figure is as good as that for the CR line, it is only an indication of the maximum variation in sensitivity, and does not show that the CR line is superior over much of the central region (67%). See figure 54 for comparison. Note also that the
Figure 62 Local sensitivity function for the 190-wire GD cathode, for $h = 4 \text{ mm}$ (a), $6 \text{ mm}$ (b) and $8 \text{ mm}$ (c).
overall oscillatory pattern observable in figure 60a is again apparent.

Noise Charge. Two attempts were made to measure the noise charge given by the Canberra 2001A preamplifiers when loaded by the 0.5 mm-pitch GD cathode, each employing the analog processing units, with \( T_a = 1\mu s \). The first, carried out with bipolar shaping, yielded a value of 780 electrons rms, while the second, with unipolar shaping, gave approximately 460 electrons rms; these figures were then compared with those expected from the capacitive load.

The grid capacity to ground, \( C \), was \( \approx 32 \text{ pF} \), giving a value of load capacity, \( C_e \), of \( 82 \text{ pF} \). This led to expected noise figures of about 500 electrons rms for the case of bipolar shaping, and about 360 for the case of unipolar shaping, by use of expressions (A2.7) and (A2.8) of Appendix 2.

The disagreement between the predicted and observed figures is a little disappointing, especially in the case of bipolar shaping, and it is not possible to find an explanation by taking into account the thermal noise from the biasing resistors (the biasing resistors were 22 M\( \Omega \) to each half of the cathode, and contributed no more than about 165 electrons rms: see Appendix 2) and flicker noise, for either mode of shaping. It may be that the greater disagreement between result and prediction in the bipolar case is in part due to poor electrical isolation or shielding, because the results were taken at different stages in the work; however, these effects are thought to be very unlikely to be responsible for the remaining discrepancy.

Correlation Coefficient. Given the values \( C = 32 \text{ pF} \) and \( C_e = 66 \text{ pF} \) for the 0.5 mm-pitch GD cathode, expression (41) (see section (ii) above) yields a value of \( r_{ab} = -0.973 \) for the correlation coefficient. However, the end-capacities to ground, \( C/2 \) (see figure 57b), must strictly be considered as being modified by the preamplifier static input capacity,
which, for the Canberra preamplifiers, is about 30 pF. This changes equation (41) such that \((C/2)\) is replaced by \((C/2 + C_i)\), and the expected values of \(r_{ab}\) becomes \(-0.875\). To test this, \(r_{ab}\) was measured by the method described in Chapter 7 (see equation (33)), employing 1 \(\mu\)s unipolar shaping, and a simulation of the cathode, comprised of silver mica capacitors of appropriate values. Care was taken in considering the capacities of the leads and small housing-box necessary in this experiment. The measured values of signal-to-noise ratio gave \((dq_{\text{sum}}^2)^{1/2} = 199\) electrons rms and \(q_n = 380\) electrons, resulting in a value for \(r_{ab}\) of \(-0.863\), in good agreement with the expected figure. It was assumed that the real grid would behave similarly.

**Spatial Resolution Results.** Having measured the electronic noise and correlation coefficient of the system, the spatial resolution performance can be predicted for a range of charge levels, given the uncertainty expected from other sources. The available experimental results were obtained when the measured noise charge was 780 electrons rms (bipolar case), and the predictions of spatial resolution are therefore based upon this figure. Using this value, and \(r_{ab} = -1\), the front-end electronic contribution to spatial uncertainty should be \(\sigma_e = (23.7/q_o) \mu m\), where \(q_o\) is the anode charge level, in picocoulombs. The beam verticality was checked, and the other contributions to the spatial uncertainty were therefore the same as the CR line measurements: \(\sigma_r = 65 \mu m\), \(\sigma_p = 30 \mu m\), \(\sigma_w = 40 \mu m\), and \(\sigma_D = 30 \mu m\) (P50 gas, and no drift region), totalling approximately 87 \(\mu\)m. Thus, for instance, the overall uncertainty, including the front-end noise, would be 90 \(\mu\)m at \(q_o = 1\) pC, and about 480 \(\mu\)m at \(q_o = 0.05\) pC. Again the value of \(h\) was 6 mm.

The experimental results are compared with these predictions in figure 63. The overall agreement is quite good, but there are small discrepancies: the turn-up of the experimental curve at high charge levels is almost certainly due to an increase in the importance of secondary
Figure 63  rms spatial uncertainty (μm) vs. anode charge level. The full curve is the expected total uncertainty, the points are the experimental results, and the broken line gives the value of $\sigma_e$ alone.
avalanches (see reference 43, and Chapter 6), but no explanation can yet be found for the slight discrepancy at lower charge levels. Comparison between these results and similar ones for the RC and CR systems would not be meaningful at this stage, because of the unaccountably-high noise value; however, an assessment of the noise performance of all readout methods here discussed, assuming identical operating conditions, will be given later.

v) Discussion

Comparison with RC and CR lines. Like the CR lines, then, a GD system offers superior noise performance to that of an RC line, at the cost of some linearity. Although its integral nonlinearity compares well with that of the RC and CR lines, its performance is markedly poorer in terms of differential nonlinearity. It is also slightly inferior to the CR line in noise performance, but against these disadvantages must be weighed the simplicity and low cost of GD electrode production, and the convenience of an "in-house" manufacturing process.

It must be remarked that these comparisons refer to the particular CR and GD electrodes tested here, which are not necessarily optimised; it may be, for instance, in the case of larger detectors, that the CR line noise performance becomes inferior to that of a GD system (if the number of cathode strips increases, so that the shunt conductance $G$ increases). It may also be possible to reduce the effective load of a GD electrode by reducing the wire diameter (see expression (47): increasing $s_w$ would be more effective, but detrimental to linearity). Indeed, an attempt was made to do this, with a cathode whose wires were of $8 \mu m$ diameter. This alteration should reduce $C_e$ by a factor of $\approx 2.3$, making the load capacity, $C_e$, about 45 pF, and making the expected noise (with $1 \mu s$ unipolar shaping) 230 electrons rms, instead of 360. The use of this particular
grid, however, made normal operation of the chamber impossible (the energy resolution was degraded), though an explanation for this has yet to be found.

Like the CR cathode again, the GD cathode is not subject to the angular localisation risetime effects which appear with RC readout, and it may still be possible, therefore, to recover the position information stored by angular localisation. The time unit to which the pulse development time must be compared is again $C/G$, with $C$ and $G$ as defined in the previous chapter, and with the GD electrode considered as an $N$-node CR line, with $N = 1$. With $C = 32 \, \text{pF}$ and $G = 4.5 \times 10^{-2} \, \text{M}\Omega^{-1}$, $C/G$ is about 700 $\mu$s.

**Application to Microchannel Plate Detectors.** One area which may also benefit from the use of GD electrodes is in the use of high-resolution microchannel plate detectors, for which resistive sheets\textsuperscript{113} or crossed RC grids\textsuperscript{98} have often been employed, although capacitive charge division\textsuperscript{94} and other methods have also been used. Again, the prime advantages are those of good spatial resolution and simplicity. Research conducted in this Department\textsuperscript{114} has shown that operation with GD cathodes is indeed possible, with similar performance to that of an RC sheet system, and while further work needs to be done in this field, it seems likely that the GD electrode will be suitable for certain applications.

In connection with the subject of MCPs, one can mention the recent development of some special progressive-geometry electrodes ("wedge-and-strip" electrodes), of similar design to the MWPC "backgammon" electrode of Allemand and Thomas\textsuperscript{95} but giving two-dimensional information, by Martin et al.\textsuperscript{115}. These designs supersede the previous quadrant electrodes\textsuperscript{30,116}, which produce large nonlinearities.
i) Justification for Subdivision

General Comments. The factor which limits the electronic noise performance of the graded-density cathode is the size of its capacitive loading: we have seen that the existing grids offer only moderate linearity, for a capacitive load of 82 pF. In order to improve spatial resolution, it would be advantageous to reduce this loading still further, and this chapter describes how this aim has been achieved, by means of subdivision.

Before the particular method applied in this investigation is described, the general concept of subdivision of a GD electrode will be examined. "Subdivision" in fact implies a cascading of two or more GD electrodes: suppose we have $M$ GD electrodes, each having linear density grading, in groups A and B, as described in the previous chapter, and that each is of width $1/M$, such that $1/M \gg 1/s$ (where $s$ is the wire pitch). We now connect the B-group of the first electrode to the A-group of the second, and repeat the process until all are joined together in this way. The resulting electrode is of unit width and is an $M$-section graded-density electrode. It has $(M + 1)$ nodes, including the end nodes 0 and $M$.

We can now formulate expressions for the density gradings in the $m^{th}$ section, from equations (37), using the coordinate $x(0 \leq x \leq 1)$ for the fractional position along the entire electrode. In the present discussion, only full sensitivity ($b = 1$) is considered, and $N$ is the total number of wires in the whole electrode. At position $x$, in the $m^{th}$ section, then,

$$V_A = N (m - Mx)$$

and

$$V_B = N (Mx + 1 - m).$$

(48)
If a charge is now induced on only the $m^{th}$ section of the electrode, the charges $q_{m-1}$ and $q_m$ appearing at the $(m - 1)^{th}$ and $m^{th}$ nodes respectively are proportional to $v_A$ and $v_B$ respectively. The centroid of these two node charges is

$$\frac{[q_{m-1}(m-1)/M + q_m/M]}{[q_{m-1} + q_m]}$$

which can easily be shown, from equations (48), to be equal to $x$. The charge $q_m$ at the $m^{th}$ node will be, in the general case where charge is induced on several sections, the sum of the $B$ charge from the $(m - 1)^{th}$ section and the $A$ charge from the $m^{th}$. In this case the above relationship can be written more generally

$$\bar{x} = \frac{\sum_{o} m_q / M}{\sum_{o} q_m}.$$  

**Signal Acquisition.** One way of obtaining the centroid position would be simply to connect each of the $(M + 1)$ nodes to a preamplifier, a procedure which would involve more complex processing, and would also increase the number of preamplifier noise contributions. The approach actually adopted makes use of the large capacity which exists between the two components of a GD electrode (or section). Because this is present, the subdivided electrode can act as a capacitive divider and allow charges to appear at the end nodes, so that only two preamplifiers are necessary. The loading on the preamplifiers is smaller than in the case of the undivided grid, so that by subdividing a GD cathode and allowing it to behave as a CR line in this way, its noise performance can be improved. Once again, however, in order to gain this improvement, some linearity has been sacrificed; now that the system is operating as a CR line it is subject to the nonlinearity associated with that form of signal location.
ii) Theory

Compensation for Nonlinearity. The CR-associated nonlinearity may be counteracted to an extent, in the special case \( M = 2 \), by exploiting one of the advantages of the GD concept: if the systematic nonlinearity is known, a specific nonlinear grading can be used to compensate for it.

Figure 64 shows a capacitive model for a two-section GD cathode. Here \( C'_c \) is the inter-component capacity of each section; the value of \( p \) (see Appendix 1) in this case is given by \( p = C/2C'_c \), where \( C \) is, as usual, the total grid capacity to ground. Suppose that the charges received by the left-hand (A) and the right-hand (B) components of the first section are \( q_A \) and \( q_B \) respectively, and that the charges delivered to the end nodes 0 and 2 are \( q_0 \) and \( q_2 \) respectively. We consider the case where all the charge is received by the first section. If all gradings are linear,

\[
q_o = q_A + q_B C'_c / (2C'_c + C/2) \quad \Rightarrow \quad q_o = q_A + q_B (2 + p),
\]

then

\[
Q = q_2/(q_o + q_2) = q_B/(2q_B + q_A(2 + p)),
\]

i.e. \( Q(x) \) is a nonlinear function. In order to compensate for this, we need to replace \( q_A \) and \( q_B \) by \( q'_A \) and \( q'_B \), such that

\[
x = q'_B/(2q'_B + q'_A(2 + p)) \quad ;
\]

in other words,

\[
x = \nu_B/(2\nu_B + \nu_A(2 + p)), \quad (50a)
\]

where \( \nu_A \) and \( \nu_B \) are the compensating nonlinear density functions.

The other constraint is that the total density should not be a function of position, which may be written
Figure 64 Capacitive model for a two-section GD cathode. $C'$ is the inter-component capacity of each section, and $C$ is the total grid capacity to ground. If the gradings are nonlinear, the end-node capacities are slightly different from $C/4$.

Figure 65 Density variations for uncorrected and corrected versions of the two-section GD cathode (see text).
\[ v_A + v_B = N \]  \hspace{1cm} (50b)

Solution of the simultaneous equations (50) yields
\[
\frac{v_A}{1+px} = \frac{N(1-2x)}{(1+px)} \quad \text{and} \quad \frac{v_B}{1+px} = \frac{N(2+p)x}{(1+px)} , \hspace{1cm} (51)
\]

and replacing \( q_A \) and \( q_B \) in equations (49) by \( \frac{q_D}{N} v_A \) and \( \frac{q_D}{N} v_B \) respectively, where \( q_D \) is the received charge, we find that \( Q = x \), i.e. linearity has been restored.

Figure 65 shows the density variation with position for the A and B components of both sections. Note that the above analysis only applies to the first section, \( 0 \leq x \leq 0.5 \); for the second section \( (0.5 \leq x \leq 1) \), the above expressions for \( v_A \) and \( v_B \) must be exchanged and \( x \) must be replaced by \( (1-x) \), where \( x \) is once again the position expressed as a fraction of the total cathode width.

The real wire connections are determined in a manner similar to that already described: the density functions (51) are integrated to give expressions for \( n_A \) and \( n_B \), but this time the values of \( x_A \) and \( x_B \) are set to integer multiples, \( x_1 \), of \( s \), the wire spacing. The component (A or B) to which each wire belongs is that which possesses a value of \( n(n_A \) or \( n_B) \) closer to an integer, for the particular value of \( x_1 \) concerned. The expressions for \( n_A \) and \( n_B \) are:
\[
n_A = \frac{N}{p^2} \left\{ (2+p) \ln(1+px_1) - 2px_1 \right\} \\
\text{and} \quad n_B = \frac{N(2+p)}{p^2} \left\{ px_1 - \ln(1+px_1) \right\} . \hspace{1cm} (52)
\]

Electronic Noise. In addition to the considerations of capacitive load and correlation coefficient, another factor influences spatial resolution: the total denominator signal, \( q_D + q_2 \), is position-dependent.

In the case where the grading has been corrected, we have,
\[
q_{\text{sum}} = q_0 + q_2 = q_D \frac{(v_A + v_B)}{N} ,
\]
and by inserting \( V_A \) and \( V_B \) from expressions (51), we find that

\[
q_{\text{sum}} = \frac{q_D}{(1 + px)}
\]

(53)

This variation with position may be expressed by altering equation (30), such that \( \Delta x = D(x) \cdot q_n/q_D \), where now

\[
D(x) = \left\{ 1 - 2(1 + r_{o2})(1 - x)x \right\}^{\frac{1}{2}}(1 + px)
\]

(54)

For \( 0.5 \leq x \leq 1 \), \( x \) is to be replaced by \((1 - x)\).

The correlation coefficient, \( r_{o2} \), and the capacitive load, \( C_e \), may be predicted from equations (A1.10) and (A1.11) of Appendix 1; in this case,

\[
r_{o2} = \frac{2 \cosh 2Y}{(1 + \cosh^2 2Y)}
\]

(55)

and

\[
C_e = C_c' \coth 2Y \sqrt{\frac{p(1 + p/4)}{2}}
\]

(56)

where \( \cosh Y = 1 + p/2 \) and \( p = C/2C_c' \). The noise charge, \( q_n \), may be calculated from a knowledge of \( C_e \) and the relevant preamplifier characteristics (see Appendix 2).

iii) Experimental Performance

Construction. The two-section subdivided grids were assembled in the same manner as the undivided ones, except that the copper strips at each end of the cathode were divided in two, allowing the wires to be connected into four groups. Two of these (the adjacent A and B components) were connected together, and the signals were extracted from the two outer groups. Each of the three groups was fed from the drift voltage supply via a large resistor (44 M\( \Omega \)).
Table 14 gives the gradings for a linear subdivided grid (a), and grids with gradings corrected for \( p = 0.65 \) (b) and for \( p = 0.44 \) (c). The grid with linear grading was constructed as a control and to allow measurement of \( C' \) and \( C \), so that the value of \( p \), for construction of the "corrected" grids, could be determined. Capacities were measured using the Q-meter already mentioned: values of \( C_e \) and \( C \) could be obtained in this way, and consideration of the capacitive model could then give a value for \( C' \), and hence for \( p \). Measurements on the "uncorrected" grid, then, gave \( C = 32 \text{ pF} \) and \( C' = 38 \text{ pF} \), so that \( p = 0.44 \).

For comparison and prediction, numerical calculations of the expected behaviour of the position signal were carried out. A gaussian distribution of charge was assumed (cf. Chapter 6) and the appropriate amount of charge injected at each wire position; the known pattern of wire connections, and the various capacity values, could then be used to determine

\[
\begin{align*}
00010 & \quad 0,0,0,0,0,0,0,0,0,0,0 \quad (a) \\
00020 & \quad 0,0,0,1,0,0,0,0,0,1,0 \\
00030 & \quad 0,0,0,1,0,0,0,1,0,0,0 \\
00040 & \quad 1,0,0,1,0,0,1,0,0,0,0 \\
00050 & \quad 0,1,0,1,0,0,1,0,0,1,0 \\
00060 & \quad 0,1,0,1,0,1,0,1,0,0,0 \\
00070 & \quad 1,1,0,1,0,1,0,1,0,0,0 \\
00080 & \quad 1,1,0,1,1,1,0,1,1,0,1 \\
00090 & \quad 1,1,0,1,1,1,0,1,1,1,1 \\
01000 & \quad 1,1,0,1,1,1,1,1,1,1,1 \\
01100 & \quad 1,1,0,1,1,1,2,1,1,1,1 \\
01120 & \quad 1,2,1,1,2,1,1,1,1,1,1 \\
01130 & \quad 2,1,1,2,1,1,2,1,1,1,1 \\
01140 & \quad 2,1,2,2,1,2,2,2,2,2,2 \\
01150 & \quad 2,1,2,2,2,2,2,2,2,2,2 \\
01160 & \quad 2,1,2,2,2,2,2,2,2,2,2 \\
01170 & \quad 2,1,2,2,2,2,2,2,2,2,2 \\
01180 & \quad 2,1,2,2,2,2,2,2,2,2,2 \\
01190 & \quad 2,1,2,2,2,2,2,2,2,2,2 \\
\end{align*}
\]

Table 14. Wire connections for the two-section GD cathodes. The 0's represent the A-type wires of the first section, the 2's the B-type wires of the second section, and the 1's the remaining wires (those forming the central element of the cathode). (a) linear grading; (b) corrected for \( p = 0.65 \); (c) corrected for \( p = 0.44 \).
Q for a particular source position, using the expressions developed above. When Q had been calculated for several source positions, nonlinearity predictions could be made.

Due to a mistake concerning the appropriate value of p for the grading, the first corrected grid was constructed with gradings corresponding to p = 0.65 (see Table 14), but it was felt that the linearity would still be better than that obtained with the uncorrected grid.

Integral Nonlinearity. For examination of these systems, the chamber was again irradiated with the collimated X-ray beam, which was moved in 4 mm intervals across the central 64 mm (~67%) of the cathode. The position signal was recorded at each source location, the best-fitting straight line through the data points (x,Q) was calculated, and the deviation, δQ, of each data-point from the straight line was found. The modular analog units were employed for processing, with unipolar shaping at Ta = 1 μs, while the chamber gas was P50 and the charge level was 1 pC.

The δQ results for the uncorrected and corrected (p = 0.65) cathodes are shown in figure 66, (a) and (b) respectively, and the definite improvement in linearity achieved by correction can be seen. The continuous curves are the theoretical predictions, based on a gaussian charge distribution having the same standard deviation (σ_i = 0.8 h = 4.8 mm) as the real distribution, and the agreement is rather satisfactory. The rms values of δQ (the integral nonlinearity) are compared in Table 15, and that for the undivided GD cathode is also given.

<table>
<thead>
<tr>
<th></th>
<th>undivided</th>
<th>two-section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>uncorrected</td>
</tr>
<tr>
<td>C_e (pF)</td>
<td>82</td>
<td>30</td>
</tr>
<tr>
<td>δ(%)</td>
<td>0.33</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 15. Capacitive loading and rms nonlinearity for the single-section and two-section GD cathodes.
Figure 66 Deviation, $\delta Q$, of position signal data points from a best-fitting straight line for the uncorrected and corrected for $p = 0.65$ versions of the two-section GD cathode. The full curves are theoretical calculations (see text).
Spatial Resolution. Table 15 also gives the respective values of \( C_e \), the measured capacitive load on the preamplifiers. The great advantage of subdivision is demonstrated, for the loading has been greatly reduced; the expected rms noise charge, for 1 \( \mu \)s unipolar shaping, has fallen to about 190 electrons (from 360). The value of \( C_e \) calculated from expression (56) is, not surprisingly, 31 pF (for \( C'_c = 38 \) pF and \( p = 0.44 \)).

The correlation coefficient was measured in the manner previously described, and found to be \(-0.5\), in conflict with the prediction, from equation (55), that \( r_{o2} = -0.81 \). It might be suggested that as the capacities involved in the calculation of \( r_{o2} \) become smaller, the argument developed in Appendix 1, which ignores the effect of resistances, becomes less valid; but this is not borne out by later results, and the disagreement is still unexplained.

It is interesting that with these particular values, of \( r_{o2} = -0.5 \) and \( p = 0.44 \), the expression (54) yields a value of \( D \) which is almost independent of \( x \) : at \( x = 0.5 \), for instance, \( D(x) = 1.06 \). This means that as in the case of the undivided GD cathode, the spatial resolution should have an rms value equal to \( q_n / q_D \) over the whole cathode width, for this particular combination of parameters.

Finally, we note that as in the case of the undivided grid, the measured electronic noise exceeds the value expected from the capacitive loading alone. The measured noise charge, \( q_n \), was in this case 360 electrons rms, so that the electronic contribution to the spatial resolution should be given by the expression \( \Delta x = (10.94) / q_o \) \( \mu m \), where \( q_o \) is again the anode charge level in picocoulombs; \( \Delta x \) should maintain this value over the whole cathode width.

The performance, then, of the subdivided GD cathode was encouraging. The linearity appeared to be good, and the electronic noise charge was the
lowest yet obtained in these investigations. Note that the figure quoted above refers to measurements for which unipolar shaping was used. For a purely capacitive preamplifier load, the noise charge should in that case be smaller than that obtained with bipolar shaping, by a factor of 1.38 (see Appendix 2). This allows comparison of values of $q_n$ obtained with either type of shaping.

iv) The Low-Loss Cathode

Excess Noise Charge. As in the case of the single-section GD cathode, then, the noise charge was larger than expected. A capacitive mock-up of the detector was again constructed and examined, and once more produced a noise charge value close to the predicted one; the correlation coefficient was $r_{o2} = -0.47$, in good agreement with the actual value.

With regard to the noise charge figures, it is interesting to note that the discrepancies between measured and expected values are approximately the same in the present and single-section cases: for the two-section cathode we have $(360)^2 - (190)^2 = (306)^2$, while for the single-section grid we have $(460)^2 - (360)^2 = (286)^2$. The fact that the capacitive simulations produced noise values agreeing with predictions leads one to suppose, therefore, that the remaining contribution, which is the same in both cases, is attributable to the presence of resistances in the real system. When the mock-up of the two-section cathode was accordingly modified to include the 44 MΩ biasing resistors, the noise charge was found to be about 205 electrons rms, implying that the resistance-associated noise was only partly accounted for by these resistors.

It was suspected that the remaining contribution was due to a non-zero conductance between the various components of the GD cathode, arising from their small physical separation. In GD cathode construction, all the wire ends are connected to the copper end-strips, so that the physical
separation between the components in the araldite is only the size of the
gap produced by the punch when breaking the wire; and there are many wires.
Presumably, over these distances the small conductance of the epoxy becomes
important.

These suppositions were upheld by measurements of the resistance
between the two components of a single-section GD cathode. Previous d.c.
measurements* of the resistance had yielded a value in excess of $10^{12} \Omega$,
but when this was measured at the correct a.c. frequency (160 kHz; see
Chapter 8), a value of only $4 \text{ M}\Omega$ was obtained. Insertion of this and
the other appropriate values into equation (A2.8) of Appendix 2 gives a
noise charge which is of the correct order of magnitude (~500 electrons
rms) to account for the discrepancies observed. A similar resistance
value must exist for the two-section grid.

Construction of the Low-Loss Cathode. In order to test this theory,
a cathode was designed and constructed\(^{118}\) which would, hopefully, reduce
the losses through the various small conductances; this time the grading
was properly corrected ($p = 0.44$). The assembly process remained the
same, except that this time the A and C (edge) sections and the B
(central) section were mounted on two separate frames, all surplus wires
being removed from each frame (see figure 67). Small and flexible plastic
wedges were then placed between the inner edges of the end-members and
the wire planes, in order to raise the level of the latter; this ensured
that when the two frames were subsequently araldited together, the two sets
of wires became coplanar.

Of course, this process is rather unsatisfactory, because the two frames
must be aligned very accurately indeed in order to preserve pitch uniformity;

\* made using a Kiethley Instruments model 602 Electrometer.
Figure 67  The two frames of the low-loss cathode, separated to reveal the gradings on each section.
however, since the experiment was chiefly concerned with reduction of the resistive losses, it was felt that some degree of non-uniformity could be tolerated. The prototype low-loss grid also suffered from some deformation of the frames, and as a result gave a highly nonlinear readout. The second version, though still suffering from some pitch non-uniformity, was more successful, and was used in the experiments described here.

Attention was also paid to the value of the biasing resistors. To reduce leakage (due to surface defects, any grease and dirt, etc.), some glass-sealed "Morganite" resistors of approximate value $2 \times 10^9 \Omega$ were employed to bias the cathode sections to the drift voltage. In this condition, the equivalent noise charge was found to be about 240 electrons rms, a figure much closer to the expected value than obtained with the previous design. The remaining discrepancy may perhaps be accounted for by extraneous factors such as microphonic or radio signal pickup, but improvement beyond this performance is probably not possible without redesigning the preamplifier/detector interface. Further reduction of the capacitive load would not otherwise be advantageous, because of the remaining noise contributions.

**Performance.** Little attention was paid to the linearity of this system, in view of the observed pitch irregularities; however, a measurement of the rms integral nonlinearity was made in the normal way, and resulted in a value of only 0.34%. This shows how small local nonlinearities are not exposed when integral nonlinearity is measured in this way.

Unfortunately, it was still not possible to find agreement between the measured and predicted values of the noise correlation coefficient, and in fact for this low-loss grid, the value of $r_{02}$ was -0.2. The fact that this figure is further from the predicted one than is the same figure for the original two-section grid, in spite of the increased value of the biasing resistors, seems to refute any argument based on the inclusion of the latter in the theoretical model (Appendix 1). As already stated,
this problem remains unresolved.

Combining a knowledge of the noise charge and correlation coefficient allows use of expressions (30) and (54), to arrive at a figure for the resolution at the line centre: this is \( \Delta x = \left(\frac{6.89}{q_0}\right) \mu m \), where, as usual, \( q_0 \) is the anode charge level in picocoulombs. The additional electronic contribution (the divider unit denominator-dependence, \( \sigma T \)) had been reduced by further adjustment of the divider unit, to a value of about 30 \( \mu m \) rms, and the total electronic contribution to spatial uncertainty gives the solid curve shown in figure 68. For confirmatory measurements, a very linear portion of the grid was located, and the chamber irradiated there with the fully-collimated X-ray beam. The chamber gas was P50, no drift region was employed, and as usual, the anode-cathode separation, \( h \), was 6 mm. The signal shaping was unipolar \( (T_a = 1 \mu s) \). The experimental data is shown in figure 68, and these uncertainty values may be completely accounted for by the various contributions \( (\sigma_D = 30 \mu m, \sigma_p = 35 \mu m, \sigma_W = 40 \mu m, \text{ plus } \sigma_T \) and \( \sigma_{\theta} ) \), except at the higher charge levels, where secondary avalanches are thought to be responsible for the worsening resolution.

The range of high performance of a system equipped with low-loss two-section GD cathodes is apparent from these measurements: the electronic contribution to spatial uncertainty is less than 150 \( \mu m \) FWHM \( (i.e. 64 \mu m \text{ rms}) \) over a range greater than a full decade of charge levels. Further discussion will be given in the next chapter.

Finally, we note that the present low-loss cathode structure was found to be mechanically unstable: after a moderate amount of use (repeated removal from and insertion into the chamber), the two frames began to separate. No attempt has yet been made to improve on this design, but it is felt that a suitable one may be found without great difficulty.
Figure 68  Spatial uncertainty vs. anode charge level, for a system equipped with low-loss two-section GD cathodes. The full curve gives the total electronic contribution.
v) **Differential Nonlinearity.**

Once again, the improvement in spatial resolution is achieved at the cost of some linearity; and once again, the measurement of differential nonlinearity reveals the extent of this sacrifice. The chamber was uniformly irradiated by use of the $^{55}$Fe source and helium column as before, with signal processing by the Harwell 3769 digital system ($T_a = 1 \, \mu s$); conditions were otherwise as described above, except that a drift region was employed, to improve efficiency. The cathode was the "lossy" corrected grid (gradings corrected for $p = 0.65$). Figure 69 shows the result of this experiment, with a disappointingly large nonlinearity of 21%, due to the feature in the centre. If this feature is ignored, the rest of the cathode shows an improvement over the case of the single-section grid for the same value of $h$, with $\epsilon < -9\%$.

![Figure 69](image)  
*Figure 69  Local sensitivity function for the corrected two-section GD cathode.*
It is unfortunate that differential nonlinearity measurements are not available for a properly corrected grid \( p = 0.44 \); however, calculations\(^\text{17} \) suggest that the improvement over the case of \( p = 0.65 \) would not be great. More effective modifications may be made, by increasing the number of wires, \( N \), or the anode-cathode separation, \( h \). The following table gives approximate comparison between the values of \( \varepsilon \) for a single-section GD grid, for totals of 190 and 380 wires, and for \( h = 6 \) and 8 mm, and provides an indication of the improvements obtainable. The figures for 190 wires are experimental results, while those for 380 wires are the result of the calculations employing a gaussian charge distribution.

<table>
<thead>
<tr>
<th>N</th>
<th>190</th>
<th>380</th>
</tr>
</thead>
<tbody>
<tr>
<td>h(mm)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10.5</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>8.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 16. Differential Nonlinearity, \( \varepsilon(\%) \), for a total number of wires, \( N \), of 190 and 380, and for \( h = 6 \) and 8 mm.

It is more difficult to draw conclusions from calculations involving the subdivided grids, because of disagreement between the predicted differential nonlinearity and that observed. This presumably arises because a gaussian charge distribution has been employed, rather than the real one (see Chapter 6).

Further work on this subject is required in order to find out whether further reduction of the nonlinearity of the subdivided GD cathode is feasible. Such work is certainly justified in view of the potentially good resolution.
CHAPTER 11

CONCLUDING REMARKS

i) Summary of Results

The results of the work reported in this thesis are such as to leave several pathways to be explored, in attempts to improve the position-sensing capability of the MWPC. The programme has partially achieved its initial objective, in showing that with further study it may be possible to exploit angular localisation for position sensing; however, the additional developments made in the field of readout methods could prove more significant. In this section, the major results of this work will be briefly recalled.

Angular Localisation. To begin with, observations were made which confirmed the presence and expected extent of angular localisation effects. Measurements of signal modulation firstly showed rough quantitative agreement with predictions\(^5\) (e.g. the dependence upon processing time constant), and secondly exposed the interesting effect of non-proportionality on the collected charge, which occurs when the electron cloud divides between two wires. Comparison between shaped pulse zero-cross time predictions and measurements also produced satisfactory agreement, except in respect of the zero-cross time of the shaped pulses from the plain cathode grids. These did not behave as expected, and were also in conflict with corresponding measurements of pulse height ratios. However, the conclusion could be drawn that angular localisation effects were present to roughly the expected degree.

Measurements performed with RC lines, which appear to have been the first attempt to exploit angular localisation by use of a transmission line, showed that these effects can actually be detrimental to detector performance. While this is disappointing, it is useful information for any
future work with RC lines. The importance of these effects appears to depend upon the relationship between the charge collection time, $t_c$, and the line diffusion time constant, $RC/\pi^2$: to minimise these detrimental effects, the relationship must be $^{100}t_c << RC/\pi^2$. (In fact, the requirement is that the major part of the pulse development should occur in a time much shorter than $RC/\pi^2$, but the definition of "major part" is of necessity arbitrary, and it is therefore convenient to retain the rather severe condition given above.) Thus it should be that for larger detectors with larger values of resistance than at present, such effects are negligible, so allowing proper exploitation of angular localisation.

The true position-sensing potential of angular localisation was demonstrated by detailed examination using a high-sensitivity electrode, when a position-sensitivity of more than 0.3 was achieved without optimisation of processing time constant.

**New Methods of Position Encoding.** Two position encoding systems, hitherto untried in MWPCs and possessing superior noise performance to that of RC line systems, were investigated, with encouraging results. Although both suffer from some nonlinearity, neither is afflicted with the detrimental effects of angular localisation experienced by the RC line.

The CR line method is an extension of one already developed for micro-channel plate cameras, and has been shown to have the advantages of much reduced thermal noise and good local linearity. The nonlinearity which does exist is a systematic variation over large distances, and is by no means prohibitive. However, as yet the physical structure is rather unwieldy, although this problem may be overcome in the future.

The graded-density (GD) cathodes are cheap and simple to construct, and also show good spatial resolution. Both the undivided and subdivided forms have been investigated, and the general principles whereby linearity may be improved (increasing the induced charge spread and reducing the
cathode wire pitch) have been demonstrated. In the present forms, local readout linearity is poor, and a certain amount of resistive noise exists; but results indicate that these problems may be at least partially overcome.

The two-section subdivided GD cathode, acting as a series capacitance line, can have its grading adjusted so as to compensate for the nonlinearity associated with its line behaviour. It is possible to construct such a cathode giving a very low noise charge (240 electrons rms has been obtained).

The very good spatial resolution obtainable with these devices may prove to outweigh the disadvantage of increased image processing for removal of nonlinearities, especially in view of the more stringent requirements of future detectors. These developments, then, are rather important, and well worth further study.

**Drift and Diffusion.** In addition to the above measurements, a study of the diffusion experienced by an electron cloud during its drift towards the anode yielded results which were in satisfactory agreement with theory. However, examination of the resulting interpolation between anode wires revealed an effect which improves linearity, beyond that expected from the diffusion, by an amount which is independent of the diffusion occurring. This discovery remains puzzling, although it is possible that the effect is associated with the initiation of secondary avalanches by ultra-violet photons.

**ii) Performance Evaluation**

This section discusses the spatial resolution and linearity capabilities of the argon IPC, in the light of the results gained from the work reported here, especially that recorded in Part III.

**Comparison of Readout Methods.** The performance characteristics of the various position readout methods examined in this work are compared with those of an RC line in Table 17. The spatial uncertainty has been
NOISE, RESOLUTION AND LINEARITY FOR
VARIOUS READOUT METHODS (9.5 cm DETECTOR)

<table>
<thead>
<tr>
<th>Readout Method</th>
<th>$q_n$ (electrons, rms)</th>
<th>$\Delta x, q_o$ ((\mu)m, pC)</th>
<th>Nonlinearity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BIPOLAR</td>
<td>UNIPOLAR</td>
<td></td>
</tr>
<tr>
<td>RC (250 kΩ)</td>
<td>1750**</td>
<td>1270*</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>&lt; 0.7**</td>
</tr>
<tr>
<td>CR</td>
<td>600 (500)</td>
<td>440 (360)</td>
<td>9.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.30</td>
</tr>
<tr>
<td>GD (single)</td>
<td>630 (500)</td>
<td>460 (360)</td>
<td>11</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.33</td>
</tr>
<tr>
<td>GD (double)</td>
<td>400 (260)</td>
<td>360 (190)</td>
<td>-</td>
</tr>
<tr>
<td>(uncorrected)</td>
<td></td>
<td></td>
<td>0.47</td>
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<tr>
<td>GD (double)</td>
<td>330 (260)</td>
<td>240 (190)</td>
<td>21</td>
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<tr>
<td>(corrected)</td>
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<td></td>
<td>0.23</td>
</tr>
<tr>
<td>GD (double)</td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(low-loss)</td>
<td></td>
<td></td>
<td>0.34</td>
</tr>
</tbody>
</table>

* rms resolution taken over entire electrode width.

** from the work of Fraser et al.\textsuperscript{80}

Notes.

- * estimate.
quantified by the parameter $\Delta x_q$, the product of the front-end electronic contribution to the uncertainty (with unipolar shaping) and the anode charge level, and is given for the cases of charge injection at the electrode edge and at its centre, and as an rms value. The figures for $\Delta x_q$ in brackets refer the RC and CR line figures to a detector length of 9.0 cm, the figures for 9.5 cm being inferred from these. The equivalent noise charges are measured values, with the theoretical values in brackets, and are given in electrons (rms) for both kinds of pulse shaping considered here.

The three values of $\Delta x_q$ for each electrode show the effect of the relevant correlation coefficients, $r_{ab}$; for example, although the CR line has a larger value of $q_n$ than the two-section GD grid (lossy version), it gives a smaller value of $\Delta x_q$ at $x = 0.5$. For this table, the experimentally-determined values of $r_{ab}$ have been employed, rather than those expected.

Comparison of the rms values of $\Delta x_q$ shows that of these particular grids, the resolution obtainable with the CR cathode is inferior only to that of the low-loss version of the two-section GD cathode, while the differential nonlinearity of the former is only bettered by that of the RC line. As already pointed out, however, the choice of readout system depends on the constraints and requirements of the particular experiment. It must be also emphasised that Table 17 refers to the specific electrodes examined here, and must serve only as a guide.

**Nonlinearity.** It is useful to make some observations concerning the definitions of integral and differential nonlinearities as used here. An important point to note is that whereas the integral nonlinearity, $\delta$, is an rms value (over a certain portion of the detector length), the differential nonlinearity, $\epsilon$, is a maximum value, based on the worst deviation figures (this definition appears to be that employed by Radeka and Boie). It is felt that a better way of assessing the performance of the whole
grid would be to quote the differential nonlinearity as an rms value (the rms deviation of the function $dx/dQ(x)$ from its mean value). However, a complete conception of the nonlinearity can only be gained by observing the entire function, as, for instance, in the photographs already shown. The differential nonlinearity is the more demanding figure of merit of the two: it may be thought of as being associated with the fluctuations of the deviation of the position calibration function, $Q(x)$, from its mean value $\delta$. It is thus a second-order differential, rather than a first-order one, and is therefore a more sensitive parameter.

Detector Scaling. Some attempt was made, in Chapter 8, to assess the effects of increasing detector size upon effective load capacity, $C_e$. It is more advisable to consider specific cases than to apply any generalised rules, in view of the number of variables concerned. The particular relationship applicable to the readout methods examined here is that given in Appendix 1, equation (A1.11); which may be used to examine either $CR$ or $\gamma$-method scaling properties:

$$C_e = C_1 \coth \gamma N \sqrt{p(1 + 1/4)}.$$  

As detector size increases, the available resolution and linearity both tend to degrade. It is possible to maintain either quantity at some cost to the other; for instance, $C_L/C$ may be held fixed (where $C_L = C/N$) to preserve linearity (see Chapter 8), but if this occurs $C_e$ will inevitably increase. Conversely, if $C_e$ is held fixed by adjustment of $C_1$ and $N$, the ratio $C_L/C$ must fall, as detector size increases.

If it is decided to accept a degree of nonlinearity in order to maintain the resolution performance, one is ultimately faced with the dependence of spatial resolution on sensitivity (see expressions (30) and (32), Chapter 7); furthermore, as $C_L$ is decreased the signal charges $q_A$ and $q_B$ fall, reducing the signal-to-noise ratio even if $C_e$ can be maintained at a low value.
iii) **Specimen Resolution Performance**

**Conditions.** We shall consider in this section the hypothetical case of a MWPC of the present design equipped with low-loss two-section GD cathodes, and examine the various factors influencing the spatial resolution. In the investigations so far carried out it has not been possible to vary the X-ray energy, and it would be interesting to examine how energy variation affects resolution. The various contributions to spatial uncertainty are given in Table 18, together with the conditions which produce them. It has been assumed that the mean energy required to produce one ion pair is 30 eV.

**Table 18. Contributions to Spatial Uncertainty.**

<table>
<thead>
<tr>
<th>Contributions in μm (rms)</th>
<th>Energy in keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^2_{\text{TOTAL}} = \sigma^2_p + \sigma^2_D + \sigma^2_e + \sigma^2_r$</td>
<td></td>
</tr>
</tbody>
</table>

- **Photoelectron range:** $\sigma_p = 28.3 E^{1.23}$ (see Chapter 7 and reference 107);
- **Centroid jitter:** $\sigma_D = 73/\sqrt{E}$; Electronic (front-end): $\sigma_e = 20.7/E$;
- **Electronic (divider):** $\sigma_r = 30$ (constant)

**Conditions:** P50 gas; $d = 25$ mm; $h = 6$ mm; $q_n = 240$ electrons rms; $T_a = 1\mu$s; unipolar shaping; charge conversion = 1/3 pC/keV.

**Energy Range.** The factors which limit the useful energy range of a MWPC are the window lower transmission limit (~0.1 keV) and the worsening efficiency and image blurring at higher energies (2-3 keV). However, we shall examine here the range 0.1 to 6 keV, assuming that the electronic units have sufficient dynamic range to deal with this.

The absorption efficiency for 25 mm of P50 gas falls from 100% at 0.1 keV to about 19% at 3.2 keV, the argon K-edge; at higher energies the
efficiency improves (to 88% just above 3.2 keV) as K-shell absorption occurs, but falls again to 20% at 6 keV.

Associated with efficiency is a much more severe limitation on the maximum X-ray energy: this is the image blurring of an inclined X-ray beam. Higher-energy X-rays have a large mean absorption depth, so that the rms fluctuation in this is also large. In turn, the fluctuation in absorption depth causes a fluctuation in mean position, when the X-ray beam is inclined. For example, in the present case, the rms fluctuation in the absorption depth for 1.5 keV X-rays is about 6 mm; so for an X-ray beam inclined at 6° to the normal (a worst case for present minor designs), the associated rms spatial uncertainty is 610 μm! This, then, sets an upper limit to the useful energy range of the detector, when used, for example, with X-ray telescopes; despite this, we shall continue the discussion for energies up to 6 keV, which may be of interest in applications where parallel X-ray beams are used.

Discussion. Figure 70 shows the various contributions to spatial uncertainty, for a vertical beam of X-rays. The front-end electronic contribution is based on the value given in Table 17 for the low-loss cathode, for a source location at the electrode centre, and all other contributions are as listed in Table 18. The photoelectron range contribution has been calculated by weighting the contributions expected for argon and methane according to the relative absorption efficiencies of the two gases.

An interesting effect occurs above 3.2 keV: the spatial resolution improves, because of the presence of two photoelectrons (see Chapter 7, where a fuller explanation is given). It should be possible to test this by means of a 55Fe X-ray source, by examining firstly the photopeak events, of which most should show this behaviour (5.9 keV), and secondly escape peak events (2.7 keV), which should show a poorer resolution. Future work may include this test.
Figure 70  Spatial uncertainty as a function of X-ray energy for an argon/methane-filled IPC having electrodes of size 9.5 cm. The particular conditions are given in the text and Table 18. The various contributions are labelled as in Table 18; the effect of blurring due to inclined entry has not been included.
It must be stressed that figure 70 shows a projected performance, and is in some ways based on optimistic assumptions. Nonetheless, it does indicate how small the electronic contribution to spatial uncertainty has been made, for this size of detector.

iv) Future Work

This section points out some subjects for future investigation, which may be divided into three categories: particular problems encountered to date; proposed extension of this programme; and associated work in other programmes.

Outstanding Problems. There are three puzzling results of this work which have not yet been explained. The first of these is the disagreement between predictions of avalanche angular spread made from pulse height ratios and those made from bipolar signal zero-cross times (see Table 6, Chapter 5). It appears that this may be associated with the large dielectric constant of the anode substrates. This is a view which is supported by the observation of the peculiar up-down asymmetry described in Chapter 5, and by a more recent measurement which indicates that the cathode pulse crossover time increases as the source is moved along the anode wire direction, towards the substrate.

The second problem which requires further study is the discrepancy between the sizes of the electron charge cloud inferred by the two methods described in Chapter 6 (see figures 41 and 42 there). It may be that a further study of secondary avalanches will throw some light on this subject: perhaps gas mixtures with lower quenching power can be examined.

Finally, there remains the problem of the correlation coefficient of the two-section GD cathodes. It is difficult to see how either the results or the predictions could be wrong, but is suggested that a simulation with a scaled-up capacity values, leading to the same expected $r_{ab}$
Aims of the Present Programme. It is intended to continue the present research programme, with two aims in particular. The first is to re-examine the subject of angular localisation. By use of GD or CR cathodes, it should be possible to observe some interpolation between anode wires (see Chapter 5), and it is hoped to examine the case where long time constants (5-10 μs) are used. There should be more success this time than with the previous RC line investigation.

The second aim is to examine more closely the behaviour of the GD cathodes. In particular, attention will be paid to construction problems (e.g. of low-loss grids) to smaller wire pitches for improved linearity, and to theoretical and practical considerations of further subdivision, with a view to obtaining an optimum noise performance.

Subsidiary work will include some attempts to confirm the existing models for photoelectron range in argon-methane mixtures, by use of a high-sensitivity electrode (Chapter 6), at soft X-ray energies (1-6 keV). Besides this, it is hoped that a more compact structure for the CR line can be designed, for use in systems where good local linearity is required, and further work is also recommended to examine large-size CR line structures for resolution and linearity.

Associated and Other Work. Development of a xenon-filled IPC, such as that under development in this Department is expected to result in the reduction of the problem of image blurring, and should also reduce the photoelectron range contribution to spatial uncertainty. Such chambers are suited to higher-energy observations, and will undoubtedly succeed the argon-filled type in applications where X-rays approach the chamber at large incident angles.

The work which has already begun in this Department on GD readout for
microchannel plate cameras is expected to continue, concentrating on linearity and resolution as usual, but also on some problems particular to this type of system (e.g. certain effects of the field around the grids). Experimental comparison between this and other systems\textsuperscript{113,115} will be made.

Finally, while some attempt to measure photoelectron range may be made as the present work continues, a systematic study of this subject (with particular reference to counter gases) by more suitable techniques would be welcome.
APPENDICES
APPENDIX 1

SUMMARY OF CR LINE THEORY

i) Filter Network

Although the CR line has been previously dealt with by considering it to possess uniformly-distributed parameters, the case of N sections of lumped parameters, where N is large, is a truer representation of the physical line described in Chapter 8. The case of discrete components may be dealt with by filter network theory, a treatment which is simpler than the line approach.

The CR line, then, may be represented by N \( \pi \)-sections, each of the form shown in figure A1; the line is terminated by a load \( z_T \). From standard filter network theory, the following results may be obtained:

a) The input impedance of the line is

\[
Z_{IN} = z_k \left\{ \frac{z_T \cosh \gamma N + z_k \sinh \gamma N}{z_T \sinh \gamma N + z_k \cosh \gamma N} \right\},
\]

where \( z_k \) is the line characteristic impedance \( z_k = z_1 / \sqrt{p(1 + p/4)} \)

if \( p = z_1 / z_2 \), and \( \gamma \) is the line propagation constant, given by \( \cosh \gamma = 1 + p/2 \).

b) If the line is shorted, \( z_T = 0 \) and

\[
Z_{IN} = z_k \tanh \gamma N \quad (A1.1)
\]

c) If \( i_T \) is the current in the (short-circuit) termination and \( i_{IN} \) is the current input to the line, then

\[
i_T = i_{IN} / \cosh \gamma N \quad (A1.2)
\]
ii) Position Signal

Suppose now that a current $i$ is injected at the node between the sections $n$ and $(n + 1)$. The current will divide in accordance with the impedances seen in each direction, $z_L$ and $z_R$, as follows (see figure A2):

$$i_L = iz_R/(z_L + z_R); \quad i_R = iz_L/(z_L + z_R),$$

where, from equation (A1.1), $z_L = z_k \tanh \gamma n$ and $z_R = z_k \tanh \gamma (N - n)$.

![Diagram](image)

The currents in the terminations are, from equation (A1.2),

$$i_0 = i_L/cosh \gamma n \quad \text{and} \quad i_{N+1} = i_R/cosh \gamma (N - n)$$

These three pairs of expressions may be manipulated to give

$$i_0/i = \frac{\sinh \gamma N(1 - x)}{\sinh \gamma N} \quad \text{and} \quad \frac{i_{N+1}}{i} = \frac{\sinh \gamma Nx}{\sinh \gamma N},$$

where $x = n/N$. If now the position signal is chosen to be

$$Q = i_{N+1}/(i_0 + i_{N+1})$$

we can write

$$Q = \frac{\sinh \gamma Nx}{[\sinh \gamma N(1 - x) + \sinh \gamma Nx]} \quad \text{(A1.3)}$$

iii) Position Signal for the CR Line

Now it is necessary to particularise for the case of the CR line. Here the π-section is as shown in figure A3, with $C_1 = NC_L$ and $a = (C/C_L)^{1/2}$ as in Chapter 8, and with $C_2 = C/N$.
Now since \( p = z_1/z_2 \),
\[
\frac{1}{sC_1} = \frac{R_1}{1 + sC_2R_1}
\]
i.e. \( p = \frac{1 + st}{s(C_1/C_2)} \),
where \( \tau = C R \),
and \( s \) is the complex frequency, as employed in Laplace Transform analysis
\( s = j\omega \) here). Replacing \( C_1 \) and \( C_2 \) appropriately,
\[
p = \frac{a^2}{N^2} \left( 1 + 1/st \right)
\]
and also \( \tau = C/G \), since \( G = N/R_1 \). Because the frequencies of
interest are usually such that \( st \gg 1 \), i.e. \( T_a << \tau \) where \( T_a \) is the
processing time constant, we can say that \( p << 1 \), since it is usual that
\( a^2 << N^2 \). Also, since \( \cosh \gamma = 1 + p/2 \), we can say \( \gamma = \sqrt{p} \), so that
\[
\gamma = a/N \left( 1 + 1/st \right)^{1/2}
\]
To a first-order approximation, then, \( \gamma N = a \), so that
\[
Q = \frac{\sinh ax}{[\sinh a(1-x) + \sinh ax]}
\]
Note also that we can write the expression for \( i_{N+1} \) as:
\[
\frac{i_{N+1}}{i} = \frac{q_{N+1}}{q} = \frac{\sinh ax}{\sinh a}
\]
yielding the expression (35) given in Chapter 8.

iv) Electronic Noise

The electronic noise is determined by the preamplifier loading, so
that here we need to find \( z_{IN} \), to predict the noise charge.
Now since \( p << 1 \), we can write \( z_k = z_1/\sqrt{p} \), and so
\[ z_{IN} = \frac{z_k}{p} \tanh a(1 + l/2st) \]

taking \( Y \) from equation (A1.5). We can also substitute for \( p \) from equation (A1.4):

\[ z_{IN} = \left( \frac{1}{sC_1} \right) \frac{N/a(1 - l/2st)}{1} \tanh a(1 + l/2st) \quad (A1.6) \]

(remember that \( st \gg 1 \)). Now

\[ \tanh a(1 + l/2st) = \frac{[\tanh a + \tanh a/2st]}{[1 + \tanh a \tanh a/2st]} \]

since \( a \ll 2st \), we have

\[ \tanh a(1 + l/2st) = \left( \tanh a + a/2st \right) \left( 1 - [a/2st] \tanh a \right) \]
\[ = \tanh a + a/2st \left( 1 - \tanh^2 a \right) \]
\[ = \tanh a + \frac{a}{2st} \sech^2 a \]

Substitution of this expression into equation (A1.6) gives, after some manipulation,

\[ z_{IN} = \frac{1}{sC_1 a} \left\{ \tanh a + \frac{1}{2st} \left( \frac{a - \sinh a \cosh a}{\cosh^2 a} \right) \right\} \]

i.e.,

\[ z_{IN} = \frac{\tanh a}{sC_1 a} - \frac{1}{4s^2 C_1 a} \frac{\sinh 2a - 2a}{\cosh^2 a} \quad (A1.7) \]

again ignoring terms in \( (1/2st)^2 \). Here \( C_1 \) has been replaced by \( NC_1 \).

If we now regard \( z_{IN} \) as a parallel combination of a resistance \( R_e \) and a capacitance \( C_e \), we can write

\[ z_{IN} = R_e (1 - sR_e C_e)/(1 - s^2 R_e^2 C_e^2) \]

or, provided \( sR_e C_e \gg 1 \),

\[ z_{IN} = \frac{1}{sC_e} - \frac{1}{s^2 R_e^2 C_e} \quad (A1.8) \]

Setting (A1.7) and (A1.8) equal, we can say that

\[ C_e = C_1 a/\tanh a \]
and \[ R_e = \frac{4}{G} \cdot \frac{a \sinh^2 a}{(\sinh 2a - 2a)} \],

using \( \tau = \frac{C}{G} \) and \( a^2 = \left( \frac{C}{C_L} \right) \). These two equations are equations (36) in Chapter 8.

\section*{v) Correlation Coefficient}

For calculation of the correlation coefficient \( r_{ab} \), the model is simplified by ignoring the effect of resistance. Again, the line has effective short-circuit terminations. We recall here that if \( Y_{TR} \) is the line transfer admittance (giving the current at one end of the line resulting from a voltage signal at the other), we can write

\[ Z_{IN} Y_{TR} = 1/cosh \gamma N \quad \text{(A1.9)} \]

and since the effect of resistance is ignored, we may say that

\[ Z_{IN} = 1/sC_{in} \quad \text{and} \quad Y_{TR} = sC_{TR} \], defining \( C_{in} \) and \( C_{TR} \).

It is assumed that the dominant noise in the system comes from the capacitively-loaded preamplifier. The system is represented in figure A4, where \( v_{n1} \) and \( v_{n2} \) are the noise sources, and are uncorrelated \( (v_{n1}v_{n2} = 0) \).

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figA4}
\caption{Fig.A4}
\end{figure}
If \( v_{ij} \) represents generally the voltage output at \( i \) due to a noise source at \( j \), we have

\[
v_{11} \left( \frac{1}{C_{\text{in}}} \right) - v_{n1} = 0
\]

because the preamplifier input is a virtual earth. If, as is usual, \( C_{\text{in}} \gg C_0 \), then

\[
v_{11} = v_{n1} C_{\text{in}} / C_0
\]

we also have

\[
v_{12} = -v_{n2} C_{\text{TR}} / C_0
\]

Hence,

\[
v_1 = v_{11} + v_{12} = \left( v_{n1} C_{\text{in}} - v_{n2} C_{\text{TR}} \right) / C_0
\]

and similarly,

\[
v_2 = \left( v_{n2} C_{\text{in}} - v_{n1} C_{\text{TR}} \right) / C_0
\]

The products \( v_1^2 \) and \( v_1 v_2 \) can now be formed, bearing in mind that

\[
v_{n1} v_{n2} = 0
\]

and writing \( v_1^2 = v_{n1}^2 = v_2^2 = v_{n2}^2 \) (identical preamplifiers):

\[
\frac{v_1^2}{v_n^2} = \frac{C_{\text{in}}^2 + C_{\text{TR}}^2}{C_0^2}
\]

and

\[
\frac{v_1 v_2}{v_n^2} = -2 \frac{v_{n1} v_{n2}}{C_{\text{in}} C_{\text{TR}}}
\]

Thus,

\[
r_{12} = \frac{v_1 v_2}{r_n^2} = -2 \frac{C_{\text{in}} C_{\text{TR}}}{C_{\text{in}}^2 + C_{\text{TR}}^2}
\]

so that, by use of equation (A1.9), we find

\[
r_{12} = -2 \cosh \gamma N / (1 + \cosh^2 \gamma N)
\]

(A1.10)

The value of \( \gamma \) is available through \( \cosh \gamma = 1 + p/2 \), where, in this case, \( p = C_2 / C_1 \), having set \( z_1 = 1 / sC_1 \) and \( z_2 = 1 / sC_2 \). The expression (A1.10) is valid for all values of \( N \): in particular, we note that the cases \( N = 1 \) and \( N = 2 \) refer to the undivided and two-section GD
cathodes respectively. Expression (41), in Chapter 9, is equation (A1.10) rewritten for $N = 1$.

vi) Load Capacity

The quantity $C_{in}$, the line input capacity, is obviously the same thing as the effective load capacity, $C_e$. Now from equation (A1.1) above, we find, by substituting for $z_k$, that

$$Z_p = Z_j \tanh \frac{YN}{\sqrt{p(1 + p/4)}}$$

so that

$$C_e = C_{in} = C_1 \cdot \coth \frac{YN}{\sqrt{p(1 + p/4)}}$$

which enables us to find the effective load for any $N$ and $p$.

Examples: a) the CR line.

Using the same approximations made in section (iii) above, namely,

$YN = a$ and $a^2 \ll N^2$, and recognising that in this case $C_1 = N.C_L$

and $C_2 = -C/N$, we find that

$$C_e = C_1 \cdot \coth a \cdot \sqrt{p}$$

ignoring $p^2$ because $p \approx a^2/N^2$. This expression may be rearranged into the form already derived.

b) GD cathode (undivided).

Here $N = 1$, and using $\cosh \gamma = 1 + p/2$,

$$C_e = C_1(1 + p/2) = C_1 + C_2/2,$$

or in the terminology of Chapter 9,

$$C_e = C_c + C/2.$$
APPENDIX 2

THE EFFECTS OF RESISTIVE AND CAPACITIVE LOADING ON PREAMPLIFIER NOISE

In Appendix 1 it was shown that the capacitive line (see Chapter 8) could be represented by a parallel combination of resistance and capacity; and indeed the graded-density cathode discussed in Chapters 9 and 10 may be similarly represented. A simple treatment of preamplifier noise, showing the effects of such a load, will now be given. Note that this treatment does not consider FET flicker noise, nor the contribution of thermal noise from the FET by the gate-source capacitance (see comments at the end of this appendix).

Current Generator

![Current Generator Diagram]

Firstly, we consider the load as a current generator. For the sake of simplicity, the load capacity and resistance here are incorporated into a total input impedance, itself a parallel combination of a resistance $R_t$ and a capacity, $C_t$. In the case of the current generator, $C_t$ is effectively shorted out because the point A is a virtual ground. $\overline{d\varepsilon_n^2}$ is the mean square thermal noise current generated by $R_t$. Thus

$$\overline{d\varepsilon_n^2} = (2kT_{e}/\pi R_t)\omega$$

where $k$ is Boltzmann's constant and $T_e$ is the absolute temperature. The mean square voltage across $C_o$ is the same as $\overline{d\varepsilon^2}$, the preamplifier output mean square voltage, so that
The resultant filter amplifier output is then given by

\[
\overline{\frac{v^2}{v_0}} = \frac{1}{A_s^2} \cdot \frac{2kT}{\pi R_t^2} \cdot \int_0^\infty \frac{1}{\omega^2} |H(j\omega)|^2 d\omega ,
\]

so that

\[
q_1 = \frac{1}{A_s^2} \cdot \frac{2kT}{\pi R_t^2} \cdot \int_0^\infty \frac{1}{\omega^2} |H(j\omega)|^2 d\omega ,
\]

(A2.1)

where \(A_s\) is the peak height of the filter step response, \(H(j\omega)\) is the filter transfer function and \(q_1\) is the preamplifier noise associated with the resistive load.

Voltage Generator

In this case, we assume that \(1/\omega C_t \ll R_t\), so that we can make an approximation by ignoring \(R_t\). In this case, then,

\[
\overline{\frac{dv^2}{n}} = \frac{1}{(1/C_t + 1/C_o)} \overline{dv^2}
\]

where \(\overline{dv^2} = \frac{2kT}{\pi} R_n d\omega\). Manipulation of these expressions shows that

\[
q_2 = \frac{1}{A_s^2} \cdot \frac{2kT}{\pi R_n C_o^2} \int_0^\infty |H(j\omega)|^2 d\omega ,
\]

(A2.2)

where \(q_2\) is the preamplifier noise associated with the capacitive component of the load.
**Bipolar Shaping.** The specific filter-amplifier transfer functions must now be considered. If a bipolar filter network is used, then

\[ H(j\omega) = \left( \frac{j\omega}{j\omega + 1/T_a} \right)^2 \left( \frac{1/T_a}{j\omega + 1/T_a} \right), \]

so that

\[ |H(j\omega)|^2 = \frac{(\omega T_a)^4}{(1 + (\omega T_a)^2)^3}. \]

This allows us to perform the integrals contained in expressions (A2.1) and (A2.2), which give

\[ q_1^2 = \frac{1}{A_S^2} \cdot \frac{2kT}{\pi} e \left\{ \frac{\pi}{16} \frac{T_a}{R_t} \right\} \quad \text{(A2.3)} \]

and

\[ q_2^2 = \frac{1}{A_S^2} \cdot \frac{2kT}{\pi} e \left\{ \frac{R_n C_t^2}{T_a} \cdot \frac{3\pi}{16} \right\}. \quad \text{(A2.4)} \]

By application of standard Laplace transform techniques, the step function response of the bipolar filter can be shown to be \( A_S = 0.231 \).

**Unipolar Shaping.** In the case of the unipolar filter, the system transfer function is

\[ H(j\omega) = \left( \frac{j\omega}{j\omega + 1/T_a} \right) \left( \frac{1/T_a}{j\omega + 1/T_a} \right), \]

so that

\[ |H(j\omega)|^2 = \frac{(\omega T_a)^2}{(1 + (\omega T_a)^2)^2}. \]

The integrals may this time be performed to give:

\[ q_1^2 = \frac{1}{A_S^2} \cdot \frac{2kT}{\pi} e \left\{ \frac{\pi}{4. R_t} \right\} \quad \text{(A2.5)} \]

and

\[ q_2^2 = \frac{1}{A_S^2} \cdot \frac{2kT}{\pi} e \left\{ \frac{R_n C_t^2}{T_a} \cdot \frac{\pi}{4} \right\}. \quad \text{(A2.6)} \]

For the unipolar filter, \( A_S \) is found to be 0.368.
Discussion. It is to be remembered that the quantities $R_t$ and $C_t$ occurring in expressions (A2.3) to (A2.6) are not simply the equivalent load values. They incorporate also resistive and capacitive parameters associated with the input of the preamplifier; for instance,

$$C_t = C_e + C_i + C_o,$$

where $C_i$ is the total input capacitance, and $C_o$ the feedback capacitance, of the particular type of preamplifier. Similarly,

$$R_t^{-1} = R_e^{-1} + \left(1/R_b + eI_G/2kT_e\right),$$

where $R_b$ is the total (shunt) value of all resistors connected to the input terminal, and the final term represents the shot-noise in the FET gate leakage current, $I_G$. Besides these, the current generator representation should also include another term, due to the transfer of thermal noise into the gate circuit via the gate-source capacity, $C_{gs}$, and having a value of about $R_n C_{gs}^2 \omega^2/3^{119}$. However, because of the presence of the factor $\omega^2$, this term may be more conveniently written into the voltage-generator set of expressions, which is why it has not been included in the expression for $R_t$ above.

One final noise contribution is that of the FET flicker noise; at present, few theoretical treatments appear to be available, and an empirical approach is therefore useful. The flicker noise may be thought of as adding to the value of $R_n$ (which itself represents the FET channel thermal noise), so that one may substitute a term such as $R_n + F/\omega$ for $R_n$ in the noise expressions, where $F$ is to be determined from experiment. When all these factors are considered, the following expressions are found:
\[ q_n^2 = \frac{1}{(0.231)^2} \cdot \frac{2kT}{\pi} \left( \frac{3\pi R_n}{16 T_a} (C_t^2 + 1/3 C_{gs}^2) + \frac{\pi T_a}{16 R_t} + \frac{1}{4} FC_t^2 \right) \]  \quad (A2.7)

(BIPOLAR CASE)

and

\[ q_n^2 = \frac{1}{(0.368)^2} \cdot \frac{2kT}{\pi} \left( \frac{\pi R_n}{4 T_a} (C_t^2 + 1/3 C_{gs}^2) + \frac{\pi T_a}{4 T_t} + \frac{1}{2} FC_t^2 \right) \]  \quad (A2.8)

(UNIPOLAR CASE)

where the parameters \( R_n, C_{gs}, F \) and all the contributions to \( C_t \) and \( R_t \), except \( C_e \) and \( R_e \), are characteristics of the particular preamplifier in question. The expressions (A2.7) and (A2.8) have previously been developed by Mathieson et al., who also give magnitudes of the various preamplifier characteristics for the case of the Ortec model 118A preamplifier.

Some example figures are now given, as a rough guide to the relative magnitudes. In many cases, \( R_b \) and \( I_B \) are such that \( R_t = R_e \).

The capacities \( C_{gs} \) and \( C_i \) are usually about 30 pF, while \( C_o \) is usually about 1 pF. The equivalent resistor \( R_n \) is of the order of a few tens of ohms, while \( F \) is of the order of 50 MΩ/s. The interesting consequence of these observations is that each of the three terms in the above expressions are of similar orders of magnitude.

The above expressions will serve to predict, with moderate accuracy, the equivalent noise charge resulting from a particular load.
# REFERENCES

## Abbreviations

<table>
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<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>IEEE Trans. Nucl. Sci.</td>
<td>Institute of Electrical and Electronics Engineers: Transactions on Nuclear Science</td>
</tr>
<tr>
<td>ISA Trans.</td>
<td>Instrument Society of America: Transactions</td>
</tr>
<tr>
<td>J. Phys. E.</td>
<td>Journal of Physics, E (Scientific Instruments)</td>
</tr>
<tr>
<td>Nature</td>
<td>Nature</td>
</tr>
<tr>
<td>Nucl. Instr. Meth.</td>
<td>Nuclear Instruments and Methods</td>
</tr>
<tr>
<td>Phys. Rev.</td>
<td>Physical Review</td>
</tr>
<tr>
<td>Proc. IEEE</td>
<td>Proceedings of the Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>Rev. Mod. Phys.</td>
<td>Reviews of Modern Physics</td>
</tr>
<tr>
<td>SPIE</td>
<td>Society of Photo-optical Instrumentation Engineers (Proceedings)</td>
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<td>Space Sci. Instr.</td>
<td>Space Science Instruments</td>
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Several aspects of X-ray position location in an imaging multiwire proportional chamber (MWPC) are examined in this work.

Experimental confirmation is obtained of existing theoretical predictions which take into account the presence of avalanche angular localisation. An attempt is made to exploit the effects of the latter by means of a position-encoding RC transmission line, revealing that the variations in signal risetime caused by angular localisation are detrimental to linear operation of the line. Other results indicate that successful exploitation by different means is possible.

A brief study of electron diffusion effects in MWPC's is made, giving some agreement with predictions, but also exposing an effect which degrades resolution and improves linearity in one sensing direction only (that perpendicular to the anode wires). This effect has yet to be explained.

The series capacitance line is examined in the context of MWPC operation, and is found to offer superior spatial resolution (though slightly inferior linearity) to that obtainable with an RC line.

The concept of the graded-density (GD) cathode is introduced, and an experimental study shows that the device can be fully competitive as the encoding cathode of a MWPC. Once again, the spatial resolution is superior to that obtained with an RC line, although some small local readout non-linearity mars the present performance.

Finally, the cascading of two such electrodes to form a single "sub-divided" GD cathode is studied. In this case the cathode behaves similarly to a series capacitance line, and the facility for partially compensating for the resultant nonlinearity, by adjustment of the density gradings, is demonstrated.