Links between fluid mechanics and quantum mechanics: a model for information in economics?

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February 19, 2016

Abstract

This paper tallies the links between fluid mechanics and quantum mechanics, and attempts to show whether those links can aid in beginning to build a formal template which is usable in economics models where time is (a) symmetric and memory is absent or present. An objective of this paper is to contemplate whether those formalisms can allow us to model information in economics in a novel way.

keywords Fluid mechanics; memory; time reversibility

1 Introduction

The social sciences, especially economics and finance, have for many years been interacting, with varying degrees of intensity, with the exact sciences. We should really distinguish two important movements. First, the ‘Econophysics’ movement which applies formalisms from statistical mechanics to the social sciences and championed by Eugene Stanley and others (see for instance Stanley [1]; Plerou et al. [2]; Yakovenko and Rosser [3]; Gell-Mann and Tsallis [4]; Castellano et al. [5]). Second, the movement which applies the mathematical apparatus from quantum information to the cognitive and social sciences and championed by Andrei Khrennikov and others (see the key paper from Khrennikov [6]; Hawkins and Frieden [7]; Pothos and Busemeyer [8]; Aerts et al. [9]; Dzhafarov and Kujala [10] and Khrennikov et al. [11]). In both of the above research domains, the Philosophical Transactions of the Royal Society A had recent special issues: see Burger et al. [12] and Haven and Khrennikov [13]. There is an intense amount of work
which is underway in applying, quantum physical concepts, to other areas of (social) science, especially psychology and now also biology. The recent work by Asano and Khrennikov et al. [14] sheds light on how quantum information theory can explain information processing in biological systems.

In the rest of the paper we will consider the topic of formalisms which may allow us to model ‘information’ in an economics environment. We will consider several approaches. In section two, we consider a model, which is essentially a ‘no memory’ model with an asymmetry between past and future times. We also briefly discuss a ‘no memory’ model but which assumes symmetry between past and future times. Section three of the paper considers another model which is a memory based model. We conclude in section 4 of the paper.

2 A no memory model with asymmetry between past and future time

We commonly use in asset pricing a version of Brownian motion which allows for the normal density of price returns and the lognormal density of prices. The so called Black-Scholes PDE [15] which, when solved, yields the prices of financial option contracts, assumes exactly this process for the time evolution description of the asset which underlies the financial option. Such a Brownian motion has white noise, i.e. it does not allow for memory between price occurrences. However, Commissariat [16] reports that the usual assumption that ‘kicks’ made to a descending particle, in say a glass of water, are random, may not always be true. When the densities of the particle and fluid are similar: the kicks are not random and ‘persistent correlations’ are predicted between the motions of the fluid and the particle. This gives rise to the idea of so called ‘colored noise’. We will briefly come back to density differences between liquids in section 3 of our paper.

Brownian motions with a so called ‘Hurst exponent’ allow for the introduction of memory. Stochastic differential equations with such exponent were popular some years ago (but less so now). The paper by Rodgers [17] is an excellent example of fruitful applications in the area of arbitrage.

In some recent work (see Haven [18]; [19]) we have tried to make the case that there may be scope to use elements of the Nelson approach ([20]; [21]), which essentially looks at quantum mechanics from a Newtonian perspective. For a discussion on how a particle model can agree with quantum mechanics, see Sutherland [22].

Nelson’s approach shows the emergence of the so called quantum
potential which we will discuss somewhat below.

We want to provide for some important elements of the theory. We follow Paul and Baschnagel [23]. The theory assumes a Brownian motion: \(dx(t) = b_+(x, t)dt + \sigma dW(t)\); where \(dW(t)\) is a Wiener process; \(\sigma\) is the diffusion coefficient; \(b_+(x, t)\) is the drift function. It is assumed that \(E(dW(t)) = 0\), and one defines:

\[
D_+ x(t) = \lim_{\epsilon \to 0} E \left[ \frac{x_{n+1} - x_n}{\epsilon} \right] = b_+(x, t). \tag{1}
\]

Similarly, a Brownian motion of the type: \(dx(t) = b_-(x, t)dt + \sigma dW(t)\) is then proposed, and one defines then:

\[
D_- x(t) = \lim_{\epsilon \to 0} E \left[ \frac{x_n - x_{n-1}}{\epsilon} \right] = b_-(x, t). \tag{2}
\]

Two velocity fields are then defined:

mean velocity: \(v(x, t) = \frac{1}{2}(b_+(x, t) + b_-(x, t))\); \(\tag{3}\)

and

osmotic velocity: \(u(x, t) = \frac{1}{2}(b_+(x, t) - b_-(x, t))\). \(\tag{4}\)

In Newtonian mechanics \(b_+(x, t) = b_-(x, t)\), and hence \(u(x, t) = 0\). We note that \(u(x, t) = \frac{1}{2}(b_+(x, t) - b_-(x, t)) = \frac{1}{2}(D_+ - D_-) x(t)\) and if \(u(x, t) \neq 0\), there is asymmetry between past and future time. Thus, the model proposed here has no memory (using the Wiener process construction) but there is explicit asymmetry between past and future time. This asymmetry between future and past time is important.

One can show that

\[
u(x, t) = \nabla \ln p(x, t); \tag{5}\]

where \(p(x, t)\) is a probability density function. Say if, \(p(x, t) = \exp(2R(x, t))\), with \(R\) some amplitude function, then \(u(x, t) = \sigma^2 \nabla R(x, t)\).

We could make the argument that \(u(x, t)\) is narrowly related to Fisher information, which we already defined above and which we can also write as:

\[
I \equiv \left\langle \left( \frac{\partial \ln p}{\partial x} \right)^2 \right\rangle. \tag{6}\]

The quantum potential, which we mentioned before, can be defined as (omitting mass and Planck constant): \(\frac{1}{R} \nabla^2 R\). It seems thus quite clear, that without this asymmetry between past and future we can
not make this link to Fisher information. When Fisher information is being optimized (and that even within an economics based information setting), it has been shown by Hawkins and Frieden ([24]; [7]) to be closely linked to a Schrödinger-like PDE. The interested reader may also want to peruse the discussion on Fisher information in Heifetz and Cohen [25].

What we thus see here is an emergent structure. The asymmetry of past and future time allows us to use Fisher information which itself is closely allied to a quantum-like construction, i.e. the Schrödinger-like PDE. We also can take this in another (but related) direction. The quantum potential which emerges out of the Nelson approach, is also of use in economics, as an information device. An extended Newton law: \( m \cdot a = \nabla (V + Q) \), where \( V \) and \( Q \) are respectively the real and quantum potentials can be estimated from real data. \( Q \) can be estimated (from the probability density function on return data for instance). One tears out \( R \) from the pdf, \( p(x, t) = \exp(2R(x, t)) \), and substitutes this \( R \) into \( Q = \frac{1}{R} \nabla^2 R \). In a recent paper, Tahmasebi et al. [26] derive the quantum potential for the Standards&Poor financial index and they find a quantum potential with infinite walls for short time scales (small price variations). They find the quantum potential does change when the time scales are lengthened. This is a very intuitive finding. The real potential has now also been shown to be of great practical use in recent work by Baaquie [27] where he shows that this potential, when minimized yields an equilibrium price formulation which ends up to be richer than the traditional basic microeconomics equilibrium formulation. Furthermore, real potentials have now been estimated for real world commodities by the same author.

The above development involving the quantum potential is also known under a different name: Bohmian mechanics\(^1\) (see Bohm and Hiley [28]). We have invoked in previous work, the use of Bohmian mechanics in finance, especially from the promise such approach brings in terms of being able to formalize information in the setting of the pricing of assets. See for instance Khrennikov [29]; Hawkins and Frieden ([24]; [7]); Haven and Khrennikov [30] and Haven [18]. Bohmian mechanics brings with itself the issue of non-locality which means that the outcome of a measurement on one particle is immediately noticeable on another particle, whatever the distance between the particles. One may argue that non-locality is closely connected to the appearance of the quantum potential (such a potential is a characteristic feature of Bohmian mechanics). See Haven and Khrennikov [30] (pp. 106-107) and also Holland [31] (p. 282). The link between non-locality and

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\(^1\)We note that the Bohmian mechanics approach arrives to the quantum potential in a different way as compared to the Nelson approach.
hidden variables is well known and important recent experimental ev-

dence indicates that non-locality may really exist *without* having to

invoke hidden variables. See Hensen et al. [32].

What can we do with an approximation of non-locality at a macro-

scopic level, especially in the area of asset pricing? Consider a simple

‘Alice and Bob’ example where each of the protagonists are rocketed

away to planets which are very far removed from each other. Assume

Bob is going to planet ‘Tito’ and Alice is going to planet ‘Nito’.

Consider three simple scenarios.

- **Scenario 1.** Alice and Bob each have an envelope with either a

  white or black square inside. They know there are two squares in

  all - one black and one white. Bob opens the envelope when he is

  on planet Tito and let us assume he has a black square. Hence, he

  instantaneously knows that Alice, on planet Nito, has the white

  square. This is not precisely non-locality. Clearly, no physical

  message is sent from Alice to Bob (since this would imply there is

  a physical transmission speed in excess of the speed of light....).

  Note also there is no uncertainty here at all.

- **Scenario 2.** Assume from the start there is 1 black square

  and 5 white squares. Bob and Alice know this distribution of

  squares from the start. If Bob opens the envelope when he is on

  planet Tito and he finds 1 black square then he instantaneously

  knows Alice must have a white square. However, if he opens the

  envelope and finds a white square instead, then he is unsure what

  Alice may have. This scenario clearly exemplifies the existence

  of uncertainty.

- **Scenario 3:** Assume from the start there are 3 white and 3

  black squares. For either Bob or Alice when they open their

  envelopes, there is uncertainty as to what the other person has

  as square color. This scenario again exemplifies the presence of

  uncertainty.

Now let us consider the above scenarios again, but now with a

twist. Imagine that when both the spaceships of Alice and Bob pass

some red line in the universe, the colors of their squares instanta-

neously reverse. Alice and Bob do not know about this color reversion.

Let us re-consider the scenarios.

- **Scenario 1’:** Bob had before the ship crossed the red line a

  black square. Bob has now a white square when he opens the

  envelope. He knows instantaneously that Alice must have black.

  There was no uncertainty in scenario 1 and the ‘red line event’

  has not created any less or more uncertainty.
• **Scenario 2’**: Having crossed the red line, there are now 5 black squares and 1 white one. If Bob picks black then he infers (using scenario 2) that Alice must have white, but that is not the case in Scenario 2’. Clearly, the ‘red line’ event has augmented the uncertainty in this scenario.

• **Scenario 3’**: After the red line event, the 3 white squares have become 3 black squares and the 3 black squares have become 3 white squares. The uncertainty which we already had before the red line event has not been changed.

After considering those simple scenarios, we could argue that the presence of information reduces uncertainty. If an information event were to augment uncertainty, then in fact this information adds to dis-information, i.e. the opposite of information. A formal definition of information can be given by the so called Fisher information, $I$, defined as: $\int \frac{1}{\hat{P}} \left( \frac{d\hat{P}}{dx} \right) dx$, where $\hat{P}$ is the pdf on noise ‘$x$’ derived from: $x_{obs} : x_{obs} = x_0 + x$. Hence, in scenario 2’, the red line event creates more noise, and thus augments uncertainty. Hence, the red line event reduces (Fisher) information. The red line event does not create (or reduces) information in either scenario 1’ or scenario 3’.

Since the red line event does not change in scenarios 1, 1’, 3 and 3’, if we were to look for an approximation of instantaneous knowledge (approximating the notion of non-locality), we could produce it very fast in scenarios 1 and 1’, and somewhat slower (and with uncertainty) in scenarios 3 and 3’. But in scenarios 2 and 2’: the event space has changed unbeknownst to Alice and Bob and this change has diminished information. We can almost argue for an approximate level of ‘non-local’ dis-information.

Translating the Alice and Bob example in the so called ‘risk neutral pricing’ approach which we use in modern asset pricing, would lead to some conundrums. Risk neutral pricing essentially uses non-event based probabilities to force expected returns of risky assets to be risk free. The importance of using a risk free rate of return on risky assets provides for a superior advantage in modelling the pricing of assets: such a rate is *objectively* determinable\(^2\). The essential problem with using risky rates of return is that they reflect so called ‘preferences for risk’ by decision makers, and those preferences are of course not unique and hence they are notoriously difficult to model. In fact, the risk-neutral pricing setting would require the red line event to be much more sophisticated. Instead of simply swapping colors, one would

\(^2\) One can peruse the financial newspapers to find the risk free rate. Typically, it will be the rate of return of some very safe investment product such as a government bond from a country which has an outstanding financial position.
need to create a red line event where black and white squares would now be converted into black and white squares with varying degrees of intensity. In other words, the level of ‘non-local’ dis-information would be much higher than in scenario 2. If we want to approximate instantaneous transmission of information, then this would be even more difficult in this setting.

Here are a couple of points we may want to keep in mind whilst progressing through this paper:

- In a risk neutral price setting, non-locality when it relates to instantaneous transmission of information seems to be difficult to fathom
- Non-locality can be shown to be tightly connected to the existence of the quantum potential
- The quantum potential emerges in the Nelson approach
- The Nelson approach implicitly shows us that asymmetry of past and future time seems to be an important condition for Fisher information to exist
- Optimizing Fisher information (in an economics setting) leads to Schrödinger-like PDE’s
- The quantum and real potentials can be estimated from real data and their interpretation can be shown to be quite intuitive
- Finally, we discussed so far a no-memory model: the stochastic differential equations used here employ white noise

An important issue which we should worry about is whether asymmetry between past and future information is a palatable condition in social science. Modern academic finance has debated such asymmetry and whilst some finance academics will continue to assume that only current information in the price of an asset can be used to predict a future price, behavioral finance will think otherwise. We note the work of De Bondt and Thaler [33]; and also Barberis and Thaler [34].

An interesting model which firmly resides in modern finance and which clearly assumes symmetry between past and future times is by Detemple and Rindisbacher [35] who show that one can provide for a formalism which models, next to public information, so called private information. The authors define the ‘Private Information Price of Risk (PIPR)’: as representing the changed price of risk when private information becomes available. See also Detemple and Rindisbacher [36]. They argue that the information gain (public information with the addition of private information) can be measured with an appropriate entropy measure. The full model occurs within a memoryless
(white noise) environment and there is no asymmetry between past and future time.

3 Laying the seeds for a ‘new’ memory model?

The former section of this paper attempted to argue that the Nelson approach and the ensuing quantum potential (which is also part of Bohmian mechanics) can be used in a financial setting. We also discussed some issues which are much harder to resolve such as the asymmetry of time which is seemingly needed to be able to define Fisher information. In the introduction of the paper we also tried to highlight, in a very heuristic way, some possible problems which may surface when considering an approximation to non-locality (which itself it very much tied to the existence of the quantum potential).

All of the arguments so far have been tied to a framework where we essentially claim that we use concepts and techniques from quantum mechanics without in any way pretending that the macroscopic world is quantum mechanical. In this section, we actually consider a macroscopic version of a model which is steeped in quantum mechanics. The model which will be discussed is also supported by experimental evidence. This model derives from some of the following (recent) work by Bush [37]; Eddi et al. [38]; Fort et al. [39]; Couder [40] and Hardesty [41].
The proposed model looks at a so called ‘walking droplet’ and the explicit ‘context’ here is definable as follows:

- The preparation is necessary before time \( t = 0 \) : the droplet bounces on a surface which needs to vibrate in a certain way (droplets will coalesce otherwise with the liquid they drop on
  (the droplet is made out of the same liquid)
- The preparation affects the trajectory on \( t = 0 \) and following: damping time of the wave formed by the impact of the droplet on the vibrating surface is dependent on the vibration frequency distance from a certain threshold

Given the above, we may wonder whether we have here ‘context conditioning’ versus ‘event conditioning’? We do not want to imply that even if there were to be scope for so called context conditioning one could allocate a quantum mechanical flavour to this experiment.
We have not come forward with any critical condition(s) which could help us in deciding this.

What is interesting in the experiment which was conducted, is that the so called ‘walker’ droplet exhibits wave particle duality. In a double slit experiment, the walker droplet chooses one slit over the other, but as Bush [37] (p. 274) indicates, “the guiding wave passes through both slits and the walker droplet ‘feels’ the second slit by virtue of its pilot wave.” As Fort et al. [39] say (p. 17515): “The motion of a droplet is...driven by its interaction with a superposition of waves emitted by the points it has visited in the recent past.” This is the argument by which one can claim that there is a memory property embedded in the walker droplet process (see Fort et al. [39], p. 17516).

Bush [37] and Fort et al. [39] discuss how close this macroscopic experiment is to Bohmian mechanics. We do not expand on this in the current paper. What we do believe is that this macroscopic model is holding great potential as an explicit formalism to model information in an economics/finance setting. Please note that in any of the examples below there is a formalism established (see Bush [37] and Fort et al. [39] for instance) to describe the events contained in the examples. Thus, the promise that analogous arguments, which are formulated below, hold is that this formalism could be used in an economics or finance setting. We do not expand on the formalism in this paper.

**Example 1**

i) The droplet bounces at time \( t \) at a height \( h \) (in the bath) and;
ii) an orbital wave is generated upon impact. Two coordinates can be determined: time and position (height, \( h \)). See Fort et al. [39], p. 17518. The formalism used in the above example, can be used
in analogy within a financial setting: i) A price level is generated at time $t$ and; ii) it has an information impact. Two coordinates are determined: price level occurring at time $t$.

**Example 2:**

The time duration of the information impact (damping time, $\tau$) is dependent on the context. The preparation comes into play: the fluid frequency in the bath is close or far from the so called ‘Faraday threshold’. See Fort et al. [39], p. 17518. In a financial setting, by analogy, we could for instance wonder why initial public offerings and their ensuing price level (height) have such differing impact before or after an election?

**Example 3:**

Past bounces of the droplet affect the current trajectory and hence memory is important. Here again, by analogy, in mathematical finance this is resisted: past information is not informative of the future but in behavioral finance, memory effects can be accepted.

**Example 4:**

The gradient of the height, $\nabla h$ determines the surface slope (and will determine the way the droplet bounces) (see Fort et al. [39], p. 17518). By analogy, in a financial setting, in so called technical analysis (see section 2 of this paper), a detailed analysis of past price increments can be used to chart future price behavior. The way a price increases or decreases in a time interval may say something about future price.

It has been remarked by Fort et al. [39], p. 17520, that the path memory we mentioned above can be related to non-locality.

## 4 Conclusion

What have we tried to claim in this paper? Although, we highlight some sources of problems when we attempt to apply concepts like Fisher information and the quantum potential to finance, there is evidence that those concepts have applicability. We refer back to work by Khrennikov [29]; Hawkins and Frieden [24]; Tahmasebi et al. [26] and Haven and Khrennikov [30]. But consider again the macroscopic, experimentally tested, version of Bohmian mechanics, as briefly described in section 3. This is macroscopic (!!) and the richness of the formalism (not shown here in this paper) especially in terms of available parameters is promising. We believe that this model seems to provide for a primer: a Bohmian mechanics model adapted to a macroscopic environment. We are aware that analogies need to be taken seriously and this can only be done with the use of hard data (for instance, estimate real and quantum potentials from data (see
Tahmasebi et al. [26] and Baaquie [27]). In the analogies presented in the above section, the mission should be similar: what are hard data driven analogies: on say damping time ($\tau$); height ($h(\tau)$); $\nabla h$ and context?

**Acknowledgment**

The author wants to thank Andrei Khrennikov for the opportunity which was given to him to present this work in a special session at the QDTF conference in June 2015.

**References**


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