ESSAYS ON THE ECONOMICS OF EDUCATION

Thesis submitted for the degree of
Doctor of Philosophy
at the
University of Leicester

by

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September 2010
To my Mother, who taught me to be demanding of myself

To my Father, who taught me to follow my dreams

To Gianni, who taught me economics
Essays in Economics of Education

by

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Abstract

The first chapter proposes a theory on how students’ social background affects school teaching and job opportunities. We study a set-up where students differ in ability and social background, and we analyse the interaction between a school and an employer. Students with disadvantaged background are penalised compared to other students: they receive less teaching and/or are less likely to be hired. A surprising result is that policy aiming to subsidise education for disadvantaged students might in fact decrease their job opportunities.

The second chapter argues that assortative matching can explain over-education. Education determines individuals’ income and, due to the presence of assortative matching, the quality of the partner, who can be a colleague or a spouse. Thus an individual acquires some education to improve the expected partner’s quality. But since everybody does that, the expected partner’s quality does not increases and over-education emerges. Public intervention can solve over-education through a progressive income tax.

The third chapter examines how higher education affects job and marital satisfaction. We build up a model with assortative matching where individuals decide whether to attend university both for obtaining job satisfaction and for increasing the probability to be matched with an educated partner. The theoretical results suggest that, as assortative matching increases, the number of educated individuals increases, their job satisfaction falls while their marital satisfaction increases. We test our model using the British Household Panel Survey data for the years 2003-2006. Our empirical findings support the theoretical results.
Acknowledgements

First and foremost I thank my parents and the rest of my family, Jacopo and Valter.

To my supervisors, Prof. Gianni De Fraja and Dr Abbi Kedir, I am truly thankful for all your guidance and patience. It has been an honour and a privilege to have been your student.

I would like to thank Piercarlo Zanchettin, Ludovic Renou, Suresh Mutuswami, James Rockey, Gaia Garino and Javier Rivas for their advice, encouragement and help. I am grateful to Matteo Lippi Bruni and Piero Pasotti for inspiring me to pursue a doctorate. I am indebted to the University of Leicester for giving me the opportunity to do my PhD and for its financial support. Also I thank all the departimental staff.

A special thank to my friend Jorge Villasenor, who picked up my pieces in my most troublesome days. I am extremely grateful to Susan Murray, Lucio Morettini, Valentina Conti, Miguel Flores, Kavita Sirichand, Francesca Acacia, Andri Kyrizi, Tom Allen, Muntaz Hussain Shah, Johan Rewilak and Fuyu Yang for their friendship. I extend my thanks to the many fellow students I have had the pleasure of meeting.
Contents

1 Social Background Effects on School and Job Opportunities  5
  1.1 Introduction ....................................................... 9
  1.2 The model ......................................................... 13
    1.2.1 Employer ................................................... 13
    1.2.2 School ...................................................... 15
    1.2.3 The game between the school and the employer .......... 16
  1.3 Equilibria ....................................................... 16
    1.3.1 General case: differences in social background ....... 20
    1.3.2 One social background ................................. 24
    1.3.3 Analysis of equilibria ............................... 26
  1.4 Subsidising disadvantaged students ....................... 27
  1.5 Concluding remarks ....................................... 29

2 Assortative Matching in Partnerships and Over-Education  30
  2.1 Introduction ................................................... 32
  2.2 The model .................................................... 36
3 Sex and the Uni: Higher Education Effects in Job and Marital Satisfaction

3.1 Introduction .............................................. 51
3.2 Theoretical model ........................................ 55
  3.2.1 The matching ......................................... 57
3.3 Analysis of equilibrium ................................. 59
3.4 The data .................................................. 65
  3.4.1 Dependent variables ................................. 66
  3.4.2 Explanatory variables ............................... 67
  3.4.3 Descriptive analysis ............................... 68
3.5 The Empirical Model ................................. 70
3.6 Results .................................................. 73
3.7 Concluding remarks .................................... 79
List of Figures

1.1 Proposition 1. Differences in social background . . . . . . . . 21
1.2 Propositon 2. No differences in social background . . . . . . . . 25
2.1 Definition 5. Over-education . . . . . . . . . . . . . . . . . . . . . . 43
2.2 Proposition 7. Optimal progressive income tax . . . . . . . . . 46
List of Tables

3.1 Payoff matrix ............................................. 57
3.2 Matching mechanism ................................. 60
3.3 Computational example of equilibrium .......... 64
3.4 Descriptive statistics: full sample, men and women .... 69
3.5 Correlation between life overall satisfaction, job and marital satisfaction ................................. 70
3.6 Assortative matching: random-effects probit .... 76
3.7 Job satisfaction results: pooled ordered probit with robust estimators ............................................. 77
3.8 Marital satisfaction results: pooled ordered probit with robust estimators ............................................. 78
Chapter 1

Social Background Effects on School and Job Opportunities

This chapter proposes a theory on how students’ social background affects school teaching and job opportunities. We study a set-up where students differ in ability and social background, and we analyse the interaction between a school and an employer. Students with disadvantaged background are penalised compared to other students: they receive less teaching and/or are less likely to be hired. A surprising result is that policy aiming to subsidise education for disadvantaged students might in fact decrease their job opportunities.
List of symbols

- $\theta_H$: high ability.
- $\theta_L$: low ability.
- $a$: advantaged social background.
- $d$: disadvantaged social background.
- $p_a \in [0,1]$: probability that an advantaged student has high-ability.
- $p_d \in [0,1]$: probability that a disadvantaged student has high-ability.
- $\lambda \in [0,1]$: proportion of advantaged students.
- $\eta_H \in [0,1]$: probability that a high-ability student not receiving extra teaching will obtain a high grade.
- $\eta_L \in [0,1]$: probability that a low-ability student receiving extra teaching will obtain a high grade.
- $\Phi \in [0,1]$: job capacity.
- $g_U$: high grade.
- $g_D$: low grade.
- $\nu$: employer’s payoff for hiring a high-ability student.
- $\mu$: school’s benefit when a student is hired.
• $c$: school’s cost when a student receives extra teaching.

• $H(\theta_H)$: number of hired students with high ability.

• $H(\theta_L)$: number of hired students with low ability.

• $ET$: number of students receiving extra teaching.

• $x_{La}, x_{Ha}, x_{Ld}, x_{Hd} \in [0, 1]$: probabilities that the school gives extra teaching to an advantaged and low or high-ability student and to a disadvantaged and low or high-ability student, respectively.

• $z_{Ua}, z_{Da}, z_{Ud}, z_{Dd} \in [0, 1]$: probabilities that the employer hires an advantaged student with a high or low grade and a disadvantaged student with a high or low grade, respectively.

• $\pi(\theta_z \mid g_j, p_i, x_{zi})$: employer’s belief about a student’s ability.

• $i \in \{a, d\}$: social background.

• $z \in \{H, L\}$: ability level.

• $j \in \{U, D\}$: possible grade.

• $s \in [0, 1]$: government subsidy of school extra-teaching costs for a disadvantaged student.

• $\Pi^E_{ji}$: employer’s expected payoff of hiring a student.

• $\Pi^N_{ji}$: employer’s expected payoff of not hiring a student.
• $\Pi_{zt}^T$: school expected payoff of giving extra teaching to a student.

• $\Pi_{zt}^{NT}$: school expected payoff of not giving extra teaching to a student.
1.1 Introduction

There is substantial evidence that individuals’ social background influences their educational results and job opportunities. This chapter proposes a theoretical explanation for this by examining how school and employer behaviour changes according to the students’ social background.

We consider a signalling game between a school and an employer where the students they deal with differ in ability and social background. The school prepares students for a final exam and wants the largest number of them to be hired. We assume that teaching improves the students’ chance of obtaining a good grade but not their ability. On the other side, the employer wants to hire only high-ability students and observes the exam grade as a signal of ability.

We assume that students with advantaged social backgrounds are more likely to have higher ability. This assumption is crucial for our results and is supported by past research documenting that family and environmental factors are major predictors of cognitive skills (Cunha et al., 2006, Carneiro and Heckman, 2003, Joshi and McCulloc, 2000). The idea is that given the same distribution of innate ability within populations with differing social backgrounds, an advantaged environment can help develop skills via parental and peer pressure.

Our results suggest that students from a disadvantaged social background
receive less teaching and/or are hired with less chance. The intuition is the following. The employer prefers advantaged rather than disadvantaged and high-grade students as they are more likely to have high ability. To increase the hiring opportunities of disadvantaged students, the school may devote less teaching effort to disadvantaged and low-ability students, because this gives them less chance of obtaining a high grade, and thus the expected ability of disadvantaged and high-grade students increases. Despite that, the employer may still find it preferable to hire advantaged students. Thus disadvantaged students are penalised in school attainment and/or in job opportunities, and this clearly aggravates class differences.

Furthermore, these results are related to the phenomenon known as grade inflation, which takes place when good grades are awarded too easily. An interesting result is that the presence of grade inflation might help disadvantaged students to have the same job opportunities as advantaged students. The reason is that grade inflation diminishes the employer’s expectations about students’ ability. Since the school here devotes less teaching effort to disadvantaged rather than advantaged and low-ability students, the effect of grade inflation is stronger for advantaged students. This leads high-grade students to have the same job opportunities irrespective of their social backgrounds.

We then consider a government that subsidises the cost of teaching disadvantaged students. Such policy may diminish their chance of being hired. The reason is that more low-ability and disadvantaged students obtain a high grade, therefore the employer’s expectations about the ability high-grade and
disadvantaged students decrease.

The chapter can be related to the literature on signalling models (Spence, 1973; for a survey of the literature, see Riley, 2001). In particular, the model presents a structure which is similar to Waldmann (1984), where a game between an employer and the job market takes place and the employees are “no players”. In analogy, in our model a school and an employer interact and the students are “no players”.

The chapter is also related to the literature on grade inflation. Chan et al. (2007) propose a signalling model where employers know only the students’ grade but not the students’ ability and the state of the world (that is, the proportion of talented students). This gives rise to an incentive to help some low ability students by giving them good grades. Indeed the labour market cannot fully distinguish whether this is due to a high grading standard or whether the school has a large proportion of talented students, and this in turn hampers the signal of good students. In Chan et al. differences in social background are not considered, which instead are central in our analysis. Also, we assume that the employer knows the proportion of talented students.

Schwager (2008) examines the impact of grade inflation on the job market with students that differ in ability and social background. They are matched with firms which offer different kinds of job, according to the grade and the expected ability. Regardless of the social background, it is possible that mediocre students receive a high grade caused by grade inflation. Also, the high-ability students from advantaged backgrounds may benefit from grade inflation since
this shields them from the competition on the part of able and disadvantaged students. Compared to this analysis, we share the same assumptions on the distributions of ability with differing social backgrounds, but in our model disadvantaged students may benefit from the presence of inflation grade.

Finally, our chapter is related to De Fraja (2005) who studies the provision of education when students differ in ability and social background. In the presence of asymmetric information (the government does not know the student’s ability) and externalities (the public provision of education makes students acquire more education than they would acquire privately) the optimal provision of education is a second best result where high-ability and disadvantaged students receive more education than high-ability and advantaged students. Hence the introduction of reverse discrimination policies, like affirmative action\(^2\), are justified on efficiency grounds, and the trade-off between equity and efficiency disappears. According to our results, a policy intervention is necessary in order to obtain the optimal provision of education in the presence of differences in social background, since a school caring about the employment of its students does not devote more teaching effort to disadvantaged rather than advantaged and high-ability students.

The remainder of the chapter is organised as follows. The model is presented in Section 1.2. Section 1.3 examines the equilibria. Section 1.4 considers the\(^2\)The term “affirmative action” refers to policies that attempt to increase the presence of individuals who belong to minorities in areas of employment and education. These policies generate controversy when they involve preferential selection on the basis of race, gender or ethnicity.
government intervention. Section 1.5 concludes.

1.2 The model

We study the interaction between a single school and a single employer\(^3\). The interaction takes place since a number of students, with measure normalised to one, attends school and afterwards asks the employer for a job.

Students can have high ($\theta_H$) or low ($\theta_L$) ability. In addition to ability, students can come from advantaged ($a$) or disadvantaged ($d$) social backgrounds, which is public information: this can be interpreted as a one-dimensional measure of family environment, peer groups\(^4\) and neighbourhood. We denote as $\lambda \in [0,1]$ the proportion of advantaged students. Let $p_a, p_d \in [0,1]$ be the probability that an advantaged or disadvantaged student has high-ability, respectively. As we stressed in the introduction, we assume $p_a > p_d$.

1.2.1 Employer

The employer decides whether or not to hire a student and offers a single job type.

We define job capacity as the maximum number of students that may be hired and we denoted it as $\Phi \in [0,1]$. For the sake of simplicity, we assume that $\Phi$ is exogenous and depends on the employer’s production potential, that is the

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\(^3\)For simplicity, we abstract from factors such as competition between schools and between employers.

\(^4\)Peer groups means that students learn better if they are in a group of abler students.
size and technology of his firm. As a consequence, neither the interaction between school and the employer nor the students’ type can affect the employer’s production potential.

Still for simplicity, we rule out uncertainty in the market where the employer operates and we assume that the students’ ability determines the employer’s profit entirely. In particular, each high-ability student yields a profit of \( \nu > 0 \) while each low-ability student yields a profit of \(-1\). The choice of \( \nu \) and \(-1\) is to simplify the algebra: other normalisations would complicate the analysis without changing the results.

The assumption that a low-ability student gives negative profit can be interpreted in many ways: low-ability employees may have a marginal productivity which is lower than salary cost. In addition, the employer may want to lay off a low-ability employee but this action still comes at a cost, e.g. industrial disputes, wasted training costs and time, and so on.

The employer doesn’t know the students’ ability and observes the grade that a student obtains in a final exam\(^5\) as a signal of it. The possible exam outcomes are a low (\(g_D\)) or a high (\(g_U\)) grade.

The employer’s payoff is given by \( \nu H(\theta_H) - 1H(\theta_L) \), where \(H(\theta_H)\) and \(H(\theta_L)\) are the number of hired students with high or low ability, respectively.

\(^5\)The exams which we have in mind in the real world are the “Scholastic Assessment Test” in United States and the “National Curriculum Assessment” in United Kingdom. These exams are managed by the “Educational Testing Service” (Rourke and Ingram, 1991).
1.2.2 School

The school influences job opportunities by preparing students for the final exam\(^6\), and learns the students’ ability through their tests and assessments results.

The school obtains a benefit \( \mu > 0 \) for every hired student. The reason is that each student’s employment increases its reputation as an effective institution for obtaining a job. Of course a school might pursue this objective in different ways, for example, by having a preference for some of their students: it may derive more benefit from increases in the job opportunities of their brightest pupils, or, vice versa, from increases in the job opportunities of their weakest pupils. To depict the interaction with the employer in the most general way we abstract from these differences.

The preparation for the exam requires resources: the quantity of teaching, the quality of buildings and classroom equipment and the teachers’ effort. We refer to all these aspects as “teaching”. In addition to teaching, the school can provide some students with “extra teaching”, that is additional resources, extra tuition, trips and more facilities. We assume that, with teaching only, the student’s probability of obtaining a high grade is \( \eta_H \in (0, 1) \) if she has high ability and zero if she has low ability. With extra teaching, the student’s probability of obtaining a high grade is 1 if she has high ability and \( \eta_L \in (0, 1) \) if she has low ability. The school bears a cost \( c > 0 \) for each student receiving

---

\(^6\)Note that the school does not arrange the exam and hence it cannot manipulate the students’ grades.
Hence the school’s payoff is given by $\mu(H(\theta_H) + H(\theta_L)) - cET$, where $ET$ is the number of students receiving extra teaching.

1.2.3 The game between the school and the employer

The interaction can be described as follows. Nature draws the student types. Then, each student\(^7\) attends school and the school chooses whether to give her extra teaching. At the end of school period, students take the final exam. Finally, each student applies for a job, and the employer decides whether to hire her.

1.3 Equilibria

We study the perfect Bayesian equilibria of this game. The choice of this equilibrium concept is motivated by the employer’s missing information about students’ ability.

A perfect Bayesian equilibrium is a combination of school and employer strategies and beliefs where both agents maximise their payoff. After observing a grade, the employer has a belief about the student type, conditional on all the information he has: student’s grade, distribution of ability according to the student’s social background and school strategy. Hence his belief must be consistent with Bayes’ rule. For each grade, the employer must maximise his

\(^7\)For simplicity, we assumed away the influence of student’s effort.
expected profit, given his belief and the school strategy. In turn the school’s strategy must maximise its expected payoff, given the employer’s strategy\(^8\). Then, the job capacity constraint requires that the number of hired students is at most \(\Phi\).

We start by making the following assumptions.

**Assumption 1** \(\Phi < \lambda(p_a + \eta_L (1 - p_a)) + (1 - \lambda)(p_d + \eta_L (1 - p_d))\).

**Assumption 2** \(\mu > \max \left\{ \frac{c}{\eta_L}, \frac{c}{1 - \eta_L}, \frac{c}{\eta_H}, \frac{c}{1 - \eta_H} \right\} \).

Assumption 1 says that the job capacity is lower than the highest possible number of high-grade students. This assumption focuses the attention on the equilibria where social background plays a role in the school and employer’s decisions. When this assumption does not hold, the employer may hire all the high-grade students irrespective of their social background. In other words, the individuals’ social background would not affect their job opportunities. As we stressed in the introduction, the empirical evidence tells us in reality this is not the case, so we prefer to set this case aside.

Assumption 2 says that the school benefit of having a hired student is sufficiently higher than the cost of providing her with extra teaching. This rules out the possibility that a student would not receive extra teaching because, according to the school technology, it is too costly. Here we want to focus on the case where the school’s response depends on the employer strategy completely.

\(^8\)Note that the school has perfect information of the student types.
and disregard the role of school technology\textsuperscript{9}. After presenting Assumption 1 and 2, we introduce the notations of the school and employer’s actions:

- \( x_{La}, x_{Ha}, x_{Ld}, x_{Hd} \in [0, 1] \) are the probabilities that the school gives extra teaching to an advantaged and low or high-ability student and to a disadvantaged and low or high-ability student, respectively;

- \( z_{Ua}, z_{Da}, z_{Ud}, z_{Dd} \in [0, 1] \) are the probabilities that the employer hires an advantaged student with a high or low grade and a disadvantaged student with a high or low grade, respectively.

We then define the employer beliefs about the students’ ability. These are denoted by \( \pi(\theta_z \mid g_j, p_i, x_{zi}) \), where \( z \in \{H, L\} \) is the ability level, \( g_j, j \in \{U, D\} \) is the grade, \( p_i, i \in \{a, d\} \) is the distribution of ability and \( x_{zi} \) is the school strategy.

**Definition 1** The employer’s beliefs about the students’ ability which are consistent with the Bayes’ rule are

\[
\pi(\theta_H \mid g_j, p_i, x_{Hi}) = \frac{p_i x_{Hi}}{p_i x_{Hi} + \eta_x x_{Li}(1-p_i)}, \text{ and}
\]

\[
\pi(\theta_L \mid g_j, p_i, x_{Li}) = \frac{\eta_l x_{Li}(1-p_i)}{p_i x_{Hi} + \eta_l x_{Li}(1-p_l)}.
\]

As we will show below in the proof of Proposition 2, if Assumption 1 and 2 hold the equilibrium will be one of three types, which we label *high-employment*, *middle-employment* and *low-employment equilibrium*.

\textsuperscript{9}To relax this assumption would allow us to compare schools with different technology. This investigation can be interesting and may be considered for future work.
Definition 2 A high-employment equilibrium is an equilibrium in which the school strategy is $x_{Ha} = x_{Hd} = x_{La} = x_{Ld} = 1$, while the employer strategy is $z_{Ua} = 1$, $z_{Ud} = \frac{\Phi - \lambda(p_a + \eta_L(1-p_a))}{(1-\lambda)(p_d + \eta_L(1-p_d))} \in (0,1)$, $z_{Da} = z_{Dd} = 0$.

Definition 3 A middle-employment equilibrium is an equilibrium in which the school strategy is $x_{Ha} = x_{Hd} = x_{La} = 1$, $x_{Ld} = p_d \frac{\nu}{(1-p_d) \eta_L} \in (0,1)$, while the employer strategy is $z_{Ua} = 1$, $z_{Ud} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi - \lambda(p_a + \eta_L(1-p_a))}{(1-\lambda)(p_d + \eta_L)(1+p)} \right\} \in (0,1)$, $z_{Da} = z_{Dd} = 0$.

Definition 4 A low-employment equilibrium is an equilibrium in which the school strategy is $x_{Ha} = x_{Hd} = 1$, $x_{La} = p_a \frac{\nu}{(1-p_a) \eta_L} \in (0,1)$, $x_{Ld} = p_d \frac{\nu}{(1-p_d) \eta_L} \in (0,1)$, while the employer strategy is $z_{Ua} = z_{Ud} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi}{(\lambda p_a + (1-\lambda)p_d)(1+p)} \right\} \in (0,1)$, $z_{Da} = z_{Dd} = 0$.

In the high-employment equilibrium the employer hires $\Phi$ students (i.e., as many students as he can according to the job capacity) while the school provides every student with extra teaching. Here the employer obtains a positive expected profit from high-grade students from both advantaged and disadvantaged backgrounds. Thus, his optimal strategy is to hire all of them, but Assumption 1 prevents this possibility. Then the employer needs to choose between these two types. He will hire all the advantaged students, since they give a higher expected profit, and the disadvantaged students will be hired only for the remaining job capacity.

In the middle-employment equilibrium, the employer does not want to hire
all the disadvantaged and high-grade students\textsuperscript{10}. In turn, the school provides extra teaching to a lower number of disadvantaged and low-ability students. This strategy increases the probability that a disadvantaged and high-grade student has high ability and thus it increases her chance to be hired.

In the low-employment equilibrium, the employer does not hire all the high-grade students from both advantaged and disadvantaged backgrounds, but is indifferent between hiring one of these two types. Like in the previous equilibrium, the school provides extra teaching to fewer disadvantaged than advantaged and low-ability students.

Note that the employer never hires a low-grade student: indeed all the high-ability students receive extra-teaching in each equilibrium, and hence all of them will obtain a high grade with probability one. Thus a low-grade student has low ability with probability one.

1.3.1 General case: differences in social background

In this section, we consider $\lambda \in (0, 1)$. The following proposition shows which equilibrium occurs depending on the values of $p_d$ and $p_a$.

**Proposition 1** Let Assumptions 1 and 2 hold. The high-employment equilibrium occurs if $p_a \geq p_d \geq \frac{\eta}{\nu+\eta_L}$; the middle-employment equilibrium occurs if $p_a \geq \frac{\eta}{\nu+\eta_L} > p_d$; the low-employment equilibrium occurs if $p_d < p_a < \frac{\eta}{\nu+\eta_L}$.

**Proof.** See Appendix. ■

\textsuperscript{10}Given this strategy, the total of hired students might be higher or lower than the students’ capacity, and in the former case clearly this is $\Phi$ again.
Figure 1 illustrates Proposition 1 when a given value of $\eta_L$ is considered. The assumption $p_a > p_d$ holds above the upward-sloping 45 degrees line.

To interpret Proposition 1, begin by looking at the key assumption, $p_a > p_d$. This makes the employer obtain a higher expected payoff by hiring advantaged students, given the same school strategy for every student. However this may not happen if the school gives extra teaching to a lower proportion of disadvantaged than advantaged and low-ability students, since this would increase the expected quality of the disadvantaged and high-grade students.

When both $p_d$ and $p_a$ are higher than $\frac{\eta_L}{\eta + \eta_L}$ (high-employment equilibrium), the school maximises the amount of hired students by providing every student with extra teaching, since the employer thinks that a high grade student very likely has high ability, irrespective of her social background. In the other two
cases, the school maximises the amount of hired disadvantaged students by giving less extra teaching to low-ability and disadvantaged compared to advantaged students.

In particular, when \( p_a \) is higher and \( p_d \) is lower than \( \eta_{L} \) (middle-employment equilibrium), the employer still prefers advantaged rather than disadvantaged and high-grade students. When both \( p_d \) and \( p_a \) are lower than \( \eta_{L} \) (low-employment equilibrium), the employer is indifferent between hiring an advantaged or a disadvantaged and high-grade student.

Proposition 1 shows that disadvantaged students are penalised compared to advantaged students in each possible case: they may receive less teaching, or be hired with lower probability to the employer, or both. In particular the high and middle-employment equilibrium, where disadvantaged students are penalised in the job market, exacerbate differences in social class.

Note that the probability \( \eta_{L} \) can be interpreted as an inverse measure of educational standard. The educational standard measures the level of difficulty at school, how hard is to obtain a high grade\(^{11}\). Indeed as \( \eta_{L} \) increases, obtaining a high grade becomes “easier” for some students. Thus the higher \( \eta_{L} \), the lower the educational standard\(^{12}\).

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\(^{11}\) The literature on educational standards examines the criteria adopted by schools in evaluating students. The theoretical frameworks on educational standards are provided by Costrell (1994, 1997) and Betts (1998). In the context of educational standards, the issue of social background has been introduced by Himmler and Schwager (2007), who show that a school with a large proportion of disadvantaged students applies less demanding standards since its students have less incentives to graduate.

\(^{12}\) The educational standard can be employed as a tool for welfare analysis. A normative extension of our set-up can be considered for future work.
Thus, these results suggest some interesting considerations about grade inflation. In our model, this situation is depicted where the educational standard is low ($\eta_L$ is high), since more low-ability students obtain a high grade. We can easily observe that when $\eta_L$ is high we are very likely to be in the low-employment equilibrium, where the employer is indifferent between hiring advantaged or disadvantaged and high-grade students. Indeed the presence of grade inflation diminishes the employer’s expectations about students’ ability. Since the school devotes less teaching effort to disadvantaged rather than advantaged and low-ability students, the grade inflation effect is stronger for advantaged students, leading to the low-employment equilibrium. Therefore grade inflation may have some positive effect by helping disadvantaged students to have the same job opportunities as advantaged students. This is in contrast with other results on grade inflation (Schwager, 2008), where the job opportunities of high-ability and disadvantaged students are penalised by the grade inflation of low-ability and advantaged students.

Finally, this result can be linked to the analysis of efficient provision of education. De Fraja (2005) shows that, in the presence of differences in social background, the optimal provision of education requires that disadvantaged and high-ability students receive more education than high-ability and advantaged students. According to Proposition 1, a school caring about the employment of its students does not devote more teaching effort to disadvantaged rather than advantaged and high-ability students. Therefore a policy intervention would be necessary to reach an efficient level of education.
1.3.2 One social background

In this section we assume no differences in social background. This allow us to highlight the role of other characteristics, such as the educational standard $\eta_L$ and the distribution of ability, and the school and employer technology.

We consider a population of disadvantaged students, i.e., $\lambda = 0$. Indeed, a population of only advantaged student ($\lambda = 1$) would make the high and middle-employment equilibria to be indeterminate. The following proposition shows which equilibrium takes place according to the value of $p_d$.

**Proposition 2** Let Assumptions 1 and 2 hold and $\lambda = 0$. The high-employment equilibrium occurs if $p_d \geq \frac{\eta_L}{\nu + \eta_L}$; the middle/low-employment equilibrium occurs if $p_d < \frac{\eta_L}{\nu + \eta_L}$.

**Proof.** See Appendix.

Figure 2 illustrates Proposition 2. Assumption 1 holds below the downward-sloping area.

The equilibrium which occurs depends on $p_d$ and $\eta_L$. If the educational standard is high ($\eta_L$ low) and $p_d$ is high, the equilibrium is high-employment. If $\eta_L$ is high and $p_d$ is low, the middle/low-employment equilibrium occurs. If both are high (or vice versa), which equilibrium occurs depends on which effect prevails.

The upward-sloping line represents the points where $p_d = \frac{\eta_L}{\nu + \eta_L}$. As the profit $\nu$ increases, the employer hires more students and the threshold shifts

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13 Note that, if $\lambda = 0$ the middle or the low-employment equilibrium are equivalent for a disadvantaged student.
down. As the educational standard decreases ($\eta_L$ high), the amount of low-ability and high-grade students increases. Therefore the employer hires less students and the threshold shifts up.

Finally, some considerations are necessary about the school strategy. Note that in the high-employment equilibrium, the school gives extra-teaching to all students even though some of them will not be hired. This happens because of Assumption 2, which ensures a very high school benefit from a student’s employment. The cost of teaching a student who will not be hired is much smaller than the benefit loss incurred from a non-hired student who did not receive extra teaching.
1.3.3 Analysis of equilibria

In this section we study the properties of our equilibria. The following corollary shows some comparative statics results.

**Corollary 3** A decrease in the educational standard (an increase in $\eta_L$) diminishes the employment opportunities and the provision of extra teaching; an increase in the employer’s profit $\nu$ increases the provision of extra teaching; an increase in the proportion of advantaged students $\lambda$ increases the employment opportunities for disadvantaged students.

**Proof.** See Appendix.

An increase in $\eta_L$ makes the number of high-grade students increase. Thus their probability of being hired diminishes. In turn this makes the probability of receiving extra teaching diminish.

An increase in $\nu$ leads to more employment opportunities, hence the school gives extra teaching to more low-ability students.

An increase in $\lambda$ has two contrasting effects in a high-employment equilibrium: the amount of disadvantaged and high-grade students diminishes and the employment opportunities for disadvantaged students are lowered. With the first effect, the probability of a disadvantaged and high-grade student being hired increases, while it diminishes with the second one. Nevertheless, the first effect more than offsets the second effect. The reason is the following: a decrease in the amount of disadvantaged and high-grade students increases the job capacity relative to disadvantaged students $\left(\frac{\phi}{(1-\lambda)(\nu_L+\eta_L(1-\nu_L))}\right)$.
with more intensity than it diminishes the relative employment placements 
\((-\frac{\lambda(p_a+\eta_L(1-p_a))}{(1-\lambda)(p_d+\eta_L(1-p_d))})\). Therefore a higher proportion of advantaged students may increase the job opportunities of disadvantaged students.

### 1.4 Subsidising disadvantaged students

In many countries, governments spend substantial resources to fight unequal educational outcomes\(^\text{14}\). We can analyse such an intervention by considering a government that cannot observe the student’s ability and subsidises the school of an amount \(s \in [0,1]\) of its extra-teaching cost \(c\) for all disadvantaged students. We examine the problem from a partial equilibrium perspective, in the sense that government taxation is not integrated into education subsidies.

The following proposition shows the policy results.

**Proposition 4** Assume that the government funds \(cs\) for every disadvantaged student receiving extra-teaching:

(i) if \(p_a \geq p_d \geq \frac{\eta_L}{\nu+\eta_L}\), the high-employment equilibrium occurs (as before);

(ii) if \(p_a \geq \frac{\eta_L}{\nu+\eta_L} > p_d\), the school strategy is \(x_{Ha} = x_{Hd} = x_{La} = 1, x_{Ld} = \frac{p_d}{(1-p_d)} \frac{\nu}{\eta_L} \in (0,1)\), while the employer strategy is \(z_{Ua} = 1, z_{Ud} = \min \left\{ \frac{c(1-s)}{\mu \eta_L}, \frac{\Phi-\lambda(p_a+\eta_L(1-p_a))}{(1-\lambda)(p_d(1+\nu))} \right\} \in (0,1), z_{Da} = z_{Dd} = 0\).

\(^{14}\)To cite some example, in the United States, recent measures of funding education for disadvantaged have been considered the “No Child Left Behind Act” of 2001, and in the “American Recovery and Reinvestment Act” of 2009. In the United Kingdom, the Education Manteinance Allowance (EMA) funds low-income students who decide to keep studying after 16.
(iii) if \( p_d < p_a < \frac{\eta_L}{\nu + \eta_L} \), the school strategy is \( x_{Ha} = x_{Hd} = 1 \), \( x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta_L} \in (0, 1) \), \( x_{Ld} = \frac{p_d}{(1-p_d)} \frac{\nu}{\eta_L} \in (0, 1) \), while the employer strategy is

\[
z_{Ua} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi}{(\lambda p_a + (1-s)(1-\lambda)p_d)(1+\nu)} \right\}, \quad z_{Ud} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi}{(1-\lambda)p_d}(1+\nu) \right\}, \quad z_{Da} = z_{Dd} = 0.
\]

**Proof.** See Appendix. ■

In case (i) \( (p_a \geq \frac{\eta_L}{\nu + \eta_L} \) and \( p_d \geq \frac{\eta_L}{\nu + \eta_L} \)), the policy does not have any effect whatsoever, since the school would have given extra teaching to every student even if no policy was applied.

In cases (ii) and (iii), as \( s \) increases, the probability that a disadvantaged student is hired decreases. In the case (iii) also, the disadvantaged and high-grade student does not have hiring opportunities as good as an advantaged student, like in the case \( p_d < p_a < \frac{\eta_L}{\nu + \eta_L} \) without subsidy. Finally, when \( s = 1 \) (full subsidising), the school gives extra teaching to all disadvantaged students and the employer never hires a disadvantaged student.

Thus the policy might worsen her hiring opportunities. The reason is intuitive. The lower the school extra-teaching cost for a disadvantaged student, the lower the credibility of a high grade as a signal of high ability if \( p_d \) is low. Providing only high-ability students with extra teaching is not a credible strategy, as *ex post* the school would give it even to low-ability students.

No policy conclusion should be drawn from it. However this analysis suggests care should be taken in policy choices, since the attempt to improve the schooling attainment of disadvantaged students might in fact diminish their job opportunities.
1.5 Concluding remarks

This chapter examines how social background affects school’s teaching and an employer’s recruitment. We analysed the interaction between a school and an employer when students attend school and then apply for a job. Our results suggest that disadvantaged students are penalised compared to advantaged students, as they receive less teaching and/or are less likely to be hired.

The policy considerations can be extended in many directions. The government might impose some restriction on the employer strategy in order to favour disadvantaged students, like in the case of affirmative action. For instance, the employer might be forced to hire a certain number of disadvantaged students. In welfare analysis a policy can be considered where the educational standard \( (\eta_L) \) is set to maximize welfare.

Furthermore, the framework can be developed in several ways, two of which we discuss briefly. First, it seems natural to consider different schools for each social group by taking into account differences in quality of teaching. Second, it would be interesting to examine this framework alongside different generations for explaining segregation or inequality. The analysis of an extended model regarding these expansions is left for future work.
Chapter 2

Assortative Matching in Partnerships and Over-Education

This chapter argues that assortative matching can explain over-education. Education determines individuals’ income and, due to the presence of assortative matching, the quality of the partner, who can be a colleague or a spouse. Thus an individual acquires some education to improve the expected partner’s quality. But since everybody does that, the expected partner’s quality does not increase and over-education emerges. Public intervention can solve over-education through a progressive income tax.
List of symbols

- $\theta \in [\theta, \overline{\theta}]$: individual’s ability.
- $\theta_p \in [\theta, \overline{\theta}]$: partner’s ability.
- $e \geq 0$: education.
- $c > 0$: cost parameter of education.
- $\alpha \in [0, 1]$: relative importance of the partner’s quality in determining the individual’s utility.
- $\beta \in [0, 1]$: probability of assortative matching.
- $W$: social welfare.
- $\gamma \neq 1$: tax progression.
- $e\theta$: individual’s income.
- $\alpha \theta_p$: partner’s quality.
- $e_{ov}$: individual level of education in equilibrium.
- $e^*$: individual level of education in equilibrium with no over-education.
2.1 Introduction

In recent decades, the level of educational attainment in developed countries has surpassed the skill requirements of available jobs\(^1\). This phenomenon is known as “over-education”. There is a large empirical literature measuring over-education\(^2\), while this chapter aims to contribute to a theoretical understanding of it.

We propose an explanation for the existence of over-education based on the idea that acquiring education has two main effects. First, it improves job conditions: income, job quality, and so on. Second, it influences the quality of the future colleagues and spouses.

School and university are among the places where people create their own social networks, make friends and spend a considerable part of their youth. At school, individuals can meet their future colleagues. For instance, school or university mates can apply to the same company, decide to work in partnership or find themselves working in the same firm. Also, many people meet their spouse at school\(^3\). Colleagues and spouses who met at school share similar

\(^1\) Vaisey (2006) shows evidence that a substantial and growing number of American workers are over-qualified for their jobs along the period 1972-2002. The principal time-trend is positive and linear, and appears to be the result of the widening gap between a large expansion in educational attainment and only modest increases in job educational requirements over the past three decades. Budria and Moro-Egido (2007) find same evidence in European countries and a negative differential in salary between over-qualified individuals and their well-matched counterparts.

\(^2\) For discussions, see Hartog, 2000 and McGuinness, 2006.

\(^3\) Stevens (1991) analysed the reasons why spouses tend to have similar educational levels. In the sample considered, more than 50% of spouses attended the same school, college or university.
education levels\textsuperscript{4}. We refer to this positive correlation as “assortative matching”\textsuperscript{5}. Assortative matching reflects similarities in innate ability, since this is similar in individuals who share the same school experience. Our idea is that the presence of assortative matching may cause over-education.

We build up a model where individuals differ in ability. They study and are matched in the working period with a partner, who can be a colleague or a spouse. The partner’s ability positively affects the individual’s utility. This may be due to a variety of reasons. An individual can benefit from a colleague by informal apprenticeship, appraising or good influence, and from a spouse by sharing interests and income. Individuals maximise their expected utility by choosing their education levels and taking into account their matching.

This can be random or assortative. Random matching takes place when partners meet each other by chance. Assortative matching occurs if an individual meets the partner at school or university, or in any situation where the educational level influences the chance of a meeting. Whether matching is assortative depends on the institutions and tradition of a society: for example, the more the educational system requires that students spend time together, the more likely the matching will be assortative.


\textsuperscript{5}The expression “assortative matching” has been coined by Gary Becker (1973), and it alludes to a relationship (either positive or negative) between characteristics of spouses. We refer to the similarities in the levels of education specifically, and we apply the relationship not only to spouses, but also to colleagues.
Our results suggest that assortative matching makes the education acquired inefficient from a social point of view. In particular, individuals would reach a lower level of education in a socially optimal solution. Thus we define over-education as the difference between the actual level of education and the socially optimal level of education.

What determines these results? Assortative matching gives an incentive to study more in order to increase the partner’s quality. However, every individual with the same level of ability acquires the same quantity of education and hence is matched with a partner of the same type. This approach is in the spirit of Akerlof (1976), where workers signal their ability through their work speed. In order to look more able, workers of a given ability work faster than they would if they were not observed. In our model, individuals observe the partner’s education level as a signal of ability, and in order to look more able they acquire more education than they would if assortative matching were not present.

The chapter considers next whether public intervention can make individuals reach the socially efficient level of education by introducing a progressive income tax. This intervention can correct over-education by imposing a higher fiscal burden the higher the individual’s income. These results may justify income progressive taxation on efficiency grounds and not to answer to redistributive arguments.

To our knowledge, over-education has not been largely developed from a theoretical perspective, with few notable exceptions. Frank (1978) investigates
the differentials in wages between men and women as a consequence of female over-qualification. This is caused by family location decisions, since a family is more likely to move close to better jobs for the husband, sacrificing the wife’s opportunities. Hence the role differences between men and women are essential for his results, and over-education is generated by a job search process. Compared to this work, we do not consider differences in wages among sexes, job search nor the different role in society between men and women.

Our results are consistent with Lommerud (1989), where over-education occurs as individuals care about social status, determined by the relative income. Like in our chapter, he corrects over-education through a progressive income taxation. This can weaken the incentive to undertake education, hence subsidies might be necessary to restore this incentive.

Konrad and Lommerud (2000) explain over-education through a household bargaining model where young individuals individually choose their level of education and, once married, they sacrifice their returns to education in favour of an optimal level of family public goods (i.e., to spend time with children, partner, and so on). Over-education emerges because the educational decisions affect the threat point (i.e., the reservation utility given by being single) of spouses. To over-invest in education is inefficient in order to optimise the quantity of the family public good, but leads to an increase in the threat point so as to be in an advantaged position in the household bargaining.

This chapter shares with studies by Peters and Siow (2001), Baker and Jacobsen (2005), Iyigun and Walsh (2005), Chiappori et al. (2006) and Nosaka
(2007) the link between education and assortative matching. However, in these contributions this link does not explain over-education, and they consider assortative matching only between spouses.

The remainder of the chapter is organized as follows: Section 2.2 describes the model. Section 2.3 shows the results. Section 2.4 illustrates government intervention. Section 2.5 concludes.

2.2 The model

There is a continuum of individuals\(^6\) normalised to 1. Individuals differ in ability, denoted by \(\theta \in [\overline{\theta}, \underline{\theta}]\) and distributed according to density \(f(\theta)\) with cumulative distribution function \(F(\theta)\). We refer to ability as every innate characteristic that contributes to income potential. Individuals choose their level of education. We denote as \(e \geq 0\) the quantity of education acquired by an individual. Education is costly for individuals. We denote the utility cost of education as \(\frac{c}{2} e^2\), where \(c > 0\) is a cost parameter.

After deciding their education, individuals work and are matched with a partner. We denote as \(e\theta\) the income of an individual with education \(e\) and ability \(\theta\). The partner can be seen as a colleague or a spouse. An individual benefits from the partner’s quality\(^7\). This is represented by \(\alpha \theta_p\), where \(\alpha \in \frac{\overline{\theta}}{2} e^2\), where \(c > 0\) is a cost parameter.

\(^6\)We do not consider differences in sex. This implies that men and women behave symmetrically, and excludes the case (more credible in reality) that educational decisions change according to sex (due to a different role in society and household, childbearing and so forth). However, the message of the paper does not change by considering differences in sex and these would only complicate the analysis.

\(^7\)In teamwork, individuals find the performance of their duties easier if those they co-
[0, 1] is the relative importance of the partner’s quality in determining the individual’s utility, while $\theta_p \in [\underline{\theta}, \overline{\theta}]$ denote the partner’s ability. Thus an individual’s utility is determined by:

$$U(e, \theta, \theta_p) = e\theta + \alpha \theta_p - \frac{c}{2} e^2.$$  

(2.1)

We analyse the matching technology and then the educational problem.

2.2.1 Matching

Matching can be of two types: random or assortative. A random matching occurs when partners meet each other by chance. This happens anytime a meeting takes place in situations that are completely unrelated to the acquired education. For example, a match between a lawyer and a botanist sharing the passion for football and playing in the same team is totally casual. Two individuals meeting at the supermarket can have completely different educational backgrounds.

Assortative matching occurs when an individual meets the partner at school, university or in any situation where the educational level influences the chance of a meeting. For example when individuals attend the same social environment given by previous school friendships, or when a certain activity is related to the operate with are able, competent and dedicated. In individual jobs, a good environment improves job performance through suggestions or discussions. In love life, individuals share the advantages of a more able spouse: a better income, work flexibility (which reflects more availability in the love life), a more interesting conversation and more open mindedness.

\footnote{We assume a linear additive utility in order to keep the analysis tractable. Different formulations would complicate the algebra without adding much insight.}
studies attended, like individuals with a degree in arts meeting in a museum or in an exhibition, and so on. In all these cases, the partners’ education is positively related. For the sake of simplicity, we assume that with assortative matching, a perfect positive correlation exists in partners’ levels of education. In other words, the partner of an individual who acquires education $e$ has the same level of education $e$. Considering an imperfect correlation would not alter our results.

Let $\beta \in [0, 1]$ denote the exogenous probability that the matching is assortative. This is independent of the individual’s ability $\theta$. The value of $\beta$ depends on the customs and the educational system of the society we are considering. For instance, the more an educational system requires that students spend years at school for obtaining a certain qualification, the more the probability of assortative matching\(^9\). Another example is the role of school tracking, that is the separation of pupils by academic ability into groups for all subjects within a school (Gamoran, 1992). An educational system that postpones school tracking keeps a more heterogeneous group of pupils together for a long time, decreasing the probability of assortative matching\(^{10}\).

---

\(^9\) Blossfeld and Timm (2003) analyse the relationship between educational system and marital assortative matching in many western countries. Their results show that the more time individuals spend at school, the greater the chance of marrying a partner with similar education (i.e., the higher $\beta$).

\(^{10}\) Holmlund (2006) studies the effects of a school reform on marital assortative matching. She examines an educational reform, implemented in Sweden in the 1950s and 60s, which postponed tracking and extended compulsory education from seven to nine years. Her results show that this might have resulted in a reduction in assortative matching.
2.2.2 Educational choice

When individuals decide the quantity of education to acquire, the future matching affects their decisions. According to equation (2.1), they prefer to be matched with a high-quality partner, as this increases their benefit. With random matching, since there is no correlation in partners’ education, individuals have no information about the partner’s characteristics during the educational decisions. Thus the partner’s expected quality is determined by the average individual type, \( \theta_p = \int_{\theta} \theta_p f(\theta_p) d\theta_p \), and hence random matching does not influence the educational choice.

With assortative matching instead, individuals can observe the education of some of their potential partners (for example, their school friends) during their educational period. Thus they may want to acquire more education in order to improve the probability of being matched with a better partner. Consequently, it is possible to influence the expected partner’s type through the educational decisions.

In particular, individuals can correctly infer the partner’s ability through their education. This is shown by supposing \( E(\theta_p) \) being the education of a partner with ability \( \theta_p \), and also that \( E'(\theta_p) > 0 \). The fact that in equilibrium, education is a strictly increasing function of ability allows the individual

\[ 11 \text{In practice, we are arguing that the belief in equilibrium is that education is an increasing and monotonic function of ability. In other words, individuals believe that the abler ones study more. The equilibrium that emerges is “separating” (i.e., the level of education will be different for each level of ability). This does not exclude the existence of other equilibria that are determined by different beliefs. For instance, if the belief is that the level of education is constant irrespective of the individuals’ ability, then a pooling equilibrium must emerge. However, the belief we focus on looks more consistent to what happens in reality.} \]
to recognise the partner’s ability through her education. From a technical perspective, this happens because an increasing function can be inverted\textsuperscript{12}. Given the assumption that in assortative matching partners have the same level of education, then an individual with ability $\theta$ acquiring an amount of education $e$ will be matched with a partner whose education is $e = E(\theta_p)$. Hence the individual can infer the partner’s ability $\theta_p$ as the inverse image of $E(\theta_p)$, so $\theta_p = E^{-1}(e)$. If this holds, we can rewrite equation (2.1) as:

$$e\theta + \alpha \left( (1 - \beta) \int_{\theta_p} \theta_p f(\theta_p) d\theta_p + \beta E^{-1}(e) \right) - \frac{c}{2} e^2.$$  

(2.2)

In equilibrium we consider, all type $e$ individuals make identical choices, and so (2.2) is the expected utility in each individual type $e$. The first part of (2.2) is the total benefit given by the individual’s income, the second part is the total benefit given by the partner’s quality, and the third part is the total cost of education. The second part of (2.2) can be in turn decomposed into two parts: (i) $\alpha (1 - \beta) \int_{\theta_p} \theta_p f(\theta_p) d\theta_p$, and (ii) $\alpha \beta E^{-1}(e)$, which represent the expected benefit given by the partner with random and assortative matching, respectively.

Equation (2.2) shows that, in the presence of assortative matching, the educational choice $e$ influences not only the future income ($e\theta$) but also the partner’s expected quality ($\beta E^{-1}(e)$). In particular, an individual tries to manipulate the education signal by acquiring more education than others of similar

\textsuperscript{12}Clearly we need to verify that in equilibrium this condition holds.
ability, in order to obtain, in the future, a partner with higher ability than her. But in equilibrium, every individual takes into account assortative matching and tries to do precisely this, hence with probability $\beta$ everyone is matched with a partner of same ability.

The first order condition for the maximisation of equation (2.2) is:

$$\theta + \alpha\beta \frac{d}{de} E^{-1}(e) - ce = 0. \quad (2.3)$$

The following lemma shows the solution of equation (2.3).

**Lemma 5** The level of education chosen by type $\theta$ in equilibrium is $e_{ov} = \alpha\beta + \frac{\theta}{c}$.

**Proof.** Since an individual with ability $\theta$ acquires a level of education $e$ and with assortative matching a partner with ability $\theta_p = E^{-1}(e)$ acquires an amount of education $e$ too, then necessarily $\theta_p = \theta$. Hence we can substitute\(^\text{13}\) $\theta = E^{-1}(e)$ in equation (2.3). This is a differential equation which has solution:

$$\frac{d}{de} E^{-1}(e) = \frac{ce - E^{-1}(e)}{\alpha\beta}. \quad (2.4)$$

We consider a linear solution $E^{-1}(e) = Ae + B$. By substituting this in (2.4) we obtain $A = c$ and $B = -\alpha\beta c$. Hence $E^{-1}(e) = ce - \alpha\beta c$. By explicating

\(^{13}\)Note that we can substitute $E^{-1}(e) = \theta$ only once that $e$ has been maximised. If we do it before the maximisation is like to keep as fixed the partner’s education. But this is a simultaneous game where every individual is also a partner, so the result would not be a Nash equilibrium.
$e$, noting that $E^{-1}(e) = \theta$, so we can rewrite $e = \alpha \beta + \frac{\theta}{e}$. In order $e_{ov}$ to be invertible, it needs to be a strictly increasing function. Differentiating $e_{ov}$ with respect to $\theta$ yields $\frac{\partial}{\partial \theta} (\alpha \beta + \frac{\theta}{e}) = \frac{1}{e} > 0$. ■

2.3 Results

In the equilibrium presented in the previous section, a part of the education acquired by individuals is to improve the quality of the potential partner. But since everyone does this, the expected quality of partners does not improve. Thus although individuals choose their optimal amount of education, the overall education is not socially efficient. Indeed the part acquired for increasing the chance of a better potential partner is not helpful in it, and hence is wasted.

In this section we investigate the equilibrium where individuals exploit the socially optimal educational resources. We assume that education is determined by a planner aiming to maximise social welfare. This is given by the unweighted sum of the individual utilities when $\beta = 0$:

$$W = \int \theta + \alpha \int \theta_p f(\theta_p) d\theta_p - \frac{c}{2}e^2 \int d\theta.$$

In other words, the social welfare function considered does not take into account assortative matching, in order to rule out the cause of inefficiency from the problem. For every $\theta$, the social planner’s problem is the maximisation of equation (2.2) when $\beta = 0$. 42
The solution of Lemma 1 becomes \( e^* = \frac{\theta}{c} \). In order to have over-education, it is necessary that \( e_{ov} > e^* \), \( \alpha\beta + \frac{\theta}{c} > \frac{\theta}{c} \), which is always verified since \( \alpha\beta > 0 \). This is intuitive. In the presence of assortative matching, individuals observe the potential partners’ education and try to look more able. This extra amount of education is not considered by the social planner. Individuals obtain the same result in terms of optimal choice (i.e., same income and partner), but employing less educational resources than in the presence of assortative matching and thus optimising social welfare (Figure 2.1). Hence we refer to \( e^* \) as the first best equilibrium. Over-education is defined as the difference between \( e_{ov} \) and \( e^* \).
**Definition 5** $\Delta e = \alpha \beta$ is the level of over-education.

By looking at $\Delta e$, we can observe that an increase either in $\beta$ or in the relative importance of the partner’s quality $\alpha$ leads to an increase in over-education. Clearly, individuals acquire more education the more likely they meet their partner among their school friends ($\beta$ high). Also, they invest more in education if $\alpha$ is high, since having a high-quality partner is more valuable. This leads to more over-education.

### 2.4 Government intervention

In this section, we assume that there is a government whose objective is to reach the first best education level. To accomplish this, the government considers to levy a tax. We focus on a first best solution through a progressive taxation on income. To do that we need the strong assumption that the government is able to perfectly discriminate taxation according to individual type. This indeed implies that the government can observe individuals’ ability, which is clearly not possible in the reality.

With progressive taxation, the tax rate increases the higher the income. We denote it as $\tau = \gamma^2 \left(1 - \frac{e\theta}{e\theta_L}\right) \in [0,1]$, where $\gamma \neq 1$ represents the tax progression and $e\theta_L$ is the lowest income in the population considered (the
income of the least able individuals). For every \( \theta \), equation (2.2) becomes:

\[
e^{\theta} \left( 1 - \gamma^2 \left( 1 - \frac{e^\theta L}{e^\theta} \right) \right) + \alpha \left( (1 - \beta) \int \theta p f(\theta p) d\theta p + \beta \left( \alpha E^{-1}(e) \right) \right) - \frac{c}{2} e^2.
\]

The first order condition for the maximisation of (2.5) is:

\[
\theta + \beta \alpha \frac{d}{de} E^{-1}(e) = ce + \gamma^2 \theta,
\]

and the level of education is determined by the following lemma.

**Lemma 6** With a progressive tax on income, the education in equilibrium is 

\[
e^\tau = \frac{\theta(1-\gamma^2)}{c} + \frac{\alpha e}{(1-\gamma^2)}.
\]

**Proof.** We substitute \( \theta = E^{-1}(e) \) in equation (2.6). This is a differential equation which has solution:

\[
\frac{d}{de} E^{-1}(e) = \frac{ce + (\gamma^2 - 1)E^{-1}(e)}{\alpha \beta}.
\]

We consider a linear solutions \( E^{-1}(e) = Ae + B \). By substituting this in (2.7) we obtain \( A = \frac{c}{1-\gamma^2} \) and \( B = \frac{\alpha e}{(1-\gamma^2)} \). Hence \( E^{-1}(e) = \frac{ce}{1-\gamma^2} - \frac{\alpha e}{(1-\gamma^2)} \). By explicating \( e \) and noting that \( E^{-1}(e) = \theta \), we can rewrite \( e = \frac{\theta(1-\gamma^2)}{c} + \frac{\alpha e}{(1-\gamma^2)} \).

Note that \( \gamma \) cannot be 1, in order to have determinate solutions. ■

In order to reach the first best level of education, \( e^\tau \) needs to be equal to \( e^* \), thus: \( \frac{\theta(1-\gamma^2)}{c} + \frac{\alpha e}{(1-\gamma^2)} = \frac{\theta}{c} \). By explicating \( \gamma \) we find two positive solutions \( \gamma_1 = \left( \frac{1}{2} + \frac{\left( \theta^2 - 4\alpha^{\gamma^2} \right)^{\frac{1}{2}}}{2\theta} \right)^\frac{1}{2} \) and \( \gamma_2 = \left( \frac{1}{2} - \frac{\left( \theta^2 - 4\alpha^{\gamma^2} \right)^{\frac{1}{2}}}{2\theta} \right)^\frac{1}{2} \). The necessary condition
to define the solutions as real numbers is that \( c < \frac{\theta}{4\alpha_\beta} \). We keep \( \gamma_2 \) because it is the lowest solution, in order to have less distortion by the taxation.

**Proposition 7** For \( c < \frac{\theta}{4\alpha_\beta} \), The optimal progressive income tax is \( \tau^* = (\gamma^*)^2 \left(1 - \frac{e_{\theta L}}{e_{\theta L}}\right) \), where \( \gamma^* = \left(\frac{1}{2} - \frac{(\theta^2 - 4\alpha_\beta e_{\theta})^{\frac{1}{2}}}{2\theta}\right)^{\frac{1}{2}} \).

Figure 2.2 shows the equilibrium where the progressive income tax is levied. These results may justify the introduction of income progressive taxation on efficiency grounds, with no appeal to equity or redistributive reasons.

The following corollary illustrates the relationship between the education
in equilibrium and the tax progression $\gamma$.

**Corollary 8** For individuals with ability lower than $\frac{\alpha \beta c}{(\gamma^2 - 1)^2}$, a more progressive taxation on income makes the incentive to acquire education diminish. For individuals with ability higher or equal to $\frac{\alpha \beta c}{(\gamma^2 - 1)^2}$, a more progressive taxation on income makes the incentive to acquire education increase.

**Proof.** Differentiation of $e^\gamma$ with respect to $\gamma$ yields $\frac{\partial e^\gamma}{\partial \gamma} = 2\alpha \beta \frac{\gamma}{(\gamma^2 - 1)^2} - \frac{2}{c} \theta \gamma$. This is positive if $\theta \geq \frac{\alpha \beta c}{(\gamma^2 - 1)^2}$ and negative otherwise. ■

An increase in tax progression has ambiguous effects on the incentives of acquiring education, according to the individual’s ability. As tax progression increases, individuals with low ability have less incentives in acquiring education while individuals with high ability have more incentives.

The reason is the following. An increase in progressive taxation lowers the incentive in acquiring education for the purpose of increasing income, but gives more incentive in acquiring education to improve the partner’s quality.

This second effect occurs since tax progression makes every individual have a relative advantage in acquiring education for improving the partner’s quality compared to other individuals with higher ability. In other words, given the that the benefit obtained by the partner is the same for every individuals (since it does not depends on the ability), to acquire education for improving it is less costly the lower the individual’s ability because of tax progression.

The first effect is stronger the higher the individual’s ability, while the second effect is identical for each individual. Consequently, when ability is
high, the first effect more than offsets the second effect, and vice versa when ability is low.

This result can be compared with Lommerud (1989), where progressive income taxation corrects over-education but blunts the incentive to undertake education, irrespective of the individual’s ability.

2.5 Concluding remarks

In the presence of assortative matching, individuals increase their education to improve the quality of colleagues or spouses. But as everyone is more educated, the extra education acquired does not improve the chance of a good match. Hence over-education emerges, since individuals can obtain the same result in terms of optimal choice but exploiting less educational resources. Public intervention can solve over-education through a progressive tax on income.

An interesting extension of the chapter may be to consider assortative matching in terms of social class. Although educational and social class assortative matching are positively correlated, individuals with different social background may acquire the same level of education. Introducing assortative matching by social class may have different effects according to the social group we regard. On the one hand, the opportunity cost to acquire more education is generally higher for advantaged individuals since, for instance, they may have better job opportunities through the parental network. On the other hand, this can strengthen the effect on over-education for disadvantaged people, as
assortative matching by class is a further barrier in the attempt to improve the matching through education. The introduction of assortative matching by social class is left for future work.
Chapter 3

Sex and the Uni: Higher Education Effects in Job and Marital Satisfaction

This chapter examines how higher education affects job and marital satisfaction. We build up a model with assortative matching where individuals decide whether to attend university both for obtaining job satisfaction and for increasing the probability to be matched with an educated partner. The theoretical results suggest that, as assortative matching increases, the number of educated individuals increases, their job satisfaction falls while their marital satisfaction increases. We test our model using the British Household Panel Survey data for the years 2003-2006. Our empirical findings support the theoretical results.
3.1 Introduction

This chapter examines how educational decisions influence job and marital satisfaction. We build up a theoretical model to highlight the relationship between higher education, job and marital satisfaction, and then we test the model empirically.

Our idea is that acquiring higher education has two main effects in an individual’s life. First, it gives several advantages at work: a better kind of job, a better salary, more bargaining power in the job market, and so on. All these advantages are expressed by a greater job satisfaction. Second, it increases the chances of marrying\footnote{Throughout the paper, we will use the verb “to marry” not necessarily considering the marriage institution, but referring to the general long-term relationship between partners. Marriage indeed has undergone a process of deinstitutionalization and a weakening of the social norms that define partners’ behavior-over the past few decades (Cherlin, 2004, Schoen and Canudas-Romo, 2005).} an educated partner, as the educational levels of partners are strongly interrelated.

Why do partners tend to have similar educational levels? This may be explained by lifestyle choices: similar-educated partners are more likely to share professional duties, past time activities and view of life. Also, the “fertility intentions” are similar between partners with similarities in education: educated individuals prefer to delay conception relative to the general population (Cochrane, 1979). In contrast, large differences in the partners’ educational level have negative effects on experienced life satisfaction (Frey and Slutzer, 2002). We refer to the similarity in partner’s educational levels as “ assortative
matching\(^2\). Past research has shown strong evidence of increases in the educational resemblance of spouses since at least the 1940s in the United States (Kalmijn 1991a, 1991b; Mare 1991; Pencavel 1998; Qian and Preston 1993; Smits et al. 2000, Schwartz and Mare, 2005).

We examine two populations, one of men and one of women. In each population, the members differ in ability and decide whether to attend university or not. To attend university gives job satisfaction in the working life, which can be positive or negative according to ability. Afterwards men and women are matched in marriage. We assume that individuals prefer to marry a partner who attended university, as they generally have a better income to share, a higher social status, a more interesting conversation and so on. The matching can be random or assortative. Random matching takes place when partners meet each other by chance. Thus the partners’ levels of education are unrelated to one another. Assortative matching occurs if an individual meets the partner at the university, or in any situation where the educational level influences the chance of a meeting. In this case the partners’ education is positively related. Whether matching is assortative depends on the institutions and tradition of a society: for example, the more the educational system requires that students spend time together, the more likely the matching will be assortative.

The theoretical results show that, as the probability of assortative matching increases, university attendance increases, the expected marital satisfaction

\(^2\)The expression “assortative matching” has been coined by Gary Becker (1973), and it alludes to a relationship (either positive or negative) between characteristics of partners. Here we refer to the similarity in level of education between partners.
increases and the marginal and average job satisfaction decrease. The intuition behind these results is the following: as assortative matching increases, the probability of marrying a partner with the same level of education increases. Educated persons are preferred as partners since they give positive marital satisfaction. As a consequence, individuals might decide to attend university even if their job satisfaction will be negative, since this can be offset by the increased probability of marrying an educated partner.

To test the theoretical model, we use the British Household Panel Survey (BHPS) and we consider a subsample of couples from years 2003-2006. We consider education as a binary measure telling us whether or not an individual attended university. To verify the existence of assortative matching, we check for a positive relationship in the level of education between partners. Then, we test for a relationship between the individuals' job satisfaction and higher education. Finally, we examine the relationship between marital satisfaction and partner's higher education, to control whether in the presence of assortative matching, individuals obtain a higher marital satisfaction from an educated partner. The empirical findings are consistent with the theoretical results, although their significance changes according to gender and is not always strong.

This chapter is related to three different branches of the literature, namely the literature on pre-marital investments, the literature on job satisfaction and the literature on marital satisfaction. In the former\textsuperscript{3}, pre-marital investments

\textsuperscript{3}To cite some important contributions, Peters and Siow, 2001, Iyigun and Walsh, 2007,
in human capital influence the kind of matching in the marriage market, the decision power inside the family or the presence of assortative matching. Following this literature, we assume that a link exists between education, marriage and assortative matching.

The chapter is also related to the job satisfaction literature, and in particular to the strand that investigates relationships between job satisfaction and education\(^4\). Meng (1990) finds that education increases workers’ freedom to decide how to do the work, workers’ influence on the decisions of supervisors, and their content with the physical environment of the job. Idson (1990) reports no significant effects of education in job satisfaction. Clark (1996) shows that individuals with longer schooling have comparative lower levels of job satisfaction, as do men, middle-aged people, those working longer hours, and employees in larger establishments. Clark and Oswald (1996) find that the overall job satisfaction is declining in the level of education when income is held constant, and that satisfaction depends inversely on workers’ comparison wage rates. Most recently, Florit and Vila-Lladosa (2007) show that the effects of education on job satisfaction are mainly indirect effects transmitted though the influence of schooling on workers’ health status, wages and other observable job characteristics. Our potential contribution to this literature is to propose a theoretical framework to interpret the relationship between job satisfaction and education.

\(^4\)Previous studies analysed job satisfaction related to training (Jones et al., 2009), temporary jobs (Booth, Francesconi, Frank, 2002), unionisation (Bryson, Cappellari, Lucifora, 2004) and work environment (Gazioglu and Tansel, 2006).
Finally, the chapter is related to the marital satisfaction literature. Here the levels of education between partners are usually considered as control variables (e.g., Glenn, 1990; White and Rogers, 2000). There are a number of studies suggesting that the quality of marital relationships is positively associated with partners’ education (some examples are Stanley et al., 2006, Hahlweg and Markman, 1988, Halford et al., 2003, Sayers et al., 1998 Silliman et al., 2001). This chapter can contribute to this literature by providing both further evidence to the positive relationship between marital satisfaction and the partner’s level of education and a theoretical explanation to it.

The chapter is organised as follows. The theoretical model is developed in Section 3.2; the analysis of equilibrium is illustrated in Section 3.3; Section 3.4 describes the data and the variables used; the empirical model is presented in Section 3.5; our results are summarised in Section 3.6, and concluding remarks are in the last section.

### 3.2 Theoretical model

We study an economy with two populations, equally large, one of men and one of women. The members of each population differ in ability, labeled \( \theta_i \in [0, 1], i = m \text{ (men)}, w \text{ (women)}, \) respectively, and distributed with same density \( f(\theta_i) \) and c.d.f. \( F(\theta_i) \). In our model, ability is higher the lower \( \theta_i \).

We consider a single generation where men and women decide whether to attend university or to work immediately. We refer to individuals who acquired
higher education as “educated” individuals. The proportions of educated men and women are denoted as \( \sigma_m, \sigma_w \in [0, 1] \), respectively.

We assume that in the job market, a non-educated individual obtains a benefit normalised to zero while an educated individuals will receive an educational benefit \( y_i > 0 \), since to attend university is generally necessary to gain access to better paid or more sophisticated jobs. The educational benefit \( y_i \) can be seen as a better salary as well as an improvement in work conditions, the quality of job, hours worked, and so on. Also, we assume that the men’s educational benefit is higher than the women’s, \( y_m > y_w \). This hypothesis reflects the empirical evidence that, ceteris paribus, women generally face worse job conditions than men\(^5\). Educated individuals have a utility cost of education \( c \theta_i \), where \( c > 0 \). This represents the fact that more able individuals make less effort in attending university.

We define job satisfaction as the educational benefit net to the cost of education, \( y_i - c \theta_i \). We assume that \( c > y_m \), therefore individuals with low ability can have negative job satisfaction by attending university so that they prefer to go to work immediately. Our definition of job satisfaction is related to the job type and the necessary education to obtain it. In other words, it is the advantages of a graduate job net to the effort of acquiring a graduate

\(^5\)For example, Burchell et al. (2007) shows some evidence of it for European countries in the period 1990-2005. There is a persistent gender inequality in many aspects of working conditions. In particular women are under-represented in senior positions, are more likely to have part-time jobs, their health is most affected by their work. Women are also less likely to be the main earner in the home because they tend to be segregated into the lower-paid jobs. In addition, the gender pay gap provides an economic rationale which reinforces women’s position as the primary person responsible for the home and care responsibilities.
degree. For simplicity, we abstract away from working conditions (i.e., distance between home and job, relationship with colleagues and so on).

After the education decision, every individual marries one of the opposite sex. We assume that to marry an educated partner gives marital satisfaction $b > 0$. This occurs since a partner benefits from a more educated partner, because of a better income to share, a more interesting conversation, more open-mindedness and so on.

Given the benefits and costs for attending university and the marital satisfaction, the payoff matrix is the following:

<table>
<thead>
<tr>
<th></th>
<th>women</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>educated</td>
<td>not educated</td>
</tr>
<tr>
<td>educated</td>
<td>$y_m - c\theta_m + b, y_w - c\theta_w + b$</td>
<td>$y_m - c\theta_m, b$</td>
</tr>
<tr>
<td>not educated</td>
<td>$b, y_w - c\theta_w$</td>
<td>$0, 0$</td>
</tr>
</tbody>
</table>

Table 3.1: Payoff matrix

### 3.2.1 The matching

The expected payoff of individuals depends on the marriage matching. This can be random or assortative.

Random matching happens anytime a meeting takes place by chance. In this case, the partners’ level of education is completely unrelated. Hence the probability for a man to marry an educated woman is $\sigma_w$ (i.e., the probability that a woman is educated) and the probability for a woman to marry an edu-
cated man is $\sigma_m$ (i.e., the probability that a man is educated), regardless of the individuals’ level of education. Assortative matching occurs when an individual meets the partner at university or in any situation where the educational level influences the chance of a meeting: in this case we assume that partners have the same education with probability one.

We denote the probability of assortative matching as $\beta \in [0, 1]$. This is exogenously determined by the educational system of a certain society. For example, the more the students are required to spend time together at university, the higher the probability of assortative matching. Another aspect of an educational system is the role of school tracking, that is the separation of pupils by academic ability into groups for all subjects within a school (Gamoran, 1992).

An educational system that postpones school tracking keeps a more heterogeneous group of pupils together for a long time, by decreasing the probability of assortative matching.

In order to determine the matching mechanism we need to make some hypothesis on the proportion of educated individuals. The different role in society and family of men and women makes us think that to assume differences in educational decisions according to sex is consistent to the real world. In par-

---

6 Blossfeld and Timm (2003) analyse the relationship between educational system and marital assortative matching in many western countries. Their results show that the more time individuals spend at school, the greater the chance of marrying a partner with similar education (i.e., the higher $\beta$).

7 Holmlund (2007) studies the effects of a school reform on marital assortative matching. She examines an educational reform, implemented in Sweden in the 1950s and 60s, which postponed tracking and extended compulsory education from seven to nine years. Her results show that this might have resulted in a reduction in assortative matching.
ticular we study the case where there is a larger number of educated men than educated women\textsuperscript{8}, i.e., $\sigma_m > \sigma_w$. To assume more educated men than women\textsuperscript{9} is consistent with the previous assumption $y_m > y_w$, which makes think that, \textit{ceteris paribus}, more men will attend university than women.

According to the case $\sigma_m > \sigma_w$, with assortative matching educated men marry an educated woman with probability $\frac{\sigma_w}{\sigma_m}$ and every educated woman finds an educated partner. On the other hand, none of the uneducated men marries an educated woman, while some uneducated women will marry an educated man. Given the assumption on the matching types and $\sigma_m > \sigma_w$, the matching mechanism is the following:

\section*{3.3 Analysis of equilibrium}

The equilibrium of the interaction in educational decisions between men and women occurs when no individual wants to change his or her choice of educa-

\textsuperscript{8}Note that the choice of focusing on this case does not imply that there is no symmetric equilibrium or an asymmetric equilibrium where the number of educated women is higher than the number of educated men. Obviously the matching mechanism changes according to which equilibrium we want to examine.

\textsuperscript{9}In reality, the gap in schooling between men and women is narrowing down. Goldin \textit{et al.}, 2006 show that, in many developed countries, women now have more schooling than men. Of the 17 OECD countries with sufficient data, they document that university enrollment rates of women were below those of men in 13 countries in the 1980s, but by 2002, women university enrollment rates exceeded those of men in 15 countries. However, our empirical analysis is based on a sample of individuals who attended higher education along the past 50 years, where the gap between men and women in higher education was straightforward in favour of men.
<table>
<thead>
<tr>
<th>Men’s matching</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>edu man + edu woman</td>
<td>$(1 - \beta)\sigma_w + \beta\frac{\sigma_w}{\sigma_m}$</td>
</tr>
<tr>
<td>edu man + unedu woman</td>
<td>$1 - (1 - \beta)\sigma_w + \beta\frac{\sigma_w}{\sigma_m}$</td>
</tr>
<tr>
<td>unedu man + edu woman</td>
<td>$(1 - \beta)\sigma_w$</td>
</tr>
<tr>
<td>unedu man + unedu woman</td>
<td>$1 - (1 - \beta)\sigma_w$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Women’s matching</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>edu woman + edu man</td>
<td>$(1 - \beta)\sigma_m + \beta$</td>
</tr>
<tr>
<td>edu woman + unedu man</td>
<td>$1 - (1 - \beta)\sigma_m + \beta$</td>
</tr>
<tr>
<td>unedu woman + edu man</td>
<td>$(1 - \beta)\sigma_m + \beta\left(\frac{\sigma_m - \sigma_w}{1 - \sigma_w}\right)$</td>
</tr>
<tr>
<td>unedu woman + unedu man</td>
<td>$1 - (1 - \beta)\sigma_m + \beta\left(\frac{\sigma_m - \sigma_w}{1 - \sigma_w}\right)$</td>
</tr>
</tbody>
</table>

Table 3.2: Matching mechanism

This is represented by the pair of abilities where individuals are indifferent between studying or not: we define this as $(\theta^*_w, \theta^*_m)$.

Educated individuals have ability below $\theta^*_i$ (note that ability is higher the lower $\theta_i$), so the value of $\theta^*_i$ increases as their number increases. As a consequence, $\theta^*_i$ is equal to the probability to be educated, i.e., $\sigma_w = F(\theta^*_w)$ and $\sigma_m = F(\theta^*_m)$. Without loss of generality, we assume $F = \theta_i$, so we can rewrite the equilibrium solutions $\sigma_w = \theta^*_w$ and $\sigma_m = \theta^*_m$.

Given the payoff matrix, the matching mechanism and the assumptions on the distribution of ability, men and women decide to attend university if their expected payoff of studying is higher than the expected payoff of going to work. This is shown by the following lemma.

**Lemma 9** A man attends university if and only if:

\[
\left( (1 - \beta)\theta^*_w + \beta\frac{\theta^*_w}{\theta^*_m} \right) (y_m + b) +
\]
\[
\left( 1 - \left( (1 - \beta)\theta_w^* + \beta \frac{\theta_w^*}{\theta_m^*} \right) \right) y_m - c\theta_m \\
\geq (1 - \beta)\theta_w^* b,
\]

while a woman attends university if and only if:

\[
((1 - \beta)\theta_m^* + \beta) (y_w + b) + \\
(1 - ((1 - \beta)\theta_m^* + \beta)) y_w - c\theta_w \\
\geq \left( (1 - \beta)\theta_m^* + \beta \left( \frac{\theta_m^* - \theta_w^*}{1 - \theta_w^*} \right) \right) b.
\]

**Proof.** Given the matching mechanism, the expected payoffs for men are:

\[
E\Pi (ed.man) = \left( (1 - \beta)\theta_w^* + \beta \frac{\theta_w^*}{\theta_m^*} \right) (y_m + b) + \\
\left( 1 - \left( (1 - \beta)\theta_w^* + \beta \frac{\theta_w^*}{\theta_m^*} \right) \right) y_m - c\theta_m,
\]

and

\[
E\Pi (non - ed.man) = (1 - \beta)\theta_w^* b,
\]

respectively, where the first part of both equations represents the expected payoff of marrying an educated woman and the second part of the first equation is the expected payoff of marrying a non-educated woman. The expected payoffs for women are:

\[
E\Pi (ed.woman) = ((1 - \beta)\theta_m^* + \beta) (y_w + b) + 
\]
\[(1 - ((1 - \beta)\theta_m^* + \beta)) y_w - c \theta_w,\]

and

\[E \Pi (non - ed.\, woman) = \left((1 - \beta)\theta_m^* + \beta \left(\frac{\theta_m^* - \theta_w^*}{1 - \theta_w^*}\right)\right) b,\]

respectively, where first part of both equations represents the expected payoff of marrying an educated man and the second part of the first equation is the expected payoff of marrying a non-educated man. Men and women will prefer to study until the expected payoff of attending university is higher than expected payoff of going to work at once:

\[E \Pi (ed.\, man) \geq E \Pi (non - ed.\, man),\]

and

\[E \Pi (ed.\, woman) \geq E \Pi (non - ed.\, woman),\]

which gives the lemma. ■

The following proposition shows the equilibrium in educational choices.

**Proposition 10** For \(\sigma_m > \sigma_w\), an equilibrium in educational choices exists and it is given by the pair \((\theta_m^*, \theta_w^*)\) which is solution of the system:

\[\begin{cases}
\theta_m^* = \frac{(1 - \theta_m^*)(y_w - c \theta_m^*) + b_3}{b_3} \\
\theta_w^* = \frac{c \theta_m^{*2} - \theta_m^* y_m}{b_3}.
\end{cases}\] (3.1)

Following that \(\sigma_m > \sigma_w\), we need to verify that \(\theta_m^* > \theta_w^*\): in other words, a woman who is indifferent between studying or not is more able than a man who
is indifferent between studying or not. This is shown by the following corollary.

**Corollary 11** Given \( y_m > y_w \), then \( \theta_m^* > \theta_w^* \).

**Proof.** Since \( \theta_m^* \) and \( \theta_w^* \) are probabilities, they need to be higher than zero. If \( \theta_m^* > 0 \), then \( (1 - \theta_m^*) (y_w - c\theta_w^*) + b \beta \geq 0 \), and hence if \( y_w \geq c\theta_w^* \). If \( \theta_w^* > 0 \), then \( \theta_m^* (c\theta_m^* - y_m) > 0 \). This holds only if \( y_m < c\theta_m^* \). Given that \( y_m > y_w \), we have \( c\theta_m^* > y_m > y_w > c\theta_w^* \), then \( \theta_m^* > \theta_w^* \). ■

To interpret Proposition 11, we need to analyse the effects of a variation in assortative matching. To do that, we study the comparative statics through a computational example of equilibrium. The parameter values are chosen in such a way that the following assumptions hold: \( \theta_m^* \), \( \theta_w^* \in [0, 1] \), and \( c > y_m > y_w \). In particular, we assign the following values: educational benefit, \( y_m = 0.2 \), \( y_w = 0.15 \), marital satisfaction, \( b = 0.4 \), cost of education \( c = 1 \).

We consider the effects of the presence of assortative matching on marginal and average job satisfaction and on expected marital satisfaction. The marginal job satisfaction (i.e., the job satisfaction of the individual being indifferent between studying or not) is \( y_m - c\theta_m^* \) for men and \( y_w - c\theta_w^* \) for women. The average job satisfaction is denoted as \( \theta_j \), and is obtained by assuming a uniform distribution, \( \theta_j = \frac{y_i - \bar{\theta}_i + y_i - c\theta_i^*}{2} \), where \( \bar{\theta}_i \) is the highest level of ability of an individual. Since \( \bar{\theta}_i = 0 \) for every \( i \), then \( \theta_j = \frac{2y_i - c\theta_i^*}{2} \).

The expected marital satisfaction is denoted by \( E(b) \), and depends on the probability of an educated individual to marry an educated partner. According to Lemma 1, this is \( E(b)_m = \left( (1 - \beta)\theta_w^* + \beta \frac{\theta_m^*}{\theta_m^*} \right) b \) for educated men and
Table 3.3: Computational example of equilibrium

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( y_m = 0.2, y_w = 0.15, c = 1, b = 0.4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assortative matching</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Men’s marginal ability</td>
<td>( \theta_m )</td>
</tr>
<tr>
<td>Women’s marginal ability</td>
<td>( \theta_w )</td>
</tr>
<tr>
<td>Marginal job satisfaction</td>
<td>men</td>
</tr>
<tr>
<td></td>
<td>women</td>
</tr>
<tr>
<td>Average job satisfaction</td>
<td>men</td>
</tr>
<tr>
<td></td>
<td>women</td>
</tr>
<tr>
<td>Expected marital satisfaction</td>
<td>men</td>
</tr>
<tr>
<td></td>
<td>women</td>
</tr>
</tbody>
</table>

\[ E(b)_w = ((1 - \beta)\theta^*_m + \beta) b \] for educated women.

Table 3.3 illustrates the results. As assortative matching increases, both marginal and average job satisfaction diminish. Moreover, while the marginal job satisfaction is always negative, the average satisfaction becomes negative for high probabilities of assortative matching. On the other hand, the expected marital satisfaction increases the higher the probability of assortative matching.

These results may be explained in the following way. As assortative matching increases, the probability of marrying a partner with the same level of education increases. Educated persons are preferred as partners since they give positive marital satisfaction. As a consequence, individuals might decide to attend university even if their job satisfaction is negative, as this can be offset by the increased probability of marrying an educated partner.
3.4 The data

The dataset used in our analysis is the British Household Panel Survey (BHPS). This is a nationally representative random sample survey of households in Britain, which began in 1991. The BHPS was designed as an annual survey of each adult (16+) member of a sample of more than 5,000 households, making a total of approximately 10,000 individual interviews. The same individuals are interviewed in successive waves and, if they leave from original households, all adult members of their new households will also be interviewed.

Unlike the previous contributions to the job satisfaction literature, which focus on cross-sectional analysis\textsuperscript{10}, we consider a four-years sample for 2003-2006, including 5406 couples (10812 individuals) of men and women aged between 23 and 65 years who provided complete information at the interview dates, who are married or in a relationship and live in the same household.

These restrictions have two effects. First, they guarantee that the individuals in the sample considered are at a working age. This is necessary in order to obtain information for job satisfaction. Second, they allow us to highlight the relationship between the educational choices of individuals in a couple. Nonetheless the choice of a sample of couples may raise concerns about self-selection and marital satisfaction, since individuals who live in a couple may

achieve more satisfaction by being in a relationship than the ones that prefer to remain single.

3.4.1 Dependent variables

We consider higher education, job satisfaction and marital satisfaction as dependent variables. A positive relationship between higher education and the partner’s higher education would indicate a high probability of assortative matching. The BHPS asks individuals which educational degree they obtained. We construct a binary variable taking the value of the unity if individuals have obtained any degree higher than college (A-level) and zero otherwise.

According to the theoretical results, a high probability of assortative matching has two effects. First, job satisfaction diminishes as the probability of obtaining higher education increases.

The BHPS asks to rate the job satisfaction levels with four items: “pay”, “job security”, “kind of work” and “hours worked”. Each of these was to be given by the worker a number from 1 to 7, where 1 corresponded to “not satisfied at all”, 7 corresponded to “completely satisfied”. Individuals were then asked a final question, after they had rated their levels of contentment with the list of topics, worded as: “All things considered, how satisfied or dissatisfied are you with your present job overall using the same 1-7 scale?”. The way the question was asked suggests that individuals’ replies weigh up many attributes of the job package\textsuperscript{11}. Hence the data may approximate total well-being from work

\textsuperscript{11}To control that, we performed the analysis of job satisfaction with the specific indicators:
rather better than can a narrow question about job satisfaction. Also, in this choice we follow Clark and Oswald (1996).

The second effect of a high probability of assortative matching is that marital satisfaction increases as the probability that the partner obtains higher education increases. The BHPS asks individuals the following question: “How dissatisfied or satisfied are you with your husband/wife/partner?” Respondents could answer on a scale from one (totally unsatisfied) and seven (very satisfied). For some values, like 1 or 2, we have an amount of answers which is lower than 1%. Hence we regroup it by creating a new variable: if marital satisfaction is 1, 2 or 3, we assign the value zero (“unsatisfied”), if marital satisfaction is 4, we assign the value one (“neutral”) and finally if it is 5, 6 or 7 we assign the value two (“satisfied”).

3.4.2 Explanatory variables

As explanatory variables, we consider a specific explanatory variable for each dependent variable, and then a number of control variables for every dependent variable. For the analysis of assortative matching and marital satisfaction, the explanatory variable is the partner’s higher education, while for the job satisfaction analysis, the explanatory variable is higher education.

The control variables are sex, age, age squared, regions, and professions. The variable sex takes values of zero for men and one for women.

As regions we consider five macro areas: Northern England, Middle Eng-
land, Southern England, Scotland and Wales. For each of them we create a dummy variable. We exclude from the analysis individuals from North Ireland, for the strong segregation in marriages between Catholics and between Protestants in this area (Jerkins, 1997), which causes distortions in the analysis of assortative matching.

Finally, we sort individuals according to their job. We use five main job qualifications, derived by the Standard Occupational Classification 2000 (SOC 2000): professional, manager, administrative, technician and manual. For every qualification, we create a dummy variable.

### 3.4.3 Descriptive analysis

Table 3.4 shows the descriptive statistics of the full sample, men and women. The mean for job satisfaction is 5.34 for the full sample, 5.21 for men and 5.48 for women. If women had, on average, a higher job satisfaction than men, since in the theoretical model we assumed $y_m > y_w$ and $\sigma_m > \sigma_w$, necessarily we would expect that the number of educated women is lower than the number of men (that is, educated women are in average abler than educated men, by which they obtain a higher job satisfaction). Indeed the amount of men who acquire higher education is approximately 5% higher.

The mean marital satisfaction is 1.89 for the full sample, 1.90 for men and 1.88 women. The average age around 42 years for men and 40 for women. The most part of couples (around 26%) are from South England and the least part comes from the Midlands (around 12%).
Table 3.4: Descriptive statistics: full sample, men and women

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Min</td>
</tr>
<tr>
<td>Job satisfaction (not at all=1, complete=7)</td>
<td>5.34</td>
<td>1.22</td>
<td>1</td>
</tr>
<tr>
<td>Marital satisfaction</td>
<td>1.89</td>
<td>0.39</td>
<td>0</td>
</tr>
<tr>
<td>Age</td>
<td>41.37</td>
<td>10.01</td>
<td>23</td>
</tr>
<tr>
<td>Regions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wales</td>
<td>0.16</td>
<td>0.37</td>
<td>0</td>
</tr>
<tr>
<td>Scotland</td>
<td>0.2</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>Southern England</td>
<td>0.26</td>
<td>0.44</td>
<td>0</td>
</tr>
<tr>
<td>Middle England</td>
<td>0.12</td>
<td>0.33</td>
<td>0</td>
</tr>
<tr>
<td>Northern England</td>
<td>0.21</td>
<td>0.41</td>
<td>0</td>
</tr>
<tr>
<td>Professions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manager</td>
<td>0.16</td>
<td>0.36</td>
<td>0</td>
</tr>
<tr>
<td>Professional</td>
<td>0.12</td>
<td>0.33</td>
<td>0</td>
</tr>
<tr>
<td>Technician</td>
<td>0.15</td>
<td>0.36</td>
<td>0</td>
</tr>
<tr>
<td>Administrative</td>
<td>0.14</td>
<td>0.35</td>
<td>0</td>
</tr>
<tr>
<td>Manual</td>
<td>0.41</td>
<td>0.49</td>
<td>0</td>
</tr>
<tr>
<td>Higher Education (Yes=1, No=0)</td>
<td>0.59</td>
<td>0.49</td>
<td>0</td>
</tr>
<tr>
<td>Observations</td>
<td>10812</td>
<td>5406</td>
<td>5406</td>
</tr>
</tbody>
</table>
Table 3.5: Correlation between life overall satisfaction, job and marital satisfaction

<table>
<thead>
<tr>
<th>Variable</th>
<th>Life overall satisfaction</th>
<th>Job satisfaction</th>
<th>Marital satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Life overall satisfaction</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job satisfaction</td>
<td>0.47</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Marital satisfaction</td>
<td>0.31</td>
<td>0.06</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Manual jobs are the most common for both genders, followed for men by management and for women by administrative jobs. Finally, in Table 3.5 we compare job and marital satisfaction to life overall satisfaction, so to examine their relative value. If an individual rates job or marital satisfaction with a high value but this is lower to the rating of life overall satisfaction, then job/marital satisfaction are relatively low although their absolute value is high. According to Table 3.5, the correlation between job/marital satisfaction and life overall satisfaction is quite low.

### 3.5 The Empirical Model

In this section we present the empirical specification. In order to test the implication of the theoretical model, first we need to verify the presence of assortative matching through a positive relationship between partners’ education\(^\text{12}\).

\(^{12}\)The literature on assortative matching focuses on trends in the positive relationship between education level of partners through time (see Schwartz and Mare, 2005, for a dis-
If assortative matching were present, according to our theoretical results, we expect a negative relationship between job satisfaction and higher education, as more low-ability individuals attend university, by diminishing the average job satisfaction. At the same time, we expect a positive relationship between marital satisfaction and the partner’s higher education. Indeed this explains why some individuals attend university although this will give them a negative job satisfaction. Therefore we estimate an equation for job satisfaction and an equation for marital satisfaction, in order to verify these implications. The equation of assortative matching is:

\[ uni_{it} = \gamma_1 sex_{it} + \gamma_2 age_{it} + \gamma_3 age^2_{it} + \gamma_4 regions_{it} + \gamma_5 unip_{it} + \varepsilon_{it}, \quad (3.2) \]

where \( i = 1, \ldots, n \) denote individuals and \( t = 1, 2, 3, 4 \) the ages considered, \( uni_{it} \) represents higher education, \( sex_{it}, age_{it} \) and \( age^2_{it} \) denote sex, age and age square, \( regions_{it} \) collects the control variables about regions and \( unip_{it} \) is the partner’s level of education. We perform a binary random-effects probit model (Guilkey and Murphy, 1993), which assumes unobserved heterogeneity to be constant over time. An alternative empirical strategy such as the fixed-effects probit model is hampered by both a very low time variation and the almost exclusive presence of binary explanatory variables. Instead we just check for the existence of a positive relationship in the partners’ education to prove the correctness of our assumption.
Second, we estimate an equation for job satisfaction:

\[ \text{jobsat}_it = \gamma_1 \text{sex}_it + \gamma_2 \text{age}_it + \gamma_3 \text{age}^2_{it} + \gamma_4 \text{regions}_it + \]
\[ + \gamma_5 \text{professions}_it + \gamma_6 \text{uni}_it + \varepsilon_{it}, \tag{3.3} \]

where \( \text{jobsat}_it \) is job satisfaction and \( \text{professions}_it \) is a vector of the control variables about job qualification. Because the ordered nature of job satisfaction scores in most surveys, the typical estimation technique performed is ordered probit estimation\(^\text{13}\). Nonetheless, the panel nature of the data impedes performance at an ordered analysis\(^\text{14}\). We sidestep the issue by keeping the ordered nature of the job satisfaction scores and perform a pooled ordered probit. This allow us to take into account that job satisfaction can change for the same individuals.

Finally, we investigate the relationship between marital satisfaction and the partner’s level of education:

\[ \text{maritalsat}_it = \gamma_1 \text{sex}_it + \gamma_2 \text{age}_it + \gamma_3 \text{age}^2_{it} + \gamma_4 \text{regions}_it + \]
\[ + \gamma_5 \text{professions}_it + \gamma_6 \text{uni}_it + \varepsilon_{it}, \tag{3.4} \]

where \( \text{maritalsat}_it \) is marital satisfaction. Even in this case we perform a

\(^{13}\)Most studies make use of ordered probit estimation but Florit and Lladosa (2007), whose work actually criticises the use of ordered choice models and compares this analysis with a Structural Equation Model (SEM).

\(^{14}\)A cross-sectional ordered probit analysis has been performed by considering years 2003, 2004, 2005, and 2006: the results are qualitative similar to the panel results. Upon request, we can provide these findings.
pooled ordered probit analysis. We take into account heteroskedasticity through robust standard errors in both job and marital satisfaction analysis.

One may argue that education can be potentially endogenous. However, this aspect is not investigated in both literatures of job and marital satisfaction. Irrespective of it, we did not control for endogeneity of education due to the absence of valid instruments in our data\(^\text{15}\). Another concern may refer to the fact that job and marital satisfaction are not necessarily simultaneously determined. Nevertheless, this identification would have been incorrect. By the theoretical model, we expect that on average job satisfaction is negatively related to education and marital satisfaction is positively related to partner’s education. But it is not necessarily true that an educated individual needs to have negative job satisfaction and positive marital satisfaction. For example, for a very able individual these may both be positive.

### 3.6 Results

Table 3.6 shows the results of assortative matching for random effects probit model. The relationship between ages and higher education is increasing but concave. This information is probably distorted by self-selection, since the sample is formed only by spouses or live-in partners. Indeed young individuals

\(^{15}\text{Only for 2003, BHPS has parents’ education, number of siblings and school type as potential instrumental variables candidates. However, the introduction of these variables reduces greatly the number of observations. An alternative instrument might be the period of the year when an individual is born. However, this is a valid instrument for compulsory school, while there is no evidence of seasonal patterns in education in colleges and graduate school competition rates (Angrist and Krueger, 1991).}\)
who are married or live with the partner usually do not attend university, as they could not bear the expenses. The region of living is not relevant. There is a positive and significant relationship between the partners’ levels of education indicating the presence of assortative matching. Table 3.7 presents the results for job satisfaction. The dummy variables omitted are: (i) for region, “Southern England” and (ii) for professions, “Manual”. Job satisfaction is positively related to age and negatively related to age square. This is significant for the full sample and men but not for women. This result is in line with the previous evidence with British data (Clark, 1996). This result can be explained in the following way. As the job years go by, generally the working skills, the wage and the responsibility increase, and a more important role is acquired. All these aspects make working more fulfilling. On the other hand, as individuals grew old, they become more and more tired of working, by increasingly offsetting the benefits of a more experienced job. Workers in Wales are more satisfied with their jobs, as are male workers in Middle England and Scotland. A possible explanation can be that, in relatively poorer regions, the presence of unemployment, lower income and less job opportunities makes the individuals’ job expectations to be lower. Hence, *ceteris paribus*, the same job is more appreciated in a poor rather than a rich area.

Also, workers are relatively more satisfied by working as managers or technicians. Male workers are more satisfied if they do professional jobs, while female workers are relatively more satisfied with manual jobs. An interpretation could be that women, apart from working, generally deal with household tasks and
look after children. A manual job generally is less stressful and it might help to manage better all these duties.

Educated workers, both men and women, are relatively less satisfied. This is significant for the entire sample and women, but not for men. According to the theoretical model, the interpretation of lower job satisfaction for educated individuals is the following: given the presence of assortative matching, some individual will attend university even if he or she will obtain a negative job satisfaction. This is optimal if the expected marital satisfaction increases by attending university.

In the literature of job satisfaction, Blanchflower and Oswald (1992) analyse the National Children Development Study (NCDS) for 1981. Unlike our results, their findings show a positive relationship between job satisfaction and higher education. Meng (1990) estimates disaggregated job satisfaction for 1981 in the Social Change in Canada Survey (SCCS). He finds significance for a negative relationships between higher education and “payment” and “surround” (i.e., job environment), and a positive relationship between higher education and “free” and “influence”. Idson performs his analysis with the Quality of Employment Survey (QES), which considers US data for 1977. He did not find any significant relationship between education and job satisfaction. Finally, Florit and Lladosa (2007), by the Spanish Household Survey Panel (SHPS) for 1998, finds a positive relationship between job satisfaction and education.
Table 3.6: Assortative matching: random-effects probit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>-0.546 ***</td>
</tr>
<tr>
<td></td>
<td>(Man=0, Woman=1) (0.184)</td>
</tr>
<tr>
<td>Age</td>
<td>0.359 ***</td>
</tr>
<tr>
<td></td>
<td>(0.070)</td>
</tr>
<tr>
<td>Age squared</td>
<td>-0.004 ***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

Regions (dummy variable omitted: Southern England)

<table>
<thead>
<tr>
<th>Region</th>
<th>Full Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wales</td>
<td>-0.317</td>
</tr>
<tr>
<td></td>
<td>(0.288)</td>
</tr>
<tr>
<td>Middle England</td>
<td>-0.068</td>
</tr>
<tr>
<td></td>
<td>(0.291)</td>
</tr>
<tr>
<td>Scotland</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.242)</td>
</tr>
<tr>
<td>Northern England</td>
<td>0.084</td>
</tr>
<tr>
<td></td>
<td>(0.24)</td>
</tr>
<tr>
<td>Partner's education</td>
<td>4.272 ***</td>
</tr>
<tr>
<td></td>
<td>(0.187)</td>
</tr>
</tbody>
</table>

Log-Likelihood       | -3440.11    |
Wald chi2             | 552.56      |
(Prob>chi2)           | 0           |
Observations          | 10812       |

Notes: The dependent variable is the individual's higher education (1=yes, 0=no). Values of standard errors are presented in parenthesis. Significance at the 1%, 5% and 10% levels is indicated by ***, ** and * respectively.
Table 3.7: Job satisfaction results: pooled ordered probit with robust estimators

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex</td>
<td>0.284 ***</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>(Man=0,Woman=1)</td>
<td>(0.021)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age</td>
<td>-0.031 ***</td>
<td>-0.055 ***</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.012)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Age squared</td>
<td>0.001 ***</td>
<td>0.001 ***</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Regions (dummy variable omitted: Southern England)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wales</td>
<td>0.109 ***</td>
<td>0.168 ***</td>
<td>0.054</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.045)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>Middle England</td>
<td>0.026</td>
<td>0.064 **</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.046)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Scotland</td>
<td>0.017</td>
<td>0.073 *</td>
<td>-0.034</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.041)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Northern England</td>
<td>-0.030</td>
<td>0.016</td>
<td>-0.069 *</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.041)</td>
<td>(0.041)</td>
</tr>
<tr>
<td>Professions (dummy variable omitted: manual)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professional</td>
<td>0.041</td>
<td>0.128 ***</td>
<td>-0.055</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.046)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Manager</td>
<td>0.051 *</td>
<td>0.151 ***</td>
<td>-0.102 **</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.038)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>Technician</td>
<td>0.053 *</td>
<td>0.122 ***</td>
<td>-0.017</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.045)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Administrative</td>
<td>-0.060 *</td>
<td>0.002</td>
<td>-0.123 ***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.064)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Education</td>
<td>-0.069 ***</td>
<td>-0.040</td>
<td>-0.091 ***</td>
</tr>
<tr>
<td>(No=0,Yes=1)</td>
<td>(0.022)</td>
<td>(0.031)</td>
<td>(0.033)</td>
</tr>
<tr>
<td>Partner's education</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Log Pseudo-likelihood: -15354.69 | -7933.64 | -7377.963
Wald chi2: 226.17 | 55.87 | 41.33
(Prob>chi2): (0.000) | (0.000) | (0.000)
Observations: 10812 | 5406 | 5406

Notes: Values of standard errors are presented in parenthesis. Significance at the 1%, 5% and 10% levels is indicated by ***,** and * respectively.
Table 3.8: Marital satisfaction results: pooled ordered probit with robust estimators

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full sample</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sex (Man=0, Woman=1)</td>
<td>-0.119 ***</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Age</td>
<td>-0.064 ***</td>
<td>-0.043 **</td>
<td>-0.085 ***</td>
</tr>
<tr>
<td>Age squared</td>
<td>0.001 ***</td>
<td>0.001 ***</td>
<td>0.001 ***</td>
</tr>
<tr>
<td>Regions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wales</td>
<td>0.148 ***</td>
<td>0.312 ***</td>
<td>0.005</td>
</tr>
<tr>
<td>Middle England</td>
<td>0.012</td>
<td>0.073</td>
<td>-0.049</td>
</tr>
<tr>
<td>Scotland</td>
<td>0.190 ***</td>
<td>0.252 ***</td>
<td>0.129 *</td>
</tr>
<tr>
<td>Northern England</td>
<td>0.121 **</td>
<td>0.309 ***</td>
<td>-0.038</td>
</tr>
<tr>
<td>Professions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professional</td>
<td>0.188 ***</td>
<td>0.063</td>
<td>0.314 ***</td>
</tr>
<tr>
<td>Manager</td>
<td>0.086</td>
<td>0.105</td>
<td>0.056</td>
</tr>
<tr>
<td>Technician</td>
<td>0.094 *</td>
<td>0.079</td>
<td>0.118</td>
</tr>
<tr>
<td>Administrative</td>
<td>0.088</td>
<td>0.052</td>
<td>0.110 *</td>
</tr>
<tr>
<td>Education</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Partner’s education</td>
<td>0.078 **</td>
<td>0.122 **</td>
<td>0.041</td>
</tr>
<tr>
<td>Log Pseudo-likelihood</td>
<td>3323.659</td>
<td>-1524.598</td>
<td>-1787.582</td>
</tr>
<tr>
<td>Wald chi2</td>
<td>59.86</td>
<td>34.99</td>
<td>39.63</td>
</tr>
<tr>
<td>Observations</td>
<td>10812</td>
<td>5406</td>
<td>5406</td>
</tr>
</tbody>
</table>

Notes: Values of standard errors are presented in parenthesis. Significance at the 1%, 5% and 10% levels is indicated by *** , ** and * respectively.
The results on marital satisfaction are reported in Table 3.8. The dummy variables omitted are the same used in the job satisfaction analysis. The region with lower marital satisfaction is Southern England. A possible interpretation can be the higher cost of life in Southern England and London, and a more stressful lifestyle which has recoils on the couple’s life.

Any worker enjoys higher marital satisfaction compared to manual workers, even though this is significant only for professional women. The reason can be that a non-manual worker might feel professionally more accomplished. This can reflect positively in the couple’s life.

There is a positive relationship between marital satisfaction and the partners’ levels of education of the full sample, men and women, even though this is not significant for women. This is in line with the previous evidence in the literature of marital satisfaction (some examples are Stanley et al., 2006, Hahlweg and Markman, 1988, Halford et al., 2003, Sayers et al., 1998, Silliman et al., 2001). These results on marital satisfaction are consistent with the findings of the theoretical model, and thus they may explain why some individuals attend university even though they are going to obtain a negative job satisfaction.

3.7 Concluding remarks

This chapter examines the impact of higher education in marital and job satisfaction. As assortative matching increases the proportion of both educated men and women increases. This makes both marginal and average job satisfac-
tion fall and marital satisfaction increase. The empirical test with the British Household Panel Survey for years 2003-2006 confirm the existence of assortative matching. Job satisfaction diminishes the higher the educational qualification, while marital satisfaction increases the higher the partner’s level of education, as expected by the theoretical model.

One critique to our approach can be that we do not take divorce into account. This can be relevant only if we assume a grade of relationship between the level of education and the probability of being divorced. In the case that there is no correlation or the probability of being divorced is negatively related to the amount of education, the “divorce effect” can be normalised to zero. Indeed in this case the assumption of positive marital satisfaction given by an educated partner still holds. On the contrary, in the case that the probability of being divorced is positively related to the amount of education, our analysis holds as long as the expected marital satisfaction (net of the negative increased expected divorce) is positive.

Some other information, such as parents’ job and education, ethnic and income differences, would have added more insights to the analysis. However, the price to pay was to reduce greatly the number of observations caused by the lack of data along the survey. Future work could investigate whether these theoretical findings are confirmed in datasets from other countries.
Appendix

Proofs of chapter 1

Proofs of Proposition 1 and 2

The proof follows Proposition 1. By setting $\lambda = 0$ we obtain the proof of Proposition 2.

Case 1. $p_a \geq p_d \geq \frac{\eta_L}{\nu + \eta_L}$

Employer. The employer strategy is $z_{Ua} = 1$; $z_{Da} = 0$, $z_{Ud} = \frac{\Phi - \lambda(p_a + \eta_L(1 - p_a))}{(1 - \lambda)(p_d + \eta_L(1 - p_d))}$; $z_{Dd} = 0$. The employer’s beliefs for advantaged students are $\pi(\theta_H \mid g_U, a) = \frac{p_a}{p_a + \eta_L(1 - p_a)}$ and $\pi(\theta_L \mid g_U, a) = \frac{\eta_L(1 - p_a)}{p_a + \eta_L(1 - p_a)}$, if the student has a high grade and $\pi(\theta_H \mid g_D, a) = 0$ and $\pi(\theta_L \mid g_D, a) = 1$ if the student has a low grade.

Thus the expected profit\(^1\) for hiring an advantaged and high-grade student is

\[
\Pi^E_{Ua} = \frac{p_a}{p_a + \eta_L(1 - p_a)}\nu - \frac{\eta_L(1 - p_a)}{p_a + \eta_L(1 - p_a)}.
\]

This must be \[
\frac{p_a}{p_a + \eta_L(1 - p_a)}\nu - \frac{\eta_L(1 - p_a)}{p_a + \eta_L(1 - p_a)} \geq 0
\]

\(^1\) The superscript of the employer’s expected profit indicates the action performed by the employer, where $E$ indicates “to hire” and $N$ “to not”. The subscript specifies the student’s grade, where $U$ indicates a high grade and $D$ a low grade, while $a$ and $d$ indicates the student’s social background.

81
and, after few passages, \( p_a \geq \frac{\eta_L}{\nu+\eta_L} \). The expected profits for hiring and not hiring an advantaged and low-grade student are \( \Pi_{D_a}^E = -1 \) and \( \Pi_{D_a}^N = 0 \), respectively, thus \( \Pi_{D_a}^E < \Pi_{D_a}^N \).

The employer’s beliefs for disadvantaged students are \( \pi(\theta_H \mid g_U, d) = \frac{p_a}{p_a+\eta_L(1-p_a)} \) and \( \pi(\theta_L \mid g_U, d) = \frac{\eta_L(1-p_a)}{p_a+\eta_L(1-p_a)} \) if the student has a high grade and \( \pi(\theta_H \mid g_D, d) = 0 \) and \( \pi(\theta_L \mid g_D, d) = 1 \) if the student has a low grade.

The expected profit for hiring one disadvantaged and high-grade student is
\[
\Pi_{U_d}^E = \frac{p_d}{p_a+\eta_L(1-p_a)} \nu - \frac{\eta_L(1-p_d)}{p_d+\eta_L(1-p_d)} \nu - \frac{\eta_L(1-p_a)}{p_a+\eta_L(1-p_a)} \geq 0
\]
and, after few passages, \( p_d \geq \frac{\eta_L}{\nu+\eta_L} \). The expected profits for hiring and not hiring a disadvantaged and low-grade student are \( \Pi_{D_d}^E = -1 \) and \( \Pi_{D_d}^N = 0 \), respectively, thus \( \Pi_{D_d}^E < \Pi_{D_d}^N \).

Then the employer needs to compare the expected profit obtained by high grade students with different social background\(^{17}\): this is \( \Pi_{U_a}^E > \Pi_{U_d}^E \), as \( p_a > p_d \). As a consequence, the employer admits all the advantaged and high-grade students and the disadvantaged ones only for the remainder of the job capacity.

Given the restrictions on the job capacity, the number of hired disadvantaged and high grade students is \( \frac{\Phi - \lambda p_a + \eta_L(1-p_a))}{(1-\lambda)(p_d+\eta_L(1-p_d))} \).

**School.** The school strategy is \( x_{L_a} = 1; x_{H_a} = 1; x_{L_d} = 1; x_{H_d} = 1 \). The expected payoffs\(^{18}\) for giving or not giving extra teaching to an advantaged and high-ability student are \( \Pi_{H_a}^T = \mu - c \) and \( \Pi_{H_a}^{NT} = \mu \eta_H \), respectively. This

\(^{17}\)This is not necessary for low-grade students as none of them are admitted.

\(^{18}\)The superscript of the school’s expected payoff indicates the action performed by the school, where \( T \) indicates “to give extra-teaching” and \( NT \) “to not”. The subscript specifies the student’s ability, where \( H \) indicates high ability and \( L \) low ability, while \( a \) and \( d \) indicates the student’s social background.
must be $\Pi^T_{Ha} > \Pi^{NT}_{Ha}$, that is $\mu - c > \mu_H$, and therefore $\mu \geq \frac{c}{1-H}$. The expected payoffs for giving or not giving extra teaching to an advantaged and low-ability student are $\Pi^T_{La} = \mu L - c$ and $\Pi^{NT}_{Ha} = 0$, respectively. This must be $\Pi^T_{La} \geq \Pi^{NT}_{La}$, that is $\mu L - c \geq 0$, and therefore $\mu \geq \frac{c}{\eta_L}$.

The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi^T_{Hd} = \mu Z_{Ud} - c$ and $\Pi^{NT}_{Hd} = \mu_Z H Z_{Ud}$, respectively. This must be $\Pi^T_{Hd} \geq \Pi^{NT}_{Hd}$, that is $\mu Z_{Ud} - c \geq \mu Z_{Ud} \eta_H$, and therefore $\mu \geq \frac{c}{Z_{Ud}(1-H)}$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are $\Pi^T_{Ld} = \mu L Z_{Ud} - c$ and $\Pi^{NT}_{Hd} = 0$, respectively. This must be $\Pi^T_{Ld} \geq \Pi^{NT}_{Ld}$, that is $\mu L Z_{Ud} - c \geq 0$, and therefore $\mu \geq \frac{c}{Z_{Ud} \eta_L}$.

**Demand constraint.** The total number of hired students is:

$$
\lambda(p_a + \eta_L (1-p_a)) + (1-\lambda) (p_d + \eta_L (1-p_d)) \Phi - \lambda(p_a + \eta_L (1-p_a)) \frac{\Phi - \lambda(p_a + \eta_L (1-p_a))}{(1-\lambda) (p_d + \eta_L (1-p_d))} \equiv \Phi.
$$

**Case 2.** $p_a \geq \frac{\eta_L}{\nu + \eta_L} > p_d$

As $p_a \geq \frac{\eta_L}{\nu + \eta_L}$, the employer and school strategy for advantaged students does not change compared to the previous case.

**Employer.** The employer strategy is $z_{Ua} = 1; z_{Da} = 0, z_{Ud} = \min \left\{ \frac{c}{\mu_n L}, \frac{\Phi - \lambda(p_a + \eta_L (1-p_a))}{(1-\lambda)(p_d + \nu_U L_L)} \right\}$; $z_{Dd} = 0$. The employer’s beliefs for disadvantaged students are $\pi (\theta_H \mid g_{Ud}, d) = \frac{p_d}{p_d + \eta_L X_{Ud}(1-p_d)}$ and $\pi (\theta_L \mid g_{Ud}, d) = \frac{\eta_L X_{Ud}(1-p_d)}{p_d + \eta_L X_{Ud}(1-p_d)}$, if the student has a high grade and $\pi (\theta_H \mid g_{Dd}, d) = 0$ and $\pi (\theta_L \mid g_{Dd}, d) = 1$ if the student has a low grade. Thus the expected profit for hiring an advantaged
and high-grade student is \( \Pi_{Ud}^E = \frac{p_d}{p_d + \eta_L x_{Ld} (1 - p_d)} \nu - \frac{\eta_L x_{Ld} (1 - p_d)}{p_d + \eta_L x_{Ld} (1 - p_d)} \). This must be \( \frac{p_d}{p_d + \eta_L x_{Ld} (1 - p_d)} \nu - \frac{\eta_L x_{Ld} (1 - p_d)}{p_d + \eta_L x_{Ld} (1 - p_d)} = 0 \) and, after few passages, \( x_{Ld} = \frac{p_d}{(1 - p_d) \eta_L} \nu \). To be a probability, then \( \frac{p_d}{(1 - p_d) \eta_L} < 1 \), by which \( p_d < \frac{\eta_L}{\nu + \eta_L} \). The expected profits for hiring and not hiring a disadvantaged and low-grade student are \( \Pi_{Dd}^E = -1 \) and \( \Pi_{Dd}^N = 0 \), respectively, thus \( \Pi_{Dd}^E < \Pi_{Dd}^N \).

Then the employer needs to compare the expected profit obtained by high grade students with different social background: this is \( \Pi_{Ua}^E > \Pi_{Ud}^E \), as \( \Pi_{Ua}^E > 0 \), while \( \Pi_{Ud}^E = 0 \).

**School.** The school strategy is \( x_{La} = 1; x_{Ha} = ; x_{Ld} = \frac{p_d}{(1 - p_d) \eta_L} \nu; x_{Hd} = 1 \). The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are \( \Pi_{Hd}^T = \mu z_{Ud} - c \) and \( \Pi_{Hd}^{NT} = \mu \eta_H z_{Ud} \), respectively. This must be \( \Pi_{Hd}^T \geq \Pi_{Hd}^{NT} \), that is \( \mu z_{Ud} - c \geq \mu \eta_H z_{Ud} \), and therefore \( \mu \geq \frac{c}{z_{Ud} (1 - \eta_H)} \). The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are \( \Pi_{Ld}^T = z_{Ud} \mu \eta_L - c \) and \( \Pi_{Ld}^{NT} = 0 \), respectively. This must be \( \Pi_{Ld}^T = \Pi_{Ld}^{NT} \), that is \( z_{Ud} \mu \eta_L - c = 0 \), and therefore \( z_{Ud} = \frac{c}{\mu \eta_L} \).

**Demand constraint.** The total number of hired students\(^{19}\) is:

\[
\lambda (p_a + \eta_L (1 - p_a)) + (1 - \lambda) (p_d (1 + \nu)) \frac{c}{\mu \eta_L} \leq \Phi,
\]

thus the job capacity constraint implies \( z_{Ud} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi - \lambda (p_a + \eta_L (1 - p_a))}{(1 - \lambda) (p_d (1 + \nu))} \right\} \).

\(^{19}\) Note that the number of disadvantaged and high-grade students in this equilibrium is \( (1 - \lambda) (p_d + \eta (1 - p_d) x_{Ld}) \), in this equilibrium \( x_{Ld} = \frac{p_d}{(1 - p_d) \eta_L} \nu \), by substituting we obtain \( (1 - \lambda) \left( p_d + \eta (1 - p_d) \frac{p_d}{(1 - p_d) \eta_L} \nu \right) \), which can be simplified in \( (1 - \lambda) (p_d (1 + \nu)) \).
Case 3. \( pd < pa < \frac{\eta_L}{\nu + \eta_L} \)

As \( pd < \frac{\eta_L}{\nu + \eta_L} \), the employer and school strategy for disadvantaged students does not change compared to the previous case.

**Employer.** The employer strategy is

\[
\begin{align*}
    z_{Ua} &= \min \left\{ \frac{c}{\mu_{Ua} L}, \frac{\Phi_L}{\nu + (1-\lambda)pd(1+\nu)} \right\}; \\
    z_{Da} &= 0; \\
    z_{Dd} &= 0.
\end{align*}
\]

The employer’s beliefs for advantaged students are

\[
    \pi(\theta_H | g_U, a) = \frac{\eta_L x_{La}(1-p_a)}{p_a + \eta_L x_{La}(1-p_a)}, \quad \text{if the student has a high grade}
\]

and

\[
    \pi(\theta_L | g_U, a) = \frac{\eta_L x_{La}(1-p_a)}{p_a + \eta_L x_{La}(1-p_a)}, \quad \text{if the student has a low grade.}
\]

Thus the expected profit for hiring an advantaged and high-grade student is

\[
    \Pi_{Ua}^E = \frac{p_a}{p_a + \eta_L x_{La}(1-p_a)} \nu - \frac{\eta_L x_{La}(1-p_a)}{p_a + \eta_L x_{La}(1-p_a)}.
\]

This must be

\[
    \frac{p_a}{p_a + \eta_L x_{La}(1-p_a)} \nu - \frac{\eta_L x_{La}(1-p_a)}{p_a + \eta_L x_{La}(1-p_a)} = 0
\]

and, after few passages, 

\[
    x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta_L}.
\]

To be a probability, it is necessary that

\[
    \frac{p_a}{(1-p_a)} \frac{\nu}{\eta_L} < 1,
\]

by which

\[
    p_a < \frac{\eta_L}{\nu + \eta_L}.
\]

The expected profits for hiring and not hiring an advantaged and low-grade student are \( \Pi_{Da}^E = -1 \) and \( \Pi_{Da}^N = 0 \), respectively, thus \( \Pi_{Da}^E < \Pi_{Da}^N \).

Then the employer needs to compare the expected profit obtained by high grade students with different social background: this is \( \Pi_{Ua}^E = \Pi_{Ud}^E \), as both \( \Pi_{Ua}^E = 0 \) and \( \Pi_{Ud}^E = 0 \).

**School.** The school strategy is

\[
    x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta_L}; \quad x_{Ha} = 1; \quad x_{La} = \frac{p_a}{(1-p_a)} \frac{\nu}{\eta_L}; \quad x_{Hd} = 1.
\]

The expected payoffs for giving or not giving extra teaching to an advantaged and high-ability student are \( \Pi_{Ha}^T = \mu z_{Ua} - c \) and \( \Pi_{Ha}^{NT} = \mu z_{Ua} \), respectively. This must be \( \Pi_{Ha}^T \geq \Pi_{Ha}^{NT} \), that is \( \mu z_{Ua} - c \geq \mu z_{Ua} \), and therefore

\[
    \mu \geq \frac{c}{z_{Ua}(1-\eta)}.
\]

The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are \( \Pi_{La}^T = \mu z_{Ua} \eta_L - c \) and \( \Pi_{La}^{NT} = 0 \), respectively. This must be \( \Pi_{La}^T = \Pi_{La}^{NT} \), that is \( \mu z_{Ua} \eta_L - c = 0 \), and therefore

\[
85
\]
\[ z_{Ua} = \frac{c}{\mu \eta_L}, \]

**Demand constraint.** The total number of hired students\(^{20}\) is:

\[
\frac{c}{\mu \eta_L} (1 + \nu) (\lambda p_a + (1 - \lambda) p_d) \leq \Phi,
\]

thus the job capacity constraint implies \( z_{Ua} = z_{Ud} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi}{(\lambda p_a + (1 - \lambda) p_d)(1 + \nu)} \right\} \).

---

**Proof of corollary 1**

**High-employment equilibrium.** Differentiation of \( \Phi = \lambda (p_a + \eta L (1 - p_a)) \) with respect to \( \eta_L \), and \( \lambda \) yields:

\[
\frac{\partial}{\partial \eta_L} \Phi = \frac{\lambda (1 - p_a)}{p_a + \eta L (1 - p_a)}, \quad \lambda = \frac{1 - p_a}{p_a + \eta L (1 - p_a)} + \frac{1 - p_a}{\lambda (1 - p_a)(1 - \lambda)} \frac{p_a + \eta L (1 - p_a)}{(p_a + \eta L (1 - p_a))^2} < 0,
\]

and:

\[
\frac{\partial}{\partial \lambda} \Phi = \frac{1 - p_a}{p_a + \eta L (1 - p_a)} \frac{p_a + \eta L (1 - p_a)}{(p_a + \eta L (1 - p_a))^2} < 0,
\]

respectively.

**Middle-employment equilibrium.** Differentiation of \( \frac{c}{\mu \eta_L} \) with respect to \( \eta_L \) yields:

\[
\frac{\partial}{\partial \eta_L} \frac{c}{\mu \eta_L} = \frac{\partial c}{\partial \eta_L} = \frac{c}{\mu \eta_L^2} < 0.
\]

Differentiation of \( \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)(1 - \lambda)^2} \) with respect to \( \eta_L \) and \( \nu \) yields:

\[
\frac{\partial}{\partial \eta_L} \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)^2} = \frac{\nu p_a}{\eta L (1 - p_a)}, \quad \frac{\partial}{\partial \nu} \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)^2} = \frac{1}{\eta L} \frac{p_a}{1 - p_a} > 0,
\]

respectively.

**Low-employment equilibrium.** Differentiation of \( \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)^2} \) with respect to \( \eta_L \) and \( \nu \) yields:

\[
\frac{\partial}{\partial \eta_L} \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)^2} = \frac{\nu p_a}{\eta L (1 - p_a)} < 0,
\]

and:

\[
\frac{\partial}{\partial \nu} \frac{p_a + \eta L (1 - p_a)}{(1 - p_a)^2} = \frac{1}{\eta L} \frac{p_a}{1 - p_a} > 0,
\]

respectively.\(^{20}\)

\(^{20}\)Note that the number of advantaged and high-grade students in this equilibrium is \( \lambda (p_a + \eta L (1 - p_a)) \), in this equilibrium \( x_{La} = \frac{p_a}{(1 - p_a) \eta} \), by substituting we obtain \( \lambda \left( p_a + \eta L (1 - p_a) \frac{p_a}{(1 - p_a) \eta} \right) \), which can be simplified in \( \lambda (p_a (1 + \nu)) \).
Proof of Proposition 3

For each disadvantaged student, the government subsidises $cs$. This does not change anything in the case 1, as the school strategy was $x_{Ld} = 1; x_{Hd} = 1$.

Case 2. $p_a \geq \frac{\eta_H}{\nu + \eta_L} > p_d$

For advantaged students we refer to the proof (case 2) of Proposition 2.

**Employer.** The employer strategy is $z_{Ua} = 1; z_{Da} = 0, z_{Ud} = \min \left\{ \frac{c(1-s)}{\mu \eta_L}, \frac{\Phi - \lambda (p_a + \eta_L (1-p_a))}{(1-\lambda)(p_d(1+\nu))} \right\}; z_{Dd} = 0$. The employer’s beliefs for disadvantaged students are $\pi (\theta_H | g_U, d) = \frac{p_d}{p_d + \eta_L x_{Ld} (1-p_d)}$ and $\pi (\theta_L | g_U, d) = \frac{\eta_L x_{Ld} (1-p_d)}{p_d + \eta_L x_{Ld} (1-p_d)}$, if the student has a high grade and $\pi (\theta_H | g_D, d) = 0$ and $\pi (\theta_L | g_D, d) = 1$ if the student has a low grade. Thus the expected profit for hiring an advantaged and high-grade student is $E^U_{Ed} = p_d \nu + \eta_L x_{Ld} (1-p_d) \nu - \frac{\eta_L x_{Ld} (1-p_d)}{p_d + \eta_L x_{Ld} (1-p_d)}$. This must be $\frac{p_d}{p_d + \eta_L x_{Ld} (1-p_d)} \nu - \frac{\eta_L x_{Ld} (1-p_d)}{p_d + \eta_L x_{Ld} (1-p_d)} = 0$ and, after few passages, $x_{Ld} = \frac{p_d \nu}{(1-p_d) \eta_L}$. To be a probability, then $\frac{p_d \nu}{(1-p_d) \eta_L} < 1$, by which $p_d < \frac{\eta_L}{\nu + \eta_L}$. The expected profits for hiring and not hiring a disadvantaged and low-grade student are $\Pi^E_{Dd} = -1$ and $\Pi^N_{Dd} = 0$, respectively, thus $\Pi^E_{Dd} < \Pi^N_{Dd}$.

Then the employer needs to compare the expected profit obtained by high grade students with different social background: this is $\Pi^E_{Ua} > \Pi^E_{Ud}$, as $\Pi^E_{Ua} > 0$, while $\Pi^E_{Ud} = 0$.

**School.** The school strategy is $x_{La} = x_{Ha} = 1; x_{Ld} = \frac{p_d \nu}{(1-p_d) \eta_L}; x_{Hd} = 1$. The expected payoffs for giving or not giving extra teaching to a disadvantaged and high-ability student are $\Pi^T_{Hd} = \mu z_{Ud} - c(1-s)$ and $\Pi^N_{Hd} = \mu \eta_H z_{Ud}$, respectively. This must be $\Pi^T_{Hd} \geq \Pi^N_{Hd}$, that is $\mu z_{Ud} - c(1-s) \geq \mu \eta_H z_{Ud}$, and there-
fore \( \mu \geq \frac{c(1-s)}{z_{Ud}(1-\eta_H)} \). The expected payoffs for giving or not giving extra teaching to a disadvantaged and low-ability student are \( \Pi_{Ld}^T = z_{Ud}\mu \eta_L - c(1-s) \) and \( \Pi_{Hd}^{NT} = 0 \), respectively. This must be \( \Pi_{Ld}^T = \Pi_{Ld}^{NT} \), that is \( z_{Ud}\mu \eta_L - c(1-s) = 0 \), and therefore \( z_{Ud} = \frac{c(1-s)}{\mu \eta_L} \).

**Demand constraint.** The total number of hired students\(^{21}\) is:

\[
\lambda(p_a + \eta_L (1 - p_a)) + (1 - \lambda) (p_d (1 + \nu)) \frac{c(1-s)}{\mu \eta_L} \leq \Phi,
\]

thus the job capacity constraint implies
\[
z_{Ud} = \min \left\{ \frac{c(1-s)}{\mu \eta_L}, \frac{\Phi}{(1-\lambda)(p_d(1+\nu))} \right\}.
\]

**Case 3.** \( p_d < p_a < \frac{\eta_L}{\nu + \eta_L} \)

For advantaged students we refer to the proof (case 3) of Proposition 2. As \( p_d < \frac{\eta_L}{\nu + \eta_L} \), the employer and school strategy for disadvantaged students does not change compared to the previous case.

**Demand constraint.** The total number of hired students is:

\[
\frac{c}{\mu \eta_L} (\lambda p_a + (1-s) (1-\lambda) p_d) (1 + \nu) \leq \Phi,
\]

thus the job capacity constraint implies
\[
z_{Ua} = \min \left\{ \frac{c}{\mu \eta_L}, \frac{\Phi}{(\lambda p_a + (1-s) (1-\lambda) p_d)(1+\nu)} \right\}.
\]

and
\[
z_{Ud} = \min \left\{ \frac{c(1-s)}{\mu \eta_L}, \frac{\Phi}{(1-\lambda)(p_d(1+\nu))} \right\}.
\]

---

\(^{21}\)Note that the number of disadvantaged and high-grade students in this equilibrium is \( (1-\lambda) (p_d + \eta (1-p_d) x_{Ld}), \) in this equilibrium \( x_{Ld} = \frac{p_d}{(1-p_d) \frac{\nu}{\eta}}, \) by substituting we obtain \( (1-\lambda) \left( p_d + \eta (1-p_d) \frac{p_d}{(1-p_d) \frac{\nu}{\eta}} \right), \) which can be simplified in \( (1-\lambda) (p_d (1 + \nu)) \).
Bibliography


96


