A desert of gas giant planets beyond tens of au: from feast to famine

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ABSTRACT

It is argued that frequency of gravitational fragmentation of young massive discs around FGK stars may be much higher than commonly believed. Numerical simulations presented here show that survival of gas giant planets at large separations from their host stars is very model dependent. Low-mass clumps in slowly cooling discs are found to accrete gas very slowly and migrate inward very rapidly in the well-known type I regime (no gap open). They are either tidally disrupted or survive as planets inwards of about 10 au. In this regime, probability of clump survival at large separations is extremely low, perhaps as low as 0.001, requiring up to a dozen clumps per star early on to explain the observed population. In contrast, initially massive clumps or low-mass clumps born in rapidly cooling discs accrete gas rapidly. Opening deep gaps in the disc, they migrate in the much slower type II regime and are more likely to survive beyond tens of au. The frequency of disc fragmentation in this case is at the per cent level if the clump growth saturates at brown dwarf masses but may be close to 100 per cent if clumps evolve into low stellar mass companions. Taking these theoretical uncertainties into account, current observations limit the number of planet mass clumps hatched by young massive discs around FGK stars to between 0.01 and ∼10. A deeper theoretical understanding of such discs is needed to narrow this uncertainty down.

Key words: accretion, accretion discs – planets and satellites: formation – planet-disc interactions – Planetary Systems.

1 INTRODUCTION

Evolution of protoplanetary discs is an active area of research (for recent reviews, see Haworth et al. 2016; Kratter & Lodato 2016). One of the open questions is the self-gravitational instability (Toomre 1964) of protoplanetary discs, which may (Kuiper 1951; Boss 1997) or may not (e.g. Kumar 1972; Goldreich & Ward 1973; Laughlin & Bodenheimer 1994) lead to formation of gas giant planets by fragmentation of the discs on clumps of a few Jupiter mass (see Helled et al. 2014, for a review).

Gammie (2001) has shown that self-gravitating gas discs fragment only when the disc cooling time, $t_{\text{cool}}$, is shorter than a few local dynamical times, e.g. $t_{\text{cool}} \ll \beta / \Omega_k$, where $\Omega_k$ is the local Keplerian frequency and $\beta \sim 3$–6 (see also Johnson & Gammie 2003; Rice, Lodato & Armitage 2005; Cossins, Lodato & Clarke 2009). Rafikov (2005) used this fact to constrain the location of disc fragmentation to separations $R \gtrsim 50$ au (see also Durisen et al. 2007; Boley 2009; Rogers & Wadsley 2012).

More recent simulations suggest that gas discs may fragment at higher values of $\beta$ (e.g. Meru & Bate 2011, 2012; Paardekooper 2012), and it is currently not clear whether there is a well-defined value of critical $\beta$ (Michael et al. 2012; Rice, Forgan & Armitage 2012; Rice et al. 2014) or the fragmentation is stochastic in nature (Lin & Kratter 2016; Young & Clarke 2016). Nevertheless, there is a general agreement that discs are not likely to fragment closer than tens of au to the host star (see Kratter & Lodato 2016).

The outcome of gravitationally unstable disc fragmentation is even more controversial. Most authors (e.g. Stamatellos & Whitworth 2008; Kratter, Murray-Clay & Youdin 2010; Forgan & Rice 2011, 2013a; Tsukamoto et al. 2015) find that the initial fragment mass is typically too high to be relevant to formation of gas giants that are observed to be dominated by planets with masses $\sim 1 M_J$ (e.g. Mayor et al. 2011), leading to formation of brown dwarfs rather than planets. This may however be debatable since the current uncertainty in the exact disc fragmentation conditions translates into a much larger uncertainty in the initial fragment mass (section 4.3 in Nayakshin 2017); some authors do find $\sim 1 M_J$ fragments forming in their discs (e.g. Boley et al. 2010).

Evolution of gas clumps after their formation is also highly uncertain. Gas clumps of planetary mass are found to migrate inward on time-scales of just a few thousand years, depending on the disc and planet masses and other parameters (Mayer et al. 2004; Vorobyov & Basu 2006; Boley et al. 2010; Inutsuka, Machida & Matsumoto 2010; Baruteau, Meru & Paardekooper 2011; Cha & Nayakshin 2011; Machida, Inutsuka & Matsumoto 2011; Michael, Durisen & Boley 2011). Further, the clumps may evolve not only in separation but also in mass. If they accrete gas rapidly (Kratter et al. 2010;
Zhu et al. 2012a; Forgan & Rice 2013b), they transit into the brown dwarf or even the low-mass stellar companion category. The reverse is possible too. If clumps do not accrete gas and contract slowly, then they are tidally disrupted (Boley et al. 2010; Nayakshin 2010a) after they migrate too close to the host star, leaving behind a solid core (if grain sedimentation within the clump was sufficiently rapid; see Boss 1997; Helled & Schubert 2008; Helled, Podolak & Kozvet 2008; Nayakshin 2011). This channel of clump evolution is the base for the tidal downsizing scenario of planet formation (see the recent review in Nayakshin 2017).

In this paper, the role of the cooling rate of the gas inside the Hill sphere of the clump in determining its evolutionary path is investigated. First, a simple analytical argument is made. Accretion of gas on to the embedded clumps must be inefficient when the gas cooling time is long compared with the clump’s cooling time. The validity of the argument is then investigated numerically, and confirmed, in the setting of the ‘β-cooling’ self-regulating discs.

The β-cooling model is useful in its clarity and simplicity of comparison to analytical arguments, and has been used by a great number of authors, but real astrophysical flows have cooling times depending on the local gas density and temperature rather than gas distance from the host star. Therefore, in the next part of the paper, I consider a protoplanetary disc with an approximate but physically better motivated cooling model. The long and the short cooling time regimes are explored in this model by varying a dust opacity multiplier, meant to capture the significant microphysical uncertainties in dust grain physics (e.g. Semenov et al. 2003; Dullemond & Dominik 2005; Helled & Bodenheimer 2011). In addition, both pre-collapse (low density) and post-collapse (high density) gas giant planets are considered (see Section 3.1.2 for detail). A wide range of initial gas clump masses, from 0.5 to 16 M_\text{Jup}, is covered.

## 2 ANALYTICAL ARGUMENTS

### 2.1 Maximum accretion rate

We are interested in the maximum accretion rate for the planet, \( M_{\text{max}} \), which would indicate how important gas accretion on to the planet could be. It is convenient to define the minimum growth time-scale for the gas clump as

\[
\tau_{\text{acc}} \equiv \frac{M_\text{p}}{M_{\text{max}}}. \tag{1}
\]

To make progress here, let the planet be on an approximately circular Keplerian orbit around the star a distance \( R \) away from it. We shall further focus on the type I migrating planets embedded into massive self-gravitating discs. In this regime, the Hill radius of the planet,

\[
R_{\text{Hill}} = R \left( \frac{M_\text{p}}{3M_\odot} \right)^{1/3} \tag{2}
\]

is smaller than the disc vertical scaleheight, \( H \). I estimate the maximum accretion rate on to an embedded planet as

\[
\dot{M}_{\text{max}} \sim 4\pi\rho_{\text{mid}}c_\text{s}R_{\text{cap}}, \tag{3}
\]

where \( \rho_{\text{mid}} \) is the mid-plane disc density, \( c_\text{s} \) is the isothermal sound speed and \( R_{\text{cap}} \) is the gas capture rate that I define by the condition

\[
\frac{GM_\text{p}}{R_{\text{cap}}} = \frac{3}{2}c_\text{s}^2. \tag{4}
\]

where mono-atomic gas equation of state is assumed for the specific energy of the gas \( \gamma = (\gamma/2)c_\text{s}^2 \). This condition requires that the thermal energy of the gas is equal to the gravitational potential from the planet the distance \( R_{\text{cap}} \) away. For a marginally stable self-gravitating disc, the mid-plane disc density is about equal to the tidal density, defined as

\[
\rho_{\text{tid}} = \frac{M_\text{p}}{2\pi R^3}. \tag{5}
\]

Using this in equation (3), assuming hydrostatic balance for the disc, \( H/R = c_\text{s}/v_\text{K} \), where \( v_\text{K} = (GM_\odot/R)^{1/2} \), we obtain,

\[
\tau_{\text{acc}} \sim \frac{1}{\Omega_\text{K}} \left( \frac{H}{R_{\text{Hill}}} \right)^3, \tag{6}
\]

which can be very fast. For example, consider the planet \( M_\text{p} = 3 \text{M}_\text{J} \) mass, for which \( R_{\text{Hill}} = 0.1R \) for \( M_\odot = 1\text{M}_\odot \). The disc scaleheight \( H \) for the discs studied below is \( 0.2-0.3 \) at \( R \sim 100 \text{au} \), and \( 1/\Omega_\text{K} \approx 160 \text{yr} \). With these parameters, the minimum planet growth time-scale is of the order of one thousand to a few thousand years. These estimates for maximum planet accretion rates are in a rough agreement with those of Zhu et al. (2012a) but are some two orders of magnitude higher than accretion rates found for gas giant planets in the type II regime (Lubow, Seibert & Artymowicz 1999) when they open a deep gap in their disc.

Since clumps embedded in self-gravitating discs are found to migrate in on time-scales of a few thousand years, we see that gas accretion on to the clumps could be in principle an even faster process that leads to the planets ‘running away’ into the brown dwarf mass domain (e.g. Zhu et al. 2012a).

On the other hand, equation (6) is based on the absolute maximum gas accretion rate on to the planet. A reduction in gas accretion rates by factors of many is possible if the gas is too hot within the Hill sphere to be captured by the planet; this may lead to the planet migrating inward at approximately constant mass instead (see Nayakshin & Cha 2013; Stamatellos 2015). It is hence important to understand when the process of gas accretion is efficient and when not.

### 2.2 Cooling time switch for gas accretion

In the non-inertial system of coordinates rotating together with the planet, the relative azimuthal velocity of a circular Keplerian flow at radius \( R + \Delta R \) is \( \Delta v = -(3/2)\Omega_\text{K}\Delta R \). This implies that the time-scale for the differential gas flow to cross the Hill sphere of the planet is

\[
\tau_{\text{cross}} = \frac{R_{\text{Hill}}}{\Delta v(R_{\text{Hill}})} \sim \frac{1}{\Omega_\text{K}}. \tag{7}
\]

The gas enters the planet Hill sphere with a positive total energy. To become gravitationally bound to the planet, this energy needs to be reduced by radiative cooling and become negative before the gas leaves the Hill sphere on the other end. Defining a cooling time-scale for the gas, \( \tau_{\text{cool}} \), as

\[
\tau_{\text{cool}} \equiv \frac{u}{\Gamma}, \tag{8}
\]

where \( u \) is gas specific energy and \( \Gamma \) is the specific energy loss rate by radiation, we should expect that gas accretion on to the planet will be efficient if

\[
\tau_{\text{cool}} \ll \tau_{\text{cross}}, \tag{9}
\]

and inefficient in the opposite case, \( \tau_{\text{cool}} \gg \tau_{\text{cross}} \). In the latter limit, the gas enters the Hill sphere only temporarily. After entering the sphere, it is compressed and heats up nearly adiabatically. It is very
likely to leave the Hill sphere since it is too hot to be bound to the planet.

Note that this analytical argument only applies to the gas giant planets migrating inward rapidly in young very massive discs. Planets growing in much less massive discs via the core accretion mechanism (e.g. Pollack et al. 1996) usually do so inward of \( \sim 10\) au (e.g. Mordasini et al. 2009). When they reach the mass of a fraction of Jupiter mass, they typically open a deep gap in the disc and then start to migrate much slower in the type II regime (e.g. Crida, Morbidelli & Masset 2006). These planets have much more time to grow and therefore their atmospheres may contract on those much longer time-scales.

3 MASSIVE SELF-GRAVITATING \( \beta \) DISCS

3.1 Numerical setup

3.1.1 Gas discs

Baruteau et al. (2011) studied migration of massive gas giant planets in self-gravitating discs in which radiative cooling of the disc is described by the widely used ‘\( \beta \)-cooling’ model (e.g. Gammie 2001; Rice et al. 2005):

\[
\frac{du}{dt} = -\frac{u}{t_{\text{cool}}},
\]

where the cooling time \( t_{\text{cool}} \) is a function of radius \( R \) only, described by

\[
t_{\text{cool}}(R) = \beta \Omega_R \rho_{\text{crit}}^{-1},
\]

where \( \beta \) is a positive constant and \( \Omega_R = (GM_*/R^3)^{1/2} \) is the Keplerian angular frequency at radius \( R \). They considered \( \beta = 10, 15 \) and 30, for which their discs are in a self-regulating but not fragmenting gravito-turbulent state (Gammie 2001).

I follow a similar physical setup here with an added focus on capturing gas accretion on to the planets. Smoothed particle hydrodynamics (SPH, see Price 2012) is used to perform the simulations, and in particular the cosmological code GADGET-3 (Springel 2005), adapted for simulations of protoplanetary discs as previously detailed in Nayakshin & Cha (2013). The disc has an initial disc surface density profile \( \Sigma(R) \propto R^{-3/2} \) initially defined in the radial range \( 20 \text{ au} \leq R \leq 200 \text{ au} \), with the total gas mass \( M_d = 0.4 \text{ M}_\odot \). The disc is orbiting the central star of mass \( M_*= 1 \text{ M}_\odot \), which is modelled as a sink particle with accretion radius \( r_a = 5 \text{ au} \). Initially, no planet is present in the disc, and the disc is relaxed for tens of orbits on the outer edge, so that it settles into the self-regulating quasi-steady state (Gammie 2001). During this relaxation, some of the disc material is accreted on to the star, so that the disc mass decreases to \( M_d \approx 0.35 \) (typically). The equation of state is that of an ideal mono-atomic gas for both the disc material and the planet.

3.1.2 Treatment of the planet

Radiative contraction of a gas giant planet formed by gravitational disc instability is conveniently divided into the pre-collapse and the post-collapse phases (Bodenheimer 1974). The former is relevant to the gas clumps that have only recently formed out of the surrounding protoplanetary disc. They are dominated by molecular hydrogen, spatially extended and thus have low gas densities. They are therefore susceptible to the tidal disruption by the host star at separations as large as a few au to tens of au (Boley et al. 2010).

After the clumps get rid of their excess formation heat and their central temperature reaches \( \sim 2000 \text{ K} \), H\(_2\) molecules dissociate, and the planet collapses dynamically to much higher densities. This process is similar to the second collapse in star formation (Larson 1969). The post-collapse clumps (planets) are orders of magnitude denser and hotter. Such planets may be tidally disrupted only at \( \sim 0.1 \text{ au} \) separations from the star, which we do not study here.

For the \( \beta \)-cooling discs, only pre-collapse planets are studied. The clumps are modelled explicitly with SPH rather than with a sink particle approach (e.g. Bate, Bonnell & Price 1995). The latter is best suited for problems in which gas is accreting on to the sink at nearly the free-fall velocity, such as the classical Bondi–Hoyle accretion problem; for accurate results the sink radius, \( r_a \), must be picked much smaller than the Bondi capture radius (see Cuadra et al. 2006). Radius of pre-collapse planets is typically only a factor of a few times smaller than the Hill radius. Gas captured by the planet from the surrounding disc settles on to it at sub-sonic velocities rather than at free-fall, as we shall see later on. For both of these reasons, the sink particle approach is not desirable.

The evolution of pre-collapse planet internal structure depends on many physical variables and physical effects (e.g. Helled & Bodenheimer 2011; Vazan & Helled 2012; Nayakshin 2015b) that cannot be yet simulated self-consistently in 3D. A very simple model is therefore employed to model the clump and its contraction. To initialize the planet, a uniform density gas sphere is inserted in the gaseous disc that was relaxed as described in the previous section. The mass of the sphere is \( M_0 = 3 \text{ M}_J \) and its radial extent is \( r_0 = 3 \text{ au} \). The initial clump temperature is set to 10 K, which is characteristic of the temperature in the outer cold protoplanetary discs. The initial density of the clump is \( \approx 1.5 \times 10^{-12} \text{ g cm}^{-3} \). This initial value of density puts the clump in the parameter space where radiative hydrodynamics simulations of first core collapse predict that radiative cooling becomes stifled by dust opacity and the clump starts evolving approximately adiabatically (see fig. 4 in Masunaga & Inutsuka 2000). Our clump is however a factor of \( \sim 2–3 \) cooler and therefore it is strongly gravitationally bound and initially collapses dynamically to higher density. The collapse slows when adiabatic compression heats the gas up so that the pressure gradient is sufficiently large to oppose clump self-gravity. Further contraction of the clump is governed by the rate at which it loses its thermal energy content.

Detailed stellar evolution calculations show that the contraction time of pre-collapse Jovian planets is of the order of \( \sim 10^7–10^8 \) yr (e.g. Bodenheimer 1974; Helled et al. 2008). The simple \( \beta \)-cooling law (equations 10 and 11) is clearly not appropriate inside the planet. Therefore, to avoid an unphysically fast contraction of the planet, the cooling law is modified at high densities,

\[
t_{\text{cool}} = \beta \Omega_R \rho_{\text{crit}}^{-1} \left(1 + \frac{\rho}{\rho_{\text{crit}}}ight),
\]

where \( \rho_{\text{crit}} \) is set to \( 10^{-10} \text{ g cm}^{-3} \) for most tests in this section. This value of the density is \( \sim 1000 \) times higher than the tidal density (that is, the maximum density of a \( Q = 1 \) disc) of the disc at \( R = 100 \text{ au} \), so that the modification is only significant at the surface or inside the planet.

The clump is inserted into the disc on a local circular orbit in the disc mid-plane at separation \( R_0 = 100 \text{ au} \) from the star. For definiteness, the current mass of the planet is defined in the simulations as the total mass of SPH particles within distance \( R_c = 0.5R_0 \) from the clump centre. This definition is chosen empirically, based on the result that the material found within that region is usually strongly bound to the planet and does not become unbound, whereas gas farther away from the planet centre is much more likely to be unbound.
3.2 The fiducial $\beta = 7.5$ case

3.2.1 Gas and velocity fields

I begin by studying the case with $\beta = 7.5$. This relatively small number of $\beta$ is interesting because marginally stable protoplanetary discs may be expected to have cooling times about equal to that of the model (e.g. Rice et al. 2005). The SPH particle number is $N_{\text{sp}} = 400000$ and the disc mass is $M_d = 0.4 M_\odot$ before the disc relaxation. The SPH particle mass is hence $m_{\text{sp}} = 10^{-6} M_\odot$. The planet is inserted at the location $(x, y, z) = (100, 0, 0)$ au at time $t = 0$.

The left-hand panel of Fig. 1 shows the map of the column density of the disc with the embedded planet after the latter made one revolution around the star, which corresponds to time 670 yr. The right-hand panel shows the same at much later time, $t = 5600$ yr. The star is located in the centre of both panels. The overlying white vector field shows the gas velocity flow map, normalized on the largest velocity found in the figure.

The clump migration rate is very high in this simulation (the separation and the mass versus time tracks of the planets are shown in Fig. 3). The planet has already migrated to radius $R = 71.4$ au in the left-hand panel of Fig. 1, after just one rotation. In this particular simulation, the planet migrates close to the inner boundary of the disc (20 au) in some $\sim 1500$ yr, but then practically stalls there. We shall later see that this stalling is only because there is little gas material there to interact with at radii less than $\sim 30$ au. This stalling of the planet is thus due to the initial conditions and numerics of our setup – the planet is likely to continue to migrate inward had the disc been present at smaller radii. On the other hand, 1D disc-planet migration calculations show that gas giant planets usually open deep gaps in the disc when they reach the radius of $\sim 10–20$ au (e.g. see figs 2 and 3 in Nayakshin 2015a), and then continue to migrate slower in the type II regime. Therefore we should expect the planet migration to slow down after reaching these radii in a full disc as well, although perhaps at slightly smaller radii. Understanding the eventual fate of the planet requires simulations on much longer time-scales of disc removal, $\sim 3$ Myr, which is not achievable due to numerical limitations in 3D simulations.

Fig. 2 shows a zoom-in view of the region around the planet at time $t = 670$ yr, with the white dotted circles showing the circles of radii $R_H$ and $0.5R_H$ centred on the densest part of the planet. For the latter plot, only SPH particles within a slab $|z| < 3$ au are used. The gas mass within the white circle is $4.4 M_J$, which is almost 50 per cent larger than the planet initial mass. Thus, the planet not only migrated but also increased in mass significantly in just one revolution around the star.

The velocity vectors, while showing prograde circulating motions around the centre of the planet, decrease rather than increase towards the centre of the planet. The velocity field is hence not Keplerian. This is because the material around the planet’s location is supported against planet self-gravity mainly by the gas pressure gradient rather than by rotation.

3.2.2 Numerical resolution

Fig. 3 shows the results of the already presented fiducial simulation, $\beta 7.5$, and two other simulations that are different only by the number of the SPH particles used. The dotted red curves are for simulations with twice as many SPH particles whereas the dashed blue ones are for twice as few. The top and the bottom panels of the figure show the planet mass and the planet–star separation, respectively, versus time. It should be noted that there is some ambiguity here in where to define the end of the planet and beginning of the
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Figure 2. Zoom in on the region close to the planet at time $t = 670$ yr. The circles drawn in the figure are centred on to the densest part of the planet and have radii of $R_{\text{H}}/2$ and $R_{\text{H}}$. For detail, see the text in Section 3.2.1.

Figure 3. Planet mass (top panel) and separation to the host star (bottom panel) versus time in a resolution study. The previously studied case, $\beta = 7.5$, with SPH particle mass $m_{\text{SPH}} = 10^{-6} \ M_{\odot}$, shown with the solid black curves, is compared with two simulations differing from it only by the number of particles. There are twice as many and twice as few SPH particles for the dotted and the dashed curves, respectively.

disc, since the outer planet envelope can be lost more easily than deeper, denser layers. For definitiveness, in Fig. 3 the planet mass is defined as the total mass of SPH particles within half of the Hill sphere.

We see that planet separation shrinks from 100 to about 25 au in just 1500 yr in all three of the simulations. There are some differences between the three sets of curves, but the variation is at the level of ~10 per cent between the three simulations. The planet mass changes are slightly larger than that. On the whole, while these differences are not negligible, they are acceptably small because it will be seen later on that both accretion and migration rates of the planet vary much more significantly when the physical parameters of the problem, such as $\beta$, $\rho_{\text{crit}}$, planet’s initial radius or density, and even the starting azimuthal angle, are varied. This suggests that numerical resolution of the fiducial run ($N_{\text{sph}} = 400 000$) is sufficient for our purposes here.

3.2.3 The planet and its envelope

The top panel in Fig. 3 shows an intriguingly abrupt mass loss episode for the planet around 1500 yr at all three resolutions. At $t = 1500$ yr, the planet mass is around $5.5 \ M_{\text{J}}$, but ~300 yr later it is reduced to about $4 \ M_{\text{J}}$. To investigate this further, Fig. 4 shows the gas surface density and the velocity map around the planet at two times corresponding to the peak in the planet mass before the mass loss episode and one of the following minima, for the simulation $\beta 7.5N2$.

The most striking difference between the two snapshots is the reduction in the physical size of the high-density part of the planet, which we define somewhat arbitrarily here by $\Sigma \geq 10^{3}$ g cm$^{-3}$. This reduction is all due to the planet separation shrinking from 40 to 25 au between the two snapshots, which severely reduces the size of the Hill sphere. The white circles in the figure show radii half and one Hill sphere, respectively.

Fig. 5 analyses the abrupt mass loss episode further by presenting the planet density profile at several times versus the distance from the planet centre (top panel) and versus the enclosed mass (bottom panel). The vertical lines in the top panel show the Hill radius distance from the planet centre, whereas the horizontal lines show the tidal density at the planet location (equation 5). Both lines indicate where the material is bound to the planet (to the left of the vertical and above the horizontal lines, respectively).

The solid black curve is for time $t = 50$ yr, which is soon after the perturbation inserted into the disc collapses on itself but not soon enough for the surrounding disc to react to the planet insertion. We can see this by noting that the planet, defined as a high-density region, ‘ends’ abruptly at mass $3 \ M_{\text{J}}$, the mass of the perturbation inserted. This abrupt density drop occurs very deep inside the Hill sphere; the density of the gas filling the rest of the sphere is barely above the tidal density (see specifically the bottom panel). That should be expected because the unperturbed density of a marginally stable self-gravitating disc should indeed be close to the tidal density (e.g. Gammie 2001).

The red dotted curve in Fig. 5 shows the planet at the same time as in the left-hand panel of Fig. 4, that is, very close to the peak in the planet mass. The planet has now significantly migrated, from 100 to about 40 au. Despite the Hill sphere shrinking, the planet gained an extended and massive envelope that contrasts with the sharp outer boundary present in the initial conditions for the simulation. This growth spur is possible because initially the planet is much smaller than its Hill sphere and so there is a significant volume to fill with accreting gas.
Figure 4. The region near the planet location in the simulation $\beta7.5N2$ at the peak of the planet mass and at one of the minima following it (cf. Fig. 3). The white dotted circles indicate a half and the full Hill sphere radius.

Figure 5. Projected gas density profiles on radial shells for simulation $\beta7.5N2$ at three selected times, as a function of either distance from the planet centre (top panel) or the enclosed mass (bottom panel). Note appearance and disappearance of the extended planetary envelope (cf. Section 3.2.3 for more detail).

We can see from the bottom panel of Fig. 5 that, if we include all the gas mass inside the Hill sphere, then the mass of the planet can be estimated at as much as $\sim 9M_J$ at this point. This is significantly higher than the $\approx 5.5M_J$ contained within the inner half of the Hill sphere (which is our standard definition of the planet mass and is used in the top panel of Fig. 3). This again shows the ambiguity in defining the planet mass in a simulation where the planet is extended and is migrating inward rapidly.

However, the mass of the accreted envelope is finite and is controlled by the entropy of the gas in the disc. Consider a toy model for the planet atmosphere which obeys $P = K\rho^{\gamma} = K\rho^{\gamma + \frac{1}{m}}$ equation of state with $K$ being a constant and $\gamma$ the polytropic index for the gas. Assuming that the gas speed of sound at the Hill radius is $c_0$, equal to that in the mid-plane of the unperturbed disc, and that the weight of the atmosphere is negligible compared to that of the planet, one can solve for the density profile in the atmosphere:

$$\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{\gamma}} - 1 = \frac{GM_p}{(n + 1)c_0^2} \left[ \frac{1}{r} - \frac{1}{R_H} \right],$$

where $\rho_0$ is gas density at the Hill radius, and $r$ is the distance from the planet centre. Neglecting unity on the left-hand side of this equation, the mass of the atmosphere is given by

$$M_{\text{atm}} = 4\pi\rho_0 \left[ \frac{GM_p}{(n + 1)c_0^2R_H} \right] \int_{r_{\text{atm}}}^{R_H} r^2 \left( \frac{R_H}{r} - 1 \right)^n dr.$$  \hspace{1cm} (14)

One can see that this integral is finite for both $n = 3/2$ (mono-atomic gas) and $n = 5/2$ (molecular hydrogen gas) even if $r_{\text{atm}}$ is formally set to zero. This then follows that the only way for the atmosphere to accrete on to the planet is to cool down, which would reduce the polytropic constant $K$. This result is not surprising at all; it is well known that core accretion atmospheres must be able to cool radiatively in order to reach the runaway gas accretion phase (e.g. Pollack et al. 1996).

The inner regions of the planet in Fig. 5 contracted to a density higher by a factor of $\sim 2$ than the initial one. This relatively moderate contraction is mainly due to radiative cooling rather than addition of more mass from the disc, as will become clearer in Section 3.2.4.

Comparing the tidal density horizontal cuts in the bottom panel of Fig. 5 at the two later times, we see that the planet should stand...
to lose at least $\sim 2 M_\oplus$ of mass when it migrates from 40 to 25 au in just $\sim 300$ yr. In fact, most of the extended planetary envelope is lost in that rapid plunge. The blue dashed curve in Fig. 5, corresponding to the time when the planet reaches the separation of 25 au, features a rather abrupt outer planetary surface at about $4.5 M_\oplus$. The sudden removal of the outer envelope unbinds material further in, so that the gas density actually drops below the tidal density at about two thirds of the Hill radius rather than at the Hill radius. Later evolution shows that the envelope is gradually rebuilt by accretion of more gas into the Hill sphere. This becomes once again possible because the planet no longer migrates rapidly into the inner disc once it reaches its inner edge (which in itself is due to our numerical setup and limitations, see Section 3.2.6).

The fact that the planet can accrete and then lose its extended envelope should prepare us for the thought that the outcome of planet migration should also depend on the planet physical parameters, such as its size (mean density), and not just the planet mass. It also suggests that one should be careful quoting mass for objects formed in the simulations of fragmenting gaseous discs – the fragment mass can change not only up, by gas accretion, but also down by the loss of the accreted envelope.

### 3.2.4 Dependence on the planet size and contraction rate

To investigate the uncertainties in the outcome of the simulations with regard to the internal physics of the planet, e.g. the planet physical size and the planet contraction rate, the initial conditions for the perturbation inserted into the disc were varied. In simulation $\beta 7.5R0.5$, the outer radius of the perturbation at time $t = 0$ was halved compared to $\beta 7.5$, whereas in simulation $\beta 7.5R2$, the initial radius is doubled, to $r_0 = 6 \, \text{au}$.

To test how the contraction rate of the planet due to radiative losses affects the outcome, simulation $\beta 7.5C9$ is started from the same initial condition as simulation $\beta 7.5$, but the cooling time prescription (equation 12) uses $\rho_{\text{cr}} = 10^{-9} \, \text{g cm}^{-3}$ instead of $10^{-10} \, \text{g cm}^{-3}$. This allows the planet and the immediate envelope to contract more rapidly. All three of these simulations are done in the ‘double’ resolution ($N_{\text{SPH}} = 0.8 \, \text{Million}$).

Fig. 6 shows how these three simulations differ from the previously shown simulation $\beta 7.5N2$, reproduced with the black curve for comparison in the figure. We see that an increase in the planet initial radius causes a relatively small decrease in the planet mass. This is driven by a reduction in the volume available for the planet envelope building within the Hill sphere. This change in the accreted mass is especially small if one considers the fact that the mean density of the initial perturbation varies by the factor $4^{3/2} = 64$ between the blue, the black and the red curves. However, based on equation (14), a change in $r_{\text{anm}}$ could be expected to lead to a small change in the planet mass only, because for $\gamma = 5/3, n = 3/2$, the integral becomes insensitive to the exact value of $r_{\text{anm}}$ once $r_{\text{anm}} \ll R_H$ (the atmosphere mass is dominated by the region $r \lesssim R_H$, not $r \sim r_{\text{anm}}$). On the other hand, if the initial planet radius was closer to the initial Hill radius, the results of the varying planet radius could be more severe. Similarly, if the planet migrated inward more, then the radius where it can be tidally disrupted does depend on the initial perturbation strongly. Therefore, while the tests analysed here show a rather weak dependence on the initial perturbation properties, there may be situations where this will not be the case.

In simulation $\beta 7.5C9$ however, the planet manages to accrete far more gas than it does in the fiducial simulation. The reason for this can be understood by noting that the densest layers of the envelope accreted by the planet have densities of the order of $\rho \sim 10^{-10} \, \text{g cm}^{-3}$ (see the red and the blue curves in the top panel in Fig. 5). This value coincides with the critical density at which radiative cooling is quenched in the fiducial simulation (equation 12) and in all other simulations except for $\beta 7.5C9$, where the critical density is one order of magnitude higher. Therefore, while radiative cooling is insufficiently rapid for a significant envelope contraction in simulation $\beta 7.5$, the same envelope is able to cool down significantly in $\beta 7.5C9$. The contraction of the envelope, which has a mass comparable to the initial perturbation mass, causes the central parts of the planet to contract as well. Analysis of the internal structure of the planet shows that the central density of the planet is almost an order of magnitude higher in the simulation $\beta 7.5C9$ compared to $\beta 7.5$.

These numerical experiments show that the physical size of the planet and its contraction rate do matter in determining the accretion rate on to the planet, with the latter being particularly important. This is ‘unfortunate’ as far as the big picture is concerned but is a key part of the physical problem at hand. On a more positive note, these physical uncertainties are larger than those due to numerical artefacts (Section 3.2.2), telling us that we should be worried less about numerics and more about including additional physics.

### 3.2.5 Stochasticity and dependence on the initial separation of the planet

The left-hand panel of Fig. 7 shows how the results of the simulations in which the cooling rate parameter is fixed at $\beta = 7.5$ change when the azimuthal angle of the planet initial position in the disc, $\phi$, is varied. The solid black curves are same as for the fiducial
There is a significant dependence of the results on the angle \( \phi \). This is consistent with the results of Baruteau et al. (2011) who found strong stochastic fluctuations in the planet migration patterns as they varied the starting \( \phi \)-angle of their planets. This is not surprising since the gas surface density and the velocity fields are highly non-uniform in strongly self-gravitating discs.

The starting angle \( \phi = 90^\circ \) yields the track that differs from the rest the most. The planet reverses the direction of its migration in this case, wanders out to separation \( R \approx 110-120 \) au before migrating in. It takes the planet \( \sim 3 \) times longer to eventually migrate into the inner disc than in the three other simulations displayed in the figure. The amount of material accreted by the planet in the \( \phi = 90^\circ \) case is \( \sim 25 M_J \), which is an order of magnitude more than in the other cases. This clearly demonstrates that the length of the time available to the planet to accrete more gas and for that gas to contract to higher densities before it is plunged into the inner disc region, where the Hill radius is much smaller, is the key determinant of gas accretion on to the planet.

The right-hand panel of Fig. 7 shows the same experiment but for the clump dropped into the disc at \( a = 140 \) au. We observe a similar stochasticity in the outcome as in the left-hand panel. Interestingly, the most massive clump in this case forms not from the clump that spends the longest time away at larger radii (the \( \phi = 270^\circ \) clump), but from the clump that started at \( \phi = 0^\circ \), which spends second longest amount of time at large distances. This may indicate that gas accretion on to a planet also depends on the relative speed between the planet and the gas entering the Hill sphere; a very high relative speed effectively reduces the size of the Hill sphere. In general, however, the clumps in the right-hand panel of the figure end up more massive than their counterparts in the left-hand panel, reinforcing the conclusion that the final mass attained by a clump increases with the amount of time the clump can accrete gas before it plunges too close to the host star.

### 3.2.6 How far in will the clump go, and will it survive?

It is notable that none of our simulated planets migrated inward closer than \( R \approx 20 \) au. This is significantly larger than the inner edge at \( R_{in} = 10 \) au in our initial pre-relaxation disc. The observed stalling of the planets far from the initial inner edge thus begs an explanation.

In all of the numerical experiments presented above, the sink radius of the star was set to \( r_{sink} = 5 \) au. We shall now see that this and \( R_{in} = 10 \) au choices influence the disc structure at radii out to \( \sim 30 \) au significantly. The reason for this is numerical. The artificial viscosity of the SPH, required to deal with shocks, is especially large in the innermost disc region where there are few SPH particles (e.g. Price 2012). This speeds up angular momentum transfer in the vicinity of the star artificially, so that particles initially located at radii much larger than \( r_{sink} \) end up in the star during the simulation. For the planet migration problem, this implies that we cannot trust our results in the innermost region, up to the distance of \( \sim 5-6 \) times the sink radius, as will be seen below.

To investigate this issue, we set up an additional simulation, \( \beta 7.5R_{in}5 \), which has an inner edge of the disc set at the pre-relaxation
stage to a smaller value, $R_{\text{in}} = 5$ au, and the sink radius for the host star is also significantly reduced from the default value, to $r_{\text{sink}} = 1$ au. The initial disc surface density profile was set proportional to $\propto R^{-2}$ rather than $R^{-3/2}$ (this difference is inconsequential as far as the main point of this section goes), and it is then relaxed as described in Section 3.1.1.

Fig. 8 compares the planet mass and separation evolution of the fiducial simulation, $\beta 7.5$, with $\beta 7.5 R_{\text{in}} 5$. We see that the fragment now migrates closer in, to $R \approx 15$ au, at which point it is tidally disrupted. At that point, the planet stops to exist and its material rejoins the disc.

To understand this result better, the azimuthally averaged surface density of the gas, $\Sigma$, multiplied by area of the disc $\pi R^2$, is plotted in Fig. 9 for several different times. In this section, we call the quantity $\Sigma(R)\pi R^2$ the mass of the disc, $M_{\text{disc}}$, even though it is defined per ring. The top panel shows $M_{\text{disc}}$ versus radius for the simulation $\beta 7.5 R_{\text{in}} 5$, whereas the bottom panel shows the same but for simulation $\beta 7.5$. The vertical lines show positions of the planet at the respective times. Note that the mass of the planet is included in this plot and is seen as (usually) a sharp peak in $M_{\text{disc}}$.

Fig. 9 makes it abundantly clear that the place where the planet stops in these simulations is when the disc ‘ends’, so that there is no more gas in the planet vicinity to continue to push it further in. In the fiducial simulation, where the planet survives as a massive $M_p \sim 6 M_J$ gas clump at the radius of about 23 au, the planet actually opens a deep gap. This can be seen by comparing the dotted red and the dashed blue lines in the bottom panel of Fig. 9: due to angular momentum exchange between the gas and the planet needed to enable planet migration, the gas initially located at $R \sim 30$ au was redistributed further out as the planet migrated through that region. Such stalling of planets on the inner edge of the disc hole (opened in a circumbinary disc by the binary torques) was previously found by Pierens & Nelson (2008).

The planet in the simulation $\beta 7.5 R_{\text{in}} 5$ was in the process of performing a similar feat, albeit probably stalling at about 10 au, but was unable to complete the manoeuvre because it filled its Hill radius completely and therefore was tidally destroyed. The dashed blue curve in the top panel of Fig. 9 shows the clump at the time when it is already partially destroyed. This is not surprising since the Hill radius for a planet of mass $3 M_J$ at 15 au is only 1.5 au, which really cuts quite deep into the planet proper (e.g. see the top panel in Fig. 5).

Tidal disruption of the planet aside, this additional experiment shows convincingly that when the planet is not disrupted and stalls at the inner edge of the disc at $R \sim 20$ au, this is a numerical artefact of our simulations. In better resolved simulations, we should expect the planet to continue to migrate inward. Whether it is disrupted or not depends on a number of additional factors (the rate of planet contraction, the disc structure and whether a deep gap in the disc is eventually opened).

### 3.3 Dependence of results on $\beta$

Fig. 10 shows the clump mass and separation versus time in calculations identical to $\beta 7.5$ but for four larger values of the cooling
rate parameter $\beta$. These discs were set up and relaxed in a similar manner to that of $\beta=7.5$.

We see that the simulations for the two smaller values of $\beta$ (10 and 15) are qualitatively similar to the $\beta=7.5$ set of runs. In all of these three cases, one of the four starting values of $\phi$ led to the planet migrating outward first, or at least not migrating in as much (e.g. the case $\phi=0$ for $\beta=10$). In each of the simulations where the planet hang at wide separations for longer it managed to accrete much more gas than it did in the other runs with same value of $\beta$. This again indicates that the longer the period of time that the planet

Figure 10. Same as Fig. 7, but for four larger values of the cooling parameter $\beta$ as shown above the respective panels. Note a general trend: the larger the $\beta$, the less gas is accreted by the planet in general.
spends at wide separations before it migrates inward significantly, the more gas is accreted by the planet. This reiterates the role of radiative cooling in allowing the atmosphere around the planet to contract more and hence become more massive before it plunges into the inner disc (cf. Section 3.2.3).

Compared to the $\beta$7.5$\phi$90 case, the planet in the simulations $\beta$10$\phi$40 and $\beta$15$\phi$270 however lose much of the mass they accreted while at large separations when they migrate in rapidly. This can be understood by noting that it takes longer for the accreted atmospheres to contract at higher $\beta$, so the outer parts of the accreted atmospheres are ‘shaven off’ when the planets plunge to smaller separations rapidly. The higher the value of $\beta$, the more planet accretion is a reversible process.

Another conclusion that can be drawn from Figs 7 and 10 is that the higher the value of $\beta$, the smaller the mass of the accreted gas on to the planet in general. In addition to that, the stochastic fluctuations in the planet migration tracks practically disappear for the two largest values of $\beta$. This is driven by the decrease in the density and mass of the spiral density arms in the discs with increasing $\beta$. In other words, the discs are much more uniform in the high $\beta$ cases.

4 DISCS WITH MORE REALISTIC COOLING

Tests with the $\beta$-cooling model presented in the previous section reveal the importance of the gas cooling rate in determining the accretion rate on to the planet. However, this cooling model is unsatisfactory for a number of reasons. The cooling rate of real astrophysical flows must depend on the gas density, temperature, optical depth and the presence of external irradiation (from the star, the planet and/or the larger scale parent molecular cloud). For the problem at hand, this implies that the (effective) $\beta$ parameter changes as a function of separation from the star (usually decreasing with increasing separation), the distance from the planet and the planet mass.

4.1 The cooling model

A simple yet widely used model for radiative cooling of discs is based on a plane-parallel geometry and an assumption that heat escapes via radiative diffusion (e.g. Shakura & Sunyaev 1973; Johnson & Gammie 2003; Rafikov 2005; Boley et al. 2010; Vorobyov & Basu 2010). In particular, we follow Galvagni et al. (2012) and Nayakshin & Cha (2013), and set the energy loss rate per unit volume as

$$\Lambda = (36\pi)^{1/3} \frac{\sigma_{SB}}{s} \left( T^4 - T_{eq}^4 \right) \frac{\tau}{\tau^2 + 1},$$

(15)

where $s$ is a length-scale explained below and $\sigma_{SB}$ is the Stefan–Boltzmann constant. The equilibrium temperature $T_{eq}$ is assumed to be established by the irradiation from the star and is set to

$$T_{eq} = 20 \text{ K} \left( \frac{R}{100 \text{ au}} \right)^{-1/2}.$$  

(16)

Most of the results presented below depend weakly on this choice for $T_{eq}$.

The length-scale $s$ is the disc scaleheight, and is approximated by $s = 0.1 R$. The disc optical depth is then estimated as

$$\tau = \kappa \rho s,$$

(17)

where $\rho$ is the local gas density and $\kappa$ is disc opacity. The latter is assumed to be dominated by dust, and is given by

$$\kappa = f_{op} \kappa_0(\rho, T),$$

(18)

where $\kappa_0(\rho, T)$ is the interstellar dust opacity as given in table 1 of Zhu, Hartmann & Gammie (2009). The positive coefficient $f_{op}$ is a free parameter, introduced to account for uncertainties such as grain growth, gas metallicity different from Solar, and other microphysical uncertainties in the dust opacity modelling, such as dust composition (e.g. Semenov et al. 2003).

4.2 Initial conditions

The initial conditions for the simulations presented in this section are created by starting with a gas disc of mass $M_d = 0.2 M_\odot$ with the surface density profile $\Sigma(R) \propto R^{-3/2}$ defined between the inner radius of 10 au and the outer radius of 140 au. The gas is set on the local circular orbits initially. The initial number of SPH particles is 0.5 Million, which corresponds to the SPH particle mass of $4 \times 10^{-15} M_\odot$. There is no embedded planet in the disc, and the disc is relaxed for about 8000 yr using the cooling model explained above with $f_{op} = 0.1$. With this relatively low value of $f_{op}$, the disc cools down to about the equilibrium temperature (equation 16) everywhere but is not self-gravitating due to its lower mass than in Section 3. The host star mass is initially $M_\star = 1 M_\odot$, as previously, but its accretion (sink) radius is reduced to 2 au.

Note that the initial disc mass is lower for these simulations than the one we used earlier for the $\beta$-cooling discs. These discs are insufficiently massive to develop strong spiral features before the planet is added to the disc. This is therefore a significantly different regime from the one investigated earlier, although the disc mass is still rather large. The practical reason for picking less massive discs here is that for the non-linear cooling law we study now it is hard to predict when the discs are close to the marginally stable regime and yet will not produce additional fragments when the planet is inserted. While that parameter space is important to investigate, in this paper we shall try to avoid additional stochasticity arising from having more than one clump in the disc (fragmentation can be triggered by planets, see e.g. Nayakshin & Cha 2013; Meru 2015). Furthermore, looking at these less massive discs with a completely different cooling law is also useful to test whether the simple analytical argument of Section 2 on the growth of planets is general enough to apply to these models too.

4.3 Two contrasting cases

Figs 11 and 12 show gas surface density maps from two simulations that are started from the same initial condition, with the initial planet mass of 2 $M_\oplus$, but assume two very different opacity factors $f_{op} = 10$ and 0.01, respectively. For both of the figures, all of the panels are coeval, but the time is not the same between the two figures. The two cases were chosen to be compared at the time when the planets reached the same separation, $R = 59$ au, rather than time. For the case of $f_{op} = 10$ this separation corresponds to time 2630 yr, whereas for the $f_{op} = 0.01$ case the time is nearly exactly 4000 yr.

The left-hand panels of the figures show the global disc structure in the face-on view. The middle and the right-hand panels are centred on the planet location, and show the face-on and the edge-on views of gas flows around the planets, respectively.

There is a striking difference between the two cases. This is driven by the differences in the accretion rate on to the planets. In the $f_{op} = 10$ case, the planet mass (defined as the total mass of gas within half the Hill sphere of the planet) is $M_p = 2.06 M_\oplus$ at the time of the figure, hardly higher than the initial planet mass. The low-opacity case, $f_{op} = 0.01$, is a very different story – here
Figure 11. Snapshots from a simulation of a gas clump of initial mass of 2 M\(_J\) with opacity factor \(f_{\text{op}} = 10\). The clump has migrated to separation \(a = 59\) au attracting hardly any gas from the surrounding disc. Left: Top view of the protoplanetary disc with the embedded clump. Middle: Zoom-in on to the clump location. Right: Same but a side-view projection of the region around the clump.

Figure 12. Same as Fig. 11 but for the opacity factor of \(f_{\text{op}} = 0.01\). Due to the lower opacity, the clump accretes gas much more rapidly and hence becomes a massive brown dwarf, opening a wide gap in the parent disc. The contracting atmosphere around the clump has a rotationally supported disc geometry.

the planet mass is \(M_p = 43.06\) M\(_J\). These differences reflect the fact that opacity strongly influences the rate at which the gas cools. Qualitatively, the high \(f_{\text{op}}\) case reflects the very long cooling time (large \(\beta\)) regime, whereas the low \(f_{\text{op}}\) case corresponds to the short cooling time cases (small \(\beta\)).

The high-opacity case planet, remaining too low mass, continues to migrate in the type I regime (Baruteau et al. 2011), affecting the disc structure around it much less than the low-opacity case planet which actually runs away well into the brown dwarf mass regime. A deep broad gap is opened in the gas disc in this case (Fig. 12). The planet mass is about half of the isolation mass (Lissauer 1987), \(M_{\text{iso}} \approx M_\odot \sqrt{M_p/M_*} \approx 0.09\) M\(_\odot\), suggesting that the growth of the planet cannot go on at this high rate for much longer. Due to the gap and also due to its large mass, the planet migrates inward slowly in this regime.

The morphology of gas around the planets is also different in these two cases. In the high-opacity case, the atmosphere around the planet is quasi-spherical, being hot and weakly bound to the planet. Note that velocity vectors are very small inside the half \(R_H\) radius of the planet, suggesting that gas is supported against gravity in that region by pressure gradient rather than rotation. In the opposite case of \(f_{\text{op}} = 0.01\), the thermal pressure support is minimal because the radiative cooling of the gas captured by the planet is rapid and so the gas is sub-virial inside the Hill sphere. The Hill sphere of the planet is also larger due to the high mass that the planet achieved by the time of the snapshot. For this reason, the angular momentum of the gas captured by the planet from the disc is larger in Fig. 12, so the gas settles into a disc that is supported mainly by rotation around the planet rather than thermal pressure.

These two contrasting cases show that the rate of cooling remains the key determinant of what happens to a clump embedded in a massive protoplanetary disc in the more realistic although approximate radiative cooling model for the disc. As with \(\beta\)-cooling discs, the longer the cooling rate, the less efficient the gas accretion on the planet is.

4.4 A suite of simulations

Fig. 13 shows the mass-separation tracks for a number of simulations with set-up and initial conditions identical to those explained...
Figure 13. Mass versus separation for pre-collapse gas clumps of different masses, all starting on a circular orbit at $a = 100$ au initially. The violet colour curve is the planet mass needed to open a deep gap in the disc (equation 19), and it is the same in all the panels. In each panel, the lines of different style refer to the different opacity factor $f_{\text{op}}$, as explained in the legend, which controls the cooling rate. For low $f_{\text{op}}$, gas cools rapidly, fuelling rapid gas accretion on to the planet. For large $f_{\text{op}}$, gas accretion is inefficient. See Section 4.4 for detail.

above, but now for a range of the initial planet mass and several values of $f_{\text{op}}$. In particular, the initial masses explored here are 0.5, 1, 2, 4, 8 and 16 $M_J$; the opacity factor $f_{\text{op}}$ is set to 0.01, 0.1, 1 and 10. The vertical axis shows the clump mass in Jupiter masses while the horizontal axis shows the clump separation in au. The violet (parabola-like) curve is the gap-opening planet mass, $M_{\text{gap}}$, plotted versus radius. It is defined following Crida et al. (2006), who have found that parameter

$$C_p = \frac{3}{4} \frac{H}{R_i} + \frac{50 \pi H^2 M_*}{R^2 M_p}$$

(19)
needs to be smaller than unity for the planet to open a gap. Here $\alpha$ is the viscosity parameter that we set to $\alpha = 0.06$ in this equation, which is appropriate for strongly self-gravitating discs (Rice et al. 2005). In computing $C_p$, the disc vertical scaleheight is found from $H/R = (k_b T_{eq} R / GM_\ast m_p)^{1/2}$, where $T_{eq}$ is the equilibrium irradiation temperature given by equation 16.

The figure shows that the planets grow in mass rapidly, becoming massive brown dwarfs, if dust opacity is low, e.g. for $f_{\text{dust}} \ll 1$, when the radiative cooling is rapid. This result is particularly striking for the planet with the initial mass of 0.5 $M_\odot$ (black solid track in the top left panel), which increased in mass by a factor of about 100 during the simulation with $f_{\text{dust}} = 0.01$, but migrated in only by a factor of $\sim 2$. As explained previously, the physical reason for the very massive objects to not migrate much in their discs during these simulations is the gap that they open in the disc. Due to that gap, they switch to the slower type II migration.

Slowly cooling planets, that is high dust opacity cases (green and blue curves), however, do not grow in mass significantly except for the most massive planet case. In this limit, the planets are unable to accrete gas rapidly and so continue to migrate in quickly in the type I regime. As a result, they eventually migrate ‘too far’, where their Hill radius is about the physical size of the planet, so that the planets become unbound.

Comparing now the planet migration tracks to the $M_{\text{gap}}(R)$ curve, we note that the curve does a reasonably accurate job at predicting when the migration rate of a planet drops significantly (this will also be discussed further in Section 5). Indeed, once a planet runs away towards masses of $\sim 20$–$30$ $M_\odot$, the migration tracks become approximately vertical, implying a much slower migration rate. Additionally, planet tracks get ‘smudged’ in Fig. 13 when the planets open deep gaps because the planet spends in that nearly stalled location more time. Some eccentricity growth is also apparent after the planets open gaps. In comparing the migration tracks to equation (19), it should also be noted that this equation is derived assuming a planet on a fixed orbit. Malik et al. (2015) have recently showed that a gap opening planet must have a migration time-scale which is shorter than the time-scale for the disc viscous flow to close the gap, which makes gap opening more difficult, increasing the estimate above $M_{\text{gap}}$ somewhat.

A secondary effect can be glimpsed from the figure: the higher the initial mass of the clump, the less likely it is to be tidally disrupted and the more likely it is to end up as a massive brown dwarf. The highest initial mass case, $M_p = 16$ $M_\odot$, is not disrupted and grows more massive for all of the values of $f_{\text{dust}}$ considered. In contrast, for the lowest initial planet mass, $M_p = 0.5$ $M_\odot$ case, only the smallest $f_{\text{dust}} = 0.01$ clump ‘runs away’ into the brown dwarf domain, with the three higher opacity cases leading to tidal disruption.

This sensitivity to the initial planet mass is due to more massive gas clumps contracting much more rapidly (Nayakshin 2010b; Helled & Bodenheimer 2011; Nayakshin 2015b). As we have seen in Section 3.2.4, the smaller the physical size of the planet, the larger is the accretion rate on it.

5 ACCRETION ON POST-COLLAPSE PLANETS

5.1 Using sinks for gas accretion

So far only the pre-collapse gas clumps were considered in the study. That is a very important case to consider. However, there may be processes not included or not resolved in our study that allow gas fragments to contract and then collapse more rapidly than our simple radiative cooling models allow. For example, grain growth may be much more rapid inside the planet than in the disc since the planet is much denser than the disc (e.g. Cha & Nayakshin 2011). If grain opacity of the planet is reduced by grain growth sufficiently, then the planet may collapse much faster (e.g. Helled & Bodenheimer 2011) than our cooling prescriptions allow. Post-collapse planets are multiple orders of magnitude denser than the pre-collapse ones. Processes such as small-scale convection inside the planet must be resolved in detail in order to follow their evolution on Million year or longer time-scales carefully (e.g. Burrows et al. 2001), which is clearly not possible here due to numerical simulations.

There is therefore little choice but to resort to the sink particle approach (Bate et al. 1995) to model post-collapse planets in 3D simulations of protoplanetary discs. The simplest capture-all within sink radius $r_{\text{sink}}$ approaches creates an artificial vacuum in that region and an unphysical pressure gradient force, almost certainly leading to overestimating accretion rate unless $r_{\text{sink}}$ is very (usually numerically uncomfortably) small (e.g. Cuadra et al. 2006). To prevent the development of the unphysical positive pressure gradient at $r \approx r_{\text{sink}}$, I soften the gravitational potential of the planet with $h_{\text{fp}} = 0.2$ au. The sink radius for the planet used for simulations in this section is set to $r_{\text{sink}} = 0.3$ au, which is much smaller than the sink radius for the star (set to 5 au here).

In addition, only the gas particles that are (a) inside distance $r_{\text{sink}}$ from the planet/sink and (b) whose gas density exceeds $\rho_{\text{cut}} = 5 \times 10^{-12}$ are accreted. Numerical experiments showed that these conditions prevent accretion of hot particles not physically bound to the sink. Gas captured by the planet’s potential forms a dense ‘atmosphere’ around the planet which results in development of a negative pressure gradient, as physically expected. This atmosphere prevents particles from the disc entering the Hills sphere of the planet only temporarily from being artificially accreted. The newly arrived gas gets bound to the planet only if it cools rapidly. In that case, the mass and the density of the atmosphere around the sink increases with time and eventually both of the above conditions for gas accretion are satisfied. This approach therefore provides a barrier and a time delay in accretion of gas particles on to the sink and hence differentiates between the physical and unphysical accretion of gas on to the sinks.

In the Appendix, I show numerical experiments that test performance of the sink particle approach compared to resolving the planet directly for pre-collapse planets. It is found that the sink particle prescription generally overestimates the accretion rate on to the planet, but that usage of a sufficiently small sink radius leads to reasonably accurate gas accretion rates. The value of the sink radius where the accretion rates saturated at acceptably accurate values was found to be $r_{\text{sink}} \sim 0.1$ au, which is smaller than the value we use below. However, numerical experiments with sink radius as small as $r_{\text{sink}} = 0.1$ au showed additional numerical difficulties for the case of rapidly cooling discs. In that limit, the gas cools rapidly enough to fuel vigorous accretion on to the planet. The accreted gas possesses a large amount of specific angular momentum with respect to the planet and first settles into a disc inside the Hill sphere. Keeping $r_{\text{sink}}$ at small values allows one to resolve that disc better, but then the disc itself becomes gravitationally unstable and requires yet higher resolution to be modelled properly. I therefore choose to compromise and set the sink particle radius at somewhat larger value or $r_{\text{sink}} = 0.3$ au, with which the issue of massive circum-planetary disc is far less severe. Note that this choice is unlikely to influence the final mass of the simulated planets significantly because in this rapidly cooling limit the planet opens a deep gap (see below) and essentially stops migrating on the timescale of the simulations. The situation when the circum-planetary
curves) result in the planet accreting less or no gas at all. These
fixed mass (see Section 5 for discussion).
whereas high-opacity cases do not accrete gas and instead migrate in at a
before, low-opacity cases accrete gas rapidly and run away to high masses
inward too much is thus unlikely to be realized in practice in this
regime. Most of the circum-planetary disc mass will eventually end
up in the planet (or more massive object).
Concluding on the sink particle approach, it is expected to be rea-
sonably accurate in the rapidly cooling regime, when the accretion
rate is large. At lower cooling rates, we should expect that accretion
rates found here are overestimate. These numerical limitations do
not jeopardize our main conclusions.

5.2 Results
The initial conditions for the disc and the planet used in this section
are identical to those used in Section 4 except that sink particles
are used to model the planets. The planet mass measured in the
runs presented here is the sink mass rather than the mass within
half of the Hill sphere (as in Section 4). This does not lead to
noticeable differences in results since for post-collapse planets the
sink accounts for nearly all mass within the Hill sphere anyway.

The suit of numerical experiments with the initial planet mass
ranging from 0.5 to 16 M\(_J\) and the disc opacity factor varied from
f\(_{\text{op}}\) = 0.01 to 10 (all as in Section 4) is then run.

Fig. 14 shows simulations performed for the initial planet mass
of \(M_p = 2 M_J\). We observe that, as previously, low-opacity cases
result in a rapid gas accretion on to the planet, so that it runs away to
brown dwarf masses. Larger values of f\(_{\text{op}}\) (dashed and dot–dashed
curves) result in the planet accreting less or no gas at all. These
planets migrate in faster and further because they do not open a gap
in the parent disc. Also note that when the planets open deep gaps in
the discs they slow down and so this is where the separation versus
time plot becomes nearly horizontal in Fig. 14. For the two low-
opacity cases, \(f_{\text{op}} = 0.01\) and 0.1, this occurs at around mass \(M_p \approx\)
20M\(_J\), when the planets are at separation \(\sim\)40–60 au. This is very
close to what equation (19) predicts for \(M_{\text{gap}}\) at such separations
(cf. Fig. 13). For the two higher opacity cases, the planets stall at separation \(\sim\)10–20 au. Their masses are only 2 and 4 M\(_J\) at that
point, which is a factor of \(\sim\)2–3 smaller than equation (19) predicts.
However, the sink radius of the star is 5 au in these simulations,
implying that the structure of the disc at around 10 au is probably
significantly affected by it. It is likely that this makes gap opening
somewhat easier, explaining why the planets tend to stall near the
inner edge of the disc even if their mass is somewhat below \(M_{\text{gap}}\).
Additionally, the lower mass of the disc near the inner edge also
means that planets migrate in slower than they would if the flow
was better resolved to much smaller radii.

Fig. 15 shows the same grid of post-collapse planet simulations as
was shown in Fig. 13 for pre-collapse planets. To improve visibility
of the similar curves in Fig. 15, the curves are shifted by the factor
of 1.1 vertically from one another.

The results of the sink particle runs are overall very similar to
those obtained in Section 4 as far as gas accretion is concerned. We
again notice that low-opacity cases (low \(f_{\text{op}}\)) encourage much faster
accretion of gas on to the planets. We also note that there is a trend
with the planet initial mass. The higher the mass of the planet, the
more likely it is to run away into the massive brown dwarf territory
by accreting gas rapidly. Minor differences do appear, such as the
fact that none of the post-collapse runs for the lowest planet mass,
\(M_p = 0.5 M_J\), resulted in a massive object, whereas the \(f_{\text{op}} = 0.01\)
run from the respective panel in Fig. 13 did runaway towards a
brown dwarf mass. This minor difference may be sensitive to the
sink particle prescription parameters, such as the sink radius.

The largest difference between the post-collapse case and the
pre-collapse calculation is that in the former case the planets are a
factor of \(\sim\)100 more compact and are therefore not tidally disrupted.
The end result of the runs that do not accrete gas in Fig. 15 is a gas
planet that migrated to the inner \(\sim\)10–20 au. We would need to
follow these cases right to the moment of their disc dissipation
to ascertain the outcome, but the range of outcomes is quite clear.
If the disc is removed rapidly then these giant planets may be cold
or warm gas giants (with separation from sub au to \(\sim\)10 au). If the
disc is removed later then the planet may become a hot Jupiter or
even be plunged all the way into the star.

6 OBSERVATIONAL IMPLICATIONS

6.1 Hypothesis I: Fragmentation of discs on planet mass
clumps is very rare

Although there are well-known massive gas giant planets orbiting
their hosts at separations of tens of au, such as HR 8799 system
(Marois et al. 2008, 2010), statistically there is a significant lack of
gas giant planets found at wide separations (e.g. Vigan et al.
2012; Bowler et al. 2015; Chauvin et al. 2015; Rice et al. 2015).
Biller et al. (2013) find that no more than a few per cent of stars
host 1–20 M\(_J\) companions with separations in the range 10–150 au.
Galicher et al. (2016) find that \(\sim\)1–2 per cent of Solar type stars have
a gas giant planet with mass between 0.5 and 14 M\(_J\) and separations
20–300 au. Vigan et al. (2017) have recently combined results of
their VLT/NaCo large direct imaging programme with the results of
12 past imaging surveys to obtain a limit on the frequency of a much wider mass range of substellar companions, from 0.5 to 75 $M_J$, at large separations from their hosts. They find that the frequency of these objects cannot exceed a few percent for nearby FGK stars.

The most natural interpretation of the severe observational constraints on the frequency of sub-stellar companions is that conditions in the young star forming discs are rarely right for fragmentation of the discs by the gravitational instability (Biller et al. 2013; Rice et al. 2015; Vigan et al. 2017). For example, most discs could be smaller than $\sim 50$ au in radial extent needed for the discs to fragment (Rafikov 2005). However, observational estimates of pre-stellar core rotation rates (Caselli et al. 2002) do not currently support the claim that most discs (99 per cent) need to be smaller than $\sim 50$ au and so cannot fragment. Vorobyov (2011), using hydrodynamical simulations of collapsing pre-stellar cores and observational constraints on their rotation rates, find that typical disc sizes are in hundreds of

Figure 15. Same as Fig. 13 but for post-collapse planets modelled as sink particles. As previously, planets accrete gas rapidly when the opacity factor $f_{op}$ is low, allowing gas to cool rapidly. In the opposite case, planet growth stalls and then migrate inward quickly instead. Since post-collapse planets are very dense, they are not disrupted tidally in the simulations even if they migrate to the inner boundary of the computational domain.
au range in both class 0 and class I. There is observational support
for such large-scale discs in some of the youngest systems (e.g.
see Liu et al. 2016). On the other hand, Vorobyov (2011)’s simu-
lations do not include effects of magnetic fields which could reduce
the disc linear size significantly; the statistics on the frequency of
large-scale (more than tens of au) massive protoplanetary discs is
currently missing.

Further support for low incidence of gravitational fragmentation
of discs may come from theoretical modelling. Rice et al. (2015)
assume one substellar object per star at large separations from the
host star surviving after the protoplanetary disc is removed. They
then show that N-body scattering with the secondary stars (most
stars form in binaries) would put a surprisingly large fraction of the
planetary mass objects into the inner au region of the host:
∼5 per cent. Since gravitational fragmentation is not expected to
correlate strongly with metallicity of the host, these objects would
contradict the strong positive metallicity correlation observed for
gas giant planet hosts (e.g. Fischer & Valenti 2005).

This argument does not however constrain the population of ob-
jects born by gravitational instability while the disc was around.
For example, suppose there is one fragment born by the instability
per star at early times, and that by the time the disc is dissipated
there remains only 0.01 fragment per star at these large separations.
The frequency of these objects scattered into the sub-au region per
star will be 0.01 × 0.05 = 5 × 10⁻⁴, far too small to be detectable
compared with the ‘normal’ population of hot Jupiters (∼1 per cent,
see Santerne et al. 2016).

Finally, population synthesis models of gravitational disc frag-
mentation can be used to estimate the expected fraction of gas giant
planets and brown dwarfs at large separations. Comparing the mod-
els of Forgan & Rice (2013b) to the observations, Vigan et al. (2017)
find a large disagreement with the data unless disc fragmentation is
rare.

However, population synthesis models are simplified compared
with 3D calculations and contain many approximations and as-
sumptions. The freedom in choosing the parameters and physics
employed – e.g. initial disc masses and sizes, exact masses of the
fragments, migration rates and switch-over conditions from type I
to type II, rates of gas accretion on to the planets, dust opacity and
grain sedimentation physics – may lead to widely different conclu-
sions. For example, population synthesis of Nayakshin & Fletcher
(2015) and Nayakshin (2016) predicts that only ∼5 per cent of gas
clumps born at wide separations survive there by the age of a few
Million years when the disc is dispersed, in stark contrast to the
results of Forgan & Rice (2013b).

6.2 Hypothesis II: Gravitational fragmentation is widespread
but most objects are destroyed

The data on frequency of sub-stellar objects around FGK stars
remaining at wide separations after the disc dispersal cannot be
argued with: these objects are quite rare (less than a few percent per
star). However there are very solid theoretical reasons to argue that
the initial population of these objects was much larger.

Rapid inward migration of wide separation (tens to hundreds of
au) planetary mass clumps has been found in simulations by many
independent groups (Mayer et al. 2004; Vorobyov & Basu 2006;
Boley et al. 2010; Inutsuka et al. 2010; Vorobyov & Basu 2010;
Baruteau et al. 2011; Cha & Nayakshin 2011; Machida et al. 2011;
Michael et al. 2011; Nayakshin & Cha 2013; Malik et al. 2015;
Stamatellos 2015; Tsukamoto et al. 2015). The defining character-
istic of the process is the fact that a few Jupiter mass
clumps are not massive enough to open deep gaps in their pro-
toplanetary discs at these wide separations (Baruteau et al. 2011;
Malik et al. 2015). Therefore, they migrate in the type I regime
(Tanaka, Takeuchi & Ward 2002; Bate, Bonnell & Bromm 2003)
which results in migration time-scales, t_mig, of just a few thousand
years for such massive discs.

Planet migration stops when protoplanetary discs are dissipated.
The discs are believed to exist for t_loc ∼ 3 Myr (e.g. Haisch, Lada &
Lada 2001) before being dispersed. It then follows that planets are
overwhelmingly likely to have migrated in by the time their host discs
are gone. In a very rough estimate, we can expect that the fraction of
planets survived on the wide separation orbits is ∼t_mig / t_loc ∼ 0.001.
The observed fraction of a percent then implies that there was a
dozens of planetary mass clumps per star to begin with.

These simple estimates are however inadequate and ideally one
needs to perform a complete calculation from the disc formation
to fragmentation and until it is dispersed. Very few of the detailed
planet/brown dwarf migration simulations were able to do this due
to a very large dynamical range of the problem. The few exceptions
that did model the disc for extended times overproduced the number
of fragments at large separations. For example, Zhu et al. (2012b)
found that 3 out of 13 fragments formed in their simulations be-
came brown dwarf mass objects; they opened a gap and essentially
stalled at large separations. Vorobyov (2013) found that 6 out of
his 60 simulations formed giant planets or brown dwarfs on stable
wide separation orbits. While this outcome is a minority in both
publications, it nevertheless significantly overpredicts the number
of observed objects of this type.

The other potential problem with Hypothesis II is that many of the
models quoted above tested only one or several disc configurations
were not quite self-consistent (e.g. ‘injecting’ planet mass objects
into the disc rather than forming them from first principles, for
example see Baruteau et al. 2011; Nayakshin & Cha 2013; Malik
et al. 2015) or neglected important processes such as gas accretion
on to the fragments.

Thus, the potential ability of migration to remove lots of initial
fragments still needs to be confirmed with more work.

6.3 Work presented in this paper

The contribution of this paper to testing Hypothesis II is in trying
to understand one important but yet insufficiently studied process
– gas accretion on to migrating gas clumps. While the processes of
disc fragmentation and clump migration have by now been studied
in dozens of publications specifically designed to test those two
processes, the process of gas accretion on to the clumps has been
sometimes avoided artificially (by introducing a large gravitational
softening radius for the planet, see e.g. Baruteau et al. 2011). In
other studies the process is present but not analysed in detail (e.g.
Boley et al. 2010; Cha & Nayakshin 2011; Vorobyov 2013).

In this paper, we explored gas accretion on to the clumps in rela-
tively simple settings to uncover what controls the efficiency of the
process. Simple order of magnitude arguments (Section 2) showed
that gas can accrete on to the clumps faster than they migrate. In
this case, the planetary mass clumps are rapidly turned into brown
dwarfs or even more massive objects. However, simple arguments
also suggest that gas accretion may become inefficient when gas
entering the Hill sphere of the clump cools on a time-scale longer
than a few local dynamical times.
In Section 3, simple but well-known and widely used $\beta$-cooling models for marginally stable self-gravitating discs were investigated. These discs are self-gravitating by construction at all radii except the inner region close to the star and beyond the outer disc radius. Pre-collapse gas planets were specifically studied, and were modelled directly by inserting a higher density gas perturbation of fixed mass $3 \, M_J$ in the disc at time zero (Section 3.1.2). Numerical resolution and numerical artefacts were probed to find limitations of the simulations. It was found that a few Jupiter mass clumps tend to migrate all the way in through the simulated discs as close as our numerical models allowed. In a simulation that extended the disc further in the clumps did not survive the tidal disruption (Section 3.2.6).

These fixed cooling time models confirmed an utmost importance of gas thermodynamics in controlling the rate of gas accretion on the clumps. There is a competition between the process of clump migration, which shrinks the size of the Hill sphere and hence leads to planet envelope mass loss, and the process of gas accretion. Faster radiative cooling usually allows the clumps to become more massive (see Section 3.3). Additionally, denser planets, planets migrating in from further out, or those that were kicked further out by stochastic interactions with the spiral density waves, usually stand a greater chance of growing by gas accretion.

In Sections 4 and 5 we considered more realistic models of massive discs in which radiative cooling depends on the local disc temperature, density and dust opacity. The opposite rapidly cooling and the slowly cooling regimes in these models are covered by varying the poorly constrained dust opacity of the disc. To do so, the interstellar dust opacity of Zhu et al. (2009) was multiplied by a factor $f_{\text{op}}$, which was varied in a broad range (from 0.01 to 10). We also considered a broad range in the initial planet mass, from 0.5 to $16 \, M_J$.

The models in Sections 4 and 5 are quite different from the idealized cooling disc models in that (a) their discs are massive but yet not strongly self-gravitating so that no strong spiral density waves are present; (b) they are passively heated by irradiation from the host star; (c) due to these differences, these discs may be in the regime where the local cooling time is shorter than dynamical, which cannot be tested in the ideal $\beta$ models because such discs cannot exist (Gammie 2001).

### 6.4 Summary of main results from the grid of models

Fig. 16 summarizes the results of the grid of models calculated in Sections 4 and 5. The panels on the left-hand and on the right-hand show the pre-collapse planets (studied in Section 4) and the post-collapse ones (Section 5), respectively. Each line in the plot represents one simulation and connects the initial planet mass and separation with the final ones. The different initial planet mass cases are plotted with different symbols and line styles and colours. The lines of the same type differ by the opacity parameter $f_{\text{op}}$. For example, for the four dotted red lines, starting from the triangle at initial values $(a, M_p) = (100 \, \text{au}, 1 \, M_J)$ in the left-hand panel, three simulations with $f_{\text{op}} = 10, 1$ and 0.1 led to the fragment tidal disruption at a few tens of au separation (cf. the top right-hand panel in Fig. 13), whereas the $f_{\text{op}} = 0.01$ simulation resulted in the planet becoming a massive brown dwarf by accreting gas rapidly.

We can see that where the clump will end up in the mass-separation phase space in the end of a simulation depends very sensitively on the opacity of the run, which unfortunately remains very uncertain for protoplanetary discs. However, there is one opacity independent outcome for all of the simulations performed in this paper. There is a desert of gas giant planets with separation greater than $\sim 20 \, \text{au}$ (more realistically $\sim 10 \, \text{au}$, see below). The desert is depicted by a rectangular box in the figure. The desert existence is due the trio of processes – migration, accretion and disruption – always removing the gas clumps from that region. Variations in the input physics or parameters of the simulation control which of the three processes is dominant in removing the gas giants from the desert. This therefore affects where a gas clump starting at $a = 100 \, \text{au}$ ends up in the diagram, but all of the possible outcomes from the simulations presented here are outside the desert.

If gas clumps are in the pre-collapse (youngest) state, hydrogen is molecular, and the clump density is $\sim 7$–10 orders of magnitude lower than that of the present day Jupiter (Nayakshin 2010b). The clump is then found to evolve in one of two ways.

**Disruption.** If gas opacity is high, then the cooling rate of gas captured into the Hill sphere of the planet is low, and hence gas accretion on to the clump is inefficient (see analytical arguments in Section 2). The clump also does not contract significantly for the same reason. It migrates inward at more or less constant mass.

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**Figure 16.** The initial and the final positions of the simulated planets in the mass versus separation parameter space. For each simulation, the initial mass and planet position is connected with the final quantities by a straight line. Note that not a single simulation ended up within the boxed region which is termed a desert. The desert is due to the clumps being taken out of that region by the inward migration, gas accretion or tidal disruption of pre-collapse planets. In the latter case, it is assumed that the clumps made massive solid cores which remain behind after the disruption of the clump. The mass of the remaining core (plus any envelope if it retains one) is set to $0.1 M_J$ arbitrarily for clarity of the figure.
until it fills its Roche lobe and gets disrupted by the tidal forces from the host star (Vorobyov & Basu 2006; Boley et al. 2010). Gas sedimentation inside the clump (McCrea & Williams 1965; Boss 1997; Helled & Schubert 2008; Helled et al. 2008; Nayakshin 2010b, 2011), not taken into account here, may form massive solid cores that are left behind after the gas clump is disrupted (Boley et al. 2010; Nayakshin 2010a; Cha & Nayakshin 2011). These massive cores are shown as symbols below the box in the left-hand panel in Fig. 16. The mass of the core is arbitrarily set at 0.1 M J in the figure but can be significantly less than that, especially in metal-poor environments.

Runaway accretion into the brown dwarf phase. When the gas opacity factor is low, gas accretion on to the clump is generally rapid. The clump grows in mass in a runaway fashion until it opens a gap in the disc. The clump exits the desert box through the upper boundary, becoming a massive brown dwarf. Its further evolution depends on the disc evolution on ~1 Myr time-scales, not modelled here, and is hence not clear. It could become even more massive by accreting more gas from the disc and/or it may migrate inward, potentially all the way in the inner 1 au from the host star.

On the other hand, grain growth inside the gas clumps or metal loading on to the clumps, both not studied here, may allow the clumps to collapse much faster (Helled & Bodenheimer 2011; Nayakshin 2015b). In this case, we should focus on the simulations with post-collapse planets, studied in Section 5, the results of which are shown in the right-hand panel of Fig. 16. These planets are much denser, with typical central densities ~10^{-3} g cm^{-3} or higher. These planets are too dense to be tidally disrupted outside the inner 1 au from the star.

Two outcomes are possible for these planets. If they accrete gas rapidly, they become brown dwarfs or even low-mass stellar companions, just as explained above.

Migration – formation of planets inward of ~10 au. In the opposite case, post-collapse planets do not accrete gas. These planets migrate rapidly in but they are not tidally disrupted because they are quite dense to begin with. In the short simulations presented here, the planets migrated to separations of a ~ 20 au and then stalled there. This is however the artefact of our inner boundary condition which requires removal of gas from the region closest to the star. A test with the inner boundary set closer to the host star showed that the planet then migrates further in (Section 3.2.6). Gas giant planets usually open deep gaps in the protoplanetary discs when they get to ~10 au (Nayakshin 2015c) radii because the discs are geometrically thinner there than at the planet’s birthplace and these discs are also not self-gravitating. These planets thus could migrate closer in to the star, but at a much slower (type II) migration rate. Where the planets end up inside the inner 10 au region is dependent on the disc evolution on longer time-scales not modelled here.

6.5 Further evolution of brown dwarf fragments

It is important to note that the brown dwarf fragments shown in Fig. 16 will continue to evolve and so will their positions in the diagram. At the end of our simulations, the mass doubling time of these objects is typically of the order of 50,000 yr, which is very short compared with the expected disc lifetime. These objects may thus gain much more mass, becoming low-mass stars, since the mass of the gaseous discs at the end of the simulations is still around 0.1 M J. In realistic discs (e.g. Vorobyov 2013), continuing gas infall from the outer envelope may deposit more gas not only outside but also inside brown dwarf orbit, making further growth of these objects possible. Likewise, the orbital separation of the brown dwarfs in Fig. 16 may continue to decrease, and perhaps significantly so, albeit in the slower type II migration regime. Further modelling, including the infalling protostellar envelope, is needed to address these issues. In the absence of such modelling, it is hard to say whether these simulations contradict the strict limits imposed on brown dwarf mass objects by the observations (see Vigan et al. 2017).

In terms of implications for the interpretation of the observed frequency of companions to nearby stars, the outcome divides into two possibilities. If brown dwarf mass objects remaining at tens of au separation in our study remain such and, additionally, do not migrate much closer in, then we would arrive at conclusions similar to that of Vigan et al. (2017): disc fragmentation must be very rare to explain the observed limits.

On the other hand, if the objects do evolve into the stellar mass regime and become secondaries to their stars, then gravitational fragmentation may be much more widespread. Raghavan et al. (2010) finds that roughly ~50 per cent of Solar type stars are in multiple systems. Given that multiplicity of the systems is likely to decrease with time due to gravitational interactions between the components or other nearby stars, we should expect that most solar type stars were in multiple systems early on. Further, because the separation distribution of the companions peaks at around 100 au, the ‘sweet spot’ for disc fragmentation (e.g. Rafikov 2005), most of these systems have likely formed via disc fragmentation rather than other means. This then argues that gravitational fragmentation of discs must be very common, with frequency of order unity.

6.6 Comparison to previous work

The results presented here are not very unexpected especially in terms of gas accretion on to clumps. Ormel, Kuiper & Shi (2015a) and Ormel, Shi & Kuiper (2015b) performed simulations of gas accretion on to massive solid cores embedded in gas discs on ~1 au scales from the host star. Despite the very different mass and length-scales, their study has similarities to the work presented here. Their results qualitatively agree with those summarized in Section 6.4. For long cooling times, the authors found that the atmosphere captured by the planet had a transient character and was not truly bound to the planet. This resulted in a significant reduction of gas accretion rate on to the planet compared with the rate of gas inflow into the Hill sphere of the planet. The same effect is seen in the simulations presented here.

The difficulty of gas clumps surviving as gas giants on wide orbits has also been discussed previously. Zhu et al. (2012a) simulated fragmentation of large-scale gas discs in 2D, allowing the discs to be built up self-consistently by the inflow of material from larger scales. Rather than varying the dust opacity as was done here, these authors instead used a fixed opacity (equivalent to setting f_{opt} = 1) but varied the deposition rate of gas into the disc. Their main conclusion is very similar to that of this paper: ‘... fast migration, accretion and tidal destruction of the clumps pose challenges to the scenario of giant planet formation by GI (gravitational instability) in situ...’. This is despite the fact that Zhu et al. (2012a) study uses very different numerical methods (fixed grid rather than SPH), initial disc set-up and assumptions. This seems to indicate a broad support to the thesis made in Section 6.4: numerical and physical detail do affect which of the three evolutionary paths the clumps take when they evolve out of the desert but the fact that they are not likely to survive in the desert as gas giant planets is independent of those detail.
Nayakshin & Cha (2013) and Stamatellos (2015) have shown that the outcome of gas clump formation in a massive gas disc may depend strongly on the thermal state of the gas inside the Hill sphere of the clump. Both studies considered the effects of radiative feedback of the clump/planet on to the surrounding gas. The former study focused on pre-collapse gas clumps, whereas the latter looked at post-collapse planets, but both found that radiative preheating of the surrounding disc by the radiation emanating from the planet may significantly reduce the accretion rate on to the planet. The strong radiative preheating regime is effectively same as the long cooling time cases studied here, preventing gas from accreting on to the planet.

The radiative feedback effect is not included in the simulations presented here in order to not overcomplicate the present study. However results of Nayakshin & Cha (2013) and Stamatellos (2015) add weight to the main conclusions of this paper. Both of these studies showed that when gas accretion is unimportant the planets migrate in rapidly whereas when accretion is fast the planets run away into the brown dwarf regime and stall at wide separations.

6.7 On the origin of disagreement with models by Forgan and Rice

Population synthesis modelling of Forgan & Rice (2013b) leaves most of gas fragments formed by fragmentation of gravitationally unstable discs behind at large separations (Fig. 3 in Vigan et al. 2017). In this scenario, the observed lack of such sub-stellar companions can only be understood if gravitational fragmentation is very rare. Numerical simulations presented in this paper however show that, independently of the initial fragment mass and gas cooling rate, none of them end up as massive gas giants at wide separations. I also argued (Section 6.5) that even the massive brown dwarfs may eventually evolve either inward or significantly up in their mass from their ‘final’ positions in Fig. 16.

Could these two opposing results and views be reconciled with one another? The following numerical experiment may help.

Fig. 17 shows the evolution of the gas clump mass (top panel) and its separation (lower panel) for three simulations that use the same setup as the fixed β simulations of Section 3 but for a varying disc mass. The black curves show the β = 7.5 fiducial simulation (Section 3.2), which was performed with the initial disc mass of $M_d = 0.4 M_\odot$. The red dotted and the blue dashed curves show simulations identical to the fiducial one but in which the initial disc mass is 0.2 and 0.1 $M_\odot$, respectively. We see that with reduction in disc mass the migration time-scale increases, first in approximate proportion to 1/$M_d$, but then makes a discontinuous jump to much longer time-scales. In particular, the migration time is $\sim 2000$ and $\sim 4000$ yr for $M_d = 0.4$ and 0.2 $M_\odot$, respectively, but becomes at least as long as $\sim 10^4$ yr for $M_d = 0.1 M_\odot$.

This can be understood based on type I and type II migration rate estimates. Using equations 8 and 9 from Nayakshin (2010a), and setting $H/R \approx M_d/M_\star$, for a marginally stable self-gravitating disc, we get for the types II and I migration time-scales at $R = 100$ au,

$$t_{\text{II}} = \frac{R^2}{\alpha \Omega_e H^2} = 4 \times 10^5 \text{ yrs } \alpha^{-1}_o \left(\frac{0.2 M_\star}{M_d}\right)^2,$$

(20)

where $\alpha_o = \alpha/0.01$ is the disc viscosity parameter, and

$$t_1 = \Omega^{-1} \frac{M_d M_\star}{M_d M_\star} = 4 \times 10^4 \text{ yrs } \frac{M_d}{0.2 M_\star} \frac{3 M_\star}{M_p}.$$

(21)

This shows that switching from type I to type II will dramatically slow down planet migration. Furthermore, although now superseded by more accurate conditions for the switch-over from type I to type II migration (e.g. Crida et al. 2006; Malik et al. 2015), the older simpler expression for the gap opening mass (in fact used by Forgan & Rice 2013b) is due to Bate et al. (2003):

$$M_\text{III} \geq M_\text{crit} = 2 M_\star \left(\frac{H}{R}\right)^3 \approx 2 M_\star \left(\frac{M_d}{M_\star}\right)^3 \biggr),$$

(22)

where the last equality hold approximately for marginally stable self-gravitating discs.

Equation (22) shows that the planet of a few Jupiter masses will migrate in type I regime in the discs with mass $M_d = 0.2 M_\odot$ and $M_d = 0.4 M_\odot$ but will open a deep gap and migrate in type II regime in the less massive, $M_d = 0.1 M_\odot$, disc. This shows an extreme sensitivity of the planet-disc system to the mass of the protoplanetary disc as indicated in the legend.

Figure 17. Comparison of planet mass (top panel) and planet separation (lower panel) evolution for three identical simulations that differ only by the mass of the protoplanetary disc as indicated in the legend.

The differences between the results of Forgan & Rice (2013b) and those presented here could be therefore due to a relatively small differences in the parameters of the problem, such as $M_d$.

Additionally, Forgan & Rice (2013b) set their eyes on a deeper, more ambitious and more constrained problem than we did in this paper. They present a complete model (albeit population synthesis which does have free parameters or approximations) for the disc and the fragment evolution. In this model there is a one to one connection between the mass of the fragment and the local disc

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conditions. They conclude that fragments are usually quite massive ($M_{\text{init}} \sim 10 M_\oplus$ or more, see Forgan & Rice 2013a). Vigan et al. (2017) then use these models to connect to the observations. This approach requires the disc fragmentation to be rare.

In contrast, here we ask a less constrained question: given a planetary mass object in an outer massive disc, how will it evolve? We find that the probability of that fragment surviving at its birthplace as a planet in actually very low for the massive discs considered here. This then suggests that the initial population of planetary mass clumps could have been much larger, perhaps up to a dozen clumps per star.

The approach in this paper is less self-consistent than that of Forgan & Rice (2013b). However, my personal view is that the initial fragment mass is uncertain by about one order of magnitude due to large uncertainties in the disc physics and initial conditions at fragmentation (see a more detailed discussion of this in section 4.3 of Nayakshin 2017). The simulations presented in this paper show that, with these physical uncertainties taken into account, the results of Vigan et al. (2017) permit another interpretation: gravitational fragmentation of $\sim 100$ au discs on planetary mass gas clumps is widespread but very few of them survive by the time the disc is dispersed.

Furthermore, there now exists observational evidence that extreme migration of planetary and brown dwarf mass objects – the main driver of the clump destruction – does occur. Core Accretion theory predicted a very strong positive metallicity correlation for the frequency of these objects appearance with metallicity of the host star (Mordasini et al. 2012). However, recent APOGEE survey observations showed that brown dwarf frequency of appearance is very insensitive to the host star metallicity (Troup et al. 2016). The objects in the APOGEE sample circle their host stars at separations of only $\sim 0.1$ to 1 au. Santos et al. (2017) recently showed that planets above the mass of $4 M_\oplus$ form a population that does not correlate with the host star metallicity. This behaviour is distinctly different from that of the well know hot Jupiter population (Fischer & Valenti 2005). Disc fragmentation followed by rapid migration of gas clumps in fact predicted (Nayakshin & Fletcher 2015) that planets with mass greater than $\sim 5 M_\oplus$ and brown dwarfs will not correlate with metallicity of the host star (see also sections 9.4 and 9.5 of Nayakshin 2017). Therefore, the most natural interpretation of these observations is that at least some of the objects made by disc fragmentation at separations of $\sim 100$ au did migrate in, and in fact all the way to separations of $\sim 0.1$ to 1 au. This should serve as a strong indication that migration of the objects born by disc fragmentation cannot be neglected when interpreting the observed population of objects in the planetary and brown dwarf mass range beyond tens of au, and that therefore that their initial population could have been much greater.

7 DISCUSSION AND CONCLUSIONS

Simulations presented in this paper show that gas giant mass clumps born by the gravitational instability in the outer cold disc are very unlikely to remain there after the disc dissipation if the mass of the disc when the clumps form is high, $M_0 \gtrsim 0.2 M_\odot$. These clumps are extremely efficiently removed from that region by a trio of processes: (1) inward planet migration; (2) runaway accretion making them brown dwarfs or even low-mass stars; (3) tidal disruption downsizing them into sub-giant planetary domain. This implies that the observed fraction of just $\sim 1$ per cent of gas giant planets orbiting their hosts outside $\sim 10$ au region may be just a tip of an iceberg of the initially numerous population of planetary mass clumps formed by gravitational instability in that region.

These results are in stark contrast to those of Forgan & Rice (2013b) and Vigan et al. (2017) who find many planetary and brown dwarf objects left after the disc dissipation, and therefore suggest that gravitational fragmentation of protoplanetary discs is rare. It is possible that the main source of differences lies in relatively mundane differences in assumptions about the fragment and disc masses at the time the latter fragment and in the treatment of fragment migration (see Fig. 17).

Current theoretical uncertainties in clump evolution thus translate the observed $\sim 1$ per cent frequency of planets into a very broad and uncertain range of frequency of disc fragmentation on to clumps, from 0.01 to a dozen per star. It is crucially important to beat this uncertainty down. To resolve the issue, we need more detailed theoretical models and also more observations of stellar companions in the massive planet and the brown dwarf regimes. If gravitational disc instability only makes brown dwarfs then that population is likely to be completely different from the population of massive planets in terms of mass versus orbital separation distributions, host metallicity correlations, bulk metallicity content, etc. If, on the other hand, gravitational disc fragmentation produces planetary mass clumps and they then evolve into the three groups of objects as shown in Fig. 16, then planets and brown dwarfs must be related to one another and thus share some characteristics.

Given the rapid inward migration of planets, planets detected in systems such as HR 8799 (Marois et al. 2008, 2010) must have evolved differently than most other gravitationally unstable protoplanetary discs. Perhaps N-body effects of planet–planet interactions slowed down planet migration or even reversed it in this system, or perhaps the parent protoplanetary disc was removed extremely rapidly by a nearby OB star ionizing radiation (e.g. Clarke 2007), which is no longer there; swept away by a supernova explosion, as hypothesized for the Solar system, or perturbed by a close star passage, etc.

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REFERENCES

Bodenheimer P., 1974, Icarus, 23, 319
Boss A. P., 1997, Science, 276, 1836
MODELLING ACCRETION ON TO PLANETS

APPENDIX A: USING SINK PARTICLES FOR MODELLING ACCRETION ON TO PLANETS

In the interest of technical detail in modelling gas accretion on to planets, we consider here how the accretion rates on to pre-collapse planets change if we used sink particle prescription for the planet instead of actually modelling it as a clump of gas. To this end, we repeat the run \( \beta'7.5N2 \) with a sink particle inserted into the disc instead of a gaseous clump. The initial sink particle mass is also not significantly off the fiducial gas particles only simulation. This suggests that using smaller values for the sink radius may be a reasonable strategy to model gas accretion on to planets in some circumstances. This also implies that gas accretion rate on to the planet in these cases is indeed controlled by what is happening at large radii (of order the Hill sphere). The sink particle prescription then seems to yield the maximum possible gas accretion rate on to the planet.
Gas accretion on to planets

Figure A1. Comparison of the fiducial $\beta7.5N2$ simulation that modelled the planet via SPH formalism with identical simulations where a sink particle of different sink radii was used.

One should not read too much into the planet migration tracks being all similar in Fig. A1. The initial rapid plunge of the planet is similarly rapid in all cases because the planet mass is not that different between the different runs yet, and the final position where planet stagnates is controlled by the position of the inner edge of the disc not the planet properties (Section 3.2.6).

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