To my father, Fei, and mother, Min, for their enduring love and support
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To my paternal grandmother's little brother, Yankui, for his constructive
advice and encouragement
Essays on Wage Inequality and Poverty

Rui Luo

Abstract

This thesis sets out to study the impact of growth and technological development on inequality. This thesis focuses on two types of inequality, the skill premium (wage inequality between skilled and unskilled labour) and the poverty. First, this thesis sets out to analyze the interaction between the skill premium and the growth and development in England in the very long run (circa 1329-1660); Second, it studies the poverty-reduction effect of the interaction between institutional development and remittances. These two topics are covered by Chapters 2, 3 and 4 respectively.

Chapter 2 studies the evolution of the skill premium in England from 1329 to 1660. It develops a unified model that accounts for the growth and development in the historical period that ranging from late medieval ear to industrial revolution. This model unveils insightful linkage between the historical development in this period and the skill premium. It shows that the growth of the relative abundance of physical capital to human capital from an initially low level to a sufficiently high level, which is triggered by demographic change and the growth of agricultural productivity, brings about the decline of the skill premium from 1329 to 1450s, and the stabilization of the skill premium afterwards.

On the basis of the analytical findings in Chapter 2, Chapter 3 brings the proposed unified model of growth into data. Through calibration and simulation, it shows that the with the simulated skill premium declining to a low and stable level, the simulated GDP per capita grows to a high and stable level and the simulated primary sector labour share declines from a high level at the beginning to a low level in the end, both of which are consistent with the actual data. The simulated skill premium, GDP per capita and the primary sector labour share jointly show the gradual ‘modernization’ of the English economy takes place well before the industrial revolution.

Chapter 4 analyzes the impact of institutional development of the poverty-reduction effect of remittances. Considering remittances and remittances’ facilitating effect on financial development, it shows that institutional development attract more remittances. This then reduces credit market imperfection and enables a larger fraction of the population to invest in human capital and escape poverty trap, which results in the reduction of poverty. This chapter is the first to analytically study the change in poverty in response to remittances and institutional development.
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Rui Luo
University of Leicester
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Chapter 2 has been presented in the following conferences:

- 2017 North America Meeting of the Econometric Society at Washington University in St. Louis
- 2017 Royal Economic Society Annual Meeting at University of Bristol
- 2017 Royal Economic Society Junior Symposium at University of Bristol (poster presentation)
- 2017 German Economic Association Development and Policy Annual Meeting at University of Göttingen
- 2017 China Meeting of the Econometric Society at Wuhan University
- III PhD Conference in Economics (in 2016) at University of Leicester
- Midland Economic Theory and Applications Meeting at University of Manchester (in 2016)

Chapter ?? has also been cited in the following sources of media:

- voxeu.org:  
  http://voxeu.org/article/historical-impact-technology-skill-premium

- LSE Business Review:  
  http://blogs.lse.ac.uk/businessreview/category/economics/business-history/

- Royal Economic Society Media Briefing:  
Chapter 1: Introduction

Inequality has been of central consideration in the study of economic growth and development. An important aspect of the studies on inequality is the interrelation between inequality and growth (i.e. whether economic growth enhances or reduce inequality). This interrelation is important in that it reflects the change in income distribution and social welfare brought about by growth and development, which is crucial to policy making. As inequality can be measured in different ways, there are different types of inequality. This thesis specifically focuses on two types of inequality: 1) the skill premium, which reflects the wage inequality between skilled and unskilled labour, and 2) poverty.

Skill Premium

The skill premium, which is measured by the wage of skilled labour relative to that of unskilled labour, represents the wage inequality between the skilled labour and the unskilled labour. As can be seen, the structural changes brought about by the economic growth and technological change are likely to alter the payment to different working groups, which then alters the skill premium. For instance, technological progress may give rise to new industries that increases the relative demand for skilled labour. This will then increase the payment to skilled worker relative to unskilled worker. The formation of the skill premium is thus a vivid reflection of the impact that economic growth and technological change have on the society. Analyzing the evolution of the skill premium over time will shed interesting light on how growth and technological change affect inequality.

Existing studies on the skill premium shows the linkage between the skill premium and technological change. The main focus of the existing literature on the skill premium is to study the evolution of the skill premium after the 1970s. The representative work that explains the simultaneous increase in the skill premium and the relative supply of skilled labour is Acemoglu, 2002, who proposes the hypothesis of skilled biased technological change (SBTC). It shows
that increasing relative supply of skilled labour, while lowering down the skill premium at the beginning, induces SBTC that generates excessive demand for skilled labour. The skill premium then goes up following the initial decline. The SBTC hypothesis captures the feature of the post-1970s technological change and successfully adopt this feature to the analysis of the pattern of the skill premium after 1970s, which declines first and increases sharply afterwards.

While the SBTC literature already well explains the post-1970s skill premium, the evolution of the skill premium in the historical period (circa 1329 to 1660) is left to be studied. Take England as an example. This long historical period contains different stages of development. From 1329 to 1450, there is little technological progress and the population declines by up to 58% due to the spread of the bubonic plague to England in 1349 and the subsequent epidemics. This period, which is referred to as the “late medieval era”, is the primitive stage of development. From 1450 to 1660, the population starts to recover, surpasses the pre-plague level and reaches the peak as of 1653. Along with the population recovery is the technological progress featured with the growth of land productivity. It is slow before 1500, speeds up a bit after 1500 and grows to a high level as of 1580. After 1580, it slows down again. England in this period is believed to undergo gradual economic and social transitions that facilitate the formation of modern England (Pamuk, 2007). The 1450-1660 period is thus is referred to as the “early modern era”. Because the 1329-1660 period covers two different stages of development, the analysis on the skill premium in this period should be based upon a model that unifies growth, development and transition in the two stages of development. In the existing literature on long-run growth, Galor and Zeira, 1993, Galor and Moav, 2004 and Galor, Moav, and Vollrath, 2009 analyze how inequality affects the outcome of growth in the very long run considering different stages of development and the transition in between, but these studies take inequality as exogenously given and do not emphasize the case in which growth and development may in turn affect the inequality. This leaves the study on the impact that growth and technological change in the very long run have on wage inequality blank.

This thesis intends to fill the blank. Chapter 2 studies the skill premium and technological change in England from 1329 to 1660. According to the data on the wages of craftsmen and labourers from Allen, 2001, the skill premium
in England declines from an average of more than 120% in the 1320s to a low level of approximately 58% as of 1450. After 1450, despite some fluctuations, the skill premium in England stays more or less stable around this level of 58% till 1660. On the basis of Galor and Moav, 2004 and Galor, Moav, and Vollrath, 2009, Chapter 2 develops a unified framework that accounts for the growth and development in this long historical period to explain this pattern of the skill premium. Using this model, we show that the growth of the human capital investment and that of capital-human capital ratio, both of which are the driving force of long-run growth and development, affect the skill premium. The growth of human capital investment has a positive impact on the skill premium whereas the growth of the capital-human capital ratio has a negative impact on the skill premium. Which effect dominates the other depends on the level of the capital-human capital ratio. When the capital-human capital ratio is low, the negative effect dominates. When the capital-human capital ratio becomes relatively higher, the negative effect cancels with the positive effect. The proposed explanation on the evolution of the skill premium is that with the economy in England starting at a status in which capital-human capital ratio is low, the increase in capital-human capital ratio during and after the plague causes no growth in human capital investment. The skill premium declines to a low level as the ratio goes up. As the capital-human capital ratio becomes sufficiently high (around 1450), the skill premium seizes to declines and stays stable at this low level. While the population starts to recover after 1450, the capital-human capital ratio may go down, but the growing land productivity, which composites the technological progress, prevents the capital-human capital ratio from becoming too low. Thus the skill premium stays stable at the level of 58% from the 1450s to 1660.

Chapter 3 presents the simulation the evolution of the skill premium in England from 1329 to 1660 as well as the evolution of the other key variables, GDP per capita and Primary Sector Labour share (PSLS in short), both of which reflect the development of the economy. The simulated skill premium is consistent with the historical data: in the event of population decline, the skill premium declines from over 120% at the beginning to a low and stable level of 58% sometime later. This corresponds to the 1329-1450 period. Later, as the population goes up, the skill premium may fluctuate beyond the stable level.
Introduction

But the fluctuation is suppressed by the subsequent growth of land productivity and soon falls back to the stable level. This corresponds to the 1450-1660 period.

The simulated GDP per capita and PSLS are also consistent with actual data. The simulated GDP per capita goes up by some 40%-50% during the population decline, which fits the surge of GDP per capita from 1348, the year right before the plague, to 1450, the year when the population reaches the trough, well. And, despite the population recovery, the GDP per capita stays at more or less the same high level as before. The simulated PSLS falls from around 80% at the beginning to around 55% in the end. Consistent with the actual data in history, the simulated GDP per capita and PSLS indicate a slow “modernization” of the English economy in the 16th and 17th century, with GDP per capita staying high despite the population growth and the majority of the labour force concentrating into the skill-intensive (industrial) sector. The calibration and simulation results support the claim that along with the “first declining then stabilizing skill premium” is the gradual formation of modern England, which takes place well before industrialization. (Van Zanden, 2009)

Poverty

Poverty is another important aspect of inequality. The expenditure required to cover the necessities of life is referred to as the “poverty line”. The poverty rate is measured by the fraction of the population whose income is below the “poverty line”. High poverty rate means that a large fraction of the population with low income. This implies higher degree of inequality in income distribution, with most of the income concentrated onto the hands of the rich people. Developing countries such as those in sub-Saharan Africa have high poverty rate. The reduction of poverty rate in these countries has captured a lot of attention from researchers on development economics and policy makers.

The inflow of money from foreign countries is believed to play a key role in poverty reduction. There are several types of foreign money: FDI (Foreign Direct Investment), Foreign Aid and Remittance. This thesis focuses on remittance and its poverty-reduction effect. Chapter 3 analyzes the influence that
the interaction between remittances and institutional development exerts on poverty. There are many indices measuring the degree of institutional development. These indices include financial development, corruption and democracy. Chapter 3 focuses on financial development only.

The study in Chapter 3 begins with analyzing the interactive effect between remittances and institutional development. On one hand, financial development lowers down the transaction costs in the process of remitting, which encourages senders’ to send more remittances back to their home country. On the other hand, remittances can serve as collateral for those who want to borrow money from banks for human capital investment. This facilitates financial development by reducing the credit market imperfection, raising the likelihood of borrowing money for human capital investment. The more the remittances, the less the credit market imperfection and the better the financial development.

Considering such interaction between remittances and development, Chapter 3 further studies the impact of such interaction on poverty. By incorporating the remittances and its effect on reducing credit market imperfections into the long-run distribution model developed by Galor and Zeira, 1993, it demonstrates that higher level of financial development attracts more remittances from senders. This then reduce the credit market imperfections by lowering down the interest rate on loans, enabling a larger fraction of the population to be able to invest in human capital, receive high income and escape the poverty trap eventually. This then reduces the poverty rate.

The facilitating effect of remittances on financial development has been empirically studied by Gupta, Pattillo, and Wagh, 2009, Aggarwal, Demirgüç-Kunt, and Peria, 2011 and Nyamongo et al., 2012. And the influence that the interaction between remittances and institutional development on poverty reduction is empirically studied by Akobeng, 2017. On the analytical side, Mundaca, 2009 studies the effect of the interaction between financial development and remittances on the economy. But this study focuses on the effect of remittances and financial development on growth only. To the best of my knowledge, this thesis is the first to analytically study the impact that the interaction between institutional development and remittances has on poverty. The formulation of senders’ motivation to remit follows the “pure altruism” hypothesis in Lucas
and Stark, 1985. The difference is that in Chapter 4, remitters care about recipients’ asset holding whereas in Lucas and Stark, 1985, remitters care about recipients’ consumption. This difference captures the fact that remitters would prefer recipients to use the remittances for investment such as constructing new houses.

The rest of the thesis is organized as follows: Chapter 2 presents the unified model that characterizes the growth and technological change in England from late-medieval era to early modern era (i.e. the 1329-1660 period) and shows that how the model can be adopted to explaining the evolution of the skill premium in England in this long historical period. All the results in this chapter are analytical. Chapter 3 presents the calibration results of the relevant parameters of the model and the simulation results of the skill premium, GDP per capita and PSLS in England from 1329 to 1660. Chapter 4 presents the analytical results on the changes in poverty incurred by the interaction between institutional development and remittances. Chapter 5 concludes.

**Bibliography**


Abstract

This paper lays out a two-sector model of development and growth over the very long run (ca. 1330 to 1660) that is capable of explaining the dramatic decline in the skill premium ca. 1330-1450 (Allen (2001), Clark (2005a), Van Zanden (2009)) as well as its subsequent stability in the subsequent periods ca. 1450-1660. In studying the evolution of the skill premium, the model simultaneously accounts for the profound demographic, economic, and sectoral changes taking place from the mid-15th century into the late 17th century (Broadberry et al. (2015), Shaw-Taylor and Wrigley (2014)). The model and the quantitative analysis endogenize both the demand and supply of skilled and unskilled workers, and the sectoral division into primary and secondary production. This paper shows that the plague affected relative prices, but due to structural imperfections in the market for human capital accumulation related to the guild and apprentice system, it had little effect on the population’s skill acquisition on the intensive margin (human capital level per skilled worker). However, relative prices changes had profound consequences on the extensive margin (the relative supply of skilled workers) and so for the sectoral composition. Thus we explain why the sectoral transformation was completed by 1700, well ahead of the industrial revolution.

Keywords: skill premium, demographic change, sectoral transformation, intensive margin, extensive margin

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1 Introduction

The spread of the bubonic plague from Asia to Europe in the early 1350s decimated populations across Europe dramatically. After the plague, population did not recover smoothly back to the pre-plague level due to the repetitive outbreak of small epidemics following the plague (Pamuk (2007)). Instead, it continued to decline until the mid-15th century. This paper mainly focuses on England in the period from 1330 to 1660. The population in 1352, the year right after the plague, dropped sharply by 60% compared with 1330. The population continued to decline and as of 1450, it declined by 56% compared with 1330. After 1450, the population in England started to grow slowly and it recovered back to the pre-plague level in 1330 in the early 15th century. After that, the population in England continued to grow and as of 1660, it surpassed the pre-plague level in 1330 by approximately 1.2 times.

The plague is believed to bring about dramatic changes to the economies of the affected countries. The demographic decline raises both the total and agricultural output per worker, reducing the relative price of food and raising income per capita. In England, income-per-capita grew by 40%-50% from 1330 to 1450 (Figure 1b). Changes also happened to the interest rate and the skill premium in England in this period: the interest rate fell by about 40% to 50% (Clark (1988)), and the skill-premium in England fell from a level of over 125% in 1329 to a relatively low level of 58% as of 1450 (Figure 1a).

![Figure 1: The Skill-Premium and GDP per capita of England (indexed) 1300-1760. Sources: Skill-premium calculated from Allen (2001). GDP per capita from Broadberry et al. (2015) (Table A21 of Bank of England (2017) (reindexed to base year 1300))](image)

(a) The salary of skilled craftsmen/masons relative to the salary of unskilled construction workers from 1300 to 1914.
(b) GDP per Capita from 1300 to 1760 (1300=100)

Curiously, in the next 210 years (ca. 1450-1660), while population was slowly recovering back and latterly surpassing the pre-plague level in 1330, the skill premium remained...
trendless over time, staying at approximately 58% in the period from 1450 to 1660 (Figure 1a). It can be seen that while the change in population is one of the factors that influence the skill premium, it is not the sole determinant of the skill premium. Other changes that transformed the structure of the economy must have taken place in this period. The evolution of two other variables further implies that the structural transformation of the economy may have happened in this period. First, the GDP per capita in this period appeared to be trendless over time, staying at approximately the same level as in 1450 (1b). The interest rate, which heavily depends on capital-labour ratio, stayed at 40% to 50% below the pre-plague level in this period (Clark (1988)).

One noticeable change that could potentially influenced the skill premium is the transformation of composition of the labour market. This transformation is featured with the decline of labour share in the agricultural (primary) sector. According to 2, the agricultural labour share in England and Wales declined from around 76% in the 14th century to approximately 53% in 1660. In an economy that consists of the agricultural (primary) sector and the industrial (secondary) sector, the decline in the agricultural sector labour share implied that the increase in the labour share of the industrial (secondary) sector. Thus the decline in the agricultural (primary) sector labour share could be seen as the formation of modern labour market in Europe in the 16th-17th century. Such formation of modern labour market is attributed to the labour shortage, which was caused by the plague, and the revolt, which led to the demise of serfdom (Allen (2008), p.956). The decline of the agricultural labour share implied an increase in the relative supply of the labour, which would naturally lower down the skill premium.

Another noticeable change that happened before the plague (i.e. 1340s) and lasted till after 1660 period was the growth of land productivity. The driving forces of the growing land productivity were social structural changes such as the Enclosure, which replaced
many different small family farms into a few large farms, and the rise of Cistercian, which advocated the spirit of hardworking and thrift [Andersen et al. (2017)]. This development of the land productivity made it possible for the growth of aggregate output to outperform the recovery of population, maintaining the GDP per capita in this period stayed as high as in the 1450s. The question is whether such development also contribute to stabilizing the skill premium. If this were the case, then we would establish a link between the skill premium and the level of socioeconomic development. We would then be able to decipher the level of development of an economy by looking at its skill premium. It is thus interesting to examine whether the growing land productivity, which was powered by the cultural and structural changes from 1450 to 1660, contributed to stabilizing the skill premium.

This paper’s objective is to analyze the skill premium from 1330 to 1660, which declined from above 120% in 1330 to a lower level of approximately 58% in 1450 and stayed stable at the lower level from 1450 to 1660 both qualitatively and quantitatively, accounting for the mentioned developments in this period. The central question that this paper is targeting is: while the demographic decline from 1330 to 1450 caused by the bubonic plague and the subsequent epidemics lowered down the skill premium, how come that the skill premium stayed stable in the subsequent period from 1450 to 1660 while the population first recovered then surpassed the pre-plague level? To answer this question, this paper develops a unified growth framework that analyzes the evolution of the skill premium taking the mentioned changes and developments into account.

Within the unified growth framework, we show that there are two the key determinants of the skill premium: 1) The degree of the abundance of physical capital relative to human capital, which is measured by the capital-human capital ratio and 2) The proposed human capital acquisition system, which affects occupational licensure system (i.e. whether to become skilled or unskilled worker). We propose that the human capital acquisition system operates in the following way: an individual has to invest a minimum amount of money to acquire the basic amount of human capital in order to become a skilled worker. This amount of money can be referred to as the “minimum investment” and corresponds to a low and fixed level. If the individual increases the investment beyond the “minimum investment”, the human capital he or she possesses will increase accordingly. And we formulate individuals’ optimal human capital investment as being affected by the interest rate, which relates to the level of the capital-human capital ratio. When the capital-human capital ratio is low (i.e. beyond the threshold level), the interest rate is so high that investing in physical capital is more rewarding relative to investing in human capital. Thus the human capital investment per person stays at the low and fixed level, which equals to the “minimum investment”. And when the capital-human capital ratio becomes sufficiently high (i.e. beyond the threshold level), the interest rate is low.
enough so that investing in human capital becomes relatively more rewarding relative to investing in physical capital. The human capital investment per person starts increasing with the capital-human capital ratio.

How does such pattern of human capital acquisition affect the skill premium? We show that when the capital-human capital ratio grows from a sufficiently low level (i.e. below the threshold level), the human capital investment of each individual stays at the level of “minimum investment”. The skill premium declines as the capital-human capital ratio goes up. The intuition here is that the growth of capital-human capital ratio raises the relative demand for skilled worker. This then induces a larger fraction of the population to invest in human capital and become skilled labour, which raises the relative supply of skilled labour. And the increasing relative supply of the skilled labour exceeds the demand for skilled labour, even though the latter is also increasing due to the growing wage of skilled labour triggered by the increasing capital-human capital ratio. The reason that the relative supply of skilled labour exceeds the relative demand is that the human capital investment per person stays at its minimum level, which fix the amount of human capital per person at the minimum level. This prohibits the relative demand for skills from growing in pace with the relative supply. The excessive relative supply of skilled labour lowers down the skill premium. After the capital-human capital ratio surpasses the threshold level and becomes sufficiently large, the human capital investment per person starts to grow, which triggers the growth of the human capital possessed by each skilled worker. This raises the wage of the skilled worker and enables the growth of the relative demand for skilled worker to keep pace with the that of the relative supply. And the skill premium, which already declined to a low level before the capital-human capital ratio reaches the threshold, stays at that level afterwards. This analysis shows that the low and stable skill premium implies that the labour markets operates more efficiently, with the relative demand for skills being able to cope with the relative supply. This is consistent with the empirical fact that the labour market was efficient to balance the demand and supply of skilled labour in the period after 1450 (Van Zanden (2009)) and shows that behind the low and stable skill premium was the better developed labour market that helped shape modern Europe.

The change in the capital-human capital ratio is negatively affected by the demographic change but positively affected by the growth of land-productivity. Here is the mechanism we propose: the demographic decline from 1350 to 1450 coupled with the growth of land productivity jointly lowered down the relative price of food. The reduction of relative price of food induced households to save a larger fraction of income. This then raised the capital-human capital ratio. As the capital-human capita ratio grew from an initially low level to a level above the threshold, the skill premium to decline first and stay stable once the capital-human capital ratio becomes sufficiently large. As the
population started to recover, so did the relative price of food. The fraction of the income households saved started to decrease, but the further growth of the land productivity prevented it from decreasing too much. As a result, the capital-human capital ratio, though falling to some extent, continued to stay beyond the threshold level. Then the skill premium continues to stay stable at the same low level. It can be concluded that the low and stable skill premium from 1450 to 1660 was the consequence of the development in this period.

In addition to the skill premium, we further show that within the same framework, the growth of the land productivity and the demographic change incur the evolution of GDP per capita, interest rate and primary sector labour share, all of which follow a pattern that is consistent with the historical data. Since these variables are highly correlated with the historical developments featured with growing land productivity, our model demonstrates that the skill premium evolves consistently with these relevant variables, which strengthens the link between the skill premium and the historical development.

Deeply rooted in the existing literature, this work strengthens and improves the existing study on the historical evolution of the skill premium. Our model shows that what is behind the low and stable skill premium after 1450 is that the labour market was able to adjust the demand for skilled labour efficiently and keep it in pace with the relative supply of skills. A similar claim is made by Van Zanden (2009). This work improves the existing explanation on the declining part of the skill premium. According to Van Zanden (2009), the reason behind the declining skill premium from 1350 to 1450 is that the households had stronger incentive to invest in human capital due to the decline in interest rate caused by the plague. In his view, it is the increase in the intensive margin (human capital per person) that reduced the skill premium. Nevertheless, according to the unified growth theory developed by Galor and Weil (2000), the economy in the preliminary stage of development such as the medieval era was in a “poverty trap” with zero (or very low level) human capital investment. The increase in human capital investment after the plague proposed by Van Zanden (2009) appears to contradict the feature of the economy in the late medieval era. This work shows that while the plague did not change the intensive margin, which is reflected by human capital per person, it altered the extensive margin, which is reflected by the relative supply of skilled labour. After the plague, a larger fraction of households chose to invest in human capital, with the amount of human capital investment of each household staying the same as before the plague. This caused excessive supply of skilled labour and as a result, the skill premium declined just to prevent too many individuals/households from becoming skilled labour. The explanation proposed by this work is consistent with the gradual decline of the primary (agricultural) labour share while capturing the feature of the economy in late medieval era.

This subsequent sections in this chapter are organized as follows: section 2 discusses
the model, section 3 analyzes the equilibrium and the skill premium on the basis of the model and section 4 concludes. The numerical calibration and the simulation of the dynamic adjustment are displayed in the next chapter.

2 The Model

Our model may be viewed as extending the formulation of Galor et al. (2009). In Galor et al. (2009), both sectors produce homogeneous good with identical prices. In our model, we set different prices for different sectors and specify the consumption of different goods. More importantly, the secondary sector in our model uses capital, unskilled, and human capital/skilled workers for production, whereas its counterpart in Galor et al. (2009) uses capital and skilled labour/human capital only. While our model retains the feature that young individuals decide whether to remain unskilled or train to become skilled workers, it is the young individuals who decide the optimal amount of human capital investment. This differs from the human capital investment decision formulated in Galor et al. (2009) but is consistent with the formulation in Galor and Moav (2004).

2.1 Production

Output (or GDP) at date $t$, $Y_t$, is the sum of primary sector output $Y_t^P$ and secondary sector output $Y_t^S$. The price of the secondary sector’s output is the numeraire and the relative price of the primary sector’s output is $p_t$.

$$Y_t = p_t Y_t^P + Y_t^S.$$  (1)

2.1.1 The Primary Sector

The primary sector uses unskilled labor $U_t$ and land $X_t > 0$ for production.\(^2\)

$$Y_t^P = (A_{X,t}X_t)^\theta U_t^{1-\theta}.$$  (2)

In equilibrium, the unskilled labor wage $w_U^t$ is

$$w_U^t = p_t(1 - \theta)(A_{X,t}X_t)^\theta U_t^{-\theta},$$  (3)

and the rate of return to land $\rho_t$ is

$$\rho_t = p_t \theta A_{X,t}^\theta X_t^{\theta - 1} U_t^{1-\theta}.$$  (4)

\(^2\)The primary sector formulated here corresponds to the “agricultural sector” Galor and Zeira (1993) and Galor et al. (2009).
In (2), $A_{X,t}$ denotes agricultural productivity. Its growth reflects the social structural change and the cultural change as mentioned in the introduction. Here we model the $A_{X,t}$ as exogenously growing. In this way, we consider cultural and social structural change as exogenous factors that affect the process of development. $X_t$ denotes the amount of arable land in $t$.

### 2.1.2 The Secondary Sector

The secondary sector has three inputs: capital $K_t$, unskilled labor $U^S_t$, and human capital $H_t = \sum_{i=1}^{S_t} h^i_t$ where $S_t$ is the number of skilled workers and $h^i_t$ the human capital/skill level of worker $i$. Note that skilled workers are indexed such that $\{1, \ldots, L_t\} = \{1, \ldots, S_t, S_t+1, \ldots, L_t\}$ where $L_t$ is the total number of workers. The production function is Cobb-Douglas:

\[
Y^S_t = A_t K_t^\alpha H_t^{1-\alpha-\beta} (U^S_t)^\beta
\]

It can be seen from (5) that unskilled workers are able to enter the secondary sector. This captures the fact that unskilled workers can be employed in either the primary sector (which corresponds to agriculture in this period) or the secondary sector such as the construction industry in this period.

Denote the capital-to-human capital ratio at date $t$ by

\[
k_t = \frac{K_t}{H_t},
\]

and the (secondary sector’s) skill intensity at date $t$ by

\[
i_t = \frac{H_t}{U^S_t}.
\]

When labor markets clear, the price of a human capital unit can then be expressed as

\[
w^H_t = (1 - \alpha - \beta) A_t k^\alpha_i t^{-\beta},
\]

and the wage of unskilled workers who work in the secondary sector must equal

\[
w^U_t = \beta A_t k^\alpha_i t^{1-\beta}.
\]

When the capital market clears, the rental rate is

\[
r_t = \alpha A_t k^\alpha_t t^{-\beta}.
\]

Throughout we assume that capital depreciates completely between periods and consequently, the rental rate is equal to the gross interest rate in the economy because the secondary sector’s price is the numeraire.\footnote{Full depreciation simplifies the model but is also realistic since one time period in our model is half a life-time (see below), and also because a substantial fraction of secondary production in this period embodied most of the capital inputs in the final product (shipbuilding, construction).}
Note that worker $i \in \{1, \ldots, S_t\}$ earns the salary $w_t^H h_i$, and more generally, a skilled worker with human capital level $h$ earns

$$w_t^S(h) \equiv w_t^H h.$$  

The last thing to note here is that the unskilled workers in the secondary sector earn the same wage as unskilled workers in primary sector. This is supported by wage data (e.g., Clark (2005b), Figure 3), which shows that agricultural workers and unskilled construction workers earned almost exactly the same daily wage in the entire 1300-1650 period. On interpretation for this is that given flexible labor flows between urban and rural areas, the equilibrium is such that the unskilled workers working in the construction industry receive the same payments as the unskilled workers working in agriculture.

### 2.2 Labor Supply and Population

In the baseline model we treat the labor supply at date $t$, $L_t$ as exogenous. This description is reasonably consistent with historical facts in the 1300-1500 period where the main factor that influence the population size (and thus labor supply assuming a constant participation rate) was the plague and the wars. Treating population size as exogenous starts to become inconsistent with history in the 16th century, since the population in the 16th century boom was Malthusian in nature, that is, productivity growth was immediately translated into population growth. We manage to capture this feature through simulation. When simulating the period between 1500 and 1660, we shock the land productivity at the beginning, which is then followed by a demographic boom.

### 2.3 Individuals

Individuals’ lifespans are divided into two periods, youth/adolescence and adulthood. Throughout the paper $L_t$ denotes the number of adults at date $t$. A date $t$, young individual $i \in \{1, \ldots, L_{t+1}\}$ receives bequest $b_i^t$ and decides how much $e_i^t \geq 0$ to invest in developing her skills (human capital). If $e_i^t$ is invested, she attains non-basic human capital (skills),

$$h_i^{t+1} = B(e_i^t)^\gamma, \quad \gamma > 0, e_i^t \geq \bar{e} > 0 .$$

To enter the skilled workforce the individual’s human capital level $h_i^{t+1}$ must be above the skilled workforce entry requirement $B > 0$. Hence the individual becomes eligible for skilled work if $e_i^t \geq 1$, and if $e_i^t < 1$ she joins the unskilled workforce. Accordingly, when reaching adulthood the worker will earn either the skilled salary $w_{t+1}^S(h_i^{t+1})$ in (11), or the unskilled salary $w_{t+1}^U$ of (3) or (9) (an unskilled worker earns the same in the primary and secondary sectors).
As we shall see, this set-up captures both the occupational choice (“extensive margin”) and individual skill decision (“intensive margin”) of human capital accumulation. Since in our model, human capital is costly and bestows no direct utility on individuals (see below), agents will never choose $0 < e_i < 1$. This discontinuity is, as we now argue, strongly supported by the historical evidence: In medieval Europe, skilled workers were concentrated in building and construction (masons, architects), shipbuilding (shipwrights), and various mercantile trades. To enter these professions an individual would enter into a guild regulated apprenticeship contract, typically of 7 years duration (Wallis (2008), Van Zanden (2009)). Anyone who did not complete the apprenticeship was barred from the profession, and this was vigorously enforced by the guilds. Furtermore, until the secondary sector’s rapid expansion in the 16th and 17th centuries, there were for practical purposes no alternative occupational options available to individuals; so the decision was between remaining unskilled or taking on and completing, an apprenticeship.\(^4\) Note that on the intensive margin (i.e., conditioned on $e_i + 1 \geq 1$), (12) maintains all key features of the human capital production function of Galor and Moav (2004) (p.1007; see also Galor et al. (2009)). But crucially, a strictly positive skilled workforce entry requirement implies the co-existence of both skilled and unskilled workers in equilibrium.\(^5\)

In adulthood at date $t + 1$, individual $i \in \{1, \ldots, L_{t+1}\}$ receives income from past savings (or repays any debt taken on), and also receives income from any inherited land holdings $X_i \geq 0$. As mentioned previously, the individual then works as either a skilled or an unskilled worker. If the individual is skilled ($e_i \geq 1$), income is

$$I_{t+1}^{S} = w_{t+1} B(e_i) + r_{t+1} (b_t - e_t) + \rho_{t+1} X_i,$$

where $w_{t+1}$ is the wage per human capital unit (see (8)), and $r_{t+1}$ is the bond (or coupon) rate which equals the rental rate.\(^6\) Note that in this formulation, the input into the human capital production function is the secondary good.\(^7\) If the individual is unskilled,

\(^4\)In the year 1500, the secondary sector employed 20 % of the population, mostly in building, construction, textiles and tools and machinery production (iron, steel, and wool being the main inputs from the primary sector). In the year 1600, the number stood at 30 %, and in 1700 it was 40 % and clothes and textiles production had by then become highly dominant (Broadberry et al. 2009). Table 10 puts the output weight of “woollens” in industry at 64.5 % and building at 9.9 %). Note that the tertiary sector was of insignificant (and stable) size until the 19th century and we therefore ignore it. At about the same time as the secondary sector expanded, guilds’ influence also weakened (especially outside of towns which were guilds’ centers of power and influence). This allowed individuals to work as “semi-professionals” especially if they were willing to move to areas outside of guilds’ geographical reach (a fascinating read in this connection is http://resources.thegospelcoalition.org/library/the-rise-and-decline-of-the-english-gilds).

\(^5\)With (ex-ante) identical agents, the model in Galor and Moav (2004) implies that either everyone invests in human capital or no one does (there is no extensive margin).

\(^6\)A bond’s face value is 1 and the annual coupon is $r_{t+1}$.

\(^7\)Our description is obviously simplistic in this regard.
he necessarily chooses \( e_t = 0 \), and income is therefore

\[
I_{t+1}^U = w_{t+1}^U + r_{t+1}b_t^i + \rho_{t+1}X_{t+1}^i, \tag{14}
\]

where \( w_{t+1}^U \) is the unskilled wage from (3). In adulthood, the individual also gives birth to children and allocates income between consumption of the primary sector’s output (food) \( c_{t+1}^i \), the secondary sector’s output (e.g. textiles) \( c_{t+1}^{S,t} \), and bequest \( b_{t+1}^i \) to be shared among the offspring along with the land holdings.

Crucially, how bequest and land holdings are shared between the offspring has no impact on the economy’s aggregate development. This can be seen by simply adding up everything together across individuals:

\[
\sum_{i=1}^{L_{t+1}} (I_{t+1}^S + I_{t+1}^U) = w_{t+1}^H L_{t+1} + \sum_{i=1}^{L_{t+1}} B(e_t^i) + r_{t+1} \left( \sum_{i=1}^{L_{t+1}} b_t^i - \sum_{i=1}^{L_{t+1}} e_t^i \right) + \rho_{t+1} \sum_{i=1}^{L_{t+1}} X_{t+1}^i + U_{t+1} w_{t+1}^U \tag{15}
\]

Clearly, \( \sum_{i=1}^{L_{t+1}} B(e_t^i) \) is the aggregate human capital, which equals to \( H_{t+1} \).

The aggregate physical capital stock in \( t+1 \), \( K_{t+1} \) is affected by two factors in the previous period: 1) the bequest aging individuals left to the young, 2) the fraction of the bequests spent on human capital investment. So we have

\[
K_{t+1} = \sum_{i=1}^{L_{t+1}} b_t^i - \sum_{i=1}^{L_{t+1}} e_t^i
\]

No matter whether an individual has his or her own land or not, the total arable land \( X_{t+1} \) always equal to adding land holdings across individuals:

\[
\sum_{i=1}^{L_{t+1}} b_t^i - \sum_{i=1}^{L_{t+1}} e_t^i X_{t+1}^i = X_{t+1}
\]

Therefore we have:

\[
\sum_{i=1}^{L_{t+1}} (I_{t+1}^S + I_{t+1}^U) = w_{t+1}^H H + r_{t+1} K_{t+1} + \rho_{t+1} \sum_{i=1}^{L_{t+1}} X_{t+1}^i + U_{t+1} w_{t+1}^U
\]

Insert the formulations of \( w_{t+1}^H, r_{t+1}, \rho_{t+1} \) into the equation above and we have:

\[
\sum_{i=1}^{L_{t+1}} (I_{t+1}^S + I_{t+1}^U) = p_{t+1} Y_{t+1}^P + Y_{t+1}^S = Y_{t+1} \tag{16}
\]

Equation (16) is not affected by either the disposal of land across individuals or that of bequest across individuals.
Hence we not need to specify how \((b_{i+1}^{L_{t+1}})_{i=1}^{L_{t+1}}\) maps into \((b_{i+1}^{L_{t+2}})_{i=1}^{L_{t+2}}\) or how the land holdings \((X_{t+1}^{i})_{i=1}^{L_{t+1}}\) map into \((X_{t+1}^{i})_{i=1}^{L_{t+2}}\).

Let \(I_{t+1}^{i}\) equal \(I_{t+1}^{S}\) if \(e_{t}^{i} \geq 1\), and \(I_{t+1}^{U}\) if \(e_{t}^{i} < 1\). The individual’s objective is to choose \(e_{t}^{i}, c_{t+1}^{i}, c_{t+1}^{S,i} \geq 0, b_{t+1}^{i} \geq 0\) that maximize utility subject to the lifetime budget constraint:

\[
p_{t+1}c_{t+1}^{i} + c_{t+1}^{S,i} + b_{t+1}^{i} \leq I_{t+1}^{i}.
\]

We return to the specific choice of consumption and bequest below but note that as long as utility is increasing in \(c_{t+1}^{i}, c_{t+1}^{S,i}, b_{t+1}^{i}\), it is also increasing in income \(I_{t+1}^{i}\). Hence the individual’s decision problem can be split into two stages: first (when young), choose \(e_{t}^{i}\) so as to maximize \(I_{t+1}^{i}\), then (when an adult) choose \(c_{t+1}^{i}, c_{t+1}^{S,i}, b_{t+1}^{i}\) to maximize utility conditioned on \(I_{t+1}^{i}\). As mentioned previously, agents who choose to remain unskilled necessary choose \(e_{t}^{i} = 0\) when young. The following key proposition characterizes the decision of young agents who choose to enter the skilled workforce. The reader is reminded that \(k_{t}\) denotes the capital-to-human capital ratio (Section 2.1.2).

**Proposition 1.** Any agent who at date \(t\) decides to become a skilled worker will invest

\[
e(k_{t+1}) = \max \{\bar{e}, \left[ \frac{\gamma B(1 - \alpha - \beta)k_{t+1}}{\alpha} \right]^{\frac{1}{\gamma}} \},
\]

in human capital. In particular, skilled workers’ human capital investments are independent of land holdings and bequests, and their levels of human capital strictly greater than the skilled workforce entry requirement \(B\) if and only if

\[
k_{t+1} > \bar{k}^{e} \equiv \frac{\alpha}{\gamma B(1 - \alpha - \beta)}.
\]

**Proof.** Conditioned on \(e_{t}^{i} \geq \bar{e}\), the objective is to maximize second period income \([13]\). This income depends on \(e_{t}^{i}\), on the going rate per human capital unit \(w_{t+1}^{H} = p_{t+1}(1 - \alpha - \beta)A_{t+1}k_{t+1}^{\alpha}i_{t+1}^{-\beta}\), and the interest rate \(r_{t+1} = p_{t+1}\alpha A_{t+1}k_{t+1}^{\alpha-1}i_{t+1}^{-\beta}\) (see \([8]\) and \([10]\)). Inserting into \([13]\) we get,

\[
p_{t+1}(1 - \alpha - \beta)A_{t+1}k_{t+1}^{\alpha}i_{t+1}^{-\beta}B(e_{t}^{i})^{\gamma} + p_{t+1}\alpha A_{t+1}k_{t+1}^{\alpha-1}i_{t+1}^{-\beta}(b_{t}^{i} - e_{t}^{i}) + \rho_{t+1}X_{t}^{i}
\]

As a function of \(e_{t}^{i}\), this expression is non-negative, strictly concave, and uniquely attains its maximum at

\[
\left[ \frac{\alpha}{\gamma B(1 - \alpha - \beta)k_{t+1}} \right]^{\frac{1}{\gamma}}.
\]

From this, all of the proposition’s conclusions follow straight-forwardly. \(\square\)
To understand this result, note that when \( k_t \leq \tilde{k}^c \) and skilled workers therefore invest the minimum requirement \( e_t = 1 \), they would be able to strictly increase their lifetime income (and utility) were they able to invest \( 0 < e_t < 1 \) into human capital and find an associated occupation paying the going rate per human capital unit \( w_{t+1}^H \). So workers who choose the skilled occupation finish their apprenticeships because this is their only option — if they could stop earlier and find a job at the acquired (lower) skill level, they would do so. At the same time, when \( k_t \leq \tilde{k}^c \), workers who choose to remain unskilled would strictly prefer to acquire a positive amount of skills below the skilled workforce entry requirement \( B \) if the option were available to them. This tension/imbalance — whereby guild enforced constraints on occupational choices force agents to make decisions that are not optimal absent these constraints — is, as we shall see in Section 3.1, resolved in equilibrium by an adjustment in the skill premium, and more specifically, when \( k_t < \tilde{k}^c \) skilled work commands an excess premium over unskilled work which intuitively is there to incentivize a positive fraction of agents to take on and finish apprenticeships rather than remain unskilled. As one would expect, the premium declines as the capital-to-human capital ratio gets closer to the cut-off \( \tilde{k}^c \), and disappears altogether when \( k_t \geq \tilde{k}^c \) and the described imbalances seize to exist.

The exact utility objective we adopt is not particularly important for the qualitative side of our results. We can however, formulate a CES utility function which individual \( i \) maximizing by choosing the consumption of food \( c_{t+1}^{i} \), the consumption of capital good \( c_{t+1}^{S,i} \) and bequest \( b_{t+1}^{i} \).

\[
U(c_{t+1}^{i}, c_{t+1}^{S,i}, b_{t+1}^{i}) = \left[ a_1(c_{t+1}^{i})^{\frac{1}{\rho}} + a_2(c_{t+1}^{S,i})^{\frac{1}{\rho}} + a_3(b_{t+1}^{i})^{\frac{1}{\rho}} \right]^\rho
\]

Since the period we are studying is the pre-modern Malthusian era, it can be reasonably assumed that any individual consumes a quantity of primary good that depends on \( p_{t+1} \):

\[
c_{t+1}^{i} \equiv \bar{c}(p_{t+1}) > 0, \text{ with } \bar{c}(p_{t+1}) \text{ decreasing with respect to } p_{t+1}.
\]

The individual then spends the rest of the income on the bequest and secondary good. Therefore the individual’s budget constraint is

\[
c_{t+1}^{S,i} + b_{t+1}^{i} \leq I_{t+1}^{i} - p_{t+1} \bar{c}(p_{t+1}).
\]

---

8 This is seen from the fact that the maximum as derived in the proof of Proposition 1 is strictly smaller than 1 when \( k_t < \tilde{k}^c \).

9 This follows from the observation in footnote 8 together with the fact that in equilibrium, skilled and unskilled workers’ lifetime incomes must be the same (see the next section).

10 This is because production is non-joint so in steady state, the relative price is independent of demand patterns.
Setting up the Lagrangian first order condition, we can get:

\[ b_{i+1}^t = \frac{\alpha_2^{\rho}}{\alpha_2^\rho + \alpha_3^\rho} (I_{i+1}^t - p_{t+1} \bar{c}(p_{t+1})) \]

\[ c_{s,i+1}^t = \frac{\alpha_3^{\rho}}{\alpha_2^\rho + \alpha_3^\rho} (I_{i+1}^t - p_{t+1} \bar{c}(p_{t+1})) \]

We set

\[ \sigma = \frac{\alpha_2^{\rho}}{\alpha_2^\rho + \alpha_3^\rho} \]

and assume that \( \sigma \) is decreasing in \( p_{t+1} \) (This is possible provided that \( a_2 \) and \( a_3 \) are correlated with \( p_{t+1} \), which is possible in the real world.) We thus set

\[ \sigma = \sigma(p_{t+1}) \]

Note that \( \sigma(p_{t+1}) \) is the fraction of income deducted by the basic consumption, \( I_i^t - p_t \bar{c}(p_t) \), that is bequeathed. The idea is that as the relative price of food \( p_t \) falls, i.e., as the relative price of the second good increases, the individual’s disposable income after necessity consumption \( I_i^t - p_t \bar{c}(p_t) \) increases (all-else-equal). Since the relative price of the second good increases, however, as \( p_t \) decreases, the agent spends a relatively lower fraction \( 1 - \sigma(p_t) \) of the total disposable income on secondary consumption, and a relatively larger fraction \( \sigma(p_t) \) on bequest. In this way, the decreasing relative price of food raises the fraction of income devoted to bequest.

With bequest \( b_i^t \) satisfying

\[ b_i^t = \sigma(p_t)(I_i^t - p_t \bar{c}(p_t)) \]

, we have aggregate bequest as:

\[ \sum_{i=1}^{L_t} b_i^t = \sum_{i=1}^{L_t} \sigma(p_t)(I_i^t - p_t \bar{c}(p_t)) = \sigma(p_t)(\sum_{i=1}^{L_t} I_i^t - p_t \bar{c}(p_t)L_t) \]

\[ = \sigma(p_t)(Y_t - p_t \bar{c}(p_t)L_t) \]

3 Equilibrium and the Skill-Premium

3.1 The Skill-Premium and the Secondary Sector’s Skill Intensity

The production functions of both the skilled and primary sectors satisfy Inada conditions, hence both sectors must be active at all dates in equilibrium. It follows that a positive
number of individuals must always choose to become skilled and a positive number of
individuals must choose to become unskilled. This in turn implies that the lifetime in-
comes of unskilled and skilled workers must be equal, which in our model holds if and
only if:

\[
w_{t+1}^S = w_{t+1}^H (e(k_{t+1}))^\gamma = w_{t+1}^U + r_{t+1}e(k_{t+1})
\]

where \(w_{t+1}^S\) is the wage of a skilled worker at date \(t + 1\) (see (11)), and \(e(k_{t+1})\) is skilled
workers’ optimal human capital investment as determined in Proposition 1. Note that
this notation and terminology rely on Proposition 1’s observation that all skilled workers
choose the same human capital investment so that, in particular, we can talk about the
wage of a skilled worker and therefore below, the skill-premium.

Since by (8), \(w_{t+1}^H = (1 - \alpha - \beta)A_{t+1}k_{t+1}^\alpha i_{t+1}^\beta\) and by (10), \(r_{t+1} = \alpha A_{t+1}k_{t+1}^{\alpha - 1}i_{t+1}^\beta\), we
obtain from (22) the following formulas for the skilled and unskilled wages in equilibrium
conditioned on the capital-to-human capital ratio \(k_{t+1}\) and the secondary sector’s skill-
intensity \(i_{t+1}\):

\[
w_{t+1}^S = (1 - \alpha - \beta)A_{t+1}k_{t+1}^\alpha i_{t+1}^\beta B(e(k_{t+1}))^\gamma
\]

and,

\[
w_{t+1}^U = (1 - \alpha - \beta)A_{t+1}k_{t+1}^\alpha i_{t+1}^\beta B(e(k_{t+1}))^\gamma - \alpha A_{t+1}k_{t+1}^{\alpha - 1}i_{t+1}^\beta e(k_{t+1})
\]

It is already evident from these two formulas, that the skill-premium (the ratio of the
two) does not depend on the skill-intensity of the secondary sector \(i_{t+1}\) and also not on
the relative price \(p_t\). Setting \(e_t = e(k_{t+1})\) to simplify expressions, and time-shifting,
we can therefore derive a formula of the skill-premium as a function of the capital-to-human
capital ratio only:

\[
\frac{w_{t+1}^S}{w_{t+1}^U} = \frac{(1 - \alpha - \beta)A_{t+1}k_{t+1}^\alpha i_{t+1}^\beta B e_t^\gamma}{(1 - \alpha - \beta)A_{t+1}k_{t+1}^\alpha i_{t+1}^\beta B e_t^\gamma - \alpha A_{t+1}k_{t+1}^{\alpha - 1}i_{t+1}^\beta e_t} = \frac{1}{1 - e_t^{1-\gamma} \frac{\alpha}{B(1-\alpha-\beta)k_{t+1}}}.
\]

Using Proposition 1 to substitute for \(e_t = e(k_{t+1})\) we arrive at the following key obser-
vation.

**Proposition 2.** Let \(\tilde{k}^c\) denote the cut-off capital-to-human capital ratio of Proposition 1. At date
t, the skill premium equals

\[
\left[1 - \frac{\alpha}{B(1-\alpha-\beta)k_t} \right]^{-1} \quad \text{if} \quad k_t < \tilde{k}^c, \quad \text{and} \quad \frac{1}{1-\gamma} \quad \text{if} \quad k_t \geq \tilde{k}^c.
\]

In particular, the skill premium is a continuous and strictly decreasing function of the capital-
to-human capital ratio below the cut-off and constant above the cut-off.
Proof. Insert (17) into (25) and evaluate to get (26). \[
[1 - \frac{\alpha}{\beta}]^{-1} \text{ is strictly decreasing in } k_t < \tilde{k}_c \text{ and as a function of } k_t, \text{ the skill-premium is continuous because } \lim_{k_t \to \tilde{k}_c} [1 - \frac{\alpha}{\beta}]^{-1} = \frac{1}{1-\gamma}. \]

To understand this result, note that the cut-off precisely marks the point where the skilled workforce entry requirement no longer imposes a binding constraint on individuals (see the discussion following Proposition 1). In particular, when \( k_t \geq \tilde{k}_c \), the minimum entry requirement no longer prevents unskilled workers from acting on their incentive to become skilled; and skilled workers no longer have an incentive to acquire skills below the minimum requirement. This causes the excess premium on skilled work \([1 - \frac{\alpha}{\beta}]^{-1} \) to disappear since it is not needed to incentivize skilled workers to complete a full apprenticeship rather than remain unskilled (in fact, a strictly positive excess premium would now cause everyone to become skilled). Of course, this description (indeed all of this paper’s results) requires a very long run perspective because the relative supply of skilled and unskilled workers is fully endogenous and flexible. During any adjustment to equilibrium — easily anywhere between 20-40 years in our set-up — there is no question that the relative supply of skilled and unskilled workers will be partially fixed and other mechanisms play key roles in the skill premium’s determination.

Turning next to the secondary sector’s skill-intensity \( i_t \), combine (9) and (24) to see that

\[
i_t = i(k_t) = \frac{B(1 - \alpha - \beta)(e(k_t))^{-\gamma} - \frac{\alpha}{\beta} k_t^{-1} e(k_t)}{\beta}.
\]

Inserting \( e(k_t) \) from Proposition 1 then determines the equilibrium skill-intensity conditioned on \( k_t \):

**Proposition 3.** Let \( \tilde{k}_c \) denote the cut-off capital-to-human capital ratio of Proposition 1. The secondary sector’s skill-intensity at date \( t \) is given by:

\[
i_t = i(k_t) = \begin{cases} 
\frac{B(1 - \alpha - \beta)}{\beta} k_t^{-1} - \frac{\alpha}{\beta} k_t^{-1} e(k_t) & \text{if } k_t < \tilde{k}_c \\
(\gamma^{-1} - 1) \frac{\alpha}{\beta} \left[ \frac{\alpha}{B(1 - \alpha - \beta)} \right]^{1-\gamma} k_t^{1-\gamma} & \text{if } k_t \geq \tilde{k}_c
\end{cases}
\]

In particular, the secondary sector’s skill intensity is a continuous and strictly increasing function of the capital-to-human capital ratio.

**Proof.** Insert (17) into (27) and evaluate to get (28). By (27), \( i(k_t) \) is the composition of continuous functions and therefore continuous. That \( i(k_t) \) is strictly increasing in \( k_t \) is obvious. \qed
3.2 The Relative Price, Labor Allocation and Sectoral Shares

Proposition 3 allows us to express the wage of unskilled workers as a function of the capital-to-human capital ratio by inserting (28) into (9):

\[ w^U_t(k_t) = \beta A_t k_t^\alpha (i(k_t))^{1-\beta} \]

Recall from previously that we also have:

\[ w^U_t = (1 - \alpha - \beta)A_t k_t^\alpha i_t^{-\beta} B(e(k_t))\gamma - \alpha A_t k_t^\alpha i_t^{-\beta} e(k_t) \]

Since \( i(k_t) \) is continuous and strictly increasing in \( k_t \), we immediately see that the unskilled wage is continuous and strictly increasing in the capital-to-human capital ratio.

From (3), \( p_t(1 - \theta)(A_{X,t} \bar{X}_t)^\theta U_t^{-\theta} \), the primary sector’s labor demand is calculated as:

\[ p_t^{\theta-1}(1 - \theta)^{\theta-1} A_{X,t} \bar{X}_t (w^U_t)^{-\theta-1} \]

Inserting (29), we get the supply of unskilled labour working in the primary sector, which is conditioned on \( k_t \) and \( p_t \):

\[ U_t(k_t, p_t) = p_t^{\theta-1}(1 - \theta)^{\theta-1} A_{X,t} \bar{X}_t (\beta A_t k_t^\alpha (i(k_t))^{1-\beta})^{\theta-1} \]

From the definition of the secondary sector’s skill-intensity \( i_t = \frac{S_t h_t}{U_t^S} \), and using that \( h_t = B(e(k_t))\gamma \), we see that in equilibrium given \( k_t \), the supply of unskilled worker in the secondary sector satisfies:

\[ U_t^S = (i(k_t))^{-1} S_t B(e(k_t))\gamma \]

Since the total population \( L_t \) satisfies

\[ L_t = U_t + U_t^S + S_t \]

it follows that \( L_t - U_t = [1 + ((i(k_t))^{-1}B(e(k_t)))\gamma]S_t \), and we may then insert (31) and rearrange to obtain:

\[ S_t(k_t, p_t) = \left[ \frac{1}{1 + ((i(k_t))^{-1}B(e(k_t)))\gamma} \right] \cdot [L_t - p_t^{\theta-1}(1 - \theta)^{\theta-1} A_{X,t} \bar{X}_t (\beta A_t k_t^\alpha (i(k_t))^{1-\beta})^{\theta-1}] \]

And we can derive \( U_t^S \) as a function of \( (k_t, p_t) \):

\[ U_t^S(k_t, p_t) = (i(k_t))^{-1} S_t(k_t, p_t) B(e(k_t))\gamma \]

And the labour market clearing condition now holds:

\[ L_t = U_t(k_t, p_t) + U_t^S(k_t, p_t) + S_t(k_t, p_t) \]
The market for the primary good clears (and by Walras’ law, the market for the secondary good therefore clears) if and only if:

\[
\bar{c}(p_t)L_t = (A_{X,t}\bar{X}_t)^\theta U_t^{1-\theta}
\]

Using (31) this becomes:

\[
\bar{c}(p_t)L_t = (A_{X,t}\bar{X}_t)^\theta (p_t^{\theta-1}(1-\theta)^{\theta-1}A_{X,t}\bar{X}_t(\beta A_t k_t^\alpha (i(k_t))^{1-\beta})^{-\theta-1})^{1-\theta}
\]

In this way we can write \( p_t \) as:

\[
(34) \quad p_t = (\bar{c}(p_t))^{\frac{1}{\theta-1}} \left( \frac{A_{X,t}\bar{X}_t}{L_t} \right)^{\frac{1}{1-\theta}} (1-\theta)^{-1-\beta}A_t k_t^\alpha (i(k_t))^{1-\beta}
\]

Define

\[
\omega(p) = (\bar{c}(p))^{\frac{1}{\theta-1}}
\]

It is straightforward to see that \( \omega(p) \) is a decreasing function of \( p \). We can then rewrite (34) as

\[
(35) \quad p_t = \omega(p_t) \left( \frac{A_{X,t}\bar{X}_t}{L_t} \right)^{\frac{1}{1-\theta}} (1-\theta)^{-1-\beta}A_t k_t^\alpha (i(k_t))^{1-\beta}
\]

Plug (35) into (31) and we can derive the primary sector labour share as:

\[
U_t(k_t,p_t) = \frac{p_t^{\theta-1}(1-\theta)^{\theta-1}A_{X,t}\bar{X}_t(\beta A_t k_t^\alpha (i(k_t))^{1-\beta})^{-\theta-1}}{L_t}
\]

\[
(36) \quad = \omega(p_t)^{\theta-1} \left( \frac{A_{X,t}\bar{X}_t}{L_t} \right)^{\frac{1}{1-\theta}}
\]

When doing the simulations, we set the parametric values and the evolution of the variables \( A_{X,t}, \bar{X}_t, L_t \), as such that \( \frac{U_t(k_t,p_t)}{L_t} \) starts at a level of around 0.76% and declines afterwards. This guarantees that the value of \( U(k_t,p_t) \) as well as those of \( U_t^S(k_t,p_t) \) and \( S_t(k_t,p_t) \) do not exceed \( L_t \), the total population.

### 3.3 Equilibrium and the Evolution of the Capital-to-Human Capital Ratio

Finally, we formulate the evolution of the capital-human capital ratio. This is the fundamental to both the evolution of the skill premium and the economic development in the period.

We begin with the aggregate stock of capital \( K_{t+1} \). It equals to the aggregate bequest deducted by the aggregate human capital investment. The number of individuals who
invest a positive amount in human capital in period \( t \) must equal the number of skilled workers \( S_{t+1} \) in period \( t + 1 \), therefore aggregate human capital investment is \( S_{t+1} \). It follows from (??) that

\[
(*) \quad K_{t+1} = \sum_{i=1}^{L_t} \tilde{b}_t^i - \epsilon(k_{t+1})S_{t+1}
\]

According to (21), we have \( \sum_{i=1}^{L_t} \tilde{b}_t^i = \sigma(p_t)(Y_t - p_t\bar{c}(p_t)L_t) \). Also, the clearing of the primary good market implies that the supply of the primary good, which is \( (A_X \bar{X}_t)^\theta U_t^{1-\theta} \), should equal to its demand, which is \( \bar{c}(p_t)L_t \). So aggregate output, or GDP, \( Y_t \), can be expressed as follows:

\[
Y_t = p_t(A_X \bar{X}_t)^\theta U_t^{1-\theta} + A_t k_t^\alpha S_t Be(k_t)^\gamma(i(k_t))^{-\beta} = p_t \bar{c}(p_t)L_t + A_t k_t^\alpha S_t Be(k_t)^\gamma(i(k_t))^{-\beta}
\]

GDP per capita can be denoted as:

\[
\frac{Y_t}{L_t} = \frac{p_t \bar{c}(p_t)L_t + A_t k_t^\alpha S_t Be(k_t)^\gamma(i(k_t))^{-\beta}}{L_t} = \frac{p_t \omega(p_t)\theta^{\theta-1}L_t + A_t k_t^\alpha S_t Be(k_t)^\gamma(i(k_t))^{-\beta}}{L_t}
\]

and therefore we can rewrite aggregate bequest in (21) as:

\[
\sum_{i=1}^{L_t} \tilde{b}_t^i = \sigma(p_t)(Y_t - p_t\bar{c}(p_t)L_t)
\]

(38)

And (*) can be rewritten as:

\[
K_{t+1} = \sigma(p_t)A_t k_t^\alpha S_t Be(k_t)^\gamma(i(k_t))^{-\beta} - \epsilon(k_{t+1})S_{t+1}
\]

The equation above can be written in terms of \( (k_t, p_t) \) as:

\[
[k_{t+1}B + \epsilon(k_{t+1})^{1-\gamma}]S_{t+1}(k_{t+1}, p_{t+1})e(k_{t+1})^\gamma = \sigma(p_t)A_t k_t^\alpha S_t(k_t, p_t)Be(k_t)^\gamma(i(k_t))^{-\beta}
\]

(39)

Since \( p_t \) can be expressed as a function of \( k_t \) (see equation (35)), equation (39) is the one that captures the evolution of \( k_t \), the capital-human capital labour ratio, from one period to another. (39) is thus the fundamental equation for simulation.

Hold variables \( A_t \) and \( B \) constant, we will generate steady states for \( k_t \) and \( p_t \). This will be shown in the simulation.

At steady state we have

\[
k_{t+1} = k_t = k
\]

\[
p_{t+1} = p_t = p
\]

Therefore, when \( k_t \) and \( p_t \) reach their steady states, (39) can be written as

\[
k = \sigma(p)A_t k^\alpha(i(k))^{-\beta} - B^{-1}(e(k))^{1-\gamma}
\]

(40)
3.4 Appendix: The Skill-premium in the long run

When markets are complete and agents have equal access to different occupations, the present value of lifetime earnings must in equilibrium be the same net of entry costs:

\[ w_{t+1}^H h(e_t^i) - r_{t+1}e_t^i = w_{t+1}^U \]

Note that this equilibrium of lifetime earnings in (41) is key to formulating the skill premium as in (26), which enables the skill premium to first decrease with respect to capital-human capital ratio \( k \) and become indifferent with it later on. In case that this equilibrium condition breaks down, the skill premium can no longer be formulated as in (26). Suppose there is a sudden destruction of aggregate capital stock \( K \), which reduces \( k \) to zero. Then (41) does not hold any more. This is because when \( k = 0 \), \( w_{t+1}^H = 0 \), \( e_t^i \) (thus \( h(e_t^i) \)) stays finite and \( r_{t+1} \to +\infty \), which make the left hand side of (41) go to \(-\infty\). The right hand side of (41) still stays finite. Since (41) does not hold anymore, the skill premium at the time when aggregate capital stock suddenly goes to zero can NOT be calculated based on (26). The last emphasis: (26) holds when individuals’ lifetime earnings reach equilibrium, which is most likely to be achieved in the long run. Even though the equality in (41) may be disrupted by a sudden evaporation of physical capital, it will be restored in the long run.

4 Conclusion

This chapter proposes a unified growth framework which captures two different stages of historical development in England as well as the evolution of the skill premium from one to the other. Given that the development of the economy from one stage to the other is driven by growing capital-human capital ratio, i.e. raising the abundance of physical capital relative to human capital, this chapter analytically proves skill premium will decline to a low level in the first stage of development and stay at the same low level after the economy enters the second stage.

What is interesting is the driving forces of the growth of capital-human capital ratio. As mentioned in the introduction, the growth of the relative abundance of physical capital relative to human capital is likely to be correlated with the major changes that happened to England from 1330 to 1660. These changes include the demographic decline and recovery, the growth of land productivity as well as the arable land. We will demonstrate that such changes do alter the capital-human capital ratio through simulation in the next chapter.
References


Abstract

This chapter brings the unified growth framework in the previous chapter and simulates the evolution of the skill premium and other relevant variables (GDP per capita and primary sector labour share) from 1328 to 1660. Through calibration and simulation, this chapter generates time series that captures the evolution of the skill premium over time and shows that it is consistent with historical data. The simulated evolution of GDP per capita, primary sector labour share and relative price of food are all in line with historical data. The calibration and simulation results in this chapter provide numerical evidence that the analytical framework in the previous chapter is good to explain the evolution of the skill premium in Western Europe from 1329 to 1660.

1 Introduction

In the previous chapter we develop a unified growth framework to study the evolution of the skill premium in England from 1329 to 1660. The unified growth framework proposes that the pattern of the skill premium in England was influenced by the capital-human capital ratio, which measured the abundance of physical capital relative to human capital. The changes in population, land productivity and arable land that took place in England from 1329 to 1660 were likely to influence the skill premium in this period. Within the framework in the previous chapter, the skill premium is influenced by the capital-human capital ratio, which will then be influenced by the mentioned changes in population, the land productivity and arable land. (Equation (38), (39)) in the previous chapter. The question that needs to be answered is: how do the mentioned changes in population, the land productivity and arable land affect the capital-human capital ratio, which then results in the “first declining then stabilizing” pattern of the skill premium?

This chapter answers this question by bringing the model into data. To begin with, we make several calibrations on the basis of the data on two indices. The first one is land-embodied agricultural productivity, $A_{X,t} \bar{X}_t$, which reflects the land productivity $A_{X,t}$ and arable land $\bar{X}_t$. It can be referred to as the “effective amount of land” (which will continued to be used in the rest of the chapter) and its growth over time captures the change in land productivity and arable land in the 1329-1660 England. The second index is the
Figure 1: Population in England

population, which equals to the total labour force $L_t$. It can be seen that the change of these two indices captures the changes in land productivity, arable land and population in the 1329-1660 England.

We partition the 1329-1660 period into two periods. The first period starts from 1329 and ends in 1500. In this period, the evolution of both the effective amount of land $A_{X,t} \bar{X}_t$ and the population $L_t$ is affected by exogenous factors. $A_{X,t} \bar{X}_t$ is influenced by the change in arable land and other socioeconomic factors such as the Enclosure and the rise of the spirit of hardworking (which raised working hours). The population $L_t$ is influenced by disease and wars. The outbreak of the bubonic plague from 1349 to 1351 incurred a dramatic demographic decline and the subsequent epidemics and wars induces a further decline of the population. The population reached the trough in 1450 and stayed low until 1500 despite some small recoveries (Figure 1).

The period between 1500 and 1660 is the second period. In this period, things started changing. $A_{X,t} \bar{X}_t$ continued to grow and its growth was determined by exogenous factors mentioned before. The population started to recover in this period (Figure 1), but its recovery was mainly the consequence of growing $A_{X,t} \bar{X}_t$, which was the Malthusian feature. That is, the growth of $A_{X,t} \bar{X}_t$ was translated into population growth. This style is reflected in the simulation of this period. We first shock $A_{X,t} \bar{X}_t$, then the population.

Our calibration and simulation results show that the growth of $A_{X,t} \bar{X}_t$ and the pop-
ulation decline (decreasing \( L_t \)) in the first period (corresponding to 1329-1500) raise the capital-human capital ratio. In the second period (1500-1660), \( A_{X,t} \bar{X}_t \) continues to grow and triggers the population recovery. As population starts to recover, the capita-human capita ratio may either decline or increase more slowly than before at the beginning, but it will eventually continue to increase until it reaches a sufficiently high level. Due to the continuing growth of the capital-human capital ratio, the skill premium then exhibits the ‘first decline then stabilizing” pattern.

In addition to successfully simulating the historical evolution of the skill premium, our calibration and simulation results shows that the evolution of GDP per capita, primary sector labour share is consistent with historical data. This brings the evolution of the skill premium in line with the evolution of the other key variables, which supports the interrelation between the skill premium and development (i.e. the skill premium goes down as GDP per capita goes up and primary sector labour share goes down) proposed by economic historians. This means that our proposed unified growth framework is good to explain the evolution of the skill premium and capture the evolution of the other key variables simultaneously.

The rest of the paper is organized as follows: section 2 discusses the calibration method and results. Section 3 discusses the simulation results and compares them with historical data. Section 4 concludes and discusses the implications of the findings for future work.

2 Calibration

As mentioned before, we partition the period into two parts: the first one ranges from 1329 to 1500 and the second one ranges from 1500 to 1600. We consider three steady states. As can be seen in the subsequent analysis, all the steady states depend on the “effective amount of land” \( A_{X,t} \bar{X}_t \), which I will simplify as “\( AX \)” and the population \( L_t \). the change in either of them will change the steady state value of variables such as capital-human capital ratio and the price, which will further affect skill premium and the other variables.

The first steady state is in the pre-plague period from 1329 to 1348. This is numbered as steady state 0 and is determined by the pre-plague \( AX \) and \( L \) which are denoted as \( AX0 \) and \( L0 \) respectively. After the plague and till 1500, there is exogenous growth of \( AX \) and exogenous decline in \( L \). We can use \( AX1 \) and \( L1 \), which are the values of \( AX \) and \( L \) in 1500 respectively, to generate another steady state, which we call steady state 1. From 1501 to 1660, there is further growth in \( AX \) and population starts to recover. So we can generate the third steady state, using \( AX2 \) and \( L2 \), which are the values of \( AX \) and \( L \) in 1660. We call it steady state 2.

\[1\] By doing this, we are basically treating the pre-plague economy in a steady state over time
We calculate the steady state capital-human capital ratio \( k \) using the equation in the previous chapter:

\[
(1) \quad k = \sigma(p)Ak^\alpha(i(k))^{-\beta} - B^{-1}(e(k))^{1-\gamma}
\]

With the steady state value \( k \) derived from (1) and given \( A_{X,t}, \bar{X}_t \) and \( L_t \), we assign values to the relevant parameters and calibrate the steady state skill premium (SP), relative price of food \( (p) \), GDP per capita (GDPpercap) and primary sector labour share(PSLS) using the following equations in the previous chapter:

\[
(2) \quad SP = \left[ 1 - e_t^{1-\gamma} \frac{\alpha}{B(1-\alpha-\beta)k} \right]^{-1}
\]

\[
(3) \quad p = \omega(p) \left( \frac{A_{X,t}X_t}{L_t} \right)^{\frac{1}{1-\theta}} (1-\theta)^{-1} \beta A_t k^\alpha(i(k))^{-\beta}
\]

\[
(4) \quad GDPpercap = \frac{p \omega(p) (1-\theta)^{-1} L_t + A_t k^\alpha SBe(k)\gamma(i(k))^{-\beta}}{L_t}
\]

\[
(5) \quad PSLS = \omega(p) (1-\theta)^{-1} \left( \frac{A_{X,t}X_t}{L_t} \right)^{1-\theta-1}
\]

The assigned parametric values are listed in Table 1 and the calibrated variables are listed in Table 2.

Note that in Table 1, \( AX0 \) and \( L0 \) are normalized to 1.5 and 1, respectively. The multipliers of \( AX1, AX2, L1, L2 \) are calculated in reference to agricultural output and total population from Broadberry et al. (2015), agricultural share of total population from Apostolides et al. (2008) (Table 17), land data from Broadberry et al. (2011). Normalizing \( L0 \) to 1 is done by Voigtländer and Voth (2013). Also, setting \( \theta \), the share of land in agricultural production, as 0.4 is to ensure that the labour share in agriculture is 0.6 which is consistent with Voigtländer and Voth (2013).

One important improvement in this revised chapter is the value of \( \sigma i(i = 1, 2, 3) \), which denotes individuals’ fraction of income devoted to saving. It is set at a reasonably low level i.e. no greater than 11% and captures the fact that households spent most of the income on consumption and devoted only a small fraction of income to saving in pre-modern era.

Using the calibrated steady state relative price of \( pi(i = 0, 1, 2) \) and the assigned values on \( \sigma i(i = 0, 1, 2) \), we formulate \( \sigma(p) \) as:

\[
(6) \quad \sigma(p) = \begin{cases} 
\sigma2 - \sigma1(p - p1) + \sigma1 & \text{if } p1 \leq p \leq p2 \\
\sigma2 - \sigma0(p - p0) + \sigma0 & \text{if } p2 \leq p \leq p0
\end{cases}
\]

It can then be seen that \( (AXXi, Li)(i = 0, 1, 2) \) determine the steady state capital-human capital ratio \( ki(i = 0, 1, 2) \). This is because the relative price of food \( pi(i = 0, 1, 2) \) is
a function of \((AXXi, Li)(i = 0, 1, 2)\) and \(ki(i = 0, 1, 2)\), which then enable \((AXXi, Li)(i = 0, 1, 2)\) to affect \(\sigma(p)\) through (6). This will then affect \(k\) through (1).

We can see from Table 2 that the calibrated values of the skill premium and primary sector labour share in steady states 0 and 2 as well as the ratio of GDP per capita in steady state 2 to that in 0 are all close to historical values. The calibrated values of these variables in steady state 1 may differ from historical values, but this does not matter much since the economy may never reach steady state 1. This is because not long after \(AXX\) and \(L\) reach \(AXX1\) and \(L1\) in 1500, new shocks on \(AXX\) may occur before the steady state is achieved, which then triggered population recovery. The simulation section will discuss the dynamic adjustment between these steady states.

One last thing to note from Table 2 is the pre-plague interest rate. The pre-plague interest rate is calibrated based on the following equation:

\[
r_0 = \alpha A k_0^{\alpha - 1} i(k0)^{-\beta} - \delta
\]

\(\delta\) is the depreciation rate in Table 1 and equals to 1. In this way we have pre-plague rental rate as

\[
\alpha A k_0^{\alpha - 1} i(k0)^{-\beta} = r_0 + 1 = 6.9413
\]

Note that (Clark 1988) estimate the pre-plague yearly interest rate is around 0.1. Then with \(\delta = 1\), the implied yearly rental rate before the plague (in around 1348) is

\[
0.1 + 1 = 1.1
\]

Clark (1988). Considering the compound effect of rental rate over time, the compound rental rate after \(n\) years satisfies

\[
(1.1)^n = 6.9413
\]

This yields

\[
n \approx 20
\]

This implies that the calibrated pre-plague interest rate \(r_0\) covers up to 20 years before the plague, which ranges from 1329 to 1348. This is why we set the starting year as 1329. We then calculate the variables \(AXX, L\) in this period by taking their averaging level treating the economy in this period as in a steady state.

3 Simulation of Dynamic Process

We now simulate the dynamic adjustment process. First we show the process of population \(L_t\) and the “effective amount of land” \(AXX_t\). For \(L_t\), we first shock it by reducing it from the level of \(L0\) by 51% to \(L1\), keep it at the low level \(L1\) for a while, then gradually
raise it to $L_2$. $AXX_t$ is shocked at two different points of time, which corresponds to its growth from 1329 to 1500 and from 1500 to 1660. The evolution of the $L_t$ and $AXX_t$ are depicted in Figure 2a and 2b respectively.

$A XX$ and $L$ starts at $A XX_0$ and $L_0$ respectively. We give a shock to both of them simultaneously in period 2 by making them equal to $A XX_1$ and $L_1$. This method captures the evolution of $A XX$ and $L$ before 1500. Both variables evolved exogenously: the growth of $A XX$ is driven by cultural and structural change and $L$ continued to decline due to the epidemics and wars after the plague. They are likely to change simultaneously. We refer to this shock as “the first shock”.

We keep $A XX_1$ and $L_1$ for some periods. Then in period 10 we shock $A XX$ again, making it equal to $A XX_2$. We do not shock $L$ simultaneously this time but keep it unchanged for one more period. In period 12 we shock $L$ and make it equal to $L_2$. This aims at capturing the Malthusian style of growth which featured with the growth of land productivity and arable land translating into population growth. Therefore, the growth of $L$ should come after the growth of $A XX$. We refer to this shock as the “second shock”.

We now simulate the process of capital-human capital ratio $k$ and human capital investment $e$. The latter is influenced by the former. Figure 3a shows that the first shock brings about the growth of $k$ and enable it to surpass the critical level $k_c$. As $k$ grows beyond $k_c$, the human capital investment $e$ grows beyond the baseline level $\bar{e}$ and starts to adjust flexibly with $k$ (i.e. $e$ goes up when $k$ grows, goes down when $k$ declines). (Figure 3b).

Before $k$ reaches the level at the first steady state $k_1$, the second shock takes place. The shock on $A XX$ causes $k$ to first decline then goes up, yet $k$ continues to be above $k_c$. Then with the population recovery which occurs after the growth of $A XX$, Figure 3a shows that $k$ goes slightly below $k_c$ and Figure 3b shows that $e$ goes back to $\bar{e}$. But this does not last long, shortly after $L$ reaches the new level $L_2$, $k$ goes up again, passing the threshold $k_c$ and reaches the new steady state level $k_2$. $e$ starts growing also.

With the evolution of $k$ and $e$ as such in Figures 3a and 3b, we can simulate the evolution of the skill premium. Comparing Figure 4a with Figure 3a, we can see that as $k$ goes up all the way beyond $k_c$, the skill premium declines to a low level and remains at that level as long as $k$ stays beyond $k_c$. In the period of population growth, $k$ goes slightly below $k_c$, which raises the skill premium a bit. But this situation does not last long. Right after the population recovery, $k$ goes beyond $k_c$ again and the skill premium goes back to the low and stable level. This slight change in the skill premium is so small that it does little to affect the stable trend of the skill premium after the initial decline. (Figure 4a). Comparing Figure 4a with Figure 4b, we can see that the historical evolution of the skill premium is similar to the simulated one.

We simulate the change in the ratio of GDP per capita after the plague to that before
the plague in Figure 5a. The pre-plague GDP per capita is normalized to 1. Figure 5a shows that after the first shock the simulated GDP per capita grows up to a high level that is approximately 1.45-1.55 times the original level and stays stable at that level after the population recovery. This is consistent with its historical counterpart depicted in Figure 5b.

Finally we simulate the evolution of the Primary Sector Labour Share (PSLS in short). Figure 6a shows that the simulated PSLS declines from an initial level of approximately 80% to a low level of 53%. This is consistent with its historical counterpart in Figure 5b.

4 Conclusion and Discussion of Future Work

This paper adopts data to testing the unified growth framework developed in the previous chapter. The simulation of the historical development in England from 1339 to 1660 shows that the skill premium declines from an initially high level to a low level and remains stable at that level afterwards. The simulation also shows that coupled with the such pattern of the skill premium are the growth of GDP per capita from an initially low level to a high level and the decline of primary sector labour share from an initially high level to a low level. These findings indicates that behind the “first declining then stabilizing” pattern of the skill premium are the growth of income per capita and the shift of labour force from the labour-intensive agricultural sector to the capital-intensive secondary sector. All these changes are the formation of the prototype of modern Europe (Van Zanden (2009)).

According to the findings in this chapter, the growth of the “effective amount of land” which consists of land productivity and arable land, and population decline jointly lowered down the skill premium in England to a low level at the beginning. The continuing growth of the “effective amount of land” prevented the skill premium from going up when the population is recovering and eventually going beyond the pre-plague level. This finding, which shows that the growth of the “effective amount of land” prevented the skill premium moving in tandem with population, not only unveils the insightful linkage between the skill premium and historical development, but also has insightful implication for future research on regional variation of the skill premium. According to Van Zanden (2009), the skill premium in Southern Europe, after declining to a low level during and after the plague like its counterpart in England, started to increase from the 16th century on, which corresponds to the time of the population recovery. And it is generally believed by historians that growth of “effective amount of land” in England outperformed that in Southern Europe in the 16th and 17th century. On the basis of the findings in this chapter, the proposition which can explain the regional variation of the skill premium between England and Southern Europe is that the growth of “effective amount of
“land” in Southern Europe is not strong enough to prevent the skill premium from moving in tandem with population growth. This proposition can be numerically tested through the calibration and simulation in this chapter once the data on the “effective amount of land” in Southern Europe from 1329 to 1660 are available.

References


Table 1: Assigned Values to Parameters of the Model

<table>
<thead>
<tr>
<th>Illustration of Variables</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical Capital Share in Secondary Sector</td>
<td>( \alpha )</td>
<td>0.34</td>
</tr>
<tr>
<td>Unskilled Labour Share in Secondary Sector</td>
<td>( \beta )</td>
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</tr>
<tr>
<td>Land Share in Primary Sector</td>
<td>( \theta )</td>
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</tr>
<tr>
<td>Marginal Rate of Return to Human Capital Investment</td>
<td>( \gamma )</td>
<td>0.73</td>
</tr>
<tr>
<td>Depreciation Rate of Physical Capital</td>
<td>( \delta )</td>
<td>1</td>
</tr>
<tr>
<td>Productivity of Secondary Sector</td>
<td>( A )</td>
<td>3.9</td>
</tr>
<tr>
<td>Average Effective Land 1329-1348</td>
<td>AX X0</td>
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<tr>
<td>Effective Land 1500</td>
<td>AX X1</td>
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<tr>
<td>Effective Land 1660</td>
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<tr>
<td>Average Population 1329-1348</td>
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<tr>
<td>Population 1500</td>
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<td>Population 1660</td>
<td>L2</td>
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<td>Fraction of Income denoted to Savings in Steady State 0</td>
<td>( \sigma_0 )</td>
<td>0.08</td>
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<td>Fraction of Income denoted to Savings in Steady State 1</td>
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<td>Fraction of Income denoted to Savings in Steady State 2</td>
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<td>Demand for food in Steady State 0</td>
<td>( \omega_0 )</td>
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<tr>
<td>Demand for food in Steady State 1</td>
<td>( \omega_1 )</td>
<td>1.232</td>
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<tr>
<td>Baseline Human Capital Investment per Skilled Worker</td>
<td>( \bar{\epsilon} )</td>
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Table 2: Calibrated Values of Relevant Variables

<table>
<thead>
<tr>
<th>Illustration of Variables</th>
<th>Variable</th>
<th>Value</th>
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<tr>
<td>Capital-Human Capital Ratio in Steady State 0</td>
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<td>Relative Price of Food in Steady State 0</td>
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<td>Interest Rate in Steady State 0</td>
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<td>Relative Price of Food in Steady State 1</td>
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<td>Relative Price of Food in Steady State 2</td>
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<td>GDP per capita Steady State Ratio (1 / 0)</td>
<td>GDPpercap₁/GDPpercap₀</td>
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<td>GDP per capita Steady State Ratio (2 / 0)</td>
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<td>Critical Value for Changing Human Capital Investment</td>
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Figure 2: Simulated $L$ and $AXX$

(a) $L$

(b) $AXX$

Figure 3: Simulated $k$ and $e$

(a) Capital-Human Capital Ratio $k$

(b) Human Capital Investment $e$
Figure 4: Skill Premium (Simulation vs. Real Data)

(a) Skill Premium (Simulated)  
(b) Skill Premium (Real Data)

Figure 5: GDP per capita (Simulation vs. Real Data)

(a) GDP per capita (Simulated)  
(b) GDP per capita (Real Data)
Figure 6: Primary Sector Labour Share (Simulation vs. Real Data)
Abstract

This chapter develops a theoretical framework that studies the influence of institutional development (financial development) on the poverty-reduction effect of remittances. I extend the model on long-run income distribution developed by Galor and Zeira (1993) and show that financial development increases the inflow of remittances, which in turn facilitates financial development. This interaction between remittances and financial development facilitates remittances’ poverty-reduction effect.

1 Introduction

The alleviation of poverty rate in developing and underdeveloped countries is the major concern in contemporary studies on development. To effectively alleviate poverty, it is important to understand the determination of poverty. In a seminal paper, Galor and Zeira (1993) proposes a model which unveils the determination of an economy’s income distribution in the long run. Because income distribution determines the poverty of an economy, this model is key to understanding factors that determine and influence poverty. Galor and Zeira (1993) show that there exists a threshold of individuals’ initial bequest, which reflects individuals’ asset holdings, that determines whether the individual will become rich or poor in the long run. If the initial bequest is beyond the threshold level, the individual will enjoy high income and hold high asset in the long run. Vice versa the individual will slip into the “poverty trap” with low income and low asset in the long run. The percentage of the population whose initial bequest is below the threshold level equals to the poverty rate.

This chapter extends this model to study how poverty is determined by remittances and institutional development (mainly financial development). While inheriting the same formulation of the financial development as in Galor and Zeira (1993), the model developed in this chapter is extended in two steps. First I incorporate the minimum consumption for subsistence into the model. The extended model can still generate two stable steady states for the dynamics of individuals’ bequest: a low steady state with a low level of bequest, which can be referred to as the “poverty trap” and a high steady state with a high level of bequest. This is similar to Galor and Zeira (1993). The difference is that
the bequest at the low steady state in this extended model is zero whereas that in Galor and Zeira (1993) goes to a low but positive level. This modification captures the feature of the people who live in poverty better: poor people in developing and underdeveloped countries usually have zero asset.

After deriving the modified poverty rate in the extended model without remittances, I then incorporate remittances into the model, which is the second step of extension. I show that remittances facilitating effect on financial development. Similar to Galor and Zeira (1993), there is credit market imperfection in my model, with the interest rate for loans being higher than interest rate for savings. The inflow of remittances reduces this gap by acting as collaterals for borrowers, thus reducing the credit market imperfections. On the other hand, the amount of remittances inflow is influenced by financial development: better financial conditions reduces the transaction costs and increases the optimal amount of remittances remitters send back home. I then derive the threshold level of initial bequest that determines whether an individual will slip into poverty trap. The result show that with financial development, the inflow of remittances generates a lower threshold level of bequest. This means that holding the initial income distribution across individuals the same, a larger fraction of the population manages to escape the poverty trap and the poverty rate goes down on a larger scale.

This work contributes to existing literature on remittances, institutions and poverty. The previous analytical study that looks at the effect of remittances and financial development on development focuses on the effect of remittances and financial intermediary on growth only (Mundaca (2009)). This work, however, does not specify how poverty (or income distribution) change in response to financial intermediary and remittances in the long run. Neither does it consider the possible change in the amount of remittances across recipients with different endowments. There are empirical studies which show that remittances facilitate financial development (Gupta et al. (2009), Aggarwal et al. (2011) and Nyamongo et al. (2012)). They propose that the remittances facilitate financial development by acting as collaterals so that more individuals could participate into financial market and Gupta et al. (2009) also mentioned the possible bi-directional causality between remittances and financial development, pointing out that financial development would possibly attract more remittances. Yet there is no analytical study that supports this claim. Akobeng (2017) studies the institutional impact on the poverty-reduction strength of remittances empirically without any analytical study. To the best of my knowledge, this chapter is the first analytical study that examines the influence of institutional development featured with financial development on remittances’ poverty-reduction effect.

The rest of the chapter is organized as follows: Section 2 presents the model and the results; Section 3 concludes and discusses the policy implications.
2 The Model

Following Galor and Zeira (1993), we consider a small open economy with two sectors for production. The first sector hires skilled labour $L_S$ and capital $K$ for production. And the production follows the constant returns to scale:

$$Y_S = F(K, L_S)$$

The equilibrium wage of skilled labour $w_S$ and interest rate $r$ equal to the marginal product of $L_S$:

$$w_S = F_{L_S}(K, L_S)$$
$$r = F_K(K, L_S)$$

Given the constant return to scale production function, both $F_{L_S}(K, L_S)$ and $F_K(K, L_S)$ are determined by the capital-skilled labour ratio $K/L_S$. And as in Galor and Zeira (1993), the small open economy has a constant interest rate $r$, which is determined by the international rate. This means the capital-skilled labour ratio $K/L_S$ is constant. A constant capital-skilled labour ratio $K/L_S$ yields a constant wage for skilled labour $w_S$.

The other sector is set to hire unskilled labour $L_U$ for production with a constant wage rate $w_U$, which also follows Galor and Zeira (1993). The production function is

$$Y_U = w_U L_U$$

And the wage of the unskilled labour is $w_U$.

Similar to Galor and Zeira (1993), an individual $i$ is bestowed with a specific amount of bequest in period $t$, which is denoted as $b_i^t$. The individual needs to invest a constant amount of money $h$ to become skilled labour. If $b_i^t < h$, the individual has to borrow. The interest for loans is $r_i^t$. And the bank impose a cost of $z$ to prevent the individual from default. So the equilibrium of the credit market satisfies

$$d_i(1 + r_i^t) = d_i(1 + r) + z$$

$d_i$ is the amount of money the individual has to borrow to make human capital investment. The cost the individual has to undertake to default is $\beta z (\beta > 1)$. Bank will set cost $z$ as such that

$$d_i(1 + r_i^t) = \beta z$$

Combine 1 with 2 and we have

$$r_i^t = \frac{1 + \beta r}{\beta - 1} \equiv i$$
\[ r_i^t = \frac{1 + \beta r}{\beta - 1} \equiv i \]

Since \( \beta > 1 \), it is easy to verify \( i > r \), and there is credit market imperfection, with borrowers facing a higher rate than savers. This formulation is exactly the same as [Galor and Zeira (1993)]

I set an overlapping generation model for individuals who live for two periods \( t \) and \( t + 1 \). In the first period of life an individual decides whether to invest in human capital to become skilled worker in the future. If yes, the individual will invest \( h \) in the first period and receive payment \( w_s \) in the second period. If not, the individual will work as unskilled worker for two periods and receive \( w_u \). In the second period of life, the individual allocate the income \( I_{t+1}^i \) between consumption \( c_{t+1}^i \) and bequest \( b_{t+1}^i \) to maximize the following utility \( u_{t+1}^i \):

\[ u_{t+1}^i = (1 - \theta) \log c_{t+1}^i + \theta \log b_{t+1}^i \]

The budget constraint is

\[ c_{t+1}^i + b_{t+1}^i \leq I_{t+1}^i \]

. All of these come from the formulation in [Galor and Zeira (1993)].

The consumption at the subsistence level is \( c \). Individual \( i \) with income \( I_{t+1}^i \) chooses consumption and bequest as follows:

\[
\begin{align*}
    c_{t+1}^i &= I_{t+1}^i & b_{t+1}^i &= 0 & \text{if } I_{t+1}^i \leq c \\
    c_{t+1}^i &= c & b_{t+1}^i &= I_{t+1}^i - c & \text{if } c < I_{t+1}^i \leq c/(1 - \theta) \\
    c_{t+1}^i &= (1 - \theta) I_{t+1}^i & b_{t+1}^i &= \theta I_{t+1}^i & \text{if } I_{t+1}^i > c/(1 - \theta)
\end{align*}
\]

(4) shows that with the introduction of subsistence consumption \( c \), individual’s optimal choices of consumption and bequest is now different from [Galor and Zeira (1993)].

Now we consider remittances. The remitter’s optimize the amount of remittances based on the following utility function:

\[ u = [w - rem_i^t(1 + c)]^{1-\rho}(rem_i^t - b_i^t)\rho \quad 0 < \rho < 1 \]

In (5), \( w \) is the remitter’s income, which is influenced by exogenous factors such as GDP growth, inflation and and unemployment in the country they migrate to. \( c \) denotes the transaction cost remitters face when sending money back home. The decrease in \( c \) reflects the degree of financial development: the less the transaction cost faced by remitters, the better the financial development and vice versa.

\( b_i^t \) is the bequest of individual \( i \). Solve (5) and we have the optimal amount of remittances individual \( i \) with bequest as:

\[ rem_i^t = \frac{w\rho}{1 + c} + b_i^t(1 - \rho) \]
shows that the optimal amount of remittances indicate that the remitters’ motivation to send remittances is positively influenced recipients’ bequest (amount of asset the recipient holds). The example that supports this claim is that remitters wish the recipients to invest the remittances to constructing new houses in addition to consumption. Recipients with more asset are more likely to do so than recipients with less assets. According to Gupta et al. (2009), West Africa, the area that experiences inflow of remittances, sees the growing construction of large homes by migrant workers.

also shows that the optimal amount of remittances is negatively related to the transaction cost $c$. Since the decrease in $c$ reflects financial development, we can see that financial development helps attract more remittances.

As mentioned before, remittances can serve as collaterals for borrowers. If an individual wants to borrow money for human capital and uses the remittances as collateral, the bank will forfeit the remittances if the individual chooses to default. An individual with bequest $b_i$ needs to borrow $h - b_i$ for human capital investment. The we can rewrite (1), the credit market equilibrium condition as:

$$(h - b_i)(1 + r_i) = (h - b_i)(1 + r) + z$$

In (7), $z$ is still the cost banks impose to prevent default. And the cost the individual has to undertake to default is still $\beta z (\beta > 1)$. With remittances as collateral, bank will set cost $z$ as such that

$$(h - b_i')(1 + r_i) = \beta z + rem_i(1 + r)$$

Combine (7), (8) and (6), we can solve for the interest rate for loans $r_i$ as:

$$r_i = \frac{1 + \beta r}{\beta - 1} - \frac{1 + r}{(h - b_i)(\beta - 1)} \left[ \frac{w \rho}{1 + c} + b_i(1 - \rho) \right]$$

Comparing (9) with (3) we have

$$r_i - i = -\frac{1 + r}{(h - b_i)(\beta - 1)} \frac{w \rho}{1 + c} + b_i(1 - \rho) < 0$$

This shows that with remittances reduce credit market imperfections by lowering down the loan rate, which facilitates financial development.

With the remittances as (6), if the individual invests on human capital and is a borrower, the second period income $I_{t+1}$ satisfies

$$I_{t+1} = w_S + (b_i - h)(1 + r_i) + rem_i(1 + r)$$

$$= w_S + \frac{\beta(1 + r)(2 - \rho)}{\beta - 1} b_i + \frac{\beta(1 + r)}{\beta - 1} \left( \frac{w \rho}{1 + c} - h \right)$$
If the individual does not need to borrow, then the second period income satisfies

$$I^{i,S2}_{t+1} = w_S + (b^i_t - h + rem^i_t)(1 + r)$$

$$= w_S + (2 - \rho)(1 + r)b_t + \left(\frac{w\rho}{1 + c} - h\right)(1 + r)$$

If the individual does not invest in human capital, the income will be

$$I^{i,U}_{t+1} = w_U(2 + r) + (b^i_t + rem^i_t)(1 + r)$$

$$= w_U(2 + r) + \left[\frac{w\rho}{1 + c} + (2 - \rho)b^i_t\right](1 + r)$$

We now check the dynamics of $b_t$. We impose assumptions on parameters as follows:

**Assumption 1.** The parametric values are assumed to satisfy the following inequalities:

(10) \[ h > \frac{w\rho}{1 + c} \]

(11) \[ (1 - \theta)w_S < c < w_S \]

(12) \[ w_S - c < h \]

Different levels of $b^i_t$ influence the level of $I^{i,S1}_{t+1}$, $I^{i,S2}_{t+1}$ and $I^{i,U}_{t+1}$, which further influence the formulation of $b_{t+1}$:

- **Case 0)**

  \[ 0 \leq b_t < \frac{\beta - 1}{(1 + r)(2 - \rho)} \left[w_U(2 + r) + \frac{\beta(1 + r)}{\beta - 1}h - w_S - \frac{1 + r}{\beta - 1}w\rho\right] \]

  Individuals whose bequests satisfy this case do not invest in human capital and remain unskilled. And we have

  \[ b_{t+1} = 0 \]

Individual whose bequests satisfies one of the following four cases will invest in human capital and become skilled workers.

- **Case 1)**

  \[ \frac{\beta - 1}{(1 + r)(2 - \rho)} \left[w_U(2 + r) + \frac{\beta(1 + r)}{\beta - 1}h - w_S - \frac{1 + r}{\beta - 1}w\rho\right] \leq b_t \]

  \[ < \frac{\beta - 1}{(2 - \rho)(1 + r)} \left(\frac{1 + r}{\beta - 1}h + \frac{c - w_S}{\beta} - \frac{1 + r}{\beta - 1}w\rho\right) \]

  In this case we have

  \[ b_{t+1} = 0 \]
• Case 2)

\[
\frac{\beta - 1}{(2 - \rho)(1 + r)} \left( \frac{1 + r}{\beta - 1} h + \frac{c - w_S}{\beta - 1} - \frac{1 + r}{\beta - 1} w_{\rho} \right) \leq b_t < \frac{1}{2 - \rho} \left( h - \frac{w_{\rho}}{1 + c} \right)
\]

In this case, we have

\[
b_{t+1} = I_{t+1}^{i,S1} - \xi
\]

\[
= w_S + \frac{\beta(1 + r)(2 - \rho)}{\beta - 1} b_t + \frac{\beta(1 + r)}{\beta - 1} \left( \frac{w_{\rho}}{1 + c} - h \right) - \xi \tag{13}
\]

• Case 3)

\[
\frac{1}{2 - \rho} \left( h - \frac{w_{\rho}}{1 + c} \right) \leq b_t < \frac{1}{2 - \rho} \left( h - \frac{w_{\rho}}{1 + c} \right) + \frac{1}{(2 - \rho)(1 + r)} \left( \frac{c}{1 - \theta} - w_S \right)
\]

In this case, we have

\[
b_{t+1} = I_{t+1}^{i,S2} - \xi
\]

\[
= w_S + (2 - \rho)(1 + r)b_t + \left( \frac{w_{\rho}}{1 + c} - h \right) (1 + r) - \xi \tag{14}
\]

• Case 4)

\[
b_t \geq \frac{1}{2 - \rho} \left( h - \frac{w_{\rho}}{1 + c} \right) + \frac{1}{(2 - \rho)(1 + r)} \left( \frac{c}{1 - \theta} - w_S \right)
\]

Then we have

\[
b_{t+1} = \theta I_{t+1}^{i,S2}
\]

\[
= \theta w_S + \theta \left( 2 - \rho \right)(1 + r)b_t + \theta \left( \frac{w_{\rho}}{1 + c} - h \right) (1 + r) \tag{15}
\]

\(\theta(2 - \rho)(1 + r) < 1\) is assumed to hold so that the bequest will converge to a high but constant level rather than grow unbounded.

In order for \(b_t\) to converge to a high but constant level, we need

\[b_{t+1} > b_t\]

when

\[
b_t = \frac{1}{2 - \rho} \left( h - \frac{w_{\rho}}{1 + c} \right) + \frac{1}{(2 - \rho)(1 + r)} \left( \frac{c}{1 - \theta} - w_S \right)
\]

This implies that the function that monotonically maps \(b_t\) to \(b_{t+1}\) need to cross the 45-degree line \(b_{t+1} = b_t\) in either Case 2) or Case 3).

We can now have the following proposition characterizing the impact of financial development on the poverty-reduction effect of remittances:
Proposition 1. Suppose the c.d.f that characterizes the distribution of bequest across individuals is \( F \) and \( F \) satisfies all the properties of c.d.f. The financial development featured with the reduction of transaction cost \( c \) results in lower poverty rate at steady state.

Proof. We consider two cases:

• Case a)

If the function cross the 45-degree line \( b_{t+1} = b_t \) in Case 2), then by setting \( b_{t+1} = b_t \) we have \( b_C^2 \), the level of \( b_t \) that corresponds to the crossing point as:

\[
(16) \quad b_C^2 = \frac{(2 - \rho) - \frac{\beta - 1}{\beta(1+r)}}{(2 - \rho) - \frac{\beta - 1}{\beta(1+r)}} - \frac{w\rho}{1+c} \]

Then the poverty rate equals to the fraction of the population whose bequest is below \( b_C^2 \). Using \( F \), the c.d.f that characterizes the distribution of bequest across individuals, we can derive the poverty rate \( PR_2 \) as:

\[
(17) \quad PR_2 = F(b_C^2) = F \left[ \frac{(2 - \rho) - \frac{\beta - 1}{\beta(1+r))}}{(2 - \rho) - \frac{\beta - 1}{\beta(1+r))}} - \frac{w\rho}{1+c} \right] \]

\( F \) is increasing with respect to \( b_C^2 \).

Furthermore, because \( 0 < \rho < 1, 0 < r < 1 \) and \( \beta > 1 \), it is straightforward to verify

\[
(2 - \rho) - \frac{\beta - 1}{\beta(1+r)} > 0
\]

Therefore using (16) we have:

\[
\frac{\partial b_C^2}{\partial c} = \frac{w\rho}{1+c} \frac{1}{(2 - \rho) - \frac{\beta - 1}{\beta(1+r))}} > 0
\]

So \( b_C^2 \) is increasing in \( c \). A reduction of \( c \) then lowers down \( b_C^2 \), which then reduce \( F(b_C^2) \), the poverty rate.

• Case b)

If the function cross the 45-degree line \( b_{t+1} = b_t \) in Case 3), then by setting \( b_{t+1} = b_t \) we can derive \( b_C^3 \), the level of \( b_t \) that corresponds to the crossing point as:

\[
(18) \quad b_C^3 = \frac{h(1+r) + \xi - wS}{(2 - \rho)(1 + r) - 1} - \frac{w\rho(1+r)}{1+c} \frac{1}{(2 - \rho)(1 + r) - 1}
\]

The poverty rate \( PR_3 \) becomes

\[
(19) \quad PR_3 = F(b_C^3) = F \left[ \frac{h(1+r) + \xi - wS}{(2 - \rho)(1 + r) - 1} - \frac{w\rho(1+r)}{1+c} \frac{1}{(2 - \rho)(1 + r) - 1} \right]
\]
Given $0 < \rho < 1$ and $0 < r < 1$, it is straightforward to verify

$$(2 - \rho)(1 + r) - 1 > 0$$

Then using (18) we have:

$$\frac{\partial b^3_C}{\partial c} = \frac{w\rho(1 + r)}{(2 - \rho)(1 + r) - 1} \frac{1}{(1 + c)^2} > 0$$

So $b^3_C$ is increasing in $c$. A reduction of $c$ then lowers down $b^3_C$, which then reduce $F(b^3_C)$, the poverty rate.

The proof of Proposition 1 shows that financial development lowers down the poverty rate and achieve this by raising the amount of remittances (the term $w\rho/(1 + c)$ is related to the amount of remittances). This shows financial development’s facilitating impact on the poverty-reduction effect of remittances.

We can see the change in the critical level of bequest that distinguishes the rich from the poor in the long run graphically. Suppose $b_{t+1} = b_t$ is achieved in case 2). Figure 1 shows that when the transaction cost $c$ reduces from $c^0_2$ to $c^1_2$, the critical level of bequest that distinguishes the rich from the poor int he long run drops from $(b^0_C)$ to $(b^1_C)$. Suppose $b_{t+1} = b_t$ is achieved in case 3), Figure 2 shows that when the transaction cost $c$ reduces from $c^0_3$ to $c^1_3$, the critical level of bequest that distinguishes the rich from the poor int he long run drops from $(b^0_C)$ to $(b^1_C)$.

### 3 Conclusion

This chapter develops a theoretical framework that studies the influence of institutional development on the poverty-reduction effect of remittances. It shows that the financial development facilitate remittances’ poverty-reduction by increasing the inflow of remittances and reducing the credit market imperfections (i.e. lower down the interest rate for loans). This finding has insightful policy implications. It shows that the attempt to increase the efficiency of banking by reducing the transaction cost raises the inflow of remittances, which yields more reduction of poverty. Moreover, this finding shows a side effect of increasing remittances inflow, which is to relieve the credit market imperfection. Hence the financial development featured with the reduction of transaction cost raises the amount of remittances, which facilitates other aspects of financial development. This improves the efficiency of the entire financial system and alleviates poverty.
References


Figure 1: Change in Critical Level of Bequest in Case 2
Figure 2: Change in Critical Level of Bequest in Case 3
Chapter 5: Conclusion

This thesis studies two types of inequality, wage inequality and poverty, in a long span of time and in different regions: First, it studies the evolution of the skill premium in England from 1329 to 1660, the long historical period that ranges from late medieval era to early modern era. This study unveils how inequality changes in response to economic development and transition in pre-modern era. Second, it analyzes how the interaction between remittances and institutional development (mainly financial development) contributes to the alleviation of poverty. This study has the potential to facilitating the understanding of how to alleviate poverty and foster development in contemporary developing and underdeveloped countries.

Chapter 2 proposes a unified model that characterizes the growth, development and transition in England from 1329 to 1660. With this model, it proposes an analytical explanation on the evolution of the skill premium in this period, which declines from an initially high level of over 120% to a low and stable level of 58%. The study shows that the abundance of physical capital relative to human capital, which is measured by the capital-human capital ratio, is the key determinant of the skill premium. At the beginning, the scarcity of physical capital means low capital-human capital ratio. When the capital-human capital ratio starts growing, the increase in the relative demand for skilled labour is less than that in the relative supply of skilled labour. This incurs excessive relative supply of skilled labour and causes the skill premium to decline. As the capital-human capital ratio grows to a sufficiently high level, the change in relative demand for skilled labour is able to keep pace with that in the relative supply of skilled labour. As a result, the skill premium stays stable. It can be seen that behind the low and the stable skill premium from 1450 onwards was the formation of an efficient labour market in England, which symbolizes the high degree of modernity of the English economy in pre-modern era.

Chapter 3 uses data to test the analytical finding in Chapter 2. The simulation result shows that with the demographic decline of more than 55%, the capital-human capital ratio increases to a sufficiently high level. Then the skill
premum declines from an initial level of over 120% towards the low and stable level of 58%. The subsequent recovery of the population lowers the capital-human capital ratio below the critical level, which then raises the skill premium beyond the stable level. But the degree to which the skill premium goes beyond the stable level is suppressed by the growing land productivity. As a result, the skill premium stays stable at the low level of 58 % in general.

Chapter 3 also displays the simulation of the GDP per capita and the primary sector labour share (PSLS in short). The simulated GDP per capita goes up by 40% on average in the period of population decline. In the subsequent period of population recovery, the GDP per capita stays at approximately the same level as during the population decline. And the simulated PSLS declines all the way from around 80% at the beginning to around 55% in the end. Consistent with the historical data, the simulated GDP per capita and PSLS further symbolize the gradual formation of modern English economy well before the industrialization.

Chapter 4 studies the impact of remittances and institutional development (mainly financial development) on poverty. First, it analytically studies on remittances' facilitating effect on financial development. It analytically shows that institutional development increases the inflow of remittances, and the growing amount remittances in turn facilitates the financial development by reducing the credit market imperfections borrowers have to face. This lends analytical support to the existing empirical studies on the bidirectional causality link between remittances and financial development. Secondly, and more importantly, this chapter analytically studies the influence that the interaction between remittances and financial development exerts on poverty. It explicitly shows that the financial development, which aims at increasing the inflow of remittances, facilitates the alleviation of poverty directly and indirectly. The direct impact is that remittance recipients have more money at hand, as senders are incentivized to send more money back home. The indirect impact is that more remittances reduce the interest rate for loans that are imposed on borrowers. in turn facilitates financial development, with banks imposing a lower interest rate for loans on the borrowers. Both impacts increase individuals' likelihood to invest in human capital for high income. As a result, a larger fraction of the population will end up with earning high income and
leaving the poverty trap. While analytically backing up the existing empirical studies that highlight the impact of remittances and financial development on poverty-reduction, this finding has insightful policy implications for developing and underdeveloped countries. It shows that policies aimed at attracting more remittances would result in healthier financial environment, higher literacy rate and lower poverty rate.

It is generally agreed that low inequality and poverty is accompanied with high income and literacy rate. With a large fraction of the population investing in human capital and earning high income, the degree of inequality and poverty can be reduced. We can come to a general conclusion from the studies in the previous three chapters that the effective way to incentivize individuals to invest in human capital is to enhance their capital stock. According to Chapters 2 and 3, as the abundance of physical capital stock relative to human capital stock increases, individuals will increase the amount of human capital investment they make. This results in 1) a larger fraction of the population concentrating into the high-income sector and 2) growing income individuals earned in the high-income sector. Both of these would effectively reduce inequality and increase the average income of the economy. Chapter 4 shows that an individual will investment in human capital and receive high income only when he or she has sufficient assets. Financial development incurs more inflow of remittances. This then increases the amount of individuals’ assets, enabling them to offer the bank more collaterals when borrowing money for human capital investment. Hence the credit market imperfection individuals are faced is relieved, which leads more individuals to invest in human capital and earn high income. The entire economy will then end up with lower poverty, higher literacy rate as well as a healthier financial environment.