Spanwise Domain Effects on Streamwise Vortices in the Plane Turbulent Mixing Layer

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Abstract

Large Eddy Simulation is used to assess the influence of the spanwise domain extent on the evolution of the spatially stationary streamwise structure that exists in the simulated plane turbulent mixing layer. The mixing layers originate from a physically-correlated inflow condition, which produces accurate mixing layer mean flow statistics. For all three spanwise domains considered a spatially stationary streamwise structure is present. The streamwise structure is artificially confined when its wavelength matches that of the spanwise domain extent, and a criterion for confinement is postulated. The confinement has no significant negative impact on either the computed flow statistics, or the growth of the large-scale spanwise structures. These results demonstrate that the streamwise structure rides passively on the large-scale spanwise vortex structure. A simulation lacking in a spanwise direction produces poor turbulence statistics, and is not a reliable representation of the real mixing layer flow.

Keywords: Mixing Layer, Large Eddy Simulation, Coherent Structures, Flow Confinement, Vortices

1. Introduction

Numerical simulation of the plane turbulent mixing layer has been a topic of academic research for over thirty years. The mixing layer is viewed as an important canonical test case in numerical simulation as its geometric simplicity permits its careful study on high-fidelity meshes. Simulation methods such as Direct Numerical Simulation (DNS) and Large Eddy Simulation...
(LES) should be able to capture the salient large-scale features of the mixing layer, as the flow is dominated by large-scale, spanwise-orientated vortex structures. These structures were first observed experimentally by Brown & Roshko [1]. Subsequent experimental flow visualisations showed that a ‘streaky’ streamwise structure develops in the mixing layer [2]. This streaky structure is a visual manifestation of spatially stationary streamwise vortices [3, 4]. Whilst these spatially stationary streamwise vortices persist far into what is normally defined as the self-similar region of the flow, their evolution has been the subject of some controversy. In some experiments the streamwise vortices maintained a constant spacing over many generations of pairing interactions between primary structures [5, 6] whilst in other studies the streamwise structure spacing increased in a stepwise manner in conjunction with the spanwise structure interactions [3, 7]. The origin of these streamwise vortices was linked to flow conditions upstream of the trailing edge of the splitter plate [4]. Small changes in the mixing layer initial conditions (note that initial conditions in experiments are termed inflow conditions in simulations, and these two terms are used interchangeably in this article) affected by changing the screens in the upstream wind tunnel section [3, 7], or switching the legs in which the freestreams are generated [8], can produce quantifiable changes in the measured properties of the streaky structure.

As noted above, numerical simulation should be able to capture these salient flow features in the mixing layer. For reasons of computational cost the earliest simulations of the flow type were of the temporal form. In a temporal mixing layer the freestreams are set in opposition and the flow develops temporally within a doubly-periodic computational domain. The roll-up of the mixing layer [9], the pairing of primary vortices [10], and transition to turbulence in the mixing layer [11], were all studied in the temporal form. The influence of the initial conditions on the temporal mixing layer was observed in both Direct Numerical Simulation (DNS) [12] and Large Eddy Simulation (LES) investigations [13]. The work of Balaras et al. [13] noted that the spanwise domain extent had an influence on the vortical structures present in the flow - smaller domains enforce two-dimensionality on the flow, whilst larger domains result in a more three-dimensional flow-field.

Modern computing power now permits the simulation of the spatially-developing mixing layer. The streamwise vortex structure that forms in simulations of the spatially-evolving mixing layer is, however, a function of the imposed inflow conditions. When low-level three-dimensional random perturbations are superposed on a mean inflow velocity profile, a helical
structure is observed in the mixing layer [14]. These oblique primary rollers undergo localised or helical pairings, which can lead to the vortex structure attaining a chain-link fence type appearance [15]. The imposition of highly two-dimensional random perturbations, however, resulted in the formation of thin streamwise vortices between the primary spanwise rollers [14]. In these simulations, however, evidence for the presence of a spatially stationary streamwise vortex structure was lacking. Recent research by McMullan and Garrett [16] has shown that the imposition of low-level, physically-correlated fluctuations in an initially-laminar mixing layer, leads to the formation of spatially stationary streamwise vortices. The evolution of these streamwise vortices agreed extremely well with comparable experimental data [4], and their origin was associated with residual streamwise vorticity in the upstream laminar boundary layers.

In three-dimensional simulations of turbulent flows it is important that the spanwise domain does not artificially impose a maximum spanwise wavelength on the flow. For mixing layers with a white noise inflow fluctuation environment the local aspect ratio of the flow, defined as the ratio of spanwise domain extent to local momentum thickness, must be greater than ten throughout the computational domain in order to prevent artificial confinement by the spanwise domain [17]. Artificial confinement of the flow has an adverse effect on the computed flow statistics [18, 19], and leads to an alteration of the coherent structure dynamics [17]. As described above, simulations of the mixing layer with a white noise disturbance environment leads to oblique spanwise vortices that undergo localised interactions. It is not clear if the aspect ratio criterion postulated by McMullan [17] also holds for simulations of the mixing layer where spatially stationary streamwise vortices are present.

As the mixing layer is considered a statistically two-dimensional flow, it is often common practice to simulate the mixing layer in a two-dimensional box. Two-dimensional mixing layer simulations originating from a white noise disturbance environment can produce reasonable mean flow statistics when compared to experiment [20, 21], but the predicted turbulence statistics agree poorly with reference data [20, 22, 23]. No data have been reported for two-dimensional, initially-laminar mixing layer simulations originating from a physically-correlated inflow condition.

This paper assesses the effect of the spanwise domain extent on the spatially stationary streamwise vortices that form in simulations of the plane turbulent mixing layer. A series of Large Eddy Simulations of the plane
turbulent mixing layer with varying spanwise domain lengths are performed. Mixing layers originating from initially laminar conditions are simulated, with the flow conditions based on the experimental conditions of Browand & Latigo [24]. The physically-correlated inflow condition is specified through the use of a recycling and rescaling method [25]. A further simulation is performed with the flow confined to a pseudo-two-dimensional box, which investigates the accuracy of a simulated mixing layer where the formation of a spatially stationary streamwise vortex structure is inhibited.

A brief overview of the research code is given in Section 2. The reference experiment details are described in Section 3. The simulations performed in fully-three-dimensional computational domains are presented in Section 4. The two-dimensional simulation results are described in Section 5, and conclusions are drawn in Section 6.

2. Code overview

The research code used here was described in detail in McMullan [17], and a brief overview is provided below.

The code is based on the low-Mach number approximation of the spatially-filtered governing equations, permitting the simulation of variable density flows in an incompressible computational framework. This code has been used extensively for mixing layer simulation research [16, 17, 22, 26], and has produced accurate results over a range of Reynolds numbers. The primitive variables are discretised on a staggered-cell, finite-volume mesh. Second-order central-differencing schemes are used for the convection and diffusion terms in the solution of the momentum equation, and a flux-limited, second-order accurate upwinding scheme is used for scalar fluxes [27]. This choice of scheme for the scalar terms minimises out of bounds errors in the calculations of the passive scalar. Time advancement is achieved through the second-order accurate Adams-Bashforth method. The pressure equation is solved using a multi-grid method. The outflow boundary condition is a convective condition similar to that used in many previous studies [28], and has been shown to be passive. The transport equation for a passive scalar, \( \tilde{\xi} \) is solved, and is closed through the commonly-used gradient-diffusion model. The Schmidt number of the resolved flow is \( Sc = 0.7 \), and the turbulent Schmidt number is set to \( Sc_t = 0.3 \).

The unresolved scales of motion are modelled through a subgrid-scale model, and in this study the WALE model is employed [29]. The WALE
model is attractive for the simulation of mixing layers as it predicts zero eddy viscosity in the presence of pure shear. Previous studies have shown that the WALE model can produce improved predictions of the mixing layer bulk evolution when compared to the standard Smagorinsky model [26].

The inflow condition is generated by a recycling and rescaling method [25]. This method is similar to that of other rescaling methods [30], where flow is recycled within a domain upstream of the main computational domain of interest. In this virtual domain, the flow is periodically rescaled to a target set of flow statistics.

3. Reference Experiment details

The simulations presented here are based on the initially laminar flow conditions reported by Browand and Latigo [24]. The flow conditions outlined in Table 1 produced a low-speed, high Reynolds number turbulent mixing layer. The guidewalls of the test section were fixed horizontal, resulting in an adverse pressure gradient being present in the flow. This pressure gradient reduced the low-speed freestream velocity with increasing streamwise distance. Reference values of the freestream velocities are estimated at the trailing edge of the splitter plate and produce a velocity ratio parameter, defined as

\[
R = \frac{U_1 - U_2}{U_1 + U_2}
\]  

of \( R = 0.66 \) at this location.

The flow conditions reported by Browand and Latigo [24] are a good candidate for numerical simulation as a large amount of statistical information is available for comparison. Unusually for initially-laminar mixing layer experiments, both the mean streamwise velocity and its fluctuating component were recorded in both streams. The velocity fluctuation measurement, however, was subject to a 1% error, and no information was recorded on the vertical or spanwise velocity fluctuations.
4. Three-dimensional Simulations

4.1. Simulation Setup

Three distinct computational domains are used in this study. These mixing layer domains and grids match those found in a previous study of spanwise domain confinement [17]. Validation of the mesh and subgrid-scale model employed here has been reported elsewhere [16], and are not repeated here.

The parameters of the computational domains are shown in Table 2. The data are presented in a non-dimensional form, with the normalisation parameter, \( \theta_i \), being the initial momentum thickness of the flow. In the experiments \( \theta_i \) was equivalent to the high-speed side boundary layer momentum thickness as the flow departs the splitter plate, and this relationship is assumed here.

The main mixing layer domain has an extent of \( 1630 \theta_i \times 1326 \theta_i \) in the streamwise (\( x \)), and vertical (\( y \)) directions respectively. Case RMD1 has the narrowest spanwise domain, with a spanwise (\( z \)) extent of \( L_z/\theta_i = 98 \). The spanwise domain extent is successively doubled, with \( L_z/\theta_i = 196 \) in Case RMD2, and \( L_z/\theta_i = 392 \) in Case RMD3. The domain begins at the trailing edge of the splitter plate. The mesh resolution in the mixing layer domain is held constant between all three runs in order to permit valid comparisons. The domain is discretised into \( 768 \times 256 \) cells in the streamwise and vertical directions respectively. The mesh is highly-refined in the vicinity of the splitter plate, with minimum grid spacings of \( \Delta x_{\text{min}}/\theta_i = 0.46 \), and \( \Delta y_{\text{min}}/\theta_i = 0.086 \). The high-speed side boundary layer is resolved with 36 points in the vertical direction, and the low-speed side boundary layer is resolved with 48 points. Grid-stretching in the streamwise and vertical directions produces a reduced cell count in regions of freestream flow far from the horizontal plane of the splitter plate. The grid points across the span of the domain are uniformly distributed, with \( \Delta z/\theta_i = 1.53 \). The WALE subgrid-scale model coefficient is set to \( C_w = 0.56 \) for all runs, as a previous validation study has shown that the large scale evolution of is not affected.

<table>
<thead>
<tr>
<th>Case</th>
<th>( L_x/\theta_i )</th>
<th>( L_y/\theta_i )</th>
<th>( L_z/\theta_i )</th>
<th>( N_x )</th>
<th>( N_y )</th>
<th>( N_z )</th>
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<tbody>
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<td>1326</td>
<td>98</td>
<td>768</td>
<td>256</td>
<td>64</td>
</tr>
<tr>
<td>RMD2</td>
<td>1630</td>
<td>1326</td>
<td>196</td>
<td>768</td>
<td>256</td>
<td>128</td>
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<tr>
<td>RMD3</td>
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<td>1326</td>
<td>392</td>
<td>768</td>
<td>256</td>
<td>256</td>
</tr>
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</table>

Table 2: Mixing layer computational domains.
by a choice of WALE model constant in the range of $C_w = 0.3 - 0.56$ on a grid of comparable resolution [26].

The non-dimensional time step for the simulations is $\Delta t / (\theta_i / U_c) = 0.0205$, where $U_c = (U_1 + U_2)/2$ is the convection velocity of the flow. Once the simulation has attained a statistically stationary state, flow statistics are accumulated at every time step over a period of 20 convective flow through times. The total number of statistical samples is increased through the spanwise averaging of the statistics, as it has been shown that this procedure removes the potential for reported statistics to be skewed by the streamwise vortex structure [31]. Flow visualisation images are recorded at a sampling rate of 1.667 kHz. In the present simulations the maximum CFL number does not exceed 0.35. The passive scalar is assigned a value of unity in the high-speed stream, and zero in the low-speed stream. A vanishingly small density difference is introduced between the freestreams to facilitate numerical schlieren flow visualisation. The density ratio of the flow, $s = \rho_2 / \rho_1$, is set to 0.9996, where $\rho_2$ and $\rho_1$ are the low-speed side and high-speed side densities respectively. The density ratio of the two streams is sufficiently close to unity that the flow simulations here should be considered as ostensibly uniform density.

A recycling and rescaling method provides the time-dependent inflow condition for these simulations. This method requires ‘virtual’ computational domains to be included in each calculation. These virtual domains extend a streamwise distance of $x/\theta_i = 333$ upstream of the trailing edge of the splitter plate. The recycling plane is located $10\theta_i$ upstream of the trailing edge. The mesh resolution in these virtual domains matches that found at the plane of $x/\theta_i = 0$ in the main mixing layer domain. The splitter plate that separates the freestreams is modelled as a no-slip solid boundary of infinitesimal thickness. Information is required for the mean and fluctuating velocity profiles.
of all three velocity components to produce the inflow condition. Only the mean and fluctuating profiles of the streamwise velocity were recorded in the experiment [24], and as such it is impossible to precisely re-create the experimental conditions. The flow conditions shown in Figure 1 are applied in both streams, with the momentum thickness of the boundary layers at the trailing edge of the splitter plate matching that found in the experiment. The root mean squared (r.m.s.) fluctuations should be considered as representative of those that may have existed in the experiment, and it has been shown elsewhere that these inflow conditions produce a simulated mixing layer that agrees well with the experimental data [16]. The nomenclature $\langle \rangle_z$ denotes a quantity that has been spanwise averaged. The upper and lower guide-walls of the computational domain are modelled as slip walls. The spanwise boundaries are periodic in nature. A standard advective boundary condition is employed at the outflow plane of the domain.

4.2. Results

In the experiment, the decrease in low-speed side freestream velocity with increasing streamwise distance caused by the adverse pressure gradient requires that the normalisation of flow statistics by the velocity difference across the mixing layer, $\Delta U = U_1 - U_2$, is performed with local freestream velocity values. The same procedure is applied to the simulation results.

4.2.1. Mean Flow Statistics

A measure of the growth of the mixing layer can be obtained from the integral thickness defined as
This quantity is commonly referred to as a momentum thickness, but does not represent a momentum deficit in the same sense as for a boundary-layer. The momentum thickness distributions from each of the three-dimensional simulation are shown in Figure 2a, along with the reference experimental data. The predicted momentum thickness distributions are very similar for all three cases, and are in good agreement with the experiment. The slight tailing-off of the momentum thickness at the far downstream end of the computational domain is caused by the passage of continuously growing structures through the outflow plane, and is common in numerical simulations of shear flow [26, 32]. The momentum thickness growth rate of the flow can be determined from the distributions in Figure 2a by computing the linear gradient in the region where the flow is self-similar. In all three simulations a linear gradient is produced in the region of \( x/\theta_i \geq 350 \). For a uniform density mixing layer the momentum thickness growth rate is expected to be a linear function of the velocity ratio parameter, given by \( d\theta/dx = k_m R \), where \( k_m \) is a constant. The constant obtained from each simulation is shown in Table 3, and the values obtained from the simulations all agree well with both each other and the reference data value of \( k_m = 0.0357 \).

The local aspect ratio of the mixing layer is defined as the ratio of spanwise domain extent to the local mixing layer momentum thickness, \( A = L_z/\theta \). The evolution of the local aspect ratio for each simulation is shown in Figure 2b. In a counterpart study it was observed that, for a mixing layer originating from a white-noise fluctuation environment, a local aspect ratio of ten resulted in spanwise confinement of the simulated mixing layer [17]. This spanwise confinement resulted in a decrease in the momentum thickness growth rate. In the present simulations both cases RMD1 and RMD2 evolve

<table>
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<tr>
<th>Case</th>
<th>( k_m )</th>
</tr>
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<tr>
<td>RMD1</td>
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</tr>
<tr>
<td>RMD2</td>
<td>0.0359</td>
</tr>
<tr>
<td>RMD3</td>
<td>0.036</td>
</tr>
<tr>
<td>Expt</td>
<td>0.0357</td>
</tr>
</tbody>
</table>

Table 3: Momentum thickness growth rate constants.
Figure 3: Flow statistics gathered at $x/\theta_i = 1000$.

to local aspect ratios that fall below $A = 10$ at $x/\theta_i = 332, 781$ respectively, but the momentum thickness distributions in Figure 2a show no evidence of a reduction in growth rate when this aspect ratio value is attained.

The normalised mean streamwise velocity of each simulation recorded at $x/\theta_i = 1000$ is shown in Figure 3a. The mean streamwise velocity data from each simulation agree extremely well with the experimental data. The r.m.s. streamwise velocity fluctuation profiles at $x/\theta_i = 1000$ are shown in Figure 3b - each simulation produces almost matching data, and the agreement with the experimental data is good. Profiles for the vertical velocity r.m.s. fluctuations, $v_{r.m.s.}$, and spanwise velocity r.m.s. fluctuations, $w_{r.m.s.}$, were not recorded in the experiment, hence only simulation data for these quantities are shown in Figures 3c-d respectively. Slight changes in the peak magnitude
of the vertical and spanwise velocity fluctuations are evident at this streamwise location. Similarly small changes in the peak magnitude of the primary Reynolds shear stress are visible in Figure 3e. Overall, however, all three simulations produce velocity fluctuation, and primary Reynolds shear stress, statistics that are within the range of values reported experimentally [33].

Figure 4: Variation of maximum streamwise velocity fluctuation with streamwise distance.

The variation of the peak r.m.s. streamwise velocity fluctuation with streamwise distance is shown in Figure 4. All three cases produce closely matching data, and are in good agreement with the reference data. Other mixing layer studies have shown that, for simulations originating from a white-noise disturbance environment, a drop in the computed peak magnitude of $u_{r.m.s.}$ occurs downstream of the point of spanwise domain confinement, where $A = 10$. [17, 19]. As noted above, Cases RMD1 and RMD2 reach a local aspect ratio of ten at $x/\theta_i = 332, 781$ respectively, but the peak $u_{r.m.s.}$ evolution plot shows no evidence for a sudden reduction in this quantity at these locations.

Time series of streamwise velocity are recorded along the horizontal plane of the splitter plate ($y/\theta_i = 0$) in each simulation, from which power spectral density plots are produced. Spectral plots from several streamwise locations in Case RMD1 are shown in Figure 5. At $x/\theta_i = 44$ peaks in the spectra are visible - the dominant peak has a frequency of 1060Hz, close to the value of 1140Hz predicted by stability theory [34]. At the measurement locations further downstream the roll-off in the spectra approach the -5/3 exponent indicative of fully-developed turbulence. Very similar spectra are also recorded from Cases RMD2 and RMD3, and are not shown here.
The results presented above demonstrate that, for initially-laminar mixing layer simulations with low-level physically-correlated inflow disturbances, the local aspect ratio $A = 10$ criterion is not a suitable metric to quantify spanwise domain confinement. Indeed, from the data presented above it is not clear if any of the three simulated flows have been artificially confined by the spanwise domain extent. The evolution of the spanwise wavelength in the present simulations must therefore be not necessarily linked to the local integral thickness, and some other criterion for spanwise domain confinement must be determined. The appropriate choice of spanwise lengthscale is investigated below.

4.2.2. Flow Structure

Typical instantaneous snapshots of numerical schlieren taken from Cases RMD1 and RMD2 are shown in Figures 6 & 7. The numerical analogue to schlieren is computed from the divergence of the density field. As schlieren images are effectively averaged representations along the direction of interrogation of the optical beam, the side- and plan-view images presented here are averaged along the spanwise and vertical directions respectively. The images are then rendered using the scaling proposed by Quirk [35]. In the side views (lower part of Figures 6 & 7) the large-scale spanwise-orientated structures are present throughout the entire streamwise extent of each simulated flow. In the laminar region of the flow (approximately $0 \leq x/\theta_i \leq 320$ in these
Figure 6: Instantaneous numerical schlieren of case RMD1. Top image, plan view. Bottom image, side view.

Figure 7: Instantaneous numerical schlieren of case RMD2. Top image, plan view. Bottom image, side view.
images) the vortical structures are the familiar laminar Kelvin-Helmholtz (K-H) vortices. In the turbulent region of the large vortical structures occupy the entire local visual thickness of the flow and are therefore of dynamical significance. The evolution of these large-scale spanwise vortex structures is discussed in Section 4.2.5.

In the plan view of the flow visualisations (the upper part of both Figures 6 & 7), the pre-transition K-H vortices extend across the entire span of the computational domain, and run parallel to the spanwise axis. In addition to the primary spanwise rollers, a ‘streaky’ structure is visible in both simulations. The streaky structure present in these images is qualitative evidence for the presence of a streamwise vortex structure in the mixing layer. Beyond the transition the streaky structure appears slightly more irregular in the plan view. These images bear striking resemblance to the flow-visualisations of the uniform density mixing layer experiments of Konrad, where both a quasi-two-dimensional coherent vortex structure in the side-view, and a streaky streamwise structure in the plan view, were observed [2].

Instantaneous cross plane \((y-z)\) maps of the filtered passive scalar are shown in Figure 8, recorded in Case RMD1 at \(x/\theta_i = 328\). Figure 8a shows the scalar distribution as the interconnecting braid region between two spanwise-orientated vortices passes through the sampling plane. Two ‘mushroom-shaped’ patterns are clearly visible at \(z/\theta_i \approx 15, 80\), and are footprints of two pairs of counter-rotating, streamwise orientated vortices in the mixing layer. Figure 8b shows the scalar distribution in the core of a spanwise-orientated vortex passing through the sampling plane. The scalar shows evidence for two rows of streamwise vortices - one that sits on the upper side of the primary vortex, and the other that sits on the under side.
of the primary vortex. Tracing the vortices from image to image over a long sampling period reveals that these streamwise vortices have a geometry matching that proposed by Bernal and Roshko [3]. This pattern of scalar distribution also occurs in Cases RMD2 and RMD3.

4.2.3. Streamwise Vortex Structure

The flow visualisation images presented above provide qualitative evidence that streamwise vortices are present in the mixing layer. The statistical nature of this streamwise structure is investigated through cross-plane data obtained at seven streamwise locations. These measurement stations are detailed in Table 4, and are expressed in terms of the normalised distance from the splitter plate trailing edge, the local value of the pairing parameter [36], and the local Reynolds number. The local Reynolds number is based on the local visual thickness, and the velocity difference across the layer. In each simulation 1,200 samples are obtained at a rate of 1.7kHz at every measurement station. An ensemble averaged flow-field is then obtained, and the mean streamwise vorticity is computed directly from the mean velocity field, such that \( \Omega_x = \partial w_t / \partial y - \partial v_t / \partial z \), where \( v_t \) and \( w_t \) are the mean vertical and spanwise velocities respectively.

Mean streamwise vorticity contours recorded at stations 2, 3, 5, and 6 in Case RMD1 are shown in Figure 9. Note that the axes extents, and contour levels, change between the images. At station 2 the streamwise vorticity is organised into clusters along the spanwise direction. Three tiers of streamwise vorticity are stacked vertically at a given spanwise location - the outer vortices being of one sign, sandwiching another vortex of alternate sign. This pattern of stacking alternates across the span of the mixing layer. Cluster-

<table>
<thead>
<tr>
<th>Station</th>
<th>( x/\theta_i )</th>
<th>( x_i^* )</th>
<th>Re</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>44</td>
<td>0.96</td>
<td>6,600</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>2.4</td>
<td>16,700</td>
</tr>
<tr>
<td>3</td>
<td>217</td>
<td>4.8</td>
<td>33,500</td>
</tr>
<tr>
<td>4</td>
<td>326</td>
<td>7.2</td>
<td>50,600</td>
</tr>
<tr>
<td>5</td>
<td>652</td>
<td>14.4</td>
<td>102,000</td>
</tr>
<tr>
<td>6</td>
<td>978</td>
<td>21.52</td>
<td>154,000</td>
</tr>
<tr>
<td>7</td>
<td>1304</td>
<td>28.8</td>
<td>209,000</td>
</tr>
</tbody>
</table>

Table 4: Cross-plane measurement station locations.
ing of this type in the near-field region has been observed experimentally [4, 37], and is attributed to the mean representation of the rib structures and opposite-signed streamwise vorticity generated in the primary spanwise vortex core [9]. At station 3 these clusters have evolved into a single row of alternating sign streamwise vortices across the span of the mixing layer. Some variability in the strength of each streamwise vortex is evident, but it is clear that this streamwise vortex structure is statistically stationary. The positive-signed vortex at $z/\theta_i \approx 80$ is undergoing an interaction with its weaker like-signed neighbour at $z/\theta_i \approx 50$, enveloping the sandwiched negative streamwise vortex at $z/\theta_i \approx 62$. Further downstream the number density of the streamwise structures has reduced, and their spacing has increased.

At measurement station 5, shown in Figure 9c, two large streamwise vortices are clearly visible, but two smaller structures are present at the outer edge of the domain. At station 6 these two weaker streamwise vortices have been removed from the flow, and only the two large vortices from station 5 remain. A further increase in structure spacing is impossible in the simulation.

The mean streamwise vorticity contours at the measurement stations 2, 3, 5, and 6 in Case RMD2 are shown in Figure 10. At station 2 (Figure 10a) the distribution is qualitatively similar to that observed at the same station in Case RMD1 - three-tiered clusters of streamwise vorticity are stacked vertically, with the pattern of clustering alternates along the span of the mix-
Figure 10: Mean streamwise vorticity maps recorded in Case RMD2.

At station 3 (Figure 10c) the streamwise vorticity has developed into a single row of alternating-sign stationary streamwise vortex structures. Further downstream at station 5 the streamwise vortices have increased in spacing, with six structures visible along the span of the flow. The large negative vortex at $z/\theta_i \approx 180$ is interacting with its neighbouring negative and positive streamwise structures at $z/\theta_i \approx 120, 150$ respectively. This interaction has completed by station 6, shown in Figure 10d, where a single large negative streamwise vortex now occupies the region of $120 \leq z/\theta_i \leq 190$. At station 6 the spanwise domain is occupied by two large streamwise vortices, and no further increase in structure spacing is possible in the simulation.

Case RMD3 also contains a spatially stationary streamwise vortex structure, the evolution of which is similar to that of Case RMD2 with the exception that multiple pairs of counter-rotating streamwise vortex structures are present at the far downstream end of the computational domain. These data are not shown here.

The presence of a spatially stationary streamwise structure causes wrinkling and distortion of the mixing layer across its span. One means of quantifying this wrinkling is through plotting the locus of the mixing layer centreline along the span at each measurement station. The centreline of the mixing layer is defined as the vertical location where the mean streamwise velocity is equal to the local convection velocity. The centreline loci at each
measurement station are shown in Figure 11 for all three simulations. The peaks and troughs in the centreline locus align with the interfaces between the streamwise vortices - a peak in the locus aligns with a pair of streamwise vortices with a common upflow, whilst a trough aligns with a pair of streamwise vortices that have a common downflow. This trend is in agreement with observations from experiments [4]. The centreline loci of both Cases RMD1 and RMD2 evolve to a single wavelength by station 6, indicating that the span is occupied by a pair of counter-rotating streamwise vortices. In Case RMD3 at least two wavelengths are clearly visible at station 7, with the flat region in the locus between $0 \leq z/\theta_i \leq 150$ caused by the interaction between streamwise vortices.

The streamwise vortex spacing, $s$, can be determined from inspection of the cross-plane mean streamwise vorticity contours. The individual positive, and negative, streamwise vortices are counted, and their average spacing is computed. The evolution of the average structure spacing with normalised streamwise distance is shown in Figure 12a. In the near-field of all three simulations the structure spacing decreases between stations 1 to 3. This is owing to the unwrapping of the three-tiered streamwise vortex structures into a single row of alternating sign streamwise vortices [4]. Downstream of
this minimum the vortex spacing in each simulation increases in a stepwise fashion, as interactions between neighbouring structures reduces the overall number of streamwise vortices. From Figure 12a it can be seen that the structure spacing in both Cases RMD1 and RMD2 is constant downstream of station 6. As two streamwise vortices of opposite sign occupy the entire spanwise extent, it is not possible for any further increases in spacing to occur in both of these simulations. In case RMD3 the structure spacing evolves in a similar manner to Case RMD2 up to station 5. An increase in structure spacing is observed at station 6, but the mixing layer still contains multiple pairs of streamwise vortices at this location. That the structure spacing does not increase between stations 6 and 7 is a consequence of the natural evolution of the flow, rather than a maximum structure spacing having been attained in the simulation.

4.2.4. Spanwise domain confinement

In Cases RMD1 and RMD2 the streamwise structure evolves to a pair of counter-rotating vortices that occupy the entire span. As no further increase in structure spacing can occur in these simulations, it is reasonable to conclude that Cases RMD1 and RMD2 have been artificially confined by the computational domain. Some criterion to quantify this confinement must now be established.

The most appropriate length-scale on which to base this criterion is the streamwise structure wavelength, $\Lambda$, defined as twice the streamwise vortex spacing, $\Lambda = 2s$. The streamwise structure aspect ratio, $A_\Lambda$, can then be defined as the ratio of the spanwise domain length to the streamwise structure wavelength, $A_\Lambda = L_z/\Lambda$. This variation of this quantity with streamwise
distance is plotted in Figure 12b. The streamwise structure aspect ratio of both Cases RMD1 and RMD2 attain a value of unity at station 6. A structure aspect ratio of unity indicates that the spanwise length-scale has become artificially confined. It is interesting to note that the spanwise length-scales of Cases RMD1 and RMD2 differ by a factor of two upon artificial confinement. The periodic spanwise boundary conditions imposed on these simulations mean that there will always be an integer number of wavelengths across the span of the domain, and the structure spacing will, if necessary, adjust to satisfy this requirement. This is clearly the situation that arises in Case RMD2, where a single pair of counter-rotating streamwise vortices occupy the span at station 6, with a wavelength double that present in Case RMD1.

In Case RMD3, where the streamwise structure aspect ratio does not reach unity, the streamwise structure evolves naturally as no cofinement occurs. The average streamwise vortex spacing in Case RMD3 increases in a stepwise fashion, but the overall evolution of the vortex spacing scales approximately with the local integral thickness. Assuming that the vorticity thickness, $\delta_\omega = 5\theta$ [24], the average ratio of streamwise structure wavelength to local vorticity thickness, $\Lambda/5\theta$, is approximately unity in the self-similar region of the mixing layer in Case RMD3. This is in the range of other values for this ratio reported experimentally [3, 4, 7]. For a uniform density mixing layer the linear growth of the momentum thickness follows the relationship $\theta = k_m R(x - x_0)$, where $x_0$ is the virtual origin, and $k_m$ is a constant. Therefore the streamwise structure wavelength is related to the distance from the virtual origin by

$$\Lambda \approx 5k_m R(x - x_0)$$

An *a priori* estimation of the downstream distance, $x_c$, at which the streamwise structure wavelength matches the spanwise domain, $\Lambda = L_z$, can then be obtained from

$$L_z \approx 5k_m R x_c$$

assuming that the virtual origin is located at the trailing edge of the splitter plate. For Case RMD1, equation 4 yields $x_c/\theta_i \approx 848$, and for Case RMD2, $x_c/\theta_i \approx 1697$. The estimate for Case RMD1 is in good agreement with the simulation data, as flow will have attained its maximum streamwise wavelength between stations 5 and 6. The estimation for Case RMD2, however,
is significantly further downstream than the streamwise location at which a single streamwise structure wavelength is attained in the simulation. As discussed above this is caused by the requirement for an integer number of wavelengths to be present in the simulation, owing to the use of imposed periodicity by the spanwise boundary condition. For Case RMD3, equation 4 produces a streamwise distance estimate of $x_c/\theta_i \approx 3321$, far in excess of the downstream end of the computational domain.

The confinement criterion postulated in equation 4 is of course an estimate, as there is no clear evidence, either experimental or numerical, to directly link the evolution of the stationary streamwise vortices with the distance from the virtual origin [4, 5, 6, 8]. From a physical standpoint, it is clearly undesirable for the streamwise structure in the simulated mixing layer to evolve to a single wavelength in the computational domain, therefore it is prudent to assert that a simulation should contain a minimum of two wavelengths of the streamwise structure across the span at the far downstream end of the computational domain. From this argument the minimum spanwise domain extent for mixing layer simulations can then be estimated from

$$L_z(\text{min}) \approx 10k_m R L_z$$

For the current simulation parameters, Case RMD3 satisfies equation 5, and the centreline locus plot at station 7 in Figure 11c confirms that at least two wavelengths of streamwise structure are present in the domain. Cases RMD1 and RMD2 do not satisfy this criterion, and are confined by the spanwise domain length. Note that this criterion is valid for simulations where a spatially stationary streamwise structure forms; a comprehensive study of the effect of inflow fluctuations on the development of the streamwise structure is beyond the scope of the current research.

4.2.5. Effect of confinement on flow dynamics

In a simulated initially-laminar mixing layer originating from a white noise disturbance environment the flow is artificially confined when the local aspect ratio of the mixing layer attains a value of $A = L_z/\theta = 10$ [17]. It has been shown that such simulations do not contain a statistically stationary streamwise vortex structure [16]. Instead the large-scale coherent structures have an inherently three-dimensional internal geometry, and artificial confinement of the maximum spanwise wavelength in the flow effectively enforces
two-dimensionality on the coherent structures in those simulations. The continuous linear growth of the coherent structures present in those simulations is disrupted, leading to the deterioration of the computed flow statistics.

In the present study Cases RMD1 and RMD2 have their maximum spanwise length-scale artificially confined by the domain. Neither simulation, however, displays any evidence of this confinement in their computed flow statistics. The momentum thickness variations of these simulations compare well with their unconfined counterpart, and there is no sudden decrease in the peak magnitude of the streamwise velocity fluctuation once the spanwise lengthscale is confined. This statistical evidence suggests that the dynamics of the large-scale structures are largely unaffected by this confinement. This hypothesis can be tested by tracking the growth of the spanwise-orientated coherent vortex structures in the flow. This analysis is performed in the same manner as other numerical studies [16, 26] where the polar points of the vortex structures are tracked from frame to frame in the side-view visualisations, with the visual thickness of the structures defined by the $\langle \xi \rangle_z = 0.01, 0.99$ contour lines. Figure 13 shows the growth tracks of representative structures throughout their lifetimes from each simulation. Whilst the precise details of the growth of each structure varies slightly, the general pattern of growth is that each structure grows continuously with the square-root of distance from their birth, and therefore with the square-root of time given that the structures convect downstream at the convection velocity, $U_c$. This type of growth mechanism has been attributed to vortical structures growing though either
irrotational roll-up [38], or turbulent diffusion [39]. The overall mean linear growth of the mixing layer is a result of a combination of continuous root-time growth of the structures, and interactions that occur between the large-scale vortex structures [16]. This pattern of growth holds in all three simulations throughout the entire post-transition flow, which demonstrates that the artificial confinement of the streamwise structure has no significant effect on the dynamics of the large-scale spanwise turbulent vortex structures. This lack of alteration of the flow dynamics post-confinement in RMD1 and RMD2 is attributable to the geometry of the large-scale structures themselves. In a mixing layer simulation containing a spatially stationary streamwise structure, it has been shown that the post-transition vortex structures have a quasi-two-dimensional internal geometry [16], where the streamwise structure rides passively along with the primary spanwise-orientated turbulent vortices [40]. Imposing a maximum spanwise length-scale on the streamwise structure in Cases RMD1 and RMD2 does not adversely affect the simulated flow, as the streamwise vortices themselves do not play an active role in the bulk flow evolution.

5. Two-dimensional simulations

As described above the spatially stationary streamwise structure present in all of the fully-three-dimensional simulations plays essentially a passive role in the dynamics that govern mixing layer growth. It is therefore tempting to surmise that the accuracy of a mixing layer simulation does not depend on the presence of this streamwise structure. A simple extension of logic would suggest that a simulation confined to a (pseudo-)two-dimensional box will produce an adequate representation of the real mixing layer flow. This hypothesis is tested here through a further Large Eddy Simulation (Case 2D) performed in a computational domain with an extremely narrow span of \( L_z/\theta_i = 0.5 \). The span is discretised into 8 uniformly-spaced cells. This narrow span is significantly smaller than the structure spacing at station 1 in all three simulations presented above - the streamwise structure does not form, and the simulation is ostensibly confined to a two-dimensional box. All simulation setup parameters are held constant with respect to Case RMD3, with the exception of the total number of simulated time steps. As spanwise-averaging of the flow statistics has no particular meaning here, the simulation is performed over a period of 200 convective flow through times in order to ensure statistical convergence.
The mean streamwise velocity recorded at $x/\theta_i = 1000$ in Case 2D is shown in Figure 14a. The predicted mean streamwise velocity in Case 2D is in good agreement with both experimental data, and the profile extracted from Case RMD3. As the mean velocity field is well-predicted in Case 2D, it is not surprising that the momentum thickness variation, shown in Figure 14b is also in reasonable agreement with both experiment and Case RMD3. The momentum thickness growth rate constant obtained from a best-fit line to the self-similar region of the flow is $k_m = 0.0384$.

A simplistic analysis of the mean velocity field would lead to the conclusion that a 2D simulation is a sufficiently accurate representation of the real flow. Interrogation of higher-order statistics, however, reveals serious discrepancies with experimental data. The predicted streamwise r.m.s. velocity fluctuation in Case 2D is in extremely poor agreement with the experimental data. Figure 15a shows that the peak value of this quantity is over-predicted by 45% when compared to the experiment. There are slight shoulders in the profile at $y/\theta = \pm 2.5$ that are typical of mixing layer simulations confined to a two-dimensional box [21, 22]. The fluctuations also persist much further into the freestreams than is the case in both the experiment and in Case RM3D. The r.m.s. vertical velocity fluctuation profile at $x/\theta_i = 1000$ is shown in Figure 15b. When compared with Case RMD3, this quantity is over-predicted in Case 2D by a factor of two, and is far outside of the range of values reported experimentally [33]. This gross over-prediction of both the streamwise and vertical velocity fluctuations is caused by the lack of a spanwise direction, as there is a lack of a degree of freedom into which turbulent energy can be transferred. This is apparent in Figure 15c, where the r.m.s. spanwise velocity fluctuation is necessarily zero for Case 2D.

Figure 15: Case 2D turbulence statistics gathered at $x/\theta_i = 1000$. Case RMD3 and experimental data shown for reference where appropriate.

The evolution of the peak r.m.s. streamwise velocity fluctuation in Case 2D is shown in Figure 16a. It is clear that the over-prediction in $u_{r.m.s.}$ pervades the entire self-similar region of Case 2D. It is also evident that Case 2D attains a markedly different self-similar state to that of its counterpart fully-three-dimensional simulation. Typical power spectral density plots of the streamwise velocity fluctuation in the plane of $y/\theta_i = 0$ are shown in Figure 16b. The roll-off in the spectra approach a $-5/2$ slope, and the flow is therefore not fully turbulent, even at the high local Reynolds numbers attained in the simulation.

The extremely poor prediction of turbulence statistics from a mixing layer confined to a two-dimensional box has been observed elsewhere in simulations originating from a white-noise disturbance environment [21, 22, 41], and the use of a physically-correlated inflow does not lead to an improvement in the predicted turbulence statistics from a 2-D simulation. Basing the assessment of a two-dimensional simulation on the quality of the mean velocity statistics alone will lead to a misleading interpretation of the accuracy of such a calculation. A 2-D simulation is not representative of the high Reynolds number
6. Conclusions

The effect of the spanwise domain extent on the spatially stationary streamwise structure that exists in a turbulent plane mixing layer has been assessed through Large Eddy Simulation. It has been shown that artificial confinement of the flow occurs when the wavelength of the streamwise structure matches that of the spanwise domain length. The bulk mean flow statistics of a confined mixing layer are largely unaffected by the artificial confinement of the flow, indicating that the spatially stationary streamwise structure rides passively on the large quasi-two-dimensional spanwise vortex structures present in the flow. A criterion for the minimum spanwise domain extent required is developed, which agrees well with the simulation data. Simulations confined to a two-dimensional box can produce superficially good comparisons to three-dimensional simulations in terms of the mean velocity statistics, but fail to produce realistic turbulence statistics owing to the lack of spanwise dimension.

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References


