Essays on Exchange Rates Behaviour

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by

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To my beloved father and mother,
Abbas and Ensieh
Proclaim! In the name of thy Lord, Who created ⋅⋅⋅

 ⋅⋅⋅ Who taught (the use of) the pen ⋅⋅⋅

 ⋅⋅⋅ Taught man that which he knew not ⋅⋅⋅

Quran; Al-Alaq 1,4,5
Essays on Exchange Rates Behaviour
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Abstract

This thesis aims to investigate the exchange rate behaviour and its abnormal movements. By doing so, I introduce a novel instrument which contributes to analyse the exchange rate behaviour. I then apply the new instrument, called wavelet analysis, to investigate the relation of exchange rate to price ratio (PPP proposition) and the relation of exchange rate and interest rates (UIRP proposition). Finally, I concentrate on the specific movements in exchange rate that leads to currency crises.

The second chapter introduces wavelet analysis which has been extensively applied to many situations with favourable results. Many researchers are expanding wavelet application in a variety of fields such as signal processing, physics and astronomy. It has remarkable potential properties that can be applied in the disciplines of economics and financial. The first attempt of this chapter is to introduce wavelet analysis in an intuitive manner. The next step involves reviewing the potential and possible contributions of wavelet analysis in empirical economic and finance literature. I also examine the validity of short-run and long-run purchasing power parity (PPP) hypothesis applying wavelet analysis. The results indicate that the impact of price ratio on exchange rates are positive and close to unity. The findings confirm that the PPP holds for long-run horizon.

The third chapter deals with examining the relationship between spot exchange rates and the interest rate differentials (UIRP) for ten bilateral currencies against the Pound Sterling in short and long time horizons, simultaneously. The distinguishing feature of this study is to apply wavelet transform to decompose the time series into short-run and long-run time horizons. I find out both negative and positive relationships between exchange rates and interest rate differentials. The former is supported by the fixed-price model in short-run and the latter is supported by flexible-price model in long-run.

In the forth chapter, I evaluate the potential leading indicators of a currency crash by applying a quarterly panel of 26 developing and developed countries. I split the definition of currency crashes according to different generations of the currency crises in the literature. Based on two different definitions, I use two binomial logit models, which provide estimations of the probability of a currency crash occurring. The empirical results reveal that domestic credit growth rate, ratio of reserves to GDP, current account, output growth rate, and ratio of national debt to GDP are consistently associated with the early warning theory. According to definitions provided by this chapter, the findings show that current account and GDP growth rate in the developing countries and current account and national debt in the developed countries are significantly related to the crash incident. This chapter also criticizes the previous papers for their construction of exchange rate overvaluation indicator and proposes a recursive Kalman filter to express overvaluation. The findings confirm that overvaluation of exchange rate is not an appropriate predictor of currency crashes unlike previous studies.
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Chapter 1

Introduction

Background and Motivation

Exchange rate is a cornerstone in international economics. Particularly, it plays a significant role in open economy system, deregulation and global integration worldwide. Exchange rate is characterised by its high level of fluctuations and risk exposures. Thus, there has been a long time struggle in modelling exchange rate movements due to its unpredictable volatile behaviour, based on different exchange rate theories such as flexible-price monetary model, fixed-price Mundell-Fleming model, and sticky price Dornbusch overshooting model. Numerous empirical studies have been conducted to find links between exchange rates and economic fundamentals, ever since Meese and Rogoff (1983) challenged the long-held idea that the economic fundamentals determine exchange rate and suggested a random walk model to predict exchange rate movements. They, in fact, believe that the parameters are not constant and vary over time in the prediction of exchange rate models. According to Meese and Rogoff, the random walk model provides the best forecast of exchange rate.

However, researchers tried to compare the structural exchange rate models and random walk models (see Rossi, 2005). They continued to apply various combinations
of economic variables and econometric models to predict exchange rate. Their attempts cannot completely disproved the random walk model, but they found evidence that economic fundamentals are still crucial under some conditions, (Wang, 2008). For instance, longer time frames improve predictability in standard exchange rate models. (see Mark, 1995) Therefore, paying particular attention to different time horizons could explain some anomalies in standard exchange rate models. Chinn and Meredith (2004) and Hacker et al. (2012) are examples of research that distinguish time horizons (short-run and long-run) in the links between economic fundamentals and exchange rates.

In terms of predicting exchange rate movements by economic fundamentals, this thesis particularly focuses on the two important relationships in exchange rate determination theories, purchasing power parity and uncovered interest rate parity.

Motivated by these facts, this thesis in Chapters 2 and 3 attempts to contribute to the empirical literature by introducing a new methodology to probe the relationship between economic fundamentals and exchange rates regarding time horizons, short-run and long-run. Finally, Chapter 4 focuses on the exchange rate episodes, which include high volatile fluctuations, that cause currency crises.

The organisation of this thesis is as follows: Chapter 2 introduces wavelet transform and provides a review of the empirical literature to which wavelet contributes. It is followed by applying wavelet analysis to examine purchasing power parity proposition. Chapter 3 examines the exchange rate behaviour under uncovered interest rate parity doctrine applying wavelet analysis. Chapter 4 considers a particular episode of exchange rate behaviour which is called currency crises. This chapter investigates probability of a currency crisis in advance by identifying leading indicators. Chapter 5 draws conclusions.
Introduction to Wavelet Transform and its Application in Testing Purchasing Power Parity Hypothesis

This chapter aims to shed light on wavelet analysis by demonstrating its properties and contributions to economics and finance literature. (see Ramsey, 2002; Gençay et al., 2002; Schleicher, 2002; Crowley, 2007; Rua, 2012) I then apply wavelet analysis to consider the validity of purchasing power parity in different time horizons to answer the question of whether wavelet analysis is capable of solving this long-held puzzle in exchange rate models.

Wavelet analysis is a refinement of Fourier analysis. Gençay et al. (2002) state that the Fourier basis function is very appealing when working with stationary time series. Nevertheless, in the real world, most economic and financial time series are nonstationary and varying over time. So, Fourier transform cannot efficiently capture the events that happen during a particular time. However, wavelet analysis is able to extract both time and frequency information from nonstationary time series.

Empirically, wavelet analysis has shown strong applicability in economics and finance due to its desirable properties. The basic premise of wavelet analysis offers insight into dynamics of economic and financial time series beyond what the current methodology offers. For instance, wavelets are able to tackle with the concepts of nonstationary and time-varying parameters, which are found in most real time series as the problematic issues. So, the stationary assumption for analysing time series could be relaxed.

A well-known wavelet property is the localization of wavelet function. This means that the wavelet transform adjusts itself to capture the feature of the series across a wide range of frequencies and also has the ability to capture events that are local in time, (Gençay et al., 2002). This property makes the wavelet analysis a particularly useful vehicle to analyse nonstationary time series. (see Percival and Walden, 2000)

Another remarkable feature of wavelets is to uncover the multiresolution properties
of a process. While an economic or financial time series may not follow the same relationship in time horizon (scale), wavelet analysis provides a desirable method to identify the time series properties in different time horizons. It, in fact, decomposes a time series into different time horizons to distinguish seasonalities, structural breaks and volatility clustering.

Wavelet transform is based on two basis functions: father and mother wavelets. The father wavelet represents the smooth (low frequency) trend part of the series. While, mother wavelet represents the detailed (high frequency) part of the series. The wavelet transform utilises mother wavelet, that is stretched and shifted in time to capture features that are local in time and local in frequency, (Gençay et al., 2002). In other words, wavelet transform could be considered as a decomposition of time series into high frequency (noisy components) and low frequency (trend components).

Orthogonality is another useful property of wavelet analysis. Orthogonality of basis wavelet functions across scales provides wavelet analysis the ability to decompose time series into its constituent scale, (Ramsey and Lampart, 1998). However, this property does not hold for all wavelet classes. There are several types of wavelet transforms. The division based on orthogonality provides the use of orthogonal basis in the discrete wavelet transform (DWT) and the non-orthogonal basis in the maximal overlap discrete wavelet transform (MODWT) and the continuous wavelet transform (CWT), (Lan, 2011).

The DWT is implemented by a pyramid algorithm on which multiresolution analysis (MRA) is built. The multiresolution (multiscale) analysis provides the decomposition of a time series into several layers of orthogonal sequences of scales. Each scale can be analysed individually. Furthermore, the relationship of each scale with the corresponding scale from different series can be examined.

Empirically, the potential applications of wavelet analysis could point out into several grounds, namely nonstationary series analysis (Durai and Bhaduri, 2009; Fan and
Gençay, 2010), multiresolution (Ramsey and Lampart, 1998; Vuorenmmma, 2005; Durai and Bhaduri, 2009), and forecasting (Conejo et al., 2005; Fernandez, 2007; Rua, 2011). Indeed, there are remarkable studies that have used wavelet analysis in finance. (see Lee, 2004; Gençay et al., 2004, 2005; In and Kim, 2005, 2006, 2007; Gallegati and Gallegati, 2007; Gallegati, 2008; Barunik et al., 2011)

Following to the introduction to wavelet transform, the last part of the chapter investigates the behaviour of exchange rates in different time horizons. I take advantage of the multiresolution property of wavelet analysis and examine purchasing power parity relation in 7 time-scale levels using maximal overlap discrete wavelet analysis (MODWT).

The Uncovered Interest Rate Parity Puzzle and Wavelet Analysis

UIRP is an important basis of the economic mainstreams in the exchange rate determination theories. It precisely presents the key relation of the exchange rates and interest rate differentials. Technically, URIP assumes that a regression of expected exchange rate changes on the interest rate differentials should give a slope coefficient of unity, (Chaboud and Wright, 2005).

However, there is a notable evidence in the US economy in the late twentieth century that denotes an instability in the interest rates and exchange rates relationship. This means that the correlation between interest rates and exchange rates can move in considerably different directions for an economy.

There are ample theoretical and empirical studies, trying to justify this contradiction. Generally, in the literature, several mainstreams explain exchange rate behaviour, such as flexible-price monetary model, fixed-price model and sticky price Dornbusch overshooting model.

In the flexible-price monetary model, the inflation level is positively related to the
nominal interest rate. Hence, when the domestic interest rate increases relative to the foreign interest rate, it is expected that the domestic currency loses value by inflation. Once the demand of domestic currency decreases relative to the foreign currency, it depreciates rapidly. Therefore, this model represents a positive correlation between interest rate and exchange rate.

On the other hand, the fixed-price Mundell-Fleming model explains that a higher domestic interest rate than a foreign one attracts a capital inflow leading to instantaneous appreciation of domestic currency. Therefore, this model implies a negative relationship between exchange rates and interest rate differentials.

Sticky price Dornbusch overshooting model (1976) is a hybrid of the two extremely opposite models. It refers to the fixed-price model in short-run and to the flexible-price monetary model in long-run.

In addition, the bulk of empirical studies in the literature have been testing the UIRP hypothesis. An overwhelming number of papers argue against UIRP and fail to support the value of unity of interest rate differentials in exchange rate prediction equations. (see Bekaert and Hodrick, 1993; Lewis, 1995; Engel, 1996; Baillie and Bollerslev, 2000; Flood and Rose, 2001; Leon et al., 2006)

On the other hand, some empirical evidence reveal that the sign of slope coefficient holds positively unity under UIRP condition. (see Mussa, 1979; Froot and Thaler, 1990; Chaboud and Wright, 2005) These considerably opposite findings lead to further research to find a coherent explanation for the UIRP puzzle.

Chinn and Meredith (2004) state that the reason for the empirical failure of UIRP is that a majority of papers evaluate the UIRP hypothesis using financial instruments with relatively short maturities, generally less than 12 months. Based on their assertion, they provide strong evidence supporting UIRP in longer horizon. Their results confirm the earlier studies of Mussa (1979) and Froot and Thaler (1990) that UIRP may work better at longer horizons. Therefore, time horizon obviously is an important issue in
investigating the UIRP proposition. It can conclude that those surveys, which reject the UIRP hypothesis, do not distinguish between the short-run and long-run relationship between exchange rates and interest rate differentials. Thus, this issue causes the rejection of UIRP.

This study relies on Chinn and Meredith (2004) reasoning and examines the UIRP condition in different time horizons, namely short-run and long-run. The main contribution of this chapter is to apply wavelet transform in considering the relationship between exchange rates and interest rate differentials under the UIRP discipline in different time horizons.

To examine the UIRP proposition, the quarterly exchange rate of Pound Sterling (GBP) is used against the currencies of ten UK trade partners, namely US dollar (USD), Japanese yen (JPY), Euro (EUR), Chinese Yuan, German Mark, French Franc, Danish Krone, Netherlands Guilder, Swedish Krona, Norwegian Krone.

Two variables of the UIRP proposition, logarithm of nominal exchange rate differential $\Delta \ln s$, and nominal interest rate differential $\Delta IR$ are decomposed into 4 time-scale levels by MODWT analysis for each country. Each time-scale level relating to a range of frequencies, such that, when the time-scale levels become longer, the oscillation of the time series are smoother. In other words, the time scale rises the time between consecutive peaks and the time between consecutive troughs gets longer.

In order to investigate the effect of the interest rate differentials on the expected exchange rate change, in different time horizons, I estimate the following equation using OLS method in four time-scale levels for each pair of countries.

$$\Delta \ln s(d_j)_t = a_t + \beta \Delta IR(d_j)_t + \epsilon_{j,t}$$
New Approach of Currency Crashes Definition in Early Warning System

This chapter aims to evaluate the potential leading indicators of a currency crises. A currency crisis is an abrupt devaluation in a domestic currency, inevitably leading to speculative attacks. As Krugman (1979) stated, in general, the mechanism of a currency crisis usually starts with a shift in expectations of speculators in the exchange market when they buy and hold the foreign currency. Meanwhile, the central bank sells the foreign currency reserves in order to stabilize the currency. However, tragedy occurs when the central bank exhausts its reserves. At this time, a sudden devaluation of domestic currency happens and speculators attack the exchange rate market and force an abandonment of the fixed exchange rate. The speculative attacks result in large scale selling of domestic currency assets that creates more devastating effects on the economic fundamentals. Speculative attacks on the UK exchange rate market in 1992 is a proper example of currency crises.

The purpose of this study is to answer the question of whether the potential causes and symptoms of currency crises can be detected sufficiently in advance to allow governments to adopt pre-emptive measures. There are numerous benefits of developing a warning system not only to financial market participants who want to avoid loss of profit, but also to policy-makers and academics. Policy-makers want to reduce the large costs of such crises and also researchers seek to identify variables (leading indicators) which are able to predict an approaching crisis.

The theoretical literature on the early warning signals attempts to explain the crises and delineate the reasons of crises occurrence. The currency crisis literature is classified into three generations. In the first generation models, the deterioration in economic fundamentals causes the crises, (Krugman, 1979; Flood & Garber, 1984) such as what happened in Latin America in the 1960s and 1970s. The second generation examine the investors’ expectation on government behaviour, (Obstfeld, 1994) as well as the
EMS crises of the early 1990s. The third generation models explain the spillover and contagion effect, under development of the banking sector and market segmentation like the Tequila crisis Latin America in 1994 and the Asian Flu crisis in South-east Asia in 1997.

These broad possibilities to choose useful indicators result in ample empirical studies in parallel. For instance, Kaminsky et al. (1998) and Esquivel and Larran (1998), Rose and Spiegel (2011), Frankel and Saravelos (2012), and Babecký et al. (2013) identify a large set of various possible leading indicators in their studies.

To investigate the probability of the occurrence of a crisis, the empirical surveys apply two different methodologies, signalling approach (see Kaminsky et al., 1998; Bruggemann and Linne, 1999; Edison, 2003) and limited dependent variable probit/logit model (see Eichengreen et al., 1995; Frankel and Rose, 1996; Berg and Pattillo, 1999; Kumar et al., 2003; Lau and Yan, 2005; Bussière and Fratzcher, 2006; Ari, 2012). In the limited dependent variable probit/logit model, dependent variable is defined as a discrete choice and macroeconomic and financial variables are utilised to explain discrete crisis events.

A novelty of this study is classification of the currency crashes definition based on different generations of currency crises in early warning literature. Assuming that, a currency crash is defined as a sharp depreciation of the nominal exchange rate, I consider the behaviour of exchange rate of those countries which have been involved in crises for almost two decades. This suggests that the exchange rates in the emerging and developing countries behave differently from those in developed countries, once a crisis happens and especially afterwards. The first and third generations are associated with emerging and developing countries and the second generation are related to the developed countries.

Given that, I split the currency crashes into two different definitions. Definition \( I \), related to the emerging market with weak fundamentals, shows that the exchange
rates require a longer recovery process after a sharp devaluation in domestic currency. While, definition II states that a crisis in the developed countries has been immediately recovered and gets back to normal level e.g. within two quarters.

These definitions capture only the successful speculative attacks crises, (Frankel and Rose, 1996). Next, I adopt the discrete choice approach in the current survey and apply a logit model which considers binary outcome variables using a linear combination of predictor variables.

To identify potential leading indicators in this study, I look at the ranking of the indicators in the earlier literature and rely on systematic literature reviews. I select seven potential indicators for a panel of 26 emerging markets and developed countries at quarterly frequency. The potential leading indicators of this survey include rate of growth domestic credit, government budget deficit as a fraction of GDP, ratio of reserve to GDP, current account as a percentage of GDP, growth rate of real output, overvaluation of exchange rate, ratio of debt to GDP.

The leading indicator overvaluation of exchange rate in a majority of studies is defined as the exchange rate deviations from its trend applying HP filter. (see Goldfajn and Valdés, 1998; Ari, 2012). However, current study shows that HP filter is not an appropriate filter to construct the overvaluation, because HP filter compromises the past and future values to construct the current value. That means it is not inherently a backward looking filter, while overvaluation as a predictor of currency crises should not contain the future value. To overcome this misconception of previous papers, this study suggests to use Kalman filter recursively. Thus, I apply Kalman filter to detrend the exchange rate and in turn construct the overvaluation.

The main contribution of this chapter is to develop a multivariate empirical model that is able to improve upon existing empirical models. As such, it departs from the majority of empirical studies in some aspects: first I categorise the definition of the currency crashes according to the different generations of the currency crises occurrence
and the country groups which are involved in those generations, because each crisis
generation has a different nature. It is intuitive to define a currency crash in different
manners in order to obtain the accurate features of each generation by my definition.

Secondly, I argue about the definition of exchange rate overvaluation in the litera-
ture. Previous studies could be criticised on their inappropriate usage of HP filter in
constructing the overvaluation. The current chapter suggests to apply a Kalman filter
recursively to construct overvaluation as a leading indicator in early warning literature.
Chapter 2

Introduction to Wavelet Transform and its Application in Testing Purchasing Power Parity Hypothesis

2.1 Introduction

Wavelet analysis is a refinement of Fourier analysis. The Fourier analysis processes a time series by using a mathematical transformation. Simply put, it is a linear combination of sines and cosines. Each of these sines and cosines is a function of frequency. Thus, Fourier transform could be viewed as a decomposition on a frequency-by-frequency basis. In fact, it transforms a time series from time domain into frequency domain.

Gençay et al. (2002) state that the Fourier basis function is very appealing when working with stationary time series. Nevertheless, in the real world, most economic and financial time series are nonstationary and varying over time. So, the Fourier transform can not efficiently capture the events that happen during a particular time. It is not
simply applicable for nonstationary series. Hence, it is necessary to explore a new tool such as wavelet analysis, because it extracts both time and frequency information from nonstationary time series.

Empirically, wavelet analysis due to desirable properties has shown strong applicability in economics and finance. A well-known property is the localization of wavelet function. This means that wavelet transform adjusts itself to capture the feature of the series across a wide range of frequencies and also has the ability to capture events that are local in time, (Gençay et al., 2002). This property makes wavelet analysis a particularly useful vehicle to analyse nonstationary time series which the Fourier analysis is not capable of. Moreover, wavelets have significant advantages over basic Fourier analysis when the object under study is locally nonstationary and inhomogeneous. (see Percival and Walden, 2000; Gençay et al., 2002)

Wavelet transform is based on two basis functions: father and mother wavelets. The father wavelet represents the smooth (low frequency), trend part of the series. While, mother wavelet represents the detailed (high frequency) part of the series.

The wavelet transform utilises mother wavelet, that is stretched and shifted in time to capture features that are local in time and local in frequency, (Gençay et al., 2002). In other words, wavelet transform could be considered as a decomposition of time series into high frequency (noisy components) and low frequency (trend components).

Orthogonality is another useful property of wavelet analysis. Orthogonality of basis wavelet functions across scales provides wavelet analysis the ability to decompose time series into its constituent scale, (Ramsey and Lampart, 1998). However, this property does not hold for all wavelet classes.

There are several types of wavelet transforms. The division based on orthogonality provides the use of orthogonal basis in the discrete wavelet transform (DWT) and the non-orthogonal basis in the maximal overlap discrete wavelet transform (MODWT) and the continuous wavelet transform (CWT), (Lan, 2011).
The DWT is implemented by a pyramid algorithm on which multiresolution analysis (MRA) is built. The multiresolution (multiscale) analysis provides the decomposition of a time series into several layers of orthogonal sequences of scales. Each scale can be analysed individually. Furthermore, the relationship of each scale with the corresponding scale from different series can be examined.

This chapter aims to shed light on wavelet analysis by demonstrating its contribution to economics and financial analysis and provide an overview of the studies that have applied this methodology. Ramsey (2002), Gençay et al. (2002), Schleicher (2002), Crowley (2007) and recently Rua (2012) attempt to introduce wavelet analysis to the realm of economics and discuss its potential ability to analyse time series. Empirically, there is an extensive research that has applied wavelet analysis into several grounds, namely nonstationary series analysis (Durai and Bhaduri, 2009; Fan and Gençay, 2010), multiresolution (Ramsey and Lampart, 1998; Vuorenmm, 2005; Durai and Bhaduri, 2009), and forecasting (Conejo et al., 2005; Fernandez, 2007; Rua, 2011). Indeed, there are remarkable studies that have used wavelet analysis in finance. (see Lee, 2004; Gençay et al., 2004, 2005; In and Kim, 2005, 2006, 2007; Gallegati and Gallegati, 2007; Gallegati, 2008; Barunik et al., 2011)

The last part of this chapter presents an empirical application of wavelet analysis in which I examine purchasing power parity theory using maximal overlap discrete wavelet analysis, in order to investigate the behaviour of exchange rates in different time-scale levels.

This chapter is organised as follows: Section 2.2 provides a brief overview of Fourier analysis. In Section 2.3, the wavelet transform is comprehensively introduced and also two types of wavelet transform, discrete wavelet transforms (DWT) and continuous wavelet transform (CWT) are explained. Section 2.4 describes the empirical wavelet literature in economics and finance. Section 2.5 represents the application of wavelet analysis to illustrate the use of that in purchasing power parity proposition. Section 2.6
concludes.

2.2 Fourier Analysis

Since Fourier analysis is the root of wavelet analysis, I provide a brief overview of Fourier analysis. Using jargon signal is meant to be a time series. As a signal, by definition is a time series, there are a few differences between them. For example, signals are continuous over both time and frequency domains, whereas most time series are taken over a discrete time domain like days, weeks, months, quarters, years.

A French mathematician, Jean Baptiste Joseph Fourier in 1807 asserted that any periodic function can be expressed as an infinite sum of sine and cosine functions of various frequencies. This idea is a building block of the well-known Fourier transform, (Rua, 2012).

The Fourier transform is an alternative representation of the raw signal (original time series). It summarises information in the data in terms of frequency instead of time, the opposite of what the raw signal presents, where no frequency resolution is provided, (Gençay et al., 2002). In other words, in Fourier analysis, the original series is decomposed into underlying sine and cosine functions of different frequencies, in order to determine those that appear particularly strong or important.

The processed time series explains how many frequencies and how much (energy) of each frequency exists in the original series. However, it does not provide the time information where a particular frequency appears in the time domain.

The Fourier transform is a conventional method for studying the frequency content of a signal. More precisely, Fourier analysis is able to identify the seasonal fluctuations of different lengths. While in the time series analysis, the length of the seasonal component is usually known (or guessed) a priori and then included in some theoretical model of moving averages or autocorrelations.
In mathematical terms, the Fourier transform is represented as basic sines and cosines of different frequencies to determine how much of each frequency the signal contains. The Fourier transform of the time series \( x(t) \) is given by:

\[
F_x(\omega) = \int_{-\infty}^{+\infty} x(t) (\cos(\omega_j t) - i \sin(\omega_j t))
\]

(2.1)

where \( \omega_j \) denotes the frequency.

On the other hand, the main weakness of Fourier transform is that it does not allow the frequency content of the signal to change over time and thus it has trouble reproducing signals that have time-varying features. This means that Fourier transform provides information of how much of each frequency exists in the signal, however, it does not give any information of when in time these frequency components exist.

The short-time Fourier transform is a key solution to deal with such a drawback. The idea behind is to use Fourier transform for short periods of time. To be more precise, Fourier transform has been applied within a short-time window of the signal as it slides across all the data.

However, the short-time Fourier transform is limited by fixed length windows. These constant length windows provide equal partition of time-frequency space. When a wide range of frequencies is involved, the constant time window tends to incorporate a large number of high frequency cycles and a few low frequency cycles. This results in an over-representation of the high frequency components and an under-representation of the low frequency components. Therefore, while the signal is considered under a fixed time-frequency window with constant intervals in the time and frequency domains, the short-time Fourier transform does not represent a sufficient resolution for all frequencies, (Rua, 2012).

Generally, any time-frequency suffers from the Heisenberg uncertainty principle. This principle declares that the location and the velocity of an object cannot both be measured exactly at the same time. That is, it is impossible to detect simultaneously
the precise frequency and the precise time of occurrence of this frequency in a signal. In other words, there is trade-off between time and frequency resolution. Particularly, for narrow windows one gets good time resolution but poor frequency resolution while for wide windows one gets good frequency resolution and poor time resolution.

2.2.1 Fourier versus Wavelet Analysis

Comparing Fourier transform with wavelet transform sheds more light on the wavelet analysis concept. There are some similarities and differences between wavelets and Fourier analysis. Similarly, Fourier transform and wavelet transform are orthogonal expansions. Orthogonality, in mathematics, displays the relation of two lines at right angles. In general, it means uncorrelated or independent objects.\(^1\)

The Fourier base functions (sine and cosine) are desirable to analyse the stationary signal. However, most signals show quite complicated patterns over time e.g. trend, structural break, and volatility clustering. Since the signals are nonstationary, Fourier transform is not applicable. In contrast, wavelet analysis has an ability to deal with time-varying characteristic of a series. That is, wavelets are able to analyse both stationary and nonstationary processes.

In addition, Fourier transform decomposes the signals in terms of sine and cosine basis functions. Conversely, wavelet transform uses a new waveform as a basis function. Similar to other orthogonal expansions, a wavelet expansion of a square integrable function \(f \in \mathcal{L}_2\) generally expresses the expanded function as a complete, orthonormal set of basis functions for the Hilbert Space of square integrable functions.\(^2\) As in the Fourier domain, which is more familiarly recognized as frequency domain, the analysis

\(^1\)The functions \(f(x)\) and \(g(x)\) are orthogonal if their inner product \(\langle f, g \rangle\) is zero for \(f \neq g\), such that:

\[
\langle f, g \rangle = \int_a^b f(x)g(x)dx = 0 \tag{2.2}
\]

\(^2\)The most convenient space of infinite dimension for representing a function of a continuous variable is the Hilbert space, \(\mathcal{L}_2 (\mathcal{R})\), which is the space of all square integrable functions over \((-\infty, \infty)\).
of signals in wavelet domain is conducted by analysing the transform coefficients.

While Fourier analysis maps a one-dimensional function of a continuous variable into a one-dimensional sequence of coefficients, the wavelet analysis maps it into a two-dimensional array of coefficients. The two-dimensional representation allows localizing the signal in both time and frequency, as the Fourier series expansion only localizes in frequency. As an illustration, if a Fourier series expansion of a signal has only one large coefficient, then the signal is essentially a single sinusoid at the frequency determined by the index of the coefficient. However, a wavelet representation provides the location in both time and frequency simultaneously, as the simple time-domain representation of the signal itself gives the localization in time, (Percival & Walden, 2000).

For Fourier series and transform and for most signal expansion systems, the expansion functions (bases) are chosen, then the properties of the resulting transform are derived and analysed. As in the wavelet system, these properties are mathematically required, then resulting basis functions are derived. Because these constraints do not use all the degrees of freedom, other properties can be required to customize the wavelet system for a particular application.

In the case of time series, every point in the time domain consists of information about all frequencies. On the contrary, for Fourier transform in the frequency domain, each point consists of information from all points in the time domain. Whereas, the short-time Fourier transform uses a fixed length window, for the wavelet transform the window width is adjusted to the frequency.

### 2.3 Wavelet Transform

A wave is usually defined as an oscillating function of time such as a sinusoid. A wavelet is a small wave which is similar to sine or cosine in that it also fluctuates about zero. Wavelet analysis involves using wavelets to decompose and analyse the fluctuation in
a time series. The basic step in wavelet analysis is wavelet transform. The wavelet transform breaks down a time series into high frequency or noisy components and low frequency or trend components.

In Fourier analysis, a signal is taken and sines and cosines are fitted to it, whereas in wavelet analysis, a basis function, so-called mother wavelet is fitted to a signal. A mother wavelet is quite different from a sine or cosine function in one particular way: it is a finite wave, whereas sines and cosines are infinite. This feature implies that when mother wavelet is fitted to the signal, the result is localised rather than global. So, instead of obtaining a single coefficient for each sine and cosine as in Fourier analysis, a series of coefficients that are varying over time is attained.

Wavelet Transform is divided into two main parts: the discrete wavelet transform (DWT) and the continuous wavelet transform (CWT). The discrete wavelet transform makes it possible to decompose a time series into its constituent multiresolution components (see Percival and Walden, 2000). High-frequency components reflect the short-term behaviour, whereas the low-frequency component captures the long-term dynamics of the variable. However, in the last couple of years, the continuous wavelet analysis is also becoming popular in economic applications. Important part of the continuous wavelet analysis is the ability to study interactions or comovement between two time series in time-frequency domain using the cross-wavelet tools (see Aguiar-Conraria et al., 2008; Rua and Nunes, 2009).

2.3.1 Wavelet Theory

This section elaborates wavelet analysis in mathematical terms. There are two wavelets which form a pair in a wavelet family: father wavelets $\phi(t)$ and mother wavelets $\psi(t)$. The father wavelet (scaling function) integrates to unity and is used for the smooth and lowest frequency parts of a signal (trend components). While, the mother wavelet (wavelet function) integrates to zero and describes the detailed and higher frequency
parts of the signal. In fact, it is suitable for detection of all deviations from the trend.

\[
\int \phi(t) dt = 1 \quad \text{Father wavelets} \quad (2.3)
\]
\[
\int \psi(t) dt = 0 \quad \text{Mother wavelets} \quad (2.4)
\]

Therefore, the wavelet transform of \( f(x) \) can be built by the sum of father and mother wavelets, such that:

\[
f(t) = \nu \phi(t) + \sum_{j=0}^{J-1} \sum_{k=0}^{2^{j}-1} \omega_{j,k} \psi_{j,k}(t) \quad (2.5)
\]

where the parameters \( j \) and \( k \) scale and translate the function and \( \nu \) and \( \omega_{j,k} \) are scaling and wavelet coefficients, respectively. As seen in the equation, while a sequence of mother wavelets is used to represent a function, only one father wavelet is used. The mother wavelet is compressed or dilated in a time domain to generate cycles to fit the actual time series.

In the wavelet mechanism, a mother wavelet typically goes through the original signal in order to discover the frequency and the time of the special events. The process operates by scaling and translating the mother wavelet. Scaling means changing the frequency by either shrinking (compressing) or stretching (dilating) of a wavelet in time. Translating means changing the time by shifting or sliding a wavelet. Generally, wavelet transform utilises the dilation and translation of the mother wavelet \( \psi(t) \) to capture features that are unique along two dimensions; frequency and time, (Lan, 2011).

To estimate the coefficients of wavelet function, Mallat (1989) represents the multiresolution analysis (MRA). Mallat’s methodology is able to provide a simple interpretation of the wavelet decomposition. As seen equation (2.5) is an additive decomposition which enable us to reconstruct the series even after decomposition the series to its constitute components. Essentially, the MRA struggles to find averages (means, sometimes
weighted) and then differences from those averages. It starts with amounts in the series which are nearest to each other i.e. the lowest scale. Then it repeats that process with the last average series, next it slowly expands how much of the original data is encompassed in each consecutive average, i.e. rising the scale. The multiresolution decomposition of the signal \( x(t) \) is given by:

\[
\{V_J, W_J, W_{J-1}, ..., W_1\}
\]  

(2.6)

The coefficients of a wavelet analysis must satisfy three properties:

- have zero mean
  \[
  \sum_{j=0}^{J-1} \psi_j(t) = 0
  \]

- have unit energy
  \[
  \sum_{j=0}^{J-1} \psi_j^2(t) = 1
  \]

- be orthogonal to their even shifts
  \[
  \sum_{j=0}^{J-1} \psi_j(t)\psi_{j+2k}(t) = 0
  \]

2.3.2  Discrete Wavelet Transform (DWT)

For a stochastic time series \( x(t) \) with the dyadic sample size \( N = 2^J \) and a number of resolution scales \( j \) such that \( j = 1, ..., J \), the sequence of the scaling and wavelet functions \( \phi_{J,k}(t) \) and \( \psi_{j,k}(t) \) are obtained as follows:

\[
\phi_{J,k}(t) = \frac{1}{2^J} \phi \left( \frac{t - 2^J k}{2^J} \right)
\]  

(2.7)

\[
\psi_{j,k}(t) = \frac{1}{2^j} \psi \left( \frac{t - 2^j k}{2^j} \right)
\]  

(2.8)
where \( j \) and \( k \) denotes the scale (dilation) and translation, respectively.

The discrete wavelet coefficients are computed by:

\[
\begin{align*}
v_{J,k} & \approx \sum x(t) \phi_{J,k}(t) dt \\
\omega_{j,k} & \approx \sum x(t) \psi_{j,k}(t) dt \quad j = 1, ..., J
\end{align*}
\] (2.9) (2.10)

where \( v_{J,k} \) is the coefficient of the father wavelet at the maximal scale, \( 2^J \) known as a smooth coefficient and \( \omega_{j,k} \) are the detail coefficients obtained from the mother wavelet at all scales from 1 to \( J \).

A multiresolution representation of \( x(t) \) is given by:

\[
x(t) = \sum_{k} v_{J,k} \phi_{J,k}(t) + \sum_{k} \omega_{J,k} \psi_{J,k}(t) + \sum_{k} \omega_{J-1,k} \psi_{J-1,k}(t) + ... + \sum_{k} \omega_{1,k} \psi_{1,k}(t)
\]

Alternatively, it could be represented by:

\[
x(t) = V_J + W_J + W_{J-1} + ... + W_j + ... + W_1
\] (2.11)

where

\[
\begin{align*}
V_J & = \sum_{k} v_{J,k} \phi_{J,k}(t) \\
W_j & = \sum_{j} \sum_{k} \omega_{j,k} \psi_{j,k}(t) \quad j = 1, ..., J
\end{align*}
\] (2.12) (2.13)

As an example, the Haar function (Haar, 1910) is the simplest discrete orthogonal symmetric wavelet which is defined by:

\[
\psi_{j,k}(t) = \begin{cases} 
1 & 0 \leq t < 1/2 \\
0 & \text{otherwise} \\
-1 & 1/2 \leq t < 1 
\end{cases}
\] (2.14)
In this case, the father wavelet is just a flat line equal to one. So, the equation could be written as:

\[ f(t) = \nu + \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \omega_{j,k} \psi_{j,k}(t) \]  

(2.15)

where \( \nu \) is the overall mean of the data and \( \omega_{j,k} \) amounts show the wavelet coefficients.

The above equation is an additive decomposition which implies a discrete wavelet. Figure (2.1) displays the Haar mother wavelet when it is scaled (dilated and compressed) and translated (shifted).

![Haar Wavelet Transform](image)

Figure 2.1: Haar Wavelet Transform

The DWT has some properties. First, it is an orthogonal transformation. Secondly, the relevant MRA has no redundancy, preparing just enough information to rebuild the original series. However, it uses a certain number of data samples, i.e., dyadic number of observations, \( 2^J \).

On the other hand, the orthogonal property of basis (scaling and wavelet functions) across scale provides an ability to decompose a time series in terms of its constituent scales. However, the orthonormal DWT has two main drawbacks: the dyadic length requirement (i.e. a sample size divisible by \( 2^J \)) and the fact that the scaling and wavelet coefficients are not shift invariant due to their sensitivity to circular shifts because of the decimation operation. Hence, the maximal overlap discrete wavelet transform (MODWT) is represented with a non-orthogonal basis as an alternative to DWT. The MODWT goes under several names in the wavelet literature, such as non-decimated DWT and stationary DWT (Nason and Silverman, 1995), translation-invariant DWT.
(Coifman and Donoho, 1995), and time-invariant DWT (Gançay et al., 2002).

### 2.3.3 Maximal Overlap Discrete Wavelet Transform (MODWT)

Percival & Walden (2000) define the maximal overlap discrete wavelet transform (MODWT) as a modification of the ordinary discrete wavelet transform. This transform gives up orthogonality of the DWT. However, it preserves the property of DWT by rescaling the scaling and wavelet coefficients.

In the orthonormal DWT the wavelet coefficients are concerned with non-overlapping differences of weighted averages from the original observations. More information on the variability of the signal could be obtained considering all possible differences at each scale, this means considering overlapping differences. So, the intuition behind the maximal overlap algorithm is computation of all possible shifted time intervals. As the orthogonality of the transform is lost, the number of scaling and wavelet coefficients at every scale remains the same as the number of observations. Thus, the MODWT coefficients is considered the result of a simple modification in the pyramid algorithm used in computing DWT coefficients. The DWT coefficients is considered as a subset of the MODWT coefficients.

The MODWT scaling and wavelet coefficients \( v_{j,k} \) and \( \omega_{j,k} \) could be written as:

\[
v_{j,k} = \frac{1}{2^j} \sum_{k=0}^{2^j-1} \phi_{j,k}(t)x(t-k)
\]

and

\[
\omega_{j,k} = \frac{1}{2^j} \sum_{k=0}^{2^j-1} \psi_{j,k}(t)x(t-k)
\]

where father and mother wavelets \( \phi_{j,k} = \frac{\phi_{j,k}}{2^j} \) and \( \psi_{j,k} = \frac{\psi_{j,k}}{2^j} \) are obtained by rescaling the DWT father and mother wavelets.

The properties of the MODWT could be summarised as follows:
• The MODWT can get any sample size $N$, whereas the DWT confines the sample size to a multiple of $2^J$.

• The detail and smooth coefficients of a MODWT multiresolution analysis corresponds to zero phase filters. This means events that feature in the original time series may be properly aligned with features in the multiresolution analysis.

• The MODWT is invariant to circularly shifting the original time series. Therefore, shifting the time series by an integer unit will shift the MODWT wavelet and scaling coefficients the same amount. There is no such a property for the DWT.

• Both the DWT and MODWT can carry out an analysis of variance on a time series. However, the MODWT wavelet variance estimator is asymptotically more efficient than the same estimator based on the DWT, (Gençay et al., 2002).

• The MODWT has the number of amounts for the averages at every scale level equal to the number of amounts in the original series compared to DWT, (Hacker et al., 2012).

Many studies applied MODWT in their research such as Crowley and Lee (2005), In and Kim (2005, 2006), Gallegati and Gallegati (2007), Gallegati (2008).

### 2.3.4 Continuous Wavelet Transform (CWT)

A Continuous Wavelet Transform (CWT) generates a wavelet amplitude function that is continuous in both time and frequency, (see Aguiar-Conraria and Soares, 2011; Rua, 2012; Tiwari et al., 2013). Its advantage is that it gives a better graphical representation of the structure of a continuous data function than does the discrete wavelets transform (DWT), which depends on a particular choice for the phase location of the sample points. If the data function is frequency-limited and if the sample is taken at the
critical Nyquist rate, then only the sample interval will be determined. The precise location of the observations within such an interval is arbitrary.

Besides, the CWT provides an excellent overview of the signal. It allows us to identify transient events and to show the time, the frequency, and the general shape of the event (by comparing the CWTs obtained by using different wavelets). Through implementing CWT, the scales could completely control. Indeed, every possible stretching and sliding of the wavelet correlated with the signal could also be observed, because the wavelets used in the CWT are not required to conform to the stringent requirements of those used in the DWT (orthogonality, alias cancellations). In fact, the basis wavelets could be invented.

The continuous wavelet transform of a time series $x(t)$ is given by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t) \psi_{t,s}(t) dt$$ (2.18)

It implies that the wavelet transform decomposes a time series $x(t)$ in terms of some basis function (wavelets) $\psi_{t,s}(t)$. These basis functions are derived from mother wavelet $\psi(t)$ which is defined as:

$$\psi_{t,s}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t - \tau}{s} \right)$$ (2.19)

where $\tau$ and $s$ is the time location and scale, respectively.

There are some disadvantages in the use of CWT. First, CWT does not have a viable inverse transform. CWT is usually used to transform and manipulate the signal, then an inverse transform is taken to construct the original signal. Secondly, the CWT is extremely redundant. Since every possible scale and time is analysed, it produces tremendous amounts of data. Therefore, due to the lack of appropriate inverse transform and data redundancy it cannot be an efficient tool for compression and denoising.
2.3.5 Multiresolution Analysis (MRA)

The most interesting property of wavelet analysis is that it allows us to separate out a signal into its constituent multiresolution components. Multiresolution is able to provide a simple interpretation of the wavelet decomposition. It is obviously designed to represent signals, where a single event is decomposed into finer and finer details. (see Lee, 2004; Shrestha and Tan, 2005; Yogo, 2008; Rua, 2011)

Suppose a sequence \( \{V_j\}_j \in \mathcal{I} \) of closed subspace of \( L_2(\mathbb{R}) \) constitutes a multiresolution analysis if and only if the following properties shown in (2.20) and (2.21) are satisfied.

\[
\forall j, k \in \mathcal{I}, \quad f(t) \in V_j \iff f(t - 2^j k) \in V_j \quad (2.20)
\]

This condition declares that if the function \( f(t) \) belongs to the space \( V_j \), such that it can be expressed in terms of the basis of that space, then the same must be true of its translated version \( f(t - 2^j k) \). In other words, it ensures elements in a space are simply scaled versions of the elements in the next space.

\[
\forall j \in \mathcal{I}, \quad V_{j+1} \subset V_j, \quad f(t) \in V_j \iff f\left(\frac{t}{2}\right) \in V_{j+1} \quad (2.21)
\]

The two conditions above assert that every subspace \( V_j \) contains a subspace \( V_{j+1} \) comprising the functions that have twice the dilation of those of \( V_j \).

Suppose that there is a space \( V_0 \). First, the range of frequencies \([0, \pi]\) of the space \( V_0 \) are divided in equal subintervals \([0, \pi/2]\) and \((\pi/2, \pi]\). The upper frequency interval; will have \( N/2 \) wavelet functions, denoted by \( \psi_{1,k}(t); k = 0, 1, \ldots, [N/2] - 1 \), separated one from the next by two time intervals. These wavelets will constitute a basis for a space denoted by \( \mathcal{W}_1 \).

The lower frequency interval will have the same number \( N/2 \) of scaling functions, denoted by \( \phi_{1,k}(t); k = 0, 1, \ldots, [N/2] - 1 \), also separated by two intervals. These scaling functions will constitute a basis for a space denoted by \( \mathcal{V}_1 \). The division of \( V_0 \) is such
that its two subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$ are mutually orthogonal. Their orthogonality, which can be denoted by $\mathcal{V}_1 \perp \mathcal{W}_1$, entails the fact that $\mathcal{V}_1 \cap \mathcal{W}_1 = 0$.

The direct sum of the two subspaces is $\mathcal{W}_1 \oplus \mathcal{V}_1 = \mathcal{V}_0$. This means that any elements in $f(t) \in \mathcal{V}_0$ can be expressed as $f(t) = \omega_1(t) + \nu_1(t)$ with $\omega_1(t) \in \mathcal{W}_1$ and $\nu_1(t) \in \mathcal{V}_1$ which is the sum of two orthogonal functions.

In the next stage of decomposition of $\mathcal{V}_0$, the lower interval is subdivided into the intervals $[0, \pi/4]$ and $[\pi/4, \pi/2]$, which are filled, respectively, with $N/4$ scaling functions, denoted by $\phi_{2,k}(t); k = 0, 1, ..., [N/4] - 1$, and $N/4$ wavelets, denoted by $\psi_{2,k}(t); k = 0, 1, ..., [N/4] - 1$, separated by four time intervals. These will constitute the basis functions, respectively, of the spaces $\mathcal{V}_2$ and $\mathcal{W}_2$, which are mutually orthogonal subspaces of $\mathcal{V}_1$.

The process of subdivision continues, by dividing successively the lower subintervals, until it can go no further. If there are $N = 2^j$, $(j = 1, ..., J)$ points in the sample, then $N$ can be divided $J$ times, and there will be a total of $J + 1$ horizontal bands.

The process of subdivision generates a nested sequence of vector spaces, each of which is spanned by a set of scaling functions.

$$\mathcal{L}_2 \supset \mathcal{V}_0 \supset \mathcal{V}_1 \supset ... \supset \mathcal{V}_j$$

(2.22)

The $j$th stage of the process, which generates $\mathcal{V}_j$, also generates the accompanying space $\mathcal{W}_j$ of wavelet functions, which is its orthogonal complement within $\mathcal{V}_{j-1}$. The
complete process can be summarised by displaying the successive decompositions of $V_0$:

\[
V_0 = W_1 \oplus V_1 \\
= W_1 \oplus W_2 \oplus V_2 \\
= W_1 \oplus W_2 \oplus ... \oplus W_J \oplus V_J \quad (2.23)
\]

Given the decomposition of $V_0$ as a sum of mutually orthogonal subspaces, represented by equation (2.23), and given that $f(t) \in V_0$, it is possible to represent the function $f(t)$ as a sum of orthogonal components residing in the subspaces.

A multiresolution analysis (MRA) could be written as:

\[
f(t) = \omega_1(t) + \omega_2(t) + ... + \omega_J(t) + v_J(t) \quad (2.24)
\]

with $\omega_j(t) \in W_j$ for $j = 1, ..., J$ and with $v_J(t) \in V_J$.

The generic component of this decomposition may be represented, relative to the basis functions $\psi_{j,k}(t); k = 0, 1, ..., [N/2^j-1]$ of $W_j$, by:

\[
W_j(t) = \sum_{k=0}^{[N/2^j-1]} \omega_{j,k} \psi_{j,k}(t) \quad (2.25)
\]

Here, $\omega_{j,k}$ is the amplitude coefficient of the $k$th wavelet function. The two final elements of the decomposition of (2.24) are the wavelet function $W_J(t) = \omega_{J,0} \psi_{J,0}(t)$ and the scaling function $V_J(t) = v_{J,0} \phi_{J,0}(t)$, which have been scaled by the amplitude coefficients $\beta_{n0}$ and $\gamma_{n0}$, respectively.

In general, the MRA could be considered as a filtered version of the series that retain
substantial parts of the series. However there are some points that would be considered in the case of interpretation of MRA using DWT:

- The number of observations dictates the number of scales. Given the number of observations, $N \geq 2^j$, only $j - 1$ scales can be produced. For example, if $N = 1024 = 2^{10}$ is the monthly observations, then the maximum number of scales would be $j = 9$.

- The multiresolution decomposition assumes that data are sampled at equally spaced intervals. The frequency resolution interpretation is more difficult with tick-by-tick data, as those data are not evenly sampled.

- Existing stylized facts need to be taken into account when applying a multiresolution decomposition to economic data. For example, as economists know that business cycles last for a decade at the most, it does not make sense to decompose a series beyond this level. So with annual data it should not use anything more than the level 3 decomposition $d3$, doing so would cause redundancy.

2.3.6 Matrix Formation of a Wavelets Analysis

In this section, in order to expand the wavelet concept, I denote the matrix formation of a wavelet analysis using slightly different notation than used in previous section. Suppose $y = [y_0, ..., y_{N-1}]'$, where $N = 2^j$ ($j = 1, ..., J$) represents the vector of observations, which are associated with the scaling functions of the initial basis, and let $\beta = [\beta_0, ..., \beta_{N-1}]'$ represent the vector of the coefficients associated with the wavelets of the final basis. Here, $\beta_{N-1} = \gamma_{J,0}$ is the coefficient associated with the single scaling function in the ultimate subdivision of the frequency range. The mapping from $y$ to $\beta$, denoted by $\beta = Q'y$, is effected by an orthonormal matrix $Q$ such that $QQ' = Q'Q = I_N$.

Since $(Q')^{-1} = Q$, it follows that there is an inverse transformation from the wavelet coefficients to the data of the form $Q\beta = y$. This mapping from $\beta$ to $y$ affects a wavelet
synthesis. If $\beta$ contains a single non-zero element representing the amplitude coefficient of a solitary wavelet, then the mapping of $\beta$ via $Q$ will generate the vector, corresponding to a signal column of $Q$, containing elements that approximate the ordinates of that wavelet, sampled at unit intervals.

The $N$ elements of the vector $\beta$ can be ordered in a manner that correspond to a dyadic decomposition. Within $\beta$, there is a succession of wavelet functions of the final basis. The subvectors of

$$\beta = [\beta'(1), \beta'(2), ..., \beta'[J], \gamma'(J)]'$$  \hspace{1cm} (2.26)

are

$$\begin{align*}
\beta(1) &= [\beta_1, \beta_2, ..., \beta_{N/2-1}]' \\
\beta(2) &= [\beta_{20}, \beta_{21}, ..., \beta_{2[N/4]-1}]' \\
&\vdots \\
\beta(J-1) &= [\beta_{J-1,0}, \beta_{J-1,1}]' \\
\beta(J) &= [\beta_{J,0}] \\
\gamma(J) &= [\gamma_{J,0}] 
\end{align*}$$  \hspace{1cm} (2.27)

A linear filter can be applied to a finite data sequence via a matrix transformation of the vector $y$ of the data. Let the $z$-transform of a causal filter be represented by the polynomial $c(z) = c_0 + z_1 z + ... + c_{M-1} z^{M-1}$ and assume that the filter is applied to the data via a process of circular convolution.

Then, the matrix transformation that implements the filter can obtained by replac-
ing the powers of \( z \) by powers of a circulant matrix \( K_N = [e_1, e_2, ..., e_{N-1}, e_0] \). This matrix is formed from the identity matrix \( I_N = [e_0, e_1, e_2, ..., e_{N-1}] \) by moving the leading vector \( e_0 \) to the end of the array. The resulting matrix is

\[
c(K_N) = c_0 I_N + z_1 K_N + ... + c_{M-1} K_N^{M-1}
\]  

(2.28)

and the filter vector is given by \( c(K_N)y \).

A process of down sampling can also be affected by a matrix transformation. The down sampling matrix is \( V = \Lambda' = [e_0, e_2, e_4, ..., e_{N-2}]' \), which is obtained by deleting alternate rows from the identity matrix \( I_N \).

To see in detail how the wavelet amplitude coefficients can be generated in this manner, it is best to take a specific example. In the example, there are \( N = 8 = 2^3 \) data points and there are \( M = 4 \) coefficients in the dilation equations. Each stage of the process that converts the data into the wavelet coefficients involves the application of a linear filter followed by a process of down sampling.

The highpass filter that is to be applied to the data in the first round of the wavelets decomposition has the following matrix representation:

\[
H_{(1)} = \begin{bmatrix}
h_0 & 0 & 0 & 0 & h_3 & h_2 & h_1 \\
h_1 & h_0 & 0 & 0 & 0 & h_3 & h_2 \\
h_2 & h_1 & h_0 & 0 & 0 & 0 & h_3 \\
h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 \\
0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\
0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 \\
0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\
0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0
\end{bmatrix}
\]

Premultiplying this by the down sampling matrix is a matter of deleting alternate
When this matrix is combined with the matrix $VG_{(1)}$, which is the down sampled version of the lowpass filter matrix, and when the data vector $y$ is mapped through the combined matrix, the result is:

$$
\begin{bmatrix}
\beta_{10} \\
\beta_{11} \\
\beta_{12} \\
\beta_{13} \\
\gamma_{10} \\
\gamma_{11} \\
\gamma_{12} \\
\gamma_{13}
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 0 & 0 & h_3 & h_2 & h_1 \\
h_2 & h_1 & h_0 & 0 & 0 & 0 & h_3 \\
0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\
0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \\
g_0 & 0 & 0 & 0 & g_3 & g_2 & g_1 \\
g_2 & g_1 & g_0 & 0 & 0 & 0 & g_3 \\
0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 \\
0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 \\
\end{bmatrix}
\begin{bmatrix}
y_0 \\
y_1 \\
y_2 \\
y_3 \\
y_4 \\
y_5 \\
y_6 \\
y_7
\end{bmatrix}
$$

The transformation can be represented, in summary notation, by:

$$
\begin{bmatrix}
\beta_{(1)} \\
\gamma_{(1)}
\end{bmatrix}
= 
\begin{bmatrix}
VH_{(1)} \\
VG_{(1)}
\end{bmatrix}
\begin{bmatrix}
y
\end{bmatrix}
$$

In the second round of the wavelets decomposition, the coefficients associated with the level-1 wavelets are preserved and the coefficients associated with the level-1 scaling
functions are subject to a further decomposition:

\[
\begin{bmatrix}
\beta_{10} \\
\beta_{11} \\
\beta_{12} \\
\beta_{13} \\
\beta_{20} \\
\beta_{21} \\
\gamma_{20} \\
\gamma_{21}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & h_0 & h_3 & h_2 & h_1 \\
0 & 0 & 0 & 0 & h_2 & h_1 & h_0 & h_3 \\
0 & 0 & 0 & 0 & g_0 & g_3 & g_2 & g_1 \\
0 & 0 & 0 & 0 & g_2 & g_1 & g_0 & g_3
\end{bmatrix}
\begin{bmatrix}
\beta_{10} \\
\beta_{11} \\
\beta_{12} \\
\beta_{13} \\
\gamma_{10} \\
\gamma_{11} \\
\gamma_{12} \\
\gamma_{13}
\end{bmatrix}
\]  \tag{2.31}

The summery notation for this is:

\[
\begin{bmatrix}
\beta_{(1)} \\
\beta_{(2)} \\
\gamma_{(2)}
\end{bmatrix} =
\begin{bmatrix}
I & 0 \\
0 & VH_{(2)} \\
0 & VG_{(2)}
\end{bmatrix}
\begin{bmatrix}
\beta_{(1)} \\
\gamma_{(1)}
\end{bmatrix}
\]  \tag{2.32}

The effect of the down sampling upon the circular filter can be seen in equation (2.31). The two filters are defined on four points and, at this level, only four data points are available. There are no zeros remaining within the matrices $VH'_{(2)}$ and $VG'_{(2)}$.

In the next round of filtering, there are only two data points to be mapped through the filters. The consequence is that $\gamma_{20}$ and $\gamma_{21}$ must be used twice in the third and final transformation. This can be represented equally by:

\[
\begin{bmatrix}
\gamma_{30} \\
\beta_{30}
\end{bmatrix} =
\begin{bmatrix}
h_0 & h_3 & h_2 & h_1 \\
g_0 & g_3 & g_2 & g_1
\end{bmatrix}
\begin{bmatrix}
\gamma_{20} \\
\gamma_{21} \\
\gamma_{20} \\
\gamma_{21}
\end{bmatrix}
\]  \tag{2.33}
or by

\[
\begin{bmatrix}
\gamma_{30} \\
\beta_{30}
\end{bmatrix} =
\begin{bmatrix}
h_0 + h_2 & h_3 + h_1 \\
g_0 + g_2 & g_3 + g_1
\end{bmatrix}
\begin{bmatrix}
\gamma_{20} \\
\gamma_{21}
\end{bmatrix}
\]

(2.34)

On the LHS is a vector containing the amplitude coefficients, respectively, of a wavelet and a scaling function stretching the length of the data sequence.

A general expression can now be given for the set of amplitude coefficients associated with the rejection of the function \(f(t)\) onto the basis of the subspace \(w_j\). These coefficients are contained in the \(j\)th vector of the sequence of (2.27), which is given by

\[
\beta_j = VH_{(j)}VG_{(j-1)}...VG_{(1)}y = Q'_{(j)}y
\]

(2.35)

In order to relieve the burden of notation, the subscripts have been omitted from the succession of downsampling matrices that would indicate their orders. Reading from right to left, the first downsampling matrix is \(V_{(1)}\) of order \(N/2 \times N\). The penultimate matrix is \(V_{(j-1)}\) of order \(N/2^{j-1} \times N/2^{j-2}\) and the final matrix is \(V_{(j)}\) of order \(N/2^j \times N/2^{j-1}\).

On the RHS of (2.35) is the matrix \(Q'_{(j)}\), which represents a submatrix formed from a set of adjacent rows of the matrix \(Q'\), which is entailed in mapping \(\beta = Q'y\) from the sampled ordinates of \(f(t)\) to the amplitude coefficients of the final basis. Given that \(QQ' = I_N\), it follows that \(Q\beta = y\) represents the synthesis of the vector \(y\) from the amplitude coefficients.

The vector \(\beta_{(j)}\) of (2.35) is entitled in the synthesis of the component vector \(\omega_j = [\omega_{0j}, \omega_{1j}, ..., \omega_{N-1,j}]'\) of decomposition of \(y = \omega_1 + ... + \omega_J + \nu_J\).

The synthesis can be represented by

\[
\omega_j = Q_{(j)}\beta_{(j)} = G'_{(1)}\Lambda...G'_{(j-1)}\Lambda H'_{(j)}\Lambda\beta_{(j)}
\]

(2.36)
where $\Lambda = V'$ represents the upsampling matrix, which interpolates zeros between the elements of any vector that it premultiplies.

### 2.4 Wavelets in Economics and Finance

Wavelet analysis has been very popular in different areas of science, in particular, electrical engineering, physics and other disciplines due to its capable properties. Although, the application of wavelet in economics is increasing in the recent years. (see Aguiar-Conraria et al., 2010; Rua, 2012; Tiwari et al., 2013)

By reviewing the existing papers that have applied wavelets in economics and finance research, it seems that the interest of applying wavelets in economics literature began in 1990s and later is expansively introduced by Ramsey (1999), Gençay et al. (2002) and Crowley (2007).

#### 2.4.1 Wavelet Literature

Broadly speaking, reviewing the extensive numbers of papers that have been applying wavelet analysis in economics drives me to categorise the applications of wavelets into three main groups. The first and largest category denotes the wavelet property corresponding with nonstationary series. The second group is concerned with the time-scale (multiresolution) decompositions. The last category represents the wavelet’s ability in forecasting.

**Nonstationarity and Complex Functions**

The assumption of stationarity of time series is not always justified in practice. There are numerous examples of nonstationarity process, like foreign exchange rate and financial data that would need suitable and flexible tool for analysing. The analysis of nonstationary time series cannot be accomplished by classical time domain representati-
tions such as correlation methods, or by frequency domain representations based on the Fourier transform. Fourier analysis is not flexible enough, because it is only applied for stationary signals within infinite data range and also it provides a poor representation of signals that are tightly localized in time.

However, wavelet transform provides a natural platform to deal with the time-varying characteristics in time series. Thus, the assumption of stationarity may be avoided. The wavelet transform intelligently adapts itself to capture features across a wide range of frequencies. Hence, it has the ability to capture events that are local in time. This feature makes the wavelet transform an ideal tool for analysing nonstationary time series. The following studies examine the nonstationarity concept in terms of existence of a unit root process:

Pan and Wang (1998) introduce an estimator based on the wavelet transform to explore nonlinear and complex relationships in the financial markets, because they believe the traditional spectral analysis cannot preserve the time dependence of relevant patterns when the series are nonstationary. They affirm that their estimator captures dynamic and stochastic elements of the data by combining the state space model with the wavelet transform. The results show that the new estimator works better with complex problems involving nonlinear, dynamic, multivariate relations than it does with problems involving linear, simple, univariate relations. Their findings also illustrate that the wavelet-based estimator can be applied to any regression problem without constraints on the number of data points.

von Sachs and Neumann (2000) design a wavelet-based unit root to test stationarity in locally stationary process. Simply, the null hypothesis is that the time series is covariance stationary. They try to detect the changes in the covariance structure of a stochastic process. They assert that wavelet-based test with mixed scale indices have rarely been used in the literature. Therefore, the use of this test has crucial advantages for detecting localized deviations from a time-constant spectral density. They state
that it is quite important to understand the dynamics with respect to possibility of very sudden changes in the variance of stochastic stock return models.

Fan and Gençay (2010) who propose a wavelet-based unit root to test stationarity, design wavelet-based unit root test by decomposing the variance of the underlying process. In fact, they decompose the variance into the variance of its low frequency components and that of its high frequency components via the discrete wavelet transformation (DWT). The findings show that their test has good size and comparable power against unit root alternative in finite sample.

Nason (2013) develops von Sachs and Neumann stationary test. He, however, constructs the test over a finite set of wavelet scales rather than a smoothed set of frequencies. The results denote that the test is well-designed to work with not only Gaussian time series but also non-Gaussian time series. He claims that his test works better on heavy tailed data compared with existing stationary tests.

**Time-Scale Decomposition**

One of the useful properties of the wavelet analysis is the ability to decompose a time series into its time-scale components. It is well known that economic relations have different behaviour across different time scales and they may even show the opposite behaviour at the aggregate level. Thus, it is a crucial issue for economists to know about changes in relationships in different time scales like short-run and long-run. As an illustration of short and long time horizons, in the securities market there are traders who take a very long view (i.e. years). These traders ignore ephemeral phenomena. In contrast, other traders trade on a much shorter time-scale (i.e. a few months to a year). Yet other traders are in the market for whom a day is a long time.

The empirical literature exhibits that the wavelet decomposition ability is the most applicable property of wavelet in analysing economics and financial time series. More specifically, this feature has been used to analyse several grounds, namely complex
relationship between variables (Ramsey and Lampart, 1998; Ramsey, 2002; Dalkir, 2004; Duchesne, 2006), measuring comovement (Rua and Nunes, 2009; Rua, 2010; Akoum and et al., 2012; Uddin et al., 2014), filtering and business cycles (Gençey et al., 2001; Yogo, 2008), and wavelet variance in finance (Lee, 2004; Gençay et al., 2004, 2005; Fernandez, 2005; In and Kim, 2005, 2006, 2007; Gallegati and Gallegati, 2007; Gallegati, 2008; In et al., 2008; Barunik et al., 2011).

According to the intuition that economic relationships demonstrate different behaviour through different time horizons, the wavelet decomposition is an useful analysis to recognise the relationships between economic variables at different time scales (disaggregate level) rather than at the aggregate level. Ramsey and Lampart (1998) stress the importance of time-scale analysis in economic relationships. They examine two relationships, namely consumption-income and money-income relationships over six different time scales by applying wavelet analysis. The results display that the relationship between consumption and income vary and the relevant variables differ across different scales. The real interest rate is discovered as an important factor only for longest time scales and for durable goods. They also reveal an answer for a long debate in money-income relationship, as such income causes money at the shortest time, and money causes income, at the intermediate time.

Shrestha and Tan (2005) empirically investigate the long-run and short-run relationships between real interest rates in G7 countries. They believe that the real interest rate in different countries must be identical. A wavelet transform is applied to indicate the existence of both short-run and long-run relationships. Their findings show the strong evidence of existence of long-run relationship between the U.S. real interest rate and the real interest rates in the other six G7 countries. They also find that there are significant short-run relationships between the US real interest rate and real interest rates in Canada, Japan, Germany, and Italy.

Aguiar-Conraria et al. (2008) consider the dependencies between monthly interest
rates and industrial production, inflation and monetary aggregates M1 and M2 of the US economy. They apply continuous wavelet power spectrum and three cross-wavelet instruments to uncover the economic time-frequency relationships. They show that the relationships between economic variables change in time and are not homogeneous across frequencies.

Wavelet analysis is able to provide a unified framework to measure comovement in the time-frequency space. Rua and Nunes (2009) examine the comovements between the stock markets of the USA, the UK, Germany and Japan between 1973 and 2007. Their paper shows that the interdependence changes in time and also varies across frequencies. The comovements between the US and the European markets are the strongest whereas Japan is independent at almost all times and frequencies. The interdependence of Germany with the USA and the UK increases in time and from 2000 onwards. The wavelet coherence is significant for all frequencies for US-Germany and UK-Germany pairs. The same results are found for separate sectors of the markets.

In the case of filtering, wavelet filtering is an alternative to band-pass filtering. As a matter of the fact, wavelet analysis provides better resolution in the time domain since wavelet basis functions are time-localized (in addition to being scale-localized), which is useful for capturing the changing volatility of the business cycle.

Gençay et al. (2001) propose a wavelet-based decomposition filter for intraday seasonality extraction. The proposed method is based on a wavelet multi-scaling approach which decomposes the data into its low and high-frequency components. They assert that their method is simple to implement, because it does not depend on a particular model selection criterion or parameter choices. Their findings include an increase of correlation from intraday scale towards the daily time-scale and the stabilization of correlation for longer time scales.

Crowley and Lee (2005) consider the frequency components of European business cycles with wavelet multiresolution analysis. They use a real GDP as a proxy for the
business activity of European countries. The results show significant differences between European countries in the degree of integration. Some countries like Germany, France and Belgium have strong correlations with the Euro zone aggregate. But, Finland, Ireland, Sweden and the UK have much lower correlation with the Euro zone aggregate.

Yogo (2008) states wavelet analysis is a natural way to decompose an economic time series into trend, cycle, and noise. He uses the multiresolution wavelet to filter the US real GDP in the post-war period. His findings reveal business-cycle component of the wavelet-filtered series closely resembles the series filtered by the approximate band-pass filter.

Raihan et al. (2005), Gallegati et al. (2008), Crowley and Mayes (2008), and Aguiar-Conraria et al. (2011) also resort to wavelets for business cycle analysis.

A board range of studies in finance have been using the wavelet variance, wavelet correlation and cross-correlation. Gençay et al. (2004) propose a simple yet powerful method to explore the relationship between a stock market return and volatility on multiple time scales using wavelet decomposition. The results show that the leverage effect is weak at high frequencies but becomes prominent at lower frequencies. Also the positive correlation between the current volatility and future returns becomes dominant on the time-scales of one day and higher, providing evidence that risk and return are positively correlated.

Barunik et al. (2011) present dynamics of cross-correlations between Central European and Western European stock markets using wavelet power spectrum and wavelet coherence. They try to show how correlations are changing in time and across frequencies, continuously. The findings provide a possibility of using wavelet analysis in financial risk modelling. That is, they could have strong implications to portfolio management.

There are more papers that apply the wavelet multiresolution property such as Davidson et al. (1998), Whitcher et al. (2000), Gençay et al. (2002), Capobianco
Forecasting

This section discusses the potential ability of wavelet in forecasting time series. Broadly speaking, the idea behind the forecasting approach is relatively simple and direct. First the series to be forecast is decomposed into its constituent time-scale components. Then for each time-scale, a model is fitted and used for forecasting. Finally, forecast components are recombined to obtain the overall forecast of the series. There are some papers which applied wavelets for forecasting with promising results. (see Renaud et al., 2002; Conejo et al., 2005; Fernandez, 2007)

Wong et al. (2003) attempt to investigate the modelling and forecasting method for nonstationarity by using wavelet approach. They consider the US dollar against DM exchange rate data from August 1989 to July 1991. They propose a modelling procedure that decomposes the series as the sum of three separate components, namely trend, harmonic and irregular components. They also compare the various methods with their one-step-ahead prediction errors. The results illustrate that all the proposed estimators to detect trends and hidden frequencies are strongly consistent. They also claim that forecasting based on wavelets is a vital alternative to existing method.

Yousefi et al. (2005) investigate the future market efficiency for oil market based on wavelet methodology. A wavelet-based prediction procedure is introduced and used to provide forecasts on crude oil market over different forecasting horizons. The results illustrate a persistent pattern displaying that the wavelet-based forecast procedure outperforms the future market in average. Their results confirm the idea that the futures market might not be efficiently priced.

Since the previous paper forecast univariate models for modelling each time-scale components, Rua (2011) extends his modelling framework by considering factor-augmented
models. His method enables to handle large dataset model. In fact, he proposes a wavelet approach for factor-augmented forecasting and put to test for forecasting GDP growth for the major Euro area countries. He evaluates the out-of-sample performance of several alternatives for forecasting one- and two-quarter-ahead GDP growth. The results show that the forecasting performance is enhanced when wavelets and factor-augmented models are used together.

Table (2.1) shows some of the research in economics and finance that have applied wavelet analysis.
Table 2.1: List of Papers that Applied Wavelet Transform

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2.5 Investigation of Purchasing Power Parity Applying Wavelet Analysis

This section provides an empirical application of wavelet analysis to illustrate the use of the described tool.

2.5.1 Introduction

The oldest building block and the most controversial issue in the theory of exchange rate is the purchasing power parity (PPP) doctrine which expresses the relationship between exchange rates and prices. The origins of the doctrine can be discovered in the writing of Wheatley and Ricardo in 19th century and then followed by Cassel’s writing mainly during 1920s, (Frenkel, 1981).

Simply put, the purchasing power parity is defined as the nominal exchange rate between two currencies which should be equal to the ratio of aggregate price levels between the two countries, so that a unit of currency of one country will have the same purchasing power in the foreign country, (Taylor and Taylor, 2004).

Balassa (1964) describes the purchasing power parity in two different versions, namely the “absolute” and “relative” interpretation. The former version calculates purchasing power parity as a ratio of consumer goods prices for any pair of countries tend to approximate the equilibrium rates of exchange. However, the latter version states that changes in relative prices would indicate the necessary adjustments in exchange rates, given prevailing equilibrium rates.

The cornerstone of any variation of purchasing power parity is the law of one price (LOP). It declares that, absent natural or government imposed trade barriers, a commodity should sell for the same price everywhere. The critical factor to hold this equality is arbitrage. On the other hand, a huge number of empirical evidence shows that the law of one price fails considerably in practice. (see Froot and Rogoff, 1995)
The law of one price denotes that for any good $i$:

$$ P_{i,t} = S_t P_{i,t}^* $$

where $P_{i,t}$ is the price of good $i$ in terms of the domestic currency at time $t$, $P_{i,t}^*$ is the price of good $i$ in terms of the foreign currency at time $t$, and $S_t$ is the nominal exchange rate, defined as the ratio of domestic currency to foreign currency at time $t$.

However, policy makers require a comprehensive measure of international price differentials. So, the absolute purchasing power parity is designed.

$$ \sum P_{i,t} = S_t \sum P_{i,t}^* $$

Absolute PPP holds when the purchasing power of a unit of currency is exactly equal in the domestic economy and in a foreign economy, once it is converted into foreign currency at the market exchange rate. It predicts that real exchange rates should equal to one, or at least have a tendency to return quickly to one when that long-run ratio is disturbed by some reasons. Nevertheless, from the empirical viewpoint several issues may be raised. First, it is difficult to determine whether the same basket of commodities is available in two different countries. Secondly, the measures of consumer prices are typically computed as indexes relative to a base year. That means they only measure the rate of changes of the price level from the base year, not its absolute level, (Obstfeld and Rogoff, 1996).

Alternatively, the relative version of PPP could be tested.

$$ \frac{\sum P_{i,t+1}}{\sum P_{i,t}} = (S_t + 1)\left(\frac{\sum P_{i,t+1}^*}{\sum P_{i,t}^*}\right) $$

Relative PPP holds the percentage change in the exchange rate over a given period which offsets the difference in inflation rates in the countries over the same period.
2.5.2 Brief Literature of Purchasing Power Parity

In the economic theoretical framework, purchasing power parity has been discussed initially by classical economists such as John Stuart Mill, Viscount Goschen, Alfred Marshall, and Ludwig von Mises. Right after the World War I, when the world financial system collapsed during the war, countries confronted the real problem of deciding how to reset the exchange rate with minimal disruption to prices and government finances. Simply returning to pre-war gold standard exchange rate system made no sense due to hugely differing inflation experience of hostile countries over the war. Under these circumstances, PPP is initiated by Cassel (1921, 1922) as an empirical tool for setting the relative gold parities, (Rogoff, 1996).

PPP during the 1920s played an important role in shaping the current PPP doctrine. Frenkel (1981) investigates the PPP under hyperinflationary conditions during the 1920s. His findings indicate some support for PPP proposition, such that in most cases the results are consistent with the hypothesis that the changes in the exchange rate with respect to price ratio is about unity. Furthermore, the exchange rate and price ratio under PPP affect each other in the same manner. He also emphasises that short-run deviations from PPP may reflect the fact that not all markets adjust at the same speed.

Since the Bretton Woods agreement is terminated in 1971 and the world began using the floating exchange rate system in March 1973, the short-run relationship between changes in exchange rates and inflation rates divergences from PPP during the 1970s. The evidence on PPP over the 1970s contrasts with the evidence from the other time period such as the 1920s when the PPP doctrine held up reasonably well. For instance, Frenkel (1981) compares the results of PPP in two periods, 1920s and 1970s. He finds extremely poor results and imprecise estimations to support PPP during the 1970s.

Rogoff (1996) describes the “purchasing power parity puzzle” as the question of how to reconcile high short-term volatility of real exchange rates with extremely slow
convergence to purchasing power parity (PPP). Since Rogoff (1996) first noted the PPP puzzle by saying while some empirical economists take PPP seriously as a short-term preposition, most instinctively believe in changes in purchasing power parity as an anchor for long-run real exchange rates, researchers have sought to address this issue in their research. Some studies reveal that in the long-run, real exchange rate is inclined towards purchasing power parity along with extremely slow speed of convergence to PPP. Whereas, in the short-run, deviations from PPP are large and volatile.

A massive empirical literature on PPP attempts to solve this controversy. However, after many years of intensive research, the empirical validity of PPP remains far from conclusive. An extensive variety of papers focus on the econometrics techniques to examine whether PPP holds. Hence, it is useful to classify the testing procedures of the validity of PPP into three groups.

First, in the earlier empirical literature, testing PPP is based on estimates of the following simple equation. They do not introduce dynamics in the estimated equation in such a way as to distinguish between short-run and long-run effects.

\[ s_t = \alpha + \beta p_t + \beta^* p_t^* + \omega_t \]  

(2.40)

where \( s_t \), \( p_t \) and \( p_t^* \) denote the log of nominal exchange rates, log of domestic price level and log of foreign price level, respectively. \( \omega_t \) is a disturbance term.

The restrictions of \( \beta = 1 \) and \( \beta^* = -1 \) are tested as a test of PPP hypothesis. It is expected that the PPP holds in the long-run. Precisely, there are two distinguishing tests for those coefficients \( \beta \) and \( \beta^* \). First, the test that \( \beta \) and \( \beta^* \) are equal and of opposite sign (the symmetry condition). Secondly, the test that \( \beta \) and \( \beta^* \) are equal to unity and minus unity, respectively (the proportionality condition), (Sarno and Taylor, 2002).

However, the estimates of equation (2.40) suggest rejection of the PPP hypothesis.
Nevertheless, Frenkel (1978) ascertains that the estimates of $\beta$ and $\beta^*$ are very close to unity. He states that PPP represents an important benchmark for long-run. He also asserts that the rejection of PPP may be due only to temporary real shocks and price stickiness in the goods market, but convergence to PPP is expected to occur in the long-run.

On the other hand, a main weakness of the early studies is they do not consider the prospect of nonstationarity of exchange rates and price levels. If both nominal exchange rates and price levels are nonstationary variables (and are not cointegrated) then equation (2.40) is a spurious regression. So, the conventional OLS-based statistical interference is invalid. Hence, researchers in the new development attempt to examine the issue of nonstationarity of the PPP equation. They believe that the relative version of PPP, the change in exchange rates offsets the differential in the relative change in prices between countries, implies the real exchange rate should be stationary. Thus, they test the stationarity of real exchange rate using unit root tests. Unit root tests are typically based on the Dickey-Fuller (DF) tests. However, the results indicate the low power of the DF tests to support the PPP hypothesis. It implies that the deviations from PPP are permanent. (see Sarno and Taylor, 2002; Taylor and Taylor, 2004; Taylor, 2006) Long (2010) states that the main reason of rejecting PPP using unit root tests is that time series data used had not been long enough for the unit root tests to have power.

On the other hand, using nonlinear KSS unit root tests is the latest expansion in testing the PPP hypothesis. Su et al. (2014) propose a nonlinear unit root test to evaluate the stationarity of real exchange rates. The results are not able to reject the null hypothesis of unit root in the real exchange rates. Their findings are relatively similar with earlier studies and implies that the PPP does not hold and arbitrage opportunities exist.

As the studies using unit root tests (linear or nonlinear) have criticized on the
ground that they do not employ enough observations over longer period, hence they suffer from low power and misleading results, other studies employ panel unit roots. Chortareas and Kapetanios (2009) stress that using panels in testing unit root of real exchange rates is popular partly because the results of such studies tend to uncover more evidence for PPP. However, a major drawback of the unit root panel methodology is that the null hypothesis of nonstationarity is a joint hypothesis for all the real exchange rates in the panel. Thus, the null hypothesis will be violated even if only one of the real exchange rate series in the panel is in fact stationary, (Taylor and Sarno, 1998). Generally speaking, panel framework studies provide mixed results in testing validity of the PPP. (See Taylor and Sarno, 1998; Chortareas and Kapetanios, 2009; Bahmani-Oskooee et al., 2014)

Third approach to test the PPP hypothesis is cointegration analysis as initiated by Engle and Granger (1987). According to this approach, PPP is commonly interpreted as the comovement of the exchange rate and the relative price level of two countries. The emphasis of cointegration analysis is to establish the stationarity of the residuals of estimated regression models since the nominal exchange rate and relative prices are shown to be nonstationary. In other words, while short-run deviations from the equilibrium level are admissible, a necessary condition for long-run PPP to hold is that the equilibrium error is stationary over time. If this is not the case then the nominal exchange rate and the relative price permanently tend to deviate from each other. Kim (1990) examines long-run PPP preposition in the bilateral exchange rates of the dollar against five currencies applying cointegration technique. He finds out when cointegration is confirmed, the cointegrating coefficient between that exchange rate and the price ratio is close to one for a very large sample of data, suggesting that the PPP holds in long-run. However, other studies applying the same technique could not provide much support for PPP such as Bahmani-Oskooee (1993).
2.5.3 Methodology and Estimation Results

The absolute PPP relationship can be written as:

\[ \ln S_t = \alpha + \beta \ln \left( \frac{P}{P^*} \right)_t + u_t \]  

(2.41)

where \( S_t \) and \( \left( \frac{P}{P^*} \right)_t \) show the nominal exchange rate (defined as the ratio of domestic currency over foreign currency), and the ratio of domestic to foreign price indexes, respectively. \( u_t \) denotes the error term. The relative PPP also can be written as:

\[ \Delta \ln S_t = \beta \Delta \ln \left( \frac{P}{P^*} \right)_t + v_t \]  

(2.42)

where \( \Delta \ln S_t \) displays the percentage change in the spot exchange rate and \( \Delta \ln \left( \frac{P}{P^*} \right)_t \) shows the percentage change in the ratio of the consumer price indexes. \( v_t \) denotes the error term.

As the short-run version of PPP is rejected by numerous studies such as Frenkel (1981), I examine that whether such a relationship holds in the longer run. In this section, I apply new analysis - wavelet analysis - to develop the PPP consideration. Wavelet analysis allows to identify different relationship between exchange rate and price ratio. This means wavelets enable us to observe the behaviour of purchasing power parity proposition in long-run and short-run simultaneously. One of the advantages of applying wavelet analysis is that there is no need to test the stationarity of variables. The crucial critique in the early literature is that they do not investigate the stationary of the residuals in the estimated PPP equation, (Sarno and Taylor, 2002). Applying wavelet overcomes this old issue of estimate in time series. In addition, Taylor (2001) addresses that high frequency dataset is an ideal to solve the problems in testing PPP and also LOP, but we have been forced to use low frequency data particularly for long-span, because that was all what was available. Under these circumstances, wavelets could be a solution. Because it provides more details of given data sequence. In fact, it sheds light
on both dimensions of given sequence; frequency and time domains, contemporarily.

The empirical work in this section is based on monthly data for the period January 1993 to April 2014. Prior to apply wavelet decomposition, I estimate both absolute and relative PPP equations at level for the monthly Pound/Dollar exchange rates using the consumer price indexes for the UK and the US. I use the robust standard errors of OLS estimation method, due to serial correlation in the error terms.

To consider the PPP in short-run and long-run simultaneously, I first apply wavelet analysis (specifically MODWT) to decompose the bilateral exchange rates and the aggregate price ratio individually, in order to examine whether there are distinct differences in the relationship between these variables at different time scales. Next, I estimate both absolute and relative PPP equations for each level using the robust standard errors of OLS method. Table (2.2) reports the estimates of equation (2.41). As may be seen, the change in the exchange rate with respect to price ratio is decomposed into seven levels $d_1, d_2, d_3, d_4, d_5, d_6$ and $d_7$ applying maximal overlapping discrete wavelet analysis. Each level corresponds to different frequency and it usually starts from high frequency ($d_1$) to low frequency ($d_7$). However, the time scale grows from 1 month to 64 months which is equivalent to 5 years and 4 months. Time scale variation in the second column clearly indicates that the time horizons move from short time spans to long time spans.

The results indicate that the effect of price ratio on nominal exchange rate in 1 month and 2 months are almost zero. This means the change in price ratio does not affect the exchange rate in short-run. Nevertheless, the coefficients of price ratio grow up in the middle-run, i.e., 4 months and 8 months and diverge from unity. It again goes to zero in 16 months (1 years and 4 months). At 32 months (2 years and 6 months) is far from unity. However, at the longest time period, 64 months (5 years and 4 months)

---

3The reasons of choosing this time period are as follows: first, Pound has experienced a crash in 1992, in order to have a smooth sample, I choose a year after the currency crash. Secondly, applying wavelet analysis imposes to use a dyadic number of data. So in this case I take 256 observations equivalent to $2^8$. 

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the coefficient of price ratio is close to one and R-squared is 51% which means the price ratio could explain 51% of exchange rate variations in long-run. So, the results provide evidence that the PPP holds in long-run.

Table 2.2: Absolute Purchasing Power Parities at Seven Levels: monthly data during 1993-2014

<table>
<thead>
<tr>
<th>Level</th>
<th>Sample period</th>
<th>Coefficient</th>
<th>s.e.</th>
<th>t-stat.</th>
<th>p-value</th>
<th>D.W.</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>1 month</td>
<td>0.643</td>
<td>0.423</td>
<td>-0.841</td>
<td>0.4</td>
<td>3.224</td>
<td>0.019</td>
</tr>
<tr>
<td>d2</td>
<td>2 months</td>
<td>0.922</td>
<td>0.497</td>
<td>-0.155</td>
<td>0.876</td>
<td>1.163</td>
<td>0.047</td>
</tr>
<tr>
<td>d3</td>
<td>4 months</td>
<td>2.582</td>
<td>0.768</td>
<td>2.06</td>
<td>0.04</td>
<td>0.255</td>
<td>0.218</td>
</tr>
<tr>
<td>d4</td>
<td>8 months</td>
<td>3.156</td>
<td>0.804</td>
<td>2.679</td>
<td>0.007</td>
<td>0.088</td>
<td>0.296</td>
</tr>
<tr>
<td>d5</td>
<td>16 months</td>
<td>0.277</td>
<td>0.663</td>
<td>0.678</td>
<td>0.277</td>
<td>0.019</td>
<td>0.001</td>
</tr>
<tr>
<td>d6</td>
<td>32 months</td>
<td>2.869</td>
<td>1.266</td>
<td>1.476</td>
<td>0.141</td>
<td>0.004</td>
<td>0.067</td>
</tr>
<tr>
<td>d7</td>
<td>64 months</td>
<td>1.314</td>
<td>0.177</td>
<td>1.776</td>
<td>0.077</td>
<td>0.002</td>
<td>0.517</td>
</tr>
</tbody>
</table>

Table 2.3: 95% Confidence Interval for Absolute PPP at Seven Levels

<table>
<thead>
<tr>
<th>Level</th>
<th>Sample period</th>
<th>Coefficient</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>1 month</td>
<td>0.643</td>
<td>-0.18608</td>
<td>1.47208</td>
</tr>
<tr>
<td>d2</td>
<td>2 months</td>
<td>0.922</td>
<td>-0.05212</td>
<td>1.89612</td>
</tr>
<tr>
<td>d3</td>
<td>4 months</td>
<td>2.582</td>
<td>1.07672</td>
<td>4.08728</td>
</tr>
<tr>
<td>d4</td>
<td>8 months</td>
<td>3.156</td>
<td>1.58016</td>
<td>4.73184</td>
</tr>
<tr>
<td>d5</td>
<td>16 months</td>
<td>0.277</td>
<td>-1.02248</td>
<td>1.57648</td>
</tr>
<tr>
<td>d6</td>
<td>32 months</td>
<td>2.869</td>
<td>0.38764</td>
<td>5.35036</td>
</tr>
<tr>
<td>d7</td>
<td>64 months</td>
<td>1.314</td>
<td>0.96708</td>
<td>1.66092</td>
</tr>
</tbody>
</table>

Table (2.3) displays the 95% confidence interval at seven different levels. In Figure (2.2) the confidence intervals are simply plotted. As the level increases in the horizontal axis in the Figure (which indicates that the time moves from short time period to longer time period), the confidence intervals rises. It implies that hardly anything is significantly different from one, but they are highly uncertain. However, level d7 which is the longest time horizon illustrates the smallest confidence interval. This means the coefficient is not significantly different from one as more certain. As may be seen in Table (2.3), the coefficient magnitude at 64 months time length (5 years and 4 months) is 1.314 which is close to unity.
Figure 2.2: 95% Confidence Interval of the Absolute PPP Coefficients at Seven Levels

Table (2.4) demonstrates the estimation of relative PPP in equation (2.42) as same time scale as the absolute PPP estimations in Table (2.2). As may be seen, the results are extremely poor and imprecise. Only coefficient on the price ratio at 16 months (1 years and 4 months) length is relatively close to one. But, it does not support the long-time horizon.

Table 2.4: Relative Purchasing Power Parities at Seven Levels: monthly data during 1993-2014

<table>
<thead>
<tr>
<th>Level</th>
<th>Sample period</th>
<th>Coefficient</th>
<th>s.e.</th>
<th>t-stat.</th>
<th>p-value</th>
<th>D.W.</th>
<th>R-squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>d1</td>
<td>1 month</td>
<td>0.084</td>
<td>0.458</td>
<td>-1.995</td>
<td>0.047</td>
<td>3.358</td>
<td>0.0002</td>
</tr>
<tr>
<td>d2</td>
<td>2 months</td>
<td>-0.008</td>
<td>0.498</td>
<td>-2.022</td>
<td>0.044</td>
<td>1.364</td>
<td>0.0000</td>
</tr>
<tr>
<td>d3</td>
<td>4 months</td>
<td>1.872</td>
<td>0.933</td>
<td>0.934</td>
<td>0.351</td>
<td>0.296</td>
<td>0.114</td>
</tr>
<tr>
<td>d4</td>
<td>8 months</td>
<td>5.486</td>
<td>1.338</td>
<td>3.353</td>
<td>0.00</td>
<td>0.106</td>
<td>0.3963</td>
</tr>
<tr>
<td>d5</td>
<td>16 months</td>
<td>1.267</td>
<td>1.361</td>
<td>0.196</td>
<td>0.844</td>
<td>0.021</td>
<td>0.016</td>
</tr>
<tr>
<td>d6</td>
<td>32 months</td>
<td>-0.868</td>
<td>0.956</td>
<td>-1.952</td>
<td>0.051</td>
<td>0.004</td>
<td>0.015</td>
</tr>
<tr>
<td>d7</td>
<td>64 months</td>
<td>2.476</td>
<td>0.312</td>
<td>4.718</td>
<td>0.000</td>
<td>0.002</td>
<td>0.433</td>
</tr>
</tbody>
</table>


2.6 Conclusion

It is undeniable that economics does not usually cope with long high frequency data series with natural periodicity. Time series analysis is well suited to analysis of shorter time series with variable non-natural periodicity, which also places its emphasis on causation and underlying processes. Fourier analysis also does not allow the frequency content of a series to change over time and thus it is unable in reproducing series that have time-varying features.

Wavelet analysis has its roots in classical Fourier analysis. However, wavelet analysis differs from Fourier analysis in the sense that it deals with both time and frequency domains, thereby it allows identification of both time period and scale.

Wavelet analysis has the potential abilities to offer much to empirical economic research. Its ability to work with nonstationary data is particularly advantageous, as most econometric methodology assumes stationarity and they are not capable to deal with time-varying characteristic of a series. Its ability to separate out a time series over a variety of different time horizons. This ability specially enable us to evaluate when there is a distinction between short and long-run relationships between variables. Its ability to analyse business cycles, which naturally lends itself to analysis of periodicities, and where filtering has been a particularly controversial issue; and in any part of macro or monetary economics where theoretical long-run and short-run relationships can be distinguished.

The ability to decompose a time series into its constituent scale is one of the useful properties of wavelet analysis for examining economic relationships. According to the intuition that economic relationships demonstrate different behaviour through different time horizons, the wavelet decomposition is an useful analysis to recognise the relationships between economics variables at different time scales (disaggregate level) rather than at the aggregate level.

The purchasing power parity theory (PPP) asserts that the long-run exchange rate
between two national currencies reflects only the ratio of their price levels. Old studies try to test this theory by regressing the nominal exchange rate on relative prices. They are criticized on the ground that the results suffered from a spurious regression problem. Combining the nominal exchange rate and relative prices together provides only one variable, i.e., the real exchange rate. More recent studies, therefore, have investigated the stationarity properties of the real exchange rates as a test of PPP. If the real exchange rate is said to be stationary, it supports the PPP doctrine. Otherwise, it may contain a unit root and reject PPP doctrine.

While empirical studies in favour of PPP have been dealing with testing the stationarity of real exchange rates, I suggest to apply a new approach -wavelet analysis- in examining the PPP doctrine. Applying wavelet analysis remedies the substantial concern of nonstationarity of exchange rates and price ratio under the PPP. I decompose the PPP equation into seven different levels using wavelet analysis, in order to observe the behaviour of exchange rate in different time scales. Then I estimate the PPP equation for each level using OLS method. The results display that the impact of price ratio on exchange rates are positive and close to unity. The findings show that the PPP holds for long-run horizon.

In the next chapter, I examine the relationship between exchange rate and interest rate under UIRP proposition applying the same method, wavelet analysis. As Taylor and Taylor (2004) point out using a dataset with more countries increases the amount of information employed in the test across the exchange rate, the power of the test should be raised. So, I apply bilateral currency for 10 different countries.
3.1 Introduction

Interest rates and exchange rates are important components of implementing monetary and fiscal policies in the economy. Policy-makers are interested in learning how interest rates and exchange rates are related, in order to design a policy and meet their targets by implementing economic policies. There is a notable evidence in the US economy in the late twentieth century that denotes an instability in the interest rates and exchange rates relationship. US interest rates and the foreign exchange rate in the 1970s and 1980s moved in opposite directions, such that, the short-term interest rates doubled from 1976 to 1979, whereas the value of dollar decreased by 17 percent. However, there has been a positive correlation between the interest rates and exchange rate in much of the 1980s period, as seen in Figure (3.1). It clearly implies that the correlation between interest rates and exchange rate can move in considerably different directions for an economy. A basic question that runs through the mind is whether the opposing changes in the linkage between interest rates and exchange rates can raise from the
There are ample theoretical and empirical studies, trying to justify this contradiction. Generally, in the literature, several mainstreams explain exchange rate behaviour, such as flexible-price monetary model, fixed-price model and sticky price Dornbusch overshooting model. The flexible-price monetary model is an aspect of the asset market approach for the exchange rate determination. It postulates that the product prices are perfectly flexible and also the bonds of different countries are perfectly substitutable. The main assumption of the flexible-price monetary model is that it relies mainly upon the purchasing power parity (PPP) hypothesis. As a consequence of this model, the inflation level is positively related to the nominal interest rate. In other words, when the domestic interest rate increases relative to the foreign interest rate, it is expected that the domestic currency loses value by inflation. When the demand of domestic currency decreases relative to the foreign currency, it depreciates rapidly. Therefore, this model represents that the interest rate and exchange rate are positively correlated in long horizon.
On the other hand, the fixed-price model which is introduced by Mundell-Fleming relies on a zero balance of payments. The exchange rate has a role of equilibrating variable to preserve the balance of payment. This model deals with determining the short-run relationship between interest rates and exchange rates. It explains that a higher domestic interest rate than a foreign one attracts a capital inflow leading to instantaneous appreciation of domestic currency. Therefore, a sustainable outcome of this model could be a negative relationship between exchange rates and interest rate differentials.

However, sticky price Dornbusch overshooting model (1976) is a hybrid of the two extremely opposite models. This approach compromises with the fixed-price model and financial markets that compensate in short-run by over adjusting. In particular, in the short-run, it refers to the fixed-price model and in the long-run it refers to the flexible-price monetary model, (Copeland, 2005).

Simply put, uncovered interest rate parity (UIRP) condition could represent the correlation between exchange rate and interest rates. UIRP is an important basis of the economic mainstreaks in the exchange rate determination theories. It precisely presents the key relation of the exchange rates and interest rate differentials. The UIRP relation assumes that the interest rate differentials between two countries should equal the expected exchange rate changes. Such that, a regression of exchange rate returns on the interest differentials should give an intercept of zero and a slope coefficient of unity, (Chaboud and Wright, 2005).

Generally speaking, the bulk of empirical studies in the literature have been testing the UIRP hypothesis in order to consider the interaction of exchange rates and interest rates. An overwhelming number of papers argue against UIRP and fail to support the value of unity of interest rate differentials in exchange rate prediction equations. (see Bekaert and Hodrick, 1993; Lewis, 1995; Engel, 1996; Baillie and Bollerslev, 2000;  

\footnote{Which is an intercept of zero and a slope coefficient of unity}
On the other hand, some empirical evidence reveals that the sign of slope coefficient holds positively unity under UIRP condition. (see Mussa, 1979; Froot and Thaler, 1990; Chaboud and Wright, 2005) These considerably opposite findings lead to further research to find a coherent explanation for the UIRP puzzle. Baillie and Bollerslev (2000) and Bekaert (2001), who provide some evidence of the failure of UIRP, assert that doubtful statistical inferences may have contributed to the strong rejection of UIRP at higher frequencies. They argue that this statistical phenomenon happened because of the high persistence of autocorrelation in the forward premium along with the small sample size. (see Roll and Yan, 2000; Maynard and Pillips, 2001; Maynard, 2006)

However, Chinn and Meredith (2004) state that the reason for the empirical failure of UIRP is that a majority of papers evaluate the UIRP hypothesis using financial instruments with relatively short maturities, generally less than 12 months. Based on their assertion, they provide strong evidence supporting UIRP in longer horizon. Their results confirm the earlier studies of Mussa (1979) and Froot and Thaler (1990) that UIRP may work better at longer horizons. Therefore, time horizon obviously is an important issue in investigating the UIRP proposition. It can conclude that those surveys, which reject the UIRP hypothesis, do not distinguish between the short-run and long-run relationship between exchange rates and interest rate differentials. Thus, this issue causes the rejection of UIRP.

Current study relies on Chinn and Meredith (2004) reasoning and examines the UIRP condition in different time horizons, namely short-run and long-run. To examine the different time horizons of UIRP condition, I use wavelet decomposition. Hacker et al. (2012) investigate the relationship between exchange rates and nominal interest rate differentials for seven pairs of countries using wavelet transform. They ignore the expected exchange rate in their estimated model. However, contribution of this study is to consider the relationship between exchange rates and interest rate differentials under
the UIRP discipline in different time scales applying wavelet transform.

As explained in the earlier chapter, wavelet decomposition breaks down a time series on a scale-by-scale basis, each scale relating to a range of frequencies. Thus, wavelets involve the possibility of decomposing a time series into various layers of orthogonal sequences of scales using Mallat’s multiscale analysis. Wavelet allows us to evaluate these scales separately and compare them across different series.

To examine the UIRP proposition, the quarterly exchange rate of Pound Sterling (GBP) is used against the currencies of ten UK trade partners, namely US dollar (USD), Japanese yen (JPY), Euro (EUR), Chinese Yuan, German Mark, French Franc, Danish Krone, Netherlands Guilder, Swedish Krona, Norwegian Krone.

Two variables of the UIRP proposition, logarithm of nominal exchange rate differential $\Delta \ln s$, and nominal interest rate differential $\Delta IR$ are decomposed into 4 time-scale levels by MODWT analysis for each country. Each time-scale level relating to a range of frequencies, such that, when the time-scale levels become longer, the oscillation of the time series are smoother. In other words, the time scale rises the time between consecutive peaks and the time between consecutive troughs gets longer.

In order to investigate the effect of the interest rate differentials on the expected exchange rate change, in different time horizons, I estimate equation (3.1) using OLS method in four time-scale levels for each pair of countries.

\[
\Delta \ln s(d_j)_t = a_t + \beta \Delta IR(d_j)_t + \epsilon_{j,t}
\]  \hspace{1cm} (3.1)

where $\Delta \ln s$ shows the logarithm of difference of the spot exchange rate $s$ and the expected exchange rate $s^e$ which is actual future value of spot exchange rate in the next period. $\Delta IR$, the nominal interest rate differential is expressed as the difference between the logarithm of one plus the interest rate of the UK and the logarithm of one plus the foreign interest rate.
The structure of the chapter is as follows: Section 3.2 outlines the theoretical and empirical literature of the relationship between interest rate differentials and the exchange rates. Section 3.3 provides the methodology which explains the new approach, wavelet analysis, in examining the relationship on a scale-by-scale basis. Section 3.4 describes the dataset and the model. The empirical results are shown in Section 3.5. Section 3.6 concludes.

### 3.2 Literature Review

Modern exchange rate models are founded on financial-asset markets. In other words, the exchange rate is viewed as adjusting to equilibrate international trade in financial assets, while the traditional view is that exchange rates adjust to equilibrate international trade in goods market. In this part, I glance through the mainstream existing exchange rate determination theories.

#### 3.2.1 Theoretical Literature

**Flexible-Price Monetary Model**

The monetary model is a primitive approach to explain the exchange rate variations and the balance of payments. The monetary model provides some useful insights of the long-run equilibrium. Theoretically, the basic framework of the monetary model is the purchasing power parity (PPP) relationship and the demand for money.

The monetary model is based on the three main assumptions which are as follows:

1. The aggregate supply curve is vertical. It implies the perfect price flexibility in all markets.

2. The demand for money is a stable function of only a few domestic macroeconomic variables, according to the Cambridge quantity equation. It is given by:
\[ M^d = kPy \] (3.2)

where \( y \) is real national income and \( k \) is a positive parameter. Given a money stock, \( M_0^S \), equilibrium in money market is:

\[ M_0^s = M^d \]
\[ M_0^s = kPy = kY \] (3.3)

It states that nominal income \( Y \) is constant along the aggregate demand curve, assuming a given value of money stock. This means the change in price and income have the same magnitude, due to keeping \( M_0^s \) constant. For example, a fall in the price level is of the same proportion as the rise in income. Moreover, if a change in the money stock occurs, it leads to a change in the price level at the same proportion subject to given real income.

3. Purchasing power parity (PPP) holds at all times. The PPP hypothesis is obtained as:

\[ SP^* = P \] (3.4)

where \( S \) is the nominal exchange rate, \( P^* \) is a foreign price and \( P \) is a domestic price. Equation (3.4), which is broadly explained in the previous chapter, states that nothing could be acquired by transporting goods from country to the other one. In other words, the purchasing power parity of each currency must be equivalent whether spent on the domestic market or changed into the foreign currency and spent overseas.

Substituting (3.4) to (3.3) displays the equilibrium of the monetary model:

\[ M_0^s = kPy = kSP^*y \] (3.5)
solving for $S$:

$$S = \frac{M_0^s}{kP^*y}$$  \hspace{1cm} (3.6)

Hence, the exchange rate in this model is the ratio of money stock to aggregate money demand, measured at the foreign price level. So, whatever leads to an increase in this ratio, for instance a rise in the money stock or a decrease in real income, will cause the price of foreign exchange rate to go up, that is, the domestic currency to depreciate.

Monetarism believes that if a monetary expansion happens, at the old price level, there is obviously an excess supply of money. Thus, it is a good reason for economic agents to raise spending so as to decline their money balances assuming the other exogenous variables (real income, $y$, and the foreign price level, $P^*$) are unchanged. Therefore, there is an excess supply of money which is corresponding to excess demand for goods. The extra spending must boost prices, in turn the price of foreign currency must increase, and equivalently the domestic currency depreciates. Conclusively, in the monetary model, a given percentage rise in domestic money supply follows, ceteris paribus, to a depreciation of the same percentage in the amount of the domestic currency.

Interest rates in the monetary model are introduced in the demand for money, such that:

$$\frac{M^d}{p} = ky - lr \quad k, l > 0$$  \hspace{1cm} (3.7)

It imparts that the impact of an increase in the interest rates is to generate a temporary surplus supply of money and excess demand for goods and consequent inflation, given the money stock. In terms of money market, the higher prices encourage the agents’ attempt to spend more money to sustain their real balances. In this process, they raise prices until the real money stock has been decreased adequately to return equilibrium at the new point, and a higher interest rate. Since equilibrium reaches at
the new price level, nominal income has increased.

Considering (3.4) and (3.6), if the exchange rate is floating, the higher interest rate is associated with a higher amount of $S$. In other words, when an exogenous rise in the domestic country’s interest rate occurs (not because of money supply decline), ceteris paribus, demand for money decreases in that country and goes up its aggregate demand. It then leads to higher prices in that country. Assuming purchasing power parity, the exchange rate will be driven up. That means the domestic currency depreciates. The same argument is applied to a change in the foreign interest rate.

As a result, according to the monetary model, an increase in domestic interest rates relative to those in the foreign country is associated with depreciation in the domestic currency, since the nominal money stocks and real incomes are constant. In other words, if something happens that leads agents to expect the exchange rate to depreciate more quickly, the domestic interest rates increase and the exchange rate does depreciate, instantaneously. Therefore, monetary model implies a positive relationship between exchange rates and interest rates for the long-run period.

Fixed-Price Mundell-Fleming Model

In the monetary model, once the price level is completely flexible, the exchange rate is determined, while the Mundell-Fleming (MF) model looks at the changes in exchange rate since the price level is perfectly fixed. The MF model follows the Keynesian tradition that states the aggregate supply has the passive role of fixing the price level, whereas aggregate demand changes specify the level of economic activity. The distinctive feature of the MF model is to describe an open economy that it has different conditions in determining the current balance and the net capital inflow. The MF model is designed by the following assumptions:

1. The aggregate supply is horizontal. This means it focuses only on the demand side of the economy which is supported by IS-LM framework. The price is completely
2. PPP does not hold, even in the long-run. The current account surplus is the function of the real exchange rate and real income. It is given by:

\[ B = B(y, Q) = B(y, S) \quad B_y > 0, B_s > 0 \]  

(3.8)

where \( B \) is the current account surplus, \( y \) is the real income. \( Q \) and \( S \) are the real and nominal exchange rates, respectively. They are identical since both domestic and foreign prices are fixed.

3. Exchange rate expectations are static.

4. Capital mobility is less than perfect.

Considering the balance of payments approach, the interest rate has a crucial role in the MF model, such that:

\[ K = K(r - r^*) = K(r) \quad k' > 0 \]  

(3.9)

where \( K \) is the home country’s net capital inflow and \( r^* \) is the exogenously given foreign interest rate.

Adding the equations (3.8) and (3.9) represents the balance of payments by:

\[ B(y, S) + K(r) = 0 \]  

(3.10)

\[ F(y, S, r) = 0 \quad F_y < 0, F_S > 0, F_r > 0 \]  

(3.11)

Balance of payments (BP) at equilibrium is defined when the flow of capital across the exchanges is sufficient to fund the current account deficit or take in the surplus. Hence, under the floating exchange rate system, the overall balance of payments must
be in equilibrium at all times, that is, a surplus on one account must be balanced by a
deficit on the other.

As a result, the MF model states that an increase in $S$ (depreciation of the domestic
currency) is followed by a higher current account surplus or lower deficit at any level of
economic activity. So, it needs a more moderate net capital inflow, and consequently a
smaller interest rate.

Under the fixed-price level when a monetary expansion occurs, it is equivalent to a
rise in the real money stock. Within the IS-LM framework, a fall in the interest rates is
expected. Given a floating exchange rate regime, the exchange rate must depreciate to
pay off the current account and ultimately balance of payments. In the meantime, the
competitiveness of domestic production improves and demand for home country output
rises.

On the other hand, under the fixed exchange rate regime an increase in money
supply has two different effects. First, the short term, when the capital is not completely
mobile, the interest rate decreases, the real income rises and the balance of payments
get worse on both current and capital accounts. Secondly, in the long term, the foreign
currency reserves decrease, but the income, the interest rate or the balance of payments
stay constant.

By and large, given the assumption of the MF model in the short-run in terms of
the relationship between exchange rate and interest rate, it could be deduced that a
decrease in domestic interest rates by a monetary expansion leads to capital outflow
from the domestic country which create a deficit in the balance of payments. A rise
in the net exports through an increase in the exchange rate i.e. depreciation of the
domestic currency resolves the deficit. Therefore, the MF model in short-run indicates
a negative correlation between exchange rates and interest rates.
**Sticky Price Dornbusch Overshooting Model**

Dornbusch overshooting model is a combination of the Keynesian and classical traditional models. In fact, it is a hybrid of fixed-price MF model and flexible-price monetary model. The Dornbusch model constitutes both the fixed-price in goods and labour markets as a short-run feature and price adjustment in long-run as a feature of the monetary model. While, both the monetary and MF models ignore the role of expectations, the Dornbusch overshooting model includes expectations in the model.

An intuition behind the Dornbusch model is when goods markets adjust only slowly, financial markets adjust far more rapidly. This means the financial markets have to over-adjust to disturbance due to the compensation for the stickiness of prices in goods markets.

Given initially fixed goods prices, a monetary expansion policy leads to an increase in the real money stock in the short term. It is followed by a fall in the interest rates through an instant change in money demand because money market should be cleared. Thus, the changes in the money stock have a “liquidity effect”.

On the other hand, if domestic interest rates temporarily deviate from foreign interest rates, the real money stock starts to change towards the real values immediately where they started, while goods prices begin to postpone response.

The “overshooting” is a phenomenon which reacts to a real disturbance instead of monetary expansion. That is, when a disturbance such as the oil shocks occur, the exchange rate ought to over-adjust to the disturbance, which has potentially detrimental consequences for the domestic industry and employment.

Hence, the fundamental assumptions of the Dornbusch overshooting model are as follows:

1. Aggregate demand is determined by the standard open economy model, IS-LM system.
2. Financial markets adjust immediately. Investors are risk neutral, so uncovered interest rate parity (UIRP) holds at all times.

\[ r = r^* + \Delta s^e \]  \hspace{1cm} (3.12)

where \( r \) and \( r^* \) are the domestic interest rate and the exogenously given foreign interest rate, respectively. \( \Delta s^e \) is the expected rate of depreciation in the value of the domestic currency relative to the foreign currency.

Similar to monetary model, the exchange rate in Dornbusch model is defined in a long-run equilibrium at any moment and determined by the level of the home country’s money stock, national income and interest rate. Only difference is that, in the Dornbusch model, the exchange rate in the long-run is at its equilibrium, while in the short-run it moves away from its equilibrium, because the goods prices respond to a disturbance extremely slow.

Dornbusch believes that when the exchange rate is below its long-run equilibrium level, there is a natural presumption that enforces the exchange rate to the future path upwards to the equilibrium. The similar mechanism works when the exchange rate is above its long-run equilibrium. All in all, it is expected that the exchange rate converges very quickly to its long-run level, (Copeland, 2005).

Fundamentally, the expectations’ mechanism is given by:

\[ \Delta s^e = \theta(\bar{s} - s) \quad \theta > 0 \]  \hspace{1cm} (3.13)

where \( \bar{s} \) is the exchange rate in the long-run equilibrium. The RHS shows the gap between the logarithm of the current real exchange rate, \( s \), and the logarithm of its current equilibrium magnitude, \( \bar{s} \). The parameter \( \theta \) is a coefficient that denotes the sensitivity of market expectations to the over or undervalue
the currency relative to the equilibrium.

Substituting equation (3.13) into (3.12) results in the new UIRP equation which is given by:

\[ r - r^* = \theta(\bar{s} - s) \] (3.14)

3. The price level is sticky. The aggregate supply curve is elastic in the immediate impact phase, upward-sloping in the adjustment phase and, eventually, inelastic in long-run equilibrium.

In long-run, the exchange rate is at its equilibrium level when domestic and foreign price levels are given, then the market is clear in real value. Nevertheless, in short-run, the price level is fixed due to the inherent rigidities of the labour and goods markets. In general, shocks that move the nominal exchange rate are related to changes in a real exchange rate and they ultimately affect the current account deficits or surpluses. Over time, if more disturbances occur, the economy shifts back to its long-run real exchange rate, because of the consequence of movements in both the nominal exchange rate and the price level.

In a nutshell, the sticky price Dornbusch overshooting model at its long-run equilibrium is subject to three conditions. First, the aggregate demand is equal to aggregate supply i.e. there is no ascending and descending pressure on the price level. Secondly, the domestic and the foreign interest rates are equal, so that the exchange rate is stable without any expectation of either depreciation or appreciation. Finally, the real exchange rate is at its long-run level implying there is neither a surplus nor a deficit in the current account.
Portfolio Balance Model

Portfolio balance approach focuses on the demand for a variety of assets instead of the demand for money. This model allows relative bond supplies and demands to determine the exchange rate, as well as relative money-market conditions. The portfolio balance model assumes perfect capital mobility. This means that the covered interest rate parity holds. However, it relaxes the assumption of the monetary model that domestic and foreign bonds are perfect substitutes. This fails to hold the uncovered interest rate parity (UIRP) i.e. there is no foreign exchange risk premium.

Based on this approach, domestic residents allocate their wealth among three types of assets: domestic money, domestic bonds and foreign bonds which are issued by a foreign government or central bank. Given the circumstances and clearing financial markets at all times, the short-run equilibrium happens when the exchange rate and the domestic interest rate are at level, in which demand is equal to supply for assets. In fact, the exchange rates play a role of balancing the assets’ demand and supply, (Copeland, 2005). The assertion of this model is that the demand for foreign assets has a negative relationship with the domestic interest rate. According to the UIRP equation, it implies that the nominal exchange rate \(s\) is negatively related to the interest rate differentials \(r - r^*\) when the supply of foreign bonds equal the demand for foreign bonds.

Dooley and Isard (1979) criticise the portfolio balance model by saying that this model, under the neutrality of risk premium, does not provide a role for the current account. In order to refine the model, they insert the interest rates into the equations. Sarno and Taylor (2002) also enhance the portfolio balance approach appending both interest rates and rational expectations.
Redux Model

The initial Redux model is introduced by Obstfeld and Rogoff in 1995. The model proposes intuitive forecasts about exchange rates and current accounts. Redux model sometimes differs severely from those of either modern flexible-price inter-temporal models or traditional sticky price Keynesian models, (Obstfeld and Rogoff, 1995). This means it takes a general equilibrium in all the main sectors of the economy like exchange rate determination model. Obstfeld and Rogoff (1995) stress that price rigidity is an essential factor to describe exchange rate behaviour without ignoring the insights of the intertemporal approach to the current account. More precisely, the basic model assumes a two country model with monopolistic competition and sticky nominal prices. It also supposes that there are two sets of the economic agents, the households who maximise their utilities and the producers who maximise their profits in each country. They conclude that the nominal exchange rate overshoots its long-run level in response to a money-supply rise.

Generally speaking, the Redux model is considered as a focus on a household choice between consumption and saving as the determining factor of current account and exchange rate, (Copland, 2005).

Obstfeld and Rogoff (1995) believe that the prices move immediately to clear goods, labour and assets markets under certain conditions. These conditions are namely the closed economy, perfect competition, uncertainty and smoothly functioning goods and assets markets. When all these conditions hold, prices take action quickly. At this point, the relative prices, the real wage, the real interest rate and the real exchange rate never change. Therefore, if a deviation from this benchmark occurs, then it creates different sorts of price level stickiness and hence different patterns of real exchange rate behaviour.

Another assumption of Rudex model is that uncovered interest rate parity holds but monetary shocks easily lead nominal interest rates to change by the same quantity.
in both countries. This change does not alter any expectations about exchange rate depreciation or appreciation.

3.2.2 Empirical Literature

One of the most puzzling issues in the exchange rate behaviour literature since the advent of floating exchange rate, is the opposite comovement of exchange rates and interest rates. As such, the periods of the 1970s and 1980s are the periods that instability in exchange rates and interest rates relationship (the persistent appreciation of the dollar) was observed. For example, countries with high interest rates tried to appreciate currency rather than depreciate it as UIRP would suggest. The UIRP, as an important building block of exchange rates determination theories, predicts the exchange rate movement when the interest rate differential is changed. The UIRP puzzle, known as the forward premium puzzle in the literature, has been a controversial issue in empirical investigations for years. In this study, I focus on the model of exchange rates in which uncovered interest parity holds. In the following section, I describe the UIRP hypothesis and discuss the existing empirical studies.

To investigate the UIRP hypothesis, it is convenient to start with the covered interest rate parity (CIRP) condition. CIRP is defined in an open economy under assumptions of free capital mobility, no transaction cost (no arbitrage), and risk-free. If the conditions for risk-free arbitrage hold, then the ratio of the forward to the spot exchange rate is equal to the interest rate differential. So, the CIRP can be written as:

$$\frac{F_{t,t+k}}{S_t} = \frac{I_{t,k}}{I^*_{t,k}}$$ (3.15)

where $S_t$ is the price of domestic currency in units of foreign currency at time $t$. $F_{t,t+k}$ is the forward value of $S$ for a contract expiring $k$ periods in the future. $I_{t,k}$ is one plus the $k$-period yield on the domestic instrument. $I^*_{t,k}$ is the corresponding yield on the
foreign instrument. Equation (3.15) in logarithm form is given by:

\[ f_{t,t+k} - s_t = i_{t,k} - i_{t,k}^* \]  

(3.16)

Above equation shows a risk-free arbitrage condition that holds regardless of investor preferences. Assume the investors are risk averse, then the forward rate can differ from the expected future spot rate by a premium that compensates for the perceived riskiness of holding domestic versus foreign assets. The risk premium is defined as:

\[ f_{t,t+k} = s^e_{t,t+k} - rp_{t,t+k} \]  

(3.17)

where \( f_{t,t+k} \) is forward exchange rate, \( s^e_{t,t+k} \) is expected exchange rate, and \( rp_{t,t+k} \) is risk premium.

Substituting equation (3.17) into (3.16) gives:

\[ \Delta s^e_{t,t+k} = (i_{t,k} - i_{t,k}^*) - rp_{t,t+k} \]  

(3.18)

This implies that the expected change in the exchange rate from period \( t \) to period \( t + k \) is a function of the interest rate differential and the risk premium. The risk premium equal to zero indicates the risk neutrality of investors. This means equation (3.18) could be the basic equation of UIRP, in which the expected exchange rate change equals the current interest rate differential.

Estimation of equation (3.18) is not convenient due to the absence of the expected exchange rate movements. However, the UIRP proposition usually assumes the rational expectation in exchange markets. Thus, actual future value of \( s_{t+k} \) equals the value expected at time \( t \) plus a white noise error term \( \xi_{t,t+k} \) that is uncorrelated with all
information known at \( t \), such that:

\[
s_{t+k} = s^r_{t,t+k} + \xi_{t,t+k}
\]  

(3.19)

where \( s^r_{t,t+k} \) is the rational expectation of the exchange rate at time \( t + k \) formed at time \( t \).

Substituting equation (3.19) into (3.18) results in the following relationship:

\[
\Delta s_{t,t+k} = (i_{t,k} - i^*_{t,k}) - r p_{t,t+k} + \xi_{t,t+k}
\]  

(3.20)

where the LHS indicates the realized change in exchange rate from \( t \) to \( t + k \).

In general, the UIRP regression which is usually estimated to test the unbiasedness hypothesis is given by:

\[
\Delta s_{t,t+k} = \alpha + \beta (i_{t,k} - i^*_{t,k}) + u_{t,t+k}
\]  

(3.21)

By simplifying, the UIRP regression is represented by:

\[
s_{t+1} - s_t = \alpha + \beta (i_t - i^*_t) + u_{t+1}
\]  

(3.22)

where \( s_t \) and \( s_{t+1} \) are the logarithms of the spot nominal exchange rates at time \( t \) and time \( t + 1 \). \( i_t \) and \( i^*_t \) are the logarithms of one plus domestic and one plus foreign nominal interest rates, respectively.

According to equation (3.22), the UIRP states that the interest rate differential is, on average, equal to the expected exchange rate change. Assuming rational expectations in exchange markets and risk neutrality of investors, \( \alpha \) should equal zero and \( \beta \) should equal one. In this sense, the interest rate differential is an unbiased predictor of exchange rates.
There are extensive empirical studies that consider the UIRP unbiasedness hypothesis. Some surveys show promising results supporting the UIRP hypothesis. (see Mussa, 1979; Froot and Thaler, 1990; Fujii and Chinn; 2000; Alexius, 2001; Chaboud and Wright, 2005)

On the other hand, a bulk of empirical research reveals the biased estimation of the UIRP proposition, that is, the estimated slope coefficient is negative. This means that the currency with the higher interest rate tends to appreciate and in turn UIRP does not hold. Bilson (1981) and Fama (1984), who initially run the UIRP regression, find the negative slope estimates of the UIRP regression. (see Baillie and Bollerslev, 2000; Bekaert, 2001; Backus et al., 2010)

Fujii and Chinn (2000) estimate the UIRP regression over long horizons. Exchange rate returns from $t$ to $t + k$ are regressed on the difference in yields on $k$-period government bonds at time $t$. They find that as the horizon $k$ increases, the rejection of the UIRP hypothesis becomes less decisive.

In contrast, Chaboud and Wright (2005) examine UIRP over extremely short horizons. They look at high frequency exchange rate movements during the overnight period when interest does accrue. They use a large dataset of 5 minute exchange rate data. Their results support the UIRP hypothesis over very short windows of data. That is, they find that the slope coefficient in the UIRP regression is close to one, over short windows of high frequency data.

Chinn and Meridith (2004) consider the UIRP condition to explain exchange rate movements. They examine the changes in logarithm exchange rate which are linearly related to the interest rate differentials. They apply the short-maturity bonds and more creatively long-maturity bonds for the set of G7 countries. Their results denote a positive sign for all coefficients on interest rate differentials with instruments with maturities ranging from 5 to 10 years regarded as the long-horizon. Moreover, almost all of the coefficients are closer to unity than to zero. Conversely, there is a negative
relationship in the short-horizon, generally less than one year. These findings follow the line of earlier studies of Mussa (1979) and Froot and Thaler (1990) that UIRP may work better in the long-run.

In the same vein, Hacker et al. (2012) emphasise the importance of time scale in discussing the relationship between exchange rates and interest rate differentials. They study the coexistence of the short-run and long-run relationships between the same two variables. For this purpose, they apply new methodology, wavelet methodology, to decompose spot exchange rates and nominal interest rate differentials series into different time-scale levels. They then estimate the relationship using wavelet-decomposed time series. They find that the exchange rate is negatively related to the interest rate differential at the shorter time horizons (a half year or less), and positively related at the longer time horizons (over a year). They believe that their results are consistent with traditional approaches (flexible-price and the sticky price) in explaining exchange rate movements in terms of interest rate differential.

3.3 Methodology

As mentioned above, there are numerous empirical literatures to explain the behaviour of exchange rates. However, they come up with various results, such that some studies fail the results of each other and further studies try to justify the weakness of previous papers. As it is noticed, the time period is a crucial issue in the exchange rates behaviour discussion. Specifically, discussion about the coexistence of the short-run relation and the long-run relation between the exchange rates and interest rate differentials, (Hacker et al., 2012).

The current study focuses on the UIRP relation which investigates the exchange rates and the interest rates linkage, empirically. Similar to Hacker et al. (2012), this chapter suggests the wavelet analysis (MODWT) to investigate the relationship in dif-
different time horizons, in order to find an explanation for such an anomaly in the UIRP proposition. By doing so, the wavelet methodology is applied for the UK currency against ten different currencies. Then, I estimate the relationship between the exchange rates and the interest rate differentials in the short-run and long-run, simultaneously applying OLS method. Hacker et al. (2012) exclude exchange rate expectation in their model, while this chapter includes the expected exchange rate as an actual future value of exchange rate under the UIRP regression.

3.3.1 Wavelet Transform

Wavelet transform has been extensively explained in the second chapter. However, this part is devoted to describing the multiresolution analysis wavelet analysis in details. Essentially, wavelet analysis is based on Fourier analysis. Fourier transform is interpreted as a bridge between the time domain and frequency domain. Similarly, wavelets are an ideal tool for the frequency domain in economic and financial time series. However, there are some significant differences between Fourier and wavelet analysis. For example, Fourier series is represented as a sum of sine and cosine functions at different wavelengths over an infinite domain. In addition, it has a strong assumption which states frequency content of the function should be stationary along the time domain. While, wavelet transform is a function of both the frequency and time over a finite domain. This allows researchers to observe and analyse a series at different scales and also enable them to study nonstationary time series taking long term movements and high-frequency details at the same time.

A useful property of wavelets is the “multiresolution” analysis (MRA). This means it enables wavelets to break down a time series on a scale-to-scale basis, each scale corresponding to a range of frequencies. Every function in a wavelet basis is a dilated and translated version of one mother wavelet. So, the wavelet bases are self-similar. A mother wavelet provides properties that are local in time and local in scale. A wavelet
basis also entails a father wavelet that denotes the smoothed baseline trend. In general, the wavelet transform smartly matches itself to capture the features across a wide range of frequency.

In terms of mathematical base, the wavelet transform $f(x)$ for a time series $x(t)$ in the domain $[0, 1]$ is given by:

$$f(x) = v_0 \phi(t) + \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \omega_{j,k} \psi_{j,k}(t)$$  \hspace{1cm} (3.23)

where $j$ and $k$ are the scale and shift parameters. $v_0$ and $\omega_{j,k}$ are scaling and wavelet coefficients. $\phi(t)$ is a father wavelet which represents the lowest frequency trend components and $\psi(t)$ is a mother wavelet which is used for the higher frequency detail components.

As seen, the equation (3.23) is an additive decomposition of discrete wavelet transform (DWT)$^{2}$. Thus, it is an appealing transform as it is able to differentiate seasonalties, reveals structural breaks and volatility clusters, and identifies local and global dynamic properties of a process at different time-scale, (Gencay, 2002). It also enables researchers to reconstruct the original series after decomposing it.

The multiresolution analysis (MRA), which is introduced by Mallat (1989), provides an intuitive interpretation of the wavelet decomposition. It involves the operations of translation and dilation to provide a representation of wavelet coefficients, easily. Simply put, the MRA tries to find the averages (means, sometimes weighted) and differences from those averages. In doing so, it starts with values in the series which are closest to each other i.e. the lowest scale. Then it repeats that process with the previous average series. Next, it slowly expands how much of the original data is encompassed in each consecutive average by increasing the scale.

Discrete wavelet transform (DWT) is a type of wavelet analysis which has some

---

$^{2}$In contrast to integrable decomposition which indicates the other type of wavelet analysis, continuous wavelet transform (CWT)
desirable properties, namely orthogonality, no data redundancy in its multiresolution analysis. However, applying DWT to decompose a series is limited to the sample size $2^j$. The maximal overlap discrete wavelet (MODWT), as a modification of the ordinary discrete wavelet transform, is free of this restriction. While, it preserves the property of DWT by rescaling the scaling and wavelet coefficients, it loses the orthogonal property of the coefficients, (Percival & Walden, 2000).

Simply in practical sense, if the mother wavelet is defined such that $\psi(j, k, x)$, then $\psi$ is being larger as the scale level $j$ gets larger and the interval is being shifted rightward as the shift parameter $k$ rises. A scale level $j$ is a positive integer and shows that there is a pattern in the original series which is corresponding to movements with a frequency of $2^{j-1}$. For instance, if $j = 1$ then changes in the scale are associated with what occurs after one period. If $j = 2$ then changes in the scale are associated with what occurs after two periods. If $j = 3$ then changes in the scale are associated with what occurs after four periods and so on. A linear combination of various forms of mother wavelet is appropriately shifted (translated) through $k$ and scaled (compressed or dilated) through $j$.

At every scale level $j$, the MRA for MODWT generates the detail series, $d_j$, and the smooth series, $s_j$. The detail series describe the differences of the smooth series at the next-lower level from the current-level’s smooth series. The smooth series denotes a series of the averages at that scale, in which the averages are over values in the next-lower scale level’s smooth series.

Given the Haar function, suppose that $y$ is a vector of actual time series with its elements ordered according to uniform units of time. The smooth and detail series, $s_j$ and $d_j$ are respectively set by $s_{j,t}$ and $d_{j,t}$ for time $t$. The level-zero smooth series $s_0$ is defined to be the same as the vector of actual observations $y$, such that $s_{0,t} = y_t$.

The following shows how the smooth $s_j$ and detail series $d_j$ of the MRA of the MODWT are calculated at scale level $j$: 

80
\[ s_{j,t} = \frac{s_{j-1,t-2^{j-1}} + 2s_{j-1,t} + s_{j-1,t+2^{j-1}}}{4} \]  
(3.24)

\[ d_{j,t} = \frac{-s_{j-1,t-2^{j-1}} + 2s_{j-1,t} - s_{j-1,t+2^{j-1}}}{4} \]  
(3.25)

Table (3.1) illustrates \( s_{j,t} \) and \( d_{j,t} \) which are calculated from equations (3.24) and (3.25) at four scale levels.

Table 3.1: The Detail and Smooth Series at Scale Level 1 to 4

<table>
<thead>
<tr>
<th>scale level</th>
<th>detail series</th>
<th>smooth series</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( d_{1t} = \frac{-y_{t+1} + 2y_t - y_{t-1}}{4} )</td>
<td>( s_{1t} = \frac{y_{t-1} + 2y_t + y_{t+1}}{4} )</td>
</tr>
<tr>
<td>2</td>
<td>( d_{2t} = \frac{s_{1t-2} + 2s_{1t} - s_{1t+2}}{4} )</td>
<td>( s_{2t} = \frac{-s_{1t-2} + 2s_{1t} - s_{1t+2}}{4} )</td>
</tr>
<tr>
<td>3</td>
<td>( d_{3t} = \frac{s_{2t-4} + 2s_{2t} - s_{2t+4}}{4} )</td>
<td>( s_{3t} = \frac{-s_{2t-4} + 2s_{2t} - s_{2t+4}}{4} )</td>
</tr>
<tr>
<td>4</td>
<td>( d_{4t} = \frac{s_{3t-8} + 2s_{3t} - s_{3t+8}}{4} )</td>
<td>( s_{4t} = \frac{-s_{3t-8} + 2s_{3t} - s_{3t+8}}{4} )</td>
</tr>
</tbody>
</table>

Technically, by having the smooth and detail series, the original series \( y \) can be reconstructed. As equation (3.26) denotes below, the reconstruction of \( y \) is possible by adding the smooth series of the largest scale level, \( \Lambda \), and the sum of the detail series from level 1 to level \( \Lambda \), such that:

\[ y = s_{\Lambda} + \sum_{j=1}^{\Lambda} d_{\Lambda} \]  
(3.26)

In this study, if the data vector \( y \) displays the spot exchange rate, the wavelet details, \( d_1 \) to \( d_{\Lambda} \) are the decompositions of the spot exchange rate at different time-scale levels. \( s_{\Lambda} \) refers to the long-term trend of the spot exchange rate at time-scale \( \Lambda \). The same process is performed for the interest rates series.
3.4 Data and Variables

To examine the UIRP proposition, I collect the quarterly base of spot exchange rates and interest rates. The exchange rate of Pound Sterling (GBP) is used against the currencies of ten UK trade partners, namely US dollar (USD), Japanese yen (JPY), Euro (EUR), Chinese Yuan, German Mark, French Franc, Danish Krone, Netherlands Guilder, Swedish Krona, Norwegian Krone. The raw data for quarterly spot exchange rates are collected from Thomson Reuters Datastream. The quarterly interest rates data are collected from IMFs International Financial Statistics (IFS) database as the three-month Treasury bills of each country. The starting point of gathered data for each currency in quarter base depends on the availability of data in the floating period of the exchange rate. Thus, for US dollar, Japanese yen (JPY), Euro (EUR), Chinese Yuan, French Franc, Danish Krone, Swedish Krona sample period are taken from 1997 to 2013. However, German Mark, Netherlands Guilder, Norwegian Krone are considered in different time intervals, 1996-2012, 1994-2010 and 1993-2009, respectively.

The variables of The UIRP proposition considered by this study are $\Delta \ln s$ and $\Delta IR$. $\Delta \ln s$ shows the logarithm of difference of the spot exchange rate $s$ and the expected exchange rate $s^e$ which is actual future value of spot exchange rate in the next period. $\Delta IR$, the nominal interest rate differential is expressed as the difference between the logarithm of one plus the interest rate of the UK and the logarithm of one plus the foreign interest rate. The UIRP equation is given by:

$$\Delta \ln s(d_j) = a_t + \beta \Delta IR(d_j) + \epsilon_{j,t}$$  (3.27)

Prior to estimation of the UIRP regression, I break each variable into four different time-scale levels. As the number of observations dictates the number of levels, I choose four scale levels. For this purpose, the R software is applied to decompose both the
exchange rates and the interest rate series for each country. I apply la8\(^3\) wavelet filter and MODWT analysis to decompose the series.

![Wavelet Decomposition](image)

**Figure 3.2: UK Interest Rate and its Wavelet Decomposed Levels**

Figures (3.2), (3.3) and (3.4) display the original series and wavelet decomposed series of the expected exchange rate change for Pound in terms of Dollar and the interest rate of the UK and US, respectively. As seen, the series are decomposed into four different time-scale levels. As such, \(s\) shows the smoothest level and \(d_j\) denotes the details in various levels. When the time scales become longer, the oscillation of the time series are smoother. In other words, the time scale rises the time between consecutive peaks and the time between consecutive troughs gets longer.

According to the construction of scale levels, the wavelet scale refers to the movement frequencies. Therefore, it is concluded that \(d_1\) corresponds to a one-period movement frequency, that is, changes can happen between successive periods. \(d_2\) is also related to a two-period movement frequency, i.e. changes occur every two periods. Similarly, \(d_3\) is

\(^3\text{la8 is the least asymmetric family developed Daubechies with width 8. (see Gallegati & Gallegati (2007), Percival & Walden (2000) and Yihui Lan (2011))}\)
associated with a four-period movement frequency, $d_4$ with an eight-period movement frequency, $d_5$ with a sixteen-period frequency, and $d_6$ with a thirty two-period movement frequency. Besides, the smooth wavelet, $s_6$, indicates the nonstationary component
(trend) of the time series, (Hacker et al., 2012).

In order to investigate the effect of the interest rate differentials on the expected exchange rate change, the MODWT is implemented for each variables. Then, I estimate equation (3.27) using OLS method in four time-scale levels for each pair of countries.

3.5 Empirical Results

This section supplies the results of UIRP estimation. First, I estimate the UIRP equation at the aggregate level. The results are reported in Table (3.2) along with the R-squared associate with each country. In all cases except Netherlands, the estimated slope coefficients are negative with the values lying closer to zero than to unity. In two cases, Japanese yen and Danish krone, not only the negative effects are statistically significant but also the R-squared of those regressions are relatively higher than the others.

Almost all estimations confirm the failure of UIRP over the aggregate level, similar to majority of studies that use the standard UIRP model. However, for the Netherlands currency (Dutch guilder), the point estimate is positive and very close to zero. The value of R-squared statistic is also negligible and equal to zero.

Next, I take advantage of the wavelet analysis to decompose the UIRP relation into four different time-scale levels. A multiresolution analysis (MRA) with 4 levels is performed using maximal overlap discrete wavelet transform (MODWT) and la8 Daubechies wavelets filter. Then, I use the decomposed data to estimate the UIRP equation at different (disaggregate) levels. Each level \( j \) is corresponding to specific time interval \( (2^{j-1}) \).

As shown in Table (3.3), moving from low to high level means shifting from high frequency to low frequency. High frequency shows short time interval (short-run) and low frequency displays large time interval (long-run), simultaneously. Given the quarter
Table 3.2: The Uncovered Interest Rate Parity at Aggregate Level

<table>
<thead>
<tr>
<th>Currency</th>
<th>Coefficients</th>
<th>R - squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese yuan</td>
<td>-0.004</td>
<td>0.37</td>
</tr>
<tr>
<td></td>
<td>(-0.48)</td>
<td></td>
</tr>
<tr>
<td>Danish krone</td>
<td>-0.019</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>(-1.42)</td>
<td></td>
</tr>
<tr>
<td>Deutch mark</td>
<td>-0.039***</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>(-2.29)</td>
<td></td>
</tr>
<tr>
<td>Dutch guilder</td>
<td>-0.026</td>
<td>2.1</td>
</tr>
<tr>
<td></td>
<td>(-1.17)</td>
<td></td>
</tr>
<tr>
<td>Euro</td>
<td>-0.027</td>
<td>2.66</td>
</tr>
<tr>
<td></td>
<td>(-1.3)</td>
<td></td>
</tr>
<tr>
<td>French franc</td>
<td>-0.025</td>
<td>2.73</td>
</tr>
<tr>
<td></td>
<td>(-1.32)</td>
<td></td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.007</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td></td>
</tr>
<tr>
<td>Norwegian krone</td>
<td>-0.002</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(-0.15)</td>
<td></td>
</tr>
<tr>
<td>Swedish Krona</td>
<td>-0.007</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>(-0.68)</td>
<td></td>
</tr>
<tr>
<td>U.S dollar</td>
<td>0.003</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: figures in brackets show t-statistics values. Coefficients are evaluated at 95% confidence intervals.

The results in Table (3.3) illustrate the performance of both short-run and long-run UIRP relation for top ten trade partners of the UK. As seen the estimates of most cases are negative. This implies the rejection of UIRP hypothesis. However, the positive significant estimates, in some cases under eight quarters interval, are reported which indicate the UIRP hypothesis cannot be rejected.

More specifically, in the cases of US dollar, Euro and French franc, both negative and positive relationship in different time horizons are seen, at the same time. These results confirm that not only UIRP holds over long-run, but also it is against over
short-run, simultaneously.

Table 3.3: Estimates of Uncovered Interest Rate Parity at Different Levels in Wavelet Domain

<table>
<thead>
<tr>
<th>Currency</th>
<th>1 quarters (level 1)</th>
<th>2 quarters (level 2)</th>
<th>4 quarters (level 3)</th>
<th>8 quarters (level 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese yuan</td>
<td>-0.047 (-0.89)</td>
<td>0.117*** (2.41)</td>
<td>0.023 (1.37)</td>
<td>0.025*** (4.05)</td>
</tr>
<tr>
<td>Danish krone</td>
<td>-0.131*** (-2.02)</td>
<td>-0.041* (-1.71)</td>
<td>0.004 (0.43)</td>
<td>-0.025*** (-3.37)</td>
</tr>
<tr>
<td>Deutch mark</td>
<td>-0.129*** (-2.12)</td>
<td>-0.014 (-0.61)</td>
<td>-0.068*** (-4.32)</td>
<td>-0.026*** (-3.34)</td>
</tr>
<tr>
<td>Dutch guilder</td>
<td>-0.17*** (-3.53)</td>
<td>-0.037*** (-2.08)</td>
<td>0.112*** (6.95)</td>
<td>-0.053*** (-5.88)</td>
</tr>
<tr>
<td>Euro</td>
<td>-0.292*** (-3.21)</td>
<td>-0.057* (-1.64)</td>
<td>0.000 (0.000)</td>
<td>-0.039*** (-3.52)</td>
</tr>
<tr>
<td>French franc</td>
<td>-0.146*** (-1.76)</td>
<td>-0.015 (-0.5)</td>
<td>-0.006 (-0.34)</td>
<td>-0.042*** (-4.47)</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.039 (0.74)</td>
<td>0.048 (1.23)</td>
<td>0.096*** (7.75)</td>
<td>0.054*** (4.45)</td>
</tr>
<tr>
<td>Norwegian Krone</td>
<td>0.005 (0.08)</td>
<td>0.022 (1.03)</td>
<td>0.026 (1.49)</td>
<td>-0.034*** (-9.94)</td>
</tr>
<tr>
<td>Swedish Krona</td>
<td>-0.118*** (-2.16)</td>
<td>0.049*** (2.12)</td>
<td>0.039*** (3.17)</td>
<td>-0.021*** (-6.99)</td>
</tr>
<tr>
<td>U.S dollar</td>
<td>-0.116 (-0.94)</td>
<td>0.263*** (3.93)</td>
<td>0.082*** (4.47)</td>
<td>-0.006 (-0.9)</td>
</tr>
</tbody>
</table>

Notes: figures in brackets show t-statistics values.

*** indicates coefficients are evaluated at 95% confidence intervals.

** indicates coefficients are evaluated at 90% confidence intervals.

To sum up, the empirical results with quarter base display that the exchange rates and interest rate differentials are positively and negatively related under UIRP condition. It could conclude that the negative significant coefficients turn up in the shorter horizons, while the positive significant coefficients do appear in the longer horizons.
3.6 Conclusion

According to Frankel (1979), the two approaches, the flexible-price monetary model and the fixed-price model are different in the variation of the prices. That is, when variation in the prices is small, then it is in regard to shorter time scales (based on Keynesian tradition) when variation in the prices is large, it means prices typically move more quickly over longer time scales. This is the reason why roughly two distinct theoretical approaches to elucidate the linkage between exchange rate and interest rate, exist.

According to the difference between these two approaches, this chapter examines the exchange rate behaviour under UIRP discipline. I apply wavelet analysis to survey the relationship between expected exchange rate changes and interest rate differentials over various time horizons. Two notable properties of wavelet, decomposing a time series to different layers and coping with nonstationary economic time series and time-varying characteristic of a series, allow to study time series on a scale-by-scale basis.

Applying wavelet decomposition time series data (in quarterly base) and examining the UIRP regression for ten bilateral currencies provide distinct results which show negative and also positive relationships between the expected exchange rate change and the interest rate differentials in short-run and long-run horizons. It could conclude that the negative result is supported by the fixed-price MF model and the positive result is consistent with the flexible-price monetary model which is similar to Hacker et al. (2012).
Chapter 4

New Approach of Currency Crashes
Definition in Early Warning System

4.1 Introduction

A currency crisis is an abrupt devaluation in a domestic currency, inevitably leading to speculative attacks. The currency crises are most often considered as real economic crises compared to other financial crises like debt crises and banking crises.

As Krugman (1979) stated, in general, the mechanism of a currency crisis usually starts with a shift in expectations of speculators in the exchange market when they buy and hold the foreign currency. Meanwhile, the central bank sells the foreign currency reserves in order to stabilize the currency. However, tragedy occurs when the central bank exhausts its reserves. At this time, a sudden devaluation of domestic currency happens and speculators attack the exchange rate market and force an abandonment of the fixed exchange rate.

The speculative attacks result in large scale selling of domestic currency assets that creates more devastating effects on the economic fundamentals. Figure (4.1) is a proper example illustrating the speculative attacks on the UK exchange rate market in 1992.
The sharp spike shows a sudden depreciation of Pound Sterling and when the speculators attack the exchange rate market with the intention of making more profit. The end of the story is a crisis that forced the UK to suspend its ERM membership which consequently collapsed the fixed exchange rate regime.

Figure 4.1: The United Kingdom Nominal Exchange Rate (Pound against Dollar)

The purpose of this study is to answer the question of whether the potential causes and symptoms of currency crises can be detected sufficiently in advance to allow governments to adopt pre-emptive measures. There are numerous benefits of developing a warning system not only to financial market participants who want to avoid loss of profit, but also to policy-makers and academics; policy-makers want to reduce the large costs of such crises and also researchers seek to understand the causes of crises. Precisely, researchers focus on identifying variables (leading indicators) which are able to predict an approaching crisis.

The theoretical literature on the early warning signals attempts to explain the crises and delineate the reasons of crises occurrence. Thus, it provides us an initial guide to identify potential indicators, which should be represented by macroeconomic fundamentals, market expectations, and also contagion effect. Kaminsky (2003) based on
the causes of speculative attacks classifies currency crisis literature into three different
generations. The first generation models are based on monetary and fiscal imbalances.
Krugman (1979) and Flood & Garber (1984) believe that the deterioration in economic
fundamentals causes the crises, such as what happened in Latin America in the 1960s
and 1970s. The second generation examine the investors’ expectation on government
behaviour (Obstfeld, 1994) as well as the EMS crises of the early 1990s. The third
generation models explain the spillover and contagion effect, under development of the
banking sector and market segmentation like the Tequila crisis Latin America in 1994
and the Asian Flu crisis in South-east Asia in 1997. These broad possibilities to choose
useful indicators result in ample empirical studies in parallel.

Kaminsky et al. (1998), who initiated the empirical study in the early warning
literature, analyse a large set of various possible leading indicators. They, in turn,
identify some relevant potential leading indicators: the amount of foreign exchange re-
serves, the real exchange rates, domestic credit, credit to the public sector and inflation.
Other studies such as Esquivel and Larram (1998), Rose and Spiegel (2011), Frankel
and Saravelos (2012), and Babecký et al. (2013) also introduce the potential leading
indicators.

To investigate the probability of the occurrence of a crisis, the empirical surveys
apply different methodologies. The two main methods are the signalling approach and
the discrete choice approach. The former monitors the behaviour of potential leading
indicators; suggesting that, if the level of the indicators deviate from their specific levels,
a crisis will occur. Kaminsky et al. (1998), Bruggemann and Linne (1999), and Edison
(2003) apply the signal approach in their papers.

The latter is a limited dependent variable probit/logit model. The dependent vari-

\footnote{The leading indicators are trade balance, growth in exports, terms of trade, real exchange rates
(as indicators are related to the current account), reserves, short-term capital flows, foreign direct
investment (as indicators are related to the capital flows), total debt, debt service, short-term debt (as
debt indicators), credit expansion, M2, stock price indicators, interest rates (as financial indicators),
GDP, investment, inflation, and public deficit (as macroeconomic indicators).}
able is defined as a discrete choice and macroeconomic and financial variables are utilised to explain the discrete crisis events. Eichengreen et al. (1995), Frankel and Rose (1996), Berg and Pattillo (1999), Kumar et al. (2003), Lau and Yan (2005), Bussière and Fratzcher (2006), and Ari (2012) use this approach.

A novelty of this study is classification of the currency crashes definition based on different generations of currency crises in early warning literature. Assuming that, a currency crash is defined as a sharp depreciation of the nominal exchange rate, I consider the behaviour of exchange rate of those countries which have been involved in crises for almost two decades. This suggests that the exchange rates in the emerging and developing countries behave differently from those in developed countries, once a crisis happens and especially afterwards. The first and third generations are associated with emerging and developing countries and the second generation are related to the developed countries.

Given that, I split the currency crashes into two different definitions. Definition $I$, related to the emerging market with weak fundamentals, shows that the exchange rates require a longer recovery process after a sharp devaluation in domestic currency. While, definition $II$ states that a crisis in the developed countries has been immediately recovered and gets back to normal level e.g. within two quarters.

The definitions capture only the successful speculative attacks crises similar to Frankel and Rose (1996). Next, I adopt the discrete choice approach in the current survey and apply a logit model which considers binary outcome variables using a linear combination of predictor variables.

To identify potential leading indicators in this study, I look at the ranking of the indicators in the earlier literature and rely on systematic literature reviews. A set of potential leading indicators, which cause the currency crisis, includes rate of growth domestic credit, government budget deficit as a fraction of GDP, ratio of reserve to GDP, current account as a percentage of GDP, growth rate of real output, overvaluation of
exchange rate, ratio of debt to GDP. Regarding the issue of data availability for some countries and limited time period, I select seven potential indicators for a panel of 26 emerging markets and developed countries at quarterly frequency.

The leading indicator overvaluation of exchange rate in a majority of studies is defined as the exchange rate deviations from its trend\(^2\) applying HP filter. (see Goldfajn and Valdès, 1998; Ari, 2012). However, current study shows that HP filter is not an appropriate filter to construct the overvaluation, because HP filter compromises the past and future values to construct the current value. That means it is not inherently a backward looking filter, while overvaluation as a predictor of currency crises should not contain the future value. To overcome this misconception of previous papers, this study suggests to use Kalman filter recursively. Thus, I apply Kalman filter to detrend the exchange rate and in turn construct the overvaluation.

The main contribution of this chapter is to develop a multivariate empirical model that is able to improve upon existing empirical models. As such, it departs from the majority of empirical studies in some aspects: first I categorise the definition of the currency crashes according to the different generations of the currency crises occurrence and the country groups which are involved in those generations, because each crisis generation has a different nature. It is intuitive to define a currency crash in different manners in order to obtain the accurate features of each generation by my definition. It could be stated that the currency fluctuations in the crises period are considered through different windows. Each window, in fact, is the definition of the currency crises based on the different generations. It is regarded as a novelty of this chapter to which other papers do not pay attention.

Secondly, I argue about the definition of exchange rate overvaluation in the literature. Previous studies could be criticised on their usage of HP filter in constructing the overvaluation, inappropriately. The current chapter suggests to apply a Kalman filter

\(^2\)Goldfajn and Valdès (1998)
recursively to construct overvaluation as a leading indicator in early warning literature.

Finally, I use quarterly data, contrary to many previous studies which applied annual or monthly datasets. The annual data is aggregate data which ignores the part of data information while weekly, monthly, even quarterly are able to represent more information. Besides, data availability is a main restriction of monthly data particularly in the case of macroeconomics fundamental dataset like GDP. Interpolation of the monthly data is a solution to overcome this drawback that is suggested in Ari (2012). However, applying those methods cause a weak robustness problem. Hence, quarterly data is an appropriate time interval for investigating the consequences of the sudden changes in economic indicators.

The chapter is organised as follows: Section 4.2 is devoted to the literature review including theoretical and empirical studies on the early warning literature. In Section 4.3, I define two different definitions of currency crises. Section 4.4 explains the methodology and identifies the leading indicators then estimates the probability of a currency crisis in my expanded sample. Section 4.5 concludes.

4.2 Literature Review

In order to provide a fairly comprehensive review of the early warning literature, I review both theoretical and empirical studies, separately.

4.2.1 Theoretical Literature

The objective of this section is to provide the possible causes of crises and indicators that would best explain them from a theoretical point of view. As such, I represent those theoretical papers which mainly explain the speculative attacks and balance of payments crises.

Currency crises are often preceded by, coincide with, or follow liquidity squeezes
and banking crises. So, there is a possibility to discover what economic variables are involved in a crisis period. The seminal paper of Krugman endorsed this thought in 1979. His research could be regarded as the first spark of academic work on the crisis literature. Over 34 years, the currency crisis literature has been enhanced and an increasing number of papers have been written in this field. Thereby, it is fair to classify them into three generations with respect to their attitudes.

**First Generation Models of Currency Crises: Canonical Models**

The first generation models follow Krugman’s (1979) paper entitled “A model for balance of payment crises”. The paper indicates the relationship between fiscal and monetary policy and currency crises. More specifically, he focuses on inconsistencies between domestic macroeconomic policies, such as an exchange rate commitment and a persistent government budget deficit that eventually must be monetised. This literature underlines that the cause of currency crisis is poor economic fundamentals and the rational arbitrage by speculators. This kind of class is based on the monetary approach of exchange rate determination in which the investors’ behaviour is added to the model. Thus, the speculators’ behaviour is not triggering the crisis, it is just an accelerator of the process.

The mechanism of the first generation explains that when a government confronts a fiscal deficit, the policy-maker tries to cover the deficiency by expansion of domestic credit or by creation of money. These continuing expansions, under a fixed exchange rate regime, will lead to inflation, depreciation expectations, and capital outflow. Therefore, international reserves gradually and persistently decrease. In other words, when the reserves fall below a certain threshold level, a sudden speculative attack on currency happens that wipes out the reserves till abandonment of the parity. A certain threshold level is determined at some point before depleting all the reserves. The process ends with an attack because economic agents perceive that the government is no longer able
to defend the currency and the fixed exchange rate regime will ultimately collapse. In this sense, the attack is called a “successful” speculative attack.

Krugman (1979) and Flood & Garber (1984) stress that within a monetary framework, the rational, forward looking speculators accelerate the breakdown of the fixed exchange rate regime. In order to discover the implication of a fixed exchange rate regime, they rely on the three ingredients of a monetary model; money market equilibrium, purchasing power parity (PPP) and uncovered interest rate parity (UIRP).

The Krugman-Flood-Garber model assumes a small open economy with perfect capital mobility and a single tradable good. Market participants hold three assets: domestic money, foreign bonds and domestic bonds. The underlying assumption of the model is perfect foresight i.e. certainty conditions. To describe the model, they use the following equations:

\[
\begin{align*}
m_t - p_t & = \alpha_0 - \alpha_1 i_t + \alpha_2 y_t \quad \alpha_1, \alpha_2 > 0 \quad (4.1) \\
m_t & = g_t + c_t \quad (4.2) \\
i_t & = i_t^* + \hat{\epsilon}_t + rp_t \quad (4.3) \\
p_t & = p_t^* + e_t \quad (4.4) \\
def_t & = \dot{c}_t = \mu \quad (4.5)
\end{align*}
\]

where \( m_t \) in nominal domestic money supply; \( y_t \) is real output, or income (assumed to be constant); \( p_t \) is domestic price level; \( p_t^* \) is the foreign price level; \( g_t \) is foreign assets; \( c_t \) is domestic assets (in nominal terms); \( i_t \) is domestic real interest rates; \( i_t^* \) is foreign real interest rates; \( \hat{\epsilon}_t \) is the expected rate of depreciation; \( e_t \) is the exchange rate (in the foreign currency terms); \( def_t \) is the fiscal deficit; \( \mu \) is the rate of credit growth and constant. \( \alpha_0, \alpha_1 \) and \( \alpha_2 \) are non-negative parameters.

Equation (4.1) is the real money demand function which is positively related to
income and negatively related to interest rate. Equation (4.2) shows that the money supply equals domestic assets plus foreign assets. Equation (4.3) is the international asset market arbitrage condition, that is, uncovered interest parity theorem plus a country or currency risk premium \((r_{pt})\), the latter is taken to be exogenous and constant. Equation (4.4) displays the purchasing power parity (PPP) condition. Equation (4.5) states that the fiscal deficit is monetized at a constant rate of \(\mu\).

By substituting (4.3) into (4.1), I get:

\[
m_t - p_t = \alpha_0 - \alpha_1 i^*_t - \alpha_1 \hat{e}_t - \alpha_1 r_{pt} + \alpha_2 y_t \tag{4.6}
\]
\[
p_t = m_t - \alpha_0 + \alpha_1 i^*_t + \alpha_1 \hat{e}_t + \alpha_1 r_{pt} - \alpha_2 y_t \tag{4.7}
\]

substituting (4.7) into (4.4) derives:

\[
p^*_t + e_t = m_t - \alpha_0 + \alpha_1 i^*_t + \alpha_1 \hat{e}_t + \alpha_1 r_{pt} - \alpha_2 y_t \tag{4.8}
\]

where \(\gamma = (\alpha_0 - \alpha_1 i^*_t - \alpha_1 r_{pt} + p^*_t + \alpha_2 y_t)\)

\[
e_t = m_t + \alpha_1 \hat{e}_t - \gamma \tag{4.9}
\]

substitute (4.2) into (4.9):

\[
e_t = g_t + c_t + \alpha_1 \hat{e}_t - \gamma \tag{4.10}
\]

Under a fixed exchange rate regime \(\hat{e} = 0\), then by differentiating equation (4.10), I get:

\[
\dot{g}_t = -\dot{c}_t = -\mu \tag{4.11}
\]

where \(\dot{g}_t\) is the foreign assets or international reserves growth.
Equation (4.11) expresses that the simultaneous combination of a fixed exchange rate regime and an open capital account must imply the loss of monetary autonomy, that is, the impossible (or inconsistent) trilogy principle.

Suppose that agents have perfect information and are aware that the peg is unmaintainable. Hence, they expect that reserve \((g_t)\) to decrease eventually to some minimum point, and the currency depreciates \((\hat{e}_t)\). The domestic interest rate rises according to the international-interest-parity condition (4.3), so that it leads to a decrease in real money based on equation (4.1). Assuming the domestic prices are held by the PPP condition, equation (4.7) implies that there is only one way to re-equilibrate the system such that there is a discontinuous upwards jump in the real exchange rate. Nevertheless, such jumps are excluded, in that they likely increase the possibility of the capital gains (losses). Therefore, the nominal money base must jump, with respect to the constraint of continuity (Rajan, 2001).

While domestic credit increases unceasingly (by \(\mu\)), a fall in the nominal monetary base is taken into account by a drop in foreign reserve \((g_t)\). In other words, presence of perfect information for speculators reveals that international reserves will go down to their minimum level. Precisely, the loss of reserves \((\Delta g)\) must be equal to the fall in money demand as follows:

\[
\Delta g_t = \alpha_1 (i^* - i_t) = -\alpha_1 \hat{e}_t
\]

(4.12)

Second Generation Models of Currency Crises: Escape Clause Models

The European Monetary System (EMS) crisis in 1992-93 and Latin American crisis in 1994-95 are two phenomena that reveal the inability of the first generation models to explain the crises. Unlike the earlier models, second generation models take into account the policy adjustment by the policy-makers in response to the attack. In fact, this class attempts to find an answer for the question of how distortions in financial markets
and banking systems can lead to currency crises. The second generation models, best described by Obstfeld (1986, 1994), indicate that in the presence of insignificant changes in the macroeconomic fundamentals, a crisis may occur. The key idea behind these models is that economic policies are not predetermined. This means that the policymakers weigh the costs and benefits of defending the currency and are willing to give up the exchange rate target even if the costs of doing so exceed the benefits. Based on this assumption, economic agents are able to form their expectations and this affects how some economic variables respond to economic policy. Thus, this set of behaviour creates multiple equilibria. The economy may move from one equilibrium to another while the economic fundamentals are stable. Therefore, it is true to say that agents’ expectations could be described as a factor which leads to changes in policies. It, in turn, results in a collapse of the exchange rate. Hence, a key role is played by changes in expectations. To sum up, the expectations are the main cause of a currency crisis and economic fundamentals have a smaller role in generating currency crisis in this sort of models.

This generation is quite similar to the first one, except for two distinct basic assumptions of these sorts of model: (1) policy-makers are tempted to reject defending the parity due to the high costs of defending the peg, and (2) policy-makers want to maintain the fixed exchange rate in order to gain political credibility and reputation. It is clear that there is a conflict between the two assumptions. Therefore, releasing the peg is a policy decision because exchange rate is used as an optimisation instrument for the policy-makers. Generally speaking, it could be stated that the second generation explicitly optimises a policy-maker’s loss function and has multiple equilibria.

Third Generation Models of Currency Crises

The first and second generation models concentrate on macroeconomic policy. They stress that the collapse of the exchange rate regime is caused by inconsistent economic
policy or an inconsistent decision by the policy-maker responsible for the evaluation of the costs and benefits of maintaining the fixed exchange rate regime (Krznar, 2004). Third generation models, however, consider that a currency crisis is caused by contagion effects. Contagion effects first came up after the Mexico crisis spread to Argentina, Chile and other emerging markets in 1995 which are dubbed the “Tequila” effect and recently the “Vodka” effect. The Asian crisis or “Asian flu” in 1997 is another example of contagion effects.³ This experience has shown that some countries may suffer from contagion effects, in spite of relatively good fundamentals.

The models of this generation, more specifically, emphasize the capital account, while the first two generations focus on the current account. Many scholars believe that this generation is an inevitable side effect of increasing globalisation of capital market.

The contagion effect is the spread of such attacks across countries as well as across time. There are various channels of contagion from one country to another which can be identified as follows:

A first channel is the matter of price competition. Under sticky prices, when prices of domestic goods produced in country A decrease in country B, the exports of country A rise and its imports fall. That is, country A reaps the benefits of the price effects. On the other hands, this makes country B suffer from a decrease in their competitiveness with country A. Hence, devaluation for country B is inevitable. The loss of country B depends on the level and magnitude of its bilateral trade with country A. Intensity of trade between countries usually is related to their geographical situation. When two countries are in the same region and neighbourhood, they are good partners for trade. Then a crisis is more likely to spread among these two countries. Thus, the change in the real effective exchange rate of partner countries will be an appropriate measurement

³The reason for naming these crises in particular words in the literature is to highlight the special features of these sorts of crises i.e. the domino effects. The domino effect is observed in the crises in a majority of Latin America following the collapse of Mexico in December 1994 and the crises in Thai baht in July 1997 which affected a number of the East Asian countries.
of the price effect.

A second channel is in terms of volume, the income effect. As the imports of country A suddenly drop, this immediately causes the currency crisis not only from the price effects, but also from the fall in income that coincides with the balance of payments crisis. Income falls and leads to deficit in the balance of payment. Even if the price ratio of A and B are constant, the imports of A would have fallen due to a fall in A’s GDP and thus the sudden decrease in the demand addressed to the partner country B. The magnitude of these effects can be evaluated by the size of B’s exports to A relative to its GDP.

The third channel is the financial interdependence which comes from two different roots. First, direct financial linkages spread the crisis across the countries. That is, financial creditors of country A who already invested in country B are experiencing the financial loss in the crisis. Second, indirect financial linkages are concerned with a situation that a crisis in one country may persuade common financial creditors (from any country) to call loans and refuse to provide new credit, not just to countries that have already experienced a crisis but also to others, thus spreading the crisis across countries.

Furthermore, the exogenous effect on investors’ belief could be a further cause of the contagion phenomenon. In other words, the expectations of investors are able to amplify crises. However, this effect could be reflected in escape clause models and give rise to multiple equilibria that are partly or completely self-fulfilling.

### 4.2.2 Empirical Literature

Following the theoretical literature, this section is a description of the numerous different empirical studies in the early warning literature. As Babecký et al. (2013) stated, the first golden era of early warning literature started from the 1990s when wide-ranging methodological discussions came on board; while the theoretical literature had already
begun to explain the crises in the late 1970s.

The existing empirical studies typically approach the topic according to different criteria, such as the countries sample (developing, emerging, industrialised); the periodicity of the time series (monthly, quarterly, annual); the crisis indicator used (exchange rate pressure, actual devaluation); the time horizon of the prediction (several months, one year, 2 years); the methods implemented (signal approach, logit/probit and etc.)

The abundant empirical findings can be broadly grouped into four methodological categories. The first and most popular category is the discrete choice approach. It mainly investigates the probability of the occurrence of a crisis, that is, a crisis alarm is issued when the probability reaches a particular threshold. It basically applies linear regression or limited dependent variable probit/logit techniques. Generally, in the class of discrete choice models, macroeconomics and financial data are employed to explain discrete crisis events across-the-board, using different countries favouring panel data. Eichengreen et al. (1995), Frankel and Rose (1996), Sachs et al. (1996) and Berg and Pattillo (1999) conduct primary studies that apply the classical econometric analyses including the multinominal logit model, the binary probit model, and the ordinary least squares (OLS) regression model, respectively.

The second category of empirical studies use the signalling approach, which was initiated by Kaminsky et al. (1998) and extended by Bruggemann and Linne (1999) and Edison (2003). The signal approach is also known as the nonparametric or leading indicators approach. This approach involves monitoring the behaviour of a number of variables as leading indicators of a crisis, and recording the signals issued by these indicators. By determining the threshold levels for each variables, the signals can be tested. Thus, if the normal level of indicators deviate from their given threshold levels, it is stated that a crisis is imminent and deviations are taken as warning signal of a currency crisis within a determined period of time.

\footnote{A threshold level for variables is defined to minimize the noise to signal ratio.}
Kaminsky, Lizondo and Reinhart (1998) present a comprehensive review of the literature on balance of payments crises. They evaluate 76 currency crises in 20 countries covering the period from 1970 through 1995. Their definition of a crisis is based on an exchange market pressure index that consists of changes in the nominal exchange rate and in the international gross reserves. Based on their results, they propose an early warning system to prevent future currency crises.

The third category of papers try to assess the behaviour of different variables around crisis occurrence applying a qualitative and quantitative approach. First, the considered countries are divided into two groups: crisis group and non-crisis (or tranquil) control group. Then, in order to examine whether there are systematic differences between the pre-crisis episodes and the control group, parametric and nonparametric tests were used. (see Kamin, 1988; Edwards, 1989)

The fourth category is outlined within more recent studies that attempt to apply innovative methods to define the crises. Frankel and Saravelose (2012) mention three types of these new techniques as follows: binary recursive trees to specify leading indicator crisis thresholds (Ghosh and Ghosh, 2003; Frankel and Wei, 2005), artificial neural indicators (Nag and Mitra, 1999; Apoteker and Barthelemy, 2000), and Markov switching models (Cerra and Sexena, 2002; Martinez Peria, 2002).

The applied methodology in this chapter is the discrete choice approach. The reason of choosing this approach is that the regression approach has several advantages compared to the signals approach. First, the prediction of the model is easily interpreted as the probability of a crisis. Second, as the method considers the significance of all the variables simultaneously, the additional information of new variables is easily checked. Thus, I focus on the relevant empirical literature under the discrete choice methodology.

Applying a panel of annual data for 105 developing countries covering 1971-1992, Frankel and Rose (1996) analyse the determinants of currency crashes. They define
a currency crash as being associated with large currency depreciation. They find 69 crashes in 780 observations. In their study, they follow two different methodologies. They start with a graphical approach. Through this analysis, they find evidence suggesting that several economic variables behave quite differently in tranquil periods as compared to crises periods. Interestingly, the authors find that neither the current account deficit nor the fiscal deficit behave significantly differently during tranquil and crisis episodes. They point out that, while a graphical analysis has the features of accessibility, it imposes no parametric structure on the data and impose few assumptions for estimation, the main disadvantage of this approach is being univariate and it does not allow one to assess the way in which the selected explanatory variables interact.

As an alternative, Frankel and Rose estimate a probit model to assess which variables are statistically significant indicators of forthcoming currency crashes. The regression results are somewhat sensitive to the specification. But, they find that low levels of foreign direct investment (FDI), low international reserves (as a share of imports), high domestic credit growth, high foreign interest rates and overvaluation of the real exchange rate increase the probability of a currency crash within the following two years. Consistent with their previous results, they also find that neither the current account nor the fiscal balance has a significant role in the occurrence of currency crashes. Moreover, they consider 7 debt composition regressors in their model. The finding, however, shows that the debt composition variables have a weak but non-negligible effect on crash incidence overall. Although many of Frankel and Rose’s results are consistent with both theory and previous empirical evidence, they should be interpreted with care. Furthermore, the explanatory power of their results is weak and their regression has almost no significant predictive power.

Berg and Pattillo (1999) evaluate the results of Frankel and Rose (1996). They assert that making several revisions to the Frankel & Rose paper and correcting an error in the calculation of the overvaluation variable, provide a better performance of the model
and lead to some improvement. They argue that one of the reasons for their rather poor results might be that the country group is too diverse. They proceed with a smaller group of larger (emerging) markets (41 developing countries) over the sample period of 1970-1996. The findings display similar results to Frankel and Rose. On the other hand, they find that the corrected overvaluation variable has a much stronger and more significant effect. Higher northern (OECD) growth now significantly decreases the risk of crisis, and the effect of foreign interest rates is smaller and insignificant. Moreover, the ratio of reserves to imports is no longer significant while the current account and the fiscal balance now are.

In addition, Berg & Pattillo (1999) in their outstanding paper compare three different methodologies in the early warning signal literature. They attempt to evaluate three prominent models for predicting the currency crises based on Kaminsky et al. (1998), Frankel & Rose (1996), and Sachs et al. (1996).^5

More precisely, they re-estimate the three models in order to forecast the 1997 Asian crises. They conclude that the three approaches adhere to the statement that the probability of a currency crisis increases when domestic credit growth is high, the real exchange rate is overvalued relative to trend, and the ratio of M2 to reserves is high. Furthermore, both Frankel & Rose (1996) and Kaminsky et al. (1998) also propose that a large current account deficit is a crucial determinant of currency crisis. This results in re-estimating the panel-based on these two models over different samples of countries and longer time periods to preserve most of the economically important results. After all comparisons, they apply a probit model to the same data and crisis definition as in Kaminsky et al. with the addition of some plausible variables to improve performance of the model. Their findings show that the probit models provide generally better

^5Sachs, Tornell and Velasco (1996) consider the spillover effects of the Mexican crisis of 1994-95 on a 20 emerging market economies. They define the currency crisis as a composite index of the change in reserves and the nominal depreciation. Their finding indicates that low international reserves relative to broad money, real exchange rate appreciation, and a weak banking system play the most role of the variation of their “crisis index”.

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forecasts than the signalling models.

Kumar et al. (2003) choose logit models rather than probit models on 32 developing countries from January 1985 to October 1999. The main difference of Kumar’s research amongst the others is to employ lagged variables. They add lagged financial and macroeconomic variables as explanatory variables in order to strengthen their results. They specify the trading strategies to evaluate their out-of-sample performance. Kumar et al. (2003) define the currency crashes as well as Frankel & Rose (1996). However, the exchange rate devaluations in their model exceed specific cut-off levels (5% and 10%). Their finding affirms that simple logit models have significant explanatory power. All in all, their results confirm that the most important determinants of incidence of currency crashes are low reserves, exports, economic fundamentals, and specifically contagion effects.

Bussière and Fratzscher (2006) attempt to use a multinomial logit model instead of using a binomial probit model for 32 open emerging markets covering 1993-2001. A distinguishable characteristic of Bussière’s study is to identify a post-crisis bias. They claim that failing to distinguish between tranquil periods and crisis/post-crisis episodes may cause a substantial bias in the estimation results and hence deteriorate the ability to predict the currency crises. Thus it represents a different definition of exchange market pressure (EMP). To illustrate, they extend the discrete choice of crises definition from two states (Yes/No) to include more states which are crisis, post-crisis, and tranquil (pre-crisis) and set the threshold for the currency crisis indicator at two standard deviations above its mean. Moreover, they assert that a multinomial logit is able to present the distinction between more than two regimes and allows solving for the post-crisis bias. Their findings show significantly stronger results than that of any previous models.

In order to illustrate the essential causes of the currency crises in Turkey, Ari (2012) examines the predictive ability of 16 economic and financial indicators by developing a
binomial and multivariate logit model for the period January 1990 to December 2002. His paper evaluates the out-of-sample forecast performance of the model in the period January 2003 to December 2008 in which two other currency crises occurred in May 2006 and October 2008. The finding reveals that the Turkish crises are mainly due to excessive budget deficits, high money supply growths, real exchange rate overvaluations, sharp rises in short-term external debt, growing riskiness of the banking system (specially currency and liquidity mismatches), and external adverse shocks. The distinguishing feature of Ari’s paper amongst others is that he considers a particular case i.e. Turkey instead of a panel data. He justifies his idea by stating that each country has its circumstances and it is difficult to define the homogeneous characters that different countries share. Therefore he focuses on one case study.

4.3 The Definition of Currency Crisis

Simply put, a currency crisis is defined as a speculative attack on the foreign exchange value of a currency that either results in a sharp depreciation or depletion of the foreign exchange reserves or the domestic interest rate rises.

There are various definitions of what a currency crisis is in the empirical literature. The preliminary step in this section is to consider the currency crisis definitions in the literature clearly. The definition of a currency crisis could be categorised into two different types. The first type of definition expresses a currency crisis as a sharp depreciation of the foreign exchange rate. Assuming a fixed exchange rate regime, once speculators decide to attack the market, it is a situation in which the economy is under pressure to give up the prevalent exchange rate peg. It is called successful attack due to the depreciation of currency. This kind of definition only captures successful attacks. For instance, Frankel and Rose (1996) define a currency crash (a subset of currency crises); a depreciation of the nominal exchange rate of more than 25%, that is also at

---

6The difference between the two concepts is that currency crashes deal with successful speculative
least 10% increase in the rate of nominal depreciation from the previous year.

Their index is given by:

\[
CC_{it} = \begin{cases} 
1 & \text{if } e_t > 25\% \text{ and } e_t > e_{t-1} + 10\% \\
0 & \text{otherwise}
\end{cases}
\] (4.13)

where \(CC_{it}\) is the currency crash index for country \(i\) at time \(t\) and \(e\) is the natural logarithm of nominal foreign exchange rate.

Milesi-Ferretti and Razin (2000) refine Frankel and Rose work (1996). They state that the Frankel and Rose definition includes the large exchange rate fluctuations corresponding to high inflation rather than a speculative attack. They consider, in addition to a 25% depreciation, at least a doubling in the rate of depreciation with respect to the previous year and a rate of depreciation the previous year below 40%. In order to find the episodes where the exchange rate was relatively stable in the preceding year, they create two other definitions which are slightly different compared to their previous definition. They assume a 15% minimum rate of depreciation, a minimum 10% increase in the rate of depreciation with respect to the previous year, and a rate of depreciation of below 10 percentage points in the previous year. Their last definition is similar to the previous one with the additional requirement that the exchange rate be pegged the year before the crisis. In a similar vein, Esquivel and Larran (1998) and Kumar et al. (2002) identify currency crashes in their papers.

Alternatively, the second type of definition describes a currency crisis as the high pressure on the foreign exchange rate that leads to a vigorous increase in domestic interest rates and/or to a shrinkage in the foreign international reserves. As the successful attacks on fixed exchange market explained above, there are unsuccessful attacks which may cause the unchanged exchange rate. However, those unsuccessful attacks lead to a depletion of international reserves and raise the interest rate because the authorities try attacks alone, while currency crises incorporate both successful and unsuccessful speculative attacks.
to defend the currency when the speculators decide to attack the market. The intuition is that a currency crisis could occur even if it did not cause a devaluation.

In this category the first attempt is to create an index which is called exchange market pressure (EMP). The index is a weighted average of the change of foreign exchange rate, the change in the international reserves loss alongside the change in the interest rate. As mentioned before, the key advantage of this measure is that it allows capturing both successful and unsuccessful speculative attacks. The next step is to define the currency crisis based on the EMP index. Therefore, a currency crisis is defined as an event when the EMP index exceeds a particular threshold value, that is, a crisis is represented as a binary variable.

Eichengreen, Rose and Wyplosz (1995) -pioneer researchers in this field- try to construct an indicator of the exchange rate pressure (speculative pressure) and define an exchange market crisis when the indicator moves at least two standard deviations above its mean.

\[ EMP_t = \triangle s_t + w_i \tilde{i}_t + w_c \tilde{c}_t \] (4.14)

where \( \triangle \) is the first-difference operator of \( s_t \). \( \triangle s_t, \tilde{i}_t, \) and \( \tilde{c}_t \) are the EMP components based on \( s_t, i_t, \) and \( c_t \). \( w_i \) and \( w_c \) are the EMP weights.

\[ CC_{it} = \begin{cases} 1 & \text{if } EMP > \overline{EMP} + 2SD(EMP) \\ 0 & \text{otherwise} \end{cases} \] (4.15)

The currency crisis definition of Glick and Hustchison (2000, 2005, 2006) and Bussi`ere and Fratzscher (2006) are analogous to Eichengreen et al. (1995). They apply a two standard deviation threshold. Furthermore, Kaminsky et al. (1997) employ a three standard deviation. There are three drawbacks of defining the EMP index. First, finding the appropriate weighted averages that are attached to each variable is difficult and there is no obvious way to compute them. Second, this measure is plagued by a
series of time aggregation problems that cast doubt on the effects it is capturing. Third, the index is defined in such a way that it tends to select situations which are largely unpredictable from a “bad fundamentals” perspective, (Flood and Marion, 1998).

In addition, as Kumar et al. (2002) point out, investors, managers of foreign reserve positions or a macro policy-maker, care primarily about the large depreciations of exchange rates i.e. successful attacks on the currency.

In this chapter, the definition of a currency crash is based on the sheer exchange rate changes expressed as the ratio of domestic currency to foreign currency. The aim is to capture only “successful” speculative attacks. That is, I focus on episodes that lead to a collapse of an exchange rate regime and exclude unsuccessful speculative attacks from my definition of crisis. However, I represent two different definitions according to the different generations of currency crises in the literature. To be more precise, the first and third generations explain the crises of the 1980s in Latin America and the 1997 in South-east Asia. These types of crises happened in the developing countries in which some of them suffer from weak economic fundamentals. While the second generation analyses the crisis in European countries which collapse due to investors’ expectations. Those developed economies do not suffer from weak economic fundamentals. Therefore, they have been recovered immediately from collapse, let’s say in six months. These distinctive characters of each generation in developing and developed regions contribute to define the different currency crash indexes.

Figure (4.2) displays the movements of nominal exchange rate\(^7\) in five selective emerging markets from 1990 to 2011. Two vertical lines show the occurrence of South-east Asia currency crisis. These economies are a sample of currency crises which are consistent with the first and third generations.

Figure (4.3) demonstrates the variations of nominal exchange rate in 4 developed

\(^7\)To compare the exchange rates between range of different countries, the exchange rate has been standardised. I divide the value of exchange rate by the maximum value of the exchange rate of each country, separately. The results (\(\text{exr}_p\)) provide the amounts between zero and one which are comparable.
countries between 1991 to 1997 when the crisis happened in September 1992. These countries are associated with the second generation and have strong fundamentals.

These two Figures clearly illustrate the difference in the two kinds of crises. By focusing on the episodes of crises between two vertical lines, Figure (4.2) indicates that crisis in the developed countries has been immediately recovered and gets back to normal level within two quarters. Whereas, the emerging market in Figure (4.2) shows a longer recovery process in those countries with weak fundamentals. This distinct feature of two group countries from different generation types provide a fact that I split the currency crises definition based on the different generations, more specifically based on the behaviour of exchange rates in the emerging and developed countries.

In order to make the countries comparable, first, I standardise all the continuous explanatory variables by subtracting the sample mean and dividing by the standard
deviation. Then I split the currency crises definition which is able to demonstrate the distinctive characteristic, I initially construct the indices as follows:

\[
\begin{align*}
dt_1 &= \text{exr}_t - \text{exr}_{t+1} \\
 dt_2 &= \text{exr}_t - \text{exr}_{t+2} \\
 dt_3 &= \text{exr}_t - \text{exr}_{t+3}
\end{align*}
\]

where \( dt_1 \) is the difference of the nominal foreign exchange rate at time \( t \) and time \( t + 1 \) (first quarter). \( dt_2 \) shows the difference of the nominal foreign exchange rate at time \( t \) and time \( t + 2 \) (including second quarter), and \( dt_3 \) is the difference of the nominal foreign exchange rate at time \( t \) and time \( t + 3 \) (including third quarter).

Using the above indices, the definition of currency crashes according to the first and the third generations is to express that a currency crash occurs if the depreciation of difference in the nominal exchange rate in the current quarter is less than 15% in the
first quarter, which has to keep increasing at most by 20% in the next two quarters. Thus, the definition \( I \) could be written as:

\[
CC_{13it} = \begin{cases} 
1 & \text{if } dt1 \leq 15\% \text{ and } dt2 \leq 20\% \text{ and } dt3 \leq 20\% \\
0 & \text{otherwise}
\end{cases}
\]  \tag{4.16}

where \( CC_{13it} \) is the currency crash index of the first and third generations for country \( i \) at time \( t \).

A currency crash of the second generation is defined, such that if a sharp depreciation of the difference in the nominal exchange rate is at least 15% in the first quarter and jumps up quickly at most by 8% during the two quarters ahead. So, the definition \( II \) is given by:

\[
CC_{2it} = \begin{cases} 
1 & \text{if } dt1 \geq 15\% \text{ and } dt2 \leq 8\% \text{ and } dt3 \leq 8\% \\
0 & \text{otherwise}
\end{cases}
\]  \tag{4.17}

where \( CC_{2it} \) denotes the currency crash index for country \( i \) at time \( t \) based on the second generation.

\section*{4.4 Methodology}

\subsection*{4.4.1 Panel Data Analysis}

I intend to take advantage of using a panel data approach which contains much information than other methodology. As Hsiao (2006) points out, the general benefits of panel data analysis are that (i) panel data usually contain more degrees of freedom and more sample variability than cross-sectional data or time series data. Thus, it improves the efficiency of econometric estimates. (ii) It is frequently argued that the reason for finding (or not finding) certain effects is due to ignoring the effects of certain variables in a model specification that are correlated with the included explanatory
variables. Panel data contain information on both the intertemporal dynamics and the individuality of the entities that may control the effects of missing or unobserved variables. (iii) Panel data generate more accurate predictions for individual outcomes by pooling the data. If individual behaviours are similar, conditional on certain variables, panel data provide the possibility of learning an individual’s behaviour by observing the behaviour of others. Thus, it is possible to obtain a more accurate description of an individual’s behaviour by supplementing observations of the individual in question with data on other individuals. (iv) Panel data contribute to uncover dynamic relationships, whereas the estimation of time-adjustment patterns using time series data often has to rely on arbitrary prior restrictions such as Koyck or Almon distributed lag models because time series observations of current and lagged variables are likely to be highly collinear. With panel data, inter-individual differences reduce the collinearity between current and lag variables, thereby assisting in the estimating unrestricted time-adjustment patterns.

However, a potential disadvantage of panel data is that it ignores some available information of the data. In country-based studies, it could be the case that some crucial information on a particular country is omitted from the pooled model. For instance, the capital account of a given country may not be as open as that of other countries, which would persuade the model to overestimate the probability of a crisis. On the other hand, the political situation of a country could be such that it makes an underestimation of this probability. There are two useful instruments which are able to fix these issues: fixed effect and random effects. In this sense, specifying between these two different effects is essential in that the fixed effects consider the “within” information only, whereas the random effects focus on the “between” information. Fixed effects are equivalent to country dummies that compare the country specific mean ignoring inter-country comparison. Despite having unbiased estimates, fixed effects result in a loss of information. On the other hand, random effects are more efficient but potentially
biased because these assumes that individual specific effects are uncorrelated with the independent variables.

4.4.2 Binary Logit for Panel Data

In this study, I apply binomial logit model in order to find the probability of crisis occurrence. First, I consider the theoretical framework of binomial logit models.

Models of binary outcomes concentrate on the determinants of the probability, \( p \), of the occurrence of one outcome rather than an alternative outcome that occurs with the probability of \( 1 - p \).

Logit is a standard binary outcome models that generally specifies different functional forms for \( p \) as a function of regressors, and the models are fitted by maximum likelihood (ML).

Suppose the outcome variable, \( y \), takes one of two values:\(^8\)

\[
y = \begin{cases} 
  1 & \text{with probability } p \\
  0 & \text{with probability } 1 - p 
\end{cases} \tag{4.18}
\]

The probability mass function for the observed outcome, \( y \), is \( p^y(1 - p)^{1-y} \), with \( E(y) = p \) and \( Var(y) = p(1 - p) \).

In a binary logit model that uses panel data, the conditional probability has the form:

\[
P_{it} = Pr[y_{it} = 1 | X_{it}, \alpha_i, \beta] = F(\alpha_i + X_{it}'\beta) \tag{4.19}
\]

\(^8\)The dependent variable, \( y \), takes only two values, so its distribution is unambiguously the Bernoulli, or binomial with one tail, with a probability of \( p \).
For logit model $F(.) = \Lambda(.)$

$$P_{it} = \Pr[y_{it} = 1|X_{it}, \alpha_i, \beta] = \Lambda(\alpha_i + X'_{it} \beta)$$

$$= \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \quad (4.20)$$

where $F(.)$ denotes the cumulative distribution function (c.d.f) on $(-\infty, \infty)$. This ensures that the bounds $0 \leq p \leq 1$ are satisfied. $\Lambda(.)$ is the c.d.f. of the logistic distribution.

The density for a single observation $i$ can be derived as:

$$f(y_{it}|X_{it}, \alpha_i, \beta) = \prod_{t=1}^{T} P_{it}^{y_{it}}(1 - P_{it})^{1-y_{it}} \quad (4.22)$$

$$f(y_{it}|X_{it}, \alpha_i, \beta) = \prod_{t=1}^{T} F(\alpha_i + X'_{it} \beta)^{y_{it}}(1 - F(\alpha_i + X'_{it} \beta))^{1-y_{it}} \quad (4.23)$$

$$f(y_{it}|X_{it}, \alpha_i, \beta) = \prod_{t=1}^{T} \left( \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{y_{it}} \left( 1 - \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{1-y_{it}} \quad (4.24)$$

In order to estimate a binary model by ML, the likelihood function is given by:

$$L(\theta) = \prod_{i=1}^{n} f(y_{it}|X_{it}, \alpha_i, \beta) \quad (4.25)$$

$$= \prod_{i=1}^{n} \prod_{t=1}^{T} \left( \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{y_{it}} \left( 1 - \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{1-y_{it}} \quad (4.26)$$

then log-likelihood function is written:

$$l(\alpha, \beta) = log(\alpha, \beta) = \sum_{i=1}^{n} log(f(y_{it}|X_{it}, \alpha_i, \beta))$$
\begin{align*}
(4.27) \quad & = \sum_{i=1}^{n} \sum_{t=1}^{T} \log \left[ \left( \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{y_{it}} \left( 1 - \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right)^{1-y_{it}} \right] \\
& = \sum_{i=1}^{n} \sum_{t=1}^{T} \left[ y_{it} \log \left( \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right) + (1 - y_{it}) \log \left( 1 - \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right) \right] \\
(4.28) \quad & = \sum_{i=1}^{n} \sum_{t=1}^{T} \left[ y_{it} \log \left( \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right) + (1 - y_{it}) \log \left( 1 - \frac{e^{(\alpha_i + X'_{it} \beta)}}{1 + e^{(\alpha_i + X'_{it} \beta)}} \right) \right] \\
\end{align*}

The MLE attempts to find the $\hat{\beta}$ that maximises above log-likelihood function. It is obtained by iterative methods and is asymptotically normally distributed.

4.4.3 The Dataset

In contrast to other papers, which so far used monthly and yearly datasets, I use a quarterly dataset which is similar to Burkart and Coudert (2000) and Babecký et al. (2013). The idea is that quarterly data is able to delineate the feature of a crisis more accurately and evaluates the leading indicators’ behaviour as the actual crisis approaches. Although more convenient, annual frequency datasets have several important drawbacks, for example, they are an aggregate data, i.e., they are a set of compact information which come from smaller intervals as well as quarterly, monthly, weekly, even daily. Thus, taking annual dataset is similar to ignoring part of the information, particularly harmful in financial market in which transactions are recorded moment by moment. The exchange rate market and stock market are examples of high frequency data. Moreover, annual datasets do not have sufficient capacity to predict values of leading indicators which is another disadvantage.

On the other hand, some papers applied monthly data such as Ari (2012). A question that comes to mind is whether macroeconomic fundamental (non-financial) datasets such as GDP are available for high frequency interval; the answer is no. In order to work out this shortcoming, some papers tried to ignore the explanatory variables which may cause a bigger problem of omitting explanatory variables. Several authors, nevertheless, have used the methods to transfer quarterly into monthly datasets. Their techniques may mean that the model is confronted by a weak robustness problem. Burkart
and Coudert (2000) also stress that a monthly time-frame introduces the problem of autoregressive effects and possibly complicated lag structures.

The independent variables are composed of seven quarterly macroeconomic and financial variables for 26 emerging and developed countries around the world.\(^9\) The sample period of the emerging market is between 1990:Q1 to 2012:Q4. Apart from United Kingdom, the developed countries are considered from 1990:Q1 through to 1998:Q4, because their currencies have been changed to Euro from 1998. I use nearly 22 years for the present study because one limitation of some empirical studies is that the sample is restricted to crisis episodes. This restriction prevents detection of “false signals” i.e. when the explanatory variable predicts a crisis that does not actually occur. This argument calls for setting the sample large enough to include both crisis and non-crisis episodes to guarantee the neutrality of the results. The dataset has mainly been collected from the IMF International Financial Statistics database, Data Stream, and the countries’ central banks. The panel data thus contains a set of 22 years for 26 countries, which makes a total of 2288 potential observations.

### 4.4.4 Potential Leading Indicators

There are three approaches to identify what variables are eligible to be included as potential leading indicators. First, some authors follow the theoretical papers and determine the potential leading indicator as Kaminsky \textit{et al.} (1998) did. Second, the studies come after systematic literature reviews. They rely on published empirical research for useful and reliable leading indicators and generate a large dataset including all detected indicators and sometimes different transformations, such as Rose and Spiegel (2011), Frankel and Saravelos (2012), Babecký \textit{et al.} (2013). Finally, some research

---

\(^9\)The emerging markets consist of Latin America (Argentina, Brazil, Chile, Colombia, Mexico, Venezuela), Europe, Middle East and Africa (Czech Republic, Hungary, Poland, Russia, Slovakia, South Africa, Turkey) and Non-Japan, Asia (China, Hong-Kong, India, Indonesia, Malaysia, Philippines, Singapore, South Korea, Thailand). The developed countries contain United Kingdom, Spain, Italy and France.
takes all the available variables and add various transformations.\textsuperscript{10}

I choose the second approach and rely on systematic literature surveys as owing to the restrictions in data availability and limitations in time periods.

Considering papers and following Frankel and Rose (1996) and Bussière and Fratzcher (2006), I select seven variables which are the most relevant potential leading indicators.\textsuperscript{11} They are listed as follows with a brief description:

- \textit{Rate of growth domestic credit}

Frankel and Rose (1996) take domestic credit growth as a measure of monetary policy. Whereas abundant empirical studies consider it as a measure of financial liberalisation as well as Kaminsky \textit{et al.} (1998). On the other hand, Sachs \textit{et al.} (1996) argue that it is a good proxy for banking system vulnerability, as swift credit growth is likely associated with a decrease in lending standards. Overall, it is concluded that the credit growth could be regarded as a substantial indicator in the prediction of currency crises. Obstfeld (2012) and many others admit that rapid increase in domestic credit plays an economically and statistically significant role in predicting subsequent crises. Besides, Berg and Pattillo (1999) demonstrate that the domestic credit growth is an informative indicator in all

\textsuperscript{10}See Lau and Yan (2005) who represent a comprehensive survey of the leading indicators applied in the existing empirical studies of speculative attacks and currency crises.

\textsuperscript{11}Several papers have shown that contagion effects are important as an explanatory factor of currency crisis, empirically. For instance, Eichengreen, Rose and Wyplosz (1996), Glick and Rose (1998) represent the trade linkage and Fratzcher (2002) stress the real financial interdependence as a vital explanatory factor of currency crisis contagion, while Kruger, Osakwe and Page (1998) and Burkart (2002) find out that the evidence of regional contagion is strong. I adopt a regional contagion instead of using the general indicator of contagion, similar to Kruger, Osakwe and Page (1998) and Burkart (2002). I have constructed a contagion indicator that captures regional contagion. The value of this indicator for country $j$ is equal to one if and only if at least one country is experiencing a crisis in the preceding quarter in the same geographical region. If this condition is not satisfied, the contagion indicator takes a value of 0 for that period. More formally, the regional contagion indicator $R$ is given by:

$$
R = \begin{cases} 
1 & \text{if } CC_{it} = 1 \text{ for } i \neq j \text{ and } i, j \in \text{same region} \\
0 & \text{otherwise}
\end{cases}
$$

(4.30)

However, the result of this indicator is insignificant and not consistent with theory. So, it has been removed.

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three approaches for anticipation of crises. Therefore, I would expect that the probability of a currency crisis increases when domestic credit growth is high.

- **Government budget as a fraction of GDP**

  It is defined as a measure of fiscal policy within the first generation in line with Frankel and Rose (1996) and Ari (2012). Excessive budget deficits reduce available national savings level. So, it may cause high inflation and interest rates. National investments, in turn, lowers investors’ confidence which makes a situation more vulnerable to speculative attacks. Hence, I would expect that high budget deficits will raise the crisis probability.

- **Ratio of reserves to GDP**

  In order to assess international illiquidity, this indicator is introduced. A high ratio of short-term international liquefiable liabilities to foreign exchange reserves is recognised by the third generation model as a crucial predictor of currency crises after the Mexican debt crisis. This indicator is defined as the quarterly change in reserve money as a percentage of GDP, and attempts to capture Krugman’s insight that monetisation of the government deficit is key to explaining exchange rate collapses. I would expect this variable to have a negative effect on the probability of a crisis.

- **Current account as a percentage of GDP**

  According to the first and second generation theories of the early warning signal, a deterioration of the current account balance is expected in anticipation of a currency crisis. Reinhart and Reinhart (2009) argue that large current account deficits prepare the circumstances that lead emerging market economies to collapse. In other words, a rise (fall) in this ratio is generally associated with large external capital inflows (outflows). This indicates a diminished (high) probability to devalue and thus to lower (increase) the probability of a crisis. Therefore, I
expect to find a negative relationship between the current account balance and the probability of currency crisis.

- **Growth rate of real output**

Based on the second generation explanation which points out the periods of economic slowdown often precede currency crisis episodes. I use the real output growth in the model, consistent with many papers in this area. It is expected that the growth of production lowers the crisis probability.

- **Ratio of debt to GDP**

Frankel and Rose (1996) believe that the ratio of debt to GDP displays the level of international debt and measures the vulnerability to external shocks. Higher debt decreases the crisis probability.

- **Overvaluation of exchange rate**

The overvaluation of exchange rate has been considered as the best traditional predictor of the currency crises since the early empirical studies, (Lau and Yan, 2005). (see Frankel & Rose, 1996; Kaminsky and Reinhart, 1996; Goldfain and Valdès, 1998) The expected exchange rate overvaluation is expected to be associated with an increased probability of currency crisis. The intuitive idea behind the power of the overvaluation of real exchange rate comes from the first generation crisis theory.

On the one hand, to attract short-term foreign capital, which would contribute to the overheating of the domestic economy, and on the other hand to decrease the international competitiveness of a country compared to its commercial competitors, that can generate unsustainable external positions. An overvalued exchange rate would thus imply a rise in the likelihood of a crisis.

Frankel & Rose (1996) define the degree of overvaluation as the deviation from
the average bilateral real exchange over the period. Later, other studies have enhanced the overvaluation definition. For instance, Bussière and Fratzcher (2006) define the exchange rate overvaluation as the deviation of the real exchange rate from its trend. Goldfajn and Valdès (1998) and Ari (2012) express it as the deviation of the real exchange rate from a Hodrick-Prescott trend. Many studies including Kaminsky et al. (1998), Abied (2003), and Ari (2012) find a significant effect of exchange rate overvaluation in predicting the crises.

Definition of the exchange rate overvaluation is given by:

\[
\text{overvaluation of exchange rate}_{it} = \left( \frac{\text{REER}_{it} - \text{Trend}_{it}}{\text{Trend}_{it}} \right) \times 100 \tag{4.31}
\]

**Argument about Overvaluation**

As stated above, majority of papers express the overvaluation of exchange rate as a deviation of exchange rate from the HP trend\(^{12}\) which causes a serious misconception in early warning system literature. The HP filter is a two-sided filter, that is, it uses the past and future information in order to construct the smoothed magnitude for the current time point \(t\). Thus, using Hp filter to construct any predictor of crisis in the early warning literature causes inconsistent results.

The leading indicators in the literature are applied to predict the crises. By default, the predictive variables do not know about what is happening in the future and do not carry any future information at the current time. On the other hand, to construct the overvaluation of exchange rate at current time \(t\), the HP filter uses the observation of future time \(t + i, i > 0\). Since the HP filtered magnitudes consist of the future magnitudes, HP filter could not be an appropriate predictor, because it knows the changes of exchange rate in the future.

\(^{12}\)Alessi and Detken (2011) compute several transformations of the variables in order to check for their forecasting performance. Variables are used deviations from slowly adjusting, recursive Hodrick-Prescott filter trend with a \(\lambda\) of 100000 instead of the usual 1600 for quarterly data.
Figure 4.4: Smoothed and Cyclical components of Thailand Exchange Rate

Figure (4.4) displays the trend and cyclical components of Thailand exchange rate which calculated by Hp filter. As clearly shown in the Figure, given the sharp spike of exchange rate series in 1998Q1, the trend series starts to go up gradually before the exchange rate’s spike happens. The reason of rise in trend is that it is aware that the currency crisis (sudden jump) is coming soon. So, the overvaluation of exchange rate constructed by HP filter is not an appropriate leading indicator for forecasting the currency crises, while is extensively applied in the literature.

In order to overcome this problem, a filter with a recursive procedure is required. A filter that does not use the future value at each point of time. I use Kalman filter because the distinct property of Kalman filter is recursive algorithm. The recursive setting assumes that only current and past states influence the current observation. Kalman filter, in fact, employs backwards filtering for a dataset and updates earlier predictions by recursive calculations backwards from time $t$. Therefore, I apply Kalman filter recursively to deterend the real effective exchange rate of each country. I then compute overvaluation of exchange rate based on (4.31) formula, similar to previous studies.
4.4.5 Model Specification

To identify the potential determinants of currency crashes and consider the probability of their occurrence, I express two different definitions (I and II) of the currency crashes based on the different generations over the period 1990 to 2012 for a sample of 22 emerging countries and the period 1990 to 1998 for 4 developed countries. The country sample includes 6 Latin American countries, 9 Asian (not including Japan) countries, 7 Europe, the Middle East and Africa, and 4 developed countries.

To provide probabilities of a currency crisis over the sample period, I fit binary logit model specification to the quarterly data. The estimated logit model takes the following form:

\[
Prob(CC_{it+2} \mid X_{it}\beta) = F(X_{it}\beta) = \frac{e^{X_{it}\beta}}{1 + e^{X_{it}\beta}}
\]  

(4.32)

\(CC_{it}\) takes value 1 when a currency crisis occurs at time \(t\). \(F\) is the logistic cumulative distribution function. \(X_{it}\) displays a set of potential leading indicators that cause the currency crisis in those countries. These indicators are namely rate of growth domestic credit (\(rdomcrd\)), government budget as a fraction of GDP (\(budg\)), ratio of reserve to GDP (\(res_{ \text{GDP}}\)), current account as a percentage of GDP (\(curact\)), growth rate of real output (\(lngdp\)), overvaluation of exchange rate (\(dev_{ bw}\)), ratio of debt to GDP (\(Gov_{ debt}\)).

Considering a panel of 26 countries leads us to estimate a binary logit fixed effect and random effects models. Checking within and between variation of indicators shows that 6 indicators including rate of growth domestic credit, government budget deficit, current account, growth rate of real output, overvaluation of exchange rate, ratio of debt to GDP have greater within variation than between variation. Whilst the ratio of reserve to GDP has a lower within variation than between variation. In other words, most of the variation is within variation rather than between variation. I therefore apply fixed effect estimators that rely on within variation and would be efficient.
This chapter is related to the paper by Frankel and Rose (1996) which focuses on developing countries and adopts a probit regression approach. However, this study differs from their study in two significant aspects. First, unlike Frankel and Rose, it focuses on binary logit model instead of probit model because probit model assumes that the cumulative distribution function (CDF) is normal while a logit model uses a cumulative logistic function which is closer to reality and the nature of crises. Next, I use a different sample period and my analysis takes into consideration the possibility of contagion effects. Eichengreen, Rose and Wyplosz (1996) also incorporate contagion effects into their analysis. However, they focus on industrial countries and define a global contagion, whereas I work on developing countries and a local contagion effect.

Moreover, the exchange rate is one of the crucial informative determinants of currency crisis shown in most of the empirical studies in the literature, I attempt to improve the computation of the overvaluation indicator. A property of this study is to use the new definition of overvaluation. Commonly, the majority papers describe overvaluation as the deviation of real exchange rate from its average. However, some studies (Bussière and Fratzcher, 2006; Ari, 2008) calculate the deviation from its trend. The trend is computed by a HP filter.\footnote{I also use an alternative filter, Butterworth filter, to define the overvaluation, because Butterworth filter is more flexible than Hodrick-Prescott in capturing common features of economic data. But it also keeps the weakness of HP filter.} I use a recursive Kalman filter in order to eliminate the forward-looking impact of HP filter.

In addition, I apply lagged indicators. The intuition is that the changes in the leading indicators do not necessarily coincide with the crisis incident particularly in a short interval.
4.4.6 Estimation Results

To evaluate stationarity of leading indicators, I apply Fisher-type test which represents the unit root test for panel data.\textsuperscript{14} The advantage of Fisher-type test among the other panel unit root tests is that it does not require strongly balanced data, and the individual series can have gaps which fits my panel dataset. The result denotes that all leading indicators are stationary.

According to the two different definitions of currency crashes, two logit models are estimated. Not only the method of the estimation of these binary logit models are similarly maximum likelihood, but also the potential leading indicators initially resemble each other. However, the indicators are applied with different lagged orders. In the following Tables, the estimated coefficients of logit model, log likelihood values and marginal effects are reported. The negative numbers next to the indicators’ name denote the lagged order of the indicators.

Table (4.1) shows the estimates of the currency crashes model which is defined by the first and third generations’ models. In the model, the signs of nearly all indicators are generally consistent with a priori expectations except overvaluation of exchange rate and budget deficit. Two indicators, current account and output growth are statistically significant. These two indicators imply that a crisis is approaching. The sign of coefficients can be interpreted. For instance, a deterioration in the current account leads to low capital inflows, which, in turn, increases the probability of currency crisis.

As budget deficit has the opposite sign, I eliminate it and reestimate the model without it. The result is displayed in Table (4.2). As may be seen, no large differences

\textsuperscript{14}Generally, to implement a unit root test (or stationarity) in panel datasets, an autoregressive model is assumed:
\begin{equation}
y_{it} = a_{it} + \rho_{i}y_{i,t-1} + e_{it}
\end{equation}

where $e_{it}$ is a mean-zero regression error term and $a_{it}$ represents the deterministic part of the model. $i = 1, \ldots, N$ indexes panels, and $t = 1, \ldots, T$ indexes time. $a_{it}$ may include panel-specific intercepts (fixed effects), a panel-specific time trend, or nothing, in which case $y_{it}$ is predicated to have mean zero for all panels.

Fisher-type test investigates null hypotheses of the general form $H_0 : \rho_i = 1$ versus $H_a : \rho_i < 1$. 
Table 4.1: Estimates of Currency Crashes based on First and Third Generations
(Definition I)
22 Emerging Markets with Weak Fundamentals

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>z</th>
<th>P ≥</th>
<th>z</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Overvaluation of exchange rate</td>
<td>-0.068</td>
<td>0.021</td>
<td>-3.14</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Budget deficit (-1)</td>
<td>-0.012</td>
<td>0.081</td>
<td>-0.15</td>
<td>0.878</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current account (-2)</td>
<td>-0.228***</td>
<td>0.091</td>
<td>-2.48</td>
<td>0.013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Domestic credit growth rate (-1)</td>
<td>0.003</td>
<td>0.02</td>
<td>0.15</td>
<td>0.882</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output growth rate (-3)</td>
<td>-0.037***</td>
<td>0.017</td>
<td>-2.18</td>
<td>0.029</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Foreign currency reserve (-1)</td>
<td>-1.427</td>
<td>1.403</td>
<td>-1.02</td>
<td>0.309</td>
<td></td>
<td></td>
</tr>
<tr>
<td>National debt (-4)</td>
<td>-0.017</td>
<td>0.018</td>
<td>-0.91</td>
<td>0.361</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood: -61.228
Prob. (LR stat.): 0.000
No. of observations: 946
Marginal effect: 0.241
(0.053)

95% confidence interval: [-0.003, 0.486]

*** shows significant coefficients in the 95% confidence interval.

in results are observed. Current account and output growth rate are still statistically
significant and exchange rate overvaluation still has the opposite sign. Thus, I exclude
overvaluation of exchange rate to examine whether the results get improved. The
estimation is reported in Table (4.3).

Excluding overvaluation of exchange rate shown in Table (4.3), the results are quite
similar to the previous results shown in Table (4.1) and (4.2), but the absolute values
of log likelihood rise from -61.228 to -61.402 and then reach -67.227. This means that
the last estimation which has excluded exchange rate overvaluation and budget deficit
is the fitted model for the definition \( I \). Therefore, two significant indicators, current
account with two lags and output growth rate with three lags, are the signals to warn
of an approaching currency crisis in the first and third generations.

Table (4.4) displays the estimates of the leading indicators according to the definition
\( II \) of the second generation of the currency crises. The signs of all leading indicators
are supported by theory except overvaluation of exchange rate. As shown in the Table,
Table 4.2: Estimates of Currency Crashes based on First and Third Generations (Definition I), excluding budget deficit
22 Emerging Markets with Weak Fundamentals

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>z</th>
<th>P ≥</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overvaluation of exchange rate</td>
<td>-0.068</td>
<td>0.021</td>
<td>-3.17</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>Current account (-2)</td>
<td>-0.231***</td>
<td>0.091</td>
<td>-2.54</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>Domestic credit growth rate (-1)</td>
<td>0.001</td>
<td>0.012</td>
<td>0.16</td>
<td>0.876</td>
<td></td>
</tr>
<tr>
<td>Output growth rate (-3)</td>
<td>-0.037***</td>
<td>0.017</td>
<td>-2.12</td>
<td>0.034</td>
<td></td>
</tr>
<tr>
<td>Foreign currency reserve (-1)</td>
<td>-1.38</td>
<td>1.37</td>
<td>-1.01</td>
<td>0.314</td>
<td></td>
</tr>
<tr>
<td>National debt (-4)</td>
<td>-0.017</td>
<td>0.018</td>
<td>-0.95</td>
<td>0.34</td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood                                  -61.402
Prob. (LR stat.)                                 0.000
No. of observations                              963
Marginal effect                                  0.235
(0.045)
95% confidence interval                          [0.005, 0.465]

*** shows significant coefficients in the 95% confidence interval.

Table 4.3: Estimates of Currency Crashes based on First and Third Generations (Definition I), excluding overvaluation of exchange rate and budget deficit
22 Emerging Markets with Weak Fundamentals

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>z</th>
<th>P ≥</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current account (-2)</td>
<td>-0.241***</td>
<td>0.088</td>
<td>-2.73</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Domestic credit growth rate (-1)</td>
<td>0.014</td>
<td>0.031</td>
<td>0.46</td>
<td>0.643</td>
<td></td>
</tr>
<tr>
<td>Output growth rate (-3)</td>
<td>-0.033***</td>
<td>0.016</td>
<td>-2.01</td>
<td>0.044</td>
<td></td>
</tr>
<tr>
<td>Foreign currency reserve (-1)</td>
<td>-1.75</td>
<td>1.338</td>
<td>-1.31</td>
<td>0.191</td>
<td></td>
</tr>
<tr>
<td>National debt (-4)</td>
<td>-0.022</td>
<td>0.019</td>
<td>-1.18</td>
<td>0.237</td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood                                  -67.227
Prob. (LR stat.)                                 0.0003
No. of observations                              1001
Marginal effect                                  0.209
(0.075)
95% confidence interval                          [-0.021, 0.439]

*** shows significant coefficients in the 95% confidence interval.

two indicators, current account and national debt are statistically significant at 95%
and 90% confidence intervals, respectively.

I remove the overvaluation of exchange rate from the model under definition II,
due to its opposite sign, then estimate the model. The result is demonstrated in Table (4.5). By doing so, the estimation results relatively stay the same. However, comparing the absolute values of log likelihood in Tables (4.4) and (4.5) indicates an increase from -315.218 to -317.064 which implies that the model shown in Table (4.5) is the best fitted. Therefore, two significant indicators, current account with two lags and national debt with three lags, are the signals to warn of an approaching currency crisis in the second generation.

Table 4.4: Estimates of Currency Crashes based on Second Generation (Definition II)
4 Developed Countries with Strong Fundamentals

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>z</th>
<th>P ≥</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overvaluation of exchange rate</td>
<td>-0.002</td>
<td>0.011</td>
<td>-0.18</td>
<td>0.854</td>
<td></td>
</tr>
<tr>
<td>Budget deficit (-1)</td>
<td>0.038</td>
<td>0.031</td>
<td>1.22</td>
<td>0.222</td>
<td></td>
</tr>
<tr>
<td>Current account (-2)</td>
<td>-0.062***</td>
<td>0.031</td>
<td>-1.99</td>
<td>0.046</td>
<td></td>
</tr>
<tr>
<td>Domestic credit growth rate (-2)</td>
<td>0.01</td>
<td>0.014</td>
<td>0.74</td>
<td>0.459</td>
<td></td>
</tr>
<tr>
<td>Output growth rate (-2)</td>
<td>-0.002</td>
<td>0.01</td>
<td>-0.22</td>
<td>0.825</td>
<td></td>
</tr>
<tr>
<td>Foreign currency reserve</td>
<td>-0.789</td>
<td>0.545</td>
<td>-1.45</td>
<td>0.148</td>
<td></td>
</tr>
<tr>
<td>National debt (-3)</td>
<td>-0.01**</td>
<td>0.006</td>
<td>-1.67</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-314.114</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob. (LR stat.)</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of observations</td>
<td>1714</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal effect</td>
<td>0.277</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>95% confidence interval</td>
<td>[0.132, 0.423]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** shows significant coefficients in the 90% confidence interval.
*** shows significant coefficients in the 95% confidence interval.

In general, the results confirm that current account, rate of growth domestic credit, growth rate of real output, ratio of reserves to GDP and ratio of national debt to GDP are known as the potential leading indicators in both types of currency crashes definitions. While, output growth rate is an informative predictor in the first and third generations (definition I), in the second generation (definition II), national debt is an informative predictor. Similarly, in all three generations (definition I and II) current account is a substantial predictor of currency crises. Overvaluation of exchange rate
Table 4.5: Estimates of Currency Crashes based on Second Generation (Definition II)
4 Developed Countries with Strong Fundamentals

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Coef.</th>
<th>Std. Err.</th>
<th>z</th>
<th>P ≥</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget deficit (-1)</td>
<td>0.035</td>
<td>0.031</td>
<td>0.14</td>
<td>0.253</td>
<td></td>
</tr>
<tr>
<td>Current account (-2)</td>
<td>-0.06***</td>
<td>0.03</td>
<td>-1.96</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Domestic credit growth rate (-2)</td>
<td>0.011</td>
<td>0.014</td>
<td>0.78</td>
<td>0.435</td>
<td></td>
</tr>
<tr>
<td>Output growth rate (-2)</td>
<td>-0.002</td>
<td>0.009</td>
<td>-0.26</td>
<td>0.795</td>
<td></td>
</tr>
<tr>
<td>Foreign currency reserve</td>
<td>-0.632</td>
<td>0.532</td>
<td>-1.19</td>
<td>0.234</td>
<td></td>
</tr>
<tr>
<td>National debt (-3)</td>
<td>-0.01***</td>
<td>0.006</td>
<td>-1.65</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-317.064</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prob. (LR stat.)</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of observations</td>
<td>1758</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marginal effect</td>
<td>0.299</td>
<td>(0.00)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

95% confidence interval [0.148, 0.449]

** shows significant coefficients in the 90% confidence interval.
*** shows significant coefficients in the 95% confidence interval.

also could not predict the currency crises in the generations. However, budget deficit
is not a predictor for the first and third generations, it might predict a crisis for the
second generation.

The results also show a variety of lagged orders for two estimated models. The
high (low) lagged order shows the lower (greater) influence of indicator in explaining
the model. Nevertheless, an early warning indicator with high lagged order is able
to inform us soon enough before a currency crash occurs, whereas it is not true for
the indicator with less lagged order. Comparing the lagged orders of foreign currency
reserve for the definitions I and II of Tables (4.1) and (4.5), which are 1 and 0, reveals
that the foreign currency reserve in the definition I has a negligible impact on the
crises event, as expected by theory. But, it could warn of an approaching crisis at least
one quarter before. However, in the definition II based on the speculative attacks on
exchange market, foreign currency reserve plays the main role in the crises occurrence
at the same time of crisis.

In terms of the argument about calculating the overvaluation of exchange rate, the
finding clearly supports my critique of the previous literature. Overvaluation in the present study is calculated by Kalman filter recursively. The results of the two logit models show that the overvaluation of exchange rate cannot be a desirable predictor of currency crises, because the sign of its coefficient is not consistent with theory. Thus, it is excluded from my estimated models.

According to this finding, the calculation of overvaluation by HP filter in previous papers (Goldfajn and Valdés, 1998; and Ari, 2012) is flawed. In order to compare my results with HP filter in terms of calculation of overvaluation, I apply HP filter and also BW filter. I then estimate the two models. The results display highly significant overvaluation coefficients, as expected, because HP filter intrinsically includes the future values at the current values. Therefore, exchange rate overvaluation calculated by HP filter is not an appropriate predictor, because by applying Kalman filter which eliminates the impact of the future values on the current values, the result turns out to be insignificant.

4.5 Conclusion

This chapter aims to detect the determinants of the currency crises before they happen. It investigates a number of variables as potential leading indicators of currency crisis. The novelty of this chapter is the classification of the currency crashes definition according to the different generations of currency crises in the literature. Two different currency crashes are defined as successful attacks on the exchange market where there is a sudden depreciation in the exchange rate market. To analyse the model, a binary logit model is applied over 22 developing (with weak fundamentals) and 4 developed (with strong fundamentals) countries with a quarterly base.

The contribution of the current study is to define the currency crisis as a dependent variable of logit model into two different disciplines. Definition I is based on the
exchange rate depreciations in the first and third generations which is associated with emerging markets and developing countries, while definition II is based on exchange rate depreciations in the second generation which is associated with developed countries.

The potential leading indicators as the independent variables of logit model are overvaluation of exchange rate, budget deficit, current account, domestic credit growth rate, output growth rate, ratio of reserve to GDP, and ratio of debt to GDP.

The results of estimating two different models according to two different definitions show that current account and output growth rate are sensible predictors for emerging markets and developing countries in the first and third generation’s model, whereas current account and national debt are appealing predictors for developed countries in the second generation’s model.

In the same vein as Frankel and Rose (1996), in this study, GDP growth rate is detected as a determinant of currency crashes and budget deficit appears not to play a role in predicting the currency crashes in emerging markets with weak fundamentals under the definition I. However, national debt is detected as an informative determinant for the definition II of crashes in developed countries, while Frankel and Rose (1996) examine several debt composition indicators in their model and in turn find statistically insignificant debt coefficients for emerging markets (same as my result for the definition I). These results clearly show the accuracy of my definitions. One such result is that national debt is able to warn of crises in developed countries and not in emerging markets.

The findings indicate that current account is a leading indicator of currency crises for all generations in emerging and developed countries. On the other hand, the results reveal that the sign of exchange rate overvaluation in the models is not consistent with the early warning theory. In other words, the finding does not confirm that the overvaluation of exchange rate has an important role in explaining the incidence of currency crashes in two type of definitions, contrary to Frankel and Rose (1996),
Kaminsky *et al.* (1998), Abied (2003), and Ari (2012), because the calculation of overvaluation in this study differs from other empirical studies.

Calculation of exchange rate overvaluation in this survey differs from other empirical studies, in one respect: I apply Kalman filter recursively. Overvaluation is usually expressed as a deviation of exchange rate from its trend, such that the trend is estimated by HP filter. HP filter is not an appropriate predictor, because it includes the future values, while Kalman filter employs the past values to find the current values. Therefore, the finding confirms the misconception of previous studies.
Chapter 5

Conclusions

This thesis examines exchange rate behaviour in three distinct topics, purchasing power parity, uncovered interest rate parity and currency crises.

First, in order to analyse exchange rate movements, a new instrument, wavelet analysis, is introduced. It is followed by investigation of purchasing power parity (PPP) proposition applying wavelet analysis, in Chapter 2. Second, given the mixed results of testing uncovered interest rate parity (UIRP) hypothesis in the empirical literature, the UIRP condition is examined employing wavelet analysis, in Chapter 3. Finally, given the immanent volatile exchange rate series, Chapter 4 focuses on the sharp devaluation of exchange rate fluctuations and try to identify the potential leading indicators of the currency crises.

Describing wavelet analysis and its properties in Chapter 2 reveals that wavelets easily deal with nonstationary time series and time-varying parameters of the series. It is also able to break down a time series into its constituent components, with different frequencies (multiresolution analysis). This feature of wavelet analysis is very appealing in examining economic relationships, particularly where short-run and long-run relationships could be distinguished.

Applying this property of wavelet to examine the PPP doctrine for a pair of coun-
tries, UK and US, results in a positive (and close to unity) impact of price ratio on exchange rates. Therefore, the findings show that the PPP holds for long-run horizon.

Implementing wavelet decomposition on the UIRP variables, and using bilateral currency of ten different countries, the findings in Chapter 3 suggest that there is both negative and positive relationship between expected exchange rate changes and interest rate differentials. A possible conclusion could be drawn that the negative effect, associated with short-run, is supported by the fixed-price MF model and the positive effect, associated with long-run, is consistent with the flexible-price monetary model. Findings reported in this chapter are similar to Hacker et al. (2012).

Finally, separating the definition of currency crises based on three different generations, and applying a binary logit model over 22 emerging markets (with weak fundamentals) and 4 developed countries (with strong fundamentals) with a quarterly base in Chapter 3 provide two different estimation results. The results suggest that current account and output growth rate are sensible predictors for emerging markets and developing countries in the first and third generation models, whereas current account and national debt are appealing predictors for developed countries in the second generation model.

The findings indicate that current account is a leading indicator of currency crises for all generations in emerging and developed countries. Moreover, the results reveal that the sign of exchange rate overvaluation in the models is not consistent with the early warning theory. In other words, the finding does not confirm that the overvaluation of exchange rate has an important role in explaining the incidence of currency crashes in two type of definitions.

In this chapter, calculation of exchange rate overvaluation is criticized and the finding confirms the misconception of previous studies. In order to overcome this problem, the finding suggests using a Kalman filter recursively to construct the exchange rate overvaluation indicator.
References


Fleming, J. M. (1962). Domestic financial policies under fixed and under floating ex-

Frankel, J. and Saravelos, G. (2012). Can leading indicators assess country vulner-
ability? evidence from the 200809 global financial crisis. *Journal of International


European Central Bank.


*European Economic Review*, 16(1):145–165.

rates. In Grossman, G. M. and Rogoff, K., editors, *Handbook of International Eco-
nomics*, volume 3 of *Handbook of International Economics*, chapter 32, pages 1647–
1688. Elsevier.


Papers 7880, National Bureau of Economic Research, Inc.


