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DEPARTMENT OF ECONOMICS

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Theodore Palivos, University of Macedonia, Greece

Dimitrios Varvarigos, University of Leicester, UK

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Theodore Palivos[a]
Department of Economics
University of Macedonia
Greece

Dimitrios Varvarigos[b]
Department of Economics
University of Leicester
England

Abstract

In a two-period overlapping generations model with production, we consider the damaging impact of environmental degradation on health and, consequently, life expectancy. The government's involvement on policies of environmental preservation proves crucial for both the economy's short-term dynamics and its long-term prospects. Particularly, an active policy of pollution abatement emerges as an important engine of long-run economic growth. Furthermore, by eliminating the occurrence of limit cycles, pollution abatement is also a powerful source of stabilisation.

Keywords: Growth; Cycles; Environmental quality; Pollution abatement

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[a] **Address:** 156 Egnatia Street, Thessaloniki GR-540 06, Greece. **Email:** tpalivos@uom.gr. **Tel:** ++30 2310 891 775

[b] **Address:** Astley Clarke Building, University Road, Leicester LE1 7RH, UK. **Email:** dv33@le.ac.uk. **Tel:** ++44 (0) 116 252 2184

1 Introduction

In recent years, environmental issues have gained prominence in both academic and political discussions. At the same time, they have received considerable media attention. Problems such as the emission of greenhouse gases and their impact on global warming, the depletion of natural resources, and the hazardous chemicals/toxins that contaminate the food and water supplies and deteriorate the quality of air, are major issues of concern. This is, of course, not surprising given their significant direct and indirect repercussions for our health characteristics and, therefore, our overall quality of life (e.g., Pimentel *et al.*, 1998; Donohoe, 2003; Lacasaña *et al.*, 2005).[1]

Naturally, economic growth has been an indispensable aspect of all this focus, attention and discussion – after all, environmental degradation is a by-product of economic activities such as production and consumption. One point of view focuses on this latter idea, as well as the economic importance of a prosperous natural environment, so as to suggest that societies, and their policy makers in particular, should shift their attention away from economic growth and towards policies and actions that preserve environmental quality (e.g., Daly and Cobb, 1989; Arrow *et al.*, 1995). Otherwise, the reckless and short-sighted quest for economic prosperity today will deteriorate the quality of the environment bestowed to future generations to the extent of severely undermining their prospects for economic prosperity, as well as their ability to support a meaningful quality of life. Another point of view discards the aforementioned arguments. It is based on empirical analyses (e.g., Grossman and Krueger, 1995; Hilton and Levinson, 1998; Millimet *et al.*, 2003; Aslanidis and Xepapadeas, 2008) that derive environmental Kuznets curves (EKCs) as well as theoretical analyses which, in a similar vein, imply that economic activity may actually represent a benefit, rather than a cost, for environmental quality (e.g., John and Pecchenino, 1994; Ono, 2003; Mariani *et al.*, 2010).[2]

Careful inspection into the ideas and mechanisms behind each of the aforementioned points of view can reveal that both of them have their shortcomings. The proponents of the former view fail to acknowledge the implications of their suggestions for persistent unemployment, poverty, lack of investment in infrastructure, education, health services etc. – issues of particular importance for both developed and developing economies. Sadly,

[1] Pimentel *et al.* (1998) estimate that the direct and indirect impacts of environmental degradation can account for almost 40% of deaths worldwide.

[2] The EKC is an inverse-U-shaped relationship between measures of pollution and per capita GDP.

however, the shortcomings of the opposing view seem to be equally serious. To begin with, the EKC, rather than being generally accepted as a stylised fact, is probably the single most contested issue in the environmental economics literature. A significant number of analyses have criticised both the methodological framework and the interpretation of the results supporting the EKC, while others have failed to reproduce co-movements in measures of pollution and income that resemble EKCs (e.g., Perman and Stern, 2003; Dijkgraaf and Vollebergh, 2005; Azomahou *et al.*, 2006). Furthermore, many of the existing theoretical analyses assume that the environmental impacts of pollutant emissions and activities such as environmental maintenance and pollution abatement are additively separable. Coupled with the assumption that individuals internalise the environmental effects of their own (polluting) consumption and environmental maintenance decisions, additive separability allows the latter to dominate the former. Consequently, given that both consumption and maintenance are proportional to income, the dynamics of environmental quality actually improve with higher incomes.

Of course, such results invite criticism because outcomes in which the environmental benefit of activities targeted at environmental support could be greater than the overall environmental cost of pollution – a cost that they are supposed to mitigate in the first place – appear to be unrealistic. In fact, other papers that employ additively separable effects for pollution and abatement/environmental maintenance, recognise this shortcoming and address it by imposing a non-negativity constraint that requires the environmental cost of emissions to dominate the benefit from abatement. Roussillon and Schweinzer (2010) justify this restriction on the basis that "requiring non-negative differences in the damage function…ensures that reductive efforts cannot substitute productive efforts" (p. 4, footnote 5). Economides and Philippopoulos (2008) use a similar restriction, arguing that the scenario for which environmental maintenance is stronger than the polluting effect of production is "too good to be true" (p. 213).

In this paper, we show that an equilibrium with (environmentally) sustainable long-run growth is possible, despite the fact that economic growth has a net damaging effect on environmental quality (irrespective on whether pollution is abated or not) and even though the quality of the environment is essential for supporting longevity and, therefore, saving and capital accumulation. We build a two-period overlapping generations model in which labour productivity is enhanced by an aggregate learning-by-doing externality. Despite the fact that this type of externality is the source of aggregate constant returns that could potentially allow an equilibrium with positive growth rate in the long-run, when pollution is left unabated in our model, the economy cannot achieve such an equilibrium. Instead, as long as there is a sufficient initial endowment of capital stock, the economy will either converge to a positive stationary level for capital per worker or to a stable cycle in which capital per worker oscillates permanently around its (non-stationary) equilibrium. Nevertheless, when resources are devoted towards pollution abatement, then equilibrium outcomes change drastically. In this case, an economy that is sufficiently endowed with capital in the initial period can achieve an equilibrium in which both capital per worker and output per worker grow constantly in the long-run. Economic growth is environmentally sustainable, since a positive level of environmental quality is maintained. This occurs in spite of the non-separable environmental effects of pollutant emissions and abatement – meaning that economic activity still entails environmental costs, notwithstanding the resources devoted to pollution abatement.[3]

In the last main section of our analysis, we endogenise the government's expenditure allocation. In particular, we consider the case where the public sector allocates its spending between public health care and environmental activities so as to maximise the life expectancy of the economy's population. The first main outcome from this procedure echoes the result of Stokey (1998) in that the government finds optimal to initiate any spending towards environmental support only after the economy's capital resources exceed a certain threshold. Casual empirical observation suggests that actual economies tend to engage in active environmental preservation only at later stages of their development process – hence, providing support for our theoretical result. We also show that, once the government supports pollution abatement activities optimally, the economy may sustain economic growth in the long-run while the dynamics do not converge to endogenous cycles.

Our results can be viewed as addressing the shortcomings of the two opposite views on the environment-economic growth nexus to which we alluded earlier. On the one hand, we show that sustainable economic growth is possible even though growth is detrimental to environmental quality, for which some sufficient degree is essential for a meaningful human existence. On the other hand, we show that environmentally sustainable growth is not

[3] The possibility of endogenous fluctuations in models of capital accumulation and environmental quality has been also identified by Zhang (1999), Ono (2003) and Seegmuller and Verchère (2004). All of them employ the John and Peccherino (1994) framework to introduce environmental quality; thus, the mechanism of endogenous cycles differs from ours. In our model, cycles may emerge because unbounded environmental degradation, and its impact on longevity, introduces non-monotonicity in the dynamics of capital accumulation.

achieved because economic growth will miraculously solve all environmental problems or because the benefit from activities of environmental maintenance is inexplicably stronger that the environmental cost of pollution. Sustainable growth can be achieved on the condition that societies devote some of their resources towards actions and policies of environmental preservation, even if the utmost that such policies and actions can achieve is just to moderate the extent of environmental degradation. As long as they can achieve this, they are still crucial for deterring economic prosperity from being associated with the kind of unbounded environmental decay that could gravely undermine human existence.

The rest of the paper is organised as follows: Section 2 sets-up the economic model. In Section 3 we analyse the different equilibrium outcomes of the model, according to whether pollution abatement is active or not. In Section 4, we discuss some important implications from our analysis and in Section 5 we consider the case where the government's expenditure towards pollution abatement is determined optimally. Section 6 summarises and concludes.

2 The Economic Framework

We construct an overlapping generations economy in which time, indicated by $t = 0, 1, 2, \ldots$, is measured in discrete intervals that represent periods. The economy is populated by an infinite sequence of agents who face a potential lifetime of two periods. In particular, an agent will live during the period following her birth, i.e., her youth, but she may or may not survive to her old age. We assume that, before her survival prospect is realised, each agent reproduces asexually and gives birth to an offspring. Thus, the prospect of untimely death does not have any repercussions for the population mass of newly-born agents, whose size we normalise to one.

During youth, each agent is endowed with one unit of labour. She supplies her labour to firms (inelastically), which compensate her by providing a salary denoted by w_t. Even if she survives to maturity, nature does not bestow to her the ability to work when old, therefore w_t is her only source of income during her lifetime. For this reason, and in order to satisfy her possible future consumption needs, she deposits an amount s_t, when young, to a financial intermediary that promises to repay it next period, augmented by the gross interest rate r_{t+1}.

As mentioned earlier, survival to maturity is not certain. Particularly, we assume that a young person will survive to maturity with probability $\beta_t \in [0,1)$ whereas with probability $1-\beta_t$ she dies prematurely. Furthermore, we assume that life expectancy is endogenous in the sense that the agent's survival prospect depends on her health characteristics (or health status), denoted as h_t, according to[4]

$$\beta_t = \mathrm{B}(h_t), \tag{1}$$

where $\mathrm{B}'(h_t) > 0$, $\mathrm{B}''(h_t) < 0$, $\mathrm{B}(0) = 0$, $\mathrm{B}(\infty) = \lambda$, $\lambda \in (0,1)$, $\mathrm{B}'(0) = \psi$, $\psi \in (0,1)$, and $\mathrm{B}'(\infty) = 0$. Thus, we employ essentially the same assumptions used by Chakraborty (2004) in his seminal analysis of endogenous lifetime and economic growth.[5]

We delve further into the determinants of life expectancy by assuming that an agent's health status depends positively on the extent to which the government supports the provision of health services g_t (e.g., public hospitals, the presence of a national health system, preventive measures, funding and support of medical research, the design and implementation of health and safety rules etc.), and on the quality of the natural environment e_t (e.g., the cleanliness of air, soil and water, the relative abundance of natural resources such as forestry and other forms of plantation etc.). Formally, these ideas are captured by

$$h_t = g_t^{\varphi} e_t^{\chi}, \tag{2}$$

where $0 < \varphi < 1$ and $0 < \chi < 1$.[6]

All choices made by an agent during her lifetime are governed by her *ex ante* (i.e., expected) lifetime utility function

$$V^t = \ln c_t^t + \beta_t \ln c_{t+1}^t, \tag{3}$$

where c_t^t and c_{t+1}^t denote the levels of consumption during youth and old age respectively.[7] It should be noted that we employ the notational standard of using subscripts to indicate the period of birth and subscripts to indicate the period at which events take place.

[4] An agent's expected lifetime at birth is equal to $\beta_t 2 + 1 - \beta_t = 1 + \beta_t$ periods. For this reason, we shall be using such terms as 'life expectancy', 'longevity' and 'survival probability' interchangeably.

[5] Other analyses that incorporate life expectancy in this manner include Blackburn and Cipriani (2002) and Bhattacharya and Qiao (2007) among others.

[6] The limiting case for which $\varphi = 1$ and $\chi = 0$ is examined by Chakraborty (2004). In his paper, he does not consider issues pertaining to the natural environment.

[7] We assume that child rearing costs are incorporated in a person's consumption expenditures when young.

There is a single, perishable commodity through which agents can satisfy their consumption needs. It is produced by perfectly competitive firms who combine physical capital, K_t (which they rent from financial intermediaries at a price of R_t per unit), and labour, L_t, so as to produce Y_t units of output according to

$$Y_t = K_t^{\gamma}(\mathrm{A}_t L_t)^{1-\gamma},\ 0<\gamma<1, \tag{4}$$

where A_t is assumed to be positively related to the economy's average amount of capital, $\bar{K}_t$ (e.g., Frankel, 1962; Romer, 1986). Thus, it captures the idea that workers gain knowledge and become more productive by handling more capital goods – knowledge that spreads costlessly over the whole economy in the manner of an externality. Formally,

$$\mathrm{A}_t = \tilde{\mathrm{A}}\bar{K}_t,\ \tilde{\mathrm{A}} > 0. \tag{5}$$

One unfortunate by-product from firms' activities is pollution. We assume that one unit of produced output generates $p > 0$ units of pollutant emissions, therefore total pollution is given by

$$P_t = pY_t. \tag{6}$$

Although pollution is the major determinant of environmental degradation, D_t, the latter can be mitigated by government-funded activities that are designed and implemented so as to reduce the extent of environmental damage for given levels of pollutant emissions. We may think of recycling facilities, wastewater management facilities, installation and operation of renewable energy techniques that reduce the emission of greenhouse gases and toxic pollutants (e.g., wind turbines, hydroelectric plants and solar photovoltaics), clean-up operations, etc. For the purposes of our analysis, we shall refer to them as pollution abatement activities, and denote them by $a_t \geq 0$. Environmental degradation is, hence, formally given by

$$D_t = \frac{P_t}{1+a_t}. \tag{7}$$

Given the aforementioned arguments, the quality of the natural environment, $e_t \geq 0$, depends on the extent of environmental degradation. We capture this idea through

$$e_t = \begin{cases} E - D_t & \text{if } D_t < E \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

where $E > 0$.[8]

Note that, according to (7), the environmental impacts of pollution and abatement are not separable. Given that, in equilibrium, both of them are proportional to income, higher production will always entail environmental degradation and net environmental costs – irrespective on whether pollution is abated ($a_t > 0$) or not ($a_t = 0$). This is an important deviation of our paper in comparison to some existing models on the relationship between economic growth and the environment (e.g., John and Pecchenino, 1994; Ono, 2003; Mariani *et al.*, 2010).

We complete our analysis of the economy's structure with a discussion on the process under which the government finances its activities. We utilise the widely-used assumption that the government imposes a flat tax rate $\tau \in (0,1)$ on firms' production revenues. Assuming that the government abides by a balance budget rule in each period, our previous assumptions imply that $g_t + a_t = \tau Y_t$. If we denote the fixed fraction of revenues devoted towards pollution abatement by $\upsilon \in [0,1)$, it is straightforward to establish that

$$g_t = (1-\upsilon)\tau Y_t, \tag{9}$$

and

$$a_t = \upsilon \tau Y_t, \tag{10}$$

give the levels of public health spending and pollution abatement activities in relation to the economy's total output, respectively.

3 Temporary Equilibrium

We begin our analysis with a description of the economy's temporary equilibrium. This is provided in the form of

Definition 1. *The temporary equilibrium of the economy is a set of quantities* $\{c_t^{t-1}, c_t^t, c_{t+1}^t, s_t, L_t, Y_t, A_t, \beta_t, h_t, e_t, D_t, L_t, P_t, a_t, g_t, K_t, K_{t+1}\}$ *and prices* $\{w_t, R_t, R_{t+1}, r_{t+1}\}$ *such that:*

[8] To maintain analytical convenience, we abstract from the dynamics of environmental quality by assuming that nature has the ability to completely regenerate and restore itself within a period. With a two-period overlapping generations setting, in which a period may include many years, this is not a very restrictive assumption. Moreover, it has been used in the analyses of Stokey (1998), Jones and Manuelli (2001) and Hartman and Kwon (2005) among others.

(i) *Given* w_t, r_{t+1} *and* β_t, *the quantities* c_t^t, c_{t+1}^t *and* s_t *solve the optimisation problem of a worker born at time* t;

(ii) *Given* w_t *and* R_t, *all firms choose quantities for* L_t *and* K_t *in order to maximise profits;*

(iii) *The labour market clears, i.e.,* $\mathrm{L}_t = 1$;

(iv) *The goods market clears, i.e.,* $Y_t = c_t^t + \beta_{t-1} c_t^{t-1} + s_t + g_t + a_t$;

(v) *The financial market clears;*

(vi) *The government's budget is balanced.*

The objective of a young agent is to choose the levels of consumption, in both periods, and saving so as to maximise V^t subject to $c_t^t = w_t - s_t$ and $c_{t+1}^t = r_{t+1} s_t$ respectively. Alternatively, given (3), the problem can be modified to $\max\limits_{0 \le s_t \le 1} \{\ln(w_t - s_t) + \beta_t \ln(r_{t+1} s_t)\}$. The solution to this problem is

$$s_t = \frac{\beta_t}{1+\beta_t} w_t . \tag{11}$$

Naturally, the prospect of premature death modifies an agent's saving behaviour. In terms of intuition, an increase in longevity raises the (expected) marginal utility of an agent's consumption when old; therefore, to restore the equilibrium, the marginal utility derived from her first period consumption must increase as well. She can achieve this by choosing to save more and consume less while she is young.

Profit maximisation by firms entails that each input's marginal product is equal to its respective price. Formally,

$$w_t = (1-\tau)(1-\gamma) K_t^{\gamma} L_t^{-\gamma} \mathrm{A}_t^{1-\gamma} = (1-\tau)(1-\gamma) k_t^{\gamma} \mathrm{A}_t^{1-\gamma}, \tag{12}$$

and

$$\mathrm{R}_t = (1-\tau)\gamma K_t^{\gamma-1} L_t^{1-\gamma} \mathrm{A}_t^{1-\gamma} = (1-\tau)\gamma k_t^{\gamma-1} \mathrm{A}_t^{1-\gamma}, \tag{13}$$

where $k_t = K_t / L_t$ is the amount of capital per worker. Using (5) together with the labour market clearing condition, $L_t = 1$, implies that $k_t = K_t = \bar{K}_t$. Consequently, using the notation $\Gamma = \widetilde{\mathrm{A}}^{1-\gamma}$, we can write (12) and (13) as

$$w_t = (1-\tau)(1-\gamma)\Gamma k_t , \tag{14}$$

and

$$R_t = (1-\tau)\gamma\Gamma \equiv \hat{\mathrm{R}} \ , \tag{15}$$

respectively.

There are two conditions that describe the financial market equilibrium. We assume that perfectly competitive financial intermediaries undertake the task of channelling capital from depositors to firms. Specifically, they transform saving deposits into capital by accessing a technology that transforms time-t output into time-$t+1$ capital on a one-to-one basis. They, subsequently, supply this capital to firms that manufacture the economy's single commodity. Hence, $K_{t+1} = L_t s_t$ or, in intensive form,

$$k_{t+1} = s_t \, . \tag{16}$$

To resolve the issue of saving under an uncertain lifetime, we assume, following among others Chakraborty (2004), that financial intermediaries represent mutual funds that offer contingent annuities. Specifically, when accepting deposits, intermediaries promise to offer retirement income (in our case, $r_{t+1}s_t$) provided that the depositor survives to old age. Otherwise, the income of those who die is shared equally among surviving members of the mutual fund. Considering this assumption, and the fact that financial intermediaries operate under perfect competition, we have

$$\beta_t r_{t+1} = \mathrm{R}_{t+1} = \hat{\mathrm{R}} \, , \tag{17}$$

which translates into the equilibrium condition requiring costs (i.e., the total return to all surviving savers) to be equal to revenues (i.e., the revenues they receive from firms who rent capital) – the reason being that financial intermediaries make zero economic profits from their activities.

Next, we can use the labour market clearing condition, together with (5), in equation (4) so as to obtain an expression for output per worker $y_t = Y_t / L_t$. That is,

$$y_t = \Gamma k_t \, . \tag{18}$$

If we combine the expression in (18) together with (1), (2), (6), (7), (8), (9) and (10), and substitute together with (11) and (14) in equation (16), we can eventually derive

$$k_{t+1} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)} k_t = z(k_t) \, . \tag{19}$$

Thus, we have reduced our model into a dynamical system of one first-order difference equation for capital per worker. The analysis of this equation will facilitate us in understanding the dynamics and the long-run equilibrium of the economy. This is the issue to which we now turn our attention.[9]

3 Dynamic Equilibrium

The economy's dynamic equilibrium is formally described through

Definition 2. *For* $k_0 > 0$*, the dynamic equilibrium is a sequence of temporary equilibria that satisfy* $k_{t+1} = z(k_t)$ *for every* t.

We can facilitate our subsequent analysis by defining a new variable, θ_{t+1}, which denotes the growth rate of physical capital per worker. That is,

$$\theta_{t+1} = \frac{k_{t+1}}{k_t} - 1. \tag{20}$$

Furthermore, our subsequent results will be further clarified with the use of

Definition 3. *Consider* $k_0 > 0$*. Then:*

(i) *If* $\lim_{t \to \infty} \theta_{t+1} = 0$*, an equilibrium with* $k_{t+1} = k_t = \hat{k} > 0$ *is a 'no growth' steady-state equilibrium;*

(ii) *If* $\lim_{t \to \infty} \theta_{t+1} = \hat{\theta} > 0$*, an equilibrium with* $\frac{k_{t+1}}{k_t} = 1 + \hat{\theta}$ *is a 'long-run growth' equilibrium;*

(iii) *If* $k_{t+1} = k_t = 0$ *and* $\lim_{t \to \infty} k_t = 0$*, the equilibrium is a 'poverty trap'.*

Our purpose is to examine two scenarios which differ with respect to the government's provision of pollution abatement services. As we shall see, the public sector's stance on environmental protection has significant repercussions for both the economy's dynamics and its long-term prospects. Notice that all proofs to our subsequent results are relegated to an Appendix. Furthermore, the subsequent analysis will be utilising

[9] It is straightforward to establish that all the results are consistent with the economy's resources constraint.

Assumption 1. $(1-\tau)(1-\gamma)\Gamma\dfrac{\mathrm{B}(\Omega)}{1+\mathrm{B}(\Omega)}>1$ *where* $\Omega=\left(\dfrac{\varphi\tau}{p}\right)^{\varphi}\chi^{\chi}\left(\dfrac{E}{\varphi+\chi}\right)^{\varphi+\chi}$,

as well as

Assumption 2. $\chi\leq\varphi$.

The first restriction is essential for the existence of a meaningful long-run equilibrium (see Footnote 10). The second one is not essential for our results and is employed purely for expositional purposes (see Footnote 12). It is actually relaxed in Appendix A5, where we show that our results still remain qualitatively similar.

3.1 Dynamic Equilibrium without Pollution Abatement

We begin our analysis with the case for which $\upsilon=0$ – a case which translates into a scenario where the government is not actively engaged in policies of environmental preservation. Given (19), we have

$$z(k_t)=(1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E-p\Gamma k_t)^{\chi}\right)}k_t\,. \tag{21}$$

First, we are interested in obtaining the model's steady-state equilibria. These are fixed points of the map $z(\cdot)$, i.e., values $\hat{k}$ of capital per worker that satisfy $\hat{k}=z(\hat{k})$. A formal analysis of (21) allows us to derive

Lemma 1. *There exist three steady-state equilibria* $\hat{k}_1$, $\hat{k}_2$ *and* $\hat{k}_3$, *such than* $\hat{k}_1=0$ *and* $\hat{k}_3>\hat{k}_2>0$. *The steady state* $\hat{k}_1$ *is locally asymptotically stable,* $\hat{k}_2$ *is an unstable steady state, while* $\hat{k}_3$ *may be either locally asymptotically stable or unstable.*

The result from Lemma 1 facilitates us in tracing the economy's dynamic behaviour and transitional dynamics.[10] We can formally present these ideas in the form of

Proposition 1. *Consider* $k_0 > 0$. *Then:*

(i) *If* $k_0 < \hat{k}_2$, *the economy will converge to the poverty trap* $\hat{k}_1 = 0$*;*

(ii) *If* $k_0 > \hat{k}_2$, *the economy will converge to a 'no growth' equilibrium. Particularly, if* $\hat{k}_3$ *is locally asymptotically stable, then it will also be the stationary equilibrium for the stock of capital per worker – otherwise, the economy will asymptotically converge to an equilibrium where capital per worker displays permanent cycles around* $\hat{k}_3$.

The different possible scenarios are depicted in Figures 1-3. In all different cases, we see that the point $\hat{k}_2$ acts as a natural threshold which allows history (approximated by the initial capital endowment) to determine the long-term prospects of the economy. The model's ability to generate multiple steady-state equilibria rests on the beneficial effect of publicly provided health services on saving behaviour – an effect that lies on the idea that health services promote longevity. Specifically, for some levels of k_t, capital accumulation and saving complement each other. Thus, for relatively low levels of initial capital endowment, saving is not sufficient enough to guarantee a positive rate of capital accumulation: capital per worker declines constantly until it rests on an equilibrium which is, essentially, a poverty trap. If, however, initial endowments are sufficient enough, the economy can escape the poverty trap because saving allows the economy to grow at positive (albeit declining) rates during the early stages of its transition.

[10] When Assumption 1 does not hold, the only steady-state equilibrium is $\hat{k} = 0$. This is because the graph of (21) lies below the 45-degree line for all $k_t > 0$. Due to its limited interest, we choose not to discuss this case in detail. Also, notice that, for this particular scenario (i.e., $\upsilon = 0$), relaxing Assumption 2 has no effect whatsoever on the results.

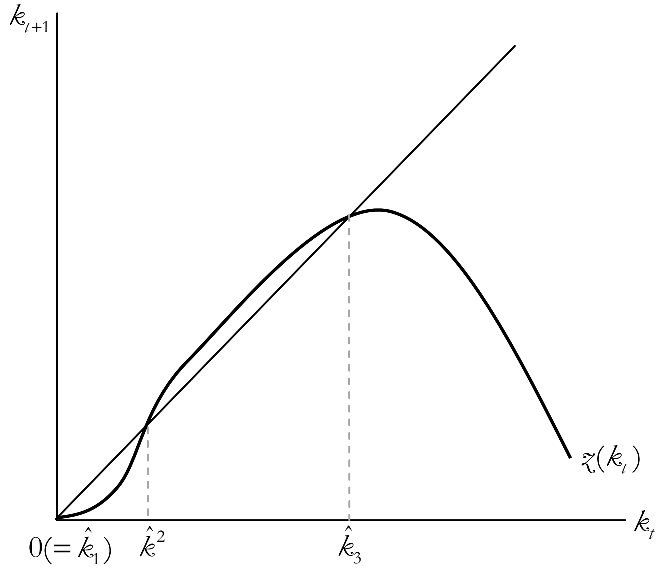

Figure 1. $\upsilon = 0$ and $0 < z'(\hat{k}_3) < 1$

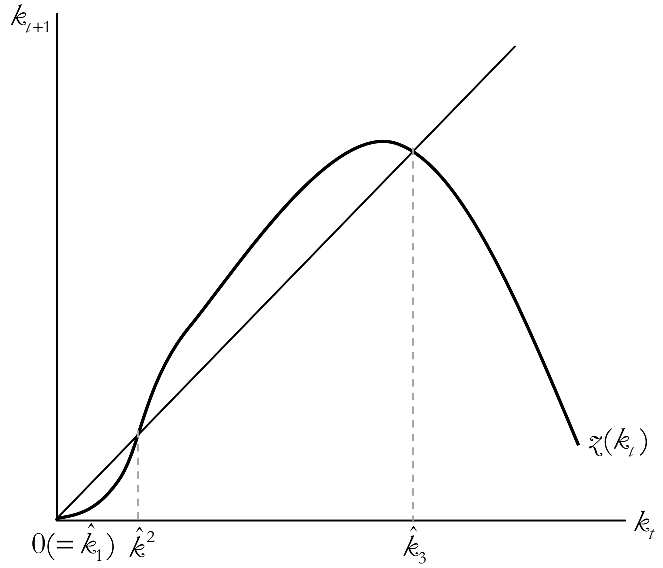

Figure 2. $\upsilon = 0$ and $-1 < z'(\hat{k}_3) < 0$

So far, the results and their intuition are similar to those discussed in Chakraborty (2004). Nevertheless, our model is able to generate richer implications for the dynamics of an economy whose history allows it to move on the right side of the natural threshold $\hat{k}_2$. The reason for such implications is economic activity's contribution to environmental degradation and the corresponding repercussions for health status and longevity. Particularly, for sufficiently high values of k_t the negative effect of pollution on life expectancy and saving dominates the positive effect of publicly provided goods and services on health. Hence, the dynamics of capital accumulation are non-monotonic and $\hat{k}_3$ may actually lie on the downward sloping part of $z(k_t)$. Furthermore, as Figure 3 illustrates, when the slope of the graph at the steady state $\hat{k}_3$ is steep enough, the economy may converge to an equilibrium in which capital per worker oscillates permanently around $\hat{k}_3$ – i.e., an equilibrium with a permanent, endogenously determined cycle. In terms of intuition, a relatively high level of capital per worker implies relatively high pollution. The health status is affected negatively and, consequently, saving is reduced. Capital accumulation is mitigated, but this also implies that the extent of environmental degradation is mitigated as well. Next period's health status improves and so is saving which promotes capital accumulation. This sequence of events may eventually become self-repeating, thus generating an equilibrium with persistent cycles.

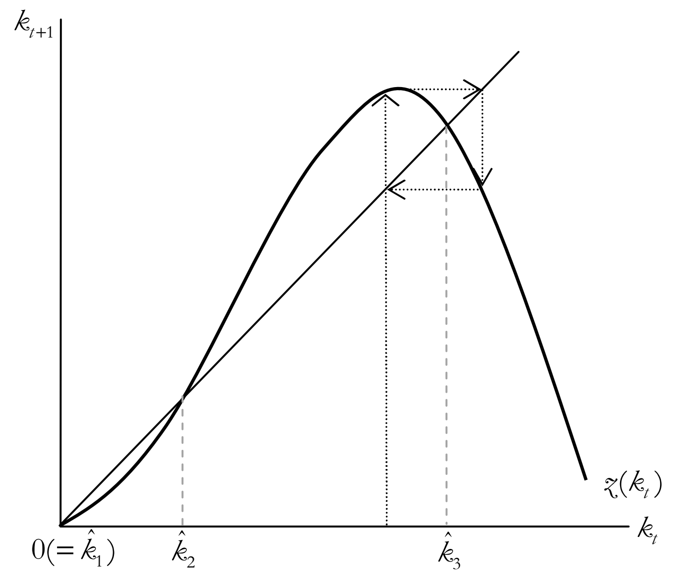

Figure 3. $\upsilon = 0$ and $z'(\hat{k}_3) < -1$: an example with a period-2 cycle

We can illustrate these results by means of a simple numerical example. Suppose that

$$\mathrm{B}(h_t) = \frac{\lambda h_t}{1+h_t}.$$

Let also $\tau = 0.2, \gamma = 0.3, p = 0.3, \Gamma = 10, E = 1, \varphi = 0.7, \chi = 0.2$. Then at $\lambda = 0.682$ a saddle-node bifurcation occurs (see Devaney 2003, p.82); that is, the number of fixed points (steady states), except from the origin, is none for $\lambda < 0.682$, one for $\lambda = 0.682$ and two for values of $\lambda > 0.682$. In particular, if $\lambda < 0.682$ the origin is the only steady-state equilibrium (Assumption 1 is not satisfied). At $\lambda = 0.682$ the function $z(k_t)$ is tangent to the 45^0 degree line and hence there is only one interior steady state. If $\lambda > 0.682$ there are two interior steady-state equilibria, say $\hat{k}_2$ and $\hat{k}_3$. The lower equilibrium, $\hat{k}_2$, is repelling, whereas the stability of the higher equilibrium, $\hat{k}_3$, depends on the value of λ. For example, if $\lambda = 0.7$ then any orbit that starts in the neighbourhood of $\hat{k}_3$ converges to it monotonically, since $0 < z'(\hat{k}_3) < 1$. On the other hand, if we let $\lambda = 0.75$, then the convergence to $\hat{k}_3$ occurs through damped oscillations since $0 > z'(\hat{k}_3) > -1$. Next, suppose that we let $\lambda = 0.78$. Simple calculations show that the stability of the equilibrium $\hat{k}_3$ changes since $z'(\hat{k}_3) < -1$; i.e., $\hat{k}_3$ becomes a repelling equilibrium. At the same time there is a period-2 cycle $\{0.306, 0.326\}$, which is stable since its multiplier is $z^{2\prime}(0.306) = z^{2\prime}(0.326) = z'(0.306)z'(0.326) = -0.452 > -1$ (z^2 denotes the second iterate of z, i.e., $z^2(k_t) = z(z(k_t))$). Next, suppose that we raise λ to 0.8. Then again simple calculations reveal that, while $\hat{k}_3$ remains a repelling equilibrium, the period-2 cycle has become an unstable one (the value of its multiplier is lower than -1). Instead, there is a period-4 cycle now, which is stable. This process continues as λ increases. In other words, the system undergoes a sequence of period-doubling bifurcations (see Devaney 2003, p. 90); that is, there is an increasing sequence of bifurcation points, such that for values of λ between any two consecutive members of the sequence λ_n and λ_{n+1} the prime $2^n -$period solution is stable, while the periodic solutions of all other periods $2, 4, \ldots, 2^{n-1}$ become unstable.

3.2 Dynamic Equilibrium with Active Pollution Abatement

The scenario we analyse now allows the government to actively pursue a policy of environmental preservation – i.e., we assume $0 < \upsilon < 1$. Therefore, the dynamics of capital accumulation are represented by the difference equation we originally obtained in (19).

Once more, we shall begin our formal analysis with the derivation of the model's steady-state equilibrium. The steady-state implications are summarised in

Lemma 2. *Suppose that $\upsilon\tau E > p$ holds. Then, there exist two steady-state equilibria $\hat{k}_1$ and $\hat{k}_2$, such than $\hat{k}_1 = 0$ and $\hat{k}_2 > 0$. The steady state $\hat{k}_1$ is locally asymptotically stable, while the steady state $\hat{k}_2$ is unstable.*

Using Lemma 2, we can identify the economy's dynamic behaviour and transitional properties in the long-run. We do this through

Proposition 2. *Consider $k_0 > 0$. Then:*

(i) *If $k_0 < \hat{k}_2$, the economy will converge to the poverty trap $\hat{k}_1 = 0$;*

(ii) *If $k_0 > \hat{k}_2$, the economy will eventually converge to a 'long-run growth' equilibrium in which both capital per worker and output per worker grow at the rate $\hat{\theta} = (1-\tau)(1-\gamma)\Gamma\dfrac{\lambda}{1+\lambda} - 1$.*[11]

The dynamics of the economy are illustrated in Figure 4. Similarly to the previous scenario, the steady state $\hat{k}_2$ emerges as an endogenous threshold that determines long-term prospects according to the initial stock of capital per worker. Once more, an economy which is initially endowed with resources below this threshold will degenerate towards the poverty trap. Naturally, the intuition behind this result is identical to the one provided in the case without pollution abatement.

What is particularly interesting, is the situation that occurs when the economy kick-starts its transition from a point that lies above the endogenous threshold $\hat{k}_2$. Contrary to the case

[11] Naturally, we assume that the value of Γ is sufficiently above unity so as to render the growth rate positive.

where $\upsilon = 0$, in which capital per worker converges to an equilibrium with zero growth (that is, either a positive level for the stock of capital or a limit cycle), in this case the economy is able to sustain a positive rate of economic growth in the long-run. The reason is that pollution abatement limits the extent to which economic activity causes environmental damage. Thus, pollution abatement protects the population's health against the damage from environmental degradation and, therefore, the saving behaviour of workers is not impeded as the economy grows. Combined with the effect of the learning-by-doing externality in the production technology, a policy of environmental preservation allows the social marginal return of capital to be high enough so as to guarantee a positive rate of capital accumulation that, eventually, allows the economy to achieve balanced growth as an equilibrium outcome.[12]

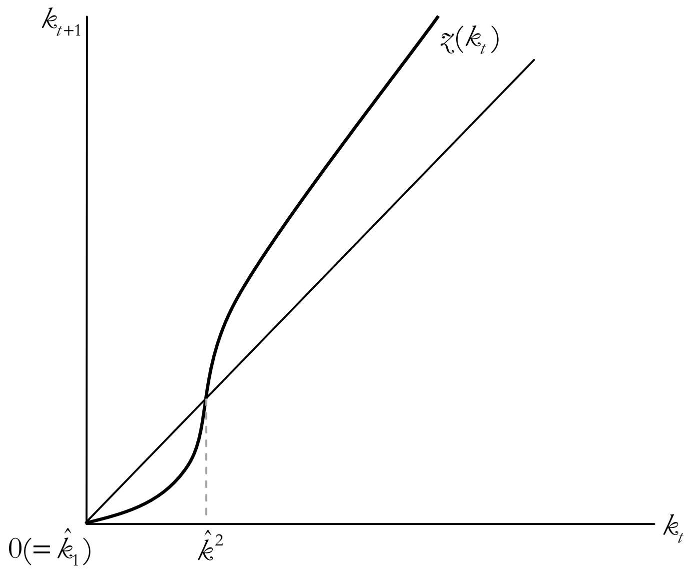

Figure 4. $0 < \upsilon < 1$

4 Some Important Implications

In the preceding sections of this paper, we have examined the transitional dynamics and the long-term equilibrium of an economy under two opposite scenarios concerning the

[12] If the condition $\upsilon \tau E > p$ does not hold, the dynamic equilibrium of the economy resembles the one derived for $\upsilon = 0$. The restriction imposed with Assumption 2 is sufficient but not necessary for the results of Lemma 2 and Proposition 2. Effectively, it ensures that only one endogenous threshold separates the two opposite convergence scenarios. In Appendix A5 we show that when this assumption is relaxed, it is possible that more equilibria emerge between the poverty trap and the long-run growth equilibrium. Nevertheless, the implication regarding the economy's ability to sustain a positive growth rate in the long-run remains intact.

government's engagement in policies that are designed to mitigate pollution and promote environmental quality. Apart from the common theme of multiple equilibria and the existence of poverty traps (an outcome related to the positive complementarities between saving and investment for some levels of the capital stock), the two scenarios' predictions concerning the long-term prospects of economies that escape such poverty traps are strikingly different. The purpose of this section is to compare and contrast these predictions in order to derive important implications that arise as a result of the government's stance on activities of pollution abatement.

We begin with the implications concerning economic growth. As we have seen from equations (4) and (5), the labour's contribution to aggregate production is augmented by a productivity variable which is driven by the presence of an economy-wide, learning-by-doing externality similar to that used by Romer (1986). It is well known that, in standard dynamic general equilibrium models with production, such externalities allow the emergence of an equilibrium with ongoing output growth (e.g., Romer, 1986; Aghion and Howitt, 1998). In our framework, however, we have established that the learning-by-doing mechanism is not by itself sufficient to guaranteed growth in the long-run. Indeed, such an equilibrium exists only when the government commits sufficient resources towards activities that abate pollution. Therefore, one significant implication from our analysis is given in

Corollary 1. *For an economy that avoids the poverty trap, pollution abatement is a complementary engine of long-run economic growth.*

This idea comes in stark contrast to previously held views concerning the macroeconomic repercussions of pollution. In her influential paper, Stokey (1998) argued that the prospects of long-run growth may be hampered as a result of the society's need to implement policies that support the quality of the natural environment – policies that are costly and, therefore, reduce the marginal product of capital to the extent that capital accumulation cannot be permanently sustained. Her model, however, does not incorporate the significant, and well-documented, effects of environmental quality to the overall health characteristics of the population. By taking account of these effects and their consequence for saving behaviour, our model has reached a different conclusion: policies that preserve some degree of environmental quality are, actually, essential for the existence of an equilibrium with ongoing output growth.

Another important implication of our analysis is related to the existence of limit cycles. As we have seen, when pollution abatement is absent, it is possible for capital per worker to oscillate permanently around its positive steady state. Of course, such persistent fluctuations are different in nature from cycles whose impulse sources may be exogenous demand and/or supply disturbances – the type of disturbances considered in the RBC and New-Keynesian literatures. In our model, both the impulse source and the propagation mechanism of cycles rest on the presence of non-monotonicity in the dynamics of capital accumulation. Thus, our framework shares more common features, among others, with the well-known papers of Grandmont (1985) and Matsuyama (1999) – both of whom discuss and derive cycles as endogenously determined phenomena whose existence depends on an economy's structural characteristics.

Naturally, policies that could eradicate such fluctuations are policies that would address the source of non-monotonicities rather than counter-cyclical rules designed to mitigate temporary shifts from a given trend. With this in mind, a straightforward comparison between our two different scenarios allows us to infer

Corollary 2. *For an economy that avoids the poverty trap, pollution abatement is a source of stabilisation, in the sense that it eliminates the possibility of permanent cycles.*

Given that environmental policy has an indirect positive effect on health and, consequently, life expectancy, our model derives implications which differ from those of Bhattacharya and Qiao (2007). In their model, the positive complementarities between private and public health spending implies that there is a trade-off between saving and private health expenditures. This trade-off generates non-monotonic capital dynamics, hence rendering health-enhancing public policy a source of endogenous fluctuations. In our model, a policy that facilitates health improvements (albeit indirectly through pollution abatement) actually eliminates such fluctuations.

Finally, by contrasting the results of our two different scenarios, it is possible to provide a novel explanation on the relationship between cycles and economic growth. We summarise this implication in

Corollary 3. *The government's stance on pollution abatement can generate a negative relationship between growth and cycles, in the sense that a policy supporting sustained long-run growth automatically eliminates the likelihood of persistent cycles.*

To the best of our knowledge, the only other theoretical analysis that derives implications on the relationship between cyclical fluctuations and economic growth, within a framework of (endogenous) limit cycles, is the model of Palivos and Varvarigos (2010). In their analysis, strategic interactions in the determination of human capital generate multiple equilibria, one of them being associated with permanent cycles. They conclude that, in the presence of such cycles, the growth rate is strictly lower compared to the one obtained under a stationary equilibrium. The present paper's view on the issue is rather different: we argue that an economy that displays persistent fluctuations will not be able to achieve long-run growth or, alternatively, an economy that sustains a positive growth permanently will not be subjected to cycles. In any case, it is the government's engagement in environmentally-friendly policies that, not solely but to a large extent, determines macroeconomic performance in the long-term.

5 Endogenous Allocation of Government Expenditure

In this Section we analyse the case where the government allocates its spending between public health services and pollution abatement optimally. To simplify the algebra we restrict our attention to the case where $\varphi = \chi \leq 1$. Accordingly, suppose that in every period the government allocates its spending so as to maximize the health status of the citizens. That is,

$$\max_{0 \leq \upsilon_t \leq 1} \left\{ h_t = [(1-\upsilon_t)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon_t \tau \Gamma k_t} \right)^{\varphi} \right\}. \quad (22)$$

The solution to this maximisation problem is formally described in

Proposition 3. *Suppose that* $\tau E > p$. *Then, there exists a threshold* $\underset{\frown}{k} = \frac{1}{2\tau\Gamma}\left(\sqrt{\frac{4E\tau}{p}+1} - 1 \right)$ *such that*

$$\upsilon_t = \begin{cases} \dfrac{-E + \left[Ep\Gamma k_t (1+\tau\Gamma k_t) \right]^{\frac{1}{2}}}{E\tau\Gamma k_t} \in (0,1) & \text{if } k_t > \underset{\frown}{k} \\ 0 & \text{if } k_t \leq \underset{\frown}{k} \end{cases}.$$

The result from Proposition 3 states that the government will find it optimal to initiate its efforts towards environmental preservation only at later stages of its development process. A similar result emerges in the analysis of Stokey (1998) where a central planner optimally decides to spend resources towards pollution abatement after the economy exceeds a threshold level of income. However, the major difference of our framework, in comparison to Stokey's (1998) is, once more, related to the prospects of long-run growth under environmental spending. This becomes apparent in

Proposition 4. *Consider* $k_0 > 0$. *If* υ_t *is chosen endogenously, there is always a threshold level, say* $\bar{k}$, *such that, as long as* $k_0 > \bar{k}$, *the economy will eventually converge to a 'long-run growth' equilibrium in which both capital per worker and output per worker grow at a positive rate* $\hat{\theta} = (1-\tau)(1-\gamma)\Gamma \dfrac{\lambda}{1+\lambda} - 1$.

In Appendix A7 we show that there may be two cases leading to the result of Proposition 4. These two cases depend on whether parameter values satisfy $\tau E > 2p$ or $2p > \tau E > p$.

In the former case, the dynamics of capital accumulation are monotonically increasing and there is only one non-trivial steady-state equilibrium, labelled as $\hat{k}_2$, which is unstable. Once again, this steady state emerges as an endogenous threshold that determines long-term prospects according to the initial stock of capital per worker (in terms of Proposition 4, it is $\hat{k}_2 = \bar{k}$). Countries that start with an initial capital stock below this threshold will decline monotonically towards a poverty trap where the (stable) steady state is $\hat{k}_1 = 0$. On the other hand, countries that start above this threshold level will experience smooth long-run growth. Diagrammatically, equilibrium outcomes resemble those presented in Figure 4.

In the latter case, however, outcomes may be slightly different in the sense that an additional (stable) steady-state equilibrium may emerge between the poverty trap and the long-run growth equilibrium. If this happens, then an economy for which $k_0 < \bar{k}$ need not necessarily fall into a poverty trap; instead, it may converge to a positive steady-state level of capital per worker. Still, however, this will be a stationary equilibrium with no long-run growth; achieving long-run growth requires that $k_0 > \bar{k}$. Diagrammatically, the equilibrium will either resemble the one presented in Figure 4 or the one presented in Figure 5.

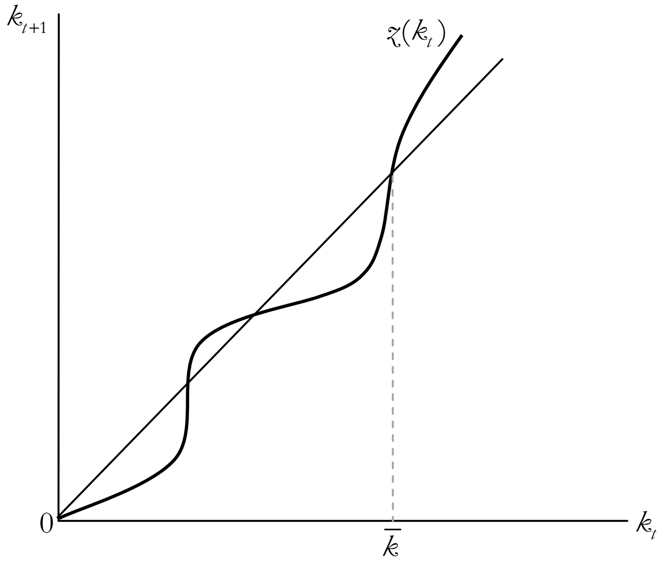

Figure 5

These details notwithstanding, we can conclude that, even with endogenous allocation of government resources, the commitment of some of these resources towards pollution abatement can allow some economies to achieve long-run growth. Furthermore, notice that, in comparison to the case where υ is set (permanently) equal to zero, the endogenous allocation of public spending eliminates the possibility of endogenous fluctuations. Hence, it verifies the role of pollution abatement as a tool for stabilisation in our framework.

6 Summary and Conclusion

We constructed and presented a two-period, overlapping generations model where life expectancy is positively affected by the provision of public health services and by the quality of the natural environment. Environmental quality declines due to pollution – a by-product of economic activity. We showed that, despite the presence of an aggregate learning-by-doing externality, the economy cannot sustain a positive growth rate in the long-run if resources are not devoted towards environmental preservation. As the environment deteriorates without bound, the negative impact on life expectancy causes a reduction in saving and, therefore, the rate of capital formation: the economy's capital stock either converges to a stationary level or oscillates permanently. An equilibrium with ongoing output growth is possible only if the government commits a sufficient amount of resources towards pollution abatement. Given that the possibility of cycles disappears in the latter scenario, we concluded that an active policy of environmental preservation in not only an important, complementary engine of long-run growth but a powerful tool of stabilisation as well.

Our model showed that environmentally sustainable economic growth is possible even if the quality of the environment, which actually deteriorates with higher levels of production, is essential for economic outcomes via its importance for longevity. Moreover, our analysis did not resort to the questionable outcome whereby the additively separable benefit of pollution abatement exceeds the environmental cost of pollutant emissions, thus rendering economic growth a net contributor to environmental quality. In our model we used the far less restrictive assumption according to which abatement aims at reducing the extent of pollution: in overall, economic activity is still a net contributor to environmental degradation in spite of the resources committed to abatement. Nevertheless, pollution abatement is critical in preserving a degree of environmental quality that is significant in maintaining a high enough social marginal product of capital that allows ongoing output growth.

We view our analysis, and its results, as pinpointing the possible weaknesses in the prevailing, opposing views concerning the relationship between economic growth and environmental quality. This is achieved by providing a moderate view according to which economic growth can be consistent with environmental sustainability and vice versa – without the need to overstress the potential environmental benefits of economic growth. If anything, it is the preservation of environmental quality that is vital in supporting ever increasing levels of income over time.

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Appendix

A1 Proof of Lemma 1

Using equation (21), we define the function

$$J(k_t) = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E - p\Gamma k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau\Gamma k_t)^{\varphi}(E - p\Gamma k_t)^{\chi}\right)}. \quad \text{(A1.1)}$$

Clearly, any interior steady state must satisfy $J(\hat{k}) = 1 \Leftrightarrow \hat{k} = z(\hat{k})$. From (A1.1), we have $J(0) = 0$ and, by virtue of (8), $J(k_t) = 0 \; \forall k_t \geq E / p\Gamma$. Thus, for an interior steady state to exist, there must be at least one $\tilde{k}$ such that $J(\tilde{k}) \geq 1$. When this condition holds with strict inequality then there will be at least two interior steady states; otherwise, there will not be any interior equilibrium at all (see Figure A1).

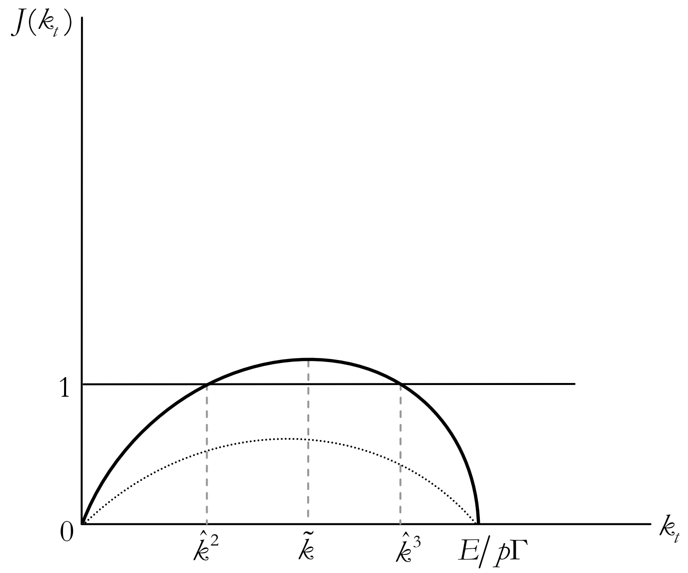

Figure A1. Interior solutions require $J(\tilde{k}) > 1$

Combining (A1.1) with (1), (2), (7), (8) and (9) allows us to derive

$$J'(k_t) = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2} \frac{dh_t}{dk_t}, \quad \text{(A1.2)}$$

where

$$\frac{\partial h_t}{\partial k_t} = \varphi\tau\Gamma(\tau\Gamma k_t)^{\varphi-1}(E - p\Gamma k_t)^{\chi} - p\Gamma\chi(\tau\Gamma k_t)^{\varphi}(E - p\Gamma k_t)^{\chi-1}. \qquad \text{(A1.3)}$$

For $0 \le k_t \le E / p\Gamma$, the sign of (A1.3) determines the sign of $J'(k_t)$. Straightforward factorisation allows us to write (A1.3) as

$$\frac{\partial h_t}{\partial k_t} = (\tau\Gamma k_t)^{\varphi}(E - p\Gamma k_t)^{\chi}\left(\frac{\varphi}{k_t} - \frac{\chi p\Gamma}{E - p\Gamma k_t}\right),$$

which means that $\frac{\partial h_t}{\partial k_t} \ge 0$ iff

$$\frac{\varphi}{k_t} \ge \frac{\chi p\Gamma}{E - p\Gamma k_t} \Rightarrow$$

$$\varphi E - \varphi p\Gamma k_t \ge \chi p\Gamma k_t \Rightarrow$$

$$k_t \le \frac{\varphi}{\varphi + \chi}\frac{E}{p\Gamma} \equiv \tilde{k}.$$

The preceding analysis implies that there exists a unique $\tilde{k} \in (0, E / p\Gamma)$ such that

$$J'(k_t)\begin{cases} > 0 & \text{for} \quad k_t < \tilde{k} \\ = 0 & \text{for} \quad k_t = \tilde{k}, \\ < 0 & \text{for} \quad k_t > \tilde{k} \end{cases}$$

i.e., $J(\tilde{k})$ is a global maximum. We can use this result to identify the parameter combination that allows the existence of interior equilibria. Particularly, we can solve $(\tau\Gamma\tilde{k})^{\varphi}(E - p\Gamma\tilde{k})^{\chi}$ using $\tilde{k} = \varphi E / (\varphi + \chi)p\Gamma$. Doing so, we derive $(\varphi\tau / p)^{\varphi}\chi^{\chi}[E / (\varphi + \chi)]^{\varphi+\chi} = \Omega$. Hence, by the Intermediate Value Theorem, Assumption 1 is a sufficient condition for the existence of interior equilibria. Moreover, if this condition holds, then there exist two interior steady-state equilibria $\hat{k}^3 > \hat{k}^2 > 0$ satisfying $\hat{k}^3 > \tilde{k} > \hat{k}^2$, i.e., $J'(\hat{k}^2) > 0$ and $J'(\hat{k}^3) < 0$.

Using (A1.1) we can derive

$$J'(k_t) = \frac{z'(k_t)k_t - z(k_t)}{(k_t)^2}. \qquad \text{(A1.4)}$$

Given (A1.4), $J'(\hat{k}^2) > 0$ implies

$$z'(\hat{k}^2) > \frac{z(\hat{k}^2)}{\hat{k}^2} \Rightarrow$$

$$z'(\hat{k}^2) > J(\hat{k}^2) \Rightarrow$$

$$z'(\hat{k}^2) > 1,$$

because $J(\hat{k}^2) = 1$. Thus, $\hat{k}^2$ is an unstable equilibrium.

Similarly, (A1.4) implies that $J'(\hat{k}^3) < 0$ is equivalent to $z'(\hat{k}^3) < 1$. In this case, however, we cannot make any definite conclusions concerning the stability of this equilibrium as we do not yet know whether the dynamics generated by equation (21) are monotonic. For this reason, let us return to the transition equation $k_{t+1} = z(k_t)$. Given (21), we can see that $z(0) = 0$, $z(k_t) = 0 \; \forall k_t \geq E / p\Gamma$ and $z(k_t) > 0$ for $k_t \in (0, E / p\Gamma)$. Thus, the dynamics of capital accumulation may not be non-monotonic which means that, indeed, the stability properties of $\hat{k}^3$ cannot be determined with certainty. Particularly, $\hat{k}^3$ is a stable long-run equilibrium if $z'(\hat{k}^3) > -1$; otherwise, i.e., if $z'(\hat{k}^3) < -1$, the equilibrium $\hat{k}^3$ is an unstable one.

In our preceding analysis, we have established that $z(0) = 0$. Of course, this result indicates that $\hat{k}^1 = 0$ is a steady state. Moreover,

$$z'(k_t) = J'(k_t)k_t + J(k_t),$$

and since, from equations (A1.2) and (A1.3),

$$\lim_{k \to 0} \left(\frac{dh_t}{dk_t} k_t \right) = 0 \quad \text{and} \quad J'(k_t)k_t = 0,$$

it follows that $z'(\hat{k}^1) = z'(0) = 0$, i.e., $\hat{k}^1 = 0$ is a super-stable equilibrium. ∎

A2 Proof of Proposition 1

The first part of Proposition 1 follows from the results of Lemma 1 in which we have shown that $\hat{k}_1 = 0$ is an asymptotically stable equilibrium while $\hat{k}_2 > 0$ is an unstable one. Hence, given $\hat{k}_2 > \hat{k}_1$, we can safely conclude that, for any $k_0 < \hat{k}_2$, it is $k_{t+1} = z(k_t) < k_t$, i.e., the economy's capital per worker will constantly decline until it converges to the poverty trap $\hat{k}_1 = 0$.

For the second part of Proposition 1 we can once more utilise the results from Lemma 1. In particular, let us consider the case where $\hat{k}_3$ is an asymptotically stable equilibrium, i.e., the case for which $\left|z'(\hat{k}_3)\right| < 1$. Given $\hat{k}_3 > \hat{k}_2$, we may conclude that for $k_0 > \hat{k}_2$ the transitional dynamics imply that $\lim_{t\to\infty} k_t = \hat{k}_3$. Also, using (20), we have $\theta_{t+1} = \frac{k_{t+1}}{k_t} - 1$ and, thus,

$$\lim_{t\to\infty} \theta_{t+1} = \lim_{t\to\infty}\left(\frac{k_{t+1}}{k_t}\right) - 1 = \lim_{t\to\infty}\left(\frac{z(k_t)}{k_t}\right) - 1 = \lim_{t\to\infty} J(k_t) - 1 = J(\hat{k}_3) - 1 = 0 . \qquad \text{(A2.1)}$$

Therefore, the economy will converge (either monotonically or through damped oscillations) to a long-run equilibrium with a positive stock for capital per worker, but zero growth.

Now, let us consider the possibility that $z'(\hat{k}_3) \leq -1$. Although $\hat{k}_3$ is an unstable steady-state equilibrium, it is well known that when the transition equation is non-monotonic and its slope at the steady state is negative and sufficiently steep (that is, below −1), then the dynamical system may exhibit periodic equilibria. In terms of our model, consider a sequence of n discrete points along the 45^0 line, denoted $\breve{k}_\eta$ for $\eta = \{1, 2, ..., i-1, i, i+1, ...n\}$, such that $\breve{k}_1 < ... < \breve{k}_{i-1} < \breve{k}_i < \hat{k}_3 < \breve{k}_{i+1} < ... < \breve{k}_n$ and

$$z(k_t)\begin{cases} > k_t & \text{for} \quad \eta \in [1, i] \\ \\ < k_t & \text{for} \quad \eta(i, n] \end{cases} .$$

If, for $k_0 > \hat{k}_2$, the capital stock passes repeatedly through the points $\breve{k}_\eta$ during its transition, then the economy converges to a period-n cycle where the sequence $\breve{k}_\eta$ represents periodic (rather than stationary) equilibria. Indeed, as long as $z'(\hat{k}_3) < -1$, the function $z(k_t)$ satisfies the following

Theorem (Azariadis, 1993, 86-88). *Suppose* 0 *and* $\hat{k} > 0$ *are fixed points of the scalar system* $k_{t+1} = z(k_t)$ *in which* $z : \mathbb{R}_+ \supseteq X \to \mathbb{R}_+$ *and* $z \in C^1$. *Suppose also that there exists a* $b > \hat{k}$ *such that* $b > z(b)$ *and* $b > z^2(b)$, *where* z^2 *is the second iterate of* z. *Then* $z'(\hat{k}) < -1$ *is a sufficient condition for the existence of a period-2 cycle* $\left\{\breve{k}_1, \breve{k}_2\right\}$ *that satisfies* $\breve{k}_1 < \hat{k} < \breve{k}_2 < b$.

Thus the system $k_{t+1} = z(k_t)$ exhibits (at least) a period-2 cycle. To apply this Theorem to our case, let $\hat{k} = \hat{k}_3$ and $b = E / p\Gamma$. Naturally, the growth rate θ_{t+1} will be positive during phases of the transition for which $\eta \in [1, i]$ but negative during phases of the transition for which $\eta \in (i, n]$. Hence, a long-run equilibrium with a constantly positive growth rate does not exist. ■

A3 Proof of Lemma 2

Consider again the function

$$J(k_t) = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi} \right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi} \right)}. \qquad \text{(A3.1)}$$

Given the properties of $\mathrm{B}(h_t)$ and the restriction $\upsilon\tau E > p$, it can be easily established that $J(0)=0$ and $J(\infty) = (1-\tau)(1-\gamma)\Gamma\lambda / (1+\lambda)$. An interior steady state must satisfy $J(\hat{k}) = 1 \Rightarrow \hat{k} = z(\hat{k})$. Therefore, Assumption 1 represents a sufficient condition for the existence of an interior equilibrium. This is because $\mathrm{B}(\infty) = \lambda$ and $\mathrm{B}(h_t)/[1+\mathrm{B}(h_t)]$ is increasing in h_t; therefore $\lambda > \mathrm{B}(\Omega)$.

Differentiating (A3.1) yields

$$J'(k_t) = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2} \frac{dh_t}{dk_t},$$

where

$$\frac{dh_t}{dk_t} = \varphi(1-\upsilon)\tau\Gamma[(1-\upsilon)\tau\Gamma k_t]^{\varphi-1} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi}$$
$$- \chi[(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi-1} \frac{p\Gamma}{(1+\upsilon\tau\Gamma k_t)^2}. \qquad \text{(A3.2)}$$

Substituting (A3.2) in (A3.1) gives us

$$J'(k_t) = \frac{(1-\tau)(1-\gamma)\Gamma\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2} [(1-\upsilon)\tau\Gamma k_t]^{\varphi} \left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t} \right)^{\chi} \Xi(k_t). \qquad \text{(A3.3)}$$

where

$$\Xi(k_t)=\frac{\varphi}{k_t}-\frac{\chi p\Gamma}{(1+\upsilon\tau\Gamma k_t)^2}\frac{1}{E-\dfrac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}}. \tag{A3.4}$$

Obviously, the sign of $J'(k_t)$ depends on the sign of $\Xi(k_t)$ in (A3.4). Particularly, for this to be non-negative, it must be $\Xi(k_t)\geq 0$. After some algebraic manipulation, the inequality $\Xi(k_t)\geq 0$ is reduced to a quadratic expression

$$(k_t)^2+\frac{(\upsilon\tau E-p)+\left(\upsilon\tau E-\dfrac{p\chi}{\varphi}\right)}{(\upsilon\tau E-p)\upsilon\tau\Gamma}k_t+\frac{E}{(\upsilon\tau E-p)\upsilon\tau\Gamma^2}\geq 0. \tag{A3.5}$$

As long as $2\upsilon\tau E>p(\varphi+\chi)/\varphi$, which is true for $\upsilon\tau E>p$ and $\chi\leq\varphi$ (Assumption 2), the above expression holds with strict inequality and, by virtue of (A3.3) and (A3.4), $J'(k_t)>0\ \forall k_t$. Hence, there is only one interior steady state $\hat{k}^2$ with $J'(\hat{k}^2)>0$. Moreover, it can be easily checked that $J'(\hat{k}^2)>0\Rightarrow z'(\hat{k}^2)>1$, i.e., the interior steady state is unstable.

Next, notice from equation (19) that $z(0)=0$; therefore $\hat{k}^1=0$ is a steady state. Moreover

$$z'(k_t)=J'(k_t)k_t+J(k_t),$$

and, since from equations (A3.3) and (A3.4)

$$\lim_{k\to 0}\Xi(k_t)k_t=0 \quad \text{and} \quad J'(k_t)k_t=0,$$

it follows that $z'(\hat{k}^1)=z'(0)=0$, i.e., $\hat{k}^1=0$ is a super-stable equilibrium. ■

A4 Proof of Proposition 2

The first part of Proposition 2 is mainly a by-product of results established in Lemma 2. Specifically, given that $\hat{k}_1=0$ is an asymptotically stable equilibrium and $\hat{k}_2>0$ is an unstable one, for any $k_0<\hat{k}_2$, we have $k_{t+1}<k_t$ for all subsequent steps of the transition. Hence, the economy's stock of capital per worker will constantly decline until it converges to the poverty trap $\hat{k}_1=0$.

For the second part, we can begin by using (19) and (20) so as to write the gross growth rate as

$$\frac{k_{t+1}}{k_t} = 1+\theta_{t+1} = (1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}, \tag{A4.1}$$

for which Appendix A3 establishes that $k_{t+1} > k_t \Rightarrow 1+\theta_{t+1} > 1$ (as long as $k_0 > \hat{k}^2$) because the dynamics of capital accumulation are monotonic. Therefore, (A4.1) can be eventually written as

$$k_t = \prod_{\varepsilon=0}^{t}(1+\theta_\varepsilon)k_0 . \tag{A4.2}$$

From equation (A4.2) we can verify that $\lim_{t\to\infty} k_t = k_\infty \to \infty$. Therefore, we can use equation (A4.1) to establish that

$$\lim_{t\to\infty}\theta_{t+1} = \theta_\infty =$$

$$\lim_{t\to\infty}\left[(1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_t]^{\varphi}\left(E - \frac{p\Gamma k_t}{1+\upsilon\tau\Gamma k_t}\right)^{\chi}\right)} - 1\right] =$$

$$(1-\tau)(1-\gamma)\Gamma \frac{\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_\infty]^{\varphi}\left(E - \frac{p\Gamma k_\infty}{1+\upsilon\tau\Gamma k_\infty}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\Gamma k_\infty]^{\varphi}\left(E - \frac{p\Gamma k_\infty}{1+\upsilon\tau\Gamma k_\infty}\right)^{\chi}\right)} - 1 =$$

$$(1-\tau)(1-\gamma)\Gamma\frac{\lambda}{1+\lambda} - 1 = \hat{\theta}\,.$$

Since $(1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda) > 1$ holds by assumption, then $\hat{\theta} > 0$: asymptotically, the economy will converge to a balanced growth path where capital per worker (and, therefore, output per worker) grow at a rate $\hat{\theta}$. ■

A5 Analysis of the Model when Assumption A2 is Relaxed

Our basic analysis utilised the restriction $\chi \le \varphi$. In this part of the Appendix, we shall demonstrate that all the main implications of our model survive even when this restriction is relaxed. To begin with, we can readily verify that this restriction has no bearing at all for the analysis and results of the case with no pollution abatement ($\upsilon = 0$). Indeed, Assumption 2 was not used in the proofs of Lemma 1 and Proposition 1. For this reason, we shall focus on the case where policies of pollution abatement are active.

The main repercussion from relaxing $\chi \leq \varphi$ relates to the possibility that we may have $2\upsilon\tau E < p(\varphi + \chi)/\varphi$. Therefore, the inequality $\Xi(k_t) \geq 0$ which we examined in equation (A3.4) (see Appendix A3, proof of Lemma 2) may not hold for every k_t. Using obvious definitions, we can rewrite the left-hand side of (A3.5) in the form $(k_t)^2 + \zeta k_t + \delta \Rightarrow (k_t - k_-)(k_t - k_+)$, where

$$k_- = \frac{-\zeta - \sqrt{\zeta^2 - 4\delta}}{2}, \tag{A5.1}$$

$$k_+ = \frac{-\zeta + \sqrt{\zeta^2 - 4\delta}}{2}. \tag{A5.2}$$

Notice that, for $2\upsilon\tau E < p(\varphi + \chi)/\varphi$, it is $\zeta < 0$. Moreover, after some tedious but straightforward algebra it can be shown that $\zeta^2 - 4\delta > 0$, i.e., both roots are real and positive. Therefore, we can use (A5.1) and (A5.2) in (A3.4) so as to infer that, given (A3.3), we have

$$J'(k_t)\begin{cases} > 0 & \text{for} \quad k_t < k_- \\ < 0 & \text{for} \quad k_- < k_t < k_+ \\ > 0 & \text{for} \quad k_t > k_+ \end{cases}.$$

Given $J(0) = 0$ and $J(\infty) = (1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda)$, the preceding analysis shows that k_- corresponds to local maximum while k_+ corresponds to a local minimum. Consequently, there may be three interior steady-state solutions from which the lowest and the highest are unstable. Thus, the difference with the results of Section 3.2 is that we may have an additional, asymptotically stable steady state for the stock of capital per worker, separating the poverty trap and the long-run growth equilibrium. Furthermore, in this case we would have two endogenous thresholds – one separating the poverty trap and the no-growth equilibrium while the other separating the no-growth and the long-run growth equilibria. Figures 6 and 7, below, illustrate such outcomes.

Notice that, although the situation illustrated in Figures A2 and A3 is possible, under certain conditions the model's equilibrium may still be qualitatively identical to the one derived in Section 3.2. Particularly, this happens if either $J(k_{-})<1$ or $J(k_{+})>1$ (see Figures A4 and A5, respectively). In both cases, there can only be one steady state, $\hat{k}^2$, with $J'(\hat{k}^2)>0 \Rightarrow z'(\hat{k}^2)>1$, i.e., an unstable steady state. Therefore, the model's behaviour resembles the one described in the main part of the paper.

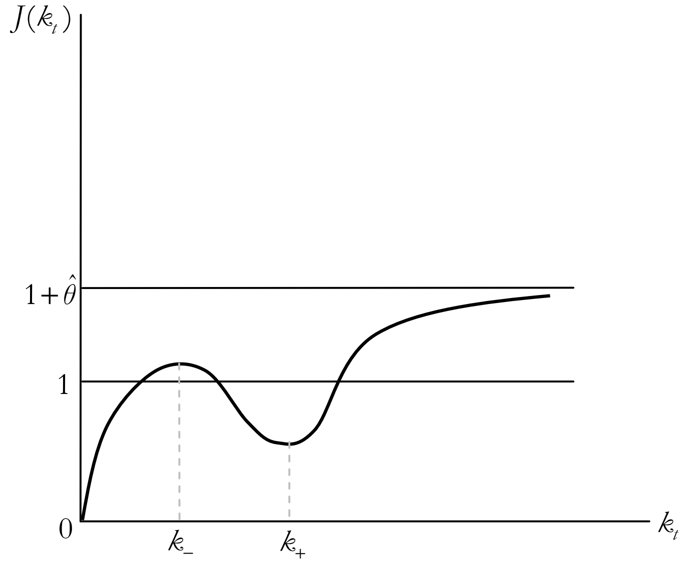

Figure A2. Three interior steady states

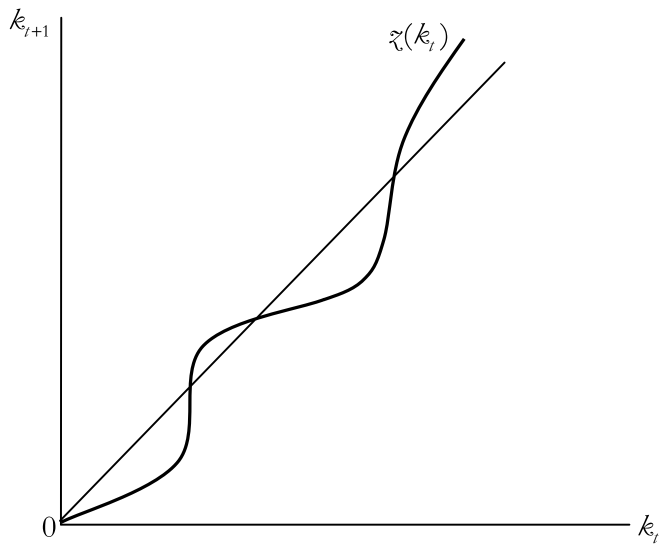

Figure A3. The dynamics of capital accumulation with three interior steady states

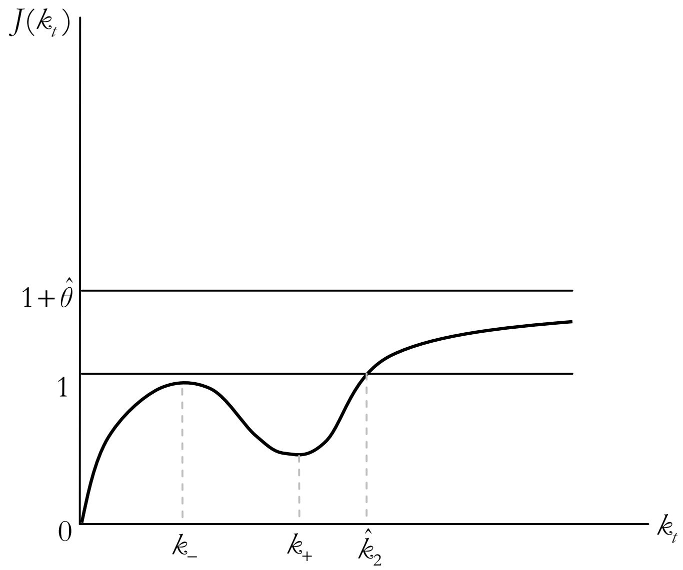

Figure A4. $J(k_-)<1$

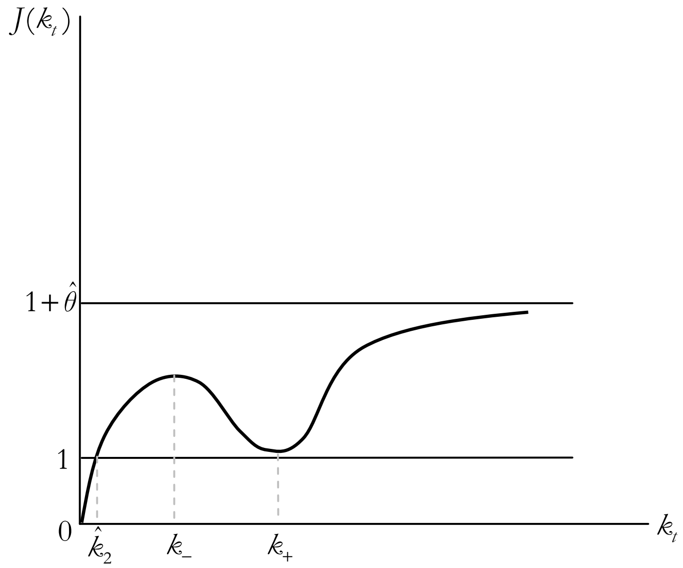

Figure A5. $J(k_+)>1$

A6 Proof of Proposition 3

The maximisation problem in (22) leads to

$$\upsilon_t^* = \frac{-E + \left[Ep\Gamma k_t \left(1+\tau\Gamma k_t\right)\right]^{1/2}}{Ep\tau\Gamma k_t}. \tag{A6.1}$$

Note that a sufficient condition for $\upsilon_t^* < 1$ is $\tau E > p$. It is also straightforward to establish that the non-negativity constraint $\upsilon_t^* \geq 0$ is satisfied for $k_t \geq \underset{\frown}{k}$, where

$$\underset{\frown}{k} = \frac{1}{2\tau\Gamma}\left(\sqrt{\frac{4E\tau}{p}+1}-1\right). \tag{A6.2}$$

■

A7 Proof of Proposition 4

Using the result in Proposition 3 and substituting (A6.1), together with (8), (9) and (10), in (2) we derive

$$h_t = \begin{cases} \left[(\tau\Gamma k_t)(E - p\Gamma k_t)\right]^{\varphi} & \text{if } k_t \leq \underset{\frown}{k} \\ \left\{[E(1+\tau\Gamma k_t)]^{1/2} - (p\Gamma k_t)^{1/2}\right\}^{2\varphi} & \text{if } k_t > \underset{\frown}{k} \end{cases}. \tag{A7.1}$$

Appropriate substitution of (A6.2) in (A7.1) reveals that the function h_t is continuous; therefore, the function $z(k_t)$ is continuous as well. Also note that $\lim_{k_t\to\infty} h_t = \lim_{k_t\to\infty}(p\Gamma k_t)^{\varphi} \lim_{k_t\to\infty}\left\{\left[\frac{E(1+\tau\Gamma k_t)}{p\Gamma k_t}\right]^{1/2} - 1\right\}^{2\varphi} = \infty$ since $\tau E > p$ implies $E(1+\tau\Gamma k_t)/p\Gamma k_t > 1$. Thus, $\lim_{k_t\to\infty} \mathrm{B}(h_t) = \mathrm{B}(\infty) = \lambda$.

Consider

$$J(k_t) = \frac{k_{t+1}}{k_t} = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}(h_t)}{1+\mathrm{B}(h_t)}.$$

Obviously, for $k_t \leq \underset{\frown}{k}$ the properties of this expression are identical to those analysed in Appendix A1. Now let us examine the properties for $k_t > \underset{\frown}{k}$. First of all, we can use the previous analysis to establish that $J(\infty) = (1-\tau)(1-\gamma)\Gamma\lambda/(1+\lambda) > 1$. Furthermore, it is

$$J'(k_t) = (1-\tau)(1-\gamma)\Gamma\frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}\frac{dh_t}{dk_t},$$

where

$$\frac{dh_t}{dk_t} = 2\varphi h_t^{(2\varphi-1)/2\varphi}\left\{[E(1+\tau\Gamma k_t)]^{-1/2} E\tau\Gamma - (p\Gamma k_t)^{-1/2} p\Gamma\right\}.$$

Hence,

$$J'(k_t) > 0 \quad \text{iff} \quad k_t > \frac{p}{\tau\Gamma(E\tau - p)} \equiv \varkappa .$$

In Appendix A1 we showed that the expression $J(k_t)$ is increasing for $k_t < \tilde{k}$ where $\tilde{k} = \frac{\varphi}{\varphi+\chi}\frac{E}{p\Gamma}$. Now, since $\varphi = \chi$, the corresponding value is $\tilde{k} = \frac{E}{2p\Gamma}$. Of course, as long as $\tilde{k} > \underset{\frown}{k}$, the switch in regime from $\upsilon_t^* = 0$ to $\upsilon_t^* > 0$ occurs in the upward sloping part of $J(k_t)$. After some straightforward algebra, we can show that $\tilde{k} > \underset{\frown}{k} > \varkappa$ iff $\tau E > 2p$.

Assume for the moment that $\tau E > 2p$. Notice that if $k_t < \underset{\frown}{k}$, then the function $J(k_t)_{\upsilon=0}$ is monotonically increasing since $k_t < \underset{\frown}{k} < \tilde{k}$. Also, if $k_t > \underset{\frown}{k}$, then the function $J(k_t)_{\upsilon=\upsilon^*}$ is again monotonically increasing because $k_t > \underset{\frown}{k} > \varkappa$. Thus, as long as $\tau E > 2p$, it is $J'(k_t) > 0$ for every $k_t > 0$. Given that $J(0) = 0$ (recall that for $k_t \leq \underset{\frown}{k}$ it is $\upsilon_t^* = 0$) and $J(\infty) > 1$, there is only one steady-state equilibrium, say $\bar{k}$, which is clearly unstable. An analysis similar to that in Appendix A4 suffices to establish that for $k_0 > \bar{k}$, the economy can achieve long-run economic growth.

Next, let us consider the case where $2p > \tau E > p$. In this case $\tilde{k} < \underset{\frown}{k} < \varkappa$ and the behaviour of the system may or may not be qualitatively identical to the one described above. Based on the previous results we can infer that the function $J(k_t)$ is increasing over the interval $(0, \tilde{k})$, decreasing over the interval $(\tilde{k}, \varkappa)$ and increasing for values of k_t greater than $\varkappa$. Hence, if $J(\varkappa) > 1$, then there is again one unstable interior steady state, $\bar{k}$, and for $k_0 > \bar{k}$, the economy will achieve long-run economic growth. Nevertheless, if $J(\varkappa) < 1$, then it is easy to check that, in addition to the stable steady-state $\hat{k}_1 = 0$, there will be three interior steady-states $\hat{k}_2 < \hat{k}_3 < \bar{k}$ from which $\hat{k}_2 \in (0, \tilde{k})$ and $\bar{k} > \varkappa$ will be unstable (because $J'(\hat{k}_2), J'(\bar{k}) > 0$) but $\hat{k}_3 \in (\tilde{k}, \varkappa)$ may be stable since $J'(\hat{k}_3) < 0$. Once more, for $k_0 > \bar{k}$ the economy will attain positive growth in the long-run. For $k_0 < \bar{k}$, however, the economy may converge to $\hat{k}_3 > 0$ instead of the poverty trap. ■