THE IMAGING PERFORMANCE

OF

MULTIWIRE PROPORTIONAL CHAMBERS

by

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ABSTRACT

A simplified description is given of the operation of a multiwire proportional chamber (MWPC) in the soft X-ray imaging application. Expressions are developed to allow the calculation of the distribution of induced charge on the cathodes of an MWPC. With extensions to permit direct comparison, the calculations are subjected to detailed experimental verification. A generalised, approximate formulation of the distribution with one independent parameter is described. The prediction of cathode system position response using the theoretical distributions is demonstrated.

The available MWPC position readout methods are reviewed, and where possible their differential non-linearity is measured experimentally. A new position-sensitive cathode of good linearity and spatial resolution is presented. The effect of the wires of an MWPC on its imaging performance is briefly considered.

An attempt is made to assess the contribution to MWPC spatial resolution of the range of the electrons produced initially by an X-ray absorption event in argon-methane mixtures. In conclusion, the important causes of MWPC imaging imperfection are noted and classified.
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This work is dedicated to my Mother 

and my late Father.
LIST OF PUBLICATIONS

Some of the work reported here has been described in the following publications. The relevant chapters of the thesis are indicated.


This last publication contains a full mathematical treatment of the calculation of cathode induced charge distributions in the anode wire direction. This is covered in outline in Chapter 3.
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TABLE OF SYMBOLS

The symbols in this table are used throughout. Other symbols are defined in the text, or have their conventional meanings.

\( x \): general co-ordinate parallel to the anode wires
\( z \): general co-ordinate orthogonal to the anode wires
\( y \): general co-ordinate orthogonal to the anode plane
\( x_c \): \( x \) co-ordinate along a cathode
\( z_c \): \( z \) co-ordinate along a cathode
\( \lambda \): \( x_c/h \) or \( z_c/h \)

\( h \): anode-cathode spacing
\( s_a \): anode wire pitch
\( r_a \): anode wire radius
\( s_c \): cathode wire pitch
\( r_c \): cathode wire radius
\( d_1, d_2 \): spacing between wire cathodes and earth planes

\( \alpha \): azimuthal angle around an anode wire; specifically the avalanche angle centroid
\( \sigma \): specifically the rms avalanche angular spread

\( \mu \): positive ion mobility
\( E \): electric field
\( P \): potential function
\( V_a \): anode potential
\( V_c \): cathode potential
$\rho_x (\lambda)$: cathode induced charge density distribution in the x axis
$\rho_z (\lambda)$: cathode induced charge density distribution in the z axis
$q_z (\lambda)$: induced charge distribution in the z axis on a wire cathode
$\Gamma$: normalised cathode charge distribution (see Section 3.4.6)
$q_a$: anode signal charge

$E_x$: X-ray energy
$F$: Fano factor
$f$: relative variance in the size of the avalanche due to one electron

$C_p$: wire array shunt capacity
$C_a$: individual node (or section) shunt capacity
$C_m$: capacitive load presented to a pre-amplifier
$C_{ni}$: capacitive line inherent longitudinal capacity
$C_{na}$: capacitive line added longitudinal capacity
$C_n = C_{ni} + C_{na}$

$Q$: position signal
$S$: position sensitivity
$G(Q)$: uniform irradiation response function
$\delta$: rms percentage differential non-linearity (Section 6.1.3)
$\epsilon$: max/min percentage differential non-linearity (Section 6.1.3)

$q_a$: equivalent noise charge at a pre-amplifier input
$r_{AB}$: noise correlation coefficient for two terminal systems
$\sigma_e$: rms contribution of electronic noise to position resolution
1.1 The birth and application of the multiwire proportional counter

The interaction of ionizing radiation with a gas produces free electric charge which may be used as the basis of the detection of that radiation. The movement of this charge in an applied electric field constitutes an electric current, but except for the most heavily ionizing radiations, the signal is too small to be of practical use in the detection of individual particles or photons. Some physical amplification is required.

Rutherford and Geiger [1] introduced a counter in which the gaseous avalanche multiplication process described by Townsend [2], occurring in the high electric field near a fine wire, was used to obtain the required gain. The real usefulness of this device, the proportional counter, was only realised with the availability of pulse processing electronics in the 1930s, however. By the end of the Second World War, Rossi and Staub [3] were able to describe a wide variety of proportional counters that had been developed under the impetus of the atomic weapons programme. It is interesting to note that one of these designs included a multiwire anode plane, but that the full promise of such an arrangement was not apparently realised at the time.

Further significant progress of the multiwire proportional counter (or chamber) (MWPC) thus awaited the work of Charpak et al.
[4], who were concerned with the localisation of charged particles produced by the accelerators at the CERN laboratories. The prototype counter is shown in Fig 1.1. Attachment of an amplifier to each wire in the plane of constant pitch anode wires confirmed that the wires operated independently. A particle track could be located in one dimension by observing on which wire an avalanche had occurred. The CERN group quickly realised the potential of the device, proposing simplified readout methods [5] and the possibility of two-dimensional imaging using the charge induced on the cathode planes [6]. Other high-energy physics groups were soon using MWPCs, and this early work has been well described by Rice-Evans [7].

A close relative of the MWPC, the multiwire drift chamber [6,8] has in fact proved to be the workhorse of high-energy physics, playing a central role in some large and highly complex detector systems (for example the CERN UA1 detector [9]). Great flexibility in the choice of geometry and gas fillings, a good rate capability, and the ability to image individual particles and photons in real time has led to many other areas of application for the MWPC, however. These have included

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Fig 1.1 The prototype position-sensitive MWPC of Charpak et al. (from [4]). Note that negative high voltage was applied to the cathodes, rather than positive high voltage to the anode as in the present work.
X-ray diffraction [10,11,12,13], biophysics and medicine (well reviewed by Charpak [14] and Faruqui [15]), and X-ray astronomy [16,17]. The advent of new synchrotron radiation sources will surely increase the number and diversity of applications in the areas of material studies, biophysics and medicine, and as Bateman et al. [18] have pointed out, the rapidly decreasing cost of computer hardware will make MWPC based systems more and more attractive.

Physical adaptation of the basic MWPC has made possible further uses. High energy gamma rays, notably those resulting from positron annihilation, may be imaged by MWPCs if a high density converter material, usually lead, is incorporated in them [19,20,21]. Alternatively, the radiation may be stopped in a scintillator such as BaF$_2$. The emitted ultra-violet radiation can then produce charge in the MWPC through absorption by a low photoionization potential molecule, the most useful of which is tetrakis(dimethylamino)ethene [22]. Thermal neutrons can generate charge in the counter through the reaction

$$n + ^3\text{He} \rightarrow p + ^2\text{H} + 0.76\text{MeV}$$

and thus be localised in the same manner as ionizing radiation [23,24].

Much recent work on gas filled radiation detectors has concentrated on parallel plate structures, in which the amplifying region extends between conducting planes, instead of being confined to the vicinity of fine wires. Of particular interest are the multistep chambers, in which the overall gain is obtained in more than one amplification region [25]. Charpak has summarised some advantages of multistep chambers over MWPCs for particle tracking [26], but it should be appreciated that much MWPC technology, particularly readout
systems, is directly applicable to the new devices. In fact, MWPCs are conveniently incorporated as position readout stages both in multistep chambers \[25,27\], and in other new devices such as hybrids with gas scintillation counters \[28\], and time projection chambers \[29\].

Overall it may thus be seen that the study and development of MWPCs will remain an area of considerable importance. Although it may be considered a fairly mature scientific instrument after over fifteen years of development as an imaging device, it is probably true to say that a good understanding of the working of MWPCs has only been built up in the last five to six years. The combination of the inherent simplicity and flexibility of the basic device, with the availability of models allowing the optimisation of design details prior to construction, should ensure a useful future.

1.2 Scope of the thesis

In using the MWPC as an imaging device, it is clearly desirable that the images obtained be as free of distortion and as high in resolution as possible. The present work is concerned with the optimisation of MWPC imaging performance. Chapter 2 gives a simplified description of the operation of the device in the soft X-ray imaging application, and includes some practical experimental details. The following three chapters (Chapters 3 to 5) relate to the distribution of induced charge on MWPC cathodes. Knowledge of this distribution is necessary for the optimisation of the design of many position readout methods, and the most important examples of these are discussed in Chapter 6. A new position-sensitive cathode system of
good linearity and resolution is described in Chapter 7. The wires that are characteristic of MWPC construction introduce their own imaging distortions as is shown in Chapter 8. In Chapter 9, the effect of the range of the initial electrons arising from the X-ray absorption on the position resolution is investigated. A concluding chapter summarises the results obtained and notes their implications. Some areas requiring further investigation are then outlined. An appendix to the thesis contains some mathematical details relevant to Chapter 3.
2.1 A simplified description of MWPC operation

2.1.1 Introduction

The present work concerns MWPCs employed as soft (<10keV) X-ray detectors. This should be kept in mind, although many of the results will be directly applicable, or easily extended to MWPCs designed for other radiations. The distinguishing characteristic of the soft X-ray application is that incident photons interact directly with the chamber gas to produce a relatively small number of electron-ion pairs (<350) in a localised region (<1mm^3). The general picture of MWPC operation given in the first half of this chapter is pertinent to this application, and emphasises the implications the various physical factors have for imaging performance.

A consistent co-ordinate system is employed throughout, based on the anode orientation (Fig. 2.1). Standard geometrical and other symbols are defined in the table at the beginning of the thesis. The centres of the anode wires, radius $r_a$, are located at $z = 0$ (the avalanche wire), $+/s_a$, $+/2s_a$...; $y=0$, and the two cathode planes at $y = +/-h$. If one of the cathodes is under specific consideration, it is taken to be at $+h$. If the cathodes are wire arrays, then the ground planes behind them at $+(h+d_A)$ and $-(h+d_A)$ are significant.
Fig. 2.1 Standard co-ordinate system for the present work, based on the anode wire direction. The avalanche position determines the origin (the x axis passes along the centre of the avalanche wire).
2.1.2 The electric field

The operation of an MWPC is dependent upon the electric field established in it, and this is primarily determined by its geometry. In normal operation, this field may be considered to be essentially unaffected by the radiation detection process. Very high counting rates, or high chamber gains can produce sufficient space charge of positive ions to cause significant local modification of the field, however. Under certain conditions, such field modifications may result in operating modes dramatically different from the normal proportional mode, as for example in self-quenching streamer operation [30]. In the present work space charge effects were negligible, so that field calculations only need consider the known arrangement of conductors.

Practical MWPCs usually have overall dimensions in the x and z axes large compared to h and s\(_a\). Furthermore, the regions close to the edges of the electrodes are not commonly used in imaging applications, as the field becomes non-regular and difficult to predict. Thus we may represent the basic MWPC geometry by regularly spaced, infinitely long parallel conducting wires equidistant between conducting planes (Fig. 2.2). With no other (significant) charges present, the arrangement is essentially two-dimensional. In other words, the electric field has no x component, which is obviously beneficial for imaging. In a general z-y plane the field may be conveniently described by a complex potential function, \( W(p) \); \( p = z + jy \), with the conjugate real and imaginary parts representing the mutually orthogonal potential function, \( P \), and flux (or field line) function, \( \phi \). Weber [31] has shown that if the wires are represented by lines (a good approximation if \( r_\phi \ll s_\phi \)) and \( h > s_\phi /2 \), a suitable
The nomenclature and subsequent treatment are adapted from Mathieson and Harris [32]. The real constants $K_0$ and $V_o$ are determined from the boundary conditions $P = V_o$, the known anode potential, when $z^2 + y^2 = r_\alpha^2$; and $P = 0$ at the grounded cathode planes, where $y = +/-h$. Taking $h/s_\alpha$ large compared to unity, we can obtain

$$K_0 = \frac{V_o}{(\pi h/s_\alpha - \ln 2 \pi r_\alpha/s_\alpha)}$$

$$V_o = \frac{(\pi h/s_\alpha - \ln 2) V_o}{(\pi h/s - \ln 2 \pi r_\alpha/s_\alpha)^*}$$

Substituting in Eq. 2.1 and taking the real part as the potential function eventually yields

$$W = P + j \Phi = -K_0 \ln \sin(\pi p/s_\alpha) + V_o$$  \hspace{1cm} \text{Eq. 2.1}
The electric field components are available from partial differentiation of Eq. 2.2, the potential function being defined as
\( E = -\nabla P \). As mentioned before, \( E_x = 0 \) for this case, and the resultant of \( E_z \) and \( E_y \) may be shown to be

\[
E = K \frac{V_\alpha}{h} \left( \frac{\cosh 2\pi y/s_a + \cos 2\pi z/s_a}{\cosh 2\pi y/s_a - \cos 2\pi z/s_a} \right)^{\frac{1}{2}}
\]

where

\[
K = \left( 1 - \frac{s_a}{\pi h} \ln \frac{2\pi r_a/s_a}{1} \right)^{-1}
\]

Near the anode wire, when \( y, z \ll s_a/2 \), and \( \sqrt{z^2+y^2} \) may be replaced by \( r \), the radial distance from the wire centre, this expression reduces to

\[
E = K \frac{V_\alpha}{h} \frac{s_a}{\pi r}
\]

Thus very close to the wires, the field has the same form as in the simple coaxial counter [33].

The flux function is obtained as the imaginary part of Eq. 2.1, and can be expressed as

\[
\phi = -KV_\alpha \tan^{-1} \left( \frac{\tanh \pi y/s_a}{\tan \pi z/s_a} \right)
\]

The field line equations, \( \phi = \text{constant} \), can therefore be written

\[
\tanh \pi y/s_a = \tan \alpha \tan \pi z/s_a
\]
where the field line starts at \( z = r_\alpha \cos \alpha \), \( y = r_\alpha \sin \alpha \) on the wire surface, and \( \alpha \) is the angle as shown in Fig. 2.2. The left hand side of this equation becomes almost constant (=unity) for \( y > s_\alpha \), when Eq. 2.5 may be re-written

\[
\alpha = \pi/2 \left( 1 - 2z/s_\alpha \right) \quad \text{Eq. 2.6}
\]

The line runs parallel to the y axis, and there is a linear relationship between \( \alpha \) and \( z \) which is significant in imaging, as is shown in Chapter 8. The predicted features can be observed in Fig 2.3, where equipotentials and field lines are plotted for the geometry \( h/s_\alpha = 2 \), \( r_\alpha/s_\alpha = 3.75 \times 10^{-3} \). More complex geometries, for example with wire cathode arrays, are more conveniently treated by the superposition of the potentials due to individual wires, as explained in Chapter 3. Fig 2.4 shows how the field lines in one half of the chamber are affected by the presence of cathode wires.

2.1.3 X-ray absorption in the chamber

X-rays in the energy range of present interest give up their energy to standard chamber gases essentially exclusively by the photoelectric process. The cross section for the competing process, Compton or incoherent scattering, is typically at least \( 10^3 \) times smaller [34]. In photoelectric absorption, an atom takes the entire X-ray photon energy, \( E_\gamma \), and ejects an electron with kinetic energy \( E_\gamma - E_\beta \), where \( E_\beta \) was the binding energy of the electron

\[
A + E_\gamma \to A^+ + e^- + (E_\gamma - E_\beta)
\]

The electron is most likely to leave the innermost shell for which the binding energy is less than the photon energy, so the ionized atom is
Fig. 2.3 Calculated field lines (starting at intervals of $\alpha = \pi/6$) and equipotentials (at intervals of $V_\alpha/11$) for the geometry $h/s_\alpha = 2$, $r_\alpha/s_\alpha = 3.75 \times 10^{-3}$ with continuous cathodes. The rapid transition from the almost radial field to the almost parallel field is notable.
Fig. 2.4 Calculated field lines on one side of the anode plane in a chamber with a wire cathode array and a drift space. There is one anode wire, and three cathode wires. The plotting was stopped short of the wire positions to prevent crowding. The fact that the field has a $z$ component near the cathode wires can be significant in imaging applications. The parameters are: $s_c/s_a = 0.5$, $h/s_a = 2$, $d_1/s_a = 2$, $V_a = +2400V$, $V_c = +250V$, earth plane at $y = h + d_1$. 
generally left in an excited state, with an inner shell vacancy. It
de-excites by a series of internal transitions, until vacancies are
confined to the lowest energy orbitals. For example, a K shell
vacancy in argon, created by an X-ray photon with energy greater than
the K edge, 3.20keV, is most likely to be filled by an L electron.
The transition energy appears either as the characteristic X-ray, or
through the ejection of an Auger electron, the average likelihood of
the former process being the fluorescent yield.

\[
\begin{align*}
\text{Ar}^+ & \rightarrow \text{Ar}^+ + \hbar\nu (\text{Ar K}_{\alpha} = 2.96\text{keV}) \\
\text{Ar}^+ & \rightarrow \text{Ar}^{2+} + e^- (\text{k.e.} = 2.96\text{keV} - \text{Ar L binding energy})
\end{align*}
\]

When the fluorescent X-ray escapes the active volume, its energy is
lost. For such events, the total energy available for ionization in
argon is reduced to \((E_X - 2.96\text{keV})\), resulting in the appearance of an
escape peak in the anode pulse height spectrum.

The remaining available energy of the ion (about 500eV for
argon) appears essentially entirely in further low energy Auger
emission [35]. In contrast to these latter Auger electrons, which may
be considered confined to the location of the X-ray interaction, the
initial photoelectron and Auger electron (if produced) can penetrate
some distance in the gas as they loose energy by collision. The
implications of this range for imaging performance are considered in
Chapter 9. A proportion of the collisions between the ejected
electrons and gas atoms produce ionization (about 70% in Ar) as
opposed to excitation, and the relative variance in the total number
of ion pairs, \(N\), is given by [36]
Eq. 2.7

\[
\left( \frac{\sigma}{N} \right)^2 = \frac{(N-\bar{N})^2}{N^2} = \frac{F}{N}
\]

where \( \bar{N} \) is the mean number of ionizations for the particular X-ray energy. The Fano factor, \( F \), would be unity if Poisson statistics were obeyed, but is in general smaller than this; for example \( F = 0.17 \) for argon \([37]\). The small value reflects the fact that the greater proportion of energy loss collisions result in ionization. It is also well known that the energy required to form each ion-electron pair is remarkably constant for a given gas (26.4eV for Ar), independent of the X-ray energy. Thus there is the basis of useful proportional operation. An increase in \( \bar{N} \) and decrease in \( F \) may be achieved if a second gas is present which can be ionized by the excited atoms of the first, and the variance therefore be reduced. This occurs in Penning mixtures.

2.1.4 Electron drift

The result of the X-ray absorption is a relatively compact cloud of primary electrons at close to thermal energies. This cloud will spread by diffusion. Superimposed on the thermal motion is the much slower drift due to the electric field. This eventually takes the electrons to the anode, providing of course that the field lines in the region where the ionization occurred started on the anode. The electrons will remain free in the gas so long as there are negligible concentrations of molecules with significant electron affinities. For this reason the noble gases are good counter gases, the process

\[ A + e^- \rightarrow A^- \]

being energetically unfavourable in their case.
The problem of electron drift in gases under the influence of an electric field has been treated by several workers [38-42]. The useful parameters describing the electron motion are the mean drift velocity of the centre of the cloud, $w$, and the coefficient of diffusion, $D$, both functions of the reduced field, $E/p$, $p$ being the pressure, for a given gas or gas mixture. An electron mobility coefficient, $\mu_e$, a function of the gas composition, temperature and pressure, and the field, is defined by

$$w = \mu_e E$$

An important consideration for imaging MWPCs is the lateral extent of the electron cloud in the $x$ and $z$ axes prior to the avalanche process. For $E$ constant, as is approximately the case in much of the chamber, the rms dispersion of the cloud after moving a distance $d$ is given by

$$\sigma_{\text{eff}} = \left( \frac{2Dd}{w} \right)^{1/2}$$

$$\sigma_{\text{eff}} = \left( \frac{2\varepsilon/E}{eE} \right)^{1/2}$$

where $\varepsilon = eD/\mu_e$ is the characteristic energy of the electrons and $e$ the electron charge. The quantity $(D/\mu)/E$ is roughly constant for the counter gases in the present work [43], giving $\sigma_{\text{eff}}^2 \propto d$.

2.1.5 The avalanche

At some critical distance from the anode wire surface, in practice only a few wire radii away, where the field is adequately described by Eq. 2.4, the electrons gain sufficient energy between collisions to cause ionization. An avalanche builds up exponentially in the rapidly increasing field [44]. The relative variance in the size of the avalanche produced by each individual primary electron is
where $A$ is the mean gain, and is quite large ($f \approx 0.6$ to 0.7 for argon [45]), being sensitive to the details of the first few ionizations. However, the overall variance for $\bar{N}$ independently developing avalanches is reduced to $f/N$. Thus, combining with Eq. 2.7 (assuming the primary ionization and the avalanche are independent) and introducing the mean energy to form an ion-electron pair, $W = E_x / \bar{N}$, we obtain for the variance in the total number of electrons generated, $P$, [45, 46, 47]

\[
\left( \frac{\sigma_p}{\bar{P}} \right)^2 = \frac{(F + f) W}{E_x}, \quad (\bar{P} = \bar{N}A) \tag{Eq. 2.10}
\]

For argon, a value of around 0.15 is predicted at the X-ray energy 5.9keV. In order to predict the actual energy resolution of a counter, the possible further contributions of the not quite truly radial field of MWPCs near the anode wires, mechanical imperfections (for example varying wire radius), and the loss of electrons to species with significant electron affinity should be considered. Additional fluctuation can arise as a further result of the non-truly radial MWPC field when the signals are processed (Section 2.1.6).

The spatial distribution of the avalanche reflects the distribution of primary electrons, resulting from the finite range of photo- and Auger electrons, and diffusion during their drift. In the $z$ axis, the lateral spread translates into an angular spread according to Eq. 2.6. Avalanche spread in the $x$ axis has been measured by Sanada [48], and the spread in $\alpha$ by Okuno et al. [49] and Harris and Mathieson [50]. The intrinsic avalanche spread at low gains is
increased at high gains by the build-up of positive ion space charge near the wire, and more particularly for the soft X-ray application by the emission and nearby absorption of uv radiation.

Noble gas atoms collisionally excited during the avalanche emit this uv radiation and are fairly transparent to it. As a result the photons can reach other metal surfaces and eject electrons [51]. These can produce secondary avalanches which degrade the energy and position [52, 53] resolutions of the chamber. Fig. 2.5(a) shows the anode pulse height spectrum for an MWPC operating with pure argon filling, and illuminated by an iron-55 (5.9keV) source. Although electronic noise has broadened considerably the peaks at the low gas gain (less than 400), the escape peak mentioned in Section 2.1.5 is evident. Any attempt to increase the gain degraded the energy resolution, as shown in Fig 2.5(b), as secondary avalanches became significant. This problem is circumvented by adding a quench gas which can both collisionally de-excite noble gas atoms and absorb the uv radiation. Methane is commonly used with argon, and stable operation at avalanche charges of well over one picocoulomb is easily achieved.

2.1.6 Positive ion drift and measurable charges

Because of the exponential nature of the avalanche, the majority of electrons and ions are created very close to the anode wire. The electrons are collected quickly by the anode with little change of potential. For the typical geometries and gains used in the present work, the electron movement contributes less than 2% to the total
Fig. 2.5 Anode pulse height spectra for an MWPC operating with pure argon gas filling.
(a) At a gas gain of around 400, showing full energy and escape peaks.
(b) The result of the attempt to obtain a further significant increase in gain in the absence of a quenching agent.
measurable induced signal. Only for the very shortest signal processing times (the order of a nanosecond) is the simplifying assumption unreliable that induced signals arise entirely from the motion of the positive ions, starting at \( r^d \). This assumption is made in the present work. As with the motion of the primary electrons, the ionic motion can be characterised by a drift velocity

\[
\mathbf{w} = \mu \mathbf{E}
\]

with \( \mu \) the positive ion mobility, and a diffusion coefficient, \( D \). Both \( w \) and \( D \) are typically two to three orders of magnitude less than for electrons [54,55]. The small size of \( D \), together with the large numbers of positive ions, mean that the effect of ion diffusion on MWPC imaging performance (in producing uncertainties in the ion cloud centroid) may be neglected. Also in contrast to electron drift, the ion mobility, \( \mu \), is relatively constant with \( E/p \), except at very high values of the latter [56]. Polyatomic quench gases have lower ionization potentials than noble gases, so charge exchange will quickly occur, and the drifting ions will be those of the quench gas [57].

The positive ion drift is along field lines (Section 2.1.2) and the path of the ion cloud centroid essentially reverses that of the primary electron cloud centroid, but of course the ions continue to the collecting electrode. At all times in the ion motion, from start (\( t=0 \)) to finish (\( t=t_f \)), the ions induce charge on all chamber electrodes, the relative magnitudes of the charges on various electrodes depending only on the ions' position and the chamber geometry. The calculation of induced charges is covered in Chapter 3, but a general description is useful here.

Field lines from the ions must terminate on equal and opposite
charges on the conductor surfaces, so we must have for a closed system

\[ q_o^+ = - \sum_{\text{all electrodes}} q_i^- (t) \]  
Eq. 2.11

where \( q_o^+ \) is the total ion charge and \( q_i^- \) the induced charges. The superscripts indicate the sign of the charge. In practice, all conductors are generally connected to low impedance nodes (the virtual earths of pre-amplifiers, true earth, or large capacitances to such earths), so the changing induced charges \( q_i^- (t) \) result in currents flowing into or out of these nodes. These currents are integrated by charge-sensitive pre-amplifiers to give observable charges, which must of course be opposite in sign to the actual induced charges. Thus positive charges, \((-q_i^- (t))\), are observed on all electrodes from which signals are taken, with the exception of the anode. In this case, the observed signal, \( q_A(t) \), is the resultant of the collected charge of avalanche electrons, which is equal to \(-q_o^+\), and the positive observable induced charge, \(-q_A^-(t)\)

\[ q_A(t) = -q_o^+ + (-q_A^- (t)) \]  
Eq. 2.12

The combined observable signal from all other electrodes

\[ q_c(t) = - \sum q_i^- (t) - (-q_A^- (t)), \quad i \neq A \]

\[ = +q_o^+ - (-q_A^- (t)) \]

where we have referred to Eq. 2.11. Thus at all times

\[ -q_A(t) = +q_c(t) \]  
Eq. 2.13

It is clear that \( q_A^- (t) \) should be minimised to maximise the signal \( q_A(t) \). Connecting all the anode wires together is detrimental in this respect. As this is often done for operational convenience, however, we should then try to avoid large values of \( h/s_A \).
Consider the arrangement and ion path shown in Fig 2.6. At t=0
(position A) effectively all induced charge is on the avalanche wire,
so from Eqs. 2.12 and 2.13 there are no measurable signals. As the
ions move quickly away from the wire in the high field region (B) all
$q^{-}$ increase except $q_{A}$ which decreases correspondingly. All signals
show a rapid rise. The exact partitioning between $q_{A}$ and all other $q_{i}$
at given t will depend upon the ion path, and thus the avalanche angle
centroid, $\alpha$. For a particular signal processing time constant this
produces a slight variation with $\alpha$ in the signal derived from $q_{A}(t)$.
Currents are smaller as the ions move more slowly in the uniform field
field region (C) with the $q_{i}^{-}$ increasing or decreasing according to
whether the ions are approaching or moving away from the electrode.
Just before $t_{c}$ (D), all charge is effectively induced on the
collecting electrode, and we have

$$-q_{A}(t_{c}) = q_{c}(t_{c}) = q_{j}$$

Some graphs of the time development of MWPC induced charges are shown
by Gilvin [43].

Some of the features of the signal development are reflected in
Fig. 2.7. The output of a charge-sensitve pre-amplifier connected to
a single wire, $j$, of a wire cathode array, located at $z = 0, y = h$, was passed through a shaping filter with a time to unipolar peak of
5.5 $\mu$s. The two pulses are for ions moving towards (upper trace) and
away from the wire, corresponding to avalanche angle centroids
$\alpha = +\pi/2$ and $-\pi/2$. Both indicate the initial rapid rise in $q_{j}$ as
the ions move away from the anode wire. For $\alpha = -\pi/2$, the signal
then drops below zero, reflecting the fact that the ions are inducing
decreasing charge on the wire as they move towards the other cathode.
For $\alpha = +\pi/2$, the effect of $q_{j}$ going on increasing, but more slowly
than the initial rise can be seen. The large pulse at the end arises
Fig. 2.6 Positive ion trajectory. For a given geometry, the relative sizes of the charges induced on the various electrodes at any time depend only on the position of the ion.
Fig. 2.7 Experimental pulse shapes from a single wire of a 1mm pitch cathode, after processing by a 2\(\mu\)s unipolar Canberra 2022 filter. Avalanche angle centroids +\(\pi/2\) and \(-\pi/2\), with the ions travelling directly towards (upper trace) and away from (lower trace) the sensing wire. In the former case the ions were collected by the wire.

\(s_x = 2\text{mm}, \ h = 4\text{mm}, \ V_x = 2400\text{V}, \ V_c = -200\text{V}, \) P50 gas.
as the ions fall through the higher field around the wire itself (Fig. 2.4).

At around 5 μm from the surface of the collecting electrode, the ions neutralise by extracting an electron from the metal surface [33]. The remaining energy, the difference between the ionization energy and the metal work function, is dissipated variously according to the ionic species. Typical polyatomic quench gases are thought to usually dissociate. If the charge carriers were noble gas atoms, then inelastic collision with the electrode, uv emission or ejection of further electrons are the possibilities, with the latter two clearly undesirable.

2.2 Experimental details

2.2.1 Multiwire chambers

Much of the present work was carried out with multiwire arrays installed in basic detector bodies, incorporating the gas volume, signal and high tension (HT) feedthroughs, and wire array supports, of a type previously used in X-ray astronomical observations [16,58]. For convenience these are referred to throughout as Leicester detector bodies. Fig. 2.8 illustrates the arrangement of the chamber; a good and detailed description has been given by Gilvin [43]. The chamber was supported horizontally on plastic foam (to reduce microphonic pickup by the pre-amplifiers), thus defining an upper and lower half of the detector, the upper half being that nearest to the X-ray entrance window. The multiwire arrays ("grids") were located at their
Fig. 2.8 Leicester detector body. Plan view (with lid removed) and cross section. Scale 0.5:1.
corners by four studs with ceramic sleeves, and clamped down with threaded collars to give a rigid detector. The grids could be oriented with their wires parallel or orthogonal to those of the other grids, and considerable flexibility in the vertical location of the individual grids was provided by the use of ceramic or G10 (fibreglass epoxy) spacing washers. The limited space around the edges of the grids required cathode readout arrangements to be compact. This is of course a common need, so the constraint was in fact beneficial in ensuring that new systems would be practical. The X-ray entrance window was usually 4mm aluminium foil, supported if necessary by a tungsten mesh. This gave good transmission at the X-ray energies used, and proved relatively rugged.

Other chamber and detector designs were used in some parts of the work. A detector body (Fig. 2.9) which accepted the same multiwire arrays was used during visits to AERE Harwell to demonstrate the two-dimensional imaging capability of a new cathode design (Chapter 7), and in measurements of anode wire modulation effects (Chapter 8). This chamber had the capability to operate with a sealed gas volume, and is referred to as the Harwell detector body. Small detectors with very low anode-cathode spacing were used at the Brookhaven National Laboratory and at Leicester to investigate limiting position resolutions; these are described in Chapter 9.

2.2.2 Chamber gases

All counters were operated as gas flow detectors, with consequent ease of commissioning and alteration compared to sealed gas volumes [59]. This made necessary the use of relatively cheap gases,
Fig. 2.9 Cross section through the circular Harwell detector body. Scale 0.5:1. The geometrical parameters $h = 5.5\text{mm}$ and $d_1 = d_2 = 30\text{mm}$ were not adjustable in this design.
with argon-methane mixtures most used. Some properties of the commercially available mixtures that were used are given in Table 2.1.

Table 2.1 Standard counter gases used in the present work

<table>
<thead>
<tr>
<th>Gas</th>
<th>Density (kg/m³)</th>
<th>μ (cm²/Vs)</th>
<th>1/e depth at EX keV (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ar 10%CH₄ (P10)</td>
<td>1.68</td>
<td>1.90</td>
<td>5 20 49</td>
</tr>
<tr>
<td>Ar 50%CH₄ (P50)</td>
<td>1.25</td>
<td>2.05</td>
<td>8 36 87</td>
</tr>
<tr>
<td>Ar 75%CH₄ (P75)</td>
<td>0.99</td>
<td>2.15</td>
<td>11 60 145</td>
</tr>
<tr>
<td>100%CH₄</td>
<td>0.72</td>
<td>2.26</td>
<td>22 1100 2620</td>
</tr>
</tbody>
</table>

The data are for one atmosphere pressure. From the comparatively large 1/e depths (the depth of gas required to attenuate an X-ray flux to 1/e times its initial intensity) it may be appreciated that the high methane mixtures are of limited value for imaging any but the softest X-rays, unless the incident radiation is normal to the detector. Electron mobility in methane mixtures is high, which implies that their susceptibility to loss of electrons to impurities with significant electron affinity (such as oxygen) is small. The continuous absorption spectrum of methane below 145nm is well placed to absorb uv emitted by excited argon (≈130nm [60]) so that it is also a good quench gas.

2.2.3 Multiwire electrodes

A number of anode and cathode grids produced for earlier studies [43,59,61], which had standardised frame dimensions to fit the chamber
described above, were used throughout the work, often in modified form. These wire arrays had been constructed using two techniques:

(i) The wire was wound continuously by hand between the pegs of a special winding frame, and could be tensioned if required by the application of gas pressure to pneumatic pistons attached to some of the pegs. The constant wire pitch was well defined by laying accurately machined sawtooth templates onto the wire plane before the wires were affixed to a complete frame by epoxy adhesive or solder.

(ii) The second method had been developed by J. E. Spragg (Leicester University X-ray Astronomy Group) for the production of reliable wire arrays with pitches as small as 0.25mm (the smallest pitch practicable using the first method was 1mm). The wire was spooled onto a specially prepared mandrel running on a small lathe. The mandrel had a thread of the appropriate pitch and a recess to take the frame end members. After winding, the wires were attached to the two end members with epoxy resin, then cut free of the mandrel and trimmed. Addition of the side members completed the frame, and the wires could be tensioned using grub screws integral to the frame.

Both construction methods have been described by Gilvin [43] and Sims [61], the latter including photographs of the equipment. Several new cathodes were produced for the present work, using the first technique exclusively. This was for reasons of convenience and flexibility, given that wire pitches of less than 1mm were not required. The completed cathodes exhibited consistent pitching and good planarity.

Cathodes had wire diameters in the range 20\(\mu\)m to 125\(\mu\)m and the wire material was usually copper 2% beryllium alloy, chosen for its relative strength and ease of handling. Active wires on the anode
Fig. 2.10 Typical multiwire arrays: a 0.5mm pitch cathode (top) and a 2mm pitch anode (bottom).
grids were of gold-plated tungsten, chosen for its strength, and typically 12 or 15 \( \mu \text{m} \) diameter. The outermost anode wires were of gradually increasing diameter (50, 125, 250 \( \mu \text{m} \)) in order to prevent the occurrence of high electric fields at the edges (the field around an MWPC anode wire is suppressed by its two neighbours). Anode wire pitch was mostly 2 mm, although larger pitches were sometimes used for anticoincidence counting.

The electrode frames were of a fibreglass epoxy material ("G10"), sometimes augmented with ceramic substrates for wire attachment, and were accurately machined such that the wire plane lay 1 mm below the top surface and 3 mm above the bottom surface. Thus the minimum value of \( h \) possible for a symmetric chamber was 4 mm. Wire plane surface areas were 90x90 mm\(^2\) up to 95x105 mm\(^2\) depending on detailed design. Fig. 2.10 shows two typical electrodes; a 2 mm pitch anode produced by the first technique, and a 0.5 mm pitch cathode produced by the second.

2.2.4 Chamber electrical arrangements

All counters in the present work were operated with positive high voltage supplied to the anode, which had all wires connected together. Cathodes were held at ground, or at a positive or negative voltage to control electron drift in the regions behind wire cathodes.

A typical arrangement of high voltage supply and pre-amplifier connections to a wire array is shown in Fig. 2.11. High voltage was provided by standard NIM modules. All components except sometimes \( R_L \) were outside the gas volume. \( R_1 \) and \( C_1 \) constitute a low pass filter.
(usually of time constant $10 \times 10^6 \times 1500 \times 10^{-12}$ s in this work) to remove any ripple on the high voltage line. $R_L$ (typically $100 \times 10^2$ ohm) prevents signal loss through $C_1$ or the high voltage supply, and $R_1$ and $R_L$ together stop large currents flowing in the event of a breakdown. The long time constant for maintaining the bias voltage on the electrode, of the order $R_L C_D$, where $C_D$ is the shunt capacity of the wire array, was not a problem in the present work, as count rates and charge levels, and therefore average detector currents, were low. Good noise performance from the pre-amplifier required that $R_L$ and any other resistance to signal ground be large, and $C_D$ and any other capacitive load on the pre-amplifier be small.

![Diagram](image)

**Fig. 2.11 High voltage and pre-amplifier connections to grids.**

Where the pre-amplifier design required a decoupling capacitor $C_d'$ to block the high voltage, this was made as large as possible, up to 100,000pF for cathodes where the voltages were limited to +/-250V, and 9000pF for anodes. This avoided unnecessary reduction of the pre-amplifier dynamic input capacity, and thus degradation of the virtual earth.
2.2.5 Analogue electronics

All pre-amplifiers were of the charge-sensitive configuration. The compact designs built into the detector housings for the original application were found to offer poor noise performance by modern standards (around 1000 electrons rms unloaded noise, referred to the input, plus 20 electrons per picofarad loading) and low dynamic input capacity (~350pF). They were therefore only used in non-critical applications. The best results were obtained from Canberra 2001 model pre-amplifiers, with a measured performance of 86 electrons rms unloaded noise plus 2.5 electrons per picofarad with 2μs unipolar shaping. Dynamic input capacity was found to be in excess of 14,000pF, and the rapid risetime (~30ns) meant that this good virtual earth was established quickly. To eliminate unnecessary shunt capacity, co-axial input connectors were removed from the pre-amplifiers, and the cases bolted directly to the detector housing. This also provided good screening. The feedback and test capacitors of all pre-amplifiers were the subject of measurement.

Absolute charge level (and thus detector gain) calibration was achieved by injecting known voltage pulses from a precision tail pulse generator into the test capacitor of the appropriate pre-amplifier. The X-ray signal was then matched to the test signal at the output of a shaping amplifier. Because a given filter responds somewhat differently to the test and counter pulses, there is an error in the value of avalanche charges measured in this way [62]. The method is well established, however; quoted values should be understood as conventional, or effective anode charge levels.

Canberra and EG&G Ortec NIM standard, and Harwell 6000 series
filter amplifiers were used with unipolar shaping for accurate pulse height analysis. Simpler units built in house having faster times to peak were used with bipolar shaping for timing and gate generation. Care was taken at all times to minimise DC offsets on analogue signals. A wide variety of standard NIM modules were used to complete the electronic systems as appropriate to the particular experiment, the signals being monitored with Tektronix oscilloscopes of adequate bandwidth (>60MHz) and analysed as necessary with Canberra and Harwell 6000 series multichannel analysers.

2.2.6 Digital electronics

The only signal processing algorithms required by the present work were simple division, A/B, and the algorithm for position output from the pulse heights from two terminal cathode systems, A/(A+B). Initially these were implemented using an analogue division circuit based upon the Intersil 8013 device. This was found to suffer from considerable output non-linearity and instability, however, and digital processing was subsequently used exclusively.

The Harwell 3769 system, which was used extensively, incorporates two eleven bit analogue to digital converters (ADCs) to digitise the signals A and B. A fast hardware digital divider then calculates A/B or A/(A+B) to the required accuracy. Fig. 2.12 shows a complete electronic set-up, with two such systems arranged to provide two-dimensional imaging. Tests of the 3769 system with a precision ramp generator and pulse generators showed it to have differential non-linearity \( S \), defined in Chapter 6) better than 1% over its entire range, and good stability. Later in the work a low cost
Fig. 2.12 View of a complete electronic set-up, with the Harwell detector in the background, the two-dimensional analyser to the right of it, and two 3769 digital signal processing systems in the centre.
equivalent system was assembled, with Silena twelve bit ADCs providing digitised pulse heights to a BBC microcomputer. Based on some initial work [63], programs were written to perform the algorithms and display the data. Again the system showed good linearity and stability, with the added advantage of flexibility, and disadvantage, not significant in the present work, of relatively low rate capability (<1-2kHz).

Digital signal processing was well suited to the experimental work, with only a narrow range of X-ray energies present at one time. Its limitations should be kept in mind, however, if a wide range of pulse heights, some not very large compared to the digitisation interval, $\Delta$, are present. The output is usually represented by a number of channels, $N_0 + 1$, sufficiently large that the resolution of the detector system is not compromised. Consider now that $a$ and $b$ are position signal pulse heights arising from illumination of the detector with a well collimated X-ray source. With digital signal processing, the output would be a number

$$N = \text{int} \left[ N_0 \left( \frac{\text{int}(b/\Delta)}{\text{int}(a/\Delta) + \text{int}(b/\Delta)} \right) \right]$$

where int means to take the integral part. When $\text{int}(a/\Delta)$ and $\text{int}(b/\Delta)$ are small, some values of $N$ cannot be produced, and others are correspondingly overproduced. Fig. 2.13 shows how a Gaussian position peak is distorted by the effect. A single wire proportional counter with a resistive anode was used to generate pulse heights such that $\text{int}(a/\Delta)$ and $\text{int}(b/\Delta)$ were about 16 with a variance of 3. $N_0 + 1$ was 2048.

The problem was avoided in the experimental work by keeping the denominator of the right hand side of Eq. 2.14 not small compared
A mathematical correction procedure that can be applied during later processing of the data has been described by Nikutta et al. [64]. A preliminary evaluation of a new design of divider for the Harwell 3769 system [65] showed that the distortion can be effectively eliminated in real time by introducing a small (less than one times the least significant bit) pseudo-random variation in \( \text{int}(a/\Delta) \). Although this must represent a loss of resolution, it is difficult to measure this anyway from outputs such as Fig. 2.13.

![Fig. 2.13 Digital distortion of a Gaussian position peak](image)

2.2.7 X-ray sources and collimation

A number of sources were used, providing X-rays at energies below 10keV. Where area illumination was required, the iron-55 radioisotope, supplied in sealed form by the Radiochemical Centre (latterly Amersham International), was most useful. The source provides a line emission spectrum primarily at the Mn K\(\alpha\) (5.90keV)
energy, but with about 15% of the output at Mn Kβ (6.49keV). The mounting and shielding arrangement produced for a new source procured during the work is shown in Fig. 2.14.

Fig. 2.14 Iron-55 source assembly (scale 2:1)

Although these sources could be collimated, the resulting intensities were low, and X-ray tubes proved more convenient when narrow beams were required. Some details of two of the sealed tubes used in the work are given in Table 2.2. The aluminium target tube, which was used extensively, has been described by Gilvin [43].
<table>
<thead>
<tr>
<th>Target material (anode)</th>
<th>Al</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anode potential</td>
<td>1.9kV</td>
<td>10kV</td>
</tr>
<tr>
<td>Characteristic line</td>
<td>1.5keV</td>
<td>8.0keV</td>
</tr>
<tr>
<td>X-ray exit window</td>
<td>75μm Be</td>
<td>100μm glass</td>
</tr>
<tr>
<td>Supplier</td>
<td>Centronic Ltd., Croydon</td>
<td>Teltron Ltd., London *</td>
</tr>
</tbody>
</table>

* On loan from Oxford Analytical Instruments Ltd.

The combination of the low overpotential applied to the aluminium target tube and the inherent filtration of low energy X-rays by the window meant that the output was fairly well defined at the Al K energy. A helium transmission path was provided between the tube exit window and the detector entrance window to maximise the transmission of these soft X-rays. The X-ray energy from the copper target tube was less well-defined, with a considerable continuum component even after the introduction of a nickel filter, and this limited its usefulness.

With only one sensing axis under detailed investigation at one time, slit collimators were used to furnish narrow X-ray beams. The initial collimation arrangement consisted of two brass discs with slits 6mm long by a minimum of about 50μm wide. The tubular enclosure separating the discs was at least 40mm long. In the later stages of the work, a more flexible collimator system was designed. After milling out internal recesses to prevent internal reflection, flat mating surfaces were ground on two steel blocks. A shallow cut
5mm wide was then ground down the centre of one face, coincident with the deeper recess, such that collimation slits were obtained when the two halves were bolted firmly together. Vertical adjustment of the assembled block was possible to minimise the distance between the exit slit and the detector window. Blocks with 200, 100, 50 and 20\(\mu\)m cuts were made. The two collimation arrangements are shown schematically in Fig. 2.15.

**Fig. 2.15 Collimator arrangements (scale 1:1 but with slit widths exaggerated).**
(a) Using narrow slits in brass discs.
(b) Using a shallow groove in a steel block.
The collimator assemblies could be fixed securely to the moving portion of a traversing table, and the whole table was in turn bolted firmly to the Leicester detector body. The whole arrangement, including the later collimator type, the aluminium target tube, the detector and various pre-amplifiers is shown in Fig. 2.16. The large diameter graduated dial at the end of the accurate 1mm pitch screw allowed the collimator to be positioned to better than 50μm (providing backlash was allowed for), and moved in steps as small as 12.5μm.

Alignment of the collimators with respect to the detector was achieved by minimising the width of the output position peak. A more sensitive check of the vertical alignment was provided by the ability to discriminate between events absorbed above and below the anode plane. Shims could be added at the place where the collimator and traversing table were bolted together until the position peaks for both types of event were coincident.
Fig. 2.16 Leicester detector body, showing aluminium target X-ray tube, collimator and traversing table assembly.
CHAPTER THREE

CALCULATION OF CATHODE INDUCED CHARGE DISTRIBUTIONS

3.1 Introduction

As mentioned in Chapter 1, although the earliest realisations of the imaging MWPC obtained position readout by identification of the avalanche wire, the usefulness of the signals induced on the cathodes was soon appreciated. Design considerations for position readout systems were clearly different, however, with the distribution of induced charge being in general much broader than the distribution of collected charge. A position-sensitive cathode must sample the induced charge distribution in space, and return signals from which the centroid of the distribution can be determined. Unless a particular sampling arrangement is completely continuous, the linearity of its response will depend upon the shape of the charge distribution in relation to the sampling elements. It is often desirable to reduce the density of these elements as much as possible while not exceeding a certain amount of response non-linearity. Apart from keeping complexity and cost down, this can also reduce the overall electronic noise of the system. It may thus be appreciated that a good knowledge of the cathode induced charge distribution is necessary for the optimum design of such systems.

The present chapter is concerned with the calculation of cathode induced charge distribution using electrostatic theory and given the physical parameters of the MWPC. The calculations are subjected to experimental test in Chapter 4. In many situations, particularly when some of the physical parameters are not well known, approximate or
empirical forms of the distribution can be more useful. Chapter 5 deals with such formulae, using the models of Chapter 3 as a basis. The use of calculated distributions to predict the linearity of a cathode system is demonstrated in Chapter 6.

3.2 Previous descriptions of the cathode induced charge distribution

The underlying trend in previously published descriptions has been the removal of various simplifying assumptions concerning MWPC geometry. An early description due to Fischer and Pich [66], for instance, neglects both the anode wires and the second cathode plane (Fig. 3.1(a)). By further assuming that all the positive ion charge is located at a point, the problem is reduced to one found in standard sources (for example [31]). The predicted distribution is broader than that found in real MWPCs [67,68]. Lee et al. [69] introduced a second plane close to the point charge to represent the anode (Fig. 3.1(b)). This gives a distribution narrower than those typically found in MWPCs [67,68], but the formula could be of use in calculating distributions for parallel plate geometries. Another simple geometry for which an exact analytical solution is possible is that of a point charge midway between two cathode planes. This was used by Endo et al. [70] and is shown in Fig. 3.1(c).

Prior to this last work, more realistic and essentially complementary calculations had been presented by Mathieson [71] and Gatti et al. [72], in which the plane of anode wires was taken into account. The symmetry of the simplified geometries was thus lost in that distributions in the z and x axes now had to be considered separately. Exact analytical solutions were also no longer possible.
Fig. 3.1 Early, analytical formulations of the MWPC cathode induced charge distribution in one axis, $\rho$, expressed in the nomenclature of the present work.
(a) Fischer and Plch [66]
(b) Lee et al. [69]
(c) Endo et al. [70].
Mathieson showed how the distribution could be calculated in the z axis on individual cathode wires, but no account was taken of the earthed conductor that must be present behind the cathode in a practical system. Time development, and dependence of the distribution on the angular co-ordinate of the starting point of ion travel, $\varphi$, was investigated by allowing a point charge representing the positive ion cloud to move away from the anode along an electric field line with velocity $\mu E$. The methods of obtaining the distribution in the z axis presented later in this chapter were obtained as extensions of this model.

Gatti et al. were able to calculate the distribution in the x axis on continuous cathodes. By making a simplifying assumption during the calculation, the dependence on $\varphi$ was eliminated. This allowed a good fit to the calculated distributions to be obtained with a convenient empirical formula, with the three parameters determined by the chamber geometry. Mathieson and Thompson [73] have since shown that the simplifying assumption can be removed, and this work is summarised in Section 3.3.5. The Gatti empirical formula has been adopted as a useful standard, and Chapter 5 describes how it is possible to reduce it to a simple one-parameter form.

An alternative approach to the models mentioned previously, all based on established results of electrostatic theory, has been reported by Van der Graaf and Wagenaar [74]. Individual electrons were allowed to move along the surfaces of conductors under the influence of an equal and opposite amount of point charge. Using only Coulomb's law and the constraints of the electrode geometries, the electrons redistributed themselves until equilibrium was reached. Although this model could in principle cope with any three-dimensional
arrangement of conductors, it makes extreme demands on computing power. With only 900 electrons (giving very poor statistics), 10000 iterations, taking one week of computer time, were needed to complete a calculation for one positive ion location.

3.3 Calculation of induced charge distributions in MWPCs

3.3.1 General points

The cornerstone of all the calculations of induced charge summarised in the present work is the reciprocity theorem, originally due to Maxwell [75], and which may be summarised in terms appropriate to MWPCs [32]:

"If a conductor raised to a unit potential produces at an external point a potential $P$, then a unit charge placed at that point will induce on the conductor, if at zero potential, a charge $-P$."

By the means of this theorem, the task of determining the induced charge on a particular electrode is reduced to the calculation of the potential function due to that electrode at unit potential, all other electrodes being earthed. A point charge, $q_0$, may then be considered to be introduced at any position between the electrodes, $(x,y,z)$, where the potential is $P(x,y,z)$. We may say at once that the charge induced on the electrode in question is

$$q_i = -P(x,y,z) q_0$$

Eq. 3.1

To illustrate the ease of application of this approach, it may be seen that the charge induced on the whole anode in the geometry of Fig. 2.2 can be obtained for any positive ion location from Eq. 2.2. If the distribution of cathode induced charge is required, then the potential function corresponding to each wire, or narrow strip of the cathode
being raised to unit potential while all other conductors are grounded has to be found. Time development of the distribution can be included by allowing the point charge to move away from the anode along field lines, and the finite angular spread of the avalanche, and thus of the positive ion cloud may be modelled by including several trajectories, suitably weighted.

It should be noted that the reciprocity theorem is not the only approach to the problem, although it is probably the most convenient. Erskine [76] has obtained expressions for the field in an MWPC with continuous cathodes due to the positive ion charge itself, although it was first necessary to show that the point charge could be replaced by a line charge, and thus reduce the problem to a two-dimensional one in the z-y plane. The induced charge on any conductor is then given by the Gauss electric flux theorem as the integral of the flux density over the relevant surface area. The results of Erskine have been used by Bellazini et al. [77] to calculate induced charge distributions, as seen by sensing cathode strips, for a number of multiwire chamber arrangements.

3.3.2 $q_2(\lambda)$: the distribution of charge orthogonal to the anode wires on a wire cathode

The geometry assumed in this calculation is shown in Fig. 3.2. $q_2(\lambda)$ will represent the charge induced on a cathode wire at position $\lambda = z_c/h$. The position $\lambda = 0$ will be specified as being opposite the centre of the anode wire on which the avalanche occurred. The subscript 2 was assigned during the development of the theory and measurements to distinguish from distributions parallel to the anode
Bearing in mind the eventual application of the reciprocity theorem, we must develop the potential function for an array of parallel wires between infinite conducting planes. Consider firstly that all the wires are absent in Fig. 3.2, and that a straight line charge of linear density $q_0$ is introduced at general position $(z_0, y_0)$ where $-(h+d_2) < y_0 < (h+d_1)$, perpendicular to the $z,y$ plane. Use of a conformal mapping and the method of images (Appendix A.1) reveals that the resulting potential is given by

$$P_o = -\frac{q_0}{2\pi\varepsilon_0} L(z, z_0, y, y_0)$$

Eq. 3.2

Fig. 3.2 Geometry for the calculation of $q_2(\lambda)$

wires which have the subscript 1.
where

\[ L = \ln \left[ \frac{\cosh(a(z-z_0)) - \cos(a(y-y_0))}{\cosh(a(z-z_0)) + \cos(a(y+y_0-d))} \right]^{1/2} \]

and \( a = \pi/21, \ l = h+(d_1+d_2)/2, \ d = d_1-d_2 \)

Now, replacing all the anode and cathode wires at positions \((z^*,y^*)\) by uniform line charges, we may use the principle of superposition (see for example [78]) to obtain the general potential function

\[ P = -\sum_{i=1}^{N} C_i L_i \quad \text{Eq. 3.3} \]

where

\[ L_i = L(z,z^*,y,y^*) \]

and the summation is taken over all \( N \) wires. The representation of the wires by line charges is a good approximation so long as the wire radii are small compared to interelectrode dimensions (i.e. \( r_a \ll s_a, h; r_c \ll s_c, h, d_1, d_2 \)) which is generally the case for a practical chamber.

The coefficients \( C_i (= q_i/2\pi\varepsilon_0) \) can be determined if we know the potentials \( P_j, \ j = 1, \ldots N \) at \( N \) positions. But we are going to define the values of equipotential cylinders around the line charges (i.e. the wire surface potentials) in the application of the reciprocity theorem. At the wire surfaces, then

\[ P_j = -\sum C_i L_{ij} \quad \text{Eq. 3.4} \]

where

\[ L_{ij} = L(z_j,z^*,y_j,y^*) \]
and \[ L_{ij} = \ln \frac{a r_c}{2 \cos(a(h-d/2))} \]

for the top cathode

\[ = \ln \frac{a r_a}{2 \cos(a d/2)} \]

for the anode

\[ = \ln \frac{a r_c}{2 \cos(a(h+d/2))} \]

for the bottom cathode

The elements \( L_{ij} \) are obtained by expanding the \( \cosh \) and \( \cos \) terms, and letting \( z \to z_i \) and \( y \to y_j \). For all other elements the wire radii are neglected since we have taken them to be small compared to the interelectrode spacing. If we now set \( P_K = 1 \) and all other \( P_j = 0 \), the vector of coefficients \( C_k \) can be determined by numerical matrix inversion. Then Eq. 3.3 may be used to obtain the potential \( P(z,y) \) at any point \( z,y \) when the \( k \)th wire is at unit potential with all other conductors grounded. The charge induced on that wire by unit charge at \( z,y \) is then simply \( -P(z,y) \) from Eq. 3.1. Repeating the procedure for a sufficient number of wires on a given cathode therefore yields \( q_z \).

3.3.3 \( \rho_z(\lambda) \): the charge density distribution orthogonal to the anode wires on a continuous cathode

This geometry is indicated in Fig. 3.3. \( \rho_z(\lambda)d\lambda \) will represent the charge induced on a narrow strip of width \( d\lambda \) at position \( \lambda = z \zeta/h \) along the cathode. The potential function \( dP_S(z,y) \) occurring between the cathodes due to such a strip at unit potential, the remainder of that cathode and the other cathode being at earth, and the anode wires absent is (see Appendix A.2)
\[ \frac{dP_s}{d\lambda} = \frac{1}{4} \frac{\cos ay}{\cosh(a(z-h\lambda)) - \sin ay} \]  
\text{Eq. 3.5}

where \( a = \pi/2h \).

Fig. 3.3 Geometry for the calculation of \( \rho_z(\lambda) \)

We now introduce uniform line charges of linear density \( 2\pi \epsilon_\alpha d\zeta \) to represent the anode wires (the charge being that induced by the cathode strip), and obtain the total potential function

\[ dP = dP_s + dP_w \]  
\text{Eq. 3.6}

where

\[ dP_w = -\sum d\zeta_i L_i \]

with \( L_i \) as given in Eq. 3.3 but with \( d_1 = d_2 = 0 \) and the summation taken over all the anode wires.

In the same manner as before, the vector \( d\zeta_i \) is determined by the boundary condition that the potential is zero at the wire surfaces:
\[
\frac{d\mathcal{P}(z_j,y_j)}{d\lambda} = \sum \frac{dC_i}{d\lambda} L_{ij} \quad \text{Eq. 3.7}
\]

with \( L_{ij} \) as given in Eq. 3.4 (with \( d_1 = d_2 = 0 \)) and \( L_{ij} = \ln(\alpha_\gamma/2) \). After solving Eq. 3.7 for \( dC_i \), Eq. 3.6 may be used to obtain \( d\mathcal{P}/d\lambda \), and so by the reciprocity theorem

\[
\rho_2(\lambda) = -\frac{d\mathcal{P}}{d\lambda}
\]

3.3.4 \( \rho_1(\lambda) \): the charge density distribution parallel to the anode wires on a continuous cathode

The problem in this case is inherently three-dimensional, and thus more complex than in the previous two sections. The theory was developed at a later stage, and is unable at present to deal with the case of wire cathodes, and so has not been the subject of detailed experimental verification. An outline only of the method will therefore be given based on the extension by Thompson et al. [79] of the work of Gatti et al. [72]. As will be shown, there are strong indications that \( \rho_1 \) and \( \rho_2 \) distributions are experimentally almost indistinguishable.

Let \( \rho_1(x_c)dx_c/h \) be the charge induced on a narrow strip of width \( dx_c \) at position on the cathode \( x_c \) by point charge at general position \((x,y,z)\) (Fig. 3.4). From the reciprocity theorem, this is equal to \(-d\mathcal{P}(x_c,x,y,z)\), the potential at \((x,y,z)\) due to the strip at unit potential with all other conductors at earth. From the symmetry
of the situation, it is evident that \( \rho_1 \) must be an even function of \((x_c - x)\), so that it is convenient in this case to define \( \lambda = (x_c - x)/h \) and \( d\lambda = dx_c/h \).

![Fig. 3.4 Geometry for the calculation of \( \rho_1(\lambda) \)](image)

With the anode wires removed, the potential function due to the strip at unit potential may be written (Appendix A.2)

\[
dP_s = \frac{1}{4} \frac{\cos ay}{\cosh \pi \lambda/2 - \sin ay} \ d\lambda
\]

We must now introduce non-uniform line charges of linear density \( dq \) (so written because they are induced by the narrow strip \( d\lambda \)) to represent the anode wires. Again from symmetry \( dq \) must be an even function of \( x_c - x_a \), where \( x_a \) is position along the anode wire, so we
can define $\chi' = (x_c - x_a)/h$ with $d\chi' = -dx_a/h$. In order to describe the potential $dP_w(x_c, x, y, z)$ due to these line charges, the method of images is used. The cathode planes may be replaced by an infinite number of planes of parallel line charges at $y = 2nh$, $(n = +/-1, +/-2, ...)$ with line charge densities $dq \cos\pi n$. It may then be shown that the resulting potential function is given to a good approximation $(r_a \ll s_a, h)$ by

$$dP_w = \int \sum_{n=\pm\infty} \sum_{k=\pm\infty} \frac{dq}{4\pi\varepsilon_0} \frac{\cos\pi n}{[(\lambda - \chi')^2 + (y/h - 2n)^2 + (z/h - ks_a/h)^2]^{\frac{1}{2}}} \text{Eq. 3.8}$$

where we are summing over all the wires and planes, and integrating along the wires. Eq. 3.8 may be written as a convolution integral

$$\frac{dP_w}{d\lambda} = \int_{-\infty}^{\infty} f(\lambda') h(\lambda - \lambda') d\lambda' \text{ Eq. 3.9}$$

where

$$f(\lambda) = \frac{1}{4\pi\varepsilon_0} \frac{dq}{d\lambda}$$

and

$$h(\lambda) = \sum_{n} \sum_{k} \frac{\cos\pi n}{(\lambda^2 + b^2)^{\frac{1}{2}}} \text{ Eq. 3.10}$$

with $b^2 = (y/h - 2n)^2 + (z/h - ks_a/h)^2$

$\text{dq}(\lambda')$ is found from the boundary condition that the potential is zero at the anode wire surfaces. This may be written

$$\int_{-\infty}^{\infty} f(\lambda') h_o(\lambda - \lambda') d\lambda = g(\lambda) \text{ Eq. 3.11}$$

where

$$h_o(\lambda) = \sum_{n} \sum_{k} \frac{\cos\pi n}{(\lambda^2 + b_o^2)^{\frac{1}{2}}}$$

with $b_o^2 = (r_a/h)^2 + (2n)^2 + (ks_a/h)^2$
and \( g(\lambda) = -\frac{1}{4} \text{sech} \frac{\pi \lambda}{2} \)

Eq. 3.11 implies that the Fourier transform, \( F(\omega) \), of \( f(\lambda) \) is given by

\[
F(\omega) = \frac{G_o(\omega)}{H_o(\omega)}
\]

Eq. 3.12

\( G_o(\omega) \) and \( H_o(\omega) \) are the transforms of \( g(\lambda) \) and \( h_o(\lambda) \) respectively, and it is possible to show that these are

\[
G_o(\omega) = -\frac{1}{2} \text{sech} \omega
\]

and

\[
H_o(\omega) = \sum_n \sum_n 2 \cos \pi n K_0(b_o \omega)
\]

\( K_0 \) is the zero order modified Bessel function of the second kind.

d\( P_y/d\lambda \) may be found, referring to Eq. 3.9, as the inverse transform of \( F(\omega)H(\omega) \), where

\[
H(\omega) = \sum_n \sum_n 2 \cos \pi n K_0(b \omega)
\]

is the transform of the expression in Eq. 3.10. The evaluation of \( H_o(\omega) \) and \( H(\omega) \) requires careful treatment by complex variable analysis [79]. The induced charge distribution is finally obtained after the addition of \( dP_s/d\lambda \)

\[
\rho_i(\lambda) = -\left( \frac{dP_s}{d\lambda} + \frac{dP_y}{d\lambda} \right)
\]
3.4 Extending the calculations to permit direct comparison with experiment

3.4.1 The necessary extensions to the basic electrostatic theory

In order that we may directly compare predictions, concentrating now on the distributions \( q_1 \) and \( \rho_4 \), with experimental measurements and with one another, we must take into consideration the following factors:

(i) The statistical fluctuation in the size of the inducing charge (the number of positive ions (Eq. 2.10))

(ii) The movement of the ions in the field of the chamber, resulting in the time development of the induced charge

(iii) The spatial distribution of the ions

(iv) The effect of signal processing electronics on the induced charge waveform

(v) The difference between continuous and wire cathode distributions.

These will be considered in turn.

3.4.2 Fluctuations in the number of positive ions: calculation of the anode signal charge

The effect of the limited energy resolution of the counter may be removed experimentally by dividing the signal from the sampling electrode by that from the anode. The problem thus becomes the theoretical prediction of the observable charge on the anode. It has already been indicated (Section 3.3.1) how this may be achieved for the case of continuous cathodes. However, the result is easily
obtained for both continuous and wire cathode cases using the expressions developed in Sections 3.3.2 and 3.3.3, which allow the calculation of the induced charge on any group of wires.

Consider the wire cathode case. First we solve Eq. 3.4 with $P_j = 1$ for all anode wires and $P_j = 0$ for all other wires. Then Eq. 3.3 and the application of the reciprocity theorem gives the charge induced on all the anode wires connected together by unit point charge at $(z,y)$ as $-P(z,y)$. Following Eq. 2.12, the observable charge is thus given as $q^a = -1 + P$.

A similar procedure applies when the cathodes are continuous. We solve

$$P_j = 1 = -\sum_{i=1}^{N} C_i L_i$$

for $C_i$, with $L_i$ as defined in Eq. 3.4 but with $d_1 = d_2 = 0$, and with the summation taken over the $N$ anode wires. The potential function is then given by

$$P = -\sum C_i L_i$$

Eq. 3.13

with $L_i$ as in Eq. 3.3 but with $d_1 = d_2 = 0$. The net charge is $q^a = -1 + P$ as before.

3.4.3 Time development

The expressions developed so far will allow the calculation of electrode induced charges at any time $t$, and therefore complete charge waveforms, if we can construct the positive ion trajectory, $z(t), y(t)$. For this, knowledge of the electric field, $E$, in the operating chamber, and the ion mobility, $\mu$, is required.
The potential function relevant to the operating chamber may be found by calculating the vector $C_i$ in Eq. 3.4 when $P_j = 1$ for all anode wires and $P_j = V_a/V_c$ for all cathode wires. $V_a$ and $V_c$ are the anode and cathode voltages as specified by experiment. The required potential function is just $V_aP$, with $P$ given by Eq. 3.3. The electric field components, $E_x$, $E_y$, are then found by partial differentiation of the potential function:

$$E_x = V_a \sum C_i \frac{\sinh a(z-z_i) \cos a(y-d/2) \cos a(y_i-d/2)}{[\cosh a(z-z_i) + \cos a(y+y_i-d)][\cosh a(z-z_i) - \cos a(y-y_i)]}$$

$$E_y = V_a \sum C_i \frac{\cos a(y_i-d/2)[\cosh a(z-z_i) \sin a(y-d/2) - \sin a(y_i-d/2)]}{[\cosh a(z-z_i) + \cos a(y+y_i-d)][\cosh a(z-z_i) - \cos a(y-y_i)]}$$

When the cathodes are continuous, the function for the operating chamber is simply $(V_a-V_c)P$, with $P$ obtained from Eq. 3.13.

Increments in the ion position in a small time interval $dt$ are

$$dz = \mu E_x dt$$
$$dy = \rho E_y dt$$

Starting at $z = r_\alpha \cos \alpha$, $y = r_\alpha \sin \alpha$ on the surface of the avalanche anode wire located at the origin, the positive ion charge is allowed to move out along the field line that is defined by taking sufficiently small increments $dt$. Field line plots such as Fig. 2.4 were generated in this way. In the present work, the ion mobility has been assumed independent of the field, and the value taken from the work of Schultz et al. [57]. In fact, $\mu$ is field dependent [56] although little suitable data appears to exist for counter gases. Trial calculations with the ion mobility in the form

$$\mu = \mu_0 (1 + E/E_0)^{-\nu}$$

revealed that an experimentally significant change to the predicted distribution only occurred when $E_0$ was chosen below 25 kV cm$^{-1}$ for P50 gas at one atmosphere. In view of the good agreement over a wide
range of conditions between measurement, and predictions which took $E_0$ as infinite, this probably represents a lower limit for $E_0$. The inclusion of field dependent ion mobility is of course straightforward, should it prove necessary for unusual geometries or gas mixtures.

### 3.4.4 Spatial distribution of the positive ions

Clearly, all the positive ions created in an avalanche cannot be considered to originate at one angular position on the wire. It is necessary to model the angular spread of the ions. This is achieved by following a number of ion trajectories from a range of initial angular positions, and summing the results to represent an initial Gaussian angular distribution. Thus if $P(z,y)$ is the potential function due to the electrode (wire, group of wires, strip of continuous cathode etc.) of interest, the induced charge at time $t$

$$q(t) = - \sum w_m P(z(t), y(t))$$  \hspace{1cm} Eq. 3.14

where

$$w_m = \frac{1}{2} \left[ \text{erf}(u_2) - \text{erf}(u_1) \right]$$

with

$$\text{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-v^2} dv$$

and

$$u_{2,1} = \left( \frac{4.71}{\sqrt{2}} \right) \left( m \pm \frac{1}{2} \right) / N$$

$N$ is the number of ion trajectories. The $m$th trajectory has initial angle

$$\alpha_m = \alpha + 4.71 \frac{m \sigma}{N}$$

where $\alpha$ is the avalanche centroid angle and $\sigma$ the rms angular spread. $N = 5$ was found to be a sufficiently dense representation for
the signal processing times employed in the measurements. Where the
exact signal processing arrangement is undecided or unknown, then Eq.
3.14 may be used to calculate an instantaneous charge distribution for
any time during the ion travel.

There is no direct evidence justifying the choice of a Gaussian
distribution, but given the (uncorrelated) effects which produce the
spread, and the fact that avalanche saturation was avoided, it must be
the most sensible one. For a complete calculation of $P_i$, it would be
straightforward to include a spread along the avalanche wire as
necessary.

3.4.5 Signal processing

For the present work, direct prediction of experimental results
obtained with known signal processing was required, so the complete
charge waveforms $q(t, \lambda, \alpha)$ and $q_\alpha(t, \alpha)$ for the sampling electrode
and anode respectively were necessary. Having obtained these, for a
given set of physical parameters (chamber geometry, $V_{p, \alpha}$, $V_{e, \alpha}$, $\mu$, $\sigma$),
the actual voltage pulse shapes $v(t, \lambda, \alpha)$ and $v_\alpha(t, \alpha)$ were predicted
by convolution with the appropriate filter impulse response $h(t)$

$$v(t, \ldots) = \frac{1}{C_F} \int_{-\infty}^{t} q(t', \ldots) h(t-t') \, dt'$$

$C_F$ is the conversion gain ($V \, C^{-1}$). The maximum pulse heights $v(\lambda, \alpha)$
and $v_\alpha(\alpha)$ (both considered to have the same sign) as would be
detected by an ADC were then simply found.
3.4.6 Comparison of wire and continuous cathode distributions

The predictions of the wire cathode model could at this stage be compared directly with experimental measurement. However, it is interesting to compare the distributions on wire cathodes with those on continuous cathodes. This may be done if the wire cathode distribution is divided by the normalised cathode wire pitch, \( s_c / h \), thus effectively treating it as a continuous distribution which has been integrated into strips of width \( s_c / h \). Calculations with the cathode wires out of spatial phase with the anode wires by various amounts justified this treatment. Thus the functions compared with experiment were

\[
\Gamma = \frac{v(\lambda, \alpha)}{v_a(\alpha)} \frac{1}{s_c / h} \quad \text{for wire cathodes}
\]

\[
\Gamma = \frac{V_c(\lambda, \alpha)}{V_a(\alpha)} \quad \text{for continuous cathodes}
\]
4.1 Experimental strategy

The aim of the experimental programme was to provide a set of accurate cathode induced charge distributions as occur in a working MWPC, which would test the usefulness of the theoretical models. It is notable that although a few previous measurements have been reported, usually as asides to other work [67,68,80,81,82], detailed comparisons of this sort do not appear to have been made. It was soon found that very careful definition of experimental parameters, some of which would not generally be known in a practical imaging system, was required in order to make such comparisons. In particular the chamber geometry had to be known accurately (making the use of wire cathodes desirable), the impulse response of the processing filters was needed, the avalanche angle centroid, $\alpha$, had to be defined and its angular spread, $\sigma$, controlled. This results in rather specific operating conditions, and may explain the apparent lack of earlier work.

The present approach was to show that the models could predict the measurements successfully as various parameters were altered within typical ranges. They might then be used with confidence to predict the range of distributions that occur in imaging detectors, and also to test the worth of simplified descriptions seeking to describe average behaviour. To this end, a set of typical "standard conditions" were defined. The effect on the measured distributions of changes in particular parameters could then be easily ascertained.
4.2 Experimental apparatus and method

4.2.1 Introduction; placing results in the co-ordinate system of the theoretical models

The experimental method involved measuring the signals from an isolated cathode wire (or narrow strip of a continuous cathode) as the avalanche position, and therefore the origin of the mathematical co-ordinate axes, was moved relative to it. Thus the X-ray beam was moved from one anode wire position to the next for measurements in the z axis, or along the wires for the x axis. The majority of the measured distributions were symmetrical around the co-ordinate axis (although the data were not in general symmetrically disposed around it) so that the $\lambda$ values for the data could be assigned simply once the peak of the distribution (assumed effectively continuous) had been found. This was done by differentiating a cubic spline smoothing function which had been fitted to the data. Alternatively, for the distributions in the z axis only, $\lambda$ values could be obtained simply from knowledge of the anode wire positions.

4.2.2 Sampling electrodes

Most measurements made use of wire cathodes, these being found to give the best available definition of the geometry. For early experiments, the sensing cathode was modified from one existing in which wires of 1mm pitch were supported by ceramic substrates. All wires were connected together with the exception of one at the centre of the grid which was isolated and provided with a connection to a pre-amplifier. It was discovered, however, that the ceramic
substrates introduced a small constant systematic error, which became significant at the tails of measured distributions. A possible explanation was thought to be that the comparatively high dielectric constant of the ceramic material \( (\varepsilon_r \approx 9\) to \(10) \) made it act as to a small extent as a floating electrode. Its resulting voltage excursions when charge was induced on it in turn caused extraneous charge induction on the sensing wire. Use of G10 fibreglass epoxy material \((\varepsilon_r \approx 4)\) only for the frame of the sensing cathode eliminated the problem.

![Sensing Wire Diagram](image)

Fig. 4.1 Electrical connections to the wires of the sensing cathodes. The black areas show where copper was left on the connecting strip.

Two new cathode arrays with wire pitches 1 and 2mm were therefore constructed, using G10, as described in Section 2.2.3. A narrow strip of copper clad circuit board was suitably etched to make the necessary connections, as illustrated in Fig. 4.1. It was found
to be most important to have $C_g$, the capacitor connecting the other wires of the cathode to ground, very large compared to the interwire capacitance to prevent unwanted charge induction on the sensing wire by voltage excursions of the others. A value of $1 \mu F$ was used.

In addition to the specially constructed wire arrays, one of 0.5mm pitch was produced by modification of a IGD cathode (see Chapter 6) which contained no ceramic material. Most measurements employed a 2mm pitch anode such as is shown in Fig. 2.10. Wire pitches were checked with a travelling microscope which could be read to $20 \mu m$. Wire diameters and the support frame thicknesses (which determined the values $h, d_1, d_2$) were checked with a micrometer to $\pm 1 \mu m$. The geometry of the sensing cathodes is summarised in Table 4.1.

Table 4.1 Geometry of wire cathodes used to sense the cathode induced charge distribution

<table>
<thead>
<tr>
<th>$s_c$ (mm)</th>
<th>$r_c$ (\mu m)</th>
<th>Number of wires</th>
<th>Wire plane area (mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>25</td>
<td>47</td>
<td>$94 \times 103$</td>
</tr>
<tr>
<td>1.0</td>
<td>25</td>
<td>95</td>
<td>$94 \times 103$</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
<td>190</td>
<td>$95 \times 104$</td>
</tr>
</tbody>
</table>

A continuous sensing cathode was constructed, but provided results of lower quality than the wire cathodes due to greater uncertainty in the effective width of the sampling element and a poorer noise performance. It was made from copper clad G10 board with two fine parallel breaks in the conductor produced by a grinding tool. It was found necessary to coat the exposed G10 with a high
resistivity coating of graphite to prevent local field distortion. Using the travelling microscope, the average width of the gaps was determined as 0.2mm and of the isolated strip as 0.8mm. The effective width \( h \delta \lambda \) over which the \( \rho_z \) distribution was integrated was taken as 1.0mm.

Cathode 1, the cathode sampling the distribution, was generally the lower cathode in the chamber, farthest from the X-ray entrance window. Cathode 2 would be of the same wire pitch, with all wires connected together, or continuous (but transparent to the X-rays) as appropriate. For wire cathodes, the two spaces of depths \( d_1 \) and \( d_2 \) were defined by an earthed foil stretched on a standard frame, and the X-ray window.

4.2.3 Electronics

The overall electrical and electronic arrangement is illustrated schematically in Fig. 4.2. The signal processing channel connected to the sampling element was identical to that connected to the anode, except for the inclusion of a high quality wide-band attenuator. An equivalent noise charge of around 200 electrons rms with \( 2\mu s \) unipolar shaping was obtained for this channel when connected to a single cathode wire. With the strip of the continuous cathode, the noise was an order of magnitude worse, mainly due to the (unoptimised) resistive coating.

A pair of well matched Canberra 2022 shaping amplifiers were used for most measurements. These provided a wide range of time constants and high maximum gain. The elements of the filter are a
differentiator and a two stage active integrator, giving an approximation to a Gaussian pulse shape. The impulse response of the circuit, necessary for the calculations, was determined from the circuit diagram as [83]

\[ h(t) = \frac{T_2}{(T_2 - \lambda_1^2)} e^{-\lambda_1^2} \left[ \left( 1 - \frac{\omega T_2}{2} \right) \cos \omega t + \frac{\omega T_2}{2} \left( 1 + \frac{t}{T_2} \right) \sin \omega t - 1 \right] \]

where \( \omega = \left( \frac{T_2}{T_1} - 1 \right)^{1/2} / T_2 \)

and \( T_1 \) and \( T_2 \) are set by the nominal time constant, \( T_a \), according to Table 4.2.

**Table 4.2** Canberra 2022 filter nominal and circuit time constants

| \( T_a \) (\( \mu s \)) | 0.5 | 2.0 | 4.0 |
|\|\|\|\|
| \( T_1 \) (\( \mu s \)) | 0.29 | 1.15 | 2.30 |
|\|\|\|\|
| \( T_2 \) (\( \mu s \)) | 0.43 | 1.69 | 3.38 |
Some results were also taken with simpler equal time constant single
differentiation, single integration filters, for which

$$h(t) = \frac{1}{T_a} e^{-t/T_a} (1 - t/T_a)$$

After shaping, the two signals were presented to the two ADCs of
the Harwell 3769 system, and the ratio of sense element pulse height
to anode pulse height formed by the digital divider was used to
increment one of the analyser channels 0 to 511. The wide band
attenuator was adjusted as necessary to maintain the results in the
approximate range 0.2 x 511 to 0.9 x 511. In order to convert the
peak channel number, $N$, to a value $\Gamma(\lambda)$, it was necessary to know
the relative conversion gains of the two channels. The measurement of
the required factor, $F_0$, presented problems during the experimental
programme until a reliable method was developed. This involved
determining the relative sizes of the two pre-amplifier test
capacitors under carefully controlled conditions, then finding the
setting of a precision pulse generator necessary to give the same
output through either channel. The random error associated with the
measurement was less than 1% relative. The values to be compared with
prediction for a given $\lambda$ were

$$F_0 \left(\frac{N}{511}\right) / \left(\sigma N / h\right) \quad \text{for wire cathodes}$$

$$F_0 \left(\frac{N}{511}\right) / \left(\delta \lambda / h\right) \quad \text{for the continuous cathode}$$
4.2.4 Control of the avalanche angle centroid, \(\alpha\)

This parameter is not generally controlled in an imaging MWPC, but is predicted, and was found, to affect the distributions significantly. It therefore had to be defined for the experimental measurements. It has been shown [84,85] that the crossover time of a doubly-differentiated cathode signal varies with \(\alpha\), and is thus a two-valued function of the position of a collimated X-ray beam moving in the \(z\) axis. In other words, for a given source position, \(z\), essentially two avalanche centroid angles are produced, \(+\alpha\) and \(-\alpha\), with \(\alpha\) given by Eq. 2.6.

For the present measurements, a bipolar filter processed the signals from cathode 2. A crossover detector provided "stop" pulses to a time to amplitude converter, the "start" pulse having been derived from the anode signal. Fig. 4.3 shows typical variation of cathode 2 crossover times with a 1\(\mu\)s filter as the source was moved in the \(z\) axis. Anode wire positions simply corresponded to those source positions for which the difference in crossover time was greatest.

Having located the anode wires, \(q_{z}\) and \(\varphi_{z}\) type distributions corresponding to avalanche angle centroids \(+\pi/2\) and \(-\pi/2\) could be measured by only sampling at positions coincident with anode wire positions. At the tails of the distributions, where the difference between \(\Gamma(\alpha = +\pi/2)\) and \(\Gamma(\alpha = -\pi/2)\) is small, gating derived from the cathode 2 crossover time spectrum allowed selection of one or the other. Fig. 4.4 shows a typical crossover spectrum with \(+\pi/2\) and \(-\pi/2\) events clearly distinguished. Such gating also allowed measurement of \(\varphi_{1}\) type distributions, where physical limitation of
Fig. 4.3 Crossover times of cathode 2 signals processed with a 1 μs bipolar filter, measured as a function of X-ray beam position. The arrows indicate the anode wire positions.
\( \alpha \) is not possible, given a slit collimator, although the definition of \( \alpha \) was not as good.

Fig. 4.4 Spectrum of crossover times from cathode 2 signals processed by a one \( \mu s \) bipolar filter; collimated source coincident with an anode wire.

4.2.5 Control of the avalanche angular spread, \( \sigma \)

This parameter could not in fact be controlled directly by experimental method. As mentioned in Section 2.1.5, with soft X-rays the amount of spread is determined principally by primary electron diffusion, with a second process, thought to be uv mediated spreading within the avalanche, becoming dominant at higher gains. For the present measurements, therefore, two precautions were taken to limit \( \sigma \) and to keep its value relatively constant. Application of a negative potential to the wire cathodes made the counter insensitive to X-rays absorbed in the drift regions, and so reduced variation in \( \sigma \) arising from different amounts of electron diffusion. Secondly, the chamber was operated at a charge level such that uv spreading of
the avalanche was a minor contribution. The available collimation for the X-ray beam was easily sufficient to prevent any lack of definition of $\alpha$ appearing as a perceived increase in $\sigma$.

The degree of avalanche localisation is reflected in the difference between $\Gamma(\alpha = +\pi/2)$ and $\Gamma(\alpha = -\pi/2)$, with the obvious limit that there is no distinction if the avalanche spreads evenly around the wire. Fig. 4.5 shows how the ratio of the heights of the centres of the two distributions, $\Gamma(0,+\pi/2) / \Gamma(0,-\pi/2)$, measured under the standard conditions (Table 4.3) varies with anode charge level. Cathode and anode wires had been aligned so that there was a cathode wire at $\lambda = 0$. The effect of rapidly increasing angular spread above about 0.1pC is apparent. The anode and cathode potentials employed to produce the charge levels shown were used in theoretical calculations of this ratio, with various values of $\sigma$ to give the continuous lines shown. Fig. 4.6 presents the equivalent data for P10 gas, instead of P50 as used in the standard conditions. The cathode wire was at $\lambda = 0.06$ in this case, this small offset being included in the calculated ratios. The combination of slightly lower ionic mobility (Table 2.1) and considerably lower anode potential required for a given charge level, meant that the $+\pi/2$ and $-\pi/2$ distributions were less differentiated at the effective sampling time imposed by the filter. The ions had not travelled as far as in P50, and this is reflected in the lower ratio values observed. However, the angular spread is obviously greater in P10 at the low charge levels, as would be expected from the greater electron diffusion in the low methane mixture [42]. The rate of spreading around the wire at higher gains is also greater in P10, which might be explained by less efficient absorption of uv radiation.
Fig. 4.5 Determination of rms avalanche spread in P50. Theoretical predictions (continuous curves) and experimental measurements (crosses) of the ratio of the maxima of the cathode distributions for $\alpha = +\pi/2$ and $\alpha = -\pi/2$. 
Fig. 4.6 Determination of rms avalanche spread in P10. Theoretical predictions (continuous curves) and experimental measurements (crosses) of the ratio of values close to the maxima of the cathode distributions for $\phi = + \pi/2$ and $\phi = - \pi/2$. 

$$\Gamma(0.06, +\pi/2) / \Gamma(0.06, -\pi/2)$$

Effective anode charge (pC)
Experimental distribution measurements were all made at a charge level of 0.1pC with Al K X-rays, this being a compromise between stability in the value of \( \sigma \) and electronic noise spreading \( \Gamma(\lambda) \). For wire cathodes, \( \Gamma(\lambda = \pm 3.0) \) represented the practical extremes with the noise performance available. The value of \( \sigma \) adopted in the calculations was 40° for P50 and 50° for P10.

4.3 Results

4.3.1 Presentation

A range of results are presented which test the predictions of the \( q_2 \) calculations, or in one case the \( p_2 \) calculations. The \( p_1 \) theory cannot make predictions for wire cathodes that can be directly compared with experiment in this manner, but some measurements in the \( x \) axis are plotted together with calculations for the equivalent distribution in the \( z \) axis. Corresponding \( \alpha = +\pi/2 \) and \( \alpha = -\pi/2 \) distributions that were obtained concurrently are presented together, and the relevant predictions are shown as curves, rather than discrete points (Section 3.4.6), as this makes the comparisons clearer.

Stochastic errors in \( \Gamma \), arising from the location of the centroids of \( \Gamma(\lambda, +\pi/2) \) and \( \Gamma(\lambda, -\pi/2) \) at given \( \lambda \) and the measurement of the relative channel gains, and in \( \lambda \), arising from the location of \( \lambda = 0 \), were smaller than could be shown clearly in the figures. The size of the symbols used for the data points was arranged to indicate maximum error boxes.
4.3.2 "Standard conditions" and the effect of cathode 2 orientation

The experimental standard conditions were as given in the first column of Table 4.3. The remaining columns of the table indicate which parameters were altered to obtain the results described below. Fig. 4.7 shows experimental data and theoretical predictions for these standard conditions. The two sets of experimental points were taken with cathode 2 wires parallel (as in Fig. 3.2) and orthogonal to those of the anode and cathode 1. The predictions agree equally well with both sets of data, and as the second arrangement is the one usually employed in imaging counters, all subsequent readings were taken with the two sets of cathode wires mutually orthogonal.

4.3.3 The effect of anode-cathode spacing

The width of the cathode induced charge distribution has long been known to increase with h [66], and indeed for some of the simple analytical formulae it is the only independent parameter taken into account [66,69,70]. For the results shown in Fig. 4.8, the anode-cathode spacing was increased to 6mm, but otherwise standard conditions applied. Although the distributions are broader at larger h in absolute units, the difference in terms of $\lambda$ can be seen to be small.

4.3.4 Time development

Figs. 4.9 and 4.10 show the effect of reducing the processing time constant to a nominal 0.5 $\mu$s or increasing to 4 $\mu$s respectively.
Table 4.3 Experimental parameters for cathode induced charge measurements

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<td>s</td>
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</tr>
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<td>-</td>
</tr>
<tr>
<td>r₀ (µm)</td>
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The standard conditions are defined in the first column. The symbol s in any other column indicates that the particular parameter had the same value as in the standard conditions.
Fig. 4.7 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3). The experimental results indicated by the box symbols were taken with cathode 2 wires parallel to those of cathode 1 and the anode; the results indicated by the star symbols were taken with cathode 2 wires orthogonal to those of cathode 1 and the anode.
Fig. 4.8 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with $h = 6\text{mm}$.
Fig. 4.9 Cathode distribution functions $\Gamma (\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with $T_\alpha = 0.5\mu s$. 
Fig. 4.10 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with $T_\alpha = 4\mu$s.
The increase in the difference between $\Gamma(+\pi/2)$ and $\Gamma(-\pi/2)$ as the distributions are effectively sampled later in time by the longer time constant may be noted. For a given nominal time constant, the simple CR-RC filter actually sampled the distribution earlier. The results in Fig. 4.11 were taken with simple filters in the sensing wire and anode channels set at $2\mu s$ time constant, giving times to unipolar peak of around $2\mu s$ (by comparison with around $5.5\mu s$ for the Canberra 2022 amplifiers on the same nominal setting). Substitution of P10 gas also caused the distribution to be effectively sampled earlier (Fig. 4.12), as already discussed in Section 4.2.5.

4.3.5 The influence of earth planes behind wire cathodes

Measurements made with the continuous cathode, for which there can obviously be no "leakage" of induced charge, are illustrated in Fig. 4.13. The effect of charge loss to the earth planes behind wire cathodes may be seen if these results are compared with those for cathodes of 0.5mm (Fig. 4.14) and 2mm pitch (Fig. 4.15).

The theoretical model places no restrictions on the drift depths $d_1$ and $d_2$, apart from the requirement $d_1, d_2 \gg r_e$. For the measurements presented in Fig. 4.16, the spaces were made very assymetric ($d_1 = 1mm$, $d_2 = 6mm$), and the significant loss of charge to the earth plane close behind cathode 1 is apparent.
Fig. 4.11 Cathode distribution functions $\Gamma (\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with the simple CR-RC filter, $T_\alpha = 2\mu s$. 
Fig. 4.12 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with P10 gas.
Fig. 4.13 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with continuous cathodes.
Fig. 4.14 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with $s_e = 0.5\text{mm}$. 

\[ s_c/s_a = 0.25 \]
Fig. 4.15 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements and theoretical predictions (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3), except with $s_c = 2\text{mm}$. 
Fig. 4.16 Cathode distribution functions $\Gamma (\lambda )$. Comparison between experimental measurements and theoretical predictions (upper curve $\phi = + \pi /2$, lower curve $\phi = - \pi /2$). Standard conditions (see Table 4.3), except with $d_1 = 1\text{mm}$ and $d_2 = 6\text{mm}$.
4.3.6 Distributions measured in the x axis

The results presented so far show that the theoretical models for \( q_2 \) and \( \Phi_1 \) type distributions may be used to make predictions of some accuracy. Comparison of \( \Phi_1 \) and \( q_2 \) calculations for \( \alpha = \pm \pi/2 \) suggested that the cathode induced charge distribution is very similar in its projection onto the x or z axis. Fig. 4.17 shows experimental points taken as the X-ray beam was moved in the x axis, the wires of cathode 1 thus being orthogonal to those of the anode, but with standard conditions otherwise applying. As indicated earlier, the definition of \( \alpha \) was not as satisfactory in this measurement. However, the agreement with equivalent calculations for the z axis is quite good, as anticipated. Even when the anode wire pitch was increased by a factor of ten, an extreme case, the \( q_2 \) calculations still provide a fair description (Fig. 4.18).
Fig. 4.17 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements in the x axis (parallel to the anode wires) and theoretical predictions for the z axis (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3) otherwise apply.
Fig. 4.18 Cathode distribution functions $\Gamma(\lambda)$. Comparison between experimental measurements in the x axis (parallel to the anode wires) and theoretical predictions for the z axis (upper curve $\alpha = +\pi/2$, lower curve $\alpha = -\pi/2$). Standard conditions (see Table 4.3) otherwise apply, except that $s_a = 20\text{mm}$.
5.1 The need for a simplified description

The results of Chapter 4 have provided good support for the calculations of distributions in the z axis, and given indication that z and x axis distributions are very similar in form. The full calculations may often be inconvenient for the prediction of cathode system non-linearity, however, dependent as they are on parameters that are not always well known. Additionally, they demand a considerable amount of computing and produce numerical, rather than convenient analytical, distributions. There is clearly a requirement for a simple analytical formulation, which, for a given basic chamber geometry, represents an average of the behaviour predicted by the full calculations.

If, during the application of the reciprocity theorem, the potential function around the anode wires is assumed to be purely radial, then the dependence on \( \alpha \) is removed. Moreover, the ratio \( \frac{\rho_1}{q_a} \) or \( \frac{\rho_x}{q_a} \) becomes independent of time because cathode and anode charges will develop identically with time. Thus this ratio can be identified directly as \( \Gamma \), without the need to consider signal processing, assuming the filters to be linear. This rather drastic simplification was employed by Gatti et al. [72], and allowed them to fit a convenient three parameter empirical formula to the results of the calculations. Before investigating the use of this simplification, we shall mention some of the specific predictions of
the full calculations that are not made by the simplified treatment.

5.2 Some specific predictions of the full calculations

5.2.1 Wire and continuous cathodes

The results of the previous chapter showed the significant "leakage" of charge that can occur through wire cathodes. As can be seen more clearly from Fig. 5.1, this loss does not produce a simple linear scaling, but also changes the shape of the distribution, it being relatively broader with higher tails for wire cathodes.

5.2.2 $\pm \pi/2$ and $-\pi/2$ distributions

Charge distributions for $\alpha = +\pi/2$ and $-\pi/2$ will represent the worst case departures from an average distribution taking no account of $\alpha$. It is obvious, and clear from the results of Chapter 4, that more charge is induced on the cathode which faces the side of the anode wire on which the avalanche is occurring. Fig. 5.2 shows calculations for the standard conditions which reveal that $+\pi/2$ and $-\pi/2$ distributions can differ significantly in shape also. The differences in both integrated area and shape at a given sampling time increase as $\sigma$ decreases, up to a limit when the angular spread is zero.
Fig. 5.1 Cathode distribution functions \( \Gamma(\lambda) \). Theoretical predictions (standard conditions) for \( \alpha = +\pi/2 \) and \( -\pi/2 \) for continuous cathodes (upper and lower continuous curves respectively) and for wire cathodes (upper and lower broken curves).
Fig. 5.2 Predicted cathode distribution functions for $\alpha = +\pi/2$ and $-\pi/2$ for standard conditions but with continuous cathodes, normalised to equal area (unity). The bottom curve is the difference $\Gamma_{\text{norm}}(\lambda, +\pi/2) - \Gamma_{\text{norm}}(\lambda, -\pi/2)$. 
5.2.3 Time development of the distribution shape

Different parts of the distribution develop differently with time, or equivalently, the shape is time dependent. This is illustrated in Fig. 5.3. Instantaneous distribution functions $\rho_2(t)/q_\alpha(t)$, normalised to equal area, for continuous cathodes and of course without signal processing, but otherwise standard conditions (Table 4.3), are shown at $t = 0.1$, $1.0$, and $10.0\mu s$. As can be seen, shape development has limited significance at normal processing filter timescales in this case.

5.2.4 Comparison of $\rho_1$ and $\rho_2$

In general the $\rho_2$ type distribution is displaced from, and therefore asymmetric about, the co-ordinate origin. A strict mathematical comparison between $\rho_1$ and $\rho_2$ distributions is therefore only possible for the cases $\alpha = +/- \pi/2$. Table 5.1 gives the calculated rms differences between $\rho_1$ and $\rho_2$ expressed as a percentage of their peak value for four positions of a single positive point charge.

<table>
<thead>
<tr>
<th>$h/s_\alpha$</th>
<th>$y/s_\alpha$</th>
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<th>1.0</th>
<th>2.0</th>
</tr>
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<tbody>
<tr>
<td>$+0.20$</td>
<td>0.86</td>
<td>0.45</td>
<td>0.10</td>
<td></td>
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<tr>
<td>$+0.05$</td>
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<tr>
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<tr>
<td>$-0.20$</td>
<td>5.15</td>
<td>1.22</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 5.3 Predicted instantaneous cathode distribution functions \( \rho_\alpha / q_\alpha \), normalised to equal area (unity), for \( \alpha = +\pi/2 \) (right hand side) and \( -\pi/2 \) (left hand side) for standard conditions but with continuous cathodes.
- continuous curves: distributions after 0.1\( \mu \)s
- short dashes: distributions after 1.0\( \mu \)s
- long dashes: distributions after 10.0\( \mu \)s
Fig. 5.4 Predicted instantaneous cathode distribution functions $\rho / q_a$ at $t = 4 \mu s$, for $\alpha = +\pi/6$ and continuous cathodes, but otherwise standard conditions.
Fig. 5.5 The difference curve resulting when the \( \rho_a \) curve in Fig. 5.4, with its centroid removed to the origin, is subtracted from the \( \rho_a \) curve. The vertical scale is multiplied by a factor of 1000 to reveal the slight asymmetry of \( \rho_a \) about its own centroid.
As may be seen, an experimentally significant difference only occurs at values of $h/a$ smaller than are usual in MWPC geometry. Fig 5.4 shows the instantaneous charge distributions $\rho/q$ due to a single point positive ion charge after 4$\mu$s of drift along the field line starting at $\alpha = \pi/6$. The standard conditions otherwise apply. It may be seen that the difference between $\rho_1$ and $\rho_2$ when $\alpha \neq +/-\pi/2$ is mainly their relative lateral displacement. Such shifts of the $\rho_2$ centroid provide non-zero position sensitivity over each anode wire (Chapter 8). If the $\rho_2$ centroid is removed to the origin in the present case, then the rms difference between $\rho_1$ and $\rho_2$, normalised to the $\rho_1$ peak is only 0.2%; of the same order as those given in Table 5.1. As may be seen from Fig. 5.5, which shows the result of subtracting the shifted $\rho_2$ curve from the symmetrical $\rho_1$ curve, some of this difference is due to a small asymmetry of $\rho_2$ about its own centroid. This is predicted when $\alpha \neq +/-\pi/2$. An attempt was made to measure this skewness for $\alpha = \pi/6$, but it proved too small to be clearly seen.

5.3 The radial field approximation

5.3.1 Anode signal

In the derivation of simplified cathode induced charge distributions using the radial field approximation, we shall further simplify the situation by only considering the case of continuous cathodes. Proceeding in the manner described in Chapter 3, we obtain the potential function in the chamber due to the array of line charges representing the anode wires between earthed cathode planes as Eq. 3.13, which we shall restate as
\[ P_w = - \sum_{n=1}^{2N+1} C_n L_n \quad \text{Eq. 5.1} \]

where
\[ L_n = \ln \left[ \frac{\cosh(a(z - n\alpha)) - \cos \alpha y}{\cosh(a(z - n\alpha)) + \cos \alpha y} \right]^{1/2}, \quad a = \pi/2h \]

The summation is taken over the 2N+1 wires. Again, we find \( C_n \) by solving
\[ 1 = - \sum C_n L_{nm} \]
where
\[ L_{nm} = |n| \tanh(a(m - n) s_\alpha/2) | \]
\[ L_{mm} = \ln (a r_\alpha/2) \]

Now, instead of using the correct potential function, Eq. 5.1, in the application of the reciprocity theorem, we assume that the potential around the avalanche wire, for which \( C_n = C_\alpha \), say, is
\[ P_w = 1 - C_\alpha \ln (r/r_\alpha) \]
and then by the reciprocity theorem and Eq. 2.12
\[ q_\alpha = - C \ln (r/r_\alpha)^2, \quad C = C_\alpha/2 \quad \text{Eq. 5.2} \]
which is of course insensitive to \( \alpha \). If the actual anode potential is \( V_\alpha \), then the radial electric field would be
\[ E = 2V_\alpha C/r \]
and the movement of positive ions of constant mobility, \( \mu \), would be described by
\[ \frac{dr}{dt} = \frac{2 \mu V_\alpha C}{r} \]

Integrating, with \( r = r_\alpha \) when \( t = 0 \), we obtain
\[ \frac{(r/r_\alpha)^2}{4C/V_\alpha} = 1 + \frac{t}{t_\alpha} \]
where \( t_\alpha = \frac{r_\alpha^2}{4C/V_\alpha} \), so the charge development on the anode is given by
\[ q_\alpha(t) = - C \ln (1 + \frac{t}{t_\alpha}) \quad \text{Eq. 5.3} \]

This is the same form as for the simple coaxial proportional counter [33], for which the field is of course truly radial. The comparison
may be pursued further by defining an equivalent coaxial cathode radius for the MWPC, $r_{ce}$, so that $C = 1/\ln(r_{ce}/r_{a})^2$. It may then be shown that

$$r_{ce} = \frac{s_a}{2\pi} e^{\pi h/s_a} \quad \text{for } e^{2\pi h/s_a} \gg 1 \quad \text{Eq. 5.4}$$

$$r_{ce} = 4 h/\pi \quad \text{for } s_a \gg h \quad \text{Eq. 5.5}$$

5.3.2 The $\rho_z$ distribution

Referring back to Section 3.3.3, we proceed in the same manner until we have solved Eq. 3.7 for the vector of coefficients $dC_\lambda/d\lambda$. The assumption is then made that the potential function around the avalanche wire, for which $dC_\lambda = dC_o$, say, is purely radial:

$$dP = -\frac{2\pi e_o}{2\pi e_o} dC_o \ln \frac{r}{r_o} = -\frac{1}{2} dC_o \ln \left(\frac{r}{r_o}\right)^2 \quad \text{Eq. 5.6}$$

The charge induced on the cathode strip $d\lambda$ at $\lambda = z_c/h$ by unit charge at $r = \sqrt{z^2+y^2}$ is thus

$$-dP = (\rho_z(\lambda)) d\lambda$$

We may form the time independent distribution function $\Gamma_z$ by normalising to the anode signal charge from Eq. 5.2.

$$\Gamma_z(\lambda) = \frac{Q_z(\lambda)}{q_o} = \frac{dC_o/d\lambda}{2C}$$

Since there is no angular dependence, we must have

$$\int_{-\infty}^{\infty} \Gamma_z(\lambda) d\lambda = 1/2 \quad \text{Eq. 5.7}$$
5.3.3 The $\rho_i$ distribution

Here we proceed as described in Section 3.3.4 as far as Eq. 3.12. The inverse transform $f(\lambda)$ of $F(\omega)$ then gives us the linear charge density of the line charge representing the avalanche wire, $dq(\lambda') = 4\pi\varepsilon_0 f(\lambda) d\lambda$, due to the unit potential cathode strip $d\lambda$ at $x_\omega$. Introducing the radial field assumption, and having defined the wire surface as zero potential, the potential function around the wire may be written

$$dP = -\frac{dq}{2\pi\varepsilon_0} \ln \frac{r}{r_\omega} = -f(\lambda) d\lambda \ln \left(\frac{r}{r_\omega}\right)^2$$

Then from the reciprocity theorem

$$-dP = \rho_i(\lambda) d\lambda$$

The time independent distribution function is

$$\Gamma_i(\lambda) = \frac{\rho_i(\lambda)}{q_\omega} = -\frac{f(\lambda)}{\zeta}$$

and again we must have

$$\int_{-\infty}^{\infty} \Gamma_i(\lambda) d\lambda = 1/2$$

Eq. 5.8

5.4 Asymptotic expressions for $\Gamma$

5.4.1 The asymptotic limits

With angular dependence removed by the radial field assumption, analytical forms of the distribution occur at the asymptotic limits $s_\omega \to \infty$ (single wire chamber) and $s_\omega \to 0$ (parallel plate chamber).
Correct reduction at these limits was a factor considered by Gatti et al. [72] in choosing a suitable analytical form of the distribution.

5.4.2 \( s_\alpha \rightarrow \infty \)

Considering this as a limiting case of the \( \rho_2 \) calculation, the condition of zero potential at the wire surface (Eq. 3.7) becomes

\[
\frac{1}{4} \sech \frac{\pi \lambda}{2} = \frac{dC_0}{d\lambda} \ln \left( \frac{a r_\alpha}{2} \right)
\]

We obtain \( dC_0 \) directly, and thus the potential function from Eq. 5.6.

From Eqs. 5.2 and 5.4 we have

\[
2C = \frac{1}{\ln(4h/\pi r_\alpha)} = -\frac{1}{\ln(a r_\alpha/2)}
\]

so that for the single wire chamber

\[
\Gamma_2(\lambda) = \frac{1}{4} \sech \frac{\pi \lambda}{2}
\]

Eq. 5.9

It is interesting to note that this is the same functional form as the distribution obtained by Endo et al. for a point charge between two conducting planes [70] (Fig. 3.1(c)). Gatti et al. report this also to be the limit of \( \rho_1 \) as \( s_\alpha \rightarrow \infty \), although no proof is given.

5.4.3 \( s_\alpha \rightarrow 0 \)

In this limit the anode becomes a continuous conducting plane. To preserve symmetry it must therefore be assumed that the charge induced on each cathode is due to positive ion charge of \(+1/2\) close to, and on the appropriate side of the anode plane. The potential
function due to a narrow strip $d\lambda$ at $\lambda$ on the top cathode at unit potential, all other conductors at earth is (see Appendix A.2)

$$dP = \frac{1}{2} \frac{\sin 2ay}{\cosh(2a(x-h\lambda)) + \cos 2ay} d\lambda$$

$x$ and $z$ axes must be equivalent at this limit. For unit positive ion charge at $x = 0$ and $y \ll h$

$$dP = \frac{1}{2} \frac{ay}{\cosh^2(\pi \lambda / 2)} d\lambda = -2\varphi(\lambda) d\lambda$$

with the factor 2 applied for the reason given above. Now for an avalanche yielding unit positive charge (half above and half below the anode plane), and referring to Eq. 2.13

$$q_\lambda = -2 \int_0^{\infty} \varphi(\lambda) d\lambda = -y/h$$

So we obtain the distribution whose form was first given by Lee et al. [69]

$$\Gamma(\lambda) = \frac{\pi}{8} \text{sech}^2 \frac{\pi \lambda}{2} \quad \text{Eq. 5.10}$$

5.5 An empirical formula for the cathode induced charge distribution

5.5.1 The Gatti empirical formula

There are a number of bell-shaped analytical forms which might prove to be satisfactory for fitting to calculated distributions. Bellazzini et al. [86] for example propose the use of the lorentzian form. However it is desirable that a formula can reduce to the exact analytical forms at the asymptotic limits (Eqs. 5.9 and 5.10). The formula proposed by Gatti et al. [72] has this attribute, and proved
convenient in use:

\[ \Gamma (\lambda) = K_1 \frac{1 - \tanh^2 K_2 \lambda}{1 + K_3 \tanh^2 K_2 \lambda} \]

Eq. 5.11

The correct reduction at the limits is obtained when the parameters are set according to Table 5.2

Table 5.2 Gatti formula parameters at the asymptotic limits

<table>
<thead>
<tr>
<th>h/s</th>
<th>K_1</th>
<th>K_2</th>
<th>K_3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/4</td>
<td>( \pi /4 )</td>
<td>1</td>
</tr>
<tr>
<td>( \infty )</td>
<td>( \pi /8 )</td>
<td>( \pi /2 )</td>
<td>0</td>
</tr>
</tbody>
</table>

For all intermediate values of h/s, the parameters lie within the indicated ranges. The function has full widths at half and tenth maximum

\[ \text{FWHM} = \frac{2 \tanh^{-1} \frac{1}{(2 + K_3)^{1/2}}}{K_2} \quad \text{FWTM} = \frac{2 \tanh^{-1} \frac{3}{(2 + K_3)^{1/2}}}{K_2} \]

and may be integrated

\[ \int_{\lambda_1}^{\lambda_2} \Gamma (\lambda) d\lambda = \frac{K_1}{K_2 \sqrt{K_3}} \left[ \tan^{-1}(\sqrt{K_3 \tanh(K_3 \lambda)}) \right]_{\lambda_1}^{\lambda_2} \]

Eq. 5.12

Eq. 5.11 may of course be used without regard to actual chamber geometrical parameters as an analytical representation of any full calculation or experimental measurement of the cathode charge distribution. The parameters K_1, K_2, K_3 would be simply determined by best fit analysis.
5.5.2 Reduction of the Gatti formula to one parameter

As originally proposed, the three parameters of Eq. 5.11 were to be independently chosen according to the actual geometrical parameters of the MWPC. However we may see at once that only two parameters are necessary because we normalise to the anode charge and we know that the integrated charge on either cathode is 1/2 (Eqs. 5.7 and 5.8). From Eq. 5.12, then

\[ K_1 = K_2 \sqrt{K_3} / (4 \tan \sqrt{K_3}) \]

leaving only \( K_2 \) and \( K_3 \) as independent parameters. Clearly, if these two parameters could be shown to be related, then a very convenient single parameter form would result.

Distributions \( \Gamma_1 \) and \( \Gamma_2 \) were calculated as described in Sections 5.3.2 and 5.3.3 for a range of chamber geometries, and the best fitting parameters \( K_2 \) and \( K_3 \) found. \( K_3 \) was found to be very sensitive to the shape of the distribution, whereas \( K_2 \) changed smoothly between the limits given in Table 5.2, and slowly over the usual range of MWPC geometries. Fig. 5.6 shows how \( K_3 \) varied with \( h/s_0^\alpha \) for the case \( r_\alpha/s_0^\alpha = 1.5 \times 10^{-3} \). It should be remarked that the difference between \( \Gamma_1 \) and \( \Gamma_2 \) distributions is actually very small, as noted earlier. The parameter \( K_3 \) is very sensitive to these small differences. The fact that it does not quite reach 1.0 for \( \Gamma_1 \) as \( s_0 \to \infty \) is interesting, as it implies that \( \Gamma_1 \) does not reduce correctly in the limit, but this is of no practical significance.

Using the \( \Gamma_1 \) results, the relation

\[ K_2 = (\pi/2)(1 - \frac{1}{2} K_3^p) \]

with a form chosen to preserve the asymptotic limits was found to give
Fig. 5.6 Variation of the best fitting parameter $K_3$ with $h/s_\alpha$ for the case $r_\alpha/s_\alpha = 1.5 \times 10^{-3}$. $x$ axis ($\Gamma_1$) and $z$ axis ($\Gamma_2$) calculations using the radial field assumption.
good fits (rms deviations < 0.1% of peak values) with $p = 0.5$ for $h/s_\alpha > 1.5$ and over a representative range of $r_\alpha/s_\alpha$. Essentially the same value of $p$ applied for the $\Gamma_1$ case. Thus from a single parameter $K_3$, which may be tabulated for normal chamber geometries, a complete simplified cathode induced charge distribution may be obtained. Fig. 5.7 summarises the formulae and gives $K_3$ values for a range of $h/s_\alpha$ and $r_\alpha/s_\alpha$. For $h/s_\alpha < 1.5$ the single parameter fit is not reliable, and full calculations should be employed.

The fact that this distribution provides an average description may be seen from Fig. 5.8. Full calculations for the standard conditions (Table 4.3) and $\alpha = +/- \pi/2$ bracket the appropriate curve obtained from Fig. 5.7.
Fig. 5.7 Best fit parameters $K_3$ for a range of MWPC geometries.
Fig. 5.8 Cathode distribution functions $\Gamma$. Full calculations for the standard conditions (Table 4.3) except with continuous cathodes, and $\alpha = +\pi/2$ and $-\pi/2$, compared with the approximate one parameter Gatti empirical formula (curve G) obtained from Fig. 5.7.
CHAPTER SIX
MWPC POSITION READOUT METHODS

6.1 MWPC position response

6.1.1 Introduction

The present work concerns MWPCs in which the position readout is derived from the charge induced on the cathodes. This approach now appears to be generally preferred, except for systems in which a very good time resolution is the prime requirement, where reading out individual anode wires remains the method of choice [87]. There are a number of reasons for this:

- Identical readout arrangements may be used in x and z axes to provide two-dimensional imaging
- Interpolation between the anode wires is possible for the z axis without additional complication. This can be the result of the angular localisation of the avalanche, or avalanching occurring on more than one wire (Chapter 8)
- The distributed nature of the charge means that the resolution is not primarily determined by the density of the sampling elements.

6.1.2 The calculation of position response

In order to define terms relating to position response, we shall consider how the output of a cathode readout system can be calculated for the x axis. Firstly a suitable induced charge distribution is obtained (Chapters 3 and 5), and expressed in analytical form
(Eq. 5.11). It is assumed that the centroid of the distribution reflects the X-ray interaction position. Chapter 8 gives some instances of when this may not be the case, the most important being a systematic modulation across the anode wires in the z axis.

For a given centroid position \( x \), where \( 0.0 \leq x \leq 1.0 \), with the sensing length of the cathode 1.0, the proportion of the total charge on each sensing element may be found from Eq. 5.12. Charge in the portions of the distribution that lie beyond the ends of the cathode may be ignored or added to the end elements as is most appropriate to the real system. The set of charge samples \( q_i \) is then presented to a model of the particular readout system, from which an output \( Q \), where \( 0.0 \leq Q \leq 1.0 \), is obtained. If this could be repeated for all \( x \), then the position response \( Q(x) \) would be obtained. The sensitivity, \( S(x) \), is simply the slope \( \frac{dQ}{dx} \). In practice, in calculations or experimental measurements of \( Q \), the number of separate \( x \) must be finite, but if increments \( \delta x \) small compared to the width of the distribution are made, then a good representation of \( Q(x) \) can be gained.

For a perfectly linear system, \( Q = x \) and \( S = 1 \) for all \( x \). It may be seen at once that such a system cannot be realised, because position information from the tails of the charge distribution beyond the ends of the cathode is lost or corrupted, and this becomes significant when \( x \) approaches 0.0 or 1.0. Practical imaging systems are generally arranged so that these extreme values of \( x \) are not produced. The integral non-linearity of a cathode system is defined in the present work over the central two thirds of the sensing length, as the rms deviation of \( Q(x) \) from \( S_o x + \text{const} \), where \( S_o \) is the average sensitivity. It is expressed as a percentage fraction of the total
sensing length.

If the physical sampling of the charge distribution is too coarse, then a significant systematic non-linearity results [68,70,72,88]. While this may in principle be compensated in the subsequent processing, it must be remembered that the regions of low and high sensitivity implied in the charge samples \( q_i \) are real. Low sensitivity regions are more prone to give random fluctuations in \( Q \) due to electronic noise in the readout system [43]. A system with uniform sensitivity is usually the most desirable.

6.1.3 Differential non-linearity

A plot of \( Q(x) \) against \( x \) is not often very revealing of imaging performance; the first differential \( S(x) \) is more informative. To obtain \( S(x) \) from \( Q(x) \) we really require the latter to be continuously known in the region of interest. While this ideal may be approached in calculations by making sufficiently small increments in the charge centroid position, this is inefficient as an experimental method. Instead, essentially all values of \( x \) within a given range may be generated by exposing the relevant area of the detector to a uniform flux of radiation for a sufficient length of time. The result is a plot of number of counts, \( n \), per channel, \( N, 0 \leq N \leq N_\star \), where \( N_\star \) is the highest channel. The output device should have all channels of equal width, so that \( N = \text{int}(QN_\star) \).

If \( N_\star \) is large, then we may consider the output function \( n(N) \) to be a close representation of the continuous function \( G(Q) \), say. Thus if the incident radiation distribution is \( F(x) \)
\[ G(Q) \, dQ = F(x) \, dx \]

or

\[ \frac{G(Q)}{F(x)} = \frac{dx}{dQ} = \frac{1}{S} \]

But \( F(x) \) has been arranged constant (\( = U_0 \), say), so what we observe is

\[ G(Q) = U_0 / S \]

Eq. 6.1

This uniform irradiation response gives a clear overall indication of the varying (inverse) sensitivity of the system, remembering of course that it is plotted against \( Q \), not \( x \). For the hypothetical ideal linear system, \( G(Q) = U_0 = \text{constant} \).

In this work, the differential non-linearity, \( \delta \), is defined as the rms deviation of \( G(Q) \) (or \( n(N) \) for experimental measurements) about its mean value over the central 70% of the position output. It is expressed as a percentage of that mean. If the full output \( n(N) \) was not available for computation, the definition

\[ \varepsilon = \left[ \left( \frac{n_{\text{max}} - n_{\text{min}}}{n_{\text{max}} + n_{\text{min}}} \right) \times 100 \right] \%
\]

was adopted. This is not so representative of the overall response, but highlights sharp local variations in \( S \).

6.1.4 Obtaining \( Q(x) \) from a uniform irradiation response

From Eq. 6.1 we have \( U_0 = G(Q) \, dQ/dx \), so that

\[
\int_{x_1}^{x_2} U_0 \, dx = \int_{Q_1}^{Q_2} G(Q) \, dQ \approx \sum_{N_i} n(N)
\]

Then if we know \( x_1 \) and \( x_2 \) at \( N_i \) and \( N_{i+1} \),

\[ U_0 = \frac{1}{x_2 - x_1} \sum_{N_i}^{N_{i+1}} n(N) \]
Thus we can obtain $x$ corresponding to general $N$

$$x = x_1 + \frac{1}{U_o} \sum_{N_i}^N n(N)$$

Using this equation, a table of $x$ corresponding to $N$ (and hence $Q$) can be produced which could be used to deconvolve systematic modulation arising in an imaging system. However, there are two provisos that must be kept in mind. Firstly, a uniform irradiation response does not give direct information on the position resolution of the system. An apparently good, flat output may in fact be due to small spatial frequency structure being smoothed out by poor resolution. Secondly, the assumption has been made that $Q$ is a single valued function of $x$. Although this is desirable, it is not necessarily the case: the dependence of the cathode charge distribution on the avalanche angle centroid can produce multiple images if the sampling is coarse, or if a significant fraction of the distribution lies beyond the end of the cathode. Such effects were encountered during the measurements described in Chapter 9.

6.1.5 Experimental uniform irradiation response

In the present work, the uniform irradiation response was used as a sensitive measure of cathode system linearity, both in the assessment of cathodes developed during earlier research at Leicester, and in the optimisation of new systems (Chapter 7). A standard method was developed for obtaining the uniform irradiation responses of cathodes which could be operated in the Leicester detector body.

Several precautions were taken to ensure that the measured response function was that of the cathode system itself. For all the
tested cathodes, the algorithm \( B/(A+B) \), implemented by the digital divider of the Harwell 3769 electronics, was used to form the position output. The conversion gains of the two channels were carefully balanced to within 0.1% of each other. It may be shown that should there be a difference \( \alpha \) in the gains, a slope of approximately \( 2\alpha \) in \( G(Q) \) (and so \( n(N) \)) would result. Statistical fluctuations in \( n(N) \) were made smaller than +/-1% by accumulating an average of at least 40,000 counts per channel. Anode charge levels of 0.4pC in P10 gas for the Mn K X-rays gave near optimum position resolution so that there was no chance of details of the responses being lost through poor resolution.

Fig. 6.1 Arrangement for uniform detector irradiation in the x axis

The detector was arranged so that the cathode under test was the lower one, sensing in the x axis to avoid modulation at the anode wire pitch. A standard value of \( h = 6 \text{mm} \) was adopted, so that all cathodes were sampling essentially similar induced charge distributions.
Modulation of the output caused by the wires of the test cathode (Chapter 8) was avoided by ensuring that X-rays were absorbed above it, either between the cathodes or in a 9mm drift region. This was achieved by illuminating the detector obliquely, but perpendicular to the x axis, so that the lower cathode was in shadow (Fig. 6.1). The one metre distance between the iron-55 source and the detector window produced a flux difference of less than 0.1% between the centre and edges of the 70mm square window.

Uniform irradiation responses are presented as plots of $G(Q)$ against $Q$, where the actual channel number, $N = \text{int}(255xQ)$, and $G(Q) = n(N)/\bar{n}$. $\bar{n}$ is the mean of $n$ over the central 70% of $N$.

6.2 Limitation of MWPC position resolution by electronic noise in the readout system

MWPCs which have an amplifier per anode wire, and discriminators arranged to identify the avalanche wire, can obviously resolve to one wire pitch at best. They are inherently digital, so that $Q(x)$ and the electronic position resolution, $\sigma_e(x)$, are not continuous functions. With cathode readout by contrast, $Q$ is continuous: there has to be an analogue stage in the readout process. The position resolution of the readout system is thus determined by the signal to noise ratio; we may write the rms spatial uncertainty due to the noise as

$$\sigma_e = \left(\frac{q_n}{q_s}\right)D$$

Eq. 6.2

where $q_s$ is the total signal charge appearing at the pre-amplifier inputs, $q_n$ is an equivalent noise charge referred to the individual pre-amplifier inputs, and $D$ is a dimensionless resolution parameter. The noise charge comprises the uncorrelated contributions from the
loaded pre-amplifiers and any thermal noise arising in the cathode or associated components. The latter is usually small unless the readout method involves resistive elements. The parameter $D$ must take into account noise correlation between the readout nodes and the position algorithm used. For the two-terminal systems and the algorithm $B/(A+B)$ used in the majority of this work it follows that

$$\frac{dQ^2}{\bar{d}q^2} = \frac{Q^2(dq_A)^2 + (1-Q)(dq_B)^2 - 2(1-Q)Q \bar{dq}_A \bar{dq}_B}{(q_A + q_B)^2}$$

where $dq_A$ and $dq_B$ are the noise fluctuations on the pre-amplifier input charges $q_A$ and $q_B$. Assuming $\bar{dq}_A^2 = \bar{dq}_B^2 = q_n^2$, a noise correlation coefficient may be defined as

$$\gamma_{AB} = \frac{\bar{dq}_A \bar{dq}_B}{q_n^2}$$

Thus

$$dQ^2 = (1 - 2Q(1-Q)(1 + \gamma_{AB})) \left(\frac{q_n^2}{q_s^2}\right)$$

and since

$$\sigma_e^2 = \bar{d}x^2 = \frac{dQ^2}{S}$$

the resolution parameter is obtained as

$$D = \frac{1}{S} (1 - 2Q(1-Q)(1 + \gamma_{AB})^/2 \quad \text{Eq. 6.3}$$

For two terminal systems, $-1 \leq \gamma_{AB} \leq 0$, a value of $-1$ indicating complete anticorrelation, and a value of 0, no correlation.

Some important general points arise from Eqs. 6.2 and 6.3.

Firstly the effect of sensitivity variation must be kept in mind for systems with appreciable non-linearity. An apparently good position resolution, measured as $\sqrt{dQ^2}$, may in fact be due instead to low local sensitivity. Secondly, it might appear that a simple way to improve
resolution would be to increase the chamber gain, so increasing $q_5$; but there are many reasons why this should be resisted:

- The higher voltages required can give rise to operating difficulties.

- In many common operating gases the avalanche process leads to polymerisation of the quench molecules, and eventual contamination of the anode wires [89,90,91]. Low charge levels therefore enhance the counter lifetime.

- High counter gains employed to reduce the electronic noise contribution to position resolution can be higher than those giving optimum energy resolution [12,53], and can lead to loss of energy proportionality.

- Higher charge levels imply greater space charge of positive ions. Space charge can cause a dependence of pulse height on count rate.

- The position resolution degrading effect of secondary avalanches increases with charge level [52,92].

Operation at low charge levels is obviously desirable, so the onus must be to minimise the electronic noise. This was the motive for the development of many of the readout systems mentioned in the rest of this chapter. In general the requirements, not all mutually compatible, are:

- The use of low noise pre-amplifiers

- Minimisation of the capacitive loading on the pre-amplifiers

- The avoidance of resistance from the pre-amplifier inputs to signal ground

- The use of signals from as few pre-amplifier outputs as possible in the evaluation of $Q$.

These requirements must also contend with considerations of linearity and sensitive area in the design of an MWPC.
6.3 Transmission line methods for position readout

6.3.1 The RC line

The use of resistive charge division for event localisation was a well established technique in solid state detectors (well summarised by Owen and Awcock [93]) before the advent of the imaging MWPC. Kuhlmann et al. [94] applied the method to a single wire proportional counter, attaching voltage amplifiers to both ends of the resistive anode wire and forming the position signal $A/(A+B)$. The need to divide pulse heights was avoided by Borkowski and Kopp [95]. The inherent shunt capacity of the resistive electrode meant that it could be treated as an RC transmission line, with the propagation distance obtainable from the risetime of the charge appearing on terminating capacitors. The approach was soon extended to read out position from MWPC anodes [96] and cathodes [97].

The relative simplicity in implementation of resistive lines, and the fine sampling possible (continuous with resistive electrodes, or at the electrode pitch when the electrodes are linked by resistors) ensured a continued interest in the method [23,99-101]. However Gilvin et al. [85] and Sims [61] showed that RC lines used to read out MWPC cathodes are subject to some unusual position responses when the characteristic time of the line, $RC_p/\pi^2$, is not long compared to the input signal risetime. They should therefore be used with caution in MWPCs.

Fig. 6.2 illustrates the electronic arrangements for an RC line in timing (Fig. 6.2(a)) and pulse division (Fig. 6.2(b)) modes. Fraser et al. [102,103] showed that the latter offers a resolution
Fig. 6.2 Electronic arrangements for position readout from an RC line.
(a) Timing method
(b) Pulse height ratio method.
Fig. 6.3 Experimental uniform irradiation response showing the differential non-linearity of the 250kA RC line cathode.
advantage. Linearity and resolution can be independently optimised for a given total line resistance and shunt capacity by suitable choice of processing filter time constant. However, in all practical cases the thermal noise generated in the resistive line dominates the electronic resolution of RC line systems, and is their major drawback.

A 1mm pitch cathode with its 90 wires attached to a 250kΩ resistive line was used as a linearity standard in the present work. The resistive strip had been etched to produce equal interwire resistances, and allied with the fine sampling of the induced charge distribution, this produced excellent linearity. This is revealed in the uniform irradiation response (Fig. 6.3).

6.3.2 The CR line

Node: 0 1 .... n .... N

Fig. 6.4 Schematic representation of a CR line. Electrode strips or wire groups, shunt capacity $C_a$, are connected to the N+1 nodes. The N longitudinal capacitors $C_n$ generally comprise inherent, $C_{ni}$, and added, $C_{na}$, capacitance. The position output is formed as a pulse height ratio, as in Fig. 6.2(b).

By comparison with the other transmission line methods, the CR line (Fig. 6.4) has received little attention as an MWPC readout
Fig. 6.5 Experimental uniform irradiation response showing the differential non-linearity of the 22 node CR line cathode.
system. However Gilvin et al. [104] showed that it could achieve improved noise performance over RC lines in small chambers, at the expense of linearity and some physical clumsiness. Position information is contained only in the size of the charges delivered to the output nodes, as the propagation is essentially instantaneous. This means, however, that the problems that can arise in both RC and LC (delay) lines when the pulse development time is not short compared to the propagation time in the line are avoided.

The CR line described by Gilvin [43] was used to obtain the uniform irradiation response in Fig. 6.5. The wires of a 1mm pitch cathode were grouped in fours, and the added components $C_{\text{a}}$ and $R_L$ were 660pF and nominally 100MΩ respectively. Note that the sampling of the charge distribution is fine enough to prevent noticeable modulation in $G(Q)$. The overall bowed shape of the response is characteristic of uniform CR lines, arising because of the finite value of $C_{\text{a}}/C_{\text{a}}$. The proportion of signal charge lost to earth depends on how far along the line it must propagate. This non-linearity can be reduced, at the cost of increased capacitive loading on the pre-amplifiers, by increasing $C_{\text{a}}$, or eliminated, as shown in Chapter 7, by removing the requirement that $C_{\text{a}}$ is constant.

6.3.3 The LC line

The LC delay line (illustrated in lumped parameter form in Fig. 6.6) is the line method most widely applied to MWPC position readout [69,82,105-113], although it is probably also the most difficult from which to obtain optimum performance. The many factors which require consideration, including for example coupling signals to the line, the
Fig. 6.6 Schematic representation of a lumped parameter LC line. Charge can be coupled to the nodes directly or capacitively. The capacities $C_i$ to earth at the nodes generally comprise inherent shunt capacity plus added discrete components. Lines made up solely of the inherent distributed inductance and capacitance of an electrode arrangement are also possible. Position output is formed by timing methods, as in Fig. 6.2(a).

Impedance, delay and risetime of the line, line termination and signal processing, have been well summarised by Boie et al. [113]. These authors also show that good differential non-linearity ($\epsilon < 2\%$) and high spatial resolution (70 $\mu$m FHWM for 5 to 8keV X-rays in xenon mixtures at one atmosphere) may be achieved in a carefully designed detector.

6.4 Centroid finding methods

6.4.1 Digital centroid finding

If a signal processing chain is dedicated to each individual cathode strip or wire group, then it becomes possible to compute the centroid of the induced charge distribution directly from the suite of signals. Thus
Fig. 6.7 Schematic electronic arrangement for a portion of a cathode with digital centroid finding position readout.
where $q_i$ are the charge samples and $x_i$ are the readout node positions.

This approach was developed at CERN [114,115]; a typical arrangement is shown in Fig. 6.7. The comparatively slow computer calculation of the centroid was later implemented in hardware [116], and very small electronic noise contributions obtained (typically $10 \mu$m FWHM at normal chamber gains). The cost, of course, is the considerable electronic complexity.

By suitable choice of a discrimination level, only those channels contributing a significant signal need be included in the evaluation of $Q$ [68]:

$$Q = \frac{\sum q_i x_i}{\sum q_i}, \quad (q_i - B) > 0$$

where $B$ is a bias level expressed in terms of charge. Thus in contrast to "global" methods where there are only two readout nodes per sensing axis and the resolution degrades with increasing area, the sensing length may be increased, by introducing extra channels, with no loss of resolution. Systematic non-linearity occurs if the sampling strips are too broad with respect to the charge distribution, or if too few samples are included in the algorithm. This non-linearity can therefore be made arbitrarily small at the expense of electronic noise.

### 6.4.2 The centroid-finding filter

Local readout methods such as described in the previous section can suffer from discontinuities in output if the fiducial points $x_i$ are introduced independently into the centroid finding process.
Fig. 6.8 Schematic electronic arrangement for a portion of a cathode using the centroid-finding filter for position readout.
Radeka and Boie [117] described a centroid finding system that combines the attributes of local and global methods to avoid this problem, while retaining the independence of resolution from sensing length. The system was used briefly during the present work (Chapter 9). The charge samples, after filtering, are stretched and held, then transferred to the time domain by a fast sequential switch (Fig. 6.8).

The sampled charge distribution may at this stage therefore be viewed on an oscilloscope (Fig. 6.9). This waveform is passed through a linear filter which has the property of providing a measure of the centroid in the zero crossing of its output. If the sequential switch and the time digitiser that gives the final position output are started together and remain synchronous, the output discontinuities are avoided. The number of charge samples included in the evaluation of Q is simply adjusted by altering the switch rate, so that the required number fall within the width of the filter weighting function. In order to improve the sampling of the charge distribution without increasing the number of amplifier channels, the method has been augmented by resistive [24] and capacitive [118] charge division.

Fig. 6.10 shows a uniform irradiation response which was not made under the standard conditions of Section 6.1.5, but which illustrates well the effect of using cathode strips which are too broad with respect to the width of the induced charge distribution. The ratio of strip centre spacing, w, to h (for simplicity treating the anode-cathode spacing as the parameter having most direct effect on the width of the distribution) was 1.2, whereas for the CR line which gave rise to Fig. 6.5, w/h was 0.7. Gatti et al. [119] have shown that this non-linearity can be cancelled out while still in the time domain by suitably arranging the response of the centroid finding filter circuit.
Fig. 6.9 Output of the sequential switch of a centroid-finding filter system. The detector was illuminated by a collimated source.

Fig. 6.10 Uniform irradiation response from a Brookhaven small detector (see Chapter 9) with the centroid-finding filter, showing the effect of physical undersampling of the induced charge distribution by a strip cathode. $\epsilon \simeq 6\%$. 
6.5 Progressive geometry methods

6.5.1 The "backgammon" electrode

The simplest way of obtaining a position signal from a distribution of induced charge is to arrange that the charge is induced on electrodes whose area in some strip $\delta x$ increases or decreases continuously with $x$. Allemand and Thomas [120] described an MWPC cathode divided into two electrically isolated components of equal area as illustrated in Fig. 6.11. $Q$ is simply formed as $B/(A+B)$ where $A$ and $B$ are signals from the two components.

*Fig. 6.11 The backgammon electrode. The white and black areas represent the two electrode components, electrically isolated from each other.*
The design is related to that of the "wedge and strip" electrode [121,122] which provides two-dimensional readout from one plane through the use of three or more components, and is thus particularly suited to microchannel plate or parallel plate proportional counter readout [123].

It should be noted that there is once again a compromise between linearity and resolution. The pattern repetition period must be small compared to the charge distribution for constant sensitivity, but decreasing this period increases the capacitive load presented by the electrode, and therefore the noise.

6.5.2 The graded density electrode

Obviously, one of the backgammon cathodes mentioned above would need to be transparent to radiation for two-dimensional imaging in an MWPC. This would present fabrication difficulties. Also, it would not be possible to drift electrons into the detector proper from a separate absorption region. The same would apply to a wedge and strip system unless about half the signal charge was unused. These problems were overcome by Mathieson et al. [124,125] with a simple and elegant adaptation of normal constant pitch wire cathodes. As shown schematically in Fig. 6.12, the wires are electrically connected into two groups, or components, A and B, such that the linear density of A, \( \nu_A \), decreases with \( x \), while the linear density of B, \( \nu_B \), increases correspondingly.
As it is only ever approximately possible to have the linear densities varying linearly with $x$ for wire arrays of constant, finite pitch, we write

$$\mathcal{V}_A \approx N \left( \tfrac{1}{2} - a(x - \tfrac{1}{2}) \right)$$ \hspace{1cm} \text{Eq. 6.4}$$

$$\mathcal{V}_B \approx N \left( \tfrac{1}{2} + a(x - \tfrac{1}{2}) \right)$$ \hspace{1cm} \text{Eq. 6.5}$$

where $N$ is the total number of wires and $a$ defines the rate of change of $\mathcal{V}_A$ and $\mathcal{V}_B$. Provided that the induced charge distribution is wide compared to the wire pitch, but not wide compared to the total sensing length, a position signal may be obtained from the two node charges.
Fig. 6.13 Experimental uniform irradiation response showing the differential non-linearity of the prototype 1mm pitch single section graded density cathode.
Thus $S = dQ/dx \approx a$. For practical electrodes with a finite number of wires at constant pitch, it is impossible to have $a = 1.0$ for all $x$, so non-linearity will result. This is illustrated in Fig. 6.13 which shows the poor differential non-linearity of a graded density (GD) cathode having 90 wires at 1mm pitch, arranged as in Table 6.1.

Table 6.1 Wire connections for the prototype single section GD cathode

<table>
<thead>
<tr>
<th>Wire connections</th>
<th>Number of wire in RH column</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0 0 0 0 0 0 0</td>
<td>10</td>
</tr>
<tr>
<td>0 0 0 1 0 0 0 1 0 1 0</td>
<td>20</td>
</tr>
<tr>
<td>0 0 0 1 0 0 1 0 0 0 0</td>
<td>30</td>
</tr>
<tr>
<td>1 0 1 0 0 1 0 1 0 0 0</td>
<td>40</td>
</tr>
<tr>
<td>1 0 1 0 1 0 1 0 1 0 0</td>
<td>50</td>
</tr>
<tr>
<td>1 1 0 1 0 1 1 0 1 0 0</td>
<td>60</td>
</tr>
<tr>
<td>1 1 1 0 1 1 0 1 1 1 1</td>
<td>70</td>
</tr>
<tr>
<td>1 1 1 0 1 1 1 0 1 1 1</td>
<td>80</td>
</tr>
<tr>
<td>1 1 1 1 1 1 1 1 1 1 1</td>
<td>90</td>
</tr>
</tbody>
</table>

The wires are connected to node 0 or 1 as indicated.

Such wire patterns are decided by integrating Eqs. 6.4 and 6.5 to give the numbers $n_A$ and $n_B$ of A and B wires that lie between 0 and $x$:

\[
\begin{align*}
n_A &= N/2 \left[ (1 + a)x - ax^2 \right] \\
n_B &= N/2 \left[ (1 - a)x + ax^2 \right]
\end{align*}
\]

Setting $a$ equal to unity and requiring that $n_A$ and $n_B$ are integers, we obtain provisional positions for A and B wires

\[
\begin{align*}
x_A &= 1 - (1 - 2n_A/N)^{1/2} \\
x_B &= (2n_B/N)^{1/2}
\end{align*}
\]
Since the constant pitch restraint actually defines the wire positions, the final grading is found by setting the wires closest to $x_A$ as A type and those closest to $x_B$ as B type.

Reduction of the wire pitch allows a to be made more constant. Differential non-linearity was measured at 7.8% for a 0.5mm pitch cathode in contrast to 18.0% for the 1mm pitch version of similar area. However, further reduction introduces problems, and so was not pursued in the present work. There are obvious fabrication difficulties. Because the wire radius cannot be reduced much below 5 to 10 $\mu$m, the array must become a more effective electrostatic screen as $s_c$ decreases, making the use of a drift region more difficult. The transparency to soft X-rays must also decrease. Most significantly, the intercomponent capacitance, $C\alpha$, will increase, roughly as $1/s_c \ln(s_c)$, so once more linearity would be gained at the expense of electronic noise. This capacitance, when it can no longer be neglected compared to the pre-amplifier dynamic input capacity, $C_{\text{pl}}$, also decreases the sensitivity by a factor $C_{\text{pl}}/(C_{\text{pl}} + 2C\alpha)$ [43].

6.5.3 The subdivided GD electrode

The problem of capacitive loading on the pre-amplifiers was addressed by Gilvin et al. [126] by subdivision of a GD cathode into two sections (referred to as a 2GD cathode in the present work). Charge induced on the central node (1) is coupled to the readout nodes (0 and 2) by the interwire capacitance. However for practical wire pitches, the ratio of internode longitudinal to node 1 shunt capacity, $C\alpha/C\alpha$, in this inherent capacitive line (Fig. 6.14) is not large, resulting in considerable overall bowing of the uniform irradiation
Node: 0 1 2

\[ C_1 = C_2 = C_{ni} = C_n \]

\[ C_d/2 \]

\[ C_d \]

\[ C_d/2 \]

**Fig. 6.14** Capacitance model for a 2GD electrode. The longitudinal capacitance in the capacitive line is comprised only of the inherent internode capacitance.

response. The cathode which produced the response of Fig. 6.15 had linear grading of its two sections, with \( s_e = 0.5\)mm and \( C_e/C_d \approx 38\)pF/16pF (h=6mm). This bowing could be compensated by adding longitudinal capacitance, but this would spoil the improvement in the capacitive load presented to the pre-amplifiers (a factor of 2.5 reduction was obtained).

Gilvin et al. instead used the possibility of adjusting the wire grading away from a linear one to compensate the overall non-linearity (Fig 6.16), thus preserving one of the major attributes of the GD system: its simplicity and compactness resulting from freedom from
Fig. 6.15 Experimental uniform irradiation response showing the differential non-linearity of the two section 0.5mm pitch GD cathode with linear wire grading.
Fig. 6.16 Schematic illustration of how the linear density of wires attached to each of the three nodes of a 2GD electrode is altered to compensate the non-linearity of the inherent CR line.

added components. As shown in Fig. 6.17, this results in a halving of differential non-linearity. However, there remains sufficient structure in the response that images can be noticeably distorted. This is well illustrated in Fig. 6.18, which shows a two-dimensional uniform irradiation response in isometric projection. The two axes were read out by a pair of 2GD cathodes with the same wire pattern as the one which gave rise to Fig. 6.17, installed in the Harwell detector body. The detector was illuminated normally by an iron-55 source, and the anode charge level was 0.4pC with P10 gas. The outputs of two Harwell 3769 systems were suitably arranged and presented to a two-dimensional analyser (a view of the electronics set-up was shown in Fig. 2.12). Serious modulation in the z axis at the anode wire pitch was suppressed by the long drift region (d₁ = d₂ = 30mm) (Chapter 8). The cathode wire pattern is given in Table 6.2; the regions which produce low sensitivity near the centre may be noted.
Fig. 6.17 Experimental uniform irradiation response showing the differential non-linearity of the two section 0.5 mm pitch GD cathode with wire grading arranged to compensate the inherent CR line non-linearity.
Fig. 6.18 Two-dimensional uniform irradiation response in isometric projection with compensated 2GD cathodes. h = 5.5mm. The image consists of 64x64 pixels, representing a range of Q of 0.25 to 0.75 in each axis. The circular outline is that of the detector window.

Table 6.2 Wire connections for the compensated 2GD cathode

<table>
<thead>
<tr>
<th>Wire connections</th>
<th>Number of wire in RH column</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 0 0 0 0 0 0 0 0 0</td>
<td>10</td>
</tr>
<tr>
<td>0 0 1 0 0 0 0 1 0 0</td>
<td>20</td>
</tr>
<tr>
<td>0 1 0 0 1 0 0 1 0 0</td>
<td>30</td>
</tr>
<tr>
<td>1 0 1 0 0 1 0 1 0 1</td>
<td>40</td>
</tr>
<tr>
<td>0 1 0 1 0 1 0 1 1 0</td>
<td>50</td>
</tr>
<tr>
<td>1 0 1 0 1 0 1 1 0 1</td>
<td>60</td>
</tr>
<tr>
<td>0 1 1 0 1 1 0 1 1 1</td>
<td>70</td>
</tr>
<tr>
<td>1 1 0 1 1 1 1 0 1 1</td>
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<tr>
<td>1 2 1 1 1 1 2 1 1 1</td>
<td>120</td>
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<tr>
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<tr>
<td>2 1 1 2 1 2 1 2 1 2</td>
<td>150</td>
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<tr>
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<td>160</td>
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<tr>
<td>2 2 1 2 2 1 2 2 1 2</td>
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<td>180</td>
</tr>
<tr>
<td>2 2 2 2 2 2 2 2 2 2</td>
<td>190</td>
</tr>
</tbody>
</table>

The wires are connected to node 0, 1 or 2 as indicated.
Fig. 6.19 Experimental uniform irradiation response showing the differential non-linearity of the compensated two section 0.5mm pitch GD cathode when $h = 8\text{mm}$.
The linearity of all progressive geometry systems can of course be improved by increasing the width of the induced charge distribution. Fig. 6.19 shows that a factor of two improvement in differential non-linearity over that of Fig. 6.17 may be achieved by increasing \( h \) from 6 to 8 mm. This approach requires higher operating voltages, however, and the useful sensing length of the cathode becomes smaller because of loss of charge from the broader distribution over the ends.

6.6 Grouped wire methods

Kitamoto [127,128] introduced a readout arrangement in which the number of processing channels increases more slowly with detector size than direct centroid finding methods. Wire groups or strips are attached to two sets of pre-amplifiers such that an unambiguous coarse position is obtained (Fig. 6.20). The appropriate charge samples must then be ordered correctly and the centroid computed. A detailed study of linearity was not presented, but the principles should be similar to those applying to the direct centroid approach.

Still more computation to obtain \( Q \) than is required in the Kitamoto method is needed for the grouped wire method described by van Beek et al. [129] in which wires are arranged in coarse and fine sections. Quoted non-linearities are poor (\( \epsilon = 15 \) to \( 30\% \)), but the method does allow the width of the charge distribution to be assessed, as well as its centroid, which could prove useful for background discrimination.
Fig. 6.20 Schematic representation of the grouped wire position readout arrangement of Kitamoto [127,128]. The sensing elements could also be cathode strips.
6.7 Testing the prediction of differential non-linearity

Highly non-linear cathode systems such as the 1mm pitch single section GD cathode provide a severe test of the accuracy of theoretical induced charge distributions. Figs. 6.21 to 6.23 show attempts to predict the response shown earlier as Fig. 6.13 using the methods described in Chapters 3 and 5, and Section 6.1. For Fig. 6.21, the relevant value of the parameter \( K_3 \) was obtained from Fig. 5.7 and the resulting distribution used in the calculation of \( G(Q) \). As can be seen, the general arrangement of features in the experimental response is well reproduced, but the empirical distribution appears to be slightly too narrow. The discrepancy would be insignificant in systems with small non-linearity, or with only slowly changing sensitivity.

For Fig. 6.22, \( \Omega_1 \) was calculated for a fan of five positive charges at the position they had reached at \( t = 4 \mu s \), corresponding to the time to peak of the filter used. The calculation is only possible at present for continuous cathodes. A suitable value of \( \sigma, 68^\circ \), was chosen using Fig. 4.6, and \( \alpha \) was chosen as \(-\pi/2\), as these type of events would predominate in the experimental set-up (Section 6.1.5).

Finally, a full calculation of the distribution on a wire cathode was made, with all parameters chosen to match those of the experimental situation. Although the calculation was necessarily of the \( \Omega_2 \) type, the good prediction of the uniform irradiation response (Fig. 6.23) may be taken as a further indication of the similarity of \( \Omega_1 \) and \( \Omega_2 \) for normal geometries.
Fig. 6.21 Uniform irradiation response of the single section 1mm pitch GD cathode. Experimental measurement (points) and theoretical prediction (continuous curve). A single parameter Gatti type cathode charge distribution, obtained from Fig. 5.7, was used for the prediction.
Fig. 6.22 Uniform irradiation response of the single section 1mm pitch GD cathode. Experimental measurement (points) and theoretical prediction (continuous curve). An instantaneous continuous cathode charge distribution was used for the prediction.
Fig. 6.23 Uniform irradiation response of the single section 1mm pitch GD cathode. Experimental measurement (points) and theoretical prediction (continuous curve). A $\phi_2$ wire cathode charge distribution was used for the prediction.
THE CR-GD CATHODE SYSTEM

7.1 The requirement for a new position-sensitive cathode

A consideration of the MWPC readout systems described in the previous chapter suggests that a charge position encoder combining the attributes of simplicity and low cost, good position resolution, and good linearity may prove a useful addition. Such a system was produced in the present work by combining the GD principle with a non-uniform "tailored" CR line. The number of added components was reduced by comparison to the standard CR line, and complete cathode systems were built without recourse to special or difficult techniques.

7.2 The use of GD sections to reduce the number of nodes in position-sensitive cathode systems

Most cathode systems benefit in reduced complexity and cost if the number of nodes at which charge is coupled to pre-amplifiers or transmission lines can be kept to a minimum. There may also be a noise advantage if capacitive loading of the pre-amplifier is small. However, as mentioned in the last chapter, if sensing strips, or wire groups, are too wide with respect to the induced charge distribution, then a periodic non-linearity results. For the purposes of illustration, a cathode with 96 wires at 1mm pitch arranged in seven groups of 12 and two outer groups of 6 was built. It was read out by a uniform CR line with the parameters given in Table 7.1
Table 7.1 CR line parameters

\[
\begin{align*}
\text{C}_{ni} &= 3 \text{pF} \\
\text{C}_{na} &= 133 \text{pF} \quad \text{C}_n = 136 \text{pF} \quad \text{(constant)} \\
\text{C}_D &= 30 \text{pF} \\
\text{R}_L &= 100 \text{M\Omega} \quad \text{(DC)}
\end{align*}
\]

The cathode gave the uniform irradiation response shown in Fig. 7.1, with low and high sensitivity regions corresponding to the centres and edges of the wire groups respectively, superimposed on the bowed response of the CR line. With \( h = 6 \text{mm} \), the crude linearity parameter \( w/h \) (Section 6.4.2) was 2.0. Effective elimination of the periodic modulation simply by reducing the sensing strip width would require \( w/h \) of the order 0.8 ([113], Figs. 6.5 and 6.10), more than doubling the number of nodes. However, by applying the graded density method, the sampling may be improved with no increase in the number of nodes. As long as the response of each resulting section is linear, then the centroid of the induced charge distribution will be given by the centroid of the node charges, so the systematic error is removed. Table 7.2 indicates the wire pattern in a portion of the test cathode before and after a simple, approximately linear grading was introduced. In the graded arrangement, the A wires of the first section shown are connected to node \( j \), the B wires to node \( k \). The A wires of the following section are also connected to node \( k \), and the B wires to node \( l \). The wire pattern was determined as described in Section 6.5.2.
Fig. 7.1 Experimental uniform irradiation response showing the differential non-linearity of the 9 node uniform CR line cathode.
Table 7.2 Wire grading for a 12 wire section

| Ungraded | ...jjjjjkkkkkkl111111... |
| Graded   | ...jjkjjkkkkklkkll111... |

GD section GD section

Fig. 7.2 reveals the resulting improvement in linearity (constructional details are given in Section 7.4.2). The technique was arrived at as an extension of the subdivision of GD cathodes. It should be noted that Gatti et al. [130] have proposed interweaving just the outer pair of adjacent wire groups to reduce the number of readout nodes (Table 7.3), and that this may be considered a limiting case of the present approach.

Table 7.3 The interwoven wire arrangement of Gatti et al.

...jjkjjkkkl111...

GD sections should also be applicable in reducing the number of nodes in resistive line and centroid finding methods. There is of course an increase in internode capacity, and this may preclude application to LC delay lines [113].
Fig. 7.2 Experimental uniform irradiation response showing the differential non-linearity of the 8 section CR-GD cathode with a uniform CR line.
7.3 The non-uniform capacitive line

7.3.1 Removal of CR line non-linearity

The remaining non-linearity evident in Fig. 7.2 can be removed essentially completely if the longitudinal capacitance values, $C_n$, may be chosen independently of each other. Consider the schematic representation of a cathode with GD sections and capacitive line readout shown in Fig. 7.3(a), and the capacitance model of a single section, Fig. 7.3(b). There are $N$ sections and $N+1$ nodes. The B component wires of section $n$ and the A component wires of section $n+1$ are connected to the $n$th node. The longitudinal capacitances can be expressed in terms of the constant section shunt capacities (equal to the shunt capacities of nodes 1 to $N-1$), $C_d$

$$C_n = f_n C_d$$  \text{Eq. 7.1}

![Diagram](attachment:diagram.png)

Fig. 7.3 (a) Representation of a CR-GD electrode  
(b) Capacitance model of a single section.
A position signal may be formed from the end node charges as $Q = q_{n}/(q_{n} + q_{N})$. Non-linearity in $Q$ (the rms difference between the actual position of the node at which charge is injected, and $Q$) may be minimised by suitable choice of the factors $f_{n}$, and thus $C_{n}$ for known $C_{A}$.

7.3.2 Calculation of the end node charges

Unlike the uniform capacitive line, an analytical treatment is impossible for the non-uniform line. For finite length lines of practical interest, however, recurrence relationships may be established which are suitable for numerical handling. Suppose that the right hand terminals of the individual section shown in Fig. 7.3(b) are loaded by a capacitance $C_{n}^{r}$, where the superscript $r$ indicates capacitance looking right. Then the capacitance looking into the left hand terminals is given by

$$
\frac{C_{n-1}}{C_{A}} = \frac{\gamma_{n} + f_{n} + (C_{n}^{r}/C_{A})(f_{n} + \frac{1}{2})}{\gamma_{n} + f_{n} + (C_{n}^{r}/C_{A})} \quad \text{Eq. 7.2}
$$

But we know that $C_{n}^{r}$ is very large, as this is the input of a charge sensitive pre-amplifier. Thus taking $C_{n}^{r} = \infty$, Eq. 7.2 can be used to obtain $C_{n}^{r}$ for successive nodes n to the left. The charge on $C_{n}$, $(V_{n-1} - V_{n})C_{n}$, where $V_{n-1}$ and $V_{n}$ are the voltages on nodes $n-1$ and $n$, must come from $C_{A}/2$ and $C_{n}^{r}$, so we have

$$
\frac{V_{n}}{V_{n-1}} = \frac{f_{n}}{C_{n}^{r}/C_{A} + f_{n} + \frac{1}{2}} \quad \text{Eq. 7.3}
$$

This recurrence formula can be used to determine, in terms of $V_{n-1}$, the voltages at successive nodes to the right. The final output charge,
\[ q_N, \text{ at node } N, \text{ is developed across } C_N \text{ (node } N \text{ is attached to a virtual earth) so that} \]

\[ q_N = V_{N-1} f_N C_d \]

Identical arguments looking left along the line yield the following formulae, with the superscript 1 indicating capacitance looking left

\[ \frac{C^l_n}{C_d} = \frac{\gamma_n + f_n + (C^l_{n-1}/C_d)(f_n + \frac{1}{2})}{\gamma_2 + f_n + (C^l_{n-1}/C_d)} \]

Eq. 7.5

and

\[ \frac{V_{n-1}}{V_n} = \frac{f_n}{C^l_{n-1}/C_d + f_n + \frac{1}{2}} \]

Eq. 7.6

We take \( C_\infty = \infty \), so that values of \( C^l_n \) can be obtained from Eq. 7.5, and the output charge is

\[ q_\infty = V_1 f_1 C_d \]

Eq. 7.7

If charge \( q_p \) is injected at the \( m \)th node, the voltage on this node will be

\[ V_m = q_p / (C^l_m + C^r_m) \]

Use of the recurrence formulae Eqs. 7.3 and 7.6, together with Eqs. 7.4 and 7.7 then allows calculation of \( q_\infty \) and \( q_N \).

It should be noted that the above treatment is not the only one for the non-uniform capacitive line. An alternative is to relate the node charges and potentials by means coefficients of capacitance and inductance

\[ q_i = \sum_{j=0}^{N} c_{ij} v_j \quad i = 0, 1, \ldots N \]
It can be shown that \( c_{ij} \) is always positive, \( c_{ij}, (i \neq j) \) is always negative and that \( c_{ij} = c_{ji} \). Because nodes 0 and \( N \) are connected to charge sensitive pre-amplifiers,

\[
C_{oo} = \infty \quad C_{NN} = \infty
\]

Fig. 7.3(b) and Eq.7.1 show that

\[
C_{mm} = (1 + f_m + f_{m+1}) C_d \quad 1 \leq m \leq N - 1
\]

\[
C_{m(m+1)} = -f_m C_d = C_{(m+1)N}
\]

with all other \( c_{ij} \) zero. If charge \( q_p \) is injected at node \( m \), then the \( N-1 \) unknown voltages \( V_j \) (we know \( V_0 = V_N = 0 \)) can be found from

\[
q_i = \sum_{j=1}^{N-1} c_{ij} V_j \quad i = 1, 2 \ldots N-1
\]

where \( q_i = 0 \) for \( i \neq m \) and \( q_m = q_p \). Knowing the vector \( V_j \), the end node charges are given by

\[
q_o = \sum_{j=1}^{N-1} c_{0j} V_j
\]

and

\[
q_N = \sum_{j=1}^{N-1} c_{Nj} V_j
\]

The approach gives numerically identical answers to the one using recurrence formulae, but has the advantage of being able to deal conveniently with capacitance between non-adjacent nodes. It was found that even quite high values of "second neighbour" capacitance, up to 20% of that between adjacent nodes, had minimal effect on predicted cathode response.

7.3.3 Calculation of longitudinal capacitances

We are now able to calculate \( Q \) for charge injected at any node on the line, with \( f_n \) as adjustable parameters. Using standard library routines, we can minimise non-linearity in \( Q \) by choice of \( f_n \). In order
that the trivial solution (all $f_n \to \infty$) is not returned, suitable constraints must be applied. For the CR lines developed in the present work, these were

(i) The capacitive loading presented by the cathode and line at each end node should not exceed the total cathode shunt capacitance, $C_P$

(ii) For charge $q_p$ injected at the central node, the total charge, $q_o + q_m$, appearing at the end nodes should not be less than $0.85q_p$.

These constraints ensured that the linear performance was not gained at the expense of a significant loss of position resolution.

In order to demonstrate the effect of "tailoring" a capacitive line in isolation, the crude CR line cathode without GD sections described in Section 7.2 was modified. Details of the calculated factors $f_n$ and the actual capacitances are given in Table 7.4.

<table>
<thead>
<tr>
<th>Internode</th>
<th>$f_n$</th>
<th>Required $C_n$</th>
<th>Actual $C_n$ ($C_{lh} + C_{na}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,8</td>
<td>37.98</td>
<td>121.5pF</td>
<td>121.3pF</td>
</tr>
<tr>
<td>2,7</td>
<td>44.29</td>
<td>141.7pF</td>
<td>142.1pF</td>
</tr>
<tr>
<td>3,6</td>
<td>47.99</td>
<td>153.6pF</td>
<td>153.1pF</td>
</tr>
<tr>
<td>4,5</td>
<td>50.00</td>
<td>160.0pF</td>
<td>159.9pF</td>
</tr>
</tbody>
</table>

$C_p = 30pF \quad C_{lh} = 31pF$

$C_a = 3.2pF \quad C_{na} = 3pF$

The resulting uniform irradiation response is shown in Fig. 7.4; the bowing seen in Fig. 7.1 has been removed.
Fig. 7.4 Experimental uniform irradiation response showing the differential non-linearity of the 9 node tailored CR line cathode.
7.4 Construction of tailored CR-GD cathodes

7.4.1 General features

A number of CR-GD cathodes for use in the Leicester and Harwell detector bodies were constructed during the present work. All had 96 copper 2% beryllium wires of 50μm diameter wound at 1mm pitch. The wire plane areas were 96 x 10^4 mm^2. The first example was arranged in six GD sections ("6CR-GD") of 16 wires. Subsequent cathodes were made with eight sections ("8CR-GD"), representing the culminating stage of the development illustrated by the cathode systems described earlier in this chapter.

7.4.2 Wire connections

A method was devised for making the appropriate wire connections simply by hand. The wire array was first wound and attached to the frame following the first technique described in Section 2.2.3. A thin smooth rod was then threaded through the wires where they had been left protruding beyond the frame at one end. The rod was passed below the wires which were to be connected to odd-numbered nodes, and above those which were to be connected to even nodes. A narrow strip of double-sided copper clad circuit board could then also be threaded through the wires and affixed to the frame with epoxy resin. The wires were soldered to the strip of circuit board, on which a suitable pattern of nodes had been produced by chemical etching. Fig. 7.5 shows the copper pattern on the strip for an eight section cathode. The attachment points for the discrete components are evident.
Upper surface

<table>
<thead>
<tr>
<th>Node:0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wires connected along this edge</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Lower surface

Fig. 7.5 Copper pattern on the wire connection strip for an 8CR-GD cathode. The dark areas are where the copper was left after etching.

7.4.3 Addition of components

With the factors $f_n$ calculated as described above for the appropriate number of sections, it remained to measure the total cathode shunt capacity, $C_{D=NCd}$, and the inherent internode capacities, $C_{n\bar{n}}$, before suitable capacitors, $C_{n\alpha}$, could be added. Having made the wire connections, the half-completed cathode was installed in a detector body, with the second cathode and anode arrays present and grounded.

To measure the total shunt capacity, the nodes were simply shorted together and the capacity to ground measured with a capacitance bridge. Obviously the result depended upon the particular geometry. Table 7.5 shows the effect of alterations in geometry on the shunt capacity of some grids. The anode and second cathode were fixed at 2mm and 1mm pitch respectively, and the anode wire plane area was $90 \times 90mm^2$. 


Table 7.5 The effect of detector geometry on grid shunt capacity

<table>
<thead>
<tr>
<th>Cathode area (mm²)</th>
<th>$s_c$ (mm)</th>
<th>Wire orientation with respect to anode wires</th>
<th>$h$ (mm)</th>
<th>$d_2$ (mm)</th>
<th>$d_1$ (mm)</th>
<th>$C_p$ (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 x 90</td>
<td>1.0</td>
<td>Orthogonal</td>
<td>4</td>
<td>9</td>
<td>30</td>
<td>25.6</td>
</tr>
<tr>
<td>90 x 90</td>
<td>1.0</td>
<td>Parallel</td>
<td>4</td>
<td>9</td>
<td>30</td>
<td>28.0</td>
</tr>
<tr>
<td>90 x 90</td>
<td>1.0</td>
<td>Orthogonal</td>
<td>6</td>
<td>9</td>
<td>26</td>
<td>21.9</td>
</tr>
<tr>
<td>95 x 100</td>
<td>0.5</td>
<td>Orthogonal</td>
<td>4</td>
<td>9</td>
<td>30</td>
<td>35.8</td>
</tr>
</tbody>
</table>

For the half completed CR-GD cathodes, the measurement was made with the wires orthogonal to those of the anode and $h = 6$ mm, yielding values around 30pF. Obviously the addition of components will in itself alter the shunt capacity, and a process of iteration might seem necessary. However, as will be seen, it was found that the overall linearity was not greatly affected by moderate changes in $C_p$ (changes for example of the size produced by the alteration of $h$ in Table 7.5 had little effect).

Inherent longitudinal capacity, $C_{ni}$, was measured by connecting the probe of the bridge to one node, and finding the capacity to ground with all other nodes and electrodes earthed. As shown in Fig. 7.6, this resulted in values of $(C_{ni} + C_{mni} + C_{A})$ measured at node $n$, and $(C_{ni} + C_{A}/2)$ and $(C_{ni} + C_{A}/2)$ for the two end nodes, with $C_A = C_p/N$. In practice it was found that the shunt capacity of the two end nodes was somewhat greater than $C_A/2$ (due perhaps to the detail of the copper pattern on the strip, and the nearby detector walls), but consistent values of $C_{ni}$ were obtained when this was allowed for. Typical values for cathodes with wires arranged as in Table 7.2 were $C_{ni} = C_{pi} = 6.8$ pF and $C_{A} = C_{2i} = ... C_{7i} = 6.9$ pF.
Fig. 7.6 Measurement of inherent internode capacity.

Miniature capacitors were soldered across the nodes to satisfy the condition

\[( C_{ni} + C_{n\alpha} ) = f_n \cdot C_d \]

with the practical limitation that no more than three preferred values (with typical tolerance +/-5%) were used in parallel between any pair of nodes. Miniature ceramic plate capacitors were used initially because of their size, low cost and easy availability, and gave good performance. A later cathode demonstrated the use of ceramic chip capacitors, which are very compact (typically 2x1.5x1.5mm³ for 100pF), and ultimately offer the possibility of being precisely trimmed. In order to achieve the best possible linearity with the available resources, however, precision capacitive trimmers with a range of 2 to 10pF were added between the central seven nodes of one 8CR-GD cathode.

An individual resistor was connected between each node and a common line which was raised to the required cathode bias voltage in operation. As described in Section 2.2.4, there are two conflicting considerations in choosing the value, \( R_L \), of these resistors:
(i) It should be as large as possible at typical signal processing frequencies to prevent undue thermal noise.

(ii) It should be low enough that the leakage time for collected charge ($\sim C_p/(N/R_l)$) is not very long compared to the average time between counts. If this is not the case, then the potential of nodes collecting charge would be modified, and field distortion leading to degradation of imaging performance would be expected. Note that this only applies if the electrode actually collects positive ions; systems could be devised where this might be avoided.

Since high count rates were not available in the present work, the highest value resistors (100M$\Omega$ nominal at DC) of suitably compact dimensions were employed. This implied $(N/R_l)/C_p \sim 2.7 \times 10^3 \text{ s}^{-1}$ for the 8CR-GD systems. No ill effects were observed with average rates up to this order.

7.5 The six section CR-GD cathode (6CR-GD)

The grading pattern employed in each of the six sections of sixteen wires in the first CR-GD cathode constructed is shown in Table 7.6.

Table 7.6 Wire grading for a 16 wire section

...jjjkkjjjjkkkk...  
\underline{one GD section}

At the time when the capacitance measurements were made on this grid, the fact that there was extra capacity on the end nodes was not
Fig. 7.7 Experimental uniform irradiation response showing the differential non-linearity of the six section CR-GD cathode.
appreciated. This is why the fitted values $C_{na}$, and thus the actual values $C_n$ shown in Table 7.7 were somewhat too large.

Table 7.7 CR line parameters for the 6CR-GD cathode

<table>
<thead>
<tr>
<th>Internode</th>
<th>$f_n$</th>
<th>Required $C_n$</th>
<th>Actual $C_n$ ($C_{ai} + C_{na}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,6</td>
<td>6.83</td>
<td>32.1pF</td>
<td>34pF</td>
</tr>
<tr>
<td>2,5</td>
<td>10.17</td>
<td>47.8pF</td>
<td>52pF</td>
</tr>
<tr>
<td>3,4</td>
<td>12.06</td>
<td>56.7pF</td>
<td>59pF</td>
</tr>
</tbody>
</table>

$C_d = 31.5pF$

$C_{ai} = 4.7pF$ \hspace{1cm} $C_{na} = 12pF$

As may be seen from the uniform irradiation response (Fig. 7.7), however, the non-linear response of the individual GD sections was contributing more to the total non-linearity than any inaccuracy in the non-uniform CR line. As calculations using the Gatti induced charge formula (Eq. 5.11) had indicated that superior linearity would be available with an eight section CR-GD cathode, further effort was concentrated there.

7.6 Eight section CR-GD cathodes (8CR-GD)

7.6.1 Linearity

The twelve wires in each section of the 8CR-GD cathodes were arranged as previously shown in Table 7.2. The full electrical arrangement of the first example built, including capacitive trimmers,
is shown in Fig. 7.8. The combinations of fixed and variable capacitors were selected to embrace the theoretically required values.

Fine tuning of the differential non-linearity was then achieved by making small alterations to the trimmer settings, and observing the effect on the uniform irradiation response. An increase in a given $C_n$ by $\theta \%$ was found to produce a localised relative increase in $G(Q)$ of approximately $2\theta \%$.

The response that was obtained after four sets of trimmer adjustments is shown in Fig. 7.9. It should be compared with Figs. 7.1, 7.2 and 7.4, demonstrating how the introduction of GD sections and the tailored CR line together produced excellent linearity. An integral linearity figure of $0.06\%$ of the line length was measured by moving a collimated source in $2\text{mm}$ steps over the central two thirds of the sensing length. The final values $C_n$ were measured, and are included in Table 7.8.

Table 7.8 CR line parameters for the trimmed 8CR-GD cathode

<table>
<thead>
<tr>
<th>Internode</th>
<th>$f_n$</th>
<th>Required $C_n$</th>
<th>Actual $C_n$ ($C_{ni}+C_{n\alpha}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.98</td>
<td>117.7pF</td>
<td>118.5pF</td>
</tr>
<tr>
<td>8</td>
<td>37.98</td>
<td>117.7pF</td>
<td>118.0pF</td>
</tr>
<tr>
<td>2</td>
<td>44.29</td>
<td>137.3pF</td>
<td>136.5pF</td>
</tr>
<tr>
<td>7</td>
<td>44.29</td>
<td>137.3pF</td>
<td>137.0pF</td>
</tr>
<tr>
<td>3</td>
<td>47.99</td>
<td>148.8pF</td>
<td>148.9pF</td>
</tr>
<tr>
<td>6</td>
<td>47.99</td>
<td>148.8pF</td>
<td>150.7pF</td>
</tr>
<tr>
<td>4</td>
<td>50.00</td>
<td>155.0pF</td>
<td>158.1pF</td>
</tr>
<tr>
<td>5</td>
<td>50.00</td>
<td>155.0pF</td>
<td>158.1pF</td>
</tr>
</tbody>
</table>

$C_p = 30pF$  
$C_\alpha = 3.1pF$  
$C_i = 6.8pF$  
$C_{ni} = 34pF$  
$C_{n\alpha} = 3pF$  
$C_n = C_{ni} = \ldots = 6.9pF$
Fig. 7.8 Schematic illustration of the electrical arrangement of the 8CR-GD cathode incorporating capacitive trimmers.
Fig. 7.9 Experimental uniform irradiation response showing the differential non-linearity of the eight section CR-GD cathode.
Fig. 7.10 Experimental uniform irradiation response showing the differential non-linearity of the eight section CR-GD cathode, with $h = 4\text{mm}$.
The small discrepancies between theoretical and actual $C_R$ values must be seen in the light of the fact that the added components, particularly the trimmers, will have modified slightly the original $C_R$ values.

Having been trimmed for a detector with $h = 6\text{mm}$, the cathode was then operated in one with $h = 4\text{mm}$ to assess the tolerance to such changes. As can be seen from Fig. 7.10, the differential non-linearity was degraded, but remained quite acceptable. As noticed before with the 6CR-GD cathode, small scale structure due to the non-linear response of the GD sections appears more significant than any overall effects due to the CR line.

7.6.2 Position resolution

Having achieved a degree of linearity ($\delta = 0.8\%$) with the 8CR-GD cathode close to the best obtained with an RC line system ($\delta = 0.6\%$), it was necessary to demonstrate that this had not been obtained at the expense of position resolution. As mentioned before, it is desirable that the contribution of electronic noise to the overall position resolution be as small as possible, to allow operation at low chamber gains. Both the total X-ray resolution and the electronic contribution were measured for the 8CR-GD cathode as a function of anode charge level, with the results shown in Fig.7.11. The experimental parameters are given in Table 7.9: the electronic resolution was measured by matching test pulses to the cathode signals at the given anode charge level.
Fig. 7.11 Measured position resolution as a function of anode charge level for the 8CR-GD cathode and the 250kΩ RC line cathode. The points are X-ray resolutions, the dashed lines are drawn through measured electronic contributions to position resolution.
Table 7.9 Chamber parameters for position resolution measurements

8CR-GD lower cathode, sensing x axis

\[ h = 6\text{mm}, \quad s_\alpha = 2\text{mm}, \quad \text{Drift depth (d_2)} = 9\text{mm} \]

\[ V_c = +200\text{V} \]

P50 gas

1.5keV X-rays, 50\text{\mu m} collimation

The measurements were repeated using the cathode with the 250k\Omega RC line readout (Section 6.3.1), but otherwise identical conditions. These results are also shown in the figure. The considerably lower electronic noise contribution of the CR-GD system is evident. The deviation of the X-ray resolution from the electronic line occurs when other resolution contributions (see Section 9.1) start to become significant. The best X-ray resolution, 82\text{\mu m} \text{rms} (or 193\text{\mu m FWHM}) at 0.5pC anode charge, was fully comparable with values previously obtained for GD and CR cathodes [43].

An attempt was made to improve this result further by reducing \( h \) to 4mm and repelling events from the drift region in an attempt to minimise electron diffusion, and using 20\text{\mu m} collimation. At 0.5pC anode charge, source positions 500\text{\mu m} apart were well resolved (Fig. 7.12), and an average resolution of 79\text{\mu m} \text{rms} was obtained.

The quoted resolutions were measured near the centre of the cathode; the only serious degradation occurred near the ends, and in fact appeared to be due to \( Q(x) \) becoming multivalued. This was thought to be due to the dependence of the proportion of the induced charge "lost" over the end on \( \alpha \).
It is straightforward to check that the measured electronic noise contributions are well predicted by Eqs. 6.2 and 6.3. The electronic noise in a Canberra 2001 pre-amplifier loaded by the cathode system was typically $q_n = 300$ electrons rms. The correlation coefficient was obtained by also measuring the noise in the sum signal $A+B$, that is

$$\bar{d}_{q_{\text{sum}}}^2 = 2q_n^2 + 2q_n^2 r_{AB}$$

An average value of $r_{AB} = -0.14$ was obtained from several readings, so for the electronic centre ($Q = 0.5$), Eq. 6.3 gives $D = 0.75$. If we now make the crude assumptions that the charge induced on each cathode is 50% of the anode charge, and that 85% of this is delivered to the end nodes, then the predicted electronic contributions to resolution given in Table 7.10 are obtained. The relative errors in the last two columns are around $+/-15\%$. 

Fig. 7.12 Demonstration of the X-ray resolution of an 8CR-GD cathode. Collimated source positions 500 $\mu$m apart.
Table 7.10 The electronic contribution to position resolution of the 8CR-GD cathode

<table>
<thead>
<tr>
<th>Charge level (pC)</th>
<th>Predicted contribution (µm rms)</th>
<th>Measured contribution (µm rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>760</td>
<td>744</td>
</tr>
<tr>
<td>0.05</td>
<td>152</td>
<td>165</td>
</tr>
<tr>
<td>0.10</td>
<td>76</td>
<td>83</td>
</tr>
<tr>
<td>0.50</td>
<td>15</td>
<td>14</td>
</tr>
</tbody>
</table>

It should be noted that the measured electronic noise cannot be explained solely in terms of the capacitive load on the pre-amplifiers. For the measured values of unloaded noise and load line slope, and $C_{IM} = 34\, \text{pF}$, a noise figure of around 170 electrons would be expected. However, account must be paid to the fact that the effective resistance to earth presented by the cathode system will be considerably lower at signal frequencies than at DC. Measurements with a Marconi "Q" meter at 160kHz, equivalent to a $1\, \mu\text{s}$ filter time constant, revealed an effective resistance of $3.2\, \text{M}\Omega$. Gilvin [43] has shown that such values can explain the extra noise, and that it is generated in the epoxy resin used to secure the wires as well as in the discrete resistors.

7.7 Two-dimensional imaging with CR-GD cathodes

Two further 8CR-GD cathodes were constructed, but using preferred capacitor values only. These were installed in the Harwell detector body. Fig. 7.13 shows the assembled wire arrays; the relatively simple and compact nature of the readout method may be
Fig. 7.13 Harwell detector body with the gas enclosure removed to show the detector arrangement with two 8CR-GD cathodes.
appreciated. The cathodes operated satisfactorily, and were used to make measurements of wire modulation patterns (Chapter 8), for which constant cathode sensitivity is desirable. The lack of detailed attention to the longitudinal capacitor values was reflected in some large scale structure in the uniform irradiation responses in the individual axes ($\delta = 4.3\%$). However, the absence of rapid local sensitivity fluctuations produced a quite acceptable two-dimensional uniform irradiation response (Fig. 7.14). It should be compared with that previously obtained for two-section GD cathodes (Fig. 6.18).

Fig. 7.14 Two-dimensional uniform irradiation response in isometric projection with 8CR-GD cathodes. Other details are the same as for Fig. 6.18.
8.1 Introduction

The fact that wires are inherent in the construction of MWPCs offers a number of important advantages. The use of thin anode wires allows very high electric fields to be established close to the wire (typically > 300V cm⁻¹ torr⁻¹), but the approximately 1/r dependence means that the resulting avalanche region is highly localised. The majority of X-ray interaction positions thus allow the full gain of the chamber to be realised. This gain may be high (>10⁵) with the chamber still stable with respect to spontaneous breakdown. Wire cathodes are advantageous in being relatively transparent to radiation and in allowing electrons to be drifted into the detector from absorption and drift regions. There is a price to be paid in imaging performance for the use of wires, however. They must produce local departures from the ideal imaging field (which would have no x or z components anywhere), so we must anticipate that there will be image distortion.

8.2 Anode wire modulation

8.2.1 Effect on imaging

Consider an MWPC irradiated uniformly, with a cathode system of good linearity sensing the z axis (perpendicular to the anode wire direction). If the amount of diffusion of primary electrons is small,
such that the majority of avalanches are confined to one wire, then the output function $G(Q)$ will show a series of peaks and troughs referred to here as anode wire modulation. The pattern reflects the stepped nature of $Q(z)$ (Fig. 8.1) which arises because avalanches confined to a wire at $z_i$ can be caused by primary electrons arising anywhere in the band $z_i + s_a / 2$.

![Diagram of $Q(z)$](image)

**Fig. 8.1** Position sensitivity function $Q$ in the $z$ axis (across the anode wires).

8.2.2 Reduction by electron diffusion

The established technique for reducing anode wire modulation is to encourage avalanches to occur on more than one anode wire for a single event, by allowing sufficient lateral diffusion of the primary electrons, usually in a low field drift region [16,131]. The effectiveness of this approach was demonstrated with the Harwell
Fig. 8.2 Removal of anode wire modulation by diffusion of primary electrons. Two-dimensional uniform irradiation response from the Harwell detector with compensated 2GD cathodes. A mask having the letter L cut in it was placed in front of the detector window. PO gas.
Top: No drift region
Bottom: 30mm drift region.
detector set up for two-dimensional imaging with 2GD cathodes (Fig. 8.2). The price of this improved linearity is loss of position resolution. The centroid position of the primary electron cloud has an increasing statistical chance of deviating from its initial value as the lateral spread becomes greater (Sections 2.1.4 and 9.1). Additionally, a large active depth of the counter may be undesirable in some applications where parallax errors could occur.

8.2.3 Reduction by signal processing

With a negligible lateral spread of the primary electrons, there would be no position sensitivity in the region \( z_i + \Delta z \), where \(-s_\alpha/2 < \Delta z < +s_\alpha/2\) and \( z_i \) is a wire position, if the avalanche were to spread evenly around the wire. Avalanche angular localisation produces non-zero sensitivity in this region, however [132,133]. Primary electrons having centroid \( z_i + \Delta z \) in the parallel field part of the detector will produce an avalanche with angular centroid \( \alpha \) where from Eq. 2.6

\[
\alpha = \frac{1 - 2\Delta z/s_\alpha}{2} \pi/2 \quad \text{for electrons created at positive } y
\]
\[
\alpha = -\frac{1 - 2\Delta z/s_\alpha}{2} \pi/2 \quad \text{for electrons created at negative } y
\]

As the resulting positive ions drift back along field lines centred on \( \alpha \), their centroid will reflect more and more the relative position \( \Delta z \). This was revealed as the offset of the centroids of \( \rho_z \) distributions from \( z_i \) (that is \( \lambda = 0 \)) when \( \alpha \neq \pm \pi/2 \) in Chapter 5. \( S(z) \) will thus become more constant as the cathode induced charge distribution is sampled later in time.
The simplest way to achieve this reduction in anode wire modulation using conventional signal processing electronics (charge-sensitive pre-amplifiers followed by filter amplifiers) is simply to increase the filter time constant, so including more information from later in the ions' travel. Fig. 8.3 shows the reduction in anode wire modulation obtained by increasing the time constant of a single differentiation, four integration filter from 0.5 to 4 $\mu$s, as revealed in the uniform irradiation response. Differential non-linearity, $\epsilon$, (Section 6.1.3) was calculated for adjacent peaks and troughs, and the average over twelve wires, $\bar{\epsilon}$, obtained for a range of time constants. Fig. 8.4 shows $\bar{\epsilon}$ plotted against time to peak of the filter for two chamber gases, P10 and the mixture 75% Xe/22% Ar/3% CO$_2$. The latter gas was used to fill the Harwell detector when it was operated briefly as a sealed device. It is evident that the xenon mixture gives somewhat more electron diffusion, but that the ion drift velocity is lower than for P10. As the operating potentials were similar in both cases, this must reflect a lower ion mobility.

Examination of Fig. 8.4 also shows that further substantial decrease in anode wire modulation would require impractically long time constants. It may prove possible to improve this situation by a suitable revision of the processing electronics. Consider a system in which the currents flowing into or out of virtual earths at readout nodes as the ions drift in the chamber, are simply integrated. Then at some point in the ion travel, the resultant charges could be sampled and held, thus allowing the centroid of the induced charge distribution at that moment to be determined. A practical realisation might consist of a current sensitive pre-amplifier followed by some delay to allow the gate for a gated integrator to be generated. In
Fig. 8.3 Reduction of anode wire modulation by increasing processing filter time constants. Uniform irradiation response in the z axis from the Harwell detector with SCR-GD cathodes. P10 gas, no drift region.

Top: 0.5 $\mu$s time constant (2 $\mu$s to unipolar peak)
Bottom: 4 $\mu$s time constant (16 $\mu$s to unipolar peak)
The base line is indicated.
Fig. 8.4 Anode wire modulation, measured as $\bar{\varepsilon}$, as a function of the time to peak of the shaping amplifiers in the cathode channels.
order for the instantaneous induced charge distribution centroid to be constantly determinable from the integrator outputs, changes in the distribution must result in the corresponding current flows occurring simultaneously at the output nodes. Of the readout methods mentioned in Chapter 6, all except those incorporating RC or LC lines could be adapted.

Fig. 8.5 Signal processing arrangement for the attempted reduction of anode wire modulation.

A preliminary attempt was made to realise this signal processing with available electronics, as illustrated in Fig. 8.5. Induced currents in the two sections of a single section GD cathode flowed in the 50Ω input impedences of fast NIM amplifiers (NE4634 type, 2ns risetime). One hundred metres of 50Ω cable provided a delay of around 400ns, allowing gates for the Harwell 2144 gated integrators [134] to be generated. Although the current pulses were clearly seen on the oscilloscope, the available gain proved too low for successful imaging, with noise and DC offsets obscuring the true signals. Considerable progress might be possible with more recent and suitable electronic units however. Millard et al. [135] have described a current sensitive pre-amplifier with a gain of $10^4$, and high
performance devices are becoming available commercially. More advanced gated integrator designs have also been described [136].

8.2.4 Compensation of anode wire modulation

The approaches to reducing anode wire modulation mentioned in the previous two sections attempt to make the real sensitivity in the z axis more constant. Provided that there is some sensitivity in the regions \( z_i + \Delta z \), non-linearity can in theory be removed if \( Q(z) \) is known, with the proviso that position resolution in regions of low sensitivity is more affected by electronic noise. Van der Graaf and Wagenaar [137] have described digital electronics incorporating look-up tables which allow compensation to be carried out in real time.

\[ \text{Cathode} \]
\[ \text{Anode} \]

Fig. 8.6 Detector geometry with cathode strip sensing elements arranged for physical compensation of anode wire modulation.

An alternative to electronic correction may be to physically compensate the modulation by deliberate introduction of non-linearity in the appropriate cathode response. Fig. 8.6 shows a possible arrangement, with the induced charge distribution being undersampled so as to give high sensitivity over the anode wires and low sensitivity between them. The method would impose restrictions on the
counter geometry \((h < s_\lambda)\), but some preliminary calculations have shown that a reduction in the perceived anode wire modulation is possible.

8.3 Cathode wire modulation

![Cathode wire modulation of G(Q) by the wires of a 1mm pitch cathode. \(\mathcal{E} = 20\%\).](image)

The modulation pattern in \(G(Q)\) shown in Fig. 8.7 was obtained when the Harwell chamber, fitted with 8CR-GD cathodes with \(V_c = -200\text{V}\), was illuminated normally to the x-z plane with Mn K X-rays. The response is that of the cathode sensing the x axis, to avoid the extra complication of anode wire modulation. The troughs correspond to the wire positions of the sensing cathode, and the peaks to interwire positions. This is the opposite to anode wire modulation. Such cathode wire modulation can be understood if we consider the field around the cathode wires, which will of course depend on the value of \(V_c\) with
respect to \( V_a \) and earth. Some field line plots, obtained as described in Section 3.4.3, are shown for one half of the chamber and one cathode wire pitch in Fig. 8.8(a) to (d). The plots show the z-y plane, but the field around the cathode wires should be essentially the same in the x-y plane.

It is clear from the figures that primary electrons produced close to the cathode wires can undergo a lateral shift of up to \( s_c/2 \) in moving to the parallel field region of the chamber. \( Q \) is a multivalued function of \( x \) or \( z \) (the position of a collimated source) for interactions near the wires, as it depends also on the position of the interaction in the y axis. Compensation such as is possible for anode wire modulation is thus impossible. The amount of cathode wire modulation can be reduced by ensuring that the non-parallel field zones represent only a small fraction of the total active depth. It might appear from Figs. 8.8(a) and (b) that the use of a drift region to accomplish this would in fact introduce more modulation as the electrons are "funnelled" between the cathode wires. Evidence of such funneling having an effect in isolation was not seen in the present work, however. If the drift region was of sufficient depth to ensure that the majority of X-rays were absorbed there, then it also allowed sufficient lateral diffusion for interpolation to occur as the electrons passed between more than one pair of wires.

Where repelling voltages are required on the cathodes, Figs. 8.8(c) and (d) suggest that the volume of the non-parallel field region can be minimised by making \( V_c \) more negative. This was confirmed by the measurements given in Fig. 8.9. The modulation due to a 1mm pitch cathode, calculated as \( \Xi \), was emphasised by selecting, by crossover discrimination, only those events from the half of the
Fig. 8.8 Calculated field lines starting on an anode wire (bottom) in the upper half of an MWPC. The cathode wires are parallel to the anode wires. $h/s_a = 2$, $d_1/s_a = 2$, $s_c/s_a = 0.5$, $V_a = +2400V$, earth plane at $y = h+d_1$.
(a) $V_c = +150V$; (b) $V_c = 0V$; (c) $V_c = -100V$; (d) $V_c = -200V$
Fig. 8.9 Variation of cathode wire modulation, measured as $\varepsilon$, with cathode potential. The anode potential was 2940V at $V_c = 0$V, and was varied by similar absolute amounts as $V_c$ to maintain the counter gain. $h = 4$mm, $s_a = 2$mm, $s_e = 1$mm, $d_f = 3$mm.
detector closest to it. A very short drift depth \( (d_i = 3\text{mm}) \) meant that there may in fact have been some "funnelling" modulation before the repelling field was established between \( V_c = -100\text{V} \) and \(-150\text{V}\), but this effect could not really be distinguished from the modulation due to interactions near the wires.

The most effective way of reducing cathode wire modulation was to make the cathode a closer approximation to a continuous plane. 0.5mm pitch cathodes did not produce a noticeable modulation in the present work. It should be noted that positioning errors analogous to those caused by the cathode wires would be expected in the \( z \) axis for X-rays absorbed close to the anode wires. In practice, any resulting modulation pattern is invariably obscured by the previously described anode wire modulation effect.
CHAPTER NINE

THE CONTRIBUTION OF ELECTRON RANGE TO MWPC POSITION RESOLUTION

9.1 The limitations to MWPC position resolution

Consider an infinitesimally narrow X-ray beam normally incident on an MWPC at x, where the sensitivity is S. The position output Q will show a variance $\sigma_Q^2$ about some Q and thus there must be an uncertainty, expressed by $\sigma_x = \sigma_Q / S$, associated with the recording of x from a single X-ray event. The factors which limit spatial resolution have been considered by several authors [92,131,138-140]; the most significant of these factors are:

(i) $\sigma_p$: Describing the variation in the distribution of primary electrons generated by photo-electrons and Auger electrons. This will be simply called the electron range contribution.

(ii) $\sigma_d$: Describing the variation in the centroid of the primary electron cloud resulting from the effects of diffusion on the limited number of primary electrons. In the x axis, for N electrons, this may be shown to be given by [92,141]

$$\sigma_d = \sigma_{\text{diff}} \left( \frac{1 + f}{N} \right)^{1/2}$$

Eq. 9.1

where $\sigma_{\text{diff}}$ is given in Eq. 2.8 and f has been defined in Eq. 2.9

(iii) $\sigma_e$: Describing the contribution of electronic noise from the position readout system (Eq. 6.3).

Assuming the contributions to be independent, we have

$$\sigma_x^2 = \sigma_p^2 + \sigma_d^2 + \sigma_e^2$$

Eq. 9.2

Secondary avalanches can provide a considerable extra contribution to position uncertainty, but were not significant for the gas mixtures and low charge levels used in the present investigation. The subject
has been studied in some detail by Thomas [59].

Advances in readout systems have led to $\sigma_{a}$ becoming less and less significant in Eq. 9.2. At the expense of quantum efficiency, $\sigma_{d}$ may be reduced by reducing the effective detector depth. The electron range contribution, $\sigma_{p}$, always dominant at high X-ray energies (>10keV), can therefore limit detector resolution even at lower energies, and indeed appears to do so in some recent designs [113]. An assessment of this contribution is therefore required.

Work was started at Leicester, in co-operation with G.C.Smith who subsequently moved to the Brookhaven National Laboratory. Measurements were continued at Brookhaven with a variety of X-ray energies and gases [142], leading to resolutions down to 22\,\mu\text{m FWHM} in xenon mixtures at high pressure [143]. Work at Leicester has concentrated on the details of the electron range contribution in various argon-methane mixtures.

9.2 Distribution of ionization after X-ray absorption in argon-methane mixtures

Following on from the description given in Section 2.1.3, Fig. 9.1 gives the energies of electrons with significant range that may be produced in argon-methane mixtures as a function of X-ray energy. The relative cross sections for the various absorption possibilities are shown in Fig. 9.2 (derived from McMaster et al. [34]). The fluorescent yield for argon K shell vacancies (in other words the probability of the 2.7keV Auger electron not being emitted) was measured by comparing the relative integrated areas of escape and main peaks in the pulse height spectrum of a small detector illuminated by
Fig. 9.1 Energies of the electrons of significant range that can be produced by X-ray interactions in argon-methane mixtures, as a function of the X-ray energy.
Fig. 9.2 Cross sections for X-ray interactions giving rise to photoelectron emission from argon K and L shells, and carbon K shell, as a function of X-ray energy (derived from McMaster et al. [34]). One barn is $10^{-28}$ m$^2$. 
Mn K X-rays. A value of 15% was obtained.

The amount of ionization per unit length of the electron track will be assumed constant at the relatively low energies of current interest, although the relevant Bragg curve should be used at higher energies [139]. The actual path of the electron is tortuous as it undergoes successive collisions, so it is usual to introduce a practical range, R, and to take the distribution of points R from an initial interaction position as being effectively isotropic [140]. Therefore, following Gilvin [43] with the severe simplifying assumption that ionization is produced uniformly along the line to R, the distribution of ionization centroids for interactions at a given point will be a sphere of radius R/2. As shown by Gilvin, this results in a rectangular distribution in one axis (the x axis, say)

\[ \gamma(x) = \frac{1}{R} \quad |x| \leq R/2 \]

with standard deviation \( \sigma = R/2\sqrt{3} \).

From Section 2.1.3 and Fig 9.1 it can be seen that there are two basic types of event that need to be considered for argon-methane mixtures:

(i) Those in which there is one energetic electron, giving a rectangular distribution of ionization, \( \gamma_1(x) \), the remainder of the ionization being localised at the absorption position (\( \gamma_2(x) = \delta(x) \), a delta function)

(ii) Those in which there are two energetic electrons, giving two uncorrelated rectangular distributions, \( \gamma_1(x) \) and \( \gamma_2(x) \), the remaining ionization again forming a delta function, \( \delta(x) \).

Consider firstly that two electrons with energies \( E_1 \) and \( E_2 \) are emitted, with \( E_1 > E_2 \), having practical ranges \( R_1 \) and \( R_2 \) respectively.
Assuming \( R \) increases in some simple fashion with \( E \), then \( R_1 \geq R_2 \). If the individual centroids of ionization are \( x_1 \) and \( x_2 \), then the resultant, \( x \), must be

\[
x = \frac{(E_1 x_1 + E_2 x_2)}{E_T}
\]

where \( E_T = E_1 + E_2 \). Obviously the distribution of \( x \) must be narrower than if all of \( E_T \) were concentrated in one electron. In fact, using the above constraint, it may be shown that the convolution of the two rectangular distributions is a trapezoidal distribution

\[
\gamma(x) = \begin{cases} 
\frac{E_2^2}{(R_1 R_2 E_1 E_2)} (x_m - x) & x_i < x < x_m \\
\frac{E_1^2}{(R_1 R_2 E_1 E_2)} (x_m - x_i) & x < x_i
\end{cases}
\]

where

\[
x_m = \frac{(E_1 R_1 + E_2 R_2)}{2E_T}
\]
\[
x_i = \frac{(E_1 R_1 - E_2 R_2)}{2E_T}
\]

Fig. 9.3 The resultant of two uncorrelated rectangular distributions.

The FWHM is simply \((x_i + x_m) = E_1 R_1 / E_T\), determined by the range of the more energetic electron. This is at a minimum when \( E_1 = E_2 \) (so \( R_1 = R_2 \)), the distribution then being triangular. The standard
deviation may be shown to be given by
\[ \sigma^2 = \frac{E_1}{E_T} \sigma_1^2 + \left( \frac{E_2}{E_T} \right) \sigma_2^2 \]
where \( \sigma_1 \) and \( \sigma_2 \) are the standard deviations of \( \gamma_1 \) and \( \gamma_2 \) respectively.

For case (i) above, we have \( R_2 = 0 \), so that \( x^m = x^i = E_1 R_1 / 2 E_T \).
The resultant distribution is rectangular
\[ \gamma (x) = \frac{E_T}{E_1 R_1} \]
with FWHM
\[ \frac{E_1 R_1}{E_T} \]

Case (ii) must therefore result in a trapezoidal distribution, and we have
\[ \gamma (x) = \frac{(E_2^2/(R_1 R_2 (E_1 + E_3) E_2)) (x_m - x)}{x_i < x < x_m} \]
\[ \gamma (x) = \frac{(E_T^2/(R_1 R_2 (E_1 + E_3) E_2)) (x_m - x_i)}{x < x_i} \]
where \( E_1 \) is the energy of the more energetic electron, \( E_2 \) the energy of the less energetic one, and \( E_3 \) the energy associated with the delta function of ionization. \( E_T = E_1 + E_2 + E_3 \), and \( R_1' \) is the width of the distribution resulting from the convolution of \( \gamma_i(x) \) and \( \delta(x) \). We have
\[ x_m = ((E_1 + E_3) R_1' + E_2 R_2) / 2 E_T = (E_1 R_1 + E_2 R_2) / 2 E_T \]
\[ x_i = ((E_1 + E_3) R_1' - E_2 R_2) / 2 E_T = (E_1 R_1 - E_2 R_2) / 2 E_T \]
The FWHM is thus
\[ \frac{E_1 R_1}{E_T} \]
again determined by the most energetic (longest ranged electron).

Early work on the range-energy relationship, mostly concerned with absorption by metal foils, has been reviewed by Katz and Penfold [144]. For the limited energy range of interest, an empirical relation of the form
\[ R \propto a E^n \]
has been found suitable, with a and n constants. As the range is
determined by the number of collisions, we must also expect it to vary
inversely with density, \( \rho \), giving

\[
R = aE^n/\rho
\]

Eq. 9.3

When \( E_x \gg E_K \), where \( E_x \) is the X-ray energy and \( E_K \) the K edge energy of
the absorbing atom, \( E \) may be simply replaced by \( E_x \) in Eq. 9.3. By
this means Smith et al. [142] obtained \( n = 1.78 \) for simple hydrocarbon
gases. This compares well with the value of 1.8 obtained by Williams
for oxygen [145]. Using this data, the electron energies from Fig.
9.1, and the expressions for ionization distribution full width half
maxima obtained above, we may predict how the electron range
contributions for argon-methane mixtures vary with X-ray energy in the
range 1 to 10keV. This is shown in Fig. 9.4, where the plotted
contribution is to be multiplied by \( (a/\rho) \) for a specific gas. The
simplifying assumption has been made that the delta function of
ionization arises from energy \( (E_x - \sum E_i) \) where \( E_x \) is the X-ray energy
and \( E_i \) the energies of electrons with significant range.

A full experimental investigation of the behaviour predicted by
this simple model would require a good selection of well defined X-ray
energies. This was not available in the present work, although the
advent of synchrotron sources should change this. Important features
could be checked, however. These included the very different
contributions for full energy events above the argon K edge that are
predicted for interactions involving the argon K shell, compared with
interactions involving the argon L or carbon K shells. The former
should be numerically dominant (Fig. 9.2), and should give better
resolution than the corresponding escape peak events for X-ray
energies above about 5.3keV. The simple \( 1/\rho \) dependence should be
observable.
Fig. 9.4 Expected FWHM electron range contributions to position resolution in argon-methane mixtures, as a function of X-ray energy. Values from the graph should be multiplied by a/φ for a particular gas.
9.3 Measurement of the electron range contribution to position resolution

9.3.1 Minimisation of other contributions

Referring to Eqs. 6.3 and 6.4, it can be seen that the electronic noise contribution, $\sigma_e$, may be made small both by reducing the noise charge, $q_A$, and also by having high sensitivity, $S$. This was achieved by using very simple split cathode systems (Fig. 9.5). As the centroid, $x$, of an induced charge distribution, $Q$, is moved relative to the division between the two cathode halves $A$ and $B$, a position response $Q(x) = q_B/(q_A + q_B)$ may be obtained. $q_A$ and $q_B$ are the charges induced on the two halves, and $Q$ is effectively the normalised integral of $Q$. For a limited region around $Q = 0.5$, $dQ/dx = S$ is very large and approximately constant. $q_A$ can be made small as there is no resistive element, and the electrode area can be reduced to that necessary to provide a useful calibration curve, so keeping capacitive loads small. An initial system had $A$ and $B$...
comprising groups of eight wires each at the centre of a standard 1mm pitch cathode operated in the Leicester detector body. The measured electronic contribution to position resolution was already below 10μm FWHM at 0.1pC, the anode charge level at which most measurements were made. This was negligible compared to the expected magnitudes of the other contributions, and subsequent arrangements produced still better performance.

A shortcoming of this sensing method becomes apparent when it is recalled that the shape of Q depends upon the avalanche angle centroid, ω. Multiple calibration curves result, as illustrated in Fig. 9.6. Although the present arrangement is an extreme case, it should be clear that any cathode system with regions of rapidly changing sensitivity will be susceptible to such problems of multiple imaging. Three solutions were adopted in the measurements of electron range contribution to position resolution. Where the resolutions to be measured were small, the region close to the electronic centre (Q = 0.5) where Q(x) is essentially single valued was found to give usable imaging. Alternatively, a narrow range of ω could be selected by crossover discrimination (Section 4.2.4), but at the expense of useful count rate. Finally, in the Leicester detector bodies, the problem could be avoided by the electrode arrangement shown in Fig. 9.7. Combining the signals from upper and lower cathode wire groups eliminated the dependence on ω and also essentially doubled the signal size. Since crossover discrimination was still required for X-ray beam alignment (Section 2.2.7), a signal was taken from the foil electrode below the lower cathode, being effectively the "leakage" of induced charge through the wire cathode. Perfectly acceptable crossover time spectra were obtained at anode charge levels around 0.3pC. Also evident in Fig. 9.7 is a second anode plane. Signals
Fig. 9.6 Multiple position calibration curves obtained with the simple split cathode system (Fig. 9.5) due to the dependence of $Q$ on $\alpha$ as well as $x$. $Q(x, \alpha = +\pi/2)$ and $Q(x, \alpha = -\pi/2)$ are shown.
from this were used to generate anticoincidence pulses for the rejection of cosmic ray background, which proved necessary for measurements with iron-55 sources.

Fig. 9.7 Detector arrangement with dual split cathode position sensing and anticoincidence volume.

From Eqs. 2.8, 2.9 and 9.1 it is apparent that the contribution of diffusion to position resolution is greatest at low X-ray energies, when the number of primary electrons, N, is small, and that for a given gas mixture the most effective way of reducing the contribution is to reduce the drift distance. In the Leicester detector body, the minimum value of h (4mm) was employed, and a negative potential was applied to the cathodes to repel electrons not produced between the cathodes. For the lowest energy X-rays used (Al K for which N≈55), a small detector of a type described by Smith et al. [144] having h = 0.8mm was obtained from the Brookhaven National Laboratory. The
geometry is shown in Fig. 9.8. Connecting the sensing strips to form a split cathode arrangement gave a range of $x$ where $Q(x)$ was approximately linear of about 3mm. The electronic noise contribution was less than 5$\mu$m FWHM at 0.1pC anode charge with 200ns unipolar shaping.

![Diagram of Brookhaven small detector geometry](image)

Fig. 9.8 Geometry of the Brookhaven small detector.

This small detector type was also used with its intended readout method, the centroid finding filter (Section 6.4.2) during a visit to the Brookhaven National Laboratory, when preliminary measurements at gas pressures above atmospheric were made. The electronic noise contribution was 20$\mu$m FWHM at 0.1pC.

9.3.2 X-ray beamwidth

In practical resolution measurements using a collimated X-ray beam, the non-infinitesimal width of the beam will be included in the results. If a variance $\sigma_b^2$ can be associated with the beamwidth,
uncorrelated with the other contributions, then Eq. 9.2 may be extended
\[ \sigma_x^2 = \sigma_e^2 + \sigma_d^2 + \sigma_f^2 + \sigma_b^2 \]
Obviously \( \sigma_b \) must be minimised just as \( \sigma_e \) and \( \sigma_d \) in order that the measured resolution reflects strongly the electron range contribution.

If the X-ray collimator slits of width \( w \) are separated by \( l \), then at a distance \( d \) from the exit slit the distribution of possible \( x \) may be shown to be trapezoidal, with
\[ \text{FWHM} = w \left( 1 + \frac{d}{l} \right) \]
d was kept as small as was allowed by the mechanical details of the detectors (around 8 mm on average for both Leicester and Brookhaven detectors). Decreases in \( w \) or increases in \( l \) had to be weighed against loss of count rate, with the corresponding demands on the stability of the system. 50 \( \mu \)m slits were the minimum tolerable for the iron-55 source, with \( l = 40 \) mm, using the collimator shown in Fig. 2.15(b). This gave rates of accepted counts of less than two per minute for some escape peak measurements. Using the aluminium target X-ray tube, \( w = 20 \mu \)m was practicable. The highest resolution measured with this source was in P50 gas. Fig. 9.9, in which the measured resolution has been plotted against the estimated beamwidth at the anode plane, indicates that the beamwidth was not a significant contributor in this measurement.

Vertical alignment of the beam was carried out as described in Section 2.2.7. Any remaining vertical misalignment would have broadened a (parallel) beam of width \( b \) to a maximum width of
\[ \frac{(2h \sin \theta + b)}{\cos \theta} \]
with \( \theta \) the angle between the beam and the \( x \) axis. Again the reduction of the detector depth, \( 2h \), is seen to be beneficial.
Estimated beamwidth ($\mu$m FWHM)

- Measured resolution for 1.5keV X-rays in P50 gas related to the estimated beamwidth at the anode plane due to various collimator slit widths. Other contributions to the resolution have become more significant than the beamwidth for the leftmost point (20$\mu$m collimation).
9.3.3 X-ray sources and chamber gases

Measurements were made of the position resolution for X-rays from the aluminium target tube and the iron-55 source in the standard gas mixtures listed in Table 2.1. Anode pulse height selection was used to select the full energy or escape peak events when the higher energy X-rays were used with argon containing mixtures. As mentioned in Chapter 2, the copper target X-ray tube proved less useful, and only a limited number of measurements were made with it. The measured resolution was found to depend strongly on the window which was set in the anode pulse height spectrum. This was considered to be due to the continuum component of the X-ray energy spectrum being at least of similar integrated intensity to the Cu K emission lines, even after passing through a 25 \( \mu \)m nickel filter. A good example of the problem was provided by an attempt to measure the resolution for the Cu K escape peak in P75 (Fig. 9.10). Two distinct resolutions are seen,

Fig. 9.10 Evidence of interference from the continuum radiation from the copper target X-ray tube. "Cu K" X-rays in P75, with attempted pulse height selection of escape events.
the larger one corresponding to true escape events, where there is one electron of significant range, and the smaller to continuum X-rays in the region of $(8-3)$ keV giving photo- and Auger electrons. Fig. 9.4 predicts a FWHM ratio of 4.9 to 1 for the electron range contributions alone for this case, which compares reasonably well with the approximate observed values of 4.5 to 1 for P75 and 4.6 to 1 for P50.

In order to supplement the commercially available argon-methane mixtures, a gas flow control system was set up, allowing the two gases to be mixed in arbitrary proportions. Fig. 9.11 shows the general arrangement.

![Fig. 9.11 Schematic illustration of the gas mixing system. The mixing volume was 5000 cm$^3$, and total flow rates of the order 25 cm$^3$ min$^{-1}$.](image)

Control circuits used the difference between the flow rate sensed by thermal mass flow sensors and the required rate as set on a precision potentiometer to control solenoid valves. The scales were calibrated to compensate the differing specific heats of argon and methane, and the argon (or methane) proportion could be set to around
one per cent with good stability. No independent check of the gas composition was available, although the anode voltage required for a given charge level gave a crude indication. By producing mixtures with high methane contents, the less frequent interactions with carbon atoms could be highlighted, and the relative cross-sections predicted by Fig. 9.2 therefore be checked.

9.4 Results

9.4.1 Treatment of data

The generally low count rates available meant that the integrated sizes of position peaks were often only a few thousand counts, even after several days for the worst cases. In order to make maximum use of the data, Gaussian distributions were fitted by least squares analysis except when the peak was obviously non-Gaussian. A constant background was allowed as a variable parameter, and where dual resolutions were observed, the fitting function comprised two superimposed coincident Gaussian distributions plus a background.

For the broadest responses, obtained in cases where the initial electron energies were highest, the relative error in estimating the FWHM by eye was sufficiently small. These peaks showed noticeable departures from Gaussian form, presumably as other contributions to resolution had become negligible compared to that of electron range. The example shown in Fig. 9.12 was observed when only the largest anode pulse heights were selected with the copper target tube in use. As mentioned above, the X-ray energy was not well defined with this source, but was presumably in the approximate range 8 to 10keV in this
case. A trapezoidal shape is evident.

Fig. 9.12 Position peak obtained with the detector arrangement of Fig. 9.7 for 8 to 10 keV X-rays in P75.

9.4.2 Experimental electron range contributions for dominant interactions in standard argon-methane mixtures

The measured resolutions in \( \mu \text{m FWHM} \) for the commercially obtained argon-methane mixtures are given in Table 9.1. In Table 9.2 the measured electronic noise contributions and estimates of the diffusion and beamwidth contributions made using the relevant expressions given earlier in this chapter have been subtracted in quadrature. Repeat measurements of the Mn K full energy resolutions in P10, P50 and P75 gave values within plus or minus three microns. It should be noted, however, that an earlier series of measurements with the Mn K radiation, while also giving apparently repeatable results, were around 20 \( \mu \text{m} \) greater than those reported. Since any unknown contribution, underestimate of known contributions, or lack of
refinement in the experiment will tend to increase the values, those
given in Table 9.2 should be regarded really only as upper limits to
the electron range contribution. Despite this, some observations may
be made with reference to the measurements and the simple model
presented in Section 9.2.

Although the data are very limited, a rough calculation of the
value of the exponent n in Eq. 9.3 may be made from the cases
involving a single electron with significant range. From the Al K, Mn
K escape and "Cu K" escape results in P50 and P75, the value 1.7 is
obtained, in comparison with 1.78 reported by Smith et al. [142].
Adopting the latter figure as more reliable, however, the constant a
in Eq. 9.3 was evaluated for each value in Table 9.2, using Fig. 9.4.
If the range contribution is in \( \mu \text{m FWHM} \), E is in keV and \( \rho \) in kg
m\(^{-3}\), then the values in Table 9.3 are obtained. The average is 47,
but there is an obvious lack of consistency. In the cases where the
electron range might be expected to be most dominant, \( a \) is around 35,
the value reported by Smith et al. It seems most likely that the
effect of diffusion requires more careful evaluation.

Despite this, most relative resolutions are approximately as
given by Fig. 9.4, with the exception of the Mn K full energy and
escape peak results. While the escape peak shows poorer resolution as
predicted, it is only by a factor of about 1.2, rather than around 2.
This may be further evidence that other resolution contributors were
not well estimated or made negligible, although it might possibly
indicate some correlation between photo- and Auger electron emission
directions.
Table 9.1 Measured resolutions ($\mu$m FWHM) for dominant interactions in standard counter gases

<table>
<thead>
<tr>
<th>Gas</th>
<th>X-ray line</th>
<th>A1 K</th>
<th>Mn K (full)</th>
<th>Mn K (escape)</th>
<th>&quot;Cu K&quot; (full)</th>
<th>&quot;Cu K&quot; (escape)</th>
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</thead>
<tbody>
<tr>
<td>P10</td>
<td>74</td>
<td>148</td>
<td>176</td>
<td>275</td>
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<td></td>
</tr>
<tr>
<td>P50</td>
<td>67</td>
<td>154</td>
<td>189</td>
<td>316</td>
<td>474</td>
<td></td>
</tr>
<tr>
<td>P75</td>
<td>71</td>
<td>163</td>
<td>208</td>
<td>387</td>
<td>540</td>
<td></td>
</tr>
<tr>
<td>Methane</td>
<td>72</td>
<td>1100</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9.2 Estimated electron range contributions ($\mu$m FWHM)

<table>
<thead>
<tr>
<th>Gas</th>
<th>X-ray line</th>
<th>A1 K</th>
<th>Mn K (full)</th>
<th>Mn K (escape)</th>
<th>&quot;Cu K&quot; (full)</th>
<th>&quot;Cu K&quot; (escape)</th>
</tr>
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<tbody>
<tr>
<td>P10</td>
<td>60</td>
<td>144</td>
<td>171</td>
<td>273</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P50</td>
<td>64</td>
<td>152</td>
<td>187</td>
<td>315</td>
<td>473</td>
<td></td>
</tr>
<tr>
<td>P75</td>
<td>69</td>
<td>162</td>
<td>207</td>
<td>386</td>
<td>540</td>
<td></td>
</tr>
<tr>
<td>Methane</td>
<td>70</td>
<td>1100</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 9.3 Derived values of the constant $a$ (keV$^{-1}$ kg$^{-1}$ m$^3$ $\mu$m)

<table>
<thead>
<tr>
<th>Gas</th>
<th>X-ray line</th>
<th>A1 K</th>
<th>Mn K (full)</th>
<th>Mn K (escape)</th>
<th>&quot;Cu K&quot; (full)</th>
<th>&quot;Cu K&quot; (escape)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10</td>
<td>72</td>
<td>80</td>
<td>48</td>
<td>46</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P50</td>
<td>57</td>
<td>63</td>
<td>39</td>
<td>39</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>P75</td>
<td>49</td>
<td>53</td>
<td>34</td>
<td>38</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Methane</td>
<td>36</td>
<td>34</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
9.4.3 Electron emission from argon L and carbon K shells

Although photoelectron emission from the argon K shell is the dominant process for X-ray energies above the argon K edge (Fig. 9.2), the ejection of electrons from argon L and carbon K shells remains a finite possibility which may be significant in some applications. In both cases the electron has an energy of about $(E_x - 0.28)\text{keV}$, and as this is greater than the individual energies of the corresponding K shell photoelectron and Auger electron, a poorer resolution is expected (Fig. 9.4). The effect is most evident in low proportions of argon, and Fig. 9.13 shows the clear dual resolution for the full energy peak in P95 (argon 95% methane), as produced by the gas flow control system. While the fitting of a Gaussian to the broad peak could not really be justified, it did allow fairly good assessments of the relative integrated areas of the two peaks. These ratios could be compared with those predicted by the cross sections of Fig. 9.2 and the measured fluorescent yield of 15%. Table 9.4 gives the number of counts in the broad peak as a fraction of the total integrated counts in both peaks as predicted and measured for a wide range of gas mixtures. The agreement is seen to be good, giving confidence in the photoelectric cross sections.

Table 9.4 Percentages of 5.9keV X-ray interactions in argon-methane mixtures which eject electrons from Ar L or C K shells

<table>
<thead>
<tr>
<th>Gas</th>
<th>Predicted</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10</td>
<td>13.1</td>
<td>11.2</td>
</tr>
<tr>
<td>P50</td>
<td>14.1</td>
<td>11.0</td>
</tr>
<tr>
<td>P75</td>
<td>16.2</td>
<td>16.8</td>
</tr>
<tr>
<td>P80</td>
<td>17.3</td>
<td>18.4</td>
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<tr>
<td>P85</td>
<td>18.9</td>
<td>19.9</td>
</tr>
<tr>
<td>P90</td>
<td>22.1</td>
<td>23.0</td>
</tr>
<tr>
<td>P95</td>
<td>30.2</td>
<td>29.8</td>
</tr>
<tr>
<td>P99</td>
<td>61.9</td>
<td>62.4</td>
</tr>
</tbody>
</table>
Fig. 9.13 Measured position peak for 5.9keV X-rays in P95 gas showing the effect of two distinct electron range contributions to position resolution. The vertical scale was expanded for the lower picture.
5.7.4 The effect of increased gas pressure

The anticipated inverse dependence of electron range with density may be most simply investigated by increasing the gas pressure. This is a most practical measure, for as well as being expected to improve position resolution when electron range effects are significant, it will also increase detector efficiency and can reduce the image blurring that results if the incident radiation is not normal to the wire planes. Fig. 9.14 shows measured resolutions, made with the Brookhaven detector with centroid finding filter and 20\(\mu\)m collimation, for X-rays from chromium and copper target tubes in P10 gas, plotted against reciprocal pressure (\(\propto 1/\rho\)). The expected relationship is observed, and the measurements also produced the best resolution obtained in the present work; 50\(\mu\)m FWHM for X-rays of around 5.4keV in P10 at three atmospheres.
Fig. 9.14 Measured position resolution for X-rays of around 8keV (upper line) and around 5.4keV (lower line) as a function of the reciprocal pressure of the PIO chamber gas.
10.1 Summary of the causes of imperfect imaging and review of the present results

It is useful at this stage to summarise the known causes of MWPC X-ray imaging imperfection, and then review the present work, indicating where it might be of use in predicting or reducing imaging distortion. Ideally an MWPC would record the co-ordinate, in one or two axes, of each incident X-ray photon; thus \( Q(x) = x \) and \( Q(z) = z \). In practice this is not the case, for reasons which may be broadly grouped into three categories:

(a) Non-linearity in \( Q \). Imperfections of this type may in principle be compensated if the function \( Q \) is known, provided \( S (= \frac{dQ}{dx} \text{ or } \frac{dQ}{dz}) \) is always greater than zero. The first example is readout system non-linearity. For cathode readout this may be due to sampling the induced charge distribution too coarsely. This may be because sensing elements or the patterns of progressive geometry methods are too large in scale with respect to the distribution. Systematic non-linearity also occurs inevitably when a significant portion of the charge distribution is "lost" beyond the ends of the sensing cathode. The methods of finding the centroid of the charge samples can in their turn introduce non-linearity. Uniform RC and LC lines may be arranged for an optimum linearity, but the uniform capacitive line has inherent non-linearity. Direct centroid finding methods will show non-linearity if too few samples are included in the algorithm. The second important cause of non-linearity in \( Q \),
occurring in the z axis only, is the restriction of avalanches to anode wire positions, leading to anode wire modulation.

(b) Multiple values of Q. Although strictly systematic, these imaging imperfections cannot be compensated because they involve a dependence of Q on some variable which is not recorded. This unknown parameter is the position of absorption in the y axis (normal to the electrode planes) in the cases of the cathode wire modulation effect, and the "blurring" that arises when the incident X-rays are not travelling parallel to the y axis. The parameter is the avalanche angle centroid in the case of the multiple imaging that can occur with highly non-linear cathode systems.

(c) Random fluctuations in Q. These are due to the effects which limit the spatial resolution of the detector: photo- and Auger electron range, re-absorption of fluorescent X-rays in the counter gas, diffusion of the limited number of primary electrons, secondary avalanches due to electrons ejected from metal surfaces by uv photons, and electronic noise in the readout.

For cathode readout systems, which are widely used, a good knowledge of the induced charge distribution is required for the prediction of the systematic non-linearity of type (a). Expressions were developed in Chapter 3 which allow the calculation of the charge induced on any wire or group of wires between conducting planes by point charges. Apart from the requirements that the wires and planes all be parallel to one another and that interelectrode distances are much greater than the wire radii, the arrangement of wires may be made arbitrarily. A straightforward extension of the theory permitted the calculation of the charge density distribution (orthogonal to the
wires) on the continuous planes. It was also shown how the basic electrostatic calculations could be extended, by consideration of avalanche angle and spread, ion motion and signal processing, to predict pulse heights such as would be obtained in experimental measurement of induced charge distributions. Chapter 4 described detailed experimental verification of the calculations. The orientation of the other (wire) cathode was found to have negligible effect, and it appeared that the cathode charge distribution parallel to the anode wires was very similar to the orthogonal distribution.

The more complex theoretical problem of the cathode charge distribution parallel to the anode wires was covered in outline in Chapter 3 for the case of continuous cathode planes. A fuller account has been presented by Thompson et al. [79]. Calculations with this model tended to confirm that distributions in the two axes are very similar in shape for practical circumstances. Normalised to equal area, changes in shape of the distribution with time were found to be fairly insignificant at normal processing timescales. The influence of the avalanche angle centroid was marked, however.

Chapter 5 showed that a generalised description of the cathode charge distribution, independent of time and avalanche angle centroid, could be obtained using a simplifying assumption originally due to Gatti et al. [72]. It was also shown that a single parameter analytical description of this generalised distribution is possible for normal MWPC geometries. Consequently there are three useful levels of description of the distribution available, the one chosen to be determined by the degree of prior knowledge of the relevant chamber parameters and the required accuracy:

(i) The simple one parameter analytical formula

(ii) A full electrostatic calculation for the positions reached by
the positive ions after a given time

(iii) A complete description including calculation of charge waveforms and modelling of signal processing.

The distribution on wire cathodes as opposed to continuous cathodes is at present only specifically predictable for the z axis, in other words for cathodes with their wires parallel to those of the anode, although, as mentioned above, there is evidence that this is not a major shortcoming. The one parameter formula is strictly only relevant to continuous cathodes. It was shown that when wire and continuous cathode distributions are presented so that a comparison is possible, the latter is somewhat narrower.

The use of the three options for predicting cathode system linearity was demonstrated in Chapter 6. It was also shown that the function Q, if required for non-linearity compensation, can also be conveniently obtained from an experimental measurement of the response to uniform irradiation, providing imaging imperfections of types (b) and (c) are small.

The test of the response to uniform irradiation under controlled conditions was used to investigate critically the linearity of cathode systems used in previous work at Leicester. The need for a simple cathode system of good linearity and resolution was noted, and addressed in the development of the tailored CR-GD cathode. The tailored CR line and the use of GD sections effectively eliminated systematic non-linearity (type (a)) over the central two-thirds of the cathode without undue complexity. Measured differential non-linearity, $S$, of better than 1% and FWHM spatial resolution better than 0.2% of the sensing length were achieved.
In Chapter 8, it was shown that anode wire modulation could be reduced, as expected, by simply increasing filter time constants. Type (b) imaging imperfection arising when X-rays are absorbed close to cathode wires was briefly investigated.

Another multiple imaging effect, thought to be due to the dependence of the cathode charge distribution shape on the avalanche angle centroid, was observed during attempts to measure electron range contributions to MWPC position resolution. The problem was circumvented, however, and the measurements gave some support for a simple description of the electron range contribution, and established upper limits for the contribution in argon-methane mixtures. FWHM resolutions below 70\(\mu\)m were measured for 1.5keV X-rays at one atmosphere gas pressure.

10.2 Areas requiring further investigation

10.2.1 Extension and application of the present work

An indirect measure of the angular spread of the MWPC avalanche was demonstrated in Section 4.2.5. It seems possible, however, that the large signals observed on an individual cathode wire as positive ions arrive (see Fig. 2.7) could provide more direct information. The spread along the anode wire might also be investigated.

The primary aim in providing well-founded models of the cathode induced charge distribution was to allow accurate prediction of linearity. However, it will also be possible to investigate the suggestion of van Beek et al. [129] that the shape of the distribution
might be used to provide discrimination against background events leaving long primary ionization trails. If position sensing is not also required, then the cathode arrangement could be quite simple. An investigation has been started at Leicester in this area.

It is hoped that the tailored CR-GD electrode system will find application. Apart from MWPC readout, an obvious application is in microchannel plate readout. Simple GD electrodes have been successfully demonstrated in this application [146], but showed significant non-linearity. The potential problem of the high rate performance of the CR line still remains to be investigated, and the basic technique of using GD sections to reduce the number of nodes might be tried with other readouts.

The two suggestions for the reduction of anode wire modulation effects made in Chapter 8, namely a modification of signal processing, and the deliberate use of cathode non-linearity, might repay further study.

There is still a need for reliable electron range measurements in chamber gases. High intensity X-ray beams of well-defined energy, such as are provided by synchrotron sources would facilitate these measurements.

Generally, it should be noted that the present work has been limited to low avalanche charges, and more significantly, low counting rates. Many of the results might require reconsideration in applications where this is not the case.
There would appear at present to be three aspects of X-ray imaging MWPC performance in which a significant improvement would be desirable. The requirements would be:

(i) High quality imaging in both axes, in the direction across as well as along the anode wires

(ii) A very high rate capability, to deal with the fluxes available from synchrotron sources and to allow fast time resolved measurements

(iii) An improvement in energy resolution.

It seems likely that a combination of the first two requirements may in fact only be physically achievable in a parallel plate geometry, although the signal processing problems are probably at least as formidable as the physical ones. The attractions of the multiwire structure (Section 8.1) are considerable, though, and a high rate MWPC with good two-dimensional imaging properties would be a very useful device.

The parallel plate structure also seems to be the most promising for the improvement of energy resolution through the use of Penning gas mixtures [147-149]. There have been two recent advances in proportional counter energy resolution which might be applied to MWPCs, however. Kosarev et al. [150] have demonstrated that when a large number of counts can be collected from a given X-ray source spectrum, then that spectrum may be reconstructed to a very good resolution using statistical techniques, based upon knowledge of the attenuation of the radiation in the counter. It is conceivable that a stack of thin MWPCs could combine this method with an imaging capability. Siegmund et al. [151], using a parallel plate chamber,
showed that it is possible to discriminate avalanches due to individual primary electrons for low energy X-rays, through the detection of light emitted by excited species in the avalanche process. Thus by counting the number of electrons, the factor f is removed from Eq. 2.10 and the energy resolution is thereby improved. Anderson and Charpak [152] demonstrated the technique in an MWPC, but this promising line does not appear to have received further attention. As some preliminary calculations have indicated that with the fastest available amplifiers, individual electron avalanches might also be discriminated in the normal charge signals, this area could well repay further study.
APPENDIX

POTENTIAL FUNCTIONS BETWEEN PARALLEL CONDUCTING PLANES

A.1 Potential function due to a uniform line charge

\( q \) = \text{uniform line charge}

\( p \)-plane

\( w \)-plane

\( \omega = U_0 + jV_0 \)

\( w_0 = U_0 - jV_0 \)

Fig. A1 Conformal mapping for the problem of a uniform line charge between infinite parallel planes. (a) \( p \)-plane (b) \( w \)-plane

The geometry is illustrated in Fig. A1(a), and may be conveniently described using complex co-ordinates, \( p = x + jy \), or \( p = z + jy \) if the final potential function is to be in the plane orthogonal to the anode wires. Both cases are equivalent at this stage. The problem is made soluble by use of the mapping function

\[
W = je^{\pi p/2l} = U + jV
\]

which maps both planes onto the \( u \) axis, and the space between into the upper half plane, as shown in Fig. A1(b).

Consider a line charge of linear density \( q_0 \), passing through the point \( p_0 \). This will map to a line charge \( q_\omega \) passing through the point \( w_\omega = je^{a p_0} \), where \( a = \pi/2l \). Now we may apply the theory of images to find the complex potential in the \( w \) plane as
\[ W = P + j \phi = -\frac{q_o}{2\pi \varepsilon_o} \ln (w - w_o) + \frac{q_o}{2\pi \varepsilon_o} \ln (\bar{w} - w_o^*) \]

where \( P \) and \( \phi \) are the potential function and flux function respectively. \( \ln (w - w_o) \) and \( \ln (w - w_o^*) \) are the respective distances to the general point \( w \) from the line charge and from its image, which has linear charge density \(-q_o\) and passes through the point \( w_o^* \). Reverting to the \( p \) plane, we obtain

\[ W = -\frac{q_o}{2\pi \varepsilon_o} \ln \frac{\sinh(a(p - p_o)/2)}{\cosh(a(p - p_o^*))/2} + \frac{q_o}{2\pi \varepsilon_o} j \frac{a(p - p_o^*)}{2} \]

Taking the real part as the potential function, we eventually obtain

\[ P = -\frac{q_o}{2\pi \varepsilon_o} \left[ \frac{\cosh(a(x - x_o)) - \cos(a(y - y_o))}{\cosh(a(x - x_o)) + \cos(a(y + y_o))} \right]^{1/2} \]

A.2 Potential function due to a narrow strip of one of the planes

This situation is illustrated in Fig. A2(a). We may again use the mapping function

\[ W = je^{\alpha p} \]

to simplify the problem (Fig. A2(b)). In order to use the reciprocity theorem, the narrow strip is considered to be at unit potential, and the remainder of that plane and the other plane (together becoming the remainder of the \( u \) axis after the conformal mapping) to be grounded. This will give rise to a complex potential, and we may choose the potential function to be the imaginary part of this. We must be able to satisfy the boundary conditions that \( P = 0 \) on the \( u \) axis except between \( u_1 \) and \( u_2 \) where \( P = 1 \). Then if we can find a solution, we know from the uniqueness theorem (see Smythe [78], for example)
that it is the unique solution.

**(a) p-plane**

**(b) w-plane**

![Diagram](image)

Fig. A2 Conformal mapping for the problem of a narrow strip of one plane at unit potential, the remainder of that plane and the other plane at earth. The mapping is the same as used in Fig. A1.

(a) p-plane (b) w-plane

---

We choose a suitable form

\[ W = \phi + jP = A \ln |w-u,| + B \ln |w-u,| + jC \]

\[ = A \ln |w-u,| + B \ln |w-u,| + j(A \Theta_1 + B \Theta_2 + C) \]

The constants follow immediately from the boundary conditions

\[ \Theta_1 = \Theta_2 = 0, \quad P = 0 \quad \Rightarrow \quad C = 0 \]

\[ \Theta_1 = \pi, \quad \Theta_2 = 0, \quad P = 1 \quad \Rightarrow \quad A = 1/\pi \]

\[ \Theta_1 = \Theta_2 = \pi, \quad P = 0 \quad \Rightarrow \quad B = -1/\pi \]

so that

\[ W = \frac{1}{\pi} [\ln(w-u,) - \ln(w-u,)] \]

Now let the width \( x_2-x_1 \) become the infinitesimal width \( dx_1 \), since we eventually require a continuous charge distribution,

\[ dW = \frac{1}{\pi} \frac{du_i}{w-u_i} \]

where

\[ du_i = a_u_i dx_1 \]

so

\[ \frac{dW}{dx_1} = \frac{a}{\pi} \frac{u_i}{w-u_i} \]
Reverting to the $p$ plane, after some manipulation we obtain:

\[
\frac{dW}{d x_i} = \frac{a \sin ay - e^{a(x_i - x)}}{2\pi} + j \cos ay
\]

Taking $dP/dx_i$ as the imaginary part finally gives:

\[
\frac{dP}{d x_i} = \frac{1}{4} \frac{\cos ay}{\cosh(a(x_i - x)) - \sin ay}
\]

For the case where there is a conducting plane at $y = 0$ and we wish to find the potential function due to a strip of a parallel plane at $y = +1$, the same procedure is employed, except in that the mapping function

\[ w = e^{\pi p/\ell} \]

is used to map the space into the upper half plane. The potential function in the $x$-$y$ plane is obtained in this case as:

\[
\frac{dP}{d x_i} = \frac{1}{2} \frac{\sin 2ay}{\cosh(2a(x_i - x)) + \cos 2ay}
\]
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The following abbreviations are used:

IEEE Trans. Nucl. Sci. : Institute of Electrical and Electronic Engineers Transactions on Nuclear Science
ISA Trans. : Instrument Society of America Transactions
J. Appl. Cryst. : Journal of Applied Crystallography
Nucl. Instr. and Meth. : Nuclear Instruments and Methods
Phil. Mag. : Philosophical Magazine
Rev. Mod. Phys. : Reviews of Modern Physics
Rev. Phys. Appl. : Revue de Physique Appliquee


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THE IMAGING PERFORMANCE OF MULTIWIRE PROPORTIONAL CHAMBERS

by J.S. Gordon

ABSTRACT

A simplified description is given of the operation of a multiwire proportional chamber (MWPC) in the soft X-ray imaging application. Expressions are developed to allow the calculation of the distribution of induced charge on the cathodes of an MWPC. With extensions to permit direct comparison, the calculations are subjected to detailed experimental verification. A generalised, approximate formulation of the distribution with one independent parameter is described. The prediction of cathode system position response using the theoretical distributions is demonstrated.

The available MWPC position readout methods are reviewed, and where possible their differential non-linearity is measured experimentally. A new position-sensitive cathode of good linearity and spatial resolution is presented. The effect of the wires of an MWPC on its imaging performance is briefly considered.

An attempt is made to assess the contribution to MWPC spatial resolution of the range of the electrons produced initially by an X-ray absorption event in argon-methane mixtures. In conclusion, the important causes of MWPC imaging imperfection are noted and classified.