Essays on Coordination Games

Thesis submitted for the degree of
Doctor of Philosophy
at the University of Leicester

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2019

Declaration

I, Zahra Gambarova, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

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Abstract

This thesis consists of three chapters on coordination games.

The first chapter studies how the existence of Imitators in population affects the success of the threshold public good production. Threshold models require at least m out of n players to form a team to produce a public good. We model the dynamics of contribution decisions when the population consists of Best-responders and Imitators. We show that the existence of Imitators improves the feasibility of production only when Imitators are willing to join the team in the early stages. The production even when achieved is not necessarily optimal. When the number of Imitators is high, contributions exceed the efficient level.

In the second chapter, we test how subjects in lab divide attention between public and private information when they need to coordinate (or anti-coordinate) their actions on the unobserved state. Subjects are provided with one public and one private piece of information about the state in the form of noisy signals and asked to assign a weight to each information type. The public signal is perfectly correlated, whereas private signals are independent or partially correlated. We find that subjects weigh private signal less when they need to coordinate. However, they do not increase the weight in the anti-coordination game. Even though subjects do not play the optimal strategy in the anti-coordination game, they react to correlation in private signals as predicted by theory. Correlated signals attract higher weight in coordination, and lower weight in the anti-coordination game, relative to the independent private signals.

The third chapter is a model of information supply to players who want to coordinate their actions on the unobserved state. A profit-maximizing monopolist designs a discriminating mechanism to extract higher surplus from the buyers with different valuations for the information. We show that the buyer with the highest valuation always receives a fully informative signal. The quality of the signal offered to the low valuation buyer is a continuous and increasing function of the frequency of low types. Correlation in actions causes externalities in prices; each type is also willing to pay for the precision of the signal received by their partner. When there is a high probability of meeting the low type, accuracy of low type's decision increases the high type's payoffs. Hence a low type receives a positive amount of information.

Acknowledgements

First of all, I thank Prof. Chris Wallace, the person who showed me how to research, teach, and most importantly, how to think. Thanks for not giving up, and encouraging me when I doubted myself.

Thanks to my supervisors Dr. Subir Bose and Dr. Piercarlo Zanchettin, who helped me to complete this thesis. Thanks for your patience and guidance in crucial times, and for teaching how to be critical about my work and how to present it.

I want to express my sincere gratitude to Prof. John Hey, who provided access to the facilities of EXEC at the University of York and helped with all arrangements to run my experiment.

Thanks to my friends who helped to run the experiment, proofread instructions, test the program and gave comments, to Hamideh, Nikitha, Valeria, Mehman, Akbar, Yongli, Dr. Martin Hoskins, and many others.

I have also received helpful comments on different chapters in this thesis from Tomasz Strzalecki, Heinrich Nax, Alex Rigos, and Egor Starkov among others.

Thanks to Eric Livny and all professors at ISET, who introduced me to economics.

Thanks to my students at UoL and UCL who taught me more than learnt from me.

Thanks to Junaid and Jingyi, for not only being friends but also parenting us throughout PhD.

Thanks to Scott and Peter for our philosophy reading group, for all interesting discussions on music and art, and for all good times spent together.

I am grateful to my family for their support and unconditional love. I dedicate this thesis to my father who always wanted me to become a Doctor of Philosophy.

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CHAPTER 1

Stochastic Evolutionary Dynamics of Public Goods Provision by Selfish and Reciprocal Types

Abstract. Experimental evidence suggests that some subjects hold reciprocal, non-selfish preferences in public good games. This paper studies the dynamics of threshold public good production when the population consists of both selfish and reciprocal types. Threshold production models require at least m out of n players to join a team to produce a good which benefits everybody. With selfish players production takes place only if it is privately feasible; it is optimal for one player to pay the cost alone. We show that the existence of reciprocal types can improve the feasibility of production relative to the homogeneous population with selfish types only, or actually can make it worse under certain circumstances. The production, even if achieved, will not necessarily be optimal.

1. Introduction

The production of certain public goods requires a specific number of people to coordinate to form a team. One example of this is the threshold production of public goods with binary actions (Palfrey and Rosenthal, 1984). Public goods are provided only if critical number, exactly m out of n players contribute money, time, effort and other resources towards production. A self-interested player has an incentive to join the team only if his contribution is pivotal for success. Incentives not to contribute are associated with the "the fear of losing", if less than $m-1$, and "incentives to free ride" if more than m players contribute.

Empirical evidence reveals the existence of heterogeneously motivated subjects in voluntary contribution games. While some participants in experiments are material payoff maximizers, the majority are reciprocal types whose contribution decision depends on what others do. In this paper, we study the dynamics of the threshold public goods game when the population is heterogeneous. Some play best-response to the current distribution of strategies to maximize payoff, whereas, the second group are reciprocators, who choose to conform to a certain type of behaviour. The analysis of the stochastic process provides answers to the following questions. Can the team coordinate to provide the public good? If yes, then who are the members of the successful team? Does production take place at an efficient level?

Threshold games have one equilibrium where no one, and multiple equilibria where exactly m out of n players contribute towards the production. Static solution concepts applied to coordination games face an equilibrium selection problem. Stochastic evolutionary dynamics can predict which of these equilibria is played in the long run (Foster and Young, 1990; Kandori, Mailath, and Rob, 1993; Young, 1993): small mistake probabilities allow a dynamic strategy revision process to move from any configuration of strategy choices to any other with positive probability. In the long run, as the probability of a mistake vanishes, the process is locked into a ”stochastically stable state”.

Myatt and Wallace (2008a,b, 2009) study the stochastic evolutionary game dynamics of threshold production models. Each period one player is randomly chosen and given a chance to update his strategy choice. The strategy played is a myopic best-reply, that is, the player best-replies to the current strategy profile. The player is also allowed to make a mistake and choose a non-best-reply with some small probability. Under this specification, the success of teamwork depends on the relative ease of forming the team versus its destruction. They show that the good is produced if and only if it is ”privately feasible”. A good is privately feasible if it would be optimal for one person to pay the costs of all m players. The benefits from individual consumption outweigh the total cost.

Vega-Redondo (1997) argues that evolutionary dynamics should not be viewed simply as an instrument for equilibrium-selection. He demonstrates that under dynamics other than best-response, evolutionary models can produce an interesting behaviour which does not correspond to Nash equilibrium. Results of experiments support this argument by revealing the existence of behavioural types in the population.

There are two persistent findings across experiments on public good games [1]: 1) subjects contribute $40\% - 60\%$ of their endowment in the first round of the one-shot public good game. Contributions decrease in further rounds, but stay above 0; 2) subjects contribute more when they believe others' will do so. The decay in the contributions is explained by mistake in choices and other-regarding preferences (Andreoni, 1995; Palfrey and Prisbrey, 1997; Kurzban and Houser, 2002).

There is vast evidence in the literature showing cooperation decision is affected by others' actions. Keser and Van Winden (2000); Ashley, Ball, and Eckel (2002); Croson and Marks (2000); Neugebauer, Perote, Schmidt, and Loos (2009) find a correlation between beliefs about others and cooperative behaviour in the lab experiment. Moreover, this correlation is persistent across belief elicitation methods, cultural and other dimensions [2] Evidence from field experiments also consistent with these results. Frey and Meier (2004); Heldt (2005); Martin, Randal, et al. (2005); Shang and Croson (2009) find that subjects donate more when they believe others did as well.

Experimental results on the public good game cannot be explained by one type of behaviour. Findings suggest the existence of heterogeneous types. Fischbacher, Gächter, and Fehr (2001) observe that in a linear public good game a quarter of subjects can be classified as free riders, whereas 50 percent are conditional cooperators, and the rest shows more complicated behavioural patterns. Kurzban and Houser (2005); Burlando and Guala (2005); Muller, Sefton, Steinberg, and Vesterlund (2008); Fischbacher and Gächter (2010) also find similar results. Even though the proportion of types differs across experiments, the general finding is that the population consists of heterogeneous types.

The results from a few experiments run on the threshold public good games were similar to the linear models. Isaac, Walker, and Thomas (1984); Marwell and Ames (1979, 1980, 1981); van de Kragt, Orbell, and Dawes (1983); Dawes, Orbell, Simmons, and Van De Kragt (1986) find that subjects voluntarily contribute $40 - 60\%$ of their endowments in the public good. They also find that subjects are more likely to cooperate if they estimate the probability of being critical is high. Schram,

[1]See Davis and Holt (1993); Ledyard (1995); Offerman (1997); Ostrom (2000) for a survey of public good experiments.

[2]See Chaudhuri (2011); Gächter (2006) for a survey of experiments.

Offerman, and Sonnemans (2008) classify subjects taking part in threshold public good experiment as individualists and cooperators.

Taking into account the evidence from experiments we model threshold public good production with 2 types of the population; selfish and reciprocal types. Selfish (S) types maximize material payoff. They contribute to production only if pivotal. Reciprocal (R) types contribute only if l others do as well. Several variations of behavioural types are considered in the literature. Our model can accommodate several types by changing the value of parameter l. If to set $l = 0$, R types are altruists(A) who gets satisfaction from the act of contribution. If $0 < l < m$, R types are conditional cooperators (CC) who contribute given l others contribute as well. If $l \geq m$, R types are in-group cooperators (IC) who gets satisfaction if good is provided and they are members of the team providing the good.

Does the presence of R types increase the success of production? As stated before, with only S types, collective action is successful under the private feasibility condition. As it turns out, having a single R type in the population changes this result. When R types are altruists and can produce good, production always takes place, private feasibility condition is not required for success. When altruists can not provide the good, or R types are conditional cooperators, production takes place under different conditions than private feasibility. Production can fail even if private feasibility is satisfied. When R types are in-group cooperators, and cannot provide the good alone production never takes place.

How does the existence of R types affect the efficiency of production? R types can cause two types of inefficiency. When production takes place, a higher number of R types cause overcontribution. All reciprocal players join the team causing an inefficient level of contribution. Overcontribution does not necessarily increase the quality or quantity of the good. From the examples provided above, more than the required number of appeals to police or votes do not improve the result. Or imagine the donation of money for some cause. Once the target level has been raised, the additional amount could be contributed to another cause. If reciprocal types form a team which does not satisfy the production threshold, there is undercontribution. The good cannot be produced and contributions are wasted.

The rest of the paper is organized as follows: Section 2 is a description of the underlying game and characterization of the game dynamics. The evolving play is characterized by the state space, strategy revision rule and transition probabilities between the states. Section 3 starts with the introduction of solution concepts used to predict the long-run outcome of the process. We also present the outcome of the model with homogeneous selfish types which are used as a benchmark model. Later, we show how results change relative to the benchmark model when few Imitators are added to the population. Section 4 presents the welfare analysis of the public good provision with a heterogeneous population. The results are summarized in the conclusion section.

2. The evolution of the discrete public goods provision

Game. The underlying game is a threshold public goods game. Threshold public goods require a specific number of players to coordinate to produce a good which is public in nature (Palfrey and Rosenthal, 1984). Let $N = \{1, 2, ..., n\}$ be set of players who want to coordinate to produce a discrete public good. Each player i chooses $z_i \in \{0, 1\}$, where $z_i = 1$ is to contribute and $z_i = 0$ is not to contribute towards the production. Decision made by all players generates a pure strategy profile $z \in Z \equiv \{0, 1\}^n$. In the proceeding discussion of the process z is a "state of a play" from the state space Z.

There is a production threshold m which shows the minimal level of participation required to produce the good. Let's also denote $|z| = \sum_{i \in N} z_i$ for the total number of players who choose to contribute. Production takes place only if $|z| \geq m$, exactly m or more players out of n coordinate to produce a good which gives equal benefit v to all players. The cost of contribution towards public good production is c, where by assumption $v > c > 0$. The payoff to a utility-maximizing player i generated by the strategy profile z takes the form:

$$u_i(z) = v \times \mathcal{I}_{|z| \geq m} - z_i c,$$

where $\mathcal{I}$ is the indicator function, which takes the value 1 if $|z| \geq m$ and 0 otherwise. Payoffs are positive when production is successful; $u_i(z \mid |z| \geq m) > 0$. If less than m players choose to contribute each of them receives $-c$, while payoff to others is 0. If more than m players choose to contribute, each of them receives $v - c$, while others, who free-ride, receive v.

This game has $\binom{n}{m} + 1$ pure-Nash equilibria, where either no one or exactly any m players out of n join the "team of contributors".

Next, we describe the dynamics of interaction among players. Noisy participation decisions generate a stochastic process. The long-run outcome of the process is determined from the analysis of the stochastic process.

One-step-at-a-time strategy revision process. Play evolves according to a one-step-at-a-time strategy revision process. At each period t a player i is randomly chosen from the population and given a chance to revise his strategy. The probability of more than one player being chosen is equal to 0. The chosen player selects a strategy from the strategy set $z_i \in \{0, 1\}$ based on the current strategy distribution $z_t \in Z$ and the strategy revision rule, which will be described next. The decision made by the chosen player i generates the strategy profile for the next period z_{t+1}.

Strategy revision rules. There are two types of players in the population according to the rule they revise strategy. The set of players is divided into 2 subsets and defined as $N = \{S, R\}$. Players from the subset $S = \{1, 2, ..s\}$ are selfish types (S types). Players chosen from the subset $R = \{s + 1, s + 2, ..n\}$ are reciprocators (R types).

If a revising player is of type S, he chooses myopic best-reply to the current strategy profile. S types choose to contribute if they achieve higher utility from the contribution than from deviation: $\Delta u_i = u_i(z_i = 1) - u_i(z_i = 0) > 0$ for $i \in S$. It is optimal for such player to join the team if it already has $m-1$ members, $|z| = m-1$, and his contribution is pivotal to form a team of m members. Production process with S types is described in figure 1.

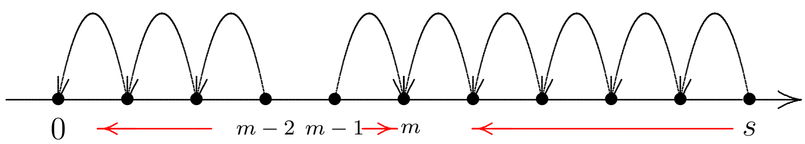


FIGURE 1. Dynamics with S types
s - number of S, m - production threshold.

R are behavioural types and they do not optimize material payoff. Instead they make a decision according to what others do. R type's decision is a binary choice to join the team or not depending on the current state and the threshold l. If l or more contributors are already in the team, the player $i \in R$ also joins them, $z_i = 1$, otherwise does not, $z_i = 0$. Production process with R types is described in figure 2.

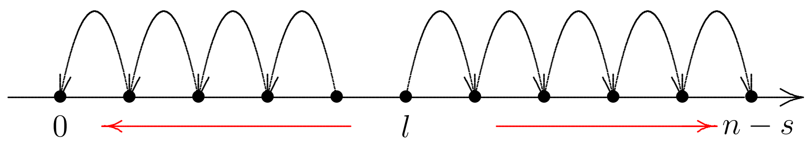


FIGURE 2. Dynamics with R types
(n-s) - number of R, l- reciprocation threshold.

The state space. To keep track of the strategy distribution over time it is useful to describe the state space Z. There are only two action choices available to players, therefore keeping track of the players who play $z_i = 1$, for each type, is enough to infer the number of players who play $z_i = 0$. The space Z is partitioned into a different set of states

$$Z_{x,y} = \{z_t \in Z : \sum_{i \in S} z_{it} = x, \text{and } \sum_{i \in R} z_{it} = y\},$$

where z_t is a current state of play, y is the number of R types and x is the number of S types who play $z_i = 1$. (x, y) shows the number of contributors by each type, but doesn't reveal the identity of the contributors; any x out of s S types, or any y out of $n - s$ R types can be the contributors. Each state z with exactly (x, y) number of contributes belongs to the set $z \in Z_{x,y}$. The total number of contributors is $|z| = x + y$.

Birth-death process. Each player $i \in N$ receives an opportunity to revise his strategy with an equal probability $1/n$. After each revision, the state is updated. If a revising player $i \in N$ who was not in the team at time t is chosen and decides to join, $z_{it} = 1$, the state moves "up" by one step. On the other hand, if one of the team members decides to drop out, $z_{it} = 0$, the state moves "down". Strategy revision process moving up and down between states by one step each period yields birth-death process.

The state can also transfer to itself if a player who is already contributing is chosen and decides to contribute again, or a non-contributing player chooses not to contribute. Player i making a choice is locked-in to that choice until the next random revision opportunity arrives.

Transition probabilities. The transition probabilities are characterized by two events: who gets the revision opportunity and which strategy is chosen by the revising player.

The state moves up in the x dimension, if an S type player is chosen from outside the team with probability $\frac{s-x}{n}$, and decides to contribute. This player chooses $z_{it} = 1$ with probability $1 - d_S$ if the current state is $z_t = \{z_t \in Z_{x,y} | x + y = m - 1\}$. For all other states, the contribution is not a best-reply, occurs with a small birth probability b_S.

$$Pr[z_{t+1} \in Z_{x+1,y} \mid z_t \in Z_{x,y}] = \frac{s-x}{n} \times \begin{cases} 1 - d_S & x + y = m - 1, \\ b_S & \text{otherwise.} \end{cases}$$

The state moves down in the x dimension when an S type player from the team is chosen to revise with probability $\frac{x}{n}$, and decides to quit. If the current state is $z_t = \{z_t \in Z_{x,y} | x + y = m\}$ choosing $z_i = 0$ is not a best-response, and happens with a small death probability d_S. For all other states, to quit is a best-response and happens with a probability $1 - b_S$.

$$Pr[z_{t+1} \in Z_{x-1,y} \mid z_t \in Z_{x,y}] = \frac{x}{n} \times \begin{cases} d_S & x + y = m, \\ 1 - b_S & \text{otherwise.} \end{cases}$$

When a player is chosen to revise his strategy is of type R, the state $Z_{x,y}$ changes in y dimension. Contribution incentive of R types is determined by the reciprocation threshold l. If the current state is $z_t = \{z_t \in Z_{x,y} | x + y \geq l\}$ they choose $z_i = 0$, and they choose $z_i = 1$ for all $z_t = \{z_t \in Z_{x,y} | x + y < l\}$.

The state moves up in y dimension when the revising player of R type starts to contribute. There is $\frac{n-s-y}{n}$ probability that an R type player is chosen from outside the team. If the current state is $z_t = \{z_t \in Z_{x,y} | x + y \geq l\}$, he plays $z_i = 1$ with probability $1 - d_R$. In all other states, $z_i = 1$ is played with a small birth probability b_R.

$$Pr[z_{t+1} \in Z_{x,y+1} \mid z_t \in Z_{x,y}] = \frac{n-s-y}{n} \times \begin{cases} 1 - d_R & x + y \geq l, \\ b_R & \text{otherwise.} \end{cases}$$

The probability of selecting an R type from the team is $\frac{y}{n}$. He leaves the team with a small death probability d_R if the current state is $z_t = \{z_t \in Z_{x,y} | x + y > l\}$. In all other states to quit is an optimal strategy and happens with probability $1 - b_R$.

$$Pr[z_{t+1} \in Z_{x,y-1} \mid z_t \in Z_{x,y}] = \frac{y}{n} \times \begin{cases} d_R & x + y > l, \\ 1 - b_R & \text{otherwise.} \end{cases}$$

A state transits to itself in two cases: 1) with probability $\frac{x}{n}$ an S type, and with probability $\frac{y}{n}$ an R type is chosen from the team and they choose $z_{it} = 1$ again;

$$Pr[z_{t+1} \in Z_{x,y} \mid z_t \in Z_{x,y}] = \frac{x}{n} \times \begin{cases} 1 - d_S & x + y = m, \\ b_S & \text{otherwise.} \end{cases} + \frac{y}{n} \times \begin{cases} 1 - d_R & x + y > l, \\ b_R & \text{otherwise.} \end{cases}$$

2) with probability $\frac{s-x}{n}$ S type, and with probability $\frac{n-s-y}{n}$ R type is chosen from outside the team and they choose $z_{it} = 0$ again.

$$Pr[z_{t+1} \in Z_{x,y} \mid z_t \in Z_{x,y}] = \frac{s-x}{n} \times \begin{cases} d_S & |z_t| = m - 1, \\ 1 - b_S & \text{otherwise.} \end{cases} + \frac{n-s-y}{n} \times \begin{cases} d_R & |z_t| < l, \\ 1 - b_R & \text{otherwise.} \end{cases}$$

Birth and death probabilities arise from noisy responses. Noiseless process ($\varepsilon = 0$) is path-dependent. The process is locked in one of the absorbing states depending on the first state it started from. Having noise allows the process to escape absorbing set of states with positive probabilities b_S, b_R, d_S, d_R. Therefore, any state can be reached with positive probability (irreducibility) and any state can transfer to itself (aperiodicity). The resulting Markov process $\mathcal{P}^\varepsilon$ is ergodic. For ergodic chains, there exists a unique ergodic distribution over the state space Z, irrespective of where the play started,

$$p_z = lim_{t \to \infty} Pr[z_t = z],$$

where p_z is the frequency the state z is observed in the long run. With small, but non-zero mutation probabilities system still spends most of the time in its limit states (absorbing states of the noiseless process).

When the error probability vanishes, that is $\varepsilon \to 0$, distinctive behaviour emerges. As noise vanishes some set of states are easier to reach and harder to leave than others. There exists z such that $\lim_{\varepsilon \to 0} p_z \to 1$ in the limiting distribution, such state z is called a stochastically stable state (SSS) (Foster and Young, 1990; Kandori, Mailath, and Rob, 1993; Young, 1993).

Birth and death rates. In the limit, as $\varepsilon \to 0$, these small birth and death probabilities vanish with different birth and death rates, which causes state-dependent mutations as in Bergin and Lipman (1996). Hence, only probabilities that disappear slowly matter.

Myatt and Wallace (2003) assume that transition probabilities vanishes at exponential rate [3], and call these transition rates "exponential cost" of a probability, $\mathcal{E}$. In the current model, $\mathcal{E}(b_i) = \beta_i$, where β_i is the "birth cost" of a revising player $i \in \{S, R\}$ moving up the state by mistake; and $\mathcal{E}(d_i) = \delta_i$, where δ_i is the "death cost" of a player $i \in \{S, R\}$ moving down the state by mistake. The transition probabilities that are positive under the noiseless process do not vanish. These transitions have zero exponential cost: $\mathcal{E}(1 - b_i) = \mathcal{E}(1 - d_i) = 0$.

Let's take two neighbouring states $z \in Z_{x,y}$, and $z \neq z'$, such that z' differs from z by only one revision process in either dimension. Denote the cost of transition between these two neighbouring states by $\mathcal{E}_{zz'} \equiv \mathcal{E}(Pr[z \to z'])$. Then by application of the properties of $\mathcal{E}$, transition probabilities decline with the following rates (Please see appendix A.1 for the properties of the exponential cost function):

$$z' = Z_{x+1,y} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} 0 & x + y = m - 1, \\ \beta_S & \text{otherwise.} \end{cases}$$

$$z' = Z_{x,y+1} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} 0 & x + y \geq l, \\ \beta_R & \text{otherwise.} \end{cases}$$

$$z' = Z_{x-1,y} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} \delta_S & x + y = m, \\ 0 & \text{otherwise.} \end{cases}$$

$$z' = Z_{x,y-1} \Rightarrow \mathcal{E}_{zz'} = \begin{cases} \delta_R & x + y > l, \\ 0 & \text{otherwise.} \end{cases}$$

The relative size of β_i and δ_i defines which transitions are more likely in the limit.

[3] If the state space is finite, and the chance of observing any state is positive, the probability distribution over the state space can be represented as an odd ratio (Blume, 2003). Whereas odd ratios can be written in an exponential form. Myatt and Wallace (2003) represents the odd of ratios by the following exponential form:

$$\frac{1 - b_i}{b_i} = \exp\left(\frac{\beta_i}{\varepsilon}\right) \quad \text{and} \quad \frac{1 - d_i}{d_i} = \exp\left(\frac{\delta_i}{\varepsilon}\right) \quad \text{where } i \in \{S, R\}. \tag{1}$$

This implies, as $\varepsilon \to 0$ the odd of ratios behave as the exponential function does.

3. Public good provision in the long run

There are two methods used in literature to identify SSS. One of them is the "tree surgery" method popularized by Kandori, Mailath, and Rob (1993); Young (1993). The second is "radius-coradius" technique proposed by Ellison (2000).

Tree-surgery technique is based on the analysis of "the set of trees rooted at z". Let each limit state to be the nodes of the complete directed graph. Imagine all states $z' \neq z$ are joined through the directed graph in a way that each of them has a unique successor. All sequences lead to the root z which has no successor. Denote such a tree h. There are several ways a tree rooted at z can be constructed. For example, take three nodes a, b, c. It is possible to construct three a-trees: (1) $b \to a$, and $c \to a$, (2) $c \to b \to a$, and (3) $b \to c \to a$. The set of all possible trees rooted at z is denoted by H_z.[4]

Exponential cost of each tree is the sum of the transition costs $\mathcal{E}_h = \sum_{(z,z')\in h} \mathcal{E}_{z,z'}$. With abuse of notation, denote the exponential cost of the least cost tree rooted at z by $\mathcal{E}(z) = \min_{h\in H_z} \mathcal{E}_h$. The stochastic potential of the state z is the least cost among all z-trees $\mathcal{E}(z)$. After the stochastic potential is calculated for each state, they are compared. If $\mathcal{E}(z) < \mathcal{E}(z')$ for all $z \neq z'$, then z is the stochastically stable state of dynamics.

Ellison (2000) offers an alternative "radius-coradius" technique to find SSS. The "radius" of a limit state is the cost of escaping from it. The "coradius" is the cost of returning to the limit state from other states. The difference between radius and coradius of the state determines the "attractiveness" of the state. The most attractive state among all limit states is a SSS.

"Radius-coradius" method provides only sufficient conditions, whereas, "tree surgery" method also provides a necessary condition for the stability of the limit state.[5] Nevertheless, the later can provide better intuition to explain the results. In this paper, the results are proved in Appendix A.2 with "tree surgery" method, and in the main text, we provide an intuition of some results with the "radius-coradius" method.

[4]The formal description of the tree-surgery method can be found in Appendix A.1.
[5]Myatt and Wallace (2008b) provide an example, where the former method fails to predict the outcome.

Benchmark model. When there are no R types in the population, $n = s$, the play is characterized by best-response dynamics only. The limit states of dynamics are $z \in Z_{0,0}$ and all states $z \in Z_{m,0}$, where a different combination of m players form a team. Myatt and Wallace (2008a,b, 2009) shows that the teamwork succeeds if $\lim_{\varepsilon \to 0} \sum_{z \in Z_{m,0}} p_z \to 1$.

To find the condition for team success radius and coradius of a state $z \in Z_{0,m}$ are compared. The formation of a team from $z \in Z_{0,0}$ requires $m-1$ players to contribute when it is not optimal, and for the last m^{th} player, it is a best-response to join them. The radius of the state is $(m-1)\beta_S$. It takes only one member to leave by mistake to disrupt the team. Once he leaves, it is optimal for the rest to quit. The coradius of such team is δ_S. From the comparison, teamwork succeeds when $(m-1)\beta_S < \delta_S$.

If to model the behaviour of selfish types by quantal-response function, stochastic stability is related to the private feasibility condition (Myatt and Wallace, 2008b). Under this specification $\beta_B = c$, and $\delta_B = v$. The stochastic stability condition is equivalent to the "private feasibility" condition, $v > mc$. It means the production is successful only if it is optimal for one player to provide the good alone.

New model. In the new environment, $n - s$ of R types with a reciprocation threshold l are added to the population of S types. All possible teams that can emerge in this environment are revealed from the analysis of limit sets. The limit sets of the system are presented in the following proposition.

LEMMA 1. *Limit sets of the system are $Z_{0,0}$, $Z_{m,0}$, $Z_{0,n-s}$ and $Z_{m-(n-s),n-s}$.*

1 *for $0 = l < m$ the possible limit sets are $Z_{0,n-s}$ and $Z_{m-(n-s),n-s}$.*
2 *for $0 < l < m$ the possible limit sets are $Z_{0,0}$, $Z_{0,n-s}$ and $Z_{m-(n-s),n-s}$.*
3 *$m \leq l$ the possible limit sets are $Z_{0,0}$, $Z_{m,0}$, and $Z_{0,n-s}$.*

PROOF OF LEMMA 1. First, we show that all possible limit states belong to the sets $z^* \in \{Z_{0,0}, Z_{m,0}, Z_{0,n-s}, Z_{m-(n-s),n-s}\}$. Then we show how the set changes according to the relative size of m and l.

1) If it is optimal for an S type to contribute then all of them join the team. Otherwise, all of them stay out. Hence there cannot exist a limit state with the number

of S types different than 0 and $n - s$. The limit states belong to the set $z \in Z_{x,y}$, where either $y = 0$, or $y = n - s$.

If no S type joins the team, $y = 0$, R types form a team with exactly 0 or m members. The limit sets are $Z_{0,0}$ and all $Z_{m,0}$, where exactly m out of s players of S type join the team.

When all R types are already in the team $y = n - s$, S types stay out when $n - s > m$ and the good is already successfully produced. The limit set $Z_{0,n-s}$ emerges from this case. Only if $n - s < m$, R types are not able to produce the good alone, a different combination of $m - (n - s)$ S types join the team to support them. This leads dynamics to the limit states in $Z_{m-(n-s),n-s}$.

Hence, the limit states belong to the set $z^* \in \{Z_{0,0}, Z_{m,0}, Z_{0,n-s}, Z_{m-(n-s),n-s}\}$.

2) When $l = 0$, all R types join the team with zero cost. Hence the states with 0 number of R types cannot be limit states. The set of limit states is $z^* \in \{Z_{0,n-s}, Z_{m-(n-s),n-s}\}$.

When $l \leq m$, $Z_{m,0}$ can't be a limit state, because $|z| = m \geq l$; it is optimal for any R type the team. The set of limit states is $z^* \in \{Z_{0,0}, Z_{0,n-s}, Z_{m-(n-s),n-s}\}$.

When $l \leq m$, $Z_{m-(n-s),n-s}$ can't be a limit set, because $|z| = m < l$; it is optimal for any R type to quit such team. The set of limit states is $z^* \in \{Z_{0,0}, Z_{m,0}, Z_{0,n-s}\}$. ■

When the population is heterogeneous, one of the teams of all S types, of all R types or the joint team of all R and some S types can emerge. The success of each team depends on the exponential cost of forming versus destruction of this team.

Let's denote the following notations for convenience:

- $z \equiv z^0$, for all $z \in Z_{0,0}$, is a state where no one participates in production;
- $z \equiv z^\dagger$, for all $z \in Z_{m,0}$, is a state where exactly m S types are in the team;
- $z \equiv z^\diamond$, for all $z \in Z_{0,n-s}$, is a state where all R types are in the team;
- $z \equiv z^\ddagger$, for all $z \in Z_{m-n+s,n-s}$, is a state where S and R types form a team together.

There are several theories about the behaviour of reciprocal types. Three of them can be captured in this model by the change in parameter l. If $l = 0$, R types are altruists (A). A types get utility from the act of contribution, irrespective of the success of the production. If $0 < l < m$, R types are conditional cooperators (CC). CC types contribute to the production only if others do as well. If $l \geq m$, R types are in-group cooperators (IC). IC get utility only if the good is produced and they contributed to production themselves. We discuss the results for each type separately.

Altruists: When $l = 0$, R types contribute to the production process irrespective of what others do. The state space is described in figure 3:

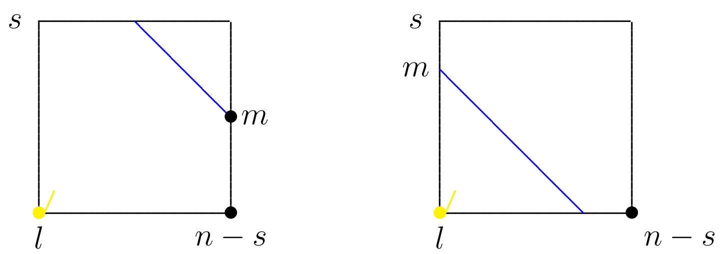


FIGURE 3. Construction of the team with A types
s - number of S, (n-s) - number of R, m - production threshold, l- reciprocation threshold, • *- limit set.*

PROPOSITION 1. *When R types are Altruists,* $l = 0$*,*

(1) *For* $m \geq n - s$*, the good is produced if* $[(m-1)-(n-s)]\beta_S < \min[\delta_S, \delta_R]$;
(2) *for* $m \leq n - s$*, the good is always produced.*

From lemma 1, when R types are A, the limit states are $z^* \in \{z^\diamond, z^\ddagger\}$. When $m \leq n - s$, the second set disappears. The only limit state is $z^\diamond$, where the good is provided by the team of all R types irrespective of the birth-death costs. A types are always willing to contribute towards the production, and they from the successful team if there are enough number of them, or the public good is is not expensive.

When $m > n - s$, the limit states are $z* \in \{z^\diamond, z^\ddagger\}$. The team at $z^\diamond$ cannot provide the good as the production threshold has not been met. Only the joint team of all R types and $m - (n - s)$ S types can provide the good if $[(m-1)-(n-s)]\beta_S < \min[\delta_S, \delta_R]$.

First, we show that the joint team $z^\ddagger$ consisting of all R types, and $m - (n - s)$ S types dominates any other team with m members. Next, we examine conditions that guarantee the success of this team.

Let's take any state $z^\ddagger$, and $z' = \{z' \in Z_{x,y} | x + y = m, z' \neq z^\ddagger\}$. There is a zero cost path from z' to $z^\ddagger$. By construction $l < m$, hence it's optimal for an R type from outside to join the team and replace one of the S members. This iteration continues until there are no R types left outside. At this stage the state is $z^{\ddagger}$. The path in the opposite direction requires the R members to leave overcoming δ_R, and being replaced by S players from outside. From the comparison, the state $z^{\ddagger}$ is more attractive. The good is produced by the joint team of S and R types.

To form the team from z^0, initially, all R types join with zero cost forming a team of R types at state $z^{\diamond}$. Next, $(m-1)-(n-s)$ S types join, overcoming β_S. Since there are exactly $m-1$ players in the team now, it is optimal for the last S type to join them as well. The radius of such a team is $[(m-1)-(n-s)]\beta_S$.

The team collapses when one of the members leave overcoming the death cost. Since there left $m-1$ players, it is optimal for all S types to leave as well. The cheapest way out of the state $z^{\ddagger}$ is the death of type with the least death cost. The coradius of such a state is $\min[\delta_S, \delta_R]$.

Comparison of the radius and coradius reveals that the good is produced only if the condition $[(m-1)-(n-s)]\beta_S < \min[\delta_S, \delta_R]$ is satisfied. Otherwise, the SSS is $z^{\diamond}$, where production fails as not enough members joined the team.

The existence of A types helps to team formation relative to the homogeneous population. Especially, when A types can provide the good by themselves, $m \leq n-s$, production is always successful. When $m > n-s$, the success of production requires $[(m-1)-(n-s)]\beta_S < \min[\delta_S, \delta_R]$ to hold. If $\min[\delta_S, \delta_R] = \delta_S$, this condition is a milder version of private feasibility. If $\min[\delta_S, \delta_R] = \delta_R$, production can fail even under private feasibility condition, unless $[(m-1)-(n-s)]\beta_S < \delta_R$ is satisfied.

Conditional Cooperators. When $0 < l < m$, R types are conditional cooperators (CC). The state space is described in figure 4.

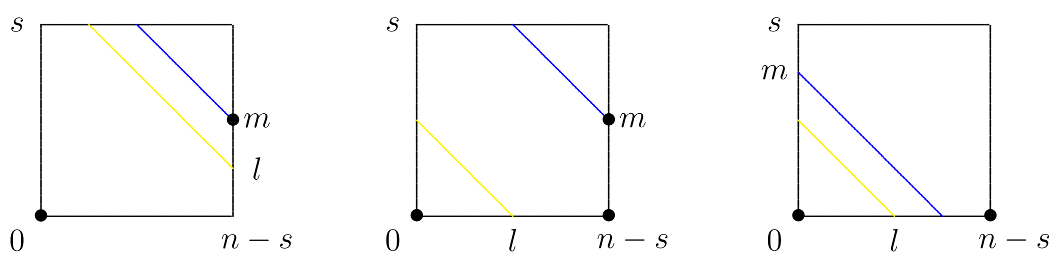


FIGURE 4. Construction of the team with CC types
s - number of S, (n-s) - number of R, m - production threshold, l- reciprocation threshold, • - limit set.

PROPOSITION 2. *When R types are conditional cooperators, $l < m$,*

(1) *For $l \geq n - s$, the good is produced if*

$$[(m-1)-(n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l-(m-1)+(n-s)] \min[\beta_S, \beta_R] < \min[\delta_S, \delta_R];$$

(2) *For $l < n - s < m$, the good is produced if*

$$[(m-1)-(n-s)]\beta_S < \min[\delta_S, \delta_R], \quad \textit{and}$$

$$[(m-1)-(n-s)]\beta_S + l \min[\beta_S, \beta_R] < \min[\delta_S, \delta_R] + (n-s-l)\delta_R;$$

(3) *for $l < n - s$, and $m \leq n - s$, the good is produced if*

$$l \min[\beta_S, \beta_R] < (n-s-l)\delta_R.$$

Here we only discuss part (3) of proposition 2. Proofs for other results can be found in Appendix A.2.

From lemma 1, when R types are CC, the limit states are $z^* = \{z^0, z^\diamond, z^\ddagger\}$. When $l < n - s$ and $m \leq n - s$ the last set of states disappears. The good either produced by the team of all R types or not produced at all. Next, we examine conditions required for the success of such a team.

To form the cheapest team from z^0, initially, l players with the cheapest birth cost join the team, overcoming $\min[\beta_S, \beta_R]$. Since now there are exactly l members, it is optimal for the rest of R types to join them. Once all R types have joined, S types quit if $\min[\beta_S, \beta_R] = \beta_S$. The radius of such a team is $l \min[\beta_S, \beta_R]$.

This team consists of R types only. It collapses when $n - s - l$ members leave overcoming the death cost δ_R. Once only l members left in the team, any R type chosen from the team observes $l - 1$ and chooses to quit with zero cost followed by others. The coradius of such team is $(n - s - l)\delta_R$.

Comparison of the radius and coradius reveals that the good is produced if

$l \min[\beta_S, \beta_R] < (n - s - l)\delta_R$. Otherwise, the SSS is z^0, where no type contributes to production.

In this example, the effect of CC types on production is not directly comparable. Even if production is privately feasible, teamwork can fail unless the new condition holds.

In-group cooperators. When $l \geq m$, R types are in- group cooperators (IC), join the team after m players are ready to produce the good. The state space is described in figure 5.

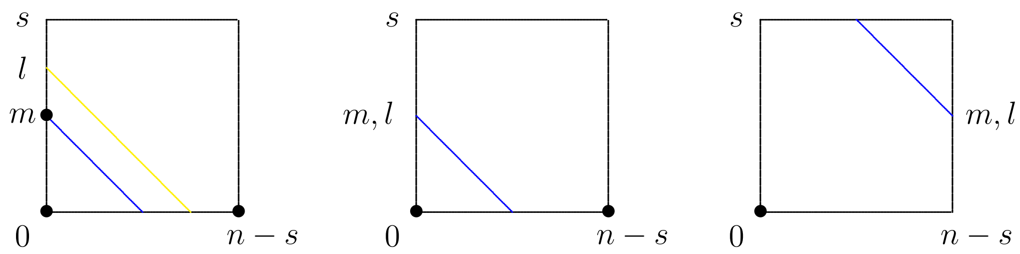


FIGURE 5. Construction of the team with IC
s - number of S, (n-s) - number of IC, m - production threshold, l- reciprocation threshold, • - limit set.

PROPOSITION 3. *When R types are in-group cooperators, $l \geq m$,*

(1) *For $m \leq s$ and $m < l$, the good is produced if either $(m-1)\min[\beta_S, \beta_R] < \min[\beta_R, \delta_S]$, or $(l-1)\min[\beta_S, \beta_R] < (n-s-l)\delta_R + \min[\beta_R, \delta_S]$;*
(2) *For $l < n-s$, and $m > s$ or $m = l$, the good is produced if $(m-1)\min[\beta_S, \beta_R] < (n-s-l)\delta_R$;*
(3) *For $l \geq n-s$, and $m > s$ or $m = l$, the teamwork always fails.*

From proposition 3, collective action fails when $l \geq n-s$, and $m > s$ or $m = l$. Under this parameter values, the only limit state is z^0, and it attracts all probabilities in the limit as errors vanishes.

To understand the reason behind let's assume $l = m$, and exactly m players formed a team with some positive cost, and the current state is any $z = \{z \in Z_{x,y} | x + y = m\}$. As $m = l$ any R type can join the team with zero cost. After each R types joins, it is optimal for an S type to leave. The process repeats until all R types are already in the team. Then any R type chosen from the team to revise drops out with zero cost, and the state moves to z^0 with zero cost. The coradius of the team is 0. The comparison reveals that the team cannot be stable under any conditions. Hence when R types are IC, $l \geq n-s$, and $m > s$ or $m = l$, production never takes place.

Let's assume the private feasibility condition is satisfied and S types can form a team to provide the good. In this example, adding even one IC type to the population can fail otherwise successful team. This example shows that the existence of behavioural types can have a damaging effect on the production of the public good.

4. Social welfare analysis

In this section, we discuss how behavioural types affect the efficiency of production. Let's denote the utilitarian welfare of the state z by $w(z)$. In the long run, each z is observed with probability $p(z)$. The expected welfare is $W = \sum_{z \in Z} p_z \times w(z)$. In the limit, SSS attracts all probability. The expected welfare reduces to $W = w(z)$. The outcome of the discrete public goods game with heterogeneous population induces the following welfare function:

$$w(z) = \begin{cases} 0 & \text{if } z = z^0, \\ nv - mc & \text{if } z = \{z^\dagger, z^\ddagger\} \\ nv - (n-s)c & \text{if } z = z^\diamond \text{ and } m < n - s, \\ -(n-s)c & \text{if } z = z^\diamond \text{ and } m \geq n - s. \end{cases}$$

Social welfare is zero when players fail to coordinate. With homogeneous S players, production fails only if it is not privately feasible. With R types, production can require additional conditions, or can fail irrespective of the cost. This is the first type of inefficiency caused by the existence of the behavioural types.

The public good is provided at an efficient level when exactly m players form a team of producers. This outcome is achieved when the states $z^\dagger$, and $z^\ddagger$ are SSS, all S types or joint team of S and R types form a team to produce the good.

When the good is provided by R types only, at $z^\diamond$, social welfare is less than optimal. This means there is an overcontribution of the public good. Overcontribution is the second type of inefficiency caused by the existence of R types, as it is not observed in the benchmark model. For $nv > (n-s)c$, this outcome is still desirable, but less efficient than the production with exactly m contributors.

When $n - s < m$, social welfare is negative. R types form a team, but cannot meet the production threshold. All contributions are wasted. Undercontribution is the fourth type of inefficiency related to R types.

With homogeneous S type population, the good is either produced by exactly m players or not produced at all. Hence successful production takes place at an efficient level. With R types, the success of team formation is not the only concern. Reciprocator dynamics are extreme, either all R types join the team, or none does. There is a potential problem of "overcontribution", which arises if more than m members contribute, $n - s > m$, and "undercontribution" of the public goods, if less than m members contribute towards the production, $n - s < m$. In the game of discrete public goods without reimbursement, any cost above m is wasted. Hence, the success of collective action is not the only concern of a social planner.

5. Conclusion

Experiments run on public good games reveals that players do not contribute at the equilibrium level. This evidence is explained by the existence of behavioural types who do not hold selfish preferences. In this paper, we analysed the evolution of collective action under heterogeneous population.

When everybody in the population optimizes monetary payoff, players manage to produce the public good if it is privately feasible. The production, if successful, takes place at an efficient level. When behavioural types are added to the population the result changes.

The first message of the paper is that the existence of behavioural types with unselfish preferences does not always improve the provision of the public good. We find that when behavioural types are altruists or conditional cooperators, the public good can be produced under more relaxed conditions than private feasibility, but it can also require additional assumptions. With in-group cooperators, the production can fail irrespective of the cost-benefit. Hence, behavioural types do not always increase the chances of production, but can also harm an otherwise successful team. Even when the production is successful it comes at a cost. When the number of R types is sufficient to provide the good alone, it is produced at less than efficient level.

Experiments on public good games test how the change in the number of the overall population, the value of the public good, and monetary cost of contribution affects the provision process. In our model, the production is affected by the level of noise in best-responses, the composition of the population, and the threshold levels.

The noise level in best-responses affects the state observed in the long-run. Hence it is useful to model how noise is added to the decision process. Noise can arise from mistakes. One natural assumption is that S players make more mistakes in low-income states, and less in high-income states. Quantal-response function captures this assumption naturally, hence it is a candidate to model S dynamics. The behaviour of R types can be modelled by mistake models, where a player choose a non-best-reply action with some small fixed mistake probability. The detailed discussion of the choice rules which incorporate mistake can be found in Blume (1995, 2003).

Contribution decision of S types is affected by the production threshold m. Reciprocal types, however, based their choices on the reciprocation threshold. Hence the relative magnitude of both parameters can change the results.

Not only threshold levels, but also the number of players of each type has an impact on the outcome. With homogeneous S population, like in (Myatt and Wallace, 2008b), as long as $s > m$, the number of players do not have any influence on the stability conditions. In this model, however, even when $s > m$ the number of R types affects the results.

CHAPTER 2

Experiment on the Value of Information in Beauty-Contest Game

Abstract. This paper tests the social value of information in the Keynesian beauty contest game in the lab. Players have two objectives, to be as close as possible to the unobserved fundamental and, at the same time, to coordinate (anti-coordinate) with other players. We test in the laboratory how subjects divide attention between public and private information when choosing an action under different strategic environments. We find that when subjects want to coordinate they weigh private information less, which is consistent with theoretical predictions. In contrast, we fail to observe the behaviour described by theory in the anti-coordination game, where private information attracts more significance. Even though subjects do not play best-response strategy in the anti-coordination game, under both environments they react to the correlation in private signals following the theoretical predictions.

1. Introduction

This paper tests the social value of information in the economic environment requiring its participants to coordinate on some common objective. We ask subjects to play a Keynesian beauty contest game where they are provided with one public and one private piece of information about the unknown state in the form of noisy signals. The public signal is commonly observed by everyone, whereas private signals can be independent or partially correlated. After the signals realized, subjects are asked to choose an action based on the information available to them. The strategy faced by subjects is to assign the weight to each information type. The experiment aims to understand how subjects in the lab use information under different strategic environments, and if they internalize correlation in their signals.

We find that subjects in the lab assign equal weights on the public and private information when the task is to guess the fundamental state. When the task is to guess the state and to be closer to their partner at the same time, they reduce the weight on the private information consistent with theory. However, the subjects in the lab do not increase the weight on private information when the task is to avoid the partner. The reaction to the correlation in private signals is consistent with the theory. Correlated signals attract higher weight in coordination, and lower weight in the anti-coordination game, relative to the independent private signals.

Most of the daily life activities require people to coordinate on some common objective. These situations can be represented by a beauty-contest game. The terminology "Beauty contest" comes from the parable introduced by Keynes (1936). The reader of the newspaper should choose the prettiest faces among the introduced photos, but at the same time, they need to guess the most popular answer given by others. There are two forms of uncertainty in this game: to guess the state correctly (fundamental motive) and to predict the behaviour of others (coordination motive). When subjects face uncertainty about the state and others' belief about it, the success of their actions depends on the use of the information available to them.

According to the theory, when players form beliefs about unobserved fundamental they weigh information from different sources according to their precisions. However, when decision-makers are also interested in the actions of others, they overreact to public information because of its dual nature. The public information conveys guidance about the fundamental, and at the same time, about the beliefs of others. Overreaction to low-quality public information can reduce the welfare of society by causing bubbles and financial crises. The central bank and other government bodies disclosing important information related to public policy should consider the effect of their announcements. Hence, it is essential to understand how individuals perceive the difference in the nature of information and how they react to those differences.

Our experiment is based on the model of beauty contest game described in Morris and Shin (2002), and Myatt and Wallace (2014). The game is captured by a linear-quadratic pay-off function. Players need to guess the unobserved state of the world, at the same time to be as close as possible (or as far as possible) to other participants. They are given information about the state in the form of noisy signals. The signal observed by everyone is public, and the signal observed by each player alone is private information. Players choose how much they value each information. They choose an action which is the linear combination of two pieces of information they have. The weight put on signals depends on the relative importance of fundamental and coordination (anti-coordination) motives.

In Morris and Shin (2002) public and private signals are used to form a belief about the unknown state. When the motive is to guess the state they attract equal weights in equilibrium. In the presence of coordination motive players also want to know what others do. It is optimal to attach a higher weight to the public information. When the motive is to anti-coordinate with others, then private information becomes more valuable, and attracts higher weight in equilibrium.

Myatt and Wallace (2014) present a model with correlated private signals which contains Morris and Shin (2002) as a special case. Once the private information becomes correlated in Myatt and Wallace (2014), the previous results are weakened. The correlated private signals attract higher weight than independent in the coordination game. In the anti-coordination game, correlated private signals attract lower weight in comparison to the independent signals.

This is one of the recently evolving topics in microeconomic theory and has found broad application in fields, such as oligopolistic competition, frictions in financial markets, central bank communication, and many others. Since the existing theories have the potential of being applied in a variety of economic situations it is useful to test their predictions in the controlled lab environment.

Our goal is to test the value of public information in the controlled lab environment. Subjects are given one public and one private piece of information and asked to assign importance to each. The decision is made in three different environments; when their actions are not related, strategic complements or substitutes. Private signals were independent at some sessions and partially correlated at others.

The predictions of the theory are stated as three hypotheses. The first hypothesis is the subjects form Bayesian beliefs, they assign equal weights to public and private information when the task is to guess the state. We can not reject this hypothesis. The second hypothesis is the weight on private information is decreasing with coordination motive. This hypothesis can not be rejected for the coordination game. Subjects decrease the weight assigned to the private information compared to the control treatment. We cannot find the evidence for the predictions in the anti-coordination game. Subjects increase the weight assigned to the private information, but this effect is not significant. The last hypothesis is that subjects internalize the correlation in their signals. This hypothesis cannot be rejected. The weight on independent signal decreases faster than the weight on the correlated private signal.

Related literature. This paper is related to several strands of literature on lab experiments. First, it's a contribution to the previous tests of the K-beauty contest game with the exogenously given information. Second, we test how subjects react to the correlation in their signals. And the last, we contribute to the discussion of the difference in the ability to learn to play equilibrium between the games of strategic substitutes and complements.

Cornand and Heinemann (2014) test the hypothesis that subjects attach larger weight to the public than to private information if they have the incentive to coordinate their actions in the lab. They find that, indeed, subjects assign a larger weight to public information as coordination motive increases, but the effect is smaller than equilibrium predictions. They lean towards coordinating on the public signal but do not achieve full coordination during the session. Similar results obtained by Dale and Morgan (2012), the low-quality public signal decreases social welfare, but less than predicted by theory.

After confirming that the weight on public information increases with coordination motive, next Baeriswyl and Cornand (2016) test how this weight changes with the precision of the signal. The precision of the signal is measured by its variance. The lower variance implies higher precision. Consistent with the first experiment, the weight assigned to the public information tends to increase in the coordination game. The weight increases further when the signal is more precise.

Boun My, Cornand, and dos Santos Ferreira (2017) proposes the theory of endogenous coordination motive and tests the reverse hypothesis that the coordination motive increases in the precision of the public signal. More precise public signals decrease the coordination uncertainty and hence increase the preference to play a coordination game. The results are consistent with the theory; when fundamental uncertainty is larger than coordination uncertainty, players tend to put more weight on coordination motive. However, the reverse hypothesis does not hold, as players put more weight on coordination motive even when coordination uncertainty is bigger.

The first aim of this experiment is to test the theory in a different environment. The existing theory provides unique predictions given the normality of the distribution signals are drawn from and the linearity of the strategies used by players. In previous experiments on K-beauty contest game signals are drawn from the uniform distribution. In our design, all signals are taken from a normal distribution. In previous experiments, subjects submit their action choice and weights on the signals are estimated from the observed behaviour. We control for the strategies by asking to assign the weights to the signal realizations. Hence all subjects use the linear strategy and we observe the weights directly.

Additionally, we test the behaviour in Keynesian beauty contest game when actions are strategic complements, as well as when they are strategic substitutes. The importance of each motive depends on the environment the theory is applied to. An important application of the beauty contest game is a firm choosing actions when his profit depends on the unobserved demand, his price and price set by other firms competing in the industry. The action of firms competing on price as in Bertrand competition are strategic complements because the firms are willing to set a high price, but also they want to do it together (Myatt and Wallace, 2015). Cournot competition is the game of strategic substitutes, as firms competing on quantities are willing to produce more given the other firm produces less (Myatt and Wallace, 2018). According to Fehr and Tyran (2008) subjects learn to play equilibrium faster in the game of strategic substitutability, than in complementarity. Previous experiments on the value of information concentrate on the game of compliments only. Taking into account the striking difference between the two games and the importance of substitutes games on its own it is important to analyse the predictions of the model in the anti-coordination dimension as well.

Finally, we test the reaction to the partially correlated information. Our information environment is closer to the Goeree and Offerman (2003) and Bayona, Brandts, and Vives (2016), who show that subjects do not internalize the correlation in their values while bidding. Our experiment is different in several dimensions. First, in previous experiments, subjects should react to the realization of several signals at each period. We design an environment where the public information is fixed, only the realization of the private signal differs across the rounds. Hence, within a treatment, subjects need to react to the changes in the realization of private signal only. The difference in the weights between independent and correlated signals captures the effects of correlation only. The second difference arises from the complexity of the strategic environment. In models tested previously, the choice is affected by the number of market participants and risk attitudes of the subjects. In K-beauty contest game, the decision parameter is not affected by either of these factors. This allows observing the effect of correlation directly. Theoretically, the results with any number of players are expected to hold generally.

Another difference from previous experiments is the subject pool. In previous experiments, only the subjects from the technical background were recruited. Our participants are students majoring in diverse areas, such as music, archaeology, finance, and mathematics, and so forth. The diverse subject pool makes the predictions of the experiment applicable to a broad range of economic situations.

The rest of the chapter is organized as follows. The theoretical background and the hypothesis tested in the experiment are described in section two. The experiment design is presented in section three. The results from the experiment are introduced in section four. The chapter ends with the conclusion section.

2. Theoretical Background

The model of interest is a quadratic pay-off "beauty contest" game presented in Myatt and Wallace (2014). There are L players, each $l \in L$ simultaneously chooses an action $a_l \in R$ and receives payoff according to utility function:

$$u_l = \bar{u} - (1-\gamma)(a_l - \theta)^2 + \gamma(a_l - \bar{a})^2. \tag{2}$$

where, θ is a fundamental state common to everybody, and $\bar{a} = \frac{\sum_{l' \neq l} a_l}{L-1}$ is the average action taken by all $l' \in L$, such that $l' \neq l$.

Players' objective is to be close to the underlying state variable and to the average action taken by others. Parameter $\gamma \in (-1; 1)$ determines the preference for matching with others relative to being closer to the true state. If $\gamma = 0$, the only objective is to guess the true state θ. If $\gamma = 1$, it becomes a pure coordination game, where the only motive is to match with other players, and θ becomes irrelevant. For other values of γ, both fundamental and coordination motives are present. The sign of γ also affects the nature of the game. For $\gamma \in (0;1)$ it is the game of strategic complementaries, where matching others is payoff increasing. For the $\gamma \in (-1;0)$ incentive is to take the action different from others, which is the game of strategic substitutes.

If θ is common knowledge, for $\gamma \geq 0$ the optimal action is to choose $a_l = \theta$, which generates the maximum achievable payoff. However, in the absence of common knowledge, each player l forms belief about the fundamental, $E_l[\theta]$, and the average action taken by others, $E_l[\bar{a}]$.

The expected utility of a player l depends on the proximity of his action a_l to the unobservable state variable θ, and to the action chosen by other players $\bar{a} = \frac{\sum_{l' \neq l} a_l}{L-1}$. From the optimization of (2) the best-reply strategy for each l is:

$$a_l = (1-\gamma)E_l[\theta] + \gamma E_l[\bar{a}]. \tag{3}$$

Information. All players share the common belief $\theta \in (-\infty, +\infty)$ about the state variable. Additionally, each receives signals $x_{il} \in R^n_+$ from n different information sources.[1] A signal from the information source i observed by player l has the following structure:

$$x_{il} = \theta + \eta_i + \varepsilon_{il}, \quad \text{where} \quad \eta_i \sim N(0, \kappa_i^2) \quad \text{and} \quad \varepsilon_{il} \sim N(0, \xi_i^2). \tag{4}$$

Each signal is observed with some common error η_i, and private error ε_{il}. Hence signals are distributed normally $x_{il} \sim N(\theta, \sigma_i^2)$, where $\sigma_i^2 = \kappa_i^2 + \xi_i^2$. The precision of each signal is measured by $\frac{1}{\sigma_{il}^2}$; lower σ_{il}^2 implies better precision.

Conditional on θ, signals are independent across n information sources. But observation of each source is correlated across players;

$$cov[x_{il}, x_{il'}|\theta] = \rho_i \sigma_i^2, \quad \text{where} \quad \rho_i = \frac{\kappa_i^2}{\kappa_i^2 + \xi_i^2}.$$

Given (4), expectation about fundamental is $E[\theta|x_l] = \sum_{i=1}^{n} \frac{1}{\sigma_i}^2 x_{il}$. The precision term is sufficient to form beliefs about the fundamental. However, to form beliefs on $\bar{a}$ the knowledge of correlation among signals is required as well. Correlation coefficient, $\rho_i \in [0,1]$ is the measure of the publicity of information source. $\rho = 1$ when the signal is commonly observed by everyone, hence it is public in nature.

[1]The assumption of improper prior simplifies calculations. It is without loss of generality, and can be avoided by making one of n signals perfectly correlated for everyone.

This case is obtained by setting $\xi_i^2 = 0$. $\rho_i = 0$ when signals are independent, and obtained by setting $\kappa_i^2 = 0$. For the interim values $\rho \in (0,1)$ the signals are partially correlated across players.

Equilibrium analysis. By taking expectations of (3), the best-reply action $a_l \equiv A(x_l)$, is a linear combination of two expectations:

$$A_l(x_l) = (1-\gamma)E[\theta|x_l] + \gamma E[A_l(x_l')|x_l]. \tag{5}$$

From normality assumption the first term is linear. For symmetric players using linear strategies there exists a unique linear equilibrium of the game.

The linear strategy can be represented as:

$$A_l(x_l) = \sum_{i=1}^{n} \omega_i x_{il}, \text{where } \sum_{i=1}^{n} \omega_i = 1. \tag{6}$$

Given the linear specification, the strategy of player l is to choose $\omega_i \in R^n$, the weight assigned to each signal i.

The equilibrium weight assigned to each signal can be found by taking expectations of (2) and maximizing with respect to ω_i. Maximizing (2) is identical to minimizing the Loss function:

$$\min(1-\gamma)E[(a_l-\theta)^2] + \gamma E[(a_l-\bar{a})^2] \tag{7}$$

After substituting the strategy (6) in (7) and taking expectations the optimization problem becomes:

$$\min_{\omega} \sum_{i=1}^{n} \omega_i^2 \Big((1-\gamma)\kappa_i^2 + \xi_i^2\Big) + \gamma \sum_{i=1}^{n} \kappa_i^2 \left[\omega_i - \frac{\sum_{l'\neq l}\omega_i}{L-1}\right]^2 \text{ subject to } \sum_{i=1}^{n} \omega_i = 1. \tag{8}$$

Here we skip derivations and present the results directly. More detailed calculations can be found in Appendix B.1. The optimal weights ω_i which solve the problem (8) are:

$$\omega_i = \frac{\widehat{\psi}_i}{\sum_{j=1}^{n} \widehat{\psi}_j}, \text{ where } \widehat{\psi}_i = \frac{1}{(1-\gamma)\kappa_i^2 + \xi_i^2}. \tag{9}$$

From (9), signals with higher variance attract lower weight in equilibrium. γ influences w through the common noise parameter κ_i^2. Recall that, for $\kappa_i^2 \neq 0$ signals are correlated, and independent signals are obtained by setting $\kappa_i^2 = 0$. As γ increases, the weight assigned to the correlated signal also increases. Whereas, the change in γ does not influence the weight assigned to the independent signal. In equilibrium, players who want to coordinate their action choices, assign a higher weight to the correlated relative to the independent signals.

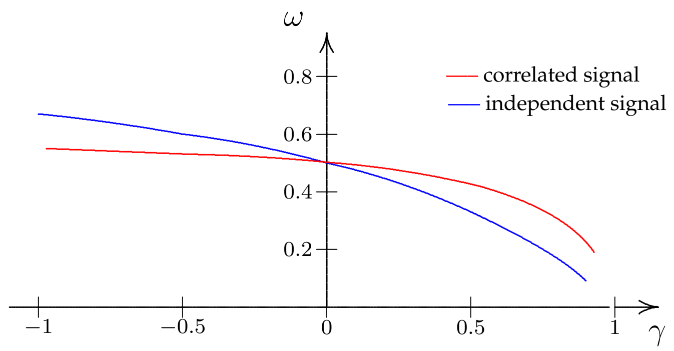


FIGURE 1. The relation between the equilibrium weights and coordination motive.

The figure represents equilibrium weight on private signal for the parameter values $i \in \{1, 2\}$, and $\sigma_1^2 = \sigma_2^2$. Blue line represents the weight on independent signal, $\rho = 0$. Red line represents the weight on correlated signal, $\rho = 0.6$.

Figure 1 shows the relation between the coordination motive, and the weight assigned to private signal. There is only one public and one private signal available to the players, $i \in \{1, 2\}$, and signals are similar in precisions, $\sigma_1^2 = \sigma_2^2$. y axis represents the weight assigned to the private signal, ω, and x axis is the coordination parameter γ. Red line shows the equilibrium weight assigned to correlated ($\rho = 0.6$), blue line shows the equilibrium weight assigned to independent ($\rho = 0$) private signal. As can be seen from the figure, ω is decreasing in γ. The function of ω is steeper for independent relative to the correlated signal.

The model gives clear predictions under the assumption of rationality and ex-ante symmetry of players, which can be tested in an experiment. We run an experiment to test the following hypotheses:

H1**:** The first hypothesis is, given precisions are equal when the strategic motive is absent subjects put equal weights on public and private information.

This hypothesis could not be rejected by the previous experiments on K-beauty contest game with a different design (Cornand and Heinemann, 2014). We will use this hypothesis as a benchmark to compare our results with the previous work. It is also a control treatment to test the next hypotheses.

H2: The second hypothesis is the weight assigned to private information decrease in coordination motive. According to the theory, as coordination motive increases, players assign lower weight on private information. In contrast, the weight on private information increases when anti-coordination motive increases.

According to Fehr and Tyran (2008) subjects in the lab learn to play the equilibrium strategy in the games of strategic substitutes faster, than the games of complements. They explain this finding by the difficulty of optimization in coordination games. They divide subjects into the high types, who are rational, and low types, who cannot optimize their strategies. In the game of complements, rational players want to mimic low types, therefore, there is a deviation from equilibrium. However, in the games of substitutes, it is optimal to respond to the low types taking optimal actions. Cornand and Heinemann (2014) show that subjects decrease the weight on private information in the K-beauty contest game with coordination motive. We are going to test if subjects play equilibrium strategies in both domains.

H3: According to the theory, the weight on independent signal decreases faster than the weight on the correlated signal. The last hypothesis is, players can internalize the correlation in their signals.

This hypothesis was tested mainly by the literature on bidding in auctions, and recently, by Bayona, Brandts, and Vives (2016) testing how sellers submit supply function in the oligopoly competition when their costs are correlated. The previous literature cannot find evidence to support this hypothesis. We check a similar hypothesis with a different underlying game.

3. Experimental Design

In this section, we present the values assigned to the parameters and describe the hypothesis formally.

We run an experiment to test if ω observed in the lab behaves as predicted by theory in (9). The data on ω is collected for different values of treatment parameters γ and ρ. The values assigned to each parameter are described below.

$L = 2$; there are only two players. This restriction affects neither qualitative nor quantitative predictions of the model. Instead, it simplifies the environment and helps to observe the treatment effect better.

$n = 2$; each subject receives two signals. The first, $i = 1$, is a perfectly correlated (public) signal, and the second, $i = 2$, is a partially correlated or independent (private) signal. As there are only two signals, the notation ω_i can be simplified by denoting the weight on the public signal by $1 - \omega$, and the weight on the private signal by ω.

The public signal, $\rho_1 = 1$, is generated by setting $\kappa_1^2 = 100$, $\xi_1^2 = 0$, and $\sigma_1^2 = \kappa_1^2 + \xi_1^2 = 100$. These specifications do not change throughout the experiment. Whereas, ρ_2 is a treatment parameter, so we can simplify the notation by setting $\rho_2 \equiv \rho$.

ρ can take one of two possible values $\rho \in \{0, 0.6\}$. The independent signal, $\rho = 0$, is generated by setting $\kappa_2^2 = 0$, $\xi_2^2 = 100$, and $\sigma_2^2 = \kappa_2^2 + \xi_2^2 = 100$. The partially correlated signal, $\rho = 0.6$, is generated by setting $\kappa_2^2 = 64$, $\xi_2^2 = 36$, and $\sigma_2^2 = \kappa_2^2 + \xi_2^2 = 100$. As can be seen, the precision is equal for all signals, correlation is the only treatment parameter. We choose the same correlation level as in Bayona, Brandts, and Vives (2016) to make predictions comparable across two experiments.

The second treatment parameter is $\gamma \in \{-1, 0, 0.9\}$. When the strategic motive is absent $\gamma = 0$. For the anti-coordination game $\gamma = -1$, and the coordination game $\gamma = 0.9$.[2]

Treatments are described in table 1. To test how ω reacts to the change in γ, we fix ρ. $T0$ and $T3$ become control treatments. The data from $T1$ and $T2$ are compared to $T0$, and $T4$ and $T5$ are compared to $T3$. To test the reaction of ω to change in ρ, γ is fixed, and $T0$, $T1$ and $T2$ become control treatments. The data from $T3$, $T4$ and $T5$ is compared to the observations from control treatments.

TABLE 1. Treatments

	$\gamma = 0$	$\gamma = 0.9$	$\gamma = -1$
$\rho = 0$	$T0$	$T1$	$T2$
$\rho = 0.6$	$T3$	$T4$	$T5$

[2]The optimisation problem has a solution for $\gamma = -1$. When $\gamma = 1$ equilibrium is not unique, hence we could not choose this parameter value.

Let's denote the weight observed in the lab by $\omega_{\gamma,\rho}$; where γ, ρ takes parameter values chosen for the treatment. Simple ω stands for the weight taken from any treatment. Theoretical predictions are denoted by $\omega^*_{\gamma,\rho}$. Given the chosen parameter values, theoretical predictions of w^* (in percentage) for each treatment, are shown in Table 2.

TABLE 2. Equilibrium weights

w^*	$\gamma = 0$	$\gamma = 0.9$	$\gamma = -1$
$\rho = 0$	50	9	67
$\rho = 0.6$	50	19	55

Given the parameters described above, we test the following hypothesis:

H1. For all values of ρ, the following is true: $\omega_{0,\rho} = 0.5$. To test if subjects form Bayesian beliefs in the simple environment $(H1)$, at first, the average weight from $T0$ and $T3$ is compared to the equilibrium predictions, $w^*_{0,\rho} = 0.5$. The null hypothesis is the average weights from theory and data are equal. The alternative is the weights are different.

H0: $\omega_{0,\rho} = 0.5$;
Ha: $\omega_{0,\rho} \neq 0.5$.

H2. ω decreases as γ increases. For $H2$, the average weight from $T0$ is treated as control data. To test the behaviour in coordination domain, the weight from $T1$ is compared to the control data. For the anti-coordination game the weight from $T2$ is compared to the control. The same procedure is done for the correlated signals. $T3$ is treated as a control treatment, the data from $T4$ and $T5$ are compared to $T3$. The null hypothesis is there is no difference between the weights. The alternative hypothesis is, weights decrease in the coordination, and to increase in the anti-coordination treatment.

H0: $\omega_{0.9,\rho} = \omega_{0,\rho}$ and $\omega_{0,\rho} = \omega_{-1,\rho}$ for each value of ρ;
Ha: $\omega_{0.9,\rho} < \omega_{0,\rho}$ and $\omega_{0,\rho} < \omega_{-1,\rho}$ for each value of ρ.

H3. $\omega_{0.9,0} < \omega_{0.9,0.6}$; and $\omega_{-1,0} > \omega_{-1,0.6}$. $H3$ is tested by comparing the average weights between $T1$ and $T4$ for coordination, and between $T2$ and $T5$ for the anti-coordination treatments. This test is valid if there is no significant difference between $T0$ and $T3$. The null hypothesis is there is no difference between the weights.

The alternative hypothesis is the average weight increases from $T1$ to $T4$ and decreases from $T2$ to $T5$.

H0: $\omega_{0.9,0} = \omega_{0.9,0.6}$; and $\omega_{-1,0} = \omega_{-1,0.6}$;
Ha: $\omega_{0.9,0} < \omega_{0.9,0.6}$; and $\omega_{-1,0} > \omega_{-1,0.6}$.

To test hypotheses we run the following experiment. Subjects are paired in groups of two and play a K-beauty contest game. Before making a decision they observe two signals. The first signal is a common prior about the fundamental. At each period the fundamental is chosen randomly from $N(100, 10)$. This prior is public information. No one knows the exact value of the fundamental, but everybody knows the distribution it is drawn from. Each player also receives a private signal, which shows the fundamental with some error drawn from $N(0, 10)$. Subjects are asked to assign weights on the public and private information they observe. The game is played under the treatment parameters described above. (Instructions can be found in appendix B.3.)

The public signal is 100 for all rounds because the fundamental is always chosen from the normal distribution with a mean 100. The realization of the private signal is different at each round because a new state and error is drawn every period. As public information is fixed, the change in the weights is affected by the realization of the private signal only. This design was chosen to simplify the decision-making process by reducing the changes in the environment. Subjects are required to react to the change in the private signal only.

The experiment runs for 4 sessions. Each session consists of three stages, each running for 10 rounds. The treatments in each session are described in Table 3.

TABLE 3. The overview of the sessions

Sessions	Stage 1	Stage 2	Stage 3
1	$T0$	$T1$	$T1$
2	$T3$	$T4$	$T4$
3	$T0$	$T2$	$T2$
4	$T3$	$T5$	$T5$
Rounds	10	10	10

In stages 1 and 2, participants are paired with a computer always playing the optimal strategy and asked to make a decision. There is no strategic interaction in stage 1. It is designed to check if subjects form Bayesian beliefs. The difference in behaviour between stages 1 and 2 reveals the reaction to the strategic environment. In stage 3 subjects are paired with another participant anonymously and play the next 10 rounds as a team. The strategic motive is the same as in stage 2, while the partner is a real player.

Previous experiments (Cornand and Heinemann, 2014; Bayona, Brandts, and Vives, 2016) do not observe the equilibrium outcome in the lab, they also cannot find any pattern of learning. They suggest that subjects best-respond to the level-k beliefs they hold about their partners. We introduce another explanation in this experiment. In previous experiments, subjects face new signals and new partners every period. Learning to best-respond and forming a belief about a partner who also learns at the same time is challenging. It is not clear whether subjects fail to best-respond or to form correct beliefs. We choose the computer as a partner to control for beliefs and observe the change strategy only in stages 1 and 2. The comparison of stages 2 and 3 shows if subjects manage to form beliefs about their partners.

After each round participants are given information about the true state, other players' actions, and their payoffs. Feedback is presented to induce learning effect. After the experiment participants are asked to fill in the questionnaire about personal information and were paid in private.

4. Experimental Results

The experiment was run at the EXEC laboratory at the University of York. Participants were students randomly chosen from various disciplines across the university. In the start of the session, instructions were read aloud. Students were asked to answer comprehension questions before proceeding to the choice stage. Overall, 60 students participated in 4 sessions. Each session lasted for 90 minutes and subjects earned 11 GBP on average. All sessions were run using the experimental software z-tree (Fischbacher, 2007).

One limitation of the experiment is the number of subjects. We cannot make strong predictions due to the small sample size. Nevertheless, each subject is an independent observation as they play against a computer in the first two stages.

At first, we test the main hypotheses. Next, we analyse the learning pattern from the dynamics of the observed behaviour.

4.1. Analysis of behaviour. In this subsection, we compare the observed w across treatments to test the hypotheses $1-3$. We also compare the average w from experiment to equilibrium predictions to find the direction of deviations.

For H1, at first, we check if the average $\omega_{0,\rho}$ from all $T0$ and $T3$ are statistically similar. Then the average weights are compared to the theoretical predictions.

Result 1.1: For $\gamma = 0$, there is no significant difference among the weights attached to the private signals among all sessions.

Kruskal-Wallis test cannot reject the equality on weights among four sessions at a p-level of 0.32.

To test H1, the average $\omega_{0,\rho}$ is compared to the $\omega^*_{0,\rho} = 50$.

Result 1.2: For $\gamma = 0$, subjects assign equal importance to private and public information consistent with theoretical predictions.

For the overall data, this hypothesis cannot be rejected using two-tailed Wilcoxon signed-rank test at a p-level of 46.6% and signtest at a p-level of 51.2% (Please see Table 1 in appendix B.2 for p-values).

RESULT 1. *when $\gamma = 0$ the weights assigned on public and private information are consistent with theory. Subjects form Bayesian beliefs in a simple environment.*

This result has two practical implications. In the simple environment, $\gamma = 0$, the data is statistically similar to the theoretical predictions. Based on this result, we use $T0$ and $T3$ as control and check the deviations from control in the coordination, and anti-coordination treatments.

The result for $H1$ is consistent with the predictions of Cornand and Heinemann (2014). Both experiments cannot reject the hypothesis that subjects assign equal weights on public and private information. In general two experiments differ in the design. Given result 1, behaviour in the coordination treatment can be compared across experiments.

For H2, we check how ω reacts to the change in γ, for coordination and anti-coordination motives separately.

Result 2.1: In the coordination domain the weight on private information decreases, but less than predicted by theory.

To test the behaviour in the coordination domain, the weights from coordination treatment are compared to the baseline ($\gamma = 0$). One-sided signtest rejects the equality of weights in favour of the alternative hypothesis. The weight on private information decreases in the coordination treatment relative to the baseline for both independent (at a p-level of 0%) and correlated information (at a p-level of 5.6%) sessions (See Table 2 in appendix B.2 for p-values).

Even though subjects decrease weights in coordination domain, one-sided signtest rejects the equality of weights to the equilibrium predictions in favour of the alternative. The weight on private information is higher than the predictions at a p-level of 0% (See Table 1 in appendix B.2 for p-values).

Behaviour in the anti-coordination domain is checked similarly, $\omega_{0,\rho}$ is compared to $\omega_{-1,\rho}$.

Result 2.2 In contrast with theory, the weight on private information does not increase with anti-coordination motive. Moreover, the weight assigned to private information is significantly different from the weight predicted by equilibrium strategy.

Two-sided signtest cannot reject the equality of weights between the control and anti-coordination treatments at a p-level of 12.1% for independent, and 33.1% for correlated signal treatments (See Table 2 in appendix B.2). Hence subjects do not adjust the weight in the anti-coordination domain accordingly.

In the anti-coordination game, one-sided signtest rejects the equality of weights from the equilibrium predictions in favour of the alternative hypothesis. Subjects assign lower weights than predicted at a p-level of 1.1% for independent and 9% for correlated information treatments (See Table 1 in appendix B.2).

From the results, weights on private signals are lower than the predictions. It means weights in the anti-coordination game are closer to the baseline treatment. Hence we also cannot reject the equality between control and treatment weights.

RESULT 2. *Subjects decrease the weight on the private signal in the game of coordination. The adjustment of weights is not statistically significant in the anti-coordination game.*

Overall, the behaviour in the coordination domain is consistent with the results of Cornand and Heinemann (2014). Subjects decreased weight on private information, but do not play equilibrium. The behaviour in the anti-coordination treatment has not been tested, hence we cannot compare the results directly to any previous test.

Finally, $H3$ is tested in 3 steps. Correlation in signals does not affect the weights in the baseline treatment. The first step is to check if the data satisfy this requirement.

Result 3.1: For $\gamma = 0$, there is no significant difference between the weights attached to the independent and correlated private signals.

The equality of weights between independent and correlated information, for the baseline treatments, could not be rejected by two-sided MW test at a p-level of 71.1%. (See Table 3 in appendix B.2).

Next, we check if the subjects internalize correlation in their signals in the coordination game. By theoretical predictions, $\omega_{0.9,0} > \omega_{0.9,0.6}$.

Result 3.2: For $\gamma = 0.9$, the weight attached to the independent signal is higher than the weight on the correlated signal, as predicted by theory.

Two-sided Mann-Whitney rank-sum test rejects equality of weights on independent and correlated information in coordination treatment at a p-level of 1%. Kolmogorov-Smirnov test also rejects the equality in favour of the alternative hypothesis. The weights are higher in correlated treatment at a p-level of 2.7% (See Table 3 in appendix B.2).

The third step is to verify the effect of correlation in the anti-coordination game.

Result 3.3: For $\gamma = -1$, the weights attached on independent signals are smaller than the weights on correlated signals as predicted by theory.

Two-sided Mann-Whitney rank-sum test rejects the equality on weights between independent and correlated information in anti-coordination treatment at a p-level of 2%. Kolmogorov-Smirnov test also rejects the equality in favour of the alternative hypothesis that weights are lower in correlated treatment at a p-level of 0.9% (See Table 3 in appendix B.2).

Combining all 3 steps, we get the following result for the H3.

RESULT 3. *Subjects react to correlation in their signals as predicted by theory. In coordination domain, correlated signals attract higher, in anti-coordination domain lower weights than independent signals.*

Result 3 contradicts to the findings of Bayona, Brandts, and Vives (2016) who does not observe adjustments in actions when signals are correlated. One possible reason for the difference in the results is the complexity of the strategic environment. In the oligopolistic market presented in of Bayona, Brandts, and Vives (2016) the decision is affected by the number of players and risk attitudes of players. In K-beauty contest game the number of the players and the risk attitudes do not affect the equilibrium behaviour. The observed behaviour can be generalized to the game with any number of players.

4.2. Learning. In this subsection, we study the evolution of weights in the data. For that reason, the data from the sessions are divided into blocks of 5 periods. There are two blocks in each stage, and 6 blocks in general. We check the changes between each block, and the direction of the adjustment, for coordination and anti-coordination sessions separately.

The evolution of w in the coordination session is shown in Figure 2, separately for private and correlated signal treatments.

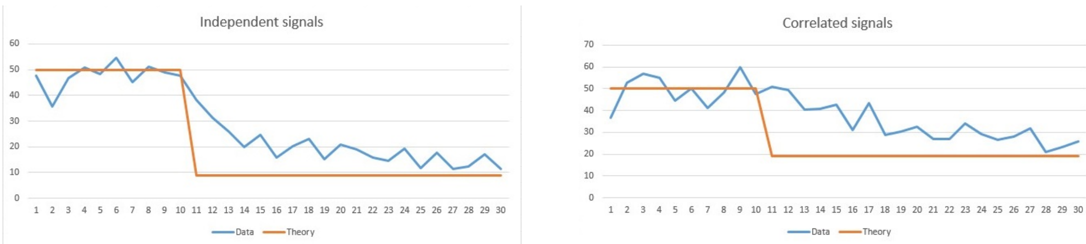


FIGURE 2. Evolution of weights in the coordination game

The aggregated weights from the coordination sessions are compared and p-values are presented in table 4 in appendix B.2. The table reveals two significant adjustments in the coordination sessions. First, the weights from the third block are lower than the second block with a p-level of 0%. This is the effect of adjusting weights between baseline to coordination treatment. Even though the weights are decreased, they are still higher than the theoretical predictions. The weights decrease further from the third to the fourth block with a p-level of 0%. The second adjustment is a sign of learning in the second stage. There are no significant adjustments in the last stage.

The evolution of w in the anti-coordination session is shown in Figure 3, separately for private and correlated signal treatments.

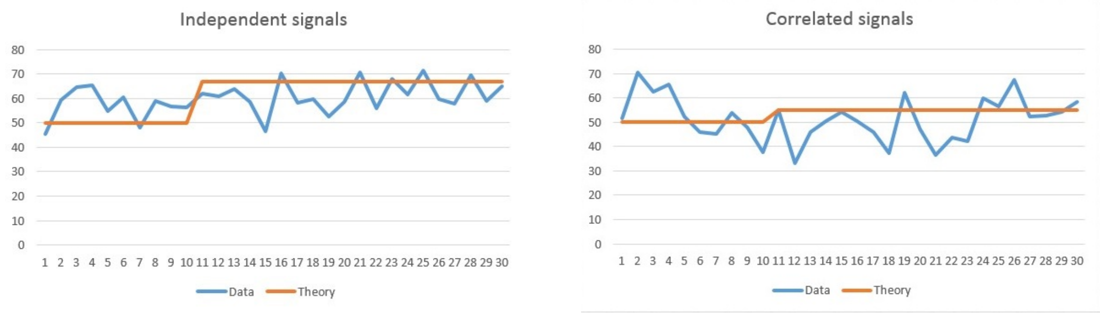


FIGURE 3. Evolution of weights in the anti-coordination game

The aggregated weights from the anti-coordination sessions are compared and p-values are presented in table 5 in appendix B.2. According to the table there is only one significant adjustment between the first and the second blocks. There is no other statistically significant evidence of adjustment.

RESULT 4. *Subjects learn to play equilibrium in the coordination games, however, they do not show a clear learning pattern in the anti-coordination games.*

The data shows that subjects demonstrate a clear pattern of learning in the coordination game but fail to do so in the anti-coordination game. This contradicts results from similar experiments, where subjects did not learn to best-response. In our experiment, leaving the partner stable across rounds creates a better environment for learning. Even when faced with real partners, subjects continue to learn and play the optimal strategy in the last rounds.

Our results also contradict to the findings in Fehr and Tyran (2008), where subjects demonstrate better learning patterns in the games of strategic substitutes. However, the results are not directly comparable across two experiments, as the underlying games are different. It suggests that the results from the previous experiment could be specific to the underlying game. The second possible explanation for our results is the choice of parameters. In the coordination game, the difference in optimal weights between baseline and coordination treatments are relatively larger, than in the anti-coordination treatments. The third explanation is a failure to best respond in the anti-coordination game. In the questionnaire, some subjects write that, private information does not help them to understand what their partner is doing, hence they assign more importance to public information in the anti-coordination treatment. This shows that subjects want to know what others know to be able to avoid them.

5. Conclusion

We test the value of information in the K-beauty contest game. Theory predicts that in the presence of coordination motive players overuse public signal. While with the anti-coordination motive players value private signal more. These results weaken when private signals become partially correlated.

The data from our experiment confirms the predictions for the coordination game. However, we fail to find evidence for the behaviour in the anti-coordination game. It suggests that the effect of public announcements can be adjusted by changing the degree of commonality in the signals in the coordination environment.

For future research, we plan to test the theory when information is costly as in Myatt and Wallace (2012), and Hellwig and Veldkamp (2009). In this models, the strategy of each player is to choose "how much" private information to purchase (i.e., to pay to increase the precision of the private signal) and what weight to assign to the purchased signal. In an experimental setting, the decision is made in two rounds. At first, subjects decide on how much information they are going to acquire (choose the precision of the private signal), then assign the weight to the realized signal in the next round. We acknowledge that the environment becomes complex for the subjects with a non-technical background, and consider two possible solutions. The underlying game can be simplified by considering a game with a binary state $\{\theta_1, \theta_2\}$ and binary action choice $\{a_1, a_2\}$. Or the participants can be restricted to the pool of students with a technical background. The second solution does not violate the generality of the predictions, as people making a decision in markets are professionals who understand technical details.

Some subjects in experiment write in the questionnaire that private signal does not help to guess the behaviour of the partner, hence they assign a lower weight to that signal in the anti-coordination game. This contradiction is also observed in the predictions of Myatt and Wallace (2012), and Hellwig and Veldkamp (2009). The first paper predicts players want to know what others don't when actions are strategic substitutes. Whereas, predictions of the second is that players want to know what others know to avoid them. This is another open question for future research.

CHAPTER 3

How to Sell Information in Coordination Games

Abstract. A profit-maximizing monopolist sells information to the data buyers who want to coordinate their actions on the unobserved state. The seller constructs a discriminating mechanism to extract higher surplus from the buyers with heterogeneous valuations for the information. We show the buyer with the highest valuation always receives the fully informative signal. But the quality of the signal offered to the low valuation buyer is a continuous and decreasing function of the frequency of high types.

1. Introduction

This chapter studies the incentives of a monopolist data seller to provide information on the strategic environment. Two players are required to coordinate their actions on the unobserved state. Players do not know the true realization but can acquire information from the monopolist who perfectly observes the state. The monopolist designs a revenue-maximizing mechanism which maps the quality of the traded information to the price. We ask what quality of information is traded in the market and what price is charged for each quality.

Players face two problems; they need to form beliefs about the state, and at the same time to coordinate their actions. The monopolist observes the state and can act as a coordination device. Using the language of communication games, players (agents) communicate through the monopolist (principal). Interaction between the principal and agents can take a complex form. According to the Revelation principle if there exists an outcome of the game with some possibly complicated interactive mechanism, then this outcome can also be implemented by a direct revelation mechanism (Myerson, 1979, 1982; Gibbard, 1973). In this simple mechanism, players report their types, transfer payments according to the revealed type and receive action recommendations from the principal which determines the outcome.

In our model, the principal is a revenue-maximizing monopolist, who charges a price for the revealed information, and agents are data buyers. Information is supplied in the form of statistical experiments. Each experiment is a test which can confirm or reject the hypothesis about the state with some probability. The fully informative experiment reveals the state perfectly, hence considered of the highest quality.

The buyers have independent prior information about the state. They acquire supplemental information from the monopolist to improve the quality of decision making. The buyers with better prior information have lower willingness to pay for the fully revealing information. Hence, buyers can be divided into the types with high and low valuations according to the priors they hold. The monopolist does not observe types but knows the distribution of priors.

The monopolist faces the problem of second-degree price discrimination. Buyers have different valuations for the experiments, and each prefers high-quality experiment to low. If monopolist knows types, she sells perfect information and charges valuation of each type. When types are not verifiable, the monopolist constructs the incentive-compatible mechanism to screen the types. However, the results of the trade with traditional goods do not translate one-to-one into the market for information goods. There is a difference in the nature of the goods which requires an alternative treatment. We discuss some of these differences below. The broader discussion of the difference in the nature of the traditional and information goods can be found in Varian (1999).

Buyers have preferences over the quality of goods. The quality of information can be measured in precisions. As the experiment is provided for each state separately, quality becomes a multidimensional parameter. For example, a player who assigns a higher probability to state 1 with his prior only, has higher valuations for the supplemental information about the state 2. Revelation principle doesn't necessarily hold for the multi-dimensional types. When the value of information is measured by the change in the utility from acquiring supplemental information, types become one-dimensional (Babaioff, Kleinberg, and Paes Leme, 2012). The monopolist's problem can be reduced to designing an incentive-compatible direct mechanism.

Unlike traditional goods, where a consumer receives utility only from the quality he purchases, utility from the consumption of information good is determined by how much it reveals to the buyer together with his prior information. Even partial information can reveal the state perfectly coupled with the prior information. It is also possible to design an experiment in such a way it reveals the state to one type, but not to another (Bergemann, Bonatti, and Smolin, 2018).

Another difference comes from the nature of the interactions. Because of the coordination motive, acquisition of information by one player has positive externalities for another. It increases the chances of coordinating on the correct state. These externalities are reflected in the prices. Willingness to pay for the information can increase in the precision of another players signal. To the best of our knowledge, we are the first to study the supply of information with positive externalities.

All differences stated above affect the nature of the information goods supplied in the market. We find that monopolist always provides high types with fully informative experiment. Both types prefer high quality, but the high type has a higher willingness to pay for it. Hence, the profits are maximized by selling the perfect information to the high type. Low types also receive a fully informative experiment when they are frequent in the population, and it is not optimal to exclude them. This result is well-known from the mechanism for traditional goods.

The second reason for the low types to receive positive information is the coordination motive. In the market without strategic interaction, low types receive either perfect or no information. In our model, the quality of the experiment received by low types is a continuous function. Coordinating players value not only their own but also the precision of the signal received by another player. Correlation in actions causes externalities in prices; each type is also willing to pay for the precision of the signal received by their partner. When there is a high chance of meeting a low type, the accuracy of the low type's decision has a significant impact on the high type's payoffs. Hence the quality of the experiment received by low types is a continuous and increasing function of their frequency.

The third reason for low types to receive positive information is the nature of the information goods. When priors of types do not agree on the most likely state, low types always receive some information, because it is possible to design the experiment which benefits low types, but does not reveal the state to the high type.

For example, if high type values information about state 1, but low type requires information about the state 2, the seller can reveal some information about the state 2 to the low type without violating incentives of the high type. In this case, low types receive some information even when their frequency is low. This result stems from the nature of the information good, and observed in the market without strategic interactions as well (Bergemann, Bonatti, and Smolin, 2018).

This model can be applied to the situations requiring coordination between market participants. Coordination problem stands in the centre of many economic interactions. One example is the speculative attack games, where traders want to attack the currency at the time when the economy is weak. For a successful attack, they need information about the state of the economy and coordinate their actions with other participants. Another example is the firms selling complementary goods. Firms need to get information about the demand for their goods, at the same time, the good is valuable only if provided together with its complement. Other potential applications are the matching market, where two sides are willing to coordinate, but unaware of the potential of their match, consumers coordinating on some standard, and so forth. Bergemann and Bonatti (2018) give an overview of the situations where statistical experiments are traded.

Literature review. The initial attempts to design information as an object of trade is observed in Persico (2000). In this paper, players have a prior x about the random variable v. They can pay to increase the informativeness of x, which raises the expected payoffs. The risk sensitivity of the outcome determines willingness to pay for information. They show that in the first price auction buyers are willing to pay more for the information than the second price auction.

Currently, there is evolving literature on the information markets, which designs information directly as the object of the trade. Some papers on global games literature analyses the demand side of the information market. Myatt and Wallace (2012, 2015, 2018); Hellwig and Veldkamp (2009) and others identify the demand and willingness to pay for the information products. The characteristic finding is that players who want to coordinate their action want to know what others know, i.e., acquire the same or correlated information. Whereas, we analyse the supply side of the information market. Our work is related to the literature that studies the sale the information to the imperfectly informed buyers. Bergemann and Bonatti

(2018) gives an overview of this literature, and some of these works are discussed below.

Admati and Pfleiderer (1986, 1990) analyze the sale of information to buyers trading common value assets. Unlike in our model, agents have homogeneous priors, the seller knows the individual utility functions, and actions of agents are strategic substitutes. They show that optimal design involves selling noisy and heterogeneous information to enable agents to differentiate their actions. It is also optimal to sell the heterogeneous information in our model, but the underlying reasons are different. In Admati and Pfleiderer (1986, 1990) it steams from the anti-coordination motive in the underlying game, whereas in our model differentiated information is provided to screen buyers for their willingness to pay.

Esö and Szentes (2007) consider a consultant who is selling information about the value of the assets to the clients trading in the financial market. Consultant charges the price contingent on the information and the action taken by the buyer. In our model transfers happen before the action takes place, it is contingent on the information only.

The closest to our work are Babaioff, Kleinberg, and Paes Leme (2012), and Bergemann, Bonatti, and Smolin (2018). Babaioff, Kleinberg, and Paes Leme (2012) model the principal providing agents with supplemental information. The principal knows the state of the world, and the agents know their types. They ask what mechanism can generate higher revenue to the principal. They prove that with committed buyers the Revelation Principle holds for the information goods as well. They also show that the optimal mechanism can generate positive and negative transfers. The results of the direct mechanism are compared to other mechanisms. They show that monopolist can extract full surplus when priors of buyers and seller are correlated similarly to the findings of Crémer and McLean (1985, 1988).

Bergemann, Bonatti, and Smolin (2018) considers the same environment as in Babaioff, Kleinberg, and Paes Leme (2012) but with specific utility functions and they control for the negative transfers. They show that when agents have a preference for information about the same state, i.e., they hold congruent priors, the monopolist sells either full or zero information to the low types. In our model, players' actions are strategic complements; there exist positive externalities in their purchase decisions. Even with congruent beliefs, low types can receive partial information.

The rest of the chapter is structured as follows. In section 2, the model is presented by describing the underlying game, and the problem faced by a mechanism designer. In section 3, we describe the optimal mechanism and its properties. The optimal level of information received by players is fully described in section 4. We present the results in section 5 and conclude the chapter with the Discussion section.

2. Model

State-coordination game. There are 2 states of the world $\Omega = \{\omega_1, \omega_2\}$. Two players simultaneously choose an action a_i from action set $A = \{a_1, a_2\}$, and receive payoffs according to the following table:

ω_1	a_1	a_2
a_1	$1;1$	$0;0$
a_2	$0;0$	$0;0$

ω_2	a_1	a_2
a_1	$0;0$	$0;0$
a_2	$0;0$	$1;1$

This game is a combination of the pure coordination game, where players need to coordinate their action choice, and match the state game, where players need to guess the realized state. If they match their actions, but not the true state each of them receives payoff 0. And if one of them guesses the state they still receive 0. The payoff is positive only when both conditions are satisfied; players coordinate on the true state.

Prior information. Players hold the prior belief $\theta \in \Theta \triangleq \triangle\Omega$ about the state of the world, where $P(\omega_1) = \theta$, and $P(\omega_2) = 1 - \theta$. Priors can take one of the two values $\Theta = \{q, p\}$, and distributed across players independently according to $F \in \triangle\Theta$. Distribution function F assigns $P(\theta = p) = \gamma$, and $P(\theta = q) = 1 - \gamma$, where $\gamma \in (0, 1)$.

Players can choose an action according to their prior belief $\theta \in \{q, p\}$, but can also acquire supplemental information from the principal to improve the quality of their decision.

Supplemental information. Players can communicate through the profit-maximizing monopolist. The monopolist (she) observes the true state, and supplies players (he) with supplemental information in the form of statistical experiments E at a price $t(E)$. Thus, the monopolist is a data seller and players are data buyers. Data seller cannot observe the buyers' beliefs but knows the distribution of θ.

A statistical experiment $E = (S, \pi)$ consists of a set of signals $S \in \{s_1, s_2\}$ [1] and distributed according to the likelihood function $\pi : \Omega \to \Delta S$. Each signal reveals the true state with probability $\pi_{ik} = P[s_k|w_i]$, where i is the state, and k is the signal indicator. π is the precision of the signal; the higher the precision the more informative the signal is. The state is revealed perfectly if $\pi_{ii} = P[s_i|w_i] = 1$, for all i. The fully informative experiment which reveals the state perfectly will be denoted by $\bar{E}$. Experiment is partially informative if $\pi_{ii} = P[s_i|w_i] < 1$, at least for some i. In terms of classic monopoly problem, informativeness of experiment is a quality of the product supplied by the monopolist.

The set of signals consists of two signals only $S = \{s_1, s_2\}$, therefore the notation π_{ik} can be simplified by dropping k from the index. The experiment is described by the following matrix:

E	s_1	s_2
ω_1	π_1	$1-\pi_1$
ω_2	$1-\pi_2$	π_2

The monopolist designs a menu of experiments $\mu = \{\mathcal{E}, t\}$, which is a collection $\mathcal{E}$ of experiments $E \in \mathcal{E}$ and associated tariff function $t : \mathcal{E} \to \mathbb{R}_+$.

The timing of events is as follows:

1) The seller posts a menu μ;

2) The true state Ω and types of buyers α are realized;

[1] In Bayesian games, it is always possible to reduce the size of the signal space to the size of the action space recommended in equilibrium by the revelation principle (Myerson, 1982). Therefore the signal space can be restricted to the size of the state space.

3) The buyers simultaneously choose an experiment $E \in \mathcal{E}$ and pay the fee $t(E)$;

4) The buyers observe the signal $s \in E$ and choose an action a.

The cost of producing information for the data seller is zero. We concentrate on the selling mechanism only. The data seller commits to the menu of experiments before the state and players' types are realized. Data buyers make payment according to the quality of the experiment they receive. The ex-post utility from stage 4 does not affect the payment.

3. Mechanism

In this section, we describe the data seller's problem.

The monopolist's problem is to maximize her profit by choosing the quality of the experiment offered to each type and the associated price for each information level. Monopolist designs $\mu = \{E(\theta), t(\theta)\}$ to maximize the profit:

$$\Pi = \max_{\mu} \sum_{\theta} t(\theta).$$

We concentrate on the incentive-compatible (IC) direct mechanism μ. Following Dasgupta, Hammond, and Maskin (1979), the mechanism is direct if agents report their types to the principal, $\theta \in \{p, q\}$, and principal sends back an action recommendation from the probability distribution according to the reported types, $E(\theta) = (S(\theta), \pi(\theta))$. Under a direct mechanism, each signal corresponds to a different action choice, $a(s_i|\theta) = a_i$ for all $s_i \in S$.

μ induces a communication game where the agent's strategy is to choose (θ, a). In the first stage, agents choose the type to report θ, they receive an experiment $E(\theta)$, and make a transfer $t(\theta)$ according to the reported type. After the acquisition stage, agents play the underlying state-coordination game, where they choose a according to their types and signals received from the principal. In principle, a type θ can misreport his type $\theta' \neq \theta$, and deviate from the recommended action by playing a_i after observing s_j, where $i \neq j$. The strategy is truthful if the player is honest, and obedient if the player follows the recommendation.

Let's denote the experiment designed for each type by $E(q) = (S, \pi)$, and $E(p) = (S, \tilde{\pi})$. If each agent is honest and obedient, then the net expected utility of type θ from μ is $U(\mu, \theta) - t(\theta)$.

Before the transfer, each type receives the following gross utilities $U(\mu, \theta)$ from μ:

$$U(\mu, q) = q\pi_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \tag{10}$$

$$U(\mu, p) = p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \tag{11}$$

Each type participates in the mechanism only if the utility from μ is higher than an outside option $u(\theta)$. The agent who doesn't participate in μ chooses a_i which maximizes his expected utility $u(\theta)$, given every other agent is honest, obedient, and participates in μ.

$$u(q) = \max\{q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\}$$

$$u(p) = \max\{p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\} \tag{12}$$

Type θ participates in mechanism μ if IR constraint holds:

$$U(\mu, \theta) - t(\theta) \geq u(\theta).$$

The honest and obedient strategies form a Bayesian equilibrium of the communication game if a direct mechanism μ is incentive-compatible (IC).[2]

Suppose a type θ is obedient, but reports $\theta' \neq \theta$, and receives $E(\theta')$. Given all other agents are honest and obedient, the utility of θ is $U(E(\theta'), \theta)$:

$$U(E(p), q) = q\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \tag{13}$$

$$U(E(q), p) = p\pi_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \tag{14}$$

A type θ is honest if the following IC constraint holds:

$$U(\mu, \theta) - t(\theta) \geq U(E(\theta'), \theta) - t(\theta').$$

Under IC constraint, the net utility of the obedient and honest type θ is at least as good as the net utility he would get from misreporting his type. However, the misreporting type can still find the deviation profitable.

[2]According to Myerson (1982), the payoff each player receives under IC mechanism is at least as good as the payoff they would receive under more general mechanism . Hence, there is no loss of generality in concentrating on IC mechanism.

Value of information. Players acquire information to enhance the quality of their decision making. From the IR constraint, the value of an experiment $E(\theta)$ for an honest and obedient type θ is

$$V(\mu, \theta) = U(\mu, \theta) - u(\theta). \tag{15}$$

$$V(\mu, q) = q\pi_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] -$$

$$- \max\{q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$$

$$V(\mu, p) = p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] -$$

$$- \max\{p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\}$$

$V(\mu, \theta)$ function reveals the following result:

LEMMA 2. *For all types, the value of experiment increases in the precision of own signal.*

The terms $[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq 0$, and $[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \geq 0$ are always positive. Increasing π_1 and π_2 raises the valuation of the type L. And increasing $\tilde{\pi}_1$, and $\tilde{\pi}_2$ raises valuation of the type H. The formal proof of lemma 2 is presented in appendix C.1.

If the monopolist could observe the types, she could send a fully informative experiment $\bar{E}$ to each type and charge

$$V(\mu, \theta) = U(\mu, \theta) - u(\theta) = 1 - \max\{\theta, (1-\theta)\} = \min\{\theta, (1-\theta)\} \tag{16}$$

Each type $\theta = \{q, p\}$ has a different WTP for the fully informative experiment. Let's assume:

ASSUMPTION 1.

$$q > p > 1 - q \quad \textit{and} \quad q > \frac{1}{2}.$$

The informativeness of prior is measured by $|\frac{1}{2} - \theta|$. From assumption 1, type p has more uncertainty about all states relative to type q, $|\frac{1}{2} - p| < |\frac{1}{2} - q|$. The less informed player p has a higher willingness to pay for the $\bar{E}$,

$$V(\mu, p) = \min\{p, (1-p)\} > V(\mu, q) = 1 - q.$$

Hence, player p is called the high type (H), and player q is called the low type (L).

To simplify the optimization process we further assume:

CONJECTURE 1.

$$u(q) = \max\{q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\} = q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1].$$

Under this conjecture, a type q player chooses a_1 without supplemental information,

$$q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]. \tag{17}$$

Given assumption 1, the conjecture (1) is not very unusual. In appendix C.1, we substitute the optimal mechanism to verify the conjecture holds. There is no restriction on $u(p)$.

From conjecture 1, the valuation of type q is,

$$V(\mu, q) = -(1-\pi_1)q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \tag{18}$$

Optimization problem. Under IC mechanism every honest type is obedient, but we still need to control for the action choice of misreporting type. At first, we assume that a misreporting type is also obedient and solve the relaxed optimization problem. Later we check the obedience constraint of the misreporting type to verify that the honest and obedience strategies are the equilibria of the communication game.

The principal designs a direct IC mechanism μ to maximize the expected profit according to $F \in \triangle\Theta$:

$$\begin{aligned}\Pi &= 2\gamma^2 t(p) + 2\gamma(1-\gamma)(t(p)+t(q)) + 2(1-\gamma)^2 t(q) = \\ &= 2[\gamma^2 t(p) + \gamma t(p) + \gamma t(q) - \gamma^2 t(p) - \gamma^2 t(q) + t(q) - 2\gamma t(q) + \gamma^2 t_L] = \\ &= 2[\gamma t(p) + (1-\gamma)t(q)]\end{aligned} \tag{19}$$

The profit function[3] (19) is maximized subject to the following constraints:

IC of L: $V(\mu, q) - t(q) \geq V(E(p), q) - t(p)$

IC of H: $V(\mu, p) - t(p) \geq V(E(q), p) - t(q)$

IR of L: $V(q) \equiv V(\mu, q) - t(q) \geq 0$

IR of H: $V(p) \equiv V(\mu, p) - t(p) \geq 0$

Binding constraints. Next, we identify the binding constraints.

LEMMA 3. *In an optimal menu:*

i) *The IR constraint of type L binds;*
ii) *The IC constraint of type H binds;*
iii) *The IC constraint of type L is slack.*

PROOF OF LEMMA 3.

i) The IR constraint of type L must bind. The proof follows in two steps, at first, we show at least one of the participation constraints should bind, then verify IR of L binds.
From IC of type H: $V(\mu, p) - V(E(q), p) \geq t(p) - t(q)$.
From IC of type L: $t(p) - t(q) \geq V(E(p), q) - V(\mu, q)$.
IC of type H and L hold simultaneously if,

$$V(\mu, p) - V(E(q), p) \geq t(p) - t(q) \geq V(E(p), q) - V(\mu, q) \tag{20}$$

The monopolist can increase prices charged to each type until one of the IR binds.
Now assume IR of type H binds, but not L, hence $V(\mu, p) = t(p)$. The equation (20) becomes

$$V(\mu, p) - V(E(q), p) \geq V(\mu, p) - t(q) \geq V(E(p), q) - V(\mu, q) \tag{21}$$

Because IR of L does not bind, the monopolist can further increase $t(q)$ without violating the equation (21), which is a contradiction. Hence it must be that IR of type L binds, $t(q) = V(\mu, q)$.

[3]For simplicity the multiplier 2 in profit function is ignored in further calculations as it does not affect the choice of optimal signal quality.

ii) The IC constraint of type H must bind. By contradiction assume IC constraint of H does not bind, and consider 2 possible cases:
1) The IR of type H does not bind: $V(\mu, p) - t(p) > V(E(q), p) - t(q)$. Then it is possible to increase $t(p)$ until either of IC or IR of type H binds.
2) The IR of type H binds, but not IC: $0 > V(E(q), p) - V(\mu, q)$. By definition, type p values full information more than type L, $V(\bar{E}, p) > V(\bar{E}, q)$. This implies $V(\bar{E}, p) - V(\bar{E}, q) > 0$. If the difference between the two valuations changes from negative to positive, for some level of information it must be equal to zero, $0 = V(E(q), p) - V(\mu, q)$.
The monopolist will increase the informativeness of information type L receives until IC of H binds. Hence IC of type H must bind.

iii) The IC of type L is slack. By contradiction assume IC of type L binds:
$V(\mu, q) - t(q) = V(E(p), q) - t(p)$.
From part i) the IR of type L binds, hence the LHS of the IC (L) is zero,
$0 = V(E(p), q) - t(p)$.
The monopolist can always raise $t(p)$ by increasing the quality of $E(p)$ to maximize her profit without violating IC(L). Hence IC of type L is slack.

■

From lemma 3 two out of four constraints are binding. We use these binding constraints to find the prices charged to each type and IR of type H.

To find $t(q)$, (10) and (17) are substituted into the IR of type L:

$$\begin{aligned} t(q) &= V(\mu, q) = U(\mu, q) - u(q) = \\ &= -(1-\pi_1)q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \end{aligned} \tag{22}$$

To find $t(p)$, (11) and (14) are substituted into the IC of type H:

$$\begin{aligned} t(p) &= V(\mu, p) - V(E(q), p) + t(q) = U(\mu, p) - U(E(q), p) + t(q) = \\ &= p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] - \\ &- p\pi_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] - (1-p)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] + t(q) = \\ &= p(\tilde{\pi}_1 - \pi_1)[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)(\tilde{\pi}_2 - \pi_2)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] + t(q) \end{aligned}$$

Substituting $t(q)$ from (23) the price charged to the type H becomes:

$t(p) = U(\mu, p) - U(E(q), p) + U(\mu, q) - u(q) =$

$= [\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1][p(\tilde{\pi}_1 - \pi_1) - (1-\pi_1)q] + [\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2][(1-p)(\tilde{\pi}_2 - \pi_2) + (1-q)\pi_2] =.$

$= [\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1][p\tilde{\pi}_1 + (q-p)\pi_1 - q] + [\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2][(1-p)\tilde{\pi}_2 - (q-p)\pi_2]$

4. Optimal menu of experiments

In this section we characterize the optimal menu of experiments μ.

The profit function (19) is a linear combination of prices. The maximization of profit implies the maximization of each price. This leads to the next proposition.

PROPOSITION 4. *(In an optimal menu):*

i) *type H receives a fully informative experiment* $\bar{E}$*;*
ii) *the experiment designed for type L should satisfy* $\pi_1 = 1$.

Below, we present a quick proof for this proposition. The formal proof is presented in appendix C.1.

At first, let's check $t(q)$,

$$t(q) = -(1-\pi_1)q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$$

The terms $[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq 0$, and $[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \geq 0$ are positive. $t(q)$ is increased by setting $\pi_1 = \pi_2 = \tilde{\pi}_2 = 1$. Once we set $\pi_1 = 1$ the value of $\tilde{\pi}_1$ does not have any effect on $t(q)$.

Now let's examine $t(p)$,

$$t(p) = [\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1][p\tilde{\pi}_1 + (q-p)\pi_1 - q] + [\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2][(1-p)\tilde{\pi}_2 - (q-p)\pi_2]$$

The terms $[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq 0$, and $\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] \geq 0$ are positive. The value of the function clearly increases by setting $\pi_1 = \tilde{\pi}_1 = \tilde{\pi}_2 = 1$. However, the effect of π_2 is not clear.

Combining two results together, setting $\pi_1 = \tilde{\pi}_1 = \tilde{\pi}_2 = 1$ raises $t(q)$ and $t(p)$, and therefore, increases the profit. Hence, in the optimal menu, the type H receives the fully informative experiment $E(p) = \bar{E}$, and type L receives $\pi_1 = 1$.

From lemma 2, the willingness to pay for the experiment increases in the precision of own signal. The monopolist can increase the price by increasing the precision of the signal. The type H, who has a higher willingness to pay, receives a fully informative experiment. At the same time, the monopolist always informs the type L about the state he is likely to choose without supplemental information, in order not to decrease his incentives to purchase the experiment. In our case, the type L is well-informed about the state ω_1, hence an experiment where $\pi_1 = 1$.

The monopolist's problem simplifies down to choosing π_2 only, which can be denoted by π from now on. Combining the results from Lemma 3 and proposition 4, the monopolist charges the following prices:

$$t(q) = (1-q)\pi[\gamma + (1-\gamma)\pi] \tag{23}$$

$$t(p) = (1-p)(1-\pi)[\gamma + (1-\gamma)\pi] + t(q) \tag{24}$$

Characterization of the full menu. From the structural properties, it is already established that in the optimal menu the type H receives the fully informative experiment $\bar{E}$, and type L is always informed about the most likely state it is going to choose with according to his prior π_1. In this section, we solve the optimization problem to find $\pi_2 \equiv \pi$.

From lemma 3, IC of type L is slack. The prices charged to each type are found from the IR of type L, and IC of type H, and substituted into the profit function (19). At first, we maximize profit to find the optimal π. Later we verify that the optimal menu satisfies the IR constraint of type H and the obedience constraint of misreporting type H.

To solve the optimization problem, prices found in (23) and (24) are substituted into the profit function (19), and maximized subject to the IR constraint of type H.

After (23) and (24) are substituted into (19), the maximization problem is:

$$\begin{aligned}
\max_{\pi} \quad & \Pi = t_L + \gamma(t_H - t_L) = \\
& = (1-q)\pi[\gamma + (1-\gamma)\pi] + \gamma(1-p)(1-\pi)[\gamma + (1-\gamma)\pi] = \\
& = [(1-q) - \gamma(1-p)][\gamma\pi + (1-\gamma)\pi^2] + \gamma(1-p)[\gamma + (1-\gamma)\pi]
\end{aligned}$$

An interior solution to maximization problem should satisfy FOC:

$$\frac{\partial \Pi}{\partial \pi} = 2(1-\gamma)[(1-q)-\gamma(1-p)]\pi + \gamma[(1-q)-\gamma(1-p)+(1-\gamma)(1-p)] = 0,$$

and SOC:

$$\frac{\partial^2 \Pi}{\partial \pi^2} = 2(1-\gamma)[(1-q)-\gamma(1-p)] \leq 0.$$

To verify the sign of FOD and SOD with respect to π, let's define:

$$A \equiv [(1-q)-\gamma(1-p)], \quad \text{and} \quad B \equiv (1-\gamma)(1-p).$$

Then, FOC becomes,

$$\frac{\partial \Pi}{\partial \pi} = 2(1-\gamma)A\pi + \gamma(A+B) = 0. \tag{25}$$

As $B > 0$, if:

$$A > 0 \Longrightarrow (1-q) > \gamma(1-p) \Longrightarrow \gamma < \frac{1-q}{1-p},$$

then both $\frac{\partial \Pi}{\partial \pi}$ and $\frac{\partial^2 \Pi}{(\partial \pi)^2}$ are positive. That is, when Π is convex, it is also monotonically increasing in π. In this case, we clearly have the corner solution $\pi^* = 1$.

Now let's reverse the above inequality :

$$A < 0 \Longrightarrow (1-q) < \gamma(1-p) \Longrightarrow \gamma > \frac{1-q}{1-p},$$

so that $\frac{\partial^2 \Pi}{(\partial \pi)^2} < 0$, i.e. Π is concave. Clearly, we can have an interior solution in π only if $\frac{\partial \Pi}{\partial \pi}|_{\pi=0} > 0$ and $\frac{\partial \Pi}{\partial \pi}|_{\pi=1} < 0$.

$\frac{\partial \Pi}{\partial \pi}|_{\pi=0} > 0$ holds when:

$$A + B > 0 \Longrightarrow B > -A \Longrightarrow \gamma < \frac{(1-q)+(1-p)}{2(1-p)} = \frac{1}{2} + \frac{1}{2}\frac{1-q}{(1-p)} < 1,$$

otherwise the solution would be at the opposite corner $\pi^* = 0$, as $\frac{\partial \Pi}{\partial \pi}$ would be negative for any $\pi \in [0,1]$.

As $\frac{\partial \Pi}{\partial \pi}$ is monotonically decreasing in π now, to have an interior solution we just need to impose an additional condition $\frac{\partial \Pi}{\partial \pi}|_{\pi=1} < 0$, that is:

$$2(1-\gamma)A+\gamma(A+B) = 2A+\gamma(B-A) < 0 \Longrightarrow \gamma(B-A) < -2A \Longrightarrow \gamma > \frac{2(1-q)}{(1-q)+(1-p)}.$$

Clearly, we can have an interior solution in π^* only if: $\gamma \in (\frac{2(1-q)}{(1-q)+(1-p)}; \frac{1}{2} + \frac{1}{2}\frac{1-q}{(1-p)})$ and from (25) this solution is:

$$\pi^* = -\frac{\gamma(A+B)}{2(1-\gamma)A} = -\frac{\gamma[(1-q) - \gamma(1-p) + (1-\gamma)(1-p)]}{2(1-\gamma)[(1-q) - \gamma(1-p)]} =$$
$$= \frac{\gamma(1-\gamma)(1-p) - \gamma[\gamma(1-p) - (1-q)]}{2(1-\gamma)[\gamma(1-p) - (1-q)]} = \frac{1}{2}[\frac{\gamma(1-p)}{\gamma(1-p) - (1-q)} - \frac{\gamma}{1-\gamma}].$$

π^* is a continuous function of γ. It takes negative value for $\gamma \geq \frac{(1-q)+(1-p)}{2(1-p)}$, and $\pi^* > 1$ for $\gamma \leq \frac{2(1-q)}{(1-q)+(1-p)}$. Combining all results together the optimal π^* is:

- $\pi^* = 1$ for $\gamma \leq \frac{2(1-q)}{(1-q)+(1-p)}$,
- $\pi^* = \frac{1}{2}[\frac{\gamma(1-p)}{\gamma(1-p)-(1-q)} - \frac{\gamma}{1-\gamma}]$ for $\gamma \in (\frac{2(1-q)}{(1-q)+(1-p)}, \frac{(1-q)+(1-p)}{2(1-p))}$,
- $\pi^* = 0$ for $\gamma \geq \frac{(1-q)+(1-p)}{2(1-p)}$.

Next, we check if π^* satisfies the participation and obedience constraints of type H.

Checking for the participation and obedience constraints. In this section, we check if the price charged to H type from equation (24), satisfies the IR constraint of H, and the obedience constraint (OC) of the non-truthful H type.

From the equation (24),the price charged to H type is

$$t(p) = (1-p)(1-\pi)[\gamma + (1-\gamma)\pi] + (1-q)\pi[\gamma + (1-\gamma)\pi] \tag{26}$$

The IR of type H is:

$$V(p) = V(\mu, p) - t(p) = U(\mu, p) - u(p) - t(p) =$$
$$= p + (1-p)[\gamma + (1-\gamma)\pi] - \max\{p, (1-p)[\gamma + (1-\gamma)\pi]\} - t(p) \geq 0$$

The participation constraint of type H contains the expected utility term from the outside option: $u(p) = \max\{p, (1-p)[\gamma + (1-\gamma)\pi]\}$.

We will analyse each action choice separately. At first, assume,

$$p > (1-p)[\gamma + (1-\gamma)\pi]. \tag{27}$$

Type H chooses action a_1 without supplementary information, and receives $u(p) = p$. This is the case when type L and type H choose the same action a_1 with their prior information only. Borrowing the terminology from Bergemann, Bonatti, and

Smolin (2018), their beliefs are congruent. With congruent beliefs, IR constraint of type H is satisfied if,

$$t(p) \leq (1-p)[\gamma + (1-\gamma)\pi].$$

From equation (26) this inequality always holds. Hence, the IR constraint of type H is satisfied for all

$$\pi^C = \max\{0, \min\{1, \pi^*\}\}.$$

Now assume,

$$p < (1-p)[\gamma + (1-\gamma)\pi]. \tag{28}$$

Type H chooses action a_2, and receives the expected utility $u(p) = (1-p)[\gamma + (1-\gamma)\pi]$. This is the case when types L and H choose different actions with their prior information only. Their beliefs are non-congruent. Then IR constraint of type H is satisfied if,

$$t(p) \leq p.$$

The price charged to H type satisfies the IR constraint when,

$$p \geq (1-p)(1-\pi)[\gamma + (1-\gamma)\pi] + (1-q)\pi[\gamma + (1-\gamma)\pi]. \tag{29}$$

Otherwise, it is optimal for the monopolist to set $\pi = 1$.

COROLLARY 1. *When beliefs are non-congruent, the participation constraint of type H is satisfied, $V^N(p) \geq 0$, for all*

$$\bar{\pi}^N \geq \frac{\gamma(1-p) - p}{\gamma(q-p) - (1-\gamma)(1-p)}.$$

The proof of Corollary 1 can be found in Appendix C.1. Hence, the IR constraint of type H is satisfied for all

$$\pi^N = \max\{\bar{\pi}, \min\{1, \pi^*\}\}.$$

From the IC contract, every honest type is also obedient. We need to check the obedience constraint (OC) for the non-truthful type. Non-truthful L type does not have an incentive to deviate from the recommended action, as he receives full information. Type H doesn't have an incentive to deviate from the recommended action after s_2. He obeys the recommendation after observing s_1 if his utility from choosing a_1 is higher than from a_2. With probability $P(\omega_1|s_1) = p$, he receives payoff 1 if chooses a_1, because all other players obey the recommendation of s_1 as well. If he deviates and chooses a_2, with probability $P(\omega_2|s_1) = (1-p)(1-\pi)$ he receives $[\gamma + (1-\gamma)\pi]$, because with probability γ he meets another H type, and with probability $(1-\gamma)$ he plays the underlying game with an obedient L type. The obedience constraint becomes:

$$p \geq (1-p)(1-\pi)[\gamma + (1-\gamma)\pi].$$

From (27), the OC is satisfied with congruent beliefs. From (29), it is satisfied with non-congruent beliefs as well.

5. Results

The optimal menu was partially characterized in proposition 4. In this section, we complete the characterization of the optimal menu by describing the information received by type L. The results are presented in three propositions which show the optimal menu for congruent beliefs, non-congruent beliefs and the general comparative static analysis.

PROPOSITION 5. *With congruent beliefs, in an optimal menu the type L receives:*

i) *Fully revealing information* $\pi = 1$, *when* $\gamma < \frac{2(1-q)}{(1-q)+(1-p)}$;
ii) *Partial information* $\pi = \frac{1}{2}[\frac{\gamma(1-p)}{\gamma(1-p)-(1-q)} - \frac{\gamma}{1-\gamma}]$, *when* $\gamma \in (\frac{2(1-q)}{(1-q)+(1-p)}; \frac{1}{2} + \frac{1}{2}\frac{1-q}{(1-p)})$;
iii) *Zero information* $\pi = 0$, *when* $\gamma > \frac{1}{2} + \frac{1}{2}\frac{1-q}{(1-p)}$.

Type L receives full information only if the proportion of type H in the population is low. As the number of type H increases, the informativeness of the experiment type L receives decreases, and after some point, L receives no information. There are two explanations for this result.

The first explanation is that when the proportion of type H in population is significant enough the monopolist can extract full surplus by excluding type L from the menu. As the frequency of type H decreases, discriminating against type L becomes less attractive, and the monopolist starts to sell information to type L as well. However, the difference caused by the proportions of the types can only explain the extreme cases, where type L receives either full or zero information. In the coordination game, type L receives partial information for certain parameters, which is caused by the second effect.

The second reason for the change in information is the externalities in prices. The underlying game requires players to coordinate in the correct state. Players receive a positive payoff only if their partner also manages to guess the state. Hence, the willingness to pay for the information increases not only in own but also in the precision of the partner's signal. When the proportion of type L is high there is a high chance for type H to be matched with type L. Hence, the informativeness of signal type L receives should also be higher not to decrease WTP of type H. As the proportion of L decreases, its effect on the WTP of type H also reduces and becomes zero at some point.

PROPOSITION 6. *With non-congruent beliefs, in an optimal menu the type L receives:*

i) *Fully revealing information* $\pi = 1$, *when* $\gamma < \frac{2(1-q)}{(1-q)+(1-p)}$;
ii) *Partial information* $\pi = \max\{\bar{\pi}, \frac{1}{2}[\frac{\gamma(1-p)}{\gamma(1-p)-(1-q)} - \frac{\gamma}{1-\gamma}]\}$, *when* $\gamma > \frac{2(1-q)}{(1-q)+(1-p)}$;
iii) *But never zero information* $\pi \neq 0$.

As in the congruent beliefs case type L receives full information only if the proportion of type H in the population is low. As the frequency of type H increases, the informativeness of the experiment type L receives decreases. However, type L is never excluded from the optimal menu.

The first explanation is when the proportion of type H in the population is low, discriminating against type L is not attractive, and the monopolist sells information to all types. As the proportion of type H increases, the data seller deteriorates the quality of information offered to type L to extract more surplus from type H.

The second explanation is the positive externalities in prices. When the frequency of type H decreases, the probability of being matched with type L increases. Hence, the expected utility of type H also depends on the quality of experiment type L receives.

There is also a difference between the results under congruent and non-congruent priors. When priors are non-congruent, type L always receives some information, and never excluded from the menu. Types value the information about states differently. Under congruent priors, all types are better informed about the state ω_1. Hence they want to improve the information about the state ω_2. In this case, the monopolist restricts information supplied to type L, in order not to distort the incentives of type H. Under non-congruent priors, type L is better informed about

ω_1, and needs additional information about ω_2. Type H, in the opposite, is better informed about ω_2 and requires information about ω_1. Hence the monopolist does not need to exclude type L from the menu completely. She can always provide type L with a level of information about ω_2, that makes type H indifferent between purchasing this experiment and not. This result is also observed in the market without strategic interactions (Bergemann, Bonatti, and Smolin, 2018).

The information type L receives is further characterized in the next proposition.

PROPOSITION 7. *The quality of the experiment type L receives is:*

i) *decreasing in the frequency of type H, γ;*
ii) *decreasing in prior of type L, q;*
iii) *increasing in prior of type H, p, when beliefs are congruent or the menu is discriminatory.*

PROOF OF PROPOSITION 7.

To prove the proposition, π^* is differentiated with respect to each parameter.

$$\pi^* = \frac{1}{2}[\frac{\gamma(1-p)}{\gamma(1-p)-(1-q)} - \frac{\gamma}{1-\gamma}]$$

$$\frac{\partial \pi^*}{\partial \gamma} = \frac{1}{2}\frac{(1-p)[\gamma(1-p)-(1-q)]-(1-p)\gamma(1-p)}{[\gamma(1-p)-(1-q)]^2} - \frac{1-\gamma+\gamma}{(1-\gamma)^2} =$$
$$= -\frac{1}{2}[\frac{(1-p)(1-q)}{[\gamma(1-p)-(1-q)]^2} + \frac{1}{(1-\gamma)^2}] < 0$$

$$\frac{\partial \pi^*}{\partial q} = -\frac{1}{2}\frac{\gamma(1-p)}{[\gamma(1-p)-(1-q)]^2} < 0$$

$$\frac{\partial \pi^*}{\partial p} = \frac{1}{2}\frac{-\gamma[\gamma(1-p)-(1-q)]+\gamma[\gamma(1-p)]}{[\gamma(1-p)-(1-q)]^2} = \frac{1}{2}\frac{\gamma(1-q)}{[\gamma(1-p)-(1-q)]^2} > 0$$

■

Part (i) is a confirmation of the previous two results, the quality of the experiment type L receives increases as the frequency of type H decreases. From part (ii) when q increases, type L becomes better informed about ω_1. Type L values supplemental information less, hence, excluding him from the menu increases the surplus of the monopolist. Part (iii) should be interpreted separately for each prior case.

Under congruent priors, increasing p implies the difference between informativeness of types, $q - p$, decreases. Reducing the quality of experiment for type L is optimal only when types are sufficiently different in their valuations. Under non-congruent priors, increasing p also implies type H becomes less informed about the state. The monopolist can increase the quality sold to type L, without distorting the incentives of type H.

6. Discussion

The literature on the global games that consider the demand side of the information market shows that players who want to coordinate their actions on the unknown state acquire common information, to increase the correlation in their actions. Our model shows that the monopolist data seller has an incentive to provide differentiated information to the buyers with heterogeneous priors.

Monopolist selling information to the buyers with different valuations can sell a differentiated product to extract a full surplus. Previously, Bergemann, Bonatti, and Smolin (2018) have characterized the optimal menu supplied by the monopolist in the market where buyers do not interact. In this chapter, we derived the properties of the information supplied in the strategic environment. We show that the buyers with the highest valuation for the quality always receive fully informative signals perfectly revealing the state. Buyers with low willingness to pay can receive full information if there is a small proportion of high types in the population. Otherwise, the profit-maximizing data seller deteriorates the quality of information offered to type L.

The coordination motive changes the structure of the information offered to the type L relative to the non-strategic environment. A buyer who wants to guess the state is also concerned with the decision of another player. Coordinating players value not only their own but also the precision of the signal received by another player. Correlation in actions causes externalities in prices; each type is also willing to pay for the precision of the signal received by their partner. Therefore, type L receives a positive amount of informative signal with a lower proportion of type H in the population. When the frequency of type H decreases, the informativeness of the signal type L receives decreases incrementally, because type H does not value the information about the state only, but also how well types L are informed about it.

In this chapter, we considered a model where perfect coordination was required to obtain positive payoffs. Our plan for the future is to analyse a more general model, where the strategic environment can require coordination or anti-coordination of actions. We plan to analyse how the change in the coordination motive affects the optimal menu provided by the monopolist.

APPENDIX A

Appendices for Chapter 1

A.1. The Characterization of the Ergodic Distribution

The content of this section is based on Appendix A page 518 from Myatt and Wallace (2008b).

Adding mutations to the process guarantees the uniqueness of limit distribution, and letting noise to vanish results with the unique outcome prediction called stochastically stable state (SSS). SSS can be found by application of the "tree surgery" method proposed by Foster and Young (1990); Kandori, Mailath, and Rob (1993); Young (1993). This technique is based on the analysis of "the set of trees rooted at z."

Let all states $z \in Z$ be the nodes of the complete directed graph in Z. A z-tree h is the set of directed edges leading to z, in such a way that, each $z' \neq z$ has a unique successor. There are several possible ways to construct a tree rooted at z. The set of all possible trees rooted at z is denoted by H_z. According to Freidlin and Wentzell (1998) the unique ergodic distribution over states is defined as:

$$p_z = q_z / \sum_{z' \in Z} q_{z'}, \text{where } q_z = \sum_{h \in H_z} \prod_{(s,s') \in h} Pr[s \to s'].$$

p_z is demanding to calculate, instead SSS can be found from the relative likelihood $\frac{p_z}{p_{z'}} = \frac{q_z}{q_{z'}}$. When $\varepsilon \to 0$, some probabilities $Pr[s \to s']$ become zero. Long run outcome depends on the rate this probabilities vanish.

The rate at which the transition probabilities vanish is the "exponential cost" $\mathcal{E}$ of a probability (Myatt and Wallace, 2003). It is denoted as $p(\varepsilon) = \tilde{o}(\mathcal{E})$, or $\mathcal{E}(p(.)) = \mathcal{E}$ and means $p(\varepsilon)$ behaves as $\exp \frac{-\mathcal{E}}{\varepsilon}$ does, as $\varepsilon \to 0$. For any set of transition probabilities $\{p_l(\varepsilon)\}$ with exponential cost $\mathcal{E}_l$ the following properties hold:

$$\prod \tilde{o}(\mathcal{E}_l) = \tilde{o}(\sum \mathcal{E}_l), \sum \tilde{o}(\mathcal{E}) = \tilde{o}(\min \mathcal{E}_l), a \times \tilde{o}(\mathcal{E}) = \tilde{o}(\mathcal{E}), \text{and} \mathcal{E}_l > \mathcal{E}_{l'} \Rightarrow \lim_{\varepsilon \to 0} \frac{\tilde{o}(\mathcal{E}_l)}{\tilde{o}(\mathcal{E}_{l'})} = 0. \quad (30)$$

Let's denote the cost of transition from z to z', where z and z' differ just by one step as $\mathcal{E}_{zz'} \equiv \mathcal{E}\big(Pr[z \to z']\big)$, and the exponential cost of the tree h by $\mathcal{E}_h \equiv \sum_{(z,z')\in h} \mathcal{E}_{z,z'}$. From properties (30):

$$\mathcal{E}(q_z) = \mathcal{E}\Bigg(\sum_{h\in H_Z}\prod_{(s,s')\in h} Pr[s \to s']\Bigg) = \min_{h\in H_z} \mathcal{E}\Bigg(\prod_{(s,s')\in h} Pr[s \to s']\Bigg) = \min_{h\in H_z} \mathcal{E}_h.$$

Further, $\mathcal{E}(q_z) < \mathcal{E}(q'_z) \Rightarrow \lim_{\varepsilon\to 0}[q'_z/q_z] = 0$; the state with the least exponential-cost-rooted tree attracts all probability when noise vanishes away. Such a state is called "stochastically stable" .

Denote the limit set of the process by z^*. The following lemma holds:

LEMMA 4. *The states in $z \in z^*$ attract all probability in the limit.*

$$\lim_{\varepsilon\to 0}\sum_{z\in z^*} p_z = 1, \textit{such that } Z^* = \{z \in Z : \min_{h\in H_z}\{\mathcal{E}_h\} \leq \min_{z'\in Z}\min_{h'\in H_{z'}}\{\mathcal{E}_{h'}\}\}$$

PROOF OF LEMMA 4. Abusing notations denote the least exponential cost tree rooted at z by $\mathcal{E}(z)$. To prove lemma we need to show that $\mathcal{E}(z) < \mathcal{E}(z')$, for all $z' \neq z$.

There exists a zero cost path to the states $z \in z^*$ from all other states $z' \neq z$. A tree rooted at one of the limit states $z \in z^*$ is constructed by adding the costly path from all other states in z^* to the state z.

Lets take any state z' from the basin of attraction of z. To construct a tree rooted at z', one of the zero cost path leading from z' to z is deleted and replaced with a new costly path in the opposite direction. It follows that $\mathcal{E}(z) < \mathcal{E}(z')$.

By the same logic, all states $z \in z^*$ have lower exponential cost than the states chosen from their basin of attraction. It implies that, limit states stochastically dominate the transient states, and one of them attracts all probability in the long run when $\varepsilon \to 0$. ■

A.2. Results

Altruists. Following lemmas are useful to prove the Proposition 1.

LEMMA 5. *When R types are A the limit states are:*

$$\begin{cases} m > n - s, \Longrightarrow z^* \in \{z^\diamond, z^\ddagger\}; \\ m \leq n - s, \Longrightarrow z^* \in \{z^\diamond\}. \end{cases}$$

PROOF OF LEMMA 5. From lemma 1, all possible limit states are $z^* \in \{z^0, z^\dagger, z^\diamond, z^\ddagger\}$.

For $l < m$ the state $z^\dagger$ cannot be one of the limit states. In this case $x + y = m > l$, all R types join the team with zero cost. For $l = 0$ the state z^0 cannot be one of the limit states. In this case $x + y = 0 = l$, all R types join the team with zero cost. Hence the set of limit states is $z^* \in \{z^\diamond, z^\ddagger\}$.

For $m \leq n - s$, the state $z^\ddagger$ can not be a limit state, as $m - (n - s) \leq 0$. Hence the set of limit states is $z^* \in \{z^\diamond\}$. ■

A. Before proceeding to lemmas, let's denote the least cost exit from the set of states $z^\ddagger \in Z_{m-(n-s),n-s}$ by $A = \sum_{z^\ddagger \in Z_{m-(n-s),n-s}} \min[\beta_S, \delta_S, \delta_R]$.

There are three possible ways to escape a state $z^\ddagger$. It is either through the death of one S type with a cost δ_S. In this case the state moves to $Z_{m-n+s-1,n-s}$, from where it is optimal for all S types to leave without any cost. The second way is through the death of R type with a cost δ_R. In this case the state moves to $Z_{m-n+s,n-s-1}$, from where it is optimal for all S types to leave without any cost. The third way to escape a state $z^\ddagger$ is through the birth of S type with cost β_S. In this case one of the S types leaves the team being replaced with a new S type from outside. Hence, the cheapest way out from any $z^\ddagger$ costs $\min[\beta_S, \delta_S, \delta_R]$, and the cost of moving out of all states in $Z_{m,0}$ is $A \equiv \sum_{z^\ddagger \in Z_{m-n+s,n-s}} \min[\beta_S, \delta_S, \delta_R]$.

A can also be used to find the cost of transition among the states $z^\ddagger \in Z_{m-(n-s),n-s}$. If to begin in one of the states $z^\ddagger$ each time one of the members leaves at a cost $\min[\beta_S, \delta_S, \delta_R]$, a new member from outside can join without any cost. If to subtract the cost of exit from the last state in the transition chain, the cost of moving between the states of the set $Z_{m-(n-s),n-s}$ is $A - \min[\beta_S, \delta_S, \delta_R]$

LEMMA 6. *When $m > n - s$, the least exponential-cost tree rooted at $z^\ddagger$ satisfies:*

$\mathcal{E}(z^\ddagger) = [(m-1) - (n-s)]\beta_S + A - \min[\beta_S, \delta_S, \delta_R]$.

PROOF OF LEMMA 6.

From lemma 5, when $m > n - s$, the limit states are $z^* \in \{z^\diamond, z^\ddagger\}$.

The tree rooted at $z^\ddagger$ has branches leading to $z^\ddagger$ from $z^\diamond$ and from other states $z^\ddagger$.

The cheapest way to construct the link $z^\diamond \to z^\ddagger$ is through the costly birth of $(m-1)-(n-s)$ S type players. Then it is optimal for the last S type to join them. This brings the process to state $z^\ddagger$ at a cost $[(m-1)-(n-s)]\beta_S$. The cost of moving between the states $z^\ddagger$ was shown above $A-\min[\beta_S,\delta_S,\delta_R]$. Overall cost of this tree is

$$\mathcal{E}(z^\ddagger) = [(m-1)-(n-s)]\beta_S + A - \min[\beta_S,\delta_S,\delta_R].$$

■

LEMMA 7. *The least exponential-cost tree rooted at $z^\diamond$ satisfies:*

$$\mathcal{E}(z^\diamond) = \begin{cases} A-\min[\beta_S,\delta_S,\delta_R]+\min[\delta_S,\delta_R], & \textit{if } m > n-s; \\ 0, & \textit{if } m \le n-s. \end{cases}$$

PROOF OF LEMMA 7.

From lemma 5, when $m > n-s$, the limit states are $z^* \in \{z^\diamond, z^\ddagger\}$.

One way of constructing a tree rooted at $z^\diamond$ is adding branches leading to $z^\diamond$ from all $z^\ddagger$. The cost of this tree is $\sum_{z^\ddagger \in Z_{m-n+s,n-s}} \min[\delta_S,\delta_R]$.

Yet the cheaper way to construct the tree is having transitions between the states $z^\ddagger$, and adding a branch from last to $z^\diamond$. The cost of such tree is the stochastic potential of $z^\diamond$.

$$\mathcal{E}(z^\diamond) = A-\min[\beta_S,\delta_S,\delta_R]+\min[\delta_S,\delta_R].$$

■

PROOF OF PROPOSITION 1.

1) When $l < n-s$ and $m > n-s$, the good is produced only if $\min\{\mathcal{E}(z^\diamond),\mathcal{E}(z^\ddagger)\} = \mathcal{E}(z^\ddagger)$. From lemmas 6 and 7,

$$\mathcal{E}(z^\ddagger) < \mathcal{E}(z^\diamond) \Longrightarrow [(m-1)-(n-s)]\beta_S < \min[\delta_S,\delta_R].$$

2) When $m \le n-s$, the good is always produced. The limit set of such dynamics contain a single state $z^* \in \{Z_{0,n-s}\}$.

The claim is $\mathcal{E}(z^\diamond) < \mathcal{E}(z)$ for all $z = \{z \in Z_{x,y} | z \neq z^0\}$.

There exists zero cost path from all states $z \neq z^\diamond$ to $z^\diamond$. Hence $\mathcal{E}(z^\diamond) = 0$.

To construct a tree rooted at any $z \in Z$, where $z \neq z_\diamond$, the zero cost branch leading from z to $z^\diamond$ is deleted and replaced with a costly path in opposite direction. Hence, $\mathcal{E}(z_\diamond) < \mathcal{E}(z)$ for all $z \neq z_\diamond$. ■

Conditional cooperators. Following lemmas are useful to prove the Proposition 2.

LEMMA 8. *When R types are CC the limit states are:*

$$l < n - s \begin{cases} m > n - s, \Longrightarrow z^* \in \{z^0, z^\diamond, z^\ddagger\}; \\ m \leq n - s, \Longrightarrow z^* \in \{z^0, z^\diamond\}; \end{cases}$$
$$l \geq n - s \Longrightarrow z^* \in \{z^0, z^\ddagger\}.$$

PROOF OF LEMMA 8. From lemma 1, all possible limit states are $z^* \in \{z^0, z^\dagger, z^\diamond, z^\ddagger\}$.

For $l < m$ the state $z^\dagger$ cannot be one of the limit states. In this case, $x+y = m > l$, all R types join the team with zero cost. Hence, the set of limit states is $z^* \in \{z^0, z^\diamond, z^\ddagger\}$.

For $m \leq n - s$, the state $z^\ddagger$ can not be a limit state. In this case, $m - (n - s) \leq 0$, R types produce the good. Hence, the set of limit states is $z^* \in \{z^0, z^\diamond\}$.

For $l \geq n-s$, the state $z^\diamond$ can not be a limit state. In this case $x+y = n-s \leq l$, R types leave the team with zero cost. Hence, the set of limit states is $z^* \in \{z^0, z^\ddagger\}$. ■

LEMMA 9. *The least exponential-cost tree rooted at z^0 satisfies:*

$$\mathcal{E}(z^0) = \begin{cases} A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n - s - l)\delta_I, & \textit{if } l < n - s, \textit{ and } m > n - s; \\ A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R], & \textit{if } l \geq n - s; \\ (n - s - l)\delta_R, & \textit{if } l < n - s, \textit{ and } m \leq n - s. \end{cases}$$

PROOF OF LEMMA 9.

From lemma 8, when $l \geq n - s$, the limit states are $z^* \in \{z^0, z^\ddagger\}$.

The cheapest exit cost from any $z^\ddagger$ is $\min[\beta_S, \delta_S, \delta_R]$. If $\beta_S > \min[\beta_S, \delta_S, \delta_R]$ this exit leads to the state z^0. If $\beta_S = \min[\beta_S, \delta_S, \delta_R]$, one of existing S types is replaced by the new S type from outside. This exit leads to another state $z^\ddagger$. To move to state z^0 exit should happen through the death of a team member. This costs $\min[\beta_S, \delta_S, \delta_R] - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R]$. Therefore, the cheapest transition to z^0 from all $z^\ddagger$ costs $A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R]$. Hence,

$$\mathcal{E}(z^0) = A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] \quad \text{when } l \geq n - s.$$

From lemma 8, when $l < n - s,$ and $m \leq n - s$, the limit states are $z^* \in \{z^0, z^\diamond\}$.

The least cost path $z^\diamond \to z^0$ involves $n - s - l$ type R to leave the team overcoming the cost δ_R. Since only l members left in the team, the rest R members leave without any cost. The cost of this tree is $(n - s - l)\delta_R$. Hence,

$$\mathcal{E}(z^0) = (n - s - l)\delta_R \quad \text{when } l < n - s, \text{ and } m \leq n - s.$$

From lemma 8, when $l < n - s,$ and $m > n - s$, the limit states are $z^* \in \{z^0, z^\diamond, z^\ddagger\}$. There are three ways to construct a tree rooted at z^0. It is either by $z^\diamond \to z^0$, and $z^\ddagger \to z^0$, $z^\diamond \to z^\ddagger \to z^0$ or $z^\ddagger \to z^\diamond \to z^0$.

The first tree is constructed by adding the links $z^\diamond \to z^0$, and $z^\ddagger \to z^0$. The cost of the first link is $(n-s-l)\delta_R$ from above. The second transition is $z^\ddagger \to z^0$. The cost of leaving all states $z^\ddagger$ was denoted by $A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R]$ earlier. This exit leads to the state $z^\diamond$. The path $z^\diamond \to z^0$ costs $(n-s-l)\delta_I$ as was shown before. Hence the cost of such tree is $(n - s - l)\delta_R + A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n - s - l)\delta_R$.

The second tree, consisting of the links $z^\ddagger \to z^\diamond \to z^0$. The overall cost of such tree is

$A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n - s - l)\delta_R$.

The third tree is constructed by $z^\diamond \to z^\ddagger \to z^0$. The first transition $z^\diamond \to z^\ddagger$ requires the $(m - 1) - (n - s)$ number of S types to join the team overcoming β_S. Now there are $m - 1$ members of the team, and it is optimal for the last to join. The cost of this branch is $[(m-1) - (n-s)]\beta_S$. The cost of the second link was shown above. Hence the cost of the third tree is $[(m - 1) - (n - s)]\beta_S + A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n - s - l)\delta_R$.

The cost of the second tree is the stochastic potential of the tree rooted at z^0:

$$\mathcal{E}(z^0) = A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n-s-l)\delta_R \quad \text{when } l < n-s, \text{ and } m > n-s.$$

∎

LEMMA 10. *The least exponential-cost tree rooted at $z^\ddagger$ satisfies:*

$$\mathcal{E}(z^\ddagger) = A - \min[\beta_S, \delta_S, \delta_R] + [(m - 1) - (n - s)]\beta_S + \\ + \begin{cases} l \min[\beta_S, \beta_R], & \text{if } l < n - s, \text{ and } m > n - s; \\ \mathcal{I}_{l > (m-1)-(n-s)} \times [l - (m - 1) + (n - s)] \min[\beta_S, \beta_R], & \text{if } l \geq n - s. \end{cases}$$

PROOF OF LEMMA 10.

From lemma 8, when $l \geq n - s$, the limit states are $z^* \in \{z^0, z^\ddagger\}$.

There are two possible cases. If $(m-1)-(n-s) \geq l$, the cheapest way to construct this link is through the costly birth of $(m-1)-(n-s)$ S type players. Then it becomes optimal for R types to join them. After all $n-s$ type R players joined, some of S members leave. This brings the process to state $z^\ddagger$ at a cost $[(m-1)-(n-s)]\beta_S$. If $(m-1)-(n-s) < l$, additional to the costly birth of $(m-1)-(n-s)$ S types, $l-[(m-1)-(n-s)]$ least cost types need to join overcoming $\min[\beta_S, \beta_R]$. Only after this the reciprocation threshold is met, and the rest of members join the team with zero cost. The cost of this branch is $[(m-1)-(n-s)]\beta_S + [l-(m-1)+(n-s)]\min[\beta_S, \beta_R]$. Combining all, the cost of the branch $z^0 \rightarrow z^\ddagger$ is $[(m-1)-(n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l-(m-1)+(n-s)]\min[\beta_S, \beta_R]$.

From above the cost of moving between the states $z^\ddagger$ without leaving the last state is $A - \min[\beta_S, \delta_S, \delta_R]$. Hence the cost of the tree is

$$\mathcal{E}(z^\ddagger) = [(m-1)-(n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l-(m-1)+(n-s)]\min[\beta_S, \beta_R] + \\ + A - \min[\beta_S, \delta_S, \delta_R] \quad \text{when } l \geq n-s.$$

From lemma 8, when $l < n-s,$ and $m > n-s$, the limit states are $z^* \in \{z^0, z^\diamond, z^\ddagger\}$. There exists three ways to construct the tree rooted at $z^\ddagger$. It is either by $z^\diamond \rightarrow z^\ddagger$, and $z^0 \rightarrow z^\ddagger$, or $z^\diamond \rightarrow z^0 \rightarrow z^\ddagger$, or $z^0 \rightarrow z^\diamond \rightarrow z^\ddagger$.

The first tree is constructed by $z^\diamond \rightarrow z^\ddagger$, and $z^0 \rightarrow z^\ddagger$. The overall cost of this tree is

$[(m-1)-(n-s)]\beta_S + [(m-1)-(n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l-(m-1)+(n-s)]\min[\beta_S, \beta_R] +$
$+ A - \min[\beta_S, \delta_S, \delta_R]$.

The second tree is constructed by $z^\diamond \rightarrow z^0 \rightarrow z^\ddagger$. The cost such tree is

$(n-s-l)\delta_R + [(m-1)-(n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l-(m-1)+(n-s)]\min[\beta_S, \beta_R] +$
$+ A - \min[\beta_S, \delta_S, \delta_R]$.

The third tree is constructed by $z^0 \rightarrow z^\diamond \rightarrow z^\ddagger$. The first transition requires l least cost types to join the team overcoming $\min[\beta_S, \beta_R]$. Since now the the number of members is exactly l, it is optimal for all I types to join the team. The cost of this branch is $l \min[\beta_S, \beta_R]$. Overall, the cost of the tree is

$lmin[\beta_S, \beta_R] + [(m-1) - (n-s)]\beta_S + A - \min[\beta_S, \delta_S, \delta_R]$.

The third tree has the least exponential cost. Hence its cost is the stochastic potential of state $z^{\ddagger}$.

$$\mathcal{E}(z^{\ddagger}) = lmin[\beta_S, \beta_R] + [(m-1)-(n-s)]\beta_S + A - \min[\beta_S, \delta_S, \delta_R] \quad \text{when } l < n-s, \text{ and } m > n-s.$$

■

LEMMA 11. *The least exponential-cost tree rooted at $z^{\diamond}$ satisfies:*

$$\mathcal{E}(z^{\diamond}) = \begin{cases} l\min[\beta_S, \beta_R] + A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R], & \textit{if } l < n-s, \textit{ and } m > n-s; \\ l\min[\beta_S, \beta_R], & \textit{if } l < n-s, \textit{ and } m \leq n-s. \end{cases}$$

PROOF OF LEMMA 11.

From lemma 8, when $l < n-s$ and $m \leq n-s$, the limit states are $z^* \in \{z^0, z^{\diamond}\}$.

The cheapest way to construct the tree rooted at $z^{\diamond}$ is through the birth of l least cost types. The cost of such tree is the stochastic potential of the state.

$$\mathcal{E}(z^{\diamond}) = l\min[\beta_S, \beta_R] \quad \text{when } l < n-s \text{ and } m \leq n-s.$$

From lemma 8, when $l < n-s,$ and $m > n-s$, the limit states are $z^* \in \{z^0, z^{\diamond}, z^{\ddagger}\}$. There exists three ways to construct the tree rooted at $z^{\diamond}$. It is either by $z^0 \to z^{\diamond}$, and $z^{\ddagger} \to z^{\diamond}$, or $z^{\ddagger} \to z^0 \to z^{\diamond}$, or $z^0 \to z^{\ddagger} \to z^{\diamond}$.

The first tree is constructed by $z^0 \to z^{\diamond}$, and $z^{\ddagger} \to z^{\diamond}$. Overall cost of this tree is

$lmin[\beta_S, \beta_R] + A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R]$.

The second tree is constructed by $z^{\ddagger} \to z^0 \to z^{\diamond}$. Overall cost of this tree is

$A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] + (n-s-l)\delta_R + lmin[\beta_S, \beta_R]$.

The third tree is constructed by $z^0 \to z^{\ddagger} \to z^{\diamond}$. Overall cost of this tree is

$[(m-1) - (n-s)]\beta_S + \mathcal{I}_{l>(m-1)-(n-s)} \times [l - (m-1) + (n-s)] \min[\beta_S, \beta_R] + A - \min[\beta_S, \delta_S, \delta_R]$.

The first tree has the least exponential cost. Hence, stochastic potential of the state $z^{\diamond}$ is

$$\mathcal{E}(z^{\diamond}) = lmin[\beta_S, \beta_R] + A - \min[\beta_S, \delta_S, \delta_R] + \min[\delta_S, \delta_R] \quad \text{when } l < n-s, \text{ and } m > n-s.$$

■

PROOF OF PROPOSITION 2.

1) When $l \geq n-s$, the good is produced if, $\mathcal{E}(z^{\ddagger}) < \mathcal{E}(z^0)$. From lemmas 9 and 10,

$$\max\{[(m-1)-(n-s)]\beta_S, [(m-1)-(n-s)](\beta_S - \min[\beta_S,\beta_R]) + l\min[\beta_S,\beta_R]\} < \min[\delta_S,\delta_R]$$

2) When $l < n-s$ and $m > n-s$, the good is produced if $\min\{\mathcal{E}(z^0), \mathcal{E}(z^{\diamond}), \mathcal{E}(z^{\ddagger})\} = \mathcal{E}(z^{\ddagger})$. From lemmas 9, 10, and 11,

$$\mathcal{E}(z^{\ddagger}) < \mathcal{E}(z^0) \Longrightarrow [(m-1)-(n-s)]\beta_S + l\min[\beta_S,\beta_R] < \min[\delta_S,\delta_R] + (n-s-l)\delta_R$$

and

$$\mathcal{E}(z^{\ddagger}) < \mathcal{E}(z^{\diamond}) \Longrightarrow [(m-1)-(n-s)]\beta_S < \min[\delta_S,\delta_R]$$

3) When $l < n-s$ and $m \leq n-s$, the good is produced if $\min\{\mathcal{E}(z^0), \mathcal{E}(z^{\diamond})\} = \mathcal{E}(z^{\diamond})$. From lemmas 9 and 11,

$$\mathcal{E}(z^{\diamond}) < \mathcal{E}(z^0) \Longrightarrow l\min[\beta_S,\beta_R] < (n-s-l)\delta_R.$$

■

In-group cooperators. Following lemmas are useful to prove the Proposition 3.

LEMMA 12. *When R types are IC the limit states are:*

$$l < n-s \begin{cases} m \leq s & \textit{and } m < l, \Longrightarrow z^* \in \{z^0, z^{\dagger}, z^{\diamond}\}; \\ m > s & \textit{or } m = l, \Longrightarrow z^* \in \{z^0, z^{\diamond}\}; \end{cases}$$

$$l \geq n-s \begin{cases} m \leq s & \textit{and } m < l, \Longrightarrow z^* \in \{z^0, z^{\dagger}\}; \\ m > s & \textit{or } m = l, \Longrightarrow z^* \in \{z^0\}. \end{cases}$$

PROOF OF LEMMA 12. From lemma 1, the possible limit states are $z^* \in \{z^0, z^{\dagger}, z^{\diamond}, z^{\ddagger}\}$.

For $l \geq m$ the state $z^{\ddagger}$ cannot be one of the limit states. In this case, $x+y = m \leq l$, R types leave the team with zero cost. Hence, the set of limit states is $z^* \in \{z^0, z^{\dagger}, z^{\diamond}\}$.

For $m = l$, the state $z^{\dagger}$ can not be a limit state as well. In this case, $|z| = m = l$, R types either leave the team or all of them joins with zero cost. Hence, the set of limit states is $z^* \in \{z^0, z^{\dagger}\}$.

For $l \geq n-s$, the state $z^{\diamond}$ can not be a limit state. In this case, $|z| = n-s \leq l$, R types leave the team with zero cost.

For $m > s$, the state $z^\dagger$ can not be a limit state. In this case, there are not enough S types to form a team. ■

A′. Before proceeding to lemmas, let's denote the least cost exit from the set of states $z^\dagger \in Z_{m,0}$ by $A' \equiv \sum_{z^\dagger \in Z_{m,0}} \min[\beta_S, \beta_R, \delta_S]$.

There are three possible ways to escape a state $z^\dagger$. It is either through the death of one S type with a cost δ_S. In this case the state moves to $Z_{m-1,0}$, from where it is optimal for all S types to leave without any cost. The second way is through the birth of R type with a cost β_R. In this case the state moves to $Z_{m,1}$, from where it is optimal for one S type to leave with zero cost. The state moves to $Z_{m-1,1}$, and since $m \geq l$, it is optimal for R type to leave followed by the rest S types with zero cost. The third way to escape a state $z^\dagger$ is through the birth of S type with cost β_S. In this case one of the S types leaves the team being replaced with a new S type from outside. Hence, the cheapest way out from any $z^\dagger$ costs $\min[\beta_S, \beta_R, \delta_S]$, and the cost of moving out of all states in $Z_{m,0}$ is $A' \equiv \sum_{z^\dagger \in Z_{m,0}} \min[\beta_S, \beta_R, \delta_S]$.

A' can also be used to find a cost of transition among the states $z^\dagger \in Z_{m,0}$. If to begin in one of the states $z^\dagger$ each time one of the members leaves at a cost $\min[\beta_S, \beta_R, \delta_S]$, a new member from outside can join without any cost. If to subtract the cost of exit from the last state in the transition chain, the cost of moving between the states of the set $Z_{m,0}$ is $A' - \min[\beta_S, \beta_R, \delta_S]$.

LEMMA 13. *The least exponential-cost tree rooted at z^0 satisfies:*

$$\mathcal{E}(z^0) = \begin{cases} A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] + (n-s-l)\delta_R & \textit{if } l < n-s, \textit{and } m \leq s \textit{ and } m < l; \\ (n-s-l)\delta_R, & \textit{if } l < n-s, \textit{and } m = l, \textit{ or } m < s; \\ A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] & \textit{if } l \geq n-s, \textit{and } m \leq s \textit{ and } m < l; \\ 0 & \textit{if } l \geq n-s, \textit{ and } m = l, \textit{ or } m < s. \end{cases}$$

PROOF OF LEMMA 13.

From lemma 12, when $l \geq n-s$, $m \leq s$, and $m < l$, the limit states are $z^* \in \{z^0, z^\dagger\}$.

The cheapest exit cost from any $z^\dagger$ is $\min[\beta_S, \beta_R, \delta_S]$. If $\beta_S = \min[\beta_S, \delta_S, \delta_R]$, one of existing S types is replaced by the new S type from outside. This exit leads to another state $z^\dagger$. The transition from the last state to z^0 requires either death of the existing S type, or birth of an R type from outside the team. The cheapest transition costs $\min[\beta_R, \delta_S]$. Therefore, the cheapest transition to z^0 from all $z^\ddagger$ costs

$$\mathcal{E}(z^0) = A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] \quad \text{when } l \geq n-s, \text{and } m \leq s \text{ and } m < l.$$

From lemma 12, when $l < n-s,$ and $m = l,$ or $m < s$, the limit states are $z^* \in \{z^0, z^\diamond\}$.

The least cost path $z^\diamond \to z^0$ involves $n-s-l$ type R to leave the team overcoming the cost δ_R. Since only l members left in the team, the rest R members leave without any cost. The cost of this tree is the stochastic potential of z^0.

$$\mathcal{E}(z^0) = (n-s-l)\delta_R \quad \text{when } l < n-s, \text{and } m = l, \text{ or } m < s.$$

From lemma 12, when $l < n-s,$ and $m \leq s$ and $m < l$, the limit states are $z^* \in \{z^0, z^\dagger, z^\diamond\}$. There are three possible ways to construct a tree rooted at z^0. It is either by $z^\dagger \to z^0$ and $z^\diamond \to z^0$, or $z^\diamond \to z^\dagger \to z^0$, or $z^\dagger \to z^\diamond \to z^0$.

The first tree consists of the links $z^\dagger \to z^0$ and $z^\diamond \to z^0$. The cost of such tree is

$A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] + (n-s-l)\delta_R.$

The second tree consists of the links $z^\diamond \to z^\dagger \to z^0$. Exit from the state $z^\diamond$ leads to $z^\dagger$ and costs $(n-s-l)\delta_R$. Th cost of the second branch is shown above. The overall cost of such tree is

$A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] + (n-s-l)\delta_R.$

The third tree consists of the links $z^\dagger \to z^\diamond \to z^0$. For the first transition $z^\dagger \to z^\diamond$, initially the process moves between the states in $Z_{m,0}$ with cost $A' - \min[\beta_S, \beta_R, \delta_S]$. From the last $z^\dagger$, the process moves up, by $l-m$ least cost types joining team. Once there are exactly l members, it is optimal for the rest of R types to join. The cost of such branch is $A' - \min[\beta_S, \beta_R, \delta_S] + (l-m)\min[\beta_S, \beta_R]$. The overall cost of such tree is

$A' - \min[\beta_S, \beta_R, \delta_S] + (l-m)\min[\beta_S, \beta_R] + (n-s-l)\delta_R.$

From observation $\min[\delta_S, \beta_R] < (l-m)[\beta_S, \beta_R]$, the first and second trees have the least cost.

$$\mathcal{E}(z^0) = A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] + (n-s-l)\delta_R \quad \text{when } l < n-s, m \leq s \text{ and } m < l.$$

From lemma 12, when $l < n-s$, and $m < s$, or $m = l$, the limit states are $z^* \in \{z^0\}$. The stochastic potential of z^0 is

$$\mathcal{E}(z^0) = 0 \quad \text{when } l \geq n-s, \text{ and } m = l, \text{ or } m < s.$$

∎

LEMMA 14. *When $l < n-s$, the least exponential-cost tree rooted at $z^\diamond$ satisfies:*

$$\mathcal{E}(z^\diamond) = \begin{cases} (l-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S], & \textit{if } m \leq s \textit{ and } m < l; \\ (l-1)\min[\beta_S, \beta_R], & \textit{if } m = l, \textit{ or } m > s. \end{cases}$$

PROOF OF LEMMA 14.

From lemma 12, when $l < n-s$, and $m > s$ or $m = l$, the limit states are $z^* \in \{z^0, z^\diamond\}$.

The path $z^0 \to z^\diamond$ passes through the set of states $z = \{z \in Z_{x,y} | x+y = m\}$. Initially, the cost $m-1$ players join the team overcoming the cost $(m-1)\min[\beta_S, \beta_R]$. It is optimal for the m^{th} S type to join. After the birth of $l-m$ the least cost players, all R types join the team replacing the existing S types. The cost of such tree is the stochastic potential of $z^\diamond$.

$$\mathcal{E}(z^\diamond) = (l-1)\min[\beta_S, \beta_R] \quad \text{when } l < n-s, \text{ and } m > s \text{ or } m = l.$$

From lemma 12, when $l < n-s$, $m \leq s$, and $m < l$, the limit states are $z^* \in \{z^0, z^\dagger, z^\diamond\}$. There are three ways to construct a tree rooted at $z^\diamond$. It is either by $z^0 \to z^\diamond$, and $z^\dagger \to z^\diamond$, or $z^\dagger \to z^0 \to z^\diamond$, or $z^0 \to z^\dagger \to z^\diamond$.

The first tree is $z^0 \to z^\diamond$, and $z^\dagger \to z^\diamond$. The cost of the this tree is

$(l-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] + (l-m)\min[\beta_S, \beta_R]$.

The second is $z^\dagger \to z^0 \to z^\diamond$. The cost of such tree is

$A' - \min[\beta_S, \beta_R, \delta_S] + \min[\beta_R, \delta_S] + (l-1)\min[\beta_S, \beta_R]$.

The third tree is $z^0 \to z^\dagger \to z^\diamond$. The cost of the tree is

$(m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] + (l-m)\min[\beta_S, \beta_R]$.

The third tree has the least exponential cost. Hence, the stochastic potential of $z^\diamond$ is

$$\mathcal{E}(z^\diamond) = (l-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] \quad \text{when } l < n-s, m \leq s, \text{ and } m < l.$$

■

LEMMA 15. *When $m \leq s$ and $m < l$, the least exponential-cost tree rooted at $z^\dagger$ satisfies:*

$$\mathcal{E}(z^\dagger) = \begin{cases} (m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] + (n-s-l)\delta_R, & \textit{if } l < n-s; \\ (m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S], & \textit{if } l \geq n-s. \end{cases}$$

PROOF OF LEMMA 15.

From lemma 12, when $l \geq n-s$, $m \leq s$, and $m < l$, the limit states are $z^* \in \{z^0, z^\dagger\}$.

The least cost path $z^0 \to z^\diamond$ involves $m-1$ least cost types to join the team overcoming $\min[\beta_S, \beta_R]$. Once $x + y = m-1$, it is optimal for m^{th} S type to join them. Since now there are exactly l members, the rest of R types join the team without any cost. Together with the cost of transition among the states $z^\dagger$, the cost of this tree is

$$\mathcal{E}(z^\dagger) = (m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] \quad \text{when } l \geq n-s, m \leq s, \text{ and } m < l.$$

From lemma 12, when $l < n-s$, $m \leq s$, and $m < l$, the limit states are $z^* \in \{z^0, z^\dagger, z^\diamond\}$. There are three ways to construct a tree rooted at $z^\dagger$. It either by $z^0 \to z^\dagger$, and $z^\diamond \to z^\dagger$, or $z^\diamond \to z^0 \to z^\dagger$, or $z^0 \to z^\diamond \to z^\dagger$.

The first tree is $z^0 \to z^\dagger$, and $z^\diamond \to z^\dagger$. It costs

$(m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] + (n-s-l)\delta_R$.

The second tree is $z^\diamond \to z^0 \to z^\dagger$. It costs

$(m-1)\min[\beta_S, \beta_R] + A' - \min[\beta_S, \beta_R, \delta_S] + (n-s-l)\delta_R$.

The third tree is $z^0 \to z^\diamond \to z^\dagger$. It costs $(l-1)\min[\beta_S, \beta_R] + (n-s-l)\delta_R + A' - \min[\beta_S, \beta_R, \delta_S]$.

The stochastic potential of $z^\dagger$ is the cost of the first/second tree:

$$\mathcal{E}(z^\dagger) = (m-1)\min[\beta_S, \beta_R] + (n-s-l)\delta_R + A' - \min[\delta_S, \beta_R] \quad \text{when } l < n-s, m \leq s, \text{ and } m < l.$$

■

PROOF OF PROPOSITION 3.

1) When $m \leq s$ and $m < l$, the good is produced if either

$$\mathcal{E}(z^\dagger) < \mathcal{E}(z^0) \Longrightarrow (m-1)\min[\beta_S, \beta_R] < \min[\beta_R, \delta_S] \tag{31}$$

and

$$\mathcal{E}(z^{\dagger}) < \mathcal{E}(z^{\diamond}) \Longrightarrow (l-m)\min[\beta_S, \beta_R] > (n-s-l)\delta_R \tag{32}$$

or,

$$\mathcal{E}(z^{\diamond}) < \mathcal{E}(z^0) \Longrightarrow (m-1)\min[\beta_S, \beta_R] + (l-m)\min[\beta_S, \beta_R] < \min[\beta_R, \delta_S] + (n-s-l)\delta_R \tag{33}$$

and

$$\mathcal{E}(z^{\diamond}) < \mathcal{E}(z^{\dagger}) \Longrightarrow (l-m)\min[\beta_S, \beta_R] < (n-s-l)\delta_R. \tag{34}$$

Assume (31) holds, then there are two possible cases:

- (32) holds and $\mathcal{E}(z^{\dagger}) = \min[\mathcal{E}(z^0), \mathcal{E}(z^{\dagger}), \mathcal{E}(z^{\diamond})]$,
- (34) holds and $\mathcal{E}(z^{\diamond}) = \min[\mathcal{E}(z^0), \mathcal{E}(z^{\dagger}), \mathcal{E}(z^{\diamond})]$.

Now assume (33) holds, then there are two possible cases:

- (32) holds and $\mathcal{E}(z^{\dagger}) = \min[\mathcal{E}(z^0), \mathcal{E}(z^{\dagger}), \mathcal{E}(z^{\diamond})]$,
- (34) holds and $\mathcal{E}(z^{\diamond}) = \min[\mathcal{E}(z^0), \mathcal{E}(z^{\dagger}), \mathcal{E}(z^{\diamond})]$.

Hence, one of the conditions (31) and (33) is sufficient for the good to be produced.

2) When $l < n-s$ and $m > s$ or $m = l$, the good is produced if $\min\{\mathcal{E}(z^0), \mathcal{E}(z^{\diamond})\} = \mathcal{E}(z^{\diamond})$. From lemmas 13, and 14,

$\mathcal{E}(z^{\diamond}) < \mathcal{E}(z^0) \Longrightarrow (l-1)\min[\beta_S, \beta_R] < (n-s-l)\delta_R + \min[\beta_R, \delta_S]$.

3) When $l \geq n-s$ and $m > s$ or $m = l$, the good is never produced. The limit state of such dynamics contain a single state $z^* \in \{Z_{0,0}\}$.

The claim is $\mathcal{E}(z^0) < \mathcal{E}(z)$ for all $z = \{z \in Z_{x,y} | z \neq z^0\}$.

There exists zero cost path from all states $z \neq z^0$ to z^0. Hence $\mathcal{E}(z^0) = 0$.

To construct a tree rooted at any $z \in Z$, where $z \neq z_0$, the zero cost branch leading from z to z^0 is deleted and replaced with a costly path in opposite direction. Hence, $\mathcal{E}(z_0) < \mathcal{E}(z)$ for all $z \neq z_0$. ■

APPENDIX B

Appendices for Chapter 2

B.1. Derivation of the model

In this appendix, we derive the equilibrium outcome for the presented model. Optimal weights assigned to the signals are solution to the the following minimization problem:

$$\min_{\omega} \quad (1-\gamma)E[(a_l-\theta)^2]+\gamma E[(a_l-\bar{a})^2] \tag{35}$$

The linear strategy for player l is assumed to be $a_l \equiv A(x_l)=\sum_{i=1}^n \omega_i\big(\theta+\eta_i+\varepsilon_{il}\big)$.

The restriction $\sum_{i=1}^n \omega_i = 1$, implies $a_l-\theta=\sum_{i=1}^n \omega_i\big(\eta_i+\varepsilon_{il}\big)$. Hence, the first expectation is

$$E\big[(a_l-\theta)^2\big]=\sum_{i=1}^n \omega_i^2\Bigg(\kappa_i^2+\xi_i^2\Bigg).$$

Since players use symmetric strategies, the average action is $\bar{a}=\frac{\sum_{i=n}^n\sum_{l'\neq l}\omega_i\big(\theta+\eta_i+\varepsilon_{il'}\big)}{L-1}$. Then,

$$\begin{aligned}
a_l-\bar{a}=a_l-\frac{\sum_{l'\neq l}a_l}{L-1} &= \sum_{i=n}^n \omega_i\big(\theta+\eta_i+\varepsilon_{il}\big)-\frac{\sum_{i=n}^n\sum_{l'\neq l}\omega_i\big(\theta+\eta_i+\varepsilon_{il'}\big)}{L-1} = \\
&= \theta-\frac{(L-1)}{L-1}\theta+\sum_{i=1}^n \eta_i\Bigg[\omega_i-\frac{1}{L-1}\sum_{l'\neq l}\omega_i\Bigg]+\sum_{i=1}^n \omega_i\varepsilon_{il}-\frac{1}{L-1}\sum_{i=n}^n\sum_{l'\neq l}\omega_i\varepsilon_{il'} = \\
&= \sum_{i=1}^n \eta_i\frac{(L-1)\omega_i-\sum_{l'\neq l}\omega_i}{L-1}+\sum_{i=1}^n\Bigg[\frac{(L-1)\omega_i\varepsilon_{il}-\sum_{l'\neq l}\omega_i\varepsilon_{il'}}{L-1}\Bigg]
\end{aligned}$$

The second expectation becomes,

$$\begin{aligned}
E\big(a_l-\bar{a}\big)^2 &= E\Bigg[\sum_{i=1}^n \eta_i\frac{(L-1)\omega_i-\sum_{l'\neq l}\omega_i}{L-1}\Bigg]^2+E\Bigg[\sum_{i=1}^n\frac{(L-1)\omega_i\varepsilon_{il}-\sum_{l'\neq l}\omega_i\varepsilon_{il'}}{L-1}\Bigg]^2 = \\
&= \sum_{i=1}^n \kappa_i^2\Bigg[\frac{(L-1)\omega_i-\sum_{l'\neq l}\omega_i}{L-1}\Bigg]^2+\frac{(L-1)^2\sum_{i=1}^n\omega_i^2\xi_i^2}{(L-1)^2}+\frac{\sum_{i=n}^n\sum_{l'\neq l}\omega_i^2\xi_i^2}{(L-1)^2}
\end{aligned}$$

Substituting these expectations back into the (35), yields,

$$\min_{\omega}(1-\gamma)\sum_{i=1}^{n}\omega_i^2\left(\kappa_i^2+\xi_i^2\right)+\gamma\sum_{i=1}^{n}\kappa_i^2\left[\frac{(L-1)\omega_i-\sum_{l'\neq l}\omega_i}{L-1}\right]^2+\gamma\sum_{i=1}^{n}\omega_i^2\xi_i^2+$$
$$+\gamma\frac{\sum_{i=n}^{n}\sum_{l'\neq l}\omega_i^2\xi_i^2}{(L-1)^2}\quad \text{subject to } \sum_{i=1}^{n}\omega_i=1.$$

From the symmetry assumption the second term becomes zero, and the last term is irrelevant to the optimization. Denote $\widehat{\psi}_i=\frac{1}{(1-\gamma)\kappa_i^2+\xi_i^2}$, and substitute in the (35), the problem simplifies to,

$$\min_{\omega}\sum_{i=1}^{n}\frac{\omega_i^2}{\widehat{\psi}_i}\text{ subject to }\sum_{i=1}^{n}\omega_i=1.$$

From FOC with respect to ω_i: $2\frac{\omega_i}{\widehat{\psi}_i}=\lambda$.

Adding constraint, the solution to the optimization problem is:

$$\omega_i=\frac{\widehat{\psi}_i}{\sum_{j=1}^{n}\widehat{\psi}_j}.$$

B.2. Tables for hypotheses testing

Tables with p-values from the tests are presented below.

The average weights from the experiment are compared to the theoretical predictions, and p-values are presented in table 1.

TABLE 1. Comparison of the average weight from data and the theoretical predictions (p-values)

$H0:\omega=\omega^*$	$\omega_{0,\rho}=50$		$\omega_{0.9,0}=9$	$\omega_{0.9,0.6}=19$		$\omega_{-1,0}=67$	$\omega_{-1,0.6}=55$
Signrank							
$H1:\omega\neq\omega^*$	0.466		0.002	0.005		0.049	0.022
Signtest							
$H1:\omega\neq\omega^*$	0.512		0.021	0.002		0.021	0.180
$H1:\omega>\omega^*$	0.256		0.011	0.001		0.998	0.971
$H1:\omega<\omega^*$	0.821		0.998	0.999		0.011	0.090

Table 2 shows p-values for testing hypothesis 2. The average weights from $T0$ and $T1$ are compared to the average weights from $T1$ and $T4$, to test if the weights decrease in coordination game. The average weights from $T0$ and $T1$ are compared to the average weights from $T2$ and $T5$ to test if the weights increase in anti-coordination game.

TABLE 2. p-values for the $H2$ test

$H0: \omega_{0,\rho} = \omega_{\gamma,\rho}$	$\gamma = 0.9$ $\omega_{0,0} = \omega_{0.9,0}$	$\omega_{0,0.6} = \omega_{0.9,0.6}$	$\gamma = -1$ $\omega_{0,0} = \omega_{-1,0}$	$\omega_{0,0.6} = \omega_{-1,0.6}$
Signrank				
$H1: \omega_{0,\rho} \neq \omega_{\gamma,\rho}$	0.000	0.056	0.121	0.331
Signtest				
$H1: \omega_{0,\rho} \neq \omega_{\gamma,\rho}$	0.000	0.180	0.455	0.791
$H1: \omega_{0,\rho} > \omega_{\gamma,\rho}$	0.000	0.089	0.895	0.395
$H1: \omega_{0,\rho} < \omega_{\gamma,\rho}$	1.000	0.971	0.227	0.788

Table 3 shows p-values from the comparison of average weights between independent and correlated signal treatments. MW stands for the Mann-Whitney rank-sum test, and KS stands for the Kolmogorov-Smirnov test.

TABLE 3. p-values for the $H3$ test

$H0: \omega_{\gamma,0} = \omega_{\gamma,0.6}$	$\omega_{0,0} = \omega_{0,0.6}$	$\omega_{0.9,0} = \omega_{0.9,0.6}$	$\omega_{-1,0} = \omega_{-1,0.6}$
MW			
$H1: \omega_{\gamma,0} \neq \omega_{\gamma,0.6}$	0.711	0.010	0.002
KS			
$H1: \omega_{\gamma,0} \neq \omega_{\gamma,0.6}$	0.882	0.055	0.018
$H1: \omega_{\gamma,0.6} < \omega_{\gamma,0}$	0.503	1.000	0.009
$H1: \omega_{\gamma,0} < \omega_{\gamma,0.6}$	0.825	0.027	1.000

Table 4 shows the evolution of average weights across blocks of 5 rounds for the aggregate data from coordination sessions. 30 rounds are divided into the block of 6 sessions. The average weights from each block compared to the next one. P-values from the comparison are reported in the table below.

TABLE 4. Evolution of weights in the coordination sessions

$H0: \omega^t = \omega^{t+1}$	$1-2$	$2-3$	$3-4$	$4-5$	$5-6$
Signrank					
$H1: \omega^t \neq \omega^{t+1}$	0.453	0.001	0.001	0.434	0.168
Signtest					
$H1: \omega^t \neq \omega^{t+1}$	0.572	0.005	0.002	0.851	0.442
$H1: \omega^t > \omega^{t+1}$	0.828	0.003	0.001	0.425	0.221
$H1: \omega^t < \omega^{t+1}$	0.286	0.999	0.999	0.714	0.876

Table 5 shows the evolution of average weights across blocks of 5 rounds for the aggregate data from coordination sessions. 30 rounds are divided into the block of 6 sessions. The average weights from each block compared to the next one. P-values from the comparison are reported in the table below.

TABLE 5. Evolution of weights in the anti-coordination sessions

$H0 : \omega^t = \omega^{t+1}$	$5-10$	$10-15$	$15-20$	$20-25$	$25-30$
Signrank					
$H1 : \omega^t \neq \omega^{t+1}$	0.028	0.629	0.805	0.453	0.249
Signtest					
$H1 : \omega^t \neq \omega^{t+1}$	0.136	0.851	1.000	0.362	0.362
$H1 : \omega^t > \omega^{t+1}$	0.068	0.425	0.645	0.900	0.900
$H1 : \omega^t < \omega^{t+1}$	0.969	0.714	0.500	0.181	0.181

B.3. Instructions

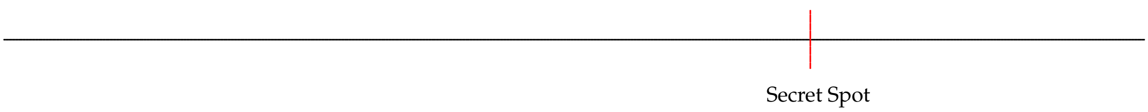
Instructions from the session 1 are presented below.

General Information

Thank you for participating in an experiment. During this experiment you will be matched with another player and asked to make a decision as a team under 3 different scenarios. Each scenario will be repeated for 10 periods. As a result of your and your partner's decision you will earn some points. At the end of the session, the total points you earned will be converted to pounds and paid to you in cash. The conversion rate is $£1 = 500$ points. You will also receive $£2.5$ as a show up fee, regardless of your performance.

Read all the instructions carefully. You can read instructions as many times as you want by navigating between pages with BACK and NEXT buttons. Please, do not talk to others during the experiment. Raise your hand if you have a question and one of the instructors will approach you.

Game



You see a horizontal line on your screen. There is a secret spot on this line which is not known to anyone. Based on the information you will be given later, You and your partner need to choose a location on that line closer to the Secret Spot.

There are 3 scenarios. Each of them will be repeated for 10 periods. You are given 200 points in the start of every period. You will lose points if you ignore the task given in each scenario.

Scenario 1: Your partner is a computer which always chooses the optimal location. Your task is to be as close as possible to the secret spot. Your partner's location do not have any impact on your profit.

$$YourProfit = 200 - (You - SecretSpot)^2$$

You lose the square of the distance between You and the Secret Spot.

Scenario 2: Your partner is a computer which always chooses the optimal location. Your task is to be close to the secret spot, and at the same time, to be close to your partner's location. However, being close to your partner is 9 times more important.

$$YourProfit = 200 - 0.1 \times (You - SecretSpot)^2 - 0.9 \times (You - Partner)^2$$

You lose the square of the distance between You and the Secret Spot, and between You and your partner. You lose 9 times more if you are far away from your partner.

Scenario 3: Your profit is calculated exactly like in the second scenario. However, instead of a computer, you will be matched with another participant, but you will not know each other's identity.

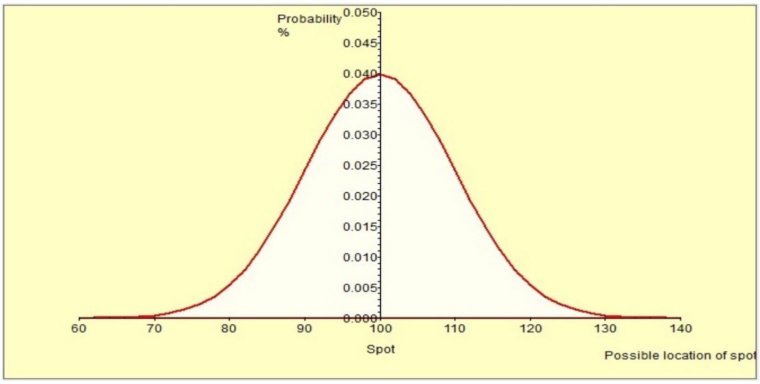
You will receive one public and one private piece of information about the location of the Secret spot.

Public Information

The location of the spot will change every period. Each time the spot is more likely to be 100, but also can be any other location close to 100. The figure below displays the probability of each location being the secret spot.

The horizontal displays the possible locations and the vertical is the chance of each location being the secret spot.

You can see from the figure that the most frequent location is 100. There is a very small chance that the secret spot will be smaller than $100{-}30$ or bigger than $100{+}30$.

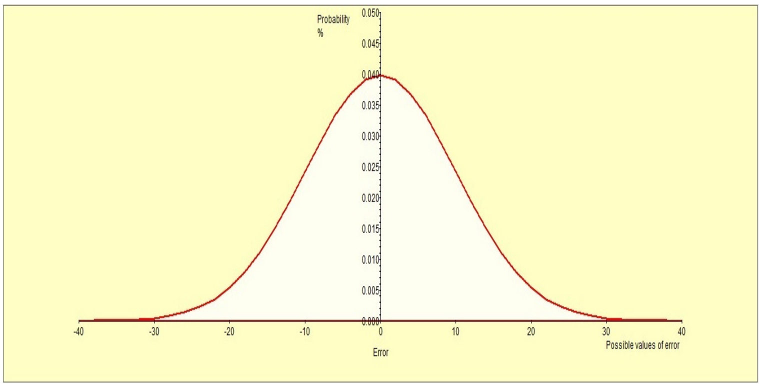


This information is public. Everybody knows, the secret spot is somewhere around 100, but no one knows the actual location.

Private Information

You will also receive a hint privately. The hint shows the location of the Secret Spot with some error: $Hint = SecretSpot + Error$.

The error is always more likely to be 0, but also can be any other number close to 0. The figure below displays the probability of each error to be chosen.



The horizontal displays the possible errors and the vertical is the chance of each error to be chosen.

You can see that the most frequent error is zero. There is a very small chance that the error will be smaller than -30 or bigger than $+30$.

The hint is your private information. Your partner's hint will be different, because, the size and the sign of your errors can be different.

Every period, a new error will be chosen for each one of you. You will not see each other's hints.

Choose a location

Above, you see the location of public information (100) and private information (hint) as blue vertical lines. Two information you receive are equally accurate, but different in publicity. Public information is perfectly observable by everyone, but private information is different for each person. You can choose a location anywhere between them.

You will express the importance of being close to public or private information by dragging the red box below. As you change the position of the box, you can see the importance assigned to each information and your location from the small box in the left. You will also see your location marked as a red vertical line above. Once you confirm your location profit is calculated based on that choice.

Examples:

Scenario 1

Your task is to be as close as possible to the secret spot. Your partner's location do not have any impact on your profit.

Example

Location of the spot	93
Your location	90
Your partner's location	99

$YourProfit = 200 - (90 - 93)^2 = 191$

Scenario 2 and 3

Your task is to be close to the secret spot, and at the same time, to be close to your partner's location. However, being close to your partner is 9 times more important.

Example

Location of the spot	93
Your location	90
Your partner's location	99

$YourProfit = 200 - 0.1 \times (90 - 93)^2 - 0.9 \times (90 - 99)^2 = 126.2$

Feedback:

After you and your partner submit location preferences, the computer will calculate your profit according to the formulas presented before. In the feedback screen you will see the following:

- The actual location of the secret spot for the current period;
- The location chosen by You for the current period;
- The location chosen by your partner for the current period;
- The profit you made for the current period;
- History box, which shows the same information for the previous periods.

End of Instructions

This is the end of the instructions. You can use the Back button to read the instructions again if you need. When you feel ready, press the START button. You will be directed to the next screen where you are asked to answer some comprehension questions. The experiment will start after you answer them correctly.

At the end of the experiment you will be asked to fill in a personal questionnaire. Any information provided will remain secret.

APPENDIX C

Appendices for Chapter 3

C.1. Omitted proofs

PROOF OF LEMMA 2. To prove that, for each θ, $V(\mu,\theta)$ is increasing in the precision of own signal, we differentiate the function $V(\mu,\theta)$ with respect to precision variables.

$$V(\mu,q) = q\pi_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] -$$
$$- \max\{q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$$
$$V(\mu,p) = p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] -$$
$$- \max\{p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\}$$

Two functions react to the change in own precision similarly. We prove the lemma for $V(\mu,p)$ only. Similar results holds for $V(\mu,q)$.

The value of $V(\mu,p)$ depends on the action choice of type p without supplementary information.

$$u(p) = \max\{p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1], (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]\}$$

We check the sign of the FOD for each action choice separately.

Type H chooses action a_1 if $p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] > (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$. Then,

$$V(\mu,p) = p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] - p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] =$$
$$= -(1-\tilde{\pi}_1)p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$$

From FOD:

$\frac{\partial V(\mu,p)}{\tilde{\pi}_1} = p[2\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1 - \gamma]$

$\frac{\partial V(\mu,p)}{\pi_1} = -p(1-\gamma)(1-\tilde{\pi}_1) \leq 0$

$\frac{\partial V(\mu,p)}{\tilde{\pi}_2} = (1-p)\left[2\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2\right] \geq 0$

$\frac{\partial V(\mu,p)}{\pi_2} = (1-p)(1-\gamma)\tilde{\pi}_2 \geq 0$

The Hessian matrix of the SOD is:

$$H(p,a_1) = \begin{vmatrix} 2\gamma p & (1-\gamma)p & 0 & 0 \\ (1-\gamma)p & 0 & 0 & 0 \\ 0 & 0 & 2\gamma(1-p) & (1-\gamma)(1-p) \\ 0 & 0 & (1-\gamma)(1-p) & 0 \end{vmatrix}$$

The first principal minor is positive, $D_1 = 2\gamma p > 0$, and the second principal minor is negative, $D_2 = -(1-\gamma)^2 p^2 < 0$. The sign pattern shows that there is neither maxima, nor minima of the function. Maximization problem has a corner solution.

From FOD, $V(\mu,p)$ is increasing in $\tilde{\pi}_1$, π_2, and $\tilde{\pi}_2$, and decreasing in π_1. Once $\tilde{\pi}_1 = 1$, it is non decreasing in π_1.

Type H chooses action a_2 if $p[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] < (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$. Then,

$$V(\mu,p) = p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] + (1-p)\tilde{\pi}_2[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] - (1-p)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2] =$$
$$= p\tilde{\pi}_1[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] - (1-p)(1-\tilde{\pi}_2)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2]$$

From FOD:

$\frac{\partial V(\mu,p)}{\tilde{\pi}_1} = p[2\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq 0.$

$\frac{\partial V(\mu,p)}{\pi_1} = p(1-\gamma)\tilde{\pi}_1 \geq 0.$

$\frac{\partial V(\mu,p)}{\tilde{\pi}_2} = (1-p)\left[2\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2 - \gamma\right]$

$\frac{\partial V(\mu,p)}{\pi_2} = -(1-p)(1-\gamma)\tilde{\pi}_2 \leq 0$

The Hessian matrix of the SOD is:

$$H(p,a_2) = \begin{vmatrix} 2\gamma p & (1-\gamma)p & 0 & 0 \\ (1-\gamma)p & 0 & 0 & 0 \\ 0 & 0 & 2\gamma(1-p) & (1-\gamma)(1-p) \\ 0 & 0 & (1-\gamma)(1-p) & 0 \end{vmatrix} = H(p,a_1)$$

The Hessian is similar to the previous case. From FOD, $V(\mu,p)$ is increasing in $\tilde{\pi}_1$, π_1, and $\tilde{\pi}_2$, and decreasing in π_2.

Hence, the value of experiment is increasing for each type in the precision of own signal. ■

PROOF OF PROPOSITION 4. To prove this proposition we show that both prices achieve their maximum value when all conditions in the proposition are satisfied, $\tilde{\pi_1} = \tilde{\pi_2} = \pi_1 = 1$.

At first, the FOD are found to check how function reacts to the change in each parameter. Then, the SOD are checked to determine the shape of the function. $t(p)$, and $t(q)$ are multivariate functions, hence the SOD generates the Hessian matrix.

$$t(q) = -(1-\pi_1)q[\gamma\tilde{\pi_1} + (1-\gamma)\pi_1] + (1-q)\pi_2[\gamma\tilde{\pi_2} + (1-\gamma)\pi_2]$$

FOD:

$\frac{\partial t(q)}{\partial \pi_1} = \gamma q\tilde{\pi_1} + 2(1-\gamma)q\pi_1 - (1-\gamma)q = q\big[\gamma\tilde{\pi_1} + (1-\gamma)(2\pi_1 - 1)\big]$

$\frac{\partial t(q)}{\partial \tilde{\pi_1}} = q\gamma\pi_1 - \gamma q = -(1-\pi_1)\gamma q \leq 0$

$\frac{\partial t(q)}{\partial \pi_2} = \gamma(1-q)\tilde{\pi_2} + 2(1-\gamma)(1-q)\pi_2 \geq 0$

$\frac{\partial t(q)}{\partial \tilde{\pi_2}} = \gamma(1-q)\pi_2 \geq 0.$

The Hessian matrix of the SOD is:

$$H_t^q = \begin{vmatrix} 2(1-\gamma)q & \gamma q & 0 & 0 \\ \gamma q & 0 & 0 & 0 \\ 0 & 0 & 2(1-\gamma)(1-q) & \gamma(1-q) \\ 0 & 0 & \gamma(1-q) & 0 \end{vmatrix}$$

The first principal minor is positive, $D_1 = 2(1-\gamma)q > 0$, the second principal minor is negative, $D_2 = 0 - \gamma^2 q^2 = -\gamma^2 q^2 < 0$. The sign pattern shows there is neither maxima, nor minima of the function. The maximization problem has a corner solution.

From FOD, $t(q)$ is increasing in π_1, π_2, and $\tilde{\pi_2}$. Once $\pi_1 = 1$, it is non-decreasing in $\tilde{\pi_1}$. Hence $t(q)$ is maximized by setting $\pi_1 = \tilde{\pi_1} = \pi_2 = \tilde{\pi_2} = 1$.

$$t(p) = [\gamma\tilde{\pi_1} + (1-\gamma)\pi_1][p\tilde{\pi_1} + (q-p)\pi_1 - q] + [\gamma\tilde{\pi_2} + (1-\gamma)\pi_2][(1-p)\tilde{\pi_2} - (q-p)\pi_2]$$

FOD:

$\frac{\partial t(p)}{\partial \pi_1} = (1-\gamma)p\tilde{\pi_1} - (1-\gamma)q + \gamma(q-p)\tilde{\pi_1} + 2(1-\gamma)(q-p)\pi_1$

$\frac{\partial t(p)}{\partial \tilde{\pi_1}} = 2\gamma p\tilde{\pi_1} + (1-\gamma)p\pi_1 + \gamma(q-p)\pi_1 - \gamma q$

$\frac{\partial t(p)}{\partial \pi_2} = (1-\gamma)(1-p)\tilde{\pi}_2 - \gamma(q-p)\tilde{\pi}_2 - 2(1-\gamma)(q-p)\pi_2 = -2(1-\gamma)\pi_2 - \big[\gamma(q-p) - (1-\gamma)(1-p)\big]\tilde{\pi}_2$

$\frac{\partial t(p)}{\partial \tilde{\pi}_2} = 2\gamma(1-p)\tilde{\pi}_2 - \gamma(q-p)\pi_2 + (1-\gamma)(1-p)\pi_2$

The sign of the first derivatives are not clear. The Hessian matrix of the SOD is:

$$H_t^p = \begin{vmatrix} 2(1-\gamma)(q-p) & (1-\gamma)p+\gamma(q-p) & 0 & 0 \\ (1-\gamma)p+\gamma(q-p) & 2\gamma p & 0 & 0 \\ 0 & 0 & -2(1-\gamma)(q-p) & (1-\gamma)(1-p)-\gamma(q-p) \\ 0 & 0 & (1-\gamma)(1-p)-\gamma(q-p) & 2\gamma(1-p) \end{vmatrix}$$

The first principal minors is positive, $D_1 = 2(1-\gamma)(q-p) > 0$, and the second principal minor is negative,

$$D_2 = 4\gamma(1-\gamma)p(q-p) - (1-\gamma)^2p^2 - 2\gamma(1-\gamma)p(q-p) - \gamma^2(q-p)^2 =$$
$$= 2\gamma pq - 2\gamma p^2 - 2\gamma^2pq + 2\gamma^2p^2 - p^2 + 2\gamma p^2 - \gamma^2p^2 - \gamma^2p^2 + 2\gamma^2qp - \gamma^2q^2 =$$
$$= 2\gamma pq - p^2 - \gamma^2q^2 = -(p-\gamma q)^2 < 0$$

The sign pattern shows the maximization problem has a corner solution. FOD are positive at $\pi_1 = \tilde{\pi}_1 = 1$, the function is increasing in both precisions. Setting $\tilde{\pi}_2 = 1$, $\frac{\partial t(p)}{\partial \tilde{\pi}_2} > 0$ for any values of π_2. The optimal value for π_2 is not clear. Hence $t(p)$ is maximized by setting $\pi_1 = \tilde{\pi}_1 = \tilde{\pi}_2 = 1$.

Combining two results , the monopolist sets $\pi_1 = \tilde{\pi}_1 = \tilde{\pi}_2 = 1$ to increase both prices. ∎

PROOF OF CONJECTURE 1. We check the conjecture 1 is satisfied under the optimal menu.

$$q[\gamma\tilde{\pi}_1 + (1-\gamma)\pi_1] \geq (1-q)[\gamma\tilde{\pi}_2 + (1-\gamma)\pi_2].$$

After substituting the results from proposition 4, the inequality becomes:

$$q > (1-q)[\gamma + (1-\gamma)\pi]$$

From assumption 1, $q > \frac{1}{2}$. The above inequality is satisfied for all values of π, as $1-q < \frac{1}{2}$, and $[\gamma + (1-\gamma)\pi] \in [0,1]$.

Thus, type L chooses action a_1 without supplemental information. ∎

PROOF OF COROLLARY 1. We solve $V^N(p) \geq 0$, to find the $\bar{\pi}$ that satisfies the participation constraint of type H.

$$V^N(p) = p - (1-p)[\gamma + (1-\gamma)\pi] + (q-p)\pi[\gamma + (1-\gamma)\pi] \geq 0.$$

Let's look at the FOD and SOD, to find how $V^N(p)$ reacts to the changes in π.

$$\frac{\partial V^N(p)}{\partial \pi} = \gamma(q-p) - (1-\gamma)(1-p) + 2(1-\gamma)(q-p)\pi$$

$$\frac{\partial^2 V^N(p)}{\partial \pi^2} = 2(1-\gamma)(q-p) > 0$$

$V^N(p)$ it takes negative value at $\pi = 0$, and positive value at $\pi = 1$. The second order derivative with respect to π is positive, which means, $V^N(p)$ is strictly increasing in $\pi \in [0,1]$. Hence there is a value of π such that $V^N(p) = 0$, lets denote it by $\bar{\pi}$. It is a minimum value the optimal π can take under the non-congruent beliefs case: $\pi^N = \max\{\bar{\pi}, \min\{1, \pi^*\}\}$.

$V(p)$ can be rearranged as follows,

$$(1-\gamma)(q-p)\pi^2 + [\gamma(q-p) - (1-\gamma)(1-p)]\pi - \gamma(1-p) + p \geq 0$$

$$(1-\gamma)(q-p)\Big[\pi^2 + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{(1-\gamma)(q-p)}\pi\Big] - \gamma(1-p) + p \geq 0$$

$$\Big[\pi^2 + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{(1-\gamma)(q-p)}\pi\Big] \geq \frac{\gamma(1-p) - p}{(1-\gamma)(q-p)}$$

Add $\left(\frac{\gamma(q-p)-(1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\right)^2$ to both sides of the inequality,

$$\pi^2 + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{(1-\gamma)(q-p)}\pi + \Big(\frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\Big)^2 \geq$$

$$\geq \frac{\gamma(1-p) - p}{(1-\gamma)(q-p)} + \Big(\frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\Big)^2$$

$$\Big[\pi + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\Big]^2 \geq \frac{\gamma(1-p) - p}{(1-\gamma)(q-p)} + \Big(\frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\Big)^2$$

Next, take the square root of both sides. The square root of the RHS is found by approximation. If $x = x_0 + h$, then by approximation $f(x) \approx f(x_0) + f'(x_0)(x - x_0)$. Let's set $x_0 = \left(\frac{\gamma(q-p)-(1-\gamma)(1-p)}{2(1-\gamma)(q-p)}\right)^2$, $h = \frac{\gamma(1-p)-p}{(1-\gamma)(q-p)}$, and $f(x) = \sqrt{x}$. Then, $f(x_0) = \sqrt{x_0} = \frac{\gamma(q-p)-(1-\gamma)(1-p)}{2(1-\gamma)(q-p)}$, and $f'(x_0) = \frac{1}{2}\frac{1}{\sqrt{x_0}} = \frac{(1-\gamma)(q-p)}{\gamma(q-p)-(1-\gamma)(1-p)}$.

Hence the inequality becomes,

$$\pi + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)} \geq \frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)} +$$

$$+ \frac{(1-\gamma)(q-p)}{\gamma(q-p) - (1-\gamma)(1-p)} \times \frac{\gamma(1-p) - p}{(1-\gamma)(q-p)}$$

$$\pi + \frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)} \geq \frac{\gamma(q-p) - (1-\gamma)(1-p)}{2(1-\gamma)(q-p)} + \frac{\gamma(1-p) - p}{\gamma(q-p) - (1-\gamma)(1-p)}$$

$$\bar{\pi} \geq \frac{\gamma(1-p) - p}{\gamma(q-p) - (1-\gamma)(1-p)}$$

$\blacksquare$

References

ADMATI, A. R., AND P. PFLEIDERER (1986): "A monopolistic market for information," *Journal of Economic Theory*, 39(2), 400–438.

——— (1990): "Direct and Indirect Sale of Information," *Econometrica*, 58(4), 901–928.

ANDREONI, J. (1995): "Cooperation in Public-Goods Experiments: Kindness or Confusion?," *The American Economic Review*, 85(4), 891–904.

ASHLEY, R., S. BALL, AND C. ECKEL (2002): "Analysis of experimental public goods data using dynamic panel regression Models," Discussion paper, Working Paper.

BABAIOFF, M., R. KLEINBERG, AND R. PAES LEME (2012): "Optimal Mechanisms for Selling Information," in *Proceedings of the 13th ACM Conference on Electronic Commerce*, EC '12, pp. 92–109. ACM.

BAERISWYL, R., AND C. CORNAND (2016): "The Predominant Role of Signal Precision in Experimental Beauty Contests.," *The BE Journal of Theoretical Economics*, 16(1), 267–301.

BAYONA, A., J. BRANDTS, AND X. VIVES (2016): "Supply Function Competition, Private Information, and Market Power: A Laboratory Study," Discussion paper no. 11378, CEPR.

BERGEMANN, D., AND A. BONATTI (2018): "Markets for Information: An Introduction," Discussion paper no. 2142, Cowles Foundation.

BERGEMANN, D., A. BONATTI, AND A. SMOLIN (2018): "The Design and Price of Information," *American Economic Review*, 108(1), 1–48.

BERGIN, J., AND B. L. LIPMAN (1996): "Evolution with state-dependent mutations," *Econometrica*, 64(4), 943–956.

BLUME, L. E. (1995): "The statistical mechanics of best-response strategy revision," *Games and Economic Behavior*, 11(2), 111–145.

——— (2003): "How noise matters," *Games and Economic Behavior*, 44(2), 251–271.

BOUN MY, K., C. CORNAND, AND R. DOS SANTOS FERREIRA (2017): "Speculation rather than enterprise? Keynes? beauty contest revisited in theory and experiment," Working paper, GATE Lyon Saint-Etienne.

BURLANDO, R. M., AND F. GUALA (2005): "Heterogeneous agents in public goods experiments," *Experimental Economics*, 8(1), 35–54.

CHAUDHURI, A. (2011): "Sustaining cooperation in laboratory public goods experiments: a selective survey of the literature," *Experimental Economics*, 14(1), 47–83.

CORNAND, C., AND F. HEINEMANN (2014): "Measuring agents reaction to private and public information in games with strategic complementarities," *Experimental Economics*, 17(1), 61–77.

CRÉMER, J., AND R. P. MCLEAN (1985): "Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent," *Econometrica*, 53(2), 345–361.

——— (1988): "Full Extraction of the Surplus in Bayesian and Dominant Strategy Auctions," *Econometrica*, 56(6), 1247–1257.

CROSON, R. T., AND M. B. MARKS (2000): "Step returns in threshold public goods: A meta-and experimental analysis," *Experimental Economics*, 2(3), 239–259.

DALE, D. J., AND J. MORGAN (2012): "Experiments on the social value of public information," Working paper, University of California.

DASGUPTA, P., P. HAMMOND, AND E. MASKIN (1979): "The Implementation of Social Choice Rules: Some General Results on Incentive Compatibility," *The Review of Economic Studies*, 46(2), 185–216.

DAVIS, D. D., AND C. A. HOLT (1993): *Experimental economics*. Princeton university press.

DAWES, R. M., J. M. ORBELL, R. T. SIMMONS, AND A. J. C. VAN DE KRAGT (1986): "Organizing Groups for Collective Action," *American Political Science Review*, 80(4), 1171–1185.

ELLISON, G. (2000): "Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution," *The Review of Economic Studies*, 67(1), 17–45.

ESÖ, P., AND B. SZENTES (2007): "The price of advice," *The RAND Journal of Economics*, 38(4), 863–880.

FEHR, E., AND J.-R. TYRAN (2008): "Limited Rationality and Strategic Interaction: The Impact of the Strategic Environment on Nominal Inertia," *Econometrica*, 76(2), 353–394.

FISCHBACHER, U. (2007): "z-Tree: Zurich toolbox for ready-made economic experiments," *Experimental Economics*, 10(2), 171–178.

FISCHBACHER, U., AND S. GÄCHTER (2010): "Social Preferences, Beliefs, and the Dynamics of Free Riding in Public Goods Experiments," *American Economic Review*, 100(1), 541–556.

FISCHBACHER, U., S. GÄCHTER, AND E. FEHR (2001): "Are people conditionally cooperative? Evidence from a public goods experiment," *Economics Letters*, 71(3), 397 – 404.

FOSTER, D., AND P. YOUNG (1990): "Stochastic evolutionary game dynamics," *Theoretical Population Biology*, 38(2), 219–232.

FREIDLIN, M. I., AND A. D. WENTZELL (1998): *Random perturbations of dynamical systems*. Springer-Verlag, Berlin/New York.

FREY, B. S., AND S. MEIER (2004): "Social comparisons and pro-social behavior: Testing" conditional cooperation" in a field experiment," *American Economic Review*, 94(5), 1717–1722.

GÄCHTER, S. (2006): "Conditional cooperation: Behavioral regularities from the lab and the field and their policy implications," Discussion paper, CeDEx Discussion Paper Series.

GIBBARD, A. (1973): "Manipulation of voting schemes: a general result," *Econometrica*, 41), number = 4, 587–601.

GOEREE, J. K., AND T. OFFERMAN (2003): "Winner's curse without overbidding," *European Economic Review*, 47(4), 625–644.

HELDT, T. (2005): "Conditional Cooperation in the Field: Cross-Country Skiers," in *Sweden." Mimeo, Uppsala University*. Citeseer.

HELLWIG, C., AND L. VELDKAMP (2009): "Knowing What Others Know: Coordination Motives in Information Acquisition," *The Review of Economic Studies*, 76(1), 223–251.

ISAAC, R. M., J. M. WALKER, AND S. H. THOMAS (1984): "Divergent evidence on free riding: An experimental examination of possible explanations," *Public Choice*, 43(2), 113–149.

KANDORI, M., G. J. MAILATH, AND R. ROB (1993): "Learning, mutation, and long run equilibria in games," *Econometrica*, 61(1), 29–56.

KESER, C., AND F. VAN WINDEN (2000): "Conditional Cooperation and Voluntary Contributions to Public Goods," *The Scandinavian Journal of Economics*, 102(1), 23–39.

KEYNES, J. M. (1936): *The General theory of employment, interest and money*. Macmillan, London.

KURZBAN, R., AND D. HOUSER (2002): "Revisiting Kindness and Confusion in Public Goods Experiments," *American Economic Review*, 92(4), 1062–1069.

——— (2005): "Experiments investigating cooperative types in humans: A complement to evolutionary theory and simulations," *Proceedings of the National Academy of Sciences*, 102(5), 1803–1807.

LEDYARD, J. (1995): "Public Goods: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, ed. by J. Kagel, and A. Roth, vol. 2, chap. 6, pp. 111–194. Princeton University Press, New Jersey.

MARTIN, R., J. RANDAL, ET AL. (2005): "Voluntary contributions to a public good: a natural field experiment," *Unpublished manuscript, Victoria University, New Zealand*.

MARWELL, G., AND R. E. AMES (1979): "Experiments on the Provision of Public Goods. I. Resources, Interest, Group Size, and the Free-Rider Problem," *American Journal of Sociology*, 84(6), 1335–1360.

——— (1980): "Experiments on the Provision of Public Goods. II. Provision Points, Stakes, Experience, and the Free-Rider Problem," *American Journal of Sociology*, 85(4), 926–937.

——— (1981): "Economists free ride, does anyone else?: Experiments on the provision of public goods, IV," *Journal of Public Economics*, 15(3), 295–310.

MORRIS, S., AND H. S. SHIN (2002): "Social Value of Public Information," *The American Economic Review*, 92(5), 1521–1534.

MULLER, L., M. SEFTON, R. STEINBERG, AND L. VESTERLUND (2008): "Strategic behavior and learning in repeated voluntary contribution experiments," *Journal of Economic Behavior & Organization*, 67(3-4), 782–793.

MYATT, D. P., AND C. WALLACE (2003): "A multinomial probit model of stochastic evolution," *Journal of Economic Theory*, 113(2), 286–301.

——— (2008a): "An evolutionary analysis of the volunteer's dilemma," *Games and Economic Behavior*, 62(1), 67–76.

——— (2008b): "When does one bad apple spoil the barrel? An evolutionary analysis of collective action," *The Review of Economic Studies*, 75(2), 499–527.

——— (2009): "Evolution, Teamwork and Collective Action: Production Targets in the Private Provision of Public Goods," *The Economic Journal*, 119(534), 61–90.

——— (2012): "Endogenous Information Acquisition in Coordination Games," *The Review of Economic Studies*, 79(1), 340–374.

——— (2014): "Central bank communication design in a Lucas-Phelps economy," *Journal of Monetary Economics*, 63, 64–79.

——— (2015): "Cournot competition and the social value of information," *Journal of Economic Theory*, 158(Part B), 466–506.

——— (2018): "Information Use and Acquisition in Price-setting Oligopolies," *The Economic Journal*, 128(609), 845–886.

MYERSON, R. B. (1979): "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47(1), 61–73.

——— (1982): "Optimal coordination mechanisms in generalized principal-agent problems," *Journal of Mathematical Economics*, 10(1), 67–81.

NEUGEBAUER, T., J. PEROTE, U. SCHMIDT, AND M. LOOS (2009): "Selfish-biased conditional cooperation: On the decline of contributions in repeated public goods experiments," *Journal of Economic Psychology*, 30(1), 52–60.

OFFERMAN, T. (1997): *Beliefs and Decision Rules in Public Goods Games: Theory and Experiments*. Kluwer Academic Publishers, Princeton.

OSTROM, E. (2000): "Collective Action and the Evolution of Social Norms," *Journal of Economic Perspectives*, 14(3), 137–158.

PALFREY, T. R., AND J. E. PRISBREY (1997): "Anomalous Behavior in Public Goods Experiments: How Much and Why?," *The American Economic Review*, 87(5), 829–846.

PALFREY, T. R., AND H. ROSENTHAL (1984): "Participation and the provision of discrete public goods: a strategic analysis," *Journal of Public Economics*, 24(2), 171–193.

PERSICO, N. (2000): "Information acquisition in auctions," *Eonometrica*, 68(1), 135–148.

SCHRAM, A., T. OFFERMAN, AND J. SONNEMANS (2008): "Explaining the comparative statics in step-level public good games," *Handbook of experimental economics results*, 1, 817–824.

SHANG, J., AND R. CROSON (2009): "A field experiment in charitable contribution: The impact of social information on the voluntary provision of public goods," *The Economic Journal*, 119(540), 1422–1439.

VAN DE KRAGT, A. J. C., J. M. ORBELL, AND R. M. DAWES (1983): "The Minimal Contributing Set as a Solution to Public Goods Problems," *American Political Science Review*, 77(1), 112–122.

VARIAN, H. (1999): "Markets for information goods," Discussion paper, Institute for Monetary and Economic Studies, Bank of Japan.

VEGA-REDONDO, F. (1997): "The evolution of Walrasian behavior," *Econometrica*, 65(2), 375–384.

YOUNG, H. P. (1993): "The evolution of conventions," *Econometrica*, 61(1), 57–84.