Essays in Financial Economics and Computational Finance

Thesis submitted for the degree of Doctor of Philosophy at the University of Leicester

by

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To my family and friends
This thesis contains three chapters. In particular, it examines the opaqueness of financial securities, risk contagion among asset markets and learning behaviours and trading skills in financial markets.

Chapter 1, “Opacity in Security Design: The Role of Derivative Markets”, builds a theoretical model to explain the contribution of financial derivatives, especially Credit Default Swap, to the opaqueness of financial securities. We find a trade-off in the opacity design of financial securities: opacity increases adverse selection in the primary market while giving the originator information advantage in the derivative market, the former decreasing the originator’s profits while the latter increasing his profits. Hence, the optimal opacity involves a balance between the two effects, implying derivative markets do play a role in deciding the opacity of financial securities. In addition, a liquid derivative market tend to induce more opacity as the originator makes more profits on the derivative market, which makes him more willing to sacrifice his profits in the primary market and increase opacity.

Chapter 2, “Statistical Arbitrage and Risk Contagion”, builds a computational model to investigate the risk contagion mechanism provided by statistical arbitrageurs among asset markets. Statistical arbitrageurs arbitrage mispricings but also follow a market-neutral rule. We find that statistical arbitrageurs help stabilise markets in normal periods. However, they may also act as the mechanism for the spread of shocks, making the whole system expose to extreme events. While statistical arbitrageurs may play a role in risk contagion, we find the effects are limited. The reason is that statistical arbitrageurs in markets not shocked trade against shocks, which helps the system to recover fast. In addition, statistical arbitrageurs persistently make positive profits from trading, consistent with the fact that financial institutions heavily rely on this strategy.

Chapter 3, “The Role of Heterogeneous Beliefs in Trading Skill Acquisition”, also creates a computational model to examine how chartists, who are effectively noise traders, affect fundamentalists’ learning and trading skill acquisition, where fundamentalists do not have the skill to fairly price financial assets and learn to do so from trading. We find that the presence of chartists can facilitate the learning of fundamentalists by stabilising the market price. However, chartists tend to reduce the accuracy of the learning outcome by misleading the market price. Hence, an increase in the amount of chartists increases the proportion of fundamentalists holding trading skills, which makes markets more resilient. However, the less accurate learning outcome of fundamentalists tend to increase market volatility.
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Declaration

I declare that chapter 1: “Opacity in Security Design: The Role of Derivative Markets” is my joint paper with Doctor Adriani Fabrizio and Doctor Subir Bose. Chapter 2: “Statistical Arbitrage and Risk Contagion” and chapter 3: “The Role of Heterogeneous Beliefs in Trading Skill Acquisition” are my joint papers with Professor Daniel Ladley. Chapter 1 has been presented at the IFABS Asia conference 2017 (Ningbo, China). Chapter 3 has been presented at the CEF conference 2018 (Milan, Italy) and the IFABS conference 2018 (Porto, Portugal).
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Introduction

This thesis contains three themes in financial markets. The first theme is about the role of derivative markets in security design in terms of the transparency of the security, in the context of the prevalence of the opaque structured financial securities before the 2007-2008 global financial crisis. The theoretical results show that derivative markets, especially the Credit Default Swap markets, do play an important role in increasing the incentive to make financial securities opaque. The magnitude of this effect depends on the liquidity in the derivative markets in the sense that a liquid derivative market induces more opaque securities. This is because a liquid derivative market makes the originator’s profits in the derivative market more important compared to his (negative) profits in the primary market. Hence, the originator is more willing to give up his profits in the primary market to reap more profits in the derivative market. In the model, an originator holds a risky asset and wants to securitise it and sell the risky security to potential buyers via first price auction. If trade of the risky security occurs in the primary market, a derivative security, which is to hedge the default risk of the risky security, will be traded and the originator is allowed to trade both the underlying and derivative securities, a wide practice among financial institutions before the global crisis. The originator is a profit maximiser and optimally chooses ex-ante the opacity of the underlying security before he knows the type of the risky asset. After opacity is decided, the originator observes the type of the asset while everyone else observes a public (noisy) signal. Then, trade of the risky security and the derivative security occurs sequentially. Finally, either security pays off. Opacity has opposite effects on the originator’s profits in the primary and derivative markets. In the primary market, opacity increases adverse selection, decreases the price buyers willing to pay and reduces the originator’s profits. In the derivative market, opacity gives the originator information advantage and increases his profits. Put it simple, opacity hurts the originator in the primary market but benefits him in the derivative market. When deciding the optimal opacity, the originator faces a trade-off and needs to find a balance between the two effects. The more liquid the derivative market, the more profits the originator can get from the derivative market, which makes the losses in the primary market less of an issue and motivates the originator to make the under-
lying security more opaque. In addition, we find that opacity can increase liquidity by discouraging the acquisition of private information. This creates a feedback loop between opacity and liquidity, through which the effects of any change in either opacity or liquidity will be amplified. This study contributes to the literature on opacity and liquidity by providing an innovation that investigates the relationship between opacity in the primary market and liquidity in the derivative market, instead of focusing on a single market. Also, this study enriches the growing literature arguing that opacity may not necessarily decrease but could increase liquidity. In addition, this study provides a new explanation for the diffusion of opaque financial securities in recent years, which is a result of the speculation in the derivative markets. The results in this study also support the policy that restricting the proprietary trading of financial institutions after the recent global financial crisis.

The second theme is concerned with the networks among asset markets, especially stock markets. We build a numerical model to look into the role of statistical arbitrage in linking stock markets and in risk contagion. This idea comes from the contagion of liquidity dry-ups among stock markets in the Flash Crash on 6 May 2010. Statistical arbitrage is a market-neutral strategy widely used by financial institutions. It involves the exploitation of short-term deviations from a long-run equilibrium between assets. When deviations occur, statistical arbitrageurs take long positions in assets underperformed and short positions in assets outperformed, in a way such that the resulting portfolio is market neutral. In this study, we consider multiple markets populated by fundamentalists, chartists and statistical arbitrageurs. Fundamentalists and statistical arbitrageurs are informed of the fundamental value of the stocks and share the same beliefs. Chartists do not care about fundamental values but only rely on past price trends. Fundamentalists and chartists trade in one market while statistical arbitrageurs cross trade among markets, through which markets are linked. The results show that in normal periods, statistical arbitrage can help stabilise the markets by driving prices back to the fundamentals. However, when extreme events occur, for example, when an unexpected shock occurs in one market, it may act as the mechanism for the transmission of the shock. In our test, we find that when statistical arbitrageurs do not trade, a shock occurring in one market does not propagate to other markets. However, when they trade, a shock can result in significant and systematic price decrease in other markets, as what has happened in the Flash Crash. This finding confirms the role of statistical arbitrage in risk transmission. Intuitively, when a shock occurs in one market, statistical arbitrageurs trading that stock and stocks linked to the shocked one will adjust their demand for each stock in their portfolio to keep market neutrality. This affects the price in these markets. Statistical arbitrageurs trading these stocks and other stocks linked will also adjust their demand, which changes prices
again and triggers larger scale of demand adjustment and price change etc. Hence, statistical arbitrageurs provide the route for risk contagion with which a shock in a single market can lead to a systematic collapse. Our study is first consistent with the existing literature arguing the double-edged role of networks in financial stability, that is, financial networks are able to stabilise market in normal periods but may act as a mechanism for risk contagion. More importantly, our study raises the concern about the potential hazard from advanced and modern trading techniques and strategies, among which statistical arbitrage is one with great representativeness by heavily relying on advanced computational capacity. It is true that the generation of advanced trading techniques and strategies has greatly improved trading efficiency. However, the downside of it should also be noticed and attract more attention.

The third theme focuses on the learning behaviours and trading skill acquisition in financial markets. The aim of this study is to investigate how noise traders (chartists) affect the learning of fundamentalists. Specially, we revisit the question of how heterogeneous beliefs affect market performance. However, we assume fundamentalists do not have the skill to fairly price financial assets but learn to do so through trading, different from most of the relevant literature assuming fundamentalists are rational by nature. We consider markets populated by fundamentalists and chartists and learning occurs only among fundamentalists. Traders trade 3-month European call options. Black-Scholes price gives the fundamental value of the options. Each trader has a pricing function deciding the quote at which he is indifferent between buying and selling one option. Transaction price is the median of all quotes. Pricing function of fundamentalists is randomly generated computer programs while pricing function of chartists is weighted average of past prices. After a trading round, all fundamentalists can review and with the same probability replace their pricing functions with a one generating more accumulated wealth, which is how learning occurs. In particular, replacement of pricing functions follows the rule of nature selection. Through evolving, pricing functions generating more accumulated wealth will survive which give quotes close to the Black-Scholes price. The numerical results show that most price quotes lie within three clusters: one at left extreme (0), one in the middle around the Black-Scholes price and one at the right extreme (over 15). As the amount of chartists increases, the middle cluster becomes bigger and fatter while the other two clusters becomes smaller, implying that as there are more chartists, more fundamentalists are quoting close to the Black-Scholes price, however, their evolved pricing functions are also more dispersed around the Black-Scholes price. This indicates that chartists promotes learning, reducing extreme quotes and increasing the proportion of fundamentalists holding skill, but also decrease the accuracy of the learning outcome. In addition, chartists reduce market stability but increase market resilience. Trading skills are normally rewarded higher
returns in practice, which is confirmed in this study by the finding that fundamentalists holding higher level of skill tend to make more profits from trading. Intuitively, the evolving of fundamentalists’ pricing functions adds uncertainties to the market price. Chartists, by taking past prices into their pricing functions, may play the role of stabilising market price, which facilitates learning. Hence, the proportion of skilled fundamentalists increases with the amount of chartists (middle cluster becomes bigger). However, chartists still add noises to the market price, which can mislead fundamentalists, resulting in low accuracy of the learning outcome (middle cluster becomes more dispersed). Skilled traders are able to absorb shocks where shocks in this study come from the entry of traders with extreme quotes (0 and 40). Hence, markets with more chartists tend to be more resilient to shocks. However, since the evolved pricing functions are more dispersed, market price becomes more volatile.
Chapter 1

Opacity in Security Design: The Role of Derivative Markets

Abstract

The period before the 2007-2008 financial crisis witnessed an incredible growth both of highly opaque financial securities and of credit derivatives. We present a model to explain the prevalence of opaque securities by considering the role of derivative markets in the design of financial securities. More opaque securities imply larger information asymmetries in the primary market, but increase the originator’s profits from informed trading in the derivative market. We find a self-reinforcing positive relationship between opacity of the security and liquidity of the derivative market: a small exogenous increase in liquidity of the derivative market can, through a feedback loop, lead to a large increase of the opacity of the security and liquidity itself.
1.1 Introduction

The period preceding to 2007-2008 financial crisis witnessed an incredible growth of opacity in financial markets, especially of structured financial products (Furfine, 2014; Sato, 2014; Brunnermeier and Oehmke, 2009).\(^1\) For instance, from 2000 to 2003, the global annual issuance of Collateralized Debt Obligations (CDOs), a widely traded class of structured securities, rose slightly from $68 billion to $86.6 billion, while growing sharply to $520 billion in 2006.\(^2\) After the crisis, both the academia and the popular press have widely documented that before the crisis, structured securities were so opaque that even top financial experts such as investment banks and rating agencies could not understand their payment structure. We observe financial companies aggressively bet against the default of structured securities and rating agencies gave inappropriate ratings to them.\(^3\) What motivates the high opacity design of structured securities and what fuelled the success of the markets for structured securities just right before the crisis?

During the same period, the markets for credit derivatives experienced spectacular growth. For instance, between 2004 and 2008, the total nominal amount of Credit Default Swap (CDS), the main form of credit derivatives, grew from just $6 trillion to $57 trillion (Stulz, 2009). Obviously, the markets for opaque structured securities and credit derivatives experienced a simultaneous period of prosperity.

While after the crisis the opacity of structured securities has been blamed for concealing risks (Gorton, 2008; Pagano and Volpin, 2012; Siegert, 2014), little attention has been paid to the incentives to design opaque securities. Traditional models imply that opacity worsens adverse selection and destroys liquidity in asset markets. If this is the case, why did originators make structured securities opaque? By doing so, what did they attain? Augustin et al. (2016) and also, in the popular press, Michael Lewis\(^4\) point out that before the crisis, the speculation in the credit derivative markets of some hedge funds and investment banks motivated these financial institutions to intentionally create fragile and opaque securities. One good example is the famous “ABACUS” case. In this case, Paulson & Co., a New York hedge fund, on the one hand picked the most risky (subprime) mortgage loans to originate the very fragile and opaque (subprime) mortgage-backed securities ‘Abacus 2007-AC1’,

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\(^{1}\)People can think mortgage-backed securities as an example of structured financial products. Since structured financial products are financial securities, we use structured securities instead of structured financial products in the rest of this paper.


\(^{4}\)In his popular book “The Big Short”, Michael Lewis provides a number of anecdotes showing that before the crisis, some financial institutions intentionally design fragile and opaque structured securities to facilitate their speculation in the credit derivative markets.
on the other hand, bet on the default of these securities by taking positions in the credit derivative markets.\textsuperscript{5}

Observing the interesting simultaneous growth of the markets for opaque structured securities and credit derivatives and the numerous anecdotes, we ask: Is the simultaneous growth in the two markets before the crisis a coincidence? Does the growth of the market for credit derivatives partly explain the high opacity of structured securities before the crisis? Or more generally, can the derivative markets play a role in the design of securities? In this paper, we answer these questions.

In our baseline model, we consider the problem of an informed originator who is a profit maximiser and designs securities to be issued in a primary market. Different from most existing models, however, we allow the originator to engage in speculative trading in the derivative market. This setting matches key features of widespread practice (see the “ABACUS” case for example). Also, it allows us to investigate how the trading activities of the originator in the credit derivative market affect his incentives in security design, which is the task of our model. The derivative asset can be interpreted as a Credit Default Swap (CDS).\textsuperscript{6} Speculative buyers of a CDS profit when the underlying asset defaults. Speculative sellers of CDS profit when the asset is safe. Obviously, if allowed to design the underlying assets, these speculators would do it to their benefits. Hence, we can expect that the derivative markets can play a role in security design.

When the originator is able to participate in both the primary and derivative markets, a trade-off arises: opacity increases information asymmetries and hence lowers the originator’s profits in the primary market. On the other hand, it increases the originator’s information advantage and raises his profits in the derivative market. Put simply, opacity hurts the originator in the primary market but benefits him in the derivative market. Without a derivative market, the originator would make the securities as transparent as possible. In contrast, in the presence of a derivative market, optimal opacity will be determined by trading off the two effects, which implies the derivative markets do change the originator’s incentive to design opaque securities. More importantly, our key prediction is that opacity is increasing in the liquidity of derivative markets. In other words, the more liquid the derivative market, the more opaque the securities will be. Intuitively, more liquid derivative markets imply greater scope for informed speculation. As liquidity increases, the originator is thus more willing to sacrifice profits in the primary market.

In an extension of the baseline model, we add speculators (different from the

\textsuperscript{5}For more details, please see https://uk.reuters.com/article/us-goldmansachs-abacus-factbox/factbox-how-goldmans-abacus-deal-worked-idUSTRE63F5CZ20100416.

\textsuperscript{6}As CDS is the main form of credit derivatives, in the following context, we focus on CDS. The speculation of CDS before the crisis was huge. As pointed out by Zabel (2008), by the end of 2007, among the $45 trillion of notional value of CDS, at least $20 trillion were speculative “bets”.

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originator) to the derivative market who can become informed at some costs. We endogenise the liquidity of the derivative market by assuming that information acquisition costs increase in opacity. This is consistent with the fact that as opacity rising, speculators need more time, resources and skills to collect and process information. Interestingly, this creates a feedback loop between opacity of the primary asset and liquidity of the derivative market, through which opacity and liquidity are interconnected. For instance, any change in opacity of the primary asset would affect liquidity in the derivative market, which further impacts the opacity of the primary asset. Through this feedback loop, the effects of any change in either opacity or liquidity would be amplified. This result shows that the simultaneous period of prosperity of the opaque structured securities and credit derivatives before the 2007-2008 financial crisis is not a coincidence, instead, it is the outcome of the interaction between opacity and liquidity. In addition, this result enriches the growing literature showing that opacity may not necessarily reduce but could improve liquidity (Farhi and Tirole, 2015; Stenzel and Wagner, 2015).

Opacity and liquidity are at the center of financial literature. However, most related research focuses only on a single market, that is, how the opacity of an asset affects the liquidity of the market for that asset. Our paper provides an innovation by investigating the interaction of opacity in the primary market and liquidity in the derivative market. A similar paper to ours is the one by Pagano and Volpin (2012) where they also consider a trade-off problem between the effects of opacity on the primary and secondary markets. However, in their paper, opacity is good in the primary market but is bad in the secondary market, while in our paper, the reverse applies.

The rest of this paper is organised as follows. Section 1.2 discusses relevant literature on security design, opacity and liquidity and market microstructure. Section 1.3 present our baseline model where liquidity is exogenous. Section 1.4 provides an extension with endogenous liquidity. Section 1.5 concludes.

1.2 Literature review

Fistly, this paper is related to the literature on security design. The security design literature is mostly interested in the question of which type of security, debt or equity, is better able to make the security information insensitive, as information insensitivity reduces adverse selection and makes the security market liquid. Generally, the consensus in this literature is that debt contracts are usually less sensitive to information and are thus optimal (Demarzo and Duffie, 1999; DeMarzo, 2005; Dang et al., 2015). Unlike the above literature focusing on the design of the security cash flows, this paper studies the design of a security’s transparency.
Sato (2014) explores why and how opaque financial assets are designed, which is quite similar to our work in this paper. In his model, financial engineers are able to transform transparent assets into opaque assets with the incentive to obtain an “opacity price premium” from the asset market. In our paper, the originator’s incentive for greater opacity comes from his ability to trade in the derivative market. Also, in Sato’s paper, in the primary market, opacity is good for the engineers, while in our paper opacity is harmful as no one benefits from it.

In addition, our paper is related to the literature of opacity and liquidity (Pagano and Volpin, 2012; DeMarzo, 2005; Stenzel and Wagner, 2015; Siegert, 2014; Kaplan, 2006). Asymmetric information is one contributor to the bid-ask spreads and thus market liquidity (Kyle, 1985; Glosten and Milgrom, 1985). The stronger the information asymmetry, the larger the bid-ask spreads and the less liquid the market. Hence, opacity and liquidity cannot be separated. In general, opacity provides incentives for private information acquisition, which results in greater information asymmetry between traders and reduces market liquidity (Welker, 1995; Diamond and Verrecchia, 2001). However, a growing literature argues opacity may facilitate trading and hence improve market liquidity. Siegert (2014) finds a non-linear relation between opacity and liquidity. Stenzel and Wagner (2015) point to a hump-shaped relation between opacity and liquidity. In their paper, high opacity does not necessarily reduce but could increase market liquidity. When opacity goes up, both parties of a trade may end up being symmetrically uninformed (see also Farhi and Tirole (2015)). This “common ignorance” facilitates trades and liquidity. In their paper, the costs of transparency are exogenous while in our paper, the costs of transparency are represented by the originator’s profits in the derivative market, which are endogenous opportunity costs.

Pagano and Volpin (2012) show that opacity weakens the incentives for private information acquisition in the primary market, but leaves large scope for private information acquisition in the secondary market. Thus, opacity increases liquidity in the primary market while decreases liquidity in the secondary market. Hence, the determination of optimal opacity involves a balance between the two effects. While our paper is similar to theirs in spirit, important differences exist between the two papers. First of all, as mentioned above, in their paper, opacity is good in the primary market but is bad in the secondary market. In our paper, the reverse is true. Secondly, in their paper, only the two extreme cases, full transparency and full opacity, are considered. In our case, opacity is a continuous variable, which enables us to solve for the optimal opacity and do comparative statics. Finally,
originators in their paper are unsophisticated in the sense that they cannot process information, while in our paper the originator has an information advantage and is fully sophisticated.

The most significant difference between the current literature on opacity and liquidity and our paper is that most of the current literature puts much attention into the asset market while we explore the relation between opacity of the underlying asset and liquidity of the associated derivative market, which allows us to provide a different angle to the prevalence of opaque securities before the crisis.

Another literature this paper is related to is voluntarily withholding of information (Diamond, 1985; Madhavan, 1995; Siegert, 2014; Kaplan, 2006; Boot and Thakor, 2001). This literature focuses on whether and what kind of information should be released. Dang et al. (2015) argue that banks should hide some information to make the financial system stable. Boot and Thakor (2001) point out that complementary information should be released to strengthen investors’ incentive to acquire private information. Again, this literature does not consider the effects of derivative markets. In this paper, the incentive for the originator to withhold information is to make the derivative market liquid, enabling him to speculate in it.

Finally, this paper is related to the literature on market microstructure (Glosten and Milgrom (1985); Kyle, 1985). The baseline model in this paper is built on the market microstructure model by Glosten and Milgrom.

1.3 Model

1.3.1 Setting

We consider a three period setting where in period 1, an informed originator $O$ is endowed with a risky asset of size 1. $O$ wants to securitise the risky asset and sell the security to a potentially uninformed buyer $B$. Let $A$ denote the security paying the underlying asset’s cash flows. The originator optimally chooses ex-ante the opacity of security $A$, which determines the extent to which information about the underlying asset is public or private. In period 2, $O$ has the chance to trade a derivative security $\mathcal{A}$ which hedges the default risk of security $A$.\footnote{Security $A$ can be seen as Asset-Backed Securities (ABS) where receivables such as mortgage loans and credit card loans could be the underlying risky asset. Security $\mathcal{A}$ can be seen as Credit Default Swap (CDS).} In the baseline model, we assume that liquidity in the derivative market is exogenous. A competitive market maker observes the order flow, sets prices and makes zero expected profits. The order flow comes from either noise traders, informed speculators or the informed originator. In an extension, we endogenise liquidity by endogenising the presence of informed speculators. Assets pay in period 3.
Securities and states
There are two possible states of nature \( \omega \in \{ \alpha, \beta \} \). The underlying risky asset could be of two types, which is denoted by \( e \in \{ a, b \} \). If the risky asset is of type \( a \), it pays 1 in state \( \alpha \) and 0 otherwise. If the risky asset is of type \( b \), it pays 1 in state \( \beta \) and 0 otherwise. Accordingly, security \( A \) pays 1 whenever the underlying risky asset pays, while the derivative security \( \bar{A} \) pays 1 whenever the risky asset does not pay. The payment structure of \( A \) and \( \bar{A} \) is summarised in the table below.

<table>
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<th>( \omega = \alpha )</th>
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<tr>
<td>( e = a ) ( A ) pays 1, ( \bar{A} ) pays 0</td>
<td>( A ) pays 0, ( \bar{A} ) pays 1</td>
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<tr>
<td>( e = b ) ( A ) pays 0, ( \bar{A} ) pays 1</td>
<td>( A ) pays 1, ( \bar{A} ) pays 0</td>
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We denote with \( \gamma \in (0, 1) \) the probability of state \( \alpha \) (and hence, \( 1 - \gamma \) is the probability of state \( \beta \)). Without loss of generality, we set \( \gamma \in [0, 1/2] \), which implies that without knowing the state of nature, the expected return of type \( b \) is larger than that of type \( a \). In this sense, we call type \( b \) the high type and type \( a \) the low type. For simplicity, we assume all traders have a prior over \( e \) such that probability \( \text{Pr}(e = a) = \text{Pr}(e = b) = 1/2 \).

Opacity
Before trading, information about the underlying asset’s type becomes available. This information is perfectly revealing to the originator - so that \( O \) is able to perfectly observe \( e \in \{ a, b \} \) - but the usefulness of the information to other participants depends on security \( A \)’s transparency.\(^9\) To model this, we assume everyone except \( O \) observes a (possibly noisy) public signal \( \sigma \in \{ 0, 1 \} \) about \( e \). In particular, we define \( \sigma_\tau, \tau \in \{ a, b \} \), as the probability \( \text{Pr}(\sigma = 1|e = \tau) \). If \( \sigma = 1 \), the probability of type \( a \) and \( b \) is respectively

\[
\text{Pr}(e = a|\sigma = 1) = \frac{\sigma_a}{\sigma_a + \sigma_b}; \quad \text{Pr}(e = b|\sigma = 1) = \frac{\sigma_b}{\sigma_a + \sigma_b}
\]

If \( \sigma = 0 \), the probability of type \( a \) and \( b \) is respectively

\[
\text{Pr}(e = a|\sigma = 0) = \frac{1 - \sigma_a}{1 - \sigma_a + 1 - \sigma_b}; \quad \text{Pr}(e = b|\sigma = 0) = \frac{1 - \sigma_b}{1 - \sigma_a + 1 - \sigma_b}
\]

Without loss of generality, we assume \( \sigma_a \leq \sigma_b \). We say security \( A \) is fully transparent if \( \sigma_a = 0, \sigma_b = 1 \) and fully opaque if \( \sigma_a = \sigma_b \). In the first case, \( \sigma = 0 \) reveals type \( a \) while \( \sigma = 1 \) reveals type \( b \). Thus, the public signal \( \sigma \) is perfectly informative. In the second case, the public signal does not help uninformed participants to update

\(^9\)For instance, if the underlying risky asset is based on credit card loans, information about labor market conditions is potentially relevant for predicting the future asset’s cash flow. However, the extent to which labor market information may be used to price the security will depend on the investors’ grasp of the details of the security they bought, which is determined by the security’s opacity.
their prior over \( e \) and hence is perfectly uninformative. The degree of opacity of security \( A \) is chosen by \( O \) ex-ante, i.e. before he observes \( e \). To make things simple, we set \( \sigma_b = 1 \) and let \( \sigma_a \) vary. Note that signal \( \sigma = 0 \) fully reveals the low type (type \( a \)). However, signal \( \sigma = 1 \) is only partially revealing as long as \( \sigma_a \in (0,1) \). Furthermore, signal \( \sigma = 1 \) is more likely when the underlying risky asset is of the high type (type \( b \)) in the absence of full opacity (\( \sigma_a = \sigma_b = 1 \)). Thus, we call \( \sigma = 0 \) the “bad” signal and \( \sigma = 1 \) the “good” signal.

**Originator and buyers** The originator is risk neutral. \( O \) is initially endowed with the risky underlying asset and securitises it to \( A \). There is a large number of potential buyers (strictly greater than one), who are also risk neutral. They can choose to buy security \( A \) offered by \( O \) or to invest in a risk-free asset paying a gross return equal to one. We assume trade between \( O \) and buyers occur via a first price auction. Buyers’ expectation about the value of security \( A \) is given by \( A \)’s expected time 3 cash flow. \( O \) faces an opportunity cost \( k \geq 0 \) when \( A \) is not sold to buyers. Hence, by trading \( A \), \( O \) realises a surplus \( k \). \( k \) can be interpreted as the net present value of a fixed size project funded by the cash raised from trading \( A \). Alternatively, \( k \) can be seen as an invisible regulatory benefit \( O \) can get from selling \( A \), which enables him to (temporarily) remove the risks associated with the risky underlying asset from his balance sheet.

**Primary market** The primary market works as follows. In the first period, after the public signal \( \sigma \) is revealed, security \( A \) is auctioned to potential buyers through a first price auction. If more than one buyer bids the same price, a lottery is used to decide the actual buyer of the security. Competition between buyers forces the transaction price to equal to buyers’ expectation about \( A \)’s value. A key assumption of our model is that \( O \) cannot commit ex-ante - i.e. prior to obtaining information about \( e \) - to trade at a given price. This conforms to practice that investment banks informally contact potentially interested parties to solicit bids for the security on offer. Typically, if the bank is not satisfied with the price, it always retains the option not to sell the security. In our setting, if \( O \) is holding the high type risky asset, that is type \( b \) given \( \gamma < 1/2 \), he may thus choose not to sell if adverse selection is severe.

**Derivative market** In the second period, the state of nature \( \omega \in \{a,b\} \) is publicly revealed. Since security \( A \) is the underlying asset of the derivative security \( \mathcal{A} \), the derivative market opens only if security \( A \) has been issued in the primary market. Otherwise, the derivative market does not exist. The derivative market is a one-trade version of the model by Glosten & Milgrom (1985). A competitive market maker, observing the order submitted, decides the bid/ask price of the derivative security \( \mathcal{A} \) based on all other public information including the public signal \( \sigma \) and state of nature \( \omega \). The derivative market is populated with noise traders, informed speculators and
the informed originator, each group of traders being chosen to submit an order of size \( s \) to the market maker with some probability. Noise traders are uninformed. Once chosen to trade, they submit a buy or a sell order with equal probability. While the one-trade assumption is not necessary for our results, it considerably simplifies the analysis in the derivative market and enables us to provide some insights into the role of derivative markets in opacity design of securities but avoid repeating complex calculations.

**Timing**

The timing of our model is as follows,

- **Period 1**
  - Information about the underlying risky asset is released, \( O \) observes the asset’s type \( e \in \{a, b\} \), everyone else observes a public signal \( \sigma \in (0, 1) \)
  - Trade of security \( A \) occurs between \( O \) and \( B \)

- **Period 2**
  - State of nature \( \omega \in \{\alpha, \beta\} \) is revealed.
  - The market for derivative security \( A \) opens, the market maker observes the order submitted and sets the bid/ask price.
  - Trade of \( A \) occurs.

- **Period 3**
  - Either security \( A \) or \( A \) pays off.

<table>
<thead>
<tr>
<th>( \sigma_a ) is chosen by ( O ) before observing ( e \in {a, b} )</th>
<th>( T=1 )</th>
<th>( T=2 )</th>
<th>( T=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O ) observes ( e \in {a, b} ); ( \omega ) is publicly revealed; Payoffs are realised; Derivative market for ( A ) opens</td>
<td>( B ) observes ( \sigma \in {0, 1} ); Trade of ( A ) occurs</td>
<td>Consumption occurs</td>
<td></td>
</tr>
</tbody>
</table>

Timeline of the baseline model.

### 1.3.2 Equilibrium Analysis

Let \( t \in \{\text{buy, sell}\} \) denote the type of order submitted to the competitive market maker in the derivative market. If \( t = \text{buy} \), the market maker sets an ask price, however, if \( t = \text{sell} \), market maker sets a bid price. Generally, we denote by \( p^A(t, \sigma, \omega) \) the transaction price of security \( A \).

So far, we have shown the whole setting of our model. The originator chooses ex-ante the level of opacity \( \sigma_a \), participates first in the primary market and then have the chance to trade in the derivative market. \( O \) is a profit maximiser and hence
the level of opacity \( \sigma_a \) chosen ex-ante maximises his ex-ante overall profits. In the primary market, the transaction price is given by a pricing function \( p^A(\sigma) \). At price \( p^A(\sigma) \), \( O \) chooses whether to trade, especially when he is holding the high type, based on his information and the expected profits he can obtain from each action. If trade occurs in the primary market, in the derivative market, the transaction price is given by a pricing function \( p^A(t, \sigma, \omega) \) at which the market maker makes zero expected profits. \( O \), once chosen to trade, submit either a buy or a sell order based on his information.

In summary, \( O \)'s strategy comprises the probability \( \sigma_a \) chosen ex-ante and an interim strategy mapping \( \{a, b\} \times \{0, 1\} \) into an action \{trade, not\} in the primary market and \( \{a, b\} \times \{0, 1\} \times \{\alpha, \beta\} \) into an action \{buy order, sell order\} in the derivative market. Equilibrium is defined by pricing functions \( p^A(\sigma) \) and \( p^A(t, \sigma, \omega) \) and \( O \)'s strategy as below.

**Definition 1.1.** An equilibrium of the overall game is a pricing function \( p^A(\sigma) \) for the primary market trade, a pricing function \( p^A(t, \sigma, \omega) \) for the derivative security \( A \) and a strategy for \( O \) such that:
1. \( \sigma_a \) maximises \( O \)'s ex-ante expected profits.
2. \( O \)'s interim strategy maximises interim expected profits at each stage.
3. \( p^A(\sigma) \) is consistent with a Nash equilibrium in the primary market's auction given the public information.
4. \( p^A(t, \sigma, \omega) \) ensures zero expected profits for the market maker conditional on the order submitted and all other public information.

We will proceed backward, by analysing the derivative market first.

### 1.3.3 Derivative Market

Upon observing the bad signal \( \sigma = 0 \), the market maker knows that \( e = a \) and hence is able to fairly price the derivative security \( A \), in which case \( O \) makes zero profits in the derivative market. Thus, \( O \) has an information advantage only when \( \sigma = 1 \) and \( \sigma_a > 0 \).

We start by deriving the prices of the derivative security and \( O \)'s profits in the derivative market in each state. As mentioned before, the derivative market exists only if security \( A \) has been issued in the primary market. Thus, we focus on the scenario where trade of \( A \) has occurred in the primary market. There are three groups of traders - noise traders (\( N \)), informed speculators (\( S \)) and the originator (\( O \)). In period 2, before trade occurs in the derivative market, each group of traders is chosen to submit an order of size \( s \) with probability \( \lambda_N, \lambda_S \) and \( \lambda_O \), respectively and \( \lambda_N + \lambda_S + \lambda_O = 1 \). \( \lambda_O \) (\( \lambda_S \) and \( \lambda_N \)) measures the participation of \( O \) (\( S \) and \( N \))
in the derivative market in the sense that the larger $\lambda_O$, the more likely $O$ is chosen to trade.

Assuming trade has occurred in the primary market, the market maker sets the price at which he earns zero expected conditional profits. The price is conditional on: a) the type of order submitted $t \in \{\text{buy}, \text{sell}\}$, b) the state of nature $\omega \in \{a, b\}$, c) the public signal $\sigma \in \{a, b\}$. Let $V_A(e, \omega)$ denote the value of security $A$.

\[
V_A(e = a, \omega = \beta) = V_A(e = b, \omega = \alpha) = 1; \quad (1.3)
V_A(e = a, \omega = \alpha) = V_A(e = b, \omega = \beta) = 0
\]

After observing all available information, while he still does not know if $e = a$ or $e = b$, the market maker can assign a probability $Pr(e = a|t, \sigma, \omega)$ and $Pr(e = b|t, \sigma, \omega)$ respectively to the two cases. Based on the analysis above, the price of $A$ will be

\[
p^A(t, \sigma, \omega) = V_A(e = a, \omega) \times Pr(e = a|t, \sigma, \omega) + V_A(e = b, \omega) \times Pr(e = b|t, \sigma, \omega)
\]

For illustrative purpose, we fix $\sigma = 1$ and $\omega = \alpha$. Note that

\[
Pr(e = a|\sigma = 1) = \frac{\sigma_a}{1 + \sigma_a}, \quad Pr(e = b|\sigma = 1) = \frac{1}{1 + \sigma_a} \quad (1.5)
\]

In the absence of full transparency ($\sigma_a > 0$), $0 < p^A(t, \sigma, \omega) < 1$. Hence, it is always optimal for informed traders (including $O$) to trade in the derivative market, that is, to sell $A$ when $e = a$ and to buy $A$ when $e = b$. The reverse is true for $\omega = \beta$ (see the table below).

<table>
<thead>
<tr>
<th>$e$</th>
<th>$\omega = \alpha$</th>
<th>$\omega = \beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>sell $A$</td>
<td>buy $A$</td>
</tr>
<tr>
<td>$b$</td>
<td>buy $A$</td>
<td>sell $A$</td>
</tr>
</tbody>
</table>

The tree below shows how the market maker updates his beliefs based on all available information. $i$ denotes informed traders and $u$ denotes uninformed traders.
Based on the above, we obtain

\[
p^A(buy, 1, \alpha) = \frac{2(\lambda_S + \lambda_O) + \lambda_N}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1)}
\] (1.6)

\[
p^A(sell, 1, \alpha) = \frac{\lambda_N}{2(\lambda_S + \lambda_O)\sigma_a + \lambda_N(\sigma_a + 1)}
\] (1.7)

Note that the bid-ask spread is

\[
p^A(buy, 1, \alpha) - p^A(sell, 1, \alpha)
\] (1.8)

which acts as the liquidity measure of the derivative market. In the absence of full opacity \((\sigma_a < 1)\), keeping \(\lambda_O\), \(\lambda_S\) and \(\lambda_N\) constant, the bid-ask spread increases with opacity. Intuitively, as opacity goes up, information asymmetries between the market maker and informed traders \((O\) and \(S)\) increase. As a consequence, the market maker will increase the bid-ask spread to avoid being cheated by informed traders.

In the extreme case of full opacity \((\sigma_a = 1)\), the bid-ask spread collapses into \(\frac{\lambda_S + \lambda_O}{\lambda_S + \lambda_O + \lambda_N}\), in which case, opacity is so high that the market maker gets no information from the public signal and his beliefs rely heavily on the proportion of informed traders in the total population. In other words, the bid-ask spread widens if there are more informed traders while narrows if there are more noise traders.

For the originator, the narrower the bid-ask spread, the higher the expected profits from derivative market trading.

To see this, note that the originator expects a profit

\[
\lambda_O s [1 - p^A(buy, \sigma, \omega)]
\] (1.9)
when he wants to buy and

$$\lambda_O s[p_A^A(sell, \sigma, \omega)]$$  \hspace{1cm} (1.10)

when he wants to sell. Recall $\lambda_O$ is the probability $O$ is chosen to trade and $s$ is the order size $O$ can submit. Again, let us focus on the case $(\sigma = 1, \omega = \alpha)$. Replace the prices in the profit expressions and rearrange to get profits

$$\pi_a^A(sell, 1, \alpha) = \lambda_O s \frac{\lambda_N}{2(\lambda_S + \lambda_O)\sigma_a + \lambda_N(\sigma_a + 1)}$$  \hspace{1cm} (1.11)

when $O$ wants to sell and

$$\pi_b^A(buy, 1, \alpha) = \lambda_O s \frac{\lambda_N \sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1)}$$  \hspace{1cm} (1.12)

when $O$ wants to buy. Since the bad signal $\sigma = 0$ is fully revealing, $O$ makes zeros profits in the case of $\sigma = 0$. Hence

$$\pi_a^A(t, 0, \omega) = 0; \pi_b^A(t, 0, \omega) = 0$$  \hspace{1cm} (1.13)

In the extreme case of full transparency ($\sigma_a = 0$), all information is public and derivative market profits are zero in all states $(e = a, \sigma = 0)$ and $(e = b, \sigma = 1)$. $O$ can make higher profits if the realisation of the signal has misleading effects on market maker’s beliefs. For instance, $O$’s profits are higher in the case of $e = b$ when $\sigma_a$ is higher because a higher $\sigma_a$ makes the market maker believe that $e = a$ happens with a high probability, which lowers the market maker’s beliefs in state $\omega = \alpha$ but increases $O$’s profits.

Moving one step backward, now we look at the derivative market from the originator’s perspective in period 1 when $\omega$ is not known but the originator already knows $e$. We denote the expected profits from derivative market trading with

$$\pi^{*A}_{e\sigma} = E[\pi^A_e(t, \sigma, \omega)|e, \sigma]$$  \hspace{1cm} (1.14)

where the expectation is taken with respect to $\omega \in \{\alpha, \beta\}$ and $t \in \{buy, sell\}$. Intuitively, the originator’s profits in the derivative market depend on the liquidity of the market which is provided by noise traders and consumed by the originator and speculators. On the one hand, the more noise traders, the more liquid the market and the better the originator will be able to hide his order and his information advantage. On the other hand, the more speculators, the less liquid the market and the less favourable the market price will be.
1.3.4 Primary market

Before trade occurs in the primary market, $O$ directly observes the type of the underlying risky asset while others including $B$ observe the public signal $\sigma$. As mentioned before, potential buyers’ surplus from buying $A$ is competed away and hence trade occurs at buyers’ expectation. Let $p_\sigma$ denote the price of security $A$ given the signal $\sigma \in \{0,1\}$ received. Upon observing $\sigma = 1$,

$$p_1 = \frac{\gamma \sigma_a}{\sigma_a + 1} + \frac{1 - \gamma}{\sigma_a + 1}$$

(1.15)

Similarly, upon observing $\sigma = 0$,

$$p_0 = \gamma$$

(1.16)

For the informed originator, let $V_e$ denote his expectation about the value of security $A$ after observing type $e \in \{a,b\}$. If he observes $e = a$, $V_a = \gamma - k$, otherwise, $V_b = 1 - \gamma - k$. Obviously, $V_a < p_0 < p_1$ for $k > 0$, which implies that trade always occurs for the low type (type $a$). However, for the high type (type $b$), trade may not always occur, depending on how severe adverse selection is in the market. If $O$ decides to sell $A$, his profits are

$$p_\sigma - V_a + \pi^A_{a\sigma}$$

(1.17)

when $e = a$ and

$$p_\sigma - V_b + \pi^A_{b\sigma}$$

(1.18)

when $e = b$. However, if $O$ chooses not to sell $A$, again, since $A$ is derived from $A$, the derivative market will not exist.

Remember when $\sigma = 0$, $O$ always earns zero profits in the derivative market. As a result, when $\sigma = 1$, there are two interim participation constraints for the originator (each for the cases $\{a,b\}$).

$$k + \frac{1 - 2\gamma}{1 + \sigma_a} + \pi^A_{a1} \geq 0 (PCa1)$$

(1.19)

$$k - \frac{(1 - 2\gamma)\sigma_a}{1 + \sigma_a} + \pi^A_{b1} \geq 0 (PCb1)$$

(1.20)

Since the transaction price in the derivative market is between 0 and 1, the originator always makes non-negative profits from trading the derivative security $A$. Given $\gamma < 1/2$, (PCa1) always hold. Intuitively, the low type always benefits from pooling, which makes the buyers believe the underlying asset could be of the high type and thus increases its expected value. However, this may not be necessarily true for
(PCb1). In other words, trade of the high type suffers from adverse selection. So long as opacity is not zero, the information asymmetry between $O$ and buyers lowers the price buyers are willing to pay. As shown in (PCb1), an increase in $\sigma_a$ results in less profits for $O$ in the primary market, given by the lower value of $k - \frac{(1-2\gamma)\sigma_a}{1+\sigma_a}$, keeping $k$ constant. When $k$ is small, $O$ may make losses from trading the high type. However, at the same time, opacity increases $O$’s information advantages and he can make gains from informed trading in the derivative market.

Hence, when designing the opacity of security $A$, the originator is facing a trade-off. By choosing full transparency ($\sigma_a = 0, \sigma_b = 1$), the originator guarantees non-negative profits in the primary market but zero profits in the derivative market in all states with positive probability, i.e. $(e = a, \sigma = 0)$ and $(e = b, \sigma = 1)$. However, full opacity imposes severe adverse selection on the primary market. As a result, the optimal opacity requires the originator to trade off the benefits from reducing information asymmetry in the primary market with the benefits from informed trading in the derivative market. The next section will talk about this trade-off.

1.3.5 Optimal Opacity

As mentioned above, when designing the opacity of security $A$, the originator is facing a dilemma: before he knows the type $e$, he wants security $A$ to be opaque, which enables him to do informed trading in the derivative market. However, opacity imposes adverse selection in the primary market, increasing the costs of trading the high type. Thus, after he knows that the underlying risky asset is of the high type, he tends to quit the primary market. Optimal opacity involves a balancing act between the two effects. Let us start with two benchmarks.

1.3.5.1 Two Benchmarks

Suppose first $O$ makes zero profits in the derivative market. This could be the case where the market maker is fully informed and $O$ can only trade at the fair value of $A$, or, it could be the case where there are no noise traders, in which case the trading behaviour of the originator reveals his private information.

**Proposition 1.1.** If there are no rents from informed trading in the derivative market, then full transparency is always ex-ante optimal.

From an ex-ante point of view, opacity only affects the expected profits in the primary market when it causes the high type to quit. When this does not happen, the extra gains from trading the overpriced type (type $a$) are perfectly offset by the underpricing of the high type (type $b$), so the overall opacity does not matter. Looking at (PCb1), if $O$ earns zero from the derivative market ($\pi^A_{01} = 0$) but $k$ is large
enough to still make (PCb1) hold, then opacity is immaterial in the sense that any value of $\sigma_a$ gives the same expected profits in the derivative market. Transparency matters only when (PCb1) does not hold, in which case, the optimal strategy for $O$ is to reveal the type $e$ to eliminate adverse selection. In this case, full transparency weakly dominates.

Now, let us look at the other extreme case where there is no adverse selection in the primary market. Suppose that $O$ can eliminate the information asymmetry by telling buyers his private information and is able to contractually bind them not to reveal what they know. In this case, $O$ can avoid adverse selection in the primary market but retains his information advantage in the derivative market. Then, how much information would $O$ like to make public?

**Proposition 1.2.** In the absence of adverse selection in the primary market, full opacity is always ex-ante optimal.

Ideally, $O$ makes the maximum profits if he can systematically make the market maker believe the underlying risky asset is of the opposite type compared to the real type he knows, for instance, he makes the market maker believe $e = a$ while the truth is $e = b$ and vice versa. However, this is impossible in equilibrium. The maximum “misleading” $O$ can impose on the market maker is with full opacity. Hence, optimally, $O$ will choose full opacity when there is no adverse selection in the primary market.

### 1.3.5.2 The right amount of opacity

Now, let us move to the more interesting case where $O$ trades off the two opposite effects of opacity on the primary and derivative market. First, let us look at the two participation constraints (PCa1) and (PCb1). As mentioned before, (PCa1) always holds and we focus on (PCb1).

**Lemma 1.1.** In equilibrium, participation constraint (PCb1) holds under full opacity whenever

$$k + \lambda_O s \frac{\lambda_n}{2(\lambda_s + \lambda_O + \lambda_N)} > \frac{1}{2} - \gamma$$  \hspace{1cm} (1.21)

If the reverse of the above inequality holds, then there exists a unique value $\sigma_a^* \in [0, 1]$ that makes (PCb1) bind, where $\sigma_a^*$ gives the optimal opacity.

If condition (1.21) is satisfied, all participation constraints are satisfied under full opacity. Hence, in equilibrium, full opacity is the optimum. The left-hand-side of (1.21) is the gains the originator obtains from the primary market, measured by $k$ and the expected profits the originator can obtain from the derivative market which
increases with order size $s$ and $\lambda_N$. The right-hand-side of (1.21) measures adverse selection. Intuitively, when $\gamma$ is close to 1/2, there is no much ex-ante difference between $O$’s and $B$’s beliefs. In other words, adverse selection is less of an issue, in which case, the incentive for full opacity is strong according to Proposition 1.2. The strong incentive is represented by the fact that (1.21) is more easily satisfied as $\gamma$ is closer to 1/2. However, if $\gamma$ gets closer to zero, adverse selection becomes more severe and (1.21) is more difficult to be satisfied and when (1.21) can not hold, the originator has to introduce some transparency to the market. When condition (1.21) is violated, Lemma 1.1 actually tells that the optimal opacity is given by the binding participation constraint (PCb1). While ex-ante the originator wants $\sigma_a$ to be as close to 1 as possible, this may hurt him in the primary market when the good signal ($\sigma = 1$) is received and he is holding the high type (type $b$), in which case, the originator may want to drop out of the primary market. Hence, to balance the two effects, optimal opacity, $\sigma_a^*$, is the solution to the equation

$$k - \frac{(1 - 2\gamma)\sigma_a}{1 + \sigma_a} + \frac{\lambda_O s \lambda_N \sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N (\sigma_a + 1)} = 0$$

which is the binding (PCb1). Let $G(\sigma_a) \equiv k - \frac{(1 - 2\gamma)\sigma_a}{1 + \sigma_a} + \frac{\lambda_O s \lambda_N \sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N (\sigma_a + 1)}$. As Figure 1.1 shows, when $\sigma_a < \sigma_a^*$, $G(\sigma_a) > G(\sigma_a^*)$, so that (PCb1) is satisfied. However, by increasing $\sigma_a$, the originator could obtain higher profits and thus has an incentive to further increase opacity. When $\sigma_a > \sigma_a^*$, (PCb1) does not hold and according to Lemma 1.1, the originator has the incentive to introduce more transparency by lowering $\sigma_a$. Thus, optimal opacity $\sigma_a^*$ satisfies $G(\sigma_a^*) = 0$, that is (1.22).

---

10Figure 1.1 depicts only one possible shape of $G(\sigma_a)$. We know $G(\sigma_a = 0) > 0$ and $G(\sigma_a = 1) < 0$ with (1.21) not holding. For $\sigma_a \in (0, 1)$, $G(\sigma_a)$ could be of any form with $G(\sigma_a) = 0$ crossing the horizontal axis only once. The similar argument applies to Figure 1.2 in the next section.
The equilibrium level of opacity/transparency is characterised by the following Proposition:

**Proposition 1.3.** The equilibrium of the overall game is characterized by

1. Full opacity \( \sigma_a = \sigma_b = 1 \) whenever (1.21) holds.
2. Partial transparency with \( \sigma_a = \sigma^*_a \) and \( \sigma_b = 1 \) if (1.21) does not hold.

To conclude, we provide some thoughts on the assumption \( \sigma_b = 1 \). In our setting with \( \sigma_b = 1 \), the probability of \( (e = b, \sigma = 0) \) is zero and the originator gets zero profits from derivative trading. This is equivalent to the case where \( (e = b, \sigma = 0) \) may occur with positive probability but the loss in the primary market is larger than the gains from the derivative market and hence the originator does not trade in the primary market, which reveals his private information. In this case, \( \sigma_b = 1 \) (weakly) dominates \( \sigma_b < 1 \). Conditional on the above case, given any level of opacity, denoted by \( \sigma_a/\sigma_b = r \) where \( r \) is constant, \( \sigma_b = 1 \) gives \( O \) the maximum interim overall profits. The intuition is the following. Since \( O \)'s profits from derivative trading is decided by opacity, once the level of opacity is given, his profits from the derivative market are fixed. In other words, any pair of \( \sigma_a \) and \( \sigma_b \) satisfying \( \sigma_a/\sigma_b = r \) gives \( O \) the same profits in the derivative market. However, in the primary market, \( \sigma_b = 1 \) is the optimum because it enables \( O \) to minimise adverse selection when he is holding the high type and hence to maximise the price buyers are willing to pay. Thus, as long as trade of the high type (type \( b \)) does not occur with the bad signal (\( \sigma = 0 \)), \( \sigma_b = 1 \) is the optimum.

Recall that when designing the optimal opacity, the originator balances the two opposite effects of opacity on the primary and derivative market. This balance is
reflected in the interior solution given by $\sigma^*_a \in (0, 1)$. For opacity lower than $\sigma^*_a$, overall, increasing opacity brings more gains from the derivative market than the losses from the primary market. While for opacity higher than $\sigma^*_a$, it is the reverse. Hence, at $\sigma = \sigma^*_a$, the two effects trade off.

**Remark 1.1.** The interior solution given by $G(\sigma_a) = 0$ reflects the trade-off between the effects of opacity in the primary market and the derivative market.

### 1.3.6 Opacity and Liquidity

The main comparative statics of the model concerns the relationship between derivative market liquidity and opacity. Note that noise traders provide liquidity while speculators and the originator consume liquidity. The higher $\lambda_N$, the more liquid the market. Recall that $\lambda_O + \lambda_S + \lambda_N = 1$. Since we are interested in $O$’s participation in the derivative market, whenever we change $\lambda_N$, we keep $\lambda_O$ constant. That is to say, all changes in $\lambda_N$ are from the corresponding changes in $\lambda_S$. For instance, an increase in $\lambda_N$ is accompanied by a same amount of decrease in $\lambda_S$ such that $\lambda_O + \lambda_S + \lambda_N = 1$. We now show that in the partial transparency regime, high liquidity induces higher opacity of security $A$. First of all, we show that when opacity is fixed, the originator’s expected profits in the derivative market increase with the order size $s$ and probability $\lambda_N$.

**Lemma 1.2.** For $\sigma_a \in [0, 1]$,

$$\pi_{b1}^A(\sigma_a) = \lambda_O s \frac{\lambda_N \sigma_a}{2(\lambda_O + \lambda_S) + \lambda_N(1 + \sigma_a)}$$

increases with liquidity measured by the order size $s$ and probability $\lambda_N$.

The reason for this result is easy to understand. When the originator can trade more in the derivative market with a larger order, he definitely makes more profits. In addition, an increase in $\lambda_N$ enables the originator to better hide his order in the order of the uninformed noise traders and thus he can better remain his information advantage and make more profits in the derivative market.

Note also that liquidity of the derivative market does not affect the primary market. Now suppose there is an increase in $\lambda_N$ (or $s$). Then, according to Lemma 1.2, the originator can make more profits in the derivative market, which implies that at the initial optimal opacity level, (PCb1) must be slack. As a result, optimal opacity must increase to a new level at which (PCb1) binds. Intuitively, the rise of liquidity allows $O$ to make greater profits in the derivative market. This makes $O$’s costs of adverse selection in the primary market more easily be covered by his gains in the derivative market. Thus, as the derivative market becomes more liquid,
O has the incentive to make the underlying asset more opaque. However, if the equilibrium is already full opacity, a change in liquidity does not affect the optimal opacity.

**Proposition 1.4.** Unless the equilibrium already involves full opacity, optimal opacity strictly increases as the derivative market becomes more liquid.

Another interesting question is how the optimal opacity changes with adverse selection measured by $\frac{1}{2} - \gamma$ where the smaller $\gamma$, the larger adverse selection. To answer this, consider the left-hand-side of (PCb1). As $\gamma$ decreases, at the initial optimal opacity, (PCb1) does not hold any more. This implies that the initial optimal opacity is too high with the now smaller $\gamma$, which will induce $O$ not to trade in the case ($e = b, \sigma = 1$). Thus, the originator will reduce opacity to ensure (PCb1) is binding. Therefore, as there is more adverse selection, $O$’s incentive for more transparency/less opacity becomes stronger.

**Proposition 1.5.** Unless the initial optimum is full opacity, optimal opacity strictly decreases with the adverse selection in the primary market, measured by $\frac{1}{2} - \gamma$.

Finally, let us look at how the optimal opacity changes with $k$, the gain from trade in the primary market. A change in $k$ does not affect the ex-ante profits in the derivative market. On the left-hand-side of (PCb1), the expression $k - \frac{(1-2\gamma)\sigma_{a}}{1+\sigma_{a}}$ measures the ex-post (after observing $e = b, \sigma = 1$) net profits the originator can obtain from the primary market. Keeping $\sigma_{a}$ constant, that is, keeping opacity constant, a larger $k$ makes adverse selection less important to $O$. In other words, a larger $k$ makes (PCb1) more easy to be satisfied. Thus, a larger $k$ reduces the negative effects of adverse selection on the originator and hence encourages more opacity.

**Proposition 1.6.** Optimal opacity increases with the gains from trading in the primary market measured by $k$.

### 1.4 Extension: Endogenous Liquidity

Opacity and liquidity are always at the center of financial studies. So far, we have shown that liquidity in the derivative market can induce more opacity of the underlying security. At the same time, does opacity also affect the liquidity of the derivative market? If yes, what is the mechanism for these effects to happen? In this section, we answer these questions.
1.4.1 Rationale

In the current literature, opacity always relates to the scope of private information acquisition. A traditional consensus in this area is that opacity encourages private information acquisition, which increases information asymmetries and hinders trade. However, the recent development in this field argues that high opacity may discourage private information acquisition, which makes traders equally uninformed and thus facilitates trade. One reason for this finding is the high costs that may be incurred in information acquisition: traders may stop collecting private information if the costs of doing so can not be compensated by the gains that they can obtain with the private information. Based on this idea, in this section, we assume speculators’ costs of becoming informed rise with opacity and they have heterogeneous cost functions. This is plausible in the sense that as opacity goes up, more time, skills and resources have to be provided to collect and process information. Consequently, as opacity increases, it will be less profitable to acquire private information. Once speculators cannot make positive profits from information acquisition, they quit the derivative market. This is equivalent to a reduction in $\lambda_S$. Hence, we assume $\lambda_S$ is a decreasing function of opacity.

**Assumption 1.1.** $\lambda'_S(\sigma_a) < 0$

Still, as in the exogenous case, we assume $\lambda_O + \lambda_S + \lambda_N = 1$ and keep $\lambda_O$ constant. This enables us to check how $O$’s profits in the derivative market change when the liquidity in that market changes, without affecting $O$’s trade opportunity in the derivative market. Hence, Assumption 1.1 implies that keeping $\lambda_O$ constant, an increase in opacity will result in a decrease in $\lambda_S$ and an equal immediate increase in $\lambda_N$. In other words, an increase in opacity will reduce informed trading and increase liquidity in the derivative market. This is consistent with the anecdotal evidence in the real world, especially some facts that have occurred during the 2007-2008 financial crisis. For instance, facing the new and complex structured financial products such as CDOs, even the world’s top financial institutions could not fully understand these securities. Similarly, credit rating agencies gave inappropriate ratings to these products and financial companies such as the American International Group (AIG) aggressively bet against the default of these products, which however turned out to be wrong. From these facts, we see that high opacity does affect traders’ abilities of getting informed. Traders such as AIG actually traded as uninformed traders providing liquidity to the CDS market before the crisis. This is similar to our setting that speculators, when stay uninformed due to high opacity, reduce informed trading and increase the liquidity in the derivative market. A microfoundation of Assumption 1.1 is provided in Appendix A.9.
From the perspective of the market maker, overall liquidity in the derivative market is provided by noise traders while consumed by informed speculators and the originator. As shown above, when \( \lambda_S \) is exogenous, an increase in opacity enlarges the bid-ask spread. In contrast, with endogenous \( \lambda_S \), we see mixed effects. Suppose first that \( \lambda_S, \lambda_O \) and \( \lambda_N \) are kept constant. An increase in opacity will increase the bid-ask spread. However, if we allow \( \lambda_S \) to correspondingly decrease with opacity, this reduces the probability of informed trading and narrows the bid-ask spread. We call the former the direct effect and the latter the indirect effect of opacity on bid-ask spread. Overall, the bid-ask spread increases when the direct effect outweighs the indirect effect, it shrinks when the indirect effect outweighs the direct effect and remains constant if the two effects cancel out. Hence, Assumption 1.1 allows us to endogenize liquidity as a function of opacity.

1.4.2 The right amount of opacity

The two benchmarks we have described in Propositions 1.1 and 1.2 also apply here. In this section, we start by looking at the interior optimal amount of opacity. Replace \( \lambda_S \) in Lemma 1.1 with \( \lambda_S(\sigma_a) \). The optimal opacity, denoted by \( \sigma_a^{**} \), is the solution to the equation

\[
 k - \frac{(1 - 2\gamma)\sigma_a}{1 + \sigma_a} + \frac{\lambda_O s(\sigma_a)}{2(\lambda_S(\sigma_a) + \lambda_O) + \lambda_N(\sigma_a + 1)} = 0 \tag{1.24}
\]

which is the binding participation constraint (PCb1) with endogenous \( \lambda_S(\sigma_a) \). Let \( G(\sigma_a, \lambda_S(\sigma_a)) \) denote the left-hand-side of (1.24). As the graph below shows, optimal opacity is given by \( \sigma_a = \sigma_a^{**}, \sigma_b = 1 \).

**Remark 1.2.** From Lemma 1.1, optimal opacity \( \sigma_a^{**} \) is given by the largest root of (1.24).

![Figure 1.2: Optimal opacity with endogenous \( \lambda_S(\sigma_a) \).](image-url)
In the endogenous case, expected profits of \( O \) in the state \((e = b, \sigma = 1)\) are

\[
\pi^A_{b1}(\sigma_a) = \lambda_O s \frac{\lambda_N \sigma_a}{2(\lambda_S(\sigma_a) + \lambda_O) + \lambda_N(\sigma_a + 1)} \tag{1.25}
\]

All the relevant results we have obtained in section 1.3.6 also apply here. In particular, in the absence of full opacity, a more liquid market induces more opacity. In this case, the liquid market not only comes from the increase in noise traders’ trading, but from the decrease in the trading of the informed speculators, which can be seen more clearly in the next section.

1.4.3 Comparison between \( \sigma^*_a \) and \( \sigma^{**}_a \)

While we have shown that qualitatively, with both the exogenous and endogenous \( \lambda_N \), optimal opacity increases with liquidity, quantitatively, the magnitude of these effects are different. In particular, optimal opacity in the endogenous case is more responsive to liquidity compared to the optimal opacity in the exogenous case. In this section, we investigate this difference.

We want to analyse how the comparative statics of a change in \( \lambda_N \) differ in the two cases. Let \( \sigma_a \) and \( \hat{\sigma}_a \) denote the optimal opacity in the exogenous case and endogenous case, respectively. Assume that, initially, \( \sigma_a = \sigma^0_a \) and \( \hat{\sigma}_a = \hat{\sigma}^0_a \). Recall that optimal opacity in the exogenous case and endogenous case is the solution to (1.22) and (1.24), respectively. Hence, \( \sigma^0_a \) and \( \hat{\sigma}^0_a \) must satisfy

\[
k - \frac{(1 - 2\gamma)\sigma^0_a}{1 + \sigma^0_a} + \frac{\lambda_O s \lambda_N \sigma^0_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma^0_a + 1)} = 0 \tag{1.26}
\]

and

\[
k - \frac{(1 - 2\gamma)\hat{\sigma}^0_a}{1 + \hat{\sigma}^0_a} + \frac{\lambda_O s \lambda_N \hat{\sigma}^0_a}{2(\lambda_S\hat{\sigma}^0_a + \lambda_O) + \lambda_N(\hat{\sigma}^0_a + 1)} = 0 \tag{1.27}
\]

Suppose in (1.26) and (1.27), \( \lambda_N = \lambda^0_N \) and \( \lambda_S = \lambda_S(\hat{\sigma}^0_a) = \lambda^0_S \) where \( \lambda^0_N + \lambda^0_S + \lambda_O = 1 \). Then the left-hand-side of (1.26) and (1.27) are the same and \( \sigma^0_a = \hat{\sigma}^0_a \). Suppose now \( \lambda_N \) increases to \( \lambda^1_N \), keeping \( \lambda_O \) constant. Since \( \lambda_N + \lambda_S + \lambda_O = 1 \), in the exogenous case, the increase in \( \lambda_N \) results in an equal decrease in \( \lambda_S \), in which case, the market becomes more liquid and the optimal opacity adjusts to a higher level (Proposition 1.4). Let \( \sigma^1_a \) denote the new level of opacity in the exogenous case, so that \( \sigma^1_a > \sigma^0_a \). Changes in liquidity and opacity in the exogenous case are one-shot. However, the same change in \( \lambda_N \) in the endogenous case generates more complicated effects. Suppose first that \( \lambda_S(\sigma_a) \) does not change with opacity. Then, as in the exogenous case, the increase in \( \lambda_N \) leads \( \lambda_S(\sigma_a) \) to decrease by the same amount, which generates an equal increase in opacity as in the exogenous case and
the new level of opacity is also $\sigma_{1a} > \sigma_{0}$. Now, we allow $\lambda_S(\sigma_a)$ to decrease with opacity (Assumption 1.1). Then, at the new and higher level of opacity $\sigma_{1a}$, $\lambda_S(\sigma_a)$ must shrink further, which increases $\lambda_N$ more and the market becomes more liquid. This pushes opacity to increase again and results in another decrease in $\lambda_S$ and increase in $\lambda_N$. This self-reinforcing process between liquidity and opacity continues until new opacity satisfies (1.24). Let $\tilde{\sigma}_a^1$ denote the new optimal opacity in the endogenous case and $\tilde{\sigma}_a^1 > \sigma_{1a}$.

From the above discussion, we observe two effects of liquidity on opacity. One is the one-shot direct effect. The other is a self-reinforcing indirect effect brought caused by the interaction between opacity and information acquisition. As mentioned above, an increase in liquidity first induces more opacity (direct effect). The higher opacity reduces the trading of informed speculators by decreasing $\lambda_S(\sigma_a)$ ($\lambda_S'(\sigma_a) < 0$) and hence increases market liquidity further. This generates a new round of increase in opacity, decrease in $\lambda_S(\sigma_a)$ and increase in market liquidity. This dynamic process ends when new opacity satisfies (1.24). Through the feedback loop between opacity and liquidity, any change in liquidity or opacity will be amplified. In the exogenous case, only the direct effect appears while in the endogenous case, both the direct and indirect effects appear and the overall effect is greater than the direct effect. Therefore, opacity in the endogenous case is more responsive than that in the exogenous case, that is, for a same change in liquidity, opacity in the endogenous case changes more than that in the exogenous case.

**Proposition 1.7.** Opacity in the endogenous case is more responsive to changes in liquidity than opacity in the exogenous case.

### 1.5 Implications and Conclusion

Our paper first suggests that derivative markets do play a role in securities design. More specifically, they affect the originator’s incentive to design opaque securities. Our results also imply that the observed simultaneous success of the markets for opaque structured securities and credit derivatives preceding to the recent crisis was not an coincidence. We explain it as the outcome of the interaction between opacity and liquidity via a feedback loop. With this feedback loop, any change in either opacity or liquidity will lead the two to reinforce each other. The effects of the initial change will be accordingly amplified.

Many commentators point out that high opacity hindered the discovery of risks in financial markets and hence contributed to the 2007-2008 financial crisis. Our paper shows that originators’ ability to speculate in the derivative markets could fuel opacity. Thus, policies restricting the ability of investment banks to take large positions on the derivative markets are expected to help reduce opacity in financial
markets. For instance, the Dodd-Frank Act issued after the crisis restricts banks’ proprietary trading. This, based on the results of our paper, would reduce originators’ incentive to design opaque securities and hence help increase transparency in financial markets.

Credit derivatives are altering the financial landscape, especially by influencing the incentives of economic agents (Augustin et al., 2016; Hu and Black, 2008; Campello and Matta, 2013; Danis and Gamba, 2018; Kim, 2013). For instance, Bolton and Helmkic (2010) and Kim (2013) present the incentive effects of CDS on lender-borrower relationship. Our paper enriches this incentive literature in the sense that we show how credit derivative markets could change originators’ incentive in security design.

Future work could address welfare and regulatory aspects of credit derivatives. In particular, the costs of opacity in the current version of the model are ultimately borne by noise traders. In this respect, it may be important to understand whether uninformed trading is motivated by genuine liquidity needs or by underlying cognitive biases. This would inform a welfare analysis and policy prescriptions. It would also be interesting to study the effects of credit derivatives on securitisation in general. For instance, how the presence of the markets for credit derivatives affect originators’ decisions of putting correlated or uncorrelated assets into the asset pool in the process of securitisation.
Chapter 2

Statistical Arbitrage and Risk Contagion

Abstract

Contagions among financial intermediaries are believed to play a significant role in market stability while contagions among asset markets receive less attention. This paper fills this gap by investigating the role of statistical arbitrage, a widely used trading strategy among financial institutions, in building networks among stock markets and in stock market stability. We find that statistical arbitrage can stabilise markets in normal periods, however, may also act as a mechanism for risk contagion when shocks occur. More interestingly, we find a double-edged role of statistical arbitrageurs in risk contagion: while they provide the routes for risk contagion, the effects are limited and disappear exponentially. This is because statistical arbitrageurs in markets unshocked trade against the shock by arbitraging mispricings.
2.1 Introduction

The 6 May 2010 witnessed a large scale collapse in the US stock market, which is known as the Flash Crash. The crash first occurred in the market for futures on the S&P 500 index. It soon spread to other index products and individual stocks (Menkveld and Yueshen, 2017). This event triggered a new round of extensive discussions regarding liquidity fluctuations and comovements, especially the transmission of liquidity dry-ups among asset markets. In recent decades, liquidity comovements happen more frequently. According to Kamara et al.(2008), liquidity comovements among large-cap stocks have increased greatly since 1960s. In terms of the reasons for this phenomenon, one stream of this literature attributes to common components in the liquidity across assets. For instance, changes in macroeconomic conditions can affect liquidity in multiple markets simultaneously and hence result in liquidity comovements (Anshuman and Viswanathan, 2005; Coughenour and Saad, 2004; Kyle and Xiong, 2001). However, more and more evidence is now pointing to another possible explanation, which is the propagation of liquidity dry-ups in one market to other markets (Cespa and Foucault, 2014; Goldstein and Yang, 2014; Menkveld and Yueshen, 2017), as what has happened in the Flash Crash. This possibility comes from the phenomenon that during the last two decades asset markets have become more connected than ever. The rapid development in information technology and computation capacity have greatly enlarged the scope of trading. Hence, we observe the generation of new trading techniques and strategies such as index trading and algorithmic trading. These techniques and strategies involve wide range of financial assets, which to some extent contributes to the close connections among asset markets. Based on this, our paper investigates the role of statistical arbitrage, a widely used algorithmic trading strategy by investment banks and hedge funds, in establishing networks among asset markets and in the contagion of liquidity crashes.

Statistical arbitrage is a long/short market neutral strategy. It involves the exploitation of short-term deviations from a long-run equilibrium between assets. The simplest form of statistical arbitrage involves only two assets and is known as “pairs trading”. In the case of pairs trading, when a deviation occurs, statistical arbitrageurs take long position in the asset underperformed and short position in the asset overperformed, in a way such that the resulting portfolio is market neutral and with the expectation that prices will revert back to the long-run equilibrium (Avellaneda and Lee, 2010; Caldeira and Moura, 2013). For a liquidity shock occurring in one market, statistical arbitrageurs adjust their demand in the shocked market and also in the other market to restore market neutrality, through which the shock spreads to the other market. Statistical arbitrage usually involves broadly diversified portfolios of securities, among which networks can be established. As
the shock spread to the other market in pairs trading, a shock in the network can transmit to the whole system.

Generally, our paper lies in the literature on financial networks. However, a large majority in this literature study the networks among banks (Georg, 2013; Acemoglu, Ozdaglar and Tahbaz-Salehi, 2015; Ladley, 2013). Our paper instead investigates the networks among stock markets. Especially, we investigate the role of a modern trading strategy – statistical arbitrage – in networks. Because of its high returns and low risks,\(^1\) statistical arbitrage has become one of the strategies financial institutions most heavily rely on. A mainstream of the literature on statistical arbitrage focuses on the exploitation of arbitrage opportunities with econometric models (Bondarenko, 2015; Caldeira and Moura, 2013; Alexander and Dimitriu, 2005). Another stream is concerned about the test of market efficiency with the strategy (Hogan, 2004). However, its market-linking effects and implications on the market stability have received less attention. Our paper fills this gap.

We consider a numerical model where stock markets are populated by fundamentalists, chartists and statistical arbitrageurs. Fundamentalists and chartists trade in one market while statistical arbitrageurs are allowed to cross trade, which creates a network: two markets are linked if statistical arbitrageurs trade the two assets. All three groups of traders simultaneously submit their orders and equilibrium price is achieved when demand equals supply. For statistical arbitrageurs, if they adjust their demand in one market, to keep their portfolio market neutral, they have to adjust their demand for other stocks in the portfolio, which affects the equilibrium price in each market involved. This provides the routes for the contagion of shocks.

The results of our paper show that in normal times, statistical arbitrageurs help stabilise markets. However, when shocks occur, the network built by them can act as a mechanism for the transmission of shocks, resulting in systematic decrease in market prices. This result is consistent with the current literature in that financial networks play a double-edged role in market stability (Ladley, 2013; Georg, 2013). In addition, we find statistical arbitrageurs persistently make positive trading profits, which echoes with the practice that statistical arbitrage generates huge profits for financial institutions in their daily business.\(^2\) Intuitively, the nature of the strategy is mispricing arbitrage. Hence, it helps drive prices back to the long-term fundamental levels, which makes prices less volatile. However, the existence of the network exposes the whole system to unexpected shocks, making the system fragile.

The rest of this paper is organised as follows. Sections 2.2 is the literature our paper most relates to. Section 2.3 describes the model of this paper. Section 2.4

\(^{1}\)In a study of Caldeira and Moura (2013), statistical arbitrage exhibits excess returns of 16.38% per year and Sharpe Ratio of 1.38.

\(^{2}\)For more details, please see https://www.quantinsti.com/blog/statistical-arbitrage/ for example.
summarises our results. Section 2.5 concludes this paper.

2.2 Literature Review

Our paper relates to the long existing literature on liquidity comovements and the rapidly growing literature on the Flash Crash. The Flash Crash brought people’s attention back to liquidity comovements. In terms of the reasons for liquidity comovements, a stream in the literature focuses on common components among the liquidity of different assets (Chordia and Subrahmanyam, 2000; Hasbrouck and Seppi, 2001; Huberman and Halka, 2001; Coughenour and Saad, 2004). For instance, Brunnermeier and Pedersen (2009) argue that tight funding constraints may make financial intermediaries reluctant to provide liquidity especially when big shocks occur, which can lead to systematic illiquidity.

However, the observed spread of liquidity dry-ups during the Flash Crash\(^3\) has triggered hot discussions about another possible explanation for liquidity comovements, that is, the contagion of illiquidity among markets. As for the channel for the propagation, Cespa and Foucault (2014) and Goldstein and Yang (2014) give theoretical possibilities. Cespa and Foucault (2014) find a feedback loop from liquidity shocks. When dealers abstract information from the price of other markets, a liquidity shock in one market may make that price less informative. This increases the costs of providing liquidity and hence less liquidity will be provided, which lowers the informativeness of more prices and further reduces the provision of liquidity. This argument is confirmed by Borkovec et al. (2010) who find that before the Flash Crash the price discovery in the market for Exchange Traded Funds (ETF) failed dramatically and the cause was an extreme deterioration in liquidity.

Arbitrage is talked widely for its role in the Flash Crash (Menkveld and Yueshen, 2017; Ben-David et al., 2012; Goldstein and Yang, 2014). In Menkveld and Yueshen (2017), cross-market arbitrage can transfer trading pressures among markets. However, before the Flash Crash, cross arbitrageurs failed to do so in the futures market due to the shortfalls in liquidity provision, which contributes to the big selling pressure and large price drop in the futures market. In Ben-David et al. (2012), arbitrage of ETF is believed to facilitate the transmission of liquidity shocks from the futures market to the stock market during the Flash Crash. Arbitrage of ETF is against both the futures and the underlying portfolio. A liquidity shock in the futures market first lead arbitrageurs to buy the futures and sell the ETF, resulting in a decrease in the ETF price, which then propagates to the stock market through

\(^3\)As mentioned in the previous section, the Flash Crash is the result of a crash spreading from the market for futures on the S&P 500 index to the markets for other index products and individual stocks. For more details, please see Menkveld and Yueshen (2017) and Ben-David et al. (2012) for example.
the hedging of arbitrageurs in the stock market.

Our paper is in the spirit of Ben-David et al. (2012) but differs from theirs in the following points. Arbitrageurs in their paper follow the law of one price. Ideally, the fair price of ETF is the same with the price of the underlying index and they arbitrage the price difference between ETF and the underlying index. Arbitrageurs in our paper exploit the deviations from a long-term equilibrium between assets and importantly, they follow a market neutral strategy. In addition, arbitrageurs in their paper trade assets of different classes – futures, ETF and the underlying index. Arbitrageurs in our paper trade only stocks and we consider networks among stocks and show a clearer idea about the propagation of shocks. In terms of methodology, they collect data from the Flash Crash to empirically test their hypotheses about the role of ETF arbitrage in the Flash Crash. With a numerical model, we investigate the role of statistical arbitrage in risk contagion in a more general setting.

Our paper also relates to the literature on statistical arbitrage, which currently focuses on the profit exploitation with econometric models (Bondarenko, 2015; Caldeira and Moura, 2013; Alexander and Dimitriu, 2005) and the test of market efficiency (Hogan, 2004). For example, Alexander and Dimitriu (2005) find that cointegration-optimal strategy performs better than the tracking error variance minimisation strategy for all the statistical arbitrage strategies in portfolio optimisation. Hogan et al. (2004) test the existence of statistical arbitrage, which provides evidence against the hypothesis of market efficiency. Our paper, however, looks at another aspect of this strategy by looking into its role in linking asset markets and in risk contagion. This role of statistical arbitrage widely exists but is often ignored.

Finally, our paper contributes to the literature on financial networks. Financial networks attract much more attention after the 2007-2008 financial crisis because the liquidity dry-ups in the interbank market before the crisis is believed to be one of the reasons for the crisis. The mainstream in this literature studies how networks affect financial stability (Ladley, 2013; Georg, 2013; Allen and Gale, 2000). For example, Allend and Gale (2000) find that relative to incomplete networks, complete networks make financial system more stable. Ladley (2013) and Georg (2013) discuss the double-edged role of networks in financial stability. However, most of the study in this area is based on the networks among banks. Our paper, instead, focuses on the networks among asset markets, especially among stock markets where the network is established by the cross trading of statistical arbitrageurs. So our paper in particular contributes to the literature on the networks among asset markets.
2.3 Model

2.3.1 Assets

We assume there are $N$ risky assets and a risk-free asset. Risk-free asset pays a constant net rate of return $r \in (0, 1)$ (hence gross rate of return is $R = 1 + r$). Asset $i \in \{1, 2, ..., N\}$ pays regular dividends $y_{i,t}$. In each market for the $N$ risky assets, three types of traders – fundamentalists, chartists and statistical arbitrageurs – trade the risk-free asset and the risky asset. In addition, we have a fairly priced market index $M$ following a geometric random walk process.

$$\frac{dM_t}{M_t} = (r + \mu_m)dt + \sigma_m dB_t$$

(2.1)

where $\mu_m$ is the market premium; $\sigma_m$ is the market volatility; $B_t$ is a standard Brownian motion. The price of the market index $M$ is public information. Let $F_{i,t}$ denote the fundamental value of asset $i$ after dividends at time $t$. According to the Capital Asset Pricing Model (CAPM), we assume $F_{i,t}$ satisfies the following geometric Brownian motion

$$\frac{dF_{i,t}}{F_{i,t}} = \beta_i[(r + \mu_m)dt + \sigma_m dB_t] + \sigma_i dV_{i,t}$$

(2.2)

The dividend process of asset $i$ follows IID.

$$y_{i,t} = \bar{y}_i + \epsilon_{i,t}$$

(2.3)

The noise term $\{\epsilon_{i,t}\}$ is IID stochastic process with zero mean.

2.3.2 Traders

As mentioned above, in each risky asset market, there are three types of traders: fundamentalists (F), chartists (C) and statistical arbitrageurs (A). Chartists ignore the public information about the market index and hence are uninformed of the fundamental value of the asset. Chartists are also uninformed of the dividend process. They trade based on past prices and dividends. Fundamentalists observe the market index and are able to infer the fundamental value. They are also informed of the dividend process. They trade based on the fundamental value of each asset. Both fundamentalists and chartists trade in one market. Statistical arbitrageurs share the same information and beliefs with fundamentalists but they cross trade among markets with market neutral strategy. In each market, the three types of traders trade the risk-free asset and the risky asset. All trader are myopic mean variance
maximisers and have identical risk aversion measured by $a$. We start from analysing the demand of fundamentalists and chartists who trade only one asset and then the demand of statistical arbitrageurs.

### 2.3.2.1 Fundamentalists and Chartists

Suppose for a fundamentalist or a chartist, his demand for a risky asset at time $t$ is $Q_t$ and his wealth, denoted by $W_t$, evolves as

$$W_{t+1} = RW_t + (S_{t+1} + y_{t+1} - RS_t)Q_t$$

where $S_t$ is the price of the risky asset traded at time $t$; $y_{t+1}$ is the dividend at time $t + 1$ of the risky asset traded at time $t$; $R = 1 + r$ is the gross risk-free rate of return; $(S_{t+1} + y_{t+1} - RS_t)$ is the excess return relative to the risk-free return at time $t + 1$ of the risky asset traded at time $t$. At time $t$, the trader has his expectation about the excess return of the risky asset in period $t + 1$, denoted by $E_t(S_{t+1} + y_{t+1} - RS_t)$ and his expectation about the variance of the excess return, denoted by $V_t(S_{t+1} + y_{t+1} - RS_t)$. We assume $V_t(S_{t+1} + y_{t+1} - RS_t) = \theta^2$ for all types of traders. Since all traders are myopic mean variance maximisers who share the same risk aversion $a$, this trader will choose $Q_t$ that solves

$$\max_{Q_t} E_t W_{t+1} - \frac{a}{2} V_t(W_{t+1})$$

The optimal demand of this trader at time $t$ is

$$Q_t = \frac{E_t(S_{t+1} + y_{t+1} - RS_t)}{a\theta^2}$$

### 2.3.2.2 Statistical Arbitrageurs

Statistical arbitrage is a long/short market neutral trading strategy. We assume statistical arbitrageurs (SAs) do pairs trading among different markets. That is, statistical arbitrageurs taking positions in one market must take opposite positions in another market such that his portfolio is market neutral. Through the hedging of SAs, any two markets involved in a pairs trading are linked together. Hence, if a SA trades assets $i$ and $j$, the two markets are in the link $ij$. Let $Q_{i,t}^{ij}$ and $Q_{j,t}^{ij}$ respectively denote this trader’s demand for asset $i$ and asset $j$ in this link. Then his wealth evolves as

$$W_{t+1} = W_t R + Q_{i,t}^{ij}[S_{i,t+1} + y_{i,t+1} - S_{i,t} R] + Q_{j,t}^{ij}[S_{j,t+1} + y_{j,t+1} - S_{j,t} R]$$
Since statistical arbitrage is a market-neutral strategy, the portfolio beta is zero, that is

\[ Q_{i,t}^j S_{i,t} \beta_i + Q_{j,t}^j S_{j,t} \beta_j = 0 \]  \hspace{1cm} (2.8)

For simplicity, let \( \hat{R}_{i,t} \) denote the excess return of asset \( i \).

\[ \hat{R}_{i,t} = S_{i,t+1} + y_{i,t+1} - S_{i,t} R \]  \hspace{1cm} (2.9)

For a SA taking positions in markets \( i \) and \( j \), his demand \( Q_{i,t}^j \) and \( Q_{j,t}^j \) solves

\[
\max_{Q_{i,t}^j,Q_{j,t}^j} \{ E_t W_{t+1} - \frac{a}{2} V_t(W_{t+1}) \}
\]  \hspace{1cm} (2.10)

where

\[
E_t W_{t+1} = W_t R + Q_{i,t}^j E_t \hat{R}_{i,t} + Q_{i,t}^j E_t \hat{R}_{j,t}
\]  \hspace{1cm} (2.11)

\[
V_t(W_{t+1}) = (Q_{i,t}^j)^2 \theta_i^2 + 2Q_{i,t}^j Q_{j,t}^j \text{cov}(\hat{R}_i, \hat{R}_j) + (Q_{j,t}^j)^2 \theta_j^2
\]  \hspace{1cm} (2.12)

subject to

\[ Q_{i,t}^j S_{i,t} \beta_i + Q_{j,t}^j S_{j,t} \beta_j = 0 \]  \hspace{1cm} (2.13)

In expression (2.12), \( \text{cov}(\hat{R}_i, \hat{R}_j) \) is the covariance between excess returns of asset \( i \) and asset \( j \) and is assumed to be constant over time. Solving the problem given by (2.10)-(2.13), we have

\[
Q_{i,t}^j = \frac{(S_{i,t} \beta_i)^2 E_t \hat{R}_{i,t} - S_{i,t} \beta_i E_t \hat{R}_{j,t}}{a(S_{i,t} \beta_i)^2 \theta_i^2 + a \theta_i^2 - 2a S_{i,t} \beta_i \text{cov}(\hat{R}_i, \hat{R}_j)}
\]  \hspace{1cm} (2.14)

\[
Q_{j,t}^j = \frac{-S_{j,t} \beta_j E_t \hat{R}_{i,t} + E_t \hat{R}_{j,t}}{a(S_{i,t} \beta_i)^2 \theta_i^2 + a \theta_j^2 - 2a S_{i,t} \beta_i \text{cov}(\hat{R}_i, \hat{R}_j)}
\]  \hspace{1cm} (2.15)

2.3.2.3 Beliefs

From the above analysis, we know the demand of fundamentalists and chartists and that of SAs. Each type’s expectation about the excess return, \( E_{ht}(S_{t+1} + y_{t+1} - S_t R) \), decides their demand for the asset. In terms of the beliefs of traders, we consider the model by Jackson and Ladley (2016). Fundamentalists trade based on
the deviation of the price from the fundamental value. They purchase the asset when it is undervalued and sell the asset when it is overvalued compared to the fundamental value. The further the price deviates from the fundamental value, the larger fundamentalists’ positions in that asset. Their one-period-ahead expectation about the price follows

$$E_{Ft}[S_{i,t+1}] = S_{i,t} + \gamma_i(F_{i,t} - S_{i,t})$$

(2.16)

where $\gamma_i \in (0, 1)$ measures the speed at which price converts back to the fundamental value in market $i$. From (2.3), we know

$$E_{Ft}[y_{i,t+1}] = \bar{y}_i$$

(2.17)

SAs have the same information and beliefs with fundamentalists, that is

$$E_{At}[S_{i,t+1}] = S_{i,t} + \gamma_i(F_{i,t} - S_{i,t}); \quad E_{At}[y_{i,t+1}] = \bar{y}_i$$

(2.18)

For chartists, their one-period-ahead expectation follows

$$E_{Ct}[S_{i,t+1}] = (1 + r + \frac{I_t \cdot SR_{annual} \cdot \sigma_{daily}}{\sqrt{252}})S_{i,t}$$

(2.19)

where $I_t$ is the value of the trading signal $\in \{-1, 0, 1\}$; $SR_{annual}$ is an annualized Sharpe ratio from daily returns; $\sigma_{daily}$ is the volatility of daily returns. One difficulty in the literature on technical trading is to map the trading signals ‘buy’, ‘sell’ and ‘hold’ to the quantity to be traded. According to Jackson and Ladley (2016), with an acceptable risk-adjusted profitability measure, technique trading rules can be mapped into price expectations. Based on this, the authors provide the expression in (2.19) as a possible solution to the difficulty. Sharpe ratio, as a widely used measure of profitability in the finance industry, enters (2.19) as the risk-adjusted profitability measure, which combines with the trading signal $I_t$ to decide chartists’ expectation about the future price. As for the trading signal, we assume

$$I_t = \begin{cases} +1 & \text{if } S_{t-1} > \max(S_{t-2}, S_{t-3}, \ldots, S_{t-n}); \\ 0 & \text{if } S_{t-1} = \max(S_{t-2}, S_{t-3}, \ldots, S_{t-n}); \\ -1 & \text{if } S_{t-1} < \max(S_{t-2}, S_{t-3}, \ldots, S_{t-n}) \end{cases}$$

(2.20)

That is to say, we assume the trading signal at time $t$ is given by a comparison between the most recently realised price ($S_{t-1}$) and the maximum price during the last $n$ days. This is reasonable in the sense that chartists heavily rely on past price trends to make the current decision. In addition, we assume chartists’ expectation
about dividends follows

\[ E_{Ct}[\frac{y_{i,t+1}}{S_{i,t}}] = g \frac{y_t}{S_{i,t-1}} + (1 - g) E_{Ct-1}[\frac{y_{i,t}}{S_{i,t-1}}] \]  

(2.21)

where \( g \) is the weight chartists put on the most recent dividend yield.

### 2.3.2.4 Equilibrium Price

In each asset market, equilibrium price clears the market by equating demand and supply. Let \( n_h \) denote the proportion of type \( h \in \{C, F, A\} \) trader in a risky asset market and \( \sum n_h = 1 \). We assume outside supply is 0. Given SAs trading in markets \( i \) and \( j \), in equilibrium, price \( S^*_{i,t} \) solves

\[ n^*_i F Q_{i,t} + n^*_i C Q_{i,t} + n^*_i A Q_{ij} = 0; \]  

(2.22)

and price \( S^*_{j,t} \) solves

\[ n^*_j F Q_{j,t} + n^*_j C Q_{j,t} + n^*_j A Q_{ij} = 0; \]  

(2.23)

From (2.14) and (2.15), we see that the cross-trading of SAs in markets \( i \) and \( j \) links the two markets and the price in each market depends on the price of the other market. In equilibrium, prices in the two markets are determined simultaneously by solving the system of equations (2.22) and (2.23).

### 2.3.3 Network

Without SAs, the \( N \) asset markets are independent of each other. However, with SAs, links between markets are established by the cross trading of SAs. Hence, in our model, a network consists of \( N \) nodes (\( N \) markets) and edges representing the cross trading of SAs, through which a shock in one market can spread to other markets.

In each market, we assume the population of fundamentalists and chartists is fixed at \( P = P_F + P_C \). Also, in each possible link between markets, the amount of SAs is given by \( P_A \). Each node links to others randomly. More formally, we assume a uniform Poisson random graph in which each possible link in the graph is present with probability \( p \). Let \( ij \) denote a link between markets \( i \) and \( j \).

\[ ij = \begin{cases} 
1, & \text{if } i \text{ and } j \text{ are connected.} \\
0, & \text{if } i \text{ and } j \text{ are disconnected.}
\end{cases} \]  

(2.24)

For \( i = j \), we assume \( ij = 0 \). Since in our network, each link is undirected, \( ij = 1 \)
and \( ji = 1 \) implies the same link between nodes \( i \) and \( j \). Once nodes \( i \) and \( j \) are connected, SAs take long position in one market and short position in the other market. We see from (2.14) and (2.15) that if \( Q_{ij,t} > 0 \) \(( Q_{ij,t} < 0 \)), then \( Q_{ji,t} < 0 \) \(( Q_{ji,t} > 0 \)), implying that SAs long (short) asset \( i \) and short (long) asset \( j \).

The amount of traders in each market depends on the markets it links to. Let \( T_i \) denote the total population in market \( i \) and let \( \kappa_i \) denote the index of markets linked to market \( i \). Then,

\[
T_i = P + \sum_{\kappa_i} P_{A,\kappa_i}^{i_\kappa_i} \tag{2.25}
\]

where \( P_{A,\kappa_i}^{i_\kappa_i} \) is the amount of SAs in link \( L_{i_\kappa_i} \). Hence

\[
n_i^F = \frac{P_i^F}{T_i}; n_i^C = \frac{P_i^C}{T_i}; n_A^{i_\kappa_i} = \frac{P_A^{i_\kappa_i}}{T_i} \tag{2.26}
\]

The equilibrium condition in market \( i \) is

\[
n_i^F Q_{1,t}^F + n_i^C Q_{1,t}^C + \sum_{\kappa_i} n_A^{i_\kappa_i} Q_{1,t}^{i_\kappa_i} = 0 \tag{2.27}
\]

Hence, in each trading period \( t \), equilibrium prices \([S_{1,t}, S_{2,t}, ..., S_{N,t}]\) solve the following system of equations.

\[
n_1^F Q_{1,t}^F + n_1^C Q_{1,t}^C + \sum_{\kappa_1} n_A^{1_\kappa_1} Q_{1,t}^{1_\kappa_1} = 0 \tag{2.28}
\]

\[
n_2^F Q_{2,t}^F + n_2^C Q_{2,t}^C + \sum_{\kappa_2} n_A^{2_\kappa_2} Q_{2,t}^{2_\kappa_2} = 0 \tag{2.29}
\]

\[\vdots\]

\[
n_N^F Q_{N,t}^F + n_N^C Q_{N,t}^C + \sum_{\kappa_N} n_A^{N_\kappa_N} Q_{N,t}^{N_\kappa_N} = 0 \tag{2.30}
\]

\[\]

### 2.4 Results

#### 2.4.1 Parametrization

In this section, we summarise the parametrization in our model. The time step in our model is assumed to be one trading day, \( \Delta t = 1/264 \). Net risk free rate is \( r = 0.01 \) (gross rate of return \( R = 1 + r = 1.01 \)). Networks are established among 20 markets \((N = 20)\). The market premium is \( \mu_m = 0.05 \) and market volatility is \( \sigma_m = 0.35 \). For each asset, we assume \( \beta_i \in [0.5 1.5] \). Speed of reversion \( \gamma \) is assumed to be within \([1\% 2\%]\). Mean dividend of each asset is \( \overline{y}_i \in [0.8 2.0] \). Risk aversion \( a \) is assumed to be 1. Standard deviation of excess return \( \theta \) is within \([0.25 1]\).
Table 2.1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size $P$</td>
<td>100</td>
</tr>
<tr>
<td>Amount of arbitrageurs $P_A$</td>
<td>[10 30]</td>
</tr>
<tr>
<td>Net interest rate $r$</td>
<td>0.01</td>
</tr>
<tr>
<td>Mean dividend $y_i$</td>
<td>Uniformly draw from [0.8 2.0]</td>
</tr>
<tr>
<td>Dividend noise term $\epsilon$</td>
<td>U(-1,1)</td>
</tr>
<tr>
<td>Market premium $\mu_m$</td>
<td>0.05</td>
</tr>
<tr>
<td>Market volatility $\sigma_m$</td>
<td>0.35</td>
</tr>
<tr>
<td>Systemic risk $\beta$</td>
<td>Uniformly draw from [0.5 1.5]</td>
</tr>
<tr>
<td>Speed of reversion $\gamma$</td>
<td>Uniformly draw from [1% 2%]</td>
</tr>
<tr>
<td>Risk aversion $a$</td>
<td>1</td>
</tr>
<tr>
<td>Variance of expected return $\theta$</td>
<td>Uniformly draw from [0.25 1]</td>
</tr>
<tr>
<td>Probability of a link $p$</td>
<td>0.4</td>
</tr>
<tr>
<td>Weight on recent dividend yield $g$</td>
<td>0.5</td>
</tr>
<tr>
<td>Daily volatility of returns $\sigma_{daily}$</td>
<td>0.01</td>
</tr>
<tr>
<td>Annualised Sharpe ratio $SR_{annual}$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Each pair of assets has correlation coefficient $\rho_{ij} \in [-1 1]$. Given $\theta_i^2$ and $\rho_{ij}$, we can calculate the covariance between the assets in a pair. The weight chartists put on the most recent dividend yield is $g = 0.5$. Any link between two nodes is present with probability $p = 0.4$. The population of fundamentalists and chartists is fixed at $P = 100$. The amount of SAs in each link is $P_A \in [10 30]$. Parameter values are summarised in Table 2.1.

In our analysis, we investigate how the presence of SAs affect the market performance. Hence, our model without SAs acts as a benchmark, which is our baseline model. In the following sections, we look into the effects of SAs on market stability, traders’ wealth and risk contagion.

### 2.4.2 Market Stability

In this section, we show the effects of statistical arbitrage on market stability. There are long-standing arguments regarding the proportion of traders using technical trading rules in financial markets. For instance, Hoffmann and Shefrin (2014) and Lewellen et al. (1980) have similar argument that technical traders account for around 30 percent of the trading population. A survey by Menkhoff and Taylor (2007) concludes that the proportion of technical traders ranges between 30% and 70%. Based on the literature and the real situation in our setting, we consider two compositions of traders in our baseline model: 60% fundamentalists and 40% chartists; 80% fundamentalists and 20% chartists.

In multiple tests, we get more accurate solutions of the equilibrium prices in all markets when there are more fundamentalists in markets. The two compositions of the trading population in our model is an outcome of the balance between the compositions in real financial markets and the
Table 2.2: Moments of Asset Prices

<table>
<thead>
<tr>
<th></th>
<th>Fundamentalists:Chartists 60:40</th>
<th>Fundamentalists:Chartists 80:20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>Value SD</td>
<td>Value SD</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.09703 (0.00176)</td>
<td>0.03510 (0.00057)</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.37005 (0.04065)</td>
<td>0.12824 (0.03843)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.61341 (0.07351)</td>
<td>2.43181 (0.05207)</td>
</tr>
<tr>
<td>With SAs</td>
<td>Value SD</td>
<td>Value SD</td>
</tr>
<tr>
<td>Volatility</td>
<td>0.03369 (0.00120)</td>
<td>0.01568 (0.00037)</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.10274 (0.08532)</td>
<td>0.03443 (0.05208)</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.76343 (0.13046)</td>
<td>2.79116 (0.09527)</td>
</tr>
<tr>
<td>Improvement</td>
<td>65.27% (0.01240)</td>
<td>55.32% (0.01091)</td>
</tr>
</tbody>
</table>

Note: Fundamentalists:Chartists is the proportion of fundamentalists to chartists in the initial population without SAs. There are 100 time steps in each model run. Statistics of the markets are averaged over the 20 markets and 200 model runs. Standard deviations across simulations are in parenthesis. All values are significantly different at 95% confidence level.

Table 2.2 summarises the results examining market stability. The top half of the table summarises market statistics of the baseline model while the bottom half is the market statistics with SAs.

Results show that the volatility of market price with 60% fundamentalists is much higher than that with 80% fundamentalists, either with or without SAs, implying that markets become more volatile as there are more chartists. This result is consistent with the literature on the role of heterogeneous beliefs on market stability (DeLong et al., 1991, Lux, 1998, Chiarella, 1992, Vigfusson, 1997). A consensus in this literature is that fundamentalists stabilise while chartists destabilise markets. For instance, Chiarella et al. (2009) find that increasing the degree of usage of technique trading rules in trading strategies reduces market stability. The intuition behind this result is that fundamentalists arbitrage mispricings and drive prices back to the fundamentals while chartists add noises, which deviates prices from fundamentals. The more chartists, the more noises and the less stable the market price.

More importantly and interestingly, we find that SAs help stabilise markets. Price volatilities decrease greatly when SAs trade in markets. Recall that SAs are taking long/short strategy to trade assets when short-term deviations from the long-term level occur. They long the underperformed assets and short the overperformed assets, keeping their portfolio market neutral. The nature of the trading strategy is also mispricing arbitrage. Hence, when SAs trade, market prices can better trace the fundamental values and therefore become less volatile. As a result, they help stabilise markets. Furthermore, the stabilising effects from SAs are more significant when accuracy of the solutions.
there are more chartists. The improvement in market stability with 40% chartists is about 10% higher than that with 20% chartists. Intuitively, more chartists drive prices further away from fundamentals, which creates more scope for SAs to trade and thus their market-stabilising role is more significant.

While markets with SAs tend to be more stable, these markets are more prone to extreme volatilities by giving higher kurtosis values. This is due to the potential that shocks may spread to other markets through the network established by SAs. Extreme volatilities occur when unexpected shocks occur. The network provides a mechanism for the transmission of shocks. Hence, an unexpected shock in one market may result in systematic turbulence. The markets as a whole are exposed to extreme events when SAs trade. More details about this result are provided in section 2.4.4.

2.4.3 Wealth Effect

In this section, we investigate the wealth change of traders when SAs are present. The literature regarding the wealth of fundamentalists and chartists is vast. Friedman (1953) argues that irrational traders will eventually be driven out of the market by rational traders who know better the fundamental and make positive profits. However, DeLong et al. (1991) find that noise traders can significantly increase the volatility of price, which discourages rational traders to bet against noise traders. This may result in rational traders not being able to take over the market. As a result, noise traders can actually make positive profits and persistently trade in the market. In our test, wealth of fundamentalists and chartists is summarised in Table 2.3 and that of SAs in Table 2.4.

From the two tables, we obtain the following findings in traders’ wealth. First, in the baseline model fundamentalists make positive profits while chartists make negative profits. However, when SAs are present, the result may be different. In the case of 20% chartists, fundamentalists make negative profits (-25.69) while chartists make positive profits (27.42), which is contrary to the result in the baseline model. SAs always earn positive wealth. Also, wealth of fundamentalists and SAs increases while wealth of chartists decreases as there are more chartists in the markets. Intuitively, more chartists may lead prices to deviate more from fundamental values, leaving greater arbitrage scope for fundamentalists and SAs, which results in more wealth transfer from chartists to fundamentalists and SAs. However, the extent of the above-mentioned change in wealth tend to reduce when SAs trade. For instance, as Table 2.3 shows, with SAs, when chartists increase from 20% to 40%, we can still observe increase in the wealth of fundamentalists and decrease in the wealth of chartists, however, compared with the baseline model without SAs, the extent of
Table 2.3: Wealth of Fundamentalists and Chartists

<table>
<thead>
<tr>
<th></th>
<th>Fundamentalists:Chartists 60:40</th>
<th>Fundamentalists:Chartists 80:20</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F C</td>
<td>F C</td>
</tr>
<tr>
<td>Mean</td>
<td>1419.04 -2128.56</td>
<td>67.19 -268.74</td>
</tr>
<tr>
<td>SD</td>
<td>74.44 111.66</td>
<td>9.98 39.94</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.04022 -0.04022</td>
<td>-0.14317 0.14317</td>
</tr>
<tr>
<td>With SAs</td>
<td>F C</td>
<td>F C</td>
</tr>
<tr>
<td>Mean</td>
<td>285.24 -656.08</td>
<td>-25.92 28.75</td>
</tr>
<tr>
<td>SD</td>
<td>56.48 86.43</td>
<td>8.98 36.78</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.49578 -0.37370</td>
<td>0.18623 -0.08113</td>
</tr>
</tbody>
</table>

Note: Fundamentalists:Chartists is the proportion of fundamentalists to chartists in the initial population without statistical arbitrageurs. There are 100 time steps in each model run. Statistics of the markets are averaged over the 20 markets and 200 model runs. All values are significantly different at 95% confidence level.

The reason behind these findings is that after SAs trade, the magnitude of wealth transfer decreases and SAs compete with fundamentalists in reaping wealth. First, since the trading strategies of SAs help drive prices back to fundamentals, the presence of SAs enables informed traders (fundamentalists and SAs) to better dominate the markets, in which case, market prices will be closer to fundamental values and there is less scope for arbitrage, which results in less wealth transfer from chartists to informed traders. This explains why after SAs trade, we observe smaller change in fundamentalists’ and chartists’ wealth as the amount of chartists increases. In addition, SAs compete with fundamentalists in collecting wealth. While both fundamentalists and SAs arbitrage mispricings, a significant difference exists. Fundamentalists trade in a single market and what matters for them is the absolute mispricings. SAs are doing pairs trading where the relative mispricings between the two assets are more important to them. For example, suppose two assets are both undervalued, but asset 1 is more undervalued than asset 2, in which case, fundamentalists will take long positions in both assets while SAs will take long position in asset 1 and short position in asset 2, keeping market neutrality of the portfolio. Because of this, fundamentalists and SAs can take the same and opposite positions in one market. In both possibilities, total wealth is divided between fundamentalists and SAs and in the second possibility, SAs may even reduce the dominance of fundamentalists in the market, in which case fundamentalists can make losses. It explains the negative wealth of fundamentalists and positive wealth of chartists in Table 2.3 when SAs are present (-25.92 for fundamentalists and 28.75 for chartists). This finding supports the argument by DeLong et al. (1991) that noise traders (chartists) may not always make losses but could make gains from trading. We observe this only with 20% chartists but not with 40% chartists. This is because when chartists are
in small amount, arbitrage scope is limited and fundamentalists are more likely to make losses. SAs’ caring about the relative mispricings between assets guarantees them non-negative profits from trading. Hence, their wealth is always positive. The positive wealth of SAs is consistent with real life in that financial institutions make huge profits with statistical arbitrage and rely heavily on it.

Another interesting finding in Table 2.3 involves the skewness of wealth distribution. In the baseline model, the skewness of wealth distribution of fundamentalists and chartists are symmetric, while in the model with SAs, this symmetry disappears. In the baseline model, the losses of chartists all become the gains of fundamentalists. Hence, we observe the inverse values of skewness of fundamentalists’ and chartists’ wealth distribution. However, when SAs trade, as mentioned before, total wealth is divided between fundamentalists and SAs and the symmetry disappears.

2.4.4 Market Resilience

In section 2.4.2, we mention that because of the network established by SAs, markets are more prone to extreme volatilities which occur when shocks arrive. In this section, we provide more detailed explanation and evidence to this result by investigating the contagion of shocks through the network. Unless otherwise stated, the data we collect to do statistical analysis in this section is from four models: the baseline model (Model 1), the baseline model with shocks (Model 2), the model with SAs but no shocks (Model 3) and the model with SAs and shocks (Model 4). A shock is described as a sudden and sharp decrease in the price of an asset. Shock size is between 0 and 1 which measures how greatly the market is shocked. For example, if the shock size is 20%, then the price of the shocked market will decrease by 20% from the level without the shock. We randomly shock a market and measure the effects of the shock on other markets. Percentage price change (PPC) is employed to measure the effects of a shock. In particular, if we shock a market at time step $t$, 

<table>
<thead>
<tr>
<th></th>
<th>Fundamentalists:Chartists 60:40</th>
<th>Fundamentalists:Chartists 80:20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>52.00</td>
<td>8.91</td>
</tr>
<tr>
<td>SD</td>
<td>2.94</td>
<td>0.68</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.18220</td>
<td>0.10527</td>
</tr>
</tbody>
</table>

Note: Fundamentalists:Chartists is the proportion of fundamentalists to chartists in the initial population without statistical arbitrageurs. There are 100 time steps in each model run. Statistics of the markets are averaged over the 20 markets and 200 model runs. All values are significantly different at 95% confidence level.
Table 2.5: Risk Contagion

<table>
<thead>
<tr>
<th></th>
<th>Fundamentalists:Chartists</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>60:40</td>
<td>80:20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPC</td>
<td>0.569%</td>
<td>-0.648%</td>
<td>0.635%</td>
<td>0.097%</td>
<td>0.115%</td>
</tr>
<tr>
<td>SD</td>
<td>0.02204</td>
<td>0.08207</td>
<td>0.08813</td>
<td>0.00798</td>
<td>0.03937</td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPC</td>
<td>-1.456%</td>
<td>-0.648%</td>
<td>0.635%</td>
<td>-1.903%</td>
<td>0.115%</td>
</tr>
<tr>
<td>SD</td>
<td>0.02184</td>
<td>0.08207</td>
<td>0.08813</td>
<td>0.00796</td>
<td>0.03937</td>
</tr>
<tr>
<td>Change in PPC</td>
<td>3.56</td>
<td>0</td>
<td>0</td>
<td>20.62</td>
<td>0</td>
</tr>
<tr>
<td>Model 3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPC</td>
<td>0.098%</td>
<td>-0.309%</td>
<td>-0.00032%</td>
<td>0.036%</td>
<td>0.115%</td>
</tr>
<tr>
<td>SD</td>
<td>0.02195</td>
<td>0.02778</td>
<td>0.01981</td>
<td>0.00819</td>
<td>0.01969</td>
</tr>
<tr>
<td>Model 4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPC</td>
<td>-4.960%</td>
<td>-2.775%</td>
<td>-3.081%</td>
<td>-4.450%</td>
<td>-0.667%</td>
</tr>
<tr>
<td>SD</td>
<td>0.02130</td>
<td>0.02737</td>
<td>0.01976</td>
<td>0.00848</td>
<td>0.01951</td>
</tr>
<tr>
<td>Change in PPC</td>
<td>51.42</td>
<td>7.98</td>
<td>9644.07</td>
<td>125.20</td>
<td>6.80</td>
</tr>
</tbody>
</table>

Note: Fundamentalists:Chartists is the proportion of fundamentalists to chartists in the baseline model. Size of shock is 40%. There are 100 time steps in each model run. Statistics of the markets are averaged over 200 model runs. All values are significantly different at 95% confidence level. The first “Change in PPC” describes the change in PPC from Model 1 to Model 2 while the second “Change in PPC” describes the change in PPC from Model 3 to Model 4.

PPC in each model can be expressed as

\[
PPC = \frac{S_t - S_{t-1}}{S_{t-1}} \times 100\%
\]

where \(S_t\) is the market price at time step \(t\). \(PPC > 0\) (\(PPC < 0\)) implies an increase (decrease) in price. The greater the magnitude of PPC in a market, the greater the price change and the more significant the effects of the shock on that market. PPCs of the four models are compared to see the effects of a shock and are summarised in Table 2.5. For illustrative purpose, in each model, we show the average PPCs and the PPCs of the least and most significantly affected markets. A general finding is that an unexpected shock can result in large scale of significant decrease in the price of other markets, which is consistent with our expectation that shocks can spread through the trading activities of SAs.

First, comparing Model 1 and Model 3, we see that the magnitude of PPCs in markets with SAs are much lower than those in markets without SAs, implying that prices change less when SAs trade. This provides evidence for our previous finding that SAs help drive prices back to fundamentals and stabilise market prices. Now, let us compare Model 1 and Model 2, Model 3 and Model 4 as two pairs to see
the role of SAs in risk contagion. We observe that when a shock occurs, in the baseline model (Model 1 and Model 2), on average, the markets as a whole has a decrease in price. For instance, on average, prices in markets with 40% chartists change from increasing by 0.569% in Model 1 to decreasing by 1.456% in Model 2, a more than 3.5 times of change. But PPCs in other markets without the shock remain the same (changes in PPC are 0), implying that the shock does not affect the prices in other markets. In the model with SAs (Model 3 and Model 4), we observe a systematic price crash, both the average price and price in individual markets decreasing significantly. For example, in markets with 40% chartists, on average, prices change from increasing by 0.098% in Model 3 to decreasing by 4.960% in Model 4. In the case of 20% chartists, the price of the least affected market changes from increasing by 0.115% to decreasing by 0.667% and that of the most affected market changes from increasing by 0.0087% to decreasing by 4.906%. It indicates a significant and large scale decrease in the prices in all other markets. All changes in price in the second pair is much greater than those in the first pair.

From the comparison of the two pairs, we observe the role of SAs in transmitting shocks. In the baseline model, individual markets are independent of each other since SAs are not trading. Hence, shocks occurring in one market will not spread to other markets whose prices therefore remain unchanged. However, when SAs trade, they link markets together, creating a network among markets, which provides the route for the contagion of shocks. The property of market neutrality of statistical arbitrage plays the key role in risk contagion. As mentioned before, when a market is shocked, SAs in that market will adjust their demand for that asset and also the demand for the other asset in their portfolio to keep their portfolio market neutral. The change in demand affects the equilibrium prices in the two markets. SAs trading the two assets and other assets will respond by adjusting their demand, affecting other prices, which will further affect SAs trading these assets and more other assets etc. Through this way, shocks spread to the whole system. This result confirms the contribution of SAs in risk contagion and echoes with our discussions in section 2.4.2, especially the greater kurtosis values when SAs trade. While SAs make market prices more stable, they make the whole system sensitive to extreme events such as unexpected shocks. When a shock occurs in one market, it may propagate to other markets through the trading of SAs. In extreme cases, it can lead to a large scale collapse in market prices, just as the case in the Flash Crash. The risk contagion role of SAs, together with their market-stabilising role, is consistent with the finding in network literature in that in normal periods, networks help stabilise markets, however, they may also act as a mechanism for risk contagion when shocks occur (Acemoglu and Tahbaz-Salehi, 2015; Haldane, 2009; Georg, 2013; Gai and Kapadia, 2010). In our paper, the network is established by SAs.
2.4.5 Propagation of Shocks

While from section 2.4.4, we show that shocks can propagate through the network established by SAs, we still do not know the pattern of the propagation. For simplicity, we consider a straight-line network with 20 markets where the 20 markets link one by one through the trading of SAs. A shock is still described as a sudden and sharp decrease in the price of a market. As in the baseline model shocks do not spread, we only consider shocks when SAs trade. More generally, we consider three models: the baseline model with no shocks where the 20 markets are independent (Model 1), the model with SAs but no shocks (Model 2) and the model with SAs and shocks (Model 3). Again, percentage price change (PPC) is used to measure the effects of shocks. We shock the terminal market (market 20) and compare the price change in other markets in all the three models. Figure 2.1 and Figure 2.2 show our findings.

Comparing the two graphs in Figure 2.2, we can see that when there are more fundamentalists, the PPC curves of Model 1 and Model 2 track each other better, implying less volatile market prices. Also, in each graph, we observe that the magnitudes of PPCs in Model 2 are smaller than those in Model 1, the PPC curve of Model 2 below that of Model 1. This is more obvious in the top graph with 40% chartists. In both findings, the increase in the amount of fundamentalists and the presence of SAs increase the dominance of informed traders in markets and prices hence better track fundamental values and become more stable.

Also, Figure 2.1 shows that when a shock occurs, markets close to the shocked market are more affected than markets less close to the shocked markets. The effects of shocks on other markets decrease exponentially and disappear after some point. In our case, as Figure 2.2 shows, in market 15, we can not observe significant differences between the PPC curves of Model 2 and Model 3 with different shock sizes and the effects of shocks almost eliminate after market 15. Hence, our findings show that while the trading activities of SAs may act as a mechanism for risk contagion, the effects are limited.

The limited effects may come from a double-edged role of SAs in risk contagion. On the one hand, trading activities of SAs link markets together, providing routes for the contagion of shocks. On the other hand, since SAs arbitrage mispricings, SAs in markets not shocked will trade in the opposite direction of the effects of the shock, which helps stop the contagion of the shock. For example, suppose the shock in market 20 results in a decrease in the price of market 19. SAs trading in markets 18 and 19 will expect the price in market 19 to increase. Hence, they will either increase their long position or decrease their short position or change from short to long position in market 19, all of which help counteract the effects of the shock on
Note: The two graphs illustrate the overall trends of PPCs in Models 1, 2 and 3. Four sizes of shock are considered: 10%, 20%, 30% and 40%. The top graph is for the markets with 40% chartists. The bottom graph is for the markets with 20% chartists.
Note: The two graphs are the bigger pictures of Figure 2.1 and illustrate the details of the changes in PPCs for markets 12 to 18. The top graph is for the markets with 40% chartists. The bottom graph is for the markets with 20% chartists.
market 19. The similar mispricing arbitrage happening in market 19 also happens in market 18 and the markets ahead, which to some extent prevents the contagion of the shock and limits the shock effects. In summary, we can say that while SAs establish the networks for risk contagion, they also play a role in absorbing the effects of shocks.

The shock-absorbing role of arbitrageurs also appear in Menkveld and Yueshen (2017) where cross-arbitrageurs act as a buffer for the spread of shocks. The authors attribute the Flash Crash to the fact that cross-arbitrageurs were not able to trade enough due to the liquidity shortfalls before the event, which prevents them to transfer selling pressures to other markets, implying the positive role of cross-arbitrageurs in absorbing shocks.

2.5 Conclusion

The spread of the liquidity dry-ups in the Flash Crash has attracted much attention. Our paper provides a new possible mechanism for the contagion of illiquidity and hence a new explanation for the Flash Crash, which is the trading of statistical arbitrageurs. Statistical arbitrage is a trading strategy that heavily rely on advanced computation capacity. As the rapid development in computer technology over the last two decades, it has become one of the strategies most used by investment banks and hedge funds in their daily business. Given the dominant positions of these financial institutions in the finance world, the influential affects of this strategy on financial markets are huge and are worth to investigate.

In this paper, we consider the role of statistical arbitrage in linking asset markets and investigate the effects of statistical arbitrageurs’ trading activities on market performance and risk contagion. We find that statistical arbitrageurs reduce market volatilities in normal periods while at the same time make the whole system more sensitive to shocks as their trading activities may act as the mechanism for the spread of shocks. However, in the investigation of the patter of the spread, we find the contagion effects are limited and disappear exponentially among markets. We attribute it to the arbitrage nature of the strategy, which means that after a shock, statistical arbitrageurs in markets not shocked will trade against the shock which to some extent prevents the spread of the shock to larger scale.

Statistical arbitrage is one representative example of the modern trading strategies that are the result of the rapid development in computer technology. Over the last two decades, these strategies have become the dominant strategies among financial institutions. During the same period, financial markets have become more linked to each other and more frequent price crashes have been observed. Our paper, by linking computational trading strategies with risk contagion, provides some insights
into the overall effects of the advances in trading strategies on financial markets. While advanced trading strategies do make trading more efficient, at the same time, they may make financial system fragile. The positive sides of these strategies are always praised, however, the negative sides are always ignored but should attract more attention. This echoes with the recent discussion regarding the role of high frequency trading on market stability.
Chapter 3

The Role of Heterogeneous Beliefs in Trading Skill Acquisition

Abstract

Trading skills bring higher returns in practice and receive much attention in the existing literature on financial markets. However, the process of acquiring trading skills draws less attention. This paper investigates the effects of heterogeneous beliefs on the acquisition of trading skills and the resulting market performance. We consider a computational model where markets are composed of fundamentalists and chartists with the former developing pricing skills from trading and the latter being effectively noise traders. Learning of fundamentalists is achieved through natural selection. We find that an increase in the amount of chartists promotes learning but also reduces the accuracy of the learning outcome. Furthermore, markets with more chartists tend to be more volatile but are also more resilient to shocks. Intuitively, the evolving process of fundamentalists’ acquiring skills adds uncertainties to the market and makes the market price more volatile, which deters learning. However, chartists, by taking past prices into their decision-making, are able to stabilise price and facilitate learning.
3.1 Introduction

Trading skills are important in identifying mis-priced assets and are able to bring abnormal returns (Coval et al., 2005; Puckett and Yan, 2011; Cremers and Petajisto, 2009). However, in the current finance theory literature, they are often ignored. In traditional finance theory, traders are assumed to be completely rational in the sense that they have all the skills needed to do financial analysis. However, as some behavioural economics studies point out, in real life, people are bounded rational (Cremers and Petajisto, 2009; Selten, 1990; Conlisk, 1996; Eliaz and Spiegler, 2006). They are bounded in the ability to understand information and they are bounded in the skills of doing complex calculations etc, even if with complete information. The fact that top financial institutions made huge losses during the 2007-2008 financial crisis by failing to understand the then popular structured financial securities provides evidence for bounded rationality in real life.¹

Numerous pieces of evidence in the academic work show that people are bounded in trading skills (Locke et al., 1999; Chang and Loche, 1996; Leaver and Reader, 2016). Also, multiple studies prove that by accumulating trading experience, people are able to improve and acquire trading skills (Coates and Page, 2009; Nicolosi et al., 2009). To capture the process of trading skill acquisition in financial markets, in this paper, we start by considering a scenario with complete information. Traders differ in trading skills and acquire trading skills through evolutionary learning by natural selection. This guarantees the survival of superior trading strategies. In particular, we investigate how the presence of noise traders affects the learning outcome and the resulting market performance. Trading skill in our paper refers to the ability to accurately price a financial asset, with complete information. For instance, a trader knowing some asset pricing model can accurately identify mispricings and make correct trading decisions and hence is more skilled.

The basic setting of our paper, as described above, distinguishes our paper from other literature on learning, especially the longstanding literature on learning by information procession (Akerlof, 1970; Kyle, 1985; Glosten and Milgrom, 1985; Dang et al., 2015; Grossman and Stiglitz, 1980). In this literature, information intransparency provides agents the incentive to acquire private information which results in information asymmetry in which case Bayesian decision theory is the mainstay for rational decision making (Asparouhova et al., 2009; Morellec and Schurhoff, 2011; Sciubba, 2005). This literature assumes traders are born with the skills and

¹Before the 2007-2008 financial crisis, people failed to understand the complex structure of the then popular structured securities Collateralised Debt Obligations (CDOs) (Carlin et al., 2013; Arora et al., 2009; Brunnermeier and Oehmke, 2009). Financial institutions used wrong models to evaluate these securities and made wrong decisions as a result. From this, we see that facing new financial products, even the world’s top financial professionals need to learn the skills to do correct analysis.
knowledge to do Bayesian updating given new information. Hence, the focus of this literature is information acquisition rather than skill. However, this literature has been challenged by a growing literature arguing that agents do imperfect learning in the sense that they may not follow the Bayes’ rule to update their beliefs (Kuhnen, 2015; Ellison and Fudenberg, 1993; Binmore, 2008; Spiegler, 2006; DellaVigna and Malmendier, 2004). For example, Spiegler (2006) argues that firms tend to complicate consumers’ understanding of actual values of products, making use of their better understanding of markets. Consumers then find it difficult to grasp the structure and thus resort to simplifying heuristics. DellaVigna and Malmendier (2004) argue that rational firms may design contracts that exploit consumers’ misperceptions of their time-inconsistent preferences, in which case consumers get welfare loss. Going beyond Bayesian rationality, Ellison and Fudenberg (1993, 1995) make progress to consider the evolutionary learning of bounded rational agents. For instance, in Ellison and Fudenberg (1993), naive agents observe the experience of others which is then used in their own future decision making. Following this simple learning rule, efficient long-run social states can be realised. Our learning mechanism is consistent with theirs and this feature also distinguishes our paper from the literature on learning by experience reflection (Daudelin, 1996; Lundgren et al., 2017). This literature emphasizes the importance of reflection in transforming experience into learning, which is different from the mechanism of learning in our paper. In addition, this literature focuses on the use of reflection as a learning mechanism to train managers in firms, which is also different from our aim in this paper mentioned above.

We consider a centralised market populated by fundamentalists and chartists. Fundamentalists are ignorant of trading skills and learn to acquire them from trading. Chartists do not learn and rely on past price trends to make trading decisions and therefore are effectively noise traders. Each trader has a pricing function determining his quote on a financial asset. Learning of fundamentalists is achieved by the evolving of their pricing functions. A fundamentalist is more skilled if his quote after learning is closer to the fundamental value of the asset. The expectation of our paper is as follows. As fundamentalists learning, the evolving of their pricing functions adds uncertainties to the market price. However, chartists, by taking past prices into consideration in their pricing functions, can stabilise market price. Learning is more doable in a market with stable price. Hence, an increase in the proportion of chartists will increase the proportion of fundamentalists holding skills. Skilled traders increase market resilience (Ladley et al., 2015). So, in expectation, a market with more chartists relative to fundamentalists tend to be more resilient. However, by adding noises to the market price, chartists tend to mislead fundamentalists, reducing the accuracy of the learning outcome of fundamentalists. The more chartists relative to fundamentalists, the greater the misleading effects. Thus, when
there are more chartists, the market price is less able to reflect the fundamental value of the asset and it is therefore expected to be more volatile.

Starting with the work by DeLong et al. (1990), heterogeneous beliefs have been documented in numerous settings (DeLong et al., 1991; Vigfusson, 1997; Panchenko, 2013). A consensus in this literature is that fundamentalists stabilise markets while chartists destabilise markets (Brocka and Hommes, 1998; Chiarella et al., 2009; Bauer and Herz, 2004; Lux, 1998). These papers assume fundamentalists are rational while chartists are irrational and add additional risk to the markets. We consider this issue from a different angle by assuming fundamentalists do not have the skills to accurately price financial assets and learn to do so. Then we investigate the effects of noise traders (chartists) in the learning process. An important innovation of our paper is that we consider learning in the context of heterogeneous beliefs and investigate the resulting learning outcome.2

The model in our paper extends that of Ladley et al. (2015). In their paper, the authors consider how market fragmentation affects traders’ learning and the associated market performance. Our paper differs from theirs in the sense that we consider a centralised market and investigate the effects of heterogeneous beliefs on the learning of traders.

This paper is organised as follows. Section 3.2 is literature review. Section 3.3 describes the model. Section 3.4 discusses data collection and performance measurements. Section 3.5 is a regression analysis. The last section concludes the paper.

### 3.2 Literature review

Trading skills bring high returns in practice but are often ignored in the finance theory literature. It is widely documented that skills enable a trader to persistently outperform the markets (Oliven and Rietz, 2004; Barras, 2010; Fama and French, 2010; Makarov and Plantin, 2011). For instance, Barber et al. (2014) finds that among the most successful day traders in the Taiwan Stock Exchange, a small group of skilled day traders earn predictably high returns. Similarly, Grinblatt et al. (2011) show that traders with high IQ, a measure of cognitive skill, tend to participate more in the stock market and earn higher Sharp ratios than those with low IQ. Learning plays a significant role in traders’ acquisition of trading skills. As pointed out by Lo et al. (2005), given proper instruction and practice, traders of different personality types can perform trading functions equally well. Nicolosi et al. (2009) document that traders learn from previous trading experience and adjust their trading beha-

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2There are papers by LeBaron who also studies the effects of heterogeneous beliefs on learning (see LeBaron (2001a), LeBaron (2001b) and LeBaron (2002) for examples). However, in his papers, heterogeneous beliefs refer to the different memory lengths traders use in the decision making while in our paper, heterogeneous beliefs refer to traders’ being fundamentalists or chartists.
viours. This helps them to obtain better investment performance. Thus, traders will participate in unprofitable trades at the beginning of their trades as long as the short-term loss can be compensated after they obtain the skills and ability to submit profitable orders (Pastor and Veronesi (2009)). Imitation is one of the main forms of learning (Shleifer and Summers, 1990; Lieberman and Asana, 2006). Literature shows that by imitating more successful traders, less successful traders can earn higher returns. In Grinblatt et al. (2012), high IQ traders have superior stock picking skills and imitation of their trades generates abnormal returns. In Coval et al. (2005), strategies long in firms purchased by previously successful traders and short in firms purchased by previously unsuccessful traders earn abnormal returns of 5 basis points per day.

Most of the above literature emphasizes the outcome of being skilled at individual level, that is, being skilled helps individuals make better investment decisions. Our paper analyses not only the performance of skilled individuals, but also the effects of skilled traders on the performance of the markets as a whole. More importantly, with genetic programming, our paper considers the details of the learning process, in particular, we investigate the role of noise traders in the process of learning and the resulting outcome.

Our paper also contributes to the small but growing literature studying the role of heterogeneous beliefs on the learning and acquisition of trading skills (LeBaron, 2001a; LeBaron, 2001b; LeBaron, 2002). For instance, LeBaron (2002) investigates the effects of short-memory traders on long-memory traders’ evolutionarily learning of an optimal investment strategy. He finds that the existence of short-memory traders hinders the learning of long-memory traders and thus it is difficult for long-memory traders to take over the market, which leads to persistence in market volatility.

Our paper is in the same spirit as of his but differs from his in the following points. First, the heterogeneous beliefs in his paper come from the past prices traders use to make the current decision. In his paper, all traders are memory traders who have different memory length. However, in our paper, heterogeneous beliefs come from the trading strategies traders use: fundamentalist or chartist. Chartists in our paper correspond to the memory traders in his paper. Second, in his paper, all memory traders learn while in our paper only fundamentalists learn with memory traders (chartists) disturbing the learning process. Finally, in his paper, traders seek to solve an optimisation problem through learning while fundamentalists in our paper do not solve optimisation problem but seek to accurately price assets through learning.

Finally, our paper relates to the literature of trading technologies. The rapid development in trading technologies in recent decades has made the financial world
more accessible for the public. Hence, various types of traders have entered into the financial markets, especially some individual traders who just rely on past price trends to speculate (Zhang and Zhang, 2015; Peri et al., 2014). This background makes our paper important in the sense that we study how this increase in noise traders in financial markets would affect the functioning of the markets. Our paper shows that an increase in noise traders can encourage learning but affect the learning accuracy by misleading prices.

3.3 Model

The model constructed in this paper follows the framework of Ladley et al. (2015). A centralised market is populated by two types of traders: fundamentalists and chartists, each in the amount of $I_f$ and $I_c$, respectively, where $I = I_f + I_c$ is the total population in the market and is assumed to be fixed. To better illustrate the role of chartists in learning, we consider different compositions of fundamentalists and chartists in a market. These agents trade options during a sequence of trading rounds (denoted by $T = 1, 2, ..., T$). In each trading round $T$, traders trade a randomly generated option $C_T$. Traders daily hedge their positions in the option until it expires at $T + 1$, after which, a new trading round starts and the same traders trade a new randomly generated option.

To collect data, we assume $I = 100$ and 5 possible values of $I_f \in \{100, 90, 80, 70, 60\}$. Note that a decrease in fundamentalists is equivalent to an equal increase in chartists and hence correspondingly, $I_c \in \{0, 10, 20, 30, 40\}$, giving 5 population compositions in total. When all traders are fundamentalists ($I_f = I = 100, I_c = 0$), without noise from chartists, learning in the market is expected to be the most efficient in the sense that transaction price best describes the fundamental value compared with other population compositions. However, if there are 40 chartists ($I_f = 60, I_c = 40$), the misleading effects from chartists on learning are expected to be the maximum and transaction price deviates the most from the fundamental value.\(^3\)

*Market.* All options in our model are 3-month European call options.\(^4\) Thus, each trading round lasts for 66 days (from the first to the last trading day, we have $t_n = n * \Delta t$ with $n = 0, 1, ..., 65$ and $\Delta t = 1/264$). The price of the underlying asset follows a Geometric Brownian Motion. Let $S(t)$ denote the price of the underlying asset.

\(^3\)Tests show that when chartists are more than fundamentalists, influences from chartists are so huge that the market can not be normally functioning, which makes the study of fundamentalists’ learning pointless. Thus, we keep $I_c$ smaller than $I_f$ to guarantee the normal functioning of the market.

\(^4\)For each year, we assume there are 264 trading days and the 3-month duration of each option accounts for a quarter of the annual trading days, that is 66 days.
asset at time $t$, then

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t)$$

where $\mu$ and $\sigma$ are respectively the annual drift and annual volatility of the asset.

Without loss of generality, we normalise the initial price of the underlying asset in each trading round to 100, that is $S(0) = 100$. In addition, a money market exists for traders to borrow and lend at the same annual rate $r$. An option is defined by $(K, \sigma, r)$ where $K$ is the strike price of the option. We use $C$ to denote a set of options including all options determined by $K \in [80, 120]$, $\sigma \in [0.1, 0.3]$ and $r \in [0.01, 0.06]$. So,

$$C := [80, 120] \times [0.1, 0.3] \times [0.01, 0.06]$$

Trade of options occurs in the following way. In the beginning of any trading round $T$, a random draw from $C$ gives the option for that trading round $C_T$. Then, traders simultaneously post their quotes at which they are indifferent between buying and selling this option. The median of their quotes is the transaction price, denoted by $P_T$.\footnote{In the following context, transaction price and market price may be used interchangeably.} Traders quoting higher than the transaction price have higher valuation and hence will buy the option while traders quoting lower will thus sell the option. Traders quoting the transaction price will buy or sell the option with probability $1/2$. Each trader is allowed to trade only one unit of the option and hence at the transaction price, market clears. On each trading day, traders participate in the underlying asset market and the money market to delta hedge their position in the option. In particular, traders buying (selling) one unit of the call option need to sell (buy) the underlying asset in a way such that the value of their portfolio does not change with the price of the underlying asset.

**Pricing Function.** Each trader has a pricing function which determines his quote. Unlike the traditional finance literature assuming fundamentalists have the knowledge and ability to accurately identify mispricings, fundamentalists in our model are assumed to have to learn to do this. More specifically, each fundamentalist is randomly assigned a pricing function at the start of our model. Through trading, they learn and improve their pricing functions. More details about the pricing functions of fundamentalists can be seen when we talk about learning, as can be seen later.

Chartists are traders who do not care about the fundamental value of the options but rely only on past price trends to decide their pricing quotes. In our model, chartists are effectively noise traders. They base their decisions on a fictitious time
series of past prices. Since each option is traded only once, the realised past prices constitute a random time series which is observed by chartists and decides their quotes. This series lies within the pricing bounds of the set of considered options. Using this series leads to chartists adding noise to the market. Fictitious time series are very often used in the finance literature (Bonanno, Valenti and Spagnolo, 2007a & 2007b). For instance, Bonanno, Valenti and Spagnolo argue that fundamental traders, speculators and noise traders make the financial market a complex system. To model the stock price closer to the real financial markets with normal activity and extreme days, they use a random walk to replace the geometric Brownian motion, in which case, the random walk represents a fictitious “Brownian particle” moving. Their papers provide rationale for the chartists in our setting using fictitious time series to decide their quotes.

Suppose chartist \( j \) has a memory length of \( L_j \). Let \( V_{j,T} \) denote this chartist’s quote on trading round \( T \). According to Chiarella and He (2003), \( V_{j,T} \) is a function of past transaction prices: \( V_{j,T} = H(\overline{P}_{T-1}) \), where \( \overline{P}_{T-1} = (P_{T-1}, P_{T-2}, ..., P_{T-L_j}) \) is a vector of transaction prices on the past \( L_j \) trading rounds. Chartists differ first in their memory length and also in the extent to which they respond to past price movements, which is the extrapolation rate. Chiarella and He (2004) assume \( V_{j,T} \) follows Geometric Decay Process (GDP), that is,

\[
V_{j,T} = H(\overline{P}_{T-1}) = \exp(g_j \sum_{k=1}^{L_j} b_j \omega^k P_{T-k})
\]

where \( b_j = 1/\sum_{k=1}^{L_j} \omega^k \), \( \omega \in [0, 1] \) measures the decay rate of memory; \( b_j \omega^k \) with \( k = 1, 2, ..., L_j \) is the weight chartist \( j \) puts on the transaction price of trading round \( T - k \) in deciding his quote on round \( T \), measuring the influence of that price on the current decision and \( \sum_{k=1}^{L_j} b_j \omega^k = 1 \); \( g_j \in [g, 0) \cup (0, \overline{g}] \) is the extrapolation rate of chartist \( j \), which measures his response to past price movements. If \( g_j > 0 \), then chartist \( j \) is a trend follower as he believes the observed price trend will continue. If \( g_j < 0 \), chartist \( j \) is a contrarian as he believes the future price will move in the opposite direction to the current price trend. For \( g_j > 0 \) (\( g_j < 0 \)), as \( g_j \) rises (declines), chartist \( j \) gets more aggressive in their expectation extrapolating from past prices.

**Gains and losses.** All traders have zero initial wealth. In trading round \( T \), after trading the option, on each trading day, traders delta hedge their position in the option via trading the underlying asset and the money market. A trader selling (buying) one unit of the option in that trading round needs to buy (sell) \( \Delta_{t_n} \) units
of the underlying asset on day \( t_n \) where

\[
\Delta t_n = \Phi\left( \frac{\log(S(t_n)/K) + (r + \sigma^2/2)(t_{66} - t_n)}{\sigma \sqrt{t_{66} - t_n}} \right) \tag{3.4}
\]

according to Black-Scholes delta hedge. \( \Phi \) is the standard normal cumulative distribution function; \( t_{66} - t_n \) is time to maturity on day \( t_n \). On trading day \( t_{n+1} \), the accumulated net cash flow of a seller from delta hedging is

\[
v(t_{n+1}) = e^{r \Delta t_n} [v(t_n) - \Delta t_n S(t_n)] + \Delta t_n S(t_{n+1}) \tag{3.5}
\]

with \( v(0) = 0 \). On the expiration of the option, that is on day \( t_{65} \), the payoff of a seller, denoted by \( \Pi^s \), is

\[
\Pi^s = P_T e^{rt_{65}} + e^{r \Delta t}[v(t_{64}) - \Delta t_{64} S(t_{64})] + \Delta t_{64} K \tag{3.6}
\]

if the option is in the money and

\[
\Pi^s = P_T e^{rt_{65}} + e^{r \Delta t}[v(t_{64}) - \Delta t_{64} S(t_{64})] + \Delta t_{64} S(t_{65}) \tag{3.7}
\]

if the option is out of the money. The payoff of a buyer \( \Pi^b \) is equal to \(-\Pi^s\) because payments net to zero.

**Learning.** Learning occurs only among fundamentalists. Before any trade occurs, every fundamentalist is randomly assigned a pricing function, which decides his quote for any option drawn from set \( C \) in each trading round. Based on the above analysis of the payoffs, fundamentalists get some wealth from each trading round and we allow wealth to accumulate as trading round. After a trading round, fundamentalist can review the performance of their pricing function relative to others in terms of the accumulated wealth their pricing function generates. The more accumulated wealth a pricing function generates, the better the pricing function is judged to be.\(^6\) Each fundamentalist, with equal probability, will be chosen to replace his pricing function with a better one. A fundamentalist’s pricing function remains the same until a better one replaces it. The learning we consider in this model is the skill acquisition of fundamentalists to price an asset more accurately through the accumulation of trading experience, rather than the learning of new information as is more common in the literature. Hence, in our model, what matters is how the parameters of options enter into pricing functions rather than the update of parameter values. Learning is realised through competition and natural selection. The

---

\(^6\)Success of a pricing function is measured by the accumulated wealth it generates compared with the other pricing functions. We say a pricing function is better than the other if it generates more accumulated wealth than the other does.
Table 3.1: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>$I = 100$</td>
</tr>
<tr>
<td>Proportion of chartists $\rho_f$</td>
<td>$\in [0, 0.1, 0.2, 0.3, 0.4]$</td>
</tr>
<tr>
<td>Maximum tree depth</td>
<td>10</td>
</tr>
<tr>
<td>Tournament size</td>
<td>4 traders</td>
</tr>
<tr>
<td>Crossover probability $\chi_1$</td>
<td>0.5</td>
</tr>
<tr>
<td>Mutation probability $\chi_2$</td>
<td>0.05</td>
</tr>
<tr>
<td>3-month Discount rate $\tau$</td>
<td>0.01</td>
</tr>
<tr>
<td>Initial stock spot price $S(0)$</td>
<td>100</td>
</tr>
<tr>
<td>Stock price drift $\mu$</td>
<td>0.06</td>
</tr>
<tr>
<td>Option strike price $K$</td>
<td>Uniformly drawn from [80,120]</td>
</tr>
<tr>
<td>Stock price volatility $\sigma$</td>
<td>Uniformly drawn from [0.1, 0.3]</td>
</tr>
<tr>
<td>Interest rate $r$</td>
<td>Uniformly drawn from [0.01, 0.06]</td>
</tr>
<tr>
<td>Extrapolation rate $g_j$</td>
<td>Uniformly drawn from $[-1, 0) \cup (0, 1]$</td>
</tr>
<tr>
<td>Penalty for quote $&lt;0$ or $&gt;40$</td>
<td>5</td>
</tr>
</tbody>
</table>

Way learning happens in this model is in the spirit of Lensberg and Schenk-Hoppe (2007) and Ladley et al. (2015) in the form of genetic programming. Each pricing function is randomly generated as a tree structure. Tree structures are composed of operators and terminators. The set of operators is $\{+, -, \times, /, \exp, \sqrt{}, \log\}$.

The set of terminators consists of a subset of parameters of options $\{S(t_n), K, \sigma, r, t_{65}\}$ and a subset of constants $X = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$. Operators and terminators are randomly drawn to construct a tree structure with depth no more than 10, which gives the pricing function of a fundamentalist. The evaluation of a tree structure gives the quote. Replacement of pricing functions happens with the following algorithm:

1. **Tournament**: After each trading round, we randomly choose four traders and rank their accumulated wealth.

2. **Reproduction**: We replace the pricing functions of the two traders at the bottom of the rank with the pricing functions of the two at the top of the rank.

3. **Crossover**: With probability $\chi_1$, we exchange two randomly selected subtrees between the two top programmes.

4. **Mutation**: With probability $\chi_2$, a randomly selected operator or terminator is replaced by a new random operator or terminator.

The change of pricing functions does not affect traders’ wealth. Without any discount, traders’ accumulated wealth might increase to infinity as more trading rounds.
taking place. This would deter the survival of better pricing functions, especially when some fundamentalists holding bad pricing functions are lucky enough to earn higher accumulated wealth. To avoid this, or, to make sure the survival of better pricing functions, we discount each trader’s wealth by the 3-month discount rate $\tau$. In addition, to facilitate learning, we impose a punishment on fundamentalists quoting lower than 0 or higher than 40. 0 and 40 are the two bounds of the value of the options considered in set $C$. More specifically, we penalise fundamentalists by subtracting 5 from their accumulated wealth if they quote outside the interval [0, 40]. A summary of the values of parameters is in Table 3.1. We allow one tournament to take place after each trading round.

### 3.4 Data and performance measurement

In this section, we describe how we collect data and the measurements we use to test our hypotheses. As mentioned before, we consider a market with 5 possible population compositions of fundamentalists and chartists. For each population composition, we do 30 model runs with 30 different seeds for random number generation. At the end of each model run, the pricing function of each fundamentalist is recorded and applied to a set of options, $\hat{C}$. This set contains 27 options and is defined by all combinations of the following parameters.

$$K = 95, 100, 105; \sigma = 0.15, 0.2, 0.25; r = 0.02, 0.035, 0.05$$

Data is collected from the evaluation of fundamentalists’ pricing functions with the above 27 options and is used to do statistical analysis. Performance measures in our paper are similar to those in Ladley et al. (2015), which include trading skills, trader wealth and market performance.

*Skill* measures how accurately a fundamentalist is able to price an option. In this paper, we choose the Black-Scholes (BS) value of each option in $\hat{C}$ as a benchmark. In Ladley et al. (2015) the market mid quote is used as the benchmark rather than the BS price. The authors mention that BS price is biased downwards because traders can only do imperfect hedging and hence as a benchmark, BS price is not as good as the market mid quote. In our paper, we use BS price as the benchmark for two reasons. First, our market is a centralised market where the BS price is less biased compared with that in fragmented markets.\(^8\) Also, quotes of fundamentalists might be misled due to the impacts of chartists, which makes the bias of BS price small compared to the bias of fundamentalists’ quotes. For fundamentalist $i$, we apply his pricing function to each of the 27 options and count the options that his

---

\(^8\)See Ladley et al. (2015) for more details
quote lies within a 10 percent interval around the BS price of that option. The number \( k_i \in \{0, 1, ..., 27\} \) gives a measure of trader \( i \)'s skill and \( \sum_{i=1}^{I_f} k_i/I_f \) gives the overall trading skill that fundamentalists acquire as a group in the market (we call it market skill for simplicity). We observe one value of market skill for each of the 5 population compositions. In 30 model runs, we get \( 30 \times 5 = 150 \) observations in total.

**Trader pricing error** measures the derivation of a fundamentalist’s quote from the BS price of an option. For each option in \( \hat{C} \), we calculate the absolute difference between fundamentalist \( i \)'s quote and the BS price of that option. The resulting value \( TPE_i \) gives the individual trader pricing error and \( \sum_{i=1}^{I_f} TPE_i/I_f \) is the average trader pricing error. Hence, in the market with 5 population compositions, 27 options and 30 model runs, we get \( 27 \times 5 \times 30 = 4050 \) observations in total.

**Market pricing error** measures the distance between transaction price and the BS price. For each option in \( \hat{C} \), we compute the absolute difference between the transaction price and the BS price. Hence, we get 27 observations in the market with each of the 5 population compositions and from the overall 30 model runs, we obtain \( 27 \times 5 \times 30 = 4050 \) observations in total.

Trader pricing error and market pricing error play important roles in explaining our results, which will be seen later. Hence, we spend some time discussing the difference between the two. Trader pricing error measures pricing errors at individual level and is easily affected by extreme quotes. The more fundamentalists with quotes close to the BS value, the less extreme quotes and the smaller the trader pricing error. Hence, trader pricing error better describes the overall learning of fundamentalists. A smaller trader pricing error indicates better overall learning.

Market pricing error measures pricing errors at the market level. The more accurate the learning outcome, the closer the transaction price to the fundamental value and the lower the market pricing error. Hence, market pricing error better describes the accuracy of the learning outcome. A smaller market pricing error indicates more accurate learning.

**Trader wealth** measures the links between trader skill and trader wealth. Data is collected in the following way. For each population composition, after the model has evolved, we wipe out the wealth that fundamentalists have obtained from previous trading and ask them to trade another 1000 rounds with their evolved pricing functions. That is, in the 1000 trading rounds, fundamentalists start with zero initial wealth again and do not learn (tournament process turned off). After that, we record fundamentalists’ skill level and wealth to examine if skills generate more wealth for fundamentalists. Each fundamentalist’s skill level lies within \( \{0, 1, 2, ..., 27\} \), 28 levels in total. For each population composition, we record the skill levels that at least

---

9Extreme quotes are quotes far from the BS price of an option. For example, quotes lower than 0 or higher than 40 can be seen as extreme quotes for the options considered in \( C \).
one fundamentalist is at. Then we calculate the average wealth of fundamentalists at each recorded skill level. Hence, for each skill level, we have the average wealth it generates. After the above process, from 30 model runs and 5 population compositions, we get 1541 observations in total.

**Price volatility** measures the time-series variations of transaction price. With each population composition, we run the converged model for another 10,000 trading rounds with tournaments taking place. For every 10 trading rounds, we apply fundamentalists’ pricing functions to the 27 options, calculating the transaction price. For each option in $\hat{C}$, we get a sample with 1000 observations of transaction price. Price volatility is therefore measured by the standard deviation of that sample. Hence, with 5 population compositions, 27 options and 30 model runs, we get $5 \times 27 \times 30 = 4050$ observations in total.

**Price sensitivity** measures the market resilience in the case of shocks. Shocks happen in the form of the entry of new traders with extreme quotes 0 and 40. We use the measure designed in Ladley et al. (2015), that is

$$\text{price sensitivity} = \frac{P(J) - P(-J)}{P(0)} / J$$

where $J \in \{10\%, 20\%, 40\%, 80\%\}$ measures the size of a shock. For instance, if $J = 20\%$, there will be $I \times J$ new traders enter the market with extreme quotes. The larger $J$, the more extreme quotes and the larger the shock is. $P(0)$ is the transaction price without any shock. $P(J)$ is the transaction price with extreme quotes 40 and $P(-J)$ is the transaction price with extreme quotes 0. In the case of $P(J) = P(-J)$, price sensitivity is zero, which implies that the shock does not affect the market price and the market is therefore perfectly resilient. However, when $P(J) \neq P(-J)$, we observe some level of sensitivity of the market to shocks. The greater $P(J) - P(-J)$, the greater the effects of shocks on the market price and the market is hence less resilient to shocks. With 5 population compositions, 27 options in $\hat{C}$ and 4 sizes of shock, we get $5 \times 27 \times 4 = 540$ observations.

Our model converges after 0.8 million trading rounds. As mentioned before, all relevant data is from the application of the converged pricing functions of fundamentalists to a set of 27 options. Due to the fact that most measures are positively skewed, such as the trader pricing error and market pricing error, we log transform these measures to make them less skewed. Also, some measures are bounded in value, for instance, skill level is bounded between 0 and 27. We take logit of these measures to make them unbounded.
3.5 Results

In this section, we illustrate our results. In particular, we show the results and analysis in the effects of chartists on pricing errors, skills and wealth and market performance.

Figure 3.1 shows the histogram of the evolved quotes of fundamentalists in pricing one of the 27 options in set $\hat{C}$ with the Black-Scholes price 3.3959. Two population compositions are considered: (1) all traders are fundamentalists (black line); (2) 60% of the population are fundamentalists and 40% are chartists (grey line). We observe that generally, pricing quotes are concentrated in three clusters: the middle cluster around the Black-Scholes price and the two clusters at the extremes (0 and over 15). The middle cluster is the result of learning. One reason for the large amount of quotes at the two extremes is that some fundamentalists are lucky enough to make positive profits by quoting very high or very low quotes, in which case, these fundamentalists free ride other fundamentalists who are less lucky and have to learn to price the options more accurately.

From Figure 3.1, we can see that as the amount of chartists increases, the middle cluster becomes taller and fatter and the two clusters at the extremes become shorter. In addition, there are less quotes between each extreme and the BS price as chartists. This is more obvious for quotes lower than the middle cluster (for quotes lower than 2.5, most of the grey line lies below the black line). At the same time, the middle cluster becomes more dispersed with chartists. Hence, increasing chartists and decreasing fundamentalists can reduce the quotes at the extremes and the quotes between each extreme and the BS price, pushing quotes towards the BS price. However, it also makes the evolved quotes in the middle cluster more dispersed.

Since the total population is fixed, an increase in chartists is equivalent to an equal decrease in fundamentalists. In the following context, unless otherwise stated, the amount of chartists is represented by the amount of fundamentalists (denoted by Flists in all regression results).

3.5.1 Pricing errors

We first consider the relationship between chartists and pricing errors. As mentioned above, we use two measures of pricing errors: trader pricing error and market pricing error. Trader pricing error is the absolute difference between individual trader quotes and the Black-Scholes (BS) price while market pricing error is the absolute

\[\text{As mentioned above, the Black-Scholes price of the 27 options in } \hat{C} \text{ ranges between 1 and 9. To better illustrate the concentration of fundamentalists' evolved quotes, we classify all quotes into three categories: quotes lower than 0, quotes between 0 and 15 and quotes higher than 15. Hence, in Figure 3.1, 0 and 15 are the two extremes in the graphs.}\]
Figure 3.1: Quoted price

Note: F:C is the proportion of fundamentalists to chartists. The top graph shows traders’ quoted prices for the option \((K, r, \sigma) = (105, 0.06, 0.35)\) in two market structures: F:C = 100:0 (black line) and F:C = 60:40 (grey line). The vertical line gives the Black-Scholes price of the option, which in this example is 3.3959. Frequency is calculated from the quotes collected from the 30 model runs. The bottom graph gives the details of the middle cluster of the top graph.
Table 3.2: Pricing Errors

<table>
<thead>
<tr>
<th>Dependent variable: $\log(\text{Pricing error})$</th>
<th>Trader pricing error (Model 1)</th>
<th>Market pricing error (Model 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.227 (0.220)</td>
<td>3.943*** (0.492)</td>
</tr>
<tr>
<td>$\log(\text{Flists})$</td>
<td>0.134** (0.050)</td>
<td>-1.202*** (0.112)</td>
</tr>
<tr>
<td>$\log(\text{BS price})$</td>
<td>0.239*** (0.016)</td>
<td>-0.019 (0.036)</td>
</tr>
<tr>
<td>Observations</td>
<td>4050</td>
<td>4050</td>
</tr>
<tr>
<td>F-statistics</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: Pricing errors are the absolute difference between pricing quotes and the Black-Scholes price. Trader pricing error (Model 1) and market pricing error (Model 2) are examined. Robust standard errors are in parentheses. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

difference between transaction price and the BS price. From Figure 3.1, we observe that as chartists increase, the middle cluster gets bigger and the other two clusters at extremes get smaller, which implies that as there are more chartists, more fundamentalists will quote around the BS price and less will quote very high or very low prices. Since trader pricing error is more easily affected by extreme quotes, we expect to see that trader pricing error decreases with chartists. At the same time, the more dispersed middle cluster as chartists increase implies that transaction price, the median of all quotes, will be less able to reflect the fundamental value and we expect to observe a greater market pricing error.

Table 3.2 summarises our findings. The zero F-statistics shows that both of the two models have significant explanatory power. The coefficient on fundamentalists is significantly positive in Model 1 and significantly negative in Model 2, which confirms our expectation that chartists decrease trader pricing error but increase market pricing error. As mentioned above, trader pricing error better reflects the overall learning of fundamentalists and market pricing error better reflects the accuracy of the learning outcome. Hence, we can first say that chartists increase the proportion of fundamentalists holding skills (bigger middle cluster when chartists increase). Data shows that when chartists increase from 0% to 40%, the proportion of fundamentalists’ quotes lying within a 20% interval of the BS price increases from 34.83% to 44.83%, a 10% increase. Chartists also reduce the accuracy of the learning outcome (more dispersed middle cluster when chartists increase). As the measure of learning accuracy, the average market pricing error over the 27 options and 30 model runs increases from 0.33578 in the case of all fundamentalists to 0.86380 in the case of 60% fundamentalists and 40% chartists, implying a significant reduction
in the learning accuracy.

Intuitively, learning has costs. One cost comes from the time fundamentalists spend on learning. Another cost would be the possibility that after learning, fundamentalists get worse in their wealth. The higher the cost, the more reluctant fundamentalists are to learn. When fundamentalists cannot accurately price assets and learn to do so, the uncertainties from the evolving of their pricing functions may make transaction price volatile, increasing the costs of learning and discouraging learning. However, the trading rules followed by chartists, which are a weighted average of past prices in our model, can help stabilise market price. This lowers the costs of learning and hence facilitates fundamentalists to learn. Therefore, in equilibrium, as chartists increase, we observe larger proportion of fundamentalists quoting around the BS price. They give less extreme quotes and as a result trader pricing error becomes smaller. However, chartists always add noises to the market price, which misleads learning and reduces the accuracy of the learning outcome. The more chartists, the greater the misleading effects and the further the market price will deviate from the BS price. This explains why market pricing error increases with chartists.

In addition, the positive coefficient on the Black-Scholes price in Model 1 is consistent with the one in Ladley et al. (2015) in that when options get more expensive, it becomes more difficult to estimate their values. The opposite result appears in Model 2 but is insignificant. In Appendix B.1, we do the same regression on market pricing error but only with fundamentalists. The result shows a positive coefficient on the BS price. Hence, the negative coefficient in Model 2 can be seen as an outcome of the inaccuracy brought about by chartists.

### 3.5.2 Skill and wealth

In this section, we investigate how trading skills and wealth change with chartists. Based on the result that chartists are able to promote learning, we expect to see a positive relationship between chartists and market skill, or a negative relationship between fundamentalists and market skill. Also, according to the current literature that skilled traders are rewarded excessive trading profits, we expect to observe positive effect of trading skills on wealth. Table 3.3 summarises our finding in the relationship between trading skills and chartists.

The overall model is significant at the 95% confidence level. The negative coefficient on fundamentalists is consistent with our prediction, implying that the more chartists (and hence the less fundamentalists), the more fundamentalists will learn and the higher the overall market skill. As Figure 3.1 shows when the amount of chartists increases, there are more quotes around the BS price but less quotes at
the extremes and less quotes between each extreme and the BS price. Intuitively, as mentioned in the above section, the replacement of fundamentalists’ pricing functions adds uncertainties to the market price. The more fundamentalists, the greater the uncertainties which make the market price more volatile, increase the cost of learning and hence deter learning. On the contrary, since past prices enter chartists’ pricing functions, the presence of chartists is able to reduce the uncertainties from fundamentalists and makes the market price stable. This lowers the cost of learning and hence facilitates learning. Therefore, when the amount of chartists increases, there will be more active learning among fundamentalists and we observe larger proportion of fundamentalists holding skills. In other words, trading skill acquisition increases with the amount of chartists. In addition, our finding provides evidence for the argument in the current literature that through accumulating experience, traders are able to acquire trading skills, as fundamentalists acquire more skills when there are more chartists in our model.

Table 3.4 contains the relationship between skill and wealth. In this test, two models are considered. Model 1 investigates only skill and wealth. From Model 1, we see that skill has positive effect on wealth, consistent with the fact that skilled traders can persistently earn abnormal returns in real financial markets (Coval et al., 2005; Barber et al., 2014). This effect is more significant when there are less fundamentalists and hence more chartists in the market (Model 2). This comes from the positive effect of chartists on skill acquisition from the above analysis. That is, an increase in chartist promotes learning and trading skill acquisition. Trading skills, according to the positive coefficients on skill in Table 3.4, bring higher returns from trading and hence increase wealth.
Table 3.4: Skill and Wealth

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.002***</td>
<td>1.422***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.111)</td>
</tr>
<tr>
<td>log(1+skill)</td>
<td>0.009*</td>
<td>0.010**</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>log(Flists)</td>
<td></td>
<td>−0.096***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.025)</td>
</tr>
<tr>
<td>Observations</td>
<td>1541</td>
<td>1541</td>
</tr>
<tr>
<td>F-statistics</td>
<td>0.011</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: We turn off the the tournament process, initialise traders’ wealth to zero and run the converged model for 1000 trading rounds. For each of the 30 model runs with each market structure, we record the skill levels that at least one fundamentalist is at and calculate the average wealth of the fundamentalists at the same recorded skill level. This gives 1541 observations in total. Robust standard errors are in parentheses. 

*p < 0.05; **p < 0.01; ***p < 0.001.

3.5.3 Market performance

In this section, we revisit the role of heterogeneous beliefs on market performance, especially on market stability and market resilience. This topic has been discussed greatly. A consensus is that chartists, who act as noise traders, increase market volatility while fundamentalists, who drive prices back to fundamentals, stabilise markets. However, most of the relevant papers assume fundamentalists are rational by nature in the sense that they have all the skills required to do financial analysis and make correct decisions. The innovation of our paper is that we assume fundamentalists do not have the skills to fairly price financial assets and learn to do so. Furthermore, we investigate the market performance given the learning outcome of fundamentalists when chartists are present. Hence, compared with the current literature, our paper goes deeper and is more consistent with the presence of bounded rational agents in real financial markets and provides another possible explanation for the destabilising effects of chartists on financial markets.

In the above analysis, we observe that chartists can mislead fundamentalists by affecting the market price. When there are more chartists, misleading effects get stronger and the learning outcome becomes less accurate, which results in the wide dispersion of the evolved pricing functions around the BS price. This leads transaction price, the median of all quotes, to be volatile. Hence, we expect that an increase in chartists, or a decrease in fundamentalists, will increase market volatility. Results are shown in Table 3.5. The negative coefficient on fundamentalists confirms our
Table 3.5: Market Volatility

<table>
<thead>
<tr>
<th>Dependent variable: log(volatility)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.815*** (0.111)</td>
</tr>
<tr>
<td>log(Flists)</td>
<td>−0.340*** (0.025)</td>
</tr>
<tr>
<td>log(BS price)</td>
<td>0.023*** (0.007)</td>
</tr>
<tr>
<td>Observations</td>
<td>4050</td>
</tr>
<tr>
<td>F-statistic</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Note: We run the evolved model for 10000 trading rounds, with the tournament process turned on and record the pricing functions of fundamentalists for every 10th round, which gives a sample with 1000 observations. In each model run with each market structure, we apply the recorded pricing functions to the 27 options in Ĉ, calculating the volatility of the transaction prices in that sample. Overall, there are $30 \times 5 \times 27 = 4050$ observations. Robust standard errors are in parentheses. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

This finding echoes with the common idea in the relevant literature mentioned above that chartists play a negative role in market stability. However, by focusing on the learning process of fundamentalists, we provide a new possible explanation for these adverse effects. In the learning process of fundamentalists, the presence of chartists add noises to the market price, which can mislead fundamentalists. The misleading effects provide different directions for the evolving of pricing functions and as a result, the evolved pricing functions become more diversified and more dispersed around the BS price. The diversity in pricing functions increases the volatility of transaction price and hence the market becomes less stable. In our paper, chartists affect market stability via affecting the learning process of fundamentalists, unlike the common literature argues that noise traders destabilise markets by discouraging the trade of rational traders (see DeLong et al. (1991) for example). The positive coefficient on the BS price is consistent with the argument that it is more difficult to estimate expensive options.

In terms of market resilience, we use the measure designed in Ladley et al. (2015), which is the expression in (3.9). This measure captures how transaction price changes with shocks. Shocks are described as the entry of traders with extreme quotes 0 and 40. The lower the value of the measure, the less the effects of a shock and the more resilient the market. Results are summarised in Table 3.6.

One of the key findings in this paper is that fundamentalists can make the market less resilient, or chartists can make the market more resilient, as shown by the positive coefficient on fundamentalists in Table 3.6. Skill plays an important role in explaining this result. Recall in expression (3.9), the difference between the
Table 3.6: Market Resilience

<table>
<thead>
<tr>
<th>Dependent variable: log(Resilience)</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.166***</td>
<td>(0.511)</td>
</tr>
<tr>
<td>log(Flists)</td>
<td>0.833***</td>
<td>(0.114)</td>
</tr>
<tr>
<td>log(BS price)</td>
<td>-0.749***</td>
<td>(0.042)</td>
</tr>
</tbody>
</table>

Observations 540  
F-statistic 0.000

Note: We model shocks with the entry of traders with extreme feasible quotes into the evolved market with only fundamentalists. We consider four sizes of shock $J = [0.1, 0.2, 0.4, 0.8]$ which indicates for each value of $J$, new entrants will account for a proportion $J$ of the fundamentalists. Let $P(0), P(J)$ and $P(-J)$ respectively denote the transaction prices without shocks, with entrants with extreme quote $40$ and with entrants with extreme quote $0$. The sensitivity measure is calculated as $(P(J) - P(-J))/P(0)$. In each market structure, the sensitivity measure is applied to the 27 options in C. For 4 shock sizes and 5 market structures, we obtain $27 \times 4 \times 5 = 540$ observations. Robust standard errors are in parentheses. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$.

transaction prices with the two extreme quotes. $P(J) - P(-J)$, is the key element in measuring market resilience. The larger the difference, the greater the effects of shocks on the market price. Since transaction price is the median of all quotes, before a shock, the more quotes concentrating around the BS price, that is the more skilled fundamentalists, the smaller the value of $P(J) - P(-J)$ after the shock and the less the effects of the shock on the transaction price. In other words, the middle cluster of quotes acts as a buffer for shocks. The larger the middle cluster, the better the buffer is able to absorb the effects of shocks and the more resilient the market is. For example, in the case that all fundamentalists are perfectly skilled at quoting the BS price, all quotes will be concentrated in the middle cluster. A shock has no effects on the market price, in which case, the market is perfectly resilient. In our model, chartists facilitate learning, pushing quotes to the middle cluster and increasing the proportion of skilled fundamentalists. Therefore, markets with more chartists tend to be more resilient to shocks.

3.6 Conclusion

In this paper, we analyse the role of chartists, who are effectively noise traders, in the learning of fundamentalists. We assume fundamentalists are not born with the skills to price financial assets and learn to acquire them. Generally, we find that the presence of chartists promotes fundamentalists to learn but has negative effects on the accuracy of the learning outcome. When chartists increase, larger proportion of
fundamentalists’ quotes will be close to the Black-Scholes price. The middle cluster of quotes around the Black-Scholes price acts as a buffer for shocks and hence a market with more chartists tends to be more resilient. However, the evolved pricing functions of fundamentalists are more dispersed because of the misleading effects of chartists on the transaction price, which makes the market price more volatile.

The higher volatility because of chartists in our model is consistent with the fact that during the recent two decades, we observe more volatile market prices. For instance, the large scale price decrease during the Flash Crash in 2010. The rapid development in communication technology has greatly lowered the threshold of the financial world to the general public and various types of traders flow into the financial markets, a large proportion of them being noise traders who do not have the ability to abstract useful information but use biased and fictitious information to make decisions. Their trading adds addition risks to the financial markets. On the other hand, it has made it much more convenient for traders to exchange experience and knowledge in trading, greatly increasing the learning among traders. This background makes our paper of great significance as we investigate how this trend will contribute to the learning in financial markets and the associated market performance.

From our findings, we can see that trading skills have great influences on market performance, especially the important role of accurate trading skills in keeping market stable and resilient. Noise information can reduce the acquisition of accurate trading skills and increase market volatility. Since now various types of traders are in the financial markets, for policy makers, our paper highlights the necessity of relevant training in finance for these traders, which might be the most direct and effective way for them to gain trading skills and to avoid being misled. Given the easy and convenient channel for information swap provided by the internet, another direction for policy makers would be to increase their regulatory strengths on the filter of financial information to reduce the spread of misleading information, which would increase accurate learning.

Based on the setting in this paper, some future work can be done. For instance, we would like to extend our current model to consider daily mark-to-market in the options market. This provides real time series which chartists can use to make their decisions. This extension allows us to model chartists fully by considering different types of chartists: chartists using fictitious time series and chartists using real time series. It will give us more complete insights into the role of noise traders in learning and in market performance.
Appendix A

to Chapter 1
A.1 Basic beliefs, derivative market price and profits

Basic beliefs

\[ p_0^A(1, \beta) = 0 \ast Pr(e = b|1, \alpha) + 1 \ast Pr(e = a|1, \alpha) = \frac{\sigma_a}{1 + \sigma_a} \]  \hspace{1cm} (A.1)

Prices

\[ p^A(buy, 1, \beta) = \frac{2(\lambda_S + \lambda_O)\sigma_a + \lambda_N\sigma_a}{2(\lambda_S + \lambda_O)\sigma_a + \lambda_N(\sigma_a + 1)} \]  \hspace{1cm} (A.2)

\[ p^A(sell, 1, \beta) = \frac{\lambda_N\sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1)} \]  \hspace{1cm} (A.3)

O’s profits

\[ \pi^A_a(buy, 1, \beta) = \lambda_O s \frac{\lambda_N}{2(\lambda_S + \lambda_O)\sigma_a + \lambda_N(\sigma_a + 1)} \]  \hspace{1cm} (A.4)

\[ \pi^A_b(sell, 1, \beta) = \lambda_O s \frac{\lambda_N\sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1)} \]  \hspace{1cm} (A.5)

A.2 Proof of Proposition 1.1

Proof. Assume \( \pi^A_{e\sigma} = 0 \) for \( e \in \{a, b\} \) and \( \sigma = 1 \). Let \( \pi^A_{e\sigma} \) denote the originator’s interim profits in the primary market in each of the four \( \{0, 1\} \times \{a, b\} \) cases. If both \( (PCa1) \) and \( (PCa0) \) are satisfied, trade in the primary market always occurs and the originator’s ex-ante profits (before he observes \( e \in \{a, b\} \)) in the primary market, denoted by \( \Pi^A \), is

\[ \Pi^A = Pr(e = a, \sigma = 0)\pi^A_{e\sigma} + Pr(e = a, \sigma = 1)\pi^A_{e\sigma} + Pr(e = b, \sigma = 0)\pi^A_{b\sigma} + Pr(e = b, \sigma = 1)\pi^A_{b\sigma} = k \]  \hspace{1cm} (A.6)

which is independent of opacity \( \sigma_a \). In other words, any \( \sigma_a \) gives the same ex-ante profits in the primary market. Hence, opacity is immaterial when both \( (PCa1) \) and \( (PCb1) \) hold. However, when \( (PCb1) \) does not hold, the originator will not trade \( A \) in the case \( (e = b, \sigma = 1) \). Hence, as long as trade occurs in the primary market,
the buyer will know his type is $a$. Thus, $\pi^A_{a1} = k$ and $\pi^A_{b1} = 0$. In this case

$$\Pi^A = \frac{1}{2} k \leq k$$  \hspace{1cm} (A.7)

Also, with full transparency $\sigma_a = 0$, $(PCb1)$ always holds. Therefore, the best strategy of the originator when he makes zeros profits in the derivative market is to eliminate adverse selection by making security $A$ fully transparent.

\[\square\]

A.3 Proof of Proposition 1.2

\textit{Proof.} From the proof of Proposition 1.1, we already know when there is no adverse selection (full transparency) in the primary market, trade always occurs in the primary market and $O$‘s ex-ante profits in the primary market are $\Pi^A_{e\sigma} = k$, independent of opacity $\sigma_a$. Hence, if we can show $O$‘s ex-ante profits in the derivative market increase with opacity, we can prove Proposition 1.2. Let $\Pi^A$ denote $O$‘s ex-ante profits in the derivative market.

$$\Pi^A = Pr(e = a, \sigma = 0)\pi^A_{a0} + Pr(e = a, \sigma = 1)\pi^A_{a1}$$

$$+ Pr(e = b, \sigma = 0)\pi^A_{b0} + Pr(e = b, \sigma = 1)\pi^A_{b1}$$

$$= \frac{1}{2} \left[ \frac{\lambda Os\lambda N\sigma_a}{(2(\lambda S + \lambda O)\sigma_a + \lambda N(\sigma_a + 1))} + \frac{\lambda Os\lambda N\sigma_a}{2(\lambda S + \lambda O) + \lambda N(\sigma_a + 1)} \right]$$

Taking first order derivative of $\Pi^A$ with respect to $\sigma_a$, we get

$$\frac{\partial \Pi^A}{\partial \sigma_a} = \frac{\lambda N}{2} \left[ \frac{\lambda N}{(2(\lambda S + \lambda O)\sigma_a + \lambda N(\sigma_a + 1))^2} + \frac{\lambda N}{2(\lambda S + \lambda O) + \lambda N(\sigma_a + 1))^2} \right] > 0$$

Hence, $O$‘s ex-ante profits in the derivative market increase with $\sigma_a$ for $\sigma_a \in [0, 1]$ and thus, $O$ gets maximum profits with full opacity. Therefore, when there is no adverse selection in the primary market, $O$‘s best strategy is to make $A$ fully opaque. \[\square\]

A.4 Proof of Lemma 1.1

\textit{Proof.} From the proof of Proposition 1.2, we have already known that ignoring participation constraints, full opacity offers $O$ the maximum ex-ante profits. Hence, for full opacity to be the optimum, we just need to find out the condition under which $(PCb1)$ holds with full opacity. Recall $(PCb1)$

$$k - \frac{(1 - \gamma)\sigma_a}{1 + \sigma_a} + \pi^A_{b1} \geq 0$$  \hspace{1cm} (A.10)
Setting $\sigma_a = 1$ and rearranging $(PCb1)$, we get

$$k + \pi_{b1}^A \geq \frac{1}{2} - \gamma$$

(A.11)

Hence, as long as $k + \pi_{b1}^A > \frac{1}{2} - \gamma$, $(PCb1)$ is slack and full opacity is the optimum. Now let us show the existence of the unique optimal opacity when (1.21) fails to hold. We define

$$G(\sigma_a) \equiv k - \frac{(1 - 2\gamma)\sigma_a}{1 + \sigma_a} + \frac{\lambda_O s \lambda_N \sigma_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1)}$$

(A.12)

which can be rearranged as

$$G(\sigma_a) \equiv \frac{D(\sigma_a)}{(1 + \sigma_a)(2(\lambda_S + \lambda_O) + \lambda_N(\sigma_a + 1))}$$

(A.13)

where

$$D(\sigma_a) = \xi_1 \sigma_a^2 + \xi_2 \sigma_a + \xi_3$$

(A.14)

and

$$\xi_1 = \lambda_N \lambda_O s - \lambda_N(1 - 2\pi - k)$$

(A.15)

$$\xi_2 = 2k(\lambda_S + \lambda_O + \lambda_N) - (1 - 2\gamma)(2\lambda_S + 2\lambda_O + \lambda_N) + \lambda_N \lambda_O s$$

(A.16)

$$\xi_3 = k(2\lambda_S + 2\lambda_O + \lambda_N)$$

(A.17)

$G(\sigma_a) = 0$ is equivalent to $D(\sigma_a) = 0$. From (A.14), obviously, $D(\sigma_a)$ is a quadratic function. In addition, we know that $D(0) = k \geq 0$ and $D(1) < 0$ when (1.21) fails to hold. Thus, for $\sigma_a \in (0, 1)$, $D(\sigma_a)$ must cross the x-axis only once. That is, when (1.21) is not satisfied, there exists one unique solution to $G(\sigma_a) = 0$ which gives the optimal opacity.

A.5 Proof of Proposition 1.3

Proof. We have already shown that if (1.21) holds, full opacity is the optimum in the proof of Lemma 1.1. Also, Lemma 1.1 shows if (1.21) fails to hold, optimal opacity is given by the binding $(PCb1)$, which gives $\sigma_a = \sigma_a^*, \sigma_b = 1$. Now, we show that $\sigma_b = 1$ in our setting is the optimum. The setting in our model that the state $(e = b, \sigma = 0)$ never occurs (due to $\sigma_b = 1$) is equivalent to the case where

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\(e = b, \sigma = 0\) occurs with positive probability but the loss in the primary market is larger than the gain from the derivative market, in which case \(O\) does not trade and hence reveals the type of the underlying asset. Conditional on this, for opacity given by \((\sigma_a, \sigma_b)\), \(O\)'s ex-ante overall profits are

\[
\Pi^A = \frac{1}{2} k (1 + \sigma_b) + \frac{1}{2} \left( \frac{\lambda_O s \lambda_N \sigma_a \sigma_b}{2 \sigma_a (\lambda_O + \lambda_S) + \lambda_N (\sigma_a + \sigma_b)} + \frac{\lambda_O s \lambda_N \sigma_a \sigma_b}{2 \sigma_b (\lambda_O + \lambda_S) + \lambda_N (\sigma_a + \sigma_b)} \right)
\]  

(A.18)

Suppose opacity satisfies \(\sigma_a / \sigma_b = r\) where \(r \in [0, 1]\). Then, (A.18) can be rewritten as

\[
\Pi^A = \frac{1}{2} k (1 + \sigma_b) + \frac{1}{2} \left( \frac{\lambda_O s \lambda_N r \sigma_b}{2 r (\lambda_O + \lambda_S) + \lambda_N (r + 1)} + \frac{\lambda_O s \lambda_N r \sigma_b}{2 (\lambda_O + \lambda_S) + \lambda_N (r + 1)} \right)
\]  

(A.19)

Taking first order condition of \(\Pi^A\) with respect to \(\sigma_b\), we obtain

\[
\frac{d\Pi^A}{d\sigma_b} = \frac{1}{2} k + \frac{1}{2} \left( \frac{\lambda_O s \lambda_N r}{2 r (\lambda_O + \lambda_S) + \lambda_N (r + 1)} + \frac{\lambda_O s \lambda_N r}{2 (\lambda_O + \lambda_S) + \lambda_N (r + 1)} \right) > 0 \quad (A.20)
\]

Thus, \(O\)'s ex-ante overall profits increase with \(\sigma_b\), implying \(\sigma_b = 1\) is the optimum.

\[\square\]

A.6 Proof of Lemma 1.2

Proof.

\[
\tilde{\pi}^A_{b1}(\sigma_a) = Pr(\omega = \alpha) [1 - p^A(buy, 1, \alpha)] + Pr(\omega = \beta) p^A(sell, 1, \beta) \quad (A.21)
\]

\[
= \frac{\lambda_O s \lambda_N \sigma_a}{2 (\lambda_S + \lambda_O) + \lambda_N (\sigma_a + 1)}
\]

Since \(\lambda_S + \lambda_O + \lambda_N = 1\), we replace \(\lambda_S + \lambda_O\) in (A.21) with \(1 - \lambda_N\), which gives

\[
\tilde{\pi}^A_{b1}(\sigma_a) = \frac{\lambda_O s \lambda_N \sigma_a}{2 (1 - \lambda_N) + \lambda_N (\sigma_a + 1)} \quad (A.22)
\]

Taking first order condition of \(\tilde{\pi}^A_{b1}\) with respect to \(s\) and \(\lambda_N\), respectively, we can see that

\[
\frac{\partial \tilde{\pi}^A_{b1}}{\partial s} = \frac{\lambda_O \lambda_N \sigma_a}{2 (1 - \lambda_N) + \lambda_N (\sigma_a + 1)} > 0 \quad (A.23)
\]
\[
\frac{\partial \pi^A_{s1}}{\partial \lambda_N} = \frac{\lambda_O s \sigma_a (2 + \lambda_N)}{[2(1 - \lambda_N) + \lambda_N (1 + \sigma_a)]^2} > 0 \tag{A.24}
\]

which implies \(O\)'s profits in the case \((e = b, \sigma = 1)\) increase with liquidity measured by \(s\) and \(\lambda_N\).

\section*{A.7 Proof of Proposition 1.4}

\textit{Proof.} Suppose now (1.21) does not hold. Hence, full opacity is not the optimum and optimal opacity is given by the binding \((PCb1)\), that is \(G(\sigma_a) = 0\).

Suppose \(\sigma^*_a\) solves \(G(\sigma_a) = 0\), then \(G(\sigma^*_a) = 0\) always holds in equilibrium. Taking first order condition of \(G(\sigma^*_a)\) with respect to \(\lambda_N\), the overall effects should be zero in equilibrium, that is,

\[
\frac{\partial G(\sigma^*_a)}{\partial \lambda_N} = \frac{\partial G(\sigma_a)}{\partial \sigma_a} \frac{\partial \sigma^*_a}{\partial \lambda_N} + \frac{\partial G(\sigma_a)}{\partial \lambda_N} = 0 \tag{A.25}
\]

which can be re-written as

\[
\frac{\partial \sigma^*_a}{\partial \lambda_N} = -\frac{\partial G(\sigma_a)}{\partial \sigma_a} \frac{\partial G(\sigma_a)}{\partial \lambda_N} \tag{A.26}
\]

From the proof of Lemma 1.2, we know that \(\frac{\partial G(\sigma_a)}{\partial \lambda_N} > 0\). As Figure 1.1 shows, \(\sigma^*_a\) is achieved when \(G(\sigma_a)\) is decreasing. Hence for \(\sigma_a \in [\sigma^*_a - \epsilon, \sigma^*_a]\), we have

\[
\frac{\partial G(\sigma_a)}{\partial \sigma_a} < 0 \tag{A.27}
\]

where \(\epsilon\) is a small interval on the left-hand-side of \(\sigma^*_a\). Therefore

\[
\frac{\partial \sigma^*_a}{\partial \lambda_N} > 0 \tag{A.28}
\]

Proofs of Propositions 1.5 and 1.6 follow the same logic. \(\square\)

\section*{A.8 Proof of Remark 1.2}

\textit{Proof.} Let \(\Pi\) denote \(O\)'s ex-ante overall profit in the endogenous case.

\[
\Pi = k + \frac{\lambda_O s}{2} \frac{\lambda_N \sigma_a}{2 \lambda_S(\sigma_a) + 2 \lambda_O \sigma_a + \lambda_N \sigma_a + \lambda_N} + \frac{\lambda_N \sigma_a}{2 \lambda_S(\sigma_a) + 2 \lambda_O + \lambda_N \sigma_a + \lambda_N} \tag{A.29}
\]
Taking the derivative of $\Pi$ with respect to $\sigma_a$, we have

$$\frac{\partial \Pi}{\partial \sigma_a} = \frac{\lambda_O s}{2} \left( \frac{-2\lambda'_S(\sigma_a)\sigma_a^2 + \lambda_N}{(2\lambda_S(\sigma_a) + 2\lambda_O + \lambda_N(\sigma_a + \lambda_N))^2} + \frac{-2\lambda'_S(\sigma_a)\sigma_a + \lambda_N(2\lambda_S(\sigma_a) + 2\lambda_O + \lambda_N)}{(2\lambda_O + 2\lambda_S(\sigma_a) + \lambda_N(\sigma_a + \lambda_N))^2} \right) > 0 \quad (A.30)$$

given $\lambda'_S(\sigma_a) < 0$, which implies that $O$’s ex-ante overall profits increase with opacity. Hence, all roots of (1.24) within $[0 1]$ satisfies the binding participation constraint (PCb1). However, at the largest root, $O$ gets maximum profits. Therefore, the optimal opacity in the endogenous case is the largest root of (1.24). □

A.9 Proof of Proposition 1.7

Proof. After a change in liquidity, only the direct effect on opacity appears in the exogenous case but both the direct and indirect effects appear in the endogenous case. In this proof, we show that the overall effect, consisting of the direct and indirect effects, is greater than the direct effect.

Suppose in the exogenous case, $\lambda_S = \lambda^0_S$ and in both the exogenous and endogenous cases $\lambda_N = \lambda^0_N,\lambda_O + \lambda^0_S + \lambda^0_N = 1$. Also, suppose initially $\sigma^0_a$ and $\hat{\sigma}^0_a$ solve $G(\sigma_a) = 0$ and $G(\sigma_a, \lambda_S(\sigma_a)) = 0$, respectively:

$$k - \frac{(1 - 2\gamma)\sigma^0_a}{1 + \sigma^0_a} + \frac{\lambda_O s \lambda_N \sigma^0_a}{2(\lambda_S + \lambda_O) + \lambda_N(\sigma^0_a + 1)} = 0 \quad (A.31)$$

and

$$k - \frac{(1 - 2\gamma)\hat{\sigma}^0_a}{1 + \hat{\sigma}^0_a} + \frac{\lambda_O s \lambda_N \hat{\sigma}^0_a}{2(\lambda_S(\hat{\sigma}^0_a) + \lambda_O) + \lambda_N(\hat{\sigma}^0_a + 1)} = 0 \quad (A.32)$$

$\hat{\sigma}^0_a$ satisfies $\lambda_S(\hat{\sigma}^0_a) = \lambda^0_S$. Obviously enough, the left-hand-side of (A.31) and (A.32) are the same and hence $\sigma^0_a = \hat{\sigma}^0_a$.

We first assume $\lambda_S(\sigma_a)$ does not change with opacity. Now suppose $\lambda_N$ increases from $\lambda^0_N$ to $\lambda^1_N$, keeping $\lambda_O$ constant. So $\lambda_S$ and $\lambda_S(\sigma_a)$ decrease from $\lambda^0_S$ to $\lambda^1_S$ such that $\lambda_O + \lambda^1_S + \lambda^1_N = 1$. After these changes, the left-hand-side of (A.31) and (A.32) are still the same. Suppose $\sigma^1_a$ and $\hat{\sigma}^1_a$ are the new optimal opacity in the exogenous and endogenous case, respectively. Then, $\sigma^1_a = \hat{\sigma}^1_a$ and

$$G(\sigma^1_a) = G(\hat{\sigma}^1_a, \lambda^1_S) = 0 \quad (A.33)$$

Proposition 1.4 implies $\sigma^1_a > \sigma^0_a$ and $\hat{\sigma}^1_a > \hat{\sigma}^0_a$. This is the direct effect of liquidity on opacity which results in an increase in opacity.

Now, we allow $\lambda_S(\sigma_a)$ to change with opacity to see the indirect effect. $\lambda'_S(\sigma_a) < 0$
implies that as opacity increases from $\hat{\sigma}_a^0$ to $\hat{\sigma}_a^1$, there must be a decrease in $\lambda_S(\sigma_a)$. Taking this into consideration, at $\hat{\sigma}_a^1$, $\lambda_S(\hat{\sigma}_a^1)$ must below $\lambda_S^1$ and

$$G(\hat{\sigma}_a^1, \lambda_S(\hat{\sigma}_a^1)) > G(\hat{\sigma}_a^1, \lambda_S^1) = 0 \quad (A.34)$$

Optimal opacity hence need to rise further from $\hat{\sigma}_a^1$, which triggers a new round of change in liquidity and opacity due to $\lambda'_S(\sigma_a) < 0$. This self-reinforcing process ends when $G(\sigma_a, \lambda_S(\sigma_a))$ binds again. That is to say, the direct effect induces a one-shot increase in opacity and the indirect effect induces more increases in opacity. Hence, the overall effect, comprised of the direct and indirect effects, is greater than the direct effect.

\section*{A.10 A Microfoundation of $\lambda'_S(\sigma_a) < 0$}

Once speculators become informed, they are the same with the originator except that they incur some costs in information acquisition. Overall, we assume speculators’ costs of becoming informed increase with opacity. As mentioned before, this assumption is plausible in the sense that as opacity going up, speculators have to spend more time, learn more skills and seek more resources to collect and process information. Speculators are heterogeneous in their cost functions. This can be understood as some speculators are more skilled in information acquisition than others, which implies these skilled speculators incur less costs in process information than the unskilled ones. For speculator $i$, we assume his cost function is

$$C_i(\sigma_a) = \kappa \sigma_a - \theta_i \quad (A.35)$$

where $\kappa$ measures the marginal effects of opacity $\sigma_a$ on the costs and $\theta_i$ measures speculator $i$’s skills in information acquisition. From the above cost function, we can see that costs of information acquisition increase with opacity $\sigma_a$ while decrease with the trader’s skills. We assume $\theta_i \sim U[\underline{\theta}, \bar{\theta}]$, $\Delta \theta \equiv \bar{\theta} - \underline{\theta}$. Hence $C_i \sim U[C, \overline{C}]$ where $\underline{C} = \kappa \sigma_a - \bar{\theta}$ and $\overline{C} = \kappa \sigma_a - \underline{\theta}$. When opacity is relatively small, it could be the case that speculator $i$’s costs are negative. The interpretation for this scenario is that this trader’s skills are advanced relative to the level of opacity. This makes information acquisition less of an issue, which makes him save some skills in information acquisition and also brings him some sense of achievements, adding (non-monetary) benefits to his trading in the market.

Speculators themselves know their skills while others only know the distribution of $\theta_i$. Trading activities of speculators are the same as the originator. Let $R$ denote the revenue and $\Pi$ denote the ex-ante net profits of speculators in the derivative
market, then

\[ \Pi(\sigma_a) = R - C_i(\sigma_a) \]  \hspace{1cm} (A.36)

where from the proof of Proposition 1.2, we can easily obtain the expression of the revenue of speculators.

\[ R(\sigma_a) = \frac{\lambda_S(\sigma_a)s\lambda_N\sigma_a}{2(2\lambda_S(\sigma_a) + \lambda_O)\sigma_a + \lambda_N(\sigma_a + 1)} \]  \hspace{1cm} (A.37)

For speculators, if they can make positive profits by collecting information, they will do so and trade in the market, otherwise, they quit the market. In other words, as long as \( R(\sigma_a) > C_i(\sigma_a) \), speculator \( i \) will remain in the market and when \( R(\sigma_a) = C_i(\sigma_a) \), he will be indifferent between collecting and not collecting information. Suppose \( C^* = \sigma_a - \theta^* \) solves \( R(\sigma_a) = C_i(\sigma_a) \). Then traders with skills lower than \( \theta^* \) will quit the market. The higher \( \theta^* \), the more speculators quit. Hence, we assume the probability \( \lambda_S(\sigma_a) \) satisfies

\[ \lambda_S(\sigma_a) = \phi \frac{C^* - C}{\overline{C} - C} \]  \hspace{1cm} (A.38)

where \( \phi > 0 \) is a multiplier. Since \( C^* \) solves \( R(\sigma_a) = C_i(\sigma_a) \), \( C^* = R(\sigma_a) \). Also \( \overline{C} - C = \Delta \theta \). Thus, (A.38) can be re-written as

\[ \lambda_S(\sigma_a) = \phi \frac{R(\sigma_a) - (\kappa \sigma_a - \bar{\theta})}{\Delta \theta} \]  \hspace{1cm} (A.39)

Taking first order condition of (A.39) with respect to \( \sigma_a \) on both sides, we obtain

\[ \frac{d\lambda_S(\sigma_a)}{d\sigma_a} = \phi \frac{dR(\sigma_a)}{d\sigma_a} - \kappa \frac{\Delta \theta}{\Delta \theta} \]  \hspace{1cm} (A.40)

Since the cost function intersects with the revenue function at \( C^* \), we must have that the slope of the cost function greater than that of the revenue function at the intersection point, that is

\[ \frac{dR(\sigma_a)}{d\sigma_a} - \kappa < 0 \]  \hspace{1cm} (A.41)

which implies \( \frac{d\lambda_S(\sigma_a)}{d\sigma_a} < 0 \).
Appendix B

to Chapter 3
Table B.1: Market Pricing Error

<table>
<thead>
<tr>
<th>Dependent variable: log(Market pricing error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>log(BS price)</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>F-statistics</td>
</tr>
</tbody>
</table>

Note: Market pricing error is the absolute difference between transaction price and the Black-Scholes price. Robust standard errors are in parentheses. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$. 

Table B.2: Convergence Test

<table>
<thead>
<tr>
<th>Dependent variable: log(Market pricing error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
</tr>
<tr>
<td>log(Fundamentalists)</td>
</tr>
<tr>
<td>log(Black-Scholes price)</td>
</tr>
<tr>
<td>dum</td>
</tr>
<tr>
<td>dum*log(Fundamentalists)</td>
</tr>
<tr>
<td>dum*log(Black-Scholes price)</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Prob &gt; F</td>
</tr>
</tbody>
</table>

Test of structural break between sample periods

$H0$: dum $\times X = 0$ for all ordinary regressors $X$

F-statistic: 0.134 on 3 and 264 DF

Note: We run the evolved model for 0.84 million trading rounds and collect data from the first and last 0.1 million trading rounds for every 10th round, which gives two samples, each with 1000 observations. All observations in each sample are applied to the 27 options in set $\hat{C}$. We calculate the average market pricing error over each sample, which is the dependent variable in this table. We add a dummy variable dum to represent the second sample period. Cross-products of the dummy variable and other regressors are also used as explanatory variables. An F-test is applied to test the convergence of the model with the null hypothesis that all explanatory variables involving the dummy variables are insignificant. Robust standard errors are in parentheses. *$p < 0.05$; **$p < 0.01$; ***$p < 0.001$. 


B.1 Market pricing error without chartists

In this section, we investigate the relationship between market pricing error and the BS price with only fundamentalists to avoid the potential misleading effects from chartists. Regression results are summarised in Table B.1.

From Table B.1, we observe a positive coefficient on the BS price, which means market pricing error is larger for expensive options. This is consistent with the finding in Ladley et al. (2015) that expensive options are more difficult to evaluate. While the coefficient is positive, it is insignificant. We believe the significantly positive coefficient in Ladley et al. (2015) is attributed to the fragmentation of the markets, which greatly facilitates learning while in our paper centralised markets are considered.

B.2 Convergence

Convergence of our model is tested with the data of market pricing error across two different sample periods. From each sample period, we collect 1000 observations, one from each 10 trading rounds. The two samples are 0.2 million trading rounds apart. Based on our analysis in section 3.5.1, we add a dummy variable dum representing the second sample period, and the products of the dummy variable and other explanatory variables are also used as explanatory variables. Results of the convergence test are summarised in Table B.2.

From Table B.2, we can see that the dummy variable and its cross-products with other regressors are insignificant, implying that the market pricing errors in the two different sample periods are not significantly different, which means our model has converged after 0.8 million trading rounds.
Bibliography


