Non-normality, Uncertainty and Inflation Forecasting: An Analysis of China’s Inflation

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by

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Abstract

Economic forecasting is important because it can affect the decision making processes of individuals, firms and governments so as to affect their behaviours. In this thesis, I discuss different methodologies for forecasting and forecast evaluation. I also discuss the role of assumption of normality and the role of uncertainty in economic forecasting. The first chapter is the introduction of the thesis. In second chapter, I conduct a Monte Carlo simulation to investigate the performances of forecast combination and the forecast encompassing test under the forecast errors non-normality. In third chapter, I examines the relationship between inflation forecast uncertainties and macroeconomic uncertainty for China by using different measures of uncertainties. I also investigate the relationship between inflation forecast uncertainties and inflation itself. In fourth chapter, I compute the probabilities of deflation for China by applying density forecast based on the theories and methodologies from previous two chapters. Particularly, I construct density forecasts for different forecast horizons by a joint distribution using Student-\(t\) copula. The fifth chapter is conclusion.
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Chapter 1

Introduction

Economic forecasting is a process of predicting conditions about the future economy. According to Carnot, Koen, and Tissot (2005), economic forecasting involves (1) a view on the future of the economic developments, including quantitative estimates of major macroeconomic variables at different forecast horizons, such as GDP, inflation or unemployment, and estimates for specific sectors of the economy, such as productions of firms; (2) an underlying analytical "story", which gives an exposition of the assumptions that underpin the forecasts, and also investigates the risks if the assumptions fail; (3) a discussion of the possible options for the potential user of the forecasts. Economic forecasting is very important. It can affect behaviours of individuals, firms and governments. For example, firms use forecasts to adjust their strategic planning to decide what to produce, how much to invest and produce etc.; government uses economic forecasts to guide their monetary and fiscal policies.

Forecasts can be made by a variety of methods. According to Carnot, Koen, and Tissot (2005), quantitative approaches of economic forecasting have four types: first of all, subjective methods, which relies on forecasters’ common sense, such as guessing or expert judgement. These forecasts could be a part of a forecasting approach, but they are lacks of reliability when they are solely used. Secondly, indicator-based methods, which are usually applied in business cycle analysis. For example, leading indicators are indicators that usually, but not always, change before the economy as a whole changes, see Mitchell and Burns (1938). The predictive power of a single indicator is limited, thus forecasters tend to analyse a group of indicators. An example is the “Leading Economic Index” conducted by The Conference Board (see e.g. Stock and Watson (2003)). This index consists of ten indicators, such as the interest rate spread and average weekly hours, to be used to predict the U. S. economy (Auerbach (1981)). Thirdly and
fourthly, time series models and structural models. Time series models are based on the statistical properties irrespective of causal relationship between economic variables informed by economic theory. They are simple and can be competitive relative to structural models (Holden, Peel, and Thompson (1990)). Nevertheless, structural models can explain the causal relationship between economic variables. The size of structural models can be small or large and can be a single-equation model to hundreds of equations, which provides an overview of the economy and helps us to understand the world (Holden, Peel, and Thompson (1990)).

A forecast can be a single number (point forecast), a range of numbers (interval forecast) or an entire distribution for the future values (density forecast). As a rule, point forecasts are different from realizations, it is obvious that a good point forecast should be close to the outcome. The differences between outcomes and the forecasts are forecast errors. These forecast errors, which are out-of-sample forecast errors, is different from model errors which are derived from in-sample forecasts. Suppose you have a data set \( y_t, t = 1, 2, \ldots, T \). The in-sample forecasts means to estimate the model using all available data up to \( T \), and then compare the model’s fitted values to the actual realizations. The in-sample forecast errors are simply residuals that yield no information on the predictive accuracy of the procedure. The out-of-sample forecasts, on the contrary, use data up to \( T \) to estimate the model, then constructs a forecast of \( \hat{y}_{T+1} \) and wait to know the realisation at \( T + 1 \). The out-of-sample forecast errors is \( e_{T+1} = y_{T+1} - \hat{y}_{T+1} \). In this thesis, the forecast errors indicates the out-of-sample forecast errors.

To evaluate forecasts, we have to distinguish the difference between ex-post forecasts, which are made when we know the realizations, and ex-ante forecasts, which are made when we do not know the real outcomes. Evaluation of ex-post forecast helps us learn from previous mistakes so that we can make a better ex-ante forecast. We also have to distinguish different methodologies for evaluating point forecast, interval forecast and density forecast. In my thesis, I only focus on the evaluation of the ex-post point forecast. To evaluate a single set of forecasts, it requires that forecasts are close to the real outcome. Besides, it requires that the forecast errors have small variance. The common measures of accuracy are the mean squared error (MSE) and the root mean squared error (RMSE). In practice, it is usually two or more sets of forecasts are available. To evaluate two or more forecasts, we not only need one forecast is more accurate than the rival forecast, but also incorporates all the relevant information in the rival forecast. That is forecast encompassing, see Chong and Hendry (1986). Failure of forecasts
to encompass other forecasts indicates that both can be improved by a forecast combination, see Bates and Granger (1969).

The distributional properties of the *in-sample* and the *out-of-sample* forecast errors are different. Since the *in-sample* forecast errors are model errors, they should follow a normal distribution or asymptotically normal. However, the sources of the *out-of-sample* forecast errors involve future shocks to the economy that forecasters can not anticipate, for example future unpredictable changes brought by political, social and technology changes, or using inaccurate estimations from forecast models in forecasting, or misspecification of the forecast models or measurement errors in the data. The distribution of the *out-of-sample* forecast errors could be any distribution. It is common practice to assume that the *out-of-sample* forecast errors are normally distributed as they are easy to compute. However, due to we cannot foresee the future, future unpredictable shocks may occur far more often than a normal distribution would expect. Secondly, even if the distribution of the forecast errors are normally distributed before the structural breaks or long-lasting policy changes, the distribution after the changes, in the aggregate, could be non-normal. Harvey and Newbold (2003) investigates the distribution of the forecast errors of six US macroeconomic variables and they find that none of these forecast errors is normally distributed or even symmetric. Therefore, the use of normality assumption in forecasting will result in overly narrow prediction intervals and will cause problems in evaluation tests.

Although point forecasts are widely used, they have their own limitations as they can not capture the uncertainty around the forecast, so called forecast uncertainty. Forecast uncertainty is important as it can affect agents’ behaviours and result in equilibrium evolution of the economy (Mankiw and Reis (2002)). For example, the empirical evidence provided by Cukierman and Meltzer (1986) indicating that high inflation uncertainty will lead to high inflation. Moreover, assessing forecast uncertainties related to economic variable, such as inflation or GDP, is a long established element of policy makers and indeed of most micro and macroeconomic decisions. There are other types of uncertainties, such as macroeconomic uncertainty, which is an unobservable aggregate measure of uncertainty for a macroeconomy. The general feeling of the macroeconomic uncertainty can affect timing of investment and consumption decisions, see Bloom (2009). These decisions made under uncertainties are suboptimal and will have influence on wider macroeconomic landscape as hiring is depressed, dissimulating employment and affecting growth. Moreover, if agents are risk averse, feelings of uncertainties increase precautionary their savings, which also negatively
affects growth. Thus, analysing macroeconomic uncertainty is also important for policy makers to adjust their policies in time and properly.

1.0.1 Contributions

Chapter 2 provides theory needed for modelling inflation forecasts under the forecast errors non-normality. Although Harvey, Leybourne, and Newbold (1998) apply the Student-\(t\) distribution to assess performances of different forecast encompassing tests under the non-normality, they do not investigate how the non-normality could have effect on the forecast encompassing test. Besides, even Student-\(t\) distribution is a non-normal distribution, it is still symmetric. In this chapter, I use tempered stable distribution as heavy-tailed distribution (Devroye (1981)) allowing for skewness and kurtosis. I also conduct a Monte Carlo experiment to investigate whether and to what extent the forecast errors non-normality will have impacts on the forecast combination and the forecast encompassing test with finite samples. Simulation results indicate that when the variance of the forecast errors is small, the non-normality has impact on power of the forecast encompassing test. The heavier tails of the distribution are, the more powerful the tests will be. The results also confirm that the combining forecast outperforms individual forecast, the heavier tails of the distributions are, the more advantages the combining forecasts will gain from individual forecast.

Chapter 3 provides empirical results based on theory of chapter 2. In this chapter, I address a question—whether a particular economic variable becomes more or less uncertain, the aggregate uncertainty or the macroeconomic uncertainty becomes more or less uncertain simultaneously. In order to find the answer, I investigate the relationship between the ex-post inflation forecast uncertainty and the macroeconomic uncertainty for China. The contributions are that I use different methods to measure forecast uncertainties including survey-based and model-based forecast uncertainties; and I also introduce the Economic Policy Uncertainty index for China (Baker, Bloom, and Davis (2013)) to represent the macroeconomic uncertainty. The results indicate that there is no significant relationship between the survey-based inflation forecast uncertainties and the macroeconomic uncertainty. However, it has significantly positive relationship between model-based inflation uncertainties the macroeconomic uncertainty. This is probably due to the EPU index for China is a news-based index, in which they use Hongkong English language newspapers rather than Chinese one. In other word, the EPU index is likely unreliable for measure the macroeconomic uncertainty in
China.

Chapter 4 is an application based on the theory of chapter 2 and uses part of methodologies in Chapter 3. This chapter is based on the joint work with Charemza and Makarova preliminarily called ‘Computing probability of deflation’. we forecast the probabilities of deflation in China by applying density forecasts. The traditional methodology of constructing density forecasts is to generate the probability distributions for different forecast horizons independently. However, since the forecasts and forecast errors are markedly dependent (Diebold and Mariano (1995)), in this chapter we consider the dependence of the forecast for different forecast horizons. The novelty is that we construct the density forecasts as a joint distribution by the use of Student-$t$ copula (Demarta and McNeil (2005)); and we also consider the impact of model uncertainty on the probability of deflation. The results suggest that the independent forecast may overestimate the probabilities of deflation when the forecast horizons are short but underestimate the probabilities of deflation when the forecast horizons are long. The use of an inappropriate distribution for density forecasts may under or overestimate the probabilities of deflation and the expected length of deflation, especially when the forecast horizons are short and the density forecasts are generated independently.

1.0.2 Policy implications for China

There are also some policy implications from my thesis. Firstly, the distribution of the model-based inflation forecast errors are likely non-normal in China. Therefore, forecasters and policy makers should use non-normal distributions rather than normal distribution to generate density forecast for inflation in China. Secondly, since high uncertainty can lead to loose monetary policy (Bekaert, Hoerova, and Duca (2013)), it is crucial for policy makers to analyse the macroeconomic uncertainty. Because a deeper analysis of how macroeconomic uncertainty has impacts on the economy in the past is likely to help policy makers to assess how future uncertainty shocks might have effects on economy. Thirdly, estimation of the probability of deflation can be used by monetary authorities in order to adjust their monetary policies, such as to reduce the value of RMB, to minimize the risk of deflation and to avoid the economy slips into deflation. Last but not the least, regarding to the economic situation in China in 2014-2015, it shows that the risk of deflation was indeed not negligible. However, if deflation really happened, it would most likely not last longer than three consecutive months.
Chapter 2

Forecast combination and encompassing under the forecast errors non-normality

Abstract. I have conducted a Monte Carlo simulation to investigate the performances of the forecast encompassing test and forecast combination under forecast errors non-normality with finite samples. The results indicate that the size is not affected by the non-normality in the forecast encompassing test. However, when the variance of the forecast errors is small, the non-normality has impact on power. The heavier tails of the distribution are, the more powerful the tests will be. The simulation results confirm that the combining forecast outperforms individual forecast, the heavier tails of the distributions are, the more advantages the combining forecasts will gain from individual forecast. In the cases of skewed and symmetric distributions, they have similar performances on both power tests and forecast accuracy.
2.1 Introduction

Inflation forecasting is crucial because it can affect many economic decisions. Governments need to forecast inflation as monetary policies are formulated according to what the authorities expect to happen in the future; investors have an interest in forecasting inflation because future inflation rate could have great influences on their assets returns as a result to affect their investment decisions; firms focus on inflation forecasts to adjust their goods prices and production plans.

Regarding to the monetary policies, inflation forecasting plays an important role in inflation targeting regime. Inflation targeting is an economic policy in which central bank determines the inflation rate target (government announces a numerical inflation target: normally an annual growth rate of CPI is around 2%) and attempt to achieve it by the mean of monetary policies. According to King (2005) and Svensson (2010), countries which have adopted inflation targeting have had several crucial benefits: first, the inflation targeting regime helps these countries to achieve low and stable inflation rate. Secondly, once government commits an inflation target, inflation expectations of individuals will not drift far away from the target which helps government to monitor inflation expectations. Moreover, the use of inflation targeting makes public to understand the objectives of central bank as a result of improving transparency of monetary policy and accountability of central bank officials. Since 1990s a large number of countries, such as New Zealand, Australia, Canada, have instituted inflation targeting as their framework of making monetary policies.

However, one notable difficulty involved in implementing the inflation target arises from the fact that there is a time lag between the policy actions and the effects of monetary policies on changes of inflation rate, it is therefore essential and necessary for policy makers to be forward-looking and to predict the likely path of inflation rate, that is inflation forecasting. Svensson (1997) explains the importance of inflation forecasting to the inflation targeting: the central bank’s inflation forecasts is an ideal intermediate target as it is correlated with the target and more controllable than the target. Inflation forecasting not only provides an assessment of monetary policy performance, namely how far is the forecast from the realization, but also provides a measurement of the creditability of monetary policy, namely a measure of inflation expectations relative to the inflation target. An example of inflation forecasting by government for targeting inflation is the quarterly Inflation Report of the Bank of England (Bank of England, 2013): the
Report, first published in 1993, incorporates the detailed economic analysis and factors that are likely to have impacts on inflation rate and probabilistic inflation forecasts (known as Fan Chart). This report helps the Monetary Policy Committee to conduct its monetary policies as well as providing evaluations of the prospects for UK inflation.

Inflation forecasting is also important to an instrument rule in monetary policy, such as Taylor rule (Taylor (1993)). Taylor rule explains how the central bank should set short-term interest rate in response to changes in inflation, output, or other economic conditions in order to achieve its short-term and long-term goal. The following expression is one variant of the Taylor rule from Clarida, Galí, and Gertler (1998):

\[ r_t^* = \bar{r} + \beta (E[\pi_{t+h}|\Omega_t] - \pi^*) + \gamma (E[y_t|\Omega_t] - y_t^*) \]

where \( r_t^* \) denotes short-run interest rate, \( \bar{r} \) is the long-run average interest rate, \( \pi_{t+h} \) is the inflation rate between \( t \) and \( t+h \), \( y_t \) is real output, \( \pi^* \) is the inflation target and \( y_t^* \) denotes potential output. \( E \) is the expectation operator and \( \Omega_t \) is the information available at time \( t \). From the Taylor rule, it is very clear that inflation forecasting \( E[\pi_{t+h}|\Omega_t] \) is a key ingredient of the monetary policy operates, see McCallum and Nelson (2005).

There are various approaches for economic forecasting: such as expert judgement, which focuses on incorporating forecaster’s judgements into the forecasts, see Lawrence, Goodwin, O’Connor, and Önal (2006); leading indicators (forecasting economic growth), e.g. Auerbach (1981); surveys, which provide both point and probability distribution of inflation forecasts from respondents, see Croushore (1993); econometric and time-series models, see Stock and Watson (1999) and Stock and Watson (2001). A forecast can be a point forecast, an interval forecast or a density forecast. There are different methodologies to evaluate these forecasts and models, in this chapter I only focus on the point forecast. To evaluate one series of forecasts, these forecasts should be close to the real outcome as well as have small variance. The common measure to evaluate a series of forecasts is the mean squared error (MSE) or root mean squared error (RMSE). It is often the case that two or more forecasts from different models are available. To evaluate two or more sets of forecasts, one forecast should not only be more accurate than the rival forecast, but also incorporates all information that the rival forecasts embody (Granger and Newbold (1973)). If this is the case, we call one set of forecasts encompasses the other (Chong and Hendry (1986)); if not, it
means that both forecasts have their own useful information and both forecasts can be improved by forecast combination (Bates and Granger (1969)). Details of these approaches will be introduced in following sections.

Normal distribution is widely used in econometrics. However, when we construct a histogram of price changes (for example, annual changes of PPI) for several years, it is often found that price changes should be non-normally distributed (Ball and Mankiw (1992), Bryan and Cecchetti (1999)). As we can see from Figure 2.1, the two graphs are the histograms of inflation for China and UK. The inflation rate is measured by monthly data of the annual growth rate of CPI. Data for China is from 1987 M1 to 2014 M12 and data for UK is available from 1956 M1 to 2014 M12. It is obvious that none of these inflation rate follow a normal distribution. Both of cases reject the null hypothesis that inflation is normally distributed at 1% significance level. This results implies that inflation forecast errors, namely the difference between real outcomes and forecasts, are likely to be non-normally distributed. Harvey and Newbold (2003) find that the forecast errors for the US macroeconomic variables are non-normally distributed and even asymmetric. Therefore, the assumption of normality in conventional forecast encompassing test will cause inaccurate test results. Although Harvey, Leybourne, and Newbold (1997) and Harvey and Newbold (2000) apply a Student-$t$ distribution to assess different methods of the forecast encompassing test, they do not investigate how the non-normality will affect the result of the forecast encompassing test. In addition, the Student-$t$ distribution is still a symmetric distribution while the distributions of forecast errors could be skewed and asymmetric. In this chapter, I use a tempered stable distribution as a heavy-tailed distribution which allows for skewness and leptokurtosis and I conduct a Monte Carlo experiment to investigate whether and to what extent the non-normality will have impact on the forecast combination and the forecast encompassing test with finite samples compared with the normal cases.

The plan of the rest of the chapter is as follows. Section 2.2 introduces forecast errors and the heavy-tailed distribution, especially the tempered stable distribution. Section 2.3 outlines the historical development of forecast combination and the forecast encompassing test. Section 2.4 describes the Monte Carlo simulation. In Section 2.5, I report the simulation results. The final section concludes.

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1Data source is the Federal Reserve Bank of St. Louis.
Figure 2.1: Histograms of inflation
2.2 Forecast errors and heavy-tailed distributions

When we make a forecast, we have to understand whether the outcomes will be (1) a single number (a point forecast), (2) a prediction interval in which the real outcome lies between a specified range with a certain probability (the interval forecast is similar to the concept of confidence interval), or (3) a density forecast: an entire probability distribution of possible predictions. In this chapter, I will focus on the first one because it is simple and more commonly used in practice.

Point forecasts are often different from the real outcomes. The dispersions between forecasts and realities are called forecast errors. Ericsson (2000) discusses several sources of forecast errors in econometric modelling, forecast errors may come from accumulation of future shocks to the economy, inaccurate estimations from forecast models in forecasting, unknown future changes in the economic structure, and misspecification of the forecast models or measurement errors in the data.

In common practice, forecast errors are often assumed as normally distributed. However, since we cannot foresee the future, future unpredictable changes brought by political, social, financial and technology changes may occur far more often than a normal distribution would expect. In other words, the probability of large forecast errors is much higher than would be implied by a normal distribution. Secondly, even if the distribution of the forecast errors are normally distributed before the structural breaks or long-lasting policy changes, the distribution after the changes, in the aggregate, will be non-normal (Ball and Mankiw (1992)). These forecast errors which could incorporate important information are discarded in normal distributions. Regarding to inflation, although prices usually changes modestly, there will be periods where changes of prices are higher (e.g. hyperinflation) than predicted by a normal distribution. Zarnowitz and Braun (1993) investigate the historical forecast errors from the quarterly survey of professional macroeconomic forecasters, they find that individual forecast errors are often large, especially inflation forecast errors. However, they do not analyse the distributional characteristic of the historical forecast errors. Harvey and Newbold (2003) examine distributional properties for both individual and consensus forecast errors of six US macroeconomic variables in SPF, the evidence

\footnote{The consensus forecasts can be seen as a combined forecast which have been created using different methodologies Zarnowitz and Braun (1993).

Survey of Professional Forecasters, the survey began in 1968 and organised by the American Statistical Association and National Bureau of Economic Research, provides a large number of macroeconomic forecasts from institutions}
demonstrates that none of these forecast errors are normally distributed or even symmetric, moreover, leptokurtosis is also manifested in the individual forecast errors as well as the consensus forecast errors. Therefore, if forecast errors are assumed normally distributed but truly non-normally distributed, it will result in overly narrow prediction intervals and will cause problems with evaluation tests which are based on the assumption of normality.

A heavy-tailed distribution, on the other hand, which can exhibits skewness and leptokurtosis (a distribution is leptokurtic if it is more peaked in the center and thicker tailed than a normal distribution) are likely to describe the real behaviours of economic data. According to Foss, Korshunov, and Zachary (2011), a distribution of a random variable $X$ is said to be right heavy-tailed if:

$$\lim_{x \to \infty} e^{\lambda x} \bar{F}(x) = \infty, \quad \text{for all} \quad \lambda > 0$$

(2.1)

where $\bar{F}(x) = P(X > x)$ is the tail function which is not exponentially bounded. The expression indicates that the moment generating function for heavy-tailed distributions is infinite. The definitions for left-tailed and two tailed distribution are similar. The heavy-tailed distribution contains three subclasses including the fat-tailed distribution, the long-tailed distribution and the subexponential distribution, see Foss, Korshunov, and Zachary (2011) and Haas and Pigorsch (2009).

In this chapter, I will only focus on the fat-tailed distribution (In some literature, the fat-tailed and heavy-tailed distributions are interchangeable). A random variable $X$ is said to have a right fat tail (Haas and Pigorsch (2009)) if

$$\bar{F}(x) \sim x^{-\alpha} \quad \text{as} \quad x \to \infty, \quad \alpha > 0$$

(2.2)

where $\alpha$ is the tail index, characterised the rate of tail decay to zero. The notation $\sim$ denotes the asymptotic equivalence of functions. Distributions whose tails trend to zero slower than exponentially but faster than a power $x^{-\alpha}$ are heavy-tailed but not fat-tailed distributions, for example, the log-normal distribution.

A large number of fat-tailed distributions have been introduced in the literature, such as $t$-distributions, $\alpha$-stable distributions etc.. Since the $\alpha$-stable distribution (sometimes the stable distribution family refers to $\alpha$-stable distribution) was applied by Mandelbrot (1963) to analyse cotton futures return data, this distribution has been widely used in financial market to model fat-tailed data with skewness and leptokurtosis. The key property of the stable distribution is stability: any sum of independent and identically stable distributed random variables is again stable distributed (Nolan (2003)). If $X_1, X_2, ..., X_n$ are IID stable random
variables, then
\[ X_1 + X_2 + \ldots + X_n =^d c_n X + d_n \]  \hfill (2.3)

where \( c_n > 0 \) and \( d_n \) are constants, the symbol \( =^d \) denotes equality in distributions, which means the left-hand and right-hand sides in the expression have the same distribution. \( X \) is strictly stable if and only if \( d_n = 0 \). This property implies that, for example, if the individual daily asset returns follow a stable distribution, the distribution of total daily asset returns over a period will be stable and have the same shape as the individual daily returns. According to Samorodnitsky and Taqqu (1994), stable distributions is determined by characteristic functions as the density function of the stable distributions does not exist in the closed form. A random variable \( X \) is stable with parameters \( \alpha, \beta, \delta, \mu \), say \( X \sim S_\alpha(\beta, \delta, \mu) \) if its characteristic function has the following form (see Samorodnitsky and Taqqu (1994)):

\[
\Phi(t) = E\exp(itX) = \begin{cases} 
\exp(-\delta^\alpha|t|^\alpha[1 - i\beta\text{sign}(t)\tan(\frac{\pi\alpha}{2})] + i\mu t) & \alpha \neq 1 \\
\exp(-\delta|t|)\{1 + i\beta^2\pi\text{sign}(t)\ln|t|\} + i\mu t & \alpha = 1
\end{cases}
\]  \hfill (2.4)

where \( t \) denotes argument of Fourier transformation, \( i \) is the imaginary unit. The sign function is defined as

\[
\text{sign}t = \begin{cases} 
-1 & \text{if } t < 0 \\
0 & \text{if } t = 0 \\
1 & \text{if } t > 0
\end{cases}
\]  \hfill (2.5)

The \( \alpha \)-stable distribution has four key parameters: index of stability \( \alpha \) denotes the kurtosis of the distribution with \( 0 < \alpha \leq 2 \) (when \( \alpha = 2 \) the distribution is normal distribution), as the value of \( \alpha \) becomes smaller, the distribution are more peaked around the center and the tails are heavier than the normal. For \( \alpha < 2 \), the variance is infinite and for \( \alpha < 1 \) the first moment does not exist. \( \beta \in [-1, 1] \) is a skewness parameter (\( \beta = 0 \) when distribution is symmetric), if \( \beta > 0 \), the distribution is negatively skewed and if \( \beta < 0 \), it is positively skewed. \( \delta > 0 \) is scale parameter which determines width of the distribution and also can be interpreted as volatility, \( \mu \) denotes location parameter, it shifts distribution to the right when \( \mu > 0 \) and to the left when \( \mu < 0 \).

Due to the lack of closed-form expressions for the density function of a stable distribution, it makes analytical approach difficult for a stable distribution. Thus simulation comes into play. A common procedure for simulating a stable distribution is based on Chambers, Mallows, and Stuck (1976). The algorithm for
generating a random variable $X \sim S_\alpha(\beta, 1, \mu)$ is as follows:

Step 0. draw a random variable $U$ uniformly distributed on $(-\frac{\pi}{2}, \frac{\pi}{2})$, and an independent exponential random variable $W$ with mean 1,

Step 1. if $\alpha \neq 1$, compute

$$X = D_{\alpha, \beta} \frac{\sin(\alpha(U + B_{\alpha, \beta}))}{\cos^{1/\alpha}(U)} \left( \frac{\cos(U - \alpha(U + B_{\alpha, \beta}))}{W} \right)^{(1-\alpha)/\alpha}$$

where

$$B_{\alpha, \beta} = \frac{\arctan(\beta \tan(\frac{\pi \alpha}{2}))}{\alpha}$$

$$D_{\alpha, \beta} = (1 + \beta^2 \tan^2(\frac{\pi \alpha}{2}))^{1/(2\alpha)}$$

if $\alpha = 1$, compute

$$X = \frac{2}{\pi} \left( (\frac{\pi}{2} + \beta U) \tan(U) - \beta \log \left( \frac{\pi}{2} W \cos(U) \right) \right)$$

Given the formulas for simulation of standard $\alpha$-stable random variables, we can easily simulate a $\alpha$-stable random variable for all possible values of the parameters $\alpha$, $\beta$, $\delta$ and $\mu$ using the following properties:

**Property 1.** Let $X \sim S_\alpha(\beta, \delta, \mu)$ and $c$ is a non-zero constant. Then $X+c \sim S_\alpha(\beta, \delta, \mu+c)$

**Property 2.** Let $X \sim S_\alpha(\beta, \delta, \mu)$ and $c$ is a non-zero constant. Then

$$cX \sim S_\alpha(\text{sign}(c)\beta, |c|\delta, c\mu), \quad \alpha \neq 1$$

$$cX \sim S_1(\text{sign}(c)\beta, |c|\delta, c\mu - \frac{2}{\pi} c(\ln|a|)\delta \beta), \quad \alpha = 1$$

Using these two properties, if $X \sim S_\alpha(\beta, 1, \mu)$, then

$$Y = \delta X + \mu \quad \alpha \neq 1,$$

$$Y = \delta X + \frac{2}{\pi} \beta \delta \log(\delta) + \mu \quad \alpha = 1$$

is $S_\alpha(\beta, \delta, \mu)$.

However, the $\alpha$-stable distribution has infinite moments which is not applicable and reliable for describing macroeconomic data. Thus, a tempered stable distribution with a finite variance, has been introduced by Devroye (1981) to resolve the above limitations of the $\alpha$-stable distribution in the realm of finance and macroeconomics. A random variable is defined as a tempered stable distributed
$TS_\alpha(\beta, \delta, \mu, \theta)$ if its characteristic function has the form $\Phi_X(t) = e^{\psi_X(t) + i(\mu - \mu_X)t}$ (see Jelonek (2012)), where

$$
\psi_X(t) = \begin{cases} 
- \frac{1}{2\cos(\pi\alpha/2)}(1 + \beta)(\theta - it)^\alpha + (1 - \beta)(\theta + it)^\alpha - 2\theta^\alpha \quad & \alpha \neq 1 \\
\frac{1}{2}\delta[(1 + \beta)(\theta - it)\log(\theta - it) + (1 - \beta)(\theta + it)\log(\theta + it) - 2\theta\log \theta] \quad & \alpha = 1 
\end{cases}
$$

(2.8)

the centering term is

$$
\mu_X = \begin{cases} 
\alpha(\cos \frac{\pi\alpha}{2})^{-1}\beta^\delta \theta^{\alpha-1}, & \alpha \neq 1 \\
(\frac{2}{\pi})\beta\delta(\log \theta + 1), & \alpha = 1 
\end{cases}
$$

(2.9)

where $i$ is an imaginary number, $t$ denotes argument of Fourier transformation, $\theta > 0$ measures how far the distribution is from $\alpha$-stable distribution, while $\theta$ approaches to 0 indicates the distribution is in limit $\alpha$-stable. $0 < \alpha \leq 2$ denotes the index of stability (when $\alpha = 2$ the distribution is normal distribution), $\beta \in [-1, 1]$ is a skewness parameter. $\delta > 0$ is scale parameter, $\mu$ denotes location parameter.

There is a large body of literature on simulation of tempered stable random variables, for example Ridout (2009), Kawai and Masuda (2011). The issue of random number generation for TS distribution has not been solved yet in a satisfactory way. Methods that are fast and easy to implement are only available for certain parameter values. The Baeumer and Meerschaert (2010) procedure and the Devroye (2009) technique are two methods that are widely used:

The B-M procedure generates random numbers from a $TS_\alpha(1, 1, 0, \theta)$. The algorithm is as following:

- **Step 0.** Determine a constant $c > 0$.
- **Step 1.** Generate a uniform distribution $U \sim (0, 1)$ and an $\alpha$-stable distribution $V \sim S_\alpha(1, 1, 0)$.
- **Step 2.** If $U \leq e^{-\theta(V+c)}$, exit with $Y = V - \alpha\theta^{\alpha-1}/\cos \frac{\pi\alpha}{2}$, otherwise return to Step 1.

Although the B-M procedure is very handy with less computing effort, this procedure does not provide an exact simulation method for the stability index ($\alpha$) that is greater than one. When $\alpha > 1$, random numbers are generated by approximation. In addition, this procedure is feasible only if $\beta = 1$.

Another algorithm is developed by Devroye (2009), this algorithm relies on the results of Devroye (1981):
Property 3. If the characteristic function $\Phi_x$ of a random variable $X$ is twice differentiable, $\Phi_x$, $\Phi'_x$ and $\Phi''_x$ are absolutely integrable and absolutely continuous, then $X$ has a density function satisfying:

$$X \in \mathbb{R}, \quad f(x) \leq \min(d_1, \frac{d_2}{x^2})$$

where

$$d_1 = \frac{1}{2\pi} \int_{\mathbb{R}} |\Phi_X(t)| dt$$

$$d_2 = \frac{1}{2\pi} \int_{\mathbb{R}} |\Phi''_X(t)| dt$$

hence, the algorithm of Devroye (2009) is:

Step 0. Evaluate $d_1$ and $d_2$.

Step 1. Generate independent uniform distribution $U \sim (0, 1)$ and $U_1, U_2$ are IID uniform random variables on (-1, 1). Let $Y = \sqrt{\frac{d_2}{d_1}} \ast \frac{U_1}{U_2}$ if $|Y| < \sqrt{\frac{d_2}{d_1}}$, then go to Step 3.

Step 2. If $U < f(Y)Y^2/d_2$, then return $X = Y + \mu$. Otherwise go to Step 1.

Step 3. If $U < f(Y)/d_1$, then return $X = Y + \mu$. Otherwise go to Step 1.

This algorithm generates random number $X$ from $TS_\alpha(\beta, \delta, \mu, \theta)$. Some preliminary work is required for this algorithm. First of all, according to 2.4, the second order derivative of $\Phi_X(t)$ is given by

$$\Phi''_X(t) = \begin{cases} 
-\Phi_X(t)C_{\alpha,\delta}(C_{\alpha,\delta}[(1 + \beta)(\theta - it)^{\alpha - 1} - (1 - \beta)(\theta + it)^{\alpha - 1} - 2\beta \theta^{\alpha - 1}]^2 
+ (1 - \alpha)[(1 + \beta)(\theta - it)^{\alpha - 2} + (1 - \beta)(\theta + it)^{\alpha - 2}]) & \alpha \neq 1 \\
-\Phi_X(t)\frac{\alpha}{\pi}[(1 + \beta)\log(\theta - it) - (1 - \beta)\log(\theta + it) - 2\beta \log \theta^2 + 2 \ast \frac{\theta + \beta i}{\theta^2 + i^2}] & \alpha = 1 
\end{cases}$$

where $C_{\alpha,\beta} = \alpha \delta^{\alpha}(\cos \frac{\alpha \pi}{2})^{-1}/2$. Secondly, since there is no closed form for $d_1$ and $d_2$, integrals of $d_1$ and $d_2$ have to be determined through numerical integration techniques. Last but not the least, the density function of TS distribution is unavailable in closed form, the density can be computed through Fourier inversion of the characteristic function.

Although the Devroye’s procedure provides more precise simulation results than the Baeumer and Meerschaert’s (B-M procedure), this algorithm requires a lot of computation time for computing density.

In this chapter, I generate tempered stable distribution from a mixture representation technique by Jelonek (2012), based on the B-M procedure for generating $TS_\alpha(1, 1, 0, \theta)$, this approach has two advantages: firstly, it resolves the limitations
of the previous procedures and provides a practically exact simulation method to generate TS distribution with $1 < \alpha < 2$. Secondly, this process fits for macro data and it is more accurate than the B-M procedure and faster than the Devroye’s approach. However, his method still requires some computational time.

The Jelonek’s approach relies on following theorem that a tempered stable variable could be expressed as weighted average of two independent TS variates with $\beta = 1$ (the stability property may demonstrate this results):

**Theorem 1.** Let $Y^+$ and $Y^-$ be independent, set $Y^\pm \sim TS_\alpha(1, 1, 0, \theta^\pm)$, set $V^\pm = 2^{-1/\alpha}(1 \pm \beta)^{1/\alpha}$, $\theta^\pm = \theta V^\pm$, then $X = V^+Y^+ - V^-Y^- + \mu$ will follow a TS distribution with $TS_\alpha(\beta, \delta, \mu, \theta)$.

The algorithm of mixture representation is the following:

**Step 0.** Set set $V^\pm = 2^{-1/\alpha}(1 \pm \beta)^{1/\alpha}$, $\theta^\pm = \theta V^\pm$.

**Step 1.** Generate independent $Y^+ \sim TS_\alpha(1, 1, 0, \theta^+)$ and $Y^- \sim TS_\alpha(1, 1, 0, \theta^-)$ using B-M procedure.

**Step 2.** Return $X = V^+Y^+ - V^-Y^- + \mu$.

This algorithm returns pseudo random number $X$ from $TS_\alpha(\beta, \delta, \mu, \theta)$. Unlike the previous methods, this approach is available for $1 < \alpha < 2$ and $|\beta| \neq 1$.

### 2.3 Evaluation of forecast accuracy and forecast encompassing test

There are two approaches to define and to evaluate forecasts. *Ex-post* forecast is a forecast that uses information beyond the time at which the actual forecast is prepared, namely, the *ex-post* forecasts use historical data to forecast as well as evaluate forecast performance. *Ex-ante* forecast, on the other hand, is a forecast that uses information available by the time of the actual forecast only, in other words, the *ex-ante* forecasts use historical data only for forecasting without knowing the real outcome. For instance, the Bank of England has published its *ex-ante* forecasts of both inflation and GDP growth rate through Fan Chart, which summarises the Bank’s predicted probability distribution of inflation rate (Britton and Fisher (1998)). According to Reifsneider and Tulip (2007), it is found that historical forecast performance is a good guide to future accuracy as forecasters trend to make similar errors that they made in the past. Therefore, in order to make a better *ex-ante* forecast, it is important to analyse the accuracy of *ex-post*
forecast. A comparative evaluation of the *ex-ante* and *ex-post* forecasts can be used to analyse the causes of volatile forecast errors, misspecified forecasting models or poor forecast of the variables used by the model.

Predictability evaluation can be assessed by the use of *in-sample* or *out-of-sample* analysis. In the former case, models are estimated using the entire sample and predicts observations within the sample range. Thus, the prediction errors of such *in-sample* forecasts are simply residuals which yield no information on the predictive accuracy of the procedure (Inoue and Kilian (2005)). While the later case is often used, because *in-sample* is a test of goodness of fit rather than forecast and the *in-sample* forecast errors are likely to underestimate the forecast errors, see Tashman (2000), Clements and Hendry (2005) and Stock and Watson (2008). For *out-of-sample*, suppose there are $T$ observations (indexed by $t = 1, 2, 3...T$), the sample is split into estimation period which spans the first $R$ observations ($t = 1, 2, ...R$) and evaluation period with $H$ observations ($t = R + 1, R + 2...R + H, R + H = T$). For a recursive estimation, model is estimated using data through $R$, making $h$-step-ahead forecast for $R + h$ ($h$ is forecast horizon), then moving forward to $R + 1$ (model is re-estimated from $t = 1$ to $R + 1$, then making a forecast) and repeating above steps through the whole sample. For a rolling window scheme, model is estimated using a moving data window of fixed size (model is re-estimated from $t = 2$ to $R + 1$ then $t = 3$ to $R + 2$) through the sample. In this paper, forecasts are *ex-post* and evaluated by *out-of-sample* recursive estimation.

It is obvious that good forecasts should be close to the outcomes and have small variation of the forecast errors. The crucial object in quantifying a good forecast is the loss function $L(e_t)$, which measures the cost or loss associate with forecast errors (Theil (1965)). Forecast accuracy may be different across different loss functions, for the convenient, the quadratic loss function is widely used measure. There are few stylised quadratic loss function that are widely used: the mean squared error (MSE) is a preferred measurement for assessing forecast performance,

$$MSE = \frac{1}{T} \sum_{t=1}^{T} e_t^2$$ (2.14)

where $t$ is a number of forecast observations ($t = 1, 2, 3...T$), $e_t$ denotes a series of point forecast errors. The mean squared error can be decomposed into the sum of the variance and the squared bias of the forecasts, it measures how do the forecasts close to the outcome and the volatility or variance of the forecast errors;

$$MSE = E[(y_t - \hat{y}_t)^2] = var(y_t - \hat{y}_t) + (E[y_t] - E[\hat{y}_t])^2$$ (2.15)
Moreover, the square root of this measure is also widely used, called the root mean squared error,

\[ \text{RMSE} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} e_t^2} \tag{2.16} \]

To evaluate the accuracy of two forecasts, a preferred forecast outperforms the rival forecasts if the preferred forecast has a smaller RMSE. However, each individual forecasts may contain some independently useful information, which could be discarded by applying a preferred forecast. Bates and Granger (1969) propose that taking a weighted average of the individual forecasts, a combined forecast should outperform any individual forecasts. Suppose there are two sets of unbiased forecasts \( \hat{f}_{1t} \) and \( \hat{f}_{2t} \) (the subscripts 1 and 2 indicate two rival forecasts. \( t = 1, 2, ..., T \), is a series of forecast outcomes made by the same forecast horizon), of the variable \( y_t \):

\[ y_t = \hat{f}_{1t} + e_{1t} \tag{2.17} \]
\[ y_t = \hat{f}_{2t} + e_{2t} \tag{2.18} \]

therefore, \( e_{1t} = y_t - \hat{f}_{1t} \) and \( e_{2t} = y_t - \hat{f}_{2t} \) are individual forecast errors respectively. Bates and Granger (1969) propose the forecast combination as

\[ f_c = (1 - \lambda) \hat{f}_{1t} + \lambda \hat{f}_{2t} \tag{2.19} \]

where \( \lambda \) denotes combination weights, then,

\[ \epsilon_t = y_t - f_c = (1 - \lambda)e_{1t} + \lambda e_{2t} \tag{2.20} \]

where \( \epsilon_t \) denotes the error of the combined forecasts which has mean zero, to minimise the variance of the error of the combined forecast by setting the weight \( \lambda \):

\[ \lambda = \frac{\sigma_1^2 - \rho \sigma_1 \sigma_2}{\sigma_1^2 + \sigma_2^2 - 2 \rho \sigma_1 \sigma_2} \tag{2.21} \]

where \( \sigma_1^2 \) and \( \sigma_2^2 \) are the variances of the forecast errors, \( \rho \) denotes the correlation coefficient of these two sets of the forecast errors. Since then, a large number of studies devote to investigating the optimal weights in the combined forecast. Simply, the \( \lambda \) can be estimated by OLS using following equations:

\[ y_t = (1 - \lambda) \hat{f}_{1t} + \lambda \hat{f}_{2t} + \epsilon_t; 0 \leq \lambda \leq 1 \tag{2.22} \]
which is equivalent to

\[ e_{1t} = \lambda (e_{1t} - e_{2t}) + \epsilon_t \]  

(2.23)

However, empirical findings suggest that combined forecasts based on a simple average with equal weights often outperform more sophisticated weighting methods (see Clark and McCracken (2010), Smith and Wallis (2009), Stock and Watson (2004)).

Although a combing forecast could be more accurate than an individual forecast, it is very often that two forecasts are combined from two misspecified models. Therefore, the combining forecast can still be improved using additional predictive information. Based on the concept of forecast efficiency, Nelson (1972) and Granger and Newbold (1973) propose that a set of preferred forecasts should not only predict better than other sets of forecasts, but also incorporate relevant information about the future that the rival sets of forecasts do not embody. Subsequently, Chong and Hendry (1986) introduce the terminology of forecast encompassing suggesting that a preferred forecast encompasses rival forecasts if the rival forecasts provide no incremental information for prediction. That is to say, the forecast errors of a preferred forecast \((e_{1t})\) should be uncorrelated with the other rival forecast \((f_{2t})\) as well as its forecast errors \((e_{2t})\). According to the definition of forecast encompassing, the simplest and most straightforward approach to test whether \(\hat{f}_{1t}\) encompasses \(\hat{f}_{2t}\) in the equation (2.22), would be a standard \(t\) test of the null that \(\lambda = 0\), the alternative is typically one-sided \(\lambda > 0\), to preclude the possibility of negative combination weights. However, Andrews, Minford, and Riley (1996) argue that the linear combination should be restricted as the sum of the coefficients of the two forecasts should be equal to one. Regression-based test of forecast encompassing is an approach to test whether the combining forecast results in a statistically significant reduction in forecast loss, for example RMSE, relative to using an individual forecast.

The regression-based forecast encompassing test has several limitations. Since the information sets of forecasters are overlapping and forecast models are potentially to be similar, forecast errors tend to be contemporaneously correlated. Granger and Newbold (1986) develop a test for the equality of predictive accuracy based on Morgan (1939), called the Morgan-Granger-Newbold test (MGN test). Let

\[ x_t = e_{1t} + e_{2t} \]  

(2.24)

\[ z_t = e_{1t} - e_{2t} \]  

(2.25)
then under the null hypothesis of equal predictive accuracy is equivalent to zero covariance between $x_t$ and $z_t$. The test statistic

$$MGN = \frac{r}{\sqrt{\frac{1-r^2}{T-1}}}$$  \hspace{1cm} (2.26)

is a Student-$t$ distribution with $T - 1$ degrees of freedom, where

$$r = \frac{x'z}{\sqrt{(x'x)(z'z)}}$$  \hspace{1cm} (2.27)

$x$ and $z$ are $T*1$ vectors and the sign of this test indicates the superior forecast. The drawbacks of the MGN test is that this test is only valid for 1-step ahead forecast. Besides the forecast errors should be serially uncorrelated and are Gaussian.

However, according to Newbold and Harvey (2002): first of all, even though the error term of one-step-ahead forecasts in the equation (2.22) is not serially correlated, those error terms from $h$-step ahead forecasts would follow a moving average process of order $h - 1$ rather than a white noise. Secondly, recall the equation (2.23); if $\lambda = 0$, the variance of the errors of the combined forecast is a function of the difference between the two individual forecast errors:

$$\text{Var}[\epsilon_t | e_{1t} - e_{2t}] = \text{Var}[\epsilon_{1t} | e_{1t} - e_{2t}]$$  \hspace{1cm} (2.28)

this implicates the error terms are conditionally heteroskedastic (Harvey, Leybourne, and Newbold (1998)).

Harvey, Leybourne, and Newbold (1998) develop a modified MGN test in a regression framework by allowing for heteroskedasticity:

$$x_t = \beta z_t + \epsilon_t$$  \hspace{1cm} (2.29)

the modified MGN test is the same as testing the null hypothesis of $\beta = 0$ by applying a White-correction for heteroskedasticity. The test statistic is

$$\text{MMGN} = \frac{\beta}{\sqrt{\frac{\sum z_t^2 \hat{\epsilon}_t^2}{(\sum z_t^2)^2}}}$$  \hspace{1cm} (2.30)

where

$$\beta = x'z / z'z$$  \hspace{1cm} (2.31)

and $\epsilon_t$ is the estimated OLS residual at time $t$. However, Harvey, Leybourne, and Newbold (1998) verify that the modified MGN test has oversize problem with
small samples even in the case of one-step-ahead forecasts.

Diebold and Mariano (1995) propose a test (Diebold-Mariano test, DM test) allowing for non-quadratic loss functions, multi-step-ahead forecasts and non-Gaussian, serially correlated and contemporaneously correlated forecast errors. They define loss differential between the two forecasts is

\[ d_t = g(e_{1t}) - g(e_{2t}) \]  

where \( g(.) \) is a loss function which could be the absolute or the squared value of the forecast error. The null hypothesis of the two forecasts have equal accuracy is that the loss differential has zero expectation for all \( t \). That is \( E(d_t) = 0 \) for all \( t \). Assuming mean stationarity for \( d_t \) then the asymptotic distribution of the sample mean loss differential is normally distributed:

\[ \sqrt{T} [\bar{d} - E(d_t)] \rightarrow N(0, 2\pi f_d(0)) \]  

where \( 2\phi f_d(0) \) is the long run variance of \( d_t \). \( f_d(0) \) denotes the spectral density of \( d_t \) and \( \bar{d} \) is the sample mean differential.

\[ f_d(\lambda) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_d(k) \exp(-ik\lambda), \ -\pi \leq \lambda \leq \pi \]  

\[ \bar{d} = \frac{\sum_{t=1}^{T} [g(e_{1t}) - g(e_{2t})]}{T} \]  

where \( \gamma_d(k) \) is the autocovariance of \( d_t \) at displacement \( k \):

\[ \gamma_d(k) = E[(d_t - E(d_t))(d_{t-k} - E(d_t))] \]  

the Diebold-Mariano test statistic is

\[ DM = \frac{\bar{d}}{\sqrt{2\pi f_d(0) T}} \]  

where \( \hat{f}_d(0) \) is a consistent estimate of \( f_d(0) \). This statistic has an asymptotic standard normal distribution under the null hypothesis. However, the DM test still has oversize problem when sample sizes are small.

Harvey, Leybourne, and Newbold (1998) develop a modified Diebold-Mariano test to overcome the small-sample oversize problem. Since forecast errors from \( h \)-step ahead forecasts would follow a moving average process of order \( h - 1 \) rather than a white noise, they assume that \( h \)-step ahead forecast errors have
zero autocorrelations at order $h$ and beyond. The modified DM test statistic is

$$M_{DM} = \frac{DM}{\sqrt{\frac{(T+1-2h+h(h-1))}{T}} / T}$$

(2.38)

they suggest to use critical values from the Student-\(t\) distribution with degrees of freedom \(T-1\) rather than a standard normal distribution.

Harvey, Leybourne, and Newbold (1998) conduct a Monte Carlo simulation of these forecast encompassing tests with finite samples, they apply multi-step-ahead forecasts, forecast errors are contemporaneously correlated, serially correlated and they also generate non-Gaussian forecast errors from Student-\(t\) distributions. In all cases, the MDM test performs better than any other conventional or modified tests. Nevertheless, Student-\(t\) distribution is still inappropriate to be applied for the distribution of the forecast errors as the Student-\(t\) distribution is limit of symmetric, see Lahiri and Teigland (1987).

The aim of this chapter is to investigate to what extent the non-normality will have impacts on the forecast encompassing test and forecast combination. The forecast errors are generated from the TS distributions allowing for skewness and leptokurtosis. In practice, forecasts are usually finite, sometimes very limited. Therefore, in this chapter I only focus on the performance of forecast combination and encompassing test for the TS case with finite samples rather than asymptotics.

### 2.4 Monte carlo simulation

In a hypothesis test, we usually would like to know the probability distributions of the estimators, so called sampling distributions. In some cases it is possible to compute the sampling distribution from the model. However, sometimes this is either not possible or costly to find the sampling distributions, especially for finite (small) samples, see Mooney (1997). In this case, we have to use the Monte Carlo simulation. A Monte Carlo simulation is an approach for analysing the statistics sampling distributions and evaluating their behaviours using lots of random samples from known populations of artificial data (Hendry (1984)). According to Mooney (1997), the Monte Carlo procedure involves following steps: (1) define a pseudo-population (an artificial population from a given probability distribution) which can be used to generate samples; (2) draw a pseudo-sample of size \(N\) from the pseudo-population; (3) estimate the parameter of interest and store it
in a vector; (3) repeat steps 2 and 3 for \( R \) times; (4) construct a relative frequency distribution of \( R \) estimates values to construct a sampling distribution. In order to do a Monte Carlo simulation, we have to specify the statistical model so called Data Generating Process (DGP, see Hendry (1984)). For example, we have to define the deterministic parts of the model, the exact parameters of the distribution of the error term and the distribution of exogenous variables.

In this chapter, in order to investigate the finite sample behaviour of the forecast encompassing test and forecast combination under forecast errors non-normality, I conduct a Monte Carlo simulation to examine (1) whether the size or power of the forecast encompassing test could be affected by non-normality and (2) to what extent the non-normality will have effects on the precision of the combined forecast respect to individual forecasts. In the experiment, there are two Data Generating Processes (DGPs), in the first DGP the forecast errors are drawn from normal distribution as a benchmark, and in the other DGP, forecast errors are generated from the tempered stable distribution as the heavy-tailed distribution. We also compare the effect of estimating methods by imposing and not imposing a restriction on combination weights.

### 2.4.1 Data generating process

Suppose there are two sets of demeaned forecasts \( \tilde{f}_{1t} \) and \( \tilde{f}_{2t} \) (the two forecasts are joint unbiased, \( t = 1, 2, 3 \ldots T \). The subscripts 1 and 2 indicate two rival forecasts which have \( T \) number of forecast outcomes made by different forecast horizons, however, each pair of individual forecasts, for instance, \( \tilde{f}_{11} \) and \( \tilde{f}_{21} \) are made by the same forecast horizon), of the inflation rate \( y_t \). The demeaned forecasts refer to subtracting the mean of the two forecasts from each individual forecast so that they are mean zero. The model is based on the work of Bates and Granger (1969):

\[
y_t = (1 - \lambda) \tilde{f}_{1t} + \lambda \tilde{f}_{2t} + \epsilon_t; \quad 0 \leq \lambda \leq 1
\]  

(2.39)

the first, denoted the DGP-1 (this DGP is based on the work of West (2001)), takes the form:

\[
\begin{bmatrix}
\tilde{f}_{1t} \\
\tilde{f}_{2t} \\
\epsilon_t
\end{bmatrix}
\sim N
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},

\begin{bmatrix}
\sigma^2_1 & \rho \sigma_1 \sigma_2 & 0 \\
\rho \sigma_1 \sigma_2 & \sigma^2_2 & 0 \\
0 & 0 & \sigma^2_\epsilon
\end{bmatrix}
\]  

(2.40)

where \( \rho \) is the correlation coefficient of the two forecasts, the two forecasts are correlated if they share common information, predicted by similar models and
so on so forth. The variances of the two individual forecasts are set to be unit, \( \sigma_1^2 = \sigma_2^2 = 1 \). Therefore the covariance of the two forecasts is \( \rho \sigma_1 \sigma_2 = \rho \). The variance of the true forecast errors\(^4\) is \( \sigma_t^2 \), and \( \lambda \) denotes the combination weight.

If forecast errors are tempered stable distributed, the DGP-2 is similar to the work of Harvey, Leybourne, and Newbold (1998). However, in their paper they generated bivariate \( t \) distributions for the forecast errors, in this section the forecast errors are generated from the TS distribution which could be asymmetric. The two demeaned forecasts are generated from bivariate normal distributions, denoted the DGP-2, takes the form

\[
\begin{bmatrix}
\tilde{f}_{1t} \\
\tilde{f}_{2t}
\end{bmatrix} \sim N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & 1 \end{bmatrix} \right)
\]

(2.41)

\[ \epsilon_t \sim TS_\alpha(\beta, \delta, 0, \theta) \]

(2.42)

where the forecast errors \( (\epsilon_t) \) of the combined forecast follow a tempered stable distribution, the mean of \( \epsilon_t \) is zero when \( \mu = 0 \). Parameters \( \alpha \) and \( \beta \) denote kurtosis and skewness of the distribution, \( \theta \) describes how far the distribution is from the \( \alpha \)-stable distribution, the distribution is approached the \( \alpha \)-stable distribution if \( \theta \to 0 \). Given different values of \( \alpha \), \( \beta \) and \( \theta \), parameter \( \delta \) will be adjusted to guarantee that the mean and the variances of the error term in both DGPs are identical. Therefore, the forecast encompassing test will only be affected by the shape of distributions of the forecast errors other than the volatility of the forecast errors. We also assume that the individual forecasts \( \tilde{f}_{1t} \) and \( \tilde{f}_{2t} \) have no correlation with the true forecast errors \( \epsilon_t \). In the experiment, 176 parameter combinations are used in the DGP-2, where \( 176 = (2 \text{ values of } \alpha) \times (2 \text{ values of } \beta) \times (2 \text{ values of } \theta) \times (2 \text{ values of } \rho) \times (11 \text{ values of } \lambda) \).

\[ \alpha = 1.1, 1.5; \quad \beta = 0, 0.5; \quad \theta = 0.1, 0.5; \quad \rho = 0.1, 0.5; \quad \lambda = 0, 0.1, 0.2, ... 1 \]

(2.43)

The DGP-2 converges to GDP-1 when TS distribution converges to normal distribution, that is when \( \alpha = 2 \).

In order to compare the effect of restriction and non-restrictions imposed on the parameters, \( \lambda \) in the equation (2.22) is estimated by unconstrained OLS and constrained OLS respectively. In the experiment, I set \( \lambda = 0, 0.1, 0.2 ... 1 \). The correlation of these two forecasts is \( \rho = 0.1, 0.5 \), this is because if two forecasters use the similar models or variables or they share same information, their forecasts

\(^4\)when \( \lambda \) is not estimated, the forecast errors of a combined forecast is the true forecast errors
maybe correlated to each other. In order to examine the impact of uncertainty on the performance of forecast combination and encompassing test, the variances of the true forecast error have 3 values, which are $\sigma^2 = 0.1, 1, 10$ to indicate small, moderate and large variances. Here I assume that as uncertainty increases, the variances of the forecast errors will rise. I also generate different number of forecasts for $N=20,30,40,50$. The number of replication is 5000 in the simulation (I used the high performance computer ALICE at the University of Leicester\(^5\)). This number of replications is not very large because generating the TS distributions is time consuming even by using the high performance computer.

### 2.5 Monte Carlo results

#### 2.5.1 Size and power test

The Type 1 error occurs when we reject the null hypothesis when it is true. The probability of the Type 1 error is so called the significance level. While the Type 2 error occurs when we accept the null hypothesis when it is false. The probability of making such an error is denoted as $\beta$. The power $(1-\beta)$ computes the probability that it correctly accept the alternative hypothesis when the alternative is true. Thus, power is the ability of a test to correctly reject the null hypothesis. The alternative hypothesis usually associated with a parameter (in this chapter, the parameter is the combination weight $\lambda$) and the relationship between power and the parameter is the power curve, which is a graph of power with a set of all possible parameter values (above explanations about size and power are based on Marx and Larsen (2006)). In this chapter, using the DGP 1 and DGP 2, I draw power curves for testing the null $\lambda = 0$ against the alternative $\lambda > 0$ in the equation (2.39), the steps of computing size and power are as follows:

1. for each DGP, I estimate $\lambda$ by OLS and compare their test statistics with the corresponding Student-$t$ critical values at 5% significance level (critical values are different as the sample sizes change). The 5% significance level is so called the nominal size. If the test statistics are greater than the critical values, I save them in a vector.

\(^5\)The programme I use is Gauss programme. ALICE is a High Performance Computing (HPC) cluster based on Linux for modelling and simulations, data processing and analysis. Processes on this system are runs as batch jobs. Typically, the batch job contains one large process running for many hours or days. Alternatively a batch job could contain many short processes running simultaneously.
(2) for 5000 replications, I compute the probability that the test statistics reject
the null hypothesis, namely the probability of the test statistics are greater than
the critical values. In this step, when $\lambda = 0$ in the DGP, the probabilities are the
empirical size. When $\lambda \neq 0$, the probabilities are the power.

(3) repeat above steps by setting $\lambda = 0, 0.1, 0.2...1$ respectively and for both
OLS and constrained OLS estimations. Then we can draw a power curve with
different values of $\lambda$.

2.5.1.1 Results

Figure 2.2 reports the results of the size and the power of the forecast encompass-
ing test with sample size 20, the forecast errors are generated from normal and
the TS distributions (tails $\alpha$, skewness $\beta$ and $\theta$ are varied) with identical variances
0.1, the correlation coefficient of the two forecasts is $\rho = 0.5$. I select this Figure
because this Figure is more reliable than others if we use real data instead of simu-
lation. That is because for aggregate data, competing models have high $\rho$ and the
encompassing regression has low $\sigma^2_\epsilon$ (West (2001)). As we can see from the chart,
the empirical sizes are round 5%-level when $\lambda = 0$ for all cases, which means the
sizes are not affected by the shapes of distributions. However, with the increas-
ing of $\lambda$. When $\lambda = 0.2$, the percentage number of rejections reaches above 80%
and 55% in TS1 and TS2 cases respectively, while in the normal case this number
is less than 40%, which means the TS distribution cases are more powerful than
the normal distribution case. Moreover, in the TS distribution cases, compared
the TS1 case with the TS2, the smaller $\theta$ (TS1: $\theta = 0.1$, TS2: $\theta = 0.5$) the distribu-
tion has, the faster the line approaches to one. That is to say, the tests are more
powerful when the distribution of the forecast errors are closer to the underlying
$\alpha$-stable distribution.

Figures 2.3-2.8 report the results of size and power of the forecast encompass-
ing test for data from the DGP-1 and the DGP-2 (in these figures, distributions
are symmetric). These figures show the empirical percentage numbers of rejec-
tions of the null hypothesis for nominal 5% significance level against one-sided
alternative using Student-$t$ critical values. To evaluate the size, $\lambda$ is set to 0, to
evaluate the power, $\lambda$ is set from 0.1 to 1.0. The results from figures 2.3-2.8 can be
summarised as follows:

(1) When the combination weights $\lambda = 0$, in all cases, the empirical sizes are
all around 5%-level. The results indicate that there is no size deterioration for
different sample sizes for both non-normal and normal forecast errors. I also
investigate the size properties of the encompassing test when using different degrees of correlation ($\lambda$) of the two forecasts and different variances of the forecast errors. The results also suggest that the size are correct and not affected. These results are suspicious since Harvey, Leybourne, and Newbold (1998) did find size deterioration when errors are non-normal distributed, but I double checked my programmes and procedures, there is nothing wrong.

(2) When $\lambda \neq 0$, I estimated the powers of the forecast encompassing. Tests were conducted at 5% significance level. It is notable that the power of the forecast encompassing test (FE test) tends to rise with the increasing number of observations. For instance, in figure 2.3, when $\lambda = 0.1$, the power of the FE test under normal distributed forecast errors rises from no more than 55% when $T = 20$ to nearly 90% when $T = 50$.

(3) In the cases of small and moderate forecast errors variances (variance=0.1, 1), for the same variances and the correlation of the forecasts, the heavier tails of the distributions are, the more powerful the tests will be. For example, in figures 2.3 and 2.4, the tests under the TS1 forecast errors displays significantly superior power performance than any other distributions. However, this advantage becomes smaller with the increasing of sample observations and the combination weights ($\lambda$). On the other hand, in the cases of large forecast variances, the power performance of the FE test under the heavy-tailed distributions errors is slightly greater than the normal distributions and the advantage evaporates rapidly with
the increasing of sample sizes.

(4) With the identical variances of the forecast errors (variance=0.1, 1) and the
same tail indexes ($\alpha$), the tests are more powerful when the distributions are
closer to the underlying $\alpha$-stable distributions.

(5) The results also suggest that the restricted estimation should be preferred
to the unrestricted estimation in the FE test. That is to say, in order to have better
forecasts, the sums of weights have to be equal to one. (The results are available
in Appendix)

(6) I also compare the power performance of the FE test under skewed TS dis-
tributions errors and the symmetric TS distributions, the results show that the
performances of the symmetric and asymmetric distributions are quite similar.
(The results are available in Appendix)

2.5.1.2 Forecast accuracy

As I mentioned in section 2.3, forecast accuracy can be measured by the MSE
(mean squared error) or the RMSE (root mean squared error). Relative RMSE is
widely used Stock and Watson (1999) to evaluate forecast accuracy for two sets
of forecasts:

$$Relative \ RMSE = \sqrt{\frac{\sum \hat{e}_N^2 / N}{\sum \hat{e}_{1N}^2 / N}}$$ (2.44)

In this paper, I use relative RMSE (RRMSE) to assess the accuracy of the forecasts.
This statistic is a ratio of the RMSE of the objected forecast relative to the same
expression for the benchmark forecast. The RRMSE measure has the advantage
of being robust to positive and negative forecast errors and of large forecast er-
rors being penalised due to the quadratic form (Kapetanios, Labhard, and Price
(2008)). If this statistic is less than one, the objected forecast outperforms the
benchmark forecast, if the ratio is equal to 1, the objected forecast is as good as
the benchmark forecast, and the benchmark forecast will outperform the objected
forecast if the ratio is greater than 1. In this experiment, the object forecasts are
combining forecasts and individual forecasts are the benchmark.

Figures 2.9-2.14 report the accuracy of the combined forecasts in comparison
with an individual forecast under normal and the TS distribution forecast errors.
In these figures, the smaller the RRMSE are, the more advantages the combined
forecasts will gain from an individual forecast. The results of the Monte Carlo
simulation suggest that:
(1) In all cases, the RRMSE are all less than 100%, which means the combining forecasts do outperform an individual forecast, which is consistent with the result of Granger and Bates. However, the advantages of the combining forecasts from the individual forecasts mitigates with the increasing of sample sizes.

(2) When the forecast errors have small variances, the RRMSEs decline rapidly as the combination weight ($\lambda$) is less than 0.5, after that, the statistics reduce gradually. When the variances of the forecasts are moderate and large, the slopes of the RRMSE are constant, while the slopes are steeper for moderate forecast variances than the large forecast variances of which are much closer to one. These results indicate that the combining forecasts increase forecast accuracy dramatically when the variances of the forecast errors are small and moderate.

(3) Regarding to the cases of Gaussian and the TS distributed forecast errors, when the forecast errors do not have large variances, the combining forecasts of the TS cases gain more advantages from individual forecast than the cases of normal distribution. The heavier tails of the distributions are, the more advantages the combining forecasts will gain from individual forecast. However, when the two forecast errors have large variances, there is no obvious advantage for the combining forecast in both cases of Gaussian and the TS distributed forecast errors.

(4) When the forecast errors are generated from the skewed and symmetric distributions, the forecast precisions of both cases perform quit similar.

(5) Compared with different estimation methods, the restricted estimation outperforms the unrestricted estimation. In other word, the forecasts will be more accurate if the sum of weights is equal to one.

2.6 Conclusion

I have discussed the importance of inflation forecasting and distributional properties of forecast errors. With regard to the former, forecast combination and the forecast encompassing test are introduced to improve forecast performance. With regard to the latter, since the forecast errors are suggested to be heavy-tailed distributed rather than normal, I introduce the tempered stable distribution which admits such heavy-tailed characteristics.

I conduct a Monte Carlo simulation to examine whether and to what extent the forecast errors non-normality could have impacts on forecast combination and
the forecast encompassing test. The simulation results indicate that the forecast errors non-normality does not have impacts on the size of the forecasting encompassing test, however, the power could be affected by non-normality when the variance of the forecast errors is not large, the heavier tails of the distribution are, the more powerful the tests will be. Moreover, with the identical variances and tail indexes, the tests are more powerful if the distributions are closer to the underlying $\alpha$-stable distributions. The simulation results also advocate that the combining forecast outperforms individual forecast. Moreover, the heavier tails of the distributions are, the more advantages the combining forecasts will gain from individual forecast. In addition, I also found the evidence that the asymmetric and symmetric distributions have similar performances on both power test and the evaluations of forecast accuracy.

In this chapter, although there is nothing on inflation forecasting, in further chapters the empirical analysis of inflation will be conducted. The theories of inflation is non-normal and forecasting methods such as forecast combination and forecast encompassing, will be used in following chapters. In additions, the results and findings also can be applied to forecasting of other macroeconomic variables which might be non-normal, such as exchange rate and output.

A number of limitations need to be addressed and considered in the research, first of all, for generating the TS distribution, it has computational burden and it is still very time consuming. In practice, although data of forecast errors could be collected by surveys, it is difficult to estimate the parameters of a TS distribution precisely. Secondly, further works could be done by applying other types of heavy-tailed distributions such as split normal distribution (Wallis et al. (2014)) and skewed normal distribution (Charemza, Diaz, and Makarova (2014)) which are widely used by central banks.
Figure 2.3: Size and power test, Variance=0.1, $\rho = 0.1$

Figure 2.4: Size and power test, Variance=0.1, $\rho = 0.5$
Figure 2.5: Size and power test, Variance=1, \( \rho = 0.1 \)

Figure 2.6: Size and power test, Variance=1, \( \rho = 0.5 \)

\( \lambda \) Percentage

T=20

N
TS1
TS2
TS3
TS4

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Figure 2.7: Size and power test, Variance=10, $\rho = 0.1$.

Figure 2.8: Size and power test, Variance=10, $\rho = 0.5$. 
Figure 2.9: Forecast precision, Variance=0.1, $\rho = 0.1$

Figure 2.10: Forecast precision, Variance=0.1, $\rho = 0.5$
Figure 2.11: Forecast precision, Variance=1, $\rho = 0.1$

Figure 2.12: Forecast precision, Variance=1, $\rho = 0.5$
Figure 2.13: Forecast precision, Variance=10, $\rho = 0.1$

Figure 2.14: Forecast precision, Variance=10, $\rho = 0.5$
Chapter 3

An analysis of China’s macroeconomic uncertainty and inflation forecast uncertainty

Abstract. This chapter discusses different methodologies to measure forecast uncertainty and macroeconomic uncertainty. It investigates the relationship between inflation forecast uncertainty and macroeconomic uncertainty for China, the results indicate that there is no relationship between survey-based forecast uncertainties and the macroeconomic uncertainty, however, there is positive relationship between model-based forecast uncertainties and the macroeconomic uncertainty. This chapter also analyses the relationship between inflation and inflation forecast uncertainties for China, the results suggest that there is significantly positive relationship between survey-based forecast uncertainties and inflation, however, there is no relationship between model-based uncertainties and inflation.
3.1 Introduction

Uncertainty plays a predominate role in macro and micro economies. It fluctuates over time and it often rises when there is bad news. A low GDP growth could induce high macro uncertainty; a low growth rate of firm could lead to an increase of micro uncertainty; and wars, catastrophe could cause a rise in uncertainty (Bloom (2007), Bloom (2009)). Bloom (2014) provides empirical evidence that uncertainty rises in recession and there is a large body of theoretical literature discusses the mechanisms through recession to uncertainty. Firstly, according to Van Nieuwerburgh and Veldkamp (2006), companies are trading actively when business is good and meanwhile they have precise information about the level of technology. However, when the economic boom ends, firms have ambiguous information about the extent of the recovery and thereby uncertainty increases. Secondly, predicting the future is harder during the recessions (Orlik and Veldkamp (2014)) because recessions are rare events that people are not familiar with. As a result, uncertainty increases in the recessions. Thirdly, Pástor and Veronesi (2013) address that policy change is more likely in bad times as the current policy is deemed to be harmful. In this case, people are unsure about how the policy will be changed and when the new policy will be implemented, thus policy uncertainty rises during the recession. Empirical evidence is provided by Baker, Bloom, and Davis (2013); they find that policy uncertainty increases during the great recession. Last but not the least, according to Bachmann and Bayer (2013), firms will adopt “wait-and-see” policy when business is bad and put more resources to research and development, which lead to micro uncertainty.

Uncertainty also has both positive and negative effects on economic growth. Bernanke (1980), Brennan and Schwartz (1985) and McDonald and Siegel (1982) focus on “real options” to analyse the relationship between uncertainty and growth. The idea is that firms have a series of options to undertake their business, they can abandon, expand or postpone their investment projects. If firms become more uncertain about the future, they might wait and see to avoid any risky investment projects or they might reduce their hiring which will result in a fall in productivity growth and output. For example, after 9/11, the investment values of the last quarter of 2001 was the lowest since 1982. However, real option effects rely on the cost of delay. If delay is extremely costly and firms are unable to delay their investments, then there is no negative influence of uncertainty on investment. Moreover, the real option also has effect on consumption. If people are uncertain about the future, they may postpone their consumptions. For example,
if you are willing to purchase a new car and you are unsure about a rise or fall of your bonus this year, it makes sense that you will delay your purchase until you receive the bonus. However, delaying purchase on non-durable goods such as food is unlikely, therefore, the real option effects of uncertainty will be low on non-durable consumption. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) suggest that the real option effects of uncertainty not only reduce the level of investment and consumption, but also reduce firm’s sensitivity to the changes of business condition. He states that when there is high uncertainty the elasticity of investment with respect to interest rates might fall to a low level. When uncertainty is high, both productive firms and unproductive firms become more cautious as they are less aggressive in expanding or contracting. This caution impedes the reallocation of resources across firms which is the drive of the aggregate productivity growth (see Foster, Haltiwanger, and Krizan (2001)). As a result, high uncertainty can reduce productivity growth.

Risk premium is another factor that uncertainty will have impact on growth (Bloom (2014)). Since high uncertainty will cause an increase in risk premium, it will raise the borrowing costs and hit financial constraints of firms which will lead to a reduction on growth (Arellano, Bai, and Kehoe (2010)). On the other hand, if people are uncertain about the economic circumstance in the future, it may affect their behaviours today. The feeling of an increase in uncertainty will lead to an increase in precautionary savings for individuals which will reduce consumption expenditure (see Bansal and Yaron (2004)). However, precautionary savings can lead to an increase in investment in a closed economy which may have potential positive effect on growth. Leduc, Liu, et al. (2012) and Basu and Bundick (2012) argue that a rise in uncertainty could lead to recession even in a closed economy. The idea is that an increase of savings should reduce prices and interest rate which will encourage investment; while if prices are sticky, prices and interest rate do not fall enough to encourage investment so that output will reduce.

The ”growth option” is one of the two mechanisms through which uncertainty can have positive impacts on long-run growth. The growth option states that uncertainty could encourage investment if future prices increase. For example, Bar-Ilan and Strange (1996) proposed that investment always takes time to implement, called as ”investment lag” or “time-to-build”. The opportunity cost of waiting will increase with uncertainty, thus uncertainty will hasten investment

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1 In a closed economy investment is equal to savings, an increase in precaution savings will lead to an increase in investment
which has a positive effect on growth.

The “Oi-Hartman-Abel effect” (Oi (1961), Hartman (1972) and Abel (1983)) is the other channel that uncertainty has positive effect on growth. It states that firms may expand during good times and contract during bad times. In this case firms are risk loving, if profits-demand or profits-cost curve is convex, uncertainty will lead to an increase in expected profits. However, firms need to be flexible to expand or contract under different circumstances, therefore, the “Oi-Hartman-Abel effect” are more powerful in the long run rather than short run because of the cost changes.

The relationship between inflation uncertainty and inflation is not conclusive. Friedman (1977), Ball, Mankiw, Romer, Akerlof, Rose, Yellen, and Sims (1988) and Mankiw, Reis, and Wolfers (2003) suggest that there is a positive relationship between inflation uncertainty and inflation, that is inflation uncertainty rises with inflation rate increases. One reasonable explanation to this phenomenon is that the response of policy to high inflation causes the increase in uncertainty. According to Ball and Mankiw (1992) when inflation is high, the authorities will face a dilemma: will they take some actions to reduce the inflation rate but to take a risk of recession. Since public does not know the tastes of policy makers, it will rise the uncertainty about the future inflation. Empirical studies also indicate that higher inflation uncertainty is associated with higher inflation rate. In contrast to Mankiw’s view, the empirical evidence suggests that high inflation uncertainty will lead to high inflation, see Cukierman and Meltzer (1986), Grier and Perry (1998) and Kontonikas (2004). However, several studies address that different regimes can have effects on inflation uncertainty (see Meltzer (1985), Meltzer (1986) and Meltzer and Robinson (1989)). For example, Meltzer (1985) proposes that when the relationship between inflation uncertainty and inflation is examined across different exchange rate regimes, inflation uncertainty does not increase with inflation. He found that inflation uncertainty was the same or slightly higher during the fixed exchange rate regime as the floating exchange rate regime. In addition, inflation was much lower during the period of fixed exchange rate regime than the period of floating exchange rate regime. Golob (1994) argue that Meltzer’s results can be explained by a downward trend in inflation uncertainty which offsets the positive relationship between inflation and inflation uncertainty.

Uncertainty is a broad concept, in this chapter, uncertainty refers to two classifications: forecast uncertainty, which is the uncertainty around the point forecast of some key macroeconomic variables such as GDP and inflation; macroeconomic
uncertainty, which is an unobservable aggregate measure of uncertainty for a macroeconomy. However, there is no consensus measure about uncertainties; the existing literature has relied on proxies or indicators to measure uncertainties.

According to Clements (2014), there are two categories of forecast uncertainty: *ex-ante* forecast uncertainty is an uncertainty around the point forecast made prior to the realisation of the outcome, known as dispersion between individual forecasts or disagreement among forecasters (see e.g. Bomberger (1996), Wallis (2008), Patton and Timmermann (2010)). *Ex-post* forecast uncertainty or historical uncertainty is reflected by differences between point forecasts and outcomes. Clements (2014) proposes following explanations for illustrative purposes. Suppose the data generating process is:

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \quad \varepsilon \sim N(0, \sigma^2_{\varepsilon,t})$$

(3.1)

where $\sigma^2_{\varepsilon,t}$ is, for example, an ARCH or GARCH process. Then the conditional forecast density of $Y_t$ based on information set through $t-h (\Omega_{t-h})$ is:

$$Y_{t|t-h} \sim D(\rho^h Y_{t-h}, \sigma^2_{y,t|t-h})$$

(3.2)

where $D(.,.)$ denotes density function, $E(Y_t|\Omega_{t-h}) = \rho^h Y_{t-h}$ and variance $Var(Y_t|\Omega_{t-h}) = \sigma^2_{y,t|t-h}$. The expected squared error of the point forecast is:

$$E_{t-h}[(Y_t - Y_{t|t-h})^2] = E_{t-h}[(\varepsilon_t + \rho \varepsilon_{t-1} + \ldots + \rho^{h-1} \varepsilon_{t-h+1})^2] = \sigma^2_{y,t|t-h}$$

(3.3)

where $\sigma^2_{y,t|t-h} = E(\sigma^2_{\varepsilon,t}|\Omega_{t-h})$ assuming $E(\varepsilon_t \varepsilon_s) = 0$ for all $t \neq s$. Then the variance in the expression 3.2 is the *ex-ante* forecast uncertainty, because the density forecast made prior to the realizations. The expression 3.3 is the *ex-post* forecast uncertainty, because it computes the variance of the forecast errors between forecasts and realizations.

Knight (1921) distinguishes the concepts of uncertainty and risk. Charemza, Diaz, and Makarova (2014) suggest that this *ex-post* forecast uncertainty should be called as “Knightian risks” rather than “uncertainty” in order to distinguish with the Knightian uncertainty which is an unobservable *ex-post* phenomenon, an example of this Knightian measure is the macroeconomic uncertainty, see e.g. Bloom (2009), Jurado, Ludvigson, and Ng (2015). There are several measures for the macroeconomic uncertainty such as the VIX index which measures the volatility of stock market (Bloom (2009)). The Economic Policy Uncertainty index (EPU) proposed by Baker, Bloom, and Davis (2013), is a proxy of uncertainty that counts
the frequency of newspapers articles about economic uncertainty. This index is created mainly based on the number of citations of "economy", "uncertainty" and "economic uncertainty" in the newspapers (now they count number of papers with 'uncertainty' or the like), more details will be described in the following section.

There is an interesting question arise from above findings: whether there is a relationship between the inflation forecast uncertainty and the macroeconomic uncertainty as inflation is a key economic indicator. In other words, whether a particular economic variable becomes more or less uncertain, the aggregate uncertainty or the macroeconomic uncertainty becomes more or less uncertain simultaneously. This question also investigate whether there is a relationship between the Knightian risk and the Knightian uncertainty. The aim of this chapter is to investigate the relationship between the ex-post inflation forecast uncertainty and the macroeconomic uncertainty for China.

The plan of this paper is as follows. Section 3.2 introduces different methodologies of assessing uncertainties. Section 3.3 discusses methodologies to measure inflation forecast uncertainty and macroeconomic uncertainty for China. Section 3.4 investigate and discuss the relationship between the forecast uncertainty and the macroeconomic uncertainty for China. Finally, section 3.5 is summary and conclusion.

### 3.2 Measurement of uncertainties

In this section, we consider methods of measuring uncertainties. According to Makarova (2014), there are generally three approaches: measuring uncertainty by panels of individual forecasts, model-based measures of uncertainty and mixed approaches.

#### 3.2.1 Survey-based measures of uncertainty

Previous studies believe that forecasters always make same mistakes that they made in the past, therefore, forecast errors incorporate useful information about future uncertainties and past forecast performance provides a proxy measure of forecast uncertainties. However, this ex-post forecast uncertainties subject to structural changes when we measure the ex-ante uncertainties from the past performance. In 1968, the American Statistical Association (ASA) and the National
Bureau of Economic Research (NBER) issued the ASA-NBER survey, then the Federal Reserve Bank of Philadelphia took over the survey in 1990 known as the Survey of Professional Forecasters (SPF), which is a quarterly survey of forecasters for the views on the US macroeconomic variables, it collects point forecasts as well as density forecasts (or probabilistic forecast) from different respondents, details can be found in Croushore (1993). Since then, it has attracted studies and discussions on the methods of measuring uncertainties, examples are Zarnowitz and Lambros (1987) and Bomberger (1996). The unique feature of the survey is that it asks forecasters to provide density forecasts for several key variables (real output growth, inflation rate, etc.), in the form of histograms. Then the probabilities are averaged over all respondents to obtain the mean or aggregate histograms. An example is shown in the table 3.1, forecasters are asked to provide probabilities to each preassigned intervals for the annual core inflation rate (the annual average percentage change in prices between the current year and the previous year, and between the current year and the following year). As we can see from the table, there are 10 intervals in this survey, it asks forecasters to attach the probabilities that the inflation will grow greater than 4%, between 3.5% to 3.9%, 3% to 3.4% and also the probability of deflation etc. The mean probability is 27.24% that the inflation rate will fall in the range of 1.5% to 1.9% for the year of 2014 and with the same interval, the probability of inflation rate predicted by forecasters is 30.11% for the next year (For a Quarter 1 survey, it provide a 4-step ahead forecast of the current year’s inflation rate and an 8-step ahead forecast of next year’s inflation rate. For a Quarter 4 survey, it provide a 1-step ahead and 4-step ahead instead.).

Table 3.1: SPF mean probability attached to core CPI inflation, 2014 Quarter 1

<table>
<thead>
<tr>
<th>Probability</th>
<th>2013 Q4 to 2014 Q4</th>
<th>2014 Q4 to 2015 Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 percent or more</td>
<td>0.22</td>
<td>0.61</td>
</tr>
<tr>
<td>3.5 to 3.9 percent</td>
<td>0.42</td>
<td>0.97</td>
</tr>
<tr>
<td>3.0 to 3.4 percent</td>
<td>2.29</td>
<td>3.16</td>
</tr>
<tr>
<td>2.5 to 2.9 percent</td>
<td>9.24</td>
<td>15.08</td>
</tr>
<tr>
<td>2.0 to 2.4 percent</td>
<td>27.24</td>
<td>30.11</td>
</tr>
<tr>
<td>1.5 to 1.9 percent</td>
<td>40.91</td>
<td>33.85</td>
</tr>
<tr>
<td>1.0 to 1.4 percent</td>
<td>14.31</td>
<td>10.77</td>
</tr>
<tr>
<td>0.5 to 0.9 percent</td>
<td>3.77</td>
<td>3.79</td>
</tr>
<tr>
<td>0.0 to 0.4 percent</td>
<td>1.17</td>
<td>1.10</td>
</tr>
<tr>
<td>will decline</td>
<td>0.41</td>
<td>0.56</td>
</tr>
</tbody>
</table>

However, there are several weaknesses in the Survey of Professional Forecasters. First of all, we can not guarantee that forecasters give their best forecasts because the respondents in the survey may provide biased forecasts to make themselves stand out (Laster, Bennett, and Geoum (1997)). Secondly, forecasters appear and disappear in the survey so that it is difficult to capture the forecasts from the corresponding forecaster in a long period. Last but not the least, regarding to the density forecast, the preassigned intervals are too many and narrow, moreover, the number of intervals changes over time (Rich, Song, and Tracy (2012), Kowalczyk, Tomasz, and Stanislawska (2013)).

Since forecasters use private information to predict macroeconomic variables, heterogeneity in forecasters’ beliefs may result in various probabilistic statements. Generally speaking, high dispersion or disagreement among the forecasters indicates high forecast uncertainty. Zarnowitz and Lambros (1987) use the mean of the standard deviations of individual probabilistic forecasts as a measure of uncertainty and the standard deviations of corresponding point forecasts as a measure of disagreement. They find that the former measure is more stable and reliable and two measures are highly positively correlated. This result suggests that in the absence of direct measure of forecast uncertainty, disagreement among forecasters could be a useful proxy.

Since the probability distributions provide important information about the forecast uncertainty, the variance of the aggregate density forecast could be a possible direct measure of the forecast uncertainty. However, it requires to assume a specific distribution to fit the aggregate histogram. Giordani and Söderlind (2003) suggest to use disagreement among point forecasters as a proxy to measure uncertainty. The relationship between the variance of SPF’s aggregate histogram and the disagreement among forecasters is as follows (Wallis (2008)): suppose there are n individual density forecasts $f_i(y)\ (i = 1, 2, 3...n)$ of a random variable $Y$. The aggregate density $f_A(y)$ is a combination of individual densities which are given equal weights:

$$f_A(y) = \frac{1}{n} \sum_{i=1}^{n} f_i(y) \quad (3.4)$$

assume that individual point forecasts are the means of individual density forecasts, denoted as $\hat{y}_i$, then the mean of the aggregate density ($\mu$) is the average point forecasts

$$\mu = \frac{1}{n} \sum_{i=1}^{n} \hat{y}_i \quad (3.5)$$

if $\sigma_i^2$ denotes variances of individual histogram, the second moment about origin
of $f_A(y)$ is

$$\mu'_2 = \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i^2 + \sigma_i^2)$$ \hspace{1cm} (3.6)$$

the variance of aggregate histogram is

$$\sigma_A^2 = \mu'_2 - \mu = \frac{1}{n} \sum_{i=1}^{n} \sigma_i^2 + \frac{1}{n} \sum_{i=1}^{n} (\hat{y}_i - \mu)^2$$ \hspace{1cm} (3.7)$$

this expresses decomposes the variance of the aggregate histogram into the average variance of individual histogram and the disagreement between the individual point forecasts. The advantage of using disagreement among forecasters is that this method is easy to compute, however, it has several limitations. First of all, the dispersion among point forecasts may only reflect the difference in opinion rather than uncertainty (Diether, Malloy, and Scherbina (2002)). Second, Lahiri and Sheng (2010) argue that the validity of disagreement as a proxy for uncertainty depends on the forecasting environment and the forecast horizon, disagreement among forecasters becomes a less useful proxy for forecast uncertainty when there are large volatility of aggregate shocks and long forecast horizons. Third, this measurement is argued by Wallis (2008) and Rich and Tracy (2010); they suggest that there is little evidence that disagreement is a useful proxy for uncertainty. Last but not the least, individual forecasts may correlate to each other as panellists may use same information sources or methods to formulate forecasts, it is likely to be the joint bias of forecasts generated by different forecasters. That is to say, the disagreement in survey point forecasts reflects the differences in opinion rather than uncertainty (Mankiw, Reis, and Wolfers (2003)).

### 3.2.2 Model-based measures of uncertainty

Since the degree of uncertainty (non-Knightian uncertainty) can be reflected by the forecast errors, Ericsson (2000) divides the forecast uncertainty into predictable uncertainty and unpredictable uncertainty based on the sources of model-based forecast errors. He summarizes Clements and Hendry (1998)’s taxonomy and interprets the predictable uncertainty as “what we know that we don’t know” in the sense that the degree of uncertainty can be anticipated or even computed numerically. The sources of the predictable uncertainty are from (a) cumulation of future shocks to the economy, including the shocks which expect to occur when model was built; (b) inaccurate estimation of parameters in the forecast model. The unpredictable uncertainty is “what we don’t know that we don’t know”,
which means the uncertainty can not be anticipated. The sources of the unpredictable uncertainty are from (a) unknown future changes in the economy, such as wars, catastrophes, and technology changes; (b) misspecified forecast models; (c) mis-measurement data. He also proposes that the degree of uncertainty depends on (a) the variable being forecast, for example, exchange rate and stock prices are difficult to forecast; (b) the model being used. For instance, the ARMA model is believed to outperform other multivariate models for inflation forecasting (Mitchell, Robertson, and Wright (2014)); (c) the information available when forecasts are generated. When there is structural changes in the data and model is misspecified, additional information can result in an increase of forecast uncertainty (Clements and Hendry (2001)); (d) the underlying data generating process; (e) the forecast horizons. An example can be seen from the Bank of England’s fan chart.

Since the survey data is not always available, it is therefore to use time series models to estimate uncertainty. There are three main methods for measuring uncertainty by empirical models. The first method is to generate reliable forecasts (made by forecasting models) by applying forecast combination or encompassing, then (1) the \textit{ex-ante} forecast uncertainty is represented by the variance of the distribution which is estimated based on the \textit{ex-post} forecast errors, examples are Charemza, Diaz, and Makarova (2014) and Knppel (2014). This method is widely used by central banks. (2) the \textit{ex-post} forecast uncertainty is reflected by RMSE of the historical forecast errors. This method is sometimes called error-based uncertainty.

The second approach based on the assumption that the degree of uncertainty can be anticipated, so called “what we know that we don’t know approach”. It applies ARCH or GARCH models to compute the conditional variance of forecast errors at each date as a proxy of uncertainty, for example Fountas (2001), Kontonikas (2004) and Elder (2004). However, Giordani and Söderlind (2003) argue that model-based measures fail to capture the increase in inflation uncertainty in the early 1980s and they suggest that survey measures of uncertainty is more reliable than the time series models in the period of underlying structural changes.

The third approach is called “modelling model uncertainty”, see Onatski and Williams (2003) and Orlik and Veldkamp (2014). They assume that economists use their own models without knowing the true models, while their models are uncertain due to the uncertainties of parameters, whether relevant variables are included etc.. Thus, the true models should be estimated (say by Bayesian tech-
niques) and there will be model errors which are associated with uncertainties.

### 3.2.3 Other measures of uncertainty

Uncertainty by surveys or models can only assess the uncertainties for few economic variables such as inflation or GDP. However, it can not measure the uncertainty for a macroeconomy which is a non-measurable phenomenon, so called macroeconomic uncertainty.

With regard to the macroeconomic uncertainty, the existing literature uses proxies for this Knightian uncertainty. Some studies focus on financial market volatility as a measure of uncertainty, one of the proxies is the Chicago Board of Options Exchange Market Volatility (VIX) see Bloom (2009). However, first of all, the VIX could be driven by factors that reflects rise aversion and other non-linear pricing effects rather than the market volatility (Bekaert, Hoerova, and Duca (2013)). Secondly, volatility of stock market varies over time even if there is no change in uncertainty about economic fundamentals. For example, a change in the financial leverage causes a change in volatility of stock returns. Another approach is developed by Jurado, Ludvigson, and Ng (2015). They assess the macroeconomic uncertainty by applying a parametric stochastic volatility model\(^2\), which extracts joint forecastable component from 279 macroeconomic and financial indicators.

Baker, Bloom, and Davis (2013) develop an index to measure economic policy uncertainty (EPU). This EPU index contains three components, first component is the coverage of policy-related economic uncertainty in the newspapers. For this component, they select 10 leading newspapers in the US (they use China’s newspaper when compute the index for China) and search articles in each paper that contains terms related to economic policy uncertainty\(^3\). Then they compute the ratio of articles that mention economic policy uncertainty to the number of total articles in the same newspaper same month. Finally, they construct a normalised index of these monthly series from 1985 onwards. The second component is based on Federal tax code expirations. They compute discounted dollar-weighted (the discount rate is 0.5 per year) numbers of tax code provisions that set to expire over the next 10 years, giving a measure of the level of uncertainty regarding the future plans that the federal tax code will take. The third component

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\(^2\)The related literature about this dynamic factor model is Stock and Watson (2002)

\(^3\)They capture uncertainty about who will be elected to be the policy maker, what economic policy will be undertaken and when, and so on. They search articles that contains terms such as ‘uncertainty’, ‘economy’, ‘federal reserves’, details can be seen on http://www.policyuncertainty.com/methodology.html
is the forecaster disagreement about inflation and Government Purchases based on the data of the Survey of Professional Forecasters (SPF). They compute the forecast dispersion of individual forecasters about future inflation rate and future level of government purchases to construct indices of uncertainty about policy-related macroeconomic variables. In their overall EPU index, they weight 1/2 on news-based policy uncertainty index and 1/6 on each of the other three components including tax expirations, disagreement of inflation forecast and forecaster disagreement about government purchases. To measure economic policy uncertainty for China, the index is pure news-based without other two components. The newspapers is the South China Morning Post, which is a Hong Kong English-language newspaper. However, this newspaper has a self-censorship and is alleged to be pro-Beijing bias.

### 3.3 Uncertainty for China

This section aims to examine to what extent the ex-post inflation forecast uncertainty, which is a non-Knightian measure of uncertainty, is correlated to the macroeconomic uncertainty, which is a Knightian measure of uncertainty for China. The monthly data for China’s economic policy uncertainty index (EPU) is selected to measure the macroeconomic uncertainty. I compute the ex-post inflation forecast uncertainty based on survey data and time series models, respectively.

#### 3.3.1 Survey based forecast uncertainty for China

The monthly survey of inflation forecasts for China has been conducted by Money163 since June of 2009. The survey only reports one-step-ahead point forecasts of inflation for China from different financial institutions and I select five financial institutions’ forecasts which consistently appear in the survey and the data spans from June 2009 to September 2014, available at http://money.163.com/. The ex-post forecast uncertainty is defined as a simple 12 months moving average RMSEs of the point forecast errors and the RMSEs are measured by 3 methods as follows. The monthly EPU index is from May 2010 to September 2014, available at http://www.policyuncertainty.com/. Another source which provides SPF for China is the “Asia Pacific Consensus Forecasts” from “Consensus Economics”, available at http://www.consensus economics.com/. However, they only provide quarterly multi-step ahead inflation forecasts from 1990s which will be too short to analyse and monthly 1-step ahead inflation forecasts.
Lahiri, Peng, and Sheng (2014) provide three approaches to measure the *ex-post* forecast uncertainty: suppose that $F_{ith}$ is the point forecast of inflation at time $t$ made by agent $i$ at time $t-h$ (here, $t = 1, 2, 3...T$. $h$ is the forecast horizon). Then the forecast error of agent $i$, $e_{ith}$ can be described as:

$$e_{ith} = A_t - F_{ith} \quad (3.8)$$

where $A_t$ denotes the actual inflation at time $t$. The first measurement of the uncertainty is the root mean squared error (RMSE) of the average forecast (Bates and Granger (1969))

$$RMSE_{AF} = \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left( \frac{1}{N} \sum_{i=1}^{N} e_{ith} \right)^2} \quad (3.9)$$

where $T$ is number of forecasts of the same horizon made by an agent. $N$ denotes the number of agents. The second measurement of the uncertainty uses the average of individual RMSEs which is represented by

$$RMSE_{RT} = \frac{1}{N} \sum_{i=1}^{N} \sqrt{\frac{1}{T} \sum_{t=1}^{T} e_{ith}^2} \quad (3.10)$$

the third measurement is reported as the RMSEs of the average forecasts from $N$ agents

$$RMSE_{LPS} = \sqrt{\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} e_{ith}^2} \quad (3.11)$$

The $RMSE_{RT}$ is not the same as the conventional $RMSE_{AF}$, according to Reifschneider and Tulip (2007), if there are different groups of forecasters in the sample, the $RMSE_{RT}$ can generate a benchmark measurement of uncertainty for a group compared with other groups. Lahiri, Peng, and Sheng (2014) stress that $RMSE_{RT}$ is different from $RMSE_{LPS}$. If individual forecast errors contain heteroskedastic idiosyncratic errors, both the $RMSE_{RT}$ and $RMSE_{AF}$ ignore the uncertainty associated with the idiosyncratic errors. Namely, both measures do not fully incorporate disagreement among the forecasters. In their paper, they decompose forecast errors as follows

$$e_{ith} = \alpha_{ith} + \lambda_{ith} + \varepsilon_{ith}$$
where $\alpha_{ih}$ is an individual time-invariant deterministic bias, $\lambda_{th}$ is a common component which is the cumulative effect of all future shocks and $\varepsilon_{ith}$ denotes idiosyncratic errors due to individuals using different forecast models, interpretations or loss functions. Then it can be written as

$$
\varepsilon_{ith} = (\alpha_{ih} - \bar{\alpha}_h) + \lambda_t h^n + \varepsilon_{ith} \tag{3.12}
$$

where $\bar{\alpha}_h = \sum_{i=1}^{N} \omega_i \alpha_{ih}$ with weights $\sum_{i=1}^{N} \omega_i = 1$. $\lambda^*_{th} = \bar{\alpha}_h + \lambda_{th}$. They also make three assumptions:

1. $\lambda_{th} = \sum_{j=1}^{h} u_{tj}$, where $u_{tj}$ is a strictly stationary and ergodic stochastic process with $E(u_{tj}) = 0$ and $E(u_{tj}^2) = \sigma_{uj}^2$. Moreover, $u_{tj}$ is independent of $u_{ts}$ for any $j \neq s$.

2. $\varepsilon_{ith} = \sigma_{\varepsilon ih} v_{ith}$, where $v_{ith} \sim i.i.d$ across $i$ and $t$ for each $h$ with $E(v_{ith}) = 0$, $E(v_{ith}^2) = 1$ and $E(v_{ith}^4) < \infty$ for all $i$. $0 < \inf \sigma_{\varepsilon ih} \leq \sup \sigma_{\varepsilon ih} < \infty$.

3. $u_{tj}$ is independent of $v_{ish}$ for all $i,j,s,t$ and $h$.

The above assumptions imply that the individual forecast error is a stationary and ergodic process. Then the equation 3.9 can be derived from equation 3.12

$$
E[(\frac{1}{N} \sum_{i=1}^{N} \varepsilon_{ith})^2] = E(\lambda_{ih}^*)^2 + \frac{1}{N^2} \sum_{i=1}^{N} E(\varepsilon_{ith}^2) \tag{3.13}
$$

while the equation 3.11 can be seen as

$$
\frac{1}{N} \sum_{i=1}^{N} E(\varepsilon_{ith}^2) = E(\lambda_{ih}^*)^2 + \frac{1}{N} \sum_{i=1}^{N} E(\varepsilon_{ith}^2) + \frac{1}{N} \sum_{i=1}^{N} (\alpha_{ih} - \bar{\alpha}_h)^2 \tag{3.14}
$$

Note that the second term $\frac{1}{N^2} \sum_{i=1}^{N} E(\varepsilon_{ith}^2)$ in the equation 3.13 is always smaller than the second term $\frac{1}{N} \sum_{i=1}^{N} E(\varepsilon_{ith}^2)$ in the equation 3.14, thus the equation 3.13 is less than the equation 3.14 even $N$ is very large. In other words, the Granger’s measure will underestimate the uncertainty. In order to see the differences between these three uncertainty measures, suppose $N$ and $T$ approach to infinity and assume that $\alpha_{ih} = 0$.

1. $RMSE_{AF} \rightarrow \sigma_{\lambda h}$, where $\sigma_{\lambda h} = \sqrt{\sum_{j=1}^{h} \sigma_{uj}^2}$

2. $RMSE_{RT} \rightarrow \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} \sqrt{\sigma_{\lambda h}^2 + \sigma_{\varepsilon ih}^2}$

3. $RMSE_{LPS} \rightarrow \frac{1}{N} \sum_{i=1}^{N} \sqrt{\sigma_{\lambda h}^2 + \sigma_{\varepsilon ih}^2}$, where $\sigma_{\varepsilon ih}^2 = \lim_{N \rightarrow \infty} \sum_{i=1}^{N} \sigma_{\varepsilon ih}^2$

In this chapter, I will use all three methods to examine whether $RMSE_{AF}$, $RMSE_{RT}$ and $RMSE_{LPS}$ give different measures of model-based forecast uncer-
tainty and whether the idiosyncratic errors contribute to the measure of forecast uncertainty.

3.3.2 Modelling forecast uncertainty

The existing literature on analysing dynamic of China’s inflation mainly focuses on three aspects. Whether inflation is a monetary phenomenon: the government to create more credits to finance the state-owned enterprises in order to avoid large bankruptcy of the state-owned enterprises, as a result, such excess credits will lead to an increase in inflation rate (Brandt and Zhu (2000)); or whether inflation is caused by the fixed exchange rate regime (‘dirty float’) (see, Frankel (2005)): the People’s Bank of China(PBOC) announces the central parity rate of the RMB against the U.S. dollar each business day with reference to a basket of currencies and limits the exchange rate fluctuations within a certain range (nowadays allowed to float 1% a day). In this case, in order to retain an export surplus, the central bank manages the pace of appreciation of the RMB to keep the exchange rate undervalued and the trade surplus has been causing a rise in inflationary pressure in recent years; or China’s inflation is determined by past inflation and output gap (Gerlach and Wensheng (2006)). In this section, five variables are selected to investigate whether these factors have predictive power for China’s inflation. To measure inflation, I use the annual growth rate on the consumer price index (CPI) for China. To measure money supply, I use the broad money M2 as M3 is not published in China and the size of large and long-term deposits is limited in China. To analyse whether inflation is caused by the ‘dirty float’, I use total reserves excluding gold and broad effective exchange rate. Since total reserves excluding gold are measured by the US dollar, this data is divided by the exchange rate of China/U.S. dollar to remove the effects of depreciation of the US dollar (for simplicity, I call these variables total reserves and exchange rate). For output gap, I estimate it gap by applying Hodrick and Prescott (1997) filter and the output gap is measure by cyclical component, not the trend component. I use different $\lambda = 129600$ and $\lambda = 14400$ (suggested by Ravn and Uhlig (2002)) for monthly data of seasonal unadjusted manufacture Purchasing Manager Index (PMI). All data above is from the National Bureau of Statistics of China and the Federal Reserve Bank of St. Louis.
3.3.2.1 Data description

Figure 3.1 shows inflation rate (annual growth rate of CPI) for China from 1987M1 to 2014M9. As we can see from the chart, since the commencement of the openness reform in 1978, China’s inflation witnessed two striking peaks in the late 1980s and the mid-1990s. The inflation rate reaches above 25% in 1989 then decline sharply to less than 4% in 1990, then it rebounded to as high as 27.7% in 1994. Following a combined policy with monetary tightening and price control, the economy experienced deflation for the period from 1996 to 2004.

The past two decades can be categorised as a low inflation era of China except two local peaks in 2004 and 2007. According to Zhang and Clovis (2010), these two periods of inflation crisis is caused by the shock from real estate market and the shock from food prices (such as pork price) respectively. Brandt and Zhu (2000) state that cheap credits from the banks to state-owned enterprises leads to the two hyperinflation periods. They propose that government provided explicit official guidances to the banks on their lending operations favouring state-owned enterprises, which served crucial roles in the delivery of social services. However, the cheap credits causes inefficient state-owned enterprises which receive more than two-thirds of the financing but only generated about one-third of the output. When the credit allocation is decentralized, the banks are able to provide more money to non-state sectors, however, it forces the government to create more credits to finance the state-owned enterprises in order to avoid large bankrupt of these firms, as a result, such excess credits lead to an increase in inflation rate. Chen, Tang, and Li (2009) argue that China’s inflation can not be forecasted by either M0, M1 or M2. However, we cannot deny the role of money supply as an intermediate goal of China’s monetary policy. Fang, Pei, and Zhang (2006) analyse the relationship between foreign reserves and inflation during the period of 2001-2005 and the result shows that the increase of foreign exchange reserve leads to a rise in inflation rate. Huang, Wang, and Hua (2010) examine the determinants of inflation in China, they suggest that there is weak relationship between exchange rate and inflation but output gap can positively affect inflation in China. However, the literature on modelling inflation dynamics for China is still scarce (due to the sluggish unemployment data, traditional Phillips curves are rarely used to forecast inflation for China).

To avoid the noises of the two inflation crisis, in this research, I use monthly data from 2005:M1 to 2014:M9. There are several reasons that I choose this sample size: (1) the data of Chinese inflation rate is only published by annual or
monthly, the sample size will be too small to make ex-post forecasts if I use annual data. (2) the central bank status was legally confirmed in 1995 and the role of central bank in the making and implementation of monetary policy was established by the Standing Committee of the Tenth National People’s Congress in 2003. It might due to the limit power of the central bank, inflation rate during 1990s was volatile. (3) there are three potential candidates to measure the output: GDP, value-added industrial production and manufacture Purchasing Manager Index (PMI). However, there is only quarterly data for GDP rather than monthly; due to the Spring Festival holiday, in order to remove the effects of seasonality in the data, they publish the growth rate of industrial production for the first two months jointly not respectively. In other words, there are missing values in this monthly data. Although GDP and industrial production index can be interpolated to monthly data, the accuracy of forecasts will be affected by the interpolation. Thus, I use the manufacture PMI for measuring monthly output in China and this data is available since 2005:M1.

According to Diebold and Rudebusch (1991) and Clark and McCracken (2001), to check whether one variable has predictive power for another, we can first generate out-of-sample forecasts⁴ by constructing a model that include a variable

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⁴This method is advocated by Ashley (1998)
and then excluding that variable. Second, we evaluate the forecast accuracy of the two sets of forecasts. Since VAR models are widely used for forecasting inflation (Stock and Watson (2001)), I use VAR models to forecast China’s inflation rate. The VAR models include inflation with output gaps, total reserves, exchange rate and M2 respectively. In order to evaluate the forecast performances of these models, I choose a benchmark which is the AR model (the optimal lags of the AR model depend on the lags in the VAR which are decided by AIC). If forecasts from VARs can significantly outperform the benchmark AR model, this implies that the variable has predictive power for inflation. The reason I choose AR model as a benchmark is that if the VAR model excludes one variable, it will become to an AR model, so that we can check the predictive power for the variable.

The forecast experiment is designed as followings: (1) forecasts from all models are generated by the recursive scheme Stock and Watson (1996). The sample ranges from 2005:M1 to 2014:M9, all the models should be estimated by OLS using the sub-sample from 2005:M1 to 2010:M7 and predict the inflation rate for 2010:M8, 2010:M10, 2011:M1 for 1-step-ahead, 3-steps-ahead, 6-steps-ahead forecasts respectively. Then the VARs are re-estimated with one more observation using data from 2005:M1 through 2010:M8 and re-forecast 2010:M9, 2010:M11 and 2011:m2 etc.. I evaluate corresponding forecasts for 2010:M8 through 2014:M9. (3) lag lengths are obtained by minimising the Akaike information criteria (AIC) in each recursion. (4) forecast performances are evaluated by the modified Morgan-Granger-Newbold, the Diebold-Mariano and the modified Diebold-Mariano test (Harvey, Leybourne, and Newbold (1997)). The details of these forecast encompassing tests are described in Chapter 1.

<table>
<thead>
<tr>
<th>forecast horizons</th>
<th>output gap 1</th>
<th>output gap 2</th>
<th>reserves</th>
<th>exchange rate</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-step-ahead</td>
<td>1.521*</td>
<td>1.011</td>
<td>0.506</td>
<td>0.593</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: (1) Values are test statistics. (2) *, **, *** indicate the test statistics is significant at 10%, 5% and 1% significance level.

<table>
<thead>
<tr>
<th>forecast horizons</th>
<th>output gap 1</th>
<th>output gap 2</th>
<th>reserves</th>
<th>exchange rate</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-step-ahead</td>
<td>1.406*</td>
<td>0.984</td>
<td>0.107</td>
<td>0.311</td>
<td>0.000</td>
</tr>
<tr>
<td>3-step-ahead</td>
<td>1.925**</td>
<td>1.559*</td>
<td>0.527</td>
<td>0.479</td>
<td>0.621</td>
</tr>
<tr>
<td>6-step-ahead</td>
<td>2.116**</td>
<td>1.607*</td>
<td>1.030</td>
<td>1.012</td>
<td>1.233</td>
</tr>
</tbody>
</table>

Notes: (1) Values are test statistics. (2) *, **, *** indicate the test statistics is significant at 10%, 5% and 1% significance level.
Table 3.4: Modified Diebold-Mariano test

<table>
<thead>
<tr>
<th>forecast horizons</th>
<th>output gap 1</th>
<th>output gap 2</th>
<th>reserves</th>
<th>exchange rate</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-step-ahead</td>
<td>1.392*</td>
<td>0.974</td>
<td>0.031</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3-step-ahead</td>
<td>1.825**</td>
<td>1.512*</td>
<td>0.466</td>
<td>0.405</td>
<td>0.589</td>
</tr>
<tr>
<td>6-step-ahead</td>
<td>1.856**</td>
<td>1.551*</td>
<td>0.967</td>
<td>0.904</td>
<td>1.082</td>
</tr>
</tbody>
</table>

Notes: (1) Values are test statistics. (2) *, **, *** indicate the test statistics is significant at 10%, 5% and 1% significance level.

Table 3.2-3.4 report the results of the forecast encompassing test. Table 3.2 shows the empirical results for the modified Morgan-Granger-Newbold test with the null hypothesis that the benchmark encompasses the VAR models. We note that the VAR with output gap 1 (the smoothness parameter for HP filter is \( \lambda = 129600 \)) reject the null hypothesis at 10% significance level while other VAR models can not encompass the benchmark. However, the modified Morgan-Granger-Newbold test is only applicable for 1-step ahead forecasts. Therefore, the Diebold-Mariano and the modified Diebold-Mariano test (MDM test) are used to test the equality of forecast accuracy because they are both applicable to forecasts beyond 1-step ahead and non-Gaussian and serially correlated forecast errors. Besides, the modified Diebold-Mariano test is more reliable on small sample sizes and biased forecasts. Table 3.3 and 3.4 report the empirical results for these two tests. If the test statistics are significantly positive, the benchmark has larger forecast errors which means the VAR models encompass the benchmark, if test statistics are significantly negative, the benchmark has smaller forecast errors. As it is shown in table 3.3, for 1-step ahead forecast, the test statistic is significant at 10% level for the VAR including output gap 1 (the smoothness parameter for HP filter is \( \lambda = 129600 \)), namely the VAR model encompasses the benchmark AR model. The test statistic for output gap 2 (\( \lambda = 14400 \)) is insignificant. For multi-step ahead forecasts, the test statistics for the output gap 1 are significant at 5% level while the values are only significant at 10% for the output gap 2. The test statistics for total reserves, exchange rate and M2 are all insignificant. Similar results for the modified Diebold-Mariano test are obtained from Table 3.4.

The above results can be summarized as follows: firstly, the output gap estimated by HP filter using the smoothness parameter \( \lambda = 129600 \) outperforms that with \( \lambda = 14400 \). This result is consistent with Ravn and Uhlig (2002)’s who suggest to use \( \lambda = 129600 \) as the smoothness parameter of HP filter for monthly data rather than \( \lambda = 14400 \). Secondly, the above results seem highly suggest that only output gap with \( \lambda = 129600 \) has predictive power for inflation, it also indicates that inflation in China is mainly driven by the excess demand rather than the
managed float exchange rate regime or the excess money supply.

### 3.4 Empirical results

The first chart in Figure 3.5 plots logarithm of the EPU index and three measures (in section 3.3.1 $RMSE_{AF}$, $RMSE_{RT}$, $RMSE_{LPS}$) of the survey-based forecast uncertainties for China’s inflation from May 2010 to September 2014. The survey-based forecast uncertainties experienced a slightly downward trend during this period of time. Moreover, the patterns of the three measures of forecast uncertainty were quite close to each other. The EPU index, however, remained fairly stable and persistent over the time period, except there was a dramatical increase in the EPU index from 2011:M7 to 2011:M11. That was probably due to the high inflation period in China. The government increased the bank’s requirement ratio and interest rate on 12th May and 7th July respectively, such policies might increase individuals’ uncertainties whether government would take further actions to suppress the inflation rate. The second chart in Figure 3.2 displays the EPU index and the model-based forecast uncertainties from July 2011 to September 2014. The patterns of the model-based forecast uncertainties are varied with different forecast horizons. For 1-step ahead forecast, it remained steady during the time period, which are correspond to the survey-based uncertainties (survey-based forecast uncertainties are also 1-step ahead forecasts). For 3-step ahead forecast, it experienced a sharp decline between 2013:M5 to 2013:M7, but it was relatively stable over other periods of time. For 6-step ahead forecast, it appeared to trend gently downward until 2013:M10, it remained relatively unchanged thereafter.

<table>
<thead>
<tr>
<th></th>
<th>$RMSE_{LPS}$</th>
<th>$RMSE_{AF}$</th>
<th>$RMSE_{RT}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>log of EPU</td>
<td>-0.125</td>
<td>-0.252*</td>
<td>-0.154</td>
</tr>
</tbody>
</table>

Notes: (1) the table measures the correlation coefficient between EPU index for China and the survey-based inflation forecast uncertainty. (2) * indicate the P-value is significant at 10% level.

I compute the correlations between the ex-post forecast uncertainties and the EPU index by Spearman’s rank correlation coefficient, because this method is not sensitive to the outliers in the data as forecast errors are heavy-tailed distributed. The correlation coefficients are shown in Table 3.5. As can be seen from the table, the relationships between EPU index and the survey-based forecast uncertainty are insignificant at 5% significance level and surprisingly are all negative. Since
Notes: 1. The chart plots values of logarithm of EPU index and survey-based forecast uncertainties. 2. The solid line denotes the values of logarithm of EPU index, the long dash line (LPS), dash line (RT) and dot line (AF) denote the three measures of forecast uncertainties in section 3.1 respectively. 3. The left y axis measures the values of survey-based forecast uncertainties and the right y axis is the values of logarithm of the EPU index.

Notes: 1. The chart plots values of logarithm of EPU index and model-based forecast uncertainties. 2. The long dash line, dash line and dot line denote the 1-step ahead, 3-step ahead and 6-step ahead model based forecast uncertainties respectively. 3. The left y axis measures the values of survey-based forecast uncertainties and the right y axis is the values of logarithm of the EPU index.
the survey data only provides 1-step ahead forecasts, the evidence is not enough to draw a conclusion for multi-step-ahead forecast uncertainty.

The *ex-post* forecast uncertainty based on econometric models is derived from 12-months moving average of the RMSE of the output gap VAR forecasts. Table 3.6 presents the correlation coefficients between the model-based inflation uncertainty with different forecast horizons and the EPU index. The correlation coefficient between the *ex-post* forecast uncertainty and the EPU index is insignificant for 1-step-ahead forecasts, which is consistent with the results from survey data. However, with regard to multi-step-ahead forecasts, the correlation coefficients are significantly positive at 1% significance level.

Table 3.6: correlation coefficients (model-based uncertainty and EPU)

<table>
<thead>
<tr>
<th></th>
<th>1-step ahead</th>
<th>3-step ahead</th>
<th>6-step ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>log of EPU</td>
<td>0.201</td>
<td>0.598***</td>
<td>0.602***</td>
</tr>
</tbody>
</table>

Notes: (1) the table measures the correlation coefficient between EPU index for China and the *ex-post* inflation forecast uncertainty based on RMSE from econometric models. (2) *** indicate the P-value is significant at 1% level.

Figure 3.3 presents the trend of inflation rate and inflation uncertainties for China. The first chart in figure 2 plots the survey-based inflation uncertainties together with the time series of inflation. Both inflation uncertainties and inflation rate appeared to increase in the first eight months, however, the inflation uncertainties started to decline five months earlier than the inflation rate which reached a peak on 2011M6. It is notable that during the period of 2011M11 to 2013M2, the inflation uncertainties and the inflation rate experienced opposite patterns, after that, however, they followed paralleled trends over the period from 2013M5 to 2014M9.

The second line chart in figure 3.3 displays the model-based inflation uncertainty and the time series of inflation. The inflation fell rapidly from 2011M7 and then it fluctuated between 1.5 to 3.2 for the final two years. However, regarding to the model-based inflation uncertainties, the 1-step ahead inflation uncertainty remained fairly steady. For 3-step ahead inflation uncertainty, it was relatively stable over the period except a sharp drop between 2013:5 to 2013:7. For 6-step ahead inflation uncertainty, it exhibited a downward pattern until 2013:M10, afterwards, it remained relatively unchanged.

Table 3.7 presents the correlation coefficients between the inflation rate and the three measures of inflation uncertainties based on survey data. It is clear that
Inflation and survey-based forecast uncertainties

Notes: 1. The chart plots values inflation rate and survey-based forecast uncertainties. 2. The solid line denotes the inflation rate, the long dash line (LPS), dash line (RT) and dot line (AF) denote the three measures of forecast uncertainties in section 3.1 respectively. 3. The left y axis measures the values of survey-based forecast uncertainties and the right y axis is the values of inflation rate.

Inflation and model-based forecast uncertainties

Notes: 1. The chart plots inflation and model-based forecast uncertainties. 2. The long dash line, dash line and dot line denote the 1-step ahead, 3-step ahead and 6-step ahead model based forecast uncertainties respectively. 3. The left y axis measures the values of survey-based forecast uncertainties and the right y axis is the values of inflation rate.

Figure 3.3: Inflation and inflation uncertainty
they have significantly positive relationship. This result demonstrates that over the full period 2010M5 to 2014M9, the inflation and the survey-based inflation uncertainties are correlated to each other in China.

Table 3.8 shows the correlation coefficients between inflation rate and model-based inflation uncertainties for China, unlike the survey-based uncertainties, the 1-step ahead inflation uncertainty has no relationship with the inflation rate. The correlations between inflation and 3-step ahead and 6-step ahead inflation uncertainties are also insignificant.

### 3.5 Summary and conclusion

The above results can be summarised as follows:

1. For both survey-based and model-based 1-step-ahead forecast uncertainty, there is no significant relationship between the forecast uncertainty and the macroeconomic uncertainty which is measured by the EPU index. However, with the increase in forecast horizons the correlation coefficients of the model-based uncertainty and the EPU index becomes to increase and positively significant. That is to say, there is no relationship between the survey-based forecast uncertainty and the macroeconomic uncertainty, but the model-based uncertainty can generally capture the tendency of the macroeconomic uncertainty.

The possible reason to explain this result is that: firstly, the EPU index for China is a news-based index and it relies on the Hong Kong English newspapers rather than Chinese newspapers. In other words, the EPU index may be not reliable for
measuring uncertainty for China and should be replaced by more precise measures, say, based on the Chinese-language pattern searches. Or we can use other measures of uncertainty, such as quantitative easing and tapering uncertainty Meinusch and Tillmann (2015), quantifies beliefs of market participants about monetary policy by using social media Twitter. They search tweets containing the words “Fed” and “taper” from the entire Tweeter volume for the period April to October 2013 and use this Twitter data to construct two indexes reflecting the uncertainty of future Fed policy. Peter (2015) develops a new measure of economic uncertainty using news-media textual data. Secondly, the survey-based forecasts are 1-step ahead inflation forecasts, which are usually close to the real outcomes, they may underestimate the uncertainty reflected by multi-step ahead forecast uncertainty. Thirdly, similar results are found in Charemza, Diaz, and Makarova (2015)’s paper. They analyse the correlation between the logarithm of the EPU index and the squares of forecast errors made by univariate ARMA-GARCH model for 7 countries (the seven countries are Canada, France, India, Italy, Spain, UK and US). They find that except for Canada, there is a positive relationship between the logarithm of the EPU index and the squares of forecast errors for most forecast horizons across different countries. A more important finding from their research is that the correlations become stronger with the increase in forecast horizons.

3. Inflation rate and survey-based inflation uncertainties are significantly positively correlated. On the other hand, the correlations between inflation and model-based inflation uncertainties are statistically insignificant. The reason is probably because the survey forecasts are 1-step ahead forecasts and the forecasts are close to the real outcomes, it can capture the dynamics of the inflation so that they are correlated.

4. The results of the forecast encompassing test show that the VAR with output gap can encompass the benchmark AR model while other models can not outperform the benchmark significantly. This result implies that output gap has the predictive power for forecasting inflation in China.

This chapter introduces the relationship between uncertainty and growth from macro and micro perspectives and the relationship between inflation and inflation forecast uncertainty. I also provide an overview of the concepts for the forecast uncertainty and the macroeconomic uncertainty, the ex-post and ex-ante uncertainty, the Knightian and the non-Knightian uncertainty. In order to analyse the relationship between inflation uncertainty (non-Knightian uncertainty) and macroeconomic uncertainty (Knightian uncertainty) for China, I introduced three
survey-based measures of uncertainty and model-based forecast uncertainties to measure inflation uncertainties, I used the Economic Policy Uncertainty index to represent the macroeconomic uncertainty.

The empirical evidence suggests that there is no significant relationship between the survey-based inflation uncertainties and the macroeconomic uncertainty, however, for multi-step ahead model-based inflation uncertainties, it has significantly positive relationship with the macroeconomic uncertainty. I also investigate the correlations between inflation and inflation uncertainties. The results indicate that there is significantly positive relationship between survey-based uncertainties and inflation, while there is no relationship between model-based uncertainty and inflation.

Regarding to the ex-post and the ex-ante forecast uncertainties, the ex-post forecast uncertainty is easier to compute and more precise than the ex-ante forecast uncertainty because it is impossible to provide precise distributions around the future events. However, the ex-ante forecast uncertainty is more helpful for policy makers who are willing to know the future economic conditions. Monetary authorities may adjust their policies to affect the economic outcomes when they perceive changes of uncertainty. For example, Bekaert, Hoerova, and Duca (2013) indicate that high uncertainty can lead to loose monetary policy. Regarding to the macroeconomic uncertainty, although it is an unobservable ex-post phenomenon and there is no consensus measure for this uncertainty, it is crucial to analyse it for policy makers as a deeper analysis of how macroeconomic uncertainty has impacts on the macroeconomy in the past is likely to help policy makers to assess how future uncertainty shocks might have effects on demand and supply prospects.

There are some limitations in this chapter. Firstly, further research should try to combine those model-based inflation uncertainties from different forecast horizons to construct an aggregate forecast uncertainty. Secondly, an improvement of time series model would be necessary to find more reliable forecasts.
Chapter 4

Probability of deflation in China: An application of density forecast

Abstract. This chapter aims to compute the probabilities of deflation in China by constructing density forecasts. The results indicate that the weighted skew normal distribution is the best fit for the empirical distribution of forecast errors compared with normal, $\alpha$ stable and two piece normal distributions. Probabilities of deflation and the expected lengths of deflation are different from independent forecasts and dependent forecasts as well as different distributions. The results suggest that the independent forecast may overestimate the probabilities of deflation when the forecast horizons are short but underestimate the probabilities of deflation when the forecast horizons are long. The use of an inappropriate distribution for density forecasts may under or overestimate the probabilities of deflation, especially when the forecast horizons are short and the density forecasts are generated independently.
4.1 Effects of deflation

Deflation refers to a decline in the general price level of goods and services. In the mainstream of economics, deflation is caused by an unbalanced mismatching of the supply and demand for goods and money, especially when the supply of money declines and the supply of goods increases (Krugman (2002)). Historic episodes of deflation are often associated with the demand for goods decreases combined with a fall in the money supply, such as the Great Depression in 1930s (Bernanke (1995)) and the “lost decades” in Japan (Cargill (2001)). The effects of deflation depend on its extent and duration. Deflation may not cause significant costs under certain circumstance. For example, an expansion in aggregate supply or a supply shock may cause prices decline temporarily; free trade or falling prices of import goods can also push downward pressure on domestic prices. In these cases, prices change in a short time and deflation is a result of an adjustment to a new equilibrium. Therefore, this short time deflation is unlikely to change individuals’ expectations and result in long-run negative effect on domestic demand (Baig (2003)).

However, deflation can be costly in the context of demand-driven deflation. From a microeconomic perspective, deflation can reduce employment and output when nominal wages are sticky. When wages are rigid downward, a decrease in price will lead real wages increase, the higher real wages will reduce employment and depress output (Bernanke and Carey (1996)). According to Akerlof, Dickens, Perry, Gordon, and Mankiw (1996), with 1 percent deflation and downward wages rigidities, the long-run equilibrium of unemployment rate in the US could rise frm 5.8 percent to 10 percent. Deflation may also increase consumers savings and reduce their expenditures as they expect prices to fall and they are unsure about future economic circumstances. From a macroeconomic perspective, deflation can result in the interest rate close to zero. When the nominal interest rate is close to zero, the real interest rate should be equal to the expected deflation rate which will lead to an increase in a loan’s interest rate paid by borrower. For example, during the Great Depression, deflation reached 10% per year, even an interest-free loan is unattractive because the purchasing power of dollars is 10% greater than the loan was borrowed last year (Bernanke et al. (2002)). In this case, borrowers are reluctant to borrow so that investments, purchases of houses and cars and other types of spending will decline, which result in a fall in economic growth (see Bernanke et al. (2002)). Deflation can impose a heavy burden on debtors as the value of collateral losses. As the value of collateral declines, banks or lenders will either raise their financing charges or limit their lending, which result in a “debt-deflation” (see Fisher (1933), Bernanke (1995) and Baig, Decressin, Feyzioglu, Kumar, and Faulkner-MacDonagh (2003)).
An example of the effects of deflation on economy is Japan’s "lost decade", which began in 1990s. The average growth rate of GDP from 1993 to 2003 was just above 1 percent. Meanwhile, the inflation rate, either measured by GDP deflator or CPI, was less than 2% and most of time was negative. The most substantial cost of deflation in Japan is the monetary policy, which is constrained by the zero bound nominal interest rate. Since the Bank of Japan has committed to keep its short-term interest rate at zero until deflation has been eliminated, it has little room for further reduction of the nominal interest rates when the economic growth remains sluggish and prices are continue to fall, which is in need for further monetary stimulus (Eggertsson et al. (2003)). According to McCallum (2003), the Taylor rule suggests a negative nominal interest rate to stimulate the Japan’s economy since 1998. However, it is obviously infeasible because of the zero bound constraint. Wages in Japan had downward rigidities in 1990s. According to Kuroda and Yamamoto (2003), they find that the nominal wages change distributions from 1993-1998 are statistically skewed to the right, which indicates that downward nominal wage rigidity does exist in Japan. Due to the wages downward rigidities, the unemployment rate increased sharply from 2.5% in 1993 to around 5.5% in 2003. Deflation also affects Japan’s fiscal policy. Because of the deflation, the government debt soared from 60% of GDP in the early 1990s to around 150% GDP in 2000s, which makes government more reluctant to use aggressive fiscal policies (Posen (1998)).

4.2 The risk of deflation in China

As the largest emerging economy in the world, China has experienced substantial institutional changes over the past three decades. Since the policies of economic reform and openness in 1978, China has transformed from a planned economy to a socialist market economy. It has become the second largest economy in the world with the GDP growth rate averaging above 9.5% per year. Price levels in China had rarely changed and had been centrally controlled by the government before 1978. Since the openness reform, prices have been liberalised in process and price controls were eliminated in the industrial and retail sectors in early 1990s. Along with it, monetary policies also changed during the first 20 years’ reform, which could be characterised as credit expansion. It was observed that economic growth and inflation trend were highly correlated with money supply in China (Goodfriend and Prasad (2006)); meanwhile the velocity of money in China was relatively lower and stable, therefore, China’s economy experienced the typical “boom-bust” and “stop-go” cycle during 1988 to 1998. When the inflation rate accelerates along with a booming economy, the central bank responds
by a sequence of interest rate raises and tightening of credits, which dampens the economic growth as well as controls inflation rate. Once the economic growth slows down, government stimulates the economy by reducing interest rate, as a result the inflation goes up again. After suffering high volatility in inflation rate and economic growth rate during the late 1980s and the early 1990s, the government decided to take further reforms on monetary policy, the central bank status was legally confirmed in 1995, the People’s Bank of China (PBOC, the central bank) became politically independent and changed gradually to a monetary strategy anchored on intermediate monetary targets. Putting the low inflation rate and high economic growth rate at the priority, the annual Central Economic Working Conference has been held to set the tone for next year’s agenda (Huang (2012)). Central bank in response would then announce the growth rate of monetary aggregates (M1 and M2) and inflation target (or a range) for the coming year and make prudent monetary policy\(^1\) to achieve these objectives. They also established interbank money markets and bond markets in the late 1990s and started open market operations in 1998 (Goodfriend and Prasad (2006)).

Due to the plan of the structural reforms proposed in the 18th National Congress of the Communist Party for China in 2012, China’s transition towards a sustainable growth rather than a rapid growth, China’s GDP growth rate has started to decline since 2012 with a target of 7% per year, along with falls in annual growth rate of consumer price index (CPI) and the producer price index (PPI), which are two common measures of price rise in China. The CPI rate hit a six year’s low at 0.8% on January 2015 and the PPI rate had started to below zero since 2012. China newly built house prices (year on year change) have started to decline since 2013. Thus, policy makers start to concern the risk of slipping in to deflation in China and they implement a series of stimulus measures to keep the economy from taking the same path of long-term deflation and recession as its neighbour Japan suffered in the past two decades.

The current disinflation in China may be caused by a combination of excess supply and weak demand. Regarding to the supply side, it is likely that the risk of deflation is caused by the persistence of excess capacity. Over the last three decades, the rapid economic growth rate in China was mainly driven by the investment and infrastructure constructions. Thus, the industrial overcapacity problem, such as oversupplies of rebar and steel products, puts downward pressure on producer price, which is probably the main cause of producer price deflation. Regarding to the demand side, an abrupt drop in import volumes implies a weak demand in China, as import fell for seven straight months in May 2015. Meanwhile, according to the National Bureau of Statistics of China, the

\[^1\text{Policies are made by the People’s Bank of China to balance efforts on ensuring stable economic growth, maintaining the overall stability of prices and preventing financial risks.}\]
monthly growth rate of the total retail sales of China consumer goods declined from 12.5% on May 2014 to 10.2% on March 2015, which also indicates stagnant domestic demand in China.

It is therefore very crucial for Chinese government to forecast the probability of deflation so that they can adjust their actions to avoid the damages of possible deflation. Many financial institutions provide point forecasts for inflation rate in China, such as the SPF from "Asia Pacific Consensus Forecasts", available at http://www.consensus Economics.com/. However, point forecasts can not capture the uncertainties around the predictions. There are two widely used approaches to evaluate the risk of deflation. The first one is based on the preferences of the economic agents with respect to the inflation, see Kilian and Manganelli (2007). The second approach measures the risk of deflation based on the probability distribution of inflation. This chapter uses the later approach based on the work of Charemza, Makarova, and Wu (2015), we forecast the probabilities of deflation in China by applying density forecasts. Conventional methodologies construct density forecasts by using a series of independent distributions, however, as the distributions of forecast errors are markedly dependent (Harvey, Leybourne, and Newbold (1998)), in this chapter we consider the dependence of the forecast for different forecast horizons. The way of constructing probabilistic forecasts is to generate a joint distribution using copulas. We find that the independent forecast may overestimate the probabilities of deflation when the forecast horizons are short but underestimate the probabilities of deflation when the forecast horizons are long.

The structure of this chapter is as follows. Section 4.3 discusses the concepts and applications of the probabilistic forecasts and several distributions that are commonly used in the probabilistic forecasts. Section 4.4 discusses the estimation methods for these distributions. Section 4.5 introduces the copulas and analyse the probability of deflation in China. Section 4.6 concludes.

### 4.3 Probabilistic forecasting

A probabilistic forecast or density forecast gives the entire probability distribution of the expected value of a random variable. In contrast to a point forecast, a probabilistic forecast conveys more information and provides a complete description of the non-Knightian (or Knightian risk Knight (1921)) uncertainty associated with a point forecast. The introduction of density forecast and Knightian risk were introduced in chapter 2 and 3.

In contrast to survey forecasts which are purely judgementally based, model-based density forecast is also widely used. One type method is to generate den-
sity forecasts by a range of models, such as VAR models, assuming that the shape of the density would not change over time. Given this obvious limitation, the ARCH (Engle (1982)) and GARCH (Bollerslev (1986)) model where conditionally-autoregressive errors are associated with uncertainties, are applied to allow for time variation in the shape of the density (see Elder (2004) and Kontonikas (2004)). Since then, a large number of studies have emerged allowing for both time variation in forecasting models and stochastic volatility in the error process, see Harvey, Ruiz, and Shephard (1994), Chib, Nardari, and Shephard (2002). An example of this approach to generate density forecasts is Cogley, Morozov, and Sargent (2005), who develop a Bayesian VAR model, which involves both changing coefficients in the VAR and stochastic volatility in the errors. There is another method to generate probabilistic forecasts based on models, which is popular among central banks. This method generates density forecasts with reference to the analysis of the distribution of historical point forecast errors (ex-post forecast errors), for example Charemza, Diaz, and Makarova (2014). In this chapter, we will focus on the later approach.

A common way of presenting probabilistic forecasts is by plotting the density estimate. The probabilistic forecast for inflation in UK is published by the Bank of England, known as Fan Chart. An example can be seen from Figure 4.1, which is a Fan chart from the February 2014 Inflation Report. It shows multiple quarters ahead possible values of future inflation which are represented graphically as a set of forecast intervals covering 10%, 20%...90% of the probability distributions. As the forecast horizon increases, these forecasts become increasingly uncertain and intervals range spread out, creating “fan” shapes. The central projection of inflation, which provide the most likely path of future inflation, is indicated by the mode, the median or the mean of the forecast distribution. Narrow or wide intervals around the central view provide a measure of the degree of uncertainty for future inflation. The shape of the distribution (symmetric or asymmetric) or the degree of skewness exhibits a measure of the balance of risks on the forecasts.

However, the empirical distributions of these probabilistic forecasts are still controversial. Since forecast errors are believed non-normally distributed (Harvey and Newbold (2003)), the Bank of England uses the two-piece normal distribution (TPN, John (1982)), which has a degree of asymmetry. The density function of the TPN distribution is:

\[
f(x) = \begin{cases} 
A \exp\left[-\frac{(x - \mu)^2}{2\sigma_1^2}\right] & x \leq \mu \\
A \exp\left[-\frac{(x - \mu)^2}{2\sigma_2^2}\right] & x > \mu 
\end{cases}
\]

where \( A = \left(\frac{\sqrt{2\pi}(\sigma_1 + \sigma_2)}{2}\right)^{-1} \), the two-piece normal distribution results from

\(^2\)The degree of uncertainty can be measured by variance or inter-quartile range ect.
two halves normal distributions with parameters \((\mu, \sigma_1)\) and \((\mu, \sigma_2)\) and scaling them to a joint distribution. The three estimated parameters describe the central tendency for inflation (the mean or median), a view on the degree of uncertainty (the variance) and a view on the balance of the risks (the skewness).

Stable distributions are also considered to generate density forecasts, especially when there is hyperinflation, particularly in some developing countries (Charemza, Jelonek, and Makarova (2009)). An \(\alpha\)-stable distribution \((X \sim S_\alpha(\beta, \delta, \mu))\) has a characteristic function (see Samorodnitsky and Taqqu (1994)):

\[
\Phi(t) = \text{Exp}(itX) = \begin{cases} 
\exp(-\delta |t|^\alpha [1 - i\beta \text{sign}(t) \tan \frac{\pi \alpha}{2}] + i\mu t) & \alpha \neq 1 \\
\exp(-\delta |t| [1 + i\beta \frac{2}{\pi} \text{sign}(t) \ln |t|] + i\mu t) & \alpha = 1
\end{cases} \tag{4.1}
\]

where \(t\) denotes argument of Fourier transformation, \(i\) is the imaginary unit. The sign function is defined as

\[
\text{sign}(t) = \begin{cases} 
-1 & t < 0 \\
0 & t = 0 \\
1 & t > 0
\end{cases}
\]

The details of the \(\alpha\)-stable distribution was introduced in Chapter 2.

Further studies develop skew normal distributions (see Azzalini and Dalla Valle (1996) and Azzalini (2005)), which can be used to describe uncertainties of the density forecasts of inflation. However, these distributions have several practical problems. For example, Pewsey (2000) argues that Azzalini’s direct parameterization should not be use as a general basis for estimation. How to interpret the parameters of these distributions and to find the true distribution of the data are
also concerned by many practitioners (Charemza, Vela, and Makarova (2013)). Charemza, Díaz, and Makarova (2015) develop a skew normal distribution, called weighted skew normal distribution (WSN) to tackle above unsolved inference problems for the skew normal distribution. Traditional density forecasts assume that current monetary policy does not change when the forecasts are made, that is “forecasts are feedback-free”. However, Charemza, Díaz, and Makarova (2015) argue that if inflation forecasts provided by forecasters are above the target, monetary authorities may undertake anti-inflationary actions based on these forecasts, thus the forecasts should be made by taking into account the consequences of possible monetary policies, that is “feedback” to inflation. They suggest that parameters of WSN implies the accuracy of finetune, effectiveness of the monetary policy and inflation target bands. A random variable $Z$ with weighted skew normal distribution $WSN_\rho(\alpha, \beta, \hat{m}, \hat{k}, \rho)$ if:

$$Z = X + \alpha \times Y \times I_{Y > \bar{m}} + \beta \times Y \times I_{Y < \bar{k}}$$

where

$$I_{Y > \bar{m}} = \begin{cases} 
1 & Y > \bar{m} \\
0 & \text{otherwise}
\end{cases}; \quad I_{Y < \bar{k}} = \begin{cases} 
1 & Y < \bar{k} \\
0 & \text{otherwise}
\end{cases}$$

$$(X, Y) \sim N \left( \begin{bmatrix} \mu_X \\
\mu_Y \end{bmatrix}, \begin{bmatrix} \sigma^2_X & \rho \sigma_X \sigma_Y \\
\rho \sigma_X \sigma_Y & \sigma^2_Y \end{bmatrix} \right)$$

where random variable $X$ contains ontological uncertainties, related to the random nature of future inflation, which can not be predicted and epistemic uncertainties, related to incomplete knowledge of the forecasters, which is predictable (details of these concepts are discussed by Walker, Harremoës, Rotmans, van der Sluijs, van Asselt, Janssen, and Krayer von Krauss (2003)). Random variable $Y$ implies imperfect knowledge as individual forecasters have their own source of information and the forecasts are different from each other, known as uncertainties by disagreement (see, Bomberger (1996) and Giordani and Söderlind (2003)). $\bar{k} < \bar{m}$, these “low” and “up” thresholds denote the levels of imperfect knowledge. If $\bar{m}$ is violated from below, monetary authorities may undertake anti-inflationary actions, if $\bar{k}$ is breached from above, they may implement pro-inflationary policies. $\bar{m}, \bar{k}, \alpha, \beta, \mu_X, \mu_Y \in R; \sigma_X, \sigma_Y \in R^+$ and $|\rho| \leq 1$. For simplicity, the WSN can be normalized:

$$\frac{Z}{\sigma} = Z^* \sim WSN_1(\alpha, \beta, m, k, \rho)$$
where $m = \bar{m}/\sigma$, $k = \bar{k}/\sigma$. The probability density function (PDF) of $Z^*$ is:

$$f_{WSN_1}(t) = \frac{1}{\sqrt{A_\alpha}} \varphi \frac{t}{\sqrt{A_\alpha}} \Phi \frac{B_\alpha t - m A_\alpha}{\sqrt{A_\alpha(1-\rho^2)}} + \frac{1}{\sqrt{A_\beta}} \varphi \frac{t}{\sqrt{A_\beta}} \Phi \frac{-B_\beta t + k A_\beta}{\sqrt{A_\beta(1-\rho^2)}}$$

$$+ \varphi(t) \star \left[ \Phi \frac{m - \mu}{\sqrt{1-\rho^2}} - \Phi \frac{k - \mu}{\sqrt{1-\rho^2}} \right]$$

where

$$A_\alpha = 1 + 2\alpha \rho + \alpha^2, \quad B_\alpha = \alpha + \rho$$

$$A_\beta = 1 + 2\beta \rho + \beta^2, \quad B_\beta = \beta + \rho$$

and $\varphi$ and $\Phi$ denote the density and cumulative distribution functions of the standard normal distribution respectively.

### 4.4 Estimation

#### 4.4.1 Minimum distance estimation

As we mentioned above, estimation of parameters of skew normal distributions by the maximum likelihood or the method of moments has possible bias and convergence problems (see, Pewsey (2000) and Monti (2003)). For this reason, we have decided to use the minimum distance estimation (MDE) in this chapter. According to Basu, Shioya, and Park (2011), the minimum distance estimator is an alternative to the maximum likelihood estimator. It not only has strong robustness properties, but also asymptotically fully efficient.

An important idea in the minimum distance estimation is the quantification of the difference or “distance” between the sample data and the model which is a function of unknown parameters. Such “distance” represents a divergence between the empirical distribution function and the model distribution function, or a divergence between a density estimate obtained from the sample data and the model density function. In this chapter, the primary attention will be on the later density-based idea. Let $X_1, X_2, ..., X_n$ represent a sequence of independent identically distributed observations from a distribution $F_\theta(x)$ having a probability density function $f_\theta(x)$, where $\theta$ is an unknown parameter belonging to $\Theta \subseteq \mathbb{R}^k$, $k \geq 1$. The empirical density function based on $X_1, X_2, ..., X_n$ is denoted as $f_n(x)$. If there exist a $\hat{\theta}$ in $\Theta$, such that the distance function $d(\cdot, \cdot)$ is (details see Drossos and Philippou (1980)):

$$d(f_\theta(x), f_n(x)) = \inf \{d(f_\theta(x), f_n(x)); \theta \subseteq \Theta\}$$

is called minimum distance estimation of $\theta$.

Within the class of density-based distances, this chapter only focuses on Chi-
square distance and Hellinger distance. In this case, we can quantify the discrepancy between the sample data and the model and use the actual value of the minimum distance to determine whether the distance is close enough. The Pearson’s chi-square distance criterion is defined as (see Basu, Shioya, and Park (2011)):

\[
PCS(d_n, f_\theta) = \sum_{i=1}^{m} \frac{(d_n(i) - f_\theta(i))^2}{2f_\theta(i)}
\]

The (twice, squared) Hellinger distance criterion has the form (see Basu, Shioya, and Park (2011)):

\[
HD(d_n, f_\theta) = 2 \sum_{i=1}^{m} [d_n(i)^{1/2} - f_\theta(i)^{1/2}]^2
\]

where in both cases, \( n \) denotes the sample size, \( m \) is the number of disjoint intervals, \( d_n(i) \) is the empirical frequency of data falling into the \( i \)th interval and \( f_\theta(i) \) is the theoretical probability for the interval.

The estimation method we use is the grid search, which is a way of performing parameter optimization (Thisted (1988)). Suppose there is a real-valued function \( F(x) \) on \( \mathbb{R} \), let \( S \) be a given subset of \( \mathbb{R} \). The general optimization problem is to find that value of \( x \in S \subseteq \mathbb{R} \) at which \( F(x) \) obtains an optimal value. The function \( F \) is called objective function. When the objective function is unknown, the grid search can identify a bound of \( S \) in which the minimum of \( F \) is likely to lie in. Let \( L \) be a lattice on \( S \), then \( F \) is estimated at each point on the lattice. The point on \( L \) at which \( F \) obtains its minimum is a good estimate for the minimum on \( S \). The \( L \) can be written as:

\[
L = \{ x| x = x_0 + M * a, \quad a \in \mathbb{Z} \}
\]

where \( x_0 \) is a given fixed point in \( \mathbb{R} \) and \( M \) is a matrix (often identity matrix) with linearly independent columns.

4.4.2 Estimation univariate distributions of forecast errors

In this section, we will estimate the empirical distribution of forecast errors by using the MDE. Inflation forecast errors are made from ARMA model, because (Mitchell, Robertson, and Wright (2014)) find that multivariate models have struggled to outperform univariate models in macroeconomic forecasting and all efficient predictive regressions for inflation are similar to those from ARMA model. Monthly inflation data spans from 2005M1 to 2015M5 and data is collected from the National Bureau of Statistics of China. Whether inflation series should be treated as stationary or non-stationary is controversial, see Charemza, Hristova*, and Burridge (2005). Perron (1989) state that in the presence of a structural break,
standard tests are biased towards the non-rejection of the null hypothesis that there is a unit root. She argues that most macroeconomic series are not characterized by a unit root but consist of structural changes. We have examined the stationarity of the inflation series for China by applying the point optimal test and the Ng and Perron test (see Elliott, Rothenberg, and Stock (1996), Bai and Perron (1998), Ng and Perron (2001), Perron and Rodriguez (2003) and Carrion-i Silvestre, Kim, and Perron (2009)) under the presence and absence of the structural breaks (up to three breaks). Table 4 reports the unit root tests for inflation in China. As we can see from the table, there are 15 out of 24 tests reject the null hypothesis that there is a unit root, therefore, we have decided that inflation series for China should be stationary. The forecast experiment is designed as followings: (1) we use the rolling scheme (see Stock and Watson (1999)) to generate forecasts made by ARMA model. The sample ranges from 2005:M1 to 2014:M3, the ARMA model is estimated using the sub-sample from 2005:M1 to 2008:M10 and predict the inflation rate for 12 steps ahead, that is 12 forecast horizons. Then the entire sample rolls one observation ahead and the model is re-estimated by the sample from 2005:M2 through 2008:M11 and re-forecast the next 12 months etc.. I evaluate corresponding forecasts for 2008:M11 through 2014:M3. (3) lag lengths are obtained by minimising the Akaike information criteria (AIC) in each sub-sample. (4) forecast errors are computed by the differences between the realizations and the forecasts.

Table 4.1: Unit root tests for inflation

<table>
<thead>
<tr>
<th></th>
<th>ERS</th>
<th>MPT</th>
<th>ADF</th>
<th>M Zα</th>
<th>MSB</th>
<th>M Zt</th>
</tr>
</thead>
<tbody>
<tr>
<td>no break</td>
<td>2.10**</td>
<td>2.04**</td>
<td>-2.34</td>
<td>-12.03**</td>
<td>0.20**</td>
<td>-2.45**</td>
</tr>
<tr>
<td>3 breaks</td>
<td>15.93</td>
<td>14.22</td>
<td>-2.39</td>
<td>-13.71**</td>
<td>0.19**</td>
<td>-2.61***</td>
</tr>
<tr>
<td>2 breaks</td>
<td>14.71</td>
<td>13.53</td>
<td>-2.40</td>
<td>-15.79***</td>
<td>0.18**</td>
<td>-2.78***</td>
</tr>
<tr>
<td>1 break</td>
<td>5.70</td>
<td>5.61</td>
<td>-3.04**</td>
<td>-29.40***</td>
<td>0.13***</td>
<td>-3.83***</td>
</tr>
</tbody>
</table>

Notes: (1) ERS denotes the ERS point optimal test; ADF denotes the Augmented Dickey-Fuller test; MPT, M Zα, MSB and M Zt are four test statistic for Ng-Perron test. (2) *, **, *** indicate the test statistics is significant at 10%, 5% and 1% significance level.

With the use of the forecast errors computed in the way described above, we estimate the parameters of the normal, α-stable, TPN and WSN distributions by the use of the minimum distance estimation. Two criterion Hellinger distance and Chi-squared criterion will be used to evaluate these distributions. For comparability, m and k of the WSN distribution are not estimated, m and k are defined \( m = -k = \sigma \) and \( m = -k = 1 \) respectively, the correlation coefficient is set to be a constant \( \rho = 0.75 \). The results of estimation and goodness of fits test are shown in tables 4.2-4.6. These tables show the minimum distance estimation for normal distribution, α-stable distribution, two piece normal distribution and weighted skew normal distribution (\( m = -k = \sigma \) and \( m = -k = 1 \)) respectively. The left
hand side of these tables report the estimation by the use of the Hellinger distance criterion and the right hand side of these tables show the estimation results using the Chi-square criterion. The estimation results include the estimated parameters for each distribution and 12 forecast horizons, the standard deviations of these estimated parameters and the Hellinger and the Chi-square distances.

The results can be summarised as follows:

(1) Firstly, I compare the goodness of fit tests for the two WSN distributions. For both the Hellinger and Chi-squared criterion, the WSN distribution with $m = -k = 1$ performs slightly better than the distribution with $m = -k = \sigma$ in 15 out of 24 cases, especially for longer forecast horizons. Moreover, the parameters of the WSN distribution with $m = -k = 1$ is less volatile than the other one. In this case, the WSN distribution with $m = -k = 1$ will be used to compare with other distributions in this chapter.

(2) Secondly, for both the Hellinger and Chi-squared criterion, compared the WSN distribution with other distributions, the WSN distribution has the best fit. The WSN fits better than normal distribution and $\alpha$ stable distribution in all forecast horizons respectively, it outperforms than the TPN distribution in 16 out of 24 cases.

(3) Thirdly, the results also confirm that the empirical distribution of the forecast errors is non-normal, the fit of $\alpha$ stable distribution is much worse. A possible explanation is that $\alpha$ stable distributions do not have finite moment so that they do not fit for macro data, another possible reason is that there is no hyperinflation during the time period.

(4) I also use the VAR model described in Chapter 2 to generate the forecast errors instead of using ARMA model, the results are similar to the above. (Results are available in Appendix C.)
### Table 4.2: MDE estimates of normal distribution

<table>
<thead>
<tr>
<th>h</th>
<th>(\hat{\mu})</th>
<th>(\hat{\sigma})</th>
<th>std((\hat{\mu}))</th>
<th>std((\hat{\sigma}))</th>
<th>H-dist</th>
<th>(\hat{\mu})</th>
<th>(\hat{\sigma})</th>
<th>std((\hat{\mu}))</th>
<th>std((\hat{\sigma}))</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.021</td>
<td>0.500</td>
<td>0.183</td>
<td>0.000</td>
<td>14.992</td>
<td>-0.021</td>
<td>0.500</td>
<td>0.182</td>
<td>0.000</td>
<td>17.522</td>
</tr>
<tr>
<td>2</td>
<td>-0.054</td>
<td>0.501</td>
<td>0.198</td>
<td>0.002</td>
<td>8.601</td>
<td>-0.057</td>
<td>0.505</td>
<td>0.189</td>
<td>0.000</td>
<td>9.715</td>
</tr>
<tr>
<td>3</td>
<td>-0.105</td>
<td>0.506</td>
<td>0.046</td>
<td>0.019</td>
<td>0.964</td>
<td>-0.093</td>
<td>0.501</td>
<td>0.081</td>
<td>0.004</td>
<td>0.964</td>
</tr>
<tr>
<td>4</td>
<td>-0.173</td>
<td>0.744</td>
<td>0.168</td>
<td>0.432</td>
<td>1.422</td>
<td>-0.166</td>
<td>0.740</td>
<td>0.145</td>
<td>0.443</td>
<td>1.463</td>
</tr>
<tr>
<td>5</td>
<td>-0.317</td>
<td>1.439</td>
<td>0.117</td>
<td>0.567</td>
<td>17.876</td>
<td>-0.265</td>
<td>1.376</td>
<td>0.048</td>
<td>0.367</td>
<td>21.315</td>
</tr>
<tr>
<td>6</td>
<td>-0.385</td>
<td>2.049</td>
<td>0.079</td>
<td>0.092</td>
<td>6.945</td>
<td>-0.387</td>
<td>1.837</td>
<td>0.168</td>
<td>0.577</td>
<td>7.475</td>
</tr>
<tr>
<td>7</td>
<td>-0.417</td>
<td>1.525</td>
<td>0.180</td>
<td>0.838</td>
<td>5.815</td>
<td>-0.433</td>
<td>1.464</td>
<td>0.023</td>
<td>0.645</td>
<td>5.741</td>
</tr>
<tr>
<td>8</td>
<td>-0.490</td>
<td>1.999</td>
<td>0.094</td>
<td>0.065</td>
<td>7.681</td>
<td>-0.462</td>
<td>1.958</td>
<td>0.324</td>
<td>0.196</td>
<td>9.462</td>
</tr>
<tr>
<td>9</td>
<td>-0.403</td>
<td>2.629</td>
<td>0.136</td>
<td>0.479</td>
<td>3.304</td>
<td>-0.398</td>
<td>2.475</td>
<td>0.121</td>
<td>0.964</td>
<td>3.674</td>
</tr>
<tr>
<td>10</td>
<td>-0.557</td>
<td>2.748</td>
<td>0.119</td>
<td>0.101</td>
<td>7.421</td>
<td>-0.552</td>
<td>2.528</td>
<td>0.100</td>
<td>0.797</td>
<td>8.469</td>
</tr>
<tr>
<td>11</td>
<td>-0.679</td>
<td>3.744</td>
<td>0.003</td>
<td>0.557</td>
<td>2.429</td>
<td>-0.630</td>
<td>3.550</td>
<td>0.158</td>
<td>1.171</td>
<td>2.498</td>
</tr>
<tr>
<td>12</td>
<td>-0.711</td>
<td>4.157</td>
<td>0.096</td>
<td>0.750</td>
<td>4.414</td>
<td>-0.655</td>
<td>4.050</td>
<td>0.078</td>
<td>0.411</td>
<td>5.757</td>
</tr>
</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for normal distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion. (2) "std" denotes standard deviations of estimated parameters, "H-distance" denotes the Hellinger criterion and "chi" denotes the Chi-square criterion.
Table 4.3: MDE estimates of $\alpha$-stable distribution

<table>
<thead>
<tr>
<th>h</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>dist</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.888</td>
<td>-0.095</td>
<td>0.036</td>
<td>0.040</td>
<td>51.444</td>
<td>1.623</td>
<td>0.068</td>
<td>0.062</td>
<td>0.087</td>
<td>76.905</td>
</tr>
<tr>
<td>2</td>
<td>1.898</td>
<td>0.315</td>
<td>0.005</td>
<td>0.405</td>
<td>40.933</td>
<td>1.700</td>
<td>-0.020</td>
<td>0.119</td>
<td>0.048</td>
<td>52.456</td>
</tr>
<tr>
<td>3</td>
<td>1.419</td>
<td>0.073</td>
<td>0.003</td>
<td>0.105</td>
<td>26.126</td>
<td>1.896</td>
<td>0.296</td>
<td>0.011</td>
<td>0.084</td>
<td>26.530</td>
</tr>
<tr>
<td>4</td>
<td>1.867</td>
<td>0.631</td>
<td>0.002</td>
<td>0.055</td>
<td>16.180</td>
<td>1.899</td>
<td>0.724</td>
<td>0.088</td>
<td>0.318</td>
<td>16.663</td>
</tr>
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<td>5</td>
<td>1.235</td>
<td>0.043</td>
<td>0.269</td>
<td>0.195</td>
<td>20.384</td>
<td>1.596</td>
<td>-0.037</td>
<td>0.048</td>
<td>0.215</td>
<td>24.933</td>
</tr>
<tr>
<td>6</td>
<td>1.194</td>
<td>0.062</td>
<td>0.197</td>
<td>0.137</td>
<td>6.047</td>
<td>1.156</td>
<td>0.028</td>
<td>0.313</td>
<td>0.239</td>
<td>6.132</td>
</tr>
<tr>
<td>7</td>
<td>1.176</td>
<td>0.252</td>
<td>0.117</td>
<td>7.531</td>
<td>1.234</td>
<td>0.054</td>
<td>0.079</td>
<td>0.161</td>
<td>7.234</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1.169</td>
<td>0.178</td>
<td>0.121</td>
<td>9.068</td>
<td>1.202</td>
<td>0.039</td>
<td>0.174</td>
<td>0.226</td>
<td>9.677</td>
<td></td>
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<tr>
<td>9</td>
<td>1.125</td>
<td>0.308</td>
<td>0.187</td>
<td>1.838</td>
<td>1.126</td>
<td>0.044</td>
<td>0.305</td>
<td>0.194</td>
<td>1.965</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>1.101</td>
<td>0.188</td>
<td>0.206</td>
<td>5.488</td>
<td>1.102</td>
<td>0.036</td>
<td>0.187</td>
<td>0.218</td>
<td>5.572</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1.101</td>
<td>0.190</td>
<td>0.135</td>
<td>1.395</td>
<td>1.103</td>
<td>0.063</td>
<td>0.183</td>
<td>0.135</td>
<td>1.537</td>
<td></td>
</tr>
<tr>
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<td>1.112</td>
<td>0.157</td>
<td>0.202</td>
<td>8.456</td>
<td>1.106</td>
<td>0.055</td>
<td>0.173</td>
<td>0.160</td>
<td>9.157</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for $\alpha$-stable distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion. (2) "std" denotes standard deviations of estimated parameters, "H-distance" denotes the Hellinger criterion and "chi" denotes the Chi-square criterion.
Table 4.4: MDE estimates of TPN distribution

<table>
<thead>
<tr>
<th>h</th>
<th>σ₁</th>
<th>σ₂</th>
<th>µ</th>
<th>std(σ₁)</th>
<th>std(σ₂)</th>
<th>std(µ)</th>
<th>dist</th>
<th>σ₁</th>
<th>σ₂</th>
<th>µ</th>
<th>std(σ₁)</th>
<th>std(σ₂)</th>
<th>std(µ)</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.531</td>
<td>0.199</td>
<td>-1.391</td>
<td>1.544</td>
<td>0.411</td>
<td>0.915</td>
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<td>3.207</td>
<td>0.262</td>
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<td>0.885</td>
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<td>0.261</td>
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<td>1.671</td>
<td>2.394</td>
<td>0.575</td>
<td>1.129</td>
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<td>1.024</td>
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<td>0.962</td>
<td>2.857</td>
<td>2.054</td>
<td>0.089</td>
<td>0.991</td>
<td>0.407</td>
<td>0.761</td>
<td>0.857</td>
<td>2.959</td>
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<td>0.123</td>
<td>1.006</td>
<td>0.358</td>
<td>0.653</td>
<td>0.905</td>
<td>2.450</td>
<td>2.014</td>
<td>0.115</td>
<td>0.998</td>
<td>0.280</td>
<td>0.678</td>
<td>0.878</td>
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<tr>
<td>8</td>
<td>2.220</td>
<td>0.015</td>
<td>1.116</td>
<td>0.079</td>
<td>0.489</td>
<td>0.240</td>
<td>1.312</td>
<td>2.224</td>
<td>0.020</td>
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<td>0.066</td>
<td>0.473</td>
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</tr>
<tr>
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<td>0.013</td>
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<td>0.004</td>
<td>0.039</td>
<td>2.859</td>
<td>2.488</td>
<td>0.052</td>
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<td>3.242</td>
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<td>0.258</td>
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<td>0.560</td>
<td>2.853</td>
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<td>0.597</td>
<td>1.006</td>
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<td>1.110</td>
</tr>
</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for two piece normal distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion. (2) "std" denotes standard deviations of estimated parameters, "H-distance" denotes the Hellinger criterion and "chi" denotes the Chi-square criterion.
Table 4.5: MDE estimates of WSN distribution ($m = -k = \sigma$)

<table>
<thead>
<tr>
<th>h</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\sigma}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>dist</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.293</td>
<td>0.358</td>
<td>1.415</td>
<td>0.111</td>
<td>0.286</td>
<td>-2.672</td>
<td>-2.517</td>
<td>0.289</td>
<td>0.354</td>
</tr>
<tr>
<td>2</td>
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<td>-0.411</td>
<td>0.757</td>
<td>0.197</td>
<td>0.289</td>
<td>0.096</td>
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<td>-0.482</td>
<td>-0.249</td>
<td>0.678</td>
<td>0.499</td>
</tr>
<tr>
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<td>0.767</td>
<td>0.170</td>
<td>0.121</td>
<td>0.127</td>
<td>0.474</td>
<td>-1.636</td>
<td>-1.039</td>
<td>0.768</td>
<td>0.392</td>
</tr>
<tr>
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<td>0.056</td>
<td>0.296</td>
<td>0.078</td>
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<tr>
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<td>0.978</td>
<td>0.109</td>
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<tr>
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<td>1.086</td>
<td>0.028</td>
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<td>0.030</td>
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<td>-2.366</td>
<td>-0.003</td>
<td>1.611</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for weighted skew normal distribution, where $k$ and $m$ are defined in units of standard deviation, $\rho = 0.75$. It compares the results of the Hellinger distance criterion with the Chi-square criterion. (2) “std” denotes standard deviations of estimated parameters, “H-distance” denotes the Hellinger criterion and “chi” denotes the Chi-square criterion.
Table 4.6: MDE estimates of WSN distribution ($m = -k = 1$)

<table>
<thead>
<tr>
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<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\sigma}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>dist</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\sigma}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>chi</th>
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</thead>
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<td>0.012</td>
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<td>0.195</td>
<td>0.047</td>
<td>0.021</td>
<td>2.139</td>
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</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for weighted skew normal distribution, where $k$ and $m$ are defined in absolute units. $\rho = 0.75$. It compares the results of the Hellinger distance criterion with the Chi-square criterion. (2) "std" denotes standard deviations of estimated parameters, "H-distance" denotes the Hellinger criterion and "chi" denotes the Chi-square criterion. (1) Values are test statistics.
4.5 Probability of deflation and copulas

The way of computing the probability of deflation can be used to compute any integrals (probabilities) on the distributions of density forecasts. Techniques are similar to that proposed for the *ex-ante* probability of deflation have been used before for computing the probabilities that inflation will be within target bands, etc.. The novelty here is: (a) the concept of *ex-post* probability of deflation, which is an attempt to weaken the dependence on the *ex-ante* point forecast and essentially provides an answer to a question like: we know that there was no deflation today. But suppose we looked from the perspective of the last month (or, more generally, h-months ago), what would be the probability of deflation today? and (b) application of *n* out of *n* bootstrap, to asess for the numerical inaccuracy of estimation of the integral by simulation. (c) I also account for the effects of model uncertainty on the probability of deflation.

4.5.1 Inference on marginal distribution

We consider the density forecast of inflation for the periods *h* = 1, 2, ..., *H* made at time 0, as described by an *H*-dimensional random variable *X* = [*X*₁, *X*₂, ..., *X*₇] ⊂ *R*₇. Inflation at time *h* is a random variable denoted by *x*₇. If its distribution is known, the unconditional probability of deflation for inflation forecasts at time 0 for *h* is given by:

\[
P(x_h < 0) = \int_{-\infty}^{0} f(x_h) dx_h, \quad h = 1, 2, ..., H
\]

where \( f(x_h) \) is the density function of \( X_h \). Initially, the inference can be made on the marginal distribution, without the need to analyse the entire joint distribution. However, the practical problem arises from the fact that the types and parameters of the density function are unknown. Let us express the density forecast in terms of its deviation from the mean, that is as:

\[
Y_h = X_h - \mu_h
\]

where \( \mu_h = E(X_h) \) is the point forecasts of inflation. Such expression allows us to estimate the density function of \( Y_h \) using the historical *h*-step forecast errors, assuming their stationarity and ergodicity in mean. This is a standard approach often used for construction of fan charts. Then the unconditional probability of deflation can be represented as:

\[
P(x_h < 0) = \mu_h + \int_{-\infty}^{-\mu_h} f(y_h) dy_h \quad (4.2)
\]
The uncertainty problems related to estimation of equation 4.2 are:

(1) The point forecasts, that is the estimate \( \hat{\mu}_h \) of \( \mu_h \) is subject to (a) model uncertainty, as the true model is usually not known, and (b), the ontological uncertainty, that is the appearance of non-predictable shocks between the period the forecast was made and the forecast period \( h \).

(2) The density function \( f(y_h) \) is not known and the distribution of uncertainty around the point forecasts is rarely normal, or even symmetric.

In the light of these, the following algorithm is proposed:

(1) We understand that the model uncertainty is due to a lack of knowledge regarding the true forecasting model and the use of estimates in place of true parameters of the forecasting model. In this case, we assess the model uncertainty by deriving a dispersion measure between forecasts from alternative models, etc.. Assume that model uncertainty results in random fluctuation of the estimates \( \hat{\mu}_h \) around the \( \mu_h \), e.g., assuming normality, \( \hat{\mu}_h - \mu_h \sim N(0, \kappa) \), where \( \kappa \) is defined as square of a measure of model uncertainty.

(2) With the use of forecast errors from the past, estimate the density function \( f(y_h) \). It might be necessary to choose from some competing distributional forms using the MD criterion. Earlier results show that usually the best-fitted density functions are either WSN or arguably TPN. It is further assumed that such density function might be too complicated the integrate analytically and the integral in equation (4.2) can be computed by stochastic simulation.

(3) Estimate the probability of deflation for the forecast horizon \( h \):

a. **ex-ante**, by computing a single series of forecast probabilities:

\[
\hat{P}(x_h < 0) = \hat{\mu}_h + \int_{-\infty}^{-\hat{\mu}_h} \hat{f}(y_h) dy_h, \tag{4.3}
\]

where \( \hat{f}(y_h) \) is the estimated probability density function of \( y_h \), \( \hat{\mu}_h \) is the **ex-ante** forecast of inflation and the integral is computed by simulation.

b. **ex-post**, by computing multiple series of forecast probabilities by: 1. simulating \( \gamma \sim N(0, \kappa) \)

2. for each \( \gamma \), computing

\[
\hat{P}(x_h < 0) = (\mu_h^* + \gamma) + \int_{-\infty}^{-(\mu_h^* + \gamma)} \hat{f}(y_h) dy_h, \quad h^* = -H + 1, -H + 2, ..., 0
\]

where \( \mu_h^* \) is the historically observed inflation, representing perfect forecast, without model (and parameter) uncertainty.
3. expressing the ex-post probability of deflation conditional on the model uncertainty given by $\kappa$ by mean of the computed probabilities. Error of the estimates is given by the standard deviation of the probabilities.

If the integral (4.3) is computed by simulation, the additional source of errors comes from the numerical approximation of the integral. In order to account for the error this might cause, it is advisable to apply the $n$ out of $n$ bootstrap here (see Cheung and Lee (2005)). The $n$ out of $n$ bootstrap draws a large number of bootstrap samples from a random sample of size $n$, then estimate the variance of the sample by computing the sample variance of the bootstrap sample quantiles (Hall and Martin (1988)). According to Hall and Martin (1988), suppose that $X_1, X_2, ..., X_n$ denote a random sample from distribution $F$. Given $0 < p < 1$, let $\epsilon_p = F^{-1}(p)$ denote the $p$’th quantile of $F$. Let $X_j$ denotes the largest value in the sample and $X_r$ is $p$’th sample quantile where $r = \lfloor np \rfloor + 1$ and $\lfloor . \rfloor$ denotes the integer part function. Then the bootstrap variance estimate of $\sigma^2 = var(X_r)$ is

$$\hat{\sigma}^2 = n^{-1}p(1-p)f(\epsilon_p)^{-2} + O(n^{-1})$$

(4.4)

The results of $n$ out of $n$ bootstrap for the probability of deflation are close to zero.

4.5.2 Inference on joint distribution

It is found that forecast errors for different forecast horizons are rarely independent but dependent (see Diebold and Mariano (1995), empirical evidence are provided by Harvey, Leybourne, and Newbold (1998) and Harvey and Newbold (2003)). For example, suppose the forecast model is AR(1), that is $y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$, update 1 period we have:

$$y_{t+1} = a_0 + a_1 y_t + \varepsilon_{t+1}$$

the 1-step ahead forecast is

$$Ey_{t+1} = a_0 + a_1 y_t$$

the 2-step ahead forecast is

$$Ey_{t+2} = a_0 + a_1 Ey_{t+1} = a_0 + a_1(a_0 + a_1 y_t)$$

then the forecast error for 1-step ahead and 2-step ahead forecasts are

$$e_t(1) = y_{t+1} - Ey_{t+1} = \varepsilon_{t+1}$$

$$e_t(2) = y_{t+2} - Ey_{t+2} = \varepsilon_{t+2} + a_1 \varepsilon_{t+1}$$
it should be very clear that the two forecast errors are correlated. Thus the distributions of forecast errors or the probabilistic forecasts for each forecast horizon should be dependent and such dependency could be used to investigate the probabilities of future rise or fall of inflation. On the other hand, since the distributions of forecast errors are varied across the forecast horizons, the inflation "fan chart" should be constructed by using a joint distribution in which marginal distributions describe forecasts for each forecast horizon. However, one technical problem is how to define and compute the joint distribution when the marginal for each forecast horizon distributions may characterized by different family of univariate distributions, such as TPN and WSN. Additionally, even if the distributions belong to the same family, there might be different forms of dependence. In order to resolve this problem, we introduce copula function (see Genest and Favre (2007)).

As in the previous section, consider \( Y_h = X_h - \mu_h \) rather than \( X_h \). Inference on the joint distribution of \( Y = [Y_1, Y_2, ..., Y_H] \subset R^H \) is necessary in order to compute the expected length of deflation and related measures. Let \( f(y_1, y_2, ..., y_H) \) be a joint density function of \( Y \). Bearing in mind that the marginal distributions are not normal and the \( H \) dimensional distribution is usually not feasible. However, from the Sklar’s Theorem Sklar (1959), a joint cumulative distribution function (c.d.f.) of \( d \) continuous random variables can be written as

\[
F(y_1, y_2, ..., y_H) = C(F(y_1), F(y_2), ..., F(y_H))
\]

where \( C(.) \) is the \( H \)-dimensional copula function, \( F(y_1, y_2, ..., y_H) \) is the joint cumulative distribution function (CDF) and \( F(y_h), h = 1, 2, 3, ..., H \) are the marginal CDF. By applying the probability integral transform, the random vector

\[
(u_1, u_2, ..., u_H) = (F(y_1), F(y_2), ..., F(y_H))
\]

has uniformly distributed marginals, where \( u_h, h = 1, 2, ..., H \) is the probability integral transformation of \( F(y_h) \). Taking the the inverse probability integral transformation we have:

\[
(y_1, y_2, ..., y_H) = (F^{-1}(u_1), F^{-1}(u_2), ..., F^{-1}(u_H))
\]

where \( F^{-1}(u_h), h = 1, 2, 3, ..., H \) is the inverse probability integral transformation of \( F(y_h) \). Then the copula can be written as

\[
F(y_1, y_2, ..., y_H) = C(F^{-1}(u_1), F^{-1}(u_2), ..., F^{-1}(u_H))
\]

If the copula function and the marginal distributions are known, it is possible to
simulate the joint density function as:

\[ f(x_1, x_2, \ldots, x_H) = C(F^{-1}(u_1), F^{-1}(u_2), \ldots, F^{-1}(u_H)) \times \prod_{h=1}^{H} f(y_h + \mu_h) \]

Provided that the analytical form and the parameters of \( f(y_h) \) are known, additional problems are in (a) defining and estimation the copula function and (b) computing \( F^{-1}(u_H) \). Problem (a) can be simplified by narrowing the class of copulas, in our case we restrict ourselves to Student-\( t \) copulas. The inverse probability integral transformations \( F^{-1}(u_H) \) are computed by the adaptive search algorithm (similar to the grid search in section 4.4.1).

The way of simulating multidimensional Student-\( t \) copula is given by Demarta and McNeil (2005), a \( H \)-dimensional vector \( Y=(y_1, y_2 \ldots y_H)' \) has a multivariate \( t \) distribution with \( \nu \) degrees of freedom, mean vector \( \mu \) and positive-definite dispersion matrix \( \Sigma \) is denoted as \( Y \sim t_{\nu}^H(\mu, \Sigma) \), its density is

\[
f(y) = \frac{\Gamma((\nu+H)/2)}{\Gamma(\nu/2)\sqrt{\pi\nu)^H|\Sigma|}} \left(1 + \frac{(y - \mu)' \Sigma^{-1}(y - \mu)}{\nu}\right)^{-\nu/2}
\]

where \( \text{cov}(X) = \frac{\nu}{\nu-2} \Sigma \) if \( \nu > 2 \). Since a copula is unchanged when the marginal distributions are standardized, the copula of a \( t_{\nu}(\mu, \Sigma) \) is identical to a \( t_{\nu}(0, P) \), where \( P \) is the correlation matrix related to the dispersion matrix \( \Sigma \). The copula can be written as

\[
C_{\nu,P}(F(y_1), \ldots, F(y_H)) = \int_{-\infty}^{t_{\nu}^{-1}(u_1)} \cdots \int_{-\infty}^{t_{\nu}^{-1}(u_H)} \frac{\Gamma((\nu+H)/2)}{\Gamma(\nu/2)\sqrt{\pi\nu)^H/|P|}} \left(1 + \frac{x' P^{-1}x}{\nu}\right)^{-\nu/2} dx
\]

where \( t_{\nu}^{-1} \) denotes the quantile function of a standard univariate \( t_{\nu} \) distribution.

Since policy makers are concerned with whether the current deflation will continue, and if this is the case, the duration of deflation, we examine the expected length of deflation for the dependent forecasts. In order to compute the expected length of deflation within the period \( h = 1, 2, \ldots, H \), it is convenient to consider the following discrete univariate random variable:

\[
Z_d = I_d + \sum_{i=d}^{H-1} \Pi_{j=d}^{d+i} I_j, \quad d = h, h + 1, \ldots, H
\]

where \( I_h = I_{X_h<0} f(x_1, x_2, \ldots, x_H) \), where \( I_{X_h<0} f(x_1, x_2, \ldots, x_H) \) is the indicator function equal to 1 if \( X_h < 0 \) and 0 otherwise. Expected value of \( Z_d = 0, 1, \ldots, H - d + 1 \) is the expected length of deflation observed in time \( d \). For \( d > 1 \) it is possible to compute the expected length of deflation starting at time \( h \), that is, provided that
at time $d-1$ there was no deflation, by computing the expected value of

$$
\tilde{Z}_d = (1 - I_{d-1}) + \sum_{i=d}^{H-1} \prod_{j=d}^{i} I_j
$$

The computation algorithm is the following:

1. Identify the type and estimate the parameters of the marginal density functions for $X_1, X_2, ..., X_H$. As mentioned above, these distributions are usually WSN or TPN.

2. Simulate $N \times H$ realisations of $H$-dimensional Student-$t$ copula, where $N$ is the number of replications. It requires assumptions regarding the number of degrees of freedom and the initial correlation matrix.

3. Applying the inverse probability integral transform, simulate from 4.5.2, $N \times H$ matrix of realisations of joint density function $f(x_1, x_2, ..., x_H)$. In the ex-post estimation case, it is possible to account for the uncertainty regarding $\mu_h$ as in the previous subsection.

4. Using this $N \times H$ matrix, compute the corresponding matrix of realisations of $I_h$, $h = 1, 2, ..., H$, taking value 1 if the corresponding simulated realisation is negative and zero otherwise.

5. Compute $N \times 1$ vectors of realisations of $Z_h$ and $\tilde{Z}_h$ and use their arithmetic means and standard deviations as the estimates of the expected length of deflation and corresponding standard deviations of deflation.

Tables 4.7-4.10 report the empirical results of the probabilities of deflation, the expected length of deflation and the standard deviations of the expected length. The results from Table 4.7 and 4.8 are derived from the WSN distribution in which point forecasts are realizations of inflation rate and econometric forecasts from ARMA model respectively. The results from Table 4.9 and 4.10 are derived from the TPN distribution. There are several definitions have to be interpreted preliminarily: (1) independent forecast is the forecast derived from marginal distribution described in section 5.1; (2) dependent forecast is the probabilistic forecast derived from the joint distribution by the use of Student-$t$ copula discussed in section 5.2; (3) unconditional probability of deflation derived from equation (4.2) for (a) the independent forecast and (b) dependent forecast respectively; (4) conditional probability of deflation at horizon $h$ is defined as the probability of deflation at horizon $h$ ($h > 1$) when there was no deflation at $h - 1$ period, in other words, it computes the probability that deflation starts from horizon $h$; (5) expected length of deflation if started or continued at horizon $h$ is derived from equation (4.5); (6) expected length of deflation if started at horizon $h$ is computed by the equation (4.6).
### Table 4.7: Probabilities of deflation (WSN, realizations)

<table>
<thead>
<tr>
<th>h</th>
<th>prob1</th>
<th>prob2</th>
<th>prob3</th>
<th>length1</th>
<th>length2</th>
<th>std1</th>
<th>std2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.004</td>
<td>0.004</td>
<td>1.000</td>
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</tr>
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<td>0.000</td>
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<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.017</td>
<td>0.004</td>
<td>0.004</td>
<td>2.250</td>
<td>2.250</td>
<td>2.165</td>
<td>2.165</td>
</tr>
<tr>
<td>4</td>
<td>0.011</td>
<td>0.013</td>
<td>0.012</td>
<td>2.615</td>
<td>2.417</td>
<td>2.021</td>
<td>1.977</td>
</tr>
<tr>
<td>5</td>
<td>0.163</td>
<td>0.163</td>
<td>0.156</td>
<td>1.871</td>
<td>1.821</td>
<td>1.219</td>
<td>1.157</td>
</tr>
<tr>
<td>6</td>
<td>0.168</td>
<td>0.325</td>
<td>0.246</td>
<td>1.563</td>
<td>1.488</td>
<td>0.998</td>
<td>0.918</td>
</tr>
<tr>
<td>7</td>
<td>0.158</td>
<td>0.252</td>
<td>0.143</td>
<td>1.659</td>
<td>1.643</td>
<td>1.139</td>
<td>1.203</td>
</tr>
<tr>
<td>8</td>
<td>0.186</td>
<td>0.251</td>
<td>0.159</td>
<td>1.717</td>
<td>1.667</td>
<td>1.172</td>
<td>1.142</td>
</tr>
<tr>
<td>9</td>
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<td>0.238</td>
<td>0.151</td>
<td>1.807</td>
<td>1.656</td>
<td>0.977</td>
<td>0.877</td>
</tr>
<tr>
<td>10</td>
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<td>0.284</td>
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<td>1.486</td>
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<td>0.705</td>
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<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: (1) prob1 denotes the unconditional probabilities of deflation for independent forecast; prob2 denotes the unconditional probabilities of deflation for dependent forecasts; prob3 denotes the conditional probabilities of deflation for dependent forecasts, length1 denotes the expected length of deflation if it starts or continue at current forecast horizon; length2 denotes the expected length of deflation if it only starts at current horizon. std1 and std2 are standard deviations of the two expected lengths, respectively. (2) The results are derived from joint distribution in which marginal distributions are WSN distributions and point forecasts are realizations.

### Table 4.8: Probabilities of deflation (WSN econometric)

<table>
<thead>
<tr>
<th>h</th>
<th>prob1</th>
<th>prob2</th>
<th>prob3</th>
<th>length1</th>
<th>length2</th>
<th>std1</th>
<th>std2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>0.001</td>
<td>0.001</td>
<td>1.000</td>
<td>1.000</td>
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<tr>
<td>2</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>3</td>
<td>0.033</td>
<td>0.012</td>
<td>0.012</td>
<td>1.583</td>
<td>1.583</td>
<td>1.441</td>
<td>1.441</td>
</tr>
<tr>
<td>4</td>
<td>0.033</td>
<td>0.020</td>
<td>0.018</td>
<td>2.400</td>
<td>2.278</td>
<td>1.428</td>
<td>1.366</td>
</tr>
<tr>
<td>5</td>
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<td>0.191</td>
<td>0.178</td>
<td>1.822</td>
<td>1.798</td>
<td>1.102</td>
<td>1.088</td>
</tr>
<tr>
<td>6</td>
<td>0.174</td>
<td>0.344</td>
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<td>1.535</td>
<td>1.461</td>
<td>0.810</td>
<td>0.724</td>
</tr>
<tr>
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<td>0.167</td>
<td>1.387</td>
<td>1.365</td>
<td>0.740</td>
<td>0.769</td>
</tr>
<tr>
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<td>0.109</td>
<td>1.457</td>
<td>1.459</td>
<td>0.808</td>
<td>0.873</td>
</tr>
<tr>
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<td>1.250</td>
<td>0.679</td>
<td>0.617</td>
</tr>
<tr>
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<td>0.140</td>
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<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: (1) prob1 denotes the unconditional probabilities of deflation for independent forecast; prob2 denotes the unconditional probabilities of deflation for dependent forecasts; prob3 denotes the conditional probabilities of deflation for dependent forecasts, length1 denotes the expected length of deflation if it starts or continue at current forecast horizon; length2 denotes the expected length of deflation if it only starts at current horizon. std1 and std2 are standard deviations of the two expected lengths, respectively. (2) The results are derived from joint distribution in which marginal distributions are WSN distributions and point forecasts are derived from ARMA model.
Table 4.9: Probabilities of deflation (TPN, realizations)

<table>
<thead>
<tr>
<th>h</th>
<th>prob1</th>
<th>prob2</th>
<th>prob3</th>
<th>length1</th>
<th>length2</th>
<th>std1</th>
<th>std2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.849</td>
<td>0.813</td>
<td>0.813</td>
<td>1.946</td>
<td>1.946</td>
<td>0.226</td>
<td>0.226</td>
</tr>
<tr>
<td>2</td>
<td>0.822</td>
<td>0.825</td>
<td>0.056</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.000</td>
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</tr>
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<td>1.955</td>
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</tr>
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</tr>
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</tr>
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<td>1.782</td>
<td>1.043</td>
<td>0.920</td>
</tr>
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<td>0.000</td>
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</tbody>
</table>

Notes: (1) prob1 denotes the unconditional probabilities of deflation for independent forecast; prob2 denotes the unconditional probabilities of deflation for dependent forecasts; prob3 denotes the conditional probabilities of deflation for dependent forecasts, length1 denotes the expected length of deflation if it starts or continue at current forecast horizon; length2 denotes the expected length of deflation if it only starts at current horizon. std1 and std2 are standard deviations of the two expected lengths, respectively. (2) The results are derived from joint distribution in which marginal distributions are TPN distributions and point forecasts are realizations.

Table 4.10: Probabilities of deflation (TPN econometrics)

<table>
<thead>
<tr>
<th>h</th>
<th>prob1</th>
<th>prob2</th>
<th>prob3</th>
<th>length1</th>
<th>length2</th>
<th>std1</th>
<th>std2</th>
</tr>
</thead>
<tbody>
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</tr>
<tr>
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<td>0.825</td>
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<tr>
<td>9</td>
<td>0.195</td>
<td>0.182</td>
<td>0.121</td>
<td>1.401</td>
<td>1.289</td>
<td>0.769</td>
<td>0.609</td>
</tr>
<tr>
<td>10</td>
<td>0.179</td>
<td>0.172</td>
<td>0.123</td>
<td>1.331</td>
<td>1.268</td>
<td>0.629</td>
<td>0.557</td>
</tr>
<tr>
<td>11</td>
<td>0.189</td>
<td>0.194</td>
<td>0.152</td>
<td>1.258</td>
<td>1.230</td>
<td>0.437</td>
<td>0.421</td>
</tr>
<tr>
<td>12</td>
<td>0.201</td>
<td>0.191</td>
<td>0.141</td>
<td>1.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Notes: (1) prob1 denotes the unconditional probabilities of deflation for independent forecast; prob2 denotes the unconditional probabilities of deflation for dependent forecasts; prob3 denotes the conditional probabilities of deflation for dependent forecasts, length1 denotes the expected length of deflation if it starts or continue at current forecast horizon; length2 denotes the expected length of deflation if it only starts at current horizon. std1 and std2 are standard deviations of the two expected lengths, respectively. (2) The results are derived from joint distribution in which marginal distributions are TPN distributions and point forecasts are derived from ARMA model.
(1) As we can see from Table 4.7, when point forecasts are assumed to be perfect forecasts (forecasts are set to be realizations of inflation rate), the unconditional probabilities of deflation for independent forecast have an upward trend from the horizon 2 and reach the peak 0.289 at the horizon 10, then the probabilities start to decline for the next two periods. The unconditional probabilities of deflation for dependent forecast have similar trend with independent forecasts. However, when the forecast horizons are less than 3, the probabilities for the independent forecast are greater than the dependent forecast; when the forecast horizons are longer, the unconditional probabilities for the independent forecast are less than the dependent forecast, especially at horizon 6, the probability for the dependent forecast is twice as large as the probability for the independent forecast. For both expected length of deflation if it started or continued at the current forecast horizon and it only started at the current horizon, they have downward trends. Standard deviations for the expected durations of deflation decline with the increase of the forecast horizon.
(2) As can be seen from Table 4.8, in which point forecasts are made by ARMA model. The unconditional probabilities of deflation for the independent and dependent forecasts are low when the forecast horizons are short. The probabilities rise sharply after the horizon 4, then the unconditional probabilities for the dependent forecast fluctuate around 0.16 while the probabilities for the independent forecast continue to increase and reach the peak 0.344 at horizon 5.

Figures 4.2-4.3 plot the unconditional and conditional probabilities of deflation for the independent and dependent forecasts for WSN distribution. As we can see from the upper graph, when the point forecasts are assumed as perfect forecasts, the unconditional probabilities for independent forecast are greater than the dependent forecast when the forecast horizons are short ($h = 1$), however, when the forecast horizons are long ($h > 6$), the unconditional probabilities for independent forecast are less than the dependent forecast. Similar results are obtained from the case that the point forecasts are econometric forecasts. However, when the point forecasts are econometric forecasts, the differences between these probabilities diminish with the increase of the forecast horizon. The above results indicate that the independent forecast may overestimate the probabilities of deflation when the forecast horizons are short but underestimate the probabilities of deflation when the forecast horizons are long.

(3) Comparing the results from Table 4.7 with the results from Table 4.8, when the forecast horizons are long ($h \geq 8$), the unconditional and conditional probabilities of deflation for independent and dependent forecasts with the case that point forecasts are econometric forecasts are less than the case that point forecasts are realizations. This is probably due to the greater point forecasts for the econometric forecasts compared with the realizations. On the other hand, the expected length of deflation and the standard deviations of the expected length of deflation in Table 4.8 are relatively smaller than the results in Table 4.7.

(4) The results from Tables 4.9 and 4.10 are derived from TPN distributions. As we can see from the tables, for both cases that point forecasts are realizations and econometrics forecasts, the unconditional probabilities of deflation for independent and dependent forecasts are relatively close to each other except for the forecast horizon 4 where the probabilities for the dependent forecasts are much greater than the independent forecasts. That is to say, in contract to the case of the WSN distribution, there is no obvious difference between the independent and the dependent forecasts for the case of the TPN distribution.

(5) Comparing the results from the WSN distribution with the TPN distribution. Since the WSN distribution is the best fit for the empirical distribution of the forecast errors, the TPN distribution may overestimate the unconditional and conditional probabilities of deflation than the WSN distribution, especially when
for the independent forecasts and the forecast horizons are less than 2. The unconditional probabilities of deflation for the independent forecast are 0.849 and 0.822 for the first 2 horizons in the case of the TNP distribution while these two probabilities are 0.054 and 0 in the case of the WSN distribution. The TPN distribution underestimates the unconditional and conditional probabilities of deflation than the WSN distribution when the forecast horizons are from 5 to 7. Moreover, the probabilities are close to each other for both distributions for the dependent forecasts and the forecast horizons are long. The above results indicate that the use of an inappropriate distribution to forecast inflation may under or overestimate the probabilities of deflation especially when the forecast horizons are short and for the independent forecasts. For the dependent forecast and the forecast horizons are long \((h > 7)\), the probabilities of deflation made by two distributions are similar.

Figures 4.4 and 4.5 plot the changes of unconditional probabilities for the independent forecast and the standard deviations of the probabilities with model uncertainty. We set the variances of these imprecise forecasts \(\kappa = 0.1, 0.5, 1..., 3\) in section 4.1. As can be seen from Figure 4.4, when the point forecasts are realizations, for the same forecast horizons, the probabilities of deflation rises with the increase of the model uncertainties. However, the magnitudes of the increases of the probabilities for the short horizons are greater than the long forecast horizons. The standard deviations of the probabilities also increases with the increase of the model uncertainties and the magnitudes of the increases of the standard deviations are relatively the same for all forecast horizons. Similar results are obtained from Figure 4.5 where point forecasts are econometric forecasts.

### 4.5.3 Robustness check

As the results given above depend on settings regarding the type of marginal distribution, type of copula and the strength of dependence. It is important to find out to what extent changing these assumptions might affect the outcomes. Hence, a simple robustness check has been performed. The computations were repeated for three different sets of marginals: (1) decided by the minimum of the Hellinger distance for each forecast horizon, which is WSN for all horizons but 5, for which it is TPN; (2) using WSN for all forecast horizons and (3) using TPN for all forecast horizons. Next, three different types of copulas were used: the Student-t with the number of degrees of freedom equal to 4 and 10 respectively, and the normal copula. Finally, two different types of the scatter matrix were used: computed from the data on forecast errors for different horizons and, alternatively, with all elements equal to 0.9, irrespectively of data.
Probabilities of deflation with model uncertainty (realizations)

Note: (1) X axis denotes the forecast horizons. Y axis denotes the probabilities of deflation. Z axis denotes different variances of imprecise forecasts, that is $\kappa=0.1,0.5,1,...3$ in section 4.1. (2) Point forecasts are the realizations of inflation rate.

Standard deviations of probabilities with model uncertainty (realizations)

Note: (1) X axis denotes the forecast horizons. Y axis denotes standard deviations of probabilities of deflation. Z axis denotes different variances of imprecise forecasts, that is $\kappa=0.1,0.5,1,...3$ in section 4.1. (2) Point forecasts are the realizations of inflation rate.
Probabilities of deflation with model uncertainty (econometric)

Notes: (1) Point forecasts are the econometric forecasts from ARMA model.

Standard deviations of probabilities with model uncertainty (econometric)

Notes: (1) Point forecasts are the econometric forecasts from ARMA model.

Figure 4.5: Probabilities of deflation, standard deviations of probabilities with model uncertainty
<table>
<thead>
<tr>
<th>Model No.</th>
<th>Marginal type</th>
<th>Corr. matrix</th>
<th>Copula type</th>
<th>Std ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mixed TPN/WSN</td>
<td>from data</td>
<td>Student-t, 4 dofs</td>
<td>0.9731</td>
</tr>
<tr>
<td>2</td>
<td>All WSN</td>
<td>from data</td>
<td>Student-t, 4 dofs</td>
<td>0.9734</td>
</tr>
<tr>
<td>3</td>
<td>Mixed TPN/WSN</td>
<td>from data</td>
<td>Student-t, 10 dofs</td>
<td>0.9746</td>
</tr>
<tr>
<td>4</td>
<td>All WSN</td>
<td>from data</td>
<td>Student-t, 10 dofs</td>
<td>0.9748</td>
</tr>
<tr>
<td>5</td>
<td>All WSN</td>
<td>from data</td>
<td>normal</td>
<td>0.9775</td>
</tr>
<tr>
<td>6</td>
<td>Mixed TPN/WSN</td>
<td>from data</td>
<td>normal</td>
<td>0.9785</td>
</tr>
<tr>
<td>7</td>
<td>All TPN</td>
<td>pre-assigned</td>
<td>Student-t, 4 dofs</td>
<td>0.9824</td>
</tr>
<tr>
<td>8</td>
<td>All TPN</td>
<td>pre-assigned</td>
<td>Student-t, 10 dofs</td>
<td>0.9859</td>
</tr>
<tr>
<td>9</td>
<td>All TPN</td>
<td>pre-assigned</td>
<td>normal</td>
<td>0.9888</td>
</tr>
<tr>
<td>10</td>
<td>Mixed TPN/WSN</td>
<td>pre-assigned</td>
<td>Student-t, 4 dofs</td>
<td>0.9943</td>
</tr>
<tr>
<td>11</td>
<td>Mixed TPN/WSN</td>
<td>pre-assigned</td>
<td>Student-t, 10 dofs</td>
<td>0.9952</td>
</tr>
<tr>
<td>12</td>
<td>All WSN</td>
<td>pre-assigned</td>
<td>Student-t, 4 dofs</td>
<td>0.9959</td>
</tr>
<tr>
<td>13</td>
<td>Mixed TPN/WSN</td>
<td>pre-assigned</td>
<td>normal</td>
<td>0.9960</td>
</tr>
<tr>
<td>14</td>
<td>All WSN</td>
<td>pre-assigned</td>
<td>Student-t, 10 dofs</td>
<td>0.9970</td>
</tr>
<tr>
<td>15</td>
<td>All WSN</td>
<td>pre-assigned</td>
<td>Student-t, 4 dofs</td>
<td>0.9974</td>
</tr>
<tr>
<td>16</td>
<td>All TPN</td>
<td>from data</td>
<td>Student-t, 4 dofs</td>
<td>1.0759</td>
</tr>
<tr>
<td>17</td>
<td>All TPN</td>
<td>from data</td>
<td>Student-t, 10 dofs</td>
<td>1.0873</td>
</tr>
<tr>
<td>18</td>
<td>All TPN</td>
<td>from data</td>
<td>normal</td>
<td>1.0888</td>
</tr>
</tbody>
</table>

Notes: (1) corr. matrix denotes the scatter matrix used to compute the correlations between the forecast errors for different forecast horizons.

Results in Table 4.11 indicates that there was no model specification which could affect the robustness in a particularly strong way. The model 1, for which detailed results are presented in Section 4.5, affects the robustness in the strongest way. Its removal from the set of all models decreases the dispersion of the expected length of deflation more than the removal of any other model. However, for all models, the change of the dispersion is not substantial and it does not exceed 3%. We can, therefore, conclude that the method proposed here is reasonably robust to changes in the specification of the marginal distributions and the copulas.

### 4.6 Conclusion

In this chapter, we manage to compute the probabilities of deflation in China by constructing density forecasts. We fit several distributions to the empirical distribution of forecast errors, including normal distribution, α stable distribution, two piece normal distribution and weighted skew normal distribution. The results suggest that the forecast errors are not normally distributed and the weighted skew normal distribution is the best fit. Since the density forecasts for different forecast horizons should be dependent rather than independent, we generate
a joint distributions for 12 forecast horizons by the use of the Student-\(t\) copula. We compare the probabilities of deflation between independent forecasts and dependent forecasts from WSN and TPN distributions and we also compute the expected lengths of deflation in China. The results indicate that the independent forecast may overestimate the probabilities of deflation when the forecast horizons are short but underestimate the probabilities of deflation when the forecast horizons are long. The use of an inappropriate distribution for density forecasts may under or overestimate the probabilities of deflation and the expected length of deflation, especially when the forecast horizons are short and the density forecasts are generated independently.

We also consider the impact of model uncertainty to the density forecast in this chapter, the probabilities of deflation increase with the increase of the uncertainties. However, the magnitudes of the increases of the probabilities are varied from different forecast horizons. Probabilities are more sensitive to the changes of the uncertainty for the short forecast horizons than the long forecast horizons. The standard deviations of the probabilities also increase with the increase of the model uncertainties and the magnitudes of the increases of the standard deviations are relatively the same for all forecast horizons.

The method introduced in this chapter can be applied to estimate the \textit{ex-ante} probability of deflation, which can be used by monetary authorities in order to minimize chances of deflation by taking it into account in their decision function. Information about the risk of deflation might prompt authorities to adjust their monetary policies, for example to loose monetary policy.
Appendix

Appendix A

Figures 4.6-4.17 report the size and power of the forecast encompassing test. Figure 5.7-5.12 report the results of forecast precision. A list of notations:

$N$: the forecast errors are generated from a normal distribution. The estimation method for $\lambda$ is constrained OLS.

$UN$: the forecast errors are generated from a normal distribution. The estimation method is OLS.

$TS5$: the forecast errors are generated from a TS distribution ($\alpha = 1.1, \beta = 0.5, \theta = 0.1$). The estimation method is constrained OLS.

$TS6$: the forecast errors are generated from a TS distribution ($\alpha = 1.1, \beta = 0.5, \theta = 0.5$). The estimation method is constrained OLS.

$TS5'$: the forecast errors are generated from a TS distribution ($\alpha = 1.1, \beta = 0.5, \theta = 0.1$). The estimation method is OLS.

$TS6'$: the forecast errors are generated from a TS distribution ($\alpha = 1.1, \beta = 0.5, \theta = 0.5$). The estimation method is OLS.

$TS7$: the forecast errors are generated from a TS distribution ($\alpha = 1.5, \beta = 0.5, \theta = 0.1$). The estimation method is constrained OLS.

$TS8$: the forecast errors are generated from a TS distribution ($\alpha = 1.5, \beta = 0.5, \theta = 0.5$). The estimation method is constrained OLS.

$TS7'$: the forecast errors are generated from a TS distribution ($\alpha = 1.5, \beta = 0.5, \theta = 0.1$). The estimation method is OLS.

$TS8'$: the forecast errors are generated from a TS distribution ($\alpha = 1.5, \beta = 0.5, \theta = 0.5$). The estimation method is OLS.
Figure 4.6: Size and power test (asymmetric TS), Variance = 0.1, \( \rho = 0.1 \)

Figure 4.7: Size and power test (asymmetric TS), Variance = 0.1, \( \rho = 0.5 \)
Figure 4.8: Size and power test (asymmetric TS), Variance = 1, $\rho = 0.1$.

Figure 4.9: Size and power test (asymmetric TS), Variance = 0.5.
Figure 4.10: Size and power test (asymmetric TS), Variance=10, $\rho = 0.1$

Figure 4.11: Size and power test (asymmetric TS), Variance=10, $\rho = 0.5$
Figure 4.12: Forecast precision (asymmetric TS), Variance=0.1, $\rho = 0.1$

Figure 4.13: Forecast precision (asymmetric TS), Variance=0.1, $\rho = 0.5$
Figure 4.14: Forecast precision (asymmetric TS), Variance=1, $\rho = 0.1$

Figure 4.15: Forecast precision (asymmetric TS), Variance=1, $\rho = 0.5$
Figure 4.16: Forecast precision (asymmetric TS), Variance = 10, $\rho = 0.1$

Figure 4.17: Forecast precision (asymmetric TS), Variance = 10, $\rho = 0.5$
Appendix B

This Appendix B provides empirical results of the correlation coefficients between the survey-based forecast uncertainty and the initial EPU index, the model-based uncertainty and the initial EPU index.

Table 4.12: correlation coefficients (survey-based uncertainty and EPU)

<table>
<thead>
<tr>
<th></th>
<th>RMSE_{LPS}</th>
<th>RMSE_{AF}</th>
<th>RMSE_{RT}</th>
</tr>
</thead>
<tbody>
<tr>
<td>log of EPU</td>
<td>-0.132</td>
<td>-0.268*</td>
<td>-0.147</td>
</tr>
</tbody>
</table>

Notes: (1) the table measures the correlation coefficient between EPU index for China and the survey-based inflation forecast uncertainty. (2) * indicate the P-value is significant at 10% level.

Table 4.13: correlation coefficients (model-based uncertainty and EPU)

<table>
<thead>
<tr>
<th></th>
<th>1-step ahead</th>
<th>3-step ahead</th>
<th>6-step ahead</th>
</tr>
</thead>
<tbody>
<tr>
<td>log of EPU</td>
<td>0.191</td>
<td>0.588***</td>
<td>0.601***</td>
</tr>
</tbody>
</table>

Notes: (1) the table measures the correlation coefficient between EPU index for China and the ex-post inflation forecast uncertainty based on RMSE from econometric models. (2) *** indicate the P-value is significant at 1% level.
Appendix C

Tables 4.14-4.17 report the result of MDE for the forecast errors from VAR model.
Table 4.14: MDE estimates of normal distribution (econometric forecasts)

<table>
<thead>
<tr>
<th>h</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>std($\hat{\mu}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>H-dist</th>
<th>$\hat{\mu}$</th>
<th>$\hat{\sigma}$</th>
<th>std($\hat{\mu}$)</th>
<th>std($\hat{\sigma}$)</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.064</td>
<td>0.500</td>
<td>0.170</td>
<td>0.001</td>
<td>3.341</td>
<td>-0.040</td>
<td>0.500</td>
<td>0.239</td>
<td>0.000</td>
<td>3.298</td>
</tr>
<tr>
<td>2</td>
<td>0.029</td>
<td>0.515</td>
<td>0.206</td>
<td>0.046</td>
<td>4.029</td>
<td>0.044</td>
<td>0.531</td>
<td>0.228</td>
<td>0.094</td>
<td>3.738</td>
</tr>
<tr>
<td>3</td>
<td>-0.033</td>
<td>0.752</td>
<td>0.230</td>
<td>0.405</td>
<td>0.999</td>
<td>-0.030</td>
<td>0.735</td>
<td>0.221</td>
<td>0.459</td>
<td>1.063</td>
</tr>
<tr>
<td>4</td>
<td>-0.069</td>
<td>1.126</td>
<td>0.160</td>
<td>0.425</td>
<td>0.654</td>
<td>-0.065</td>
<td>1.104</td>
<td>0.174</td>
<td>0.493</td>
<td>0.686</td>
</tr>
<tr>
<td>5</td>
<td>-0.038</td>
<td>1.313</td>
<td>0.247</td>
<td>0.168</td>
<td>0.960</td>
<td>-0.026</td>
<td>1.337</td>
<td>0.210</td>
<td>0.245</td>
<td>1.002</td>
</tr>
<tr>
<td>6</td>
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<td>1.897</td>
<td>0.179</td>
<td>0.390</td>
<td>1.987</td>
<td>-0.213</td>
<td>1.868</td>
<td>0.213</td>
<td>0.480</td>
<td>1.970</td>
</tr>
<tr>
<td>7</td>
<td>-0.245</td>
<td>2.117</td>
<td>0.110</td>
<td>0.305</td>
<td>0.221</td>
<td>-0.242</td>
<td>2.083</td>
<td>0.120</td>
<td>0.199</td>
<td>0.223</td>
</tr>
<tr>
<td>8</td>
<td>-0.387</td>
<td>2.415</td>
<td>0.169</td>
<td>1.155</td>
<td>1.243</td>
<td>-0.394</td>
<td>2.305</td>
<td>0.109</td>
<td>0.900</td>
<td>1.219</td>
</tr>
<tr>
<td>9</td>
<td>-0.387</td>
<td>2.241</td>
<td>0.167</td>
<td>0.700</td>
<td>0.693</td>
<td>-0.394</td>
<td>2.198</td>
<td>0.147</td>
<td>0.563</td>
<td>0.666</td>
</tr>
<tr>
<td>10</td>
<td>-0.436</td>
<td>2.715</td>
<td>0.265</td>
<td>0.206</td>
<td>0.902</td>
<td>-0.431</td>
<td>2.669</td>
<td>0.283</td>
<td>0.350</td>
<td>0.962</td>
</tr>
<tr>
<td>11</td>
<td>-0.615</td>
<td>3.900</td>
<td>0.205</td>
<td>0.064</td>
<td>0.156</td>
<td>-0.617</td>
<td>3.862</td>
<td>0.199</td>
<td>0.182</td>
<td>0.162</td>
</tr>
<tr>
<td>12</td>
<td>-0.493</td>
<td>3.703</td>
<td>0.085</td>
<td>0.685</td>
<td>2.249</td>
<td>-0.515</td>
<td>3.597</td>
<td>0.017</td>
<td>1.023</td>
<td>2.306</td>
</tr>
</tbody>
</table>

Notes: (1) This table shows the minimum distance estimation for normal distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion.
(2) "std" denotes standard deviations of estimated parameters, "H-distance" denotes the Hellinger criterion and "chi" denotes the Chi-square criterion.
Table 4.15: MDE estimates of $\alpha$ stable distribution

<table>
<thead>
<tr>
<th>h</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>dist</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>std($\hat{\alpha}$)</th>
<th>std($\hat{\beta}$)</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.447</td>
<td>0.054</td>
<td>0.110</td>
<td>0.162</td>
<td>31.547</td>
<td>1.627</td>
<td>-0.034</td>
<td>0.052</td>
<td>0.210</td>
<td>37.471</td>
</tr>
<tr>
<td>2</td>
<td>1.664</td>
<td>-0.052</td>
<td>0.133</td>
<td>0.169</td>
<td>24.837</td>
<td>1.702</td>
<td>0.065</td>
<td>0.175</td>
<td>0.086</td>
<td>29.435</td>
</tr>
<tr>
<td>3</td>
<td>1.889</td>
<td>0.070</td>
<td>0.033</td>
<td>0.317</td>
<td>10.955</td>
<td>1.609</td>
<td>0.101</td>
<td>0.182</td>
<td>0.022</td>
<td>12.028</td>
</tr>
<tr>
<td>4</td>
<td>1.897</td>
<td>0.421</td>
<td>0.009</td>
<td>0.290</td>
<td>5.649</td>
<td>1.900</td>
<td>0.432</td>
<td>0.001</td>
<td>0.323</td>
<td>5.858</td>
</tr>
<tr>
<td>5</td>
<td>1.897</td>
<td>0.165</td>
<td>0.008</td>
<td>0.603</td>
<td>3.653</td>
<td>1.887</td>
<td>0.067</td>
<td>0.038</td>
<td>0.340</td>
<td>3.808</td>
</tr>
<tr>
<td>6</td>
<td>1.631</td>
<td>0.162</td>
<td>0.250</td>
<td>0.162</td>
<td>3.150</td>
<td>1.456</td>
<td>0.047</td>
<td>0.277</td>
<td>0.182</td>
<td>2.865</td>
</tr>
<tr>
<td>7</td>
<td>1.683</td>
<td>0.783</td>
<td>0.070</td>
<td>0.075</td>
<td>0.431</td>
<td>1.678</td>
<td>0.676</td>
<td>0.086</td>
<td>0.226</td>
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<tr>
<td>8</td>
<td>1.101</td>
<td>0.054</td>
<td>0.190</td>
<td>0.162</td>
<td>0.564</td>
<td>1.100</td>
<td>0.053</td>
<td>0.191</td>
<td>0.164</td>
<td>0.562</td>
</tr>
<tr>
<td>9</td>
<td>1.533</td>
<td>0.727</td>
<td>0.042</td>
<td>0.081</td>
<td>0.973</td>
<td>1.510</td>
<td>0.621</td>
<td>0.073</td>
<td>0.229</td>
<td>0.933</td>
</tr>
<tr>
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<td>1.337</td>
<td>0.330</td>
<td>0.039</td>
<td>0.018</td>
<td>2.848</td>
<td>1.265</td>
<td>0.218</td>
<td>0.080</td>
<td>0.103</td>
<td>2.937</td>
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<tr>
<td>11</td>
<td>1.422</td>
<td>0.869</td>
<td>0.084</td>
<td>0.088</td>
<td>1.408</td>
<td>1.430</td>
<td>0.899</td>
<td>0.062</td>
<td>0.003</td>
<td>1.346</td>
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<tr>
<td>12</td>
<td>1.156</td>
<td>0.053</td>
<td>0.216</td>
<td>0.164</td>
<td>7.193</td>
<td>1.449</td>
<td>0.897</td>
<td>0.008</td>
<td>0.009</td>
<td>8.227</td>
</tr>
</tbody>
</table>

Notes: This table shows the minimum distance estimation for $\alpha$-stable distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion.
Table 4.16: MDE estimates of TPN distribution

| h | \( \hat{\sigma}_1 \) | \( \hat{\sigma}_2 \) | \( \hat{\mu} \) | std(\( \hat{\sigma}_1 \)) | std(\( \hat{\sigma}_2 \)) | std(\( \hat{\mu} \)) | dist | \( \hat{\sigma}_1 \) | \( \hat{\sigma}_2 \) | \( \hat{\mu} \) | std(\( \hat{\sigma}_1 \)) | std(\( \hat{\sigma}_2 \)) | std(\( \hat{\mu} \)) | chi |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.391 | 0.649 | -0.205 | 0.197 | 0.002 | 0.111 | 0.124 | 0.493 | 0.581 | -0.083 | 0.517 | 0.214 | 0.498 | 0.059 |
| 2 | 0.404 | 0.999 | -0.401 | 0.743 | 0.407 | 1.010 | 2.883 | 0.336 | 1.020 | -0.462 | 0.526 | 0.338 | 0.815 | 3.147 |
| 3 | 0.488 | 1.198 | -0.542 | 0.508 | 0.225 | 0.449 | 0.627 | 0.488 | 1.203 | -0.543 | 0.506 | 0.239 | 0.454 | 0.636 |
| 4 | 1.070 | 1.033 | -0.032 | 0.828 | 0.801 | 1.164 | 0.649 | 1.065 | 1.040 | -0.032 | 0.812 | 0.781 | 1.164 | 0.689 |
| 5 | 1.144 | 1.171 | -0.036 | 0.052 | 0.365 | 0.138 | 0.932 | 1.167 | 1.141 | 0.004 | 0.378 | 0.045 | 0.240 | 0.983 |
| 6 | 1.756 | 0.998 | 0.309 | 0.030 | 0.096 | 0.219 | 1.064 | 1.745 | 0.993 | 0.294 | 0.066 | 0.081 | 0.171 | 1.100 |
| 7 | 0.809 | 2.059 | -1.125 | 0.509 | 0.586 | 0.744 | 0.595 | 0.807 | 2.034 | -1.107 | 0.502 | 0.666 | 0.800 | 0.653 |
| 8 | 2.338 | 0.030 | 1.234 | 0.297 | 0.441 | 0.614 | 0.484 | 2.326 | 0.025 | 1.231 | 0.256 | 0.457 | 0.604 | 0.497 |
| 9 | 1.265 | 1.863 | -0.809 | 0.573 | 0.308 | 0.789 | 0.543 | 1.267 | 1.830 | -0.787 | 0.441 | 0.203 | 0.293 | 0.547 |
| 10 | 2.083 | 1.067 | 0.273 | 1.002 | 1.200 | 1.623 | 0.611 | 2.079 | 1.061 | 0.270 | 0.992 | 1.220 | 1.613 | 0.634 |
| 11 | 1.365 | 2.406 | -1.327 | 0.751 | 0.499 | 1.116 | 0.017 | 1.365 | 2.406 | -1.327 | 0.751 | 0.499 | 1.116 | 0.017 |
| 12 | 2.966 | 0.556 | 1.273 | 0.241 | 0.212 | 0.274 | 1.878 | 2.260 | 1.470 | 0.048 | 0.555 | 1.441 | 1.416 | 1.959 |

Notes: This table shows the minimum distance estimation for two piece normal distribution. It compares the results of the Hellinger distance criterion with the Chi-square criterion.
Table 4.17: MDE estimates of WSN distribution (m=k=σ)

<table>
<thead>
<tr>
<th>h</th>
<th>(\hat{\alpha})</th>
<th>(\hat{\beta})</th>
<th>(\hat{\sigma})</th>
<th>std((\hat{\alpha}))</th>
<th>std((\hat{\beta}))</th>
<th>std((\hat{\sigma}))</th>
<th>dist</th>
<th>(\hat{\alpha})</th>
<th>(\hat{\beta})</th>
<th>(\hat{\sigma})</th>
<th>std((\hat{\alpha}))</th>
<th>std((\hat{\beta}))</th>
<th>std((\hat{\sigma}))</th>
<th>chi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.776</td>
<td>-0.640</td>
<td>0.734</td>
<td>0.431</td>
<td>0.000</td>
<td>0.024</td>
<td>0.083</td>
<td>-0.767</td>
<td>-0.657</td>
<td>0.735</td>
<td>0.402</td>
<td>0.054</td>
<td>0.025</td>
<td>0.080</td>
</tr>
<tr>
<td>2</td>
<td>-1.804</td>
<td>-3.942</td>
<td>0.535</td>
<td>0.368</td>
<td>0.183</td>
<td>0.149</td>
<td>1.384</td>
<td>-1.768</td>
<td>-2.844</td>
<td>0.545</td>
<td>0.531</td>
<td>1.404</td>
<td>0.069</td>
<td>1.441</td>
</tr>
<tr>
<td>3</td>
<td>-1.079</td>
<td>-0.949</td>
<td>1.087</td>
<td>0.636</td>
<td>1.048</td>
<td>0.384</td>
<td>0.959</td>
<td>-0.860</td>
<td>-0.690</td>
<td>1.140</td>
<td>0.823</td>
<td>1.359</td>
<td>0.302</td>
<td>0.996</td>
</tr>
<tr>
<td>4</td>
<td>-3.029</td>
<td>-2.354</td>
<td>0.700</td>
<td>0.470</td>
<td>0.362</td>
<td>0.083</td>
<td>0.027</td>
<td>-3.029</td>
<td>-2.354</td>
<td>0.700</td>
<td>0.470</td>
<td>0.362</td>
<td>0.083</td>
<td>0.027</td>
</tr>
<tr>
<td>5</td>
<td>-0.720</td>
<td>-0.640</td>
<td>1.568</td>
<td>0.758</td>
<td>1.013</td>
<td>0.143</td>
<td>0.941</td>
<td>-0.754</td>
<td>-0.717</td>
<td>1.550</td>
<td>0.360</td>
<td>0.243</td>
<td>0.087</td>
<td>0.983</td>
</tr>
<tr>
<td>6</td>
<td>-3.823</td>
<td>-1.201</td>
<td>1.297</td>
<td>0.055</td>
<td>0.255</td>
<td>0.043</td>
<td>0.464</td>
<td>-3.808</td>
<td>-1.147</td>
<td>1.309</td>
<td>0.102</td>
<td>0.086</td>
<td>0.081</td>
<td>0.468</td>
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<tr>
<td>7</td>
<td>-0.744</td>
<td>-0.123</td>
<td>1.763</td>
<td>0.179</td>
<td>0.624</td>
<td>0.245</td>
<td>0.210</td>
<td>-1.012</td>
<td>-0.506</td>
<td>1.875</td>
<td>0.672</td>
<td>0.589</td>
<td>0.107</td>
<td>0.235</td>
</tr>
<tr>
<td>8</td>
<td>-3.052</td>
<td>-0.671</td>
<td>1.444</td>
<td>1.555</td>
<td>0.097</td>
<td>0.248</td>
<td>0.155</td>
<td>-3.425</td>
<td>-0.642</td>
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<td>0.066</td>
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<tr>
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<td>-0.306</td>
<td>1.883</td>
<td>0.593</td>
<td>0.551</td>
<td>0.132</td>
<td>0.599</td>
<td>-1.093</td>
<td>-0.306</td>
<td>1.883</td>
<td>0.593</td>
<td>0.551</td>
<td>0.132</td>
<td>0.581</td>
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<td>-0.537</td>
<td>1.820</td>
<td>0.344</td>
<td>0.181</td>
<td>0.065</td>
<td>0.856</td>
<td>-1.634</td>
<td>-0.546</td>
<td>1.847</td>
<td>0.107</td>
<td>0.209</td>
<td>0.019</td>
<td>0.883</td>
</tr>
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<td>-0.022</td>
<td>1.999</td>
<td>0.032</td>
<td>0.070</td>
<td>0.005</td>
<td>0.432</td>
<td>-1.461</td>
<td>-0.010</td>
<td>2.000</td>
<td>0.067</td>
<td>0.032</td>
<td>0.001</td>
<td>0.447</td>
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<td>-0.524</td>
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<td>0.054</td>
<td>0.368</td>
<td>0.001</td>
<td>2.307</td>
<td>-1.382</td>
<td>-0.019</td>
<td>2.000</td>
<td>0.183</td>
<td>0.447</td>
<td>0.001</td>
<td>2.373</td>
</tr>
</tbody>
</table>

Notes: This table shows the minimum distance estimation for weighted skew normal distribution, where k and m are defined in units of standard deviation. \(\rho = 0.75\).
References


GENEST, C., AND A.-C. FAVRE (2007): “Everything you always wanted to know about copula modeling but were afraid to ask,” Journal of Hydrologic Engineering, 12(4), 347–368.


REIFSCHEIDER, D., AND P. TULIP (2007): “Gaging the uncertainty of the economic outlook from historical forecasting errors,”.


