Instability of political preferences and the role of mass media: a dynamical representation in a quantum framework

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We search to devise a new paradigm borrowed from concepts and mathematical tools of quantum physics, to model the decision-making process of the US electorate. The statistical data of the election outcomes in the period between 2008 and 2014 is analysed, in order to explore in more depth the emergence of the so-called divided government. There is an increasing urge in the political literature which indicates that preference reversal (strictly speaking the violation of the transitivity axiom) is a consequence of the so-called non-separability phenomenon (i.e. a strong interrelation of choices). In the political science literature, non-separable behaviour is characterized by a conditioning of decisions on the outcomes of some issues of interest. An additional source of preference reversal is ascribed to the time dynamics of the voters’ cognitive states, in the context of new upcoming political information. As we discuss in this paper, the primary source of political information can be attributed to the mass media. In order to shed more light on the phenomenon of preference reversal among the US electorate, we accommodate the obtained statistical data in a classical probabilistic (Kolmogorovian) scheme. Based on the obtained results, we attribute the strong ties between the voters non-separable decisions that cannot be explained by conditioning with the Bayes scheme, to the quantum phenomenon of entanglement. Second, we compute the degree of interference of voters’ belief states with the aid of the quantum analogue of the formula of total probability. Lastly, a model, based on the quantum master equation, to incorporate the impact of the mass media bath is proposed.
1. Introduction

Large portions of an electorate simply don’t have meaningful beliefs, even on issues that have formed the basis for intense political controversy among elites [1, p. 245].

There has been an ever-growing interest in the US political literature in the emergence of the so-called divided government. The splitting of executive and legislative powers between the two major parties is resulting in a political ‘gridlock’ (e.g. the decisions of the President are blocked by the votes of the Congress and vice versa). Such power division has been periodically occurring over the last 40 years. This phenomenon is rooted in voters’ behavioural patterns. The voters split their preferences between the Democrats and the Republicans, when casting ballots in the Presidential and the House & Senate elections.

If we look to the recent literature on the formation of political preferences, we note that the phenomenon of non-separability of choices has been outlined as a key explanation for such type of preference reversals, see [2–8]. Non-separability is defined in political science as an interaction between different choices. The definition of non-separability in Smith et al. [7, p. 739] states that ‘respondents exhibit non-separability by conditioning their views about partisan control of the Congress on hypothetical partisan outcomes for the presidency’. Another definition is provided in Lacy [4, p. 240], ‘A person has non-separable preferences when her preferences on an issue or set of issues depend on the outcome of other issues.’ The non-separability phenomenon is usually understood in a Bayesian fashion: the choices of the voters are conditioned on her own choices on other issues, or on the outcomes of related political issues. This type of behaviour makes it impossible to measure the voters’ preferences in isolation. We could claim that this effect leads to the emergence of a Divided Government in the US political system (see for a broader discussion and an analysis [3,4,7]). Furthermore, [2,4,9] show that the non-separability is manifest in humans’ ranking the different combinations of preferences in a mode that is not consistent with the axiomatic formulation of modern normative decision theories (i.e. the completeness and the transitivity axioms of Von Neumann and Morgenstern’s expected utility [10] are violated with such ranking pattern).

Non-separability as an empirical phenomenon is most often researched in the context of surveys and interviews in many fields of social science from consumer behaviour to politics. In the political literature, various opinion polling questionnaires have been developed to explore in more depth the existence of independence between respondents’ choices (see for instance [5–7] and others). If an order effect is detected, so that the decision on some issue changes, given there is some new information, it can be interpreted as a proof of the emergence of a non-separability effect in the responders’ preferences.1 A more robust approach suggested in reference [4], is to ask the respondents to perform a ranking of all possible combinations of the issues’ outcomes (events in the probabilistic framework) ranging from the most preferred to the least preferred combination. This approach is limited to the possibility of ranking only a few combinations of events and the outcomes of the events have to be certain. Otherwise, the complexity of the decision tasks (i.e. the amount of combinations and their probabilistic nature) makes it difficult to perform an accurate measurement of the respondents choice preferences.2 Morris Fiorina [3], was the first to point out that the non-separability effect may be triggering the emergence of ‘ticket-splitting’ in the US Federal elections. He advocated that the ticket-splitting is a strategic act of

1Ironically, it is implicitly assumed that a strategically thinking person is able to update new information in a Bayesian scheme, as well as correctly estimate the likelihood of baseline probabilities of outcomes in the condition of uncertainty in order to form homogeneous subjective beliefs. At the same time, it has been shown in the political literature that the new information on the issues of interest provides a clearer incentive to carry out a strategic voting scheme. See the so-called paradox of multiple elections by Brams et al. [2]. In other words, in reality it can be a more complicated task to think strategically and to form non-separable preferences in the condition of uncertainty. These findings resonate with the results of scholars in psychology, e.g. [11,12], which reveal irrational behaviour (that deviates from the assumptions of modern decision theories) in peoples’ judgements and decisions when uncertainty is present.

2The number of issues to consider adds an additional level of complexity that may result in the heuristic behaviour of voters. The voters are not able to evaluate all the events in a joint state space.
a careful conditioning of the election outcomes by some part of the US electorate, in order to achieve a desired goal in terms of political power distribution. Other scholars are questioning the purely strategic origins of the non-separability phenomenon and are pointing to other contextual factors that can contribute to the emergence of the observed results. These ‘hidden factors’ can be various biases that surface through, e.g., memory impact and the overall time dynamics, characterized by processing of the new information. For a broader discussion consult [13]. In line with the previous work, [4] confirms the undeniable impact of the time dynamics on voters’ beliefs, by putting forward that ‘Temporal shifts in survey responses may arise from sources other than the non-separable preferences (generated by the question order effect as defined above). Respondents may gather new information that shifts their induced preferences’ [4, p. 252]. The above-discussed studies unearth the potentially important contribution, which may emerge if we explore the origins of voters preference reversal, especially if we take into consideration the difficulty to empirically detect non-separability outside the remit of opinion polling studies.

(a) Non-separability: a special type of preference reversal

Thus far, we have only noted the existence and briefly sketched the nature of the phenomenon of non-separability, defined by the voters’ ability to make two or more complex decisions that are not isolated from each other. There are some additional features of the phenomenon that are important to mention: the direction of non-separability as discussed in reference [9] i.e. $A \rightarrow B$, or $B \rightarrow A$. The direction of non-separability can be further split into a positive or a negative relationship between the choices, so that a ‘yes’ answer on one issue automatically generates a ‘no’ answer on another related issue. Naturally, this can also be a two-way direction, coined reciprocity, i.e. $A \leftrightarrow B$.

An example in reference [4], illustrating voters’ preference rankings in the House and Presidential elections gives an understanding of the distinction between separability (preferences that are emerging in isolation) and non-separability (interdependence of preferences):

— Completely separable preferences of a partisan Democrat in the Presidential and House elections would entail a preference ranking: $(DD) > (DR) > (RD)$ (or $(RD) > (DR)) > (RR)$. A reverse ordering for a Republican partisan.

— Completely non-separable preference ranking of a voter who (i) prefers a Divided Government, no matter which party wins House/Presidential elections: $(RD) > (DR)$ (or $(DR) > (RD)) > (DD) > (RR)$ (or $(RR) > (DD))$ (ii) Preferences for the same party to control both powers, no matter which party it would be: $(DD) > (RR)$ (or $(RR) > (DD)) > (RD) > (DR)$ (or $(RD) > (DR))$.4

Non-separability is modelled in references [3–5,7,9], by accommodating events’ outcomes (more specifically the voters choices) in a $n$-dimensional Euclidean outcome space. This allows for the measurement of the degree of non-separability effect along the dimensions of this space, i.e. how the voters interrelate the issues’ outcomes, based on the importance for them. For instance, Lacy [4] proceeds through a depiction of possible preference sets (so-called tuples) and their ranked combinations that are manifest for separability/ non-separability, which in its turn can be classified by different levels of its strength. These models are modifications of the original spatial model of voting proposed by Kramer [8], who proceeded with the modelling of separable

\[3\] The feature of reciprocity closely resembles the quantum effect of entanglement, where a measurement on the systems’ quantum (belief states in our model) states is always producing correlated outcomes. We give a more technical definition of the complex phenomenon of entanglement and its mathematical formulation in the §5c(ii).

\[4\] The second case is more difficult to detect in the context of real elections: the voters that do not exhibit a violation of the transitivity axiom would normally be considered as voters with separable preferences, because only their most preferred outcome combination is observed. A complete ordering shows that separable preferences can be naturally distinguished from observing the partisan preferences (DD) or (RR) being ranked always as the least/most preferred options of the choice set.
preferences as points in a Euclidean vector space. Some limitations of the representation of the preference combinations in the above-mentioned models, as noted by Zorn & Smith [14] are the static nature of the models and the omission of the uncertainty associated with the events’ outcomes. Another problem with modelling non-separability is related to its true origins that would need further exploration, i.e. how exactly are the belief states of voters forming these non-separable preferences.5

The work by Zaller [15] is approaching the problem of the dynamics of the formation of mass opinion, with the development of the so-called receive-accept-sample (RAS) model. This is in a sense a dynamical model that pertains to the issue of preference reversal over time. Zaller [15] shows by analysing and interpreting the results obtained from multiple surveys and opinion studies that the beliefs of the respondents are not fixed, but are vacillating over time. The vacillation as a type of wave dynamics of the ‘pro’ and ‘con’ arguments is according to Zaller [15] rooted in the ‘existence of ambivalence in peoples’ reactions to issues’. These ideas are closely reflecting the philosophy of quantum mechanics, penetrated by the presence of contextuality, where the properties of the quantum system emerge through the act of measurement. Another complex quantum effect is the notion of entanglement characterizing a quantum non-separability that was suitably coined the ‘pseudo non-separability’ by Zorn & Smith [14] to reflect the distinction between its understanding in political science and its quantum physical meaning.

(b) Contributions of quantum (like) models

The quantum framework has at its command a very powerful mathematical and conceptual alternative to the classical probabilistic decision-making models which are related to the usage of the classical Kolmogorovian state space of additive events’ probabilities. Quantum models are appealing to the application of a complex vector space, the Hilbert space, with orthogonal bases assigned to the observables. The quantum-like models explain well the existence of an indeterminate state of the decision-maker through the quantum effect of superposition. They also represent the intrinsic inseparability of beliefs through encoding them by entangled states. The notions of superposition and entanglement that are playing a central role in the quantum formalism capture the non-additivity of probabilities and the fundamentally strong interrelation of the belief states. In addition, new information (for instance, a question or the act of voting) is irreversibly changing the belief state of a person and may be incompatible with her previous state of the mind. The origins for this effect are rooted in the incompatibility of observables associated with the different pieces of information or questions. They cannot be modelled as random variables in a joint probability space of the classical probability theory. The phenomenon of incompatibility of observables is encompassed in the famous ‘uncertainty principle’ introduced by Heisenberg [16]. Below, we will now present the main directions of the interdisciplinary application of quantum models and their generalizations so far. We focus primarily on the advances in the application to the decision-making and more specifically, decision-making in politics.

The phenomenon of non-separability of voters preferences in the context of US political elections has been accommodated in a quantum framework by a group of researchers. The pioneers in this interdisciplinary field are [14], followed by Khrennikova and co-workers [17–19]. The usage of the tools from quantum physics has been motivated by the improved explanatory

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5We can witness that the linear space representation of states is widely used in politics, as well as in cognitive science and psychology. In the quantum-like models introduced in this work, we proceed by representing the states in a Hilbert space. On first sight, the proposed framework carries some similarity with the general linear space modelling in politics. However, we have to emphasize a crucial difference. In the quantum-like model here, the state vectors bear a straightforward probabilistic interpretation in line with the ideology of quantum mechanics. The novelty of the usage of a Hilbert space representation of observables is motivated not by the use of a linear space structure by itself, but by a coupling to the calculus of probabilities (more specifically their amplitudes)—the quantum probabilities. In addition, the use of complex numbers as building blocks, makes the proposed model more powerful, allowing to encode a wider class of interference phenomena that characterize a deeper type of uncertainty than the classical uncertainty. We pinpoint that even a Hilbert space model with real numbers that is linked to a quantum probabilistic interpretation of states would allow to capture the non-trivial quantum effects, i.e. such as entanglement and superposition.
power of the quantum probabilities for depicting the information impact on voters’ belief states and the way the information is interacting with these states. The notion of superposition that entails an interference of the belief probabilities was shown to provide a competing explanation for the complex evolution of beliefs to the final state of firm opinions. The idea of using quantum concepts and modelling tools has shifted in the last 10 years from being something exotic to becoming a more established and rapidly growing field. There is some work directly pertaining to the application of quantum physical models to decision-making processes in politics: Aerts [20] planted the flag in the area of application of quantum models to political contexts, by proposing the usage of quantum random generators to establish a fair decision-making procedure in a democracy. More recently, Bagarello [21] focused on modelling decision-making of political parties in the process of formation of political alliances within a quantum field framework.

The field of applications of models inspired by quantum mechanics to decision-making is growing in many related fields of social science. Some of the studies focus on various information processing fallacies, such as order effects in surveys and opinion polls; disjunction effect and conjunction fallacies of judging the likelihoods of events; various biases such as uncertainty avoidance and cognitive dissonance that lead to violations of the axioms of modern decision theories. Some of the contributions to this rapidly developing field can be found in references [18,19,22–35]. There is one common course in the above strands of research, namely the aim to capture the contextuality of human reasoning. An evolution from a static (geometric) representation of events in the Hilbert space depicting dynamical decision-making processes with the aid of the Schrödinger model [36–38] and its more complex extension, that is incorporating the impact of the external environment by the model of open quantum systems [39,40], opened new horizons in the domain of modelling dynamical decision-making processes under uncertainty. We can also observe growing interest in explaining the puzzling features that emerge in human cognition in various contexts by an existence of features similar to quantum entanglement, [14,41–44]. For a methodological and introductory outlook, consult [23,24,29,30].

In §2, we search to address the more complex process of voting in the real US elections by performing an analysis of voters’ choice frequencies that we collect from various statistical sources. We present the summary of the last election outcomes for the years 2008–2014 for the Presidential (2008, 2012) and Congress (2008, 2010, 2012, 2014) election contests. We search to explore in more detail the emergence of non-separability, related to uncertain events as opposed to the conditioning of choices on the known outcomes of the Presidential elections. In §3, we analyse the data in order to assert if any preference reversal among voters took place. We embed the statistical data in a Kolmogorovian [45] probabilistic framework (also known as classical probability), to identify the presence of a strategic behaviour, based on a Bayesian conditioning of upcoming information. In line with the existing studies in political science, we conceive that the manifestation of non-separability in the Presidential and Congressional voting contexts can be of a more complex nature than just being the result of a Bayesian conditioning of outcomes. In particular, we detect that the conditional probabilities of the Congress’ preferences (by party) show subadditivity in respect to the marginal probability in the framework of the so-called law of total probability (LTP) [45], that conceptualizes the total probability of a choice given its various distinct realizations.

In §4, we discuss, whether the informational ‘bath’ from the mass media amplifies the preference reversal. In §5, we motivate the application of the (dynamical) quantum apparatus, borrowed from the literature of open quantum systems, to model the decision-making process of the voters, until the final decision is reached. We also motivate the applicability of this model by matching the psychological and quantum physical factors in §5b. We build a quantum representation of entangled belief states, which characterize well the phenomenon of preference non-separability. Furthermore, we apply the quantum analogue of the LTP and obtain the so-called interference term that illustrates the degree of non-classicality in voters’ beliefs. Finally, in §6, we discuss some of the considerations related to the analysed statistical data and the suggestions for further applications of the proposed framework for capturing the voters’ decision-making processes.
2. Statistics on the US election outcomes

Before we outline the methodology of this study and present the statistical data on the US elections, we highlight some of the main features of the US election process. The elections to the White House take place every 4 years and the elections to the House of Representatives and the Senate occur every 2 years. The House of Representatives, whose members are directly elected by popular vote, experience a complete turnaround from the results of each election, whereas only one-third of the congressmen in the Senate are due to be elected each 2 years. The elections to the Congress in between the Presidential elections are usually called in the literature and mass media the ‘midterm’ elections. See reference [46] for an overview of the history of American politics. We will not go in details on the features of the US voting scheme in this paper, because we are merely interested in the voters’ overall party preferences and not in their preferences for the concrete President or congressman.

In order to be able to track the changes in voters’ preferences and the degree to which the conditioning of election outcomes occurs, both in the simultaneous elections and in the midterm elections, we searched to analyse the latest voting statistics between the years 2008 and 2014. In order to obtain the concrete ‘conditional frequencies’, we considered an additional large-scale study carried out in the years 2011–2012. The data from this study enabled us to measure not only the overall changes in voting activity between the years 2008 and 2010, but also to detect how many of the voters from the 2008 Presidential elections exhibiting Republican or Democratic partisanship remained partisan or alternatively became bi-partisan. The statistics between 2012 and 2014 have been analysed in order to detect the emergence of the overall preference reversal. The data for the whole US population voting statistics are extracted from references [47–51]. The statistics on the conditional frequencies for each subgroup of voters are obtained from references [52,53].

Below, we present the summarized data that show the affiliation of voters by party. In order to simplify the analysis (i) we excluded the data on voters who cast ballots for other parties and candidates than the Republicans and Democrats, (ii) we merged together the statistics of the outcomes of the House of Representatives and the Congressional elections, to obtain the cumulative frequencies by party, based on the total amount of voters summed across these two election contests. This approximation is motivated by the rather similar connotation these elections have for the voters (i.e. in both contests the voters cast ballots for some representatives of their states that execute legislative power). The election design is also similar in respect to the regularity of these elections. The contingency tables 1 and 3 illustrate the summary of statistics for the Federal Elections in the year 2008 held for both the Presidential (with candidates, McCain and Obama) and the Congressional elections on 4 November 2008 and the year 2012 (candidates Romney and Obama) held on 6 November 2012. Tables 2 and 4 summarize the outcomes of midterm Congressional elections. The dates for these elections are 2 November 2010 and 4 November 2014. We remark that the statistical data on 2014 elections is not officially published yet. Below are the four contingency tables that summarize the number of ballots cast for either the Republicans or Democrats. The numbers below the percentages denote the actual number of voters who voted for a particular party in each election contest.

Because we are dealing with roughly the whole population of voters in the above statistics (the eligible voting population), i.e. we do not use a drawn sample, we assume that the observed

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6In appendix A there is a detailed table with the statistical data by popular vote, including the results for the Presidential elections, Senate elections and the House of Representatives elections, with an indication of all the original sources of data for each election contest.

7These are voters who voted for, e.g. ‘Libertarian’, ‘Independent’ and other parties. The number of this part of the US electorate in relative terms is not significant. In 2008, these voters accounted for 1.42% in the Presidential election, 5% in the Senate election and 4.9% in the House election. At this stage of the study we omit these voters from our analysis to reduce the complexity of election outcomes.

8As mentioned above, the number contains the total amount of voters who participated in both the House and Senate elections.
Table 1. 2008 Federal election outcomes.

<table>
<thead>
<tr>
<th></th>
<th>president (P)</th>
<th>congress (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_r$ $P_d$</td>
<td>$C_r$ $C_d$</td>
</tr>
<tr>
<td></td>
<td>46.31% 53.69%</td>
<td>45.19% 54.81%</td>
</tr>
<tr>
<td></td>
<td>59 948 100</td>
<td>82 872 183</td>
</tr>
<tr>
<td></td>
<td>69 498 100</td>
<td>100 502 644</td>
</tr>
<tr>
<td>total:</td>
<td>129 446 200</td>
<td>total: 183 374 847</td>
</tr>
</tbody>
</table>

Table 2. 2010 Congress election outcomes.

<table>
<thead>
<tr>
<th></th>
<th>congress (house + senate)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_r$ $C_d$</td>
</tr>
<tr>
<td></td>
<td>52.76% 47.24%</td>
</tr>
<tr>
<td></td>
<td>80 721 693</td>
</tr>
<tr>
<td></td>
<td>72 263 804</td>
</tr>
<tr>
<td>total:</td>
<td>152 985 497</td>
</tr>
</tbody>
</table>

Table 3. 2012 Federal election outcomes.

<table>
<thead>
<tr>
<th></th>
<th>president (P)</th>
<th>congress (C)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_r$ $P_d$</td>
<td>$C_r$ $C_d$</td>
</tr>
<tr>
<td></td>
<td>48.04% 51.96%</td>
<td>46.98% 53.02%</td>
</tr>
<tr>
<td></td>
<td>60 933 500</td>
<td>97 672 114</td>
</tr>
<tr>
<td></td>
<td>65 915 796</td>
<td>110 251 389</td>
</tr>
<tr>
<td>total:</td>
<td>126 849 296</td>
<td>total: 207 923 503</td>
</tr>
</tbody>
</table>

Table 4. 2014 Congress election outcomes.

<table>
<thead>
<tr>
<th></th>
<th>congress (house + senate)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_r$ $C_d$</td>
</tr>
<tr>
<td></td>
<td>53.18% 46.82%</td>
</tr>
<tr>
<td></td>
<td>62 406 755</td>
</tr>
<tr>
<td></td>
<td>54 932 808</td>
</tr>
<tr>
<td>total:</td>
<td>117 339 563</td>
</tr>
</tbody>
</table>

differences in the frequencies did not occur by chance and in fact reflect the changes in voters’ behaviour.9

In addition to the overall voting statistics, we present another contingency table 5 with the data obtained from the Cooperative Campaign Analysis Project. The data provide us with ‘conditional frequencies’ (that will be approximated by probabilities in §3).

9If one were to go into more depth on this level of analysis, we would have to account for the differences in the eligible voting population during different election periods, as well as the changes in their voting activity. It can be noted that some voters are voting in ‘cycles’ and their voting activity drops in the midterm elections. Unfortunately, at this stage, we are not able to ‘filter’ out the voters who change their preference for instance from preferring a Democratic Congress to not voting at all. This type of analysis can be more easily performed with statistics from opinion polls. Such an analysis is done in reference [18], based on the data from reference [7].
Table 5. Statistics on conditional preferences of US electorate in 2010. (Online version in colour.)

|                         | bi-partisan: (C₆|P₇), (C₇|P₈) | partisan: (C₉|P₉), (C₈|P₈) | did not vote |
|-------------------------|----------------------------------|-----------------------------|--------------|
| voted (P₇) in 2008 (%)  | 6                                | 65                          | 28            |
| voted (P₈) in 2008 (%)  | 6                                | 76                          | 17            |

The table is sourced from the *New York Times* [53]. The results of this study, with \( N = 45\,000 \) are reported in reference [52] as being statistically significant with respect to the whole US population.

3. Analysis: non-separability in classical probability framework

Let us look into the statistics illustrated in tables 1–5 and analyse in more detail how the conditioning of the voters preferences takes place. In order to embed the statistics in a probabilistic framework, we approximate the frequencies from tables 1–5 by probabilities.

We stress that the Bayesian approach plays a key role in classical decision theories (which is related to rational postulates of modern decision theories under objective and subjective uncertainties). Based on the axiom of rationality, each individual is acquiring and analysing the information in a systematic way, be it a complete certainty about an outcome, or its probabilistic expression. The simplest definition of the formula of conditional probability (also known as Bayes’ formula) for an event \( C₆ \) (Democratic Congress) conditioned upon the event \( P₇ \) (a Democrat President) can be written as

\[
p(C₆|P₇) = \frac{p(C₆ \cap P₇)}{p(P₇)}. \tag{3.1}
\]

Please see also [45, p. 6].

The probabilistic scheme of equation (3.1) can be used by a strategically thinking voter to update her beliefs and preferences in the light of some new information. We remind that separable preferences are characterized by statistical independence of the unconditional probabilities for the event \( C \) from the upcoming information. As such one obtains \( p(C₆,r) = p(C₆,r|P₇) = p(C₆,r|P₈) \). If the conditional probabilities are different from the marginal probabilities, we are assuming non-separability, according to the political literature, cf. [7,14].

Before we derive the total likelihood of an outcome (i.e. a voter could have chosen a particular party in the Congressional elections by a Bayesian updating of information), we search to answer the question whether any preference change occurred at all in the different election periods. Can we infer that a preference change took place among voters? First of all, we can see from tables 1 and 3 that the frequencies of voters who voted for some party in the Presidential elections differ slightly from the frequency of voters who decided for the same party in the Congressional elections. This means that in line with, e.g. [2,9], studies, the non-separability is present even in simultaneous decision-making tasks. The problem is that both elections are held on the same day. Consequently, as noted in reference [2], the voters do not have enough information on the outcomes of the Presidential elections to strategically condition their preferences for Congress upon it (or vice versa). In line with this conjecture, the voters are supposed to hold consistent preferences in the simultaneous elections, i.e. vote for the same party. An important point one has to make is that the consistent preferences can still bear a non-separable character. Remarkably, the preference reversal becomes more magnified when we look at the outcomes of the midterm Congressional elections. Such a state of affairs could imply that (i) as the outcomes of Presidential elections are known, the voters more confidently condition their preferences for Congress upon

10It is fair to say that there is a high number of opinion polling studies that can provide the voters with approximate probabilities of the election outcomes. This is one of the reasons why voters, formally being in a condition of uncertainty, still hold some implicit beliefs about the election outcomes. We are not able to analyse in depth whether such conditioning is strategic or not, because we do not have any accurate data on the voters’ subjective probabilistic estimates.
the Presidential election outcomes11; (ii) during the time interval of 2 years a large amount of new information reaches the voters that amplifies their preference reversal, i.e. the impact of time dynamics. In order to analyse in more detail if the preference reversal is a consequence of (i) or (ii), we first compute the size of preference reversal. Based on the results from the tables 1–4, there exists a slight (about 1–2%) preference reversal in terms of party affiliation in the simultaneous elections. In the midterm elections, the preference reversal becomes quite substantial. We observe that between 2008 and 2010 the party affiliation in the Congress elections switched by 7.57%, i.e. the difference between the marginal probabilities \( p(C_d(2010)) - p(C_d(2008)) \) is \( \delta_p = -0.0757 \). The change of preferences in respect to the Republican Congress is logically of the same size, with a positive sign, because we restricted our observables to dichotomous outcomes.

For the years 2012 and 2014, the difference between the marginal probabilities \( p(C_d) \) was \( \delta_p = -0.0622 \) (a positive sign for the Republican Congress outcome probability). Because we are dealing with the whole population of voters, we can articulate that these differences are large enough to indicate a substantial preference reversal. We stress that the preference reversal occurrence is in fact more complex than just switching preferences between two parties, because there is also the ‘no voting’ electorate that similarly can alter their preferences from voting to not voting in the years 2008–2014. As such, this subpopulation of ‘non-voters’ could contribute to the changes of the voting statistics, implying that not only changes in party affiliation occurred, but also changes in the overall voting activity. We hope to approach the problem of ‘non-voters’ in one of the future publications.

Next, we formulate the LTP [45] to observe if the conditional probabilities of some random variables (in the context of this analysis, the \( C \) conditioned on \( P \)) follow the principle of additivity in respect of the total probability of the realization of \( C \). LTP formalizes in rational decision theories how a classical probabilistic scheme can accommodate the probabilistic conditioning of events that occurs in a Bayesian mode. We denote the total probability of an event that should be obtained by LTP, by the letter \( q \):

\[
q(C_d) = p(C_d \cap P_d) + p(C_d \cap P_r).
\]

(3.2)

We can rewrite the LTP through

\[
q(C_d) = p(P_d)p(C_d|P_d) + p(P_r)p(C_d|P_r).
\]

(3.3)

The frequencies for the outcomes of the Presidential elections are approximated by probabilities and denoted as \( p(P_d) \) and \( p(P_r) \). Similarly, \( C_d \) denotes the outcome of the Republican Congress and \( C_d \) the outcome of a Democratic Congress. We insert the data into (3.3), to detect if the non-separability that is attributed the key role in the preference reversal among the voters is indeed an act of strategic conditioning of the probabilistic information in a Bayesian mode. We perform only such an analysis in respect to the outcome \( C_d \), where a similar check can be performed for the outcome \( C_r \). We take the 2010 year statistics (table 2) to obtain the total probability \( q(C_d) \) that should be the same as the marginal probability \( p(C_d) \) in 2010. Next, we extract the probability \( p(P_r) \) and \( p(P_d) \) of our priming variable from the 2008 Presidential elections in (table 1). In the dichotomous setting of the two parties, the marginal probabilities in each election context sum up to unity, \( p(C_d) + p(C_r) = 1 \), as well as \( p(P_r) + p(P_d) = 1 \). The conditional probabilities \( p(C_d|P_d) \) and \( p(C_d|P_r) \) for the 2010 elections are extracted from table 5.

To be more concrete, we search to explore with the aid of (3.3) whether the percentage of voters who voted, e.g., for Democratic Congress equals to the sum of the conditional frequencies of voters, who performed a conditioning of their choices on the two outcomes of Presidential elections 2 years before. If (3.3) does not hold, this means that the statistical data do not obey the rules of the classical probabilistic framework. When operating with probabilities, we can also extrapolate whether the total probability that an average voter casts ballots for a Democratic Congress (or a Republican Congress) equals to the disjunctions of the marginal probability for the

\[11\text{It can be also true that the voters in a similar way condition the preferences for the President on the outcomes of Congressional elections. Because the Congressional elections take place more often, we focus similarly to Zorn & Smith [14] only on how the preferences for Congress evolve.}\]
outcome of the Presidential elections, conditioning the probability that the voter would vote for a particular party in the subsequent Congressional elections. The $p(P_d)$ and $p(P_r)$ are the objective, given probabilities. The $p(C_d|P_d)$ and $p(C_d|P_r)$ are the conditional probabilities that are supposed to confirm the emergence of the classical origins of non-separability (non-separability = Bayesian conditioning.) By incorporating the data from tables 1, 2 and 5, we obtain

$$0.4724 \neq 0.65 \times 0.537 + 0.06 \times 0.463 = 0.377. \quad (3.4)$$

As we see from (3.4), the frequency of voters, who are conditioning the decisions to vote for the Congress on probabilistic outcomes of $P_d$ or $P_r$, is not the same as the unconditional frequency of voting for the Congress, i.e. $p(C_d) \neq q(C_d)$.$^{12}$ This means that (i) the Bayesian conditioning cannot be the source (or at least not the only source) of voters’ non-separable preferences. In the analysis for 2010, the difference lies at $\delta_p = -0.096$, where the negative sign means that the disjunctions of conditional probabilities show a subadditivity in respect to the marginal probability of the $C_d$ outcome in 2010. The violation of classicality of the statistics in the year 2010 can indicate an existence of stronger correlations between preferences for Presidential and Congressional variables than implied by classical Bayesian conditioning. Next, the time dynamics and new information, as suggested by (ii), can contribute to the preferences reversal in between the voting periods. Because the voters’ variables are not controlled in the real-world election context in the same way as in surveys and opinion polls, it is not possible to find out exactly whether the voters condition preferences solely on the basis of the outcomes of the previous Presidential elections or on some new issues that emerge during the time interval. To sum up, we are dealing with two decision-making issues that would require our attention. We are confronted with an existence of non-classical origins of ‘non-separability’ that was also confirmed in other studies (controlling for the impact of time dynamics) $^{14,30}$. We can infer that the non-separability phenomenon can be of a more complex origin than simply ‘conditioning of information’. We propose an operational representation of such non-classical statistical correlations between voters’ decision probabilities with the aid of the notion of entanglement. In the §5c(ii), we construct an illustration of how to accommodate such correlated states in a Hilbert state space.

A second factor that comes into play is the impact of time dynamics that changes the preference states of the voters. The preference state of the voter is constantly encountering new contextual factors. One of the main contextual sources, surrounding most of the voters (unless they are completely isolated from any interaction with society), can be attributed to the mass media. In §4, we will aim to scratch the surface on the extensive debate which deals with the influence that the mass media has on the beliefs and preferences of the US electorate.

4. The role of mass media in preference emergence

Media coverage is the very lifeblood of the politics [...] Media do more than depict the political environment, they are the political environment

$^{[54, p. 238]}$. The main information source in the political context that has the ability, in a dynamical way, to influence the decision formation of people is attributed to the mass media. The mass media includes a set of resources such as television, radio, newspapers and magazines. See reference $^{[46]}$ for details. More recently, the dominating role of television has been challenged by the increased usage of the internet as a main source of political information and election campaigns.$^{13}$ When

$^{12}$We are aware of the problem of the ‘non-voters’ that are not reported in the statistics of cooperative campaign analysis project $^{[52]}$. It could be the case that some of the ‘non-voters’ from 2008 might have decided to vote for the Democratic Congress in 2010. One would assume that the percentage of such voters is not high, given that the overall voter turnout always drops in the midterms election. Another motivation for not considering this problem in detail at this stage of the analysis is the analogy one can draw with the quantum physical measurement, by not considering the voters that we are not able to ‘detect’. In the experiments of quantum physics, the observer disregards the undetected particles (e.g. photons that did not arrive at the detector) from the investigation.

$^{13}$We include the shift of the parties’ political ideology as one of the parts of the mass media ‘bath’, because we are primarily concerned with how those changes are communicated to the voters.
political analysts talk about public opinion formation, the key question one asks is whether media (and in particular television because it is the predominant source of political information) is influencing the political preferences of American voters and, if so, to what degree?

Before we address how to possibly quantify the degree of the media impact, we present a brief background on the state of affairs of media promotion of political candidates and the overall process of shaping the political opinions of the electorate. We outline the two main viewpoints that appear in the political literature: (i) the media has no effect on voters preferences; (ii) the media has some (or a strong) effect on voters preferences. We start with some critique on the ability of television to shape the preferences of the potential voters.

If we look at the works of [46,54,55], we note that there is a lot of controversy surrounding the true impact of media. The work by McKeever [46] advocates that, in particular, the television campaigns set by channel owners in line with some profit targets may lack an emphasis on presenting an objective informational framework. Instead, the political programmes are targeting the so-called general public, and such programmes are designed to catch their attention by any means, by presenting straightforward, emotional and eye-catching episodes. As such, the delivered information may appear biased and superficial. These issues open a discussion on the trustworthiness of the delivered information. In this respect, [54,55] point out that the respondents in opinion polling studies think that mass media has very little effect on their political preferences. At the same time, it is noted that the voters may prefer not to acknowledge that they can be easily influenced by the mass media.

The situation is more complicated, because, as McKeever [46] points out, for the vast majority of voters the mass media is the only source of information concerning the political reality. Obviously, very few voters have the possibility to see the President and the members of Congress in person. This means that the voters are in need of the media to be able to form any preferences at all, not to mention the process of updating of new information related to the state of affairs in the economic, social and political spheres. We can compare the situation with the behaviour of financial investors, where the investors are in need of transparent and accurate information to be able to make strategic investment decisions.

After presenting the critical aspects of the media’s ability to deliver objective and complete information to the potential voters, we search to explore some statistics to see if there is any positive correlation between the interaction of voters with the media and their subsequent political decisions. In 1990, the average adult watched television around 4 h per day [56, p. 19a]. In comparison, in the year 2012, the New York Daily News [57] reported that an adult spends from 3 h 35 min. per day in front of TV (for those aged 18–24) increasing up to nearly 7 h per day for those aged 65+. At the same time, the internet usage is quite moderate, accounting for less than an hour per day for an average adult [57]. In 2014, the New York Daily News [58] reported similar figures, implying that the media habits of Americans remained stable over past 2 years, with a clear tendency of spending much more time in front of the TV for those aged 65+. If we come back to the issue of trustworthiness, we can see from the survey data in reference [59] that during the period between 1959 and 1988 the perceived trustworthiness of television (in comparison with other media sources) was gradually increasing from 29% in 1959 to 49% in 1988. Considering the brief findings that were presented here, we could argue that if we consider a long enough time interval (a year-long campaign but not a week-long election campaign), the informational context of the media does have an impact on the mental state of the voters by affecting their certain and uncertain preferences. In the model that we propose in the next section, §5, we will refer to the impact of the media as the ‘political environment’, or more technically the ‘bath’ surrounding the potential voters.15

14We do not focus on newspapers which are usually regarded as more informative and independent. However, they reach a smaller audience.

15The information field of the mass media surrounding the voters, acts in a similar way to a large ‘bath’, interacting with a quantum system. The ‘bath’ in the theory of open quantum systems is characterized by a high level of complexity and so approximations are used.
5. Construction of the dynamical decision-making model from the theory of open quantum systems

In §3, we have shown that the classical probability space exhibits a subadditivity of voters decision probabilities in respect to the statistical data that we analysed. From a probabilistic viewpoint, the problem can be rooted in either (i) general subadditivity of probabilities, i.e. the marginal and the conditional probabilities in (3.3) do not sum up to the value of the baseline probability; (ii) the non-Bayesian mode of updating with conditional probabilities, (iii) or both. The phenomenon of subadditivity is characterized as a main source of interference of quantum (belief) states, see reference [60].

We propose an alternative approach that makes use of quantum Markovian dynamics of quantum systems interacting with a large bath. The mathematical and conceptual structure is borrowed from the theory of open quantum systems. The mathematical formalism of this dynamics can be found in the book by Ohya & Volovich [61].

(a) Interference emergence in the statistics of voting preferences

We could argue that the interference that is emergent within the political decision-making environment which we have described so far can be captured, in a formal way, through an additional so-called interference term in the quantum formula of total probability (see formulation in reference [62]) (see also [17,29] for more of a discussion on the rationale for the use of probability interference in, respectively, political science and other social sciences). We write it out in a similar way as (3.3) for the \( q(C_d) \) outcome:

\[
q(C_d) = p(P_d)p(C_d|P_d) + p(P_r)p(C_d|P_r) + 2 \cos \theta \sqrt{p(P_d)p(C_d|P_d)p(P_r)p(C_d|P_r)}. \tag{5.1}
\]

A similar expression can be written for the interference of the probabilities for the \( C_r \) outcome. We insert the probabilities from (3.4) into (5.1) to show the type and the magnitude of interference that emerges for those statistics.\(^{16}\) For the 2008–2010 data, we obtain an interference equal to \( \cos \theta = 0.486 \). This is a quantum type of constructive interference that can be accommodated in a Hilbert space and a state reconstruction with the Born rule can be performed (see reference [19]). In comparison with Khrennikova’s [19] data analysis, the magnitude of interference for the statistical information considered here is moderate, i.e. less than \( \cos \theta = 1 \). This could be attributed to the fact that the frequencies for the whole population of voters were analysed and this can entail a smaller size effect. From a psychology point of view, we can interpret the interference of beliefs about the Presidential election outcomes with the beliefs about the voting for the Democratic Congress in the situation of uncertainty, corresponding to quantum superposition of states. Moreover, the interference of beliefs (the interference of probability ‘waves’ corresponding to the degree of belief) can exhibit features of entanglement, especially, when the voter performs the psychological ‘measurement’ of her beliefs simultaneously.

(b) The assumptions of applicability

In order to apply any model to existing data or to make predictions, one needs to develop assumptions that can justify the structural and conceptual suitability of a proposed model. We discuss some of those assumptions now. Please see reference [29] for more background.

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\(^{16}\)In line with the experiments in the quantum formalism, we treat the voters that make the decisions in each voting context as being initially prepared in some initial pure belief state, denoted by the \( \psi \) vector. By applying this approximation, we are not interested in the decisions of each particular voter, but rather in a construction of a general model of belief state evolution.
vector provide us with a probability of observing our observable in some eigenstate (more specifically, the squared modulus of the complex numbers correspond to the probability of observation, this is the essence of Born rule). The usage of complex numbers is very important because they allow for the formal rendering of the quantum effect of probability interference that leads to quantum wave dynamics of the quantum state which is in superposition of the different decision outcomes.

The Hilbert space is a linear inner product space over the field of complex numbers \( \mathbb{C} \). The elements of this state space are vectors. In our case, we are operating with the state vector representing the belief state of the voter, denoted as \( \psi \). This state remains pure in its quantum dynamics until it is measured or slowly stabilized as a result of the interaction with a quantum ‘bath’. In our model, we proceed with the construction of a tensor product of 2 one-qubit state spaces. Below, we will elaborate, why the variables that we aim to measure, motivate such a construction of the state space.

— **Observables as operators.** The decision evolution of the voter is associated with the Hamiltonian operator which is represented in the form of a Hermitian matrix. The eigenvalues of this operator correspond to the possible values of the observable. In the political decision-making context, we are interested in following the eigenvalues of the election outcome measurement. We deal with two operators \( \hat{C} \) and \( \hat{P} \) that each, respectively, have a basis in eigenstates \( |C_r\rangle \), \( |C_d\rangle \) and \( |P_r\rangle \), \( |P_d\rangle \).

— **Entanglement.** The voter’s belief state \( |\psi\rangle \) (a ket vector) is exhibiting features similar to a quantum entangled system. The system cannot be factorized as we show in §5c(ii) in a product of (separable) states. The state of the voters entangled belief states is captured through the notion of the superposition of eigenstates of some observables. The act of measurement (generated through a question or new information) collapses the state of the entire entangled system. This is the key feature of the non-separability phenomenon, as defined in this work. In §5c(ii), we advance with a mathematical representation of maximally entangled states, so-called Bell states. The properties of these states yield values of statistical correlations between the probabilities of their observation that are not present in classical probability theory.

— **The quantum bath.** The effect of the media campaigns and news is encoded in the quantum analogue of a large thermostat bath. The approximation is made with the usage of the Lindblad term (which is not Hermitian, in order to stabilize the pure state into the mixed state) of the Gorini–Kossakowski–Sudarshan–Lindblad (henceforth GKSL) equation for open quantum systems [61,63]. The bath plays the key role in our model, because the whole process of opinion formation and final decision-making is surrounded by the environmental information, coming primarily from the media sources. In analogy to the quantum theory of open systems, the effect is equally one sided: the environmental bath can swing the dynamics of the voters’ mental states, but the voters cannot change the dynamics of the surrounding bath.

— **Markovian dynamics.** The effect of the working memory is not long lasting. The voters’ mental state updates are taking place step by step, where all the previous information that was not used is erased by the impact of the bath. The Markov property matches well the impact of the mass media: the new incoming information essentially ‘washes out’ the preceding information. The memory characteristics related to knowledge and other personal characteristics are however sustained and incorporated through the rotation of the Hamiltonian operator.\(^{18}\)

\(^{17}\) In quantum physics, observables are for instance the spins of electrons. In decision theory, observables are referred to as the decision-making problems (questions) that the subject faces. In quantum-like models, the observables and the corresponding context are encoded in Hermitian operators that are acting upon the mental state of the decision-maker.

\(^{18}\) For a detailed treatment of the quantum formalism see [64,65].
(c) Construction of the voters’ entangled belief state

(i) One context preference state space

We represent the belief state of the voter (in each election context) in one-qubit space $H_1$, for the Congress, and $H_2$ for the President observable. In each state, an operator $\hat{C}$ or $\hat{P}$ related to the election contexts’ observables is acting upon the initial belief state of the voter.

If we explore only the impact of the Congress operator on the voters’ preference state, say $\psi_1$, we construct a state space in the basis of $|0\rangle, |1\rangle$, encoding preferences to choose, e.g., $C_d$ or $C_r$ outcomes. One of the main reasons to apply the quantum-like information description is that the preference state of the voter, $\psi_1$, can be in the superposition of voting for Democrats or Republicans. This genuine quantum feature can well explain the indeterminacy of the voter between the respective decisions and the interference effect with the new information.

Such superpositions are naturally represented in the quantum formalism as

$$|\psi_1\rangle = c_0|0\rangle + c_1|1\rangle,$$

where $c_0$ and $c_1$ are complex numbers, $|c_0|^2 + |c_1|^2 = 1$. Here $|c_0|^2, |c_1|^2$ give the probabilities $p_0, p_1$ that a voter with the mental state $\psi_1$ will vote for a particular party. As we mentioned in the §5b, there are some eigenvalues corresponding to the voters’ decision outcomes each of them associated with an eigenvector. As such, the probabilities of each decision can be obtained, by observing the squared amplitudes of the eigenvectors related to $\psi_1$. We remind that in the quantum formalism one is dealing not with the absolute measures of probabilities, but with their amplitudes that can exhibit a positive or a negative interference. We obtain the probabilities for each eigenvalue of the observable $C$:

$$p(C_d) = |\langle \psi_1 | 0 \rangle|^2 = |c_0|^2,$$

and

$$p(C_r) = |\langle \psi_1 | 1 \rangle|^2 = |c_1|^2.$$  

The squared modulus of the probability amplitudes of (B 2) and (B 3) gives us the probabilities that sum up to unity in accordance with the general rules of probability theory. We remark that complex numbers have not only amplitudes, but also phases, $c_k = |c_k|e^{i\theta_k}$. In the quantum formalism the phases, more precisely the relative phase $\theta_2 - \theta_1$, also play an important role. The presence of relative phases contributes non-trivially to the state dynamics. Either the Schrödinger dynamics describe the evolution of the preference state of a party in isolation from a social and political environment, or one can consider dynamics based on the quantum master equation taking into account interaction with the mass media environment. This type of dynamics differs crucially from say classical Markovian state dynamics, which take into account only probabilities $p_0$ and $p_1$.

A similar state space $H_2$ with a normalized state vector $\psi_2$ belief state, as a superposition of $|P_d\rangle, |P_r\rangle$ corresponding to eigenstates (the Presidential election outcomes) can be constructed and the probabilities for these outcomes obtained. The probabilities equal to the squared amplitudes of the corresponding basis vectors:

$$p(P_d) = |\langle \psi_2 | 0 \rangle|^2 = |c_0|^2,$$

and

$$p(P_r) = |\langle \psi_2 | 1 \rangle|^2 = |c_1|^2.$$  

For a voter that has to make a decision in two political contexts, the complete state space for preferences for (non)-cooperation is represented (in complete accordance with quantum information theory) as the tensor product of the two above defined one-qubit state spaces:

$$H_{12} = H_1 \otimes H_2.$$  

The compound state space is naturally a two-qubit space.
**Remark.** We could have accommodated the observables $C$ and $P$ with orthogonal bases in a joint two-dimensional Hilbert space. These observables would be incompatible. At the same time, we know that through the notion of non-separability, the states of the voter can interact with each other and become entangled. This is the reason, why we proceed with a more complicated approach: a combination of the state space for the Congressional elections and the Presidential elections through a tensor product.

**(ii) Quantum representations of separable versus non-separable belief states**

The states of the space $H_{12}$ can be separable and non-separable (entangled):

— Separable states can be represented as the tensor product of two one-qubit states in the form of a state $\psi \in H_{12} = H_1 \otimes H_2$. The composite state can be factorized as

$$\psi = \psi_1 \otimes \psi_2,$$

where

$$\psi_i \in H_i.$$

The $\psi_1$ is the state belonging to the $C$-observable state space $H_1$ and $\psi_2$ is the state belonging to the $P$-observable state space $H_2$. The $C$-factor of the compound state can be written as

$$\psi_1 = c_r |C_r\rangle + c_d |C_d\rangle,$$

which denotes the separable belief states about the choices for the Congressional elections, with $c_r$ and $c_d$ the corresponding complex number amplitudes. Then, the $P$-factor of the compound state, about the choices for the Presidential elections, can be written as

$$\psi_2 = c'_r |P_r\rangle + c'_d |P_d\rangle.$$ 

— The states, which cannot be represented in this way, are called non-separable or entangled. For instance, for a product of two qubit spaces, we would obtain the so-called Bell states which are the maximally entangled pure states:

$$\psi_+ = \frac{(|C_rP_d\rangle + |C_dP_r\rangle)}{\sqrt{2}},$$

$$\phi_+ = \frac{(|C_rP_r\rangle + |C_dP_d\rangle)}{\sqrt{2}},$$

$$\psi_- = \frac{(|C_rP_d\rangle - |C_dP_r\rangle)}{\sqrt{2}},$$

$$\phi_- = \frac{(|C_rP_r\rangle - |P_dP_d\rangle)}{\sqrt{2}}.$$ 

We see that there are in total four possibilities for the type of maximal entanglement between the voters’ belief states. The states producing consistent decision outcomes can also be entangled with each other, which might be a very important observation for decision theory.

From the interpretational viewpoint, the notion of entanglement is one of the most complicated notions of quantum mechanics. One of the features of entanglement (in the framework of our modelling) is that the voter’s cognitive entity cannot treat her preferences for the $C_d$, $P_r$, $C_r$, $P_d$ outcomes separately. The voter cannot split her integral preference state $|\psi\rangle \in H_{12}$ into the preference states related to the individual decision outcomes. For instance, the measurement
of observable $C$ cannot be performed, without the voter considering the outcomes of the observable $P$.

(d) Quantum model of state dynamics under the impact of mass media

We extend the formalism that we have outlined above in two respects. First, we search to model the dynamics of the voters’ belief state as a wave function evolution. As we have already done in reference [17], as a next step, we extend the form of the Schrödinger equation to the GKSL master equation, in order to incorporate the effect of the mass media (in terms of strength and intensity) through the additional operators that are forming the so-called Lindblad term. The Lindblad term has a non-Hermitian form, which makes the model more suitable for describing the behaviour of real voters (as opposed to the Schrödinger model which describes beliefs that evolve in isolation). Solutions of the Schrödinger equation exhibit the basic feature of transforming pure states into pure states. They fluctuate without any stabilization, whatever the initial state. Below, we introduce the more complex (however, it provides for a better mathematical and conceptual framework for the depiction of the complex environment surrounding the decision-makers throughout their decision-making process) dynamical Markovian quantum equation, see [61]. See also reference [17] for more details. This master equation describes a non-unitary evolution of a density matrix $\rho$. One of the main distinguishing features of the solutions of the Markovian quantum master equation is that a non-stationary solution $\rho(t)$ can stabilize to a stationary solution $\rho_d$, representing a slow stabilization of the pure entangled states to the mixed states, i.e. the quantum master equation can transform pure states into mixed states. Such a process is called decoherence. Based on the presented structure, this is a dynamical equation in the space of density operators. Therefore, the limiting strategy, determining the decision of voters’ party preference can be a mixed state even if the initial joint state of voters preferences was a pure state. It should be noted that the GKSL model enables to obtain only the probabilities of various pure strategies.

(i) State dynamics construction with Gorini–Kossakowski–Sudarshan–Lindblad

We now write the Markovian approximation of the quantum master equation (the GKSL). Instead of Planck’s constant, we apply a scaling parameter $\gamma = 1$ because of the difficulty to find an psychological analogue of the ‘mental energy’ of the voters’ cognitive states:

$$\frac{d\rho}{dt}(t) = -\frac{i}{\gamma}[\mathcal{H}, \rho(t)] + L(\rho(t)), \quad (5.15)$$

where $\mathcal{H}$ is a Hermitian operator acting in $H$ and $L$ is a linear operator acting in the space of linear operators $B(H)$ in $H$ (such maps are often called super-operators). Typically, the operator $\mathcal{H}$ represents the state dynamics in the absence of an environment, until the collapse, as understood by a quantum measurement, takes place. However, in general, $\mathcal{H}$ can also contain a contribution from the environmental impact. The linear operator $L$ (‘Lindblad term’) has to map density operators into density operators, i.e. it has to preserve Hermiticity, positive definiteness of the trace of the density matrix. These conditions constrain essentially the class of possible super-operators $L$. By adding some additional conditions (that can be found in reference [61]), the so-called complete positive definiteness, we can describe the class of these operators precisely, to obtain as maximally correct as possible approximations of the intensity of the mass media bath. We perform a toy simulation in the appendix B, to show how such a quantum Markovian dynamics takes place.

19In this construction of Bell states, we proceed operationally, i.e. we apply the minimalist interpretation of entanglement (without touching the issue of ‘non-locality’ associated with such correlation emergence), by simply noting that the statistical correlations of the probabilistic data, analysed in §3 are too strong to be captured in the single sample space of classical probability theory. In this respect, some concrete statistical tests (such as the test for CHSH inequality, [66] satisfaction and its different version known under ‘Suppes–Zanotti’ theorem, [67]) could provide for concrete evidence of the contextuality presence in the statistical data, cf. works by previous studies [41–44]. We plan to perform an analogous examination for contextuality of the observed probabilities in future works, when we have access to additional statistical data on voters ballot casting.
6. Conclusion

We can conclude that the non-separability as a consequence of, e.g., question or information order in a survey or opinion polling study, is a special case of the more global phenomenon of preference reversal as a result of information that is obtained, accumulated and processed through time by the cognitive systems of the decision-makers. Another feature of the non-classical mode of information processing is manifest in the non-classical correlations between voters preference probabilities. As such, the final choice of the voter as a result of the change of her belief state is not simply a matter of the interrelation of a set of event outcomes in a Bayesian mode, but it emerges through the impact of (i) non-classical correlations between the different preferences, and (ii) overtime dynamics induced by the complex informational ‘bath’ generated by the mass media. Furthermore, the more certainty the new information brings on the state of affairs, the more ability the decision-maker has to elucidate her beliefs. An important point to consider, which is related to the collected statistical data, is that we made the assumptions that the population of the voters is large enough to disregard the different amounts of voters participating in the different elections. We are aware of the fact that some voters might not only change their preferences from \( C_d \) to \( C_r \) outcomes (or vice versa), but also decide to vote for some minor party or to abstain from any voting at all. The voters who do not prefer to vote at all comprised in 2014 up to 64% of the eligible voting population and the emergence of their preferences ‘not to vote’, should be studied in more detail.

In particular, the very low voting activity in the 2014 midterm elections, with a turnout of 36.4% only, can be regarded as very surprising, given the intensive election campaigns (at least on the level of spending this was estimated at $3.67 billion which is much higher in comparison with other election periods). See [68,69]. The possible correlation that naturally emerges is that a more intensive election campaign instead of motivating the potential voters to vote for some party resulted in an opposite outcome. Namely the decision not to vote at all! A quote from reference [68] supports well this conjecture: ‘apathy, anger and frustration at the relentlessly negative tone of the campaign [that led to a reluctance to vote]’. If we analyse these results in the framework of our model, we could state that not only the intensity of the campaign plays a role, but also the way the presented information interacts with the voters belief states, i.e. either the mass media bath does not induce any stabilization of the voters’ belief states. Or if we add an additional eigenvalue to our model, corresponding to the decision outcome ‘not to vote’ (as was done in reference [18]), we can speculate that the stabilization of the belief state mimics this outcome. It would potentially be of interest to measure the emergence of non-separability for an augmented multidimensional state space, say with observables for the elections to the House of Representatives, the Senate and the Presidential election as being interdependent on each other. We are aware that the robustness of the results could be enhanced through this more in-depth analysis, because not all voters who vote for a party in the House elections, would do so in the Senate elections. These two voting contexts can exhibit quantum non-separability as well. With this in mind, one could proceed further, by expanding the proposed model from a bipartite entanglement to an entanglement of three and more states, \( n \geq 3 \), which would exhibit new features that are not appearing in the \( n = 2 \) (the two-qubit state space).

Lastly, we believe that on top of what has been studied in this work it is of vast interest to carry out further studies on voters’ decision-making dynamics in the context of real-world elections. The approximations that we applied in this work can be further refined and explored in more depth. We remind that non-separability is a complex phenomenon and it would benefit from further analysis.

Data accessibility. Supplementary data can be found in appendix A and appendix B.

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<tr>
<td>2012</td>
<td>60,933,500</td>
<td>65,915,796</td>
<td>2,236,107</td>
</tr>
<tr>
<td></td>
<td>total: 129,085,403</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>total: 43,737,552</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Appendix A

The appendix consists of a composite contingency table that provides detailed statistics on the election outcomes, 2008–2014 (table 6). It summarizes the total turnout of voters including the percentage of voters, who voted for minor parties (labelled as ‘other’). This statistical data were used to generate the summarizing tables 1–4. The statistics are obtained from these sources:

- 2008 Federal elections from references [47, 48],
- 2010 Congressional elections from reference [49],
- 2012 Federal elections from reference [50], and
- 2014 Congressional elections from reference [51].

### Appendix B. Numerical simulation with Gorini–Kossakowski–Sudarshan–Lindblad model

We illustrate how the proposed dynamics of voters’ state evolution could be obtained in practice. At this preliminary stage, we are ‘fitting’ the free parameters responsible for the internal and
external state evolution encoded in the Hamiltonian and Lindblad operators. We consider in this toy simulation only the subpopulation of voters who exhibit non-separability as conceptualized in the political literature, i.e. those who prefer ticket-splitting and thus their initial belief state is denoted as the superposition of only two basis states:

$$|\psi\rangle = c_0|DR\rangle + c_1|RD\rangle$$  \hspace{1cm} (B1)

Because we consider only the submodel of the (5.8) for the subspace of preferences of $Cd \cap Pr$ and $Cr \cap Cd$, the probabilities for these outcomes are denoted as the squared amplitudes of the corresponding complex numbers:

$$p(C_d \cap P_r) = |\langle \psi |0\rangle|^2 = |c_0|^2.$$  \hspace{1cm} (B2)

and

$$p(C_r \cap P_d) = |\langle \psi |1\rangle|^2 = |c_1|^2,$$  \hspace{1cm} (B3)

where $|c_0|^2 + |c_1|^2 = 1$; the probabilities of the partisan outcomes $C_d \cap P_d$ and $C_r \cap P_r$ are equal to zero.

As a next step, we model the dynamics of the state evolution of this cluster of voters with the aid of the presented GKSL model, (5.15). We set the parameters of Hamiltonian and Lindblad operators. The corresponding Hamiltonian matrix in the basis $|DR\rangle$ and $|RD\rangle$ is chosen as

$$\mathcal{H} = \begin{pmatrix} 0 & \lambda \\ \lambda & 0 \end{pmatrix}.$$  \hspace{1cm} (B4)

The free parameter $\lambda$ captures the speed of internal self-analysis of the voters, technically it is the intensity of flipping from the preference for $DR$ to $RD$ and back. In the absence of the interaction with the election bath such a quantum dynamics would continue for ever, unless an external ‘measurement’ is imposed on it. We set the parameter $\lambda = 1$ for this particular simulation. The Lindblad term is given by a function

$$C\rho C^* - \frac{(C^*\rho + \rho C^*)}{2} = C\rho C^* - \frac{1}{2}\{C^*C, \rho\},$$  \hspace{1cm} (B5)

where $C^*$ is a Hermitian adjoint to the operator $C$. We select the matrix for the operator $C$ as (B6). The adjoint operator $C^*$ has a corresponding matrix (B7)

$$C = \begin{pmatrix} \lambda & \lambda \\ 0 & 0 \end{pmatrix}.$$  \hspace{1cm} (B6)

and

$$C^* = \begin{pmatrix} 0 & 0 \\ \lambda & \lambda \end{pmatrix}.$$  \hspace{1cm} (B7)

The parameter $\lambda = 1$ is now responsible for the interaction of the voters’ belief states with the election campaign. In future works, we aim to refine the model, to capture the real-time dynamics of the decision-making. In this case, one would introduce an additional scaling parameter responsible for the ‘intensity of decoherence’ from a quantum belief state and set it in front of (B5).

As such, the dynamics of the quantum master equation (5.15), can be restated through (B5) as

$$\frac{d\rho}{dt}(t) = -i[\mathcal{H}, \rho(t)] + C\rho(t)C^* - \frac{1}{2}\{C^*C, \rho(t)\}.$$  \hspace{1cm} (B8)

We perform a simulation in the program Mathematica for voters that are in an initial state of strongly asymmetric superposition; in particular,

$$c_0 = \sqrt{0.9}, \ c_1 = \sqrt{0.1}.$$  \hspace{1cm} (B9)

Hence,

$$|\psi\rangle = \sqrt{0.9}|DR\rangle + \sqrt{0.1}|RD\rangle.$$  \hspace{1cm} (B10)
We see from figure 1 that the matrix elements $\rho_{11} \equiv \rho_{dr,dr}$ and $\rho_{22} \equiv \rho_{rd,rd}$ stabilize to classical probabilities or firm preferences. The initial preferences are considerably altered, as a result of the interaction with the ‘election campaign’ and arrive to the state of almost equal distribution of voters ($c_0 = \sqrt{0.5}$ and $c_1 = \sqrt{0.5}$) that prefer Democratic Congress and Republican President and vice versa. This reflects well the ticket splitting preferences observed in the 2010 elections, [52].

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