TESTING GAUSSIAN-BASED KERNELS FOR MODELLING T-WAVES AND P-WAVES IN ECG SIGNALS


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Abstract: This paper presents a comparative study of segmentation and modelling of P and T waves in electrocardiograms, using three different mathematical models: Gaussian function, a composition of two Gaussian functions and Rayleigh probability density function. In order to evaluate the adaptability and the matching degree between each model and each characteristic wave, we compute the evolution of the corresponding parameters related to the fitted kernels throughout ECG records from the well-known QT database, as well as the normalized least mean square error between each model and the analysed waves. We have found the most accurate results for the kernel derived from the composition of two Gaussian functions, for which the average of normalized relative least mean square errors for T-wave and P-wave were, respectively, 1.56% and 9.13%, considering both available leads from the QT database.

Keywords: electrocardiogram (ECG), P and T waves, mathematical models, Gaussian function, Rayleigh probability density function, least mean square error.

Introduction

The electrocardiogram (ECG) consists of the measurement of electrical activity on the body surface associated with myocardial contraction with respect to time. Each cardiac cycle in the ECG is normally characterized by a sequence of waveforms known as P wave, QRS complex and T-wave, so that both their shape and the time intervals between onset and offset of different waves are significant as they reflect physiological processes of the heart and of the autonomous nervous system.

The idea of mathematical modelling segments of the cardiac cycle is by no means new. Firstly, the purpose was to provide tools for the spectral analysis of the actual cardiac signal departing from a modelled signal. Richardson et al. [1] proposed three mathematical models for individually modelling P-wave, QRS complex and T-wave using Gaussian functions and their first and second derivatives. In more recent developments, mathematical modelling is part of a dual-stage framework. A parametric function related to a segment of the cardiac cycle is obtained and algorithms for waveform characterization are applied starting with the parameters of the model fitted [2,3].

Concerning T-wave, its detection and segmentation following QRS segmentation provide the beat-to-beat analysis of the time intervals between the QRS onset and T-wave end, known as QT intervals, whose prolongation is a marker of risk for ventricular arrhythmias [4]. The QT intervals depend on the accuracy with which onset and offset are determined. Furthermore, a recent technique, microvolt T-wave alternans (MTWA) has the ability to identify patients at high risk for sudden cardiac death. Its spectral method of analysis allows detection of beat-to-beat alternans in the microvolt range of T-wave amplitudes [5]. Concerning P-wave, it is well-known that their regular morphology is associated with a normal cardiac impulse starting in the sinoatrial node and then spreading throughout the atrial myocardium. However, when the atrial depolarization wavefront spreads in an abnormal way and/or presents accessory pathways, reentries, or conduction delays, the P-wave morphology changes [6]. Nowadays, P-wave duration is accepted as the most reliable noninvasive marker for atrial conduction characterization and its prolongation has been associated with the history of atrial fibrillation [7,8].

The purpose of the paper is to propose and compare different kernels for the modelling and segmentation of P-wave and T-wave, based on Gaussian functions and Rayleigh probability density function. The performance of each mathematical model is assessed using all the publicly available records of the annotated QT database, by computing the least mean square error between each model and the analysed waves and analysing the evolution of the corresponding parameters related to the fitted kernels throughout the records pertaining to the referred database.

Materials and methods

This paper considers only T-wave and P-wave modelling. Obviously, the R-wave and QRS onset and offset need to be accurately detected previously in order to delimit an interval that contains the T-wave and P-wave. We have adopted our already validated QRS
detection and segmentation approach, which is based on Hilbert and Wavelet transforms, first-derivative and adaptive threshold technique [9].

**Description of mathematical models**

In this work, we apply three different mathematical kernels in order to model T-waves and P-waves: a Gaussian function, a composition of two Gaussian functions and the Rayleigh probability density function.

We define a discrete-time Gaussian function \( G_\sigma[k] \), comprising the interval \(-3 \leq k \leq 3\), with a variable standard deviation \( \sigma \) and a time-step equal to 0.004s, equivalent to a 250 Hz-sampling frequency:

\[
G_\sigma[k] = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{k^2}{2\sigma^2}\right).
\] (1)

In order to derive a composition of two discrete-time Gaussian functions, named as \( G_{\sigma_1,\sigma_2}[k] \), we depart from two discrete-time Gaussian functions \( G_{\sigma_1}[k] \) and \( G_{\sigma_2}[k] \), respectively with variable standard deviations \( \sigma_1 \) and \( \sigma_2 \), both defined for \(-3 \leq k \leq 3\), such that:

\[
G_{\sigma_1,\sigma_2}[k] = \begin{cases} 
G_{\sigma_1}[k], & -3 \leq k \leq 0 \\
G_{\sigma_2}[k], & 0 \leq k \leq 3
\end{cases}
\] (2)

Later, we define a discrete-time Rayleigh probability density function \( R_\sigma[k] \), comprising the interval \( 0 \leq k \leq 10 \), with a variable standard deviation \( \sigma \) and a time-step equal to 0.004s:

\[
R_\sigma[k] = \frac{k}{\sigma^3} \exp\left(-\frac{k^2}{2\sigma^2}\right).
\] (3)

Furthermore, we define a modification \( R'_\sigma[k] \) of the Rayleigh probability density function given as its time reflection, such that:

\[
R'_\sigma[k] = R_\sigma[-k].
\] (4)

**Detection of T-wave and P-wave peaks**

In order to detect T-wave and P-wave peaks, we apply the approach developed by Martinez et al. [10], which considers, for discrete-time signals, the dyadic discrete wavelet transform (DWT) equivalent to an octave filter bank, which can be implemented as a cascade of identical cells (low-pass and high-pass finite impulse response (FIR) filters), as illustrated in Figure 1.

![Filter-bank implementation of DWT](image)

**Figure 1: Filter-bank implementation of DWT.**

As applied by Martinez et al. [10], we used as prototype wavelet \( \psi(t) \) a quadratic spline whose Fourier transform is \( \Psi(\Omega) = j\Omega \left( \frac{\sin(\Omega/2)}{\Omega/2} \right)^4 \). (5)

The wavelet can be identified as the derivative of the convolution of four rectangular pulses, i.e., the derivative of a low-pass function.

First of all, we define a search window for each beat, relative to each interval between a given QRS offset (J-point) and the subsequent QRS onset. Within this window, we look for local maxima of \( |W_2[n]| \) with opposite signs and amplitude greater than \( \gamma_\tau \), which are related to the most significant slopes of the original wave. The zero crossings between them are detected as T-wave peaks [10]. Concerning the search for P-wave peaks, we define another window between each detected T-wave peak and the subsequent QRS onset. A similar algorithm is applied over DWT using adequate thresholds \((\epsilon_\rho \text{ and } \gamma_\rho)\) [10]. We use \( \epsilon_\rho = 0.25.\text{RMS}(W_2[n]) \), \( \gamma_\tau = 0.125.\text{max}(|W_2[n]|) \), \( \epsilon_\rho = 0.02.\text{RMS}(W_2[n]) \), \( \gamma_\rho = 0.125.\text{max}(|W_2[n]|) \).

**T-wave and P-wave modelling**

After determining each T-wave peak and P-wave peak, we begin the process of wave modelling. Each of the proposed kernels has one or two specific parameters, corresponding to standard deviations \( \sigma_1 \) and \( \sigma_2 \). For each kernel, we establish a process of matching with each waveform by varying the corresponding parameters over a predefined range: \( \sigma_i \in [0.2; 0.8] \), where \( i = 1, 2 \).

Therefore, concerning the first kernel \( G_\sigma[k] \), we establish a window \( W_1[n] \) around each T-wave peak and a window \( W_2[n] \) around each P-wave peak, both beginning 120 ms before the corresponding fiducial point and ending 120 ms after that. Then, we apply a special normalization process over the kernel samples, such that:

a) its maximum amplitude should be equal to the maximum amplitude of \( |W_1[n]| \) or \( |W_2[n]| \);

b) its minimum amplitude before its maximum position should be equal to the ‘first’ minimum of \( |W_1[n]| \) or \( |W_2[n]| \), i.e., from the beginning to its maximum position;

c) its minimum amplitude after its maximum position should be equal to the ‘second’ minimum of \( |W_1[n]| \) or \( |W_2[n]| \), i.e., from the maximum position to end.

As explained, we normalize the samples of each analysed kernel in two stages. Firstly, we normalize the samples from the beginning to the maximum position according to the maximum and minimum values of \( |W_1[n]| \) or \( |W_2[n]| \), from the beginning to the maximum positions. Secondly, we normalize the samples of the analysed kernel from its maximum position to its end, according to the maximum and minimum values of \( |W_1[n]| \) or \( |W_2[n]| \), from their maximum positions to their end.

Then, the normalized kernel \( G_{\sigma_1}^n[k] \) is aligned by its peak amplitude location with T-wave peak and P-wave peak, respectively, in \( |W_1[n]| \) and \( |W_2[n]| \). Finally, we...
compute a normalized mean square error $E_{\sigma_1}^{(1)}$ for each tested parameter $\sigma_1$ and each analysed wave, given as

$$E_{\sigma_1}^{(1)} = \frac{\sum_{k=P_i-\gamma}^{P_i+\gamma} |W_i[k]-G_{\sigma_1}[k]|^2}{\sum_{k=P_i-\gamma}^{P_i+\gamma} |W_i[k]|^2},$$

where $P_i$ corresponds to the peak position (T-wave or P-wave) inside the analysis window $W_i[n]$ ($i=1, 2$), and $\gamma$ refers to the number of samples equivalent to 150 ms.

Therefore, we search for the value of $\sigma_1$ related to the minimum value of $E_{\sigma_1}^{(1)}$, named as $E_{\sigma_1}^{\text{min}}$, which refers to the normalized least mean square error between the best fitted kernel and a given characteristic wave.

Analogously, we proceed with the steps of amplitude normalization and peak alignment for the other proposed kernels, that is, $G_{\sigma_2}[k]$ , $R_{\sigma_1}[k]$ , $R_{\sigma_1}'[k]$ . The computing of the respective evaluation parameters $E_{\sigma_2,\sigma_2}^{(2)}$, $E_{\sigma_1}^{(3)}$ and $E_{\sigma_1}^{(4)}$, as well as the searching for the best fitted kernels, are also analogous, attempting to the specific case of $E_{\sigma_1}^{(2)}$, where all possible combinations of $\sigma_1$ and $\sigma_2$ should be assessed. Then, we compute, for each tested ECG signal, the average values of $E_{\sigma_1}^{(1)}$, $E_{\sigma_2,\sigma_2}^{(2)}$, $E_{\sigma_1}^{(3)}$, $E_{\sigma_1}^{(4)}$, that is, the corresponding normalized least mean square errors, for both T-wave and P-wave modelling, as well as the corresponding standard deviations. Also, we compute, for each analysed ECG signal, the average and standard deviation of the fitted parameters $\sigma_1$ and $\sigma_2$ related to each characteristic wave. As an illustration of the proposed technique, we can observe in Figures 2(a) and 2(b) the process of T-wave and P-wave modelling using kernel $G_{\sigma_1,\sigma_2}[k]$ for record sele0106 (QT database). Figures 2(c) and 2(d) present the mapping of pairs $\sigma_1, \sigma_2$ related to the best fitted kernels for the analysed waves and P-waves, respectively, using the second channel.

![Figure 2: T-wave and P-wave modelling using the composition of two Gaussian functions for record sele0106: (a) and (b) best fitted kernels for T-wave and P-wave, (c) and (d) mapping of pairs $\sigma_1, \sigma_2$ related to the best fitted kernels for the analyzed waves.](image.png)

**Results**

We evaluated the algorithm using the publicly available QT database (QTDB), which was developed for wave limits validation purposes with a total of 105 records and with two leads, each of them with a length of 15 minutes and 250 Hz sampling frequency [11].

Firstly, we compare the overall average of mean and standard deviations of normalized least mean square errors $E_{\sigma_1}^{(1)}$, $E_{\sigma_2,\sigma_2}^{(2)}$, $E_{\sigma_1}^{(3)}$, $E_{\sigma_1}^{(4)}$ for both T-wave and P-wave modelling, considering, for each record, the channel with the minimum mean value for the corresponding metric, as summarized in Table 1.

<table>
<thead>
<tr>
<th>Kernel</th>
<th>T-wave</th>
<th>P-wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\sigma_1}[k]$</td>
<td>$4.90% \pm 6.38%$</td>
<td>$12.02% \pm 15.90%$</td>
</tr>
<tr>
<td>$G_{\sigma_2}[k]$</td>
<td>$1.56% \pm 3.74%$</td>
<td>$9.13% \pm 15.04%$</td>
</tr>
<tr>
<td>$R_{\sigma_1}[k]$</td>
<td>$6.95% \pm 7.47%$</td>
<td>$28.88% \pm 24.89%$</td>
</tr>
<tr>
<td>$R_{\sigma_1}'[k]$</td>
<td>$3.20% \pm 5.85%$</td>
<td>$26.90% \pm 22.05%$</td>
</tr>
</tbody>
</table>

We also found the overall average of mean and standard deviations of the parameters $\sigma_1, \sigma_2$, for kernel $G_{\sigma_1,\sigma_2}[k]$, and $\sigma_1$, for $G_{\sigma_1}[k]$, $R_{\sigma_1}[k]$, and $R_{\sigma_1}'[k]$, concerning T-wave and P-wave modelling for the first lead of the available leads in QT database in Table 2.

| Table 1: overall average of mean (a) and standard deviations (sd) of normalized least mean square errors for each kernel (a ± sd). |

<table>
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Table 2: overall average of mean (a) and standard deviations (sd) of the parameters related to the fitted kernels: $\sigma_1$ and $\sigma_2$. |
<table>
<thead>
<tr>
<th>Kernel</th>
<th>T-wave: $\sigma_1$</th>
<th>P-wave: $\sigma_1$</th>
<th>T-wave: $\sigma_2$</th>
<th>P-wave: $\sigma_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{\sigma_1}[k]$</td>
<td>0.51±0.05</td>
<td>0.35±0.09</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$G_{\sigma_1\sigma_2}[k]$</td>
<td>0.59±0.07</td>
<td>0.33 ± 0.10</td>
<td>0.43 ± 0.07</td>
<td>0.35 ± 0.1</td>
</tr>
<tr>
<td>$R_{\sigma_1}[k]$</td>
<td>0.79±0.03</td>
<td>0.78 ± 0.04</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$R_{\sigma_2}[k]$</td>
<td>0.76±0.03</td>
<td>0.78±0.05</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

**Discussion**

Although previous works have proven that a normal P-wave resembles a Gaussian shape [12], our preliminary results indicate a predominantly asymmetric behaviour for both T-wave and P-wave, so that the kernel obtained as a composition of two Gaussian functions provided the most accurate results for modelling purposes. Also, the time-reflected version of Rayleigh probability density function behaved as an efficient alternative for T-wave modelling.

**Conclusion**

This paper evaluates the performance of four different mathematical models for segmenting and modelling T-wave and P-wave, using a well-known QT database for experimental simulations, which provides a wide range of morphologies and signal-to-noise ratios.

Based on the experiments conducted in this work and on preliminary results, we conclude that a composition of two Gaussian functions is a robust alternative for fitting a diverse set of both T-wave and P-wave morphologies, which are predominantly asymmetric. The Rayleigh probability density function, a typical asymmetric mathematical function, also presented a satisfactory performance for fitting T-waves.

In our future research we intend to investigate and compare the robustness and adaptability of the studied models in each of specific morphologies: positive, negative, upward, downward and biphasic waves.

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**References**