Three Essays in Game Theory

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June 2016
To my grandparents Ida, Raisa, Kardan and Yuri
Three Essays in Game Theory

by

Maria Kozlovskaya

Abstract

This thesis applies game theoretic methods to problems of individual choice and business strategy. It consists of three self-contained chapters. Chapters 2 and 3 explore strategic interaction between agents who are concerned about the social consequences of their actions, while Chapter 4 considers information use by firms who can spy on their competitors. By investigating these diverse topics, the thesis shows how Game Theory can be used to explain and predict outcomes in a variety of real-life scenarios of economic interest.

Chapter 2 explores the effect of “other-regarding” preferences on the outcomes of repeated social interactions, modelled as a Prisoner’s Dilemma game. The models of inequality aversion and guilt aversion are identified as compatible with the existing laboratory evidence on the game, and a novel experiment is run to test them against each other. The experiment provided support for the inequality aversion model, which shows that fairness is an important consideration in repeated social interaction.

Chapter 3 develops a formal theory of moral choice by proving a representation theorem for guilt-averse preferences. In the chapter, an axiomatic approach is used to deduce a utility function which explains how people trade off material and moral incentives. A novel logarithmic utility is proposed, which is grounded in realistic psychological assumptions and accounts well for the existing body of laboratory evidence.

Chapter 4 investigates the welfare consequences of industrial espionage by modelling it as an information acquisition game played by duopolists. In the model, firms can acquire information about unobservable demand either by conducting market research, or by spying on their competitors. The chapter shows how the two information sources affect price and quantity decisions taken by firms. It also shows that aggregate profit and social welfare grow with the precision of the intelligence technology in most environments.
Acknowledgements

I would like to thank my supervisor and co-author Professor Martin Kaæe Jensen, without whom the completion of this work would have been impossible. Martin was the most supportive, inspiring and encouraging mentor, and helped me become the person I always wanted to be – a theorist.

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I would like to thank my fiancé Jonathan for all his love and support, and for the sleepless nights he spent proofreading this thesis.

If there are any good traits or talents in me, they are all to my mother’s credit. I can’t thank her and my father enough for dedicating their life to bringing me up, and for then letting me go to the UK against their best judgement.

Last but not least, I am grateful to Dr William Pouliot, who taught me Econometrics and life wisdom, both of which I find very challenging disciplines. He said, ‘Your PhD should be a struggle. If you are having fun, something is wrong.’ I am not sure what exactly went wrong, but I had the time of my life.
Declaration

Chapter 3 is a joint work with Martin Kaae Jensen. Since first submission, a version of this chapter was published in *Journal of Economic Behavior and Organization* (volume 125 (2016), pp. 148–161). Versions of this chapter were also presented at the following conferences:

- 20th Spring Meeting of Young Economists (Ghent, Belgium, 21-23 May 2015).
- Royal Economic Society Inaugural Symposium of Junior Researchers (Manchester, 2 April 2015).
- Leicester International PhD Conference in Economics (4-5 December 2014).
- UECE Lisbon Meetings in Game Theory and Applications 2014 (6-8 November 2014).

A version of Chapter 4 was presented at the following conferences:

- Oligo Workshop: Optimal Firm Behaviour and Game-theoretic Modelling of Competition (Carlos III University of Madrid, Spain, 1-3 June 2015).
- Royal Economic Society Annual Conference (Brighton, 21-23 March 2016).
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<td>Consumer Surplus</td>
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<td>D</td>
<td>Dictator</td>
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<td>EEP</td>
<td>Equilibrium Expected Profit</td>
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<td>EPR</td>
<td>Endgame Preference Reversal</td>
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<td>PsyNE</td>
<td>Psychological Nash Equilibrium</td>
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<td>R</td>
<td>Recipient</td>
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<td>SW</td>
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Chapter 1

Introduction

The ideas investigated in this thesis contribute to two separate research programmes. One of them is concerned with developing rigorous, testable models of economic behaviour. Such models should rely on realistic assumptions and generate falsifiable predictions. Chapters 2 and 3 use the inductive and deductive methods respectively to contribute to this research goal. In particular, Chapter 3 provides axiomatic foundations for models of guilt aversion. Chapter 2 analyses existing experimental evidence on repeated social dilemmas and derives utility models which can rationalise it. These models are then put to the test in new empirical conditions.

The other research programme seeks to estimate the social value of information. The private benefits of information are indisputable, as it helps individuals and firms make better decisions. However, its public effects are not unambiguous. In particular, Morris and Shin (2002) and Angeletos and Pavan (2007) demonstrated that information can be welfare-enhancing or reducing, depending on the way it was acquired. Chapter 4 contributes to the study of information acquisition by considering its unorthodox form – industrial espionage.

Models of Other-Regarding Preferences

A recent trend in the theory of rational choice was towards incorporating psychological insights into economic analysis. The main lesson which other social sciences teach economics is that decision-makers care about other people's payoffs. At first glance, this motivation is easily modelled by suitably modified utility functions including measures of the agent’s altruism, fairness sensitivity, reciprocity inclination, etc. (Bolton and Ockenfels, 2000; Falk and Fischbacher, 2006; Fehr and Schmidt, 1999; Rabin, 1993). However, the explanatory success of the models of “social preferences” comes at the price of low predictive power. These models’ many degrees of freedom imply that they are compatible with almost any set of observations.

This unfalsifiability problem is addressed in Chapters 2 and 3, both of which provide rigorous justification for utility models in two specific examples. In chapter 2,
a technique is developed for deriving utility function forms from experimental evidence on finitely repeated Prisoner’s Dilemma. Chapter 3, in contrast, derives utility functions over money and guilt from realistic and easily interpretable assumptions about an agent’s moral compass.

Experimental Play in Repeated Prisoner’s Dilemma It is well known that cooperation in the experimental trials of repeated Prisoner’s Dilemma is neither fully unravelling nor persisting until the end of the game, which is a remaining puzzle in Game Theory (Andreoni and Miller, 1993; Bereby-Meyer and Roth, 2006; Selten and Stoecker, 1986). Chapter 2 uses both theory and experiments to shed light on this famous problem. Firstly, the formal properties of the agents’ preferences are deduced from their experimental play. Secondly, two models of motivation are found which fit these preferences – relative inequality aversion and delayed guilt. Finally, the identified theories are subjected to a novel experimental test, which provides support to the inequality aversion model. To the best of our knowledge, this is the first experiment on asymmetric repeated Prisoner’s Dilemma. Our finding highlights the role of fairness in repeated social interaction, which was not previously investigated as a possible reason behind cooperation in this game.

Axiomatic Models of Guilt Guilt aversion has been shown to be an important factor in economic decision-making, and a mediator of pro-social behaviour (Battigalli, Charness, and Dufwenberg, 2013; Charness and Dufwenberg, 2006; Geng, Weiss, and Wolff, 2011; Hopfensitz and Reuben, 2009; Ketelaar and Au, 2003). Experimentalists use utility functions over money and guilt to model tradeoff between increasing one’s income and conforming to society’s expectations (Battigalli, Charness, and Dufwenberg, 2013; Chang et al., 2011; Khalmetski, Ockenfels, and Werner, 2014). However, the choice of the functional form in these papers was dictated by technical convenience rather than any underlying behavioural theory. By taking an axiomatic approach to moral choice, Chapter 3 discovers the underlying assumptions of well known utility functions over money and guilt. Evaluating these assumptions and updating them in the direction of greater realism yields a novel representation which is logarithmic in money and linear in guilt. When applied to several laboratory games, this novel function accounts well for the existing body of experimental evidence.
Social Value of Industrial espionage   Industrial espionage is an increasingly important competition tool in today's knowledge-based economy. Its net effect on the marketplace is the subject of much debate. On the one hand, individual businesses suffer significant losses due to the illegal acquisition of their confidential information, which in the UK add up to £7.6bn annually (Cabinet Office, 2011). On the other hand, consumers benefit from the intense competition fuelled by economic spying. Moreover, industrial espionage makes information more widely available, which helps market participants better coordinate their actions and avoid inefficiencies like overproduction. Chapter 4 explores this information-transmission role of industrial espionage by focusing on its corresponding type: stealing a competitor's private information about the unknown state of the world. In the duopoly context, the state-of-the-world parameter of chief importance is demand shock. Chapter 4 looks at two competing firms who can spy on their rival's demand information. The welfare consequences of this type of spying depend on the nature of the competition and the precision of the alternative information sources. It is shown that aggregate profit and social welfare increase with the precision of the espionage technology when either of these two conditions holds: (i) the firms are competing in price; (ii) the espionage technology is more precise than the market research technology. In short, our results show that some types of espionage are beneficial for industry, and for the economy as a whole.
Chapter 2

End-Effect in Finitely Repeated
Prisoner’s Dilemma: an Experiment

Chapter Abstract

Using the combination of a revealed preference approach and laboratory tests, this paper sheds light on experimental subjects’ motivation in finitely Repeated Prisoner’s Dilemma (hereafter RPD). A consistently observed pattern of play in RPD, known as cooperation with defective end-effect, can be formulated as a stylised fact. This stylised fact generates a restriction on admissible utility functions, which helps us identify two utility models rationalising observed behaviour: inequality aversion and guilt aversion. We run an experiment to test the models and find some support for inequality aversion: players cooperate less when their opponent’s payoff is scaled up. This means that the “other-regarding preferences” affecting the subjects’ play in RPD are not of the “altruistic” form, as previously believed.
2.1 Introduction

Will you be nice to someone, at the expense of yourself, if you have to do business with them tomorrow? Will you be nice to them again tomorrow, if it is the last time you will see them in your life? This situation is famously modelled in game theory as a finitely repeated Prisoner's Dilemma, in which being “nasty” (a.k.a. defecting on your partner) pays off more than being “nice” (a.k.a. cooperating) regardless of the partner's action. As is well known, game theoretic tools predict that backward induction will wipe out all cooperation, with both players defecting throughout the game. A contrasting prediction comes from the literature on “other-regarding preferences”. Owing to their many degrees of freedom, these models admit multiple equilibria involving some level of cooperation, and the likeliest one is an equilibrium where both players cooperate throughout the game.\(^1\) The empirical truth lies in the middle: when matched with the same partner for a known number of Prisoner's Dilemma rounds, experimental subjects cooperate throughout most of the game and defect in the final rounds, as indicated by the laboratory data reviewed and analysed in Section 2.2.

A strikingly elegant solution to this puzzle was proposed by Kreps et al. (1982), who use a reputation idea: if players believe, even incorrectly, that they might be matched with an opponent committed to conditional cooperation, they will cooperate in the beginning of the game and then defect.\(^2\) Although qualitatively sound (Andreoni and Miller, 1993), the reputation hypothesis can hardly be used for quantitative predictions, as is demonstrated in Section 2.2. Indeed, for empirical payoffs, the pure equilibrium of Kreps et al. (1982) model implies defection throughout the game, whereas its mixed equilibrium does not match the observed data patterns either, as shown in Cooper et al. (1996). In fact, in Section 2.2 we argue that existing explanations for cooperation with end-effect (including Kreps et al. (1982) reputation hypothesis) do not fully embrace the available body of experimental evidence.

This motivates our search for an alternative, empirically corroborated model. In order to make a contribution to the most intensely studied problem in Game Theory, this paper utilises a combination of theoretical and experimental methods. First, we

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\(^1\)The likeliest here means supported by the biggest range of parameter values.

\(^2\)Conditional cooperation is one of the following strategies: Tit-for-tat (cooperate in the first round; play your opponent's previous choice in all subsequent rounds) and Grim Trigger (cooperate until your opponent defects; defect in all subsequent rounds).
use a revealed preference approach to identify utility models which can rationalise observed data, as described in Section 2.3. In particular, we formulate the observed experimental pattern as a stylised fact, which, informally, states that cooperation is neither unravelling nor persisting until the end of the game. This fact translates into a restriction on admissible utility function forms, which is a special case of non-linearity. We identify two utility models which fit the restriction: relative inequality aversion and guilt aversion. As a direct consequence of fitting the stylised fact, these models admit the observed behavioural pattern in a Pareto-dominant equilibrium. This feature sets them apart from most other utility models, which admit an equilibrium with cooperation in every round, and hence are incapable of explaining the prevalence of defective end-effect. Section 2.4 then presents a novel experimental design, which was used to test these models against each other. In order to test for inequality aversion, we observed how cooperation rates reacted to a change in the symmetry of the payoff matrix. The test of guilt aversion involved comparing players’ choices following unilateral and mutual defection. Section 2.5 reports the results of our experiment, which show that laboratory behaviour is affected by inequality aversion: the participants cooperate less when their opponents’ payoffs are scaled up in each cell of the payoff matrix. Note that the effect of the opponent’s payoff increase would have the opposite sign if players were motivated by altruism, which was considered a possible cause of cooperation in Prisoner’s Dilemma in the previous literature (Andreoni and Miller, 1993). Another interesting finding is that “strategically sophisticated” players (those who were able to find the best response to the tit-for-tat algorithm) cooperate more than “naïfs”. Section 2.6 discusses these results and proposes directions for future research. The Appendix contains the instructions which were given to experiment participants.

2.2 Literature Review

2.2.1 Experimental Studies of Finitely Repeated Prisoner's Dilemma

Laboratory investigation of Prisoner's Dilemma has been going on for at least half a century, with thirty thousand papers written on the topic, according to Google Scholar. This section will specifically review experimental literature on finitely repeated Prisoner's Dilemma, which is the focus of this paper.
Before proceeding with the review, it will be useful to introduce some unifying notation. A game under consideration is a $T$-round PD supergame, by which we mean a basic PD played by the same players over $T$ consecutive rounds. After each round, the players’ choices become public information.

Classical game theory predicts the unique subgame perfect equilibrium in this game, which is defection throughout all $T$ rounds. Multiple experimental studies have established a notably different outcome, characterised by a high proportion of cooperative choices (43% in Andreoni and Miller (1993), 55% in Cooper et al. (1996) and 62% in Bereby-Meyer and Roth (2006)). The subjects played the game many times against different opponents, which provided them with a learning opportunity. Towards the end of most existing experiments, the stable pattern of supergame play emerged, in which the players cooperated in the beginning of the supergame and defected in the later rounds. We call this behavioral pattern “cooperation with end-effect”.

Table 2.1 reports existing experimental results on finitely repeated Prisoner’s Dilemma. In particular, we present the difference in cooperation rates between the first half of the supergame and its last round (averaged over all supergames), which clearly indicates the presence of cooperation with end-effect. Observe that all papers, following the seminal work of Selten and Stoecker (1986), tested a 10-round supergame, which is a one-shot PD played ten times by the same two players. After playing 10 rounds of PD against the same opponent, players were rematched, with the number of supergames (i.e. rematchings) varying from 2 to 25 in different studies. Table 2.1 suggests that the dynamics of laboratory play in existing experiments depended primarily on the number of supergames, as well as on information available to the players. In the longest experiment to date (Selten and Stoecker, 1986), the number of cooperative rounds was growing in the first 10-15 supergames and then falling in the last 10 supergames. Hauk and Nagel (2001) performed only 10 supergames, and cooperation was steadily increasing from one supergame to the other. In Bereby-Meyer and Roth (2006), the subjects did not know how many rematchings they would face, which might explain why cooperation levels were not declining towards the end of the experiment. Whenever the rematching of subjects was not across all population but within groups, significant group heterogeneity in cooperation rates was reported. This suggests the diversity of agents’ preferences and/or the presence of learning.
Table 2.1: Experimental Research in Repeated Prisoner’s Dilemma

<table>
<thead>
<tr>
<th>Paper</th>
<th>No of rounds</th>
<th>No of supergames</th>
<th>Cooperation rate rounds 1-6</th>
<th>Cooperation rate round 10</th>
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<tr>
<td>Selten and Stoecker (1986)</td>
<td>10</td>
<td>25</td>
<td>90%</td>
<td>10%</td>
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<tr>
<td>Bereby-Meyer and Roth (2006)</td>
<td>10</td>
<td>20</td>
<td>78%</td>
<td>11%</td>
</tr>
<tr>
<td>Andreoni and Miller (1993)</td>
<td>10</td>
<td>20</td>
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<tr>
<td>Hauk and Nagel (2001)</td>
<td>10</td>
<td>10</td>
<td>55%</td>
<td>5%</td>
</tr>
<tr>
<td>Cooper et al. (1996)</td>
<td>10</td>
<td>2</td>
<td>60%</td>
<td>20%</td>
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Table 2.2 illustrates cooperation with end-effect by presenting the typical choices of an experienced player, who is using a “cutoff” strategy. The player cooperates either until round $n$, or until his opponent defects (whichever happens earlier), and then defects throughout the rest of the supergame. In the example in Table 2.2, Player 2 is using a 7-cutoff strategy, and Player 1 is supposedly using a $n$-cutoff strategy, where $n > 7$.

Table 2.2: Typical Individual Play Pattern in RPD

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<th>7</th>
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<tr>
<td>Player 1</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>D</td>
<td>D</td>
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<tr>
<td>Player 2</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>C</td>
<td>D</td>
<td>D</td>
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</table>

Additional insights can be drawn from data pooled across all players; most importantly, the fraction of cooperative choices (cooperation rate) can be analysed. Typically, the average frequency of cooperation reaches 0.6-0.8 in the first half of the supergame, falling to 0.1-0.2 in the later rounds. Figure 2.1 shows aggregated cooperation level data for two well known experimental papers: Andreoni and Miller (1993) and Bereby-Meyer and Roth (2006).8

In addition to a standard repeated PD, several papers report the results of modified treatments, which can shed light on the reasons behind cooperation in the “standard” game. Hauk and Nagel (2001) ran an experiment with unilateral or mutual partner selection and found that cooperation-to-defection rates were much higher in these treatments than in the standard treatment with no choice of part-

---

3 Data from the last supergame.
4 Players were unaware of the number of supergames.
5 This data is for the “Partners” treatment, which is analogous to a standard RPD.
6 In each supergame subjects played with 6 opponents simultaneously.
7 Data from the last supergame.
8 As stated before, experimental subjects in Bereby-Meyer and Roth (2006) were unaware of the number of supergames (i.e. 10-round rematchings) they were going to play.
2.2: Literature Review

Figure 2.1: Cooperation with End-effect in Well-known Experiments

Another notable laboratory study of finitely repeated PD is Andreoni and Miller (1993). Their research objective was to test Kreps et al. (1982) theory of reputation building, as well as to establish if some agents are in fact altruistic (rather than “pretending” to be so, by building an altruistic reputation). In order to test these hypotheses, the authors manipulated experimental subjects’ beliefs about the probability of being paired with a conditional cooperator. In a “Strangers” treatment, the subjects played a series of one-shot PDs and were randomly rematched after every game. The non-zero cooperation level in this treatment was attributed to altruism by the authors, since reputation building is impossible in a one-shot game. In another treatment (“Partners”) the subjects played 20 supergames consisting of 10 rounds each. Andreoni and Miller (1993) argue that if agents were building reputation in this simple treatment, their strategies would be based on “homemade” prior $\delta_b$ (i.e. an extra-laboratory belief about meeting a conditional cooperator).

\[9\] Importantly, a defect-defect payoff in their experiment was negative, so remaining unmatched was better than being paired with a defector.

\[10\] Andreoni and Miller (1993) note that the participants in the “Strangers” treatment could have built reputation as a group, since the treatment comprised only 14 subjects, who were rematched 200 times.
Cooperation levels in this treatment were significantly higher than in “Strangers”, which suggests reputation building by the subjects. In the third treatment (“Computer50”), the subjects were told that they would have a 50% chance of being matched with a computer programmed to play tit-for-tat. This design feature induced a belief about meeting a conditional cooperator ($\delta$). As expected, cooperation level in “Computer50” was significantly higher than in “Partners”. In the fourth treatment (“Computer0”), the subjects were told they had a 0.1% chance of meeting a tit-for-tat playing computer. This treatment was designed to check whether the higher cooperation rates in “Computer50” are simply due to the players’ awareness of the tit-for-tat strategy (which proved wrong, since the cooperation rate in “Computer0” was the same as in “Partners” and lower than in “Computer50”). Andreoni and Miller (1993) interpret these results as providing support for the reputation hypothesis. This is true when the qualitative prediction of Kreps et al. (1982) is considered: players who play a series of one-shot games cooperate significantly less than players in the repeated game, since the former can not form reputations.

Note that, although all papers in question were investigating the reasons behind cooperation in RPD, only Selten and Stoecker (1986) sought to explain the defective end-effect. That paper used a learning model to explain the dynamics of the observed play. The fitted parameters of the model predicted no end-effect in the long run; all cooperation will be eliminated.

### 2.2.2 Theoretical Studies of Finitely Repeated Prisoner's Dilemma

Theoretical research on finitely repeated Prisoner’s Dilemma was kindled by Robert Axelrod’s famous strategy tournament (Axelrod and Hamilton, 1981). In this round-robin contest of PD strategies submitted by game theorists, the equilibrium strategy (always defect) scored very low. This result showed that the subgame perfect equilibrium of the game perhaps should not be an endpoint of theoretical enquiry into the topic. Among the multiple models proposed to include cooperation in the equilibrium of the game, those which have received most attention in the literature are level-k thinking, limited rationality, incomplete information, ambiguity about duration, and reaction lag. The overview of theoretical research in RPD is presented in Table 2.3. The considered papers aim to explain how cooperation in the game is possible in principle; only a few of them predict defective end-effect in equilib-
rium (marked by a plus sign in the “EE” row, which stands for end-effect). None of the considered models admits cooperation with end-effect as the only equilibrium; additional equilibria are presented in the last row of Table 2.3.

Table 2.3: Theoretical literature in Discrete-Time Repeated Prisoner's Dilemma

<table>
<thead>
<tr>
<th>Paper</th>
<th>Main Idea</th>
<th>Plausibility</th>
<th>EE</th>
<th>Other equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kreps et al.</td>
<td>Rational players pretend to be conditional cooperators to exploit their opponent's belief.</td>
<td>The prediction fits the data qualitatively but not quantitatively.</td>
<td>+</td>
<td>Equilibria with longer defection</td>
</tr>
<tr>
<td>(1982)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geanakoplos et al. (1989)</td>
<td>A rational player evokes sympathy in his opponent by unexpectedly cooperating.</td>
<td>The prediction (defection followed by cooperation) does not fit the data.</td>
<td>-</td>
<td>Universal defection</td>
</tr>
<tr>
<td>Radner (1986)</td>
<td>Cooperation in RPD is an $\varepsilon$-equilibrium, since it is very close to a mutual best response.</td>
<td>The argument does not withstand backward induction logic.</td>
<td>+</td>
<td>Any strategy profile for a suitable $\varepsilon$</td>
</tr>
<tr>
<td></td>
<td>Computational complexity means that only simple strategies are played.</td>
<td></td>
<td>-</td>
<td>Universal cooperation and defection</td>
</tr>
<tr>
<td>Neyman (1999)</td>
<td>Probabilistic duration: players are unsure about the number of stages.</td>
<td>In reported experiments subjects were aware of the number of supergames.</td>
<td>+</td>
<td>Any length of cooperation for a suitable belief</td>
</tr>
</tbody>
</table>

We will now briefly describe the most famous theoretical ideas behind cooperation in RPD and discuss their predictive and explanatory power.

**Epsilon-equilibrium (Radner, 1986)** is a famous model of bounded rationality. Less famous is the fact that it was proposed specifically to explain observed cooperation in finitely repeated PD. The paper introduced the notion of *epsilon-equilibrium*, which is a strategy profile such that the corresponding payoff for each player lies within $\varepsilon$ of his best-response payoff. As with other bounded rationality models, $\varepsilon$-equilibrium theory has too many degrees of freedom: for any length of the end-effect, there exists $\varepsilon > 0$ which ensures that cooperation with such end-effect is an $\varepsilon$-equilibrium of the game (in fact, all shorter end-effects also are supported in equilibrium for such $\varepsilon$).

Apart from $\varepsilon$-equilibrium, Radner (1986) proposed two more explanations of cooperation with end-effect, which also rely on the bounded rationality argument. According to the second explanation, players believe that their opponents are using a cutoff strategy but are not sure about the timing of the first defection. The best response to a probabilistic belief about the opponent's first defection round is
also a cutoff strategy, which can be mixed for specific beliefs. However, the best response will be always shifted towards earlier rounds, compared to the opponent’s strategy, because of the “pre-emption” motive (similarly to backward induction on pure strategies). Thus, a mixed strategy suggested by Radner (1986) cannot be part of an equilibrium in RPD.

The third explanation attributes cooperation with end-effect to the computational complexity of the game. Suppose players are restricted to strategies which can be implemented by an automata of a bounded size (i.e. with a finite number of states of automation). Then, even though a theoretical strategy set for a 10-round repeated Prisoner’s Dilemma consists of $2^{349525}$ strategies, only the simplest of them are implementable.\textsuperscript{11} Since the conditional cooperation strategy requires two states of automation only, it is the easiest to implement, apart from universal cooperation and universal defection. This model is subject to the same critique as other bounded rationality theories: it is difficult to falsify, because any observed cutoff can be rationalised by a suitable size of the automata.

**Reputation model of Kreps et al. (1982)** shows that players will be willing to cooperate in the early rounds of the game in order to build a reputation of an “irrational” type. Irrational players are defined as committed to conditional cooperation. Crucially, they do not need to be present in the player pool; it is the belief that they exist which supports some degree of cooperation. The quantitative result of the paper is the lower bound on the number of defection rounds. The paper predicts multiple sequential equilibria, and the Pareto-efficient one is characterized by no defection by either player until $1 + (2a - 4b + 2\delta)/\delta$ rounds until the end, where $\delta$ is a [perceived] proportion of conditional cooperators and $a, b$ are the normalized payoffs in Prisoner’s Dilemma (Table 2.4)

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>1,1</td>
<td>b,a</td>
</tr>
<tr>
<td>D</td>
<td>a,b</td>
<td>0,0</td>
</tr>
</tbody>
</table>

However, if the formula provided in Kreps et al. (1982) is applied to a normalisation of payoffs used in existing laboratory studies, the predicted length of the

\textsuperscript{11}Generally, a player’s strategy set in a $T$-round repeated $2 \times 2$ game will contain $2^{(4^T-1)/3}$ strategies.
end-effect turns out to be greater than the number of rounds in the game (Table 2.5).

Table 2.5: Predicted Length of End-Effect in Existing Laboratory Studies

<table>
<thead>
<tr>
<th>Paper</th>
<th>Rounds of defection</th>
<th>Total rounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selten, Stoecker (1986)</td>
<td>24</td>
<td>10</td>
</tr>
<tr>
<td>Andreoni, Miller (1993)</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>Bereby-Meyer, Roth (2006)</td>
<td>35</td>
<td>10</td>
</tr>
</tbody>
</table>

The calculation in Table 2.5 was performed for a very optimistic belief that half of the population are conditional cooperators ($\delta = 0.5$); for smaller $\delta$ the defective endplay is even longer.

One might wonder if there is any belief about the proportion of conditional cooperators $\delta$ that will induce a rational player to play a pure “cutoff” strategy observed in the lab. Suppose such equilibrium exists, where rational players are cooperating for $n$ rounds out of $T$.\(^\text{12}\) If a player believes he is paired with a conditional cooperator with probability $\delta$, then his expected future payoff from cooperating in round $n+1$ equals

$$\delta(T - n - 1 + a) + (1 - \delta)b.$$  \hspace{1cm} (2.1)

His expected future utility from defecting equals $\delta a$.

A rational player prefers to defect in round $n+1$ if it brings him higher expected payoff than defecting:

$$\delta(T - n - 1 + a) + (1 - \delta)b > \delta a,$$  \hspace{1cm} (2.2)

which yields

$$\delta < \frac{-b}{T - n - 1 - b}.$$  \hspace{1cm} (2.3)

\(^{12}\)In such sequential equilibrium, an opponent’s action in round $n+1$ reveals his type: a conditionally cooperating opponent would cooperate, whereas a rational opponent would defect. Hence, if a rational player cooperated in round $n+1$ and learned he was paired with a conditional cooperator, his best response is to cooperate until the second-to-last round and then defect, thus obtaining a payoff of $T - n - 1 + a$ in rounds $n+1$ to $T$. If a player defected on a conditional cooperator in round $n+1$, or if he learned he was paired with another rational player (regardless of his own action), his best response in the rest of the game is to defect, because his opponent will defect. His payoff in rounds $n+2$ to $T$ will then be 0, and his payoff in round $n+1$ is either 0 or $a$, depending on whether he was paired with a rational or conditionally cooperating player.
In words, the player defects in round $n+1$ if the probability of being paired with a conditional cooperator (and hence getting an extra payoff $T - n - 1$ from mutual cooperation in rounds $n+1$ to $T-1$) is not high enough to compensate him for the possible loss of $b$ which he incurs if he is paired with a rational player.

Similarly, given that a rational player and all his rational opponents defect from round $n+1$ onward, he prefers to cooperate in round $n$ if

\[ \delta(1 + a) + 1 - \delta > a, \]

which yields

\[ \delta \geq \frac{a - 1}{a}. \]

Equations (2.5) and (2.3) establish an upper and a lower limit on the belief about the percentage of conditional cooperators in the population. Table 2.6 reports the values of the limits for the existing experiments and different lengths of end-effect (note that the lower limit is length-independent). Strikingly, in all considered experiments the subjects would have had to believe that about two thirds of other participants are “irrational”. Moreover, the only admitted equilibrium (where the upper limit is above the lower limit) has a 1-round-long end-effect, as compared to the experimental average of 4-round-long end-effect.$^{13}$

Table 2.6: Bounds on Irrationality Belief in Existing Experiments

<table>
<thead>
<tr>
<th>Paper</th>
<th>Lower bound on $\delta$</th>
<th>Upper bound on $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>length of end-effect, rounds</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Selten, Stoecker (1986)</td>
<td>0.63</td>
<td>1</td>
</tr>
<tr>
<td>Andreoni, Miller (1993)</td>
<td>0.63</td>
<td>1</td>
</tr>
<tr>
<td>Bereby-Meyer, Roth (2006)</td>
<td>0.7</td>
<td>1</td>
</tr>
</tbody>
</table>

Evidently, if it is reputation concerns that motivate experimental subjects to cooperate, they are not of the type suggested by Kreps et al. (1982). This has been first noted by Cooper et al. (1996), who showed that the pattern of play in the lab is different from the famous paper’s prediction. However, the qualitative hypothesis of

$^{13}$As Table 2.6 suggests, the model also admits a 2-round end-effect for the payoff table used in Bereby-Meyer and Roth (2006).
reputation building in RPD received strong empirical support (Andreoni and Miller, 1993).

In addition to the theories analysed in Table 2.3, cooperation in repeated PD was also attempted to be explained within level-k models.

**Level-k models.** Level-k theory argues that economic agents vary in their level of strategic sophistication, indexed by $k$. Level-0 agents are the most naive and do not behave strategically, level-1 are slightly more sophisticated and best-respond to the play of level-0 agents, etc. There is no universally accepted definition of level-0, hence, in applications it is usually defined arbitrarily. As a consequence, the model is compatible with a very wide range of phenomena. In repeated Prisoner’s Dilemma game, level-0 can also be assigned in a variety of ways. Heller (2015) defines $k$ as the level of foresight ability, i.e. a stage of repeated interaction at which a player is informed of its realised length. In that model, level-0 agents conditionally cooperate until the end of the game, level 1 agents defect at the very last stage and so on. Heller (2015) provides an evolutionary explanation for a specific stable distribution of types in a RPD game. In particular, he proves that a population consisting of level 1 (naive) and level 3 (moderately sophisticated) agents is the unique evolutionary stable state in which players cooperate in the early stages of the interaction. However, the empirical distribution of strategic sophistication levels, which can be inferred from RPD experimental results, in general is different from the one predicted in Heller (2015). For example, Figure 2.2 illustrates the distribution of types in Selten and Stoecker (1986), which does not include any level-1 players.

Figure 2.2: Empirical Distribution of Foresight Levels in Selten and Stoecker (1986)

[Graph showing empirical distribution of foresight levels]

It can be concluded that the considered models of experimental play in RPD have either been falsified or are unfalsifiable. In what follows, we search for a model which will not be subject to either problem. In Section 2.3, existing experimental
2.3: The Model: Endgame Preference Reversal

2.3.1 Setting

Consider a game of finitely repeated Prisoner’s Dilemma, which comprises $T$ repetitions of the underlying one-shot (stage) game $G = \langle \{1, 2\}, (A_i), (\succeq_i) \rangle$. Cooperation and defection are the two strategies in the one-shot game: $A_1 = A_2 = \{C, D\}$. Let $A = A_1 \times A_2$ denote the strategy space. Stage payoff $\pi_i : A \to R$ is defined by a payoff matrix in Table 2.7, where $t > c > d > s$\textsuperscript{14}. Note that the ordering of the outcomes according to $\succeq_i$ is not necessarily the same.

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>c, c</td>
<td>s, t</td>
</tr>
<tr>
<td>Defect</td>
<td>t, s</td>
<td>d, d</td>
</tr>
</tbody>
</table>

Table 2.7: Prisoner’s Dilemma Payoff Table

Let $\pi'_i$ denote player $i$'s payoff in round $t$. Also, let $\pi_i$ denote his overall (supergame) payoff, where $\pi_i = \sum_{t=1}^{T} \pi'_i$. As before, $\pi_i$ is a “material” payoff and does not necessarily define the player’s preferences over the supergame outcomes. Let $H = \bigcup_{t=0}^{T} A'$ be a set of histories, where $A^0 = \emptyset$ is the initial history. After each non-terminal history, information about the previous round’s choices is revealed and both players simultaneously pick an action. Thus, a strategy in a repeated game is a mapping from histories to actions: $S_i : H \to \{C, D\}$. A supergame $SG$ (a finitely repeated game of Prisoner’s Dilemma) induced by $G$ is defined as $SG = \langle \{1, 2\}, H, (S_i), (U_i) \rangle$.

This section searches for a model of motivation, i.e. a preference relation over outcomes (or, more universally, over strategy profiles) which will explain the prevalence of cooperation with end-effect in experimental RPD. If a player’s preference relation is monotone and defined over his own monetary payoffs $\pi_i$, the only equilibrium of the game is defection in all rounds. Hence we consider an extension of

\textsuperscript{14}The letters stand for temptation, cooperation, defection, sucker.
2.3: The Model: Endgame Preference Reversal

the choice set to two-dimensional payoff vectors, which represent the player’s own and his opponent’s payoffs. Standard assumption of preference rationality applies everywhere in the section.

We proceed by formulating a stylised fact which summarises agents’ choices in existing RPD experiments, holding their opponent’s strategy fixed. After that, we deduce utility function forms which admit such choices as utility-maximising.

• **Fact 1:** cooperation in all rounds of the supergame is almost never observed.
  
  – **Inference 1:** agents defect in the very last round if they believe their opponent is conditionally cooperating throughout the game.

• **Fact 2:** cooperation is not fully unravelling.
  
  – **Inference 2:** an agent does not want to defect one round before an opponent who conditionally cooperates for \( n \) out of \( T \) rounds in the game.

Inferences 1 and 2 combined imply that the preference relation over strategy profiles has to satisfy the property which we call *endgame preference reversal* (hereafter EPR) in order to be compatible with observed behaviour.

\[
\begin{align*}
(S_1 = n\text{-coop}, S_2 = n\text{-coop}) & \succeq_1 (S_1 = (n-1)\text{-coop}, S_2 = n\text{-coop}) \\
(S_1 = T\text{-coop}, S_2 = T\text{-coop}) & \preceq_1 (S_1 = (T-1)\text{-coop}, S_2 = T\text{-coop})
\end{align*}
\] (2.6)

for some \( n < T \).

“\( t\text{-coop} \)” stands for conditional cooperation until round \( t \) followed by defection until the end of the supergame (also called cutoff strategy in the literature).\(^{15}\)

A natural preference domain for experimental games is the set of payoff vectors \((\pi_i, \pi_j)\) representing player’s own and his opponent’s payoffs (then \( \succeq \in \mathbb{R}^2 \times \mathbb{R}^2 \)). Under such preference formulation, (2.6) becomes

\[
\begin{align*}
\begin{array}{ll}
(n c + (T-n)d, n c + (T-n)d) & \succeq_1 ((n-1)c + t + (T-n)d, (n-1)c + s + (T-n)d) \\
(T c, T c) & \preceq_1 ((T-1)c + t, (T-1)c + s)
\end{array}
\end{align*}
\] (2.7)

\(^{15}\)Obviously, the outcome path observed in experimental RPD (cooperation for \( n \) out of \( T \) rounds and defection afterwards) can be generated by several different strategy profiles, which can be divided into two broad categories: conditional and unconditional cooperation until round \( n \). The former, however, seems more reasonable than the latter. Indeed, in all reported experiments, when players are experienced, they both defect in a round following unilateral defection.
Thus, our research goal can be recast as describing \( U_i : R^2 \rightarrow R \) which represents a preference relation satisfying (2.7).

An important class of utility functions in repeated games are those for which the utility of final allocations equals the sum of utilities of stage payoffs:

\[
U_i \left( \sum_{i=1}^{T} \pi_i^t, \sum_{i=1}^{T} \pi_j^t \right) = \sum_{i=1}^{T} U_i(\pi_i^t, \pi_j^t)
\]  

(2.8)

Functional form (2.8) owes its wide use to technical convenience; indeed, it allows the deduction of supergame preferences from one-shot game preferences. However, it can be shown that utility functions which satisfy (2.8) do not support endgame preference reversal. Indeed, under the functional form (2.8), system (2.7) becomes

\[
\begin{align*}
(n-1)U_i(c, c) + U_i(t, s) + (T-n)U_i(d, d) &< nU_i(c, c) + (T-n)U_i(d, d); \\
(T-1)U_i(c, c) + U_i(t, s) &> TU_i(c, c).
\end{align*}
\]  

(2.9)

By cancelling out common terms we obtain

\[
\begin{align*}
U_i(t, s) &< U_i(c, c); \\
U_i(t, s) &> U_i(c, c).
\end{align*}
\]  

(2.10)

a contradiction.

A function \( u : R^2 \rightarrow R \) satisfies (2.8) if and only if it is linear (by generalization of Cauchy equation to functions of multiple variables, see Aczél (2006)):

\[
U_i = \sum_{j=1}^{N} w_j \pi_j
\]  

(2.11)

Note that most models of social preferences utilize linear utility functions and are thus incapable of explaining observed behaviour in repeated Prisoner’s Dilemma. Examples include altruism and social welfare, where utility is a weighted average of a player’s own and others players’ payoffs, as well as some models of inequality aversion. Notably, Fehr and Schmidt (1999) theory of fairness belongs to this category, because it measures inequality as an absolute difference between players’ payoffs. In that model, the allocations (1,3) and (1000,1002) have the same inequality measure.

It follows that, in order to explain observed behavior in RPD, we need to look
beyond the class of functions satisfying (2.11). Functions for which property (2.11) does not hold are either non-linear or non-static over rounds. In the next subsections, we will consider each of these divergences from (2.11).

2.3.2 Non-linear Utility

Recall that we are looking for non-linear preferences which would satisfy endgame preference reversal. Let us abstract out of the Prisoner’s Dilemma context for a moment and just examine EPR as a pair of inequalities comparing two-dimensional vectors:

\[
\begin{align*}
C, C & \succeq 1 C + a, C - b \\
U, U & \preceq 1 U + a, U - b
\end{align*}
\]

(2.12)

where \( C = n c + (T - n)d \), \( U = T c \), \( a = t - c \) and \( b = c - s \).

Both inequalities describe an agent’s choice between an equal division of total payoff and an allocation where his own payoff is increased by \( a \) while his opponent’s payoff is decreased by \( b \). EPR tells us that equal division is preferred when both payoffs are low, but not when it is high (since \( U > C \)).

In natural language, the system (2.12) describes how an agent resolves a tradeoff between equality and personal income, at different levels of total payoff.

The tradeoff between equality and income is the realm of inequality aversion theories, and we will now try and interpret EPR in their parlance. EPR tells us that inequality (the same difference between the payoffs: \( t - s \)) hurts less when the total payoff is bigger. Thus it is the relative inequality which motivates an agent (\( i.e. \) the difference in shares of total income, or the divergence of one’s share from 50%). Such motivation has already been formally described in the literature. A model of relative inequity aversion was proposed by Bolton and Ockenfels (2000), who argue that an agent is motivated by his absolute payoff and his “relative standing”. The latter is measured as the ratio of the agent’s payoff to the total payoff. Keeping relative payoff fixed, an agent’s utility is increasing in his own payoff. Keeping his own payoff fixed, the utility function reaches its maximum at equal division of the total payoff between the players. It is easy to see that this general description fits the EPR. We will now confirm it using a specific utility function form proposed in Bolton and Ockenfels (2000). Utility function in that model (which they call motivation func-
tion) in a two-player case reduces to (2.13).

\[ U_i = a_i \pi_i - b_i \left( \frac{\pi_i}{\pi_i + \pi_j} - \frac{1}{2} \right)^2 / 2 \]  

(2.13)

where \( a_i \) and \( b_i \) are weights attributed to “pecuniary” and “relative” payoffs.

Note that the function is non-linear in payoffs. Unlike Fehr and Schmidt (1999) fairness theory, Bolton and Ockenfels (2000) measure unfairness of the outcome as the divergence of one’s share of total payoff from 50%. Thus (1,3) and (1000,3000) allocations have the same inequality measure. This feature of the model is key to its success in explaining endgame preference reversal, since longer cooperation means bigger total payoff and thus less inequality is created by one defection (which always entails the same absolute difference between players’ payoffs - \( t - s \)).

Indeed, under the Bolton and Ockenfels (2000) utility function, (2.7) becomes

\[
\begin{align*}
\frac{a_i}{b_i} & \leq \frac{(t-s)^2}{8n(c-d-2c+2d+t+s)(t-c)} \\
\frac{a_i}{b_i} & \geq \frac{(t-s)^2}{8c(t-1+t+s)^2(t-c)}
\end{align*}
\]  

(2.14)

There exist values of \( \frac{a_i}{b_i} \) satisfying both inequalities. Specifically, for parameter values from Bereby-Meyer and Roth (2006) experiment, \( 0.012 < \frac{a_i}{b_i} < 0.0165 \) ensures that cooperation until rounds 5-9 of the 10-round supergame can be equilibrium, but not universal cooperation.

### 2.3.3 Non-stationary Utility

Non-stationary supergame utility implies that preferences over allocations depend on the round of the game:

\[ U_i = f(u_i^1, u_i^2, ..., u_i^T) \]  

(2.15)

In case of non-stationarity, (2.8) does not hold, which means it is possible to maintain a technically convenient additively separable supergame utility form in our search for the function satisfying EPR:

\[ U_i = \sum_{t=1}^{T} u_i(h_t), \]  

(2.16)

where \( h_t : \{1, ..., t\} \rightarrow \{C, D\} \times \{C, D\} \) is history (vector of chosen actions) up to
round $t$. We assume utility in stage $t$ is affected by present and past but not future events.

For simplicity, assume that stage utility depends on the current and the previous stage outcome (2-round memory):

$$u(A_1^t, A_2^t; A_1^{t-1}, A_2^{t-1}) : \{C, D\} \times \{C, D\} \rightarrow R$$

(2.17)

Endgame preference reversal under (2.17) becomes:

$$\left\{ \begin{array}{ll}
    u(C, C; \emptyset) + (T - 2)u(C, C; C, C) + u(D, C; C, C) > u(C, C; \emptyset) + (T - 1)u(C, C; C, C); \\
    u(C, C; \emptyset) + (n - 2)u(C, C; C, C) + u(D, C; C, C) + u(D, D; D, C) + (T - n - 1)u(D, D; D, D) < \\
    < u(C, C; \emptyset) + (n - 1)u(C, C; C, C) + u(D, D; D, C) + (T - n - 1)u(D, D; D, D).
\end{array} \right.$$ 

By rearranging we obtain

$$\left\{ \begin{array}{ll}
    u(D, C; C, C) > u(C, C; C, C); \\
    u(D, C; C, C) + u(D, D; D, C) < u(C, C; C, C) + u(D, D; C, C).
\end{array} \right.$$ 

Which implies that a player’s utility from mutual defection is smaller if it was preceded my unilateral defection on player’s part:

$$u(D, D; D, C) < u(D, D; C, C)$$

(2.18)

Let us now consider a perfect-memory case (all past events affect present utility). To save space, denote outcomes by player 1’s material payoffs: $c = (C, C)$, $t = (D, C)$, $s = (C, D)$, $d = (D, D)$. EPR becomes

$$\left\{ \begin{array}{ll}
    u(c; \emptyset) + u(c; c) + ... + u(t; c^{T-1}) > u(c; \emptyset) + u(c; c) + ... + u(c; c^{T-1}); \\
    u(c; \emptyset) + ... + u(t; c^{n-1}) + u(d; t; c^{n-1}) + ... + u(d; d^{T-n-1}; t; c^{n-1}) < \\
    < u(c; \emptyset) + ... + u(c; c^{n-1}) + u(d; c^n) + ... + u(d; d^{T-n-1}; c^n).
\end{array} \right.$$ 

By cancelling out common terms we obtain
2.3: The Model: Endgame Preference Reversal

\[
\begin{align*}
& \left\{ \begin{array}{l}
u(t; c^{T-1}) > u(c; c^{T-1}); \\
u(t; c^{n-1}) + u(d; t; c^{n-1}) + ... + u(d; d^{T-n-1}; t; c^{n-1}) < \\
< u(c; c^{n-1}) + u(d; c^{n}) + ... + u(d; d^{T-n-1}; c^{n}).
\end{array} \right.
\end{align*}
\] (2.19)

In words, \textbf{unilateral defection negatively affects the defector's future utility (of the same outcome stream)}\textsuperscript{16}. In natural language, such utility decrease is called guilt.

In Game Theory guilt has been modelled as a loss which a player inflicts on their opponent, \textit{i.e.} the difference between the opponent's actual and expected payoffs (if positive), which matches the definition of guilt from psychological literature. The first formal model of guilt was proposed by Battigalli and Dufwenberg (2009), who incorporated this emotion in the utility function:

\[
u = \pi - \theta \max\{0, E(\pi) - \pi\}. \] (2.20)

where the notation of this paper is used.\textsuperscript{17}

Note that Battigalli and Dufwenberg (2009) specification is stationary and linearly additive and thus incompatible with EPR. Indeed, (2.19) implies unilateral defection affects one's future utility, otherwise cooperating until the end of the supergame would have been an equilibrium too (and the most efficient one).

We propose the following specific (but the simplest) form of a utility function over the player's own payoff and the opponent's payoff loss (\textit{i.e.} “guilt”) with 2-round memory satisfying (2.17) and (2.19):

\[
u' = \pi - \max\{0, E(\pi) - \pi\} \] (2.21)

Observe that in (2.21) stage utility positively depends on a player's today's payoff and the opponent's payoff loss (\textit{i.e.} “guilt”) with 2-round memory satisfying (2.17) and (2.19):

Since (2.21) satisfies (2.19), it supports the endgame preference reversal and is

\textsuperscript{16}Note that it is a necessary and sufficient condition for EPR to hold in non-stationary, but additively separable supergame utility case.

\textsuperscript{17}Strictly speaking, expectation of the other player's payoff $E(\pi)$ is a function of the player's first and second-order beliefs (what player 1 thinks player 2 expects to receive, which depends on what player 1 thinks player 2 does, and what player 1 thinks player 2 believes player 1 does). Because players' payoffs depend not only on strategies but also on beliefs, the resulting game belongs to the field of psychological game theory (Geanakoplos, Pearce, and Stacchetti, 1989).
2.4: Experimental Design

thus capable of explaining observed behavior in RPD.

In what follows, we take the models (2.13) and (2.21) to laboratory and establish their predictive power in novel empirical conditions.

2.4 Experimental Design

2.4.1 Hypotheses

A revealed preference analysis performed in the previous section helped us identify two utility models compatible with existing experimental data on RPD: relative inequality aversion and delayed guilt aversion.

The intuition behind the models’ success in explaining cooperation with end-effect is simple. The inequality aversion model implies that agents cooperate in the early rounds because they dislike the payoff difference arising from unilateral defection. The end-effect exists because it is relative inequality which agents dislike (i.e. the difference in their shares of the total payoff). Cutting off cooperation by defecting unilaterally in the later rounds produces less relative inequality than defecting in the beginning of the game, because the total payoff in the former situation is bigger. In other words, the material benefit of endgame defection outweighs the equality concerns.

According to the guilt aversion model, agents cooperate in the early rounds because inflicting a payoff loss on their opponent makes them feel guilty. Defective endplay is due to the lasting nature of this emotion. Agents avoid unilateral defection at the beginning of the supergame because they anticipate feeling guilty during subsequent interactions with the betrayed opponent. However, the negative feeling is terminated at the end of the interaction with the specific opponent, so in the last round agents defect because their guilt from doing so would not last.

We test for inequality aversion by studying the effect of the payoff matrix asymmetry on cooperation levels. The presence of delayed guilt is established by offering the participants an option to exit the game, either after unilateral or mutual defection. The two models under consideration issue concrete predictions about the effect of these variables on the outcome of the game, which are formalised as the hypotheses of our study.
• **H1 (inequality aversion).** Subjects cooperate less when their opponent’s payoff is increased but the ranking of the outcomes stays the same for both players.

• **H2 (guilt).** Subjects choose to exit the game more often after being a unilateral defector than after mutual defection, assuming their payoff would be the same in both cases if they stayed in the game.

In order to test these hypotheses, we used a fractional factorial design with respect to payoff asymmetry and exit option availability (Table 2.8).

Table 2.8: Experimental Treatments

<table>
<thead>
<tr>
<th>Session</th>
<th>Treatment</th>
<th>Payoff table</th>
<th>Exit option</th>
<th>No of rounds</th>
<th>No of supergames</th>
<th>No of players</th>
<th>Matching protocol</th>
</tr>
</thead>
<tbody>
<tr>
<td>No 1</td>
<td>T1: Control</td>
<td>Sym</td>
<td>No</td>
<td>6</td>
<td>20</td>
<td>16</td>
<td>Random</td>
</tr>
<tr>
<td>No 2</td>
<td>T2: Asymmetric</td>
<td>Asym</td>
<td>No</td>
<td>6</td>
<td>20</td>
<td>16</td>
<td>Random</td>
</tr>
<tr>
<td>No 3</td>
<td>T3: Exit option</td>
<td>Sym</td>
<td>Yes</td>
<td>6</td>
<td>20</td>
<td>16</td>
<td>Random</td>
</tr>
</tbody>
</table>

2.4.2 Treatments

The game we tested in the laboratory was a 6-round repeated Prisoner’s Dilemma. The experiment was programmed in z-Tree (Fischbacher, 2007) and took place in LExEcon Laboratory in May 2015. The subjects were randomly selected from LExEcon voluntary subject pool, which consists of University of Leicester students. None of the participants had prior experience of playing Prisoner’s Dilemma. For each of the three treatments, a separate session with 16 unique participants was run (Table 2.8). In each session, the participants played the game 20 times and after each 6-round game were rematchd with another partner. The subjects earned £9.74 on average, including the show-up fee of £2.19

At the beginning of each session, the subjects were randomly assigned a role of a “red” or “blue” player. This role was only payoff-relevant in T2, where the blue

---

18The choice of supergame length (6 rounds) was determined by the existence of an insightful theoretical prediction (Nachbar, 1992) for such number of repetitions. The paper shows that cooperation with end-effect is a point of pseudoconvergence of an evolutionary process which models agents’ learning. Most existing experiments on RPD, however, were conducted with 10-round supergames, which became a tacit standard in the literature.

19The “advantaged” players in T2 earned £16.02, while all other players earned £8.48.
player’s payoffs were scaled up compared to the rest of the experiment (Table 2.10). The participants stayed in their role throughout the 20-period session, and were always matched with a player from the opposite colour role. The decision screen seen by the subjects displayed a payoff matrix, the history of past play in the current supergame, and prompted them to make a choice between options “A” and “B”, corresponding to cooperation and defection.

**Control Treatment (T1)**

T1 constituted a control treatment, in relation to which the effects of payoff asymmetry (T2) and availability of the exit option (T3) were established.

A payoff table used in T1 (Table 2.9) ensured compatibility with existing studies in terms of Rappoport’s coefficients of cooperation (Rapoport and Chammah, 1965). For our payoffs, the coefficients were $r_1 = (5-3)/(9-0) = 0.22$ and $r_2 = (5-0)/(9-0) = 0.56$, which is close to $r_1 = 0.26$, $r_2 = 0.56$ used in Selten and Stoecker (1986), $r_1 = 0.25$, $r_2 = 0.58$ in Andreoni and Miller (1993) and $r_1 = 0.18$, $r_2 = 0.59$ in Bereby-Meyer and Roth (2006).

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>5, 5</td>
<td>0, 9</td>
</tr>
<tr>
<td>Defect</td>
<td>9, 0</td>
<td>3, 3</td>
</tr>
</tbody>
</table>

**Asymmetric Treatment (T2)**

To the best of our knowledge, our experiment was the first laboratory investigation of an asymmetric finitely repeated Prisoner’s Dilemma game.\(^{20}\)

In the Asymmetric treatment T2, the colour role of a participant was payoff relevant. The “red” (disadvantaged) players were assigned the same payoffs as in the Control treatment, whereas the payoffs of the “blue” (advantaged) player were increased (Table 2.10). In this new payoff table, unilateral defection by a disadvantaged player entails a less unequal outcome than a unilateral defection in the sym-

\(^{20}\)The novelty of our design was confirmed by the members of the Economics Science Association Discussion Group.
metric case. Hence, by comparing cooperation levels in T1 and T2, it can be inferred how inequality aversion affects cooperation in the game.

Table 2.10: Payoff Matrix in the Asymmetric Treatment T2

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>5, 12</td>
<td>0, 19</td>
</tr>
<tr>
<td>Defect</td>
<td>9, 2</td>
<td>3, 8</td>
</tr>
</tbody>
</table>

The payoff structure in Table 2.10 was designed to allow disentanglement of absolute and relative inequality aversion. Indeed, the absolute inequality of outcomes (D,C) and (C,C) is the same (9−2 = 12−5). Thus, an absolute-inequality-averse player 1 would like to deviate from mutual cooperation, as it increases his material payoff while leaving the equality unchanged. Hence, if agents are motivated by absolute inequality aversion, cooperation in T2 will be completely eliminated. In contrast, a relative-inequality-averse players will still cooperate, albeit for shorter than in the symmetric case. Indeed, a unilateral defection increases inequality from 1/2 − 5/12 = 0.21 to 1/2 − 2/2+3 = 0.32, as compared to a sharper increase from 0 to 0.5 in the symmetric case. Also note that if players are motivated not by inequality aversion but by altruism (or “warm glow” from giving), their cooperation level should increase or stay unchanged in response to the increase in their opponents’ payoffs.

Exit Option Treatment (T3)

The second hypothesis we tested states that players cooperate to avoid anticipated guilt, which is triggered by unilateral defection and lasting until the end of the interaction with a specific partner.\textsuperscript{21} We are testing this hypothesis by offering subjects an option to terminate an interaction with their opponent after defection took place. We then measure the difference in the rate of acceptance of the exit option between unilateral and mutual defectors, and if the difference is significant, conclude that non-distributional considerations matter for players.\textsuperscript{22} Opting out might

---

\textsuperscript{21}Note the key difference of guilt aversion from distributional models of other-regarding preferences (i.e. where preferences are fully specified by the player’s own and others’ payoffs): it implies that utility loss from unilateral defection is lasting, or, in other words, spilling over to future rounds.

\textsuperscript{22}Choosing an exit option over continuing the game can be influenced by other factors, e.g. tiredness or boredom. Hence we are measuring the difference in acceptance rates between unilateral and mutual defectors.
be then interpreted as a choice to avoid interaction with betrayed partner due to the feeling of guilt which it would cause.

The exit option was offered to the subjects in the second half of the experimental session (supergames 11-20), after they gained experience with the game. This design feature was based on our expectation that cooperation with end-effect would establish after the first 5-10 supergames, akin to the existing experimental results. The exit payment to both players was equal to the mutual defection payoff times the number of remaining rounds. If players’ beliefs are determined by the established pattern, then their exit payment will be equal to how much they expect to receive (and expect their opponent to receive) from staying in the game, whether the defection in round \( n \) was unilateral or mutual.

### 2.4.3 Strategic Sophistication Test

At the end of the experiment we evaluated the subjects’ strategic sophistication by running an additional “restart” supergame (Andreoni and Miller, 1993), in which players were knowingly paired with a computer programmed to play tit-for-tat. The results of this treatment allowed us to isolate a subset of subjects who understand how to best respond to such strategy. Thus, their departure from the best response in the main experiment is likely to be dictated by other-regarding preferences, which obviously do not affect any subject’s play in the strategic sophistication test.

### 2.5 Results

Our experiment replicated the main finding of existing RPD studies – higher cooperation in the first half of the supergame. Using the Wilcoxon signed-rank test, we found that the subjects were significantly more cooperative in rounds 1-3, compared to rounds 4-6, in all treatments: control (\( z = 3.493, p = 0.0005 \)), asymmetric (\( z = 3.291, p = 0.001 \)) and the first half of the exit option (\( z = 3.493, p = 0.0005 \)).

---

23 Cooperation with end-effect is a consistently observed pattern of experimental play in RPD. It is defined as both players cooperating in every round until one or both of them defect, after which both defect in remaining rounds.

24 The unit of observation here is a subject (\( N = 16 \)). If a supergame is considered a unit of observation (\( N = 20 \)), the difference is still significant in control (\( z = 3.921, p = 0.0001 \)), asymmetric (\( z = 3.884, p = 0.0001 \)) and exit option treatments (\( z = 3.920, p = 0.0001 \)). It should be noted that for both types of units, the observations are not independent. Indeed, the number of participants in
In line with existing experimental results, the pattern of play for experienced players was “cut-off” cooperation (*a.k.a. cooperation with end-effect*) in 70% of all supergames, with no return to cooperation after the first defection, either unilateral or mutual (Figure 2.3).

![Figure 2.3: Fraction of Supergames where “Cutoff” Strategies Were Used](image)

However, the average level of cooperation in the control group (21%) was lower than in existing experiments with 10-round supergames (43% in Andreoni and Miller (1993), 55% in Cooper et al. (1996) and 62% in Bereby-Meyer and Roth (2006)), falling from 49% in the first supergame to 5% in the last (Table 2.11). Given the small sample size in our experiment (one session with 16 players), this idiosyncratic result might be attributed to chance. Alternatively, it could be the case that a less complex game (*i.e.* containing fewer repetitions) makes it easier for subjects to learn backward induction. Indeed, even in 10-round experiments (Bereby-Meyer and Roth, 2006; Selten and Stoecker, 1986), an early trend of growing cooperation rates went into reverse in the last 5 supergames. Selten and Stoecker (1986) attributed this dy-

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25 This datum is for the second half of the sessions, when all players had sufficient experience in playing the game.
namic to learning and proposed a model which predicted complete elimination of cooperation in the long run.

Table 2.11: Decline in Cooperation Rates in All Treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Cooperation (first supergame)</th>
<th>Cooperation (last supergame)</th>
<th>Fraction of “cutoff” supergames</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control</td>
<td>49%</td>
<td>5%</td>
<td>74%</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>28%</td>
<td>17%</td>
<td>60%</td>
</tr>
<tr>
<td>Exit Option</td>
<td>59%</td>
<td>10% or 17%</td>
<td>72%</td>
</tr>
</tbody>
</table>

Figure 2.4: Average Cooperation as a Function of the Supergame

2.5.1 Payoff Asymmetry

Our experiment results imply that inequality aversion is an important concern in RPD. Indeed, the asymmetry of payoffs had a marked effect on the subjects’ play,

\[\text{The fraction of “cutoff” supergames in the second half of the experiment, after players gained experience.}\]

\[\text{The second number reports average cooperation by only those pairs of players who rejected the exit option.}\]
with cooperation level among advantaged players in T2 (22.5%) being higher than among disadvantaged players (14%).\footnote{This difference is not statistically significant, possibly because of the small sample size (8 players of each type). Additional experimental sessions for this treatment will be run in the future in order to establish whether the difference in play is likely to be coincidental.} Moreover, “red” (disadvantaged) players in T2 cooperated less than “red” players in T1 (14% versus 16.5%), although their payoffs in two treatments were the same. This result suggests that altruism, which is considered a likely reason behind cooperation with end-effect in the established literature (Andreoni and Miller, 1993), cannot be its main cause. Indeed, an altruistic player would either cooperate more or as much as before, when his opponent’s payoff is scaled up.\footnote{According to Andreoni and Miller (1993), there are several types of altruistic preferences. A pure altruist, whose utility equals a weighted average of his and his opponent’s payoffs, would cooperate weakly more if his opponent’s payoff is scaled up. A reciprocal altruist, whose payoff from mutual cooperation equals his monetary payout plus a constant (pleasure from successful cooperation), would cooperate as much as before.} However, according to our results, neither is inequality aversion the main driving force behind cooperation with end-effect, even if it does affect the subjects' play. Indeed, overall cooperation levels in control (18.5%) and asymmetric (18.2%) treatments were not significantly different (Mann-Whitney $z = -0.692, p = 0.4889$).

### 2.5.2 Non-distributional Preferences

The exit option treatment was designed to test whether players have a strict preference towards one of the payoff-equivalent alternatives, which would indicate the presence of non-distributional preferences. The main result is that such effect is present in the subject pool, but its sign is opposite to the predicted one.

Indeed, players accepted the exit option significantly more often after a mutual defection than after a unilateral one (Wilcoxon signed-rank test $z = -2.110, p = 0.0349, N = 14$). What would motivate the player to reject the exit option offer? If he is quite sure that his opponent is going to defect throughout the rest of the supergame after the first defection occurred (at which point the exit option is offered), rejecting the offer is in fact weakly payoff-decreasing.\footnote{There are two reasons to believe that the players would place a high probability on their opponent never cooperating after the first defection. Firstly, in supergames 1-9, where no exit option was offered, the fraction of players who followed this pattern of play rose from 38% to 88% as players gained experience; this should have formed the players’ expectation of post-defection play as they entered an exit-option stage. Secondly, these hypothetical beliefs must have been reinforced by the actual play in supergames 11-20, where only 21% of non-deciding players cooperated after the first defection.} The only reason to reject the
offer is that staying in the game provides an opportunity to increase the opponent’s payoff by unilaterally cooperating in the post-offer rounds. Indeed, many unilateral defectors (19 out of 40) cooperated post-offer, at the expense to themselves. In contrast, mutual defectors only cooperated in 3 out of 22 cases of exit option rejection. This suggests that unilateral defection creates an incentive to help the opponent, which can be attributed to inequality aversion (and a resulting desire to mitigate the inequality created by unilateral defection). The same behaviour can be described in terms of emotions: the defector is seeking to placate his guilty conscience by doing a favour to the victim. Note that our original guilt-aversion hypothesis (H2) implied that players would seek to avoid anticipated guilt. The play induced by this motive would be observationally distinguishable from inequality aversion; however, it found no confirmation in the data.

### 2.5.3 Strategic Sophistication Test

When knowingly paired with a conditionally cooperating computer, only half of the subjects played the best response. Interestingly, in the main experiment these sophisticated players were cooperating more than naïfs (Table 2.12). The difference in cooperation rates was significant even within treatments (Mann-Whitney \( z = -2.472, p = 0.0134 \) in control treatment). This result suggests that cooperation in Prisoner’s Dilemma cannot be attributed to bounded rationality.

<table>
<thead>
<tr>
<th>Session</th>
<th>Cooperation by sophisticates</th>
<th>Cooperation by naïfs</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1 control</td>
<td>0.27</td>
<td>0.10</td>
</tr>
<tr>
<td>T2 asymmetric</td>
<td>0.28</td>
<td>0.13</td>
</tr>
<tr>
<td>T3 exit option(^{31})</td>
<td>0.52</td>
<td>0.41</td>
</tr>
</tbody>
</table>

### 2.6 Conclusion and Discussion

Using a revealed preference approach, we identified two utility models which are compatible with observed play in the finitely Repeated Prisoner’s Dilemma: relative defection.

\(^{31}\)Cooperation rates in the first half of the session (pre-exit) are reported.
inequality aversion and delayed guilt aversion. We tested the models in a novel experimental design and found mild support for the inequality aversion model. Laboratory behaviour was sensitive to payoff differences; in particular, players cooperated less when their opponents’ payoffs were scaled up, leaving the outcome ranking the same. Another implication of this result is that altruism is unlikely to be the reason for cooperation in RPD, as altruistic players would have cooperated more with a better-off opponent.

Several issues raised by our results deserve further investigation. First, cooperative behaviour was almost completely eradicated in the second half of the experiment, with only 19% of all choices being cooperative. This result is in contrast with previous laboratory studies, which used 10-round supergames and obtained cooperation levels of 40-50% (as compared to 6-round supergames in our experiment). Since we only ran one session of the treatment with 16 participants, more experimental runs are needed to establish whether this difference was coincidental. Alternatively, it could be caused by shorter supergames, which make it easier for the subjects to learn backward induction.

Another intriguing finding of our experiment was that players who were able to best-respond to a computer algorithm also cooperated more when paired with other human subjects. Evidently, cooperation in RPD is not a result of naivety or a reasoning fault. Future research might examine the link between strategic sophistication and cooperation frequency. Are sophisticated players more “pro-social” than naive players? Or perhaps they are able to reach a tacit cooperation agreement with similarly sophisticated players, in line with the “superrationality” idea of cooperation in RPD.
Chapter Abstract

Guilt aversion has been shown to play an important role in economic decision-making. In this paper, we take an axiomatic approach to guilt by deducing a utility representation from a list of easily interpretable assumptions on an agent's preferences. It turns out that our logarithmic representation can mitigate the problem of multiplicity of equilibria to which psychological games are prone. We apply the model in three well-known games and show that its predictions are consistent with experimental observations.
### 3.1 Introduction

Guilt is the experience of discomfort that follows when we violate a personal or social standard. If an action raises income but disappoints our own or other individuals’ expectations of us, it may trigger our guilty conscience. Any individual who is sufficiently averse to this discomfort may therefore refrain from taking the action in the first place. Guilt aversion is able to explain a vast array of behaviours, including cooperation (Miettinen and Suetens, 2008), altruism (Andreoni and Rao, 2011), conformism (Khalmetski, 2014), group favouritism (Güth, Ploner, and Regner, 2009) and reciprocity (Chang et al., 2011), and economic experiments indicate that it is indeed an important determinant in a variety of different situations (Battigalli, Charness, and Dufwenberg, 2013; Charness and Dufwenberg, 2006; Geng, Weiss, and Wolff, 2011; Hopfensitz and Reuben, 2009; Ketelaar and Au, 2003). More recently, guilt averse behavior has also made its way into macroeconomic modelling. Thus Ahrens and Snower (2014) incorporates guilt and envy into a dynamic stochastic equilibrium model and shows that when these emotions are experienced by workers, a Phillips curve relationship between inflation and output can be generated.

A popular way to model emotions, including guilt, is to include them as inputs in agents’ utility functions. Particularly important for experimental work are linear utility representations with money and guilt as the inputs (Battigalli, Charness, and Dufwenberg, 2013; Battigalli and Dufwenberg, 2007; Chang et al., 2011; Khalmetski, Ockenfels, and Werner, 2014; López-Pérez, 2010). This paper’s main theoretical objective is to axiomatise utility representations of guilt-averse preferences. Specifically, axioms are presented that are necessary and sufficient for (i) the linear representation mentioned a moment ago, (ii) a representation that is logarithmic in money and linear in guilt, and (iii) a general additively separable utility representation of money and guilt. Call the sacrifice ratio between money and guilt (“how much money an agent is willing to pay to lower guilt by one unit”) the *price of a clear conscience* (PCC). For well-behaved preferences we find that (i) obtains if and only if the PCC is constant for all money-guilt combinations; (ii) holds if and only if for any two levels of income the relative PCC equals the relative income, and (iii) derives whenever a suitably redefined “double cancellation condition” (Debreu, 1960) is satisfied.

By tracing specific utility representations to the level of preferences, we are able
to shed light on the deeper psychological conditions that they entail vis-a-vis the previously mentioned personal or social standards. In doing so, we quickly end up concluding that the assumptions about an agent’s moral compass embodied in (i) are problematic. While (iii) is not subject to this critique, it has — as will become clear from the following discussion — too many degrees of freedom to provide a useful alternative in strategic settings. This motivates our introduction of (ii) as the simplest realistic alternative — and it is important to stress, this is not an ad hoc alternative but one grounded in moral/psychological considerations. With this in hand we then — in what is arguably the paper’s main contribution to existing literature — reanalyse a number of famous laboratory games, namely the Dictator game, the Public Good Provision game, and the Prisoners’ Dilemma. This exercise provides further support for model (ii), but we postpone the specifics until section 3.4.

To the best of our knowledge, the only existing paper concerned with the axiomatisation of guilt-income representations is López-Pérez (2010). López-Pérez (2010) proposes a utility function exhibiting guilt aversion and provides axiomatic foundation for it. The study also features a discussion of the psychological foundation of guilt and shame and links the feeling of guilt to internalization of a social norm. The paper differs from ours in a number of ways, however, most importantly in the definition of guilt. In López-Pérez (2010), guilt is binary (-1 if the social norm was breached and 0 otherwise), and for the value of guilt to be determined, an exogenous social norm must be specified. By contrast, in our setting guilt is a real number with the standard interpretation as the difference between an opponent’s actual and expected payoff (see e.g. Battigalli and Dufwenberg (2007) as well as the discussion in section 3.2). Finally, the properties of the preference relation in López-Pérez (2010) depend on what other players do, hence any given representation is only defined within a specific game. By contrast, our preference relation is set on an abstract guilt-money space and thus can be applied to both decision and game theory. An axiomatic approach to the broader field of other-regarding preferences has been pursued by several authors, most notably Neilson (2006) and Sandbu (2008). Both papers axiomatise general function forms: additively separable reference-dependent utility in the former and CES-utility in the latter. What sets these studies apart from the results of the current paper is our focus on specific functional forms with few enough free parameters to be testable in the laboratory.

The rest of the paper proceeds as follows. Section 3.2 introduces money-guilt
utility functions. Section 3.3 develops a theory of moral choice and presents our axiomatisation results. Section 3.4 studies the experimental evidence in the three games mentioned above as well as further discussion. The Appendix contains proofs.

3.2 Existing Literature and the Logarithmic Alternative

The first formal model of guilt aversion was proposed by Battigalli and Dufwenberg (2007). They define guilt as the perceived payoff loss inflicted on another player, i.e., as the difference between an opponent’s expected payoff $E(m_j)$ and actual payoff $m_j$:

$$G(m_j, E(m_j)) = \max\{0, E(m_j) - m_j\}.$$  

(3.1)

To be precise, since a player $i$ does not know exactly how much his opponent $j$ expects, $E(m_j)$ is $i$’s belief about $j$’s expectation. That makes guilt, and a guilt-averse agent’s utility, a function of second-order beliefs (cf. Geanakoplos, Pearce, and Stacchetti (1989), Attanasi and Nagel (2007), Battigalli and Dufwenberg (2009)). Battigalli and Dufwenberg (2007) also propose a utility function over money and guilt (3.2), which has been extensively used in subsequent theoretical and experimental research.\footnote{Examples are Battigalli and Dufwenberg (2009), Chang et al. (2011), Battigalli, Charness, and Dufwenberg (2013), Khalmetski (2014).}

$$u_i(m_i, G) = m_i - \theta G.$$  

(3.2)

Here $m_i$ is the decision-maker’s monetary payoff, $G$ is the guilt he experiences, and $\theta$ is a guilt sensitivity parameter. A key advantage of such an approach is that it endogenises the reference point $E(m_j)$ which with a formulation such as (3.2) is implicitly solved for in equilibrium. A constant marginal rate of substitution (MRS) between money and guilt arguably has a drawback, however: it can explain nearly any observed behavior. To illustrate with an often studied example, consider the so-called Dictator game, in which one player (the Dictator, hereafter D) decides upon the division of the total endowment $T$ between himself and the other player (the Recipient, hereafter R). His donation to R, $m_R$, is hence his strategy. In Psychological Nash equilibrium of the game, D’s donation will maximize his utility, given his belief about what R expects from him ($E(m_R)$), and this belief will be cor-
3.3: Moral Choice and Axiomatization

It is easy to see that, if D’s utility is defined as in (3.2): \( u = T - m_R - \theta \max\{0, E(m_R) - m_R\} \), either the set of equilibria coincides with the strategy set (if \( \theta \geq 1 \)), or giving zero is the only equilibrium (if \( \theta < 1 \)). The second case is falsified by experimental evidence of positive giving in the Dictator game (Engel, 2011). The first case is consistent with experimental data but — and this is our main point — it is unfalsifiable in the sense that it is consistent with any set of empirical/experimental observations.

In section 3.4, we return to the Dictator game as well as two other games that suffer from related difficulties and explain how these shortcomings are overcome if we instead use the following logarithmic specification:

\[
    u_i(m_i, G) = \log m_i - \theta G. \tag{3.3}
\]

To be sure, it is rather obvious that one way out of the previously described predicament is not to assume a constant MRS. At the same time, one cannot pass to arbitrary utility representations, however, including arbitrary additively separable representations, since that merely compounds the explanatory richness. In brief, one must commit to a specific functional form for falsification to be possible. In the next section we will argue from deeper moral axioms that (3.3) is a proper alternative to linear specifications.

3.3 Moral Choice and Axiomatization

Consider a decision-maker who has a preference relation \( \succeq \) over a two-dimensional choice set \( \mathcal{M} \times \mathcal{G} \subseteq \mathbb{R}_+^2 \), where \( m \in \mathcal{M} = \mathbb{R}_{++} \) is his strictly positive monetary payoff, and \( G \in \mathcal{G} = \mathbb{R}_+ \) is the guilt he experiences. \( (m_1, G_1) \succeq (m_2, G_2) \) reads “a payoff of \( m_1 \) accompanied by guilt of size \( G_1 \) is at least as good as a payoff of \( m_2 \) accompanied by guilt of size \( G_2 \)”.

We assume throughout that \( \succeq \) is complete and transitive (rational), monotone in the sense that money is desirable whereas guilt is undesirable (\( m_1 > m_2 \Rightarrow (m_1, G) > (m_2, G) \) and \( G_1 < G_2 \Rightarrow (m, G_1) > (m, G_2) \)), and continuous (for all \( m_1, G_1 \)), the lower and the upper contour sets, \( \{(m, G) : (m, G) \succeq (m_1, G_1)\} \) and \( \{(m, G) : (m, G) \succeq (m_1, G_1)\} \) are closed). These assumptions are of course completely standard.

The previous assumptions together with convexity of the choice set \( \mathcal{M} \times \mathcal{G} \) imply
the existence of a continuous utility representation, i.e., a continuous function $u : \mathfrak{M} \times \mathfrak{G} \to \mathbb{R}$ so that $u(m_1, G_1) \geq u(m_2, G_2) \iff (m_1, G_1) \succeq (m_2, G_2)$ (Debreu, 1954). A simple adaptation of another contribution by Debreu, immediately provides us with necessary and sufficient conditions for $u$ to be additively separable, i.e., for $u$ to take the form $u(m, G) = f(m) + g(G)$ (here $f$ must be strictly increasing and $g$ strictly decreasing under our monotonicity condition). What we have in mind is Debreu’s “double cancellation condition” (Debreu, 1960) which in the current setting can be cast as follows: if $(m_1, G_1) \succeq (m_2, G_2)$ and $(m_2, G_3) \succeq (m_3, G_1)$, then $(m_1, G_3) \succeq (m_3, G_2)$.

In words, the decision-maker’s marginal disutility of guilt does not depend on how wealthy he is, and vice versa.²

Everything that has been said so far is either well-known or trivial in light of existing literature. In contrast, the next concept is new. Consider an agent who is indifferent between two options, one of which offers more money and more guilt (the greedy option), while the other offers less money and less guilt (the conscientious option). Formally, consider a pair of distinct alternatives $\{(m_1, G_1), (m_2, G_2)\}$ such that:

$$(m_1, G_1) \sim (m_2, G_2). \quad (3.4)$$

We call any $\{(m_1, G_1), (m_2, G_2)\}$ that satisfies (3.4) a moral dilemma. For a given moral dilemma, the agent is willing to give up $m_1 - m_2$ in order to reduce his level of guilt by $G_1 - G_2$. The sacrifice ratio between the two,

$$\frac{m_1 - m_2}{G_1 - G_2}, \quad (3.5)$$

is referred to as the price of a clear conscience. Our first moral axiom makes the (bold) postulate that the price of a clear conscience is independent of the moral dilemma the agent faces:

**Axiom 3.1. (Constant Price of a Clear Conscience)** For any $(m_1, G_1) \sim (m_2, G_2)$ and $(m'_1, G'_1) \sim (m'_2, G'_2)$, it holds that $\frac{m_1 - m_2}{G_1 - G_2} = \frac{m'_1 - m'_2}{G'_1 - G'_2}$.

The linear utility representation $u(m, G) = m - \theta G$ of equation (3.2) is easily shown to imply Axiom 3.1. In addition, the underlying preferences are clearly rational, monotone, and continuous. More surprisingly, the converse is true as well:

²For further interpretation and discussion of this condition as well as equivalent independence type conditions see Debreu (1960) as well as Vind and Grodal (2003).
Theorem 3.2. Consider a rational, monotone, and continuous preference relation $\succeq$ on $\mathcal{M} \times \mathcal{G}$. Then $\succeq$ admits the utility representation $u(m, G) = m - \theta G$, $\theta > 0$ if and only if $\succeq$ satisfies Axiom 3.1.

**Proof:** in the Appendix.

Theorem 3.2 tells us that assuming a linear utility representation amounts to assuming that the mental tradeoff between money and guilt captured by the price of a clear conscience remains the same regardless of the level of income and load of sin an agent faces. This, it may be argued, is not necessarily a realistic description of actual behaviour. We would arguably expect an agent to care less about giving up a unit of income the richer he is. He will therefore be willing to pay more to clear his conscience than a relatively poorer version of himself. The next axiom formalizes this description of behaviour in a specific way by requiring the relative price of a clear conscience to always equal the relative income. This statement is of course only meaningful if relative income is well defined, i.e. if the increase in income is the same for both the greedy and the conscientious options.

**Axiom 3.3. (Income Effect in Moral Choice)** For any $(m_1, G_1) \sim (m'_1, G'_1)$ and $(m_2, G_2) \sim (m'_2, G'_2)$, such that $m_1/m_2 = m'_1/m'_2$, it holds that $m_1/m_2 - m'_1/m'_2 = m'/m_1$.

Axiom 3.3 implies the double cancellation condition. Indeed, in the Appendix we prove the following lemma:

**Lemma 3.4.** Suppose a rational, monotone and continuous $\succeq$ on $\mathcal{M} \times \mathcal{G}$ satisfies Axiom 3.3. Then $(m_1, G_1 + a) \sim (m_2, G_2 + a)$ whenever $(m_1, G_1) \sim (m_2, G_2)$, and $(m_1, G_1 + a) > (m_2, G_2 + a)$ whenever $(m_1, G_1) > (m_2, G_2)$ for any $a \in [\max\{-G_1, -G_2\}, +\infty]$.

Applying Lemma 3.4 to the antecedent statements of the double cancellation condition, we obtain:

\[(m_1, G_1) \succeq (m_2, G_2) \Rightarrow (m_1, G_3) \succeq (m_2, G_2 + G_3 - G_1) \quad (3.6)\]
\[(m_2, G_3) \succeq (m_3, G_1) \Rightarrow (m_2, G_3 + G_2 - G_1) \succeq (m_3, G_2) \quad (3.7)\]

which by transitivity yields $(m_1, G_3) \succeq (m_3, G_2)$. In particular, Axiom 3.3 implies that $u$ must be additively separable. One can show that Axiom 3.1 also implies the double cancellation condition and therefore an additively separable utility function.
It turns out that Axiom 3.3 is necessary and sufficient for a representation which is logarithmic in money and linear in guilt.

**Theorem 3.5.** Consider a rational, monotone and continuous preference relation \( \succeq \) on \( M \times G \). Then \( \succeq \) admits the utility representation \( U(m, G) = \log m - \theta G, \theta > 0 \) if and only if \( \succeq \) satisfies Axiom 3.3.

**Proof:** in the Appendix.

Intuitively, the proof of Theorem 3.5 proceeds by showing that the indifference curves of \( \succeq \) satisfying Axiom 3.3 are related by parallel displacement along the guilt axis and proportional expansion along the money axis, which leads to the desired representation.

### 3.4 Reinterpreting Existing Literature

In this section, we continue the discussion of section 3.2 with the representation (3.3) in hand. We study three well-known laboratory games in which experimental subjects appear to be motivated by considerations other than their monetary payoff. Non-monetary motivation is manifested in positive giving in the Dictator game, non-zero contributions in the Public Good Provision game, and cooperation in the Prisoner’s Dilemma. We show that this seemingly irrational behaviour can be explained by guilt aversion, as modelled by functions (3.2) and (3.3). Indeed, both models admit observed experimental outcomes in equilibrium. However, we also demonstrate that the explanatory power of model (3.2) stems from its unfalsifiability, and prove that the logarithmic representation (3.3) achieves sharper predictions without sacrificing the goodness of fit.

The rest of this section proceeds as follows. First, for each game, we introduce the classic structure: players, strategies and “material” payoffs corresponding to experimental payouts. Second, we replace these payoffs with utility functions (3.2) and (3.3), which depend not only on the strategies (via “material” payoff \( m \)), but also on pre-game beliefs (via guilt \( G \)). The extended utility function domain means that the resulting structure is a “psychological game” (Geanakoplos, Pearce, and Stacchetti, 1989). If a player’s preferences are represented by such a utility function, his preference ordering over the outcomes of the game, *e.g.* possible Dictator’s dona-
tions to the Recipient, depends on his pre-game beliefs, e.g. what he thought the Respondent expected to receive. Third, the new game is solved for Psychological Nash equilibria, in which all players best-respond to their beliefs, and these beliefs are correct. Finally, the equilibria of the game under utility functions (3.2) and (3.3) are compared to experimental outcomes, which constitutes an empirical test of the models.

3.4.1 The Dictator Game

In the Dictator game, one of the players (the Dictator) determines how to split a total endowment $T$ between himself and a passive player (the Recipient). D’s donation to R, which is his strategy, is denoted by $m_R$. If D is selfish, i.e. if his preferences are reflected by his material payoff $T - m_R$, the only Nash equilibrium of the game is zero donation: $m_R = 0$. We would expect a guilt-averse D to donate a positive amount to R. This is formally confirmed below, when we find equilibria in psychological games induced by the linear (3.2) and the logarithmic (3.3) guilt models. Denote D’s belief about R’s expectation of the donation by $E(m_R)$. The amount of guilt that he experiences from donating $m_R$ is thus $G = \max\{0, E(m_R) - m_R\}$, and Psychological Nash equilibrium (hereafter PsyNE) is a strategy $m^*_R$ which solves (3.8).

$$
\begin{align*}
U(m^*_R, E(m_R)) &\geq U(m_R, E(m_R)) \text{ for all } m_R \in [0, T]; \\
m^*_R &= E(m_R). 
\end{align*}
$$

(3.8)

First, consider D’s utility under the linear guilt model (3.2):

$$
u = T - m_R - \theta \max\{0, E(m_R) - m_R\}.\]$$

(3.9)

PsyNE is determined from the system (3.10), which is obtained by applying the utility function (3.9) to the equilibrium condition (3.8):

\[\text{Strictly speaking, Psychological Nash Equilibrium consists of a strategy profile and a belief profile (Geanakoplos, Pearce, and Stacchetti, 1989). However, since these profiles are required to coincide (in equilibrium, strategies match beliefs), hereafter we denote PsyNE by its constituent strategy profile, implying that it is accompanied by the matching belief profile.}\]
\[
\begin{align*}
T - m^*_R - \theta \max\{0, E(m_R) - m^*_R\} &\geq T - m^*_R - \theta \max\{0, E(m_R) - m_R\} \quad \text{for all } m_R \in [0, T], \\
m^*_R &= E(m_R).
\end{align*}
\] (3.10)

which yields

\[
m^*_R \in \begin{cases} [0, T] & \text{if } \theta \geq 1; \\ \{0\} & \text{otherwise}. \end{cases}
\] (3.11)

Equation (3.11) characterizes the set of equilibria in the game. This set, depending on D’s guilt sensitivity \( \theta \), either coincides with the strategy set \([0, T]\), or consists of the unique equilibrium where D gives zero.

**Observation 1.** *In the Dictator game with the linear utility function (3.2), all possible Dictator’s donations are PsyNE of the game if \( \theta \geq 1 \), and zero donation is the only PsyNE if \( \theta < 1 \).*

Figure 3.1, left illustrates the equilibria in the Dictator game under the linear guilt function, as solved from the system (3.10). The first equation in (3.10) solves for D’s optimal donation \( m^*_R \) as a function of his belief about R’s expectation \( E(m_R) \). Depending on D’s guilt sensitivity, it is either a 45° line from the origin \( m^*_R = E(m_R) \) (if \( \theta \geq 1 \), solid graph), or a horizontal line \( m^*_R = 0 \) (if \( \theta < 1 \), dashed graph). The second equation in (3.10) is a 45° line from the origin, and the solution is their intersection, which is the whole line for \( \theta \geq 1 \) or a single point \((0, 0)\) for \( \theta < 1 \).

![Figure 3.1](image-url)
3.4: Reinterpreting Existing Literature

Let us compare this prediction with the laboratory evidence. In a meta study of 129 different Dictator Game experiments (a total of 41,433 observations), Engel (2011) reports a mean contribution of 28.35% of the initial endowment. In fact, only 36.11% of all participants give nothing. Thus, if the linear guilt model (3.2) is correct, then for the majority of players guilt sensitivity must be larger than one. The model then suggests that these people will be giving out donations of all sizes, but the experimental data indicates that dictators are more likely to give little. Indeed, the distribution of average giving compiled in Engel (2011) is left skewed. Large donations are very rare. In particular, less than 10% of all participants surveyed in the meta-study gave more than 60% of the pie. In short, the linear model (3.2) does not account for the main stylized fact implied by the experimental evidence: the prevalence of moderate donations.

It turns out that the logarithmic model (3.3) embraces this stylized fact, predicting that for big expectations $E(m_R)$, D’s optimal donation to R will be less than such expectation: $m^*_R(E(m_R)) < E(m_R)$. It follows that such $E(m_R)$ cannot be part of an equilibrium and will not be observed. Indeed, PsyNE under model (3.3) solves the following system:

$$\begin{cases} 
\log(T - m^*_R) - \theta \max\{0, E(m_R) - m^*_R\} \geq \log(T - m_R) - \theta \max\{0, E(m_R) - m_R\} \text{ for all } m_R \in [0, T]; \\
m^*_R = E(m_R); 
\end{cases}$$

which yields

$$m^*_R \in [0, \max\{0, T - \frac{1}{\theta}\}].$$  \hspace{1cm} (3.13)

Expression (3.13) tells us that if D’s preferences are represented by the logarithmic utility function (3.3), he will only satisfy expectations $E(m_R)$ up to a maximum of $T - \frac{1}{\theta}$ (Figure 3.1, right). This threshold expectation is increasing with D’s guilt sensitivity.

**Observation 2.** In the Dictator game with the logarithmic utility function (3.3), the Dictator’s donations up to $\max\{0, T - \frac{1}{\theta}\}$ are PsyNE of the game.

**Proof:** in the Appendix.

Observation 2 implies that smaller donations are more likely to be observed,
which is in line with the existing body of laboratory evidence. Note that, apart from
the equilibrium prediction, the logarithmic model describes how D best-responds
to a big $E(m_R)$: the amount he gives will be less than what he believes is expected
from him. This implication of the logarithmic model validates the guilt aversion
hypothesis, which has been put into question by some experimental results, most
notably Ellingsen et al. (2010) who show that players’ donations in the Dictator game
do not always match their beliefs. Ellingsen et al. (2010) data is indeed inconsistent
with the linear guilt model (3.2), which predicts that, for any belief $E(m_R)$, the Dictator
will either grant it in full or give nothing. This leads Ellingsen et al. (2010) to
refute the guilt aversion hypothesis. However, the logarithmic model, as we just ar-
gued, accounts for “sub-belief giving” by the Dictators, thus explaining most of their
data.

3.4.2 The Public Good Provision Game

In a 2-player Public Good Provision game, each player $i = 1, 2$ is endowed with $w_i$
and decides upon the amount of his contribution to a common fund $x_i \in [0, w_i]$, which
is hence his strategy. The money in the fund is multiplied by a number $2a$ and
shared equally among the players. The final payoff $m_i$ is then determined as
follows:

$$m_i = w_i - x_i + a(x_i + x_j), \quad (3.14)$$

where $1 > a > 0.5$. The restriction on $a$ makes contributions collectively efficient
but not individually rational.

If utilities equal material payoffs, the only equilibrium is zero contribution by
both agents (“free-riding”). This sharp prediction is refuted by laboratory tests of
the game, which report an average contribution of 40-60% of the initial endowment
(Ledyard, 1995). Can the phenomenon of positive contributions be attributed to
guilt aversion? It has been shown that non-binding promises during pre-play com-
munication helps sustain high contribution levels in Public Good Provision experi-
ments (Ledyard (1995) and more recently Denant-Boemont, Masclet, and Noussair
(2011)), which suggests the players’ desire to meet expectations, i.e. guilt aversion.
We formally confirm this intuition below, demonstrating that the models of guilt
aversion (3.2) and (3.3) indeed admit positive contributions in equilibrium.
Let $E(x_i)$ denote $i$’s second-order belief about $x_i$, i.e. what he thinks $j$ expects him to contribute. First, consider the linear guilt model (3.2), under which $i$’s utility becomes

$$u_i(x_i, x_j, E(x_i)) = w_i - x_i + a(x_i + x_j) - \theta_i \cdot \max \{0, (w_j - x_j + a(E(x_i) + x_j)) - (w_j - x_j + a(x_i + x_j))\}$$

$$= w_i - (1 - a)x_i + a x_j - \theta_i \cdot \max \{0, a(E(x_i) - x_i)\}.$$ 

Observe that utility is linear in the choice variable, which implies a corner solution. Indeed, it is easy to show that, if $\theta_i \geq \frac{1-a}{a}$, a player $i$ will maximize his utility by contributing as much as expected from him ($x_i = E(x_i)$), regardless of $j$’s contribution. In other words, any belief about his contribution is self-fulfilling, which means that the set of equilibria coincides with the set of strategy profiles. If the player is not guilt-averse enough ($\theta_i < \frac{1-a}{a}$), he will contribute zero.

**Observation 3.** Consider a Public Good Provision game with the linear utility function (3.2). A contribution profile $(x_1, x_2)$ is a PsyNE if it satisfies the following: (i) $x_i = 0$ for any $i = 1, 2$ such that $\theta_i < \frac{1-a}{a}$; (ii) $x_i \in [0, w_i]$ for any $i = 1, 2$ such that $\theta_i \geq \frac{1-a}{a}$.

**Proof:** in the Appendix.

The linear guilt model thus predicts that some agents will be giving out positive contributions, but remains agnostic about their size, their correlation with the opponent’s contribution, and the effect of the parameters of the model on the amounts given. However, the existing body of experimental evidence from Public Good Provision games has some clearly identifiable patterns. In an early meta study, Ledger (1995) observes that contributions positively depend on Marginal per Capita Return ($a$ in our model). In a survey of post-1995 experimental literature, Chaudhuri (2011) emphasizes two stylized facts: first, heterogeneity of players in terms of social preferences, and second, the prevalence of conditional cooperators in the subject pool, whose contributions positively depend on the average contribution in the group (in a 2-player setting considered here, this is equivalent to dependence on the opponent’s contribution). We will now show that the logarithmic guilt model (3.3) accounts for all three of these stylized facts.\(^4\)

\(^4\)Heterogeneity of players is also implied by the linear model, which suggests that the players fall
3.4: Reinterpreting Existing Literature

Under the logarithmic model (3.3), agent $i$’s utility becomes:

$$U_i(x_i, x_j, E(x_i)) = \log(w_i - (1 - a)x_i + ax_j) - \theta_i \cdot \max\{0, a(E(x_i) - x_i)\}. \quad (3.15)$$

PsyNE is a pair of contributions $(x^*_1, x^*_2)$, where each $x_i$ satisfies the following conditions:

$$\begin{align*}
U_i(x^*_i, x^*_j, E(x_i)) &\geq U_i(x_i, x^*_j, E(x_i)) \text{ for all } x_i \in [0, w_i] \text{ for } i = 1, 2; \\
x^*_i & = E(x_i) \text{ for } i = 1, 2.
\end{align*} \quad (3.16)$$

Solving the system (3.16) yields Observation 4.

**Observation 4.** Consider a Public Good Provision game with the logarithmic utility function (3.3). A contribution profile $(x^*_1, x^*_2)$ is a Psychological NE if, for $i = 1, 2$, either $x^*_i = 0$ or $0 < x^*_i \leq \left(\frac{w_i + ax^*_j}{1 - a}\right)^{1/\theta_i}a^{-1}$.

**Proof:** in the Appendix.

In words, for given values of players’ guilt sensitivities, admitted in equilibrium are contributions up to a certain limit. The player’s maximum contribution positively and continuously depends on the marginal return $a$, his guilt sensitivity $\theta_i$ and his opponent’s contribution $x_j$, which is an exact match of the stylized experimental facts discussed above. The predictions of the model are illustrated in Figure 3.2 for the cases of big (left) and small (right) initial endowments: shaded areas are Psychological Nash equilibria of the Public Good Provision game.

### 3.4.3 The Prisoner's Dilemma Game

In the Prisoner’s Dilemma game, two players choose between cooperation and defection. The latter strategy is dominant but leads to an inefficient outcome, as described in the payoff table below:

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>$(R, R)$</td>
<td>$(0, S)$</td>
</tr>
<tr>
<td>Defect</td>
<td>$(S, 0)$</td>
<td>$(T, T)$</td>
</tr>
</tbody>
</table>

into one of the two groups, depending on their guilt sensitivity: those who contribute nothing and those who can contribute anything. The logarithmic model provides a sharper prediction, since it implies a continuum of contribution behaviour, where the player’s maximum contribution positively and continuously depends on his guilt sensitivity.
Mutual defection is the only Nash equilibrium of Prisoner’s Dilemma. Contrary to this prediction, existing experiments report non-negligible proportion of cooperative choices, around 20% in most studies (Andreoni and Miller, 1993; Cooper et al., 1996).

The presence of cooperation in laboratory Prisoner’s Dilemma has been attributed to guilt aversion by some experimental papers, notably Miettinen and Suetens (2008). In Miettinen and Suetens (2008), it is argued that players who cooperate do so to avoid the feeling of guilt which comes with unilateral defection. We validate this intuition by applying the formal models of guilt aversion (3.2) and (3.3) to Prisoner’s Dilemma and comparing their predictions with the experimental findings.

First, let us find pure strategy equilibria under the linear guilt model (3.2).

**Observation 5.** Consider a Prisoner’s Dilemma game with the linear utility function (3.2). Mutual defection is a PsyNE for any values of \( \theta \); mutual cooperation is a PsyNE. \(^5\)

\[^5\]The letters stand for temptation, cooperation, defection, sucker.
\( \text{iff } \theta_i \geq \frac{t-c}{t-s} \) (for \( i = 1,2 \)) and unilateral cooperation, where \( i \) is the cooperator, is a PsyNE \( \text{iff } \theta_i \geq \frac{d-s}{t-d} \).

**Proof:** in the Appendix.

Observation 5 tells us that all strategy profiles can be equilibria, including, perhaps surprisingly, unilateral cooperation. Indeed, sufficiently guilt averse players (with \( \theta_i \geq \frac{d-s}{t-d} \)) will be willing to cooperate even when they think their opponent is going to defect. In order to see how extreme this guilt sensitivity threshold is, we consider a specific payoff table used in one of the most famous Prisoner’s Dilemma experiments, conducted by Bereby-Meyer and Roth (2006):

<table>
<thead>
<tr>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>0.105, 0.105</td>
</tr>
<tr>
<td>Defect</td>
<td>0.175, 0.005</td>
</tr>
</tbody>
</table>

Let us calculate the threshold guilt sensitivities for mutual and unilateral defection under the linear guilt model, using Bereby-Meyer and Roth, 2006 payoffs. It turns out that the thresholds coincide:

\[
\frac{t-c}{t-s} = \frac{0.175 - 0.105}{0.105 - 0.005} = \frac{d-s}{t-d} = \frac{0.075 - 0.005}{0.175 - 0.075} = 0.7.
\]

This means that, for the payoff table considered, unilateral cooperation is an equilibrium under strictly larger set of parameter values than mutual cooperation, and hence is more likely to be observed, as illustrated in Figure 3.3, left (Zones B,C,D VS Zone C only).

**Figure 3.3:** Equilibria in the Prisoner’s Dilemma game under the linear (left) and logarithmic (right) guilt models.

Contrary to this prediction, existing experimental research suggests that most
players who cooperate in Prisoner’s Dilemma do so conditionally on their opponent also cooperating. Cooper et al., 1996 do not identify any unconditional cooperators in their subject pool, and estimate the fraction of conditional cooperators to be 12.5-22%. Croson, 2000 finds that at least 51% of subjects reciprocate their counterpart’s expected action i.e. play a conditional strategy. Brosig, 2002 reports that in a face-to-face experiment 90% of cooperators switch to defection when they think the opponent is going to defect.

Let us compare this evidence with the predictions of the logarithmic guilt model (3.3).

**Observation 6.** Consider the Prisoner’s Dilemma game with the logarithmic utility function (3.3). Mutual defection is a PsyNE for any values of \( \theta \); mutual cooperation is a PsyNE when \( \theta_i \geq \log(\frac{t}{c}) \) (for \( i = 1, 2 \)) and unilateral cooperation, where \( i \) is a cooperator, is a PsyNE when \( \theta_i \geq \log(\frac{d}{s}) \).

**Proof:** in the Appendix.

We can now calculate the threshold values of \( \theta \) using Bereby-Meyer and Roth (2006) payoff table. Mutual cooperation is an equilibrium if both players have guilt sensitivity greater or equal to \( \log(\frac{5}{3}) \) \( \sim \) 5.1. For unilateral cooperation to be an equilibrium, the cooperating player must have guilt sensitivity of at least \( \log(\frac{15}{0.1}) \) \( \sim \) 27. Note that although the cardinal values of \( \theta \) are not comparable between the linear and the logarithmic models, the order of the thresholds is meaningful: under the latter model, a necessary condition for mutual cooperation is strictly weaker than for unilateral cooperation.

Figure 3.3, right shows that, although all strategy profiles can still be equilibria under the logarithmic model, unconditional cooperation is limited to the extreme end of guilt sensitivity distribution. As a result, (C,C) is a more likely equilibrium outcome than (D,C) and (C,D) combined; which is in line with the experimental evidence.

**3.5 Concluding Comment**

By developing a formal theory of moral choice, we provided axiomatic foundations for two utility representations of guilt aversion. First, we proved a representation
theorem for a frequently used linear utility, which enabled us to carefully examine and question its underlying axioms. We then proposed a logarithmic representation grounded in more realistic assumptions about the way agents trade off material and moral considerations. By applying the logarithmic representation to laboratory games, we showed that it can better account for the existing body of experimental evidence than a linear representation. Moreover, the novel functional form is able to address experimentally informed criticism of a guilt aversion hypothesis, e.g. it predicts Ellingsen et al. (2010) observation that actions do not always match beliefs. Our framework gives experimentalists tools to pick the most appropriate utility model by considering the axioms of moral choice. Future theoretical research can draw on our approach to discover the grounding of other utility models of non-selfish preferences.
Chapter 4

Industrial Espionage in Duopoly Games

Chapter Abstract

This paper considers an information acquisition game between duopolists, who can learn about unobservable demand from two sources: market research and industrial espionage. The latter provides a noisy signal of the competitor’s market research results. In equilibrium firms pick an action which is a weighted average of the signals they have received. The weight on espionage increases in the complementarity of the players’ actions, but is lower than the weight placed on the market survey signal, unless the espionage system has perfect precision. If the coordination motive is not strong enough, or the market research technology is sufficiently precise, the players will not engage in espionage at all, fully relying on their market research instead. Spying under price competition is shown to be beneficial for the industry, and the economy as a whole: aggregate profit and social welfare increase as the precision of the spying device improves. The same is true under quantity competition if and only if the espionage technology is already more accurate than the alternative market research technology.
4.1 Introduction

Industrial espionage is the illicit appropriation of a firm’s valuable or confidential information by its competitors. Although espionage is sometimes called the second oldest profession in the world, during the Internet era it has been catapulted to a position of utmost strategic importance. Businesses are spending vast amounts of money gathering information about their rivals, with 10% of US corporate intelligence budgets exceeding one million dollars (Fuld Insights, 2013). For those companies being spied upon, the financial loss is also significant: American businesses lose $100-250bn annually due to illegal appropriation of their sensitive information (Barrachina, Tauman, and Urbano, 2014). In the UK, cyber crime costs the economy £27bn a year, including £7.6bn lost to industrial espionage (Cabinet Office, 2011). According to the Cabinet Office report, the sectors of the British economy most vulnerable to industrial espionage are financial services (annual loss £2bn), mining (£1.6bn), aerospace & defense (£1.2bn) and software & computer services (£0.9bn). Global losses to cybercrime are estimated to be between $375-575bn a year, with a substantial proportion of crimes belonging to the industrial espionage category (Center for Strategic and International Studies, 2014).

However, there are reasons to believe industrial espionage is not an unambiguously destructive phenomenon. In particular, the above empirical studies calculate the cost of espionage to the economy as a sum of direct revenue losses to individual businesses. This approach ignores the potential benefit of information exchange between firms as a consequence of industrial espionage, which has been emphasized by some commentators. Notably, the Oxford Encyclopaedia of Economic History criticises the attempts to guard against spying as unsustainable information mercantilism: “Secrecy and need-to-know regimes impede information flow and transactions in personal networks, and would ultimately force organisations to innovate in isolation” (Macdonald, 2003). Indeed, economic spying can be beneficial to industry as a whole if coordination between firms’ actions is socially desirable while the incentives for voluntary information sharing are absent.

1 Indeed, references to economic spying go as far back as the Old Testament: “And Joshua the son of Nun sent two men secretly from Shittim as spies, saying, ‘Go, view the land, especially Jericho.’ And they went, and came into the house of a harlot whose name was Rahab, and lodged there.” (Joshua 2:1)
2 These figures do not include intellectual property theft, which is a separate category of cybercrime.
This paper studies a theoretical model of such an environment: a coordination game with an unknown state of the world (e.g. demand conditions). Naturally, the object of espionage in this setting is the competitor's private information about demand. This type of industrial espionage is notably different from intellectual property theft, because it does not result in the complete loss of competitive advantage by the targeted firm. Indeed, shared knowledge of the state of the world helps the firms better predict each other's action and hence avoid inefficiencies like overproduction. Thus, this study contributes to the debate about costs and benefits of industrial espionage by demonstrating that a certain type of spying can have a positive effect on aggregate profit and social welfare. Further discussion of the difference between information espionage and technological espionage and their contrasting welfare consequences is presented in Section 4.2, which also reviews existing research on the topic and explains this paper's contribution.

Turning to the model, the game considered here is a quadratic payoff coordination game. In particular, we analyse a beauty-contest game, as well as Bertrand and Cournot duopoly models. In a beauty-contest game, an agent is rewarded for picking an action close to some unobserved fundamental $\theta$, as well as to the other players' actions. This structure concisely captures many real-life economic and social scenarios, such as stock market trading, creation of corporate culture, political activism, and output fluctuations under money supply shocks. It also can be shown that the Bertrand and Cournot games are strategically equivalent to the beauty-contest, with $\theta$ corresponding to a (scaled) unobserved demand shock.

In all of these settings, a more precise estimate of the state of the world will help the player; hence he may wish to acquire information about it. For example, in order to learn about demand, a duopolist may conduct a market survey. In this paper, such “primary” information about the state of the world is modelled as a private noisy $\theta$-centered signal, which both players receive before they take their action. Another way of learning about demand would be getting hold of your competitor's market survey results, which qualifies as industrial espionage. Perhaps, such actions will not be welcomed by the firm who conducted the survey. In this paper, this “secondary” source of information is modelled as a noisy signal of the other

---

$^3$Although it will in most cases reduce the targeted firm's profit, which is why it would not share its demand information voluntarily. See Section 4.4.4 for details.

$^4$This presumption is reexamined in Section 4.4.4, where a case is presented in which the victim of espionage would have shared the targeted information voluntarily.
firm’s “primary” signal.

After listening to their two information sources, the firms take signal-contingent actions and receive payoffs. As shown in Section 4.4.2, it turns out that in equilibrium firms choose actions which are weighted averages of their signals. The weight placed on a signal depends on its precision and on how important it is for the players to align their actions, relative to the importance of matching the state of the world (called “coordination motive” in the paper). Since the espionage signal conveys more information about the other player’s action than about the state of the world, its weight is bigger than its relative precision when players value coordination. However, firms always rely more on the results of independent market research, unless the spying device has perfect precision.

Section 4.4.3 reports welfare results, which confirm the intuition stated in the first paragraphs of this paper; indeed, information espionage is less detrimental to the economy than technology espionage. In fact, the effect of spying precision on aggregate profit and social welfare is always positive in the Bertrand duopoly. In the Cournot case, the precision of intelligence technology is welfare-enhancing if and only if it is greater than the precision of the alternative market survey technology; otherwise a reduction in spying device noise makes the players over-rely on its reports. Section 4.4.4 shows that a Cournot duopolist would always prefer his opponent to have a more noisy spying technology. This should not be surprising: more precise espionage results in more strongly correlated production quantities, which a Cournot duopolist is trying to avoid.

More surprisingly, a Bertrand duopolists would not always like its competitor to have a more precise estimate of its private information, even though their ultimate goal is to align their actions. In particular, if a Bertrand firm’s market survey technology is much noisier than its competitor’s, it would rather they both relied more on the latter’s superior market research. However, the competitor puts more trust in its espionage if its precision improves, thus decreasing its reliance on its market research. As a result, an increase in the competitor’s espionage precision makes both prices noisier and more dispersed, which harms the firm. However, in all cases, it benefits the player to have a more precise espionage technology himself; consequently, he will be willing to pay for noise reduction. In Section 4.5 it is demonstrated that a firm will indeed pay to improve the precision of its espionage technology, unless the alternative market research technology is precise enough, or
unless the coordination motive is too low. Otherwise it will abstain from spying and
g以免合作动机太低。否则它将停止间谍活动，完全依赖于市场调研结果来估计需求。

4.2 Related literature

This paper is related to several research areas. The most closely related strand of

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is costly. As this paper shows, allowing the less informed party to choose the noise level and making them pay for it generates a contrasting result with respect to the information flow. In the equilibrium outcome of this model, a positive finite amount of private information is transmitted, as compared to the full revelation (Bertrand) or no-revelation (Cournot) in Vives (1984) and Gal-Or (1985).

Industrial espionage also has been modelled as a resolution to imperfect information in a dynamic game (Barrachina, Tauman, and Urbano, 2014; Solan and Yariv, 2004). Spying there means obtaining a noisy signal about the opponent’s past move. That approach and the present study share the definition of espionage intensity as the precision of this noisy signal, but differ in the object of spying. This paper is concerned with spying on what others know, and not on what they did; consequently its main focus is a tradeoff between the value of the different information sources available to the players.

Finally, some papers consider industrial espionage in the context of R&D, imitation and technological spillovers. There, a firm spies in order to acquire a cost-saving technology (“process innovation”), or a profit-shifting technology (“product innovation”) already employed by its competitor. The strategy space in these papers is modelled in a variety of ways: In Marjit and Yang (2015), firms have a binary option of either imitating the rival’s technology or investing in their own R&D. Whitney and Gaisford (1999) let players choose the degree of spying effort, which translates into the probability of discovery. Schneider (2008) assumes that espionage is not a choice, since a follower will always try to imitate the leader’s technology, but the latter can take costly effort to protect its innovation. In d’Aspremont and Jacquemin (1988) and Petit and Tolwinski (1997) the firms can either passively receive cost-saving technology spilling over from their competitor, or collude in their R&D efforts. Similarly, in Žigić (2000) and Petit and Sanna-Randaccio (2000) technological spillover is a costless by-product of foreign trade. Some papers embed espionage in the principal-agent framework, e.g. Ho (2008) considers moral hazard and double-crossing problems when hiring a private investigator to collect the competitor’s sensitive information. Another example is Cooper (2001), who models a two-period labour market where workers can translate their former employer’s trade secrets to their new workplace.

In all the above models, the object of espionage – a business rival’s technology – makes it isomorphic to stealing. The implied welfare consequences of economic
spying are hence similar to how the risk of being robbed discourages accumulation of property. The present paper complements these results by focusing on a different type of espionage – spying on the competitor’s information about the state of the world. Shared knowledge of the fundamental is valuable in coordination games, thus such spying in general has a positive effect on aggregate profit. Hence, our analysis highlights that the welfare implications of industrial espionage depend crucially on its object.

### 4.3 The Model

In this paper the interaction between firms is modelled as a quadratic coordination game. Its special cases considered here are beauty-contest and duopoly models. The former is general enough to allow comparison with existing literature (Angeletos and Pavan, 2007; Morris and Shin, 2002; Myatt and Wallace, 2012), while the Bertrand and Cournot models are a natural application for the analysis of industrial espionage.

#### 4.3.1 A Two-player Beauty-Contest

The beauty-contest specification used in this paper is due to Morris and Shin (2002), who based it on Keynes’s famous parable.

In a two-player beauty-contest game, strategies are real numbers $a_i \in R$ for $i = 1, 2$, and payoffs are determined as follows:

$$u_i = -(1 - r)(a_i - \theta)^2 - r(a_i - a_j)^2,$$

where $|r| \leq 1$.

The following subsection shows that Bertrand and Cournot duopoly models can be captured by a beauty-contest framework, where $r \in [0, \frac{1}{2}]$ in Bertrand and $r = -\frac{1}{2}$ in Cournot.

#### 4.3.2 Bertrand Duopoly

Consider a game of Bertrand duopoly with differentiated products and an unknown demand parameter $\theta'$. Both players pick a price $p_i$ and face the demand $q_i = \theta' -
4.3: The Model

$p_i + b p_j$, where $p_j$ is their opponent’s price and $0 < b < 1$. Suppose both firms incur constant marginal cost of $c'$. A firm’s profit is thus

$$\pi_i = p_i q_i - c' q_i = (p_i - c')(\theta' - p_i + b p_j). \quad (4.2)$$

Rescaling the demand parameter $\theta = \frac{\theta'}{2-b}$ and unit cost $c = \frac{c'}{2-b}$ and redefining the choice variable as $\bar{p}_i = p_i - c$ obtains

$$\pi_i = \left(-\frac{b}{2}\right)(\bar{p}_i - \theta)^2 - \frac{b}{2}(\bar{p}_i - \bar{p}_j)^2 + f(\bar{p}_j), \quad (4.3)$$

where

$$f(\bar{p}_j) = \frac{b}{2}(\bar{p}_j - (1-b)c)^2 + \left(1 - \frac{b}{2}\right)(\theta - (1-b)c)^2. \quad (4.4)$$

Observe that the term $f(\bar{p}_j)$ does not contain the choice variable $\bar{p}_j$, i.e. is strategically irrelevant. Hence, the model is equivalent to the beauty-contest for $a_i = \bar{p}_i$ and $r = \frac{b}{2}$.

### 4.3.3 Cournot Duopoly

In a Cournot duopoly game, firms simultaneously and independently choose production quantities of a homogeneous good $q_i$ for $i = 1, 2$. The price of the product is then determined from the market demand, which, as above, includes an unobserved fundamental $\theta'$. Let the firms face constant marginal cost $c'$. A firm’s profit can then be written as follows:

$$\pi_i = p_i q_i - c' q_i = q_i(\theta' - q_i - q_j) - c' q_i. \quad (4.5)$$

Rescaling the demand parameter $\theta = \frac{\theta'}{3}$ and unit cost $c = \frac{c'}{3}$ and redefining the choice variable as $\bar{q}_i = q_i + c$ obtains

$$\pi_i = -\frac{3}{2}(\bar{q}_i - \theta)^2 + \frac{1}{2}(\bar{q}_i - \bar{q}_j)^2 + f'(\bar{q}_j), \quad (4.6)$$

where

$\quad ^5$Here the first inequality means that the firms’ products are imperfect substitutes, whereas the second one ensures that an equal increase in $p_i$ and $p_j$ decreases the demand.
4.4 Basic Case: Costless Spying

The analysis of the game starts with a basic case of exogenous spying precision. In this framework, the firms have no influence on the noise in their espionage technology; they take an action after observing their market research and spying signals, characterised by given variance. In Section 4.5, the case of endogenous precision is considered, where firms can pay to reduce the noise in their spying devices.

4.4.1 Information Structure

The players in the game share a uniform improper prior about the unobserved demand parameter $\theta$, and each receives two noisy signals of its value: a primary signal $x_i$, interpreted as a piece of information he obtained himself via market research, and a secondary signal $\bar{x}_i$ which he obtained spying on the other player’s primary signal. The composition of the signals is as follows:

$$x_i = \theta + \epsilon_i,$$

$$\bar{x}_i = x_j + \phi_i = \theta + \epsilon_j + \phi_i,$$

where $\epsilon_i \sim N(0, \xi^2)$ and $\phi_i \sim N(0, \psi^2)$ for $i = 1, 2$. All noises are distributed independently from one another and from $\theta$. The primary signal includes an idiosyncratic noise term ($\epsilon_i$). If $x_i$ is interpreted as an outcome of market research, $\epsilon_i$ can be thought of as the sampling error of the market survey. The noise term in the secondary signal ($\phi_i$) is generated by the players’ spying devices which have imperfect precision.
### 4.4.2 Equilibrium

Consider a player’s expected utility under model (4.1):

\[
E[u_i|x_i, \bar{x}_i] = -(1 - r)E[(a_i - \theta)^2] - rE[(a_i - a_j)^2] 
\]

(4.10)

\[
= -(1 - r)[a_i^2 - 2a_iE(\theta) + E(\theta^2)] - r[a_i^2 - 2a_iE(a_j) + E(a_j^2)]. 
\]

(4.11)

Differentiating (4.10) yields:

\[
\frac{\partial E[u_i|x_i, \bar{x}_i]}{\partial a_i} = -2a_i + 2(1 - r)E(\theta) + 2rE(a_j). 
\]

(4.12)

Equation (4.10) is strictly concave in \(a_i\), and thus \(i\)’s best response is uniquely determined by (4.13).

\[
a_i(x_i, \bar{x}_i) = (1 - r)E[\theta|x_i, \bar{x}_i] + rE[a_j|x_j, \bar{x}_j]. 
\]

(4.13)

It has been established in the literature that a beauty-contest game has a linear equilibrium which is unique within a large class of strategies (Myatt and Wallace, 2012). Existing research has thus considered the linear case, and this paper follows suit. Suppose a player \(i\) believes that his opponent \(j\) follows a strategy linear in his signals: \(a_j(x_j, \bar{x}_j) = \kappa_1 x_j + \kappa_2 \bar{x}_j\). His best reply then satisfies

\[
a_i(x_i, \bar{x}_i) = (1 - r)E[\theta|x_i, \bar{x}_i] + rE[a_j|x_j, \bar{x}_j]. 
\]

(4.14)

It can be shown (and is indeed demonstrated in the Appendix) that the players’ estimates of the fundamental and the opponent’s signals are as follows:

\[
E_i(\theta|x_i, \bar{x}_i) = \frac{(\xi^2 + \psi^2)x_i + \xi^2 \bar{x}_i}{2\xi^2 + \psi^2}; 
\]

(4.15)

\[
E_i(x_j|x_i, \bar{x}_i) = \frac{\psi^2 x_j + 2\xi^2 \bar{x}_j}{2\xi^2 + \psi^2}; 
\]

(4.16)

\[
E_i(\bar{x}_j|x_i, \bar{x}_i) = x_i. 
\]

(4.17)
Observe that, due to the noise normality assumption, a player’s estimates of $\theta$, $x_j$ and $\tilde{x}_j$ are weighted averages of his signals. In the expectation of the fundamental, information sources are weighted by their precision. In particular, the primary signal always receives at least as big a weight as the secondary signal, because its variance is smaller ($\xi^2 \leq \xi^2 + \psi^2$). By contrast, when forming an expectation of the opponent’s primary signal $x_j$, a player gives twice as much credence to the data provided by espionage as when estimating $\theta$.

Turning to the equilibrium, observe that the best response (4.14) is linear in expectations (4.15)-(4.17), which, in their turn, are linear in the player’s signals. It follows that the equilibrium strategy is a linear combination of a primary and a secondary signal (in fact, a convex combination, as demonstrated in the Appendix). The characterisation of equilibrium use of two information sources constitutes the first result of this paper.

**Proposition 4.1.** In the unique linear equilibrium, players will choose actions which are weighted averages of their signals:

$$a_i = (1 - w)x_i + we \tilde{x}_j \quad \text{for } i = 1, 2, \quad (4.18)$$

where

$$w = \frac{\xi^2(r + 1)}{2\xi^2(r + 1) + \psi^2}. \quad (4.19)$$

**Proof:** in the Appendix.

The equilibrium strategy (4.18) describes how the firms trade off available information sources: industrial espionage and market research. Several observations readily follow. First, the equilibrium weights on the signals depend on their precision and the strength of coordination motive $r$. Moreover, if players’ actions are strategic substitutes ($r > 0$) the weight on the secondary signal is larger than its relative precision, as it carries valuable information about the other player’s action. However, the players always put more weight on the primary signal, unless the spying device has perfect precision. Finally, it can be shown that the weight put on the secondary signal is increasing in the coordination motive.

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6A straightforward interpretation of (4.15) is that each signal is weighted by the variance of the other signal, but the expression can be rearranged into a precision-weighted average.
4.4.3 Welfare Properties of the Equilibrium

This section investigates the welfare properties of the equilibrium in a beauty-contest game and duopoly markets with spying. The result of this exercise will, on one hand, be policy relevant, as it establishes the net effect of information espionage on the industry and the economy as a whole. On the other hand, the analysis in this section contributes to the study of the welfare properties of information use, first addressed in Morris and Shin (2002). In that paper, the authors showed that, strikingly, more precise information about the value of the fundamental can be welfare-reducing. Later, Angeletos and Pavan (2007) identified exact conditions under which the precision of public or private signal can decrease welfare. In particular, they showed that more precise public (private) information can be welfare-reducing in the Cournot (Bétrand) duopoly. The principal novelty of the present paper is a new type of information source – a secondary “spying” signal, which does not fall into either public or private category. Hence, it is not straightforward to extend existing welfare results to this paper’s setting. Would this world be a better place if everyone knew everyone else’s secrets? It turns out that the answer depends crucially on whether the players want to align or misalign their actions. In a game of strategic complementarity, more information is always better. In a game of strategic substitutability, players are better off increasing their reliance on a more accurate source of information. Thus, they will benefit from an improvement in the intelligence technology if and only if it is already superior, in terms of precision, to the alternative market research technology.

Before presenting the welfare analysis of duopoly, the beauty-contest model (4.1) is considered. Recall that the beauty-contest is a general coordination game of either strategic substitutability or complementarity, depending on the sign of coordination motive $r$. Since in the symmetric case $a_i = a_j$, the sum of the agents’ expected utilities in the beauty-contest can be rewritten as a weighted average of two terms:

$$E(U) = E(u_i) + E(u_j) = -2(1-r)\text{Var}(a_i - \theta) - 2r\text{Var}(a_i - a_j).$$ (4.20)

Intuitively, more noise in the spying device prevents the players from learning about each other’s actions and from forming a more precise expectation of the fundamental. As a result, bigger $\psi^2$ increases both volatility $\text{Var}(a_i - \theta)$ and dispersion...
4.4: Basic Case: Costless Spying

Var\((a_1 - a_2)\) of the players’ actions, as is shown in the Appendix.\(^7\) This will reduce welfare when \(0 < r < 1\), as can be clearly seen from (4.20). The effect of spying technology noise in case of strategic substitutability between actions \((-1 < r < 0)\) is ambiguous: (4.20) tells us that in this case volatility is still welfare-reducing, whereas bigger dispersion is beneficial for the players.

**Proposition 4.2.** *Aggregate expected utility in the beauty-contest game increases as the precision of the spying device improves if actions are strategic complements \((r > 0)\).*

Turning to the Bertrand duopoly model (4.3), we would expect any information in the game of strategic complementarity to be welfare-increasing. This is because knowing what the opponent knows results in a more precise and more strongly correlated expectations of the fundamental, hence helping the player to align his action both with \(\theta\) and the other player’s action. The following analysis confirms this intuition. Aggregate expected profit in the Bertrand duopoly game can be written as follows:

\[
E(\Pi) = \overline{E(\Pi)} - 2(1 - r)\text{Var}(a_i - \theta) - 2r\text{Var}(a_i - a_j) + 2r\text{Var}(a_j), \tag{4.21}
\]

where \(\overline{E(\Pi)}\) is the sum of all terms independent from \(\psi^2\).

Recall that a coordination coefficient in the Bertrand duopoly model is positive: \(r \in [0, 0.5]\). The expression (4.21) implies that the firm’s profit is decreasing in the volatility of its price choice \(\text{Var}(a_i - \theta)\) and the the dispersion of prices in the market \(\text{Var}(a_i - a_j)\). The former effect is explained by the firm’s objective to match the true (scaled) value of the demand parameter \(\theta\). The latter is due to the strategic complementarity of firms’ actions under the Bertrand model. The positive effect of a competitor’s price variance is again explained by the profit function of a Bertrand duopolist. Recall that profit is increasing in the opponent’s [positive] action, thus it is also increasing in the expectation of its square. Moreover, in the symmetric equilibrium price variances of the firms are equal: \(\text{Var}(a_j) = \text{Var}(a_i) = \text{Var}(a_i - \theta) + \text{Var}(\theta)\). Hence, (4.21) can be simplified:

\[
E(\Pi) = \overline{E(\Pi)} - 2(1 - 2r)\text{Var}(a_i - \theta) - 2r\text{Var}(a_i - a_j), \tag{4.22}
\]

\(^7\)The terms “volatility” and “dispersion” in the context of coordination games were introduced in Angeletos and Pavan (2004), although our definition does not match theirs exactly.
where \( E(\Pi) = E(\Pi) + 2r \text{Var}(\theta) \).

Observe that a firm’s profit is a convex combination of volatility and dispersion (indeed, \( 1 - 2r \in (0, 1) \) in the Bertrand case). Larger noise in the spying device \( \psi^2 \) increases both volatility and dispersion; hence its inverse (the precision of the signal) has a positive effect on profit.

**Proposition 4.3.** *Aggregate expected profit in the Bertrand duopoly game increases as the precision of the spying device improves.*

Finally, comparative statics in the Cournot model are established. Consider the sum of expected profits in model (4.6):

\[
E(\Pi) = E(\Pi) - 3\text{Var}(a_i - \theta) + \text{Var}(a_i - a_j) - \text{Var}(a_j).
\]  

(4.23)

The interpretation of (4.23) is analogous to the argument above. Profit decreases with volatility, because it results in incorrect predictions of demand. It increases with dispersion, because the firms would like their quantity choices to be far apart. Turning to the last component of (4.23), recall that the Cournot duopolist’s profit is decreasing in its opponent’s action \( a_j \), and hence also in the expectation of its square. In the symmetric equilibrium, (4.23) can be transformed analogously to the Bertrand case:

\[
E(\Pi) = E(\Pi) - 4\text{Var}(a_i - \theta) + \text{Var}(a_i - a_j).
\]  

(4.24)

As stated above, both volatility and dispersion are increasing in the spying noise, hence its effect on profit in the Cournot case is ambiguous. It is shown in the Appendix that the total effect of spying noise depends on the relative precision of information sources: if the intelligence technology is noisier than the market survey technology, then the negative indirect effect of more precise spying (via increased confidence in and hence over-reliance on the noisy spying device) outweighs the positive direct effect (via decreased volatility of actions), and vice versa.

**Proposition 4.4.** *Aggregate expected profit in the Cournot duopoly game increases with the precision of the spying device if and only if the market survey technology is noisier than the spying technology \( \xi^2 > \psi^2 \).*

The intuition behind Proposition 4.4 can be gained by considering the direct and the indirect effects of \( \psi^2 \) on profit. Recall that the equilibrium action \( a_i \) is a weighted
average of noisy signals; hence, the marginal effect of the spying noise $\psi^2$ on $a_i$ is equal to the spying weight $w$. Consequently, the marginal effect of spying noise on the variance of the equilibrium action is its squared weight: $w^2$. The effect of $\psi^2$ on dispersion is twice as big, because dispersion is a variance of the sum of two actions. Overall, the direct effect of spying noise on profit is negative: $-4w^2 + 2w^2 = -2w^2$. Recall that $w^2$ is proportional to $\xi^2$ (the bigger the noise in market research technology, the more trust the firm puts in the espionage technology). Turning to the indirect effect of spying noise on profit, it operates through the player’s equilibrium weight on spying. Naturally, noisier spying device means smaller $w$. In its turn, the effect of $w$ on profit is negative in the Cournot case: if firms put more weight on spying, the resulting increase in volatility is bigger than in dispersion. Moreover, the effect of $w$ on profit is proportional to $\psi^2$, since $w$ enters the profit equation as a weight placed on the spying signal. Hence, if $\xi^2 > \psi^2$, the negative direct effect offsets the positive indirect effect, and overall profit is decreasing in spying noise.

The positive effect of spying on the aggregate profit under price competition can raise concerns about consumer welfare; in particular, is this increase achieved at their expense? In order to complete the analysis, the influence of espionage on consumer surplus needs to be investigated. Consumers benefit from “overproduction” (or “underpricing”) by imperfectly informed firms, and suffer from “underproduction” (or “overpricing”). It turns out that in the Bertrand case the benefit of the former always outweighs the loss implied by the latter, hence less spying and less precise estimate of demand by firms increases the consumer surplus. In the Cournot case, consumers benefit from increasing spying precision until it matches that of market research, after which it becomes welfare-reducing.

**Proposition 4.5.** *Consumer welfare in the Bertrand duopoly game decreases as the precision of the the spying device improves. The same is true in the Cournot duopoly if and only if $\psi^2 > \xi^2$.*

Since the increase in profits due to industrial espionage comes at the consumers’ expense, one might wonder about the total welfare effect of more precise spying. The result of such investigation could inform government legislative action on industrial espionage. As is shown in the Appendix, the effect of espionage on the pro-

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8To be precise, the marginal effect of $w$ on dispersion is less than four times its effect on volatility, hence the total effect is negative.
ducers always outweighs the loss to the consumers, and thus the welfare effects of more precise spying coincide with its profit effects.

**Proposition 4.6.** Spying precision is welfare-increasing if $\xi^2 > \psi^2$ under the Cournot model, and always under the Bertrand model.

Proposition 4.6 provides a case in favour of less stringent regulation of demand espionage, at least for the cases of price competition, or developed existing corporate intelligence practices ($\xi^2 > \psi^2$).

**4.4.4 Asymmetric Spying Technology**

So far it was assumed that the duopolists employ the same spying technology ($\text{Var}(\phi_i) = \text{Var}(\phi_j)$) and established that, in most cases, aggregate profits increase as the precision of the spying device improves. One might wonder if this optimistic result is reversed when one looks at how individual profit responds to changes in an opponent’s intelligence system precision. It turns out that the answer depends on the relative precision of the market survey and espionage technologies, as well as the nature of competition. A Cournot duopolist will always prefer its competitor to spy less, while a Bertrand duopolist will only benefit from closer spying if its market research technology is precise enough. This result seems counterintuitive: since Bertrand duopolists want to coordinate their price choices, they should welcome all forms of information transmission between them. The indirect effect of spying holds the key to our counterintuitive finding. A more precise spying device makes the competitor rely more on its reports, which a firm wouldn’t like to happen if its market survey (the source of the espionage signal) is very noisy. Indeed, the firm would rather the competitor relied more on its own, more precise market research, which would lead to less dispersed actions.

Let us first establish equilibrium actions with differentiated precision of spying. Since $\text{Var}(\phi_i) \neq \text{Var}(\phi_j)$, the spying variance is now indexed with the player’s number: $\text{Var}(\phi_i) = \psi_i^2$.

In equilibrium of the asymmetric espionage game, players will choose an action which is a weighted average of their signals (4.25).

$$a_i = (1 - w_i)x_i + w_i\tilde{x}_i,$$ (4.25)
where the weight on the spying signal is determined as follows:

$$w_i = \frac{2\xi^2(1-r^2) + \xi^2\psi^2_j(1+r)}{4\xi^4(1-r^2) + 2\xi^2(\psi^2_i + \psi^2_j) + \psi^2_i \psi^2_j}. \quad (4.26)$$

It is easy to show that (4.25) collapses to the symmetric equilibrium when $\psi^2_i = \psi^2_j$.

Recall that the firm’s expected profit in the Bertrand and Cournot duopolies is a linear combination of three variance terms (4.27). Note that in the asymmetric case it cannot be simplified since $\text{Var}(a_i) \neq \text{Var}(a_j)$.

The expected profit equation can be recast in terms of the firms’ choice variables $w_i, w_j$ and variances of noise terms:

$$E(\pi_i) = \overline{E(\pi_i)} - (1-r)\text{Var}(a_i - \theta) - r\text{Var}(a_i - a_j) + r\text{Var}(a_j). \quad (4.27)$$

where $r \in [0, 0.5]$ in the Bertrand case, $r = -0.5$ in the Cournot case and $\overline{E(\pi_i)}$ is a sum of terms independent of the firm’s or its competitor’s strategy and thus irrelevant for equilibrium welfare analysis.

The expected profit equation can be recast in terms of the firms’ choice variables $w_i, w_j$ and variances of noise terms:

$$E(\pi_i) = \overline{E(\pi_i)} - (1-r)\left[(1-w_i)^2 \xi^2 + w_i^2(\xi^2 + \psi^2_i)\right] - r\left[(1-w_i) - w_j\right]^2 \xi^2 + \left(w_i - (1-w_j)\right)^2 \xi^2 + w_i^2 \psi^2_i + w_j^2 \psi^2_j + r\left[(1-w_j)^2 \xi^2 + w_j^2(\xi^2 + \psi^2_j)\right],$$

where $\overline{E(\pi_i)}$ is the sum of terms independent of $\psi^2_j$.

Recall that the main focus of this section is the marginal effect of the opponent’s spying noise $\psi^2_j$ on the firm’s equilibrium expected profit (hereafter $\text{EEP}_i$). Observe that the opponent’s spying noise enters a player’s profit directly, as well as via the equilibrium weights $w_i$ and $w_j$:

$$\text{EEP}_i(\psi^2_j) = E(\pi_i)\left(w_i(\psi^2_j), w_j(\psi^2_j), \psi^2_j\right). \quad (4.28)$$

The total change in the equilibrium expected profit is the sum of the direct effect and two indirect effects, through the firm’s and its opponent’s equilibrium weights respectively:
\[ \frac{d \text{EEP}_i}{d \psi^2_j} = \frac{\partial E(\pi_i)}{\partial \psi^2_j} + \frac{\partial E(\pi_i)}{\partial w_j} \frac{\partial w_j}{\partial \psi^2_j} + \frac{\partial E(\pi_i)}{\partial w_j} \frac{\partial w_j}{\partial \psi^2_j}. \] (4.29)

By the envelope theorem, the indirect effect via the choice variable \((w_i)\) is zero, and (4.29) can be simplified:

\[ \frac{d \text{EEP}_i}{d \psi^2_j} = \frac{\partial E(\pi_i)}{\partial \psi^2_j} + \frac{\partial E(\pi_i)}{\partial w_j} \frac{\partial w_j}{\partial \psi^2_j}. \] (4.30)

In the Appendix it is established that for both Bertrand and Cournot cases, (i) the precision of the opponent’s spying has no direct effect on profit; and (ii) the noisier the espionage technology, the less trust the opponent puts in its reports: \(\frac{\partial w_j}{\partial \psi^2_j} < 0\). The effect of the opponent’s espionage weight on the firm’s profit \(\frac{\partial E(\pi_i)}{\partial w_j}\) is negative in the Cournot case and variable in the Bertrand case. Consequently under the Cournot duopoly the total effect of the opponent’s spying precision (equal to the indirect effect) is negative.

**Proposition 4.7.** A firm’s profit in the Cournot duopoly decreases with the precision of the opponent’s spying device.

Turning to the Bertrand duopoly, the opponent’s spying precision is profit-increasing if the noise in the firm’s own spying technology is sufficiently high, compared to its market research technology (Figure 4.1). In the special case where the precision of both espionage technologies is the same \((\psi^2_i = \psi^2_j)\), the opponent’s spying precision is always profit-increasing in Bertrand. This result is formally presented in Proposition 4.8.

**Proposition 4.8.** A firm’s profit in the Bertrand duopoly increases with the precision of the opponent’s spying device if and only if \(\xi^2_j > \frac{\xi^2_i}{\xi^2_i} - 0.5\).

In the shaded area of Figure 4.1, the firm’s market research is so noisy that it would rather the competitor did not listen to it too much via the spying device. Improved precision of spying implies increased reliance on its report and hence noisier and more dispersed price choices, which a Bertrand duopolist wants to avoid.

According to Propositions 4.7-4.8, more precise spying by the firm’s opponent can be profit-reducing. One might reasonably ask if the same ambiguity holds for the firm’s own spying activities: will it always prefer a clearer signal of its competi-
4.4: Basic Case: Costless Spying

Figure 4.1: Under the Bertrand model, the opponent’s spying precision is profit-decreasing in the shaded area.

A firm’s profit in the Bertrand and the Cournot duopoly increases as the precision of its spying device improves.

Proposition 4.9 claims that a player would always like to know what his opponent knows, whether he is trying to align or misalign their actions. Indeed, either goal is facilitated by a more precise estimate of the opponent’s action, which results from a less noisy espionage technology.

Turning to the general beauty-contest game, if the actions are strategic complements ($r > 0$), then the player’s payoff increases both with his own and his opponent’s spying precision. This is because the players’ incentives are perfectly aligned, so if one of them possesses clearer information he will use it to their mutual advantage. However, if the actions are strategic substitutes ($r < 0$), a more precise espionage technology can harm the player through its indirect effect on payoff. Indeed, an improvement in the player’s spying precision can increase his opponent’s reliance on his own spying (in order to avoid being a less informed and hence disadvantaged party), which, in turn, implies a more strongly correlated estimate of the fundamental $E(\theta)$ and more closely aligned actions, which is precisely what the player want to avoid when $r < 0$. By the same token, a player is harmed by the other player’s more precise spying device in the beauty-contest game of strategic substitutability.
Proposition 4.10. If the actions in a beauty-contest game are strategic complements, a player’s payoff increases as the precision of either his own or the other player’s spying technology improves. If the actions are strategic substitutes, the player’s payoff decreases with the precision of the other player’s spying technology.

4.5 Costly Industrial Espionage

In the previous section it was shown that a more precise espionage technology always benefits the duopolist. The firm may thus be willing to pay for noise reduction. This section drops the assumption that the precision of spying is exogenously given at \( 1/\psi^2 \) and explores how duopolists balance the cost of spying against its benefit. It turns out that firms will only acquire spying services if their per unit cost is sufficiently low (equivalently, if the coordination motive \( r \) is strong enough or if the exogenous noise in spying \( \psi^2 \) is sufficiently low, compared to the noise in market research \( \xi^2 \)). Otherwise, the duopolists will rely on their independent market research only.

Let the noise in the espionage technology be distributed as follows:

\[
\phi \sim N\left(0, \frac{\psi^2}{z}\right),
\]

where \( z \) is the volume of spying activities. One concrete interpretation of \( z \) is the number of documents stolen from the competitor, or sample size. The spying signal \( \tilde{x}_i \sim N(x_j, \psi^2/z) \) can then be viewed as the unbiased estimator of the other firm’s primary signal \( x_j \). Note that \( \psi^2/z \) is sampling variance, which, as is well known, is inversely proportionally related to sample size for most widely used estimators.

If espionage was free, the firm would choose its maximum possible amount in order to perfectly observe their competitor’s private information. However, if stealing the other firm’s commercial secrets is costly, the benefits from more precise spying should be traded off against their price. In this section, the basic case is considered when the costs of spying are linear: \( C(z) = \text{const} \cdot z \); this could be the case if the firm is using an external organisation’s services for spying on its competitors, with fixed price per spying job. It has been reported that businesses indeed prefer to delegate their grey corporate intelligence to third parties in order to mitigate legal risks in case of a revelation. In a report on the ethics of industrial espionage, Crane (2003)
observed, “Companies who want to engage in dubious practices simply contract the work out to independent operators.”

Informal accounts of industrial espionage quote both the costliness of economic spying and the widespread practice of its outsourcing:

*Corporate surveillance is out there, with an estimated £11 million a year spent on bugging devices alone in the UK. A long string of subcontracted security firms is not enough to keep a company’s name out of the headlines, so firms need to be sure of their legal ground regardless of who is actually doing the work for them. If they don’t they could find themselves, like Hewlett Packard, with the information that they want, but at a very high price.*

Now that the firms decide not only on their action but also on the amount of spying they would like to purchase, the game structure becomes more elaborate:

- **Step 1.** The firms $i = 1, 2$ choose amounts of espionage they would like to engage in: $z_i \in R^+$.  
  
- **Step 2.** Each firm observes a “primary” ($x_i = \theta + \epsilon_i$) and a “secondary” ($\bar{x}_i = x_j + \phi_i$) noisy signal about the value of the market fundamental $\theta$. The precision of the primary signal is exogenously given: $\epsilon_i \sim N(0, \xi^2)$, while the amount of purchased spying determines the precision of the secondary signal: $\phi_i \sim N(0, \frac{\psi^2 z_i}{z_j})$.

- **Step 3.** The firms take signal-contingent actions $a_i \in R$. In the context of duopoly models, which are strategically equivalent to beauty-contest, $a_i$ is either price (Bertrand) or output (Cournot).

- **Step 4.** Payoffs are obtained according to the the payoff function of the game minus espionage costs $c z_i$. For example, the final payoff in the beauty-contest is determined as follows:

$$u_i = -(1 - r)(a_i - \theta)^2 - r(a_i - a_j)^2 - c z_i,$$

where $r \in (-1, 1)$ is a coordination motive and $c \in R_+$ is a price per “spying job” charged by a subcontracted intelligence firm.

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9http://www.out-law.com/page-7385
As argued in previous sections, the linear equilibrium in beauty-contest is unique under relatively mild restrictions. Moreover, recall that for the basic asymmetric game defined in section 4.4.4, in the unique linear equilibrium the actions are convex combinations of signals. In the game considered here, the action picking stage (Step 3) coincides with the game in section 4.4.4 (for $\psi_i^2 = \psi_j^2 \mid z_i$). It follows that the equilibrium actions will have the same structure, i.e. the coefficients will sum up to one. Hence they are denoted by $w_i$ (weights):

$$a_i = (1 - w_i)x_i + w_i\bar{x}_i.$$  

(4.33)

Thus, a firm’s strategy in the game is a pair $(z_i, w_i) \in R_+ \times [0, 1]$, where $z_i$ is its chosen amount of spying activities and $w_i$ is the weight it puts on the spying signal when picking its final action (i.e. output or price).

The information structure of the signals implies that $E(a_i) = E(a_j) = \theta$. The expectation of a player’s expected payoff (4.32) can then be recast in terms of weights and noise terms:

$$E(u_i) = -(1 - r)\text{Var}[(1 - w_i)e_i + w_i\epsilon_j + w_i\phi_j] - r\text{Var}[(1 - w_i - w_j)e_i + (w_i - 1 + w_j)e_j + w_i\phi_i - w_j\phi_j] - cz_i.$$  

(4.34)

Expanding variances by making use of noise term distributions obtains:

$$E(u_i) = -(1 - r)\left[\xi^2(1 - 2w_i + 2w_i^2) + w_i^2\frac{\psi_i^2}{z_i}\right] - r\left[\xi^2((1 - w_i - w_j)^2 + (w_i - 1 + w_j)^2) + w_i^2\frac{\psi_i^2}{z_i} + w_j^2\frac{\psi_j^2}{z_j}\right] - cz_i.$$  

(4.35)

Expression (4.35) is obviously not defined when $z_i = 0$ or $z_j = 0$; the noise term in this case has an infinite variance and the spying signal is uninformative. However, if a profit-maximising firm chose not to purchase any spying ($z_i = 0$), it will put zero weight on the spying signal ($w_i = 0$), because a positive weight will result in an infinite loss. Also note that if $w_i = 0$, then $a_i = x_i$, and expected utility (4.34) is well-defined. Intuitively, if a firm chooses not to spend any resources on spying it

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10To be precise, linear equilibrium is unique if the player’s actions $a_i = a_i(x_i, \bar{x}_i)$ are bounded.
will not obtain the secondary signal at all; its estimate of $\theta$ and $a_j$ are thus equal to its primary signal, and so is its best reply, which is their weighted average.

It can be shown that the payoff (4.35) is concave; hence, a first-order approach can be used to find the mutual best response in the game. As is demonstrated in the Appendix, the game has a unique linear equilibrium, characterised in the following proposition.

**Proposition 4.11.** In the unique linear equilibrium of the costly espionage game, either both or neither firm will engage in spying. In particular, if $\xi^2(1 + r) < \psi \sqrt{c}$ (spying is too expensive, or not precise enough compared to market research, or the coordination motive is not strong enough), then $z_i = w_i = 0$ for $i = 1, 2$. If the converse is true ($\xi^2(1 + r) > \psi \sqrt{c}$), then the equilibrium consists of positive espionage policy:

$$
z_i = \frac{\xi^2 \psi (1 + r) - \psi^2 \sqrt{c}}{2 \xi^2(1 + r) \sqrt{c}},
$$

and actions which are weighted averages of the market research and spying signals:

$$
a_i = \frac{\xi^2(1 + r) + \psi \sqrt{c}}{2 \xi^2(1 + r)} x_i + \frac{\xi^2(1 + r) - \psi \sqrt{c}}{2 \xi^2(1 + r)} \tilde{x}_i.
$$

**Proof:** In the Appendix.

According to Proposition 4.11, the players will only purchase spying (pick $z_i > 0$) if its per unit cost is sufficiently low (equivalently, if the coordination motive $r$ is strong enough or if the exogenous noise in spying $\psi^2$ is sufficiently low, compared to the noise in market research $\xi^2$). Otherwise, the firms will choose not to engage in costly espionage, and fully rely on their market research instead: $a_i = x_i$.

If the firms engage in spying activities (pick $z_i > 0$), their equilibrium strategies are a weighted average of their signals. The weights depend on the precision of the signals (for the spying signal, this precision is endogenously determined as $z_i/\psi^2$ and the strength of coordination motive $r$.

Proposition 4.11 also implies that the firms will always put bigger weight on their market research signal, unless spying has perfect exogenous precision ($\psi^2 = 0$) or is costless ($c = 0$). The amount of spying purchased $z_i$ and its equilibrium weight $w_i$ are increasing in the strength of coordination motive $r$. Indeed, the stronger the players’ desire to coordinate, the greater their interest in their opponent’s private
information. The weight put on the spying signal is decreasing in $\psi^2$, because a noisier information source is trusted less.

### 4.5.1 Welfare Properties of Costly Espionage

This section establishes the welfare effects of costly spying. In particular, it is interesting to see how the cost of spying affects social welfare, as this is the parameter of the model indirectly influenced by government policy on industrial espionage. Indeed, if illegal corporate intelligence is outsourced, then the toughening of anti-espionage legislation should raise the price of these services.

Similarly to the costless spying framework, it can be shown that under price competition, aggregate profit and social welfare always decrease with the cost of spying. The same is true under quantity competition if and only if the market survey technology is less accurate than the industrial espionage technology, as summarised in Propositions 4.12-4.13.

**Proposition 4.12.** *In the Bertrand duopoly game with costly espionage, aggregate profit and social welfare are increasing in the precision of the intelligence technology and decreasing in the cost of spying.*

Firms competing in price are better off when more information about market conditions is shared, because it lets them align their prices more closely. Similarly to the baseline costless case, the profit increase here comes at the consumers’ expense. However, their loss is fully offset by the benefit to the producers.

**Proposition 4.13.** *In the Cournot duopoly game with costly espionage, aggregate profit and social welfare increase with the precision of the spying device and decrease with the cost of spying if and only if the intelligence technology is more accurate than the alternative market survey technology.*

The intuition behind Proposition 4.13 is as follows. The firm would benefit from relying on a superior (i.e. more accurate) information source, and lose out from trusting the inferior source. In equilibrium, the firm increases its reliance on the signal if its precision goes up. Hence, the marginal effect of an inferior source’s increased accuracy is negative.

Proposition 4.13 implies that social welfare troughs at the point where the information sources are equally (im)precise, but grows as $\psi^2 \psi z$ moves away from the
equality point in either direction. This raises a question of socially optimal noise (and cost) of spying. As is shown in the Appendix, welfare is the same under costless espionage ($c = 0$), and the regime where the cost is big enough to discourage the duopolists from spying ($c \geq \xi^4(1 + r)^2/\psi^2$). Note that by completely deregulating industrial espionage the central planner can only reduce $c$ to its non-negative minimum – the competitive price of corporate intelligence. Hence, from the practical point of view, it is easier to approach maximum social welfare from the high-cost end, i.e. by manipulating the strictness of anti-espionage legislation in order to achieve $c = \xi^4(1 + r)^2/\psi^2$.

### 4.6 Conclusion

This paper models industrial espionage in a coordination game with information acquisition. In equilibrium firms’ actions are weighted averages of the signals provided by their independent market research and by espionage. Simultaneous improvement in the firms’ intelligence technology always raises aggregate profit and social welfare under the Bertrand model, and has the same effect under the Cournot model if the market research technology is noisier than the espionage technology. A firm’s profit always decreases with the precision of its opponent’s spying device under quantity competition, and increases under price competition if the firm’s own corporate intelligence is insufficiently precise. A firm will pay to reduce the noise in its spying technology if it is affordable enough (equivalently, if the coordination motive is strong enough or if the market research precision is low enough). Otherwise it will not spy at all and will fully rely on its market research instead.
Chapter 5

Conclusion

This thesis demonstrated the power of Game Theory in modelling social interaction and market competition.

Chapter 2 examined the agents’ motivation to cooperate in repeated Prisoner’s Dilemma. Using a combination of theoretical and experimental methods, it showed that the main “other-regarding” motivation affecting the agents’ behaviour in the game is inequality aversion, and not altruism, as was previously believed. Another intriguing laboratory finding reported in Chapter 2 is that players who can best-respond to a pre-specified strategy are also more cooperative. This finding runs counter to conventional “bounded rationality” explanations of cooperation, and definitely deserves future investigation.

Chapter 3 explored a trade-off between material and moral incentives faced by decision-makers. It proposed a logarithmic utility function over money and “guilt”, where the latter is proportional to harm inflicted on some other agent. A novel utility function is deduced from realistic assumptions about a decision-maker’s moral compass. Consequently, it has a greater predictive power than the currently used linear model and can better explain observed play in popular experimental games.

Chapter 4 investigated a duopoly market with industrial espionage. Drawing on the established literature in information acquisition (Angeletos and Pavan, 2007; Morris and Shin, 2002; Myatt and Wallace, 2012), it modelled the content of espionage as the noisy signal of a competitor’s private information. The main contribution of this chapter is the assessment of the welfare consequences of information espionage. Unlike intellectual property theft, spying on another firm’s information is welfare-enhancing in most environments. However, a firm would typically be disadvantaged by a unilateral improvement in its competitor’s spying precision, which means the incentives for voluntary information sharing are weak. In this case, industrial espionage is beneficial for industry and for the economy.

The main methodological tool employed in this thesis is equilibrium analysis.
Conclusion

Although this approach is theoretical in nature, the resulting findings can be applied to diverse real-life strategic situations. In particular, Chapter 2 predicts how the longevity of a business partnership is affected by the fairness of the division of total profit. The utility model developed in Chapter 3 quantifies the price of a clear conscience, which can be employed to estimate the amount of charity donations. Finally, the analysis in Chapter 4 can be used to advise government on whether a blind eye should be turned to industrial espionage.
Appendices
Appendix A

Appendix to Chapter 2

A.1 Instructions for Experimental Subjects (Exit Option Treatment)

Welcome!

Thank you for taking part in this experiment.

You are not allowed to communicate during this experiment. All mobile phones are to be switched off, and all of your belongings are to be stored under the desk.

If you have any questions at any time, raise your hand and the experimenter will come to your desk.

At the end of the experiment you will be paid £2 for turning up plus whatever you earn in the experiment. You can earn a considerable amount of money depending on your choices. So, it is in your interest to pay attention to the instructions.

Decisions You Will Need to Make

The experiment will consist of a series of rounds. At every round, you will be paired with another player in the room via computer network. This pairing is anonymous - your identity will not be revealed at any point during or after the experiment.

At every round, you will be asked to choose between two options: A and B. To make a choice, click next to your chosen option and press the “OK” button. The player you will be paired with will have the same options. Both you and the other player will make your choices simultaneously and not knowing what the other have chosen. After you both make your choice, you will earn points, depending on your and the other player's action.

You will be randomly assigned a role of either a red or a blue player. You will stay in this role throughout the experiment. All other participants will also be assigned
a role of either a blue or a red player. That is, half of the people in this room will be blue players and the other half will be red players.

If you are a blue player, you will always be paired with a red player. If you are a red player, you will always be paired with a blue player. The points you earn in each round depend on your and the other player's choice, as summarized in the table below. The points which a blue player earns are in blue, in the first position of every cell. The points which a red player earns are in red, in the second position.

<table>
<thead>
<tr>
<th>Blue player's action</th>
<th>Red player's action</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5, 5</td>
</tr>
<tr>
<td>B</td>
<td>9, 0</td>
</tr>
</tbody>
</table>

• If both players choose A, the blue player will earn 5 points, and the red player will earn 5 points.

• If both players choose B, the blue player will earn 3 points, and the red player will earn 3 points.

• If the blue player chooses A and the red player chooses B, the blue player will earn 0 points, and the red player will earn 9 points.

• If the blue player chooses B and the red player chooses A, the blue player will earn 9 points, and the red player will earn 0 points.

The points you earn in each round will be summed up, converted to cash and paid to you confidentially at the end of the experiment. The conversion rate is 100 points = £1.40.

The experiment consists of 20 sets of rounds. A set is a series of rounds during which you are paired with the same participant. Each set consists of 6 rounds. That is, you will play a total of 120 rounds, but after every 6 rounds, you will be re-matched with a new player. The matching is done randomly by a computer at the beginning of each 6-round set. If you are a blue player, the computer will randomly pick one of the red players in the room and match him or her with you. If you are a red player, the computer will randomly pick one of the blue players in the room and match him or her with you. You and this partner will play 6 rounds and then
be unpaired at the end of the set. The points you and the other player earn in each round will always be determined by the table on the previous page.

After each round, you will be told your decision, the other player’s decision and how many points you both earned. The information about your choices and earnings in the current set will be displayed on the screen. At the end of the experiment you will be informed about your total earnings.

**An Option to Cut the Set Short**

In the last 10 sets of the experiment, you might be given an option to cut a 6-round set short before all 6 rounds are played. After both you and the other player make your choices, the message will appear telling you how many rounds out of 6 are still left to play with the same participant, and asking if you would like to terminate the set and receive some amount of points instead of playing the remaining rounds. This amount of points, which is the same for you and the other player, will be displayed on the screen before you make your decision.

If you chose to terminate the set, you will **not** play the remaining rounds of the set. You and the other player will both get paid the stated amount and then wait to be rematched with different participants for a new 6-round set. Note that if you are given the termination option, the other player will not be asked whether they want to terminate the set; the decision is yours.

If you choose to go on with the set, the game will continue as before and you and the same player will finish the remaining rounds. In that case, the other player will not even know that you were given an option to terminate the set.

Similarly, the player you are matched with might be offered to terminate a set at some point. If he or she chooses to terminate it, you will be informed about it, and then you will both get paid the same amount of points and wait to be rematched with different players. If he or she chooses to go on with the set, you will not know that he or she was offered a decision to terminate it.

**After the Experiment**

At the end of the experiment, after you finish playing 20 sets of rounds with other participants, you will play one extra 6-round set with a computer. The computer is programmed to follow a simple strategy which will be explained to you on the screen before you start the final 6-round set.
After you finish playing the final set of rounds with a computer, you will be asked to fill in a short questionnaire. Your answers to this questionnaire are completely anonymous and have no effect on the amount of money you will be paid at the end of the experiment.

If you have a question please raise your hand and an experimenter will come to your desk. Please do not ask any questions out loud.

Thank you and good luck!
Appendix B

Appendix to Chapter 3

B.1 Representation Theorems

In order to establish Theorems 3.2 and 3.5, we prove several auxiliary results, which allow us to construct new indifference sets from existing ones. This follows from continuity of $\succeq$ and Axioms 3.1-3.3, which describe how given indifference sets are related to one another.

We first establish Lemmas 3.4-B.3 which are used in the proof of Theorem 3.5.

Our first lemma demonstrates that the indifference sets of $\succeq$ satisfying Axiom 3.3 are related by parallel displacements along the guilt axis.

Proof of Lemma 3.4. First, observe that Axiom 3.3 implies that the PCC is constant for moral dilemmas with fixed amounts of money: $\frac{m_2-m_1}{G_2-G_1} = \frac{m_2-m_1}{G_2-G_1}$ for any $(m_1, G_1) \sim (m_2, G_2)$ and $(m_1, G_1') \sim (m_2, G_2')$.

Second, we prove the symmetric ($\sim$) part of the lemma. Consider $(m_1, G_1) \sim (m_2, G_2)$. WLOG, let $m_2 > m_1$ (then $G_2 > G_1$ by monotonicity). Suppose $(m_1, G_1 + a) \not\sim (m_2, G_2 + a)$. First, suppose $(m_1, G_1 + a) \succ (m_2, G_2 + a)$. By monotonicity, $(m_2, G_2 + a) \succ (m_1, G_1 + a)$. Since by continuity the lower and the upper contour sets of $(m_2, G_2 + a)$ are closed, their intersections with a closed connected interval $\{(m_1, G) : G \in [G_1 + a, G_2 + a]\}$ are also closed. Moreover, they are non-empty, since the ends of the interval belong to the upper and the lower contour sets respectively, as shown above. Recall that a connected space cannot be divided into two disjoint non-empty closed sets, hence there is a point $(m_1, G_3)$, with $G_1 + a < G_3 < G_2 + a$ (*) which belongs to both intersections, i.e. $(m_1, G_3) \sim (m_2, G_2 + a)$. Hence we have two moral dilemmas which, by Step 1, should entail the same price of clear conscience: $\frac{m_2-m_1}{G_2-G_1} = \frac{m_2-m_1}{G_2+a-G_3}$ which implies $G_3 = G_1 + a$, a contradiction to (*). Now suppose that $(m_1, G_1 + a) \prec (m_2, G_2 + a)$. There are two cases to consider. If $a > 0$, then $(m_2, G_2 + a) \prec (m_2, G_2) \sim (m_1, G_1)$. Thus, $(m_1, G_1) \succ (m_2, G_2 + a) \succ (m_1, G_1 + a)$. By connectedness and con-
Step 1, (**). Thus, our supposition was wrong, and continuity again, there exists a point \((m_1, G_4)\) such that \((m_2, G_2 + a) \sim (m_1, G_4)\), where \(G_1 + a > G_4 > G_1\) (**) . Then by Step 1, \(G_2 - G_1 = G_2 + a - G_1\), i.e. \(G_4 = G_1 + a\), which contradicts (**). The case when \(a < 0\) is demonstrated analogously.

Third, we prove the asymmetric (> part of the lemma. Let \((m_1, G_1) > (m_2, G_2)\). We need to show that \((m_1, G_1 + a) > (m_2, G_2 + a)\). Case 1. \(G_1 > G_2\) (hence \(m_1 > m_2\)) and \(a > 0\). Suppose, conversely, that \((m_1, G_1 + a) \leq (m_2, G_2 + a)\). By monotonicity and transitivity, \((m_1, G_1 + a) \leq (m_2, G_2 + a) < (m_2, G_2) < (m_1, G_1)\). By connectedness of \([(m_1, G) : G \in [G_1, G_1 + a]]\) and continuity of \(\succeq\), there exists such \(G_3\) that \((m_2, G_2 + a) \sim (m_1, G_3)\) and \(G_1 < G_3 \leq G_1 + a\) (*). Now consider \((m_2, G_2)\). By connectedness and continuity again, there exists \(G_4\) such that \((m_2, G_2) \sim (m_1, G_4)\) and \(G_1 < G_4 < G_1 + a\). By Step 1, \(G_4 - G_2 = G_3 - (G_2 + a)\) and thus \(G_3 = G_4 + a > G_1 + a\), but by (*) \(G_3 \leq G_1 + a\), which is a contradiction. Thus, our supposition was wrong, and \((m_1, G_1 + a) > (m_2, G_2 + a)\).

The remaining three cases are demonstrated analogously.

In order to be able to construct new indifference sets from existing ones by proportional expansion along the money axis, we need to prove existence of an indifference curve passing through any two money levels, which is demonstrated in the following lemma.

**Lemma B.1.** If a rational, monotone and continuous \(\succeq\) satisfies Axiom 3.3, then for any \((m, G)\) and any \(m' > m\), there exists \(G'\) such that \((m, G) \sim (m', G')\).

**Proof.** The logic of the proof follows Hara, Segal, and Tadelis (1997, p.3-6), and relies on continuity of \(\succeq\), connectedness of \(\mathcal{M}\) and quasilinearity in guilt which follows from Axiom 3.3.

First, we show that for two arbitrary levels of money \(m_1\) and \(m_2\), where \(m_1 > m_2\), one of the following is true: (i) for any \(G_2\) there exists such \(G_1\) that \((m_2, G_2) \sim (m_1, G_1)\) (ii) there does not exist such \(G_1\) for any \(G_2\). In the former case, we write \(m_1 \succ m_2\). In the latter case, we write \(m_1 \not\succ m_2\). Suppose there exists \(G_2\) such that we cannot find \(G_1\) to make an agent indifferent between \((m_1, G_1)\) and \((m_2, G_2)\). Note that it implies that \((m_1, G) > (m_2, G_2)\) for all \(G\). Indeed, suppose there exists such \(G_3\) that \((m_1, G_3) \leq (m_2, G_2)\). Observe that \((m_1, G_2) > (m_2, G_2) \geq (m_1, G_3)\). Then, by connectedness of \([(m_1, G) : G \in [G_2, G_3]]\) and continuity of \(\succeq\), there exist \(G_4 \in (G_2, G_3)\) such that \((m_2, G_2) \sim (m_1, G_4)\), which contradicts our assumption. Thus, \((m_1, G) > (m_2, G_2)\) for all \(G \in \mathcal{G}\), in particular, \((m_1, G_2 + G' - G'') > (m_2, G_2)\) for any \(G' > G''\) (so
that $G_2 + G' - G'' \in \emptyset$). By lemma 3.4, it implies that $(m_1, G') > (m_2, G'')$. If $G' \leq G''$, then $(m_1, G') > (m_2, G'')$ by monotonicity. We have just shown that $(0, m_1)$ consists of two disjoint subsets: $\{ m \in \{0, m_1\} : m \succ m \}$ and $\{ m \in \{0, m_1\} : m \not\sim m \}$.

Second, we show that for any $m_1$ the set of all $m \in (0, m_1)$ such that $m \succ m$ is open in $(0, m_1)$. Denote such set as $E(m_1) \equiv \{ m \in (0, m_1) : m \succ m \}$. Consider $m_2 \in E(m_1)$ and some $G_2 > 0$ and $G_1$ such that $(m_2, G_2) \sim (m_1, G_1)$, where $G_1 > G_2$ by monotonicity. Such $G_1$ exists by the definition of $E(m_1)$. Pick a value $\varepsilon \in (0, \min\{m_2, m_1 - m_2, G_2\})$. Such $\varepsilon$ can always be found, because $m_2 \in \mathbb{M} = R^{++}$. Observe that $(m_2 - \varepsilon, G_2) < (m_2, G_2) < (m_2, G_2 - \varepsilon)$. A line segment between the points $(m_2 - \varepsilon, G_2)$ and $(m_2, G_2 - \varepsilon)$ is a closed connected interval, hence by continuity of $\succ$ there exists $(m_3, G_3)$ on the line segment such that $(m_3, G_3) \sim (m_2, G_2)$ and $m_3 > m_3 > m_2 - \varepsilon$. Define $\delta \equiv m_2 - m_3$. Now, for all $m$ such that $m_3 < m < m_2$, it holds that $(m, G_3) > (m_1, G_1) > (m, G_1)$, where the first relation follows from transitivity: $(m_1, G_1) \sim (m_2, G_2) \sim (m_3, G_3) < (m, G_3)$. Hence, by continuity and connectedness again, there exists $G_4$ for all $m$ such that $(m_1, G_1) \sim (m, G_4)$. Also, for all $m \in (m_2, m_2 + \delta)$ it holds that $(m, G_2) > (m_1, G_1) > (m, G_1)$, where the second relation follows from the fact that $\delta < \varepsilon < m_1 - m_2$. Thus, there exists $G_5$ for all such $m$ so that $(m_1, G_1) \sim (m, G_5)$. To summarize, for any $m_2 \in E(m_1)$, all the points $m$ such that $|m_2 - m| < \delta$ also belong to $E(m_1)$, which means $E(m_1)$ is open in $(0, m_1)$. Moreover, $E(m_1)$ is non-empty for all $m_1$. Indeed, consider any point $(m_1, G)$. Its upper and lower contour sets are closed by continuity and non-empty (e.g. they contain points $(m_1 + 1, G)$ and $(m_1, G + 1)$ respectively), thus their intersection is non-empty, i.e. $\exists (m', G')$ such that $(m_1, G) \sim (m', G')$.

Third, we show that for any $m_1$ the set of all $m \in (0, m_1)$ such that $m \not\sim m$ is open. Recall that, by Step 1, this set can denoted as $(0, m_1) \setminus E(m_1)$. Note that $(0, m_1) \setminus E(m_1)$ is equal to the union of such $E(m)$ that $m \not\in E(m_1)$. Indeed, for any $m \in (0, m_1) \setminus E(m_1)$, it is true that $m \in E(m)$. Conversely, for any $m \not\in (0, m_1) \setminus E(m_1)$, it is true that $m \in E(m_1)$. Thus, there does not exist such $\overline{m}$ that $\overline{m} \in (0, m_1) \setminus E(m_1)$ and $m \in E(\overline{m})$. As a union of open sets, $(0, m_1) \setminus E(G_1)$ is open itself.

Finally, since $(0, m_1)$ is connected for all $m_1 \in \mathbb{M}$, it cannot be separated into two disjoint non-empty open sets, hence we conclude that the set $(0, m_1) \setminus E(m_1)$ is empty, thus, $m_1 \not\sim m$ for all $m \in (0, m_1)$, q.e.d.

We can now use Lemma B.1 to show that indifference sets of $\geq$ satisfying Axiom 3.3 are related by proportional expansion along the money axis. This is established
Similarly to the asymmetric part of Lemma 3.4, this proof consists of 4 cases. However, in the rest of the Appendix we will only be using the result for one case:

\[ \parallel_{G} \]

first of these shows that indifference curves of \( m \) and \( G \) whenever \( m_{1}, G_{1} \sim (m_{2}, G_{2}) \) and \( m_{1}, G_{1} \succ (m_{2}, G_{2}) \)

whenever \( (m_{1}, G_{1}) \succ (m_{2}, G_{2}) \) for any \( b \in R_{+} \).

**Proof.** First, we prove the symmetric part of the lemma. Consider \((m_{1}, G_{1}) \sim (m_{2}, G_{2})\). WLOG, let \( m_{1} > m_{2} \). By Lemma B.1, \( \exists G_{3} \) so that \((b m_{2}, G_{2}) \sim (b m_{1}, G_{3})\). By Axiom 3.3, 

\[
\frac{b m_{1} - b m_{2}}{G_{3} - G_{2}} = b,
\]

which implies that \( G_{3} = G_{1} \). Hence, \((b m_{1}, G_{1}) \sim (b m_{2}, G_{2})\).

Second, we prove the asymmetric part of the lemma. Let \((m_{1}, G_{1}) \succ (m_{2}, G_{2})\). **Case 1:** \( m_{1} > m_{2} \). By Lemma B.1, \( \exists G_{3} \) and \( G_{4} \) so that \((m_{2}, G_{2}) \sim (m_{1}, G_{3})\) and \((b m_{2}, G_{2}) \sim (b m_{1}, G_{4})\). By monotonicity, \( G_{1} < G_{3} \). By Axiom 3.3, 

\[
\frac{b m_{1} - b m_{2}}{G_{3} - G_{2}} = b,
\]

which implies that \( G_{4} = G_{3} \). Hence, by monotonicity, \((b m_{2}, G_{2}) \sim (b m_{1}, G_{3}) \prec (b m_{1}, G_{1})\). The remaining case \((m_{2} > m_{1})\) is demonstrated analogously.

In the proof of Theorem 3.5, we will reduce the comparison of points on the money-guilt plane to the comparison of points on the \( G \) axis. In order to show it is possible, in Lemma B.3 we prove that every point in \( M \times G \) belongs to an indifference set which has a nonempty intersection with any line parallel to the \( G \) axis (including the \( G \) axis itself).

**Lemma B.3.** If a rational, monotone and continuous \( \succeq \) satisfies Axiom 3.3, then for any \((m, G)\) and any \( G' \), there exists \( m' \) such that \((m, G) \sim (m', G')\).

**Proof.** The proof follows that of Lemma B.1 with two divergences. First, the roles of the two variables \((m, G)\) are switched. Second, we are able to demonstrate Step 1 for two cases: \( G_{1} > G_{2} \) and \( G_{1} < G_{2} \), thus for all \( G_{1}, G_{2} \) (and not just \( G_{1} > G_{2} \)) and for any \( m_{1} \) there exist \( m_{2} \) such that \((m_{1}, G_{1}) \sim (m_{2}, G_{2})\). In the proof we require Lemma B.2 in the same way as Lemma 3.4 was required to prove Lemma B.1.

Finally, we establish two auxiliary results necessary to prove Theorem 3.2. The first of these shows that indifference curves of \( \succeq \) satisfying Axiom 3.1 are related by parallel displacement along the money axis.

**Lemma B.4.** If a rational, monotone and continuous \( \succeq \) on \( M \times G \) satisfies Axiom 3.1, then \((m_{1} + a, G_{1}) \succ (m_{2} + a, G_{2})\) whenever \((m_{1}, G_{1}) \succ (m_{2}, G_{2})\) for any \( a \in \max\{-m_{1}, -m_{2}\}, +\infty\).

**Proof.** Similarly to the asymmetric part of Lemma 3.4, this proof consists of 4 cases. However, in the rest of the Appendix we will only be using the result for one case:
$m_2 > m_1$ and $a > 0$, which is considered here. The remaining 3 cases are proved analogously. Let $(m_1, G_1) > (m_2, G_2)$, where $m_1 < m_2$ (hence $G_1 < G_2$ by monotonicity). We need to show that $(m_1 + a, G_1) > (m_2 + a, G_2)$, where $a > 0$. Suppose, conversely, that $(m_1 + a, G_1) \preceq (m_2 + a, G_2)$. By monotonicity, $(m_1 + a, G_1) > (m_1, G_1)$ and hence by transitivity $(m_1 + a, G_1) > (m_2, G_2)$. By connectedness of $\{(m, G_2) : m \in [m_2, m_2 + a]\}$ and continuity of $\succeq$, there exists such $m_3$ that $(m_1 + a, G_1) \sim (m_3, G_2)$ and $m_2 < m_3 \leq m_2 + a$ (*). Now consider $(m_1, G_1)$ and observe that $(m_2, G_2) < (m_1, G_1) < (m_2 + a, G_2)$, where the last relation follows from monotonicity and transitivity. By connectedness and continuity again, there exists $m_4$ such that $(m_1, G_1) \sim (m_4, G_2)$ and $m_2 < m_4 < m_2 + a$. By Axiom 3.1, $m_4 - m_1 = m_3 - (m_1 + a)$ and thus $m_3 = m_4 + a > m_2 + a$, but by (*) $m_3 \leq m_2 + a$, which is a contradiction. Thus, our supposition was wrong, and $(m_1 + a, G_1) > (m_2 + a, G_2)$, q.e.d.

In the proof of Theorem 3.2, we will require a result similar to Lemma B.1 for preferences satisfying Axiom 3.1, i.e. that for any initial money-and-guilt situation $(m, G)$ and any large guilt $G' > G_1$ there exist a sum of money large enough to “seduce” the agent by making him indifferent between the status quo and the large guilt.

**Lemma B.5.** If a rational, monotone and continuous preference relation $\succeq$ on $\mathbb{M} \times \mathcal{G}$ satisfies Axiom 3.1, then for any $(m, G) \in \mathbb{M} \times \mathcal{G}$ and any $G' > G$, there exists $m' \in \mathbb{M}$ such that $(m, G) \sim (m', G')$.

**Proof.** The proof follows that of Lemma B.1, with the exception that the roles of the variables ($m$ and $G$) are switched. In the proof we require Lemma B.4 in the same way as Lemma 3.4 was required to prove Lemma B.1.

We are now ready to establish our main representation theorems.

**Proof of Theorem 3.2.** Necessity is established in the text before the theorem. We now show sufficiency. Suppose $\succeq$ satisfies Axiom 3.1. Consider the point $(1, 0)$. By Lemma B.5, there exists $m^* \in \mathbb{M}$ such that $(1, 0) \sim (m^*, 1)$. By monotonicity of $\succeq$, such $m^*$ is unique. Let $\theta = {m^* - 1 \over a - 1} = m^* - 1$. Assign $u(m, G) = m - \theta G$ uniquely for all $(m, G) \in \mathbb{M} \times \mathcal{G}$. We now show that $u$ represents $\succeq$, i.e. that $u(m_1, G_1) \geq u(m_2, G_2)$ iff $(m_1, G_1) \succeq (m_2, G_2)$. First, suppose $(m_1, G_1) \geq (m_2, G_2)$. Pick a $G_3 > \max\{G_1, G_2\}$. By Lemma B.5, there exist $\mu_1$ and $\mu_2$ such that $(m_1, G_1) \sim (\mu_1, G_3)$ and $(m_2, G_2) \sim (\mu_2, G_3)$. By monotonicity, $\mu_1 < \mu_2$, and thus $u(m_1, G_1) < u(m_2, G_2)$. Therefore, $u(m_1, G_1) \geq u(m_2, G_2)$, as required.
By Axiom 3.1, \(\frac{\mu_1 - m_2}{g_1 - g_2} = \frac{\mu_2 - m_3}{g_1 - g_2} = 0\). Then \(u(m_1, G_1) = m_1 - \theta G_1 = \mu_1 - \theta(G_1 - G_1) - \theta G_1 = \mu_1 - \theta G_1 \geq \mu_2 - \theta G_1 = \mu_2 - \theta(G_1 - G_2) - \theta G_2 = m_2 - \theta G_2 = u(m_2, G_2)\). Second, suppose \(u(m_1, G_1) \geq u(m_2, G_2)\) and consider \(\mu_1, \mu_2\) defined as above. Then \(\mu_1 = m_1 + \theta(G_3 - G_1) = u(m_1, G_1) + \theta G_3 \geq u(m_2, G_2) + \theta G_3 = m_2 + \theta(G_3 - G_2) = \mu_2.\) By monotonicity it follows that \((\mu_1, G_3) \succeq (\mu_2, G_3)\) and then by transitivity \((m_1, G_1) \succeq (m_2, G_2)\), q.e.d.

**Proof of Theorem 3.5.**  
**Necessity.** Suppose \(U(m, G) = \log m - \theta G\) represents \(\succeq\) for some \(\theta > 0\). Consider \((m_1, G_1) \sim (m_2, G_2)\) and \((m'_1, G'_1) \sim (m'_2, G'_2)\), where \(m_1/m'_1 = m_2/m'_2\). Then \(\log m_1 - \theta G_1 = \log m_2 - \theta G_2\) and \(\log m'_1 - \theta G'_1 = \log m'_2 - \theta G'_2\), which implies \(\theta(G_1 - G'_2) = \log m'_1 = \log \frac{m'_1}{m'_2} = \theta(G'_1 - G'_2)\). The relative PCC then becomes \(\frac{m_1 - m_2}{G_1 - G_2} = \frac{m'_1 - m'_2}{G'_1 - G'_2} = \frac{m_1 - m_2}{m'_1 - m'_2} = \frac{m_1}{m_2}\), which establishes Axiom 3.3.

**Sufficiency.** Step 1. Consider any \((m_1, G_1) \in \mathcal{M} \times \mathcal{G}\). If \(G_1 = 0\), let \(U(m_1, G_1) = \log m_1\), which is uniquely defined. If \(G_1 > 0\), by Lemma B.3, \(\exists \mu_1 \in \mathcal{M}\) s.t. \((m_1, G_1) \sim (\mu_1, 0)\). Moreover, such \(\mu_1(m_1, G_1)\) is unique. Indeed, suppose \((m_1, G_1) \sim (\mu_1, 0)\) and \((m_1, G_1) \sim (\mu'_1, 0)\), where \(\mu_1 \neq \mu'_1\). WLOG, let \(\mu_1 > \mu'_1\). Then, by monotonicity, \((\mu_1, 0) \succ (\mu'_1, 0)\), which contradicts transitivity of \(\succeq\). Assign \(U(m_1, G_1) = \log \mu_1(m_1, G_1)\).

We now show that \(U\) defined as above represents \(\succeq\). Suppose \((m_2, G_2) \succeq (m_1, G_1)\). As shown above, there exist unique numbers \(\mu_1\) and \(\mu_2\) such that \((m_2, 0) \sim (m_2, G_2) \succeq (m_1, G_1) \sim (\mu_1, 0) \Rightarrow \mu_2 \geq \mu_1 \Rightarrow U(m_2, G_2) = \log \mu_2 \geq \log \mu_1 = U(m_1, G_1)\) by transitivity and monotonicity. Following the same steps backwards proves that \((m_2, G_2) \succeq (m_1, G_1)\) whenever \(U(m_2, G_2) \geq U(m_1, G_1)\).

It is left to be demonstrated that \(U\) is of the specified form, i.e. there exists a well-defined function \(f(G) = U(m, G) - \log m, \forall G\). In order to do so, consider \((m_1, G_1)\) and \((m_3, G_3)\) such that \(G_1 = G_3\). We need to show that \(f(G_1) = f(G_3)\). WLOG, let \(m_1 > m_3\). By Lemma B.3, there exist \(\mu_1\) and \(\mu_3\) such that \((m_3, G_3) \sim (\mu_3, 0)\) and \((m_1, G_1) \sim (\mu_1, 0)\). By Lemma B.2, \((m_3, G_3) \sim (\mu_3, 0)\) implies \((m_1, G_1) \sim (\mu_1, 0)\), thus \(\mu_1 = \frac{m_3}{m_1} \mu_3\). Thus, \(f(G_1) = U(m_1, G_1) - \log m_1 = \log \mu_1 - \log m_1 = \log \frac{\mu_1}{m_1} = \log \frac{m_3 \mu_3}{m_1} = \log \mu_3 - \log m_3 = U(m_3, G_3) - \log m_3 = f(G_3)\).

Step 2. We need to demonstrate that \(f\) defined as above is linear in \(G\). It suffices to show that, for any \(G_1, G_2, G_3\) and \(G_4 \in \mathcal{G}\) such that \(G_2 - G_1 = G_4 - G_3\), it is true that \(f(G_2) - f(G_1) = f(G_4) - f(G_3)\).

Consider arbitrary \(G_1, G_2, G_3\) and \(m\), as well as \(G_4 = G_3 + G_2 - G_1\). WLOG, let \(G_2 > G_1\) and \(G_3 > G_1\). As shown in Step 1, there exist unique \(\mu_1\) and \(\mu_2\) such that \((m, G_1) \sim
B.2: Applications

(µ₁, 0) and (m, G₂) ∼ (µ₂, 0). Thus, \( f(G₁) = \log µ₁ - \log m \) and \( f(G₂) = \log µ₂ - \log m \), which implies \( f(G₂) - f(G₁) = \log \frac{µ₂}{µ₁}. \) Lemma B.3 ensures that there exists \( m^* \) such that \((m, G₁) ∼ (m^*, G₃)\). Thus, by transitivity, \((m^*, G₃) ∼ (µ₁, 0)\), i.e. \( U(m^*, G₃) = \log µ₁ \) and \( f(G₃) = \log µ₁ - \log m^* \).

By Lemma 3.4, \((m, G₁) ∼ (m^*, G₃) \Rightarrow (m, G₂) ∼ (m^*, G₃ + (G₂ - G₁)), \) where \( G₃ + (G₂ - G₁) = G₄ \). We can now calculate \( U(m^*, G₄) = U(m, G₂) = \log µ₂ \) and \( f(G₄) = \log µ₂ - \log m^* \). It is now easy to see that an arbitrary change in \( G \) produces the same change in \( f \) at any two values of \( G \): \( f(G₄) - f(G₃) = (\log µ₂ - \log m^*) - (\log µ₁ - \log m^*) = \log \frac{µ₂}{µ₁} = f(G₂) - f(G₁) \).

Thus, \( f \) can be written as \( f = a + bG \). Observe that \( a = f(0) = \log m - \log m = 0 \).

In order to calculate \( b \), consider a point \((1, 1)\). Note that \( f(1) = \log µ₀ - \log 1 = \log µ₀ \), where \((1, 1) ∼ (µ₀, 0)\), and \( µ₀ \) is unique for a given preference relation. Hence \( b = µ₀ \).

We can now write \( f(G) = µ₀G \). Observe that, by monotonicity, \( µ₀ < 1 \) and \( \log µ₀ < 0 \).

Assign \( \theta = -b = -\log µ₀ \geq 0 \).

\( U(m, G) = \log m - \theta G \) represents \( ≥ \), q.e.d.

B.2 Applications

Proof of Observation 2. Under the logarithmic guilt model (3.3), D’s utility becomes

\[ U(m_R, E(m_R)) = \log(T - m_R) - \theta \max\{0, E(m_R) - m_R\}. \] (B.1)

D maximizes his utility by choosing an optimal donation \( m^*_R \), given his second-order belief \( i.e. \) his belief about R’s expectation of the donation \( E(m_R) \). If the optimal donation equals the belief, it is a PsyNE of the game.

Let \( m^*_R(E(m_R)) \) denote D’s optimal donation, as a function of his belief.

Observe that, due to the maximum operator present in the function form, \( U(m_R) \) admits a kink at \( m_R = E(m_R) \), where \( U(m_R) \) is continuous, but not differentiable.

Note that the kink point \( m_R = E(m_R) \) separates the function into two halves:

\[ U = \begin{cases} 
\log(T - m_R) - \theta(E(m_R) - m_R) & \text{if } m_R \leq E(m_R); \\
\log(T - m_R) & \text{if } m_R > E(m_R). 
\end{cases} \] (B.2)

Let \( U_l(m_R) = \log(T - m_R) - \theta(E(m_R) - m_R) \) and \( U_r(m_R) = \log(T - m_R) \).
Observe that $\frac{\partial U_i}{\partial m_R} = \frac{-1}{T-m_R} + \theta$. Denote by $m_R^l$ the value of $m_R$ maximizing the function $U_i$:

$$m_R^l = \begin{cases} T - \frac{1}{\theta} & \text{if } T - \frac{1}{\theta} > 0; \\ 0 & \text{if } T - \frac{1}{\theta} \leq 0. \end{cases} \tag{B.3}$$

Observe that utility to the right of the kink is decreasing: $\frac{\partial U_i}{\partial m_R} = \frac{-1}{T-m_R} < 0$. Also, utility to the left of the kink is concave: $\frac{\partial^2 U_i}{\partial m_R^2} = \frac{-1}{(T-m_R)^2} < 0$. The value of $m_R$ maximizing the whole function is thus the smaller of the two: the kink $E(m_R)$, or the value $m_R^l$ maximizing the left-hand-side utility $U_i$:

$$m_R^s(E(m_R)) = \begin{cases} 0 & \text{if } T - \frac{1}{\theta} \leq 0; \\ T - \frac{1}{\theta} & \text{if } 0 < T - \frac{1}{\theta} < E(m_R); \\ E(m_R) & \text{otherwise}, \end{cases} \tag{B.4}$$

which proves Observation 2.

**Proof of Observation 4.** Recall the equilibrium condition on contributions and beliefs in the Public Good Provision game:

$$x_i^* = \begin{cases} U_i(x_i^*, x_j^*, E(x_i)) \geq U_i(x_i, x_j^*, E(x_i)) \text{ for all } x_i \in [0, w_i] \text{ for } i = 1, 2; \\ E(x_i) \text{ for } i = 1, 2. \end{cases} \tag{B.5}$$

Similarly to the Dictator game application, the function $U_i(x_i)$ admits a kink at $x_i = E(x_i)$; it is strictly decreasing to the right of it $(\frac{\partial U_i}{\partial x_i} = \frac{\partial \log(w_i-(1-a)x_j+a x_j)}{\partial x_i} = \frac{-a-1}{w_i-(1-a)x_j+a x_j} < 0)$ and strictly concave to the left of it $(\frac{\partial^2 U_i}{\partial x_i^2} = \frac{\partial^2 \log(w_i-(1-a)x_j+a x_j)-\theta a E(x_i)-x_j)}{\partial x_i^2} = \frac{-a}{(w_i-(1-a)x_j+a x_j)^2} < 0)$. Hence, best response contribution is equal to the expectation if and only if either of the two conditions holds: (i) utility is increasing to the left of the kink: $\frac{\partial U_i}{\partial x_i}(E(x_i)) \geq 0$ or (ii) expectation is zero $E(x_i) = 0$. Below we solve for equilibrium contribution $x_i^*$ under (i).

$$x_i^* = \begin{cases} \frac{\partial \log(w_i-(1-a)x_j+a x_j)-\theta a E(x_i)-x_j)}{\partial x_i}(E(x_i)) \geq 0; \\ E(x_i). \end{cases} \tag{B.6}$$

$$x_i^* = \begin{cases} (a-1)(w_i-(1-a)E(x_i)+a x_j)+\theta a \geq 0; \\ E(x_i). \end{cases} \tag{B.7}$$
\[
\begin{align*}
\begin{cases}
x_i^* \leq (w_i + a x_j)/(1 - a) - (\theta_i a)^{-1}; \\
x_i^* = E(x_i).
\end{cases}
\end{align*}
\] (B.8)

As one can see from the formula, player \(i\)'s maximum equilibrium contribution is increasing with his initial endowment \(w_i\), his guilt sensitivity \(\theta_i\) and the other player's contribution \(x_j\). It is also easy to show that \(x_i^*\) is increasing with return on investment \(a\).

**Proof of Observation 5.** We will establish the values of parameters \(\theta_i, i = 1, 2\) under which each pure strategy profile (with corresponding beliefs) is a Psychological Nash equilibrium.

1. (Defect, Defect; \(E(m_1) = d, E(m_2) = d\)) is the only NE of the game and hence also an equilibrium in the psychological game with guilt aversion (deviations from it will neither increase the players' material payoffs nor decrease their guilt, which is already zero).

2. (Cooperate, Defect; \(E(m_1) = s, E(m_2) = t\)) is an equilibrium if:

\[
\begin{align*}
\begin{cases}
u_1(C, D) & \geq u_1(D, D); \\
u_2(C, D) & \geq u_2(C, C).
\end{cases}
\end{align*}
\] (B.9)

Applying utility function (3.2) and solving for \(\theta_1, \theta_2\) we obtain

\[
\begin{align*}
\begin{cases}
\theta_1 & \geq \frac{d - s}{t - d}; \\
\theta_2 & \in \mathbb{R}.
\end{cases}
\end{align*}
\] (B.10)

3. (Cooperate, Cooperate; \(E(m_1) = c, E(m_2) = c\)) is an equilibrium if \(u_i(C, C) \geq u_i(D, C)\) for \(i = 1, 2\). Applying utility function (3.2) we obtain \(c - \theta_i(c - s) \geq t - \theta_i(c - s)\), which yields \(\theta_i \geq \frac{t - c}{c - s}\).

**Proof of Observation 6.** Similarly to the previous proof, we write down conditions ensuring that unilateral deviation is unprofitable for each candidate pure strategies equilibrium.

1. (Defect, Defect; \(E(m_1) = d, E(m_2) = d\)) is always an equilibrium in games with guilt aversion as argued above.

2. (Cooperate, Defect; \(E(m_1) = s, E(m_2) = t\)) is an equilibrium if
\[
\begin{align*}
\log s - \theta_1(t - t) & \geq \log d - \theta_1(t - d); \\
\log t - \theta_2(s - s) & \geq \log c - \theta_2 \cdot 0;
\end{align*}
\]
which yields
\[
\begin{align*}
\theta_1 & \geq \frac{\log(d/s)}{t - d}, \\
\theta_2 & \in R.
\end{align*}
\]

2. (Cooperate, Cooperate; \(E(m_1) = c, E(m_2) = c\)) is an equilibrium if \(\log c - \theta_i(c - c) \geq \log t - \theta_i(c - s)\) for \(i = 1, 2\), which yields \(\theta_i \geq \frac{\log(t/c)}{c - c}\).
Appendix C

Appendix to Chapter 4

C.1 Conditional Expected Values

The player's expectation of the fundamental value $\theta$ is calculated from its conditional distribution, using properties of univariate and multivariate normal distributions.

First, observe that $\text{Var}(x_i|\theta) = \xi^2$ and $\text{Var}(\bar{x}_i|\theta) = \xi^2 + \psi^2$.

Moreover, $\text{Cov}(x_i, x_j|\theta) = \text{E}[(x_i-\theta)(x_j-\theta)] = \text{E}[\epsilon_i(\epsilon_j+\phi_j)] = 0$ and thus $\rho(x_i, \bar{x}_i|\theta) = 0$.

The expectation of $\theta$, conditional on signals received, can be calculated from its conditional distribution: $E_i(\theta|x_i, \bar{x}_i) = \int_\theta \theta f(\theta|x_i, \bar{x}_i) d\theta$. Consider the conditional distribution of $\theta$:

$$f(\theta|x_i, \bar{x}_i) = \frac{f(x_i, \bar{x}_i|\theta)f(\theta)}{f(x_i, \bar{x}_i)}.$$  \hfill (C.1)

Observe that, since the prior is uniform improper and the values of $x_i, \bar{x}_i$ are realized, $f(\theta|f(x_i, \bar{x}_i) = c_1$ is constant.

Conditional density of $\theta$ can then be written down as follows:

$$f(\theta|x_i, \bar{x}_i) = c_1 f(x_i, \bar{x}_i|\theta).$$  \hfill (C.2)

Conditional on $\theta$, the signals are independently distributed, thus their joint density is equal to the product of their marginal densities:

$$f(x_i, \bar{x}_i|\theta) = f(x_i|\theta)f(\bar{x}_i|\theta).$$  \hfill (C.3)

The probability density function of a joint conditional distribution thus equals
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\[ f(x_i, \bar{x}_i|\theta) = f(x_i|\theta) \cdot f(\bar{x}_i|\theta) \]
\[ = \frac{1}{\sqrt{2\pi} \xi} \exp\left(-\frac{(x_i - \theta)^2}{2\xi^2}\right) \cdot \frac{1}{\sqrt{2\pi}(\xi^2 + \psi^2)} \exp\left(-\frac{(\bar{x}_i - \theta)^2}{2(\xi^2 + \psi^2)}\right) \]
\[ = \frac{1}{\xi \sqrt{\xi^2 + \psi^2} 2\pi} \exp\left(-\frac{\left(\theta - (x_i(\xi^2 + \psi^2) + \bar{x}_i\xi^2)/(2\xi^2 + \psi^2)\right)^2}{2\xi^2(\xi^2 + \psi^2)/(2\xi^2 + \psi^2)}\right) \exp\left(-\frac{(x_i - \bar{x}_i)^2}{2(\xi^2 + \psi^2)}\right) \]
\[ = c_2 \frac{\sqrt{2\xi^2 + \psi^2}}{\xi \sqrt{2\pi(\xi^2 + \psi^2)}} \exp\left(-\frac{\left(\theta - (x_i(\xi^2 + \psi^2) + \bar{x}_i\xi^2)/(2\xi^2 + \psi^2)\right)^2}{2\xi^2(\xi^2 + \psi^2)/(2\xi^2 + \psi^2)}\right), \quad \text{(C.4)} \]

where the expression which is constant given the realised signal values is denoted by \(c_2\) for compactness: \(\exp\left(-\frac{(x_i - \bar{x}_i)^2}{2(\xi^2 + \psi^2)}\right) \sqrt{2\pi(\xi^2 + \psi^2)} = c_2\). Note that the right-hand side of (C.4), apart from the constant, is a density of normally distributed variable \(\theta\).

Substituting (C.4) back into (C.2) obtains the expression for the conditional density function of \(\theta\):

\[ f(\theta|x_i, \bar{x}_i) = c_1 c_2 \frac{\sqrt{2\xi^2 + \psi^2}}{\xi \sqrt{2\pi(\xi^2 + \psi^2)}} \exp\left(-\frac{\left(\theta - (x_i(\xi^2 + \psi^2) + \bar{x}_i\xi^2)/(2\xi^2 + \psi^2)\right)^2}{2\xi^2(\xi^2 + \psi^2)/(2\xi^2 + \psi^2)}\right) \quad \text{(C.5)} \]

Observe that the expressions on both sides, apart from constants, are probability density functions, thus, integrating (C.5) with respect to \(\theta\) yields \(c_1 c_2 = 1\). It can be concluded that the posterior of the fundamental is normally distributed, with mean \((x_i(\xi^2 + \psi^2) + \bar{x}_i\xi^2)/(2\xi^2 + \psi^2)\) and variance \(\xi^2(\xi^2 + \psi^2)/(2\xi^2 + \psi^2)\).

In the main body of the paper, it was claimed that a player's estimate of \(\theta\) is an average of his signals, weighted by their precision (the inverse of conditional variance). It is straightforward to show that \(E_j(\theta|x_i, \bar{x}_i)\) obtained above is equal to such a weighted average:

\[ \frac{x_j(\xi^2 + \bar{x}_i)/(\xi^2 + \psi^2)}{1/(\xi^2 + \psi^2)} = \frac{(\xi^2 + \psi^2)x_j + \xi^2 \bar{x}_i}{2\xi^2 + \psi^2}. \quad \text{(C.6)} \]

**Player's expectation of his opponent's signals** is solved for similarly.

First, consider the opponent's primary signal \(x_j\).
The substitutions were made for density values that are constant, and the decomposition of joint conditional distribution is due to the fact that $x_i = x_j - \epsilon_j + \epsilon_i$ and $\bar{x}_i = x_j + \phi_i$ are independent given $x_j$ (because of the independence of the noise terms).

Observe that, given $x_j$, $x_i = x_j - \epsilon_j + \epsilon_i$ is a sum of two independent normally distributed variables with mean 0 and variance $\xi^2$ each, and a constant $x_j$. A sum of independent normally distributed variables is normally distributed with the mean and variance equal to the sum of means and variances of constituent variables correspondingly (Ross, 2013). Thus, conditional on $x_j$, $x_i \sim N(x_j, 2\xi^2)$.

Recall that $\bar{x}_i = x_j + \phi_i$. Thus, conditional on $x_j$, $\bar{x}_i \sim N(x_j, \psi^2)$. Expression (C.8) becomes

$$f(x_j|x_i, \bar{x}_i) = c_1 \frac{1}{\sqrt{2\pi} \sigma_x \psi^2 \pi} \exp\left(-\frac{(x_i - x_j)^2 \psi^2 + (\bar{x}_i - x_j)^2 2\xi^2}{4\xi^2 \psi^2}\right).$$ (C.9)

Similarly to the argument above, it is concluded that, conditional on $x_i$, $\bar{x}_i$ the opponent’s primary signal $x_j$ is normally distributed, and $E(x_j|x_i, \bar{x}_i) = (x_i \psi^2 + \bar{x}_i 2\xi^2)/(2\xi^2 + \psi^2)$.

Finally, the conditional expectation of the opponent’s secondary signal is simply equal to the player’s primary signal, as shown below:

$$E(\bar{x}_j|x_i, \bar{x}_i) = E(x_i - \phi_j|x_i, \bar{x}_i) = E(x_i|x_i, \bar{x}_i) - E(\phi_j|x_i, \bar{x}_i) = x_i.$$ (C.10)
C.2 Proofs of Propositions

Proof of Proposition 4.1.

The equilibrium is solved for from system (C.11), which ensures that the players’ strategies satisfy the best reply condition (4.13), given that the opponent is playing a strategy linear in his signals \((a_1 = \lambda_1 x_1 + \lambda_2 \tilde{x}_1\) and \(a_2 = \kappa_1 x_2 + \kappa_2 \tilde{x}_2\) respectively).

\[
\begin{align*}
    a_1(x_1, \tilde{x}_1) &= (1 - r) \left( \frac{x_1 + \psi}{2} \right) + r \kappa_1 \left( \frac{x_1 + \psi}{2} \right) + r \kappa_2 x_1; \\
    a_2(x_2, \tilde{x}_2) &= (1 - r) \left( \frac{x_2 + \psi}{2} \right) + r \lambda_1 \left( \frac{x_2 + \psi}{2} \right) + r \lambda_2 x_2;
\end{align*}
\]

(C.11)

Rearranging the equations in (C.11) obtains the best replies which are linear in signals:

\[
\begin{align*}
    a_1(x_1, \tilde{x}_1) &= \frac{(1 - r)(x_1 + \psi) + r \kappa_1 r + r \kappa_2}{2} \left( \frac{x_1 + \psi}{2} \right) + x_1 + \frac{(1 - r)(r + 1) \kappa_2}{2} \left( \frac{x_1 + \psi}{2} \right); \\
    a_2(x_2, \tilde{x}_2) &= \frac{(1 - r)(x_2 + \psi) + r \lambda_1 r + r \lambda_2}{2} \left( \frac{x_2 + \psi}{2} \right) + x_2 + \frac{(1 - r)(r + 1) \lambda_2}{2} \left( \frac{x_2 + \psi}{2} \right).
\end{align*}
\]

(C.12)

Equating the player’s linear strategy coefficients with the opponent’s belief about them generates system (C.13).

\[
\begin{align*}
    \frac{(1 - r)(x_1 + \psi) + r \kappa_1 r + r \kappa_2}{2} &= \lambda_1; \\
    \frac{(1 - r)(x_2 + \psi) + r \lambda_1 r + r \lambda_2}{2} &= \lambda_2; \\
    \frac{(1 - r)(x_1 + \psi) + r \lambda_1 r + r \lambda_2}{2} &= \kappa_1; \\
    \frac{(1 - r)(x_2 + \psi) + r \lambda_1 r + r \lambda_2}{2} &= \kappa_2.
\end{align*}
\]

(C.13)

It turns out from solving (C.13) for coefficients \(\kappa_1, \kappa_2, \lambda_1, \lambda_2\) that the equilibrium strategies are (i) symmetric: \(\kappa_1 = \lambda_1\) and \(\kappa_2 = \lambda_2\) (ii) convex combinations of signals: \(\kappa_1 = 1 - \kappa_2\). Hence, they are denoted as \(w = \kappa_2 = \lambda_2\) and \(1 - w = \kappa_1 = \lambda_1\).

The unique linear equilibrium of the game consists of both players choosing actions which are weighted averages of their signals:

\[
a_i = \frac{(x^2 + 1 + \psi^2)x_i + x^2(r - 1) \tilde{x}_i}{2x^2(r + 1) + \psi^2},
\]

(C.14)

which completes the proof of Proposition 4.1.

Turning to the interpretation of the equilibrium, observe that, given the model’s restriction on coordination motive \(|r| < 1\), (C.14) is a weighted average of signals
Consider the sum of expected utilities under the beauty-contest model:

\[ E(U) = E(u_1) + E(u_2) = -(1 - r) \left[ E[(a_1 - \theta)^2] + E[(a_2 - \theta)^2] \right] - 2r E[(a_1 - a_2)^2]. \]  
(C.15)

Observe that, by symmetry, \( E[(a_1 - \theta)^2] = E[(a_2 - \theta)^2] \). Since the players are not systematically wrong in their evaluation of the fundamental (\( E(a_1) = E(a_2) = \theta \)), expectations of the squares can be replaced with variances:

\[ E(U) = -2(1 - r)\text{Var}(a_i - \theta) - 2r \text{Var}(a_i - a_j), \]  
(C.16)

where the last equality is due to the fact that \( \text{Var}(a_i - \theta) = \text{Var}(w(\epsilon_j + \phi_j) + (1 - w)\epsilon_i] = (2w^2 - 2w + 1)\xi^2 + w^2\psi^2 = \text{Var}(a_j - \theta) \) and \( \text{Var}(a_i - a_j) = \text{Var}(w(\epsilon_j + \phi_j) + (1 - w)\epsilon_i - w(\epsilon_i + \phi_j) - (1 - w)\epsilon_i] = 2(1 - 2w)^2\xi^2 + 2w^2\psi^2 = \text{Var}(a_j - a_i) \).

Denote the volatility of \( i \)'s action by \( A = \text{Var}(a_i - \theta) \) and dispersion of actions by \( B = \text{Var}(a_i - a_j) \). The aggregate payoff in the beauty-contest game can be compactly rewritten as follows:

\[ E(U) = -2(1 - r)A - 2r B. \]  
(C.17)

First, consider the direct effects of \( \psi^2 \) and \( w \) on volatility and dispersion:
C.2: Proofs of Propositions

\[ \frac{\partial A}{\partial \psi^2} = w^2; \quad \text{(C.18)} \]

\[ \frac{\partial A}{\partial w} = 2w(\psi^2 + 2\xi^2) - 2\xi^2; \quad \text{(C.19)} \]

\[ \frac{\partial B}{\partial \psi^2} = 2w^2; \quad \text{(C.20)} \]

\[ \frac{\partial B}{\partial w} = 4w(\psi^2 + 4\xi^2) - 8\xi^2. \quad \text{(C.21)} \]

The direct effects of these variables on aggregate payoff are now easy to find:

\[ \frac{\partial E(U)}{\partial \psi^2} = -2(1-r) \frac{\partial A}{\partial \psi^2} - 2r \frac{\partial B}{\partial \psi^2} = -(1+r)w^2. \quad \text{(C.22)} \]

Indeed, the direct effect of spying noise on payoff is negative, as it increases both volatility and dispersion.

\[ \frac{\partial E(U)}{\partial w} = -2(1-r) \frac{\partial A}{\partial w} - 2r \frac{\partial B}{\partial w} = -4w\psi^2(1+r) - 8w\xi^2(1 + 3r) - 4\xi^2(1 + 3r). \quad \text{(C.23)} \]

Next, in order to find the indirect effect of \( \psi^2 \) on the aggregate payoff, calculate the effect of noise in intelligence technology on the weight attributed to the spying signal:

\[ \frac{\partial w}{\partial \psi^2} = \frac{\xi^2(r+1)}{(2\xi^2(r+1) + \psi^2)^2}. \quad \text{(C.24)} \]

Although the increase in noise always makes the players rely less on their spying source, the resulting indirect effect of noise on payoff can be either positive or negative. If \( r > 0 \), it is always positive, as noisier spying means less weight on the spying signal and consequently smaller dispersion and variance.

Finally, the total effect of spying noise on aggregate payoff can be calculated:
\[
\frac{dE(U)}{d\psi^2} = \frac{\partial E(U)}{\partial \psi^2} + \frac{\partial E(U)}{\partial w} \cdot \frac{\partial w}{\partial \psi^2} = \frac{2\xi^4(1+r)}{D^3} \psi^2(r^2-1-4r) - 2\xi^2(1+r)^3,
\]

where \(D\) is the denominator of \(w\). Observe that (C.25) has variable sign; however, if the actions are strategic complements, the effect is always negative, i.e. more precise spying is payoff-enhancing.

Now consider the sum of payoffs (i.e. aggregate profit) in the Bertrand duopoly model (4.3):

\[
E(\Pi) = E(\Pi) - 2(1-r)E[(a_i - \theta)^2] - 2rE[(a_i - a_j)^2] + 2rE[a_i^2]
= E(\Pi) - 2(1-r)A - 2rB + 2rC,
\]

where \(C = E[a_i^2]\). Also observe that \(E[a_i^2] = \text{Var}[a_i] + E(\theta)^2 = (2w^2 - 2w + 1)\xi^2 + w^2\psi^2 + E(\theta^2) = \text{Var}(a_j - \theta) + E(\theta^2) = A + E(\theta^2)\). Denote the sum of terms independent of \(\psi^2\) by \(E(\Pi)' = E(\Pi) + 2rE(\theta^2)\). Aggregate profit in the Bertrand model becomes:

\[
E(\Pi) = E(\Pi)' - 2(1-2r)A - 2rB.
\]

Using the effects of \(w\) and \(\psi\) on volatility and dispersion obtained above, calculate the direct effects of these variables on aggregate profit:

\[
\frac{\partial E(\Pi)}{\partial \psi^2} = -2(1-2r) \frac{\partial A}{\partial \psi^2} - 2r \frac{\partial B}{\partial \psi^2} = -2w^2.
\]

\[
\frac{\partial E(\Pi)}{\partial w} = -2(1-2r) \frac{\partial A}{\partial w} - 2r \frac{\partial B}{\partial w} = -4w\psi^2 - 8w^2\xi^2(1+2r) + 4\xi^2(1+2r).
\]

The total effect of spying noise in the Bertrand model turns out to be negative (recall that \(r \in (0,0.5)\)):
Turning to the Cournot model, the aggregate profit in it has the same structure, but different coefficients:

\[ E(\Pi) = E(\Pi) - 3E[(a_i - \theta)^2] + E[(a_i - a_j)^2] - E[a_i^2] \]
\[ = E(\Pi) - 3A + B - C = E(\Pi)' - 4A + B, \]

where \( E(\Pi)' \) is the sum of terms independent of \( \psi^2 \).

Similarly to the Bertrand model, the direct, indirect and total effects of spying noise on aggregate profit are readily calculated:

\[ \frac{\partial E(\Pi)}{\partial \psi^2} = -4 \frac{\partial A}{\partial \psi^2} + \frac{\partial B}{\partial \psi^2} = -2w^2. \]  
(C.30)

\[ \frac{\partial E(\Pi)}{\partial w} = -4 \frac{\partial A}{\partial w} + \frac{\partial B}{\partial w} = -4w\psi^2. \]  
(C.31)

\[ \frac{dE(\Pi)}{d\psi^2} = \frac{\partial E(\Pi)}{\partial \psi^2} + \frac{\partial E(\Pi)}{\partial w} \cdot \frac{\partial w}{\partial \psi^2} \]
\[ = \frac{\xi^4}{2D^3} \left[ \psi^2 - \xi^2 \right]. \]  
(C.32)

We conclude that the spying device noise reduces aggregate profit in the Cournot model if and only if \( \psi^2 < \xi^2 \), which proves Proposition 4.4.

**Proof of Proposition 4.5**

Consumer welfare is measured by the size of consumer surplus (CS). Recall that in all models considered here the inverse demand function has a slope of one, hence consumer surplus equals \( \frac{1}{2}Q^2 \), where \( Q \) is the aggregate equilibrium production quantity.
First, consider Cournot duopoly, where firms engage in quantity competition, and the size of consumer surplus equals half the square of equilibrium output.

\[
\text{CS} = 0.5 \mathbb{E} \left[ (q_i + q_j)^2 \right] = 0.5 \mathbb{E} \left[ (w(x_i + x_j) + (1-w)(x_i + x_j))^2 \right] = 0.5 \text{Var} \left[ x_i + x_j + w(\phi_i + \phi_j) \right] + 0.5 \left( \mathbb{E} \left[ x_i + x_j + w(\phi_i + \phi_j) \right] \right)^2 = 2\mathbb{E}(\theta^2) + \xi^2 + w^2 \psi^2.
\]

Observe that consumer surplus is increasing with the variance of the firms’ output. This is due to the fact that the benefit of “overproduction” outweighs the loss of “underproduction” of the same size. Consequently, the direct effect of spying noise on consumer surplus is positive, as it increases output variance: \( \partial \text{CS} / \partial \psi^2 = w^2 = \xi^4(1+r)^2 / D^2 > 0 \), where \( D = 2\xi^2(1+r) + \psi^2 > 0 \) is the denominator of \( w \). However, the indirect effect is negative: while the weight on the spying signal is surplus-enhancing \( \partial \text{CS} / \partial w = 2w\psi^2 = 2\xi^2(1+r)^2 / D > 0 \), the firms put less weight on a noisier signal \( \partial w / \partial \psi^2 = -\xi^2(1+r) / D^2 < 0 \). As a result, the total effect of the spying noise on consumer surplus has an ambiguous sign (recall that in the Cournot duopoly \( r = -0.5 \)):

\[
\frac{d \text{CS}}{d \psi^2} = \frac{\xi^4(1+r)^2}{D^3} \left( 2\xi^2(r+1) + \psi^2 - 2\psi^2 \right) = \frac{\xi^4}{4D^3} \left( \xi^2 - \psi^2 \right),
\]

which is negative if and only if \( \psi^2 > \xi^2 \).

In words, when spying noise is big enough the indirect effect dominates, as the marginal reduction in the firms’ reliance on their spying signal brings about bigger decrease in volatility than the increase caused by higher noise. Also observe that \( d\text{CS} / d\psi^2 > 0 \) iff \( \psi^2 < \xi^2 \), which implies that consumer welfare peaks when the precisions of spying and market research are equal. This proves the first half of Proposition 4.5.

Second, consider price competition (Bertrand duopoly), where equilibrium actions are prices: \( p_i = (1-w)x_i + w \bar{x}_i \), and equilibrium quantity is determined from demand equations: \( q_i = \theta - p_i + b p_j \) for \( i = 1, 2 \). Expected consumer surplus (CS) is the sum of surpluses in the two markets. In each market, consumer surplus equals half the square of equilibrium quantity:
The total effect of spying noise on consumer welfare is the sum of direct and
indirect effects:

\[
\frac{dCS}{d\psi^2} = \frac{\partial CS}{\partial \psi^2} + \frac{\partial CS}{\partial w} \cdot \frac{\partial w}{\partial \psi^2}.
\]  \hfill (C.34)

The expression for total effect comprises three partial derivatives (the denomi-
nator of \( w \) is denoted by \( D \)):

\[
\frac{\partial CS}{\partial \psi^2} = w^2(1 + b^2) = \frac{\xi^4(1 + r)^2(1 + b^2)}{D^2},
\]  \hfill (C.35)

\[
\frac{\partial w}{\partial \psi^2} = -\frac{\xi^2(1 + r)}{D^2},
\]  \hfill (C.36)

\[
\frac{\partial CS}{\partial w} = 2(2w - 1)(1 + b^2)\xi^2 + 2w(1 + b^2)\psi^2 = \frac{2\xi^2\psi^2(r b^2 - 2b + r)}{D}.
\]  \hfill (C.37)

Observe that, unlike the Cournot case, both the direct and indirect effects of \( \psi^2 \)
on consumer surplus are positive. Indeed, \( r b^2 - 2b + r = 4r^3 - 3r < 4r^3 - r < 0 \)
(recall that in the Bertrand case \( r = b/2 \in (0, 0.5) \)). Consequently, the total effect is
also positive:

\[
\frac{dCS}{d\psi^2} = \frac{\xi^4(1 + r)}{D^3}\left[(1 + r)(1 + 4r^2)(2\xi^2(1 + r) + \psi^2) - 2\psi^2(4r^3 - 3r)\right] > 0,
\]

where the last inequality follows from the fact that \( 1 + 7r + 4r^2 - 4r^3 = 1 + 7r + 4r^2(1-r) > 0 \). It can be concluded that a reduction in spying precision \( 1/\psi^2 \) improves
consumer surplus, which completes the proof of Proposition 4.5.
Proof of Proposition 4.6.

Social welfare (SW) is calculated by adding up consumer surplus and the duopolists’ profits. Consequently, its derivative with respect to spying precision is the sum of its effects on the consumer and producer welfare. First, consider the Cournot case:

\[
\frac{\partial SW}{\partial \psi^2} = \frac{\partial CS}{\partial \psi^2} + 2 \frac{\partial \pi}{\partial \psi^2} = \frac{\xi^4}{4D^3}(\xi^2 - \psi^2 + 2\psi^2 - 2\xi^2) = \frac{\xi^4}{4D^3}(\psi^2 - \xi^2).
\]  
(C.38)

Welfare effects in the Bertrand case are calculated analogously:

\[
\frac{\partial SW}{\partial \psi^2} = \frac{\partial CS}{\partial \psi^2} + 2 \frac{\partial \pi}{\partial \psi^2} = \frac{\xi^4(1 + r)}{D^3}(2\xi^2(1 + r)^2(1 + 4r^2) + \psi^2(1 + 7r + 4r^2 - 4r^3) - 2\psi^2(5 + 7r) - 4\xi^2(1 + r)^2) \\
= \frac{\xi^4(1 + r)}{D^3}\left(2\xi^2(1 + r)^2(4r^2 - 1) + \psi^2(-9 - 7r + 4r^2 - 4r^3)\right) < 0,
\]  
(C.39)

where the last inequality is due to the fact that $4r^2 - 1 < 0$ and $-9 - 7r + 4r^2 - 4r^3 < 0$, which, in their turn, hold since $0 < r < 0.5$. This completes the proof of Proposition 4.6.

Proof of Propositions 4.8-4.7.

To simplify notation, the substitutions introduced in the proof of Proposition 4.2 are used. Denote the volatility of a player’s action \(\text{Var}(a_i - \theta)\) by \(A\), the dispersion of actions \(\text{Var}(a_i - a_j)\) by \(B\) and the variance of the opponent’s action \(\text{Var}(a_j)\) by \(C\). Expected profit in the Bertrand duopoly model then becomes:

\[
E(\pi_i) = E(\pi_i) - (1 - r)A - rB + rC.
\]  
(C.40)

Expected profit in the Cournot duopoly has the same structure, but different coefficients:

\[
E(\pi_i) = E(\pi_i) - \frac{3}{2}A + \frac{1}{2}B - \frac{1}{2}C.
\]  
(C.41)

Recall the formula for the total effect of \(\psi_j^2\) on the equilibrium expected profit:
\[
\frac{d EEp_i}{d \psi_j^2} = \frac{\partial E(\pi_i)}{\partial \psi_j^2} + \frac{\partial E(\pi_j)}{\partial \psi_j^2} \partial w_j.
\]  \hspace{1cm} (C.42)

We will first calculate the effect of the opponent’s spying signal weight \( w_j \) and of the noise in the opponent’s spying device \( \psi_j^2 \) on the individual components of the firm’s profit \((A, B, C)\) and then establish the total effect.

Observe that neither the opponent’s strategy coefficients nor his intelligence technology noise directly affect the firm’s action volatility.

\[
\frac{\partial A}{\partial w_j} = \frac{\partial \text{Var}(a_i - \theta)}{\partial w_j} = \frac{\partial [(1 - w_i)^2 \xi^2 + w_i^2 (\xi^2 + \psi_j^2)]}{\partial w_j} = 0. \hspace{1cm} (C.43)
\]

\[
\frac{\partial A}{\partial \psi_j^2} = 0. \hspace{1cm} (C.44)
\]

Next, we establish the effect of spying weight and spying noise on action dispersion.

\[
\frac{\partial B}{\partial w_j} = \frac{\partial \text{Var}(a_i - a_j)}{\partial w_j} = \frac{\partial [(1 - w_i - w_j)^2 \xi^2 + (w_i - (1 - w_j))^2 \xi^2 + w_i^2 \psi_j^2 + w_j^2 \psi_j^2]}{\partial w_j} = 2\xi^2 (2w_j - 1) + 2\xi^2 (2w_j - 1) + 2w_j \psi_j^2,
\]

\[
\frac{\partial B}{\partial \psi_j^2} = w_j^2 > 0. \hspace{1cm} (C.45)
\]

Finally, the effect of \( w_j \) and \( \psi_j^2 \) on the opponent’s action variance is established.

\[
\frac{\partial C}{\partial w_j} = \frac{\partial \text{Var}(a_j)}{\partial w_j} = \frac{\partial \text{Var}(\theta) + (1 - w_j)^2 \xi^2 + w_j^2 (\xi^2 + \psi_j^2)}}{\partial w_j} = 2\xi^2 (2w_j - 1) + 2w_j \psi_j^2,
\]

\[
\frac{\partial C}{\partial \psi_j^2} = w_j^2 > 0. \hspace{1cm} (C.46)
\]

The dispersion of actions and the opponent’s action volatility increase as the noise in the opponent’s spying device goes up. The effect of the spying signal weight
is ambiguous.

In order to calculate the total effect of changes in the opponent's spying technology precision on profit, we need to know how it affects the weight placed by the opponent on his spying signal:

$$\frac{\partial w_j}{\partial \psi_j^2} = -\frac{\xi^2(1 + r)(2\xi^2 + \psi_i^2)(2\xi^2(1 - r) + \psi_j^2)}{(4\xi^4(1 - r^2) + 2\xi^2(\psi_i^2 + \psi_j^2) + \psi_i^2\psi_j^2)^2} < 0.$$  \hspace{1cm} (C.48)

Not surprisingly, if the noise in the spying technology rises, firm $j$ puts less trust in it.

We can now calculate the direct effect of spying noise on the firm's profit. First, consider Bertrand duopoly:

$$\frac{\partial E(\pi_i)}{\partial \psi_j^2} = -(1 - r) \cdot \frac{\partial A}{\partial \psi_j^2} - r \cdot \frac{\partial B}{\partial \psi_j^2} + r \cdot \frac{\partial C}{\partial \psi_j^2} = -r[w_j^2 - w_j^2] = 0.$$ \hspace{1cm} (C.49)

Second, we obtain this effect in the Cournot case:

$$\frac{\partial E(\pi_i)}{\partial w_j} = \frac{1}{2}w_j^2 - \frac{1}{2}w_j^2 = 0.$$ \hspace{1cm} (C.50)

Similarly, the effect of the weight that the opponent puts on his spying signal can be established. First, let us look at the Bertrand duopoly model:

$$\frac{\partial E(\pi_i)}{\partial w_j} = -2r\xi^2(2w_i - 1).$$ \hspace{1cm} (C.51)

We can now establish the overall effect of spying device precision on equilibrium expected profit in the Bertrand Duopoly.

$$\frac{d\text{EEP}_i}{d\psi_j^2} = 2r\xi^2(2w_i - 1) \cdot \frac{1}{D} \cdot \xi^2(1 + r)(2\xi^2 + \psi_i^2)(2\xi^2(1 - r) + \psi_i^2),$$ \hspace{1cm} (C.52)

where $D$ is the denominator of $w_i$ (which is the same as the denominator of $w_j$):

$$D = 4\xi^4(1 - r^2) + 2\xi^2(\psi_i^2 + \psi_j^2) + \psi_i^2\psi_j^2 > 0.$$ \hspace{1cm} (C.53)

As can be easily seen, the expression (C.52) is negative (and hence the opponent's spying is profit-increasing) if and only if $2w_i - 1 < 0$: 


\[ 2r \xi^2 \psi^2_i - 2 \xi^2 \psi^2_j - \psi^2_i \psi^2_j < 0. \]  
(C.54)

The left-hand side of (C.54) is a homogenous polynomial. Divide both sides by \( \psi^2_i \psi^2_j \) to achieve a two-variable form:

\[ 2r \xi^2 \psi^2_i - 2 \xi^2 \psi^2_j - 1 < 0. \]  
(C.55)

To simplify notation, denote \( x = \frac{\xi^2}{\psi^2_i} \) and \( y = \frac{\xi^2}{\psi^2_j} \). Recall that a signal’s precision is the inverse of its variance. Thus, \( x = \frac{1}{1/\psi^2_i} \) is relative precision of the firm’s spying technology. Similarly, \( y \) is relative precision of the competitor’s spying technology. Expression (C.55) then becomes

\[ 2r x - 2y - 1 < 0, \]  
(C.56)

or

\[ y > r x - 0.5. \]  
(C.57)

which proves Proposition 4.8. Also observe that inequality (C.57) always holds when \( x = \frac{\xi^2}{\psi^2_i} \leq 1/2r \), because \( x, y > 0 \).

Consider a simpler case where \( \psi^2_i = \psi^2_j \) i.e. \( x = y \). Since in the Bertrand case \( r < 0.5 \), inequality (C.57) always holds.

Turning to the Cournot case (recall that \( r = -0.5 \)), the effect of the opponent’s spying signal weight on profit is negative:

\[
\frac{\partial E(\pi_i)}{\partial w_j} = 0.5 \left[ 2 \xi^2 (2w_j - 1) \right] = \frac{\xi^2}{D} \left[ -\xi^2 \psi^2_j - 2 \xi^4 \psi^2_i - \xi^2 \psi^2_i \psi^2_j \right] < 0.
\]

Since the direct effect of spying noise on profit in the Cournot case is zero and the indirect effect is a product of two negative values, the total effect is positive, i.e. the noisier the opponent’s spying device the bigger the firm’s profit, which proves Proposition 4.7.
Proof of Proposition 4.9.

By the envelope theorem, the total effect of noise in own spying device $\psi_i^2$ on a firm’s expected equilibrium profit (EEP) is determined as follows:

$$\frac{d \text{EEP}_i}{d \psi_i^2} = \frac{\partial E(\pi_i)}{\partial \psi_i^2} + \frac{\partial E(\pi_i)}{\partial w_j} \cdot \frac{\partial w_j}{\partial \psi_i^2}. \tag{C.58}$$

Thus, in order to establish the total effect of $\psi_i^2$ on profit, three partial derivatives need to be calculated. The effect of $\psi_i^2$ on the opponent’s spying weight $w_j$ is readily derived:

$$\frac{\partial w_j}{\partial \psi_i^2} = \frac{2 \xi^4 r(1 + r)(2 \xi^2(1 - r) + \psi_j^2)}{(4 \xi^4(1 - r^2) + 2 \xi^2(\psi_i^2 + \psi_j^2) + \psi_i^2 \psi_j^2)^2}. \tag{C.59}$$

The above expression is positive iff $r > 0$, from which its interpretation follows.

In Bertrand (Cournot) case, the bigger the noise in the firm’s spying device, the more (less) its opponent relies on its intelligence report, because the players’ actions are strategic complements (substitutes).

Turning to the remaining partial derivatives $\frac{\partial E(\pi_i)}{\partial \psi_i^2}$ and $\frac{\partial E(\pi_i)}{\partial w_j}$, first consider the direct effects of $w_j$ and $\frac{\partial \psi_i^2}{\partial w_j}$ on the components of the profit (volatility $A$, dispersion $B$ and the opponent’s action variation $C$) and then calculate their linear combination which forms the total effect of parameter in question on profit.

The bigger the noise in spying, the bigger volatility and dispersion:

$$\frac{\partial A}{\partial \psi_i^2} = \frac{\partial B}{\partial \psi_i^2} = w_j^2. \tag{C.60}$$

The opponent’s reliance on intelligence has no effect on volatility, and the firm’s own spying precision has no effect on the opponent’s action variation:

$$\frac{\partial A}{\partial w_j} = \frac{\partial C}{\partial \psi_i^2} = 0, \tag{C.61}$$

However, $w_j$ affects both dispersion and the opponent’s action variation:

$$\frac{\partial C}{\partial w_j} = 2 \xi^2(2 w_j - 1) + 2 w_j \psi_j^2 = \frac{\partial B}{\partial w_j} - 2 \xi^2(2 w_i - 1). \tag{C.62}$$

In the Bertrand duopoly, the total effect of $\psi_i^2$ on a firm’s expected equilibrium profit is calculated as follows:
\[
\frac{d\text{EEP}_i}{d\psi^2_i} = -(1-r) \frac{\partial A}{\partial \psi^2_i} - r\left(\frac{\partial B}{\partial \psi^2_i} - \frac{\partial C}{\partial \psi^2_i}\right) + \left(-(1-r) \frac{\partial A}{\partial w_j} - r\left(\frac{\partial B}{\partial w_j} - \frac{\partial C}{\partial w_j}\right)\right) \frac{\partial w_j}{\partial \psi^2_i} \\
= -w^2_i - (2w_i - 1) \frac{4\xi^6 r^2 (1 + r)(2\xi^2(1 - r) + \psi^2_j)}{(4\xi^4(1 - r^2) + 2\xi^2(\psi^2_i + \psi^2_j) + \psi^2_i \psi^2_j)} \tag{C.63}
\]

Rearranging (C.63) and substituting (4.26) for \(w_i\) yields that the profit is decreasing with the noise in spying if and only if the following expression is positive:

\[
8\xi^6(1-r^2)^2 + 4\xi^4\psi^2_i(1-3r^2) + 4\xi^4\psi^2_j(2+r-2r^2+r^3) + 2\xi^2(\psi^2_i \psi^2_j(2+r-3r^2) + 2\xi^2 \psi^4_i(1+r) + \psi^2_i \psi^4_j(1+r).
\]

It is easy to verify that, for \(r < \frac{1}{2}\) (which is true in Bertrand case, see Section 4.3.2), all the terms in the expression above are positive. Hence, equilibrium profit is always decreasing with the noise in spying \(\psi^2_i\) and increasing with its inverse, i.e. the precision of the spying device.

Turning to Cournot duopoly, the effect of noise in the firm's spying on its profit can be written down as follows:

\[
\frac{d\text{EEP}_i}{d\psi^2_i} = \frac{3}{2} \frac{\partial A}{\partial \psi^2_i} + \frac{1}{2} \left(\frac{\partial B}{\partial \psi^2_i} - \frac{\partial C}{\partial \psi^2_i}\right) + \left(-\frac{3}{2} \frac{\partial A}{\partial w_j} + \frac{1}{2} \left(\frac{\partial B}{\partial w_j} - \frac{\partial C}{\partial w_j}\right)\right) \frac{\partial w_j}{\partial \psi^2_i} \\
= -w^2_i - \xi^2(2w_i - 1) \frac{\xi^4(3\xi^2 + \psi^2_j)}{2(3\xi^4 + 2\xi^2(\psi^2_i + \psi^2_j) + \psi^2_i \psi^2_j)^2} \\
= -\xi^4(3\xi^2 + \psi^2_j)(9\xi^6 + 2\xi^4 \psi^2_i + 7\xi^4 \psi^2_j + 3\xi^2 \psi^2_i \psi^2_j + 2\xi^2 \psi^4_i + \psi^2_i \psi^4_j) \\
\tag{4.64}
\]

which is negative for all values of parameters. It can be concluded that the firm's profit rises as the precision of its intelligence technology improves in both Cournot and Bertrand models, which completes the proof.

**Proof of Proposition 4.10.**

A player's expected payoff in the beauty contest game can be written down as follows:
\[ E(u) = -(1-r)A - rB. \] (C.65)

First, the effect of own spying on payoff is calculated, using its effect on volatility and dispersion.

\[
\begin{align*}
E(u) &= -(1-r) \frac{\partial A}{\partial \psi_i^2} - r \frac{\partial B}{\partial \psi_i^2} + \left( -(1-r) \frac{\partial A}{\partial w_j} - r \frac{\partial B}{\partial w_j} \right) \frac{\partial w_j}{\partial \psi_i^2} \\
&= -w_j^2 - r(2\xi^2(2w_i + 2w_j - 2) + 2w_j \psi_j^2) \frac{2\xi^2 r w_j}{D} \\
&= -\frac{\xi^2 w_i}{D^2} \left[ (4\xi^4(1 - r^2) + 2\xi^2(\psi_i^2 + \psi_j^2) + \psi_i^2 \psi_j^2)(2\xi^2(1 - r^2) + \psi_j^2(1 + r)) + 4\xi^2 r^2(2\xi^2(r - 1)(\psi_i^2 + \psi_j^2) - 2\psi_i^2 \psi_j^2 + 2\xi^2 \psi_j^2(1 - r^2) + \psi_i^2 \psi_j^2(1 + r)) \right] \\
&= -\frac{\xi^2 w_i}{D^2} \left[ 8\xi^6(1 - r^2)^2 + 4\xi^4 \psi_j^4(1 - r)(1 - r^2 + r(1 - r)) + 4\xi^4 \psi_j^4(1 - r)(2r^3 + r^2 + 3r + 2) \\
&+ 2\xi^2 \psi_i^2 \psi_j^2((1 - r)(2 + 3r) + 2r^3) + 2\xi^2 \psi_i^4(1 + r) + \psi_i^2 \psi_j^4(1 + r) \right]. \end{align*}
\] (C.66)

As one can see, when \( r > 0 \), the above expression is negative, hence, payoff is increasing in own spying precision when actions are strategic complements.

Second, consider the effect of the opponent's spying precision on the payoff.

\[
\begin{align*}
E(u) &= -(1-r) \frac{\partial A}{\partial \psi_i^2} - r \frac{\partial B}{\partial \psi_i^2} + \left( -(1-r) \frac{\partial A}{\partial w_j} - r \frac{\partial B}{\partial w_j} \right) \frac{\partial w_j}{\partial \psi_i^2} \\
&= -w_j^2 - r(2\xi^2(2w_i + 2w_j - 2) + 2w_j \psi_j^2) \frac{2\xi^2 + \psi_j^2) \xi^2 r w_j}{D} \\
&= -\frac{\xi^2(1+r)r}{D^3} \left[ 16\xi^8(1 - r)^3(1 + r)^2 + 8\xi^6 \psi_j^2(1 - r)^2(2r^2 + 5(1 + r)) + 8\xi^6 \psi_j^2(1 - r)^3 \\
&+ 4\xi^4 \psi_j^4(6 + (1 + r)^2) + 4\xi^4 \psi_i^2 \psi_j^2(1 - r)(r - 2)^2 + 1 + 2\xi^2 \psi_i^4(3 - r) \\
&+ 2\xi^2 \psi_i^2 \psi_j^2((1 - r)(5 - 2r) + 2) + \psi_i^2 \psi_j^2(3 - r) \right]. \end{align*}
\] (C.67)

Here, owing to \( r \) in the numerator of the first fraction, the whole expression is positive for \(-1 < r < 0\) and negative for \(1 > r > 0\). Hence, the effect of the opponent's spying precision on payoff is positive when actions are strategic complements and negative when actions are strategic substitutes.
Proof of Proposition 4.11.

First, observe that $i$'s utility as defined by (4.35) in case of $z_j > 0$ and by setting $w_j = 0$ in case of $z_j = 0$ is discontinuous in $z_j$. Consequently, equilibrium is established by considering a firm's best response in two cases: (i) the competitor choosing a positive amount of spying $z_j > 0$; (ii) the competitor purchasing no spying $z_j = 0$.

Case 1: the opponent engages in positive amount of spying $z_j > 0$ (and hence $w_j > 0$). The firm $i$'s utility depends on whether it spies itself. If it purchases no spying ($z_i = 0$), its expected utility can be derived from (4.34) as follows:

$$E(u_i)(0, 0) = -(1 - r)ξ^2 - r\left[2ξ^2(1 - w_j)^2 + w_j^2 \frac{ψ^2}{z_j}\right] - cz_i. \quad (C.68)$$

Combining and (4.35) and (C.68) and dropping the expectation operator to save on notation, yields the utility function defined for both zero and positive amounts of spying:

$$u_i(z_i, w_i) = \begin{cases} 
-(1 - r)\left[ξ^2(1 - 2w_i + 2w_i^2) + w_i^2 \frac{ψ^2}{z_i}\right] & \text{if } z_i > 0; \\
-(1 - r)ξ^2 - r[2ξ^2(1 - w_j)^2 + w_j^2 \frac{ψ^2}{z_j}] & \text{if } z_i = 0. 
\end{cases} \quad (C.69)$$

Each firm's optimization problem entails choosing $z_i$ and $w_i$ to maximize (C.69), which is equivalent to a two-stage problem where the firm first picks an optimal weight for every spying decision $w_i^*(z_i)$ and then maximizes $u(z, w_i^*(z_i))$ w.r.t. $z_i$. Interestingly, this univariate utility function is continuous in $z_i$ (unlike the original problem (C.69)), which is demonstrated in Lemma C.1.

Consider Stage 1 optimization problem (finding an optimal $w_i$ for each $z_i$). For $z_i > 0$, it entails maximizing the first line of (C.69) with respect to $w_i$. Observe that this problem is concave (indeed, $∂^2 u_i/∂ w_i^2 = -4ξ^2 - 2\frac{ψ^2}{z_i} < 0$) and defined on an open set $(R)$ with convex inequality constraints ($0 \leq w_i \leq 1$), thus, the first-order condition with respect to $w_i$ yields a global maximizer $w_i^*(z_i)$:

$$w_i^* = \frac{ξ^2(1 + r - 2rw_j)}{2ξ^2 + \frac{ψ^2}{z_i}}. \quad (C.70)$$
Equation (C.70) defines optimal weight put on a spying signal, for each possible positive amount of espionage spending.\footnote{It is easy to show that the numerator of (C.70) is positive. Indeed, $1 + r - 2r w_j = 1 + r(1 - 2w_j)$. Since $1 \geq 1 - 2w_j \geq -1$ and $1 > r(1 - 2w_j) > -1$, we have $2 > 1 + r(1 - 2w_j) > 0$.} Observe that the more spying was purchased, the bigger the influence of the spying signal on the firm’s equilibrium action. For $z_i = 0$, the only value of $w_i$ in the domain of $u_i$ is 0, hence it is also the optimal weight. Taken together, these two statements form a solution to Stage 1 optimization problem:

$$w^*_i(z_i) = \begin{cases} \frac{\xi^2(1+r-2r w_j)}{2\xi^2 + \psi^2} & \text{if } z_i > 0; \\ 0 & \text{if } z_i = 0. \end{cases} \label{C.71}$$

A global maximizer $(z^*_i, w^*_i)$ can be obtained by substituting $w^*_i(z_i)$ into the utility function (C.69) and maximizing it with respect to $z_i$ (Stage 2). First, observe that the resulting univariate utility function $u_i(z_i, w^*_i(z_i))$ is continuous in $z_i$:

Lemma C.1. $\lim_{z_i \to 0} u_i(z_i, w^*_i(z_i)) = u_i(0,0)$.

**Proof**: in the next section of the Appendix.

The continuity of $u_i(z_i)$ implies that if $d u_i/d z_i < 0$ for all $z_i$, then $z_i = 0$ is the optimal solution.\footnote{Since $u_i$ is not differentiable at $z_i = 0$, standard Kuhn-Tucker conditions cannot be applied.} Assuming that the opponent is choosing optimal $z^*_j(z_i)$ and $w^*_j(z_i)$, firm $i$ is always better off reducing its amount of spying if $\xi^2(1 + r) < \psi \sqrt{c}$, as summarized in the following Lemma.

Lemma C.2. Consider a firm $i$’s utility $u_i(z_i, z^*_j(z_i), w^*_j(z_i))$ defined at a point $z_i$, where the opponent is playing his best response $(z^*_j(z_i), w^*_j(z_i))$. Then $d u_i/d z_i < 0$ for all $z_i$ iff $\xi^2(1 + r) < \psi \sqrt{c}$.

**Proof**: in the next section of the Appendix.
Solving a system of first-order conditions with respect to \( z_i^* \) and \( z_j^* \) generates the following Lemma.

**Lemma C.3.** Consider a firm i’s utility \( u_i(z_i, z_j^*(z_i), w_j^*(z_i)) \) defined at a point \( z_i \), with the opponent playing his best response \( (z_j^*(z_i), w_j^*(z_i)) \). Suppose that \( \xi^2(1+r) > \psi \sqrt{c} \). Then \( z_i = \frac{\psi(1+r)-\psi \sqrt{c}}{2\xi^2(1+r)\sqrt{c}} \) is a global maximizer of \( u_i(z_i) \).

**Proof:** in the next section of the Appendix.

A profile \((z_1, w_1^*(z_1); z_2, w_2^*(z_2))\) where each \( z_i \) is defined as in Lemma C.3 and each \( w_i^* \) satisfies (C.70) forms a mutual best response *i.e.* the equilibrium of the game.

**Case 2: The opponent does not engage in spying** \((z_j = w_j = 0)\). Firm i’s utility becomes:

\[
u_i = -(1-r)\left[\xi^2(1-2w_1+2w_i^2)+w_i^2\psi^2\right] - r\left[2\xi^2(1-w_1)^2+w_i^2\psi^2\right] - c z_i, \quad (C.72)
\]

and the solution of FOC on \( w_i \) is a global maximizer, given the concavity of \( u_i(w_i) \):

\[
w_i^*(z_i) = \begin{cases} \frac{\xi(1+r)}{2\xi^2+\psi^2} & \text{if } z_i > 0; \\ 0 & \text{if } z_i = 0. \end{cases} \quad (C.73)
\]

Consider a univariate function \( u(z_i, w_i^*(z_i)) \). It can be readily established how the utility is affected by changes in \( z_i \). By the envelope theorem,

\[
\frac{du(z_i, w_i^*(z_i))}{dz_i} = \frac{\partial u(z_i, w_i)}{\partial z_i}(z_i, w_i^*(z_i)) = \frac{(w_i^*(z_i))^2\psi^2}{z_i^2} - c. \quad (C.74)
\]

Thus, the effect of spying \( \frac{du(z_i, w_i^*(z_i))}{dz_i} \) is negative iff \( \frac{w_i^*(z_i)^2\psi^2}{z_i} < \sqrt{c} \) iff \( \frac{\xi^2(1+r)}{2\xi^2 z_i+\psi^2} < \sqrt{c} \) iff \( \psi(\xi^2(1+r)-\sqrt{c}\psi) < 2\xi^2 \sqrt{c} \).

The last expression is true for all \( z_i > 0 \) iff \( \xi^2(1+r)-\sqrt{c}\psi < 0 \). Otherwise \( \exists z_i > 0 \) such that \( \frac{du(z_i, w_i^*(z_i))}{dz_i} = 0 \), and this optimal \( z_i \) is determined as follows:

\[
z_i^* = \frac{\psi(\xi^2(1+r)-\sqrt{c}\psi)}{2\xi^2 \sqrt{c}}. \quad (C.75)
\]

The corresponding optimal weight is then
\[ w^*_i = \frac{\xi^2(1+r) - \sqrt{c} \psi}{2 \xi^2}. \] (C.76)

Following the two-stage maximization process, we find that, if \( \xi^2(1+r) < \psi \sqrt{c} \), then \( z^*_i = w^*_i = 0 \) i.e. firm \( i \) will also choose not to spy. Otherwise it will pick \( z^*_i = \frac{\psi(\xi^2(1+r) - \sqrt{c} \psi)}{2 \xi^2 \sqrt{c}} \) and \( w^*_i = \frac{\xi^2(1+r) - \sqrt{c} \psi}{2 \xi^2 \sqrt{c}} \). Let us now check if \( z^*_j = w^*_j = 0 \) is a best response to such strategy. By the above logic, \( z^*_j = w^*_j = 0 \) is a best response to \( z^*_i = w^*_i = 0 \) iff \( \xi^2(1+r) < \psi \sqrt{c} \). If, however, firm \( i \) chooses a positive amount of spying equal to the best response to no-acquisition, firm \( j \)'s marginal utility of spying becomes

\[
\frac{d u(z_j, w^*_j(z_j))}{dz_j} = \frac{(w^*_j(z_j))^2 \psi^2}{z_j^2} - c = \frac{\psi^2(\xi^2(1+r-2r w_i))^2}{4 \xi^4 z_j^2 + 4 \xi^2 \psi^2 z_j + \psi^4} - c
\] (C.77)

Reducing the amount of spying will benefit firm \( j \) iff \( \frac{\xi^2 \psi(1+r-2r w_i)}{2 \xi^2 z_j + \psi^2} < \sqrt{c} \) iff \( \xi^2 \psi(1+r-2r \frac{\xi^2(1+r) - \sqrt{c} \psi}{2 \xi^2 \sqrt{c}}) < \sqrt{c}(2 \xi^2 z_j + \psi^2) \) iff \( \psi(1-r)(\xi^2(1+r) - \sqrt{c} \psi) < 2 \xi^2 \sqrt{c} z_j \).

Similarly to \( i \)'s best response, the last expression is true for all \( z_j > 0 \) iff \( \xi^2(1+r) - \sqrt{c} \psi < 0 \). Otherwise the optimal \( z^*_j \) is determined as follows:

\[ z^*_j = \frac{\psi(1-r)(\xi^2(1+r) - \sqrt{c} \psi)}{2 \xi^2 \sqrt{c}}. \] (C.78)

We conclude that one firm can only choose zero spying in equilibrium if the other firm does so too. Thus, if \( \xi^2(1+r) < \psi \sqrt{c} \), both firms choosing \( z_i = w_i = 0 \) is an equilibrium of the game, and an asymmetric spying profile cannot be equilibrium under any values of parameters.

**Proof of Lemma C.1.**

Observe that \( \lim_{z_i \to 0} w^*_i(z_i) = \lim_{z_i \to 0}(w^*_i(z_i))^2 = 0 \) and use it to calculate the limit of utility:
By the envelope theorem, the total effect of $z_i$ on $u_i$ is equal to the direct effect:

$$
\lim_{z_i \to 0} u_i(z_i, w^*_i(z_i)) = \lim_{z_i \to 0} \left[ (1-r) \left( \xi^2(1-2w_i^*(z_i)) + 2(w_i^*(z_i))^2 \right) \frac{\psi^2}{z_i} \right] 
$$

$$
- r \left[ \xi^2(1-w_i^*(z_i) - w_j^2 + (w_i^*(z_i) - 1 + w_j^2) + (w_i^*(z_i))^2 \frac{\psi^2}{z_i} + w_j^2 \frac{\psi^2}{z_j} \right] - c z_i 
$$

$$
= -(1-r)\xi^2 - (1-r) \lim_{z_i \to 0} \left( \frac{w_i^*(z_i))^2 \psi^2}{z_i} \right) 
$$

$$
- r[2\xi^2(1-w_j)^2 + w_j^2 \frac{\psi^2}{z_j}] - r \lim_{z_i \to 0} \left( \frac{w_i^*(z_i))^2 \psi^2}{z_i} \right) - c z_i 
$$

$$
u_i(0,0) - \lim_{z_i \to 0} \frac{\xi^4\psi^2(1 + r - 2w_j)^2}{4\xi^4z_i + 4\xi^2\psi^2 + \psi^2} = u_i(0,0).
$$

**Proof of Lemmata C.2-C.3.**

By the envelope theorem, the total effect of $z_i$ on $u_i$ is equal to the direct effect:

$$
\frac{du_i(z_i, w_i^*(z_i))}{dz_i} = \frac{\partial u_i(z_i, w_i^*(z_i))}{\partial z_i} + \frac{\partial u_i(z_i, w_i^*(z_i))}{\partial w_i^*} \frac{\partial w_i^*(z_i)}{\partial z_i} = \frac{\partial u_i(z_i, w_i^*(z_i))}{\partial z_i} = \frac{w_i^2\psi^2}{z_i^2} - c. \tag{C.79}
$$

Thus, the derivative will be negative ($du_i/dz_i < 0$) if the following condition holds:

$$
\frac{w_i\psi}{z_i} < \sqrt{c}. \tag{C.80}
$$

Also note that $u_i(z_i)$ is concave: $d^2u_i/dz_i^2 = -2w_i^2\psi^2 \left| z_i^3 \right| < 0$.

Since we are interested in the effect of $z_i$ on utility at a candidate equilibrium point, where the firm's choice of $w_i$ is optimal given $z_i$, and the other firm's choice of $w_j$ and $z_j$ is optimal given $z_i$, we derive the optimal values $w_i^*(z_i, z_j)$ and $z_j^*(z_i)$, and substitute them into (C.79).

To obtain $w_i^*(z_i, z_j)$, consider Stage 1 optimization again, now done by both firms:

$$
\begin{align*}
  w_i &= \frac{\xi^2(1+r-2r w_j)}{2\xi^2 + \psi^2} \\
  w_j &= \frac{\xi^2(1+r-2r w_i)}{2\xi^2 + \psi^2} \tag{C.81}
\end{align*}
$$

System (C.81) solves for optimal weights $w_i^*, w_j^*$, given own and opponent's spying amounts $z_i, z_j$.
C.2: Proofs of Propositions

\[ w_i^* = \frac{2\xi i^i(1 - r^2) + \xi i^i \psi i^{i(1+r)}}{4\xi i^i(1 - r^2) + 2\xi j(\frac{\psi j}{z_j} + \frac{\psi i}{z_i}) + \frac{\psi j}{z_j}}. \]  

(C.82)

where \( i = 1, 2. \)

Observe that (C.82) corresponds to equilibrium strategies in the exogenous information case (discussed in Chapter 4). Substitute (C.82) into the negative derivative condition (C.80):

\[ \frac{2\xi i^i(1 - r^2) + \xi i^i \psi i^{i(1+r)}}{4\xi i^i(1 - r^2)z_i + 2\xi j(\psi j^i + \frac{\psi j z_i}{z_j}) + \frac{\psi j}{z_j}} < \sqrt{c} \]  

(C.83)

Multiply both sides by a positive denominator of the LHS and then by \( z_j > 0: \)

\[ \xi i^i(1+r)[2\xi i^i(1-r)z_j + \psi i^i] < 2\xi i^i \sqrt{c} z_i[2\xi i^i(1-r^2)z_j + \psi i^i] + \psi i^i \sqrt{c}[2\xi i^i z_j + \psi i^i], \]  

(C.84)

which yields

\[ 2\xi i^i z_i[\xi i^i(1-r^2) - \xi i^i \sqrt{c} z_i(1-r^2) - \psi i^i \sqrt{c}] < \psi i^i[2\xi i^i \sqrt{c} z_i + \psi i^i \sqrt{c} - \xi i^2(1+r)]. \]  

(C.85)

We need to express (C.85) in terms of \( z_i \) only, in order to determine the values of parameters under which it will hold for all \( z_i \) (i.e. firm \( i \) will want to reduce its amount of spying at any point \( z_i \)). Recall we assumed firm \( j \) is choosing positive spying. Such \( z_j \) should then be a solution to \( d u_j/d z_j = 0 \) (since \( u_j(z_j) \) is concave):

\[ \frac{w_j^2 \psi^2}{z_j^2} = c \]  

(C.86)

which yields

\[ z_j^* = \frac{\xi i^i(1+r)[2\xi i^i(1-r) + \frac{\psi i^i}{z_i}] - \psi i^i \sqrt{c}[2\xi i^i + \frac{\psi i^i}{z_i}]}{2\xi i^i \sqrt{c}(\xi i^i(1-r^2) + \frac{\psi i^i}{z_i})}. \]  

(C.87)

We can now plug the opponent’s best response \( z_j^*(z_i) \) into the “negative effect of spying on utility” condition (C.85); we then multiply both sides by the positive denominator of \( z_j^*(z_i) \) and divide by \( 2\xi i^2 \psi > 0: \)
\[
\left( \xi^2(1 + r)[2\xi^2(1 - r) + \frac{\psi^2}{z^2}] - \psi \sqrt{c}[2\xi^2 + \frac{\psi^2}{z^2}] \right)[\xi^2 \psi(1 - r^2) - 2\xi^2 \sqrt{c} z_i(1 - r^2) - \psi^2 \sqrt{c}] \\
< \psi \sqrt{c}\left(2\xi^2(1 - r^2) + \frac{\psi^2}{z^2}\right)\left[2\xi^2 \sqrt{c} z_i + \psi^2 \sqrt{c} - \xi^2 \psi(1 + r)\right].
\]

After expanding all factors most terms cancel out and the above expression reduces to:

\[
2\xi^4 \psi(1 - r^2) - 4\xi^2 \psi^2 \sqrt{c} < 4\xi^4 \sqrt{c} (1 - r^2) z_i - \frac{\sigma^4 \psi^2(1 + r) - \psi_\psi \sqrt{c}}{z_i}, \tag{C.88}
\]

which can be further simplified, by multiplying both sides by \(z_i > 0\) and factoring out common factors:

\[
(z_i - \frac{\xi^2 \psi(1 + r) - \psi^2 \sqrt{c}}{2\xi^2(1 + r) \sqrt{c}})(z_i + \frac{\psi^2}{2\xi^2(1 - r)}) > 0. \tag{C.89}
\]

Expression is (C.89) true for all \(z_i\) iff \(\xi^2(1 + r) < \psi \sqrt{c}\). In this case, at any point \((z_i, w^*_i(z_i), z^*_i(z_i), w^*_j(z_i))\) firm \(i\) can increase its utility by reducing \(z_i\). Thus, no positive \(z_i\) can be part of equilibrium.

Conversely, if \(\xi^2(1 + r) > \psi \sqrt{c}\), there exist \(z_i\) such that \(d u_i/d z^*_i = 0\), determined as follows:

\[
z_i = \frac{\xi^2 \psi(1 + r) - \psi^2 \sqrt{c}}{2\xi^2(1 + r) \sqrt{c}}. \tag{C.90}
\]

**Proof of Proposition 4.12**

Consider the familiar expression for the firm’s expected profit in the Bertrand model, for the case of symmetric equilibrium \((w_i = w_j\) and \(z_i = z_j\):

\[
E(\pi_i) = E(\pi_i) - (1 - 2r)A - r B, \tag{C.91}
\]

where the profit components can be expanded as follows:

\[
A = \text{Var}(\theta - a_i) = (2w^2 - 2w + 1)\xi^2 + w^2 \psi^2 \quad z = (2w^2 - 2w + 1)\xi^2 + w \psi \sqrt{c}, \tag{C.92}
\]
\[ B = \text{Var}(a_i - a_j) = 2(1 - 2w)^2 \xi^2 + 2w^2\psi^2 \mid z = 2(1 - 2w)^2 \xi^2 + 2w\psi \sqrt{c}, \quad (C.93) \]

where \( w = \left[ \xi^2(1 + r) - \psi \sqrt{c} \right] / \left(2\xi^2(1 + r) \right) \). First, observe that the underlying noise in spying and its cost are only entering profit equation as a product \( \psi \sqrt{c} \). It is thus possible to establish the effect of either parameter by calculating \( d\pi / d\psi \sqrt{c} \), and multiplying the result by the other parameter. Denote \( \psi \sqrt{c} = \sigma \).

The total effect of \( \sigma \) on profit is the sum of direct and indirect effects.

\[ \frac{dE(\pi)}{d\sigma} = \frac{\partial E(\pi)}{\partial \sigma} + \frac{\partial E(\pi)}{\partial w} \frac{\partial w}{\partial \sigma}, \quad (C.94) \]

where \( \frac{\partial w}{\partial \sigma} = -1 / \left(2\xi^2(1 + r) \right) \).

The direct effects of \( \sigma \) and \( w \) on the profit components are as follows:

\[ \frac{\partial A}{\partial \sigma} = w; \quad (C.95) \]

\[ \frac{\partial A}{\partial w} = (4w - 2)\xi^2 + \sigma; \quad (C.96) \]

\[ \frac{\partial B}{\partial \sigma} = 2w; \quad (C.97) \]

\[ \frac{\partial B}{\partial w} = (16w - 8)\xi^2 + 2\sigma. \quad (C.98) \]

The direct effects of these variables on profit are readily calculated.

\[ \frac{\partial \pi}{\partial \sigma} = -w; \quad (C.99) \]

\[ \frac{\partial \pi}{\partial w} = 2(1 - 2w)(1 + 2r)\xi^2 - \sigma. \quad (C.100) \]

Finally, the total effect of \( \sigma \) can be easily shown to be negative:

\[ \frac{d\pi}{d\sigma} = -w - \frac{1}{D} \left[ 2\xi^2(1 - 2w)(1 + 2r) - \sigma \right] = -\frac{1}{D(1 + r)} \left[ \xi^2(1 + r)^2 + 2r\sigma \right]. \quad (C.101) \]
Now consider the consumer surplus under price competition, calculated analogously to the proof of Proposition 4.5:

$$CS = \left[1 + b^2 + 2(1 + b)^2(w^2 - w)\right] \xi^2 + w(b^2 + 1)\sigma_\phi + b^2E(\theta^2).$$

It is easy to show that \(\frac{\partial CS}{\partial \sigma_\phi} = w(1 + b^2) = (\xi^2(1 + r) - \sigma_\phi)(1 + b^2)/D\) and \(\frac{\partial CS}{\partial w} = 2\xi^2(1 + b)^2(2w - 1) + (b^2 + 1)\sigma_\phi\). Moreover, the effect of \(\sigma_\phi\) on \(w\) is negative: \(\frac{\partial w}{\partial \sigma_\phi} = -1/D^2\). The total effect turns out to be positive, and can then be calculated as follows:

$$\frac{dCS}{d\sigma_\phi} = \frac{(\xi^2(1 + r) - \sigma_\phi)(1 + b^2)}{D} + \frac{1}{D(1 + r)}\left[2(1 + b)^2(1 + r)\sigma_\phi - (b^2 + 1)\sigma_\phi\right]$$

$$= \frac{1}{D(1 + r)}\left[(\xi^2(1 + r) - \sigma_\phi)(1 + b^2)(1 + r) + 2(1 + b)^2\sigma_\phi - (b^2 + 1)(1 + r)\sigma_\phi\right]$$

$$= \frac{1}{D(1 + r)}\left[\xi^2(1 + r)^2(1 + 4r^2) + 2\sigma_\phi r(3 - 4r^2)\right] > 0.$$

The effect of \(\sigma_\phi\) on social welfare is the sum of its effects on the two firms’ profits and consumer surplus:

$$\frac{dSW}{d\sigma_\phi} = \frac{1}{D(1 + r)}\left[\xi^2(1 + r)^2(1 + 4r^2) + 2\sigma_\phi r(3 - 4r^2) - 2\xi^2(1 + r)^2 - 4r\sigma_\phi\right]$$

$$= \frac{4r^2 - 1}{D(1 + r)}\left[\xi^2(1 + r)^2 - 2r\sigma_\phi\right] < 0,$$

where the last inequality follows from the fact that \(r \in (0, 0.5)\), which implies, first, that \(4r^2 - 1 < 0\), and second, that \(1 + r > 2r\) and hence that \(\xi^2(1 + r)^2 > 2r\sigma_\phi\), which proves Proposition 4.12.

**Proof of Proposition 4.13**

Recall that in the Cournot case \(r = -\frac{1}{2}\), hence \(w = (0.5\xi^2 - \sigma_\phi)\xi^2\) and \(\frac{\partial w}{\partial \sigma_\phi} = -1/\xi^2\).

First, consider the effect of spying noise and cost on profit. Similarly to the proof of Proposition 4.12, the direct effects of \(w\) and \(\sigma_\phi\) are readily established:
\[
\frac{\partial \pi}{\partial \sigma} = -2 \frac{\partial A}{\partial \sigma} + \frac{1}{2} \frac{\partial B}{\partial \sigma} = -w; \tag{C.102}
\]

\[
\frac{\partial \pi}{\partial w} = -2 \frac{\partial A}{\partial w} + \frac{1}{2} \frac{\partial B}{\partial w} = (-8w + 4)\xi^2 - 2\sigma_\phi + (8w - 4)\xi^2 + \sigma_\phi = -\sigma_\phi. \tag{C.103}
\]

According to (C.102), costlier spying results in bigger volatility, which is profit-decreasing. However, (C.103), together with the negative effect of spying cost on the \( w \) established in the previous proof, imply that the indirect effect of \( \sigma_\phi \) on profit is positive. Costlier spying makes firms rely less on the intelligence report when taking the final decision, which, in turn, increases profit. The total effect is thus ambiguous, and needs to be explicitly calculated:

\[
\frac{d\pi}{d\sigma_\phi} = -w + \frac{\sigma_\phi}{\xi^2} = \frac{4\sigma_\phi - \xi^2}{2\xi^2}. \tag{C.104}
\]

As can be easily seen, the total effect of cost on profit is positive if and only if \( 4\sigma_\phi > \xi^2 \). Note that this condition is equivalent to the precision of market research technology being greater than the precision of intelligence technology:

\[
\xi^2 < \psi^2 \left| z = \frac{2\xi^2(1 + r)\sqrt{c}\psi}{\xi^2(1 + r) - \psi\sqrt{c}}, \right. \tag{C.105}
\]

which holds if and only if \( 2(1 + r)\sqrt{c}\psi > \xi^2(1 + r) - \psi\sqrt{c} \), that is, if and only if \( 4\sqrt{c}\psi > \xi^2 \).

In words, if the firms’ spying is already noisier than their market surveying, then the harm from over-reliance on the intelligence report, brought about by the marginal increase in its precision (decrease in cost), is greater than the benefit of decreased action volatility.

Second, the effect of costlier spying on consumer surplus is calculated:

\[
\frac{dCS}{d\sigma_\phi} = \frac{d\left(2E(\theta^2) + \xi^2 + w^2\psi^2\right|z}{d\sigma_\phi} = \frac{d\left(2E(\theta^2) + \xi^2 + w\sigma_\phi\right)}{d\sigma_\phi} = \frac{d\left(0.5\xi^2\sigma_\phi - \sigma_\phi^2\right)}{d\sigma_\phi} = \frac{\xi^2 - 4\sigma_\phi}{2\xi^2}. \]
It can be concluded that the consumers and the producers have opposing interests with respect to the espionage cost and precision. However, since there are two firms on the markets, they outweigh the consumers in social welfare calculations:

\[
\frac{dSW}{d\sigma_\phi} = \frac{dCS}{d\sigma_\phi} + 2 \cdot \frac{d\pi}{d\sigma_\phi} = \frac{4\sigma_\phi - \xi^2}{2\xi^2}. \tag{C.106}
\]

It is readily seen that social welfare falls with \(\sigma_\phi\) until it reaches \(0.25\xi^2\), and then increases. Regarding the socially optimal level of spying costs, it can be established that zero costs imply exactly the same welfare as the costs which are high enough to discourage the duopolists from spying (so that \(2\sigma_\phi > \xi^2\)).

Observe that the aggregate profit equals \(2\pi = 2\bar{\pi} - 4A + B = 2\bar{\pi} - 2\xi^2 - 2w\sigma_\phi\). Also note that if \(c = 0\), then \(\sigma_\phi = 0\); moreover, the firms will choose \(w = 0\) if \(2\sigma_\phi > \xi^2\). Let \(SW = SW(c, w)\):

\[
SW(0, w) = 2E(\theta^2) - \xi^2 + 2\bar{\pi} = SW(c, 0). \tag{C.107}
\]
Bibliography


BIBLIOGRAPHY

