MEANDERING SHALLOW ATMOSPHERIC JET AS A MODEL OF SATURN’S NORTH-POLAR HEXAGON

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ABSTRACT

The Voyager flybys of Saturn in 1980-1981 revealed a circumpolar Hexagon at ∼78° north planetographic latitude that has persisted for over 30 Earth years, more than one Saturn year, and has been observed by ground-based telescopes, Hubble Space Telescope and multiple instruments onboard the Cassini orbiter. Its average phase speed is very slow with respect to the System III rotation rate, defined by the primary periodicity in the Saturn Kilometric Radiation (SKR) during the Voyager era. Cloud tracking wind measurements reveal the presence of a prograde jet-stream whose path traces the Hexagon’s shape. Previous numerical models have produced large-amplitude, n = 6, wavy structures with westward intrinsic phase propagation (relative to the jet). However, the observed net phase speed has proven to be more difficult to achieve. Here we present numerical simulations showing that instabilities in shallow jets can equilibrate as meanders closely resembling the observed morphology and phase speed of Saturn’s Northern Hexagon. We also find that the winds at the bottom of the model are as important as the winds at the cloud level in matching the observed Hexagon’s characteristics.

Subject headings: planets and satellites: general — planets and satellites: atmospheres — planets and satellites: physical evolution

1. INTRODUCTION

Saturn’s Northern Hexagon is a planetary-scale cloud band that has six well-defined corners and encircles the north pole of Saturn at ∼78° N¹ (Fig. 1). It was discovered by Godfrey (1988) in Voyager images captured in 1980-81, and it has persisted for more than one Saturn year (∼29.5 Earth years) (Caldwell et al. 1993; Sánchez-Lavega et al. 2014). Observations by Voyager and Cassini have revealed the following key characteristics of the Hexagon: 1) it is associated with an eastward zonal jet at 78° N which has a peak speed of 100 ms⁻¹, as determined from tracking individual cloud patterns forming the Hexagon (Godfrey 1988; Baines et al. 2009; Sánchez-Lavega et al. 2014); 2) the path of the zonal jet follows the outline of the hexagonal cloud morphology; 3) there is no evidence that the Hexagon is a “vortex-street,” a series of spots of alternating vorticity staggered on the flanks of the jet; 4) measurements from Voyager images yield a speed for the vertices of the Hexagon of 0.8 ± 1.1 ms⁻¹ (Godfrey 1988). Measurements based on more recent Cassini observations yield a speed of −0.036 ± 0.004 ms⁻¹ (Sánchez-Lavega et al. 2014). Therefore, the Hexagon’s net phase speed is very small relative to the Voyager-era radio period of the Saturn Kilometric Radiation (SKR) of 10h 39m 24s (Desch & Kaiser 1981), which forms the basis for the currently defined System III rotation period for Saturn (Seidelmann et al. 2007), and corresponds to a planetary angular velocity Ω_{HI} = 1.6378 × 10⁻⁴ s⁻¹. All quantities presented in this paper are measured in this reference frame; and 5) there is a meridional temperature gradient associated with the Hexagon in the 100-800 mbar region, with the equatorial side of the jet being colder than the polar side (Fletcher et al. 2008). Here we present results of numerical simulations that reproduce all five observed characteristics of Saturn’s Northern Hexagon.

2. MODEL SETUP

Our simulations tested the evolution of Gaussian eastward jets of the form:

\[ U(\lambda) = U_0 \exp(-b\gamma^2(\lambda)/2U_0) \]  

(1)

To perturbations of their associated Montgomery stream function described by

\[ \Delta M(\phi, \lambda, \psi) = \delta e^{-(\lambda - \lambda_0)^2 + \log(\delta_0/\epsilon_0)^2} \sum_{k=1}^{L} \sin(k\phi - \psi_k) \]

(2)

where \( \lambda \) is the planetographic latitude, \( \gamma(\lambda) \) is the meridional distance from the center of the jet, \( U_0 \) is the peak velocity or amplitude of the jet, \( b \) is the peak latitudinal curvature of the jet, \( \delta \) is the amplitude of the perturbation, \( \lambda_0 \) is the planetographic latitude of the center of the jet, \( \delta_0 \) is the full-width half-maximum of the perturbation, \( P \) is the pressure, \( P_0 \) is the pressure level where the perturbation is added, \( \epsilon_0 \) is the vertical extension of the perturbation in scale heights, \( k \) is the east-west planetary wave-number, \( \phi \) is the longitude, \( \psi_k \) is a random phase offset, and \( L \) is the total number of wave-numbers included in the perturbation, which we set to be 50.

The model used in this study is the Explicit Planetary Isentropic-Coordinate (EPIC) General Circulation Model developed by Dowling et al. (1998). This model integrates the hydrostatic primitive equations on an oblate sphere. The nominal domain consists of a channel

¹ Latitudes are planetographic unless otherwise noted.
different constant target values of the Brunt-Väisälä frequency at the bottom of the model. The nominal value for $N$ at the bottom of the model is set to be $6 \times 10^{-3}$ s$^{-1}$.

With this set-up, the space of parameters explored by our simulations consists of variations in the jet amplitude ($U_0$), the jet curvature ($b$), the vertical shear of our Gaussian wind profile above 2 bars, the vertical wind shear below the same level, the zonal winds at the bottom of the model, and the static stability of the atmosphere below 1 bar. The model’s response to perturbations like the one described by Eq. 2 is then compared to observations of the Hexagon. The criteria applied to validate our model output against the observations and to guide our free parameter exploration consisted of checking for the following characteristics in the model output: 1) the jet evolves and equilibrates into a meandering state and not into a vortex-street, so that at the end of the simulation, the jet is not flanked by alternating patches of cyclonic and anticyclonic vorticity; 2) the final wind profile describing the jet is within the error bar of the observed wind profile; 3) the drift rate of the meander that forms is close to the observed propagation of the Hexagon; and 4) there is a temperature gradient associated with the Hexagon, the equatorial side of the jet being colder than the polar side.

3. SIMULATIONS

In this section, we present three series of simulations with distinct outcomes. The first series of simulations we present are analogous to those presented by Morales-Juberías et al. (2011), in which the wind structure is assumed to be deeper than the simulation domain. For this structure of zonal winds the model always equilibrates into a vortex-street, independently of the value of the other parameters. The left column of Fig. 2 shows the results of initializing the model with a Gaussian jet characterized by an amplitude of $U_0 = 100$ ms$^{-1}$ and curvature of $b = 20 \times 10^{-11}$ m$^{-1}$s$^{-1}$ that remains constant with altitude below 2 bars. When the model is seeded with a perturbation, Eq. 2, it equilibrates very rapidly (after 25 days [1 day = 24 h]) into six pairs of interlocking cyclones and anticyclones that form a vortex-street.

The time evolution of the relative vorticity at the peak of the jet shows that, after its rapid emergence, the vortex-street propagates too fast to the east. In all simulations, the growth of the instability widens and weakens the jet, reducing its peak speed (Fig. 3).

In the second series of simulations, we assume that the wind structure is shallower than the model domain. This initial wind profile equilibrates in a hexagonal, meandering jet. The middle column of Fig. 2 shows the results of initializing the model with a Gaussian jet characterized by an amplitude of $U_0 = 100$ ms$^{-1}$ and curvature of $b = 20 \times 10^{-11}$ m$^{-1}$s$^{-1}$ that decreases to zero at the bottom of the model below 2 bars. When we seed the model with a perturbation, Eq. 2, its path equilibrates into a stable meander with a dominant zonal wave-number of six as shown in the polar projected maps.
Fig. 2.— Comparative results of the three series of simulations described in the text. The left column shows the results for a simulation initialized with a Gaussian jet that remains constant with altitude below 2 bars. The middle column shows the results for a simulation initialized with a Gaussian jet that decreases to zero at the bottom of the model below 2 bars. The right column shows the results of a simulation initialized with Gaussian jet that decreases to the bottom level winds as shown in Fig. 3. In all cases the polar plots in the middle show the potential vorticity after 500 simulated days, and the bottom plots show the time evolution of the relative vorticity at the center of the jet. The values of $U_0$ and $b$ in each case are those that evolve into a dominant wave-number of six for the different vertical wind configurations, so that a morphological comparison is possible with the same wave-number. An accompanying animation shows the evolution of different variables in our third series simulation.
of potential vorticity after 500 days. The time evolution
of the relative vorticity at the center of the jet shows that
after forming, the meander propagates slowly to the east
with a speed of \( \approx 3.1 \text{ ms}^{-1} \). This speed is slower than the
vortex-street case, but it is still faster than that of the
observed Hexagon. While variations in the other param-
eters \((U_0, b, N)\) explored in this configuration can
alter the dominant wave-number and propagation rate
of the meanders formed, all the meanders produced this
way fail to meet one or more of our criteria described
above. In other words, either the wave-number at the
end of the simulation is not six, the propagation speed
is faster to the east than the observed propagation rate,
or the final shape of the jet is beyond the error of the
observed jet. In this group of simulations, the growth of
the instability also widens and weakens the jet, reducing
its peak speed (Fig. 3).

In the third series of simulations, we keep the
cloud-top 1-bar wind speed observed in the inertial frame
identical to those in the first two series, but we make it
decay towards a wind profile at the bottom of the
model as shown in top-right panel of Fig. 3. The
right column of Fig. 2 shows the results of initializing
the model in this way for a Gaussian jet characterized
by an amplitude of \( U_0 = 125 \text{ ms}^{-1} \) and curvature of
\( b = 10 \times 10^{-11} \text{ m}^{-1} \text{s}^{-1} \). For vertical structure of the
jet, after 400 simulated days, the model equilibrates into
a stable meander with a dominant wave-number of six.
This meander is practically stationary, and its ampli-
tude is smaller than those in our second series of simul-
ations. Thus when projected into a polar map, it has a
sharp hexagonal shape instead of a star-like shape (see
Fig. 8 in Antuñano et al. 2015). Furthermore, the mea-
der produced this way matches all the observed dynami-
cal properties of Saturn’s Northern Hexagon. There are
no vorticity patches associated with the meandering of
the jet (i.e. not a vortex-street), the zonal winds at the
end of the simulation are within the observed errors of
the measured winds in the location of the jet where the
Hexagon exists, and the propagation rate of the meander
(\( \approx -0.3 \text{ ms}^{-1} \)) is practically stationary.

4. DISCUSSION

Past numerical and laboratory modeling efforts have
succeeded in reproducing some, but not all, of the
Hexagon’s characteristics. Allison et al. (1990) inter-
preted this feature to be a stationary Rossby wave per-
turbed by the anticyclonic vortex observed to the south
of the eastward jet and meridionally trapped by the rela-
tive vorticity gradient of the flow itself. Barbosa-Aguia-
et al. (2010) showed how polygonal patterns, correspond-
ing to wave modes excited by the nonlinear equilibration
of a barotropically unstable zonal jet, can appear in lab-
oratory experiments of flows in a rotating tank. Those
patterns are associated with a vortex-street, and their
sharpness and rotation can be adjusted with the slope
at the bottom of the tank (which simulates a weak to-
pographic \( \beta \)-effect.) Morales-Juberías et al. (2011) pre-
4 4
sented numerical simulations like the first series of sim-
ulations described here in that the wind structure was
assumed to be deeper than the simulation domain, and
the hexagonal structure that emerged was a vortex-street
with a net phase speed too high compared to the observed
value.

At the latitude of the Hexagon, the beta parameter
(\( \beta = df/dy \), where \( f = 2\Omega \sin(\lambda) \) is the Coriolis param-
eter) is very small compared to the second derivative
of the mean zonal wind, and the necessary, but insuffi-
cient, Rayleigh-Kuo criterion for barotropic stability is
easily violated (Ingersoll et al. 1984; Read et al. 2009).
The Gaussian jets adopted in our simulations violate the
Rayleigh-Kuo criterion, and past linear stability analy-
ses demonstrate that barotropic instabilities indeed arise
in such profiles (Holland & Haidvogel 1980). Flierl et al.
(1987) studied the nonlinear evolution of barotropic beta
plane jets as a function of the beta parameter and the
dominant wave-number of a perturbation to the stream-
function. Within this space of parameters, they showed
how jets can equilibrate forming either a stable vortex-
street or a steady meander. Our Gaussian jets also have
reversals in their zonal mean potential vorticity profiles,
which represent a violation of the Charney-Stern stabil-
ity criterion (Charney & Stern 1962). Previous studies
have also shown how baroclinic instabilities can equi-
librate into meanders in such profiles (Koschmieder &
White 1981; Bastin & Read 1997; Sutyrin et al. 2001).
Other experiments have shown how polygonal patterns
can emerge in flows for a wide range of parameters and
different kinds of forcing (Sommeria et al. 1989; Vatistas
1990; Vatistas et al. 1994; Marcus & Lee 1998; Jansson
et al. 2006).

Here, we have shown that small perturbations to the
stream-function of an eastward Gaussian jet can grow
and equilibrate as a vortex-street or as a meander, de-
pending on the vertical wind shear and the wind profile
at the bottom of our model. Our Gaussian jets violate
the criteria for barotropic and baroclinic instability, but
violations of these criteria do not determine which in-
stability type will be dominant in the simulations. In
general, barotropic instabilities in quasi-geostrophic jets
are due to strong horizontal gradients of vorticity and are
associated with transfer of kinetic energy from the mean
flow to the perturbation. Baroclinic instabilities are due
to the tilting of the isobaric surfaces, and are associated
with transfer of potential energy from the zonal flow to
the perturbation. In our first two series of simulations
the jet decreases in amplitude and widens significantly
as the simulations evolve (Fig. 3) which is a character-
istic signature of barotropic instability (Pedlosky 1982).
In our third series of simulations, the jet amplitude and
shape are left relatively unaltered throughout the simu-
lation (Fig. 3), and thus we speculate that barotropic
instability is not the dominant instability in this case.

Fig. 4 shows the comparison between the tempera-
tures observed in Saturn retrieved from CIRS observa-
tions with Cassini (Fletcher et al. 2008) and the model’s
temperatures after 500 simulated days for three cases in
our third series of simulations that differ only in the ver-
tical shear of the Gaussian wind profile above 2 bars.
Between 76° N and 80° N, our model shows in all cases
a temperature gradient that corresponds to the decay of
the hexagonal jet with altitude. Changing the vertical
wind gradient in the top of the model can flatten the
model’s meridional temperature gradient in that pres-
sure range without significantly affecting the dynamical
properties of the resulting meander (i.e., dominant wave-
number stays at six and the propagation speed remains
slow), since those are fundamentally controlled by the
vertical shear and the shape of the winds at the bottom of
the model. This model behavior reproduces the seasonal
insensitivity of the Hexagon’s characteristics to seasonal
changes observed in the 70-250 mb region (Fletcher et al.
insensitivity to the seasonal effects observed by CIRS as
evidence that the Hexagon must be a deep-rooted fea-
ture. Our results show that a jet shallower than the
model domain can produce a hexagon that matches all
the observed properties of Saturn’s Hexagon and is also
insensitive to “seasonal” changes at the top of the model.
Outside of the 76° N and 80° N region, our background
wind field lacks the additional eastward and westward
jets at other latitudes to be able to reproduce the temper-
ature gradients associated with those jets. At altitudes
above the 500 mbar pressure level, our temperatures are
on average 3 K higher than the observed temperatures
which have an error of ≈ 1 K. This 3 K deviation could
be due to physical processes that are not included in our
modeling such as heating and cooling by absorbers in
those levels of the atmosphere.

Finally, to test the effect of the latitude on the evolution
of the perturbations, and to address the question of
why we do not observe a Hexagon in the Southern Hemi-
sphere jet located at ≈ 74° S, we ran a preliminary series
of simulations implemented with a Gaussian jet like the
one used in our third series of simulations placed at dif-
f erent latitudes from 75° to 30° (in 5° intervals). We do
not observe instabilities growing when the center of the
jet is placed between 75° and 40°. For the cases when the
jet center is placed at 35° and 30°, the jet equilibrates
in a state that is more reminiscent of the morphology
of the Ribbon (Godfrey & Moore 1986; Sayanagi et al.
2010) than that of the Hexagon. In a future study we
will explore this latitude variation with more detail in-
cluding more parameters, like the width and amplitude
of the jet and the structure of the jet at the bottom of
the model. Cassini proximal orbits will likely be able to bet-
ter constrain the depth and structure of the zonal flows in
Saturn at all latitudes, although being mostly near the
equator, there will be higher uncertainty towards high
latitudes of the planet like the one where the Hexagon is
present.

5. CONCLUSIONS

We have shown that small perturbations to the stream-
function of an eastward Gaussian jet can grow and equi-
librate either as a vortex-street pattern or as a meander-
ing jet depending on the vertical shear of the jet: deep
jets evolve into vortex-streets and shallow jets evolve
into meanders. In our simulations, the initial ampli-
tude ($U_0$) and curvature of the jet ($b$) determine the
dominant wave-number, similar to results in (Morales-
Juberías et al. 2011). We also find that the winds at
the bottom of the model are as important as the winds
at the cloud level in matching the observed Hexagon’s
characteristics, in particular its drift rate and its shape
sharpness. In addition, we show that the model behavior
reproduces the insensitivity of the Hexagon’s character-
istics to seasonal changes observed in the 70-250 mb re-
region, since the morphology and propagation rate of the
hexagon in our model is fundamentally controlled by the
vertical shear and the shape of the winds at the bottom
of the model.

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Fig. 4.— Top Left: Mean zonal temperatures observed in Saturn (Fletcher et al. 2008); Top: Mean zonal temperatures at 500 days for three cases in our third series of simulations that differ only in the vertical shear of the Gaussian wind profile above 2 bars. From left to right the wind profile decays to zero in 10, 14 and 18 scale heights respectively. Middle: polar projections of the potential vorticity after 500 simulated days. Bottom: time evolution of the relative vorticity at the center of the jet for the three different cases. The propagation speed of the meander in the last 100 days is \( \approx 0.1 \), and 0.2 ms\(^{-1}\) respectively.

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